Hall Ticket No | | | | | | | | | | | Question Paper Code: AEC003

# INSTITUTE OF AERONAUTICAL ENGINEERING

**(Autonomous)**

Four Year B.Tech III Semester End Examinations (Supplementary) - July, 2018

**Regulation: IARE – R16**

## PROBABILITY THEORY AND STOCHASTIC PROCESSES

**Time: 3 Hours** **(ECE)** **Max Marks: 70**

**Answer ONE Question from each Unit**

**All Questions Carry Equal Marks**

**All parts of the question must be answered in one place only**

### UNIT – I

1. (a) State and prove Baye's theorem. **[7M]**

   (b) An ordinary 52 card deck is thoroughly shuffled. When four cards are drawn then What is the probability that all four cards are seven? **[7M]**

2. (a) If A and B are independent events, prove that **[7M]**

      i. A and $\bar{B}$

      ii. $\bar{A}$ and $\bar{B}$ are independent

   (b) In a bolt factory there are four machines A,B,C,D manufacturing 20%, 15%,25%,40% of the total production of these 5%, 4%,3%,2% are found to be defective. If a bolt is drawn at random and was found to be defective what is the probability that it was manufactured by A or D. **[7M]**

### UNIT – II

3. (a) State the properties of distribution function and density function in the case of continuous random variable. **[7M]**

   (b) A random variable X can have values -5,-4,-1,2,3 and 4 each with a probability 1/6. Find mean and variance of the random variable Y=3$X^2$. **[7M]**

4. (a) Find Moment Generating Function(MGF) of the random variable with probability law $(X = a) = q^{x-1}\beta$ , X=1,2,……. Also find mean and variance. **[7M]**

   (b) Find the characteristic function of the Poisson distribution and hence find the values of first four central moments. **[7M]**

### UNIT – III

5. (a) A joint probability density function is $f_{XY}(X,Y) = \begin{cases} \frac{1}{ab}, 0 < x < a, 0 < y < b \\ 0, elsewhere \end{cases}$

   Find $f_{XY}$ (X,Y) also if a<b find $P[X + Y \leq 3/4]$ **[7M]**

   (b) Two random variables X and Y have the joint characteristic function $\varphi_{X,Y}(w_1, w_2) = EXP\left(-2w_1^2 - 8w_2^2\right)$ Show that X and Y are both zero mean random variables and They are uncorrelated. **[7M]**

6. (a) State the properties of Joint density function. **[7M]**

(b) Two Gaussian random variables $X_1$ and $X_2$ are defined by the mean and covariance matrices $\left[\bar{X}\right] = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $[C_X] = \begin{bmatrix} 5 & \frac{-2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & 4 \end{bmatrix}$. Two new random variables $Y_1$ and $Y_2$ are formed using the transformation $[T] = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ Find $\left[\bar{Y}\right]$, $[C_Y]$ and $Y_2$. **[7M]**

## UNIT – IV

7. (a) Let N(t) be a Zero-mean wide-sense stationary noise process for which $R_{NN}\left(t\right) = \left(N_0/2\right)\delta\left(\tau\right))$ where $N_0$>0 is a finite constant. Determine N(t) if it is mean-ergodic. **[7M]**

(b) Given a random process $X\left(t\right) = ACos\left(w_0 t\right) + BSin\left(w_0 t\right)$ where $w_0$ is a constant and A and B are uncorrelated non-zero random variables having different density functions but the same variances. Show that X(t) is WSS. **[7M]**

8. (a) Define random process and classify the random processes. **[7M]**

(b) Let $X\left(t\right) = ACos\left(w_0 t\right) + BSin\left(w_0 t\right)$ and $Y\left(t\right) = ACos\left(w_0 t\right) - BSin\left(w_0 t\right)$ where A and B are random variables, $w_0$ is a constant. Given X(t) and Y(t) are WSS. A and B are uncorrelated, zero-mean random variables with same variance. Find the cross correlation function and also show that X(t) and Y(t) are jointly Wide sense stationary(WSS). **[7M]**

## UNIT – V

9. (a) Define two random processes $X\left(t\right) = ACos\left(w_0 t + \theta\right)$, $Y\left(t\right) = w\left(t\right)\cos\left(w_0 t + \theta\right)$ where A and $w_0$ are constants, $\theta$ is a random variable independent of w(t) and w(t) is a random process with constant mean value $\bar{w}$. Find the cross correlation function and time average of $R_{XY}\left(t, t+\tau\right)$. **[7M]**

(b) A low pass random process X(t) has a continuous power spectrum $S_{XX}\left(w\right)$ and $S_{XX}\left(0\right) \neq 0$. Find the bandwidth w of a low pass band limited whose noise power spectrum has a density $S_{XX}(0)$ and the same total power as in X(t). **[7M]**

10. (a) A random process has the power spectrum density $S_{xx}\left(w\right) = {}^{6w^2}\!/_{1+w^4}$. Find the average power of the process. **[7M]**

(b) Find the cross correlation function corresponding to the Cross Power Spectrum $S_{xx}\left(w\right) = \frac{6}{(9+w^2)(3+jw)^2}$. **[7M]**

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