Hall Ticket No | | | | | | | | | | Question Paper Code: AECB08

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Four Year B.Tech III Semester End Examinations(Regular) - November, 2019

Regulation: IARE – R18

PROBABILITY THEORY AND STOCHASTIC PROCESS

Time: 3 Hours **(ECE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Explain probability with axioms. State and prove total probability theorem. **[7M]**

   (b) A ship can successfully arrive at its destination if its engine and its satellite navigation system do not fail en route. If the engine and satellite navigation system are known to fail independently with respective probabilities of 0.05 and 0.001, what is the probability of successful arrival. **[7M]**

2. (a) Give expressions of probability density and distribution functions of Gaussian random variable and sketch them with parameters marked. **[7M]**

   (b) A pair of fair dice is rolled 10 times. Find i) The probability that number 7 will show at least once ii) The probability that number 7 will show at least twice. **[7M]**

UNIT – II

3. (a) Define joint density function, joint probability distribution function and list its properties. **[7M]**

   (b) For uniform density function having f(x) $\begin{matrix} = \frac{1}{b-a} & a \leq x \leq b \\ = \ 0 & else \end{matrix}$.

   Determine the characteristic function and mean of X. **[7M]**

4. (a) Obtain the expression for a monotonic transformation of a single random variable. **[7M]**

   (b) Random variables X and Y have the joint density function,

   $$f(x,y) = \begin{cases} \frac{2}{43}(x+0.5y)^2 & 0 < x < 2 \ and \ 0 < y < 3 \\ 0 \ else \end{cases}.$$

   Find the second-order moments m_{02} and m_{20}. **[7M]**

UNIT – III

5. (a) Define i) Stationary random process ii) Wide sense stationary random process iii) Ergodic random process **[7M]**

   (b) Given that an ergodic process $X(t)$ has an autocorrelation function $R_{XX}(\tau) = 18 + \frac{2\,[1+4\cos(12\tau)]}{6+\tau^2}$. Determine i) $\left|\overline{X(t)}\right|$ ii) Average power in $X(t)$ **[7M]**

6. (a) Define and explain cross correlation of two random processes X(t) and Y(t). **[7M]**

   (b) Let two random processes be defined by $X(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$, $Y(t) = B\cos(\omega_0 t) - A\sin(\omega_0 t)$, where A and B are random variables and is constant. Find the cross-correlation function $R_{XY}(t,\, t+\tau)$. **[7M]**

UNIT – IV

7. (a) What is cross power density spectrum of a random process. Explain with examples. **[7M]**

   (b) Determine the average power of the random process $X(t) = A_0\cos(\omega_0 t + \theta)$ where A_0 and ω_0 are constants and θ is a uniformly distributed random variable on $\left(0,\, \frac{\pi}{2}\right)$. **[7M]**

8. (a) Deduce the relation between the cross-power density spectra of input and output of an LTI system. **[7M]**

   (b) If the auto correlation of WSS process is $R_{XX}(\tau) = Ke^{-k|\tau|}$, show that its power spectral density is $S_{XY}\,(\omega) = \frac{2}{1+(\omega/K)^2}$. **[7M]**

UNIT – V

9. (a) What is significance of the power density spectrum of a random process? Obtain the expression for it. **[7M]**

   (b) The auto correlation function of a random process is $R_{XX}(\tau)$= $e^{-|\tau|}$. Find the PSD and average power of the process X(t). **[7M]**

10. (a) List the properties of cross power density spectra of input and output random processes of a LTI system. **[7M]**

    (b) A random process has the power density spectrum S_{XX}(w)=$w^2/(w^2$+1).Find the average power of the random process X(t). **[7M]**