Hall Ticket No | | | | | | | | | | Question Paper Code: AHSB05

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Four Year B.Tech III Semester End Examinations(Regular) - November, 2019

Regulation: IARE – R18

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

Time: 3 Hours **(ECE)** **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Write the definition of ananalytic function. Also, if $f(z)$is an analytic function with constant modulus, then show that $f(z)$ is constant. **[7M]**
   (b) Prove that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied there. **[7M]**
2. (a) Find the bilinear transformation which maps the points $(\infty, i, 0)$ into the points $(0, i, \infty)$. **[7M]**
   (b) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ **[7M]**

UNIT – II

3. (a) Evaluate $\int\limits_C z\, dz$ where C is the upper semi-circle $x^2 + y^2 = 1$ taken in anti-clock wise direction. **[7M]**
   (b) Evaluate $\oint\limits_C \frac{z}{(z-1)(z^2+1)} dz$ where $C : |z - i| = \frac{1}{2}$ **[7M]**
4. (a) Evaluate using Cauchy's theorem for the function $\int\limits_c \frac{e^{-z}}{z+1}$dz where C is $|z| = 3$ **[7M]**
   (b) Evaluate the integrals
   i) $\oint\limits_C \frac{e^{2z}}{(z+1)^4} dz$ where C is $|z| = 3$
   ii) $\oint\limits_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C is $|z| = 4$ **[7M]**

UNIT – III

5. (a) Expand $f(z) = \frac{1}{z^2+3z+2}$ in i) $|z| < 1$ ii) $|z + 1| < 1$ **[7M]**
   (b) Evaluate $\oint\limits_C \frac{1}{z^3+7z^2+6z} dz$ where $C : |z| = 2$ **[7M]**
6. (a) Determine the residues of $f(z) = \frac{2z}{(z^2+1)(2z-1)}$ at each of the poles in the z-plane. **[7M]**
   (b) Using the concept of Residue theorem, prove that $\int\limits_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2} d\theta = \frac{2\pi a^2}{1-a^2},\ a^2 < 1$ **[7M]**

UNIT – IV

7. (a) Define Beta and Gamma functions and prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $m, n > 0$ **[7M]**

   (b) Express $\int\limits_0^\infty x^2 e^{-x^2} dx$ in terms of beta and gamma functions. **[7M]**

8. (a) Show that $\int\limits_0^{\pi/2} \sqrt{\cot\theta}\ d\theta = \int\limits_0^{\pi/2} \sqrt{\tan\theta}\ d\theta = \frac{\pi}{\sqrt{2}}$ **[7M]**

   (b) Prove that $\beta(n, n+1) = \frac{1}{2}\frac{[\Gamma(n)]^2}{\Gamma(2n)}$ and hence find $\beta(1/4, 5/4)$ **[7M]**

UNIT – V

9. (a) State and prove orthogonal trajectories for Bessel functions. **[7M]**

   (b) Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ **[7M]**

10. (a) Prove $e^{\frac{1}{2}x(t-\frac{1}{t})} = \sum\limits_{n=-\infty}^{\infty} t^n J_n(x)$ **[7M]**

    (b) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$ **[7M]**