INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal - 500 043, Hyderabad, Telangana

COURSE CONTENT

PROGRAMMING FOR PROBLEM SOLVING LABORATORY								
II Semester: AE / ME / CE / ECE / EEE / CSE / CSE (AI & ML) / CSE (DS) / CSE (CS) / IT								
Course Code	**Category**	**Hours / Week**			**Credits**	**Maximum Marks**		
		L	**T**	**P**	**C**	**CIA**	**SEE**	**Total**
ACSD06	**Foundation**	0	1	2	2	40	60	100
Contact Classes: Nil	**Tutorial Classes: 15**	**Practical Classes: 30**				**Total Classes: 45**		
Prerequisites: **There are no prerequisites to take this course.**								

I. COURSE OVERVIEW:

The course is designed with the fundamental programming skills and problem-solving strategies necessary to tackle a wide range of computational challenges. Through hands-on programming exercises and projects, students will learn how to write code, analyze problems and develop solutions using various programming languages and tools. The course will cover fundamental programming concepts and gradually progress to more advanced topics.

II. COURSE OBJECTIVES

The students will try to learn:

I. The fundamental programming constructs and use of collection data types in Python.
II. The ability to develop programs using object-oriented features.
III. Basic data structures and algorithms for efficient problem-solving.
IV. Principles of graph theory and be able to apply their knowledge to a wide range of practical problems across various disciplines.

III. COURSE OUTCOMES:

At the end of the course students should be able to:

CO1 Adapt programming concepts, syntax, and data structures through hands on coding exercises.
CO2 Develop the ability to solve a variety of programming problems and algorithms using python.
CO3 Implement complex and custom data structures to solve real-world problems.
CO4 Demonstrate proficiency in implementing graph algorithms to solve variety of problems and scenarios.
CO5 Develop critical thinking skills to solve the various real-world applications using graph theory.
CO6 Learn the importance of numerical methods and apply them to tackle a wide range of computational problems.

IV. COURSE CONTENT:

EXERCISES FOR PROGRAMMING FOR PROBLEM SOLVING LABORATORY

Note: Students are encouraged to bring their own laptops for laboratory practice sessions.

1. Getting Started Exercises

1.1 Two Sum

Given an array of integers nums and an integer target, return indices of the two numbers such that they add up to target. You may assume that each input would have exactly one solution, and you may not use the same element twice. You can return the answer in any order.

Input: nums = [2, 7, 11, 15], target = 9

Output: [0, 1]

Explanation: Because nums[0] + nums[1] == 9, so return [0, 1].

Input: nums = [3, 2, 4], target = 6

Output: [1, 2]

Input: nums = [3, 3], target = 6

Output: [0, 1]

Hints:

```
def twoSum(self, nums: List[int], target: int) -> List[int]:
     a=[]
     # Write code here
     …

     return a
```

1.2 Contains Duplicate

Given an integer array nums, return true if any value appears at least twice in the array, and return false if every element is distinct.

Input: nums = [1, 2, 3, 1]

Output: true

Input: nums = [1, 2, 3, 4]

Output: false

Input: nums = [1, 1, 1, 3, 3, 4, 3, 2, 4, 2]

Output: true

Hints:

```
def containsDuplicate(self, nums):
        a = set() # set can have only distinct elements
        # Write code here
        …
        return False
```

1.3 Roman to Integer

Roman numerals are represented by seven different symbols: I, V, X, L, C, D and M.

Symbol	Value
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

For example, 2 is written as II in Roman numeral, just two ones added together. 12 is written as XII, which is simply X + II. The number 27 is written as XXVII, which is XX + V + II.

Roman numerals are usually written largest to smallest from left to right. However, the numeral for four is not IIII. Instead, the number four is written as IV. Because the one is before the five we subtract it making four. The same principle applies to the number nine, which is written as IX. There are six instances where subtraction is used:
I can be placed before V (5) and X (10) to make 4 and 9.
X can be placed before L (50) and C (100) to make 40 and 90.
C can be placed before D (500) and M (1000) to make 400 and 900.

Given a roman numeral, convert it to an integer.

Input: s = "III"

Output: 3

Input: s = "LVIII"

Output: 58

Hints:

```
def romanToInt(self, s: str) -> int:
         # Write code here
        …

         return number
```

1.4 Plus One

You are given a large integer represented as an integer array digits, where each digits[i] is the i^{th} digit of the integer. The digits are ordered from most significant to least significant in left-to-right order. The large integer does not contain any leading 0's. Increment the large integer by one and return the resulting array of digits.

Input: digits = [1, 2, 3]

Output: [1, 2, 4]

Explanation: The array represents the integer 123.

Incrementing by one gives 123 + 1 = 124.

Thus, the result should be [1, 2, 4].

Hints:

```
def plusOne(self, digits: List[int]) -> List[int]:
        n = len(digits)
        # Write code here
         …

        return digits
```

1.5 Majority Element

Given an array nums of size n, return the majority element. The majority element is the element that appears more than ⌊n / 2⌋ times. You may assume that the majority element always exists in the array.

Input: nums = [3, 2, 3]

Output: 3

Input: nums = [2, 2, 1, 1, 1, 2, 2]

Output: 2

Hints:

```
def majorityElement(self, nums):
        # write code here
         …
```

1.6 Richest Customer Wealth

You are given an m x n integer grid accounts where accounts[i][j] is the amount of money the i^{th} customer has in the j^{th} bank. Return the wealth that the richest customer has. A customer's wealth is the amount of money they have in all their bank accounts. The richest customer is the customer that has the maximum wealth.

Input: accounts = [[1, 2, 3], [3,2,1]]

Output: 6

Explanation:

1st customer has wealth = 1 + 2 + 3 = 6

2nd customer has wealth = 3 + 2 + 1 = 6

Both customers are considered the richest with a wealth of 6 each, so return 6.

Input: accounts = [[1, 5], [7,3],[3,5]]

Output: 10

Explanation:

1st customer has wealth = 6

2nd customer has wealth = 10

3rd customer has wealth = 8

The 2nd customer is the richest with a wealth of 10.

Input: accounts = [[2,8,7],[7,1,3],[1,9,5]]

Output: 17

Hints:

```
def maximumWealth(self, accounts: List[List[int]]) -> int:
        # write code here
         …
```

1.7 Fizz Buzz

Given an integer n, return a string array answer (1-indexed) where:

answer[i] == "FizzBuzz" if i is divisible by 3 and 5.

answer[i] == "Fizz" if i is divisible by 3.

answer[i] == "Buzz" if i is divisible by 5.

answer[i] == i (as a string) if none of the above conditions are true.

Input: n = 3

Output: ["1","2","Fizz"]

Input: n = 5

Output: ["1","2","Fizz","4","Buzz"]

Input: n = 15

Output: ["1","2","Fizz","4","Buzz","Fizz","7","8","Fizz","Buzz","11","Fizz","13","14","FizzBuzz"]

Hints:

```
def fizzBuzz(self, n: int) -> List[str]:
        # write code here
         …
```

1.8 Number of Steps to Reduce a Number to Zero

Given an integer num, return the number of steps to reduce it to zero. In one step, if the current number is even, you have to divide it by 2, otherwise, you have to subtract 1 from it.

Input: num = 14

Output: 6

Explanation:

- 14 is even; divide by 2 and obtain 7.
- 7 is odd; subtract 1 and obtain 6.
- 6 is even; divide by 2 and obtain 3.
- 3 is odd; subtract 1 and obtain 2.
- 2 is even; divide by 2 and obtain 1.
- 1 is odd; subtract 1 and obtain 0.

Input: num = 8

Output: 4

Explanation:

- 8 is even; divide by 2 and obtain 4.
- 4 is even; divide by 2 and obtain 2.
- 2 is even; divide by 2 and obtain 1.
- 1 is odd; subtract 1 and obtain 0.

Input: num = 123

Output: 12

Hints:

```
def numberOfSteps(self, n: int) -> int:
        # write code here
         …
```

1.9 Running Sum of 1D Array

Given an array nums. We define a running sum of an array as runningSum[i] = sum (nums[0]…nums[i]).

Return the running sum of nums.

Input: nums = [1, 2, 3, 4]

Output: [1, 3, 6, 10]

Explanation: Running sum is obtained as follows: [1, 1+2, 1+2+3, 1+2+3+4].

Input: nums = [1, 1, 1, 1, 1]

Output: [1, 2, 3, 4, 5]

Explanation: Running sum is obtained as follows: [1, 1+1, 1+1+1, 1+1+1+1, 1+1+1+1+1].

Input: nums = [3, 1, 2, 10, 1]

Output: [3, 4, 6, 16, 17]

Hints:

```
def runningSum(self, nums: List[int]) -> List[int]:
      # write code here
        …
      return answer
```

1.10 Remove Element

Given an integer array nums and an integer val, remove all occurrences of val in nums in-place. The order of the elements may be changed. Then return the number of elements in nums which are not equal to val. Consider the number of elements in nums which are not equal to val be k, to get accepted, you need to do the following things:

- Change the array nums such that the first k elements of nums contain the elements which are not equal to val. The remaining elements of nums are not important as well as the size of nums.
- Return k.

Input: nums = [3, 2, 2, 3], val = 3

Output: 2, nums = [2, 2, _, _]

Explanation: Your function should return k = 2, with the first two elements of nums being 2.

It does not matter what you leave beyond the returned k (hence they are underscores).

Input: nums = [0,1,2,2,3,0,4,2], val = 2

Output: 5, nums = [0,1,4,0,3,_,_,_]

Explanation: Your function should return k = 5, with the first five elements of nums containing 0, 0, 1, 3, and 4.

Note that the five elements can be returned in any order.

It does not matter what you leave beyond the returned k (hence they are underscores).

Hints:

```
def removeElement(self, nums: List[int], val: int) -> int:
       # write code here
         …
       return len(nums)
```

2. Matrix Operations

2.1 Add Two Matrices

Given two matrices X and Y, the task is to compute the sum of two matrices and then print it in Python.

Input:

```
X= [[1, 2, 3],
    [4, 5, 6],
    [7, 8, 9]]

Y = [[9, 8, 7],
    [6, 5, 4],
    [3, 2, 1]]
```

Output:

```
Result = [[10, 10, 10],
          [10, 10, 10],
          [10, 10, 10]]
```

Hints:

```
# Program to add two matrices using nested loop
```

```
X = [[1, 2, 3],
    [4, 5, 6],
    [7, 8, 9]]

Y = [[9,8,7],
    [6,5,4],
    [3,2,1]]


result = [[0,0,0],
          [0,0,0],
          [0,0,0]]

# iterate through rows
for i in range(len(X)):
    # write code here
    …

for r in result:
    print(r)
```

TRY

1. Take input as X = [[10, 20, 30],[41, 52, 63], [47, 58, 69]] Y = [[19,18,17],[66,35,49], [13,21,11]] and verify the results.

2.2 Multiply Two Matrices

Given two matrices X and Y, the task is to compute the multiplication of two matrices and then print it.

Input:

X= [[1, 7, 3],
[3, 5, 6],
[6, 8, 9]]

Y = [[1, 1, 1, 2],
[6, 7, 3, 0],
[4, 5, 9, 1]]

Output:

Result = [[55, 65, 49, 5],
[57, 68, 72, 12],
[90, 107, 111, 21]]

Hints:

```
# Program to multiply two matrices using list comprehension

# take a 3x3 matrix
A = [[12, 7, 3],
    [4, 5, 6],
    [7, 8, 9]]

# take a 3x4 matrix
B = [[5, 8, 1, 2],
    [6, 7, 3, 0],
```

```
    [4, 5, 9, 1]]

# result will be 3x4
# write code here
    …
for r in result:
    print(r)
```

TRY

1. Take input as X = [[11, 0, 30],[-41, -2, 63], [41, -5, -9]] Y = [[19,-48,17],[-6,35,19], [13,1,-9]] and verify the results.

2.3 Transpose of a Matrix

A matrix can be implemented using a nested list. Each element is treated as a row of the matrix. Find the transpose of a matrix in multiple ways.

Input: [[1, 2], [3, 4], [5, 6]]
Output: [[1, 3, 5], [2, 4, 6]]
Explanation: Suppose we are given a matrix

[[1, 2],
[3, 4],
[5, 6]]

Then the transpose of the given matrix will be,

[[1, 3, 5],
[2, 4, 6]]

Hints:

```
# Program to multiply two matrices using list comprehension

# take a 3x3 matrix
A = [[12, 7, 3],
    [4, 5, 6],
    [7, 8, 9]]

# result will be 3x4
# write code here
    …
for r in result:
    print(r)
```

TRY

1. Take input as X = [[11, 0, 30],[-41, -2, 63], [41, -5, -9]] and verify the results.

2.4 Matrix Product

Matrix product problem we can solve using list comprehension as a potential shorthand to the conventional loops. Iterate and find the product of the nested list and at the end return the cumulative product using function.

Input: The original list: [[1, 4, 5], [7, 3], [4], [46, 7, 3]]
Output: The total element product in lists is: 1622880

Hints:

```
# Matrix Product using list comprehension + loop

def prod(val):
    # write code here
    …

# initializing list
test_list = [[1, 4, 5], [7, 3], [4], [46, 7, 3]]
```

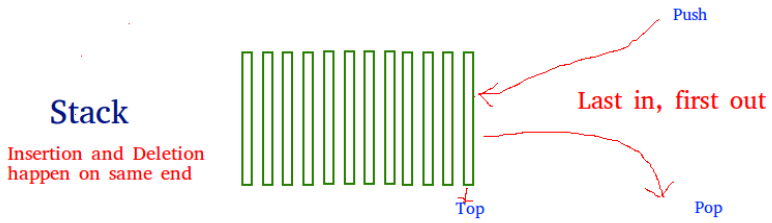
TRY

1. Take input list: [[1, 4, 5], [7, 3], [4], [46, 7, 3]] and verify the result.

3. Stack

3.1 Stack implementation using List

A stack is a linear data structure that stores items in a Last-In/First-Out (LIFO) or First-In/Last-Out (FILO) manner. In stack, a new element is added at one end and an element is removed from that end only. The insert and delete operations are often called push and pop.



The functions associated with stack are:

- **empty()** – Returns whether the stack is empty
- **size()** – Returns the size of the stack
- **top() / peek()** – Returns a reference to the topmost element of the stack
- **push(a)** – Inserts the element 'a' at the top of the stack
- **pop()** – Deletes the topmost element of the stack

Hints:

```
# Stack implementation using list
top=0
mymax=5
def createStack():
    stack=[]
    return stack
def isEmpty(stack):
    # write code here
    …
def Push(stack,item):
    # write code here
    …
```

```
def Pop(stack):
    # write code here
    …
# create a stack object
stack = createStack()
while True:
    print("1.Push")
    print("2.Pop")
    print("3.Display")
    print("4.Quit")
    # write code here
    …
```

TRY

1. Take input operations as [PUSH(A),PUSH(B),PUSH(C),POP,POP,POP,POP,PUSH(D)] and verify the result.
2. Take input operations as [POP, POP, PUSH (A), PUSH (B), POP, and PUSH(C)] and verify the result.

3.2 Balanced Parenthesis Checking

Given an expression string, write a python program to find whether a given string has balanced parentheses or not.

Input: {[]{()}}

Output: Balanced

Input: [{}{}(]

Output: Unbalanced

Using stack One approach to check balanced parentheses is to use stack. Each time, when an open parentheses is encountered push it in the stack, and when closed parenthesis is encountered, match it with the top of stack and pop it. If stack is empty at the end, return Balanced otherwise, Unbalanced.

Hints:

```
# Check for balanced parentheses in an expression
open_list = ["[","{","("]
close_list = ["]","}",")"]

# Function to check parentheses
def check(myStr):
    # write code here
    …
```

TRY

1. Take input as {[]{()}[][]} and verify the result.
2. Take input as {[]{()}[]{}} and verify the result.

3.3 Evaluation of Postfix Expression

Given a postfix expression, the task is to evaluate the postfix expression. Postfix expression: The expression of the form "a b operator" (ab+) i.e., when a pair of operands is followed by an operator.

Input: str = "2 3 1 * + 9 -"

Output: -4

Explanation: If the expression is converted into an infix expression, it will be 2 + (3 * 1) – 9 = 5 – 9 = -4.

Input: str = "100 200 + 2 / 5 * 7 +"

Output: 757

Procedure for evaluation postfix expression using stack:

- Create a stack to store operands (or values).
- Scan the given expression from left to right and do the following for every scanned element.
    - If the element is a number, push it into the stack.
    - If the element is an operator, pop operands for the operator from the stack. Evaluate the operator and push the result back to the stack.
- When the expression is ended, the number in the stack is the final answer.

Hints:

```
# Evaluate value of a postfix expression

# Class to convert the expression
class Evaluate:

    # Constructor to initialize the class variables
    def __init__(self, capacity):
        self.top = -1
        self.capacity = capacity

        # This array is used a stack
        self.array = []

    # Check if the stack is empty
    def isEmpty(self):
        # write code here
        …
    def peek(self):
        # write code here
        …
    def pop(self):
        # write code here
        …
    def push(self, op):
        # write code here
        …
    def evaluatePostfix(self, exp):
        # write code here
        …
```

```
# Driver code
exp = "231*+9-"
obj = Evaluate(len(exp))

# Function call
print("postfix evaluation: %d" % (obj.evaluatePostfix(exp)))
```

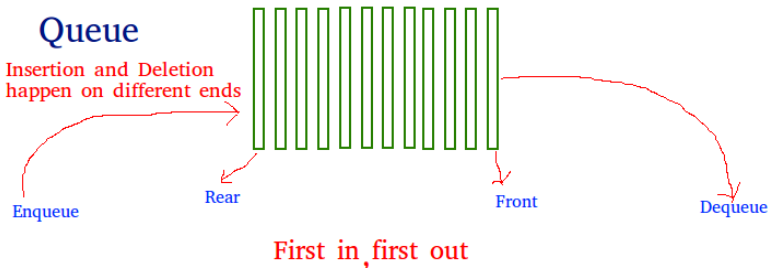
TRY

1. Take input str = "A B + C* D / E +" and verify the result.
2. Take input str = "XYZ- + W+ R / S -" and verify the result.

4. Queue

4.1 Linear Queue using List

Linear queue is a linear data structure that stores items in First in First out (FIFO) manner. With a queue the least recently added item is removed first. A good example of queue is any queue of consumers for a resource where the consumer that came first is served first.



Hints:

```
# Static implementation of linear queue
front=0
rear=0
mymax=5
def createQueue():
    queue=[]   #empty list
    return queue

def isEmpty(queue):
    # write code here
    …
def enqueue(queue,item):  # insert an element into the queue
    # write code here
    …
def dequeue(queue):  #remove an element from the queue
    # write code here
    …
```

```
# Driver code
queue = createQueue()
while True:
    print("1.Enqueue")
    print("2.Dequeue")
    print("3.Display")
    print("4.Quit")
    # write code here
    …
```

TRY

1. Take input operations as [ENQUEUE(A),DEQUEUE(),ENQUEUE(B),DEQUEUE(),ENQUEUE(C),DEQUEUE(),] and verify the result.

2. Take input operations as [ENQUEUE(A), ENQUEUE(B),DEQUEUE(),ENQUEUE(C), DEQUEUE(),ENQUEUE(D), DEQUEUE(), ENQUEUE(C),DEQUEUE(),] and verify the result.

4.2 Stack using Queues

Implement a last-in-first-out (LIFO) stack using only two queues. The implemented stack should support all the functions of a normal stack (push, top, pop, and empty).

- void push(int x) Pushes element x to the top of the stack.
- int pop() Removes the element on the top of the stack and returns it.
- int top() Returns the element on the top of the stack.
- boolean empty() Returns true if the stack is empty, false otherwise.

Input:

["MyStack", "push", "push", "top", "pop", "empty"]

[[], [1], [2], [], [], []]

Output:

[null, null, null, 2, 2, false]

Hints:

```
class MyStack:

    def __init__(self):
       # write code here
       …
    def push(self, x: int) -> None:
       # write code here
       …
    def pop(self) -> int:
       # write code here
       …
```

```
    def top(self) -> int:
        # write code here
        …
    def empty(self) -> bool:
        # write code here
        …
# Your MyStack object will be instantiated and called as such:
# obj = MyStack()
# obj.push(x)
# param_2 = obj.pop()
# param_3 = obj.top()
# param_4 = obj.empty()
```

4.3 Implement Queue using Stacks

Implement a first in first out (FIFO) queue using only two stacks. The implemented queue should support all the functions of a normal queue (push, peek, pop, and empty).

- void push(int x) Pushes element x to the back of the queue.
- int pop() Removes the element from the front of the queue and returns it.
- int peek() Returns the element at the front of the queue.
- boolean empty() Returns true if the queue is empty, false otherwise.

Input:

["MyQueue", "push", "push", "peek", "pop", "empty"]

[[], [1], [2], [], [], []]

Output:

[null, null, null, 1, 1, false]

Hints:

```
class MyQueue:

    def __init__(self):
        # write code here
        …
    def push(self, x: int) -> None:
        # write code here
        …
    def pop(self) -> int:
        # write code here
        …
    def peek(self) -> int:
        # write code here
        …
```

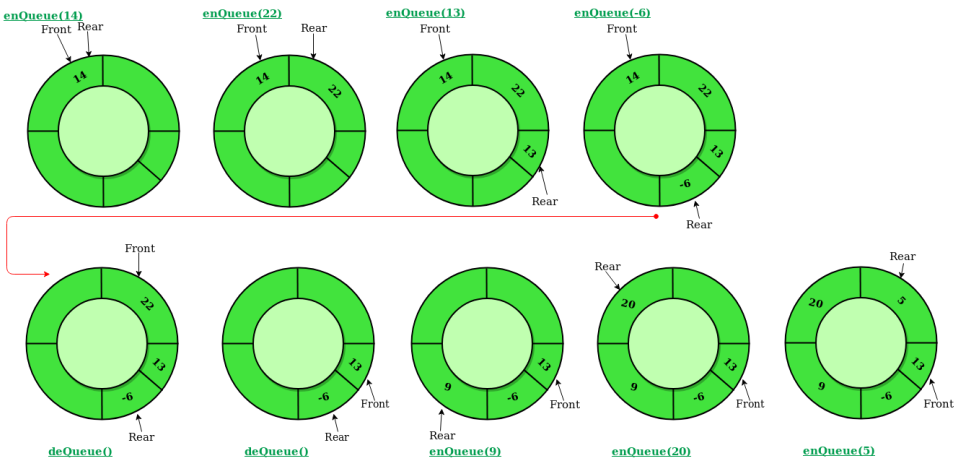
```
    def empty(self) -> bool:
        # write code here
        …
# Your MyQueue object will be instantiated and called as such:
# obj = MyQueue()
# obj.push(x)
# param_2 = obj.pop()
# param_3 = obj.peek()
# param_4 = obj.empty()
```

4.4 Circular Queue

A Circular Queue is an extended version of a normal queue where the last element of the queue is connected to the first element of the queue forming a circle. The operations are performed based on FIFO (First In First Out) principle. It is also called 'Ring Buffer'.

Operations on Circular Queue:

- **Front:** Get the front item from the queue.
- **Rear:** Get the last item from the queue.
- **enQueue(value)** This function is used to insert an element into the circular queue. In a circular queue, the new element is always inserted at the rear position.
    - Check whether the queue is full – [i.e., the rear end is in just before the front end in a circular manner].
    - If it is full then display Queue is full.
        - If the queue is not full then, insert an element at the end of the queue.
- **deQueue()** This function is used to delete an element from the circular queue. In a circular queue, the element is always deleted from the front position.
    - Check whether the queue is Empty.
    - If it is empty then display Queue is empty.
        - If the queue is not empty, then get the last element and remove it from the queue.



Implement Circular Queue using Array:

1. Initialize an array queue of size **n**, where n is the maximum number of elements that the queue can hold.
2. Initialize two variables front and rear to -1.
3. **Enqueue:** To enqueue an element **x** into the queue, do the following:
    - Increment rear by 1.
        - If **rear** is equal to n, set **rear** to 0.
    - If **front** is -1, set **front** to 0.
    - Set queue[rear] to x.
4. **Dequeue:** To dequeue an element from the queue, do the following:
    - Check if the queue is empty by checking if **front** is -1.
        - If it is, return an error message indicating that the queue is empty.
    - Set **x** to queue [front].
    - If **front** is equal to **rear**, set **front** and **rear** to -1.
    - Otherwise, increment **front** by 1 and if **front** is equal to n, set **front** to 0.
    - Return x.

Hints:

```
class CircularQueue():

    # constructor
    def __init__(self, size): # initializing the class
        self.size = size

        # initializing queue with none
        self.queue = [None for i in range(size)]
        self.front = self.rear = -1

    def enqueue(self, data):
         # Write code here
         …

    def dequeue(self):
         # Write code here
         …

    def display(self):
        # Write code here
        …


# Driver Code
ob = CircularQueue(5)
ob.enqueue(14)
ob.enqueue(22)
ob.enqueue(13)
ob.enqueue(-6)
ob.display()
print ("Deleted value = ", ob.dequeue())
print ("Deleted value = ", ob.dequeue())
ob.display()
ob.enqueue(9)
ob.enqueue(20)
ob.enqueue(5)
ob.display()
```

TRY

1. Take input operations as [ENQUEUE(A,B,C,D,E,F),DEQUEUE(),DEQUEUE(),DEQUEUE(),ENQUEUE(G,H,I] and verify the result.

2. Take input operations as [DEQUEUE(),ENQUEUE(A,B,C,D,E,F),DEQUEUE(),ENQUEUE(G,H,I] and verify the result.

5. Graph Representation

5.1 Build a graph

You are given an integer n. Determine if there is an unconnected graph with n vertices that contains at least two connected components and contains the number of edges that is equal to the number of vertices. Each vertex must follow one of these conditions:

- Its degree is less than or equal to 1.
- It's a cut-vertex.

Note:

- The graph must be simple.
- Loops and multiple edges are not allowed.

Input: First line: n

Output: Print Yes if it is an unconnected graph. Otherwise, print No.

Sample Input	Sample Output
3	No

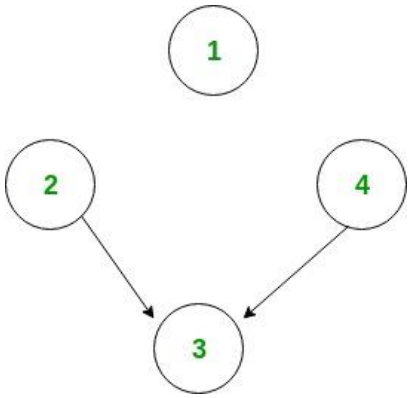
Constraints: $1 \leq n \leq 100$

Explanation: There is only one graph with the number of edges equal to the number of vertices (triangle) which is connected.

5.2 Number of Sink Nodes in a Graph

Given a Directed Acyclic Graph of n nodes (numbered from 1 to n) and m edges. The task is to find the number of sink nodes. A sink node is a node such that no edge emerges out of it.

Input: n = 4, m = 2, edges[] = {{2, 3}, {4, 3}}



Only node 1 and node 3 are sink nodes.

Input: n = 4, m = 2, edges[] = {{3, 2}, {3, 4}}

Output: 3

The idea is to iterate through all the edges. And for each edge, mark the source node from which the edge emerged out. Now, for each node check if it is marked or not. And count the unmarked nodes.

Algorithm:

1. Make any array A[] of size equal to the number of nodes and initialize to 1.

2. Traverse all the edges one by one, say, u -> v.

   (i) Mark A[u] as 1.

3. Now traverse whole array A[] and count number of unmarked nodes.

Hints:

```
# Program to count number if sink nodes

# Return the number of Sink Nodes.
def countSink(n, m, edgeFrom, edgeTo):
     # Write code here
       …

    return count

# Driver Code
n = 4
m = 2
edgeFrom = [2, 4]
edgeTo = [3, 3]

print(countSink(n, m, edgeFrom, edgeTo))
```

5.3 Connected Components in a Graph

Given n, i.e. total number of nodes in an undirected graph numbered from 1 to n and an integer e, i.e. total number of edges in the graph. Calculate the total number of connected components in the graph. A connected component is a set of vertices in a graph that are linked to each other by paths.

Input: First line of input line contains two integers' n and e. Next e line will contain two integers u and v meaning that node u and node v are connected to each other in undirected fashion.

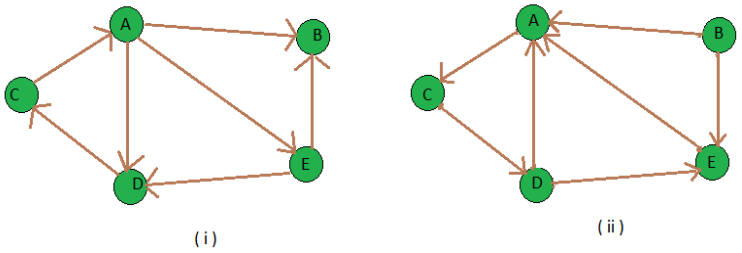
Output: For each input graph print an integer x denoting total number of connected components.

Sample Input	Sample Output
8 5	3
1 2	
2 3	
2 4	
3 5	
6 7	

Constraints: All the input values are well within the integer range.

5.4 Transpose Graph

Transpose of a directed graph G is another directed graph on the same set of vertices with all of the edges reversed compared to the orientation of the corresponding edges in G. That is, if G contains an edge (u, v) then the converse/transpose/reverse of G contains an edge (v, u) and vice versa. Given a graph (represented as adjacency list), we need to find another graph which is the transpose of the given graph.



Input: figure (i) is the input graph.

Output: figure (ii) is the transpose graph of the given graph.
0--> 2
1--> 0 4
2--> 3
3--> 0 4
4--> 0

Explanation: We traverse the adjacency list and as we find a vertex v in the adjacency list of vertex u which indicates an edge from u to v in main graph, we just add an edge from v to u in the transpose graph i.e. add u in the adjacency list of vertex v of the new graph. Thus traversing lists of all vertices of main graph we can get the transpose graph.

Hints:

```
# find transpose of a graph.
# function to add an edge from vertex source to vertex dest
def addEdge(adj, src, dest):
    adj[src].append(dest)

# function to print adjacency list of a graph
def displayGraph(adj, v):
     …
        print()

# function to get Transpose of a graph taking adjacency list of given graph and that
of Transpose graph
def transposeGraph(adj, transpose, v):
    # traverse the adjacency list of given graph and for each edge (u, v) add
    # an edge (v, u) in the transpose graph's adjacency list
    …

# Driver Code
v = 5
adj = [[] for i in range(v)]
addEdge(adj, 0, 1)
addEdge(adj, 0, 4)
addEdge(adj, 0, 3)
```

```
addEdge(adj, 2, 0)
addEdge(adj, 3, 2)
addEdge(adj, 4, 1)
addEdge(adj, 4, 3)

# Finding transpose of graph represented by adjacency list adj[]
transpose = [[]for i in range(v)]
transposeGraph(adj, transpose, v)

# Displaying adjacency list of transpose graph i.e. b
displayGraph(transpose, v)
```

TRY

1. Take input operations as addEdge(A, B), addEdge(A, D), addEdge(A, C), addEdge(C, A),addEdge(A, D), addEdge(C, B), addEdge(B, C) and verify the result.

5.5 Counting Triplets

You are given an undirected, complete graph G that contains N vertices. Each edge is colored in either white or black. You are required to determine the number of triplets (i, j, k) ($1 \le i < j < k \le N$) of vertices such that the edges (i, j), (j, k), (i, k) are of the same color.

There are M white edges and (N (N-1)/2) – M black edges.

Input:

First line: Two integers – N and M ($3 \le N \le 10^5$, $1 \le M \le 3 * 10^5$)

$(i+1)^{th}$ line: Two integers – u_i and v_i ($1 \le u_i, v_i \le N$) denoting that the edge (u_i, v_i) is white in color.

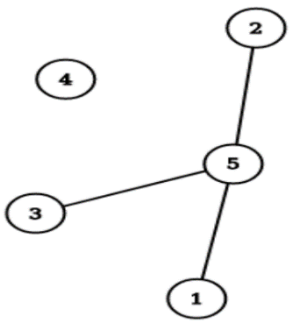
Note: The conditions $(u_i, v_i) \ne (u_j, v_j)$ and $(u_i, v_i) \ne (v_j, u_j)$ are satisfied for all $1 \le i < j \le M$.

.**Output:** Print an integer that denotes the number of triples that satisfy the mentioned condition.

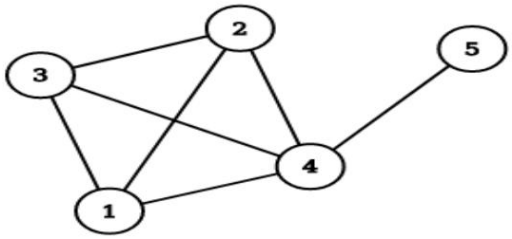
Sample Input	Sample Output
5 3	4
1 5	
2 5	
3 5	

Explanation: The triplets are: {(1, 2, 3), (1, 2, 4), (2, 3, 4), (1, 3, 4)}

The graph consisting of only white edges:



The graph consisting of only black edges:

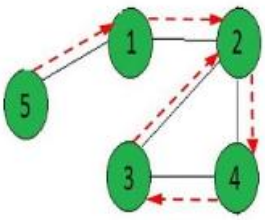


6. Graph Routing Algorithms

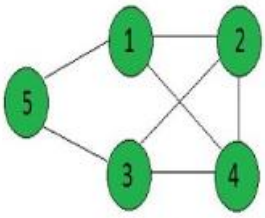
6.1 Seven Bridges of Konigsberg

There was 7 bridges connecting 4 lands around the city of Königsberg in Prussia. Was there any way to start from any of the land and go through each of the bridges once and only once? Euler first introduced graph theory to solve this problem. He considered each of the lands as a node of a graph and each bridge in between as an edge in between. Now he calculated if there is any Eulerian Path in that graph. If there is an Eulerian path then there is a solution otherwise not.

There are n nodes and m bridges in between these nodes. Print the possible path through each node using each edges (if possible), traveling through each edges only once.



Map 1



Map 2

No such path

Input: [[0, 1, 0, 0, 1],
[1, 0, 1, 1, 0],
[0, 1, 0, 1, 0],
[0, 1, 1, 0, 0],
[1, 0, 0, 0, 0]]

Output: 5 -> 1 -> 2 -> 4 -> 3 -> 2

Input: [[0, 1, 0, 1, 1],
[1, 0, 1, 0, 1],
[0, 1, 0, 1, 1],

[1, 1, 1, 0, 0],
[1, 0, 1, 0, 0]]

Output: "No Solution"

Hints:

```
# A Python program to print Eulerian trail in a
# given Eulerian or Semi-Eulerian Graph
from collections import defaultdict

class Graph:
# Constructor and destructor
def __init__(self, V):
self.V = V
 self.adj = defaultdict(list)

# functions to add and remove edge
def addEdge(self, u, v):
def rmvEdge(self, u, v):
   …
# Methods to print Eulerian tour
def printEulerTour(self):
 # Find a vertex with odd degree
  …
 # Print tour starting from oddv
self.printEulerUtil(u)
 print()
def printEulerUtil(self, u):
 # Recur for all the vertices adjacent to this vertex
for v in self.adj[u]:
 # If edge u-v is not removed and it's a valid next edge
# The function to check if edge u-v can be considered as next edge in Euler Tout
 def isValidNextEdge(self, u, v):
    # The edge u-v is valid in one of the following two cases:

    # 1) If v is the only adjacent vertex of u
        …

    # 2) If there are multiple adjacents, then u-v is not a bridge
    # Do following steps to check if u-v is a bridge
    # 2.a) count of vertices reachable from u
       …

    # 2.b) Remove edge (u, v) and after removing
    # the edge, count vertices reachable from u
```

```
        …

    # 2.c) Add the edge back to the graph
        self.addEdge(u, v)

    # 2.d) If count1 is greater, then edge (u, v) is a bridge
        return False if count1 > count2 else True

    # A DFS based function to count reachable vertices from v

    def DFSCount(self, v, visited):
        # Mark the current node as visited
        …
        # Recur for all the vertices adjacent to this vertex
        …
    # utility function to form edge between two vertices source and dest
    def makeEdge(src, dest):
        graph.addEdge(src, dest)

# Driver code
# Let us first create and test graphs shown in above figure
g1 = Graph(4)
g1.addEdge(0, 1)
g1.addEdge(0, 2)
g1.addEdge(1, 2)
g1.addEdge(2, 3)
g1.printEulerTour()

g3 = Graph(4)
g3.addEdge(0, 1)
g3.addEdge(1, 0)
g3.addEdge(0, 2)
g3.addEdge(2, 0)
g3.addEdge(2, 3)
g3.addEdge(3, 1)
g3.printEulerTour()
```

TRY

1. Take input: [[1, 0, 1, 0, 1], [1, 0, 1, 0, 0], [1, 1, 0, 1, 0], [0, 0, 1, 0, 0], [1, 0, 1, 0, 0]] and verify the result.
2. Take input: [[0, 0, 1, 0, 1], [0, 0, 1, 0, 0], [1, 0, 0, 1, 0], [1, 0, 1, 0, 0], [1, 1, 1, 0, 0]] and verify the result.

6.2 Hamiltonian Cycle

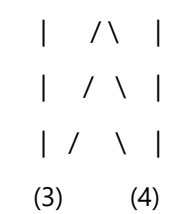
The Hamiltonian cycle of undirected graph G = <V, E> is the cycle containing each vertex in V. If graph contains a Hamiltonian cycle, it is called Hamiltonian graph otherwise it is non-Hamiltonian.
Hamiltonian Path in an undirected graph is a path that visits each vertex exactly once. A Hamiltonian cycle (or Hamiltonian circuit) is a Hamiltonian path such that there is an edge (in the graph) from the last vertex to the first vertex of the Hamiltonian path. Consider a graph G, and determine whether a given graph contains Hamiltonian cycle or not. If it contains, then prints the path. Following are the input and output of the required function.

Input: A 2D array graph [V][V] where V is the number of vertices in graph and graph [V][V] is adjacency matrix representation of the graph. A value graph[i][j] is 1 if there is a direct edge from i to j, otherwise graph[i][j] is 0.

Output: An array path [V] that should contain the Hamiltonian Path. Path [i] should represent the i^{th} vertex in the Hamiltonian Path. The code should also return false if there is no Hamiltonian Cycle in the graph.

For example, a Hamiltonian Cycle in the following graph is {0, 1, 2, 4, 3, 0}.

And the following graph doesn't contain any Hamiltonian Cycle.



Backtracking Algorithm: Create an empty path array and add vertex 0 to it. Add other vertices, starting from the vertex 1. Before adding a vertex, check for whether it is adjacent to the previously added vertex and not already added. If we find such a vertex, we add the vertex as part of the solution. If we do not find a vertex then we return false.

Hints:

```
# Python program for solution of Hamiltonian cycle problem

class Graph():
    def __init__(self, vertices):
        self.graph = [[0 for column in range(vertices)]
                            for row in range(vertices)]
        self.V = vertices

    def isSafe(self, v, pos, path):
        # Check if current vertex and last vertex in path are adjacent
      …

    # A recursive utility function to solve Hamiltonian cycle problem
    def hamCycleUtil(self, path, pos):
        …

    def hamCycle(self):
        …

    def printSolution(self, path):
```

```
        print ("Solution Exists: Following","is one Hamiltonian Cycle")
        for vertex in path:
            print (vertex, end = " ")
        print (path[0], "\n")

# Driver Code

''' Let us create the following graph
    (0)--(1)--(2)
    |    / \   |
    |   /   \  |
    |  /     \ |
    (3)-------(4) '''
g1 = Graph(5)
g1.graph = [ [0, 1, 0, 1, 0], [1, 0, 1, 1, 1],
            [0, 1, 0, 0, 1,],[1, 1, 0, 0, 1],
            [0, 1, 1, 1, 0], ]

# Print the solution
g1.hamCycle();

''' Let us create the following graph
    (0)--(1)--(2)
    |    / \   |
    |   /   \  |
    |  /     \ |
    (3)     (4) '''
g2 = Graph(5)
g2.graph = [ [0, 1, 0, 1, 0], [1, 0, 1, 1, 1],
        [0, 1, 0, 0, 1,], [1, 1, 0, 0, 0],
        [0, 1, 1, 0, 0], ]

# Print the solution
g2.hamCycle();
```

TRY

1. Take a graph = [[1, 1, 0, 1, 0], [1, 1, 1, 1, 1], [0, 1, 0, 1, 1,], [1, 1, 0, 1, 0], [0, 1, 1, 1, 0],] and verify the results.

6.3 Number of Hamiltonian Cycle

Given an undirected complete graph of N vertices where N > 2. The task is to find the number of different Hamiltonian cycle of the graph.

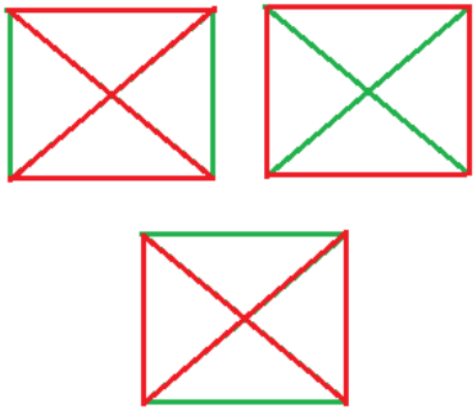
Complete Graph: A graph is said to be complete if each possible vertices is connected through an Edge.
Hamiltonian Cycle: It is a closed walk such that each vertex is visited at most once except the initial vertex. and it is not necessary to visit all the edges.

Formula: (N – 1)! / 2

Input: N = 6
Output: Hamiltonian cycles = 60

Input: N = 4
Output: Hamiltonian cycles = 3

Explanation: Let us take the example of N = 4 complete undirected graph, The 3 different Hamiltonian cycle is as shown below:



Hints:

```
# Number of Hamiltonian cycles
import math as mt

# Function that calculates number of Hamiltonian cycle
def Cycles(N):
    …

# Driver code
N = 5
Number = Cycles(N)
print("Hamiltonian cycles = ", Number)
```

TRY

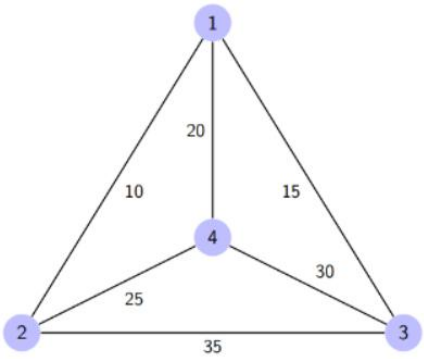
1. Take an input N=7 and verify the results.
2. Take an input N=10 and verify the results.

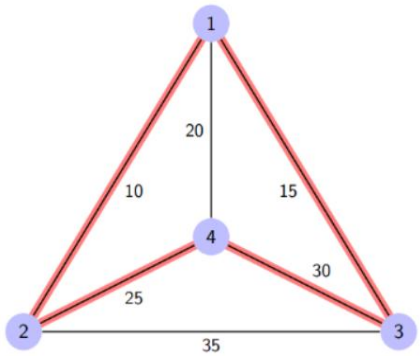
7. Shortest Path Algorithms

7.1 Travelling Salesman Problem

Given a set of cities and the distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point. The problem statement gives a list of cities along with the distances between each city.

Objective: To start from the origin city, visit other cities only once, and return to the original city again. Our target is to find the shortest possible path to complete the round-trip route.





Here a graph is given where 1, 2, 3, and 4 represent the cities, and the weight associated with every edge represents the distance between those cities. The goal is to find the shortest possible path for the tour that starts from the origin city, traverses the graph while only visiting the other cities or nodes once, and returns to the origin city.
For the above graph, the optimal route is to follow the minimum cost path: 1 – 2 – 4 – 3 - 1. And this shortest route would cost 10 + 25 + 30 + 15 =80

Algorithm for Traveling Salesman Problem: We will use the dynamic programming approach to solve the Travelling Salesman Problem (TSP).

- A graph G=(V, E), which is a set of vertices and edges.
- V is the set of vertices.
- E is the set of edges.
- Vertices are connected through edges.
- Dist(i,j) denotes the non-negative distance between two vertices, i and j.

Let's assume S is the subset of cities and belongs to {1, 2, 3, …, n} where 1, 2, 3…n are the cities and i, j are two cities in that subset. Now cost(i, S, j) is defined in such a way as the length of the shortest path visiting node in S, which is exactly once having the starting and ending point as i and j respectively.

For example, cost (1, {2, 3, 4}, 1) denotes the length of the shortest path where:

- Starting city is 1
- Cities 2, 3, and 4 are visited only once
- The ending point is 1

The dynamic programming algorithm would be:

- Set cost(i,, i) = 0, which means we start and end at i, and the cost is 0.
- When |S| > 1, we define cost(i, S, 1) = ∝ where i !=1 . Because initially, we do not know the exact cost to reach city i to city 1 through other cities.
- Now, we need to start at 1 and complete the tour. We need to select the next city in such a way-
- cost(i, S, j) = min cost (i, S−{i}, j) + dist(i, j) where i ∈ S and i ≠ j

For the given figure above, the adjacency matrix would be the following:

dist(i, j)	1	2	3	4
1	0	10	15	20
2	10	0	35	25
3	15	35	0	30
4	20	25	30	0

Now S = {1, 2, 3, 4}. There are four elements. Hence the number of subsets will be 2^4 or 16. Those subsets are-

1) |S| = Null:

{Φ}

2) |S| = 1:

{{1}, {2}, {3}, {4}}

3) |S| = 2:

{{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}}

4) |S| = 3:

{{1, 2, 3}, {1, 2, 4}, {2, 3, 4}, {1, 3, 4}}

5) |S| = 4:

{{1, 2, 3, 4}}

As we are starting at 1, we could discard the subsets containing city 1. The algorithm calculation steps:

1) |S| = Φ:

cost (2, Φ, 1) = dist(2, 1) = 10
cost (3, Φ, 1) = dist(3, 1) = 15
cost (4, Φ, 1) = dist(4, 1) = 20

2) |S| = 1:

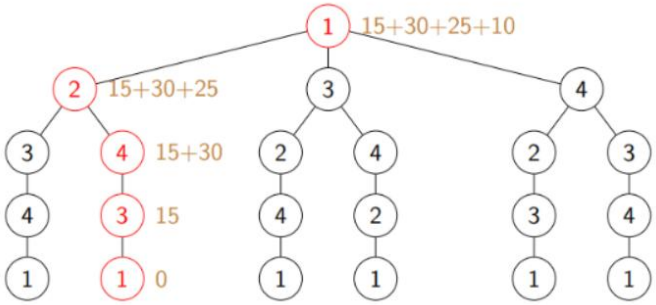
cost (2, {3}, 1) = dist(2, 3) + cost (3, Φ, 1) = 35+15 = 50
cost (2, {4}, 1) = dist(2, 4) + cost (4, Φ, 1) = 25+20 = 45
cost (3, {2}, 1) = dist(3, 2) + cost (2, Φ, 1) = 35+10 = 45
cost (3, {4}, 1) = dist(3, 4) + cost (4, Φ, 1) = 30+20 = 50
cost (4, {2}, 1) = dist(4, 2) + cost (2, Φ, 1) = 25+10 = 35
cost (4, {3}, 1) = dist(4, 3) + cost (3, Φ, 1) = 30+15 = 45

3) |S| = 2:

cost (2, {3, 4}, 1) = min [dist[2,3] + Cost(3,{4},1) = 35+50 = 85,
dist[2,4]+Cost(4,{3},1) = 25+45 = 70] = 70
cost (3, {2, 4}, 1) = min [dist[3,2] + Cost(2,{4},1) = 35+45 = 80,
dist[3,4]+Cost(4,{2},1) = 30+35 = 65] = 65
cost (4, {2, 3}, 1) = min [dist[4,2] + Cost(2,{3},1) = 25+50 = 75
dist[4,3] + Cost(3,{2},1) = 30+45 = 75] = 75

4) |S| = 3:
cost (1, {2, 3, 4}, 1) = min [dist[1,2]+Cost(2,{3,4},1) = 10+70 = 80
dist[1,3]+Cost(3,{2,4},1) = 15+65 = 80
dist[1,4]+Cost(4,{2,3},1) = 20+75 = 95] = 80

So the optimal solution would be 1-2-4-3-1



Hints:

```
from sys import maxsize
from itertools, import permutations
V = 4
def tsp(graph, s):
        …

# Driver code
graph = [[0, 10, 15, 20], [10, 0, 35, 25], [15, 35, 0, 30], [20, 25, 30, 0]]
s = 0
print(tsp(graph, s))
```

TRY

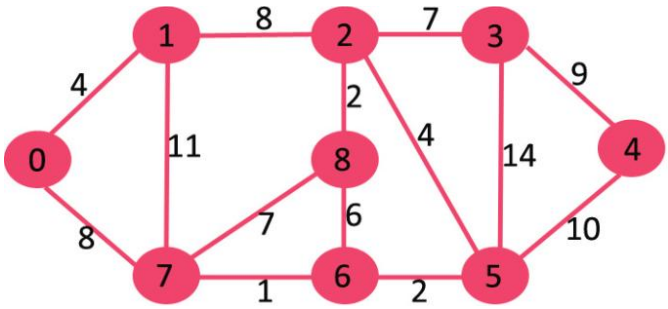
1. Take a below table values and verify the results.

dist(i, j)	1	2	3	4
1	0	40	25	40
2	20	0	35	25
3	25	35	0	60
4	40	25	30	0

7.2 Shortest Paths from Source to all Vertices (Dijkstra's Algorithm)

Given a graph and a source vertex in the graph, find the shortest paths from the source to all vertices in the given graph.

Input: src = 0, the graph is shown below.



Output: 0 4 12 19 21 11 9 8 14

Explanation: The distance from 0 to 1 = 4.
The minimum distance from 0 to 2 = 12. 0->1->2
The minimum distance from 0 to 3 = 19. 0->1->2->3
The minimum distance from 0 to 4 = 21. 0->7->6->5->4
The minimum distance from 0 to 5 = 11. 0->7->6->5
The minimum distance from 0 to 6 = 9. 0->7->6
The minimum distance from 0 to 7 = 8. 0->7
The minimum distance from 0 to 8 = 14. 0->1->2->8

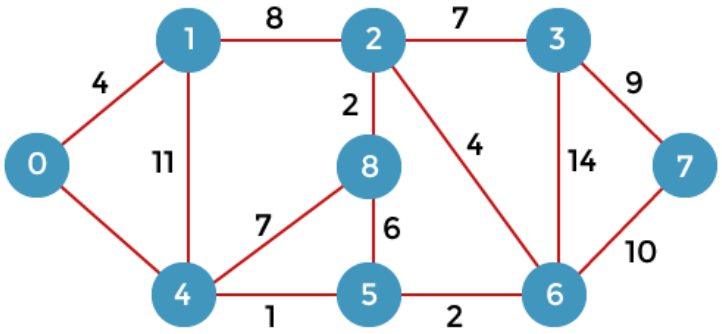
Hints:

```
# Dijkstra's single source shortest path algorithm. The program is for adjacency
matrix representation of the graph

# Library for INT_MAX
import sys

class Graph():
    def __init__(self, vertices):
        self.V = vertices
        self.graph = [[0 for column in range(vertices)]
                      for row in range(vertices)]

    def printSolution(self, dist):
        print("Vertex \tDistance from Source")
        for node in range(self.V):
            print(node, "\t", dist[node])

    # A utility function to find the vertex with minimum distance value,
    # from the set of vertices not yet included in shortest path tree
    def minDistance(self, dist, sptSet):
        …

    # Function that implements Dijkstra's single source shortest path
    # algorithm for a graph represented using adjacency matrix representation
    def dijkstra(self, src):
        …
```

```
# Driver's code
g = Graph(9)
g.graph = [[0, 4, 0, 0, 0, 0, 0, 8, 0],
           [4, 0, 8, 0, 0, 0, 0, 11, 0],
           [0, 8, 0, 7, 0, 4, 0, 0, 2],
           [0, 0, 7, 0, 9, 14, 0, 0, 0],
           [0, 0, 0, 9, 0, 10, 0, 0, 0],
           [0, 0, 4, 14, 10, 0, 2, 0, 0],
           [0, 0, 0, 0, 0, 2, 0, 1, 6],
           [8, 11, 0, 0, 0, 0, 1, 0, 7],
           [0, 0, 2, 0, 0, 0, 6, 7, 0]
          ]

g.dijkstra(0)
```

TRY

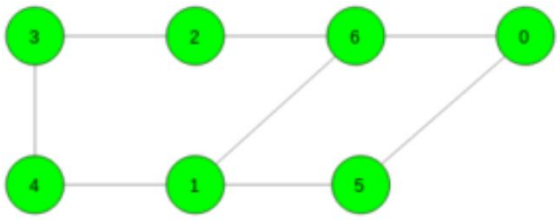
1. Take a below graph and verify the results.



7.3 Shortest Cycle in an Undirected Unweighted Graph

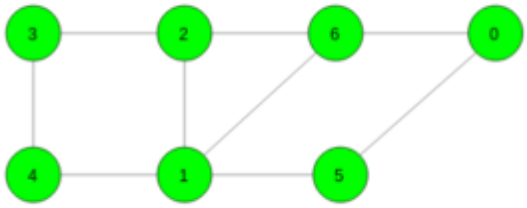
Given an undirected unweighted graph. The task is to find the length of the shortest cycle in the given graph. If no cycle exists print -1.

Input: Consider the graph given below



Output: 4
Cycle 6 -> 1 -> 5 -> 0 -> 6

Input: Consider the graph given below



Output: 3

Cycle 6 -> 1 -> 2 -> 6

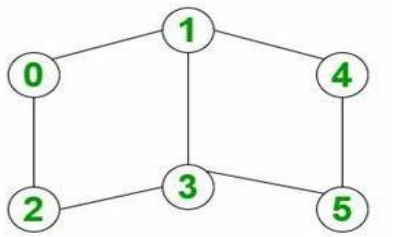
Hints:

```
from sys import maxsize as INT_MAX
from collections import deque

N = 100200

gr = [0] * N
for i in range(N):
    gr[i] = []

# Function to add edge
def add_edge(x: int, y: int) -> None:
    global gr
    gr[x].append(y)
    gr[y].append(x)

# Function to find the length of the shortest cycle in the graph
def shortest_cycle(n: int) -> int:

    # To store length of the shortest cycle
    ans = INT_MAX

    # For all vertices
    # write code here
    …

    # If graph contains no cycle
    if ans == INT_MAX:
        return -1

    # If graph contains cycle
    else:
        return ans

# Driver Code
# Number of vertices
n = 7
# Add edges
add_edge(0, 6)
add_edge(0, 5)
add_edge(5, 1)
```

```
add_edge(1, 6)
add_edge(2, 6)
add_edge(2, 3)
add_edge(3, 4)
add_edge(4, 1)

# Function call
print(shortest_cycle(n))
```

TRY

1. Take a below graph and verify the results.



7.4 Count Unique and all Possible Paths in a M x N Matrix

Count unique paths: The problem is to count all unique possible paths from the top left to the bottom right of a M X N matrix with the constraints that from each cell you can either move only to the right or down.

Input: M = 2, N = 2

Output: 2

Explanation: There are two paths
(0, 0) -> (0, 1) -> (1, 1)
(0, 0) -> (1, 0) -> (1, 1)

Input: M = 2, N = 3

Output: 3

Explanation: There are three paths
(0, 0) -> (0, 1) -> (0, 2) -> (1, 2)
(0, 0) -> (0, 1) -> (1, 1) -> (1, 2)
(0, 0) -> (1, 0) -> (1, 1) -> (1, 2)

Count all possible paths: We can recursively move to right and down from the start until we reach the destination and then add up all valid paths to get the answer.

Procedure:

- Create a recursive function with parameters as row and column index
- Call this recursive function for N-1 and M-1
- In the recursive function
    - If N == 1 or M == 1 then return 1
    - else call the recursive function with (N-1, M) and (N, M-1) and return the sum of this
- Print the answer

Hints:

```
# Python program to count all possible paths from top left to bottom right

# Function to return count of possible paths to reach cell at row number m and column
number n from the topmost leftmost cell (cell at 1, 1)

def numberOfPaths(m, n):
    …

# Driver program to test above function
m = 3
n = 3
print(numberOfPaths(m, n))
```

TRY

1. Take input : M = 3, N = 2 and verify the results.
2. Take input : M = 2, N = 1 and verify the results.

8. Graph Coloring

8.1 Graph Coloring using Greedy Algorithm

Greedy algorithm is used to assign colors to the vertices of a graph. It doesn't guarantee to use minimum colors, but it guarantees an upper bound on the number of colors. The basic algorithm never uses more than d+1 colors where d is the maximum degree of a vertex in the given graph.

Basic Greedy Coloring Algorithm:

1. Color first vertex with first color.
2. Do following for remaining V-1 vertices.
    a) Consider the currently picked vertex and color it with the lowest numbered color that has not been used on any previously colored vertices adjacent to it. If all previously used colors appear on vertices adjacent to v, assign a new color to it.

Hints:

```
# Implement greedy algorithm for graph coloring

def addEdge(adj, v, w):
    adj[v].append(w)
    # Note: the graph is undirected
    adj[w].append(v)
```

```
    return adj

# Assigns colors (starting from 0) to all
# vertices and prints the assignment of colors
def greedyColoring(adj, V):
    …

# Driver Code
g1 = [[] for i in range(5)]
g1 = addEdge(g1, 0, 1)
g1 = addEdge(g1, 0, 2)
g1 = addEdge(g1, 1, 2)
g1 = addEdge(g1, 1, 3)
g1 = addEdge(g1, 2, 3)
g1 = addEdge(g1, 3, 4)
print("Coloring of graph 1 ")
greedyColoring(g1, 5)

g2 = [[] for i in range(5)]
g2 = addEdge(g2, 0, 1)
g2 = addEdge(g2, 0, 2)
g2 = addEdge(g2, 1, 2)
g2 = addEdge(g2, 1, 4)
g2 = addEdge(g2, 2, 4)
g2 = addEdge(g2, 4, 3)
print("\nColoring of graph 2")
greedyColoring(g2, 5)
```

Output:

Coloring of graph 1
Vertex 0 ---> Color 0
Vertex 1 ---> Color 1
Vertex 2 ---> Color 2
Vertex 3 ---> Color 0
Vertex 4 ---> Color 1

Coloring of graph 2
Vertex 0 ---> Color 0
Vertex 1 ---> Color 1
Vertex 2 ---> Color 2
Vertex 3 ---> Color 0
Vertex 4 ---> Color 3

8.2 Coloring a Cycle Graph

Given the number of vertices in a Cyclic Graph. The task is to determine the Number of colors required to color the graph so that no two adjacent vertices have the same color.

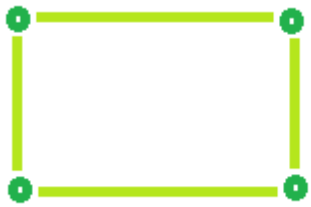
Approach:

- If the no. of vertices is Even then it is Even Cycle and to color such graph we require 2 colors.
- If the no. of vertices is Odd then it is Odd Cycle and to color such graph we require 3 colors.

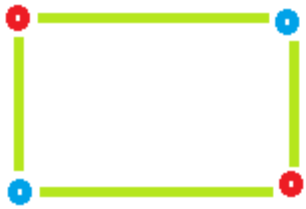
Input: Vertices = 3
Output: No. of colors require is: 3

Input: vertices = 4
Output: No. of colors require is: 2

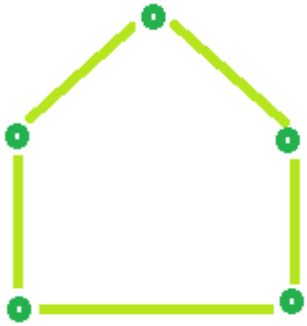
Example 1: Even Cycle: Number of vertices = 4



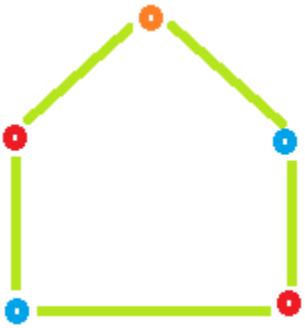
Color required = 2



Example 2: Odd Cycle: Number of vertices = 5



Color required = 3



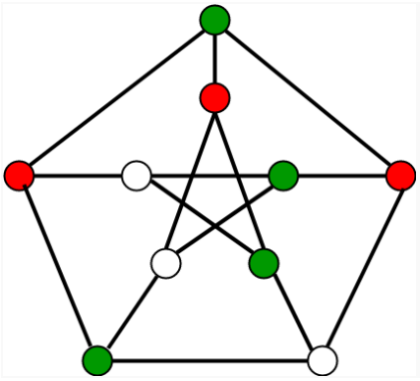
Hints:

```
# Find the number of colors required to color a cycle graph

# Function to find Color required.
def Color(vertices):
    …

# Driver Code
vertices = 3
print ("No. of colors require is:", Color(vertices))
```

8.3 m Coloring Problem

Given an undirected graph and a number m, determine if the graph can be colored with at most m colors such that no two adjacent vertices of the graph are colored with the same color.

Note: Here coloring of a graph means the assignment of colors to all vertices
Following is an example of a graph that can be colored with 3 different colors:



Input: graph = {0, 1, 1, 1},
{1, 0, 1, 0},
{1, 1, 0, 1},
{1, 0, 1, 0}
Output: Solution Exists: Following are the assigned colors: 1 2 3 2
Explanation: By coloring the vertices with following colors, adjacent vertices does not have same colors

Input: graph = {1, 1, 1, 1},
{1, 1, 1, 1},
{1, 1, 1, 1},
{1, 1, 1, 1}
Output: Solution does not exist
Explanation: No solution exits

Generate all possible configurations of colors. Since each node can be colored using any of the m available colors, the total number of color configurations possible is mV. After generating a configuration

of color, check if the adjacent vertices have the same color or not. If the conditions are met, print the combination and break the loop

Follow the given steps to solve the problem:

- Create a recursive function that takes the current index, number of vertices and output color array
- If the current index is equal to number of vertices. Check if the output color configuration is safe, i.e check if the adjacent vertices do not have same color. If the conditions are met, print the configuration and break
- Assign a color to a vertex (1 to m)
- For every assigned color recursively call the function with next index and number of vertices
- If any recursive function returns true break the loop and returns true.

Hints:

```
# Number of vertices in the graph
# define 4 4

# check if the colored graph is safe or not


def isSafe(graph, color):
    # check for every edge
    for i in range(4):
        for j in range(i + 1, 4):
            if (graph[i][j] and color[j] == color[i]):
                return False
    return True


def graphColoring(graph, m, i, color):
    # write your code here
    …

# /* A utility function to print solution */

def printSolution(color):
    print("Solution Exists:" " Following are the assigned colors ")
    for i in range(4):
        print(color[i], end=" ")


# Driver code
# /* Create following graph and test whether it is 3 colorable
# (3)---(2)
# |   /  |
# |  /   |
# | /    |
# (0)---(1)
# */
graph = [
        [0, 1, 1, 1],
        [1, 0, 1, 0],
        [1, 1, 0, 1],
```

```
        [1, 0, 1, 0],
        ]
m = 3  # Number of colors

# Initialize all color values as 0.
# This initialization is needed
# correct functioning of isSafe()
color = [0 for i in range(4)]

# Function call
if (not graphColoring(graph, m, 0, color)):
        print("Solution does not exist")
```

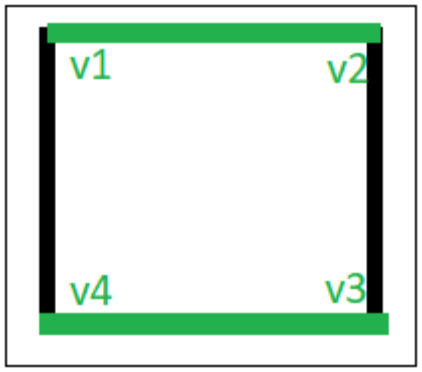
8.4 Edge Coloring of a Graph

Edge coloring of a graph is an assignment of "colors" to the edges of the graph so that no two adjacent edges have the same color with an optimal number of colors. Two edges are said to be adjacent if they are connected to the same vertex. There is no known polynomial time algorithm for edge-coloring every graph with an optimal number of colors.

Input: u1 = 1, v1 = 4
u2 = 1, v2 = 2
u3 = 2, v3 = 3
u4 = 3, v4 = 4

Output: Edge 1 is of color 1
Edge 2 is of color 2
Edge 3 is of color 1
Edge 4 is of color 2

The above input shows the pair of vertices (u_i, v_i) who have an edge between them. The output shows the color assigned to the respective edges.



Edge colorings are one of several different types of graph coloring problems. The above figure of a Graph shows an edge coloring of a graph by the colors green and black, in which no adjacent edge have the same color.

Algorithm:

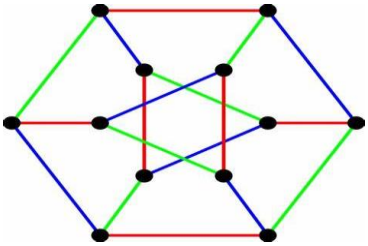
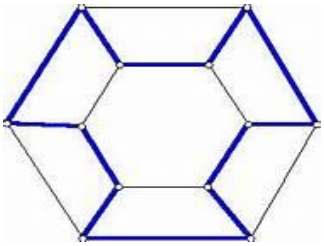
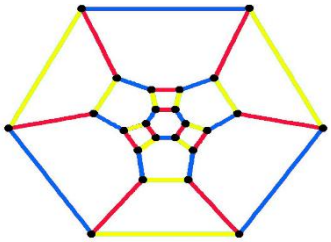
1. Use BFS traversal to start traversing the graph.
2. Pick any vertex and give different colors to all of the edges connected to it, and mark those edges as colored.
3. Traverse one of it's edges.
4. Repeat step to with a new vertex until all edges are colored.

Hints:

```
# Edge Coloring

from queue import Queue

# function to determine the edge colors
def colorEdges(ptr, gra, edgeColors, isVisited):
      # Write your code here
      …


# Driver Function
empty=set()
# declaring vector of vector of pairs, to define Graph
gra=[]
edgeColors=[]
isVisited=[False]*100000

ver = 4
edge = 4
gra=[[] for _ in range(ver)]
edgeColors=[-1]*edge
gra[0].append((1, 0))
gra[1].append((0, 0))
gra[1].append((2, 1))
gra[2].append((1, 1))
gra[2].append((3, 2))
gra[3].append((2, 2))
gra[0].append((3, 3))
gra[3].append((0, 3))
colorEdges(0, gra, edgeColors, isVisited)

# printing all the edge colors
for i in range(edge):
    print("Edge {} is of color {}".format(i + 1,edgeColors[i] + 1))
```

TRY

1. Write a program to implement graph coloring and edge coloring by consider the below graph and verify the results.

9. Graph Traversal

9.1 Breadth First Search

The **Breadth First Search (BFS)** algorithm is used to search a graph data structure for a node that meets a set of criteria. It starts at the root of the graph and visits all nodes at the current depth level before moving on to the nodes at the next depth level.
For a given graph G, print BFS traversal from a given source vertex.

Hints:

```
# BFS traversal from a given source vertex.

from collections import defaultdict

# This class represents a directed graph using adjacency list representation
class Graph:

    # Constructor
    def __init__(self):
        # Default dictionary to store graph
        self.graph = defaultdict(list)

    # Function to add an edge to graph
    def addEdge(self, u, v):
        self.graph[u].append(v)

    # Function to print a BFS of graph
    def BFS(self, s):
      # Write your code here
      …


# Create a graph given in the above diagram
g = Graph()
g.addEdge(0, 1)
g.addEdge(0, 2)
g.addEdge(1, 2)
g.addEdge(2, 0)
g.addEdge(2, 3)
g.addEdge(3, 3)

print("Following is Breadth First Traversal" " (starting from vertex 2)")
g.BFS(2)
```

Output: Following is Breadth First Traversal (starting from vertex 2)
2 0 3 1

9.2 Depth First Search

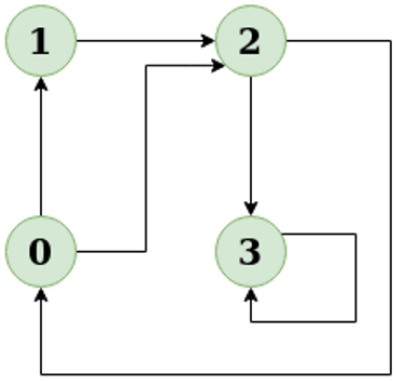
Depth First Traversal (or DFS) for a graph is similar to Depth First Traversal of a tree. The only catch here is, that, unlike trees, graphs may contain cycles (a node may be visited twice). To avoid processing a node more than once, use a boolean visited array. A graph can have more than one DFS traversal.

For a given graph G, print DFS traversal from a given source vertex.

Input: n = 4, e = 6
0 -> 1, 0 -> 2, 1 -> 2, 2 -> 0, 2 -> 3, 3 -> 3

Output: DFS from vertex 1: 1 2 0 3

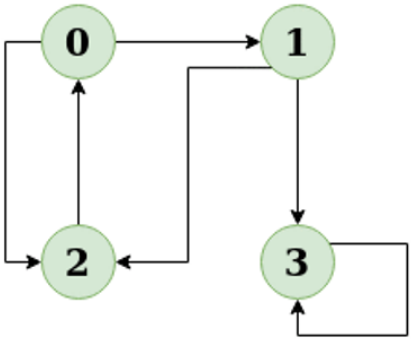
Explanation:
DFS Diagram:



Input: n = 4, e = 6
2 -> 0, 0 -> 2, 1 -> 2, 0 -> 1, 3 -> 3, 1 -> 3

Output: DFS from vertex 2: 2 0 1 3

Explanation:
DFS Diagram:

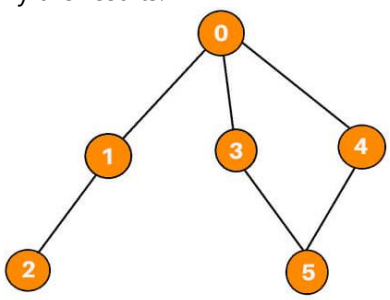


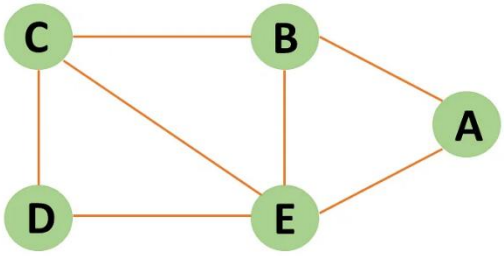
Hints:

```
# DFS traversal from a given graph
from collections import defaultdict

# This class represents a directed graph using adjacency list representation
class Graph:
    # Constructor
    def __init__(self):
        # Default dictionary to store graph
        self.graph = defaultdict(list)


    # Function to add an edge to graph
    def addEdge(self, u, v):
        self.graph[u].append(v)


    # A function used by DFS
    def DFSUtil(self, v, visited):
        …


    # The function to do DFS traversal. It uses recursive DFSUtil()

    def DFS(self, v):
       # Write your code here
        …

# Driver's code
g = Graph()
g.addEdge(0, 1)
g.addEdge(0, 2)
g.addEdge(1, 2)
g.addEdge(2, 0)
g.addEdge(2, 3)
g.addEdge(3, 3)
print("Following is Depth First Traversal (starting from vertex 2)")
# Function call
g.DFS(2)
```

TRY

1. Write a program to implement breadth first search and depth first search by consider the below graph and verify the results.





10. Minimum Spanning Tree (MST)

10.1 Kruskal's Algorithm

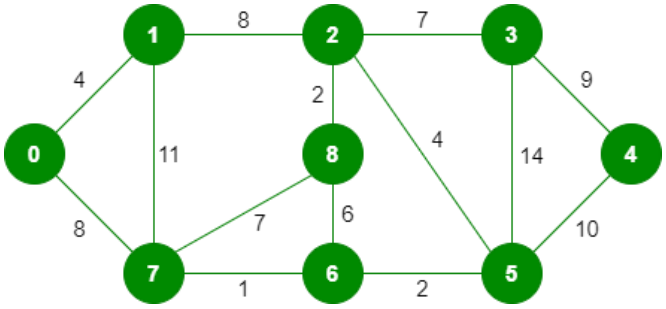
In Kruskal's algorithm, sort all edges of the given graph in increasing order. Then it keeps on adding new edges and nodes in the MST if the newly added edge does not form a cycle. It picks the minimum weighted edge at first and the maximum weighted edge at last.

MST using Kruskal's algorithm:

1. Sort all the edges in non-decreasing order of their weight.
2. Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If the cycle is not formed, include this edge. Else, discard it.
3. Repeat step#2 until there are (V-1) edges in the spanning tree.

Kruskal's algorithm to find the minimum cost spanning tree uses the greedy approach. The Greedy Choice is to pick the smallest weight edge that does not cause a cycle in the MST constructed so far.

Input: For the given graph G find the minimum cost spanning tree.



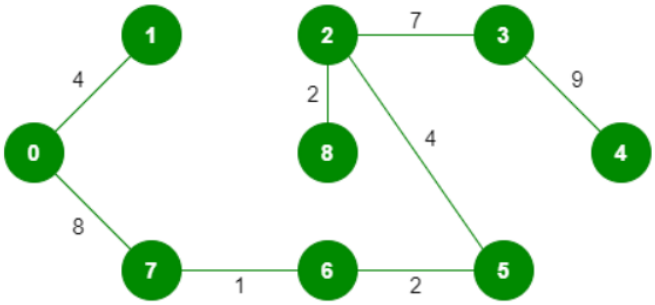
The graph contains 9 vertices and 14 edges. So, the minimum spanning tree formed will be having (9 – 1) = 8 edges.

After sorting:

Weight	Source	Destination
1	7	6
2	8	2
2	6	5
4	0	1
4	2	5
6	8	6
7	2	3
7	7	8
8	0	7
8	1	2
9	3	4
10	5	4
11	1	7
14	3	5

Now pick all edges one by one from the sorted list of edges.

Output:



Hints:

```
# Kruskal's algorithm to find minimum Spanning Tree of a given connected,
# undirected and weighted graph

# Class to represent a graph
class Graph:
    def __init__(self, vertices):
        self.V = vertices
        self.graph = []

    # Function to add an edge to graph
    def addEdge(self, u, v, w):
        self.graph.append([u, v, w])

    def find(self, parent, i):
        …

    def union(self, parent, rank, x, y):
        …

    def KruskalMST(self):
       # write your code here
       …

# Driver code
g = Graph(4)
g.addEdge(0, 1, 10)
g.addEdge(0, 2, 6)
g.addEdge(0, 3, 5)
g.addEdge(1, 3, 15)
g.addEdge(2, 3, 4)

# Function call
g.KruskalMST()
```

Output: Following are the edges in the constructed MST
2 -- 3 == 4

0 -- 3 == 5

0 -- 1 == 10

Minimum Cost Spanning Tree: 19

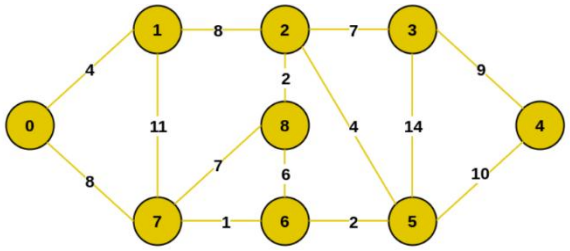
10.2 Prim's Algorithm

The Prim's algorithm starts with an empty spanning tree. The idea is to maintain two sets of vertices. The first set contains the vertices already included in the MST, and the other set contains the vertices not yet included. At every step, it considers all the edges that connect the two sets and picks the minimum weight edge from these edges. After picking the edge, it moves the other endpoint of the edge to the set containing MST.

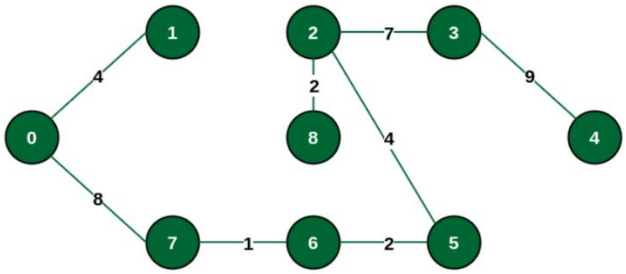
Prim's Algorithm:
The working of Prim's algorithm can be described by using the following steps:

1. Determine an arbitrary vertex as the starting vertex of the MST.
2. Follow steps 3 to 5 till there are vertices that are not included in the MST (known as fringe vertex).
3. Find edges connecting any tree vertex with the fringe vertices.
4. Find the minimum among these edges.
5. Add the chosen edge to the MST if it does not form any cycle.
6. Return the MST and exit

Input: For the given graph G find the minimum cost spanning tree.



Output: The final structure of the MST is as follows and the weight of the edges of the MST is (4 + 8 + 1 + 2 + 4 + 2 + 7 + 9) = 37.



Hints:

```
# Prim's Minimum Spanning Tree (MST) algorithm.
# The program is for adjacency matrix representation of the graph

# Library for INT_MAX
import sys

class Graph():
    def __init__(self, vertices):
        self.V = vertices
        self.graph = [[0 for column in range(vertices)]
                      for row in range(vertices)]

    # A utility function to print
    # the constructed MST stored in parent[]
    def printMST(self, parent):
        print("Edge \tWeight")
        for i in range(1, self.V):
            print(parent[i], "-", i, "\t", self.graph[i][parent[i]])

    # A utility function to find the vertex with
    # minimum distance value, from the set of vertices
    # not yet included in shortest path tree
    def minKey(self, key, mstSet):

        …
    def primMST(self):
        …


# Driver's code
g = Graph(5)
g.graph = [[0, 2, 0, 6, 0],
           [2, 0, 3, 8, 5],
           [0, 3, 0, 0, 7],
           [6, 8, 0, 0, 9],
           [0, 5, 7, 9, 0]]

g.primMST()
```

Output:

Edge Weight

0 - 1 2

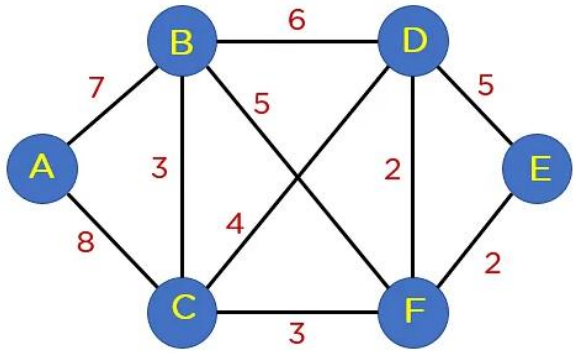
1 - 2 3

0 - 3 6

1 - 4 5

TRY

1. Write a program to implement kruskal's algorithm and prim's algorithm by consider the below graph and verify the results.



11. Roots of Equations

11.1 Bisection Method

The Bisection method is also called the interval halving method, the binary search method or the dichotomy method. This method is used to find root of an equation in a given interval that is value of 'x' for which f(x) = 0. The method is based on **The Intermediate Value Theorem** which states that if f(x) is a continuous function and there are two real numbers a and b such that f(a) * f(b) 0 and f(b) < 0), then it is guaranteed that it has at least one root between them.

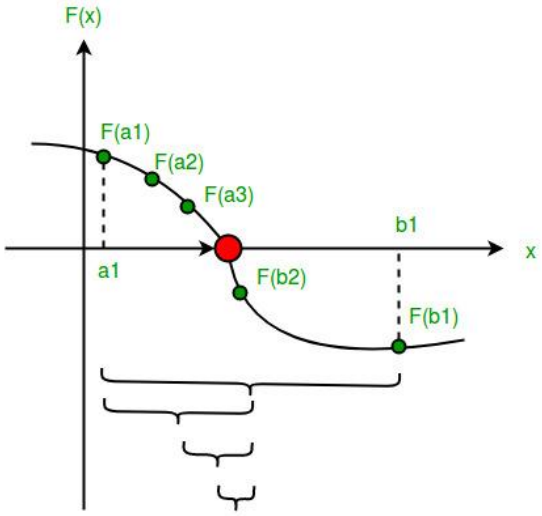
Assumptions:

1. f(x) is a continuous function in interval [a, b]
2. f(a) * f(b) < 0

Steps:

1. Find middle point c= (a + b)/2.
2. If f(c) == 0, then c is the root of the solution.
3. Else f(c) != 0
    - If value f(a)*f(c) < 0 then root lies between a and c. So we recur for a and c
    - Else If f(b)*f(c) < 0 then root lies between b and c. So we recur b and c.
    - Else given function doesn't follow one of assumptions.

Since root may be a floating point number, we repeat above steps while difference between a and b is greater than and equal to a value? (A very small value).



Hints:

```
# An example function whose solution is determined using Bisection Method.
# The function is x^3 - x^2  + 2
def func(x):
    return x*x*x - x*x + 2

# Prints root of func(x) with error of EPSILON
def bisection(a,b):
    # Write your code here
    …

# Driver code
# Initial values assumed
a =-200
b = 300
bisection (a, b)
```

Output: The value of root is : -1.0025

TRY

1. Take an input function x^2 - x^3 + 2 and verify the results.
2. Take an input function x^3 - x^3 + 4 and verify the results.

11.2 Method of False Position

Given a function f(x) on floating number x and two numbers 'a' and 'b' such that f(a)*f(b) < 0 and f(x) is continuous in [a, b]. Here f(x) represents algebraic or transcendental equation. Find root of function in interval [a, b] (Or find a value of x such that f(x) is 0).

Input: A function of x, for example x3 – x2 + 2.
And two values: a = -200 and b = 300 such that
f(a)*f(b) < 0, i.e., f(a) and f(b) have opposite signs.

Output: The value of root is : -1.00
OR any other value close to root.

Hints:

```
MAX_ITER = 1000000

# An example function whose solution is determined using Regular Falsi Method.
# The function is x^3 - x^2 + 2
def func( x ):
    return (x * x * x - x * x + 2)

# Prints root of func(x) in interval [a, b]
def regulaFalsi( a , b):
    # Write your code here
    …

# Driver code to test above function
# Initial values assumed
a =-200
b = 300
regulaFalsi(a, b)
```

TRY

1. Take an input function x^2 - x^3 + 2 and verify the results.
2. Take an input function x^3 - x^3 + 4 and verify the results.

11.3 Newton Raphson Method

Given a function f(x) on floating number x and an initial guess for root, find root of function in interval. Here f(x) represents algebraic or transcendental equation.

Input: A function of x (for example $x^3 – x^2 + 2$), derivative function of x ($3x^2 – 2x$ for above example) and an initial guess x0 = -20

Output: The value of root is: -1.00 or any other value close to root.

Algorithm:
Input: initial x, func(x), derivFunc(x)
Output: Root of Func()

1. Compute values of func(x) and derivFunc(x) for given initial x
2. Compute h: h = func(x) / derivFunc(x)
3. While h is greater than allowed error ε
   - h = func(x) / derivFunc(x)
   - x = x – h

Hints:

```
# Implementation of Newton Raphson Method for solving equations
# An example function whose solution is determined using Bisection Method.
# The function is x^3 - x^2 + 2
def func( x ):
```

```
    return x * x * x - x * x + 2

# Derivative of the above function which is 3*x^x - 2*x
def derivFunc( x ):
    return 3 * x * x - 2 * x

# Function to find the root
def newtonRaphson( x ):
    # Write your code here
    …

# Driver program
x0 = -20
newtonRaphson(x0)
```

TRY

1. Take an input function x^2 - x^3 + 2 and verify the results.
2. Take an input function x^3 - x^3 + 4 and verify the results.

11.4 Secant Method

The secant method is used to find the root of an equation f(x) = 0. It is started from two distinct estimates x1 and x2 for the root. It is an iterative procedure involving linear interpolation to a root. The iteration stops if the difference between two intermediate values is less than the convergence factor.

Input: Equation = x3 + x – 1
x1 = 0, x2 = 1, E = 0.0001

Output: Root of the given equation = 0.682326
No. of iteration=5

Algorithm

Initialize: x1, x2, E, n // E = convergence indicator
calculate f(x1),f(x2)

if(f(x1) * f(x2) = E); //repeat the loop until the convergence
  print 'x0' //value of the root
  print 'n' //number of iteration
}
else
  print "cannot found a root in the given interval"

Hints:

```
# Find root of an equations using secant method
# Function takes value of x and returns f(x)
def f(x):
    # we are taking equation as x^3+x-1
    f = pow(x, 3) + x - 1;
    return f;
```

```
def secant(x1, x2, E):
   # Write your code here
    …

# Driver code
# initializing the values
x1 = 0;
x2 = 1;
E = 0.0001;
secant(x1, x2, E);
```

TRY

1. Take an input function x^2 - x + 2 and verify the results.
2. Take an input function x^3 - x^2 + 4 and verify the results.

11.5 Muller Method

Given a function f(x) on floating number x and three initial distinct guesses for root of the function, find the root of function. Here, f(x) can be an algebraic or transcendental function.

Input: A function f(x) = x + 2x + 10x - 20 and three initial guesses - 0, 1 and 2 .

Output: The value of the root is 1.3688 or any other value within permittable deviation from the root.

Input: A function f(x) = x - 5x + 2 and three initial guesses - 0, 1 and 2.

Output: The value of the root is 0.4021 or any other value within permittable deviation from the root.

Hints:

```
# Find root of a function, f(x)
import math;

MAX_ITERATIONS = 10000;

# Function to calculate f(x)
def f(x):
    # Taking f(x) = x ^ 3 + 2x ^ 2 + 10x - 20
    return (1 * pow(x, 3) + 2 * x * x +
                            10 * x - 20);

def Muller(a, b, c):
    # Write your code here
    …


# Driver Code
a = 0;
b = 1;
c = 2;
Muller(a, b, c);
```

TRY

1. Take an input function x^2 - x^3 + 2 and verify the results.
2. Take an input function x^3 - x^3 + 4 and verify the results.

12. Numerical Integration

12.1 Trapezoidal Rule for Approximate Value of Definite Integral

Trapezoidal rule is used to find the approximation of a definite integral. The basic idea in Trapezoidal rule is to assume the region under the graph of the given function to be a trapezoid and calculate its area.

$$\int_a^b f(x)\,dx \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

Hints:

```
# Implement Trapezoidal rule

# A sample function whose definite integral's approximate value is
# computed using Trapezoidal rule
def y( x ):

    # Declaring the function
    # f(x) = 1/(1+x*x)
    return (1 / (1 + x * x))

# Function to evaluate the value of integral
def trapezoidal (a, b, n):
    # Write your code here
    …

# Driver code
# Range of definite integral
x0 = 0
xn = 1

# Number of grids. Higher value means more accuracy
n = 6
print ("Value of integral is ",
     "%.4f"%trapezoidal(x0, xn, n))
```

12.2 Simpson's 1/3 Rule

Simpson's 1/3 rule is a method for numerical approximation of definite integrals. Specifically, it is the following approximation:

$$\int_a^b f(x)dx \approx \frac{(b-a)}{6}\left(f(a)+4f\frac{(a+b)}{2}+f(b)\right)$$

Procedure:

In order to integrate any function f(x) in the interval (a, b), follow the steps given below:

1. Select a value for n, which is the number of parts the interval is divided into.
2. Calculate the width, h = (b-a)/n
3. Calculate the values of x0 to xn as x0 = a, x1 = x0 + h, …..xn-1 = xn-2 + h, xn = b.
   Consider y = f(x). Now find the values of y (y0 to yn) for the corresponding x (x0 to xn) values.
4. Substitute all the above found values in the Simpson's Rule Formula to calculate the integral value.

Approximate value of the integral can be given by **Simpson's Rule:**

$$\int_a^b f(x)dx \approx \frac{h}{3}\left(f_0 + f_n + 4 * \sum_{i=1,3,5}^{n-1} f_i + 2 * \sum_{i=2,4,6}^{n-2} f_i\right)$$

Input: Evaluate logx dx within limit 4 to 5.2.

First we will divide interval into six equal parts as number of interval should be even.

x : 4 4.2 4.4 4.6 4.8 5.0 5.2

$\log_x$: 1.38 1.43 1.48 1.52 1.56 1.60 1.64

Output: Now we can calculate approximate value of integral using above formula:

= h/3[(1.38 + 1.64) + 4 * (1.43 + 1.52 + 1.60) +2 *(1.48 + 1.56)]

= 1.84

Hence the approximation of above integral is

1.827 using Simpson's 1/3 rule.

Hints:

```
# Simpson's 1 / 3 rule
import math

# Function to calculate f(x)
def func(x):
    return math.log(x)

# Function for approximate integral
def simpsons_(ll, ul, n):
     # Write your code here
     …

# Driver code
lower_limit = 4      # Lower limit
upper_limit = 5.2    # Upper limit
n = 6                # Number of interval
print("%.6f"% simpsons_(lower_limit, upper_limit, n))
```

12.3 Simpson's 3/8 Rule

The Simpson's 3/8 rule was developed by Thomas Simpson. This method is used for performing numerical integrations. This method is generally used for numerical approximation of definite integrals. Here, parabolas are used to approximate each part of curve.

Input: lower_limit = 1, upper_limit = 10, interval_limit = 10

Output: integration_result = 0.687927

Input: lower_limit = 1, upper_limit = 5, interval_limit = 3

Output: integration_result = 0.605835

Hints:

```
# Implement Simpson's 3/8 rule

# Given function to be integrated
def func(x):
    return (float(1) / ( 1 + x * x ))

# Function to perform calculations
def calculate(lower_limit, upper_limit, interval_limit ):
     # Write your code here
     …



# driver function
interval_limit = 10
lower_limit = 1
upper_limit = 10

integral_res = calculate(lower_limit, upper_limit, interval_limit)

# rounding the final answer to 6 decimal places
print (round(integral_res, 6))
```

13. Ordinary Differential Equations

13.1 The Euler Method

Given a differential equation dy/dx = f(x, y) with initial condition y(x0) = y0. Find its approximate solution using Euler method.

Euler Method:
In mathematics and computational science, the Euler method (also called forward Euler method) is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value.

Consider a differential equation dy/dx = f(x, y) with initial condition y(x0) = y0
then a successive approximation of this equation can be given by:

y(n+1) = y(n) + h * f(x(n), y(n))
where h = (x(n) – x(0)) / n, h indicates step size. Choosing smaller values of h leads to more accurate results and more computation time.

Example:
Consider below differential equation dy/dx = (x + y + xy) with initial condition y(0) = 1 and step size h = 0.025. Find y(0.1).

Solution:
f(x, y) = (x + y + xy)
x0 = 0, y0 = 1, h = 0.025
Now we can calculate y1 using Euler formula
y1 = y0 + h * f(x0, y0)
y1 = 1 + 0.025 *(0 + 1 + 0 * 1)
y1 = 1.025
y(0.025) = 1.025.
Similarly we can calculate y(0.050), y(0.075), ...y(0.1).
y(0.1) = 1.11167

Hints:

```
# Find approximation of an ordinary differential equation using Euler method.

# Consider a differential equation
# dy / dx =(x + y + xy)
def func( x, y ):
    return (x + y + x * y)

# Function for Euler formula
def euler( x0, y, h, x ):
    # write code here
    …

# Driver Code
# Initial Values
x0 = 0
y0 = 1
h = 0.025

# Value of x at which we need approximation
x = 0.1
euler(x0, y0, h, x)
```

13.2 Runge-Kutta Second Order Method

Given the following inputs:

1. An ordinary differential equation that defines the value of **dy/dx** in the form **x** and **y**.
   $\frac{dy}{dx} = f(x, y)$

2. Initial value of y, i.e., **y(0)**.
   $y(0) = y_o$

The task is to find the value of unknown function y at a given point x, i.e. **y(x)**.
Input: x0 = 0, y0 = 1, x = 2, h = 0.2

Output: y(x) = 0.645590

Input: x0 = 2, y0 = 1, x = 4, h = 0.4;

Output: y(x) = 4.122991

Approach:
The Runge-Kutta method finds an approximate value of y for a given x. Only first-order ordinary differential equations can be solved by using the Runge-Kutta 2nd-order method.
Below is the formula used to compute the next value yn+1 from the previous value yn. Therefore:
y_{n+1} = value of y at (x = n + 1)
y_n = value of y at (x = n)
where 0 ? n ? (x - x_0)/h, h is step height
$x_{n+1} = x_0 + h$
The essential formula to compute the value of y(n+1):

K1 = h * f(x, y)

K2 = h * f(x/2, y/2) or K1/2

$y_{n+1} = y_n + K^2 + (h^3)$

The formula basically computes the next value $\mathbf{y_{n+1}}$ using current $\mathbf{y_n}$ plus the weighted average of two increments:

- **K1** is the increment based on the slope at the beginning of the interval, using y.
- **K2** is the increment based on the slope at the midpoint of the interval, using **(y + h*K1/2)**.

Hints:

```
# Implement Runge-Kutta method

# A sample differential equation
# "dy/dx = (x - y)/2"

def dydx(x, y):
    return (x + y - 2)

# Finds value of y for a given x using step size h and initial value y0 at x0.
def rungeKutta(x0, y0, x, h):
    # write code here
    …

# Driver Code
x0 = 0
y = 1
x = 2
h = 0.2
print("y(x) =", rungeKutta(x0, y, x, h))
```

14. Final Notes

The only way to learn programming is program, program and program on challenging problems. The problems in this tutorial are certainly NOT challenging. There are tens of thousands of challenging problems available – used in training for various programming contests (such as International Collegiate Programming Contest (ICPC), International Olympiad in Informatics (IOI)). Check out these sites:

- The ACM - ICPC International collegiate programming contest (https://icpc.global/)
- The Topcoder Open (TCO) annual programming and design contest (https://www.topcoder.com/)
- Universidad de Valladolid's online judge (https://uva.onlinejudge.org/).
- Peking University's online judge (http://poj.org/).
- USA Computing Olympiad (USACO) Training Program @ http://train.usaco.org/usacogate.
- Google's coding competitions (https://codingcompetitions.withgoogle.com/codejam, https://codingcompetitions.withgoogle.com/hashcode)
- The ICFP programming contest (https://www.icfpconference.org/)
- BME International 24-hours programming contest (https://www.challenge24.org/)
- The International Obfuscated C Code Contest (https://www0.us.ioccc.org/main.html)
- Internet Problem Solving Contest (https://ipsc.ksp.sk/)
- Microsoft Imagine Cup (https://imaginecup.microsoft.com/en-us)
- Hewlett Packard Enterprise (HPE) Codewars (https://hpecodewars.org/)
- OpenChallenge (https://www.openchallenge.org/)

Coding Contests Scores

Students must solve problems and attain scores in the following coding contests:

Name of the contest	Minimum number of problems to solve	Required score
• CodeChef	20	200
• Leetcode	20	200
• GeeksforGeeks	20	200
• SPOJ	5	50
• InterviewBit	10	1000
• Hackerrank	25	250
• Codeforces	10	100
• BuildIT	50	500
	Total score need to obtain	2500

Student must have any one of the following certification:

1. HackerRank - Problem Solving Skills Certification (Basic and Intermediate)
2. GeeksforGeeks – Data Structures and Algorithms Certification
3. CodeChef - Learn Python Certification
4. Interviewbit – DSA pro / Python pro
5. NPTEL – Programming, Data Structures and Algorithms
6. NPTEL – The Joy of Computing using Python

V. TEXT BOOKS:

1. Eric Matthes, "Python Crash Course: A Hands-On, Project-based Introduction to Programming", No Starch Press, 3rd Edition, 2023.
2. John M Zelle, "Python Programming: An Introduction to Computer Science", Ingram short title, 3rd Edition, 2016.

VI. REFERENCE BOOKS:

1. Yashavant Kanetkar, Aditya Kanetkar, “Let Us Python”, BPB Publications, 2nd Edition, 2019.
2. Martin C. Brown, “Python: The Complete Reference”, Mc. Graw Hill, Indian Edition, 2018.
3. Paul Barry, “Head First Python: A Brain-Friendly Guide”, O’Reilly, 2nd Edition, 2016
4. Taneja Sheetal, Kumar Naveen, “Python Programming – A Modular Approach”, Pearson, 1st Edition, 2017.
5. R Nageswar Rao, “Core Python Programming”, Dreamtech Press, 2018.

VII. ELECTRONICS RESOURCES

1. https://realPython.com/Python3-object-oriented-programming/
2. https://Python.swaroopch.com/oop.html
3. https://Python-textbok.readthedocs.io/en/1.0/Object_Oriented_Programming.html
4. https://www.programiz.com/Python-programming/
5. https://www.geeksforgeeks.org/python-programming-language/

VIII. MATERIALS ONLINE

1. Course template
2. Lab Manual