AEROSPACE STRUCTURES

Course Code: AAEB07

B TECH IV SEMESTER (R18)

Regulation: IARE R-18

by

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COURSE OUTCOMES

CO 1	**Describe the concept of Structural components, structural joints, Monocoque and semi monocoque structures and also energy methods and principles.**
CO 2	Describe the concept of thin plates subject to different types of loads and also buckling phenomena of thin plates, local instability and instability of stiffened panels.
CO 3	Understand the concept of symmetric and un-symmetric bending of beams shear stresses and shear flow distribution of thin walled sections and Torsion phenomenon.
CO 4	Explore the concept of Structural idealization and stress distribution of idealized thin walled sections.
CO 5	Discuss the concept of idealized thin walled sections, fuselage, Wing spar and box beams.

Module- 1
INTRODUCTION TO AIRCRAFT STRUCTURAL COMPONENTS AND ENERGY METHODS

Aircraft Structural components and loads

Aircraft are generally built up from the basic components of wings, fuselages, tail units and control surfaces

The structure of an aircraft is required to support

1 *ground loads*: includes all loads encountered by the aircraft during movement or transportation on the ground such as taxiing and landing loads, towing and hoisting loads

2 *air loads*: comprises loads imposed on the structure during flight by maneuvers and gusts.

Source from Aircraft Structures by T. H. G. Megson

Aircraft designed for a particular role encounter loads peculiar to their range of operation.

Carrier born aircraft, for instance, are subjected to shoot take-off and arrested landing loads

Most large civil and practically all military aircraft have pressurized cabins for high altitude flying;

Amphibious aircraft must be capable of landing on water and aircraft designed to fly at high speed at low altitude,

Ex. The Tornado, require a structure of above average strength to withstand the effects of flight in extremely turbulent air.

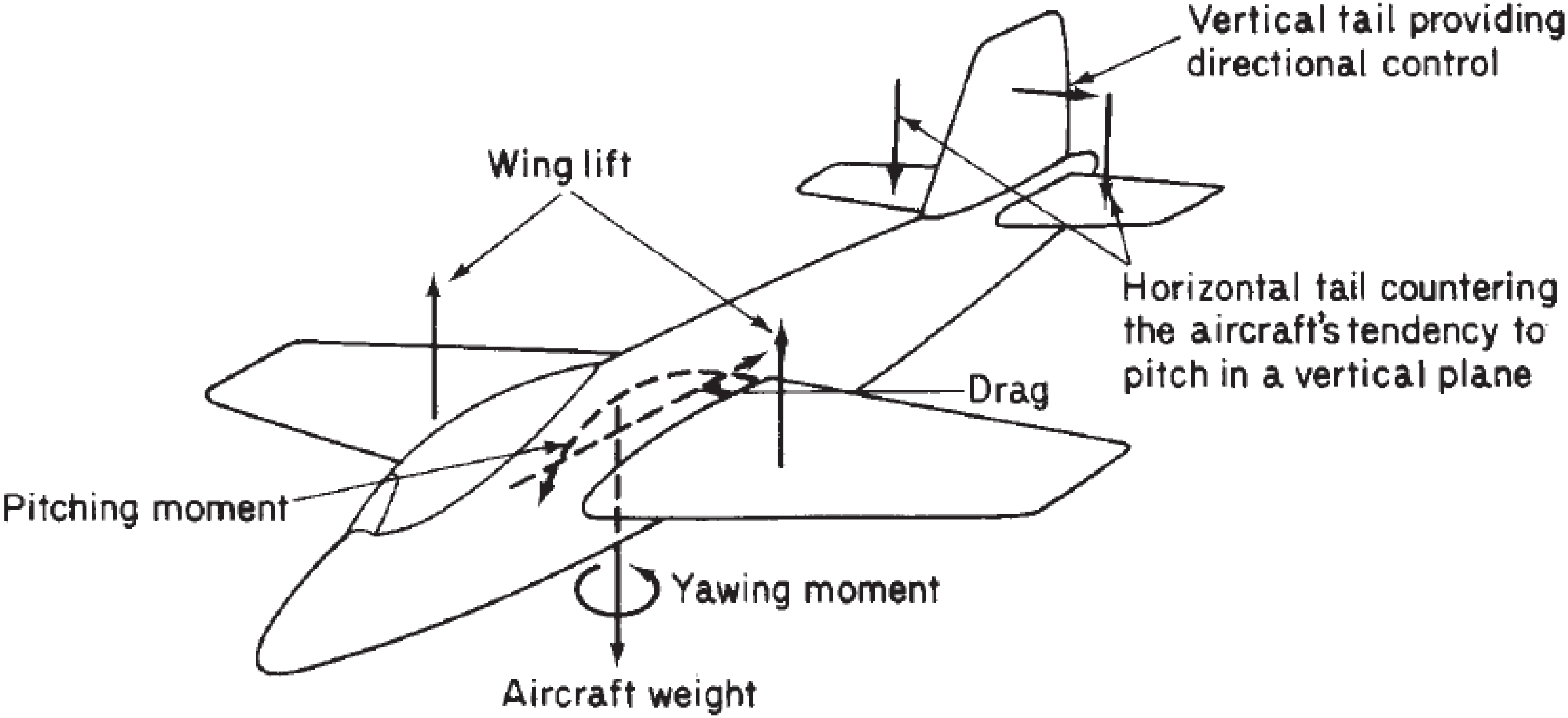
Source from Aircraft Structures by T. H. G. Megson

The two classes of loads may be further divided into *surface forces* which act upon the surface of the structure,

e.g. aerodynamic and hydrostatic pressure, and *body forces* which act over the volume of the structure and are produced by gravitational and inertial effects.

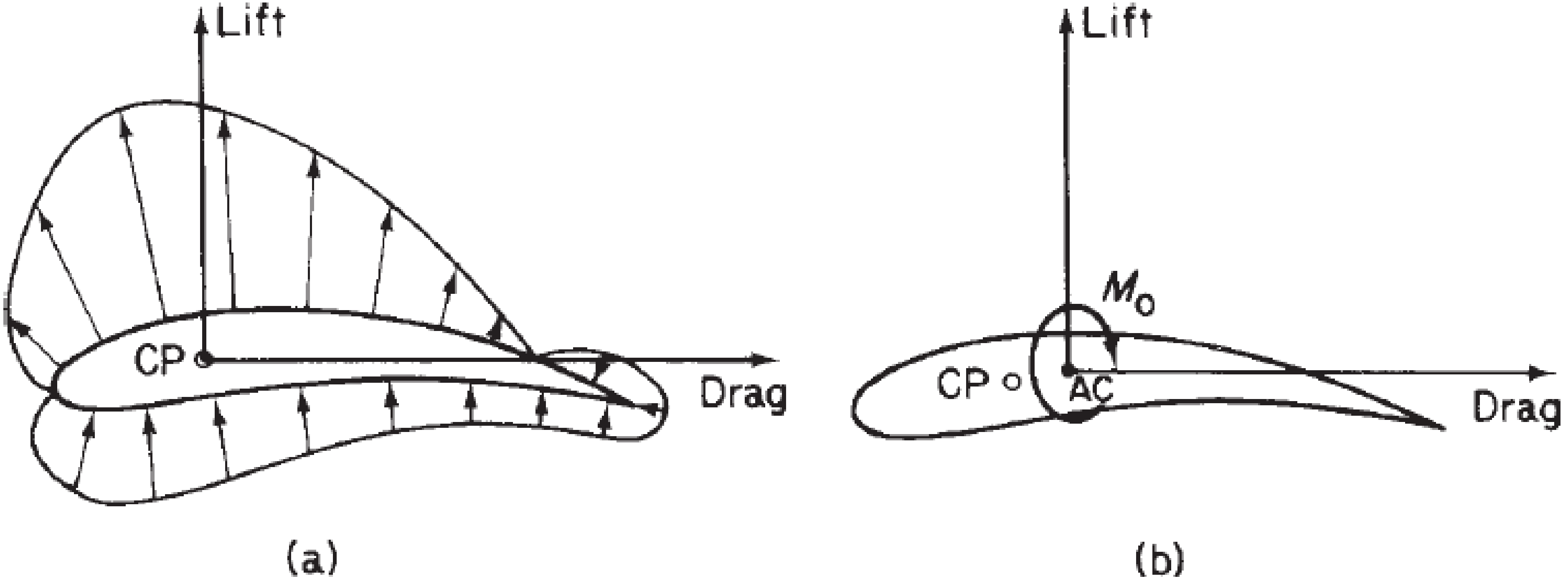
Calculation of the distribution of aerodynamic pressure over the various surfaces of an aircraft's structure is presented in numerous texts on aerodynamics and will therefore not be attempted here.

We shall, however, discuss the types of load induced by these various effects and their action on the different structural components.



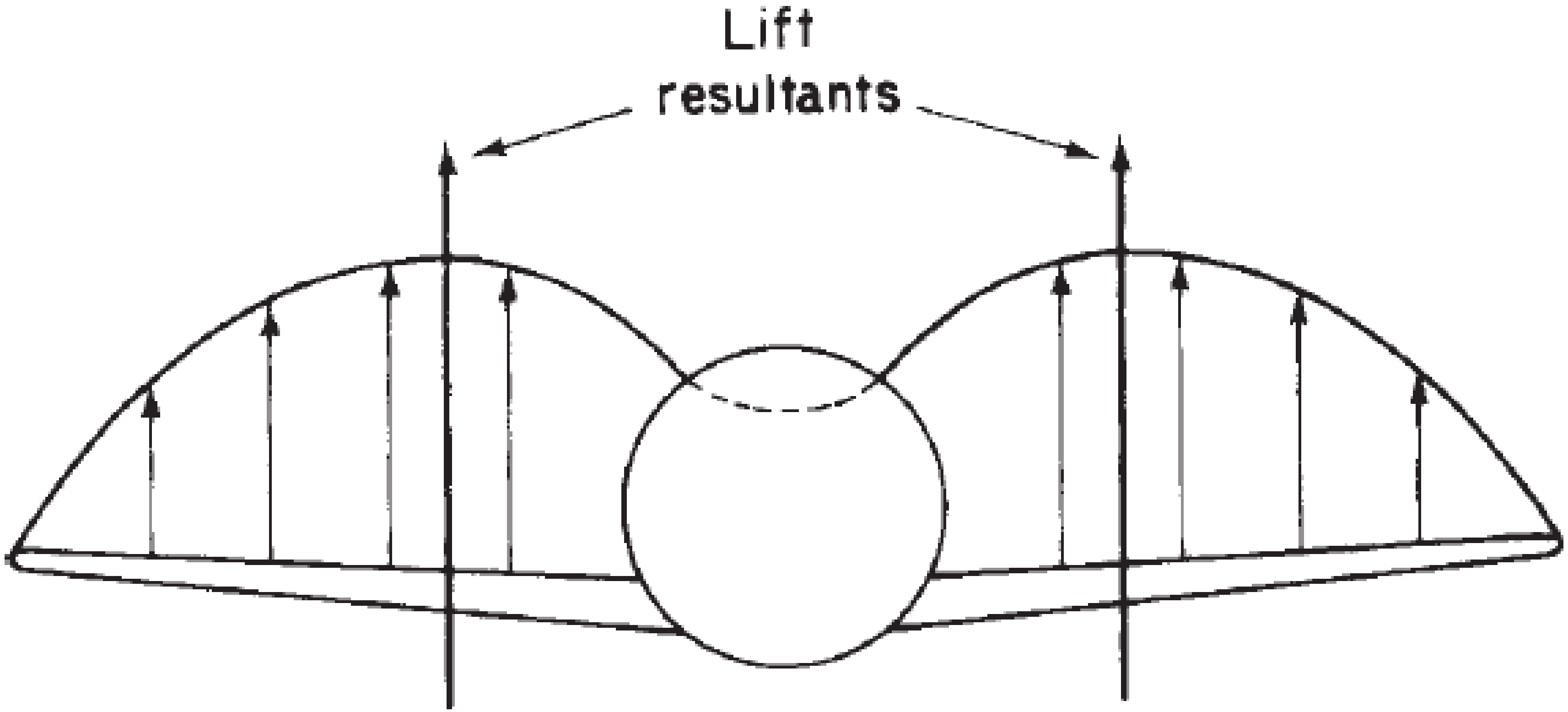
Principal aerodynamic forces on an aircraft during flight.

Source from Aircraft Structures by T. H. G. Megson



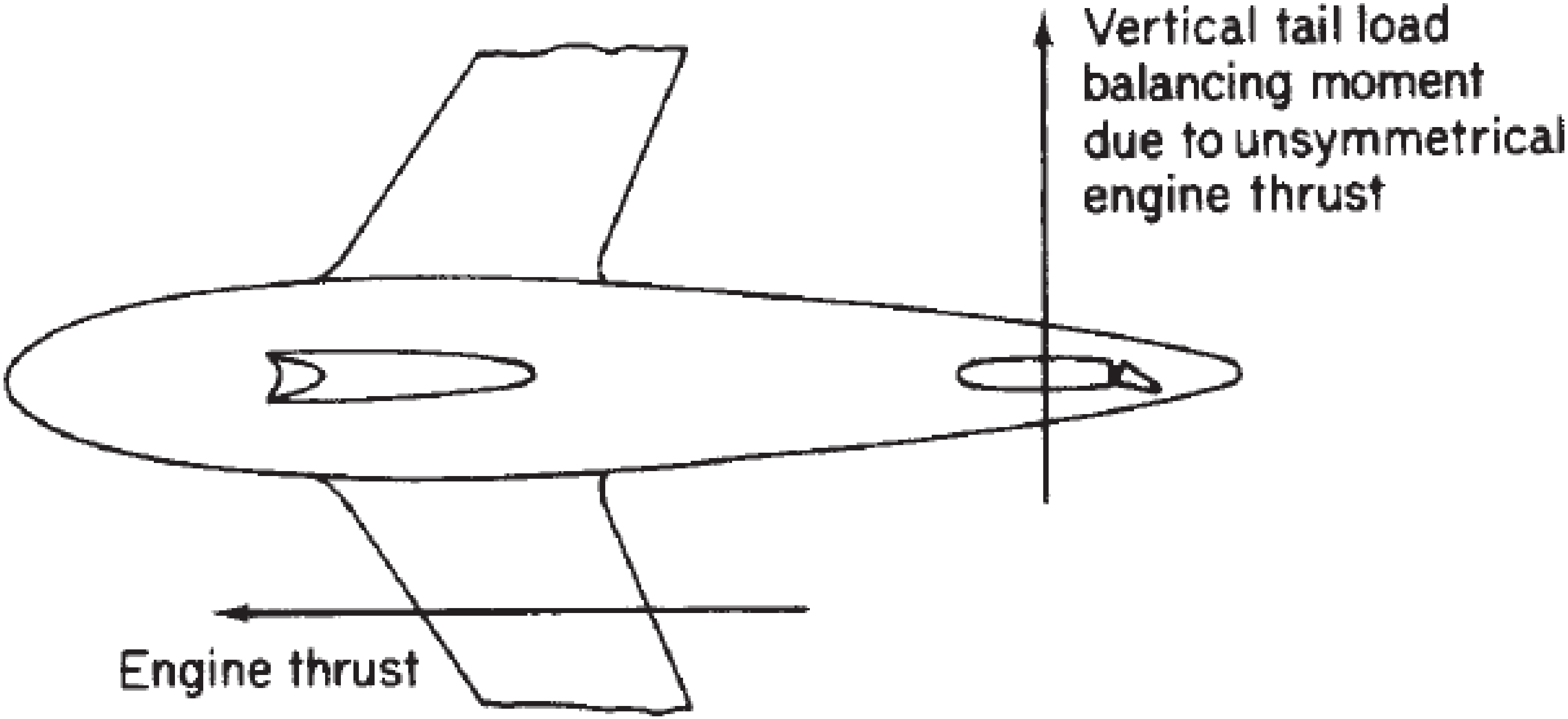
(a) Pressure distribution around an aerofoil; (b) transference of lift and drag loads to the AC.

Source from Aircraft Structures by T. H. G. Megson



Typical lift distribution for a wing/fuselage combination

Source from Aircraft Structures by T. H. G. Megson



Fuselage and wing bending caused by an unsymmetrical engine load.

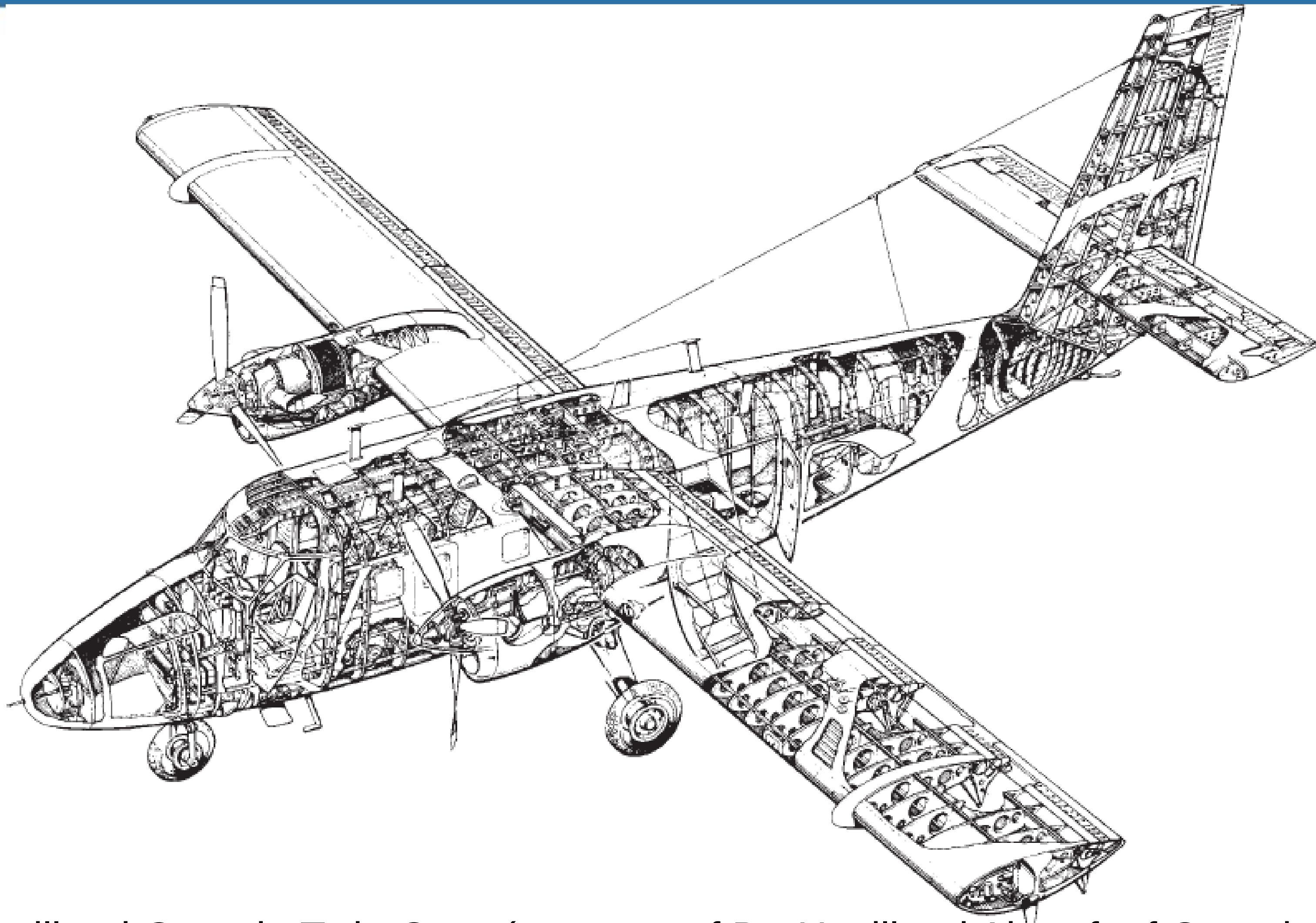
Source from Aircraft Structures by T. H. G. Megson

The basic functions of an aircraft’s structure are to transmit and resist the applied loads; to provide an aerodynamic shape and to protect passengers, payload, systems, etc. from the environmental conditions encountered in flight.

These requirements, in most aircraft, result in thin shell structures where the outer surface or skin of the shell is usually supported by longitudinal stiffening members and transverse frames to enable it to resist bending, compressive and torsional loads without buckling.

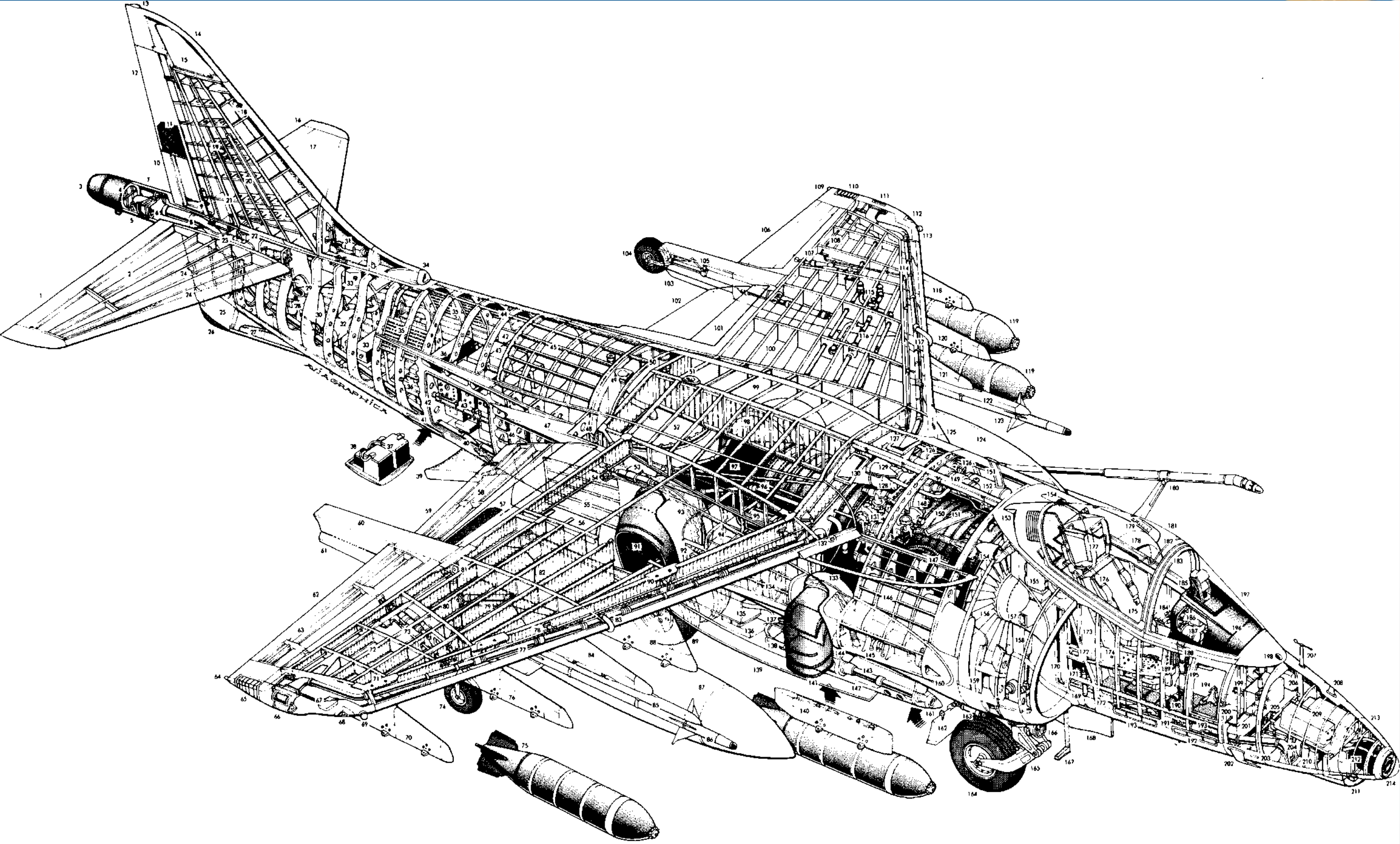
Such structures are known as *semi-monocoque*, while thin shells which rely entirely on their skins for their capacity to resist loads are referred to as *monocoque*.

Source from Aircraft Structures by T. H. G. Megson



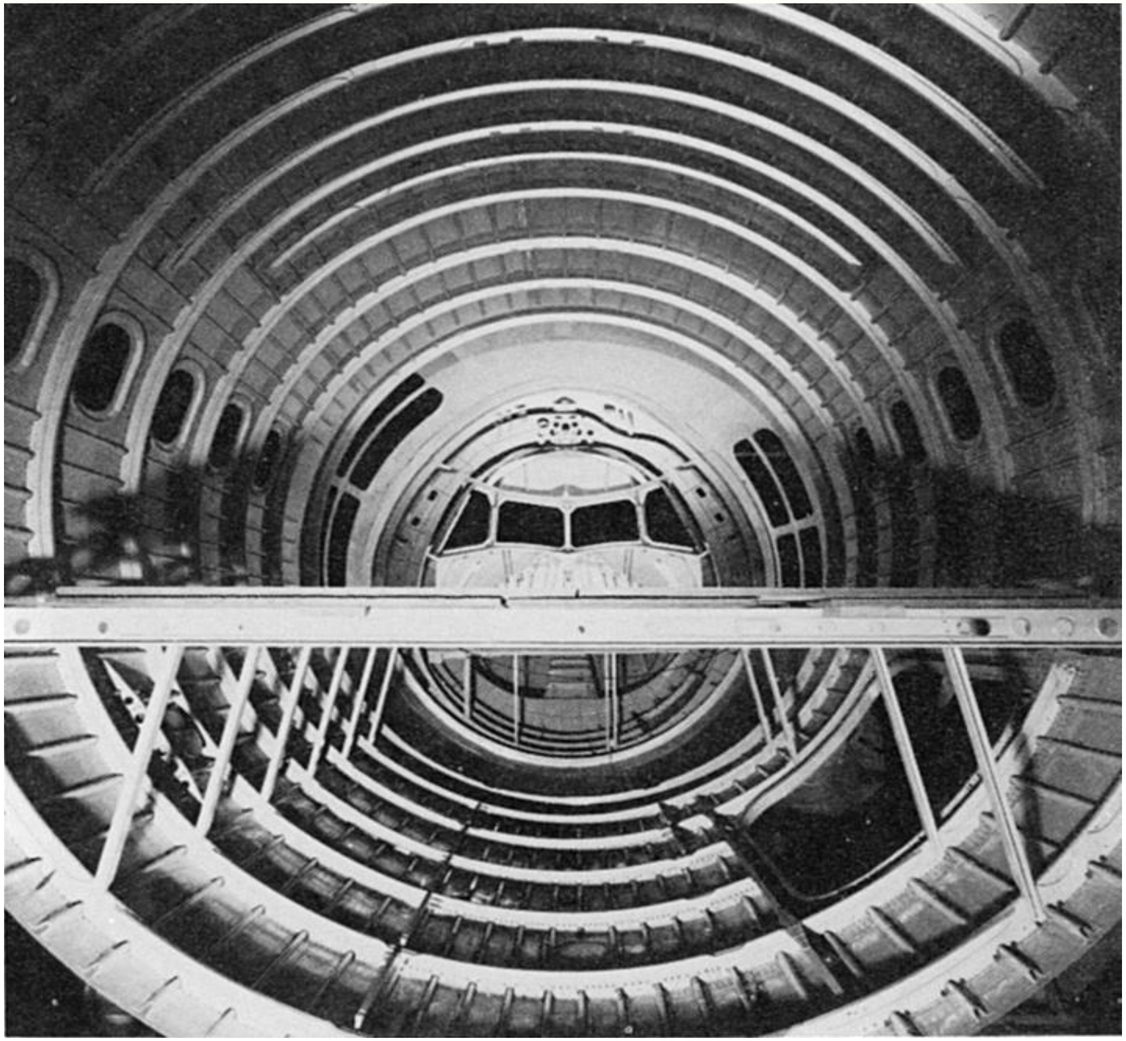
De Havilland Canada Twin Otter (courtesy of De Havilland Aircraft of Canada Ltd.).

Source from Aircraft Structures by T. H. G. Megson



Harrier (courtesy of Pilot Press Ltd.).

Source from Aircraft Structures by T. H. G. Megson



British Aerospace 146 (courtesy of British Aerospace).

Source from Aircraft Structures by T. H. G. Megson

No matter how complex the internal structural arrangement the different components perform the same kind of function.

The shape of the cross-section is governed by aerodynamic considerations and clearly must be maintained for all combinations of load; this is one of the functions of the ribs.

They also act with the skin in resisting the distributed aerodynamic pressure loads; they distribute concentrated loads

(e.g. undercarriage and additional wing store loads)

into the structure and redistribute stress around discontinuities, such as undercarriage wells, inspection panels and fuel tanks, in the wing surface.

Source from Aircraft Structures by T. H. G. Megson

Ribs increase the column buckling stress of the longitudinal stiffeners by providing end restraint and establishing their column length; in a similar manner they increase the plate buckling stress of the skin panels.

The dimensions of ribs are governed by their spanwise position in the wing and by the loads they are required to support.

In the outer portions of the wing, where the cross-section may be relatively small if the wing is tapered and the loads are light, ribs act primarily as formers for the aerofoil shape.

A light structure is sufficient for this purpose whereas at sections closer to the wing root, where the ribs are required to absorb and transmit large concentrated applied loads, such as those from the undercarriage, engine thrust and fuselage attachment point reactions, a much more rugged construction is necessary.

Source from Aircraft Structures by T. H. G. Megson

Between these two extremes are ribs which support hinge reactions from ailerons, flaps and other control surfaces, plus the many internal loads from fuel, armament and systems installations.

The primary function of the wing skin is to form an impermeable surface for supporting the aerodynamic pressure distribution from which the lifting capability of the wing is derived.

These aerodynamic forces are transmitted in turn to the ribs and stringers by the skin through plate and membrane action.

Resistance to shear and torsional loads is supplied by shear stresses developed in the skin and spar webs, while axial and bending loads are reacted by the combined action of skin and stringers.

Source from Aircraft Structures by T. H. G. Megson

Types of structural joints

The fuselage **structure** generally consists of skin panels joined directly to the **structural** members such as frames, stringers for longitudinal splices. In assembling process critical **structures** like military or commercial **aircraft**, riveted or bolted **joints** are basically used as they offer many options to the engineer.

(source:https://www.google.co.in/search?source=hp&ei=OgH8W tqUMYLI0gSpsKi4BQ&q=types+of+structural+joints+in+aircraft&oq=Types+of+structural+joints&gs_l=psy-ab.1.1.35i39k1j0j0i22i30k1l8.8846.8846.0.10736.3.2.0.0.0.0.176.176.0j1.2.0....0...1c.1.64. psy-ab..1.2.348.6...172.muPwFe8AWP8)

Source from Aircraft Structures by T. H. G. Megson

The fabrication of aircraft components generally involves the joining of one part of the component to another.

For example, fuselage skins are connected to stringers and frames while wing skins are connected to stringers and wing ribs unless, as in some military aircraft with high wing loadings, the stringers are machined integrally with the wing skin.

With the advent of all-metal, i.e. aluminium alloy construction, riveted joints became the main form of connection with some welding although aluminium alloys are difficult to weld, and, in the modern era, some glued joints which use epoxy resin.

In this section we shall concentrate on the still predominant method of connection, riveting.

Source from Aircraft Structures by T. H. G. Megson

In general riveted joints are stressed in complex ways and an accurate analysis is very often difficult to achieve because of the discontinuities in the region of the joint.

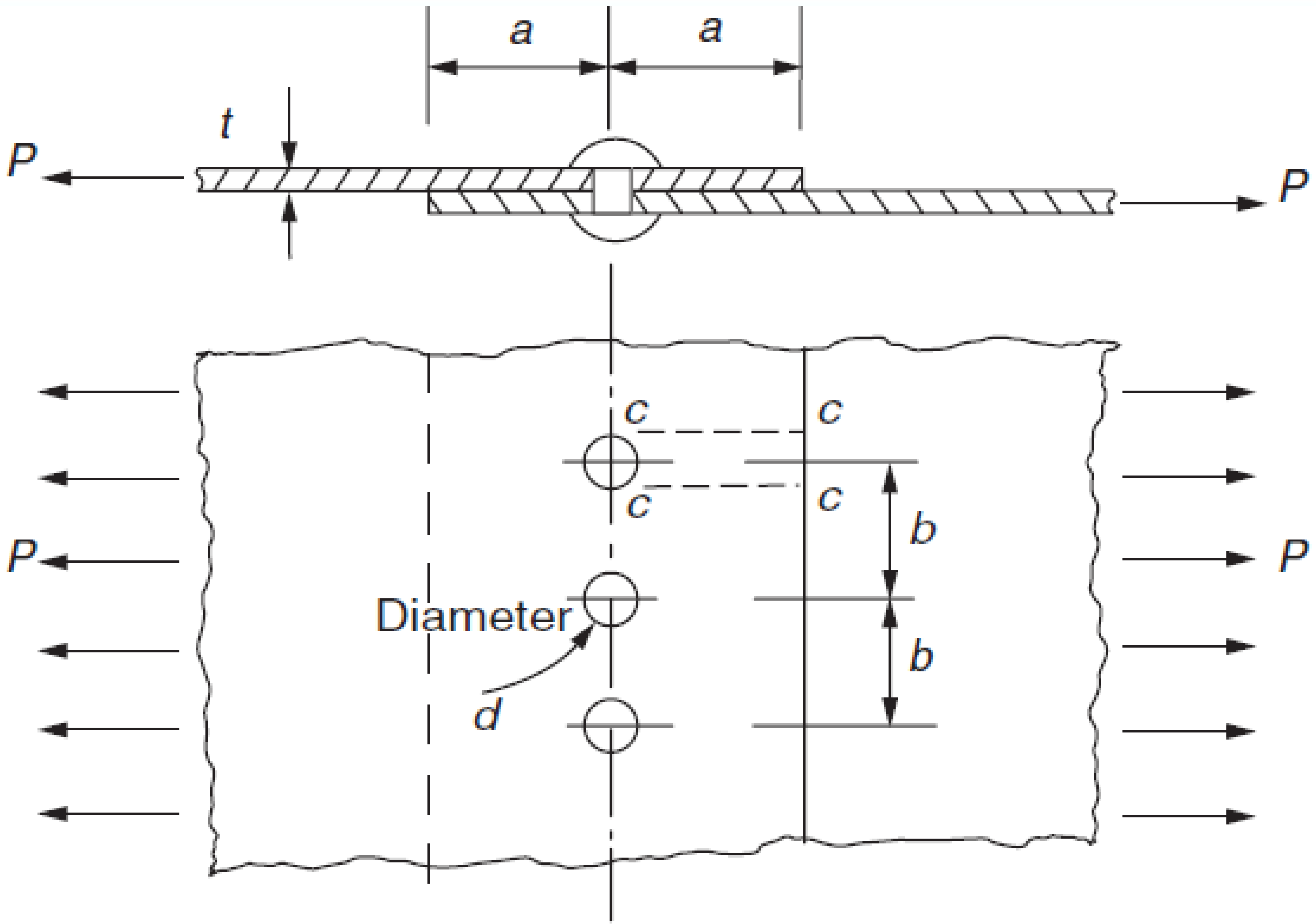
Fairly crude assumptions as to joint behaviour are made but, when combined with experience, safe designs are produced.

Figure shows two plates of thickness t connected together by a single line of rivets; this type of joint is termed a lap joint and is one of the simplest used in construction.

Suppose that the plates carry edge loads of P/unit width, that the rivets are of diameter d and are spaced at a distance b apart, and that the distance from the line of rivets to the edge of each plate is a.

There are four possible modes of failure which must be considered as follows:

Source from Aircraft Structures by T. H. G. Megson



Simple riveted lap joint.

Source from Aircraft Structures by T. H. G. Megson

Rivet shear

The rivets may fail by shear across their diameter at the interface of the plates.

Then, if the maximum shear stress the rivets will withstand is $\tau 1$ failure will occur when

$$Pb = \tau_1 \left(\frac{\pi d^2}{4} \right)$$

which gives

$$P = \frac{\pi d^2 \tau_1}{4b}$$

Source from Aircraft Structures by T. H. G. Megson

Bearing pressure

Either the rivet or plate may fail due to bearing pressure. Suppose that *pb* is this pressure then failure will occur when

$$\frac{Pb}{td} = p_b$$

$$P = \frac{p_b td}{b}$$

Source from Aircraft Structures by T. H. G. Megson

Plate failure in tension

The area of plate in tension along the line of rivets is reduced due to the presence of rivet holes.

Therefore, if the ultimate tensile stress in the plate is σult failure will occur when

$$\frac{Pb}{t(b-d)} = \sigma_{\mathrm{ult}}$$

from which

$$P = \frac{\sigma_{\mathrm{ult}}\, t(b-d)}{b}$$

Source from Aircraft Structures by T. H. G. Megson

Shear failure in a plate

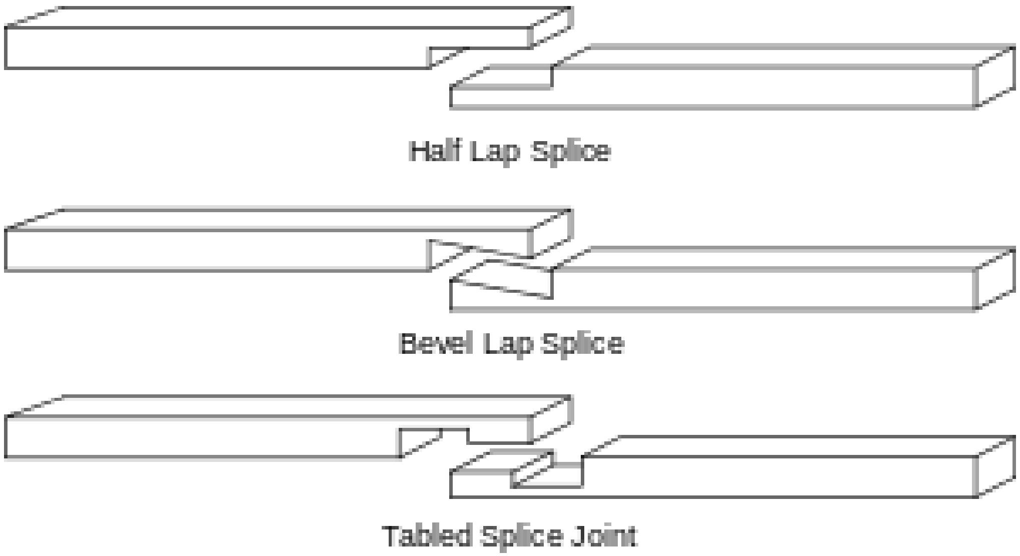
Shearing of the plates may occur on the planes cc resulting in the rivets being dragged out of the plate. If the maximum shear stress at failure of the material of the plates is τ_2 then a failure of this type will occur when

$$Pb = 2at\,\tau_2$$

which gives

$$P = \frac{2at\,\tau_2}{b}$$

Source from Aircraft Structures by T. H. G. Megson



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Airframe loads

Aircraft inertia loads

The maximum loads on the components of an aircraft's structure generally occur when the aircraft is undergoing some form of acceleration or deceleration, such as in landings, take-offs and manoeuvres within the flight and gust envelopes.

Before a structural component can be designed, the inertia loads corresponding to these accelerations and decelerations must be calculated.

For these purposes we shall suppose that an aircraft is a rigid body and represent it by a rigid mass, *m*, as shown in Fig. below.

We shall also, at this stage, consider motion in the plane of the mass which would correspond to pitching of the aircraft without roll or yaw.

Source from Aircraft Structures by T. H. G. Megson

The centre of gravity (CG) of the mass has coordinates $\bar{x}$, $\bar{y}$ referred to x and y axes having an arbitrary origin O; the mass is rotating about an axis through O perpendicular to the xy plane with a constant angular velocity ω.

The acceleration of any point, a distance r from O, is $\omega^2 r$ and is directed towards O.

Thus, the inertia force acting on the element, δm, is $\omega^2 r\delta m$ in a direction opposite to the acceleration, as shown in Fig. below.

The components of this inertia force, parallel to the x and y axes, are $\omega^2 r\delta m\cos\vartheta$ and $\omega 2r\delta m sin\ \vartheta$, respectively, or, in terms of x and y, $\omega^2 x\delta m$ and $\omega^2 y\delta m$.

The resultant inertia forces, Fx and Fy, are then given by

Source from Aircraft Structures by T. H. G. Megson

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The resultant inertia forces, F_x and F_y, are then given by

$$F_y = \int \omega^2 y \, \mathrm{d}m = \omega^2 \int y \, \mathrm{d}m$$

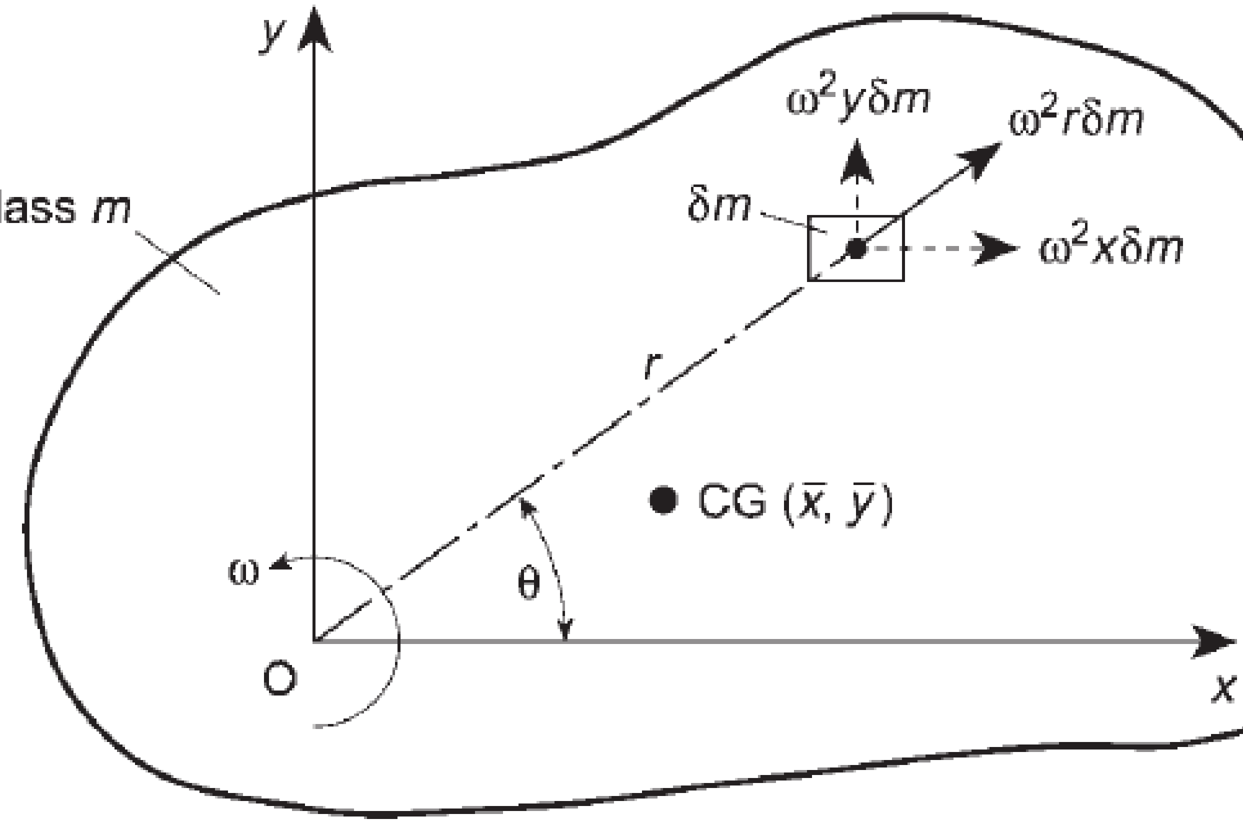
$$F_x = \int \omega^2 x \, \mathrm{d}m = \omega^2 \int x \, \mathrm{d}m$$

The angular velocity ω is constant and may therefore be taken outside the integral sign.

In the above expressions x dm and y dm are the moments of the mass, m, about the y and x axes, respectively, so that

$$F_x = \omega^2 \bar{x} m$$

$$F_y = \omega^2 \bar{y} m$$



Inertia forces on a rigid mass having a constant angular velocity.

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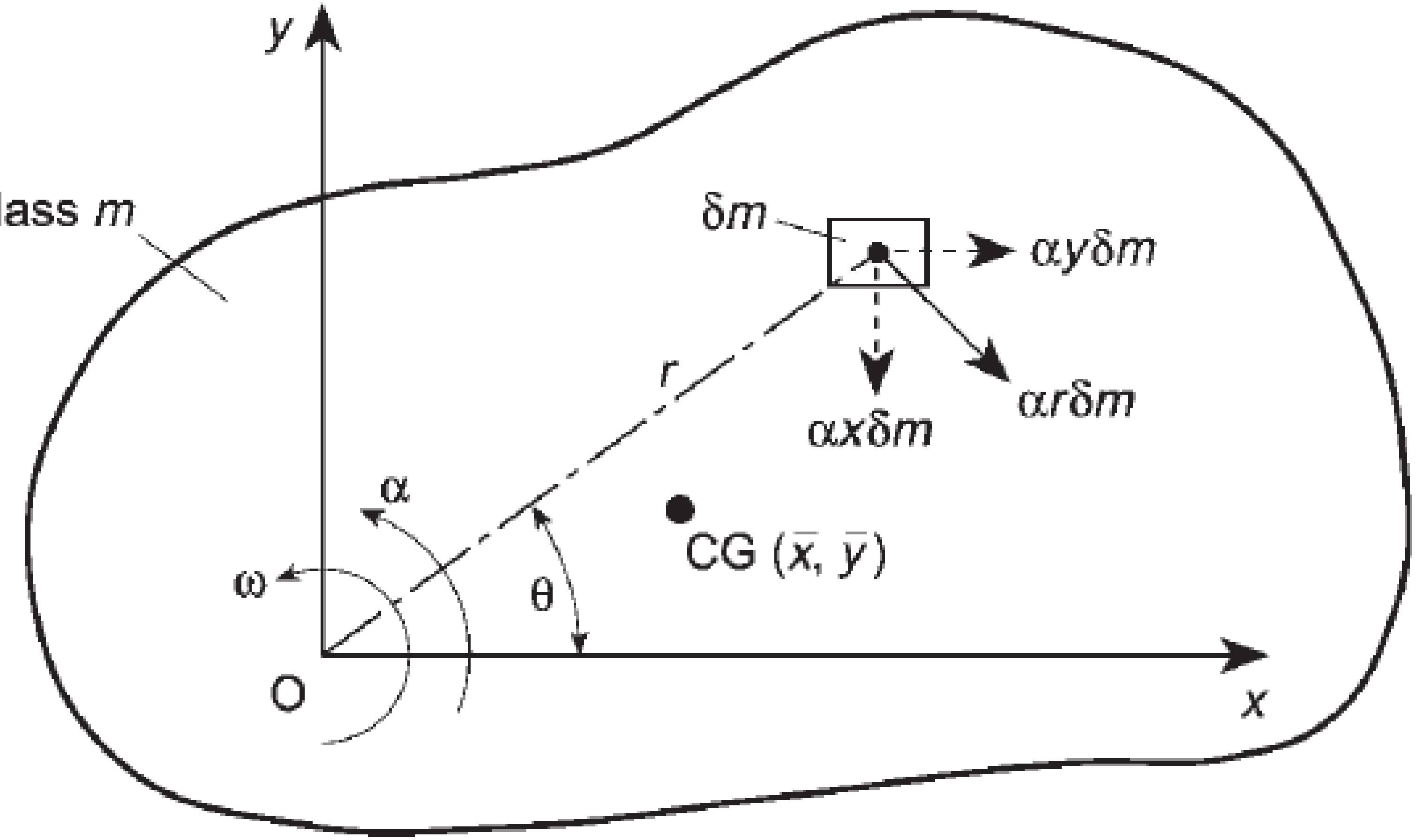


Inertia forces on a rigid mass subjected to an angular acceleration.

Source from Aircraft Structures by T. H. G. Megson

Symmetric manoeuvre loads

Consider the calculation of aircraft loads corresponding to the flight conditions specified by flight envelopes.

There are infinite number of flight conditions within the boundary of the flight envelope although, structurally, those represented by the boundary are the most severe.

In symmetric manoeuvres we consider the motion of the aircraft initiated by movement of the control surfaces in the plane of symmetry.

Examples of such manoeuvres are loops, straight pull-outs and bunts, and the calculations involve the determination of lift, drag and tailplane loads at given flight speeds and altitudes.

Source from Aircraft Structures by T. H. G. Megson

Level flight

Steady level flight is not a manoeuvre in the strict sense of the word, it is a useful condition to investigate initially since it establishes points of load application and gives some idea of the equilibrium of an aircraft in the longitudinal plane.

The loads acting on an aircraft in steady flight are shown in Fig. below, with the following notation:

L is the lift acting at the aerodynamic centre of the wing.

D is the aircraft drag.

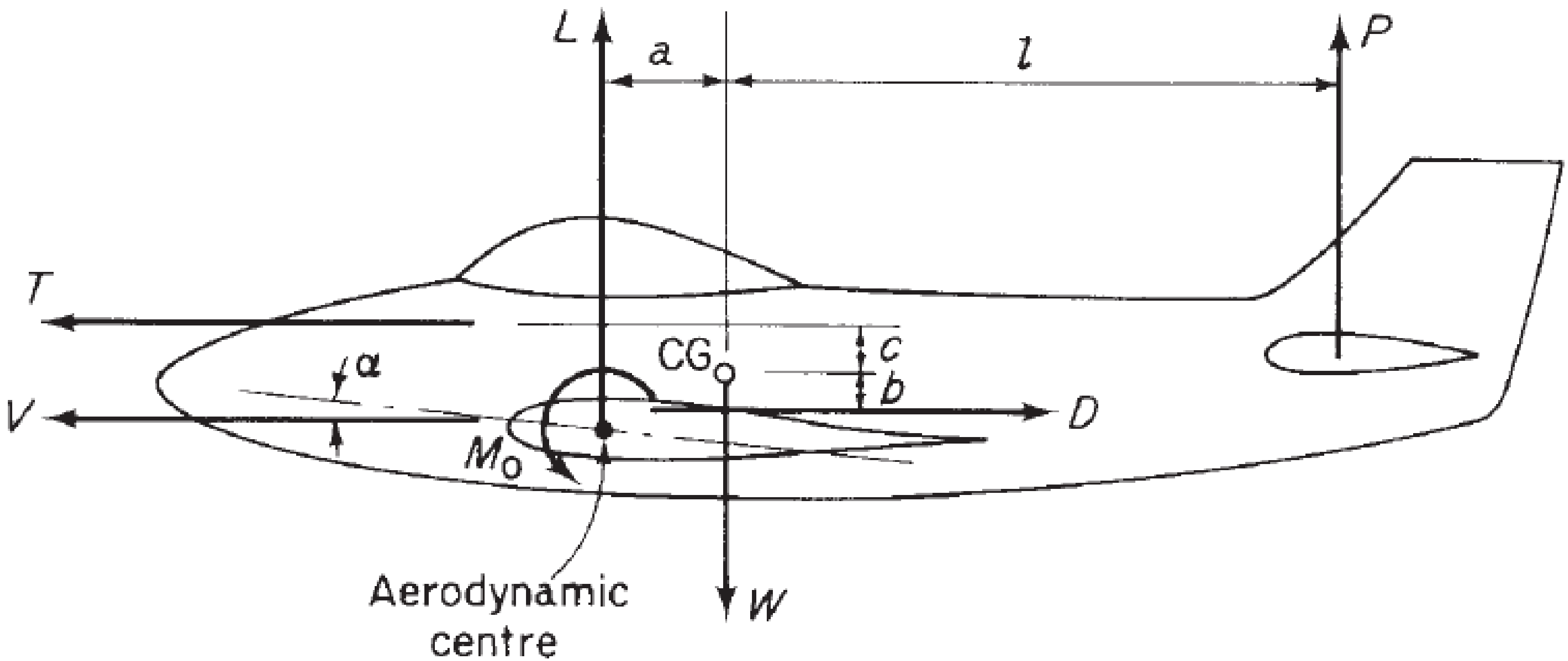
*M*0 is the aerodynamic pitching moment of the aircraft *less* its horizontal tail.

P is the horizontal tail load acting at the aerodynamic centre of the tail, usually taken to be at approximately one-third of the tailplane chord.

W is the aircraft weight acting at its CG.

T is the engine thrust, assumed here to act parallel to the direction of flight in order to simplify calculation.

Source from Aircraft Structures by T. H. G. Megson



Aircraft loads in level flight.

Source from Aircraft Structures by T. H. G. Megson

The loads are in static equilibrium since the aircraft is in a steady, unaccelerated, level flight condition. Thus for vertical equilibrium

$$L + P - W = 0$$

- $T - D = 0$

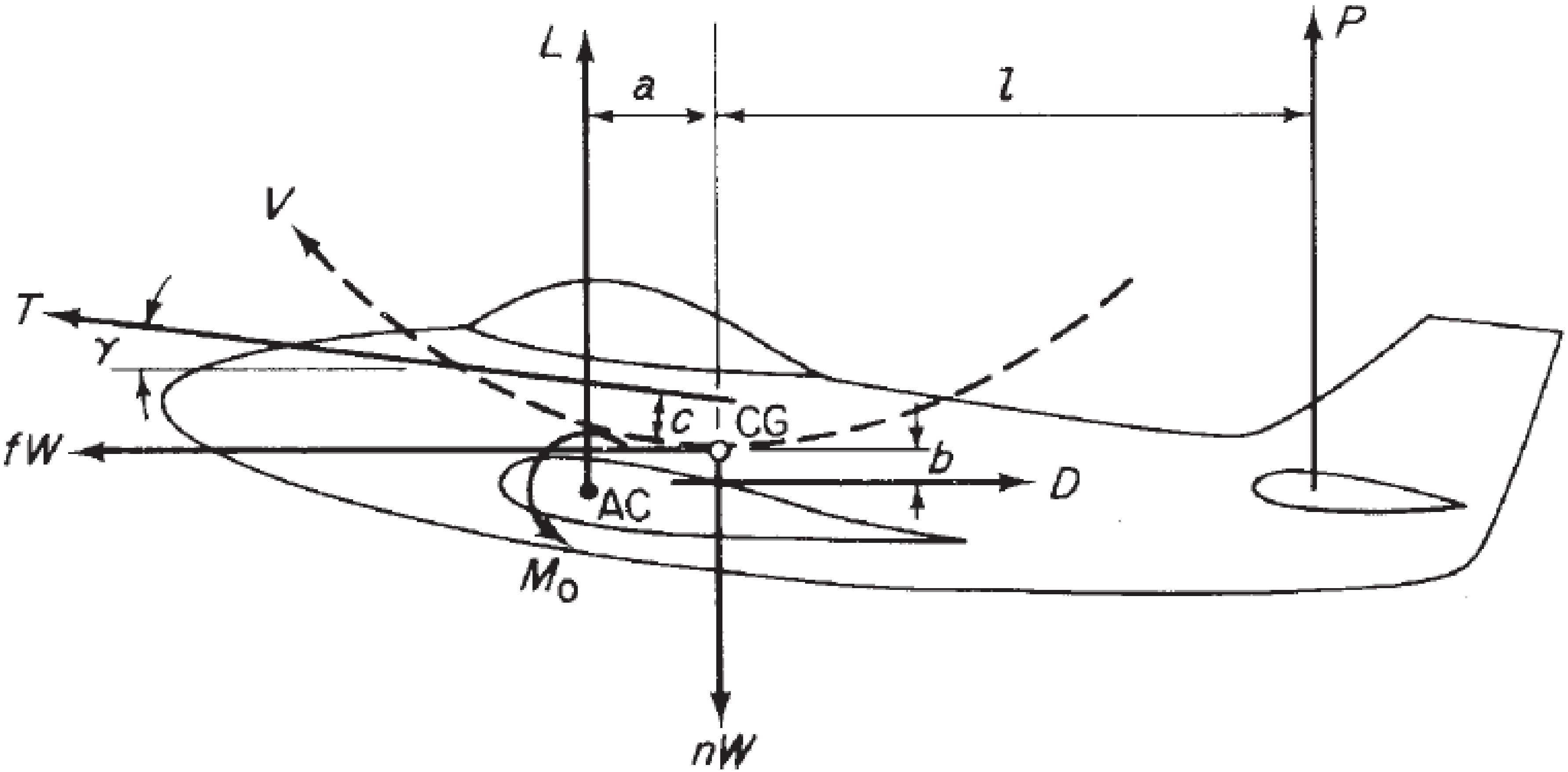
and taking moments about the aircraft's CG in the plane of symmetry

- $La - Db - Tc - M0 - Pl = 0$

As a first approximation we assume that the tail load P is small compared with the wing lift L so that, from , $L \approx W$. From aerodynamic theory with the usual

$$L = \frac{1}{2}\rho V^2 S C_{\mathrm{L}} \qquad \frac{1}{2}\rho V^2 S C_{\mathrm{L}} \approx W$$

Source from Aircraft Structures by T. H. G. Megson



Aircraft loads in a pull-out from a dive.

Source from Aircraft Structures by T. H. G. Megson

Monocoque and semi monocoque structures

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to provide an aerodynamic shape and to protect passengers, payload, systems, etc. from the environmental conditions encountered in flight.

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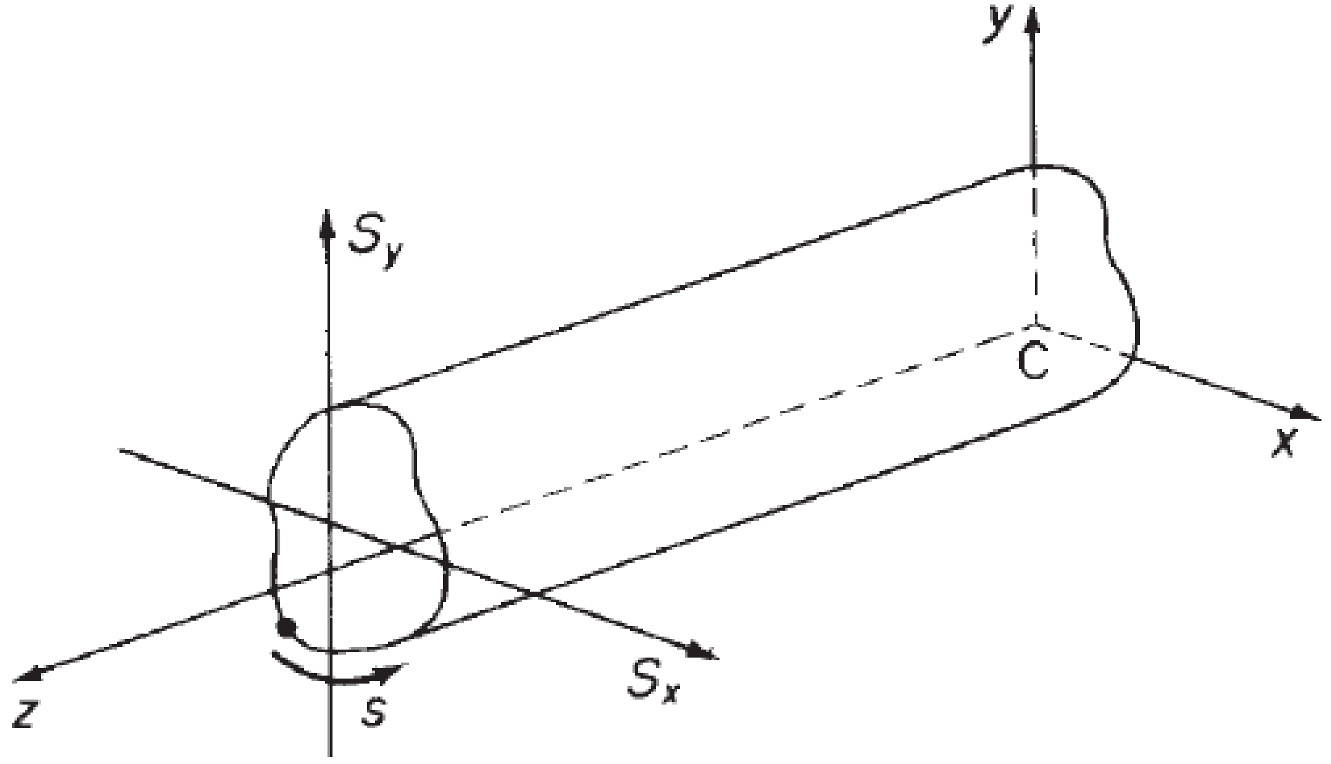

monocoque structures

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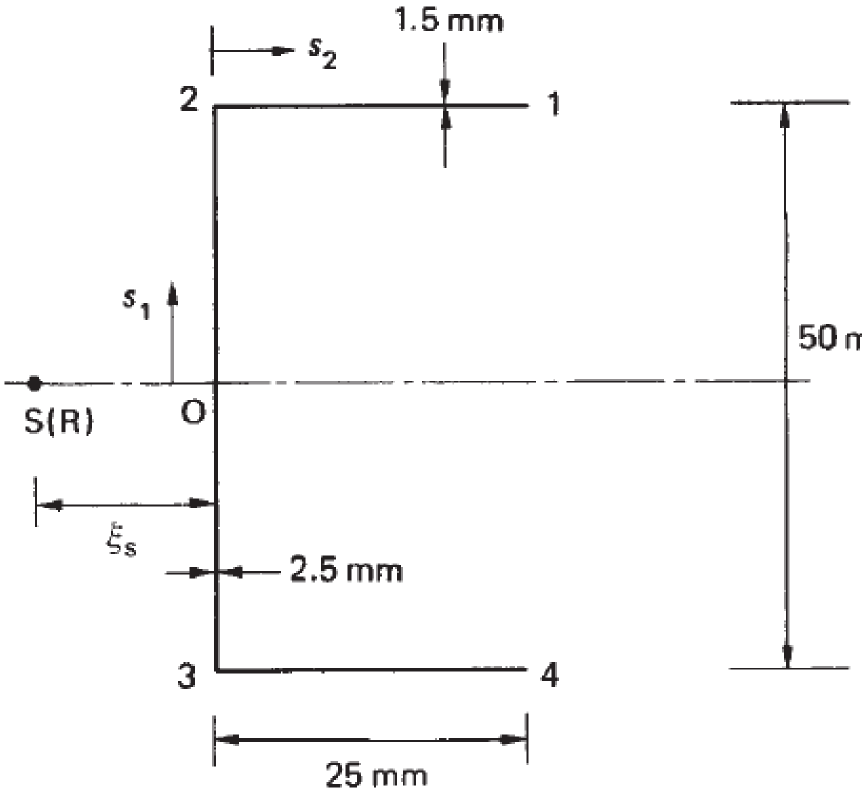

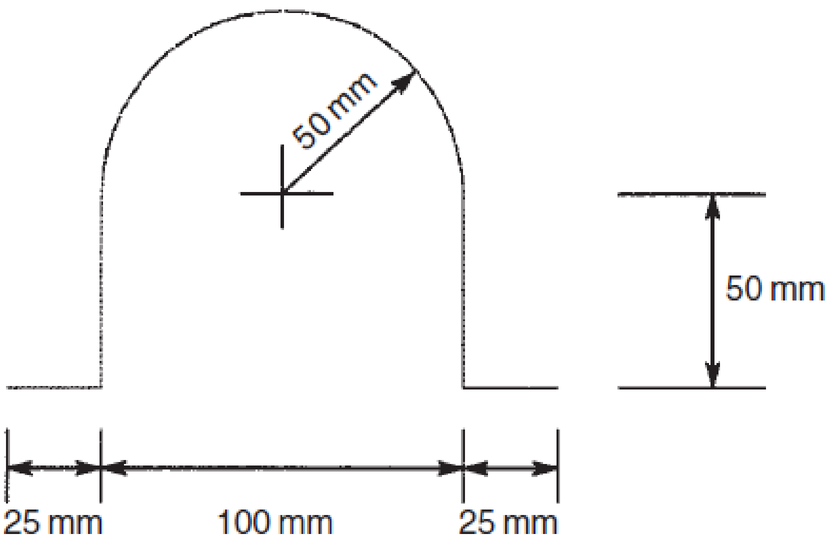

monocoque structures

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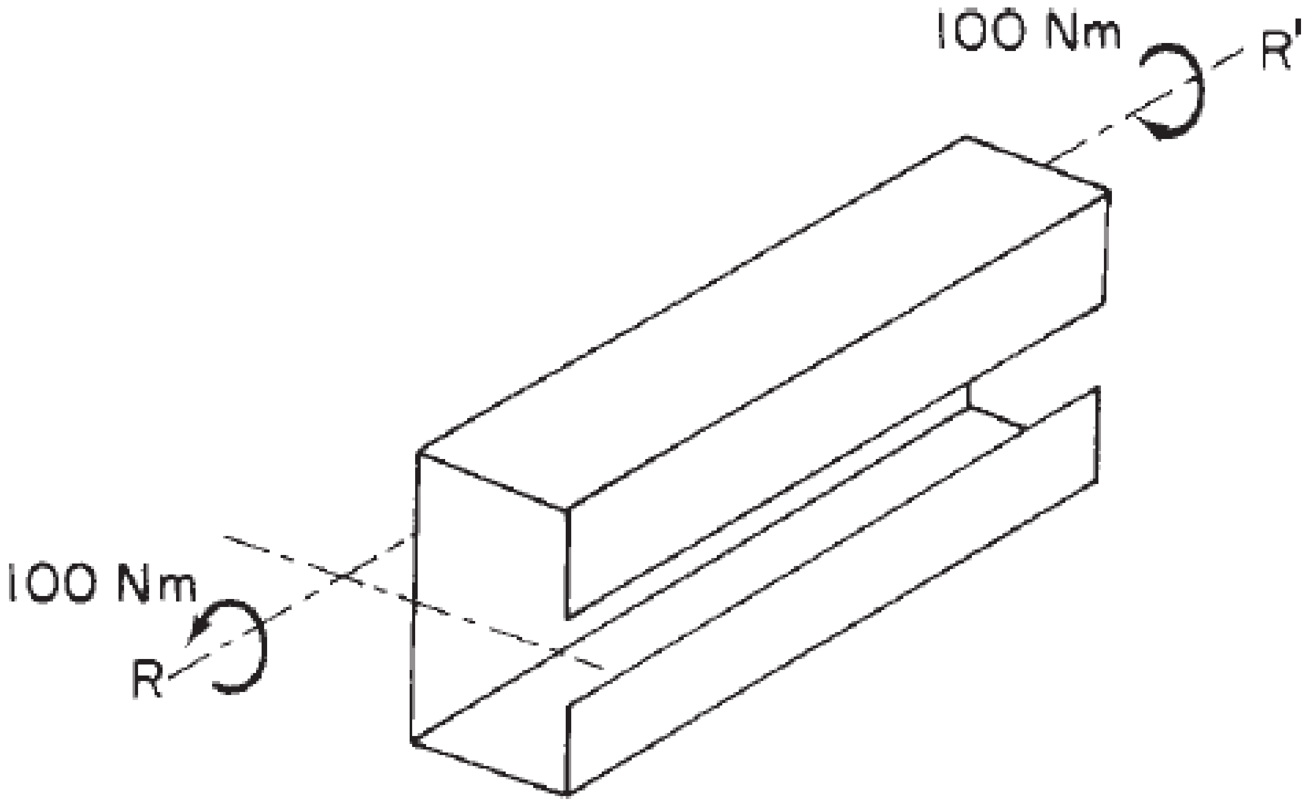

monocoque structures

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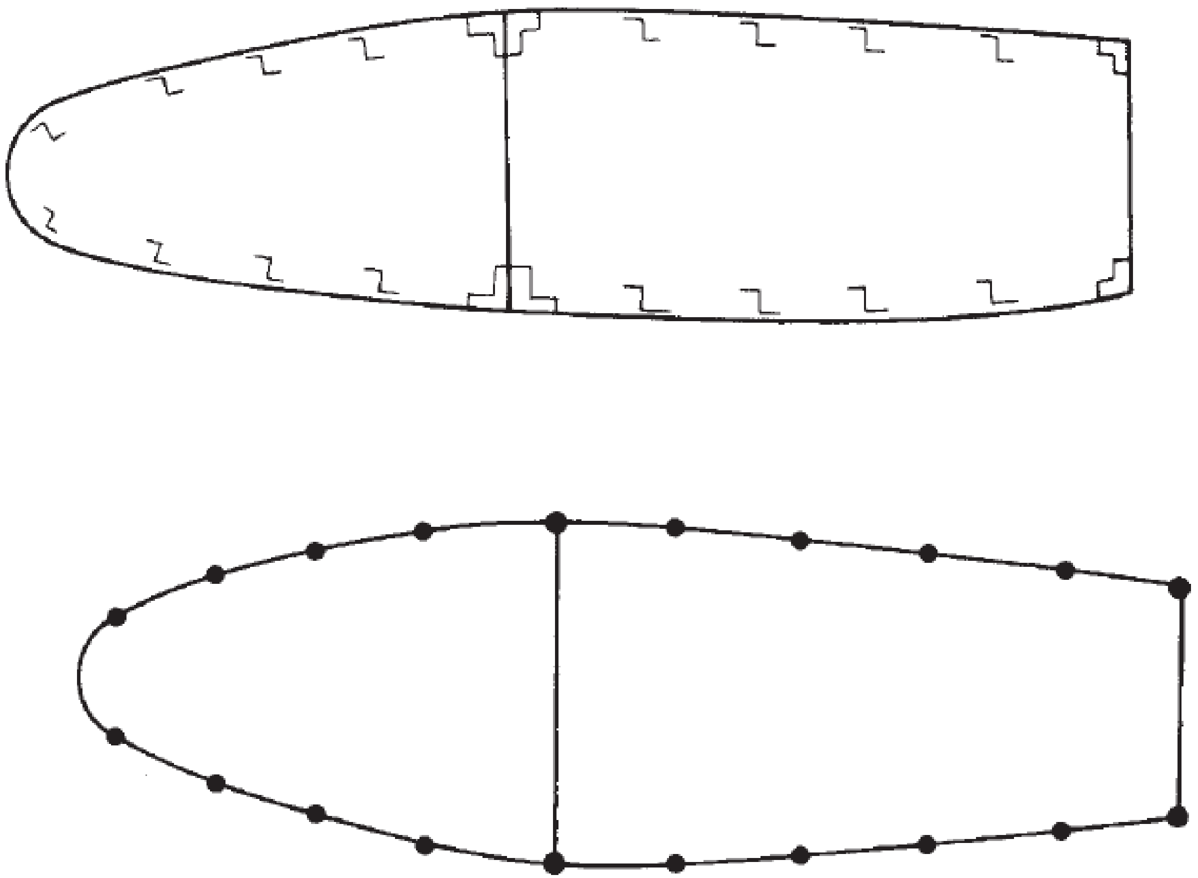

semi monocoque structures

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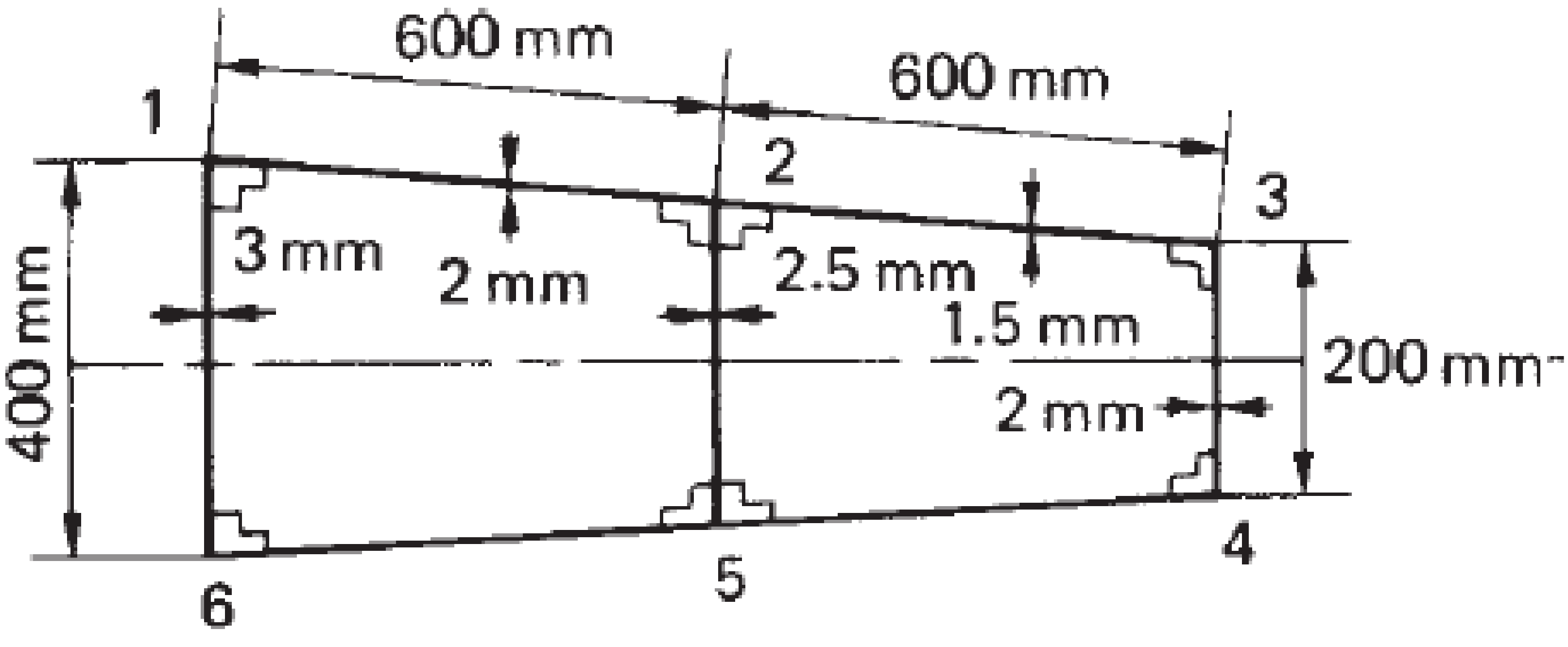

(a)

semi monocoque structures

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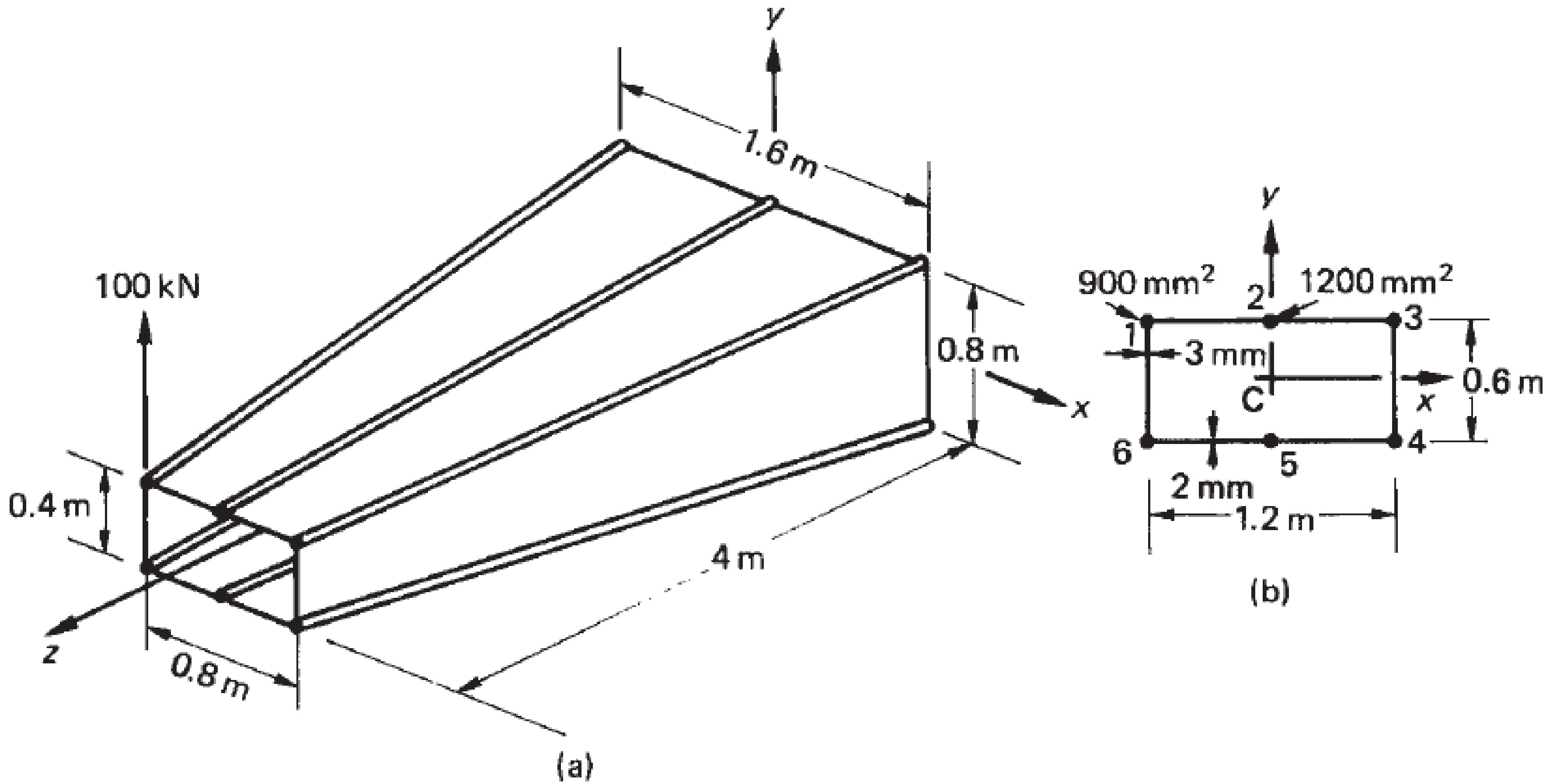

semi monocoque structures

Source from Aircraft Structures by T. H. G. Megson

THIN-WALLED PRESSURE VESSELS

- Cylindrical and spherical pressure vessels are commonly used for storing gas and liquids under pressure.

- A thin cylinder is normally defined as one in which the thickness of the metal is less than 1/20 of the diameter of the cylinder.

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THIN-WALLED PRESSURE VESSELS CONTD

- In thin cylinders, it can be assumed that the variation of stress within the metal is negligible, and that the mean diameter, D_m is approximately equal to the internal diameter, D.

- At mid-length, the walls are subjected to hoop or circumferential stress, and a longitudinal stress, .

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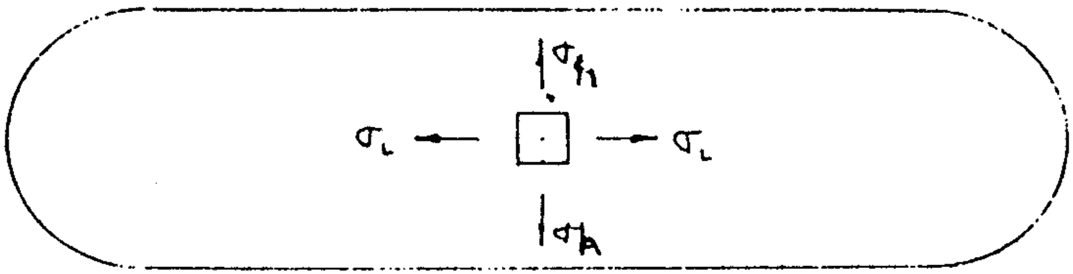
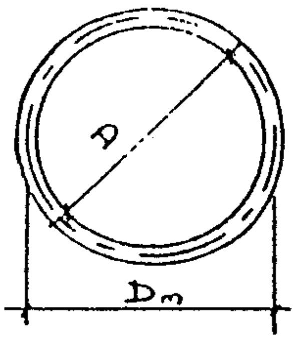
Hoop and Longitudinal Stress



(a)



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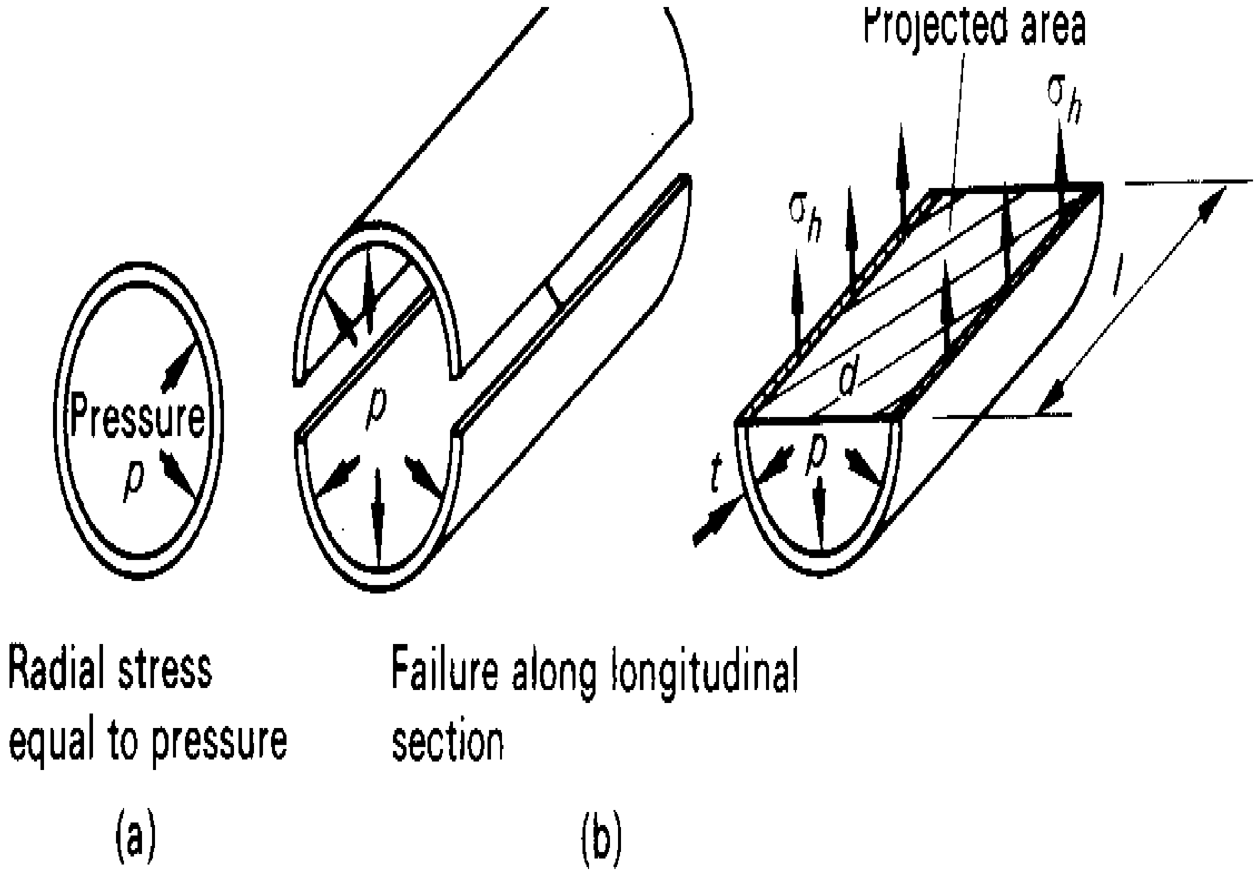
Hoop stress in thin cylindrical shell



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Hoop stress in thin cylindrical shell Contd.

- The internal pressure, p tends to increase the diameter of the cylinder and this produces a hoop or circumferential stress (tensile).

- If the stress becomes excessive, failure in the form of a longitudinal burst would occur.

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Hoop stress in thin cylindrical shell Concluded

Consider the half cylinder shown. Force due to internal pressure, p is balanced by the force due to hoop stress, σ_h .

i.e. hoop stress x area = pressure x projected area

$$\sigma_h \times 2 L t = P \times d \ L$$

$$\boxed{\sigma_h = (P\,d) / 2\,t}$$

Where: d is the internal diameter of cylinder; t is the thickness of wall of cylinder.

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Longitudinal stress in thin cylindrical shell

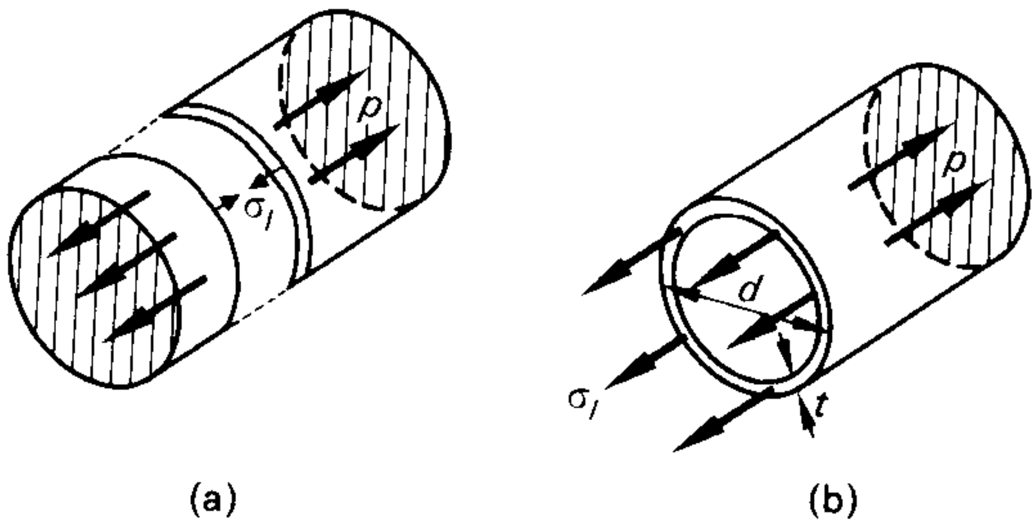

Fig. 3.15 Longitudinal stress in a thin cylindrical shell

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Longitudinal stress in thin cylindrical shell Contd.

The internal pressure, P also produces a tensile stress in longitudinal direction as shown above.

Force by P acting on an area $\frac{\pi d^2}{4}$ is balanced by

longitudinal stress, σ_L acting over an approximate area,

$\pi d t$ (mean diameter should strictly be used). That is:

$$\sigma_L \; x \, \pi \, d \, t = P \, x \, \frac{\pi d^2}{4}$$

$$\sigma_L = \frac{P d}{4 t}$$

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Note

- 1. Since hoop stress is twice longitudinal stress, the cylinder would fail by tearing along a line parallel to the axis, rather than on a section perpendicular to the axis.

- The equation for hoop stress is therefore used to determine the cylinder thickness.

- Allowance is made for this by dividing the thickness obtained in hoop stress equation by efficiency (i.e. tearing and shearing efficiency) of the joint.

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INTRODUCTION:

The thickness of the cylinder is large compared to that of thin cylinder.

i. e., in case of thick cylinders, the metal thickness ‘t’ is more than ‘d/20’, where ‘d’ is the internal diameter of the cylinder.

Magnitude of radial stress (p_r) is large and hence it cannot be neglected. The circumferential stress is also not uniform across the cylinder wall. The radial stress is compressive in nature and circumferential and longitudinal stresses are tensile in nature. Radial stress and circumferential stresses are computed by using ‘Lame’s equations’.

www.engineeringduniya.com/slide_folder/First %20Year/.../MCYLINDERS.ppt

LAME'S EQUATIONS (Theory) :

ASSUMPTIONS:

1. Plane sections of the cylinder normal to its axis remain plane and normal even under pressure.

2. Longitudinal stress (σ_L) and longitudinal strain (ε_L) remain constant throughout the thickness of the wall.

3. Since longitudinal stress (σ_L) and longitudinal strain (ε_L) are constant, it follows that the difference in the magnitude of hoop stress and radial stress (p_r) at any point on the cylinder wall is a constant.

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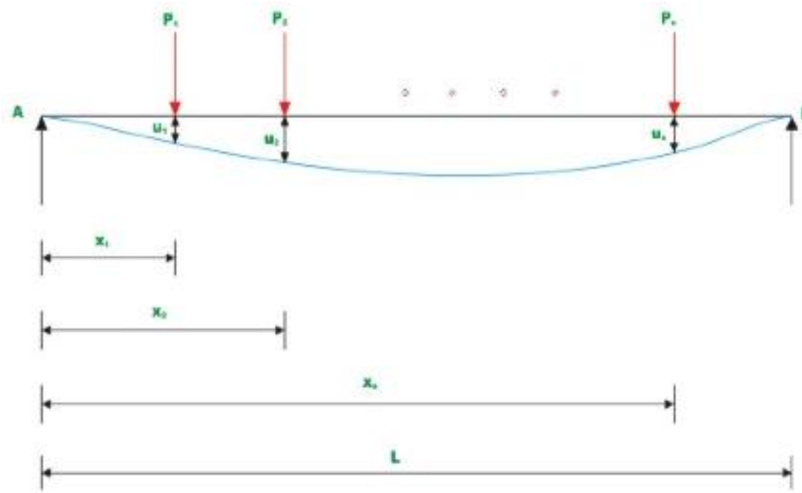
MODULE –II

THIN PLATE THEORY, STRUCTURAL INSTABILITY

Castigliono's First Theorem

- Let *P1, P2 ,...., Pn be the forces acting at x1 , x2 ,......, xn from the left end on a simply supported beam of span L .Let u1 , u2 ,..., un be the displacements at the loading P1, P2 ,...., Pn respectively as shown in figure.*



Source: https://www.slideshare.net/deepak_223/lecture-5-castiglionos-theorem

Now, assume that the material obeys Hooke's law and invoking the principle of superposition, the work done by the external forces is given by

$$W = \frac{1}{2}P_1u_1 + \frac{1}{2}P_2u_2 + \ldots. + \frac{1}{2}P_nu_n$$

Work done by external forces is stored in structure as strain energ€y.

$$U = \frac{1}{2}P_1u_1 + \frac{1}{2}P_2u_2 + \ldots. + \frac{1}{2}P_nu_n$$

https://www.slideshare.net/deepak_223/lectur e-5-castiglionos-theorem *u1 (deflection at point of application of P1) can be expressed*

As

$$u_1 = a_{11}P_1 + a_{12}P_2 + \ldots. + a_{1n}P_n$$

In general

$$u_1 = a_{i1}P_1 + a_{i2}P_2 + \ldots. + a_{in}P_n$$

aij= flexibility coeff at i due to unit force applied at j.

Work done by external forces is stored in structure as strain energy.

$$U = \frac{1}{2}P_1\left[a_{11}P_1 + a_{12}P_2 + ..\right] + \frac{1}{2}P_2\left[a_{21}P_1 + a_{22}P_2 + ..\right] + \ldots. + \frac{1}{2}P_n\left[a_{n1}P_1 + a_{n2}P_2 + ..\right]$$

https://www.slideshare.net/deepak_223/lectur e-5-castiglionos-theorem

- *In general*

$$a_{ji} = a_{ij}$$

$$U = \frac{1}{2}\left[a_{11}P_1^{\,2} + a_{22}P_2^{\,2} + .. + a_{nn}P_n^{\,2}\right] + \left[a_{12}P_1P_2 + a_{13}P_1P_3 + .. + a_{1n}P_1P_n\right]$$

- *Differentiating the strain energy with force P_1*

$$\frac{\partial U}{\partial P_1} = \left[a_{11}P_1 + a_{12}P_2 + .. + a_{1n}P_n\right]$$

- *This is nothing but displacement at the loading point*

$$\frac{\partial U}{\partial P_n} = u_n$$

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Castigliano's first theorem may be stated as the first partial derivative of the strain energy of the structure with respect to any particular force gives the displacement of the point of application of that force in the direction of its line of action.

$$\frac{\partial U}{\partial P_n} = u_n$$

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The reciprocal theorem

The reciprocal theorem is an exceptionally powerful method of analysis of linearly elastic structures and is accredited in turn to Maxwell, Betti and Rayleigh.

Before establish the theorem we first consider a useful property of linearly elastic systems resulting from the principle of superposition.

The principle enables us to express the deflection of any point in a structure in terms of a constant coefficient and the applied loads.

For example, a load *P*1 applied at a point 1 in a linearly elastic body will produce a deflection Δ_1 at the point given by

$$\Delta_1 = a_{11} P_1$$

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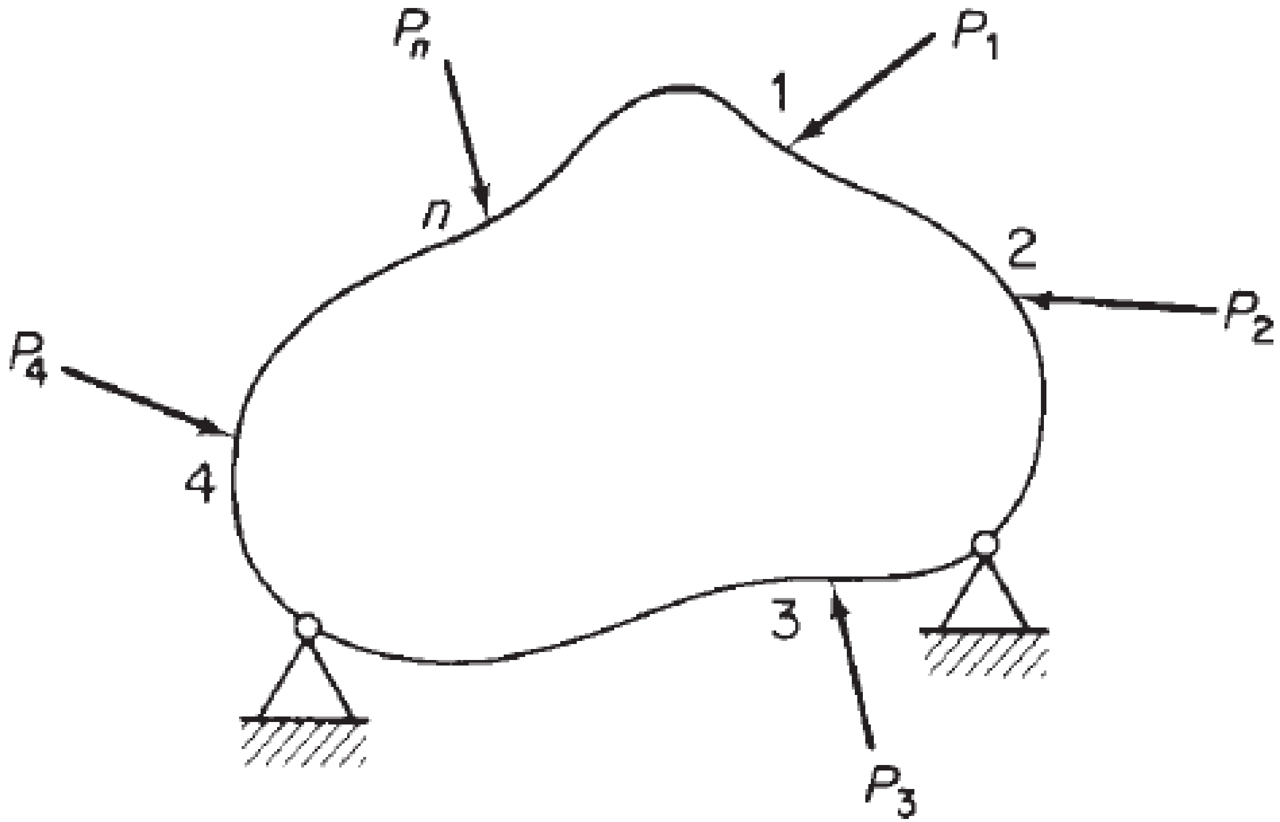

Linearly elastic body subjected to loads P_1, P_2, P_3,…, P_n.

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The *influence* or *flexibility* coeffcient a_{11} is defined as the deflection at the point 1 in the direction of P_1, produced by a unit load at the point 1 applied in the direction of P_1.

Clearly, if the body supports a system of loads such as those shown in Fig. each of the loads P_1, P_2, …, P_n will contribute to the deflection at the point 1.

Thus, the *corresponding deflection* Δ_1 at the point 1 is then

$$\Delta_1 = a_{11}P_1 + a_{12}P_2 + \cdots + a_{1n}P_n$$

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where $a12$ is the deflection at the point 1 in the direction of $P1$, produced by a unit load at the point 2 in the direction of the load $P2$ and so on.

The corresponding deflections at the points of application of the complete system of loads are then

$$\left.\begin{aligned}
\Delta_1 &= a_{11}P_1 + a_{12}P_2 + a_{13}P_3 + \cdots + a_{1n}P_n \\
\Delta_2 &= a_{21}P_1 + a_{22}P_2 + a_{23}P_3 + \cdots + a_{2n}P_n \\
\Delta_3 &= a_{31}P_1 + a_{32}P_2 + a_{33}P_3 + \cdots + a_{3n}P_n \\
&\vdots \\
\Delta_n &= a_{n1}P_1 + a_{n2}P_2 + a_{n3}P_3 + \cdots + a_{nn}P_n
\end{aligned}\right\}$$

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in matrix form

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_n \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \end{Bmatrix}$$

Source from Aircraft Structures by T. H. G. Megson

which may be written in shorthand matrix notation as

$$\{\Delta\} = [A]\{P\}$$

Suppose now that an elastic body is subjected to a gradually applied force *P*1 at a point 1 and then, while *P*1 remains in position, a force *P*2 is gradually applied at another point 2.

The total strain energy *U* of the body is given by

$$U_1 = \frac{P_1}{2}(a_{11}P_1) + \frac{P_2}{2}(a_{22}P_2) + P_1(a_{12}P_2)$$

Source from Aircraft Structures by T. H. G. Megson

The third term on the right-hand side of Eq. results from the additional work done by $P1$ as it is displaced through a further distance $a12$ $P2$ by the action of $P2$.

If we now remove the loads and apply $P2$ followed by $P1$ we have

$$U_2 = \frac{P_2}{2}(a_{22}P_2) + \frac{P_1}{2}(a_{11}P_1) + P_2(a_{21}P_1)$$

By the principle of superposition the strain energy stored is independent of the order in which the loads are applied. Hence

$$U_1 = U_2$$

$$a_{12} = a_{21}$$

Thus in its simplest form the reciprocal theorem states that:

The deflection at a point 1 in a given direction due to a unit load at a point 2 in a second direction is equal to the deflection at the point 2 in the second direction due to a unit load at the point 1 in the first direction.

In a similar manner, we derive the relationship between moments and rotations, thus: *The rotation at a point 1 due to a unit moment at a point 2 is equal to the rotation at the point 2 produced by a unit moment at the point 1.*

Finally, we have: *The rotation at a point 1 due to a unit load at a point 2 is numerically equal to the deflection at the point 2 in the direction of the unit load due to a unit moment at the point 1.*

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Unit load method

Discussed the *dummy* or *fictitious* load method of obtaining deflections of structures.

For a linearly elastic structure the method may be stream-lined as follows.

Consider the framework of Fig. in which we require, say, to find the vertical deflection of the point C.

place a vertical dummy load *P*f at C and write down the total complementary energy of the framework, i.e.

$$C = \sum_{i=1}^{k} \int_0^{F_i} \lambda_i \, \mathrm{d}F_i - \sum_{r=1}^{n} \Delta_r P_r$$

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For a stationary value of *C*

$$\frac{\partial C}{\partial P_{\mathrm{f}}} = \sum_{i=1}^{k} \lambda_i \frac{\partial F_i}{\partial P_{\mathrm{f}}} - \Delta_{\mathrm{C}} = 0$$

$$\Delta_C = \sum_{i=1}^{k} \lambda_i \frac{\partial F_i}{\partial P_{\mathrm{f}}} \quad \text{as before}$$

If instead of the arbitrary dummy load *P*f we had placed a unit load at C, then the load in the *i*th linearly elastic member would be

$$F_i = \frac{\partial F_i}{\partial P_{\mathrm{f}}} 1$$

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$$\Delta_{\mathrm{C}} = \sum_{i=1}^{k} \frac{F_{i,0}F_{i,1}L_i}{A_iE_i}$$

where $F_{i,0}$ is the force in the ith member due to the actual loading and $F_{i,1}$ is the force in the ith member due to a unit load placed at the position and in the direction of the required deflection.

Similar expressions for deflection due to bending and torsion of linear structures follow from the well-known relationships between bending and rotation and torsion and rotation.

Hence, for a member of length L and flexural and torsional rigidities EI and GJ, respectively

$$\Delta_{\mathrm{B.M}} = \int_L \frac{M_0M_1}{EI}\mathrm{d}z \quad \Delta_{\mathrm{T}} = \int_L \frac{T_0T_1}{GJ}\mathrm{d}z$$

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where $M0$ is the bending moment at any section produced by the actual loading and $M1$ is the bending moment at any section due to a unit load applied at the position and in the direction of the required deflection.

Similarly for torsion. Generally, shear deflections of slender beams are ignored but may be calculated when required for particular cases.

Of greater interest in aircraft structures is the calculation of the deflections produced by the large shear stresses experienced by thin-walled sections.

FE Modification of the Rayleigh-Ritz Method

- In the Rayleigh-Ritz method
  - A single trial function is applied throughout the entire region
  - Trial functions of increasing complexity are required to model all but the simplest problems
- The FE approach
  - uses comparatively simple trial functions that are applied piece-wise to parts of the region
  - These subsections of the region are then the finite elements

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FE Modification of the Rayleigh-Ritz Method

- Consider the problem of 1-D heat flow, the functional to be extremised is

$$\Pi(\phi)=\int_{\Omega}\left\{k\left(\frac{d\phi}{dx}\right)^{2}-Q(x)\phi\right\}dx-k\overline{\phi_r}\phi_r$$

  - where the integral over Ω corresponds to the length of the region and Neumann boundary conditions are specified at one end, Γ,of the region

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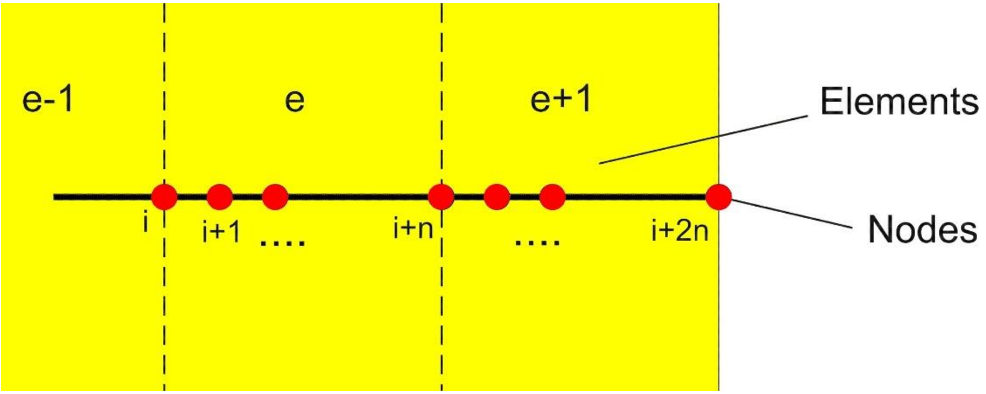
FE Modification of the Rayleigh-Ritz Method

- The length over which the solution is required, is divided up into finite elements



- In each element the value of ϕ is found at certain points called nodes
- Two nodes will mark the extremities of the element
- Other nodes may occur inside the element

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FE Modification of the Rayleigh-Ritz Method

- Let the unknown temperatures at the nodes of the element *e* be

$$(\phi)^e = \{\phi_i \quad \phi_{i+1} \dots \phi_{i+n}\}^T$$

$$\{\phi\}^e = \begin{Bmatrix} \phi_i \\ \phi_{i+1} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{i+n} \end{Bmatrix}$$

  - where *n+1* is the number of nodes in each element.

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FE Modification of the Rayleigh-Ritz Method

- The temperature at any other position in the element is represented in terms of the nodal values $\{\phi\}^e$ and shape functions associated with each node

$$\phi = \sum_{\beta} N_{\beta}\phi_{\beta} = [N]\{\phi\}^{e}$$

- where N_β is the shape function associated with the node β and $\beta = i \ldots i+n$ and $[N]$ is the corresponding row matrix.

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FE Modification of the Rayleigh-Ritz Method

- Let us write the trial function ϕ over the entire region $\wedge$ in the form

$$\phi = \sum_{\alpha} N_{\alpha}^{g} \phi_{\alpha}$$

  - where the summation is over all the nodes in Ω.

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FE Modification of the Rayleigh-Ritz Method

- The global shape functions N_α^g have been used to take into account the contribution from ϕ_α to ϕ over the entire region Ω
- The global shape functions over much of Ω will be zero
- For interior nodes of an element N_α^g will be non-zero only within that element
- End nodes of an element will have non-zero values over the two elements sharing the node.

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FE Modification of the Rayleigh-Ritz Method

For example :

- N_{i+n}^{g} is non-zero only in elements e and e+1.

- $N_{i+1}^{g},\ N_{i+2}^{g},\ \ldots .\ N_{i+n-1}^{g}$ will be non-zero only in element e.

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FE Modification of the Rayleigh-Ritz Method

- Neglecting for the moment, consideration of the first and last elements of the region
- Write the Rayleigh-Ritz statement in which the nodal values are the adjustable parameters.
- Consider the nodes *i...i+n* belonging to element *e*

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FE Modification of the Rayleigh-Ritz Method

$$\frac{\partial \Pi(\phi)}{\partial \phi_i} = \frac{\partial}{\partial \phi_i}\left[\int_{element\, e-1} + \int_{element\, e}\right] = 0$$

$$\frac{\partial \Pi(\phi)}{\partial \phi} = \frac{\partial}{\partial \phi}\left[\int_{element\, e}\right] = 0 : \beta = i+1 \ldots\ldots i+n-1$$

$$\frac{\partial \Pi(\phi)}{\partial \phi_{i+n}} = \frac{\partial}{\partial \phi_{i+n}}\left[\int_{element\, e} + \int_{element\, e+1}\right] = 0$$

where for example $\int_{element\ e}$ stands for

$$\int \left\{ k\left(\frac{d\phi}{dx}\right)^2 - Q(x)\phi \right\} dx$$ over the element *e*

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FE Modification of the Rayleigh-Ritz Method

- Since

$$\frac{\partial}{\partial \phi_i} \int_{element\, e-1}$$ is an expression involving $\{\phi\}^{e\text{-}1}$

$$\frac{\partial}{\partial \phi_i} \int_{element\, e}$$ involves $\{\phi\}^{e}$

and there is no relationship between $\{\phi\}^{e\text{-}1}$ and $\{\phi\}^{e}$,both expressions must be equal to zero

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FE Modification of the Rayleigh-Ritz Method

Let us

- focus on the terms containing an integral over the element *e*
- Drop the superscript *g* on the shape functions
- Suppose that the element extends from $x=x_e$ to $x=x_e+h$

- No loss in generality is incurred if we
  - Shift the origin to $x=x_e$
  - Take the element to extend rather from 0 to *h*

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FE Modification of the Rayleigh-Ritz Method

- The function can be written as,

$$\frac{\partial \Pi(\phi)}{\partial \phi_\alpha} = \frac{\partial}{\partial \phi_\alpha}\left\{\int_0^k \left\{\frac{k}{2}\left[\frac{d}{dx}\{N\}\{\phi\}^e\right]^2 - Q(x)\{N\}\{\phi\}^e\right\}dx\right\}$$

  - where $\alpha = i \ldots i+n$

  - Note that

$$\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left([N]\{\phi\}^e\right) = \left[\frac{dN_i}{dx}, \frac{dN_{i+1}}{dx} \ldots\ldots\right]\begin{Bmatrix}\phi_i \\ \cdot \\ \phi_{i+n}\end{Bmatrix} = \left[\frac{dN}{dx}\right]\{\phi\}^e$$

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FE Modification of the Rayleigh-Ritz Method

- Also, noting

$$\frac{\partial}{\partial \phi_\alpha}\left(\frac{\partial \phi}{\partial x}\right)^2 = 2\left(\frac{\partial \phi}{\partial x}\right)\left\{\frac{\partial}{\partial \phi_\alpha}\left(\frac{\partial \phi}{\partial x}\right)\right\} = 2\left\{\frac{dN}{dx}\right\}\{\phi\}^e \frac{dN_\alpha}{dx}$$

Since

$$\frac{\partial \phi}{\partial x} = \left\{\frac{dN}{dx}\right\}\{\phi\}^e$$

Hence

$$\frac{\partial}{\partial \phi_\alpha}\left(\frac{\partial \phi}{\partial x}\right) = \frac{dN_\alpha}{dx}$$

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FE Modification of the Rayleigh-Ritz Method

- So, differentiating under the integral sign, we have

$$\frac{\partial \Pi(\phi)}{\partial \phi_\alpha} = \int_0^h \left\{ k \left[\frac{dN}{dx} \right] \{\phi\}^e \left(\frac{dN_\alpha}{dx} \right) - Q(x) N_\alpha \right\} dx = 0$$

Hence

$$k \int_0^h \left(\frac{dN_\alpha}{dx} \right) \left\{ \frac{dN}{dx} \right\} \{\phi\}^e \, dx = \int_0^h Q(x) N_\alpha \, dx$$

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FE Modification of the Rayleigh-Ritz Method

- This equation is one in the set of *n+1* simultaneous equations obtained by letting α run through the values *i...i+n* :

$$\begin{bmatrix} k_{i,i} & k_{i,i+1} & . & . & k_{i,i+n} \\ - & k_{i+1,i+1} & . & . & k_{i+1,i+n} \\ - & - & . & . & . \\ - & - & - & . & . \\ - & - & - & - & k_{i+n,i+n} \end{bmatrix} \begin{bmatrix} \phi_i \\ \\ \\ \\ \phi_{i+n} \end{bmatrix} = \begin{bmatrix} F_i^e \\ \\ \\ \\ F_{i+n}^e \end{bmatrix}$$

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FE Modification of the Rayleigh-Ritz Method

- where

$$F_\alpha^e = \int_0^h Q(x) N_\alpha dx$$

and

$$k_{\alpha\beta} = k \int_0^t \left(\frac{\partial N}{\partial x}\right)_\alpha \left(\frac{\partial N}{\partial x}\right)_\beta dx$$

- In the end elements, where Neumann boundary conditions may have to be considered, there is an additional term

  - where $N_{\alpha r}$ is the value of N_α on the boundary Γ

$$\frac{\partial}{\partial \phi_\alpha}\left(k \overline{\phi}_r \phi_r\right) = k \overline{\phi}_r N_{\alpha,r}$$

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FE Modification of the Rayleigh-Ritz Method

- If there are two 2-noded elements, labelled *m* and *n*, with nodes *i*, i+*1* and *i+2,* assembly of the element matrices is as before. Then
  - for the first element *m*

$$\begin{bmatrix} k_{i,i}^{m} & k_{i,i+1}^{m} \\ k_{i+1,i}^{m} & k_{i+1,i+1}^{m} \end{bmatrix} \begin{Bmatrix} \phi_i \\ \phi_{i+1} \end{Bmatrix} = \begin{Bmatrix} F_i^{\,m} \\ F_{i+1}^{m} \end{Bmatrix}$$

  - and similarly for element *n*

$$\begin{bmatrix} k_{i+1,i+1}^{n} & k_{i+1,i+2}^{n} \\ k_{i+2,i+1}^{n} & k_{i+2,i+2}^{n} \end{bmatrix} \begin{Bmatrix} \phi_{i+1} \\ \phi_{i+2} \end{Bmatrix} = \begin{Bmatrix} F_{i+1}^{n} \\ F_{i+2}^{n} \end{Bmatrix}$$

FE Modification of the Rayleigh-Ritz Method

- By combining these two matrix equations

$$\begin{bmatrix} k_{i,i}^{m} & k_{i,i+1}^{m} & 0 \\ k_{i+1,i}^{m} & \left(k_{i+1,i+1}^{m} + k_{i+1,i+1}^{n}\right) & k_{i+1,i+2}^{n} \\ 0 & k_{i+2,i+1}^{n} & k_{i+2,i+2}^{n} \end{bmatrix} \begin{Bmatrix} \phi_i \\ \phi_{i+1} \\ \phi_{i+2} \end{Bmatrix} = \begin{Bmatrix} F_i^{m} \\ F_{i+1}^{m} + F_{i+1}^{n} \\ F_{i+2}^{n} \end{Bmatrix}$$

  - The global assembly matrix is built up in this way
  - The boundary conditions on the extreme elements are inserted
  - The set of equations is solved for the unknown values of ϕ

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Total potential energy Method

In the spring–mass system shown in its unstrained position in Fig. normally define the *potential energy* of the mass as the product of its weight, *Mg*, and its height, *h*, above some arbitrarily fixed datum.

In other words it possesses energy by virtue of its position. After deflection to an equilibrium state the mass has lost an amount of potential energy equal to M_{gy}.

Thus we may associate deflection with a loss of potential energy. Alternatively, we may argue that the gravitational force acting on the mass does work during its displacement, resulting in a loss of energy.

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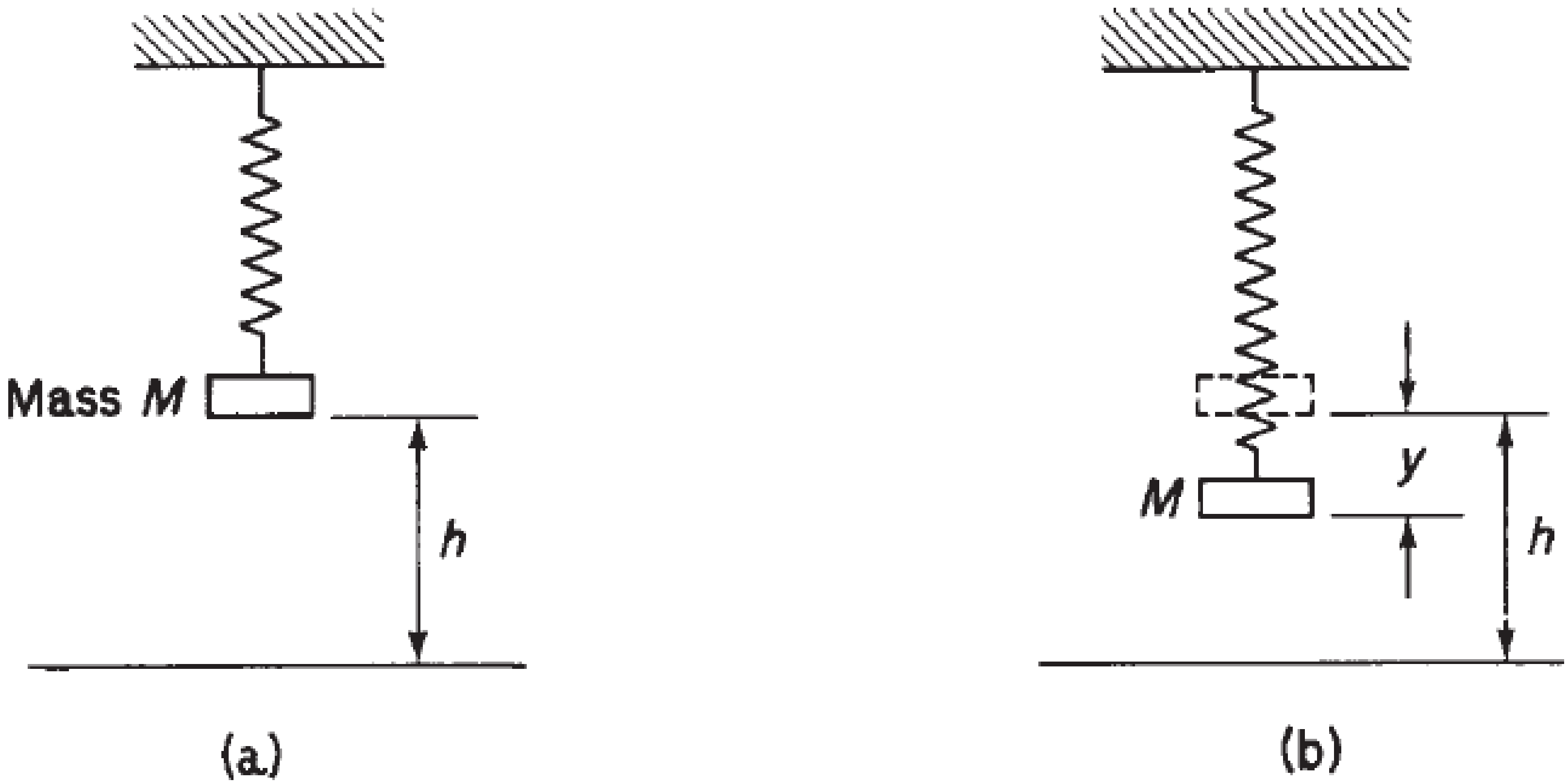
Applying this reasoning to the elastic system of Fig.

Assuming that the potential energy of the system is zero in the unloaded state, then the *loss* of potential energy of the load P as it produces a deflection y is Py.

The potential energy V of P in the deflected equilibrium state is given by

$$V = -Py$$



(a) Potential energy of a spring–mass system; (b) loss in potential energy due to change in position.

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We now define the *total potential energy* (TPE) of a system in its deflected equilibrium state as the sum of its internal or strain energy and the potential energy of the applied external forces.

for the single member–force configuration of Fig. (a)

$$\mathrm{TPE} = U + V = \int_0^y P \, \mathrm{d}y - Py$$

For a general system consisting of loads *P*1, *P*2, ..., *Pn* producing *corresponding displacements* 1, 2, ..., *n* the potential energy of all the loads is

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$$V = \sum_{r=1}^{n} V_r = \sum_{r=1}^{n} (-P_r \Delta_r)$$

and the total potential energy of the system is given by

$$\text{TPE} = U + V = U + \sum_{r=1}^{n} (-P_r \Delta_r)$$

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Flexibility method

An alternative approach to the solution of statically indeterminate beams and frames is to release the structure

i.e. remove redundant members or supports, until the structure becomes statically determinate.

The displacement of some point in the released structure is then determined by the unit load method.

The actual loads on the structure are removed and unknown forces applied to the points where the structure has been released;

the displacement at the point produced by these unknown forces must, from compatibility,

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be the same as that in the released structure. The unknown forces are then obtained; this approach is known as *the flexibility method*.

Pure bending of thin plates

The thin rectangular plate of Fig. is subjected to pure bending moments of intensity *Mx* and *My* per unit length uniformly distributed along its edges.

The former bending moment is applied along the edges parallel to the *y* axis, the latter along the edges parallel to the *x* axis.

Assume that these bending moments are positive when they produce compression at the upper surface of the plate and tension at the lower.

If we further assume that the displacement of the plate in a direction parallel to the *z* axis is small compared with its thickness *t* and that sections

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which are plane before bending remain plane after bending, then, as in the case of simple beam theory, the middle plane of the plate does not deform during the bending and is therefore a *neutral plane*.

Take the neutral plane as the reference plane for our system of axes.

Consider an element of the plate of side $\delta x \delta y$ and having a depth equal to the thickness t of the plate as shown in Fig.

Suppose that the radii of curvature of the neutral plane n are ρx and ρy in the xz and yz planes respectively Fig.

Positive curvature of the plate corresponds to the positive bending moments which produce displacements in the positive direction of the *z* or downward axis.

Again, as in simple beam theory, the direct strains εx and εy corresponding to direct stresses σx and σy of an elemental lamina of thickness δz a distance *z* below the neutral plane are given by

$$\varepsilon_x = \frac{z}{\rho_x} \quad \varepsilon_y = \frac{z}{\rho_y}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

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$$
\left.
\begin{aligned}
\sigma_x &= \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_x}+\frac{\nu}{\rho_y}\right) \\
\sigma_y &= \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_y}+\frac{\nu}{\rho_x}\right)
\end{aligned}
\right\}
$$

$$
M_x \delta y = \int_{-t/2}^{t/2} \sigma_x z \delta y \, \mathrm{d}z
$$

$$
M_y \delta x = \int_{-t/2}^{t/2} \sigma_y z \delta x \, \mathrm{d}z
$$

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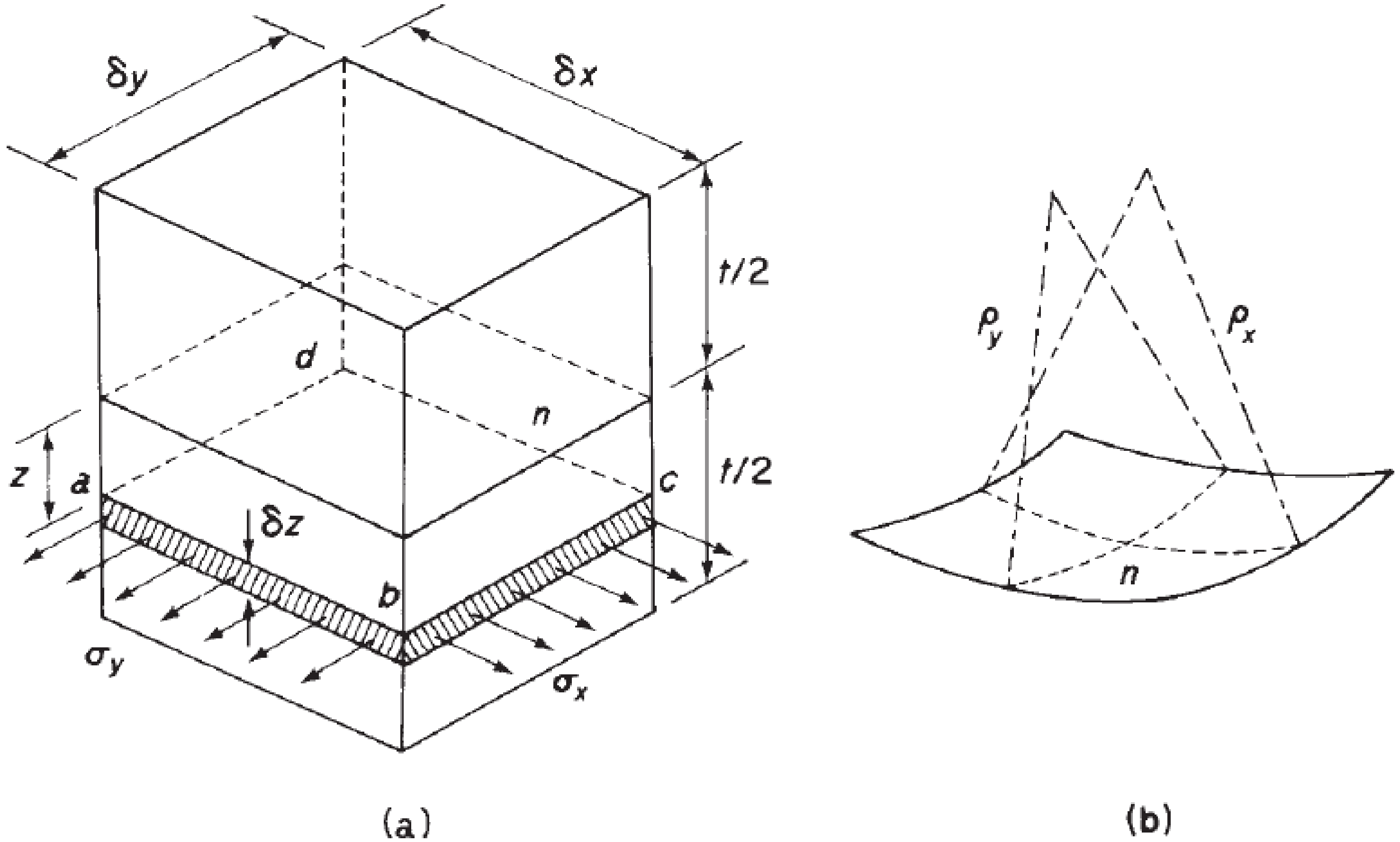

(a) Direct stress on lamina of plate element; (b) radii of curvature of neutral plane.

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$$M_x = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2}\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right) \mathrm{d}z$$

$$M_y = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2}\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right) \mathrm{d}z$$

$$D = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2} \mathrm{d}z = \frac{Et^3}{12(1-\nu^2)}$$

$$M_x = D\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right) \qquad M_y = D\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right)$$

Source from Aircraft Structures by T. H. G. Megson

D is known as the *flexural rigidity* of the plate.

If *w* is the deflection of any point on the plate in the *z* direction, then we may relate *w* to the curvature of the plate in the same manner as the well-known expression for beam curvature.

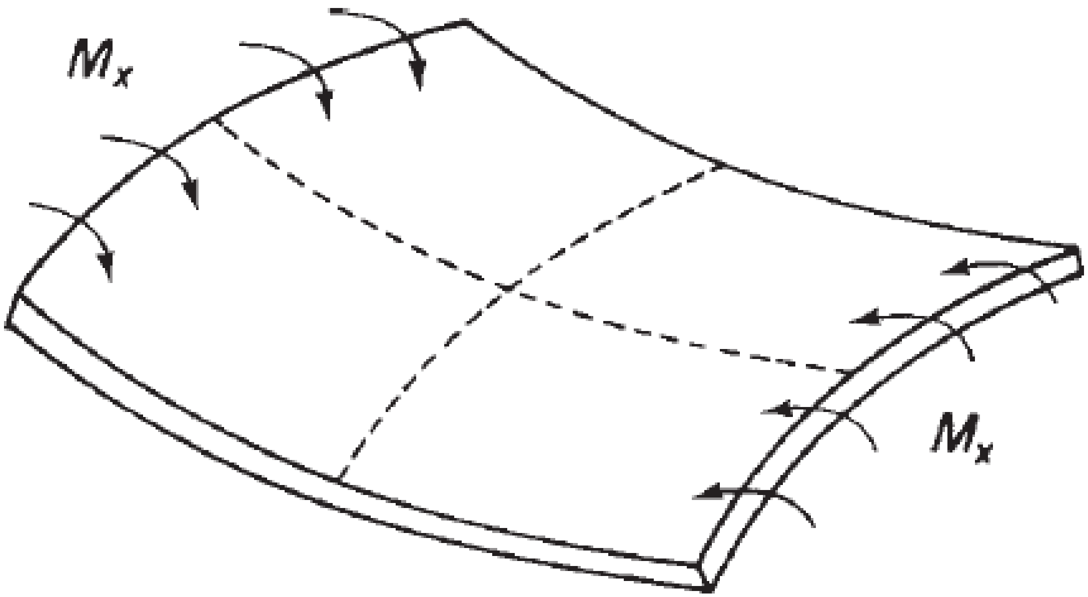
Hence

$$\frac{1}{\rho_x} = -\frac{\partial^2 w}{\partial x^2} \qquad \frac{1}{\rho_y} = -\frac{\partial^2 w}{\partial y^2}$$

The negative signs resulting from the fact that the centres of curvature occur above the plate in which region *z* is negative.

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

Source from Aircraft Structures by T. H. G. Megson



Anticlastic bending

Source from Aircraft Structures by T. H. G. Megson

The deflected shape of the plate provided that *Mx* and *My* are known.

If either *Mx* or *My* is zero then

$$\frac{\partial^2 w}{\partial x^2} = -\nu \frac{\partial^2 w}{\partial y^2}$$

$$\frac{\partial^2 w}{\partial y^2} = -\nu \frac{\partial^2 w}{\partial x^2}$$

The plate has curvatures of opposite signs. The case of *My* =0 is illustrated in Fig. A surface possessing two curvatures of opposite sign is known as an *anticlastic surface*, as opposed to a *synclastic surface* which has curvatures of the same sign. Further, if *Mx* =*My* =*M*

Source from Aircraft Structures by T. H. G. Megson

$$\frac{1}{\rho_x} = \frac{1}{\rho_y} = \frac{1}{\rho}$$

Therefore, the deformed shape of the plate is spherical and of curvature

$$\frac{1}{\rho} = \frac{M}{D(1 + \nu)}$$

Source from Aircraft Structures by T. H. G. Megson

Plates subjected to bending and twisting

Bending moments applied to the plate will not be in planes perpendicular to its edges.

Such bending moments, however, may be resolved in the normal manner into tangential and perpendicular components, as shown in Fig.

The perpendicular components are seen to be M_x and M_y as before, while the tangential components M_{xy} and M_{yx} (again these are moments per unit length) produce twisting of the plate about axes parallel to the x and y axes.

The system of suffixes and the sign convention for these twisting moments must be clearly understood to avoid confusion.

Source from Aircraft Structures by T. H. G. Megson

Mxy is a twisting moment intensity in a vertical *x* plane parallel to the *y* axis, while *Myx* is a twisting moment intensity in a vertical *y* plane parallel to the *x* axis.

Note that the first suffix gives the direction of the axis of the twisting moment.

Also define positive twisting moments as being clockwise when viewed along their axes in directions parallel to the positive directions of the corresponding *x* or *y* axis.

All moment intensities are positive.

Source from Aircraft Structures by T. H. G. Megson

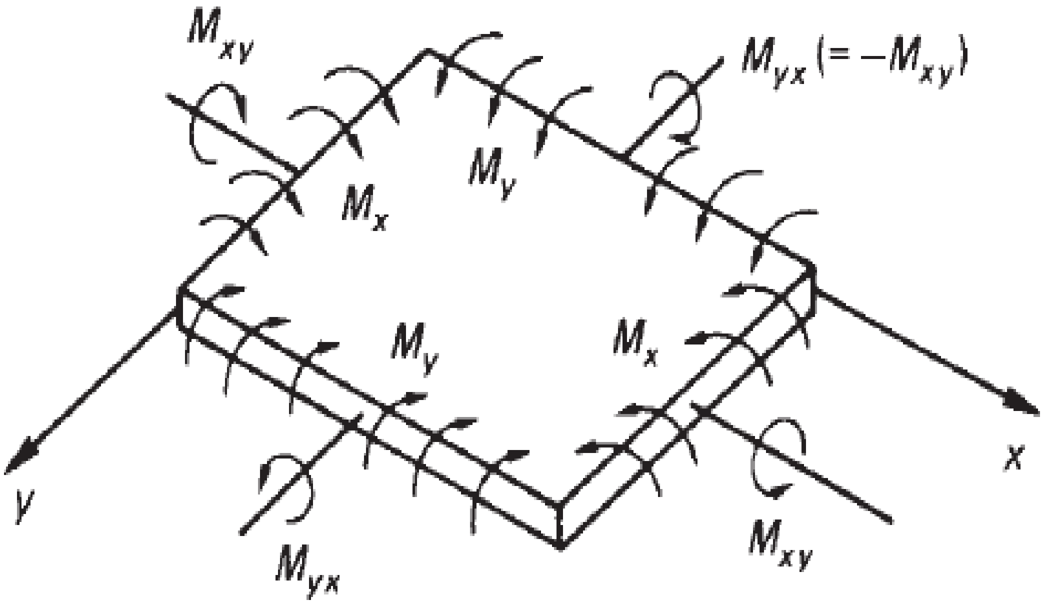
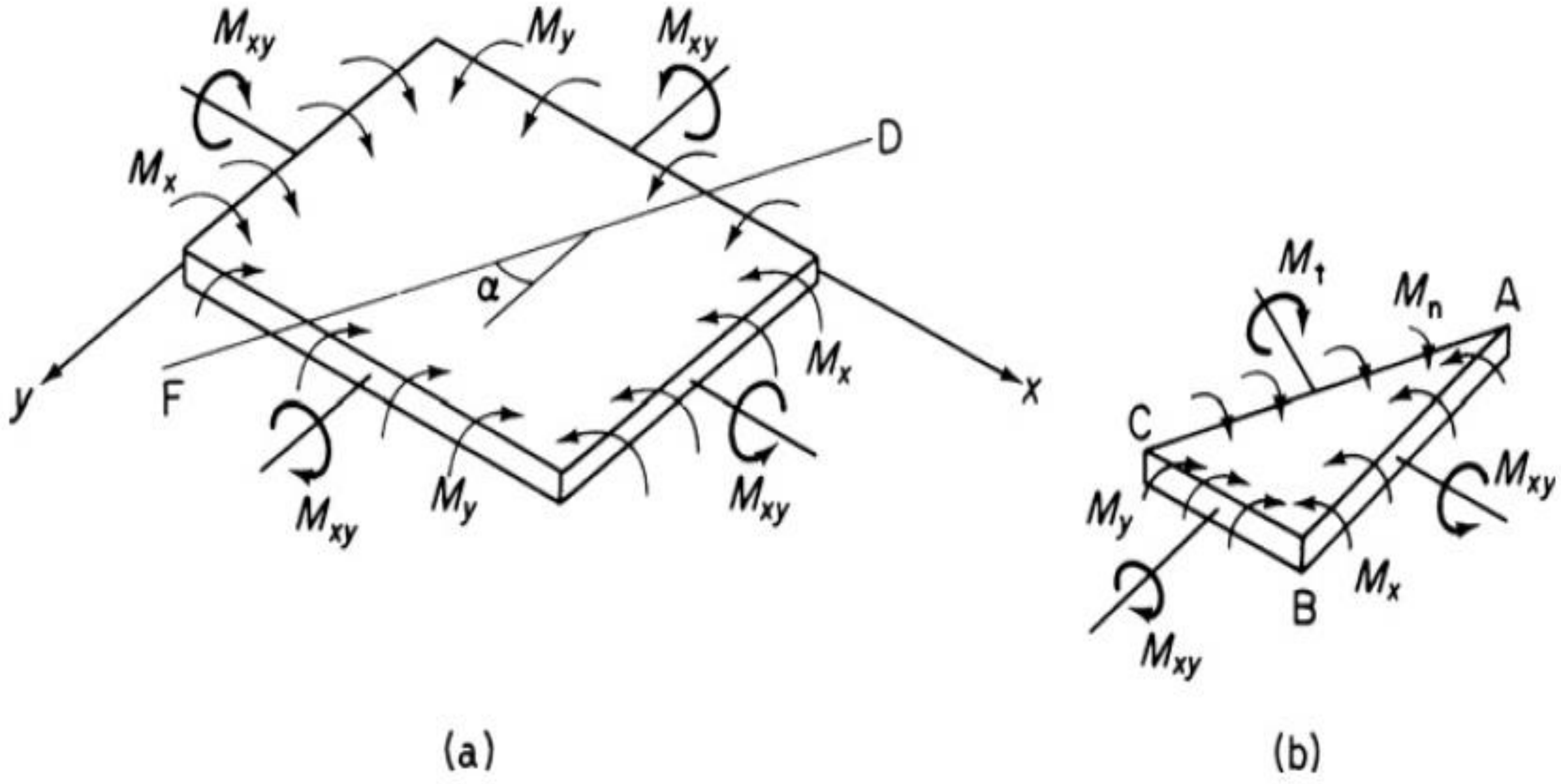


Plate subjected to bending and twisting

Source from Aircraft Structures by T. H. G. Megson



(a) Plate subjected to bending and twisting; (b) tangential and normal moments on an arbitrary plane

Source from Aircraft Structures by T. H. G. Megson

Since the twisting moments are tangential moments or torques they are resisted by a system of horizontal shear stresses τ_{xy}, as shown in Fig.

From a consideration of complementary shear stresses. $M_{xy}=-M_{yx}$, so that we may represent a general moment application to the plate in terms of M_x, M_y and M_{xy} as shown in Fig.

These moments produce tangential and normal moments, M_t and M_n, on an arbitrarily chosen diagonal plane FD.

Express these moment intensities in terms of M_x, M_y and M_{xy}. Thus, for equilibrium of the triangular element ABC of Fig. in a plane perpendicular to AC

$$M_{\mathrm{n}}\mathrm{AC} = M_x\mathrm{AB}\cos\alpha + M_y\mathrm{BC}\sin\alpha - M_{xy}\mathrm{AB}\sin\alpha - M_{xy}\mathrm{BC}\cos\alpha$$

$$M_{\mathrm{n}} = M_x\cos^2\alpha + M_y\sin^2\alpha - M_{xy}\sin 2\alpha$$

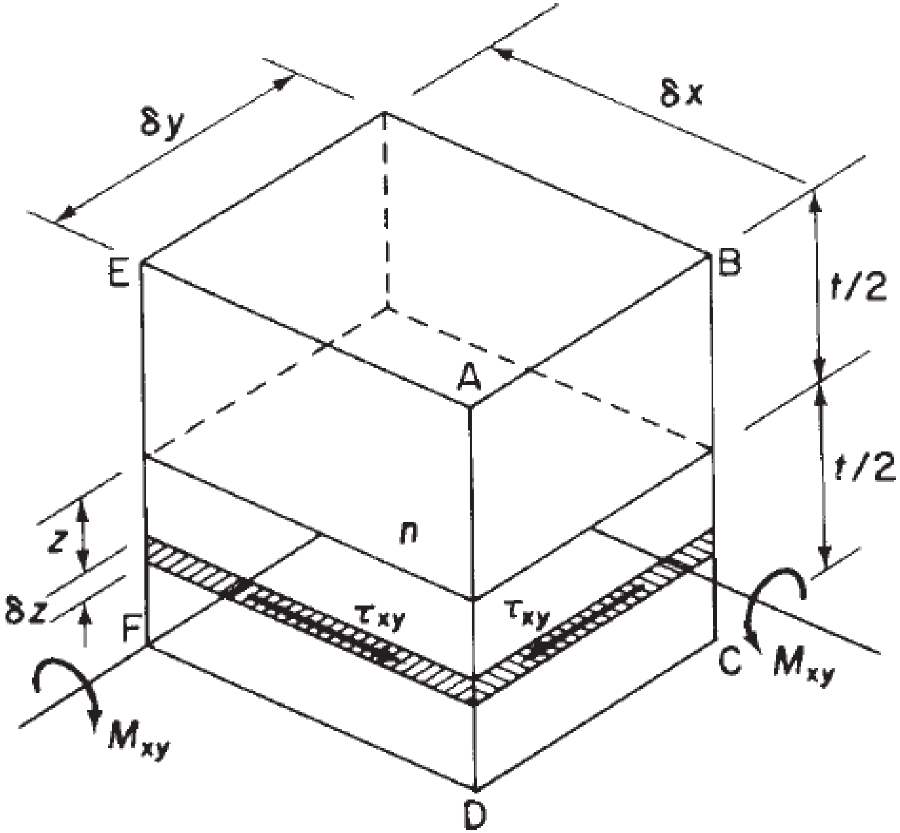
Similarly for equilibrium in a plane parallel to CA

$$M_{\mathrm{t}}\mathrm{AC} = M_x\mathrm{AB}\sin\alpha - M_y\mathrm{BC}\cos\alpha + M_{xy}\mathrm{AB}\cos\alpha - M_{xy}\mathrm{BC}\sin\alpha$$

$$M_{\mathrm{t}} = \frac{(M_x - M_y)}{2}\sin 2\alpha + M_{xy}\cos 2\alpha$$

$$\tan 2\alpha = -\frac{2M_{xy}}{M_x - M_y}$$

Source from Aircraft Structures by T. H. G. Megson



Complementary shear stresses due to twisting moments M_{xy}.

Source from Aircraft Structures by T. H. G. Megson

$$M_{xy}\delta y = -\int_{-t/2}^{t/2} \tau_{xy}\delta y z \,\mathrm{d}z$$

$$M_{xy}\delta x = -\int_{-t/2}^{t/2} \tau_{xy}\delta x z \,\mathrm{d}z$$

$$M_{xy} = -\int_{-t/2}^{t/2} \tau_{xy} z \,\mathrm{d}z$$

$$M_{xy} = -G\int_{-t/2}^{t/2} \gamma_{xy} z \,\mathrm{d}z$$

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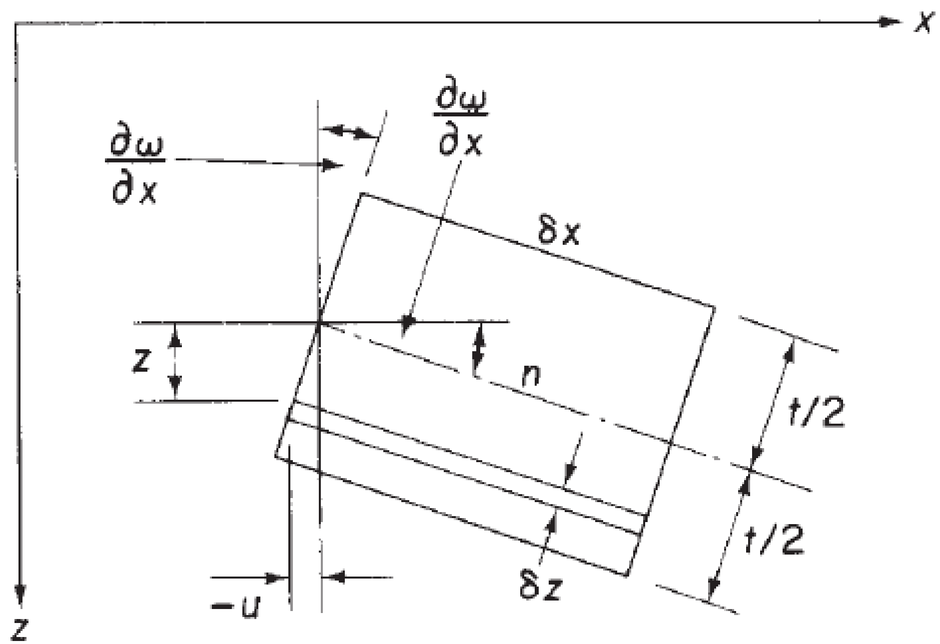
$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$



Determination of shear strain *γxy*.

Source from Aircraft Structures by T. H. G. Megson

$$u = -\frac{\partial w}{\partial x} z$$

$$v = -\frac{\partial w}{\partial y} z$$

$$\gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y}$$

$$M_{xy} = G \int_{-t/2}^{t/2} 2z^2 \frac{\partial^2 w}{\partial x \partial y} \mathrm{d}z$$

$$M_{xy} = \frac{Gt^3}{6} \frac{\partial^2 w}{\partial x \partial y}$$

Source from Aircraft Structures by T. H. G. Megson

Replacing G by the expression $E/2(1+\nu)$

$$M_{xy} = \frac{Et^3}{12(1+\nu)} \frac{\partial^2 w}{\partial x \partial y}$$

Multiplying the numerator and denominator of this equation by the factor $(1-\nu)$ yields

$$M_{xy} = D(1-\nu) \frac{\partial^2 w}{\partial x \partial y}$$

Above eqs. relate the bending and twisting moments to the plate deflection and are analogous to the bending moment-curvature relationship for a simple beam.

Source from Aircraft Structures by T. H. G. Megson

Plates subjected to a distributed transverse load

The relationships between bending and twisting moments and plate deflection are now employed in establishing the general differential equation for the solution of a thin rectangular plate, supporting a distributed transverse load of intensity *q* per unit area .

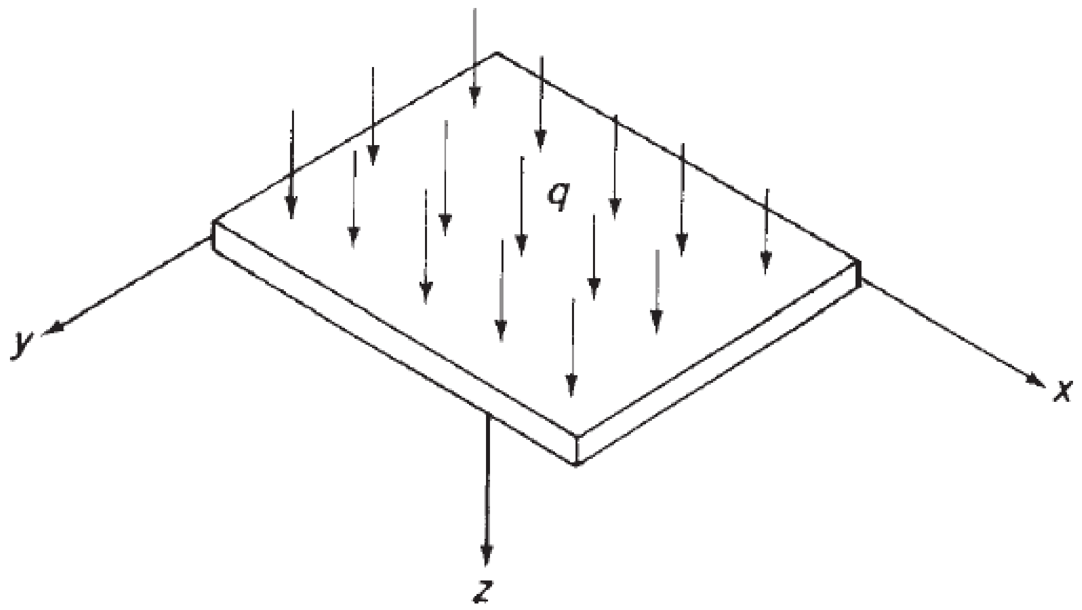


Plate supporting a distributed transverse load.

Source from Aircraft Structures by T. H. G. Megson

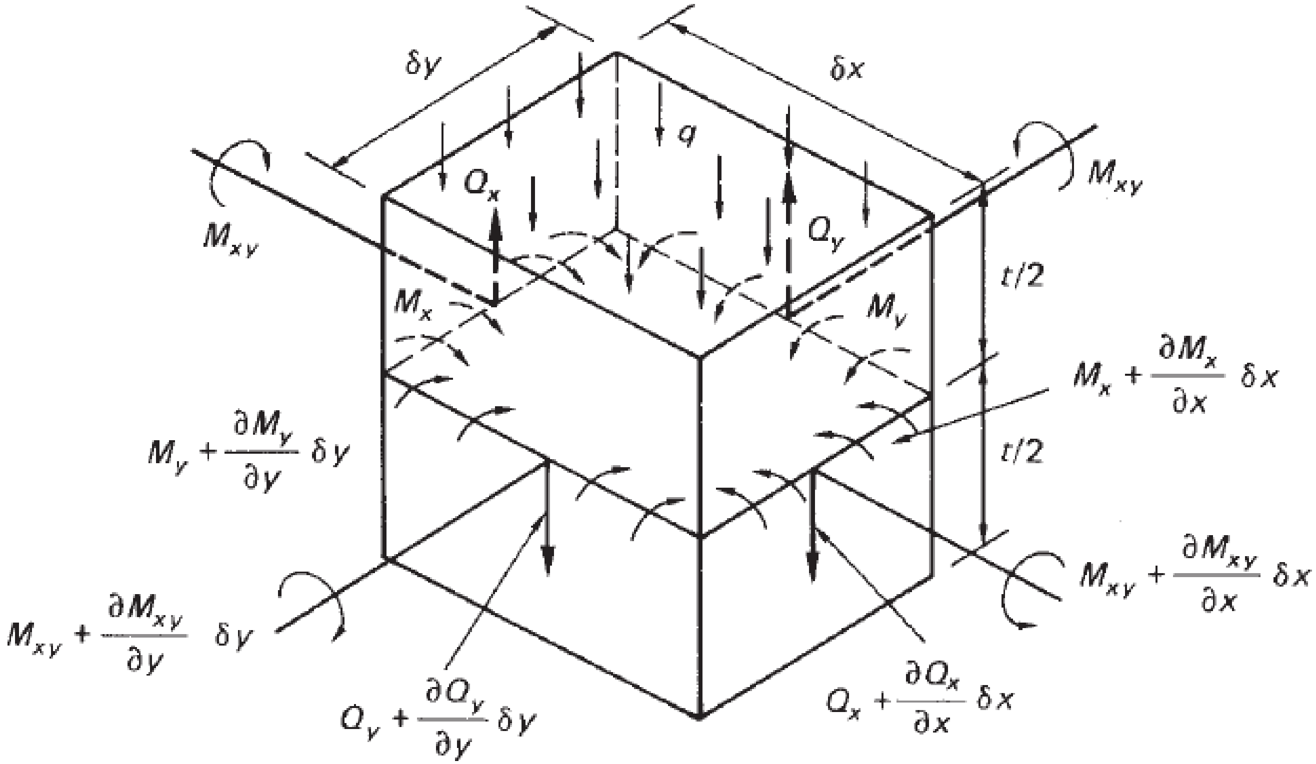


Plate element subjected to bending, twisting and transverse loads.

Source from Aircraft Structures by T. H. G. Megson

shear forces $Q_x\delta y$ and $Q_y\delta x$ are assumed to act through the centroid of the faces of the element. From the previous sections

$$M_x = \int_{-t/2}^{t/2} \sigma_x z \, \mathrm{d}z \quad M_y = \int_{-t/2}^{t/2} \sigma_y z \, \mathrm{d}z \quad M_{xy} = (-M_{yx}) = -\int_{-t/2}^{t/2} \tau_{xy} z \, \mathrm{d}z$$

$$Q_x = \int_{-t/2}^{t/2} \tau_{xz} \, \mathrm{d}z \quad Q_y = \int_{-t/2}^{t/2} \tau_{yz} \, \mathrm{d}z$$

For equilibrium of the element parallel to O*z* and assuming that the weight of the plate is included in *q*

$$\left(Q_x + \frac{\partial Q_x}{\partial x}\delta x\right)\delta y - Q_x\delta y + \left(Q_y + \frac{\partial Q_y}{\partial y}\delta y\right)\delta x - Q_y\delta x + q\delta x\delta y = 0$$

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$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0$$

$$M_{xy}\delta y - \left(M_{xy} + \frac{\partial M_{xy}}{\partial x}\delta x\right)\delta y - M_y\delta x + \left(M_y + \frac{\partial M_y}{\partial y}\delta y\right)\delta x$$
$$- \left(Q_y + \frac{\partial Q_y}{\partial y}\delta y\right)\delta x\delta y + Q_x\frac{\delta y^2}{2} - \left(Q_x + \frac{\partial Q_x}{\partial x}\delta x\right)\frac{\delta y^2}{2} - q\delta x\frac{\delta y^2}{2} = 0$$

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0$$

$$\frac{\partial M_{xy}}{\partial y} - \frac{\partial M_x}{\partial x} + Q_x = 0$$

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$$\frac{\partial^2 M_x}{\partial x^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = -q$$

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{q}{D}$$

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$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w=\frac{q}{D}$$

The operator $(\partial^2/\partial x^2+\partial^2/\partial y^2)$ is the well-known Laplace operator in two dimensions and is sometimes written as ∇^2. Thus

$$(\nabla^2)^2 w=\frac{q}{D}$$

$$Q_x=\frac{\partial M_x}{\partial x}-\frac{\partial M_{xy}}{\partial y}=-D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y=\frac{\partial M_y}{\partial y}-\frac{\partial M_{xy}}{\partial x}=-D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)$$

Direct and shear stresses are then calculated from the relevant expressions relating them to *Mx*, *My*, *Mxy*, *Qx* and *Qy*.

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Combined bending and in-plane loading of a thin rectangular plate

So far our discussion has been limited to small deflections of thin plates produced by different forms of transverse loading.

In these cases we assumed that the middle or neutral plane of the plate remained unstressed.

Additional in-plane tensile, compressive or shear loads will produce stresses in the middle plane, and these, if of sufficient magnitude, will affect the bending of the plate.

Where the in-plane stresses are small compared with the critical buckling stresses it is sufficient to consider the two systems separately; the total stresses are then obtained by superposition.

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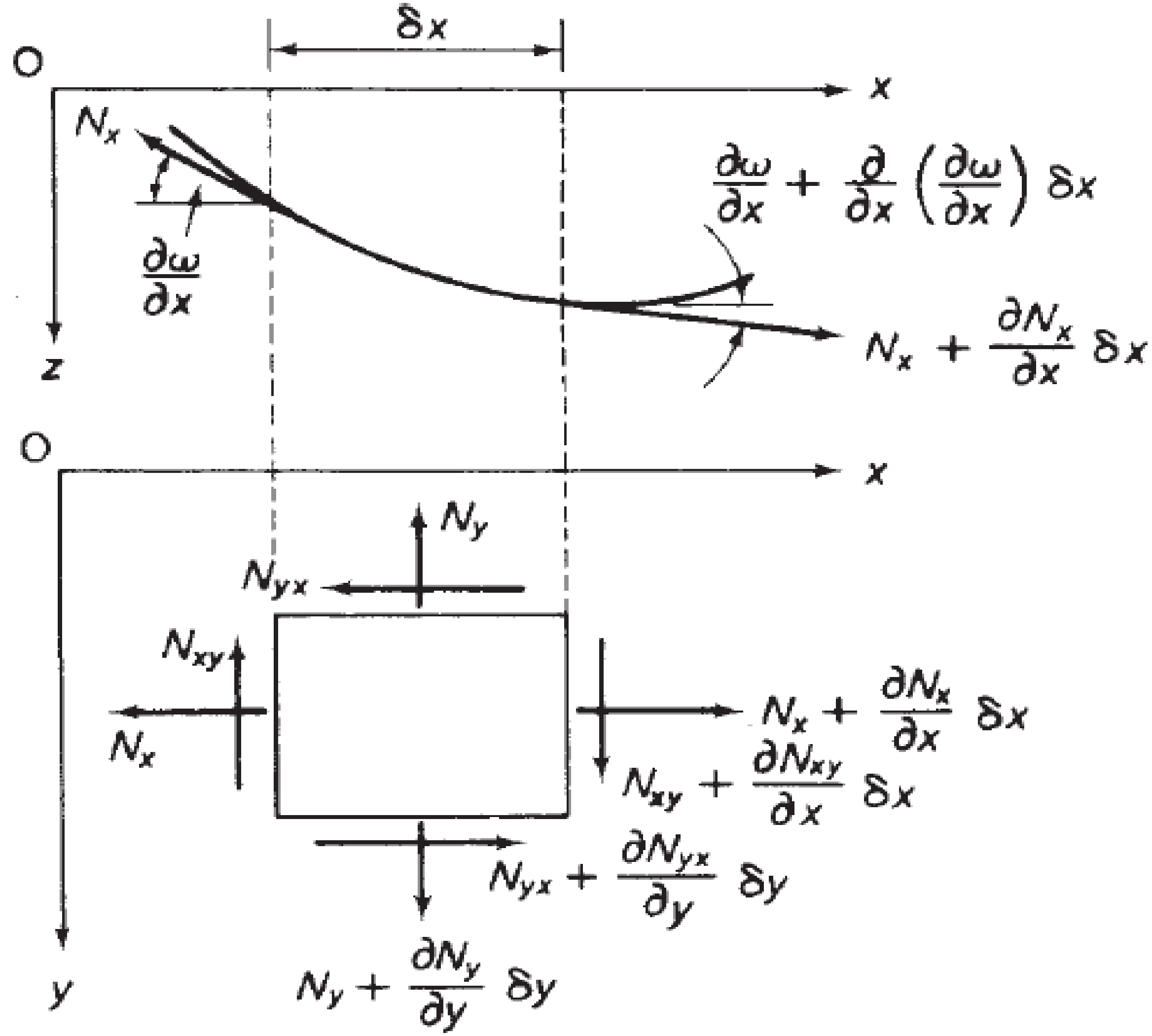

In-plane forces on plate element

Source from Aircraft Structures by T. H. G. Megson

On the other hand, if the in-plane stresses are not small then their effect on the bending of the plate must be considered.

$$\left(N_x + \frac{\partial N_x}{\partial x}\delta x\right)\delta y \cos\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\delta x\right) - N_x \delta y \cos\frac{\partial w}{\partial x}$$
$$+ \left(N_{yx} + \frac{\partial N_{yx}}{\partial y}\delta y\right)\delta x - N_{yx}\delta x = 0$$

For small deflections $\partial w/\partial x$ and $(\partial w/\partial x)+(\partial^2 w/\partial x^2)\delta x$ are small and the cosines of these angles are therefore approximately equal to one. The equilibrium equation thus simplifies to

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = 0$$

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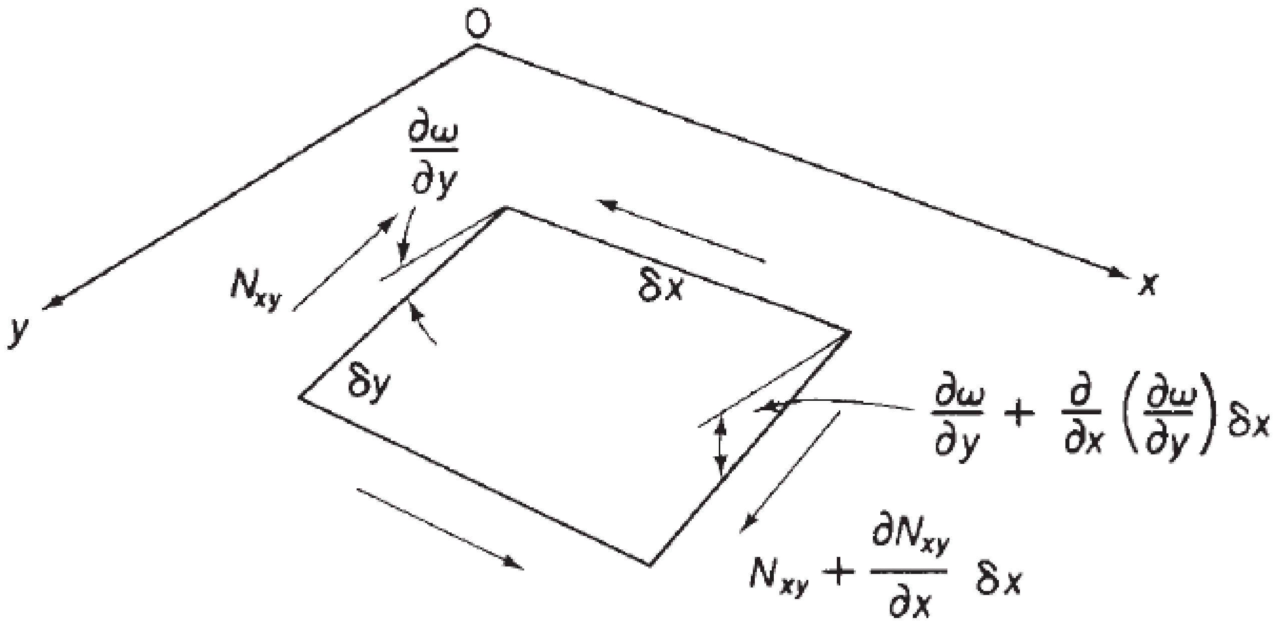

Component of shear loads in the *z* direction.

Source from Aircraft Structures by T. H. G. Megson

Similarly for equilibrium in the y direction we have

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\left(N_{xy} + \frac{\partial N_{xy}}{\partial x}\delta x\right)\delta y\left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial x\,\partial y}\delta x\right) - N_{xy}\delta y\frac{\partial w}{\partial y}$$

neglecting terms of a lower order. Similarly, the contribution of N_{yx} is

$$N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\delta x\,\delta y + \frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y}\delta x\,\delta y$$

Source from Aircraft Structures by T. H. G. Megson

$$\left(N_x + \frac{\partial N_x}{\partial x}\delta x\right)\delta y\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\delta x\right) - N_x \delta y \frac{\partial w}{\partial x}$$

$$N_x \frac{\partial^2 w}{\partial x^2}\delta x\, \delta y + \frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x}\delta x\, \delta y$$

$$N_y \frac{\partial^2 w}{\partial y^2}\delta x\, \delta y + \frac{\partial N_y}{\partial y}\frac{\partial w}{\partial y}\delta x\, \delta y$$

The total force in the *z* direction is found from the summation of these expressions and is

$$N_x \frac{\partial^2 w}{\partial x^2}\delta x\,\delta y + \frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x}\delta x\,\delta y + N_y \frac{\partial^2 w}{\partial y^2}\delta x\,\delta y + \frac{\partial N_y}{\partial y}\frac{\partial w}{\partial y}\delta x\,\delta y$$
$$+ \frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y}\delta x\,\delta y + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\delta x\,\delta y + \frac{\partial N_{xy}}{\partial y}\frac{\partial w}{\partial x}\delta x\,\delta y$$

in which N_{yx} is equal to and is replaced by N_{xy}. Reduce this expression to

$$\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)\delta x\,\delta y$$

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Since the in-plane forces do not produce moments along the edges of the element then Eqs. remain unaffected.

Modified simply by the addition of the above vertical component of the in-plane loads to $q\delta x\delta y$.

Therefore, the governing differential equation for a thin plate supporting transverse and in-plane loads is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(q + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

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Combined bending and in-plane loading of a thin rectangular plate

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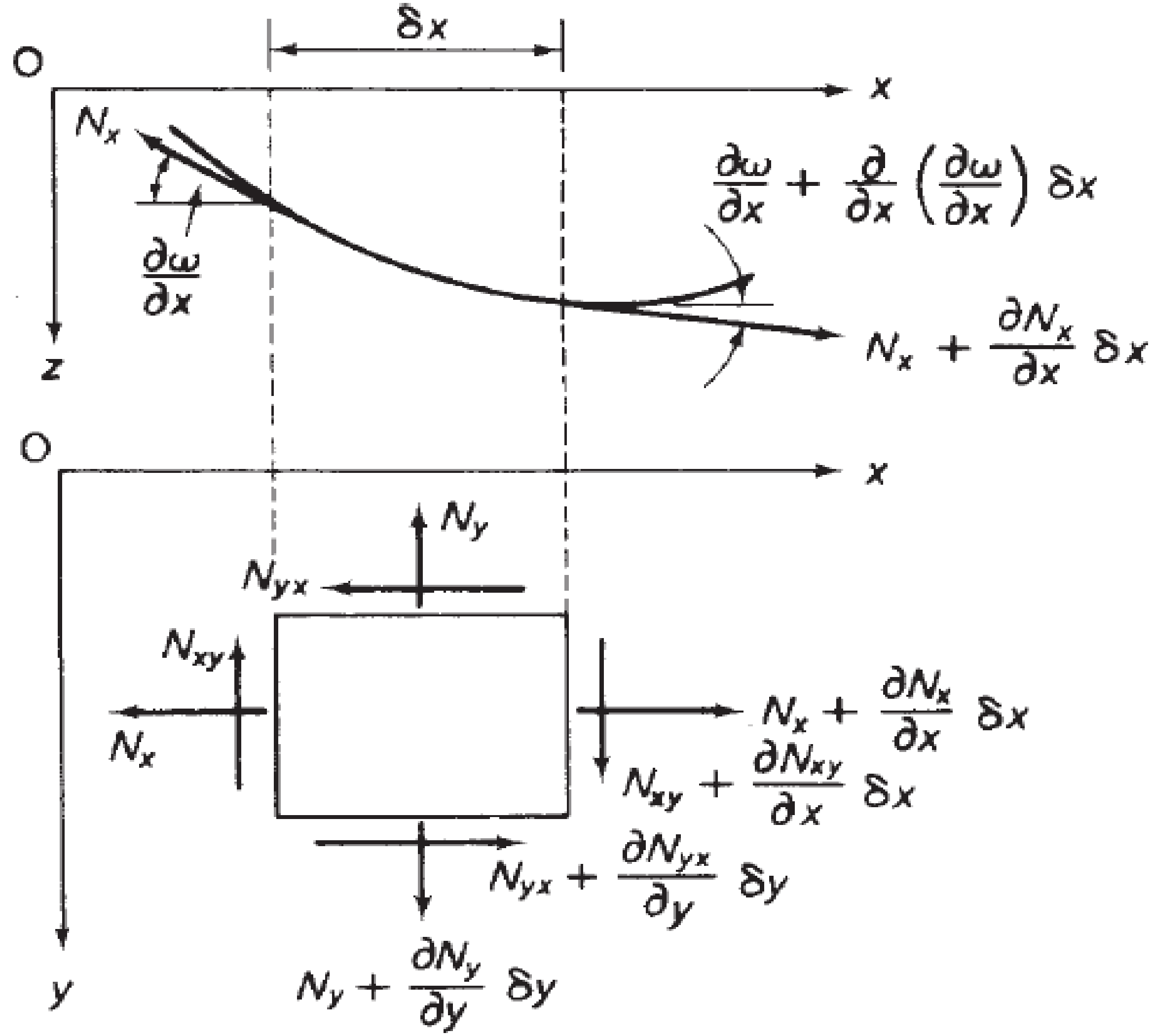

In-plane forces on plate element

Source from Aircraft Structures by T. H. G. Megson

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$$\left(N_x + \frac{\partial N_x}{\partial x}\delta x\right)\delta y \cos\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\delta x\right) - N_x \delta y \cos\frac{\partial w}{\partial x}$$

$$+ \left(N_{yx} + \frac{\partial N_{yx}}{\partial y}\delta y\right)\delta x - N_{yx}\delta x = 0$$

For small deflections $\partial w/\partial x$ and $(\partial w/\partial x)+(\partial^2 w/\partial x^2)\delta x$ are small and the cosines of these angles are therefore approximately equal to one. The equilibrium equation thus simplifies to

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = 0$$

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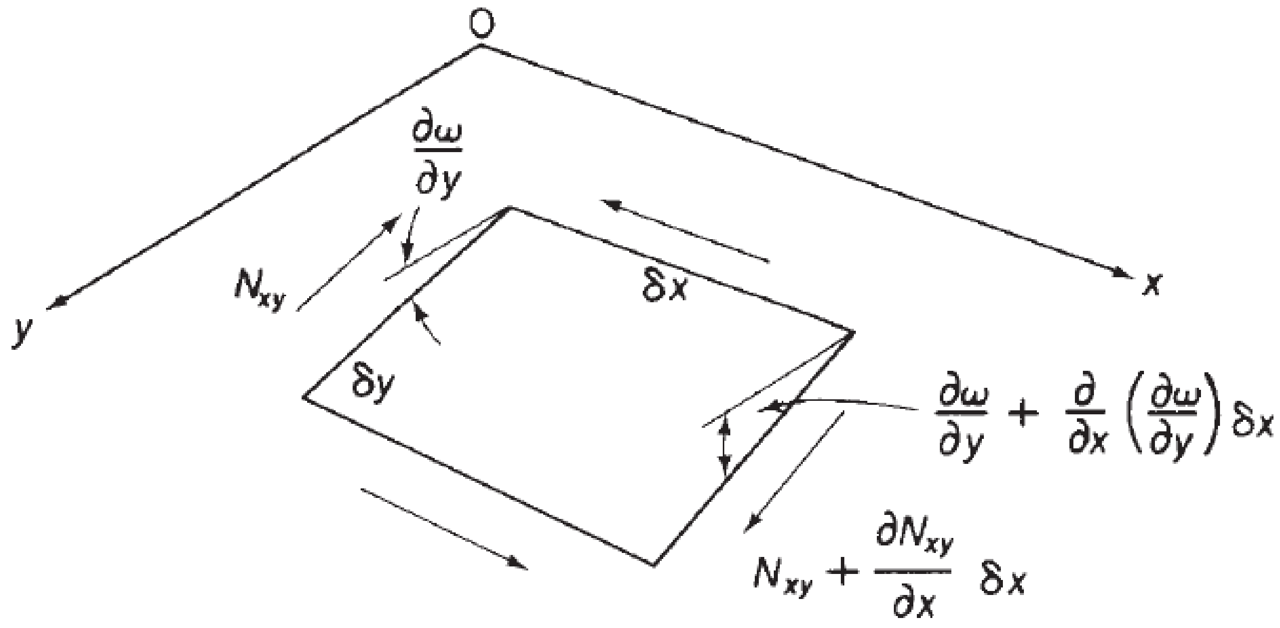

Component of shear loads in the *z* direction.

Source from Aircraft Structures by T. H. G. Megson

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neglecting terms of a lower order. Similarly, the contribution of N_{yx} is

$$N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\delta x\,\delta y + \frac{\partial N_{xy}}{\partial x}\frac{\partial w}{\partial y}\delta x\,\delta y$$

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$$\left(N_x + \frac{\partial N_x}{\partial x}\delta x\right)\delta y\left(\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\delta x\right) - N_x \delta y \frac{\partial w}{\partial x}$$

$$N_x \frac{\partial^2 w}{\partial x^2}\delta x\, \delta y + \frac{\partial N_x}{\partial x}\frac{\partial w}{\partial x}\delta x\, \delta y$$

$$N_y \frac{\partial^2 w}{\partial y^2}\delta x\, \delta y + \frac{\partial N_y}{\partial y}\frac{\partial w}{\partial y}\delta x\, \delta y$$

The total force in the *z* direction is found from the summation of these expressions and is

Source from Aircraft Structures by T. H. G. Megson

$$N_x \frac{\partial^2 w}{\partial x^2} \delta x\, \delta y + \frac{\partial N_x}{\partial x} \frac{\partial w}{\partial x} \delta x\, \delta y + N_y \frac{\partial^2 w}{\partial y^2} \delta x\, \delta y + \frac{\partial N_y}{\partial y} \frac{\partial w}{\partial y} \delta x\, \delta y$$

$$+ \frac{\partial N_{xy}}{\partial x} \frac{\partial w}{\partial y} \delta x\, \delta y + 2N_{xy} \frac{\partial^2 w}{\partial x\, \partial y} \delta x\, \delta y + \frac{\partial N_{xy}}{\partial y} \frac{\partial w}{\partial x} \delta x\, \delta y$$

in which N_{yx} is equal to and is replaced by N_{xy}. Reduce this expression to

$$\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x\, \partial y} \right) \delta x\, \delta y$$

Source from Aircraft Structures by T. H. G. Megson

Since the in-plane forces do not produce moments along the edges of the element then Eqs. remain unaffected.

Modified simply by the addition of the above vertical component of the in-plane loads to *qδxδy*.

Therefore, the governing differential equation for a thin plate supporting transverse and in-plane loads is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(q + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

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Thin plates having a small initial curvature

Suppose that a thin plate has an initial curvature so that the deflection of any point in its middle plane is w_0.

Assume that w_0 is small compared with the thickness of the plate. The application of transverse and in-plane loads will cause the plate to deflect a further amount w_1 so that the total deflection is then $w=w_0+w_1$.

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\,\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left(q+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

The derivation of Eq. the left-hand side was obtained from expressions for bending moments which themselves depend on the change of curvature.

Use the deflection $w1$ on the left-hand side, not w. The effect on bending of the in-plane forces depends on the total deflection w

Source from Aircraft Structures by T. H. G. Megson

$$\frac{\partial^4 w_1}{\partial x^4} + 2\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \frac{\partial^4 w_1}{\partial y^4}$$

$$= \frac{1}{D}\left[q + N_x \frac{\partial^2 (w_0 + w_1)}{\partial x^2} + N_y \frac{\partial^2 (w_0 + w_1)}{\partial y^2} + 2N_{xy} \frac{\partial^2 (w_0 + w_1)}{\partial x \, \partial y}\right]$$

The effect of an initial curvature on deflection is therefore equivalent to the application of a transverse load of intensity

$$N_x \frac{\partial^2 w_0}{\partial x^2} + N_y \frac{\partial^2 w_0}{\partial y^2} + 2N_{xy} \frac{\partial^2 w_0}{\partial x \, \partial y}$$

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Thus, in-plane loads alone produce bending provided there is an initial curvature. Assuming that the initial form of the deflected plate is

$$w_0 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

then by substitution in Eq. we find that if N_x is compressive and $N_y = N_{xy} = 0$

$$w_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Source from Aircraft Structures by T. H. G. Megson

$$B_{mn} = \frac{A_{mn}N_x}{(\pi^2 D/a^2)[m + (n^2a^2/mb^2)]^2 - N_x}$$

Source from Aircraft Structures by T. H. G. Megson

A thin plate may buckle in a variety of modes depending upon its dimensions, the loading and the method of support.

Buckling loads are much lower than those likely to cause failure in the material of the plate.

The simplest form of buckling arises when compressive loads are applied to simply supported opposite edges and the unloaded edges are free, as shown in Fig. A thin plate in this configuration behaves in exactly the same way as a pin-ended column so that the critical load is that predicted by the Euler theory.

Once this critical load is reached the plate is incapable of supporting any further load.

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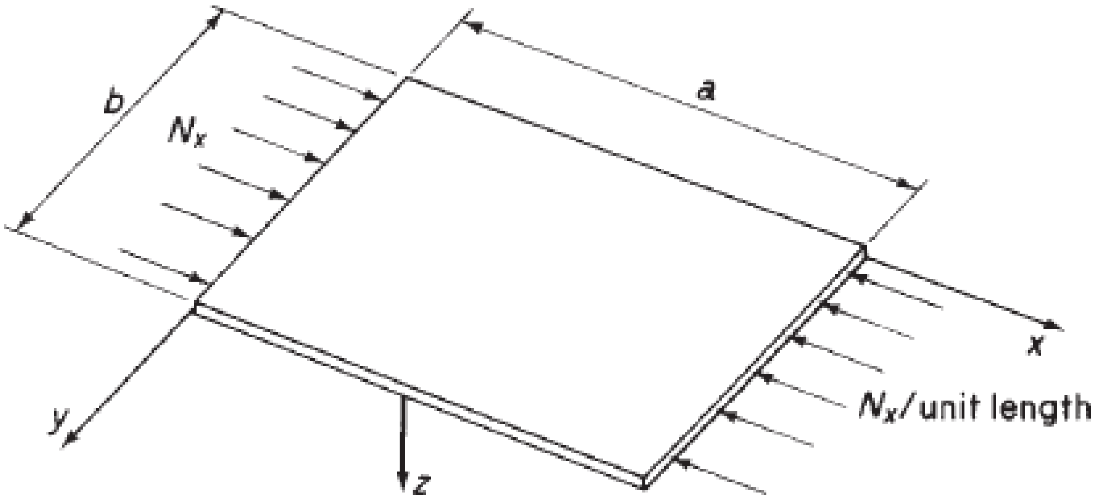
The unloaded edges are supported against displacement out of the *xy* plane.

Buckling, for such plates, takes the form of a bulging displacement of the central region of the plate while the parts adjacent to the supported edges remain straight.

These parts enable the plate to resist higher loads; an important factor in aircraft design.

Here not concerned with this post-buckling behaviour, but rather with the prediction of the critical load which causes the initial bulging of the central area of the plate.

Source from Aircraft Structures by T. H. G. Megson



Buckling of a thin flat plate

Source from Aircraft Structures by T. H. G. Megson

Consider the relatively simple case of the thin plate of Fig. , loaded as shown, but simply supported along all four edges.

Deflected shape may be represented by the infinite double trigonometrical series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Also, the total potential energy of the plate is,

$$U + V = \frac{1}{2} \int_0^a \int_0^b \left[D \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1 - \nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \, \partial y} \right)^2 \right] \right\} - N_x \left(\frac{\partial w}{\partial x} \right)^2 \right] \mathrm{d}x \, \mathrm{d}y$$

Source from Aircraft Structures by T. H. G. Megson

The integration of Eq. on substituting for *w* is

$$U + V = \frac{\pi^4 abD}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) - \frac{\pi^2 b}{8a} N_x \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 A_{mn}^2$$

The total potential energy of the plate has a stationary value in the neutral equilibrium of its buckled state (i.e. $N_x = N_{x,\mathrm{CR}}$).

Differentiating Eq. with respect to each unknown coefficient A_{mn} we have

$$\frac{\partial(U + V)}{\partial A_{mn}} = \frac{\pi^4 abD}{4} A_{mn} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 - \frac{\pi^2 b}{4a} N_{x,\mathrm{CR}} m^2 A_{mn} = 0$$

Source from Aircraft Structures by T. H. G. Megson

for a non-trivial solution

$$N_{x,\mathrm{CR}} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2$$

$$N_{x,\mathrm{CR}} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{1}{b^2}\right)^2$$

$$N_{x,\mathrm{CR}} = \frac{k\pi^2 D}{b^2}$$

where the plate *buckling coefficient k* is given by the minimum value of

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

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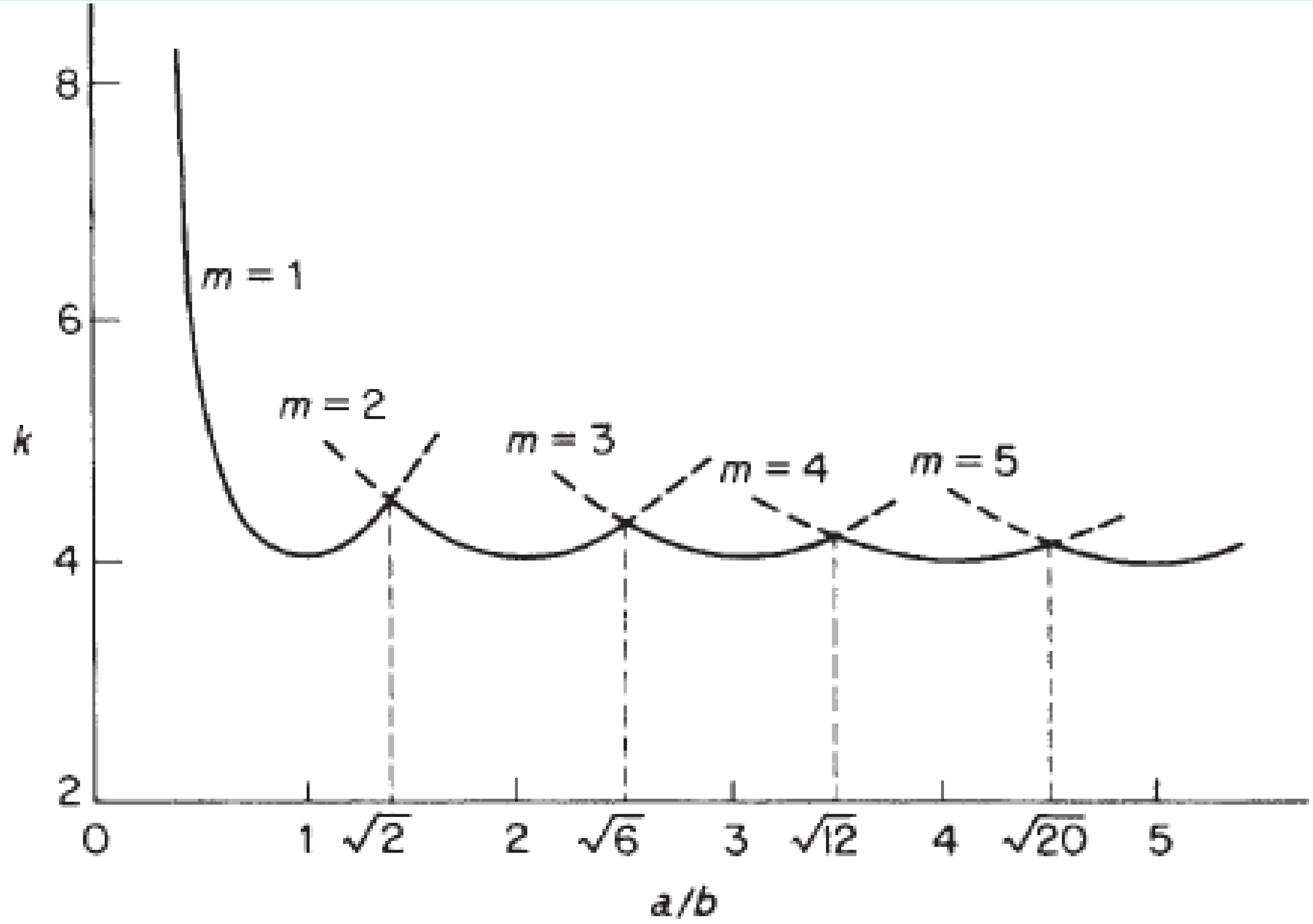
for a given value of *a/b*. To determine the minimum value of *k* for a given value of *a/b* we plot *k* as a function of *a/b* for different values of *m* as shown by the dotted curves in Fig.

The minimum value of *k* is obtained from the lower envelope of the curves shown solid in the figure.



Buckling coefficient *k* for simply supported plates.

Source from Aircraft Structures by T. H. G. Megson

It can be seen that *m* varies with the ratio *a/b* and that *k* and the buckling load are a minimum when *k* =4 at values of *a/b*=1, 2, 3, ...As *a/b* becomes large *k* approaches 4 so that long narrow plates tend to buckle into a series of squares.

The transition from one buckling mode to the next may be found by equating values of *k* for the *m* and *m*+1 curves. Hence

$$\frac{mb}{a} + \frac{a}{mb} = \frac{(m+1)b}{a} + \frac{a}{(m+1)b}$$

$$\frac{a}{b} = \sqrt{m(m+1)}$$

Source from Aircraft Structures by T. H. G. Megson

Substituting m=1, we have $a/b = \sqrt{2} = 1.414$, and for m=2, $a/b = \sqrt{6} = 2.45$ and so on.

For a given value of a/b the critical stress, $\sigma_{CR} = N_{x,CR}/t$, is found from Eqs

$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

Source from Aircraft Structures by T. H. G. Megson

Inelastic buckling of plates

For plates having small values of *b/t* the critical stress may exceed the elastic limit of the material of the plate.

In such a situation, Eq.

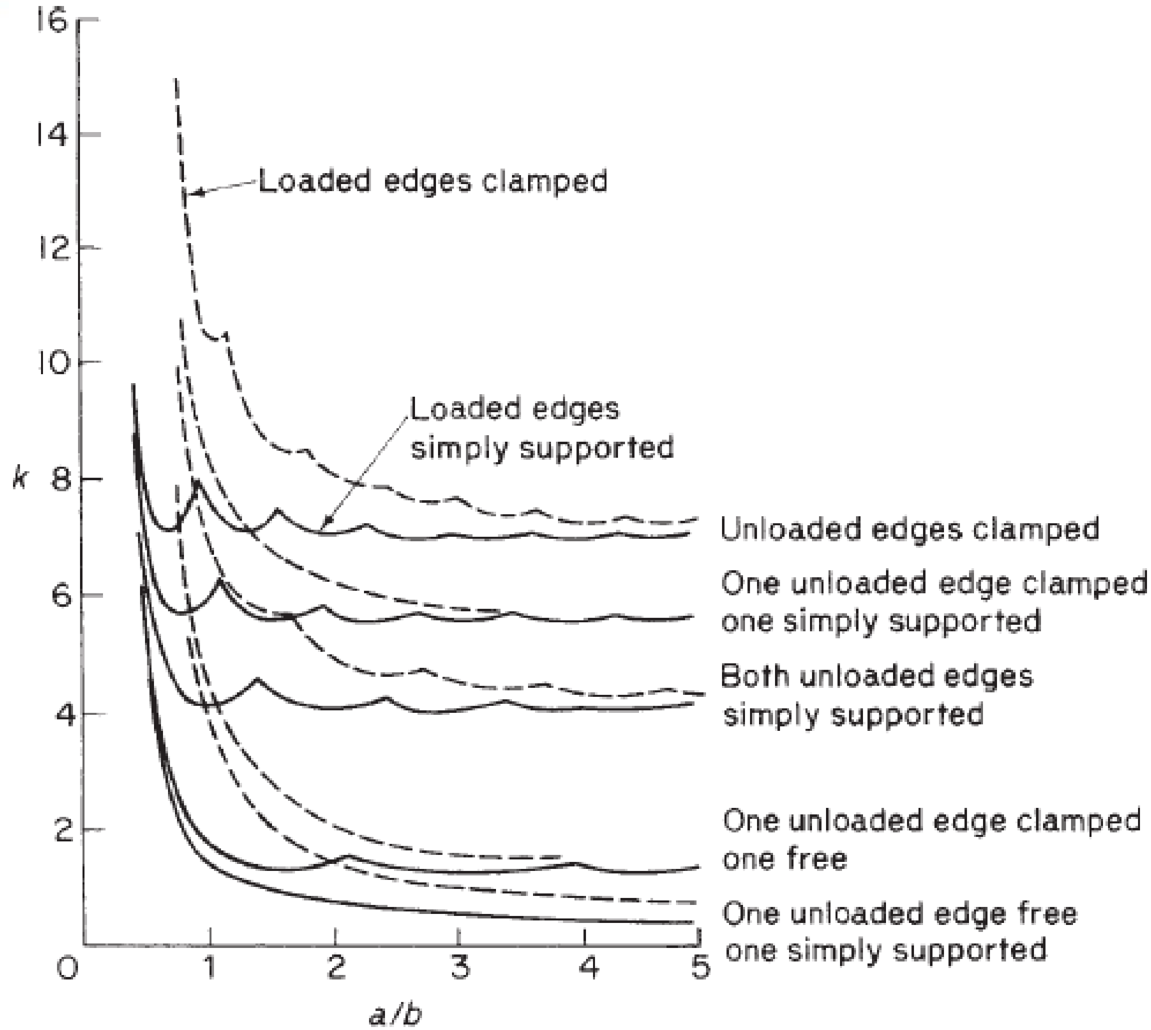
$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

is no longer applicable since, as we saw in the case of columns, *E* becomes dependent on stress as does Poisson's ratio *v*.

These effects are usually included in a plasticity correction factor *η* so that above Eq. becomes

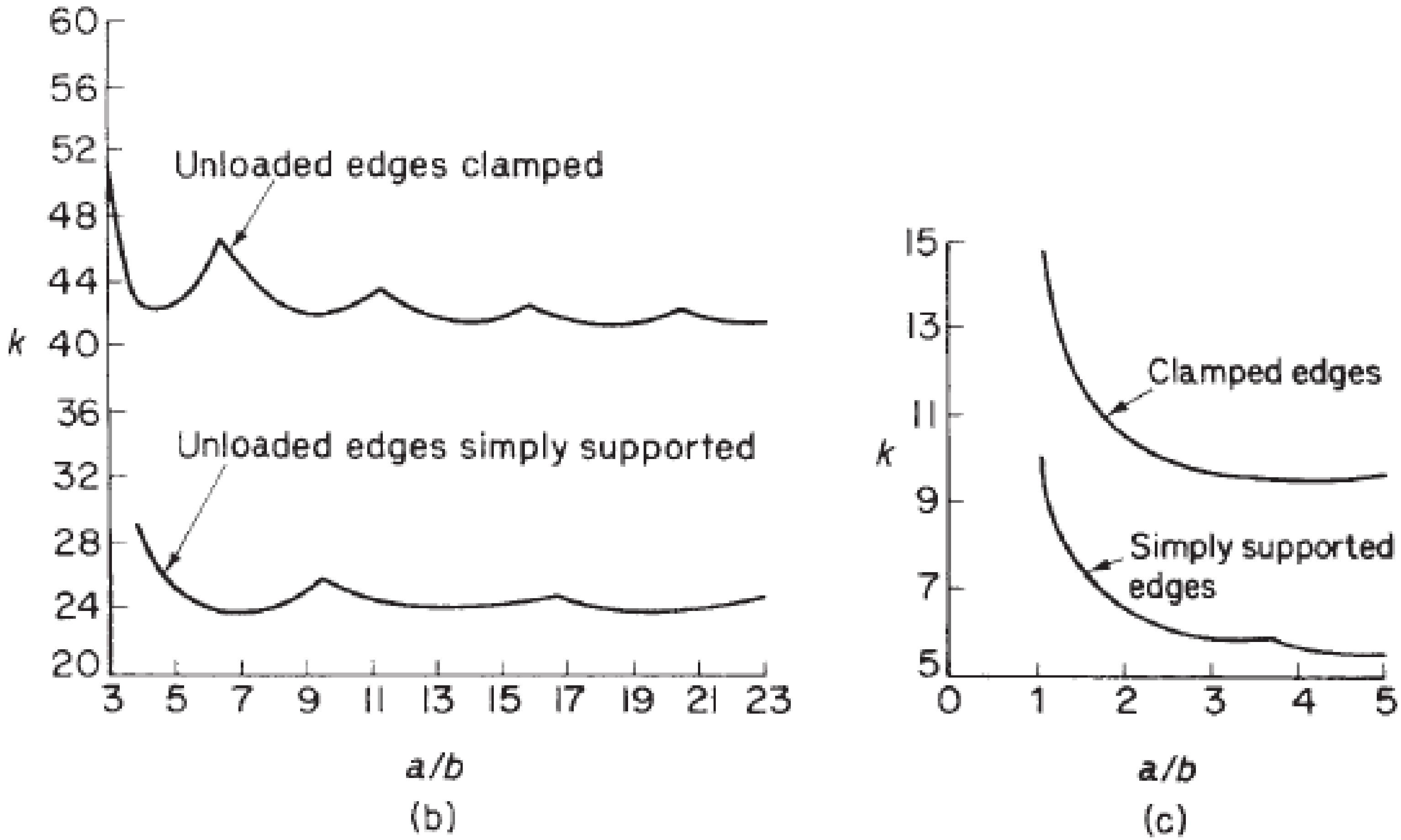
$$\sigma_{CR} = \frac{\eta k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

Source from Aircraft Structures by T. H. G. Megson



(a)

Source from Aircraft Structures by T. H. G. Megson



(a) Buckling coefficients for flat plates in compression; (b) buckling coefficients for flat plates in bending; (c) shear buckling coefficients for flat plates.

Source from Aircraft Structures by T. H. G. Megson

where E and v are elastic values of Young's modulus and Poisson's ratio.

In the linearly elastic region η=1, which means that Eq. may be applied at all stress levels.

Below eq. will giving good agreement with experiment is

$$\eta = \frac{1-\nu_e^2}{1-\nu_p^2}\frac{E_s}{E}\left[\frac{1}{2}+\frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)^{\frac{1}{2}}\right]$$

where Et and Es are the tangent modulus and secant modulus (stress/strain) of the plate in the inelastic region and ve and vp are Poisson's ratio in the elastic and inelastic ranges.

Source from Aircraft Structures by T. H. G. Megson

Experimental determination of critical load for a flat plate

The critical load for a column may be determined experimentally, without actually causing the column to buckle, by means of the Southwell plot.

The critical load for an actual, rectangular, thin plate is found in a similar manner.

The displacement of an initially curved plate from the zero load position was found

$$w_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

Source from Aircraft Structures by T. H. G. Megson

where $$B_{mn} = \frac{A_{mn}N_x}{\frac{\pi^2 D}{a^2}\left(m + \frac{n^2a^2}{mb^2}\right)^2 - N_x}$$

The coefficients *Bmn* increase with an increase of compressive load intensity *Nx*.

It follows that when *Nx* approaches the critical value, *Nx*,CR, the term in the series corresponding to the buckled shape of the plate becomes the most significant.

For a square plate *n*=1 and *m*=1 give a minimum value of critical load so that at the centre of the plate

Source from Aircraft Structures by T. H. G. Megson

$$w_1 = \frac{A_{11} N_x}{N_{x,\mathrm{CR}} - N_x}$$

or, rearranging

$$w_1 = N_{x,\mathrm{CR}} \frac{w_1}{N_x} - A_{11}$$

A graph of w_1 plotted against w_1 / N_x will have a slope, in the region of the critical load, equal to $N_{x,\mathrm{CR}}$.

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Local instability

Distinguished in the primary and secondary (or local) instability.

The latter form of buckling usually occurs in the flanges and webs of thin-walled columns having an effective slenderness ratio, *le*/*r* <20.

For *le*/*r* >80 this type of column is susceptible to primary instability.

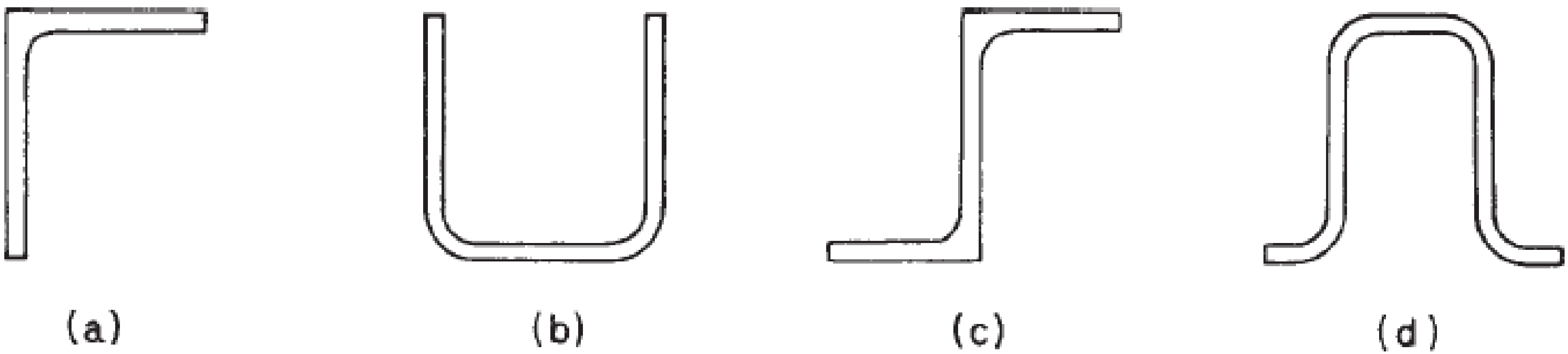
In the intermediate range of *le*/*r* between 20 and 80, buckling occurs by a combination of both primary and secondary modes.

Source from Aircraft Structures by T. H. G. Megson

Thin-walled columns are encountered in aircraft structures in the shape of longitudinal stiffeners, which are normally fabricated by extrusion processes or by forming from a flat sheet.

A variety of cross-sections are employed although each is usually composed of flat plate elements arranged to form angle, channel, *Z*- or ‘top hat’ sections, as shown in Fig. below.

The plate elements fall into two distinct categories: flanges which have a free unloaded edge and webs which are supported by the adjacent plate elements on both unloaded edges.



(a) Extruded angle; (b) formed channel; (c) extruded *Z*; (d) formed ‘top hat’.

Source from Aircraft Structures by T. H. G. Megson

Values of local critical stress for columns possessing these types of section may be found using Eq.

$$\sigma_{\mathrm{CR}} = \frac{\eta k \pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

and an appropriate value of *k*.

For example,

K for a cruciform section column is obtained from Fig(a) . Below for a plate which is simply supported on three sides with one edge free and has $a/b>3$.

Hence k =0.43 and if the section buckles elastically then η=1

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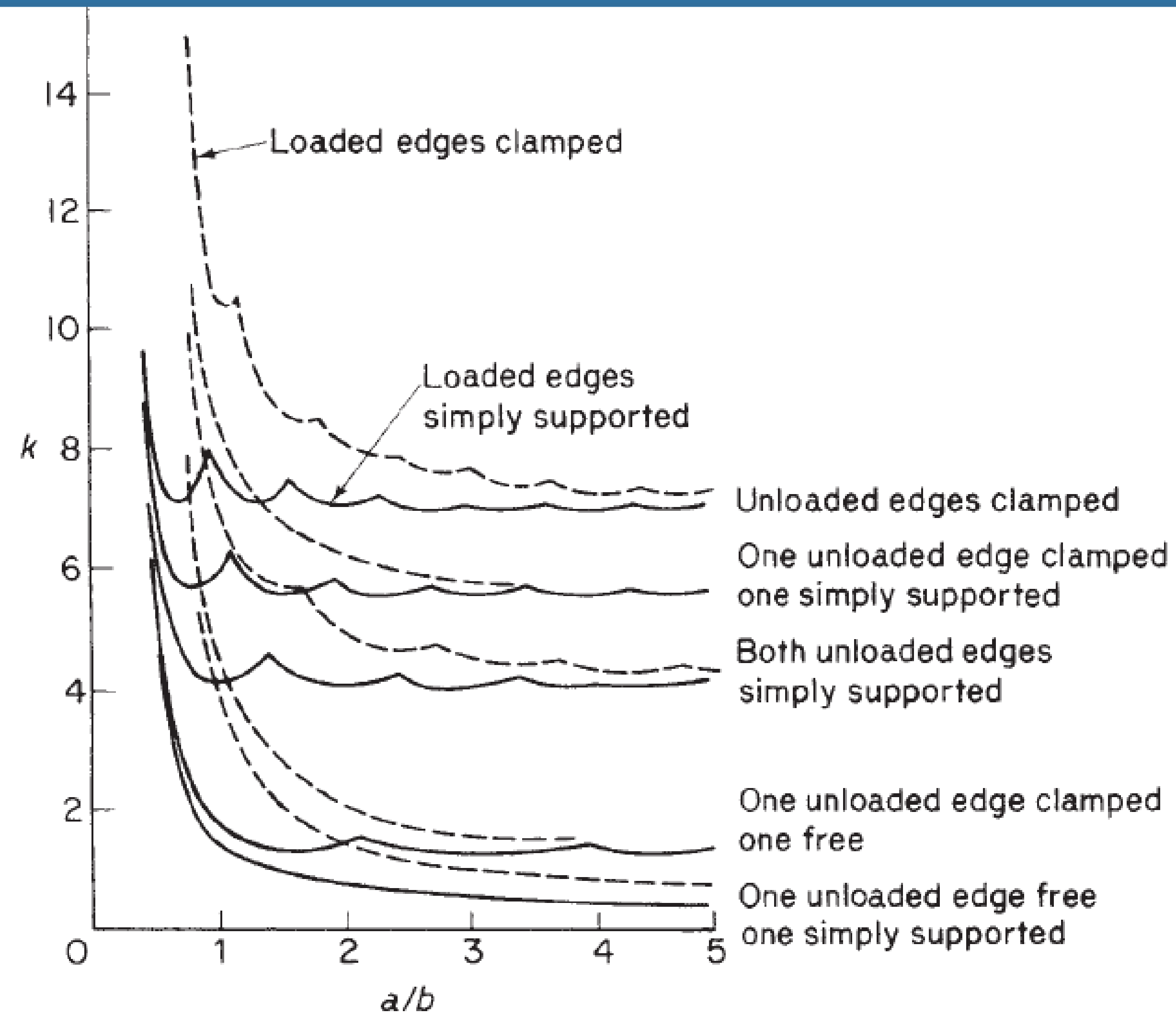

(a)

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$$\sigma_{CR} = 0.388E\left(\frac{t}{b}\right)^2 \quad (\nu = 0.3)$$

It must be appreciated that the calculation of local buckling stresses is generally complicated with no particular method gaining universal acceptance, much of the information available being experimental.

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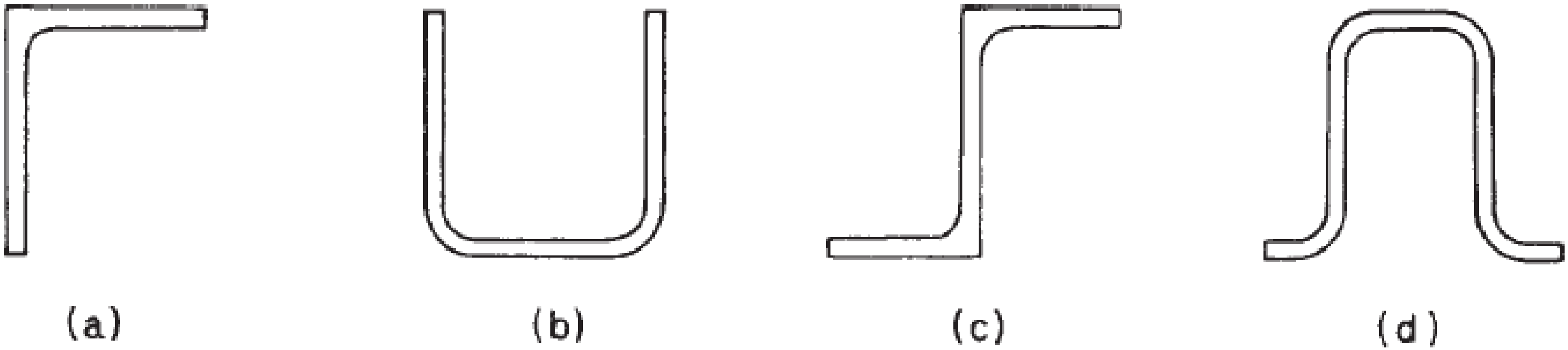

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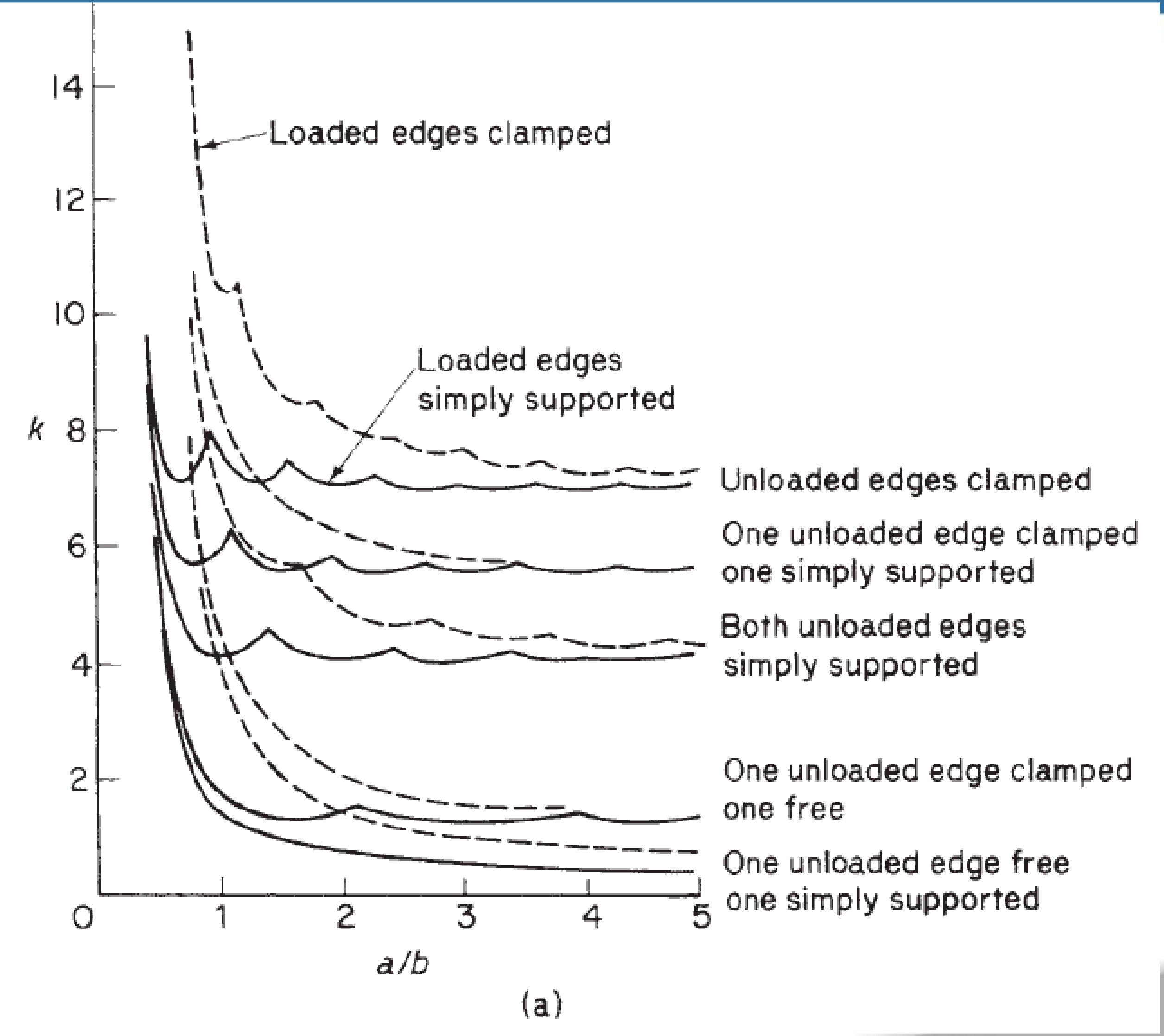
Source from Aircraft Structures by T. H. G. Megson



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Unit - III

Shear loaded thin walled beams:

General stress,

Strain and displacement relationships,

Direct stress and shear flow system,

Shear centre,

Twist and warping

General stress

The equations of equilibrium and expressions for strain which are necessary for the analysis of open section beams supporting shear loads and closed section beams carrying shear and torsional loads.

The analysis of open section beams subjected to torsion requires a different approach

The relationships are established from first principles for the particular case of thin-walled sections

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In this analysis

- Assumed that axial constraint effects are negligible,
- shear stresses normal to the beam surface may be neglected since they are zero at each surface and the wall is thin,
- direct and shear stresses on planes normal to the beam surface are constant across the thickness,
- Finally, the beam is of uniform section so that the thickness may vary with distance around each section but is constant along the beam.
- In addition, ignore squares and higher powers of the thickness t in the calculation of section properties

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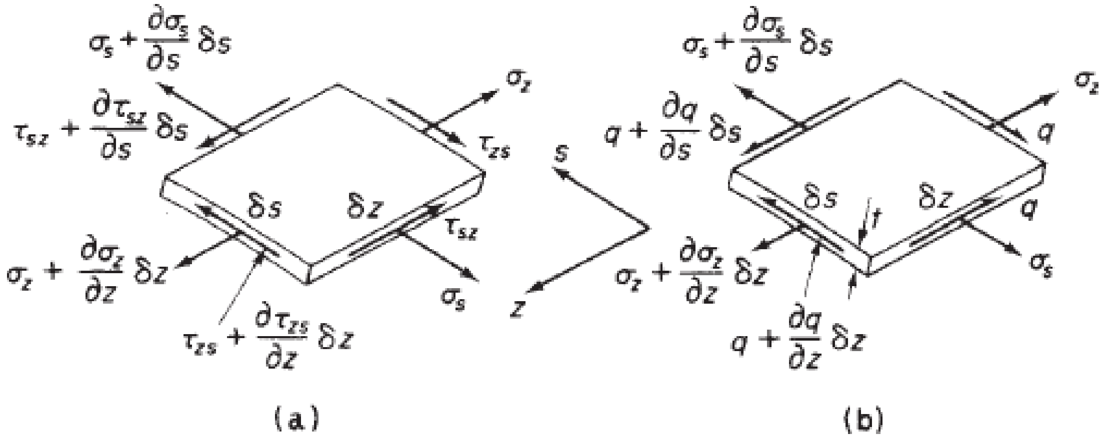
The parameter *s* in the analysis is distance measured around the cross-section from some convenient origin.

An element $\delta_s \times \delta_z \times t$ of the beam wall is maintained in equilibrium by a system of direct and shear stresses as shown in Fig. below.



(a) General stress system on element of a closed or open section beam;

(b) (b) direct stress and shear flow system on the element.

Source from Aircraft Structures by T. H. G. Megson

The direct stress σ_z is produced by bending moments or by the bending action of shear loads while the shear stresses are due to shear and/or torsion of a closed section beam or shear of an open section beam.

The hoop stress σ_s is usually zero but may be caused, in closed section beams, by internal pressure.

Specified that t may vary with s, this variation is small for most thin-walled structures

so that we may reasonably make the approximation that t is constant over the length δs. Also, deduce that $\tau_{zs} = \tau_{sz} = \tau$ say.

However, we shall find it convenient to work in terms of *shear flow q*, i.e. shear force per unit length rather than in terms of shear stress. $q = \tau t$

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Source from Aircraft Structures by T. H. G. Megson

For equilibrium of the element in the z direction and neglecting body forces

$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right) t\delta s - \sigma_z t\delta s + \left(q + \frac{\partial q}{\partial s}\delta s\right)\delta_z - q\delta z = 0$$

which reduces to

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

Similarly for equilibrium in the s direction

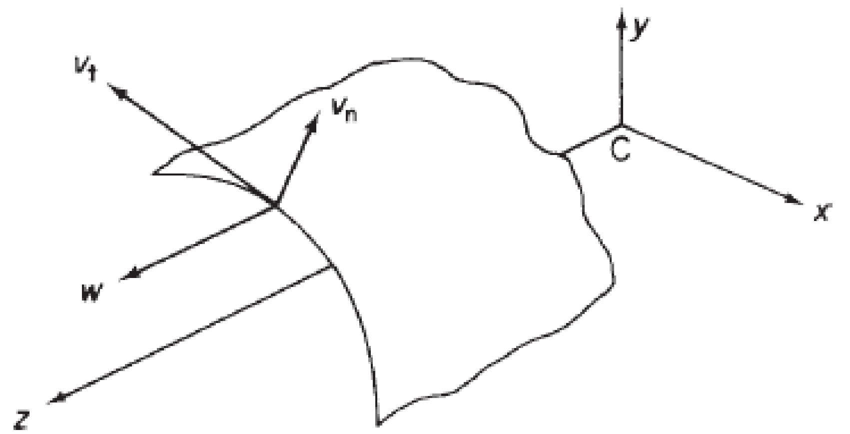
$$\frac{\partial q}{\partial z} + t\frac{\partial \sigma_s}{\partial s} = 0$$

The direct stresses σ_z and σs produce direct strains εz and εs, while the shear stress τ induces a shear strain $\gamma(=\gamma_{zs} = \gamma_{sz})$.

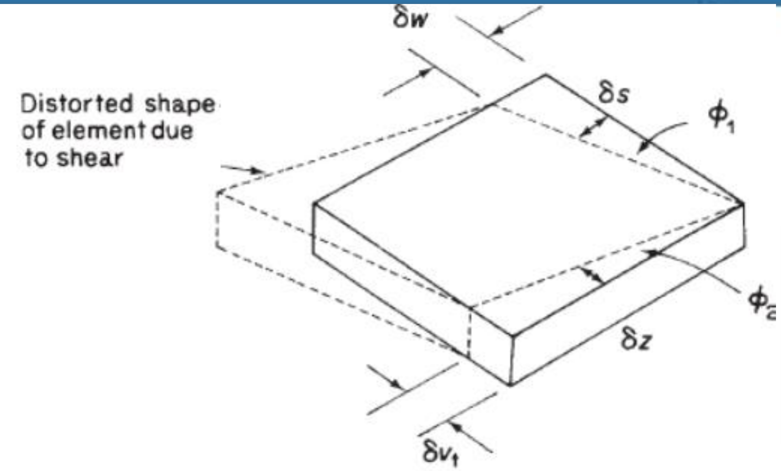
Let v_t is a tangential displacement in the xy plane and is taken to be positive in the direction of increasing s; v_n is a normal displacement in the xy plane and is positive outwards; and w is an axial displacement.

$\varepsilon_z = \partial w/\partial z$ $\varepsilon_s = [(\partial vt/\partial s) + (v_n/r)]$

Source from Aircraft Structures by T. H. G. Megson



Axial, tangential and normal components of displacement of a point in the beam wall.



Determination of shear strain γ in terms of tangential and axial components of displacement.

The shear strain γ is found in terms of the displacements w and v_t by considering the shear distortion of an element $\delta_s \times \delta_z$ of the beam wall. The shear strain is given by

$$\gamma = \varphi_1 + \varphi_2$$

or, in the limit as both δ_s and δ_z tend to zero

$$\gamma = \partial w/\partial s + \partial_{vt}/\partial z$$

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Assume that during any displacement the shape of the beam cross-section is maintained by a system of closely spaced diaphragms which are rigid in their own plane but are perfectly flexible normal to their own plane (CSRD assumption).

There is, therefore, no resistance to axial displacement *w* and the cross-section moves as a rigid body in its own plane, the displacement of any point being completely specified by translations *u* and *v* and a rotation ϑ .

The origin O of the axes in Fig. has been chosen arbitrarily and the axes suffer displacements *u*, *v* and ϑ. These displacements, in a loading case such as pure torsion, are equivalent to a pure rotation about some point R(*x*R,*y*R) in the cross-section where R is the *centre of twist.*

$$v_t = p\theta + u\cos\psi + v\sin\psi$$

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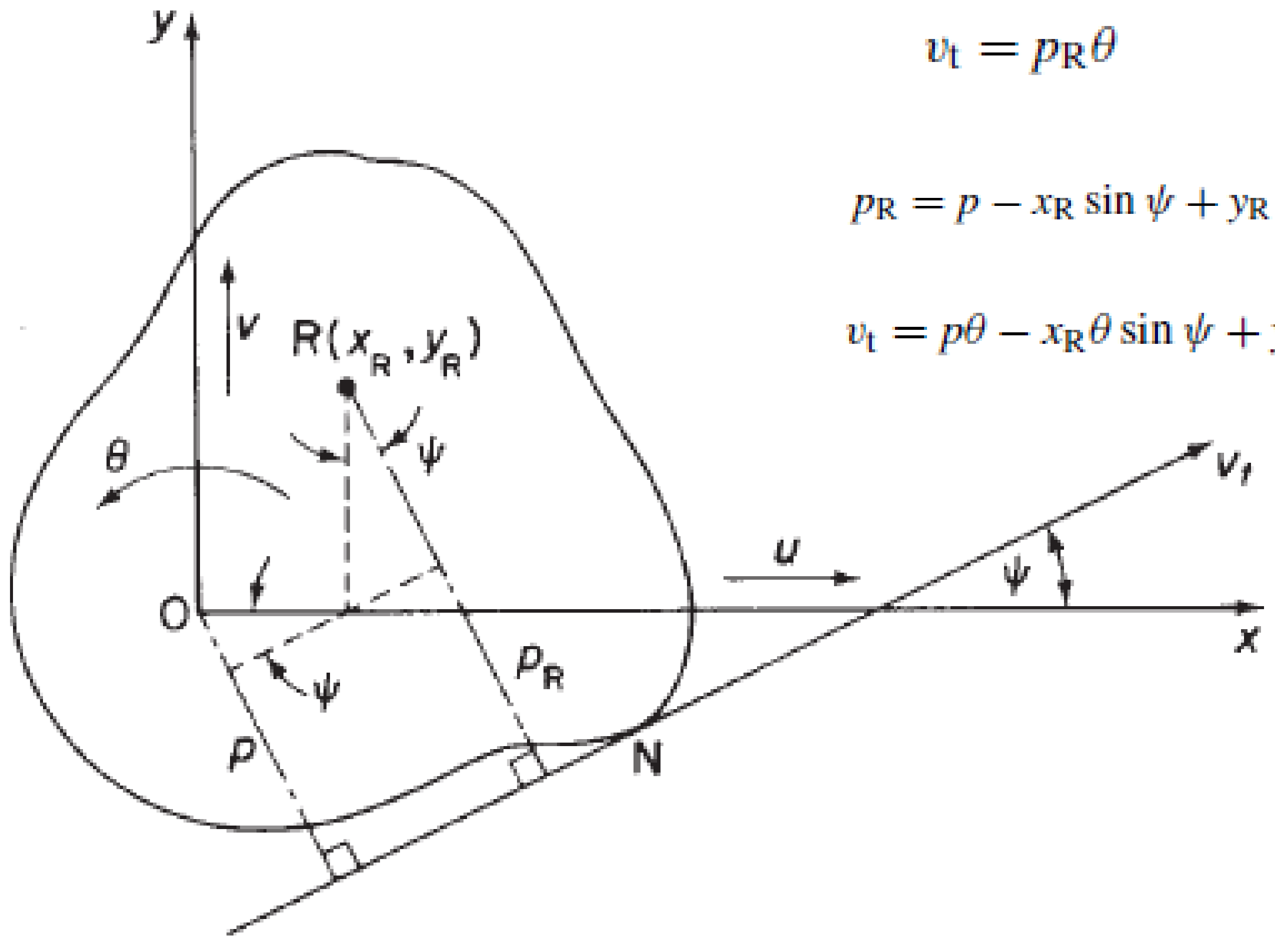

$$v_t = p_R \theta$$

$$p_R = p - x_R \sin\psi + y_R \cos\psi$$

$$v_t = p\theta - x_R\theta \sin\psi + y_R\theta \cos\psi$$

Establishment of displacement relationships and position of centre of twist of beam (open or closed).

$$v_{\mathrm{t}} = p\theta + u\cos\psi + v\sin\psi \qquad \frac{\partial v_{\mathrm{t}}}{\partial z} = p\frac{\mathrm{d}\theta}{\mathrm{d}z} + \frac{\mathrm{d}u}{\mathrm{d}z}\cos\psi + \frac{\mathrm{d}v}{\mathrm{d}z}\sin\psi$$

$$v_{\mathrm{t}} = p\theta - x_{\mathrm{R}}\theta\sin\psi + y_{\mathrm{R}}\theta\cos\psi \qquad \frac{\partial v_{\mathrm{t}}}{\partial z} = p\frac{\mathrm{d}\theta}{\mathrm{d}z} - x_{\mathrm{R}}\sin\psi\frac{\mathrm{d}\theta}{\mathrm{d}z} + y_{\mathrm{R}}\cos\psi\frac{\mathrm{d}\theta}{\mathrm{d}z}$$

$$x_{\mathrm{R}} = -\frac{\mathrm{d}v/\mathrm{d}z}{\mathrm{d}\theta/\mathrm{d}z} \qquad y_{\mathrm{R}} = \frac{\mathrm{d}u/\mathrm{d}z}{\mathrm{d}\theta/\mathrm{d}z}$$

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Shear of open section beams

The open section beam of arbitrary section shown in Fig. supports shear loads S_x and S_y such that there is no twisting of the beam cross-section.

For this condition to be valid the shear loads must both pass through a particular point in the cross-section known as the *shear centre.*

Since there are no hoop stresses in the beam the shear flows and direct stresses acting on an element of the beam wall are related by

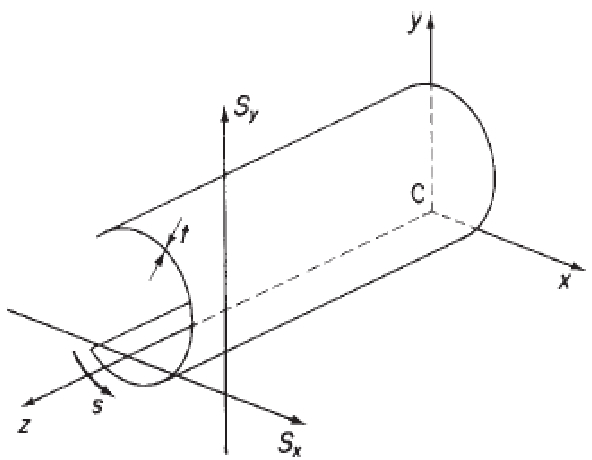
$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

Source from Aircraft Structures by T. H. G. Megson

By assume that the direct stresses are obtained with sufficient accuracy from basic bending theory

$$\frac{\partial \sigma_z}{\partial z} = \frac{[(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{[(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}]}{I_{xx}I_{yy} - I_{xy}^2}y$$

$$\partial M_y/\partial z = S_x,$$



Shear loading of open section beam.

$$\frac{\partial \sigma_z}{\partial z} = \frac{(S_x I_{xx} - S_y I_{xy})}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{(S_y I_{yy} - S_x I_{xy})}{I_{xx}I_{yy} - I_{xy}^2}y$$

shear loads *Sx* and *Sy* such that there is no twisting of the beam cross-section. For this condition to be valid the shear loads must both pass through a particular point in the cross-section known as the *shear centre.*

Since there are no hoop stresses in the beam the shear flows and direct stresses acting

on an element of the beam wall are related by

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Substituting for *∂σz/∂z*

$$\frac{\partial q}{\partial s} = -\frac{(S_x I_{xx} - S_y I_{xy})}{I_{xx} I_{yy} - I_{xy}^2} tx - \frac{(S_y I_{yy} - S_x I_{xy})}{I_{xx} I_{yy} - I_{xy}^2} ty$$

Integrating above equation with respect to *s* from some origin for *s* to any point around the cross-section, we obtain

$$\int_0^s \frac{\partial q}{\partial s} \mathrm{d}s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, \mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, \mathrm{d}s$$

If the origin for *s* is taken at the open edge of the cross-section, then *q*=0 when *s*=0 and Eq.above becomes

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$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, ds$$

For a section having either C*x* or C*y* as an axis of symmetry *Ixy* =0 and Eq. above reduces to

$$q_s = -\frac{S_x}{I_{yy}} \int_0^s tx \, ds - \frac{S_y}{I_{xx}} \int_0^s ty \, ds$$

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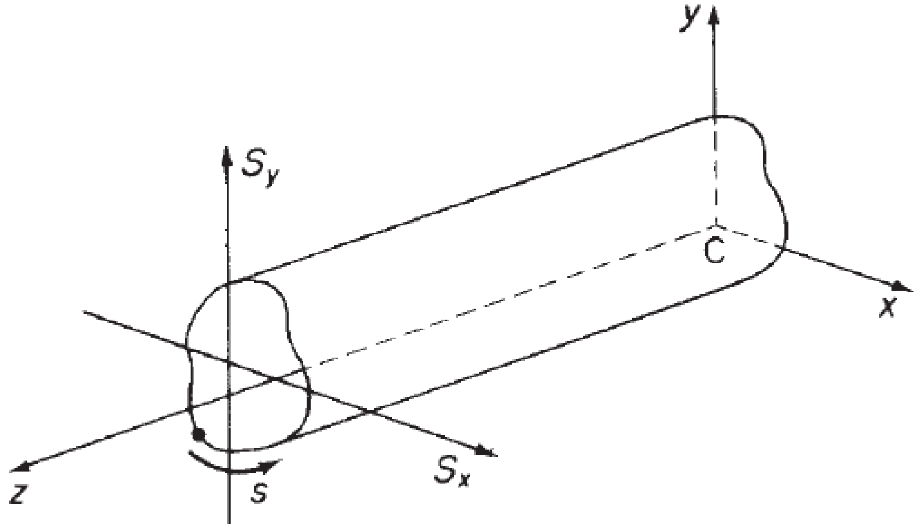
Shear of closed section beams

The solution for a shear loaded closed section beam follows a similar pattern to that described in Sectio for an open section beam but with two important differences.

First, the shear loads may be applied through points in the cross-section other than the shear centre so that torsional as well as shear effects are



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Shear of closed section beams.

This is possible since, as we shall see, shear stresses produced by torsion in closed section beams have exactly the same form as shear stresses produced by shear, unlike shear stresses due to shear and torsion in open section beams.

Secondly, it is generally not possible to choose an origin for *s* at which the value of shear flow is known.

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

$$\int_0^s \frac{\partial q}{\partial s} \mathrm{d}s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx \, \mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty \, \mathrm{d}s$$

Let us suppose that we choose an origin for *s* where the shear flow has the unknown value *qs*,0.

Source from Aircraft Structures by T. H. G. Megson

$$q_s - q_{s,0} = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s$$

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This fact indicates a method of solution for a shear loaded closed section beam. Representing this ‘open’ section or ‘basic’ shear flow by *q*b,

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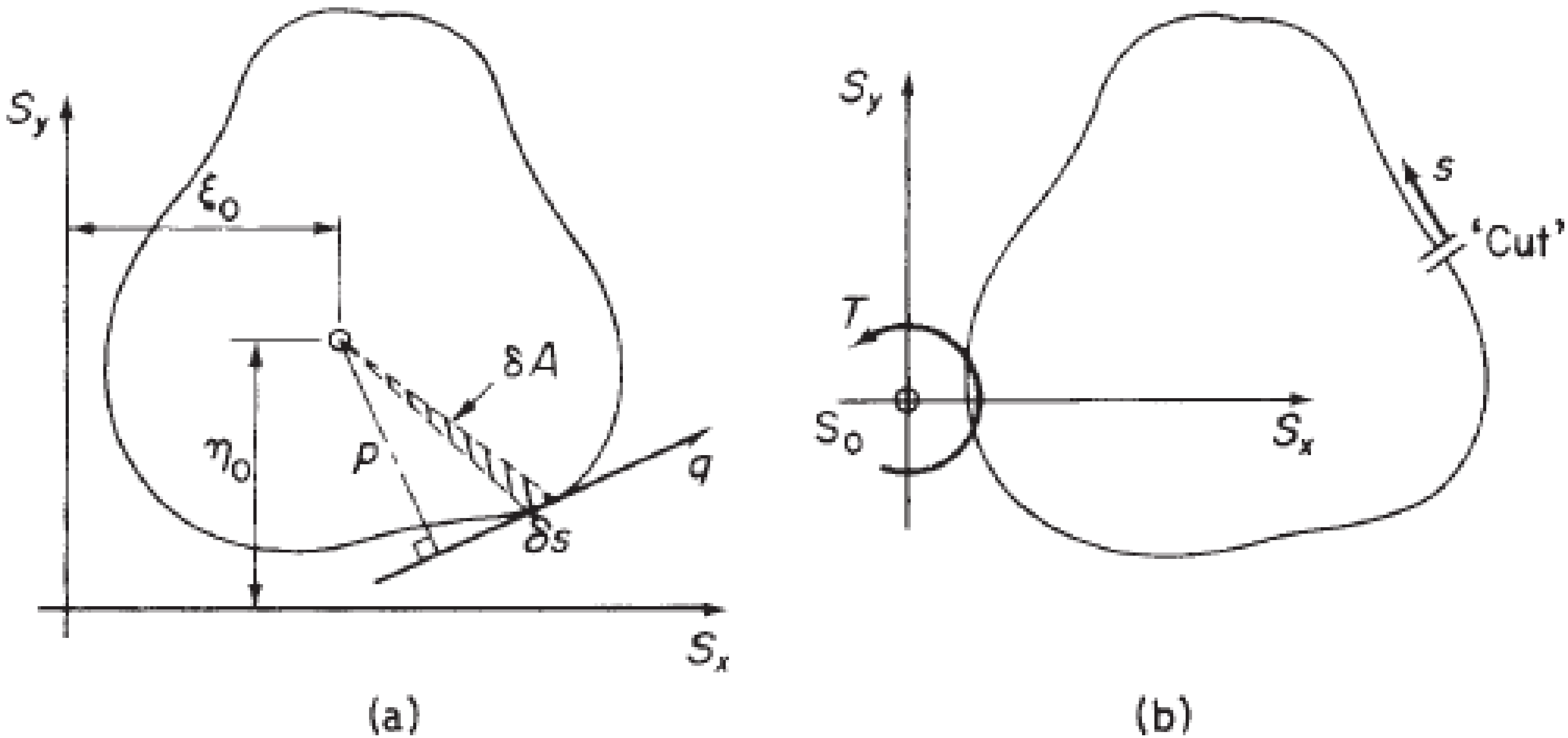

g. 17.11 (a) Determination of $q_{s,0}$; (b) equivalent loading on 'open' section beam.

$$q_s = q_b + q_{s,0}$$

$$q_b = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{xy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s$$

The value of shear flow at the cut (s = 0) is then found by equating applied and internal moments taken about some convenient moment centre.

$$S_x \eta_0 - S_y \xi_0 = \oint pq\,\mathrm{d}s = \oint pq_b\,\mathrm{d}s + q_{s,0}\oint p\,\mathrm{d}s$$

$$\delta A = \frac{1}{2}\delta s p$$

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$$\oint \mathrm{d}A = \frac{1}{2}\oint p\,\mathrm{d}s$$

$$\oint p\mathrm{d}s = 2A$$

where *A* is the area enclosed by the mid-line of the beam section wall.

Hence

$$S_x\eta_0 - S_y\xi_0 = \oint pq_{\mathrm{b}}\mathrm{d}s + 2Aq_{s,0}$$

The unknown shear flow $q_{s,0}$ follows from either of Eqs above.

If the moment centre is chosen to coincide with the lines of action of S_x and S_y then

$$0 = \oint p q_b \, ds + 2A q_{s,0}$$

SHEAR CENTRE

We have defined the position of the shear centre as that point in the cross-section through which shear loads produce no twisting.

It may be shown by use of the reciprocal theorem that this point is also the centre of twist of sections subjected to torsion.

The stresses produced by the separate actions of torsion and shear may then be added by superposition.

It is therefore necessary to know the location of the shear centre in all types of section or to calculate its position.

Where a cross-section has an axis of symmetry the shear centre must, of course, lie on this axis.

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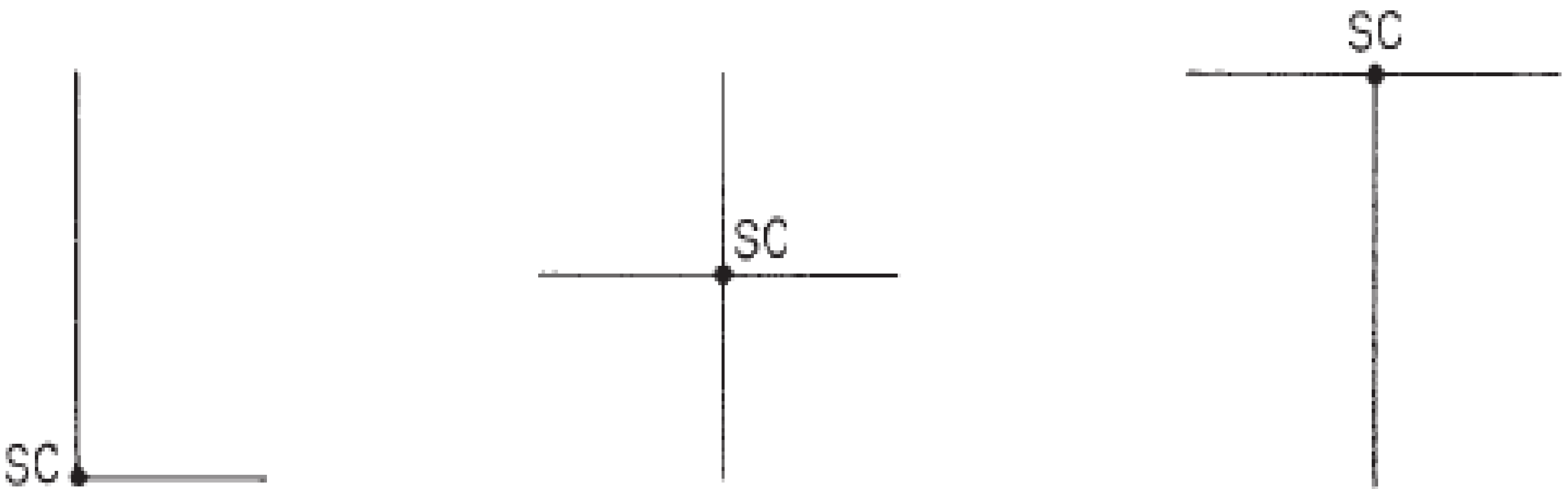


7.8 Shear centre position for type of open section beam shown.

Source from Aircraft Structures by T. H. G. Megson

The shear centre of a closed section beam is located in a similar manner to that described for open section beams.

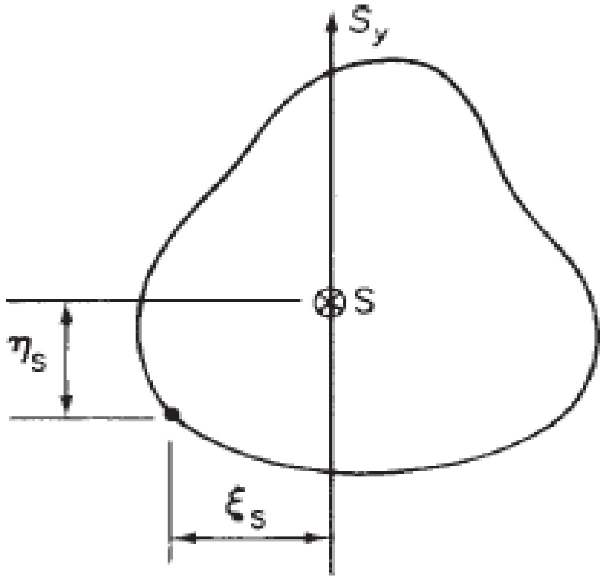
Therefore, to determine the coordinate ξ_S (referred to any convenient point in the cross-section) of the shear centre S of the closed section beam

Apply an arbitrary shear load *Sy* through S, calculate the distribution of shear flow *qs* due to *Sy* and then equate internal and external moments.

$$0 = \oint \frac{q_s}{Gt} \mathrm{d}s$$

$$0 = \oint \frac{1}{Gt}(q_b + q_{s,0}) \mathrm{d}s$$

Source from Aircraft Structures by T. H. G. Megson



Shear centre of a closed section beam.

$$q_{s,0} = -\frac{\oint q_b \, ds}{\oint ds}$$

If Gt =constant then

$$q_{s,0} = -\frac{\oint (q_b/Gt) ds}{\oint ds/Gt}$$

The coordinate η_S is found in a similar manner by applying S_x through S.

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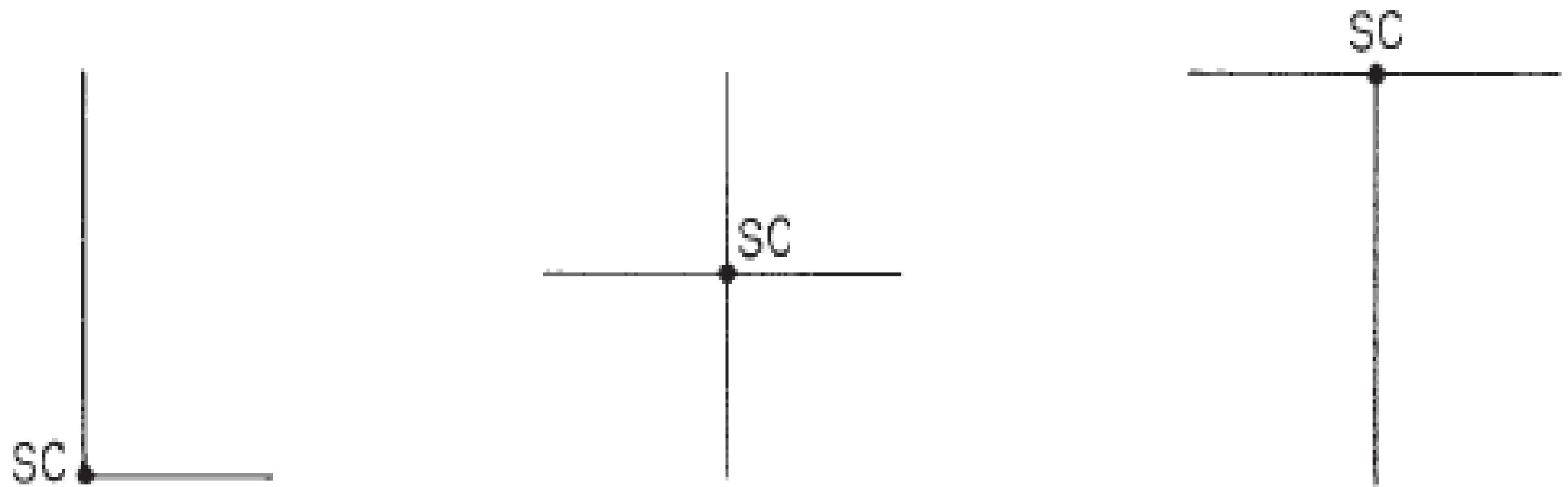

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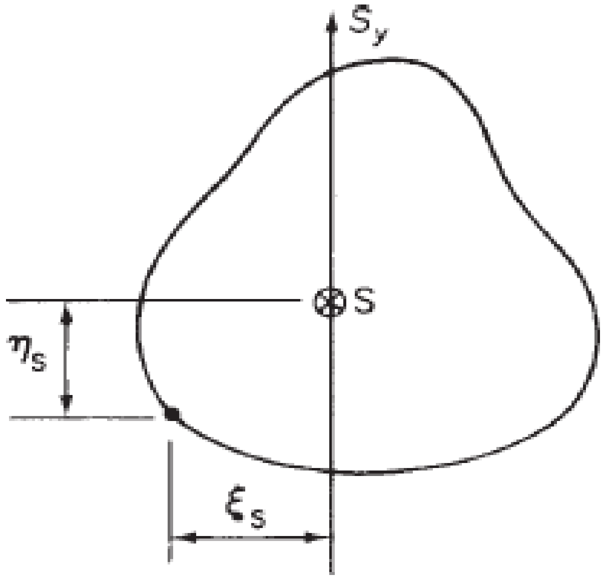
$$0 = \oint \frac{q_s}{Gt} \mathrm{d}s \qquad\qquad 0 = \oint \frac{1}{Gt}(q_{\mathrm{b}} + q_{s,0}) \mathrm{d}s$$

Source from Aircraft Structures by T. H. G. Megson



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$$q_{s,0} = -\frac{\oint (q_b/Gt) ds}{\oint ds/Gt}$$

The coordinate η_S is found in a similar manner by applying S_x through S.

Source from Aircraft Structures by T. H. G. Megson

Twist and warping of shear loaded closed section beams

Shear loads which are not applied through the shear centre of a closed section beam cause cross-sections to twist and warp, in addition to rotation, they suffer out of plane axial displacements.

Expressions for these quantities may be derived in terms of the shear flow distribution *qs* as follows.

Since $q = \tau t$ and $\tau = G\gamma$ then we can express *qs* in terms of the warping and tangential displacements *w* and *v*t of a point in the beam wall by using Eq.

$$q_s = Gt\left(\frac{\partial w}{\partial s} + \frac{\partial v_t}{\partial z}\right)$$

$$\frac{q_s}{Gt} = \frac{\partial w}{\partial s} + p\frac{\mathrm{d}\theta}{\mathrm{d}z} + \frac{\mathrm{d}u}{\mathrm{d}z}\cos\psi + \frac{\mathrm{d}v}{\mathrm{d}z}\sin\psi$$

$$\int_0^s \frac{q_s}{Gt}\mathrm{d}s = \int_0^s \frac{\partial w}{\partial s}\mathrm{d}s + \frac{\mathrm{d}\theta}{\mathrm{d}z}\int_0^s p\,\mathrm{d}s + \frac{\mathrm{d}u}{\mathrm{d}z}\int_0^s \cos\psi\,\mathrm{d}s + \frac{\mathrm{d}v}{\mathrm{d}z}\int_0^s \sin\psi\,\mathrm{d}s$$

$$\int_0^s \frac{q_s}{Gt}\mathrm{d}s = \int_0^s \frac{\partial w}{\partial s}\mathrm{d}s + \frac{\mathrm{d}\theta}{\mathrm{d}z}\int_0^s p\,\mathrm{d}s + \frac{\mathrm{d}u}{\mathrm{d}z}\int_0^s \mathrm{d}x + \frac{\mathrm{d}v}{\mathrm{d}z}\int_0^s \mathrm{d}y$$

Source from Aircraft Structures by T. H. G. Megson

$$\int_0^s \frac{q_s}{Gt}\mathrm{d}s = (w_s - w_0) + 2A_{Os}\frac{\mathrm{d}\theta}{\mathrm{d}z} + \frac{\mathrm{d}u}{\mathrm{d}z}(x_s - x_0) + \frac{\mathrm{d}v}{\mathrm{d}z}(y_s - y_0)$$

$$\oint \frac{q_s}{Gt}\mathrm{d}s = 2A\frac{\mathrm{d}\theta}{\mathrm{d}z} \qquad \frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{1}{2A}\oint \frac{q_s}{Gt}\mathrm{d}s$$

$$w_s - w_0 = \int_0^s \frac{q_s}{Gt}\mathrm{d}s - \frac{A_{Os}}{A}\oint \frac{q_s}{Gt}\mathrm{d}s - \frac{\mathrm{d}u}{\mathrm{d}z}(x_s - x_0) - \frac{\mathrm{d}v}{\mathrm{d}z}(y_s - y_0)$$

$$w_s - w_0 = \int_0^s \frac{q_s}{Gt}\mathrm{d}s - \frac{A_{Os}}{A}\oint \frac{q_s}{Gt}\mathrm{d}s - y_R\frac{\mathrm{d}\theta}{\mathrm{d}z}(x_s - x_0) + x_R\frac{\mathrm{d}\theta}{\mathrm{d}z}(y_s - y_0)$$

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$$w_s - w_0 = \int_0^s \frac{q_s}{Gt} \mathrm{d}s - \frac{A_{Os}}{A} \oint \frac{q_s}{Gt} \mathrm{d}s$$

$$\sigma = \text{constant} \times w$$

$$\text{Resultant axial load} = \oint \sigma t \, \mathrm{d}s$$

$$0 = \oint wt \, \mathrm{d}s$$

$$0 = \oint (w_s - w_0) t \, \mathrm{d}s$$

$$w_0 = \frac{\oint w_s t \, \mathrm{d}s}{\oint t \, \mathrm{d}s}$$

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Topics

Torsion of beams of closed section: Displacements associated with Bredt-Batho shear flow.

Torsion of open section beams; Warping of cross section, conditions for zero warping.

Bending, shear, torsion of combined open and closed section beams.

A closed section beam subjected to a pure torque *T* does not, in the absence of an axial constraint, develop a direct stress system.

It follows that the equilibrium conditions of Eqs

$$(\partial q/\partial s) + t(\partial \sigma z/\partial z) = 0 \text{ and}$$

$$(\partial q/\partial z) + t(\partial \sigma s/\partial s) = 0 \text{ reduce to}$$

$$\partial q/\partial s = 0 \text{ and } \partial q/\partial z = 0, \text{ respectively.}$$

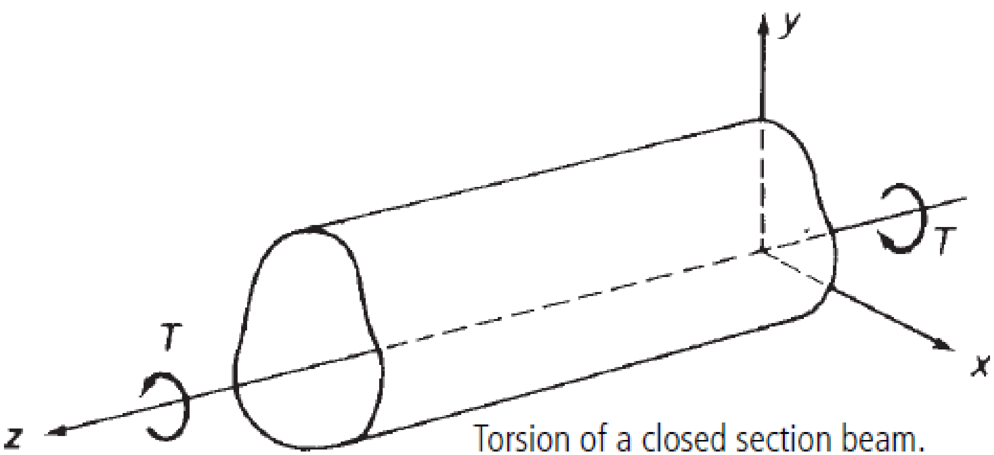
These relationships may only be satisfied simultaneously by a constant value of *q*.

Therefore, that the application of a pure torque to a closed section beam results in the development of a constant shear flow in the beam wall.

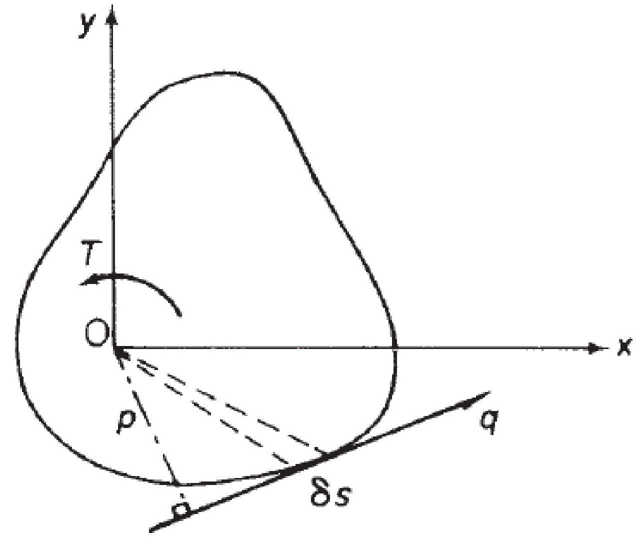
The shear stress τ may vary around the cross-section since we allow the wall thickness t to be a function of s.

The relationship between the applied torque and this constant shear flow is simply derived by considering the torsional equilibrium of the section shown in below Fig.

The torque produced by the shear flow acting on an element δs of Fig below



Torsion of a closed section beam.



Determination of the shear flow distribution in a closed section beam subjected to torsion.

the beam wall is $pq\delta s$. Hence

$$T = \oint pq\, ds$$

or, since q is constant and $\oint p\,ds = 2A$

$$T = 2Aq$$

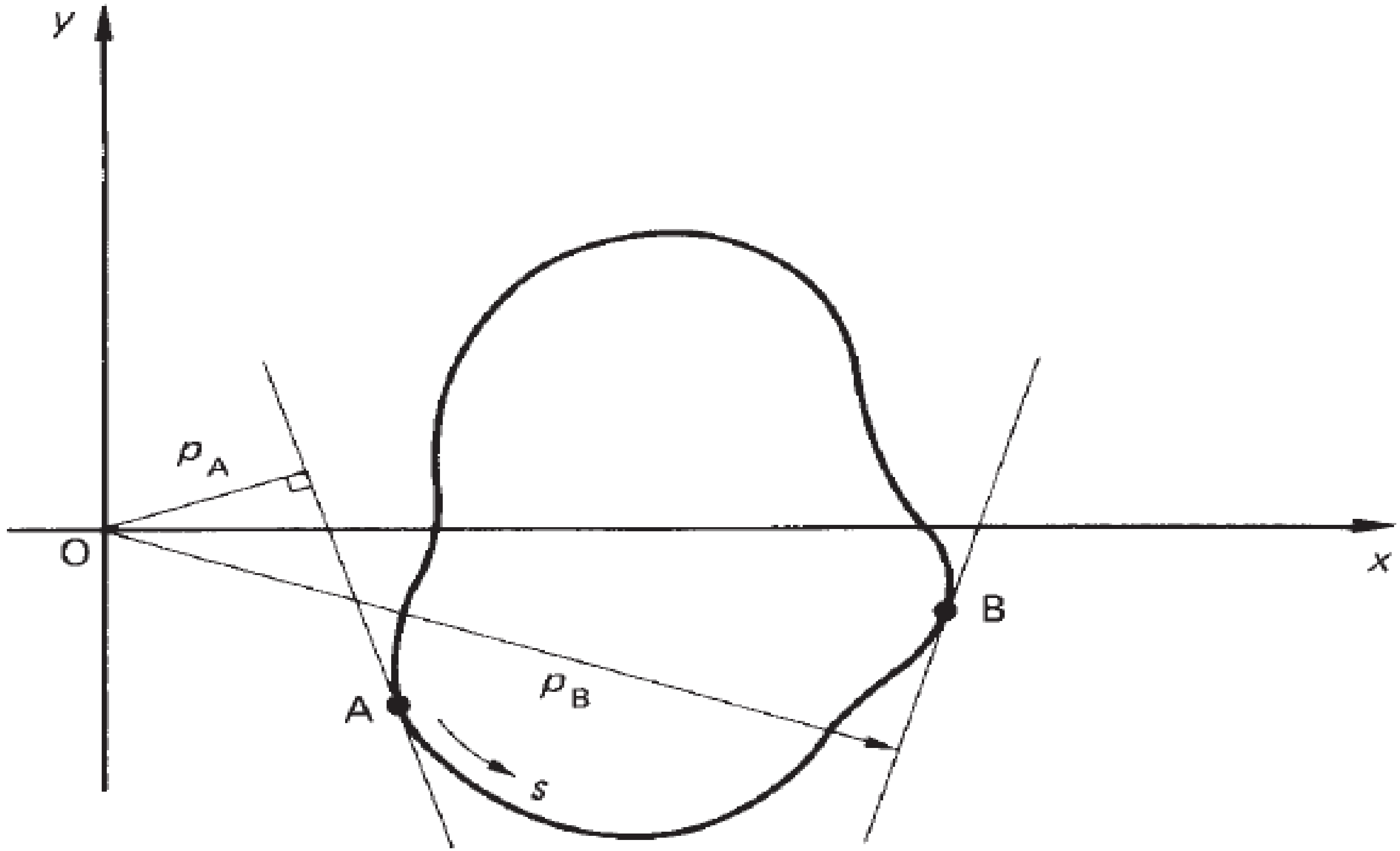
The origin O of the axes may be positioned in or outside the cross-section of the beam since the moment of the internal shear flows (whose resultant is a pure torque) is the same about any point in their plane.

For an origin outside the cross-section the term $p\, ds$ will involve the summation of positive and negative areas.

The sign of an area is determined by the sign of p which itself is associated with the sign convention for torque as follows.

If the movement of the foot of p along the tangent at any point in the positive direction of s leads to an anticlockwise rotation of p about the origin of axes, p is positive.

The positive direction of s is in the positive direction of q which is anticlockwise (corresponding to a positive torque).



Sign convention for swept areas.

In Fig, a generator OA, rotating about O, will initially sweep out a negative area since pA is negative.

At B, however, pB is positive so that the area swept out by the generator has changed sign (at the point where the tangent passes through O and p=0).

Positive and negative areas cancel each other out as they overlap so that as the generator moves completely around the section, starting and returning to A say, the resultant area is that enclosed by the profile of the beam.

The theory of the torsion of closed section beams is known as the *Bredt–Batho theory* and Eq. (18.1) is often referred to as the *Bredt–Batho formula*.

Torsion of closed section beams: Bredt-Batho Equation

A closed section beam subjected to a pure torque *T* does not, in the absence of an axial constraint, develop a direct stress system.

It follows that the equilibrium conditions of Eqs

$$(\partial q/\partial s) + t(\partial \sigma z/\partial z) = 0 \text{ and}$$

$$(\partial q/\partial z) + t(\partial \sigma s/\partial s) = 0 \text{ reduce to}$$

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These relationships may only be satisfied simultaneously by a constant value of *q*.

Therefore, that the application of a pure torque to a closed section beam results in the development of a constant shear flow in the beam wall.

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The torque produced by the shear flow acting on an element δs of Fig below

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The theory of the torsion of closed section beams is known as the *Bredt–Batho theory* and Eq. (18.1) is often referred to as the *Bredt–Batho formula*.

Displacements associated with the Bredt–Batho shear flow

The relationship between q and shear strain γ established in Eq., namely

$$q = Gt\left(\frac{\partial w}{\partial s} + \frac{\partial v_{\mathrm{t}}}{\partial z}\right)$$

is valid for the pure torsion case where q is constant. Differentiating this expression with respect to z we have

$$\frac{\partial q}{\partial z} = Gt\left(\frac{\partial^2 w}{\partial z\,\partial s} + \frac{\partial^2 v_t}{\partial z^2}\right) = 0 \quad \text{or} \quad \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial z}\right) + \frac{\partial^2 v_t}{\partial z^2} = 0$$

In the absence of direct stresses the longitudinal strain $\partial w/\partial z(=\varepsilon z)$ is zero so that

$$\frac{\partial^2 v_t}{\partial z^2} = 0$$ Hence from Eq. $$v_t = p\theta + u\cos\psi + v\sin\psi$$

$$p\frac{d^2\theta}{dz^2} + \frac{d^2 u}{dz^2}\cos\psi + \frac{d^2 v}{dz^2}\sin\psi = 0$$

For above Eq. to hold for all points around the section wall, in other words for all values of ψ

$$\frac{d^2\theta}{dz^2} = 0, \quad \frac{d^2 u}{dz^2} = 0, \quad \frac{d^2 v}{dz^2} = 0$$

It follows that $\vartheta = Az+B$, $u=Cz+D$, $v=Ez+F$, where *A, B, C, D, E* and *F* are unknown constants. Thus ϑ, *u* and *v* are all linear functions of *z*.

Equation for rate of twist is

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{1}{2A}\oint \frac{q_s}{Gt}\mathrm{d}s$$

relating the rate of twist to the variable shear flow *qs* developed in a shear loaded closed section beam, is also valid for the case *qs* =*q*=constant.

Hence

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{q}{2A}\oint \frac{\mathrm{d}s}{Gt}$$

which becomes, on substituting for *q* from Eq. T = 2Aq

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = \frac{T}{4A^2}\oint \frac{\mathrm{d}s}{Gt}$$

The warping distribution produced by a varying shear flow, as defined by Eq.

$$w_s - w_0 = \int_0^s \frac{q_s}{Gt} \mathrm{d}s - \frac{A_{Os}}{A} \oint \frac{q_s}{Gt} \mathrm{d}s$$

for axes having their origin at the centre of twist, is also applicable to the case of a constant shear flow.

$$w_s - w_0 = q \int_0^s \frac{\mathrm{d}s}{Gt} - \frac{A_{Os}}{A} q \oint \frac{\mathrm{d}s}{Gt}$$

Thus

Replacing *q* from Eq. T = 2Aq we have

$$w_s - w_0 = \frac{T\delta}{2A} \left(\frac{\delta_{Os}}{\delta} - \frac{A_{Os}}{A} \right)$$

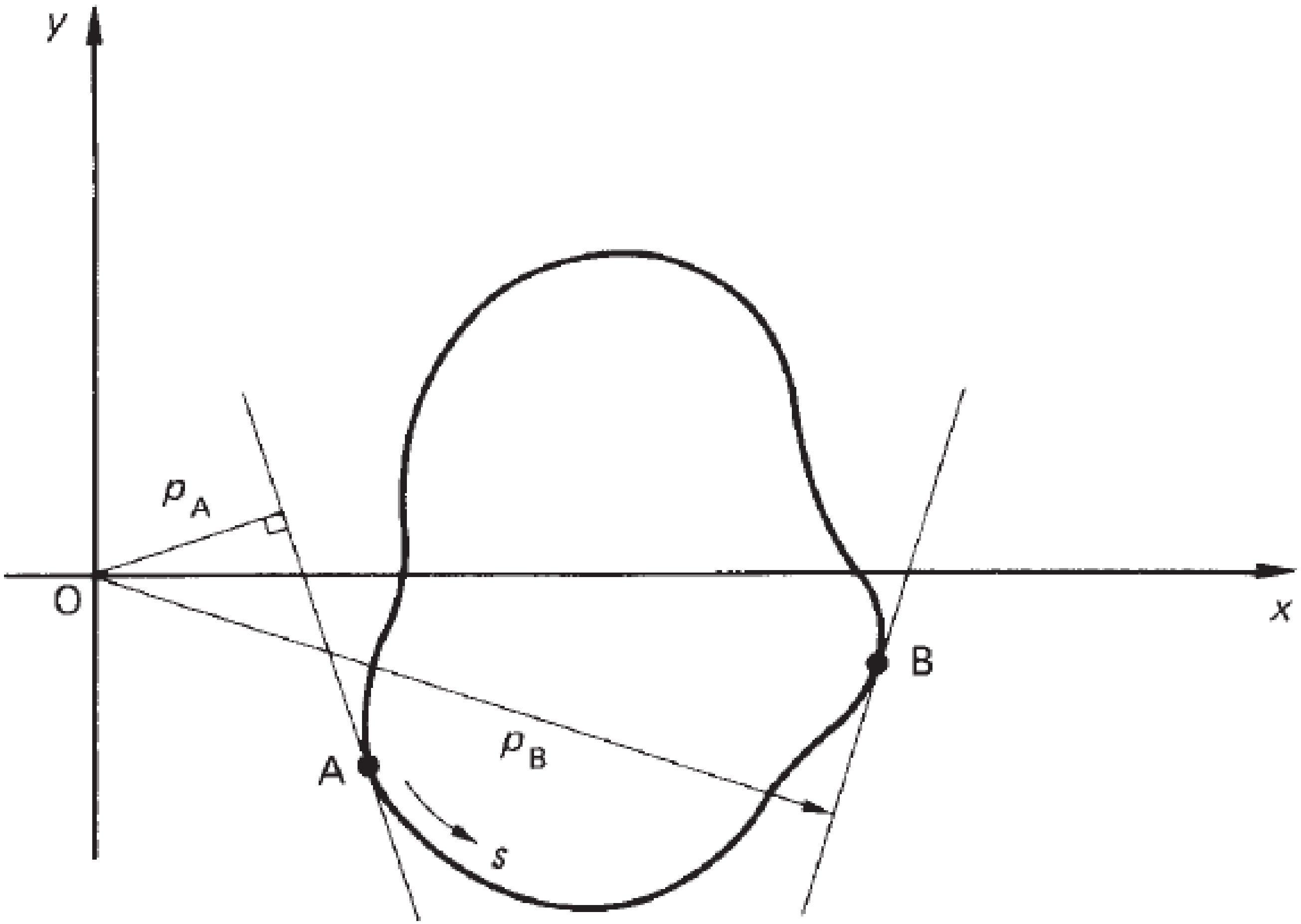
$$\delta = \oint \frac{ds}{Gt} \quad \text{and} \quad \delta_{Os} = \int_0^s \frac{ds}{Gt}$$

The sign of the warping displacement in Eq.

$$w_s - w_0 = \frac{T\delta}{2A}\left(\frac{\delta_{Os}}{\delta} - \frac{A_{Os}}{A}\right)$$

is governed by the sign of the applied torque *T* and the signs of the parameters δOs and AOs.

Having specified initially that a positive torque is anticlockwise, the signs of δOs and AOs are fixed in that δOs is positive when *s* is positive, i.e. *s* is taken as positive in an anticlockwise sense, and *Aos* is positive when, as before, *p* (Fig. below) is positive.



Sign convention for swept areas.

Noted that the longitudinal strain εz is zero in a closed section beam subjected to a pure torque.

This means that all sections of the beam must possess identical warping distributions.

In other words longitudinal generators of the beam surface remain unchanged in length although subjected to axial displacement.

Torsion of open section beams

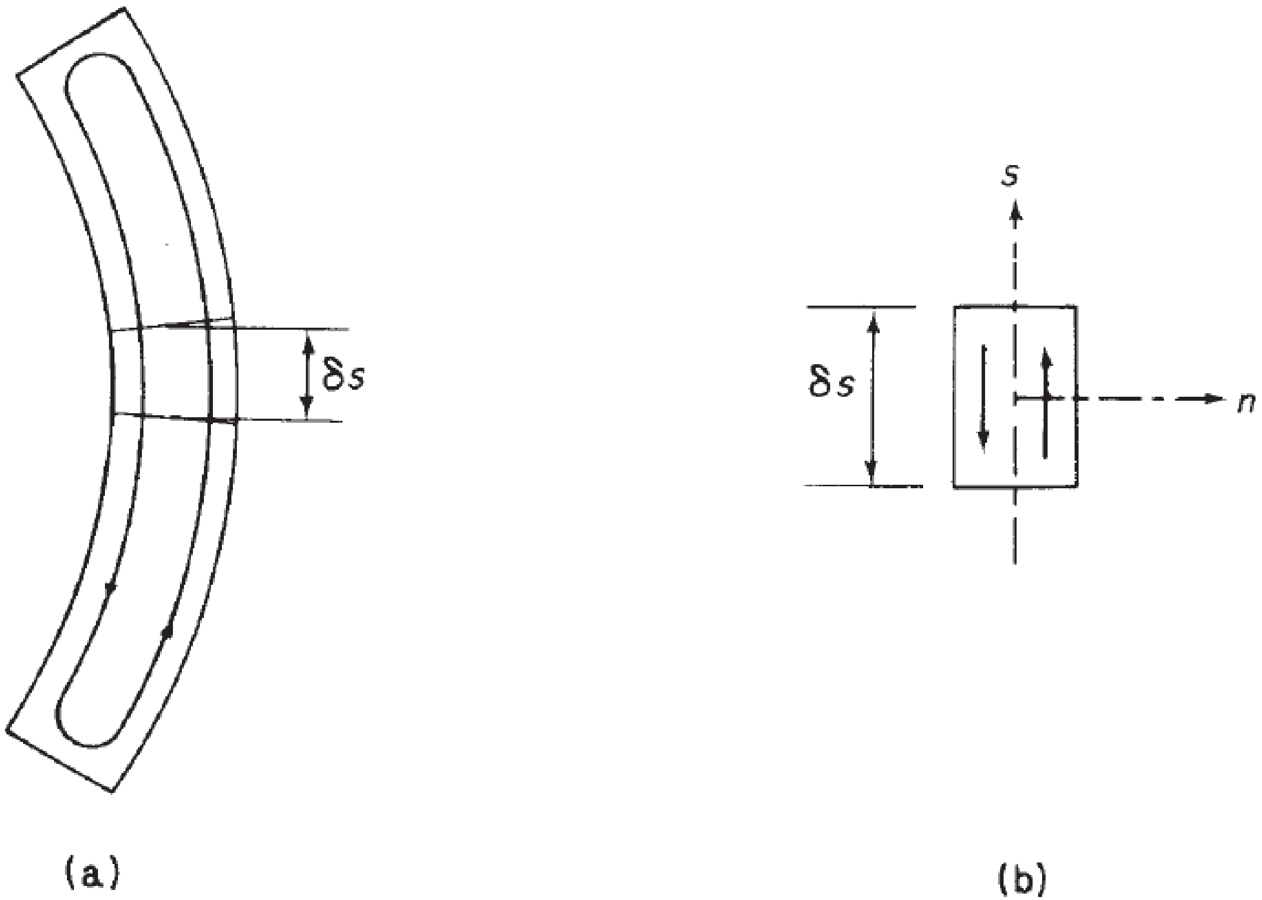
An approximate solution for the torsion of a thin-walled open section beam may be found by applying the results obtained for the torsion of a thin rectangular strip.

If such a strip is bent to form an open section beam, as shown in Fig. in next slide and if the distance s measured around the cross-section is large compared with its thickness *t* then the contours of the membrane,

i.e. lines of shear stress, are still approximately parallel to the inner and outer boundaries.

It follows that the shear lines in an element δs of the open section must be nearly the same as those in an element δy of a rectangular strip

Source from Aircraft Structures by T. H. G. Megson



(a) Shear lines in a thin-walled open section beam subjected to torsion;
(b) approximation of elemental shear lines to those in a thin rectangular strip.

Source from Aircraft Structures by T. H. G. Megson

$$\tau_{zs} = 2Gn\frac{\mathrm{d}\theta}{\mathrm{d}z}, \quad \tau_{zn} = 0$$

$$\tau_{zs,\max} = \pm Gt\frac{\mathrm{d}\theta}{\mathrm{d}z}$$

$$J = \sum \frac{st^3}{3} \quad \text{or} \quad J = \frac{1}{3}\int_{\text{sect}} t^3\,\mathrm{d}s$$

Source from Aircraft Structures by T. H. G. Megson

$$J = \sum \frac{st^3}{3} \quad \text{or} \quad J = \frac{1}{3}\int_{\text{sect}} t^3 \, \mathrm{d}s$$

Above equation the second expression for the torsion constant is used if the cross-section has a variable wall thickness.

Finally, the rate of twist is expressed in terms of the applied torque

$$T = GJ\frac{\mathrm{d}\theta}{\mathrm{d}z}$$

Source from Aircraft Structures by T. H. G. Megson

The shear stress distribution and the maximum shear stress are sometimes more conveniently expressed in terms of the applied torque.

Therefore, substituting for dϑ/dz in Eqs

$$\tau_{zs} = 2Gn\frac{\mathrm{d}\theta}{\mathrm{d}z}, \quad \tau_{zn} = 0$$

and

$$\tau_{zs,\max} = \pm Gt\frac{\mathrm{d}\theta}{\mathrm{d}z}$$

gives

$$\tau_{zs} = \frac{2n}{J}T, \quad \tau_{zs,\max} = \pm\frac{tT}{J}$$

Source from Aircraft Structures by T. H. G. Megson

We assume in open beam torsion analysis that the cross-section is maintained by the system of closely spaced diaphragms described and that the beam is of uniform section.

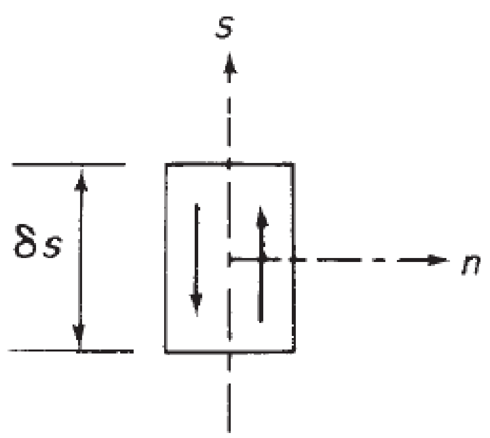
Clearly, in this problem the shear stresses vary across the thickness of the beam wall whereas other stresses such as axial constraint stresses are assumed constant across the thickness.

Warping of the cross-section

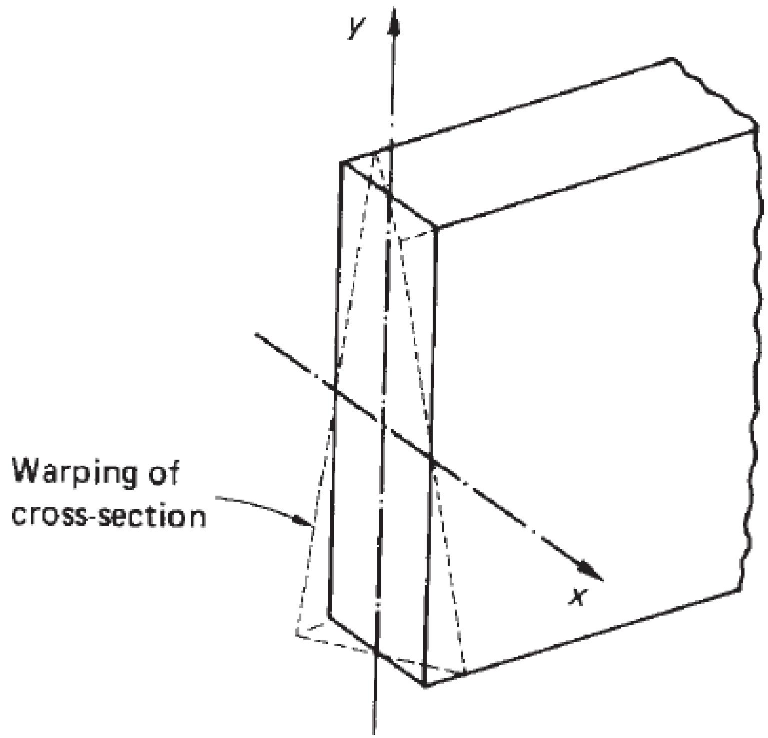
The thin rectangular strip suffers warping across its thickness when subjected to torsion.

In the same way a thin-walled open section beam will warp across its thickness.

This warping, *w*t, may be deduced by comparing Fig.



with Fig. next slide



and using Eq.

$$w = xy\frac{d\theta}{dz}$$

$$w_t = ns\frac{d\theta}{dz}$$

In addition to warping across the thickness, the cross-section of the beam will warp in a similar manner to that of a closed section beam.

$$\gamma_{zs} = \frac{\partial w}{\partial s} + \frac{\partial v_{\mathrm{t}}}{\partial z}$$

Referring the tangential displacement v_t to the centre of twist R of the cross-section we have, from Eq

$$v_{\mathrm{t}} = p_{\mathrm{R}}\theta$$

$$\frac{\partial v_{\mathrm{t}}}{\partial z} = p_{\mathrm{R}} \frac{\mathrm{d}\theta}{\mathrm{d}z}$$

Substituting for *∂vt/∂z*

$$\gamma_{zs} = \frac{\partial w}{\partial s} + p_{\mathrm{R}} \frac{\mathrm{d}\theta}{\mathrm{d}z}$$

$$\tau_{zs} = G\left(\frac{\partial w}{\partial s} + p_{\mathrm{R}} \frac{\mathrm{d}\theta}{\mathrm{d}z}\right)$$

On the mid-line of the section wall $\tau_{zs} = 0$

$$\frac{\partial w}{\partial s} = -p_{\mathrm{R}} \frac{\mathrm{d}\theta}{\mathrm{d}z}$$

Integrating this expression with respect to *s* and taking the lower limit of integration to coincide with the point of zero warping, we obtain

$$w_s = -\frac{d\theta}{dz}\int_0^s p_R \, ds$$

It can be seen that two types of warping exist in an open section beam. The above equation gives the warping of the mid-line of the beam; this is known as *primary warping* and is assumed to be constant across the wall thickness.

Equation

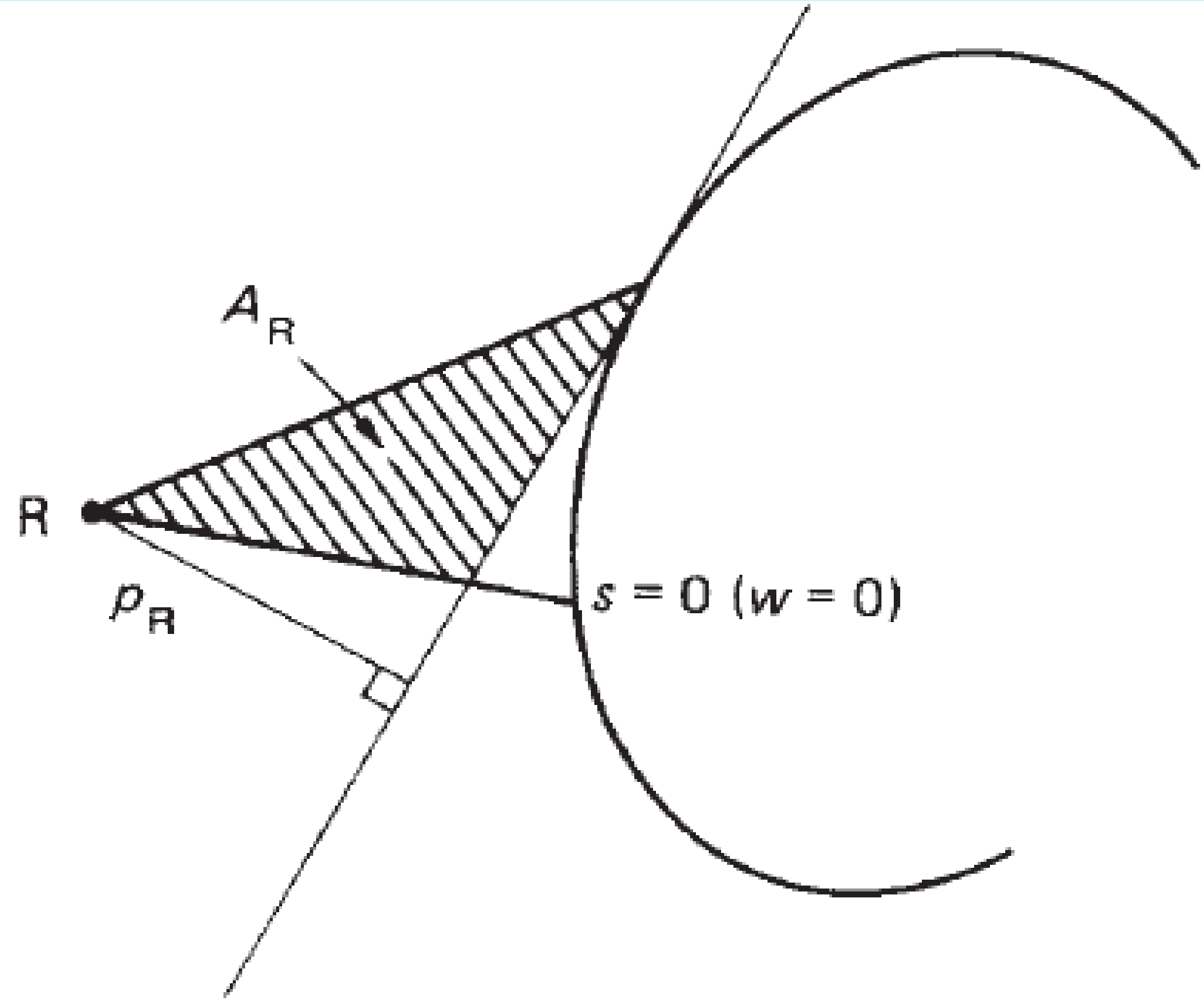
$$w_t = ns\frac{d\theta}{dz}$$

gives the warping of the beam across its wall thickness.

This is called *secondary warping*, is very much less than primary warping and is usually ignored in the thin-walled sections common to aircraft structures.

Equation $w_s = -\frac{d\theta}{dz}\int_0^s p_R \, ds$ may be rewritten in the form

$$w_s = -2A_R\frac{d\theta}{dz}$$



Warping of an open section beam.

or, in terms of the applied torque

$$w_s = -2A_R \frac{T}{GJ}$$

in which $A_R = \frac{1}{2} \int_0^s p_R \, ds$ is the area swept out by a generator, rotating about the centre of twist, from the point of zero warping .

The sign of w_s, for a given direction of torque, depends upon the sign of A_R which in turn depends upon the sign of P_R.

The perpendicular distance from the center of twist to the tangent at any point.

Again, as for closed section beams, the sign of p_R depends upon the assumed direction of a positive torque, in this case anticlockwise.

Therefore, p_R (and therefore A_R) is positive if movement of the foot of p_R along the tangent in the assumed direction of s leads to an anticlockwise rotation of p_R about the center of twist.

For open section beams the positive direction of s may be chosen arbitrarily since, for a given torque, the sign of the warping displacement depends only on the sign of the swept area A_R.

Condition for zero warping at a section

The geometry of the cross-section of a closed section beam subjected to torsion may be such that no warping of the cross-section occurs.

This condition arises when $$\frac{\delta_{\mathrm{O}s}}{\delta} = \frac{A_{\mathrm{O}s}}{A}$$ or

$$\frac{1}{\delta}\int_0^s \frac{\mathrm{d}s}{Gt} = \frac{1}{2A}\int_0^s p_{\mathrm{R}}\,\mathrm{d}s$$

Differentiating above Eq. with respect to *s* gives

$$\frac{1}{\delta Gt} = \frac{p_{\mathrm{R}}}{2A}$$

or

$$p_{\mathrm{R}}Gt = \frac{2A}{\delta} = \text{constant}$$

A closed section beam for which $p_R Gt$ =constant does not warp and is known as a *Neuber beam*.

For closed section beams having a constant shear modulus the condition becomes $p_R t$ = constant

Examples of such beams are a circular section beam of constant thickness, a rectangular section beam for which $at_b = bt_a$, and a triangular section beam of constant thickness.

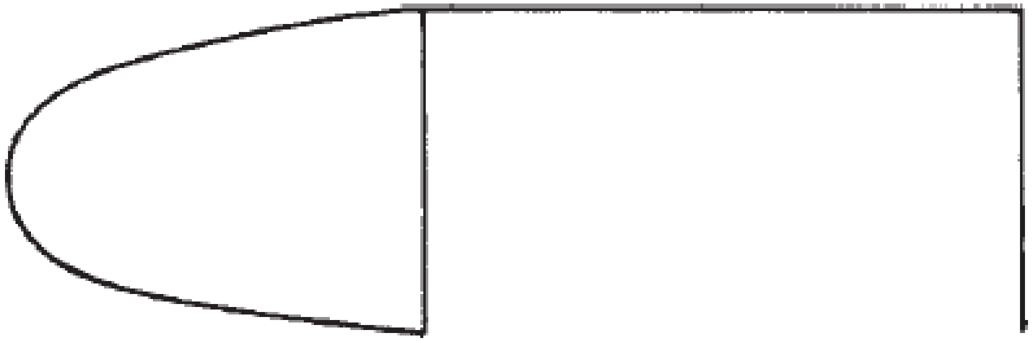
In the last case the shear centre and hence the centre of twist may be shown to coincide with the centre of the inscribed circle so that p_R for each side is the radius of the inscribed circle.

Combined open and closed section beams

So far, we have analysed thin-walled beams which consist of either completely closed cross-sections or completely open cross-sections.

Frequently aircraft components comprise combinations of open and closed section beams.

For example the section of a wing in the region of an undercarriage bay could take the form shown in Fig.



Wing section comprising open and closed components.

Clearly part of the section is an open channel section while the nose portion is a single cell closed section.

We shall now examine the methods of analysis of such sections when subjected to bending, shear and torsional loads.

Bending

It is immaterial what form the cross-section of a beam takes; the direct stresses due to bending are given by either of below Eqs.

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y \qquad \sigma_z = \frac{M_x (I_{yy} y - I_{xy} x)}{I_{xx} I_{yy} - I_{xy}^2} + \frac{M_y (I_{xx} x - I_{xy} y)}{I_{xx} I_{yy} - I_{xy}^2}$$

Shear

The methods described in **Shear of open section Beams** and **Shear of closed section Beams** are used to determine the shear stress distribution.

Although, unlike the completely closed section case, shear loads must be applied through the shear centre of the combined section.

Otherwise shear stresses of the type described in **Torsion of open section Beams** due to torsion will arise.

Where shear loads do not act through the shear centre its position must be found and the loading system replaced

By shear loads acting through the shear centre together with a torque; the two loading cases are then analysed separately.

Again we assume that the cross-section of the beam remains undistorted by the loading.

Torsion

Generally, in the torsion of composite sections, the closed portion is dominant

since its torsional stiffness is far greater than that of the attached open section portion

which may therefore be frequently ignored in the calculation of torsional stiffness;

shear stresses should, however, be checked in this part of the section.

Bending, shear, torsion of combined open and closed section beams.

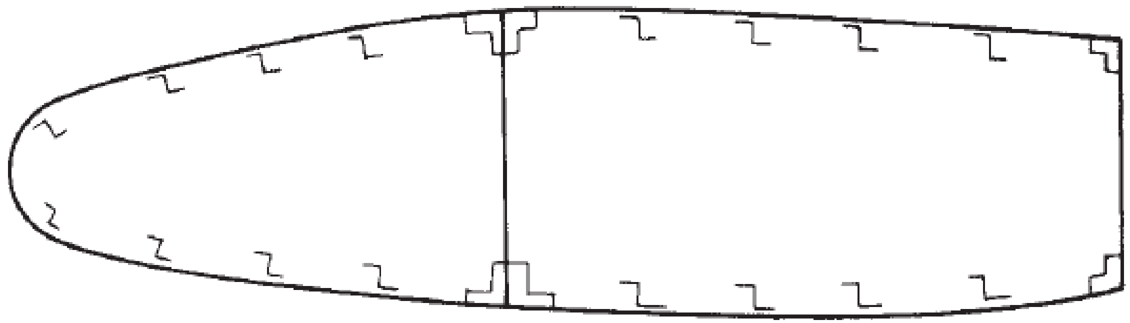
Problems

Structural idealization

So far we have been concerned with relatively uncomplicated structural sections which in practice would be formed from thin plate or by the extrusion process.

While these sections exist as structural members in their own right they are frequently used, to stiffen more complex structural shapes such as fuselages, wings and tail surfaces.

Thus a two spar wing section could take the form shown in below Fig.



Typical wing section.

Source from Aircraft Structures by T. H. G. Megson

Above fig. Z-section stringers are used to stiffen the thin skin while angle sections form the spar flanges.

Clearly, the analysis of a section of this type would be complicated and tedious unless some simplifying assumptions are made.

Generally, the number and nature of these simplifying assumptions determine the accuracy and the degree of complexity of the analysis; the more complex the analysis the greater the accuracy obtained.

The degree of simplification introduced is governed by the particular situation surrounding the problem.

For a preliminary investigation, speed and simplicity are often of greater importance than extreme accuracy; on the other hand a final solution must be as exact as circumstances allow.

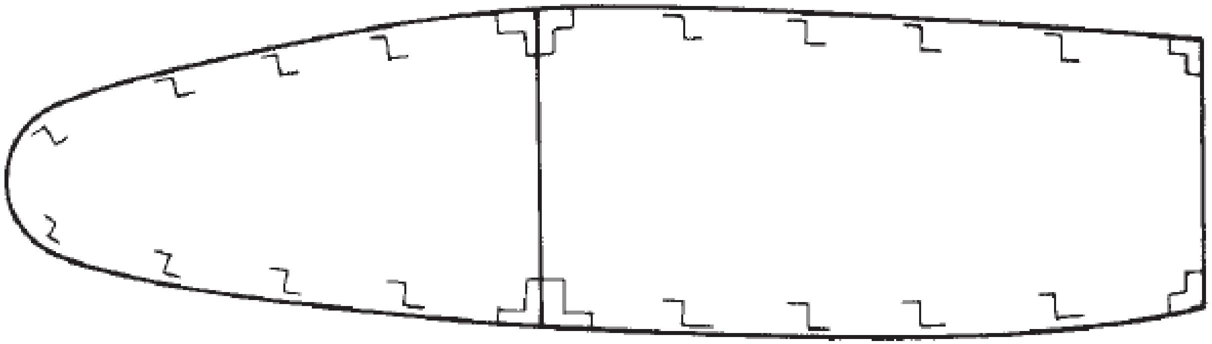
Complex structural sections may be idealized into simpler ‘mechanical model’forms which behave, under given loading conditions, in the same, or very nearly the same, way as the actual structure.

We shall see, that different models of the same structure are required to simulate actual behaviour under different systems of loading.

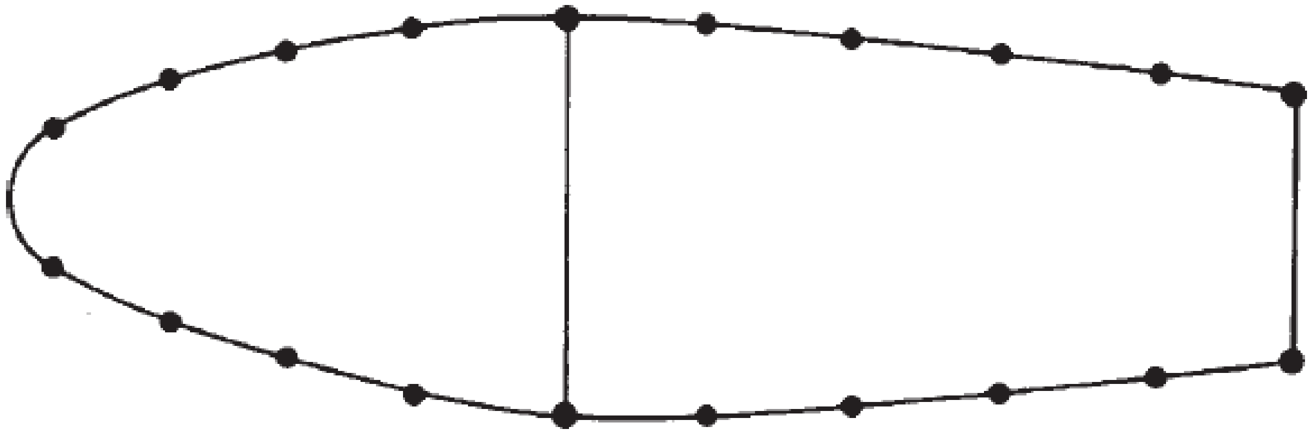
Source from Aircraft Structures by T. H. G. Megson

Principle

In the wing section of Fig. below the stringers and spar flanges have small cross-sectional dimensions compared with the complete section.



| Typical wing section.



Idealization of a wing section.

Source from Aircraft Structures by T. H. G. Megson

The variation in stress over the cross-section of a stringer due to, say, bending of the wing would be small.

The difference between the distances of the stringer centroids and the adjacent skin from the wing section axis is small.

It would be reasonable to assume therefore that the direct stress is constant over the stringer cross-sections.

We could therefore replace the stringers and spar flanges by concentrations of area, known as *booms*,

over which the direct stress is constant and which are located along the midline of the skin, as shown in above Fig.

Source from Aircraft Structures by T. H. G. Megson

In wing and fuselage sections of the type shown in above Fig., the stringers and spar flanges carry most of the direct stresses while the skin is mainly effective in resisting shear stresses although it also carries some of the direct stresses.

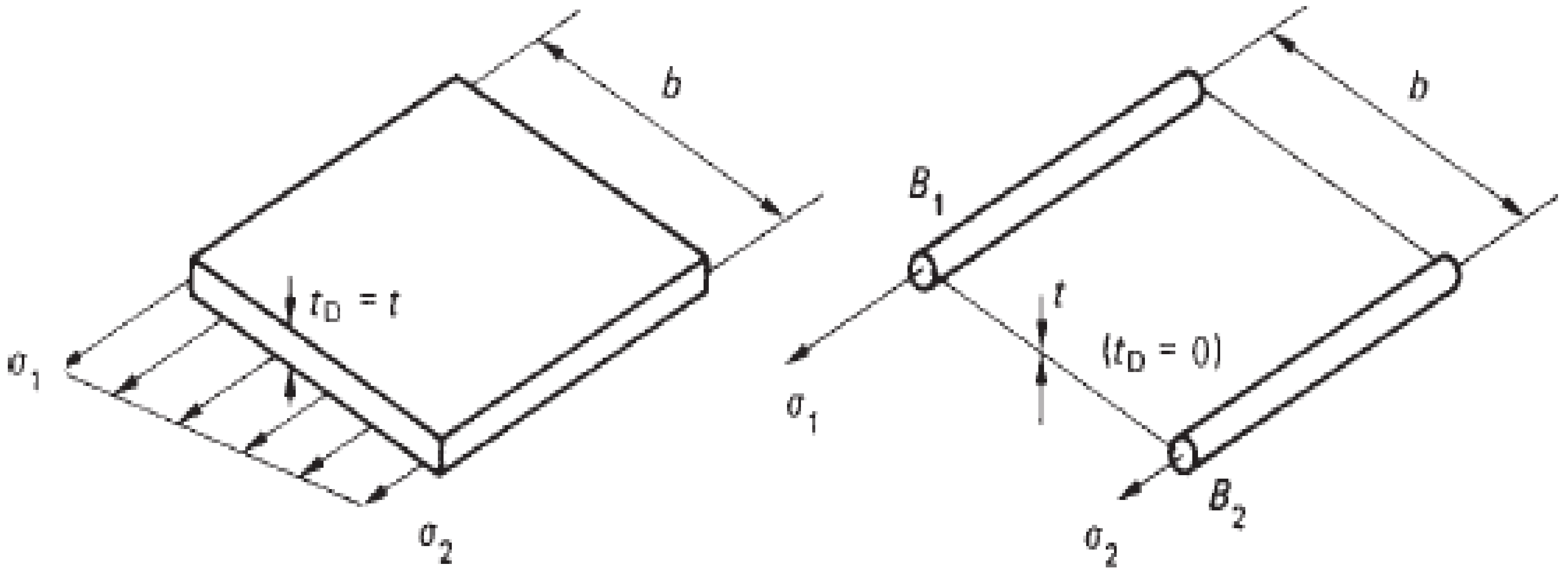
The idealization shown in above Fig. may therefore be taken a stage further by assuming that all direct stresses are carried by the booms while the skin is effective only in shear.

The direct stress carrying capacity of the skin may be allowed for by increasing each boom area by an area equivalent to the direct stress carrying capacity of the adjacent skin panels.

The calculation of these equivalent areas will generally depend upon an initial assumption as to the form of the distribution of direct stress in a boom/skin panel.

Source from Aircraft Structures by T. H. G. Megson

Idealization of a panel



(a) Actual (b) Idealized

Idealization of a panel.

Source from Aircraft Structures by T. H. G. Megson

Idealization of a panel

Wish to idealize the panel of above Fig. into a combination of direct stress carrying booms and shear stress only carrying skin as shown in above Fig.

The direct stress carrying thickness t_D of the skin is equal to its actual thickness t while t_D =0.

Suppose also that the direct stress distribution in the actual panel varies linearly from an unknown value σ_1 to an unknown value σ_2.

Clearly the analysis should predict the extremes of stress σ_1 and σ_2 although the distribution of direct stress is obviously lost.

Source from Aircraft Structures by T. H. G. Megson

Since the loading producing the direct stresses in the actual and idealized panels must be the same

we can equate moments to obtain expressions for the boom areas B_1 and B_2.

Thus, taking moments about the right-hand edge of each panel,

$$\sigma_2 t_D \frac{b^2}{2} + \frac{1}{2}(\sigma_1 - \sigma_2) t_D b \frac{2}{3} b = \sigma_1 B_1 b$$

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) \qquad B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

Source from Aircraft Structures by T. H. G. Megson

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) \qquad B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

In above Eqs, the ratio of σ_1 to σ_2, if not known, may frequently be assumed.

The direct stress distribution in above Fig. is caused by a combination of axial load and bending moment.

For axial load only $\sigma_1/\sigma_2 = 1$ and $B_1 = B_2 = t_D b/2$; for a pure bending moment $\sigma_1/\sigma_2 = -1$ and $B_1 = B_2 = t_D b/6$.

Thus, different idealizations of the same structure are required for different loading conditions.

Source from Aircraft Structures by T. H. G. Megson

Effect of idealization on the analysis of open and closed section beams

The addition of direct stress carrying booms to open and closed section beams will clearly modify the analyses presented.

Before considering individual cases we shall discuss the implications of structural idealization.

Generally, in any idealization, different loading conditions require different idealizations of the same structure.

Suppose the loading is applied in a vertical plane. If, however, the loading had been applied in a horizontal plane the assumed stress distribution in the panels of the section would have been different, resulting in different values of boom area.

Source from Aircraft Structures by T. H. G. Megson

Suppose that an open or closed section beam is subjected to given bending or shear loads and that the required idealization has been completed.

The analysis of such sections usually involves the determination of the neutral axis position and the calculation of sectional properties.

The position of the neutral axis is derived from the condition that the resultant load on the beam cross-section is zero, i.e.

$$\int_A \sigma_z \, \mathrm{d}A = 0$$

The area *A* in this expression is clearly the direct stress carrying area. It follows that the centroid of the cross-section is the centroid of the direct stress carrying area of the section, depending on the degree and method of idealization. The sectional properties, *Ixx*, etc., must also refer to the direct stress carrying area.

Source from Aircraft Structures by T. H. G. Megson

Example

The fuselage section shown in Fig. is subjected to a bending moment of 100 kNm applied in the vertical plane of symmetry. If the section has been completely idealized into a combination of direct stress carrying booms and shear stress only carrying panels, determine the direct stress in each boom.

The section has C*y* as an axis of symmetry and resists a bending moment M_x =100 kN m. Equation therefore

$$\sigma_z = \frac{M_x}{I_{xx}} y$$

The origin of axes C_{xy} coincides with the position of the centroid of the direct stress carrying area which, in this case, is the centroid of the boom areas.

Thus, taking moments of area about boom 9

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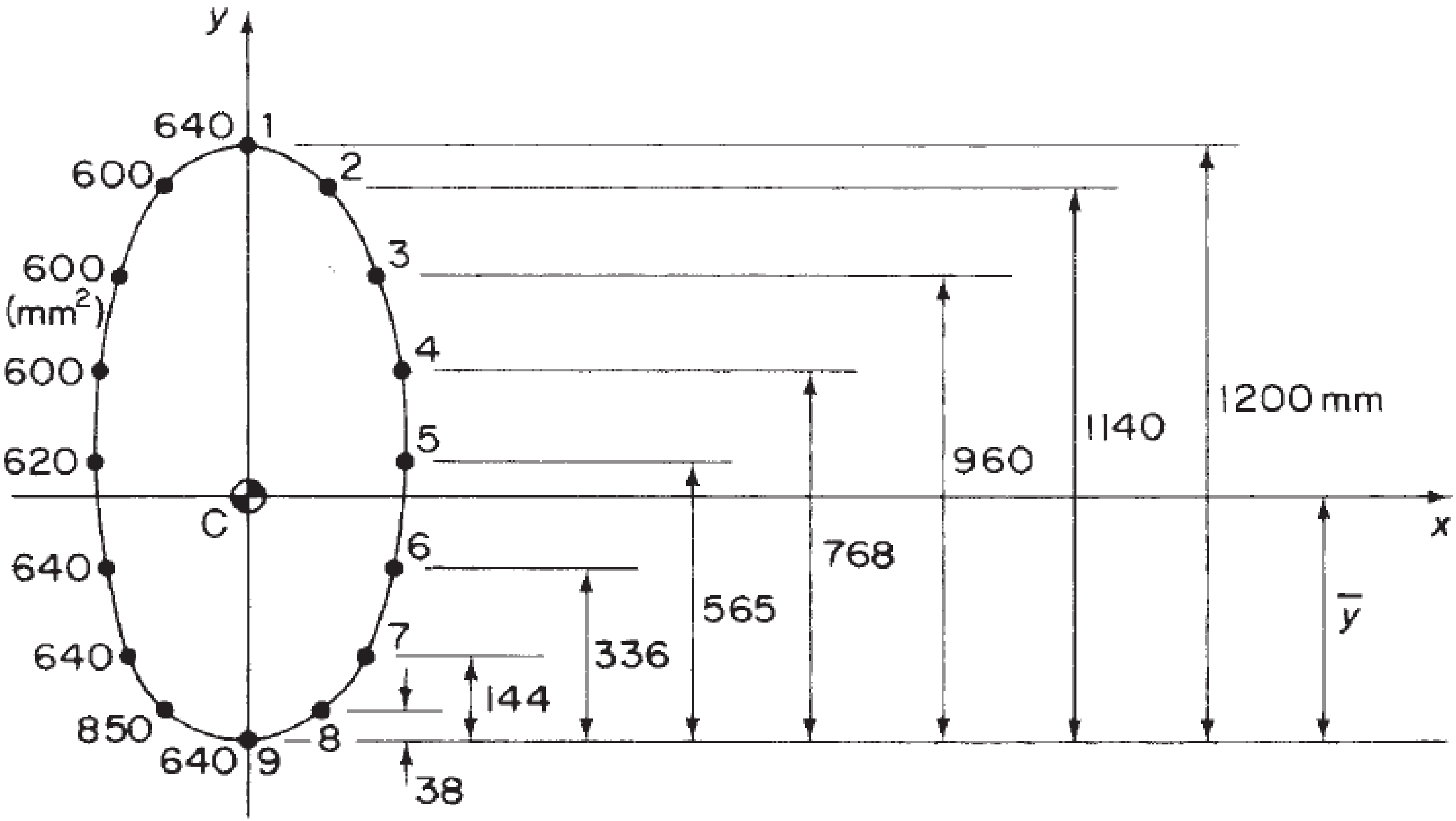


Idealized fuselage section

Source from Aircraft Structures by T. H. G. Megson

$(6 \times 640 + 6 \times 600 + 2 \times 620 + 2 \times 850)y$

$= 640 \times 1200 + 2 \times 600 \times 1140 + 2 \times 600 \times 960 + 2 \times 600 \times 768$

$+ 2 \times 620 \times 565 + 2 \times 640 \times 336 + 2 \times 640 \times 144 + 2 \times 850 \times 38$

① Boom	② y (mm)	③ B (mm^2)	④ $\Delta I_{xx} = By^2$ (mm^4)	⑤ σ_z (N/mm^2)
1	+660	640	278×10^6	35.6
2	+600	600	216×10^6	32.3
3	+420	600	106×10^6	22.6
4	+228	600	31×10^6	12.3
5	+25	620	0.4×10^6	1.3
6	−204	640	27×10^6	−11.0
7	−396	640	100×10^6	−21.4
8	−502	850	214×10^6	−27.0
9	−540	640	187×10^6	−29.0

Source from Aircraft Structures by T. H. G. Megson

which gives y = 540mm

The solution is now completed in above Table

From column ④ $I_{xx} = 1854 \times 106$ mm4

Effect of idealization on the analysis of open and closed section beams

The direct stress carrying booms to open and closed section beams .

Generally, in any idealization, different loading conditions require different idealizations of the same structure.

The loading had been applied in a horizontal plane the assumed stress distribution in the panels of the section would have been different, resulting in different values of boom area.

Suppose that an open or closed section beam is subjected to given bending or shear loads and that the required idealization has been completed.

The analysis of such sections usually involves the determination of the neutral axis position and the calculation of sectional properties

The position of the neutral axis is derived from the condition that the resultant load on the beam cross-section is zero, i.e.

$$\int_A \sigma_z \, \mathrm{d}A = 0$$

The area *A* in this expression is clearly the direct stress carrying area.

It follows that the centroid of the cross-section is the centroid of the direct stress carrying area of the section, depending on the degree and method of idealization.

The sectional properties, *Ixx*, etc., must also refer to the direct stress carrying area.

Bending of open and closed section beams

The direct stress distribution is given by any of Eqs

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y \qquad \sigma_z = \frac{M_x (I_{yy} y - I_{xy} x)}{I_{xx} I_{yy} - I_{xy}^2} + \frac{M_y (I_{xx} x - I_{xy} y)}{I_{xx} I_{yy} - I_{xy}^2}$$

depending on the beam section being investigated.

In these equations the coordinates (*x*, *y*) of points in the cross-section are referred to axes having their origin at the centroid of the direct stress carrying area.

Furthermore, the section properties *Ixx*, *Iyy* and *Ixy* are calculated for the direct stress carrying area only.

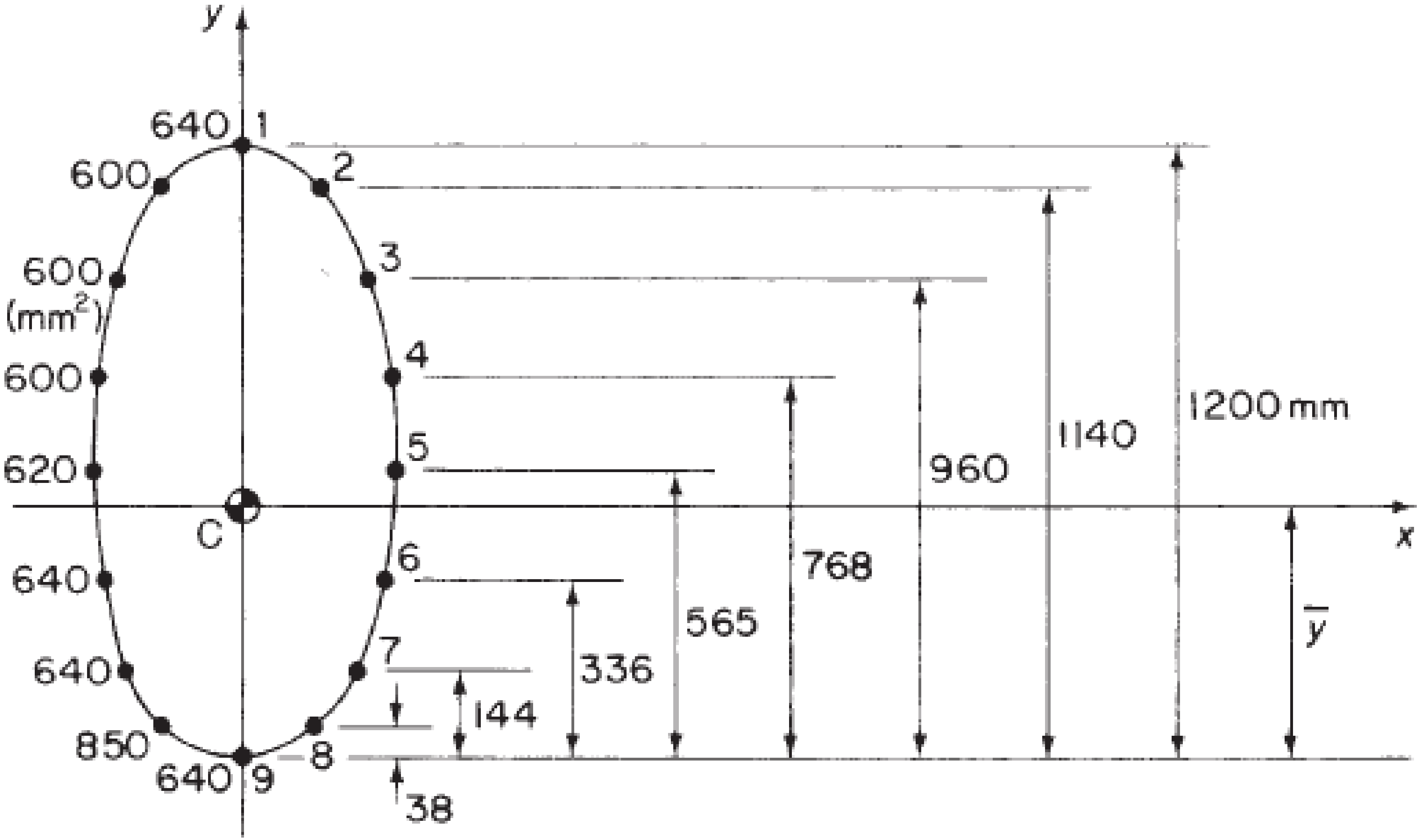
In the case where the beam cross-section has been completely idealized into direct stress carrying booms and shear stress only carrying panels, the direct stress distribution consists of a series of direct stresses concentrated at the centroids of the booms.

Ex: The fuselage section shown in Fig. below is subjected to a bending moment of 100 kNm applied in the vertical plane of symmetry.

If the section has been completely idealized into a combination of direct stress carrying booms and shear stress only carrying panels, determine the direct stress in each boom.

The section has C*y* as an axis of symmetry and resists a bending moment *Mx* =100 kN m. Equation therefore reduces to

$$\sigma_z = \frac{M_x}{I_{xx}} y$$


y
640
1
600
2
600
(mm²)
3
600
4
620
5
C
x
640
6
640
7
850
8
640
9
38
144
336
565
768
960
1140
1200 mm
$\bar{y}$

Shear of open section beams

The derivation for the shear flow distribution in the cross-section of an open section beam is based on the equilibrium equation.

The thickness t in this equation refers to the direct stress carrying thickness t_D of the skin.

Equation may therefore be rewritten

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D x \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D y \, ds$$

Source from Aircraft Structures by T. H. G. Megson

$t_D = t$ if the skin is fully effective in carrying direct stress or $t_D = 0$ if the skin is assumed to carry only shear stresses.

Again the section properties in above Eq. refer to the direct stress carrying area of the section since they are those which feature in Eqs.

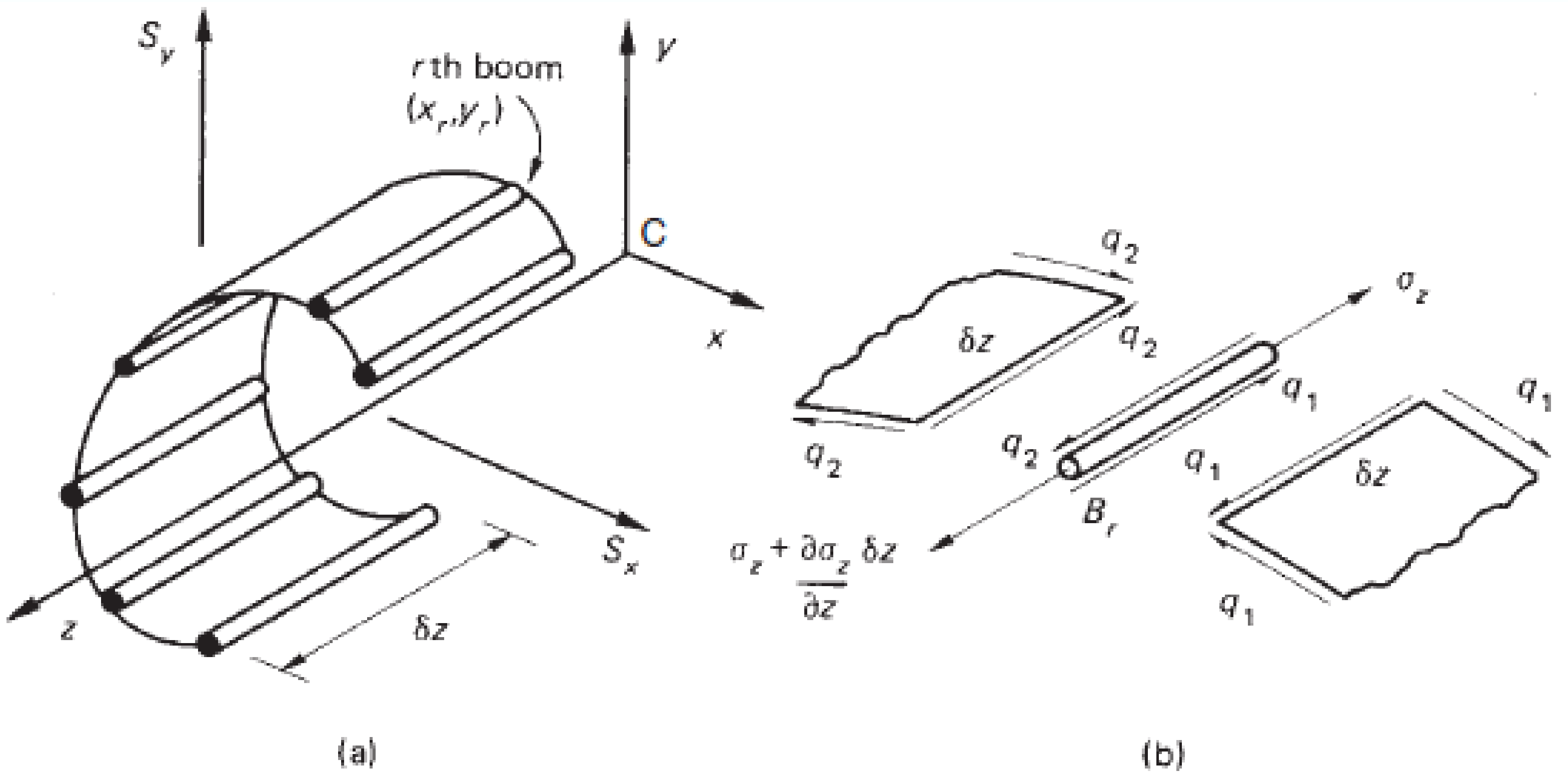
$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y \qquad \sigma_z = \frac{M_x(I_{yy} y - I_{xy} x)}{I_{xx} I_{yy} - I_{xy}^2} + \frac{M_y(I_{xx} x - I_{xy} y)}{I_{xx} I_{yy} - I_{xy}^2}$$

Equation

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D x \, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D y \, ds$$

makes no provision for the effects of booms which cause discontinuities in the skin and therefore interrupt the shear flow.

Source from Aircraft Structures by T. H. G. Megson



(a) Elemental length of shear loaded open section beam with booms; (b) equilibrium of boom element.

Source from Aircraft Structures by T. H. G. Megson

Consider the equilibrium of the *r*th boom in the elemental length of beam shown in above Fig. which carries shear loads *Sx* and *Sy* acting through its shear centre S.

These shear loads produce direct stresses due to bending in the booms and skin and shear stresses in the skin.

Suppose that the shear flows in the skin adjacent to the *r*th boom of cross-sectional area *Br* are *q*1 and *q*2. Then, from above Fig.

$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right) B_r - \sigma_z B_r + q_2 \delta z - q_1 \delta z = 0$$

which simplifies to

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_r$$

Source from Aircraft Structures by T. H. G. Megson

$$q_2 - q_1 = -\left[\frac{(\partial M_y/\partial z)I_{xx} - (\partial M_x/\partial z)I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right] B_r x_r - \left[\frac{(\partial M_x/\partial z)I_{yy} - (\partial M_y/\partial z)I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right] B_r y_r$$

$$q_2 - q_1 = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right) B_r x_r - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right) B_r y_r$$

The above Equation gives the change in shear flow induced by a boom which itself is subjected to a direct load ($\sigma_z Br$).

Source from Aircraft Structures by T. H. G. Megson

Each time a boom is encountered the shear flow is incremented by this amount so that if, at any distance *s* around the profile of the section, *n* booms have been passed, the shear flow at the point is given by

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_{\mathrm{D}} x \, \mathrm{d}s + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_{\mathrm{D}} y \, \mathrm{d}s + \sum_{r=1}^{n} B_r y_r\right)$$

Source from Aircraft Structures by T. H. G. Megson

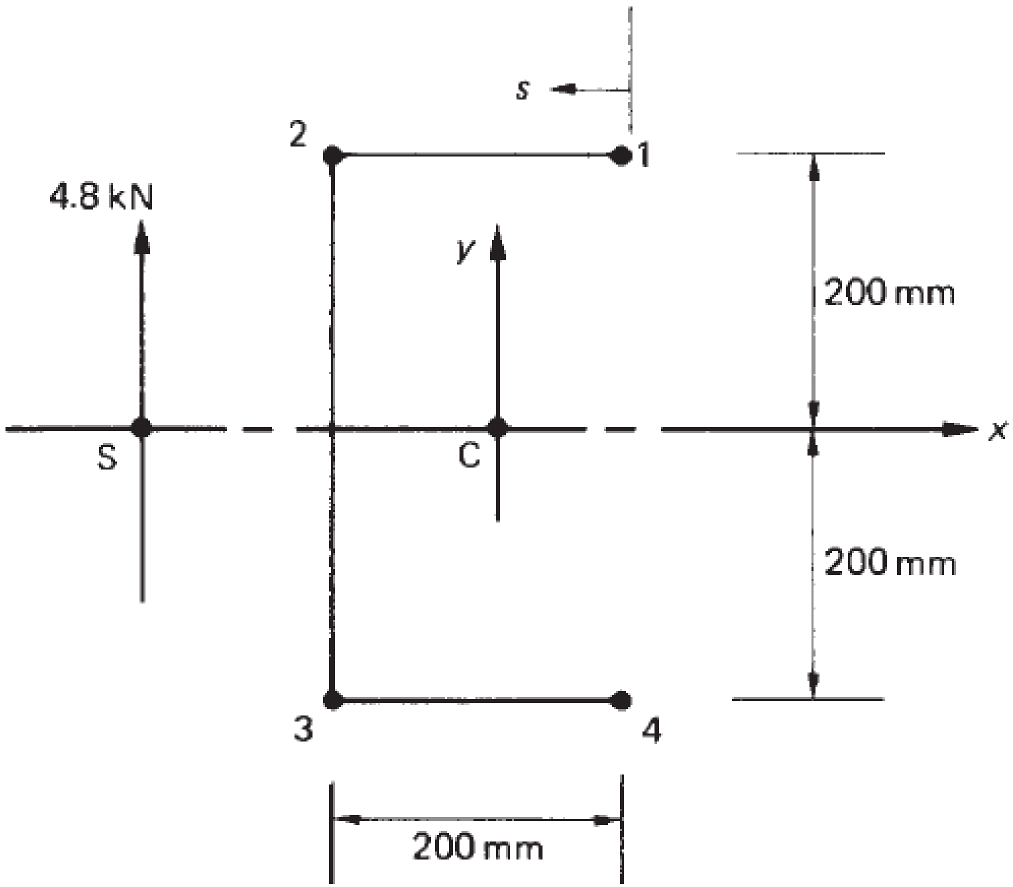
Calculate the shear flow distribution in the channel section shown in Fig. produced by a vertical shear load of 4.8 kN acting through its shear centre. Assume that the walls of the section are only effective in resisting shear stresses while the booms, each of area 300mm^2, carry all the direct stresses.

The effective direct stress carrying thickness *t*D of the walls of the section is zero so that the centroid of area and the section properties refer to the boom areas only.

Since C*x* (and C*y* as far as the boom areas are concerned) is an axis of symmetry $I_{xy} = 0$; also $S_x = 0$ and Eq. thereby reduces to

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r$$

Source from Aircraft Structures by T. H. G. Megson



Idealized channel section of Example 20.3.

Source from Aircraft Structures by T. H. G. Megson

in which $I_{xx} = 4 \times 300 \times 200^2 = 48 \times 10^6$ mm^4.

Substituting the values of S_y and I_{xx} in gives

$$q_s = -\frac{4.8 \times 10^3}{48 \times 10^6} \sum_{r=1}^{n} B_r y_r = -10^{-4} \sum_{r=1}^{n} B_r y_r$$

At the outside of boom 1, $q_s = 0$. As boom 1 is crossed the shear flow changes by an amount given by

$$q_1 = -10^{-4} \times 300 \times 200 = -6\text{N/mm}$$

Source from Aircraft Structures by T. H. G. Megson

Hence q_{12}=−6 N/mm since, it can be seen that no further changes in shear flow occur until the next boom (2) is crossed.

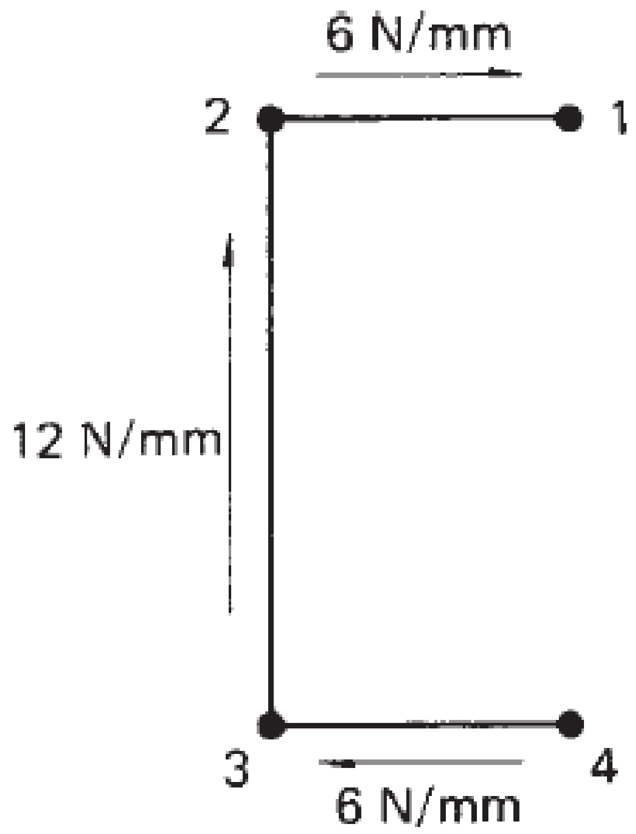
Hence q_{23} = −6 − 10−4 × 300 × 200 = −12N/mm Similarly

q_{34} = −12 − 10−4 × 300 × (−200) = −6N/mm

while, finally, at the outside of boom 4 the shear flow is

−6 − 10−4 × 300 × (−200) = 0as expected.

The complete shear flow distribution is shown in Fig. below



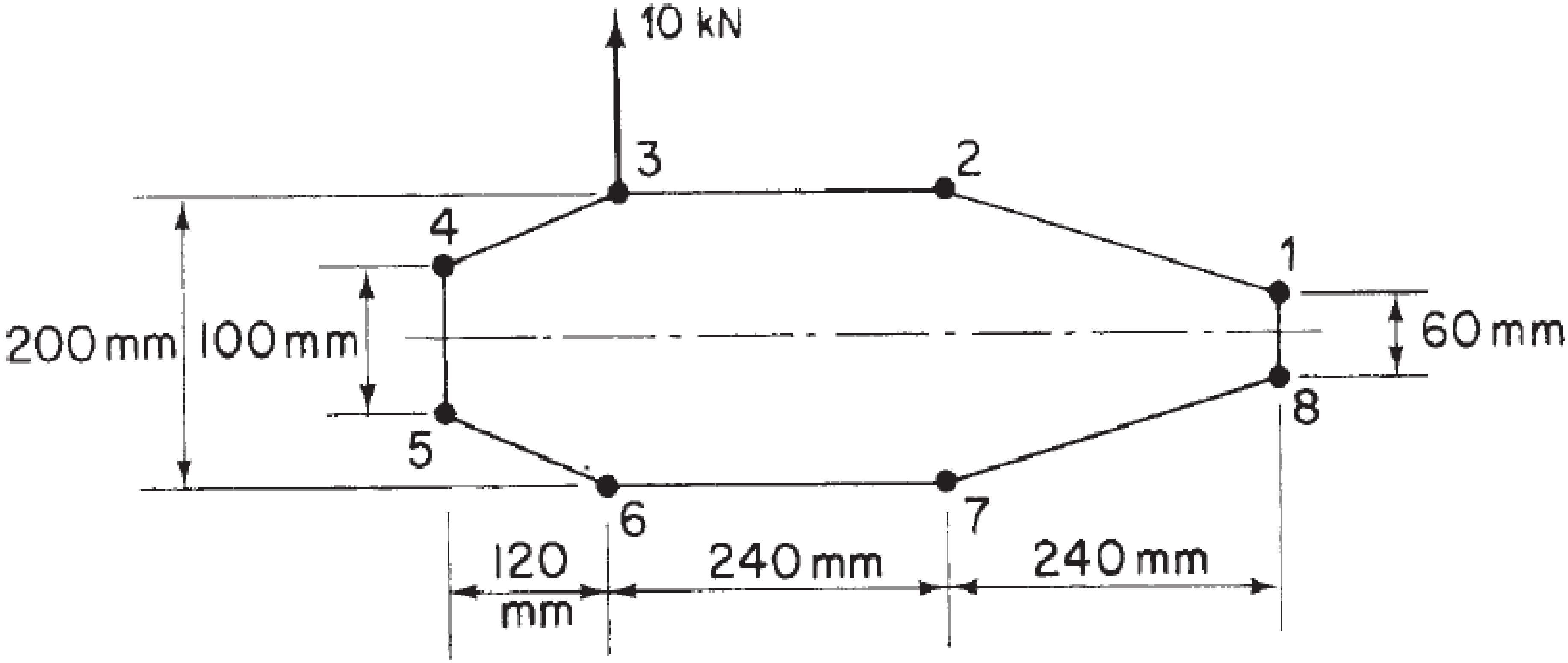
Source from Aircraft Structures by T. H. G. Megson

The thin-walled single cell beam shown in Fig. below has been idealized into a combination of direct stress carrying booms and shear stress only carrying walls. If the section supports a vertical shear load of 10 kN acting in a vertical plane through booms 3 and 6, calculate the distribution of shear flow around the section.

Boom areas: $B_1 = B_8 = 200 mm^2$, $B_2 = B_7 = 250 mm^2$, $B_3 = B_6 = 400 mm^2$, $B_4 = B_5 = 100 mm^2$.

The centroid of the direct stress carrying area lies on the horizontal axis of symmetry so that $I_{xy} = 0$. Also, since $t_D = 0$ and only a vertical shear load is applied

Source from Aircraft Structures by T. H. G. Megson



Source from Aircraft Structures by T. H. G. Megson

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

in which

$I_{xx} = 2(200 \times 30^2 + 250 \times 100^2 + 400 \times 100^2 + 100 \times 50^2) = 13.86 \times 10^6\ \text{mm}^4$

$$q_s = -\frac{10 \times 10^3}{13.86 \times 10^6} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

$$q_s = -7.22 \times 10^{-4} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

Source from Aircraft Structures by T. H. G. Megson

‘Cutting’ the beam section in the wall 23 (any wall may be chosen) and calculating the ‘basic’ shear flow distribution q_b from the first term on the right-hand side of Eq. we have

$q_{b,23} = 0$

$q_{b,34} = -7.22 \times 10^{-4}(400 \times 100) = -28.9\text{N/mm}$

$q_{b,45} = -28.9 - 7.22 \times 10{-}4(100 \times 50) = -32.5\text{N/mm}$

$q_{b,56} = q_{b,34} = -28.9\text{N/mm}$ (by symmetry)

$q_{b,67} = q_{b,23} = 0$ (by symmetry)

$q_{b,21} = -7.22 \times 10^{-4}(250 \times 100) = -18.1\text{N/mm}$

$q_{b,18} = -18.1 - 7.22 \times 10^{-4}(200 \times 30) = -22.4\text{N/mm}$

$q_{b,87} = q_{b,21} = -18.1\text{N/mm}$ (by symmetry)

Taking moments about the intersection of the line of action of the shear load and the horizontal axis of symmetry and referring to the results of Eqs .

$0 = [q_{b,81} \times 60 \times 480 + 2q_{b,12}(240 \times 100 + 70 \times 240) + 2q_{b,23} \times 240 \times 100$

$- 2q_{b,43} \times 120 \times 100 - q_{b,54} \times 100 \times 120] + 2 \times 97\,200 q_{s,0}$

Substituting the above values of q_b in this equation gives

$q_{s,0} = -5.4\text{N/mm}$

the negative sign indicating that $q_{s,0}$ acts in a clockwise sense.

In any wall the final shear flow is given by $q_s = q_b + q_{s,0}$ so that

$q_{21} = -18.1 + 5.4 = -12.7\text{N/mm} = q_{87}$

$q_{23} = -5.4\text{N/mm} = q_{67}$

Source from Aircraft Structures by T. H. G. Megson

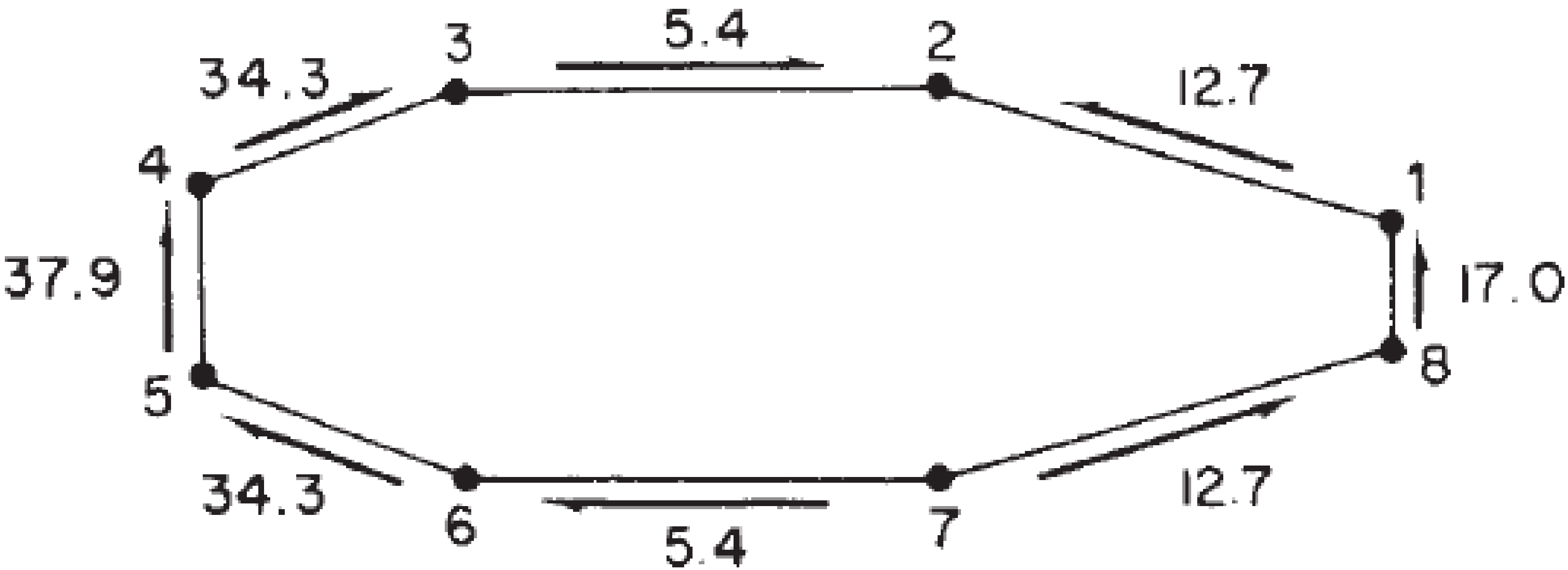
$q_{34} = -34.3 \mathrm{N/mm} = q_{56}$

$q_{45} = -37.9 \mathrm{N/mm}$

And

$q_{81} = 17.0 \mathrm{N/mm}$

giving the shear flow distribution shown in Fig. below.



Shear flow distribution N/mm in walls of the beam section

Source from Aircraft Structures by T. H. G. Megson

The fuselage of a light passenger carrying aircraft has the circular cross-section shown in Fig. The cross-sectional area of each stringer is 100mm^2 and the vertical distances given in Fig. are to the mid-line of the section wall at the corresponding stringer position. If the fuselage is subjected to a bending moment of 200 kNm applied in the vertical plane of symmetry, at this section, calculate the direct stress distribution.

The section is first idealized using the method described . As an approximation we shall assume that the skin between adjacent stringers is flat so that we may use either Eq.

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

to determine the boom areas.

Source from Aircraft Structures by T. H. G. Megson

From symmetry $B1 = B9$, $B2 = B8 = B10 = B16$, $B3 = B7 = B11 = B15$, $B4 = B6 = B12 = B14$ and $B5 = B13$.

$$B_1 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_{16}}{\sigma_1}\right)$$

i.e.

$$B_1 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{352.0}{381.0}\right) \times 2 = 216.6\,\text{mm}^2$$

Source from Aircraft Structures by T. H. G. Megson

Similarly B_2 =216.6mm^2, B_3 =216.6mm^2, B_4 =216.7mm^2.

We note that stringers 5 and 13 lie on the neutral axis of the section and are therefore unstressed; the calculation of boom areas *B*5 and *B*13 does not then arise.

For this particular section *Ixy* =0 since C*x* (and C*y*) is an axis of symmetry. Further, *My* =0 so that Eq.

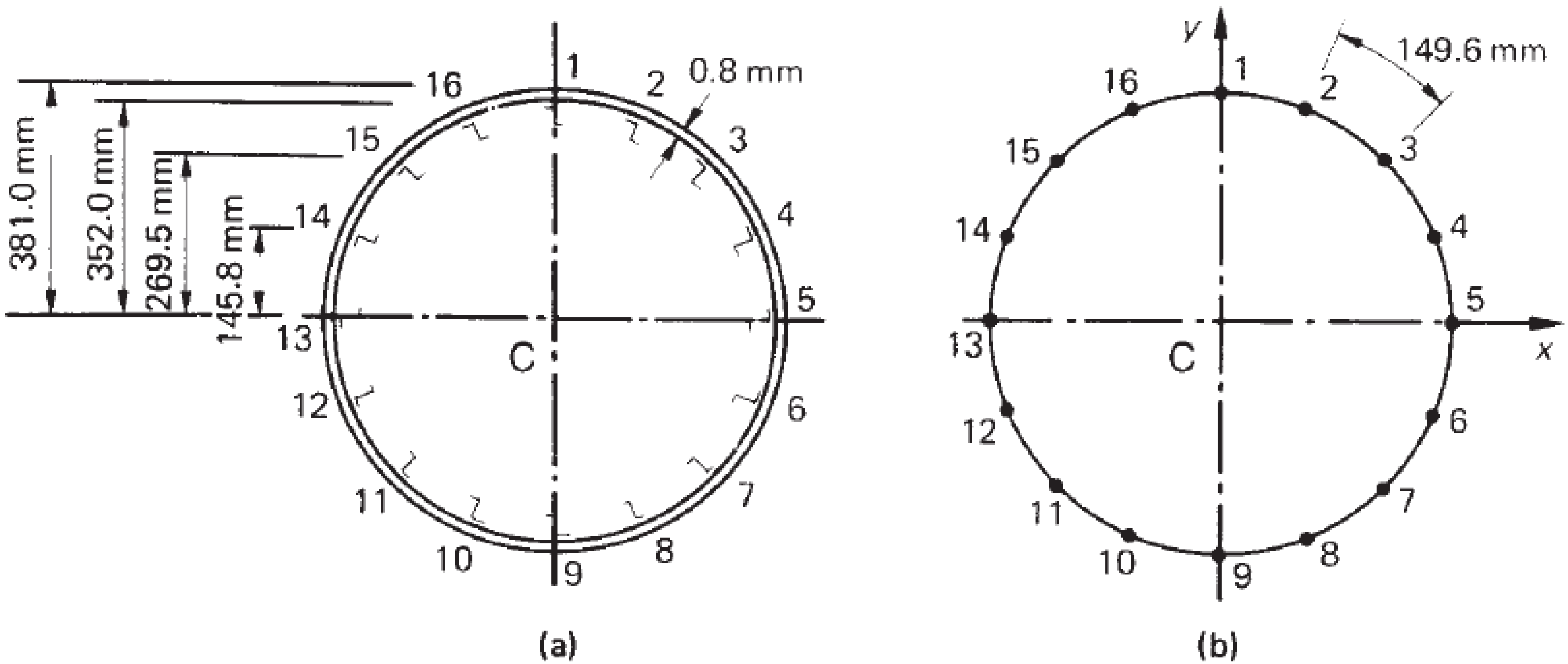
$$\sigma_z = \frac{M_x y}{I_{xx}}$$

in which

$Ixx = 2 \times 216.6 \times 381.0^2 + 4 \times 216.6 \times 352.0^2 + 4 \times 216.6 \times 2695^2$

$+ 4 \times 216.7 \times 145.8^2 = 2.52 \times 10^8$ mm4

Source from Aircraft Structures by T. H. G. Megson



(a) Actual fuselage section; (b) idealized fuselage section

Source from Aircraft Structures by T. H. G. Megson

Stringer/boom	y (mm)	σ_z (N/mm^2)
1	381.0	302.4
2, 16	352.0	279.4
3, 15	269.5	213.9
4, 14	145.8	115.7
5, 13	0	0
6, 12	−145.8	−115.7
7, 11	−269.5	−213.9
8, 10	−352.0	−279.4
9	−381.0	−302.4

Source from Aircraft Structures by T. H. G. Megson

The fuselage is subjected to a vertical shear load of 100 kN applied at a distance of 150mm from the vertical axis of symmetry as shown, for the idealized section, in Fig. 22.2. Calculate the distribution of shear flow in the section.

Ixy =0 and, since *Sx* =0, Eq.

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

in which *Ixx* =2.52×10^8 mm4 as before. Then

$$q_s = \frac{-100 \times 10^3}{2.52 \times 10^8} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

The first term on the right-hand side of Eq. is the ‘open section’ shear flow q_b.

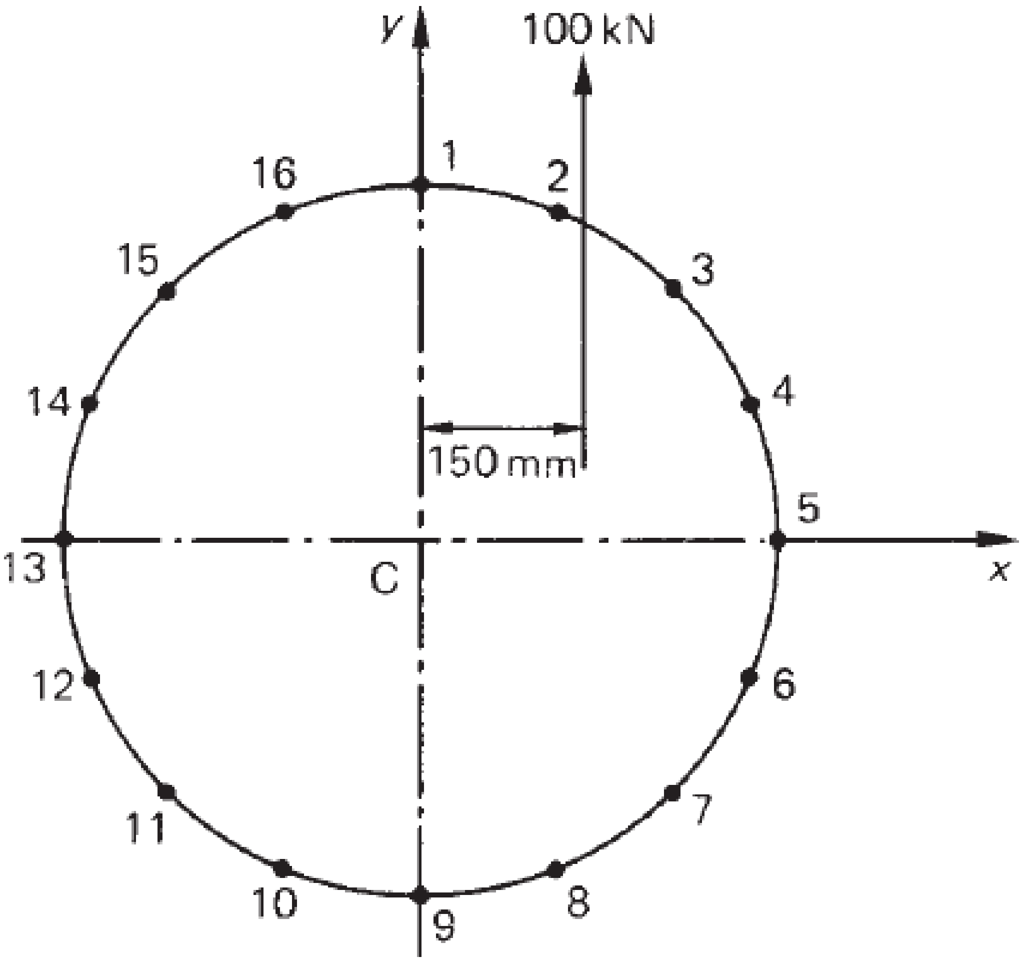
We therefore ‘cut’one of the skin panels, say 12, and calculate q_b.

The results are presented in Table. , the column headed Boom indicates the boom that is crossed when the analysis moves from one panel to the next.

Note also that, as would be expected, the q_b shear flow distribution is symmetrical about the Cx axis.

The shear flow $q_{s,0}$ in the panel 12 is now found by taking moments about a convenient moment centre, say C. Therefore from Eq.

Source from Aircraft Structures by T. H. G. Megson



Source from Aircraft Structures by T. H. G. Megson

$$q_s = -3.97 \times 10^{-4} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

$$100 \times 10^3 \times 150 = \oint q_b p \mathrm{d}s + 2Aq_{s,0}$$

in which $A = \pi \times 381.0^2 = 4.56 \times 10^5$ mm^2. Since the q_b shear flows are constant between the booms, Eq. may be rewritten in the form

$100 \times 10^3 \times 150 = -2A_{12}q_{b,12} - 2A_{23}q_{b,23} - \cdots - 2A_{161}q_{b,16\,1} + 2Aq_{s,0}$

in which $A12, A23, \ldots, A161$ are the areas subtended by the skin panels 12, 23,…, 16 l at the centre C of the circular cross-section and anticlockwise moments are taken as positive. Clearly $A12 = A23 = \cdots = A16$ l $= 4.56 \times 10^5/16 = 28\,500$mm^2. Equation then becomes

Source from Aircraft Structures by T. H. G. Megson

$100\times10^3\times150 = 2\times28\,500(-q_{b,12}-q_{b,23}-\cdots-q_{b,16}\,l\,)+2\times4.56\times10^5 q_{s,0}$

Substituting the values of q_b from Table in Eq. we obtain

$100 \times 10^3 \times 150 = 2 \times 28\,500(-262.4) + 2 \times 4.56 \times 10^5 q_{s,0}$

from which $q_{s,0}$ = 32.8 N/mm (acting in an anticlockwise sense)

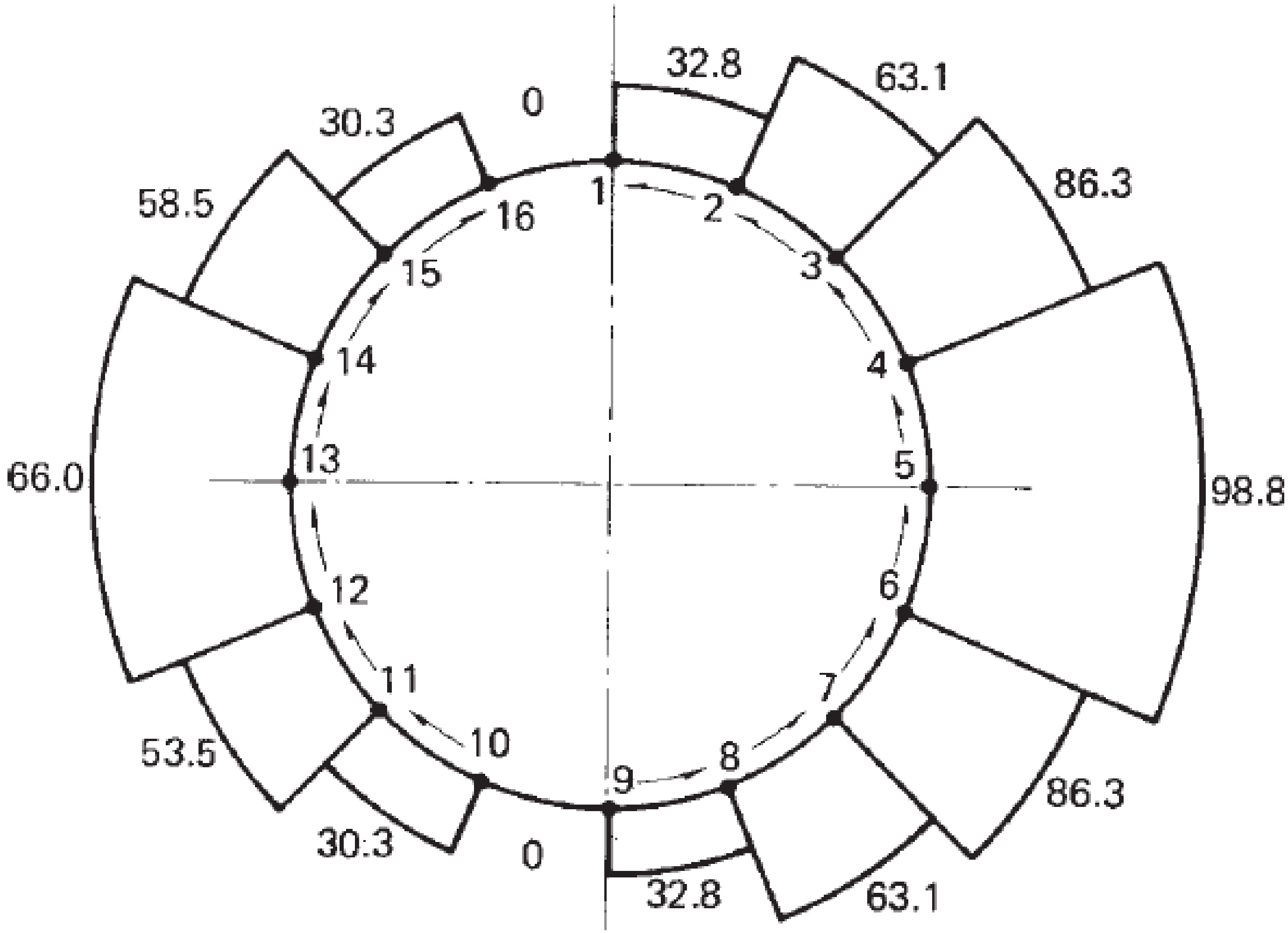
The complete shear flow distribution follows by adding the value of $q_{s,0}$ to the q_b shear flow distribution, giving the final distribution shown in Fig. The solution may be checked by calculating the resultant of the shear flow distribution parallel to the Cy axis. Thus

$$2[(98.8 + 66.0)145.8 + (86.3 + 53.5)123.7 + (63.1 + 30.3)82.5$$
$$+ (32.8 - 0)29.0] \times 10^{-3} = 99.96 \text{ kN}$$

Source from Aircraft Structures by T. H. G. Megson

Skin panel		Boom	B_r (mm^2)	y_r (mm)	q_b (N/mm)
1	2	–	–	–	0
2	3	2	216.6	352.0	−30.3
3	4	3	216.6	269.5	−53.5
4	5	4	216.7	145.8	−66.0
5	6	5	–	0	−66.0
6	7	6	216.7	−145.8	−53.5
7	8	7	216.6	−269.5	−30.3
8	9	8	216.6	−352.0	0
1	16	1	216.6	381.0	−32.8
16	15	16	216.6	352.0	−63.1
15	14	15	216.6	269.5	−86.3
14	13	14	216.6	145.8	−98.8
13	12	13	–	0	−98.8
12	11	12	216.7	−145.8	−86.3
11	10	11	216.6	−269.5	−63.1
10	9	10	216.6	−352.0	−32.8

Source from Aircraft Structures by T. H. G. Megson



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Wing spars and box beams

Established the basic theory for the analysis of open and closed section thin-walled beams subjected to bending, shear and torsional loads.

In addition, complex stringer stiffened sections could be idealized into sections more amenable to analysis.

Now extend this analysis to actual aircraft components including, wing spars and box beams.

Investigation and analysis of fuselages, wings, frames and ribs, and consider the effects of cut-outs in wings and fuselages.

Source from Aircraft Structures by T. H. G. Megson

Aircraft structural components are, complex, consisting usually of thin sheets of metal stiffened by arrangements of stringers.

These structures are highly redundant and require some degree of simplification or idealization before they can be analysed.

The analysis presented here is therefore approximate and the degree of accuracy obtained depends on the number of simplifying assumptions made.

A further complication arises in that factors such as warping restraint, structural and loading discontinuities and shear lag significantly affect the analysis,

Shall investigate these effects in some simple structural components.

Source from Aircraft Structures by T. H. G. Megson

Generally, a high degree of accuracy can only be obtained by using computer-based techniques such as the finite element method.

However, the simpler, quicker and cheaper approximate methods can be used to advantage in the preliminary stages of design when several possible structural alternatives are being investigated,

they also provide an insight into the physical behaviour of structures which computer-based techniques do not.

Major aircraft structural components such as wings and fuselages are usually tapered along their lengths for greater structural efficiency.

Source from Aircraft Structures by T. H. G. Megson

Thus, wing sections are reduced both chordwise and in depth along the wing span towards the tip and fuselage sections aft of the passenger cabin taper to provide a more efficient aerodynamic and structural shape.

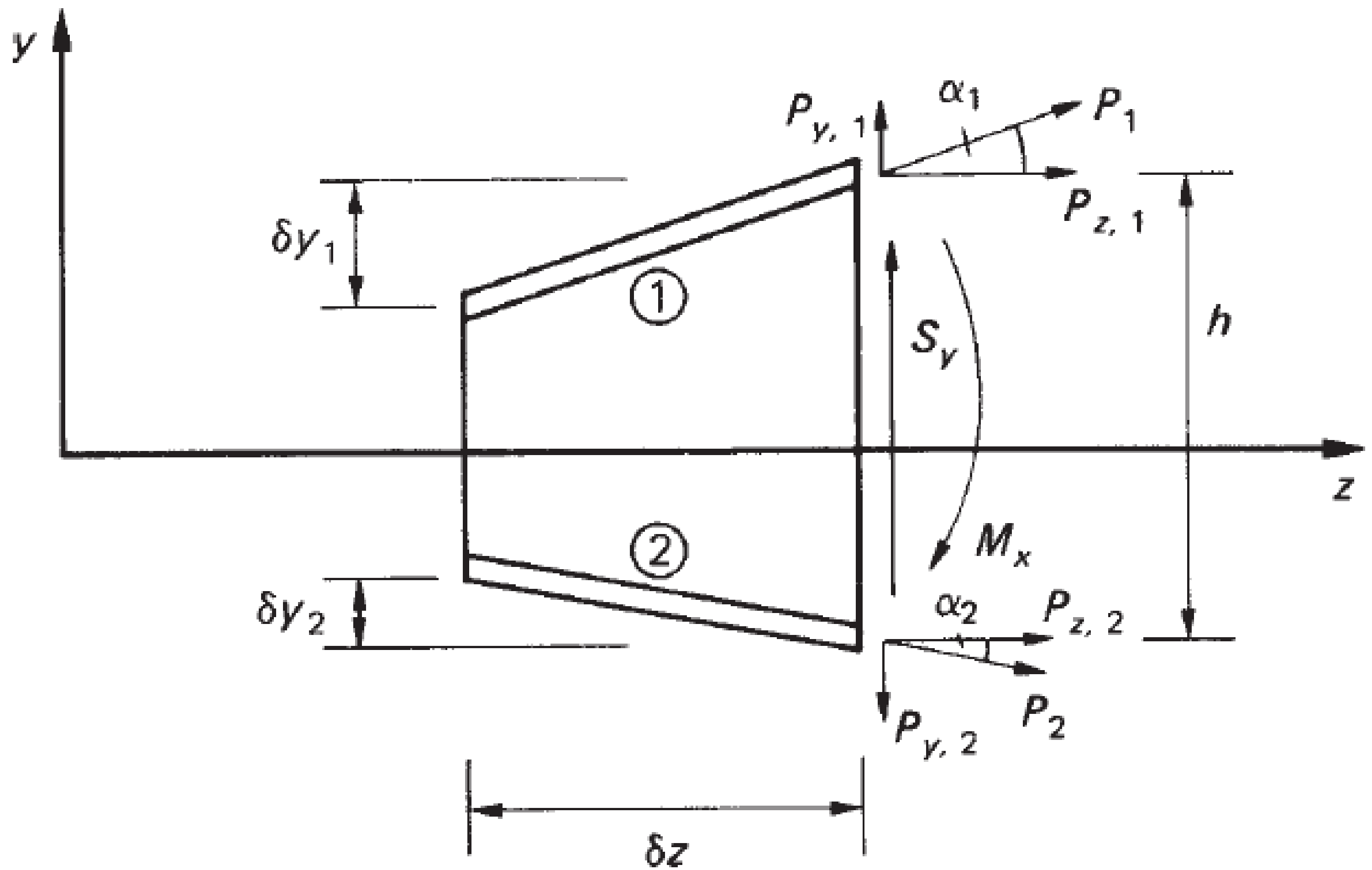
The analysis of open and closed section beams assumes that the beam sections are uniform.

The effect of taper on the prediction of direct stresses produced by bending is minimal if the taper is small and the section properties are calculated at the particular section being considered,

Calculation of shear stresses in beam webs can be significantly affected by taper.

Source from Aircraft Structures by T. H. G. Megson

Tapered wing spar



Effect of taper on beam analysis.

Source from Aircraft Structures by T. H. G. Megson

Consider the simple case of wing spar beam, positioned in the yz plane and comprising two flanges and a web: an elemental length δz of the beam is shown in Fig. above slide.

At the section z the beam is subjected to a positive bending moment M_x and a positive shear force S_y.

The bending moment resultants $P_{z,1}$ and $P_{z,2}$ are parallel to the z axis of the beam.

For a beam in which the flanges are assumed to resist all the direct stresses, $P_{z,1} = M_x/h$ and $P_{z,2} = -M_x/h$.

In the case where the web is assumed to be fully effective in resisting direct stress, $P_{z,1}$ and $P_{z,2}$ are determined by multiplying the direct stresses $\sigma_{z,1}$ and $\sigma_{z,2}$ found using Eq.

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y$$

$$\sigma_z = \frac{M_x(I_{yy} y - I_{xy} x)}{I_{xx} I_{yy} - I_{xy}^2} + \frac{M_y(I_{xx} x - I_{xy} y)}{I_{xx} I_{yy} - I_{xy}^2}$$

by the flange areas $B1$ and $B2$.

$P_{z,1}$ and $P_{z,2}$ are the components in the z direction of the axial loads P_1 and P_2 in the flanges.

These have components $P_{y,1}$ and $P_{y,2}$ parallel to the y axis given by

$$P_{y,1} = P_{z,1}\frac{\delta y_1}{\delta z} \quad P_{y,2} = -P_{z,2}\frac{\delta y_2}{\delta z}$$

in which, for the direction of taper shown, δy_2 is negative. The axial load in flange is given by

$$P_1 = (P_{z,1}^2 + P_{y,1}^2)^{1/2}$$

Substituting for *Py*,1 in above Eq. we have

$$P_1 = P_{z,1}\frac{(\delta z^2 + \delta y_1^2)^{1/2}}{\delta z} = \frac{P_{z,1}}{\cos\alpha_1}$$

$$P_2 = \frac{P_{z,2}}{\cos\alpha_2}$$

The internal shear force S_y comprises the resultant $S_{y,w}$ of the web shear flows together with the vertical components of P_1 and P_2. Thus

$$S_y = S_{y,w} + P_{y,1} - P_{y,2}$$

$$S_y = S_{y,w} + P_{z,1}\frac{\delta y_1}{\delta z} + P_{z,2}\frac{\delta y_2}{\delta z}$$

$$S_{y,w} = S_y - P_{z,1}\frac{\delta y_1}{\delta z} - P_{z,2}\frac{\delta y_2}{\delta z}$$

Again we note that $\delta y2$ is negative. The above Equation may be used to determine the shear flow distribution in the web.

For a completely idealized beam the web shear flow is constant through the depth and is given by $S_{y,w}/h$.

For a beam in which the web is fully effective in resisting direct stresses the web shear flow distribution is found using Eq.

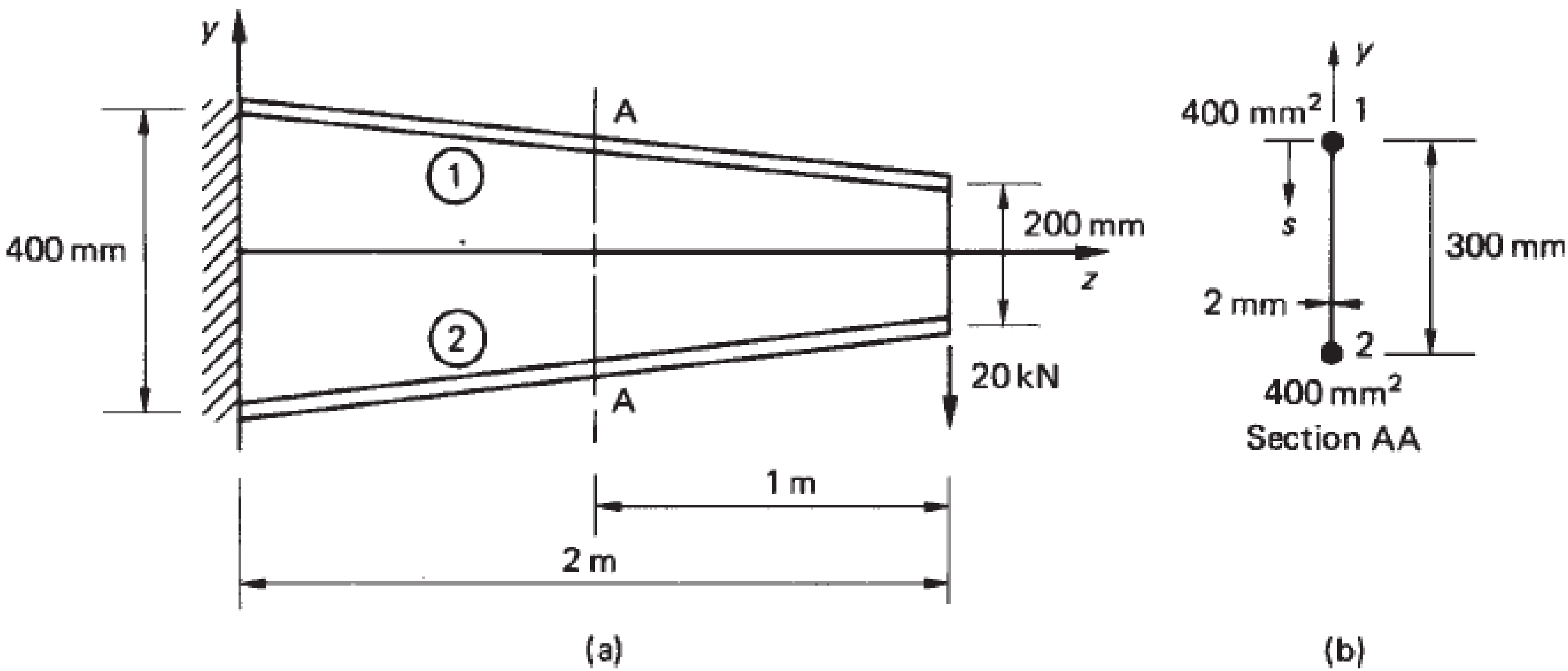
$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_{\mathrm{D}} x \,\mathrm{d}s + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_{\mathrm{D}} y \,\mathrm{d}s + \sum_{r=1}^{n} B_r y_r\right)$$

Sy is replaced by *Sy,w* and which, for the beam would simplify to

$$q_s = -\frac{S_{y,w}}{I_{xx}}\left(\int_0^s t_{\mathrm{D}} y \,\mathrm{d}s + B_1 y_1\right)$$

$$q_s = -\frac{S_{y,w}}{I_{xx}}\left(\int_0^s t_{\mathrm{D}} y \,\mathrm{d}s + B_2 y_2\right)$$

Determine the shear flow distribution in the web of the tapered beam shown in Fig. below, at a section midway along its length. The web of the beam has a thickness of 2mm and is fully effective in resisting direct stress. The beam tapers symmetrically about its horizontal centroidal axis and the cross-sectional area of each flange is 400mm².



The internal bending moment and shear load at the section AA produced by the externally applied load are, respectively

$$M_x = 20 \times 1 = 20\,\text{kN m} \quad S_y = -20\,\text{kN}$$

The direct stresses parallel to the z axis in the flanges at this section are obtained either from Eqs (16.18) or (16.19) in which $M_y = 0$ and $I_{xy} = 0$. Thus, from Eq. (16.18)

$$\sigma_z = \frac{M_x y}{I_{xx}} \tag{i}$$

in which

$$I_{xx} = 2 \times 400 \times 150^2 + 2 \times 300^3/12$$

i.e.

$$I_{xx} = 22.5 \times 10^6\,\text{mm}^4$$

Hence

$$\sigma_{z,1} = -\sigma_{z,2} = \frac{20 \times 10^6 \times 150}{22.5 \times 10^6} = 133.3\,\text{N/mm}^2$$

The components parallel to the z axis of the axial loads in the flanges are therefore

$$P_{z,1} = -P_{z,2} = 133.3 \times 400 = 53\,320\,\text{N}$$

The shear load resisted by the beam web is then, from Eq. (21.5)

$$S_{y,w} = -20 \times 10^3 - 53\,320\frac{\delta y_1}{\delta z} + 53\,320\frac{\delta y_2}{\delta z}$$

in which, from Figs 21.1 and 21.2, we see that

$$\frac{\delta y_1}{\delta z} = \frac{-100}{2 \times 10^3} = -0.05 \quad \frac{\delta y_2}{\delta z} = \frac{100}{2 \times 10^3} = 0.05$$

Hence

$$S_{y,w} = -20 \times 10^3 + 53\,320 \times 0.05 + 53\,320 \times 0.05 = -14\,668\text{ N}$$

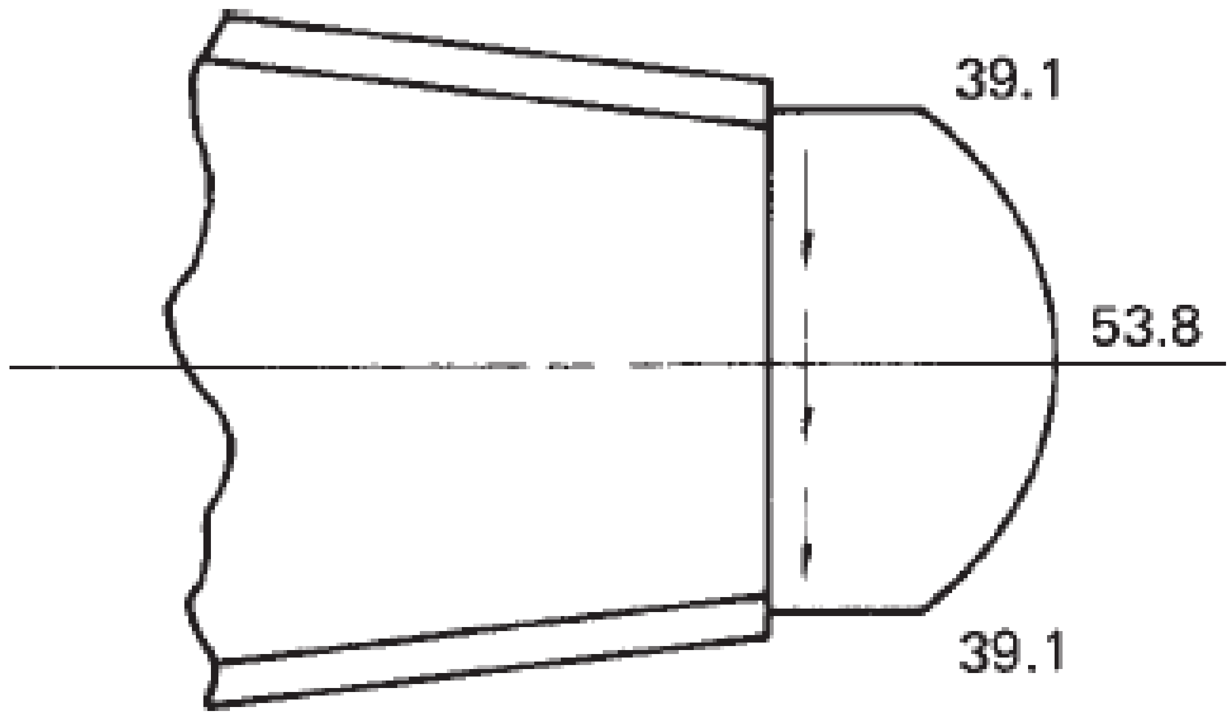
The shear flow distribution in the web follows either from Eq. (21.6) or Eq. (21.7) and is (see Fig. 21.2(b))

$$q_{12} = \frac{14\,668}{22.5 \times 10^6}\left(\int_0^s 2(150 - s)\,\mathrm{d}s + 400 \times 150\right)$$

i.e.

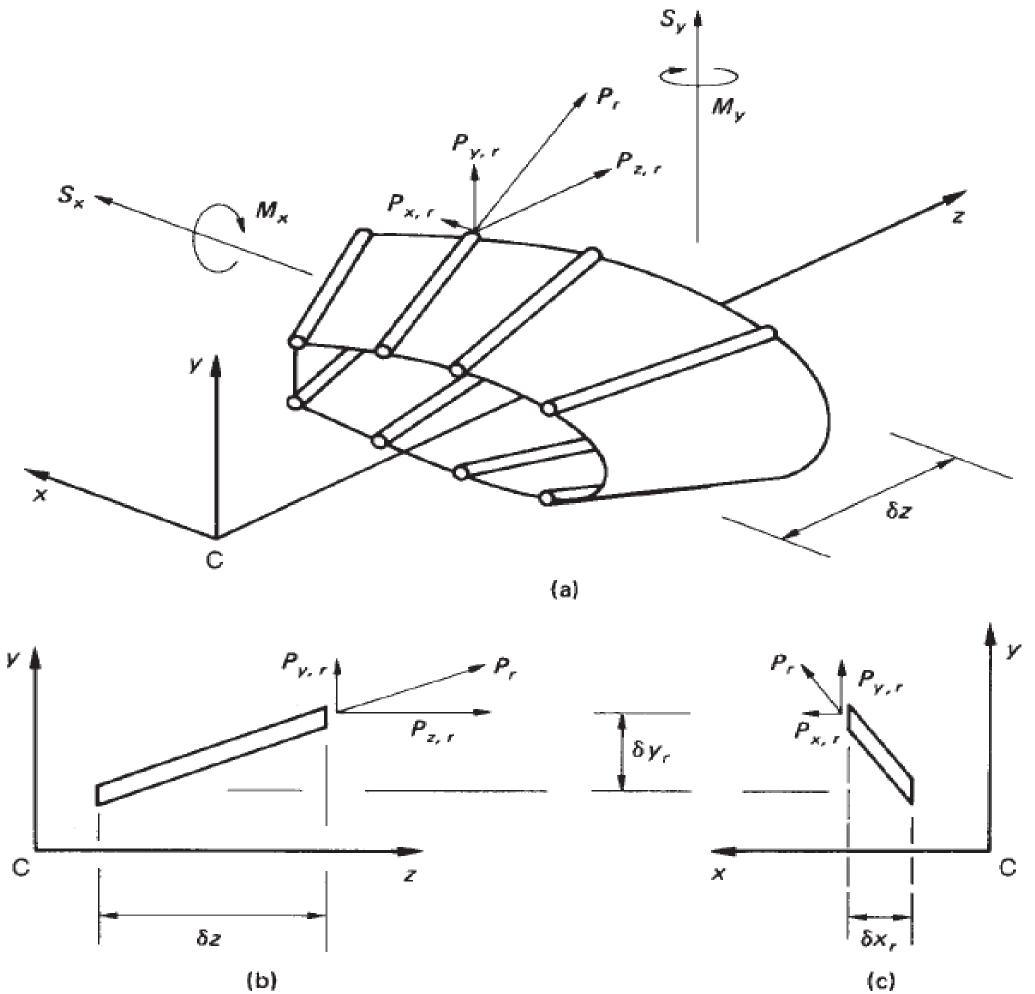
$$q_{12} = 6.52 \times 10^{-4}(-s^2 + 300s + 60\,000) \tag{ii}$$

The maximum value of q_{12} occurs when $s = 150$ mm and q_{12} (max) $= 53.8$ N/mm. The values of shear flow at points 1 ($s = 0$) and 2 ($s = 300$ mm) are $q_1 = 39.1$ N/mm and $q_2 = 39.1$ N/mm; the complete distribution is shown in Fig. 21.3.



Shear flow (N/mm) distribution at Section AA in Example 21.1.

Open and closed section beams



Effect of taper on the analysis of open and closed section beams.

Source from Aircraft Structures by T. H. G. Megson

Figure shows a short length δz of a beam carrying shear loads S_x and S_y at the section z; S_x and S_y are positive when acting in the directions shown.

Note that if the beam were of open cross-section the shear loads would be applied through its shear centre so that no twisting of the beam occurred.

In addition to shear loads the beam is subjected to bending moments M_x and M_y which produce direct stresses σ_z in the booms and skin.

Suppose that in the rth boom the direct stress in a direction parallel to the z axis is $\sigma_{z,r}$, which may be found using either σ_z Eq. The component $P_{z,r}$ of the axial load P_r in the rth boom is then given by

$$P_{z,r} = \sigma_{z,r} B_r$$

Source from Aircraft Structures by T. H. G. Megson

$$P_{y,r} = P_{z,r}\frac{\delta y_r}{\delta z}$$

$$P_{x,r} = P_{y,r}\frac{\delta x_r}{\delta y_r}$$

$$P_{x,r} = P_{z,r}\frac{\delta x_r}{\delta z}$$

$$P_r = (P_{x,r}^2 + P_{y,r}^2 + P_{z,r}^2)^{1/2}$$

$$P_r = P_{z,r}\frac{(\delta x_r^2 + \delta y_r^2 + \delta z^2)^{1/2}}{\delta z}$$

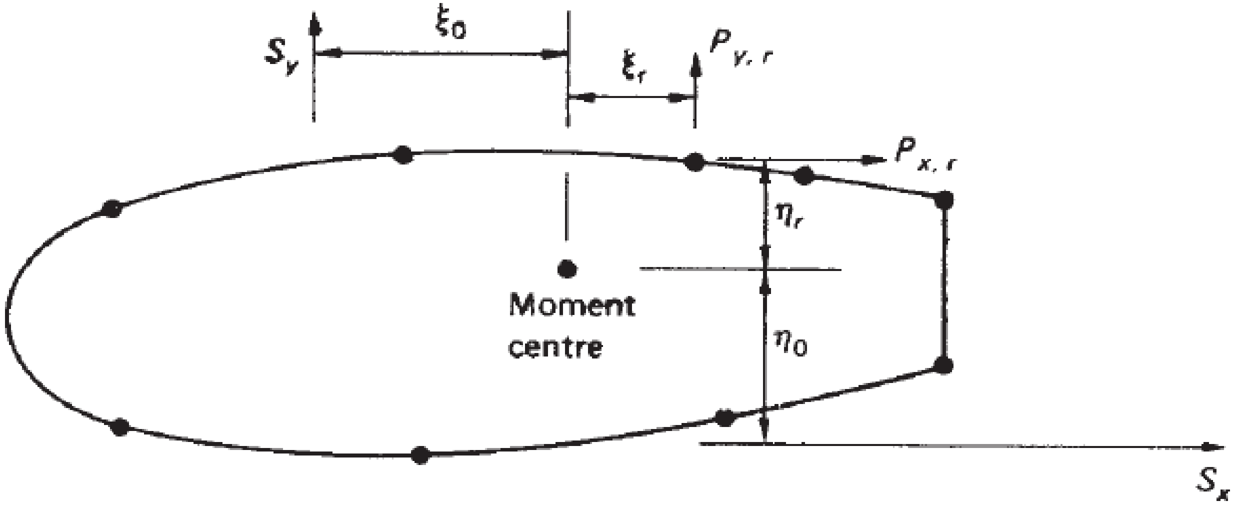
Source from Aircraft Structures by T. H. G. Megson

The applied shear loads S_x and S_y are reacted by the resultants of the shear flows in the skin panels and webs, together with the components $P_{x,r}$ and $P_{y,r}$ of the axial loads in the booms.

Therefore, if $S_{x,w}$ and $S_{y,w}$ are the resultants of the skin and web shear flows and there is a total of m booms in the section

$$S_x = S_{x,w} + \sum_{r=1}^{m} P_{x,r} \quad S_y = S_{y,w} + \sum_{r=1}^{m} P_{y,r}$$

Source from Aircraft Structures by T. H. G. Megson



Modification of moment equation in shear of closed section beams due to boom load.

$$S_x = S_{x,w} + \sum_{r=1}^{m} P_{z,r} \frac{\delta x_r}{\delta z} \quad S_y = S_{y,w} + \sum_{r=1}^{m} P_{z,r} \frac{\delta y_r}{\delta z}$$

$$S_{x,w} = S_x - \sum_{r=1}^{m} P_{z,r} \frac{\delta x_r}{\delta z} \quad S_{y,w} = S_y - \sum_{r=1}^{m} P_{z,r} \frac{\delta y_r}{\delta z}$$

Source from Aircraft Structures by T. H. G. Megson

The shear flow distribution in an open section beam is now obtained using Eq.

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D x \, ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D y \, ds + \sum_{r=1}^{n} B_r y_r\right)$$

in which S_x is replaced by $S_{x,w}$ and S_y by $S_{y,w}$ from Eq.

$$S_{y,w} = S_y - P_{z,1}\frac{\delta y_1}{\delta z} - P_{z,2}\frac{\delta y_2}{\delta z}$$

Similarly for a closed section beam, S_x and S_y in Eq. (20.11)

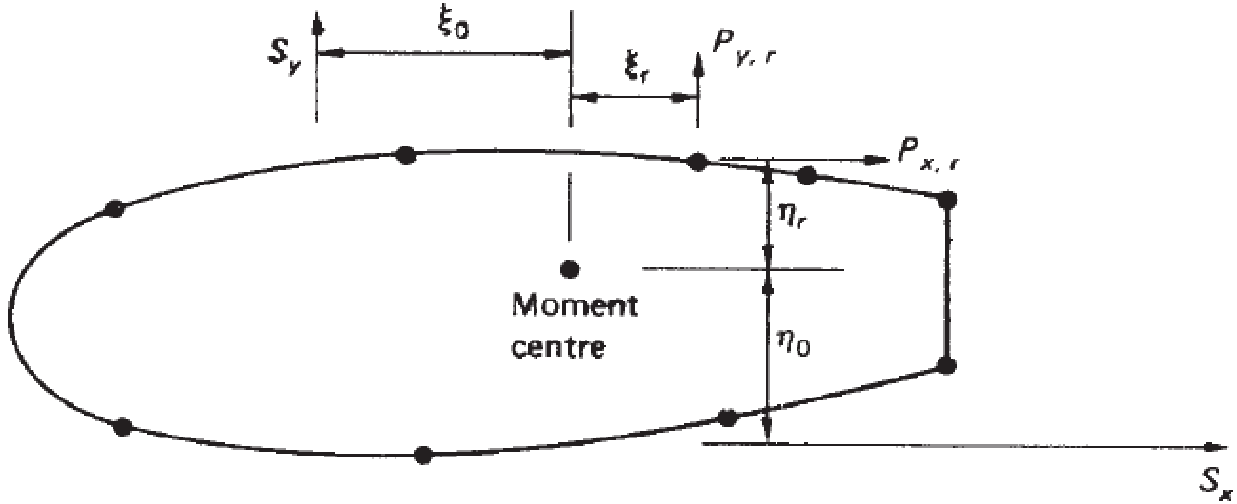
$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D x \, ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D y \, ds + \sum_{r=1}^{n} B_r y_r\right) + q_{s,0}$$

Source from Aircraft Structures by T. H. G. Megson

are replaced by $S_{x,w}$ and $S_{y,w}$. In the latter case the moment equation

$$S_x\eta_0 - S_y\xi_0 = \oint pq_{\mathrm{b}}\mathrm{d}s + 2Aq_{s,0}$$

requires modification due to the presence of the boom load components $P_{x,r}$ and $P_{y,r}$. Thus from Fig.



5 Modification of moment equation in shear of closed section beams due to boom load.

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we see that Eq.

$$S_x \eta_0 - S_y \xi_0 = \oint p q_{\mathrm{b}} \mathrm{d}s + 2A q_{s,0}$$

becomes

$$S_x \eta_0 - S_y \xi_0 = \oint q_{\mathrm{b}} p \, \mathrm{d}s + 2A q_{s,0} - \sum_{r=1}^{m} P_{x,r} \eta_r + \sum_{r=1}^{m} P_{y,r} \xi_r$$

The above Equation is directly applicable to a tapered beam subjected to forces positioned in relation to the moment centre as shown.

Care must be taken in a particular problem to ensure that the moments of the forces are given the correct sign.

Beams having variable stringer areas

In many aircraft, structural beams, such as wings, have stringers whose cross-sectional areas vary in the spanwise direction.

The effects of this variation on the determination of shear flow distribution cannot therefore be found by the methods described in previous Section which assume constant boom areas.

In fact, if the stringer stress is made constant by varying the area of cross-section there is no change in shear flow as the stringer/boom is crossed.

Source from Aircraft Structures by T. H. G. Megson

The calculation of shear flow distributions in beams having variable stringer areas is based on the alternative method for the calculation of shear flow distributions described in previous Section and illustrated in the alternative solution also.

The stringer loads $P_{z,1}$ and $P_{z,2}$ are calculated at two sections z1 and z2 of the beam a convenient distance apart.

We assume that the stringer load varies linearly along its length so that the change in stringer load per unit length of beam is given by

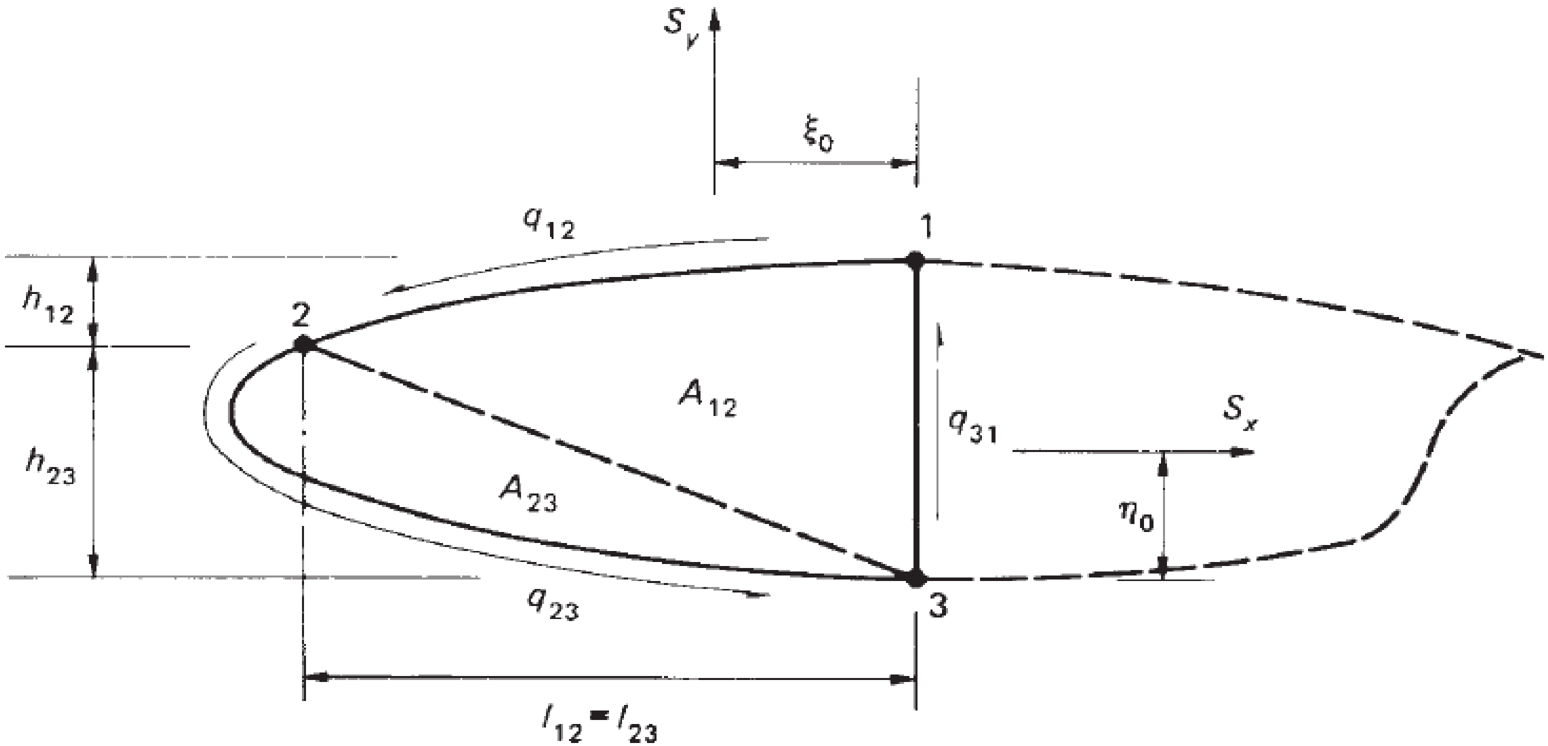
$$\Delta P = \frac{P_{z,1} - P_{z,2}}{z_1 - z_2}$$

The shear flow distribution follows as previously described.

Source from Aircraft Structures by T. H. G. Megson

Three-boom shell

The wing section shown in Fig. below has been idealized into an arrangement of direct stress carrying booms and shear–stress-only carrying skin panels. The part of the wing section aft of the vertical spar 31 performs an aerodynamic role only and is therefore



Three-boom wing section.

Source from Aircraft Structures by T. H. G. Megson

unstressed. Lift and drag loads, *Sy* and *Sx*, induce shear flows in the skin panels which are constant between adjacent booms since the section has been completely idealized.

Therefore, resolving horizontally and noting that the resultant of the internal shear flows is equivalent to the applied load, we have

$$S_x = -q_{12}l_{12} + q_{23}l_{23}$$

Now resolving vertically

$$S_y = q_{31}(h_{12} + h_{23}) - q_{12}h_{12} - q_{23}h_{23}$$

Finally, taking moments about, say, boom 3

$$S_x\eta_0 + S_y\xi_0 = -2A_{12}q_{12} - 2A_{23}q_{23}$$

In the above there are three unknown values of shear flow, q_{12}, q_{23}, q_{31} and three equations of statical equilibrium. We conclude therefore that a three-boom idealized shell is statically determinate.

We shall return to the simple case of a three-boom wing section when we examine the distributions of direct load and shear flows in wing ribs.

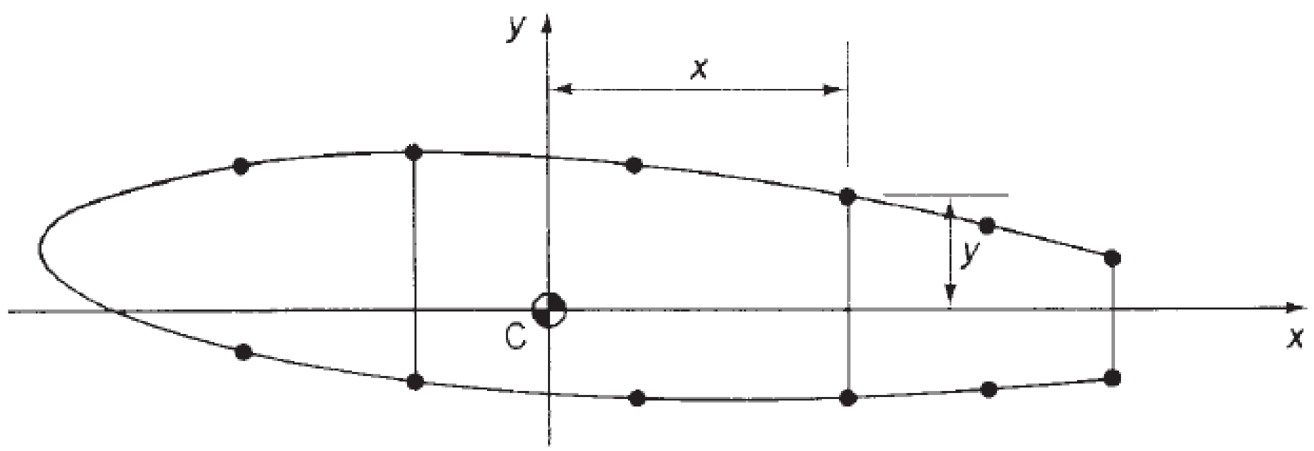
Meanwhile, we shall consider the bending, torsion and shear of multicellular wing sections.

Source from Aircraft Structures by T. H. G. Megson

Bending

Bending moments at any section of a wing are usually produced by shear loads at other sections of the wing.

The direct stress system for such a wing section below Fig. is given by σ_z Eqs in which the coordinates (x, y) of any point in the cross-section and the sectional properties are referred to axes Cxy in which the origin C coincides with the centroid of the direct stress carrying area.



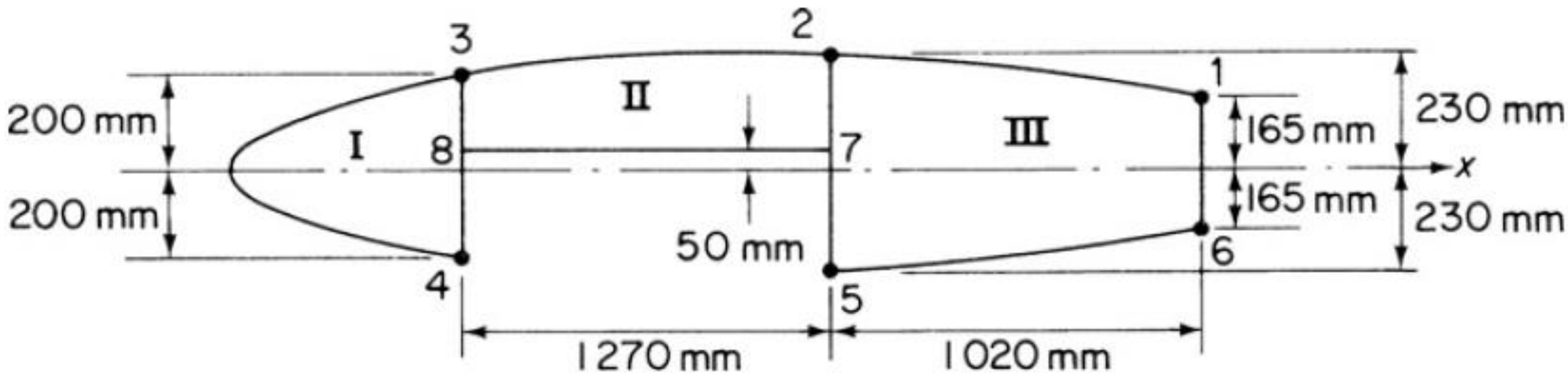
Idealized section of a multicell wing.

Source from Aircraft Structures by T. H. G. Megson

Example

The wing section shown in Fig. below has been idealized such that the booms carry all the direct stresses. If the wing section is subjected to a bending moment of 300 kNm applied in a vertical plane, calculate the direct stresses in the booms.

Boom areas: $B1 = B6 = 2580\text{mm}^2$ $B2 = B5 = 3880\text{mm}^2$ $B3 = B4 = 3230\text{mm}^2$



Source from Aircraft Structures by T. H. G. Megson

We note that the distribution of the boom areas is symmetrical about the horizontal *x* axis.

Hence, in *σz* Eq. *Ixy* =0. Further, *Mx* =300 kNm and *My* =0 so that *σz*Eq. reduces to

$$\sigma_z = \frac{M_x y}{I_{xx}}$$

$$I_{xy} = 2(2580 \times 165^2 + 3880 \times 230^2 + 3230 \times 200^2) = 809 \times 10^6 \,\text{mm}^4$$

$$\sigma_z = \frac{300 \times 10^6}{809 \times 10^6} y = 0.371y$$

Source from Aircraft Structures by T. H. G. Megson

The solution is now completed in below Table in which positive direct stresses are tensile and negative direct stresses compressive.

Boom	y (mm)	σ_z (N/mm^2)
1	165	61.2
2	230	85.3
3	200	74.2
4	−200	−74.2
5	−230	−85.3
6	−165	−61.2

Torsion

The chordwise pressure distribution on an aerodynamic surface may be represented by shear loads (lift and drag loads) acting through the aerodynamic centre together with a pitching moment $M0$.

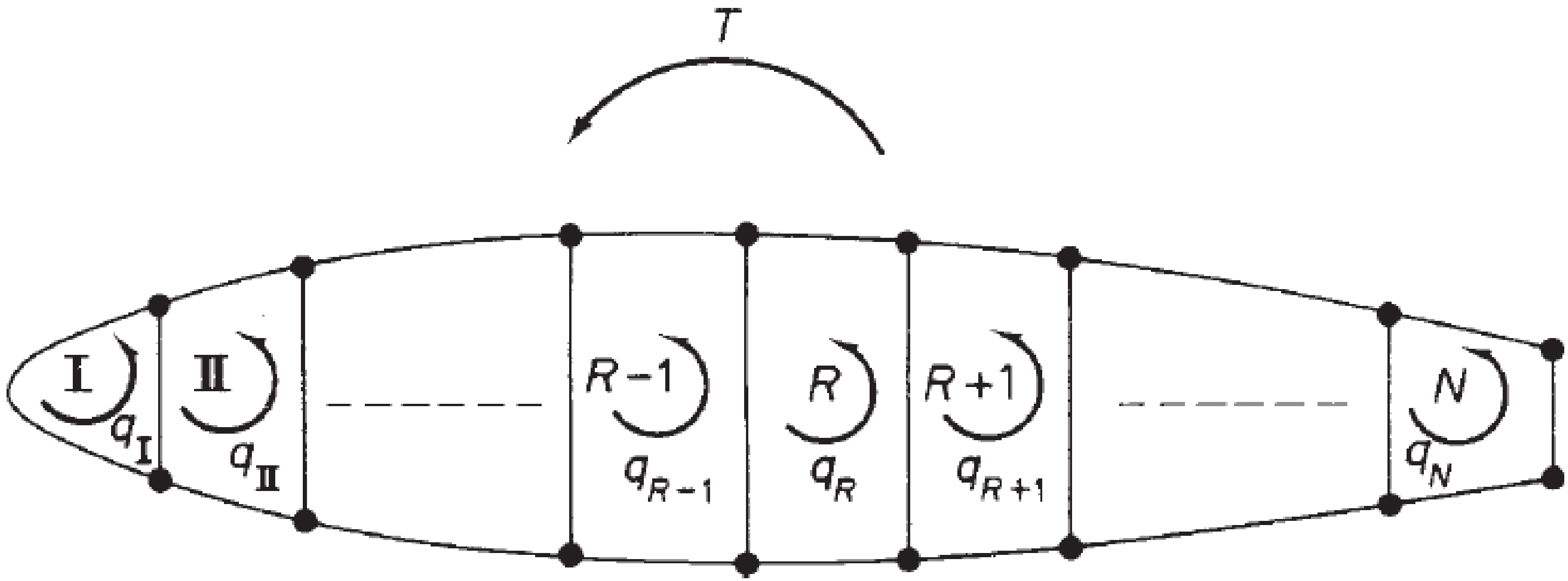
This system of shear loads may be transferred to the shear centre of the section in the form of shear loads Sx and Sy together with a torque T.

It is the pure torsion case that is considered here. In the analysis we assume that no axial constraint effects are present and that the shape of the wing section remains unchanged by the load application.

In the absence of axial constraint there is no development of direct stress in the wing section so that only shear stresses are present.

It follows that the presence of booms does not affect the analysis in the pure torsion case.

Source from Aircraft Structures by T. H. G. Megson



Multicell wing section subjected to torsion.

Source from Aircraft Structures by T. H. G. Megson

The wing section shown in Fig. 23.4 comprises N cells and carries a torque T which generates individual but unknown torques in each of the N cells. Each cell therefore develops a constant shear flow $q_I, q_{II}, \ldots, q_R, \ldots, q_N$ given by .
The total is therefore

Source from Aircraft Structures by T. H. G. Megson

Deflections

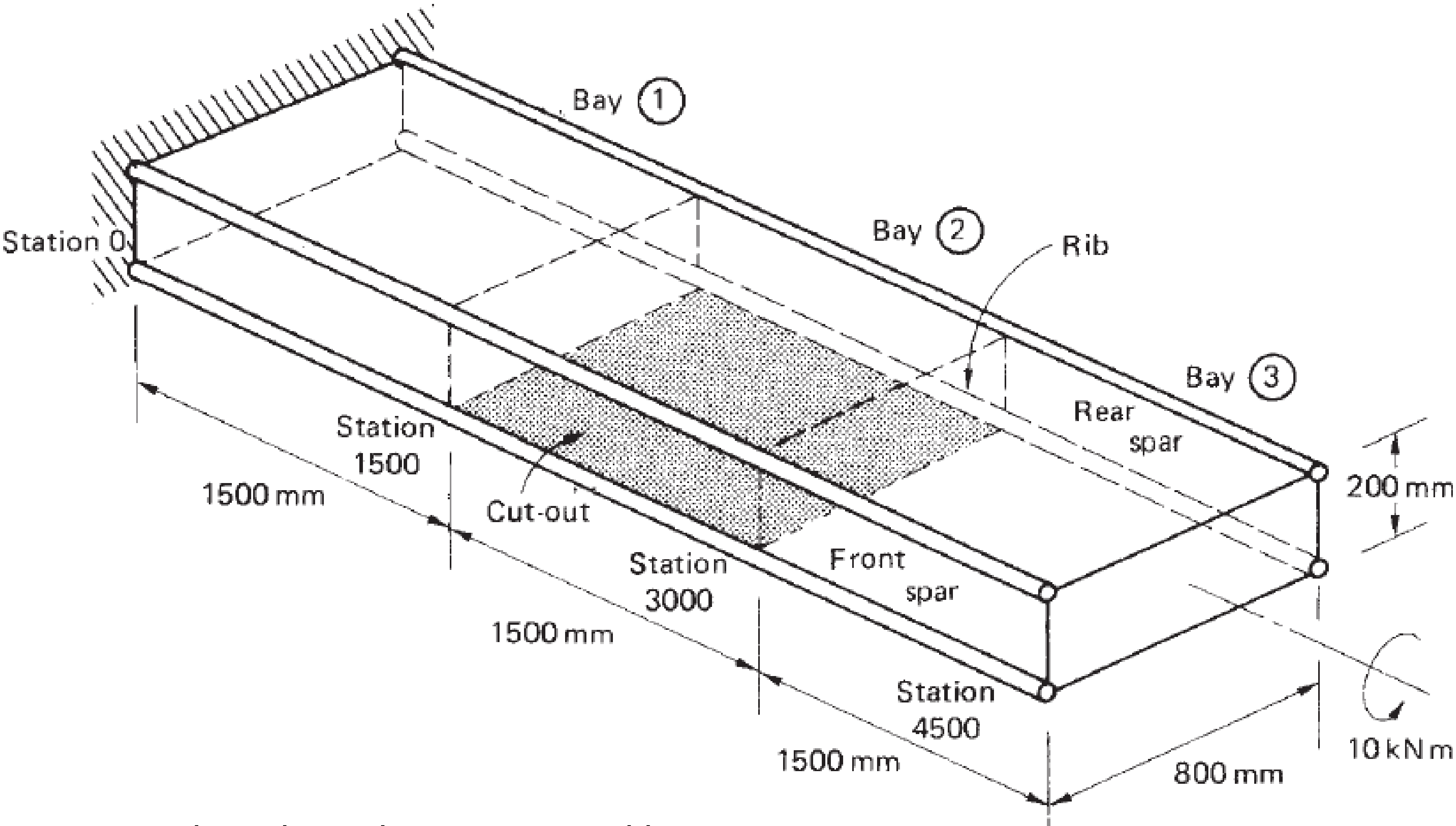
Deflections of multi-cell wings may be calculated by the unit load method in an identical manner to that described in open and single cell beams.

Cut-outs in wings

Wings, as well as fuselages, have openings in their surfaces to accommodate undercarriages, engine nacelles and weapons installations, etc.

In addition inspection panels are required at specific positions so that, as for fuselages, the loads in adjacent portions of the wing structure are modified.

Source from Aircraft Structures by T. H. G. Megson



Three-bay wing structure with cut-out

Source from Aircraft Structures by T. H. G. Megson

In practice it is necessary to provide openings in these closed stiffened shells for, for example, doors, cockpits, bomb bays, windows in passenger cabins, etc.

These openings or ‘cut-outs’ produce discontinuities in the otherwise continuous shell structure so that loads are redistributed in the vicinity of the cut-out thereby affecting loads in the skin, stringers and frames.

Frequently these regions must be heavily reinforced resulting in unavoidable weight increases.

In some cases, for example door openings in passenger aircraft, it is not possible to provide rigid fuselage frames on each side of the opening because the cabin space must not be restricted.

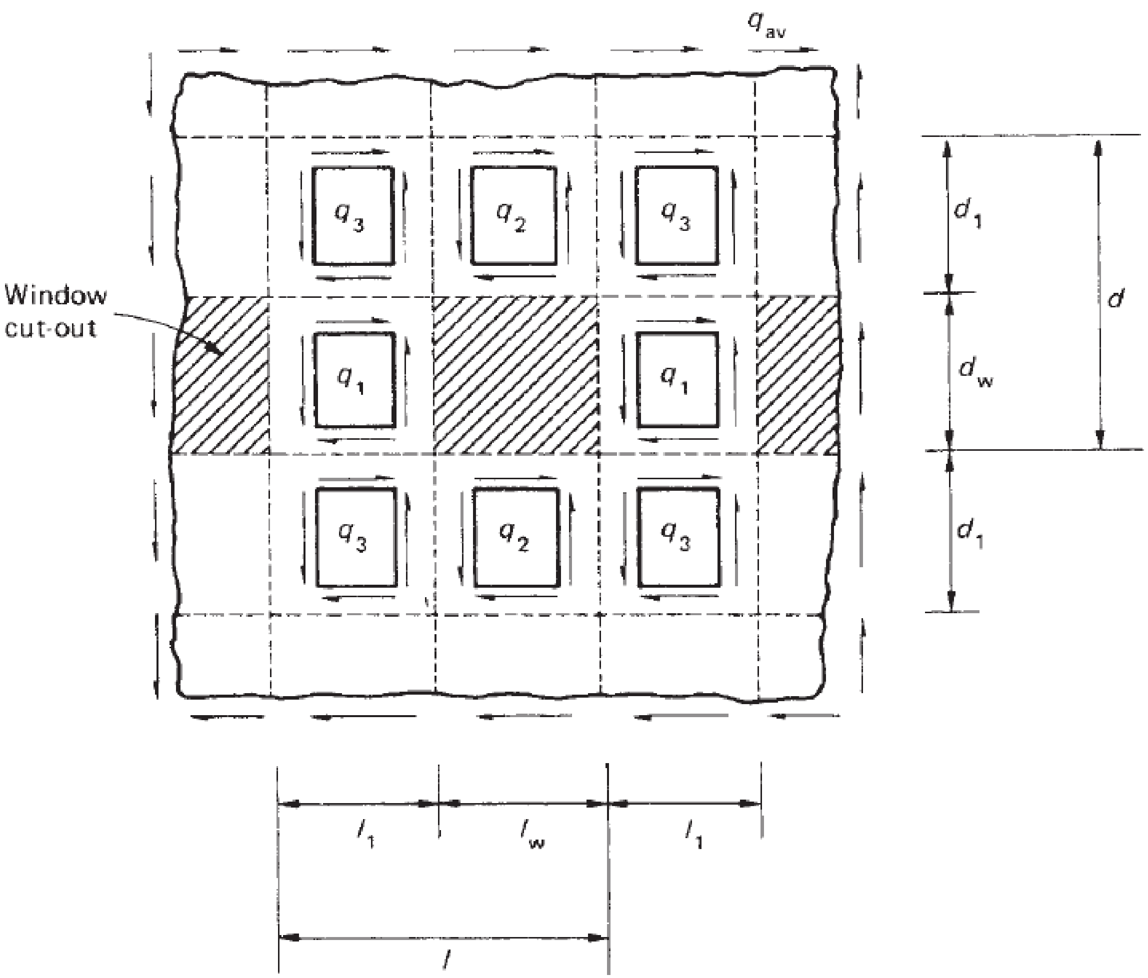
In such situations a rigid frame is placed around the opening to resist shear loads and to transmit loads from one side of the opening to the other.

The effects of smaller cut-outs, such as those required for rows of windows in passenger aircraft, may be found approximately as follows.

Figure shows a fuselage panel provided with cut-outs for windows which are spaced a distance *l* apart.

The panel is subjected to an average shear flow q_{av} which would be the value of the shear flow in the panel without cut-outs.

Considering a horizontal length of the panel through the cut-outs we see that



.5 Fuselage panel with windows.

$$q_1 l_1 = q_{\text{av}} l$$

$$q_1 = \frac{l}{l_1} q_{\text{av}}$$

Now considering a vertical length of the panel through the cut-outs

$$q_2 d_1 = q_{\text{av}} d$$

$$q_2 = \frac{d}{d_1} q_{\text{av}}$$

The shear flows q_3 may be obtained by considering either vertical or horizontal sections not containing the cut-out. Thus

$$q_3 l_1 + q_2 l_{\text{w}} = q_{\text{av}} l$$

Substituting for q_2 from Eq. (22.3) and noting that $l = l_1 + l_w$ and $d = d_1 + d_w$, we obtain

$$q_3 = \left(1 - \frac{d_{\mathrm{w}}}{d_1}\frac{l_{\mathrm{w}}}{l_1}\right) q_{\mathrm{av}}$$

Fuselage frames and wing ribs

Aircraft are constructed primarily from thin metal skins which are capable of resisting in-plane tension and shear loads but buckle under comparatively low values of in-plane compressive loads.

The skins are therefore stiffened by longitudinal stringers which resist the in-plane compressive loads and, at the same time, resist small distributed loads normal to the plane of the skin.

The effective length in compression of the stringers is reduced, in the case of fuselages, by transverse frames or bulkheads or, in the case of wings, by ribs.

In addition, the frames and ribs resist concentrated loads in transverse planes and transmit them to the stringers and the plane of the skin.

Thus, cantilever wings may be bolted to fuselage frames at the spar caps while undercarriage loads are transmitted to the wing through spar and rib attachment points.

Source from Aircraft Structures by T. H. G. Megson

Principles of stiffener/web construction

Generally, frames and ribs are themselves fabricated from thin sheets of metal and therefore require stiffening members to distribute the concentrated loads to the thin webs.

If the load is applied in the plane of a web the stiffeners must be aligned with the direction of the load.

Alternatively, if this is not possible, the load should be applied at the intersection of two stiffeners so that each stiffener resists the component of load in its direction.

Source from Aircraft Structures by T. H. G. Megson

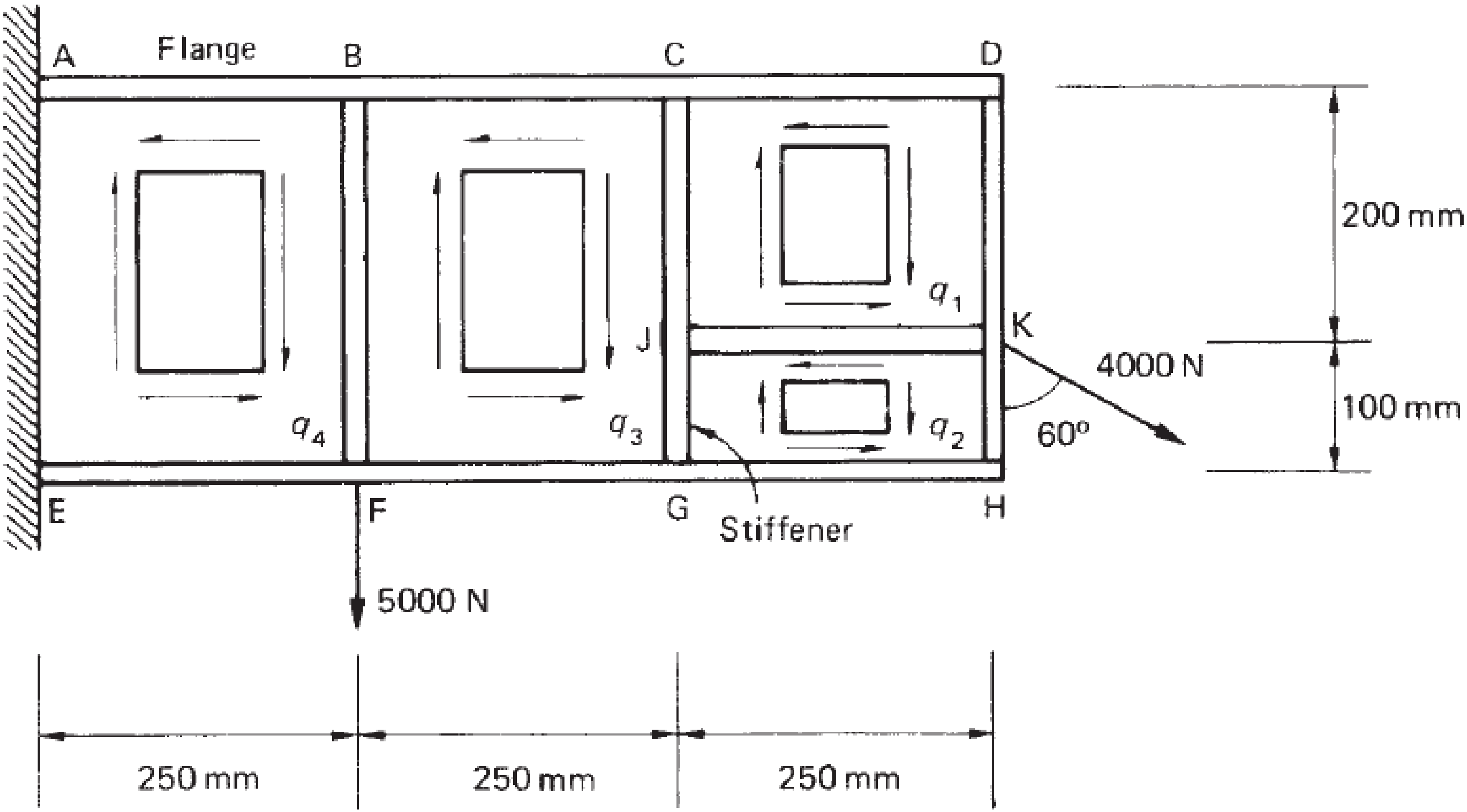
A cantilever beam Fig. below carries concentrated loads as shown. Calculate the distribution of stiffener loads and the shear flow distribution in the web panels assuming that the latter are effective only in shear.

We note that stiffeners HKD and JK are required at the point of application of the 4000N load to resist its vertical and horizontal components.

A further transverse stiffener GJC is positioned at the unloaded end J of the stiffener JK since stress concentrations are produced if a stiffener ends in the centre of a web panel.

We note also that the web panels are only effective in shear so that the shear flow is constant throughout a particular web panel; the assumed directions of the shear flows are shown in Fig.

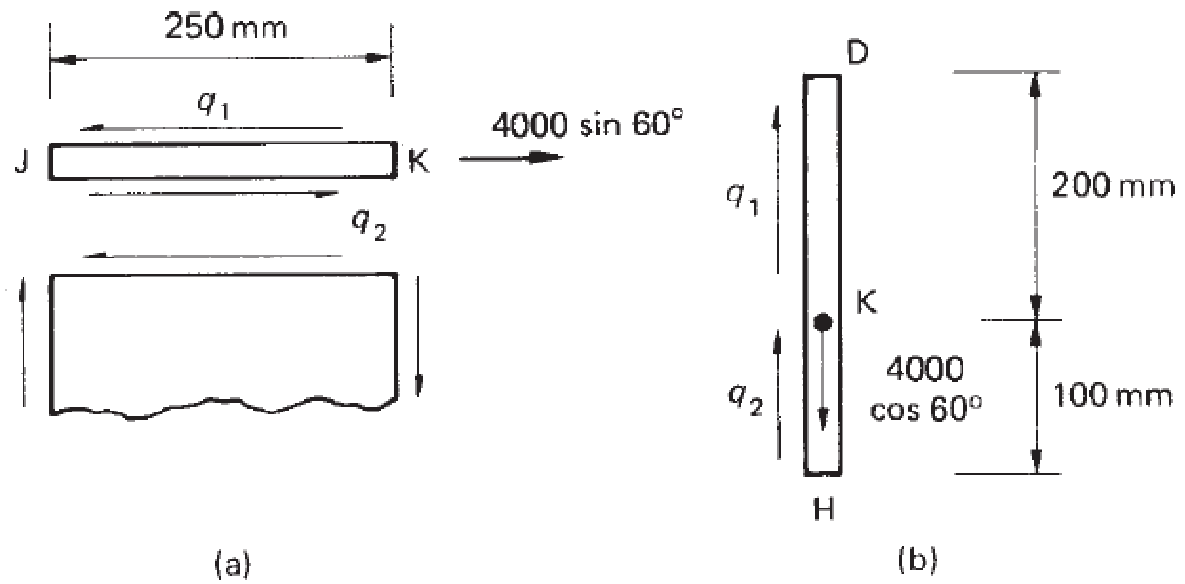
Source from Aircraft Structures by T. H. G. Megson



Source from Aircraft Structures by T. H. G. Megson

From the equilibrium of stiffener JK we have

$$(q_1 - q_2) \times 250 = 4000 \sin 60^{\circ} = 3464.1\text{N}$$



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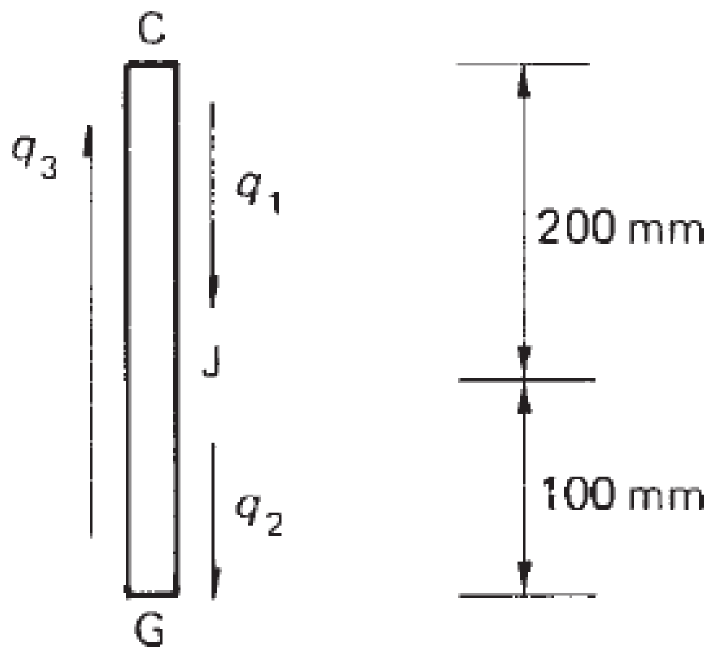


Equilibrium of stiffener CJG in the beam

Source from Aircraft Structures by T. H. G. Megson

and from the equilibrium of stiffener HKD

$200q_1 + 100q_2 = 4000 \cos 60^\circ = 2000\text{N}$ (ii)

$q_1 = 11.3\text{N/mm}$ $q_2 = -2.6\text{N/mm}$

The vertical shear force in the panel BCGF is equilibrated by the vertical resultant of the shear flow q_3. Thus

$300q_3 = 4000 \cos 60^\circ = 2000\text{N}$
Whence $q_3 = 6.7\text{N/mm}$

Alternatively, q_3 may be found by considering the equilibrium of the stiffener CJG.

$300q3 = 200q1 + 100q2$

or

$300q3 = 200 \times 11.3 - 100 \times 2.6$

from which

$q3 = 6.7\text{N/mm}$

The shear flow $q4$ in the panel ABFE may be found using either of the above methods.

Thus, considering the vertical shear force in the panel

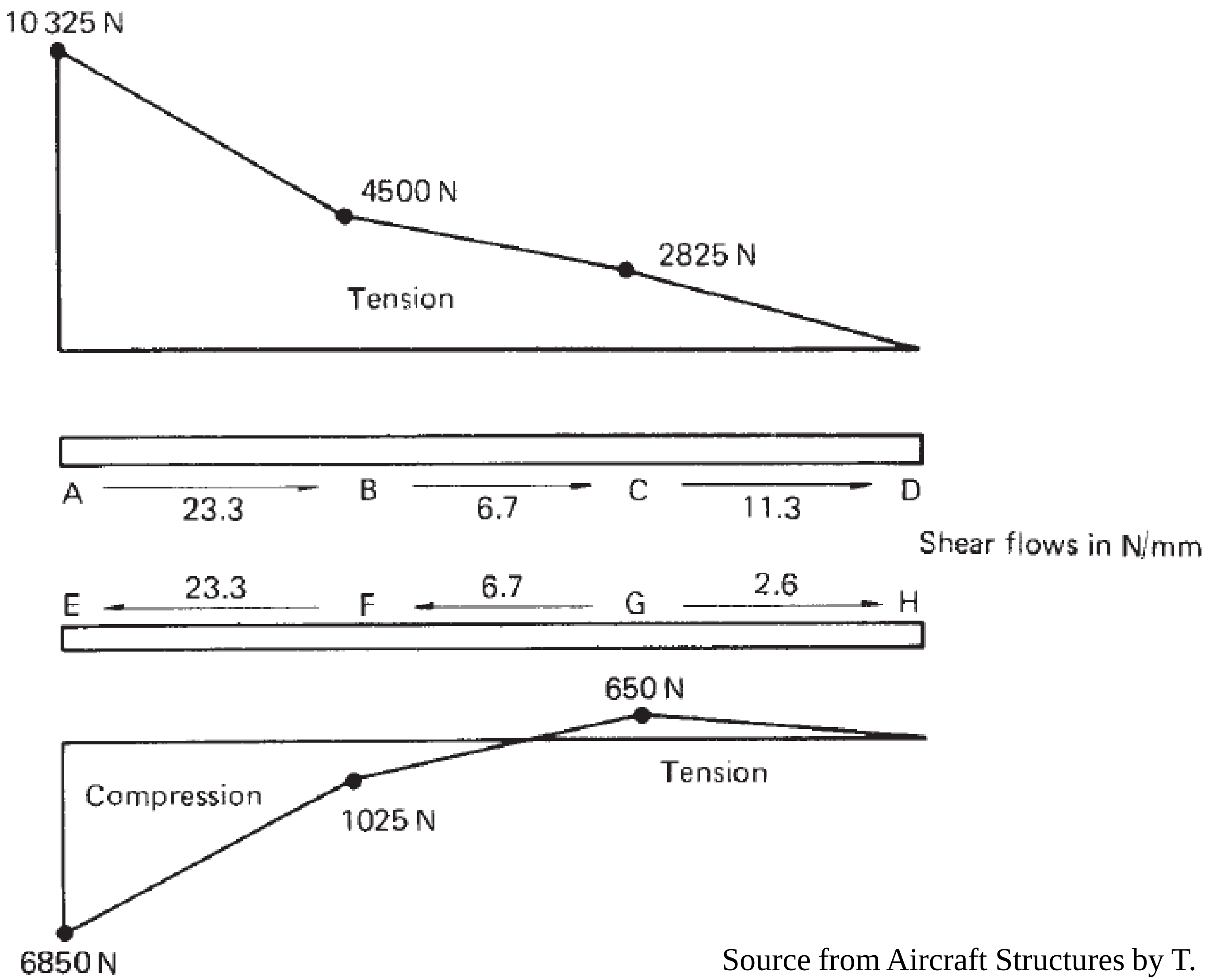
$300q4 = 4000 \cos 60^\circ + 5000 = 7000\text{N}$

whence

$q4 = 23.3\text{N/mm}$

Alternatively, from the equilibrium of stiffener BF

$300q4 - 300q3 = 5000\text{N}$



Source from Aircraft Structures by T. H. G. Megson

Landing Gear Types

Aircraft landing gear supports the entire weight of an aircraft during landing and ground operations.

They are attached to primary structural members of the aircraft. The type of gear depends on the aircraft design and its intended use.

Most landing gear have wheels to facilitate operation to and from hard surfaces, such as airport runways.

Other gear feature skids for this purpose, such as those found on helicopters, balloon gondolas, and in the tail area of some tail dragger aircraft.

Aircraft that operate to and from frozen lakes and snowy areas may be equipped with landing gear that have skis.

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Aircraft that operate to and from the surface of water have pontoon-type landing gear.

Regardless of the type of landing gear utilized, shock absorbing equipment, brakes, retraction mechanisms, controls, warning devices, cowling, fairings, and structural members necessary to attach the gear to the aircraft are considered parts of the landing gear system.

Landing Gear Arrangement

Three basic arrangements of landing gear are used: tail wheel type landing gear (also known as conventional gear), tandem landing gear, and tricycle-type landing gear.

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Tail Wheel-Type Landing Gear

Tail wheel-type landing gear is also known as conventional gear because many early aircraft use this type of arrangement.

The main gear are located forward of the center of gravity, causing the tail to require support from a third wheel assembly.

A few early aircraft designs use a skid rather than a tail wheel. This helps slow the aircraft upon landing and provides directional stability.

The resulting angle of the aircraft fuselage, when fitted with conventional gear, allows the use of a long propeller that compensates for older, underpowered engine design.

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The increased clearance of the forward fuselage offered by tail wheel-type landing gear is also advantageous when operating in and out of non-paved runways.

Today, aircraft are manufactured with conventional gear for this reason and for the weight savings accompanying the relatively light tail wheel assembly.

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Tandem Landing Gear

Few aircraft are designed with tandem landing gear. As the name implies, this type of landing gear has the main gear and tail gear aligned on the longitudinal axis of the aircraft.

Sailplanes commonly use tandem gear, although many only have one actual gear forward on the fuselage with a skid under the tail.

A few military bombers, such as the B-47 and the B-52, have tandem gear, as does the U2 spy plane.

The VTOL Harrier has tandem gear but uses small outrigger gear under the wings for support.

Generally, placing the gear only under the fuselage facilitates the use of very flexible wings.

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Tricycle-Type Landing Gear

The most commonly used landing gear arrangement is the tricycle-type landing gear. It is comprised of main gear and nose gear. *[Figure 13-6]*

Tricycle-type landing gear is used on large and small aircraft with the following benefits:

1. Allows more forceful application of the brakes without nosing over when braking, which enables higher landing speeds.
2. Provides better visibility from the flight deck, especially during landing and ground maneuvering.
3. Prevents ground-looping of the aircraft. Since the aircraft center of gravity is forward of the main gear, forces acting on the center of gravity tend to keep the aircraft moving forward rather than looping, such as with a tail wheel-type landing gear.

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