



INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad -500 043

AERONAUTICAL ENGINEERING

COURSE LECTURE NOTES

Course Name	Flight mechanics
Course Code	AAEB09
Programme	B.Tech
Semester	IV
Course Coordinator	Madhurakavi Sravani , Assistant Professor, AE
Course Faculty	Madhurakavi Sravani , Assistant Professor, AE C.Navaneetha, Assistant Professor, AE

FLIGHT MECHANICS

IV Semester: AE

Course Code	Category	Hours / Week			Credits	Maximum Marks		
		L	T	P		CIA	SEE	Total
AAEB09	Core	3	1	-	4	30	70	100
Contact Classes: 45		Tutorial Classes: 15			Practical Classes: Nil		Total Classes: 60	

OBJECTIVES:

The course should enable the students to:

- I. Learn the different Regimes of aircraft and performance requirements at different atmospheric conditions.
- II. Understand the different type of velocities and gives differences between stall velocity and maximum and minimum velocities.
- III. Estimate the time to climb and descent and gives the relation between rate of climb and descent and time to climb and descent at different altitudes.
- IV. Illustrate the velocity and radius required for different type of maneuvers like pull-up, pull down and steady turn.

MODULE -I	INTRODUCTION TO AIRCRAFT PERFORMANCE	Classes: 10
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The role and design mission of an aircraft; Performance requirements and mission profile; Aircraft design performance, the standard atmosphere; Off-standard and design atmosphere; Measurement of air data; Air data computers; Equations of motion for performance - the aircraft force system; Total airplane drag- estimation, drag reduction methods; The propulsive forces, the thrust production engines, power producing engines, variation of thrust, propulsive power and specific fuel consumption with altitude and flight speed; The minimum drag speed, minimum power speed; Aerodynamic relationships for a parabolic drag polar.

MODULE -II	CRUISE PERFORMANCE	Classes:08
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Maximum and minimum speeds in level flight; Range and endurance with thrust production, and power producing engines; Cruise techniques: constant angle of attack, constant mach number; constant altitude, methods- comparison of performance. The effect of weight, altitude and temperature on cruise performance; Cruise performance with mixed power-Plants.

MODULE -III	CLIMB AND DECENT PERFORMANCE	Classes: 10
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Importance of Climb and descent performance, Climb and descent technique generalized performance analysis for thrust producing, power producing and mixed power plants, maximum climb gradient, and climb rate. Energy height and specific excess power, energy methods for optimal climbs - minimum time, minimum fuel climbs. Measurement of best climb performance. Descent performance in Aircraft operations. Effect of wind on climb and decent performance.

MODULE -IV	AIRCRAFT MANOEUVRE PERFORMANCE	Classes: 09
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Lateral maneuvers- turn performance- turn rates, turn radius- limiting factors for turning performance. Instantaneous turn and sustained turns, specific excess power, energy turns. Longitudinal aircraft maneuvers, the pull-up, maneuvers. The maneuver envelope (V-n diagram), Significance. Maneuver boundaries and limitations, Maneuver performance of military Aircraft, transport Aircraft.

MODULE -V	SAFETY REQUIREMENTS -TAKEOFF AND LANDING PERFORMANCE AND FLIGHT PLANNING	Classes:08
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Estimation of takeoff distances. The effect on the takeoff distance of weight wind, runway conditions, ground effect. Takeoff performance safety factors. Estimation of landing distances. The discontinued landing, Baulk landing, air safety procedures and requirements on performance. Fuel planning fuel requirement, trip fuel, Environment effects, reserve, and tinkering.

Text Books:

1. Anderson, J.D. Jr., —Aircraft Performance and Design, International edition McGraw Hill, 1st Edition, 1999, ISBN: 0-07-001971-1.
2. Eshelby, M.E., —Aircraft Performance theory and Practice, AIAA Education Series, AIAA, 2nd Edition, 2000, ISBN: 1-56347-398-4.

Reference Books:

1. McCormick, B.W., —Aerodynamics, Aeronautics and Flight Mechanics, John Wiley, 2nd Edition, 1995, ISBN: 0-471-57506-2.
2. Yechout, T.R. et al., —Introduction to Aircraft Flight Mechanics, AIAA Education Series, AIAA, 1st Edition, 2003, ISBN: 1-56347-577-4.
3. Shevel, R.S., —Fundamentals of Flight, Pearson Education, 2nd Edition, 1989, ISBN: 81-297-0514-1.

Web References:

1. www.myopencourses.com/subject/flight-dynamics-i-airplane-performance
2. www.scribd.com/doc/185026212/Introduction-to-Flight-Third-Edition-by-John-D-Anderson-Jr
3. www.scribd.com/book/282507871/Performance-and-Stability-of-Aircraft
4. www.scribd.com/doc/203462287/Aircraft-Performance-NPTEL
5. www.nptel.ac.in/courses/101106041/

E-Text Books:

1. www.scribd.com/doc/97544751/Anderson-Aircraft-Performance-and-Design

MODULE –I

INTRODUCTION TO AIRCRAFT PERFORMANCE

The role and design mission of an aircraft

Performance can be used as a measure of the capability of the aircraft in many ways. Performance can be defined as a measure of the ability of the aircraft to carry out a specified task.

In the case of a civil transport aircraft it determines an element of the cost of the operation of the aircraft and hence it contributes to its economic viability as a transport vehicle. In military combat operations, time, maneuver and radius of action are some of the more critical performance parameters in the overall evaluation of the effectiveness and air superiority of the aircraft.

Performance can also be regarded as a measure of safety.

An aircraft has an excess of thrust over drag it can increase its energy by either climbing or accelerating if the drag exceeds the thrust then it will be losing energy as it either decelerates or descends.

In safe flight the thrust must not be committed to a decrease of energy that would endanger it so that at all critical points in the mission thrust must exceed the drag. This is a consideration of the performance aspect of the airworthiness of the aircraft.

The design of an aircraft starts from the statement of the flight path related performance that the aircraft is expected to achieve.

The basic statement of the performance will be concerned with the payload the aircraft will be required to carry and the mission profile it will be required to fly.

The payload of a civil transport aircraft may be defined in the terms of no. of passengers, tonnage of freight, volume of freight or as combinations of freight and passengers.

The definition of military aircraft mission payloads may cover a wide range of possibilities including personnel, troops, support equipment and supplies in transport aircraft and internally carried stores, externally carried stores and sensor pods on combat aircraft.

Specification Of Performance Requirements

Mission Profile:

Aircraft operations can be classified into civil operations, which are commercial flights transporting passengers or cargo.

Military operations which are concerned with defensive or offensive flight operations or their associated support operations.

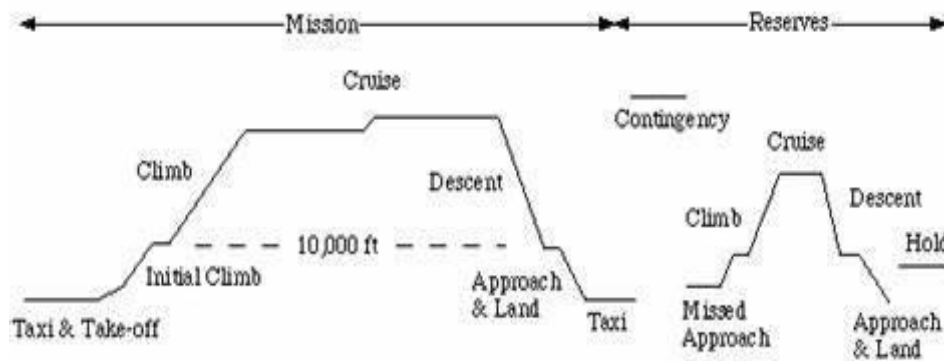


Figure:1.1 Measurement of air data; Air data computers

A typical mission profile of a civil transport aircraft is shown in the above figure.

The primary mission is to fly a payload from the departure point to the destination. This requires the aircraft to takeoff from the departure point, climb to the cruising height and cruise to the destination, where the aircraft descends and lands.

If the aircraft be unable to land the destination when it arrives, it will have to divert to an alternate airfield and the flight plan will need to include provision for the diversion.

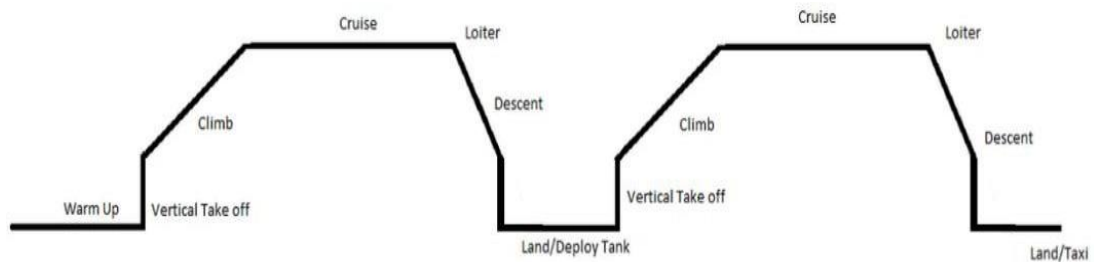


Figure:1.2 Military aircraft mission profiles

Once the mission profile and the payload of the aircraft have been specified the design process can commence. From the performance standpoint the total design process extends from the initial project design estimations right through to the delivery of the aircraft into service.

In the final phase the aircraft is prepared for its operational role. The overall procedure can be divided into 3 broad areas:

- Performance estimation
- Performance measurement
- Operational performance

Performance Estimation:

Performance estimation involves the prediction of the capabilities of the aircraft from the considerations of its aerodynamic design, power plant and operating environment.

It can be applied to

- 1) The design of new type of aircraft
- 2) Modification of existing aircraft type in respect to design changes affecting its aerodynamic characteristics or power plant
- 3) To supplement or extend the full scale measured performance of an aircraft type for conditions outside those already established

Performance estimation process begins with the proposal of some performance target.

Initially the estimation will center on individual elements of the flight path. Performance estimation is usually based on the assumption of a simple atmosphere model, the international standard atmosphere (ISA). This is a linearized model of the temperature atmosphere which represents the mean global atmosphere state with respect to seasonal changes and latitude and is used as the basis for aircraft design.

Performance Measurement:

A performance measurement is required for the three main purposes:

- 1) To verify that aircraft achieves the estimated design performance targets.
- 2) To demonstrate that the aircraft can satisfy the safety criteria set down in the worthiness requirements.
- 3) To provide validated performance data for the performance section of the flight manual.

The performance of the aircraft is measured in development trials and compared with the estimated performance where there is difference in the characteristics of the aircraft and of the power plant can be measured and compared with those used in the models.

As the design of the aircraft is developed and the flight trials show that it is meeting its performance targets, data was measured for submission to the airworthiness authority for the certification of the aircraft.

As a part of the certification process validated performance data are required for the performance section of the flight manual, known as the flight performance manual or the operating data manual(ODM), which contains the information of the performance of the aircraft needed by the operator for flight planning.

Operational Performance

The basic requirements for the safe flight are that the space required for the aircraft to maneuver should never exceed the space available, and that the aircraft carries sufficient fuel for the flight these fundamental requirements form the basis of performance planning and fuel planning.

Performance planning: It is a part of the flight plan made in advance of the flight ensures that at any point in the flight the aircraft has sufficient performance to be able to maneuver within the space available. The space required for any given maneuver is a function of the weight of the aircraft and the space required increases as the weight increases.

Fuel planning: It ensures that the aircraft carries sufficient fuel for the mission, taking into account reserves for contingencies, diversions and safety. Since the fuel required for the mission will depend on the takeoff weight of the aircraft the fuel planning must follow the flight planning.

The Atmosphere and Air data Measurement

The state of the atmosphere defined by its temperature and pressure is fundamental to both the design and operation of the aircraft. The atmospheric air provides the lift force that propulsive force that is necessary to sustain flight. These forces depend on the properties of the atmosphere and aircraft.

The Characteristics of the Atmosphere

The atmosphere consists of air which is a mixture of gases, mainly Nitrogen (78%), Oxygen (21%) with traces of argon (0.9%), carbon dioxide (0.03%) and other inert gases (0.07%) in minute quantities. There are quantities of dust particles, water vapor and moisture in variable amounts which although they do not affect the gaseous properties of the air significantly.

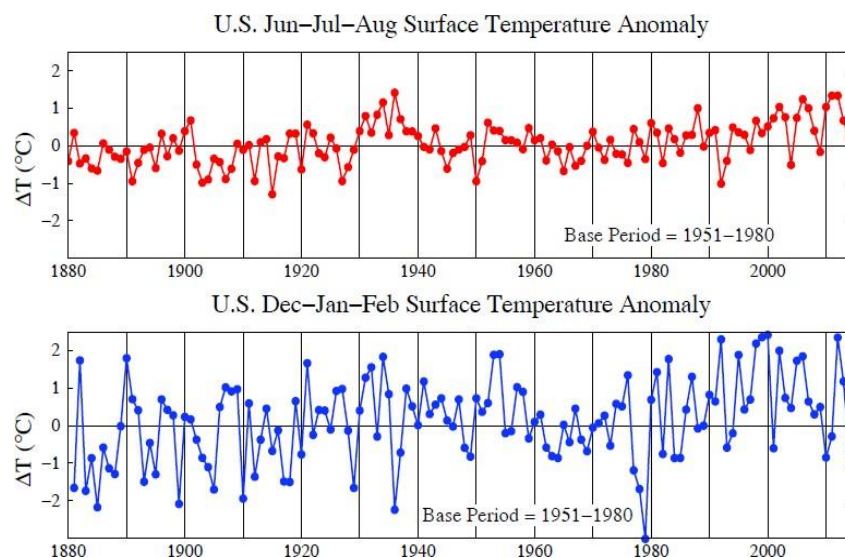


Figure:1.3 Mean seasonal global temperature distribution.

The atmosphere air can be taken to behave as a neutral gas that obeys the equation of state

$$p = \rho TR$$



Figure:1.4 General global atmosphere pressure distribution

Vertical development of the atmosphere

The radiation that is absorbed by the atmosphere is not absorbed uniformly but selectively by different layers giving rise to a complex temperature –height profile in the atmosphere.

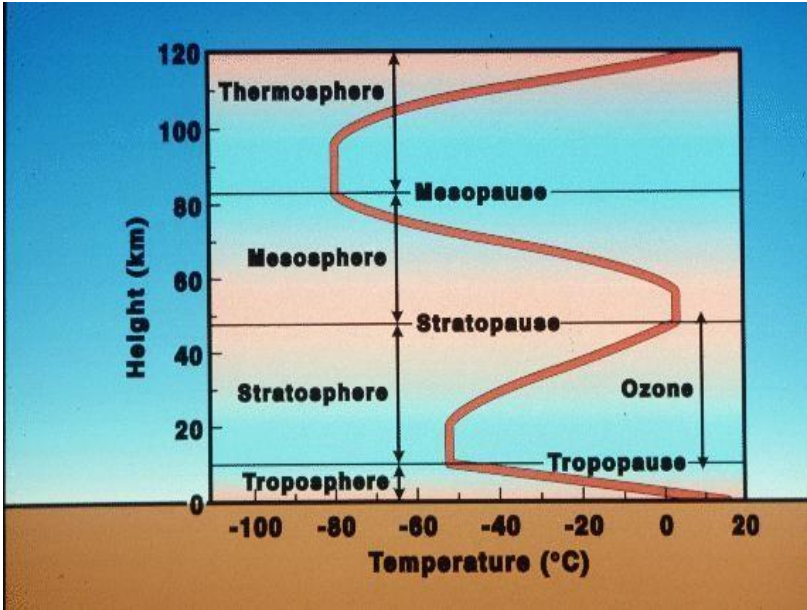


Figure:1.5 Vertical temperature structure of the atmosphere

At low levels the water vapor and carbon dioxide absorb the terrestrial radiation producing a warm air region near the ground extending upwards to about 11km, this layer is known as the troposphere. In the troposphere the temperature decreases with increasing height and the temperature-height gradient which is negative here known as the temperature lapse rate, L . Above this region there is little water vapor in the atmosphere and its absorptive is reduced, this layer is called the stratosphere which extends upwards to some 50km. The ozone content of the atmosphere increases with height up to about 80km and in the layer between 5km and 80 km. The absorptive particularly of the ultra violet spectrum increases to form further warm air layer, this is the mesosphere. Above the mesosphere is a layer of very low pressure the thermosphere, extending up to about 800km and the final layer, the exosphere forms the boundary with space.

The standard atmosphere model

The performance of the aircraft is depends on the state of the atmosphere in which it is flying. The state f the atmosphere, as defined by its pressure and temperature is viable, so that the actual performance of the aircraft will depend on its geographical location and time. In the design of the aircraft assumptions of the state of the atmosphere will have to be made in order to predict its performance. A model of structure of the atmosphere is required.

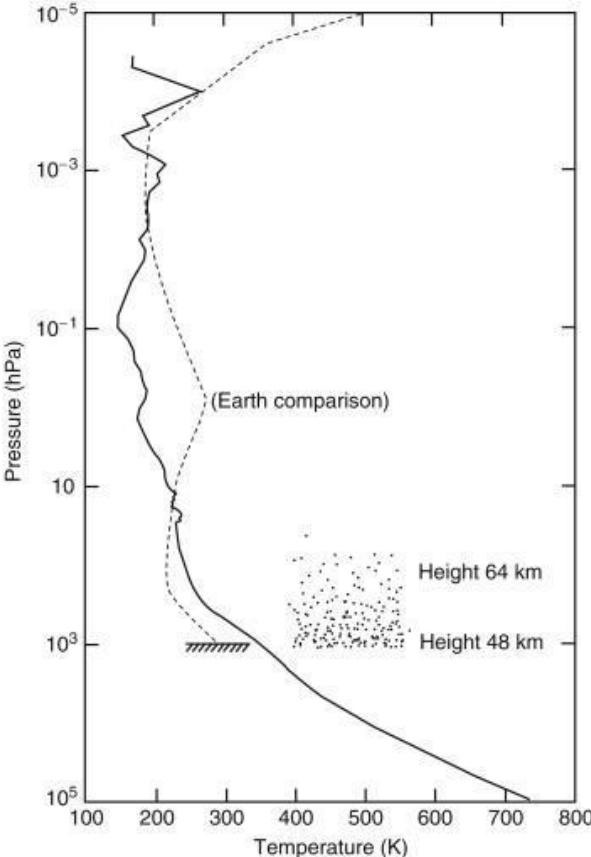


Figure:1.6 Measured temperatures – height profiles

The atmosphere model needs to represent an average atmosphere with respect to geographical and seasonal variations in pressure and temperature and to have a vertical structure which is similar to that in the real atmosphere.

An atmosphere model has been accepted by international agreement and is used as the basis for all performance work, it is known as the International Standard Atmosphere.

The reference datum values of the principal characteristics of the international standard atmosphere model are Reference pressure $P_0=101325 \text{ N/m}^2$

Reference temperature

$$T_0=288.15 \text{ K}$$

Reference density

$$\rho_0=1.225 \text{ Kg/m}^3$$

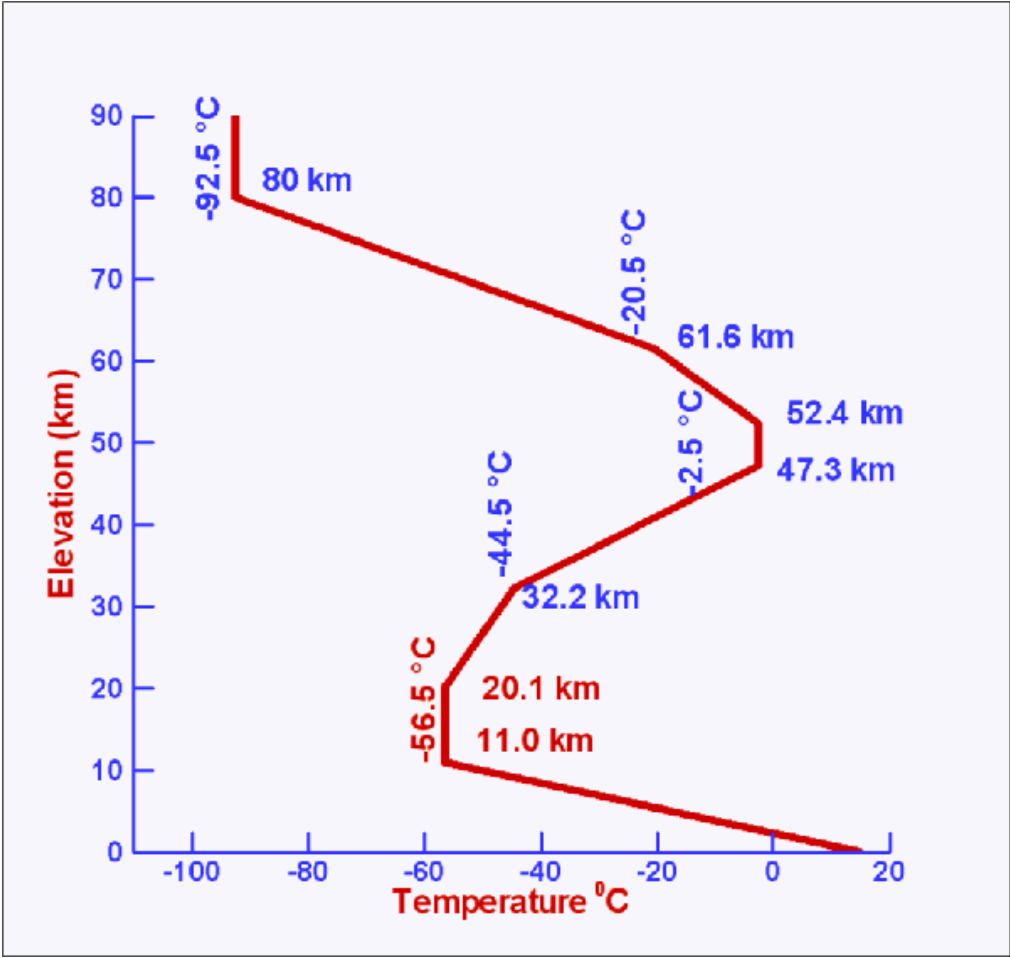


Figure:1.7 International standard atmosphere model; temperature – height profile.

The vertical structure of the atmosphere model is defined by the assumptions of a series of linear relationships between temperature and height as shown in the above figure up to a height of 32km which is the vertical extent of the ISA model used in connection with aircraft performance the model consists of three layers in each of which the temperature height profile is given by

Where the subscript I denotes the height of the layer boundary of the layer considered in kms. Thus at the datum level $i=0$ and the temperature lapse rate L_i is the rate of change of temperature, weight, height in the layer above H_i

Pressure height

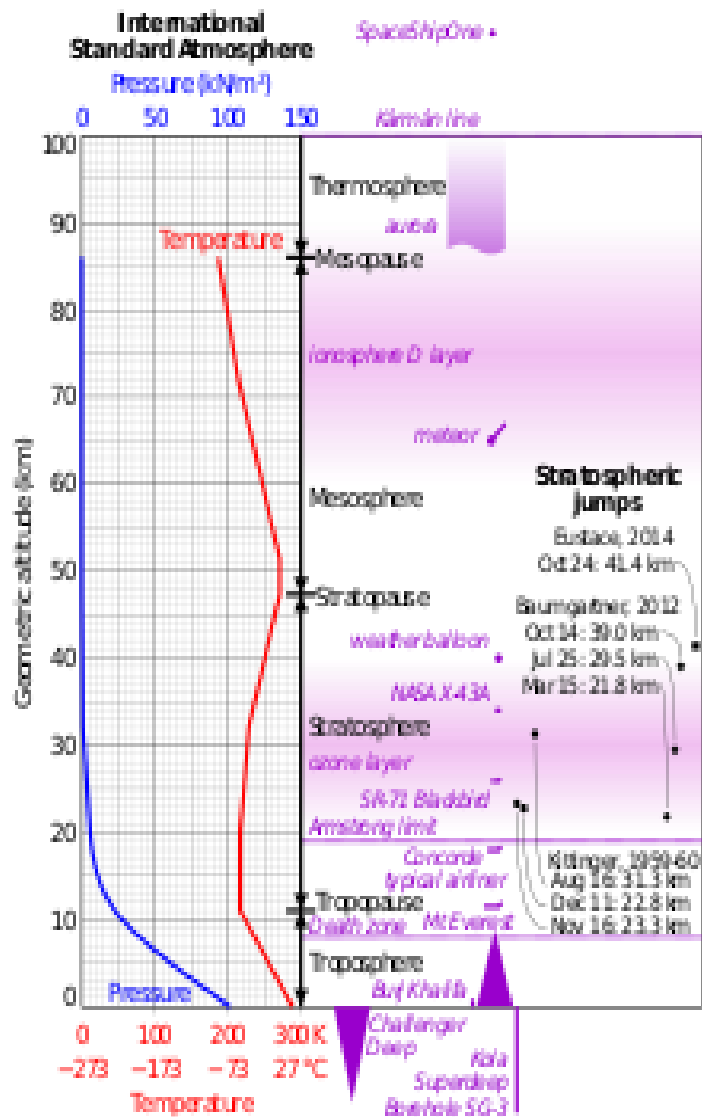


Figure:1.8 Pressure height

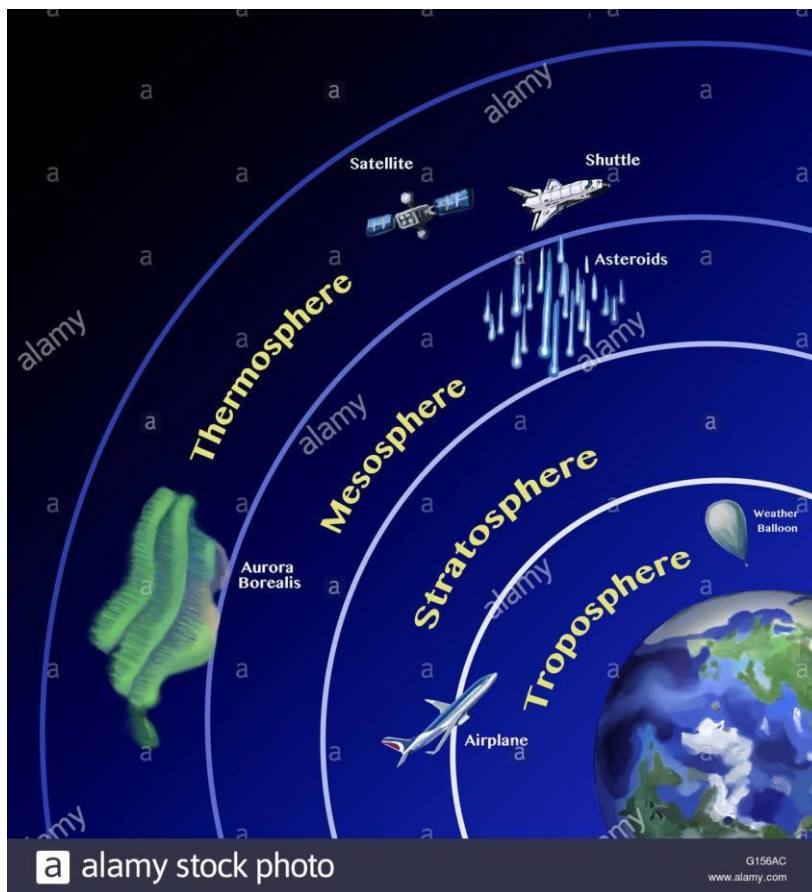
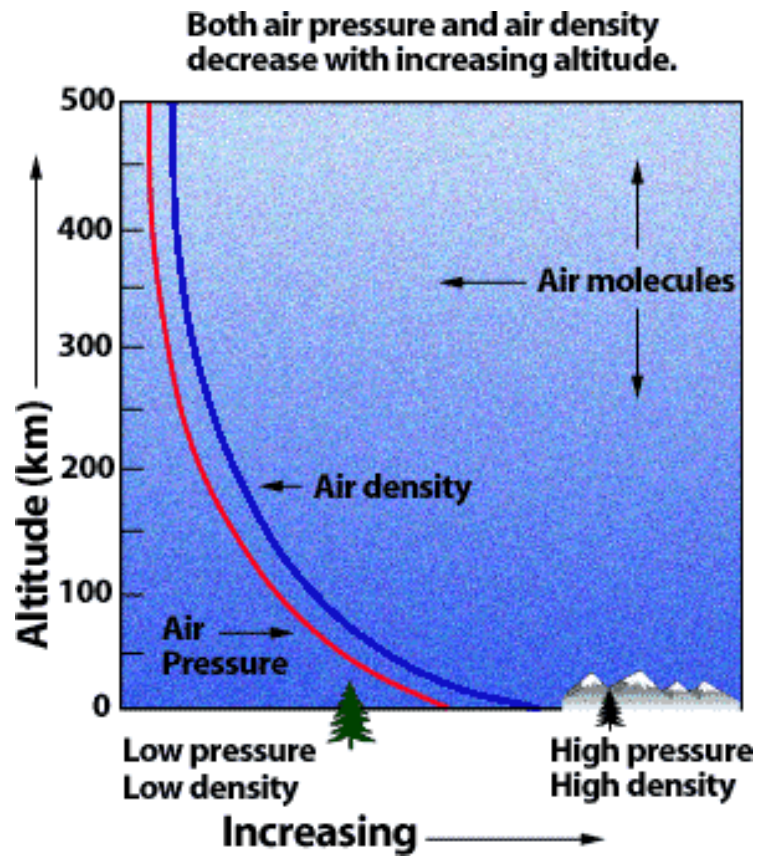


Figure:1.9 Design atmospheres. A) Pressure heights. B) Geo-potential heights.

The difference between pressure height and geo-potential height in an off standard atmosphere can be determined in any atmosphere

$$\frac{dp}{p} = -\frac{g_0}{RT} dH$$

And in the standard atmosphere since $H=H_p$

$$\frac{dP}{P} = -\frac{Mg dz}{R^* T}$$

Where T_{std} is the standard atmosphere temperature at the pressure height H_p

$$dH = \left[\frac{T}{T_{std}} \right]_{H_p} dH_p$$

Thus a geo-potential height increment dH is related to a pressure height increment dH_p by a temperature correction. This correction is used to obtain geo-potential height intervals from measured height intervals for the measurement of gradient of climb and other flight path related performance characteristics.

Relative properties of atmosphere

The atmospheric equation of state applies to all points in the atmosphere so that,

$$P = \frac{\rho}{M} \cdot R^* T$$

And at the ISA datum

$$p_0 = \rho_0 T_0 R$$

Thus

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} \frac{T}{T_0}$$

This can be written

$$\delta = \sigma \theta$$

The relative properties are a convenient means of expressing and manipulating the atmosphere properties and avoiding the need to use the gas constant.

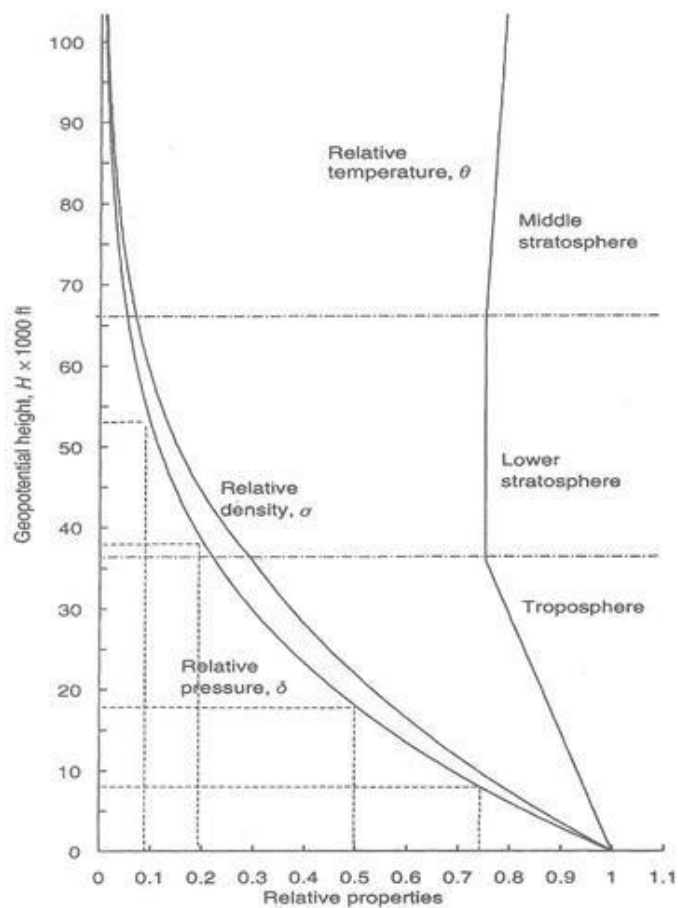


Figure:1.10 International standard atmosphere; relative properties.

Pressure height profiles;

8000 ft $\delta = 0.75$ (maximum cabin pressure height for passenger transport) 18000 ft $\delta = 0.5$ (short haul operations)

38000 ft $\delta = 0.2$ (long range transport operations)

53000 ft $\delta = 0.1$ (Concorde and some military operations)

100000 ft $\delta = 0.01$ (TR1, SR71 surveillance aircraft, 80-90 000 ft)

Density Altitude:

It is sometimes more convenient to consider the state of the atmosphere in terms of its density rather than its pressure and temperature separately. In this case, the relationship between the density and height in the standard atmosphere model is used as a datum.

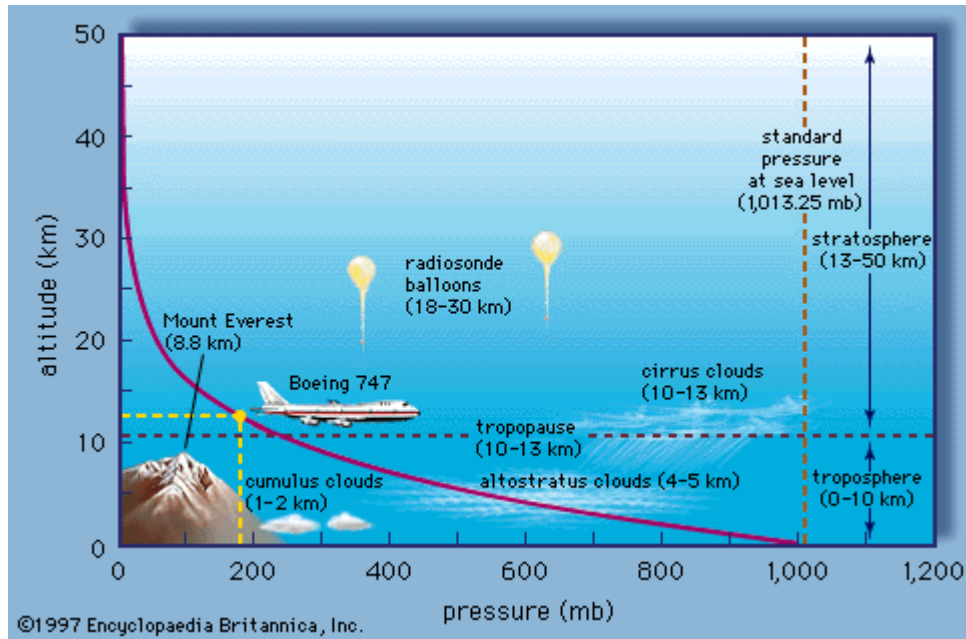


Figure:1.11 Density atmosphere

The concept of the density altitude can be illustrated by a simple example. If the observed temperature at a pressure height 15000ft is -30°C then, from the pressure height relationship, the relative pressure at 15000ft is

$$\delta = 0.56434$$

And the relative temperature is

$$\theta = 243.15/288.15 = 0.84383$$

Giving the relative density to be

$$\sigma = \delta/\theta = 0.66878.$$

Now from the properties of the standard atmosphere the pressure height at which a relative density of 0.66878 occurs is 13120ft. thus the density altitude is 13120ft since the standard atmosphere density at this height is equivalent to the actual density at a pressure height of 15000ft and a temperature of -30°C this is shown by point A in fig 1.10

Measurement of air data:

The essential requirements in the measurement of aircraft performance are first, the knowledge of the state of the atmosphere in which the aircraft is flying and secondly the relative motion between the aircraft and the air mass. This information is collected by the air data system.

The air data system of an aircraft in fig 1.11 consists of a Pitot-static installation to sense the airflow pressures from which height, airspeed and Mach number are derived. An air thermometer from which the air temperature can be determined and in some cases airflow direction detectors (ADD) which sense the local flow directions relative to the aircraft body axes are part of the system.

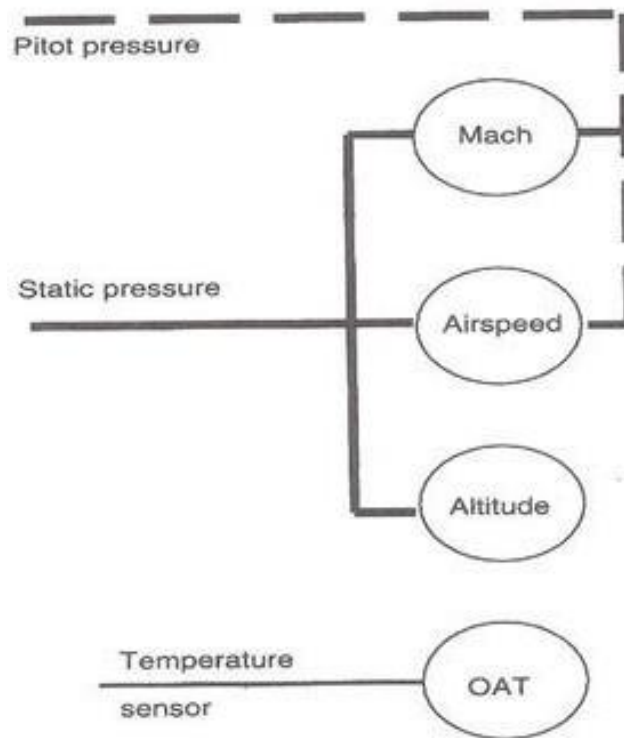


Figure:1.12 Air data system of an aircraft

Both the air data computer and the mechanical instruments use the same basic calibration equations to convert the measured data into a suitable form for operational use. The calibration equation will be developed in the subsequent sections. The modules used in the display of air data are usually the foot for measurement of height and the knot for airspeed since international regulation requires primary flight instruments to be calibrated in these modules.

Measurement of height:

In above section relationships were found the related pressure to geo-potential height in the standard atmosphere. By rearranging these equations height can be expressed in terms of pressure so that in the troposphere in which $L \neq 0$

$$H = \frac{T_0}{L_0} \left[\left(\frac{p}{p_0} \right)^{\frac{-L_0 R}{g_0}} - 1 \right]$$

And in the isothermal lower stratosphere in which $L=0$

$$H = 11\,000 - \frac{RT_{11}}{g_0} \ln \left(\frac{p}{p_{11}} \right)$$

Although the standard atmosphere model uses a static pressure of 1013mb as its datum in practice height is measured with respect to other datum pressures, one of which is mean sea level. Fig 1.12 shows the most common altimeter datum settings

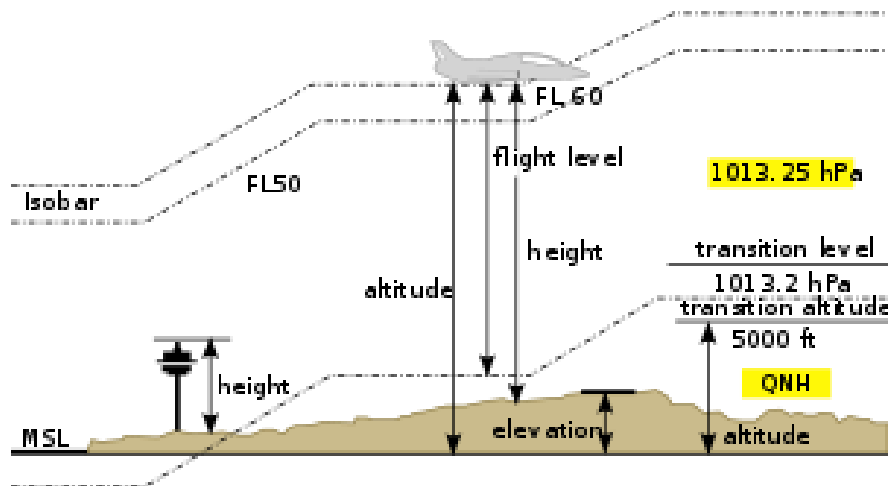


Figure:1.13 Altimeter reference pressure settings

In the measurement of height a number of corrections need to be taken into account before the pressure height and the quantities derived from pressure height can be determined. These can be summarized working back from the altimeter to the free stream flow.

(a) Altimeter reading; Alt or H_{pi}

This is the reading of an individual instrument. Since the instrument is a mechanical device driven by the static pressure, there will be errors due to mechanical tolerance.

The instrument error correction can be evaluated by calibrating the individual instrument against an accurate source of pressure and the correction applied to the altimeter reading to give indicated altitude.

(b) Indicated altitude; H_{pi}

This is the altimeter reading corrected for instrument error. The indicated altitude will be measured with reference to the appropriate altimeter datum pressure setting.

(c) Pressure altitude; H_p

This is the indicated altitude measured with respect to the appropriate datum pressure setting corrected for static pressure error. This is the error due to the location of the static pressure source within the disturbed pressure field caused by the presence of the aircraft.

(d) Geo potential height interval; dH_p

This is the pressure height interval, dH_p measured by the altimeter and corrected for temperature difference from the ISA model atmosphere.

(e) Static pressure p and relative pressure

When the altimeter datum pressure is set to 1013mb the pressure heights can be converted into atmospheric pressure or relative pressure either by reference to the atmosphere tables or from the ISA pressure height relationship.

Measurement of airflow characteristics:

Airspeed is the relative velocity between the aircraft and the air mass in which it is flying. It is one of the most important parameters in aircraft performance since the aerodynamic forces acting on the aircraft, and upon which its performance is based are functions of airspeed.

Since the total energy of the flow is constant the energy relationship neglecting the potential energy term can be written in the form

$$\frac{dp}{\rho} + V dV = 0$$

Integrating above equation for adiabatic flow in which

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2} \right)^\gamma$$

Gives

$$\frac{\gamma}{\gamma-1} \frac{p}{\rho} + \frac{V^2}{2} = \text{constant}$$

Relating the flow pressure to the flow velocity or true airspeed V Alternatively from the equation of state 2.1

$$\frac{p}{\rho} = RT$$

And expressing the gas constant R in the form

$$R = C_p \frac{\gamma-1}{\gamma}$$

Above equation can be written in the form

$$C_p T + \frac{V^2}{2} = \text{constant}$$

This relates the flow temperature to the true airspeed.

Above two equations are alternative statements of the energy equation of the adiabatic flow of an ideal gas and can be used in the measurement of the airflow characteristics.

Measurement of airspeed:

The airspeed can be measured by comparing the total and static pressures of the airflow relative to the aircraft. From above equation the energy at any two points in the flow are equal, thus

$$\frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} = \frac{\gamma}{\gamma-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2}$$

If point 1 refers to the undisturbed free stream conditions in which the pressure p_1 is the static pressure p and V_1 is true airspeed of the flow V and point 2 refers to the stagnation conditions in the pitot tube in which the airflow velocity, V_2 is zero and the pressure p_2 is the total or pitot pressure p_p then above equation becomes,

$$\frac{\gamma}{\gamma - 1} \frac{p}{\rho} + \frac{V^2}{2} = \frac{\gamma}{\gamma - 1} \frac{p_p}{\rho_p}$$

Now the speed of the sound a is given by

$$a = \sqrt{\gamma RT} = \sqrt{\frac{\gamma p}{\rho}}$$

So that above equation reduces to

$$p_p = p \left\{ 1 + \frac{\gamma - 1}{2} \left(\frac{V}{a} \right)^2 \right\}^{\frac{\gamma}{\gamma - 1}}$$

Comparing the total and static pressure provides the relationship between airspeed and the differential pressure or impact pressure p_d

$$p_d = p_p - p = p \left[\left\{ 1 + \frac{\gamma - 1}{2} \left(\frac{V}{a} \right)^2 \right\}^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$

THE FORCE SYSTEM OF THE AIRCRAFT AND THE EQUATIONS OF MOTION

The equations of motion for performance:

The equations of motion of the aircraft are statements of Newton's law, $F = ma$, in each of three mutually perpendicular axes. The general force F is the sum of the components of a system of forces acting on the aircraft, which results in the inertial force, ma . The system of forces acting on the aircraft can be categorized into four groups; the gravitational forces, F_g , the aerodynamic forces, F_a , and the propulsive forces, F_p , which result in the inertial forces, F_I , so that the statement of Newton's law becomes,

$$\underline{F_a + F_p + F_g = F_I}$$

There will also be a system of moments acting on the aircraft but, as these do not affect the flight path directly, they do not need to be taken into account in the equations of motion for performance. Each group of forces acts in its own axis system and needs to be resolved into the velocity axis system before the equations of motion for performance. Each group of forces acts in its own axis system and needs to be resolved into the velocity axis system before the equations of motion can be

developed. The axis systems are described in full in Appendix A and the full equations of motion for aircraft performance are developed in Appendix B. Only a summary of the characteristics of the forces and the equations of motion will be considered here.

$$\begin{aligned}
 & -D + [T \cos(\alpha + \tau_1) \cos \beta - D_M] - mg \sin \gamma_2 \\
 & = m\dot{V} + \dot{m}V \\
 & Y \cos \gamma_1 + L \sin \gamma_1 + T[-\cos(\alpha + \tau_1) \sin \beta \cos \gamma_1 + \sin(\alpha + \tau_1) \sin \gamma_1] \\
 & = mV\dot{\gamma}_3 \cos \gamma_2 \\
 & Y \sin \gamma_1 - L \cos \gamma_1 + T[-\cos(\alpha + \tau_1) \sin \beta \sin \gamma_1 - \sin(\alpha + \tau_1) \cos \gamma_1] + mg \cos \gamma_2 \\
 & = -mV\dot{\gamma}_2
 \end{aligned}$$

In these equations of motion some simplifying assumptions have already been made, three include the assumption that all engines are operating at equal gross thrust. For conventional aircraft, additional assumptions can be made to simplify the equations further, these are:

- That the rate of change of aircraft mass is negligible, $\dot{m} = 0$.
- That the aircraft is in symmetric flights that $\beta = 0$ and $Y = 0$.
- That the gross thrust acts in aircraft body axes, $\tau_1 = 0$.
- That the total net thrust $F_N = 0$ and
- That the thrust component $T \sin \gamma$ is small when compared with the lift force.

When these assumptions are made, the equations of motion reduced to a simplified form that can be used for most performance analysis takes:

$$\left. \begin{aligned}
 F_N - D - mg \sin \gamma_2 &= m\dot{V} \\
 L \sin \gamma_1 &= mV\dot{\gamma}_3 \cos \gamma_2 \\
 -L \cos \gamma_1 + mg \cos \gamma_2 &= -mV\dot{\gamma}_2
 \end{aligned} \right\}$$

The majority of performance analysis is based on the longitudinal equation of motion in which the term, $F_N - D$, is known as the excess thrust and provides the increase in potential energy (climb), or the increase in kinetic energy (acceleration).

The equations of motion stated above are written in terms of aircraft with thrust-producing engines. If the aircraft has power-producing engines, which drive propellers to convert the power in to thrust, then the equations must be converted into their power form; this will be considered later in the section on propulsive forces.

The aircraft force system:

In the development of the equation of motion, the forces acting on the aircraft are represented as simple force terms and appear as constants. However, the forces stated in equation and in the equations of motion are not simple forces but depend on the performance variables, aircraft weight, airspeed (or flight Mach number) and the state of the atmosphere. In particular, the aerodynamic forces and the propulsive forces are of great importance to the performance of the aircraft. Their characteristics will define, for example, the airspeeds for best climb rate and gradient and for optimum range or endurance in the cruise part of the flight.

Each group of forces can be considered in turn to determine how its characteristics vary with the flight variables.

The inertial forces, f_I

The inertial forces arise from the mass of the aircraft and its acceleration. The accelerations may be linear accelerations or result from the combination of the forward speed of the aircraft with its rates of pitch and turn. The inertial forces act in the velocity axis system, which is discussed fully in Appendix B.

The gravitational forces, f_g

The gravitational force acts downwards in the Earth axis system and is the product of the aircraft mass, m , and the acceleration due to gravity, g . It may be referred to either as weight, W , or as the product mg ; each form of reference has its own applications within the theory and practice of aircraft performance.

The aerodynamic forces, f_a

The aerodynamic forces arise from the relative motion between the aircraft and the air mass in which it is flying; they act in the wind axis system. It will be assumed that the reader is familiar with the

concepts of aerodynamics and this treatment will only consider the aerodynamic characteristics of the aircraft that are directly applicable to the study of performance.

The dynamic pressure of the airflow, q , may be considered in terms of either airspeed or Mach number,

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2}\gamma p M^2$$

Whilst either form may be used when considering the non-dimensional aerodynamic forces, the form involving the Mach number is particularly useful when considering operational performance. If the airspeed is considered in terms of the flight Mach number, then the temperature of the atmosphere is implicit in the statement of the Mach number and the atmosphere pressure can be considered independently. Since altitude is related uniquely to the static pressure of the atmosphere, the altitude becomes a basic variable of the aerodynamic forces. Therefore, the forces need to be considered only in terms of their variation with aircraft weight, flight Mach number and altitude rather than in terms of aircraft weight, airspeed, altitude and temperature.

The aerodynamic forces that concern performance are the lift, L , the drag, D , and the side force, Y . In the case of an aircraft, the speed of flight is relatively high and the non-dimensional flow variables that characterize the flow are,

$$Re = \frac{\rho V l}{\mu}$$

(i) **The Reynoldsnumber,**

Typically the flight value of Re is large, 10 to 10, and the flow can be treated as continue flow. If the aerodynamic characteristics of the aircraft have been determined from experimental sources (e.g. wind tunnels), any Reynolds number effects should have been accounted for before being used in any performance estimation process. It is unlikely that the Reynolds number will influence the analysis of the full scale flight performance of the aircraft significantly, except in extreme cases.

(ii) **The Mach number,** $M = V/a$

This may vary from almost zero up to a typical maximum of 2.2 for conventional aircraft; higher Mach number is possible but raises special problems. Since this treatment of performance is concerned mainly with subsonic flight, the supersonic flow characteristics will not be considered in depth. Only in the transonic region, where the Mach numbers Will the effects be considered. In flight up to Mach number of 0.5 the flow can be regarded as incompressible and Mach number

effects ignored; for $0.5 < M < 0.8$ compressibility becomes significant and may lead to small changes in the lift and drag force characteristics. For most subsonic aircraft the critical Mach number occurs typically around $M = 0.8$; at this Mach number the local flow at points on the aircraft becomes supersonic and shock waves begin to form. This effect starts the change from subsonic to supersonic flow and affects the characteristics of both the lift and drag forces, leading to significant effects on the performance of the aircraft. Mach number is one of the most important variables of performance and its effect on the aerodynamic forces needs to be considered.

The aerodynamic force characteristics: The lift force, l

The lift force is generated mainly by the wing, but other parts of the aircraft will also produce contributions to the overall lift. The general expression for the lift force relates the lift to the angle of attack of the airflow relative to the aircraft,

$$L = \frac{1}{2} \rho V^2 S C_L = \frac{1}{2} \rho V^2 S a (\alpha - \alpha_0)$$

Where a is the lift curve slope, $dC_L/d\alpha$ and α is the angle of attack measured from the zero-lift angle of attack,

The lift characteristic of the plain, cambered aerofoil is shown in Figure 1.13. There is an angle of attack, at which the aerofoil produces zero lift; the zero-lift angle of attack is zero if the aerofoil is symmetrical, and negative in the case of a positively cambered aerofoil. As the angle of attack increases, the lift coefficient increases in proportion and the slope of the lift characteristic is known as the lift curve slope. The lift curve slope has a theoretical value of 2 per radian if the aerofoil is a flat plate of infinite span, but this is increased by the thickness of the aerofoil section and reduced as the aspect ratio decreases. A typical range of values for the lift curve slope is between 4 and 6 per radian depending on the aerofoil section and wing geometry. A straight wing of aspect ratio around 10 and an aerofoil with a thickness of about 12% will have a lift curve slope of about 5.7/rad.

As the angle of attack increases, so the lift coefficient increases until the pressure distribution over the aerofoil section starts to cause separation of the flow. This

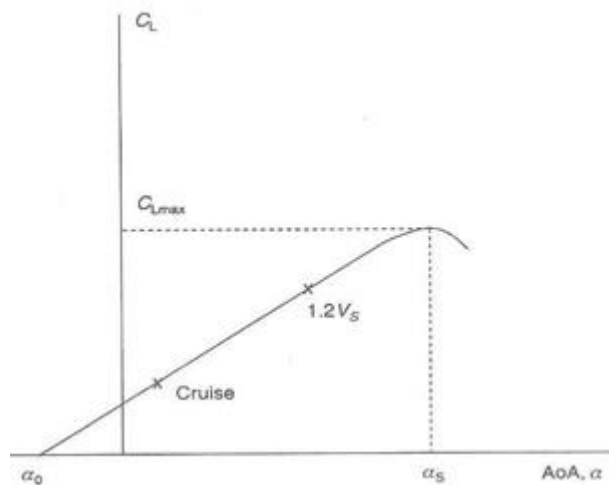


Figure:1.14 Lift Characteristic of a plain, cambered aerofoil

Causes the lift curve slope to decrease as the angle of attack increases and a point is reached when the slope becomes zero; this is the point of maximum lift coefficient, C_{Lmax} , which denotes the stall. The angle of attack at the stall, is known as the stalling angle of attack and is the greatest angle of attack at which the aircraft can be maintained in steady, 1g flight. Any further increase in angle of attack will produce a decrease in lift coefficient and the lift force is then less than the weight of the aircraft. In this state, the aircraft will sink and, usually, pitch nose-down in the stall. The stall denotes the boundary of controlled flight and defines the low speed limit of the performance envelope of the aircraft. The stall is normally preceded by aerodynamic buffeting caused by the separation of the flow. This acts as a natural stall warning and the stall buffet boundary is sometimes used as the low speed limit to performance; the airworthiness requirements contain a number of definitions of the stall and stall boundaries. Since the stall is an uncontrollable state of flight, all speeds scheduled for operational maneuvers will have a margin of safety over the stall speed.

The lift characteristic can be modified by leading edge and trailing edge flaps (and other devices), so that the aerodynamic properties of the wing are better suited to the different performance regimes. Figure 1.14 shows the general effects of leading and trailing edge flaps.

- The basic plain aerofoil is optimized for cruising flight; it has low drag and cruising flight takes place at a low angle of attack and hence a low lift coefficient. However, the stalling lift coefficient of the plain aerofoil would be too low for the take-off and landing maneuvers and would result in speeds for these maneuvers that would be too high. Assuming a safety margin of speed over the stall, the minimum speed in a maneuver will be typically $1.2V_S$ and the speed scheduled for take-off or landing will be based on a lift coefficient of $0.7C_{Lmax}$ (Fig.1.13).

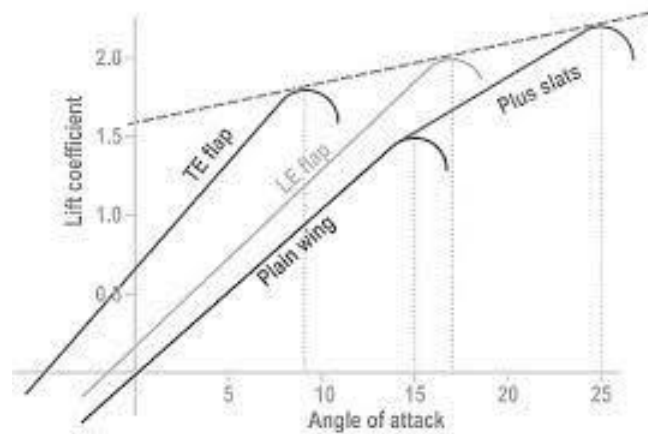


Figure:1.15 The effect of flaps on the lift characteristic

- Leading edge flap deflection has the effect of extending the lift curve to a higher stalling angle of attack, and hence lift coefficient. This would enable the take-off and landing speeds to be reduced, but it would result in a high nose-up attitude because of the large stalling angle of attack. The leading edge flap will also increase the drag, particularly at a low angle of attack.
- The deflection of the trailing edge flaps has the effect of increasing the camber of the aerofoil section and thus shifting the lift characteristic upwards as the zero lift angle of attack becomes more negative. There is also a tendency to decrease the stalling angle of attack slightly. The trailing edge flap allows higher lift coefficients to be achieved at lower angle of attack and, thus, at lower pitch attitudes. The deployment of the trailing edge flap is often made in several stages. First, a rearward translation of the flap without significant deflection extends the wing area. Effectively, this decreases the wing loading and permits increases in lift coefficient. Secondly, deflection of the extended flap increases the aerofoil camber. Effectively, this shifts the lift curve upwards and increases the lift coefficient for a given angle of attack. There may be a number of stages of deflection optimized for take-off, climb, descent, approach and landing.

Flap systems are often combined with slats and slots, and a flap extension may open a slot between the flap and wing, or expose a slat, to assist the flow over the aerofoil. A combination of leading edge and trailing edge flap can be found that permits the take-off and landing maneuvers, and other maneuvers, to be carried out at reasonable speeds and safe pitch attitudes. Fig 1.16 shows typical flap and angle of attack combinations for the principal states of flight.

Zero-lift compressibility

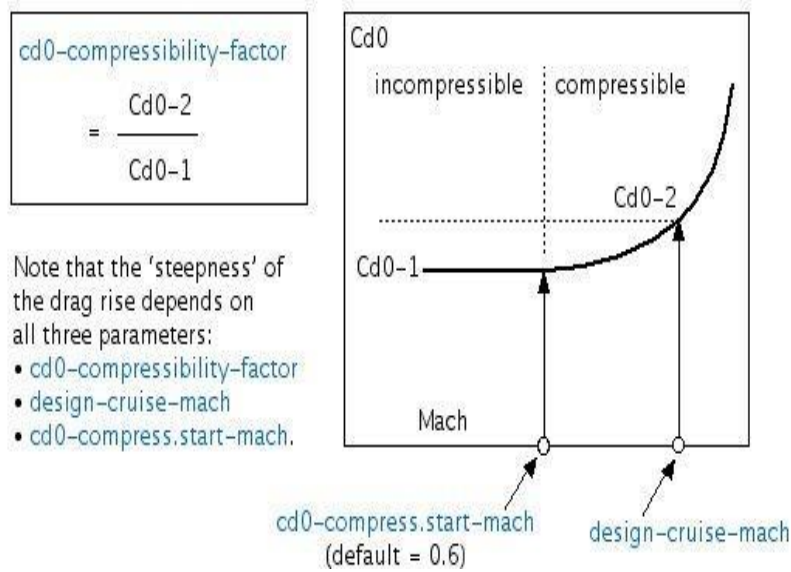


Figure:1.16The compressible lift coefficient

The effect of Mach number on lift:

The main flight variable that affects the characteristic of the lift force is the Mach number. As the Mach number of the airflow increases, so the characteristics of the flow change from those of an incompressible fluid to those of a compressible fluid.

This modifies the pressure coefficients, and hence the force coefficients, generated by the aircraft. The compressible flow coefficients are related to the incompressible flow coefficients by the Prandtl-Glauert factor, So that the compressible lift coefficient is given by,

$$C_{Lc} = C_{Li}/\beta$$

Where

$$\beta = \sqrt{(1 - M^2)} \text{ for } M < 1.$$

The ratio between the compressible and incompressible lift coefficients is shown in Fig 1.15

Whilst this effect appears to be very significant when seen in terms of the lift coefficient, its real effect is felt on the angle of attack of the aircraft. Since the aircraft flies at (almost A) constant weight, the lift coefficient decreases with Mach number on the angle of squared and, at high subsonic Mach numbers, the angle of attack of the aircraft will be small.

Figure 1.16 shows the typical effect of Mach number on the angle of attack required for steady, level, flight at constant aircraft weight in compressible flow when compared within compressible flow. It can be seen that the effect of Mach number on the angle of attack is relatively small.

Therefore, it is not likely to produce very significant effects on angle of attack dependent variables in the normal, subsonic, range of the operating Mach number.

The side force, Y

The aerodynamic side force generated by the aircraft arises from side slipping flight. It can be regarded as a lateral lift' due to the sided slip angle, which acts as a lateral

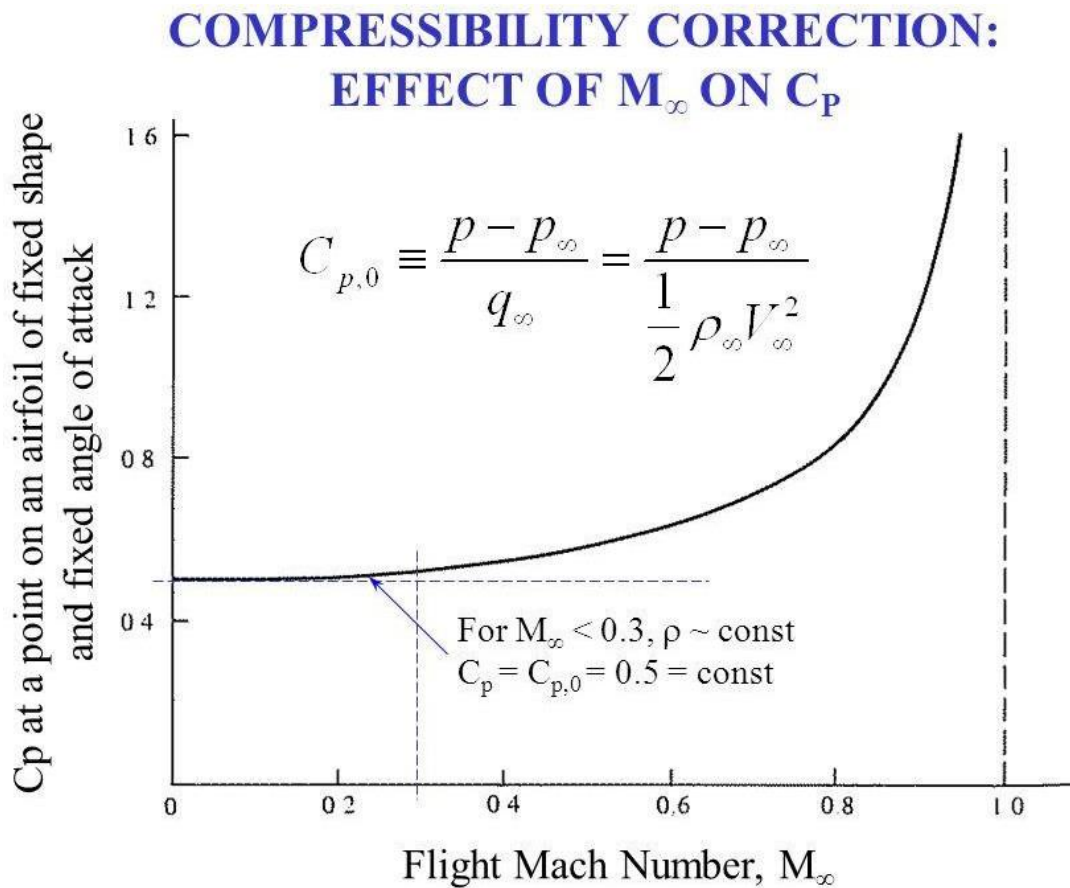


Figure:1.17 The effect of Mach number on Angle of Attack

Angle of attack; the comments on the lift force can be generally applied to the side force. In symmetric flight there is no sideslip and the aerodynamic side force will be zero. Except in special

cases in which the aircraft is in asymmetric flight, for example – flight with asymmetric thrust following an engine failure – the side force has little significance on performance.

The drag force, D

The drag force is the most important aero dynamic force in aircraft performance. In subsonic flight, it is made up of several components, each of which has its own characteristics. The components are the lift independent drag, D_z , the lift dependent drag, D_i , and, at high subsonic Mach numbers, a volume dependent wave drag, D_{wv} . The sum of the drag components makes up the total drag of the aircraft.

It is usually assumed in the analysis of subsonic performance that the drag polar of the aircraft is parabolic and represented by the lift dependent and lift independent terms only, the drag coefficient being given by,

$$C_D = C_{Dz} + KC_L^2$$

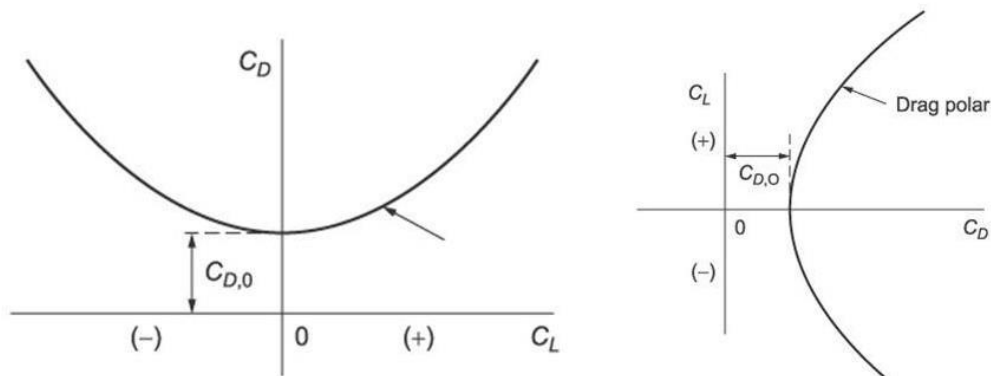
Where z and K are constants.

Whilst this approximation is used to develop the basic functions of aircraft performance it should be remembered that the real drag characteristic will not be purely parabolic but will contain terms dependent on Mach number. Moreover, particularly at the higher subsonic Mach numbers, the drag characteristic of the aircraft may deviate considerably from the parabolic approximation. In the following subsections, each element of the drag force will be considered separately and the effect of the flight variables, Mach number, weight and altitude, will be assessed on each element.

DRAG POLAR

- $C_{D,0}$ is parasite drag coefficient at zero lift ($\alpha_L=0$)
- $C_{D,i}$ drag coefficient due to lift (induced drag)
- Oswald efficiency factor, e , includes all effects from airplane
- $C_{D,0}$ and e are known aerodynamics quantities of airplane

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR} = C_{D,0} + C_{D,i}$$



Example of Drag Polar for complete airplane

4

Figure:1.18 The zero-lift drag coefficient

The lift independent drag, D_z

The lift independent drag coefficient can be broken down into two parts, the surface friction drag and the profile drag. The surface friction drag coefficient, usually accounts for about 75% of the lift independent drag and tends to decrease slightly as the Mach number increases, as the result of a Reynolds number effect.

The profile drag coefficient, which accounts for the other 25% of the lift independent drag, is a pressure dependent drag. This is affected by the Prandtl–Glauert factor in the same manner as the lift coefficient, increasing rapidly as the Mach number approaches M_{crit} , see Fig 1.17.

Here, it can be seen that the value of $C_{D,0}$ remains almost constant up to a Mach number of about 0.7; this is typical for a conventional subsonic aircraft.

When the compressible, zero-lift, drag coefficient is multiplied by the dynamic pressure, to turn it into a force, the effect of the Mach number can be seen when compared with the assumption of the constant from the parabolic drag polar, see Fig 1.18. There is good agreement between the predicted

drag forces up to a Mach number of about 0.8, above which the compressible flow drag force increases significantly.

The forces are expressed here as Drag Area, D/S , which is a convenient way of expressing the drag without involving the scale of the aircraft:

The zero-lift drag force is directly proportional to the atmospheric pressure, p , since the drag force is proportional to the dynamic pressure, q , and above equation. Thus, for flight a given Mach number, the zero-lift drag force will decrease as altitude increases since the atmospheric pressure decreases as a function of altitude.

Zero-lift compressibility

$$\text{cd0-compressibility-factor} = \frac{C_{d0-2}}{C_{d0-1}}$$

Note that the 'steepness' of the drag rise depends on all three parameters:

- cd0-compressibility-factor
- design-cruise-mach
- cd0-compress.start-mach.

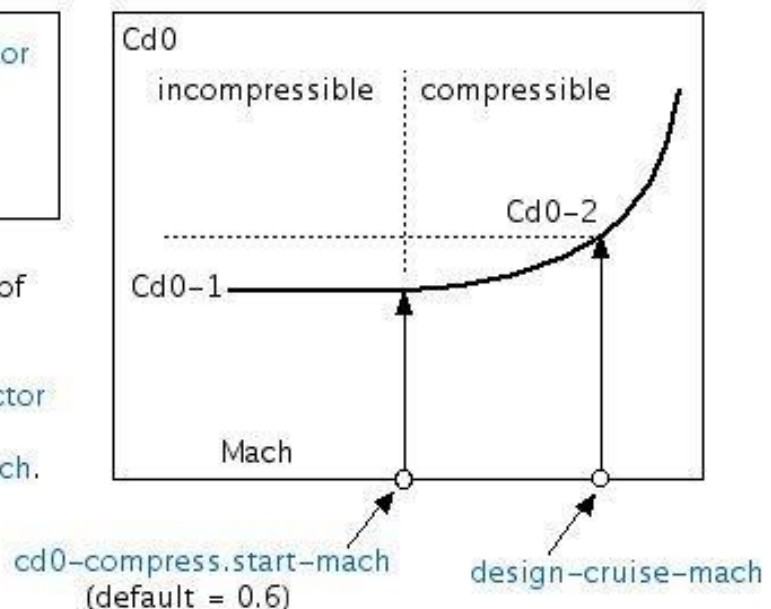


Figure:1.19 Effect of Mach number on the zero-lift drags force

Aircraft weight has no effect on the zero-lift drag force.

The lift dependent drag D_i

The lift dependent, or vortex drag coefficient, is a function of the angle of attack, and is usually taken to be

$$C_{Di} = k C_L^2$$

Where K is generally assumed to be $1/\pi A_e$ in compressible flow.

His approximation is based on the aspect ratio of the wing, A , and the span efficiency factor, e , which is a function of the span wise wing load distribution. However, there may be contributions to the lift force from parts of the aircraft other than the wing, notably the tail plane, and basing the lift dependent drag factor, K , on the wing alone is likely to be optimistic.

Flow separation at low airspeeds may also contribute to the effective value of the lift dependent drag factor; although it may not be strictly dependent on the lift force itself. In addition, the vortex drag is a function of angle of attack, and the Mach number effect on shown in Fig 1.16, will produce a further contribution to the value of K .

The value of the lift dependent drag factor, K , will usually have to be determined experimentally but it can be generally accepted as being reasonably constant over the working range of the lift coefficient.

The lift dependent drag force, D_i , is given, as a drag area, by

$$D_i/S = qKC_L^2 = \frac{K(W/S)^2}{\frac{1}{2}\rho M^2}$$

And is shown in Fig 1.19, for a given weight and altitude combination;

Since the lift dependent drag force is inversely proportional to the dynamic pressure q , it will decrease with Mach number squared and increase with increasing altitude. Increasing aircraft weight will also increase the lift dependent drag force.

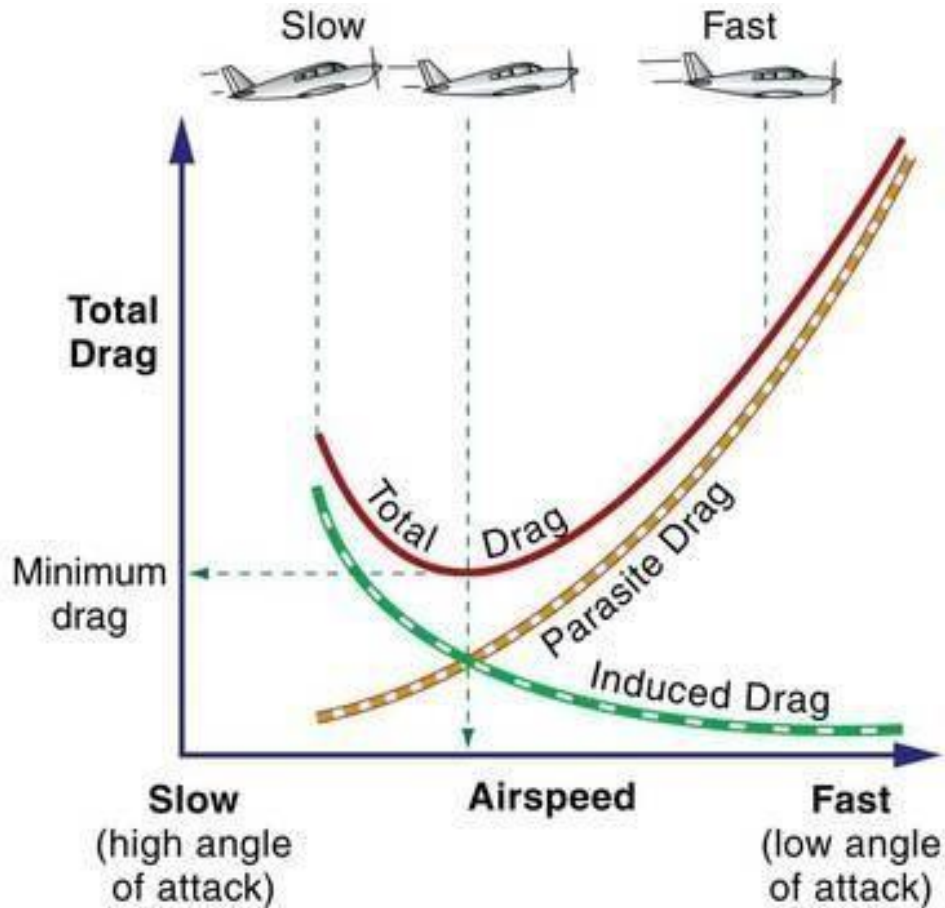


Figure:1.20 The lift dependent drag

The volume dependent wave drag, D_{wv}

As the aircraft passes through the air mass its volume displaces the flow and produces local disturbances in flow velocity. At the critical flight Mach number, M_{crit} , the local flow at points on the aircraft becomes supersonic and shock waves begin to form, growing in strength as the flight Mach number increases. The energy required to sustain these shock waves manifests itself as a drag force that increases rapidly as the flight Mach number exceeds its critical value.

There is no simple expression for the volume dependent wave drag. However, experimental results indicate that, above the critical Mach number, the volume dependent wave drag coefficient is related to the volume, and other dimensions, of the aircraft by a relationship – based on the slender body theory of the form,

$$C_{D_{wv}} \propto K_0 Vol^2$$

Where K_o is a shaping factor, which is a function of Mach number. A first-order approximation to K_o is that K_o increases as Mach number squared above M_{crit} in the transonic region. In supersonic flight beyond the transonic region, KJ_o tends to decrease. On this assumption, the volume dependent wave drag can be expected to increase as the fourth power of Mach number in the transonic region. This indicates the significance of the wave drag term in the drag characteristic of the aircraft above the critical Mach number, as shown in Fig1.20.

As in the case of the zero-lift drag, the volume dependent wave drag will decrease as altitude increases for a given Mach number and is independent of aircraft weight.

The overall drag force, D

The overall drag force is the sum of the components of the drag force, the zero-lift drag, the lift dependent drag and the volume dependent wave drag. Each component has been shown to be a function of Mach number, altitude (or pressure) and, in the case of the lift dependent drag, aircraft. The drag characteristic is shown in Fig 1.21. For a given weight and altitude combination.

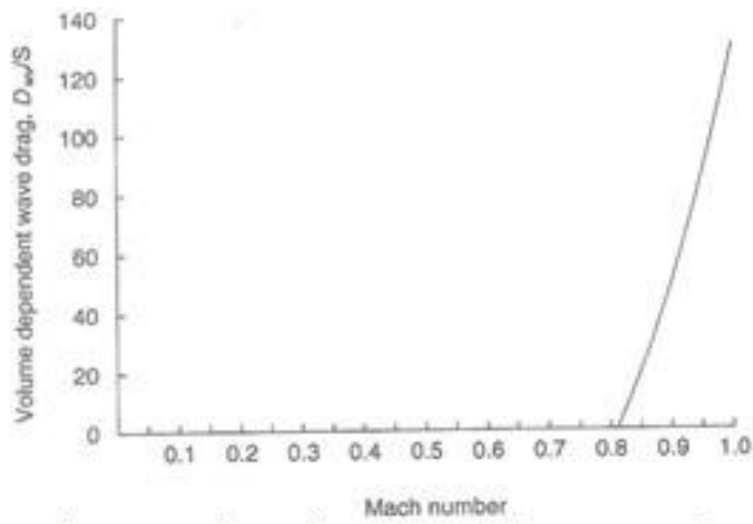


Figure:1.21 2The volume dependent wave drag

Figure 1.21 shows that, below the critical Mach number, there is a reasonable comparison between the compressible flow drag characteristic and the incompressible approximation. This justifies the use of the simple, incompressible, parabolic drag polar in the development of the basic expression of performance. However, it should be remembered that the parabolic drag polar is an approximation and that any performance characteristics estimated on the assumption of a parabolic drag polar will not be exact. In practice, it will be necessary to measure the performance of the aircraft in flight to

define the actual performance achieved. At Mach numbers above are critical value, the drag force increases rapidly and the approximation becomes invalid; any estimation of the aircraft performance above M_{crit} will need consideration of the full drag characteristic of the aircraft.

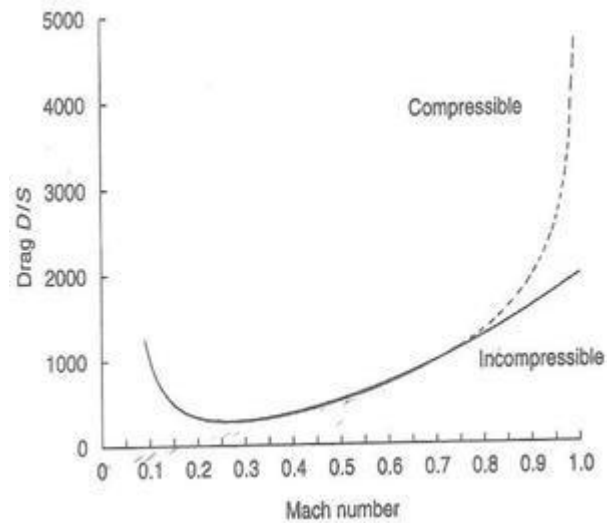


Figure:1.22 The aircraft drag polar

The minimum drag speed

In above equation the drag characteristic was taken to be

$$C_D = C_{Dz} + KC_L^2$$

and has been shown reasonably to represent the subsonic aircraft at Mach numbers below M_{crit} . If the drag characteristic is factored by $1/2 \rho_0 V_e^2 S_1$ to convert it into force modules, above equation becomes

$$D = YV_e^2 + \frac{ZW^2}{V_e^2} = D_z + D_i$$

$$V_{emd} = \sqrt[4]{\frac{ZW^2}{Y}}$$

Where $Y = 1/2$ and $Z = K/2 S$, both of which are constants. Figure 1.22 shows the total drag force, and its two components, for a given aircraft weight, W .

Differentiating above equation with respect to EAS leads to the expression for the minimum drag speed.

This occurs when the two components of the drag force are equal. The minimum drag speed is important to performance and it will be seen in later chapters that it determines the best operating speeds of aircraft with thrust producing engines. The relative magnitudes of the zero-lift drag, D_z , and the lift dependent drag, D_l will affect the minimum drag force and the minimum drag speed.

If the zero-lift drag is reduced then the total drag will be reduced but the minimum drag speed will be increased. If the lift-dependent drag is reduced then the total drag will be reduced and the minimum drag speed will be reduced. The ability to adjust the minimum drag speed in this way is an important tool in the design of the aircraft performance characteristics for different regimes of flight.

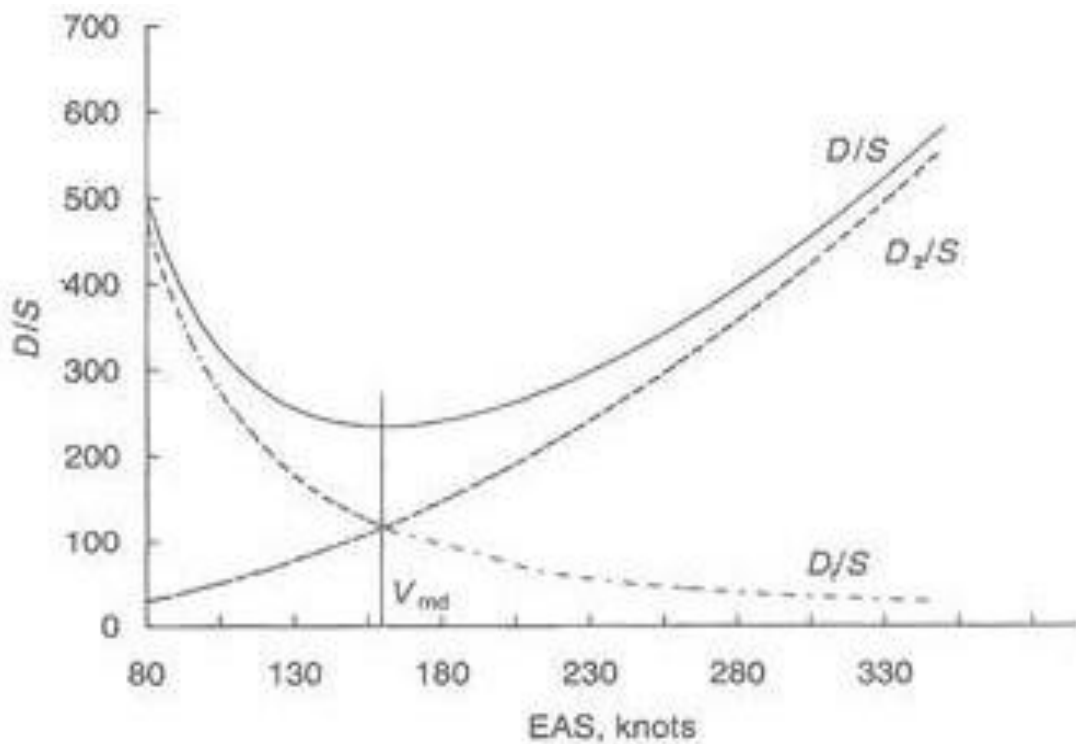


Figure:1.23 The minimum drag speed

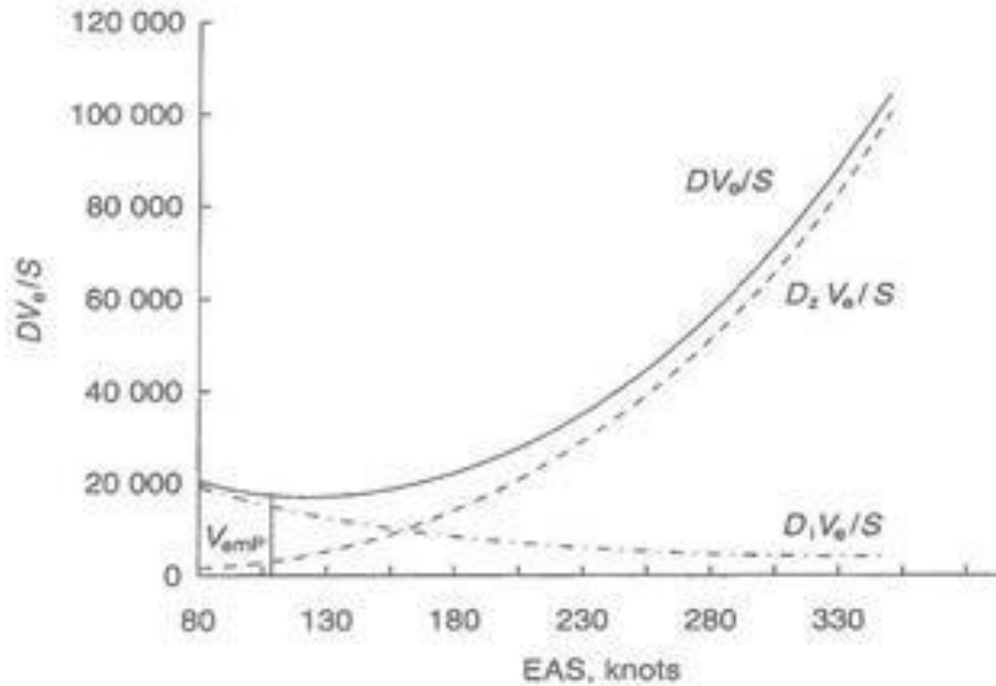


Figure:1.24 The minimum drag speed

The minimum power speed

Power is the product of force and velocity and the performance equation can be considered in terms of the thrust- power, P, required, rather than the thrust force required. The drag-power is given by multiplying the drag force, D, by the true airspeed, V, so that, in terms of EAS the drag power equation becomes,

$$P\sqrt{\sigma} = DV_e = YV_e^3 + \frac{ZW^2}{V_e}$$

and is shown in Fig 1.23. (Here it should be noted that the true airspeed, V has been converted to equivalent airspeed, before the multiplication.)

Differentiating above equation with respect to EAS leads to the expression for the minimum power speed,

$$V_{emP} = \sqrt[4]{\frac{ZW^2}{3Y}}$$

The minimum power speed is important to the performance of aircraft with power producing engines. It will be seen in later chapters that it determines the best operating speeds of aircraft with power producing engines in the same way that the minimum drag speed determines the optimum performance of aircraft with thrust producing engines. The relative magnitudes of the zero-lift drag, D_z , and the lift dependent drag, D_i , will affect the minimum drag power and the minimum power speed in the same general way as they affected the minimum drag fore and minimum drag speed.

Although the minimum drag speed and minimum power speed are related by a simple numerical factor fourth root three = 1.316, they should not be considered to be simply related in their application to aircraft performance. The minimum drag speed relates to the performance of aircraft with thrust producing engines, whilst the minimum power speed relates to aircraft with power producing engines. Some further relationships of the drag characteristic will be summarized.

The propulsive forces:

There are two basic forms of power plant used for aircraft propulsion

- The thrust-producing power plant, which produces its propulsive force directly by increasing the momentum of h_e airflow through the engine, and
- The power-producing power plant, which produces shaft power that is then turned into a propulsive force by a propeller.

Each form of power plant has different characteristics and needs to be considered separately.

The thrust-producing power plant:

The usual form of thrust-producing engine is the turbojet or turbofan, although rocket could be included in this category.

The turbojet or turbofan engine uses atmospheric air as its working fluid and, with the addition of fuel, burns the air to increase its energy. The high-energy air is then expelled through a nozzle with increased momentum to produce the thrust force. The principle is shown in Fig 1.24. Atmospheric air flows into the intake where it is slowed down to a velocity that can be accepted by the compressor.

After compression, fuels are mixed with the air and the mixture is burned in the combustion chamber. The hot gas produced is passed through a turbine that extracts energy to drive the

compressor and the exhaust is expelled through a nozzle that converts its remaining energy into thrust.

In simplified terms, the turbojet engine can be considered to produce thrust by increasing the momentum of its internal flow stream. The net propulsive force, F_N , is the difference between the stream force entering the engine and the stream force exiting the engine.

The thrusts produced at the exit plane of the nozzle is known as the gross thrust, F_G , and is equal to the rate of change of momentum of the exhaust gas flow, $F_G = \dot{m}V_j$. The flow into the intake also contributes to the engine thrust. In this case, the momentum of the flow is lost as the air enters the engine.

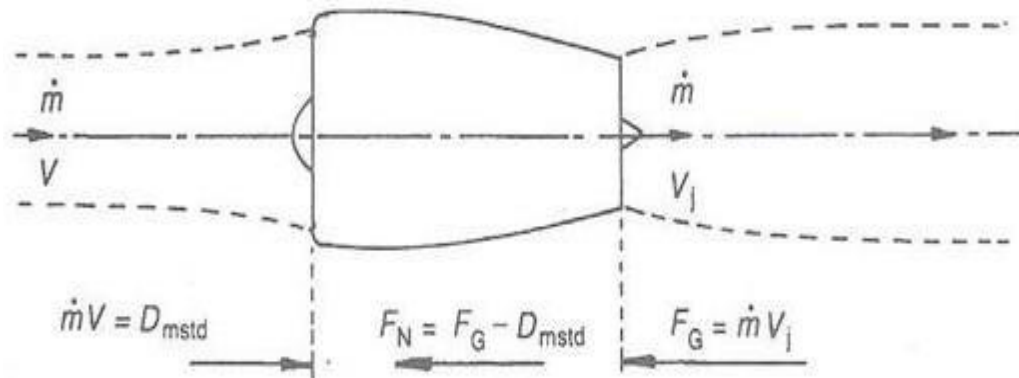


Figure:1.25 The thrust-producing power plant

The force due to the intake flow, known as the momentum drag, D_m , is equal to the rate of change of momentum in the intake airflow, $D_m = \dot{m}V$. The net propulsive thrust is given by

$$F_N = F_G - D_m = \dot{m}V_j - \dot{m}V$$

Turbofan engines may have more than one flow path, a core or hot flow and a bypass or cold flow. Strictly, each needs to be considered separately but, in this treatment, a mean, gross thrust will be assumed for the engine. It will also be assumed in the simple analysis that the intake and exhaust mass flow are equal. This is reasonable since the increase in mass flow at the exhaust due to the addition of the fuel mass flow may well be offset by compressor air bleeds for aircraft pneumatic services, e.g. pressurization and anti-icing.

The gross thrust, F_G , acts in ‘thrust axes’, which may not be parallel to the aircraft body axes. Thus, there may be a need to resolve the gross thrust into aircraft body axes before it can be used in the

equations of motion. An example is seen in the case of the vectored thrust engine in which the thrust axes are variable with respect to the aircraft body axes (Appendix B).

The momentum drags D_m , acts in velocity axes since it represents a hang of momentum of the airflow in the direction of flight. The momentum drag is the product of the engine air mass flow and the aircraft rule airspeed. Although referred to as a drag' force, the momentum drag is pastof the engine thrust as it results from the engine internal flow stream. Any forces resulting from the eternal flow to the engine will be included in the airframe drag (Appendix B). The allocation of flow forces to the airframe drag or to the propulsive thrust is known as thrust-drag accounting. It is important to distinguish between these contributions since the optimum operating airspeeds of the aircraft are determined by its drag characteristic. Allocation of a force contribution into the wrong side of the thrust-drag _balance sheet' will result in inaccurate estimations of the performance of the aircraft.

The net thrust of the power plant will be affected by the flight Mach number and altitude. It is not possible to postulate any precise function that will relate thrust to Mach number or altitude for all thrust-producing power plants. However, simple relationships can be developed that will enable the general characteristic of the thrust variation with Mach number and altitude to be deduced. From above equation the net thrust can be expressed as,

$$F_N = \dot{m}(V_j - V)$$

The turbine engine is a volumetric device and the air mass flow, \dot{m} , is the product of the volume of air passed by the engine per second (which is controlled by the engine rotational speed), and the density of the air entering the engine. Since the airflow needs to be slowed down to a Mach number of about 0.5 before it can be accepted by the compressor there will be an isentropic change to the density of the flow as it enters the engine intake. The density of the air entering the engine, ρ_i , will be givenby

$$\rho_i = \rho [1 + 0.2(M^2 - M_i^2)]^{\frac{1}{\gamma-1}}$$

Where subscript I refer to the conditions at the engine compressor face.

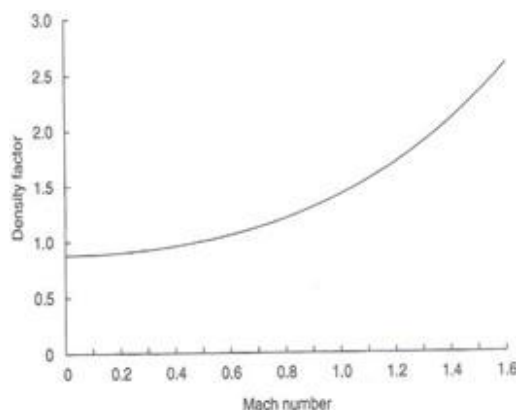


Figure 1.26The effect of Mach number on intake density

The rise in density at the compressor face is shown in Fig 1.25. The increased density will increase the air mass flow through the engine at any given engine rotational speed and hence the thrust will tend to increase with increasing flight Mach number.

From above equation, it can be seen that the net thrust is also proportional to the difference between the velocity of the engine exhaust flow, V_j , and the aircraft true airspeed, V . The velocity of the engine exhaust flow is a function of the temperature of the exhaust gas and will be determined by the engine throttle setting. For any given engine thrust setting at the exhaust gas velocity can be considered constant. The airspeed, and hence the net thrust, decreases, Fig1.26

The overall effect of Mach number on the net thrust is the product of the two functions, above equations, and is shown in Fig 1.27. Here it can be seen that the thrust characteristic is substantially influenced by the temperature of the exhaust gas. If the exhaust gas is relatively cool, as for example in the case of a high bypass ratio turbofan, then the exhaust gas Mach number will be low and the effect of the density function will be small. The thrust will decrease almost linearly with flight Mach number. A pure turbojet, which has no bypass flow, will have a relatively hot exhaust gas flow. Therefore, the density function will tend to dominate the thrust function and help to maintain the thrust level as the flight Mach number increases. Figure 1.27 shows the form of the thrust characteristics of low, medium and high bypass ratio engines.

It is emphasized that the thrust characteristics shown in Fig 1.27 have been developed to show the likely variation of thrust with Mach number and do not represent a means of calculating or estimating the thrust of an engine.

The thrust produced by an engine decreases with altitude. Empirical data show that the decrease in thrust can be reasonably approximated by a function of

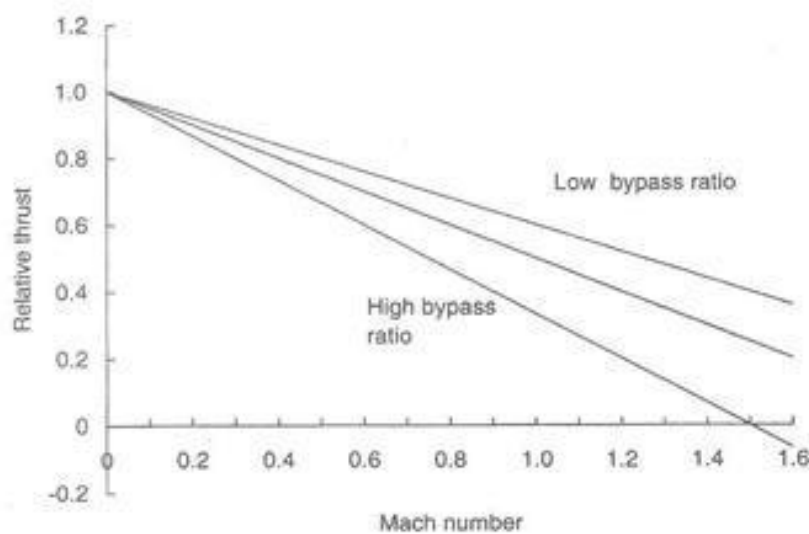


Figure:1.27 Thrust decrease due to flight Mach number

The form,

$$\frac{F_N}{F_{N0}} = \sigma^x$$

where the exponent x has a value of about 0.7 in the troposphere and moduley in the stratosphere. These values may vary with characteristics of the engine cycle, particularly the bypass ratio.

The specific fuel consumption, C , is similarly affected, in this case as a function of temperature. The values of the exponent y are about 0.5 and may be influenced by bypass ratio.

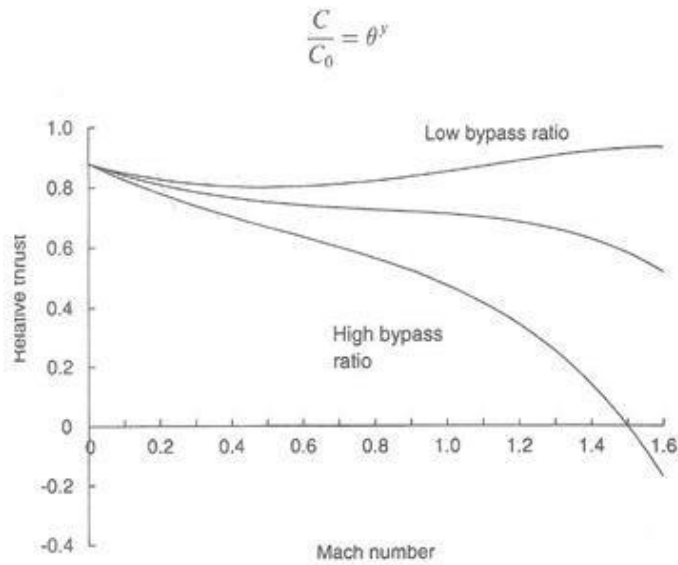


Figure:1.28 Thrust variation with Mach number and bypass ratio

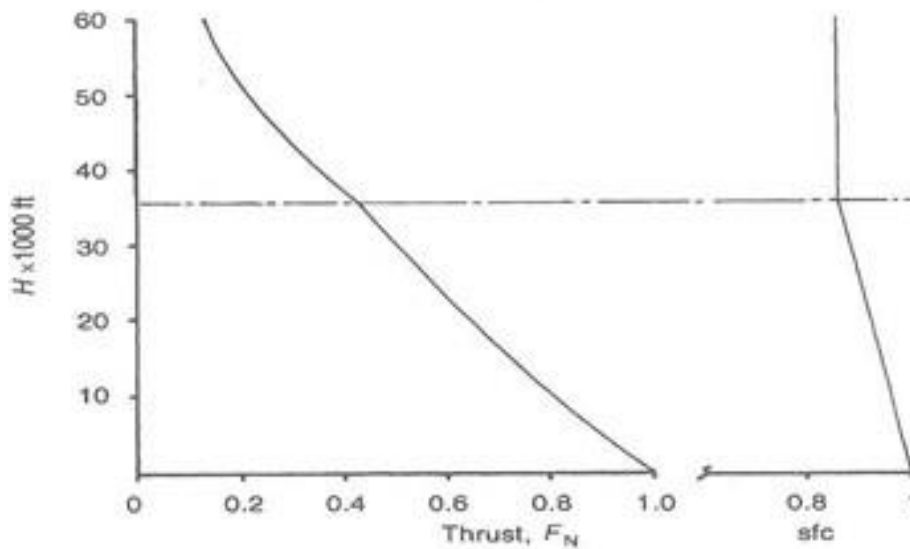


Figure:1.29 Thrust variation with Mach number and bypass ratio

consumption these functions are shown in Fig 1.28.

From equation, the longitudinal performance equation of motion for aircraft with thrust-producing engines is given by,

$$F_N - D - W \sin \gamma_2 = m\dot{V}$$

The excess thrust, $F_n - D$, which is available for climb or acceleration is the difference between the drag characteristic and the thrust characteristic of the aircraft, see Fig 1.29.

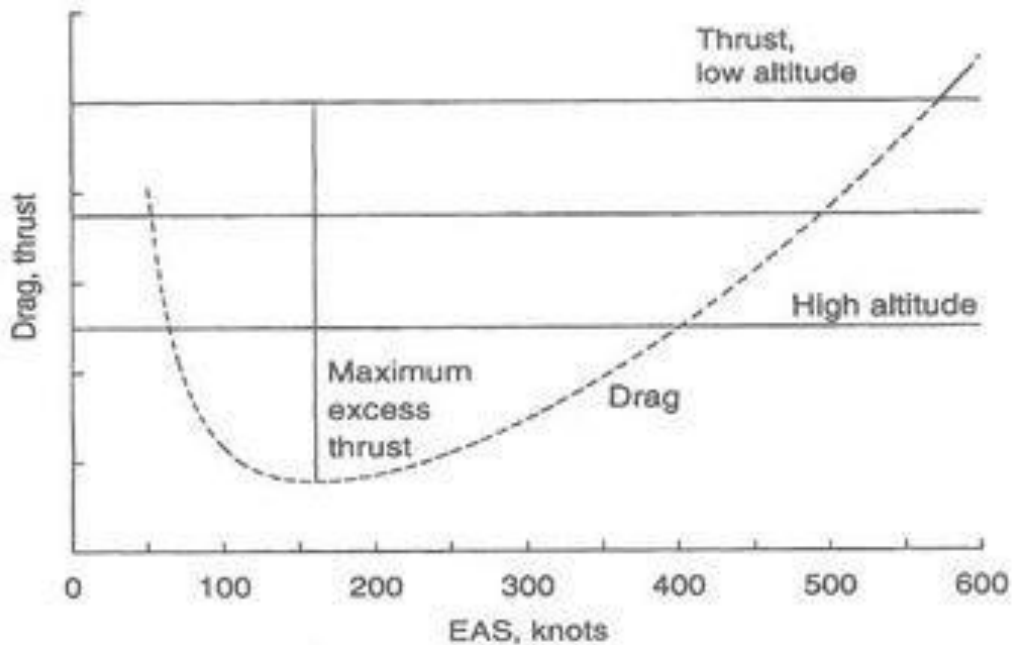


Figure:1.30 Excess thrust

Figure 1.29 is drawn in terms of equivalent airspeed so that the drag characteristic is independent of altitude. However, the thrust decreases with increasing altitude so that the excess thrust tends to decrease as altitude increases and the aircraft will eventually reach a performance ceiling at which the excess thrust is zero. This occurs at airspeed equal to the minimum drag speed of the aircraft.

In Fig 1.29, the thrust is shown as being independent of airspeed for the purpose of illustration. This is not generally the case and the thrust characteristic will be of the form shown in Fig 1.27. This will lead to a maximum excess thrust close to, but not necessarily equal to, the minimum dragspeed.

The power-producing power plant:

The power-producing power plant delivers its power through a rotating shaft to a propeller that converts the power into propulsive thrust. The power plant may be either a piston engine or a gas turbine that converts the energy of its gas flow into shaft-power rather than into thrust. In either case the shaft-power output is not greatly affected by airspeed and, to a first-order approximation, the power can be regarded as independent of airspeed.

The shaft-power, P , is converted into thrust, T , by the propeller. In the process, losses occur and the thrust-power produced will be less than the shaft-power delivered. The propeller efficiency, η , is the ratio of the thrust-power output to the shaft-power input so that,

$$\eta P = TV$$

where V is the true airspeed.

This implies that the propulsive thrust increases as airspeed decreases at constant engine power and that the thrust will become infinite at zero airspeed. In practice, the propeller efficiency will vary with airspeed and a constant engine shaft-power produces a finite thrust at zero airspeed, known as the Static Thrust, which decreases as the airspeed increases. The propeller efficiency, η , is a characteristic of an individual propeller and depends on the Advance ratio, J , of the propeller, and the Power coefficient, C_p , of the engine.

The propeller Advance ratio is given by

$$J = V/nD$$

And the engine Power coefficient by,

$$C_P = \frac{P}{\frac{1}{2}\rho n^3 D^5}$$

Where n is the rotational speed in rad/s and D is the propeller diameter.

The propeller efficiency generally can be regarded as being reasonably constant in the cruising range of airspeed in the case of a variable pitch propeller. A typical relationship between the thrust and drag of an aircraft with power-producing engines is of the form shown in Fig 1.30.

The longitudinal performance equation of motion for aircraft with power producing engines differs from that for aircraft with thrust-producing engines since

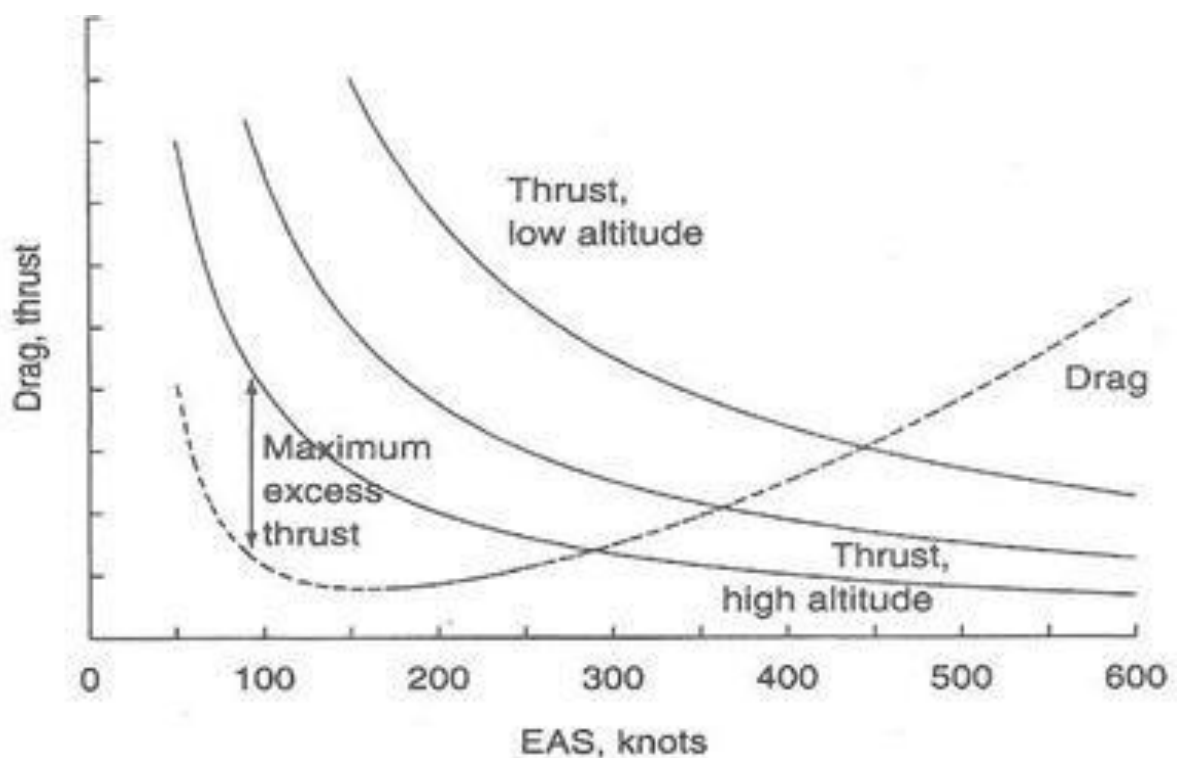


Figure:1.31 The trust characteristic of a power-producing engine

The propulsive thrust is a function of airspeed, equation. The equation is now written in the form.

$$\frac{\eta P}{V} = D + W \sin \gamma_2 + m \dot{V}$$

Or, rearranging and writing in terms of equivalent airspeed gives,

$$\eta P \sqrt{\sigma} - DV_e = WV \sqrt{\sigma} \left[\sin \gamma_2 + \frac{V}{g} \right]$$

Which leads to the conclusion that the excess power is the difference between the equivalent-thrust power, and the equivalent-drag power, DV_e , see Fig1.31?

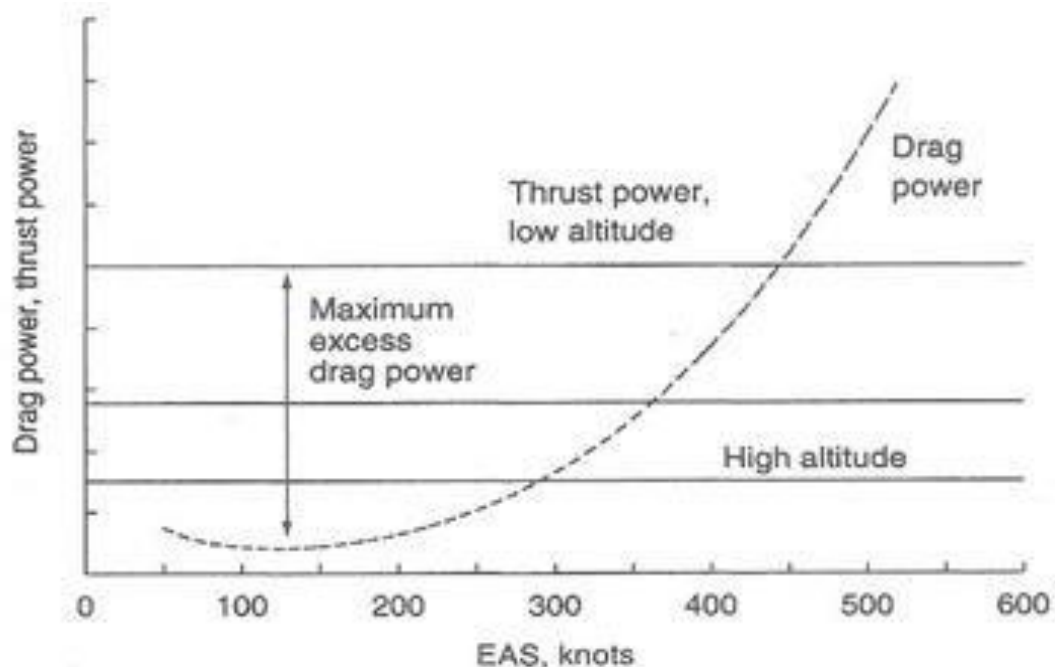


Figure:1.32 excess thrust power

Figure 1.31 is drawn in terms of equivalent airspeed so that the equivalent-drag power, DV_e , is the independent of altitude. However, the equivalent-thrust power decreases with increasing altitude so that the excess thrust power tends to decrease as altitude increases and the aircraft will eventually reach a performance ceiling at which the excess thrust power is zero. This occurs at an airspeed equal to, or very close to, the minimum power speed of the aircraft.

The simplified power plant characteristics described are intended to give a general understanding of the manner in which propulsive thrust may vary with the performance variables airspeed (or Mach number), altitude and temperature. They are not intended to be a means of calculating power plant performance or of scaling performance for the effects of those variables.

Aerodynamic relationships:

If it is assumed that the aircraft has a parabolic drag polar, above equations, then a number of relationships can be deduced that can be used in the derivation of some simplified expressions for the performance of the aircraft. These relationships, which were addressed, are well known and will be quoted without proof.

Figure 1.19 shows that the parabolic drag polar have a minimum value that is important in determining the airspeeds for optimum performance. At minimum drag, it can be shown that

$$C_{Dz} = KC_L^2$$

This implies that the lift coefficient for minimum drag is given by,

$$C_{Lmd} = [C_{Dz}/K]^{\frac{1}{2}}$$

And the minimum drag speed, and minimum drag Mach number, are given by,

$$V_{md} = [2W/\rho S]^{\frac{1}{2}} \{K/C_{Dz}\}^{\frac{1}{4}} \quad \text{and} \quad M_{md} = [2W/\gamma p S]^{\frac{1}{2}} \{K/C_{Dz}\}^{\frac{1}{4}}$$

Respectively

The power required for level flight is given by the product of the drag force and the true airspeed, DV, since in steady level flight T = D. The airspeed for minimum power, Vmp, can be shown to be given by,

$$V_{mp} = \frac{1}{\sqrt[4]{3}} V_{md}$$

Although there is a simple numerical constant linking the minimum drag speed and the minimum power speed these terms should only be applied to aircraft with the appropriate power plant. The minimum power speed relates to the performance of aircraft with power-producing engines whereas the minimum drag speed relates to the performance of aircraft with thrust-producing engines. Only in the case of the glider, which has no engines, do both minimum power speed and minimum drag speed have significance.

The lift-drag ratio, L/D, is a measure of the aerodynamic efficiency, E, of the aircraft and has a maximum value at the minimum drag speed so that,

$$\frac{L}{D_{min}} = E_{max} = \frac{1}{2[KC_{Dz}]^{\frac{1}{2}}}$$

The relative airspeed, u, is the ratio between the airspeed and the minimum drag speed.

$$u = V/V_{md}$$

Using the relative airspeed, the ratio of the drag to minimum drag can be expressed as,

$$\frac{D}{D_{min}} = \frac{1}{2} \left[u^2 + \frac{1}{u^2} \right]$$

In addition, the power plant propulsive thrust can be expressed in terms of the minimum drag, D_{min} , of the aircraft; this form of expression will be used in the generalized performance equations. In the case of thrust-producing engines the dimensionless thrust, τ , is given by,

$$\tau = F_N/D_{min}$$

and, in the case of power-producing engines, the dimensionless power, λ , is given by,

$$\lambda = \eta P/D_{min} V_{md}$$

Using above equations the performance equation, above, can now be written in a generalized, dimensionless, form

$$\left[\frac{\lambda}{u} + \tau \right] - \frac{1}{2} \left[u^2 + \frac{1}{u^2} \right] = E_{max} \left\{ \sin \gamma_2 + \frac{V}{g} \right\}$$

This equation can be used to determine the performance characteristics of aircraft with either form of power plant, or a maximum of thrust- and power-producing engines, as in the case of the turbo-prop.

These relationships will be used in the following chapters for the development of expressions for the estimation of the performance of the aircraft.

MODULE II

CRUISE PERFORMANCE

Introduction:

The cruise performance of an aircraft is one of the fundamental building blocks of the overall mission. In the cruising segment of the mission, both height and airspeed are essentially constant and the (airspeed) aircraft is required to cover distance in the most expedient manner.

Usually, the majority of the fuel carried in the aircraft will be used during the cruise.

The distance that can be flown, on the time that the aircraft can remain aloft, on a given quantity of fuel are important factors in the assessment of the cruise performance.

Maximum and minimum speeds in level flight:

In the analysis of cruise performance, the aircraft is considered to be in steady, level, straight, symmetric flight with no acceleration or maneuvers.

Under those conditions the aircraft can be taken to be in a state of equilibrium in which the forces and moments are in balance; this is referred to as being in TRIM

In practice cruising flight may involve very low levels of climb (or) acceleration.

In addition, it may be required to make gradual maneuvers associated with the mission.

For example, turning to change track and to correct errors in its course, on climbing to change cruising altitude. Usually these maneuvers can be neglected to design estimations on, if it is considered necessary, corrections applied to account for the errors produced.

In practice, an aircraft carries a fuel contingency allowance over and above the estimated cruising fuel requirement to allow for such unscheduled maneuvers.

When the aircraft is cruising in trim the equations of motion of a conventional aircraft

$$\begin{aligned}F_N - D - mg \sin \gamma_2 &= m v \\L \sin \gamma_1 &= m v \gamma_3 \cos \gamma_2 \\-L \cos \gamma_1 + mg \cos \gamma_2 &= -m v \gamma_2\end{aligned}$$

The above equations can be reduced to the simple statements

$$\begin{aligned}F_N &= D \\L &= W\end{aligned}$$

The development of the basic expression for cruising flight is based on these simplified statements. (It should be remembered that the simplified statements given in eq.2 contain a number of assumptions that must be fulfilled if the expressions developed from them are to be used to estimate the performance of the aircraft.)

Specific air range and specific endurance:

Cruising efficiency can be measured in terms of either the range or the endurance of the aircraft. The Specific Air Range (SAR) is defined as the horizontal distance flown per module of fuel consumed and the Specific Endurance (SE) is defined as the time of flight per module of fuel consumed.

The distance travelled, x , in still air is given by the time integral of the true airspeed, V , so that

$$V = dx/dt$$

And in cruising flight the true airspeed is usually quoted in knots, or nautical miles per hour (nm/hour)

In addition, during cruise, fuel is burned and the fuel mass flow, Q_f , will determine the rate of change of mass of the aircraft,

$$Q_f = -dm/dt$$

The fuel mass flow is usually quoted in kg/hour and is negative since the mass of the aircraft decreases with time as fuel is burned.

The specific air range is an expression of the instantaneous distance flown per module of fuel consumed and can thus be expressed as,

$$SAR = -dx/dm = V/Q_f$$

And has modules of length/mass. It would normally be quoted in nautical miles per kilogram (nm/kg).

The specific endurance is an expression of the instantaneous flight time per module of fuel consumed and can be expressed as,

$$SE = -dt/dm = 1/Q_f$$

and would normally be quoted in hr/kg.

Since the drag of the aircraft is a function of the aircraft weight, which is continuously decreasing as fuel is burned; the specific air range and the specific endurance will be point performance parameters, relating to the range and endurance at that point on the cruise path. To find the cruise range or endurance, the SAR or SE must be integrated over the cruise flight path as functions of the aircraft

weight. Neither the SAR nor the SE is conveniently formulated for integration in the form given in

above equations. They need to be written in terms of the performance variables before they can then be integrated to give the range or endurance of the aircraft. Because of the fundamental difference of the propulsive characteristic of thrust producing and power producing engines, the performance of the aircraft with each type of engine must be considered separately.

Range and endurance for aircraft with thrust – producing engines:

If the aircraft is powered by thrust-producing engines (turbojets or turbofans), the fuel flow is seen to be a function of engine thrust which, in cruise, is equal to aircraft drag above equation.

The specific fuel consumption, C , is defined as the fuel flow per module thrust

$$C = Q_f / F_N$$

And has modules of kg/N hr.

(NB It should be noted that, in the subsequent analysis of the cruising performance of the aircraft, dimensional consistency of the expressions might not be strictly observed. This is particularly the case where the specific fuel consumption is used; sine the modules in which it is quoted may vary. The expressions that are developed here will be kept in their simplest possible form and may not include all the terms necessary to maintain their strict dimensional consistency. Therefore, it may be necessary to include the gravitational constant, g , or other constants, to make the modules consistent, a check on modules of the expressions will show when this is needed.) Although above equation suggests that the fuel flow is proportional to thrust, the specific fuel consumption (sfc), may be a function of other performance-related parameters and a number of alternative fuel flow laws may be considered. Examples of some of the commonly assumed laws are,

(i)

$$C = C_1$$

Assuming a constant value for SFC is the simplest fuel flow law. It implies that the fuel flow is directly proportional to thrust. This law is usually accepted in the determination of the general expressions for range and endurance (e). In practice, it does not reflect of changes in engine operating conditions or in flight conditions and so it should only be applied when variations in thrust or Mach number are small and cruising conditions are constant.

(ii)

$$C = C_2 \theta^{\frac{1}{2}} M^n$$

This is a reasonable approximation to the fuel flow law of a turbojet or turbofan engine. It takes into account variations in the temperature of the atmosphere, θ , and of the effects of flight Mach number. The exponent n may vary and empirical

Data indicate values ranging between about 0.2 for a turbojet and about 0.6 for a high bypass ratio turbofan. This law is not particularly difficult to apply in the integration of SAR or SE.

(iii)

$$\left. \begin{aligned} C &= C_3 + C_4 M \\ C &= C_5 + C_6 F_N \end{aligned} \right\}$$

These are further attempts to produce approximations to empirical fuel flow data over a range of thrust and Mach numbers but they tend to be more cumbersome when introduced to the range and endurance equations.

In the following analysis of the cruise performance, the simple fuel law, equation, will be used. The effects of using equation as an alternative law will be discussed later.

Using equations in the expressions for SAR and SE, equations lead to expressions in a form suitable for integration,

$$SAR = \frac{V}{C} \frac{L}{D} \frac{1}{W}$$

and

$$SE = \frac{1}{C} \frac{L}{D} \frac{1}{W}$$

Since these point performance characteristics include the aircraft lift-drag ratio, they will have maximum values at airspeeds related to the minimum drag speed of the aircraft. Writing above equation in coefficient form, and substituting for airspeed, gives.

$$SAR = \frac{1}{C} \left[\frac{2}{\rho W S} \right]^{\frac{1}{2}} \frac{C_L^{\frac{3}{2}}}{C_D}$$

And

$$SE = \frac{1}{C} \frac{C_L}{C_D} \frac{1}{W}$$

If it is assumed that the aircraft has a parabolic drag polar and that the simple fuel flow law for thrust-producing engines, above equation, applies, then, for the instantaneous or point performance of the aircraft, the maximum SAR would be obtained by flying at an angle of attack corresponding to $C_{L/D \max}$. This gives an optimum airspeed for maximum range of $3V_{md}$ or $1.316V_{md}$.

These results apply to any point along the flight path but, in some methods of cruising, do not necessarily apply continuously along the flight path. Above equations are in a form that can be integrated to give the path performance of the aircraft in cruising flight. As the aircraft cruises, and fuel is consumed, the weight of the aircraft decreases. It can be seen that the cruise performance is a function of, firstly, the quantity of fuel available for cruise and, secondly of the effect of the change of weight on the minimum drag speed. The range and endurance are found, as path performance functions, by integrating the SAR and SE over the change in weight between the beginning and end of cruise. If the initial weight of the aircraft is W_i and the final weight is W_f then the fuel ratio, w , can be defined as,

$$w = W_i / W_f$$

So that the weight of fuel consumed can be related to the initial, or final, cruise weight,

$$W_i - W_f = W_i(1 - 1/\omega)$$

In the cruise, lift equals weights so that,

$$W = \frac{1}{2}\gamma p M^2 S C_L$$

As weight decreases during the cruise the variables on the right-hand side of above equation must vary to compensate. These are air pressure (which can be controlled by the cruise altitude), flight Mach number and lift coefficient (controlled by angle of attack). Three methods of cruise can therefore be considered, in each of which one of the variables is varied to compensate for the decrease in weight and the other two are maintained constant. Each method produces a different result and has its particular application in aircraft operations.

Cruise method 1

Constant angle of attack, constant Mach number:

In this method, the air pressure must be allowed to decrease to allow for the decrease in air craft weight as fuel is consumed, thus

$$\frac{W}{p} = \frac{1}{2}\gamma M^2 S C_L$$

This implies that the aircraft must be allowed to climb during the cr4uise to maintain the parameter W/p constant and the method is known as the Cruise-Climb technique. Since the angle of attack is constant throughout the cruise the lift coefficient, and hence lit-drag ratio. L/D , will be constant. The constant Mach number implies flight at constant true airspeed.

Let the range under cruise method 1 be then from above equation,

$$R_1 = \frac{V}{C} \frac{L}{D} \int_{W_i}^{W_f} \frac{dW}{W}$$

This becomes,

$$R_1 = \frac{V}{C} \frac{L}{D} \ln \omega$$

This expression is the best known expression for the range of an aircraft and is known as the Breguet Range expression. Although it offers the optimum performance in terms of distance flown on a given fuel load there are practical reasons that make its application to flight operations difficult, and further consideration of this cruise method is necessary.

Substituting for the true airspeed, V , and writing equation in coefficient form gives,

$$R_1 = \frac{1}{C} \left[\frac{2W}{\rho S} \right]^{\frac{1}{2}} \frac{C_L^{\frac{3}{2}}}{C_D} l_n \omega$$

This has a maximum value that occurs at an airspeed corresponding to $1.316V$. However, it may not be possible, or expedient, to cruise at the optimum airspeed and the effect of cruising at an alternative airspeed needs to be considered.

Since the cruise-climb is flown at constant angle of attack the ratio will be constant and, therefore, the relative airspeed, will be constant throughout the cruise. Also since the airspeed is constant in this method of cruise, $V = V_1 = V_f$ and therefore the initial and final relative airspeeds in the cruise are given by

$$u_i = V/V_{mdi} \text{ and } u_f = V/V_{mdf}$$

$$\text{thus } V = u_i V_{mdi} \text{ or } u_f V_{mdf}$$

so that the equation can be

written in terms of either the minimum drag speed at the beginning of cruise, or at the end of cruise, Using above equations in given equation gives

$$R_1 = \left[\frac{V_{mdi}}{C} E_{max} \right] \left\{ \frac{2u^3}{u^4 + 1} \right\} l_n \omega$$

This consists of three parts.

The square bracket is known as the range factor, It contains a term that is a function of the airframe-engine combination and can be regarded as a constant scaling factor, although it contains two variables – the initial cruise weight and air density. This term can be used to determine the effect of modifications to the aircraft (in respect of either the airframe or the power plant) on the cruise performance through the drag characteristic, the specific fuel consumption and the aircraft weight.

The curly bracket is a function of the relative airspeed. It acts as a shaping factor, which is characteristic of the method of cruise.

The third term is a function of the fuel ratio. This is also a characteristic of the method of cruise and the magnitude of this term will determine the relative range of the cruise methods.

The product of the curly bracket and the function of the fuel ratio is known as the range function of the cruise method.

Figure 2.1 shows the range function of the cruise – climb method for three values of fuel ratio representing the cruise fuel required for short, medium and long-range aircraft. Maximum range is obtained by flying at a relative airspeed $u = 1.316$ and the range penalty for operation at any airspeed other than this can be determined. The range of the aircraft, in navigational modules, can be found by multiplying the range function by the range factor.

The values of the fuel ratio shown in Fig 2.1, and subsequent figures, are typical of very long range aircraft, $w = 1.5$, medium to long range aircraft, $w = 1.3$, and short-range aircraft, $w = 1.1$.

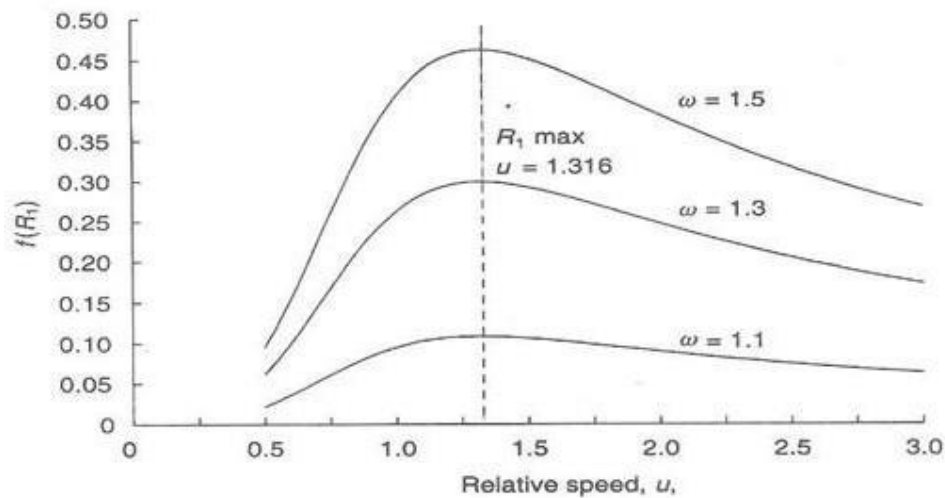


Fig 2.1 Range function cruise method1

The endurance of the aircraft can be found by integrating above equation over the weight change during the cruise, in the same manner as the integration of the SAR. This leads to the expression for the endurance of the aircraft under cruise method 1. E1.

$$E_1 = \frac{1}{C} \frac{C_L}{C_D} l_n \omega$$

This shown that maximum endurance is obtained by flying at the minimum drag speed. Writing above equation in terms of the relative airspeed, u , gives.

$$E_1 = \left[\frac{E_{\max}}{C} \right] \left\{ \frac{2u^2}{u^4 + 1} \right\} l_n \omega$$

Where the square bracket is the endurance factor for the airframe – engine combination and the relative airspeed and fuel ratio terms provide the endurance function. The product of these gives the endurance of the aircraft in hours. The endurance function is shown in Fig 2.2.

Although the cruise – climb method gives the best possible range for a given quantity of fuel, its use is limited by practical restrictions to aircraft operations. Due to the constraints of the control of air traffic and the need to provide vertical separation between flights in different directions, air craft cannot be allowed to change height freely. This means that, in practice, it is unusual to be able to

take advantage of the benefits of the cruise – climb unless the aircraft is operating in airspace with no conflicting traffic.

As a compromise, aircraft are sometimes allowed to step climb during the cruise; this involves discrete increments in height, compatible with the requirements for vertical separation, at intervals along the route to keep W/p close to the required value. In a typical transport flight, cruising at about 30000 ft a step climb of 2000 ft would be required after about 2 hours flying to bring the value of W/p back to its initial value.

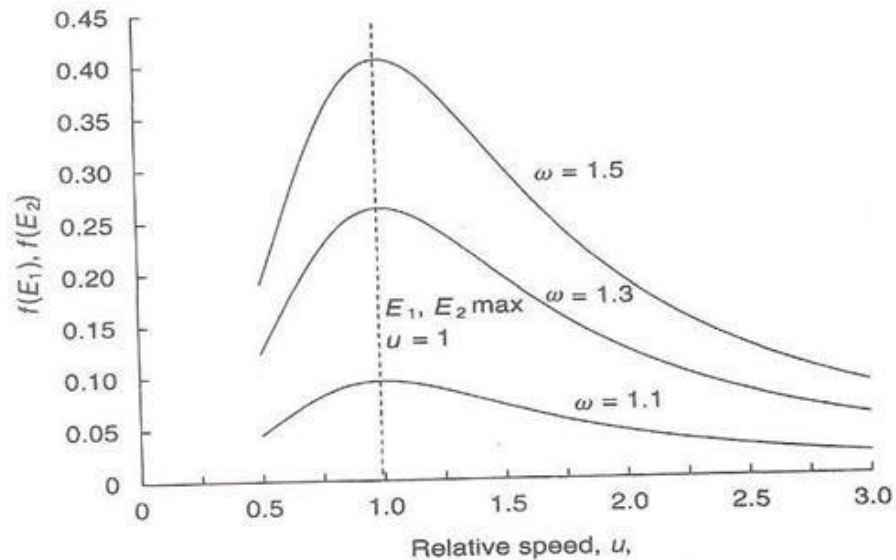


Fig. 2.2 Endurance function cruise methods 1 and 2.

The operation of the aircraft in the cruise – climb differs in the troposphere and in the stratosphere because of the effect of the structure of the atmosphere on the engine thrust characteristic. This can be explained by considering the thrust – drag balance, in parametric form (see Chapter 8). In the cruise – climb the aircraft is cruising at constant Mach number and constant angle of attack, thus the drag coefficient is constant, and

$$\frac{F_N}{\delta} = \frac{D}{\delta} = \frac{1}{2} \gamma p_0 M^2 S C_D$$

Which are constants under these cruise conditions?

Now the parametric form of the engine thrust shows that the thrust is related to the engine rotational speed, N, and the flight Mach number by a functional relationship of the form,

$$\frac{F_N}{\delta} = f \left[\frac{N}{\theta^{\frac{1}{2}}}, M \right]$$

In the troposphere, the relative temperature, θ , decreases with height so that, during the cruise – climb the parametric engine rotational speed, N/ω , increases and hence the parametric thrust increase and will cause the aircraft to climb or to accelerate. This tendency will need to be checked by a continuous reduction in engine rotational speed to maintain the parametric rotational speed constant during the cruise – limb so that, with the constant cruise Mach number, the parametric thrust – drag balance is maintained.

In the isothermal stratosphere, where the parametric engine rotational speed will remain constant if the actual engine rotational speed is constant, the rates of the change of the parametric thrust and parametric drag as height increases are equal. The cruise – climb is achieved by trimming the aircraft to the required angle of attack and setting the thrust, governed by the engine rotational speed, to give the required Mach number. The aircraft will then cruise 0 climb as weight decreases without the need for any further correction to thrust setting or to trim other than to account for any shift of the centre of gravity (CG) as fuel is burned. Cruise – climb is the ideal cruise method for operation in the stratosphere.

Cruise method 2

Constant angle of attack, constant altitude:

In this case, the Mach number, or true airspeed, must be reduced during cruise, so that

$$\frac{W}{M^2} = \frac{1}{2} \gamma \rho S C_L$$

This implies that the lift coefficient, and lift – drag ratio, will be constant during the cruise and that, substituting for airspeed in above equation, the range under cruise method 2, R_2 , and will be given by the integral.

$$R_2 = \frac{1}{C} \left(\frac{2}{\rho S C_L} \right)^{\frac{1}{2}} \frac{L}{D} \int_{W_i}^{W_f} \frac{dW}{W^{\frac{3}{2}}}$$

Which, on integration, becomes?

$$R_2 = \frac{1}{C} \left[\frac{2W_i}{\rho S} \right]^{\frac{1}{2}} \frac{C_L^{\frac{1}{2}}}{C_D} 2(1 - \omega^{-\frac{1}{2}})$$

As in cruise method 1, this gives maximum range when the cruising airspeed is 1.316V md since the angle of attack is maintained constant.

As in method 1, this is a constant angle of attack cruise method so that the relative airspeed is constant throughout the cruise and writing above equation in terms of the relative airspeed gives the range factor and range function for cruise method 2.

$$R_2 = \left[\frac{V_{mdi}}{C} E_{max} \right] \left\{ \frac{2u^3}{u^4 + 1} \right\} 2 \left(1 - \omega^{-\frac{1}{2}} \right)$$

The range factor is identical to that found in cruise method 1. The range function has a similar general form to that of cruise method 1, but its value is smaller for a given value of fuel ratio, w , so that the overall range, R_2 is less than R_1 . The range function is shown in Fig.2.3.

The endurance of the aircraft, found by integrating above equation over the weight change during the cruise, is seen to produce the same expression as that found under cruise method 1, if the constant fuel flow law is assumed. This is because it is also a constant angle of attack method.

Cruise method 2 has the disadvantage that the cruise Mach number, and hence true airspeed, is continuously reduced to compensate for the decrease in aircraft weight. This will increase the time of flight and the cost of the time penalty incurred is likely to far outweigh any fuel advantage the method may have. This method of

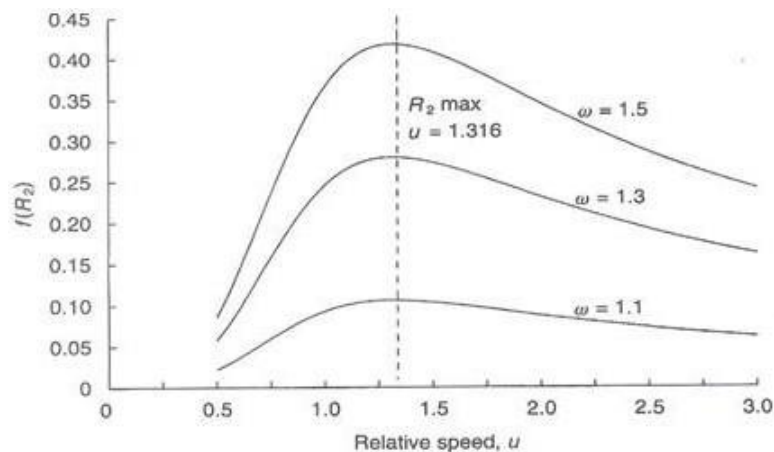


Fig. 2.3 range function cruise method 2

Cruise can be considered for petrol or surveillance operations, in which endurance is more important than distance travelled, but it will require a constant reduction in engine thrust to maintain the cruise conditions. It has the advantage of being a constant attitude cruise method that may be favorable to some surveillance sensors.

Cruise method 3

Constant altitude, constant Mach number:-

In this case, the angle of attack must be allowed to decrease as the weight decreases to maintain W/Cl constant.

$$\frac{W}{C_L} = \frac{1}{2} \gamma \rho M^2 S$$

Assuming that the aircraft is cruising at a speed greater than its minimum drag speed then the decrease in lift coefficient during the cruise will cause the drag coefficient and hence the drag force, to decrease. This will require a progressive decrease in the thrust required to maintain the Mach number or true airspeed constant. Since the lift – drag ratio will not be constant, the range will need to be found by integrating the drag over the weight change during cruise.

$$R_3 = \frac{V}{C} \int_{W_i}^{W_f} \frac{dW}{D}$$

Now for an aircraft with a parabolic drag polar.

$$D = qSC_{Dz} + \frac{KW^2}{qS}$$

$$\text{where } q = \frac{1}{2}\rho V^2.$$

Substituting in above equation and integrating gives,

$$R_3 = \frac{V}{C} \frac{1}{(KC_{Dz})^{\frac{1}{2}}} \left\{ \tan^{-1} \left[\frac{W_i}{qS} \left(\frac{K}{C_{Dz}} \right)^{\frac{1}{2}} \right] - \tan^{-1} \left[\frac{W_f}{qS} \left(\frac{K}{C_{Dz}} \right)^{\frac{1}{2}} \right] \right\}$$

Now

$$\frac{W_i}{qS} \left(\frac{K}{C_{Dz}} \right)^{\frac{1}{2}} = \frac{C_{Li}}{C_{Lmdi}} = \frac{V_{mdi}^2}{V_i^2} = \frac{1}{u_i^2}$$

And, similarly,

$$\frac{W_f}{qS} \left(\frac{K}{C_{Dz}} \right)^{\frac{1}{2}} = \frac{1}{u_f^2}.$$

Since the airspeed is constant in this method of cruise, $V = V_1 = V_f$ and, therefore, the initial and final relative airspeed in cruise are given by and thus Also, and writing above equation in terms of the relative air speed gives.

$$R_3 = \left[\frac{V_{mdi}}{C} E_{max} \right] 2u_i \left\{ \tan^{-1} \left[\frac{1}{u_i^2} \right] - \tan^{-1} \left[\frac{1}{\omega u_i^2} \right] \right\}$$

This expression for the range of the aircraft is considerably more complex than those found in the other two cruise methods, and the cruising speed for maximum range is found to be a function of the fuel ratio, w . This can be seen by considering the relative airspeed, u , at the beginning and end of the cruise. The cruising speed is maintained constant so that but the minimum drag speed will decrease with aircraft weight causing the relative airspeed, u , to increase during the cruise. This means that the optimum airspeed for maximum range will be a function of the weight change during the cruise and, therefore, of the fuel ratio, w . Figure 2.4 shows the range function

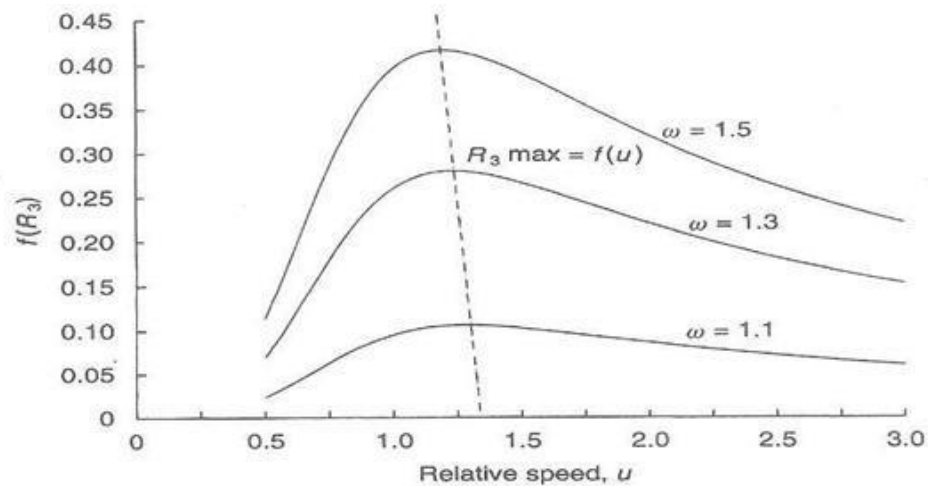


Fig: 2.4 range function cruise method 3.

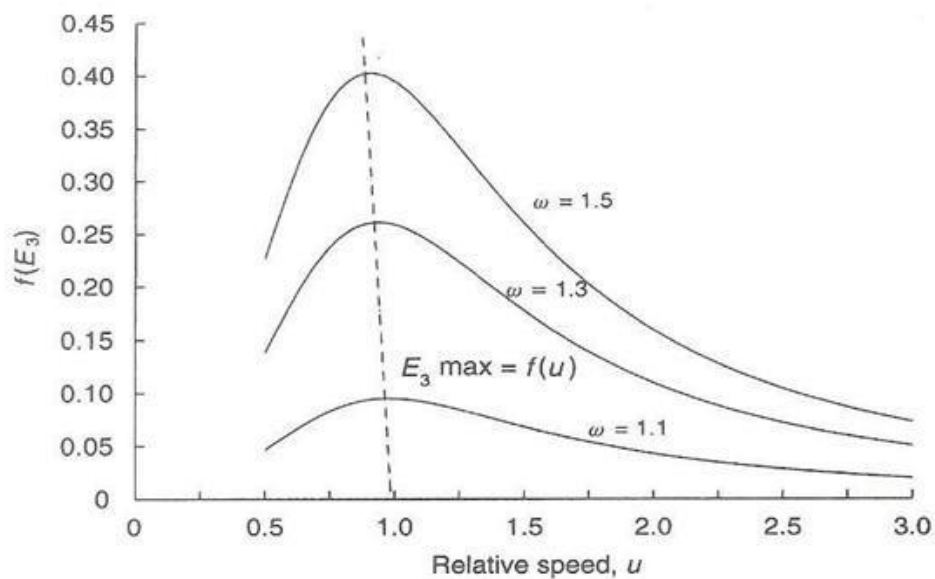


Fig: 2.5 Endurance function cruise method 3.

For cruise method 3 and the variation of the initial relative airspeed for maximum range as the fuel ratio increases.

Following the integration of above equation, the endurance under cruise at constant altitude and Mach number is given by

$$E_3 = \left[\frac{E_{\max}}{C} \right] 2 \left\{ \tan^{-1} \left[\frac{1}{u_i^2} \right] - \tan^{-1} \left[\frac{1}{\omega u_i^2} \right] \right\}$$

And the relative airspeed for best endurance will, similarly, be a function of fuel ratio; Figure 2.5 shows the endurance function that indicates that the optimum endurance can only be achieved by commencing cruise at airspeed less than the minimum drag speed. This will require cruise on the backside of the drag curve, which lends to be speed unstable. This cruise method, therefore, is not ideal for mission to be flown for endurance.

Comparison of cruise methods:

It has been possible to write the range and endurance attained by each method of cruise in the form of a product of a range factor and a range function, and of an endurance factor and an endurance function, given

$$R = \left[\frac{V_{\text{mdi}}}{C} E_{\max} \right] \{ f_R(u_i, \omega) \}$$

And

$$E = \left[\frac{E_{\max}}{C} \right] \{ f_E(u_i, \omega) \}$$

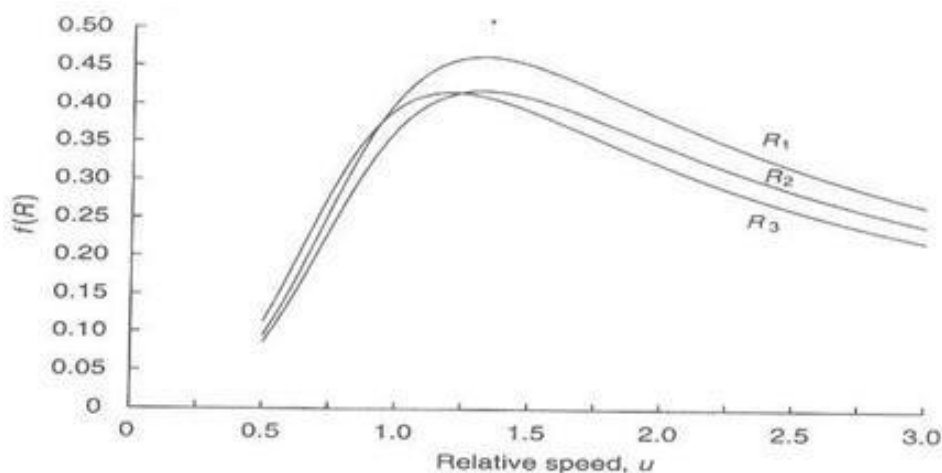


Fig: 2.6 Comparison of cruise methods for range.

This enables the methods of cruise to be compared in terms of the relative magnitudes of the range and endurance functions.

The range function are compared in Fig. 2.6 for a fuel ratio of 1.5 and show that the cruise – climb is the optimum method of cruise, indicating that, at its best, it gives about 10%\$ better range than the other methods. However, operational considerations generally demand the constant altitude, constant Mach number cruise, which tends to be the least efficient in terms of fuel consumption.

The comparison between the cruise methods for endurance, Fig. 2.7, shows less disparity but favors the constant angle of attack methods. This is particularly the case when flying the endurance, as the cruise would generally be performed at airspeed slightly above the minimum drag speed to avoid the backside of the drag curve and to give flight path stability.

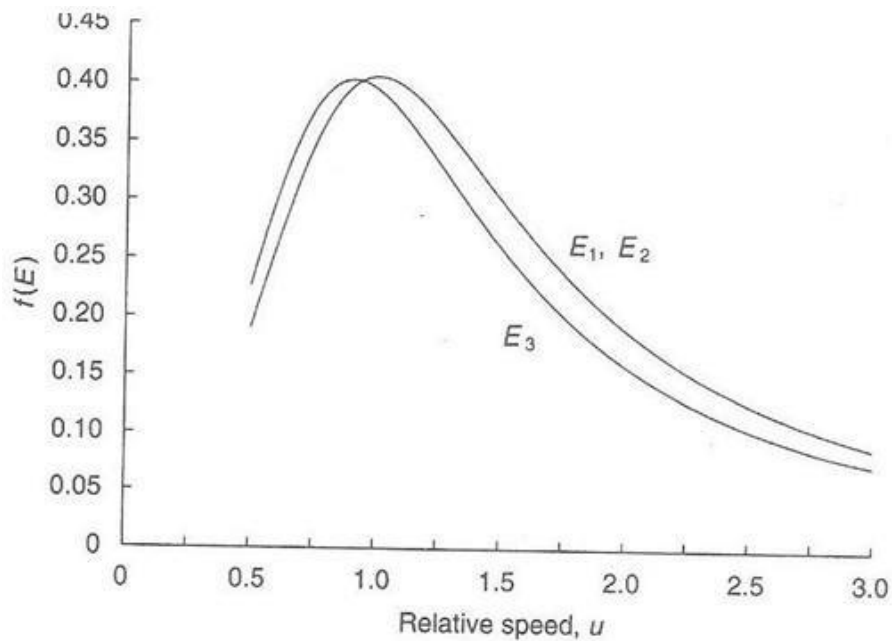


Fig. 2.7 Comparison of cruise methods for endurance

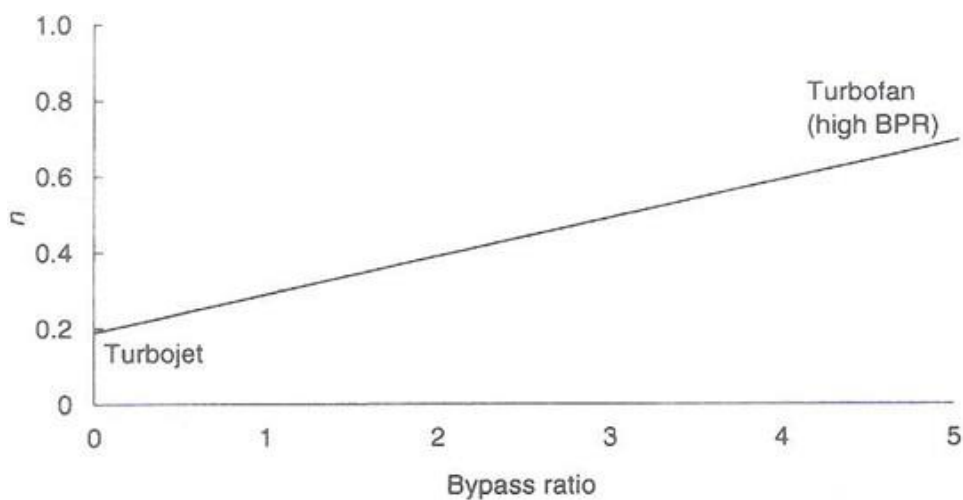


Fig. 2.8 Effect of bypass ratio on fuel flow low

The effect of alternative fuel flow laws:

The range and endurance functions developed above have used the simple fuel flow law, above equation which assumes that the fuel flow is proportional only to net thrust. It was accepted that this fuel flow law is idealized, probably not full describing the characteristics of the engine, and that a more realistic law should be applied. If an alternative fuel flow law is considered, for example above equation then it may produce different optimum operating airspeeds.

Above equation states a fuel flow law of the form:

$$C = C_2 \theta^{\frac{1}{2}} M^n$$

Where n may vary from about 0.2 for a turbojet to about 0.6 for a high bypass ratio turbofan, see Fig. 2.8

When the SAR above equations, are written in terms of Mach number and the alternative fuel flow law substituted they give.

$$\text{SAR} = \frac{a_0 \theta^{\frac{1}{2}}}{C_2 \theta^{\frac{1}{2}}} M^{1-n} \frac{L}{D} \frac{1}{W}$$

$$\text{SE} = \frac{1}{C_2 \theta^{\frac{1}{2}} M^n} \frac{L}{D} \frac{1}{W}$$

When these expressions are differentiated, they give the airspeeds for maximum SAR and SE to be,

$$V_{\text{max SAR}} = \left[\frac{3-n}{1+n} \right]^{\frac{1}{4}} V_{\text{md}}$$

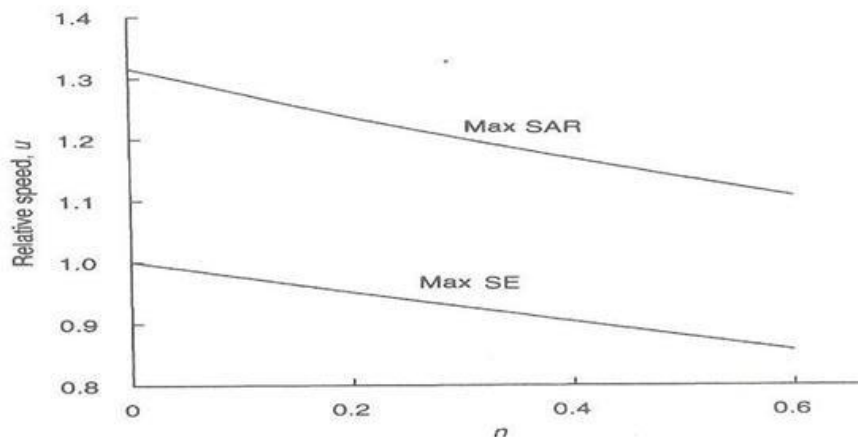


Fig: 2.9 Effect of bypass ratio on optimum speeds for range and endurance.

$$V_{\max \text{ SE}} = \left[\frac{2-n}{2+n} \right]^{\frac{1}{4}} V_{\text{md}}$$

Figure 2.9 shows how the optimum speeds vary with the bypass ratio of the engine. This implies that as the exponent, n , increases (i.e.; as the bypass ratio of the engine increases), the relative airspeed for best range or endurance will decrease. This may be advantageous when cruising for maximum range but when cruising for endurance it will demand a cruising speed below the minimum drag speed with the consequence of flight path instability.

The SAR and SE equations can be integrated to produce the full range and endurance functions under the alternative fuel flow laws. These will be similar in form to those produced by the assumption of the simple law but will show different relative speeds for optimum cruise performance.

The effect of weight, altitude and temperature on cruise performance:

The range factor determined for each of the cruise performance expressions can be written as,

$$\frac{V_{\text{mdi}}}{C} E_{\text{max}} = \frac{1}{C} \left[\frac{W_i \theta}{\delta_i} \right]^{\frac{1}{2}} \left\{ \frac{2RT_0}{\rho_0 S C_{L\text{md}}} \right\}^{\frac{1}{2}} E_{\text{max}}$$

In which the square bracket contains the variable elements of the expression.

From above equation the range factor is seen to be proportional to the square root of the initial cruising weight which appears to suggest that the range of the aircraft will increase with its weight. It should be remembered that the fuel ratio is the ratio of the initial to final cruise weights, above equation so that

$$W_i = \omega W_f$$

Therefore, for a given final weight of the aircraft, the increase in initial weight implies an increase in the fuel available for the cruise; it is this that extends the range not the weight of the aircraft itself. If the additional weight consists only of payload or of aircraft zero-fuel weight – that is, an increase in the final weight – that the fuel ratio will be decreased and hence the range will be reduced.

An increase in air temperature increases the range since the TAS is increased and the aircraft flies further in a given time, during which it burns the same quantity of the fuel. However, in above equation the specific fuel consumption is assumed to be a simple constant, which may not be the case. If the specific fuel consumption is a function of air temperature, as in above equation, then the effect of the temperature may be lessened and the range may even decrease as temperature increases.

Increasing altitude will produce an increase in the range as the ambient relative pressure decreases. In the troposphere, the effect will be reduced by the accompanying decrease of temperature, with altitude and further affected by any dependency of the specific fuel consumption on air temperature. Generally, cruising at higher altitude will lead to better range performance. However, it has been

seen that the optimum subsonic performance of an aircraft, in terms of the range or endurance it can attain from a given quantity of fuel, is a function of its minimum drag speed. Therefore, operation at airspeeds other than its optimum airspeed will incur a range or endurance penalty. The minimum drag speed in terms of equivalent airspeed is affected by altitude. However, as the altitude of operation increases, so that true Mach number will approach its critical value. The aircraft drag will then increase due to the onset of the wave drag. And the parabolic drag polar no longer describes the drag characteristic of the aircraft.

The optimum altitude for cruise will be determined by combining the optimum cruise airspeed and the critical Mach number, M_{crit} , as that best range is flown at the highest true airspeed. This gives the greatest economy of operation by minimizing the fuel consumed and the time of flight. Assuming that the best range is obtained by flying at $1.316 V_{md}$ then, from above equation,

$$M_{crit} = 1.316 \left[\frac{2}{\gamma S} \right]^{\frac{1}{2}} \left[\frac{K}{C_{Dz}} \right]^{\frac{1}{4}} \left[\frac{W}{P} \right]^{\frac{1}{2}}$$

And the parameter W/p can be evaluated. From this, the optimum cruise altitude can be found for a given aircraft weight, this is shown in Fig. 2.10.

If the aircraft is flown at a higher than optimum altitude then the critical Mach number is exceeded and the increase in drag will reduce the range. To avoid this, the aircraft must be flown at airspeed less than the optimum, again with a range penalty. If the cruise is at lower than optimum altitude then it would be usual practice to cruise at the critical Mach number to take advantage of the higher airspeeds. In this

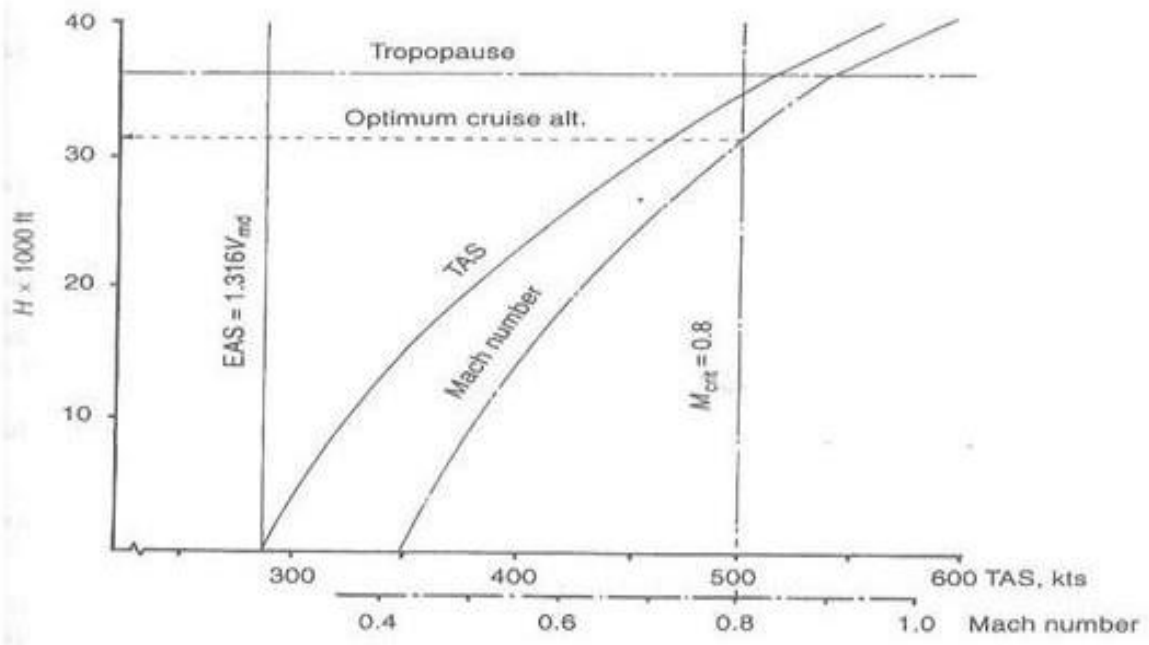


Fig. 2.10 optimum altitudes for cruise

Case, it would be necessary to accept that the relative airspeed will be greater than the optimum.

It is also assumed in equation that the specific fuel consumption does not vary with altitude or Mach number. In practice, the fuel flow law may be a function of the atmospheric variables, and of Mach number, and contribute to the altitude effects within the range factor. A further effect can be seen in above equation in which the optimum relative airspeed is seen to be a function of the bypass ratio of the engine. In this case, the fuel flow law produces an optimum relative airspeed that is less than 1.3126 Vmd. Therefore, from above equation, the optimum cruise altitude at which the best range speed the critical Mach number is balanced will be increased and better cruise economy can be achieved.

The endurance factor of the aircraft shows that the endurance is unaffected by altitude, other than through any dependence of the specific fuel consumption on the atmospheric variables.

Range and endurance for aircraft with power producing engines:

If the aircraft has power-producing engines, which produce shaft power with negligible residual thrust, the power is converted into propulsive thrust by a propeller. In this case, the performance equation is written in terms of the thrust power available and drag power required for cruising flight,

$$\eta P = DV$$

Where η is the propeller efficiency.

The specific fuel consumption is defined in power terms as,

$$C = \frac{Q_f}{P}$$

This is usually quoted in kg/kW hr.

(NB In the subsequent analysis of the cruise performance of the aircraft with power producing engines, particularly where the specific fuel consumption is used, it may be necessary to include the gravitational constant, g , and other constants, to make the modules of the expressions consistent. A check on the modules of the expressions will show when this is needed.)

The specific air range and specific endurance of the aircraft with power-producing engines are given by,

$$SAR = \frac{V}{CP} = \frac{\eta}{C} \frac{L}{D} \frac{1}{W}$$

And

$$SE = \frac{1}{CP} = \frac{\eta}{CV} \frac{L}{D} \frac{1}{W}$$

Respectively.

From these expressions it can be deduced that the SAR is a maximum when the aircraft is flying at its minimum drag speed, V and that the SE is a maximum when flying at the minimum power speed, V_{np} , since it can be further deduced that the optimum airspeeds for the performance of aircraft with power-producing engines are related to the minimum power speed in the same way

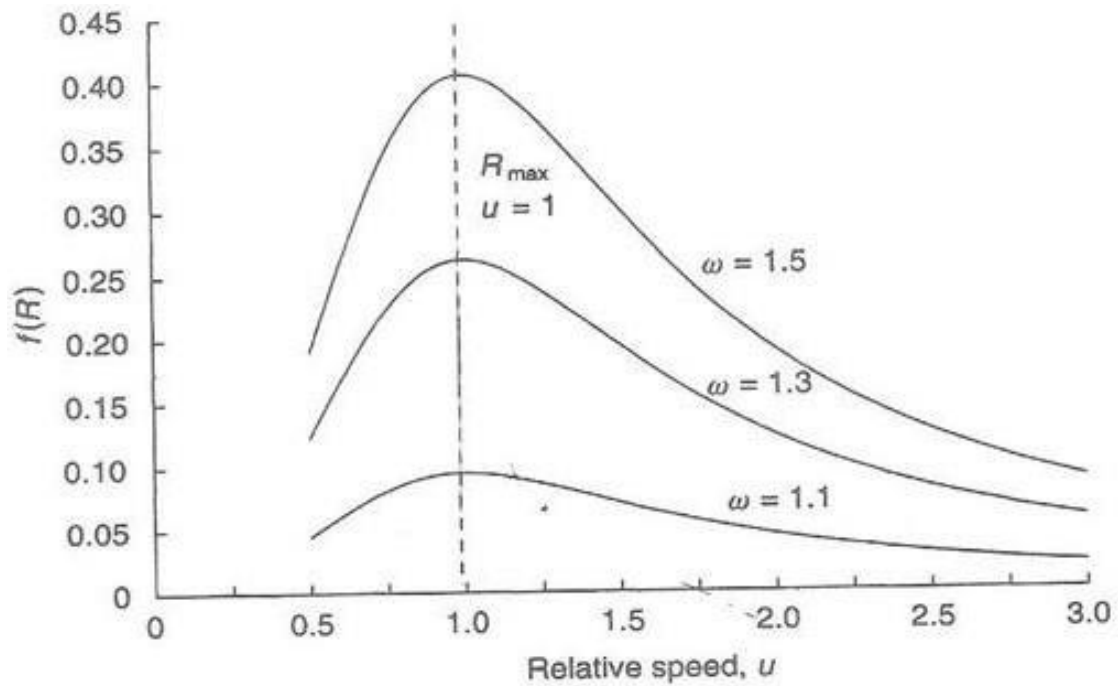


Fig. 2.11 Range function, power-producing engines.

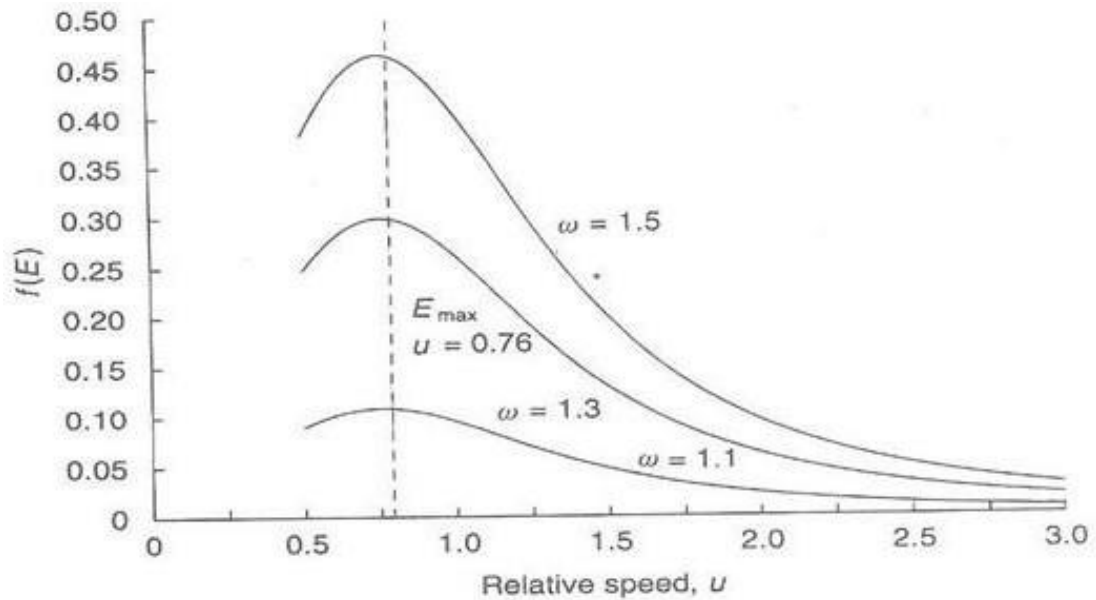


Fig. 2.12 Endurance function, power-producing engines.

as the optimum airspeeds of aircraft with thrust-producing engines are related to the minimum drag speed. Above equations can be integrated to give expressions for the cruise performance of the aircraft with power-producing engines in the same way as above equations were used to give the cruise performance for aircraft with thrust-producing engines.

For example, in the case of the rise – climb the range and endurance would be given by

$$R = \left[\frac{\eta E_{\max}}{C} \right] \left\{ \frac{2u^2}{u^4 + 1} \right\} l_n \omega$$

$$E = \left[\frac{\eta E_{\max}}{C V_{\text{md}}} \right] \left\{ \frac{2u}{u^4 + 1} \right\} l_n \omega$$

These are shown in Figs. 2.11 and 2.12.

Since the best endurance speed is very low, it may be too close to the stalling speed for safe operation. It is likely that the aircraft would have to be flown at airspeed in excess of the theoretical optimum when it is being operated for patrol missions in which maximum endurance is the primary requirement.

Aircraft with mixed power plants:

Some aircraft have power plants with both thrust-producing and power-producing characteristics. Although there have been aircraft designed with a combination of turbojet engines and piston engines, the most common example is that of the turbo-prop. Here, the shaft power of the turbine engine is converted into thrust through the propeller and the residual energy in the exhaust gas is converted into thrust by the exhaust nozzle. The cruising performance characteristic of an aircraft with mixed power plants lies between those of the aircraft with pure thrust- or pure power-producing power plants. It needs to be estimated by taking the proportion of direct thrust to thrust power produced by the engine.

Using the cruise – climb range expression as an example, the principle can be demonstrated. From above equation the range of the aircraft with thrust-producing Engines is given by

$$R_T = \left[\frac{V_{\text{mdi}}}{C_T} E_{\max} \right] \left\{ \frac{2u^3}{u^4 + 1} \right\} l_n \omega$$

And from above equation the range of the aircraft with power-producing engines is given by.

$$R_P = \left[\frac{\eta E_{\max}}{C_P} \right] \left\{ \frac{2u^2}{u^4 + 1} \right\} l_n \omega$$

Where subscript T refers to the thrust-producing engine and subscript P refers to the power-producing engine.

As an approximation to the cruise performance of an aircraft with a mixed power plant, these can be combined into a common equation.

$$R = E_{\max} \left[\Pi \frac{\eta}{C_P} + (1 - \Pi) \frac{uV_{\text{md}}}{C_T} \right] \left\{ \frac{2u^2}{u^4 + 1} \right\} l_n \omega$$

When Π is the proportion of the thrust derived from the shaft power in the overall thrust of the power plant and C_p and C_r are the specific fuel consumptions based on the shaft power and net thrust respectively. It can be seen that the expression in the square brackets proportions the thrust and power terms and, since it also contains a term in u , modifies the range function in the curly brackets (The same expression can be applied to the endurance equation.)

In the case of the turbo-prop power plant, the specific fuel consumption is usually based on the equivalent shaft horsepower, ESHP, of the engine. ESHP is the combination of the thrust output with the shaft power output to give the total output in power form as if the engine was a pure power-producing engine. In effect, the expression in the square bracket in above equation describes the combination of thrust and power into ESHP so that the performance can be estimated as if the aircraft had a pure power producing engine. However, it is unlikely that the proportions of thrust and power will be independent of speed or engine output, and so the expression will need to be calculated for each combination of engine power setting and aircraft speed. Because of this, cruise performance calculations for turbo-prop aircraft will usually need to be performed in a point-to-point manner rather than by a continuous function.

MODULE III

CLIMB AND DESCENT PERFORMANCE

Introduction:

In the overall mission of the aircraft, there will be a climb phase in which the aircraft increases its height to the required cruising level and a descent phase from the end of the cruise to the landing. In these phases of the flight, the difference between the propulsive thrust and the airframe drag is used to change the potential energy, and the kinetic energy, of the aircraft. If the thrust exceeds drag, the aircraft will climb and if the drag exceed thrust it will descend; the rate at which this occurs will depend on the relative magnitudes of the thrust and drag forces. Although climb and descent imply changes in height, they may also involve changes in true airspeed sine the air density decreases with altitude. If the rates of climb or descent are high, the acceleration of the aircraft implicit in the climbing maneuver will have to be taken into consideration.

Climb performance is important from both economic and flight safety points of view. In a climb, the potential energy of the aircraft is increased and fuel energy must be expended to achieve this. This fuel required climbing to a given height and be minimized by the use of the correct climb technique and optimum economy of operation can be attained. Economy, however, is not the only criterion of operation. The safety of the aircraft depends on is ability to climb above obstructions at all points on the flight path. Sufficient excess thrust must be available to ensure that the aircraft can meet certain minimum gradients of climb in any of the safety critical segments of the flight.

The descent is less critical than the climb economically since the aircraft will be operating at low thrust and hence low fuel flow. However, several safety-related considerations will affect the choice of the flight path in the descent. Among them are the attitude of the aircraft, the rate of change of cabin pressure and the need for the engines to supply power for airframe services. The descent strategy will need to consider all of these.

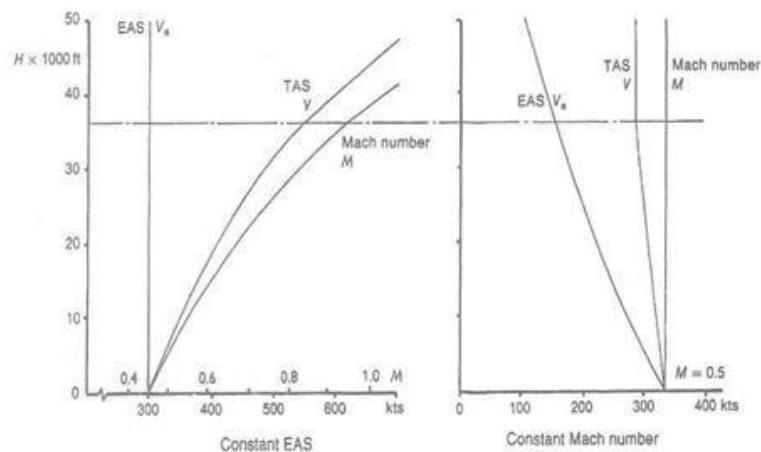


Fig 3.1 Speed relationships in climbing flight.

A climb, or a descent, will usually be performed with reference to an indication either of airspeed or of Mach number. If the climb were based on an airspeed reference then, strictly, it would be the

calibrated airspeed, assuming that any instrument error and pitot-static pressure errors had been accounted for (Chapter-2) At the typical climbing speeds of a subsonic transport aircraft the scale-altitude correction is small and the calibrated airspeed (CAS) is close to the equivalent airspeed (EAS). Therefore, for all practical purposes, the climb can be assumed to be performed at constant EAS. This implies that, as the aircraft climbs, the ambient air density will be decreasing and the true airspeed (TAS) will be increasing, thus the aircraft will be accelerating throughout the climb.

If the climb is based on Mach number then, in the troposphere, as altitude increases, the ambient temperature will be decreasing and with it the speed of sound. This implies that the true airspeed will be decreasing as the aircraft climbs. In the isothermal stratosphere, a climb at constant Mach number results in a constant TAS and there is no acceleration. Figure 3.1 shows the relationship between true airspeed, equivalent airspeed and Mach number in climbing flight. It is evident that if a climb is performed at a constant EAS then the Mach number will increase with height and the critical Mach number will eventually be reached. Alternatively, if the climb is performed at constant Mach number then the EAS will decrease towards the stalling speed as height increases.

In practice, an aircraft climb in to a height at which the Mach number would approach its critical value would usually start the climb at a constant EAS and the Mach number would be allowed to increase. In this state, the angle of attack is constant and the climb can be made at a constant, and possibly optimum, lift-drag ratio. As the Mach number increases, it becomes necessary to avoid the drag rise as the critical Mach number is reached. The climb would be converted into a constant Mach number climb allowing the EAS to decrease as the climb continues. The aircraft will no longer be climbing at the optimum aerodynamic efficiency but the drag rise will be avoided and a good compromise can be achieved between climb performance, EAS and Mach number, and see Fig. 3.2. The descending flight path is usually structured in a similar manner.

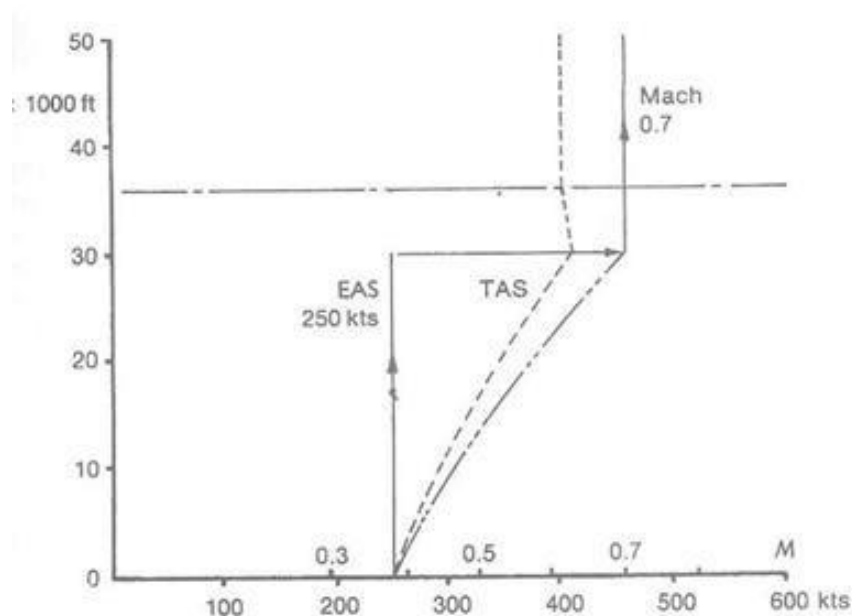


Fig. 3.2 Typical climb or descent speed/Mach number schedule

In the case of a subsonic aircraft, with a normal thrust-to-weight ratio at take-off, the rate of climb is usually low enough to allow the acceleration term in the performance equation to be neglected since the rate of change of air density, and hence TAS, with time is small. Under these conditions, the

climb can be treated as being quasi- steady for the purposes of performance analysis. Similarly, the rates of descent involved with subsonic aircraft operations are usually low enough to allow the same assumption to be made in descending flight. This chapter will deal principally with the climb and descent performance of aircraft with the moderate thrust-to-weight ratios of a transport aircraft, typically around a maximum of 0.3 at take-off, as a steady-state analysis.

If the thrust-to-weight ratio is high, as in the case of a military combat aircraft, then the acceleration during climb cannot be omitted and the simultaneous change of potential and kinetic energies must be taken into account. This is known as the total energy limb and requires a quite different approach to its analysis.

Steady-state climb and descent can be measured in terms of rate or gradient. The rate of climb is the vertical velocity, dH/dt , and is usually quoted in feet/min. The gradient of climb, γ_2 , is defined by,

$$\gamma_2 = \sin^{-1} \left[\frac{dH/dt}{V} \right]$$

Where the height, H , is geo-potential height.

The gradient is often expressed as a percentage gradient, or grade %, which is defined as,

$$\text{grad}\% = 100 \tan \gamma_2$$

Climb and descent performance analysis:

The power plant of the aircraft may be thrust producing or power producing. Thrust-producing engines, turbojets and turbofans, produce thrust that is relatively constant with small change of airspeed in subsonic flight. Power-producing engines, piston engines or turbo-shaft engines, produce shaft power, which is relatively constant with change of airspeed, and which needs to be converted into propulsive thrust by a propeller. The differing characteristics of these power plants lead to different criteria for optimum climb performance and each need to be considered separately.

3.2.2 Aircraft with thrust-producing engines:

The equations of motion of an aircraft with thrust-producing engines in a climb (or a descent) can be taken from above equation. In a straight, wings level, climb, in which the flight path gradient is constant the eqns. Of motion can be written,

$$\left. \begin{aligned} F_N - D &= W \sin \gamma_2 + m\dot{V} \\ L &= W \cos \gamma_2 \end{aligned} \right\}$$

(It should be remembered that these equations of motion contain simplifying assumptions and can only be used when those conditions apply.)

If the aircraft has a normal take-off thrust-to-weight ratio of about 0.3 then the rates of climb will be low enough to assume that the acceleration associated with the rate of climb is negligible. The climb can then be assumed to be made either at constant airspeed or at constant Mach number. Also, the gradient of climb and descent will be low enough to allow the assumption that $\cos \gamma_2 = 1$ in above equation and the equation can be simplified further to the form.

$$\left. \begin{aligned} F_N - D &= W \sin \gamma_2 \\ L &= W \end{aligned} \right\}$$

These equations will be used to derive the climb and descent performance expressions for the quasi-steady flight path.

From above equation the excess thrust ($F_N - D$) provides the gradient of climb,

$$[F_N - D] \frac{1}{W} = \sin \gamma_2$$

So that, if the thrust is constant, the best gradient of climb will be obtained by flying at the minimum drag speed. Figure 3.3 shows the ideal thrust and drag relationship (relative to the minimum drag) in which the thrust does not vary with airspeed and the maximum excess thrust occurs at the minimum drag speed. It should be noted, however, that, in practice, the airspeed is likely to influence the thrust to some extent. Therefore, the airspeed for optimum climb gradient will be found to be close to, but not necessarily at, the minimum drag speed.

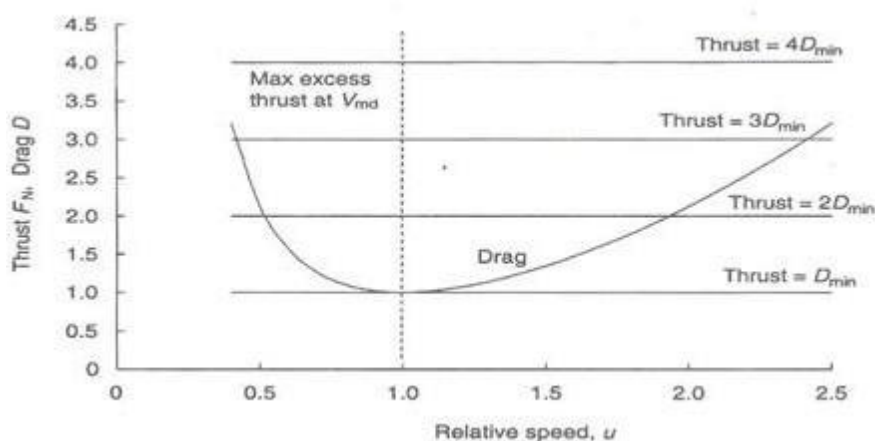


Fig. 3.3 Excess thrust, thrust-producing engines.

Using above equations leads to an expression for the best rate of climb,

$$[F_N - D] \frac{V}{W} = \frac{dH}{dt}$$

This indicates that the airspeed for the best rate of climb occurs when the excess thrust power, $F_N V$, over drag power, $D V$, is a maximum. Since the ideal thrust power increases linearly with true airspeed, the best rate of climb is predicated to be at airspeed greater than the minimum drag speed; this is seen in Fig. 3.4. In this case, there is no simple solution for the airspeed for best rate of climb, this will occur

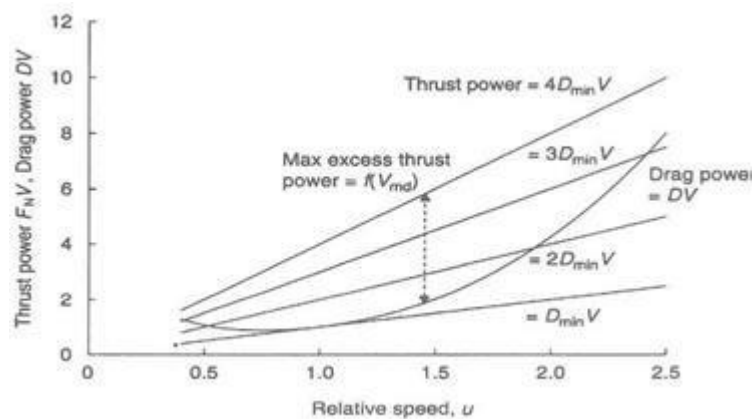


Fig.3.4 Excess thrust power, thrust-producing engines.

When the difference between the thrust power and drag power is a maximum and is a function of the excess thrust-power.

Generalized climb performance for rate and gradient of climb or descent:

The climb and descent performance can best be analyzed in a general manner by considering a dimensionless form of the performance equation, above equation. The generalized climb performance equation can be written for thrust-producing engines as.

$$\tau - \frac{1}{2}[u^2 + u^{-2}] = E_{\max} \sin \gamma_2 = E_{\max} \left(\frac{v}{u} \right)$$

Where the dimensionless rate of climb, v , is defined as,

$$v = \frac{dH/dt}{V_{md}}$$

Above equation can be applied to any aircraft for which the drag characteristic, the thrust and the weight are known. By differentiating above equation, the relative airspeeds for best climb or descent performance can be found.

Climb gradient:

From above equation the gradient of climb is given by,

$$E_{\max} \sin \gamma_2 = \tau - \frac{1}{2}[u^2 + u^{-2}]$$

For maximum gradient $dy_2/du = 0$, which occurs when $u = 1$ if T is constant, and confirms that the steepest climb occurs at the minimum drag speed above equation. Figure 3.5 shows the dimensionless climb gradient as a function of relative airspeed for several values of dimensionless thrust, combinations of τ and u that positive values of \sin produce climbing flight. Descending flight occurs when the

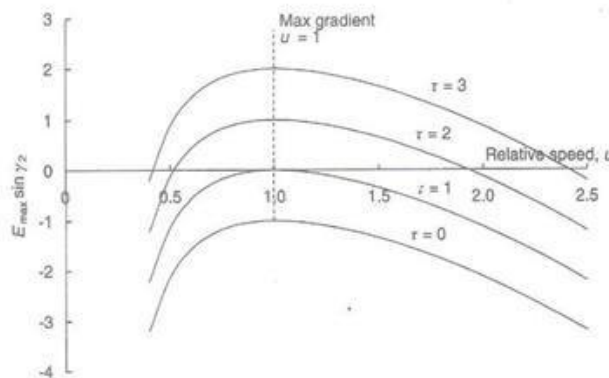


Fig. 3.5 Gradient of climb, thrust-producing engines

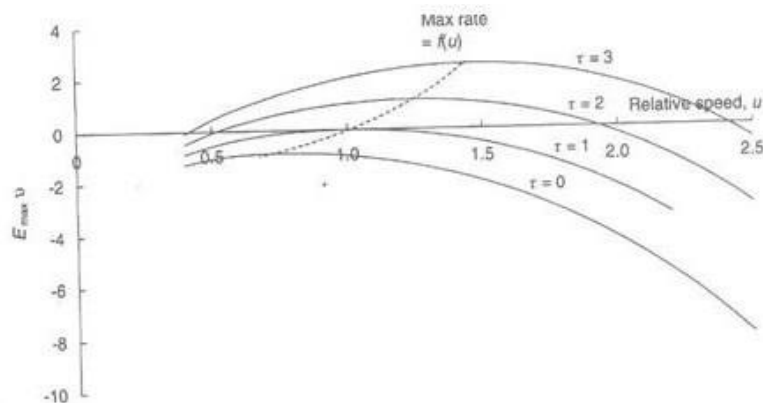


Fig. 3.6 Rate of climb, thrust producing engines.

Combination of r and u give negative values of $\sin^2 \gamma$. Typical values of r for a transport aircraft at take-off thrust usually lie between 3 and 4.

In the special case of gliding flight, in which $r = 0$, flight at the minimum drag speed will give the shallowest glide angle, which will give the greatest range of glide this speed is used when cruising between thermals. The minimum glide angle will be,

$$\gamma_2 = \sin^{-1}[1/E_{\max}]$$

Climb rate:

From above equation the rate of climb is given by,

$$E_{\max} v = \tau u - \frac{1}{2}[u^3 + u^{-1}]$$

And for maximum rate of climb $dv/du = 0$. In this case, there is no simple solution and the relative airspeed for best rate of climb is found to be a function of the dimensionless thrust,

$$u = \frac{1}{\sqrt{3}}[\tau \pm (\tau^2 + 3)^{\frac{1}{2}}]^{\frac{1}{2}}$$

Figure 3.6 shows the dimensionless rate of climb as a function of relative airspeed for several values of dimensionless thrust, and the relative airspeed for best rate of climb is seen to in tease with dimensionless thrust. In gliding flight, the minimum sink rate is attained by flying at a relative airspeed of which is the minimum power speed of the aircraft. Flying at this speed will maximize the time (or endurance) of gliding flight and is the speed used for climbing in thermals.

Aircraft with power-producing engines:

Using the same assumptions that were used in the case of the aircraft with thrust-producing engines, the equation of performance for the aircraft with power-producing

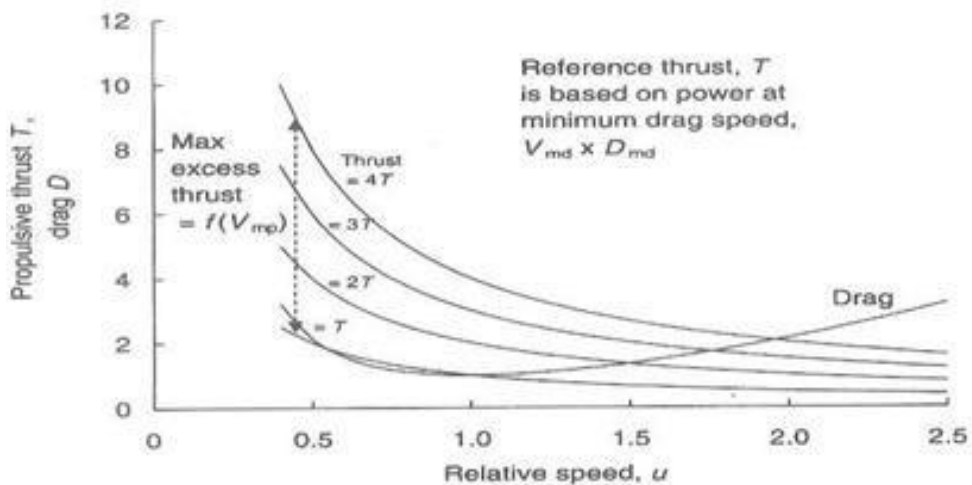


Fig. 3.7 Excess propulsive thrust, power-producing engines.

Engines above equations can be written in parallel with those for aircraft with thrust-producing engines, as

$$\frac{\eta P}{V} - D = W \sin \gamma_2$$

Where $\eta P/V$ is the propulsive force developed by the engine – propeller combination.

The gradient of climb is given by,

$$\left[\frac{\eta P}{V} - D \right] \frac{1}{W} = \sin \gamma_2$$

which has a maximum value when the excess propulsive force is a maximum. This occurs at airspeed less than the minimum drag speed in climbing flight. Figure 3.7 shows the excess propulsive force (relative to the power at the minimum drag speed, V_{md} x D_{md}). It indicates that the maximum occurs at an airspeed that is less than the minimum drag speed and which tends to decrease as the power available increases.

The rate of climb is given by,

$$[\eta P - DV] \frac{1}{W} = \frac{dH}{dt}$$

And is a maximum at the minimum power speed. Fig.3.8 has shown the excess thrust power that occurs at the minimum power speed of the aircraft, assuming that the thrust power is independent of airspeed.

Generalized performance:

The generalized performance equation for climb and descent is given, from above equation for aircraft with power-producing engines, as

$$\frac{\lambda}{u} - \frac{1}{2}[u^2 + u^{-2}] = E_{\max} \sin \gamma_2 = E_{\max} \frac{v}{u}$$

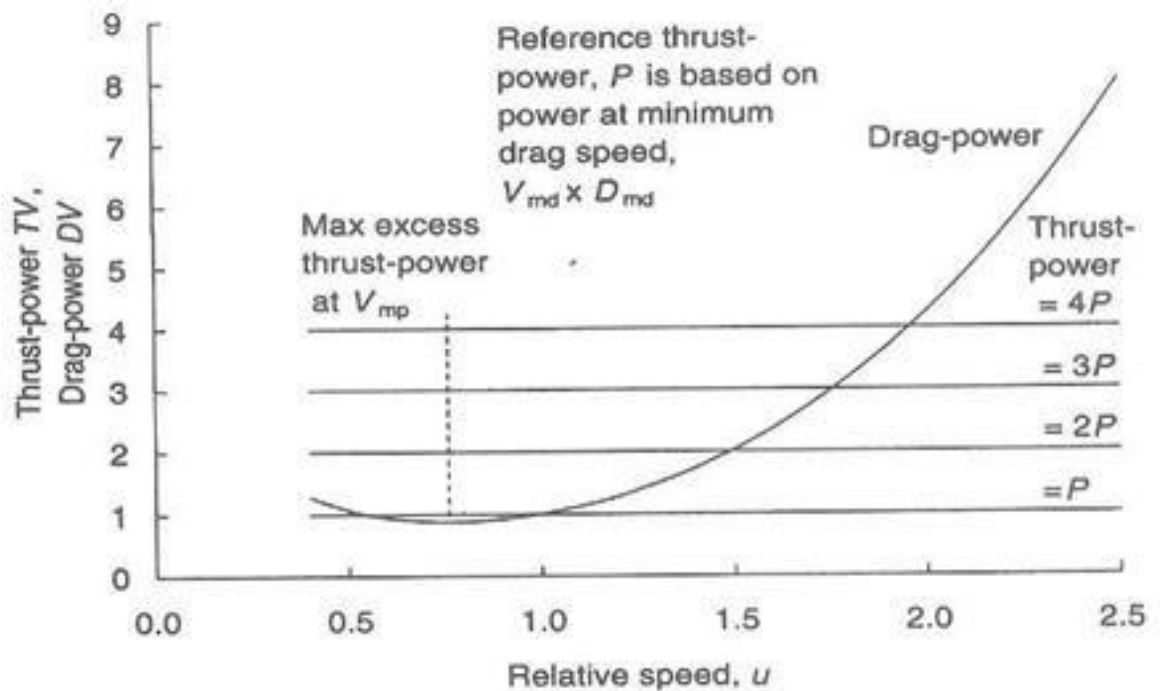


Fig. 3.8 Excess thrust power, power-producing engines.

Climb gradient:

From above equation the gradient is given by,

$$E_{\max} \sin \gamma_2 = \frac{\lambda}{u} - \frac{1}{2}[u^2 + u^{-2}]$$

By differentiating above equation the relative airspeed for best limb gradient is found to occur when $dy_2/du = 0$ which gives,

$$u^4 + \lambda u - 1 = 0$$

This has no simple solution but shows that the relative airspeed for maximum gradient of climb is a function of engine power. The dimensionless climb gradient is shown in Fig. 3.9 as a function of relative airspeed for several values of dimension-less power, In gliding flight, $\lambda = 0$, and the shallowest glide angle is given by flying at the minimum drag speed. As the power increases, the airspeed for maximum, climb gradient decreases and, for values of dimensionless power greater than unity, the best gradient is attained by flying at airspeeds less than the minimum power speed. In practice, this may be impractical since the aircraft may be operating too close to the stalling speed for safety; this will be referred to later.

Climb rate:

From above equation the rate of climb is given by,

$$E_{\max} v = \lambda - \frac{1}{2} [u^3 + u^{-1}]$$

For maximum rate $dv/du = \text{zero}$ which occurs when $u = 1$ which is the minimum power speed, above equation. Figure 3.10 shows the dimensional rate of climb as a function of relative airspeed for several values of dimensionless power. The best rates of climb are attained by flying at the minimum power speed at all levels of power. In gliding flight, the minimum sink rate occurs at the minimum power speed.

An alternative method is to measure the maximum excess thrust or power by level accelerations. In this technique, the aircraft is flown as slowly as possible in level flight; maximum thrust or power is selected and the airspeed recorded in a level acceleration to maximum airspeed. For the acceleration, the excess thrust or power can be deduced and thus so can the speeds for best climb performance. The level acceleration method is best suited to aircraft with thrust-producing engines. The partial climb method is best suited to aircraft with power-producing engines since the best climb speeds tend to be towards the lower end of their speed range.

Climb performance in aircraft operations:

In practice, the climb may be performed to give either a steep gradient of climb or a high climb rate; the choice will depend on the most critical consideration of the phase of flight.

Climb gradient:

In the take-off and initial climb phase, the most critical consideration is that of flight safety and the ended the ensure that the aircraft can avoid all known obstructions along its flight path. In the licensing of the airfield, a departure path is defined along which no obstructions are permitted and the aircraft is guaranteed a clear flight path. The definition of the departure path is complex and depends on the size of the airfield and the type of aircraft operations that are intended. For large, international, airports, the obstacle limitation surface – which defines the safe departure path – is a surface, of gradient 2% extending from the end of the take-off distance available on the runway to a distance of 15000m (A full definition can be found in ICAO International Standards and Recommended Practices, Annex 14, Aerodromes) Therefore, to guarantee a safe departure from the airfield the aircraft must be capable of climbing at a gradient of at least 2% under all conditions, including emergency conditions with one engine inoperative. Clearly, in this phase of flight the aircraft needs to be operated at an airspeed that will produce the best gradient of climb so that the departure flight path will be steep enough to exceed the minimum safe gradient specified. Therefore, the airspeed chosen for the after-take-off limb should be that for maximum gradient.

However, the airspeed for best gradient is usually a low speed and may be losing to other critical operating airspeeds, such as the stalling speed or minimum airspeeds for lateral-directional control.

Restrictions on the airspeed scheduled for the climb are based on a margin over the stalling speed and the ability to maintain lateral-directional control in the event of a sudden loss of propulsive thrust on an engine. This will often result in the scheduled airspeed for the climb being higher than that for optimum climb gradient; this is particularly true in the case of aircraft with power-producing engines.

One of the most critical parts of the flight path is the after-take-off climb. This is made with the aircraft in the take-off configuration, initially with landing gear extended, and with flaps set to optimize the take-off speed and runway distance requirement. In this state, the climb gradient with one engine inoperative will often be the critical limiting factor in determining the maximum allowable take-off weight of the aircraft.

Climb rate:

Once the aircraft has climbed to a safe height, usually taken to be 1500ft above the airfield, the need to avoid ground-based obstacles is no longer critical and the climb can continue in the most expedient manner. In the case of transport operations, this will usually be the most economic climb. This will be based on either the minimum time to climb to operating height, the minimum fuel consumed in the climb or some compromise between these which will give the best overall economy.

The maximum rate climbs will enable the aircraft to reach its operating height in the minimum time so that the cruise phase can commence. The airspeed for best climb rate is higher than that for best gradient. Therefore, following the after-take-off climb, the aircraft can be alerted to its climb speed for best climb rate in its enroute configuration and continue to climb to cruise altitude following a convenient schedule of airspeed and Mach number.

Aircraft with power-producing engines will usually climb at their airspeed for best rate of climb, which will be less than their minimum drag speed. The climb will then continue to the cruising height where the aircraft will accelerate to its cruising speed.

Aircraft with thrust-producing engines have airspeed for best rate of climb that is a function of their excess thrust; the greater the excess thrust the higher will be the airspeed for best climb rate, above equation. As the aircraft climbs, the thrust will decrease and with it the optimum airspeed for climb rate. The airspeed used in the climb will generally be a compromise based on excess thrust, which will be a function of the weight, altitude and temperature (WAT) conditions at the start of the climb. It will take into account the anticipated WAT changes during the climb to give the best average climb performance throughout the climb. As the climb continues, the flight Mach number will increase as the relative pressure of the atmosphere decreases. It may become necessary to convert the climb to constant Mach number to avoid the drag rise that would reduce the climb performance (Fig.3.2).

Minimum fuel climb:

The fuel consumed in the climb can be expressed in terms of the specific climb, SC, as,

$$SC = (dH/dt)/Q_f$$

And is expressed in terms of ft/kg

If the fuel flow is measured during the flight trials to determine the optimum limb speed, then the specific climb function can be formed to give the airspeed for a minimum fuel climb. In the simple analysis, in which the assumption of constant specific fuel consumption is made, the thrust or power is set to the maximum continuous setting. The minimum fuel climb will then occur at the airspeed for best rate of climb. However, in practice, the specific fuel consumption, together with the output of the power plant, may vary with air temperature and Mach number and airspeed – Mach number schedule may be found that will optimize the climb for minimum fuel consumption. Any analysis of the economic benefits of a minimum fuel climb will depend on the direct operating costs of the aircraft and therefore, cannot be conclusive on performance grounds alone.

Noise limitations:

Transport aircraft are required to conform to stringent noise regulations on take-off and climb-out from airports; operators of aircraft that exceed the noise limits may be subjected to the penalties. To conform to the regulations it may be necessary to reduce the thrust or power in the after-take-off climb before the aircraft has reached the noise measuring station that will be positioned at a point under the departure path. The thrust reduction will reduce the rate, and gradient, of the climb and will extend the time taken to reach the cruising altitude. Engines with a low noise signature, or which do not require a large thrust reduction to comply with the noise regulations, will provide the aircraft with a better performance in the limb. This is; because they can operate at higher thrust levels without exceeding the noise limits.

Descent performance in aircraft operations:

If the propulsive thrust is less than the airframe drag then the aircraft will decelerate or descend, as can be seen from the generalized limb performance characteristics, Figs 3.5 and 3.6. The descending flight path can be varied from a shallow descent to a very steep descent either by reducing the engine thrust or by increasing the airframe drag. The drag can be increased either by aerodynamic means or by varying the airspeed. Thus, the aircraft has a very wide range of descent path profiles available to it. In the special case of gliding flight, in which there is no propulsive thrust, the descent will be determined by the lift – drag ratio, E . In this case, the minimum rate of descent occurs at the minimum power speed and the minimum gradient occurs at the minimum drag speed.

Although in Fig. 3.5 it is apparent that a descent can be produced by flying at airspeed less than the minimum drag speed, the aircraft will not have flight path stability in this condition. Flight path stability occurs when the flight path gradient can be controlled by the use of the elevator control only, Fig. 3.9. If the aircraft is flying at airspeed greater than the minimum drag speed then the flight path gradient of descent can be increased (steepened) by increasing the airspeed. This is achieved by a nose down pitch change with no need to adjust the engine thrust setting. Conversely, a decrease in descent gradient can be made by a nose up pitch change that will decrease the airspeed; this control can be achieved by using the elevator control alone. However, on the backside of the drag curve (that is, at airspeeds less than the minimum drag speed), the rate of change of drag with airspeed is negative and the flight path gradient cannot be controlled by the elevator alone. To maintain precise

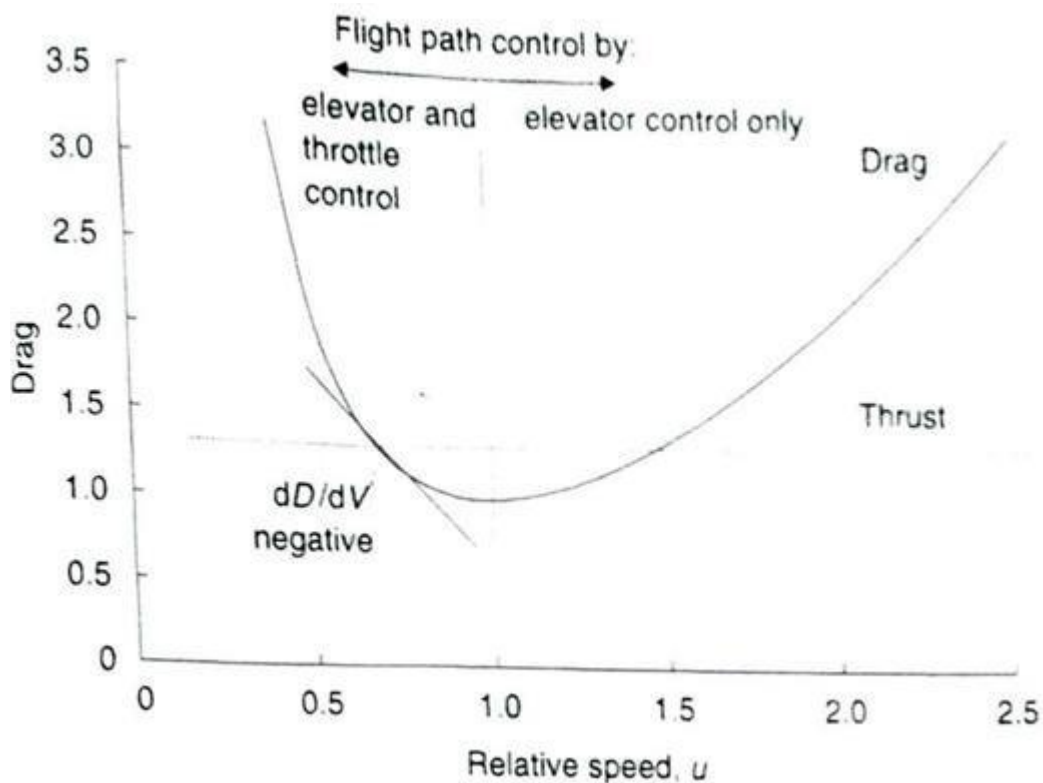


Fig. 3.9 Effect of minimum drag speed on flight path control

Control of the flight path gradient, changes in thrust setting will be necessary in addition to the elevator control inputs to control the descent gradient, otherwise; large excursions from the flight path will occur. Descents at airspeeds less than the minimum drag speed (or, in the case of aircraft with power-producing engines, below the minimum power speed), are generally to be avoided. Unless an auto-throttle is employed to maintain airspeed, accurate manual control of the flight path will be difficult.

In practice, limitations to the descent performance may be necessary. In transport operations it would be undesirable to make a very steep, high airspeed descent since this would entail a steep nose-down attitude that could be uncomfortable, if not dangerous, to persons in the cabin. In addition, the rate of increase of cabin pressure during descent must be kept to a reasonably low value to prevent discomfort due to the re-pressurization of the passengers' ear passages. The rate of change of cabin pressure should not exceed the equivalent of 300ft/min at sea level. This implies that, if the cabin is pressurized to the equivalent of 8000ft pressure height, the descent to sea level should take not less than 24 minutes regardless of the pressure height from which the aircraft commenced its descent. An exception to this general rule is the emergency descent following the loss of cabin pressure. In this case, the aircraft must descend to a safe altitude as quickly as possible and the highest rate of descent must be used.

The optimization of the descent is not as straightforward as the optimization of the climb. Since the engines will be operating at a low thrust or power, the fuels consumption will be low and the optimization based on fuel; consumed is not usually considered to be the most critical condition. Normally, the power plant will produce some propulsive force that will contribute to the performance equation and the aircraft cannot be considered to be in a true glide. A turbojet or turbofan engine will usually produce a residual thrust, even when operating at the flight idle setting. There will be a small, but not negligible, thrust contribution, which will reduce both the descent rate and the gradient that would occur in the glide. It is not

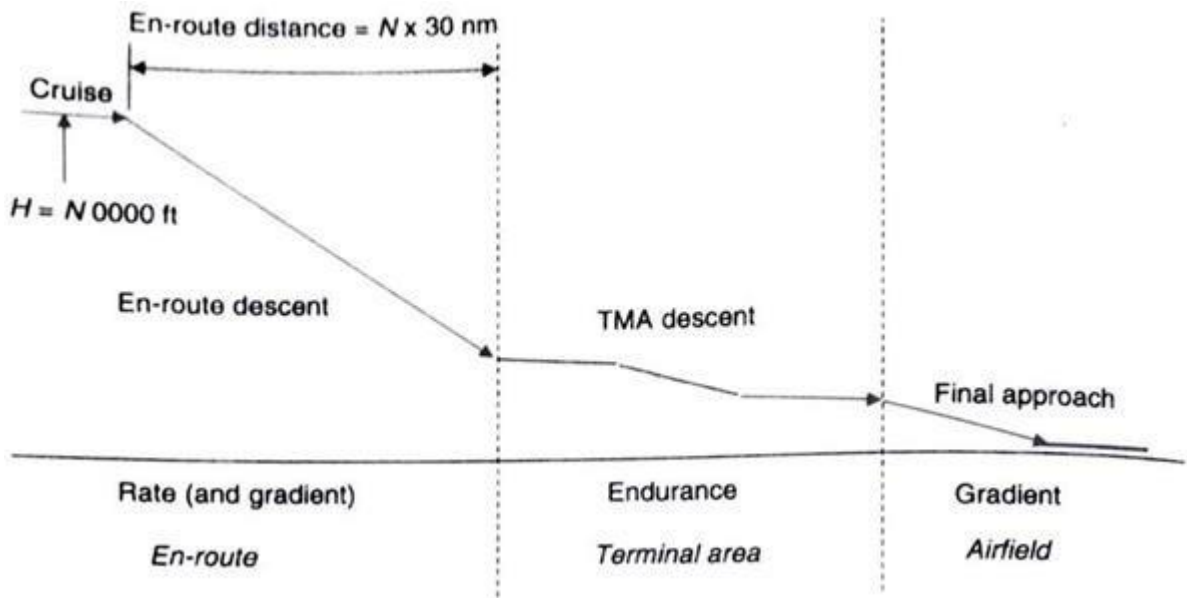


Fig. 3.10 Phases of descending flight.

Uncommon to have to increase the drag of the aircraft to enable it to descended at a sufficiently high rate to gradient. Flaps, spoilers, airbrakes and landing gear are all used as means of increasing the drag to obtain a suitable descent performance. Propeller driven aircraft can usually develop sufficient drag from the propeller to avoid the need for airbrakes or spoilers; the flaps and landing gear together with the propeller will normally produce a steep enough descent for all practical purposes.

The engine, however, must be operated in such a way that all essential aircraft systems remain fully operative. Pressurization must be maintained and generators, hydraulic pumps and engine air bleeds will need to provide the necessary output. Therefore, the minimum engine settings in the descent may be determined, for example, by the need to provide power for the anti-icing systems.

There are several phases of a typical descent flight path from cruising altitude down to landing. Each phase of the descent has different criteria that govern the manner in which the aircraft is flown. Figure 3.10 shows the phases of a typical descent.

From cruising height, the aircraft will descend towards the terminal area of the airfield in the en-route descent phase. In this phase, the descent will usually be made at a high airspeed and a rate of descent commensurate with the requirement for re-pressurization of the cabin, and engine power settings necessary to keep the aircraft systems operative. Typically, this phase may be flown by reference to a simple rule of thumb, such as commencing the descent at a distance of 30 nm from the destination for each 100 000 ft of altitude. This type of strategy will often produce a good working compromise between rate of descent and airspeed for large transport aircraft. The strategy could be improved by using a flight management system to provide a more exact optimization of the descent.

It would adjust the air speed and rate of descent to complete the descent to the required height over the reporting point at the required time. This is referred to as 4-D navigation since it combines area navigation with height and time. During then-route descent, it may be necessary to fly a schedule of Mach number and airspeed, as in the climb, to avoid the critical Mach number and stalling speed boundaries of the flight envelope.

From the boundary of the terminal area the aircraft will normally be subject to air traffic control restrictions that demand that it files at a given airspeed to maintain traffic separation as it is maneuvered onto the final approach. The descent is likely to be at a low rate and priority given to the positioning maneuvers. Since the airspeed is now constrained, the aircraft performance must be optimized by changes in its configuration. Flaps and other aerodynamic devices can be used to ensure that the necessary safety margins of airspeed are complied with and that the aircraft is being flown in the most economic manner. In this phase, the aircraft should be flown at a speed close to its maximum endurance speed for the best economy as it is maneuvered on to the final approach.

On the final approach, the gradient of the flight path is the main criterion. The gradient must be steep enough to exceed the slope of the minimum obstacle limitation surface, but not so steep that the flare to touchdown requires an excessive pitch attitude change. Typically, the gradient of the descent flown by large transport aircraft will be about 3° , which is equivalent to a 5% gradient. Smaller transport aircraft are often capable of using steeper final approach gradients for approaches into airports with restricted approach paths or to assist in the separation of airport traffic arrivals.

During the final approach to the landing, the aircraft will need to be flown at the lowest airspeed at which the safety margins can be met and at a pitch attitude that allows for a smooth flare and touchdown. In this phase of flight, the handling qualities must be such that the aircraft can be flown with accurate flight path control. This implies that the airspeed should not be less than the minimum drag speed to maintain flight path stability. The minimum drag speed is determined by the drag characteristic of the aircraft and by its weight above equation.

By increasing the zero-lift drag coefficient, C_{dz} , the minimum drag speed will be reduced although the overall drag force will be increased; doubling the zero-lift drag will usually decrease the minimum drag speed by almost 20%. The zero-lift drag increase can be produced by lowering the landing gear and flaps and by using airbrakes, spoilers or other devices specifically designed to produce high zero-lift drag forces. The final approach will usually be flown in a high drag configuration with landing gear extended, flaps fully extended and probably with airbrakes deployed. In this way, the minimum drag speed is reduced to its lowest value and the aircraft will have the necessary flight path stability at its minimum approach speed. A further benefit is that the

increased drag will require the engines to be operating at a fairly high thrust setting. In this state, should the aircraft have to abandon the approach and go around, the engines will respond quickly to the demand for maximum climb thrust?

At the same time, the high drag devices can be retracted and an excess of thrust over drag for the climb can be achieved in the minimum time. If the aircraft was operating at lower engine thrust without the high drag devices, the engine response time to achieve climb thrust would be much longer and the aircraft drag could not be reduced. This would lead to a delay in achieving the necessary climb gradient.

The emergency descent is used, for example, when the aircraft needs to descend rapidly to recover cabin pressure should the pressurization system fail. In this case, the descent to a safe altitude, below 10 000 ft, at which the ambient pressure of the atmosphere is high enough to breathe without the assistance of supplementary oxygen, must be made in the shortest possible time. Two strategies can be considered. First, a high airspeed descent using minimum thrust and with the aircraft in a clean configuration. This strategy will be limited by the maximum Mach number that can be achieved before the onset of Mach buffet or handling problems occur. If the cruising Mach number was already close to the limiting Mach number, the excess drag that would be achieved may not be sufficient to produce a high enough rate of descent.

Furthermore, the high airspeed may lead to restrictions on the maneuvering of the aircraft. Secondly, a low airspeed, high drag descent can be used. In this strategy, the aircraft must first be slowed down to an airspeed at which the flaps, landing gear and other high drag devices can be extended. The descent will then be made at the highest structural limiting airspeed and minimum thrust. In this case, time is lost in the deceleration process and the aircraft may have a very steep nose down attitude in the descent, which could cause difficulties in passenger to cargo restraint. There is no absolute rule for the emergency descent strategy, a procedure will have to be developed for each aircraft type that will minimize the descent time and keep the aircraft within its design limitations.

3.4. The effect of wind on climb and descent performance:

Wind is the relative velocity between the general air mass and the ground. Usually the wind can be assumed to have only a horizontal component of velocity. However, in close proximity to the ground, it will tend to follow the ground profile and so may have considerable vertical velocity components where sloping or undulating ground occurs, in some cases this effect may extend to considerable heights. For the purpose of this analysis, the aircraft will be taken to be operating over level ground so that only the horizontal component of the wind velocity will be considered. Close to the ground the relative velocity of the wind produces a boundary layer in which the wind speed decreases as height decreases, this will produce further effects on the flight path of the aircraft.

The flight path of the aircraft is calculated relative to the air mass and, so far, the development of the theory covering the climb and descent performance achieved by the aircraft has been assumed to occur in still air conditions. In a moving air mass the actual performance of the aircraft relative to the air mass is not affected since the reference axes for performance are velocity axes. Since their origin is at the centre of gravity (CG) of the aircraft and moves with the aircraft, it assumes zero velocity datum within the air mass. The aircraft will achieve the same rates and gradients of climb, and descent, relative to the air mass regardless of the wind. However, the performance relative to the ground will be affected by the wind. This is the perceived performance seen by the observer, from either the aircraft or the ground, and which affects the ability of the aircraft to clear ground-based obstructions.

Figure 3.11 shows the effect of a headwind or tailwind on the climb and descent gradients. The rate of climb, or descent, relative to the air mass is un-affected by the wind but the horizontal component of the true airspeed is increased by the tailwind

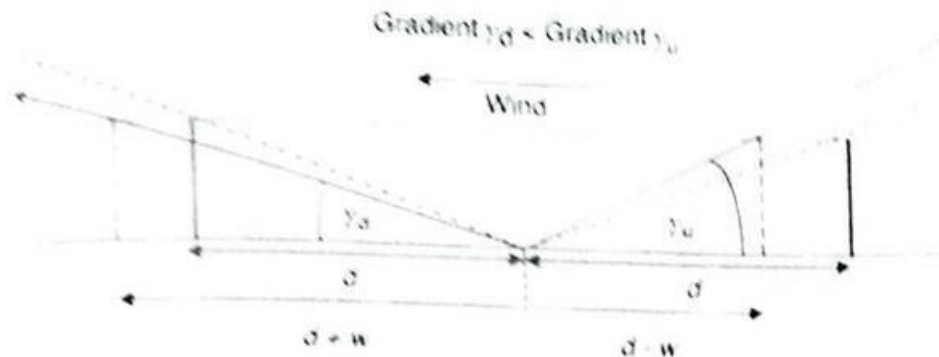


Fig 3.13 Effect of wind on climb gradient

Or decreased by the headwind the perceived gradient of the flight path relative to the ground thus becomes.

$$\gamma_p = 100 \tan \left[\frac{\sin \gamma_2}{\cos \gamma_2 - \frac{V_w}{V}} \right]$$

Where V_w is the headwind velocity component and γ_2 is the actual gradient of climb or descent relative to the air mass.

The perceived gradients are reduced relative to the actual gradients in a tailwind and increased in a headwind. Thus, if an aircraft climbs down wind, the ability to clear obstructions will be reduced although the aircraft is still producing its predicted climb gradient with respect to the air mass. The effects of wind on the perceived climb gradient has caused incidents to occur, particularly in cases where the air craft has encountered tail wind in a critical climb situation.

Spirally, the downwind approach is a well-established cause of landing incidents due to the reduction in perceived gradient of descent. The situation is made more difficult as the aircraft enters the boundary layer region in which there is a wind velocity gradient caused by the rate of change of wind-speed with height. Because of these effects, it is normal practice when taking the wind component into account to use a factor of 50% for the headwinds and 150% for tailwinds. This factor will be seen in the wind-speed correction in the take – off and climb performance of the aircraft performance manual.

An extreme case of the wind effect on the flight path is that of Wind-shear, in which the rate of change of wind velocity is very large. Wind shear is caused by severe meteorological conditions associated with rainstorm that create very strong local downdraughts, known as a microburst, which

separated out rapidly as they contact the ground. This results in localized hands in wind speed and direction that may be large and occur very suddenly. Since wind-shear occurs close to the ground, air-craft on final approach or making their after-take-off climb are particularly at risk.

On encountering wind-shear, a sudden change in airspeed will occur, together with a change in the flight path gradient through the mechanism described above. To recover to its former stage of flight the aircraft will need a repaid response in both engine thrust and single of attack which is very as bay be beyond the performance capability of the aircraft. Wind-shear warning systems, which sense changes in airspeed and vertical motion, can enable an early response to be made, which will minimize the effect of a wind-shear encounter.

MODULE IV

AIRCRAFT MANEUVER PERFORMANCE

Introduction:

An aircraft can be said to be in maneuvering flight when its flight path is in a continuous change of state and in which there is an inertial force due to acceleration. In Appendix B, it is shown that the inertial forces acting on the aircraft give rise to the statement of the accelerations acting in a general maneuver above equation. Usually the rate of change of aircraft mass can be neglected and the statement of the inertial forces can be expressed as,

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}_v = m \begin{bmatrix} \dot{V} \\ V\dot{\gamma}_3 \cos \gamma_2 \\ -V\dot{\gamma}_2 \end{bmatrix}_v$$

which describes the three linear accelerations that occur in maneuvering flight. These can be summarized as follows.

The linear acceleration, \dot{V} , arises from the imbalance of the forces in the direction of flight; this may be due to an excess of thrust or drag, or due to a component of weight in non-level flight. The linear acceleration is employed to control the airspeed in which thrust is increased or decreased to provide the necessary thrust – drag balance to achieve, or maintain, the required airspeed. When the aircraft is climbing or descending, the component of weight in the direction of flight will contribute to the accelerating force; thus, a rate of climb or descent can also be used to control the airspeed in non-level flight.

The lateral acceleration arises from the rate of turn, or rate of change of heading, $\dot{\gamma}_3$, which produces the centripetal force in a turning maneuver. The balancing centripetal force is provided by a component of the lift force by banking the aircraft into the turn. The effect of the lateral acceleration will be perceived as a normal force, g' force, acting on the aircraft during a turning maneuver.

The normal acceleration arises from the pitch rate of the aircraft, $\dot{\gamma}_2$, which produces the 'g' force experienced in a symmetric pull-up, or looping, maneuver.

The general maneuver produces a combination of these accelerations and the equations of motion for performance, developed in Appendix B, above equation, describing the general maneuver, can be expressed as,

$$\left. \begin{aligned}
 F_N - D - W \sin \gamma_2 &= m\dot{V} \\
 Y \cos \gamma_1 + L \sin \gamma_1 + T \{-\cos(\alpha + \tau_1) \sin \beta \cos \gamma_1 + \sin(\alpha + \tau_1) \sin \gamma_1\} &= mV\dot{\gamma}_3 \cos \gamma_2 \\
 Y \sin \gamma_1 - \cos \gamma_1 \\
 + T \{-\cos(\alpha + \tau_1) \sin \beta \sin \gamma_1 - \sin(\alpha + \tau_1) \cos \gamma_1\} + W \cos \gamma_2 &= -mV\dot{\gamma}_2
 \end{aligned} \right\}$$

In which the term for rather of change of aircraft mass as fuel is consumed has been omitted. The power plant thrust is expressed as net thrust F_N , and the total gross thrust component, T (Appendix B, above equation).

In a coordinated maneuver the side force, Y , and the sideslip angle, B , are both zero and it is assumed that themanoe4uveres are coordinated. The equations of motion can then be reduced to the form.

$$\left. \begin{aligned}
 F_N - D - W \sin \gamma_2 &= m\dot{V} \\
 Y \cos \gamma_1 + L \sin \gamma_1 + T \{-\cos(\alpha + \tau_1) \sin \beta \cos \gamma_1 + \sin(\alpha + \tau_1) \sin \gamma_1\} &= mV\dot{\gamma}_3 \cos \gamma_2 \\
 Y \sin \gamma_1 - \cos \gamma_1 \\
 + T \{-\cos(\alpha + \tau_1) \sin \beta \sin \gamma_1 - \sin(\alpha + \tau_1) \cos \gamma_1\} + W \cos \gamma_2 &= -mV\dot{\gamma}_2
 \end{aligned} \right\}$$

Where the term $(L + T \sin(\alpha + \tau_1))$ represents the total normal force as the sum of the lift and the normal component of gross thrust from the engines.

The load factor, n , which characterizes the g' force, is defined as the ratio of the overall normal fore produced by the air4craft to the aircraft weight, thus

$$n = \frac{L + T \sin(\alpha + \tau_1)}{W}$$

This definition of the load factor allows the normal component of the gross thrust from the power plant to be taken into account during a maneuver. This enables the equations of motion to be used to analyze the performance of vectored thrust aircraft and aircraft maneuvering at a very high angle of attack. However, in the case of conventional aircraft, with lift – drag ratios of 10 or more, operating at angles of attack up to about 10° , and having little or no downward thrust deflection, the thrust component is small enough to be neglected. The load factor can then be taken to be the ratio of the aerodynamic lift fore to the aircraft weight for all practical purposes, thus the approximation can be used.

$$n = L/W$$

Substituting above equations gives the equations of motion for coordinated flight.

$$\left. \begin{aligned} [F_N - D] \frac{1}{W} - \sin \gamma_2 &= \frac{\dot{V}}{g} \\ n \sin \gamma_1 &= \frac{V}{g} \dot{\gamma}_3 \cos \gamma_2 \\ -n \cos \gamma_1 + \cos \gamma_2 &= -\frac{V}{g} \dot{\gamma}_2 \end{aligned} \right\}$$

in this form, the equations can be used to develop the basic expressions for co-ordinate maneuvers.

The maneuver envelope:

The maneuver performance of the aircraft will be limited by the structural strength of the airframe; there are two basic reasons for this.

The pressure loading produced by the dynamic pressure of the airflow increases with the square of the airspeed and with it the air loads on structural components of the aircraft. This is particularly obvious in the case of the deflection of devices such as flaps or landing gear into the airstream. The design maximum dynamic equivalent airspeed, EAS. The normal acceleration associated with maneuvering flight produces structural loads in the airframe. The maximum allowable load factor in a maneuver is determined by the load bearing capability of the airframe structure.

These limitations do not always apply universally. Since the configuration of the airframe may be changed for certain parts or the flight, for example by the lowering of the landing gear or the deflection of flaps, the structural loading and strength limits may be affected. Therefore, different maximum airspeed and load factor limitations may exist for each configuration. The design maneuver envelope, or n-V diagram, describes the design limitations on airspeed and load factor.

The structural strength of the airframe must be capable of sustaining the structural loading generated by flight maneuvers and gusts at a given aircraft weight and by the dynamic pressure of the airflow at the maximum permissible airspeed. The load factor limits and maximum airspeeds depend on the role of the aircraft.

A military combat aircraft requires a high level of maneuverability and, therefore, the capability of sustaining a high load factor. In addition, it will be called upon to perform the maneuvers over a wide range of airspeeds. These requirements lead to the need for a strong airframe to withstand the combined aerodynamic and maneuver loading; this implies that the airframe will be relatively heavily constructed.

A large, subsonic, transport aircraft need only low maneuverability and can meet the entire maneuver and gust loading requirements with a relatively low load factor.

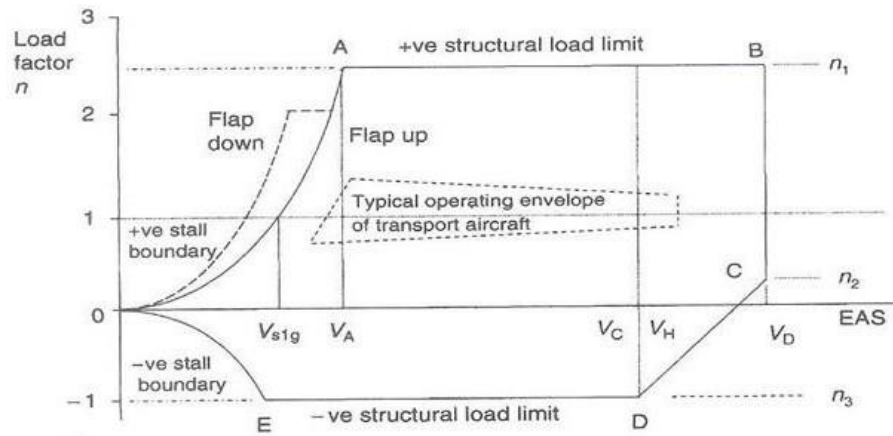


Fig. 4.1 The maneuver envelope

The airspeed is likely to be limited by a sub sonic Mach number, which may enable a relatively low maximum EAS to be scheduled. These considerations lead to a much lower structural strength requirement and, consequently, a lighter airframe construction.

Fig 4.1 shows the main elements of a typical maneuver envelope. Although the maneuver envelopes for civil and military aircraft differ in detail, the definitions of the boundaries and the principal airspeeds are similar enough to be generalized.

The structural boundaries:

The load factor limits, n_1 and n_3 , are the maximum positive and negative normal acceleration loadings respectively.

The positive load factor, n_1 (boundary A – B in Fig. 4.1), is defined by requirements for the aircraft and the minimum load factor limit is laid down by the airworthiness codes of practice. The requirement for a high structural strength of the air frame increases the weight of the aircraft. Therefore, the maximum load factor needs to be kept as low as possible, commensurate with the ability of the aircraft to maneuver and to be able to sustain the loads induced by gusts.

The positive load factor limit, n_1 , is defined in FAR/JAR as a function of aircraft weight and is given by the expression.

$$n_1 = 2.1 + \frac{24000}{0.225W + 10000}$$

When W is the weight of the aircraft in Newton's. However, n_1 need not exceed the range $23.5 < n_1 < 3.8$ for normal category aircraft, which includes large transport aircraft.

For aircraft in the utility category, n_1 is taken to be +4.4, and for aerobatic aircraft, n_1 is taken to be +6.0. The maximum n_1 for military aircraft or -0.5 n_1 for aerobatic aircraft. In the case of large transport aircraft, n_3 is usually taken to be not less than -1/0 at speeds up to V_c decreasing to zero at V_d . For military aircraft the value of n_3 is taken to be $-0.6(n_1 - 1)$ up to V_h decreasing to $n_2 = 1 - 0.3n_1$ at V_{Gd} .

The airspeed boundaries:

The airspeed limits, which are defined as equivalent airspeeds, are determined by the stall boundaries, O-A and O-E, and by the design diving speed, V_d , B – C.

The stalling speed or the minimum airspeed at which the aircraft can maintain steady flight in a specified configuration, forms the low-speed boundary of the maneuver envelope. Since the lift force is a function of the load factor, the stalling speed is defined under steady, level, straight flight conditions to be V_{s1} or V_{s1g} , the 1g stalling speed. The minimum, practical, airspeed may be set by the stall buffet, which is caused by the initial separation of the airflow as the stalling speed is approached. Stall boundaries will usually be shown for flaps up and flaps down cases.

The high-speed boundary is determined by the maximum structural dynamic pressure loading, $q = \frac{1}{2} \rho V^2$ which sets the design diving speed of the aircraft, V_d . Since V_d is defined in terms of equivalent airspeed, the Mach number associated with it will increase with altitude and may further limit the maneuver envelope. The high-speed boundary may be quoted as the lower of either V_d or M_d .

Other notable speeds within the maneuver envelope are as follows.

V_a , the design maneuvering speed,

where $V_a = V_{s1g} \sqrt{n_1}$ this is the minimum air speed at which the aircraft can achieve the maximum positive load factor in a steady maneuver.

V_{dc}/M_c , the design cruising speed or Mach number (civil transport aircraft), this is the normal cruising speed at which the structure can sustain the maximum load factors; V_c defines the corner D of the civil aircraft maneuver envelope.

V_h , the maximum speed in level flight with maximum continuous power, this airspeed defines the corner D of the military aircraft maneuver envelope.

(The relationships between these speeds depend on the size and purpose of the aircraft and are defined in more detail in the airworthiness code of practice under which the aircraft is to be certificated.)

In normal, civil, aircraft operations, a large transport aircraft uses only a portion of the available maneuver envelope, as indicated in Fig. 4.1. Operational speeds are usually limited by a safety margin over the stalling speed and by the cruise Mach number, turns do not generally exceed 30° bank angle so that load factors due to maneuver will rarely exceed 1.2g.

The longitudinal maneuver:

The longitudinal maneuver is the result of an imbalance of thrust and drag, which results in either a linear acceleration or a steady rate of climb, or in a combination of both acceleration and climb, in the direction of flight.

It does not involve directly the accelerations that result from rates of pitch or turn, although those maneuvers may produce increase in the drag force, which will have an indirect effect on the longitudinal force balance.

By expressing the gradient of climb in terms of the true rate of climb and true airspeed the longitudinal equation of motion for maneuvering flight can be written as,

$$[F_N - D] \frac{V}{W} = \frac{d}{dt} \left\{ H + \frac{V^2}{2g} \right\}$$

The term $(H + V^2/2g)$ is the specific energy, E_s of the aircraft per module weight. It is also known as the energy-height since it represents the height the aircraft would attain if all the kinetic energy were to be converted into potential energy.

The term $(F_N - D)V/W$, the product of the excess thrust and the true airspeed per module weight, is known as the specific excess power (SEP), of the aircraft and determines the rate of change of the specific energy.

The excess power can be used to increase potential energy (climb), or to increase the potential and kinetic energies in combination to achieve the maximum rate of change of total energy, the sum of the PE and KE, to minimize the time required to climb and accelerate the aircraft to its operating height and Mach number.

This principle is employed by high performance aircraft in the optimization of their climb profile through the transonic flight region where the excess power is reduced by the increase in drags. This is also discussed in under the climb performance of aircraft with a high excess thrust.

Any change in the specific excess power arising from an increment in either the thrust or the drag will produce either a rate of climb or an acceleration of the aircraft. If height is maintained constant then the airspeed will vary or, conversely, if the (true) airspeed is maintained constant the height will vary. This principle is important in the consideration of the overall effect of a maneuver on the flight path of the aircraft.

The lateral maneuver or the level turns:

In a level, constant airspeed, coordinated turn, the rate of climb, y_2 , the rate of pitch, y_2 , and the rate of change of true airspeed, V , are all zero and the equations for maneuver performance become.

$$\left. \begin{aligned} [F_N - D] \frac{1}{W} &= 0 \\ n \sin \gamma_1 &= \frac{V}{g} \dot{\gamma}_3 \\ -n \cos \gamma_1 + 1 &= 0 \end{aligned} \right\}$$

It should be noted that the bank angle used in the equations of motion is γ_1 , and the aircraft body axes relative to velocity axes, and not the Euler angle, the body axes relative to Earth axes. This enables the analysis to be applied in the general maneuvering case in which the aircraft may be in a maneuver combining g both turning and pitching motions.

The turn is shown diagrammatically in Fig 4.2. From the normal force equation the load factor, n , is seen to be a function of the bank angle of the aircraft and is given by,

$$n = \frac{L}{W} = \frac{1}{\cos \gamma_1}$$

From the lateral force equation the rate of turn is given by,

$$\dot{\gamma}_3 = \frac{V}{R} = \frac{ng \sin \gamma_1}{V} = \frac{g \tan \gamma_1}{V}$$

Where R is the radius of the turn which, using above equation, can be expressed as,

$$R = \frac{V^2}{g \tan \gamma_1}$$

From this expression, it can be seen that both the rate and radius of the turn are functions of true airspeed and bank angle only and are independent of the weight of the aircraft.

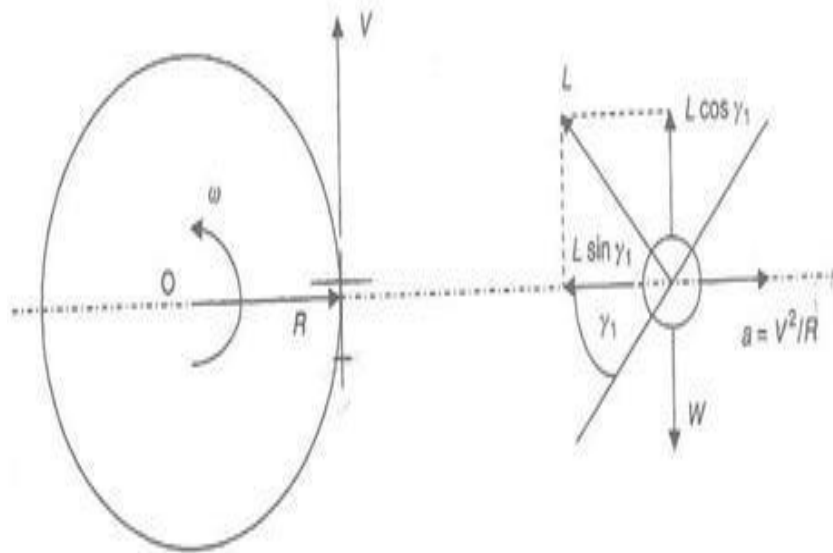


Fig.4.2 The turning maneuver

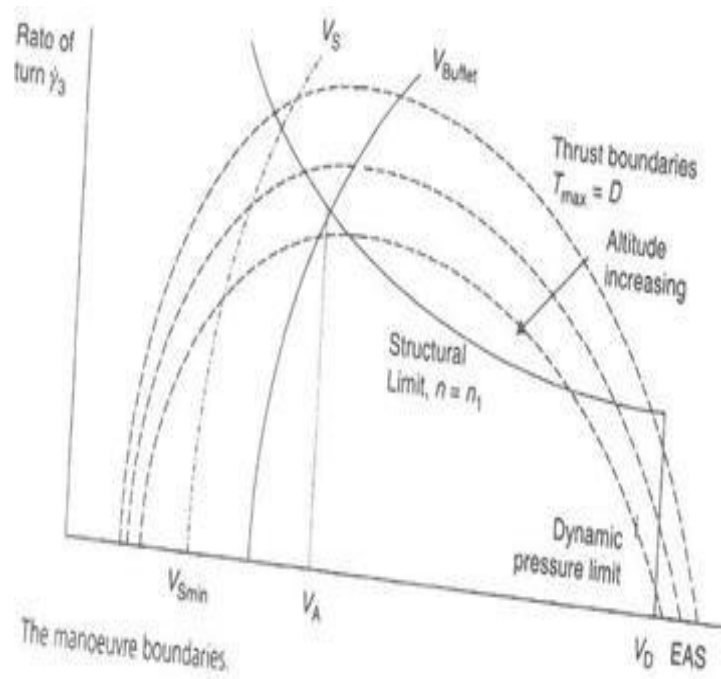


Fig. 4.3 The maneuver boundaries

The turning performances will be constrained by the aerodynamic and structural limitations of the aircraft, together with the limit imposed on the turning maneuver by the excess thrust available. These are defined by the maneuver envelope, and are shown in Fig. 4.3 in terms of the rate of turn, $\dot{\gamma}$, and EAS.

From above equation the maximum bank angle in the steady, level turn is determined by the limit load factor, n_1 . Therefore, from above equation, the maximum rate of turn permitted by the structural strength of the aircraft. The boundary will tend to move downwards as the air density decreases with increasing altitude and temperature. The maximum angle of attack at which the aircraft can be flown in steady flight is limited by the stall (or stall buffet) boundary, this will determine the minimum airspeed in the steady, level turns. From the lift equation in turning flight, the airspeed and load factor are related by the expression.

$$\frac{V_e^2}{n} = \frac{2W}{\rho_0 S C_{L_s}} = \text{constant}$$

Where C_{L_s} is the maximum steady, level-flight, lift coefficient.

The rate of turn is a function of the airspeed and load factor, above equation, so that there will be a minimum airspeed boundary set by the value of determined from above equation. The maximum airspeed boundary is set by the maximum design speed, V_d , which is a structural strength limitation determined from the maximum dynamic pressure loading. However, there may be further restrictions to the high speed boundary caused by the maximum design Mach number, M_d , and set by the aerodynamic limitation associated with changes in airflow characteristics leading to stability and handling qualities issues.

The maximum excess thrust power limitation is found from above equations in turning flight. The maximum non-maneuvering excess thrust-power is given from above equation and is reduced by the increment in drag-power resulting from the rate of turn; above equation the curves in Fig. 4.3 represent the maximum level flight maneuver boundaries for increasing WAT limits.

Figure 4.3 shows the limit of maneuver of the aircraft under these constraints. The maximum rate of turn occurs at V_a , the design maneuver speed. However, it should be noted that in steady, level flight the limiting maneuver is only possible if there is sufficient propulsive thrust to overcome the aerodynamic drag. The aircraft will be further limited in the maneuver by the thrust available if it is climbing. In the design of a combat aircraft the thrust and drag characteristics need to be developed to provide the maximum excess thrust-power as close as possible to the design maneuver speed. This is to avoid the thrust available limiting the turning performance as the most critical maneuver point.

The increase in the drag force in maneuvering flight can be found from the drag characteristic of the aircraft. The drag polar of a conventional, subsonic, aircraft is assumed parabolic so that the drag coefficient can be expressed in the form.

$$C_D = C_{Dz} + KC_L^2$$

And the drag force in level flight is given by,

$$D = \frac{1}{2}\rho S C_{Dz} V^2 + \frac{KW^2}{\frac{1}{2}\rho S} \frac{1}{V^2}$$

In the turn, the load factor of the aircraft is increased so that the lift generated by the wing will have to be increased to balance the forces. If the engine thrust component in the normal axis is negligible, the drag force becomes.

$$D = \frac{1}{2}\rho S C_{Dz} V^2 + \frac{Kn^2 W^2}{\frac{1}{2}\rho S} \frac{1}{V^2}$$

Thus, the increment in drag, D, due to the turn is the difference between equations, and

$$\Delta D = \left[\frac{KW^2}{\frac{1}{2}\rho S} \right] \frac{n^2 - 1}{V^2}$$

Which, from above equations, can be written in the form?

$$\Delta D = \left[\frac{Km^2}{\frac{1}{2}\rho S} \right] \dot{\gamma}^2$$

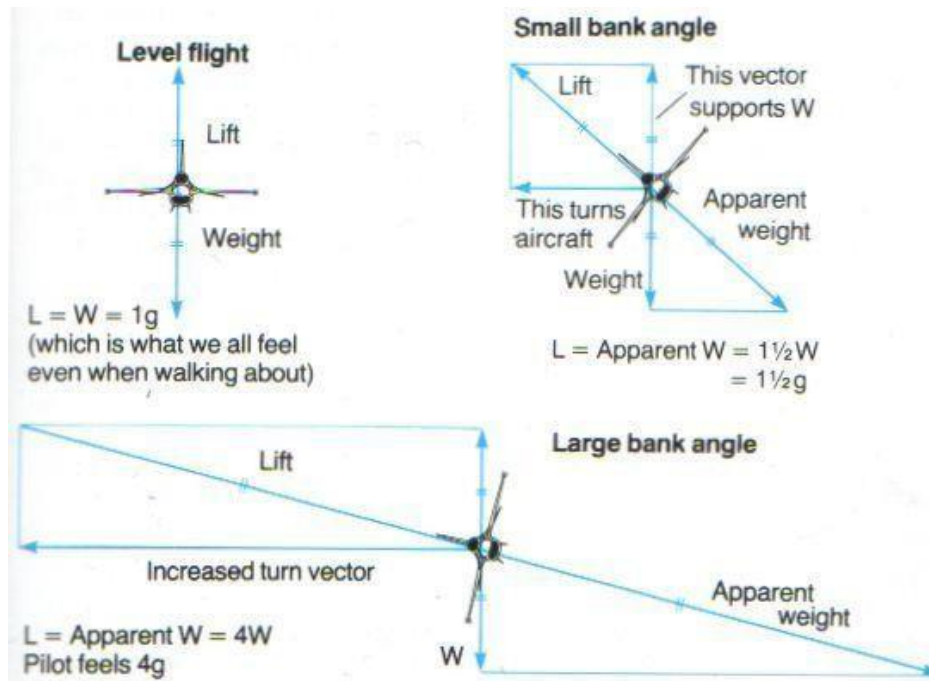
Here, the square bracket is a quasi-constant for the aircraft in the turn at a given weight and height.

The increment in drag is thus seen to be proportional to the square of the rate of turn, Now, in the turn, the drag increases and, if the thrust is not increased to compensate, the drag increment D will lead to either a rate of descent or a deceleration relative to the non-turning flight state. This can be deduced from above equation.

$$-\Delta D \frac{V}{W} = \Delta \frac{d}{dt} \left\{ H + \frac{V^2}{2g} \right\}$$

In turning flight, the increase in the lift dependent drag coefficient leads to a modified expression for the minimum drag ratio, this is now given by,

$$\frac{D}{D_{\min}} = \frac{1}{2} \left(u^2 + \frac{n^2}{u^2} \right)$$



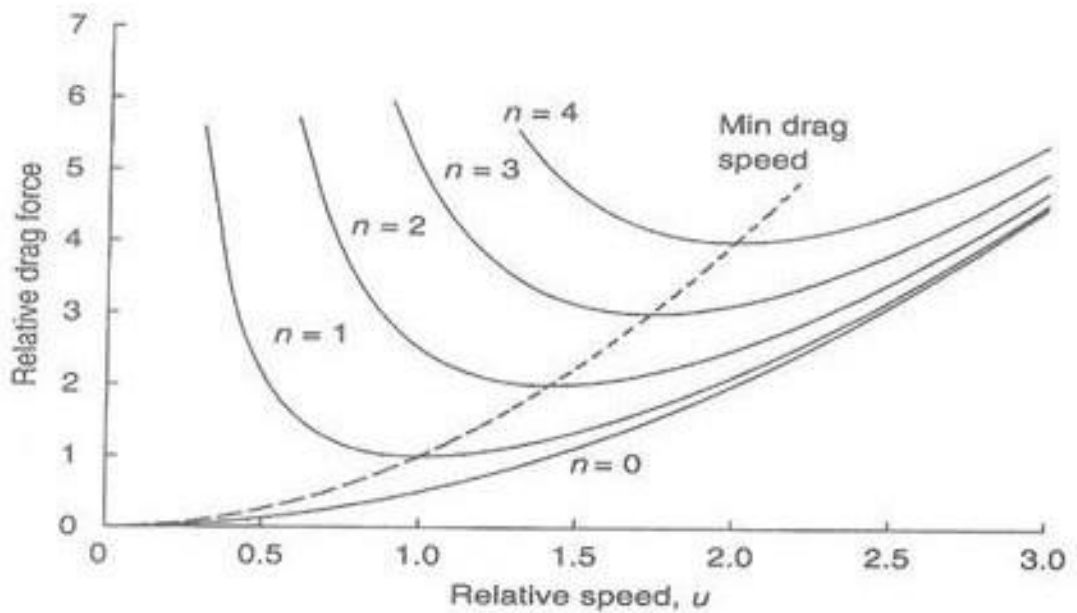
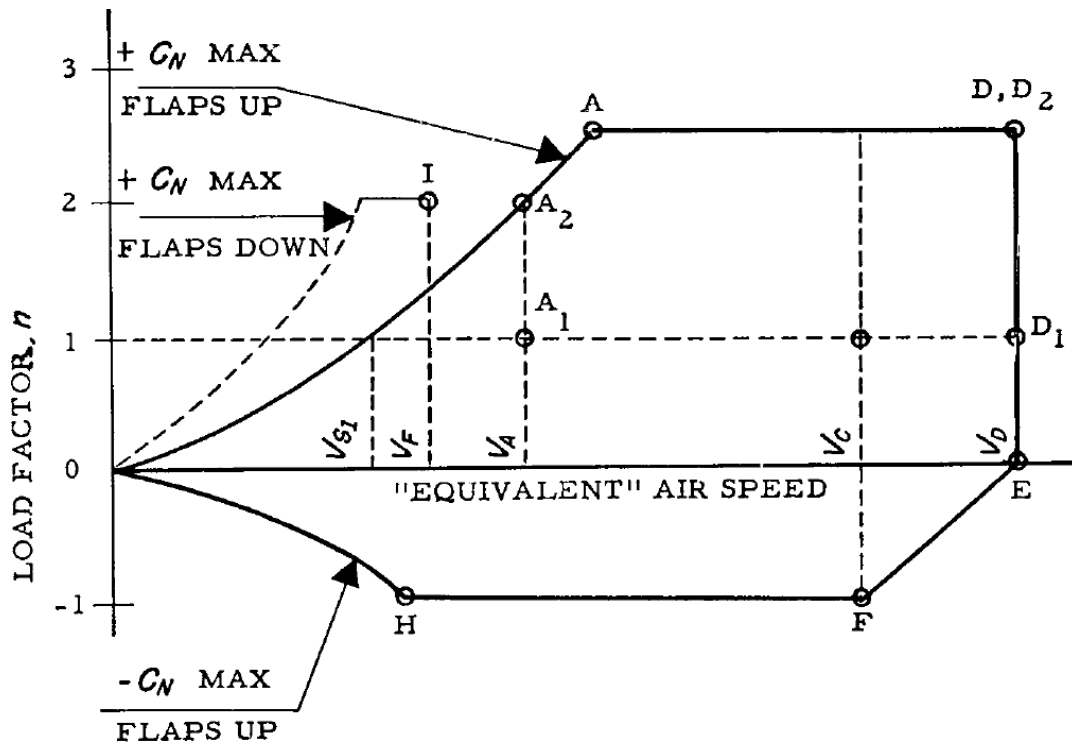


Fig. 4.4 The effect of load factor on the drag

characteristic And the minimum drag speed in the turn $V_{md(turn)}$ is given by,

$$V_{md(turn)} = \left(\frac{2W}{\rho S C_{Lmd}} \right)^{\frac{1}{2}} n^{\frac{1}{2}} = V_{md} n^{\frac{1}{2}}$$

So that the minimum drag speed increases as a function of the rate of turn. Figure 4.4 shows the increase in drag and in the minimum drag speed with load factor as the rate of turn is increased.

Unless the thrust is increased to compensate for the increased drag then the turn will cause the specific energy of the aircraft to decrease. If airspeed is maintained, then a rate of descent will occur or, if height is maintained, then the aircraft will decelerate. If the turn is initiated at an airspeed sufficiently above the minimum drag speed the airspeed will decrease, reducing the drag until the force equation is re-balanced and the level turn will continue at the lower airspeed.

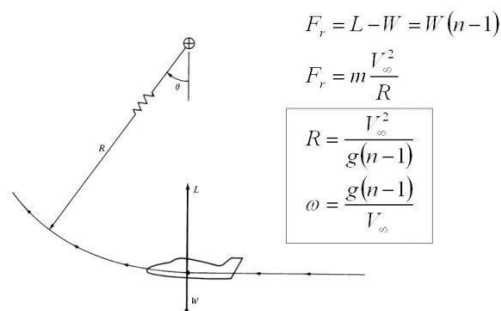
However, if the initial airspeed is close to or below the minimum drag speed, then any decrease in airspeed will lead to a further increase in drag and a consequent increase in the rate of loss of airspeed. If the thrust available is limited then the maximum airspeed in the level turn will decrease as the turn is tightened until the aircraft is at its minimum drag speed with maximum available thrust. At that point, the aircraft is performing its tightest, constant speed, level turn.

These effects can be very important in climbing turns with very little excess thrust available, for example, the after-take-off climb with one engine inoperative (Chapters 5 and 9). In such cases, the additional drag due to a turn can reduce the climb gradient to an unacceptably low level or even to a descent.

The pull-up maneuver or the loop:

The pull-up maneuver is a coordinated maneuver in the vertical or pitching, plane with no rate of turn or sideslip so that, the equations

EXAMPLE: PULL-UP MANEUVER



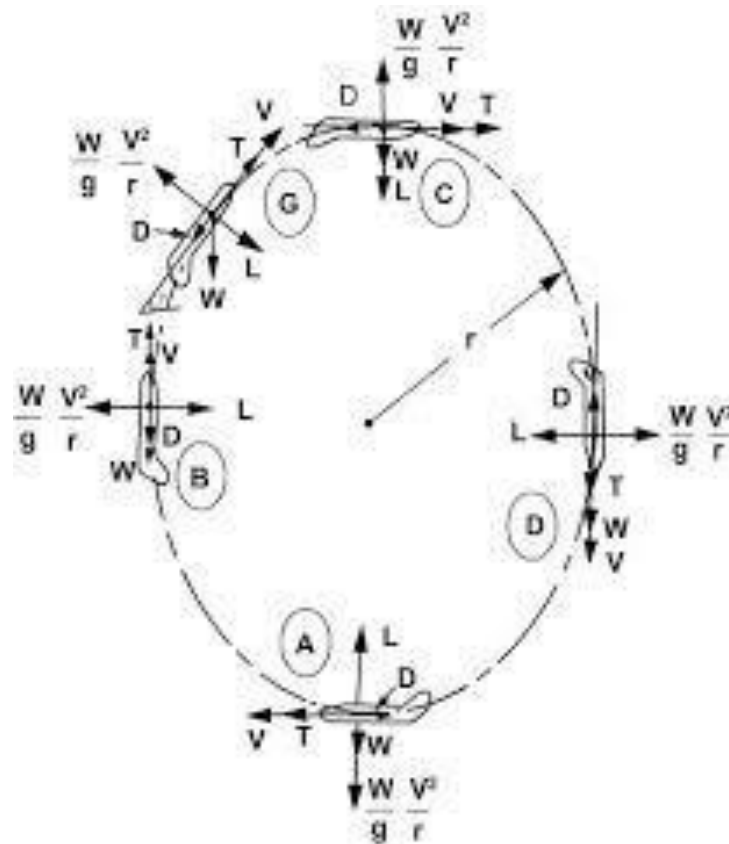
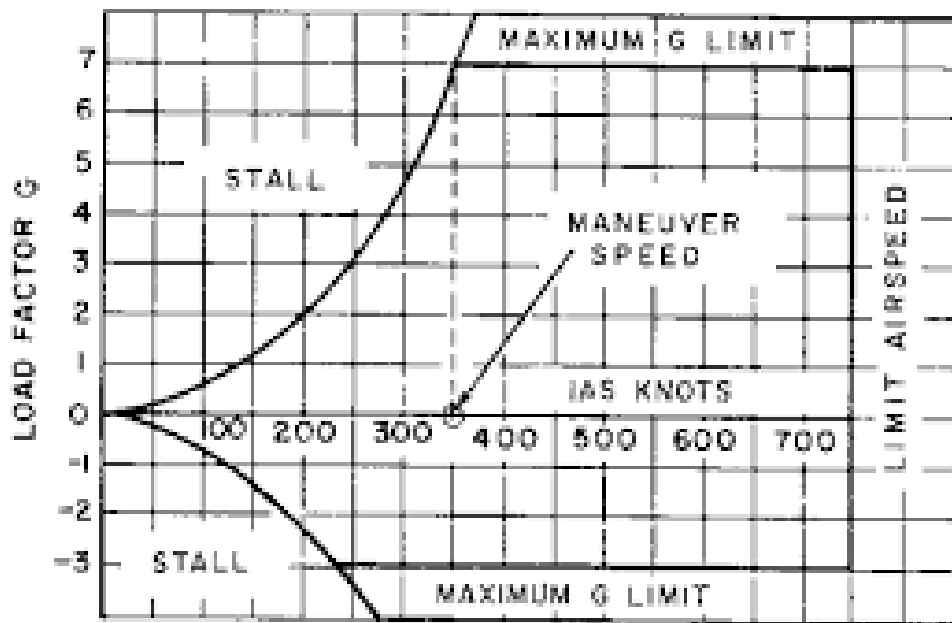


Figure 4.5 The load factor in the pull-up maneuver

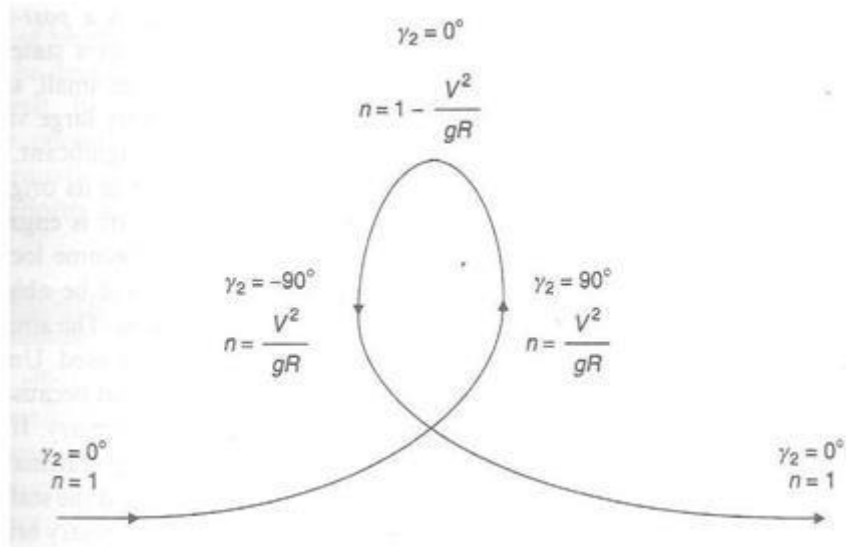


Fig.4.6 The load factor in the pull-up maneuver

Of motion thus become,

$$\left. \begin{aligned} F_N - D - W \sin \gamma_2 &= m\dot{V} \\ Y &= 0 \\ -L - T \sin(\alpha + \tau_1) + W \cos \gamma_2 &= -mV\dot{\gamma}_2 \end{aligned} \right\}$$

Using the definition of the load factor, above equation, and writing the instantaneous rate of pitch as V/R , where R is the radius of the loop, leads to the load factors in the pull-up maneuver given by,

$$n = \cos \gamma_2 + \frac{V^2}{gR}$$

And the radius of the maneuver given by,

$$R = \frac{V^2}{g(n - \cos \gamma_2)}$$

The pull-up maneuver is shown in Fig.4.5

The load factor in the loop is not uniform and will vary with airspeed and flight path angle as the aircraft progresses around the loop, Fig. 4.6. In practice, the variation is complex since the increased load factor increases the drag force, above equation, which, together with the weight component, affects the balance of the longitudinal forces acting on the aircraft. This causes a continuous change in airspeed throughout the maneuver.

To control the airspeed within acceptable limits the engine thrust must be increased in the upward segment of the loop and reduced in the downward segment, thus the loop cannot be regarded as a steady maneuver. Similarly, the radius of the loop is not uniform but tends to decrease to a minimum at the top of the maneuver and increase again on the descending path. Beyond aerobatic flight and some military aircraft combat maneuvers, there are few practical applications of the extended pull-up, or looping, maneuver.

An extreme case of the pull-up maneuver is the 'Cobra' which is a post-stall maneuver involving a rapid pitch-up to increase in angle of attack to a state far beyond the stalling angle of attack. In this state, the lift force becomes small, since the aircraft is in the stalled condition, but the drag increases to a very large value and acts in the wind axis direction. Since the lift force is no longer significant, the aircraft will not enter a looping maneuver but will tend to continue in its original direction of flight together with a rapid deceleration.

When an aircraft is engaged in combat with another aircraft of similar performance, they may become locked into a circular tail-chase, each turning at maximum rate. Neither will be able to tighten the turn to bring its adversary into line of sight to fire its weapons. The aircraft will re-burn fuel at a high rate.

Particularly if afterburners are being used, unless the statement can be broken one aircraft will have to break off the combat because its fuel state is becoming critical – that aircraft is then at risk from its adversary. If the aircraft has suitable aerodynamic characteristics, a means of breaking the stalemate is for the aircraft to be pitched up to a very large angle of attack, far beyond the stalling angle of attack, in the 'Cobra' maneuver. This enables it to bring its adversary briefly into line of sight for the weapons to be fired.

The angle of attack in the Cobra may be greater than 70° and although the aircraft is described as flying at post-stall angle of attack' this is an overstatement. The aircraft may be stable in this condition (inasmuch as it does not have a tendency to rotate uncontrollably about any of its axes), its engines may continue to produce thrust and it may possess some degree of controllability. However, it will not be producing aerodynamic lift and so cannot be described properly as flying'.

It will produce a very large drag force, which will cause the aircraft to lose energy at a very high rate. This may assist it to escape retaliatory attack since its adversary is likely to over-fly the rapidly decelerating aircraft before it can aim and fire its weapons. Once the Cobra has been performed, the loss of total energy may have left the aircraft unable to resume normal flight without a great loss of height. It is an extreme maneuver that normally would only be used where no alternative escape is available.

In combat missions, turning maneuvers are often combined with an extended pull-up and most maneuvers, whether attacking or evasive, will contain rates of turn and pitch as well as longitudinal acceleration. If the maneuver is coordinate, above equation can be used to analyzed the combined maneuvers including those of aircraft with vectored thrust. These aircraft have the ability to rotate the gross thrust vector in their Oxz plane so that the normal force consists of the aerodynamic lift and a large component of the engine gross thrust. If this facility is used in flight, the thrust component forms part of the normal force vector and some of the assumptions made in the development of the expressions for turning flight no longer apply. The rates and radii of turn will no longer be restricted by the maximum lift coefficient.

Some aircraft with thrust vectoring capability are able to perform a similar maneuver to the Cobra to avoid attack from behind by vectoring thrust in forward flight, known as 'Viffing'. In this case, the engine nozzles are rotated downwards together with some pitch-up of the aircraft. The result is a rapid deceleration that cannot be matched by the attacking aircraft, which is likely to overtake the decelerating aircraft before it can aim and fire its weapons. Since the vectored thrust aircraft must not need to have been pitched up to an angle of attack beyond the stall, it may be able to recover normal flight relatively quickly without great loss of height. This is an example of the way in which tactical combat maneuvers can be developed to take advantage of the performance characteristics of a combat aircraft. By using the aerodynamic and propulsive characteristics of the aircraft type, offensive and evasive maneuvers peculiar to that type can be developed from the equations of motion for maneuver and used to give that aircraft an area of air superiority in combat.

Transport aircraft maneuver performance:

The effect of maneuvers on the flight path performance of civil transport aircraft is generally not very significant. They spend only a very low proportion of their time in turning flight and since the maximum bank angle used in the turn is typically 20° , the turns are generally of very low rate. The effect of such turns on the overall performance is minimal. En-route turns, which are generally associated with heading changes, are usually made through angles of less than 90° . If the aircraft is stacked in the hold during the descent to landing it is required to fly an oval flight path, with a 180° turn at each turn, known as the holding pattern.

The holding pattern turn is flown at Rate 1, or $3^\circ/s$, and takes one minute. This is probably the most sustained turn that the aircraft will be required to carry out in normal operations and will be flown at a speed commensurate with flight safety and air traffic control considerations. This speed should be losing to the maximum endurance speed if possible.

The rate and radius of the turns made by aircraft with differing airspeeds can be used to advantage in control of air traffic. The smaller regional aircraft, often turbo-props, can fly at lower speeds than the big jets and can use bank angles up to 30° compared with the normal maximum of 20° in the case of the big jets. This enables the regional aircraft to turn at a higher rate and with a smaller radius. These differences in performance can be used to aid the separation and flow of traffic in airport terminal areas.

The only case in which the effect of turning would be significant to a transport aircraft, in terms of its effect on flight safety, is a turn made with marginal excess thrust available. For example, during

the after-take-off climb with one engine inoperative, the additional drag due to the turn may decrease the already small gradient of climb. Similarly, the sustained pull-up is not a maneuver associated with civil transport operations. The most significant pitching maneuvers, other than the transition at take-off and the flare on landing, will be transient maneuvers between the limb and cruising states or between cruising and descending flight.

Military aircraft maneuver performance:

Military transport aircraft may be required to maneuver more aggressively than civil aircraft and sustained turning performance may be significant in the operational profile of the mission. Although pure pull-ups are still unlikely to be more than transient maneuvers in path, they may extend, in the case of tactical transport aircraft, to pop-up⁶ maneuvers from low-level flight that would demand a higher rate of pitch than the simple gradient changing maneuver. The increased fuel burned due to the turning maneuvers may need to be accounted for in the analysis of the fuel requirement for the mission.

The combat aircraft (or the aerobatic aircraft) spends a significant proportion of its mission time in maneuvering flight and the design criterion for the aircraft is that it will be capable of sustaining high-g maneuvers with high rates of turn. In the design phase of the development of the aircraft, estimates will be needed of the thrust required to achieve the design target maneuvers combining turning with pull-ups and acceleration will also need to be considered to establish the limits of the aircraft in air-to-air combat situations.

The performance in these maneuvers will be part of the design specification of the aircraft. However, in service combat maneuvers will be developed from combinations of the three basic maneuvers to take advantage of the performance equalities peculiar to the aircraft. It is unlikely that such maneuvers could be foreseen at the design stage, but their development in service through experience of the aircraft, is a necessary phase of the maintenance of the aircraft air superiority.

MODULE V

SAFETY REQUIREMENTS- TAKE-OFF AND LANDING AND FLIGHT PLANNING

Take-off performance:

In the conventional take-off maneuver, the aircraft is accelerated along the runway until it reaches a speed at which it can generate sufficient aerodynamic lift to overcome its weight. It can then lift off the runway and start its climb. During the take-off, consideration is given to the need to ensure that the aircraft can be controlled safely and the distances required for the maneuvers do not exceed those available. The take-off maneuver is shown in Fig.5.1 and some of the principal speeds and events described.

The aircraft starts the take-off at rest on the runway, take-off thrust is set and the brakes released. The excess thrust accelerates the aircraft along the runway and, initially, the directional control needed to maintain heading along the runway would be provided by the nose-wheel steering. This is because the rudder cannot provide sufficient aerodynamic yawing moment to give directional control at very low airspeeds.

As the airspeed increases the rudder will gain effectiveness and will take over directional control from the nose-wheel steering. However, should an engine fail during the take-off run the yawing moment produced by the asymmetry loss of thrust will have to be opposed by a yawing moment produced by the rudder. There will be an airspeed below which the rudder will not be capable of producing a yawing moment large enough to provide directional control without assistance from either brakes or nose-wheel steering or a reduction in thrust on another engine.

This airspeed is known as the minimum control speed, ground, V_{mcg} ; if an engine failure occurs before this airspeed is reached, the take-off run must be abandoned.

During the ground run the nose wheel of the aircraft is held on the runway to keep the pitch attitude, and hence the angle of attack in the ground runs, low. This will keep the lift produced by the wing to a small value so that the lift-dependent drag is minimized and the excess thrust available for acceleration is maximized. As the aircraft continues to accelerate, it will approach the lift-off speed, V_{lof} , at which it can generate enough lift to become airborne. Just before the lift-off speed is reached, the aircraft is rotated into a nose-up attitude equal to the lift-off angle of attack.

The rotation speed, V_R , must allow time for the aircraft to

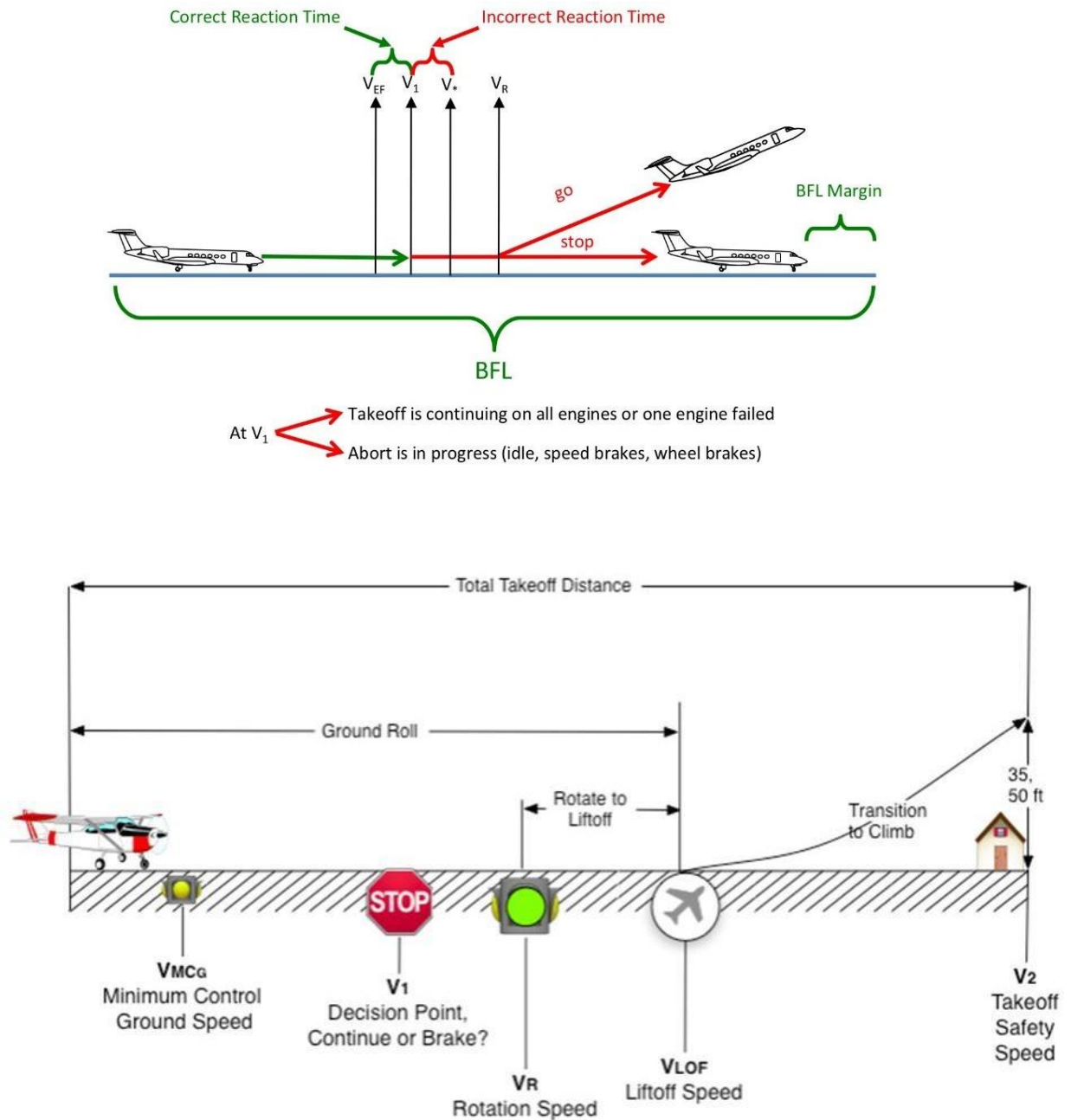


Figure 5.1 Take-off speeds and distances

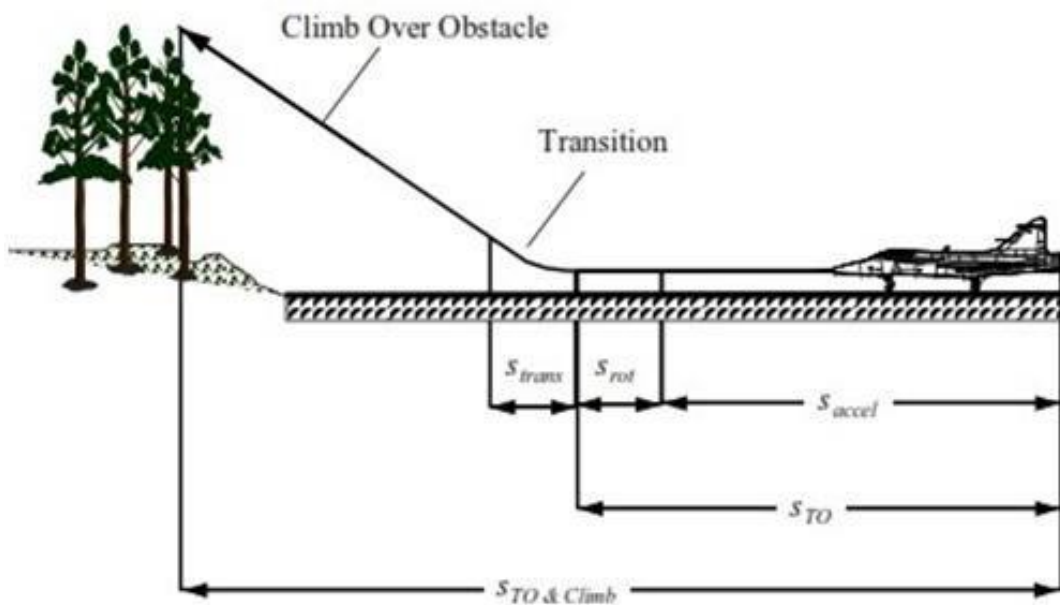


Fig.5.1 Take-off speeds and distances.

rotate into the lift-off attitude before the lift-off speed is achieved. As the aircraft continues to accelerate it reaches the lift-off airspeed and becomes airborne; this is the end of the ground run distance. So the lift-off speed must allow a sufficient margin over the stalling speed to avoid an inadvertent stall, and a consequent loss of height. This may be caused by turbulence in the atmosphere or any loss of airspeed during the maneuvering of the aircraft after the lift-off. The lift-off speed will usually be taken to be not less than $1.2V_{s1}$, where V_{s1} is the stalling speed of the aircraft in the take-off configuration. This will give a lift coefficient at lift-off of about $0.7 C_{lmax}$ and provide an adequate margin of safety over the stall.

If the aircraft is over-rotated to a greater angle of attack at the rotation speed then lift-off can occur too soon and the aircraft start the climb at too low an airspeed. This can occur if, for example, the elevator trim control is set incorrectly or turbulence produces an unexpected nose-up pitching moment. The minimum speed at which the aircraft can become airborne is known as the minimum unstuck speed, V_{mu} . It occurs when extreme over rotation pitches the aircraft up to the geometry limited angle of attack with the tail of the aircraft in contact with the runway. Tests are usually required to measure the take-off performance in this condition.

During the take-off run, should an engine fail between the minimum control speeds (ground) and the rotation speed, the decision either to abandon or continue the take-off will have to be made. This

decision is based on the distances required either to stop the aircraft or to continue to accelerate to the lift-off speed with one engine inoperative.

There will be a point during the acceleration along the runway at which the distances required by the two options are equal. This point is recognized by the indicated airspeed of the aircraft and is known as the decision speed, V_1 . The decision speed also determines the minimum safe length of runway from which the aircraft can take off. If an engine fails before the decision speed is reached then the takeoff is abandoned, otherwise the take-off must be continued.

Once the lift-off has been achieved the aircraft must be accelerated to the take-off safety speed (V_2 , one engine inoperative, V_3 , all engines operating). This is the air-speed at which both a safe climb gradient and directional control can be achieved in the case of an engine failure in the airborne state; this phase of the take-off path is known as the transition.

The ability to maintain directional control in the climb is determined by the minimum control speed, airborne, V_{mca} . The minimum control speed, airborne, will be greater than the minimum control speed, ground, V_{mcg} , since the aircraft is not restrained in roll by the contact between the landing gear and the runway.

In the event of an engine failure in the climb, the aircraft will depart in yaw, which will cause the aircraft to roll and enter a spiral dive if the yaw cannot be controlled. The take-off is complete when the lowest part of the aircraft clears a screen height of 35 ft (ore 10.5m) above the extended take-off surface. The distance between the lift-off point and the point at which the screen height is cleared is known as the air4borne distance, S_a .

The total take-off distance required will be the sum of the ground run distance, S_g , and the airborne distance, S_a . To ensure that the take-off is performed safely, the take-off distances will be suitably factored to allow for statistical variation in the take-off performance of the individual aircraft and in the ambient conditions.

This outline of the events, which take place during the take-off, applies to CTOL and RTOL operations and, broadly, to STOL operations. The relationships between the various speeds referred to above are set out in the airworthiness requirements and depend on the classification of the aircraft and the particular code of practice under which it is certificated.

Estimation of take-off distances:

At the design stage of the aircraft, estimates of the take-off distances will be needed to determine the minimum size of airfield from which the aircraft will be able to operate. Any such estimation will depend on the ability to estimate the thrust of the power plant at very low forwards speeds and the drag of the aircraft at low speeds and in ground effect. In addition, there will be other forces acting on the aircraft, for example, wheel spin-up runway friction and side-wind loads, some of which are very difficult to quantify. In practice, although reasonable estimates of the take-off distances can be made, it will be necessary to determine the actual take-off performance by measurement.

There are several methods for the estimation of the ground run and airborne take-off distances; the methods given here are among the simplest but give reasonable off distances; the methods given here are among the simplest but give reasonable results for convention aircraft with moderate thrust.

The equations of motion of the aircraft are given in Appendix B, above equation. Using the simplifying assumptions and including g the runway reaction force, the equations of motion during the take-off run can be written.

$$ma = \left(\frac{W_{TO}}{g} \right) a = T - D - \mu(W_{TO} - L)$$

Where γ_R = the runway slope, R = wheel load or runway reaction force, and μ_R = runway coefficient of rolling friction.

The ground-run distance, S_G :

The ground run distance can be estimated from the time integral of the ground speed of the aircraft.

$$S_G = \int_0^t V dt = \int_0^{V_{LOF}} \frac{mV}{F} dV$$

Where F = the accelerating force acting on the aircraft.

Now, from above equation, the accelerating force, F , can be written in the form.

$$\frac{F}{W} = \left\{ \frac{F_N}{W} - \mu_R - \sin \gamma_R \right\} - \frac{L}{W} \left(\frac{C_D}{C_L} - \mu_R \right)$$

In above equation, the curly bracket { } represents the net propulsive thrust – weight ratio, taking into account the runway rolling friction and the runway slope. The thrust force produced by the power plant is unlikely to vary substantially during the take-off and, for the a purposes of a simple estimation of the ground run, the net propulsive thrust – weight ratio can be assumed to be a constant, A.

The round bracket () is predominantly the aerodynamic drag – lift ratio at the ground run angle of attack, it also includes a runway friction term since any lift generated by the aircraft will alleviate the total runway friction force. The lift force, L, is the lift force that occurs during the take-off run and will be produced by the angle of attack, α , determined by the ground run pitch attitude of the aircraft. Since the aircraft pitch attitude is constant up to the point of rotation the angle of attack is constant, which implies that the lift and drag coefficients will be constant during the ground run. Therefore, the aerodynamic drag term can be evaluated as.

$$\frac{\frac{1}{2}\rho S C_L}{W} \left(\frac{C_D}{C_L} - \mu_R \right) V^2 = BV^2$$

Where B is a quasi-constant determined by the aircraft configuration and ground run attitude. Above equation is now being expressed in the form.

$$\frac{F}{W} = A - BV^2$$

And above equation becomes

$$S_G = \int_0^{V_{LOF}} \frac{V dV}{g[A - BV^2]}$$

This can be integrated to give

$$S_G = \frac{1}{2Bg} [\log_e(A - BV_{LOF}^2) - \log_e A]$$

If $A > BV^2$ a further simplifying approximation can be made by evaluating above equation at $0.7V_{LOF}$ and integrating above equation on the assumption that $(A - BV^2)$ is a constant, this gives

$$S_G = \frac{V_{LOF}^2}{2g(A - BV^2)_{0.7V_{LOF}}}$$

Equations above can be used to estimate the ground runoff a CTOL or RTOL aircraft up to the lift-off point. However, it may be necessary to divide the ground run into two parts. First, the ground run to the rotation, during which the ground attitude is constant. Secondly, the ground run during and after rotation in which the angle of attack increases to the lift-off angle of attack and the lift dependent drag will become significantly larger.

The airborne distance, S_A

After lift-off, the aircraft is accelerated to the safe climbing speed as it is rotated into the climb. The take-off is complete when the aircraft clears a screen height of 35ft above the extended take-off surface at a speed not less than the take-off safety speed. The flight path in the airborne phase of the take-off is not, thereof, a simple path but combines both acceleration and climb. An approximation to the airborne distance can be made by considering the change of energy of the aircraft during the airborne phase. This form of approximation does not presuppose the flight path to be a smooth curve and allows for a variation in take-off technique. The energy change is given by,

$$\Delta E = \text{excess thrust} \times \text{distance moved}$$

Giving

$$\Delta E = \int (F_N - D) dS \approx (F_N - D)_{av} S_A$$

Now, the energy change between the lift-off and the end of the airborne distance, at which the airspeed has been increased to the take-off safety speed, V_2 or V_{34} as appropriate, and the potential energy has been increased by 35 ft, is,

$$\Delta E = [KE + PE]_{35} - [KE + PE]_{LOF}$$

So that, from above equations, the airborne distance is given by

$$S_A = \frac{W}{(F_N - D)_{av}} \left\{ \frac{V_2^2 - V_{LOF}^2}{2g} + 35 \right\}$$

Assuming that the horizontal airborne distance travelled is very much greater than 35 ft.

The difference between the lift-off speed and the take-off safety speed should be as small as possible to minimize the time during which the aircraft may be unable to meet the requirement for directional control following an engine failure. In most cases, the increment required in the kinetic energy is about equal to the change in potential energy.

The operation of an aircraft usually requires the take-off distances to be as short as possible this applies to the ground run , the airborne distance and to their sum as the overall distance. For an aircraft with a given air frame- engine combination, any such optimization process will depend on the aerodynamic characteristics of the airframe, since the maximum available thrust of the engines is fixed. The ground run can be minimized by selecting the combination of high-lift devices that will produce the lowest lift-off speed together with a high lift—drag ratio.

This will need to take into account the drag is in the phase between rotation and lift-off and the performance with one engine inoperative. Similar criteria can be applied to the airborne distance and the overall distance, but will need to include the difference between the lift-off speed and the take-off safety speed. Clearly, such a problem cannot be solved by analytical means alone and, in practice, the final combination of high lift devices will be optimized by flight trails.

The effect on the take-off distances of the flight variables:

The take-off distances will be affected by the weight of the aircraft, the state of the atmosphere and the airfield conditions. Equations above enable the approximate effect of variation of the flight variables to be discussed

Aircraft weight:

If above equation is expressed for a take-off on a level runway in still air then the approximation for the ground run distance is.

$$S_{GR} = \frac{WV_{LOF}^2}{2g(F_N - D - \mu(W - L))_0 \gamma_{LOF}}$$

An increase in aircraft weight can be seen to have two direct effects on the ground run distance.

First, the ground run distance is directly proportional to aircraft weight, so that the ground run will increase in proportion to the weight increase.

Secondly, the ground run distance is directly proportional to the square of the lift-off speed, V_{tof} . Now, the lift-off speed is proportional to the stalling speed, V_{st} . Which, in turn, is proportional to the square root of the weight of the aircraft? Therefore, the take-off ground run distance will be increased, again in proportion to the aircraft weight.

In addition to the direct effects, the increased weight will increase the runway friction force acting on the aircraft, but the effect of this on the ground run will be relatively small compared with the direct effects of weight.

Summing the individual effects of a weight increase, it can be expected that increasing the aircraft weight by 10% will increase the take-off ground run distance by at least 20%

The airborne distance will be similarly affected.

First, from above equation it can be seen that the airborne distance is directly proportional to the aircraft weight.

$$S_A = \frac{W}{(F_N - D)_{\text{ex}}} \left\{ \frac{V_{\text{st}}^2}{2g} + V_{\text{tof}} + 35 \right\}$$

Secondly, both the lift-off speed and the take-off safety speed will be increased by the increase in aircraft weight. This will produce a proportional increment in the kinetic energy related term since the airborne distance is a function of the square of the airspeeds, which are proportional to the square root of the weight. However, as the potential energy and kinetic energy terms in the expression for the airborne distance are roughly equal, the effect of weight increase of 10% on the kinetic energy term is to increase the airborne distance by about 5%.

Also, the increase in the airframe drags and reduces the excess thrust available. The magnitude of this effect will depend on the excess thrust and on the lift-dependent part of the drag characteristic of the airframe. For a transport aircraft, it will probably equate to a reduction in the excess thrust of the order of 2% for a weight increase of 10%.

By summing these effects, it can be expected that increasing the weight of the aircraft by 10% will increase the airborne distance by between 15% and 20%.

Atmosphere state effects:

The state of the atmosphere has two basic effects on the take-off distances.

The take-off is performed with reference to indicated airspeed (IAS) displayed by the airspeed indicator. In the absence of pitot-static errors, the IAS will be the same as the equivalent airspeed, EAS (since the take-off is made at low speed and flow attitude and the scale-altitude correction will be negligible). However, the take-off distances are functions of true airspeed (TAS), since they are determined by the motion of the aircraft in Earth axes. Since the ground run distance is proportional to the TAS^2 it will be proportional to the inverse of the relative density. I/o, or $0/8$. This implies that hot or high (low relative pressure) conditions will increase the take-off ground run since the TAS will be increased for a given EAS determined by the aircraft weight. The airborne distance will also be increased but, because only the kinetic energy term is affected, the effect will be about halved.

The output of the power plant is roughly proportional to the relative density since the engine output is dependent point's air mass flow. Therefore, the net thrust will decrease in hot or high conditions increasing the take-off distances; the magnitude of the effect will depend on the characteristics of the power plant.

Head wind:

The effect of the headwind, V_w , is to change the datum speed of the take-off; the aircraft now only needs to be accelerated to a ground speed of V_l of $-V_w$. From above equation, it can be seen that the effect of a headwind equal to 10% of the lift-off speed is to decrease the ground run by almost 20%. Conversely, the same tailwind would increase the ground run by a little over 20%

The effect of the headwind on the airborne distance is approximately half as severe as it is on the ground run distance since it only affects the kinetic energy term.

Runway conditions:

The effects of a runway slope (uphill) and the runway friction coefficient on the ground run distance can each be accounted for by considering them as equivalent to a decrease in the take-off thrust-to-weight ratio. There is, of course, no effect on the airborne distance.

It will be seen later, that account is taken of the effect of the flight variables on the take-off performance data in their presentation in the aircraft performance manual. The effects of the weight, wind and runway slopes are each accounted for by using the corrections developed above to factor the datum performance measured at a known atmosphere state.

Landing performance:

In the landing phase of the flight, the aircraft is on a descending flight path towards the runway. As it approaches the runway, the airspeed and the rate of descent are reduced in the flare so that a touchdown is achieved at a low rate of descent. After touchdown, the nose is lowered onto the runway and the aircraft brought to a halt. During the landing, consideration is given to the need to ensure that the aircraft can be controlled safely and that the distances required for the maneuvers do not exceed those available. The landing maneuver is described in Fig. 5.2.

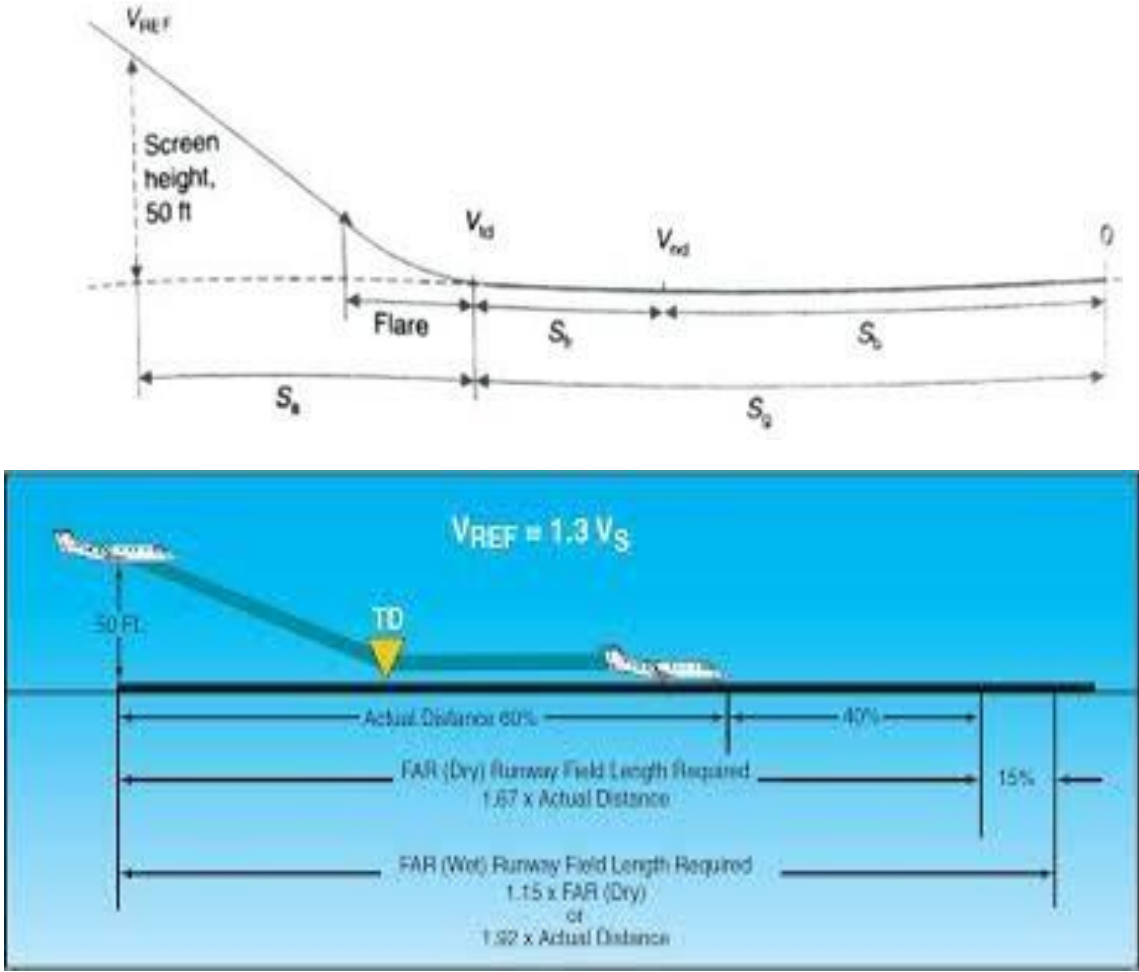


Fig.5.2 landing speeds and distances.

The approach path to the runway is protected by a safe approach sector, sloping downwards to the threshold of the runway, which is required to be clear of obstructions. The gradient of the approach sector is determined by the classification of the airfield and is a minimum of 2% for large airfields. The gradient of the approach path of the aircraft must be steeper than the minimum approach sector gradient and will usually be about 3* or 5% gradient for CTOL aircraft.

However, it may be steeper for RTOL and STOL aircraft; a STOL aircraft may be able to make an approach at a gradient of 7* or more. The landing distance commences when the aircraft just clears a screen height of 50ft above the extended landing surface.

The approach to the landing is made with the flaps and other high lift devices set to a high lift (and high drag) setting to enable the approach to be made at a low air speed. The approach airspeed will usually be not less than $1.3V$ so to provide a safe margin over the landing configuration. To maintain flight path stability, in which an increase in airspeed will produce an increase in drag, which will tend to restore the airspeed to its former value, the aircraft must be flown on the forward side of the drag curve. Since the approach speed is relatively low, it may be necessary to increase the zero-lift drag of the aircraft so that the minimum drag speed is less than the approach speed.

Extending the landing gear will increase the zero-lift drag (in some cases to twice the value with the landing gear retracted), and usually flaps will be lowered to a high-lift, high-drag setting. If additional drag is needed, airbrakes and spoilers can be used. In the high drag state, the engine thrust required to hold the approach gradient would be relatively high. This is a further advantage since, if the approach has to be abandoned and a go-around has to be initiated, the engines are already producing high thrust and the time required for them to accelerate to maximum thrust will be shortened. By retracting the landing gear and high drag devices, the drag can be reduced quickly. This enables the maximum excess thrust for the go-around to be achieved quicker than by the acceleration of the engines from idle thrust to maximum thrust with the aircraft in a low drag configuration.

As the aircraft passes the screen height, its combined airspeed and gradient of descent will produce a rate of descent that would be too high for an acceptable touch-down. It would exceed the capability of the landing gear to absorb the kinetic energy of the descent. A maneuver, the flare, is needed to reduce the rate of descent for the touchdown. In the flare, the thrust is reduced to flight idle and the nose of the aircraft is steadily raised to increase progressively the angle of attack. This will allow the

airspeed to decrease to a safe touchdown speed and the rate of descent to be reduced, ideally to almost zero, as the aircraft touches down.

The pitch attitude of the aircraft at touchdown will be equal to the angle of attack, since the flight path gradient is zero, and the touchdown is made in a nose up attitude. The height at which the flare commences is judged by the pilot or, in auto-land systems, determined by reference to a measurement of true height above the runway from a radio altimeter. Since the aircraft is in a high drag configuration there will be a large decelerating force acting after the thrust is reduced. This will cause the airspeed to bleed away quickly enough to avoid a long float during the flare as the airspeed decreases. The distance between the 50ft screen height and the touché down point is known as their borne landing distance, S_a .

At the touchdown, the aircraft will be in a nose up attitude with the nose wheel clear of the runway at the touchdown speed, V_{td} . In this attitude, the large angle of attack will produce a high lift-dependent drag that will help to decelerate the aircraft. As the airspeed decreases there will come a point at which there will be insufficient pitching moment produced by the elevator control to hold the nose-up attitude. The aircraft will then pitch down onto the nose wheel at the nose-down speed, V_{nd} . (In practice, the speed at which the nose wheel is lowered onto the ground will be controlled by the pilot.)

The ground run distance with the nose wheel clear of the ground is known as the free roll distance, S_{fr} , since it is not generally possible to use any retarding system, other than the aerodynamic drag of the aircraft in the landing configuration, to assist the deceleration. The reason for this is the possibility of producing a pitching moment that could not be controlled by the elevator. This could result in a pitch up, which might result in the aircraft becoming air borne again, or a pitch down which would result in the nose wheel slamming down onto the runway.

Once the nose wheel is on the ground, the aircraft can be decelerated to a halt in the braking distance, S_v . The deceleration in this phase can be assisted by various devices. Reverse thrust from the engines and the wheel brakes provide direct retarding forces. Flaps can be deployed to a high drag, 'lift-dump', setting in which they are deflected to a very large angle to provide a high zero-lift drag. In this setting they also act as spoilers to decrease the lift produced by the wing, thus assisting the wheel braking by increasing the load on the wheels. Spoilers, airbrakes or drag parachutes can be employed to increase the aerodynamic retarding force; in some cases the aerodynamic retarding systems and wheel brakes can be armed in flight to activate automatically as the nose wheel contacts the ground. The landing run is complete when the aircraft has been brought to a halt on the runway.

The landing ground run, S_g , is the sum of the free-roll distance and the braking distance and can be minimized by selection of the best nose-down speed. If the aircraft produce a large lift-dependent drag force in the free-roll phase; it might be advantageous to maintain the nose up attitude as long as possible. The nose can then be allowed to descend onto the runway and braking used to bring the aircraft finally to a halt. If the lift-dependent drag is small then the nose should be lowered onto the runway as soon as possible and the deceleration assisted by any other means available; this is the more usual case for transport aircraft.

The landing distance, S_{ldg} , is the sum of the airborne landing distance and the landing ground run.

The estimation of the landing distances:

The landing distances can be estimated in a very similar manner to the take-off distances.

The airborne distance, s_a :

This is the distance required for the aircraft to clear the 50ft screen height at the landing reference speed, V_{ref} , and to decelerate and descend to the touchdown. The distance can be estimated in the same way as the airborne take-off distance by considering the energy difference between the screen height and the touchdown point, this gives,

$$S_a = \frac{W}{(F_N - D)_{av}} \left\{ \frac{V_{REF}^2 - V_{td}^2}{2g} + 50 \text{ ft} \right\}$$

In applying this method of estimation, it should be noted that during the flare there would be a thrust reduction and a drag increase that may need to be taken into account. It should also be noted that in landing a large aircraft having a high kinetic energy, it might be necessary to start the flare before reaching the 50ft screen height. A small aircraft with less kinetic energy may not need to flare until well below 50ft.

The ground-run distance:

The landing ground run distance can be estimated by using the same technique that was employed for the take-off run by considering the integral,

$$S_g = \int \frac{WV'}{gR} dV'$$

Where R is the retarding force acting on the aircraft given by?

$$R = [D + \mu_R(W - L) + W \sin \gamma_R - F_N]$$

In which the drag, D, represents the aerodynamic drag force of the airframe and of any drag-producing regard action devices. Equation above can be expressed in the form,

$$\frac{R}{W} = \left[\frac{\rho S C_L}{2W} \left(\frac{C_D}{C_L} - \mu_R \right) V'^2 + \left\{ \mu_R + \sin \gamma_R - \frac{F_N}{W} \right\} \right] = BV'^2 + A$$

(The terms A and B are not the same as those used in above equation and integrating leads to a general expression for the ground run distance.

$$S_g = \frac{1}{2gB} \log_e (BV'^2 + A)$$

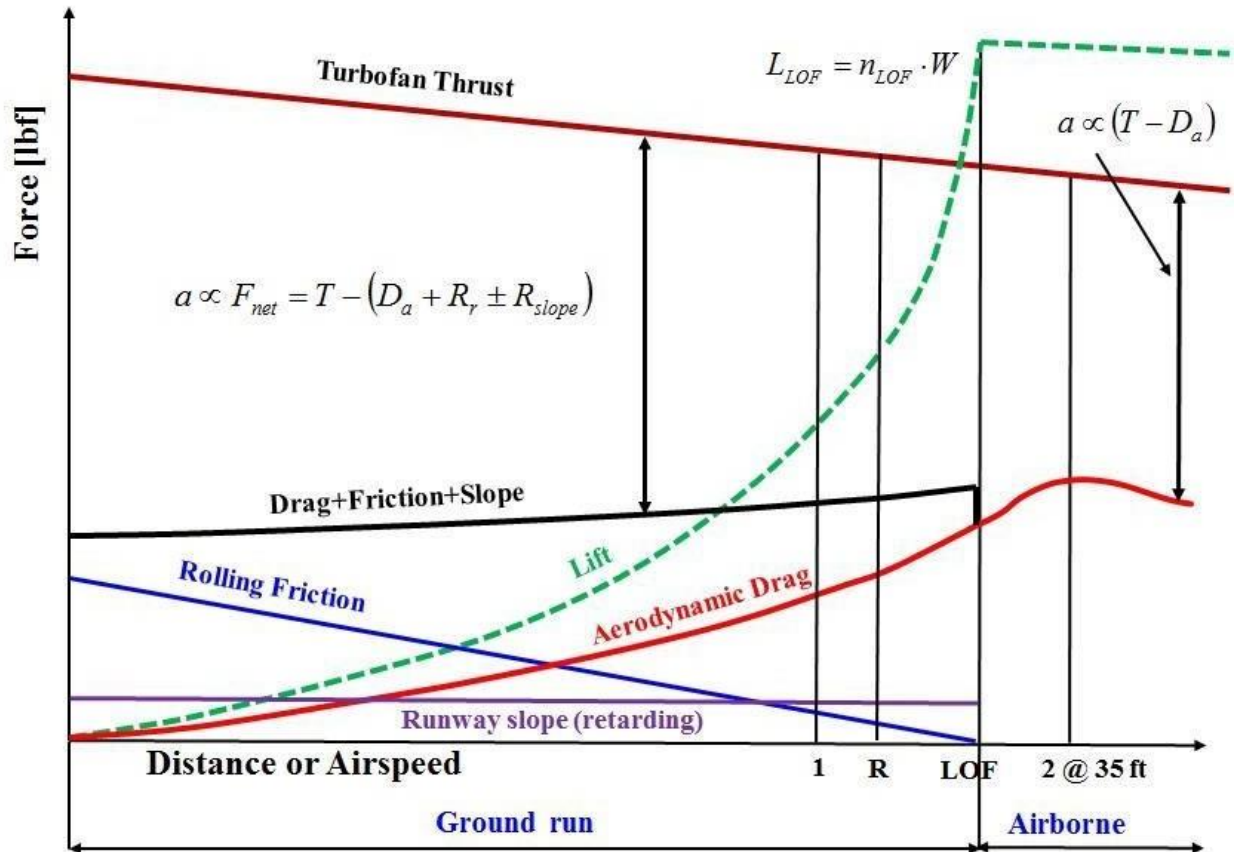


Fig. 5.3 Retarding forces acting during the landing ground run.

This will need to be evaluated between the touchdown speed and the nose-down speed, and the nose-down speed and zero, to give the free-roll and the braked distances respectively. The evaluation will depend on the determination of the nose-down speed from the relative magnitudes of the terms A and B, which will be different in the free-roll and braked ground run.

In the free-roll, the retarding force will be dominated by the aerodynamic drag of the airframe in its landing configuration and in ground effect. The rolling coefficient of friction will be small and the angle of attack, and hence the lift efficient, C_L , will be large. This will lead to a high drag-to-lift ratio, C_d/C_L , for the aircraft in its landing configuration, which may be further increased by being in ground effect; the thrust – weight ratio at flight idle thrust will be small. In this case the term A will be very small compared with BV2.

In the braked ground run the retarding force will be dominated by the non-aerodynamic retarding devices, the brakes and reverse thrust. The braking coefficient of friction will be large, probably as

high as 0.5. The angle of attack will be small, and hence the coefficient of lift will be small, but the aerodynamic drag coefficient, which includes the effects of any high drag devices, could be large. This would lead to a large drag-to-lift ratio at the higher airspeeds. If reverse thrust is used, F_n/W will be large and negative. In this case A will be of similar magnitude to, or larger than, BV^2 and will dominate at low speeds. The relative retarding forces are shown in Fig.5.3.

Comparison between the two cases will determine the optimum nose-down speed for the aircraft, see Fig. 5.3 at high airspeeds, the aerodynamic drag of the aircraft will tend to be dominant and favor the free-roll technique, whereas at low airspeeds braking will be more effective. However, the capacity of the brakes to absorb the kinetic energy of the aircraft as heat must be taken into account. If the aircraft is equipped with thrust reversers it is likely that there will be no benefit in using the free-roll retardation. The nose will be lowered onto the runway immediately after touchdown so that mechanical retardation and thrust reversal can be employed. It is not usual to pre-arm thrust reversers prior to landing so that they operate automatically when the nose wheel touches the runway.

The effect on the landing distances of the flight variables:

The landing distances will be affected by the weight of the aircraft, the state of the atmosphere and the airfield conditions, in a broadly similar manner to the take-off distances.

Aircraft weight:

In the approach phase of the landing the thrust produced by the engines will be less than the maximum thrust available. Therefore, the gradient of the approach flight path can be controlled by the engine thrust and will not be affected by the weight of the aircraft. In the flare, the thrust is reduced to idle and the aircraft decelerates as the angle of attack is steadily increased towards the touchdown angle of attack to arrest the rate of descent. Since the angle of attack on the approach and at touchdown is usually determined by the stalling speed of the aircraft in the landing configuration, the lift – drag ratio during the flare will be practically independent of the aircraft weight.

This implies that, unlike the airborne take-off distance, the only significant effect of the weight on the airborne landing distance will be due to the kinetic energy term in the flare. Because the approach is made at about $1.3V_{so}$, the kinetic energy loss during the flare will be about twice as much as the potential energy loss. Therefore, a weight increase of 10% would be expected to increase the distance in the flare by about 7%.

The effect of the weight on the landing group under distance is similar to the take-off case; it increases the ground speed at touchdown through its effect on the stalling speed. The direct effect of the increased weight on the kinetic energy will increase the ground run distance by about 20% for a weight increase of 10%. This affects both the free roll distance and the braked ground run.

5.7.3 Atmosphere state effects:

The state of the atmosphere will affect the landing distances through its effect on the TAS. There will be no significant effect through the engine thrust since the thrust is either being controlled to maintain the approach gradient or will be at idle. As in the take-off case, the ground run distances will be increased in proportion to the inverse of their density and the flare.

Headwind:

The effect of the headwind on the ground run distance is to change the datum speed of the landing since the aircraft now only needs to be decelerated from an effect touch-down speed. The effect of a headwind equal to 10% of the touch-down speed would be to reduce the landing ground run by about 20%.

The effect of the head wind the airborne distance is only felt in the flare since the approach gradient is relative to the ground and is controlled by engine thrust or power. A headwind of 10% of the touch down speed will reduce the kinetic energy loss during the flare by 20% and the distance in the flare by about 14%

Runway conditions:

The effects of a runway slope (uphill) and the runway friction coefficient on the ground run distance can each be accounted for by considering them as equivalent to an increase in the braking force. There is, of course, no effect on the airborne distance.

Landing performance:

The requirements for landing commence when the aircraft is 1500ft above the landing airfield and cover the approach and landing to bring the aircraft to a complete stop on the runway. The landing flight path was discussed and was seen to be in two parts, an airborne phase and a ground phase. Methods of calculation or the landing distances were determined for the estimation of the airfield performance at the design stage of the aircraft showing the effects of the flight variables.

As in the case of take-off, landing performance data are required for the performance manual. However, due to the nature of the landing maneuver, the provision of that data from measurement of the landing distances is not straight forward. In the measurement of the take-off distances, there was a clearly defined datum point maneuver is subject to very much greater statistical or than the take-off sine it is not possible to position the aircraft with absolute accuracy at the screen height above the runway threshold on the approach. Neither is it possible to guarantee that the aircraft is flying as exactly the correct approach speed, V_{ref} , maneuver is not well fixed, as it is for the take-off, and this leads to a statistical error in the touchdown point on the runway.

The distance required to bring the aircraft to a halt following the touchdown depends on the level of braking and other methods of retardation applied. If a minimum distance landing were attempted, the service braking required and the extreme use of retardation devices would not be avertable for normal operations. It would compromise passenger comfort and safety and cause unacceptable wear and tear on the aircraft. Defining a normal landing maneuver thus becomes a subjective problem. Due to the statistical errors implicit in the landing maneuver, large safety factors need to be applied to the landing distances required to ensure that the landing will not exceed the space available. For these reasons, the landing distances required for the aircraft flight manual are mainly determined by calculation rather than from measured data, but usually there will be some measured data to provide verification. In the landing case, calculation of performance data for the flight manual is acceptable because of the large factors applied to the calculated distances require. A parametric analysis will enable the effect of the flight variables to be assessed.

- The maximum landing weight of the aircraft is the lowest of the weights necessary to comply with the limitations imposed by,
- The maximum design structural landing weight,
- The landing distance available

The WAT knot set by the climb performance following a discontinued approach or a baulked landing.

The maximum design structural landing weight is an absolute limit that must not be exceeded. It is determined by the structural strength of the airframe needed to withstand the loads imposed by the vertical velocity of the aircraft at touchdown.

The space available:

As in the case of take-off the space available for landing is limited by the dimensions of the airfield and the approach to the runway in the landing direction, see Fig 5.4.

The approach path is protected by an obstacle limitation surface in a similar manner to the take-off net flight path. The definition of the obstacle limitation surface depends on the classification of the runway. Figure 5.4 shows the limitations for a large airfield with a precision approach category; full definitions for all runway classifications can be found in ICAO Annex 14. As in the case of the take-off obstacle free zone, it may not be possible to comply fully with the requirements, and the flight planning will need to consider any known obstructions near the airfield.

The first section of the obstacle limitation surface starts from a point 60m from the threshold of the runway and extends for a distance of 3000m along the approach path at a gradient of 2%. The second section extends a further 3600m at a gradient of 2.5%. The horizontal section extends a further 8400m with a base height of 150m above runway surface. The total length of the approach obstacle limitation surface is 15000m. There are of course, lateral dimensions associated with the obstacle limitation surface to enable the aircraft to make turning and positioning maneuvers on the approach. As these do not affect the performance considerations addressed here, they need not be considered in detail.

In the event of a missed approach or a baulked landing, in which the aircraft abandons the landing and climbs away from the airfield, there needs to be an obstacle free area in the direction of the climb-out. The baulked-landing obstacle clearance surface extends from 1800m beyond the threshold of the landing runway, or the end of the runway whichever is the lesser, at a gradient of 3.33%, to a distance of 4000m.

The landing distance available (LDA) is defined as the length of runway that is declared available and suitable for the ground run of an aircraft landing.

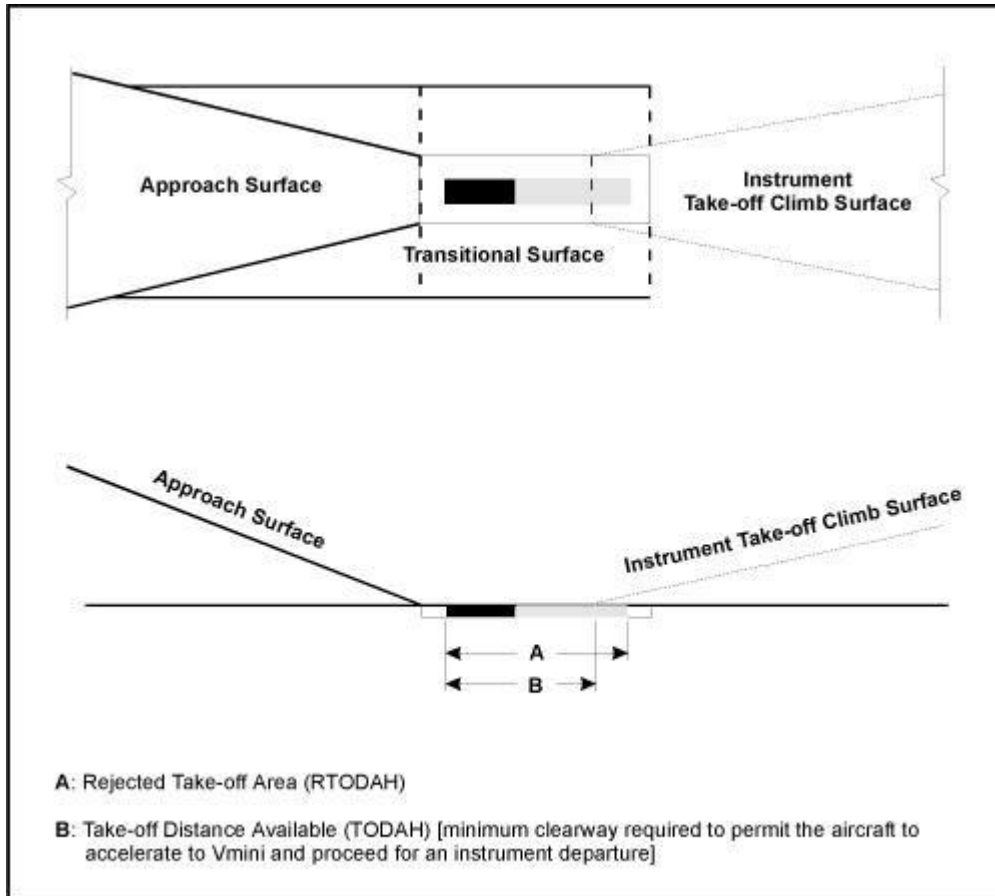


Figure 4-6. Example of obstacle limitation surfaces for a precision FATO

Figure:5.4 (a) Landing approach obstacle limitation surface.

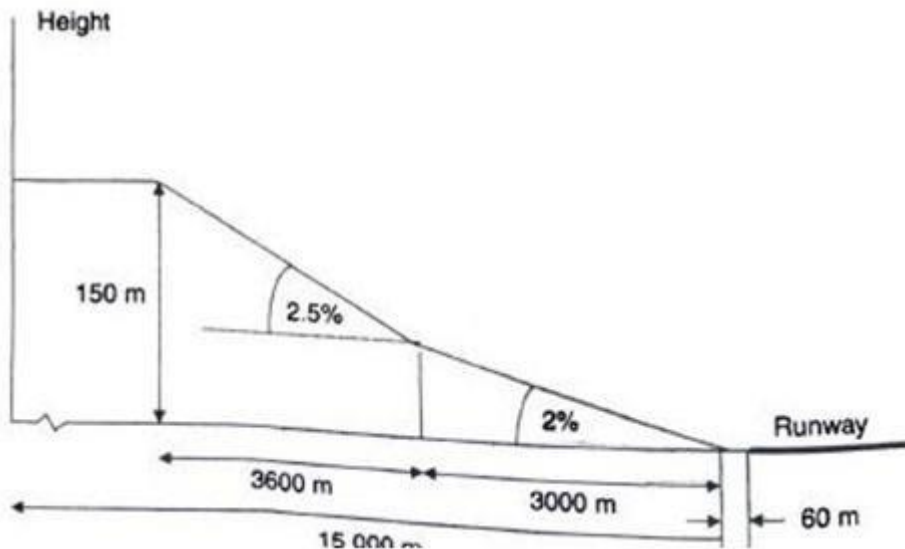


Figure:3 Landing approach obstacle limitation surface.

The space required

The landing distance required (LDR) is the gross distance required to land on a legal, smooth, hard-surfaced runway from a specified screen height at the runway threshold and to come to a complete stop, multiplied by a suitable safety factor.

The landing flight path assumes that the aircraft approaches the runway in a steady descent to a screen height of 50ft at the runway threshold. The gradient of the descent in the approach is usually 5% (or 3*), but may be steeper in some restricted airfield operations.

The airspeed at the screen height should be not less than the greater of V_{mc} or $1.3V_s$ in the landing configuration. Between the screen height and touchdown, the airspeed is reduced to a safe touchdown speed in the flare. This is done by progressively increasing the angle of attack, so that the touchdown is achieved at an acceptable vertical speed avoiding excessive vertical acceleration or any tendency to bounce, in the flare, no changes to the configuration, addition of thrust or depression of the nose should be required.

After touchdown, brakes and other means of retardation, for which a satisfactory means of operation has been approved, can be used to decelerate the aircraft to a halt. If any retarding system depends on the operation of any engine, for example a thrust reverser, then it may be necessary to assess the landing distances with that engine inoperative to determine the most critical landing distance required.

To account for wet runway conditions, the dry runway distances are factored by a function of the coefficient of friction of the runway under wet and dry conditions.

The maximum landing weight based on the landing distance required is determined by the balance between the landing distance available and the greatest of the landing distances required. This takes into account the factors for the runway surface, slope and condition, the wind and an overall safety factor to account for the statistical variation in the parameters affecting the landing performance.

The discontinued landing:

There are two cases to be considered in which the landing has to be abandoned. These are,

- The discontinued approach in which the aircraft terminates the approach before the thrust is reduced in the flare and continues the flight to a point from which a new approach can be made, and
- The baulked landing in which the aircraft is required to go-around after the thrust has been reduced in the flare.

The discontinued approach occurs, for example, when the aircraft has reached the minimum decision height on the approach to landing and the run way is not insight. At that point the approach must be discontinued and a climb initiated; it is assumed for the purpose of the requirements that the aircraft is flying with one engine in operative. In a go-around from a discontinued approach, the aircraft must be capable of climbing away, with the critical engine inoperative and in the approach configuration, to a safe height from which it can make another approach or a diversion.

The steady gradient of climb must not be less than 2.1% for two-engine aircraft, 2.4% for three-engine aircraft and 2.7% for four-engine aircraft; the different gradients reflect the effect of statistical variability in the thrust of the operating engines (For Category II operations the gradient is 2.5% for all types.) The climb gradient must be demonstrated with the critical engine inoperative and the operating engines at take-off thrust, at maximum landing weight and with the landing gear retracted. The configuration for the discontinued approach must be such that the stalling speed is not greater than $1.1V_s$ for the all-engines operating landing configuration.

In the case of the baulked landing the aircraft must be capable of climbing at a gradient that will maintain clearance from all obstructions with all engines operating in the landing configuration and with the landing gear down. When the go-around is initiated, take-off thrust is selected and the aircraft is rotated into the climb. With all engines operating, the aircraft must be capable of achieving as gradient of climb of not less than 3.2% in the landing configuration. The engine thrust or power used to calculate the gradient of climb is that which is available 8 seconds after-take-off power is selected from the flight idle condition. The airspeed used for the climb is $1.2V_s$, but must not be less than V_{mc} or more than the greater of V_{mc} and $1.3V_s$.

The gradients of climb in the discontinued landing will determine the maximum weight at which the aircraft can safely terminate the approach taking into account the WAT conditions. In most cases the limit of compliance for the discontinued approach and the baulked landing can be combined into a single WAT chart for the aircraft, this will be discussed further in Chapter 10. The maximum landing weight for the aircraft will be the lowest of the weights determined by consideration of the climb gradient in the discontinued approach and the baulked landing can be combined into a single WAT chart for the aircraft, this will be discussed further in Chapter 10. The maximum landing weight for the aircraft will be the lowest of the weights determined by consideration of the climb gradient in the discontinued land in the space available and the maximum design landing weight.

The trip fuel

The flight from the point of departure to the intended destination is made up of the basic elements of the flight path, take-off, climb, cruise, descent and landing. In addition, the aircraft will need to taxi from the ramp to the runway for take-off and from the runway to the ramp after landing.

The fuel required for each element of the intended flight is derived from flight measured data. This is reduced to a form from which the effects of weight, altitude and temperature can be interpolated separately. The data are presented in graphical or tabular form so that the fuel required for each element of the flight can be found for the particular WAT state and operational parameters. For example, the cruising segment will need a statement of the distance between the end of the climb and the beginning of the descent and the climbing segment will need the height increase between take-off and cruise altitude. In some cases, it may be sufficient to state an allowance of fuel for a maneuver or fuel burned per module time. For example, in the case of smaller aircraft it is not unusual to quote a fixed quantity of fuel for the take-off and landing and for the taxiing fuel in modules per minute. This is acceptable if the effect of variation of WAT on the fuel.

Since the fuel consumed during the trip depends on the weight of the aircraft, a starting weight' is needed before the trip fuel can be calculated. The initial weight of the aircraft will depend on the quantity of fuel that will be burned during the trip. Hence, the only weight that can be determined is the weight at the end of the diversion at which time only the reserve fuel remains. Therefore, the calculation process is reversed, so that the starting weight' for the fuel calculation is the weight at the end of the flight, rather than that at the beginning. The starting weight for the calculation of the trip fuel is, therefore, the aircraft prepared for service (APS) weight of the aircraft plus its payload and the fuel remaining at the end of the trip. The fuel required for landing descent, cruise, climb and take-off can be calculated in their reverse order and summed to give the trip fuel. This process estimates the minimum quantity of fuel required for the trip based on assumptions that include the

route to be flown, cruise altitude, atmosphere state and forecast winds. In practice, however, the ideal flight plan is rarely achieved and the actual trip will differ from the trip assumed for flight planning; the reasons for this include the following.

- The flight planning will be completed sometime before the flight and will use forecast weather (temperature profiles and winds), request the best cruise altitude and route and will be based on a specified departure time. The flight plan may not be confirmed until just before take-off, or even after- take-off, and may require changes to the route and cruise altitude to coordinate the flight with other traffic. Any such changes will usually increase the time and distance of the flight and extra fuel will be required.
- Delays in departure times may result in different air temperatures at take-off and en-route from those used for the flight plan. For example, if an early morning departure is delayed until mid-day, the increase in air temperature may increase the trip fuel required. Consequently, it could mean that the take-off WAT limit would be exceeded and the payload would then have to be reduced to comply with the scheduled performance requirements.
- On arrival at the destination, the aircraft may have to hold to await its turn to land. Flying in the hold will require additional fuel.

To account for the additional fuel requirement caused by environmental effects (temperature and wind), the trip fuel can be increased by a percentage of the calculated en-route fuel. In addition contingency allowances can be added to account for holding and unscheduled maneuvers or route changes. The percentage increase of the en-route fuel may be based on a requirement set by the regulatory authorities or decided by the operator. The contingency fuel allowances are usually determined by the operator and based on knowledge and experience of the route.

The diversion fuel:

The fuel for the diversion is calculated in the same manner as the trip fuel, but usually it assumes the same contingency fuel as the trip.

Reserves:

Minimum reserves are usually set by the regulatory authorities although the operator may increase them at his or her discretion. An aircraft should not need to use fuel from its minimum reserves except in an emergency reserve are additional to the minimum reserve.

Tankering:

In addition to the fuel required for the mission, the opportunity may be taken to carry extra fuel if it is economically advantageous to do so, this is known as tankering. Fuel can be tankered if the price at the departure point is sufficiently below the price at the destination to make it worth using any available surplus weight to carry the extra fuel. In this way, the cost of fuel uplift for the next flight can be reduced and the overall economy of the operation improved. Tankered fuel cannot be considered as fuel reserve for fuel planning purposes.

The fuel planning progression is shown in Fig 5.5. The payload weight is added to the aircraft prepared for service weight to give the zero fuel weight; this is the basic weight of the aircraft for that mission. The landing weight at the alternate airfield is the sum of the zero fuel weight and the fuel reserves that have not been used, together with any tankered fuel being carried. The landing weight at the destination is the landing weight at the alternate plus the fuel for the diversion and, possibly, a part of the percentage en-route reserve if it has not been required.

The greatest landing weight as the destination would occur if no reserves had been used; this case is shown in Fig. 5.5. The trip fuel added to the landing weight at the destination gives the take-off weight, which must not exceed the maximum scheduled take-off weight determined by the performance planning. Taxiing fuel may be carried to permit the aircraft to taxi to the take-off point and take-off as its limiting weight; this is added to the take-off weight to give the ramp weight. The fuel plan usually contains a simple self-check by adding the total fuel weight to the zero fuel weight to give the ramp weight directly.

Comparing this with the ramp weight from the detailed plan helps to eliminate any arithmetic errors that may occur in the process. Although the performance plan takes precedence over the fuel plan in flight planning, the fuel plan may need to be completed first in order to find the fuel required to transport the aircraft and payload to the intended destination. When this has been done, the take-off weight of the aircraft is known and the performance planning will be used to confirm that the aircraft can meet all the performance criteria along the intended route.

If the aircraft cannot meet any of the performance requirements then its weight must be reduced until it is able to do so. The APS weight is fixed and cannot be reduced.

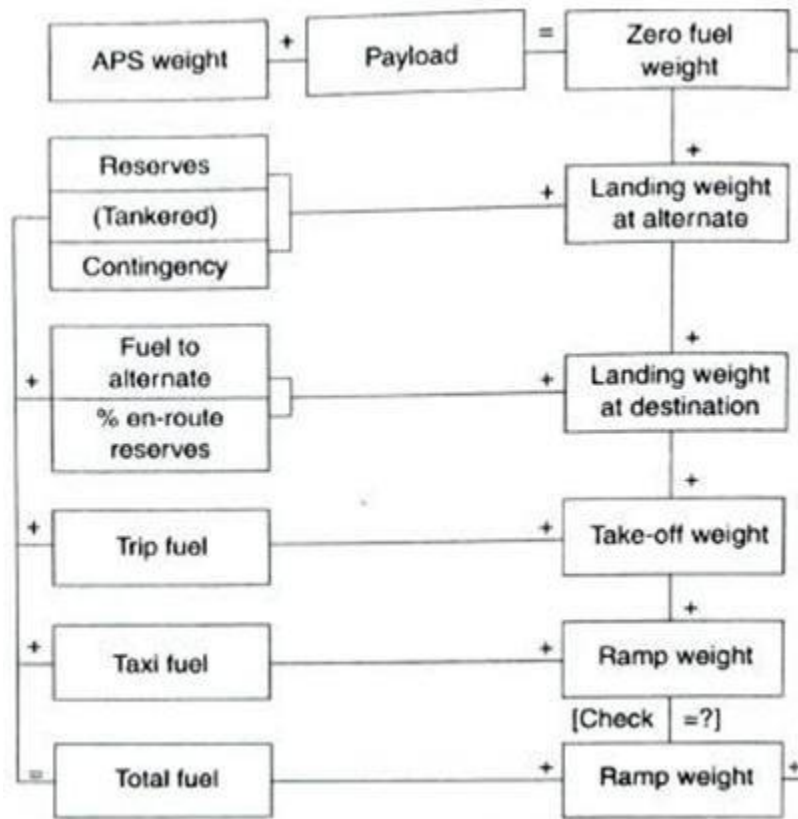


Figure: 5.5 (a) Fuel planning.

PHASES OF IN-FLIGHT PERFORMANCE

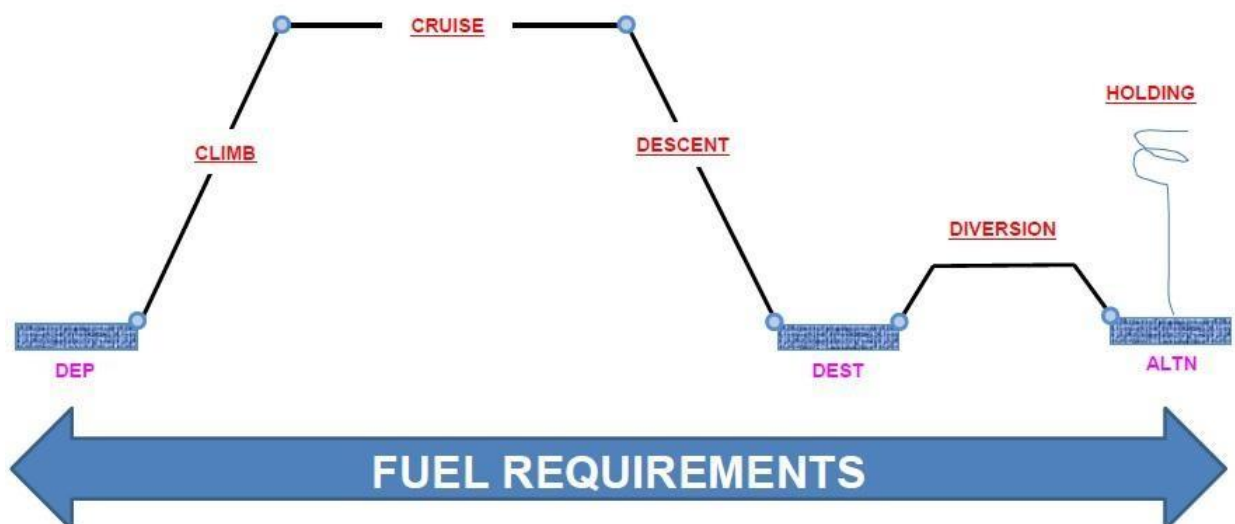


Figure: 5.5 (b) Fuel planning

The fuel load, including the reserves, required for the mission will also be fixed for that aircraft weight. Only the payload can be reduced, and to reduce the aircraft weight to comply with the

performance criteria payload will have to be off-loaded. This is very serious economically since the reduction can only be made in the first instance in the revenue-earning part of the weight of the aircraft. However, since this will reduce the total aircraft weight the fuel required for the mission will also be reduced so that the final weight reduction will not be all payloads but will include some fuel. A new weight schedule can then be made in which it may be possible to recover some of the payload reduction.

Effects and use of the control

In explaining the functions of the controls, the instructor should emphasize that the controls never change in the results produced in relation to the pilot.

The pilot should always be considered the center of movement of the airplane, or the reference point from which the movements of the airplane are judged and described. The following will always be true, regardless of the airplane's attitude in relation to the Earth.

- When back pressure is applied to the elevator control, the airplane's nose rises *in relation to the pilot*.
- When forward pressure is applied to the elevator control, the airplane's nose lowers *in relation to the pilot*.
- When right pressure is applied to the aileron control, the airplane's right wing lowers in relation to the pilot.
- When left pressure is applied to the aileron control, the airplane's left wing lowers in relation to the pilot.
- When pressure is applied to the right rudder pedal, the airplane's nose moves (yaws) to the right in relation to the pilot.
- When pressure is applied to the left rudder pedal, the airplane's nose moves (yaws) to the left in relation to the pilot.

The preceding explanations should prevent the beginning pilot from thinking in terms of "up" or "down" in respect to the **Earth**, which is only a relative state to the pilot. It will also make understanding of the functions of the controls much easier, particularly when performing steep banked turns and the more advanced maneuvers. Consequently, the pilot must be able to properly determine the control application required to place the airplane in any attitude or flight condition that is desired.

The flight instructor should explain that the controls will have a natural "live pressure" while in flight and that they will remain in neutral position of their own accord, if the airplane is trimmed properly.

The ability to sense a flight condition, without relying on cockpit instrumentation, is often called "feel of the airplane," but senses in addition to "feel" are involved.

Feel on the Airplane

Sounds inherent to flight are an important sense in developing "feel." The air that rushes past the modern light plane cockpit/cabin is often masked by soundproofing, but it can still be heard. When the level of sound increases, it indicates that airspeed is increasing. Also, the powerplant emits distinctive sound patterns in different conditions of flight. The sound of the engine in cruise flight may be

different from that in a climb, and different again from that in a dive. When power is used in fixed-pitch propeller airplanes, the loss of r.p.m. is particularly noticeable. The amount of noise that can be heard will depend on how much the slipstream masks it out. But the relationship between slipstream noise and power plant noise aids the pilot in estimating not only the present airspeed but the trend of the airspeed.

There are three sources of actual "feel" that are very important to the pilot. One is the pilot's own body as it responds to forces of acceleration. The "G" loads imposed on the airframe are also felt by the pilot. Centripetal accelerations force the pilot down into the seat or raise the pilot against the seat belt. Radial accelerations, as they produce slips or skids of the airframe, shift the pilot from side to side in the seat. These forces need not be strong, only perceptible by the pilot to be useful. An accomplished pilot who has excellent "feel" for the airplane will be able to detect even the minutest change.

The response of the aileron and rudder controls to the pilot's touch is another element of "feel," and is one that provides direct information concerning airspeed. As previously stated, control surfaces move in the airstream and meet resistance proportional to the speed of the airstream. When the airstream is fast, the controls are stiff and hard to move. When the airstream is slow, the controls move easily, but must be deflected a greater distance. The pressure that must be exerted on the controls to effect a desired result, and the lag between their movement and the response of the airplane, becomes greater as airspeed decreases.

Another type of "feel" comes to the pilot through the airframe. It consists mainly of vibration. An example is the aerodynamic buffeting and shaking that precedes a stall.

Kinesthesia, or the sensing of changes in direction or speed of motion, is one of the most important senses a pilot can develop. When properly developed, kinesthesia can warn the pilot of changes in speed and/or the beginning of a settling or musing of the airplane.

The senses that contribute to "feel" of the airplane are inherent in every person. However, "feel" must be developed. The flight instructor should direct the beginning pilot to be attuned to these senses and teach an awareness of their meaning as it relates to various conditions of flight. To do this effectively, the flight instructor must fully understand the difference between perceiving something and merely noticing it. It is a well established fact that the pilot who develops a "feel" for the airplane early in flight training will have little difficulty with advanced flight maneuvers.

Attitude Flying

In contact (VFR) flying, flying by attitude means visually establishing the airplane's attitude with reference to the natural horizon. [Figure 5.6] Attitude is the angular difference measured between an airplane's axis and the line of the Earth's horizon. Pitch attitude is the angle formed by the longitudinal axis, and bank attitude is the angle formed by the lateral axis.

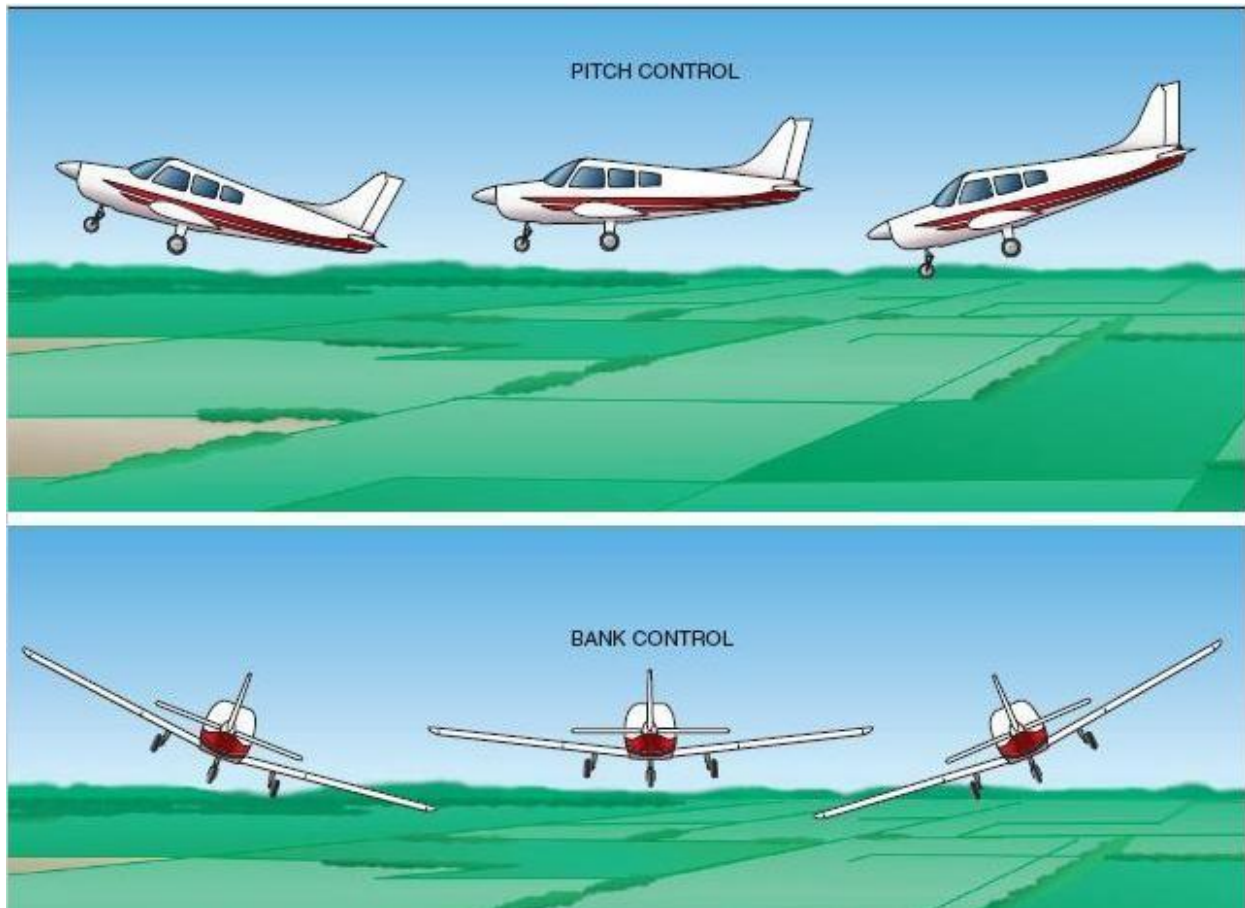


Figure 5-6. Airplane attitude is based on relative positions of the nose and wings on the natural horizon. In attitude flying, airplane control is composed of four components: pitch control, bank control, power control, and trim.

- **Pitch control** is the control of the airplane about the lateral axis by using the elevator to raise and lower the nose in relation to the natural horizon.
- **Bank control** is control of the airplane about the longitudinal axis by use of the ailerons to attain a desired bank angle in relation to the natural horizon.
- **Power control** is used when the flight situation indicates a need for a change in thrust.
- **Trim** is used to relieve all possible control pressures held after a desired attitude has been attained.

Climb and climbing turns

When an airplane enters a climb, it changes its flight path from level flight to an inclined plane or climb attitude. In a climb, weight no longer acts in a direction perpendicular to the flight path. It acts in a rearward direction. This causes an increase in total drag requiring an increase in thrust (power) to balance the forces. An airplane can only sustain a climb angle when there is sufficient thrust to offset increased drag; therefore, climb is limited by the thrust available.

Like other maneuvers, climbs should be performed using outside visual references and flight instruments. It is important that the pilot know the engine power settings and pitch attitudes that will produce the following conditions of climb.

Normal climb --Normal climb is performed at an airspeed recommended by the airplane manufacturer. Normal climb speed is generally somewhat higher than the airplane's best rate of climb. The additional airspeed provides better engine cooling, easier control, and better visibility over the nose. Normal climb is sometimes referred to as "cruise climb." Complex or high performance airplanes may have a specified cruise climb in addition to normal climb.

Best rate of climb --Best rate of climb (VY) is performed at airspeed where the most excess power is available over that required for level flight. This condition of climb will produce the most gain in altitude in the least amount of time (maximum rate of climb in feet per minute). The best rate of climb made at full allowable power is a maximum climb. It must be fully understood that attempts to obtain more climb performance than the airplane is capable of by increasing pitch attitude will result in a decrease in the rate of altitude gain.

Best angle of climb --Best angle of climb (VX) is performed at an airspeed that will produce the most altitude gain in a given distance. Best angle-of-climb airspeed (VX) is considerably lower than best rate of climb (VY), and is the airspeed where the most excess thrust is available over that required for level flight. The best angle of climb will result in a steeper climb path, although the airplane will take longer to reach the same altitude than it would at best rate of climb. The best angle of climb, therefore, is used in clearing obstacles after takeoff.

Descent and descending turns

When an airplane enters a descent, it changes its flightpath from level to an inclined plane. It is important that the pilot know the power settings and pitch attitudes that will produce the following conditions of descent.

Partial power descent --The normal method of losing altitude is to descend with partial power. This is often termed "cruise" or "enroute" descent. The airspeed and power setting recommended by the airplane manufacturer for prolonged descent should be used. The target descent rate should be 400 - 500 f.p.m. The airspeed may vary from cruise airspeed to that used on the downwind leg of the landing pattern. But the wide range of possible airspeeds should not be interpreted to permit erratic pitch changes. The desired airspeed, pitch attitude, and power combination should be preselected and kept constant.

Descent at minimum safe airspeed

A minimum safe airspeed descent is a nose-high, power assisted descent condition principally used for clearing obstacles during a landing approach to a short runway. The airspeed used for this descent condition is recommended by the airplane manufacturer and normally is no greater than 1.3 VSO. Some characteristics of the minimum safe airspeed descent are a steeper than normal descent angle, and the excessive power that may be required to produce acceleration at low airspeed should "mushing" and/or an excessive rate of descent be allowed to develop.

Glides

A glide is a basic maneuver in which the airplane loses altitude in a controlled descent with little or no engine power; forward motion is maintained by gravity pulling the airplane along an inclined path and the descent rate is controlled by the pilot balancing the forces of gravity and lift.

Gliding turns

The action of the control system is somewhat different in a glide than with power, making gliding maneuvers stand in a class by themselves and require the perfection of a technique different from that required for ordinary power maneuvers. The control difference is caused mainly by two factors--the absence of the usual slipstream, and the difference or relative effectiveness of the various control surfaces at various speeds and particularly at reduced speed. The latter factor has its effect exaggerated by the first, and makes the task of coordination even more difficult for the inexperienced pilot. These principles should be thoroughly explained in order that the student may be alert to the necessary differences in coordination.

After a feel for the airplane and control touch have been developed, the necessary compensation will be automatic; but while any mechanical tendency exists, the student will have difficulty executing gliding turns, particularly when making a practical application of them in attempting accuracy landings.

Three elements in gliding turns which tend to force the nose down and increase glide speed are:

- Decrease in effective lift due to the direction of the lifting force being at an angle to the pull of gravity.
- The use of the rudder acting as it does in the entry to a power turn.
- The normal stability and inherent characteristics of the airplane to nose down with the power off.

These three factors make it necessary to use more back pressure on the elevator than is required for a straight glide or a power turn and, therefore, have a greater effect on the relationship of control coordination.

When recovery is being made from a gliding turn, the force on the elevator control which was applied during the turn must be decreased or the nose will come up too high and considerable speed will be lost. This error will require considerable attention and conscious control adjustment before the normal glide can again be resumed.

Common errors in the performance of descents and descending turns

- Failure to adequately clear the area.
- Inadequate back-elevator control during glide entry resulting in too steep a glide.
- Failure to slow the airplane to approximate glide speed prior to lowering pitch attitude.
- Attempting to establish/maintain a normal glide solely by reference to flight instruments.
- Inability to sense changes in airspeed through sound and feel.
- Inability to stabilize the glide (chasing the airspeed indicator).
- Attempting to "stretch" the glide by applying back-elevator pressure.
- Skidding or slipping during gliding turns due to inadequate appreciation of the difference in rudder action as opposed to turns with power.
- Failure to lower pitch attitude during gliding turn entry resulting in a decrease in airspeed.

- Excessive rudder pressure during recovery from gliding turns.
- Inadequate pitch control during recovery from straight glides.
- "Ground shyness" resulting in cross-controlling during gliding turns near the ground.
- Failure to maintain constant bank angle during gliding turns.