Aerodynamics

by

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MODULE -I

INTRODUCTORY TOPICS FOR AERODYNAMICS

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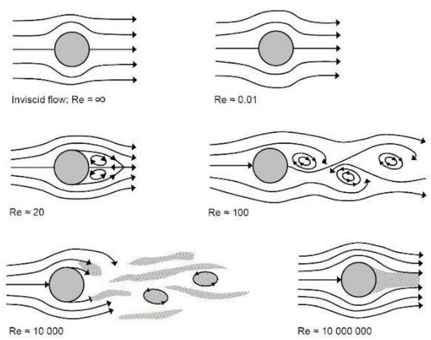
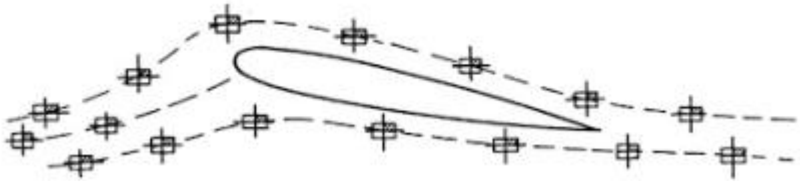
We can treat external flows around bodies as :

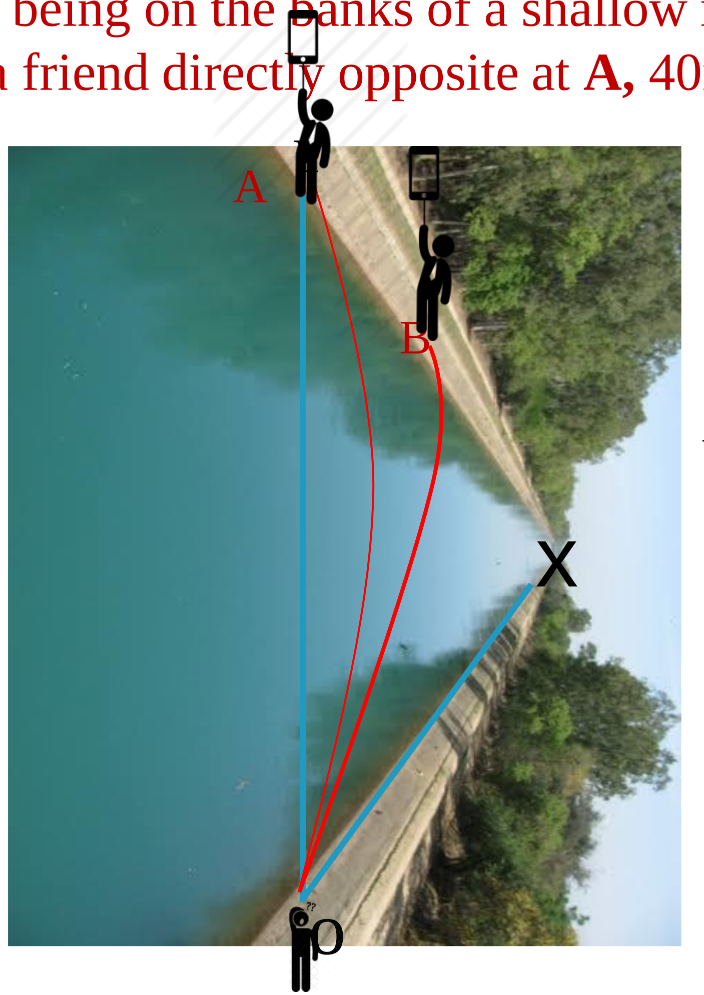
1. Invicid (i.e. Frictionless)



2. Irrotational (i.e. The Fluid Particles are Not Rotating)



Imagine being on the banks of a shallow river of a constant depth of 1 m at a position O with a friend directly opposite at **A,** 40m away.



- The bank can be represented by the ***Ox*** axis
- The line joining you to your friend at

A the *Oy* axis in the two-coordinate system

STREAM FUNCTION

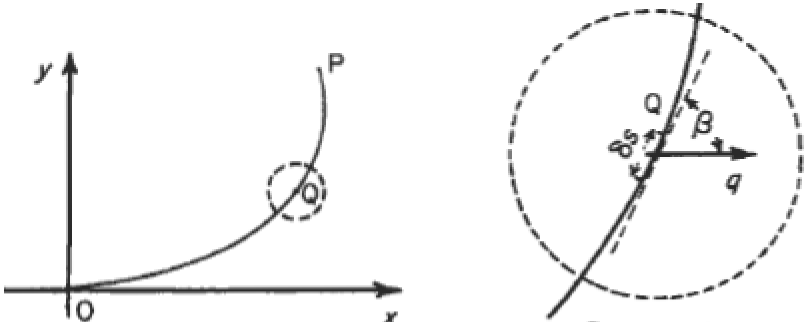
- Now if the stream speed is 2m/s the amount of water passing between you and your friend is 40 x 1 x 2 = 80 m3 s-l and this is the amount of water flowing past any point anywhere along the river which could be measured at a weir downstream.
- Suppose you now throw a buoyant rope to your friend who catches the end but allows the slack to fall in the river and float into a curve as shown. The amount of water flowing under the line is still $80m^3$ /s no matter what shape the rope takes, and is unaffected by the configuration of the rope.
- Suppose your friend moves along to a point B somewhere downstream, still holding his end of the line but with sufficient rope paid out as he goes. The volume of water passing under the rope is still only $80m^3$ /s providing he has not stepped over a tributary stream or an irrigation drain in the bank. It follows that, if no water can enter or leave the stream, the quantity flowing past the line will be the same as before and furthermore will be unaffected by the shape of the line between 0 and B.

- **The amount or quantity of fluid passing such a line per second is called the *stream function* or *current function* and it is denoted by ψ.**

Let us study the Mathematically about Stream Function

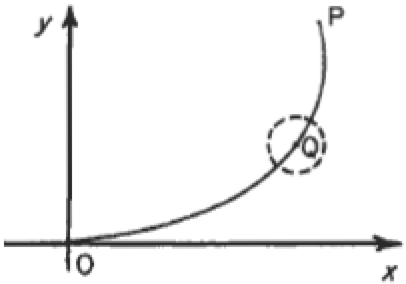
Let us study the Mathematically about Stream Function

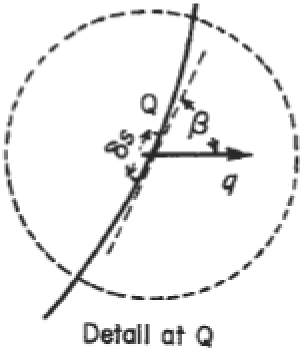
1. Consider now a pair of coordinate axes set in a two-dimensional air stream that is moving generally from left to right.
2. The axes are arbitrary space references and in no way interrupt the fluid streaming past.
3. Similarly the line joining **0** to a point **P** in the flow in no way interrupts the flow since it is as imaginary as the reference axes ***Ox*** and ***Oy***. An algebraic expression can be found for the line in *x* and *y*.



Detail at Q

1. Let the flow past the line at any point Q on it be at velocity ***q*** over a small length δs of line where direction of ***q*** makes angle β to the tangent of the curve at Q.
2. The component of the velocity ***q*** perpendicular to the element δs is ***q***sinβ and therefore, assuming the depth of stream flow to be unity, the amount of fluid crossing the element of line δs is ***q***sinβ x δs x 1 per second.
3. Adding up all such quantities crossing similar elements along the line from 0 to P, the total amount of flow past the line (***sometimes called flux***) is





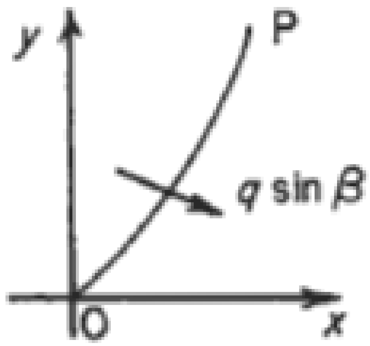
If this quantity of fluid flowing between O and P remains the same irrespective of the path of integration.

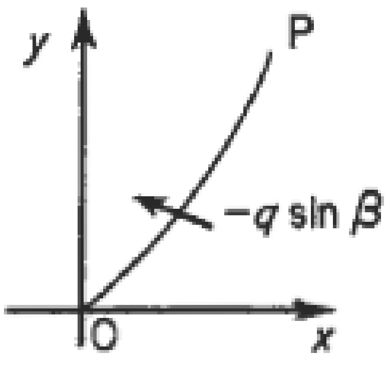
$$\int_{Op} q \sin\beta \, ds$$

which is the line integral of the normal velocity component from O to P.

STREAM FUNCTION

Sign Convention For Stream Functions

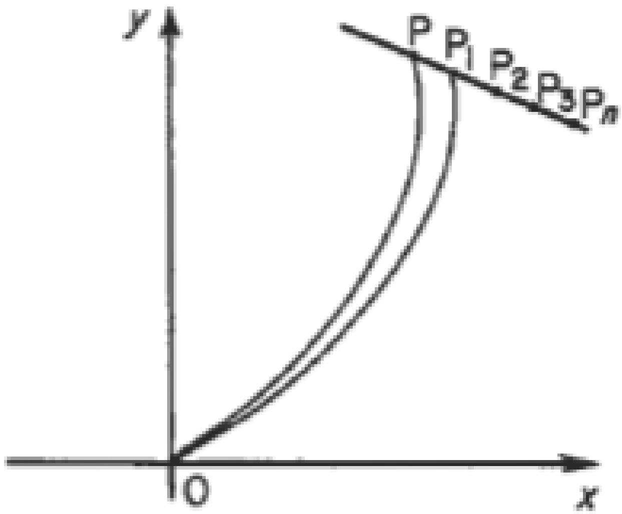




Flow ***across*** the path of integration is ***positive*** if, when looking in the direction of integration, it crosses the path from left to right and vice-versa.

A ***streamline*** is a line of constant ψ

Suppose there is a point P_1 close to P which has the same value of stream function as point P . Then the flow across any line OP_1 equals that across OP, and the amount of fluid flowing into area OP P_1O across OP equals the amount flowing out across OP_1. Therefore, no fluid crosses line PP_1 and the velocity of flow must be along, or tangential to, PP_1.

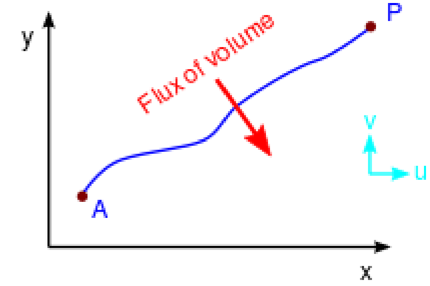


All other points $\mathbf{P_2}$, $\mathbf{P_3}$, etc. which have a stream function equal in value to that of **P** have, by definition, the same flow across any lines joining them to 0, so by the same argument the velocity of the flow in the region of $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$, etc. must be along $\mathbf{PP_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$, etc., and no fluid crosses the line $\mathbf{PP_1}$, $\mathbf{P_2}$,. . .,**Pn.**

The line **P**, $\mathbf{P_1}$, $\mathbf{P_2}$, . . . $\mathbf{P_n}$, etc. is a line of constant ψ and is called a streamline

Velocity components in terms of ψ

The stream function ψ(x,y,t)– in the point P with two dimensional coordinates (x,y) and as a function of time t for an incompressible flow.



$$\Psi=\int_A^P (u dy - v dx)$$

$$\delta\psi = \frac{\partial\psi}{\partial x}\delta x + \frac{\partial\psi}{\partial y}\delta y,$$

An infinitesimal shift $\delta P = (\delta x, \delta y)$ of the position results, In a stream function shift $\delta\Psi = u\delta y - v\delta x$ ----------------(1)

The flow velocity components have to be

$$u = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial\psi}{\partial x}$$

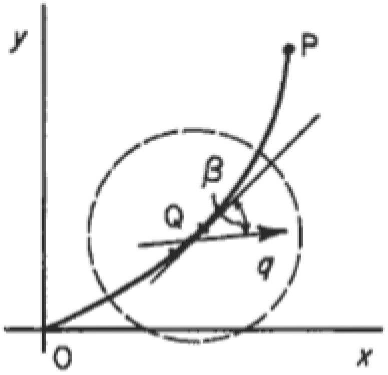
Which is an exact differential provided as follows $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

in relation to the stream function ψ.

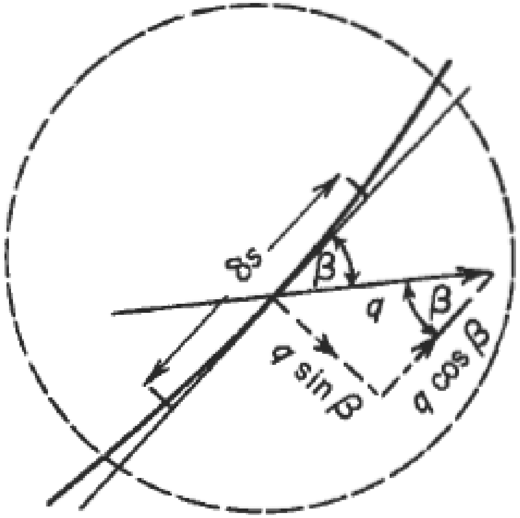
This is the condition of zero divergence resulting from flow incompressibility. Since

VELOCITY POTENTIAL

In a general two-dimensional fluid flow, consider any (imaginary) line OP joining the origin of a pair of axes to the point **P(x,y)**. Again, the axes and this line do not impede the flow, and are used only to form a reference datum.



At a point Q on the line let the local velocity q meet the line OP in β. Then the component of velocity parallel to δs is qcosβ.



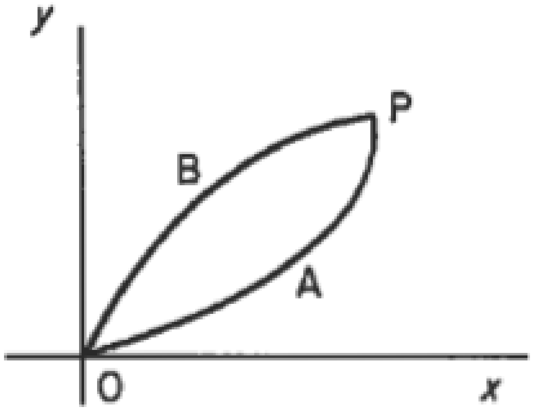
The amount of fluid flowing along δs is ***q***cosβδs

The total amount of fluid flowing along the line towards P is the sum of all such amounts, i.e.

$$\int_{Op} \boldsymbol{q} \cos\beta \, ds$$

This function is called the ***velocity potential*** of P with respect to O and is denoted by **ϕ**

Now OQP can be any line between O and P and a necessary condition for $q\cos\beta\delta s$ to be the velocity potential ϕ is that the value of ϕ is unique for the point P, irrespective of the path of integration. Then:

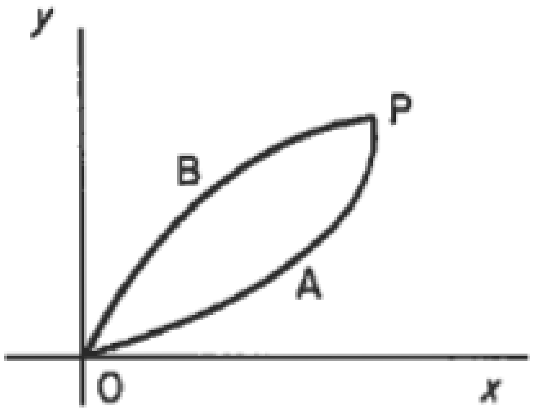


Velocity potential $\phi = \int_{Op} q \cos\beta \, ds$

Sign convention for velocity potential

The tangential flow along a curve is the product of the local velocity component and the elementary length of the curve. Now, if the velocity component is in the direction of integration, it is considered a ***positive*** increment of the velocity potential.

This in turn would imply that the fluid within the circuit possessed vorticity. The existence of a velocity potential must therefore imply zero vorticity in the flow, or in other words, a flow without circulation, i.e. an ***irrotational*** flow.

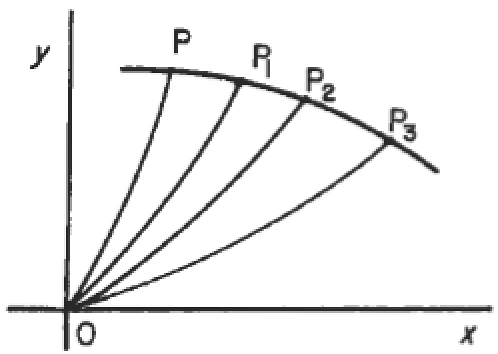


Velocity potential ϕ *in a close path* = $\oint q \cos\beta \, ds = 0$

VELOCITY POTENTIAL

The equipotential

Consider a point **P** having a velocity potential ϕ (**ϕ** is the integral of the flow component along OP) and let another point $\mathbf{P_I}$ close to **P** have the same velocity potential ϕ**. This** then means that the integral **of** flow along $\mathbf{OP_1}$ equals the integral of flow along **OP.** But by definition $\mathbf{OPP_1}$ is another path of integration from O to $\mathbf{P_I}$. Therefore



$$\text{Velocity potential } \phi = \int_{Op} q \cos\beta \, ds = \int_{Op_1} q \cos\beta \, ds = \int_{Opp1} q \cos\beta \, ds$$

Similarly for other points such as $\mathbf{P_2}$, $\mathbf{P_3}$, having the same velocity potential, there can be no flow along the line joining $\mathbf{P_I}$ to $\mathbf{P_2}$.

The Equipotential Characteristics

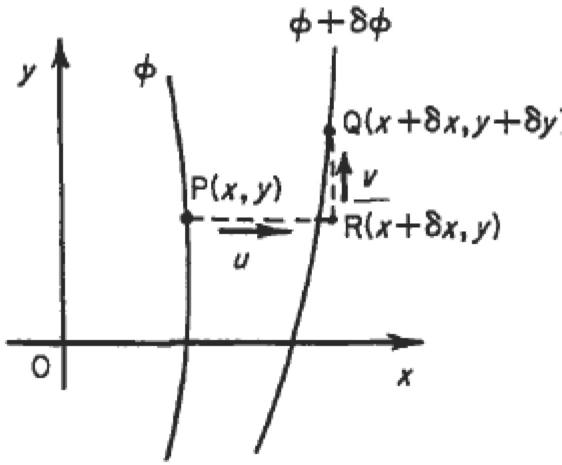
The line joining **P, P_1, P_2, P_3** is a line joining points having the same velocity potential and is called an ***equipotential*** or a line of constant velocity potential, i.e. **a** line of constant ϕ *.*

The significant characteristic of an equipotential is that there is no flow along such a line. Notice the correspondence between an equipotential and a streamline that is a line across which there is no flow.

The flow in the region of points **P** and **P_1** should be investigated more closely. From the above there can be no flow along the line **PP_1,** but there is fluid flowing in this region so it must be flowing in such a way that there is no component of velocity in the direction **PP_1. So** the flow can only be at right-angles to **PP_1,** that is the flow in the region **PP_1** must be normal to **PP_1.** Now the streamline in this region, the line to which the flow is tangential, must also be at *right-angles* to **PP_1** which is itself the local equipotential.

VELOCITY COMPONENTS IN TERMS OF Φ

Let a point P(x, *y)* be on an equipotential Φ and a neighbouring point Q(x+δx, y+δ*y)* be on the equipotential Φ + δϕ. Then by definition the increase in velocity potential from P to Q is the line integral of the *tangential* velocity component along any path between P and Q. Taking PRQ as the most convenient path where the local velocity components are *u* and *v:*



$$\delta\phi = u\delta x + v\delta y$$

LAPLACE EQUATION

The equation of continuity in two dimensions (incompressible flow)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

The equation of vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta$ (2)

The stream function (incompressible flow) .Ψ, describes a continuous flow in two dimensions where the velocity at any point is given by

$$u = \frac{\partial \psi}{\partial y} = 0 \text{ and } v = -\frac{\partial \psi}{\partial x} \qquad (3)$$

The stream function (incompressible flow) .ϕ, describes a continuous flow in two dimensions where the velocity at any point is given by

$$u = \frac{\partial \phi}{\partial x} = 0 \text{ and } v = \frac{\partial \phi}{\partial y} \qquad (4)$$

LAPLACE EQUATION

Substituting (3) in (1) gives the identity $\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$

which demonstrates the validity of (3), while substituting (4) in (2) gives the identity

$$\frac{\partial^2 \emptyset}{\partial x \partial y} - \frac{\partial^2 \emptyset}{\partial x \partial y} = 0$$

demonstrating the validity of (4), i.e. a flow described by a unique velocity potential must be irrotational.
Alternatively substituting (3) in (2) and (4) in (1) the criteria for irrotational continuous flow are that

$$\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Laplace Equation

Also written as

$$\nabla^2 \phi = \nabla^2 \psi = 0$$

Laplace Equation

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Relation between ψ and ϕ

We know that

$$v_x = -\frac{\partial \psi}{\partial y}, \qquad v_y = \frac{\partial \psi}{\partial x}.$$

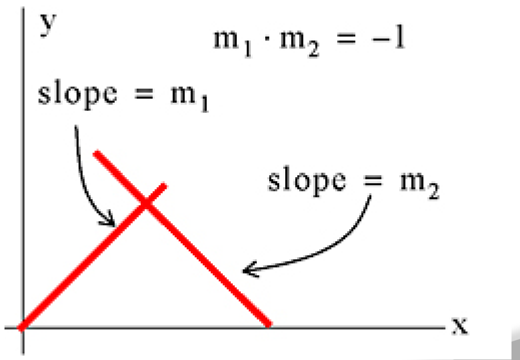
and

$$v_x = -\frac{\partial \phi}{\partial x}, \qquad v_y = -\frac{\partial \phi}{\partial y}.$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \qquad \frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y}.$$

Slope of the velocity potential as dy/dx = u/v
Slope of the velocity stream function as dy/dx = u/v

Multiply both the slop we get =-1



Two **lines** with **slopes** that are **negative reciprocals of each other** are **perpendicular** to **each other**.

FLOW SINGULARITIES

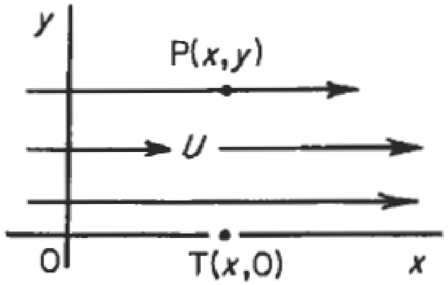
UNIFORM FLOW

Flow of a fluid properties (Temperature, pressure, density) in which each particle moves along its line of **flow** with constant speed and in which the cross section of each **stream** tube remains unchanged.

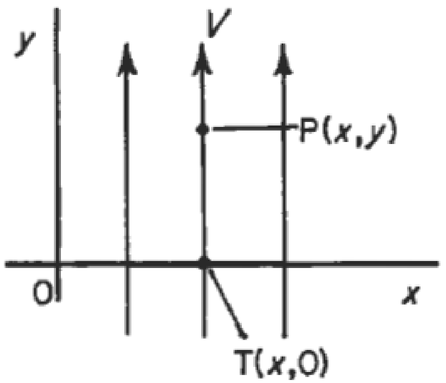
https://www.youtube.com/watch?v=AQhCGkK-hoA Curl in Hindi

UNIFORM FLOW

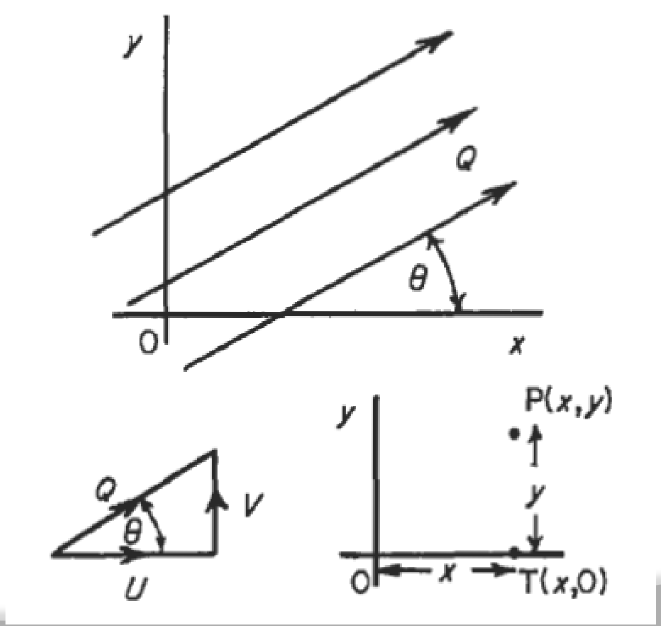
Flow of constant velocity parallel to Ox axis from left to right



Flow of constant velocity parallel to Oy axis



Flow of constant velocity in any direction

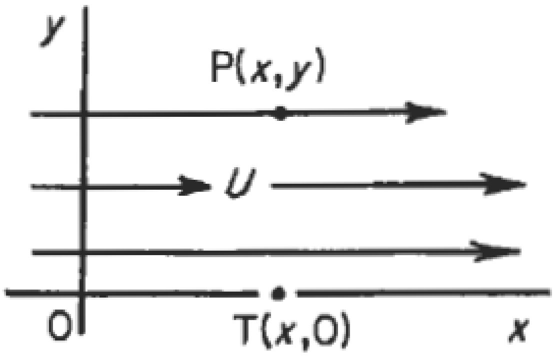


UNIFORM FLOW

Flow Of Constant Velocity Parallel To Ox Axis From Left To Right

Consider flow streaming past the coordinate axes ***Ox,*** *Oy* at velocity ***U*** parallel to ***Ox.***

By definition the stream function ψ at a point **P(x,** ***y)*** in the flow is given by the amount of fluid crossing any line between O and **P.** For convenience the contour OTP is taken where **T** is on the Ox axis x along from O, i.e. point T is given by (x, 0).



Then

ψ = flow across line OTP

= flow across line OT plus flow across line TP

= O + *U* x length TP =o + uy

Therefor; ψ = *UY*

The streamlines (lines of constant ψ) are given by drawing the curves

ψ = constant = *Uy*

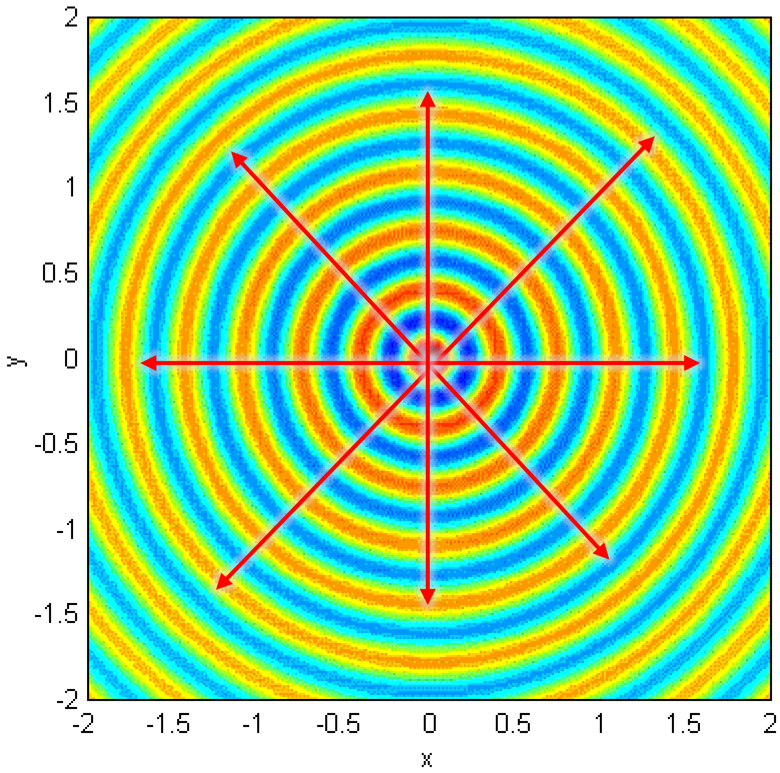
UNIFORM FLOW

$$\text{Therefore; } y = \frac{\psi}{U} = \text{constant on streamlines}$$

The lines ψ = constant are all straight lines parallel to ***Ox*.**

ϕ = flow along contour OTP
= flow along OT + flow along TP
= ux+0
Therefore; ϕ = ux
The lines of constant ϕ , the equipotentials, are given by Ux = constant, and since the velocity is constant the equipotentials must be lines of constant *x*, or lines parallel to *Oy* that are everywhere normal to the streamlines.

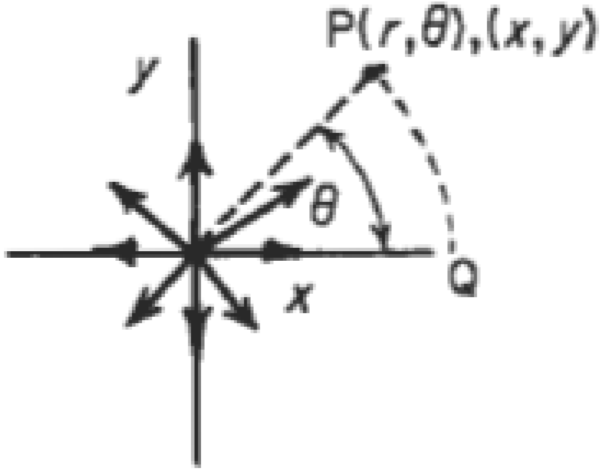
SOURCE



A source (sink) of strength m(-m) is a point at, which fluid is appearing (or disappearing) at a uniform rate of m(-m)m^2 s^{-1}.

Stream Function Ψ of a Source

Place the source for convenience at the origin of a system of axes, to which the point P has ordinates (x, *y)* and ***(r, θ)***



Putting the line along the x-axis as Ψ = 0 and taking the most convenient contour for integration as OQP where QP is an arc of a circle of radius ***r, then***

Ψ = flow across OQ + flow across QP

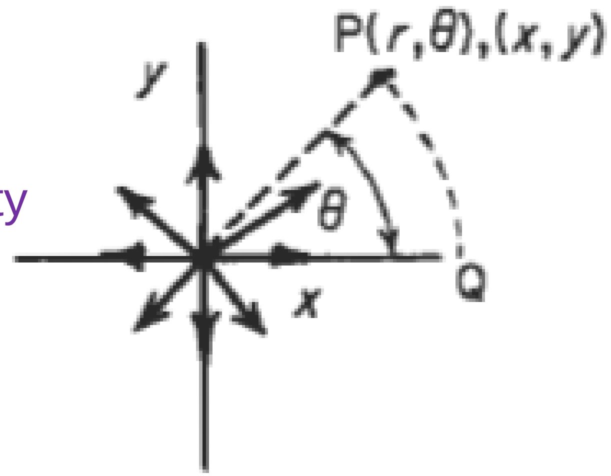
= velocity across OQ*OQ + velocity across QP * QP

$$\Psi = 0 + \frac{m}{2\pi r} * r\theta = \frac{m\theta}{2\pi}$$

or putting θ = $\tan^{-1}$ (y/x) ⟹ $\Psi = \frac{m\theta}{2\pi} \tan^{-1}(y/x)$

Velocity Potential φ of a Source

Place the source for convenience at the origin of a system of axes, to which the point P has ordinates (x, *y)* and ***(r, θ)***



The velocity everywhere in the field is radial, i.e. the velocity at any point P(r, ***θ)* is** given by

Q=$\sqrt{q_n^2 + q_t^2}$, here Q= q_n since $\mathbf{q_t}$ = 0. Integrating round OQP where Q is point ***(r,***0)

$$\phi = \int_{OQ} q\cos\beta\, ds + \int_{QP} q\cos\beta\, ds$$

$$= \int_{OQ} q_n dr + \int_{QP} q_t r\,\delta\theta = \int_{OQ} q_n\, dr + 0$$

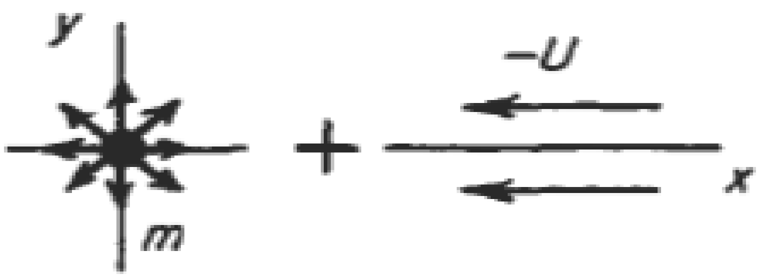
But knt, $q_n = \frac{m}{2\pi r}$ → $\phi = \int_{r_0}^{r} \frac{m}{2\pi r} dr = \frac{m}{2\pi} \ln\frac{r}{r_0}$

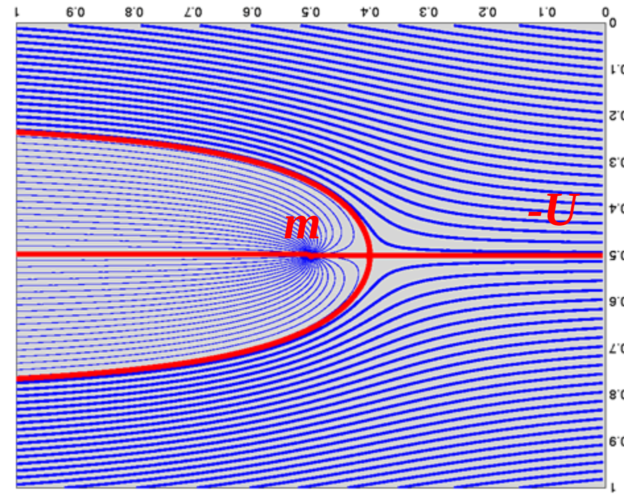
where r_o is the radius of the equipotential φ= 0.

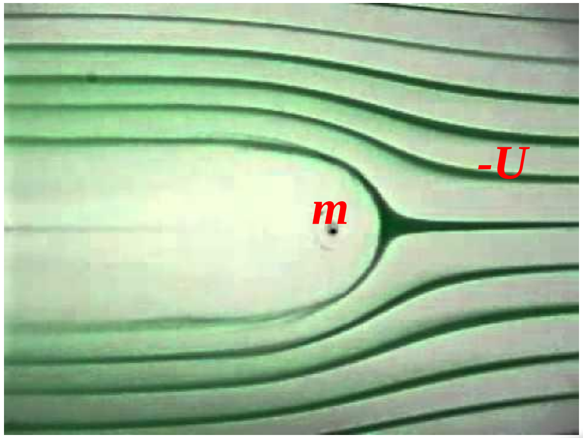
In cartesian Coordinate $\phi = \frac{m}{4\pi}\ln(x^2 + y^2)$

A SOURCE IN A UNIFORM HORIZONTAL STREAM

Let a source of strength ***m*** be situated at the origin with a uniform stream of ***-U*** moving from right to left







Then Ψ = stream function of Source + Stream Function of Uniform Flow

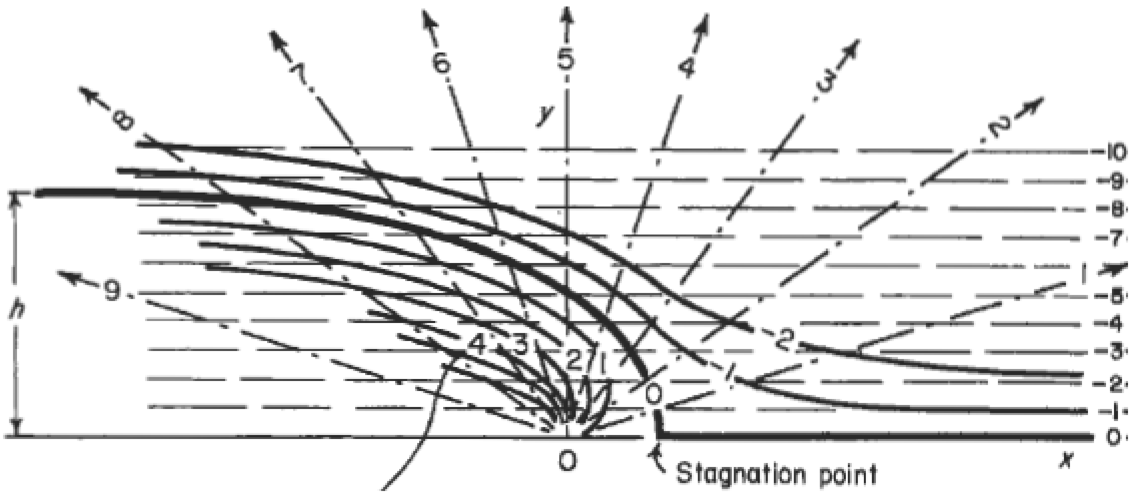
$$\Psi = \frac{m\theta}{2\pi} - Uy \qquad \phi = \frac{m}{2\pi} ln\frac{r}{r_0} - Ux$$

$$\Psi = \frac{m}{2\pi}\tan^{-1}\frac{y}{x} - Uy \qquad \phi = \frac{m}{2\pi} ln\frac{r}{r_0} - Urcos\theta$$

$$\Psi = \frac{m}{2\pi}\tan^{-1}\frac{y}{x} - Uy$$

Let us differentiate the above equation w.r t. x we get,

$$\frac{\partial\Psi}{\partial x} = \frac{m}{2\pi}.\frac{\partial tan^{-1}(\frac{y}{x})}{\partial(\frac{y}{x})}\frac{\partial(\frac{y}{x})}{\partial x}$$



If $\Psi = 0$ is replaced by a solid boundary

The vertical velocity component at any point in the flow is given by - $\frac{\partial\Psi}{\partial x}$.

Solve the differential equation and we get , $\frac{\partial\Psi}{\partial x} = \frac{m}{2\pi}\frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{-y}{x^2}$

$$v = \frac{m}{2\pi}\frac{y}{x^2+(y)^2}$$

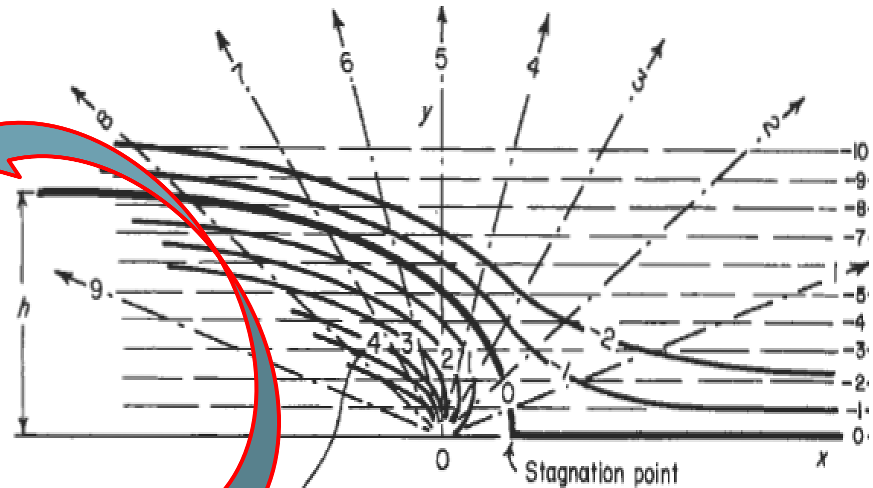
Let us analyse now for Stagnation point !!!

The position of the stagnation point and local velocity

A stagnation point is given by $u = 0$, $v = 0$

$$u = \frac{\partial \psi}{\partial y} = 0 = \frac{m}{2\pi} \frac{x}{x^2 + y^2} - U$$

$$v = -\frac{\partial \psi}{\partial x} = 0 = \frac{m}{2\pi} \frac{y}{x^2 + y^2}$$



From Eqn $\boldsymbol{v} = 0$ when $y = 0$, and substituting infirst Eqn when $y = 0$ and $\boldsymbol{x} = \boldsymbol{x_o}$

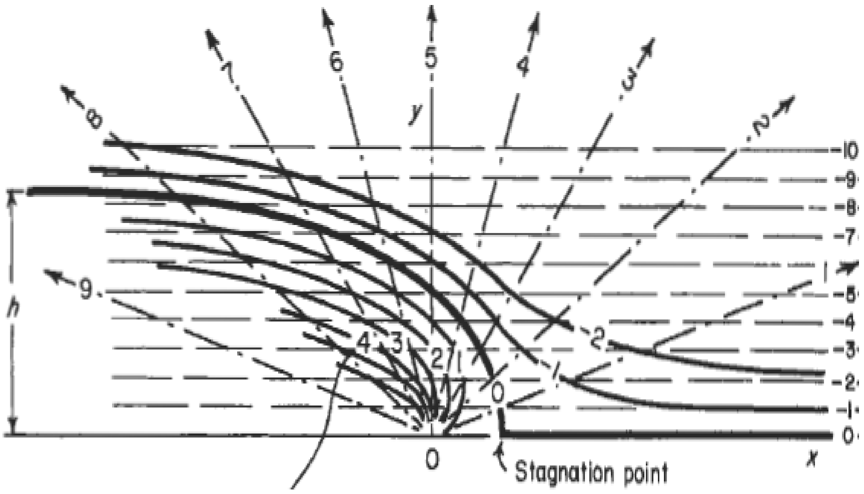
From the above two equation we can get local velocity u, v

$$u = 0 = \frac{m}{2\pi} \frac{1}{x_0} - U$$

$\boldsymbol{x_o = m/2\pi U}$

Height of Cliff *h*

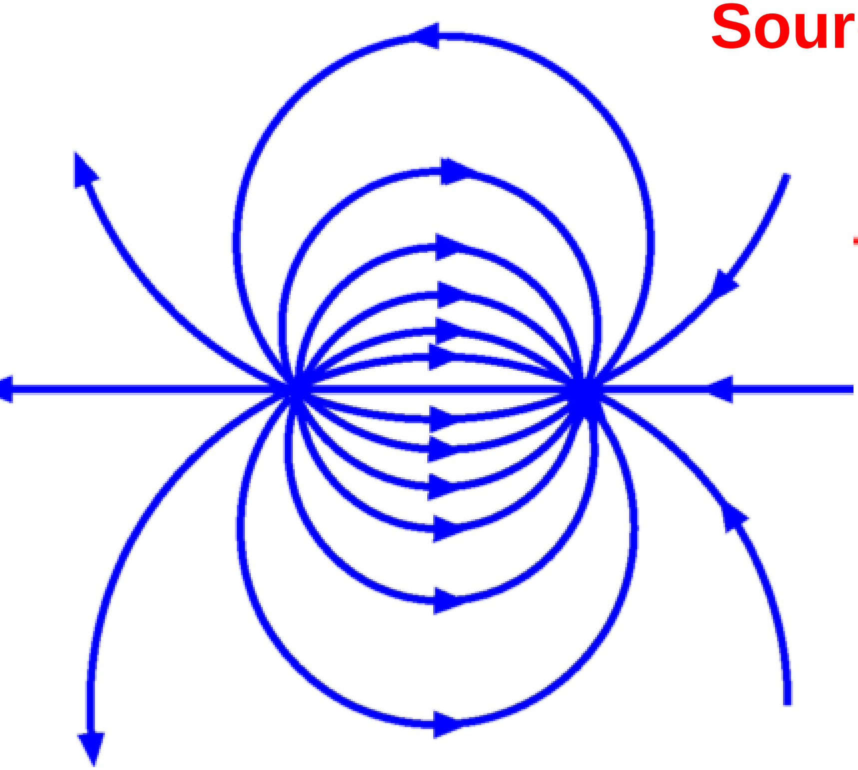
The ultimate thickness, ***2h*** (or height of cliff ***h)*** of the shape given by Ψ = 0 for this combination **is** found by putting y = ***h*** and $\theta = \pi$ in the general expression, we get,



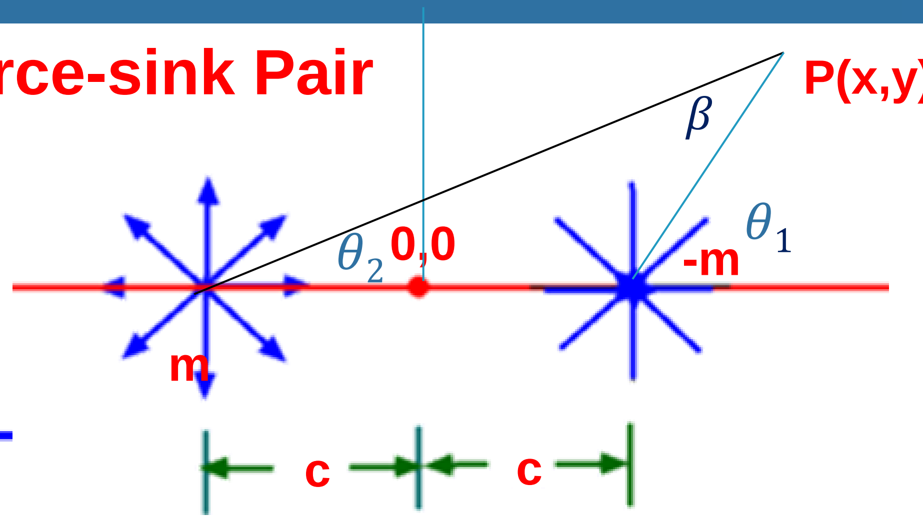
$$\psi = \frac{m\pi}{2\pi} - Uh = 0$$

Therefore, $h = m/2U$

SOURCE-SINK PAIR



Source-sink Pair

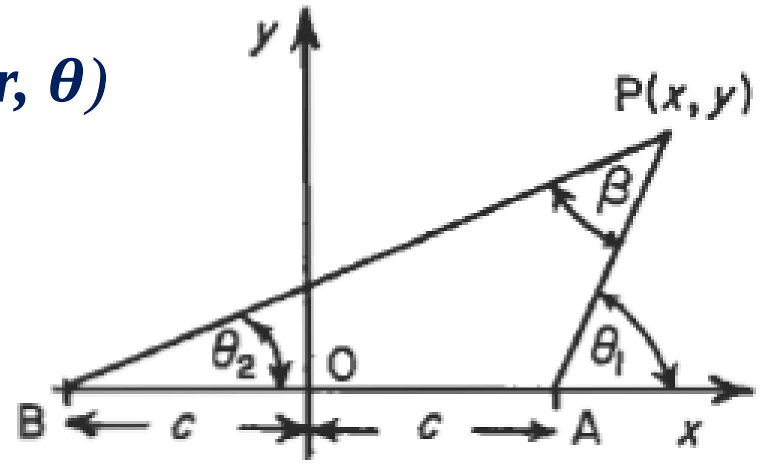


This is a combination of a source and sink of equal (but opposite) strengths situated a distance ***2c*** apart.

Let $\pm$ ***m*** be the strengths of a source and sink situated at points **A (c,**0) and B ***(-c,*** 0), that is at a distance of ***c*** m on either side of the origin.

SOURCE-SINK PAIR

The stream function at a point ***P(x, y), (r, θ)*** due to the combination is



$$\psi = \frac{m\theta_1}{2\pi} - \frac{m\theta_2}{2\pi} = \frac{m}{2\pi}(\theta_1 - \theta_2)$$

$$\psi = \frac{m}{2\pi}\beta$$

Transposing the equation to Cartesian coordinates $\tan\theta_1 = \frac{y}{x-c}$ And $\tan\theta_2 = \frac{y}{x+c}$

$$\tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1\tan\theta_2} = \frac{\frac{y}{x-c} - \frac{y}{x+c}}{1 + \frac{y^2}{x^2-c^2}}$$

Therefore $\beta = \theta_1 - \theta_2 = \tan^{-1}\frac{2cy}{x^2+y^2-c^2}$

And Substituting β in above Eqn. we get

$$\psi = \frac{m}{2\pi}\tan^{-1}\frac{2cy}{x^2+y^2-c^2}$$

SOURCE-SINK PAIR

To find the shape of the streamlines associated with **this** combination it is necessary to investigate Eqn **.**

$$\psi = \frac{m}{2\pi}\tan^{-1}\frac{2cy}{x^2+y^2-c^2}$$

By Rearranging we get,

$$\tan\left(\frac{2\pi}{m}\psi\right) = \frac{2cy}{x^2+y^2-c^2}$$

$$\Rightarrow \quad x^2+y^2-c^2 = \frac{2cy}{\tan\left(\frac{2\pi\psi}{m}\right)}$$

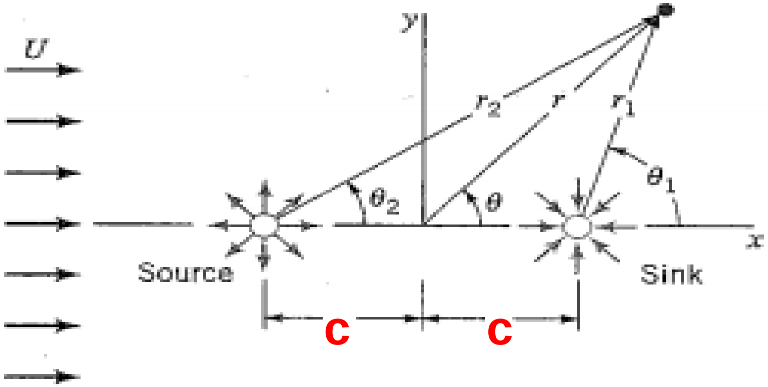
$$\Rightarrow \quad x^2+y^2-2c\cot\frac{2\pi\psi}{m}y - c^2 = 0$$

Which is the Equation of a Circle of Radius

$c\sqrt{\cot^2(2\pi\psi/m)+1}$ **and** $c\cot(2\pi\psi/m)$

Therefore, streamlines for this combination consist of a series of circles with centres on the Oy axis and intersecting in the source and sink

Consider the velocity potential at any point ***P(r, O)(x,*** y).



$$\phi = \frac{m}{2\pi}\ln\frac{r_1}{r_0} - \frac{m}{2\pi}\ln\frac{r_2}{r_0} = \frac{m}{2\pi}\ln\frac{r_1}{r_2}$$

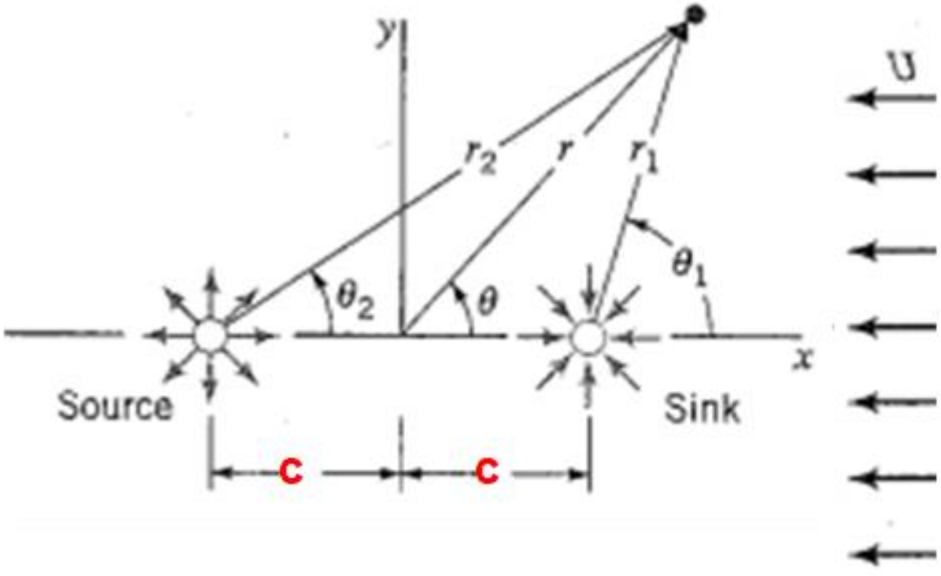
$$r_1^2 = (x-c)^2 + y^2 = x^2 + y^2 + c^2 - 2xc$$

$$r_2^2 = (x+c)^2 + y^2 = x^2 + y^2 + c^2 + 2xc$$

$$\phi = \frac{m}{4\pi}\ln\frac{x^2+y^2+c^2-2xc}{x^2+y^2+c^2+2xc}$$

The stream function due to this combination is:

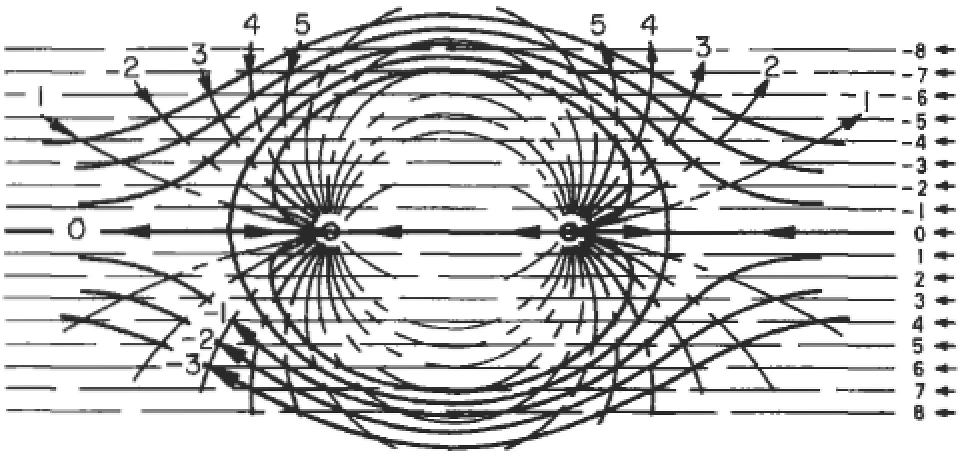
$$\psi = \frac{m}{2\pi}\tan^{-1}\frac{2cy}{x^2 + y^2 - c^2} - Uy$$



The velocity potential at any point in the flow due to this combination is given by:

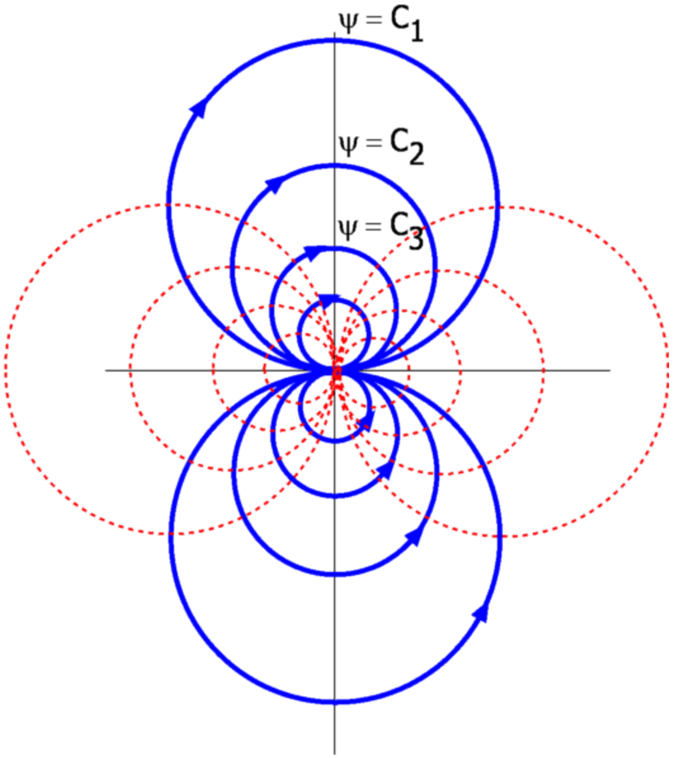
$$\phi = \frac{m}{2\pi}\ln\frac{r_1}{r_2} - Ur\sin\theta$$

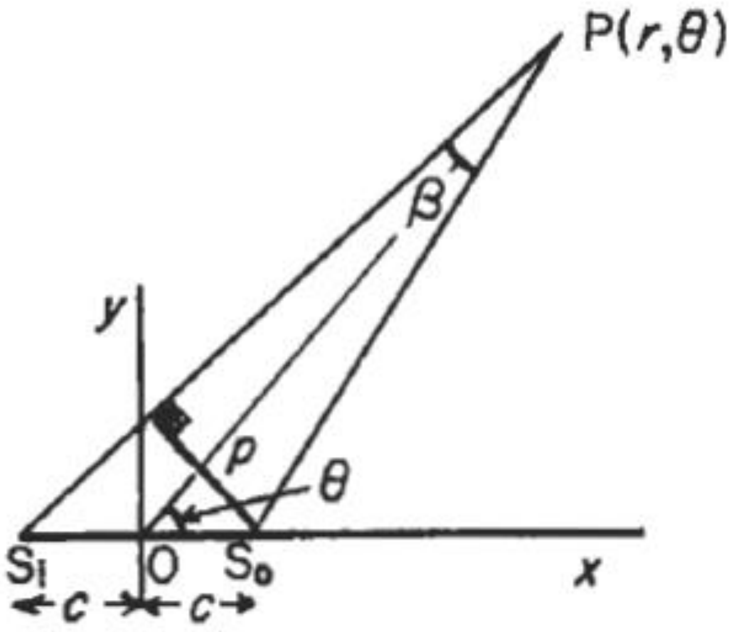
$$\phi = \frac{m}{4\pi}\ln\frac{x^2 + y^2 + c^2 - 2xc}{x^2 + y^2 + c^2 + 2xc} - Ux$$



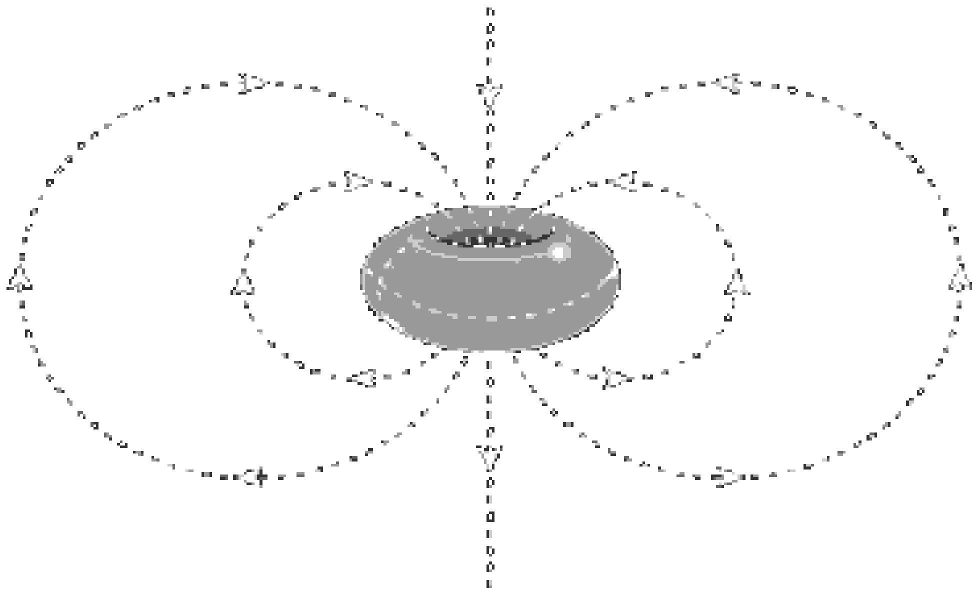
DOUBLET

A doublet is a source and sink combination, as described above, but with the separation infinitely small. A doublet is considered to be at a point.





DOUBLET



DOUBLET

For the source and sink, we the stream function as below:

$$\psi = (m/2\pi)\beta$$

By constructing the perpendicular of length ***p*** from the source to the line joining the sink and P it can be seen that as the source and sink approach

p -> 2csinθ and also ***p*** -> *rβ*

Therefore in the limit

$$2c\sin\theta = r\beta \Longrightarrow \beta = \frac{2c\sin\theta}{r}$$

$$\psi = \frac{m}{2\pi} = \frac{2c\sin\theta}{r}$$

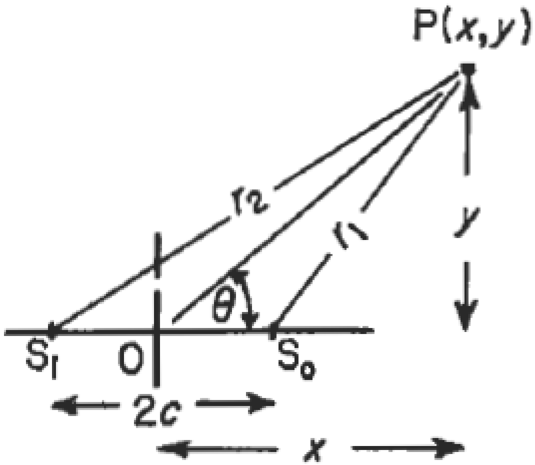
$$r = \sqrt{x^2 + y^2}, \qquad \sin\theta = \frac{y}{\sqrt{x^2 + y^2}}, \qquad \psi = \frac{\mu}{2\pi}\frac{y}{x^2 + y^2}$$

$$(x^2 + y^2) - \frac{\mu}{2\pi\psi}y = 0$$

Equation of a Circle

DOUBLET

Consider again a source and sink set a very small distance, 2c, apart



$$\phi = \frac{m}{2\pi} \ln \frac{r_1}{r_0} - \frac{m}{2\pi} \ln \frac{r_2}{r_0}$$

where $\pm m$ is the strength of the source and sink respectively. Then

$$\phi = \frac{m}{2\pi} \ln \frac{r_1}{r_2} = \frac{m}{4\pi} \ln \frac{r_1^2}{r_2^2}$$

From the Above Figure

$$r_1^2 = x^2 + y^2 - 2xc + c^2$$

$$r_2^2 = x^2 + y^2 + 2xc + c^2$$

$$\phi = \frac{m}{4\pi} \ln \frac{x^2 + y^2 - 2xc + c^2}{x^2 + y^2 + 2xc + c^2}$$

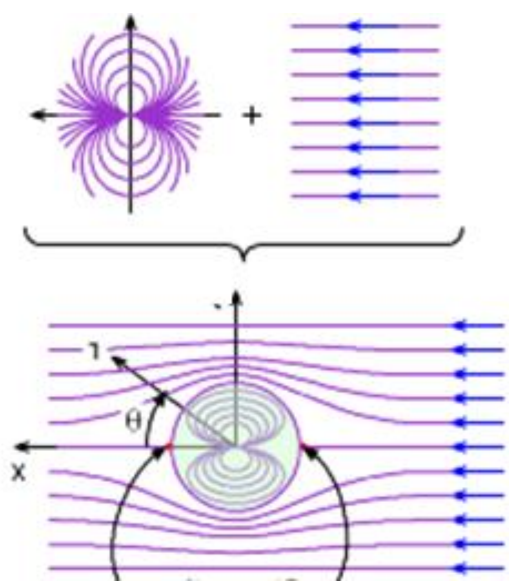
$$\phi = \frac{m}{4\pi} \ln \left(1 - \frac{4xc}{x^2 + y^2 + c^2 + 2xc}\right)$$

$$\phi = -\frac{\mu}{2\pi r} \cos\theta$$

The stream function due to this combination is: $\psi = \frac{\mu}{2\pi r}\sin\theta - Uy$

By substituting the value in the above Eqn of sinθ and r we get,

$$r = \sqrt{x^2 + y^2}, \qquad \sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$



Converted to rectangular coordinates gives us:

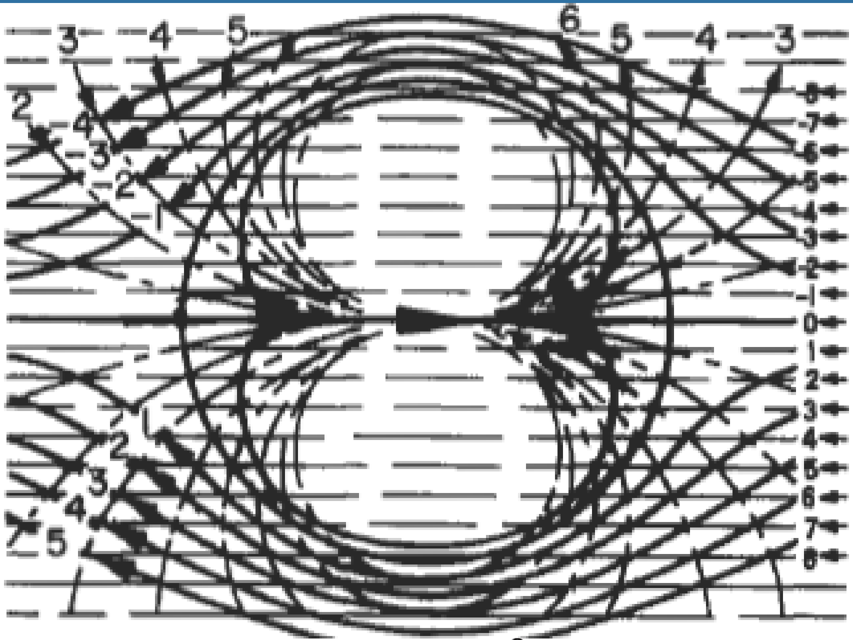
$$\psi = \frac{\mu}{2\pi}\frac{y}{x^2 + y^2} - Uy \qquad \psi = y\left(\frac{\mu}{2\pi(x^2 + y^2)} - U\right)$$

For the streamline ψ = 0; $y\left(\frac{\mu}{2\pi(x^2 + y^2)} - U\right) = 0$

$$y = 0 \quad \text{or} \quad x^2 + y^2 = \frac{\mu}{2\pi U}$$

This shows the streamline $\psi = 0$, to consist of the Ox together with a circle, of radius $\sqrt{\mu/2\pi U} = a$ (say).

DOUBLET IN UNIFORM FLOW

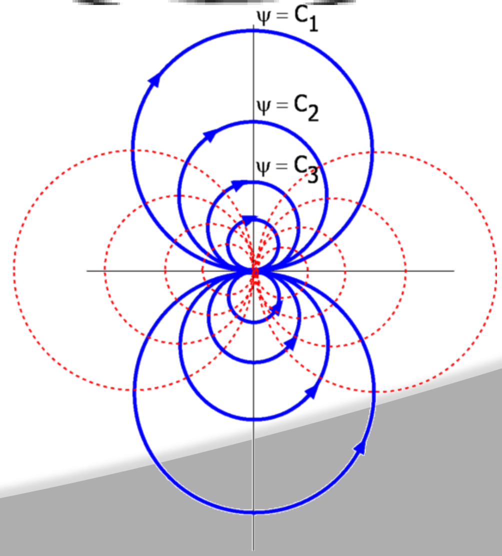


The velocity potential due to this combination is that corresponding to a uniform stream flowing parallel to the **Ox** axis, superimposed on that of a doublet at the origin. Putting **x** = r cosθ:

$$\phi = -Ur\cos\theta + \frac{\mu}{2\pi r}\cos\theta$$

$$\phi = -U\cos\theta\left(r + \frac{a^2}{r}\right)$$

where $a = \sqrt{\mu/2\pi U}$



DOUBLET IN UNIFORM FLOW

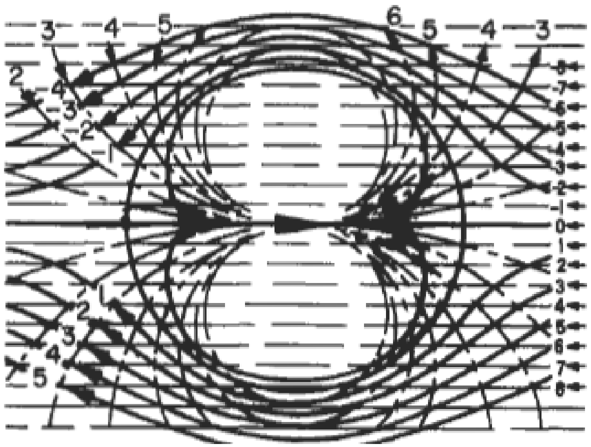
In Polar Coordinates Stream Function can be written as :

$$\psi = \frac{\mu}{2\pi r}\sin\theta - Ur\sin\theta \Longrightarrow \psi = U\sin\theta\left(\frac{\mu}{2\pi r U} - r\right) \Longrightarrow \psi = U\sin\theta\left(\frac{a^2}{r} - r\right)$$

Differentiating this partially with respect to r and θ in turn will give expressions for the velocity

$$q_n = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta\left(\frac{a^2}{r^2} - 1\right)$$

$$q_t = -\frac{\partial\psi}{\partial r} = U\sin\theta\left(\frac{a^2}{r^2} + 1\right)$$



Putting r = a, we get q_n=0 and q_t = 2Usinθ

Therefore the velocity on the surface **is** 2Usin θ and it is important to note that the velocity at the surface is independent of the radius of the cylinder.

If a long circular cylinder **is** set in a uniform flow the motion around it will, ideally, be given by the expression, $q_t = -\frac{\partial \psi}{\partial r} = U \sin\theta\left(\frac{a^2}{r^2}+1\right)$ and the velocity the anywhere on surface by

the formula ***q = 2Usinθ***

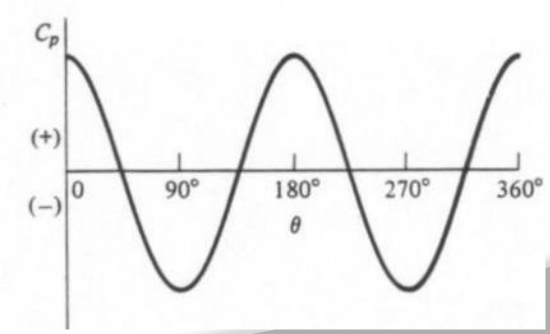
By the use of Bernoulli's equation, the pressure ***p*** acting on the surface of the cylinder where the velocity is ***q*** can be found. If p_o is the static pressure of the free stream where the velocity is *U* then by Bernoulli's equation:

$$p_0 + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho q^2$$

$$= p + \frac{1}{2}\rho(2U\sin\theta)^2$$

$$p - p_0 = \frac{1}{2}\rho U^2[1 - 4\sin^2\theta]$$

Substitute the Value of q from above equation; we get



A SPINNING CYLINDER IN A UNIFORM FLOW

This is given by the stream function due to a doublet, in a uniform horizontal flow, with a line vortex superimposed at the origin.

$$\psi = \frac{\mu}{2\pi r}\sin\theta - Uy - \frac{\Gamma}{2\pi}\ln\frac{r}{r_0}$$

Converting to homogeneous coordinates $$\psi = Ur\sin\theta\left(\frac{\mu}{2\pi r^2 U} - 1\right) - \frac{\Gamma}{2\pi}\ln\frac{r}{r_0}$$

WKT $\sqrt{\mu/2\pi U}$ = *a; If* r_0*=a we get,*

$$\psi = Ur\sin\theta\left(\frac{a^2}{r^2} - 1\right) - \frac{\Gamma}{2\pi}\ln\frac{r}{a}$$

and differentiating partially with respect to ***r*** and **θ** the velocity components of the flow anywhere on or outside the cylinder become, respectively

$$q_t = -\frac{\partial\psi}{\partial r} = U\sin\theta\left(\frac{a^2}{r} + 1\right) + \frac{\Gamma}{2\pi r}$$

$$q_n = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta\left(\frac{a^2}{r^2} - 1\right)$$

$$q = \sqrt{q_n^2 + q_t^2}$$

On the surface of the spinning cylinder ***r*** = ***a.***

Therefore, $q_n = 0$

A SPINNING CYLINDER IN A UNIFORM FLOW

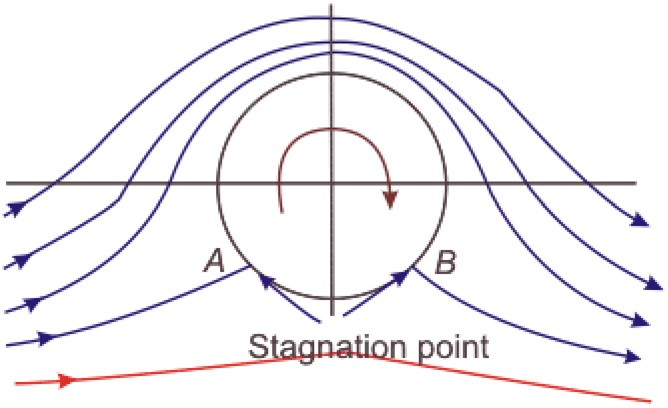
$$q_t = 2U \sin\theta + \frac{\Gamma}{2\pi a}$$

$$q = q_t = 2U \sin\theta + \frac{\Gamma}{2\pi a}$$

and applying Bernoulli's equation between a point a long way upstream and a point on the cylinder where the static pressure is ***p***

$$p_0 + \frac{1}{2}\rho U^2 = p + \frac{1}{2}\rho q^2$$
$$= p + \frac{1}{2}\rho\left(2U\sin\theta + \frac{\Gamma}{2\pi a}\right)^2$$

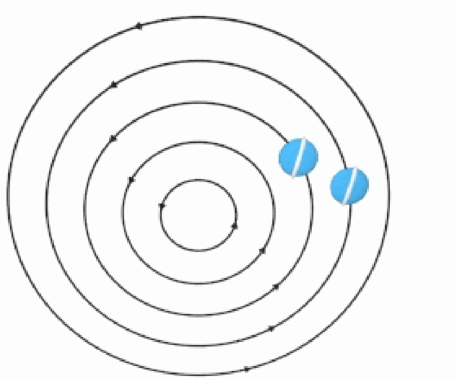
$$p - p_0 = \frac{1}{2}\rho U^2\left[1 - \left(2\sin\theta + \frac{\Gamma}{2\pi U a}\right)^2\right]$$

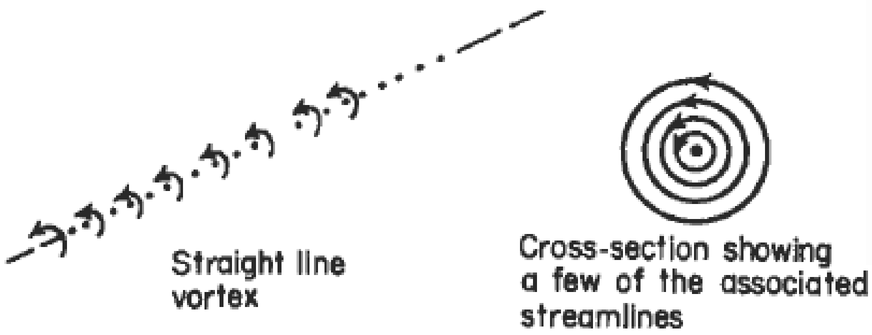


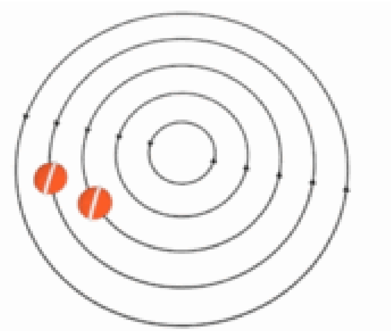
A SPINNING CYLINDER IN A UNIFORM FLOW

Line (point) vortex

This flow is that associated with a straight line vortex. A line vortex can best be described as a string of rotating particles. A chain of fluid particles are spinning on their common axis and carrying around with them a swirl of fluid particles which flow around in circles. A cross-section of such a string of particles and its associated flow shows a spinning point outside of which is streamline flow in concentric circles







A SPINNING CYLINDER IN A UNIFORM FLOW

Consider a vortex located at the origin of a polar system of coordinates. But the flow is irrotational, so the vorticity everywhere is zero. Recalling that the streamlines are concentric circles, centred on the origin, so that $q\theta = 0$, it therefore;

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(rq_t) = 0$$

By integrating we get $rq_t = C$

Integration Result equivalent to $\Gamma = \oint \vec{q} \cdot \vec{t}\,\mathrm{d}s$

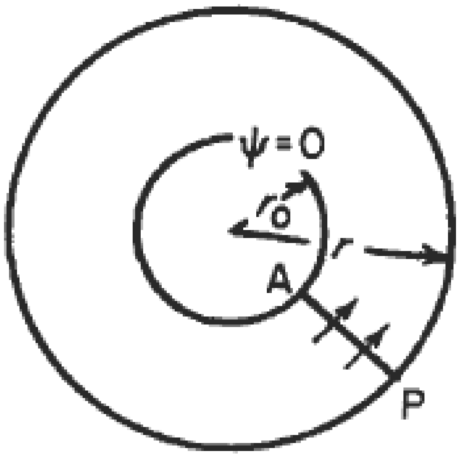
In the present example, $\vec{q}.\ \vec{t} = q_t$ and ds = rθ, let us put the values in the integration, then integration value we get,

$$\Gamma = 2\pi r q_t = 2\pi C$$

A SPINNING CYLINDER IN A UNIFORM FLOW

Thus we the integration constant as $C = \Gamma/(2\pi)$

$$q_t = -\frac{d\psi}{dr} = \frac{\Gamma}{2\pi r} \quad \Rightarrow \quad \psi = \int -\frac{\Gamma}{2\pi r} dr$$



Integrating along the boundary from radius r_o to P(r, θ)

$$\psi = -\int_{r_0}^{r} \frac{\Gamma}{2\pi r} dr \qquad \psi = -\left[\frac{\Gamma}{2\pi} \ln r\right]_{r_0}^{r} = -\frac{\Gamma}{2\pi} \ln \frac{r}{r_0}$$

the flow due to a line vortex gives streamlines that are concentric circles, the equipotential, shown to be always normal to the streamlines, must be radial lines emanating from the vortex,

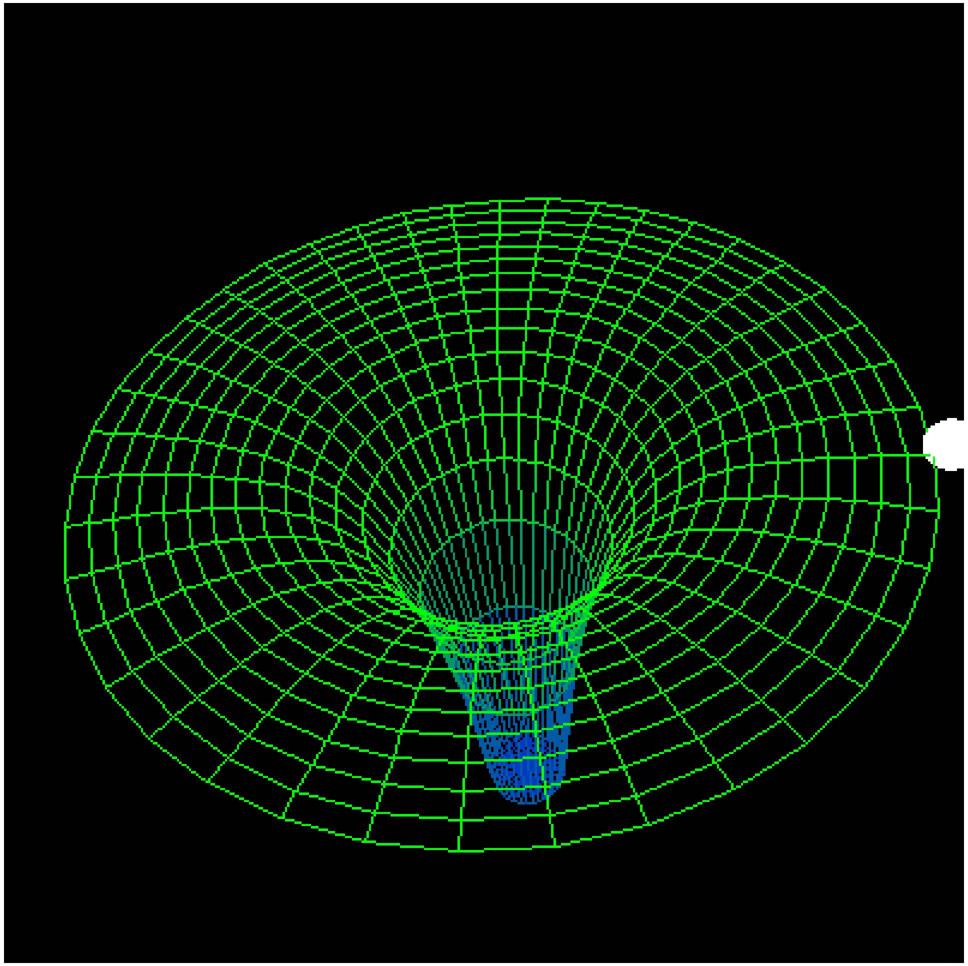
$$\phi = \frac{\Gamma}{2\pi} \theta$$

A SPINNING CYLINDER IN A UNIFORM FLOW

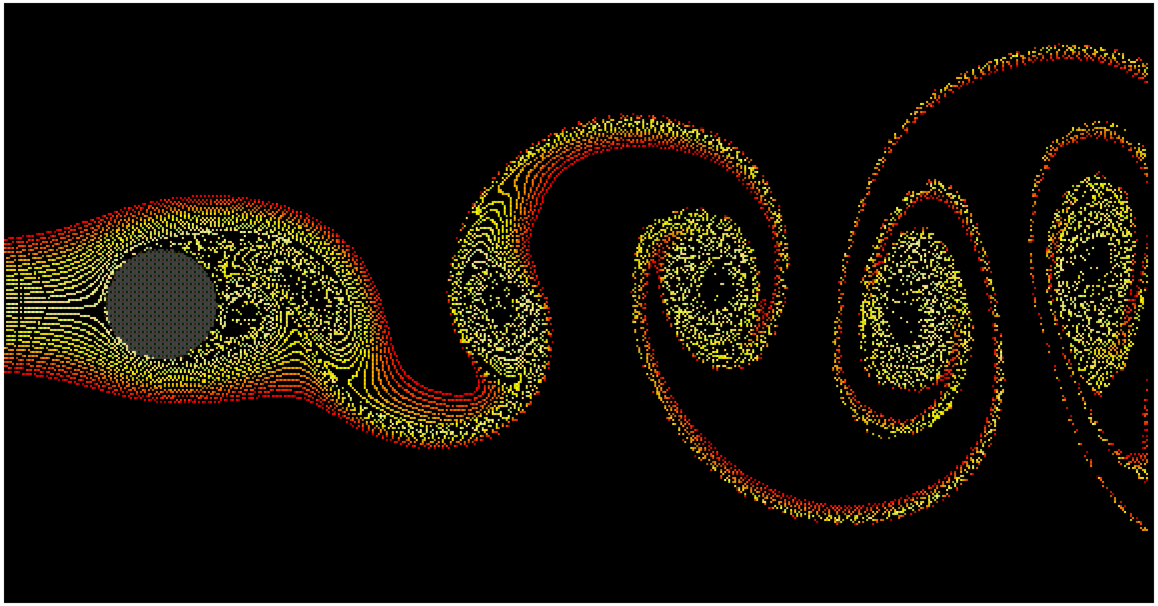
A SPINNING CYLINDER IN A UNIFORM FLOW

VORTEX

VORTEX



VORTEX



Circulation and lift

Lift force per unit span $\boldsymbol{l} = \rho U \acute{\Gamma}$

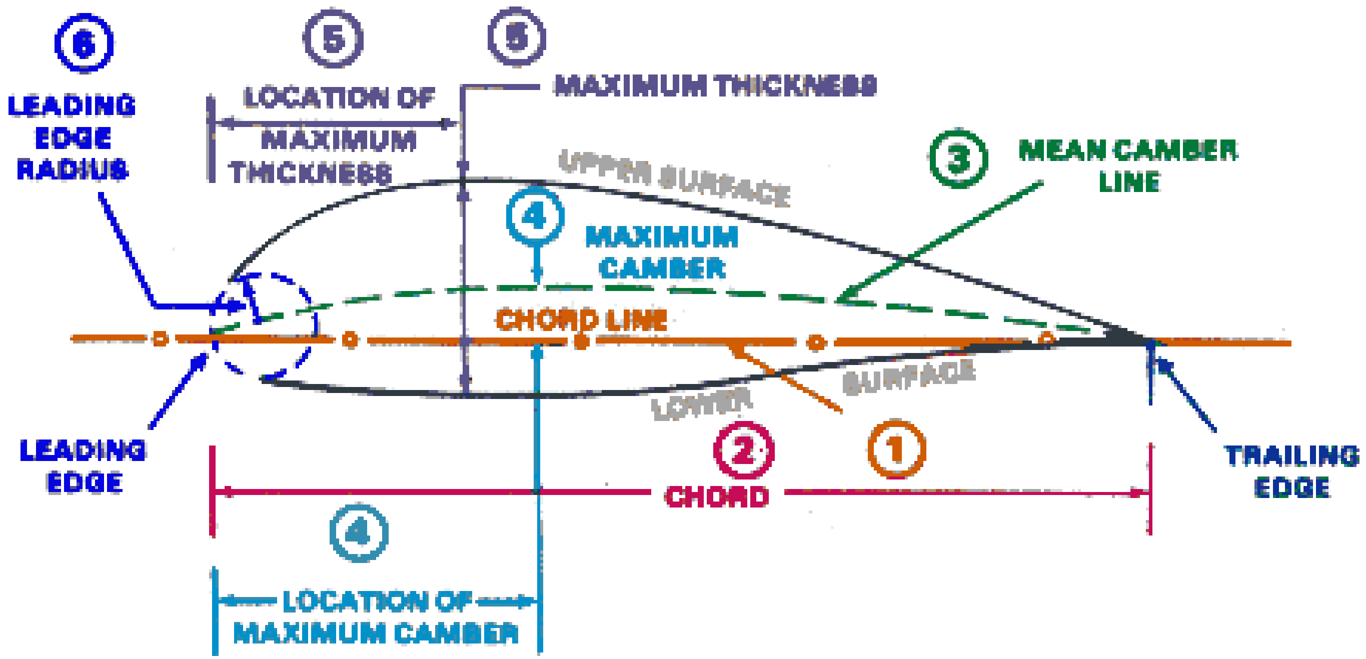
The lift per unit span in N is equal to the product of density ρ, the linear velocity U, and the circulation Γ.

This expression is the algebraic form of the Kutta-Zhukovsky theorem

MODULE -II

THIN AEROFOIL THEORY

Aerofoil Nomenclature



AIRFOIL TERMINOLOGY

Aerofoil Nomenclature

Leading Edge: The forward section of the airfoil is named the **leading edge**

Trailing Edge: The rear section of the airfoil is named the **Trailing edge**

Upper Surface ***Upper surface*** is the surface of an aerofoil between the leading and trailing edges, on the upper surface

Chord is a distance between the leading and trailing edges measured along the chord line;

Chord line is a straight line joining the leading and trailing edges of an aerofoil;

Lower surface is the surface of an aerofoil between the leading and trailing edges, on the lower surface;

Mean camber line is a line joining the leading and trailing edges of an aerofoil, equidistant from the upper and lower surfaces;

Maximum camber is the maximum distance of the mean camber line from the chord line;

Maximum thickness is the maximum distance of the lower surface from the upper

NACA Aerofoil Nomenclature

Family	Advantages	Disadvantages	Applications
4-Digit	1. Good stall characteristics 2. Small center of pressure movement across large speed range 3. Roughness has little effect NACA AEROFOIL.xlsx	1. Low maximum lift coefficient 2. Relatively high drag 3. High pitching moment	1. General aviation 2. Horizontal tails Symmetrical: 3. Supersonic jets 4. Helicopter blades 5. Shrouds 6. Missile/rocket fins
5-Digit	1. Higher maximum lift coefficient 2. Low pitching moment 3. Roughness has little effect	1. Poor stall behavior 2. Relatively high drag	1. General aviation 2. Piston-powered bombers, transports 3. Commuters 4. Business jets
16-Series	1. Avoids low pressure peaks 2. Low drag at high speed	1. Relatively low lift	1. Aircraft propellers 2. Ship propellers
6-Series	1. High maximum lift coefficient 2. Very low drag over a small range of operating conditions 3. Optimized for high speed	1. High drag outside of the optimum range of operating conditions 2. High pitching moment 3. Poor stall behavior 4. Very susceptible to roughness	1. Piston-powered fighters 2. Business jets 3. Jet trainers 4. Supersonic jets
7-Series	1. Very low drag over a small range of operating conditions 2. Low pitching moment	1. Reduced maximum lift coefficient 2. High drag outside of the optimum range of operating conditions 3. Poor stall behavior 4. Very susceptible to roughness	Seldom used
8-Series	Unknown	Unknown	Very seldom used

NACA Four-Digit Series:

The first family of airfoils designed using this approach became known as the NACA Four-Digit Series. The first digit specifies the maximum camber (m) in percentage of the chord (airfoil length), the second indicates the position of the maximum camber (p) in tenths of chord, and the last two numbers provide the maximum thickness (t) of the airfoil in percentage of chord. For example, the NACA 2415 airfoil has a maximum thickness of 15% with a camber of 2% located 40% back from the airfoil leading edge (or 0.4c).

NACA Five-Digit Series: The NACA Five-Digit Series uses the same thickness forms as the Four-Digit Series but the mean camber line is defined differently and the naming convention is a bit more complex. The first digit, when multiplied by 3/2, yields the design lift coefficient (c_l) in tenths. The next two digits, when divided by 2, give the position of the maximum camber (p) in tenths of chord. The final two digits again indicate the maximum thickness (t) in percentage of chord. For example, the NACA 23012 has a maximum thickness of 12%, a design lift coefficient of 0.3, and a maximum camber located 15% back from the leading edge.

NACA 6-Series:

Although NACA experimented with approximate theoretical methods that produced the 2-Series through the 5-Series, none of these approaches was found to accurately produce the desired airfoil behavior. The 6-Series was derived using an improved theoretical method that, like the 1-Series, relied on specifying the desired pressure distribution and employed advanced mathematics to derive the required geometrical shape. The goal of this approach was to design airfoils that maximized the region over which the airflow remains laminar. In so doing, the drag over a small range of lift coefficients can be substantially reduced. The naming convention of the 6-Series is by far the most confusing of any of the families discussed thus far, especially since many different variations exist. One of the more common examples is the NACA 64_1-212, a=0.6.

In this example, 6 denotes the series and indicates that this family is designed for greater laminar flow than the Four- or Five-Digit Series. The second digit, 4, is the location of the minimum pressure in tenths of chord (0.4c). The subscript 1 indicates that low drag is maintained at lift coefficients 0.1 above and below the design lift coefficient (0.2) specified by the first digit after the dash in tenths. The final two digits specify the thickness in percentage of chord, 12%. The fraction specified by a=___ indicates the percentage of the airfoil chord over which the pressure distribution on the airfoil is uniform, 60% chord in this case. If not specified, the quantity is assumed to be 1, or the distribution is constant over the entire airfoil.

NACA 7-Series:

The 7-Series was a further attempt to maximize the regions of laminar flow over an airfoil differentiating the locations of the minimum pressure on the upper and lower surfaces. An example is the NACA 747A315. The 7 denotes the series, the 4 provides the location of the minimum pressure on the upper surface in tenths of chord (40%), and the 7 provides the location of the minimum pressure on the lower surface in tenths of chord (70%). The fourth character, a letter, indicates the thickness distribution and mean line forms used. A series of standaradized forms derived from earlier families are designated by different letters. Again, the fifth digit incidates the design lift coefficient in tenths (0.3) and the final two integers are the airfoil thickness in perecentage of chord (15%)

NACA 8-Series:

A final variation on the 6- and 7-Series methodology was the NACA 8-Series designed for flight at supercritical speeds. Like the earlier airfoils, the goal was to maximize the extent of laminar flow on the upper and lower surfaces independently. The naming convention is very similar to the 7-Series, an example being the NACA 835A216. The 8 designates the series, 3 is the location of minimum pressure on the upper surface in tenths of chord (0.3c), 5 is the location of minimum pressure on the lower surface in tenths of chord (50%), the letter A distinguishes airfoils having different camber or thickness forms, 2 denotes the design lift coefficient in tenths (0.2), and 16 provides the airfoil thickness in percentage of chord (16%).

Aerodynamic Characteristics

Aerodynamic centre
Centre of Pressure
Pitching moment

Centre of Pressure

Centre of pressure

The aerodynamic forces on an aerofoil section may be represented by a lift, a drag, and a pitching moment. At each value of the lift coefficient there will be found to be one particular point about which the pitching moment coefficient is zero, and the aerodynamic effects on the aerofoil section may be represented by the lift and the drag alone acting at that point. This special point is termed the centre of pressure.

Aerodynamic Centre

Aerodynamic centre

If the pitching moment coefficient at each point along the chord is calculated for each of several values of ***CL,*** one very special point is found for which ***CM*** is virtually constant, independent of the lift coefficient. This point is the aerodynamic centre.

For incidences up to 10 degrees or so it is a fixed point close to, but not in general on, the chord line, between **23%** and 25% of the chord behind the leading edge.

Wing of Infinite Aspect Ratio

Aspect ratio

The aspect ratio is a measure of the narrowness of the wing planform. It is denoted by ***A*****,** or sometimes by ***(AR)*****,** and **is** given by AR=span/**SMC=b/c**

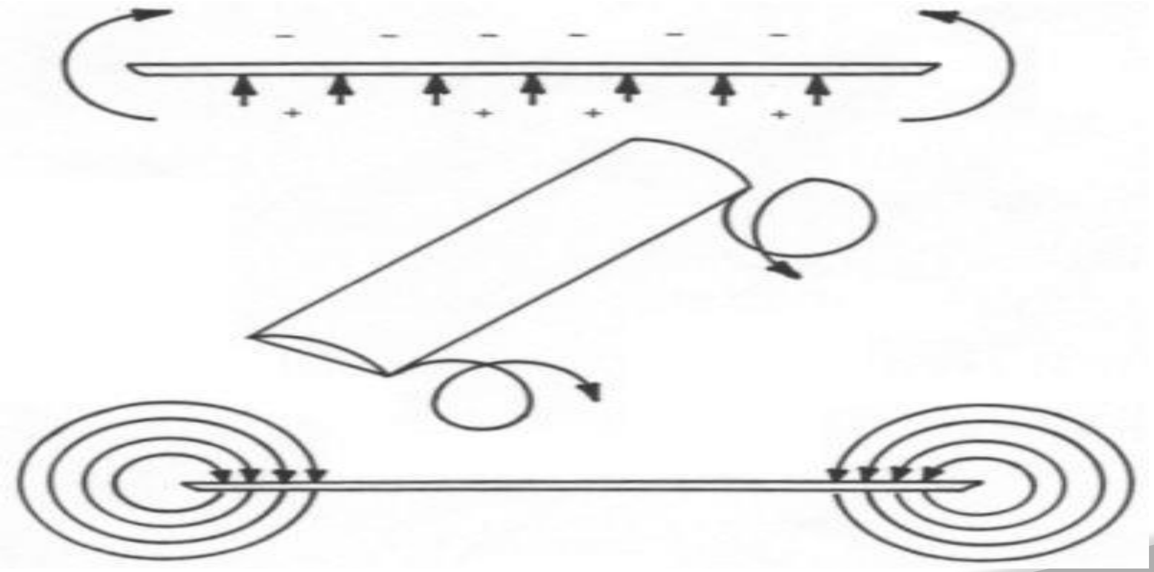
If both top and bottom of this expression are multiplied by the wing span, ***by*** it becomes

$$A = \frac{b^2}{b\bar{c}} = \frac{(\text{span})^2}{\text{area}}$$

a form which is often more convenient.

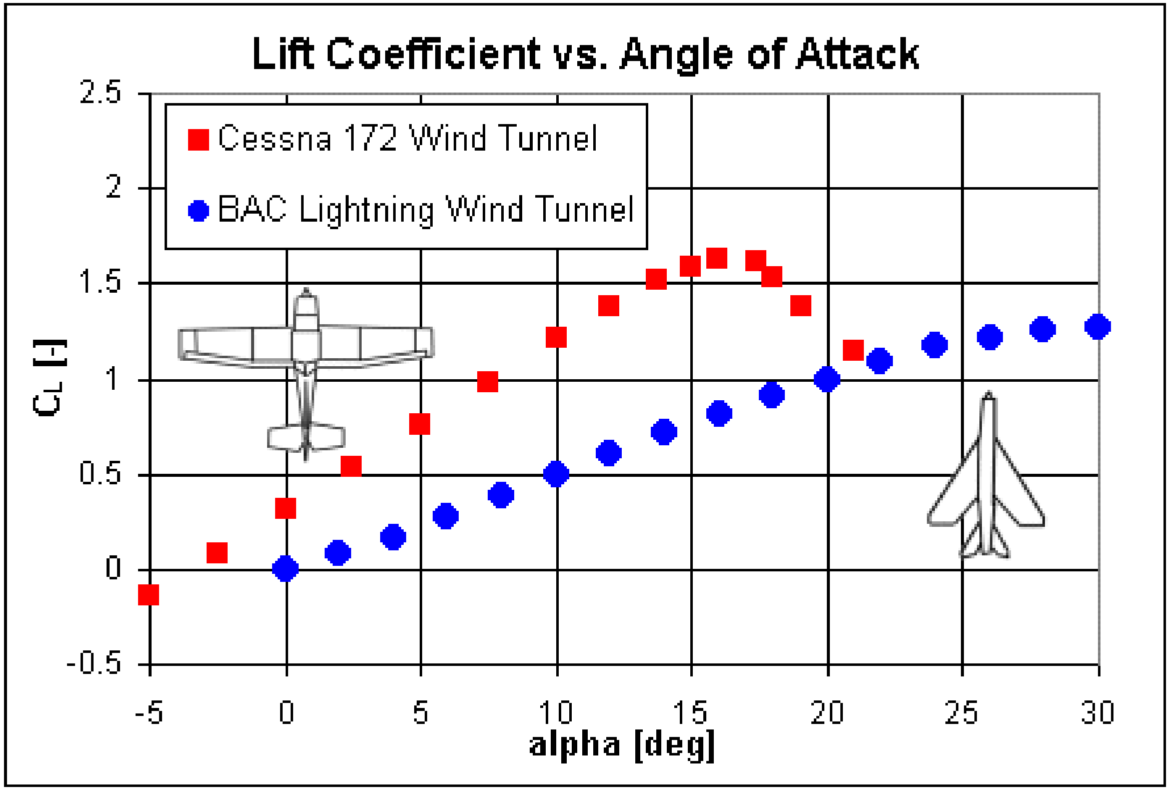
Finite and Infinite Wing

You are partially correct. As you surmised, the difference between a finite wing and an infinite wing is in that a finite wing has tips. As a result, the higher pressure air from beneath the wing tries to move around the tips towards the lower pressure above the wing. This motion creates a swirling vortex of air from each tip that trails behind the wing. For that reason, we call these vortices trailing vortices.

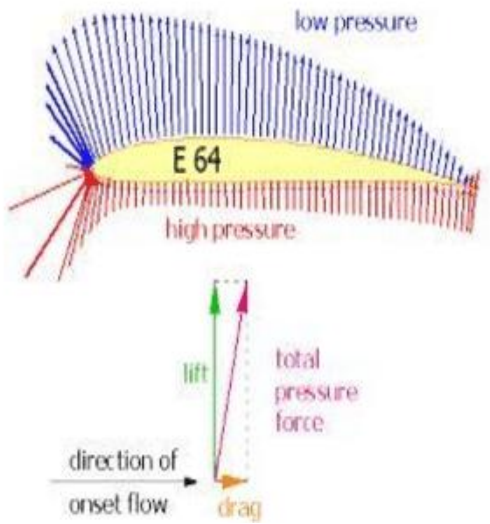


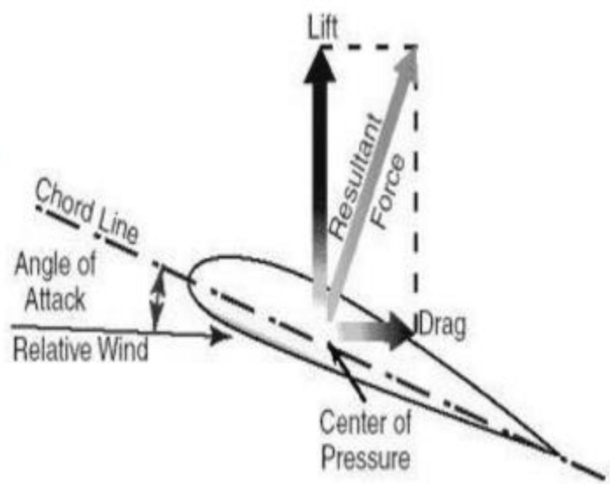
Cl-α- Diagram for a Wing of Infinite Aspect Ratio

You can see the effect of aspect ratio on the lift produced by a wing quite clearly in the following graph.



Lift occurs when a moving flow of gas is turned by a solid object. The flow is turned in one direction, and the **lift** is generated in the opposite direction, according to Newton's Third Law of action and reaction. Because air is a gas and the molecules are free to move about, any solid surface can deflect a flow.

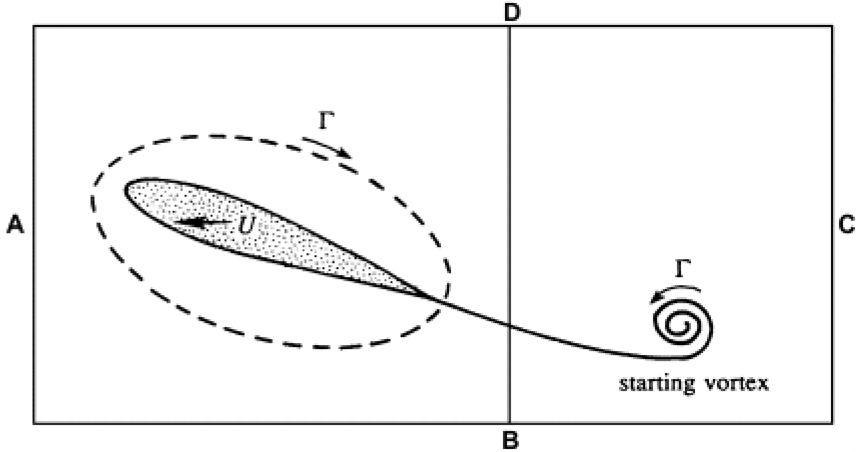




Starting Vortex

The **starting** vortex which forms in the air adjacent to the trailing edge of an airfoil as it is accelerated from rest in a fluid. It leaves the airfoil (which now has an equal but opposite "bound vortex" around it), and remains (nearly) stationary in the flow. It rapidly decays through the action of viscosity.

The **starting vortex** is significant to an understanding of the Kutta condition and its role in the circulation around any airfoil generating lift.



Kutta's Trailing Edge Condition

A body with a sharp trailing edge which is moving through a fluid will create about itself a circulation of sufficient strength to hold the rear stagnation point at the trailing edge. In fluid flow around a body with a sharp corner, the Kutta condition refers to the flow pattern in which fluid approaches the corner from both directions, meets at the corner, and then flows away from the body. None of the fluid flows around the sharp corner.

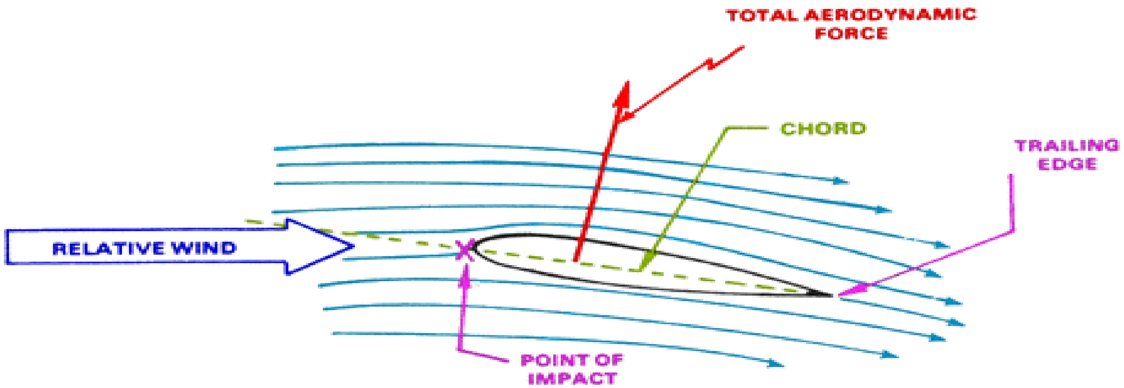


FIGURE 2-18. AIRFLOW AROUND AN AIRFOIL.

Thin Aerofoil Theory

Fundamental Equation of Thin Airfoil Theory :

$$\frac{1}{2\pi}\int_0^c \frac{\gamma(\xi)d\xi}{x-\xi} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

Coordinate Transformation

$$\xi = \frac{c}{2}(1-\cos\theta)$$

$$d\xi = \sin\theta d\theta$$

$$x = \frac{c}{2}(1-\cos\theta_0)$$

Transformed Equation

$$\frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta - \cos\theta_0} = V_\infty\left(\alpha - \frac{dz}{dx}\right)$$

- In words: Camber line is a streamline
- Written at a given point x on the chord line
- dz/dx is evaluated at that point x
- Variable x is a dummy variable of integration which varies from 0 to c along the chord line
- Vortex strength g=g (x) is a variable along the chord line and is in units of
- In transformed coordinates, equation is written at a point, q_0. q is the dummy variable of integration
  - At leading edge, x = 0, q = 0
  - At trailed edge, x = c, q =p
- The central problem of thin airfoil theory is to solve the fundamental equation for g (x) subject to the Kutta condition, g(c)=0
- The central problem of thin airfoil theory is to solve the fundamental equation for g (q) subject to the Kutta condition, g(p)=0

SUMMARY: SYMMETRIC AIRFOILS

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\gamma(\theta)\sin\theta d\theta}{\cos\theta-\cos\theta_0}=V_\infty\alpha$$

- Fundamental equation of thin airfoil theory for a symmetric airfoil (dz/dx=0) written in transformed coordinates

$$\gamma(\theta)=2V_\infty\alpha\frac{(1+\cos\theta)}{\sin\theta}$$

- Solution
  - "A rigorous solution for $\gamma(\theta)$ can be obtained from the mathematical theory of integral equations, which is beyond the scope of this book." (page 324, Anderson)

$$\gamma(\pi)=2V_\infty\alpha\frac{0}{0}$$

- Solution must satisfy Kutta condition $\gamma(\pi)=0$ at trailing edge to be consistent with experimental results

$$\gamma(\pi)=2V_\infty\alpha\frac{-\sin\pi}{\cos\pi}=0$$

- Direct evaluation gives an indeterminant form, but can use L'Hospital's rule to show that Kutta condition does hold.

SUMMARY: SYMMETRIC AIRFOILS

$$\Gamma = \int_0^c \gamma(\xi) d\xi$$

$$\Gamma = \frac{c}{2} \int_0^{\pi} \gamma(\theta) \sin\theta d\theta$$

$$\Gamma = \pi \alpha c V_\infty$$

$$L' = \rho_\infty V_\infty \Gamma = \pi \alpha c \rho_\infty V_\infty^2$$

$$c_l = 2\pi\alpha$$

$$\frac{dc_l}{d\alpha} = 2\pi$$

- **Total circulation, Γ, around the airfoil (around the vortex sheet described by $\gamma(\xi)$)**
- **Transform coordinates and integrate**
- **Simple expression for total circulation**
- **Apply Kutta-Joukowski theorem (see §3.16), *"although the result [L'=$\rho_\infty V_\infty^2 \Gamma$] was derived for a circular cylinder, it applies in general to cylindrical bodies of arbitrary cross section."***
- **Lift coefficient is linearly proportional to angle of attack**
- **Lift slope is 2π/rad or 0.11/deg**

Elements of Panel Method

- Panel methods are techniques for solving incompressible potential flow over thick 2-D and 3-D geometries.
- In 2-D, the airfoil surface is divided into piecewise straight line segments or panels or “boundary elements” and vortex sheets of strength g are placed on each panel.
  - We use vortex sheets (miniature vortices of strength gds, where ds is the length of a panel) since vortices give rise to circulation, and hence lift.
  - Vortex sheets mimic the boundary layer around airfoils.

Elements of Panel Method

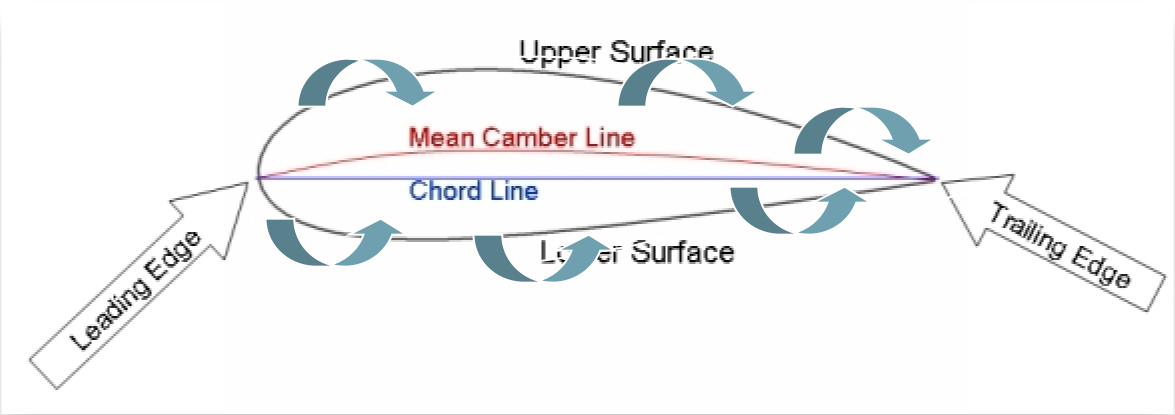


FIGURE 3

Upper surface boundary layer contains, in general, clockwise rotating vorticity

Lower surface boundary layer contains, in general, counter clockwise vorticity.

Because there is more clockwise vorticity than counter clockwise Vorticity, there is net clockwise circulation around the airfoil.

In panel methods, we replace this boundary layer, which has a small but finite thickness with a thin sheet of vorticity placed just outside the airfoil.

Elements of Panel Method

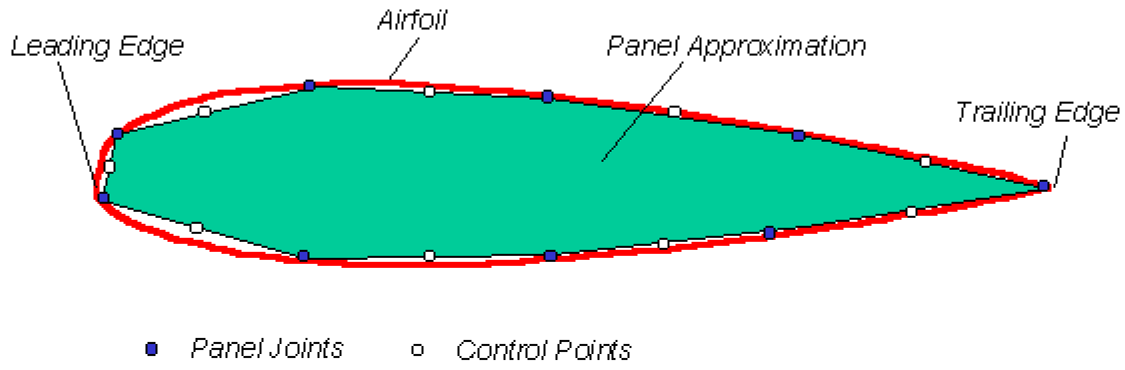


Figure 1. Vortex panel approximation to an airfoil.

On each panel, there is vortex sheet of strength DG = g_0 ds_0
Where ds_0 is the panel length.

Each panel is defined by its two end points (panel joints)
and by the control point, located at the panel center, where we will
Apply the boundary condition y= Constant=C.

The more the number of panels, the more accurate the solution,
since we are representing a continuous curve by a series
of broken straight lines

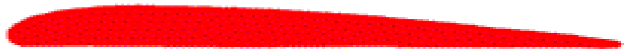
Boundary Condition

- We treat the airfoil surface as a streamline.
  - This ensures that the velocity is tangential to the airfoil surface, and no fluid can penetrate the surface.
- We require that at all control points (middle points of each panel) y= C
- The stream function is due to superposition of the effects of the free stream and the effects of the vortices $g_0\ ds_0$ on each of the panel.

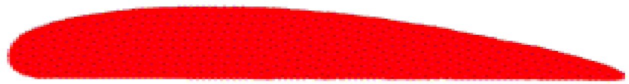
High Lift Aerofoils

CONVENTIONAL AIRFOILS

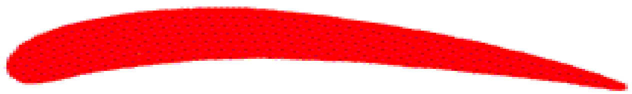
The following illustrations depict a selection of designs of airfoil sections. These are known as conventional airfoils.



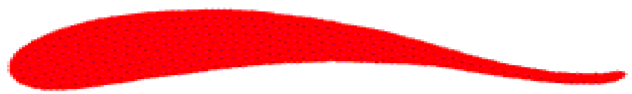
Low camber — low drag — high speed — thin wing section
Suitable for race planes, fighters, interceptors, etc.



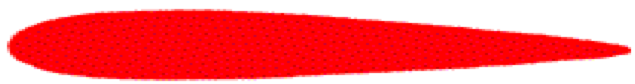
Deep camber — high lift — low speed — thick wing section
Suitable for transports, freighters, bombers, etc.



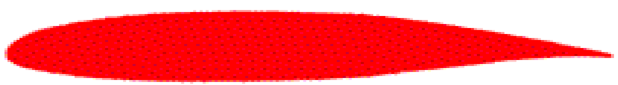
Deep camber — high lift — low speed — thin wing section
Suitable as above.



Low lift — high drag — reflex trailing edge wing section.
Very little movement of centre of pressure. Good stability.

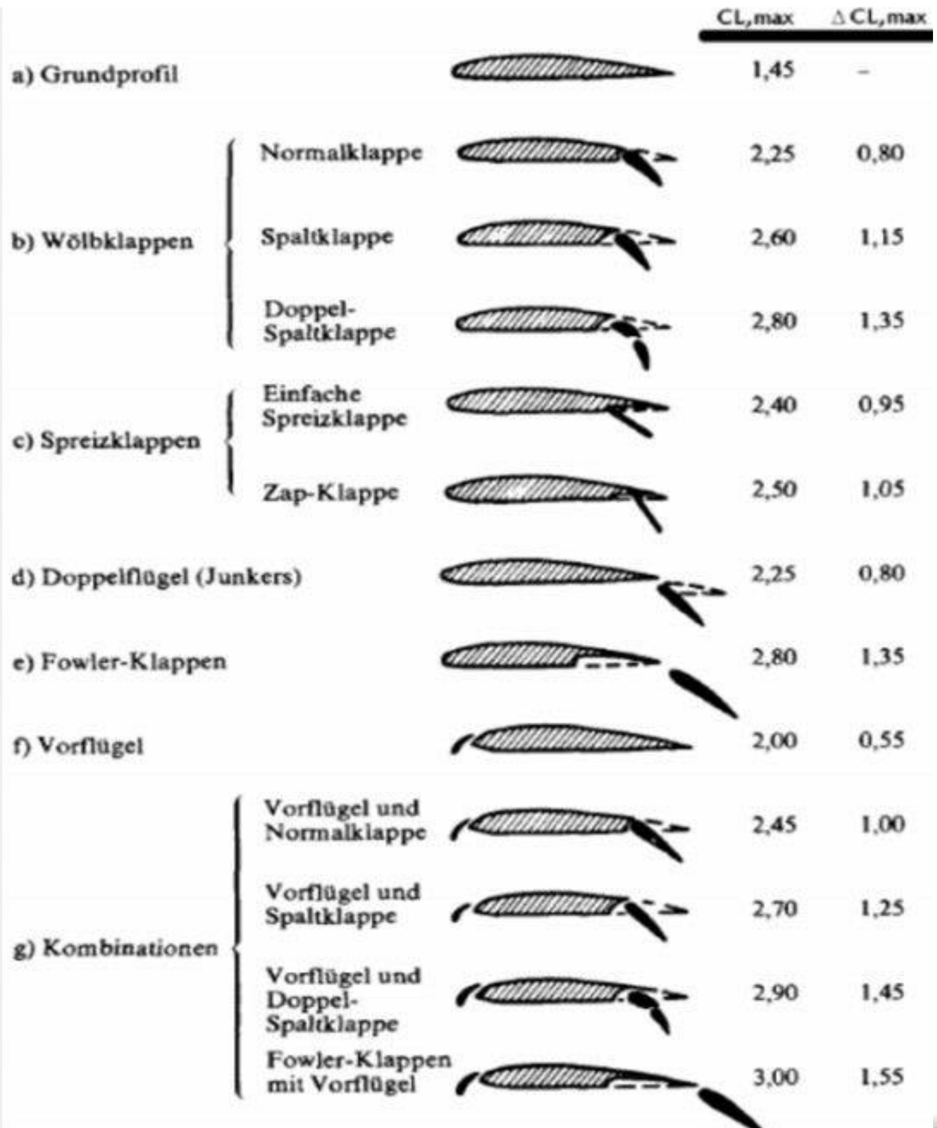


Symmetrical (cambered top and bottom) wing sections.
Similar to above.



GA(W)-1 airfoil — thicker for better structure and lower weight — good stall characteristics — camber is maintained farther rearward which increases lifting capability over more of the airfoil and decreases drag.

High Lift Devices



		$C_{L,max}$	$\Delta C_{L,max}$
a) Grundprofil		1,45	–
b) Wölbklappen	Normalklappe	2,25	0,80
	Spaltklappe	2,60	1,15
	Doppel-Spaltklappe	2,80	1,35
c) Spreizklappen	Einfache Spreizklappe	2,40	0,95
	Zap-Klappe	2,50	1,05
d) Doppelflügel (Junkers)		2,25	0,80
e) Fowler-Klappen		2,80	1,35
f) Vorflügel		2,00	0,55
g) Kombinationen	Vorflügel und Normalklappe	2,45	1,00
	Vorflügel und Spaltklappe	2,70	1,25
	Vorflügel und Doppel-Spaltklappe	2,90	1,45
	Fowler-Klappen mit Vorflügel	3,00	1,55

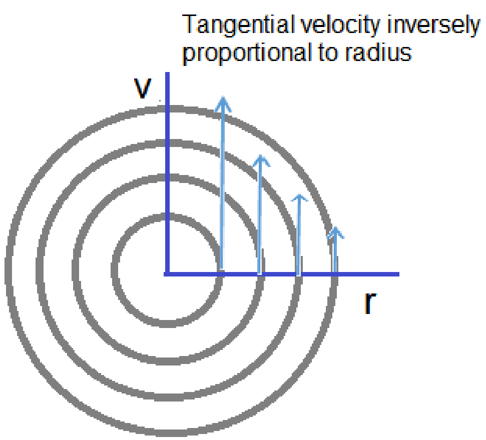
MODULE -III

FINITE WING THEORY

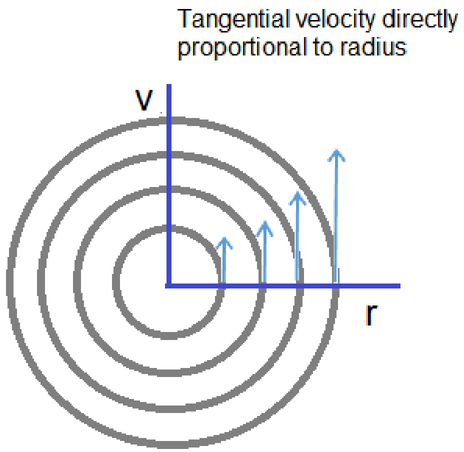
3.1 Vortex Motions
3.2 Vortex Line
3.3 Vortex Tube
3.4 Vortex Sheet
3.5 Circulation
3.6 Kelvin and Helmhotz Theorem
3.7 Biot-Savart's Law
3.8 Applications
3.9 Rankine's Vortex
3.10 Flow Past Finite Wings
3.11 Vortex Model of The Wing and Bound Vortices
3.12 Induced Drag
3.13 Prandtl's Lifting Line Theory
3.14 Elliptic Wing
3.15 Influence of Taper and Twist Applied to Wings
3.16 Effect of Sweep Back Wings
3.17 Delta Wings
3.18 Primary and Secondary Vortex
3.19 Elements of Lifting Surface Theory
3.20 Source Panel Vortex Panel and Vortex Lattice Methods

Vortex Motions

A **vortex** is commonly associated with the rotating **motion** of fluid around a common centerline. It is defined by the vorticity in the fluid, which measures the rate of local fluid rotation. In a free **vortex flow** total mechanical energy remains constant.

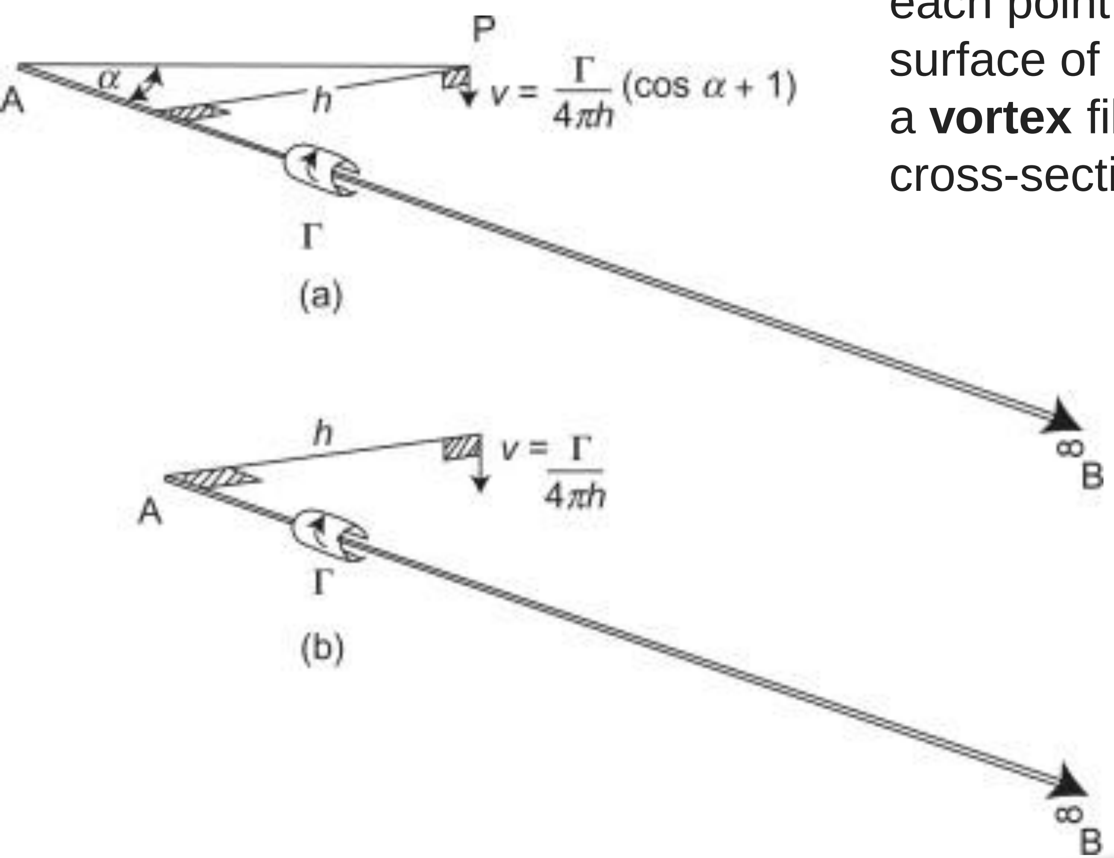


Free vortex



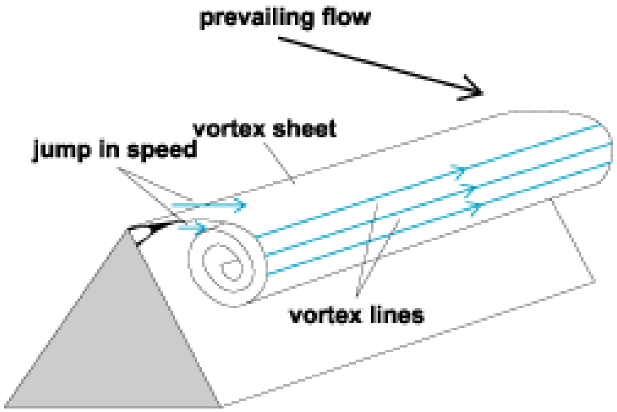
Forced vortex

Vortex Line



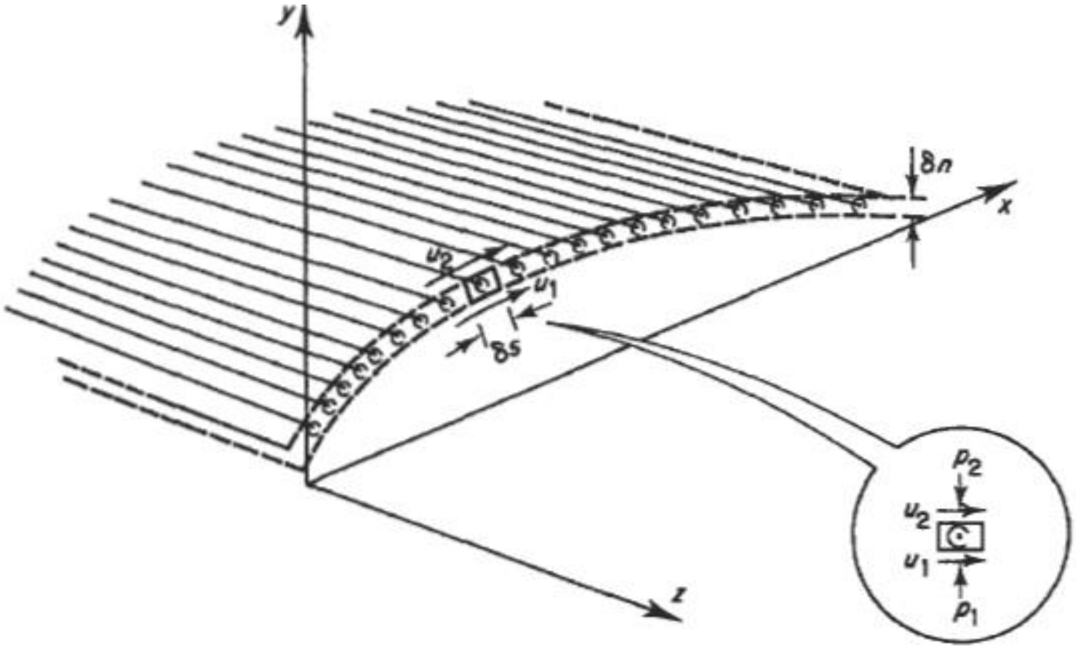
A **vortex line** is a **line** whose tangent is everywhere parallel to the local vorticity vector. The **vortex lines** drawn through each point of a closed curve constitute the surface of a **vortex** tube. Finally,
a **vortex** filament is a **vortex** tube whose cross-section is of infinitesimal dimensions

Vortex lines can be defined analogously to streamlines as lines that are tangential to the vorticity vector at all points in the flow field. Similarly the concept of the ***vortex tube*** is analogous to that of stream tube. Physically we can think of flow structures like vortices as comprising bundles of vortex tubes. **In** many respects vorticity and vortex lines are even more fundamental to understanding the flow physics than are velocity and streamlines.



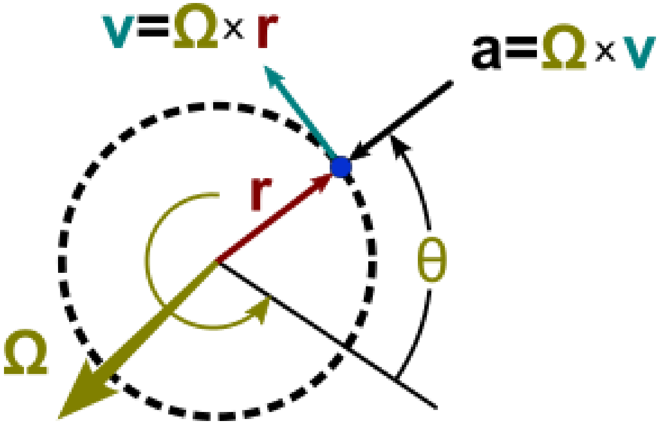
Vortex Sheet

A **vortex sheet** is a term used in fluid mechanics for a surface across which there is a discontinuity in fluid velocity, such as in slippage of one layer of fluid over another.



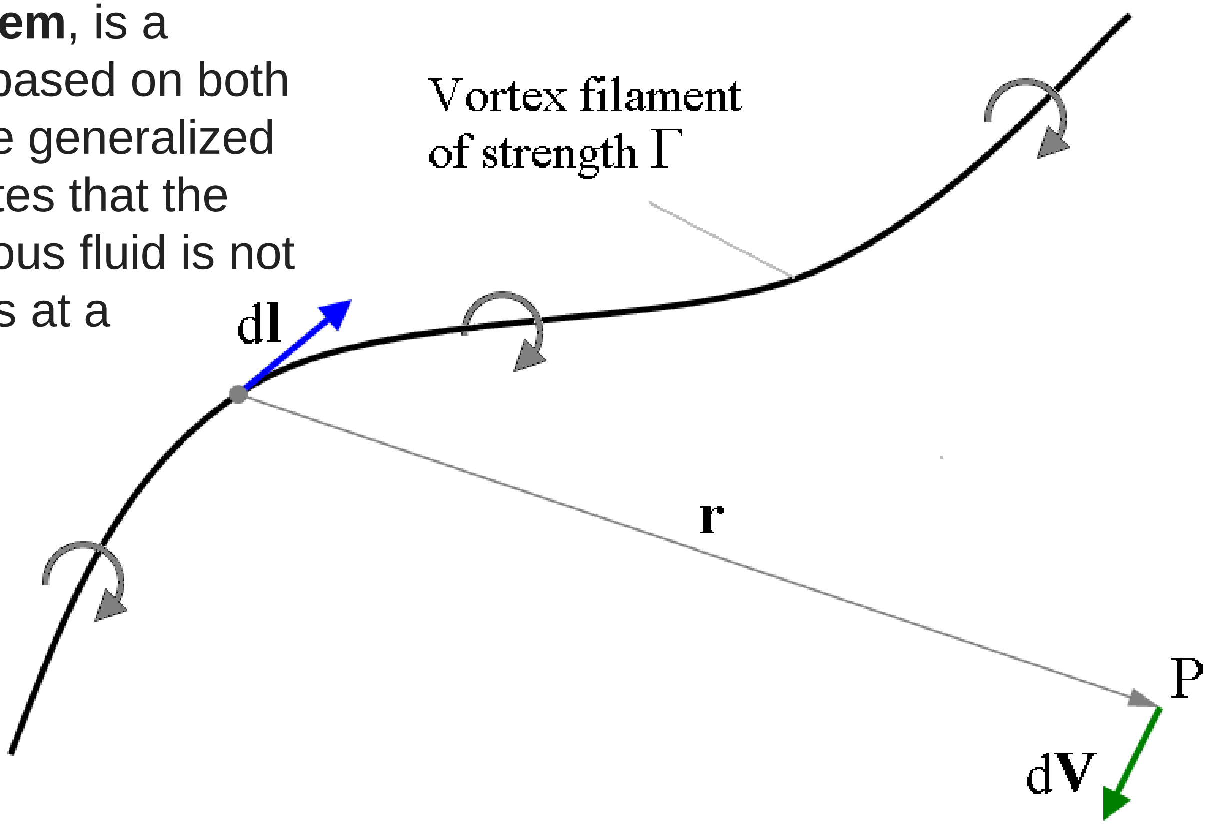
Circulation

Circular motion is a movement of an object along the circumference of a circle or rotation along a circular path. It can be uniform, with constant angular rate of rotation and constant speed, or non-uniform with a changing rate of rotation. The rotation around a fixed axis of a three-dimensional body involves circular motion of its parts. The equations of motion describe the movement of the center of mass of a body.



Kelvin and Helmhotz Theorem

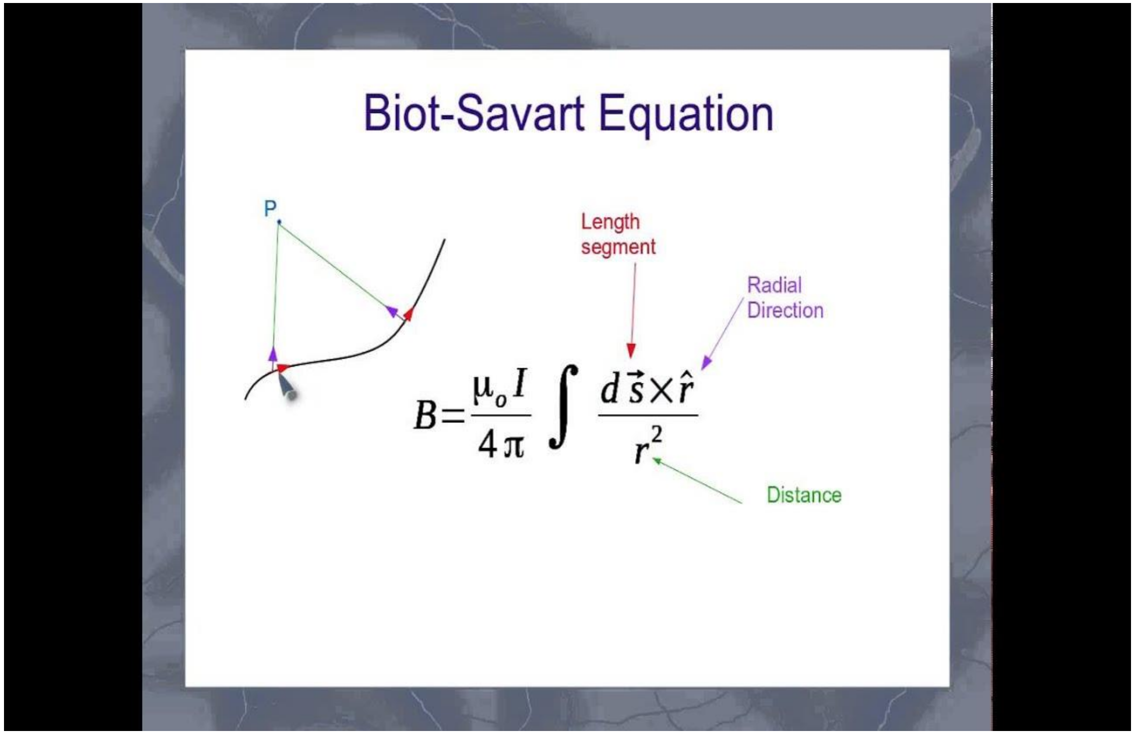
Helmholtz' second vortex **theorem**, or its equivalence **Kelvin's theorem**, is a vorticity-dynamic **theorem** based on both kinetics and kinematics. The generalized second vortex **theorem** states that the vorticity strength in the viscous fluid is not conserved in time; it diffuses at a predictable rate



Biot-Savart's Law

What is Biot Savart Law

The **Biot Savart Law** is an equation describing the magnetic field generated by a constant electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. Biot–Savart law is consistent with both Ampere's circuital law and Gauss's theorem. The Biot Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics.



Applications

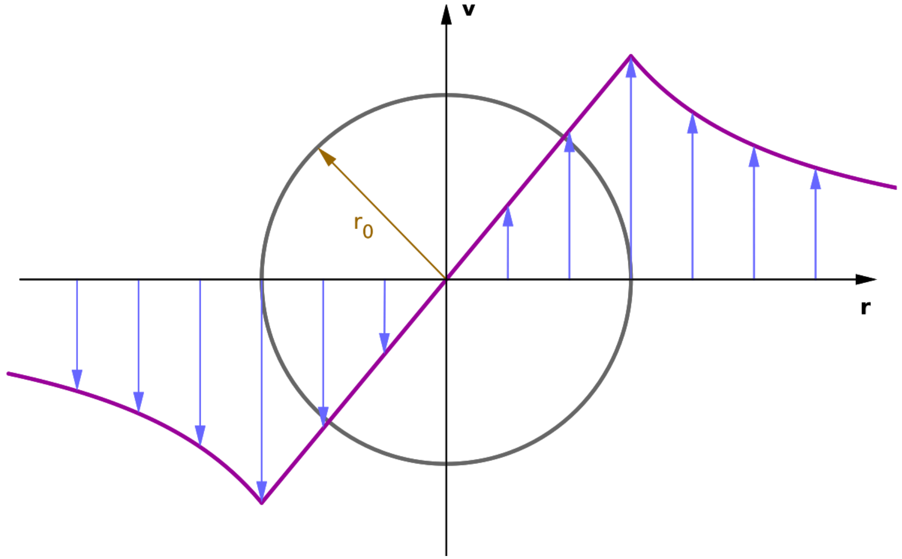
Biot Savart Law Applications

This **law** can be used for calculating magnetic reactions even on the level of molecular or atomic.

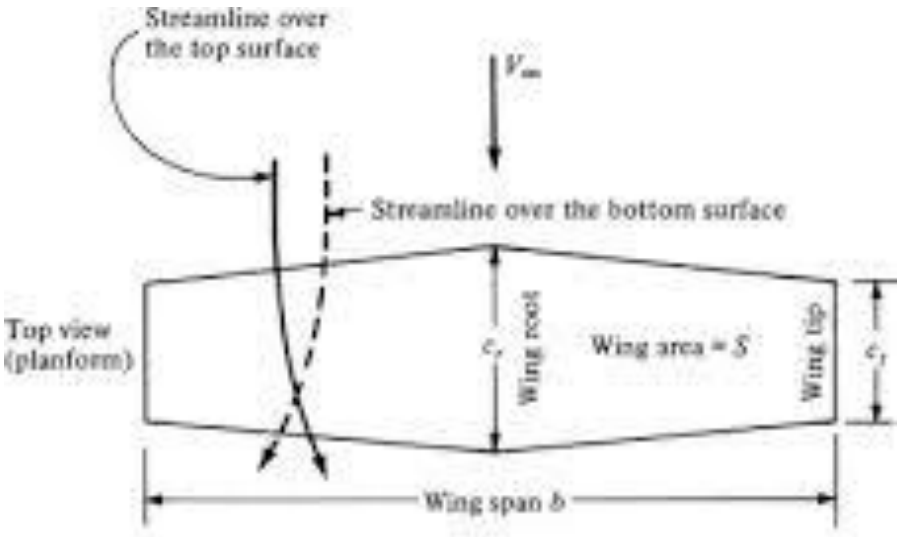
It can be used in the theory of aerodynamic for determining the velocity encouraged with vortex lines.

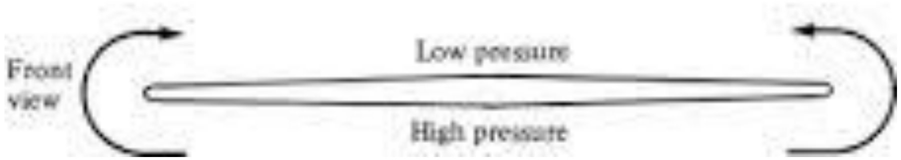
Rankine's Vortex

The Rankine vortex is a simple mathematical model of a vortex in a viscous fluid. It is named after its discoverer, William John Macquorn Rankine. A swirling flow in a viscous fluid can be characterized by a central core comprising a forced vortex, surrounded by a free vortex.



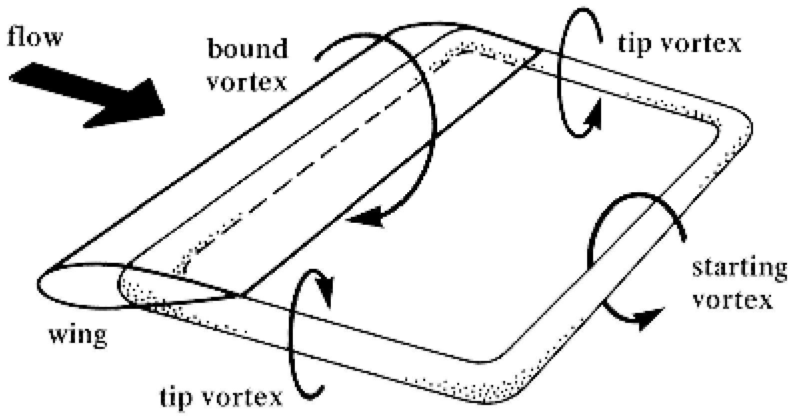
Flow Past Finite Wings





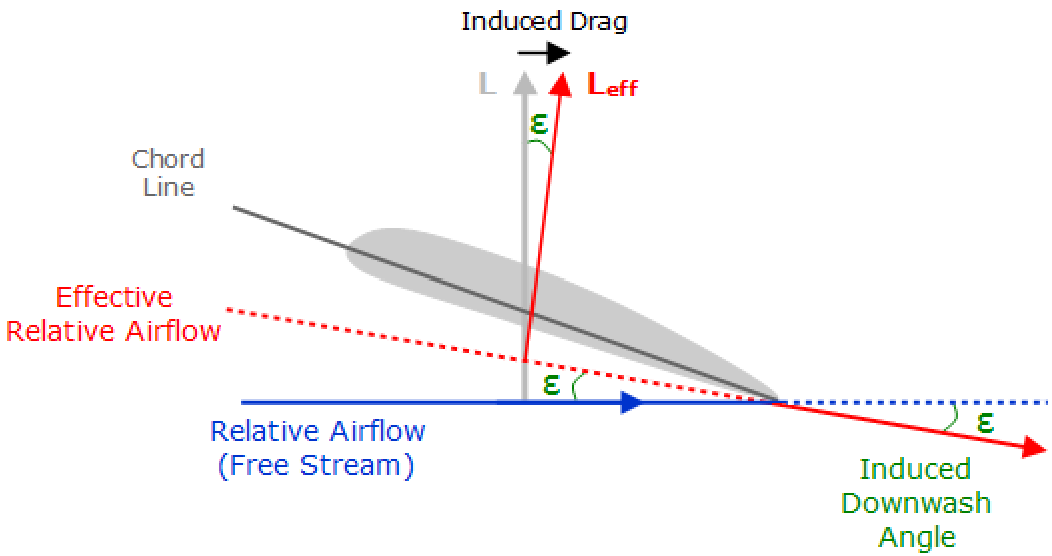
Vortex Model of The Wing and Bound Vortices

Bound Vortex. a **vortex** that is considered to be tightly associated with the body around which a liquid or gas flows, and equivalent with respect to the magnitude of speed circulation to the real vorticity that forms in the boundary layer owing to viscosity.



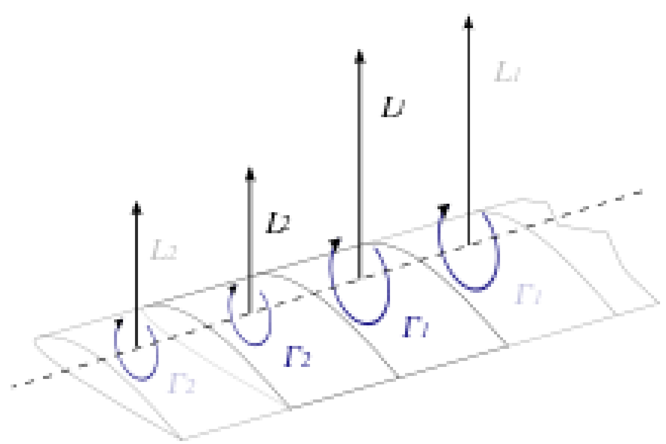
Induced Drag

In aerodynamics, **lift-induced drag**, **induced drag**, **vortex drag**, or sometimes **drag due to lift**, is an aerodynamic drag force that occurs whenever a moving object redirects the airflow coming at it. This drag force occurs in airplanes due to wings or a lifting body redirecting air to cause lift and also in cars with airfoil wings that redirect air to cause a downforce.



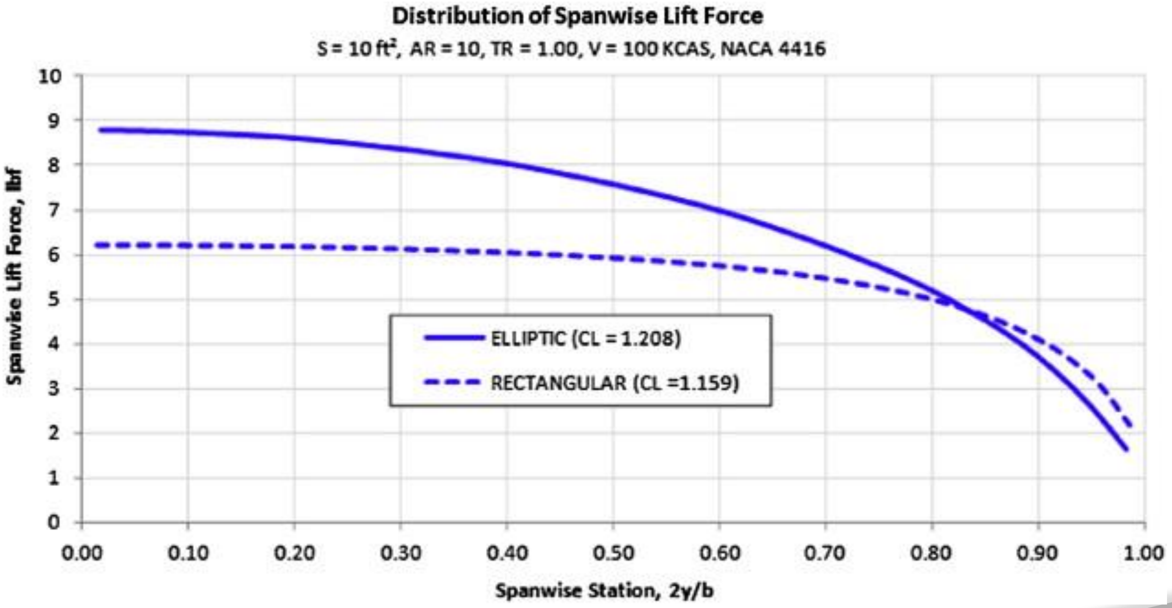
Prandtl's Lifting Line Theory

The **Prandtl lifting-line theory** is a mathematical model that predicts **lift** distribution over a three-dimensional wing based on its geometry. ... In this model, the vortex loses strength along the whole wingspan because it is shed as a vortex-sheet from the trailing edge, rather than just at the wing-tips.



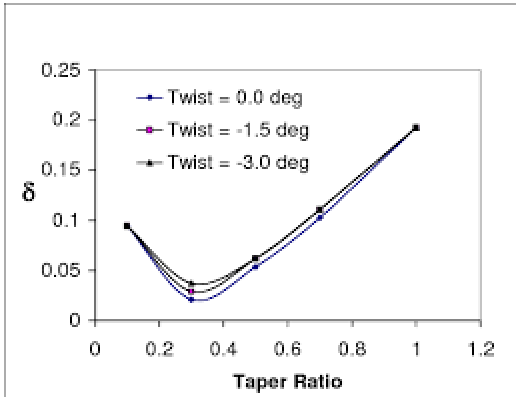
Elliptic Wing

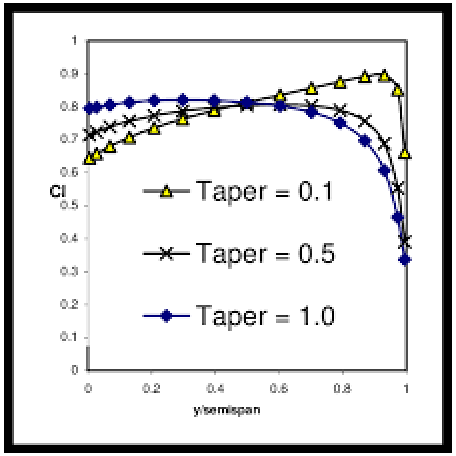
An **elliptical wing** is a **wing** planform whose leading and trailing edges each approximate two segments of an **ellipse**. Not to be confused with annular **wings**, which may be elliptically shaped.



Influence of Taper and Twist Applied to Wings

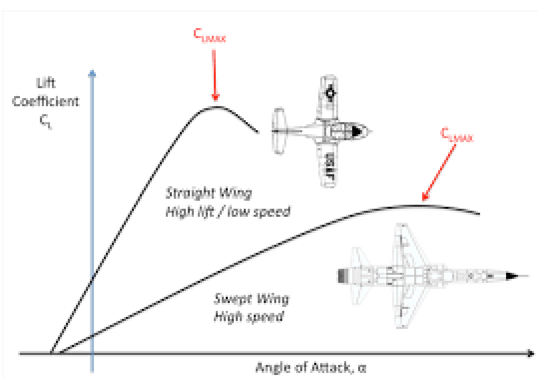
Wing twist is an aerodynamic feature added to aircraft **wings** to adjust lift ... cause the **wing** itself to be deflected and is related to compressibility **effects**; ... Hornet **Wing Twist** · **Applied** Aerodynamics: A Digital Textbook, **Wing** Design Parameters.

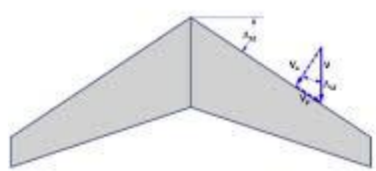


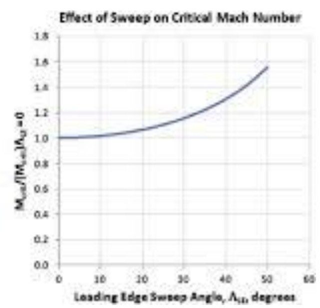


Effect of Sweep Back Wings

weeping The **Wing Back** Delays Supersonic Flow
It delays the start of supersonic flow, by reducing the amount of acceleration over the **wing**. On a straight **wing** airplane, all of the airflow over the **wing** travels parallel to the aircraft's chord line.

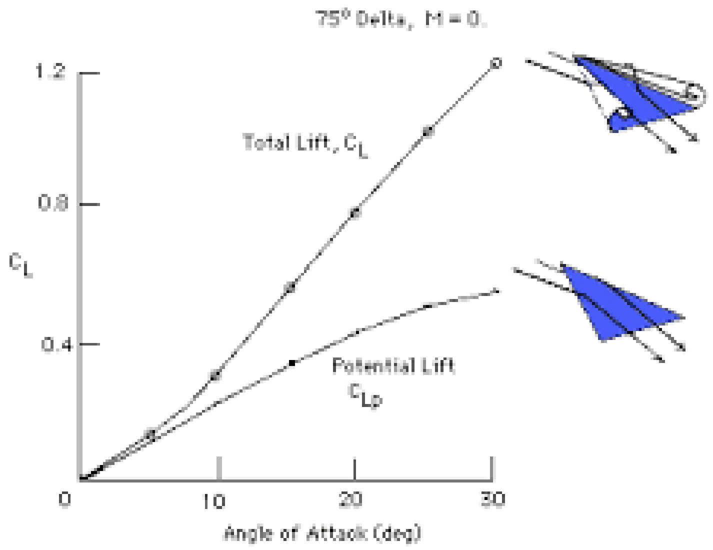






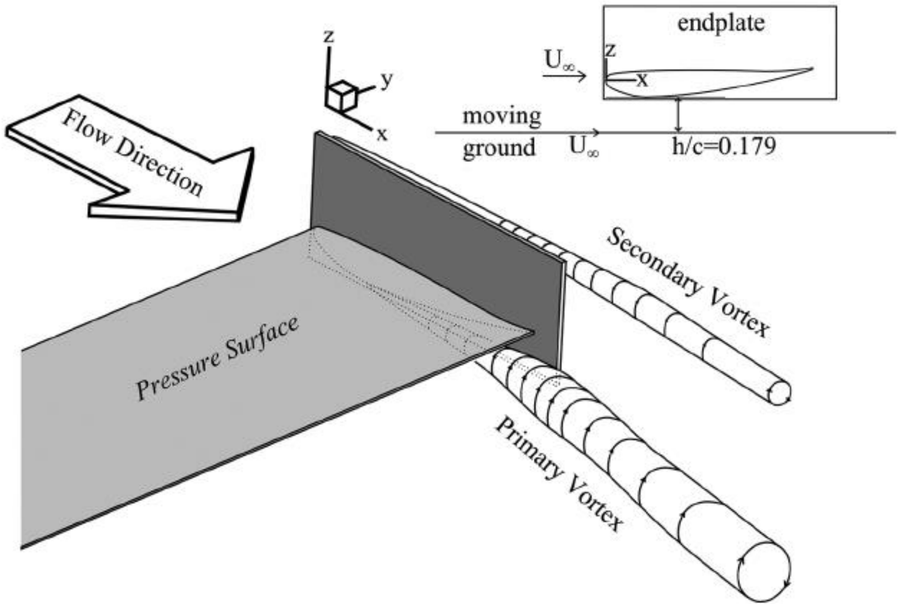
Delta Wings

The **delta wing** is a **wing** shaped in the form of a triangle. It is named for its similarity in shape to the Greek uppercase letter **delta** (Δ). Although long studied, it did not find significant applications until the jet age, when it proved suitable for high-speed subsonic and supersonic flight.

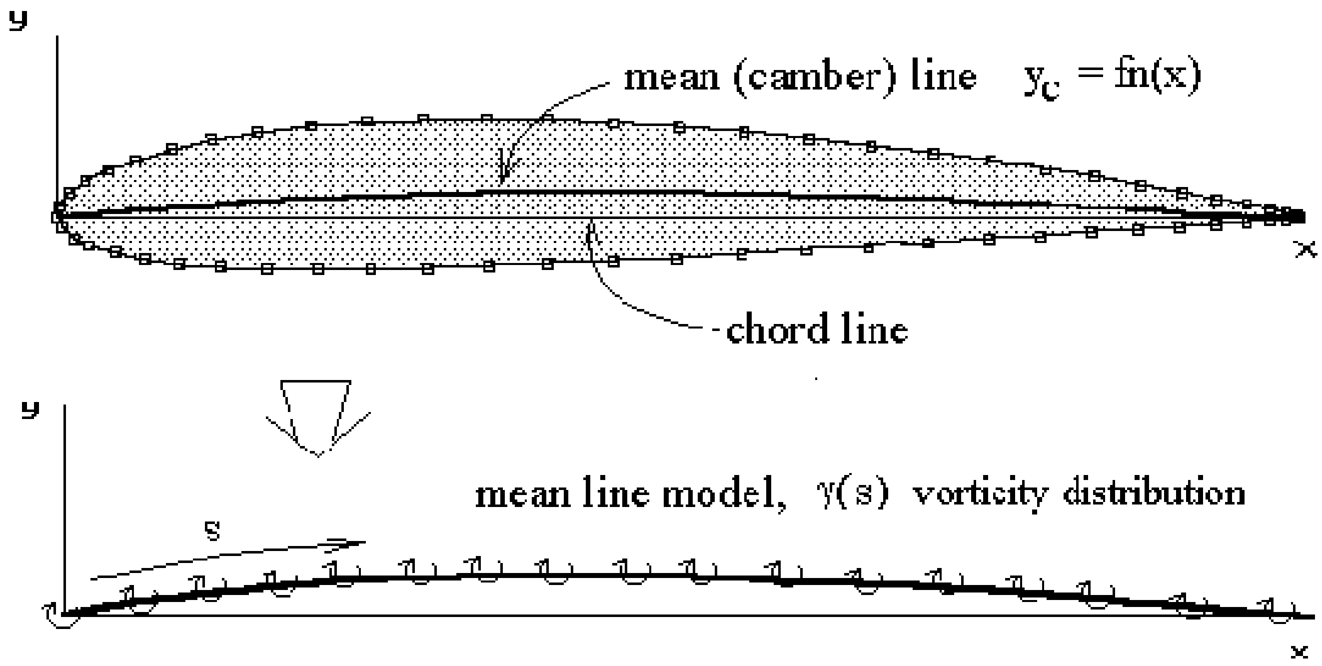


Primary and Secondary Vortex

Vortices that attain their full strength in a single oscillation are named “ primary,” whilst those which require more than one oscillation for their complete development are termed “ secondary ” vortices: in previous papers the latter were called “ residual ” vortices.

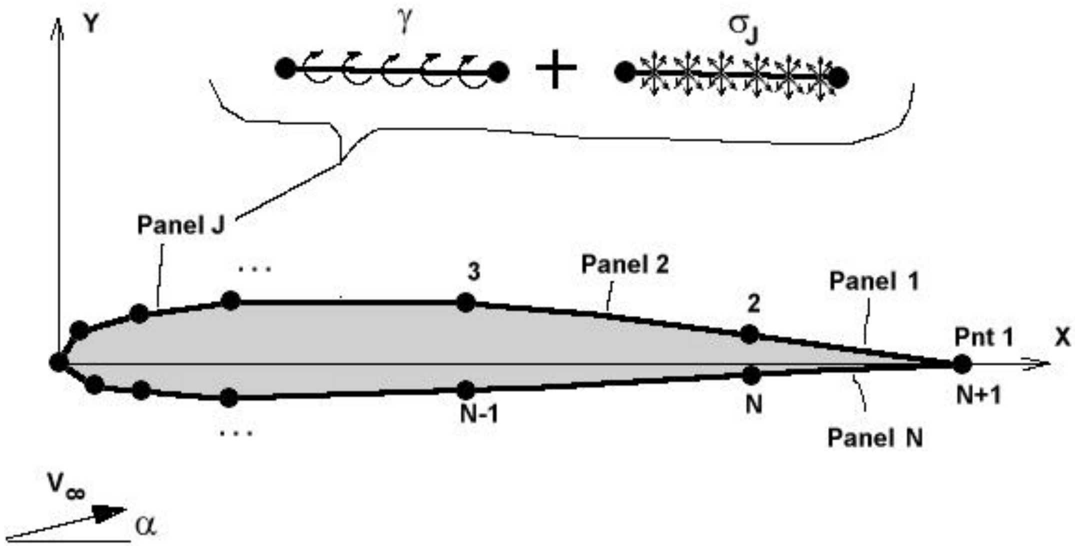


Elements of Lifting Surface Theory



Source Panel Vortex Panel and Vortex Lattice Methods

The vortex-panel method is a method for computing ideal flows - flows in which the effects of compressibility and viscosity are negligible. Ideal flow is often the first type of fluid motion that student engineers and scientists study, because it is the simplest.

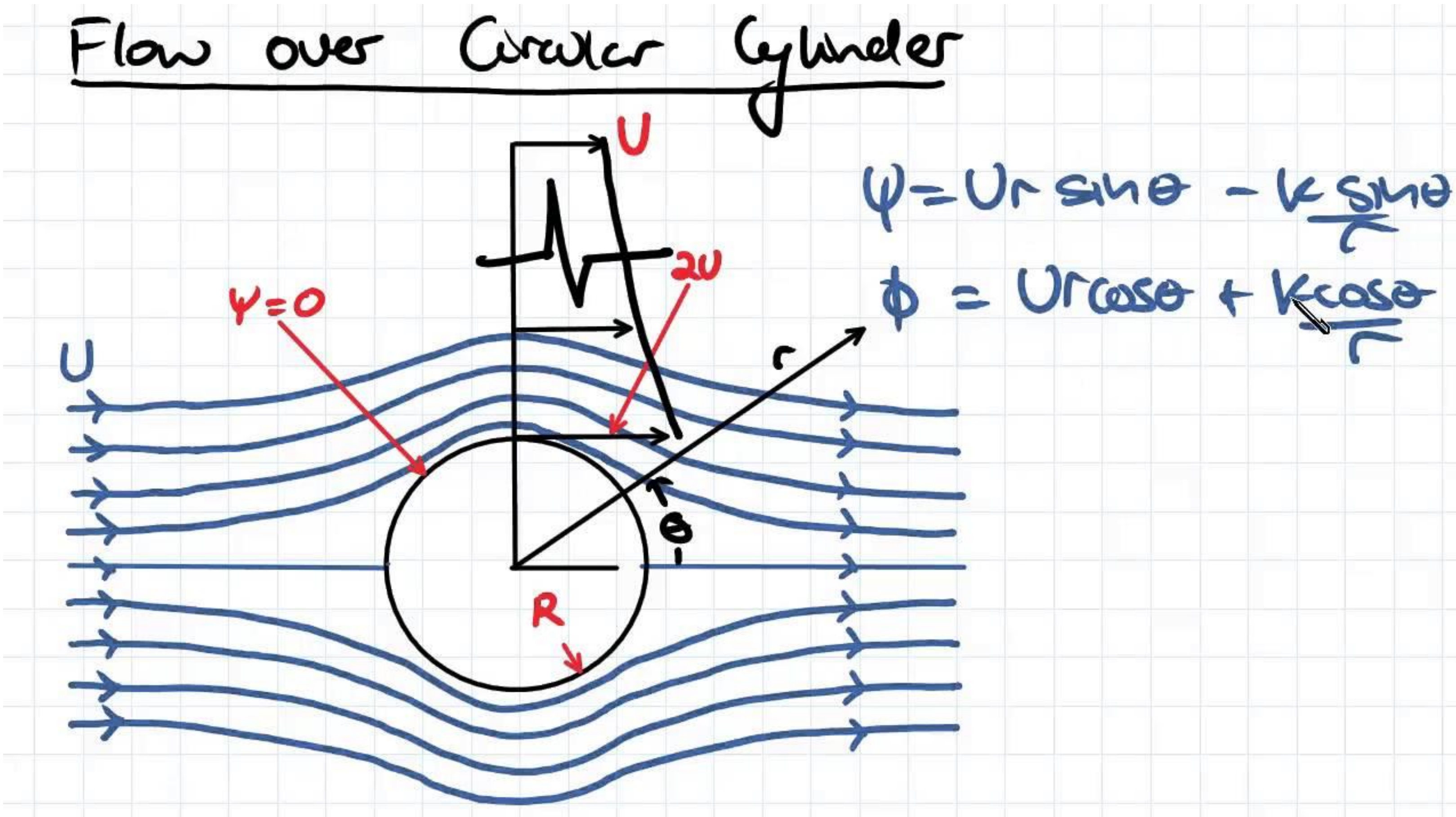


MODULE -IV

FLOW PAST NON-LIFTING BODIES AND INTERFERENCE EFFECTS

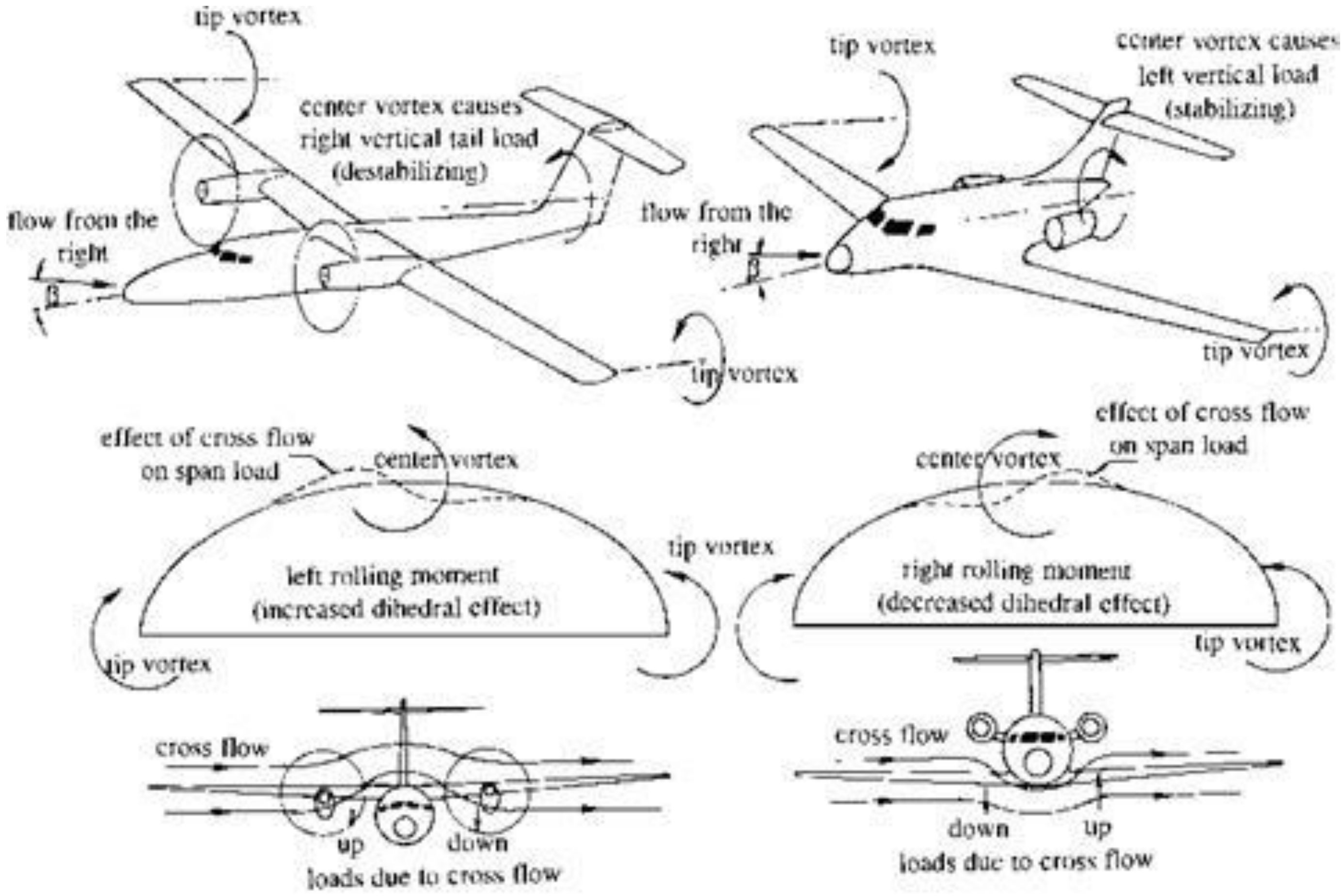
4.1 Flow Past Non-Lifting Bodies
4.2 Method of Singularities
4.3 Wing-Body Interference
4.4 Effect of Propeller on Wings and Bodies and Tail Unit
4.5 Flow Over Airplane as A Whole

Flow Past Non-Lifting Bodies

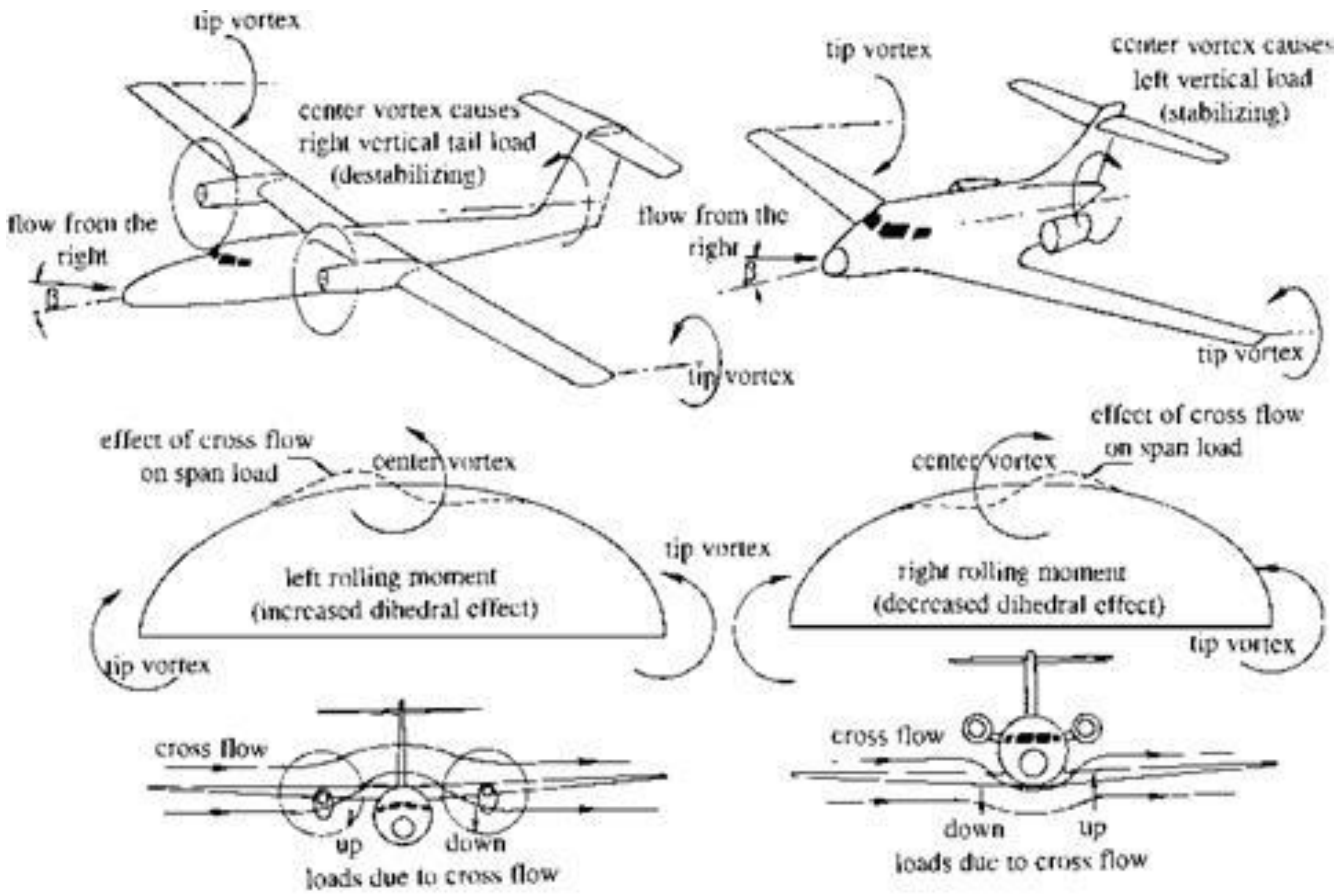


Method of Singularities

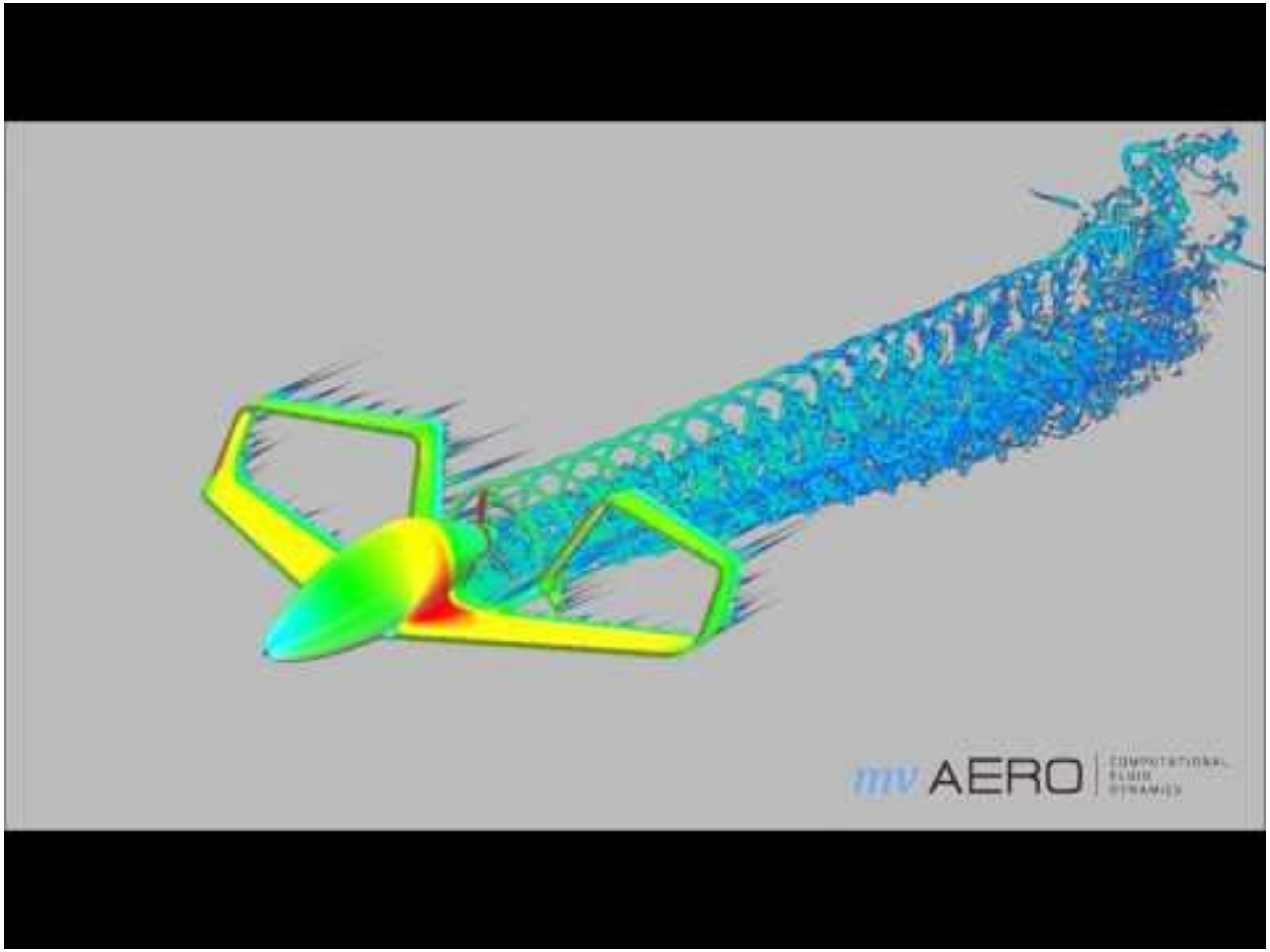
Wing-Body Interference



Effect of Propeller on Wings and Bodies and Tail Unit



Flow Over Airplane as A Whole



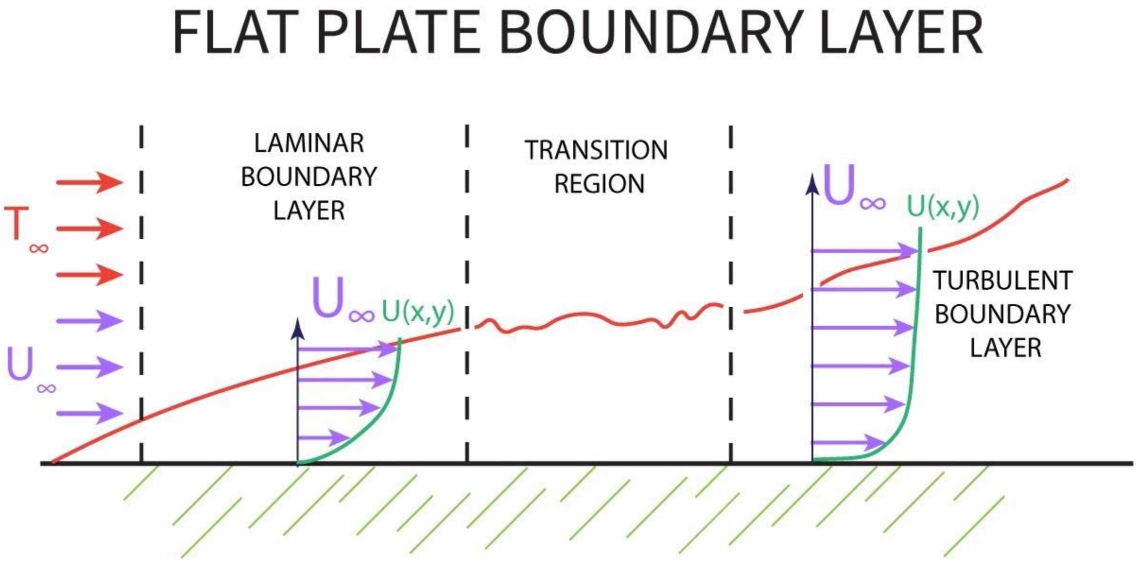
MODULE -V

BOUNDARY LAYERTHEORY

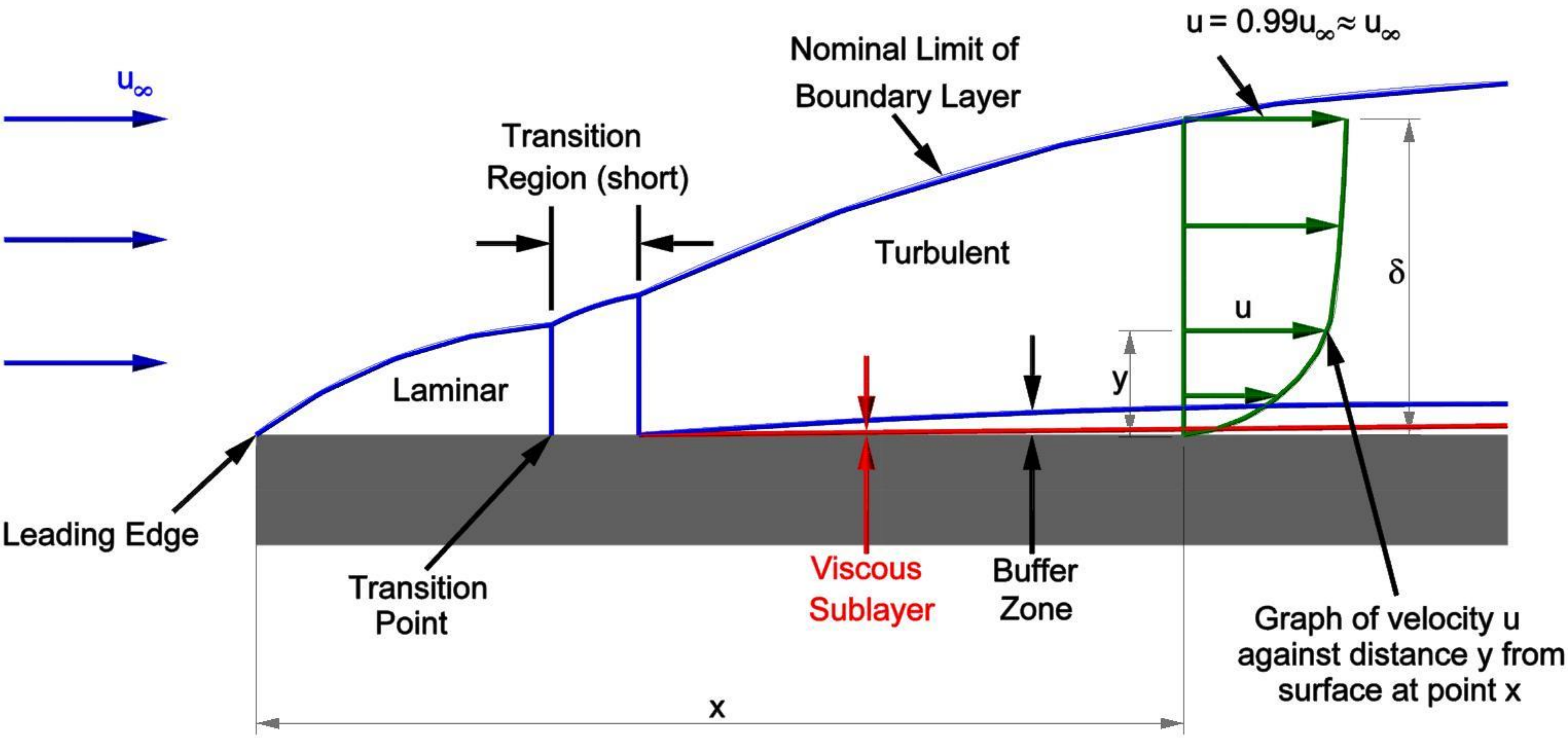
5.1 Introduction to Boundary Layer
5.2 Laminar and Turbulent Boundary Layer
5.3 Transition, Boundary Layer on Flat Plate
5.4 Displacement Thickness
5.5 Momentum Thickness
5.6 Energy Thickness
5.7 Effect of Curvature
5.8 Temperature Boundary Layer

Introduction to Boundary Layer

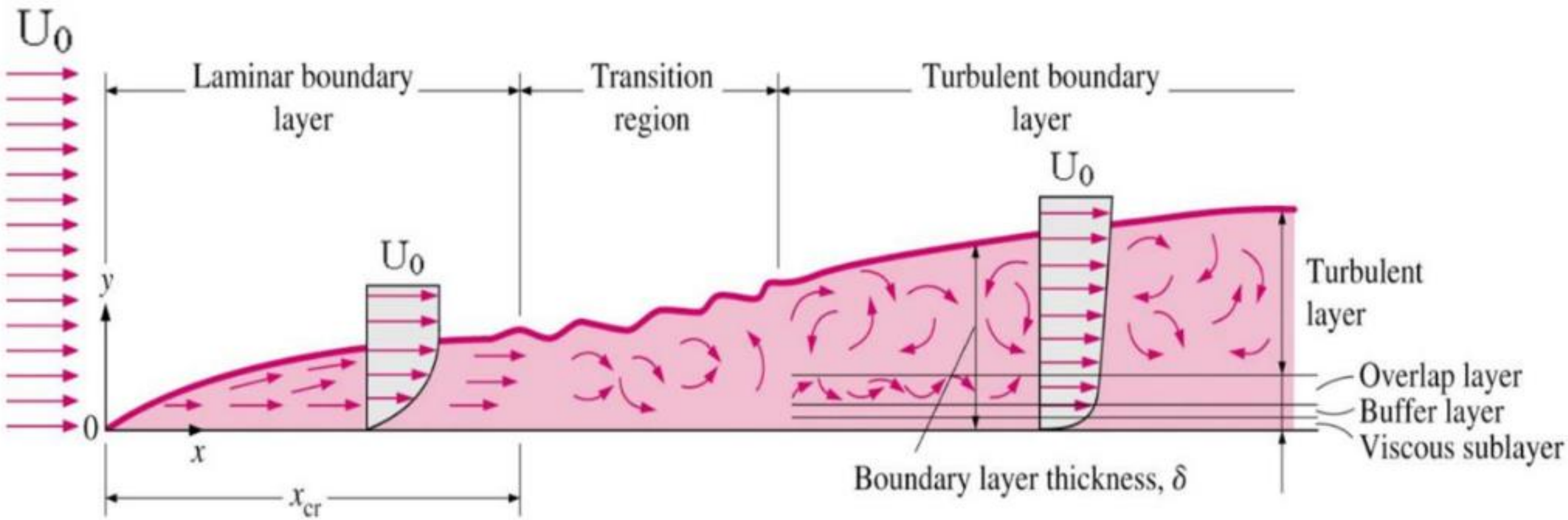
The fundamental concept of the boundary layer was suggested by L. Prandtl (1904), it defines the boundary layer as a layer of fluid developing in flows with very high Reynolds Numbers Re, that is with relatively low viscosity as compared with inertia



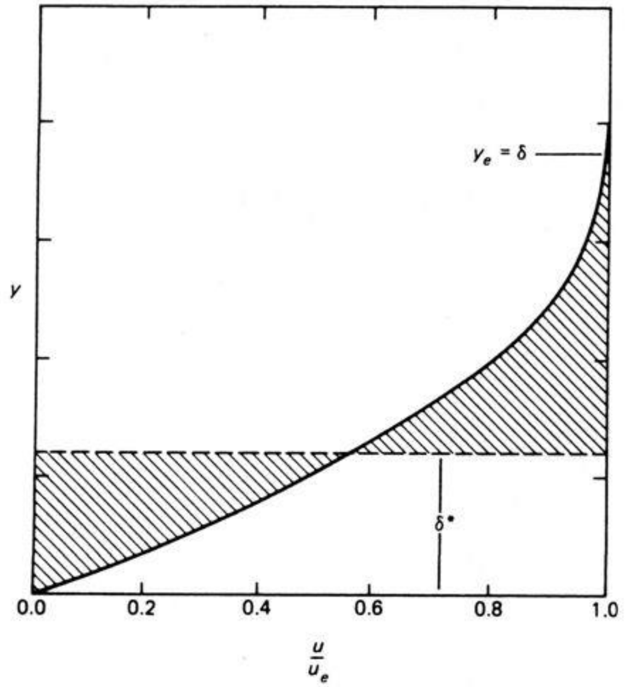
Laminar and Turbulent Boundary Layer



Transition, Boundary Layer on Flat Plate



Displacement Thickness
Laminar B.L.



$$\rho_e u_e \delta^* = \int_0^\delta \rho(u_e - u)\, dy$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dy$$

Momentum Thickness

Momentum Thickness

The rate of mass flow across an element of the boundary layer is (ρ u dy) and the mass has a momentum (ρ u^2 dy) The same mass outside the boundary layer has the momentum (ρ u u_e dy)

$$\rho \int_{BL} (uu_e - u^2)\mathrm{d}y = \rho u_e^2 \int_0^\infty \left(\frac{u}{u_e} - \frac{u^2}{u_e^2}\right)\mathrm{d}y = \rho u_e^2 \quad \theta$$

Θ is a measure of the reduction in momentum transport in the B. Layer

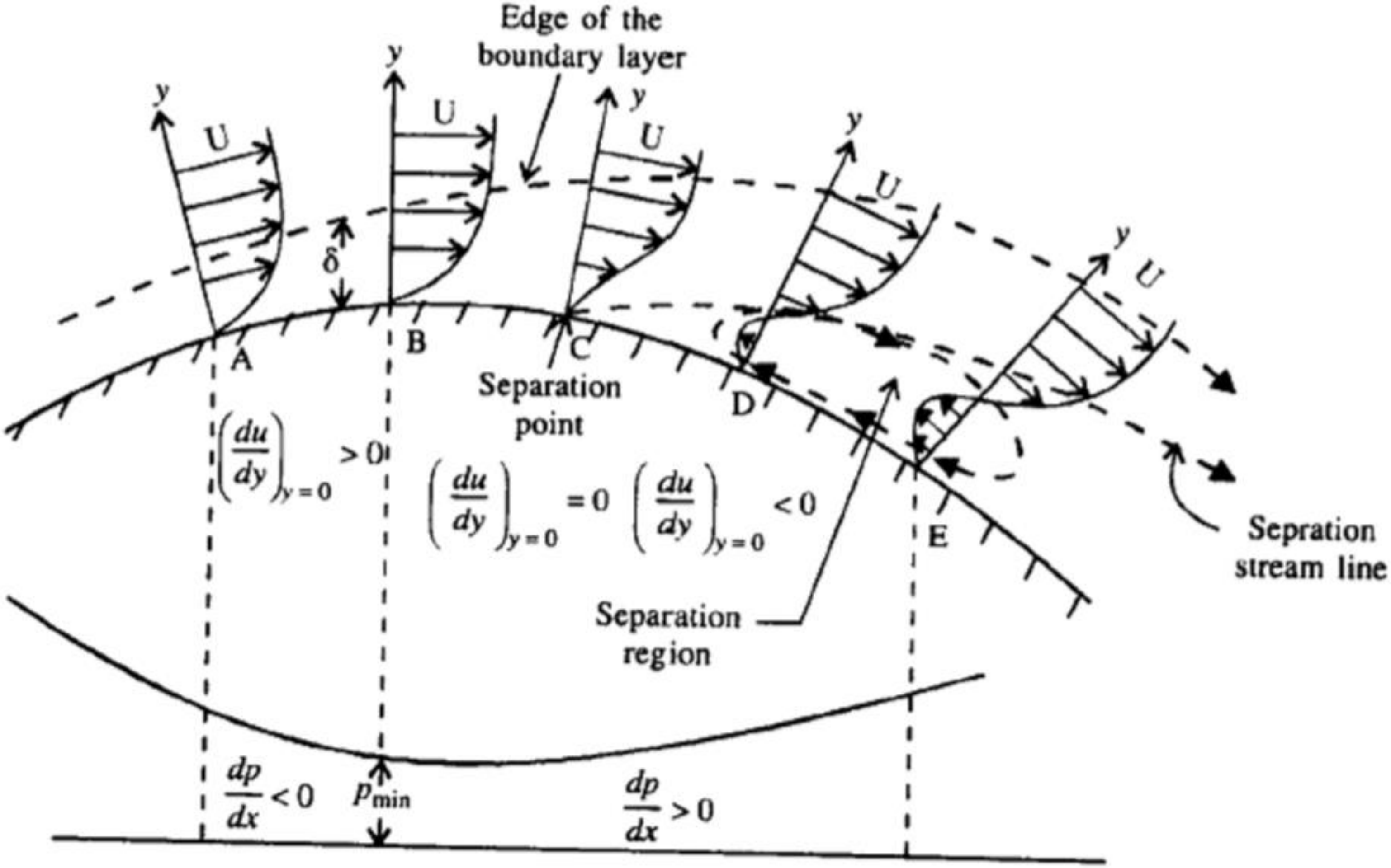
$$\theta = \int_0^\infty \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)\mathrm{d}y$$

Energy Thickness

Eqn. for Energy Thickness

- By equating the energy transport rate for velocity defect to that for ideal fluid
  - $$\frac{1}{2}\rho U^2\delta_e = \frac{1}{2}\int_0^{\delta}(\rho u dy)(U^2 - u^2)$$
- If density is constant, this simplifies to
  - $$\delta_e = \int_0^{\delta}\frac{u}{U}\left(1-\frac{u^2}{U^2}\right)dy$$

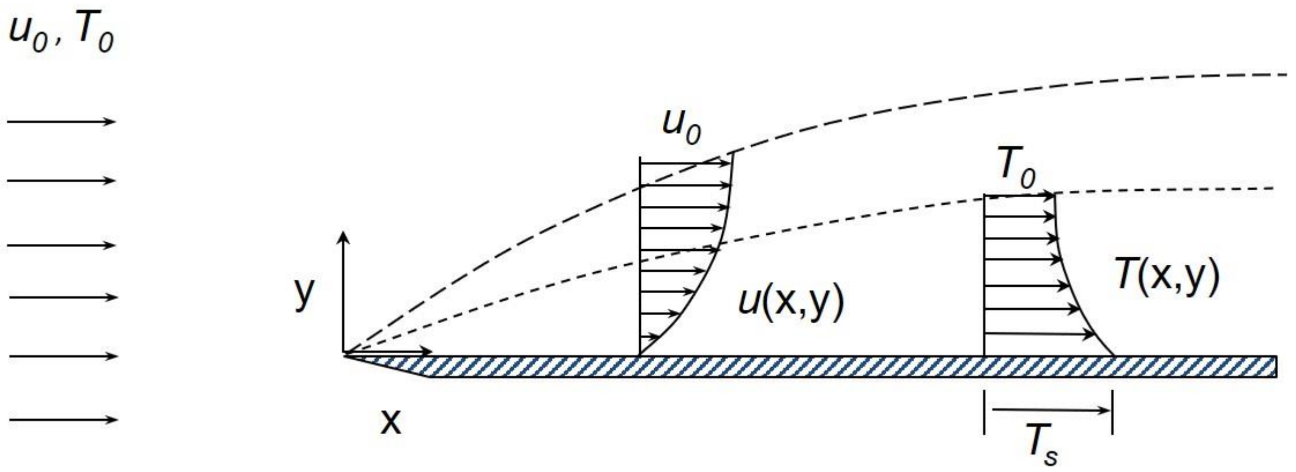
Effect of Curvature



Temperature Boundary Layer

The **thermal boundary layer thickness**, $\boldsymbol{\delta_T}$, is the distance across a boundary layer from the wall to a point where the flow temperature has essentially reached the 'free stream' temperature, T_0. This distance is defined normal to the wall in the y-direction. The thermal boundary layer thickness is customarily defined as the point in the boundary layer, y_{99}, where the temperature T(x,y) reaches 99% of the free stream value T_0

Temperature Boundary Layer



Schematic drawing depicting fluid flow over a heated flat plate.

$$\delta_T = y_{99} \text{ such that } T(x, y_{99}) = 0.99\, T_0$$

at a position x along the wall. In a real fluid, this quantity can be estimated by measuring the temperature profile at a position x along the wall. The temperature profile is the temperature as a function of y at a fixed x position.

For laminar flow over a flat plate a zero incidence, the thermal boundary layer thickness is given by

$$\delta_T = \delta_v \mathrm{Pr}^{-1/3}$$

$$\delta_T = 5.0\sqrt{\frac{\nu x}{u_0}}\mathrm{Pr}^{-1/3}$$

where

- Pr is the Prandtl Number
- δ_v is the thickness of the velocity boundary layer thickness [3]
- u_0 is the freestream velocity
- x is the distance downstream from the start of the boundary layer
- ν is the kinematic viscosity

Temperature Boundary Layer

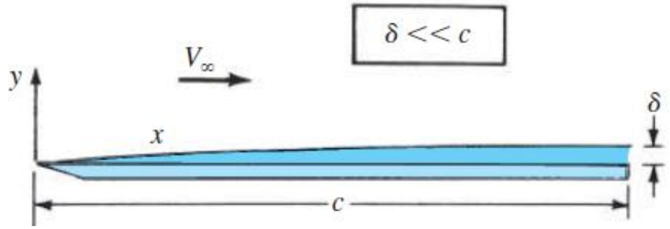
The thermal boundary layer thickness for turbulent flow does not depend on the Prandtl number but instead on the Reynolds number. Hence, the turbulent thermal boundary layer thickness is given approximately by the turbulent velocity boundary layer thickness expression given by

$$\delta_T \approx \delta \approx 0.37x/\mathrm{Re}_x{}^{1/5}$$

where

$\mathrm{Re}_x = u_0 x/\nu$ is the Reynolds number

BLASIUS' EQUATION



The basic assumption of boundary-layer theory:

A boundary layer is very thin in comparison with the scale of the body

ρ = constant
μ = constant
dpe/dx = 0 (because the inviscid flow over a flat plate at α = 0

Basic Equations for Incompressible Flow

Continuity: $$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

x momentum: $$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

y momentum: $$\frac{\partial p}{\partial y} = 0$$

Energy: $$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{dp_e}{dx} + \mu \left(\frac{\partial u}{\partial y}\right)^2$$

Boundary Layer Equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

kinematic viscosity, defined as $v \equiv \mu/\rho$.

Let us transform the independent var... (ξ, η), where

$$\xi = x \quad \text{and} \quad \eta = y\sqrt{\frac{V_\infty}{\nu x}}$$

Using the chain rule, we obtain the derivatives

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial x} \quad \text{and} \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial y} + \frac{\partial}{\partial \eta}\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \xi}{\partial x} = 1 \qquad \frac{\partial \xi}{\partial y} = 0 \qquad \frac{\partial \eta}{\partial y} = \sqrt{\frac{V_\infty}{\nu x}}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial y} = \sqrt{\frac{V_\infty}{\nu x}}\frac{\partial}{\partial \eta}$$

$$\frac{\partial^2}{\partial y^2} = \frac{V_\infty}{\nu x}\frac{\partial^2}{\partial \eta^2}$$

Let us define a stream function ψ such that

$$\psi = \sqrt{\nu x V_\infty} f(\eta)$$

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} = \sqrt{\frac{V_\infty}{\nu x}}\frac{\partial \psi}{\partial \eta} = V_\infty f'(\eta)$$ &

$$v = -\frac{\partial \psi}{\partial x} = -\left(\frac{\partial \psi}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial \psi}{\partial \eta}\right) = -\frac{1}{2}\sqrt{\frac{\nu V_\infty}{x}} f - \sqrt{\nu x V_\infty}\frac{\partial \eta}{\partial x} f'$$

The function $f\ (\eta)$ defined has the property that its derivative f gives the x compon… as

$$f'(\eta) = \frac{u}{V_\infty}$$

By substituting into the momentum equation, we get

$$V_\infty f'\left(V_\infty \frac{\partial \eta}{\partial x} f''\right) - \left(\frac{1}{2}\sqrt{\frac{\nu V_\infty}{x}} f + \sqrt{\nu x V_\infty}\frac{\partial \eta}{\partial x} f'\right) V_\infty \sqrt{\frac{V_\infty}{\nu x}} f'' = \nu V_\infty \frac{V_\infty}{\nu x} f'''$$

Simplifying, we obtain

$$V_\infty^2 \frac{\partial \eta}{\partial x} f' f'' - \frac{1}{2}\frac{V_\infty^2}{x} f f'' - V_\infty^2 \left(\frac{\partial \eta}{\partial x}\right) f' f'' = \frac{V_\infty^2}{x} f'''$$

The first and third terms cancel, and Equation becomes

$$2 f''' + f f'' = 0$$

is called *Blasius' equation*


Thank you