Hall Ticket No | | | | | | | | | | | Question Paper Code: AHSB02

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Four Year B.Tech I Semester End Examinations(Regular) - December, 2019

Regulation: IARE – R18

LINEAR ALGEBRA AND CALCULUS

Time: 3 Hours **(Common to All Branches)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Find rank by reducing it to normal form of $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. **[7M]**

   (b) Find the inverse of a matrix using Gauss-Jordan method $\begin{bmatrix} 2 & 1 & -1 \\ 5 & 2 & 2 \\ 3 & 1 & 1 \end{bmatrix}$. **[7M]**

2. (a) Solve the second order differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = cos^2 x$. **[7M]**

   (b) Solve the differential equation by variation of parameters method $\left(D^2 + 1\right) y = \sec x$. **[7M]**

UNIT – II

3. (a) Find the eigen values and eigen vectors of A= $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **[7M]**

   (b) Using Cayley - Hamilton theorem find A^{-1} for A= $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **[7M]**

4. (a) Change the order of integration $\int_0^4 \int_y^4 \frac{x\,dx\,dy}{x^2+y^2}$. **[7M]**

   (b) Find by double integration, the area lying inside the circle r=a sin θ and outside the cardioid r=a(1-Cos θ). **[7M]**

UNIT – III

5. (a) Verify Lagrange's Mean Value theorem for the function $x\,(x-1)\,(x-2)$ in $\left[0, \frac{1}{2}\right]$. **[7M]**

   (b) Verify Rolle's theorem for the function $(x-1)^2\,(x-2)^2$ in [1,2]. **[7M]**

6. (a) Using triple integration find the volume of the sphere $x^2+y^2+z^2=a^2$. **[7M]**

   (b) Evaluate the triple integral $\int\limits_0^1 \int\limits_0^{\sqrt{1-x^2}} \int\limits_0^{\sqrt{1-x^2-y^2}} xyz\,dzdydx.$ **[7M]**

UNIT – IV

7. (a) If $u = x^2 - 2y^2,\ v = 2x^2 - y^2,$ where $x = r\cos\theta,\ y = r\sin\theta$, show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3\sin 2\theta$. **[7M]**

   (b) Find the dimensions of the rectangular box, open at the top, of max capacity whose surface area is 32 sq.cm. **[7M]**

8. (a) Explain the procedure for finding the stationary and extreme points of a two independent variable function f(x,y). **[7M]**

   (b) Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. **[7M]**

UNIT – V

9. (a) Find the circulation of $\bar{F} = (2x - y + 2z)\bar{i} + (x + y - z)\bar{j} + (3x - 2y - 5z)\bar{k}$ along the circle $x^2 + y^2 = 4$ in the xy plane. **[7M]**

   (b) Find the constant 'a' so that $\bar{F} = (a\,x\,y - z^3)\ \bar{i}\ + (a-2)\,x^2\ \bar{j}\ + (1-a)\,a\,z^3\ \bar{k}$ is a conservative field. **[7M]**

10. (a) Applying Green's theorem evaluate $\int\limits_c (xy + y^2)dx + x^2\,dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^2$. **[7M]**

    (b) Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the point P(1,-2,-1) in the direction to the surface x log z - y^2 = -4 at (-1,2,1). **[7M]**