INSTITUTE OF AERONAUTICAL ENGINERRING (AUTONOMOUS)

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ADVANCED STRUCTURAL ANALYSIS AND DESIGN (ACE016)

IARE-R16 B.Tech VII SEM

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Unit I
Matrix methods of analysis

Different types of supports

1.Fixed Support(Has 3 Reactions)

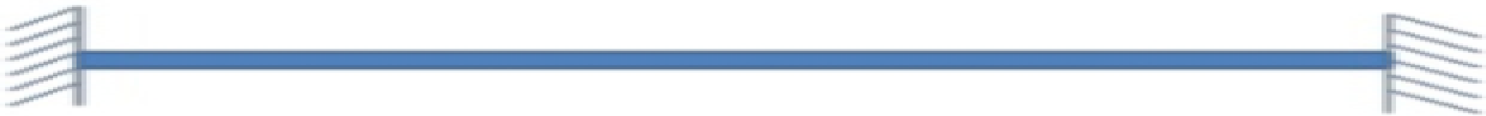
2.Hinge/Pin Support(Has 2 Reactions)

3.Roller Support(Has 1 Reactions)

Different types of beams

1.Simply Supported Beam:

2.Fixed Supported Beam:



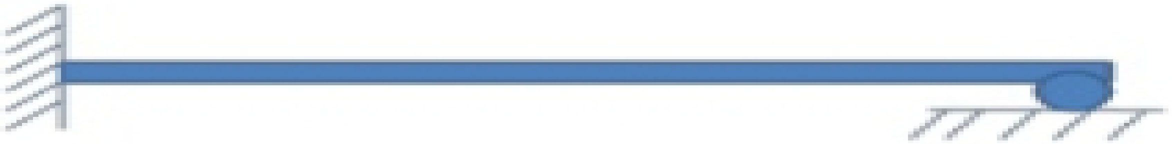
3.Cantilever Beam:

4.Overhanging beam:

Different types of beams

5.Continuous Beams:

6.Propped Cantilever Beam:



Structural analysis

DEFINITION: Analysis of a given structure subject to some given loads and to predict the response of the structure.

METHODS OF ANALYSIS

- Slope deflection method
- Moment distribution method
- Kani's method
- Stiffness method or Displacement method
- Flexibility method or Force method

Static indeterminacy(Ds):

If the number of independent static equilibrium equations are not sufficient for solving for all the external and internal forces.

Kinematic indeterminacy or degrees of freedom(Dk):

The independent joint displacements that are necessary to specify the deformed shape of the structure when subjected to loading.

Support	DOF
Fixed	0
Hinge	1
Roller	2
Free end	3

Beams:

Ds = R-3

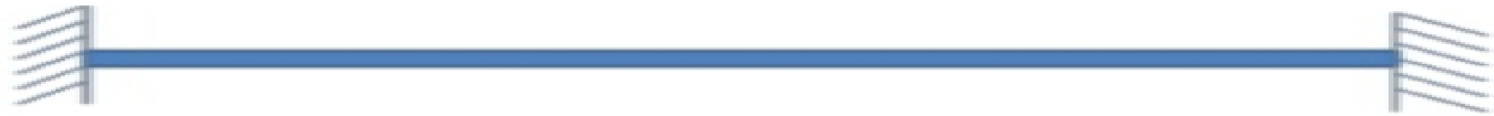
= 3-3 = 0

Dk = 0+3 = 3

Ds = 3-3 = 0

Dk = 1+2 = 3

Beams:



Ds = R-3

= 6-3 = 3

Dk = 0+ 0 = 0

Ds = 4-3 = 1

Dk = 0+2 = 2

Beams:

Ds = 2

Dk = 7

Trusses:

- **External static indeterminacy (De) =** R-3

Internal static indeterminacy (Di) = m – (2j-3)

Where R = no. of reactions

m = no. of members

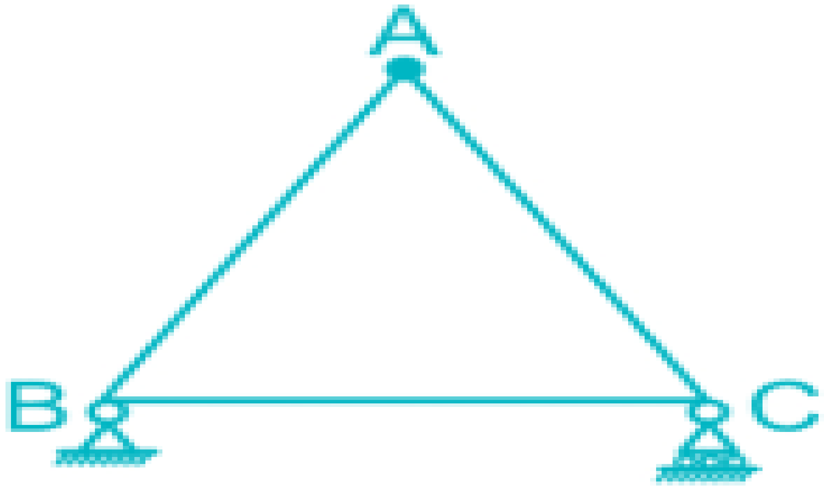
j = no. of joints

Total static indeterminacy Ds = De + Di

- **Kinematic indeterminacy Dk = 2j - R**

Static And Kinematic Indeterminacy

Truss:



De = 0

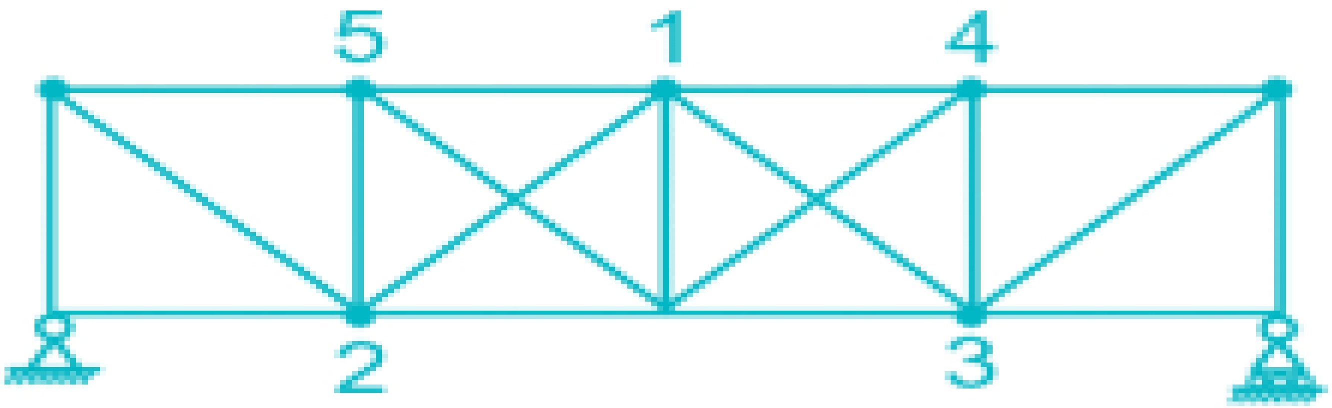
Di = 0

m = 2j-3

Ds = 0+0 = 0

Dk = 3

Truss:



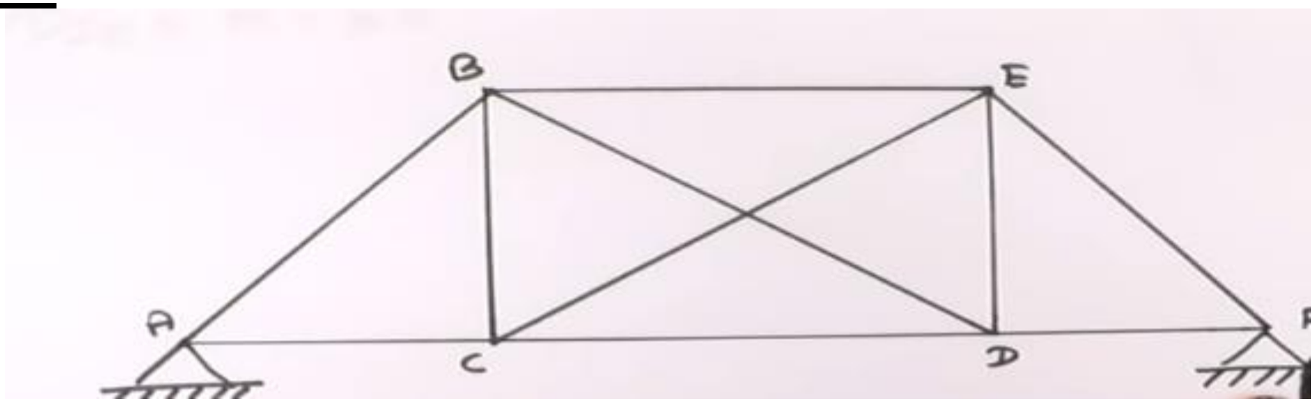
De = 0

Di = 2

Ds = 0+2 = 2

Dk = 17

Truss:



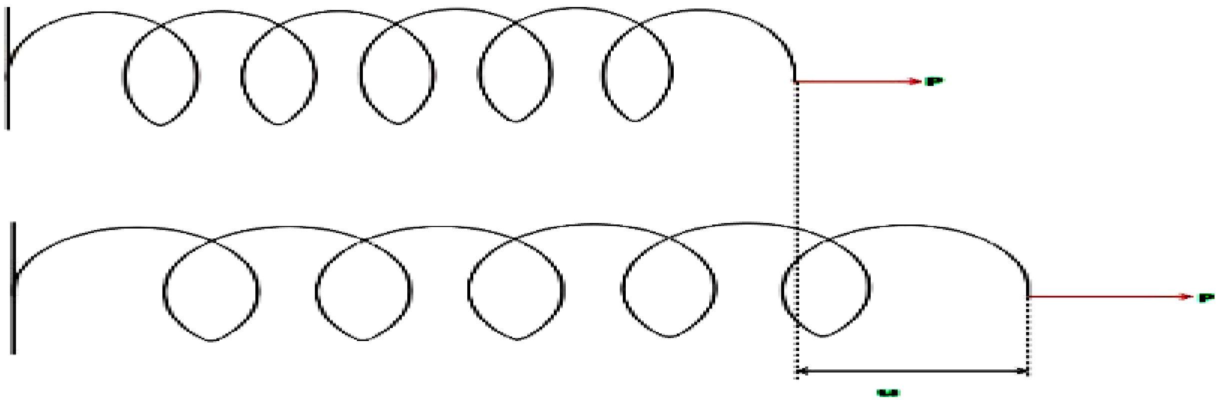
De = 1

Di = 1

Ds = 1+1 = 2

Dk = 8

Stiffness method



U α P

U = f x P

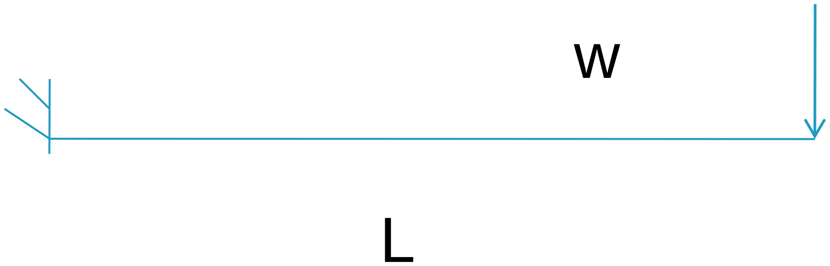
F = U/P

Flexibility is defined as displacement per unit force

Stiffness is defined as force required due to unit force

Relationship between flexibility and Stiffness

F = 1/S



<u>Example:</u>

Y = wL3/3EI

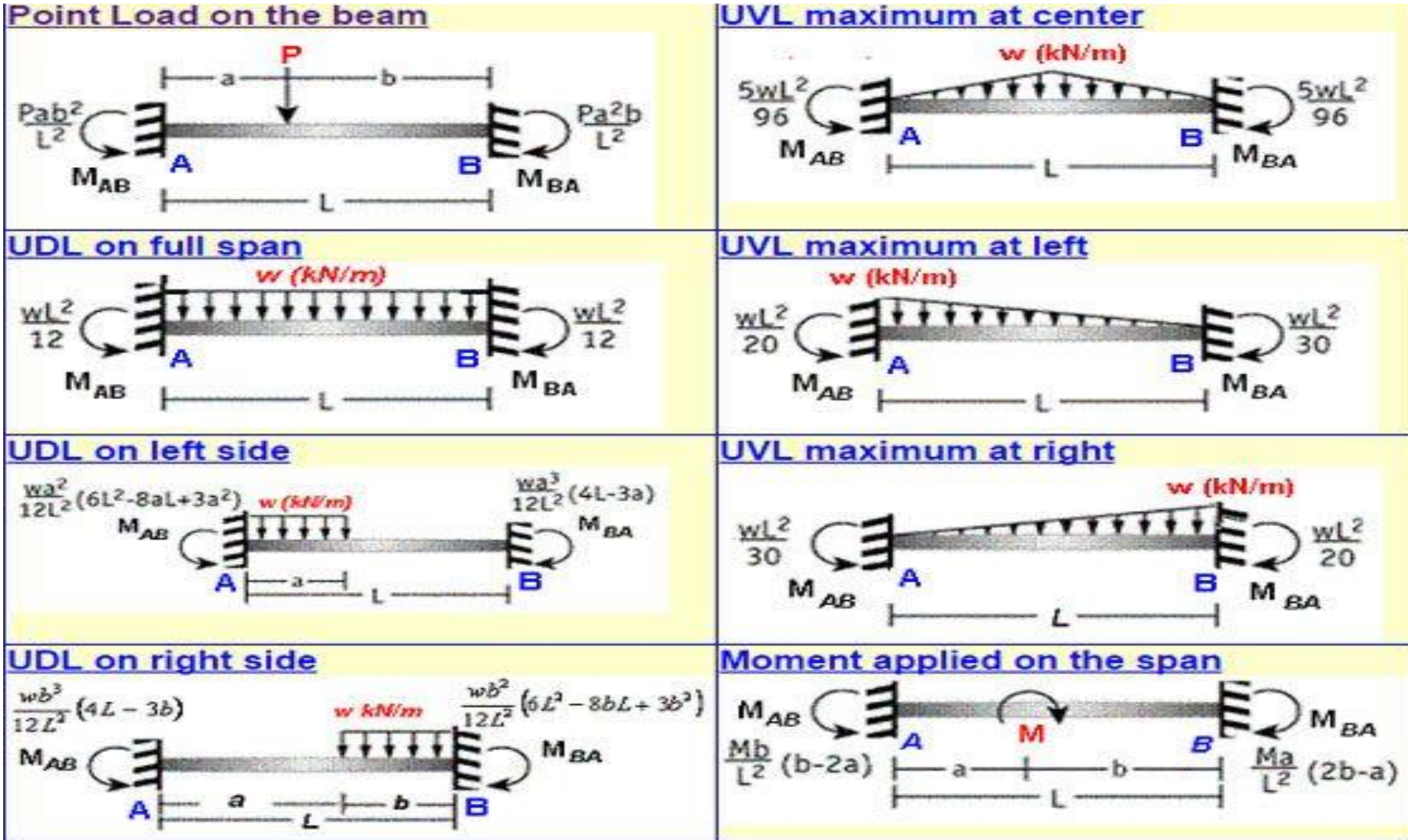
F = L3/3EI

S = 3EI/L3

Stiffness method

S.No.	Type of displacement, Δ	Flexibility, δ	Stiffness, k
1.	Axial	$\frac{L}{AE}$	$\frac{AE}{L}$
2.	Transverse		
	(a) Far-end fixed	$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$
	(b) Far-end hinged	$\frac{L^3}{3EI}$	$\frac{3EI}{L^3}$
3.	Bending or flexural		
	(a) Far-end fixed	$\frac{L}{4EI}$	$\frac{4EI}{L}$
	(b) Far-end hinged	$\frac{L}{3EI}$	$\frac{3EI}{L}$
4.	Torsional	$\frac{L}{GK}$	$\frac{GK}{L}$

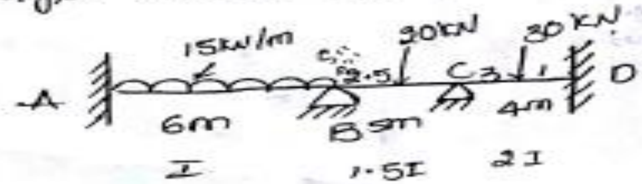
Fixed end moments



Stiffness method procedure

- Determine the kinematic indeterminacy of the given structure(neglect the horizontal forces in case of beams)
- Identify the unknowns
- Assign the co-ordinates
- Calculate the fixed end moments and net forces at joints
- Determine the external loads at the co-ordinates
- Determine the stiffness matrix
- Use the conditions of equilibrium of forces determine unknown displacements
- Compute the final end moments using slope deflection equations

① Analyse a C.B as shown in the fig. using stiffness m...



Sol:-

Step 1 :- $K \cdot I = 2$

Step 2:- fixed end moments

$$M^F_{AB} = -\frac{wl^2}{12} = \frac{-15 \times 6^2}{12} = -45\ kN \cdot m$$

$$M^F_{BA} = +45\ kN \cdot M$$

$$M^F_{BC} = -\frac{wl}{8} = -\frac{20 \times 5}{8} = -12.5\ kN \cdot m$$

$$M^F_{CB} = +12.5\ kN \cdot M$$

$$M^F_{CD} = -\frac{wab^2}{l^2} = -\frac{30 \times 3 \times 1^2}{4^2} = -5.63\ kN \cdot m$$

$$M^F_{DC} = +16.88\ kN \cdot M.$$

Example 1

Step 3:- Net forces:-

ΣFEM @ B $R_{f_1} = 45 - 12.5 = 32.50$ kN·m

ΣFEM @ C $R_{f_2} = 12.5 - 5.63 = 6.87$ kN·m

$$\begin{bmatrix} R_{f_1} \\ R_{f_2} \end{bmatrix} = \begin{bmatrix} 32.5 \\ 6.87 \end{bmatrix}$$

Step 4:- R matrix is zero because no external loads or forces

$[R] = 0$

Example 1

Step 5:- Stiffness Matrix.

$$S_{11} = \frac{4EI_1}{l_1} + \frac{4EI_2}{l_2}$$

$$= \frac{4EI_1}{6} + \frac{4E \times 1.5I}{5}$$

$$S_{11} = 1.87\,EI$$

$$S_{21} = \frac{2EI_1}{L_1} = \frac{2E \times 1.5I}{5}$$

$$S_{21} = 0.60\,EI$$

$$S_{22} = \frac{4E \times 1.5I}{5} + \frac{4E \times 2I}{4}$$

$$= 3.20\,EI$$

$$S_{12} = \frac{2 \times E \times 1.5I}{5}$$

$$= 0.6\,EI$$

Applying unit displacement @ co-ordinate 1.

Applying unit displacement @ co-ordinate 2.

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = EI \begin{bmatrix} 1.86 & 0.6 \\ 0.6 & 3.2 \end{bmatrix}$$

Example 1

$$[D] = [S]^{-1}\left([R] - [R_f]\right)$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 1.86 & 0.6 \\ 0.6, & 8.2 \end{bmatrix}^{-1}\begin{bmatrix} -32.5 \\ -6.87 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0.5722 & -0.107 \\ -0.107 & 0.3326 \end{bmatrix}\begin{bmatrix} -32.5 \\ -6.87 \end{bmatrix}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -17.86 \\ 1.1925 \end{bmatrix}$$

Example 1

Step6:- End moments

$$M_{AB} = M^F_{AB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l}$$

$$= -45 + 0 + \frac{2 \times -17.86}{6}$$

$$= -50.95 \text{ kN·m}$$

$M_{BA} = 33.09$ kN·m

$M_{BC} = -33.03$ kN·m

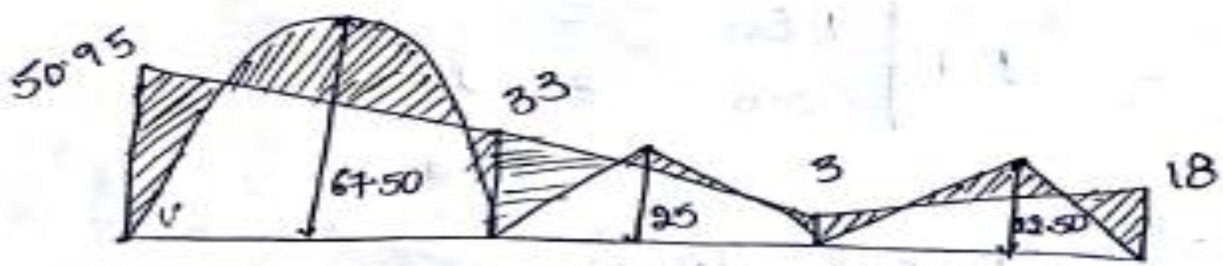
$M_{CB} = 3.4$ kN·m

$M_{CD} = -3.0$ kN·m

$M_{DC} = 18.07$ kN·m

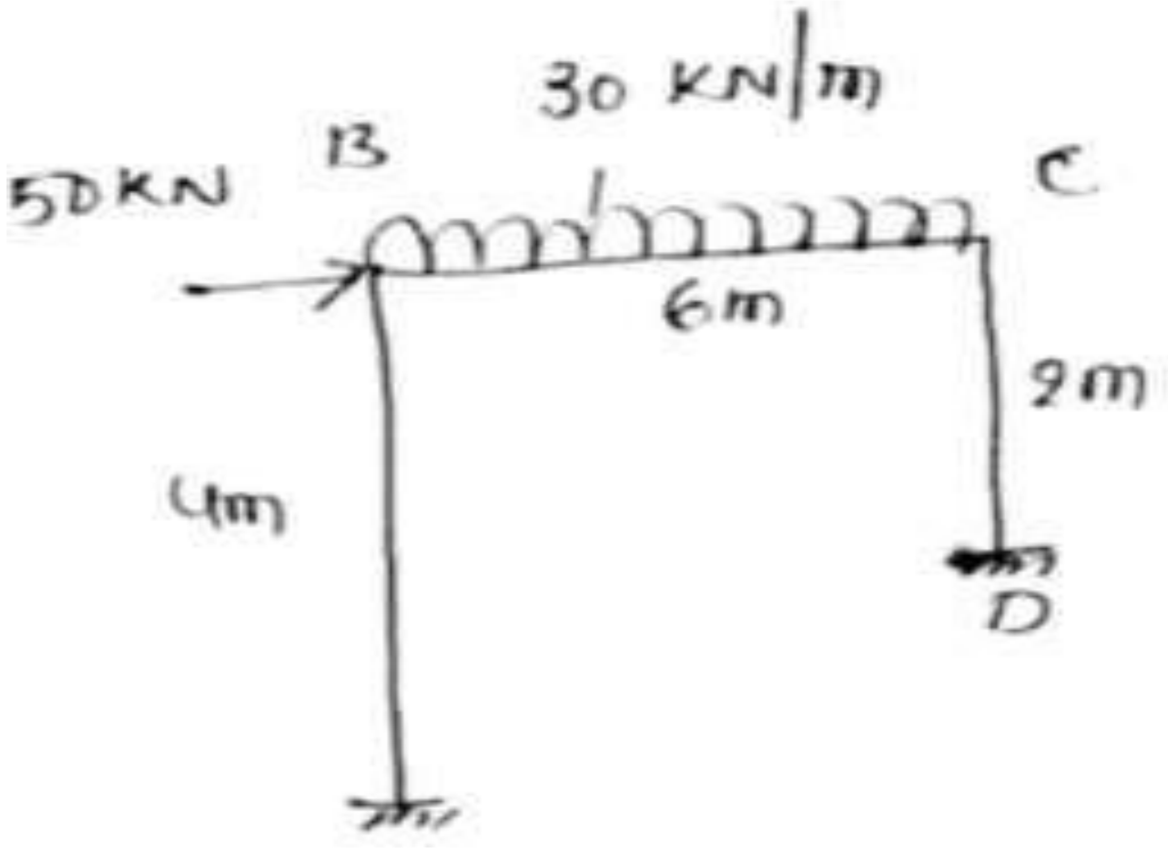
Example 1

Bending moment diagram:-



BMD (kN·m)

Analysis of frames



Analysis of frames

- **Kinematic indeterminacy = 3**

  Unknowns are Δ, θB, θC

- **Fixed end moments are**

  Mf AB = MfBA =0

  MfBC = -90 kNm

  MfCB = 90 kNm

  MfCD = MfDC = 0

Analysis of frames

- **NET FORCES AT THE COORDINATES**

  AT COORDINATE 1 RF1 = 0

  AT COORDINATE 2 RF2 = -90kNm

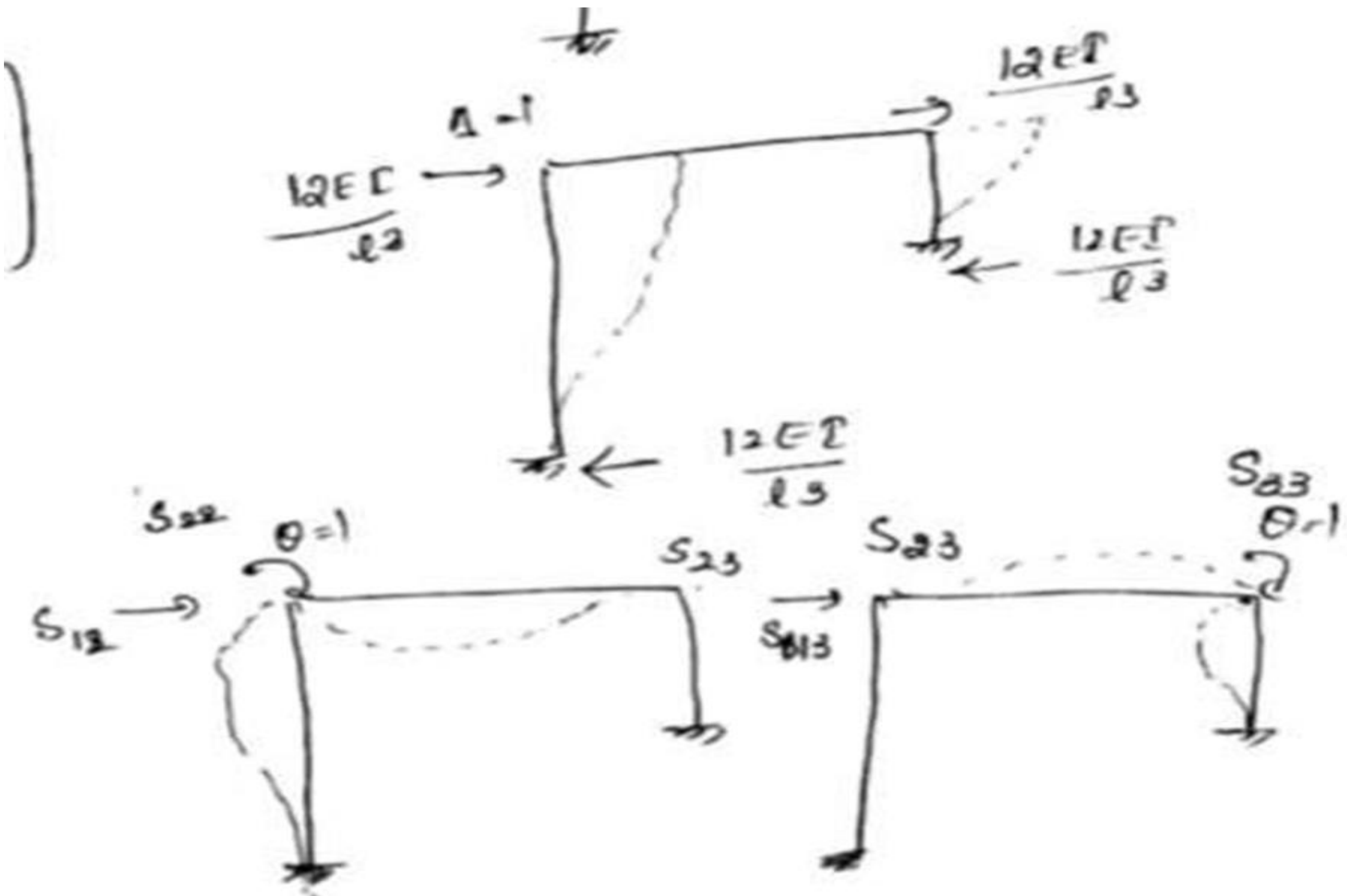
  AT COORDINATE 3 RF3 = 90kNm

- **EXTERNAL FORCES AT COORDINATES**

  AT COORDINATE 1 R1 = 50

  AT COORDINATE 2 R2 = 0

  AT COORDINATE 3 RF3 = 0


12EI/l3
12EI/l2
12EI/l3
12EI/l3
S22
θ=1
S23
S12
S23
S33
θ=1
S13

Analysis of frames

S.No.	Type of displacement, Δ	Flexibility, δ	Stiffness, k
1.	Axial	$\frac{L}{AE}$	$\frac{AE}{L}$
2.	Transverse		
	(a) Far-end fixed	$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$
	(b) Far-end hinged	$\frac{L^3}{3EI}$	$\frac{3EI}{L^3}$
3.	Bending or flexural		
	(a) Far-end fixed	$\frac{L}{4EI}$	$\frac{4EI}{L}$
	(b) Far-end hinged	$\frac{L}{3EI}$	$\frac{3EI}{L}$
4.	Torsional	$\frac{L}{GK}$	$\frac{GK}{L}$

Analysis of frames

S11 = (12EI/L3) + (12EI/L3) = 1.688

S12 = S21 = -6EI/L2 = 0.38

S13 = S31 = -6EI/L2 = 1.5

S22 = (4EI/L) + (4EI/L) = 1.67

S23 = S32 = 2EI/L = 0.33

S33 = 4EI/L + 4EI/L = 2.67

Δ = 81.62/EI

θB = 17.84/EI

θC = -63.04/EI

Analysis of frames

Final end moments

M AB = 32.335 kNm

M BA = 63.125 kNm

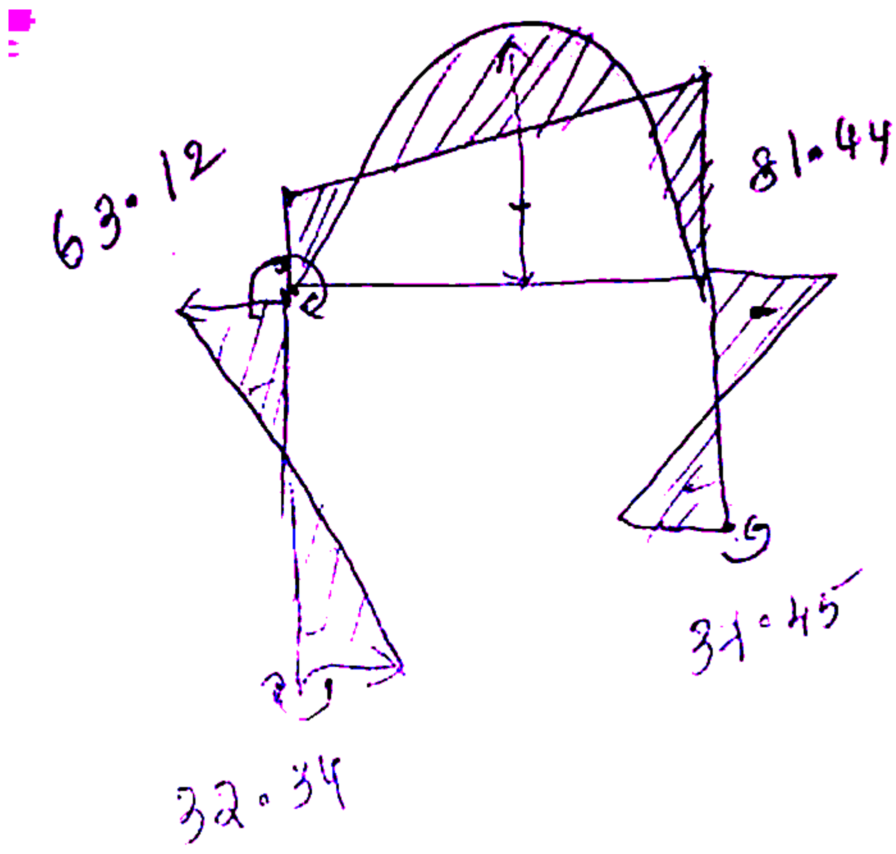
M BC = -63.49 kNm

M CB = 81.44 kNm

M CD = -81.08 kNm

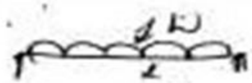
M DC = -37.45 kNm

Bending moment diagram

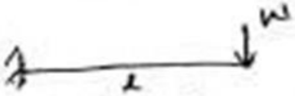


① 

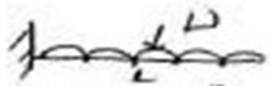
$\theta = \frac{wl^2}{16EI}$, $D = \frac{wl^3}{48EI}$

② 

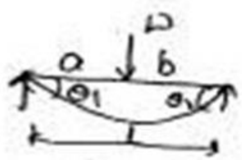
$\theta = \frac{wl^3}{24EI}$, $D = \frac{5wl^4}{384EI}$

③ 

$\theta = \frac{wl^2}{2EI}$, $\frac{wl^3}{3EI}$

④ 

$\theta = \frac{wl^3}{6EI}$, $D = \frac{wl^4}{8EI}$

⑤ 

$\theta_1 = \frac{wb}{6EIL}(l^2-b^2)$

$\theta_2 = \frac{wa}{6EIL}(l^2-a^2)$

$D = \frac{wa^2b^2}{3EIL}$

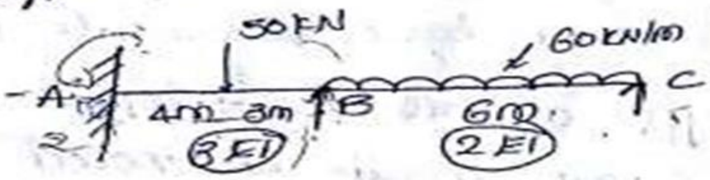
⑤ 

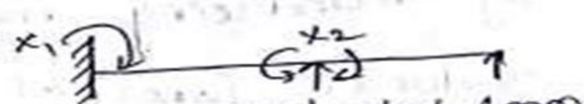
$\theta = \frac{5wl^3}{120EI}$

$D = \frac{wl^4}{30EI}$

Flexibility method

Analyse a C B as shown in fig





Redundant beam

Sol:-

Step 1:- $S.I = 2+1+1-2 = 2$

Static indeterminacy of given beam is equal to 2

∴ release two unknown forces i.e moment at A and reaction at B

Step 2:- Flexibility matrix

Applying unit force @ A

$$f_{11} = \frac{L}{3EI} = \frac{7}{9EI}$$

$$f_{21} = \frac{L}{6EI} = \frac{7}{18EI}$$

Step2:- Flexibility matrix

$f_{11} = \frac{L}{3EI} = \frac{7}{9EI}$

$f_{21} = \frac{L}{6EI} = \frac{7}{18EI}$

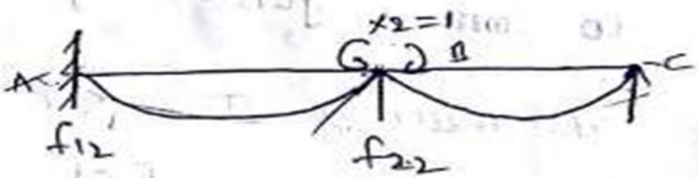
Applying unit force @ A

$f_{12} = \left(\frac{L}{6EI}\right)_{BA} = \frac{7}{18EI}$

$f_{22} = \left(\frac{L}{3EI}\right)_{BA} + \left(\frac{L}{3EI}\right)_{BC}$

$= \frac{7}{9EI} + \frac{6}{6EI}$

$= \frac{16}{9EI}$



Applying unit force @ B

step3 :- Deformation matrix.

$$\delta_1 = \frac{wb}{6EIL}(L^2-b^2)$$

$$= \frac{50\times 3}{18EI\times 7}(7^2-3^2)$$

$$\delta_1 = \frac{47.62}{EI}$$

$$\delta_2 = \frac{wb}{6EIL}(L^2-a^2) + \frac{wL^3}{24EI}$$

$$= \frac{50\times 4}{18EI\times 7}(7^2-4^2) + \frac{60\times 6^3}{48EI}$$

52.38

$$\delta_2 = \frac{322.38}{EI}$$

Flexibility method

$$[f]_e [x]_e = [\delta]_e$$

$$[x]_e = [f]_e^{-1} [\delta]_e$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{7}{9EI} & \frac{7}{18EI} \\ \frac{7}{18EI} & \frac{16}{9EI} \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{47.62}{EI} \\ \frac{322.38}{EI} \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} \frac{7}{9} & \frac{7}{18} \\ \frac{7}{18} & \frac{16}{9} \end{bmatrix}^{-1} \begin{bmatrix} \frac{47.62}{EI} \\ \frac{322.38}{EI} \end{bmatrix}$$

$$= \left(\frac{1}{EI}\right) \begin{bmatrix} 1.4436 & -0.315 \\ -0.315 & 0.635 \end{bmatrix} \begin{bmatrix} \frac{47.62}{EI} \\ \frac{322.38}{EI} \end{bmatrix}$$

$$= \begin{bmatrix} 1.4436 & -0.315 \\ -0.315 & 0.6315 \end{bmatrix} \begin{bmatrix} 47.62 \\ 322.38 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -32.80 \\ 188.58 \end{bmatrix}$$

Unit II
Approximate methods of analysis

Approximate methods

Why approximate analysis?

- Rapid check on computer aided analysis
- Preliminary dimensioning before exact analysis

Advantage?
- Faster

Disadvantage?
- Results are approximate

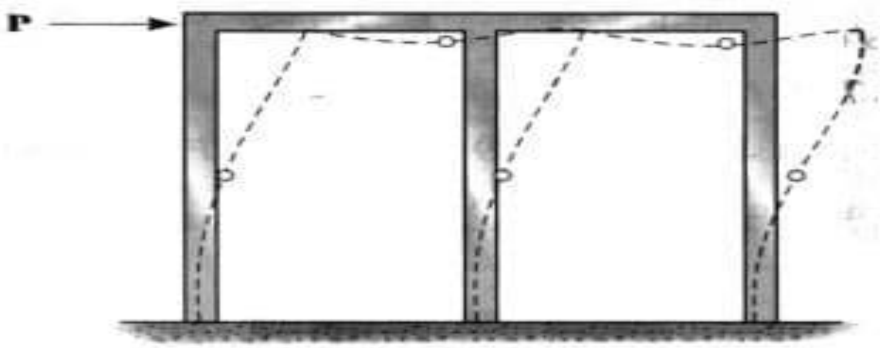
- Approximate methods are particularly useful for multi-storey frames taller than 3 storeys.

Portal frame method

The portal method is an approximate analysis used for analyzing building frames subjected to lateral loading. This method is more appropriate for low rise buildings with uniform framing.

Assumptions

- An inflection point is located at mid-height of each column, Figure.
- An inflection point is located at the centre of each beam, Figure.

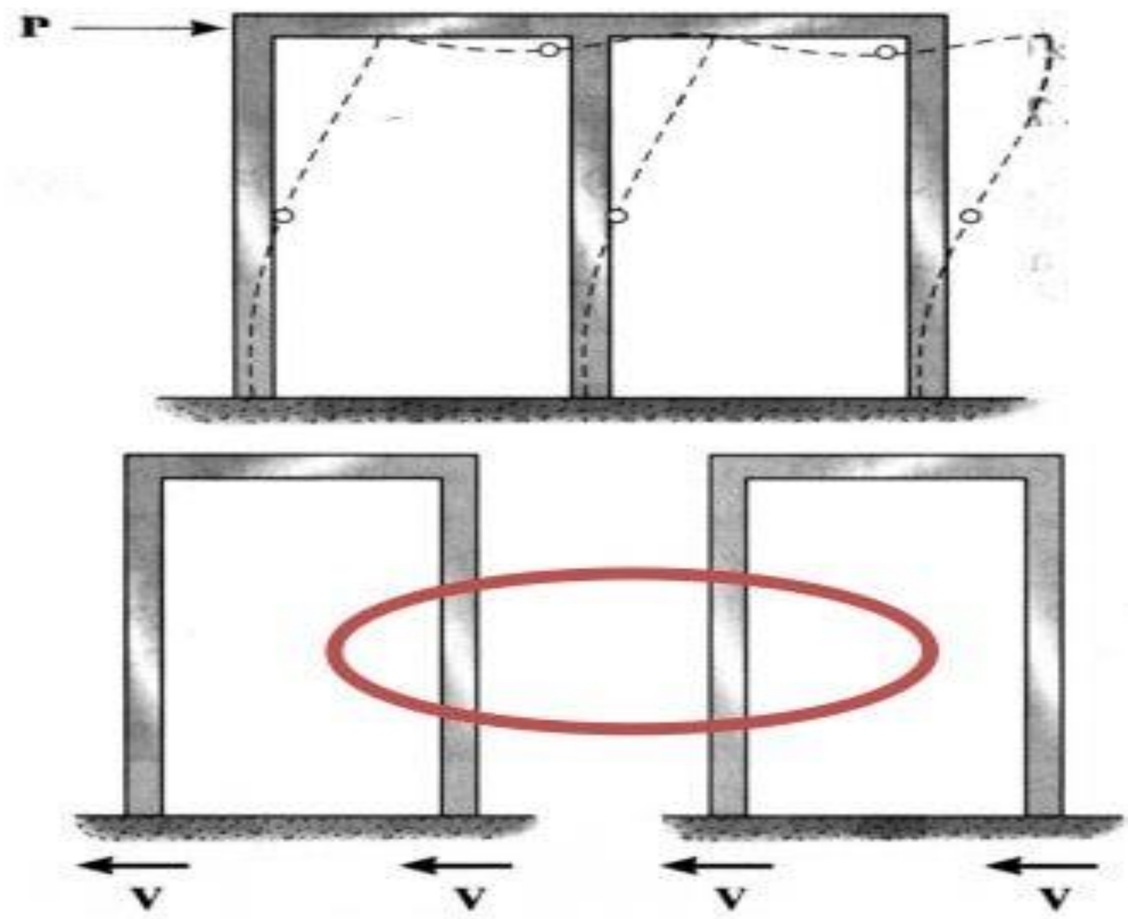


○ = inflection point

Portal frame method

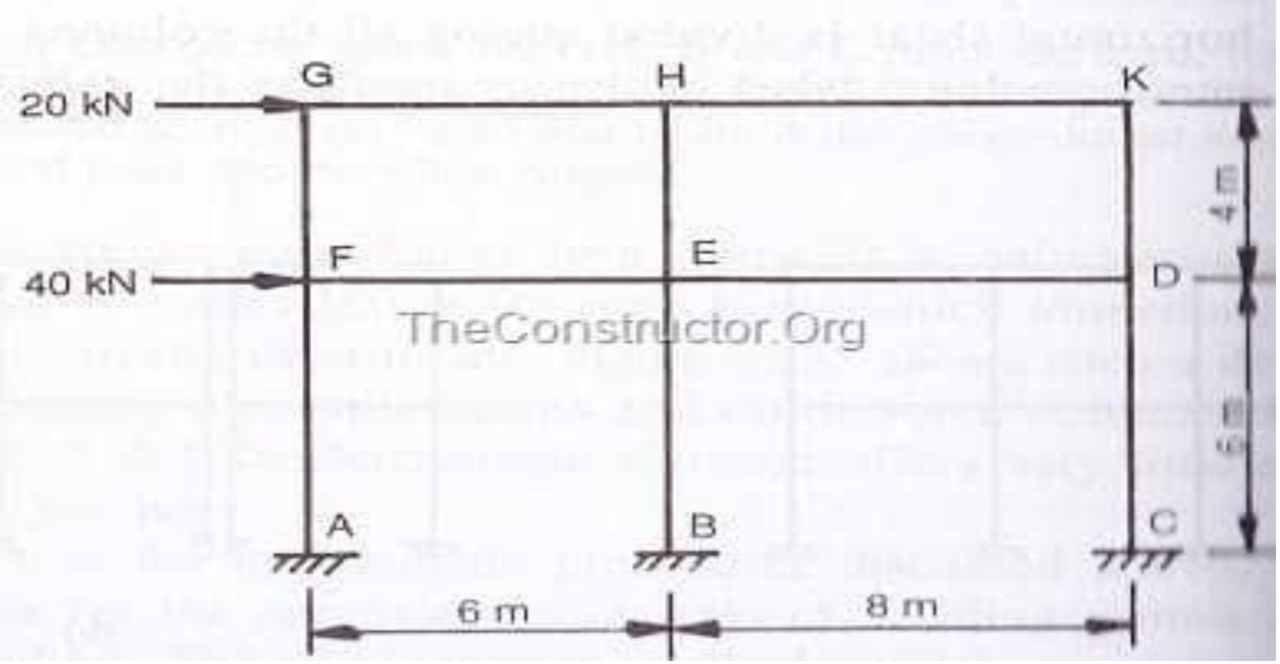
- The horizontal shear is divided among all the columns on the basis that each interior column takes twice as much as exterior column.
- The basis for the third assumption stems from the reasoning that the frame is composed of individual portals as shown in Figure.
- Obviously an interior column is in effect resisting the shear of columns of the individual portals.

Portal frame method



Example

It is required to determine the approximate values of moment, shear and axial force in each member of frame as shown in Figure, using portal method.



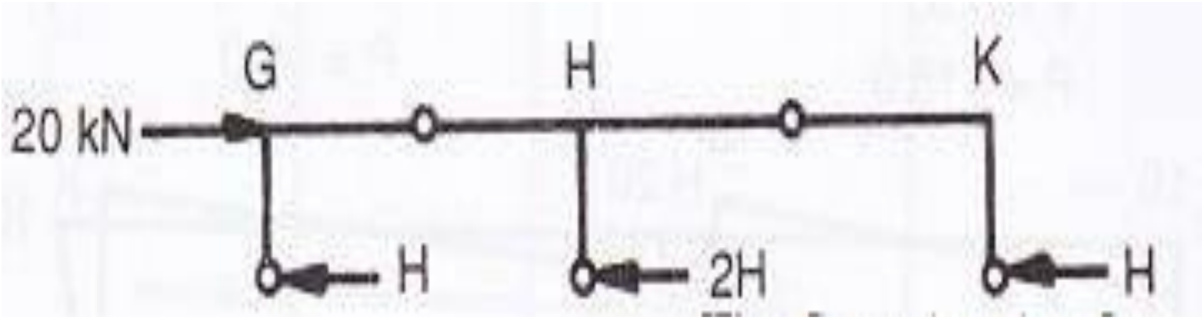
Portal frame method

Considering the first upper storey, inflection points are assumed at mid height on each column. We obtain the shear in each column from a free body diagram of the structure above the hinge level by assigning shear to the interior column equal to twice the shear in exterior column as shown in Figure

By taking summation of horizontal force, the unknow forces can be as follow

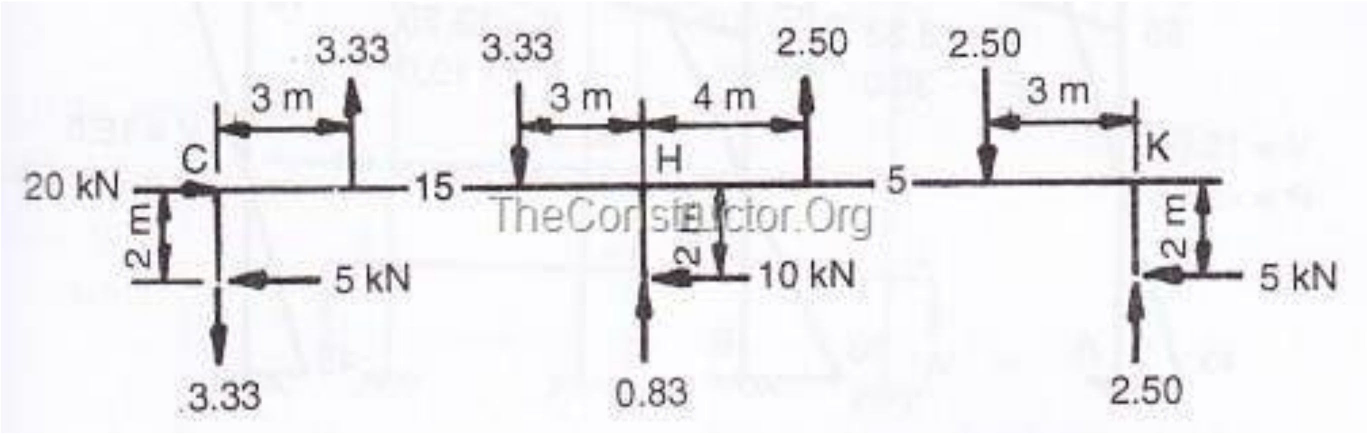
$H + 2H + H = 20 \rightarrow H = 5$ kN

Portal frame method



- Inflection points are also assumed at the centre of beams GH and HK. The member forces in the upper part of the frame can be evaluated from the free body diagram of the parts shown in Fig. beginning with G or from K and working across.
- The resulting forces must check with the free body diagram at the opposite end. The resulting forces are indicated on the diagram

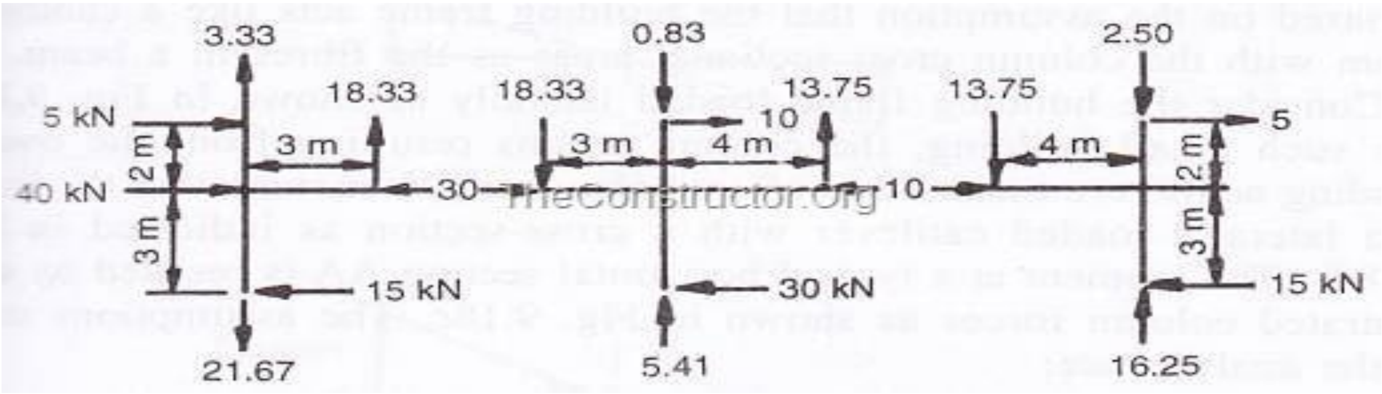
Portal frame method



- Again inflection points are assumed at mid-height of lower storey columns and the shear is distributed as in the upper storey.
- Thus, in the lower storey, the horizontal forces can be computed using the following expression:

H + 2H + H = 60 kN ==> H = 15 kN.

Portal frame method

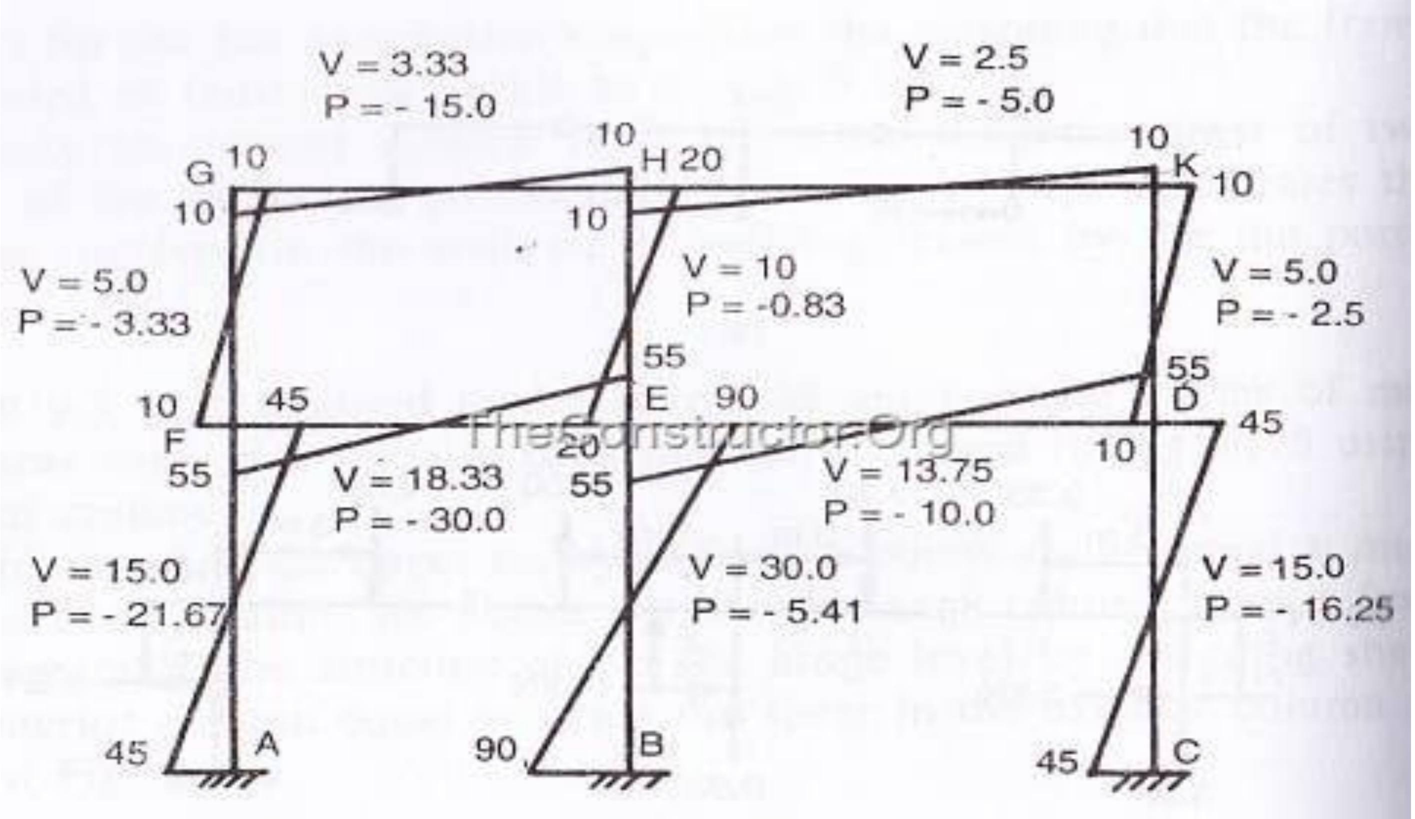


- The forces in the members of the lower storey are obtained from the free body diagrams of Fig.
- The maximum moment in each member of the structure is readily obtained once the value of shear at the inflection points have been determined.

Portal frame method

- The moment diagram drawn on the frame on the tension side of the members is shown in figure 4.
- The maximum values of shear and axial forces are indicated along each member.
- The shears are shown without any sign.
- The positive value of the axial force indicates tension.

Portal frame method



(All shear and axial forces in kN and moments in kN.m)

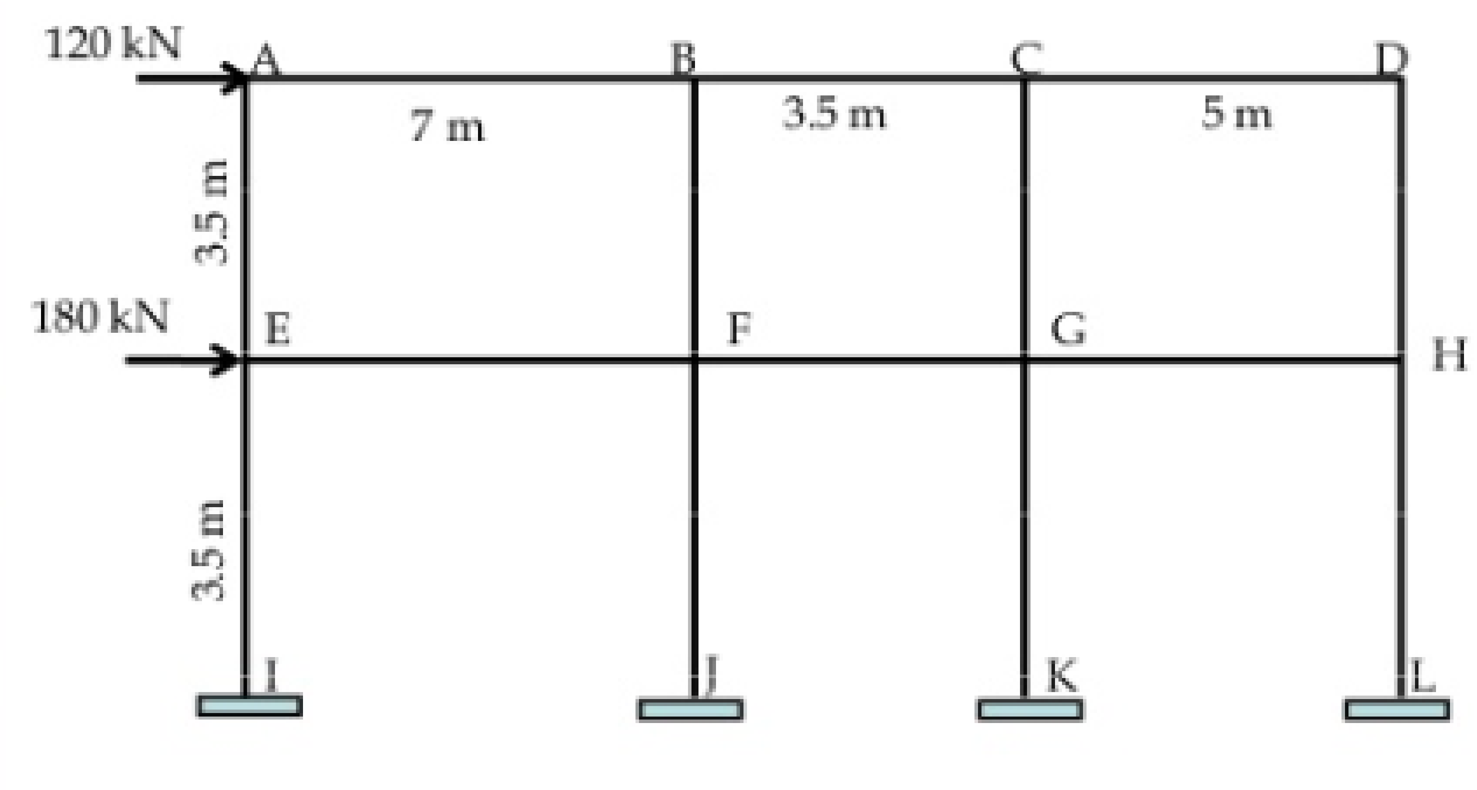
CANTILEVER METHOD

This method is applicable to high rise structures. This is based on the simplifying assumptions regarding the Axial Force in columns.

Assumptions in cantilever method

- The shear force in an interior column is twice the shear force in an exterior column.
- There is a point of inflection at the center of each column.
- There is a point of inflection at the center of each beam.

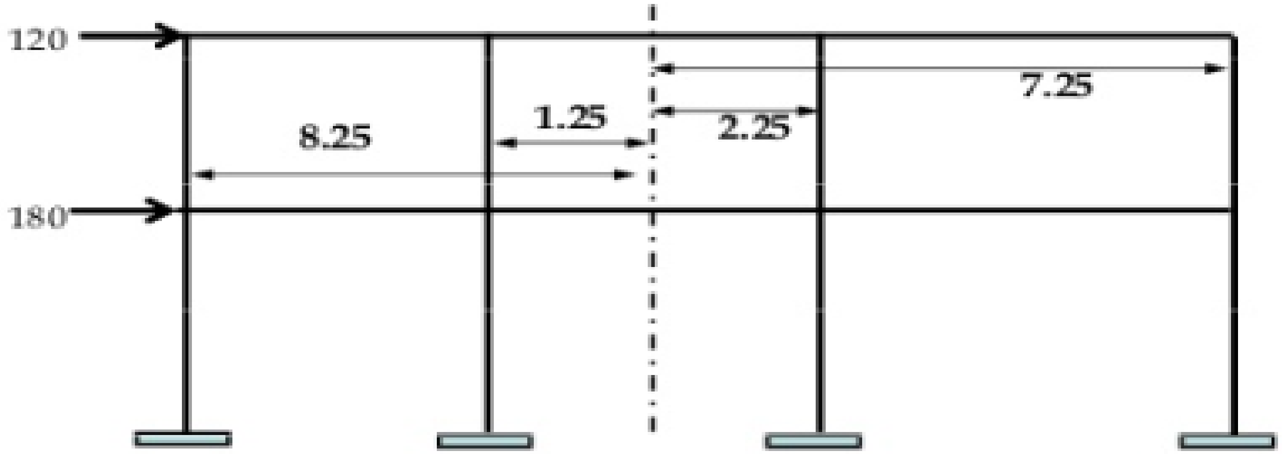
Cantilever method



Cantilever method

To locate centroidal vertical axis of the frame,

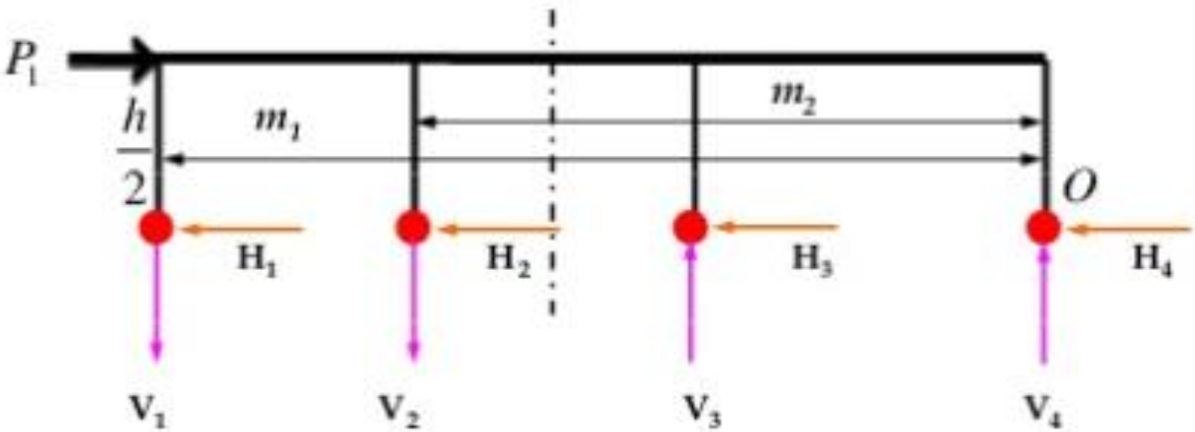
$$\bar{y} = \frac{A_1 \times 0 + A_1 \times 7 + A_1 \times 10.5 + A_1 \times 15.5}{A_1 + A_1 + A_1 + A_1} = \frac{33}{4} = 8.25m$$



Also,

$$\frac{V_1/A_1}{8.25} = \frac{V_2/A_2}{1.25} = \frac{V_3/A_3}{2.25} = \frac{V_4/A_4}{7.25} \Rightarrow \frac{V_1}{8.25} = \frac{V_2}{1.25} = \frac{V_3}{2.25} = \frac{V_4}{7.25}$$

$$V_2 = \frac{1.25V_1}{8.25},\ V_3 = \frac{2.25V_1}{8.25},\ V_4 = \frac{7.25V_1}{8.25}$$



For the top storey, $\sum M_O \Rightarrow P_1\frac{h}{2} = V_1m_1 + V_2m_2 - V_3m_3 - V_4m_4$

$$\Rightarrow 120\times\frac{3.5}{2} = V_1\times 15.5 + V_2\times 8.5 - V_3\times 5 - V_4\times 0$$

Cantilever method

$$\Rightarrow 120\times\frac{3.5}{2}=V_1\times 15.5+\left(\frac{1.25V_1}{8.25}\right)\times 8.5-\left(\frac{2.25V_1}{8.25}\right)\times 5$$

$$\Rightarrow V_1=13.615kN$$

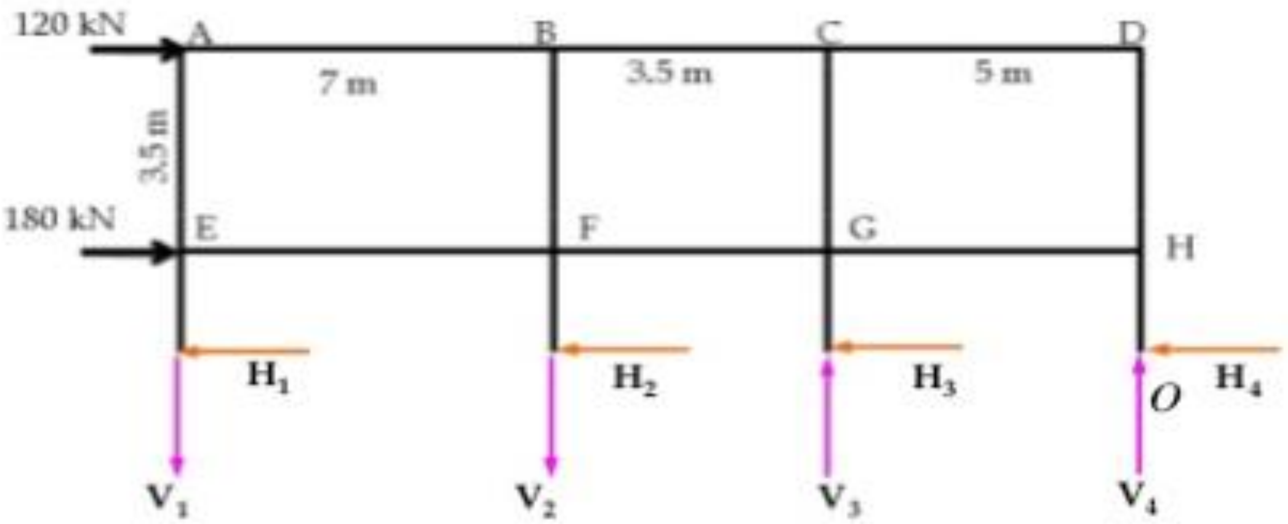
$$V_2=\frac{1.25\times 13.615}{8.25}=2.063kN,$$

$$V_3=\frac{2.25\times 13.615}{8.25}=3.713kN,$$

$$V_4=\frac{7.25\times 13.615}{8.25}=11.965kN$$

$$Check:\ 13.615+2.063-3.713-11.965=0$$

Cantilever method



For the bottom storey,

$$\sum M_o \Rightarrow 120\times\left(3.5+\frac{3.5}{2}\right)+180\times\frac{3.5}{2}=V_1\times15.5+V_2\times8.5-V_3\times5-V_4\times0$$

Cantilever method

$$\Rightarrow 120\times\left(3.5+\frac{3.5}{2}\right)+180\times\frac{3.5}{2}=V_1\times 15.5+\left(\frac{1.25V_1}{8.25}\right)\times 8.5-\left(\frac{2.25V_1}{8.25}\right)\times 5$$

$$\Rightarrow V_1 = 61.267\,kN$$

$$V_2=\frac{1.25\times 61.267}{8.25}=9.283kN,$$

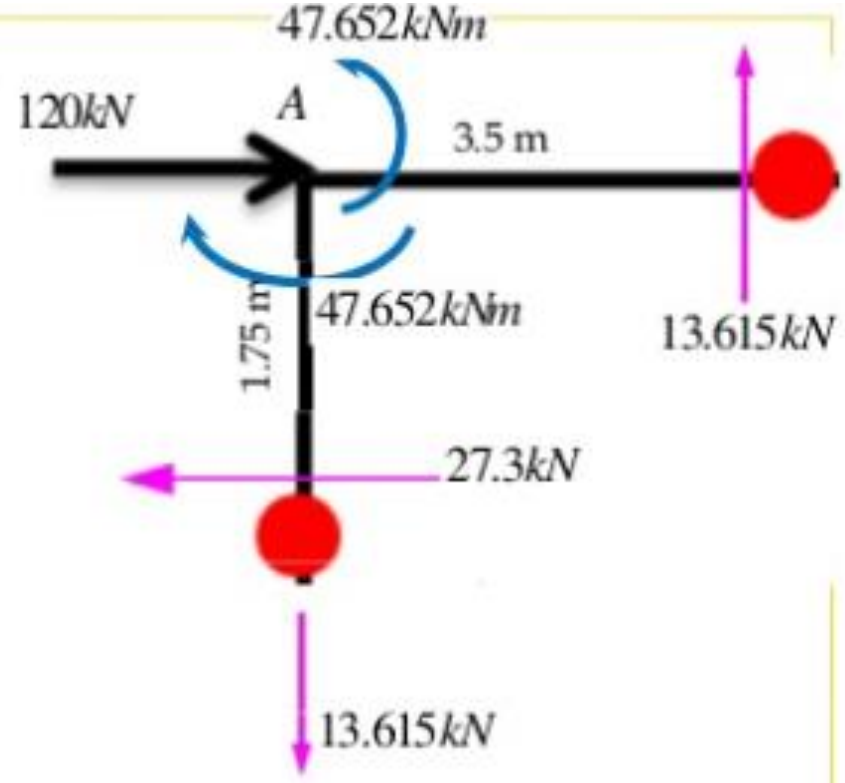
$$V_3=\frac{2.25\times 61.267}{8.25}=16.709\,kN,$$

$$V_4=\frac{7.25\times 61.267}{8.25}=53.841kN$$

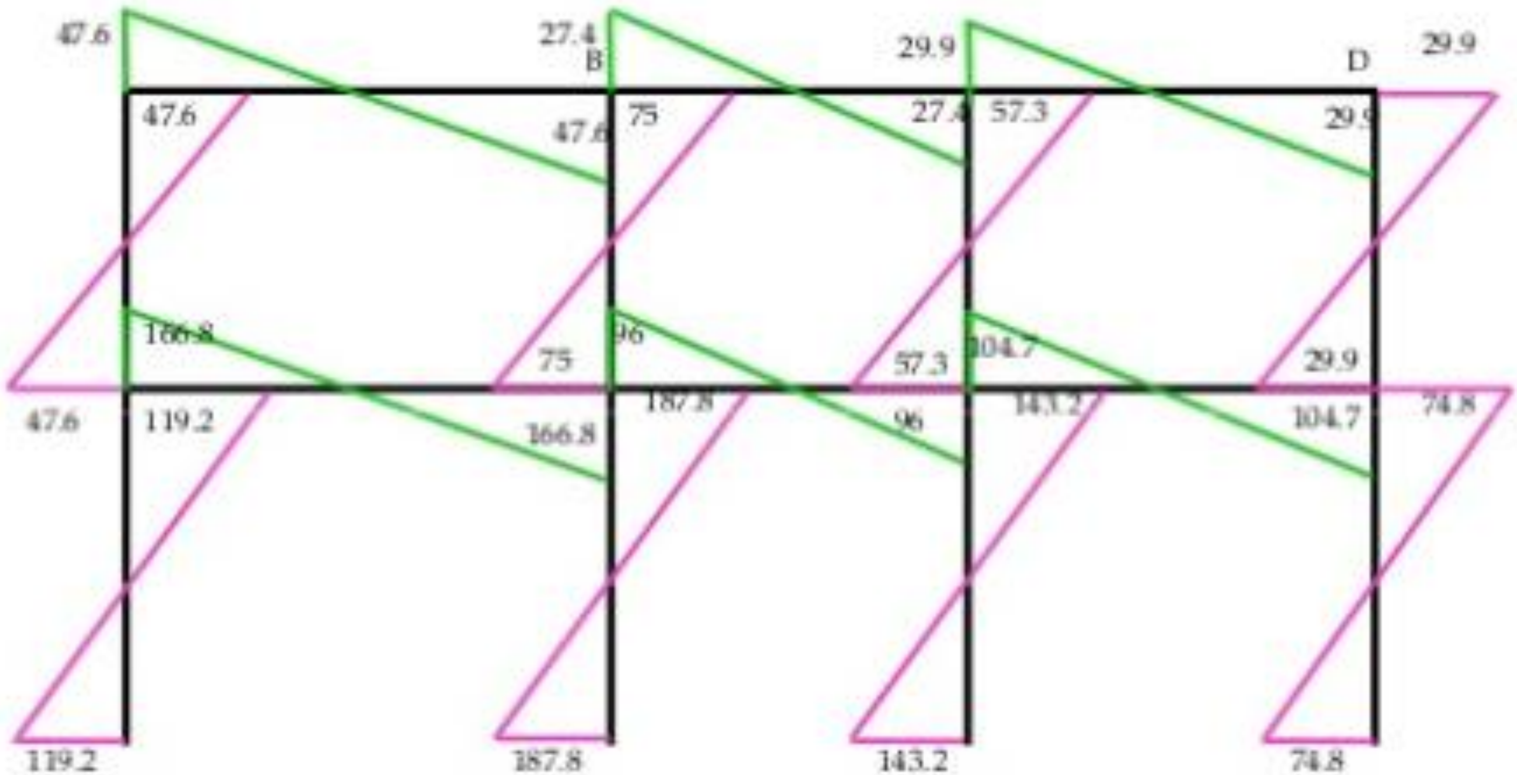
$$Check:\ 61.267+9.283-16.709-53.841=0$$

Cantilever method

Moments:



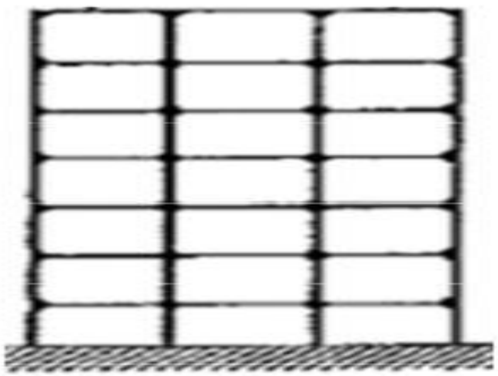
Beam and Column moments:



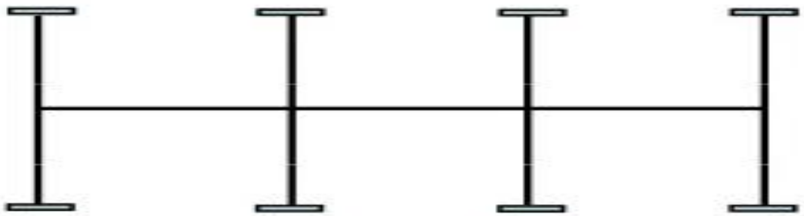
Substitute frame method

Approximate analysis for vertical loads

- Analyse only a part of the frame - substitute frame
- Carry out a two-cycle moment distribution

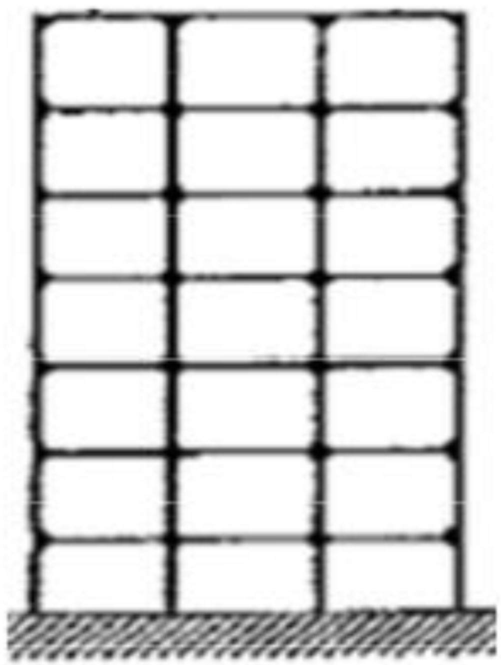


Actual frame

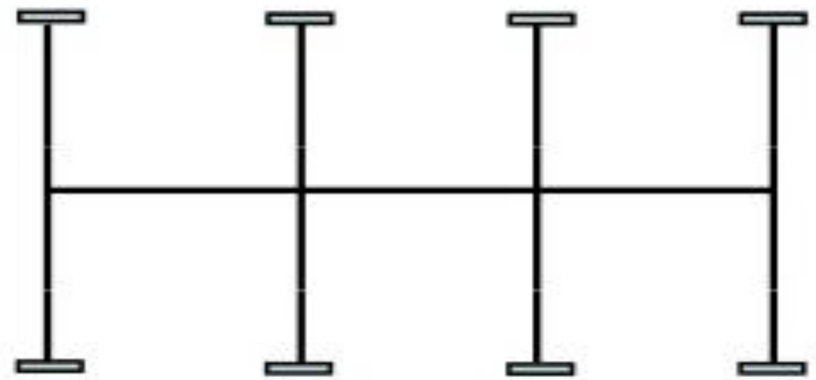


Substitute frame

Substitute frame method



Actual frame



Substitute frame

Substitute frame method

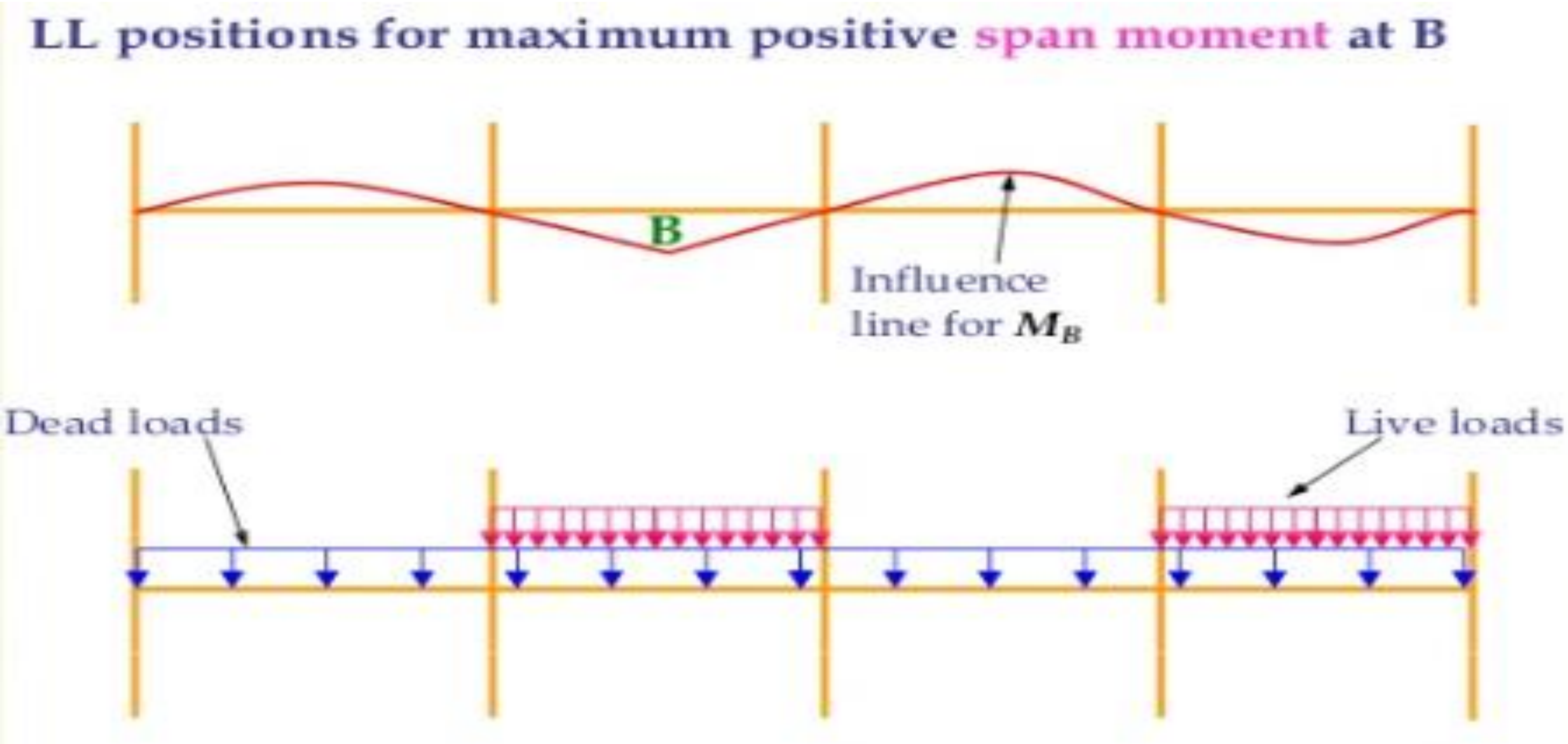
- Analysis done for

  Beam span moments

  Beam support moments

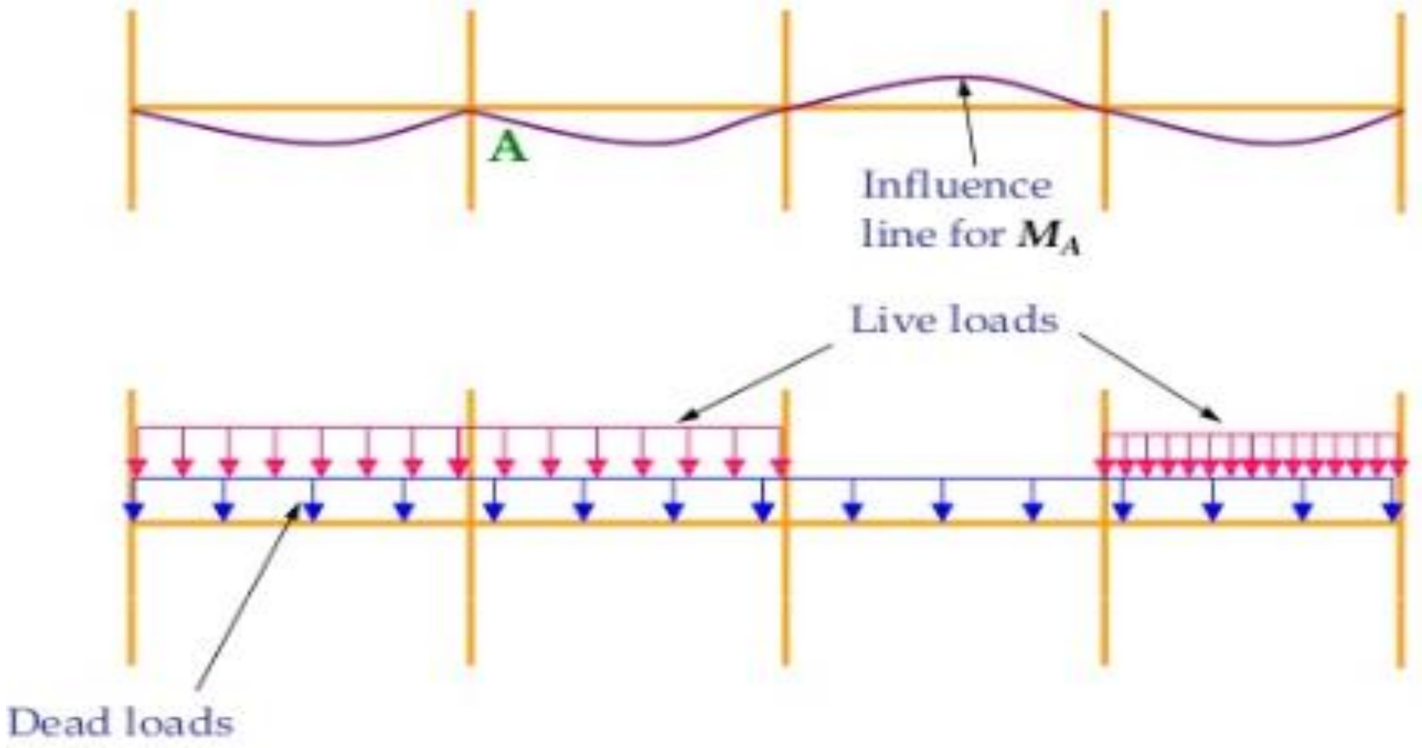
  Column moments
- Live load positioning for the worst condition
- For the same frame , live load positions for maximum span moments, support moments and column moments may be different.
- For maximum moments at different points, live load positions may be different

Substitute frame method

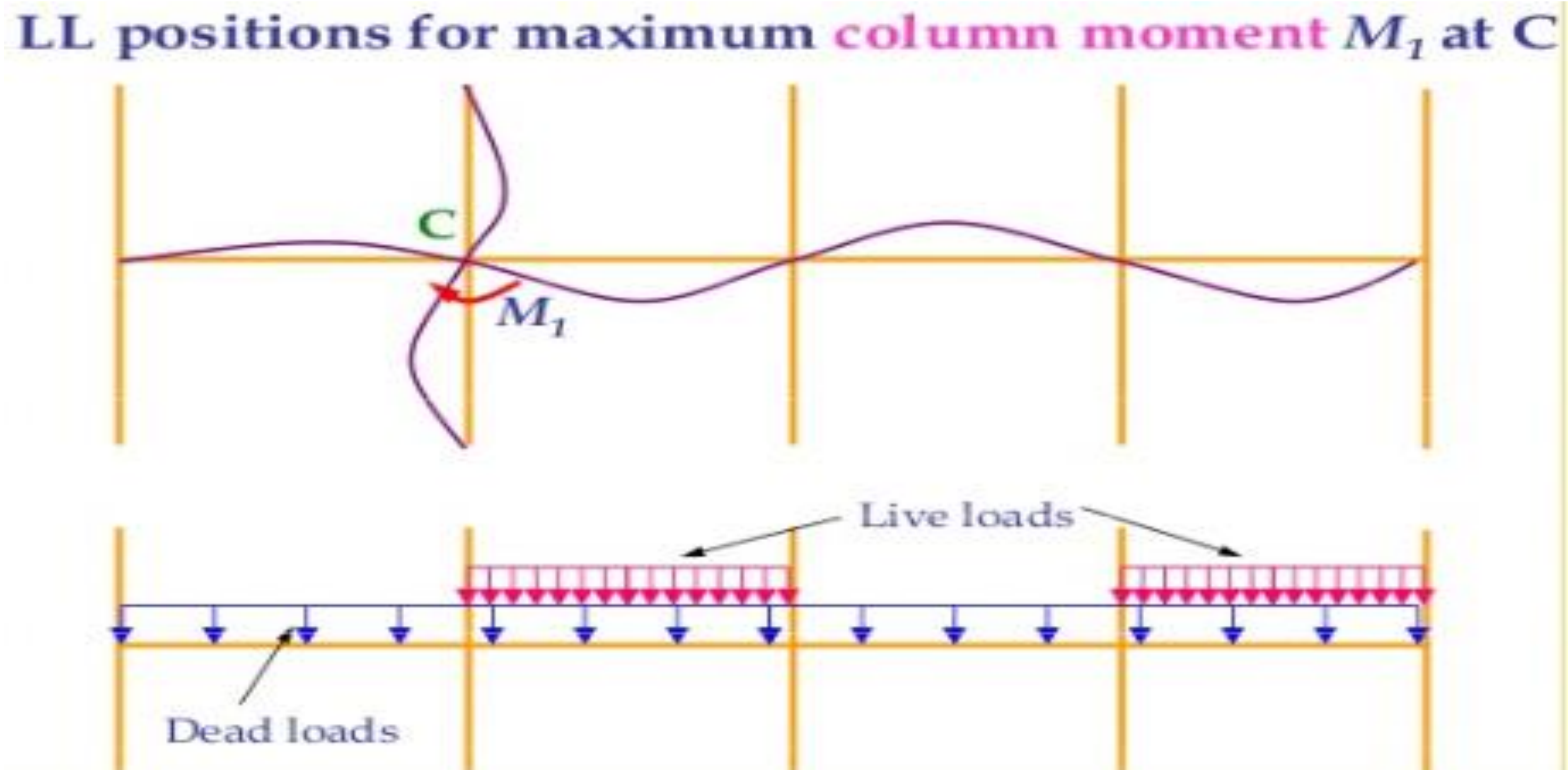


Substitute frame method

LL positions for maximum negative support moment at A

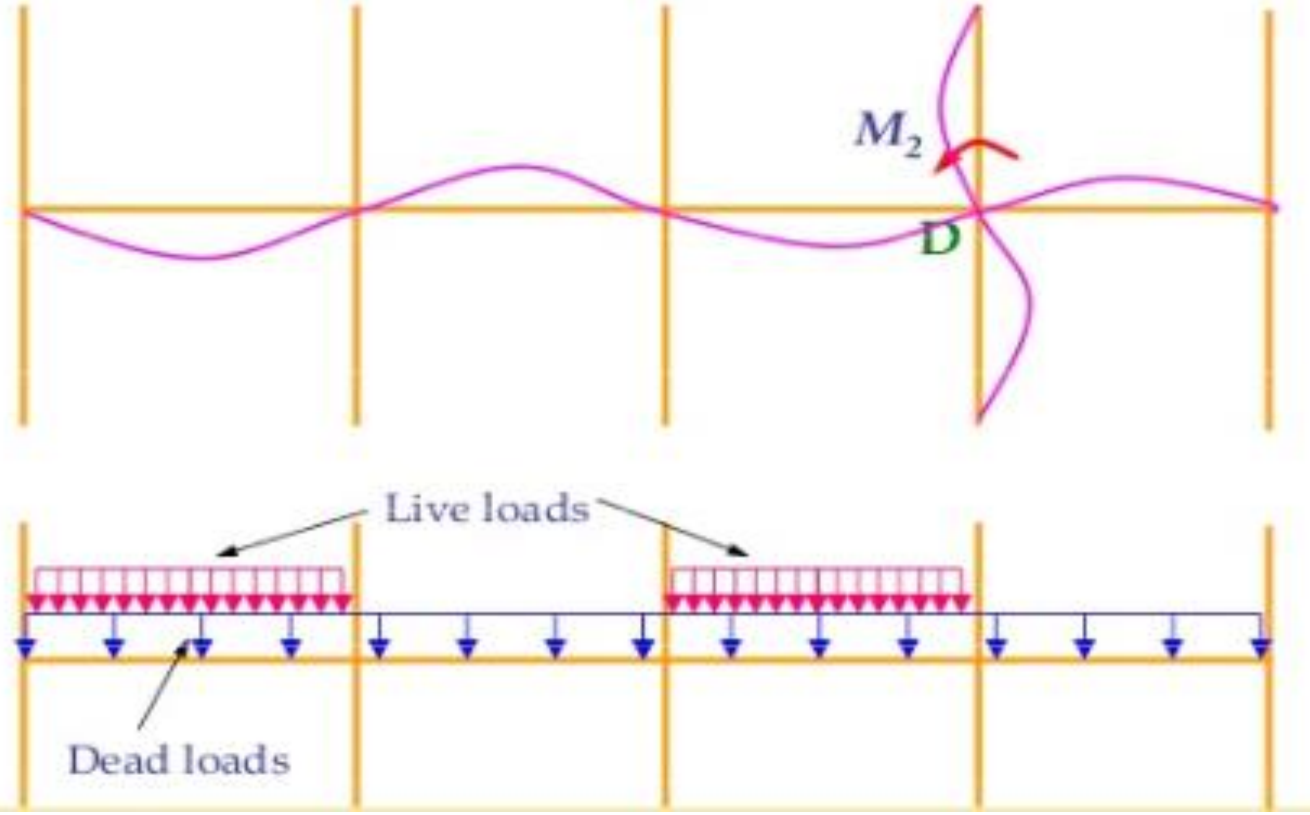


Substitute frame method



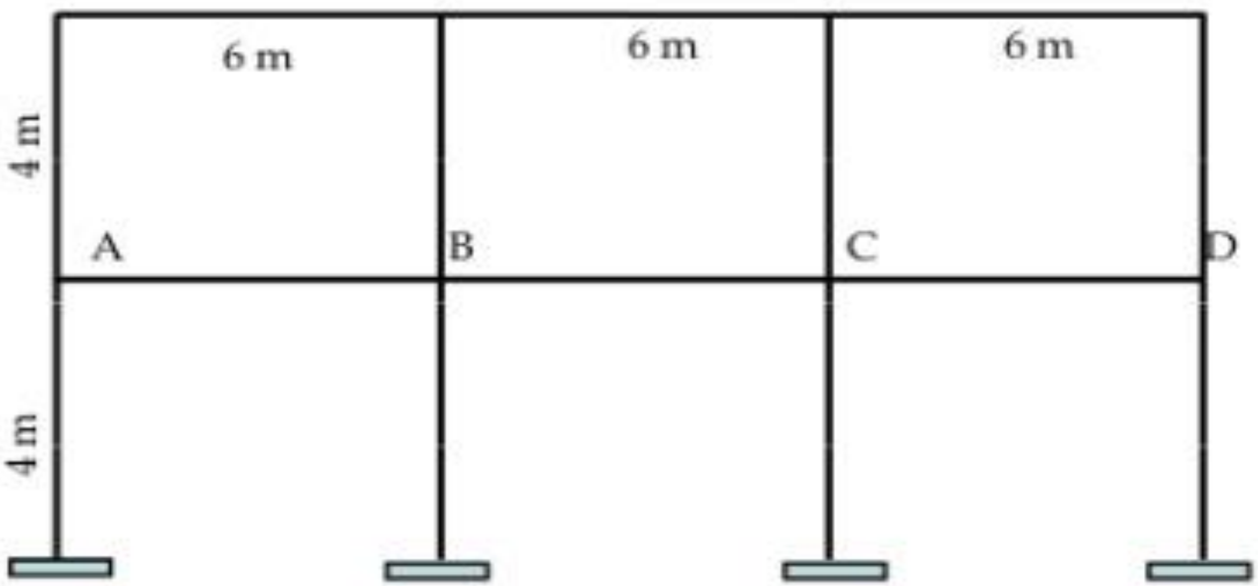
Substitute frame method

LL positions for maximum column moment M_2 at D

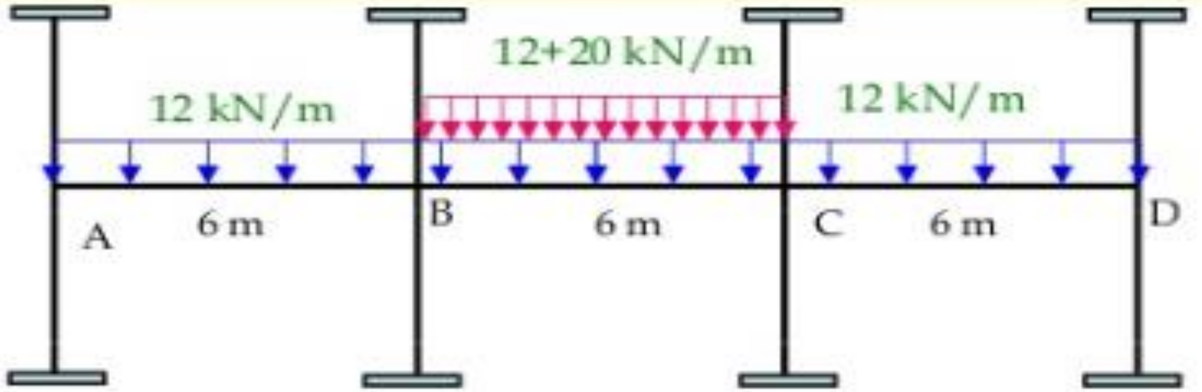


Substitute frame method

Problem 1: Total dead load is 12 kN/m. Total live load is 20 kN/m. Analyse the frame for midspan positive moment on BC.



Substitute frame method



Fixed end moments

$$FEM_{AB} = \frac{-wl^2}{12} = \frac{-12 \times 6^2}{12} = -36kNm$$

$$FEM_{BA} = 36kNm$$

$$FEM_{BC} = \frac{-32 \times 6^2}{12} = -96kNm$$

$$FEM_{CB} = 96kNm$$

$$-FEM_{CD} = FEM_{DC} = 36kNm$$

Substitute frame method

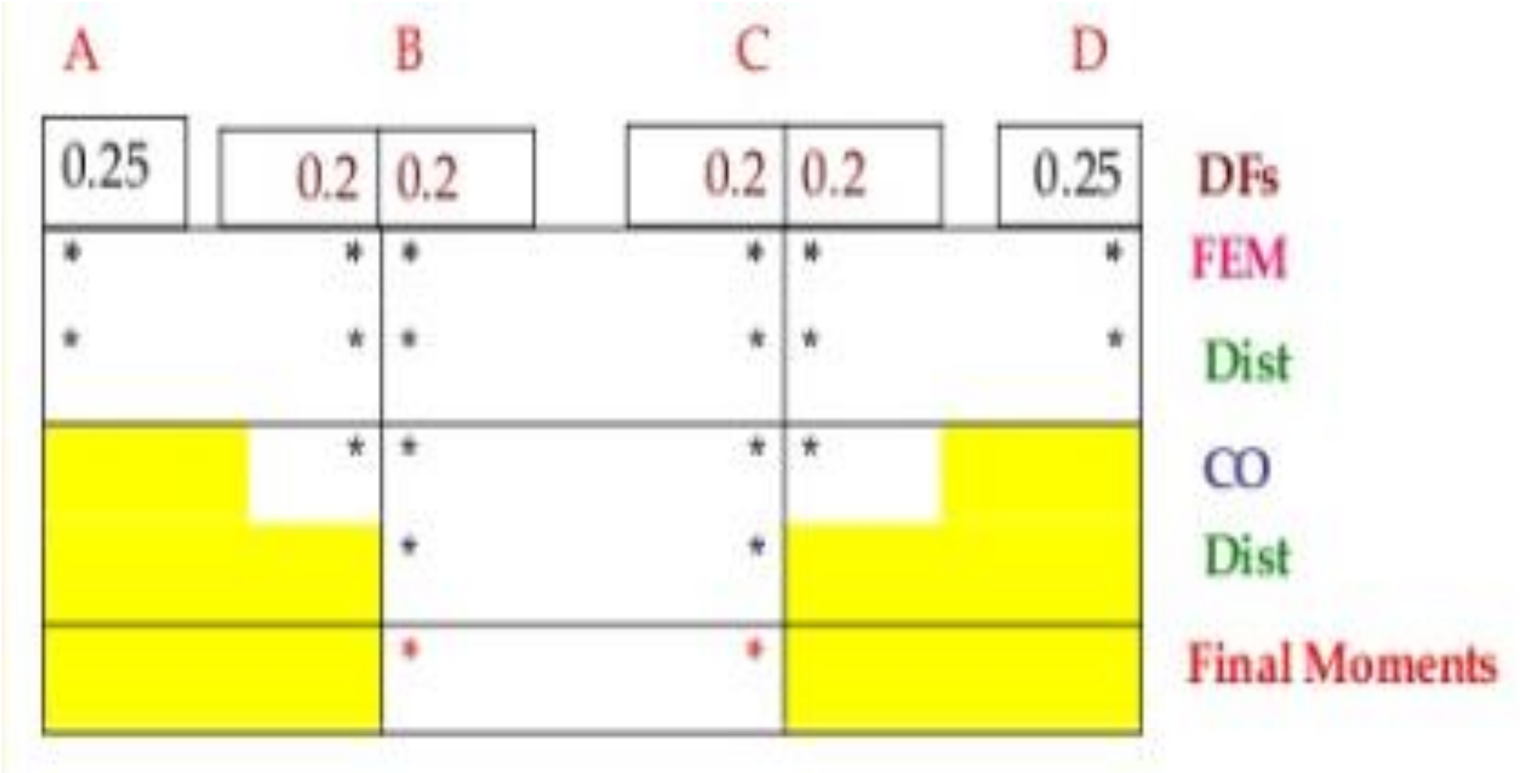
Distribution factors

$$DF_{AB} = \frac{K_1}{K_1 + K_2 + K_3} = \frac{4EI/6}{4EI/6 + 4EI/4 + 4EI/4} = 0.25 = DF_{DC}$$

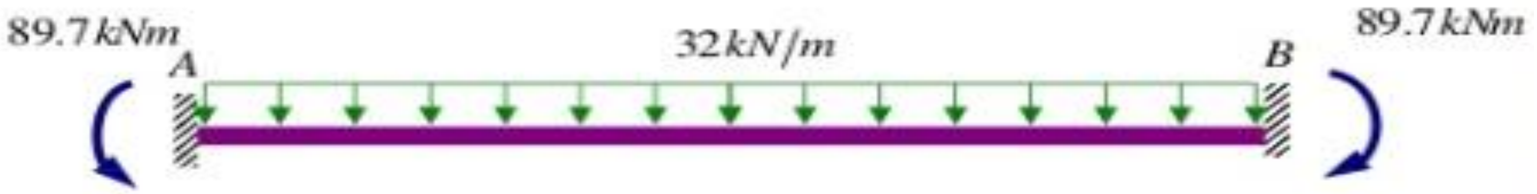
$$DF_{BA} = \frac{K_1}{K_1 + K_2 + K_3 + K_4} = \frac{4EI/6}{4EI/4 + 4EI/6 + 4EI/4 + 4EI/6} = 0.2$$

$$DF_{BC} = DF_{CD} = DF_{CB} = DF_{BA} = 0.2$$

Substitute frame method



Substitute frame method

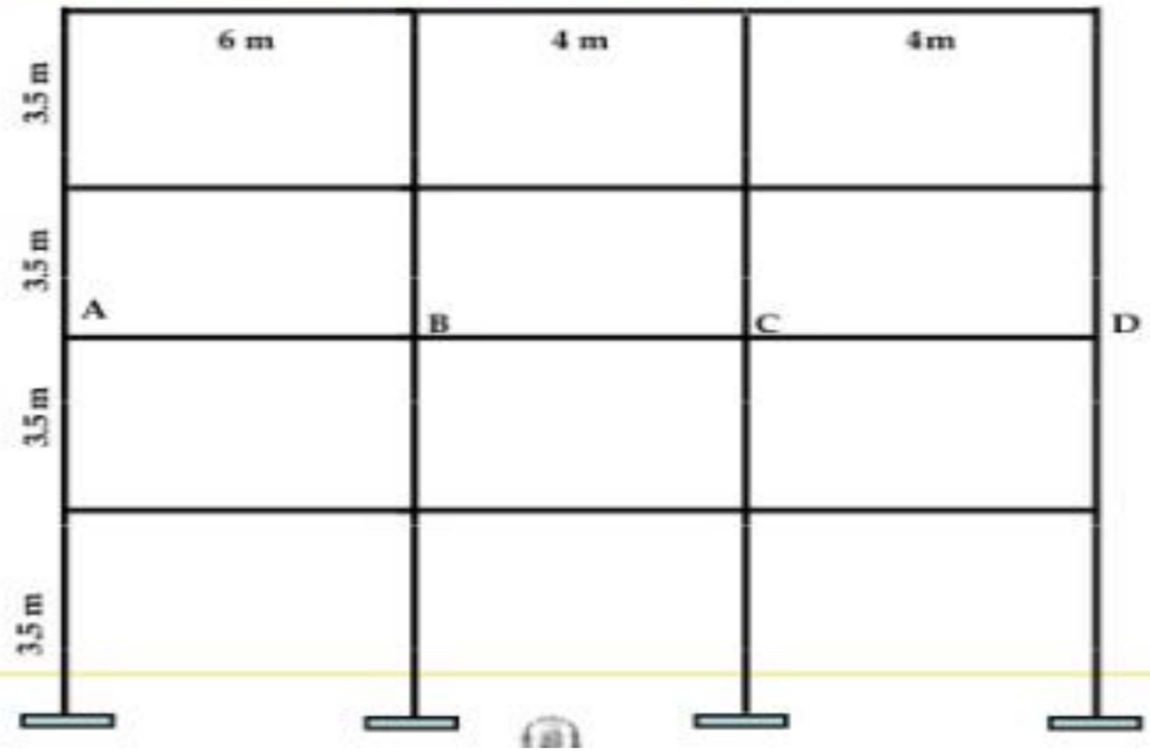


Midspan positive moment on BC,

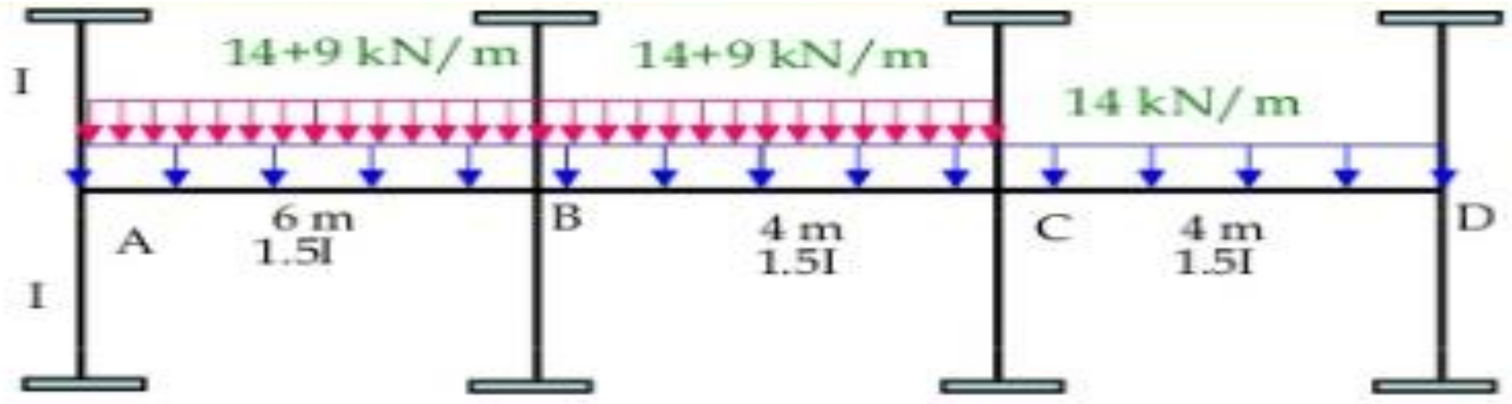
$$M_E = -89.7 - 32 \times \frac{3^2}{2} + \frac{32 \times 6}{2} \times 3 = 54.3\ kNm$$

Substitute frame method

Problem 2: Analyse the frame for beam negative moment at B. Moment of inertia of beams is 1.5 times that of columns. Total dead load is 14 kN/m and total live load is 9 kN/m.



Substitute frame method



Fixed end moments

$$FEM_{AB} = \frac{-wl^2}{12} = \frac{-23\times 6^2}{12} = -69\,kNm \qquad FEM_{BA} = 69\,kNm$$

$$-FEM_{BC} = FEM_{CB} = \frac{23\times 4^2}{12} = 30.67\,kNm$$

$$-FEM_{CD} = FEM_{DC} = \frac{14\times 4^2}{12} = 36\,kNm$$

Substitute frame method

Distribution factors

$$DF_{AB} = \frac{K_1}{K_1 + K_2 + K_3} = \frac{4E(1.5I)/6}{4E(1.5I)/6 + 4EI/3.5 + 4EI/3.5} = 0.304$$

$$DF_{BA} = \frac{K_1}{K_1 + K_2 + K_3 + K_4} = \frac{1.5I/6}{1.5I/6 + I/3.5 + I/3.5 + 1.5I/4} = 0.209$$

$$DF_{BC} = \frac{K_1}{K_1 + K_2 + K_3 + K_4} = \frac{1.5I/4}{1.5I/6 + I/3.5 + I/3.5 + 1.5I/4} = 0.313$$

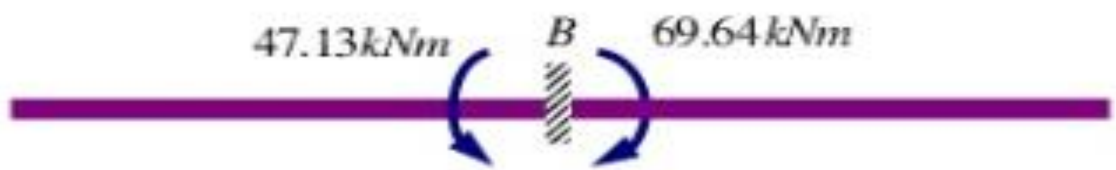
$$DF_{CB} = 0.284, \quad DF_{CD} = 0.284, \quad DF_{DC} = 0.396$$

Substitute frame method

A	B		C		D	
0.304	0.209	0.313	0.284	0.284	0.396	**DFs**
*	*	*	*	*		**FEM**
*	*	*	*			**Dist**
	*	*				**CO**
	*	*				**Dist**
	*	*				**Final Moments**

Substitute frame method

A	B		C		D	
0.304	0.209	0.313	0.284	0.284	0.396	DFs

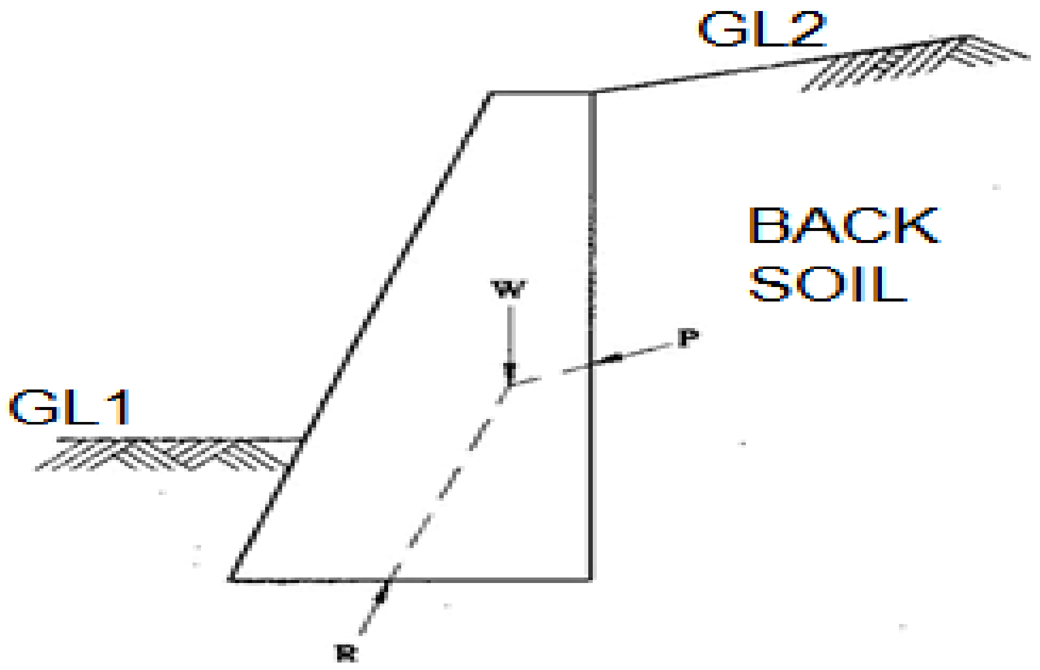


Max. beam negative moment at B $= 69.64\ kNm$

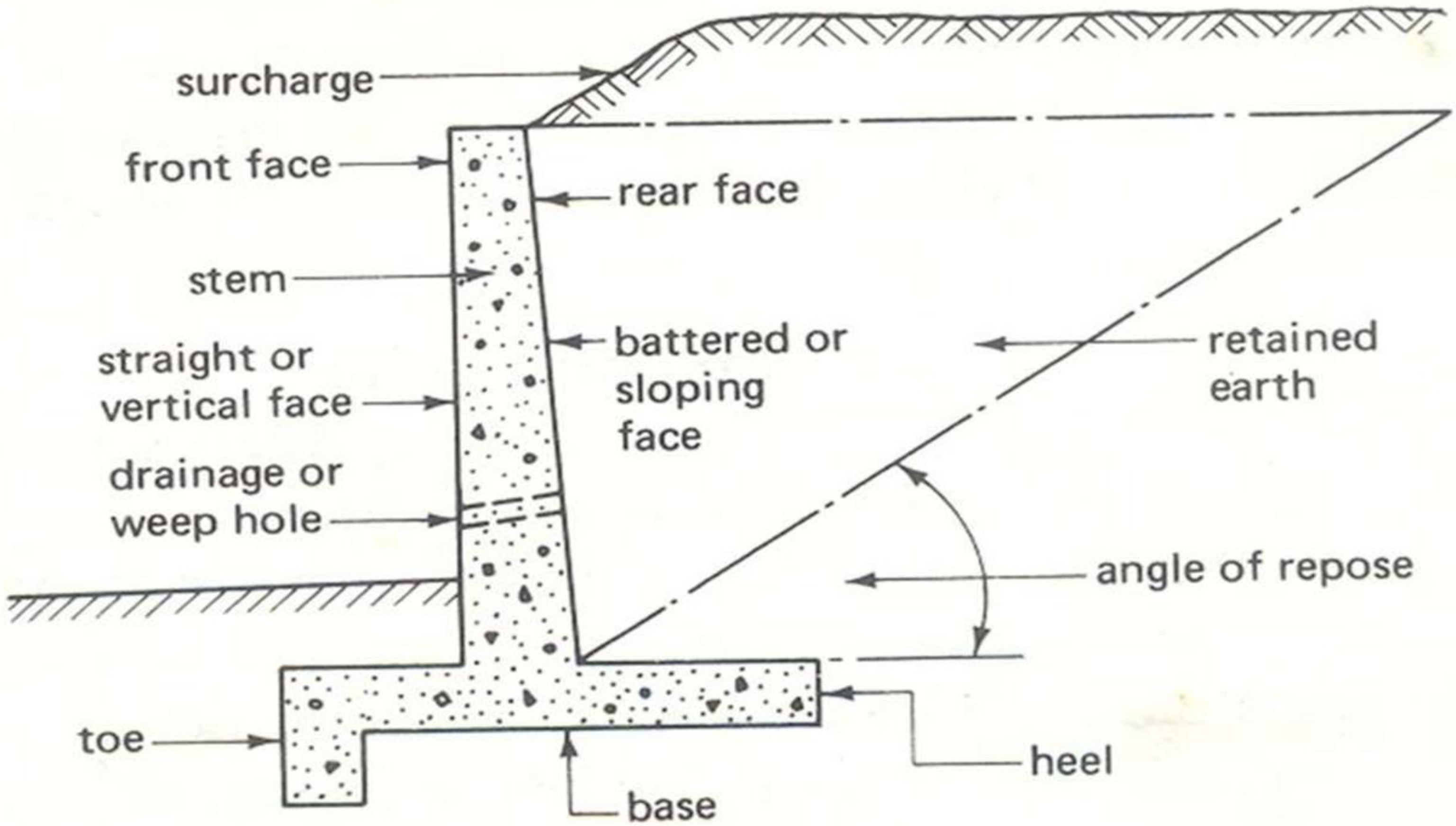
Unit III
Design of Retaining walls and Tanks

Retaining wall

Retaining walls are usually built to hold back soil mass. However, retaining walls can also be constructed for aesthetic landscaping purposes.



Retaining walls

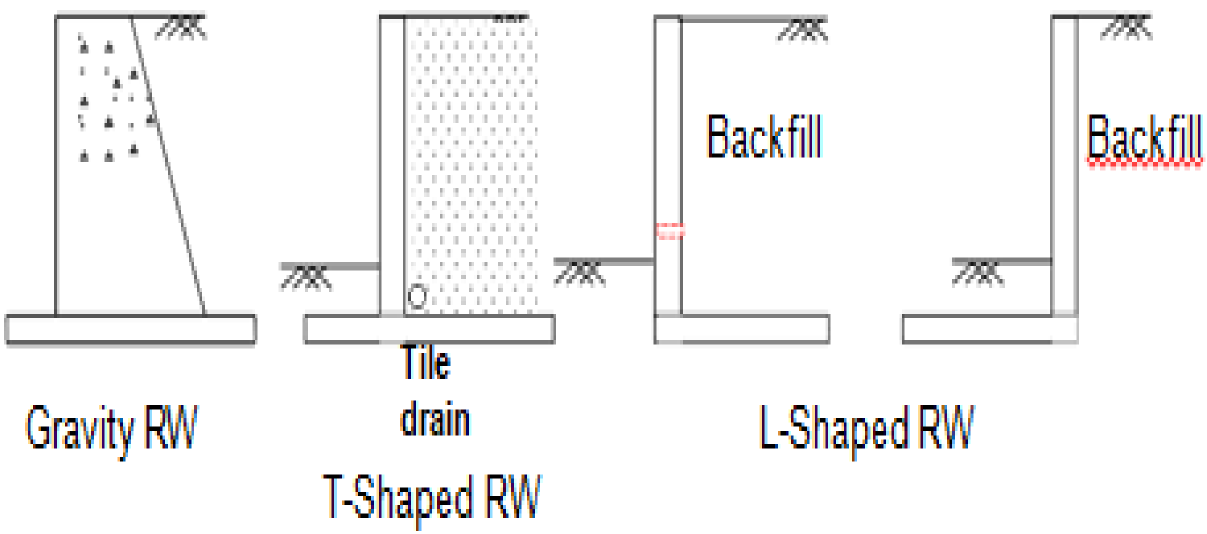


Retaining wall terminology

Retaining walls

Classification of retaining walls

1. Gravity wall-masonry or plain concrete
2. Cantilever retaining wall- RCC (inverted T and L)
3. Counterfort retaining wall- RCC
4. Butress wall-RCC



Retaining walls

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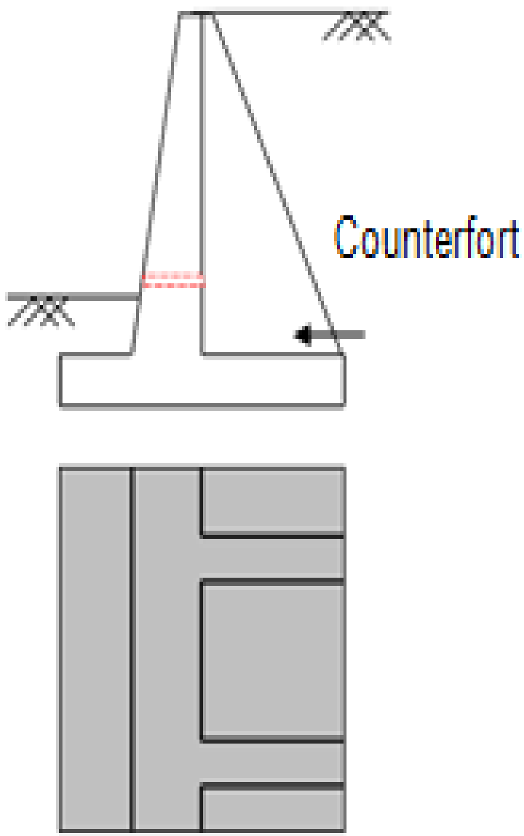
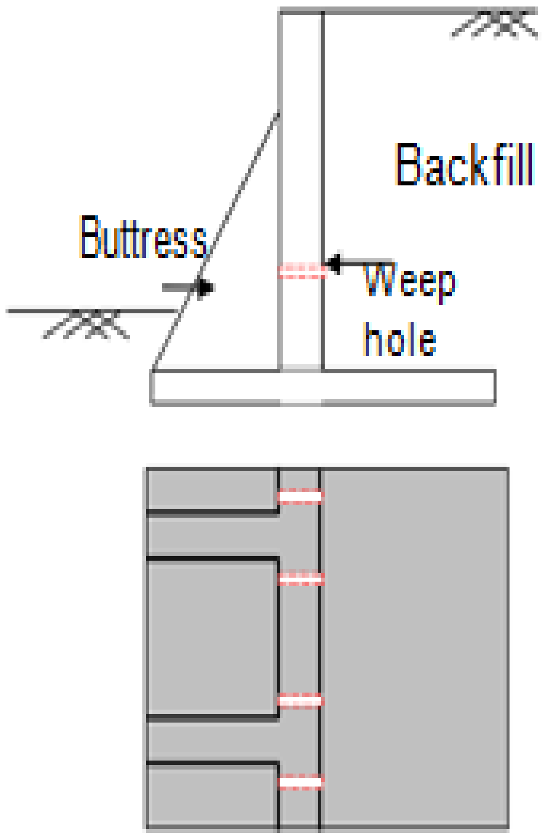
Retaining walls



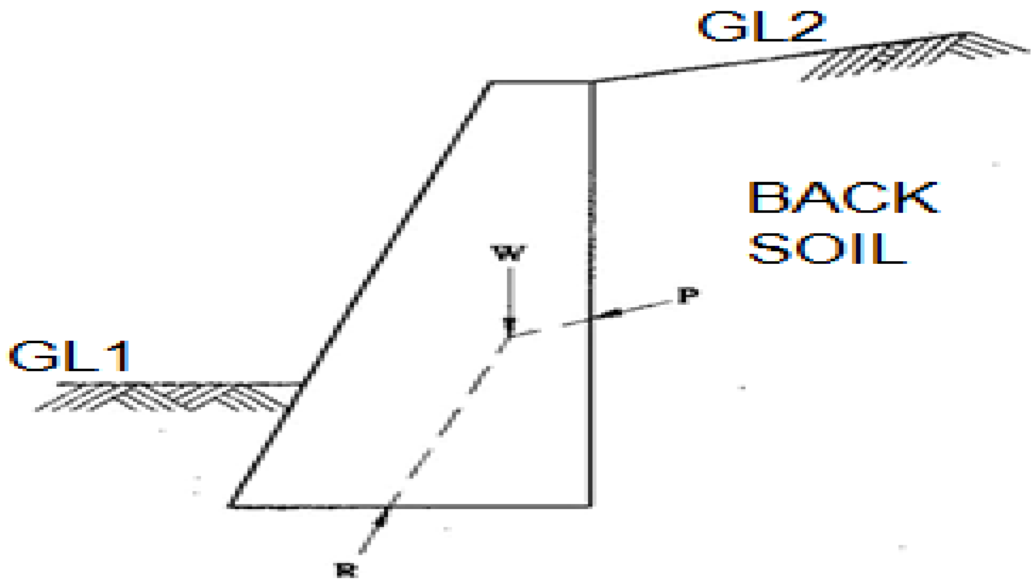
Counterfort RW



Buttress RW

Importance of retaining walls

Retaining walls are usually meant to serve a single purpose, retaining soil that may erode. However, retaining walls have become more mainstream for other reasons. Today, they are used to block off areas such as outdoor living spaces and for landscaping.

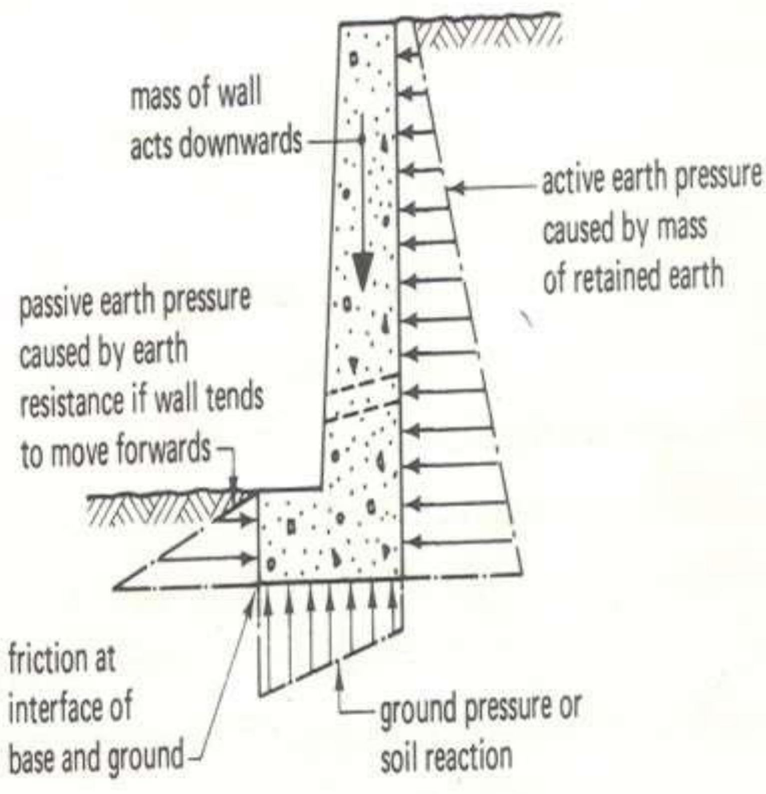


Segmental retaining walls

These consist of modular concrete blocks that interlock with each other. They are used to hold back a sloping face of soil to provide a solid, vertical front. Without adequate retention, slopes can cave, slump or slide.

Forces acting on retaining wall

- Lateral earth pressure
- Self weight of retaining wall
- Weight of soil above the base slab
- Surcharge, i.e. forces due to loads on earth surface.
- Soil reaction below base slab
- Frictional force at the bottom of base slab



Mass retaining walls

Earth pressure

Earth pressure is the pressure exerted by the retaining material on the retaining wall. This pressure tends to deflect the wall outward.

Types of earth pressure :

- Active earth pressure or earth pressure (Pa) and
- Passive earth pressure (Pp).

Active earth pressure tends to deflect the wall away from the backfill.

Factors affecting earth pressure

Earth pressure depends on type of backfill, the height of wall and the soil conditions

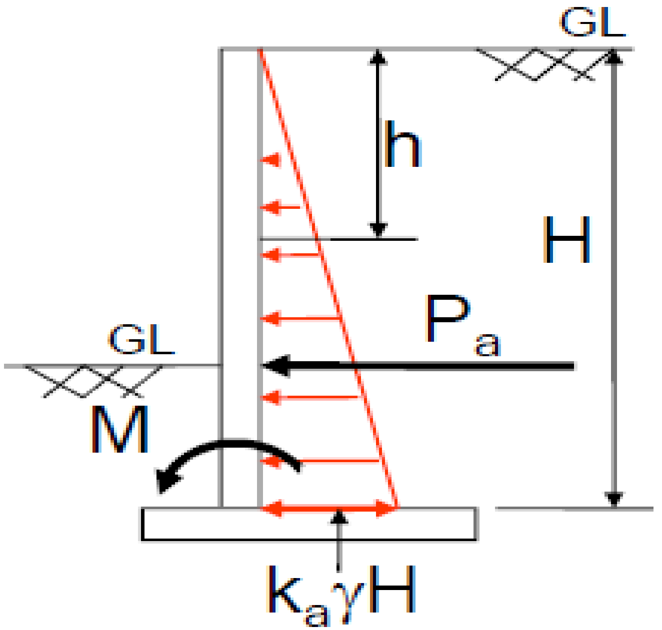
Soil conditions: The different soil conditions are

- Dry or moist backfill with no surcharge
- Submerged backfill
- Backfill with uniform surcharge
- Backfill with sloping surface
- Inclined back and surcharge

Analysis for dry back fills without surcharge

- Maximum pressure at any height, p=kaϒh
- Total pressure at any height from top,

$$P_a = \frac{1}{2} \Upsilon k \ H^2$$



H=stem height

(2)Submerged backfill

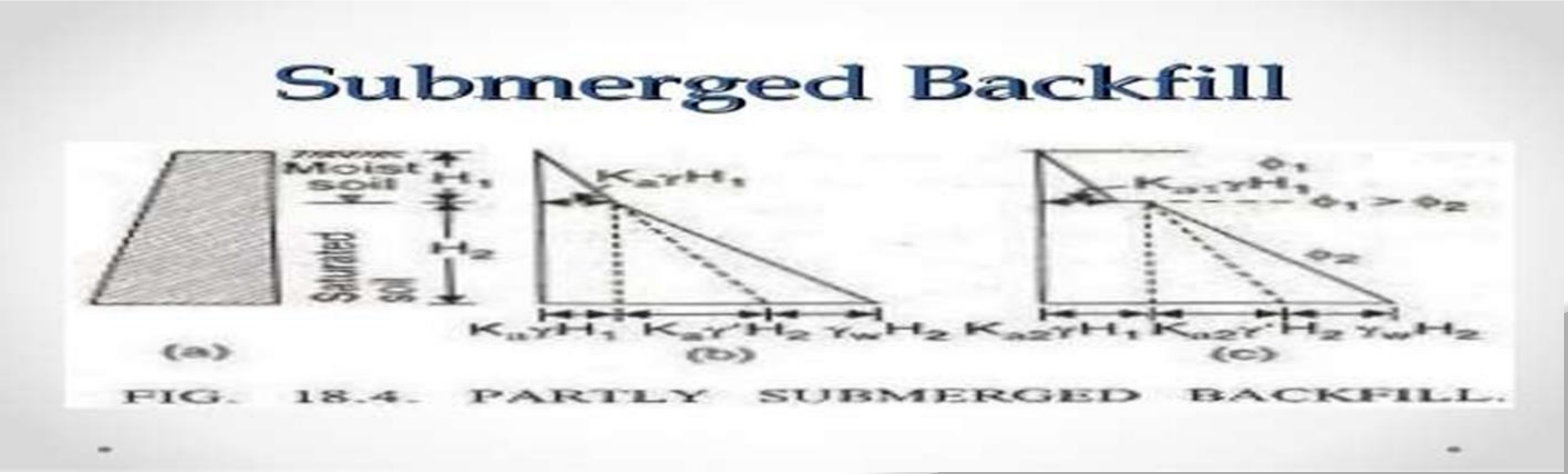
Lateral pressure due to submerged weight of soil

= $ka\Upsilon$H

Lateral pressure due to water =ΥwH

Total pressure at base,

Pa = $ka\Upsilon$H+ΥwH



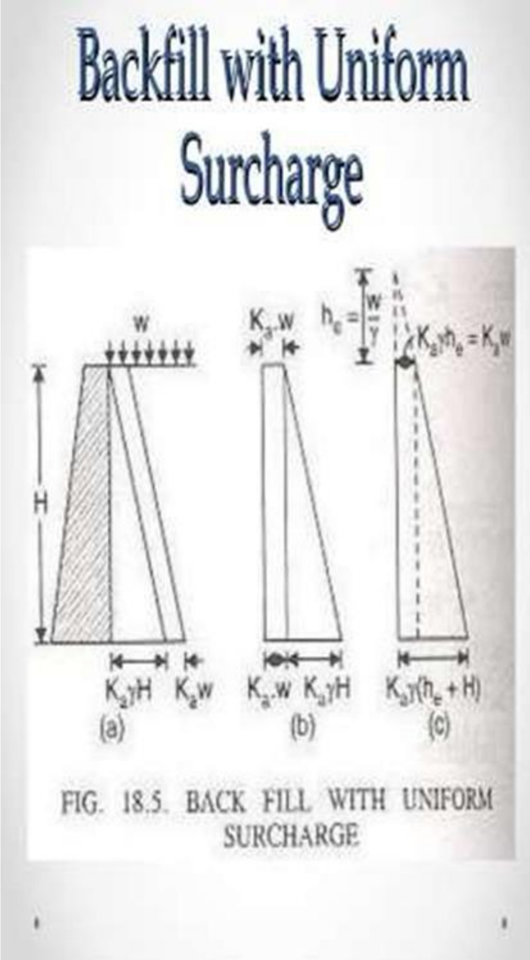
(3)Backfill with uniform surcharge

- The lateral pressure due to surcharge,

  $=ka$ q
- The lateral pressure due to backfill,

  $=ka$ ϒ H
- Lateral pressure intensity at base,

  $Pa=ka$ q + ka ϒH



(4) Backfill with sloping surface

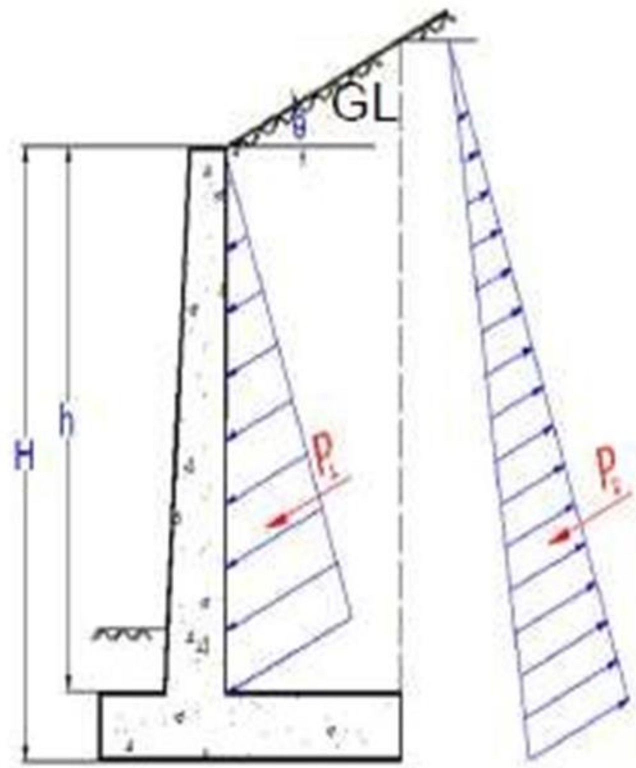
- The total earth pressure acts at an angle β with horizontal.

$$k \quad \cos\beta \frac{\cos\beta - \quad \cos^2\beta - \cos^2 \emptyset}{\cos\beta + \quad \cos^2\beta - \cos^2 \emptyset}$$

β=angle of surcharge

If surcharge is horizontal,β=0

Therefore, $k_{a=} \; {}_{1+\sin\emptyset}$



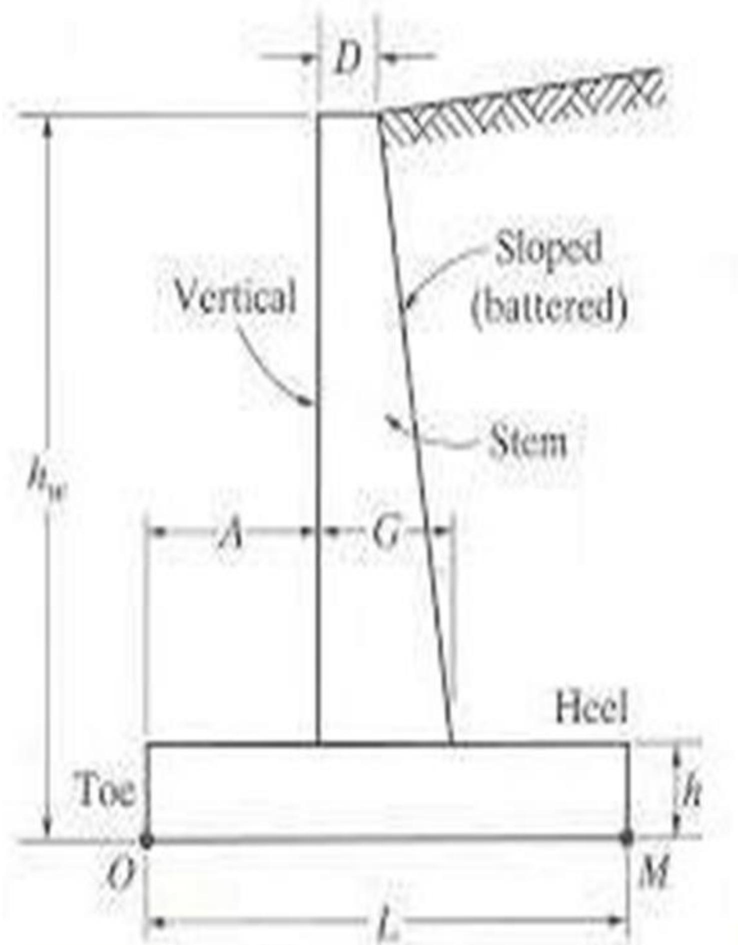
SOIL PRESSURE DUE TO INCLINED SURCHARGE

(5) Inclined back and surcharge

- Resultant of pressure P_1 and weight of soil wedge W is calculated as

P= $P_1^2 + W^2$

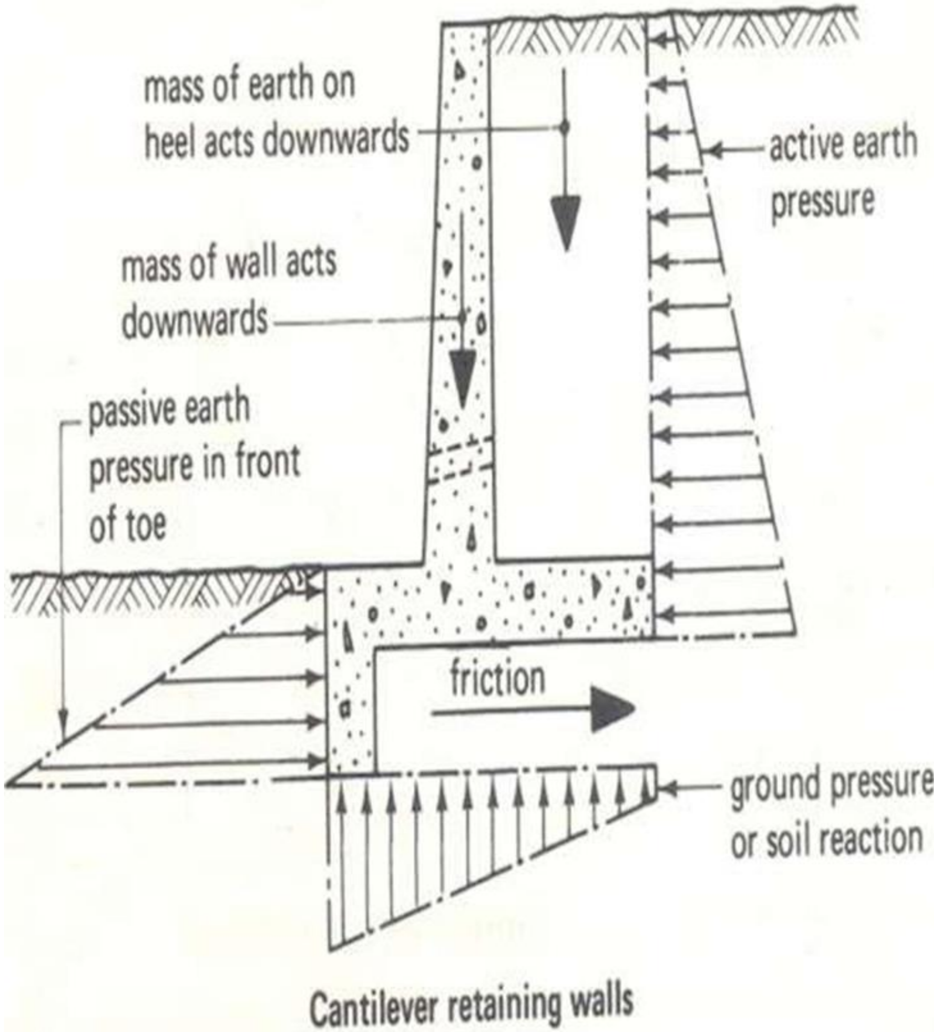
where $P_1 = \frac{1}{2} \Upsilon k \; H^2$



Stability Conditions

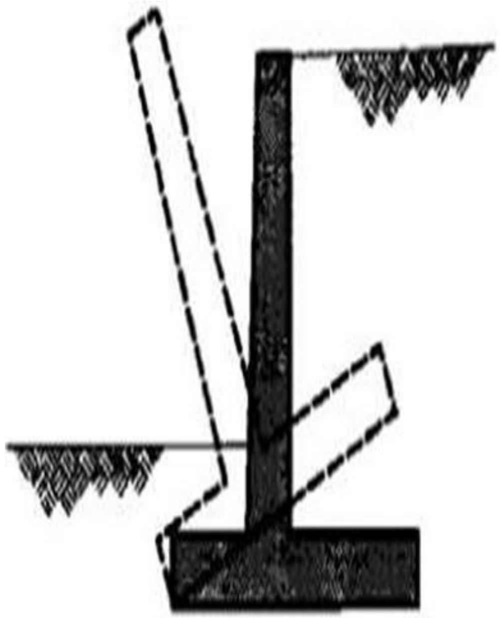
The retaining wall should satisfy the following stability condition-

- Stability against overturning
- Stability against sliding
- Maximum pressure at base should not exceed safe bearing capacity of soil.



Stability against overturning

- As per IS:456-2000, Cl.20.1, factor of safety against overturning should not be less than 1.4.In case where dead load provides the restoring moment, only 0.9 times the characteristic dead load shall be considered.



Hence ,the factor of safety is given by relation

$$F.S = \frac{0.9(restoring\ moment)}{overturning\ moment} \geq 1.4$$

$$= \frac{M_r}{M_o} \geq 1.55$$

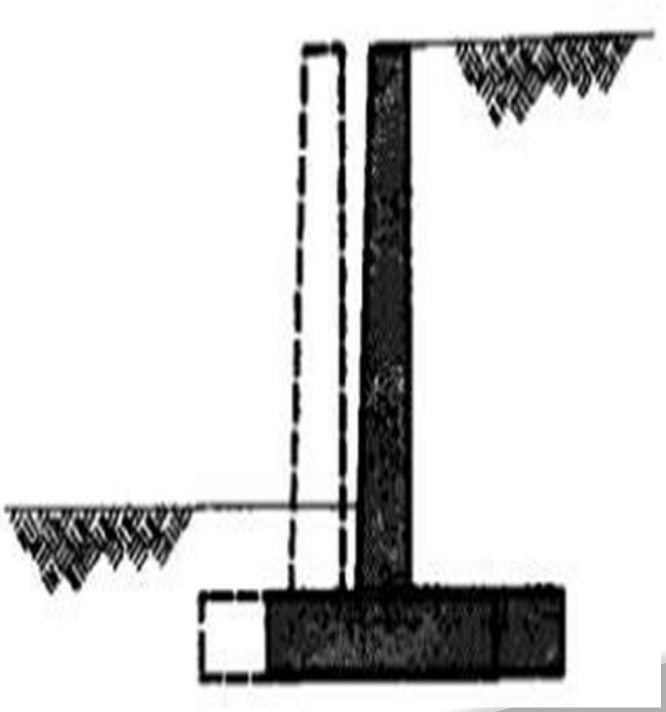
Stability against sliding

- As per IS:456-2000, Cl.20.2, factor of safety against sliding should not be less than 1.4.In this case also 0.9 times characteristic dead load shall be taken into account.

$$F.S = \frac{0.9(restoring\ force)}{sliding\ force} \geq 1.4$$

$$= \frac{0.9(\mu \cdot \sum W)}{P_{ah}} \geq 1.4$$

$$= \frac{(\mu \cdot \sum W + P_p)}{P_{ah}} \geq 1.55$$



Where,μ=coefficient of friction

Maximum pressure at base should not exceed safe bearing capacity of soil.

- The intensity of soil pressure at toe,

$P_{max} = \frac{\Sigma W}{b}[1 + \frac{6e}{b}]$..........at toe

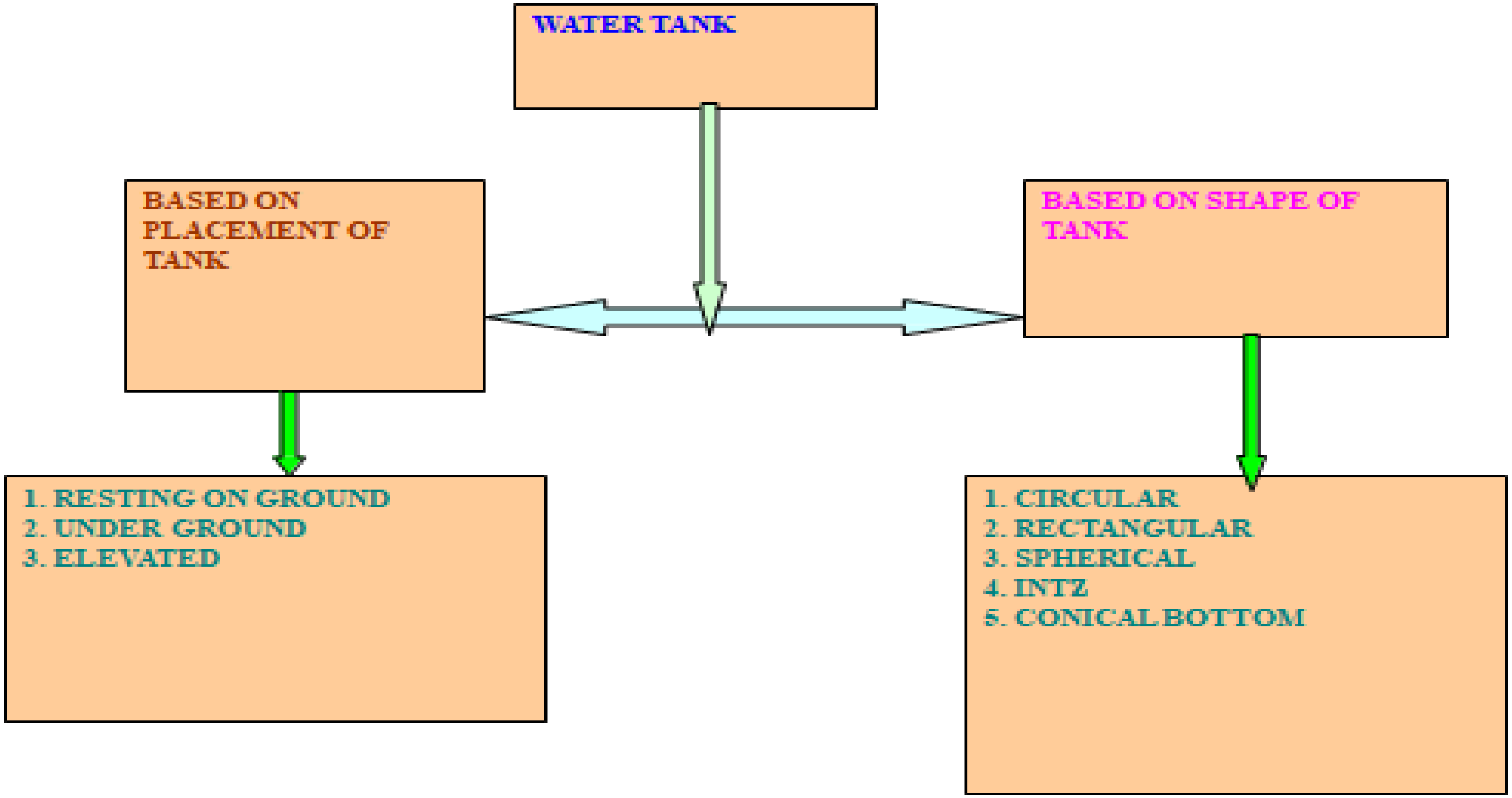
$P_{min} = \frac{\Sigma W}{b}[1 - \frac{6e}{b}]$...........at heel

- $pmax$ should not exceed safe bearing capacity(SBC) of soil.
- For no tension, $pmin$ should not be negative.

Water tanks

- **Storage tanks are built for storing water, liquid petroleum, petroleum products and similar liquids.**
- **Code of practice for plain and reinforced concrete IS 456-2000.**
- **Code of Practice for the storage of Liquids- IS3370 (Part I to IV).**

Types of Water tanks



Water tanks

Water tanks

RECTANGULAR TANK	CIRCULAR TANK	INTZE TANK	PRESTRESSED TANKS
For smaller capacities we go for rectangular tanks	For bigger capacities we go for circular tanks	Intze tank is constructed to reduce the project cost because lower dome in this construction resists horizontal thrust	For bigger tanks, prestressing is the superior choice resulting in a saving of up to 20%.

Rectangular Water tanks

- Rectangular tanks are used when the storage capacity is small.
- Rectangular tanks should be preferably square in plan from point of view of economy.
- It is also desirable that longer side should not be greater than twice the smaller side.
- Moments are caused in two directions of the wall i.e., both in horizontal as well as in vertical direction.
- Exact analysis is difficult and are designed by

  approximate methods.
- When the length of the wall is more in comparison to its height, the moments will be mainly in the vertical direction, i.e., the panel bends as vertical cantilever

Water tanks

- Storage tanks are built for storing water, liquid petroleum, petroleum products and similar liquids
- Designed as crack free structures to eliminate any leakage
- Permeability of concrete is directly proportional to water cement ratio.
- Cement content ranging from 330 Kg/m3 to 530 Kg/m3 is recommended in order to keep shrinkage low.
- Use of high strength deformed bars of grade Fe415 are recommended for the construction of liquid retaining structures
- Correct placing of reinforcement, use of small sized and use of deformed bars lead to a diffused distribution of cracks
- A crack width of 0.1mm has been accepted as permissible value in liquid retaining structures

Water tanks

- Code of Practice for the storage of Liquids- IS3370 (Part I to IV)
- Fractured strength of concrete is computed using the formula given in clause 6.2.2 of IS 456 -2000 ie., fcr=0.7fck MPa.
- Allowable stresses in reinforcing steel as per IS 3370 are

σst= 115 MPa for Mild steel (Fe250) and

σst= 150 MPa for HYSD bars(Fe415)

- In order to minimize cracking due to shrinkage and temperature, minimum reinforcement is recommended as:
- For thickness 100 mm = 0.3 %
- For thickness 450 mm = 0.2%
- For thickness between 100 mm to 450 mm = varies linearly from 0.3% to 0.2%

Water tanks

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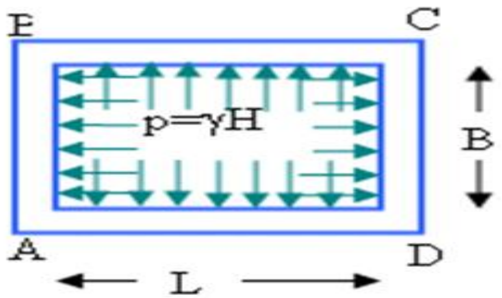
Water tanks

- For concrete thickness ⍰ 225 mm, two layers of reinforcement be placed, one near water face and other away from water face.
- Cover to reinforcement is greater of
  i) 25 mm, ii) Diameter of main bar
- For tension on outer face:
  σst=140 MPa for Mild steel and
  σst=230 MPa for HYSD bars
- For concrete thickness ⍰ 225 mm, two layers of reinforcement be placed, one near water face and other away from water face.

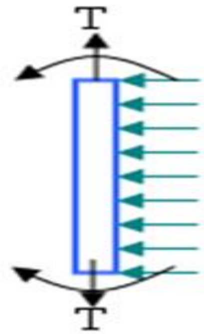
Rectangular Water tanks

- Rectangular tanks are used when the storage capacity is small
- Rectangular tanks should be preferably square in plan from point of view of economy.
- It is also desirable that longer side should not be greater than twice the smaller side.
- Moments are caused in two directions of the wall ie., both in horizontal as well as in vertical direction
- Exact analysis is difficult and are designed by approximate methods.
- When the length of the wall is more in comparison to its height, the moments will be mainly in the vertical direction, ie., the panel bends as vertical cantilever

Rectangular Water tanks

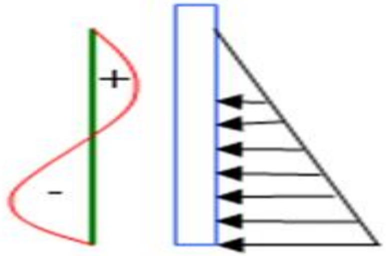


PLAN @ BASE

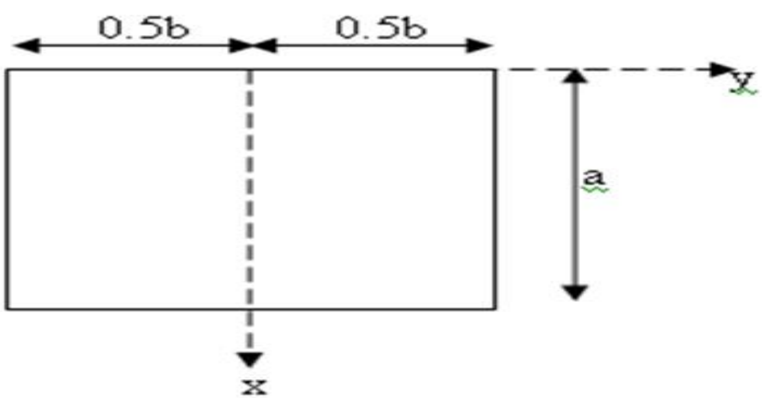


FBD OF AD
IN PLAN

FBD OF AB
IN PLAN



Bending moment diagram



Rectangular Water tanks

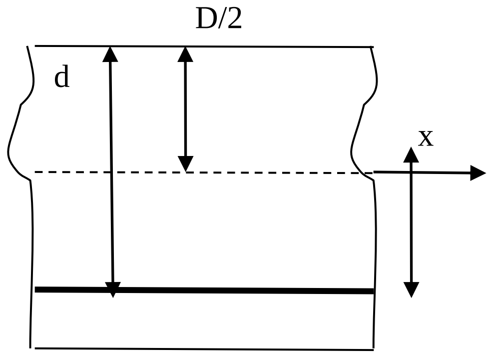
- IS3370 (Part-IV) gives tables for moments and shear forces in walls for certain edge condition. Table 3 of IS3370 provides coefficient for max Bending moments in horizontal and vertical direction.

Maximum vertical moment = $M_x \gamma_w a^3$ (for x/a = 1, y=0)
Maximum horizontal moment = $M_y \gamma_w a^3$(for x/a = 0, y=b/2)
Tension in short wall is computed as T_s=pL/2
Tension in long wall T_L=pB/2

Rectangular Water tanks

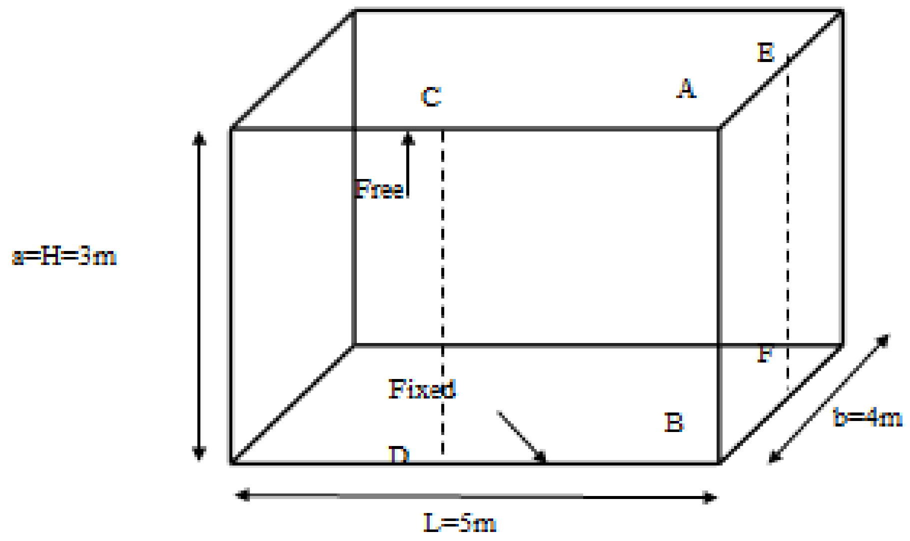
- Horizontal steel is provided for net bending moment and direct tensile force
- Ast=Ast1+Ast2;

- M'=Maximum horizontal bending moment – T x;
- x= d-D/2



Example

Design a rectangular water tank 5m x 4m with depth of storage 3m, resting on ground and whose walls are rigidly joined at vertical and horizontal edges. Assume M20 concrete and Fe415 grade steel. Sketch the details of reinforcement in the tank



Example

1. **Analysis for moment and tensile force**

i) Long wall:

L/a=1.67≈1.75; at y=0, x/a=1, M_x=-0.074; at y=b/2, x/a=1/4, M_y=-0.052

Max vertical moment = $M_x \gamma_w a^3$ = -19.98

Max horizontal moment = $M_y \gamma_w a^3$ = -14.04; $T_{long}=\gamma_w ab/2$=60 kN

i) Short wall:

B/a=1.33≈1.5; at y=0, x/a=1, M_x=-0.06; at y=b/2, x/a=1/4, M_y=-0.044

Max vertical moment = $M_x \gamma_w a^3$ = -16.2

Max horizontal moment = $M_y \gamma_w a^3$ = -11.88; $T_{short}=\gamma_w aL/2$=75 kN

Example

2. Design constants

σ_{cbc}=7 MPa, σ_{st}=150 MPa, m=13.33

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = 0.38$$

j=1-(k/3)=0.87

Q= ½ σ_{cbc}jk = 1.15

3. Design for vertical moment

For vertical moment, the maximum bending moment from long and short wall

$(M_{max})_x$=-19.98 kN-m

$$d = \sqrt{\frac{M}{Qb}} = \sqrt{\frac{19.98x10^6}{1.15x1000}} = 131.8mm$$

Example

Assuming effective cover as 33mm, the thickness of wall is

$t=131.88+33=164.8\ mm \approx 170\ mm$

$d_{provided}=170-33=137mm$

$$A_{st}=\frac{M}{\sigma_{st}jd}=\frac{19.98x10^6}{150x0.87x137}=1117.54mm^2$$

$$\text{Spacing of 12 mm diameter bar} = \frac{113x1000}{1117.54}=101.2mmc/c \ \text{(Max spacing 3d=411mm)}$$

Provide #12 @ 100 mm c/c

Distribution steel

Minimum area of steel is 0.24% of concrete area

$A_{st}=(0.24/100)\ x1000\ x\ 170 = 408\ mm^2$

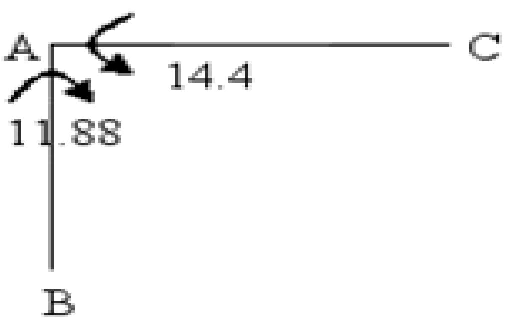
$$\text{Spacing of 8 mm diameter bar} = \frac{50.24x1000}{408}=123.19mmc/c$$

Provide #8 @ 120 c/c as distribution steel.

Provide #8 @ 120 c/c as vertical and horizontal distribution on the outer face.

4. Design for Horizontal moment

Horizontal moments at the corner in long and short wall produce unbalanced moment at the joint. This unbalanced moment has to be distributed to get balanced moment using moment distribution method.



$$K_{AC} = \frac{1}{5}; K_{AC} = \frac{1}{5}; \sum K = \frac{9}{20}$$

$$DF_{AC} = \frac{1/5}{9/20} = 0.44$$

$$DF_{AB} = \frac{1/4}{9/20} = 0.56$$

4. Design for Horizontal moment

Moment distribution Table

Joint	A	
Member	AC	AB
DF	0.44	0.56
FEM	-14	11.88
Distribution	0.9328	1.1872
Final Moment	-13.0672	13.0672

The tension in the wall is computed by considering the section at height H_1 from the base.

Where, H_1 is greater of i) H/4, ii) 1m,

ie., i) 3/4=0.75, ii) 1m; $\therefore H_1 = 1m$

Depth of water $h=H-H_1=3-1-2m$;

$p=\gamma_w h=10 \times 2= 20\ kN/m^2$

Example

4. Design for Horizontal moment

$$A_{st1} = \frac{10.4672 x 10^6}{150 x 0.87 x 137} = 585.46 mm^2$$

$$A_{st2} = \frac{50 x 10^3}{150} = 333.33 mm^2$$

$A_{st}=A_{st1}+A_{st2}=918.79\ mm^2$

Spacing of 12 mm diameter bar = $\frac{113 x 1000}{918.74} = 123 mm c/c$

(Max spacing 3d=411mm)

Provide #12@120 mm c/c at corners

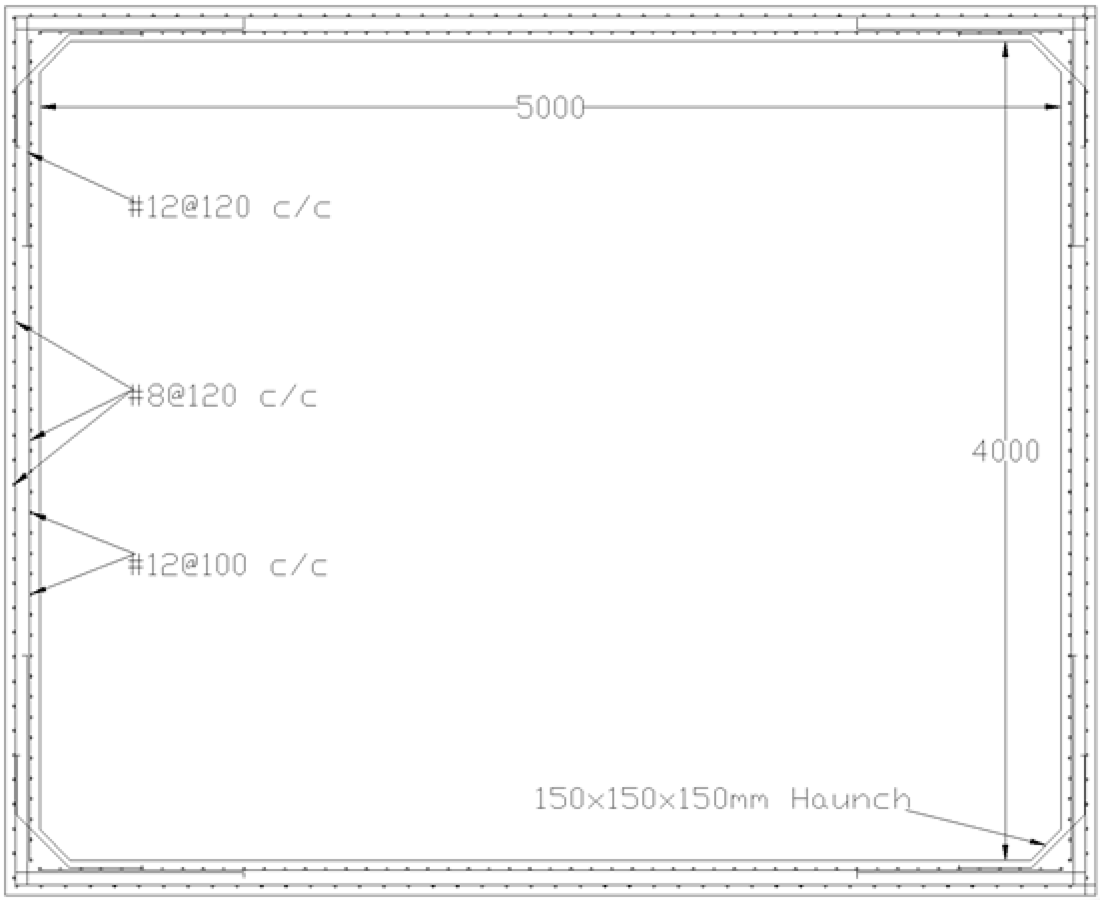
Example

5. Base slab

- The slab is resting on firm ground. Hence nominal thickness and reinforcement is provided.
- The thickness of slab is assumed to be 200 mm and 0.24% reinforcement is provided in the form of #8 @ 200 c/c. at top and bottom
- A haunch of 150 x 150 x 150 mm size is provided at all corners

Example

Reinforcement details



PLAN

Example

Reinforcement details

ELEVATION

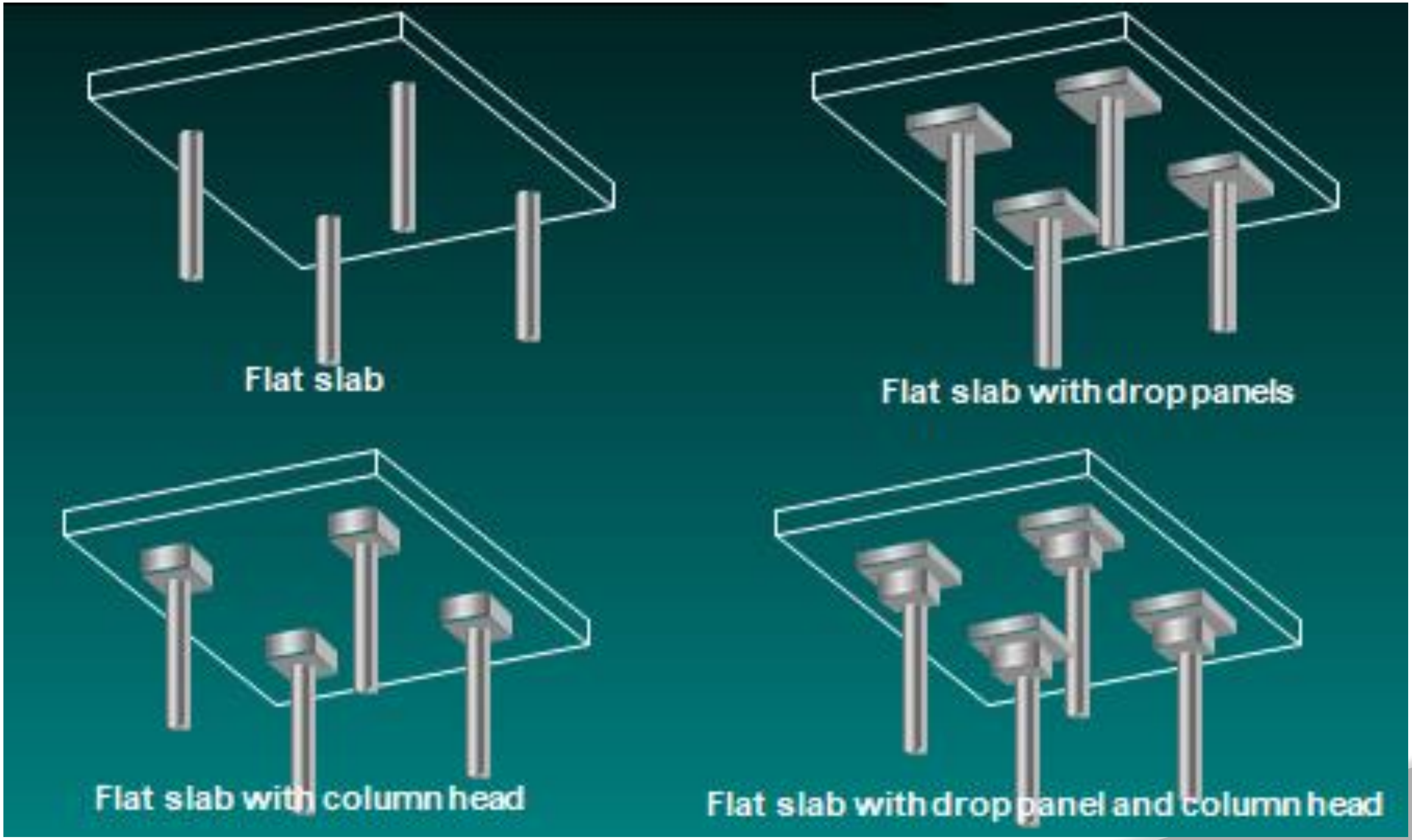
Unit IV
Design of Slabs and Foundations

Flat slabs

What is a flat slab?

A reinforced concrete slab supported directly by concrete columns without the use of beams

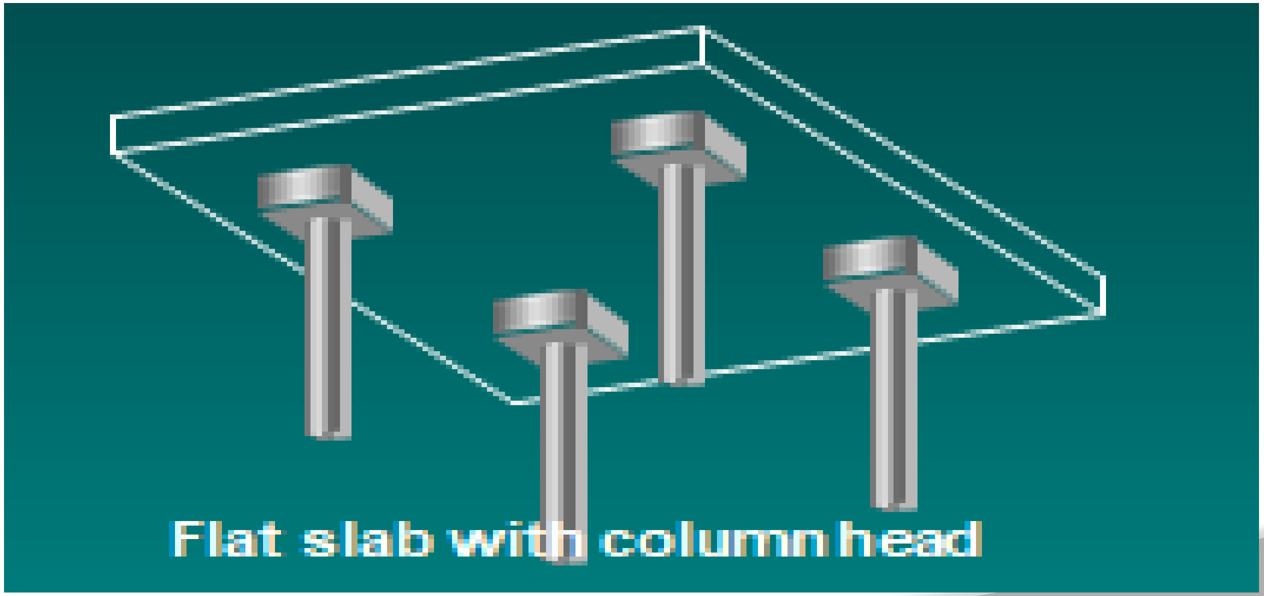
Types of Flat slabs



Flat slabs

Uses of column heads

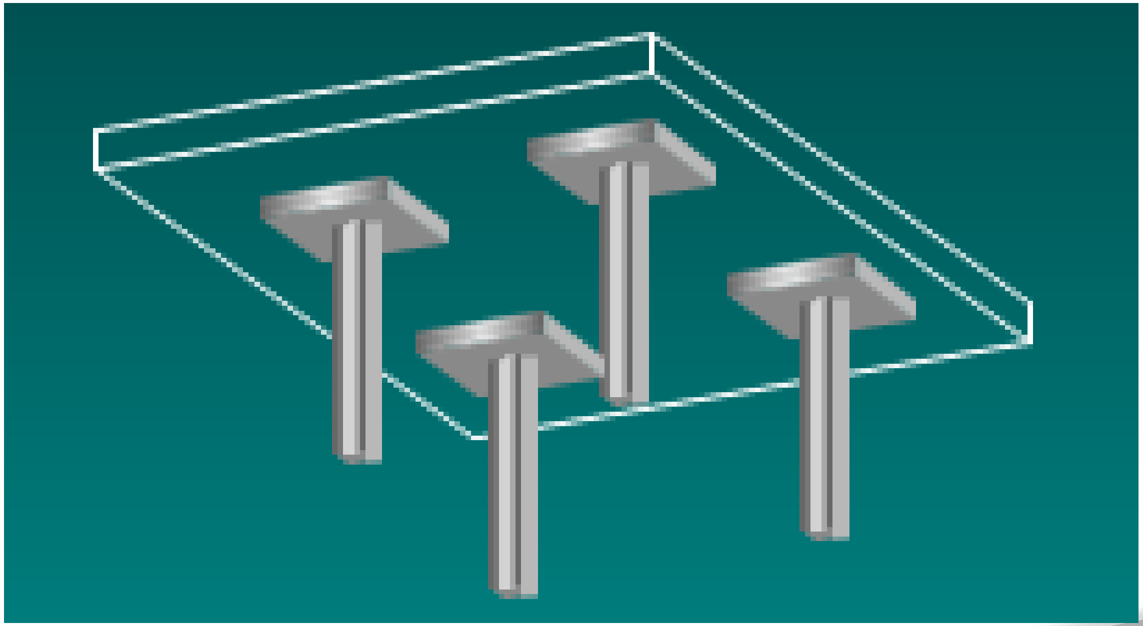
- Increase shear strength of slab
- Reduce the moment in the slab by reducing the clear or effective span



Flat slabs

Uses of drop panels

- Increase shear strength of slab
- Increase negative moment capacity of slab
- Stiffen the slab and hence reduce deflection



Design of Flat slabs

The flat slab structure is divided longitudinally and transversely into frames consisting of columns and strips of slabs with :

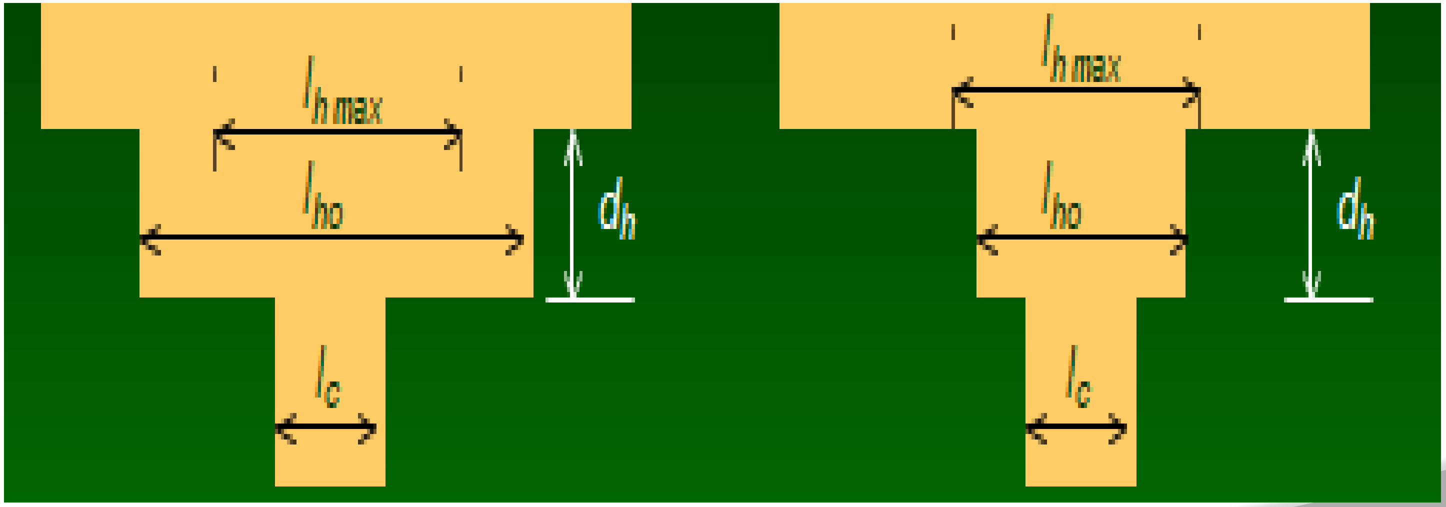
- Stiffness of members based on concrete alone
- For vertical loading, full width of the slab is used to evaluate stiffness
- Effect of drop panel may be neglected if dimension < lx/3

Column head

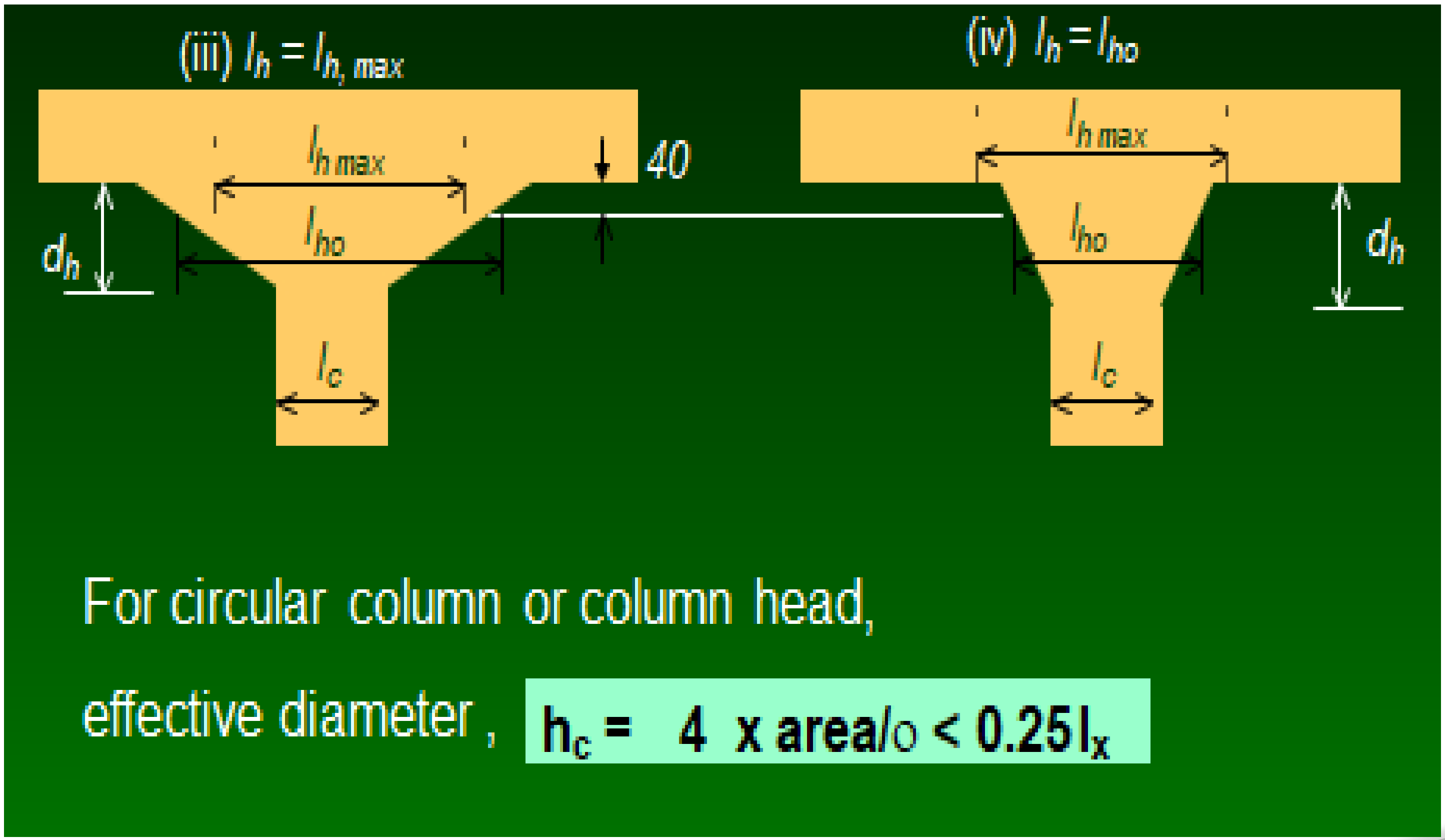
Effective dimension of a head, l_h (mm) = lesser of l_{ho} or $l_{h\,max}$

where l_{ho} = actual dimension, $l_{h\,max} = l_c + 2(d_h - 40)$

(i) $l_h = l_{h,\,max}$ (ii) $l_h = l_{ho}$



Column head

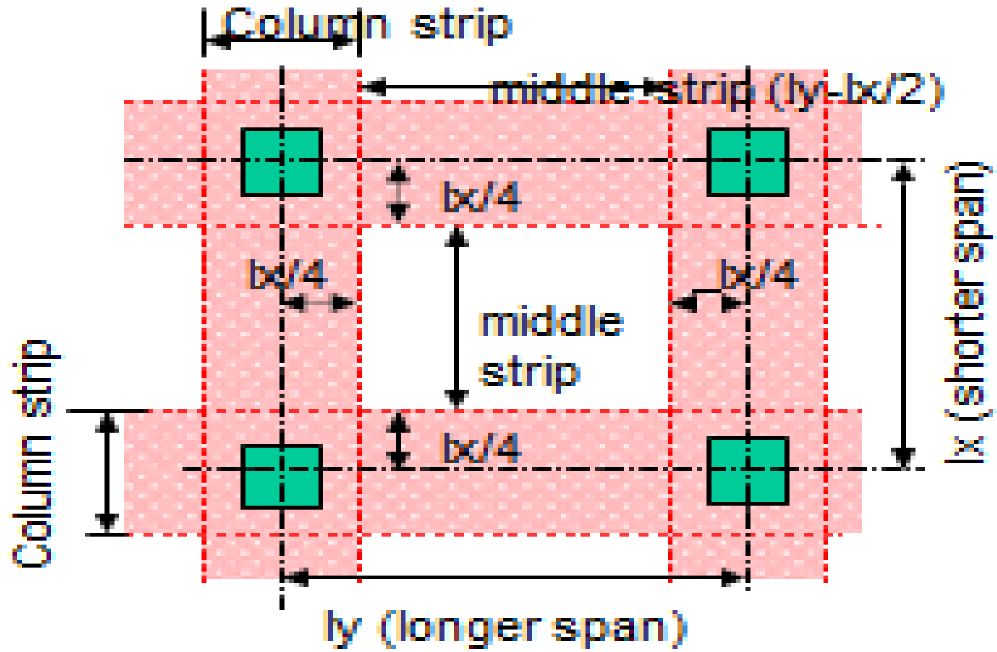

(iii) $l_h = l_{h,\,max}$
$l_{h\,max}$
40
l_{ho}
d_h
l_c
(iv) $l_h = l_{ho}$
$l_{h\,max}$
l_{ho}
d_h
l_c
For circular column or column head,
effective diameter , $h_c = 4 \times area/\pi < 0.25 l_x$

Flat slabs

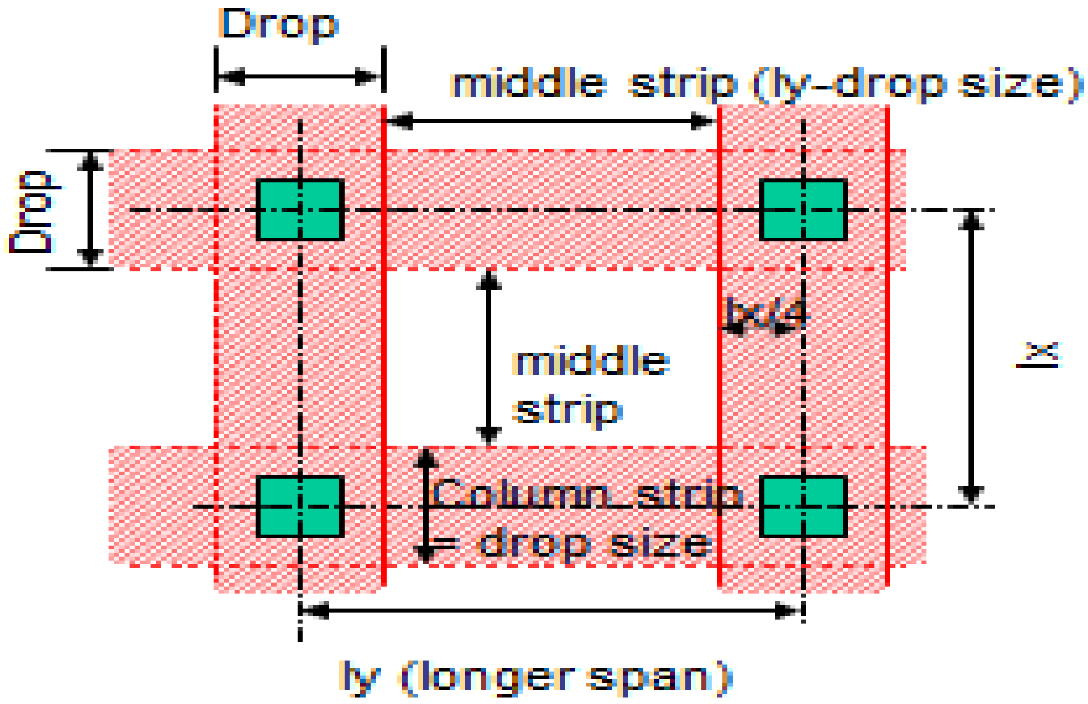
Division of panels

The panels are divided into 'column strips' and middle strips' in both direction.

(a) Slab Without Drops



(b) Slab with drops



Moment division

	Apportionment between column and middle strip expressed as % of the total negative design moment	
	Column strip	Middle strip
Negative	75%	25%
Positive	55%	45%

For slab with drops where the width of the middle strip exceeds L/2, the distribution of moment in the middle strip should be increased in proportion to its increased width and the moment resisted by the column strip should be adjusted accordingly.

Design for bending

Edge panels

apportionment of moment exactly the same as internal columns

max. design moment transferable between slab and edge column by a column strip of breadth be is

$$M_{t,\,max} = 0.15\, b_e\, d^2 fcu$$

(i) < 0.5 design moment (EFM)

(ii) < 0.7 design moment (FEM)

(iii) Otherwise structural arrangements shall be changed

Design for bending

Edge panels

apportionment of moment exactly the same as internal columns

max. design moment transferable between slab and edge column by a column strip of breadth be is

$$M_{t,\,max} = 0.15\, b_e\, d^2 f_{cu}$$

(i) < 0.5 design moment (EFM)

(ii) < 0.7 design moment (FEM)

(iii) Otherwise structural arrangements shall be changed

Flat slabs

Deflection

	Span/depth ratio
Cantilever	7
Simply supported	20
Continuous	26

i) use normal span/effective depth ratio if drop width >1/3 span each way; otherwise

ii) To apply 0.9 modification factor for flat slab, or where drop panel width < L/3

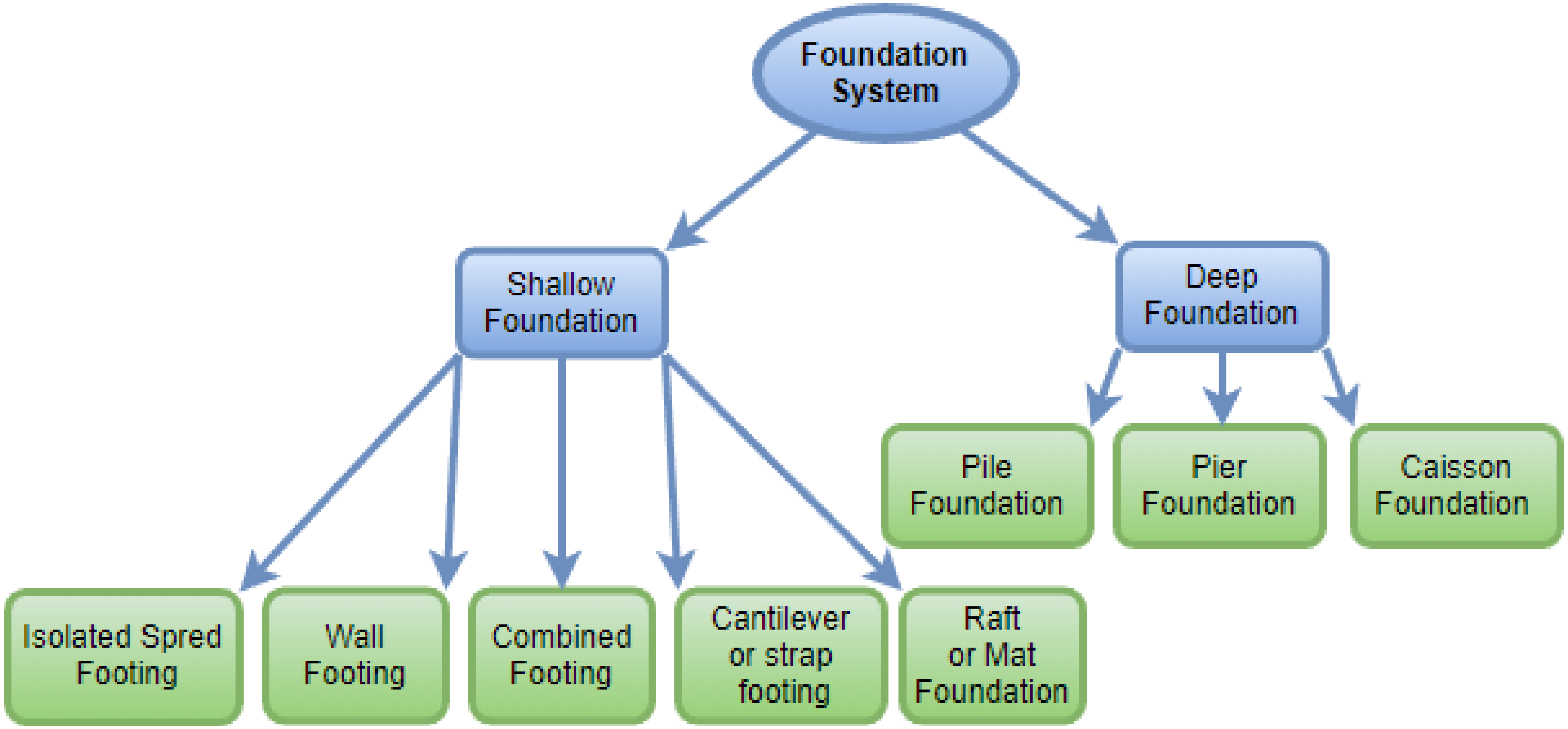
1.0 otherwise

Foundations

Foundation should fulfill the following objectives:

- Distribute the weight of the structure over a large area of soil.
- Avoid unequal settlement.
- Prevent the lateral movement of the structure.
- Increase structural stability.

Types of Foundations



Raft Foundations

- A mat foundation is a thick reinforced concrete slab supporting arrangement of columns or walls in a row and transmitting the load into the soil.
- It is used to support storage tanks, industrial equipments, chimney`s and various structures

Raft Foundations

Necessity

- The spread footings covers over 50% of the foundation area because of large column load.
- The soil is soft with low bearing capacity.
- When the expenses of deep foundation is higher than the raft foundation.
- Walls of the structure are so close that individual footings would overlap

Raft Foundations

Merits

- Raft foundation is economic due to combination of foundations & floor slab.
- Require little excavation.
- Can cope with mixed and poor ground condition.
- It reduces different settlement.

Demerits

- It require specific treatment for point loads.
- Edge erosion occur if not treated properly.

Pile Foundation

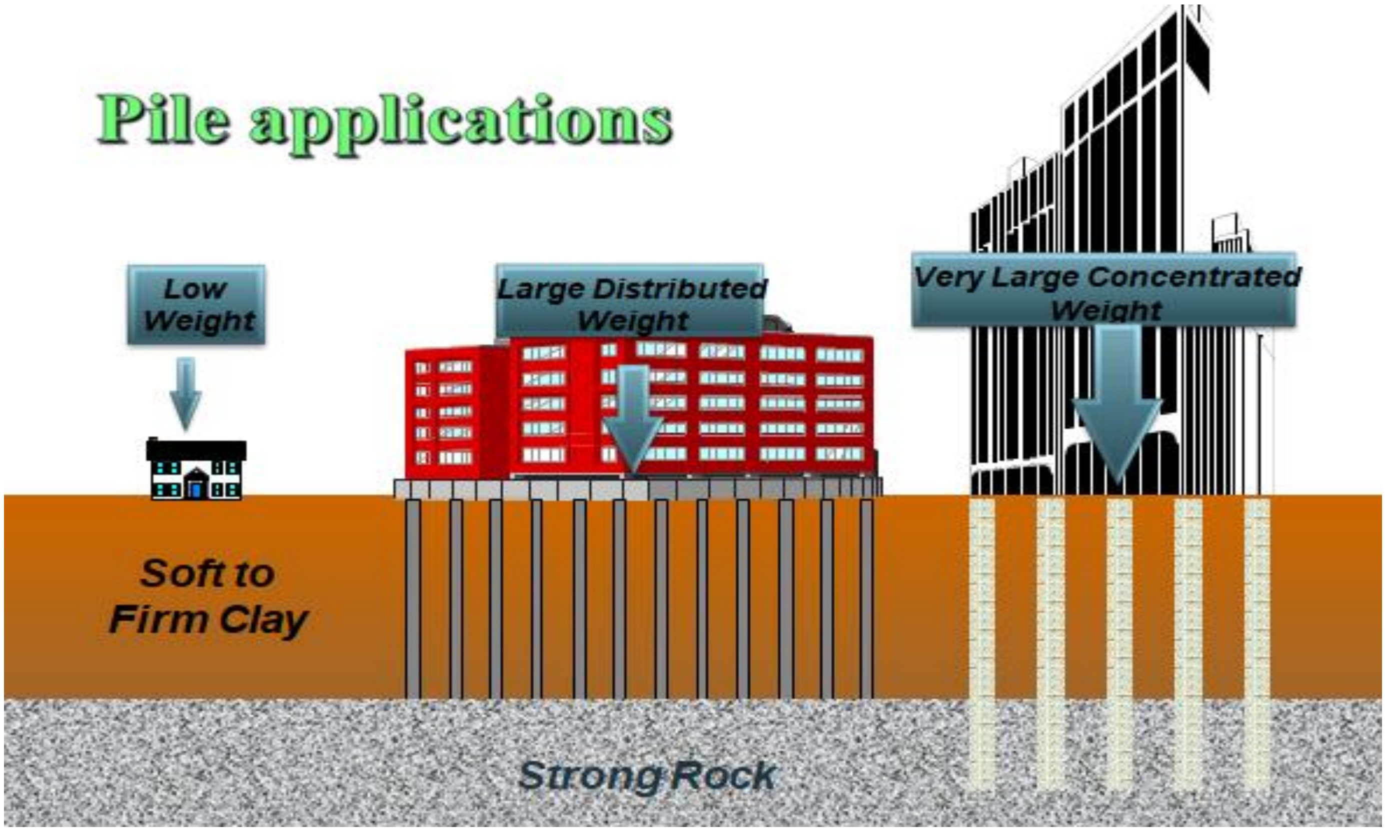
The pile foundation is used to describe a construction for the foundation of a wall or a pier, which is supported on the pile

- The piles may be placed separately or they may be placed in
- the form of cluster throughout the length of the wall.
- Piles are adopted when the loose soil extended to a great depth.
- The load of the structure is transmitted by the piles to hard
- stratum below or it is resisted by the friction developed on the sided of the piles.

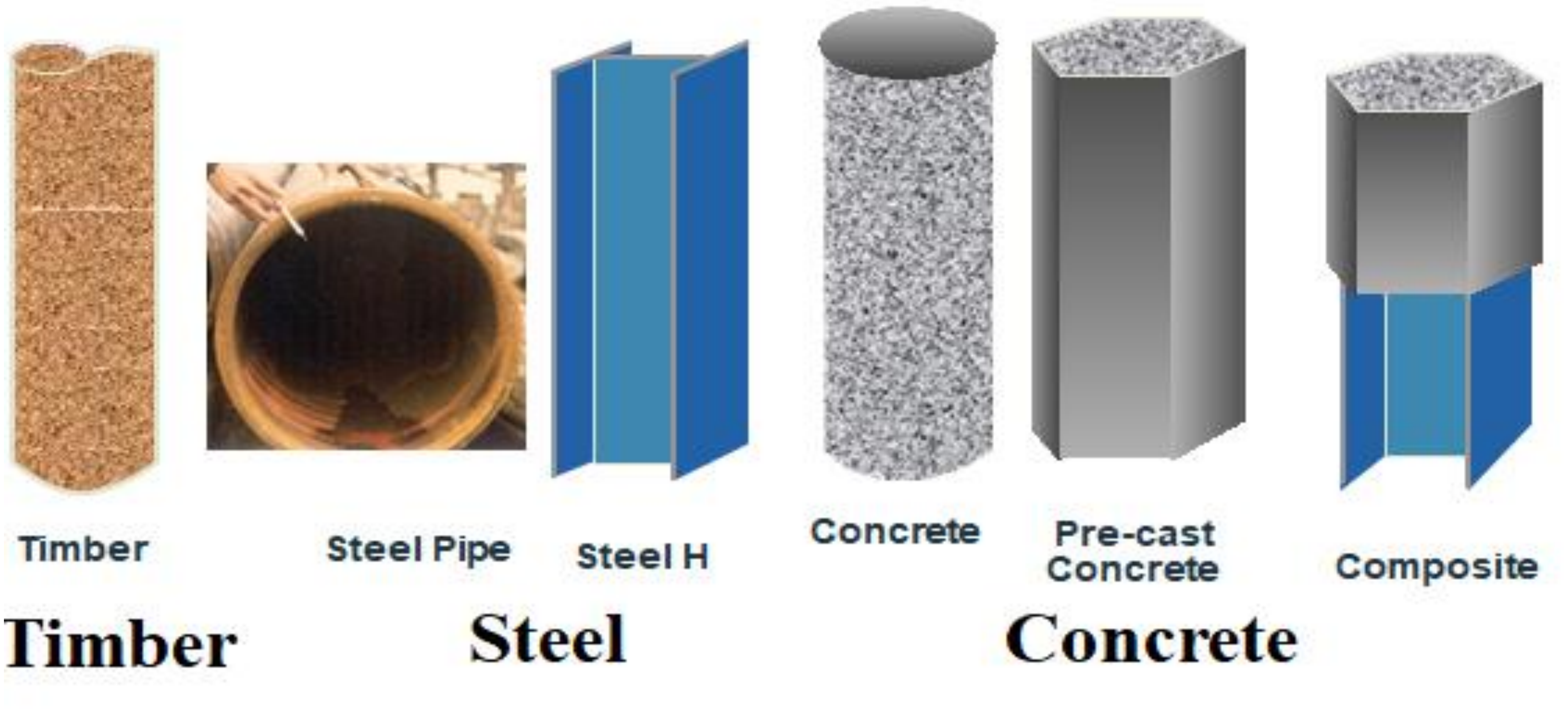
Applications of Pile Foundation

- The piles are applicable at the places where, the load coming from the structure is very high & the distribution of the load on soil is uneven.
- The subsoil water level is likely to rise or fall appreciably.
- The pumping of subsoil water is too costly for keeping the foundation trench in dry condition.
- The construction of raft foundation is likely to be very expensive or it is likely to practically impossible.
- The piles are considered to be long when their length exceeds 30 meters.
- The structure is situated on sea shore or river bed and the foundation is likely to adopted by the scouring action of water.

Applications of Pile Foundation



Pile materials



Types of piles

Load bearing piles

Non-load bearing piles

Load bearing piles :

- It bear the load coming from the structure.
- The Piles are generally driven vertical y or in near vertical position.
- When a horizontal forces to be resisted, the inclined piles may be driven in an inclined position and such inclined piles are termed the batter piles
- Load bearing piles are divided into,
  i. Bearing piles
  ii . Friction Piles

Pile spacing

The centre to centre distance of successive piles is known as pile spacing

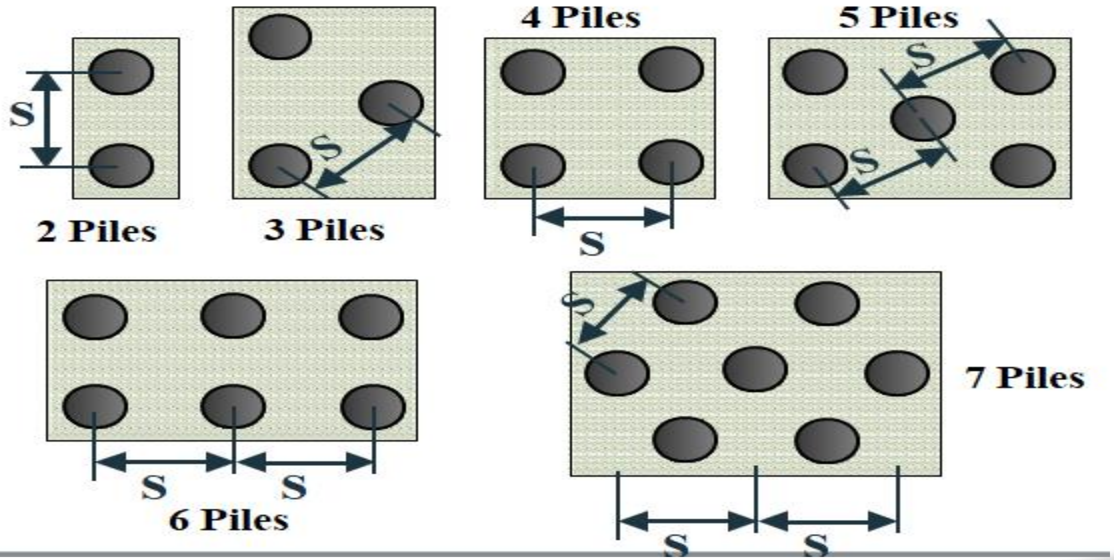
It has to be careful y designed by considering the following factors,

- Types of piles
- Material of piles
- Length of piles
- Grouping of piles
- Load coming on piles
- Obstruction during pile driving
- Nature of soil through which piles are passing.

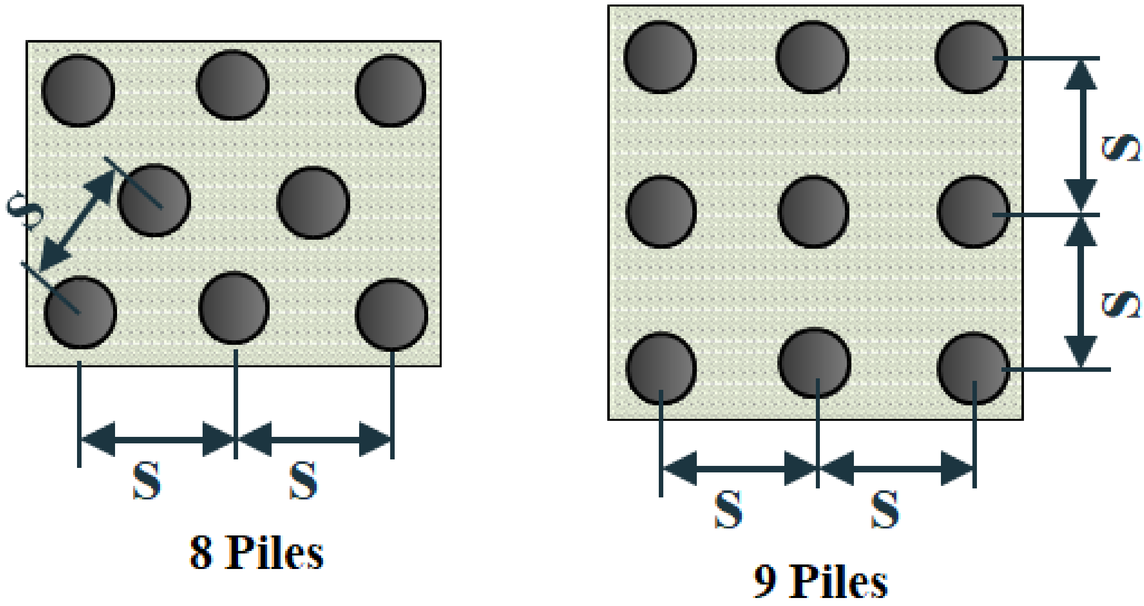
Pile spacing

The spacing between piles in a group can be assumed based on the following:

- Friction piles need higher spacing than bearing piles.
- Minimum spacing (S) between piles is 2.5.
- Maximum spacing (S) between piles is 8.0



Types of piles



Pile capacity

There are two approaches for obtain the capacity of the pile

1. Field Approach
2. Theoretical Approach

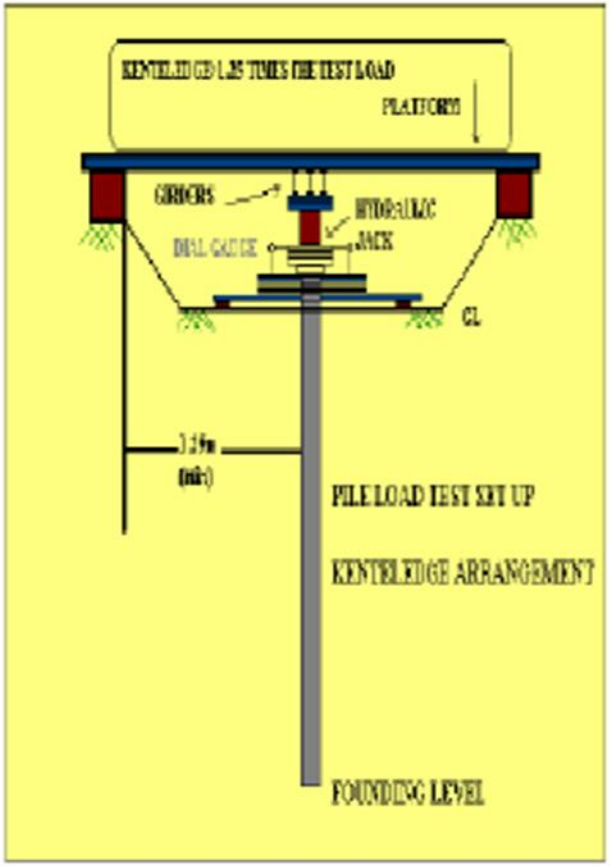
Field Approach : In this approach the pile is loaded to the desired level and its capacity is measured.

A test pile of required dimensions is constructed in the field work and load test is conducted to assess the capacity of the pile.

This approach gives more realistic estimate of pile capacity. However it is time consuming as well as costly.

Field setup for a static axial compressive load test on a single pile

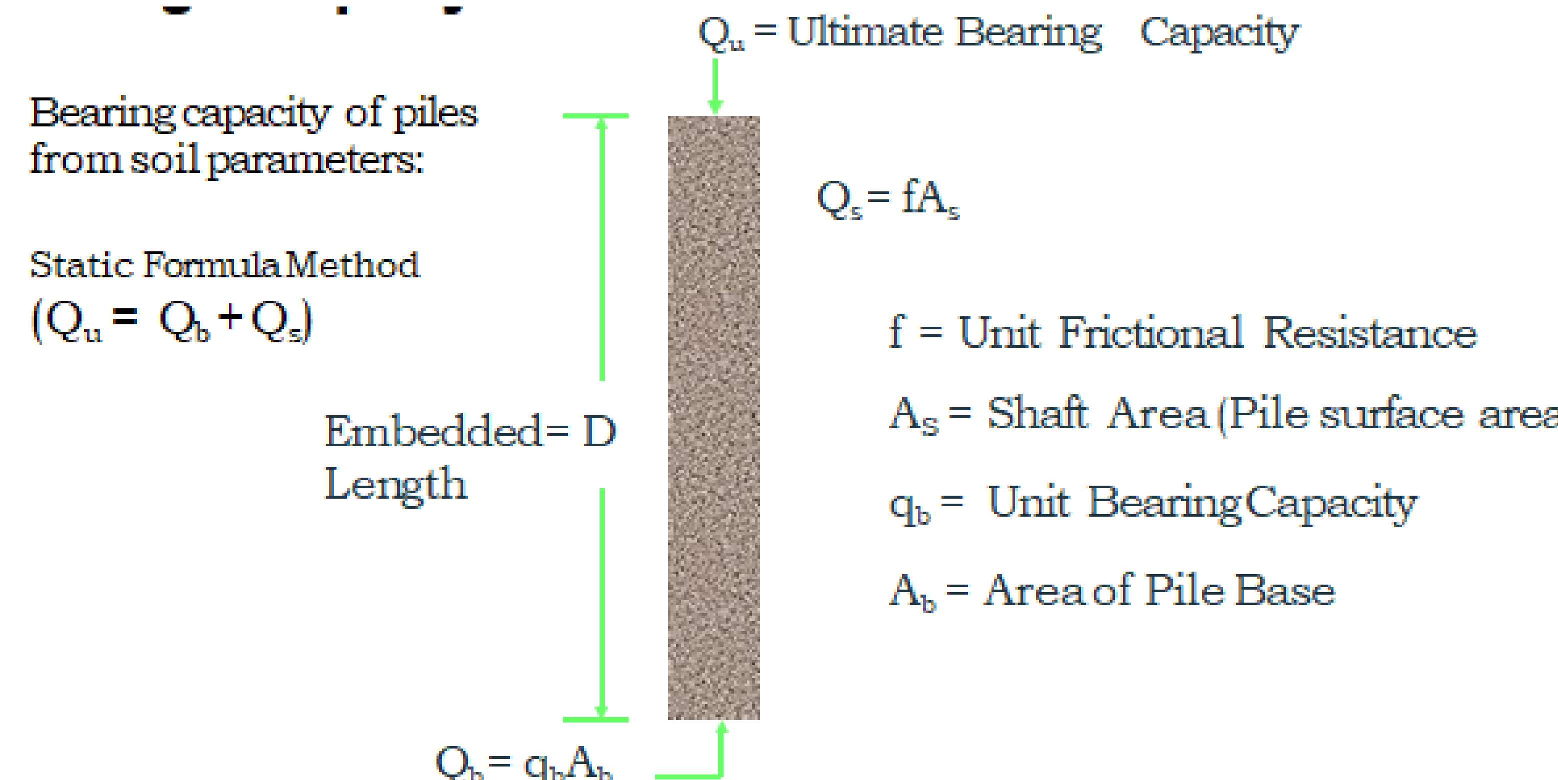
FIELD SET UP



TEST SCHEMATIC DIAGRAM

Pile capacity

Theoretical Approach : In this approach the pile capacity is calculated using some formula in which the soil data is fed for obtaining the capacity.



Factors affecting Pile capacity

- Surrounding soil
- Installation technique (like driven or bored).
- Method of construction (like pre cast or cast in situ).
- Location of pile in a group.
- Spacing of piles in a group.
- Symmetry of the group.
- Shape of pile cap
- Location of pile cap (like above soil or below soil).
- Drainage condition in soil.

Non-load bearing piles

This piles are used to function as the separating members below ground level and they are generally not designed to take any vertical load.

This piles are also known as the sheet piles. The materials used for the construction of non load bearing piles are,

i. Timber sheet piles
ii. Steel sheet piles
iii. Concrete sheet pile

Non-load bearing piles

Non load bearing piles are used for the following purposes.

- To isolate foundations from the adjacent soils. This prevents escape of soil and passage of shocks and vibrations to adjoining structure.
- To prevent underground movement of water like to construct a cofferdam, it requires a water tight enclosure in the construction of foundation under water.
- To prevent the transfer of machine vibrations to the adjacent structures.
- To construction of retaining wall.
- To protect the river banks.
- To retain the sides of foundation trenches.

Pile driving

The process of forcing the piles into the ground without excavation is termed as the pile driving.

The piles should be driven vertical y. However, a tolerance of eccentricity of 2 % of the pile length is permissible.

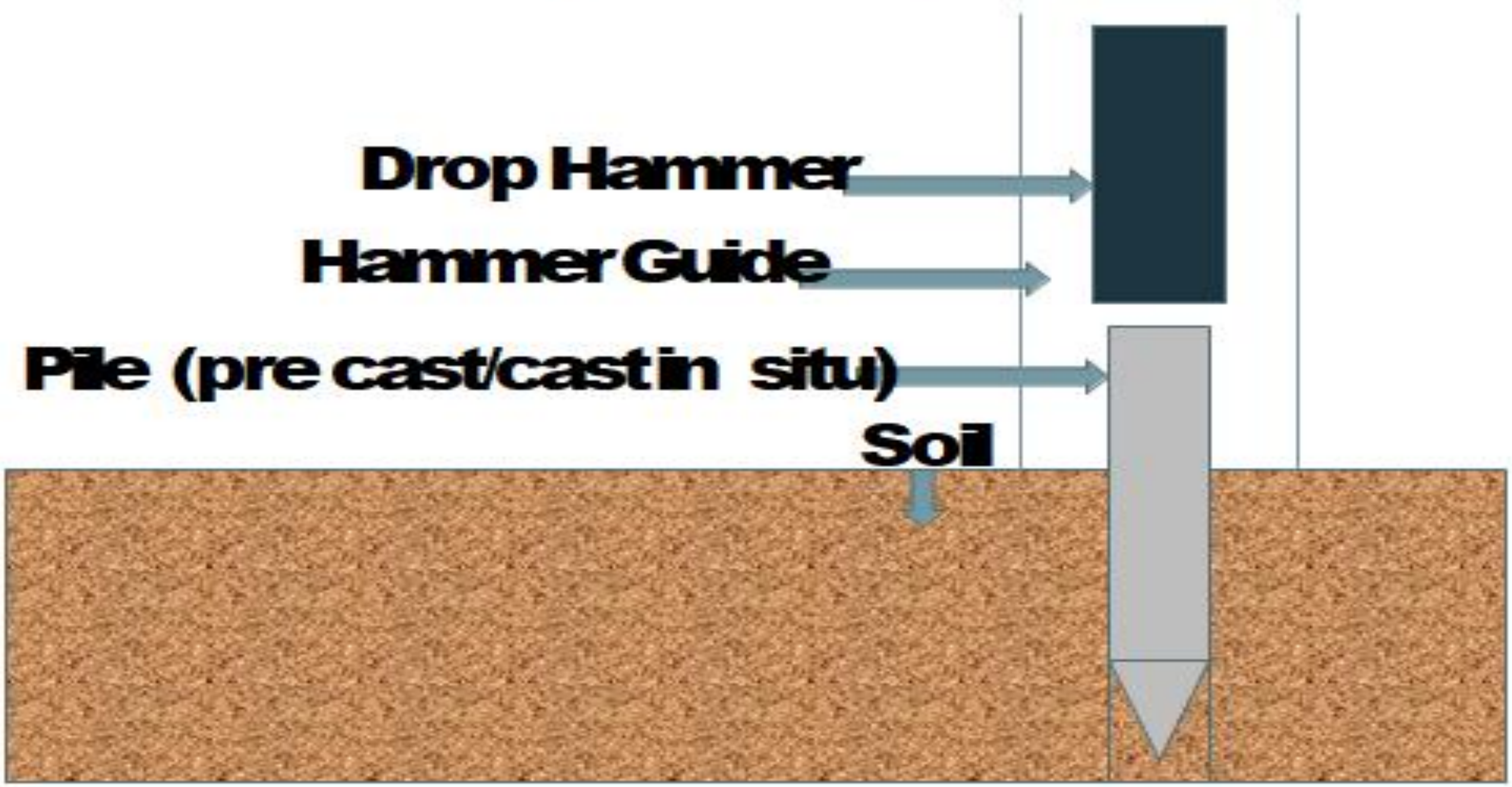
The eccentricity is measured by means of plumb bob.

The equipments required for pile driving are as follows,

- Pile fames
- Pile hammers
- Leads
- Winches
- Miscellaneous

Pile driving

Schematic sketch of pile driving with hammer



Unit V
Design of Chimney, Bunker and Silos

Chimney

Chimneys are tall and slender structures which are used to discharge waste/flue gases at higher elevation with sufficient exit velocity such that the gases and suspended solids (ash) are dispersed into the atmosphere.

DESIGN CODES

Main Codes:

IS: 4998 – Criteria for Design of Reinforced Concrete Chimneys

IS: 6533 _ Code of Practice for Design and Construction of Steel Chimneys

Classification of chimneys

1. Based on number of flues

•Single flue

•Multi flue

2. Based on material of construction

•Concrete (Chimney); Reinforced/Pre-stressed

•Steel (stack)

3. Based on structural support

•Guyed stacks (used in steel stacks for deflection control)

•Self supporting (cantilever structures)

4. Based on lining

•With Lining : Lined chimneys/stacks

•Without lining :Unlined chimneys/stacks

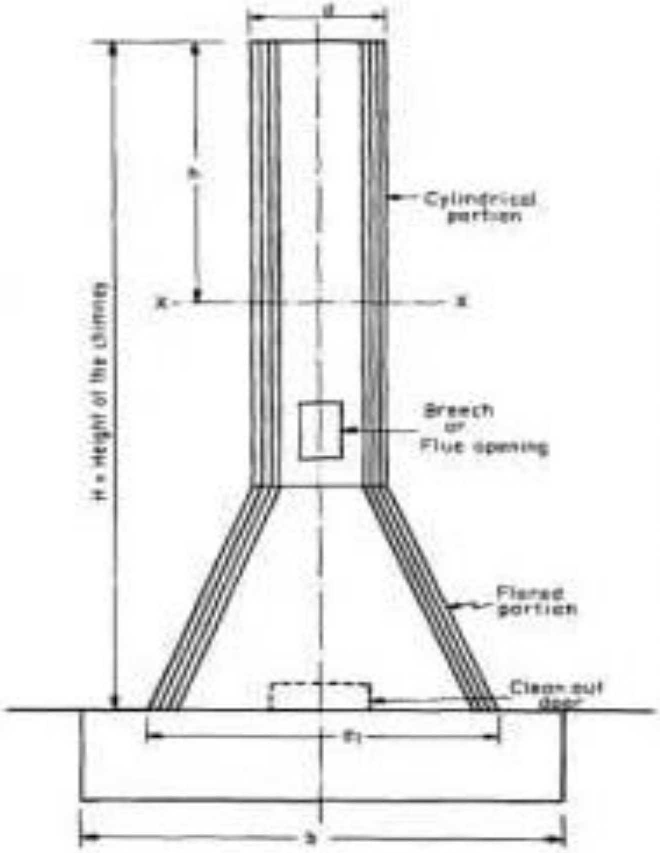
Self- supporting steel Chimney

- When the lateral forces (wind or seismic forces) are transmitted to the foundation by the cantilever action of the chimney, then the chimney is known as self-supporting chimney.

- The self-supporting chimney together with the foundation remains stable under all working conditions without any additional support.

- The self-supporting chimneys are made up to 10 m diameter and from 50 m to 100m in height.

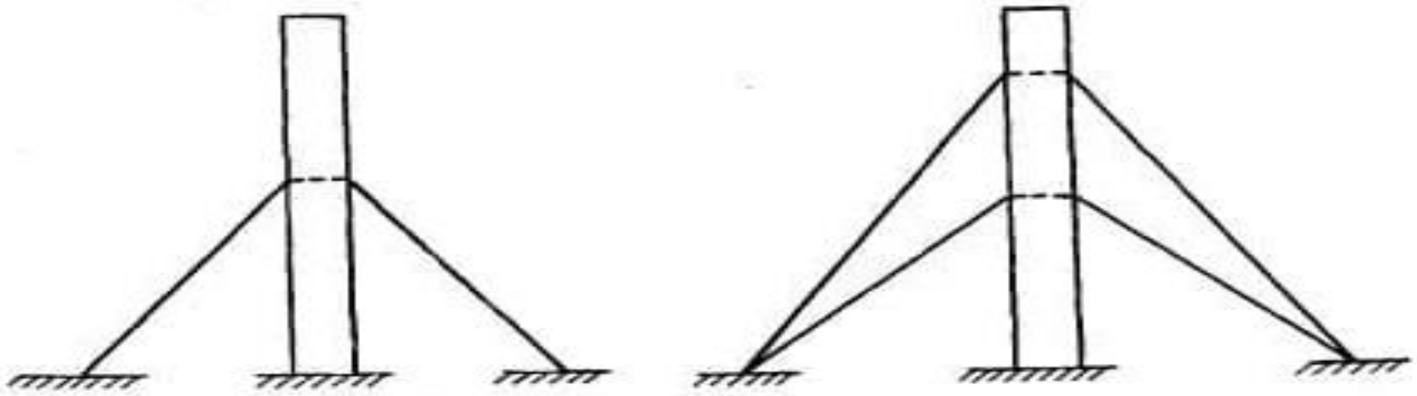
Self- supporting steel Chimney



Guyed steel Chimney

- In high steel chimneys, the mild steel wire ropes or guys are attached to transmit the lateral forces. Such steel chimneys are known as guyed steel chimneys.
- These guys or stays ensure the stability of the guyed steel chimney. These steel chimneys may be provided with one, two or three sets of guys. In each set of guys, three or four or sometimes six wires are attached to the collars.
- When one set of guy is used, then the guys are attached to a collar at one-third or one-fourth of the height from the top. When- more than one set of

Guyed steel Chimney



Forces acting on Chimney

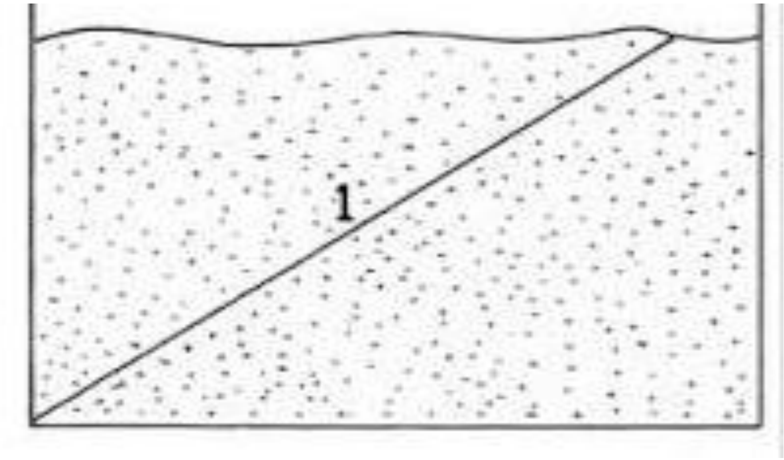
- Force due to self weight
- Force due to wind load
- Force due to Temperature variation
- Combined effect on of self weight, wind and temperature variation

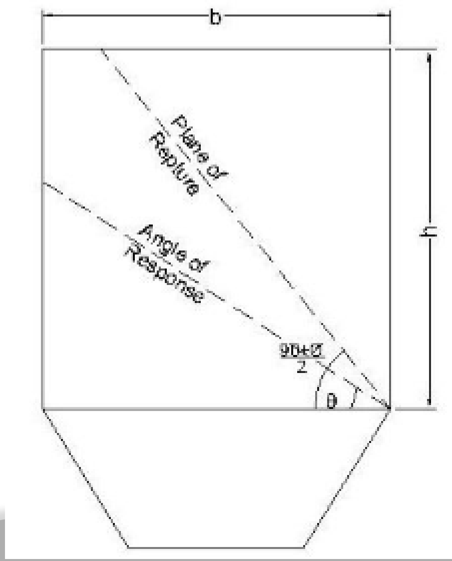
Design procedure of chimney

- Calculation of wind load
- Calculation of dimensions of equivalent steel ring
- Stresses due to section due to self-weight and wind load
- Hoop reinforcement due to shear at the base of chimney
- Check for combined stresses due to self-weight, wind and temperature
    1. leeward side
    2. windward side
- Stresses in hoop steel
- Reinforcement details

Bunkers

- Bunker is a large container or compartment for storing materials
- They are used to store materials like grain, cereals, coal cement etc.
- A bin whose relative dimensions are such that the plane of rupture meets the grain surface before it strikes the opposite side as shown in Figure is called 'shallow bin' and is shown in Fig



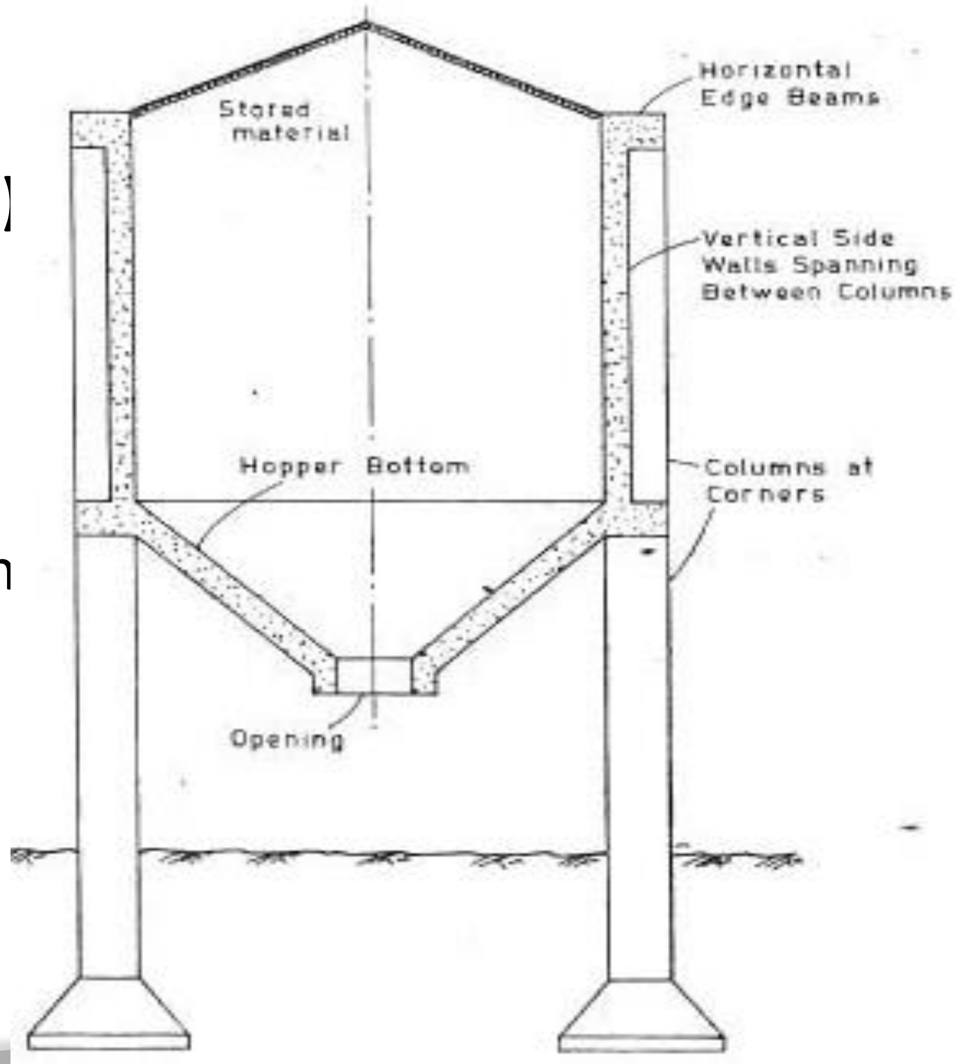


Components of Bunkers

- **Vertical walls**
- **Hopper Bottom**
- **Edge Beam (At the top level]**
- **Supporting Columns**

- **Angle of repose**

  The steepest angle at wh
  loose material is stable.

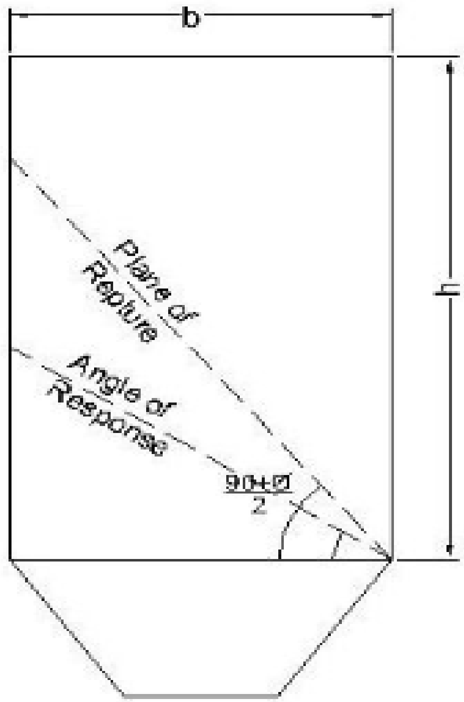


Design procedure of Bunkers

1. Design of vertical walls
2. Design moments
3. Direct tension
4. Design of hopper bottom
5. Dimensions of egde beams

Silos

In silos lot of weight of stored material gets transferred as axial compression due to friction between the material and concrete wall. It results into lateral pressure. Hence Rankine's theory of lateral pressure cannot be used.



Silos

The following two theories are available for fitting lateral pressures in the silos.

1. Janssen's theory
2. Airy's theory

Airy's theory of design of silos is based on Coulumb's wedge theory of earth pressure. The results obtained from this theory also fairly agree with experimental results although the basis of the theory is different from that of the Janssen's theory. Using Airy's formula, horizontal pressure per unit length of periphery and position of plane of rupture can be determined.

Assumptions of Janssen's theory

- Most of the weight of the material stored in the bin is supported by friction between the material and the vertical wall.
- Weight transferred to the hopper bottom is very less. (Hence Rankine's or Coulomb's lateral pressure theory cannot be applied.
- The vertical wall of the bin is subjected to vertical force and horizontal
- pressure.

Difference between bunkers and Silos

Bunker	Silo
Bunkers are shallow structures.	Silos are tall structures.
Plane of rupture meets the top horizontal surface.	Plan of rupture meets the opposite side of the structure.
The total load of material is supported by the floor of the bunker.	Only a fraction of the total load of material (due to side wall friction) is supported by the floor of the silo.
The intensity of horizontal pressure on the sidewall is determined by Rankine’s Theory.	The intensity of horizontal pressure on the sidewall is determined by Janssen’s Theory.
Bunkers are normally metallic with less storage capacity	Silos are normally built by concrete

Silos

Assumptions

- Most of the weight of the material stored in the bin is supported by friction between the material and the vertical wall.
- Weight transferred to the hopper bottom is very less. (Hence Rankine's or Coulomb's lateral pressure theory cannot be applied.
- The vertical wall of the bin is subjected to vertical force and horizontal
- pressure.

Design procedure of Silos

- **Dimensions of silos**
- **Calculation of hoop tension**
- **Calculation of vertical reinforcement**
- **Design of hopper bottom**
- **Check for stress in concrete**
- **Design for hoop tension**
- **Reinforcement details**