Hall Ticket No | | | | | | | | | | | Question Paper Code: AHS004

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

B.Tech IV Semester End Examinations (Regular / Supplementary) - May, 2019

Regulation: IARE – R16

COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTION

Time: 3 Hours **(Common to AE | EEE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Define the term Continuity of a complex variable function f(z). Justify whether every differentiable function is continuous or not. Give a valid example. **[7M]**

   (b) If $f\,(z) = u + iv = \frac{1}{z}$, then show that u(x,y)=$c_1$ and v(x,y)=c_2 the curves are intersects orthogonally. **[7M]**

2. (a) Define the term Analyticity and Differentiability of a complex variable function f (z). Prove that an analytic function f (z) with constant real part is always constant **[7M]**

   (b) Show that the function u= x^3-3xy^2 is harmonic and find the corresponding analytic function. **[7M]**

UNIT – II

3. (a) Define the term Power series expansions of complex functions. Write the Cauchy's integral formula and Cauchy's integral formula for multiple connected region. **[7M]**

   (b) Verify Cauchy's theorem for the function f(z)=z+1 in the region of c with vertices z=0, z=1, z=1+i, z=i. **[7M]**

4. (a) Define the term line integral. Evaluate $\int\limits_0^{2+i} z^2 dz$ along the real axis to 2 and then vertically to 2+i. **[7M]**

   (b) Evaluate $\int\limits_c \left(3x^2 + 4\,x\,y + i\,x^2\right) dz$ along the parabola $y = x^2$ from (0,0) to (1,1). **[7M]**

UNIT – III

5. (a) State Cauchy's Residue theorem of an analytic function f(z) within and on the closed curve, Taylor's theorem and Laurent theorem of complex power series. **[7M]**

   (b) Represent the function f(z)= $\frac{4z+3}{z(z-3)(z-2)}$ as Laurent series **[7M]**

   (i) With in |z|=1

   (ii) In the annulus region |z|=2 and |z|=3

   (iii) Exterior to |z|=3.

6. (a) Define **[7M]**
    i. The Isolated singularity of an analytic function f(z).
    ii. Pole of order m of an analytic function f(z).
    iii. Essential and removable singularity of an analytic function f(z).

   (b) Prove that $\int_0^{\pi} \frac{\cos 2\theta}{1-2a\ \cos\theta+a^2}\, d\theta = \frac{\pi a^2}{1-a^2},\ \left(a^2 < 1\right)$ using Residue theorem. **[7M]**

UNIT – IV

7. (a) Express the relation between the probability mass and cumulative mass function of a random variable. List the important properties of probability mass function **[7M]**

   (b) A random variable X has the following probability distribution as shown in Table 1. **[7M]**
   Determine (i) k (ii) Mean (iii) Variance (iv) $P\left(X < 6\right)$, (v) $P\left(0 < X < 5\right)$

Table 1

x	0	1	2	3	4	5	6	7
p(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

8. (a) Define the term probability density function. Explain mean and variance of a probability density function. Obtain the first 4 moments for the set of numbers 2, 4, 6 and 8. **[7M]**

   (b) Let X denote the maximum of the two numbers that appear when a pair of fair dice is thrown once. Find (i) Discrete probability distribution (ii) Expectation and (iii) Variance **[7M]**

UNIT – V

9. (a) Explain in detail about mean and variance of Binomial distribution. Draft the recurrence relation for the Binomial distribution. **[7M]**

   (b) Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (i)exactly10 (ii) At least 10 (iii) At most 8 (iv) At most 9 are good in mathematics. **[7M]**

10. (a) Explain the median and variance of a Normal distribution. **[7M]**

    (b) The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine (i) How many students got marks above 90%. (ii) What was the highest mark obtained by the lowest 10% of the students. **[7M]**

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