Hall Ticket No | | | | | | | | | | Question Paper Code: AHS011

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Four Year B.Tech III Semester End Examinations (Regular) - November, 2018

Regulation: IARE – R16

MATHEMATICAL TRANSFORM AND TECHNIQUES

Time: 3 Hours **(Common to AE | ECE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Expand the function from $f(x) = x - x^2$ to x= $-\pi$ as a Fourier series. Prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... \infty = \frac{\pi^2}{12}$. **[7M]**
   (b) Obtain the Fourier series expansion of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$. Deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} - ----\infty = \frac{\pi}{4}$. **[7M]**
2. (a) Find a Fourier series to represent x^2 in the interval (-l, l). **[7M]**
   (b) Expand $f(x) = \begin{cases} \frac{1}{4} - x, if 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, if \frac{1}{2} < x < 1 \end{cases}$ in the Fourier series of sine terms. . **[7M]**

UNIT – II

3. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ and hence find the value of $\int_0^\infty \left[\frac{\sin t}{t}\right]^2 dt$. **[7M]**
   (b) Find the Fourier cosine transform of $f(x) = e^{-ax} (a > 0)$ and hence find the value of $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$. **[7M]**
4. (a) If the Fourier inverse finite sine transform of f(n) = $\frac{1-\cos n\pi}{n^2\pi^2}$, $0 < x < \pi$, $find\, f(x)$. **[7M]**
   (b) Find Fourier cosine transform of f(x), where $f(x) = \begin{cases} x & , 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0 & , x > 2 . \end{cases}$ **[7M]**

UNIT – III

5. (a) Find $L\left[t\, e^{-t} \cos t\right]$ **[7M]**
   (b) Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. **[7M]**

6. (a) Find $L \; [\, f(t) \,]$, where $f(t) = \begin{cases} 1 & , \; 0 \le t < 2 \\ -1 & , \; 2 \le t < 4 \end{cases}$, $f(t+4) = f(t)$. **[7M]**

(b) Apply Laplace transforms, find the solution of the initial value problem $x'' + 9x = \sin t$ if x(0)= 1, $x\left(\frac{\pi}{2}\right) = 1$ **[7M]**

UNIT – IV

7. (a) Find Z transform of $\left[\frac{2n+3}{(n+1)(n+2)}\right]$ **[7M]**

(b) Find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ using convolution theorem. **[7M]**

8. (a) Find $Z \; \left[\, (n+1)^2 \,\right]$ **[7M]**

(b) Determine $Z^{-1}\left[\frac{2z\ (2z-1)}{(z-1)\ (z-z)^2}\right]$. **[7M]**

UNIT – V

9. (a) Apply Lagrange's method to solve the linear partial differential equation.
$x^2(y-z) + y^2(z-x) = z^2(x-y)$. **[7M]**

(b) A tightly stretched string with fixed end-points x=0 and x= l cm is initially in its equilibrium position. If each of its points is given by velocity $g(x) = \lambda x\,(l-x)$. Determine the displacement of the string at any distance x from one end at any time t. **[7M]**

10. (a) Using the method of separation of variables, solve the partial differential equation.
$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x,0) = 8\, e^{-3x}$ **[7M]**

(b) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0^0C and kept at that temperature . Find the temperature function u(x, t). **[7M]**

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