Hall Ticket No | | | | | | | | | | | Question Paper Code: BCC002

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

M.Tech I Semester End Examinations (Supplementary) - July, 2018

Regulation: IARE–R16

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS (CAD/CAM)

Time: 3 Hours **Max Marks: 70**

Answer ONE Question from each Unit
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

UNIT – I

1. (a) Explain Crank - Nicholson explicit method for solving partial differential equations. **[7M]**

   (b) Solve by Crank-Nicolson method, $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}$, subject to the conditions u(x, 0) = sin π x; $0 \leq x \leq 1$, u(0, t) = u(1, t) = 0, taking h = 1/3; k = 1/36. **[7M]**

2. (a) Give examples of parabolic, elliptic, hyperbolic, semilinear, quasi linear partial differential equations. **[7M]**

   (b) Derive explicit scheme to solve parabolic equation and using it to solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ $0 \leq x \leq 1$, $t > 0$ subject to the conditions u(x, 0) = sin πx, u(0, t) = u(1, t) = 0. **[7M]**

UNIT – II

3. (a) Explain conditions for one dimensional heat equation in cylindrical and spherical coordinates. **[7M]**

   (b) Discuss about convergence, stability, consistency of implicit methods. **[7M]**

4. (a) Explain five point formula for finite difference by alternative direction implicit formula. **[7M]**

   (b) Explain the concepts of local truncation error and global rounding error. **[7M]**

UNIT – III

5. (a) Explain the method of characteristics for the hyperbolic partial differential equation. **[7M]**

   (b) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$; $t > 0$, using explicit method given that u(x, 0) = 0; u_t (x, 0) = 0, u(0, t) = 0 and u(1, t) = 100 sin(πt). Compute u for four time steps with h = 0.25. **[7M]**

6. (a) Summarize the stability of the finite difference procedure for solving a hyperbolic equation. **[7M]**

   (b) Explain an explicit method for solving hyperbolic differential equation. **[7M]**

UNIT – IV

7. (a) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for the following square mesh boundary values as shown in the following figure 1: **[7M]**

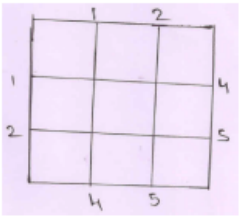

Figure 1

   (b) Solve $u_{xx} + u_{yy} = -x^2y^2$ over the square region bounded by lines x = 0; y = 0; x = 3; y = 3 given that u = 10 throughout the boundaries taking h = 1. **[7M]**

8. (a) Explain analysis of the discretization error of the five point approximation to Poissons equation. **[7M]**

   (b) Solve the Poisson equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides x = 0; y = 0; x = 3; y = 3 with u = 0 on the boundary and mesh length=1. **[7M]**

UNIT – V

9. (a) Explain about the different types of variational methods. **[7M]**

   (b) Solve the boundary value problem $y'' + y - x^2 = 0$; $0 < x < 1$, $y(0) = 0$; $y(1) = 1$ by Galerkin method. **[7M]**

10. (a) Explain Stones implicit method with an example. **[7M]**

    (b) Solve the boundary value problem $y'' + 1 = 0$; $0 < x < 1$, $y(0) = 0$; $y(1) = 1$ by weighted residual method. **[7M]**

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