MECHANICS OF SOLIDS

B Tech III semester (Autonomous IARE R-18)

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CO	COURSE OUTCOME
CO 1	Understand the basics of material properties, stress and strain.
CO 2	Apply knowledge of various kinds of beams for engineering applications.
CO 3	Ability to identify, formulate, and solve engineering & real life problems.
CO 4	Ability to design and conduct experiments, as well as to analyze and interpret data
CO 5	Ability to design a component to meet desired needs within realistic constraints of safety.

MODULE-I

Introduction to Stresses and Strains

CLOs	Course Learning Outcome
CLO 1	Calculate the stress strain relations in conjunction with elasticity and material properties
CLO 2	Describe the resistance and deformation in members which are subjected to axial, flexural and torsion loads.
CLO 3	Discuss thermal explanations in solid bars and induced thermal stresses

INTRODUCTION

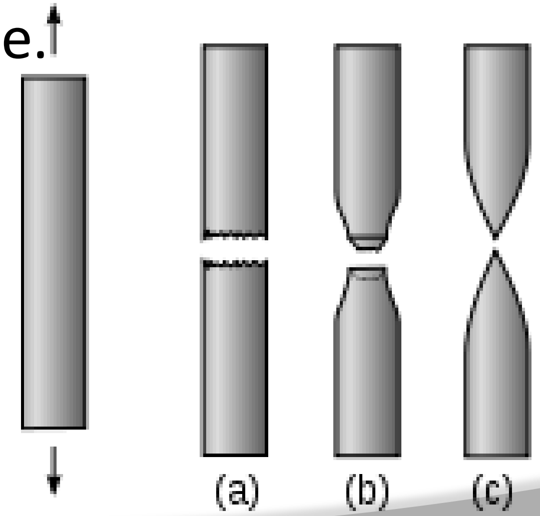
- Solid mechanics is the branch of mechanics that studies the behavior of solid materials.
- Motion and deformation of material under action of
  - Force
  - Temperature change
  - Phase change
  - Other external or internal agents
- These changes lead us to some properties that are called Mechanical properties

MECHANICAL PROPERTIES

- Some of the Mechanical Properties
  - Ductility
  - Hardness
  - Impact resistance
  - Fracture toughness
  - Elasticity
  - Fatigue strength
  - Endurance limit
  - Creep resistance
  - Strength of material

MECHANICAL PROPERTIES

- Ductility: **ductility** is a solid material's ability to deform under tensile stress.
- **Hardness of a material** may refer to resistance to bending, scratching, abrasion or cutting.
- **Impact resistance** is the ability of a material to withstand a high force or shock applied to it over a short period of time.
- **Plasticity** is ability of a material to deform permanently by the application of force.



MECHANICAL PROPERTIES

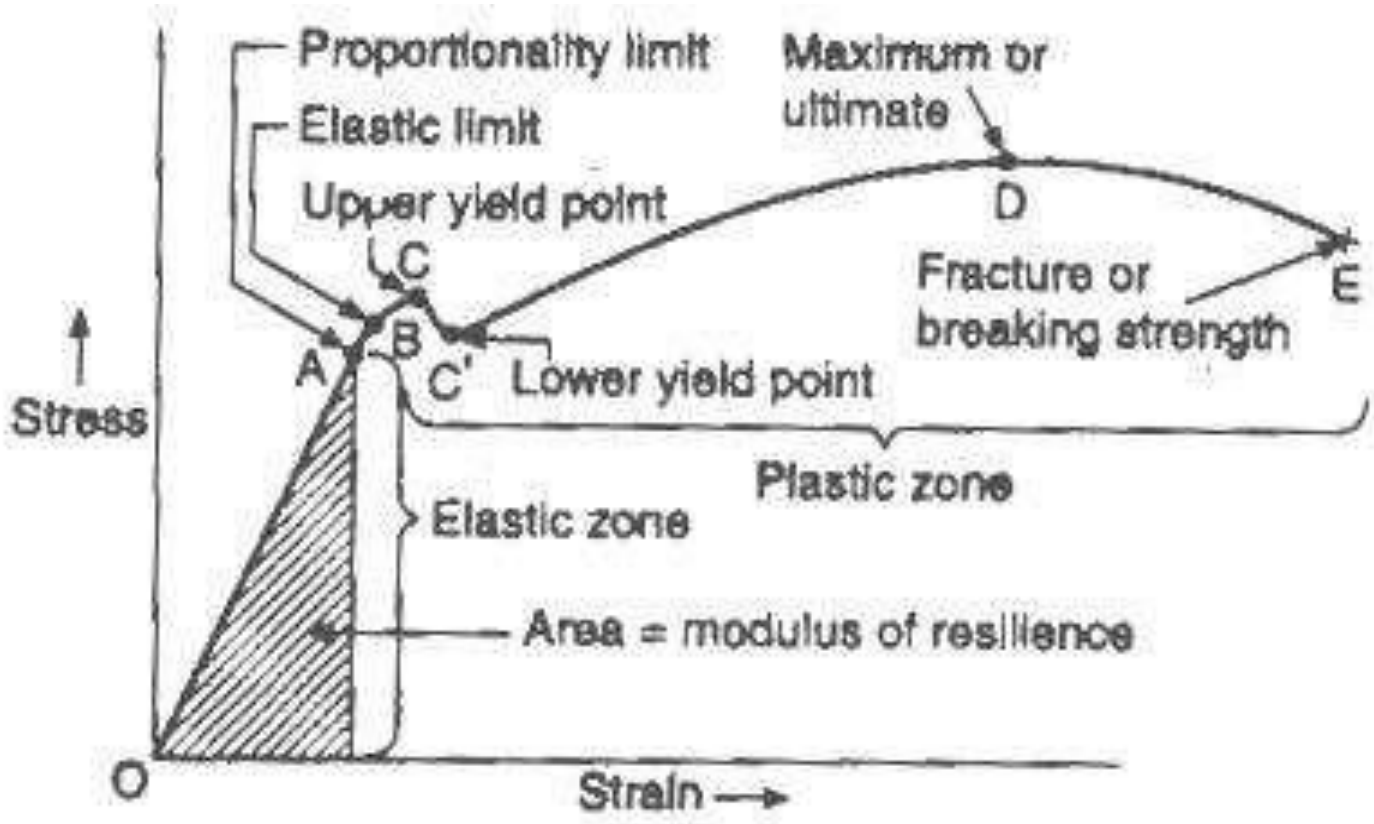
- **Fracture toughness** is a property which describes the ability of a material containing a crack to resist **fracture**
- **Elasticity** is the tendency of solid **materials** to return to their original shape after being deformed
- **Endurance strength/ Fatigue strength:** The highest stress that a material can withstand for a given number of cycles without breaking —called also **endurance strength**
- **Endurance limit:** In fatigue testing, the maximum stress which can be applied to a material for an infinite number of stress cycles without resulting in failure of the material is called Endurance limit
- **Creep Resistance:** It's the ability of a material not to deform permanently or slowly under the influence of Mechanical Stress.

STRESS AND STRENGTH

- **Stress in a material**: In solid mechanics, stress is a physical quantity that express the internal force per unit area that neighboring particles of a continuous material exert on each other.
- **Strength of material:** it is the measurement in engineering of the capacity of metal, wood, concrete, and other materials to withstand stress and strain.
- **Strain:** It is the deformation of material due to stress

- Different strengths are
  - Yield strength/ Tensile strength
  - Ultimate Tensile strength
  - Rupture strength
  - Compressive strength
  - Impact strength

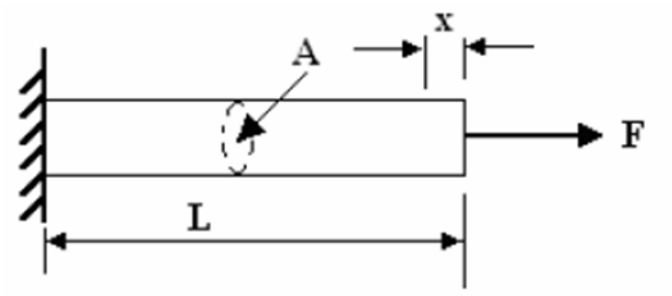
Stress strain Curve of MildIron



Types of Stresses & Strains

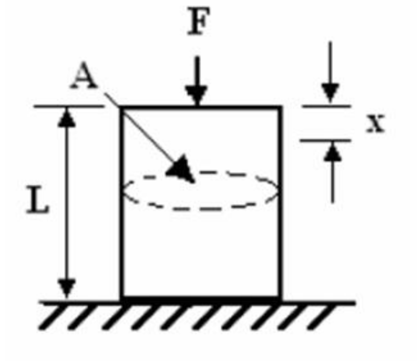
Direct Stress (σ)

When a force is applied to an elastic body, the body deforms. The way in which the body deforms depends upon the type of force applied to it.



Tensile Stress due to *tensile force*

A tensile force makes the body longer



Compressive Stress due to *compressive force*

A Compression force makes the body shorter.

- Resistance offered by the material per unit cross- sectional area is called STRESS

- Tensile and compressive forces are called DIRECT FORCES

- Stress is the force per unit area upon which it acts.

$$Stress = \sigma = \frac{Force}{Area} = \frac{F}{A} \quad N/m^2$$

LONGITUDINAL STRAIN

In each case, a force 'F' produces a deformation 'x' . In engineering, we usually change this force into stress and the deformation into strain and we define these as follows:

- Strain is the deformation per unit of the original length.

Strain, $\varepsilon = \Delta L/L$ = Change in length/ Original length
(ε is called as Epsilon)

- Strain has no unit's since it is a ratio of length to length

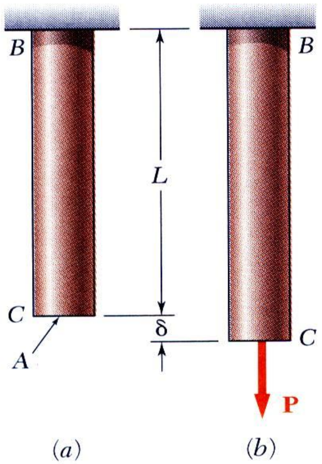


Fig. 2.1

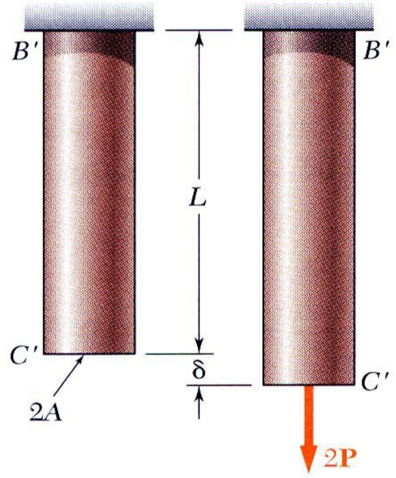


Fig. 2.3

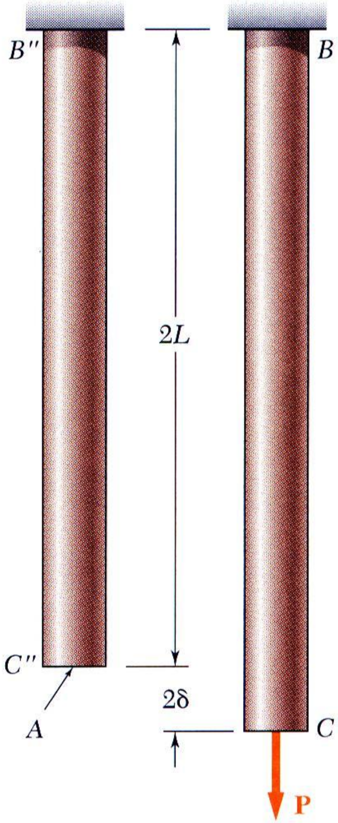


Fig. 2.4

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Hooke's law

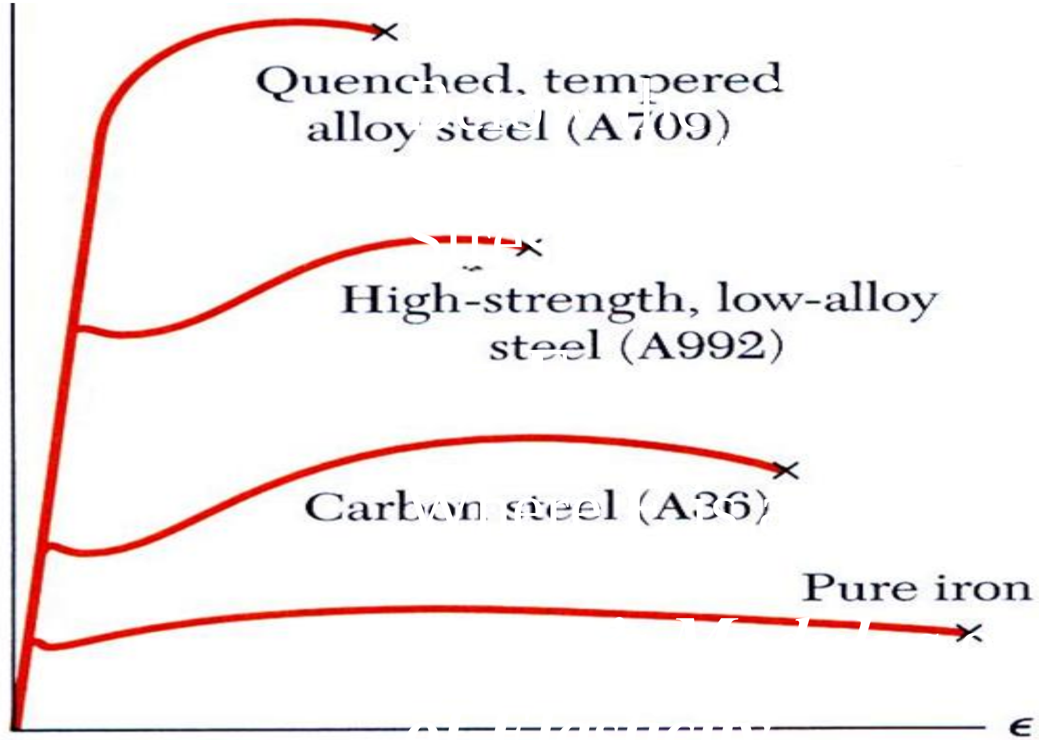


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

- Hooke's Law:-

Up to elastic limit, Stress is proportional to strain

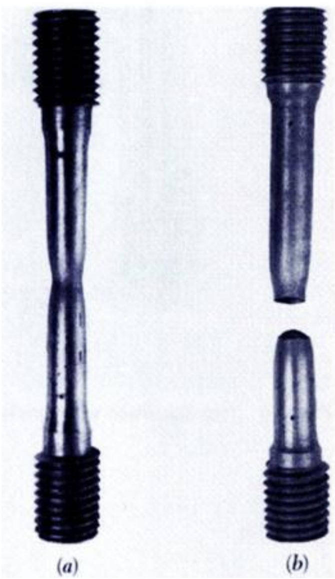
$\sigma \alpha \in$

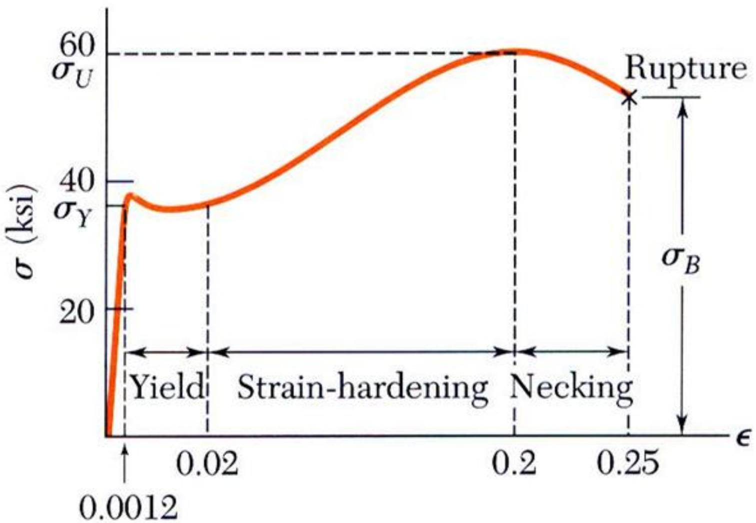
$\sigma = E \in$**;** where E=Young's modulus

$\sigma = P/A$ and $\in = \delta / L$

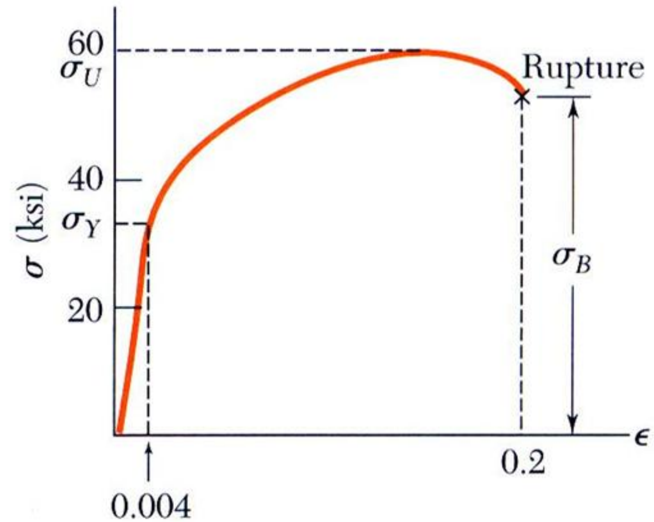
$P/A = E (\delta / L)$ **$\delta = PL / AE$**

Stress-Strain diagram



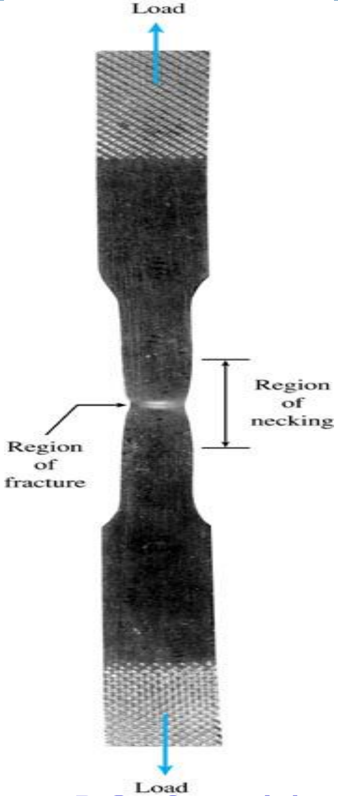


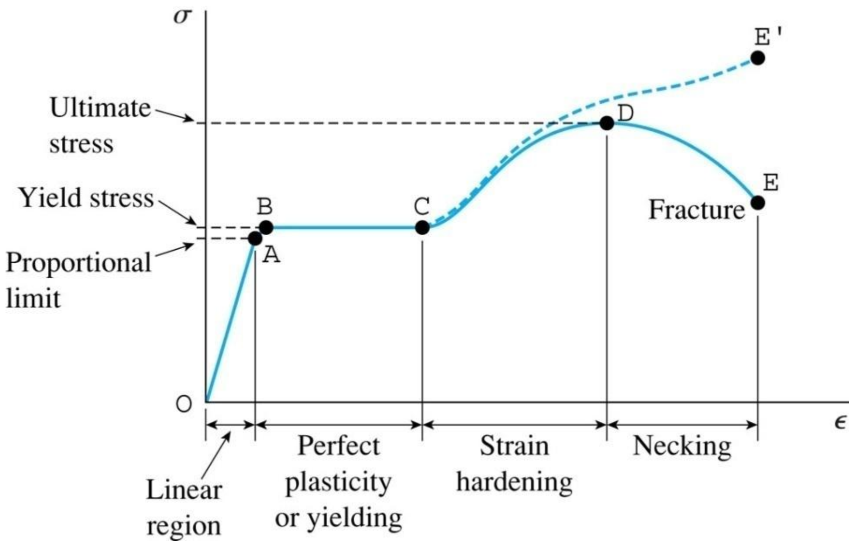
(*a*) Low-carbon steel



(*b*) Aluminum alloy

Stress-strain diagram for a typical structural steel in tension (not to scale)





OA: Initial region which is linear and proportional

Slope of OA is called modulus of elasticity

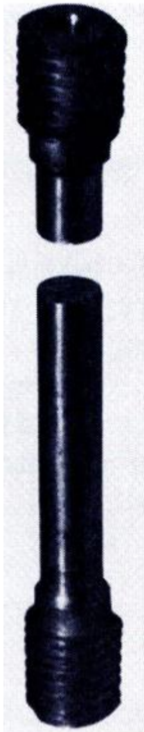
BC: Considerable elongation occurs with no noticeable increase in stress (yielding)

CD: Strain hardening – changes in crystalline structure (increased resistance to further deformation)

DE: Further stretching leads to reduction in the applied load and fracture

OABCE': True stress-strain curve

Stress (σ) – strain (ε) diagrams



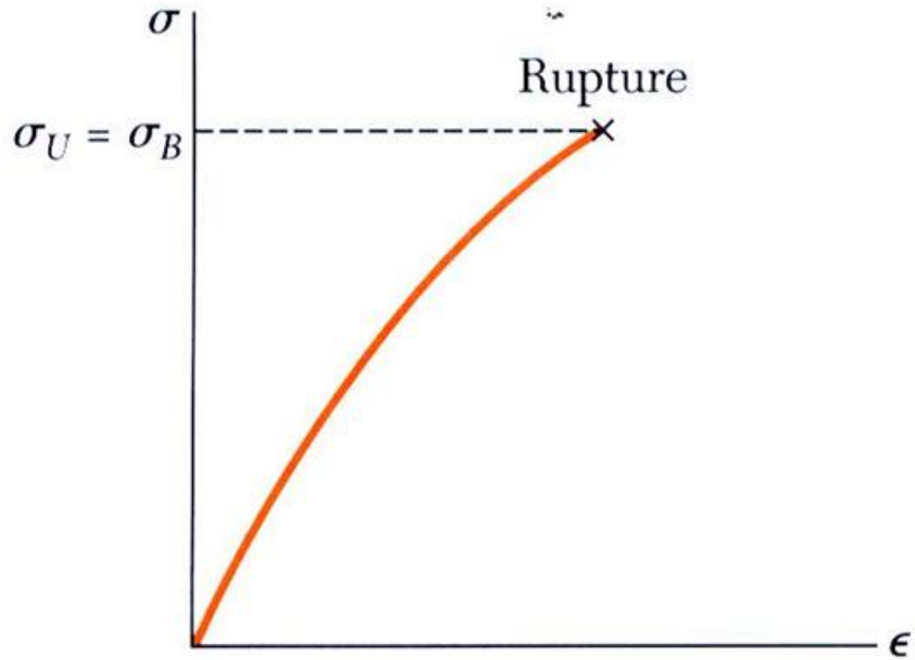


Fig. 2.11 Stress-strain diagram for a typical brittle material.

Working stress

- **The stress to which the material may be safely subjected in the course of ordinary use. Also called as Allowable Load or Allowable stress**

- **Max load that a structural member/machine component will be allowed to carry under normal conditions of utilization is considerably smaller than the ultimate load**

- **This smaller load = Allowable load / Working load / Design load**

- **Only a fraction of ultimate load capacity of the member is utilised when allowable load is applied**

- **The remaining portion of the load-carrying capacity of the member is kept in reserve to assure its safe performance**

Lateral strain

- "The strain at right angles to the direction of applied load is known as LATERAL STRAIN."

(OR)

- "Change in breadth dimensions to original dimension is also known as LATERAL STRAIN"

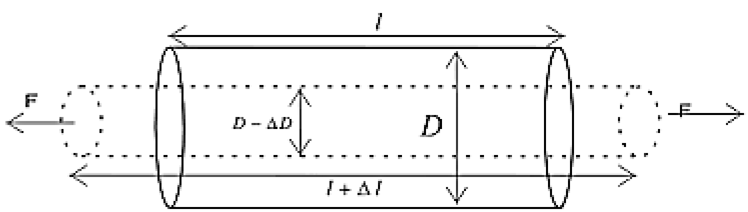
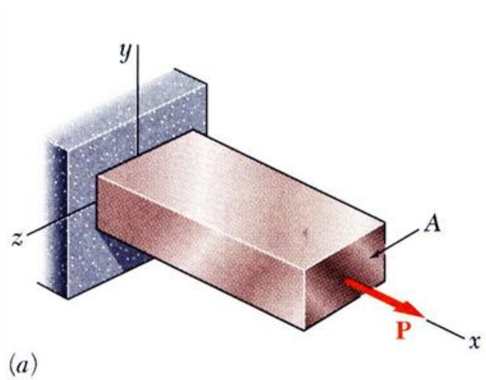
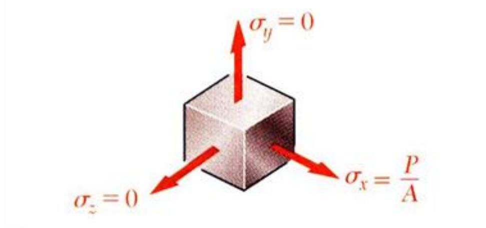


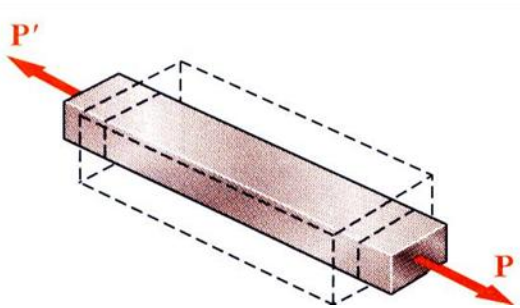
Figure 4.

- $$Lateral\ Strain = \frac{Change\ in\ Lateral\ dimensions}{Original\ dimensions}$$

Poisson’s ratio







- For a slender bar subjected to axial loading:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \sigma_y = \sigma_z = 0$$

- The elongation in the x-direction is accompanied by a contraction in the other directions. Assuming that the material is isotropic (no directional dependence),

$$\varepsilon_y = \varepsilon_z \neq 0$$

- Poisson’s ratio is defined as

$$\nu = \left| \frac{\text{lateral strain}}{\text{axial strain}} \right| = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

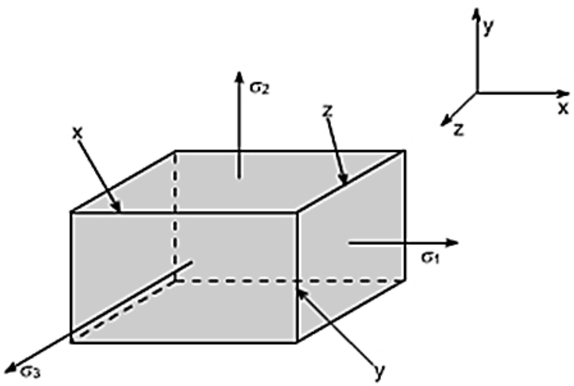
Volumetric strain

- **It is the unit change in volume due to a deformation. It is an important measure of deformation.**

Consider a rectangular solid of sides **x**, **y** and **z** under the action of principal stresses $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$, $\boldsymbol{\sigma_3}$ respectively.

Then $\boldsymbol{\varepsilon_1}$, $\boldsymbol{\varepsilon_2}$, and $\boldsymbol{\varepsilon_3}$ are the corresponding linear strains, than the dimensions of the rectangle and it becomes

$(x + \varepsilon_1 . x); (y + \varepsilon_2 . y); (z + \varepsilon_3 . z)$



$$\text{Volumetric strain} = \frac{\text{Increase in volume}}{\text{Original volume}}$$

$$= \frac{x(1+\varepsilon_1)y(1+\varepsilon_2)(1+\varepsilon_3)z - xyz}{xyz}$$

$$= (1+\varepsilon_1)y(1+\varepsilon_2)(1+\varepsilon_3) - 1 \cong \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \left[\text{Neglecting the products of } \varepsilon\text{'s}\right]$$

Torsion

- **Torsion :**It is the twisting of an object due to an applied torque (twisting moment)is expressed in N·m

  - $\tau = \overline{Tjr}$

$$\varphi = \frac{T\ell}{GJ_T}.$$

  - T = is the applied torque or moment of torsion in Nm
  - τ =is the maximum shear stress at the outer surface
  - J= Polar moment of Inertia
  - r= is the distance between the rotational axis and the farthest point
    in the section (at the outer surface).
  - l = is the length of the object
  - φ= is the angle of twist in radians.
  - G= Modulous of Rigidity

- The angular frequency can be calculated with the following formula:

$$\omega = 2\pi f$$

- The torque carried by the shaft is related to the power by the following equation:

$$P = T\omega$$

MODULE-II
Forces and deflections in Beams

CLOs	Course Learning Outcome
CLO 4	Solve for bending and shear parameters of beams under loading conditions
CLO 5	Explain for deflections of beams under loading with various approaches.
CLO 6	Determine the deflections of different beams under different loading conditions.

APPLIED AND REACTIVE FORCES

- **Forces that act on a Body can be divided into two Primary types: applied and reactive.**
- **In common Engineering usage, applied forces are forces that act directly on a structure like, dead, live load etc.)**
- **Reactive forces are forces generated by the action of one body on another and hence typically occur at connections or supports.**
- **The existence of reactive forces follows from Newton's third law, which state that to every action, there is an equal and opposite reaction.**

SUPPORTS

- To bear or hold up (a load, mass, structure, part, etc.); serve as a foundation or base for any structure.
- To sustain or withstand (weight, pressure, strain, etc.) without giving way

- It is a aid or assistance to any structure by preserve its load

- Supports are used to connect structures to the ground or other bodies in order to restrict (confine) their movements under the applied loads.

- The loads tend to move the structures, but supports prevent the movements by exerting opposing forces, or reactions, to neutralize the effects of loads thereby keeping the structures in equilibrium.

TYPES OF SUPPORTS

- Supports are grouped into three categories, depending on the number of reactions (1,2,or3) they exert on the structures.

  1) Roller support

  2) Hinge support

  3) fixed support

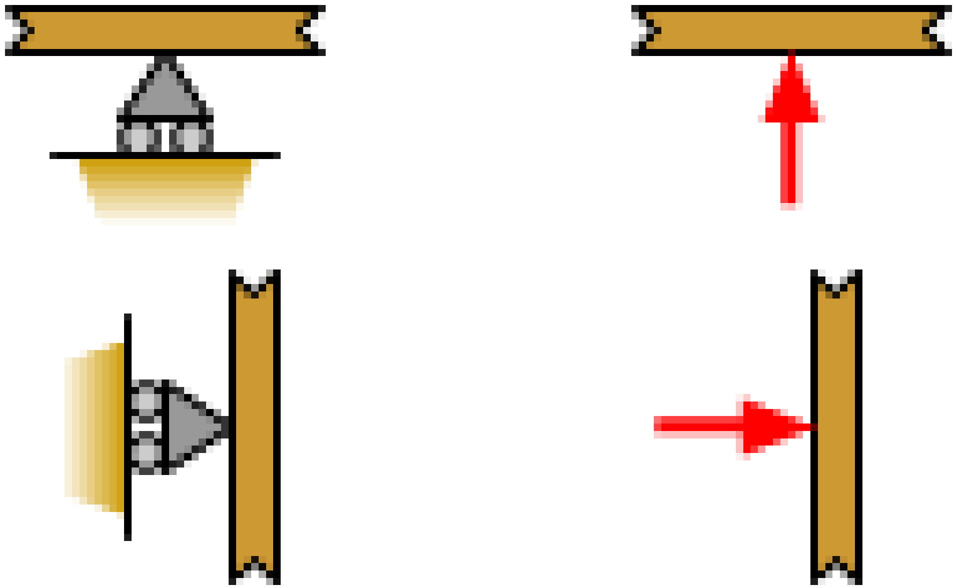
ROLLER SUPPORT

- Roller supports are free to rotate and translate along the surface upon which the roller rests.

- The surface can be horizontal, vertical, or sloped at any angle.

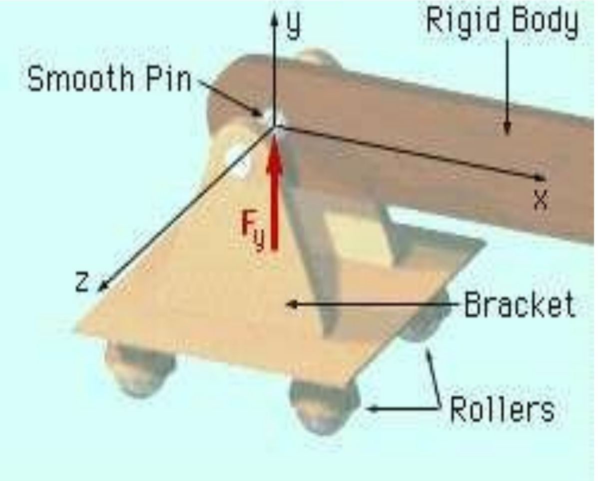
- The resulting reaction force is always a single force that is perpendicular to, and away from, the surface

ROLLER SUPPORT

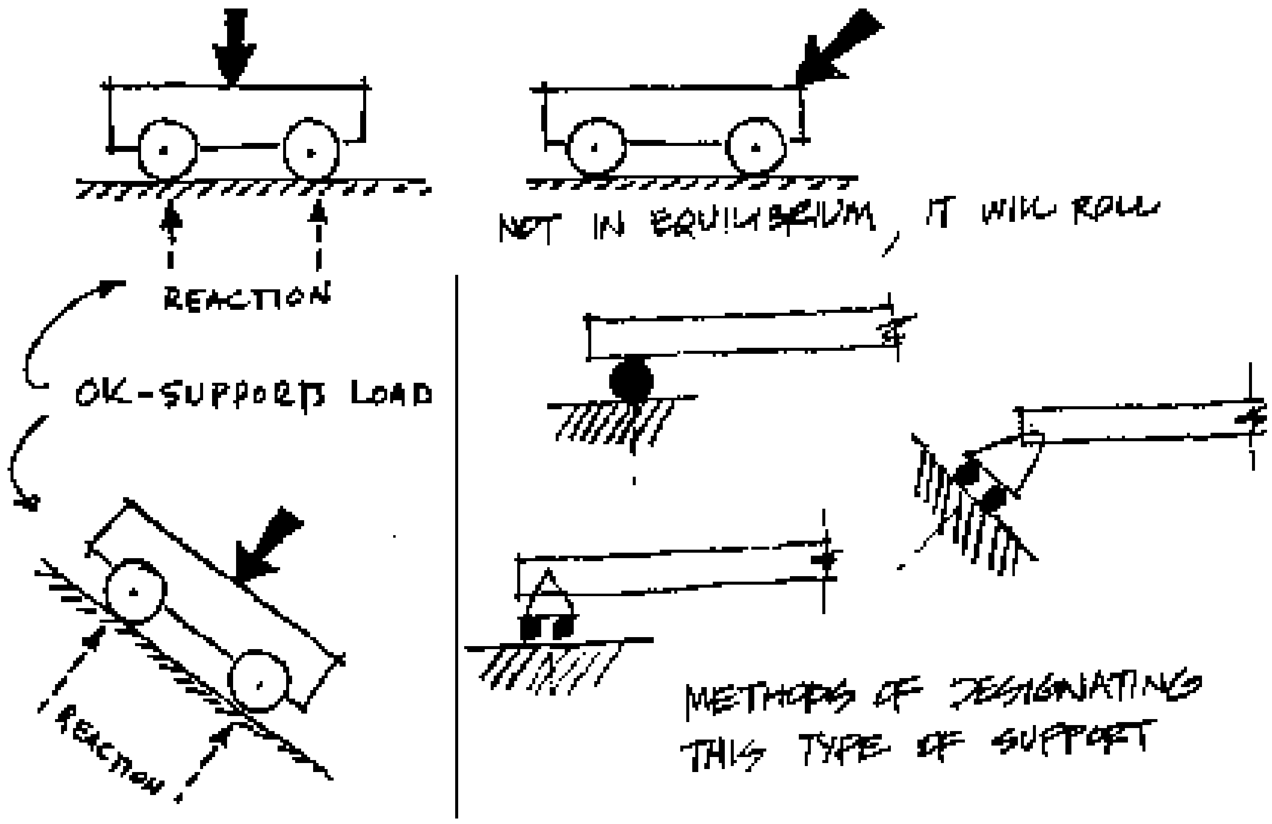


Restrains the structure from moving in one or two perpendicular directions.

ROLLER SUPPORT



ROLLER SUPPORT



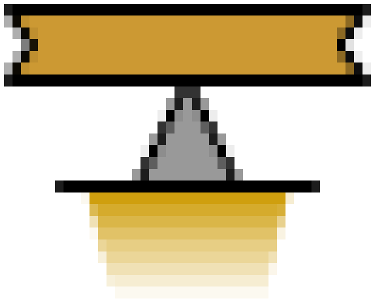
HINGE SUPPORT

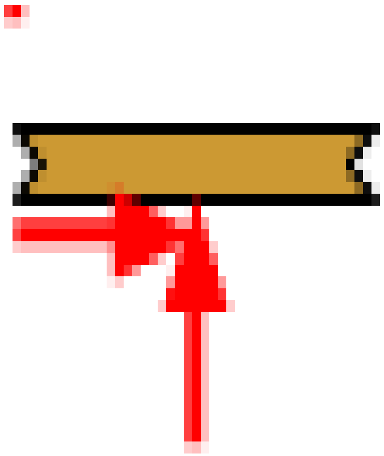
- A Hinge support can resist both vertical and horizontal forces but not a moment. They will allow the structural member to rotate, but not to translate in any direction

- Pin or hinge support is used when we need to prevent the structure from moving or restrain its translational degrees of freedom.

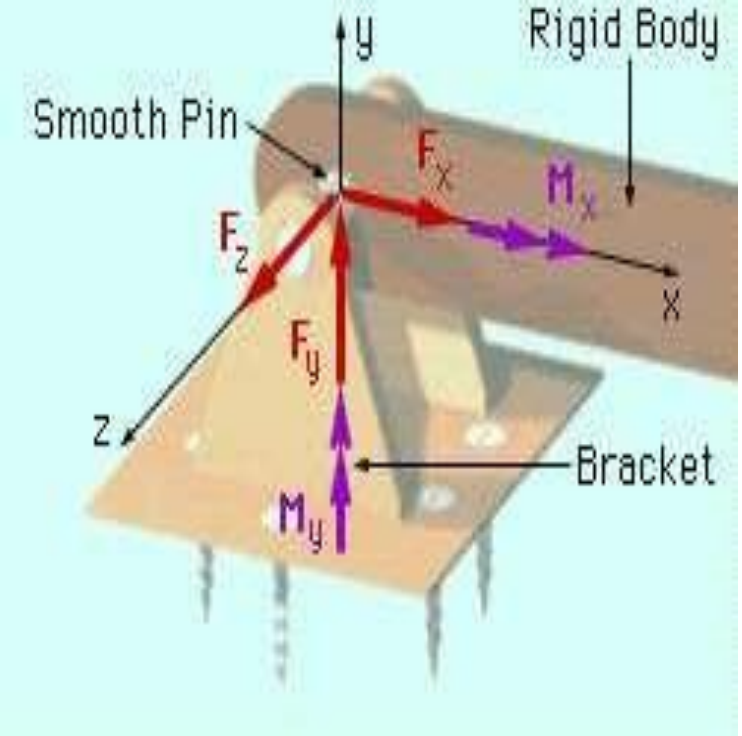
HINGE SUPPORT

- A **hinge** is a type of bearing that connects two solid objects, typically allowing only a limited angle of rotation between them. Two objects connected by an ideal hinge rotate relative to each other about a fixed axis of rotation.





HINGE SUPPORT



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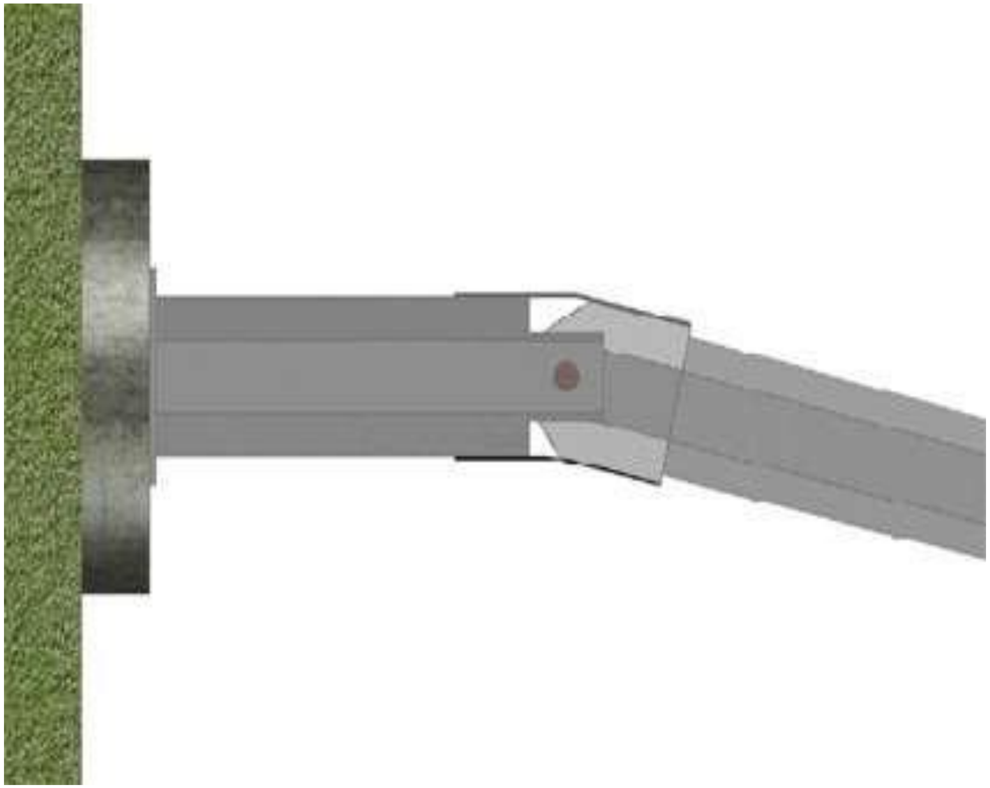

HINGE SUPPORT

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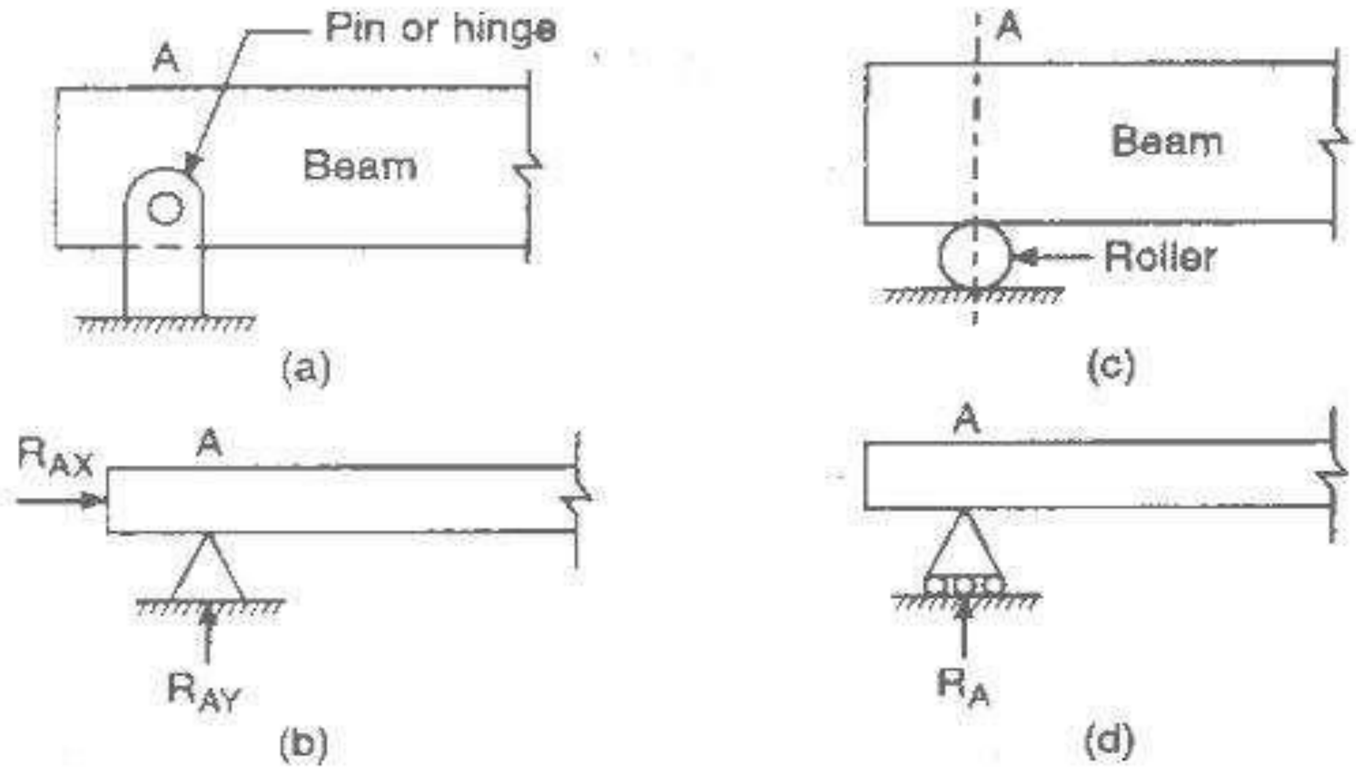
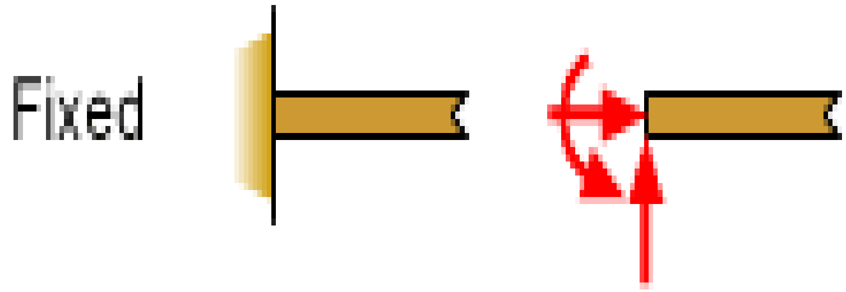
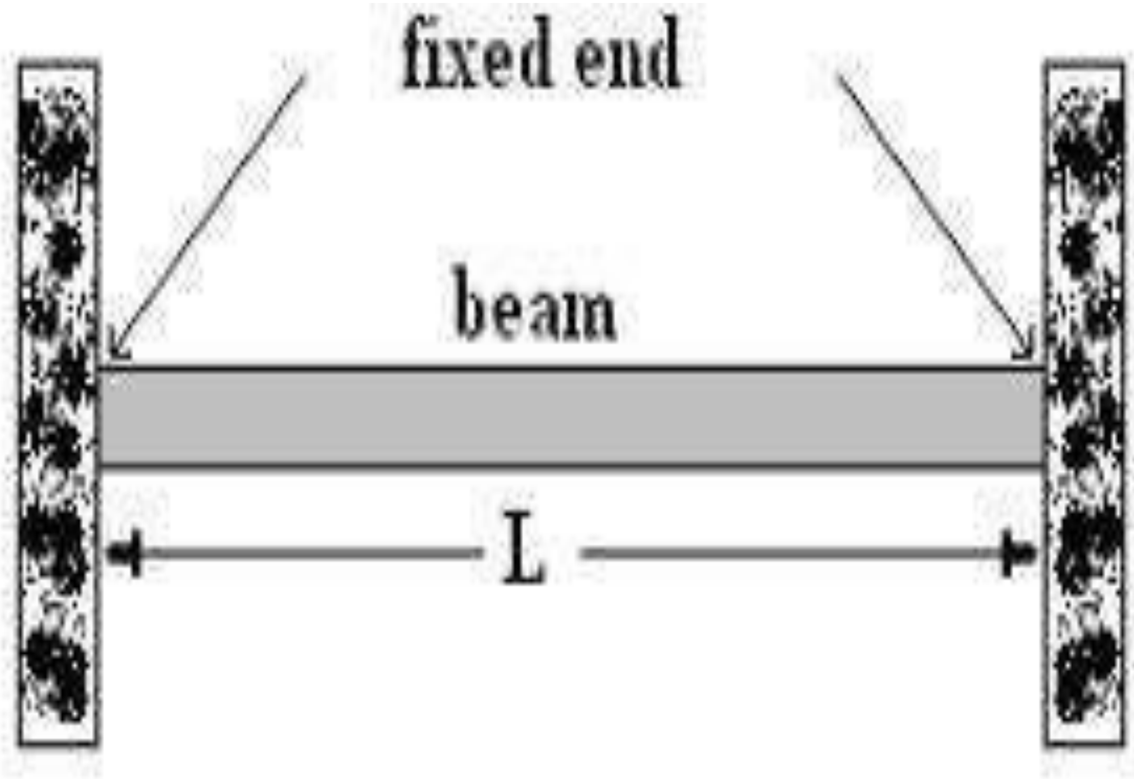


FIXED SUPPORT

- Fixed supports can resist vertical and horizontal forces as well as a moment. Since they restrain both rotation and translation, they are also known as rigid supports.



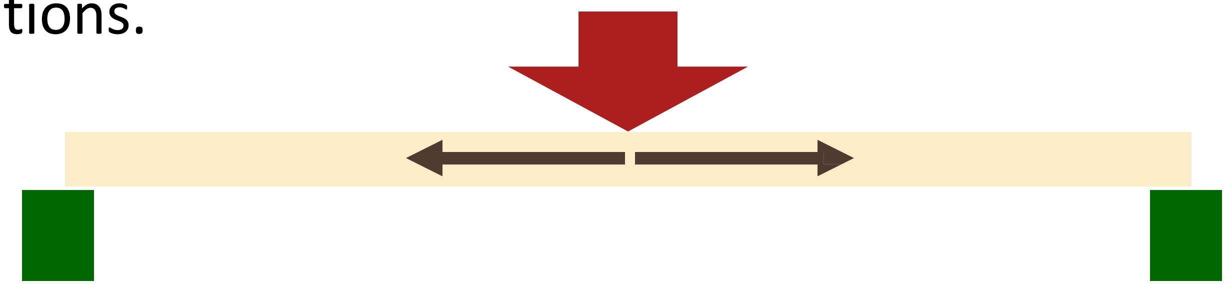
FIXED SUPPORT



BEAM

- A beam is a structural member (horizontal) that is design to support the applied load (vertical). It resists the applied loading by a combination of internal transverse shear force and bending moment.

- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.



BEAM

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- It is perhaps the most important and widely used structural members and can be classified according to its support conditions.

- In buildings majority of loads are vertical and majority of useable surfaces are horizontal
- Action of beams involves combination of bending and shear

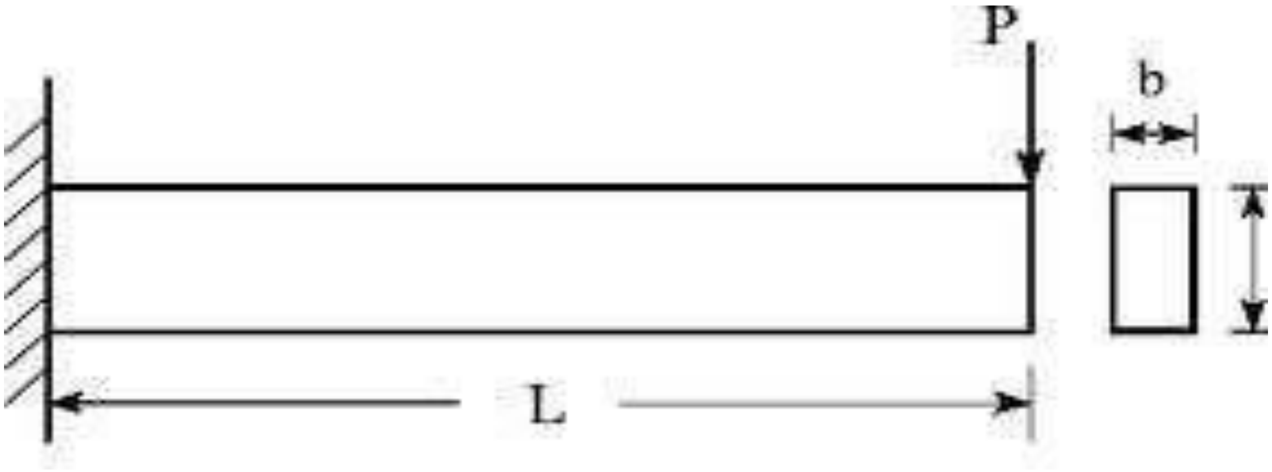
TYPES OF BEAMS

The following are the important types of beams:

1. Cantilever
2. Simply supported
3. Overhanging
4. Fixed beams
5. Continuous beam

CANTILEVER BEAM

- A beam which is fixed at one end and free at the other end is known as cantilever beam.



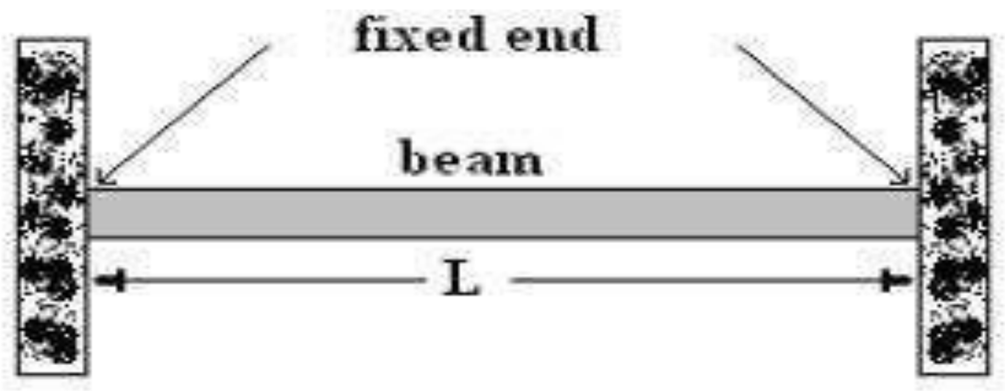
CANTILEVER BEAM

SIMPLY SUPPORTED BEAMS

- A beam supported or resting freely on the supports at its both ends,

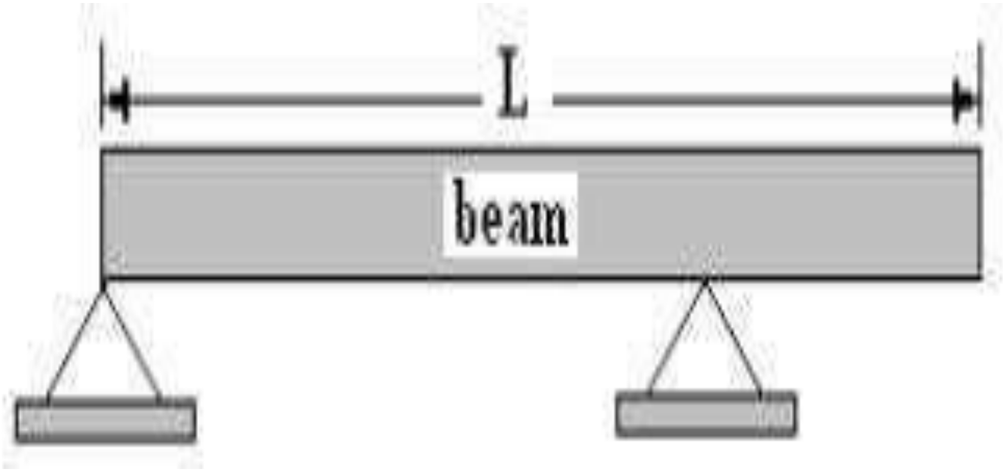
FIXED BEAMS

A beam whose both ends are fixed and is restrained against rotation and vertical movement. Also known as built-in beam or encastred beam.



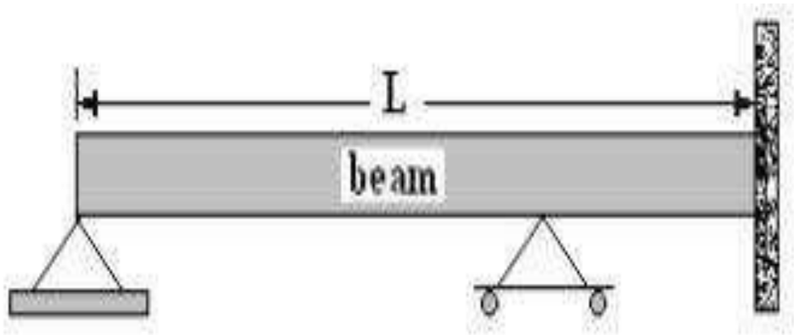
OVERHANGING BEAM

- If the end portion of a beam is extended outside the supports.



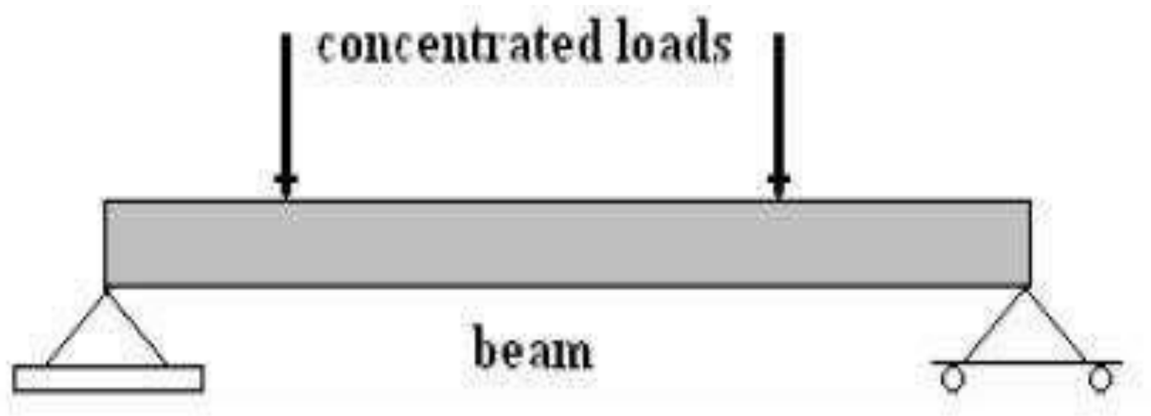
CONTINUOUS BEAMS

A beam which is provided with more than two supports.



TYPES OF LOADS

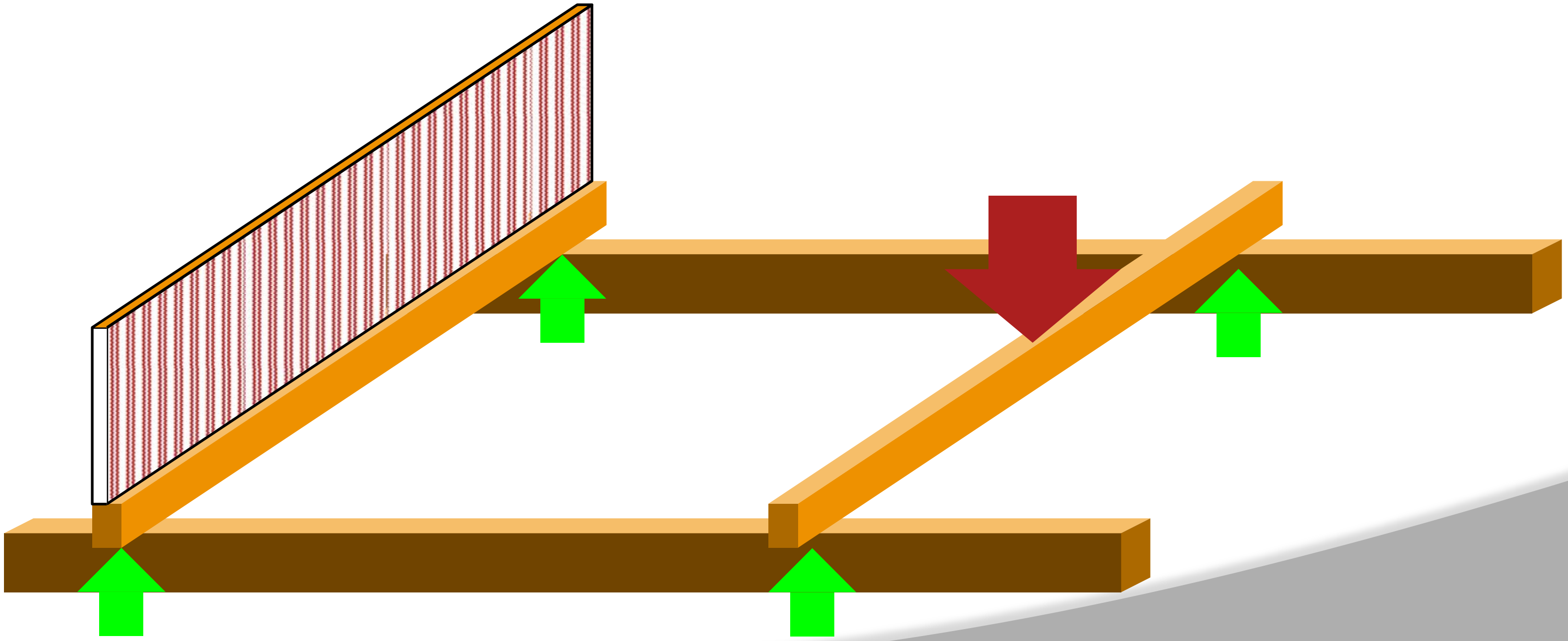
- Concentrated load assumed to act at a point and immediately introduce an oversimplification since all practical loading system must be applied over a finite area.



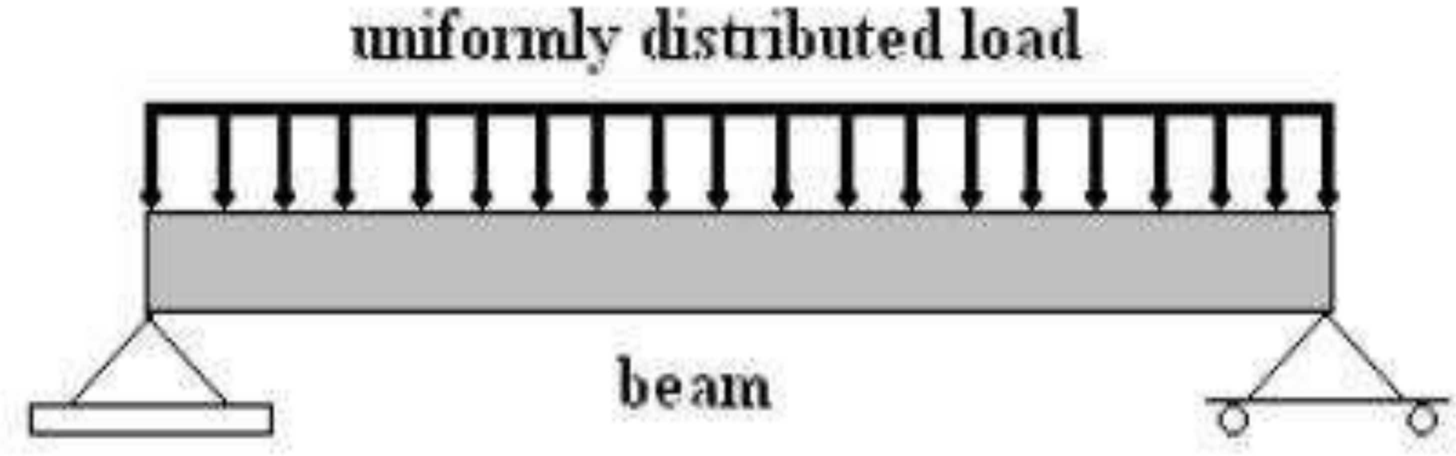
LOADS ON BEAMS

- Point loads, from concentrated loads or other beams

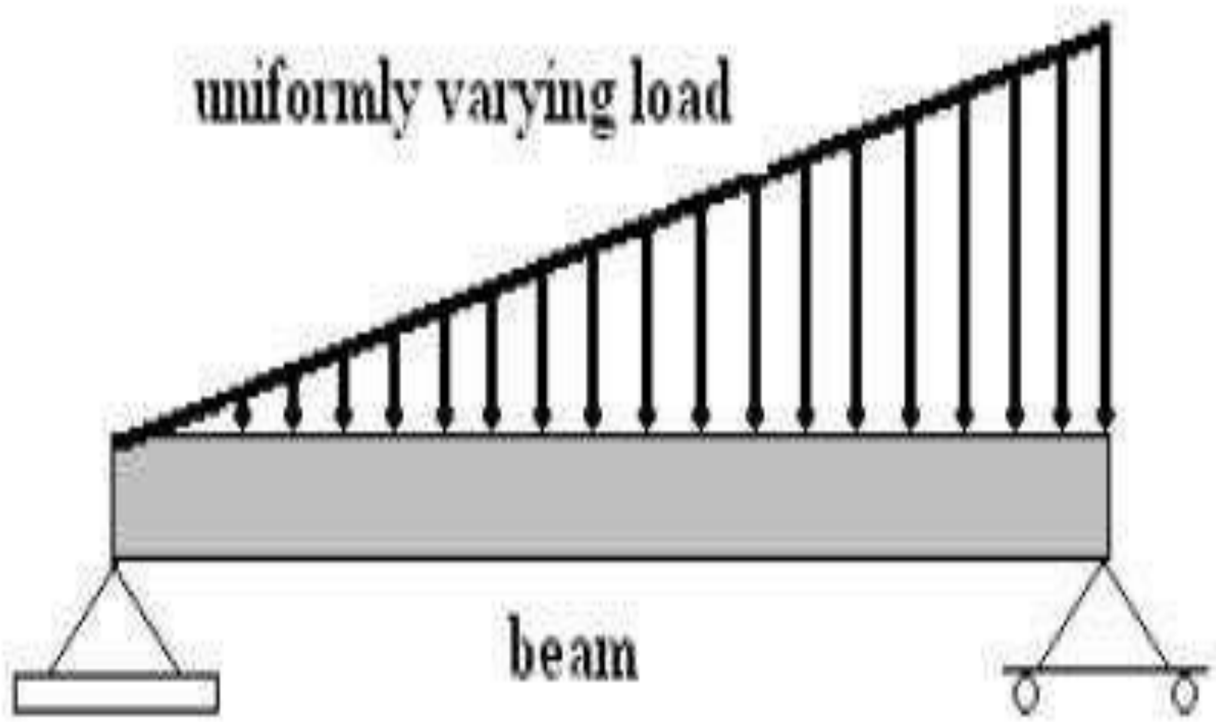
- Distributed loads, from anything continuous



UNIFORMLY DISTRIBUTED LOAD

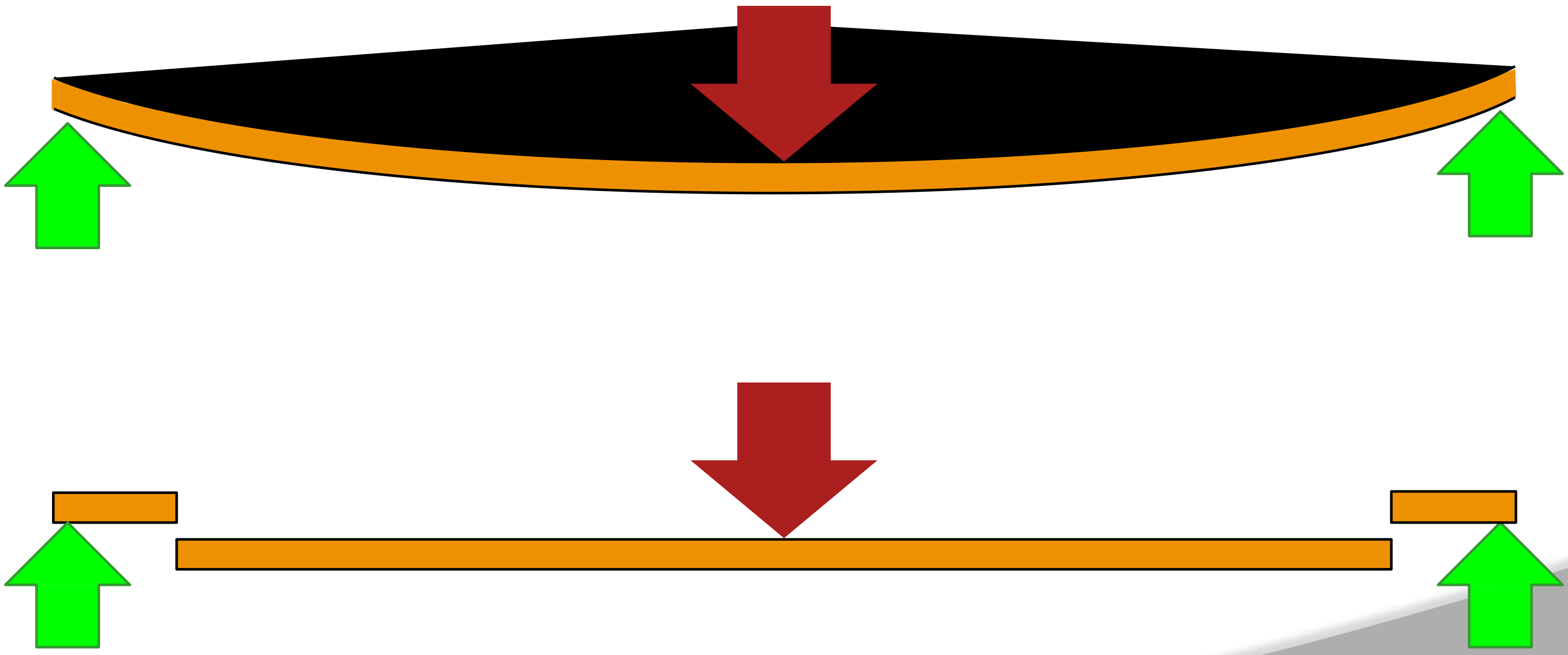


UNIFORMLY VARIYING LOAD

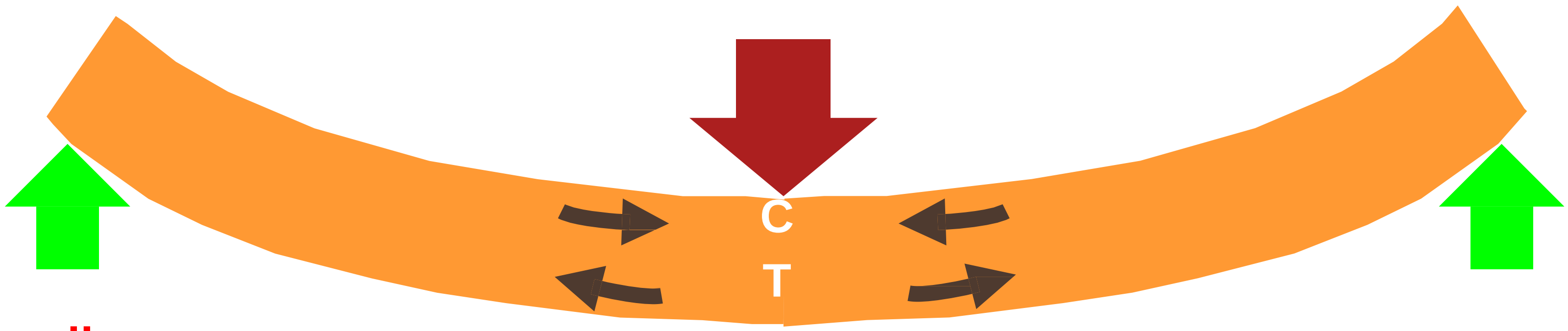


WHAT THE LOADS DO

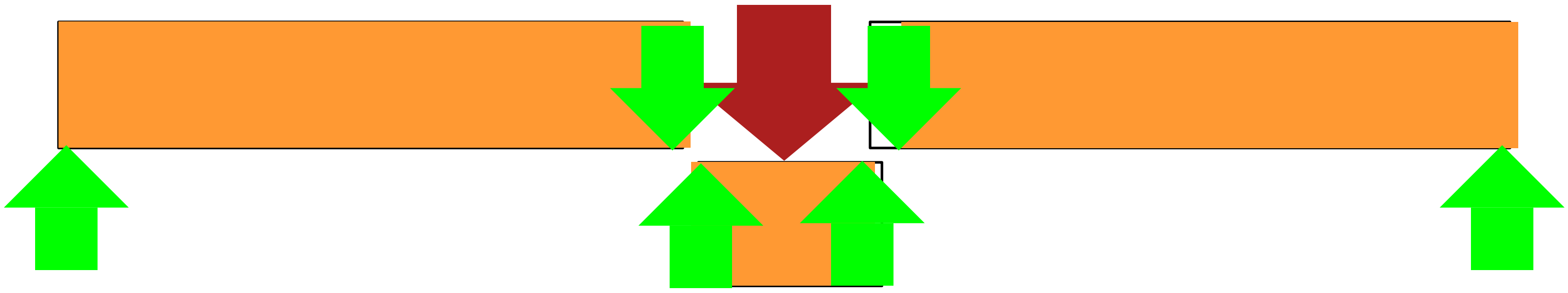
- The loads (& reactions) bend the beam, and try to shear through it



WHAT THE LOADS DO



Bending



Shear

DESIGNING BEAMS

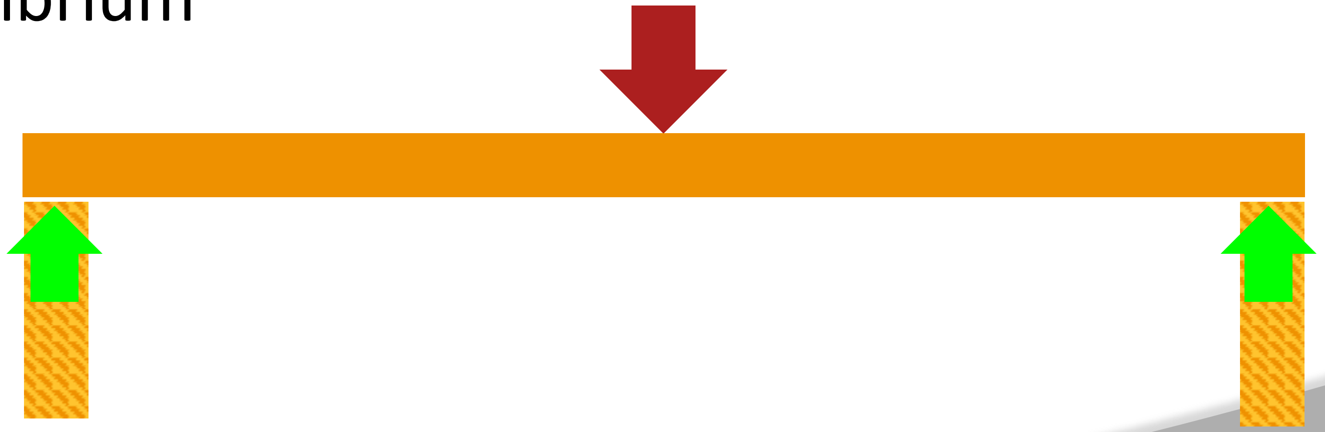
- In architectural structures, bending moment more important increases as span increases.

- Short span structures with heavy loads, shear dominant
  - e.g. pin connecting engine parts

- Beams in building designed for bending checked for shear

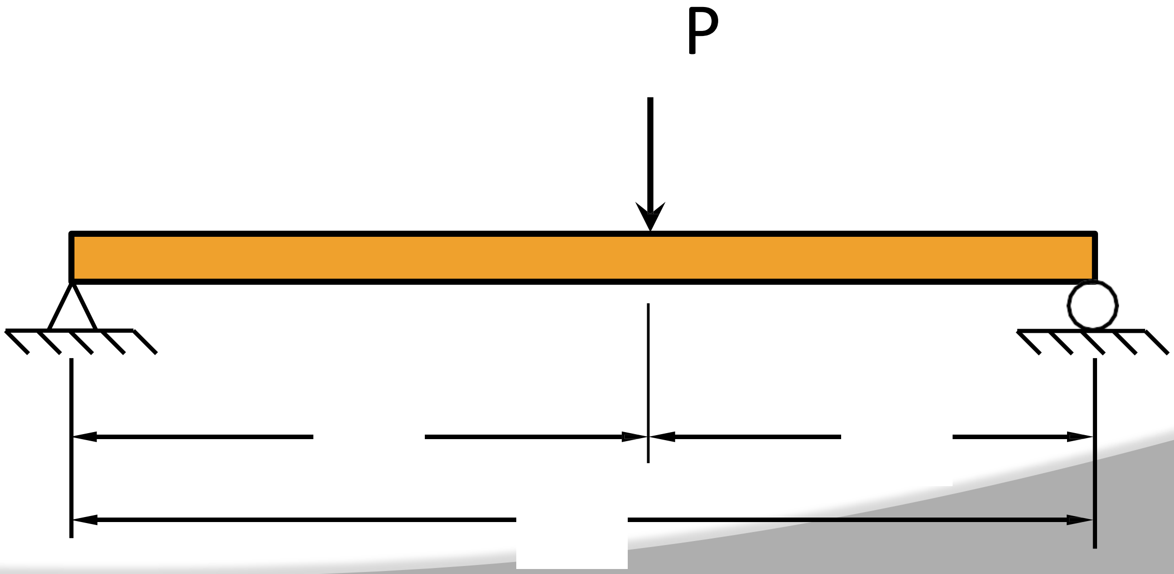
HOW WE CALCULATE THE EFFECTS

- First, find ALL the forces (loads and reactions)
- Make the beam into a free body (cut it out and artificially support it)
- Find the reactions, using the conditions of equilibrium



INTERNAL REACTIONS IN BEAMS

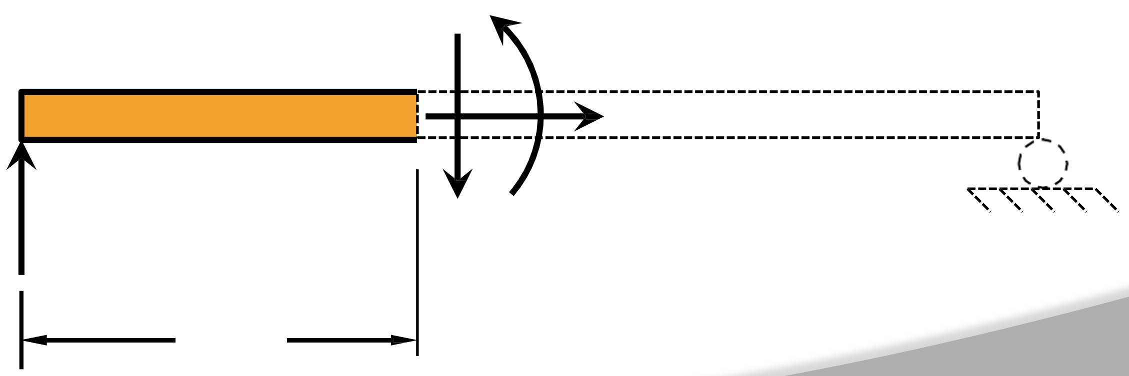
- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
  - normal force,
  - shear force,
  - bending moment.



INTERNAL REACTIONS IN BEAMS

- At any cut in a beam, there are 3 possible internal reactions required for equilibrium:
  - ➢normal force,
  - ➢shear force,
  - ➢bending moment.

Positive Directions Shown!!!



INTERNAL REACTIONS IN BEAMS

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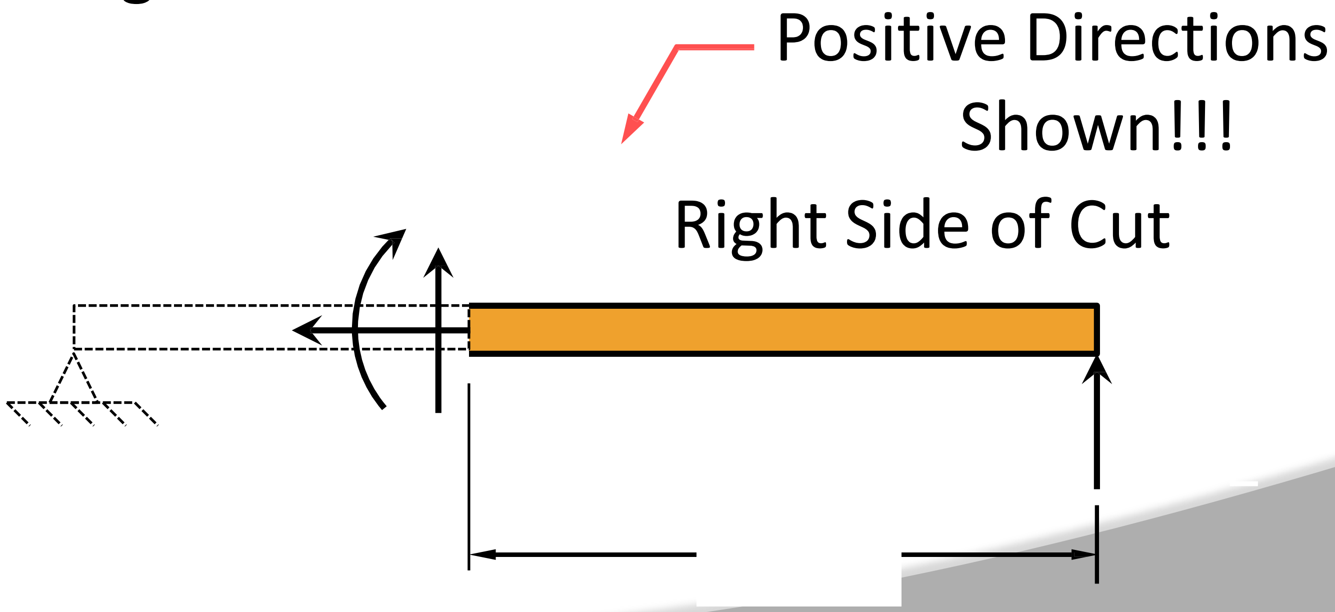
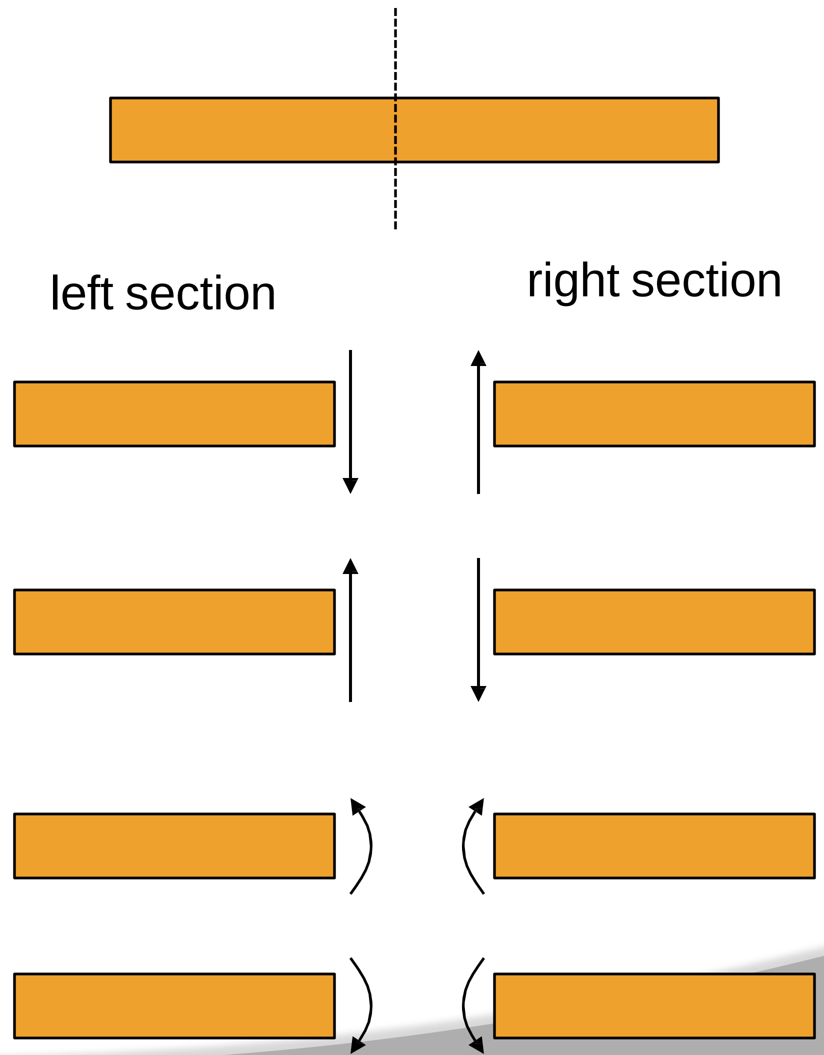


SHEAR FORCES, BENDING MOMENTS - SIGN CONVENTIONS



Shear forces:

positive shear:

Negative shear:

Bending moments:

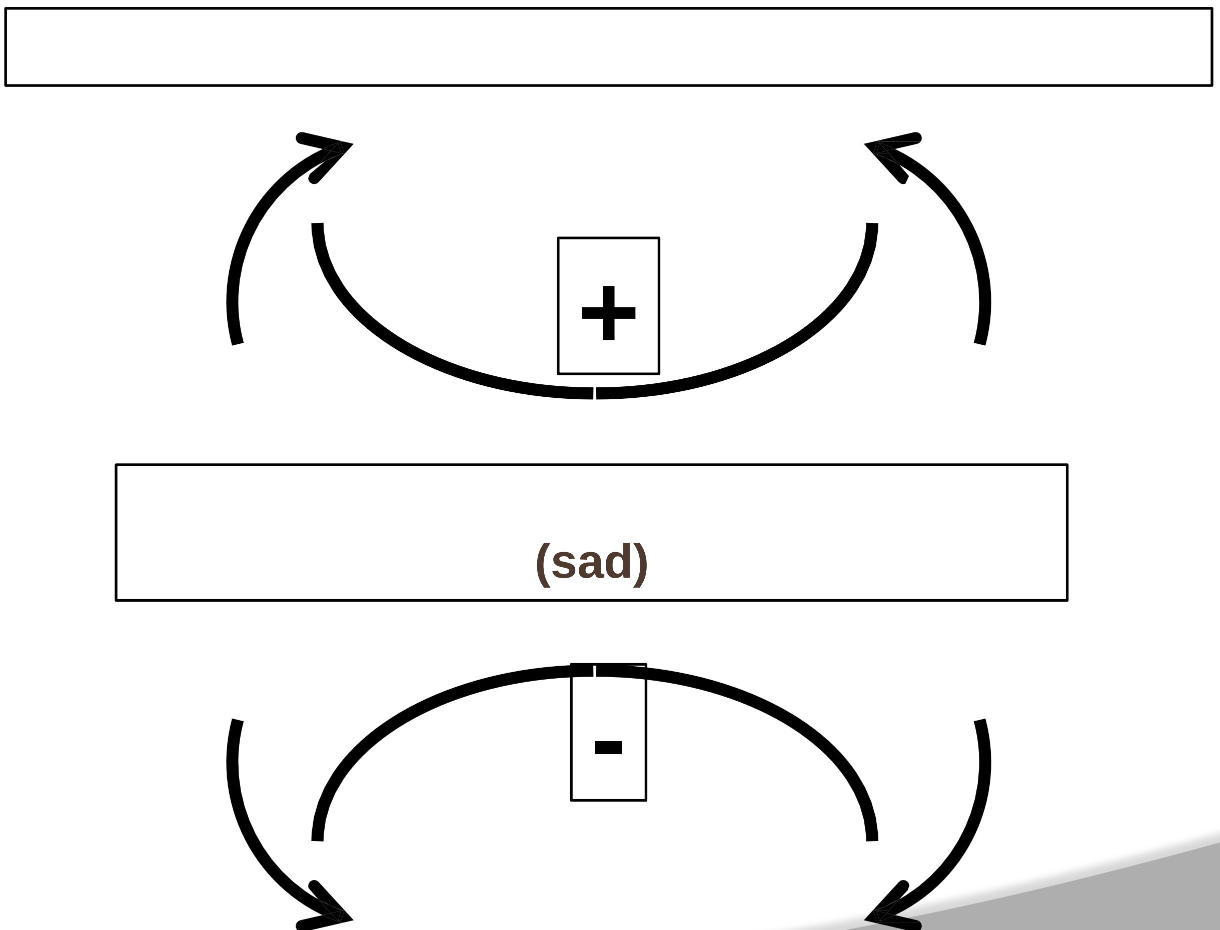
Negative moment

positive moment

C.W

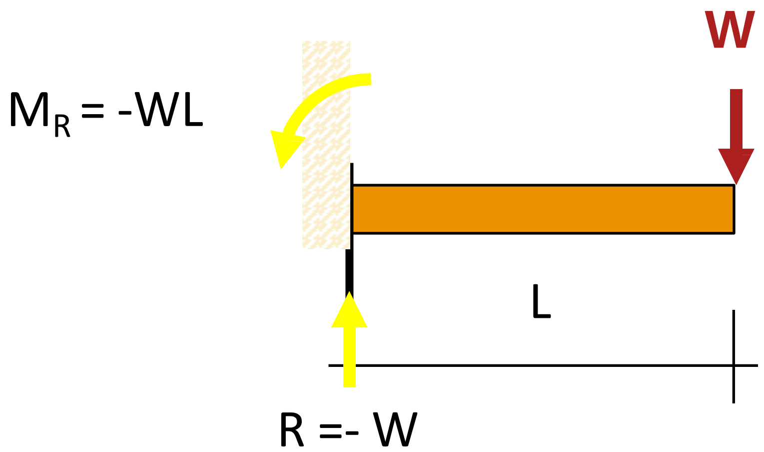
ACW

SIGN CONVECTIONS
BENDING MOMENT DIAGRAMS



CANTILEVER BEAM POINT LOAD AT END

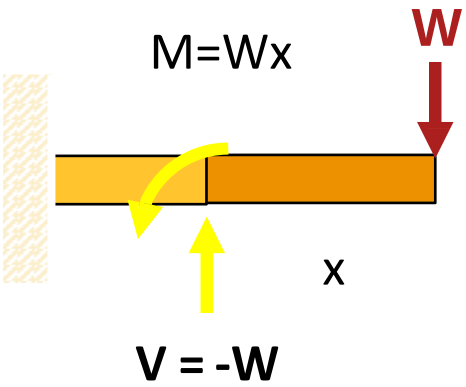
- Consider cantilever beam with point load on end



vertical reaction, R = -W
and moment reaction M_R = - WL

- Use the free body idea to isolate part of the beam
- Add in forces required for equilibrium

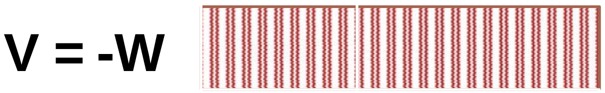
CANTILEVER BEAM POINT LOAD AT END



Take section anywhere at distance, x from end

Add in forces, V = -W and moment M = - Wx

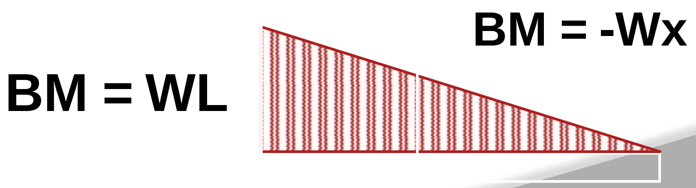
Shear V =- W constant along length



Shear Force Diagram

Bending Moment BM = -W.x

when x = L	**BM = -WL**
when x = 0	**BM = 0**



Bending Moment Diagram

UNIFORMLY DISTRIBUTED LOAD FOR CANTILEVER BEAM

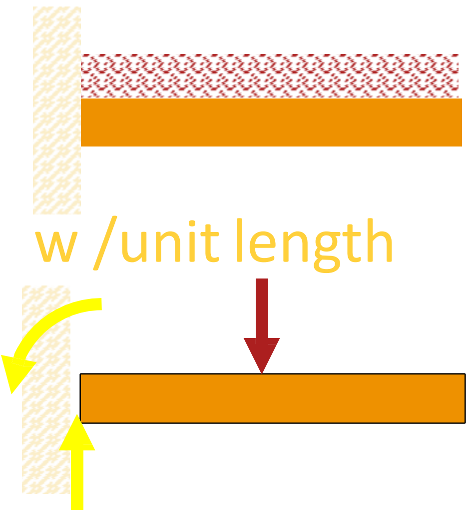
For maximum shear V and bending moment BM

Total Load = W L

M_R = -WL/2

= -wL²/2

R = W = wL



vertical reaction, R = W = wL

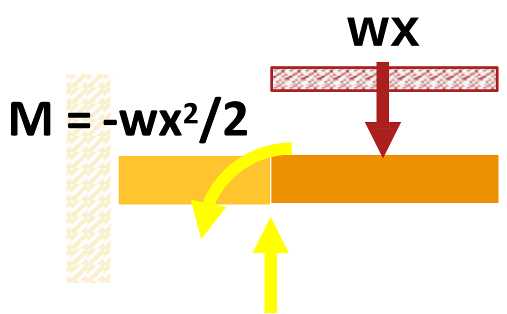
and moment reaction M_R = - WL/2 = - $wL^2/2$

CANTILEVER BEAM FOR UNIFORMLY DISTRIBUTED LOAD

For distributed V and BM

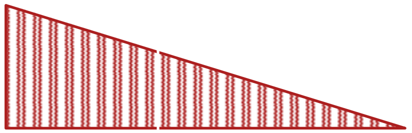
Take section anywhere at distance, x from end

Add in forces, V = w.x and moment M = - wx.x/2



Shear when V = wx
x = L when V = W = wL
x = 0 V = 0

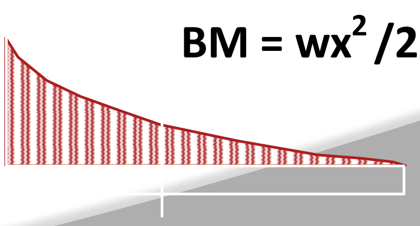
V = wL
= W



Shear Force Diagram

Bending Moment BM = w.x²/2
when x = L BM = wL²/2 = WL/2
when x = 0 BM = 0
(parabolic)

BM = wL²/2
= WL/2



Bending Moment Diagram

Fig. 6.22 shows a cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.

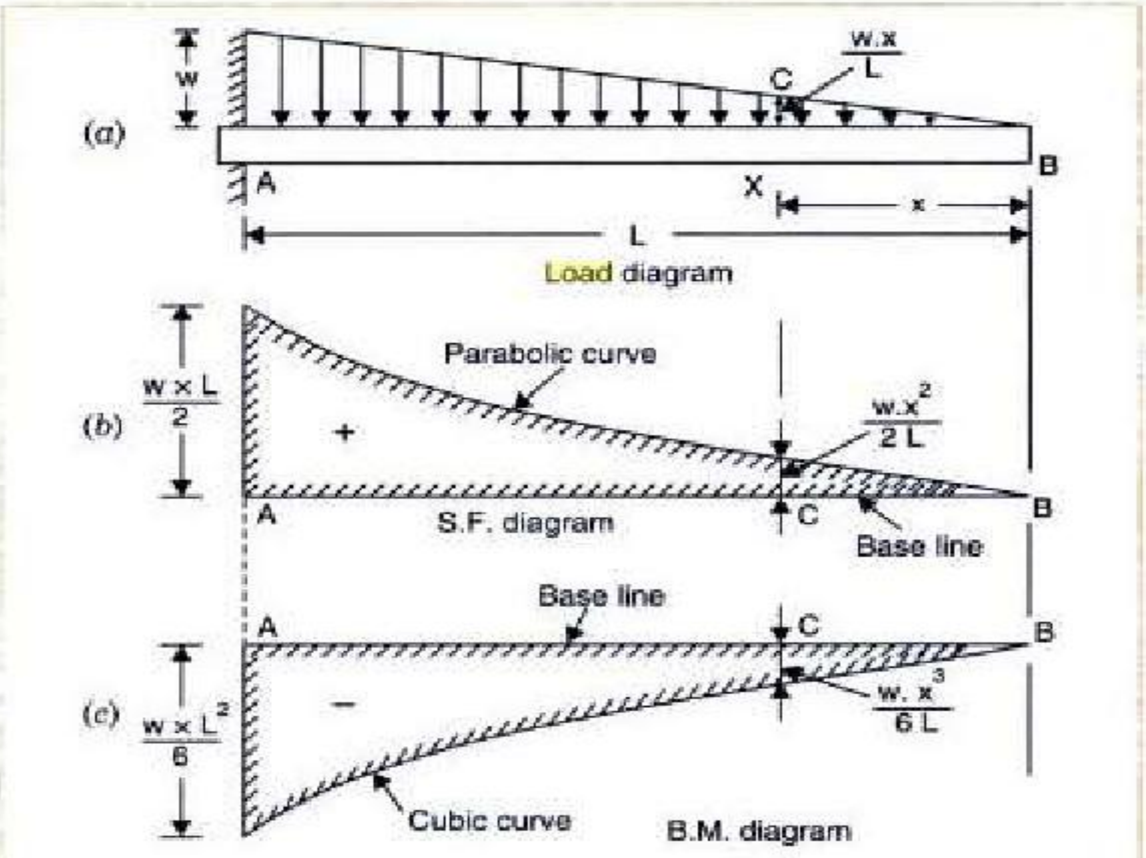


Fig. 6.22

Take a section X at a distance x from the free end B.

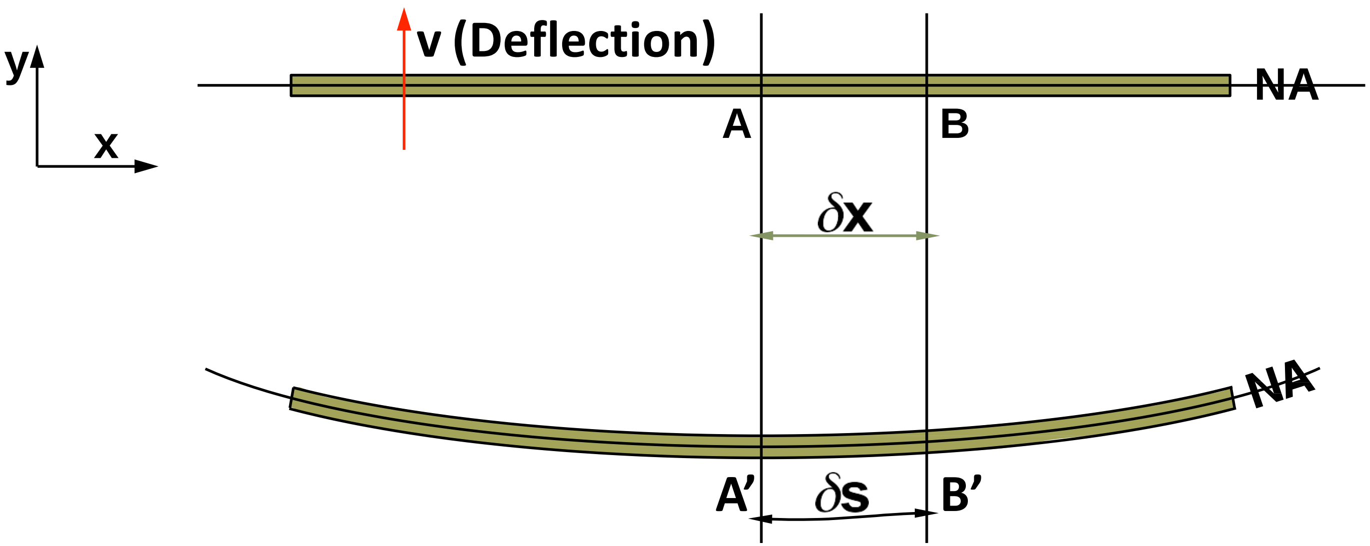
Let F_x = Shear force at the section X, and

BEAM DEFLECTION

Recall: THE ENGINEERING BEAM THEORY

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Moment-Curvature Equation



If deformation is small (i.e. slope is "flat"):

$$\delta s \approx \delta x$$

Methods to find slope and deflection

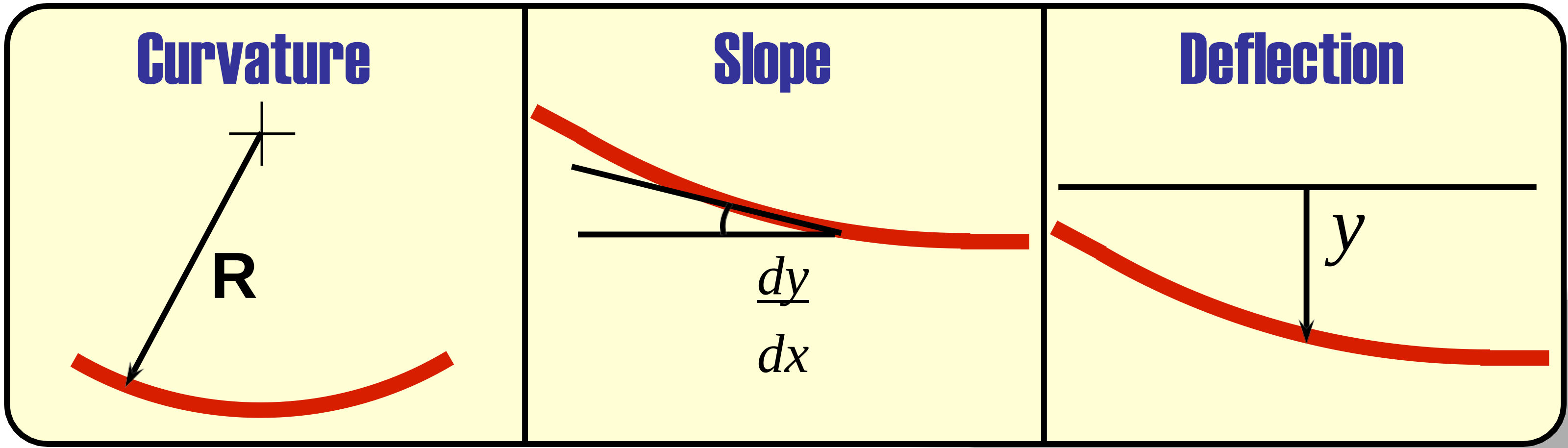
- Double integration method
- Moment area method
- Macaulay's method

Since, $$\frac{d^2y}{dx} = \frac{1}{EI}$$

$$\frac{dy}{dx} \quad \frac{1}{EI} \int M \cdot dx + C$$

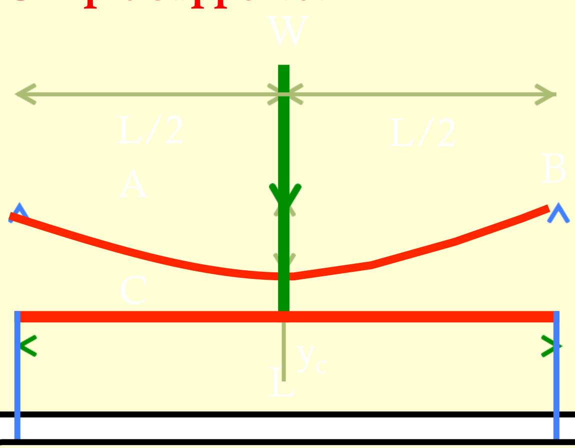
$$\Rightarrow y = \frac{1}{EI} \int \int M \cdot dx \cdot dx + \int C \cdot dx + C$$

Where C_1 and C_2 are found using the boundary conditions.



Double integration method

Simple supported



Slope

$$\text{Slope} = \frac{dy}{dx}$$

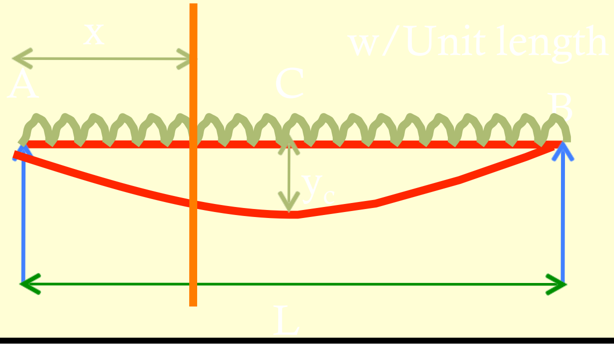
$$= \theta_A = \theta_B = -\frac{WL^2}{16EI}$$

Deflection

$$\text{Deflection} = y_c$$

$$= -\frac{WL^3}{48EI}$$

Uniform distributed load



Slope

$$\text{Slope} = \frac{dy}{dx}$$

$$= \theta_A = \theta_B = -\frac{WL^2}{24EI}$$

Deflection

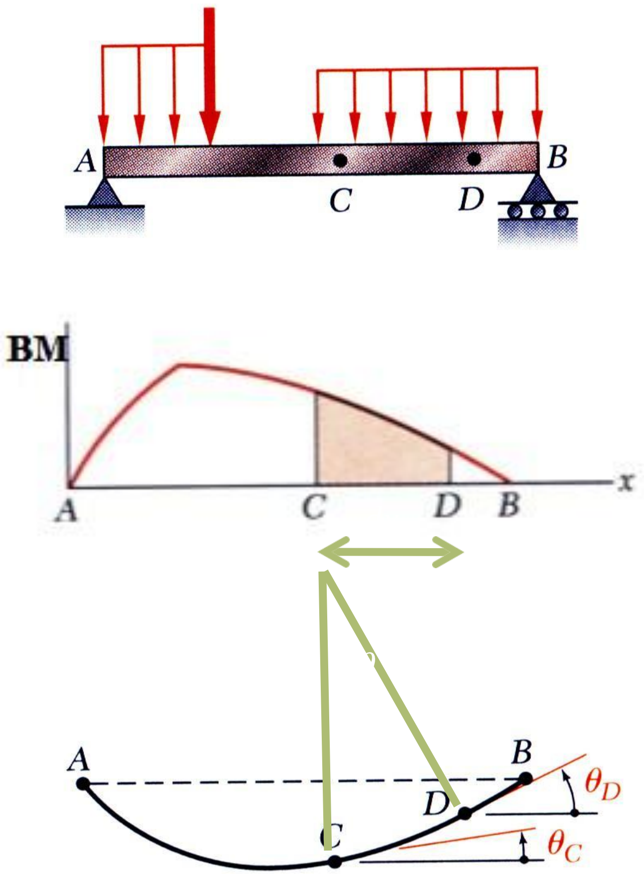
$$\text{Deflection} = y_c$$

$$= -\frac{5}{384}\frac{WL^3}{EI}$$

Macaulay’s method

- The procedure of finding slope and deflection for simply supported beam with an eccentric load is very laborious.
- Macaulay’s method helps to simplify the calculations to find the deflection of beams subjected to point loads.

Moment-Area Theorems



- Consider a beam subjected to arbitrary loading,

$$\frac{d\theta}{dx} = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\int d\theta = \int \frac{\ }{EI} dx$$

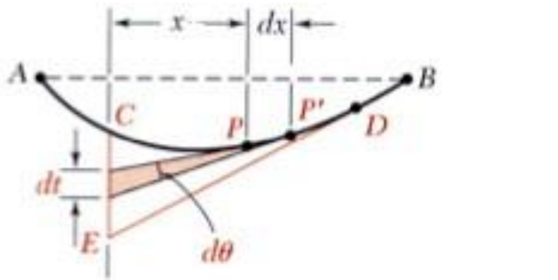
$$\theta_D - \theta_C = \int \frac{\ }{EI} dx$$

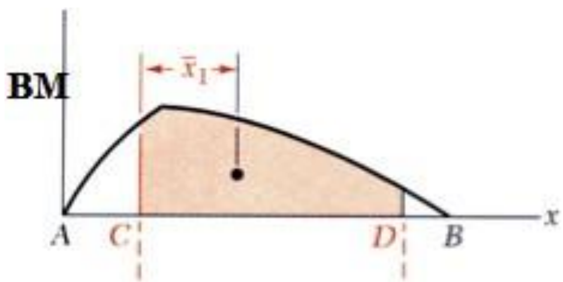
$$CD = Rd\theta = dx$$

- *First Moment-Area Theorem*:

$\theta_{D/C}$ = area under *BM* diagram between *C* and *D*.

Moment-Area Theorems





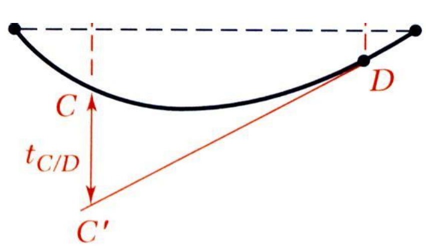
- Tangents to the elastic curve at *P* and *P'* intercept a segment of length *dt* on the vertical through *C*.

$$dt = xd\theta = x \frac{M}{EI} dx$$

$$t_{C/D} = \int x \frac{}{EI} dx = \frac{}{EI}\int xMdx = \frac{A}{EI}x$$

– A= total area of BM diagram between C & D

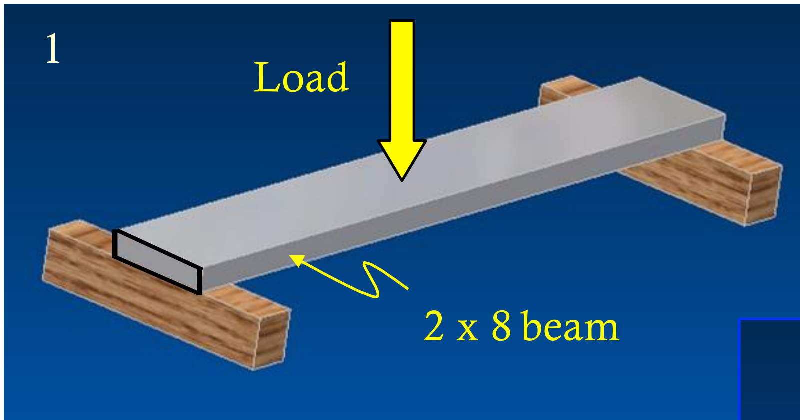
x = Distance of CG of BM diagram from C

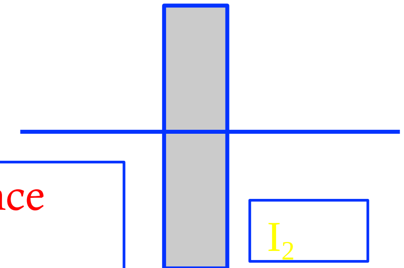


9 - 12

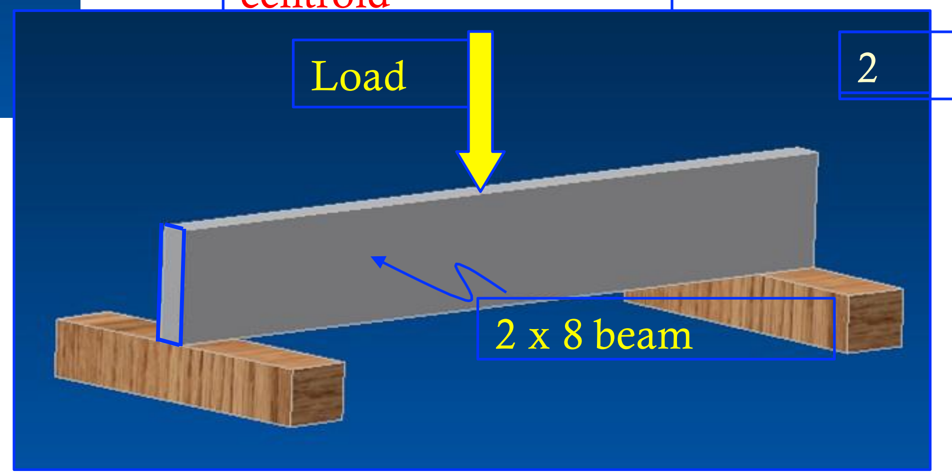
- *Second Moment-Area Theorem*:
  The tangential deviation of *C* with respect to *D* is equal to the first moment with respect to a vertical axis through *C* of the area under the BM diagram between *C* and *D*.

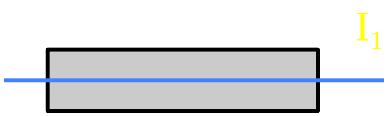
An Exercise- Moment of Inertia – Comparison





Maximum distance of 4 inch to the centroid





Maximum distance of 1 inch to the centroid

$I_2 > I_1$, orientation 2 deflects less

MODULE-III
Stresses in Beams

CLOs	Course Learning Outcome
CLO 7	Compute the bending stresses developed in various sections of beams of real field problems.
CLO 8	Apply the bending equation on various sections
CLO 9	Determine the shear stresses developed in various sections of beams

Shear force

- Any force which tries to shear-off the member, is termed as shear force.

- Shear force is an unbalanced force, parallel to the cross-section, mostly vertical, but not always, either the right or left of the section.

Shear Stresses

- To resist the shear force, the element will develop the resisting stresses, Which is known as **Shear Stresses.**

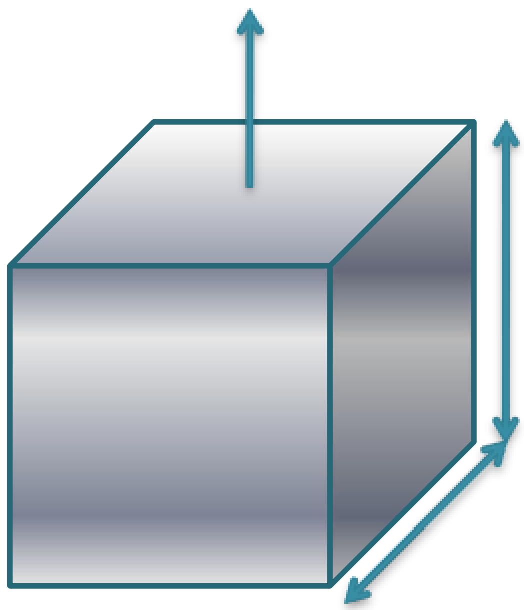
$$\tau = \frac{\text{Shear force}}{\text{Cross sectional area}} \qquad \frac{S}{A}$$

Example

- For the given figure if we want to calculate the
- Then it will be

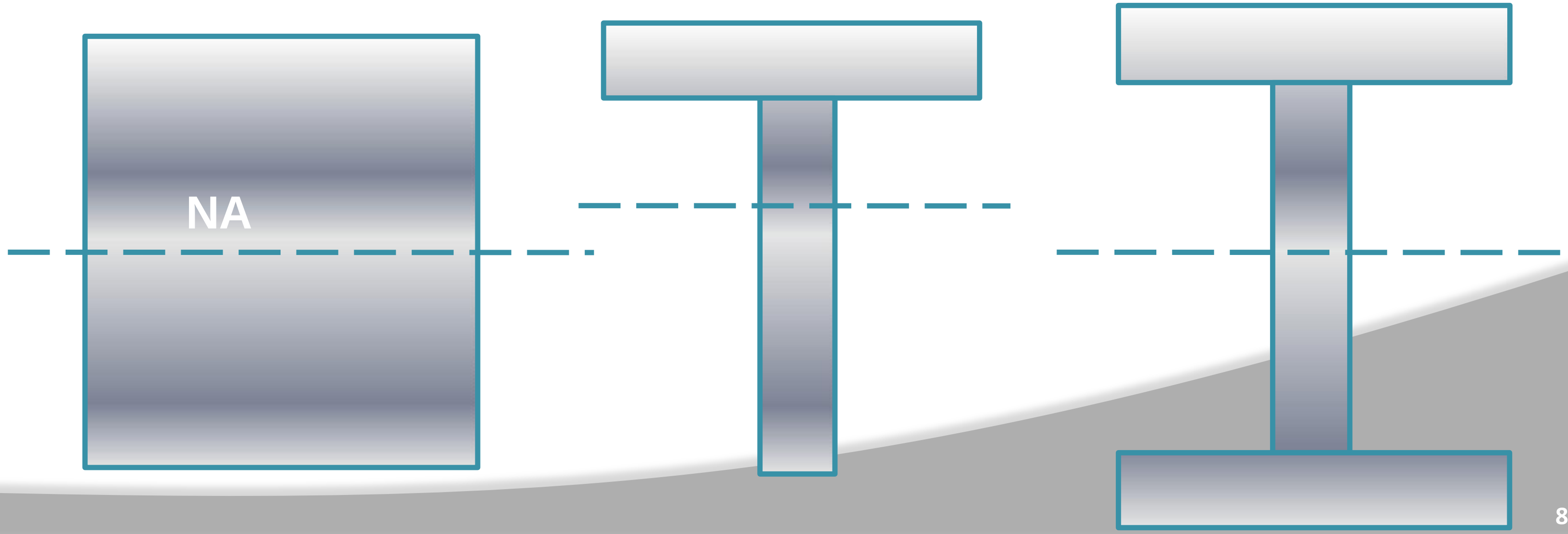
  Let shear force be S

$$\therefore \tau = S/(b \times d)$$

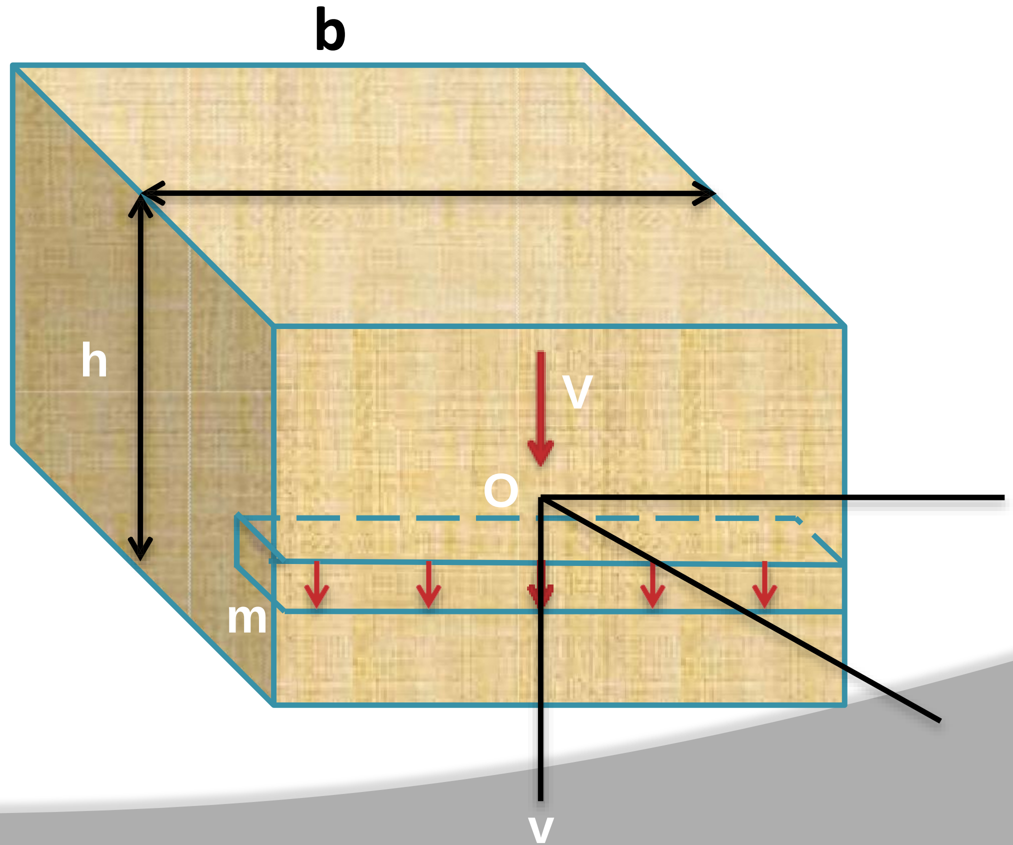


Shear Stresses In Beams

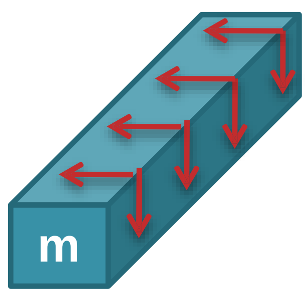
- **Shear stresses** are usually **maximum** at the **neutral axis** of a beam (always if the **thickness is constant** or if **thickness at neutral axis is minimum for the cross section**, such as for **I-beam or T-beam**), but **zero at the top and bottom** of the cross section as **normal stresses are max/min.**

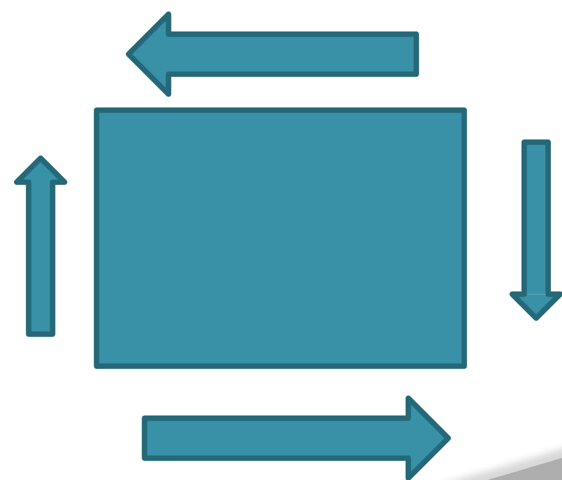


- When a beam is **subjected to a loading**, both **bending moments, M, and shear forces, V, act on the cross section**. Let us consider a beam of rectangular cross section. We can reasonably assume that the **shear stresses** τ act **parallel** to **the shear force V**.



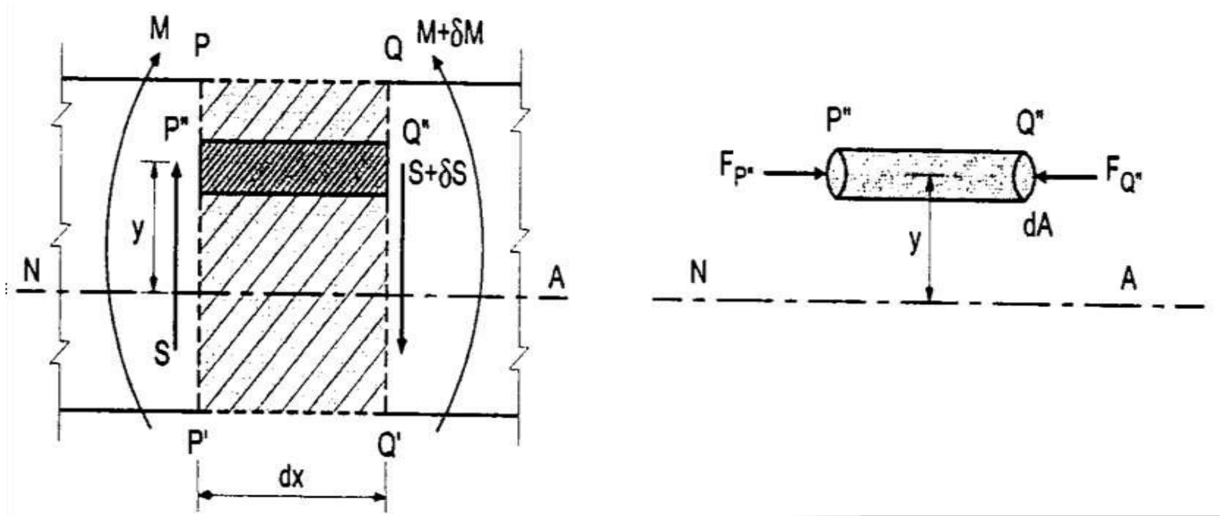
- Shear stresses on one side of an element are accompanied by shear stresses of equal magnitude acting on perpendicular faces of an element. Thus, there will be **horizontal shear stresses** between horizontal layers of the beam, as well as, **Vertical shear stresses** on the vertical cross section.





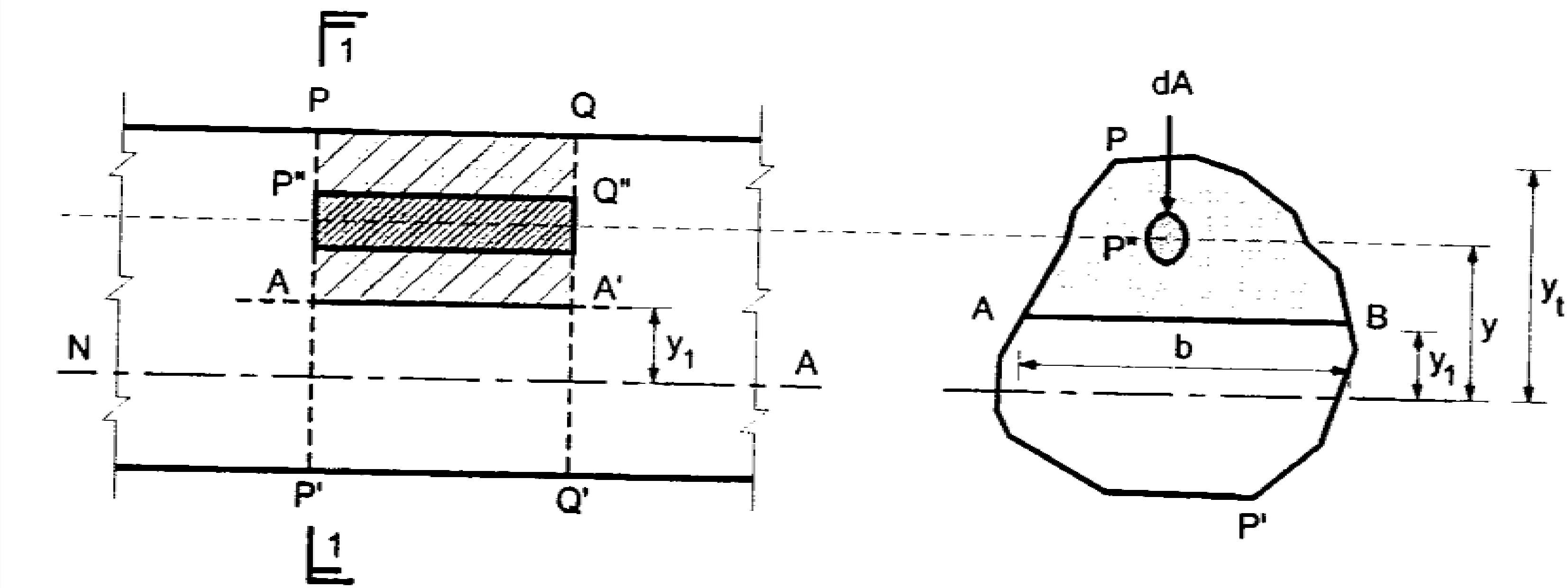
Horizontal Shear Stress

- Horizontal shear stress occurs due to the variation in bending moment along the length of beam.
- Let us assume two sections PP' and QQ', which are 'dx' distance apart, carrying bending moment and shear forces 'M and S' and 'M+ ΔM and S+ ΔS‘ respectively as shown in Fig.



DERIVATION OF FORMULA: SHEAR STRESS DISTRIBUTION ACROSS BEAM SECTION

- Let us consider section PP' and QQ' as previous.
- Let us determine magnitude of horizontal shear stress at level 'AB' which is at distance YI form neutral axis.
- The section above AA' can be assumed to be made up of numbers of elemental cylinder of area 'dA'. Then total unbalance horizontal force at level of' AS' shall be the summation of unbalanced horizontal forces



$$F_H = \sum_{y=y1}^{y=y1} \frac{dM}{I} xy \, . \, dA = \frac{dM}{I} X \sum_{y=y1}^{y=y1} . \, y \, . \, dA$$

$$FM = \frac{dM}{I} x \bar{y} a$$

- Here, y = distance of centroid of area above AB from neutral axis, And a= area of section above AB.
- This horizontal shear shall be resisted by shear area ABA'B‘ parallel to the Neutral plane. The horizontal resisting area here distance of centroid of area above AB from neutral axis and a=area of section above AB.
- Ah = AB x AA’=b x dx

  where ‘b’is width of section at AB.

$$\text{Horizontal shear stress} = \frac{\text{shear resisting}}{\text{horizontal shear force}}$$

$$\tau_H = \frac{F_M}{A_H} \text{ x } \frac{\frac{dM}{I} a\bar{y}}{b.dx}$$

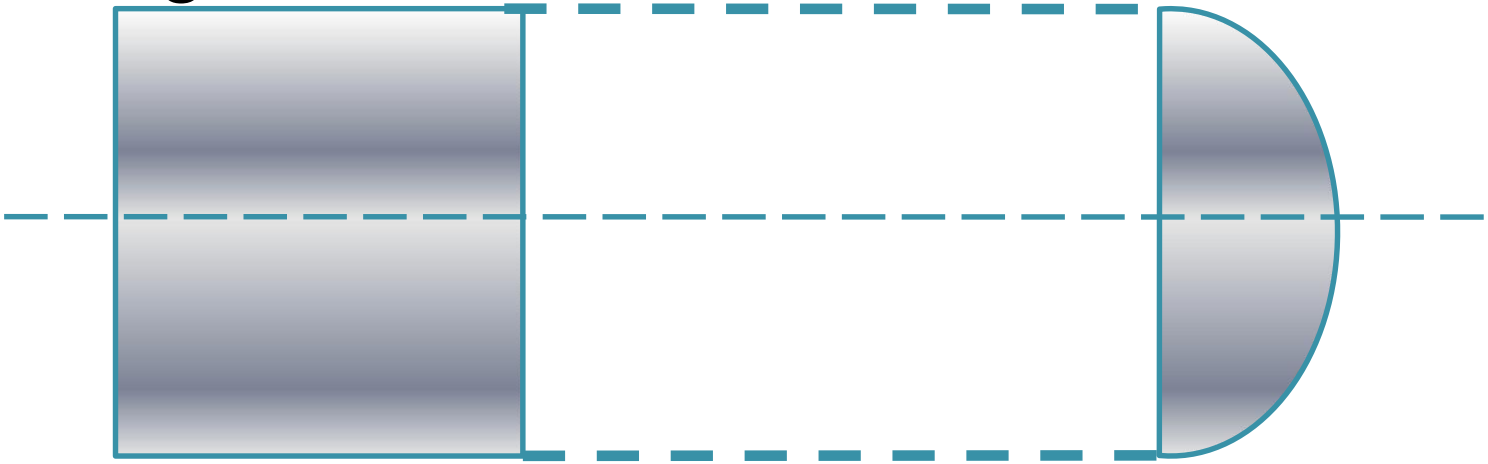
$$\tau_H = \frac{dM}{dx} \text{ x } \frac{a\bar{y}}{Ib}$$

- We know that shear force is defined as S=dM/dx
- Therefore, horizontal shear stress acting at any level across the cross sections.

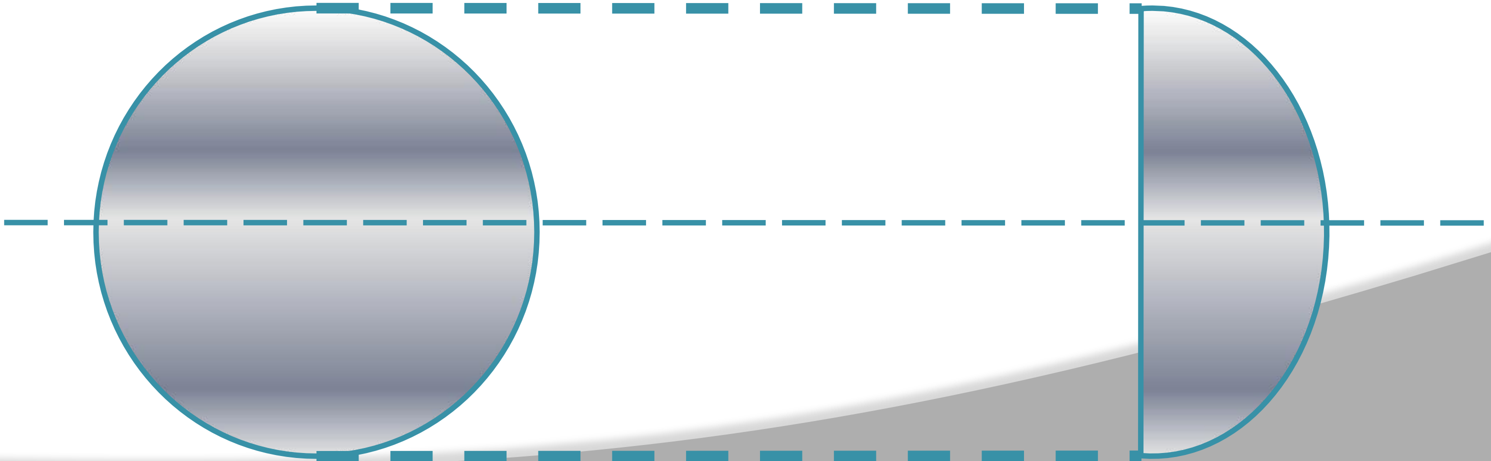
TH= Say / Ib

SHEAR STRESS DISTRIBUTION DIAGRAM

2.Rectangle section



2.Circular section

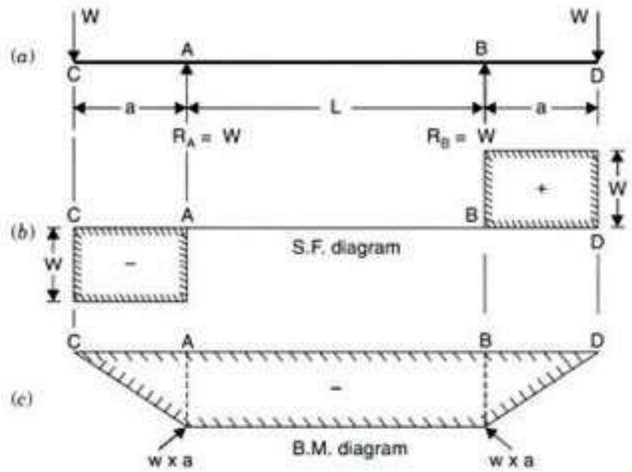


SIMPLE BENDING OR PURE BENDING

- When some external force acts on a beam, the shear force and bending moments are set up at all the sections of the beam
- Due to shear force and bending moment, the beam undergoes deformation. The material of the beam offers resistance to deformation
- Stresses introduced by bending moment are known as bending stresses
- Bending stresses are indirect normal stresses

SIMPLE BENDING OR PURE BENDING

- When a length of a beam is subjected to zero shear force and constant bending moment, then that length of beam is subjected to pure bending or simple pending.
- The stress set up in that length of the beam due simple bending stresses to pure bending is called



SIMPLE BENDING OR PURE BENDING

- Consider a simply supported beam with over hanging portions of equal lengths. Suppose the beam is subjected to equal loads of intensity W at either ends of the over hanging portion
- In the portion of beam of length l, the beam is subjected to constant bending moment of intensity w x a and shear force in this portion is zero
- Hence the portion AB is said to be subjected to pure bending or simple bending

ASSUMPTIONS FOR THE THEORY OF PURE BENDING

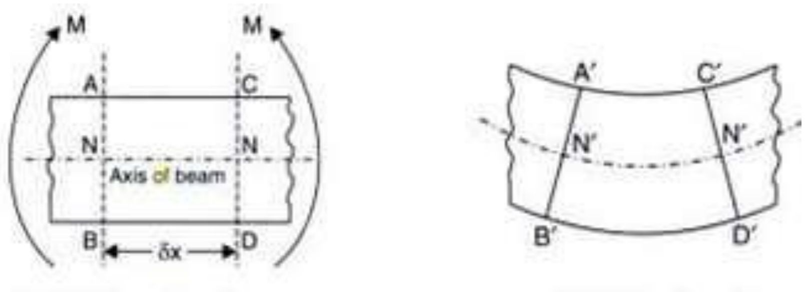
- The material of the beam is isotropic and homogeneous. Ie of same density and elastic properties throughout
- The beam is initially straight and unstressed and all the longitudinal filaments bend into a circular arc with a common centre of curvature
- The elastic limit is nowhere exceeded during the bending
- Young's modulus for the material is the same in tension and compression

ASSUMPTIONS FOR THE THEORY OF PURE BENDING

- The transverse sections which were plane before bending remain plane after bending also
- Radius of curvature is large compared to the dimensions of the cross section of the beam
- There is no resultant force perpendicular to any cross section
- All the layers of the beam are free to elongate and contract, independently of the layer, above or below it.

THEORY OF SIMPLE BENDING

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam and parallel to each other.



- Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
- The layers of the beam are not of the same length before bending and after bending .

THEORY OF SIMPLE BENDING

- The layer AC is shortened to A'C'. Hence it is subjected to compressive stress
- The layer BD is elongated to B'D'. Hence it is subjected to tensile stresses.
- Hence the amount of shortening decrease from the top layer towards bottom and the amount of elongation decreases from the bottom layer towards top
- Therefore there is a layer in between which neither elongates nor shortens. This layer is called neutral layer .

THEORY OF SIMPLE BENDING

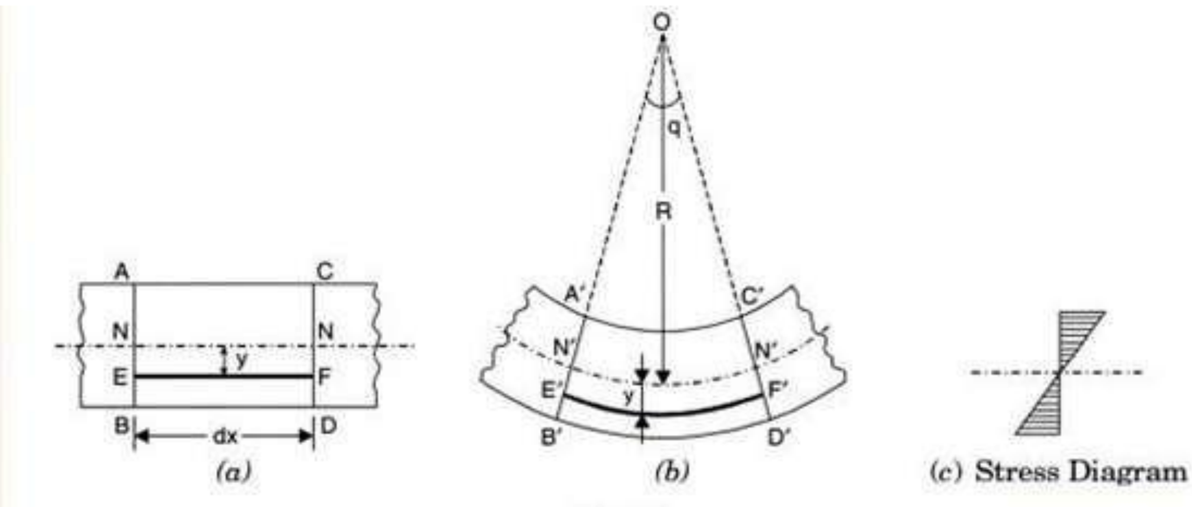
- The filaments/ fibers of the material are subjected to neither compression nor to tension
- The line of intersection of the neutral layer with transverse section is called neutral axis (N-N).
- Hence the theory of pure bending states that the amount by which a layer in a beam subjected to pure bending, increases or decreases in length, depends upon the position of the layer w.r.t neutral axis N-N.

EXPRESSION FOR BENDING STRESS

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam. Due to the bending action the element ABCD is deformed to A’B’C’D’ (concave curve).
- The lines B’A’ and D’C’ when extended meet at point O (which is the centre of curvature for the circular arc formed).
- Let R be the radius of curvature of the neutral axis.

STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Consider a layer EF at a distance y from the neutral axis. After bending this layer will be deformed to E'F'.
- Strain developed= (E'F'-EF)/EF
- EF=NN=dx=R x θ
  E'F'= (R + y) x θ



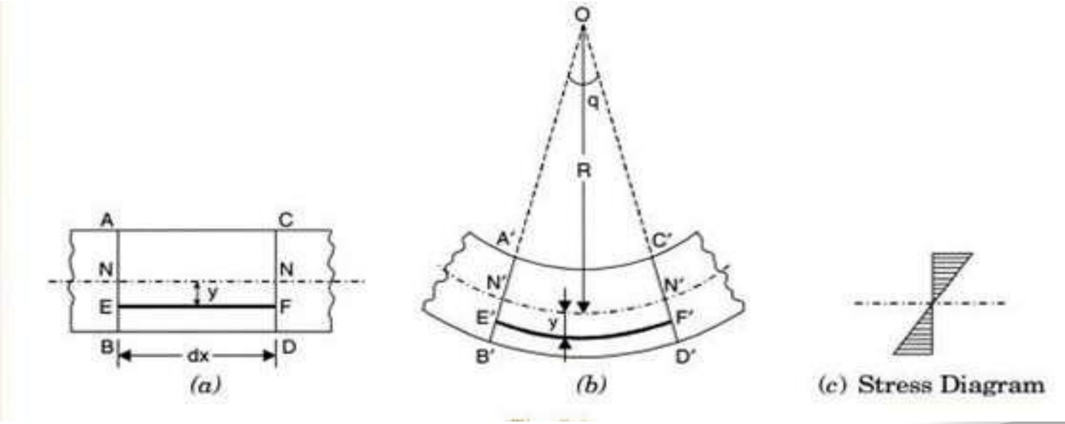
STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Strain developed eb= { (R + y) x θ - R x θ)}/ R x θ=y/R
- STRESS VARIATION WITH DEPTH OF BEAM
- σ/E= y/R or σ= Ey/R or σ/y = E/R
- Hence σ varies linearly with y (distance from neutral axis)
- Therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer

NEUTRAL AXIS

- For a beam subjected to a pure bending moment, the stresses generated on the neutral layer is zero.
- Neutral axis is the line of intersection of neutral layer with the transverse section
- Consider the cross section of a beam subjected to pure bending. The stress at a distance y from the neutral axis is given by σ/y

= E/R



NEUTRAL AXIS

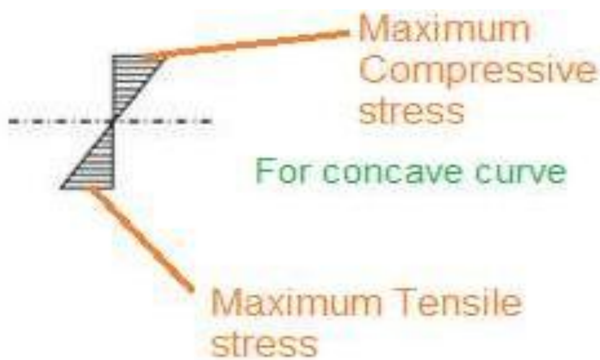
- σ= E x y/R;
- The force acting perpendicular to this section, dF= E x y/R x dA, where dA is the cross sectional area of the strip/layer considered.
- Pure bending theory is based on an assumption that "There is no resultant force perpendicular to any cross section". Hence F=0;
- Hence, E/R x $\int$ydA=0

  => $\int$ydA= Moment of area of the entire cross section w.r.t the neutral axis=0

NEUTRAL AXIS

- Moment of area of any surface w.r.t the centroidal axis is zero. Hence neutral axis and centroidal axis for a beam subjected to simple bending are the same.
- Neutral axis coincides with centrodial axis or the ce osition of neutral axis

ntroidal axis gives the p



Stress strain variation w.r.t the depth of the beam

MOMENT OF RESISTANCE

- Due to the tensile and compressive stresses, forces are exerted on the layers of a beam subjected to simple bending
- These forces will have moment about the neutral axis. The total moment of these forces about the neutral axis is known as moment of resistance of that section
- We have seen that force on a layer of cross sectional area dA at a distance y from the neutral axis,

  $dF = (E \times y \times dA)/R$

  Moment of force dF about the neutral axis= $dF \times y = (E \times y \times dA)/R \times y = E/R \times (y^2 dA)$

MOMENT OF RESISTANCE

- Hence the total moment of force about the neutral axis= Bending moment applied= $\int E/R \times (y^2 dA)$= $E/R \times I_{xx}$; I_{xx} is the moment of area about the neutral axis/centroidal axis.

  Hence $M = E/R \times I_{xx}$ Or

  $M/I_{xx} = E/R$
- Hence $M/I_{xx} = E/R = \sigma b/y$; σb is also known as flexural stress (Fb)
- Hence **$M/I_{xx} = E/R = F_b/y$**
- The above equation is known as bending equation

CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Bending equation is applicable to a beam subjected to pure/simple bending. Ie the bending moment acting on the beam is constant and the shear stress is zero
- However in practical applications, the bending moment varies from section to section and the shear force is not zero
- But in the section where bending moment is maximum, shear force (derivative of bending moment) is zero
- Hence the bending equation is valid for the section where bending moment is maximum

CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Or in other words, the condition of simple bending may be satisfied at a section where bending moment is maximum.
- Therefore beams and structures are designed using bending equation considering the section of maximum bending moment
- Flexural rigidity/Flexural resistance of a beam:-
- For pure bending of uniform sections, beam will deflect into circular arcs and for this reason the term circular bending is often used.

CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- The radius of curvature to which any beam is bent by an applied moment M is given by R=EI/M
- Hence for a given bending moment, the radius of curvature is directly related to "EI"
- Since radius of curvature is a direct indication of the degree of flexibility of the beam (larger the value of R, less flexible the beam is, more rigid the beam is), EI is known as flexural rigidity of flexural stiffness of the beam.
- The relative stiffnesses of beam sections can then easily be compared by their EI value

SECTIONAL MODULUS (Z)

- Section modulus is defined as the ratio of moment of area about the centroidal axis/neutral axis of a beam subjected to bending to the distance of outermost layer/fibre/filament from the centroidal axis
- **$Z = I_{xx}/y_{max}$**
- From the bending equation, $M/I_{xx} = \sigma_{bmax}/y_{max}$

  Hence $I_{xx}/y_{max} = M/\sigma_{bmax}$ $M =$

  $\sigma bmax$ X Z
- Higher the Z value for a section, the higher the BM which it can withstand for a given maximum stress

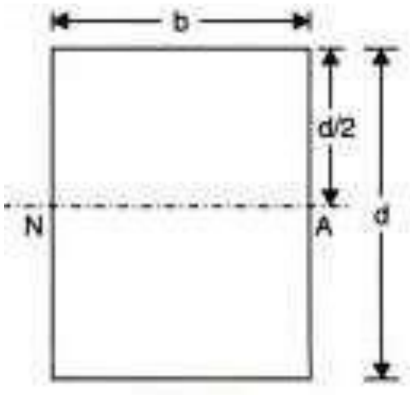
- 1) For a Rectangular section

  Z=Ixx/ymax

  $Ixx=INA= bd^3/12$

  ymax= d/2

  $Z= bd^2/6$

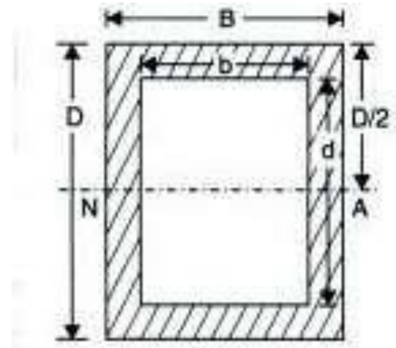


- 2) For a Rectangular hollow section

  $Ixx= 1/12 \times (BD^3/12 - bd^3/12)$

  Ymax = D/2

  $Z=(BD^3 - bd^3)/6D$



VARIOUS SHAPES OR BEAM SECTIONS

- 3) For a circular section of diameter D,

  Ixx= πD^4/64

  ymax = D/2

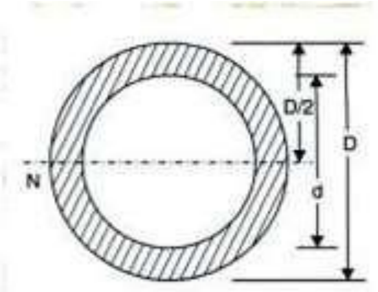
  Z= πD³/32

- 4) For a hollow circular section of outer diameter D and inner diameter d,

  Ina= (πD^4 - d^4)/64

  ymax=D/2

  Z= (πD^4 -d^4)/32D



BENDING OF FLITCHED BEAMS

- A beam made up of two or more different materials assumed to be rigidly connected together and behaving like a single piece is called a flitched beam or a composite beam.
- Consider a wooden beam re-inforced by steel plates. Let

  E_1 = Modulus of elasticity of steel plate

  E_2 = Modulus of elasticity of wooden beam

  M_1 = Moment of resistance of steel plate

  M_2 = Moment of resistance of wooden beam

BENDING OF FLITCHED BEAMS

I1= Moment of inertia of steel plate about neutral axis

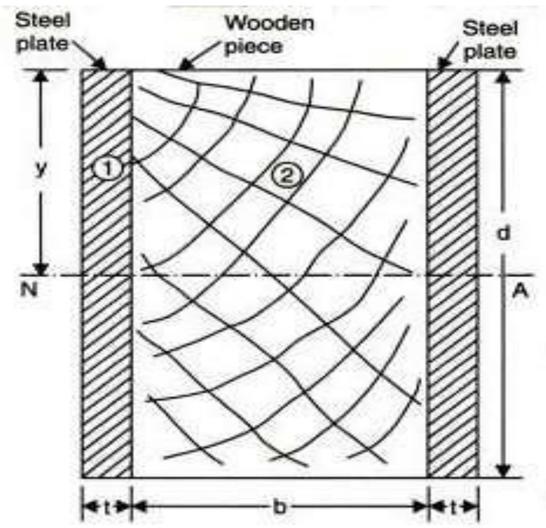
I2= Moment of inertia of wooden beam about neutral axis.

The bending stresses can be calculated using two conditions.

- Strain developed on a layer at a particular distance from the neutral axis is the same for both the materials
- Moment of resistance of composite beam is equal to the sum of individual moment of resistance of the members

BENDING OF FLITCHED BEAMS

- Let σ1 be the bending stress developed in steel at a distance y from the neutral axis and σ2 be the bending stress developed in wooden beam at th m neutral axis.
e same distance y fro



BENDING OF FLITCHED BEAMS

- Using condition-1:

$\sigma 1/E1 = \sigma 2/E2;$

$\sigma 1 = \sigma 2 \times (E1/E2)$ or

$\sigma 1 = \sigma 2 \times m;$

where m= E1/E2 is the modular ratio between steel and wood

- Using condition-2:

$M = M1 + M2;$

$M1 = \sigma 1 x\, I1/y$ $M1 = \sigma 2 x$

$I2/y$

BENDING OF FLITCHED BEAMS

- Hence $M = \sigma_1 \times I_1/y + \sigma_2 \times I_2/y$ $M = \sigma_2/y \times (I_2 + I_1 \times \sigma_1/\sigma_2)$ $M = \sigma_2/y \times (I_2 + I_1 \times m)$
- $(I_2 + I_1 \times m) = I$ = equivalent moment of inertia of the cross section;
- Hence $M = \sigma_2/y \times I$

MODULE-IV
Columns

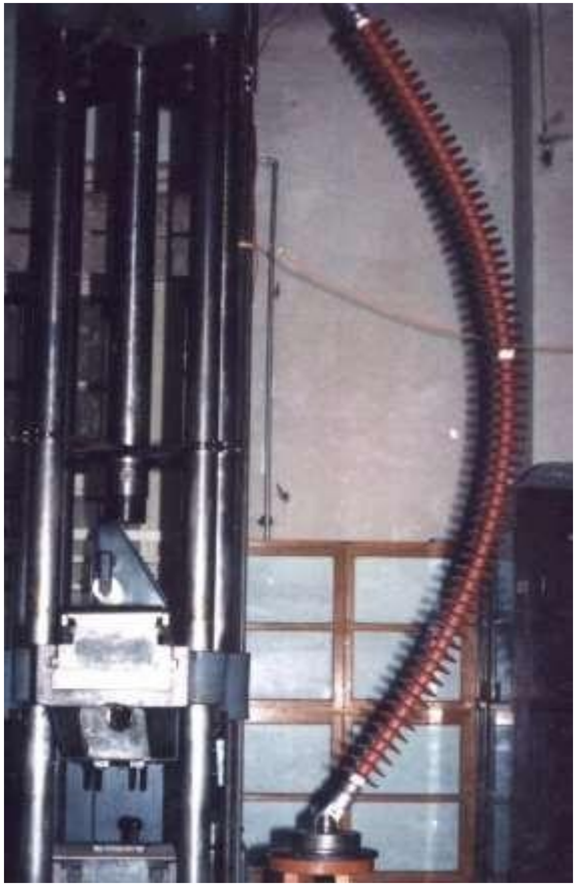
CLOs	Course Learning Outcome
CLO 10	Calculate the stability of structural elements and determine buckling loads.
CLO 11	Discuss critical buckling load for column with various loading and end conditions
CLO 12	Apply theories and to predict the performance of bars under axial loading including buckling.
CLO 13	Understand the theory of beam column & determine buckling loads on it.

What is a column?

- A structural member subjected to axial compressive force is called a column.
- Normally, columns carry heavy compressive loads.
- Columns are used in concrete and steel buildings.

Real world examples

Real world examples



Types Of Columns

- Long
- Short (Strut)
- Intermediate

Classification

- LONG COLUMN :
- When length of column is more as compared to its c/s dimension, it is called long column.

Long Column $L_e/r_{min} > 50$

Where,

L_e = effective length of column

rmin = Minimum radius of gyration

Real world example:

- Here in picture we can see long columns on front of building in “The White house” Washington D.C(USA).

SHORT COLUMN :

- When length of column is less as compared to its c/s dimension, it is called Short column.

  Short Column $L_e/r_{min}<50$

Or,

  $L_e/d < 15$

- Crushing Load : The load at which short column fails by crushing is called crushing load.

INTERMEDIATE COLUMN:

- Column is intermediate when $4d < L < 30d$ and $30 < L_e/r_{min} < 100$ or Critical slenderness ratio.

Before we move onward....

- Crippling Load (P_{cr})
- Radius of gyration (r)
- Moment of inertia (I)
- Area (A)
- Effective Length (L_e)
- Slenderness ratio (λ)
- Stronger Axis
- Weaker Axis

Types of supports

- Roller type support (1)
- Pin type support (2)
- Fixed support (3)

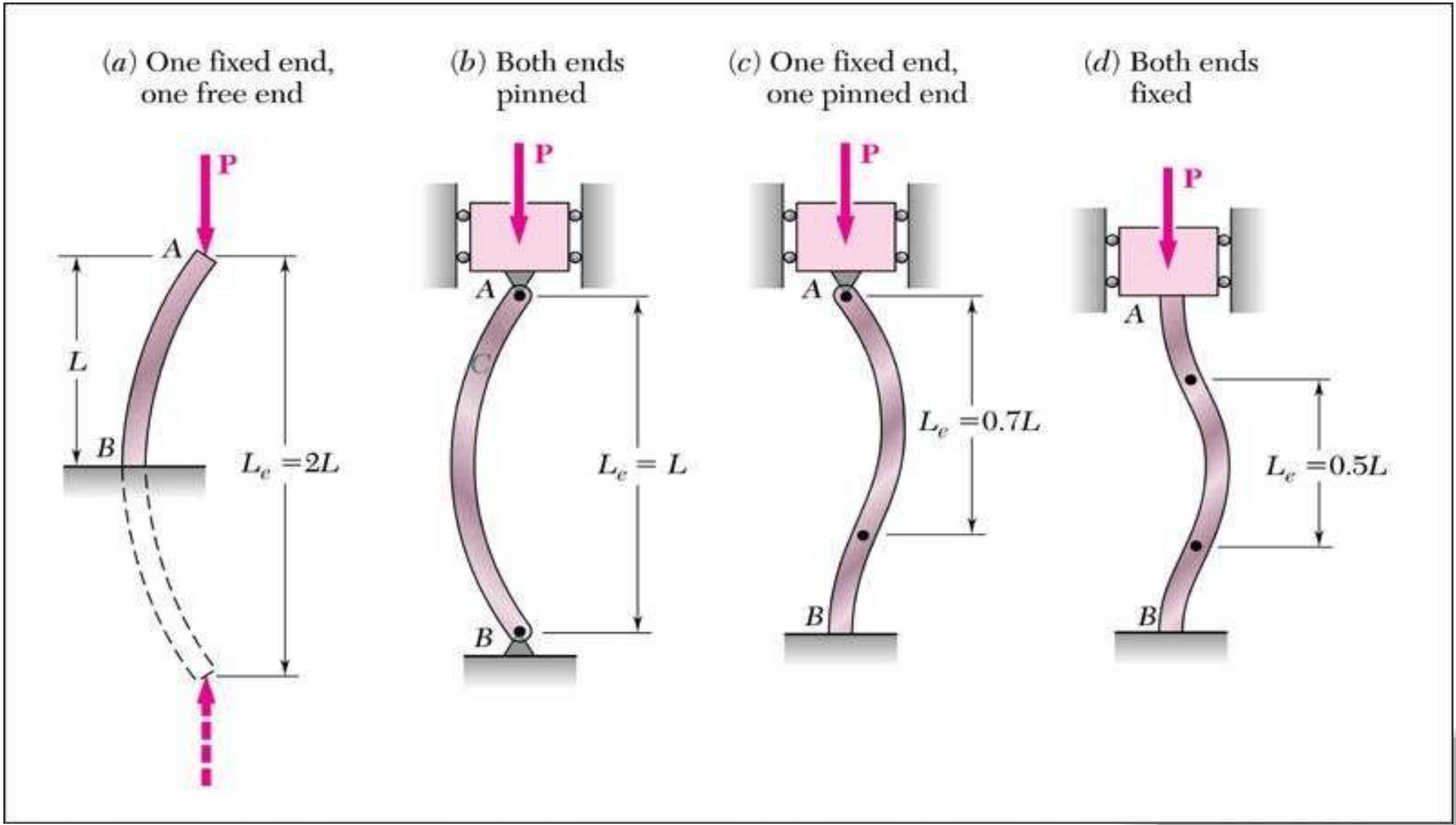
Effective Length (L_e)

- The distance between points of two zero moments.
- Depends upon the type of support conditions.

COLUMN END CONDITION &EFFECTIVE LENTH:

1. Both ends hinged.
2. Both ends fixed.
3. One end fixed and other hinged.
4. One end fixed and other free.

(L_e) for differentsupport conditions



Slenderness ratio (λ)

- **Slenderness ratio** is the **ratio** of the length of a column and the least radius of gyration of its cross section. Often denoted by lambda. It is used extensively for finding out the design load as well as in classifying various columns in short/intermediate/long.
- $\lambda = l_e/r_{min}$

Radius of Gyration(r) , $r = \sqrt{(I/A)}$ or $I = Ar^2$

K=radius of gyration

I = Moment of Inertia (mm^4)

A = Area of Section (mm^2)

Slenderness ratio,

Slenderness Ratio = effective length of column/Minimum radius of gyration

$$\lambda = le/r_{min}$$

CRIPPLING LOAD OR BUCKLING LOAD

- The load at which, long column starts buckling(bending) is called buckling load or crippling load.
- Buckling of column depends upon the following factors.

1. Amount of load.
2. Length of column
3. End condition of column
4. C/s dimensions of column
5. Material of column.

Euler's Formula

- Euler's Crippling Load, $P_E = \Pi^2 EI / le^2$
- Where, E is Modulus of Elasticity (Mpa)

I is MOI or 2nd Moment ofarea (mm^4)

Le is Effective length (mm)

Also known as Critical Buckling Load

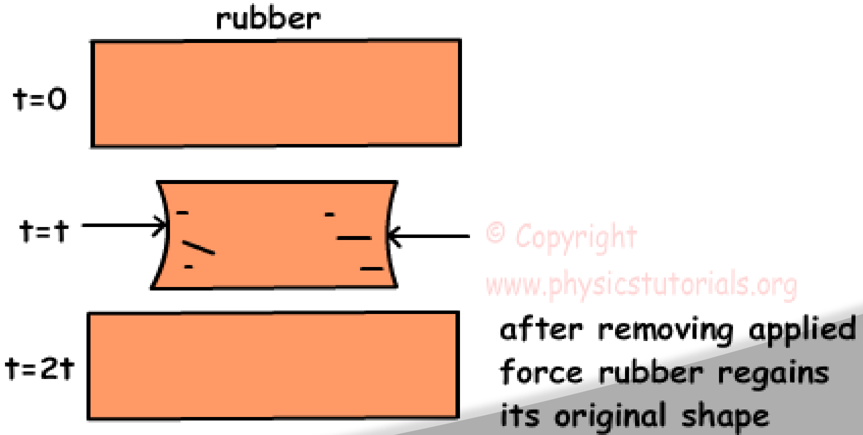
MODULE-V
Theory of Elasticity

CLOs	Course Learning Outcome
CLO 14	Solve the principal stress problems by graphical methods.
CLO 15	Explain the stress transformation and concept of principle plane and principle stresses
CLO 16	Evaluate principal stresses, strains and apply the concept of failure theories for design
CLO 17	Acquire knowledge to solve real time problems in Aircraft structure subjected loading conditions

ELASTICITY

- The property of solid materials to deform under the application of an external force and to regain their original shape after the force is removed is referred to as its elasticity.

- The external force applied on a specified area is known as stress, while the amount of deformation is called the strain.



- It is the branch of Solid Mechanics which deals with the stress and displacements in elastic solids produced by external forces or changes in temperature.
- The purpose of study is to check the sufficiency of the strength, stiffness and stability of structural and machine elements.

IMPORTANT CONCEPT IN THEORY OF ELASTICITY

- External forces
- Stresses - (internal force)
- Deformations-strain and displacements

There aretwo kinds of ***external forces*** thatact on the bodies

- ***Body forces***

  Gravitational forces

  Inertia forces(in motion)

- ***Surface forces***

  Pressure(in water, atmosphere) Contact forces

.

Internal forces: produced by external force

Deformation: By deformation we mean the change of shape of a body

- *Strain components:* completely define the deformation condition (or strain condition) at that point

- *Displacement:* The change of position, the displacement components in the x, y, z axes are denoted by u, v, w respectively.

BASIC ASSUMPTIONS IN THEORY OF ELASTICITY

- The body is continuous
- The body is perfectly elastic
- The body is homogeneous
- The body is isotropic

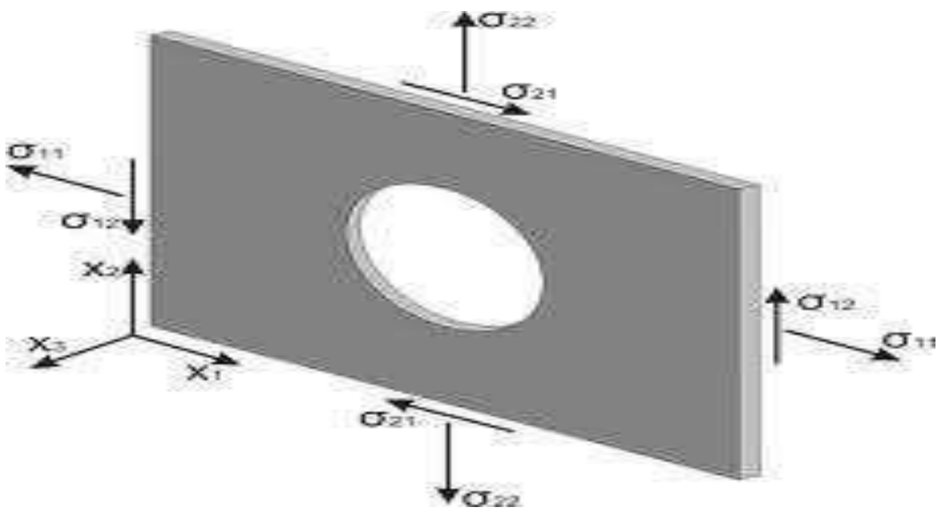
  Example: polycrystalline ceramics and steel

  wood and fiber reinforced composite
- The displacements and strains are small

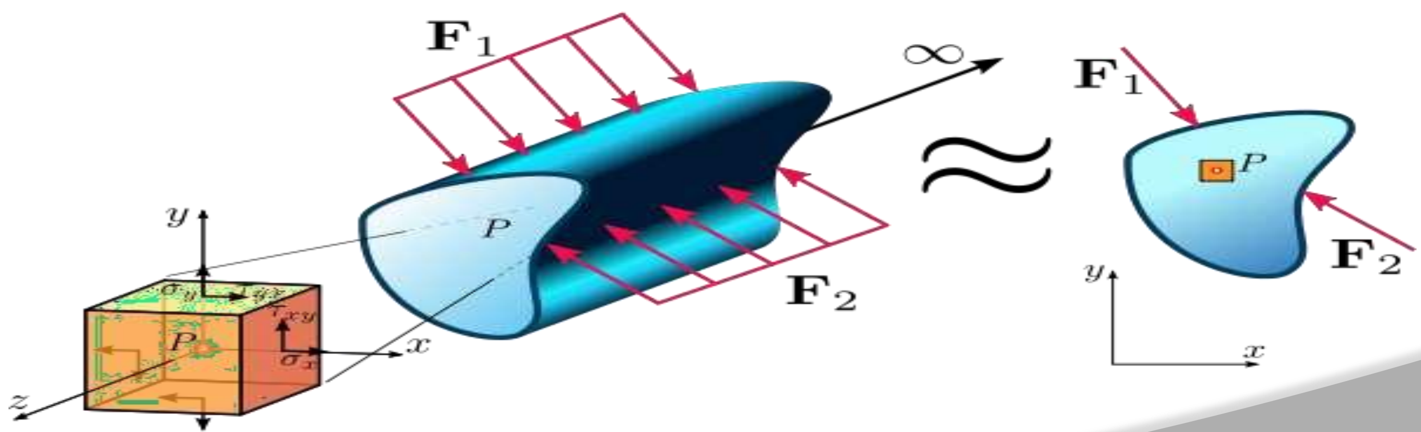
Many problems in elasticity may be treated by two dimensional or plane theory of elasticity. The purpose of application of elasticity is to analyze the stress and displacements of elements within in the elastic range. There are two general types of problems involved

- Plane stress
- plane strain

- Plane stress condition can be visualized as thin plate with stresses acting along its plane. There is no stress acting perpendicular to the plane and (∂/∂z) components in equations are zero.



- Plain strain can be idealized as long wire with stresses acting perpendicular to its length. Therefore the strain or displacement along the length is zero.
- Again(∂/∂z) components in equations are zero.



- Example for plane stress

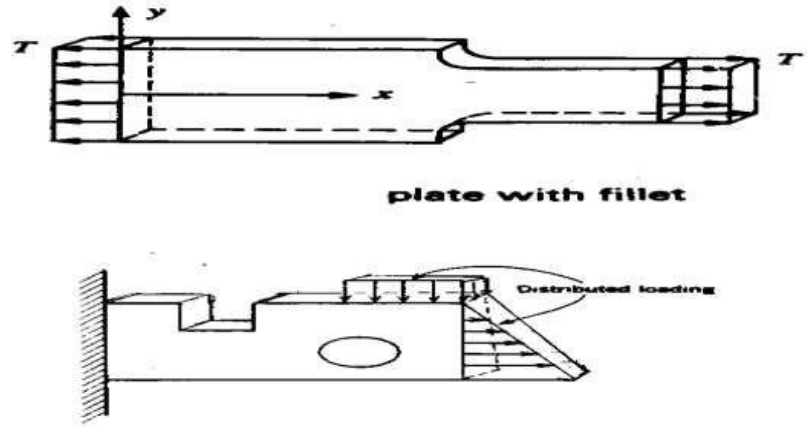
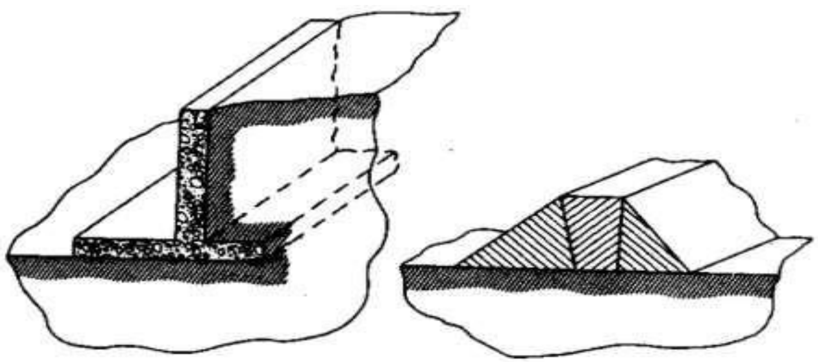


plate with fillet

plate with hole

- Example for plain strain


Retaining wall

Earth dam

Plane stress condition

$$\sigma_x = f_1(x, y)$$
$$\sigma_y = f_2(x, y)$$
$$\tau_{xy} = f_3(x, y)$$
$$\sigma_z = 0$$
$$\tau_{xz} = \tau_{yz} = 0$$

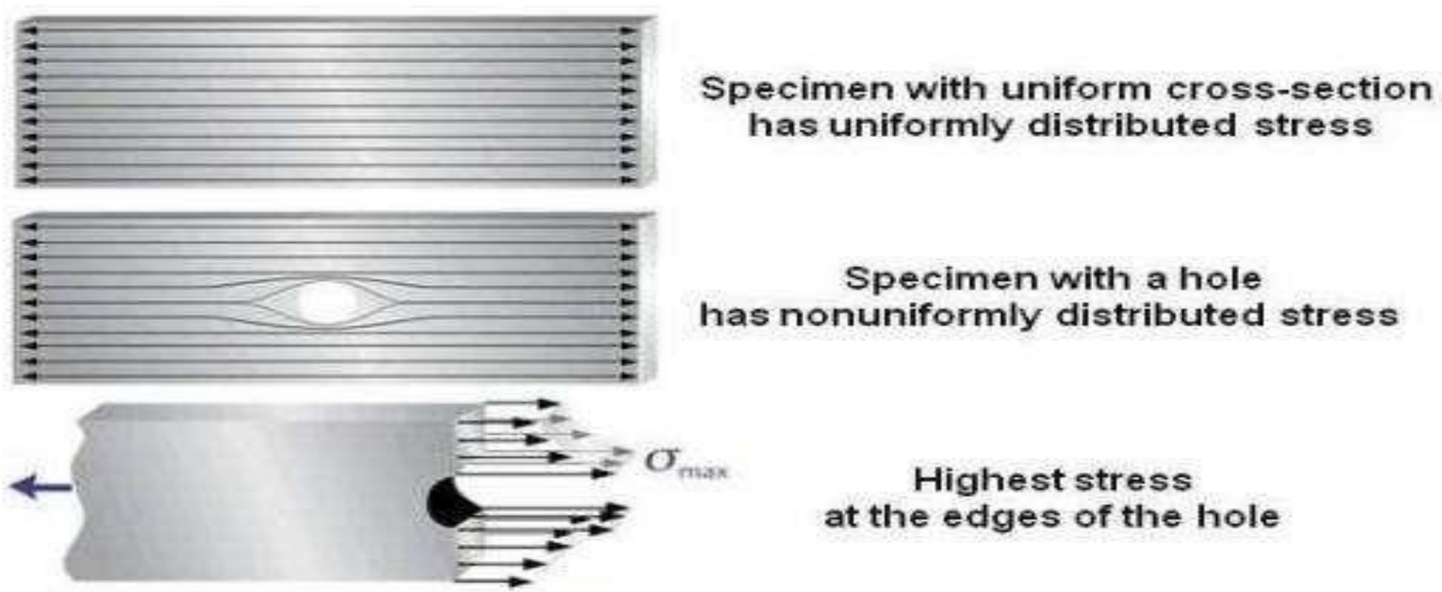
Plane strain condition

$$\varepsilon_x = f_1(x, y)$$
$$\varepsilon_y = f_2(x, y)$$
$$\varepsilon_{xy} = f_3(x, y)$$
$$\varepsilon_z = 0$$
$$\varepsilon_{xz} = \varepsilon_{yz} = 0$$

APPLICATIONS OF THEORY OF ELASTICITY

- In designing mechanical parts
- Stress calculations on beams
- Stress concentration factor

APPLICATIONS



Stress Concentration

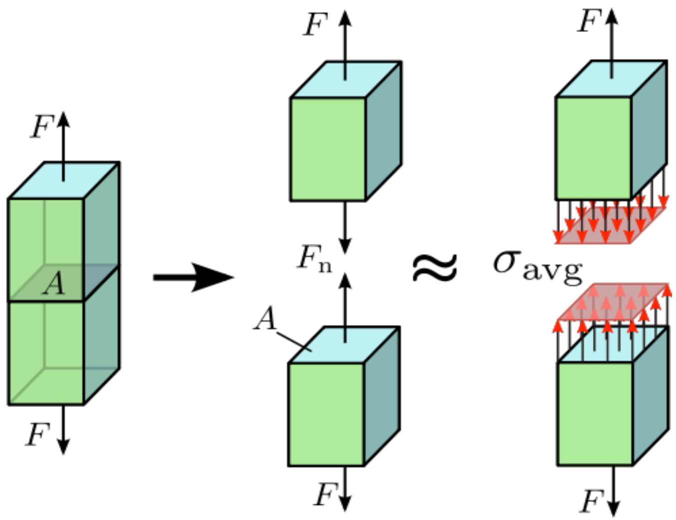
Stresses and strains

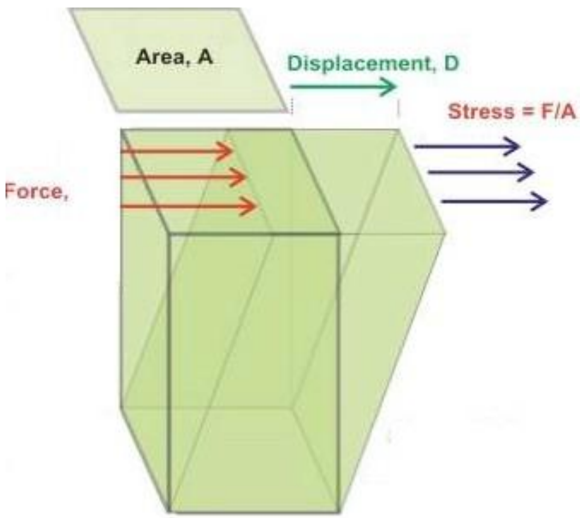
- By using strength of materials we cant predict the stress directly under a load or supports in a simple beam where theory of elasticity is applied.
- Generally factor of safety used for a mechanical members is 3or 4.But in the design of precision parts where strength to weight ratio is minimum accuracy of stress values is more important.

Stresses and strains

- In last lecture we looked at stresses were acting in a plane that was at right angles/parallel to the action of force.

Tensile Stress **Shear Stress**





Stresses and strains

Compressive load

Failure in shear

Stresses are acting normal to the surface yet the material failed in a different plane

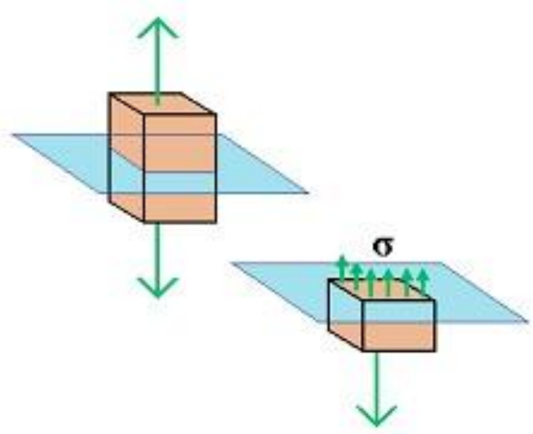
Principal stresses and strains

- What are principal stresses.
- Planes that have no shear stress are called as principal planes.
- Principal planes carry only normal stresses

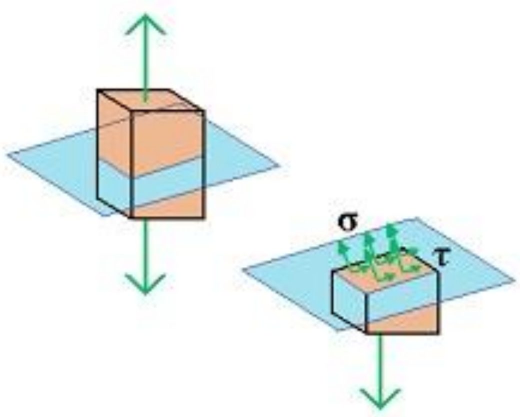
Stresses in oblique plane

- In real life stresses does not act in normal direction but rather in inclined planes.

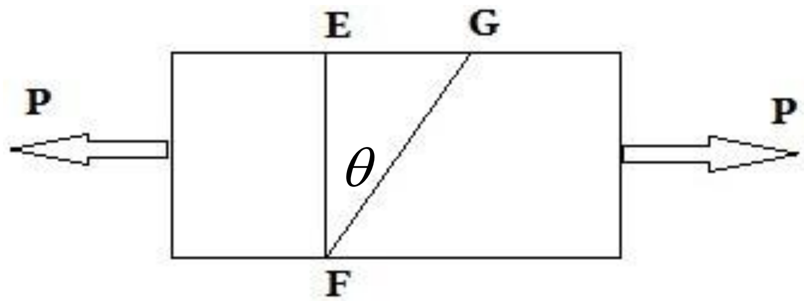
Normal Plane

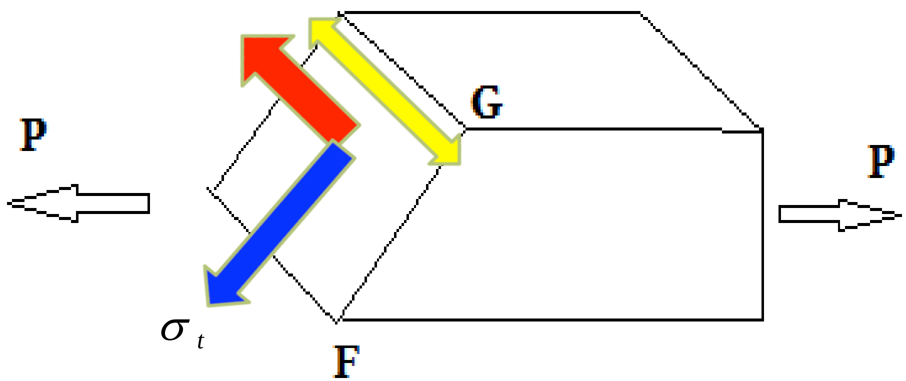


Oblique Plane



Stresses in oblique plane





$$\sigma = \frac{P}{A}$$

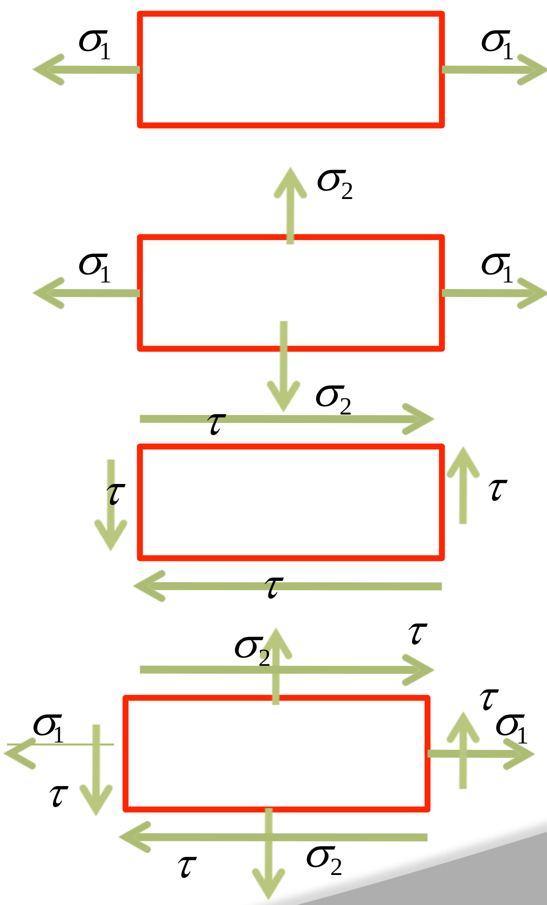
P =Axial Force
A=Cross-sectional area perpendicular to force

$$\sigma_n = \sigma \cos^2 \theta$$

$$\sigma_t = \frac{\sigma}{2} \sin 2\theta$$

Stresses in oblique plane

- Member subjected to direct stress in one plane
- Member subjected to direct stress in two mutually perpendicular plane
- Member subjected to simple shear stress.
- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress



Stresses in oblique plane

❖ **Member subjected to direct stress in two mutually perpendicular directions + simple shear stress**

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

Stresses in oblique plane

- **Member subjected to direct stress in two mutually perpendicular directions + simple shear stress**
- **Position of principal planes**
- **Shear stress should be zero**

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

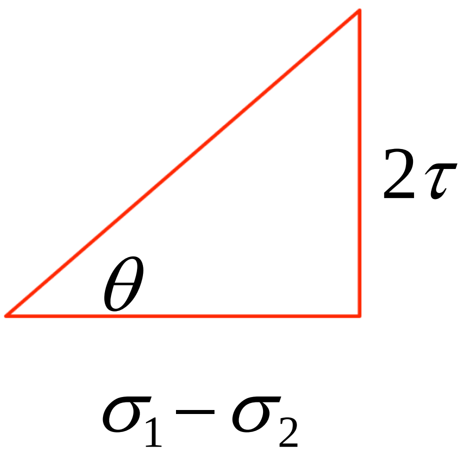
Stresses in oblique plane

- **Member subjected to direct stress in two mutually perpendicular directions + simple shear stress**
- **Position of principal planes**

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$\sin 2\theta = \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\cos 2\theta = \frac{(\sigma_1 - \sigma_2)}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$



Stresses in oblique plane

- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress

Major principal Stress = $$\frac{\sigma_1+\sigma_2}{2}+\sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$$

Minor principal Stress = $$\frac{\sigma_1+\sigma_2}{2}-\sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2+\tau^2}$$

Stresses in oblique plane

- **Member subjected to direct stress in two mutually perpendicular directions + simple shear stress**
- **Max shear stress**

$$\frac{d}{d\theta}(\sigma_t)=0$$

$$\frac{d}{d\theta}\left[\frac{\sigma_1-\sigma_2}{2}\sin 2\theta-\tau\cos 2\theta\right]=0 \quad \Rightarrow \quad \tan 2\theta=\frac{\sigma_1-\sigma_2}{2\tau}$$

Stresses in oblique plane

- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress
- Max shear stress

  Evaluate the following equation at

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$

$$\tan 2\theta = \frac{\sigma_1 - \sigma_2}{2\tau}$$

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

Stresses in oblique plane

- Member subjected to direct stress in one plane
- Member subjected to direct stress in two mutually perpendicular plane
- Member subjected to simple shear stress.
- Member subjected to direct stress in two mutually perpendicular directions + simple shear stress

Stresses in oblique plane

- Member subjected to direct stress in one plane

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

Stress in one direction and no shear stress $\quad \tau = 0 \quad \sigma_2 = 0$

$$\sigma_n = \frac{\sigma_1}{2} + \frac{\sigma_1}{2}\cos 2\theta = \sigma_1 \cos^2\theta$$

$$\sigma_t = \frac{\sigma_1}{2}\sin 2\theta$$

Stresses in oblique plane

- Member subjected to direct stress in two mutually perpendicular plane

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

Stress in two direction and no shear stress $\tau = 0$

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$$

Stresses in oblique plane

❖ Member subjected to simple shear stress.

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

No stress in axial direction but only shear stress $\sigma_1 = \sigma_2 = 0$

$$\sigma_n = \tau \sin 2\theta$$

$$\sigma_t = -\tau \cos 2\theta$$