INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
B.Tech IV Semester End Examinations (Regular) - May, 2018
Regulation: IARE – R16
COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTION
Time: 3 Hours   (Common to AE | EEE)   Max Marks: 70

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) If f(z) is an analytic function with constant modulus, show that f(z)is constant.   **[7M]**

   (b) Show that the function $f(z) = \bar{Z}$ is continuous at every point but not differentiable at any point.

        [7M]

2. (a) Find the points at which the function $f(z) = e^{|z|^2}$ is analytic.   **[7M]**

   (b) Show that $u(x, y) = \cos x \cosh y$ is harmonic and find its conjugate harmonic function.

        [7M]

UNIT – II

3. (a) Evaluate $\int_C \operatorname{Re} z \, dz$, where C is the unit circle $x^2 + y^2 = 1$.   **[7M]**

   (b) Evaluate $\int_c \frac{e^{z^2+1}}{z} \, dz$, where $c: \ z = x + iy = 5\cos(t) - 3i\sin(t), \ 0 \le t \le 2\pi$.

        [7M]

4. (a) Evaluate $\int_c Z^{-2} dz$, C is a $|z - 1| = 1$.   **[7M]**

   (b) Evaluate $\oint_C \frac{z - \sin z}{z \sin z} \, dz$, where $C : |z - 3| = 1$.   **[7M]**

UNIT – III

5. (a) Find the Taylor's series expansion of $f(z) = \log z$, about $z_0 = i$. Also find the radius of convergence   **[7M]**

   (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2 - \cos\theta}$ using contour integration.

        [7M]

6. (a) By using Cauchy's residue theorem, evaluate the integral $\int_c \frac{z^2}{(z-1)^2(z+2)} dz$ where C is the circle |z|=5/2.   **[7M]**

   (b) Find the bilinear transformation that maps the points z_1=0, z_2=-i, z_3=-1 on the points w_1=i, w_2=1, w_3=0.   **[7M]**

UNIT – IV

7. (a) A box contains 12 items of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X. Determine the mean and variance. **[7M]**

   (b) A petrol pump is supplied with petrol once a day. If its daily volume X of sales in thousands of litre is distributed by f(x)=5$(1-x)^4$, 0≤X≤1. What must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01? **[7M]**

8. (a) A continuous random variable X has p.d.f f(x)=$3x^2$,0≤x≤1. Find 'a' and 'b' such that
   i. P(x≤a)=P(x≥a)
   ii. P(X<b)=0.05.

   [7M]

   (b) If the moments of a variable X are defined by $E(x^r)$=0.6, r=1,2,3. Show that P(X=0)=0.4, P(X=1)=0.6,$P(X \geq 2) = 0$.

   [7M]

UNIT – V

9. (a) If X is a normal distribution with mean 5 and variance 2 then find P $\{|X - 1| \leq 5\}$. **[7M]**

   (b) 30% of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random
   i. None is defective
   ii. One is defective
   iii. At least 3 are defective.

   [7M]

10. (a) If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals
    i. Exactly 3 suffers a bad reaction
    ii. More than 2 individuals suffers a bad reaction
    iii. None suffers a bad reaction.

    [7M]

    (b) If the probability density function of a random variable is then $f_X(x) = \frac{1}{\sqrt{10\pi}}e^{-\frac{(x-2)^2}{10}}$ find mean, variance and $P\{-1 < X \leq 3\}$

    [7M]

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