MATHEMATICAL METHODS

CONTENTS

- Interpolation and Curve Fitting
- Algebraic equations transcendental equations, numerical differentiation & integration
- Numerical differentiation of O.D.E
- Fourier series and Fourier transforms
- Partial differential equation
- Vector Calculus

TEXT BOOKS

- Advanced Engineering Mathematics by Kreyszig, John Wiley & Sons.
- Higher Engineering Mathematics by Dr. B.S. Grewal, Khanna Publishers

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- Mathematical Methods by T.K.V. Iyengar, B.Krishna Gandhi & Others, S. Chand.
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UNIT-I

Interpolation and Curve Fitting

Finite difference methods

Let $(x_i, y_i), i=0,1,2 \ldots\ldots\ldots\ldots n$ be the equally spaced data of the unknown function $y=f(x)$ then much of the $f(x)$ can be extracted by analyzing

the differences of $f(x)$.

Let $x_1 = x_0+h$

$x_2 = x_0+2h$

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$x_n = x_0+nh$ be equally spaced points where the function value of $f(x)$

be $y_0, y_1, y_2 \ldots\ldots\ldots\ldots\ldots\ldots y_n$

Symbolic operators

Forward shift operator(E) :

It is defined as $Ef(x)=f(x+h)$ (or) $Ey_x = y_{x+h}$

The second and higher order forward shift operators are defined in similar manner as follows

$$E^2f(x)= E(Ef(x))= E(f(x+h)= f(x+2h)= y_{x+2h}$$

$$E^3f(x)= f(x+3h)$$

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$$E^kf(x)= f(x+kh)$$

Backward shift operator(E^{-1}) :

It is defined as $E^{-1}f(x)=f(x-h)$ (or) $Ey_x = y_{x-h}$

The second and higher order backward shift operators are defined in similar manner as follows

$E^{-2}f(x)= E^{-1}(E^{-1}f(x))= E^{-1}(f(x-h)= f(x-2h)= y_{x-2h}$

$E^{-3}f(x)= f(x-3h)$

.

.

$E^{-k}f(x)= f(x-kh)$

Forward difference operator (Δ) :

The first order forward difference operator of a function f(x) with increment h in x is given by

$$\Delta f(x)=f(x+h)-f(x) \quad \text{(or)} \quad \Delta f_k = f_{k+1}-f_k \; ; \; k=0,1,2\ldots\ldots\ldots$$

$$\Delta^2 f(x)= \Delta[\Delta f(x)]= \Delta[f(x+h)-f(x)]= \Delta f_{k+1}- \Delta f_k \; ; \; k=0,1,2\ldots\ldots\ldots\ldots$$

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Relation between E and Δ :

$$\Delta f(x)=f(x+h)-f(x)$$

$$=Ef(x)-f(x) \qquad [Ef(x)=f(x+h)]$$

$$=(E-1)f(x)$$

$$\Delta=E-1 \qquad E=1+\Delta$$

Backward difference operator (nabla ∇) :

The first order backward difference operator ∇ of a function f(x) with increment h in x is given by

$\nabla f(x)=f(x)-f(x-h)$ (or) $\nabla f_k = f_{k+1}-f_k$; k=0,1,2………

$\nabla^2 f(x)= \nabla[\nabla f(x)]= \nabla[f(x+h)-f(x)]= \nabla f_{k+1}- \nabla f_k$; k=0,1,2…………

………………………………

………………………………

Relation between E and nabla :

nabla f(x)=f(x+h)-f(x)

=Ef(x)-f(x) [Ef(x)=f(x+h)]

=(E-1)f(x)

nabla=E-1 E=1+ nabla

Central difference operator (δ) :

The central difference operator is defined as

$$\delta f(x)= f(x+h/2)-f(x-h/2)$$

$$\delta f(x)= E^{1/2}f(x)-E^{-1/2}f(x)$$

$$= [E^{1/2}-E^{-1/2}]f(x)$$

$$\delta= E^{1/2}-E^{-1/2}$$

INTERPOLATION : The process of finding a missed value in the given table values of X, Y.

FINITE DIFFERENCES : We have three finite differences

1. Forward Difference
2. Backward Difference
3. Central Difference

RELATIONS BETWEEN THE OPERATORS IDENTITIES:

1. $\Delta = E - 1$ or $E = 1 + \Delta$
2. $\nabla = 1 - E^{-1}$
3. $\delta = E^{1/2} - E^{-1/2}$
4. $\mu = \frac{1}{2}(E^{1/2} - E^{-1/2})$
5. $\Delta = E\nabla = \nabla E = \delta E^{1/2}$
6. $(1 + \Delta)(1 - \nabla) = 1$

Newtons Forward interpolation formula :

- $y=f(x)=f(x_0+ph)= y_0 + p\Delta y_0 + p(p-1)/2!\ \Delta^2 y_0 + p(p-1)(p-2)/3!\ \Delta^3 y_0 + \ldots\ldots\ldots\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + p(p-1)(p2)\ \ldots\ldots\ [p-(n-1)]/n!\ \Delta^n y_0$.

Newtons Backward interpolatin formula :

$y=f(x)=f(x_n+ph)= y_n + p\nabla y_n + p(p+1)/2!\nabla^2 y_n + p(p+1)(p+2)/3!\nabla^3 y_n + \ldots\ldots\ldots\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + p(p+1)(p+2)\ \ldots\ldots\ [p+(n-1)]/n!\nabla^n y_n$.

GAUSS INTERPOLATION

The Guass forward interpolation is given by $y_p = y_0 + p\,\Delta y_0 + p(p-1)/2!\ \Delta^2 y_{-1} + (p+1)p(p-1)/3!\ \Delta^3 y_{-1} + (p+1)p(p-1)(p-2)/4!\ \Delta^4 y_{-2} + \ldots\ldots$

The Guass backward interpolation is given by $y_p = y_0 + p\,\Delta y_{-1} + p(p+1)/2!\ \Delta^2 y_{-1} + (p+1)p(p-1)/3!\ \Delta^3 y_{-2} + (p+2)(p+1)p(p-1)/4!\ \Delta^4 y_{-2} + \ldots\ldots$

INTERPOLATIN WITH UNEQUAL INTERVALS:

The various interpolation formulae Newton's forward formula, Newton's backward formula possess can be applied only to equal spaced values of argument. It is therefore, desirable to develop interpolation formula for unequally spaced values of x. We use Lagrange's interpolation formula.

The Lagrange's interpolation formula is given by

$$Y = \frac{(X-X_1)(X-X_2)\ldots\ldots\ldots(X-X_n)}{(X_o-X_1)(X_o-X_2)\ldots\ldots\ldots(X_o-X_n)} Y_o +$$

$$\frac{(X-X_o)(X-X_2)\ldots\ldots\ldots(X-X_n)}{(X_1-X_o)(X_1-X_2)\ldots\ldots\ldots(X_1-X_n)} Y_1 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{(X-X_o)(X-X_1)\ldots\ldots\ldots(X-X_{n-1})}{(X_n-X_o)(X_n-X_1)\ldots\ldots\ldots(X_n-X_{n-1})} Y_n +$$

CURVE FITTING

INTRODUCTION : In interpolation, We have seen that when exact values of the function Y=f(x) is given we fit the function using various interpolation formulae. But sometimes the values of the function may not be given. In such cases, the values of the required function may be taken experimentally. Generally these expt. Values contain some errors. Hence by using these experimental values . We can fit a curve just approximately which is known as approximating curve.

Now our aim is to find this approximating curve as much best as through minimizing errors of experimental values this is called best fit otherwise it is a bad fit.

In brief by using experimental values the process of establishing a mathematical relationship between two variables is called CURVE FITTING.

METHOD OF LEAST SQUARES

Let y_1, y_2, y_3 ....y_n be the experimental values of $f(x_1)$,$f(x_2)$,.....$f(x_n)$ be the exact values of the function $y=f(x)$. Corresponding to the values of $x=x_0,x_1,x_2....x_n$ .Now error=experimental values –exact value. If we denote the corresponding errors of y_1, y_2,....y_n as e_1,e_2,e_3,.....e_n, then $e_1=y_1-f(x_1)$, $e_2=y_2-f(x_2)$

- $e_3=y_3-f(x_3)$.......$e_n=y_n-f(x_n)$.These errors $e_1,e_2,e_3,......e_n$,may be either positive or negative.For our convenient to make all errors into +ve to the square of errors i.e $e_1^2,e_2^2,......e_n^2$.In order to obtain the best fit of curve we have to make the sum of the squares of the errors as much minimum i.e $e_1^2+e_2^2+......+e_n^2$ is minimum.

METHOD OF LEAST SQUARES

- Let $S=e_1^2+e_2^2+\ldots\ldots+e_n^2$, S is minimum.When S becomes as much as minimum.Then we obtain a best fitting of a curve to the given data, now to make S minimum we have to determine the coefficients involving in the curve, so that S minimum.It will be possible when differentiating S with respect to the coefficients involving in the curve and equating to zero.

FITTING OF STRAIGHT LINE

Let $y = a + bx$ be a straight line

By using the principle of least squares for solving the straight line equations.

The normal equations are

$\sum y = na + b \sum x$

$\sum xy = a \sum x + b \sum x^2$

solving these two normal equations we get the values of a & b ,substituting these values in the given straight line equation which gives the best fit.

FITTING OF PARABOLA

Let $y = a + bx + cx^2$ be the parabola or second degree polynomial.

By using the principle of least squares for solving the parabola

The normal equations are

$\sum y = na + b \sum x + c \sum x^2$

$\sum xy = a \sum x + b \sum x^2 + c \sum x^3$

$\sum x^2 y = a \sum x^2 + b \sum x^3 + c \sum x^4$

solving these normal equations we get the values of a,b & c, substituting these values in the given parabola which gives the best fit.

FITTING OF AN EXPONENTIAL CURVE

The exponential curve of the form

$$y = a\, e^{bx}$$

taking log on both sides we get

$\log_e y = \log_e a + \log_e e^{bx}$

$\log_e y = \log_e a + bx \log_e e$

$\log_e y = \log_e a + bx$

- $Y = A + bx$
- Where $Y = \log_e y$, $A = \log_e a$
- This is in the form of straight line equation and this can be solved by using the straight line normal equations we get the values of A & b, for $a = e^{A,}$ substituting the values of a & b in the given curve which gives the best fit.

EXPONENTIAL CURVE

The equation of the exponential form is of the form $y = ab^x$

taking log on both sides we get

$\log_e y = \log_e a + \log_e b^x$

$Y = A + Bx$

where $Y = \log_e y$, $A = \log_e a$, $B = \log_e b$

this is in the form of the straight line equation which can be solved by using the normal equations we get the values of A & B for $a = e^A$

$b = e^B$ substituting these values in the equation which gives the best fit.

FITTING OF POWER CURVE

Let the equation of the power curve be

$$y = a\,x^b$$

taking log on both sides we get

$$\log_e y = \log_e a + \log_e x^b$$

$$Y = A + Bx$$

this is in the form of the straight line equation which can be solved by using the normal equations we get the values of A & B, for $a = e^A$ $b= e^B$, substituting these values in the given equation which gives the best fit.

UNIT-2

Algebraic and Transcendental equations
Linear system of equations
Numerical Differentiation and integration
Numerical solution of First order differential equations

Method 1: Bisection method

If a function f(x) is continuous b/w x_0 and x_1 and $f(x_0)$ & $f(x_1)$ are of opposite signs, then there exsist at least one root b/w x_0 and x_1

- Let $f(x_0)$ be –ve and $f(x_1)$ be +ve ,then the root lies b/w xo and x_1 and its approximate value is given by $x_2=(x_0+x_1)/2$
- If $f(x_2)=0$,we conclude that x_2 is a root of the equ $f(x)=0$
- Otherwise the root lies either b/w x_2 and x_1 (or) b/w x_2 and x_0 depending on wheather $f(x_2)$ is +ve or –ve

Then as before, we bisect the interval and repeat the process untill the root is known to the desired accuracy

Method 2: Iteration method or successive approximation

Consider the equation $f(x)=0$ which can take in the form

$x = \phi(x)$ -------------(1)

where $|\phi^1(x)|<1$ for all values of x.

Taking initial approximation is x_0

we put $x_1=\phi(x_0)$ and take x_1 is the first approximation

$x_2=\phi(x_1)$, x_2 is the second approximation

$x_3=\phi(x_2)$,x_3 is the third approximation

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$x_n=\phi(x_{n-1})$,x_n is the n^{th} approximation

Such a process is called an iteration process

Method 3: <u>Newton-Raphson method or Newton iteration method</u>

Let the given equation be $f(x)=0$

Find $f^1(x)$ and initial approximation x_0

The first approximation is $x_1 = x_0 - f(x_0)/ f^1(x_0)$

The second approximation is $x_2 = x_1 - f(x_1)/ f^1(x_1)$

.

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The n^th approximation is $x_n = x_{n-1} - f(x_n)/ f^1(x_n)$

LU Decomposition Method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

This is in the form AX=B Where

Let A=LU where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}, U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

Hence LUX=B

(a)LY=B (b)UX=Y

Solve for Y from (a) then Solve for X from (b)

LY=B Can be solved by forward substitution and UX=Y can be solved by backward substitution

Jacobi's Iteration Method

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{1}{a_1}[d_1 - b_1y - c_1z] = k_1 - l_1y - m_1z$$
$$y = \frac{1}{b_2}[d_2 - a_2x - c_2z] = k_2 - l_2x - m_2z$$
$$z = \frac{1}{c_3}[d_3 - a_3x - b_3y] = k_3 - l_3x - m_3y$$

Substituting these on the right hand side we get second approximations this process is repeated till the difference between two consecutive approximations is negligible or same

Gauss-Seidel iteration Method

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{1}{a_1}[d_1 - b_1y - c_1z] = k_1 - l_1y - m_1z$$

$$y = \frac{1}{b_2}[d_2 - a_2x - c_2z] = k_2 - l_2x - m_2z$$

$$z = \frac{1}{c_3}[d_3 - a_3x - b_3y] = k_3 - l_3x - m_3y$$

as soon as new approximation for an unknown is found it is immediately used in the next step

NUMERICAL DIFFERENTIATION

The process by which we can find the derivative of a function i.e dy/dx for some particular value of independent variable x is called Numerical Differentiation.

The problem of numerical differentiation are to be solved by approximating the function using interpolation formula and then differentiating this formula as many times as desired.

NEWTON'S FORMULA

- If the values of the argument are equally spaced and if the derivative is required for some values of given x lying in the begin of the table, we can represent the function by Newton-Gregory forward interpolation formula.
- If the value of dy/dx is required at a point near the end of the table, we have to use Newtons backward formula.

CENTRAL DIFFERENCE

If the derivative dy/dx is to be found at some point lying near the middle of the tabulated value we have to use central difference interpolation formula

While applying these formulae, it must be observed that the table of values defines the function at these points only and does not completely define the function and hence the function may not be differentiable at all.

Derivatives by using Forward Difference Formula :

The first order derivative of the function is given by

$(dy/dx)_{x=xo}$ = 1/h (Δy_o – 1/2Δ^2yo + 1/3 $\Delta^3 y_o$- ¼ $\Delta^4 y_o$ + ………)

$(d^2x/dy^2)_{x=xo}$= 1/h^2 ($\Delta^2 y_o$ – Δ^3yo + 11/12 $\Delta^4 y_o$- 5/6 $\Delta^5 y_o$ + ………)

Derivatives by using Backward Difference Formula :

The first order derivative of the function is given by

$(dy/dx)_{x=xn}$ = 1/h(yn+1/2 ^{2}yn+1/3 ^{3}yn+1/4 ^{4}yn+..................)

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$(d2y/dx2)_{x=xn}$ = 1/h(^{2}yn+ ^{3}yn+11/12 ^{4}yn+5/6 ^{5}yn+..................)

∇ ∇ ∇

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Striling's formula is given by using central difference is

$(dy/dx)_{x=x0} = 1/h$ ($\Delta y_0 + \Delta y_{-1/2} - 1/6\ \Delta^3 y_{-1} + \Delta^3 y_{-2}/2 + 1/30\ \Delta^5 y_{-2} + \Delta^5 y_{-3/2} + \ldots\ldots..$)

NUMERICAL INTEGRATION

- Numerical Integration is a process of finding the value of a definite integral.When a function y = f(x) is not known explicity. But we give only a set of values of the function y = f(x) corresponding to the same values of x. This process when applied to a function of a single variable is known as a quadrature.

NUMERICAL INTEGRATION

- For evaluating the Numerical Integration we have three important rules i.e
- Trapezoidal Rule
- Simpsons 1\3 Rule
- Simpsons 3\8 th Rule

NUMERICAL INTEGRATION

- Trapezoidal Rule : The Trapezoidal Rule of the function y = f(x) is given by
- f (x) dx= h\2 ($y_0 + y_n$) + 2 ($y_1 + y_2 + y_3 + ... + y_{n-1}$)
- f (x) dx = h\2 (sum of the first and last terms) + (sum of the remaining terms)

NUMERICAL INTEGRATION

- Simpson's 1\3 rd Rule : The Simpson's 1\3 rd Rule of the function f (x) is given by
- f (x) dx = h/3 ($y_0 + y_n$) + 4 ($y_1 + y_3 + y_5 + \ldots . y_{n-1}$) + 2 ($y_2 + y_4 + y_6 + \ldots .$)

NUMERICAL INTEGRATION

- Simpson's 3/8 th Rule : The Simpson's 3/8 th rule for the function f (x) isgiven by
- f (x) dx = 3h/8 ($y_0 + y_n$) + 2 ($y_3 + y_6 + y_9$ + ...) + 3 ($y_1 + y_2 + y_4$ +......)
- f (x) dx = 3h/8 (sum of the first and the last term) + 2 (multiples of three) + 3 (sum of the remaining terms)

INTRODUCTION

- There exists large number of ordinary differential equations, whose solution cannot be obtained by the known analytical methods. In such cases, we use numerical methods to get an approximate solution o f a given differential equation with given initial condition.

NUMERICAL DIFFERIATION

- Consider an ordinary differential equation of first order and first degree of the form
- $dy/dx = f(x,y)$ ....(1)
- with the intial condition $y(x_o) = y_o$ which is called initial value problem.
- To find the solution of the initial value problem of the form (1) by numerical methods, we divide the interval (a,b) on whcich the solution is derived in finite number of sub- intervals by the points

TAYLOR'S SERIES METHOD

- Consider the first order differential equation
- dy/dx = f (x, y)........(1)

  with initial conditions y (x_o) = y_o then expanding y (x) i.e f(x) in a Taylor's series at the point x_o we get

  $y (x_o + h) = y (x_o) + hy^1(x_o) + h^2/ 2! Y^{11}(x_o)+.......$

Note : Taylor's series method can be applied only when the various derivatives of f(x,y) exist and the value of f (x-xo) in the expansion of y = f(x) near x_o must be very small so that the series is convergent.

PICARDS METHOD OF SUCCESSIVE APPROXIMATION

- Consider the differential equation
- dy/dx = f (x,y) with intial conditions
- $y(x_0) = y_0$ then
- The first approximation y_1 is obtained by
- $y_1 = y_0 + f(x,y_0)$ dx
- The second approximation y_2 is obtained by $y_2 = y_1 + f (x,y_1)$ dx

PICARDS APPROXIMATION METHOD

- The third approximation of y_3 is obtained y by y_2 is given by
- $y_3 = y_o + f(x,y_2)$ dx……….and so on
- ………………………………..
- …………………………………
- $y_n = y_o + f\ (x,y^{n-1})$ dx
- The process of iteration is stopped when any two values of iteration are approximately the same.
- Note : When x is large , the convergence is slow.

EULER'S METHOD

- Consider the differential equation
- $dy/dx = f(x,y)$...........(1)
- With the initial conditions $y(x_0) = y_0$
- The first approximation of y_1 is given by
- $y_1 = y_0 + h\, f(x_0, y_0)$
- The second approximation of y_2 is given by
- $y_2 = y_1 + h\, f(x_0 + h, y_1)$

EULER'S METHOD

- The third approximation of y_3 is given by
- $y_3 = y_2 + h\,f\,(x_0 + 2h, y_2)$
- ……………………………
- ……………………………
- ……………………………
- $y_n = y_{n-1} + h\,f\,[x_0 + (n-1)h, y_{n-1}]$
- This is Eulers method to find an appproximate solution of the given differential equation.

IMPORTANT NOTE

- Note : In Euler’s method, we approximate the curve of solution by the tangent in each interval i.e by a sequence of short lines. Unless h is small there will be large error in y_n . The sequence of lines may also deviate considerably from the curve of solution. The process is very slow and the value of h must be smaller to obtain accuracy reasonably.

MODIFIED EULER'S METHOD

- By using Euler's method, first we have to find the value of
  $y_1 = y_0 + hf(x_0 , y_0)$
- <u>WORKING RULE</u>
- Modified Euler's formula is given by
- $y^i_{k+1} = y_k + h/2 [f(x_k ,y_k) + f(x_{k+1},y_{k+1}$
- when $i=1,y(0)_{k+1}$ can be calculated
- from Euler's method.

MODIFIED EULER'S METHOD

- When k=0,1,2,3,........gives number of iterations
- i=1,2,3,.......gives number of times a particular iteration k is repeated when
- i=1
- $Y^1_{k+1} = y_k + h/2\ [\ f(x_k, y_k) + f(x_{k+1}, y_{k+1})],,,,,,$

RUNGE-KUTTA METHOD

The basic advantage of using the Taylor series method lies in the calculation of higher order total derivatives of y. Euler's method requires the smallness of h for attaining reasonable accuracy. In order to overcomes these disadvantages, the Runge-Kutta methods are designed to obtain greater accuracy and at the same time to avoid the need for calculating higher order derivatives.The advantage of these methods is that the functional values only required at some selected points on the subinterval.

R-K METHOD

- Consider the differential equation
- $dy/dx = f(x, y)$
- With the initial conditions $y(x_o) = y_o$
- First order R-K method :
- $y_1 = y(x_o + h)$
- Expanding by Taylor's series
- $y_1 = y_o + h\, y^1{}_o + h^2/2\, y^{11}{}_o + \ldots\ldots.$

R-K METHOD

- Also by Euler's method
- $y_1 = y_0 + h\, f\,(x_0\,, y_0)$
- $y1 \quad = y_0 + h\, y^1{}_0$
- It follows that the Euler's method agrees with the Taylor's series solution upto the term in h. Hence, Euler's method is the Runge-Kutta method of the first order.

- The second order R-K method is given by
- $y_1 = y_0 + \frac{1}{2} (k_1 + k_2)$

  where $k_1 = h f (x_{0,}, y_0)$

  $k_2 = h f (x_0 + h , y_0 + k_1)$

- Third order R-K method is given by
- $y_1 = y_0 + 1/6 (k_1 + k_2 + k_3)$
- where $k_1 = h f (x_0 , y_0)$
- $k_2 = hf(x_0 + 1/2h , y_0 + ½ k_1)$
- $k_3 = hf(x_0 + ½h , y_0 + ½ k_2)$

The Fourth order R – K method :

This method is most commonly used and is often referred to as Runge – Kutta method only. Proceeding as mentioned above with local discretisation error in this method being O (h5), the increment K of y correspoding to an increment h of x by Runge – Kutta method from

$dy/dx = f(x,y), y(x_o)=Y_o$ is given by

- $K_1 = h\,f\,(x_0\,,\,y_0)$
- $k_2 = h\,f\,(x_0 + ½\,h,\, y_0 + ½\,k_1)$
- $k_3 = h\,f\,(x_0 + ½\,h,\, y_0 + ½\,k_2)$
- $k_4 = h\,f\,(x_0 + ½\,h,\, y_0 + ½\,k_3)$

  and finally computing

- $k = 1/6\,(k_1 + 2\,k_2 + 2\,k_3 + k_4)$
- Which gives the required approximate value as
- $y_1 = y_0 + k$

R-K METHOD

Note 1 : k is known as weighted mean of

k_1, k_2, k_3 and k_4.

Note 2 : The advantage of these methods is that the operation is identical whether the differential equation is linear or non-linear.

PREDICTOR-CORRECTOR METHODS

- We have discussed so far the methods in which a differential equation over an interval can be solved if the value of y only is known at the beginning of the interval. But, in the Predictor-Corrector method, four prior values are needed for finding the value of y at x_k.
- The advantage of these methods is to estimate error from successive approximations to y_k

PREDICTOR-CORRECTOR METHOD

- If x_k and x_{k+1} be two consecutive points, such that $x_{k+1}=x_{k+h}$, then
- Euler's method is
- $Y_{k+1} = y_k + hf(x_0 + kh, y_k)$,k= 0,1,2,3.......
- First we estimate y_{k+1} and then this value of yk+1 is substituted to get a better approximation of $y_{k+1.}$

MILNE'S METHOD

- This procedure is repeated till two consecutive iterated values of yk+1 are approximately equal. This technique of refining an initially crude estimate of yk+1 by means of a more accurate formula is known as Predictor- Corrector method.
- Therefore, the equation is called the Predictor while the equation serves as a corrector of yk+1.Two such methods, namely, Adams- Moulton and Milne's method are discussed.

MILNE’S METHOD

- Consider the differntial equation
- $dy/dx = f(x, y), f(0) = 0$
- Milne’s predictor formula is given by
- $Y^{(p)}_{n+1} = y_{n-3} + 4h/3(2y^{1}_{n} - y^{1}_{n-1} + 2y^{1}_{n-2})$
- Error estimates
- $E^{(p)} = 28/29 h^{5} y^{v}$

MILNE'S METHOD

- The Milnes corrector formula is givenby
- $y^{(c)}_{n+1}=y_{n-1}+h/3(y^{1}_{n+1}+4y^{1}_{n}+y^{1}_{n-1})$
- The superscript c indicates that the value obtained is the corrected value and the superscript p on the right indicates that the predicted value of y_{n+1} should be used for computing the value of
- $f(x_{n+1}, y_{n+1})$.

ADAMS – BASHFORTH METHOD

- Consider the differential equation
- $dy/dx = f(x,y)$, $y(x_0) = y_0$
- The Adams Bashforth predictor formula is given by
- $y^{(p)}_{n+1} = y_n + h/24\,[55y^1_n - 59y^1_{n-1} + 37y^1_{n-2} - 9y^1_{n-3}]$

ADAM'S BASHFORTH METHOD

- Adams – Bashforth corrector formula is given by
- $Y_{n+1}=y_n+h/24[9y^{1p}_{n+1} +19y^1_{n-5}-y^1_{n-1}+y^1_{n-2}]$

Error estimates

$E_{AB} = 251/720\ h^5$

UNIT-III

Fourier Series and Fourier Transform

INTRODUCTION

- Suppose that a given function f(x) defined in (-π,π) or (0, 2π) or in any other interval can be expressed as a trigonometric series as

$f(x)=a_0/2 + (a_1\cos x + a_2\cos 2x + a_3\cos 3x + \ldots + a_n\cos nx) + (b_1\sin x$ +

$b_2\sin 2x + \ldots\ldots\ldots b_n\sin nx) + \ldots\ldots\ldots$

$f(x) = a_0/2 + \sum(a_n\cos nx + b_n\sin nx)$

- Where a and b are constants with in a desired range of values of the variable such series is known as the fourier series for f(x) and the constants a_o, a_n,b_n are called fourier coefficients of f(x)

- It has period 2π and hence any function represented by a series of the above form will also be periodic with period 2π

POINTS OF DISCONTINUITY

- In deriving the Euler's formulae for ao,an,bn it was assumed that f(x) is continuous. Instead a function may have a finite number of discontinuities. Even then such a function is expressable as a fourier series.

DISCONTINUITY FUNCTION

- For instance, let the function f(x) be defined by
- $f(x) = \phi(x),\ c < x < x_0$
- $= \Psi(x),\ x_0 < x < c + 2\pi$
- where x_0 is thepoint of discontinuity in $(c, c+2\pi)$.

DISCONTINUITY FUNCTION

In such cases also we obtain the fourier series for f(x) in the usual way. The values of a_o,a_n, b_n are given by

$a_o = 1/\pi\ [\ \int ø(x)\ dx + \int \Psi(x)\ dx\ ]$

$a_n = 1/\pi\ [\ \int ø(x)\cos nx\ dx + \int \Psi(x)\cos nx\ dx$

$b_n = 1/\pi\ [\ \int ø(x)\sin nx\ dx + \int \Psi(x)\sin nx\ dx]$

EULER'S FORMULAE

- The fourier series for the function f(x) in the interval $C \leq x \leq C+2\pi$ is given by

$$f(x) = a_o/2 + \sum(a_n \cos nx + b_n \sin nx)$$

where $a_o = 1/\pi \int f(x)\, dx$

$$a_n = 1/\pi \int f(x) \cos nx\, dx$$

$$b_n = 1/\pi \int f(x) \sin nx\, dx$$

These values of a_o, a_n, b_n are known as Euler's formulae

EVEN AND ODD FUNCTIONS

- A function f(x) is said to be even if f(-x)=f(x) and odd if f(-x) = - f(x).
- If a function f(x) is even in (-π, π), its fourier series expansion contains only cosine terms, and their coefficients are a_o
- and a_n.
- $f(x)= a_o/2 + \sum a_n \cos nx$
- where $a_o= 2/ \pi \int f(x)\, dx$
- $a_n= 2/ \pi \int f(x) \cos nx\, dx$

ODD FUNCTION

- When f(x) is an odd function in $(-\pi, \pi)$ its fourier expansion contains only sine terms.
- And their coefficient is bn
- $f(x) = \sum b_n \sin nx$
- where $b_n = 2/\pi \int f(x) \sin nx \, dx$

HALF RANGE FOURIER SERIES

- THE SINE SERIES: If it be required to express f(x) as a sine series in (0,π), we define an odd function f(x) in (-π, π) ,identical with f(x) in (0,π).
- Hence the half range sine series (0,π) is given by
- $f(x) = \sum bn \sin nx$
- Where $bn = 2/\pi \int f(x) \sin nx \, dx$
-

HALF RANGE SERIES

- The cosine series: If it be required to express f(x) as a cosine series, we define an even function f(x) in (- (-π, π) , identical with f(x) in (o, π) , i.e we extend the function reflecting it with respect to the y-axis, so that f(-x)=f(x).

HALF RANGE COSINE SERIES

- Hence the half range series in $(0,\pi)$ is given by
- $f(x) = a_0/2 + \sum a_n \cos nx$
- where $a_0 = 2/\pi \int f(x)dx$
- $a_n = 2/\pi \int f(x) \cos nx\, dx$

CHANGE OF INTERVAL

- So far we have expanded a given function in a Fourier series over the interval $(-\pi,\pi)$and $(0,2\pi)$ of length 2π. In most engineering problems the period of the function to be expanded is not 2π but some other quantity say $2l$. In order to apply earlier discussions to functions of period $2l$, this interval must be converted to the length 2π.

PERIODIC FUNCTION

- Let f(x) be a periodic function with period 2l defined in the interval $c<x<c+2l$. We must introduce a new variable z such that the period becomes 2π.

CHANGE OF INTERVAL

- The fourier expansion in the change of interval is given by $f(x) = a_0/2+\sum a_n \cos n\pi x/l + \sum b_n \sin n\pi x/l$
- Where $a_0 = 1/l \int f(x)dx$
- $a_n = 1/l \int f(x)\cos n\pi x/l \, dx$
- $b_n = 1/l \int f(x)\sin n\pi x/l \, dx$

EVEN AND ODD FUNCTION

- Fourier cosine series : Let f(x) be even function in (-l,l) then
- $f(x) = ao/2 + \sum ancos\ n\pi x/l$
- where $ao = 2/l \int f(x)dx$
- $an = 2/l \int f(x)\cos n\pi x/l\ dx$

FOURIER SINE SERIES

- Fourier sine series : Let f(x) be an odd function in (-l,l) then
- $f(x) = \sum bn \sin n\pi x/l$
- where $bn = 2/l \int f(x) \sin n\pi x/l \, dx$
- Once ,again here we remarks that the even nature or odd nature of the function is to be considered only when we deal with the interval (-l,l).

HALF-RANGE EXPANSION

- Cosine series: If it is required to expand f(x) in the interval (0,l) then we extend the function reflecting in the y-axis, so that f(-x)=f(x).We can define a new function g(x) such that $f(x)= a_0/2 + \sum a_n \cos n\pi x/l$
- where $a_0= 2/l \int f(x)dx$
- $a_n= 2/l \int f(x)\cos n\pi x/l \, dx$

HALF RANGE SINE SERIES

- Sine series : If it be required to expand f(x)as a sine series in (o,l), we extend the function reflecting it in the origin so that f(-x) = f(x).we can define the fourier series in (-l,l) then,
- $f(x) = a_0/2 + \sum b_n \sin n\pi x/l$
- where $b_n = 2/l \int f(x) \sin n\pi x/l \, dx$

FOURIER INTEGRAL TRANSFORMS

- INTRODUCTION:A transformation is a mathematical device which converts or changes one function into another function. For example, differentiation and integration are transformations.
- In this we discuss the application of finite and infinite fourier integral transforms which are mathematical devices from which we obtain the solutions of boundary value.

- We obtain the solutions of boundary value problems related toengineering. For example conduction of heat, free and forced vibrations of a membrane, transverse vibrations of a string, transverse oscillations of an elastic beam etc.

- DEFINITION: The integral transforms of a function f(t) is defined by
- $F(p)=I[f(t) = \int f(t)\ k(p,t\ )dt$
- Where k(p,t) is called the kernel of the integral transform and is a function of p and t.

FOURIER COSINE AND SINE INTEGRAL

- When f(t) is an odd function cospt,f(t) is an odd function and sinpt f(t) is an even function. So the first integral in the right side becomes zero. Therefore we get
- $f(x) = 2/\pi\int sinpx$

FOURIER COSINE AND SINE INTEGRAL

- When f(t) is an odd function cospt,f(t) is an odd function and sinpt f(t) is an even function. So the first integral in the right side becomes zero. Therefore we get
- $f(x) = 2/\pi \int \sin px \int f(t) \sin pt \, dt \, dp$
- which is known as FOURIER SINE INTEGRAL.

- When f(t) is an even function, the second integral in the right side becomes zero.Therefore we get
- $f(x) = 2/\pi \int \cos px \int f(t) \cos pt \, dt \, dp$
- which is known as FOURIER COSINEINTEGRAL.

FOURIER INTEGRAL IN COMPLEX FORM

- Since cos p(t-x) is an even functionof p, we have
- $f(x) = 1/2\pi \int\int e^{ip(t-x)} f(t)\, dt\, dp$
- which is the required complex form.

INFINITE FOURIER TRANSFORM

- The fourier transform of a function f(x) is given by
- $F\{f(x)\} = F(p) = \int f(x)\, e^{ipx}\, dx$
- The inverse fourier transform of F(p) is given by
- $f(x) = 1/2\pi \int F(p)\, e^{-ipx}\, dp$

FOURIER SINE TRANSFORM

- The finite Fourier sine transform of f(x) when $0<x<l$, is definedas
- Fs{f(x) = Fs (n) sin (nπx)/l dx where n is an integer and the function f(x) is given by
- f(x) = 2/l∑ Fs (n) sin (nπx)/l is called the Inverse finite Fourier sine transform Fs(n)

FOURIER COSINE TRANSFORM

- We have $f(x) = 2/\pi \int \cos px \int f(t) \cos pt \, dt \, dp$
- Which is the fourier cosine integral .Now
- $Fc(p) = \int f(x) \cos px \, dx$ then
- f(x) becomes $f(x) = 2/\pi \int Fc(p) \cos px \, dp$ which is the fourier cosine transform.

PROPERTIES

- Linear property of Fourier transform
- Change of Scale property
- Shifting property
- Modulation property

UNIT-IV

Partial Differential equations

INTRODUCTION

- Equations which contain one or more partial derivatives are called Partial Differential Equations. They must therefore involve atleast two independent variables and one dependent variable. When ever we consider the case of two independent variables we shall usually take them to be x and y and take z to be the dependent variable.The partial differential coefficients

FORMATION 0F P.D.E

- Unlike in the case of ordinary differential equations which arise from the elimination of arbitrary constants the partial differential equations can be formed either by the elimination of arbitrary constants or by the elimination of arbitrary functions from a relation involving three or more variables.

ELIMINATION OF ARBITRARY CONSTANTS

- Consider z to be a function of two independent variables x and y defined by
- f (x,y,z,a,b) = 0……….(1) in which a and b are constants. Differentiating (1) partially with respect to x and y, we obtains two differential equations,let it be equation 2 &3. By means of the 3 equations two constants a and b can be eliminated.This results in a partial differential equation of order one in the form F(x,y,z,p,q)=0.

ELIMINATION OF ARBITRARY FUNCTIONS

- Let u= u(x,y,z) and v=v(x,y,z) be independent functions of the variables x,y,z and let ø(u,v)=0.............(1) be an arbitrary relation between them.We shall obtain a partial differential equation by eliminating the functions u and v. Regarding z as the dependent variable and differentiating (1) partially with respect to x and y, we get

LINEAR P.D.E

- Equation takes the form
- $Pp + Qq = R$
- a partial differential equation in p and q and free of the arbitrary function $ø(u,v)=0$ a partial differential equation which is linear. If thegiven relation between x,y,z contains two arbitrary functions then leaving a few exceptional cases the partial differential equations of higher order than the second will be formed.

SOLUTIONS OF P.D.E

- Through the earlier discussion we can understand that a partial differential equation can be formed by eliminating arbitrary constants or arbitrary functions from an equation involving them and three or more variables.
- Consider a partial differential equation of the form $F(x,y,z,p,q)=0$.......(1)

LINEAR P.D.E

- If this is linear in p and q it is called a linear partial differential equation of first order, if it is non linear in p,q then it is called a non-linear partial differential equation of first order.
- A relation of the type F $(x,y,z,a,b)=0$.....(2) from which by eliminating a and b we can get the equation (1) is called complete integral or complete solution of P.D.E

PARTICULAR INTEGRAL

- A solution of (1) obtained by giving particular values to a and b in the complete integral (2) is called particular integral.
- If in the complete integral of the form (2) we take f =(a,b).

COMPLETE INTEGRAL

- A solution of (1) obtained by giving particular values to a and b in the complete integral (2) is called particular integral.
- If in the complete integral of the form (2) we take f = aø() where a is arbitrary and obtain the envelope of the family of surfaces f(x,y,z,ø(a0) =0

GENERAL INTEGRAL

- Then we get a solution containing an arbitrary function. This is called the general solution of (1) corresponding to the complete integral (2)
- If in this we use a definite function ø(a), we obtain a particular case of the general integral.

SINGULAR INTEGRAL

- If the envelope of the two parameter fmily of surfaces (2) exists, it will also be a solution of (1). It is called a singular integral of the equation (1).
- The singular integral differs from the particular integral. It cannot be obtained that way. A more elaborate discussion of these ideas is beyond the scope.

LINEAR P.D.E OF THE FIRST ORDER

- A differential equation involving partial derivatives p and q only and no higher order derivatives is called a first order equation. If p and q occur in the first degree, it is called a linear partial differential equation of first order, other wise it is called non- linear partial differential equation.

LAGRANGE'S LINEAR EQUATION

- A linear partial differential equation of order one, involving a dependent variable and two independent variables x and y, of the form
- $Pp + Qq = R$
- Where P,Q,R are functions of x,y,z is called Lagrange's linear equation.

PROCEDURE

- Working rule to solve Pp+Qq=R
- First step: write down the subsidary equations dx/P = dy/Q = dz/R
- Second step: Find any two independent solutions of the subsidary equations. Let the two solutions be u=a and v=b where a and b are constants.

METHODS OF SOLVING LANGRANGE'S LINEAR EQUATION

- Third step: Now the general solution of Pp+Qq=R is given by f(u,v) = 0 or u=f(v)
- T o solve dx/P = dy/Q = dz/R
- We have two methods
- (i) Method of grouping
- (ii) Method of multipliers

METHOD OF GROUPING

- In some problems it is possible that two of the equations $dx/P = dy/Q = dz/R$ are directly solvable to get solutions $u(x,y)=\text{constant}$ or $v(y,z)= \text{constant}$ or $w(z,x) = \text{constant}$. These give the complete solution.

METHOD OF GROUPING

- Sometimes one of them say $dx/P = dy/Q$ may give rise to solution $u(x,y) = c_1$. From this we may express y, as a function of x. Using this $dy/Q = dz/R$ and integrating we may get $v(y,z) = c_2$. These two relations $u=c_1$, $v=c_2$ give rise to the complete solution.

METHOD OF MULTIPLIERS

- If $a_1/b_1 = a_2/b_2 = a_3/b_3 = \ldots\ldots\ldots\ldots = a_n/b_n$ then each ratio is equal to
- $l_1a_1+l_2a_2+l_3a_3+\ldots\ldots+l_na_n$
- $l_1b_1+l_2b_2+l_3b_3+\ldots\ldots+l_nb_n$
- consider $dx/P = dy/Q = dz/R$
- If possible identify multipliers l,m,n not necessarily so that each ratio is equal to

METHOD OF MULTIPLIERS

- If, l,m,n are so chosen that lP+mQ+nR=0 then ldx + mdy + ndz =0, Integrating this we get u(x,y,z) = c_1 similarlyor otherwise get another solution v(x,y,z) = c_2 independent of the earlier one. We now have the complete solution constituted by u=c_1, v= c_2.

NON-LINEAR P.D.E OF FIRST ORDER

- A partial differential equation which involves first order partial derivatives p and q with degree higher than one and the products of p and q is called a non- linear partial differential equations.

DEFINITIONS

- Complete integral: A solution in which the number of arbitrary constants is equal to the number of arbitrary constants is equal to the number of independent variables is called complete integral or complete solution of the given equation

PARTICULAR INTEGRAL

- Particular integral : A solution obtained by giving particular values to the arbitrary constants in the complete integral is called a particular integral.
- Singular integral: Let f(x,y,z,p,q) = 0 be a partial differential equation whose complete integral is ø(x,y,z,p,q) =0

STANDARD FORM I

- Equations of the form f(p,q)=o i.e equations containing p and q only.
- Let the required solution be z= ax+by+c
- Where p=a , q=b.substituting these values in f(p,q)=o we get f(a,b)=o
- From this, we can obtainb in terms of a .Let b=ø(a). Then the required solution is
- Z=ax + ø(a)y+c.

STANDARD FORM II

- Equations of the form f(z,p,q)=o i.e not containing x and y.
- Let u=x+ay and substitute p and q in the given equation.
- Solve the resulting ordinary differential equation in z and u.
- Substitute x+ay for u.

STANDARD FORM III

- Equations of the form f(x,p)=f(y,q) i.e equations not involving z and the terms containing x and p can be separated from those containing y and q.We assume each side equal to an arbitrary constant a, solve for p and q from the resulting equations
- Solving for p and q, we obtain p= f(x,p) and q= f(y,q) since is a function of x and y we have pdx + q dy integrating which gives the required solution.

STANDARD FORM IV

- CLAUIRT'S FORM : Equations of the form $z=px+qy+f(p,q)$. An equation analogous to clairaut's ordinary differential equation $y=px+f(p)$. The complete solution of the equation $z=px+qy+f(p,q)$. Is
- $z=ax+by+f(a,b)$. Let the required solution be $z=ax+by+c$

METHOD OF SEPARATION OF VARIABLES

- When we have a partial differential equation involving two independent variables say x and y, we seek a solution in the form X(x), Y(Y) and write down various types of solutions.

Heat equation

- Consider heat equation

$$u_t = c\, u_{xx}, \qquad 0 \le x \le 1, \qquad t \ge 0$$

  with initial and boundary conditions

$$u(0, x) = f(x), \qquad u(t, 0) = \alpha, \qquad u(t, 1) = \beta$$

- Define spatial mesh points $x_i = i\Delta x$, $i = 0, 1, \ldots, n+1$, where $\Delta x = 1/(n+1)$, and temporal mesh points $t_k = k\Delta t$, for suitably chosen Δt

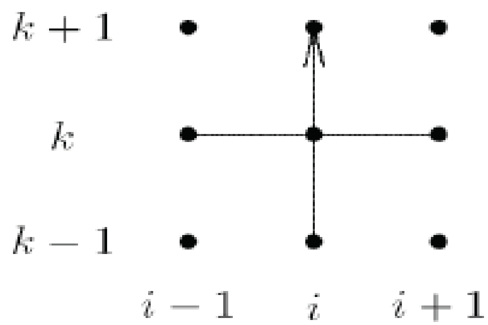
- Let u_i^k denote approximate solution at (t_k, x_i)

wave equation

- With mesh points defined as before, using centered difference formulas for both u_{tt} and u_{xx} gives finite difference scheme

$$\frac{u_i^{k+1} - 2u_i^k + u_i^{k-1}}{(\Delta t)^2} = c\,\frac{u_{i+1}^k - 2u_i^k + u_{i-1}^k}{(\Delta x)^2}, \quad \text{or}$$

$$u_i^{k+1} = 2u_i^k - u_i^{k-1} + c\left(\frac{\Delta t}{\Delta x}\right)^2 \left(u_{i+1}^k - 2u_i^k + u_{i-1}^k\right), \; i = 1, \ldots, n$$



UNIT-V

Vector Calculus

INTRODUCTION

- In this chapter, vector differential calculus is considered, which extends the basic concepts of differential calculus, such as, continuity and differentiability to vector functions in a simple and natural way. Also, the new concepts of gradient, divergence and curl are introducted.
- *Example*: i,j,k are unit vectors.

VECTOR DIFFERENTIAL OPERATOR

- The vector differential operator Δ is defined as $\Delta = i\, \partial/\partial x + j\, \partial/\partial y + k\, \partial/\partial z$. This operator possesses properties analogous to those of ordinary vectors as well as differentiation operator. We will define now some quantities known as gradient, divergence and curl involving this operator.

∂/∂

GRADIENT

- Let f(x,y,z) be a scalar point function of position defined in some region of space. Then gradient of f is denoted by grad f or Δf and is defined as

  $$\text{grad } f = i\,\partial f/\partial x + j\,\partial f/\partial y + k\,\partial f/\partial z$$

- *Example*: If f=2x+3y+5z then gradf= 2i+3j+5k

DIRECTIONAL DERIVATIVE

- The directional derivative of a scalar point function f at a point P(x,y,z) in the direction of g at P and is defined as grad g/|grad g|.grad f
- *Example*: The directional derivative of f=xy+yz+zx in the direction of the vector i+2j+2k at the point (1,2,0) is 10/3

DIVERGENCE OF A VECTOR

- Let f be any continuously differentiable vector point function. Then divergence of f and is written as div f and is defined as

  Div f = $\partial f_1/\partial x$ + j $\partial f_2/\partial y$ + k $\partial f_3/\partial z$

- *Example* 1: The divergence of a vector 2xi+3yj+5zk is 10
- *Example* 2: The divergence of a vector $f=xy^2i+2x^2yzj-3yz^2k$ at (1,-1,1) is 9

SOLENOIDAL VECTOR

- A vector point function f is said to be solenoidal vector if its divergent is equal to zero i.e., div f=0
- *Example* 1: The vector $f=(x+3y)i+(y-2z)j+(x-2z)k$ is solenoidal vector.
- *Example* 2: The vector $f=3y^4z^2i+z^3x^2j-3x^2y^2k$ is solenoidl vector.

CURL OF A VECTOR

- Let f be any continuously differentiable vector point function. Then the vector function curl of f is denoted by curl f and is defined as curl $f = i \times \partial f/\partial x + j \times \partial f/\partial y + k \times \partial f/\partial z$
- *Example* 1: If $f = xy^2 i + 2x^2 yzj - 3yz^2 k$ then curl f at (1,-1,1) is $-i-2k$
- *Example* 2: If $r = xi + yj + zk$ then curl r is 0

IRROTATIONAL VECTOR

- Any motion in which curl of the velocity vector is a null vector i.e., curl v=0 is said to be irrotational. If f is irrotational, there will always exist a scalar function f(x,y,z) such that f=grad g. This g is called scalar potential of f.
- *Example*:Thevectorf=(2x+3y+2z)i+(3x+2y+3z)j+(2x+3y+3z)k is irrotational vector.

VECTOR INTEGRATION

- INTRODUCTION: In this chapter we shall define line, surface and volume integrals which occur frequently in connection with physical and engineering problems. The concept of a line integral is a natural generalization of the concept of a definite integral of f(x) exists for all x in the interval [a,b]

WORK DONE BY A FORCE

- If F represents the force vector acting on a particle moving along an arc AB, then the work done during a small displacement F.dr. Hence the total work done by F during displacement from A to B is given by the line integral $\int F.dr$
- *Example*: If $f=(3x^2+6y)i-14yzj+20xz^2k$ along the lines from (0,0,0) to (1,0,0) then to (1,1,0) and then to (1,1,1) is 23/3

SURFACE INTEGRALS

- The surface integral of a vector point function F expresses the normal flux through a surface. If F represents the velocity vector of a fluid then the surface integral $\int F.n\, dS$ over a closed surface S represents the rate of flow of fluid through the surface.
- *Example*:The value of $\int F.n\, dS$ where $F=18zi-12j+3yk$ and S is the part of the surface of the plane $2x+3y+6z=12$ located in the first octant is 24.

VOLUME INTEGRAL

- Let f (r) = $f_1 i+f_2 j+f_3 k$ where f_1,f_2,f_3 are functions of x,y,z. We know that dv=dxdydz. The volume integral is given by $\int f\, dv=\iiint(f_1 i+f_2 j+f_3 k)dxdydz$
- *Example*: If $F=2xzi-xj+y^2k$ then the value of $\int f\, dv$ where v is the region bounded by the surfaces $x=0,x=2,y=0,y=6,z=x^2,z=4$ is128i-24j-384k

VECTOR INTEGRAL THEOREMS

- In this chapter we discuss three important vector integral theorems.
- 1)Gauss divergence theorem
- 2)Green’s theorem
- 3)Stokes theorem

GAUSS DIVERGENCE THEOREM

- This theorem is the transformation between surface integral and volume integral. Let S be a closed surface enclosing a volume v. If f is a continuously differentiable vector point function, then
- $\int \text{div}\, f \, dv = \int f.n \, dS$
- Where n is the outward drawn normal vector at any point of S.

GREEN'S THEOREM

- This theorem is transformation between line integral and double integral. If S is a closed region in xy plane bounded by a simple closed curve C and in M and N are continuous functions of x and y having continuous derivatives in R, then
- $\int M dx + N dy = \iint (\partial N/\partial x - \partial M/\partial y) dx dy$

STOKES THEOREM

- This theorem is the transformation between line integral and surface integral. Let S be a open surface bounded by a closed, non-intersecting curve C. If F is any differentiable vector point function then
- $\int F.dr=\int Curl\ F.n\ ds$