Hall Ticket No | | | | | | | | | | | Question Paper Code: AHS003

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech I Semester End Examinations (Regular) - December, 2016

Regulation: IA-R16

COMPUTATIONAL MATHEMATICS AND INTEGRAL CALCULUS

(Common for CSE/IT/ECE/EEE)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the question must be answered in one place only

UNIT – I

1. (a) Find the real root of the equation $log\ x - cos\ x = 0$, near 1.5 by using Newton-Raphson method. **[7M]**

   (b) Find the real root of the equation $x^2 - \log_e x - 12 = 0$, by false position method up to three decimal places. **[7M]**

2. (a) Find a real root of the equation $x\log_{10}x = 1.2$ by Newton Raphson method. Correct the root to three decimal places. **[7M]**

   (b) Obtain the interpolating polynomial passing through the points (0, 1), (1, 3), (2, 7) and (3, 13) and hence find $f(2.5)$

UNIT – II

3. (a) Fit a parabola $y = a + bx + cx^2$ to the following data: **[7M]**

x	-3	-2	-1	0	1	2	3
f(x)	4.63	2.11	0.67	0.09	0.63	2.15	4.58

   (b) Find $y(0.2)$, given that $\frac{dy}{dx} = 2y + 3e^x;\ y(0) = 0$ by Taylor's series method. **[7M]**

4. (a) Solve: $\frac{dy}{dx} = \log_{10}(x + y);\ y(0) = 2$ at $x = 0.4$ with $h = 0.2$ by modified Euler's method **[7M]**

   (b) Using Runge-Kutta method, find $y(1.1)$ given that $\frac{dy}{dx} = 3x + y^2;\ y(1) = 1.2$ **[7M]**

UNIT – III

5. (a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ **[7M]**

   (b) Evaluate $\int_0^1 \int_{x^2}^{2-x} xydydx$ **[7M]**

6. (a) Evaluate $\int_0^a \int_{x/a}^{\sqrt{(x/a)}} (x^2 + y^2)\, dy\, dx$, by changing to polar coordinates. **[7M]**

(b) If R is the region bounded by the planes $x = 0,\ y = 0,\ z = 1$ and the cylinder $x^2+y^2=1$, evaluate $\iiint\limits_R xyz\,dx\,dy\,dz$ **[7M]**

UNIT – IV

7\. (a) Find the directional derivative of $\phi = xy^2+yz^3$ at the point $(1, -2, -1)$ in the direction of normal to the surface $x\,\log z - y^2 = -4$ at $(-1, 2, 1)$. **[7M]**

(b) Using Divergence theorem evaluate $\iint\limits_S \overrightarrow{F}.\hat{n}\ ds$ for $\vec{F} = y\,\hat{i} + x\,\hat{j} + z^2\hat{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2,\ z = 0$ and $z = b$. **[7M]**

8\. (a) Prove that $\nabla \times \nabla \times \vec{F} = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2\vec{F}$ **[7M]**

(b) If $\vec{F} = 3y\,\hat{i} - xz\,\hat{j} + y\,z^2$ and S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$, using Stokes theorem evaluate $\iint\limits_S curl\,\vec{F} \cdot \hat{n}\,ds$ **[7M]**

UNIT – V

9\. (a) i. If m and n are real constants greater than -1, prove that $\int\limits_0^1 x^m(\log x)^n dx = \frac{(-1)^n}{(m+1)^{n+1}}\Gamma\left(n+1\right)$ **[7M]**

ii. Evaluate $\int\limits_0^1 x^4\left(\log\frac{1}{x}\right)^3 dx$

(b) Prove that $J_{1/2}\left(x\right) = \sqrt{\frac{2}{\pi x}}\sin x$ and show that $\int\limits_0^{\pi/2} \sqrt{x}\,J_{1/2}\left(2x\right)dx = \frac{1}{\sqrt{\Pi}}$ **[7M]**

10\. (a) If α and β are the two distinct roots of $J_n\left(x\right) = 0$, then show that $\int\limits_0^1 x\,J_n\left(\alpha x\right)\ J_n\left(\beta x\right)\ dx = 0$ **[7M]**

(b) Prove that $x\,J_n{}'\left(x\right) = n\,J_n\left(x\right) - x\,J_{n+1}\left(x\right)$ **[7M]**