Hall Ticket No | | | | | | | | | | | Question Paper Code: BCC002

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

M.Tech I Semester End Examinations (Regular) - February, 2017

Regulation: IARE–R16

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS (CAD/CAM)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Solve the heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the initial and boundary conditions $u\,(x, 0) = sin\pi x, 0 \leq x \leq 1 \; and \; u\,(0, t) = u\,(1, t) = 0$ using Crank-Nicolson method for $h = \frac{1}{3}$ and $k = \frac{1}{36}$ Integrate upto two time levels. **[7M]**

   (b) Explain Stability and Convergence analysis of difference schemes **[7M]**

2. (a) Discuss the classification of second order partial differential equations . **[7M]**

   (b) Show that the Crank-Nicolson method is unconditionally stable. **[7M]**

UNIT – II

3. (a) Explain five point formula for finite difference **[7M]**

   (b) Discuss the stability of heat equation using Von Neumann method. **[7M]**

4. (a) Explain conditions for one dimensional diffusion equation in cylindrical and spherical coordinates. **[7M]**

   (b) Explain Alternating Direction Implicit (ADI) methods. **[7M]**

UNIT – III

5. (a) Find the solution of the initial-boundary value $u_{tt} = u_{xx}$ $0 \leq x \leq 1$ subject to the initial and boundary conditions $u\,(x, 0) = sin\pi x, 0 \leq x \leq 1 \; and \; u\,(0, t) = u\,(1, t) = 0, t > 0$ using explicit scheme for $h = \frac{1}{4}$ and $r = \frac{3}{4}$. Integrate upto two time levels. **[7M]**

   (b) Prove that the Wenderoff's scheme is unconditionally stable. **[7M]**

6. (a)) Find the solution of the differential equation $\frac{\partial u}{\partial t} + \frac{\partial}{\partial t}\left(\frac{u^2}{2}\right) = 0$ subject to conditions $u\,(0, t) = 0, u\,(x, 0) = x, 0 \leq x \leq 1$ Using the Lax-Wenderoff's formula h=0.2 ,r=0.5 and integrate for one time step. **[7M]**

   (b) Derive Wenderoff's formula. **[7M]**

UNIT – IV

7. (a) Solve the mixed boundary value problem $\nabla^2 u = 0,\ 0 \le x, y \le 1$, u=2x ,u=2x-1,y=0,y=1 ,$0 \le x \le 1$,, $u_x + u = 2 - y, u = 2 - y, x = 0, x = 1, 0 \le y \le 1$, using five point formula with h=k=1/3. **[7M]**

   (b) Solve the boundary value problem $u_{xx}+u_{yy}-5u\left(u_x - u_y\right) = -5e^{2x}cosy\left(cosy + siny\right)$,$0 \le x, y \le 1$ using second order method with $h = \frac{1}{2}$ **[7M]**

8. (a) Solve the boundary value problem **[7M]**
   $u_{rr} + \frac{1}{r}u_r + u_{zz} = -1$,$0 \le r \le 1, -1 \le z \le 1$
   u=0, on the boundary.
   Using five point formula with h=k=1/2.

   (b) Explain Weighted Residual Methods **[7M]**

UNIT – V

9. (a) Discuss Variation methods, least square method and Galerkin method . **[7M]**

   (b) Obtain a one parameter approximate solution of the boundary value problem **[7M]**
   $\nabla^2 u = 0$,$|x| \le 1, |y| \le 1$
   u=0 ,on the boundary using Galerkin method.

10. (a) Discuss Finite element method. **[7M]**

    (b) Find a one parameter Galerkin solution of the boundary value problem **[7M]**
    $\nabla^2 u = 0$,$|x| \le 1, |y| \le \frac{1}{2}$
    u=0 , On the boundary.