LECTURE NOTES

ON

ADVANCED STRUCTURAL ANALYSIS AND DESIGN

(ACE016)

B.Tech VII Sem (IARE-R16)

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MATRIX METHODS OF ANALYSIS

INTRODUCTION:

Indeterminate structures are being widely used for its obvious merits. It may be recalled that, in the case of indeterminate structures either the reactions or the internal forces cannot be determined from equations of statics alone. In such structures, the number of reactions or the number of internal forces exceeds the number of static equilibrium equations. In addition to equilibrium equations, compatibility equations are used to evaluate the unknown reactions and internal forces in statically indeterminate structure. In the analysis of indeterminate structure it is necessary to satisfy the equilibrium equations (implying that the structure is in equilibrium) compatibility equations (requirement if for assuring the continuity of the structure without any breaks) and force displacement equations (the way in which displacement are related to forces). We have two distinct method of analysis for statically indeterminate structure depending upon how the above equations are satisfied:

1. Force method of analysis (also known as flexibility method of analysis, method of consistent deformation, flexibility matrix method)

2. Displacement method of analysis (also known as stiffness matrix method).

In the force method of analysis, primary unknown are forces. In this method compatibility equations are written for displacement and rotations (which are calculated by force displacement equations). Solving these equations, redundant forces are calculated. Once the redundant forces are calculated, the remaining reactions are evaluated by equations of equilibrium. In the displacement method of analysis, the primary unknowns are the displacements. In this method, first force -displacement relations are computed and subsequently equations are written satisfying the equilibrium conditions of the structure. After determining the unknown displacements, the other forces are calculated satisfying the compatibility conditions and force displacement relations. The displacement-based method is amenable to computer programming and hence the method is being widely used in the modern day structural analysis. In general, the maximum deflection and the maximum stresses are small as compared to statically determinate structure.

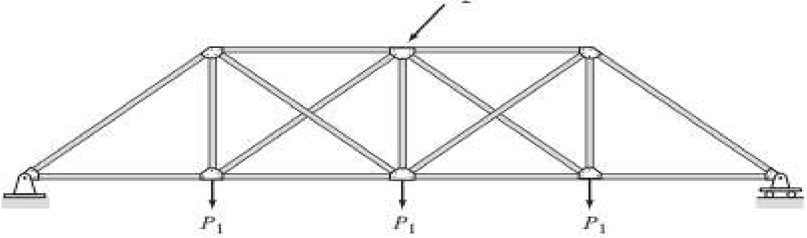
Two different methods can be used for the matrix analysis of structures: the flexibility method, and the stiffness method. The flexibility method, which is also referred to as the force or compatibility method, is essentially a generalization in matrix form of the classical method of consistent deformations. In this approach, the primary unknowns are the redundant forces, which are calculated first by solving the structure's compatibility equations. Once the redundant forces are known, the displacements can be evaluated by applying the equations of equilibrium and the appropriate member force–displacement relations.

CLASSIFICATION OF FRAMED STRUCTURES

Framed structures are composed of straight members whose lengths are significantly larger than their cross-sectional dimensions. Common framed structures can be classified into six basic categories based on the arrangement of their members, and the types of primary stresses that may develop in their members under major design loads.

Plane Trusses

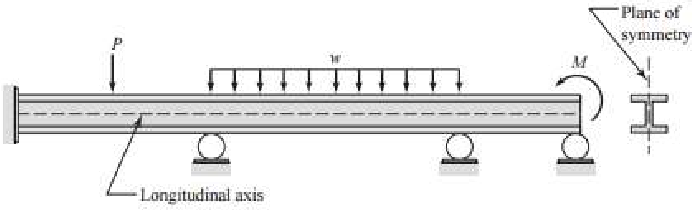
A truss is defined as an assemblage of straight members connected at their ends by flexible connections, and subjected to loads and reactions only at the joints (connections). The members of such an ideal truss develop only axial forces when the truss is loaded. In real trusses, such as those commonly used for supporting roofs and bridges, the members are connected by bolted or welded connections that are not perfectly flexible, and the dead weights of the members are distributed along their lengths. Because of these and other deviations from idealized conditions, truss members are subjected to some bending and shear. However, in most trusses, these secondary bending moments and shears are small in comparison to the primary axial forces, and are usually not considered in their designs. If large bending moments and shears are anticipated, then the truss should be treated as a rigid frame (discussed subsequently) for analysis and design. If all the members of a truss as well as the applied loads lie in a single plane, the truss is classified as a plane truss. The members of plane trusses are assumed to be connected by frictionless hinges. The analysis of plane trusses is considerably simpler than the analysis of space (or three-dimensional) trusses. Fortunately, many commonly used trusses, such as bridge and roof trusses, can be treated as plane trusses for analysis.



Plane Truss

Beams

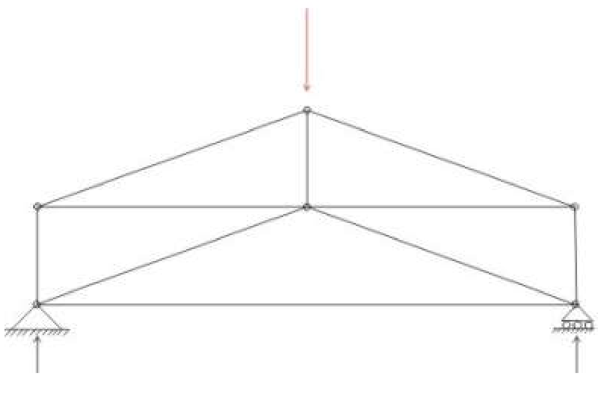
A beam is defined as a long straight structure that is loaded perpendicular to its longitudinal axis. Loads are usually applied in a plane of symmetry of the beam's cross-section, causing its members to be subjected only to bending moments and shear forces.



Beam

Space Trusses

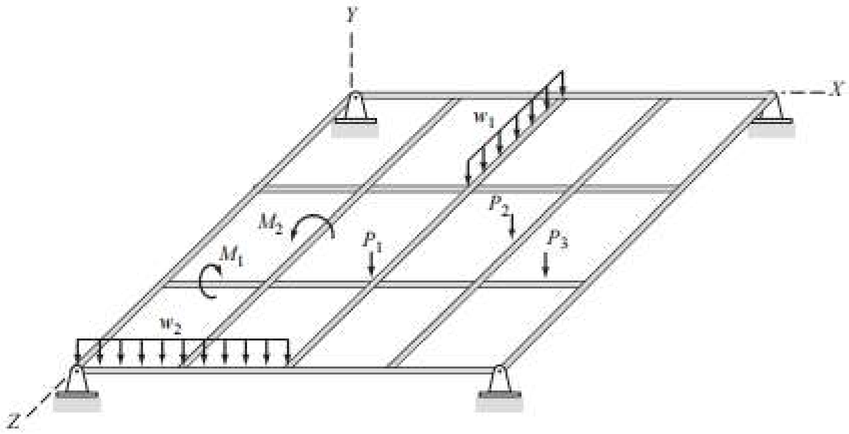
Some trusses (such as lattice domes, transmission towers, and certain aerospace structures cannot be treated as plane trusses because of the arrangement of their members or applied loading. Such trusses, referred to as space trusses, are analysed as three-dimensional structures subjected to three dimensional force systems. The members of space trusses are assumed to be connected by frictionless ball-and-socket joints, and the trusses are subjected to loads and reactions only at the joints. Like plane trusses, the members of space trusses develop only axial forces.



Space Trusses

Grids

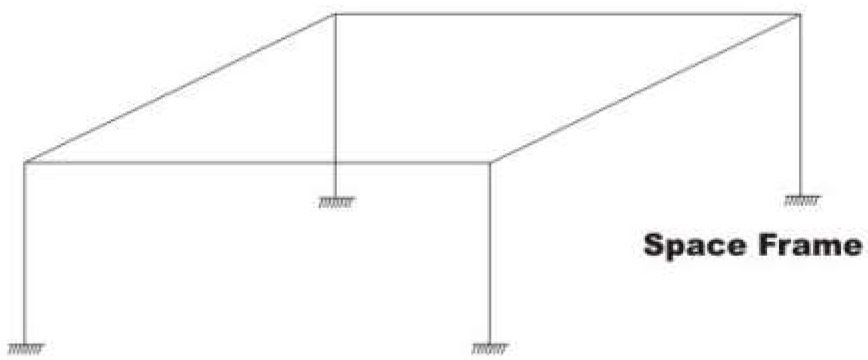
A grid, like a plane frame, is composed of straight members connected together by rigid and/or flexible connections to form a plane framework. The main difference between the two types of structures is that plane frames are loaded in the plane of the structure, whereas the loads on grids are applied in the direction perpendicular to the structure's plane. Members of grids may, therefore, be subjected to torsional moments, in addition to the bending moments and corresponding shears that cause the members to bend out of the plane of the structure. Grids are commonly used for supporting roofs covering large column-free areas in such structures as sports arenas, auditoriums, and aircraft hangars.



Grid

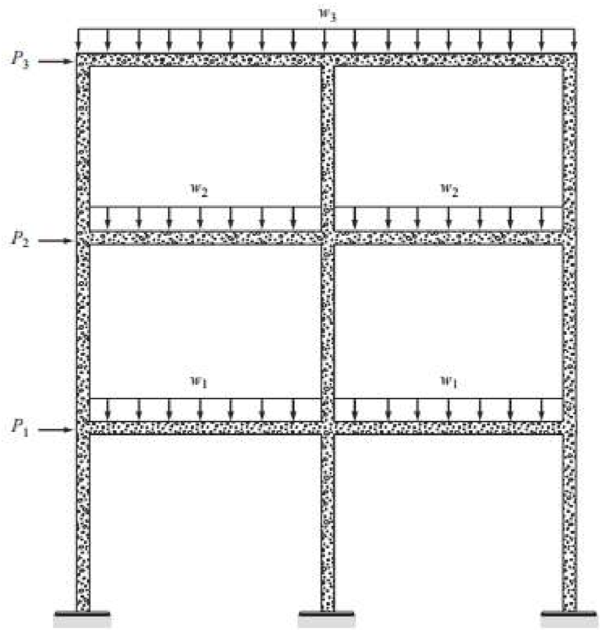
Space Frames

Space frames constitute the most general category of framed structures. Members of space frames may be arranged in any arbitrary directions, and connected by rigid and/or flexible connections. Loads in any directions may be applied on members as well as on joints. The members of a space frame may, in general, be subjected to bending moments about both principal axes, shears in principal directions, torsional moments, and axial forces.



Plane Frames

Frames, also referred to as rigid frames, are composed of straight members connected by rigid (moment resisting) and/or flexible connections. Unlike trusses, which are subjected to external loads only at the joints, loads on frames may be applied on the joints as well as on the members. If all the members of a frame and the applied loads lie in a single plane, the frame is called a plane frame. The members of a plane frame are, in general, subjected to bending moments, shears, and axial forces under the action of external loads. Many actual three-dimensional building frames can be subdivided into plane frames for analysis.



Plane Frame

FUNDAMENTAL RELATIONSHIPS FOR STRUCTURAL ANALYSIS

Structural analysis, in general, involves the use of three types of relationships:

- Equilibrium equations,
- Compatibility conditions and
- Co-ordinate systems.

Equilibrium Equation

A structure is considered to be in equilibrium if, initially at rest, it remains at rest when subjected to a system of forces and couples. If a structure is in equilibrium, then all of its members and joints must also be in equilibrium. Recall from statics that for a plane (two-dimensional) structure lying in the XY plane and subjected to a coplanar system of forces and couples, the necessary and sufficient conditions for equilibrium can be expressed in Cartesian

(XY) coordinates. These equations are referred to as the equations of equilibrium for plane structures. For a space (three-dimensional) structure subjected to a general three dimensional system of forces and couples (Fig. 1.12)

The equations of equilibrium are expressed as

FX = 0, FY = 0 and FZ = 0

MX = 0, MY = 0 and MZ = 0

For a structure subjected to static loading, the equilibrium equations must be satisfied for the entire structure as well as for each of its members and joints. In structural analysis, equations of equilibrium are used to relate the forces (including couples) acting on the structure or one of its members or joints.

Compatibility Conditions

The compatibility conditions relate the deformations of a structure so that its various parts (members, joints, and supports) fit together without any gaps or overlaps. These conditions (also referred to as the continuity conditions) ensure that the deformed shape of the structure is continuous (except at the locations of any internal hinges or rollers), and is consistent with the support conditions. Consider, for example, the two-member plane frame. The deformed shape of the frame due to an arbitrary loading is also depicted, using an exaggerated scale. When analysing a structure, the compatibility conditions are used to relate member end displacements to joint displacements which, in turn, are related to the support conditions. For example, because joint 1 of the frame is attached to a roller support that cannot translate in the vertical direction, the vertical displacement of this joint must be zero. Similarly, because joint 3 is attached to a fixed support that can neither rotate nor translate in any direction, the rotation and the horizontal and vertical displacements of joint 3 must be zero.

GLOBAL AND LOCAL COORDINATE SYSTEMS

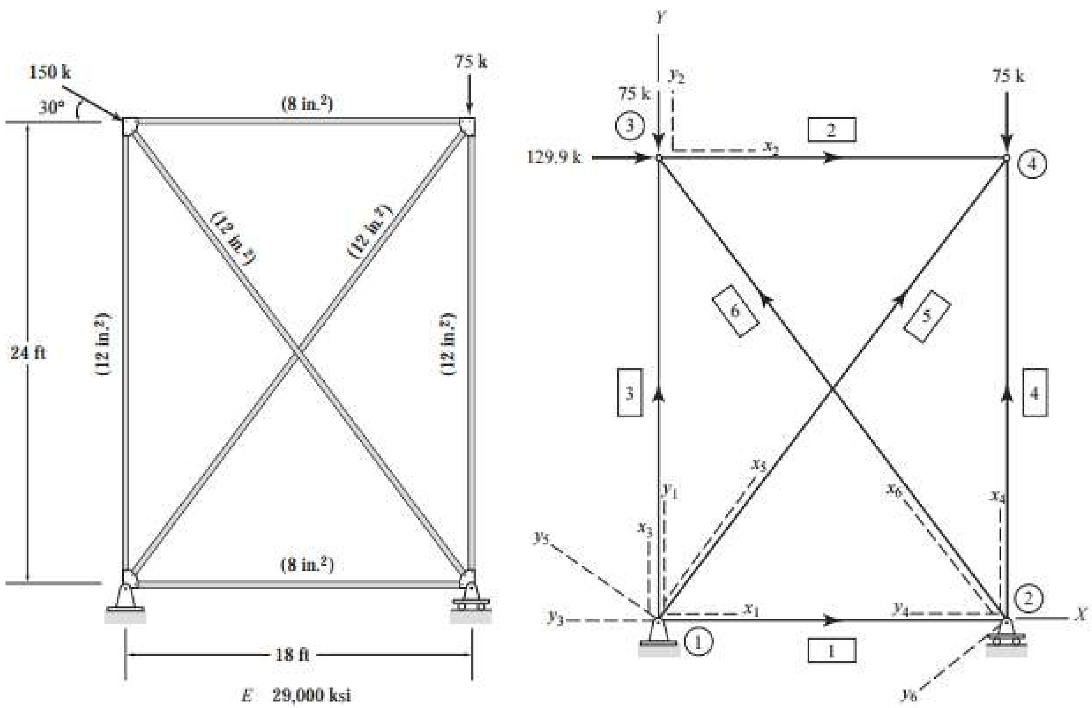
In the matrix stiffness method, two types of coordinate systems are employed to specify the structural and loading data and to establish the necessary force–displacement relations. These are referred to as the global (or structural) and the local (or member) coordinate systems.

Global Coordinate System

The overall geometry and the load–deformation relationships for an entire structure are described with reference to a Cartesian or rectangular global coordinate system. When analyzing a plane (two-dimensional) structure, the origin of the global XY coordinate system can be located at any point in the plane of the structure, with the X and Y axes oriented in any mutually perpendicular directions in the structure's plane. However, it is usually convenient to locate the origin at a lower left joint of the structure, with the X and Y axes oriented in the horizontal (positive to the right) and vertical (positive upward) directions, respectively, so that the X and Y coordinates of most of the joints are positive.

Local Coordinate System

Since it is convenient to derive the basic member force–displacement relationships in terms of the forces and displacements in the directions along and perpendicular to members, a local coordinate system is defined for each member of the structure.



(a) Actual Truss

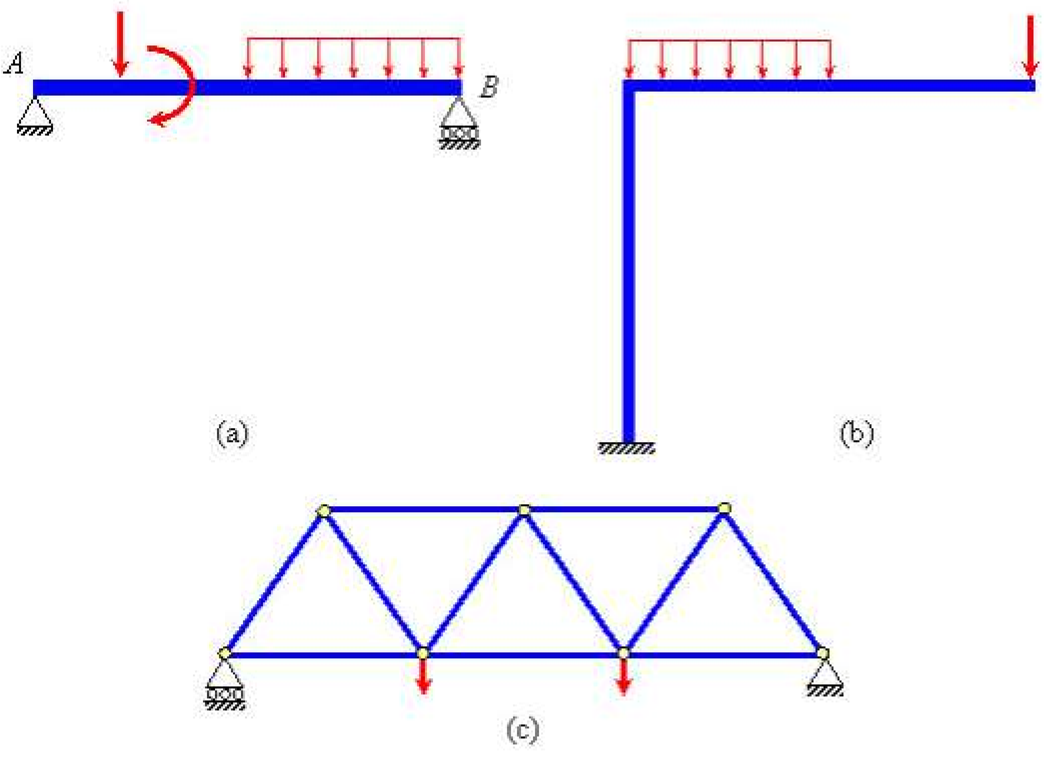
(b) Analytical Model Showing Global and Local Coordinate Systems

DEGREES OF FREEDOM

The degrees of freedom of a structure, in general, are defined as the independent joint displacements (translations and rotations) that are necessary to specify the deformed shape of the structure when subjected to an arbitrary loading. Since the joints of trusses are assumed to be frictionless hinges, they are not subjected to moments and, therefore, their rotations are zero. Thus, only joint translations must be considered in establishing the degrees of freedom of trusses. The deformed shape of the truss, for an arbitrary loading, is depicted in using an exaggerated scale. From this figure, we can see that joint 1, which is attached to the hinged support, cannot translate in any direction; therefore, it has no degrees of freedom. Because joint 2 is attached to the roller support, it can translate in the X direction, but not in the Y direction. Thus, joint 2 has only one degree of freedom, which is designated d1 in the figure. As joint 3 is not attached to a support, two displacements (namely, the translations d2 and d3 in the X and Y directions, respectively) are needed to completely specify its deformed position 3. Thus, joint 3 has two degrees of freedom. Similarly, joint 4, which is also a free joint, has two degrees of freedom, designated d4 and d5.

Static Indeterminacy of Structures

If the number of independent static equilibrium equations (refer to Section 1.2) is not sufficient for solving for all the external and internal forces (support reactions and member forces, respectively) in a system, then the system is said to be statically indeterminate. A statically determinate system, as against an indeterminate one, is that for which one can obtain all the support reactions and internal member forces using only the static equilibrium equations. For example, idealized as one-dimensional, the number of independent static equilibrium equations is just 1 while the total numbers of unknown support reactions aretwo, that is more than the number of equilibrium equations available. Therefore, the system is considered statically indeterminate. The following figures illustrate some example of statically determinate and indeterminate structures.



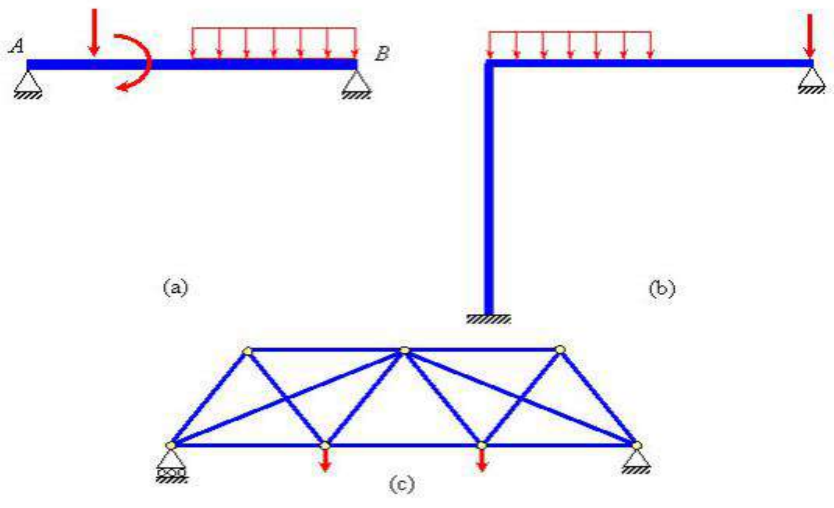
Statically determinate structures

the equilibrium equations are described as the necessary and sufficient conditions to maintain the equilibrium of a body. However, these equations are not always able to provide all the information needed to obtain the unknown support reactions and internal forces. The number of external supports and internal members in a system may be more than the number that is required to maintain its equilibrium configuration. Such systems are known as indeterminate systems and one has to use compatibility conditions and constitutive relations in addition to equations of equilibrium to solve for the unknown forces in that system. For an indeterminate system, some support(s) or internal member(s) can be removed without disturbing its equilibrium. These additional supports and members are known as redundants. A determinate system has the exact number of supports and internal members that it needs to maintain the equilibrium and no redundants. If a system has less than required number of supports and internal members to maintain equilibrium, then it is considered unstable. For example, the two-dimensional propped cantilever system in (Figure 1.13a) is an indeterminate system because it possesses one support more than that are necessary to maintain its equilibrium. If we remove the roller support at end B (Figure 1.13b), it still maintains equilibrium. One should note that here it has the same number of unknown support reactions as the number of independent static equilibrium equations.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_z (\text{about any point}) = 0$$



Statically indeterminate structures

An indeterminate system is often described with the number of redundants it contains and this number is known as its degree of static indeterminacy. Thus, mathematically:

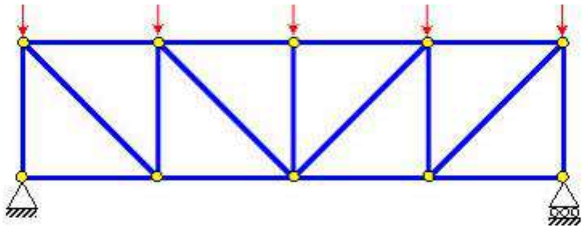
Degree of static indeterminacy = Total number of unknown (external and internal) forces - Number of independent equations of equilibrium

It is very important to know exactly the number of unknown forces and the number of independent equilibrium equations. Let us investigate the determinacy/indeterminacy of a few two-dimensional pin-jointed truss systems. Let m be the number of members in the truss system and n be the number of pin (hinge) joints connecting these members. Therefore, there will be m number of unknown internal forces (each is a two-force member) and 2 n numbers of independent joint equilibrium equations (and for each joint, based on its free body diagram). If the support reactions involve r unknowns, then:

Total number of unknown forces = m + r

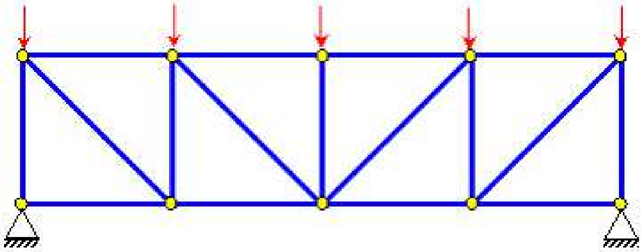
Total number of independent equilibrium equations = 2 n

So, **degree of static indeterminacy = (m + r) - 2 n**



Determinate truss

m = 17, n = 10, and r = 3. So, degree of static indeterminacy = 0, that means it is a statically determinate system.



(Internally) indeterminate truss

m = 18, n = 10, and r = 3. So, degree of static indeterminacy = 1.

Kinematic Indeterminacy of Structures

A structure is said to be kinematicaly indeterminate if the displacement components of its joints cannot be determined by compatibility conditions alone. In order to evaluate displacement components at the joints of these structures, it is necessary to consider the equations of static equilibrium. i.e. no. of unknown joint displacements over and above the compatibility conditions will give the degree of kinematic indeterminacy.

We have seen that the degree of static indeterminacy of a structure is, in fact, the number of forces or stress resultants which cannot be determined using the equations of static equilibrium. Another form of the indeterminacy of a structure is expressed in terms of its degrees of freedom; this is known as the kinematic indeterminacy, nk, of a structure and is of particular relevance in the stiffness method of analysis where the unknowns are the displacements.

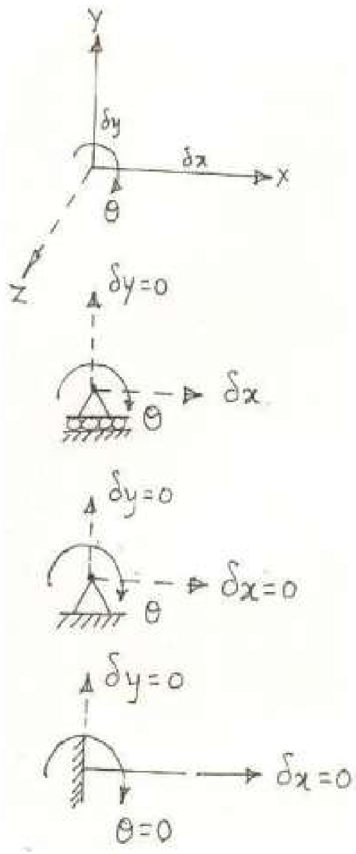
A simple approach to calculating the kinematic indeterminacy of a structure is to sum the degrees of freedom of the nodes and then subtract those degrees of freedom that are prevented by constraints such as support points. It is therefore important to remember that in three-dimensional structures each node possesses 6 degrees of freedom while in plane structures each node possesses three degrees of freedom.

For determinate structures, the force method allows us to find internal forces (using equilibrium i.e. based on Statics) irrespective of the material information. Material (stress-strain) relationships are needed only to calculate deflections. However, for indeterminate structures, Statics (equilibrium) alone is not sufficient to conduct structural analysis. Compatibility and material information are essential.

Fixed beam

Kinematicaly determinate

Simply supported beam Kinematicaly indeterminate



Any joint – Moves in three directions in a plane structure Two displacements δx in x direction, δy in y direction, θ rotation about z axis as shown.

Roller Support :
r = 1, $\delta y = 0$, θ & δx exist – DOF = 2 e = 1

Hinged Support :
r = 2, $\delta x = 0$, $\delta y = 0$, θ exists – DOF = 1 e = 2

Fixed Support :
r = 3, $\delta x = 0$, $\delta y = 0$, $\theta = 0$ DOF = 0 e = 3

Reaction components prevent the displacements no. of restraints = no. of reaction components.

Degree of kinematic indeterminacy:

Pin jointed structure: Every joint – two displacements components and no rotation

$\therefore$ **Dk = 2j – e** where, e = no. of equations of compatibility
= no. of reaction components

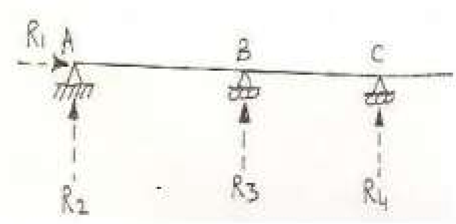
Rigid Jointed Structure:

Every joint will have three displacement components, two displacements and one rotation. Since, axial force is neglected in case of rigid jointed structures, it is assumed that the members are inextensible & the conditions due to inextensibility of members will add to the numbers of restraints. i.e to the ‘e’ value.

$\therefore$ **Dk = 3j – e** where, e = no. of equations of compatibility
= no. of reaction components +
constraints due to in extensibility

Example 1 : Find the static and kinematic indeterminacies

r = 4, m = 2, j = 3



3j

= (3 x 2 + 4) – 3 x 3 = 1

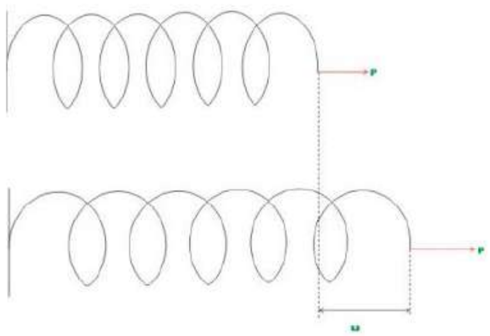
Dk = 3j – e

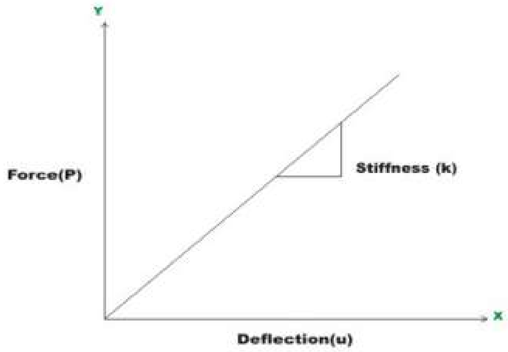
= 3 x 3 – 6 = 3

i.e. rotations at A,B, & C i.e. θa, θb & θc are the displacements.

(e = reaction components + inextensibility conditions = 4 + 2 = 6)

Force-Displacement Relationship





Consider linear elastic spring as shown in Fig. Let us do a simple experiment. Apply a force at the end of spring and measure the deformation. Now increase the load to and measure the deformation. Likewise repeat the experiment for different values of load. Result may be represented in the form of a graph as shown in the above figure where load is shown on -axis and deformation on abscissa. The slope of this graph is known as the stiffness of the spring and is represented by and is given by

$$k = \frac{P_2 - P_1}{u_2 - u_1} = \frac{P}{u}$$

$$P = ku$$

The spring stiffness may be defined as the force required for the unit deformation of the spring. The stiffness has a unit of force per unit elongation. The inverse of the stiffness is known as flexibility. It is usually denoted by and it has a unit of displacement per unit force.

$$a = \frac{1}{k} \quad P = ku$$

S.No.	Type of displacement, Δ	Flexibility, δ	Stiffness, k
1.	Axial	$\frac{L}{AE}$	$\frac{AE}{L}$
2.	Transverse		
	(a) Far-end fixed	$\frac{L^3}{12EI}$	$\frac{12EI}{L^3}$
	(b) Far-end hinged	$\frac{L^3}{3EI}$	$\frac{3EI}{L^3}$
3.	Bending or flexural		
	(a) Far-end fixed	$\frac{L}{4EI}$	$\frac{4EI}{L}$
	(b) Far-end hinged	$\frac{L}{3EI}$	$\frac{3EI}{L}$
4.	Torsional	$\frac{L}{GK}$	$\frac{GK}{L}$

Step	Force method (flexibility or compatibility method)	Displacement method (stiffness or equilibrium method)
1.	Determine the degree of static indeterminacy (degree of redundancy), *n*.	Determine the degree of kinematic indeterminacy, (degree of freedom), *n*.
2.	Choose the redundants.	Identify the independent displacement components.
3.	Assign coordinates 1, 2, ..., *n* to the redundants.	Assign coordinates 1, 2, ..., *n* to the independent displacement components.
4.	Remove all the redundants to obtain the released structure.	Prevent all the independent displacement components to obtain the restrained structure.
5.	Determine $[\Delta_L]$, the displacements at the coordinates due to the applied loads acting on the released structure.	Determine $[P']$, the forces required at the coordinates in the restrained structure due to the loads other than those acting at the coordinates.
6.	Determine $[\Delta_R]$, the displacements at the coordinates due to the redundants acting on the released structure.	Determine $[P_\Delta]$, the forces required at the coordinates in the unrestrained structure to cause the independent displacement components $[\Delta]$.
7.	Compute the net displacements at the coordiantes. $[\Delta] = [\Delta_L] + [\Delta_R]$	Compute the net forces at the coordinates. $[P] = [P'] + [P_\Delta]$
8.	Use the condtions of compatiblility of displacements to compute the reduntands. $[P] = [\delta]^{-1}\{[\Delta]-[\Delta_L]\}$	Use the conditions of equilibrium of forces to compute the displacements. $[\Delta] = [k]^{-1}\{[P] - [P']\}$
9.	Knowing the redundants, compute the internal member forces by using equations of statics.	Knowing the displacements, compute the internal member forces by using slope-deflection equations.

Examples:

Determine the degree of static indeterminacy of the pin-jointed plane frame shown in Fig. 1.8.

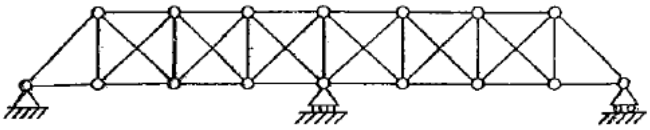


Fig. 1.8

Solution

Total number of independent external reaction components,

$$r = 2 + 1 + 1 = 4$$

Using Eq. (1.7), degree of external indeterminacy,

$$D_{se} = 4 - 3 = 1$$

Number of joints, $j = 16$

Actual number of members, $m = 35$

Using Eq. (1.8), minimum number of members required to preserve geometry of the frame,

$$m' = 2 \times 16 - 3 = 29$$

Using Eq. (1.10), degree of internal indeterminacy,

$$D_{si} = 35 - 29 = 6$$

Hence, degree of static indeterminacy

$$D_s = D_{se} + D_{si} = 1 + 6 = 7$$

Alternatively, the degree of static indeterminacy may be computed using Eq. (1.16). Substituting

$$m = 35 \quad r = 4 \quad j = 16$$

into Eq. (1.16)

$$D_s = 35 + 4 - 2 \times 16 = 7$$

Determine the degree of static indeterminacy of the rigid-jointed plane frame shown in Fig. 1.9.

Solution

Total number of independent external reaction components,

$$r = 2 \times 3 + 2 + 1 = 9$$

Using Eq. (1.7), degree of external indeterminacy,

$$D_{se} = 9 - 3 = 6$$

The number of cuts required to obtain an open configuration, $c = 12$. For instance, cuts may be made in all the beams except in the topmost beams. Using Eq. (1.12), degree of internal indeterminacy

$$D_{si} = 3 \times 12 = 36$$

Hence, degree of static indeterminacy,

$$D_s = D_{se} + D_{si}$$
$$= 6 + 36 = 42$$

Alternatively, the degree of static indeterminacy may be computed using Eq. (1.18). Substituting

$$m = 35$$
$$r = 9$$
$$j = 24$$

into Eq. (1.18),

$$D_s = 3 \times 35 + 9 - 3 \times 24 = 42$$

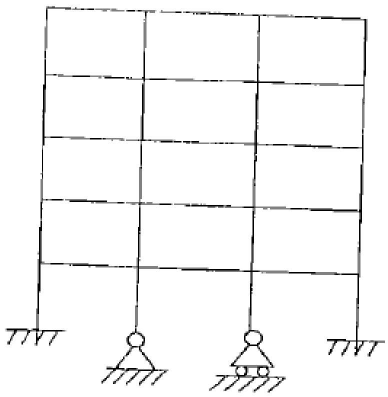


Fig. 1.9

Determine the degree of static indeterminacy of the bow-string girder shown in Fig. 1.10. Assume all joints to be rigid.

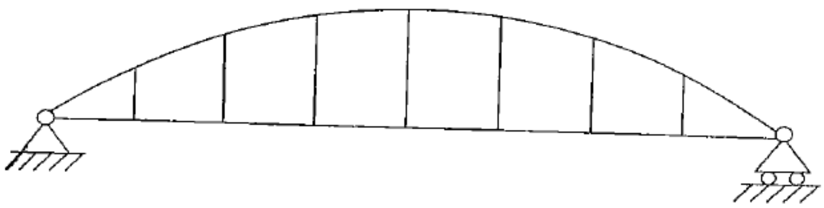


Fig. 1.10

Solution

Total number of independent external reaction components, $r = 3$. Degree of external indeterminacy,

$$D_{se} = 3 - 3 = 0$$

The number of cuts required to obtain an open configuration, $c = 8$. For instance, a cut may be made in the horizontal member in each cell. Using Eq. (1.12), degree of internal indeterminacy,

$$D_{si} = 3 \times 8 = 24$$

Hence, degree of static indeterminacy,

$$D_s = D_{se} + D_{si} = 0 + 24 = 24$$

Alternatively, the degree of static indeterminacy may be computed using Eq. (1.18). Substuting

$$m = 23 \quad r = 3 \quad j = 16$$

into Eq. (1.18),

$$D_s = 3 \times 23 + 3 - 3 \times 16 = 24$$

Determine the degree of static indeterminacy of the rigid-jointed building frame shown in Fig. 1.13(a).

Solution

Total number of independent external reaction components,

$$r = 6 \times 6 = 36$$

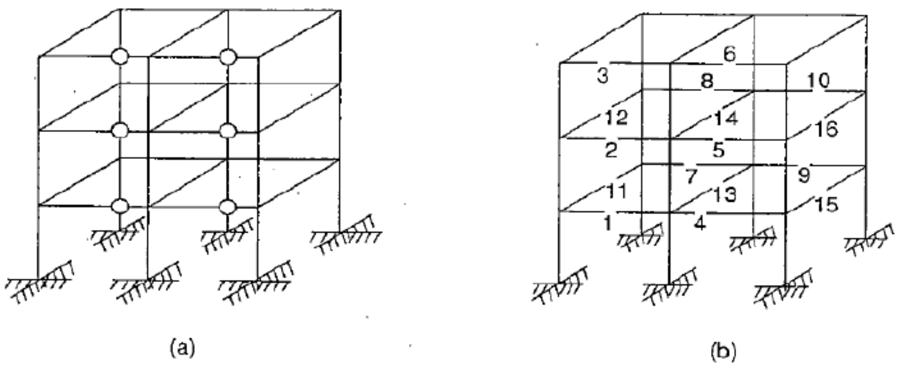


Fig. 1.13

Degree of external indeterminacy,

$$D_{se} = 36 - 6 = 30$$

Number of cuts required to obtain an open configuration, $c = 16$ [Fig. 1.13(b)].
Using Eq. (1.13), degree of internal indeterminacy,

$$D_{si} = 6 \times 16 = 96$$

Hence, degree of static indeterminacy of the frame,

$$D_s = D_{se} + D_{si} = 30 + 96 = 126$$

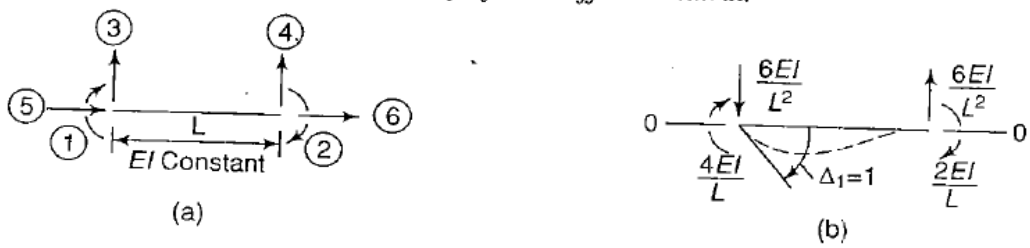
Alternatively, the degree of static indeterminacy may be computed using Eq. (1.19).
Substituting

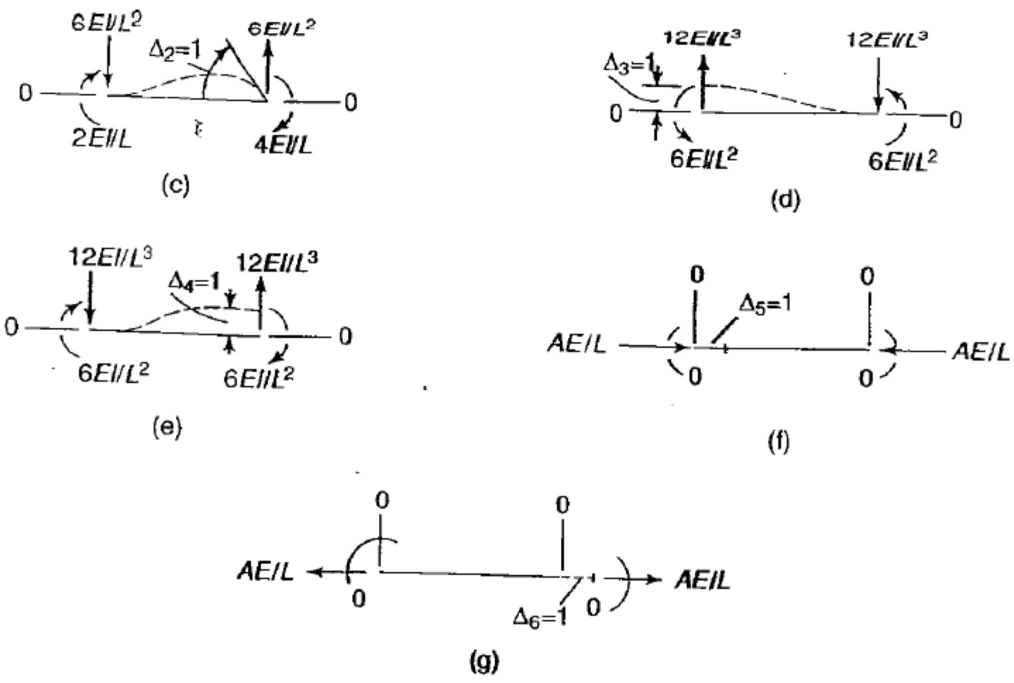
$$m = 39 \quad r = 36 \quad j = 24$$

into Eq. (1.19),

$$D_s = 6 \times 39 + 36 - 6 \times 24 = 126$$

Develop the stiffness matrix for the end-loaded prismatic member AB with reference to the coordinates shown in Fig. 4.4(a). Comment on the relevance of the chosen coordinates. Examine the reciprocity of the stiffness matrix.





The stiffness matrix of the member can be developed by giving a unit displacement successively at each coordinate without any displacement at other coordinates. The forces at coordinates 1 to 6, when a unit displacement is given successively at each of the coordinates 1 to 4, may be computed by using the equations given in Sec. 2.14. For example, when a unit displacement is given at coordinate 1, the forces at coordinates 1 to 6, which constitute the elements of the first column of the stiffnes matrix, are

$$k_{11} = \frac{4EI}{L} \qquad k_{21} = \frac{2EI}{L}$$

$$k_{31} = -\frac{6EI}{L^2} \qquad k_{41} = \frac{6EI}{L^2}$$

$$k_{51} = k_{61} = 0$$

Similarly, the elements of the second, third and fourth columns of the stiffness matrix can be determined.

When a unit displacement is given at coordinate 5 without any displacement at other coordinates, the forces evidently are

$$k_{15} = k_{25} = k_{35} = k_{45} = 0 \qquad k_{55} = \frac{AE}{L} \qquad k_{65} = -\frac{AE}{L}$$

These forces constitute the elements of the fifth column of the stiffness matrix. The sixth column of the stiffness matrix may be generated in a similar manner by giving a unit displacement at coordinate 6.

The deformed shape of the member, when unit displacement is given succesively at coordinates 1 to 6, together with the resulting forces required to sustain the deformed shape of the member, are shown in the free-boy diagrams in Fig. 4.4(b) to (g). Thus the stiffness matrix of member AB with reference to the chosen coordinates may be written as

$$[k] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} & 0 & 0 \\ -\frac{6EI}{L^2} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{12EI}{L^3} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{12EI}{L^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{AE}{L} & -\frac{AE}{L} \\ 0 & 0 & 0 & 0 & -\frac{AE}{L} & \frac{AE}{L} \end{bmatrix} \quad (4.27)$$

where A = area of cross-section of the member
L = length of the member.

Two steel bars AB and BC, each having a cross-sectional area of 20 mm^2, are connected in series as shown in Fig. 4.10. Develop the flexibility and stiffness matrices with reference to coordinates 1 and 2 shown in the figure. Verify that the two matrices are the invervse of eachy other. Take E = 200 kN/mm^2.

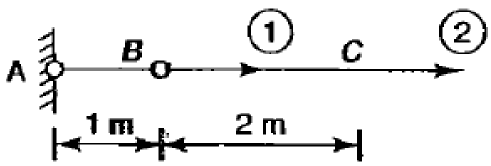


Fig. 4.10

Solution

$$\text{Axial flexibility of bar } AB = \frac{L}{AE} = \frac{1000}{20 \times 200} = 0.25 \text{ mm/kN}$$

$$\text{Axial stiffness of bar } AB = \frac{AE}{L} = 4 \text{ kN/mm}$$

$$\text{Axial flexibility of bar } BC = \frac{L}{AE} = \frac{2000}{20 \times 200} = 0.5 \text{ mm/kN}$$

$$\text{Axial stiffness of bar } BC = \frac{AE}{L} = 2 \text{ kN/mm}$$

The flexibility matrix can be developed by applying a unit force successively at coordinates 1 and 2 and evaluating the displacements at coordinates 1 and 2. To generate the first column of the flexibility matrix, apply a unit force at coordinate 1. The displacements at coordinates 1 and 2 are

$$\delta_{11} = \delta_{21} = 0.25 \text{ mm}$$

Similarly, to generate the second column of the flexibility matrix, apply a unit force at coordinate 2. The displacements at coordinates 1 and 2 are

$$\delta_{12} = 0.25 \text{ mm}$$

$$\delta_{22} = 0.25 + 0.5 = 0.75 \text{ mm}$$

Hence, the required flexibility matrix $[\delta]$ is given by the equation

$$[\delta] = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

The stiffness matrix can be developed by giving a unit displacement successively at coordinates 1 and 2 without any displacement at the other coordinate and determining the forces required at coordinates 1 and 2. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1. The forces required at coordinates 1 and 2 are

$$k_{11} = 4 + 2 = 6 \text{ kN}$$

$$k_{21} = -2 \text{ kN}$$

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 2. The forces required at coordinates 1 and 2 are

$$k_{12} = -2 \text{ kN}$$

$$k_{22} = 2\text{kN}$$

Hence, the required stiffness matrix $[k]$ is given by the equation

$$[k] = \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix}$$

Multiplying the flexibility and stiffness matrices,

$$[\delta][k] = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As the product of the two matrices is a unit matrix, the two matrices are the inverse of each other.

Develop the flexibility and stiffness matrices for prismatic member AB with reference to the coordinates shown in Fig. 4.11 (a) for the following support conditons:

(i) hinged support at A and roller support at B

(ii) fixed supports at A and B

(iii) fixed support at A and roller support at B.

Verify in each case that the flexibility and stiffness matrices are the inverse of each other.

Solution

(i) The support conditions are shown in Fig. 4.11(b). The flexibility matrix can be developed by applying a unit force successively at coordinates 1 and 2 and evaluating displacements at coordiantes 1 and 2. To generate the first column of the flexibility matrix, apply a unit force at coordinate 1. Using Eqs (A. 71) and (A.72) of Appendix A, the displacement at coordinates 1 and 2 are

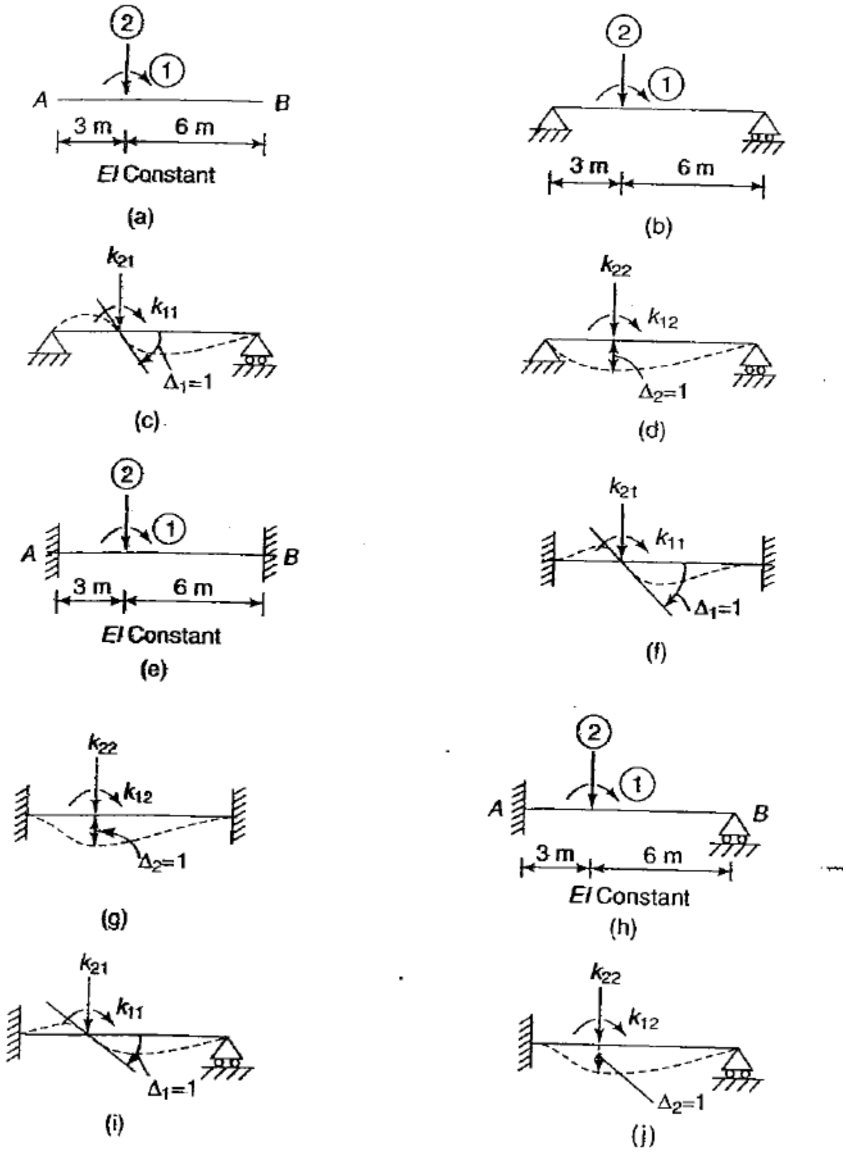


Fig. 4.11

$$\delta_{11} = \frac{1}{3 \times 9EI}[3 \times 3^2 - 3 \times 3 \times 9 + 9^2] = \frac{1}{EI}$$

$$\delta_{21} = \frac{3(9-3)(9-6)}{3 \times 9EI} = \frac{2}{EI}$$

To generate the second column of the flexibility matrix, apply a unit force at coordinate 2. Using Eqs (A.63) and (A.64) of Appendix A, the displacements at coordinates 1 and 2 are

$$\delta_{12} = \frac{3(9-3)(9-6)}{3 \times 9EI} = \frac{2}{EI}$$

$$\delta_{22} = \frac{3^2 \times 6^2}{3 \times 9EI} = \frac{12}{EI}$$

Hence, the required flexibility matrix $[\delta]$ is given by the equation

$$[\delta] = \frac{1}{EI}\begin{bmatrix} 1 & 2 \\ 2 & 12 \end{bmatrix}$$

The stiffness matrix can be developed by giving a unit displacement successively at coordinates 1 and 2 without any displacement at the other coordinate and determining the forces required at coordinates 1 and 2. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1 as shown in Fig. 4.11(c). The forces required at the coordinates are

$$k_{11} = \frac{3EI}{3} + \frac{3EI}{6} = 1.5EI$$

$$k_{21} = -\frac{3EI}{3^2} + \frac{3EI}{6^2} = -0.25EI$$

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 2 as shown in Fig. 4.11(d). The forces required at coordinates 1 and 2 are

$$k_{12} = -\frac{3EI}{3^2} + \frac{3EI}{6^2} = -0.25EI$$

$$k_{22} = \frac{3EI}{3^3} + \frac{3EI}{6^3} = 0.125EI$$

Hence, the required stiffness matrix [k] is given by the equation

$$[k] = EI\begin{bmatrix} 1.500 & -0.250 \\ -0.250 & 0.125 \end{bmatrix}$$

Multiplying the flexibility and stiffness matrices,

$$[\delta][k] = \frac{1}{EI}\begin{bmatrix} 1 & 2 \\ 2 & 12 \end{bmatrix} EI\begin{bmatrix} 1.500 & -0.250 \\ -0.250 & 0.125 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As the product is a unit matrix, the two matrices are the inverse of the each other.

(ii) The support conditions are shown in Fig. 4.11(e). The flexibility matrix can be developed by applying a unit force successively at coordinates 1 and 2 and evaluating the displacement at coordinates 1 and 2. To generate the first column of the flexibility matrix, apply a unit force at coordinate 1. Using Eqs (A.113) and (A.114) of Appendix *A*, the displacements at coordinates 1 and 2 are

$$\delta_{11} = \frac{3(9-3)(9^2 - 3\times 3\times 9 + 3\times 3^2)}{9^3 EI} = \frac{2}{3EI}$$

$$\delta_{21} = \frac{3^2}{2\times 9^3 EI} \times (9-3)^2(9-6) = \frac{2}{3EI}$$

To generate the second column of the flexibility matrix, apply a unit force at coordinate 2. Using Eqs. (A.104) and (A.105) of Appendix A, the displacements at coordinates 1 and 2 are

$$\delta_{12} = \frac{3^2}{2\times 9^3 EI}(9-3)^2(9-6) = \frac{2}{3EI}$$

$$\delta_{22} = \frac{3^3(9-3)^3}{3\times 9^3\ EI} = \frac{8}{3EI}$$

Hence, the required flexibility matrix [δ] is given by the equation

$$[\delta] = \frac{2}{3EI}\begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

The stiffness matrix can be developed by giving a unit displacement successively at coordinates 1 and 2 without any displacement at the other coordinate and determining the forces required at coordinates 1 and 2. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1 as shown in Fig. 4.11 (f). The forces required at coordinates 1 and 2 are

$$k_{11} = \frac{4EI}{3} + \frac{4EI}{6} = 2EI$$

$$k_{21} = -\frac{6EI}{3^2} + \frac{6EI}{6^2} = 0.5EI$$

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 2 as shown in Fig. 4.11(g). The forces required at coordinates 1 and 2 are

$$k_{12} = -\frac{6EI}{3^2} + \frac{6EI}{6^2} = -0.5EI$$

$$k_{22} = \frac{12EI}{3^3} + \frac{12EI}{6^3} = 0.5EI$$

Hence, the required stiffness matrix [k] is given by the equation

$$[k] = EI\begin{bmatrix} 2.0 & -0.5 \\ -0.5 & 0.5 \end{bmatrix}$$

Multiplying the flexibility and stiffness matrices,

$$[\delta][k] = \frac{2}{3EI}\begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix} EI\begin{bmatrix} 2 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As the product is a unit matrix, the two matrices are the inverse of each other.

(iii) The support conditions are shown in Fig. 4.11(h). The flexibility matrix can be developed by applying a unit force successively at coordinates 1 and 2 and evaluating the displacements at coordinates 1 and 2. To generate the first column of the flexibility matrix, apply a unit force at coordinate 1. Using Eqs (A.35) and (A.36) of Appendix A, the displacements at coordinates 1 and 2 are

$$\delta_{11} = \frac{3}{4 \times 9^3 EI}\left[4 \times 9^3 - 12 \times 9^2 \times 3 + 12 \times 9 \times 3^2 - 3 \times 3^3\right]$$

$$= \frac{11}{12EI}$$

$$\delta_{21} = \frac{3^2}{4 \times 9^3 EI}[2 \times 9^3 - 6 \times 9^2 \times 3 + 5 \times 9 \times 3^2 - 3^3]$$

$$= \frac{7}{6EI}$$

To generate the second column of the flexibility matrix, apply a unit force at coordinate 2. Using Eqs (A.30) and (A.31) of Appendix A, the displacements at coordinates 1 and 2 are

$$\delta_{12} = \frac{3^2}{4 \times 9^3 EI}\left[2 \times 9^3 - 6 \times 9^2 \times 3 + 5 \times 9 \times 3^2 - 3^3\right]$$

$$= \frac{7}{6EI}$$

$$\delta_{22} = \frac{3^3}{12 \times 9^3 EI}\left[4 \times 9^3 - 9 \times 9^2 \times 3 + 6 \times 9 \times 3^2 - 3^3\right]$$

$$= \frac{11}{3EI}$$

Hence, the required flexibility matrix $[\delta]$ is given by the equation

$$[\delta] = \frac{1}{12EI}\begin{bmatrix} 11 & 14 \\ 14 & 44 \end{bmatrix}$$

The stiffness matrix can be developed by giving a unit displacement successively at coordinates 1 and 2 without any displacement at the other coordinate and determining the forces required at coordinates 1 and 2. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1 as shown in Fig. 4.11(i). The forces required at coordinates 1 and 2 are

$$k_{11} = \frac{4EI}{3} + \frac{3EI}{6} = \frac{11EI}{6}$$

$$k_{21} = \frac{-6EI}{3^2} + \frac{3EI}{6^2} = \frac{-7EI}{12}$$

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 2 as shown in Fig. 4.11(j). The forces required at coordinates 1 and 2 are

$$k_{12} = -\frac{6EI}{3^2} + \frac{3EI}{6^2} = \frac{-7EI}{12}$$

$$k_{22} = \frac{12EI}{3^3} + \frac{3EI}{6^3} = \frac{11EI}{24}$$

Hence, the required stiffness matrix $[k]$ is given by the equation

$$[k] = \frac{EI}{24}\begin{bmatrix} 44 & -14 \\ -14 & 11 \end{bmatrix}$$

Multiplying the flexibility and stiffness matrices,

$$[\delta][k] = \frac{1}{12EI}\begin{bmatrix} 11 & 14 \\ 14 & 44 \end{bmatrix}\frac{EI}{24}\begin{bmatrix} 44 & -14 \\ -14 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Develop the flexibility and stiffness matrices for beam AB with reference to the coordinates shown in Fig. 4.12(a).

Solution

The flexibility matrix can be developed by applying a unit force successively at the coordinates and evaluating the displacements at all the coordianates. To generate the first column of the flexibility matrix, apply a unit force at coordinate 1. Using Eqs (A.14), (A.15) and (A.16) of Appendix A, the displacements at the coordinates are

$$\delta_{11} = \frac{10}{EI}$$

$$\delta_{21} = \frac{10 \times 10}{2EI} = \frac{50}{EI}$$

$$\delta_{31} = \frac{10}{EI}$$

$$\delta_{41} = \frac{10(2 \times 20 - 10)}{6EI} = \frac{150}{EI}$$

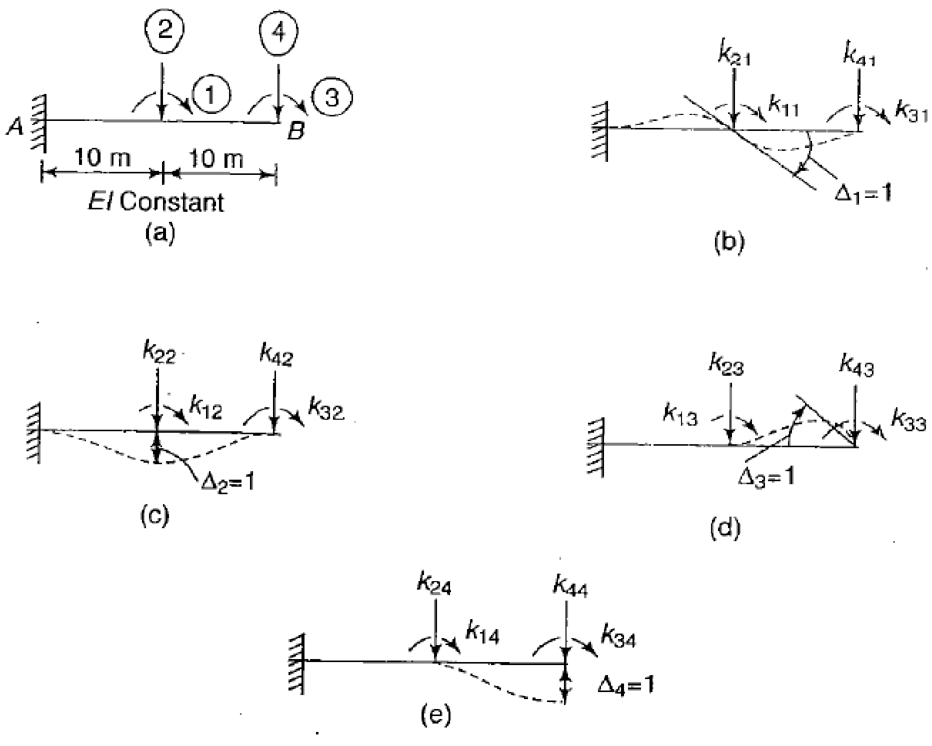


Fig. 4.12

To generate the second column of the flexibility matrix, aply a unit force at coordinate 2. Using Eqs. (A.9), (A.10) and (A.11) of Appendix A, the displacements at the coordinates are

$$\delta_{12} = \frac{10 \times 10}{2EI} = \frac{50}{EI}$$

$$\delta_{22} = \frac{10^3}{3EI} = \frac{1000}{3EI}$$

$$\delta_{32} = \frac{10 \times 10}{2EI} = \frac{50}{EI}$$

$$\delta_{42} = \frac{10^2(3 \times 20 - 10)}{6EI} = \frac{2500}{3EI}$$

To generate the third column of the flexibility matrix, apply a unit force at coordinate 3. Using Eqs (A.5) to (A.8) of Appendix A, the displacements at the coordinates are

$$\delta_{13} = \frac{10}{EI} \qquad \delta_{23} = \frac{10^2}{2EI} = \frac{50}{EI}$$

$$\delta_{33} = \frac{20}{EI} \qquad \delta_{43} = \frac{20^2}{2EI} = \frac{200}{EI}$$

To generate the fourth column of the flexibility matrix, apply a unit force at coordinate 4. Using Eqs (A.1) to (A.4) of Appendix A, the displacements at the coordinates are

$$\delta_{14} = \frac{10(2 \times 20 - 10)}{2EI} = \frac{150}{EI}$$

$$\delta_{24} = \frac{10^2(3 \times 20 - 10)}{6EI} = \frac{2500}{3EI}$$

$$\delta_{34} = \frac{20^2}{2EI} = \frac{200}{EI}$$

$$\delta_{44} = \frac{20^3}{3EI} = \frac{8000}{3EI}$$

Hence, the required flexibility matrix $[\delta]$ is given by equation

$$[\delta] = \frac{1}{3EI}\begin{bmatrix} 30 & 150 & 30 & 450 \\ 150 & 1000 & 150 & 2500 \\ 30 & 150 & 60 & 600 \\ 450 & 2500 & 600 & 8000 \end{bmatrix}$$

The stiffness matrix can be developed by giving a unit displacement successively at each coordinate without any displacement at the other coordinates and determining the forces required at all the coordinates. To generate the first column of the stiffness matrix, give a unit displacement at coordinate 1 as shown in Fig. 4.12(b). The forces required at the coordinates are

$$k_{11} = \frac{4EI}{10} + \frac{4EI}{10} = 0.8EI$$

$$k_{21} = \frac{6EI}{10^2} - \frac{6EI}{10^2} = 0$$

$$k_{31} = \frac{2EI}{10} = 0.2EI$$

$$k_{41} = -\frac{6EI}{10^2} = -0.06EI$$

To generate the second column of the stiffness matrix, give a unit displacement at coordinate 2 as shown in Fig. 4.12(c). The forces required at the coordinates are

$$k_{12} = \frac{6EI}{10^2} - \frac{6EI}{10^2} = 0$$

$$k_{22} = \frac{12EI}{10^3} + \frac{12EI}{10^3} = 0.024EI$$

$$k_{32} = \frac{6EI}{10^2} = 0.06EI$$

$$k_{42} = -\frac{12EI}{10^3} = -0.012EI$$

To generate the third column of the stiffness matrix, give a unit displacement at coordinate 3 as shown in Fig. 4.12(d). The forces required at the coordinates are

$$k_{13} = \frac{2EI}{10} = 0.2EI$$

$$k_{23} = \frac{6EI}{10^2} = 0.06EI$$

$$k_{33} = \frac{4EI}{10} = 0.4EI$$

$$k_{43} = \frac{-6EI}{10^2} = -0.06EI$$

To generate the fourth column of the stiffness matrix, give a unit displacement at coordinate 4 as shown in Fig. 4.12(e). The forces required at the coordinates are

$$k_{14} = \frac{-6EI}{10^2} = -0.06EI$$

$$k_{24} = \frac{-12EI}{10^3} = -0.012EI$$

$$k_{34} = \frac{-6EI}{10^2} = -0.06EI$$

$$k_{44} = \frac{12EI}{10^3} = 0.012EI$$

Hence, the required stiffness matrix [k] is given by the equation

$$[k] = EI\begin{bmatrix} 0.800 & 0 & 0.200 & -0.060 \\ 0 & 0.024 & 0.060 & -0.012 \\ 0.200 & 0.060 & 0.400 & -0.060 \\ -0.060 & -0.012 & -0.060 & 0.012 \end{bmatrix}$$

In this example the computational effort required for developing the flexibility matrix is approximately the same as that for the stiffness matrix.

Analysis of pin-jointed frames by Stiffness Matrix method

Unit displacement in coordinate direction j:

Consider the Figure 11.48.

$$AA' = 1$$

Therefore, the shortening of member $AB = AA' \sin\theta = \sin\theta$

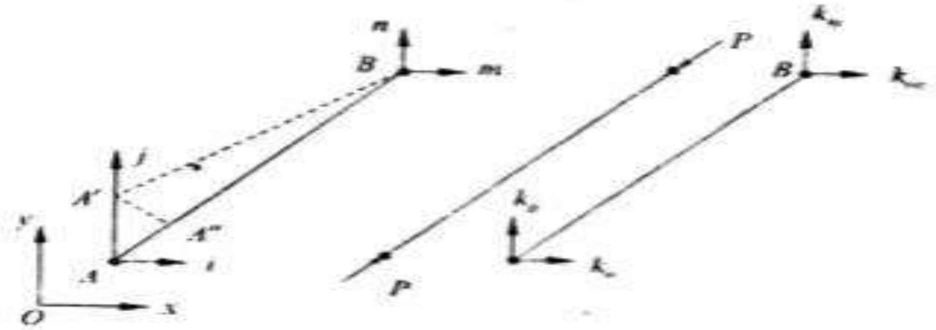


Figure 11.48: Unit displacement in coordinate direction j.

Therefore, the axial compressive force P developed is given by

$$\frac{PL}{AE} = \sin\theta$$

or
$$P = \frac{AE}{L}\sin\theta$$

∴
$$k_{ij} = P\cos\theta = \frac{AE}{L}\times\cos\theta\sin\theta$$

$$k_{ji} = P\sin\theta = \frac{AE}{L}\times\sin^2\theta$$

$$k_{mi} = -P\cos\theta = -\frac{AE}{L}\times\sin\theta\cos\theta$$

$$k_{ni} = -P\sin\theta = -\frac{AE}{L}\times\sin^2\theta$$

Joint stiffness will be

$$k_{ij} = \sum\left[\frac{AE}{L} \times \cos\theta \sin\theta\right]$$

$$k_{jj} = \sum\left[\frac{AE}{L} \times \sin^2\theta\right]$$

$$k_{mj} = -\left[\frac{AE}{L} \times \sin\theta \cos\theta\right]$$

$$k_{nj} = -\left[\frac{AE}{L} \times \sin^2\theta\right]$$

Member Forces

Let the final position of member AB be $A'B'$ as shown in Figure 11.49. Note that, for deriving the expression, $A'B'$ is selected such that all the displacements are positive.

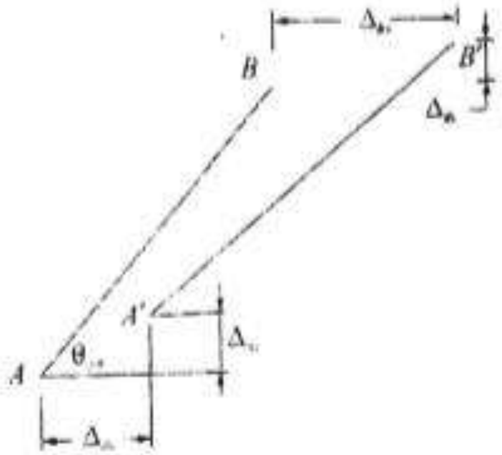


Figure 11.49: Final position of member AB.

Shortening of member due to displacement at A

$$= \Delta_{AX} \cos\theta_{AB} + \Delta_{AY} \sin\theta_{AB}$$

Extension of the member due to displacement at B

$$= \Delta_{BX} \cos\theta_{AB} + \Delta_{BY} \sin\theta_{AB}$$

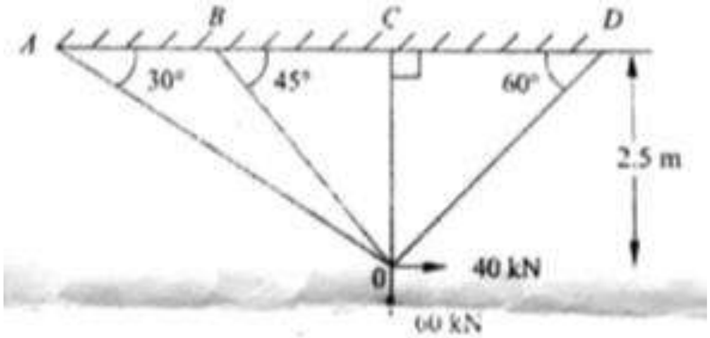
Therefore, the extension of member AB

$$= (\Delta_{BX} - \Delta_{AX}) \cos\theta_{AB} + (\Delta_{BY} - \Delta_{AY}) \sin\theta_{AB}$$

$$\therefore \qquad P_{AB} = \frac{AE}{L}[(\Delta_{BX} - \Delta_{AX})\cos\theta_{AB} + (\Delta_{BY} - \Delta_{AY})\sin\theta_{AB}]$$

Example :

Analyse the pin-jointed truss as shown in figure by stiffness matrix method. Take area od cross-section for all members = 1000 mm^2 and modulus of elasticity E = 200 kN/mm^2



Solution Degree of freedom = 2

The coordinates are selected as shown in Figure 11.50(*b*). Table 11.4 is prepared.

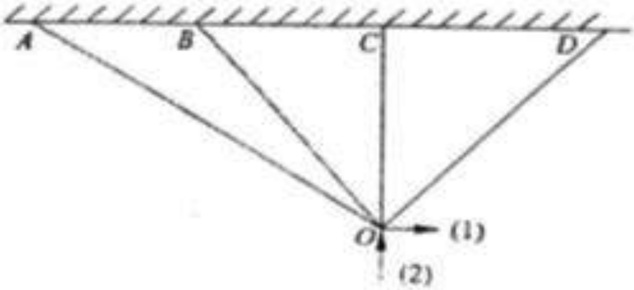


Figure 11.50(b): Coordinates selected.

Table 11.4: Calculations for assembling stiffness

Member	$\frac{AE}{L}$	θ	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\cos\theta\sin\theta$	$\frac{AE}{L}\sin^2\theta$
OA	40	150°	30	−17.321	10
OB	56.569	135°	28.285	−28.285	28.285
OC	80.0	90°	0	0	80.000
OD	69.282	60°	17.321	30	51.962
		Σ	75.606	−15.606	170.247

$$k_{11} = \sum \frac{AE}{L}\cos^2\theta = 75.606$$

$$k_{21} = k_{12} = \sum\left[\frac{AE}{L}\times\cos\theta\sin\theta\right] = -15.606$$

$$k_{22} = \sum\left[\frac{AE}{L}\times\sin^2\theta\right] = 170.247$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 40 \\ -60 \end{bmatrix}$$

Therefore, the stiffness equation is

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \left(\frac{1}{12628.12}\right)\begin{bmatrix} 170.247 & 15.606 \\ 15.606 & 75.606 \end{bmatrix}\begin{bmatrix} 40 \\ -60 \end{bmatrix}$$

$$= \begin{bmatrix} 0.465 \\ -0.310 \end{bmatrix}$$

$$P_{OA} = \frac{AE}{L}\left[(\Delta_{OX} - \Delta_{AX})\cos\theta_{OA} + (\Delta_{OY} - \Delta_{AY})\sin\theta_{OA}\right]$$

$$= 40\ [(0 - 0.465)\cos 150^\circ + (0 + 0.310)\sin 150^\circ]$$

$$= 22.308\ \text{kN}$$

$$P_{OB} = 56.569\ [(0 - 0.456)\cos 135^\circ + (0 + 0.310)\sin 135^\circ]$$

$$= 31.000\ \text{kN}$$

$$P_{OC} = 80\ [(0 - 0.465)\cos 90^\circ + (0 + 0.310)\sin 90^\circ]$$

$$= 24.8\ \text{kN}$$

$$P_{OD} = 69.282\ [(0 - 0.465)\cos 60^\circ + (0 + 0.310)\sin 60^\circ]$$

$$= 2.492\ \text{kN}$$

UNIT-II
APPROXIMATE METHODS OF ANALYSIS

The portal method

The portal method is based on the assumption that, for each storey of the frame, the interior columns will take twice as much shear force as the exterior columns. The rationale for this assumption is illustrated in fig 2.1

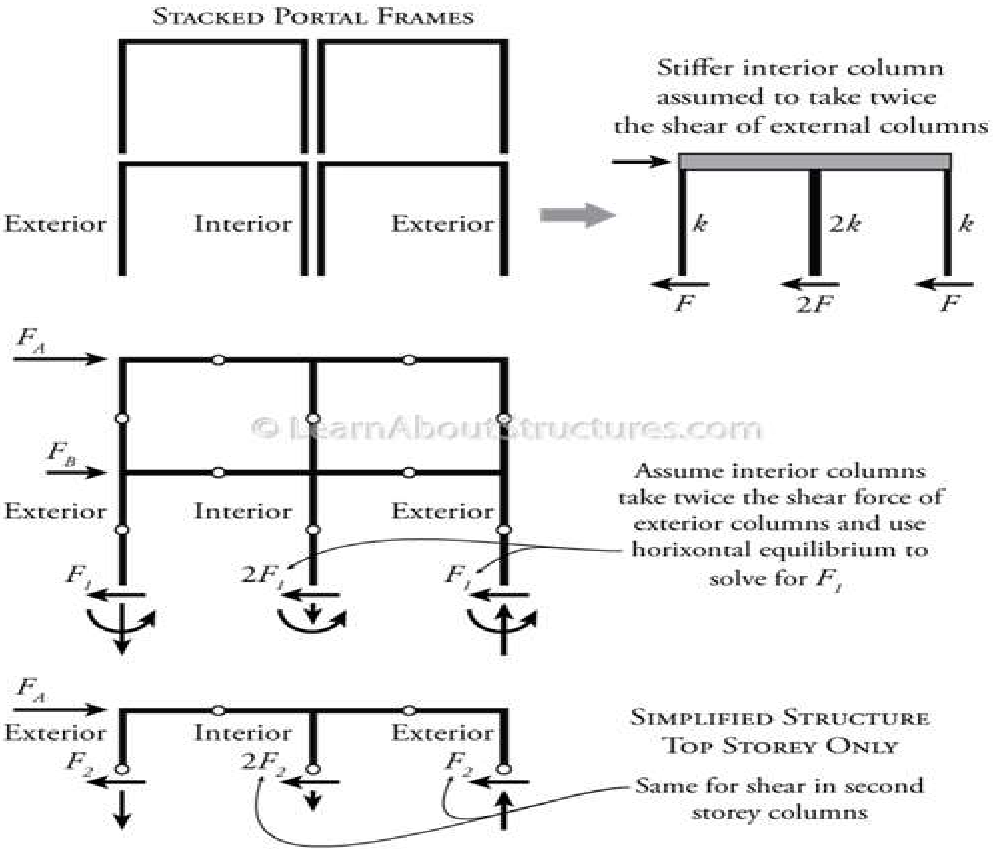


Fig 2.1: Portal Method for the Approximate Analysis of Indeterminate Frames

Let's consider our multi-storey, multi-bay frame as a series of stacked single storey moment frames as shown at the top of Figure 2.1. The columns on either end of each individual portal frame are likely similar size because they would each equally share the gravity load from above. When we join these all together into a stacked system, we can see, as in the figure, that the interior columns have two portal frame columns each since they need to take axial force from the left *and* from the right (whereas the exterior columns only take gravity loads from the left *or* right). So, if we combine all of these individual portal frames together, our interior column (the sum of the two individual portal frame columns) will need to be twice as strong as the exterior columns.

If the interior columns are twice as strong, they may also be approximately twice as stiff (as shown in the diagram at the top right of Figure 2.1). If we then have three columns in parallel as shown and they all share the total lateral load at the top as shown, then they will resist the total load in shear in proportion to their relative stiffness. A column that is twice as stiff will take twice as much load for the same lateral displacement.

So, it may be reasonable to assume that, since the interior columns are approximately twice as big, and therefore twice as stiff, as the exterior columns, those interior columns will take twice as much shear as the exterior columns. This is the basis of the portal method assumption.

This assumption is valid for the columns at every storey as shown in Figure 2.1. So, the portal method provides us with the shear force in each column at each storey in the structure. In our example structure, for any given free body diagram cutting at the hinge location at a single storey, the system will be 2°2° indeterminate. If we know the shear in the middle column in relation to the shear at the left column, that eliminates one unknown (we assume the middle column has twice as much as the left column 2F12F1). If we know the shear in the right column in relation to the shear at the left column, that eliminates another unknown (we assume they are equal). These two assumptions eliminate the remaining 2°2° of static indeterminacy, meaning that we can find the rest of the unknowns using the equilibrium equations only. The portal method assumptions do not give us three known forces because we still have to solve for the force in the left column using horizontal equilibrium before we can use that force to find the forces in the middle and right columns.

Example2.1

An example indeterminate frame that may be solved using the portal method is shown in Figure 2.2. The column areas are given for use with the cantilever method which will be discussed in the next section. For now we will only analyse this structure using the portal method.

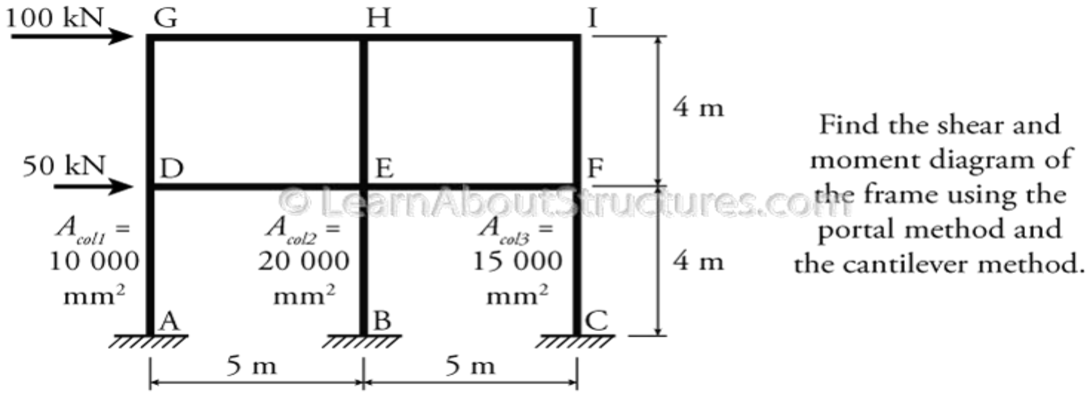


Fig 2.2: Indeterminate Frame Approximate Analysis Example

The first step in the portal method analysis is to add hinges at the centre span or height of all the beams and columns (except for the lower storey if the column bases are pinned), and then determine the column shears at each storey using the portal method assumptions. This process is illustrated in Figure 2.3. The new hinges are shown in the figure at points a through j.

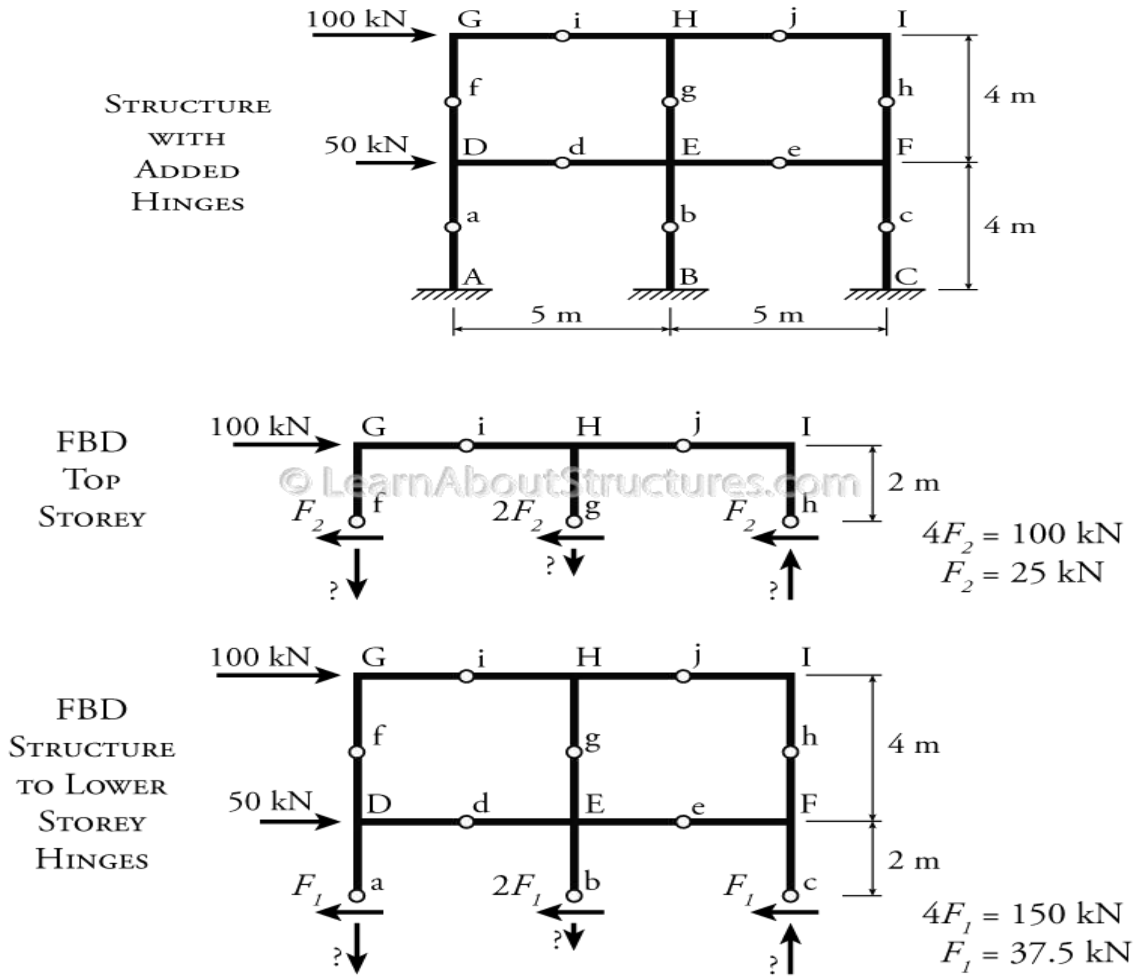


Fig 2.3: Portal Method Example - Determining Column Shears

To determine the column shears for each storey, two different section cuts are made. For the top storey (shown in the middle of Figure 2.3), a section cut is made through the hinges at points f, g, and h (although for the portal method, this cut could be anywhere along the height of the storey when finding the column shear). To find the shear force in the left column (F2F2), the force in the middle column is assumed to be equal to twice the force in the left column (2F22F2 since it is an interior column) and the force in the right column is assumed to be equal to the force in the left column (F2F2). Then, using horizontal equilibrium applied to the whole free body diagram of the top storey:

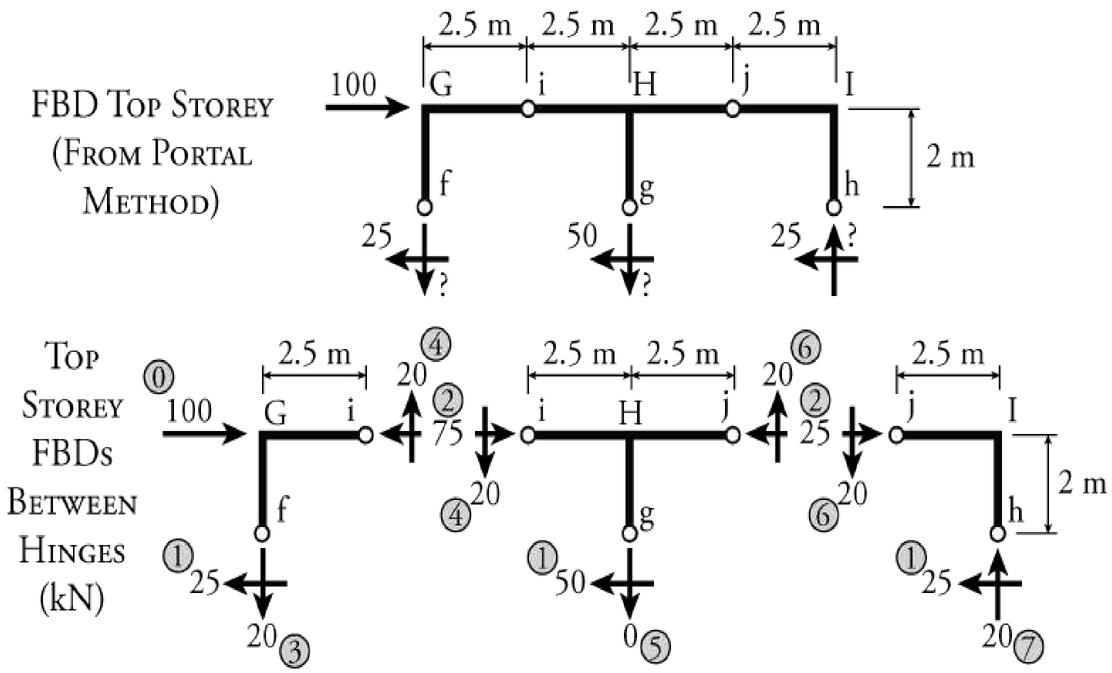
$$\rightarrow \sum F_x = 0$$
$$100 - F_2 - 2F_2 - F_2 = 0$$
$$4F_2 = 100$$
$$F_2 = 25 \text{ kN} \leftarrow$$

Therefore, the shear in the exterior columns in the second storey is 25kN and the shear in the interior column is 50kN. For the lower storey (shown in the bottom of Figure 7.5), a section cut is made through the hinges at points a, b, and c. Similarly:

$$\rightarrow \sum F_x = 0$$
$$100 + 50 - F_1 - 2F_1 - F_1 = 0$$
$$4F_1 = 150$$
$$F_1 = 37.5 \text{ kN} \leftarrow$$

Therefore, the shear in the exterior columns in the first storey is 37.5kN and the shear in the interior column is 75kN.

Now that we know the column shears, the rest of the analysis uses only equilibrium to find the rest of the forces in the frame. To do so, the entire frame is cut into separate pieces at every hinge location. This is useful because each piece of the structure between the hinges can be analyse with the knowledge that the moment at the hinge is always zero. This process is illustrated in Figure 7.6.



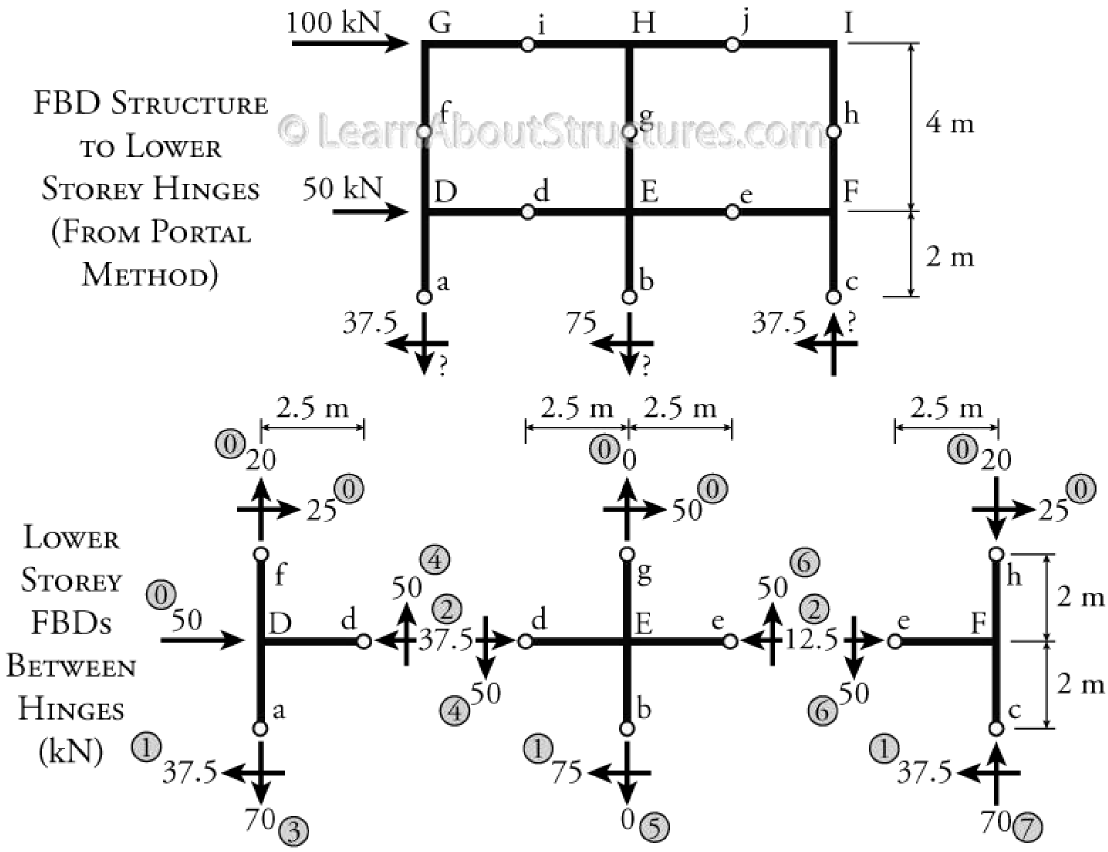


Fig 2.4 Portal Method Example - Analysis for Internal Member Forces at Hinge Locations

To analyze the frame, it is helpful to start at the top of the structure and work our way down. The previous free body diagram of the top storey from Figure 2.3 with the known column shears is shown at the top of Figure 2.4. This free body diagram is further split into three pieces as shown directly below, cutting the storey apart at the hinge locations in the beams (at points i and j). The numbers that are shown in grey circles provide a suggested order for the analysis that will be described here. This is not the only order that is possible, there are many ways to solve this structure. The goal of this analysis is to find all of the unknown vertical and horizontal loads at the hinge locations. The force for step 0 is a given: don't forget to include the external lateral load of 100kN100kN. Step 1 loads are from the portal method analysis, giving the column shears for each column at points f, g, and h (the results of which are shown at the top of the figure). Now that all of the previously known forces are included on the free body diagrams, we can use equilibrium to find the remaining unknowns. In step 2, we can use horizontal equilibrium for the left free body diagram to find the horizontal load at point i to be equal to 75kN←75kN←. Don't forget that on the other side of the cut at point i (the right side) the horizontal force at point i must point in the opposite direction (75kN←75kN←). At the same time in step 2, horizontal equilibrium of the middle free body diagram for the top storey can be used to find the horizontal load at point j (which is also in opposite directions on either side of the cut at j). In step 3, moment equilibrium around point i may be used to find the vertical load at point f. In step 4, vertical equilibrium is used to find the final unknown for the left free body diagram, the vertical load at point i. Don't forget to transfer that load to the other side of the cut at point i. Like the horizontal load, the vertical load on the other side of the cut at point i must point in the opposite direction. Moving onto the middle free body diagram for the top storey, in step 5, moment equilibrium about point j is used to solve for the vertical load at point g (which happens to be 0). Then in step 6, vertical equilibrium is used on the middle free body diagram to find the vertical load at point j, which is also transferred in the opposite direction to the other side of the cut. Last, in step 7, vertical equilibrium on the right free body diagram for the top storey is used to find the final remaining unknown, the vertical load at point h. Again, this step-by-step method is not the only order that can be used to solve for the unknowns. The important thing is to look at how you can use some equilibrium equation to solve for one of the remaining unknowns.

For the lower storey, the frame is again cut into three different pieces with cuts being made at the hinge locations (to avoid having any unknown moments in the free body diagrams), as shown in lower diagram of Figure 2.4. This time, step 0 may include the external lateral load of 50kN50kN in addition to the forces at points f, g, and h that were previously found using the top storey free body diagrams shown above. At points f, g, and h on the lower storey free body diagrams, the loads from the top storey must be applied in the opposite directions to those from the top storey free body diagrams (because they are on either side of a cut in the structure). Then in step 1, the known column shears from the portal method analysis are applied to points a, b, and c (based on the results from the previous analysis which are shown about the lower storey free body diagrams. Once all of the known forces are included, the rest of the unknown forces may be found using equilibrium as was done for the top storey. Again, one suggested solution order is shown in the figure using numbers in grey circles.

Once all of the forces at the hinge locations are known, the shear and moment diagrams may be drawn for the frame. The resulting diagrams are shown in Figure 2.5. The shear in all of the beams and columns are always constant for these types of analyses, and are simply equal to the horizontal force in the middle hinge for the columns or equal to the vertical force in the middle hinge for the beams. The maximum moment in the beams and columns is then found using the shear multiplied by half of the column height for columns or multiplied by half of the beam length for beams. This is because there is no moment at the hinge. So if we start at the hinge and move towards any beam column intersection, then the moment at the intersection will be equal to the shear multiplied by the distance between the hinge and the intersection. For example, for the moment in column AD at point D, we start with a shear in the column of 37.5kN37.5kN at point a as shown in Figure 2.4, and then the distance between point A and point D is 2m. This gives a total moment in column AD at point D of 2(37.5)=75kNm 2(37.5)=75kNm. For the moment in beam HI at point H, we start with a shear in the beam of 20.0kN20.0kN at point j as shown in Figure 2.4, and then the distance between point j and point H is 2.5m2.5m. This gives a total moment in column AD at point D of 2.5(20.0)=50kNm2.5(20.0)=50kNm.

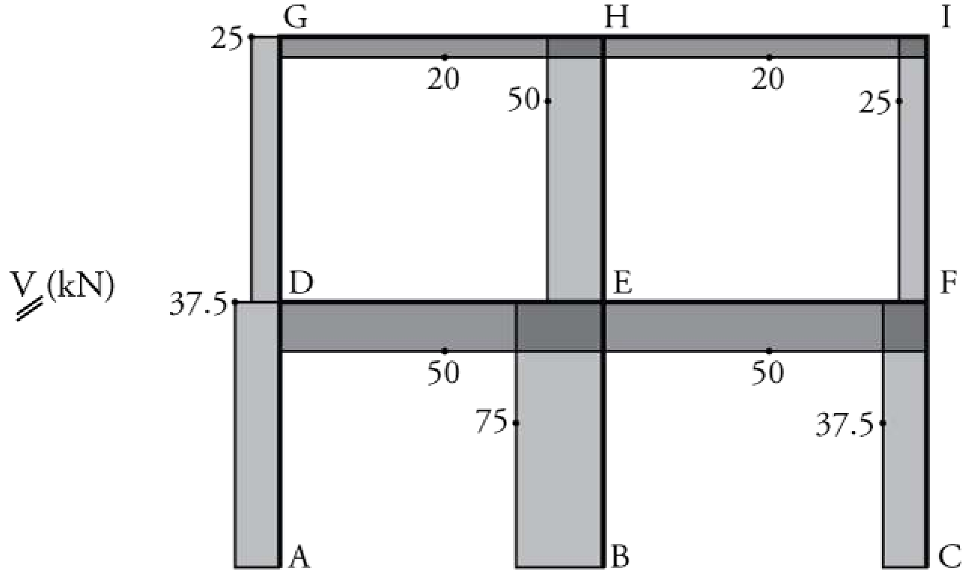


Figure 2.5 Portal Method Example - Resulting Frame Shear Diagram

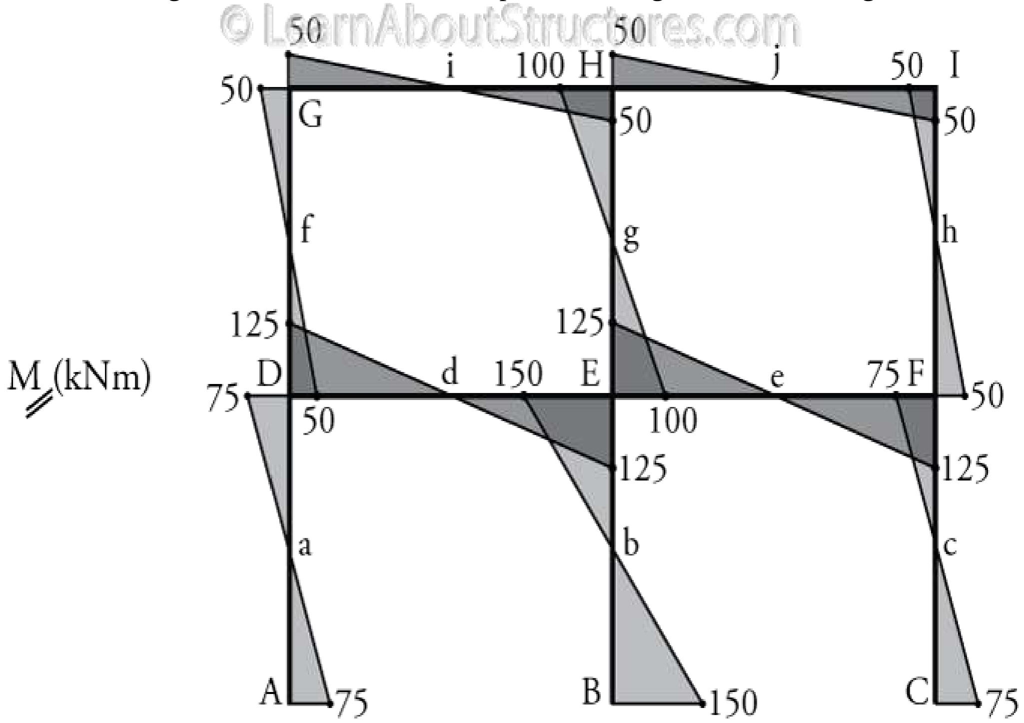


Figure 2.6: Portal Method Example - Resulting Moment Diagram

The cantilever method

The cantilever method is very similar to the portal method. We still put hinges at the middles of the beams and columns. The only difference is that for the cantilever method, instead of finding the shears in the columns first using an assumption, we will find the axial force in the columns using an assumption.

The assumption that is used to find the column axial force is that the entire frame will deform laterally like a single vertical cantilever. This concept is shown in Figure 2.7. When a cantilever deforms laterally, it has a strain profile through its thickness where one face of the cantilever is in tension and the opposite face is in compression, as shown in the top right of the figure. Since we can generally assume that plain sections remain plane, the strain profile is linear as shown. The relative values of the tension and compression strain are dependent on the location of the neutral axis for bending, which is in turn dependent on the shape of the cantilever's cross-section

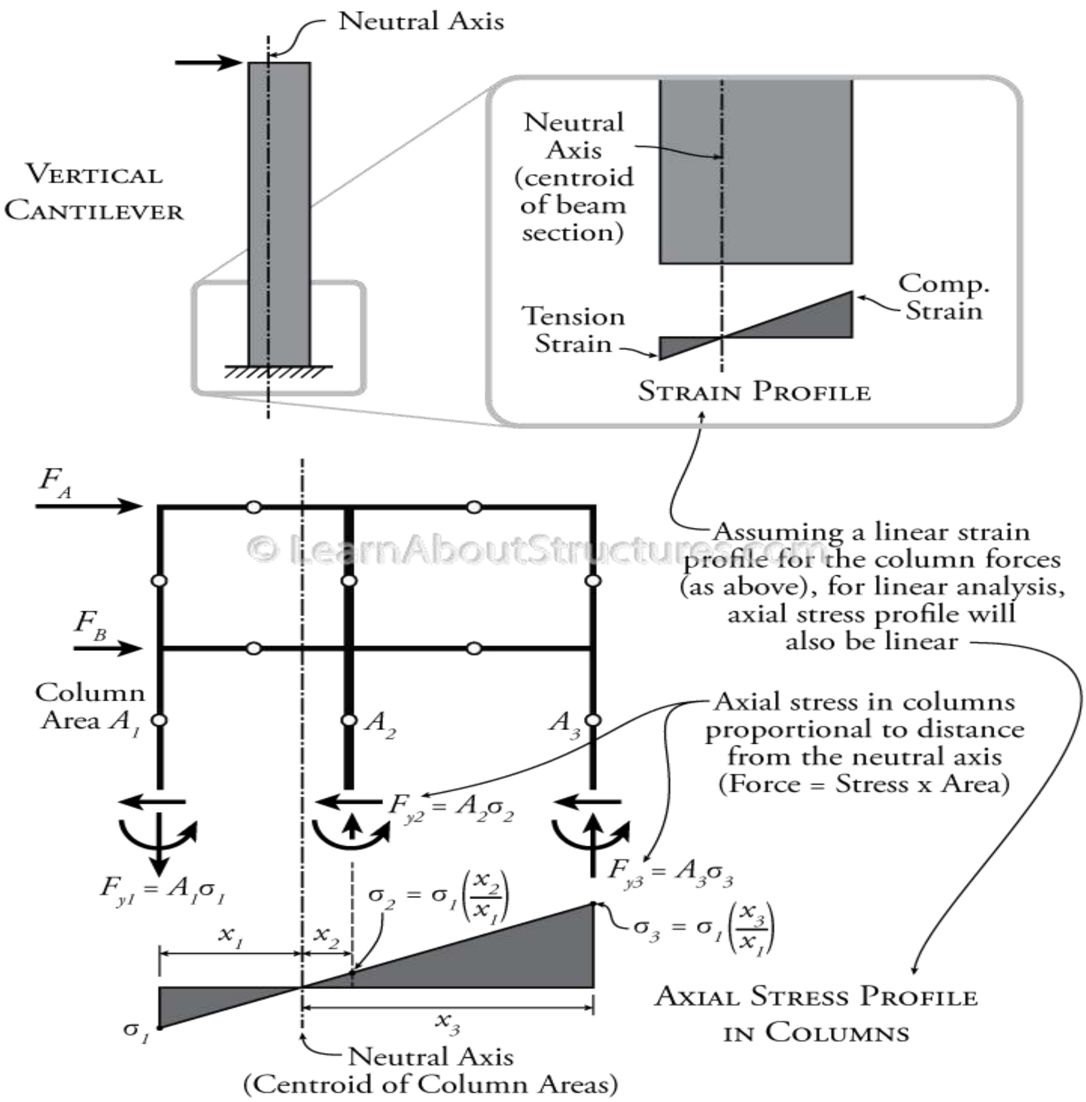


Figure 2.7: Cantilever Method for the Approximate Analysis of Indeterminate Frames

The cantilever method assumed that the whole frame will deform laterally in the same way as the vertical cantilever. The location of the neutral axis of the whole frame is found by considering the cross-sectional areas and locations of the columns at each storey:

$$\bar{x} = \frac{\sum_i (A_i x_i)}{\sum_i A_i}$$

where x¯x¯ is the horizontal distance between the location of the neutral axis and the zero point, AiAi is the area of column ii, and xixi is the horizontal distance between column ii and the zero point. The location zero does not matter, but is commonly set as the location of the leftmost column.

Once we know the location of the neutral axis, using the assumption that the frame behaves as a vertical cantilever, we know that the axial strain in each column will be proportional to that column's distance from the neutral axis, just like the strain in any fibre a distance xx away from the neutral axis of a cantilever is proportional to the distance xx. Since we are assuming that all of our materials are linear (stress is linear to strain), then this also means that the axial stress in each column is proportional to it's distance from the neutral axis of the frame. Also, columns on one side of the neutral axis will be in tension, and columns on the other side of the neutral axis will be in compression. The linear axial stress profile for a sample structure is shown at the bottom of Figure 2.7. If we assume an unknown value for the stress in the left column (σ1σ1in the figure) then the cantilever method can be used to find the stress in the other two columns as a function of their relative distance from the neutral axis as shown in the figure. From these relative stresses, we can determine the force in each column as a function of stress σ1σ1. Then, using a global moment equilibrium, we can solve for σ1σ1, and therefore for the axial force in each column. From this point, the structure is again broken into separate free body diagrams between the hinges as was done for the portal method and all of the remaining unknown forces at the hinges are found using equilibrium.

Since this method relies on the frame behaving like a bending cantilevered beam, it should generally be more accurate for more slender or taller structures, whereas the portal method may be more accurate for shear critical frames, such as squat or short structures.

Example 2.2

The details of the cantilever method process will be illustrated using the same example structure that was used for the portal method (previously shown in Figure 2.3).

The most important part of the cantilever method analysis is to find the axial forces in the columns at each storey. We will start with the top story as shown at the top of Figure 2.8

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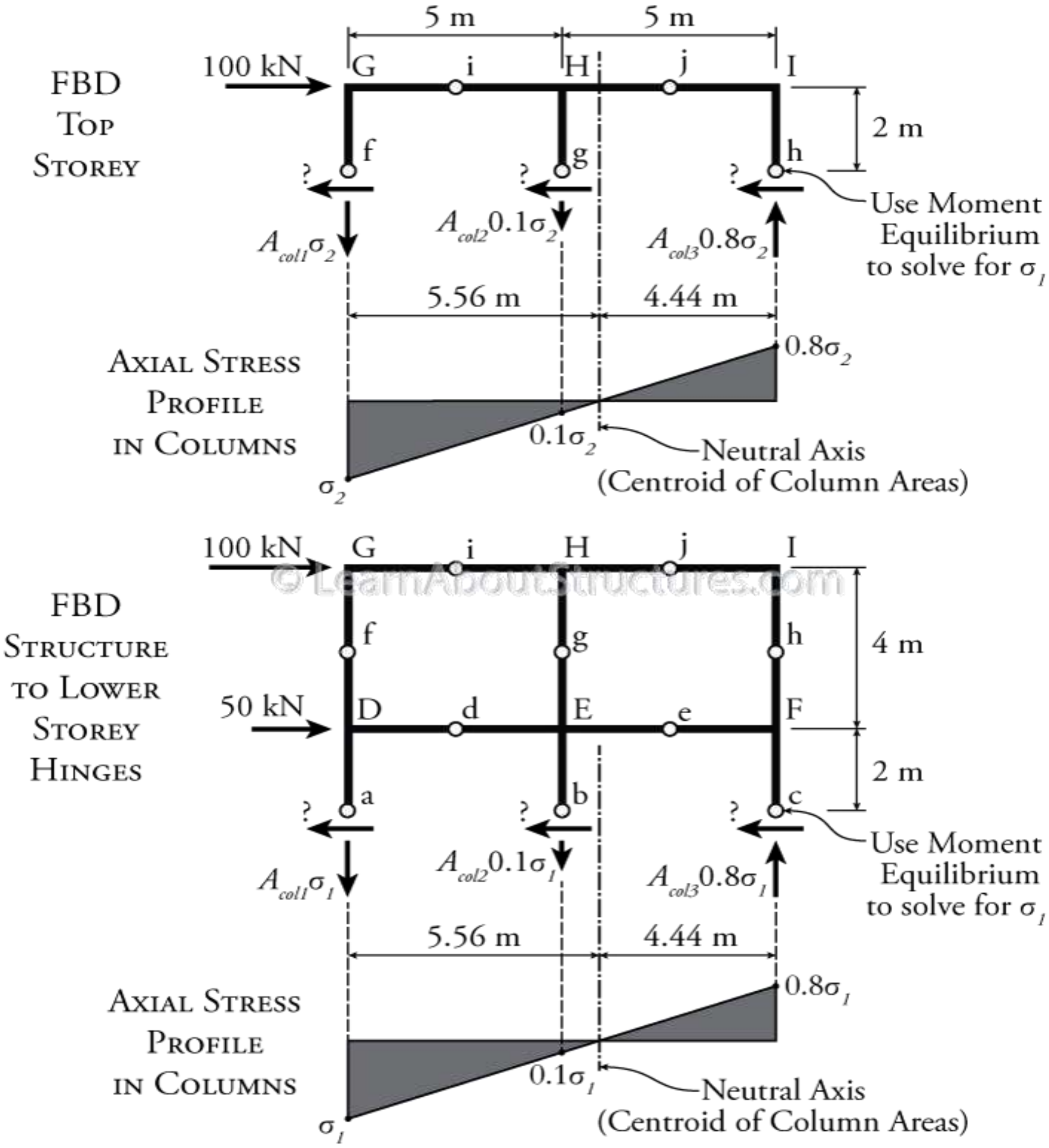

Figure 2.8: Cantilever Method Example - Determining Column Axial Forces

First, we must find the location of the neutral axis for the frame when cut at the top story using equation (1)(1) (the column cross-sectional areas are the same for both storeys and are shown in Figure 7.4):

$$\bar{x} = \frac{\sum_i (A_i x_i)}{\sum_i x_i}$$

$$\bar{x} = \frac{10\,000(0) + 20\,000(5) + 15\,000(10)}{10\,000 + 20\,000 + 15\,000}$$

$$\bar{x} = 5.555 \text{ m}$$

The location of the left column is selected as the zero point.

Knowing the neutral axis location (as shown in the top diagram of Figure 2.8), we can determine the axial stress in all of the columns in the top storey. We will do this in terms of the stress in the left column, which we will call σ2σ2 as shown. The stress in the middle column will be equal to σ2σ2 multiplied by the ratio of the distance from the second column to the neutral axis to the distance from the first column to the neutral axis:

$$\left(\frac{0.56}{5.56}\right)\sigma_2 = 0.1\sigma_2$$

Likewise, the stress in the right column will be:

$$\left(\frac{4.44}{5.56}\right)\sigma_2 = 0.8\sigma_2$$

From these stresses, we can determine the force in the columns by multiplying the stress in each column by it's cross-sectional area as shown in the top diagram of Figure 2.8. Also, the left and middle columns are on the tension side of the neutral axis, so the column axial force arrows will point down as shown (pulling on the column) and the right column is on the compression side of the neutral axis, so the column axial force arrow for that column will point up as shown.

Now, we can use a moment equilibrium on the top story free body diagram in Figure 7.9 to solve for the unknown stress. We will use the moment around point f:

$$\begin{aligned} \curvearrowleft \sum M_f &= 0 \\ -100\ \text{kN}(2\ \text{m}) - A_{col2}(0.1\sigma_2)(5\ \text{m}) + A_{col3}(0.8\sigma_2)(10\ \text{m}) &= 0 \\ -100\ \text{kN}(2\ \text{m}) - (0.02\ \text{m}^2)(0.1\sigma_2)(5\ \text{m}) + (0.015\ \text{m}^2)(0.8\sigma_2)(10\ \text{m}) &= 0 \\ \sigma_2 = 1818.2\ \text{kN}/\text{m}^2 \end{aligned}$$

This resulting stress in the left column may be subbed back into the equations for the force in each column shown in the figure to get forces of 18.2kN↓18.2kN↓ in the left column, 3.6kN↓3.6kN↓ in the middle column, and 21.8kN↑21.8kN↑ in the right column.

For the lower story, the column areas are the same, so the neutral axis will be located in the same place as shown in the lower diagram in Figure 2.8. This means that the relative stresses will also be the same. To solve for the stresses in the left column again for the lower storey (σ1σ1), we need to take a free body diagram of the entire structure above the hinge in the middle of the lower column (as shown in the figure). We should cut the lower storey at the hinge location because that way we do not have any moments at the cut (since the hinge is, by definition, a location with zero moment). If we chose to cut the structure at the base of the columns instead, we would have additional point moment reaction at the base of each column which would have to be considered in the moment equilibrium (which are unknown). Such moment reactions at the base of the columns are shown in Figure 2.7. These extra moments would make it impossible to solve the equilibrium equation for σ1σ1. So, taking the cut at the lower hinges as shown in the lower diagram in Figure 2.8, we can solve for σ1σ1using a global moment equilibrium about point a:

$$\curvearrowleft \sum M_a = 0$$

$$-100(6) - 50(2) - (0.02)(0.1\sigma_1)(5) + (0.015)(0.8\sigma_1)(10) = 0$$

$$\sigma_1 = 6363.6 \text{ kN}/\text{m}^2$$

This resulting stress in the left column may be subbed back into the equations for the force in each column shown in the figure to get forces of 63.6kN↓63.6kN↓ in the left column, 12.7kN↓12.7kN↓ in the middle column, and 76.4kN↑76.4kN↑ in the right column.

From this point forward, the solution method is the same as it was for the portal method. Split each storey free body diagram into separate free body diagrams with cuts at the hinge locations, and then work methodically through using equilibrium to find all of the unknown forces at the hinge cuts. This process is illustrated in Figure 2.9

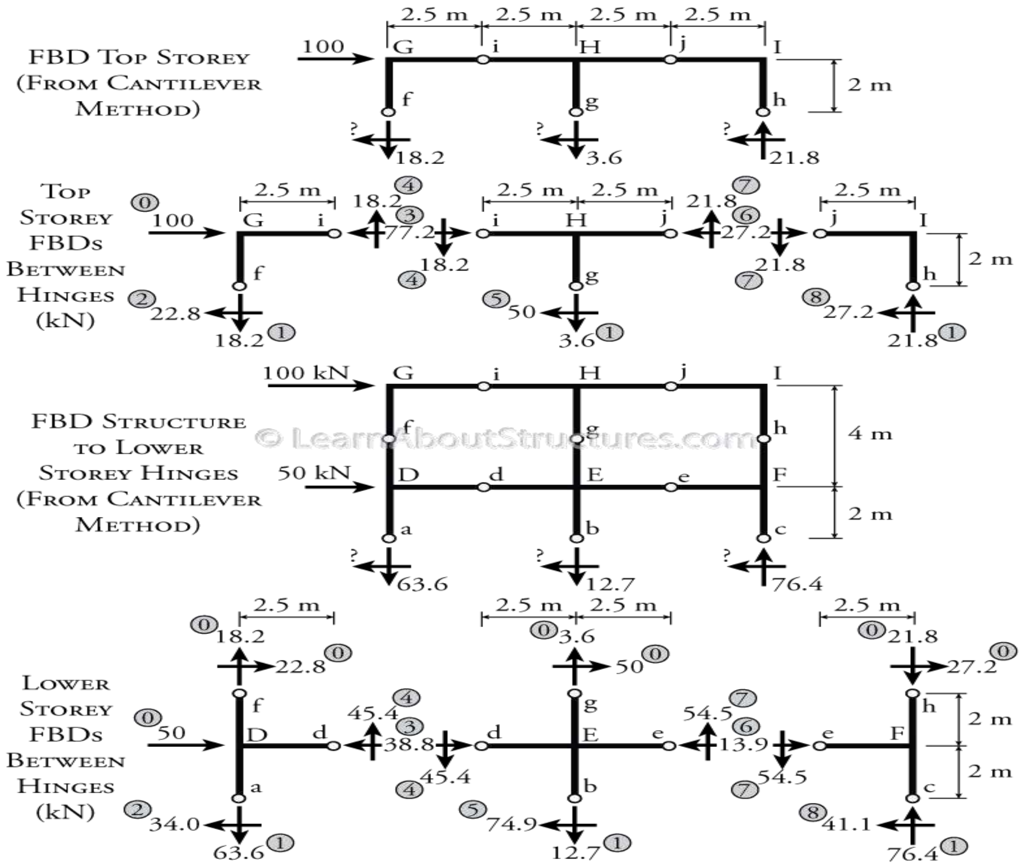

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Figure 2.9: Cantilever Method Example - Analysis for Internal Member Forces at Hinge Locations

Like the portal frame example, the free body diagrams in Figure 2.9 are annotated with numbers in grey circles to show a suggested order for solving all of the unknown forces. Of course, as before, step 0 and step 1 consist of known values, either caused by external forces or the previous storey (for step 0) or the column axial forces that were solved using the cantilever method assumptions (for step 1). The rest of the unknowns are solved for using vertical, horizontal or moment equilibrium.

Once all of the unknown forces at the hinges are found, the shear and moment diagrams for the frame may be drawn using the same methods that were used for the previously described portal method analysis example. The final shear and moment diagrams for this analysis are shown in Figure 2.10. This figure shows both the values from this cantilever method analysis compared with the previous portal method analysis example results (in square brackets). This shows that with a significantly different set of assumptions for this example frame, we get similar shear and moment diagrams using the two different methods.

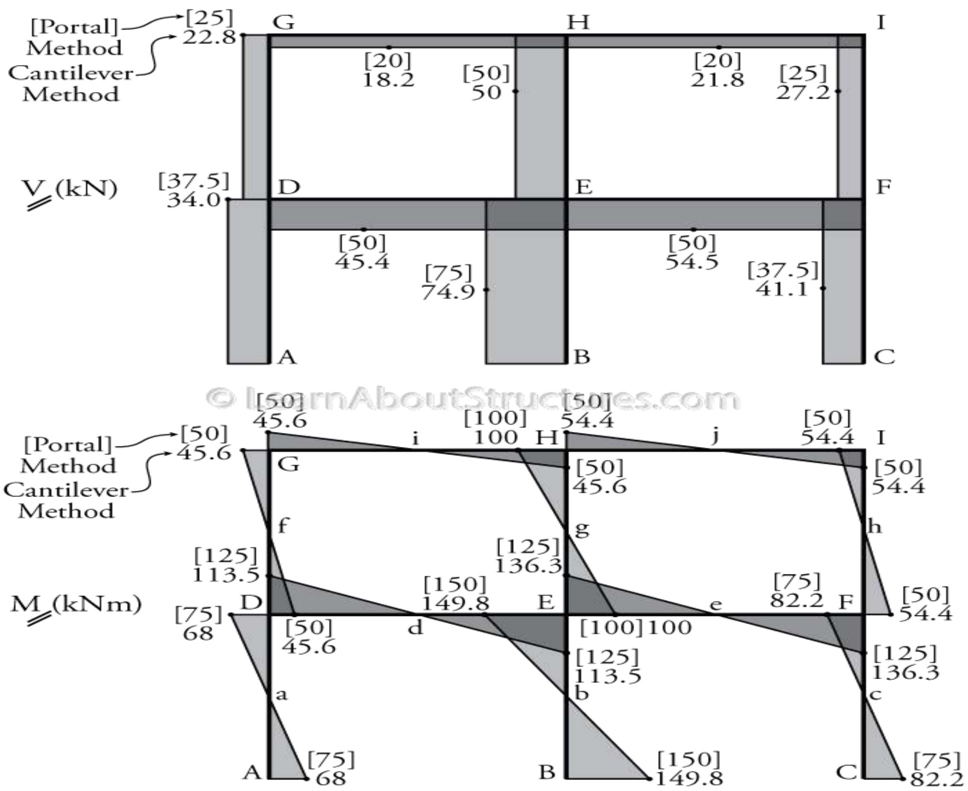
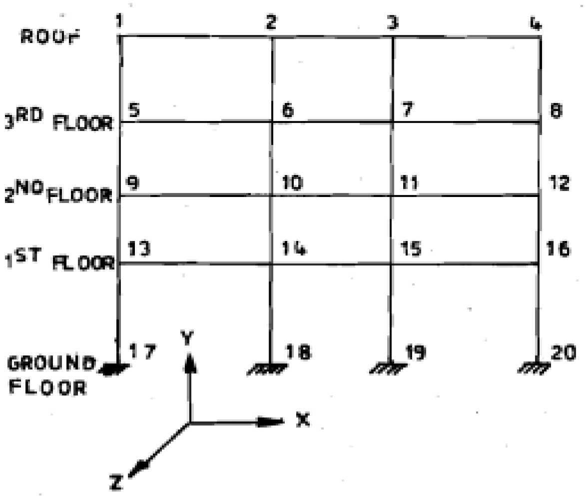


Figure 2.10: Cantilever Method Example - Resulting Frame Shear and Moment Diagrams

Substitute frame method

The building frame is a three dimensional space structure having breadth, height and length i.e. x, y and z coordinates. The manual analysis of space structure is tedious and time consuming. Therefore, approximation is made and the space frame is divided into several plane frames in x and z directions. Then the analysis of these plane frames is carried out.

Even an analysis of in multi-storey plane frame is laborious and time-consuming. Therefore, further simplified assumptions are made and analysis of roof or floor beam is made by considering this beam along with columns of upper and lower storey. Columns are considered as fixed at far ends. Such a simplified beam-column arrangement is called a substitute frame.



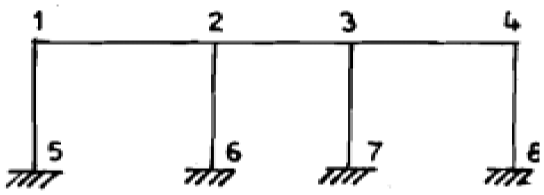


Figure 17.2 (a) : Substitute Frame at Roof Level

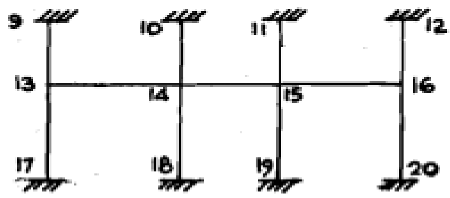
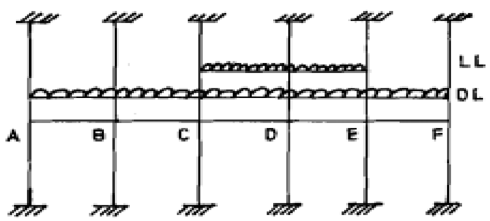


Figure 2.11: Typical Plane Frame Figure 2.12 (b): Substitute Frame at First Floor Level

Normally, a building frame is subjected to vertical as well as horizontal loads. The vertical loads consist of dead load and live load. The dead load comprises of self-weight of beams, slabs, columns, wall, finishes, water proofing course etc. The horizontal loads consist of wind forces and earthquake forces. In order to evaluate ultimate load or factored load, the dead load and live load are multiplied by a factor which is known as partial safety factor qf load or simply a load factor. This factor is 1.50. In order to evaluate minimum possible dead load on the span which is self-weight, sometimes the dead load is multiplied by a factor 0.90 for stability criteria. Therefore, Wn, in = D.L. or 0.9 D.L, and W = 1.5 (D.L+ L;.L) The effect of a loaded span on the farther spans is much smaller. Then moment, shear and reaction in any element is mainly due to loads on the spans very close to it. Therefore it is, recommended to put live load on alternate spans and adjacent spans in order to cause severe effect at a desired location or section.



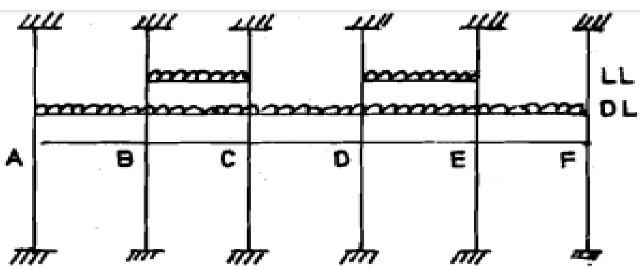


Figure 2.13(c): Maximum Sagging Moment in a Column at the Centre of CD

Figure 2.14(d): Maximum Column Force at D, i.e. Maximum Shear in Beam CD and DE

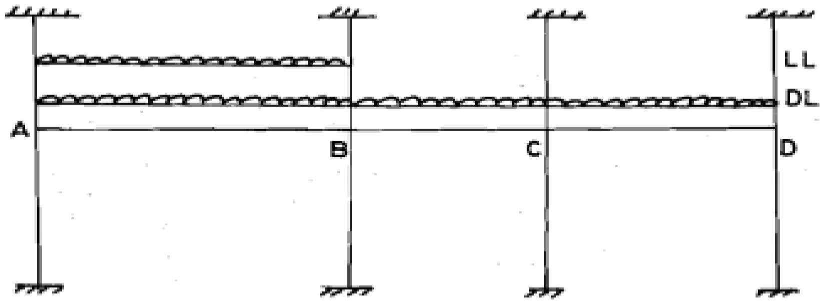


Figure 2.15 (e): Arrangement of Loads for Maximum Bending Moment in a column at B

Table 2.1 shows the arrangement of live load (LL) on spans in addition to dead load (DL) 011 all spans depending upon critical condition.

Table 2.1

S. No.	Critical Condition	Live load (LL) on spans	Reference
1.	Maximum hogging moment at D	DE and CD	Figure 17.3 (a)
2.	Maximum sagging moment at centre of B	BC and DE	Figure 17.3 (b)
3.	Minimum sagging moment at centre of CD	AB, CD, and EF	Figure 17.3 (c)
4.	Maximum axial force in a column at D, i.e. maximum shear in beam CD and DE	CD and DE	Figure 17.3 (d)
5.	Maximum moment in column at B	Longer span on one side of column	Figure 17.3 (e)

The restraining effect of any member forming a joint depends also upon the restraining condition existing at the other end. The other end may have following three conditions:

(a) Freely supported or hinged.

(b) Partially restrained. or

(c) Rigidly fixed.

In most of the framed structures the far end is considered as rigidly fixed because of monolithic construction of a joint. In a substitute frame, unbalanced moment at a joint IS distributed in columns and beams depending upon their ratio of stiffnesses.

Steps for the Analysis

(a) Select a substitute frame, by taking-floor beam with columns of lower and upper storey fixed at far ends.

(b) Cross sectional dimensions of beams and columns may be chosen such that moment of inertia of beam is 1.5 to 2 times that of a column and find distribution factors at a joint considering stiffnesses of beams and columns.

(c) Calculate the dead load and live load on beam. Live load should be placed in such a way that it causes worst effect at the section considered i.e alternate and adjacent loading should be adopted.

(d) Find the initial fixed end moments and analyse this frame by moment distribution method.

(e) Finally draw shear and moment diagram indicating values at critical section.

Limitations

(a) Height of all columns should be same in a particular storey.

(b) Sway of substitute frame is ignored even during unsymmetrical loading.

Example 2.3 Analyse the substitute frame shown in Figure 2.16 for

(a) Maximum sagging moment at centre of span BC,

(b) Maximum hogging moment at D,

(c) Minimum possible moment at centre of BC and

(d) Maximum axial force in column at D.

Assume frames are spaced at 3.5 m each. Other data is as follows:

Thickness of floor slab = 120 mm

Live load = 2 wm^2

Floor finish = 1 kN/m^2

Size of beam (overall) = 230 x 450 mm

Size of column = 230 x 375 mm

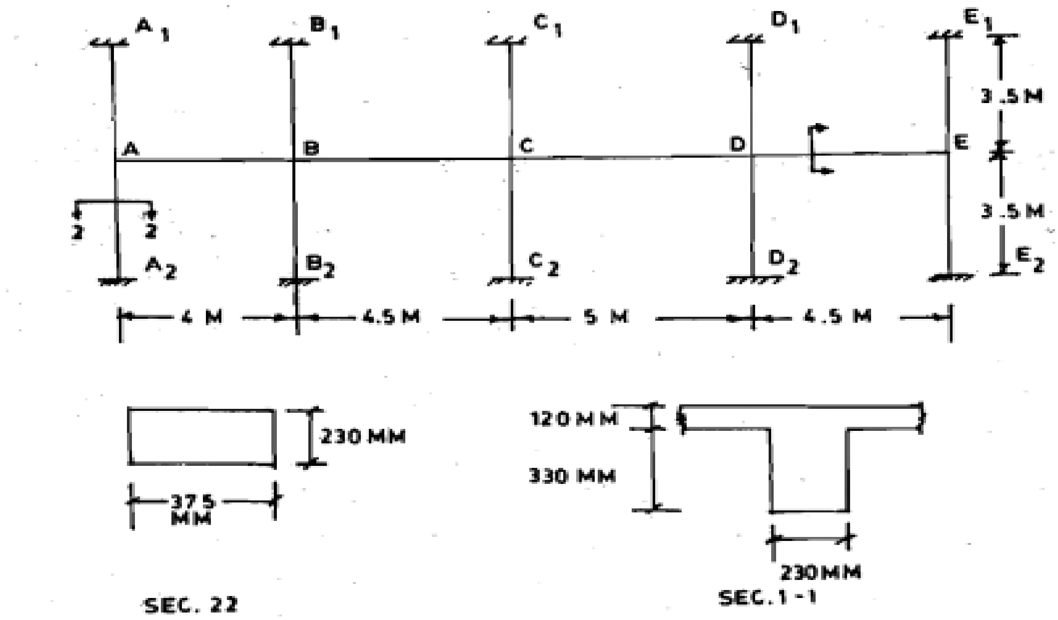


Fig 2.16

Calculation of distribution factors

I_{col} = 230 x 375^3 / 2 = 1.01 x 10^9 mm^4

I_{beam} = 230 x 450^3 / 12 = 1.75 x 10^9 mm^4

Joint	Member	Relative stiffness, $K = \frac{I}{l}$	Total Relative Stiffness ($\sum k$)	Distribution factor $= k / \sum k$
A	AA_1 AA_2 AB	288571.43 288571.43 437500.00	1014642.86	0.284 0.284 0.432
B	BA BB_1 BB_2 BC	437500.00 288571.43 288571.43 388888.89	1403531.75	0.312 0.206 0.206 0.276
C	CB CC_1 CC_2 CD	388888.89 288571.43 288571.43 350000.00	1316031.75	0.296 0.219 0.219 0.266
D	DC DD_1 DD_2 DE	350000.00 288571.43 288571.43 388888.89	1316031.75	0.266 0.219 0.219 0.296
E	ED EE_1 EE_2	388888.89 288571.43 288571.43	966031.75	0.402 0.299 0.299

Factored Loads $w_{max} = 1.5 (w_d + w_l) = 1.5 (15.8975 + 10.5)$

(a) Maximum sagging moment at centre of BC

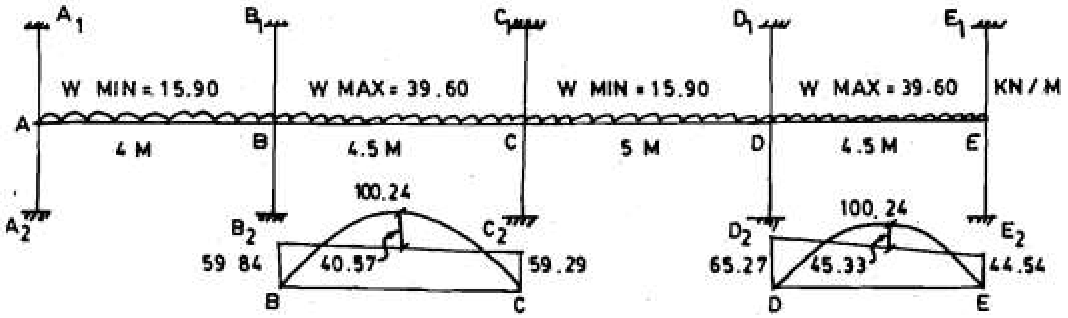


Fig 2.17

Fixed end moments are as follows:

$M_{AB} = -(15.9 \times 4^2) / 12 = -21.20$ kNm $= -M_{BA}$

$M_{BC} = -66.825$ kNm $= -M_{BA}$

$M_{CB} = -33.125$ kNm $= -M_{DC}$

$M_{DE} = -66.825$ kNm $= -M_{ED}$

Joint	Member	D.F.	F.E.M.	Fitst Dist.	C.O.	Second Dist.	C.O.	Third Dist.	Final
A	AA_1	0.284		6.02		– 2.02		– 0.017	3.98
	AA_2	0.284		6.02		– 2.02		– 0.017	3.98
	AB	0.432	– 21.20	9.16	7.12	– 3.07	0.06	– 0.036	– 7.96
B	BA	0.312	+ 21.20	14.24	4.58	0.12	– 1.53	0.97	39.58
	BB_1	0.206		9.4		0.08		0.64	10.12
	BB_2	0.206		9.4		0.08		0.64	10.12
	BC	0.276	– 66.83	12.59	– 4.98	0.11	– 1.59	0.86	– 59.84
C	CB	0.296	+ 66.83	– 9.97	6.29	– 3.19	0.05	– 0.72	59.29
	CC_1	0.219		– 7.38		– 2.36		– 0.53	– 10.27
	CC_2	0.219		–7.38		– 2.36		– 0.53	– 10.27
	CD	0.266	– 33.13	– 8.96	4.48	– 2.86	2.38	– 0.65	– 38.74
D	DC	0.266	+ 33.13	8.96	– 4.48	4.76	– 1.43	0.65	41.59
	DD_1	0.219		7.38		3.92		0.53	11.83
	DD_2	0.219		7.38		3.92		0.53	11.83
	DE	0.296	– 66.83	9.97	–13.43	5.30	– 1.0	0.72	– 65.27
E	ED	0.402	+ 55.83	– 26.86	4.98	– 2.0	2.65	– 1.06	44.54
	EE_1	0.299		– 19.98		– 1.49		– 0.8	– 22.27
	EE_2	0.299		– 19.98		– 1.49		– 0.8	– 22.27

(b)Maximum hogging moment at D in beam and (d) maximum axial force in column at D

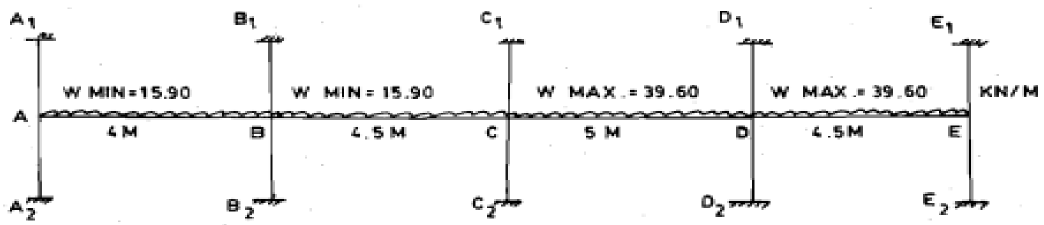


Fig 2.18

Fixed end moments are as follows :

$M_{AB} = -21.2$ kN m $= -M_{BA}$ $\qquad$ $M_{BC} = -26.83$ kN m $= -M_{CB}$

$M_{CD} = -82.50$ kN m $= -M_{DC}$ $\qquad$ $M_{DE} = -66.83$ kN m $= -M_{ED}$

Joint	Member	D.F.	F.E.M.	Fitst Dist.	C.O.	Second Dist.	C.O.	Third Dist.	Final
A	AA_1	0.284		6.02		– 0.25		0.57	6.34
	AA_2	0.284		6.02		–0.25		0.57	634
	AB	0.432	– 21.20	9.16	0.88	– 0.38	–2.00	0.86	– 12.86
B	BA	0.312	+ 21.20	1.76	4.58	– 4.00	– 0.19	0.00	23.35
	BB_1	0.206		1.16		– 2.64		0.00	– 1.48
	BB_2	0.206		1.16		–2.64		0.00	–1.48
	BC	0.276	– 26.83	1.55	8.24	– 3.54	0.19	0.00	– 20.38
C	CB	0.296	+ 26.83	16.48	0.77	0.39	– 1.77	0.09	42.79
	CC_1	0.219		12.19		0.29		0.07	12.55
	CC_2	0.219		1219		0.29		0.07	1255
	CD	0.266	– 82.50	14.81	– 2.08	0.35	1.46	0.08	– 67.88
D	DC	0.296	+ 82.50	– 4.17	7.40	2..93	0.175	– 0.17	83.66
	DD_1	0.219		– 3.43		2.41		– 0.14	– 1.16
	DD_2	0.219		–3.43		2.41		–0.14	–1.16
	DE	0.296	– 66.83	– 4.64	–13.43	3.26	0.46	– 0.19	– 81.37
E	ED	0.402	+ 66.83	– 26.86	– 2.32	0.93	1.63	– 0.65	39.56
	EE_1	0.299		– 19.98		0.69		– 0.49	– 19.78
	EE_2	0.299		–19.98		0.69		–0.49	– 19.78

* The distribution factors for upper column and lower column is same, therefore several steps in moment distribution are common to both at a joint.

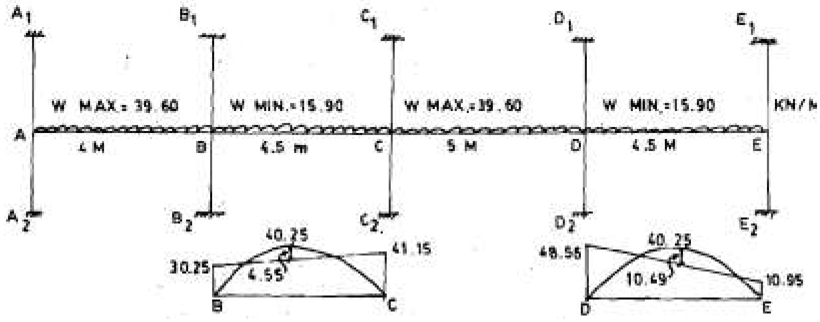
	C	D		E
	CD	DC	DE	ED
Reaction due to udl	99.00	99.00	89.10	89.10
Reaction due to moment	– 3.16	+ 3.16	+ 9.29	– 9.29
		102.16	98.39	
		R_D = 200.55 kN		

The maximum moment is at support D

(M_{DC} = 83.66 kN m and M_{DE} = – 81.37 kN m) and

maximum axial load in column is also at support, D

$R_D = 200.55$ kN.



Fixed end moments are as follows :

$M_{AB} = -52.80$ kN m $= -M_{BA}$ $\quad$ $M_{BC} = -26.83$ kN m $= -M_{CB}$

$M_{CD} = -83.50$ kN m $= -M_{DC}$ $\quad$ $M_{DE} = -26.83$ kN m $= -M_{ED}$

Joint	Member	D.F.	F.E.M.	First Dist.	C.O.	Second Dist.	C.O.	Third Dist.	Final
A	AA_1	0.284		15.00		1.15		0.87	17.02
	AA_2	0.284							
	AB	0.432	−52.80	22.81	−4.05	1.75	−3.06	1.32	−34.03
B	BA	0.312	+52.80	−8.10	11.40	−6.13	0.87	−0.78	50.06
	BB_1	0.206		−5.35		−4.04		−0.51	−9.90
	BB_2	0.206		−5.35		−4.04		−0.50	−0.90
	BC	0.276	−26.83	−7.17	8.24	−5.42	1.62	−0.69	−30.25
C	CB	0.296	+26.83	16.48	−3.58	3.25	−2.71	0.88	41.15
	CC_1	0.219		12.19		2.40		0.65	15.24
	CC_2	0.219		12.19		2.40		0.65	15.24
	CD	0.266	−82.50	14.81	−7.40	2.92	−0.26	0.79	−71.64
D	DC	0.296	+82.50	−14.81	7.40	−0.53	1.46	−0.83	75.19
	DD_1	0.219		−12.19		−0.44		−0.68	−13.31
	DD_2	0.219		−12.19		−0.44		−0.92	−13.31
	DE	0.296	−26.83	−16.48	−5.39	−0.59	1.65	−0.92	−48.56
E	ED	0.402	+26.83	−10.78	−8.24	3.31	−0.29	0.12	10.95
	EE_1	0.299		−8.02		2.46		0.09	−5.47
	EE_2	0.299		−8.02		2.46		0.09	−5.47

UNIT-III
DESIGN OF RETAINING WALLS AND TANKS

Retaining walls

Retaining walls are usually built to hold back soil mass. However, retaining walls can also be constructed for aesthetic landscaping purposes.

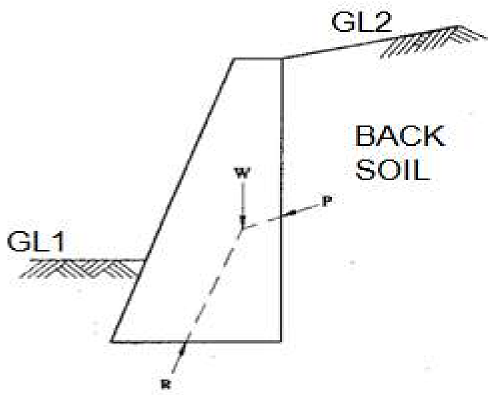


Fig3.1 Gravity retaining wall

Classification of retaining walls

1. Gravity wall-masonry or plain concrete
2. Cantilever retaining wall- RCC (inverted T and L)
3. Counterfort retaining wall- RCC
4. Butress wall-RCC

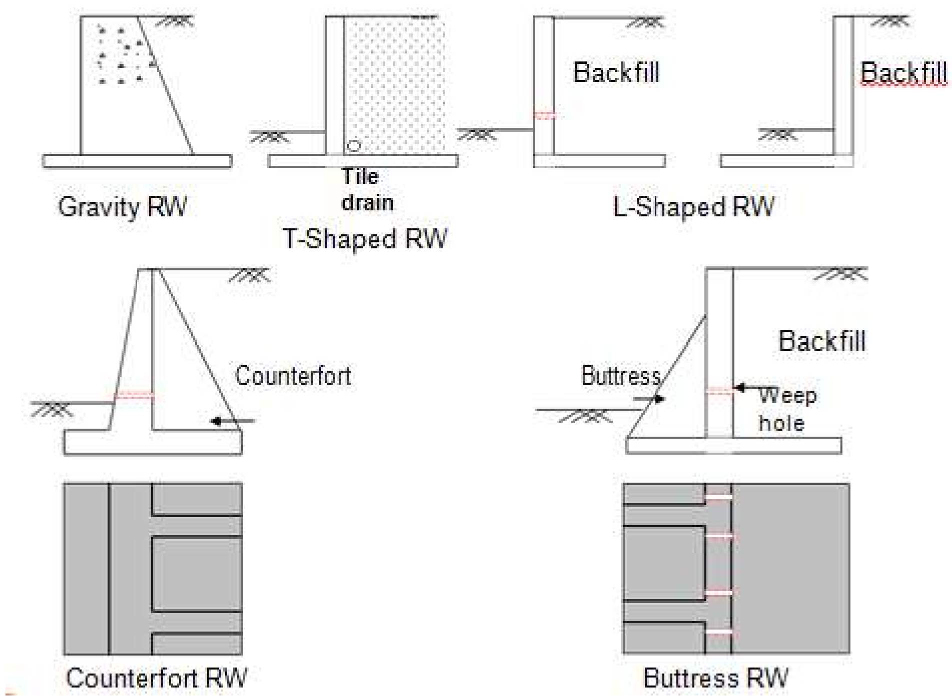


Fig3.2 Types of retaining walls

Importance of retaining walls

Retaining walls are usually meant to serve a single purpose, retaining soil that may erode. However, retaining walls have become more mainstream for other reasons. Today, they are used to block off areas such as outdoor living spaces and for landscaping.

Segmental retaining walls

These consist of modular concrete blocks that interlock with each other. They are used to hold back a sloping face of soil to provide a solid, vertical front. Without adequate retention, slopes can cave, slump or slide. With the unique construction of segmental retaining walls, higher and steeper walls can be constructed with the ability to retain the force of lateral earth pressure created by the backfill soil.

Segmental retaining walls can be installed in a wide variety of colours, sizes, and textures. They can incorporate straight or curved lines, steps, and corners. They are ideal for not only slope support, but also for widening areas that would otherwise be unusable due to the natural slope of the land. Retaining walls are often used for grade changes, and for other functional reasons such as widening driveways, walkways, or creating more space in a patio outdoor area.

Segmental retaining walls consist of a facing system and a lateral tieback system. The facing systems usually consist of modular concrete blocks that interlock with each other and with the lateral restraining members. The lateral tiebacks are usually geo-grids that are buried in the stable area of the backfill. In addition to supporting the wall, the geo-grids also stabilize the soil behind the wall. These two factors allow higher and steeper walls to be constructed.

Advantages of Concrete Segmental Retaining Walls

- Rapid construction
- Horizontal and vertical curvatures
- Easy grade changes
- A wide variety of colours, sizes and textures
- No need for a concrete footing

Some segmental systems use steel or fiberglass pins, clips or integral lips to create a continuous facing system. Some blocks are hollow, some are solid. Just about all block systems permit backfill drainage through the face joints.

Earth Pressure (P) Earth pressure is the pressure exerted by the retaining material on the retaining wall.Thispressure tends to deflect the wall outward.

Types of earth pressures

1. Active earth pressure or earth pressure (Pa) and
2. Passive earth pressure (Pp).

Active earth pressure tends to deflect the wall away from the backfill.

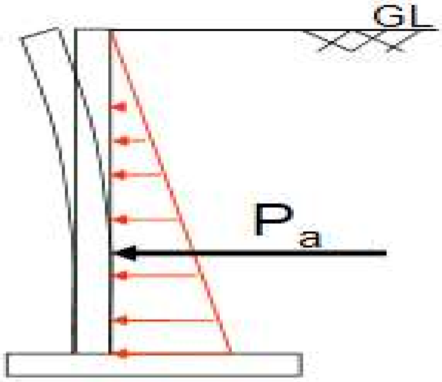


Fig 3.2 variation of earth pressure

Factors effecting earth pressure

1. Earth pressure depends on type of backfill, the height of wall and the soil conditions
2. Soil conditions are
   a. Dry levelled back fill
   b. Moist levelled backfill
   c. Submerged levelled backfill
   d. Levelled backfill with uniform surcharge
   e. Backfill with sloping surface

Earth pressure theories

1. Rankine's theory
2. Column's theory

<u>Rankine's theory:</u>

Rankine assumed that the soil element is subjected to only two types of stresses:

i. Vertical stress (σz) due to the weight of the soil above the element.

ii. Lateral earth pressure (pa)

Rankine's theory assumes that there is no wall friction $(\delta = 0)$, the ground and failure surfaces are straight planes, and that the resultant force acts parallel to the backfillslope.

In case of retaining structures, the earth retained may be filled up earth or natural soil. These backfill materials may exert certain lateral pressure on the wall. If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of elastic equilibrium. Consider the prismatic element E in the backfill at depth z, as shown in Fig.3.3.

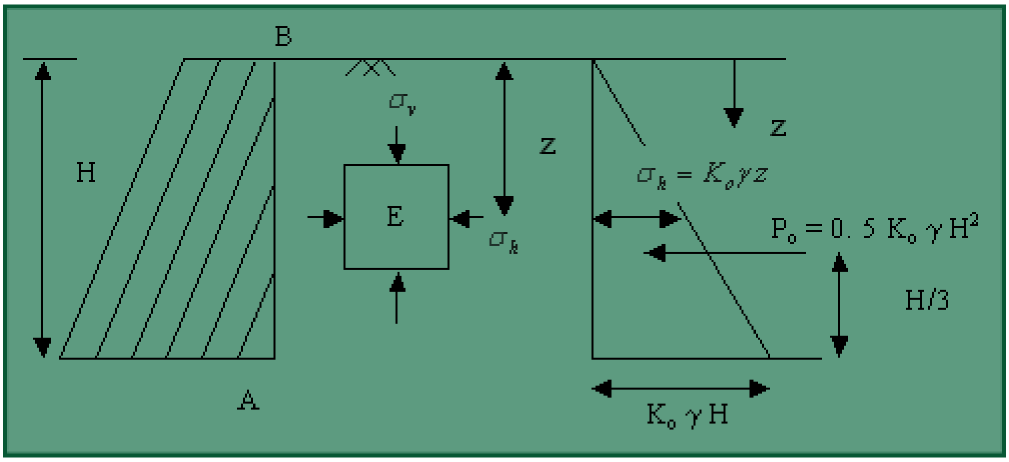


Fig. 3.3 Lateral earth pressure at restcondition.

The element E is subjected to the following pressures:Vertical pressure =

$\sigma_v = \gamma z$

Lateralpressure= σ_h , where g is the effective unit weight of thesoil.

If we consider the backfill is homogenous then both σ_v and σ_h increases rapidly with depth z.

In that case the ratio of vertical and lateral pressures remain constant with respect to depth, that is $\sigma_h / \sigma_v = \sigma_h / \gamma z$=constant = K_o, where K_o is the coefficient of earth pressure for at rest condition.

At rest earth pressure

The at rest earth pressure coefficient (K_o) is applicable for determining the active pressure in clays for strutted systems. Because of the cohesive property of clay there will be no lateral pressure exerted in the at- rest condition up to some height at the time the excavation is made. However, with time, creep and swelling of the clay will occur and a lateral pressure will develop. This coefficient takes the characteristics of clay into account and will always give a positive lateralpressure.

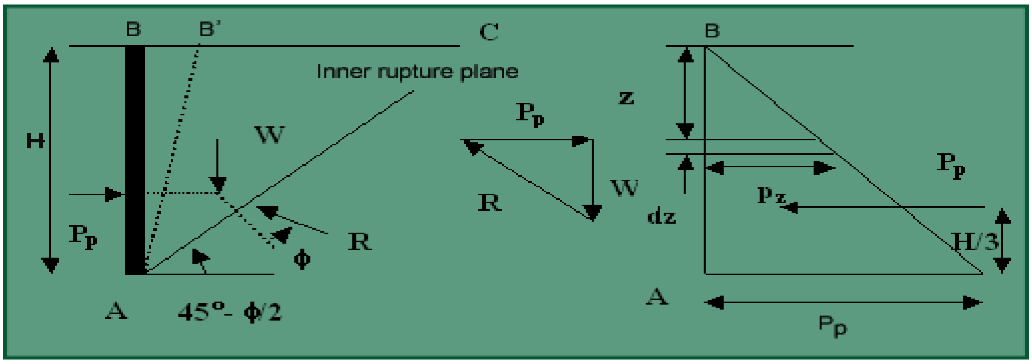
The lateral earth pressure acting on the wall of height H may be expressed as $\sigma_h = K_o \gamma H$.

The total pressure of the soil at rest condition is given by $P_o = 0.5 K_o \gamma H^2$.

The value of K_o depends on the relative density of sand and the process by which the deposit was formed. If this process does not involve artificial tamping the value of K_o ranges from 0.4 for loose sand to 0.6 for dense sand. Tamping of the layers may increase it upto0.8.

From $K_A = \tan^2(45^0 + \phi/2)$ elastic theory, $K_o = \mu/(1-\mu)$.

Passive earth pressure:



If the wall AB is pushed into the mass to such an extent as to impart uniform compression throughout the mass, the soil wedge ABC in fig. will be in Rankine's Passive State of plastic equilibrium. The inner rupture plane AC makes an angle $(45^0 + \phi/2)$ with the vertical AB.

The pressure distribution on the wall is linear as shown.

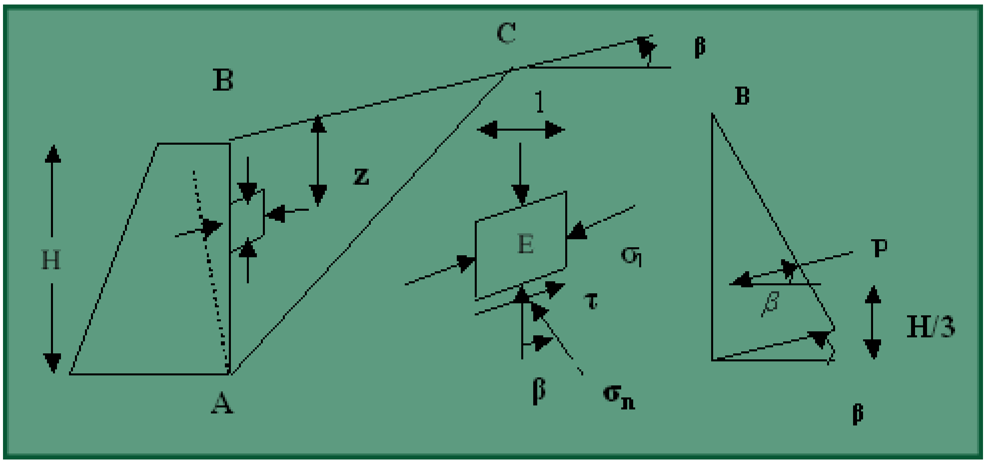
The lateral passive earth pressure at A is $P_p = K_p \gamma H$. Which acts at a height H/3 above the base of thewall.

The total pressure on AB is therefore

$P_p = \int_0^H p_z dz = \int_0^H K_p \gamma z dz = 0.5 K_p \gamma H^2$,

Rankine's active earth pressure with a sloping cohesionless backfill surface:

Fig shows a smooth vertical gravity wall with a sloping backfill with cohesionless soil. As in the case of horizontal backfill, active case of plastic equilibrium can be developed in the



backfill by rotating the wall about A away from the backfill. Let AC be the plane of rupture and the soil in the wedge ABC is in the state of plasticequilibrium.The pressure distribution on the wall is shown in fig. The active earth pressure at depth H is which acts parallel to the surface. The total pressure per unit length of the wall is which acts at a height of H/3 from the base of the wall and parallel to the sloping surface of the backfill. In case of active pressure,

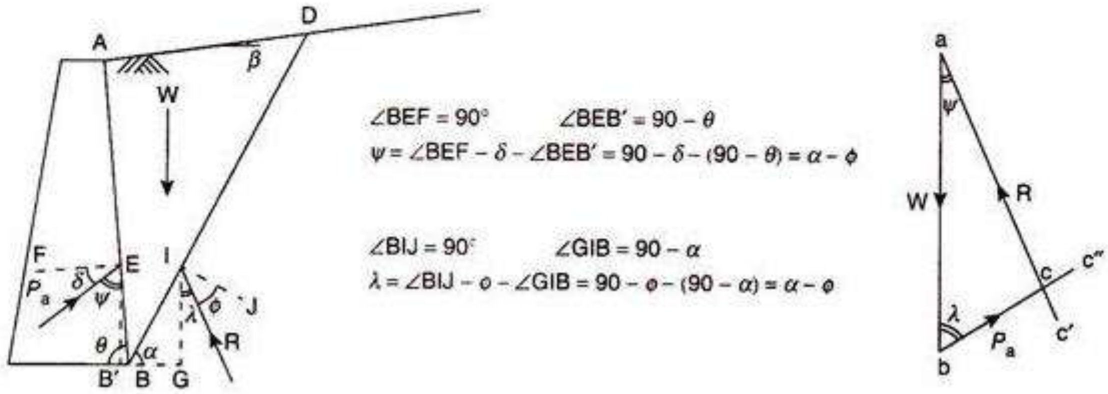
$$K_A = \cos\beta\left(\cos\beta - \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right) / \left(\cos\beta + \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right)$$

In case of passive pressure'

$$K_p = \cos\beta\left(\cos\beta + \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right) / \left(\cos\beta - \sqrt{\left(\cos^2\beta - \cos^2\phi\right)}\right)$$

Coulomb's Wedge Theory for Earth Pressure:

Coulomb (1776) developed the wedge theory for determination of lateral earth pressure on a retaining wall. Unlike Rankine's theory, which considers the equilibrium of a soil element, Coulomb's theory considers the equilibrium of a sliding wedge of soil in the backfill that separates from the rest of the backfill above a failure plane.The mass of soil in the backfill above safe/stable slope is unstable and it tends to slide as the wall moves away or toward the backfill. Coulomb stated that this wedge of soil moves outward (away from the backfill) and downward in the active case when the wall moves away from the backfill.



Retaining wall with a trial slip surface and force diagram

Expression for Coulomb's Active Earth Pressure:

Referring to the force diagram shown above and applying Lami's theorem –

Pa/sin(α – ϕ) = W/sinC …(1)

In Δabc

Substituting this value of angle C in Eq. (1), we get

$$\frac{P_a}{\sin(\alpha-\phi)} = \frac{W}{\sin(180-\alpha+\phi+\theta-\delta)} \quad \Rightarrow \quad P_a = \frac{W\sin(\alpha-\phi)}{\sin(180-\alpha+\phi+\theta-\delta)}$$

$$P_a = \frac{W\sin(\alpha-\phi)}{\sin(\alpha-\phi-\theta+\delta)} \qquad (15.85)$$

$$W = \text{Area of } \Delta ABC \times 1 \times \gamma \quad \Rightarrow \quad W = \frac{1}{2} \times BC \times AD \times \gamma \qquad (15.86)$$

In ΔABC

$$\frac{BC}{\sin A} = \frac{AB}{\sin C} \quad \Rightarrow \quad \angle A = 90+\beta-(90-\theta) = 90+\beta-90+\theta = \beta+\theta$$

In ΔABC

$$\frac{BC}{\sin A} = \frac{AB}{\sin C} \quad \Rightarrow \quad \angle A = 90 + \beta - (90 - \theta) = 90 + \beta - 90 + \theta = \beta + \theta$$

Therefore,

$$BC = \frac{AB \sin A}{\sin C} = AB \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)}$$

In ΔABD

$$\frac{AD}{\sin B} = \frac{AB}{\sin 90} \quad \Rightarrow \quad \angle B = 180 - (\theta + \alpha)$$

Therefore,

$$AD = AB \frac{\sin[180 - (\theta + \alpha)]}{\sin 90} = AB \sin[180 - (\theta + \alpha)] \quad \Rightarrow \quad AD = AB \sin(\theta + \alpha)$$

Substituting the values of BC and AD in Eq. (15.86), we get

$$W = \frac{1}{2} \times AB \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)} \times AB \sin(\theta + \alpha) \times \gamma \quad \Rightarrow \quad W = \frac{1}{2} \times AB^2 \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)} \times \sin(\theta + \alpha) \times \gamma$$

In ΔABE

$$\sin\theta = \frac{H}{AB} \quad \Rightarrow \quad AB = \frac{H}{\sin\theta}$$

$$W = \frac{\gamma H^2}{2\sin^2\theta} \frac{\sin(\beta + \theta)}{\sin(\alpha - \beta)} \times \sin(\theta + \alpha)$$

Substituting the value of W in Eq. (15.85), we get

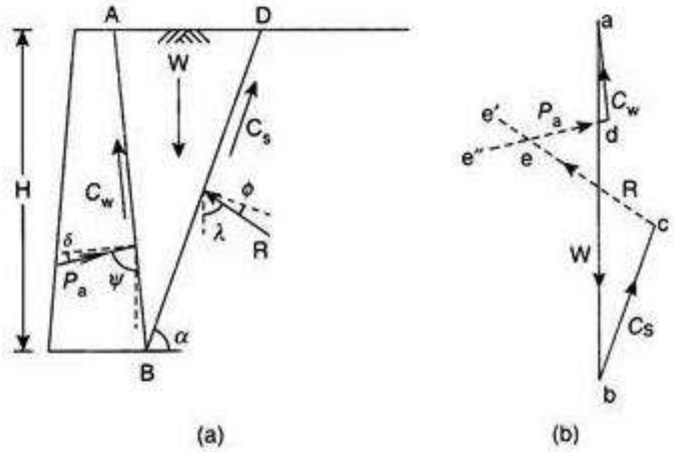
$$P_a = \frac{\gamma H^2}{2} \times \frac{\sin(\beta + \theta) \times \sin(\theta + \alpha)}{\sin^2\theta \times \sin(\alpha - \beta)} \times \frac{\sin(\alpha - \phi)}{\sin(\alpha - \phi - \theta + \delta)} \quad (15.87)$$

<u>Coulomb's Theory of Active Earth Pressure:</u>

Figure 15.51(a) shows a retaining wall of height H, with a cohesive backfill, with its surface inclined at angle β with the horizontal. The back of the wall is inclined at an angle θ with the horizontal. Consider failure plane BD at an inclination of α with the horizontal

The wedge of soil ABD tends to slide outward and downward always from the rest of the backfill in the active case. The wall resists the movement of the wedge and exerts a reaction Pa, inclined at an angle δ with the normal to the wall, where δ is the angle of wall friction. The magnitude of total active earth pressure is equal to Pa.

A line ab is drawn parallel to the line of action of W, with the length ab equal to W to some scale. From b, a line be is drawn equal in length to Cs to the same scale parallel to line of action of Cs, shown in Fig. 15.51(a). From a, a line ad is drawn equal in length to Cw parallel to the line of action of Cw. From point c, aline ce' is drawn parallel to the line of action of R, that is, at an angle (α – ϕ) with the vertical. Another line de" is drawn from point d parallel to the line of action of Pa, that is, at an angle (θ – δ) with the vertical. The two lines ce' and de" intersect at point e, which completes the force diagram abcde. The length of line de gives the value of Pa to the scale of the force diagram for the assumed trial value of α.



Coulomb's theory for active earth pressure: (a) Retaining wall with a trial slip surface and (b) force diagram, which is constructed based on Bow's notation.

The procedure is repeated for other failure planes, taking different trial values of α, and the corresponding values of Pa are determined. The maximum value of Pa, among the trial values, is taken as the active earth pressure. The corresponding trial failure plane is taken as the critical failure plane. The active earth pressure acts along the same line of action as Pa, but opposite in direction. To determine the point of application of Pa, a line is drawn from the centroid, G, of the wedge of soil ABD parallel to the critical failure plane to intersect the back of the wall at point P, which is the approximate point of application of Pa.

Coulomb's Theory for Passive Earth Pressure:

As per Coulomb's theory, a wedge of soil above a failure plane moves inward and upward in the passive case when the wall moves toward the soil on the front side of the wall due to lateral earth pressure. Figure 15.56(a) shows a retaining wall of height H, with a cohesionless backfill, with its surface inclined at an angle β with the horizontal.

The back of the wall is inclined at an angle θ with the horizontal. Consider the failure plane BC at an inclination of α with the horizontal. The wedge of soil ABC tends to slide inward and upward. A pressure is exerted on the wall, which is the passive earth pressure Pp, inclined at an angle δ above the normal to thewall, where δ is the angle of wall friction.

The total passive earth pressure is determined through Coulomb's theory by considering the equilibrium of the wedge of soil ABC.

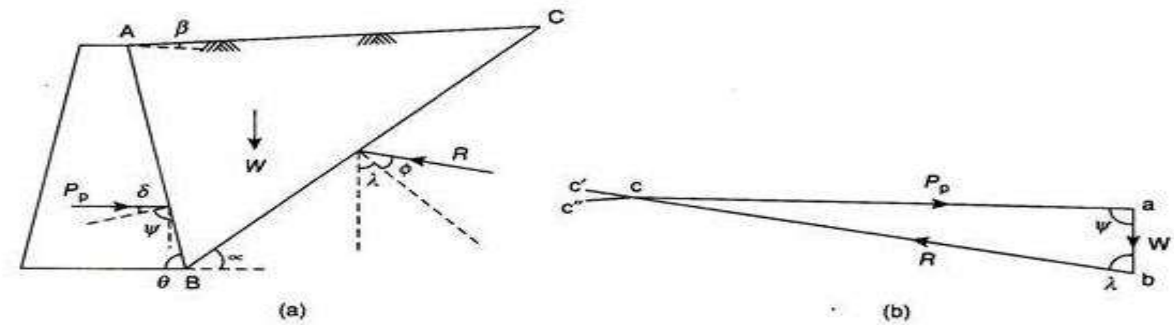
The forces acting on the wedge are as follows:

i. Weight (W) of the wedge of soil ABC acting vertically downward.

ii. The reaction (Pp) on the contact surface AB of the wall with the backfill, acting at an angle Δ above the normal to the back of the wall.

iii. The reaction (R) on the trial failure plane BC, which is the contact surface of the wedge with the rest of the back-fill. The reaction R acts at an angle ϕ above the normal to the surface BC. This reaction acts upward and outward, opposing the movement of the wedge.

A trial value of α is assumed and the force diagram is constructed. Figure 15.56(b) shows the force diagram abc. A line ab is drawn parallel to the line of action of W, with the length ab equal to W to some scale. Now, a line bc' is drawn parallel to the line of action of P, that is, at an angle (θ – δ) with ab. Another line ac" is drawn parallel to the line of action of R, that is, at an angle (α – ϕ) with ab. The two lines bc' and ac" intersect at point c, which completes the force diagram abc. The length of line be gives the value of Pp to the scale of the force diagram for the assumed trial value of α.



Column's passive earth pressure for a cohesionless backfill retaining wall and force diagram

The procedure is repeated for other failure planes, taking different trial values of α, and the corresponding values of Pp are determined. The minimum value of Pp among the trial values, is taken as the passive earth pres-sure. The corresponding trial failure plane is taken as the critical failure plane. The final expression for Coulomb's passive earth pressure is given by

$$P_P = K_P \frac{\gamma H^2}{2}$$

$$K_P = \frac{\sin^2(\theta - \phi)}{\sin^2\theta \sin(\theta + \delta)[1 - \sqrt{\{\sin(\phi + \delta)\sin(\phi + \beta)\} / \{\sin(\theta + \delta)\sin(\theta + \beta)\}}]^2}$$

Coulomb's theory assumes that the failure surface is a plane surface. The actual surface is found to be a curved surface, being either a logarithmic spiral or a circular arc. In the passive case, however, the error involved in the estimation of Pp is large when a plane failure surface is used for values of δ > (ϕ/3), which is the usual case. The value of Pp estimated is more than the actual value and is therefore on the unsafe side. Coulomb's theory is therefore generally not used for the estimation of passive earth pressure.

Analysis for dry back fills

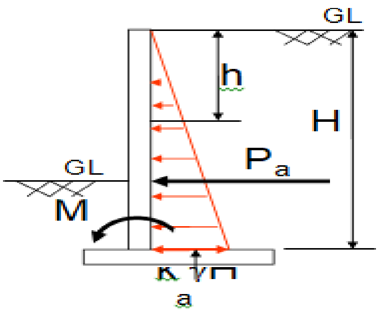
Maximum pressure at any height, p=kaϒh Total pressure at any height from top,

Pa=1/2[kaϒh]h = [kaϒh2]/2

Bending moment at any height M=paxh/3= [kaϒh3]/6

Total pressure, Pa= [kaϒH2]/2

Total Bending moment at bottom, M = [kaϒH3]/6



Where, ka = Coefficient of active earth pressure

= (1-sinϴ)/(1+sinϴ)=tan2ϴ

= 1/kp, coefficient of passive earth pressure

ϕ= Angle of internal friction or angle of repose

ϒ=Unit weigh or density of backfill

If ϕ= 30, ka=1/3 and kp=3. Thus ka is 9 times kp

Backfill with sloping surface

$p_a = k_a \gamma H$ at the bottom and isparallel to inclined surfaceofbackfill

$$k_a = \cos\theta \left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}} \right]$$

Where θ=Angle of surcharge

Total pressure at bottom

$=P_a = k_a \gamma H^2/2$

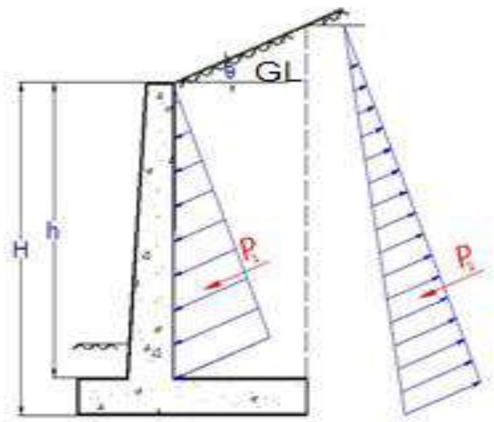


Fig.3.3 Soil pressure due to inclined surcharge

Stability requirements of retaining walls

As per IS 456-2000 following conditions must be satisfied for stability of retaining wall

1. Check against overturning

Factor of safety against overturning

$= M_R / M_O \geq 1.55 (=1.4/0.9)$

Where,

M_R =Stabilising moment

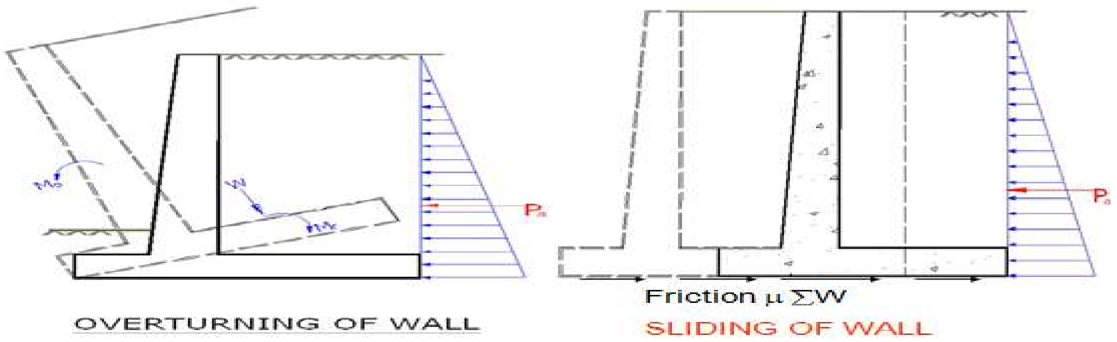
or restoring moment

M_O =overturning moment

As per IS:456-2000,

$M_R > 1.2\ M_O$, ch.DL + 1.4 M_O, ch.IL

$0.9\ M_R \geq 1.4\ M_O$, chIL



2. <u>Check against Sliding</u>

FOS against sliding

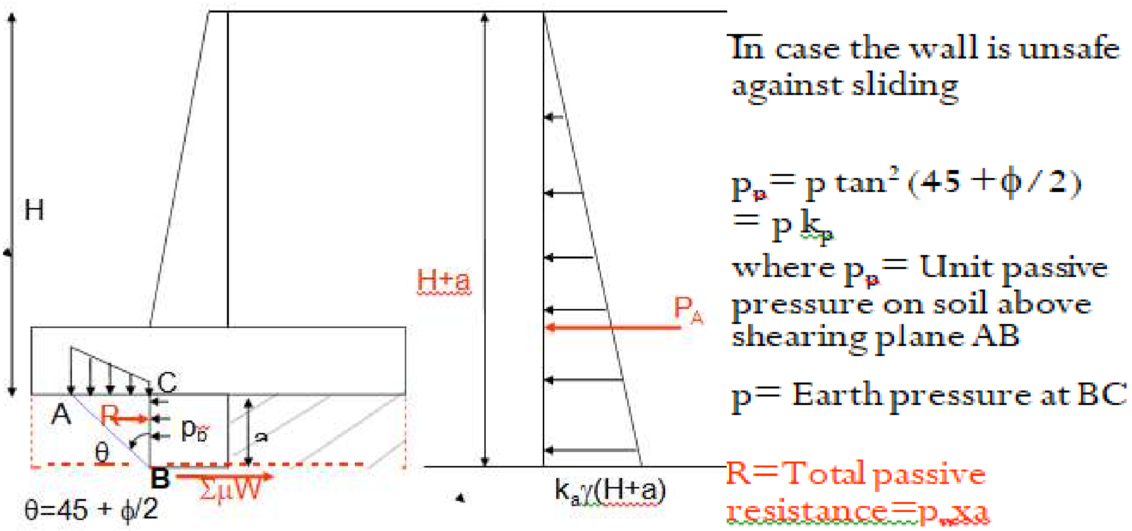
= Resisting force to sliding/Horizontal force causing sliding

= $\mu \sum W/Pa$ greater than or equal to 1.55 (=1.4/0.9)

As per IS:456:2000

$1.4 = \mu(0.9 \sum W)/Pa$

Design of shear key



If $\sum w$ = Total vertical force acting at the key base

Φ = shearing angle of passive resistance

R= Total passive force = $p_p x a$

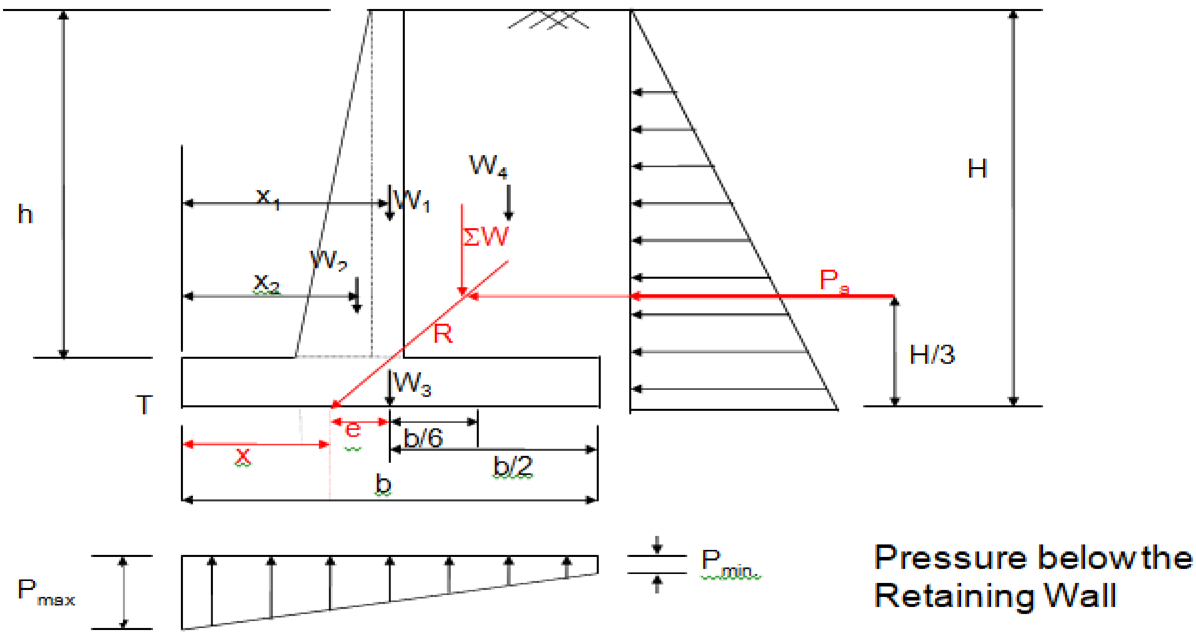
P_A=Active horizontal pressure at key base forH+a

$\mu\sum W$=Total frictional force under flatbase

For equilibrium, R + $\mu\sum W$ =FOS xP_A

FOS= $(R + \mu\sum W)/ P_A \geq 1.55$

Pressure distribution



Let the resultant Rdueto$\sum W$ andP_a

lieat a distance x from thetoe.

$X=\sum M/\sum W$,

$\sum M$ = sum of all moments abouttoe.

Eccentricity of the load = e = (b/2-x) <b/6

Minimum pressure at heel = P min = $\frac{\sum W}{b}\left[1-\frac{6e}{b}\right]$>Zero

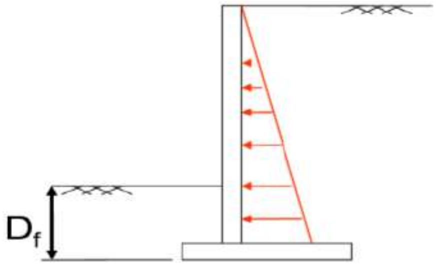
For zero pressure, e=b/6, resultant should cut the base withinthe middlethird

Maximum pressure at toe = P min = $\frac{\sum W}{b}\left[1+\frac{6e}{b}\right]$

Depth of foundation

Rankine's formula:

$$D_f = \frac{SBC}{\gamma}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2 = \frac{SBC}{\gamma}k_a^2$$



Preliminary proportioning

1. Stem:Top width 200 mm to 400 mm
2. Base slab width b= 0.4H to 0.6H, 0.6H to 0.75H for surcharged wall
3. Base slab thickness= H/10 to H/14
4. Toe projection= (1/3-1/4) Base width

Design of Cantilever retaining wall

Stem, toe and heel acts as cantileverslabs

Stem design: M_u=psf ($k_a \gamma H^3/6$)

Determine the depth d from $M_u = M_{u,lim} = Qbd^2$

Design as balanced section and findsteel

$M_u = 0.87 f_y A_{st}[d - f_y A_{st}/(f_{ck} b)]$

Heel slab and toe slab should also be designed as cantilever. For this stability analysis should be performed as explained and determine the maximum bending moments at thejunction.

- Determine thereinforcement.
- Also check for shear at thejunction.
- Provide enough developmentlength.
- Provide the distribution steel

Example 3.1 Design a cantilever retaining wall a retain an earth embankment with a horizontal top 3.5 m above ground level. Density of earth = 18 kN/m^3. Angle of internal friction ϕ = 30^0. SBC of soil is 200kN/m^3. Take coefficient of friction between soil and concrete = 0.5. Adopt m20 grade concrete and Fe 415 steel.

Solution:

$H_2 = 3.5$ m $\quad \gamma = 18$ kN/m^3 $\quad \phi = 30°$ $\quad$ SBC, $q_o = 200$ kN/m^2

$\mu = 0.5$ $\quad f_{ck} = 20$ N/mm^2 $\quad f_y = 415$ N/mm^2

∴ Coefficient of active earth pressure

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$$

∴ Minimum depth of foundation is

$$y_{min} = \frac{q_o}{\gamma} k_a^2 = \frac{200}{18} \times \left(\frac{1}{3}\right)^2 = 1.23 \text{ m}$$

Provide depth of foundation as 1.25 m

∴ Height of retaining wall = 3.5 + 1.25 = 4.75 m

Preliminary Dimensions of Retaining Wall

B = 0.48 H to 0.56 H = 2.375 m to 2.66 m

Say $\quad b = 2.5$ m

Toe projection $\quad = 0.3\, b = 0.75$ m

Thickness of base slab $\quad$ = Thickness of stem $= \frac{H}{12} = \frac{4.75}{12}$, say 0.4 m

Let top width of stem be 0.2 Fig 3.4 shows dimensions of the retaining wall selected and Fig 3.5 shows various forces on the retaining wall

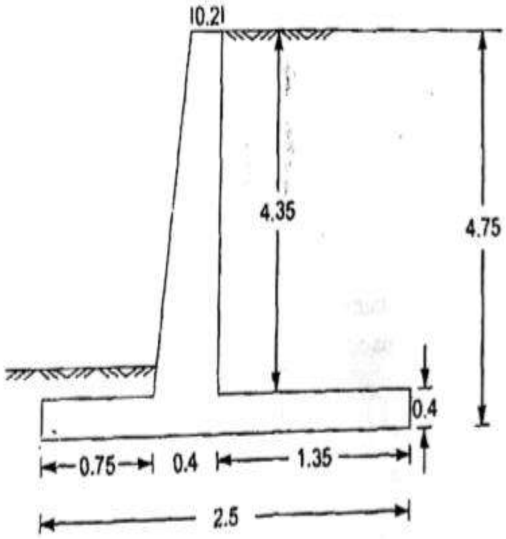


Fig 3.4

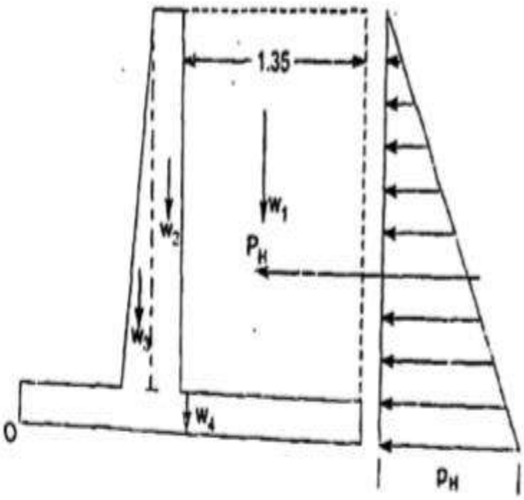


Fig 3.5

Check for stability

Various vertical loads acting on the retaining wall, their distances from overturning point O and the moment of these forces about o are shown in th table below

	Weight in kN	x in m	M_r in kN-m
Weight of backfill	$w_1 = 1.35 \times 4.35 \times 18 = 105.7$	$2.5 - \frac{1.35}{2} = 1.825$	192.9
Rectangular portion of stem	$w_2 = 0.2 \times 4.35 \times 25 = 21.75$	$0.75 + 0.4 - 0.1 = 1.05$	22.84
Triangular portion of stem	$w_3 = \frac{1}{2} \times 0.2 \times 4.35 \times 25 = 10.88$	$0.75 + \frac{2}{3} \times 0.2 = 0.88$	9.61
Base slab	$w_4 = 0.4 \times 2.5 \times 25 = 25$	$\frac{2.5}{2}$	31.25
	$\sum W = 163.33$ kN		$\sum M_r = 256.6$ kN-m

Horizontal pressure $P_H = \frac{1}{2} k_a \gamma H^2$

$$= \frac{1}{2} \times \frac{1}{3} \times 18 \times 4.75^2 = 67.688 \text{ kN}$$

Overturning moment, $M_o = P_H \frac{h}{3} = 67.688 \times \frac{4.75}{3} = 107.17$ kN-m

As per IS 456–2000, factor of safety for over turning is

$$F_1 = \frac{0.9 \times 256.6}{107.17} = 2.15 > 1.4 \quad \text{Hence O.K.}$$

$$F_2 = \frac{0.9\mu \sum W}{P_H} = \frac{0.9 \times 0.5 \times 163.33}{67.688} = 1.09 < 1.4$$

Hence shear key is to be provided.

Pressure Under Base Slab

Total moment about point O

$$= M_r - M_o = 256.6 - 107.17 = 149.43 \text{ kN-m}$$

Total vertical load $= 163.33$ kN

Horizontal distance from O where resultant intersects the base line

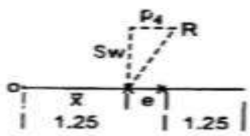


Fig. 10.12

$$\bar{x} = \frac{149.43}{163.33} = 0.915 \text{ m}$$

∴ Eccentricity $e = \frac{2.5}{2} - 0.915 = 0.335$ m

∴ Maximum pressure

$$p_1 = \frac{\sum W}{b}\left[1 + \frac{6e}{b}\right] = \frac{163.33}{2.5}\left[1 + \frac{6 \times 0.335}{2.5}\right] = 117.86 \text{ kN/m}^2$$

Minimum pressure, $p_2 = \frac{163.33}{2.5}\left[1 - \frac{6 \times 0.335}{2.5}\right] = 12.8 \text{ kN/m}^2$

Thus $p_1 <$ SBC of soil
and p_2 is positive.
Hence satisfactory.

Design of Stem

Stem acts as a cantilever of height 4.35 m subject to uniformly varying load of

$$= k_a \gamma h$$

Maximum moment at the base of cantilever

$$= \frac{1}{2} k_a \gamma h^2 \times \frac{h}{3} = \frac{1}{6} k_a \gamma h^3$$

$$= \frac{1}{6} \times \frac{1}{3} \times 18 \times 4.35^3 = 82.31 \text{ kN-m}$$

$\therefore$ $$M_u = 1.5 \times 82.31 = 123.47 \text{ kN-m}$$

Since M20 concrete and Fe-415 steel are used

$$0.138 f_{ck} bd^2 = M_u, \text{ for balanced section}$$

$$\therefore \quad 0.138 \times 20 \times 1000 \times d^2 = 123.47 \times 10^6$$

$$d = 211.5$$

$\therefore$ Depth d = 350 mm and overall depth D = 400 mm give sufficiently under reinforced section.

Area of steel required is obtained from

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st}}{bd} \times \frac{f_y}{f_{ck}}\right)$$

$$123.47 \times 10^6 = 0.87 \times 415 \times A_{st} \times 350 \left(1 - \frac{A_{st}}{1000 \times 350} \times \frac{415}{20}\right)$$

$$977 = A_{st}\left(1 - \frac{A_{st}}{16867.5}\right)$$

or $$A_{st}^2 - 16867.5 \, A_{st} + 977 \times 16867.5 = 0$$

$\therefore$ $$A_{st} = 1041 \text{ mm}^2$$

Using 12 mm bars,

$$s = \frac{\frac{\pi}{4} \times 12^2}{1041} \times 1000 = 108 \text{ mm}$$

Provide 12 mm bars at 100 mm c/c.

Distribution Steel

Average thickness of wall $= \dfrac{200 + 400}{2} = 300$ mm

$\therefore$ $$A_{st} = \frac{0.12}{100} \times 1000 \times 300 = 360 \text{ mm}^2$$

Providing 180 mm^2 on each face and using 8 mm bars

$$s = \frac{\frac{\pi}{4} \times 8^2}{180} \times 1000 = 279 \text{ mm}$$

Provide 8 mm bars at 270 mm c/c on tension face

A mesh of 8 mm bars @ 270 mm is given on compression face of the wall.

Curtailment of Vertical Bars

Bending moment is proportional to cube of depth of filling and thickness varies linearly from 200 mm at top 400 mm at a depth of 4.35 m. One third of vertical bars may be curtailed at a height of 1.5 m from base and another $\frac{1}{3}$rd at a height of 3 m from the base as shown in Fig.

Check for shear: $V = P_H = 107.17$ kN

$\therefore$ $V_u = 1.5 \times 107.17 = 160.75$ kN

$$\therefore \quad \tau_v = \frac{160.75 \times 1000}{1000 \times 400} = 0.4 \text{ N-mm}^2$$

$$p = \frac{\frac{\pi}{4} \times 12^2 \times 100}{400 \times 100} = 0.283$$

$$\therefore \quad \tau_c = 0.4 \text{ N/mm}$$

$\therefore$ No shear reinforcement is required.

Pressure diagram under the base varies from 117.86 kN/m^2 to 12.8 kN/m^2 as shown in Fi

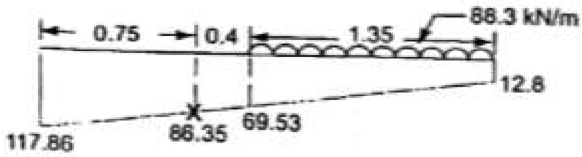


Fig. 10.15

$\therefore$ Pressure at the face of toe $= 12.8 + \frac{1.75}{2.5}(117.86 - 12.8) = 86.35$ kN/m^2

Dividing it into a *udl* of 12.8 kN/m and a linearly varying load

$$\therefore \quad M = 86.35 \times \frac{0.75^2}{2} + \frac{1}{2} \times 0.75 \times (117.86 - 86.35) \times \frac{2}{3} \times 0.75$$

$$= 30.19 \text{ kN-m}$$

$$\therefore \quad M_u = 1.5 \times 30.19 = 45.285 \text{ kN/m}$$

$$d = 350 \text{ mm}$$

$$45.285 \times 10^6 = 0.87 \times 415 \times A_{st} \times 350 \left(1 - \frac{A_{st}}{1000 \times 350} \times \frac{415}{20}\right)$$

$$358.36 = A_{st}\left(1 - \frac{A_{st}}{16867.5}\right)$$

or

$$A_{st}^2 - 16867.5\, A_{st} + 16867.5 \times 358.36 = 0$$

$$\therefore \quad A_{st} = 366 \text{ mm}^2$$

$$\therefore \quad A_{st} \text{ minimum} = \frac{0.12}{100} \times 1000 \times 400 = 480 \text{ mm}^2$$

$$\therefore \quad A_{st} = 480 \text{ mm}^2$$

Using 12 mm bars

$$s = \frac{\frac{\pi}{4} \times 12^2}{480} \times 1000 = 235 \text{ mm}$$

Provide 12 mm bars at 300 mm c/c in both directions.

Design of Heel Slab

Its width is 1.35 m. Pressure varies from 12.8 kN/m^2 on outer edge to 69.53 kN/m^2 at the face of the column as shown in Fig. 10.13.

Weight of back fill $= \gamma H_1 = 18 \times 4.35 = 78.3$ kN/m

Self weight $= 0.4 \times 1 \times 25 = 10$ kN/m

$\therefore$ Total downward load $= 78.3 + 10 = 88.3$ kN/m

$\therefore$ Maximum bending moment

$$= 88.3 \times \frac{1.35^2}{2} - 12.8 \times \frac{1.35^2}{2} - \frac{1}{2} \times (69.53 - 12.8)1.35 \times \frac{1}{3} \times 1.35$$

$$= 56.04 \text{ kN-m}$$

$$\therefore \quad M_u = 1.5 \times 56.04 = 84.06 \text{ kN-m}$$

Hence area of steel required is given by

$$84.06 \times 10^6 = 0.87 \times 415 \times A_{st} \times 350\left(1 - \frac{A_{st}}{1000 \times 350} \times \frac{415}{20}\right)$$

$$665.2 = A_{st}\left(1 - \frac{A_{st}}{16867.5}\right)$$

or $$A_{st}^2 - 16867.5\, A_{st} + 665.2 \times 16867.5 = 0$$

$$\therefore \quad A_{st} = 693 \text{ mm}^2$$

Provide minimum reinforcement of 12 mm bars at 160 mm c/c in both directions.

Design of Shear Key

Pressure at face of shear key = 86.35 kN/m

Coefficient of passive earth pressure

$$k_p = \frac{1}{k_a} = 3$$

If 'a' is the projection of shear key, resistance offered by passive earth pressure

$$= k_p \times \text{vertical pressure}$$

$$= 3 \times 86.35 \times a = 259.05a \text{ kN}$$

$\therefore$ Factor of safety against sliding

$$F_2 = \frac{0.9\mu \sum W + 259.05a}{67.688} = 1.4$$

i.e., $$\frac{0.9 \times 0.5 \times 163.33 + 259.05a}{67.688} = 1.4$$

$$\therefore \quad a = 0.085 \text{ m}$$

Provide 200 mm deep shear key.

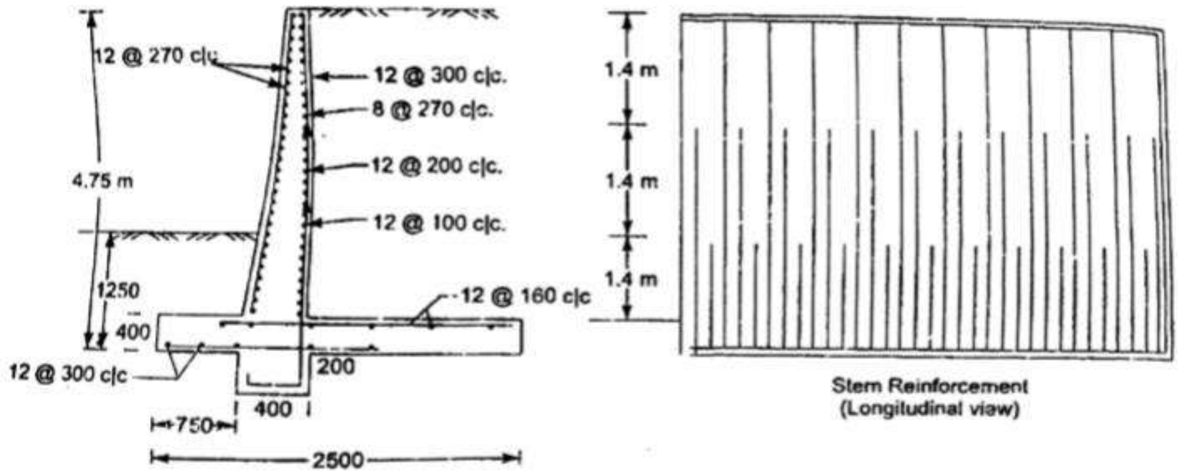
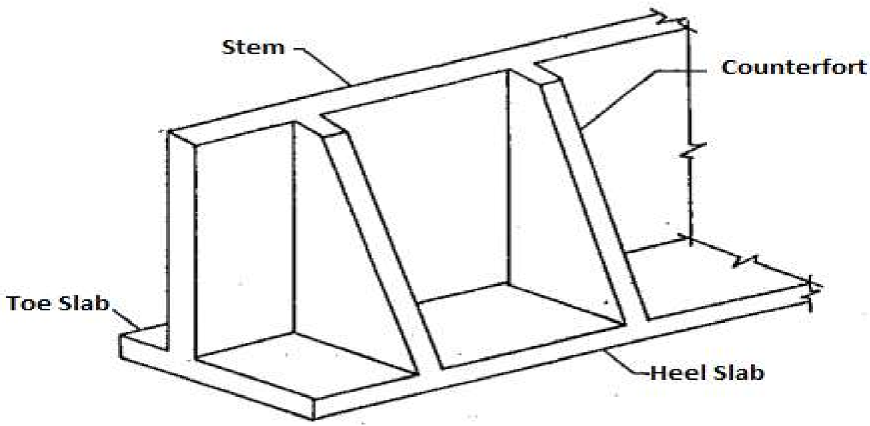


Fig 3.6 Reinforcement details

Counterfort retaining wall

Counterforts are provided at 3 to 3.5 m interval and they act as T-beam subjected to backfill, maximum value being at the base slab. The moment of $\frac{1}{6}k_a\gamma h^3 \times L$, where L is the spacing of counterforts is to be resisted by counterfort. The horizontal thrust tries to separate wall from the counterforts and hence horizontal ties are provided to connect stem and counterfort. Similarly the vertical downward load on heel slab tries to separate heel slab from counterfort and hence vertical ties are provided in the counterfort. The design procedure is illustrated with the example below:



Example 3.2 Design a counterfort retaining wall if the height of wall above ground level is 5.5 m. Unit weight of back fill = 18 kN/m^3. Angle of internal friction $\phi = 30^0$. SBC of soil is 180kN/m^3. Keep spacing of counterforts as 3 m. Take coefficient of friction between soil and concrete = 0.5. Adopt M20 grade concrete and Fe 415 steel.

Solution:

$H_2 = 5.5$ m, $\quad q_o = 180$ kN/m^2, $\quad \phi = 30°$, $\quad \gamma = 18$ kN/m^3

Coefficient of active earth pressure $k_e = \dfrac{1-\sin\phi}{1+\sin\phi}$

$$= \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$$

∴ Minimum depth of foundation

$$y_{min} = k_a^2 \frac{q_o}{\gamma} = \left(\frac{1}{3}\right)^2 \times \frac{180}{18} = 1.111 \text{ m}$$

Provide depth of foundation = 1.3 m

∴ Total height of retaining wall H = 5.5 + 1.3 = 6.8 m

Base width is kept 0.5H to 0.6H

In this case it may be from 3.4 m to 4.08 m

Let base width $\quad b = 4.0$ m

Toe projection $\quad = \frac{1}{4}$th $\frac{1}{5}$th b

Let it be $\quad = 0.8$ m

Width of counterforts $\quad = 0.03$H to 0.06 H

Let it be $\quad = 300$ mm

Thickness of stem $\quad$ = Thickness of base slab $= \dfrac{H}{25}$

Sav $\quad d = 260$ mm and D = 300 mm

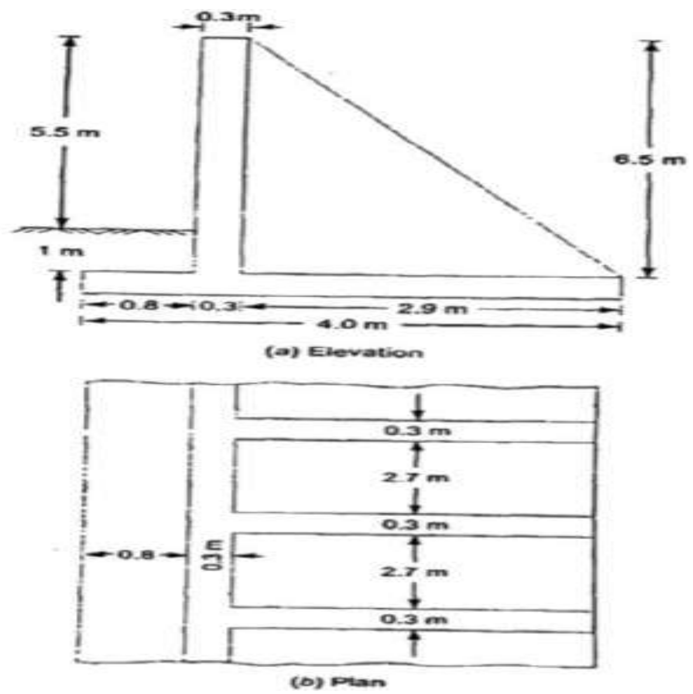


Fig 3.7 Counterfort retaining wall

Table below shows weight, distance from overturning point 'o' [toe edge] and moment about 'o' by various stabilizing forces

Sl. No.	Force	Magnitude in kN	$\bar{x}$	M_s in kN-m
1	W_1 = Weight of back fill	$2.7 \times 6.5 \times 18 = 315.9$	$4 - \frac{2.9}{2} = 2.55$	805.5
2	W_2 = Weight of stem	$0.3 \times 6.5 \times 25 = 48.75$	0.95	46.3
3	W_3 = Weight of base slab	$0.3 \times 4.0 \times 25 = 30$	2.0	60.0
		$\Sigma W = 394.65$ kN		$\Sigma M_s = 911.8$ kN

Overturning moment due to horizontal backfill earth pressure

$$M_o = k_a \frac{\gamma h^3}{6} = \frac{1}{3} \times \frac{1}{6} \times 18 \times 6.8^3 = 314.4 \text{ kN-m}$$

$\therefore$ **Factor of safety against overtuning**

$$= 0.9 \frac{M_s}{M_o} = \frac{0.9 \times 911.8}{314.4} > 1.4$$

Hence O.K.

Sliding force $\qquad P_H = k_a \frac{\gamma h^2}{2} = \frac{1}{3} \times \frac{18}{2} \times 6.8^2 = 138.72$ kN

Resisting force $\qquad = \mu\,(0.9W_1 + W_2 + W_3)$

$$= 0.5(0.9 \times 315.9 + 48.719 + 30) = 181.51$$

$\therefore$ Factor of safety against sliding

$$F_2 = \frac{181.51}{138.72} = 1.3 < 1.4$$

Hence need key. Provide a key of depth 300 mm

Pressure from Soil

Total moment about point O $\quad = M_s - M_o$

$$= 911.8 - 314.4 = 597.4 \text{ kN-m}$$

$\therefore$ Horizontal distance at which resultant intercepts base

$$= \frac{597.4}{\sum W} = \frac{597.4}{394.65} = 1.514 \text{ m}$$

$$e = \frac{b}{2} - \bar{x} = \frac{4}{2} - 1.514 = 0.486 \text{ m}$$

$\therefore$

$$p_1 = \frac{\sum W}{b}\left[1 + \frac{6e}{b}\right] = \frac{394.65}{4}\left[1 + \frac{6 \times 0.486}{4}\right]$$

$$= 170.6 \text{ kN/m}^2 < 180 \text{ kN/m}^2, \text{ Hence O.K.}$$

$\therefore$

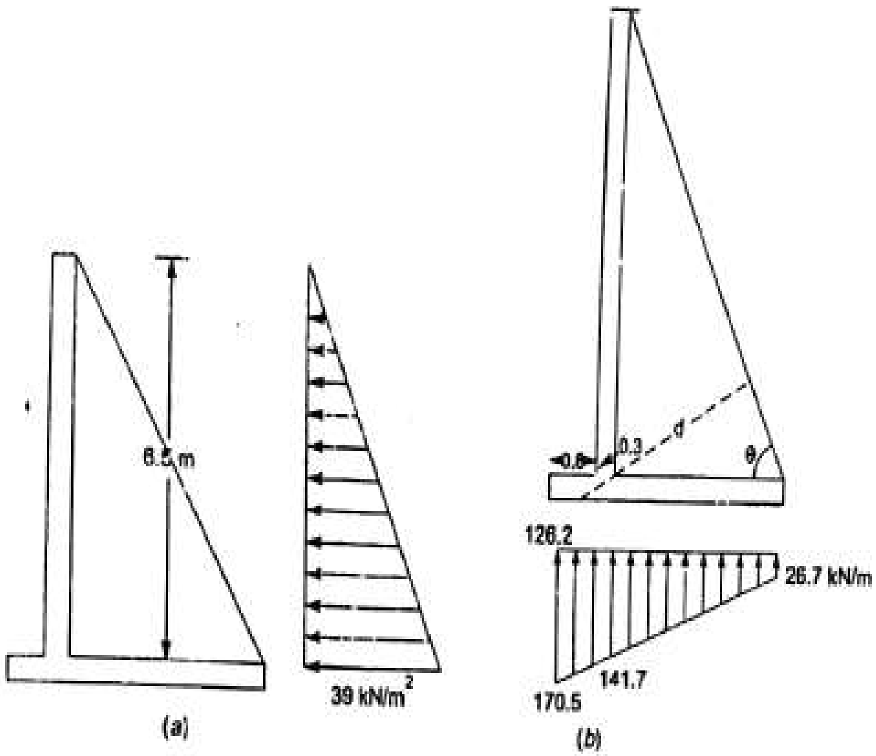
$$p_2 = \frac{\sum W}{b}\left[1 - \frac{6e}{b}\right] = 26.7 \text{ kN/m}^2, \text{ positive. Hence O.K.}$$

Design of Stem

Stem acts as a horizontal slab of span 3 m. Referring to Fig. 3.17, Maximum horizontal pressure on stem

$$= k_a \gamma H_1 = \frac{1}{3} \times 18 \times 6.5 = 39 \text{ kN/m}^2$$

Maximum moment $\qquad M = \frac{39 \times 3^2}{12} = 29.25$ kN-m



$$M_{u\ lim} = 0.138\, f_{ck}\, bd^2 = 0.138 \times 20 \times 1000 \times 260^2$$
$$= 186.57 \times 10^6 \text{ N-mm} > M_u$$

Hence thickness of stem is sufficient. Now,

$$M_u = 0.87 f_y A_{st}\, d\left(1 - \frac{A_{st}}{bd} \times \frac{f_y}{f_{ck}}\right)$$

$$43.9 \times 10^6 = 0.87 \times 415 \times A_{st} \times 260\left(1 - \frac{A_{st}}{1000 \times 260} \times \frac{415}{20}\right)$$

$$467.65 = A_{st}\left(1 - \frac{A_{st}}{12530}\right)$$

$$A_{st}^2 - 12530\, A_{st} + 467.65 \times 12530 = 0$$

$\therefore$ $$A_{st} = 486 \text{ mm}^2$$

Using 12 mm bars, spacing required is

$$s = \frac{\frac{\pi}{4} \times 12^2}{486} \times 1000 = 232 \text{ mm}$$

$\therefore$ Provide 12 mm bars at 225 mm c/c.

$$\text{Distribution steel} = \frac{0.12}{100} \times 260 \times 1000 = 312 \text{ mm}^2$$

Using 12 mm bars, spacing required is

$$s = \frac{\frac{\pi}{4} \times 12^2}{312} \times 1000 = 362 \text{ mm}$$

Provide 12 mm bars at 300 mm c/c.

12 mm bars are provided at 300 mm in both direction on the front side also. It takes care of +ve moment in the middle which is equal to $\frac{39 \times 3^2}{12}$ kN-m.

Check for Shear

Maximum shear force at the face of counterfort

$$= \frac{39 \times (3 - 0.3)}{2} = 52.65 \text{ kN}$$

$\therefore$ $V_u = 1.5 \times 52.65 = 79$ kN

$\therefore$ $\tau_v = \frac{79 \times 1000}{260 \times 1000} = 0.303$ N/mm^2

Percentage reinforcement $p = \frac{\frac{\pi}{4} \times 12^2}{225 \times 260} \times 100 = 0.193$

$\therefore$ $\tau_c = 0.32$ N/mm^2, Hence safe.

Increase the spacing to 300 mm at a height of 1.5 m, since pressure (hence bending moment) reduces linearly towards the top of stem.

Design of Toe slab

Figure 3.8 shows variations of pressure under base slab

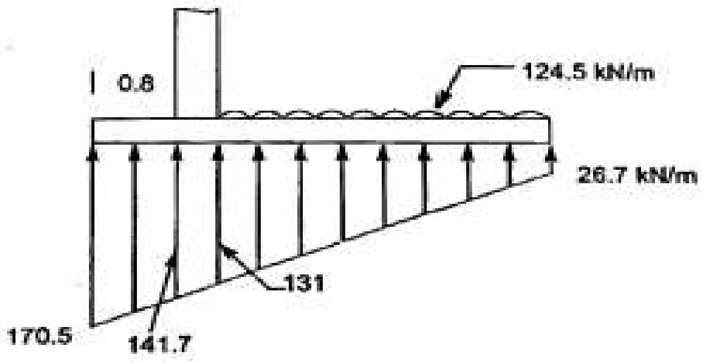


Fig 3.8

Cantilever moment $= 141.7 \times \frac{0.8^2}{2} + \frac{1}{2} \times (170.5 - 141.7) \times 0.8 \times \frac{2}{3} \times 0.8$

$= 51.488$ kN-m

$\therefore$ $$M_u = 1.5 \times 51.488 = 77.232 \text{ kN-m} < M_{u\,lim}$$

Hence depth of 260 mm is sufficient

$$77.232 \times 10^6 = 0.87 \times 415 \times A_{st} \times 260 \left(1 - \frac{A_{st}}{1000 \times 260} \times \frac{415}{20}\right)$$

$$822.7 = A_{st}\left(1 - \frac{A_{st}}{12530}\right)$$

$$A_{st}^2 - 12530\,A_{st} + 822.7 \times 12530 = 0$$

$$A_{st} = 885 \text{ mm}^2$$

Using 16 mm bars,

$$s = \frac{\frac{\pi}{4} \times 16^2}{885} \times 1000 = 227 \text{ mm}$$

Provide 16 mm bars @ 220 mm c/c.

Check for Shear

Critical section is at a distance d = 260 mm from the face of stem. Pressure at this point

$$= 26.7 + \frac{(141.7 - 26.7)}{4} \times (3.2 + 0.260) = 126.2 \text{ kN/m}^2$$

$\therefore$ Shear force per metre width of toe

$$V = \frac{1}{2}(170.5 + 126.2) \times (0.8 - 0.26) = 80.10 \text{ kN}$$

$$V_u = 1.5 \times 80.10 = 120.15 \text{ kN}$$

$\therefore$ $$\tau_v = \frac{120.15 \times 1000}{1000 \times 260} = 0.462 \text{ N/mm}^2$$

Percentage reinforcement $p = \dfrac{\frac{\pi}{4} \times 16^2}{290} \times \dfrac{100}{260} = 0.352$

From Table 19 in IS 456 – 2000,

$\therefore$ $$\tau_c = k_s \times \tau'_c = 1.08 \times 0.41 = 0.442 \text{ N/mm}^2 < \tau_v$$

$\therefore$ Not safe in shear. Increase the depth to d = 300 mm

D = 350. Since the additional load directly gets transferred to soil, without creating SF and BM, the pressure calculation need not be repeated. For d = 300 mm,

$$\tau_v = \frac{120.15 \times 1000}{1000 \times 300} = 0.4 \text{ N/mm}^2 < \tau_c$$

Hence safe.

Spacing of main bars may be increased to 250 mm c/c.

Design of Heel Slab

Here also, since additional thickness of heel slab do not create SF and BM, the same analysis is maintained but for the design the effective depth is taken as $d = 300$ mm.

Soil pressure at junction with stem

$$= 26.7 + \frac{(170.5 - 26.7)}{4} \times 2.9 = 131 \text{ N/mm}^2$$

Load from back fill $= 6.5 \times 18 = 117 \text{ kN/m}^2$

Load from 300 mm thick slab (self weight)

$$= 0.3 \times 1 \times 25 = 7.5 \text{ kN/m}^2$$

$\therefore$ Total downward load $= 117 + 7.5 = 124.5 \text{ kN/m}^2$

Maximum downward pressure intensity is at the edge

$$p_{max} = 124.5 - 26.7 = 97.8 \text{ kN/m}^2$$

$\therefore$

$$M = 97.8 \times \frac{3^2}{12} = 73.35 \text{ kN-m}$$

$$M_u = 1.5 \times 73.35 = 110.0 \text{ kN-m}$$

$\therefore$ Longitudinal main bar required for $d = 300$ mm, slab

$$110.0 \times 10^6 = 0.87 \times 415 \times A_{st} \times 300 \left(1 - \frac{A_{st}}{1000 \times 300} \times \frac{415}{20}\right)$$

$$1016 = A_{st}\left(1 - \frac{A_{st}}{14457.8}\right)$$

or

$$A_{st}^2 - 14457.8 \times A_{st} + 1016 \times 14457.8 = 0$$

$$A_{st} \doteq 1100 \text{ mm}^2$$

Using 16 mm bars, spacing required is

$$s = \frac{\frac{\pi}{4} \times 16^2}{1100} \times 1000 = 183 \text{ mm}$$

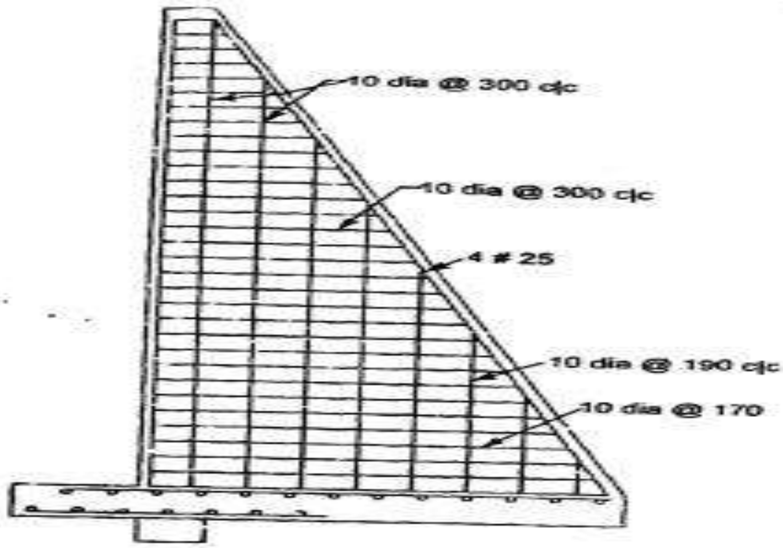
Provide 16 mm bars at 180 mm spacing near the edges. Since downward pressure reduces linearly, the spacing may be increased to 300 mm towards junction.

Distribution steel of 12 mm diameter bars @ 225 c/c is provided at right angles to main bars.

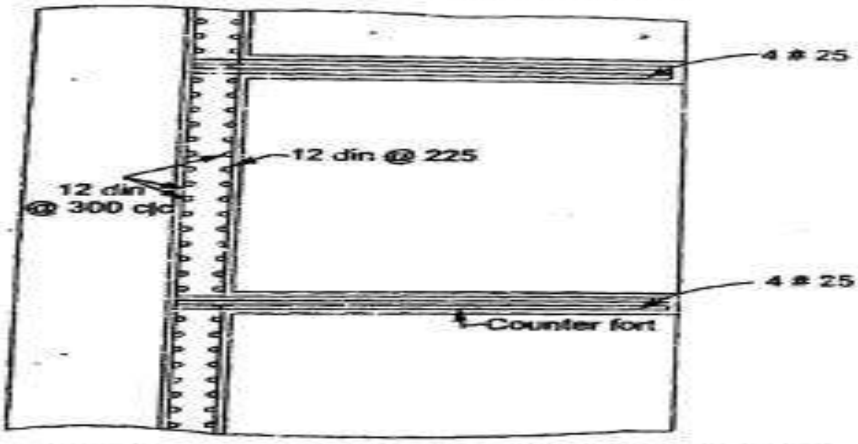
Design of Counterfort

Reinforcements are required for beam action and for, against separating force in horizontal and vertical directions.

(a) **For beam action:** Counterfort behaves as T-beam of varying section, cantilevering out of the base.



(b) Section through counterfort

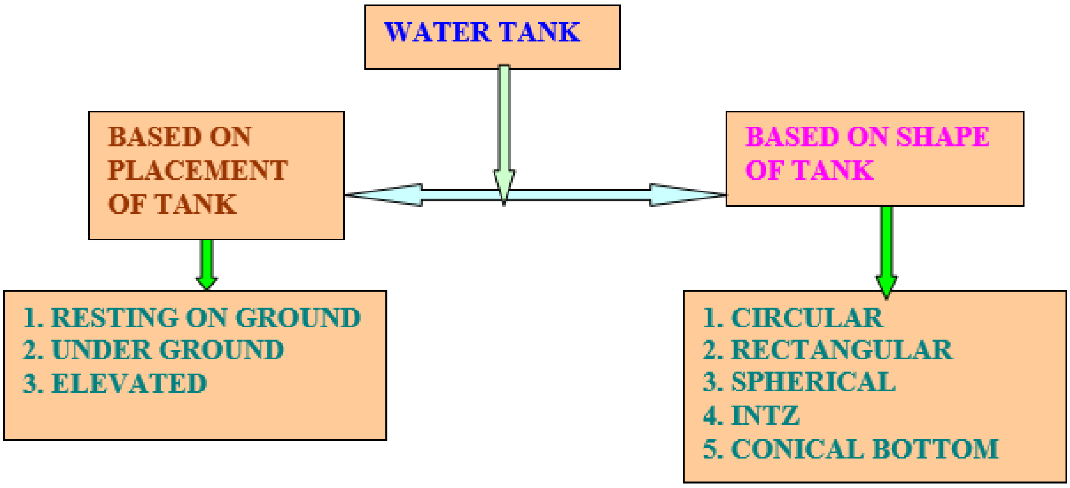


(c) Reinforcement in stem and counterfort (plan view)

CIRCULAR WATER TANK

Introduction:

Storage tanks are built for storing water, liquid petroleum, petroleum products and similar liquids. Analysis and design of such tanks are independent of chemical nature of product. They are designed as crack free structures to eliminate any leakage. Adequate cover to reinforcement is necessary to prevent corrosion. In order to avoid leakage and to provide higher strength concrete of grade M20 and above is recommended for liquid retaining structures.



To achieve imperviousness of concrete, higher density of concrete should be achieved. Permeability of concrete is directly proportional to water cement ratio. Proper compaction using vibrators should be done to achieve imperviousness. Cement content ranging from 330 Kg/m^3 to 530 Kg/m^3 is recommended in order to keep shrinkagelow.

The leakage is more with higher liquid head and it has been observed that water head up to 15 m does not cause leakage problem. Use of high strength deformed bars of grade Fe415 are recommended for the construction of liquid retaining structures. However mild steel bars are also used. Correct placing of reinforcement, use of small sized and use of deformed bars lead to a diffused distribution of cracks. A crack width of 0.1mm has been accepted as permissible value in liquid retaining structures. While designing liquid retaining structures recommendation of “Code of Practice for the storage of Liquids- IS3370 (Part I to IV)” should be considered. Fractured strength of concrete is computed using the formula given in clause 6.2.2 of IS 456 - 2000 ie., $f_{cr}=0.7\sqrt{f_{ck}}$ MPa. This code does not specify the permissible stresses in concrete for resistance to cracking. However earlier version of this code published in 1964 recommends permissible value as $\sigma_{cat}= 0.27 \sqrt{f_{ck}}$ for directtension and

$\sigma_{cbt}=0.37 \sqrt{f_{ck}}$ for bending tensile strength.

Allowable stresses in reinforcing steel as per IS 3370 are

σ_{st}= 115 MPa for Mild steel (Fe250) and σ_{st}= 150 MPa for HYSD bars(Fe415)

In order to minimize cracking due to shrinkage and temperature, minimum reinforcement is recommended as:

i) For thickness ≤ 100 mm = 0.3%

ii) For thickness ≥ 450 mm = 0.2%

iii) For thickness between 100 mm to 450 mm = varies linearly from 0.3% to 0.2%

For concrete thickness ≥ 225 mm, two layers of reinforcement are placed, one near water face and other away from water face.

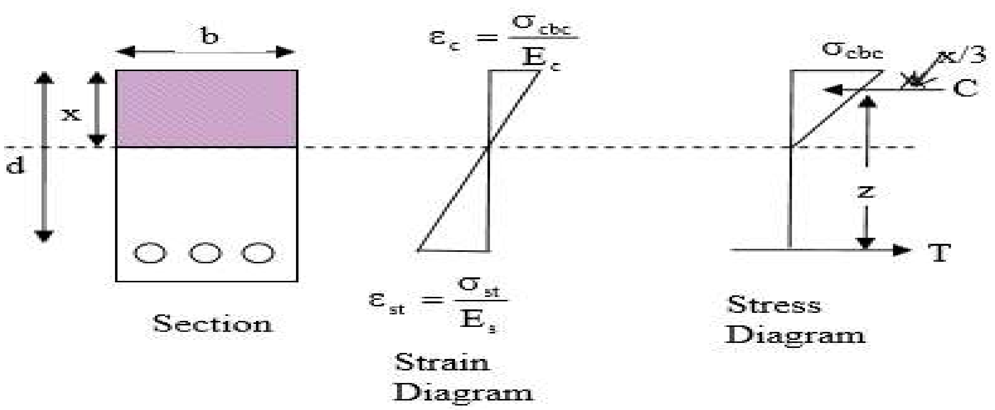
Cover to reinforcement is greater of i) 25 mm, ii) Diameter of main bar.

In case of concrete cross section where the tension occurs on fibers away from the water face, then permissible stresses for steel to be used are same as in the analysis of other sections, ie., σ_{st}=140 MPa for Mild steel and σ_{st}=230 MPa for HYSD bars.

In this method the concrete and steel are assumed to be elastic. At the worst combination of working loads, the stresses in materials are not exceeded beyond permissible stresses. The permissible stresses are found by using suitable factors of safety to material strengths. Permissible stresses for different grades of concrete and steel are given in Tables 21 and 22 respectively of IS456-2000.

The modular ratio 'm' of composite material ie., RCC is defined as the ratio of modulus of elasticity of steel to modulus of elasticity of concrete. But the code stipulate the value of 'm as m = 280/σcbc,where σbc is the permissible stress in concrete

To develop equation for moment of resistance of singly reinforced beams, the linear strain and stress diagram are shown below



$\therefore A_{st} = \frac{M}{\sigma_{st} jd}$; Let p_t be the percentage of steel expressed as

$$p_{tbal} = \frac{100A_{st}}{bd} = 100\frac{M}{\sigma_{st} jd}\frac{1}{bd} = \frac{50k\sigma_{cbc}}{\sigma_{st}}$$

The neutral axis depth is obtained from strain diagram as

$$\frac{x}{d-x}=\frac{\sigma_{cbc}/E_c}{\sigma_{st}/E_s}=\frac{m\sigma_{cbc}}{\sigma_{st}} \text{ solving for x; } x=\left[\frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}}\right]d=kd$$

where, $k=\left[\frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}}\right]$, k is known as neutral axis constant

The lever arm z=d-x/3 = d-(kd/3)= d(1-k/3) = jd, where, j=1-k/3; j is known as lever arm constant

C= ½ σ_{cbc}bx; T= σ_{st} A_{st}

Moment of resistance M= C z = T z

Consider, M=C z = (½ σ_{cbc}bx) jd = (½ σ_{cbc}bkd) jd = (½ σ_{cbc}kj) bd^2 = Q_{bal} bd^2

Where, Q_{bal} is known as moment of resistance factor for balanced section.

Now consider M= T z = σ_{st} A_{st} jd;

$\therefore A_{st}=\frac{M}{\sigma_{st}jd}$; Let p_t be the percentage of steel expressed as

$$p_{tbal}=\frac{100A_{st}}{bd}=100\frac{M}{\sigma_{st}jd}\frac{1}{bd}=\frac{50k\sigma_{cbc}}{\sigma_{st}}$$

Design constants

Concrete Grade	Steel Grade	σ_{cbc}	σ_{st}	k	j	Q_{bal}	p_{tbal}
M20	Fe250	7	140	0.4	0.87	1.21	1.00
	Fe415	7	230	0.29	0.9	0.91	0.44
M25	Fe250	8.5	140	0.4	0.87	1.48	0.68
	Fe415	8.5	230	0.29	0.9	1.1	0.533

Liquid Retaining Members subjected to axial tension only:

When the member of a liquid retaining structure is subjected to axial tension only, the member is assumed to have sufficient reinforcement to resist all the tensile force and the concrete is assumed to be uncracked.

For analysis purpose 1m length of wall and thickness'*t*' is considered. The tension in the member is resisted only by steel and hence

$$A_{st}=\frac{T}{\sigma_{st}} \text{ and } T\leq 1000\,t\,\sigma_{ct}+(m-1)A_{st}\,\sigma_{st} \text{ or } t\geq\frac{T}{1000\sigma_{ct}}\left[1-(m-1)\frac{\sigma_{ct}}{\sigma_{st}}\right]$$

Minimum thickness of the member required is tabulate in table 6.2

Table 6.2 **Minimum thickness of members under direct tension (Uncracked condition)**

Grade of concrete	Thickness of members in mm for force T in N	
	Mild steel	HYSD
M20	T/1377	T/1331
M25	T/1465	T/1423
M30	T/1682	T/1636

Liquid Retaining Members subjected to Bending Moment only:

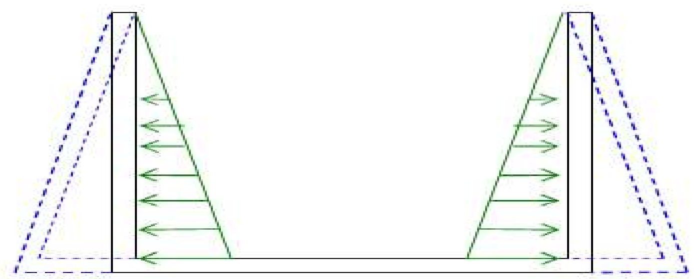
For the members subjected to BM only with the tension face in contact with water or for the members of thickness less than 225 mm, the compressive stress and tensile stresses should not exceed the value given in IS 3370. For the member of thickness more than 225 mm and for the face away from the liquid, this condition need not be satisfied and higher stress in steel may be allowed. The bending analysis is done for cracked and uncracked condition. Cracked condition: The procedure of designing is same as in working stress method except that the stresses in steel are reduced. The design coefficients for these reduced stresses in steel are given below.

Design constants for members in bending (Cracked condition)

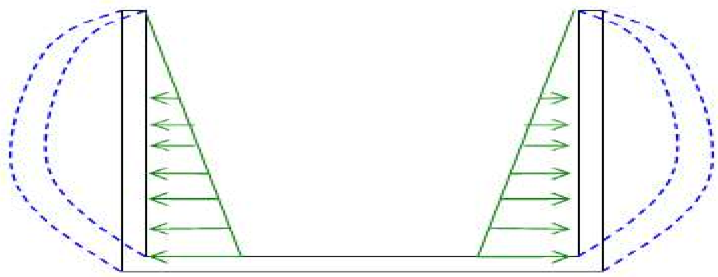
Concrete Grade	Steel Grade	σ_{cbc}	σ_{st}	k	j	Q_{bal}	p_{tbal}
For members less than 225 mm thickness and tension on liquid face							
M20	Fe250	7	115	0.445	0.851	1.33	1.36
	Fe415	7	150	0.384	0.872	1.17	0.98
For members more than 225 mm thickness and tension away from liquid face							
M20	Fe250	7	125	0.427	0.858	1.28	1.2
	Fe415	7	190	0.329	0.89	1.03	0.61

Circular Tanks resting on ground:

Due to hydrostatic pressure, the tank has tendency to increase in diameter. This increase in diameter all along the height of the tank depends on the nature of joint at the junction of slab and wall as shown in Fig6.5



Tank with flexible base

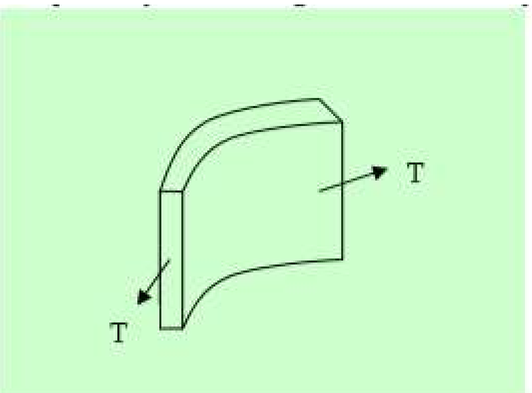


Tank with rigid base

When the joints at base are flexible, hydrostatic pressure induces maximum increase in diameter at base and no increase in diameter at top. This is due to fact that hydrostatic pressure varies linearly from zero at top and maximum at base. Deflected shape of the tank is shown in above fig. When the joint at base is rigid, the base does not move. The vertical wall deflects as shown in above fig

<u>Design of Circular Tanks resting on ground with flexible base:</u>

Maximum hoop tension in the wall is developed at the base. This tensile force T is computed by considering the tank as thin cylinder



$T = \gamma H \frac{D}{2}$; Quantity of reinforcement required in form of hoop steel is computed as $A_{st} = \frac{T}{\sigma_{st}} = \frac{\gamma HD/2}{\sigma_{st}}$ or 0.3 % (minimum)

When the thickness of the wall is less than 225 mm, the steel placed at centre. When the thickness exceeds 225mm, at each face $A_{st}/2$ of steel as hoop reinforcement is provided

In order to provide tensile stress in concrete to be less to be less than permissible stress, the stress in concrete is computed using equation

$\sigma_c = \frac{T}{A_c + (m-1)A_{st}} = \frac{\gamma HD/2}{1000t + (m-1)A_{st}}$ If $\sigma_c \le \sigma_{cat}$, where $\sigma_{cat}=0.27\sqrt{f_{ck}}$, then the section is from cracking, otherwise the thickness has to be increased so that σc is less than σcat. While designing, the thickness of concrete wall can be estimated as t=30H+50 mm, where H is in meters. Distribution steel in the form of vertical bars are provided such that minimum steel area requirement is satisfied. As base slab is resting on ground and no bending stresses are induced hence minimum steel distributed at bottom and the top are provided

Example3.3

Design a circular water tank with flexible connection at base for a capacity of 400000 liters. The tank rests on a firm level ground. The height of tank including a free board of 200 mm should not exceed 3.5m. The tank is open at top. Use M 20 concrete and Fe 415 steel.

i) Plan at base

ii) **Cross section through centre of tank**.

Solution:

Step 1: Dimension of tank

Depth of water H=3.5 -0.2 = 3.3 m Volume V = 400000/1000 = 400 m3

Area of tank A = 400/3.3 = 121.2 m2

$$\text{Diameter of tank } D = \sqrt{\frac{4A}{\pi}} = 12.42\text{m} \approx 13 \text{ m}$$

The thickness is assumed as t = 30H+50=149.160 mm

Step 2: Design of Vertical wall

$$\text{Max hoop tension at bottom } T = \gamma H \frac{D}{2} = \frac{10 \times 3.3 \times 13}{2} = 214.5\text{kN}$$

$$\text{Area of steel } A_{st} = \frac{T}{\sigma_{st}} = \frac{T}{\sigma_{st}} = \frac{214.5 \times 10^3}{150} = 1430 \text{ mm}^2$$

Minimum steel to be provided

A_{stmin}=0.24%of area of concrete = 0.24x 1000x160/100 = 384mm^2

The steel required is more than the minimum required

Let the diameter of the bar to be used be 16 mm, area of each bar =201 mm^2 Spacing of 16 mm diameter bar=1430x 1000/201= 140.6 mm c/c

Provide #16 @ 140 c/c as hoop tension steel

Step 3: Check for tensile stress

Area of steel provided $A_{stprovided}$=201x1000/140 = 1436.16 mm^2

$$\text{Modular ratio } m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$\text{Stress in concrete } \sigma_c = \frac{T}{1000t + (m-1)A_{st}} = \frac{214.5 \times 10^3}{1000 \times 160 + (13.33 - 1)1436} = 1.2 \text{ N/mm}^2$$

Permissible stress σ_{cat}=0.27$\sqrt{f_{ck}}$= 1.2 N/mm^2

Permissible stress σ_{cat}=0.27$\sqrt{f_{ck}}$= 1.2 N/mm^2

Actual stress is equal to permissible stress, hence safe.

Height from top	Hoop tension T =γHD/2(kN)	A_{st}= T/σ_{st}	Spacing of #16 mm c/c
2.3 m	149.5	996	200
1.3 m	84.5	563.33	350
Top	0	Min steel (384 mm^2)	400

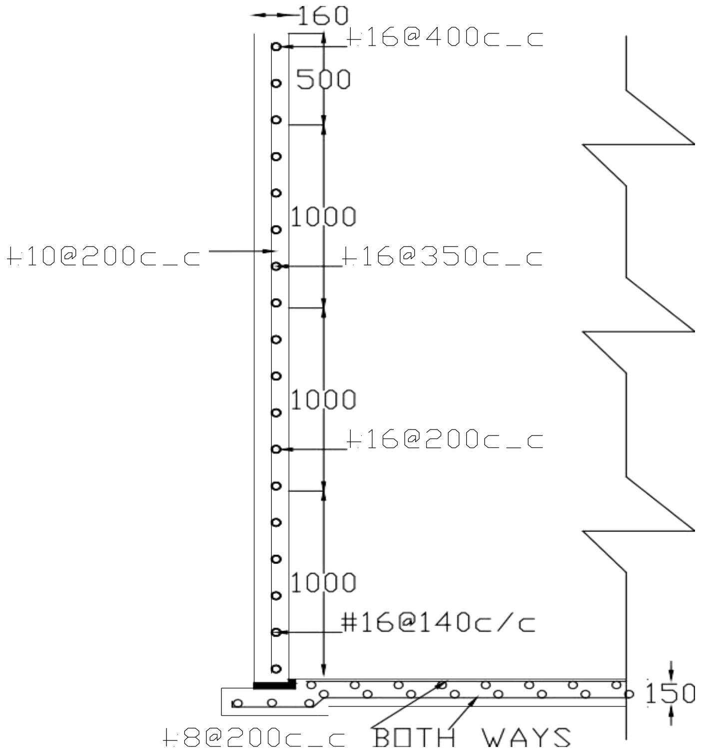
Step 5: Vertical reinforcement:

For temperature and shrinkage distribution steel in the form of vertical reinforcement is provided @ 0.24 % ie., A_{st}=384 mm^2.

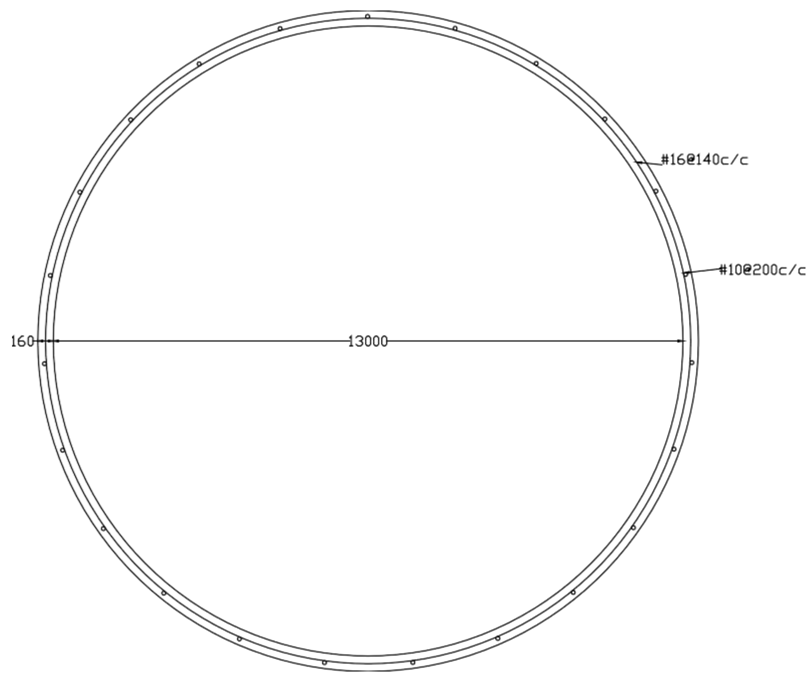
Spacing of 10 mm diameter bar = 78.54x1000/384=204 mm c/c □ 200 mm c/c

Step 6: Tank floor:

As the slab rests on firm ground, minimum steel @ 0.3 % is provided. Thickness of slab is assumed as 150 mm. 8 mm diameter bars at 200 c/c is provided in both directions at bottom and top of theslab.



Sectional Elevation



Plan at base

Example 3.4 Design a circular water tank for a storage capacity of 360000 litres. The joint between the wall and the floor of the tank is not monolithic. The tank is not monolithic. The tank is to rest at ground level. Adopt M 20 grade of concrete.

Sol. Required capacity of tank = 3,60,000 litres

$\therefore$ Volume of tank $= \frac{360000}{1000} = 360\ m^3$

Assuming the depth (H) of the water in the tank to be 3 metres, floor area of the tank

$$= \frac{360}{3} = 120\ m^2$$

Let (D) be the internal diameter of the tank

$$\therefore \quad \frac{\pi}{4} \times D^2 = 120$$

$$D = \sqrt{\frac{120 \times 4}{\pi}} = 12{\cdot}36\text{ m say } 12{\cdot}4\text{ m}$$

Max. hoop tension is given by

$$T = \frac{1}{2}\, w.H.D.$$

Let wt. of water (w) be

$$= 10\ kN/m^3$$

$$\therefore \quad T = \frac{1}{2} \times 10 \times 3 \times 12{\cdot}4$$

$$= 186\ kN = 186 \times 10^3\ N$$

Area of hoop reinforcement is given by

$$A_t = \frac{T}{\sigma_s} = \frac{186 \times 10^3}{115} = 1617\ mm^2$$

Spacing, using 16 mm ϕ hoops ($A_\phi = 201\ mm^2$)

$$= \frac{201 \times 1000}{1617} = 124\text{ mm say } 120\text{ mm c/c}$$

A_t actually provided $= \frac{201 \times 1000}{120} = 1675\ mm^2$

To fix thickness of wall :

For M 20 grade of concrete

$m = 13$

σ_{ct} = permissible direct tensile stress in concrete = $1{\cdot}2\ N/mm^2$

The thickness of the wall (t) can be obtained from equation

(*i*)
$$\frac{T}{t \times 1000 + (m-1)A_t} = \sigma_{ct}$$

or
$$\frac{186 \times 10^3}{t \times 1000 + (13-1) \times 1675} = 1{\cdot}2$$

which gives $t = 135$ mm

(*ii*) Thickness of the wall from empirical formula

$$t = 30\,H + 50 = 30 \times 3 + 50 = 140\text{ mm}$$

(*iii*) Minimum thickness at per norms = 150 mm

Hence adopt thickness of wall = 150 mm uniformly, throughout the height of tank.

Since the thickness of wall is less, the hoof reinforcement will be placed at the centre of the wall thickness.

Since the water pressure and hence the hoop tension decreases towards top, the area of reinforcement can be reduced toward top.

Curtailment of reinforcement :

A_t required at a depth of 2 m below top :

Value of hoop tension at this depth

$$T_1 = \frac{1}{2} \times 10 \times 2 \times 12.4 = 24 \text{ kN}$$

$$A_t = \frac{124 \times 10^3}{115} = 1078 \text{ mm}^2$$

Spacing of 16 mm ϕ bars $= \dfrac{201 \times 1000}{1078} = 186$ mm say 180 c/c

A_t at 1 m below top $= \dfrac{1}{2} \times 1078 = 539 \text{ mm}^2$

Spacing of 16 mm ϕ bars $= \dfrac{201 \times 1000}{539} = 372$ say 370 mm c/c

Vertical reinforcement :

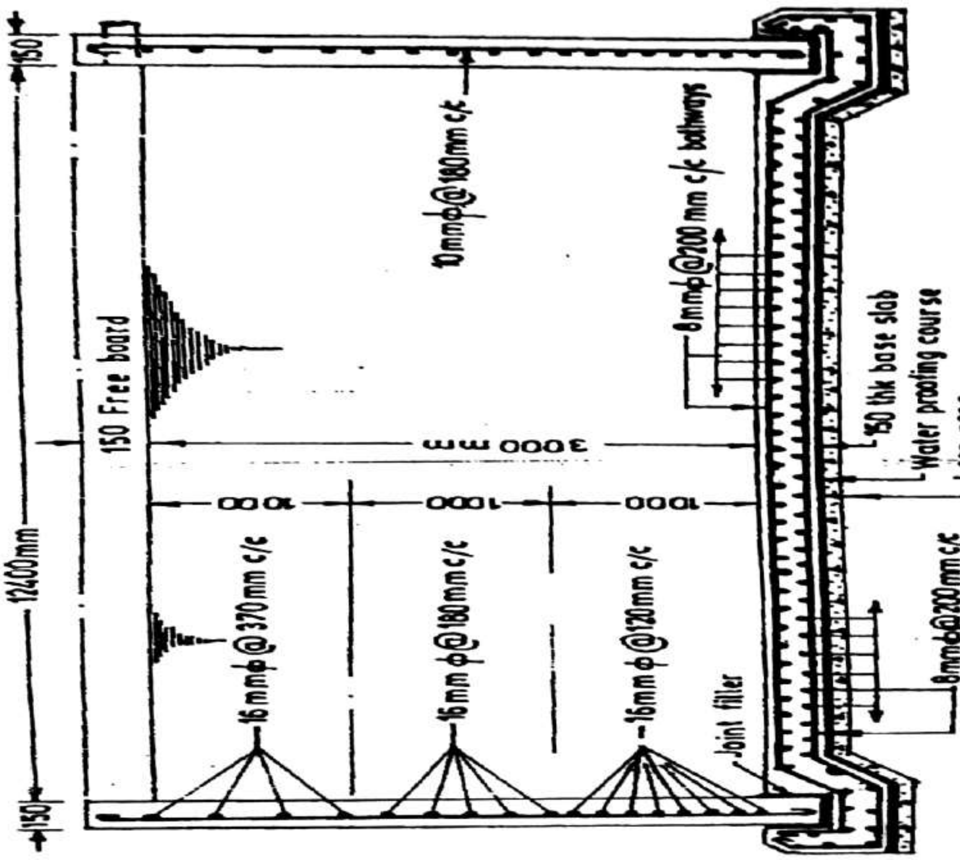
Percentage of distribution reinforcement to be provided in the vertical direction

$$= 0.3 - 0.1\left(\frac{150 - 100}{350}\right) = 0.29\%$$

Area of distribution bars $= \dfrac{0.29 \times 150 \times 1000}{100} = 435 \text{ mm}^2$

Spacing, using 10 mm ϕ bars ($A_\phi = 78.5 \text{ mm}^2$)

$$= \frac{78.5 \times 1000}{435} = 180 \text{ mm c/c}$$



Design of base of floor slab:

Adopt thickness of the base slab = 150 mm

Minimum area of reinforcement = 0.3x150x1000/100 = 450 mm^2

Area of reinforcement on each face = 450/2 = 225mm^2

Spacing, using 8 mm diameter bars = 50x1000/225 = 222 mm

Hence provide 8 mm diameter bars at 200 mm c/c both ways both at top and bottom of the slab.

Design procedure of a circular tank with rigid joint between floor and wall

Depending upon the depth of liquid (H), the thickness of the wall is assumed from the empirical formula

$$t = (30H + 50) \text{ mm}$$

or

$$t = 150 \text{ mm which is more}$$

The thickness of wall thus fixed is converted in metres. The diameter (D) of the tank and the depth of water being computed beforehand, the values of $\frac{H}{D}$ and $\frac{H}{t}$ are then calculated.

Referring to the Table 31.2, the coefficients for F and K can be obtained.

Knowing the co-efficients, the following formulae can directly be used to design the tank :

(1) Maximum circumferential or hoop tension

or

$$T = \frac{wHD}{2}(1-K)$$

(2) Maximum bending moment at the base = FwH^3

(3) Position of maximum circumferential tension

or

$$h = KH$$

Next step is to check the thickness provided for the wall from maximum bending moment consideration and provide the necessary area of vertical reinforcements. Sufficient area of steel must be provided at the height h to resist maximum tension. Above this height, the area of reinforcement can be uniformly decreased and below this, the area of steel is maintained constant.

Example 3.5 Design a circular tank of 200000 litres capacity. The joint between slab and side wall is to be rigid. Good foundation for the tank available at a depth of 0.6 metre below the ground level. Assume suitable working stresses.

Sol. Capacity of the tank = 200000 litres.

∴ Volume of tank = 200 cu.m.

Let the depth of water 4 metres

∴ Area of the base $= \frac{200}{4} = 50$ sq.m.

∴ The diameter (D) of base

$$= \sqrt{\frac{50 \times 4}{\pi}} = 7{\cdot}97 \text{ m}$$

= say 8 m.

The approximate thickness of wall for 4 metres depth of water as given by the empirical formula :

$$t = 30H + 50 = 30 \times 4 + 50 = 170 \text{ mm}$$

which is greater than 150 mm, hence adopt

$$t = 170 \text{ mm} = 0{\cdot}17 \text{ m}$$

$$\therefore \quad \frac{H}{D} = \frac{4}{8} = 0{\cdot}5$$

and

$$\frac{H}{t} = \frac{4}{0{\cdot}17} = 23{\cdot}5$$

From Table 31·2, we find

$$F = 0{\cdot}011$$

$$K = 0{\cdot}35$$

(1) Max. circumferential tension

$$T = \frac{w.H.D}{2}(1 - K)$$

$$= \frac{10 \times 4 \times 8}{2}(1 - 0{\cdot}35)$$

$$= 104 \text{ kN}$$

(2) Max. bending moment at base

$$M = F.w.H^2$$

$$= 0{\cdot}011 \times 10 \times 4^3$$

$$= 7{\cdot}04 \text{ kNm}$$

(3) Position of maximum circumferential tension

$$h = K.H.$$

$$= 0{\cdot}35 \times 4 = 1{\cdot}4 \text{ m}$$

Using M 20 grade of concrete, we have

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

$$\sigma_{st} = 115 \text{ N/mm}^2$$

$$m = 13$$

$$\therefore \quad k = \frac{n}{d} = \frac{m \times \sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}}$$

$$= \frac{13 \times 7}{13 \times 7 + 115} = 0{\cdot}442$$

$$\therefore \quad j = 1 - \frac{k_1}{3}$$

$$= 1 - \frac{0{\cdot}442}{3} = 0{\cdot}853$$

$\therefore \quad R = \frac{1}{2}\sigma_{cbc}.j.k$

$= \frac{1}{2} \times 7 \times 0.853 \times 0.442$

$= 1.32$

The required thickness of the wall from B.M. consideration

$$d = \sqrt{\frac{M}{R.b}}$$

$$= \sqrt{\frac{7.04 \times 10^6}{1.32 \times 1000}} = 73 \text{ mm}$$

Hence the assumed thickness of 170 mm is in order.

Assuming a cover 35 mm upto the centre of main bars on water side, the effective thickness of the wall

$$= 170 - 35 = 135 \text{ mm}$$

Area of steel required

$$A_t = \frac{M}{j.d.\sigma_{st}}$$

$$= \frac{7.04 \times 10^6}{0.853 \times 135 \times 115}$$

$$= 532 \text{ mm}^2$$

Spacing of 12 mm ϕ bars ($A_\phi = 113 \text{ mm}^2$)

$$= \frac{113 \times 1000}{532} = 212 \text{ say } 200 \text{ mm c/c}$$

Provide 12 mm ϕ bars @ 200 mm c/c in the form of vertical bars on the inner face of wall place at a clear cover of 25 mm upto a height $= h = 1.4$ m above the base slab. Above this height alternate bars can be curtailed.

Design of section for hoop tension :

Max. hoop tension (T) = 104 kN at 1.4 m above base

Area of hoop reinforcement

$$A_t = \frac{T}{\sigma_t} = \frac{104 \times 1000}{115}$$

$$= 904 \text{ mm}^2$$

Let up provide hoop reinforcement at both the face.

$\therefore$ Area of hoop reinforcement on each face

$$= \frac{904}{2} = 452 \text{ mm}^2$$

Spacing, using 12 mm ϕ rings ($A_\phi = 113\ mm^2$)

$$= \frac{113 \times 1000}{452} = 250 \text{ mm c/c}$$

Hence provide 12 mm ϕ rings at 250 mm c/c on both face of the wall. The spacing will be kept constant upto 1·4 m above the top of the base slab and beyond this height the spacing may be increased.

Check for tensile stress in composite section :

$$\sigma_t = \frac{T}{t \times 1000 + (m-1) A_t}$$

$$= \frac{104 \times 1000}{170 \times 1000 + (13-1)\,904}$$

$= 0·575\ N/mm^2 < 1·2\ N/mm^2$, hence safe.

Distribution reinforcement :

Percentage of distribution reinforcement

$$= 0·3 - 0·1 \times \left(\frac{170-100}{350}\right) = 0·28\%$$

Area of steel $= \frac{0·28}{100} \times 170 \times 1000$

$= 476\ mm^2$

Area of steel on each face

$$= \frac{476}{2} = 238\ mm^2$$

Spacing, using 8 mm ϕ bars ($A_\phi = 50\ mm^2$)

$$= \frac{50 \times 1000}{238} = 210 \text{ mm say } 200 \text{ mm c/c}$$

Hence provide 8 mm of vertical distribution bars @ 200 mm c/c on external face only. On the inner face vertical reinforcement provided for cantilever action will serve the function of distribution bars as well and no additional reinforcement need be provided.

Curtailment of hoop reinforcement :

At 2 m below top

Hoop tension $T = \frac{1}{2}\, w.h.D$

$= \frac{1}{2} \times 10 \times 2 \times 8 = 80$ kN

$$A_t = \frac{80 \times 10^3}{115} = 696\ mm^2$$

Area of rings on each face

$$= \frac{696}{2} = 348\ mm^2$$

Spacing of 12 mm ϕ bars

$$= \frac{113 \times 1000}{348} = 324 \text{ say } 320 \text{ mm c/c}$$

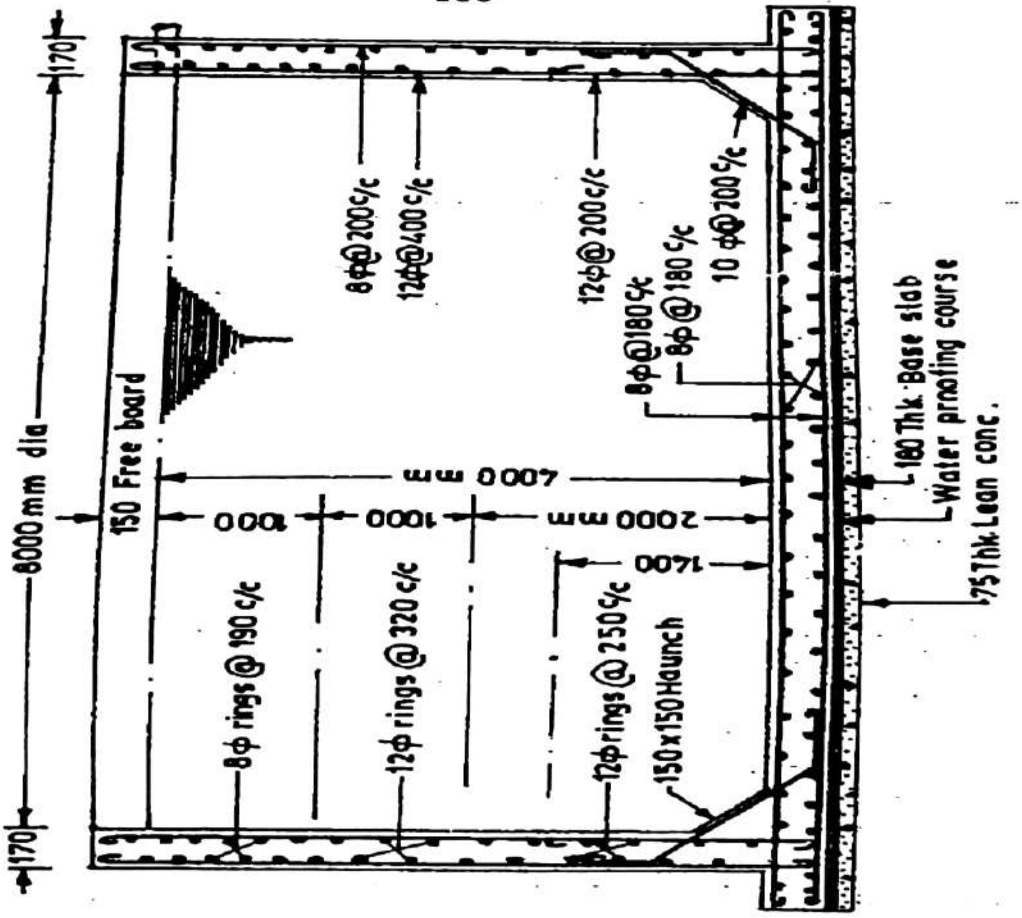
A_t 1 m below top

Hoop tension $= \frac{1}{2} \times 10 \times 8 = 40$ kN

$$A_t = \frac{40 \times 10^3}{115} = 348 \text{ mm}^2$$

Minimum area of reinforcement

$$= \frac{0.3}{100} \times 170 \times 1000 = 510 \text{ mm}^2$$



Reinforcement details

UNIT IV
DESIGN OF SLABS AND FOUNDATION

4.1 INTRODUCTION

Common practice of design and construction is to support the slabs by beams and support the beams by columns. This may be called as beam-slab construction. The beams reduce the available net clear ceiling height. Hence in warehouses, offices and public halls sometimes beams are avoided and slabs are directly supported by columns. This types of construction is aesthetically appealing also. These slabs which are directly supported by columns are called **Flat Slabs.** Fig. 4.1 shows a typical flatslab.

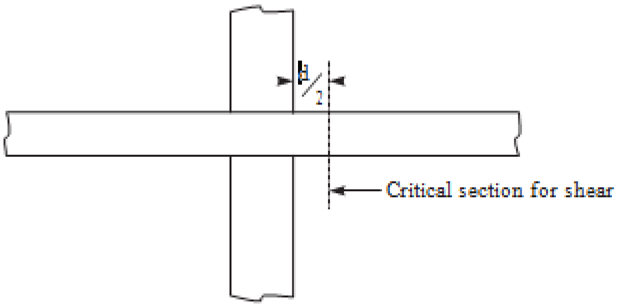


Fig. 4.1 A typical flat slab (without drop and column head)

The column head is sometimes widened so as to reduce the punching shear in the slab. The widened portions are called **column heads.** The column heads may be provided with any angle from the consideration of architecture but for the design, concrete in the portion at 45º on either side of vertical only is considered as effective for the design [Ref. Fig. 4.2].

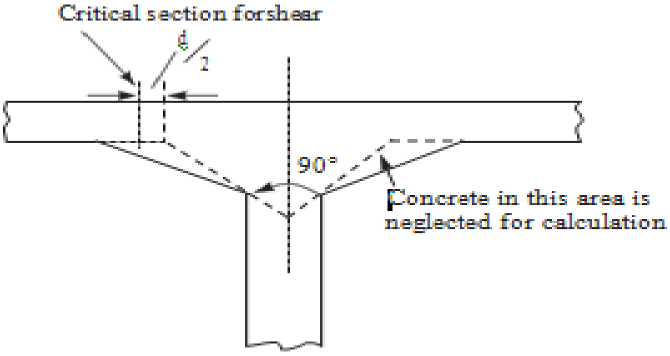


Fig. 4.2 Slab without drop and column with column head

Moments in the slabs are more near the column. Hence the slab is thickened near the columns by providing the drops as shown in Fig. 4.3. Sometimes the drops are called as capital of the column. Thus we have the following types of flat slabs:

(i) Slab without drop and without column head(Fig4.1)

(ii) Slab without drop and column with column head(Fig4.2)

(iii) Slabs with drop and column without column head(Fig.4.3)

(iv) Slabs with drop and column with column head(Fig.4.3)

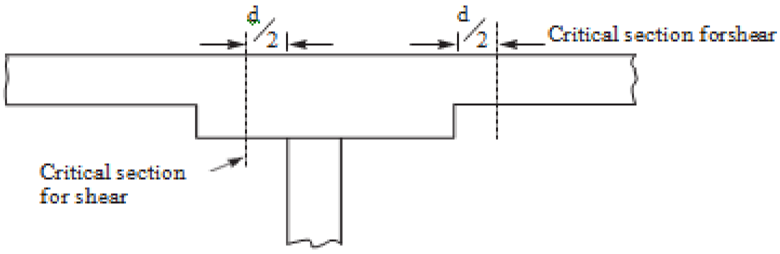


Fig. 4.3 Slab with drop and column without column head

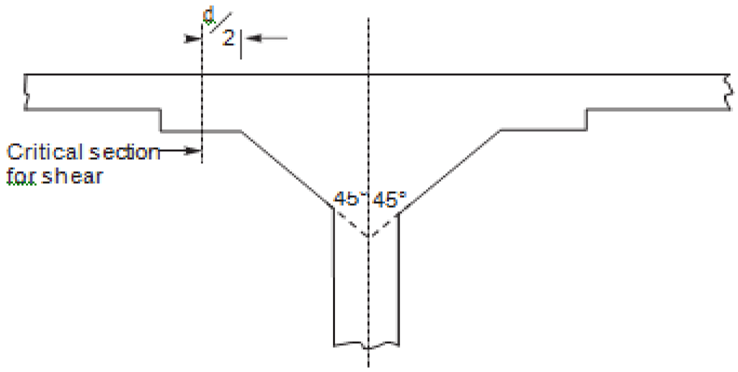


Fig. 4.4 Slab with drop and with column head

The portion of flat slab that is bound on each of its four sides by centre lines of adjacent columns is called panel. The panel shown in Fig4.5 has size L1 x L2. A panel may be divided into column strips and middle strips. Column strip means a design strip have a width of 0.25L1 x 0.25L2, whichever is less. The remaining middle portion which is bound by the column strips is called middle strip. Fig 4.5 shows the division of flat slab panel into column and middle strips in the direction y.

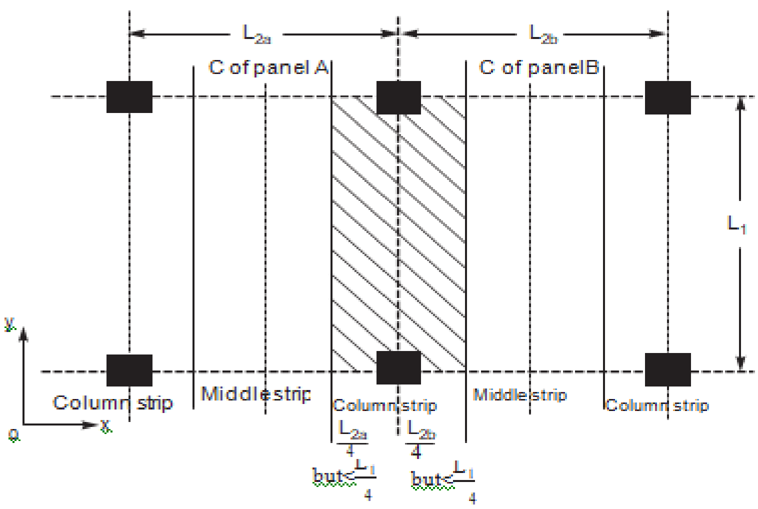


Fig. 4.5 panels, column strip and middle strip in y-direction

4.2 PROPORTIONING OF FLATSLABS

IS 456-2000 [Clause 31.2] gives the following guidelines for proportioning.

4.2.1 Drops

The drops when provided shall be rectangular in plan, and have a length in each direction not less than one third of the panel in that direction. For exterior panels, the width of drops at right angles to the non- continuous edge and measured from the centre-line of the columns shall be equal to one half of the width of drop for interior panels.

4.2.2 ColumnHeads

Where column heads are provided, that portion of the column head which lies within the largest right circular cone or pyramid entirely within the outlines of the column and the column head, shall be considered for design purpose as shown in Figs. 4.2 and 4.4.

4.2.3 Thickness of FlatSlab

From the consideration of deflection control IS 456-2000 specifies minimum thickness in terms of span to effective depth ratio. For this purpose larger span is to be considered. If drop as specified in

4.2.1 is provided, then the maximum value of ratio of larger span to thickness shall be

= 40, if mild steel is used

= 32, if Fe 415 or Fe 500 steel is used

If drops are not provided or size of drops do not satisfy the specification 4.2.1, then the ratio shall not exceed 0.9 times the value specified above *i.e.*,

= 40 × 0.9 = 36, if mild steel is used.

= 32 × 0.9 = 28.8, if HYSD bars are used

It is also specified that in no case, the thickness of flat slab shall be less than 125 mm.

4.3 DETERMINATIONOFBENDINGMOMENTANDSHEARFORCE

For this IS 456-2000 permits use of any one of the following two methods:

(a) The Direct DesignMethod

(b) The Equivalent FrameMethod

4.4 THE DIRECT DESIGNMETHOD

This method has the limitation that it can be used only if the following conditions are fulfilled:

(a) There shall be minimum of three continuous spans in each directions

(b) The panels shall be rectangular and the ratio of the longer span to shorter span within a panel shall not be greater than 2.

(c) The successive span length in each direction shall not differ by more than one-third of longer span.

(d) The design live load shall not exceed three times the design dead load.

(e) The end span must be shorter but not greater than the interior span.

(f) It shall be permissible to offset columns a maximum of 10% of the span in the direction of the offset not withstanding the provision in (b).

Total Design Moment

The absolute sum of the positive and negative moment in each direction is given by

$$Mo = WLn/8$$

Where,

M_0 = Total moment

W = Design load on the area $L_2 \times L_n$

L_n = Clear span extending from face to face of columns, capitals, brackets or walls but not less than $0.65L_1$

L_1 = Length of span in the direction of M_0; and L_2 = Length of span transverse to L_1

In taking the values of L_n, L_1 and L_2, the following clauses are to be carefully noted:

(a) Circular supports shall be treated as square supports having the same area *i.e.*, squares of size 0.886D.

(b) When the transverse span of the panel on either side of the centre-line of support varies, L2 shall be taken as the average of the transverse spans in Fig 4.5 it is given by $(L_{2a} + L_{2b})/2$.

(c) When the span adjacent and parallel to an edge is being considered, the distance from the edge to the centre-line of the panel shall be substituted for L2.

Distribution of Bending Moment in to –ve and +ve Moments

The total design moment M_0 in a panel is to be distributed into –ve moment and +ve moment as specified below:

In an interior span

Negative Design Moment 0.65 M_0
Positive Design Moment 0.35 M_0

In an end span

Interior negative design moment

$$= \left[0.75 - \frac{0.10}{1 + \frac{1}{\alpha_c}}\right] M_0$$

Positive design moment

$$= \left[0.63 - \frac{0.28}{1 + \frac{1}{\alpha_c}}\right] M_0$$

Exterior negative design moment

$$= \left[\frac{0.65}{1 + \frac{1}{\alpha_c}}\right] M_0$$

where α_c is the ratio of flexural stiffness at the exterior columns to the flexural stiffness of the slab at a joint taken in the direction moments are being determined and is given by

$$\alpha_c = \frac{\sum K_c}{\sum K_s}$$

Where

K_C = Sum of the flexural stiffness of the columns meeting at the joint; and K_S = Flexural stiffness of the slab, expressed as moment per unit rotation.

Distribution of Bending Moments across the Panel Width

The +ve and –ve moments found are to be distributed across the column strip in a panel as shown in Table 1.1. The moment in the middle strip shall be the difference between panel and the column strip moments.

Table1.1 Distribution of Moments across the Panel Width in a Column Strip

S. No.	Distributed Moment	Per cent of Total Moment
a	Negative BM at the exterior support	100
b	Negative BM at the interior support	75
c	Positive bending moment	60

Moments in Columns

In this type of constructions column moments are to be modified as suggested in IS 456–2000 [Clause No. 31.4.5].

Shear force:

The critical section for shear shall be at a distance d/2 from the periphery of the column /capital drop panel. Hence if drops are provided there are two critical sections near columns. These critical sections are shown in Figs. 4.1 to 4.4. The shape of the critical section in plan is similar to the support immediately below the slab as shown in Fig. 4.6.

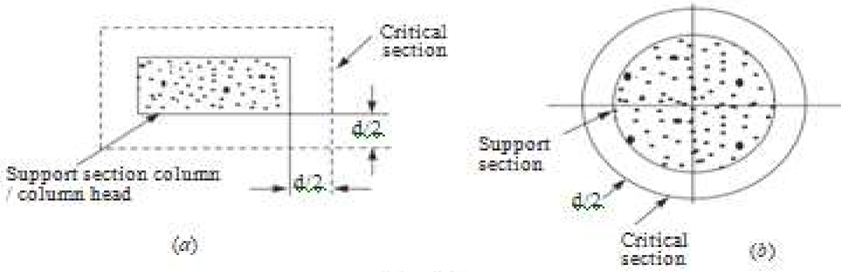


Fig 4.6

For columns sections with re-entrant angles, the critical section shall be taken as indicated in Fig. 4.7.

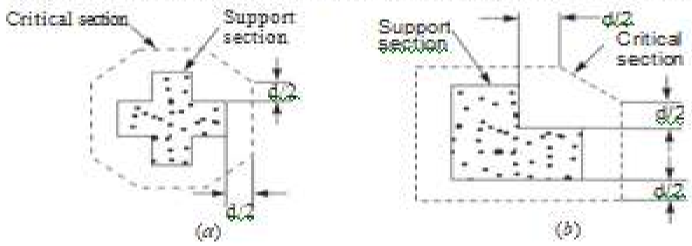


Fig.4.7

In case of columns near the free edge of a slab, the critical section shall be taken as shown in Fig. 4.8.

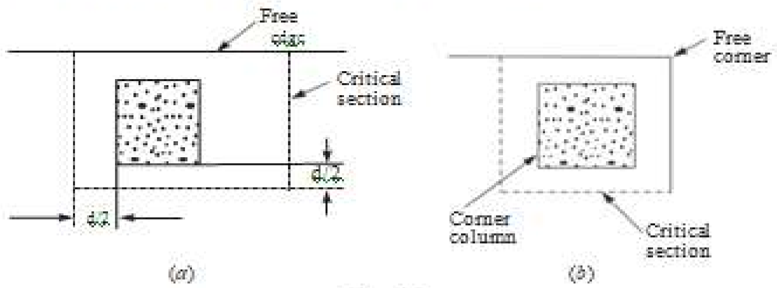


Fig.4.8

The nominal shear stress may be calculated as

$$\tau_v = \frac{V}{b_0 d}$$

where V – is shear force due todesign

b_0 – is the periphery of the critical section

d – is the effective depth

The permissible shear stress in concrete may be calculated as $k_s\tau_c$, where $k_s = 0.5 + \beta_c$ but not greater than 1, where β_c is the ratio of short side to long side of the column/capital; and

$$\tau_c = 0.25 \sqrt{f_{ck}}$$

If shear stress $\tau_v < \tau_c$ – no shear reinforcement are required. If $\tau_c < \tau_v < 1.5\tau_c$, shear reinforcement shall be provided. If shear stress exceeds 1.5 τ_c flat slab shall be redesigned.

4.5 EQUIVALENT FRAMEMETHOD

IS 456–2000 recommends the analysis of flat slab and column structure as a rigid frame to get design moment and shear forces with the following assumptions:

(a) Beam portion of frame is taken as equivalent to the moment of inertia of flat slab bounded laterally by centre line of the panel on each side of the centre-line of the column. In frames adjacent and parallel to an edge beam portion shall be equal to flat slab bounded by the edge and centre-line of the adjacent panel.

(b) Moment of inertia of the members of the frame may be taken as that of the gross section of the concrete alone.

(c) Variation of moment of inertia along the axis of the slab on account of provision of drops shall be taken into account. In the case of recessed or coffered slab which is made solid in the region of the columns, the stiffening effect may be ignored provided the solid part of the solid does not extend more than 0.15 l_{ef} into the span measured from the centre-line of the columns. The stiffening effect of flared columns head may be ignored.

(d) Analysis of frame may be carried out with substitute frame method or any other accepted method like moment distribution or matrix method.

Loading Pattern

When the live load does not exceed ¾th of dead load, the maximum moments may be assumed to occur at all sections when full design live load is on the entire slab.

If live load exceeds ¾th dead load analysis is to be carried out for the following pattern of loading also:

(i) To get maximum moment near mid span

- ¾th of live load on the panel and full live load on alternate panel

(ii) To get maximum moment in the slab near the support

- $3/4^{th}$ of live load is on the adjacent panel only.

It is to be carefully noted that in no case design moment shall be taken to be less than those occurring with full design live load on all panels.The moments determined in the beam of frame (flat slab) may be reduced in such proportion that the numerical sum of positive and average negative moments is not less than the value of total design

4.6 SLABREINFORCEMENT

Spacing

The spacing of bars in a flat slab shall not exceed 2 times the slab thickness.

Area of Reinforcement when the drop panels are used, the thickness of drop panel for determining area of reinforcement shall be the lesser of the following:

(a) Thickness of drop

(b) Thickness of slab plus one quarter the distance between edge of drop and edge of capital. The minimum percentage of the reinforcement is same as that in solid slab is 0.12% if HYSD bars used

0.15% if mild steel is used

Minimum length of reinforcement

At least 50% of bottom bars should be from support to support. The rest may be bent up. The minimum length of different reinforcement in flat slab should be as shown in Fig 4.9. If adjacent spans are not equal, the extension of the negative reinforcement beyond each face shall be based on the longer span. All slab reinforcement should be anchored property at discontinuous edges.

Example 4.1: Design an interior panel of a flat slab of size 5 m × 5 m without providing drop and column head. Size of columns is 500 × 500 mm and live load on the panel is 4 kN/m^2. Take floor finishing load as 1 kN/m^2. Use M20 concrete and Fe 415 steel.

Solution:

Thickness

Since drop is not provided and HYSD bars are used span to thickness ratio shall not exceed 1/(0.9x32) = 1/28.8

Minimum thickness required = span/28.8 = 5000/28.8 = 173.6 mm

Let d =175mm and D = 200mm

Loads

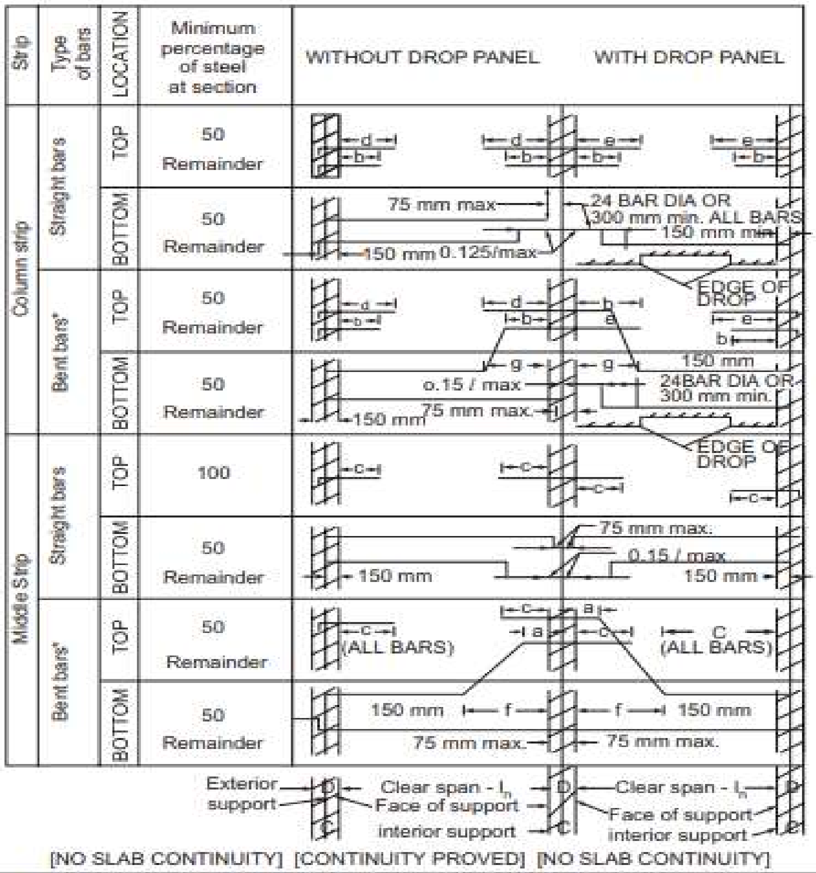
Self-weightofslab = 0.20 × 25 = 5 kN/m^2

Finishingload = 1kN/m^2

Liveload = 4kN/m^2

Totalworkingload = 10kN/m^2

Factoredload = 1.5 × 10 = 15kN/m^2



Bar Length From Face of Support							
	Minimum Length						Maximum Length
Mark	a	b	c	d	e	f	g
Length	0.14 l_n	0.20 l_n	0.22 l_n	0.30 l_n	0.33 l_n	0.20 l_n	0.24 l_n

* Bent bars at exterior supports may be used if a general analysis is made.

Fig.4.9 Minimum bend joint locations and extensions for reinforcement in flat slabs

$$L_n = 5 – 0.5 = 4.5 \text{ m}$$

Total design load in a panel W = 15 L_2 L_n = 15 × 5 × 4.5 = 337.5 kN

Moments

Panel Moment Mo = WLn/8 = 337.5x4.5/8 = 189.84 kNm

Panel–vemoment = 0.65 × 189.84 = 123.40kNm

Panel+vemoment = 0.35 × 189.84 = 0.35 × 189.84 = 66.44kNm

Distribution of moment into column strips and middle strip.

	Column Strip in kNm	*Middle Strip in kNm*
–ve moment	0.75 × 123.40 = 92.55	30.85
+ve moment	0.60 × 66.44 = 39.86	26.58

Checking the thickness selected:

Since Fe 415 steel is used,

$$M_{u\ lim} = 0.138 f_{ck} b\, d^2$$

Width ofcolumnstrip = 0.5 × 5000 = 2500mm

$$M_u\ lim = 0.138 \times 20 \times 2500 \times 175^2 = 211.3125 \times 10^6 Nmm$$

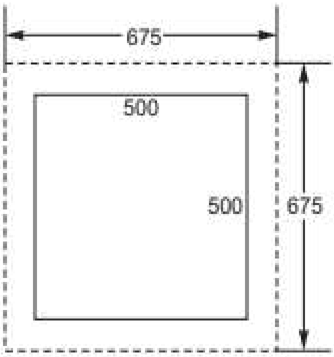
$$= 211.3125\ kNm$$

Hence singly reinforced section can be designed *i.e.*, thickness provided is satisfactory from the consideration of bending moment.

Check for Shear

The critical section for shear is at a distance $\frac{d}{2}$ from the column face. Hence periphery of critical section around a column is square of a size = 500 + d = 500 + 175 = 675 mm

Shear to be resisted by the critical section



$$V = 15 \times 5 \times 5 - 15 \times 0.675 \times 0.675$$

$$= 368.166\ kN$$

$$\therefore \quad \tau_v = \frac{368.166 \times 1000}{4 \times 675 \times 175} = 0.779\ N/mm^2$$

$$k_s = 1 + \beta_c \text{ subject to maximum of 1.}$$

$$\beta_c = \frac{L_1}{L_2} = \frac{5}{5} = 1$$

$$\therefore \quad k_s = 1$$

$$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118\ N/mm^2$$

safe in shear since $\tau_v < \tau_c$

Reinforcement

For –ve moment in column strip:

$$M_u = 92.55\ kNm$$

$$92.55 \times 10^6 = 0.87 f_y\ A_{st}\ d\left[1 - \frac{A_{st}}{bd}\frac{f_y}{f_{ck}}\right]$$

$$= 0.87 \times 415 \times A_{st} \times 175\left[1 - \frac{A_{st}}{2500 \times 175} \times \frac{415}{20}\right]$$

i.e., $$1464.78 = A_{st}\left[1 - \frac{A_{st}}{21084.3}\right]$$

i.e., $$A_{st}^2 - 21084.3A_{st} + 1464.78 \times 21084.3 = 0$$

$$A_{st} = 1583.74\ mm^2$$

This is to be provided in a column strip of width 2500 mm. Hence using 12 mm bars, spacing required is given by

$$s = \frac{\pi/4 \times 12^2}{1583.74} \times 2500 = 178\ mm$$

Provide 12 mm bars at 175 mm c/c.

For +ve moment in column strip:

$$M_u = 39.86\ kNm$$

$$\therefore \qquad 39.86 \times 10^6 = 0.87 \times 415 \times A_{st} \times 175\left[1 - \frac{A_{st}}{2500 \times 175} \times \frac{415}{20}\right]$$

$$630.86 = A_{st}\left[1 - \frac{A_{st}}{21084.3}\right]$$

or $$A_{st}^2 - 21084.3\, A_{st} + 630.86 \times 21084.3 = 0$$

$$\therefore \qquad A_{st} = 651\ \text{mm}^2$$

Using 10 mm bars, spacing required is

$$s = \frac{\pi/4 \times 10^2}{651} \times 2500 = 301.6\ \text{mm} < 2 \times \text{thickness of slab}$$

Hence provide 10 mm bars at 300 mm c/c.

Provide 10 mm diameter bars at 300 mm c/c in the middle strip to take up –ve and +ve moments.

Since span is same in both directions, provide similar reinforcement in other direction also.

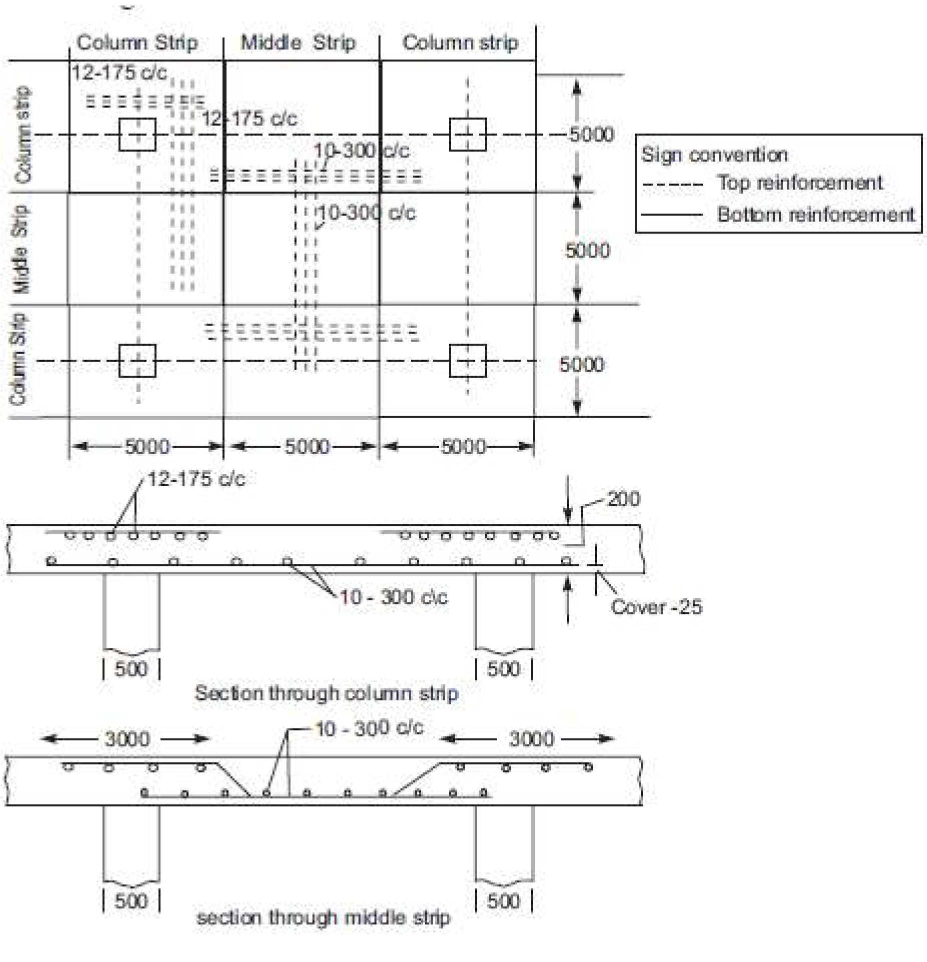


Fig 4.10 Reinforcement details (all dimensions in mm units)

Example 4.2: Design an interior panel of a flat slab with panel size 6 × 6 m supported by columns of size 500 × 500 mm. Provide suitable drop. Take live load as 4 kN/m^2. Use M20 concrete and Fe 415 steel.

Solution :

Thickness : Since Fe 415 steel is used and drop is provided, maximum span to thickness ratio permitted is 32

$\therefore$ Thickness of flat slab $= \dfrac{6000}{32} = 187.5$ mm

Provide 190 mm thickness. Let the cover be 30 mm

$\therefore$ Overall thickness D = 220 mm

Let the drop be 50 mm. Hence at column head, d = 240 mm and D = 270 mm

Size of Drop

It should not be less than $\frac{1}{3} \times 6$ m = 2 m

Let us provide 3 m × 3 m drop so that the width of drop is equal to that of column head.

$\therefore$ Width of column strip = width of middle strip = 3000 mm.

Loads

For the purpose of design let us take self-weight as that due to thickness at column strip

$\therefore$ Self-weight $= 0.27 \times 1 \times 1 \times 25 = 6.75$ kN/m^2

Finishing load $= 1.00$ kN/m^2

Live load $= 4.00$ kN/m^2

Total load $= 11.75$ kN/m^2

$\therefore$ Design (factored) load $= 1.5 \times 11.75 = 17.625$ kN/m^2

Clear span $L_n = 6 - 0.5 = 5.5$ m

$\therefore$ Design load $W_0 = W_u \times L_2 \times L_n$

$= 17.625 \times 6 \times 5.5$

$= 581.625$ kN

Design Total Moment

Total moment

$$M_0 = \frac{W_0 L_n}{8} = \frac{581.625 \times 5.5}{8} = 400 \text{ kNm}$$

$\therefore$ Total negative moment $= 0.65 \times 400 = 260$ kNm

Total positive moment $= 0.35 \times 400 = 140$ kNm

The above moments are to be distributed into column strip and middle strip

	Column Strip	*Middle Strip*
–ve moment	$0.75 \times 260 = 195$ kNm	$0.25 \times 260 = 65$ kNm
+ve moment	$0.6 \times 140 = 84$ kNm	$0.4 \times 140 = 56$ kNm

Width of column strip = width of middle strip = 3000 mm

$M_{u\,lim} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 3000 \times 240^2 = 476.928 \times 10^6$ Nmm

$= 476.928$ kNm

Thus $M_{u\,lim} > M_u$. Hence thickness selected is sufficient.

Check for Shear

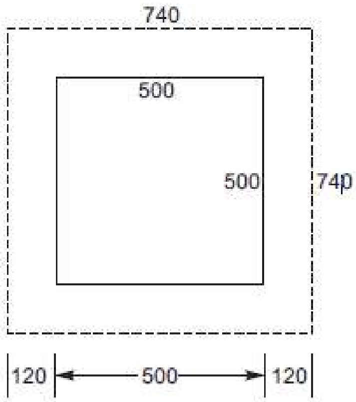
The critical section is at a distance

$$\frac{d}{2} = \frac{240}{2} = 120 \text{ mm from the face of column}$$

$\therefore$ It is a square of size $= 500 + 240 = 740$ mm

$$V = \text{Total load} - \text{load on } 0.740 \times 0.740 \text{ area}$$
$$= 17.625 \times 6 \times 6 - 17.625 \times 0.740 \times 0.740$$
$$= 624.849 \text{ kN}$$

$\therefore$ Nominal shear $= \tau_v = \frac{624.489 \times 1000}{4 \times 740 \times 240} = 0.880 \text{ N/mm}^2$



Shear strength $= k_s \, \tau_c$

where $k_s = 1 + \beta_c$ subject to maximum of 1

where $\beta_c = \frac{L_1}{L_2} = 1$

$\therefore$ $k_s = 1$

$$\tau_c = 0.25\sqrt{20} = 1.118 \text{ N/mm}^2$$

Design shear stress permitted

$$= 1.118 \text{ N/mm}^2 > \tau_v$$

Hence the slab is safe in shear without shear reinforcement also.

Shear strength may be checked at distance $\frac{d}{2}$ from drop. It is quite safe since drop size is large.

Reinforcement

(*a*) For –ve moment in column strip

$$M_u = 195 \text{ kNm}$$

Thickness $d = 240$ mm

$$\therefore \quad M_u = 0.87 f_y A_{st} d\left[1 - \frac{A_{st}}{b \times d} \times \frac{f_y}{f_{ck}}\right]$$

$$195 \times 10^6 = 0.87 \times 415 \times A_{st} \times 240 \left[1 - \frac{A_{st}}{3000 \times 240} \times \frac{415}{20}\right]$$

$$2250.38 = A_{st}\left[1 - \frac{A_{st}}{34698.8}\right]$$

$$A_{st}^2 - 34698.8 \, A_{st} + 2250.38 \times 34698.8 = 0$$

Using 12 mm bars, spacing required is

$$s = \frac{\pi/4 \times 12^2}{2419} \times 3000 = 140.26 \text{ mm}$$

Provide 12 mm bars at 140 mm c/c

(*b*) For +ve moment in column strip

$$M_u = 84 \text{ kNm} = 84 \times 10^6 \text{ Nmm. Thickness } d = 190 \text{ mm}$$

$$84 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190 \left[1 - \frac{A_{st}}{3000 \times 240} \times \frac{415}{20}\right]$$

$$1224.5 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$\therefore$ $$A_{st} = 1285 \text{ mm}^2$$

Using 10 mm bars

$$s = \frac{\pi/4 \times 10^2}{1285} \times 3000 = 183 \text{ mm}$$

Provide 10 mm bars at 180 mm c/c

(*c*) For –ve moment in middle strip:

$$M_u = 65 \text{ kNm}; \qquad \text{Thickness} = 190 \text{ mm}$$

$$65 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190\left[1 - \frac{A_{st}}{3000 \times 190} \times \frac{415}{20}\right]$$

$$947.5 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$A_{st}^2 - 27469.9\, A_{st} + 947.5 \times 27469.9 = 0$$

$$A_{st} = 983 \text{ mm}^2 \text{ in 3000 mm width}$$

Using 10 mm bars

$$s = \frac{\pi/4 \times 10^2}{983} \times 3000 = 239.7 \text{ mm}$$

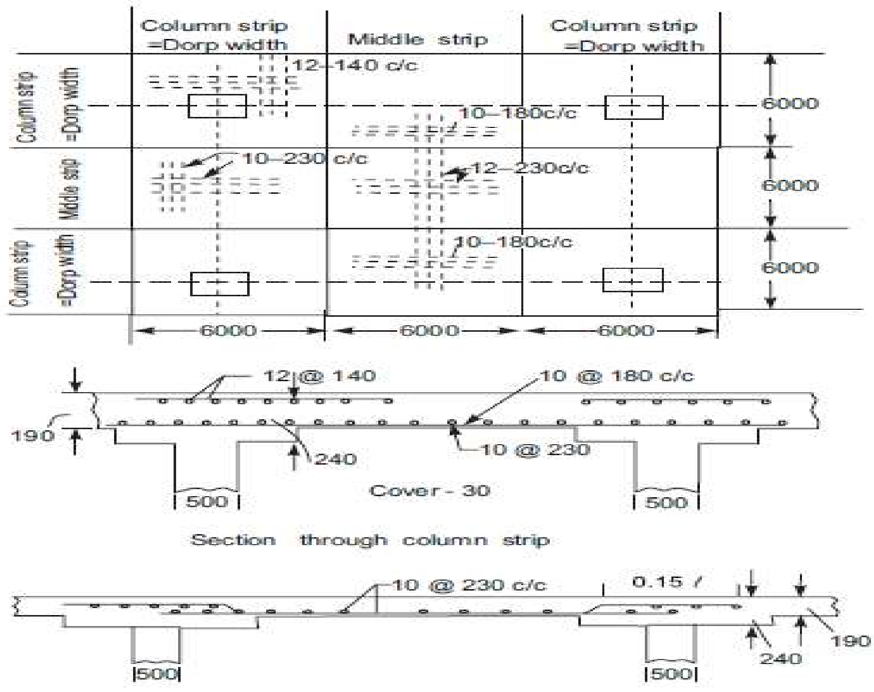
Provide 10 mm bars at 230 mm c/c

(*d*) For +ve moment in middle strip

$$M_u = 56 \text{ kNm}; \qquad \text{Thickness} = 190 \text{ mm}$$

Provide 10 mm bars at 230 mm c/c in this portion also.

Since span is same in both direction, provide similar reinforcement in both directions. The details of reinforcement are shown below



Example 4.3: design the interior panel of the flat slab in example 4.2, providing a suitable column head, if columns are of 500mm diameter

Solution:

Let the diameter of column head be

$$= 0.25L = 0.25 \times 6 = 1.5 \text{ m}$$

It's equivalent square has side 'a' where

$$\frac{\pi}{4} \times 1.5^2 = a^2$$

$$a = 1.33 \text{ m}$$

$$\therefore \quad L_n = 6 - 1.33 = 4.67 \text{ m}$$

$$W_0 = 17.625 \times 6 \times 4.67 = 493.85 \text{ kN}$$

$$M_0 = \frac{W_o L_n}{8} = \frac{493.85 \times 4.67}{8} = 288.3 \text{ kNm}$$

$\therefore$ Total –ve moment = 0.65 × 288.3 = 187.4 kNm

Total +ve moment = 0.35 × 288.3 = 100.9 kNm

The distribution of above moment into column strip and middle strips are as given below:

	Column Strip	*Middle Strip*
–ve moment	0.75× 187.4 = 140.55 kNm	0.25× 187.4 = 46.85 kNm
+ve moment	0.60× 100.9 = 60.54 kNm	0.4× 100.9 = 40.36 kNm

Width of column strip = width of middle strip = 3000 mm

$$\therefore \quad M_{u\,lim} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 3000 \times 240^2$$

$$= 476.928 \times 10^6 \text{ Nmm} > M_u$$

Hence thickness selected is sufficient.

Check for Shear

The critical section is at a distance

$$\frac{d}{2} = \frac{240}{2} = 120 \text{ mm from the face of column head}$$

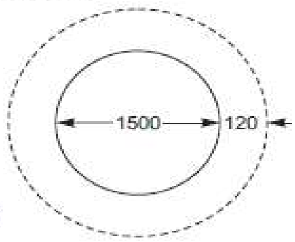
Diameter of critical section = 1500 + 240 =1740 mm

= 1.740 m

Perimeter of critical section = π D

= 1.740 π

Shear on this section



$$V = 17.625\left[6 \times 6 - \frac{\pi}{4} \times 1.74^2\right] = 592.59 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{592.59 \times 1000}{\pi \times 1740 \times 240} = 0.45 \text{ N/mm}^2$$

Maximum shear permitted = $k_s \times 0.25\sqrt{20}$

= 1.118 N/mm² Since k_s works out to be 1

Since maximum shear permitted in concrete is more than nominal shear τ_v, there is no need to provide shear reinforcement

Design of Reinforcement

(*a*) For –ve moment in column strip

$$M_u = 140.55 \text{ kNm}; \quad d = 240 \text{ mm}$$

$$\therefore \quad 140.55 \times 10^6 = 0.87 \times 415 \times A_{st} \times 240\left[1 - \frac{A_{st}}{3000 \times 240} \times \frac{415}{20}\right]$$

$$1622 = A_{st}\left[1 - \frac{A_{st}}{34698.8}\right]$$

$$A_{st}^2 - 34698.8\ A_{st} + 1622 \times 34698.8 = 0$$

$$A_{st} = 1705\ mm^2$$

Using 12 mm bars,

$$s = \frac{\pi/4 \times 12^2}{1705} \times 3000 = 199\ mm$$

Provide 12 mm bars at 190 mm c/c.

(*b*) For the +ve moment in column strip

$$M_u = 60.54\ kNm; \qquad d = 190\ mm$$

$$60.54 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190 \left[1 - \frac{A_{st}}{3000 \times 190} \times \frac{415}{20}\right]$$

$$882.51 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$A_{st}^2 - 27469.9\ A_{st} + 882.51 \times 27469.9 = 0$$

$$A_{st} = 913\ mm^2$$

Using 10 mm bars

$$s = \frac{\pi/4 \times 10^2}{913} \times 3000 = 258\ mm$$

Provide 10 mm bars at 250 mm c/c.

(*c*) For –ve moment in middle strip:

$$M_u = 46.85\ kNm; \qquad d = 190\ mm$$

$$46.85 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190\left[1 - \frac{A_{st}}{3000 \times 190} \times \frac{415}{20}\right]$$

$$683 = A_{st}\left[1 - \frac{A_{st}}{27469.9}\right]$$

$$A_{st}^2 - 27469.9A_{st} + 683 \times 27469.9 = 0$$

$$A_{st} = 701\ mm^2$$

Using 10 mm bars,

$$s = \frac{\pi/4 \times 10^2}{701} \times 3000 = 336\ mm$$

Provide 10 mm bars at 300 mm c/c.

(*d*) Provide 10 mm bars at 300 mm c/c for +ve moment in middle strip also.

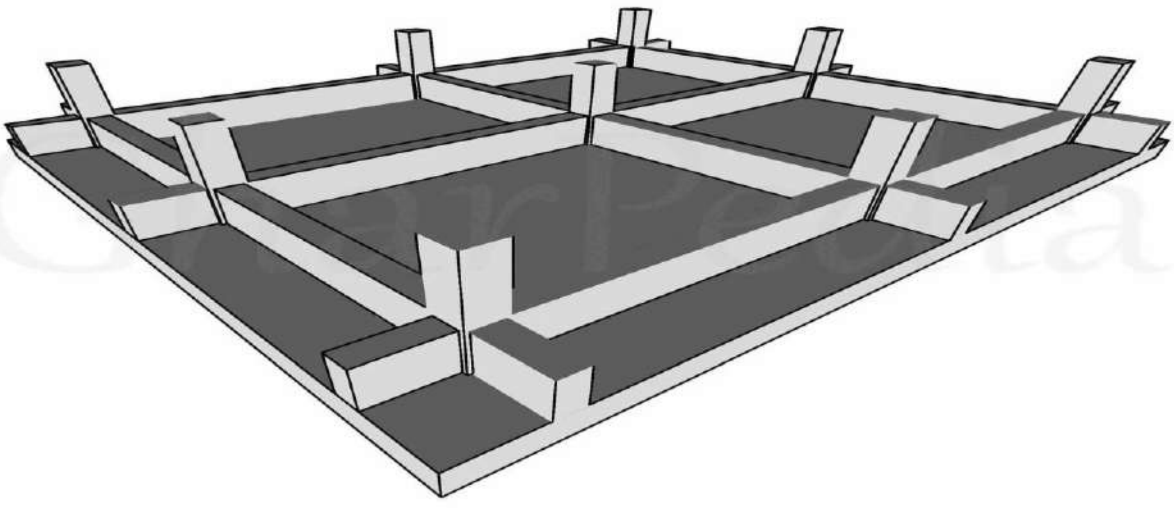
As span is same in both directions, provide similar reinforcement in both directions. Reinforcement detail may be shown as was done in previous problem.

Raft foundation

Raft foundations (sometimes known as Mat Foundations) are a large concrete slab which can support a number of columns and walls. The slab is spread out under the entire building or at least a large part of it which lowers the contact pressure compared to the traditionally used strip or trench footings. Because of the speed and volume of houses required after the second world war, the raft foundation was widely used. The raft foundation was cheaper, easier to install and most importantly, did not require as much excavation as the usual strip foundations.

A raft foundation spreads the weight of the building over the whole ground floor area of that building. The raft is laid on a hardcore or scalping bed and usually thickened at the edges, especially in very poor ground. Rafts are most suitable when the ground is of good load bearing capacity and little work is required to get a solid foundation.

Rafts are most often used these days when the strata is unstable or (because of this) a normal strip foundation would cover more than 50% of the ground area beneath the building. There are also situations (usually in areas where mining has occurred) where there may be areas of movement in the strata. They are much more commonly used in the construction of commercial building in the UK that they are for domestic homes, but can be used very successful in both situations. To understand when it is better to use raft foundations, you need to understand how they work.



Slab-beam type raft foundation

Raft Foundations are built is this following steps:

- The soil removed down to correct depth
- The foundation bed is then compacted by ramming
- Lay reinforcement on spacers over the foundation bed
- Pour the concrete over the reinforcement

- The foundation may stiffened by ribs or beams built in during construction which will add extra strength and rigidity.

<u>A raft foundation is usually preferred under a number of circumstances:</u>

- it is used for large loads, which is why they are so common in commercial building which tend to be much larger, and therefore heavier, than domestic homes
- The soil has a low bearing capacity so the weight of the building needs to be spread out over a large area to create a stable foundation
- The ratio of individual footings to total floor space is high. Typically if the footings would cover over half of the construction area then raft foundation would be used
- If the walls of the building are so close that it would cause the individual footings to overlap, then raft foundations should be used

<u>Advantages and disadvantages:</u>

Raft foundations tend to be cheaper and quicker to use than traditional footings. There are a number of reasons why this is the case:

- The foundation and floor slab is combined, which saves time and materials
- Less excavation is required

Other reasons that make raft foundations preferable to footings are due to their engineering benefits. They are ideal for poor ground condition where normal footings would not cope well as they cannot spread the load as effectively. Related to this is that raft foundations can reduce differential settlement, where settlement occurs at different rates across the ground surface of the building, which reduces cracking and other more serious problems.

The main disadvantage is that they can prone to edge erosion if they are not treated properly. They are not effective is the load of the building is going to be focused on a single point, although this is rare in domestic construction, so this isn't generally of concern.

Pile foundation

Foundations provide support to the structure, transfers the loads from the structure to the soil. But the layer at which the foundation transfers the load shall have an adequate bearing capacity and suitable settlement characteristics. There are several types of foundation depending on various considerations such as-

- Total load from the superstructure.
- Soil conditions.
- Water level.
- Noise and vibrations sensitivity.
- Available resources.
- Time-frame of the project.
- Cost.

Pile foundation, a kind of deep foundation, is actually a slender column or long cylinder made of materials such as concrete or steel which are used to support the structure and transfer the load at desired depth either by end bearing or skin friction.

Following are the situations when using a pile foundation system can be

- When the groundwater table is high.
- Heavy and un-uniform loads from superstructure are imposed.
- Other types of foundations are costlier or not feasible.
- When the soil at shallow depth is compressible.
- When there is the possibility of scouring, due to its location near the river bed or seashore, etc.
- When there is a canal or deep drainage systems near the structure

Pile foundations can be classified based on function, materials and installation process, etc. Followings are the types of pile foundation used in construction:

Based on Function or Use

- Sheet Piles
- Load Bearing Piles
- End bearing Piles
- Friction Piles
- Soil Compactor Piles

Based on Materials and Construction Method

- Timber Piles
- Concrete Piles
- Steel Piles
- Composite Piles

Load carrying capacity of piles

The ultimate load carrying capacity of a pile is defined as the maximum load which can be carried by a pile, and at which the pile continues to sink without further increase of load. The *allowable load* is the load which the pile can carry safely, and is determined on the basis of : (*i*) ultimate bearing resistance divided by suitable factor of safety (*ii*) permissible settlement and (*iii*) overall stability of the pile foundation. The load carrying capacity of a pile can be determined by the following methods: (1) dynamic formulae (2) static formulae (3) pile load tests (4) penetration tests.

Dynamic Formulae. When a pile hammer hits the pile, the total driving energy is equal to the weight of hammer times the height of drop of stroke. In addition to this, in the case of double acting hammers, some energy is also imparted by the steam pressure during the return stroke. This total downward energy is consumed by the work done in penetrating the pile and by certain losses. The various dynamic formulae are essentially based on this assumption. Following are some commonly used dynamic formulae :

(*a*) ***Engineering News formula***. The Engineering News formula was proposed by A.M. Wellington (1888), in the following form :

$$Q_a = \frac{WH}{F(S+C)}$$

where Q = allowable load ; W = weight of hammer
H = height of fall ; F = factor of safety = 6
S = final set (penetration) per blow, usually taken as average penetration, cm per blow for the last 5 blows of drop hammer, or 20 blows of steam hammer.
C = empirical constant.

Denoting W in kg, H in cm and S in cm and C = 2.5 cm for drop hammer and C = 0.25 cm for single and double acting hammers the above formula reduces to the following forms:

(*i*) Drop hammers : $$Q_a = \frac{WH}{6\,(S + 2.5)}$$

(*ii*) Single acting steam hammers : $$Q_a = \frac{WH}{6\,(S + 0.25)}$$

(*iii*) Double acting steam hammers : $$Q_a = \frac{(W + a\,p)\,H}{6\,(S + 0.25)}$$

where a = effective area of piston (cm^2) and p = mean effective steam pressure (kg/cm^2).

(*b*) ***Hiley's formula***. Indian Standard IS : 2911 (Part II) 1964 gives the following formula based on original expression by Hiley :

$$Q_f = \frac{\eta_h\, W\, H\, \eta_b}{S + C/2}$$

where Q_f = ultimate load on pile
W = weight of hammer, in kg

Group action in pile

When several closely spaced piles are grouped, together, it is reasonable to expect that the soil pressure, developed in the soil as resistance, will overlap. The bearing capacity of a pile group may or may not be equal to the sum of the bearing capacity of individual piles constituting a group. Theory and test have shown that the bearing value Q_f of a group of *friction piles*, particularly in clay, is equal to bearing capacity of individual pile multiplied by the number of piles n in a group. However, no reduction due to grouping occurs in end bearing piles. For combined end bearing and friction piles, only the load carrying capacity of the friction portion is reduced. A method of estimating the bearing capacity of a group of friction piles is to multiply the quantity nQ_f by a reduction factor called the *efficiency of the pile group* η_g.

$$Q_g = n\, Q_f \,.\, \eta_g \,.$$

The efficiency of pile group depends upon the following factors : characteristics of pile(*i.e.* length, diameter, material, etc.), spacing of piles, total number of piles (n) in a row, and number of rows (m). Out of a number of formulae for determining efficiency of a pile group, two are given below :

1. ***Converse-Labarre formula*** : $$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)\,m + (m-1)\,n}{mn}\right]$$

where $\theta = \tan^{-1}\frac{d}{s}$ (degrees)
d = diameter of piles and s = spacing of piles.

2. ***Silver Keeney formula*** : $$\eta_g = \left[1 - 0.479\left(\frac{s}{s^2 - 0.093}\right)\left(\frac{m + n - 2}{m + n - 1}\right)\right] + \frac{0.3}{m + n}$$

where s = average spacing, centre to centre in metres.

Structural design of RC pile

A R.C. pile is designed for the following : (1) total load coming on it from the structure (2) handling stresses (3) driving stresses. While designing the pile as a column, it may be considered as fixed at one end hinged at the other. The length of the pile may be taken as 2/3 rd the length embedded in firm stratum. The cross-section of pile varies with its overall length.

Stresses During Handling. Precast piles should be checked against handling stresses. When a pile is lifted by means of a derrick, it is subjected to bending moment due to its own weight. When the pile is of less than 12 metre length, it is suspended from one point at its middle. Piles longer than 12 metres are suspended at two or three points suitably spaced at its length so that handling moment is as small as possible.

(i) ***Pile suspended at one point.*** [Fig. 17.1(a)]

If w is the weight of the pile per unit length, and L is the length of the pile, it will be subjected to a maximum bending moment $= \frac{wL}{2} \times \frac{L}{4} = \frac{wL^2}{8}$ at the point of suspension.

(2) ***Pile suspended at two points.*** [Fig. 17.1(b)].

Let the distance of each point of suspension be x from the respective ends. The value of x should be such that maximum bending moment anywhere in the pile should be the least. This is possible when the hogging moment is equal to maximum sagging moment.

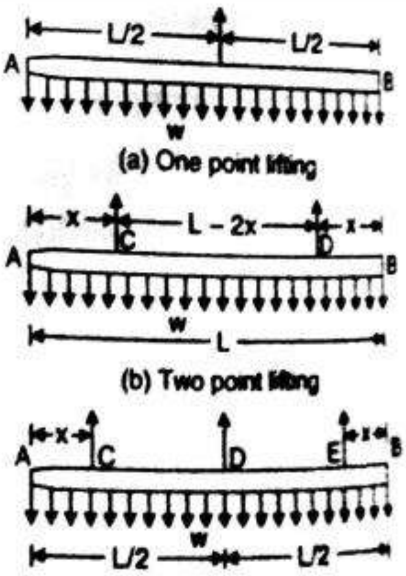
Reaction $R = \frac{wL}{2}$ at each point of suspension.

Maximum hogging moment at the end of each cantilever

$$= \frac{wx^2}{2} \qquad \ldots(i)$$

Maximum sagging moment at the middle of the pile

$$= \frac{wL}{2}\left(\frac{L}{2} - x\right) - \frac{wL^2}{8} \qquad \ldots(ii)$$



Equating the two, we get

$$\frac{wx^2}{2} = \frac{wL}{2}\left(\frac{L}{2} - x\right) - \frac{wL^2}{8} = \frac{wL}{2}\left(\frac{L}{4} - x\right)$$

$$x^2 + Lx - \frac{L^2}{4} = 0, \quad \text{which gives } x = 0.206\ L$$

Hence maximum B.M. $= \frac{w}{2}(0.206\ L)^2 = \frac{wL^2}{47}$

Design of pile cap

When a column or pier is supported on the pile only, the column should rest centrally on the pile. However, when the column is supported on more than one pile, the piles should be connected through a rigid pile cap, to distribute the load to the individual piles. The pile cap consists of a rigid, deep, reinforced concrete slab which acts monolithically with the group of piles. The piles should be arranged symmetrically about the axis of the column so that the load from column is distributed uniformly to all the columns. The pile cap slab is provided in uniform thickness. The pile cap should be extended beyond exterior piles by 10 to 15 cm. The pile should be embedded by at least 15 cm in the pile cap, and the reinforcement in the cap should be placed at least 10 cm above the pile head. Thus, effective depth of the pile cap will be equal to the total depth minus 25 cm. The pile cap, provided over the entire area of the piles, is considered to be divided into a framework of rectangular beams, along which main reinforcement is provided. The arrangement of these beams depends upon the number of piles, and the width of beam is taken equal to width of pile.

Pile cap for three piles. Fig. 17.3 (*a*) shows the pile cap for three piles. The pile cap is considered to be composed of two beams *AB* and *CD* ; *A*, *B* and *C* being the three piles placed at distance *L* centre to centre. The column *W* is placed on the beam *DC*, at the centroid of the triangle *ABC*.

Let W = total load of column.

$\therefore$ Load on each pile $= W/3$

Length of beam $CD = l = \dfrac{L\sqrt{3}}{2}$. Distance of W from $D = \frac{1}{3}l = \dfrac{L}{2\sqrt{3}}$.

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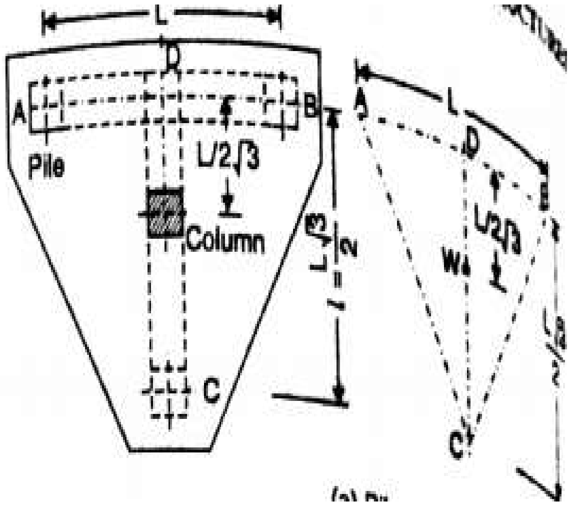
Beam *CD* is supported on *C* and *D*, and is loaded with *W*, such that reaction at *C* is *W*/3. Hence maximum bending moment is

$$M_1 = \frac{W}{3} \times \frac{2l}{3}$$

$$= \frac{W}{3} \times \frac{2}{3}\left(\frac{L\sqrt{3}}{2}\right) = \frac{WL}{3\sqrt{3}}$$

Reaction transferred to

$$D = W - \frac{W}{3} = \frac{2W}{3}$$



The beam AB is of length L, and is loaded at its middle point D with a load $(2W)/3$. Hence maximum bending moment is given by

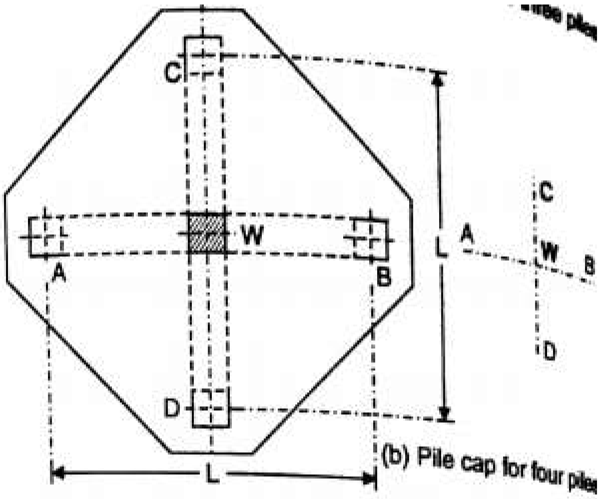
$$M_2 = \frac{W}{3}\left(\frac{L}{2}\right) = \frac{WL}{6}.$$

The reinforcement for both beams can now be computed.

Pile cap for four piles [Fig. 17.3(b)].

Load on each pile $= \frac{W}{4}$

The pile cap is considered to be composed of two beams AB and CD.



(b) Pile cap for four piles

Example4.4 Design a pile under a column transmitting an axial load of 800 kN. The pile is to be driven to a driven to a hard stratum available at a depth of 8 m. Use M 20 concrete and Fe 415 steel.

Solution:

For M 20 concrete, $\sigma_{cc} = 5\ N/mm^2$. For Fe 415 steel, $\sigma_{sc} = 190\ N/mm^2$.

Also, $m = 13.33$ for M 20 concrete.

1. ***Main reinforcement.*** Let the length of the pile above ground, including pile cap, etc. = 0.6 m. ∴ Total length of pile = 8.6 m.

Let the size of the pile be 400 mm × 400 mm

$\frac{l}{D}$ ratio $= \frac{8.6}{0.4} = 21.5$. Since this is greater than 12, the pile behaves as long column.

Hence reduction coefficient $C_r = 1.25 - \frac{l_{ef}}{48\,D} = 1.25 - \frac{8.6}{48 \times 0.4} \approx 0.8$

∴ Design load for a short column $= 800/0.8 = 1000$ kN

Load carrying capacity of column is given by $P = \sigma_{cc} A_c + \sigma_{sc} A_{sc}$

where A_c = area of concrete $= (400 \times 400) - A_{sc} = 16 \times 10^4 - A_{sc}$

∴ $1000 \times 10^3 = 5\,(16 \times 10^4 - A_{sc}) + 190\,A_{sc}$. From which $A_{sc} = 1081\ mm^2$.

Since the length of pile is less than 30 times the width, minimum reinforcement @ 1.25% of gross cross-sectional area $= \frac{1.25}{100}(400 \times 400) = 2000\ mm^2$.

However, provide 4 bars of 25 mm Φ giving total area of steel $= 4 \times 490 = 1960\ mm^2$.

Provide a nominal cover of 50 mm. Cover to the centres of main reinforcement using 8 mm φ ties $= 50 + 8 + 25/2 = 70.5$ mm..

2. ***Lateral reinforcement in the body of the pile***

Lateral reinforcement in the body of pile is provided @ 0.2% of gross volume.

∴ Volume needed per mm length

$$= \frac{0.2}{100}(400 \times 400 \times 1) = 320 \text{ mm}^3.$$

Nominal cover = 50 mm

Using 8 mm Φ ties, length of each side of tie

$$= 400 - 2 \times 50 - 8 = 292$$

$$\text{Area } A_\Phi = \frac{\pi}{4}(8)^2 = 50.3 \text{ mm}^2.$$

∴ Volume of each tie = $4 \times 292 \times 50.3 = 58750 \text{ mm}^3$

∴ Pitch = $58750/320 = 183$ mm

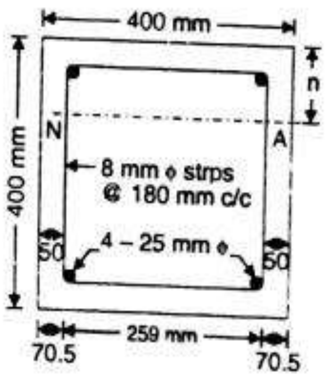


FIG. 17.4

Maximum pitch permissible $= \frac{1}{2} \times 400 = 180$ mm.

Hence provide 8 mm Φ ties @ 180 mm c/c throughout the length of the pile.

3. ***Lateral reinforcement near pile head.*** Near pile head, special spiral reinforcement is to be provided for a length of $3 \times 400 = 1200$ mm. Volume of spiral, @ 0.6% of gross volume, per mm length is $= \frac{0.6}{100}(400 \times 400 \times 1) = 960 \text{ mm}^3$.

Using 8 mm Φ spiral, having $A_\Phi = 50.3 \text{ mm}^2$, pitch is given by

$$s = \frac{\text{circumference of spiral} \times A_\Phi}{960} = \frac{\pi \times 292 \times 50.3}{960} = 48 \text{ mm}$$

Provide the spiral at 45 mm pitch. Provide 6 additional bars of 16 mm Φ vertically within the spiral. These spirals will be in addition to the normal ties.

4. ***Lateral reinforcement near pile end***

Volume of ties per mm length @ 0.6% of gross volume = 960 mm³.

∴ Volume of each tie = 58750 mm³. Hence, Pitch = 58750/960 ≈ 60 mm

Provide ties @ 60 mm c/c in a bottom length of $3 \times 400 = 1200$ mm

5. *Spacer forks and links*. Provide two pairs of 12 mm Φ spacer fork with 6 mm Φ links @ 1.5 m c/c along the length.

6. *Check for handling stresses*. Provide three holes in the pile as follows

(*i*) One hole at $0.293L = 0.293 \times 8.6 = 2.5$ m from the pile for the purpose of handling it.

(*ii*) One hole each from either end, at a distance of $0.206L = 0.206 \times 8.6 = 1.75$ m for the purpose of stacking.

(*iii*) Weight of pile per meter run $= 0.4 \times 0.4 \times 1 \times 25000 = 4000$ N/m

= 1.75 m for the purpose of stacking.

(iii) Weight of pile per meter run $= 0.4 \times 0.4 \times 1 \times 25000 = 4000$ N/m

$$M = \frac{4000\,(2.5)^2}{2} = 12500 \text{ N-m}$$

$$= 125 \times 10^5 \text{ N-mm}$$

Effective depth of pile section $= 400 - 70.5 = 329.5$ mm. Let the neutral axis be situated at n below one face.

$$\frac{b\,n^2}{2} + (m_c - 1)\,A_{sc}\,(n - d_c) = m\,A_{sc}\,(d - n)$$

$$\frac{400}{2} n^2 + (1.5 \times 13.33 - 1)\,980\,(n - 70.5)$$

$$= 13.33 \times 980\,(329.5 - n)$$

or $n^2 + 158.39\,n - 28083 = 0$

From which $n = 106.2$ mm

Taking moment of forces about tensile steel, we get

$$bn\,\frac{c}{2}\left[d - \frac{n}{3}\right] + (1.5\,m - 1)\,A_{sc}\,c'\,(d - d_c) = M$$

where $c' = \frac{(n - d_c)\,c}{n} = \frac{106.2 - 70.5}{106.2}\,c$

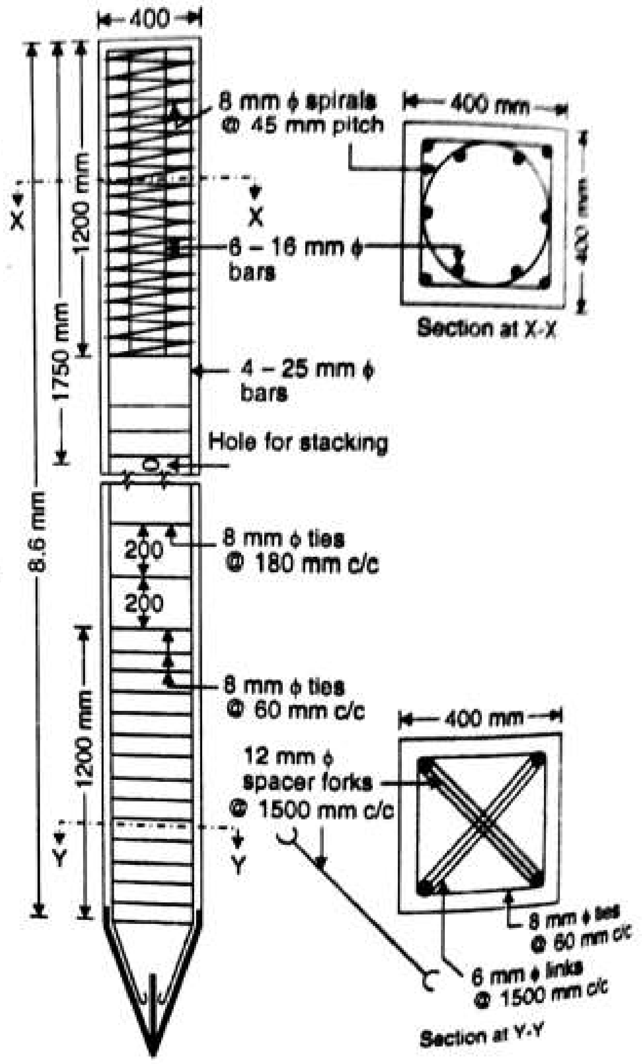
$= 0.336\,c$

$$\therefore \quad 400 \times 106.2\,\frac{c}{2}\left[329.5 - \frac{106.2}{3}\right]$$

$$+ 18.995 \times 980 \times 0.336\,c\,(329.5 - 70.5)$$

$$= 125 \times 10^5$$

which gives $c = 1.59$ N/mm².



Example 4.5A RC column 400 mm x 400 mm carrying a load of 600 kN as supported on piles 400 mm x 400 mm in section. The centre to centre distance between the piles is .5 m. Design a suitable pile cap. Use M 20 concrete and Fe 415 steel.

Solution.

1. ***Dimensions of pile cap.*** Centre to centre spacing of piles $= L = 1.5$ m

Keeping 200 mm clear projection of the cap beyond pile face. Overall length of cap along the direction $AB = 1.5 + 0.4 + 0.4 = 2.3$ m.

Length of beam $CD = l = \frac{L\sqrt{3}}{2} = 1.5\,\frac{\sqrt{3}}{2} = 1.3$ m.

$\therefore$ Length of cap in the direction DC $= 1.3 + 0.4 + 0.4 = 2.1$ m.

2. Design of beam DC

Load on each pile $= W/3 = 600/3 = 200$ kN

Let the width of beam = width of column = 400 mm

$$\therefore \quad \text{B.M. due to load} = \frac{WL}{3\sqrt{3}} = \frac{600 \times 1.5}{3\sqrt{3}} = 173 \text{ kN-m} = 173 \times 10^6 \text{ N-mm} \quad ...(i)$$

In order to calculate the bending moment due to self weight of the beam plus weight of part of slab, let us assume total thickness of slab to be 800 mm. The self-weight of the beam is calculated on the assumption that weight of slab equal to two times the width of the beam acts with the beam.

$$\therefore \quad w = \left[\frac{3 \times 400 \times 800}{10^6}\right] \times 1 \times 25000 = 24000 \text{ N/m}$$

Length of beam $l = L\sqrt{3}/2 = 1.3$ m.

$$\therefore \quad \text{Total load} = 1.3 \times 24000 = 31200 \text{ N}$$

$$\therefore \quad \text{Reaction at } C = 31200/2 = 15600 \text{ N}$$

Distance of point of application of column load $= \frac{2}{3} l = \frac{2}{3} \times 1.3 = 0.87$ m.

$\therefore$ B.M. at the centre of column, due to self-weight is $= (15600 \times 0.87) - \frac{24000}{2}(0.87)^2$

$$= 4489 \text{ N-m} = 4.489 \times 10^6 \text{ N-mm.}$$

$$\therefore \quad \text{Total B.M.} = 173 \times 10^6 + 4.489 \times 10^6 = 177.5 \times 10^6 \text{ N-mm}$$

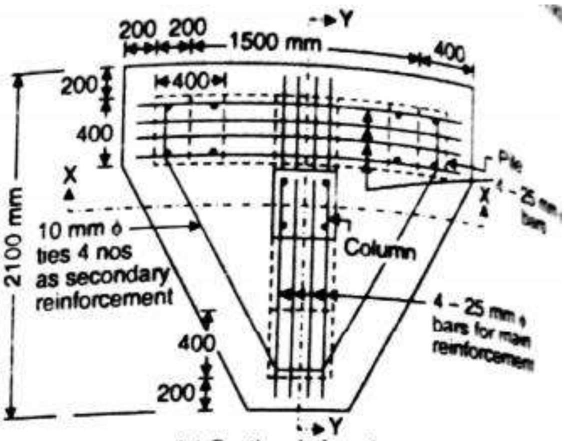
$$\therefore \quad d = \sqrt{\frac{177.5 \times 10^6}{0.914 \times 400}} = 697 \text{ mm.} \quad \text{Keep } d = 700 \text{ mm}$$

$$\therefore \quad A_{st} = \frac{175.5 \times 10^6}{230 \times 0.904 \times 700} = 1206 \text{ mm}^2$$

$\therefore$ No. of 25 mm Φ bars $= 1200/490.8 = 2.5$

However, provide 4 bars of 25 mm Φ.

Actual area of steel provided $= 4 \times 490.8 = 1963$ mm².



(a) Sectional plan of cap

3. Design of beam AB

Span $L = 1.5$ m

B.M. due to load from beam

$$CD = \frac{WL}{6} = \frac{600 \times 1.5}{6}$$

$$= 150 \text{ kN-m} = 150 \times 10^6 \text{ N-mm}$$

B.M. due to self weight

$$= \frac{24000 \, (1.5)^2}{8} = 6750 \text{ N-m}$$

$$= 6.75 \times 10^6 \text{ N-mm}$$

$\therefore$ Total B.M.

$= 150 \times 10^6 + 6.75 \times 10^6$

$= 156.75 \times 10^6$ N-mm

The reinforcement in direction *AB* will be placed below the reinforcement of *CD*. Hence available $d = 700 + 25 = 725$ mm.

$$\therefore A_s = \frac{156.75 \times 10^6}{230 \times 0.904 \times 725}$$

$= 1040$ mm^2.

However, provide the same reinforcement, *i.e.* 4 Nos. of 25 mm Φ bars. Keep total depth = 800 mm.

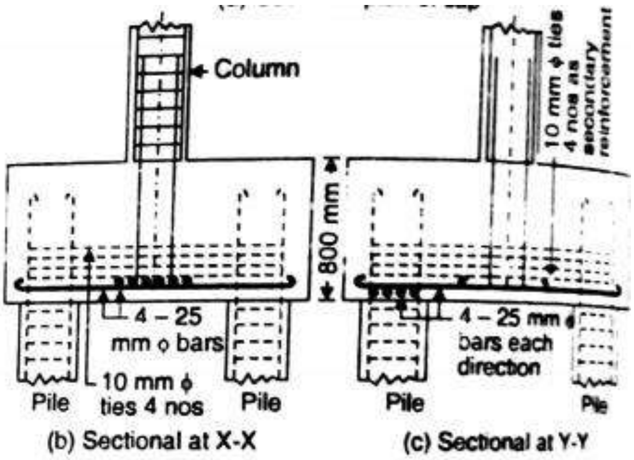


FIG. 17.6

4. ***Secondary reinforcement***

Areat of secondary reinforcement running round each pile head = $0.2 \times 1206 = 242$ mm^2

Using 10 mm Φ bars, $A_\Phi = 78.5$ mm^2.

$\therefore$ No. of bars $= 242/78.5 = 3.08 \approx 4$.

5. ***Check for shear***

Shear is tested at a distance *d* from the beam. The dispersion lines(at 45°) transfer the load directly to the column. Hence there is no possibility of diagonal tension cracks. The details of reinforcement etc. are shown in Fig. 17.6.

UNIT-V
DESIGN OF CHIMNEYS, BUNKER AND SILOS

DESIGN OF CHIMNEYS

In many industries chimneys are required to leave hot waste gases at greater heights. The chimneys of 50 – 100 m are very commonly used. The outer diameter may be kept constant throughout or may be linearly varied. The thickness of concrete shell may be varied in steps linearly Fig 5.1 shows typical chimneys.

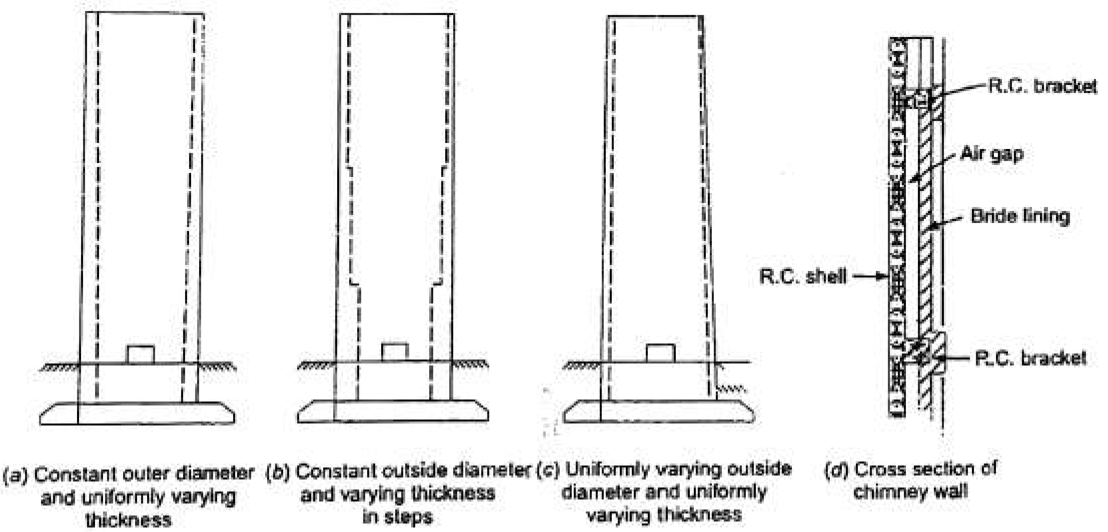


Fig 5.1 Typical chimneys

Design factors

Chimneys are to be designed to sustain the stresses due to

1. Self-weight
2. Wind load
3. Temperature variation

Using Is 875 one has to arrive the wind pressure exerted on a structure. The design wind load depends upon the shape of the structure also. Table 5.1 gives the shape factor, with which one has to multiply wind pressure in the area to get design wind pressure on the structure

Table 5.1 Shape factors for wind load calculation

Ratio of height to base width	*0 to 4*	*4 to 8*	*8 or above*
Shape of Chimney			
Circular	0.7	0.7	0.7
Octagonal	0.8	0.9	1.0
Square (Wind perpendicular to diagonal)	0.8	0.9	1.0
Square (wind perpendicular to face)	1.0	1.15	1.3

Inside temperature is higher compared to outer side. This causes differential expansion and hence stresses are induced in R.C. wall in both vertical and horizontal directions. Design should take care of these stresses also.

Design of R.C. chimneys is carried out by selecting the section first and then checking for the stresses developed. Thickness of shell wall is kept to a minimum of 200 mm at top and is increased to 300 – 400 mm at bottom depending upon the height of chimney. Vertical reinforcement of about 1% and hoop steel of 0.2% per cent is usually sufficient. The section selected is checked for stresses due to:

(*a*) Self weight and wind load

(*b*) Combined effect of self-weight, wind load and temperature variation.

Stresses in concrete and steel both in vertical and horizontal directions are to be checked. It is to be noted that permissible stresses are increased to 1.33 times when wind load is considered.

12.3 STRESSES DUE TO SELF-WEIGHT AND WIND LOAD

The following two assumptions are made:

(*i*) Reinforcement is replaced by a steel ring of equal area.

(*ii*) The stress at middle of shell is taken as the average stress in the shell.

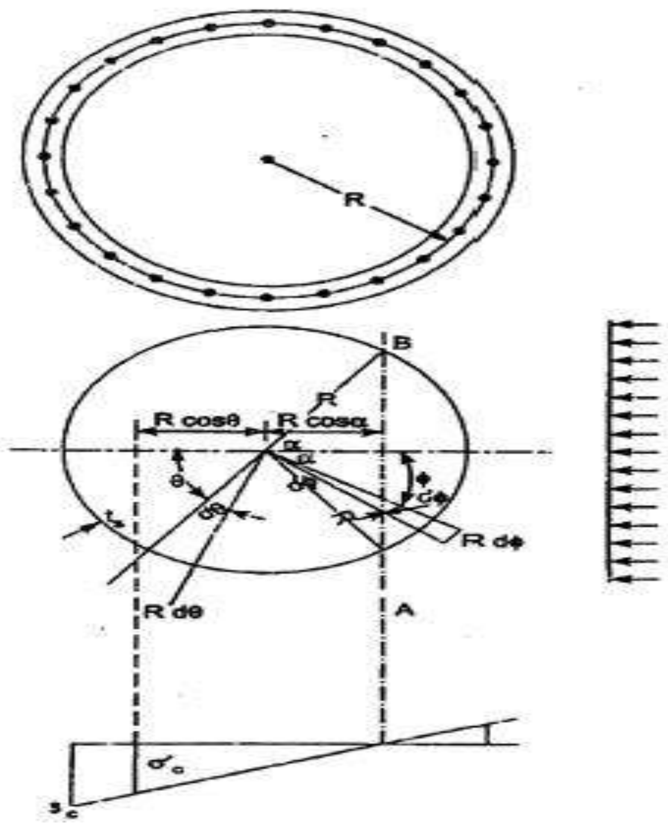


Fig 5.2The idealized section and stress variation

Let, W be self-weight and M be moment due to wind load.

A_s—Area of reinforcement in vertical direction.

R—Radius upto centre of shell.

t_s—Thickness of steel ring

$$= \frac{A_s}{2\pi R}$$

t_c—Thickness of concrete shell

AB—Neutral axis

α—Angle subtended by neutral axis at centre with *x-x* axis

m—Modular ratio

σ_c—Compressive stress in concrete

σ_s—Tensile stress in steel

To find total compressive force resisted by shell, consider the element R. $d\theta$ at an angle θ to x-axis. Let compressive stress in the element be σ'_c. Then from stress diagram, we see that

$$\sigma'_c = \frac{R\cos\theta + R\cos\alpha}{R + R\cos\alpha}\sigma_c = \frac{\cos\theta + \cos\alpha}{1+\cos\alpha}\sigma_c$$

Area of concrete in the element = R $d\theta\ t_c$

Equivalent area of steel in the element = $(m-1)\ t_s$ R $d\theta$

∴ Total compressive force

$$C = 2\int_0^{\pi-\alpha} R d\theta\ t_c\, \sigma_c \frac{\cos\theta + \cos\alpha}{1+\cos\alpha} + 2\int_0^{\pi-\alpha}(m-1)\,t_s\, R d\theta\, \sigma_c \frac{\cos\theta+\cos\alpha}{1+\cos\alpha}$$

$$= \frac{2R\sigma_c t_c}{1+\cos\alpha}\int_0^{\pi-\alpha}(\cos\theta+\cos\alpha)\,d\theta + \frac{2(m-1)t_s\sigma_c R}{1+\cos\alpha}\int_0^{\pi-\alpha}(\cos\theta+\cos\alpha)\,d\theta$$

$$= \frac{2R\sigma_c}{1+\cos\alpha}[\,t_c + (m-1)\,t_s][\sin\theta + \theta\cos\alpha]_0^{\pi-\alpha}$$

$$= \frac{2R\sigma_c}{1+\cos\alpha}[\,t_c + (m-1)\,t_s][\sin(\pi-\alpha) + (\pi-\alpha)\cos\alpha]$$

$$= \frac{2R\sigma_c}{1+\cos\alpha}[\,t_c + (m-1)\,t_s][\sin\alpha + (\pi-\alpha)\cos\alpha]$$

To find total tensile force consider elemental strip R $d\,\phi$.

$$T = 2\int_0^{\alpha} R d\phi\, t_s\, m\sigma_c \frac{(R\cos\phi - R\cos\alpha)}{R + R\cos\alpha}$$

$$= \frac{2R\sigma_c\, m\, t_s}{1+\cos\alpha}[\sin\phi - \phi\cos\alpha]_0^{\alpha}$$

$$= \frac{2R\sigma_c\, m\, t_s}{1+\cos\alpha}[\sin\alpha - \alpha\cos\alpha]$$

The equilibrium of forces in vertical direction gives.

$$W = C - T$$

$$W = \frac{2R\sigma_c}{1+\cos\alpha}[t_c + (m-1)t_s][\sin\alpha + (\pi-\alpha)\cos\alpha] - \frac{2R\sigma_c\, mt_s}{1+\cos\alpha}[\sin\alpha - \alpha\cos$$

$$= \frac{2R\sigma_c}{1+\cos\alpha}\left[(t_c - t_s)\{\sin\alpha + (\pi-\alpha)\cos\alpha\} + mt_s\,\pi\cos\alpha\right] \quad \ldots($$

Moment equilibrium condition gives,

$$M = \int_0^{\pi-\alpha} C.R.\cos\theta\, d\theta + \int_0^{\alpha} TR\cos\phi\, d\phi$$

$$= \frac{2R^2\sigma_c}{1+\cos\alpha}[\,t_c + (m-1)\,t_s]\int_0^{\pi-\alpha}(\cos\theta + \cos\alpha)\cos\theta\, d\theta + \frac{2R^2\sigma_c m t_s}{1+\cos\alpha}\int_0^{\phi}(\cos\phi - \cos\alpha)\cos\phi\, d\phi$$

$$= \frac{2R^2\sigma_c}{1+\cos\alpha}[\,t_c + (m-1)\,t_s]\int_0^{\pi-\alpha}(\cos^2\theta - \cos\theta\cos\alpha)\,d\theta$$

$$+ \frac{2R^2\sigma_c\, mt_s}{1+\cos\alpha}\int_0^{\phi}(\cos^2\phi - \cos\phi\cos\alpha)\,d\phi$$

Noting that $\cos^2 A = \frac{1+\cos^2 A}{2}$

$$M = \frac{2R^2\sigma_c}{1+\cos\alpha}\left[t_c+(m-1)t_s\right]\left[\frac{\theta}{2}+\frac{\sin 2\theta}{4}+\cos\alpha\cdot\sin\theta\right]_0^{\pi-\alpha} + \frac{2R^2\sigma_c}{1+\cos\alpha}mt_s\left[\frac{\phi}{2}+\frac{\sin 2\phi}{4}-\cos\alpha\sin\phi\right]_0^{\alpha}$$

$$= \frac{2R^2\sigma_c}{1+\cos\alpha}\left[t_c+(m-1)t_s\right]\left[\frac{\pi-\alpha}{2}+\frac{\sin 2\alpha}{4}+\cos\alpha\cdot\sin(\pi-\alpha)\right]$$

$$= \frac{2R^2\sigma_c}{1+\cos\alpha}\left[t_c+(m-1)t_s\right]\left[\frac{\pi-\alpha}{2}+\frac{1}{4}\sin 2\alpha\right] + \frac{2R^2\sigma_c}{1+\cos\alpha}mt_s\left[\frac{\alpha}{2}-\frac{1}{4}\sin 2\alpha\right]$$

$$= \frac{2R^2\sigma_c}{1+\cos\alpha}\left[(t_c-t_s)\left\{\frac{\pi-\alpha}{2}+\frac{1}{4}\sin 2\alpha\right\}+mt_s\pi\right]$$

$\therefore$ Eccentricity e is given by

$$e = \frac{M}{W} = R\,\frac{(t_c-t_s)\left[\frac{\sin 2\alpha}{4}+\frac{\pi-\alpha}{2}\right]+\frac{mt_s}{2}\pi}{(t_c-t_s)\left[\sin\alpha+(\pi-\alpha)\cos\alpha\right]+mt_s\,\pi\cos\alpha}$$

The value of alpha is to be determined by trial and error. Once this is known the stresses in concrete may be found using above equation. Then

$$\sigma_s = m\sigma_c\,\frac{R(1-\cos\alpha)}{R(1+\cos\alpha)} = m\sigma_c\,\frac{1-\cos\alpha}{1+\cos\alpha}$$

Stress in horizontal reinforcement

Due to shear, stresses developed in the horizontal reinforcement. Let horizontal shear on the section be H and is shown in the figure5.3

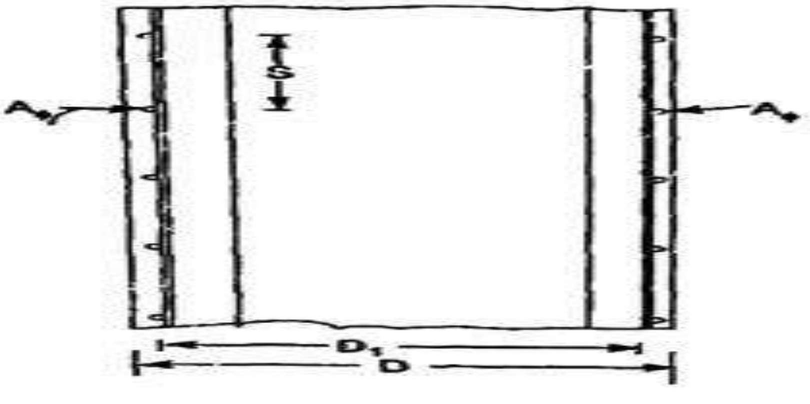


Fig 5.3

$\therefore$ Area of steel in 1 m height

$$= \frac{2A_\phi \times 1000}{s}$$

If σ_s is the stress produced in steel, shear force resisted

$$= \frac{2A_\phi \times 1000}{s}\times\sigma_s$$

Shear force per metre height

$$= \frac{H\times 1000}{\text{Lever Arm}}$$

Assuming lever arm $= D_1$

Shear per metre height $= \frac{1000\ H}{D_1}$

Equating equation (*a*) and (*b*) we get,

$$\frac{2\ A_\phi \times 1000}{s}\ \sigma_s = \frac{1000\ H}{D_1}$$

or

$$\sigma_s = \frac{H s}{2\ A_\phi\ D_1}$$

Temperature stresses

Brick lining reduces the temperature to considerate extent, but still exit some temperature difference between inner and outer surfaces of concrete shell. Let this difference be T. The drop in temperature takes linearly across the wall as shown in Fig.5.4

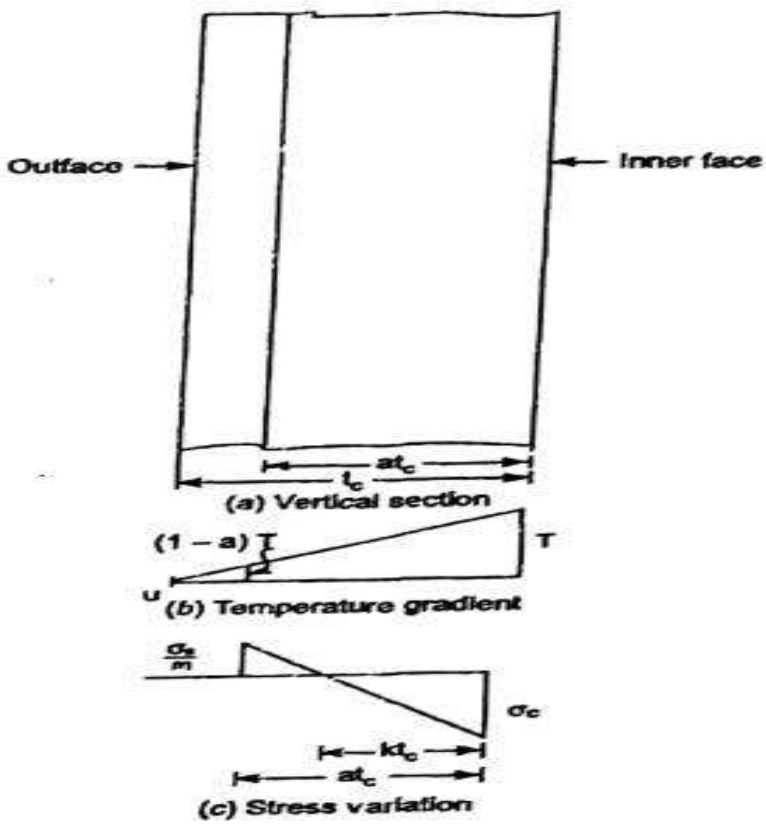


Fig.5.4

Due to higher temperature, concrete on inner face has higher free expansion while free expansion on outer face is less. Since free expansion is prevented by concrete, compressive stresses develop on inner faces and tensile stresses on the outer faces. This variation of stress is linear as shown in Fig. 12.4. Let the compressive stress at inner face be σ_c and tensile stress in steel be σ_r.

Let t_c—Thickness of concrete shell.

at_c—Distance of reinforcement from inner face.

kt_c—Distance of neutral axis from inner face

Consider unit length of circumference. Equating tensile force to compressive forces, we get

$$\frac{1}{2}kt_c \quad \sigma_c = t_s \quad \sigma_s \qquad (a)$$

If percentage reinforcement is 'p' we know

$$t_s = pt_c \qquad (b)$$

∴ From equations (a) and (b) we get

$$\frac{1}{2}kt_c\ \sigma_c = pt_c\sigma_s$$

or
$$\sigma_s = \frac{k\sigma_c}{2p} \qquad (c)$$

From stress diagram, we get

$$\frac{\frac{\sigma_s}{m}}{(at_c - kt_c)} = \frac{\sigma_c}{kt_c}$$

∴
$$\sigma_s = \frac{a-k}{k}\sigma_c \qquad (d)$$

From equations (c) and (d) we get

$$\frac{k\sigma_c}{2p} = \frac{a-k}{k}m\sigma_c$$

or
$$k^2 = 2apm - 2kpm$$

$$k^2 + 2kpm - 2apm = 0$$

∴
$$k = -mp + \sqrt{m^2p^2 + 2mpa}$$

Let α be coefficient of thermal expansion, which is almost same both for steel and concrete.

e – be the final strain experienced by the section.

∴ Free expansion of concrete per unit height of shell

$$= \alpha\ \mathrm{T}$$

Free expansion of steel per unit height of section

$$= \alpha\ (1 - a)\ \mathrm{T}$$

∴ Stress in steel $\quad \sigma_s = \mathrm{E}_s[e - (1 - a)\ \mathrm{T}\alpha] \qquad (e)$

Stress in concrete $\quad \sigma_c = \mathrm{E}_c\ [\mathrm{T}\alpha - e] \qquad (f)$

n Inner Edge

At neutral axis, free expansion = e

∴
$$e = (1 - k)\ \alpha\mathrm{T}$$

Substituting it in (e) and (f), we get

$$\sigma_s = \mathrm{E}_s\ [(1 - k)\ \alpha\mathrm{T} - (1 - a)\mathrm{T}\alpha]$$
$$= \mathrm{E}_s\alpha\mathrm{T}(a - k)$$
$$\sigma_c = \mathrm{E}_c[\mathrm{T}\alpha - (1 - k)\ \mathrm{T}\alpha]$$
$$= \mathrm{E}_c\ \mathrm{T}\ \alpha\ k$$

K can be found from above equation and then σs and σt can be determined

Combined effect of self-weight, wind load and temperature

Stresses at leeward side and windward side due to combined effects of self-weight, wind load and temperature are to be calculated and checked. The relevant expressions are derived below:

(a) Leeward side:

Fig 5.5 shows typical vertical section of shell. Across the section stresses due to self-weight and wind load are assumed uniform, value being that at middle of concrete shell wall, as shown in 5.3. Let this stress be$\sigma c1$. The temperature stress is compressive on inner side and tensile on outer side. Hence combined stress varies linearly as shown fig5.5

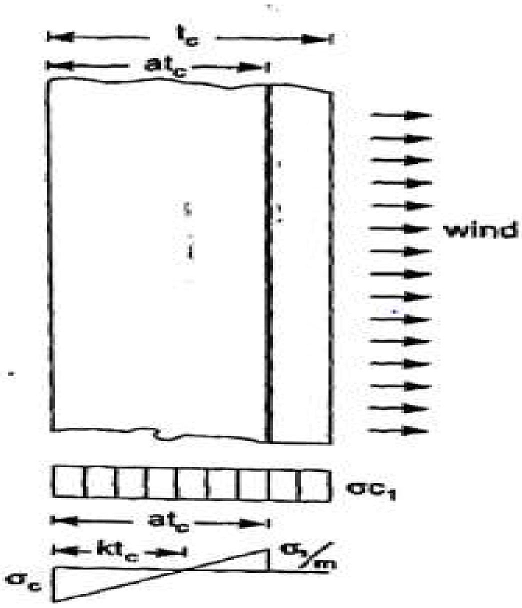


Fig 5.5

Equilibrium of forces in vertical direction gives,

$$\therefore \quad \sigma_{c1}\, t_c + (m-1)t_s\, \sigma_{c1} = \frac{1}{2}\sigma_c k t_c - t_s\, \sigma_s$$

If percentage of steel is p, then $t_s = p\, t_c$.

$$\therefore \quad \sigma_{c1}\, t_c + (m-1)\, p t_c\, \sigma_{c1} = \frac{1}{2}\sigma_c\, k t_c - p t_c\, m\sigma_c \frac{(a-k)}{k}$$

$$i.e., \quad \sigma_{c1}[1+(m-1)p] = \sigma_c\left[\frac{k}{2} - mp\,\frac{a-k}{k}\right]$$

$$\therefore \quad \sigma_c = \frac{\sigma_{c1}[1+(m-1)p]}{\frac{k}{2} - mp\,\frac{a-k}{k}}$$

$$\therefore \quad \sigma_s = m\sigma_c\left(\frac{a-k}{k}\right)$$

Change in stress on inside face

$$= \sigma_c - \sigma_{c1}$$

If ‘e’ is the final strain due to combined effect, additional compressive strain on inside face

= σ T – e

$$\frac{\sigma_c - \sigma_{c1}}{E_c} = \alpha T - e$$

or $$e = \alpha T - \frac{\sigma_c - \sigma_{c1}}{E_c}$$

Change in stress in steel $= m\sigma_{c1} + \sigma_s$

∴ Additional tensile strain $= e - \alpha\,(1 - a)\,T$

$$\therefore \quad e = \frac{m\sigma_{c1} + \sigma_s}{E_s} + \alpha(1-a)T$$

Equating ‘*a*’ and (*b*) we get,

$$\alpha T - \frac{\sigma_c - \sigma_{c1}}{E_c} = \frac{m\sigma_{c1} + \sigma_s}{E_s} + \alpha(1-a)T$$

Substituting for σ_s from equation (12.10) and $E_s = mE_c$, we get

$$\alpha\, T\, E_c - \alpha(1-a)E_c\, T = \frac{m\sigma_{c1} + \frac{(a-k)}{k} m\sigma_{c1}}{m} + \sigma_c - \sigma_{c1}$$

$$a\,\alpha\, E_c\, T = \sigma_c\left(1 + \frac{a-k}{k}\right) + \sigma_c - \sigma_{c1} = \sigma_c\left(1 + \frac{a-k}{k}\right)$$

$$\therefore \quad \sigma_{c1} = \frac{a\alpha T E_c}{1 + \frac{a-k}{k}}$$

$$\frac{\sigma_{c1}\left[1 + (m-1)p\right]}{\frac{k}{2} - mp\frac{a-k}{k}} = \frac{a\alpha T E_c}{1 + \frac{a-k}{k}}$$

$$\frac{\sigma_{c1}\left[1 + (m-1)p\right]}{0.5k^2 - mp(a-k)} = \frac{a\alpha T E_c}{a} = \alpha\, T\, E_c$$

From above equation, k can be determined. Using this value of k in above equations the final stresses due to combined effect σc and σs can be determined.

(b) Wind ward side:

As shell is in tension, due to self-weight and wind, there is no compressive stress in concrete. There is only tensile stress σs1 in steel. σs,σc are stresses due to combined effect.

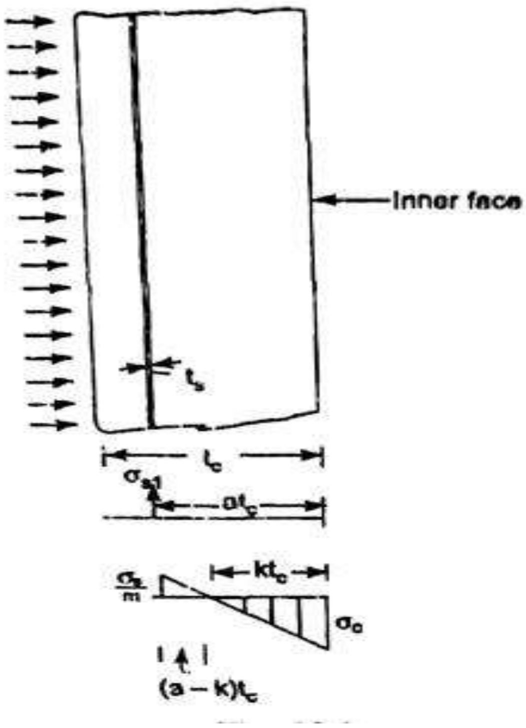


Fig.5.6

Distance of reinforcement from inner edge is at_c and distance of NA is kt_c. Equilibrium condition of forces in vertical direction gives,

$$\sigma_{s1} t_s = \sigma_s t_s - \frac{1}{2} k t_c \sigma_c$$

But
$$\sigma_s = m\sigma_c \cdot \frac{a-k}{k}$$

and $t_s = p\, t_c$ where p is percentage of steel.

Hence
$$\sigma_{s1}\, p\, t_c = m\sigma_c \frac{a-k}{k} p t_c - \frac{1}{2} k\, t_c \sigma_c$$

∴
$$p\, \sigma_{s1} = \sigma_c \left[mp\left(\frac{a-k}{k}\right) - \frac{k}{2} \right]$$

∴
$$\sigma_c = \frac{p\sigma_{s1}}{mp\left(\frac{a-k}{k}\right) - \frac{k}{2}}$$

Change in strain in concrete on inner face,

$$\frac{\sigma_{s1}}{E_s} + \frac{\sigma_c}{E_c} = \alpha T - e$$

or
$$e = \alpha T - \left(\frac{\sigma_{s1}}{mE_c} + \frac{\sigma_c}{E_c} \right) \qquad \textbf{(a)}$$

Change in strain in steel.

$$\frac{\sigma_s - \sigma_{s1}}{E_s} = e - (1 - a)\,\alpha T$$

$$\therefore \quad e = \frac{\sigma_s - \sigma_{s1}}{mE_c} + (1 - a)\alpha T \qquad (b)$$

From equation (*a*) and (*b*) we get

$$\alpha T - \frac{\sigma_{s1}}{mE_c} - \frac{\sigma_c}{E_c} = \frac{\sigma_s - \sigma_{s1}}{mE_c} + (1 - a)\,\alpha T$$

$$a\,\alpha T = \frac{\sigma_c}{E_c} + \frac{\sigma_s}{mE_c}$$

or

$$\sigma_c = a\alpha T E_c - \frac{\sigma_s}{m} \qquad (c)$$

But from stress diagram, we find,

$$\frac{\frac{\sigma_s}{m}}{(a-k)\,t_c} = \frac{\sigma_c}{kt_c}$$

Substituting it in (*c*), we get

$$\sigma_c = a\alpha T E_c - \sigma_c \frac{a-k}{k}$$

$$\therefore \quad \sigma_c = \alpha\,T\,E_c\,k$$

Value of k is determined using the following equation

$$\frac{p\sigma_{s1}}{mp(a-k) - 0.5k^2} = \alpha\,T\,E_c$$

Then from above equations σs and σc are determined.

Temperature stresses in Hoop reinforcement

Fig.5.7 shows the plan of part of cylindrical concrete shell. If Aϕ is the area of reinforcement at spacing s, total area of steel per unit height = Aϕ / s

Treating this area of steel as equal to a steel ring of thickness t_s, we get

$$2\pi R\,t_s = \frac{A_\varphi}{s}$$

Hence t_s can be found.

If percentage of steel is '*p*' then

$$t_s = p\,t_c.$$

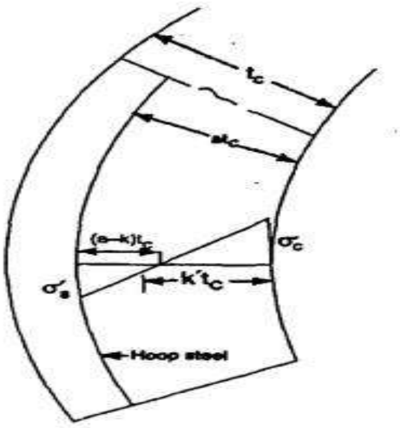


Fig.5.7

For equilibrium, compressive force in concrete is equal to tensile force in steel.

$$\therefore \qquad \frac{1}{2}\sigma_c' k' t_c = \sigma'_s t_s = \sigma'_s p t_c$$

From stress diagram, we find

$$\frac{\sigma_s'}{m} = \sigma_c' \frac{a'-k'}{k'}$$

Substituting it in (*a*) we get,

$$\frac{1}{2}\sigma_c' k' t_c = m\sigma_c' \frac{a'-k'}{k'} p t_c$$

or $$k'^2 = 2\,m\,p\,a' - 2\,m\,pk'$$

or $$k'^2 + 2\,m\,pk' - 2\,m\,p\,a' = 0$$

or $$k' = -mp + \sqrt{m^2p^2 + 2mpa'}$$

From this k' can be found.

Consider the strains in concrete and steel,

strain in concrete:

$$\frac{\sigma_c'}{E_c} = \alpha T - e$$

or $$e = \alpha T - \frac{\sigma_c'}{E_c} \qquad (c)$$

Strain in steel is

$$\frac{\sigma_s'}{E_s} = e - \alpha T(1-a')$$

$$\therefore \qquad e = \frac{\sigma_s'}{E_s} + \alpha T(1-a') \qquad (d)$$

From equations (*c*) and (*d*), we get

$$\alpha T - \frac{\sigma_c'}{E_c} = \frac{\sigma_s'}{E_s} + \alpha T(1-a')$$

or $$\alpha T a' = \frac{\sigma_c'}{E_c} + \frac{\sigma_s'}{mE_c}$$

$$\sigma'_c = \frac{E_c \alpha T a'}{1 + \frac{a' - k'}{k'}} = k' E_c \alpha T$$

Using above equations K', σ's and σ'c are determined.

Example 5.1 Design a chimney of height 70 m and check the stresses in bars. Given

External diameter (i) at top – 4 m

(ii)at base – 4.8 m

Shell thickness (i) at top – 200 mm

(iii)at base – 400 mm

Wind intensity 1.8kN/m^3, throughout,

Thickness of the fire brick lining --100 mm.

Air gap - 70^{0}C

Coefficient of thermal expansion – 11 x 10^{-6}/C^0

Es = 210 x 10^3 N/ mm^2

Unit weight of brick lined = 20kN/m^3.

Use M25 concrete and Fe 415 steel.

Solution:

For M25 concrete $\sigma_{cbc} = 8.5 \text{ N/mm}^2$

∴ $m = \frac{280}{3 \times 8.5} = 10.98 = 11.$

σ_{st} for Fe-415 steel $= 230 \text{ N/mm}^2$.

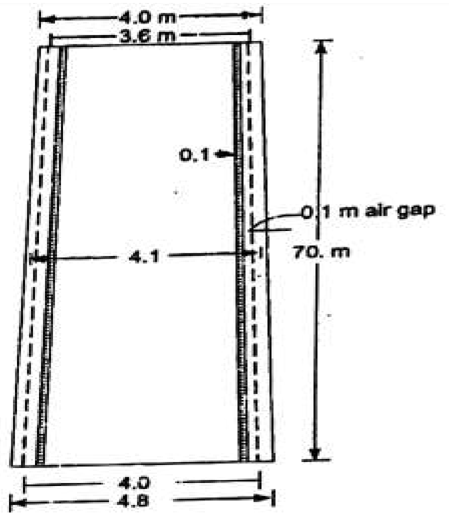


Fig 5.8

Load at Base Section:

Average diameter of shell at top = 3.8 m

Average diameter of shell at base = 4.4 m.

Average thickness of shell $= \frac{200 + 400}{2} = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Weight of shell $= \pi \frac{(3.8 + 4.4)}{2} \times 0.3 \times 70 \times 25$

$= 6762.29$ kN

Diameter of brick lining at mid-height

$= 4.1 - 0.2 - 0.1 = 3.8$ m

$\therefore$ Weight of brick lining $= \pi \times 3.8 \times 0.1 \times 70 \times 20$

$= 1671.33$ kN

$\therefore$ Total dead weight W on the base section

$= 6762.29 + 8433.62 = 8433.62$ kN

Intensity of wind pressure $= 1.8$ kN/m^2

Area obstructing wind $= \frac{4 + 4.8}{2} \times 70 = 308$ m^2

Shape factor $= 0.7$

Wind load $H = 0.7 \times 1.8 \times 308 = 388.08$ kN

Its resultant may be taken as acting at a height

$$= \frac{70}{2} = 35 \text{ m}$$

$\therefore$ $M = 388.08 \times 35 = 1382.8$ kN-m on base section.

Eccentricity $e = \frac{M}{W} = \frac{1382.8}{8433.62} = 1.610 \text{ m} = 1610 \text{ mm}$

einforcement

et us use 1 per cent vertical reinforcement

$\therefore$ A_{st} at base section

$$= \frac{1}{100} \times \pi \times 4400 \times 400 = 55292 \text{ mm}^2$$

Number of 25 mm bars required $= \frac{55292}{\frac{\pi}{4} \times 25^2} = 112.6$

Provide 115 number of 25 mm bars.

$\therefore$ Thickness of equivalent steel ring

$$= \frac{\frac{\pi}{4} \times 25^2 \times 115}{\pi \times 4400} = 4.0 \text{ mm.}$$

Stresses at base section due to self-weight and wind load:

$$e = R \frac{(t_c - t_s)\left[\frac{\sin 2\alpha}{4} + \frac{\pi - \alpha}{2}\right] + \frac{m t_s}{2} \pi}{(t_c - t_s)\left[\sin\alpha + (\pi - \alpha)\cos\alpha\right] + m\, t_s\, \pi \cos\alpha}$$

$$1610 = 2200\frac{(400-4)\left[\frac{\sin 2\alpha}{4}+\frac{\pi-\alpha}{2}\right]+11\times\frac{4}{2}\times\pi}{(400-4)\left[\sin\alpha+(\pi-\alpha)\cos\alpha\right]+11\times 4\times\pi\cos\alpha}$$

Let us try $\alpha = 70°$. Then α in radians = 1.222

$$\text{R.H.S.} = 2200\frac{396\left(\frac{\sin 140}{4}+\frac{\pi-1.222}{2}\right)+11\times\frac{4}{2}\times\pi}{396\left[\sin 70+(\pi-1.222)\cos 70\right]+11\times 4\,\pi\cos 70}$$

$$= 1660.1 \text{ mm.}$$

But L.H.S. = 1610 mm.

Let us try $\alpha = 68° = 1.187$ radians.

Then R.H.S.

$$= 2200\frac{396\left(\frac{\sin 136}{4}+\frac{\pi-1.182}{2}\right)+11\times\frac{4}{2}\times\pi}{396\left[\sin 68+(\pi-1.187)\cos 68\right]+11\times 4\times\pi\cos 68} = 1628.9$$

L.H.S. = 1610 mm.

Let us say the solution is $\alpha = 67° = 1.169$ radians

✓ $$W = \frac{2R\,\sigma_c}{1+\cos\alpha}\left[(t_1-t_s)\left\{\sin\alpha+(\pi-\alpha)\cos\alpha\right\}+m\,t_s\,\pi\cos\alpha\right]$$

$$8433.62\times 1000 = \frac{2\times 2200\,\sigma_c}{1+\cos 67}\left[(400-4)\left[\sin 67+(\pi-1.169)\cos 67\right]+11\times 4\times\pi\cos 67\right]$$

∴ $\sigma_c = 3.683$ N/mm².

∴ ✓ $$\sigma_s = m\sigma_c\frac{1-\cos\alpha}{1+\cos\alpha} = 11\times 3.683\frac{1-\cos 67}{1+\cos 67} = 17.75 \text{ N/mm}^2.$$

The stresses in steel and concrete are within the safe limits.

Hoop Reinforcement

Shear at the base of chimney

= H = 388.08 kN = 388080 N.

Mean Diameter = 4.4 m = 4400 mm.

Lever area = D_1 = distance between the centres of steel

= 4400 − 2 × cover = 4400 − 2 × 50 = 4300 mm.

Provide 10 mm bars of 200 mm c/c as hoop reinforcement.

Then $$\sigma_s = \frac{H.S}{2A_\phi D_1} = \frac{388080\times 200}{2\times\frac{\pi}{4}\times 10^2\times 4300} = 114.91 \text{ N/mm}^2 < 230 \text{ N/mm}^2.$$

Hence safe.

Check for combined stresses due to self-weight, wind and temperature effect:

(a) Leeward side

$t_c = 400$ mm and $t_s = 4$ mm.

Let cover be 50 mm. Then $a\ t_c = 350$ mm.

$$\therefore \quad a = \frac{350}{400} = 0.875.$$

Percentage steel $= p = \frac{t_s}{t_c} = \frac{4}{400} = 0.01$

$$\alpha = 11 \times 10^{-6}/C^{\circ}, \quad m = 11, \quad E_c = \frac{E_s}{m} = \frac{210 \times 10^3}{11} = 19090.9 \text{ N/mm}^2$$

$$\frac{\sigma_{c1}[1+(m-1)p]}{0.5k^2 - mp(a-k)} = 11 \times 10^{-6} \times 70 \times 19090.9 = 14.7$$

Now $\sigma_{c1} = 3.683$ N/mm^2

$$\frac{3.683[1+(11-1)0.01]}{0.5k^2 - 11 \times 0.01(0.875-k)} = 14.7$$

$$4.051 = 7.35k^2 + 1.617\,k - 1.415$$

$$k^2 + 0.22k - 0.7437 = 0$$

$$k = \frac{-0.22 + \sqrt{0.22^2 + 4 \times 0.7437}}{2} = 0.759$$

$$\therefore \quad \sigma_c = \frac{\sigma_{c1}[1+(m-1)p]}{\frac{k}{2} - mp\frac{a-k}{k}}$$

$$\sigma_{c1} = 3.683 \text{ N/mm}^2$$

$$\sigma_c = \frac{3.683[1+(11-1)0.01]}{\frac{0.759}{2} - 11 \times 0.01\frac{0.875-0.759}{0.759}} = 11.16 \text{ N/mm}^2.$$

Permissible value $= 1.33\sigma_{cbc} = 1.33 \times 8.5 = 11.305$ N/mm^2.

Hence safe.

(*b*) Windword side

$$\frac{p\sigma_{s1}}{mp(a-k) - 0.5k^2} = \alpha\, T\, E_c$$

Now, $\sigma_{s1} = 17.75$ N/mm^2

$$\frac{0.01 \times 17.75}{11 \times 0.01(0.875-k) - 0.5k^2} = 11 \times 10^{-6} \times 70 \times \frac{210000}{11}$$

$$0.1775 = 14.7[0.09625 - 0.11k - 0.055k^2]$$

i.e., $k^2 - 2k - 1.2373 = 0$

$$\therefore \quad k = 0.4957$$

$$\sigma_c = \alpha\, T\, E_c\, k = 11 \times 10^{-6} \times 70 \times \frac{210000}{11} \times 0.4957$$

$$= 7.287 \text{ N/mm}^2 \quad < 1.33 \times 8.5 \quad \text{O.K.}$$

$$\sigma_s = m\sigma_c \frac{a-k}{k} = 11 \times 7.287 \frac{0.875 - 0.4957}{0.4957}$$

$= 61.34$ N/mm^2 < 230 N/mm^2. Hence safe.

Streses in Hoop Steel

$$k' = -mp + \sqrt{m^2 p^2 + 2mpa'}$$

$$mp = 11 \times 0.01 = 0.11$$

$$\therefore \quad k' = -0.11 + \sqrt{(0.11)^2 + 2 \times 0.11 \times 0.875} = 0.330$$

$$\sigma_c' = k' E_c \alpha T$$

$$= 0.33 \times \frac{210000}{11} \times 11 \times 10^{-6} \times 70$$

$$= 4.851 \text{ N/mm}^2 \qquad \text{O.K.}$$

$$\sigma_s' = m\sigma_c \frac{a' - k'}{k'}$$

$$= 11 \times 4.851 \frac{0.875 - 0.33}{0.33} = 32.05 \text{ N/mm}^2. \qquad \text{O.K.}$$

Hence the design is safe at base

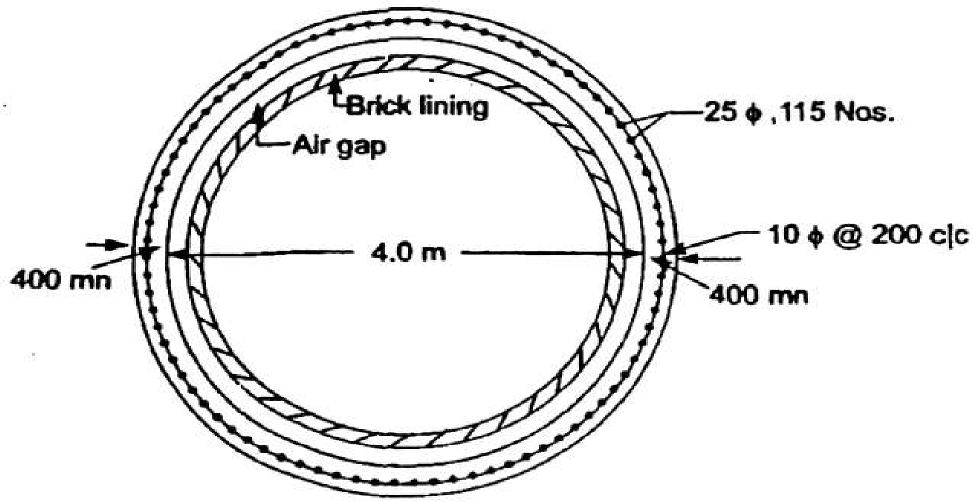


Fig 5.9 Reinforcement details.

DESIGN OF BUNKERS

Bunkers and silos are structures that are used as storage tanks. The bunkers and silos made of reinforced concrete have almost replaced the steel storage structures. Concrete bins possess less maintenance and other architectural qualities greater than steel storage tanks. They are used to store materials like grain, cereals, coal cement etc. They both serve the purpose of bins. Concept and difference between bunkers and silos are explained in the following sections:

Bunkers are mainly employed for storage of underground dwellings. These are mainly related to emergency conditions during wars. The main two characteristics that make a bin to act as a bunker is based on the

- Depth (H)
- Angle of rupture

These are characterized as shallow structures. The angle of rupture of the material in case of bunkers, will meet the horizontal surface at the top of the bin, before it touches the opposite side walls of the structure as shown in the figure-5.1. Bunkers may be circular or rectangular (or square) in plan.

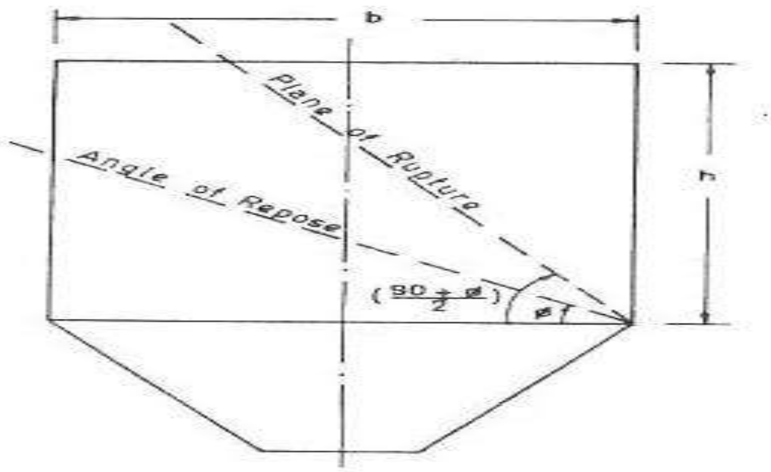


Fig.5.10 Sectional View of a Bunker

The angle of rupture is formed at $(\frac{90+\Phi}{2})$ from the horizontal as shown above. The angle Φ is called as the angle of repose. The lateral pressure form the material is resisted by the side walls. The bunker floor takes up the total load of the material.

The theory used in determination of lateral pressure in bunkers is Rankine‘s Theory.

Design Consideration of Bunkers

1. Design of Bunkers with Rectangular or Square Bottom

The main structural element that constitutes a bunker is shown in figure-5.2. They comprise of

- Vertical walls
- Hopper Bottom
- Edge Beam (At the top level)
- Supporting Columns

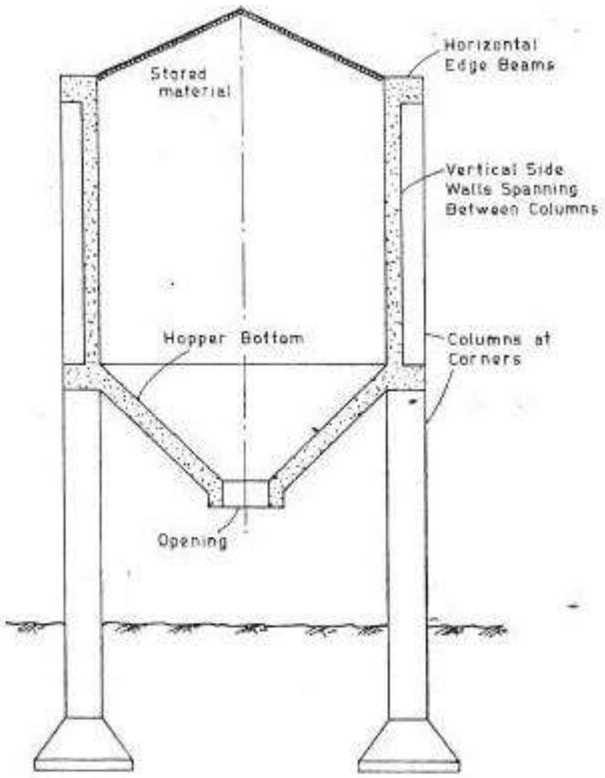


Fig.5.11 Structural Elements of a bunker

The design Procedure can be explained in following steps:

Step 1: Design of Vertical Walls

Based on Rankine's Theory, the lateral pressure applied on the vertical wall can be given by the formula

$$p_a = w \, . \, h \, . \, \cos\alpha \, \left[\frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\Phi}}{\cos + \sqrt{\cos^2\alpha - \cos^2\Phi}} \right] \rightarrow \textbf{Equation-1}$$

Where, P_a = Lateral pressure intensity that is acting at a height of 'h'.

L = Length of the bunker

B = Breadth of the bunker

a = Angle of surcharge (The material slope as shown in figure-5.3)

Φ= Angle of rupture

w = density of the material stored in the bunker

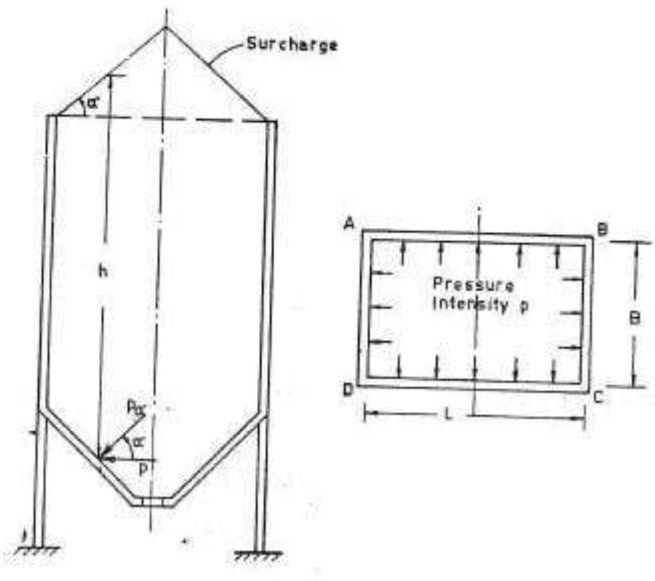


Fig.5.12 Representation of angle of surcharge (α) and pressure component acting on walls (p).

This pressure p_a is acting in the direction parallel to angle of surcharge. So, the pressure that is applied on the vertical walls are the horizontal component of p_a. Let it be p as shown in figure-5.12.

Hence $p = p_{\alpha} . \cos \alpha$

When $\alpha = \Phi$;

Equation-1 Becomes, $p_{\alpha} = w.h.\cos \Phi$;

Hence $p = w.h.\cos^2 \Phi$ →**Equation -2**

Design Moments:

a) Negative Moments at the supports

$$M_A = M_B = M_C = M_D = -\frac{P}{12}[L^2 + B^2 - BL]$$

b) Positive Bending Moment at the centre of longer sides (AB or CD)

$$= \frac{PL^2}{8} - \frac{P}{12}[L^2 + B^2 - BL]$$

c) Positive Bending Moment at the centre of shorter sides (BC or AD)

$$= \frac{PB^2}{8} - \frac{P}{12}[L^2 + B^2 - BL]$$

Direct Tension:

a) Tension in long walls

$= (\frac{pB}{2})$

b) Tension in Short walls

$= (\frac{pL}{2})$

Effective depth:

The effective depth is given by the formula

$$d = \sqrt{\frac{M - T.x}{Q.b}}$$

$$A_{st} = (\frac{M - T.x}{\sigma_m \ j.d}) + (\frac{T}{\sigma_m})$$

To resist maximum bending moment adequate thickness should be provided. The reinforcement details are provided for the vertical walls based on the maximum bending moments and the direct tension design values.

The reinforcement obtained from above equation (A_{st}), is arranged in the horizontal direction. Minimum distribution reinforcement is provided in the vertical direction.

Minimum cross section of 300mm x 300mm edge beams are provided at the top, to facilitate attachments used by conveyor supports.

Step 2: Design of Hopper Bottom

The hopper bottom is designed for direct tension caused due to:

a) Self weight of the material

b) Self weight of sloping slab

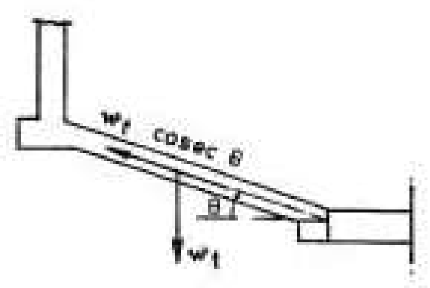


Fig. 5.13 Sloping slab in the hopper subjected to direct tension

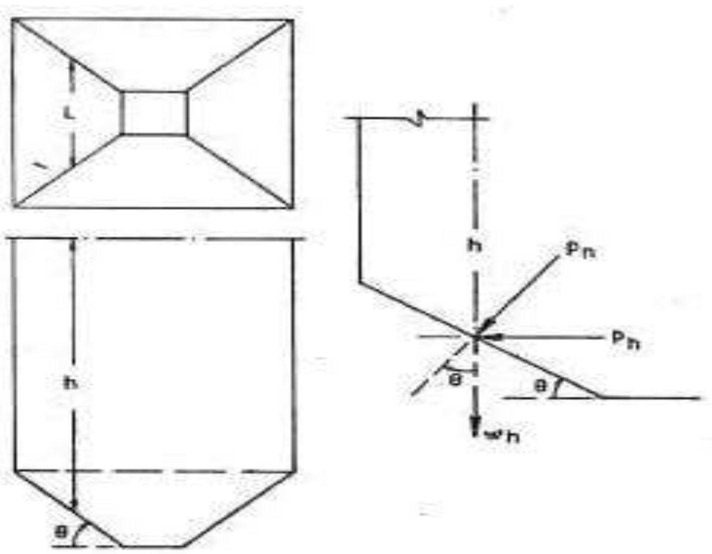


Fig.5.14 Sloping Slab in Hopper Bottom Subjected to bending

From figure-5.4 and 5.5,

wt = weight of material

Calculation of Direct tension

$= Wt \, . \, cosec \, \theta$

Where, θ is the angle between the horizontal and the sloping slab

Calculation for Bending Moment

To determine the maximum moments at the supports and the centre of the sloping slab, we need to determine the normal pressure intensity which is the sum of normal pressure due to material weight and the self-weight of the slab

a) Due to material weight

If *w* = density of the material

h= average height at the centre of the slope of bottom

L = Effective span at the centre of the slope, as shown in figure-5

Then, Normal pressure intensity for depth h is

$P_n = \text{ph} \cos^2\theta + \text{wh}\cos^2\theta$

(put, ph = $\cos^2\Phi$ from equation-2)

hence after rearranging,

$$p_n = wh \, [\, cos^2\theta + cos^2\Phi . \, sin^2\theta]$$

b) Due to self-weight of slab

Let W_d be the self-weight of slab

Its normal component with respect to plane of slab is given by,

$= W_d \, . \, cos\theta$

Hence total normal pressure intensity is given by,

$$P = (p_n + W_d \cdot cos\theta)$$

Hence,

Maximum Negative Bending Moment at Supports

$$= \frac{PL^2}{12}$$

Positive Bending Moment at the Centre

$$= \frac{PL^2}{24}$$

Example 5.1: design side walls and hopper bottom of a rectangular bunker of capacity 300kn to store coal using M20 and Fe 415 steel. Given unit weight of coal is 8kN/m^3 And angle of repose of coal φ = 25^0

Solution:

Volume of bunker = 300/8 = 37.5 m^3

The bunker can store coal to a maximum surcharge of β = φ = 25^0

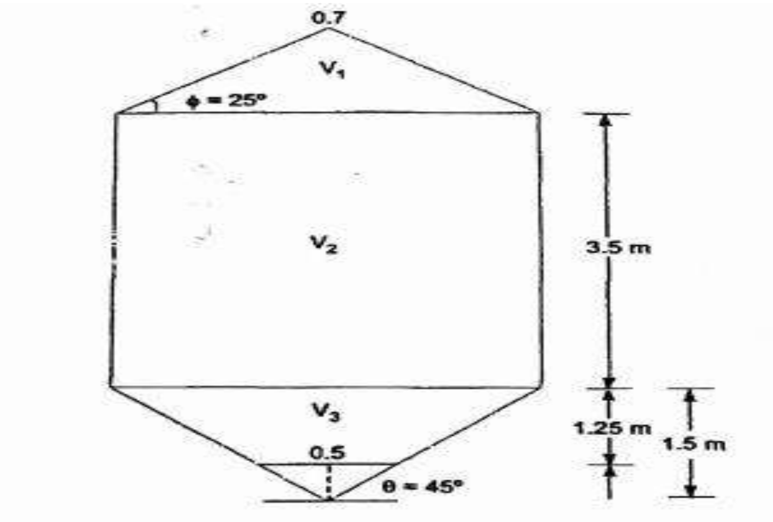


Fig..5.15 Dimensions of bunker

Volume of coal stored as surcharge

$$V_1 = \frac{1}{3} A_1 h_1$$

where A_1 is area of rectangular portion of bunker h_1 – height of surcharge.

Taking size of bunker as 3 × 3 m in plan,

$$h_1 = 1.5 \tan \phi = 1.5 \tan 25 = 0.7$$

$$\therefore \quad V_1 = \frac{1}{3}(3 \times 3 \times 0.7) = 2.1 \text{ m}^3$$

Let the Hopper bottom be at 45° to horizontal which is more than angle of friction between concrete and coal ($\phi^1 = \tan^{-1} \mu^1$). Let us keep size of opening of hopper bottom = 0.5 × 0.5 m.

$$V_3 = \frac{1}{3}\left(A_1 h_2 - A_2 h_2^1\right)$$

$$= \frac{1}{3}(3 \times 3 \times 1.5 - 0.5 \times 0.5 \times 0.25) = 4.48 \text{ m}^3$$

$\therefore$ V_2 = Volume of chamber

$$= V - V_1 - V_3$$

$$= 37.5 - 2.1 - 4.48 = 30.92 \text{ m}^3$$

$\therefore$ Height of chamber $= \frac{30.92}{3 \times 3} = 3.435 \text{ m}^3.$

Let us provide 3.5 m high chamber.

$\therefore$ V_2 provided $= 3 \times 3 \times 3.5 = 31.5 \text{ m}^3.$

Check for bunker action: $3 \times \tan\left(\frac{90+25}{2}\right) = 4.7 \text{ m} > 3.5 \text{ m}.$

Hence line of fracture intersect top surface first. Hence it can be designed as a bunke

$\therefore$ Total storage capacity

$$= (V_1 + V_2 + V_3)\gamma = (31.5 + 2.1 + 4.48) \times 8 = 304.64 \text{ kN}.$$

Design of Side Walls

Horizontal pressure on wall at the junction with hopper bottom

$$p_{n} = \gamma h \cos^2 \varphi = 8 \times 3.5 \cos^2 25 = 23 \text{ kN/m}^2.$$

Corner moment in a square frame

$$= -\frac{p_h}{12}\left[L^2 + B^2 - BL\right] = -\frac{p_h L^2}{12}$$

Assuming 180 mm thick wall

$$L = 3 + 0.18 = 3.18$$

$\therefore$ Corner –ve moment

$$= \frac{23 \times 3.18^2}{12} = 19.38 \text{ kN-m}$$

Tensile force $T = \frac{23 \times 3.18}{2} = 36.57 \text{ kN}$

Assuming 30 mm effective cover

$$x = \frac{180}{2} - 30 = 60 \text{ mm} = 0.06 \text{ m}.$$

This direct tension produces moment of Tx opposite to bending tension about centre line of the section.

$\therefore$ $M = 19.38 - 36.57 \times 0.06 = 17.19 \text{ kN-m}.$

BM at centre of span due to horizontal pressure

$$= \frac{p_h L^2}{8} - \text{Corner moment}$$

$$= \frac{23 \times 3.18^2}{8} - 19.38 = 9.69 \text{ kN-m}$$

$\therefore$ Moment at centre of span

$$= 9.68 - 36.57 \times 0.06 = 7.5 \text{ kN-m}$$

$$M_{u\,lim} = 0.138 f_{ck} bd^2 = 0.138 \times 20 \times 1000 \times 150^2 = 62.1 \text{ kN/m.}$$

But $\quad M_u = 17.19 \times 1.5 = 25.785 < M_{u\,lim}$

Hence thickness selected is satisfactory. Area of steel required for corner moment is given by

$$25.785 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \left[1 - \frac{A_{st}}{1000 \times 150} \times \frac{415}{20}\right]$$

$$476.11 = A_{st}\left[1 - \frac{A_{st}}{7228.92}\right]$$

i.e., $\quad A_{st}^2 - 7228.92\, A_{st} + 476.11 \times 7228.92 = 0$

$\therefore \quad A_{st} = 512 \text{ mm}^2.$

Area of steel required to resist direct tension

$$= \frac{1.5 \times 36.57 \times 1000}{0.87 \times 415} = 152 \text{ mm}^2.$$

$\therefore$ Total area of steel required

$$= 512 + 152 = 664 \text{ mm}^2.$$

Using 12 mm bars spacing required at corners

$$= \frac{\frac{\pi}{4} \times 12^2}{664} \times 1000 = 170 \text{ mm.}$$

Provide 12 mm bars at 160 mm c/c.

It is increased to 300 mm spacing at a height of 1.75 m. Area of steel required at mid-span to resist +ve bending moment is obtained by

$$7.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \left[1 - \frac{A_{st}}{1000 \times 150} \times \frac{415}{20}\right]$$

$$138.48 = A_{st}\left[1 - \frac{A_{st}}{7228.92}\right]$$

or $\quad A_{st}^2 - 7228.92\, A_{st} + 138.42 \times 7228.92 = 0$

$$= 141 \text{ mm}^2.$$

Steel required to take up direct tension = 152 mm².

$\therefore$ Total A_{st} required $= 293 \text{ mm}^2$

Using 12 mm bars spacing

$$s = \frac{\frac{\pi}{4} \times 12^2}{293} \times 1000 = 386 \text{ mm}$$

Provide 12 mm bars at 300 mm c/c.

Vertical reinforcement (distribution steel)

$$= \frac{0.12}{100} \times 1000 \times 180 = 216 \text{ mm}^2.$$

Using 8 mm bars

$$s = \frac{\frac{\pi}{4} \times 8^2}{216} \times 1000 = 232 \text{ mm.}$$

Provide 8 mm bars at 225 mm c/c.

Design of Hopper Bottom

Total weight of coal to be supported

$$= 304.64 \text{ kN.}$$

Assuming 180 mm thick hopper bottom.

$$\text{Self-weight} = 4 \times \frac{3 + 0.5}{2} \times 1.25\sqrt{2} \times 0.180 \times 25 = 55.69 \text{ kN.}$$

$\therefore$ Total weight of coal and self at of hopper bottom $= 304.64 + 55.69 = 360.33$ kN.

∴ Weight on each plate of hopper bottom

$$= 90.08 \text{ kN.}$$

∴ Direct tension in sloping bottom slab

$$= 90.08 \text{ cosec } 45° = 127.4 \text{ kN}$$

Design tensile force per metre width

$$= 1.5 \times \frac{127.4}{3} \doteq 63.7 \text{ kN/metre width}$$

$$\therefore \quad A_{st} = \frac{63.7 \times 1000}{0.87 \times 415} = 176.42 \text{ mm}^2.$$

But minimum reinforcement required

$$= \frac{0.12}{1000} \times 180 \times 1000 = 216 \text{ mm}^2.$$

Providing them equally on both faces

$$A_{st} \text{ on each face} = \frac{216}{2} = 108 \text{ mm}^3.$$

Using 8 mm bars

$$s = \frac{\frac{\pi}{4} \times 8^2}{108} \times 1000 = 465 \text{ mm.}$$

Provide 8 mm bars at 300 mm c/c on both faces.

Check for stress $m = \frac{280}{3 \times 7} = 13.33$. It is taken as 13.

$$\therefore \quad A_c = 180 \times 1000 + 216 \times 13 = 182808$$

$$\therefore \text{ Stress in concrete} = \frac{127.4 \times 1000}{182808} = 0.7 \text{ N/mm}^2 < 2.8. \text{ Hence safe.}$$

Design of reinforcement in horizontal direction for bending in the middle strip:

Total normal pressure

$$p_n = \gamma h \cos^2 \theta + P_n \sin^2 \theta + W_s \cos \theta$$

Now h = depth at mid point of sloping slab

$$= \frac{0.7}{2} + 3.5 + \frac{1.25}{2} = 4.475 \text{ m.}$$

$$\theta = 45°$$

$$p_h = \gamma h \cos^2 \varphi = \gamma h \cos^2 25$$

$$p_n = 8 \times 4.475 \cos^2 45 + 8 \times 4.475 \cos^2 25 \sin^2 45 + 0.180 \times 25 \times \cos 45$$

$$= 35.785 \text{ kN/m}^2.$$

$$\text{Effective span} = \frac{3 + 0.5}{2} + 0.18 = 1.93 \text{ m.}$$

$$\therefore \text{ Maximum –ve BM} = \frac{p_n L^2}{12} = \frac{35.785 \times 1.93^2}{12} = 11.1 \text{ kN-m.}$$

$$M_u = 1.5 \times 11.1 = 16.65 \text{ kN-m}$$

$$D = 180 - 30 = 150 \text{ mm}$$

$$16.65 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150 \left[1 - \frac{A_{st}}{1000 \times 150} \times \frac{415}{20}\right]$$

$$307.65 = A_{st}\left[1 - \frac{A_{st}}{7228.92}\right]$$

$$A_{st}^2 - 7228.92\, A_{st} + 307.65 \times 7228.92 = 0$$

$$A_{st} = 322 \text{ mm}^2$$

Using 10 mm bars

$$s = \frac{\frac{\pi}{4} \times 10^2}{322} \times 1000 = 244 \text{ mm}$$

Provide 10 mm bars at 240 mm c/c.

+ve moment at mid-span

$$= p_n \frac{L^2}{8} - p_n \frac{L^2}{12} = \frac{p_n L^2}{24}$$

$$= \frac{35.785 \times 1.93^2}{24} = 5.55 \text{ kN-m}$$

$$M_u = 1.5 \times 5.55 = 8.25 \text{ kN-m.}$$

$$8.25 \times 10^6 = 0.87 \times 415 \times A_{st} \times 150\left[1 - \frac{A_{st}}{1000 \times 150} \times \frac{415}{20}\right]$$

$$152.33 = A_{st}\left[1 - \frac{A_{st}}{7228.92}\right]$$

$$A_{st}^2 - 7228.92\, A_{st} + 152.33 \times 7228.92 = 0$$

$$A_{st} = 155.6 \text{ mm}^2$$

But minimum required $= \frac{0.12}{100} \times 1000 \times 180 = 216 \text{ mm}^2$.

Using 10 bars

$$s = \frac{\frac{\pi}{4} \times 10^2}{216} \times 1000 = 363 \text{ mm.}$$

Provide 10 mm bars at 300 mm c/c.

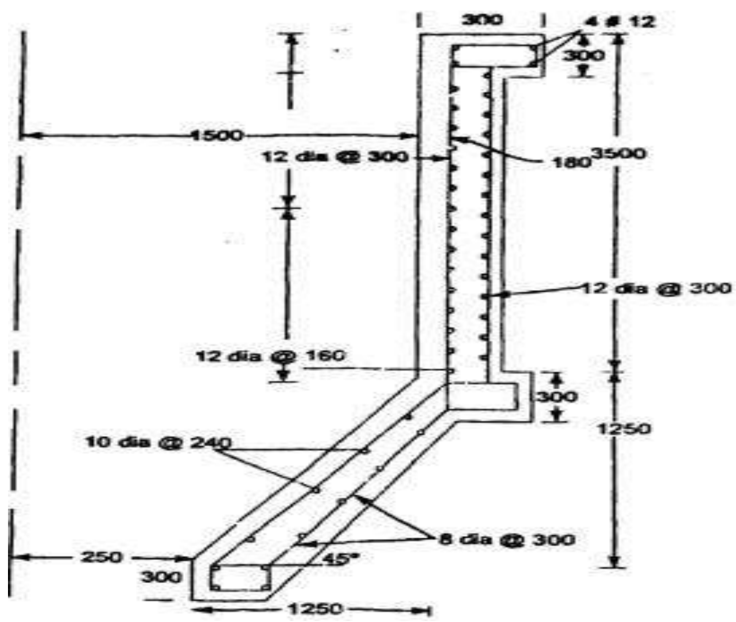


Fig 5.16 Reinforcement detail in bunker

Design consideration of Bunkers with Circular Bottom

For design of bunkers with circular cross section, vertical walls are subjected to a hoop tension along the diameter of the bunker. The value of hoop tension is given by the formula

$T_h = 0.5 p_h . D$

D = Diameter of the bunker

p_h = horizontal component of pressure at a depth *h* from the top

The reinforcement details are provided to resist the hoop tension for this a minimum thickness of 120mm is recommended.

The hopper bottom is designed for both direct and hoop tension due to normal pressure on the sloping slabs.

Minimum vertical reinforcement is provided based on the bar used.

Design of Columns

Columns are designed for compression and bending. The loads on the columns are due to:

a) Vertical loads = weight of stored material + self-weight of members

b) Horizontal loads = Wind Loads

Example 5.2 Design a circular cylindrical bunker of capacity 300kN to store coal using M20 concrete and Fe415 steel. Given unit weight of coal is 8kN/m^{3}and angle of repose of coal $\varphi = 25^0$

Solution:

∴ Volume of coal to be retained

$$= \frac{300}{8} = 37.5 \text{ m}^2.$$

Volume stored in surcharge at $\varphi = 25°$

$$V_1 = \frac{1}{3}\frac{\pi D^2}{4} h$$

where $h = \frac{D}{2} \sin 25°$

Let us select diameter of bunker = 3.5 m.

Then, $$h = \frac{3.5}{2} \sin 25 = 0.74 \text{ m}.$$

$$V_1 = \frac{1}{3} \times \frac{\pi \times 3.5^2}{4} \times 0.74 = 2.37 \text{ m}^3.$$

Let hopper bottom be at 45° with a opening of 0.5 m as shown in Fig. 11.8. Volume of coal store in hopper portion

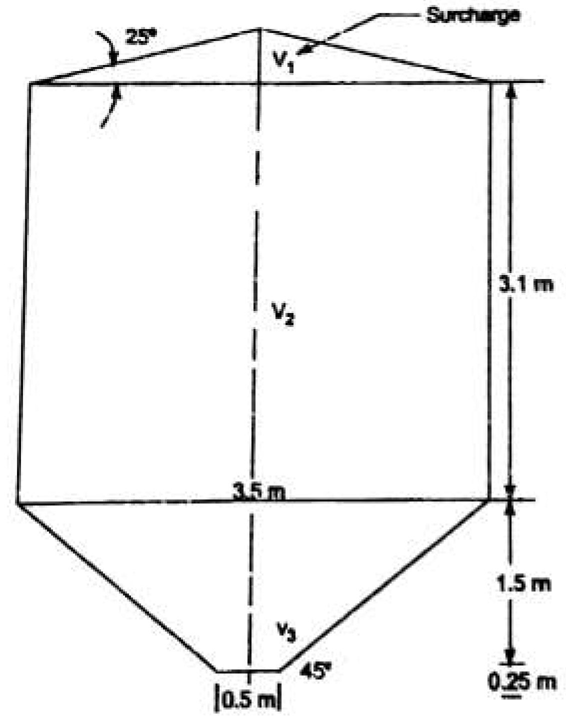


Fig.5.17 Dimensions of bunker

$$V_3 = \frac{1}{3}\pi\frac{3.5^2}{4}\times\frac{3.5}{2} - \frac{1}{3}\times\pi\times\frac{0.5^2}{4}\times\frac{0.5}{2} = 5.6\ m^3$$

∴ Volume of cylindrical portion required

$$V_2 = 37.5 - 2.37 - 5.6 = 29.53\ m^3.$$

∴ Height 'h' of cylindrical portion required is obtained by

$$\frac{\pi}{4}\times 3.5^2 h = 29.53$$

$$h = 3.07\ m.$$

Provide $h = 3.1$ m.

∴ Total volume of coal retained

$$V = V_1 + V_2 + V_3 = 2.37 + \frac{\pi}{4}\times 3.5^2 \times 3.1 + 5.6 = 37.8\ m^2$$

$$W = 37.8 \times 8 = 302.4\ kN.$$

Design of Cylindrical Walls

$$p_h = \gamma h \cos^2\phi = 8 \times 3.1 \times \cos^2 25 = 20.37\ kN/m^2$$

Hoop tension $T = 0.5 \times 20.37 \times 3.5 = 35.65$ kN

∴ $T_u = 1.5T = 1.5 \times 35.65 = 53.47$ kN

$$A_{st} = \frac{T_u}{0.87\, f_y} = \frac{53.47 \times 1000}{0.87 \times 415} = 148 \text{ mm}^2$$

Using 140 mm thick wall,

A_{st} minimum $= \frac{0.12}{100} \times 140 \times 1000 = 168 \text{ mm}^2$

Using 8 mm bars

$$s = \frac{\frac{\pi}{4} \times 8^2}{168} \times 1000 = 299 \text{ mm.}$$

∴ Provide 8 mm bars at 280 mm c/c for hoop tension. Provide the same as distribution steel in vertical direction.

Design of Hopper Bottom

Provide a sloping slab also of thickness 140 mm.

Weight of coal $= 302.4$ kN.

Mean diameter of sloping bottom

$$= \frac{3.5 + 0.5}{2} + 0.14\sqrt{2} = 2.2 \text{ m.}$$

Weight of hopper bottom $= \pi \times 2.2 \times 0.14 \times \frac{1.5}{\cos 45} \times 25 = 51.3$ kN

∴ Total vertical load $= 302.4 + 51.3 = 353.7$ kN

∴ Tension per metre run of hopper bottom

$$= \frac{353.7 \times \operatorname{cosec} 45}{2.2\pi} = 72.37 \text{ kN/m.}$$

∴ $T_u = 1.5 \times 72.37$ kN.

$$A_{st} = \frac{1.5 \times 72.37 \times 1000}{0.87 \times 415} = 300 \text{ mm}^2$$

Using 8 mm bars $s = \frac{\frac{\pi}{4} \times 8^2}{300} \times 1000 = 167.5$ mm

Provide 8 mm bars @ 160 mm c/c in the sloping direction of slab.

Check for direct stress: $m = 13$ $A_e = 140 \times 1000 + 300 \times 13 = 143900 \text{ mm}^2$

$$\text{Stress} = \frac{72.37 \times 1000}{143900} = 0.503 \text{ N/mm}^2 < 2.8 \text{ N/mm}^2\text{. Hence safe}$$

Reinforcement for Hoop Tension

Average depth of sloping bottom

$$= 3.1 + \frac{3.5}{2} \tan 25 + \frac{1.5}{2} = 4.66 \text{ m.}$$

$\theta = 45°$, $\varphi = 25°$

∴ $P_n = \gamma h \cos^2\theta + p_h \sin^2\theta$

$= \gamma h \cos^2\theta + \gamma h \cos^2\varphi \sin^2\theta$

$= 8 \times 4.66 \times \cos^2 45 + 8 \times 4.66 \times \cos^2 25 \times \cos^2 45 = 33.95 \text{ kN/m}^2$

Normal pressure due to self-weight

$$= W \cos 0 = 51.3 \times \cos 45 = 36.27 \text{ kN}$$

$\therefore$ Normal Pressure due to self-weight per unit length $= \dfrac{36.27}{\pi \times 2.2} = 5.25$ kN

$\therefore$ Normal Pressure per unit length

$$= 33.95 + 5.25 = 39.20 \text{ kN/m}$$

$\therefore$ Hoop tension per metre run

$$= 0.5 \times 39.20 \times 2.2 = 43.12 \text{ kN.}$$

$\therefore$ $$T_u = 1.5 \times 43.12$$

$\therefore$ $$A_{st} = \frac{1.5 \times 43.12 \times 1000}{0.87 \times 415} = 179 \text{ mm}^2$$

A_{st} minimum $$= \frac{0.12}{100} \times 140 \times 1000 = 168 \text{ mm}^2$$

Using 8 mm bars $$s = \frac{\frac{\pi}{4} \times 8^2}{179} \times 100 = 280.8 \text{ mm}$$

Provide 8 mm bars at 280 mm c/c.

Check for direct tension: $m = 13$, $A_c = 140 \times 1000 + 13 \times 179 = 142327 \text{ mm}^2$

$\therefore$ Stress $$= \frac{43.12 \times 1000}{142327} = 0.303 \text{ N/mm}^2 < 2.8 \text{ N/mm}^2. \text{ Hence safe.}$$

Nominal edge beams of size 300 × 300 mm with 4 bars of 12 m may be provided at junction.

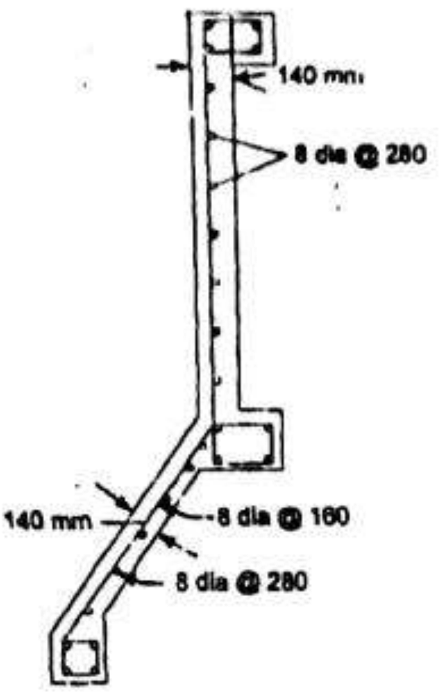


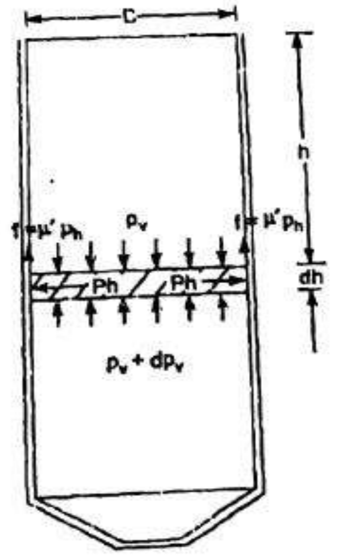
Fig.5.18 Reinforcement details of circular bunker

DESIGN OF SILOS

In silos lot of weight of stored material gets transferred as axial compression due to friction between the material and concrete wall. It results into lateral pressure. Hence Rankine theory of lateral pressure cannot be used. The following two theories are available for fitting lateral pressures in the silos.

1. Janssen's theory
2. Airy's theory

(i) Janssen's Theory: This theory is derived by considering equilibrium of material of elemental depth '*dh*' stored. Referring to Fig. 11.10, at depth *h* from top surface.



Let p_v – Vertical pressure

p_h – Horizontal intensity of pressure

μ' – Coefficient of friction between material stored and concrete. Hence

$f = \mu' p_h$ – intensity of frictional force

Let γ – Unit weight of material stored

A – Cross-sectional area of silo

U – Perimeter of section of silo

$R = \frac{A}{U}$ = Hydraulic mean radius

ϕ – Angle of repose of stored material. Fig. 11.2 shows various forces acting on an element depth *dh* of the silo. Equilibrium of vertical forces on it gives.

$$p_v A + \gamma A\, dh = (p_v + dp_v)A + f U\, dh$$
$$= (p_v + dp_v)A + \mu' p_h U\, dh$$

i.e.,
$$\gamma\, dh = dp_v + \mu' p_h \frac{U}{A} dh$$

$$\therefore \qquad dp_v = \left(\gamma - \frac{\mu' p_h}{R}\right) dh$$

Since $\frac{U}{A}$ = R, hydraulic mean radius.

There exist a constant ratio between horizontal and vertical pressure. Let

$$p_h = K p_v$$

$$\therefore \qquad dp_v = \left(\gamma - \mu' p_v \frac{K}{R}\right) dh.$$

$$\therefore \qquad \frac{dp_v}{\left(\gamma - \mu' \frac{K}{R} p_v\right)} = dh$$

∴ Integration gives

$$h = \frac{1}{-\mu' \frac{K}{R}} \log\left(\gamma - \mu' \frac{K}{R} p_v\right) + \text{Constant}$$

$$\therefore \qquad \log\left(\gamma - \mu' \frac{K}{R} p_v\right) = -\mu' \frac{K}{R} h + C$$

where C is also constant since $\mu' \frac{K}{r}$ is constant. Substituting the boundary condition that at $h = 0$, $p_v = 0$, we get

$$\log \gamma = C$$

$$\therefore \qquad \log \frac{\gamma - \mu' \frac{K}{R} p_v}{\gamma} = -\mu' \frac{K}{R} h$$

$$\therefore \qquad \frac{\gamma - \mu' \frac{K}{R} p_v}{\gamma} = e^{-\mu' \frac{K}{R} h}$$

or

$$1 - \frac{\mu' K}{\gamma R} p_v = e^{-\frac{\mu' k}{R} h}$$

$$\therefore \qquad p_v = \frac{\gamma R}{\mu' K}\left[1 - e^{-\frac{\mu' k}{R} h}\right]$$

Hence

$$p_h = K p_v = \frac{\gamma R}{\mu'}\left[1 - e^{-\frac{\mu' k}{R} h}\right]$$

Silos are normally having circular sections. If diameter of circular section is D, then

$$R = \frac{A}{U} = \frac{\pi \frac{D^2}{4}}{\pi D} = \frac{D}{4}$$

$$\therefore \qquad p_v = \frac{\gamma D}{4\mu' K}\left[1 - e^{\frac{-4\mu' K}{D} h}\right] = \frac{p_h}{K}$$

and

$$p_h = \frac{\gamma D}{4\mu'}\left[1 - e^{-\frac{4\mu' K}{D} h}\right]$$

Load Carried by Wall

Weight of grain above the lower ring beam (*i.e.*, at h = H) partly gets transferred to wall and partly to the bottom slab. This is because of friction between the wall and the grain. Hence it may be obtained as integral of frictional force $f = \mu' p_h \times$ perimeter or as a difference between total grain weight above the lower ring beam (at h = H) and the pressure intensity p_v multiplied by area. Then

$$P_w = \gamma A h - p_v A = A(\gamma h - p_v) \qquad ...(11.17)$$

(*ii*) **Airy's Theory:** W Airy suggests derivation of horizontal pressure based on the equilibrium of material stored above the line of rupture. Depending up on the plane of rupture the following two cases arise:

(*i*) Plane of rupture cuts the top horizontal surface

(*ii*) Plane of rupture cuts the opposite vertical wall.

Case I: Plane of rupture cuts the top horizontal surface:

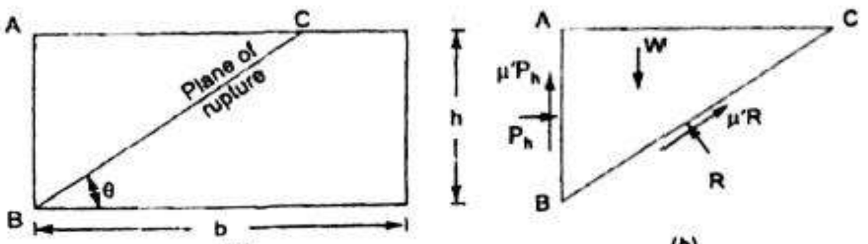
Fig. 11.11(*a*) Shows a typical situation. In this

AC is horizontal top surface

BC is line of rupture

θ – Angle of line of rupture with horizontal

b – Diameter of silo.



(a) (b)

Let

P_h – Be horizontal reaction from wall

μ' – Coeficient of friction between the material stored and concrete.

Vertical frictional force exerted by wall on material = $\mu' P_h$.

Let R – be the normal reactive force along plane of rupture

$\mu = \tan\phi$, be the coefficient of friction

$\therefore$ μR is the frictional force along the plane of rupture

W – be the weight of material in the portion ABC (weight of wedge)

Now, $\quad$ AC = $h \cot\theta$

$\therefore$ Weight of wedge

$$W = \frac{1}{2} \times AB \times AC\,\gamma$$

$$= \frac{1}{2} h\; h\; \cot\theta\,\gamma$$

$$= \frac{1}{2}\gamma h^2 \cot\theta$$

Consider the equilibrium of the wedge ABC.

Σ Forces in vertical direction = 0, gives

$\mu' P_h + \mu R \sin\theta + R\cos\theta - W = 0$

$\therefore \quad R(\mu \sin\theta + \cos\theta) = W - \mu' P_h$

or
$$R = \frac{W - \mu' P_h}{\mu \sin\theta + \cos\theta}$$

Σ Forces forces in horizontal direction = 0, gives

$P_h + \mu R \cos\theta - R\sin\theta = 0$

$\therefore \quad P_h = R(\sin\theta - \mu\cos\theta)$

or
$$R = \frac{P_h}{\sin\theta - \mu\cos\theta}$$

Equating equation (1) to Eqn. (2), we get.

$$\frac{W - \mu' P_h}{\mu\sin\theta + \cos\theta} = \frac{P_h}{\sin\theta - \mu\cos\theta}$$

or $\quad W(\sin\theta - \mu\cos\theta) - \mu' P_h(\sin\theta - \mu\cos\theta) = P_h(\mu\sin\theta + \cos\theta)$

or $\quad W(\sin\theta - \mu\cos\theta) = P_h[\mu\sin\theta + \cos\theta + \mu'\sin\theta - \mu\mu'\cos\theta]$

$$= P_h[(\mu + \mu')\sin\theta + (1 - \mu\mu')\cos\theta]$$

$$\therefore \quad P_h = W\frac{\sin\theta - \mu\cos\theta}{(\mu + \mu')\sin\theta + (1 - \mu\mu')\cos\theta}$$

Dividing numerator and denominator by cos θ, we get

$$P_h = W\frac{\tan\theta - \mu}{(1 - \mu\mu') + (\mu + \mu')\tan\theta}$$

$$= \frac{1}{2}\gamma h^2 \cot\theta \frac{\tan\theta - \mu}{(1 - \mu\mu') + (\mu + \mu')\tan\theta}$$

$$= \frac{1}{2}\gamma h^2 \frac{\tan\theta - \mu}{(1 - \mu\mu')\tan\theta + (\mu + \mu')\tan^2\theta}$$

Let $\quad u = \tan\theta - \mu$

and $\quad v = (1 - \mu\mu')\tan\theta + (\mu + \mu')\tan^2\theta$

∴ **For maximum** P_h

$$\frac{dP_h}{d\theta} = 0$$

$$\frac{dP_h}{d\theta} = \frac{1}{2}\gamma h^2\left(\frac{udv - vdu}{u^2}\right) = 0$$

i.e., $udv - vdu = 0$ *i.e.,* $udv = vdu$

i.e. $(\tan\theta - \mu)\,[(1 - \mu\mu')\sec^2\theta + (\mu + \mu')2\tan\theta\sec^2\theta]$

$= [1 - \mu\mu')\tan\theta + (\mu + \mu')\tan^2\theta]\,(\sec^2\theta)$

$\sec^2\theta$ term appears on both sides of equality and hence may be cancelled. Simplification leads to the equation.

$$\tan^2\theta - 2\mu\tan\theta - \frac{\mu(1-\mu\mu')}{\mu+\mu'} = 0$$

$\therefore$ $$\tan\theta = \mu + \sqrt{\frac{\mu(1+\mu^2)}{\mu+\mu'}}$$

Substituting it in equation (3) and simplifying, we get

$$P_h = \frac{1}{2}\gamma h^2\left[\frac{1}{\sqrt{1+\mu^2} + \sqrt{\mu(\mu+\mu')}}\right]^2$$

Noting that p_h denotes the total horizontal force per unit length of wall the pressure p_h below top ,

$$p_h = \frac{dP_h}{dh} = \gamma h\left[\frac{1}{\sqrt{1+\mu^2} + \sqrt{\mu(\mu+\mu')}}\right]^2$$

Total lateral pressure $= \pi\, b\, p_h$

Total vertical load carried by wall $= \pi\, b\, P_h\, \mu'$

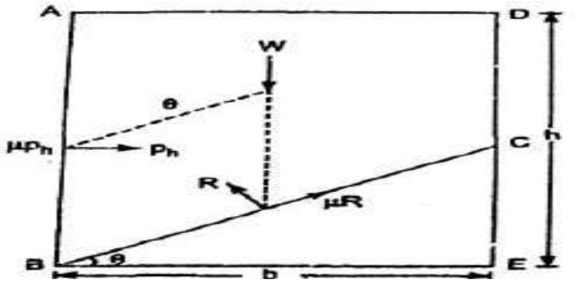
The depth upto which bin will act as shallow is given by

$$\tan 0 = \frac{h}{b}$$

i.e., $$\mu + \sqrt{\frac{\mu(1+\mu^2)}{\mu+\mu'}} = \frac{h}{b}$$

or $$h = b\left[\mu + \sqrt{\frac{\mu+(1+\mu^2)}{\mu+\mu'}}\right]$$

Case II : Plane of rupture cuts opposite wall as shown below



Now $\quad CE = b \tan\theta$

∴ Weight of wedge ABCD

$$W = \gamma\left[bh - \frac{1}{2}b.\,b\tan\theta\right] = \frac{\gamma b}{2}[2h - b\tan\theta]$$

Considering the equilibrium of forces acting on the wedge, P_h value is obtained as in equati previous case

$$P_h = W\frac{\tan\theta - \mu}{(1-\mu\mu') + (\mu + \mu')\tan\theta}$$

Substituting the value of W from equation (11.23),

$$P_h = \frac{\gamma b}{2}\frac{(2h - b\tan\theta)(\tan\theta - \mu)}{(1-\mu\mu') + (\mu+\mu')\tan\theta}$$

Differentiating P_h with respect to θ and equating that to zero for maximum P_h (as done in case), we get a quadratic in tan θ. After solving that quadratic equation we get

$$\tan\theta = -\frac{1-\mu\mu'}{\mu+\mu'} + \frac{1+\mu}{\mu+\mu}\sqrt{(1-\mu\mu') + \frac{24}{b}(\mu+\mu')}$$

Substituting the value of tan teta obtained from above equations value of Ph can be obtained

$$p_h = \frac{dP_h}{dh} = \frac{\gamma\, b(\tan\theta - \mu)}{(1-\mu\mu') + (\mu+\mu')\tan\theta}$$

Total lateral pressure $= \pi\, b\, p_h$

Total vertical load carried by wall $= \pi\, b\, \mu\, p_h$

For conical hopper bottom, the surcharge pressure

$$\frac{\frac{\pi b^2}{4}\gamma H - \pi b\, p_H\, \mu'}{\frac{\pi b^2}{4}}$$

where H is the depth from top of the silo.

Example 5.3A silo with internal diameter 5.5m, height of cylindrical portion 18 m and central opening with 0.5m is to be built to store wheat. Design the silo using M20 grade concrete and Fe 415 steel. Given

Unit weight of wheat	$= 8.5\ kN/m^3$.
Angle of internal friction	$= 28°$
Angle of wall friction	$= 0.75\ \varphi$ *while filling*
	$= 0.60\ \varphi$ *while emptying*
Pressure ratio	$= \frac{p_h}{p_v} = K = 0.5$ *while filling*

Use Janssen's theory for pressure calculations.

Solution:

The following figure 5.19 shows the dimensions of the silo to be designed. Slope of hopper is kept as 45^0

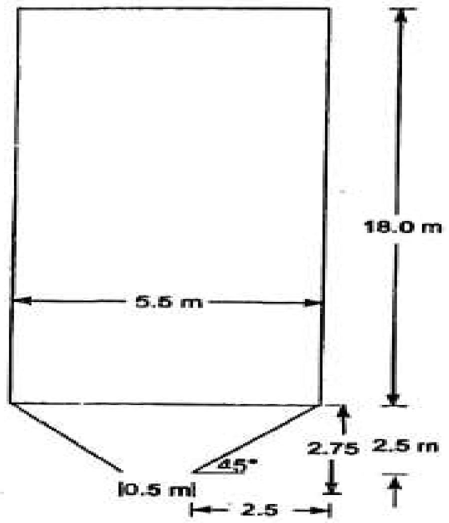


Fig.5.19

Since wheat is a granular material the loading cases to be considered for finding horizontal pressure and the load carried by the wall is emptying case: For this case:

$$\phi' = 0.6\phi = 0.6 \times 28 = 16.8$$

$\therefore$ $\mu' = 0.30$ and $\mu = \tan\varphi = \tan 28 = 0.532$

$$k = 1.0$$

In this problem $\gamma = 8.5\ \text{kN/m}^3$, $b = 5.5$ m and $h = 18$ m

Hydraulic mean radius $= R = \dfrac{\frac{\pi D^2}{4}}{\pi D} = \dfrac{D}{4} = \dfrac{5.5}{4} = 1.375$ m

$$\therefore \quad \frac{\mu' k}{R} h = \frac{0.30 \times 1.0}{1.375} h = 0.218\, h$$

$$p_h = \frac{\gamma R}{\mu'}\left[1 - e^{-\frac{\mu' k}{R} h}\right] = \frac{8.5 \times 1.375}{0.30}\left[1 - e^{-0.218h}\right] = 38.958\left[1 - e^{-0.218h}\right]$$

Hoop tension $T = p_h \dfrac{b}{2}$ and $A_{st} = \dfrac{1.5\,T}{0.87 \times 415}$.

Table below shows these values at every 3 m and the reinforcement to be provided as hoop steel. It is to be noted that minimum reinforcement to be provided for direct steel = 0.3%. Hence A_{st} minimum $= \dfrac{0.3 \times 150 \times 1000}{100} = 450\ \text{mm}^2$, since thickness is selected as 150 mm (This is the minimum thickness suggested).

h	p_h in kN/m^2	[illegible]	A_{st} in mm^2	Reinforcement details
3	18.70	51.425	213.6	Provide minimum reinforcement of 10 mm @ 170
6	28.43	78.183	324.8	
9	33.48	92.07	382.5	
12	36.11	99.30	412.5	
15	37.48	103.07	428.2	
18	38.19	105.02	436.3	

Vertical Reinforcement

Provide minimum reinforcement of 10 mm @ 170 mm c/c, since axial compression works out to be too small.

Design of Hopper Bottom

Vertical load on hopper bottom is more during filling. Hence we have to use

$$\mu' = \tan(0.7\,\varphi) = \tan 19.6 = 0.356$$

$$K = 0.5$$

$$\therefore \quad \frac{\mu' K}{R} h = \frac{0.356 \times 0.5}{1.375} h = 0.129\, h$$

$$\therefore \quad p_h = \frac{\gamma R}{\mu'}\left[1 - e^{\frac{-\mu' K}{R} h}\right] = \frac{8.5 \times 1.375}{0.356}\left[1 - e^{-0.140h}\right]$$

$$= 32.83\,[1 - e^{-0.140h}]$$

$$\therefore \quad \text{Maximum } p_h = 32.83[1 - e^{-0.140 \times 18}] = 30.436 \text{ kN/m}^2$$

$$\therefore \quad p_v = \frac{p_h}{K} = \frac{30.436}{0.5} = 60.872 \text{ kN}$$

∴ Load on hopper bottom from grain stored in cylindrical portion

$$= 60.872 \times \frac{\pi}{4} \times 5.5^2 = 1446.21 \text{ kN.}$$

Volume of hopper bottom $= \frac{1}{3}\left[\pi \times 2.75^3 - \pi \times 0.25^3\right] = 21.762 \text{ m}^3$

∴ Weight of wheat in this portion = 21.762 × 8.5 = 184.98 kN

Self-weight of hopper bottom:

Assuming the thickness to be 150 mm,

Mean diameter $= \frac{5.5}{2} + 0.150 \times \sqrt{2} = 2.96$ m.

Thickness $= 0.150$ m

Sloping length $= \left(\frac{5.5 - 0.5}{2}\right)\sqrt{2} = 2.5\sqrt{2}$

∴ Volume of concrete $= \pi \times 2.96 \times 0.150 \times 2.5\sqrt{2}$

$= 4.93\ m^3$.

∴ Self-weight of hopper bottom = 4.93 × 25 = 123.29 kN.

∴ Total weight on hopper bottom

$$W = 1446.21 + 184.98 + 123.29 = 1754.48 \text{ kN.}$$

∴ Direct tension $= W \operatorname{cosec} \theta = 1754.48 \times \operatorname{cosec} 45$

$= 2481.21$ kN.

∴ Direct tension per metre width

$$= \frac{2481.21}{\pi \times 5.5} = 143.6 \text{ kN/m.}$$

∴ A_{st} required $= \frac{1.5 \times 143.6 \times 1000}{0.87 \times 415} = 596.6\ mm^2$

Using 10 mm bars.

Spacing $= \frac{\frac{\pi}{4} \times 10^2}{596.6} \times 1000 = 131.6$ mm.

∴ Provide 10 mm bars @ 130 mm c/c in the sloping direction. Curtail 50% of bars at half th ıping length of hopper bottom.

ıeck for Stress in Concrete

$$A_c = 150 \times 1000 + 596.6 \times 13 = 157755$$

Direct tensile stress $= \frac{134.0 \times 1000}{157755} = 0.849\ N/mm^2 < 2.8\ N/mm^2$

Hence safe.

Design for Hoop Tension

At mid-height of conical bottom, $h = 18 + \frac{2.5}{2} = 19.25$ m.

Diameter $D = \frac{5.5 + 0.5}{2} = 3.0$ m,

R = hydraulic mean radius

$= \frac{D}{4} = 0.75$ m.

$\mu' = 0.356, \quad K = 0.5,$

$$\frac{\mu' K}{R} h = \frac{0.356 \times 0.5}{0.75} \times 19.25 = 4.569$$

$$\therefore \quad p_h = \frac{8.5 \times 0.75}{0.356}\left[1 - e^{-4.569}\right] = 17.721 \text{ kN/m}^2$$

$$p_v = \frac{p_h}{K} = \frac{17.721}{0.5} = 35.442 \text{ kN/m}^2$$

$$W_s = 0.15 \times 25 = 3.75 \text{ kN/m}^2$$

$\therefore$ Normal pressure $p_n = p_v \cos^2 45 + p_h \sin^2 45 + W_s \cos 45$

$= 35.442 \cos^2 45 + 17.721 \sin^2 45 + 3.75 \cos 45$

$= 31.885$ kN/m^2

Mean diameter at centre of sloping slab

$$= \frac{5.5 + 0.5}{2} + 0.15\sqrt{2} = 3.21 \text{ m}$$

Hoop tension, $T = 0.5 \times 3.21 \times 31.885$

$= 51.175$ kN per metre length

$$\therefore \quad A_{st} = \frac{1.5T}{0.87 f_y} = \frac{1.5 \times 51.175 \times 1000}{0.87 \times 415} = 212.6 \text{ mm}^2.$$

At junction with ring beam *i.e.*, at $h = 18$ m.

$p_v = 60.872$ kN/m^2, $\quad p_H = 31.885$ kN/m^2

$\therefore \quad p_n = 60.872 \cos^2 45 + 31.885 \sin^2 45 + 3.75 \cos 45$

$= 49.03$ kN/m^2.

Diameter $= 5.5 + 0.15\sqrt{2} = 5.71$ m.

$\therefore \qquad T = 49.03 \times \frac{5.71}{2} = 140 \text{ kN}$

$\therefore \qquad A_{st} = \frac{1.5 \times 140 \times 1000}{0.87 \times 415} = 581.6 \text{ mm}^2.$

Using 10 mm bars,

Spacing $= \frac{\frac{\pi}{4} \times 10^2}{581.6} \times 1000 = 135$

∴ Provide 10 mm bars at 130 mm c/c.

Increase it gradually to 300 mm by mid depth and then maintain the spacing of 300 mm.

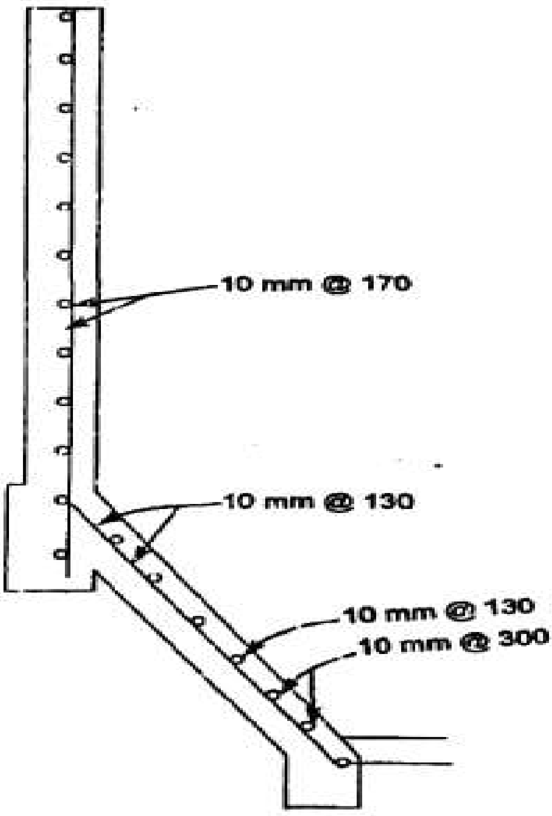


Fig 5.20 Reinforcement details