INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Four Year B.Tech II Semester End Examinations (Supplementary) - July, 2018
Regulation: IARE − R16
Complex Analysis and Probability Distribution

Time: 3 Hours   (ECE)   **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT − I

1. (a) Show that $\lim\limits_{z\to 0} \frac{x^2 y}{x^4 + 2y^2}$ does not exist even though this function approaches the same limit along every straight line through the origin. **[7M]**

   (b) If $\varphi + i\psi$ represents the complex potential of an electric field where $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$ find φ and also complex potential as a function of x. **[7M]**

2. (a) Find the analytic function whose real part is $u = e^{2x}\left(x\cos 2y - y\sin 2y\right)$. **[7M]**

   (b) If $f(z) = u + iv$ is analytic prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **[7M]**

UNIT − II

3. (a) State and Prove Cauchy's integral theorem. **[7M]**

   (b) Evaluate $\int_c \frac{dz}{(z^2+4)^2}$ where C is |z-i|=2 by using Cauchy integral formula **[7M]**

4. (a) When 'n' is negative integer, evaluate $\oint_C (z-a)^n\, dz$ where C is simple closed curve and the point z = a is (i) inside C (ii) outside C. **[7M]**

   (b) Evaluate $\int_C \frac{z}{(z-1)(z-2)^2}\, dz$, where $C : |z-2| = 0.5$. **[7M]**

UNIT − III

5. (a) Find the Taylor's series of $\log_e(1 + e^z)$ about z = 0. **[7M]**

   (b) Evaluate $\int_0^\pi \frac{d\theta}{1+a\cos\theta}$, $0 < a < 1$ by residue theorem. **[7M]**

6. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2a\cos\theta+a^2}, 0 < a < 1$ **[7M]**

   (b) Find the Bilinear transformation which maps the points 0, 1, ∞ into the points -5, -1, 3 respectively. **[7M]**

UNIT − IV

7. (a) If X has the Probability density function $f(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$, find the moment generating function of X. **[7M]**

   (b) The density function of a random variable X is given by $f(x) = Kx(2-x), 0 \le x \le 2$, Find K, mean variance rth moment. **[7M]**

8. (a) A random variable X has the probability function for Various Values of X.

i.Find K

ii.Evaluate P(x≥6) and P(3<x≤6)

iii.Find Cumulative Distribution Function   [7M]

Table 1

x:	0	1	2	3	4	5	6	7
P(x):	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

(b) If the Probability density function is given by $f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & otherwise \end{cases}$

i. Find Mean

ii. Evaluate $E[(2x+1)^2]$ [7M]

UNIT – V

9. (a) The probability that a bulb has at least 100 days of life is 0.05. Find the probability that out of 6 bulbs tested (i) at least one (ii) none will have a life of at least 100 days.   [7M]

(b) If the time to print an 8 x 10 size photograph may be looked upon as a random variable having the normal distribution with a mean of 10.1 seconds and a standard deviation of 2.7 seconds. Find the probability that it will take (i) anywhere from 8.5 to 13 seconds (ii)at least 15.1 seconds to print a photograph of this size.   [7M]

10. (a) A distributor of bean seeds determines form extensive tests that 5% of large batch of seeds will not germinate. He sells the seeds in packets of 200 and guarantees 90% germination. Use Poisson distribution to determine the probability that a particular packet violates the guarantee.   [7M]

(b) Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT?   [7M]

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