SEMICONDUCTOR PHYSICS

Course code:AHSB13

B.Tech II semester

Regulation: IARE R-18

BY

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CO’s	Course outcomes
CO1	Interpret the concept of Quantum mechanics with dual nature of matter.
CO2	Identify different types of semiconductors and dependence of their Fermi level on various factors.
CO3	To give knowledge about semiconductor physics and discus working and applications of basic devices, including p-n junctions, PIN, Avalanche photodiode, Solar cell
CO4	Ability to identify appropriate magnetic, and dielectric, materials required for various engineering applications.
CO5	Understand the working principle of different types of lasers and optical fibre communication.

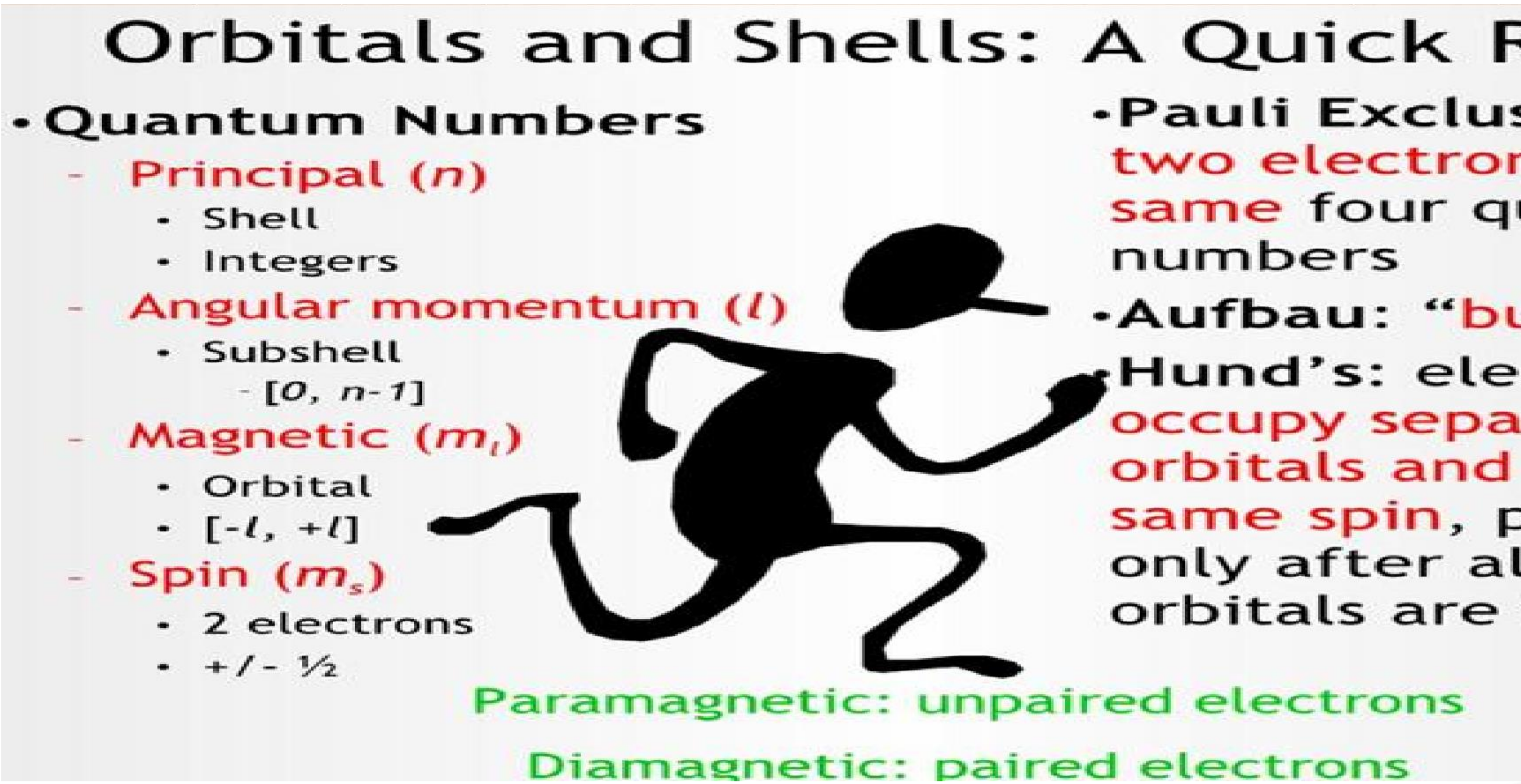
MODULE-I
QUANTUM MECHANICS

CLOs	Course Learning Outcome
CLO1	Recall the basic principles of physics and apply these concepts in solving the real-time problems.
CLO2	Acquire knowledge about fundamental in quantum mechanics.
CLO3	Interpretation of dual nature of matter wave concept using Davisson & Germer’s experiment.
CLO4	Estimate the energy of the particles using Schrödinger’s wave equation and apply it to particle in potential box.

Quantum mechanics

- **Quantum mechanics** (QM – also known as **quantum physics** or **quantum theory**) is a branch of physics which deals with physical phenomena at microscopic scales, where the action is on the order of the Planck constant. Quantum mechanics departs from classical mechanics primarily at the *quantum realm* of atomic and subatomic length scales. Quantum mechanics provides a mathematical description of much of the dual **particle-like** and **wave-like** behavior and interactions of energy and matter. Quantum mechanics is the non-relativistic limit of **Quantum Field Theory** (QFT), a theory that was developed later that combined Quantum Mechanics with Relativity.

QUANTUM MECHANICS



QUANTUM MECHANICS

Quantum Mechanics

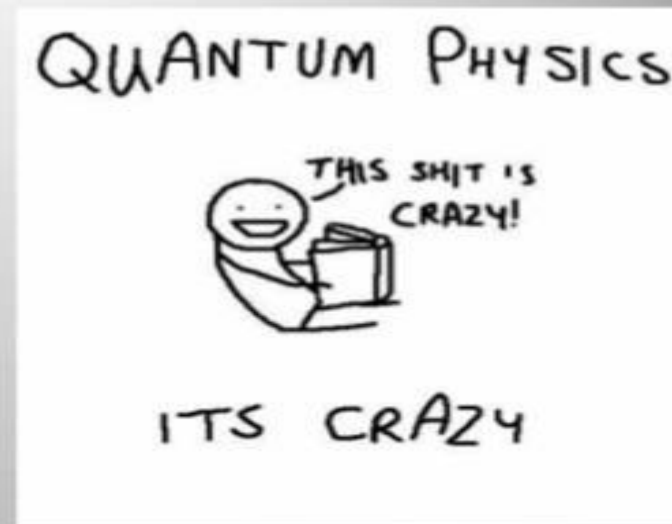
The other great theory of modern physics

- Deals with very small objects
  - → Electrons, atoms, molecules

Grew out of problems that seemed simple

- → Black-body radiation
- → Photoelectric Effect
- → Atomic Spectra

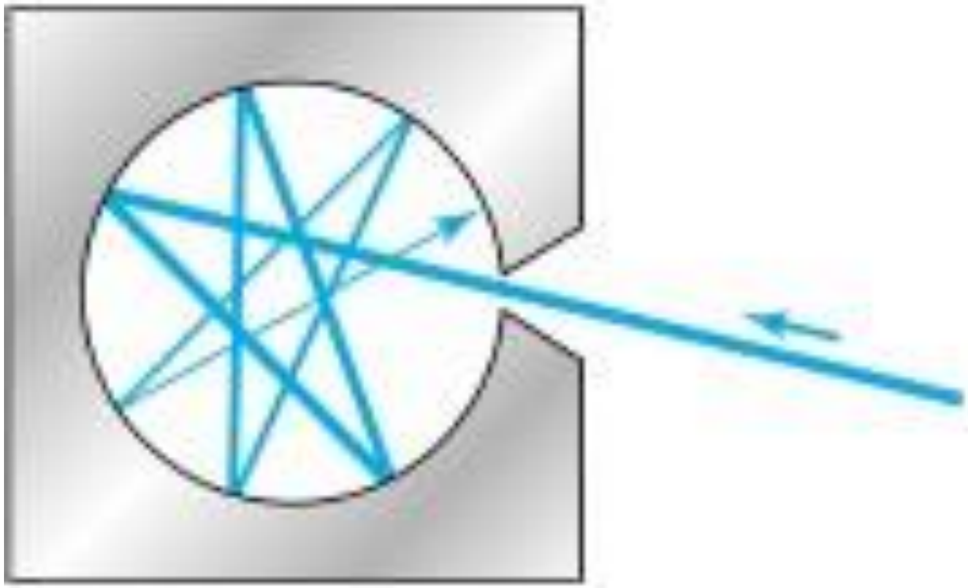
Produces some very strange results…



Blackbody?

- A material is constantly exchanging heat with its surrounding (to remain at a constant temperature):
  - It absorbs and emits radiations
  - Problem: it can reflect incoming radiations, which makes a theoretical description more difficult (depends on the environment)
- A blackbody is a perfect absorber:
  - Incoming radiations is totally absorbed and none is reflected

Blackbody Radiation



- Blackbody = a cavity, such as a metal box with a small hole drilled into it.

BLACK BODY RADIATION

Max Planck and the blackbody problem

- Max Planck 1858-1947
  - Expert in thermodynamics and statistical me
  - Around 1900: Proposes first an empirical foı (based on real physics) to reproduce both tł low wavelength parts of the emission spectr
    - → Remarkable agreement with experimen
  - Then, works on a theoretical basis of the foı

Planck's radiation law

- Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" contained in the walls. He used Boltzman's statistical methods to arrive at the following formula:

$$\mathcal{I}(\lambda,T)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

Planck's radiation law

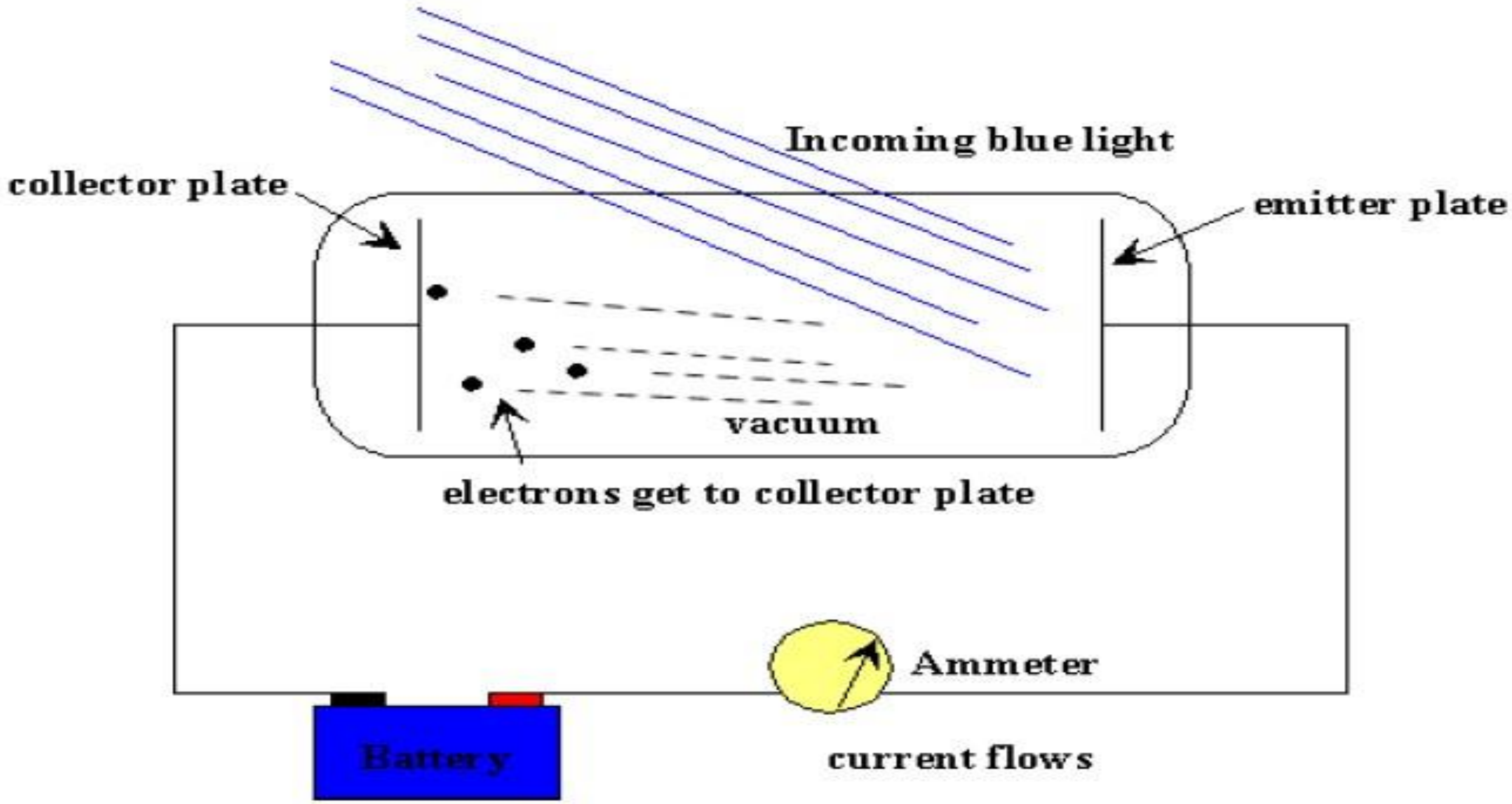
- Planck made two modifications to the classical theory:
  1) The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nh\nu$, where n is an integer, ν is the frequency, and h is called Planck's constant.

     $h = 6.6261 \times 10^{-34}$ J·s.

  2) The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

     $\Delta E = h\nu$

Photoelectric Effect



Photoelectric Effect

❖The photoelectric effect provides evidence for the particle nature of light.

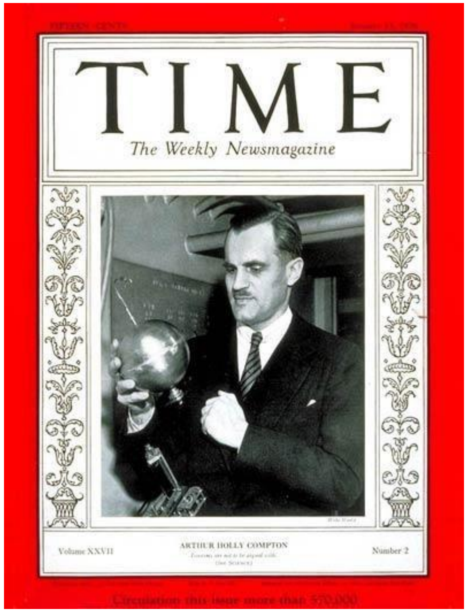
❖ It also provides evidence for quantization.

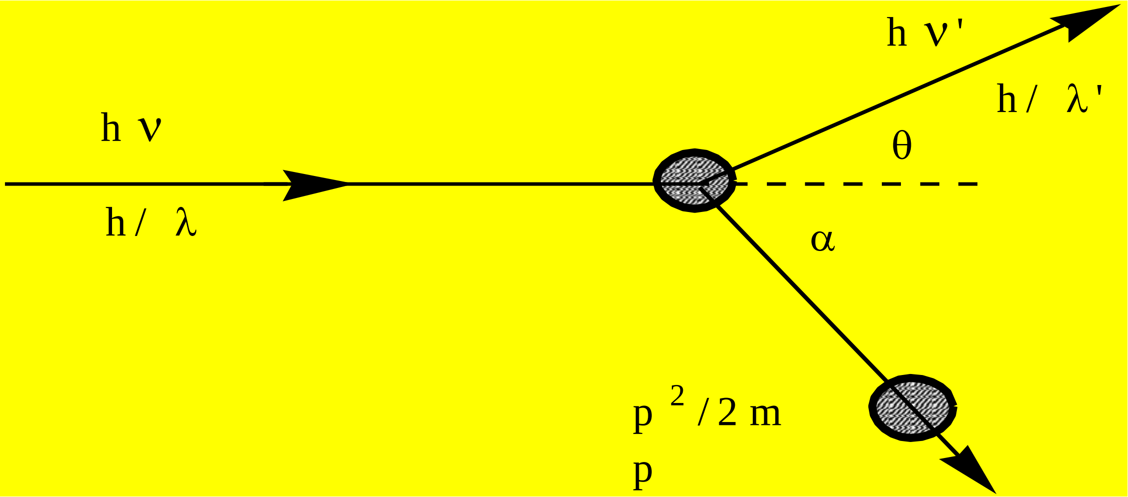
❖If light shines on the surface of a metal, there is a point at which electrons are ejected from the metal.

❖The electrons will only be ejected once the threshold frequency is reached

❖Above the threshold frequency, the number of electrons ejected depend on the intensity of the light.

Compton effect





Energy Conservation

$$h\frac{c}{\lambda} = h\frac{c}{\lambda'} + \frac{p^2}{2m} \qquad \left(\nu = \frac{c}{\lambda}\right)$$

Momentum Conservation (projection on x)

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p\cos\alpha$$

Momentum Conservation (projection on y)

$$0 = \frac{h}{\lambda'}\sin\theta - p\sin\alpha$$

Compton effect

Compton effect can't observed in Visible Light

$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta) = 0.0243\ (1-\cos\theta)\ \text{Å}$$

$\Delta\lambda$ is maximum when (1- cosθ) is maximum i.e. 2.

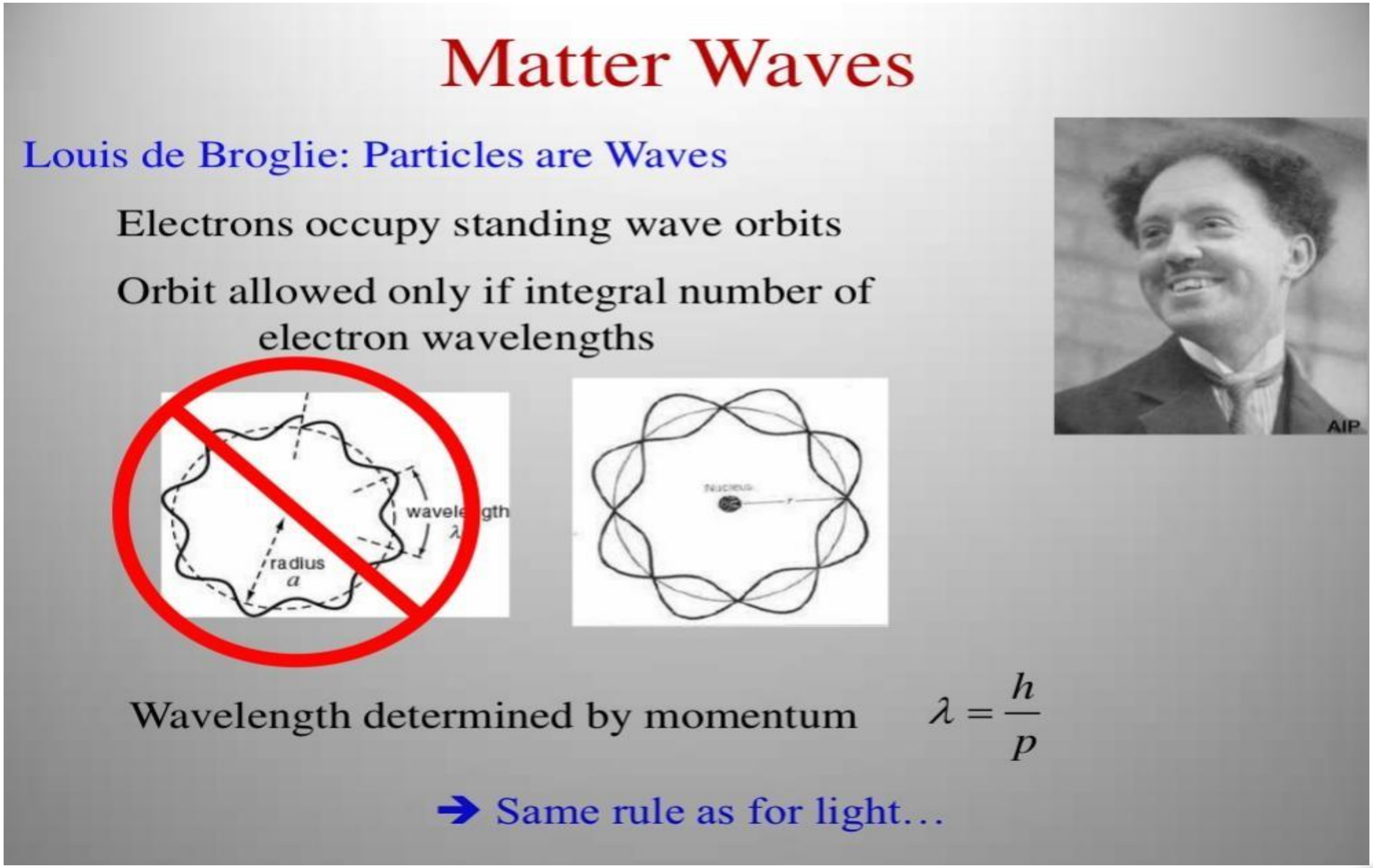
$$\Delta\lambda_{max} = 0.05\ \text{Å}$$

So Compton effect can be observed only for radiation having wavelength of few Å.

For $\lambda = 1\text{Å}$ $\Delta\lambda \sim 1\%$

For $\lambda = 5000\text{Å}$ $\Delta\lambda \sim 0.001\%$ (undetectable)

MATTER WAVES

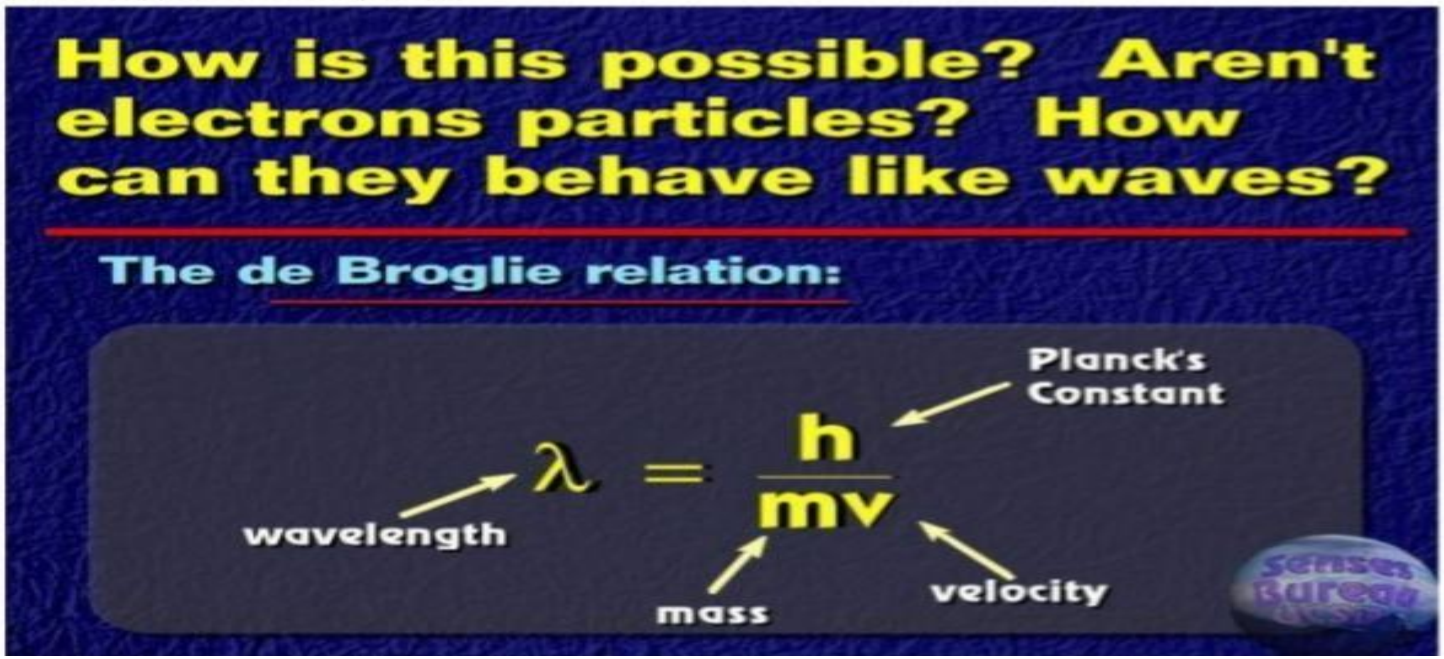


MATTER WAVES

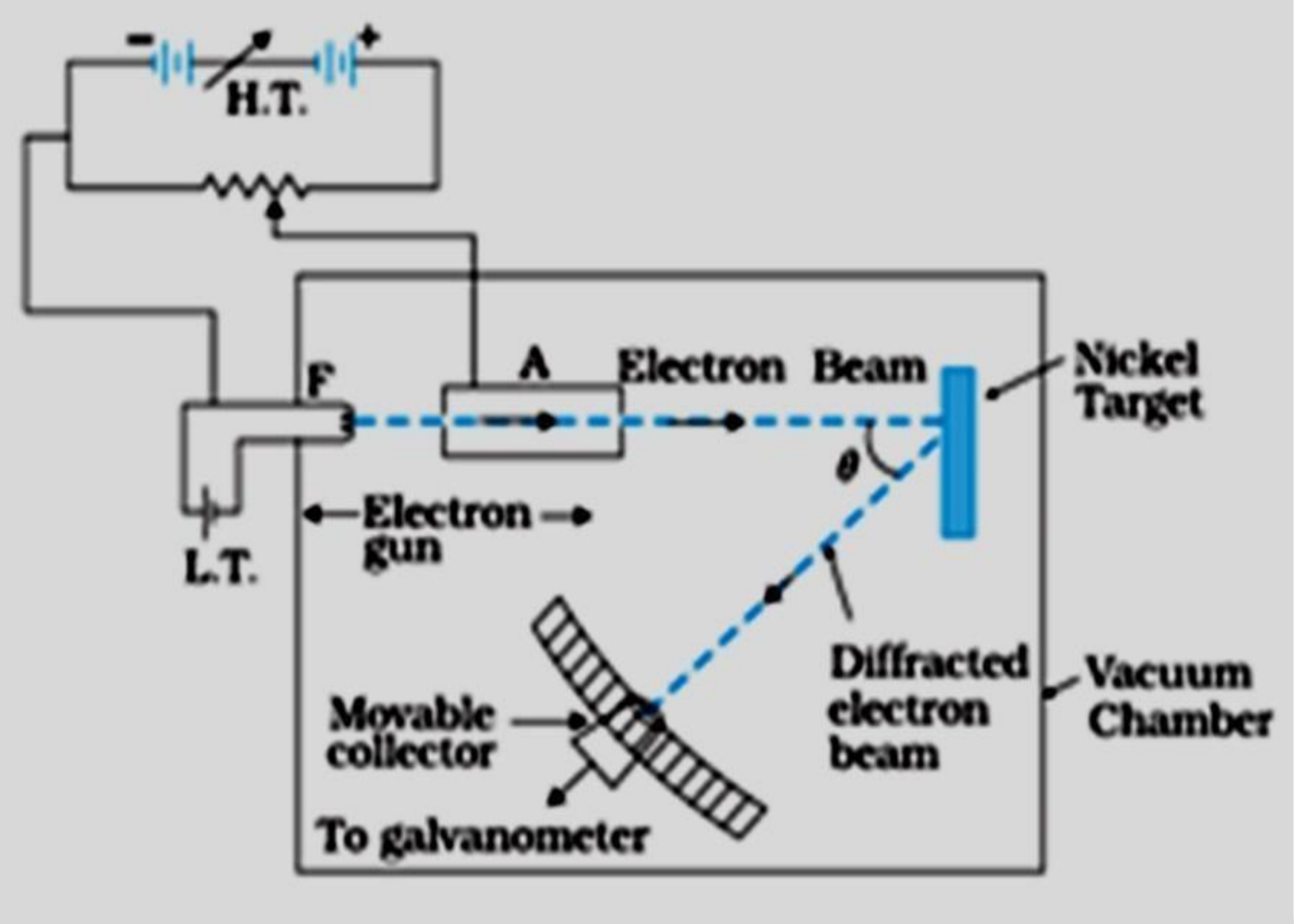
Louis de Broglie proposes that matter has wave properties and using the relation between Wavelength and Photon mass:

$$\lambda = \frac{c}{\nu} \overset{\nu=\frac{pc}{h}}{=} \frac{h}{p} \overset{p=mc}{=} \frac{h}{mc}$$

He postulate that any Particle of mass m and velocity v has an associate Wave with a Wavelength λ.



Davisson & Germer experiment

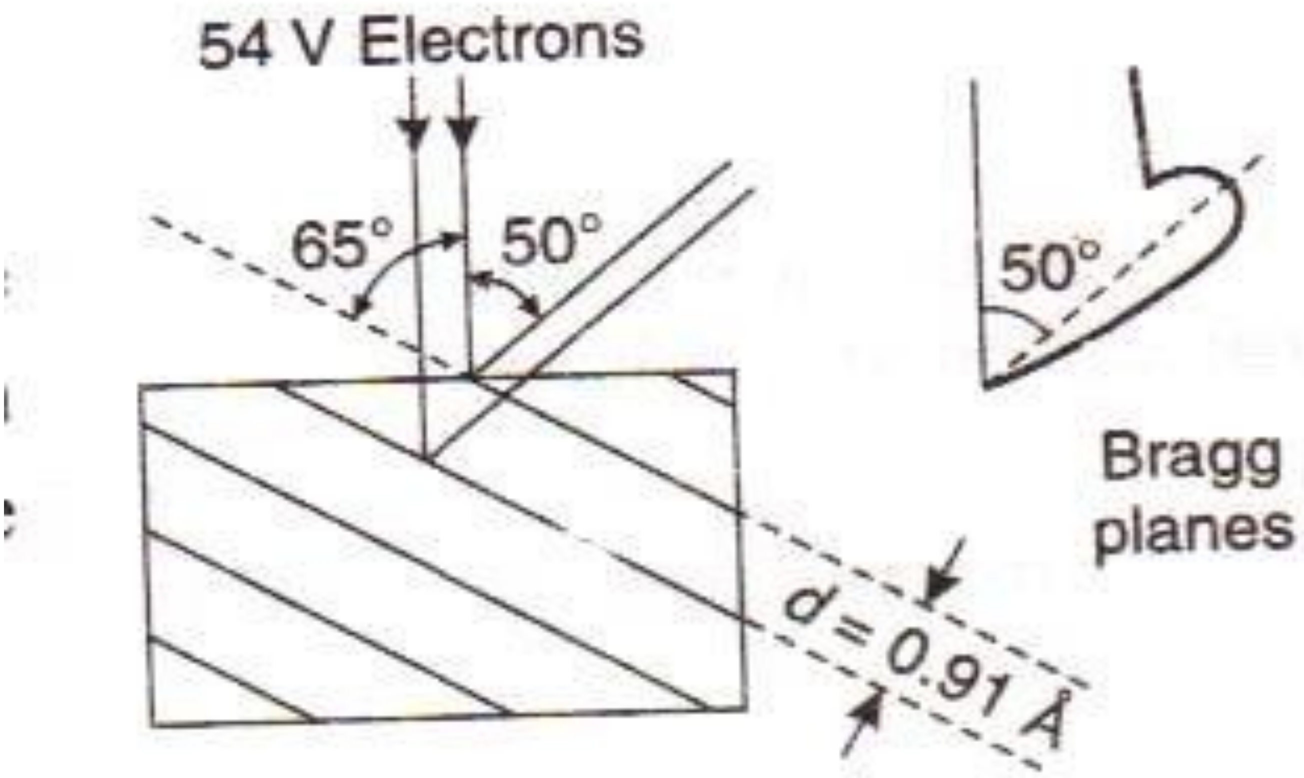


Davisson & Germer experiment

- The first experimental evidence of matter waves was given by two American physicists, Davisson and Germer in 1927. The experimental arrangement is shown in figure

- The apparatus consists of an electron gun G where the electrons are produced. When the filament of electron gun is heated to dull red electrons are emitted due to thermionic emissions. Now, the electrons are accelerated in the electric field of known potential difference. These electrons are collimated by suitable slits to obtain a fine beam which is then directed to fall on a large single crystal of nickel, known as target T..

Davisson & Germer experiment



Time independent Schrodinger equation for wave

So what can we know?

Rather than saying that a particle has a specified position and momentum, we instead describe it by a **wavefunction** which is a function of all the coordinates of the particle and of time

$$\Psi = \Psi(x, y, z, t)$$

Time independent Schrodinger equation for wave

Schrodinger developed a differential equation whose solutions yield the possible wave functions that can be associated with a particle in a given situation. This equation is popularly known as Schrodinger equation. The equation tells us how the wave function changes as a result of forces acting on the particle. One of its forms can be derived by simply incorporating the de-Broglie wavelength expression into the classical wave equation.

Time independent Schrodinger equation for wave

The total energy E of the particle is the sum of its kinetic energy k and potential energy V

i.e., $E = K + V$

But $K = mv^2$

$$E = mv^2 + V$$

$$mv^2 = E - V$$

$$m^2v^2 = 2m (E - V)$$

Time independent Schrodinger equation for wave

$$\psi = \psi_0 \, sin(\omega t - \mathrm{kx}) \text{ ------------- } (1)$$

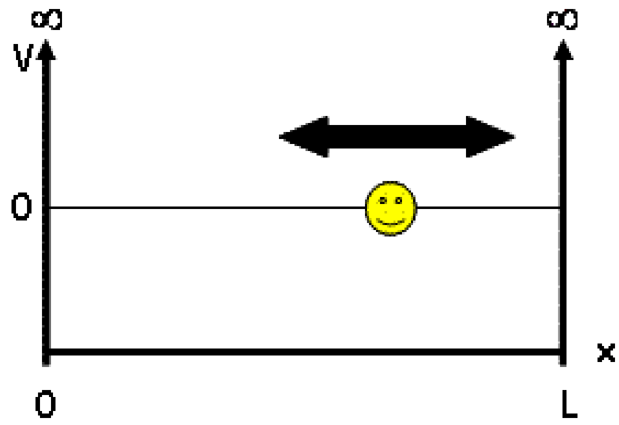
Where $\psi = \psi(x, t)$, ψ_0 is amplitude

Potential box

A particle in a 1-dimensional box is a fundamental quantum mechanical approximation describing the translational motion of a single particle conned inside an infinitely deep well from which it *cannot* escape.

The time independent Schrödinger wave equation in one dimensional case

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar}\psi = 0 \text{ -------- } (1)$$



Potential box

We are still left with an arbitrary constant 'A' in eq (7). It can be obtained by applying normalization condition i.e.; the probability of finding the particle inside the box is unity.

$$\int_0^L |\psi\, x|^2\, dx = 1$$

$$\int_0^L A^2 \sin^2\frac{n\pi x}{L}\, dx = 1$$

$$A^2 \int_0^L \frac{1}{2}\left[1 - \cos\frac{2n\pi}{l}x\right] dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{L}{2\pi n} \sin\frac{2\pi n x}{L}\right]_0^L = 1$$

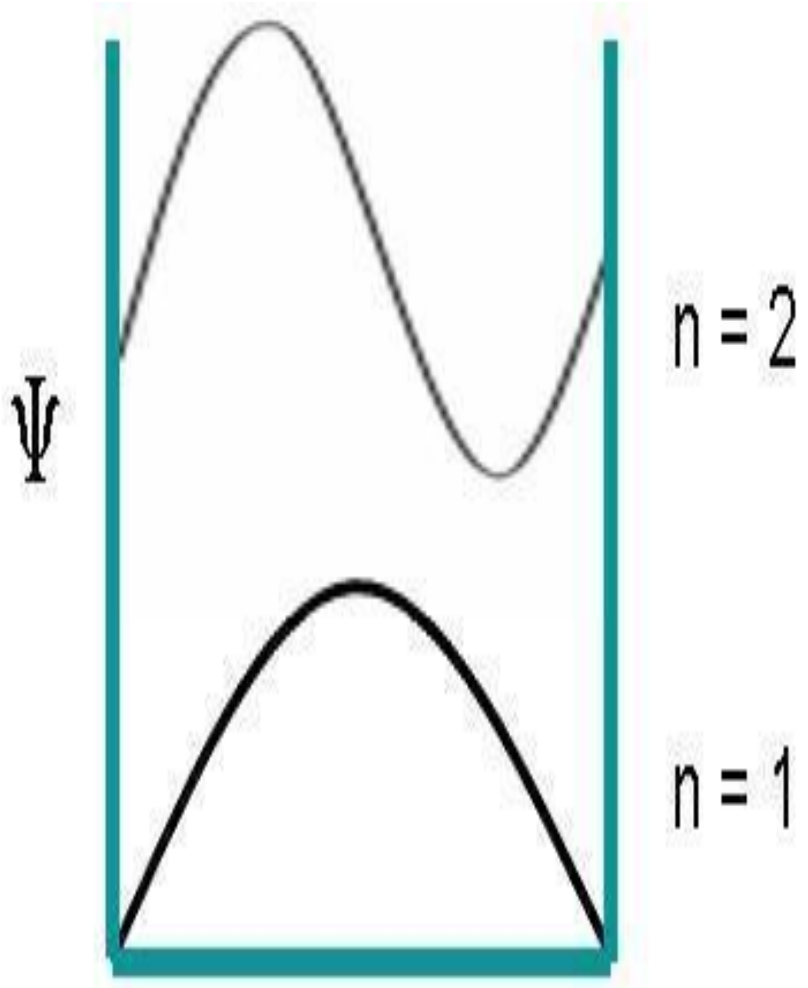
$$\Longrightarrow \frac{A^2}{2}[(L-0) - (0-0)] = 1$$

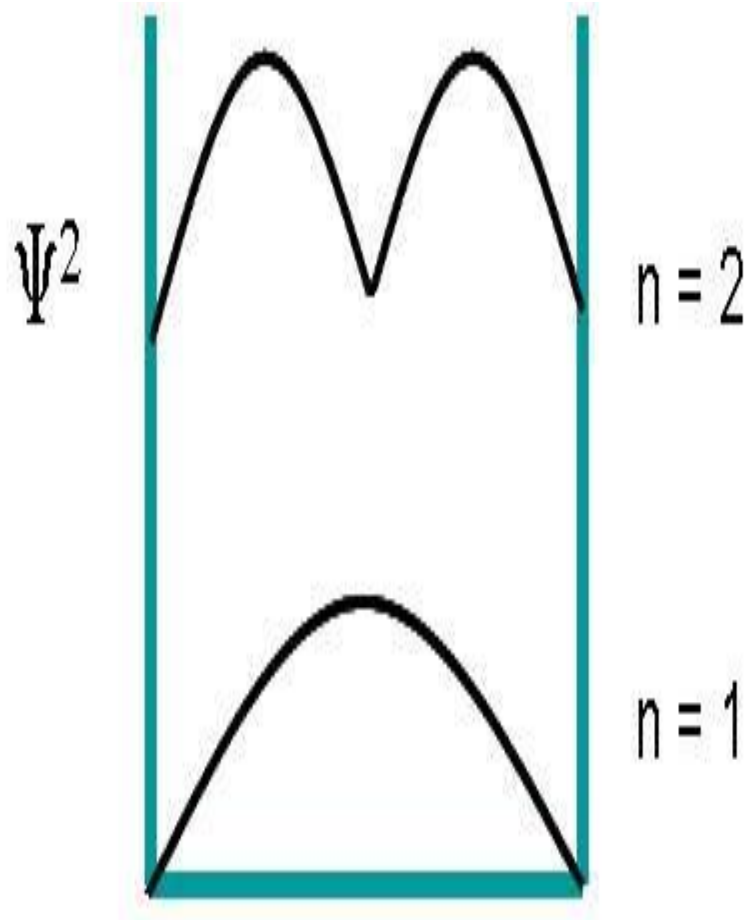
$$\frac{A^2 L}{2} = 1 \text{ or } A = \sqrt{\frac{2}{L}} \quad \text{----(10)}$$

$\therefore$ The normalized wave function is

$$\psi_n = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \quad \text{----------(11)}$$

Potential box



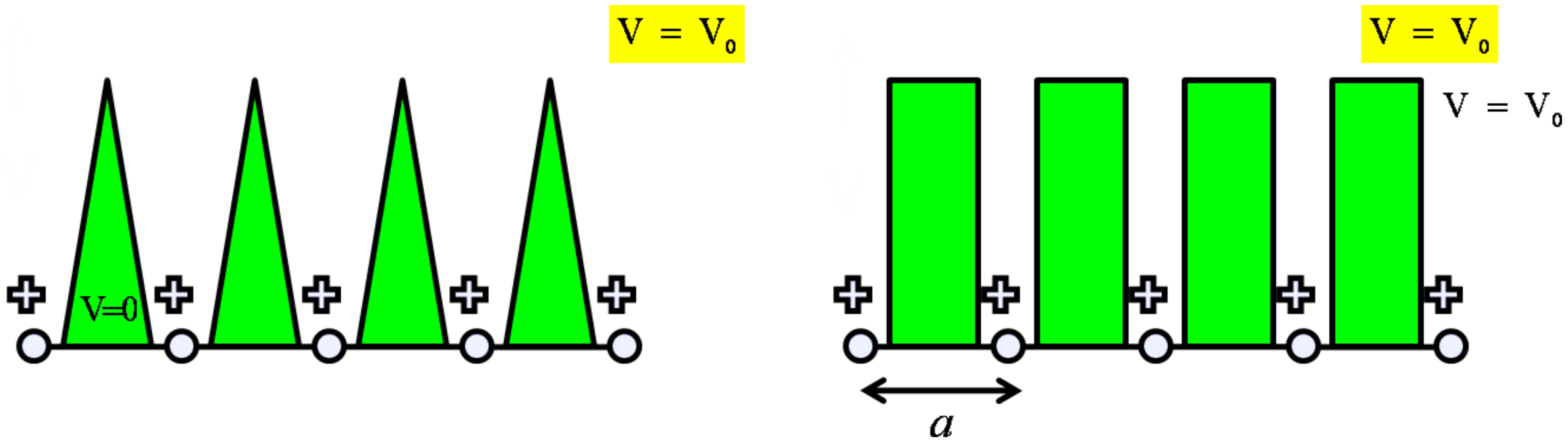


MODULE-II

INTRODUCTION TO SOLIDS AND SEMICONDUCTORS

MODULE-II

CLOs	Course Learning Outcome
CLO 5	Recollect the conductivity mechanism involved in semiconductors and calculate carrier concentrations.
CLO 6	Investigate the band structure of a solid and classify materials as metals, insulators or semiconductors and sketch a schematic band diagram for each one.

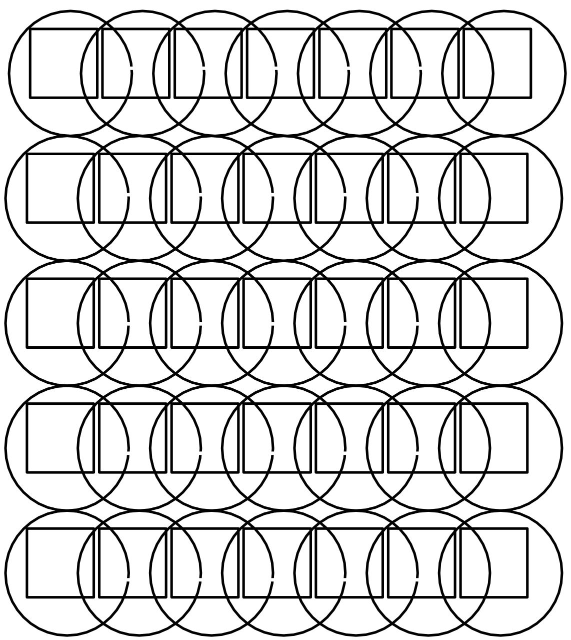


- The solution of sinusoidal type is not easily tractable, so Kronig and Penny introduced simpler model for the potential variation.

- The concept of discrete allowed electron energies that occur in a single atom and large number of allowed energies of electrons are explained by band theory of solids.

crystalline solid consists of a lattice which is composed of a large number of positive ion cores at regular intervals and the conduction electrons move freely throughout the lattice.

The variation of potential inside the metallic crystal with the periodicity of the lattice is explained by Bloch theorem.



inside metallic crystals.

ensional periodic potential in crystal.

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - V]\psi = 0$$

$$V(x) = V(x+a)$$

$$\psi(x) = U_k(x)\exp(ikx)$$

Where U_k (x) is a periodic with periodicity of a crystallattice.

$$U_k(x) = U_k(x+a)$$

$$\psi(x+Na) = U(x+Na)\exp(ik(x+Na))$$

$$\psi(x+Na) = U_k(x)\exp(ikx)\exp(ikNa)$$

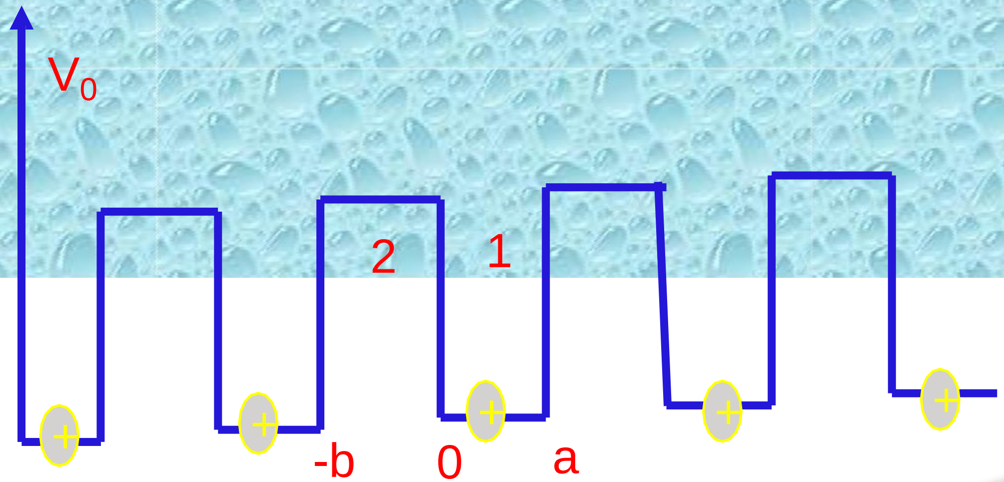
$$\psi(x+Na) = \psi(x)\exp(ikNa)$$

...to as Bloch theorem

- According to Kronig - Penney model the electrons move in a periodic potential field provided by the lattice.

- The potential of the solid varies periodically with the periodicity of space lattice.

For region 1 & 2



$$V(x) = 0 \rightarrow 0 < x < a$$

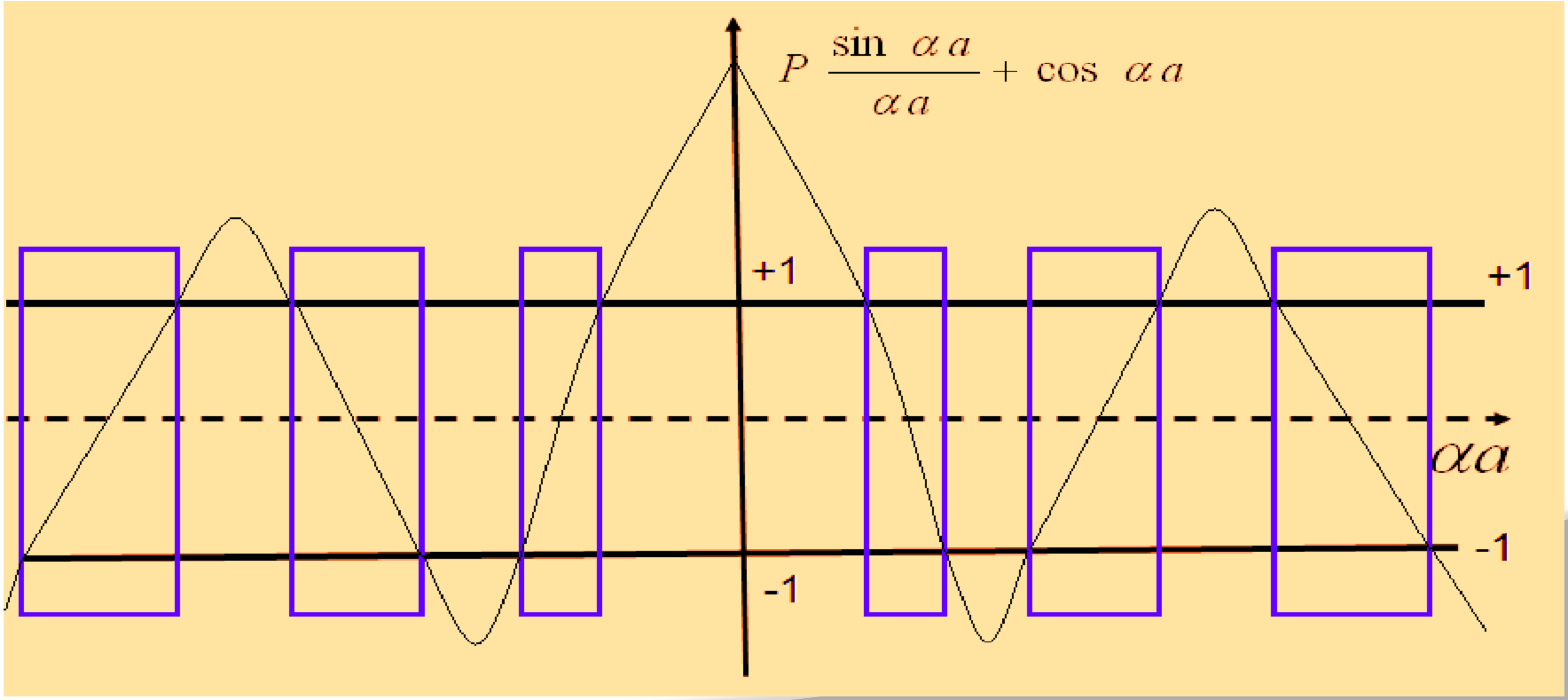
$$V(x) = v_0 \rightarrow -b < x < 0$$

$$\psi(x) = U_k(x)e_{ikx}.....(a)$$

$$\cos ka = P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a$$

$$where., P = \frac{mV_0ab}{\hbar^2}$$

$$P\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$



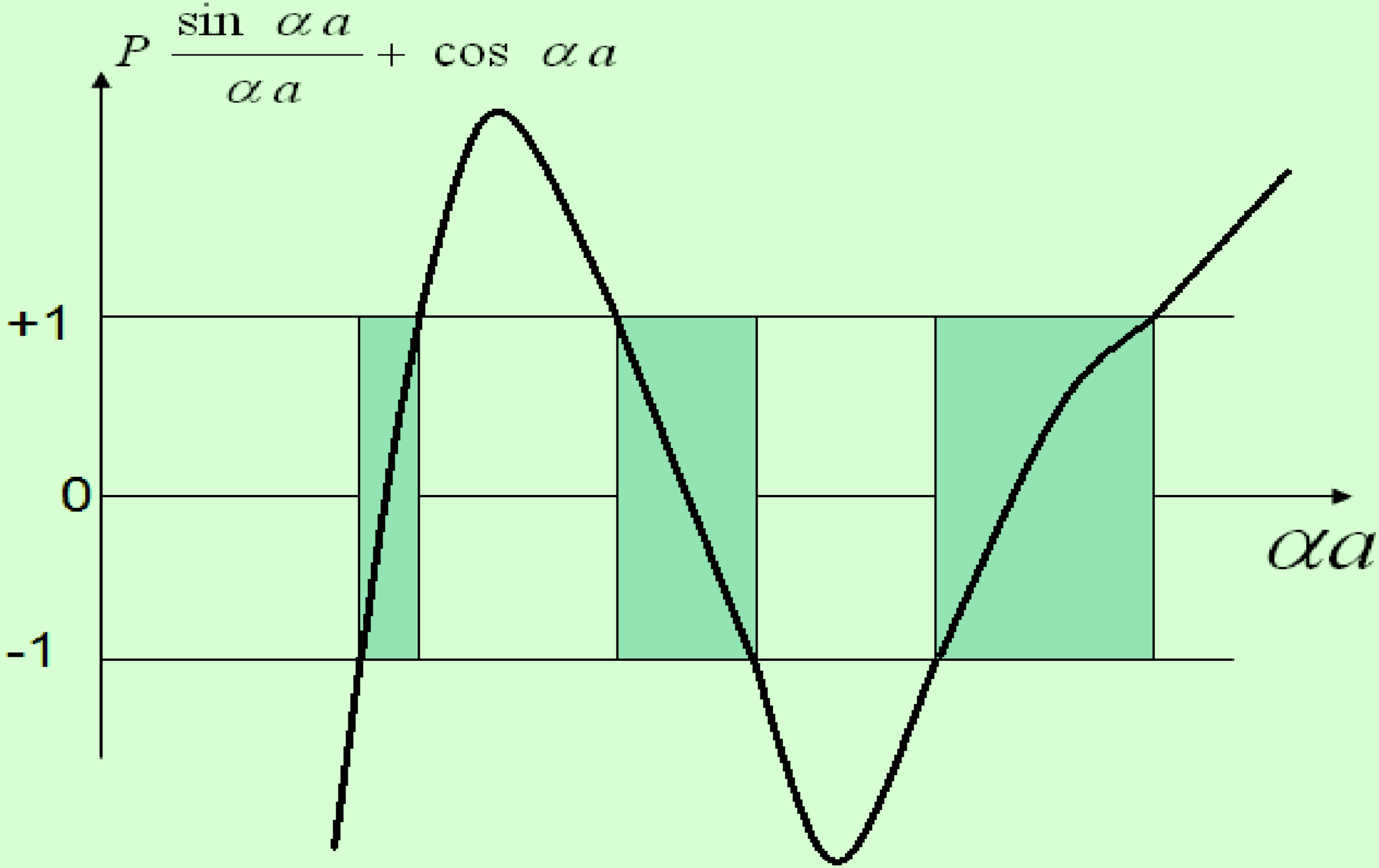
1. The motion of electrons in a periodic lattice is characterized by the bands of allowed energy bands separated by forbidden regions.

2. As the value of αa increases, the width of allowed energy bands also increases and the width of the forbidden bands decreases. i.e., the first term of equation decreases on the average with increasing αa .

3. Let us now consider the effect of varying barrier strength P. if V_0b is large ,i.e. if p is large ,the function described by the left hand side of the equation crosses +1 and -1 region as shown in figure. thus the allowed bands are narrower and the forbidden bands are wider.

when $\alpha a = \pm n\pi, \cos\alpha a = \cos Ka$ with $Ka = \pm n\pi ; K = \pm \frac{n\pi}{a}$

these values of K are points of discontinuity in the E-K curve for electrons in the crystal.


$P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a$
+1
0
-1
αa

If $P \to \infty$ the allowed band reduces to one single energy level; that is we are back to the case of discrete spectrum existing in isolated atoms.

$P \to \infty$ $p \to \infty$

then sin α $a = o$;

Or

α $a = \pm n\pi$

$$\alpha^2 = \frac{n^2 \pi^2}{a^2} = \frac{2mE}{\hbar^2}$$

0 αa

$$E = \frac{n^2 \hbar^2 \pi^2}{2ma^2} = \frac{n^2 h^2}{8ma^2}$$

Here E is independent of K

The energy levels in this case are discrete and the result is similar to the energy levels of a particle in a constant potential box of atomic dimensions.

$$P\frac{\sin\alpha a}{\alpha a}+\cos\alpha a=\cos ka$$

$P \rightarrow 0$

$$\cos\alpha a=\cos ka$$

$$\alpha=k;\alpha^2=k^2$$

$$k^2=\alpha^2=\frac{2mE}{\hbar^2}$$

$$E=(\frac{\hbar^2}{2m})k^2=(\frac{h^2}{8\pi^2 m})(\frac{2\pi}{\lambda})^2$$

$$E=(\frac{h^2}{2m})\frac{1}{\lambda^2}$$

$$E=(\frac{h^2}{2m})\frac{p^2}{h^2}=\frac{p^2}{2m}=\frac{1}{2}mv^2$$

$P \rightarrow \infty$

then sin **α**a=0;

Or

αa=±nπ

$$\alpha^2=\frac{n^2\pi^2}{a^2}=\frac{2mE}{\hbar^2}$$

$$E=\frac{n^2\hbar^2\pi^2}{2ma^2}=\frac{n^2h^2}{8ma^2}$$

Origin of energy band formation in solids

- **When we consider isolated atom, the electrons are tightly bound and have discrete, sharp energy levels.**
- **When two identical atoms are brought closer the outer most orbits of these atoms overlap and interact.**
- **If more atoms are brought together more levels are formed and for a solid of N atoms , each of the energy levels of an atom slipts into N levels of energy.**
- **The levels are so close together that they form an almost continuous band.**
- **The width of this band depends on the degree of overlap of electrons of adjacent atoms and is largest for outer most atomic electrons.**

- **The energy bands in solids are important in determining many of physical properties of solids. The allowed energy bands**

(1) Valance band

(2) Conduction band

- **The band corresponding to the outer most orbit is called conduction band and the next inner band is called valence band. The gap between these two allowed bands is called forbidden energy gap.**

Classifications of materials into Conductors, Semiconductors & Insulators:

- On the basis of magnitude of forbidden band or energy gap the solids are classified into insulators, semiconductors and conductors.

Insulators:

- In case of insulators, the forbidden energy band is very wide as shown in figure.

- Due to this fact the electrons cannot jump from valance band to conduction band.

Semi conductors:

- In semi conductors the forbidden energy (band) gap is very small as shown in a figure.

- Ge and Si are the best examples of semiconductors.

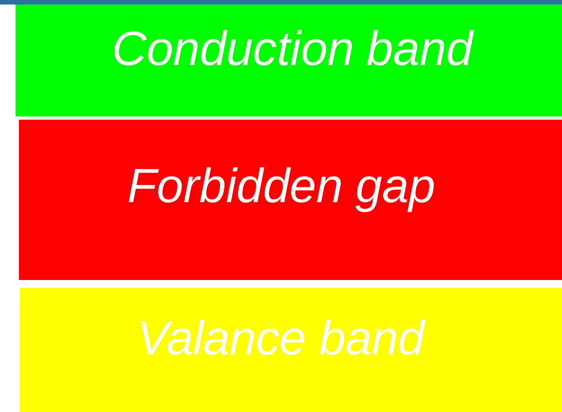
- Forbidden (band) is of the order of 0.7ev & 1.1ev.

Conductors:

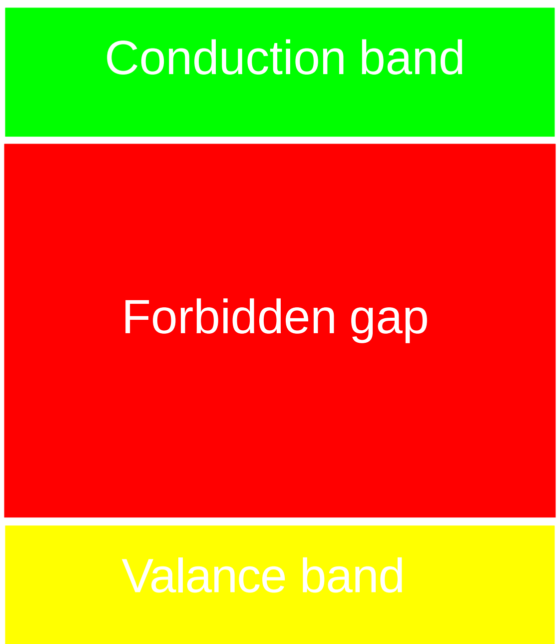
- In conductors there is no forbidden gap. Valence and conduction bands overlap each other as shown in figure above.

- The electrons from valance band freely enter into conduction band.

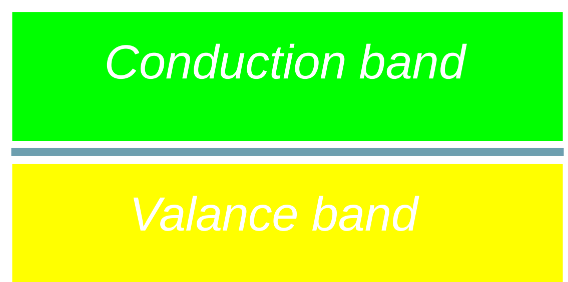
SEMI CONDUCTORS



INSULATORS



CONDUCTORS



SEMICONDUCTOR PHYSICS

INTRINSIC

1) Pure semiconductors.

- Ex: Si, Ge.

2)They behaves as a insulators at OK

3)Density of electrons in C.B=Density of holes in V.B

4)Fermi level is lies between C.B & V.B

EXTRINSIC

1)Impure semiconductors

2)Impurities(3^{rd} and 5^{th} GROUP added to convert extrinsic)

3) P.TYPE(3^{rd}Group(B, Al) (Acceptors)

4)N.type(5^{th}Group,P,As,Sb) (Donors)

III	IV	V
Boron (B)	Carbon (C)	
Aluminum (Al)	Silicon (Si)	Phosphorous (P)
Galium (Al)	Germanium (Ge)	Arsenic (As)

Acceptors (P-TYPE)

Donars (N-TYPE)

- **Semiconductors are the materials whose conductivity (or resistivity) values lie between that of insulators and conductors.**

- **There is a large gap between valance band and conduction band of an insulator where as both the bands are partially overlapped in conductors at room temperature. The gap between both the bands of a semiconductor is small.**

- The electrical conductivity of a conductor decreases with increase of temperature. But, the electrical conductivity of a semiconductor increases with increase of temperature.

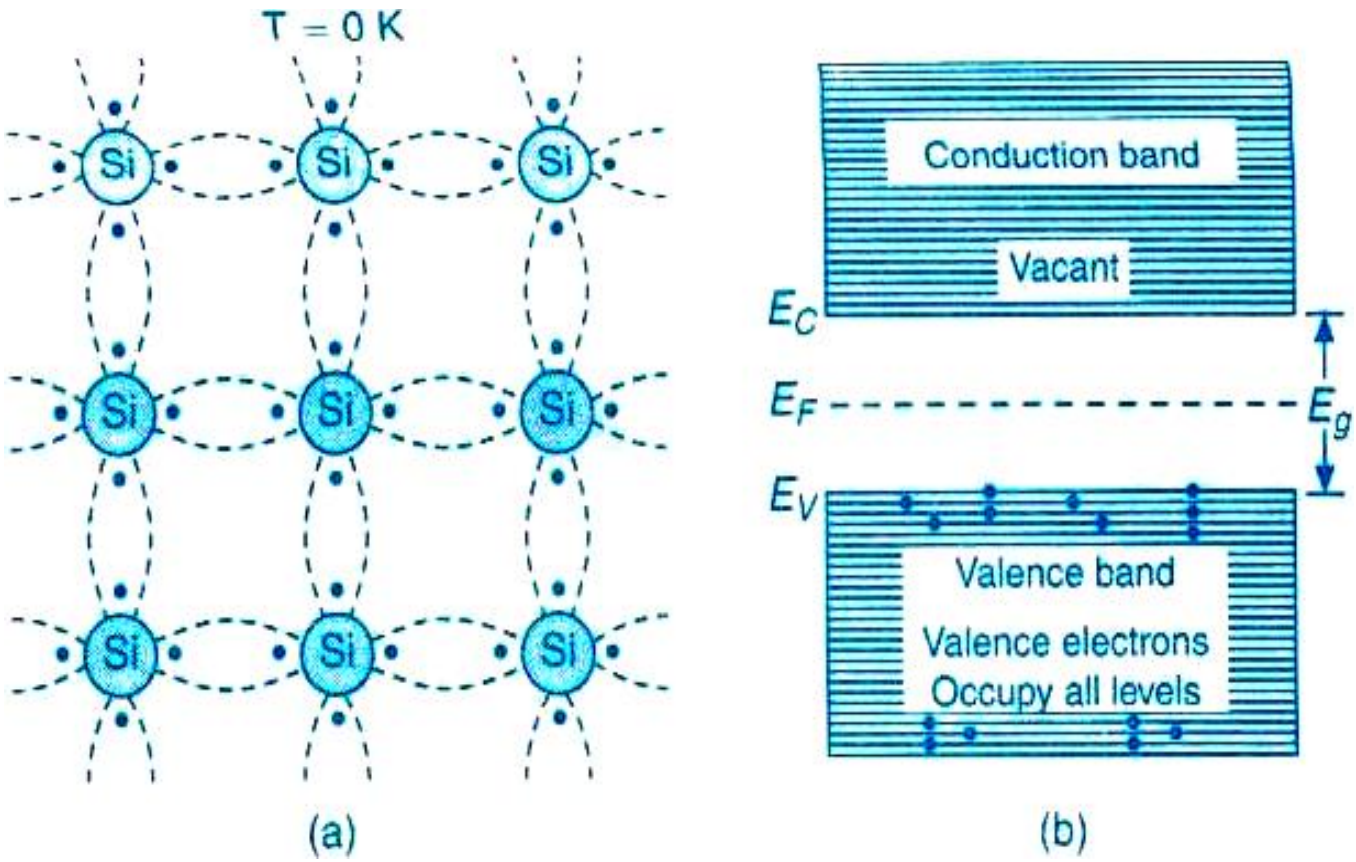
- The semiconductor in its pure form (with no impurities) is called intrinsic semiconductor or pure semiconductor.

- Germanium and silicon are the best examples of Intrinsic semi conductors.

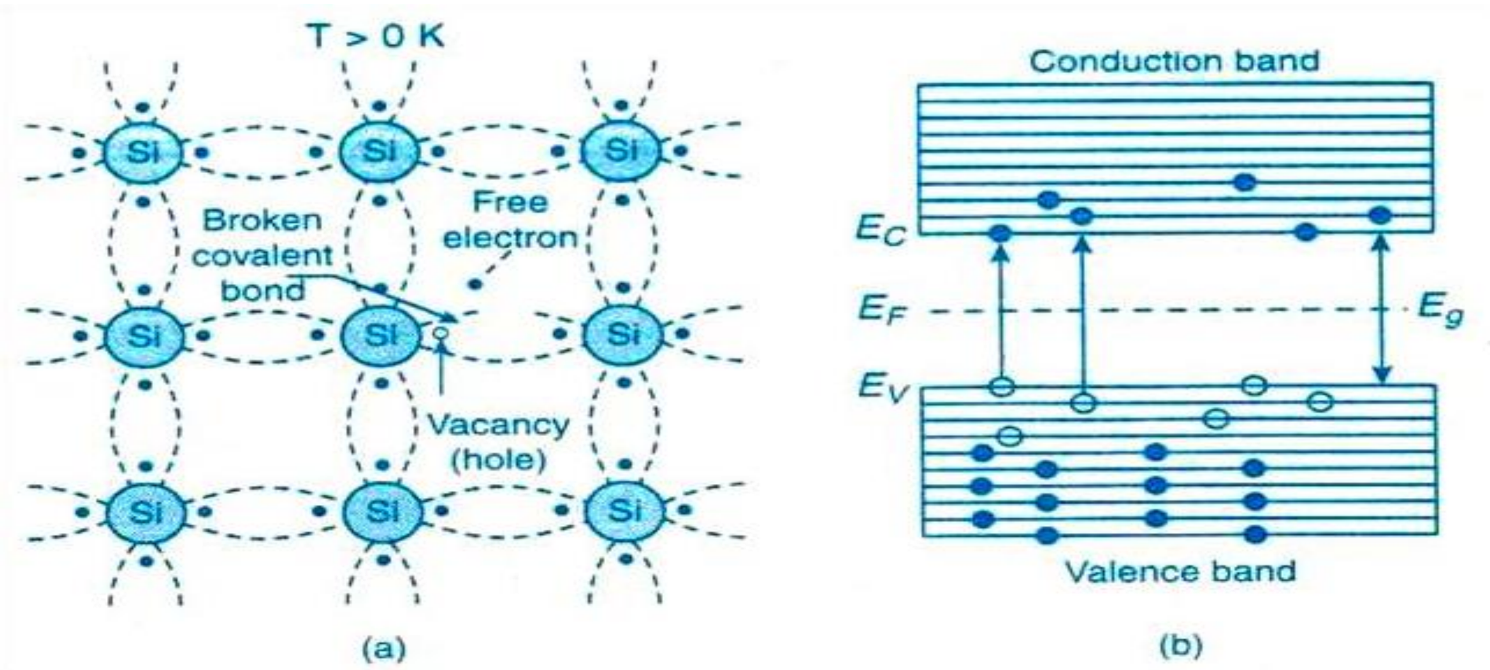
- In a pure silicon or germanium crystal, each atom is surrounded by four identical atoms. Each atom possesses four valence electrons and forms covalent bonds with its four neighboring atoms to attain closed shell configuration.

- At zero Kelvin, there are no electrons in the conduction band and hence electrical conductivity of pure semiconductor is zero.

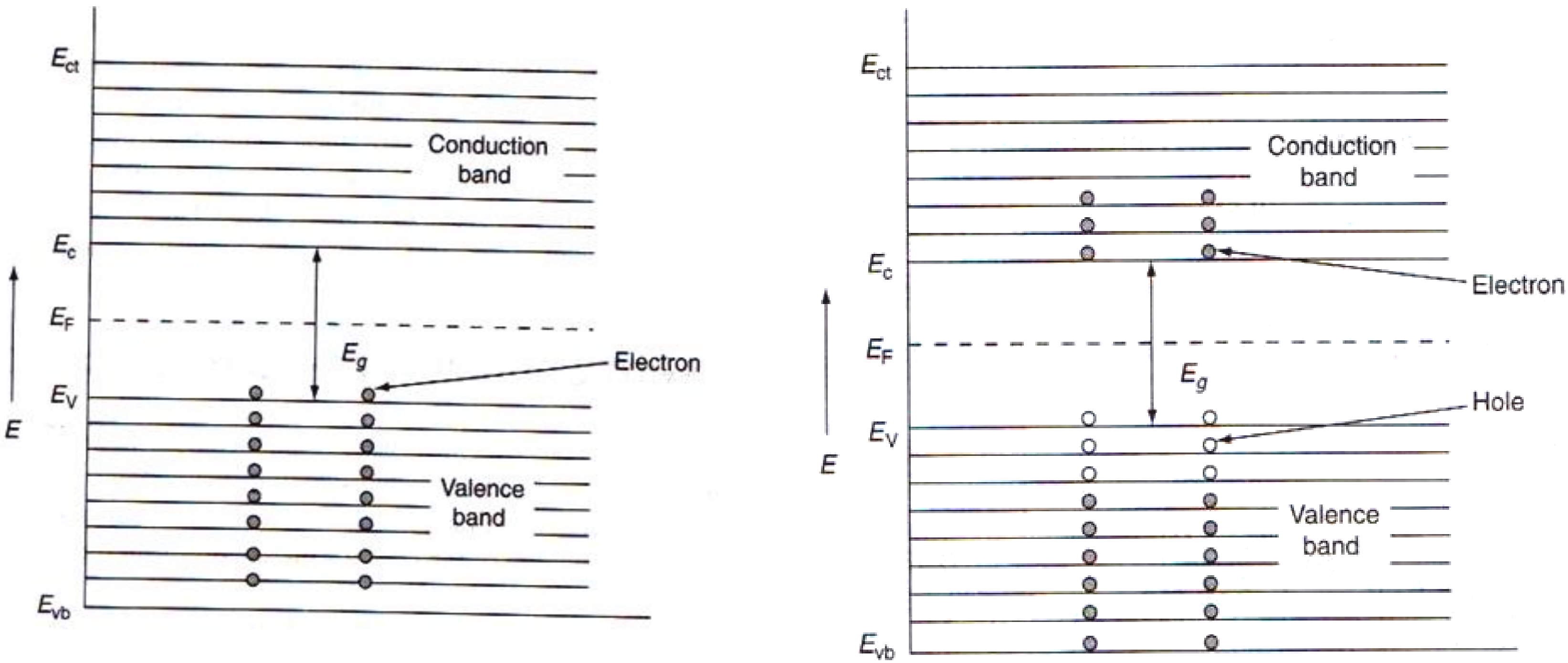
- As the temperature increases, covalent bonds break, electrons become free to conduct electricity.


T = 0 K
Si
Conduction band
Vacant
E_C
E_F
E_g
E_V
Valence band
Valence electrons
Occupy all levels
(a)
(b)

- **As the covalent bonds are broken, electrons move to conduction band, leaving behind holes in the valence band.**
- **Therefore, the number of electrons in the conduction band of a semiconductor is <u>equal to</u> the number of holes in the valence band.**


T > 0 K
Si
Broken
covalent
bond
Free
electron
Vacancy
(hole)
(a)
Conduction band
E_C
E_F
E_V
E_g
Valence band
(b)

semiconductor



(b)

at $T = 0K$ and

(b) Energy band diagram of silicon at $T > 0K$

Electron concentration in the conduction band of an intrinsic semiconductor

The expression for Electron concentration in the conduction band of an intrinsic semiconductor is given by

$$n = 2\left[\frac{2\pi m_e^{*} kT}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_C}{kT}\right] \rightarrow (1)$$

Hole concentration in the valence band of an intrinsic semiconductor

- **The expression for Electron concentration in the conduction band of an intrinsic semiconductor is given by**

$$p = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} \exp\left[\frac{E_v - E_F}{kT}\right] \rightarrow (2)$$

Fermi level in an intrinsic semiconductor

- **In an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valence band.**

$$n = p \qquad \rightarrow (3)$$

i.e., On substituting eq (1) and eq (2) in eq(3) and simplifying, we get

$$E_F = \frac{E_c + E_v}{2} + \frac{3}{4} \ln\left(\frac{m_h^*}{m_e^*}\right) \qquad \rightarrow (4)$$

From eq (4), when T = 0K, then

$$E_F = \frac{E_c + E_v}{2} \rightarrow (5)$$

- Hence, in an intrinsic semiconductor, at 0K, Fermi level is located exactly half-way between the top of the valence band and the bottom of the conduction band.

- Generally, $m_h^* > m_e^*$ and as the temperature increases, Fermi energy E_F increases slightly.

Doping- Extrinsic semiconductors

- **Generally, the electrical conductivity of an intrinsic semiconductor is low. It can be increased by adding a small quantity of impurities to an intrinsic semiconductor.**

- **"The process of adding impurities to an intrinsic semiconductor to enhance the electrical conductivity "is called <u>doping</u>.**

- **"The semiconductor formed after doping" is called extrinsic semiconductor.**

- **Extrinsic semiconductors are of <u>two</u> types namely, p-type and n-type.**

n-type semiconductor

- **When pentavalent impurity atom like antimony or phosphorous or arsenic is added to a pure silicon (or germanium) crystal, four of its five valence electrons form covalent bonds with its neighboring four silicon atoms.**

- **The fifth electron of each impurity atom is loosely held with its parent atom and can be easily detached by supplying a little amount of energy.**

- Energy levels are introduced to all the fifth electrons of impurity atoms just below the bottom of the conduction band. These levels are called donor levels and the impurities added are called donor impurities.

- At room temperature and temperatures nearer to room temperature, almost all the electrons in the donor levels jump to conduction band but only a few electrons from valence band jump in to conduction band, leaving behind few holes in valences band.

- Therefore, electrons are the majority charge carriers and holes are the minority charge carriers.

- As there are excess conducting electrons, "the semiconductor thus formed after doping pentavalent impurities to a pure semiconductor" is called <u>n-type</u> semiconductor.

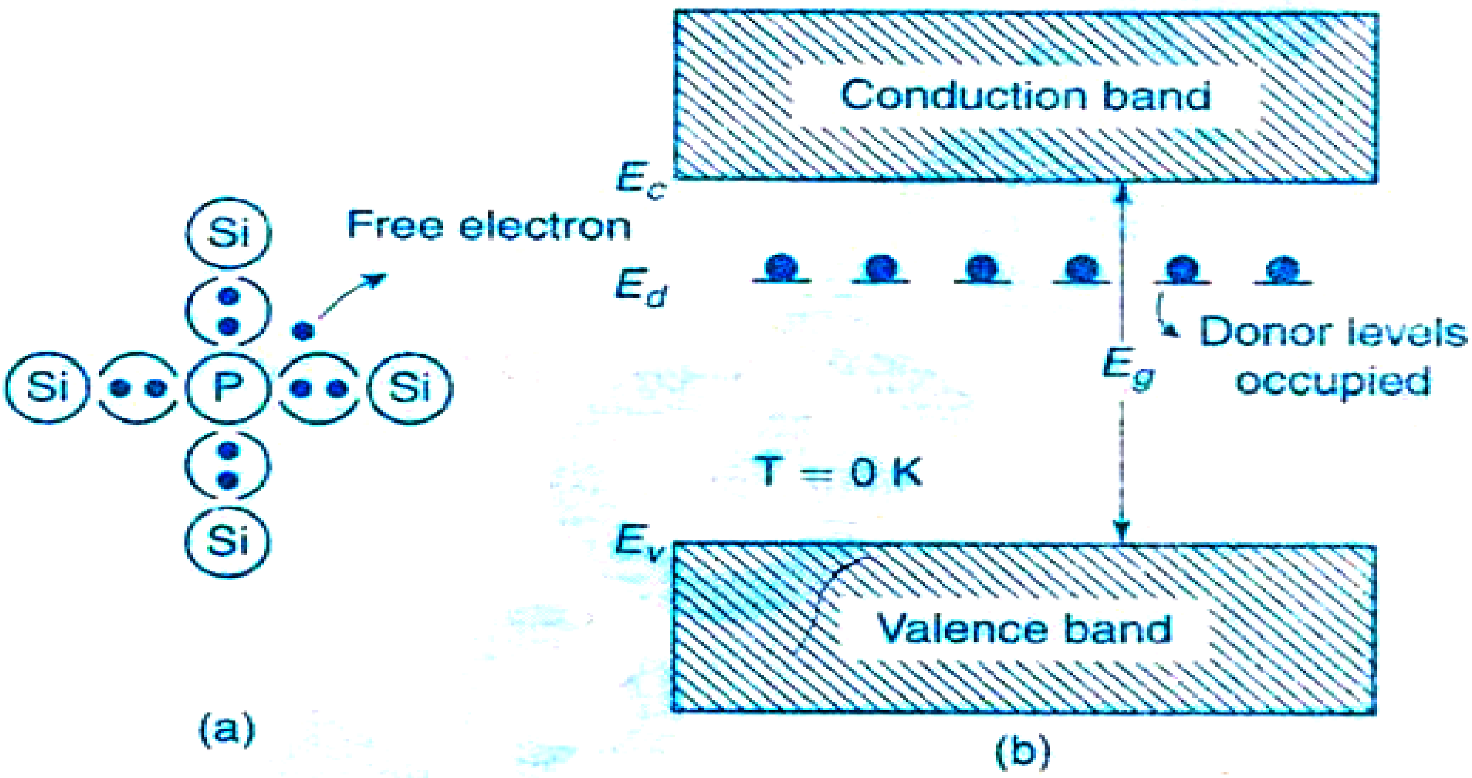


Figure 4. (a) Representation of n- type silicon at T = 0K and (b) Energy band diagram at T = 0 K

Carrier concentration in n-type semiconductor

- Electrons in conduction band are the majority charge carriers in an n-type semiconductor.

- The expression for electron concentration in the conduction band of an intrinsic semiconductor is given by

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} \exp\left[\frac{E_F - E_C}{kT}\right] \qquad \rightarrow (1)$$

- **But, in an n-type semiconductor, concentration of electrons in conduction band is equal to concentration of empty donors.**

- **The concentration of empty donors in an n-type semiconductor**

$$= \frac{N_d}{1 + e^{\left(1 + \frac{E_F - E_d}{kT}\right)}} \rightarrow (2)$$

Where N_d is the concentration of donor levels

$$\therefore 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{\left[\frac{E_F - E_c}{kT}\right]} = \frac{N_d}{1+e^{\left(1+\frac{E_F - E_d}{kT}\right)}} \rightarrow (3)$$

After simplification, we get

$$E_F = \frac{E_c + E_d}{2} + KT \ln\left[\frac{\left(\frac{N_d}{2}\right)^{\frac{1}{2}}}{\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/4}}\right] \rightarrow (4)$$

It is clear from eq(4) that, when T= 0K,

$$E_F = \frac{E_c + E_d}{2} \qquad \rightarrow (5)$$

i.e. At 0K, the Fermi level of an n-type semiconductor lies exactly half way between the bottom of the conduction band and the donor levels. But, as T increases, Fermi level falls.

Substituting eq (4) in eq (1), we get the expression for carrier concentration in the n-type semiconductor as

$$n = \left(2N_d\right)^{1/2}\left[\frac{2\pi m_e^{*}kT}{h^2}\right]^{3/4}\exp\left[\frac{E_d - E_C}{2kT}\right] \rightarrow (6)$$

P - TYPE SEMICONDUCTOR

- When trivalent impurity atoms like boron or aluminium or gallium are added to pure silicon (or germanium crystal), each impurity atom forms three covalent bonds with three of its four neighboring silicon atoms. There is deficiency of electron to form fourth covalent bond and attain closed shell configuration.

- Energy levels are created to the electron deficiencies just above the top of the valence band. These levels are called acceptor levels.

- At around room temperatures, many electrons nearer to the top of the valence band jump into acceptor levels and only few electrons jump into conduction band, leaving behind holes in the valence band.

- Therefore, the majority charge carriers are holes in the valence band and minority charge carriers are a few electrons that are present in the conduction band.

- As there are excess holes, "the semiconductor thus formed after doping trivalent impurities to a pure semiconductor" is called p-type semiconductor and the impurities are called acceptor impurities.

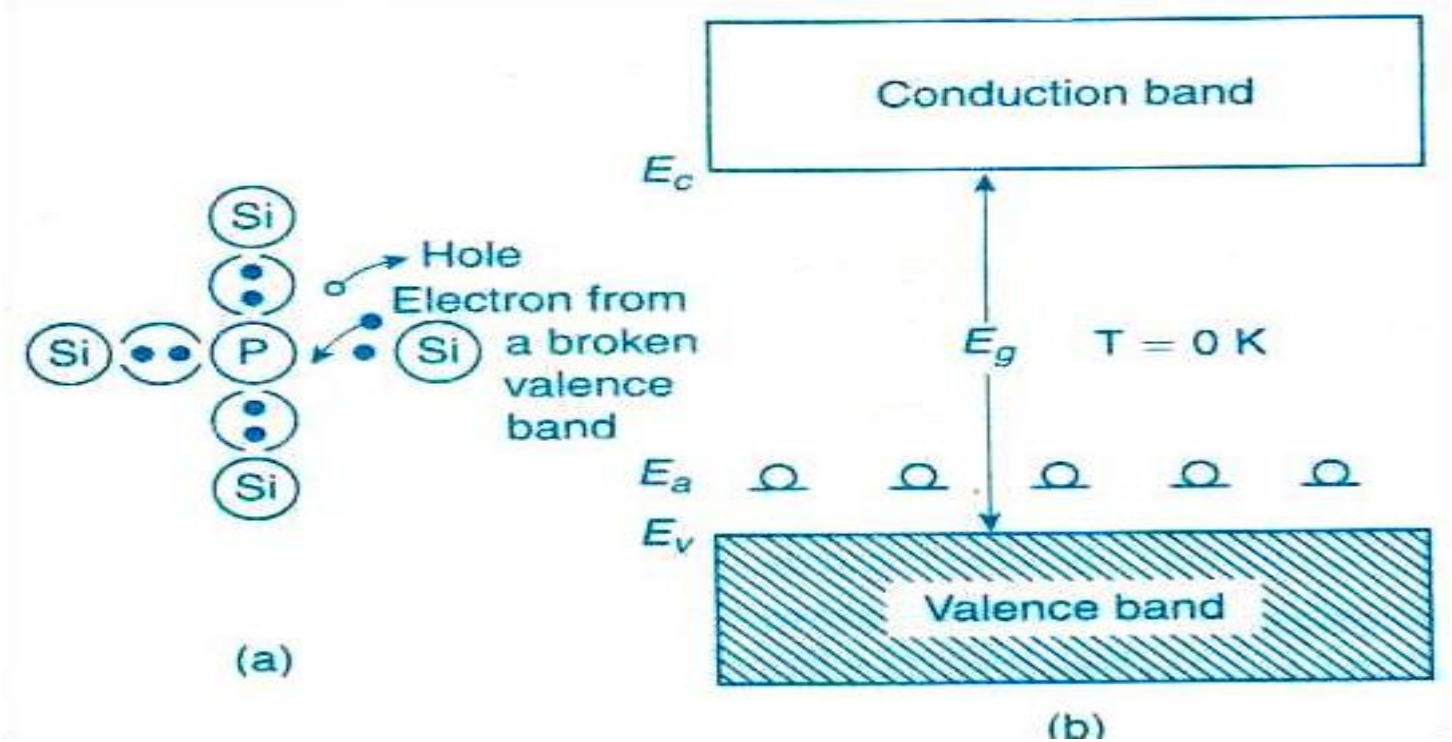


Figure 5 (a) Representation of p- type silicon at T = 0K and (b) Energy band diagram at T = 0 K

- **Holes in the valence band are the majority charge carriers in a p-type semiconductor.**

- **The expression for hole concentration in the valence band of an intrinsic semiconductor is given by**

$$p = 2\left[\frac{2\pi m_h^{*} kT}{h^2}\right]^{3/2} \exp\left[\frac{E_V - E_F}{kT}\right] \rightarrow (1)$$

But, the concentration of electrons in the acceptor levels

$$= N_a \, e^{\left[\frac{E_F - E_c}{kT}\right]} \qquad \rightarrow (2)$$

where N_a is the concentration of acceptor levels.

- Since the acceptor level is very nearer to the valence band, the effect of conduction band is negligible. So, the number of electrons in the acceptor levels is equal to the number of holes in the valence band.

$$\therefore \quad 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} \exp\left[\frac{E_V - E_F}{kT}\right] = N_a e^{\left[\frac{E_F - E_c}{kT}\right]} \rightarrow (3)$$

On further simplification, we get

$$E_F = \frac{E_a + E_v}{2} - KT \ln\left[\frac{\left(\frac{N_a}{2}\right)^{1/2}}{\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/4}}\right] \rightarrow (4)$$

clear from eq (4) that, when T = 0K,

$$E_F = \frac{E_a + E_v}{2} \rightarrow (5)$$

- Therefore, the Fermi level of a p-type semiconductor lies exactly half-way between the acceptor levels and top of the valence band. But , as T increases, the Fermi level rises.

$$p = (2N_a)^{1/2} \left[\frac{2\pi m_h^{*} kT}{h^2} \right]^{3/4} e^{\left[\frac{E_v - E_a}{2kT} \right]} \rightarrow (6)$$

Hall Effect

When a material carrying current is subjected to a magnetic field in a direction perpendicular to the direction of current, an electric field is developed in a direction perpendicular to both the directions of magnetic field and current. This phenomenon is called Hall effect and the generated voltage is called Hall voltage.

- It was generally believed that the critical current in solids is due to the flow of only electrons.

- But, in semiconductors, the external effect of the current is due to the currents of both negative and positive charge carriers inside the solid.

- It is not possible to assert the sign of the charges from the direction of current , because, it is the same whether positive charges flow in that direction or negative charges flow in the opposite direction.

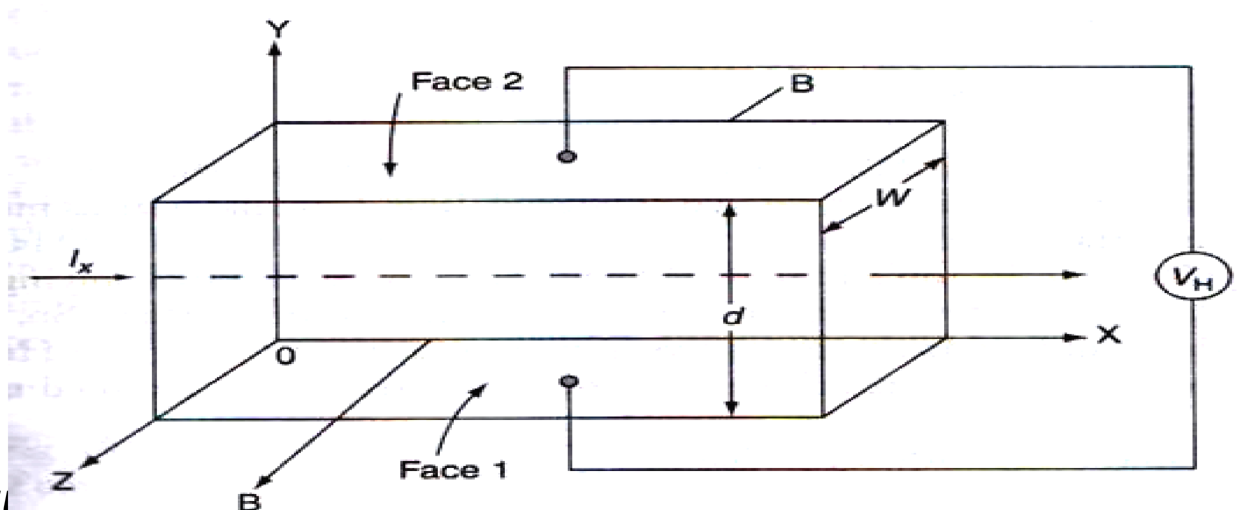
- **Hall effect discovered by E.H. Hall in 1879 helps to resolve the dilemma on the sign of the charges.**

- **When a moving charge is subjected to the influence of a magnetic field, it experiences a force called Lorentz force.**

- **<u>Fleming's left hand rule</u> states that " if the first three fingers of the left hand are stretched mutually perpendicular to each other in such a way that, the forefinger is along the magnetic Field, mIddle finger is along the current (or charge), then the direction to which Thumb is pointing indicates the direction in which the charge experiences a Thrust".**

- Consider a semiconducting specimen or a metallic specimen carrying a current along the X-direction.

- Let the specimen be subjected to magnetic field along the Z-direction. Then a force will be developed along the Y-direction.



Fig… field perpendicularly in Hall Effect

- **If the specimen is a metal, since it has only one type of charge carriers, i.e. electrons, the electrons are forced down into the bottom surface.**

- **Therefore, the bottom surface becomes more negative compared to the upper surface. Hence, a potential difference is developed between the bottom and upper surfaces.**

- **If the specimen is an n-type semiconductor, since the electrons are the majority charge carriers, they are forced down into the bottom surface. Therefore, the upper surface becomes positive, whereas the bottom surface becomes negative.**

- **In a p-type semiconductor, the bottom surface is occupied by holes and it becomes more positive compared to the upper surface.**

- **The potential difference between the upper and lower surfaces is known as Hall voltage.**

Expression for Hall Coefficient

- **The Hall coefficient R_H is inversely proportional to the charge concentration and is negative for n-type semiconductors and positive for p-type semiconductors.**

$$R_H\,(p-type) = \frac{1}{pe} \qquad \rightarrow (1)$$

$$R_H\,(n-type) = -\frac{1}{ne} \qquad \rightarrow (2)$$

Where p and n represent carrier concentration in p-type and n-type semiconductors respectively.

- **If a magnetic field 'B' is applied to a specimen carrying current 'I' and a Hall voltage V_H is developed across it, then the Hall coefficient is given by**

$$R_H = \frac{V_H t}{BI} \qquad \rightarrow (3)$$

Applications of Hall effect

1) semiconductor, R_H is positive whereas for a p-type semiconductor it is positive. Thus, from the direction of Hall voltage developed, one can find out the type of semiconductor.

2) Calculation of carrier concentration Once Hall coefficient is measured, the carrier concentration can be obtained from

$$R_H = \frac{1}{ne}$$

$$\sigma_n = ne\mu_e \left(for\ n-type\ semiconductor \right) \Rightarrow \mu_e = \frac{\sigma_n}{ne} = \sigma_n R_H$$

$$\sigma_p = pe\mu_p \left(for\ p-type\ semiconductor \right) \Rightarrow \mu_p = \frac{\sigma_p}{pe} = \sigma_p R_H$$

$$B = \frac{V_H t}{IR_H}$$

MODULE-III

LASERS AND FIBER OPTICS

MODULE-III

CLOs	Course Learning Outcome
CLO 7	Understand the basic principles involved in the production of Laser light and also real-time applications of lasers.
CLO 8	Recollect basic principle, construction, types and attenuation of optical fibers.
CLO 9	Understand the importance of optical fibers in real-time communication system.

LASERS

A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation.

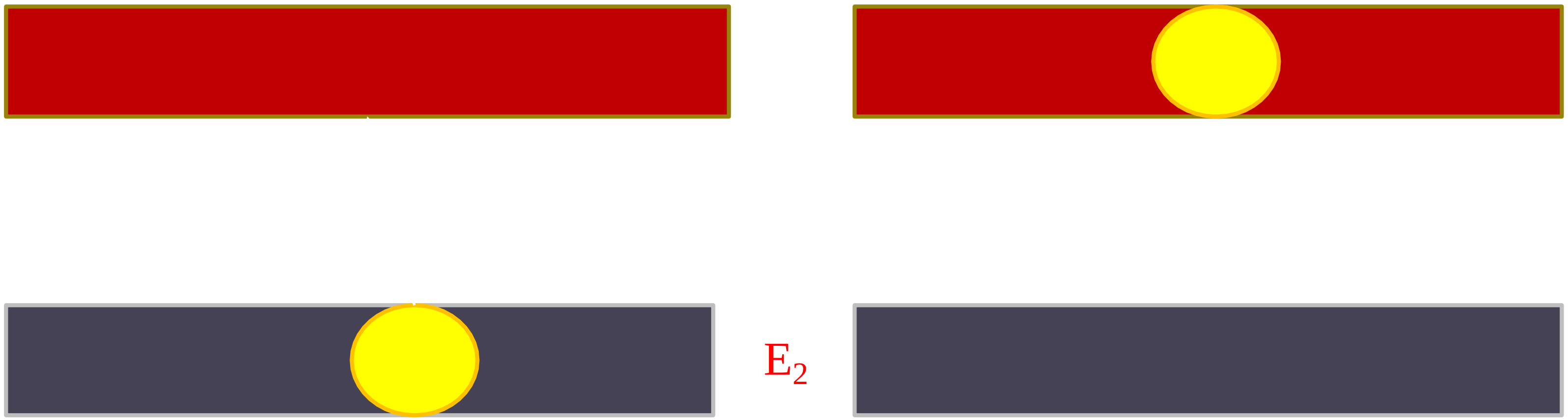
The term “LASER" originated as an acronym for "light amplification by stimulated emission of radiation".

Stimulated emission was first used by Townes and Schawlov in USA & Bosov & Prokhrov in USSR.

Maiman demonstrated the first Laser in 1960.

surrounding, the emission of light is the result of:

Absorption: If a photon of energy $h\nu_{12}(E_2\text{-}E_1)$ collides with an atom present in the ground state of energy $E_{1,}$ then the atom completely absorbs the incident photon and makes transition to excited state E_2.



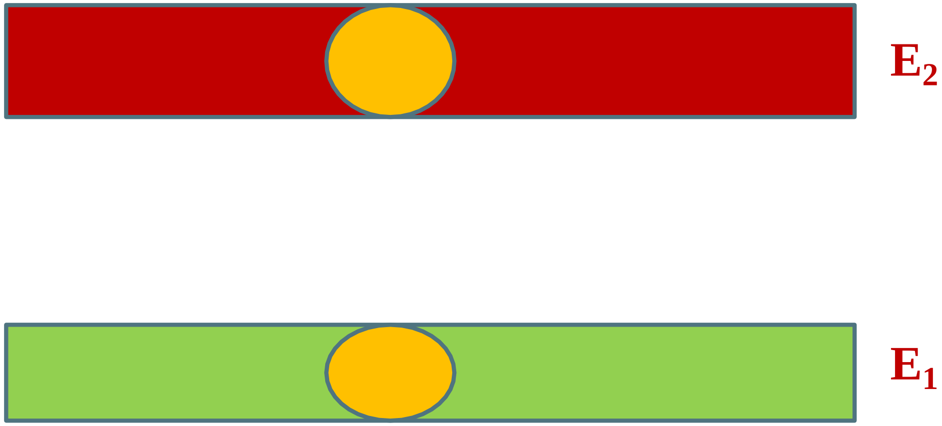
Before Absorption **After Absorption**

Spontaneous emission

atom initially present in the excited state makes transition voluntarily on its own ,without any aid of external stimulus or an agency , to the ground state and emits a photon of energy

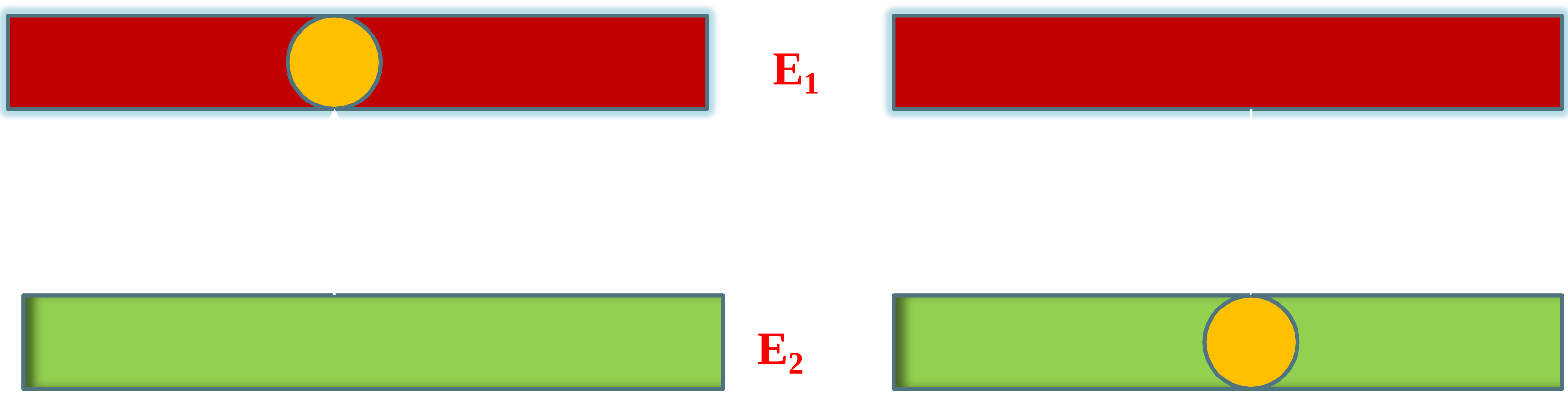
$h\nu=E_2-E_1$. The period of stay of the atom (electron) in the excited state is called its life time.

This process of emission of light is called spontaneous emission.



Stimulated Emission

A photon having energy $h\nu_{12}$ (E_2-E_1) impinges on an atom present in the excited state and the atom is stimulated to make transition to the ground state. This gives off a photon of energy $h\nu_{12}$. The emitted photon is in phase with the incident photon. These are coherent. This type of emission is known as stimulated emission.



Difference between Spontaneous and Stimulated Emission of radiation

Spontaneous Emission of Radiation	Stimulated Emission of Radiation
❖ It is a Polychromatic radiation.	❖ It is a Monochromatic radiation.
❖ It has less intensity.	❖ It has High intensity.
❖It has less directionality and more angular spread during propagation.	❖It has high directionality and so less angular spread during propagation.
❖It is Spatially and temporally incoherent radiation.	❖It is Specially and temporally coherent radiation.
❖ In this emission ,light is not amplified.	❖ In this emission , light is amplified.
❖Spontaneous emission takes place when excited atoms make a transition to lower energy level voluntarily without any external stimulation.	❖Stimulated emission takes place when a photon of energy equal to h ν_{12} (=E_2-E_1) stimulates an excited atom ,to make transition to lower energy level.
❖In a single downward transition, Spontaneous emission results in the emission of one photon.	❖In a single downward transition , Stimulated emission results in the emission of two photons.
❖Ex: Light from an ordinary electric bulb, Light from an LED.	❖ Ex: Light from a Laser source.

Characteristics of Laser light

- High Monochromaticity
- High degree of coherence
- High directionality
- High brightness

Characteristics of Laser light

1. High Monochromaticity :

- In laser radiation, all the photonsemitted between discrete energy levels will have same wavelength.
- As a result the radiation is monochromatic in nature.
- Due to the stimulated characteristic of laser light, the laser light is more monochromatic than that of a convectional light.
- laser radiation -the wavelength spread = 0.001 nm
- So it is clear that the laser radiation is highly monochromatic

Characteristics of Laser light

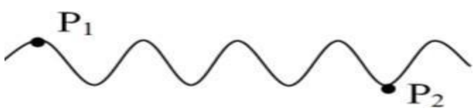
2. High degree of coherence :

- Coherence is the property of the wave being in phase with itself and also with another wave over a period of time, and space or distance. There are two types of coherence
- Temporal coherence
- Spatial coherence.
- For laser radiation all the emitted photons are in phase, the resultant radiation obeys spatial and temporal coherence.

Characteristics of Laser light

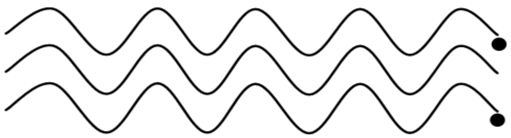
- **Temporal coherence (or longitudinal coherence):-**

The predictable correlation of amplitude and phase at one point on the wave train w .r. t another point on the same wave train, then the wave is said to be temporal coherence.



- **spatially coherence (or transverse coherence).**

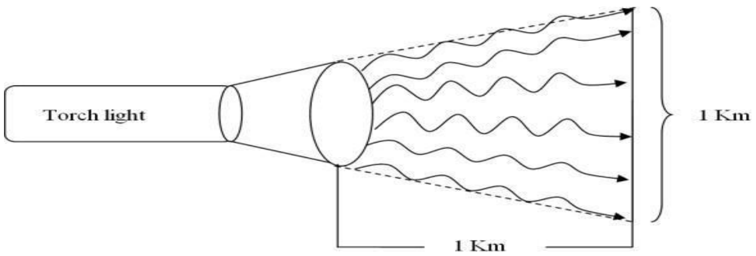
The predictable correlation of amplitude and phase at one point on the wave train w. r .t another point on a second wave, then the waves are said to be spatially coherence (or transverse coherence).

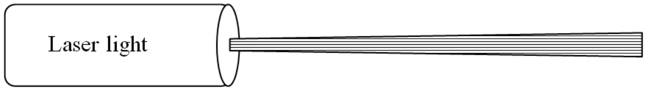


Characteristics of Laser light

3. High directionality :

- The light ray coming ordinary light source travels in all directions, but laser light travels in single direction.
- For example the light emitted from torch light travels 1km distance it spreads around 1 km wide.
- But the laser light spreads a few centimeters distance even it travels lacks of kilometer distance.





Characteristics of Laser light

- The directionality of laser beam is expressed in terms of divergence ∅

$$\emptyset = \frac{arc}{radius} = \frac{\mathrm{d2} - \mathrm{d1}}{\mathrm{s2} - \mathrm{s1}}$$

- Where d_2 and d_1 are the diameters of laser spots at distances of s_2 and s_1 respectively from laser source.
- For laser light divergence ϕ = $10^{-3} radians$.
- Since the divergence of light is very low, so we say that the laser light having highly directional.

Characteristics of Laser light

4. High Brightness:

The Laser beam is highly bright (intense) as compared to the conventional light because more light is concentrated in a small region.

It is observed that the intensity of 1mV laser light is 10,000 times brighter than the light from the sun at the earth's surface.

The number of photons coming out from a laser per second per unit area is about 10^{22} to 10^{34} where as the number of photons coming out per second per unit area of a black body at 1000K having a wavelength of 6000 is 10^{16}

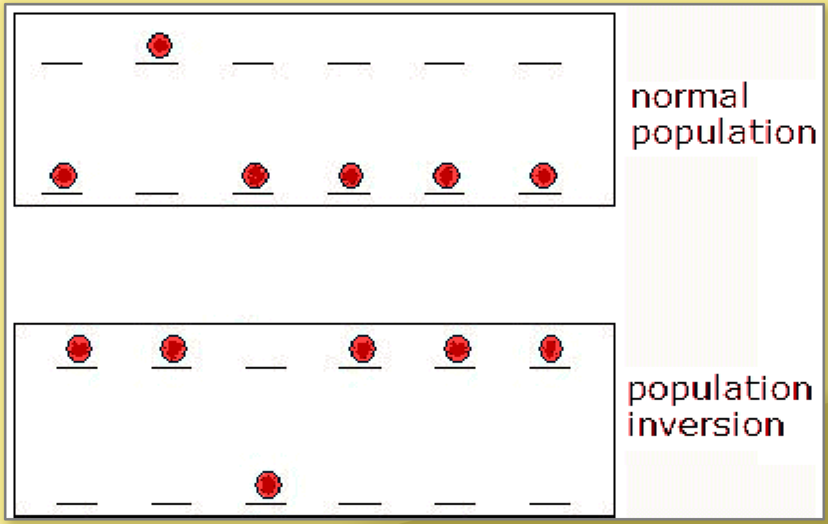
Laser light is coherent and so at a time many photons are in phase and they superimpose to produce a wave of larger amplitude.

The intensity is proportional to the square of the amplitude and hence the intensity of the resultant laser beam is very high.

Population Inversion

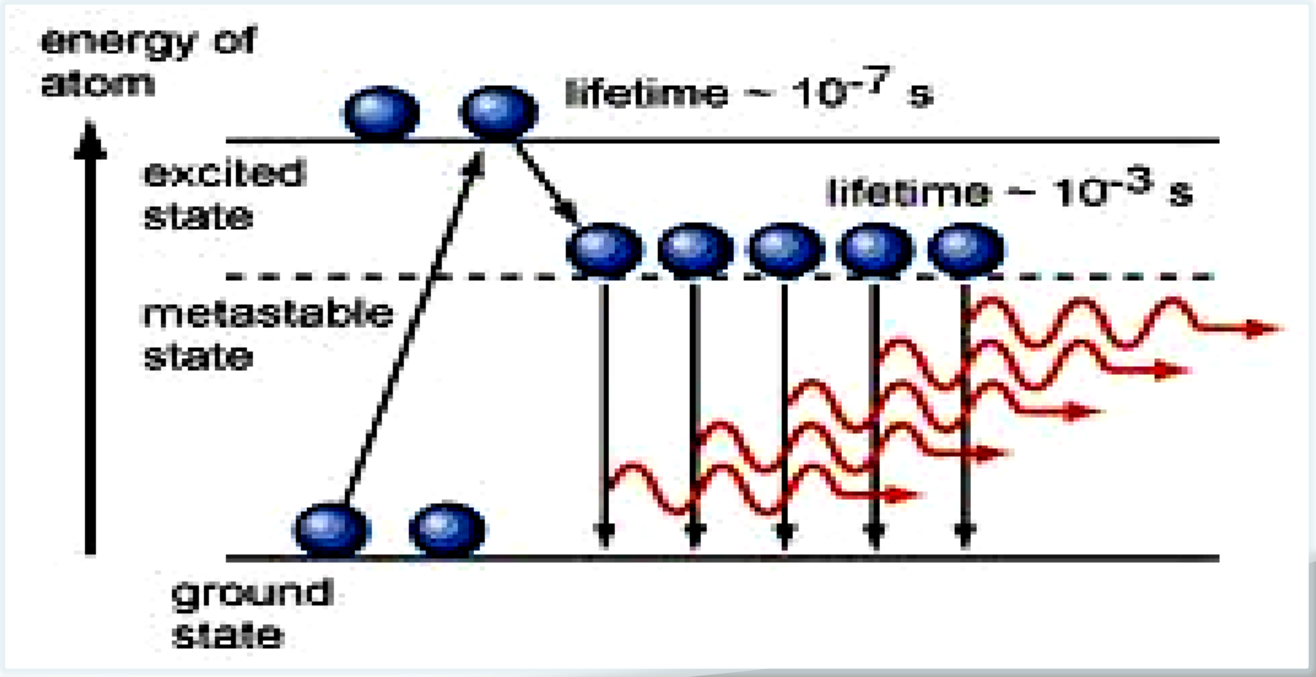
- Usually in a system, the number of atoms (N_1) present in the ground state (E_1) is larger than the number of atoms (N_2) present in the higher energy state. The process of making $N_2 > N_1$ is called population inversion.
- Conditions for population inversion are:
- The system should posses at least a pair of energy levels ($E_2 > E_1$), separated by an energy equal to the energy of a photon (hv).
- There should be a continuous supply of energy to the system such that the atoms must be raised continuously to the excited state

Population inversion can be achieved by a number of ways. Some of them are,
(i) Optical pumping
(ii) Electrical discharge
(iii) Inelastic collision of atoms
(iv) Chemical reaction and
(v) Direct conversion



Meta Stable state

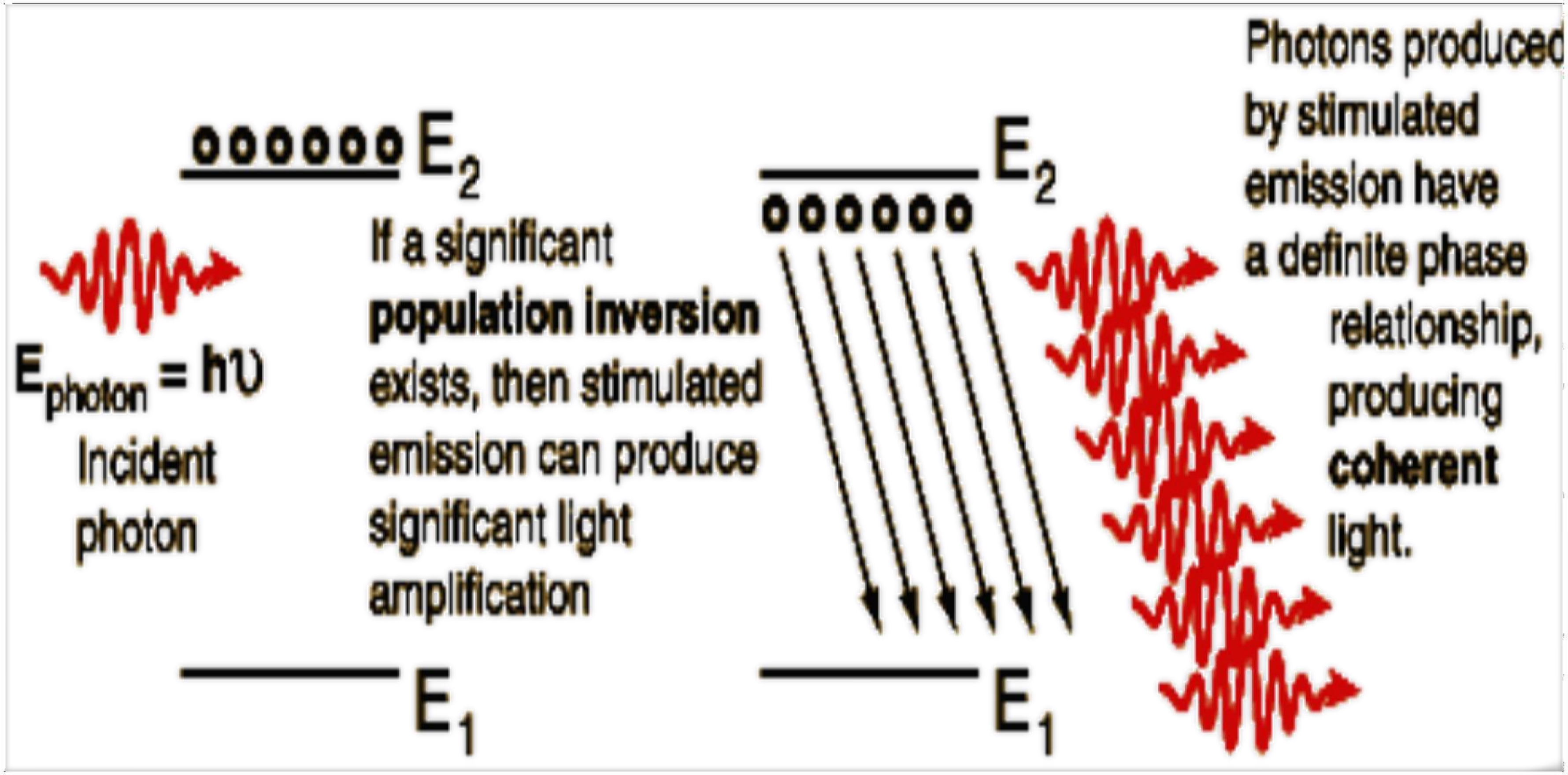
- An excited state with relatively more life time(10^{-3} sec) is called a Meta stable state.
- The necessary condition for population inversion is the presence of a meta stable state.



Lasing Action

1. Pumping: The process of sending atoms from lower energy state to higher energy state is called Pumping..Different pumping mechanisms are adopted depending on the type of the laser. For Ruby laser, Optical pumping is adopted. For He-Ne laser, the pumping mechanism is Electric discharge. In Semi-conductor laser, it is Direct conversion and in the case of CO_2 laser, the mechanism is Chemical reaction.
2. Population inversion : Population inversion can be achieved with the presence of a meta stable state.
3. Stimulated emission of radiation : Photons produced by stimulated emission are in phase and they produce coherent light.

Lasing Action

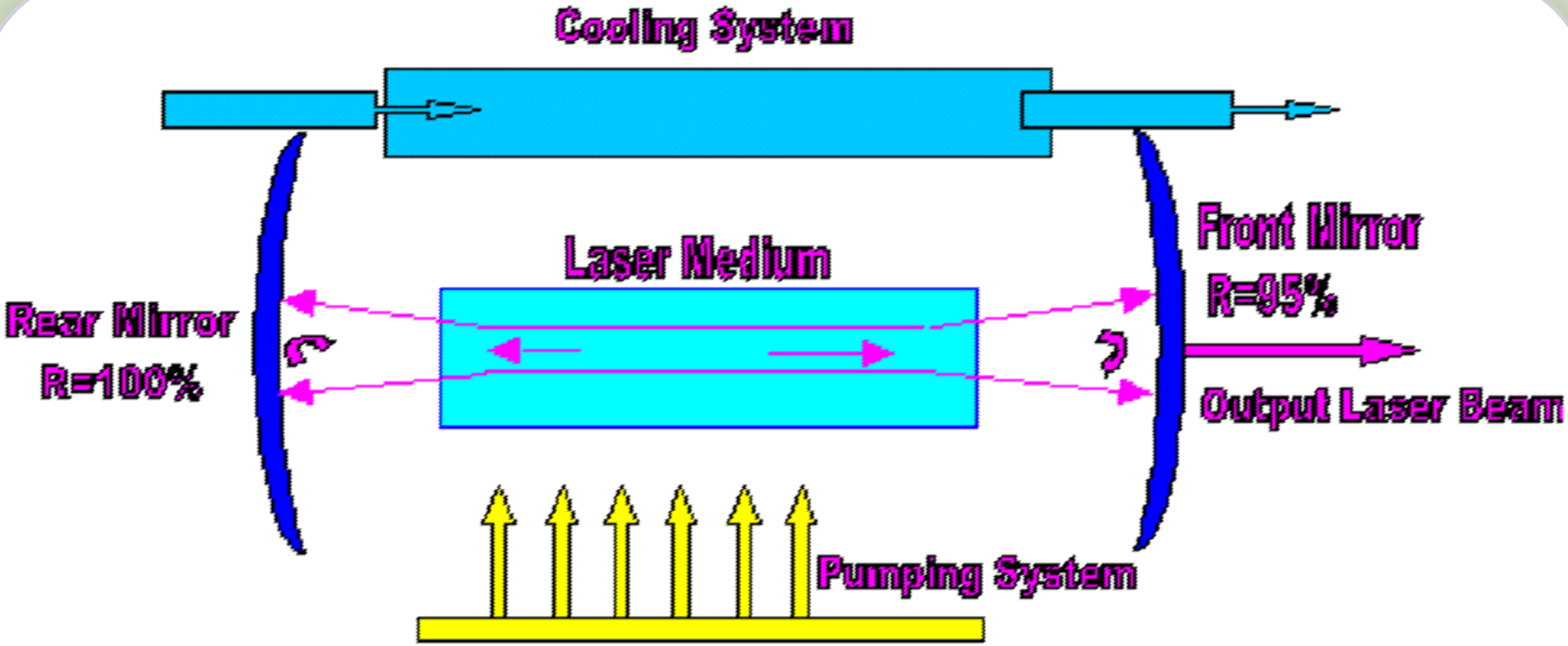


Laser System

1. An Active medium, with a suitable set of energy levels to support laser action. For example, in Ruby laser, Cr^{3+} ions are the active laser particles.

2. Energy source, (Source of Pumping) in order to establish population inversion.

3. An Optical Cavity or Resonator to introduce optical feedback and so maintain the gain of the system overcoming all losses. Depending on the type of the system, optical feedback is provided with the help of dielectric mirrors or polished and coated ends of a crystal rod or cleaved crystal face.


Cooling System
Laser Medium
Rear Mirror
R=100%
Front Mirror
R=95%
Output Laser Beam
Pumping System

Ruby Laser

- Ruby Laser is the first type of laser, demonstrated in the year 1960 by T.H.Maiman.
- Ruby Laser is a solid state laser.
- It is a pulsed three level pumping scheme.
- Active medium: The active medium in Ruby rod ($Al_2O_3+Cr_2O_3$) is Cr^{3+}ions.
- Some of the Aluminum atoms are replaced by 0.05% of Chromium atoms.
- Lasing action takes place in Chromium energy levels.
- Energy Source: The pumping of ions is through optical pumping, using Xenon flash lamp.

Ruby Laser

Construction:

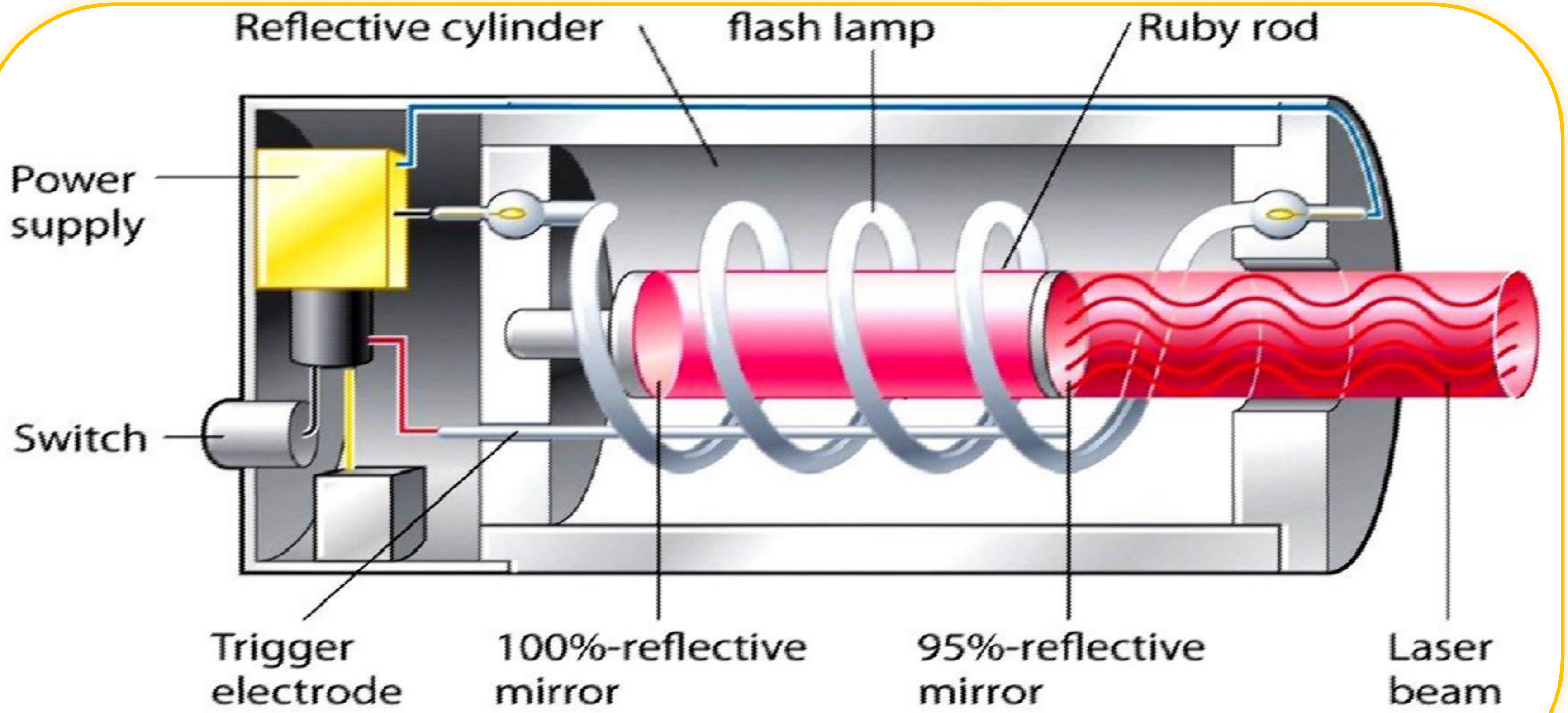
Ruby Laser consists of a cylindrical shaped Ruby crystal rod. One of the end faces is highly silvered and the other face is partially silvered so that it transmits 10-25% of the incident light and reflects the rest.

The ruby crystal is placed along the axis of a helical Xenon or Krypton flash lamp of high intensity. This is surrounded by a reflector.

The ruby rod is protected from heat by enclosing it in a hollow tube, through which cold water is circulated.

The ends of the flash lamp are connected to a pulsed high voltage source, so that the lamp gives flashes of an intense light.

Ruby Laser



Reflective cylinder
flash lamp
Ruby rod
Power supply
Switch
Trigger electrode
100%-reflective mirror
95%-reflective mirror
Laser beam

Ruby Laser

Working:

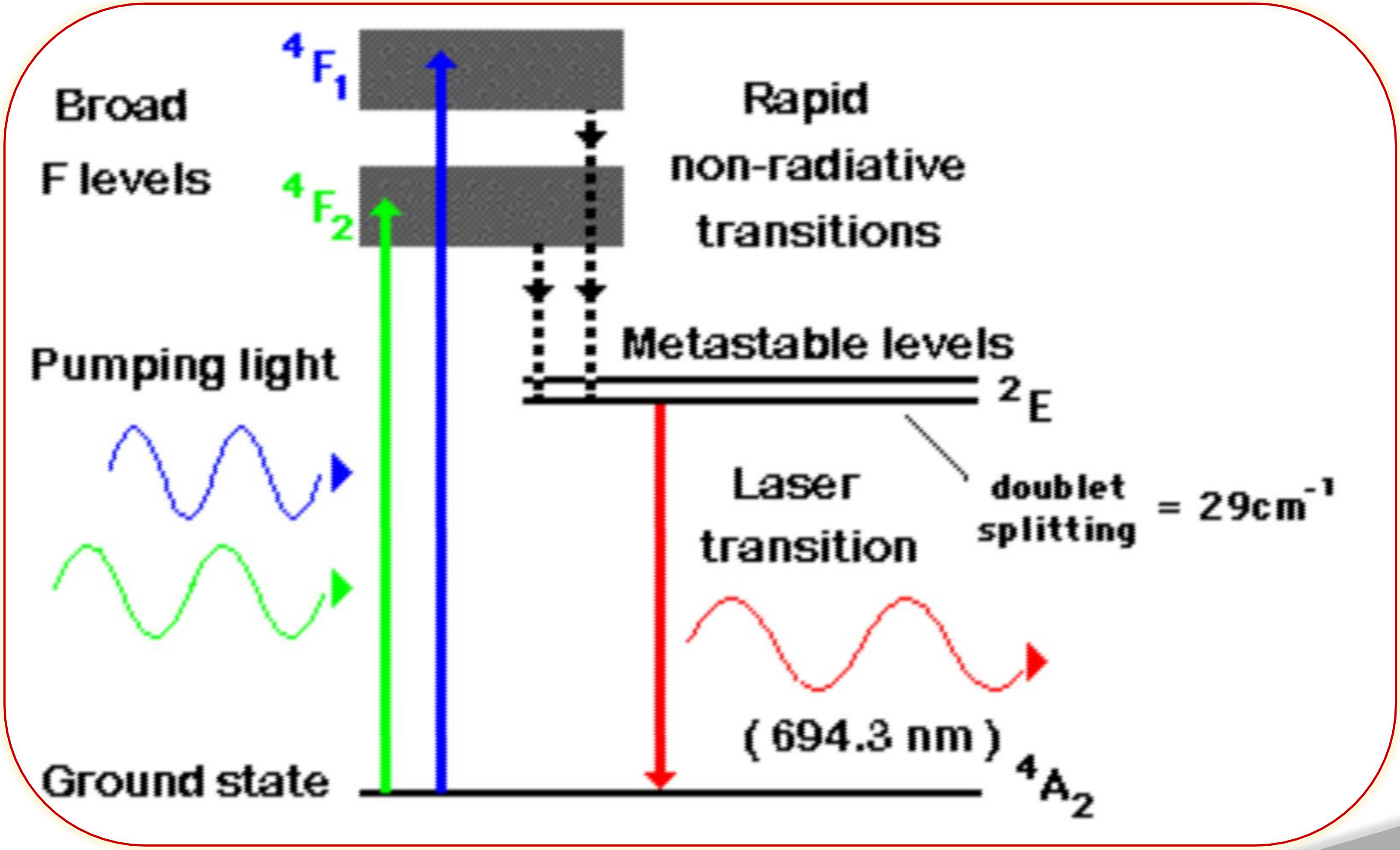
The Chromium ions are responsible for the stimulated emission of radiation, whereas Aluminum and Oxygen ions are passive, sustaining the lasing action.

The Chromium ions absorb the radiations of wavelength around 5500A^o (Green) and 4000A^o (Blue),emitted by the flash lamp and get excited to $4F_2$ and $4F_1$ energy levels respectively, from ground state.

After the life time, the ions make non- radiative transition to the metastable state ^2E, consisting of a pair of energy levels (doublet).

Population inversion takes place between metastable and ground state. As a result, stimulated emission takes place giving rise to the emission of light of wavelengths 6929A^o and 6943A^o , of which 6943A^o is the laser radiation of high intensity.

Ruby Laser



Broad
F levels
$^4F_1$
$^4F_2$
Rapid
non-radiative
transitions
Metastable levels
^2E
Pumping light
Laser
transition
doublet
splitting
= 29cm^{-1}
(694.3 nm)
Ground state
$^4A_2$

Ruby Laser - Applications

Ruby laser is used in Distance measurement using 'pulse echo' technique.

Ruby laser is used to create holograms of large objects such as aircraft tires to look for weaknesses in the lining.

Used in atmospheric ranging, scattering studies and LIDAR measurements.

Used for trimming resistors and integrated circuit masks.

Ruby lasers were used mainly in research One of the main industrial uses is drilling holes through diamond.

Used in military as target designators and range finders.

Used in research applications such as Plasma production and fluorescence spectroscopy.

Ruby lasers were used extensively in tattoo and hair removal.

Helium-Neon(He-ne)laser

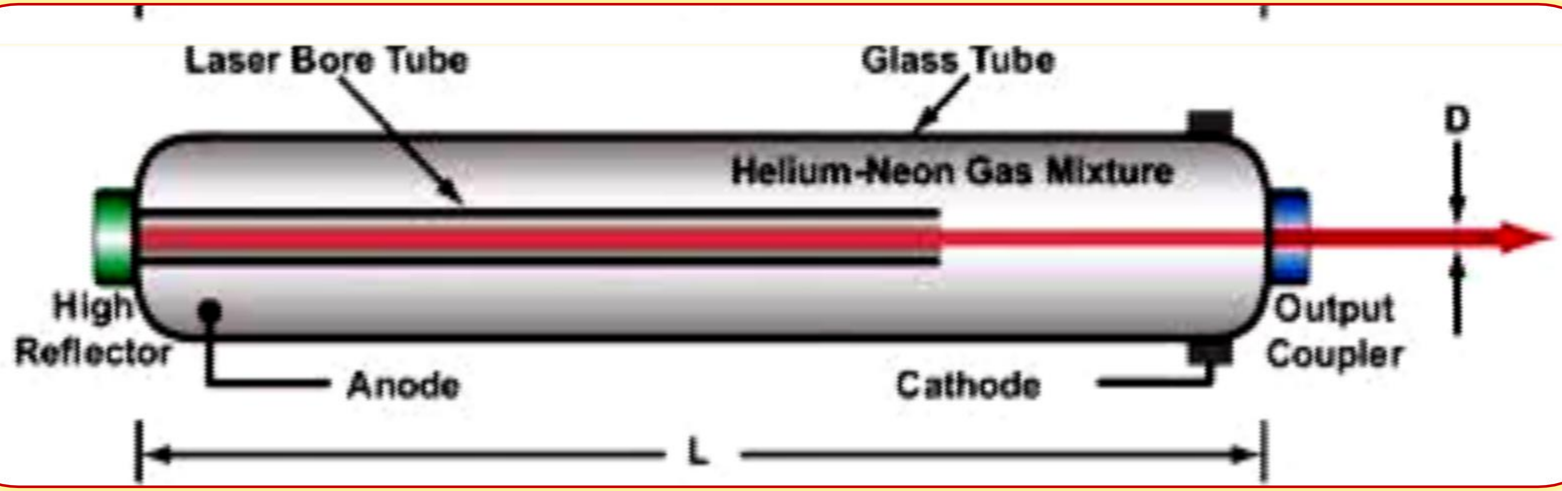
- The best-known and most widely used He-Ne laser operates at a wavelength of 632.8 nm in the red part of the visible spectrum.
- It was developed at Bell Telephone Laboratories in 1962.
- Helium-Neon is a gas laser.
- It is a continuous four level laser.
- Active medium: Helium and Neon gases in the ratio of 10:1respectively. Ne atoms are responsible for lasing action.
- Energy Source: Two electrodes are fixed near the ends of the tube to pass electric discharge through the gas.

Helium-Neon(He-ne)laser

Construction:

- He-Ne laser consists of a long, narrow cylindrical tube made up of fused quartz, of diameter around 2 to 8 mm and length around 10 to 100 cm.
- The tube is filled with helium and neon gases in the ratio of 10:1. The pressure of the mixture of gases inside the tube is nearly 1 mm of Hg.
- Two electrodes are fixed near the ends of the tube to pass electric discharge through the gas.
- Two optically plane mirrors are fixed at the two ends of the tube.
- One of the mirrors is fully silvered so that nearly 100% reflection takes place and the other is partially silvered, so that 1% of the light incident on it will be transmitted.

Helium-Neon(He-ne)laser



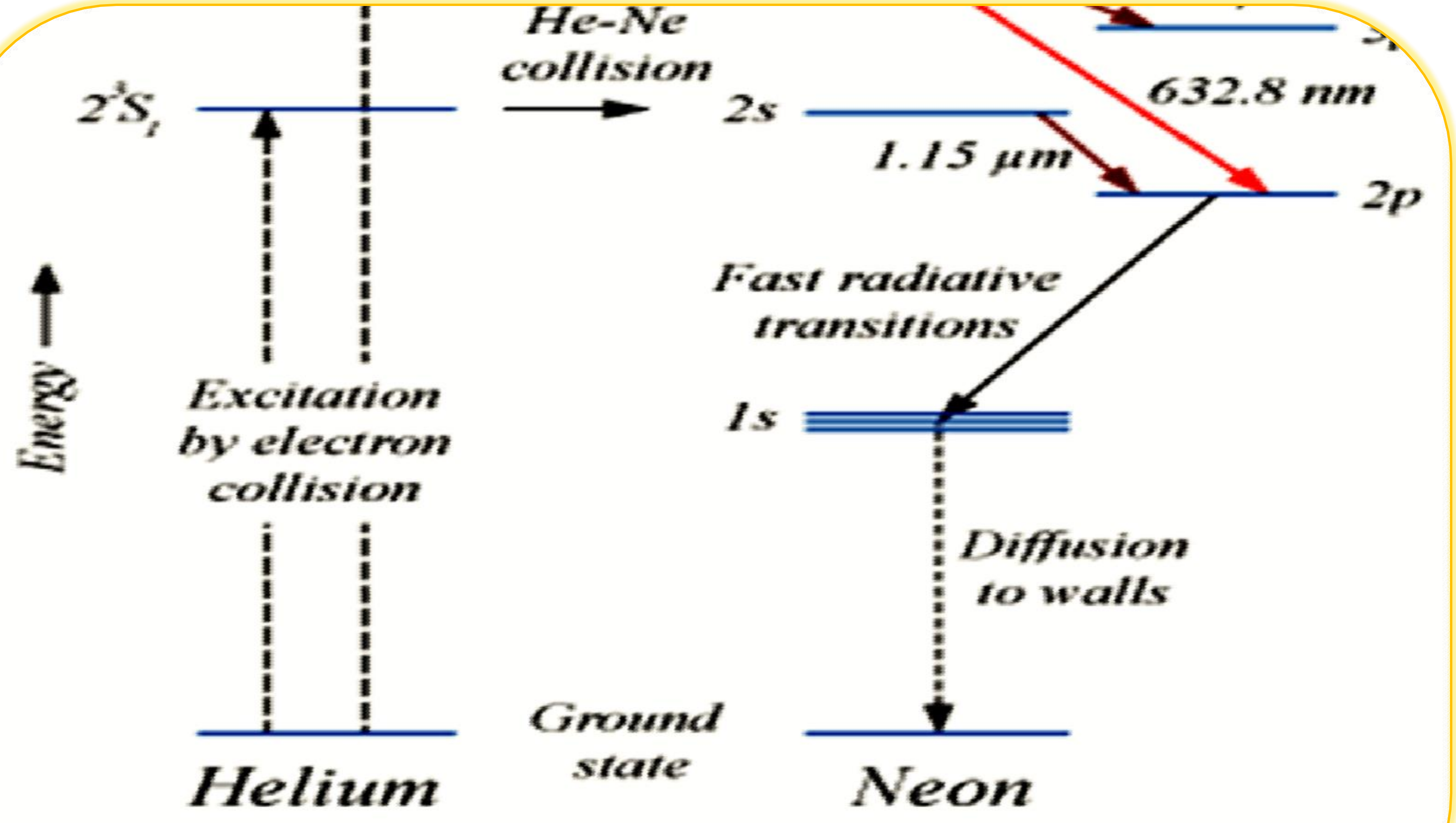
Working:

- Lasing action is due to the neon atoms. Helium is used for selective pumping of neon atoms to upper energy levels.
- When a discharge is passed through the gaseous mixture, electrons are accelerated down the tube. These accelerated electrons collide with the ground state helium atoms and excite them to two meta stable states 2^1s and 2^3s.

Helium-Neon(He-ne)laser

- The helium atoms in the meta stable state 2^1s collide with the neon atoms in the ground state and excite them to 3s level.
- Similarly, the helium atoms in the meta stable state 2^3s collide with the neon atoms in the ground state and excite them to 2s energy level
- During collisions, the helium atoms transfer their energy to neon atoms and come back to ground state.
- Since 3s and 2s levels of neon atoms are meta stable states, population inversion takes place at these levels. Any of the spontaneously emitted photon will trigger the laser action
- The excited neon atoms transit to ground state in three different ways, leading to three lasers of different wavelengths. They are,
- Transition from 3s to 3p level, giving rise to a radiation of 3.39μm, which lies in the infrared region.

Helium-Neon(He-ne)laser



Energy band diagram of He-Ne laser

Applications of Lasers

Lasers in medicine:

Lasers are used in eye surgery, especially to attach the detached retina.

- ❖Lasers are used for treatments such as plastic surgery, skin injuries and to remove moles, tattoos and tumours developed in skin tissue.

- ❖Lasers are used in stomatology- the study of mouth and its

Lasers in Eye surgery

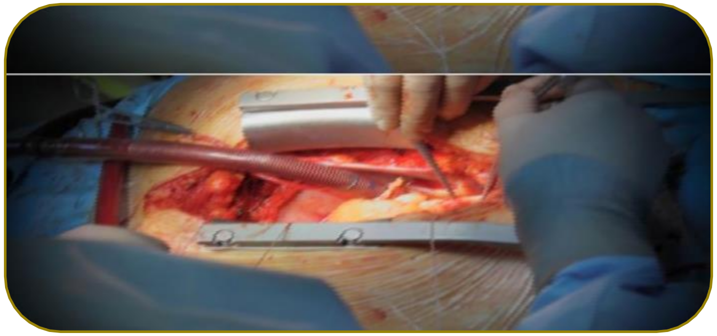
Lasers in tattoo removal

Lasers used in stomatology

Applications of Lasers

- Laser radiation is sent through optical fibre to open the blocked artery region.
- Lasers are used to destroy kidney stones and gall stones.
- Lasers are used in cancer diagnosis and therapy.
- Lasers are used in blood loss less surgery.
- Lasers are used to control hemorrhage.
- Using CO_2 laser, liver and lung treatment can be carried out.
- Lasers are used in endoscopes, to detect hidden parts.
- Laser Doppler velocimetry is used to measure the velocity of blood in blood vessels.

Red Argon laser used in throat cancer treatment



Lasers used to open artery block

Lasers used to destroy kidney stones

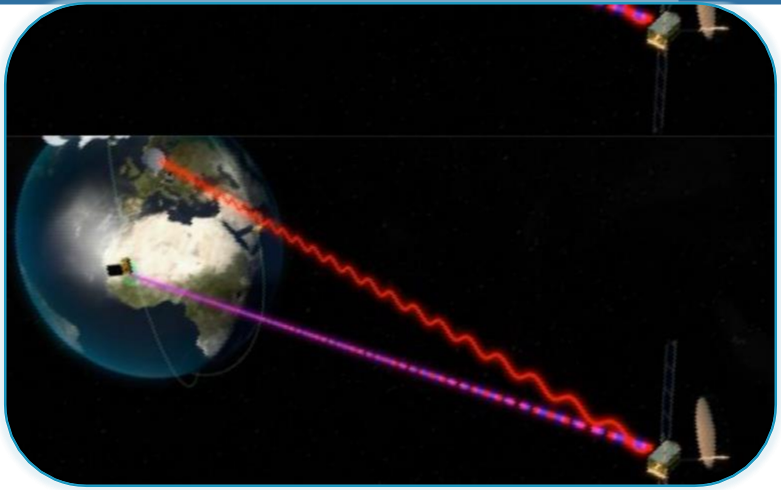
Applications of Lasers

Lasers in Communication:

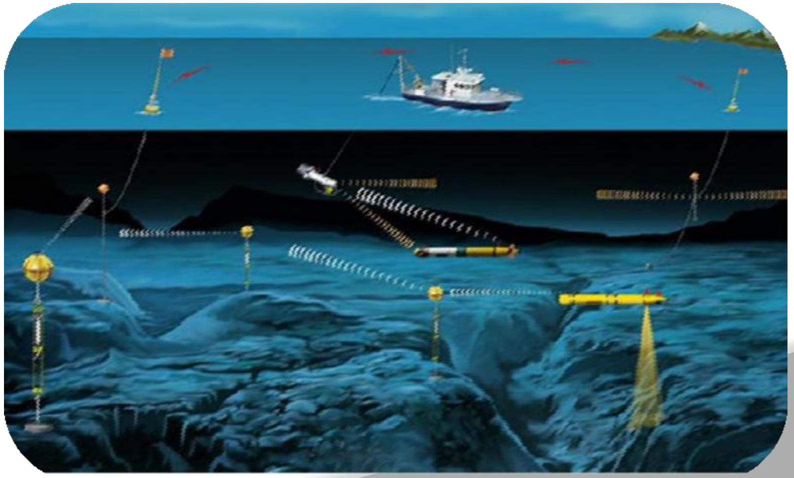
Lasers are used in Optical fibre communication as light source to transmit audio, video signals and data to long distances without attenuation and distortion.

Laser beam can be used for the communication between the earth and the moon or to other satellites.

Laser beam can be used for under water communication, as laser radiation is not absorbed by water.



Lasers in Satellite communication



Lasers in under water communication

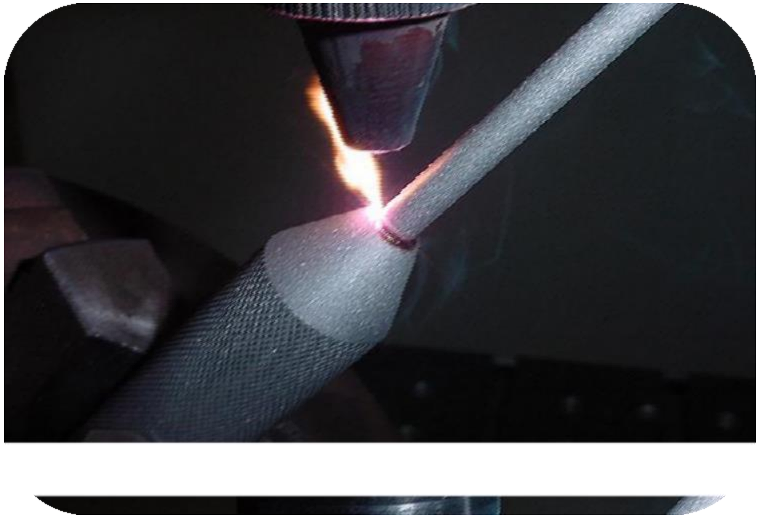
Applications of Lasers

Lasers in Industry:

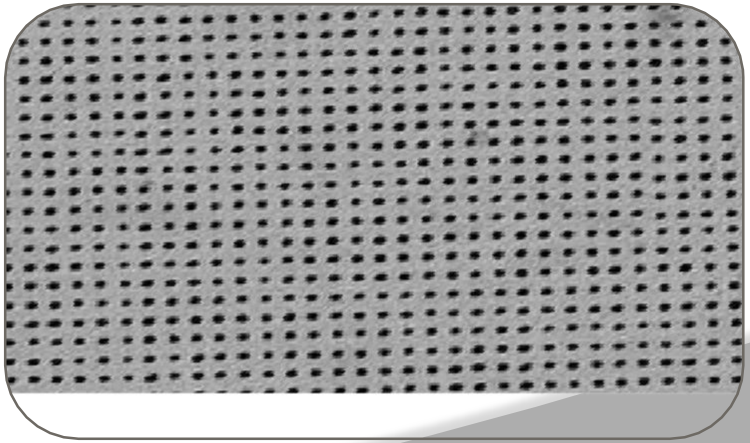
Lasers are used for welding. Dissimilar metals can be welded using lasers.

Holes with controlled precision can be drilled in steel, ceramics, diamond and alloys, using lasers.

Lasers are widely used in electronic industry in trimming the components of ICs.



Lasers used in welding



Drilling Steel foil for high density filters

Applications of Lasers

- Lasers are used in cutting metal sheets, diamond and cloths. In the mass production of stitched clothes, lasers are used to cut the cloth in a desired dimension, all at once.

- Lasers are used for surface treatment. Laser beam is used in selective heat treatment for tempering the desired parts in automobile industry.

Cutting wood using laser

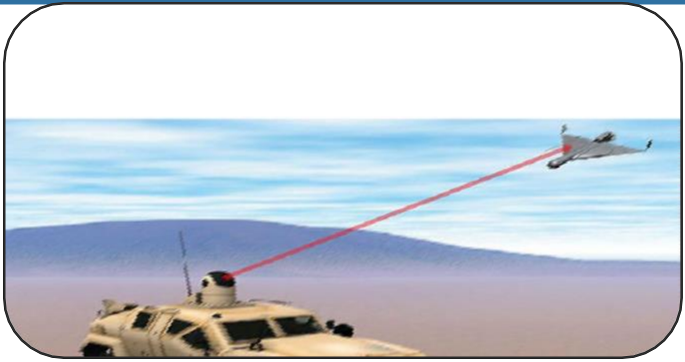
Laser surface treatment to change the micro structure of metals through controlled heating and cooling.

The world's first all-diamond ring, cut with Laser

Applications of Lasers

Lasers in Military:

- Focusing of high energetic laser beam for few seconds, destroys aircrafts, missiles, etc. These rays are called death rays.
- The vital part of the enemy's body can be evaporated by focusing a highly convergent laser beam from a laser gun.
- LIDAR (Light Detecting And Ranging) is used to estimate the size and shape of distant objects or war weapons.



Laser armed Humvees shooting a Drone

Soldiers using laser gun

Lasers beams of RMR LIDAR at ALOMAR Observatory

Applications of Lasers

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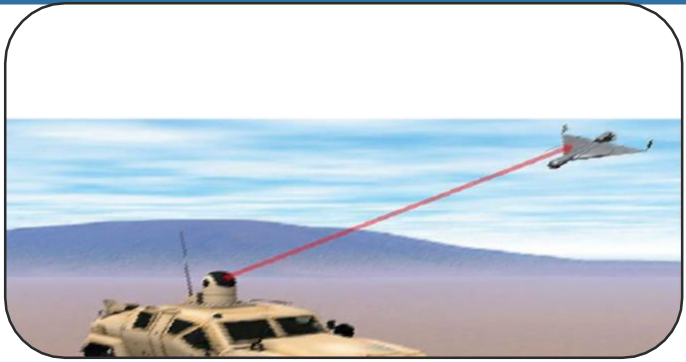

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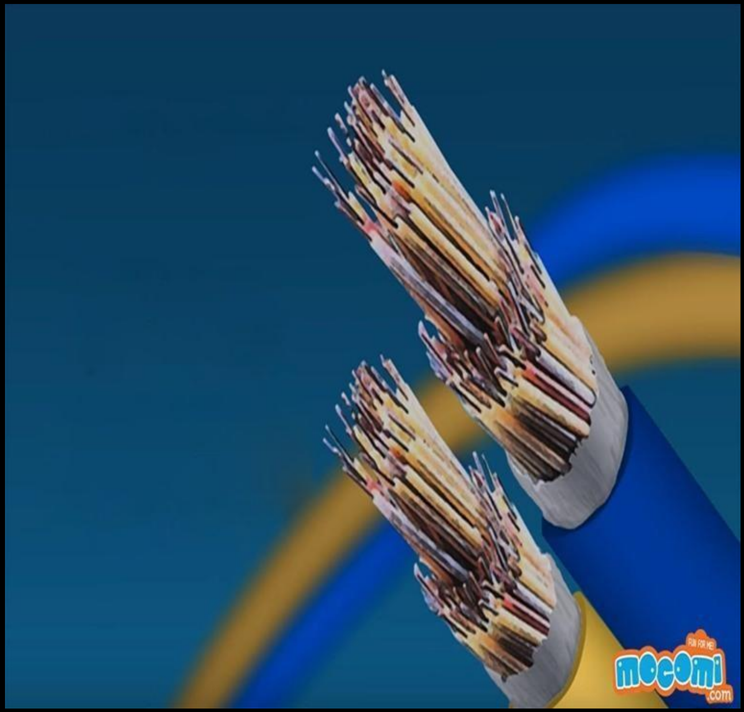
FIBER OPTICS

Optical fibers are widely used in fiber-optic communications, where they permit transmission over longer distances and at higher bandwidths(data rates) than wire cables.

Fibers are used instead of metal wires because signals travel along them with less loss and are also immune to electromagnetic interference.

Fibers are also used for illumination, and are wrapped in bundles so that they may be used to carry images.



Fibre Optic cable

Structure of an Optical Fibre

Structure of an optical fiber consists of three parts.

The core, the cladding and the coating (or buffer or outer jacket).

The core:

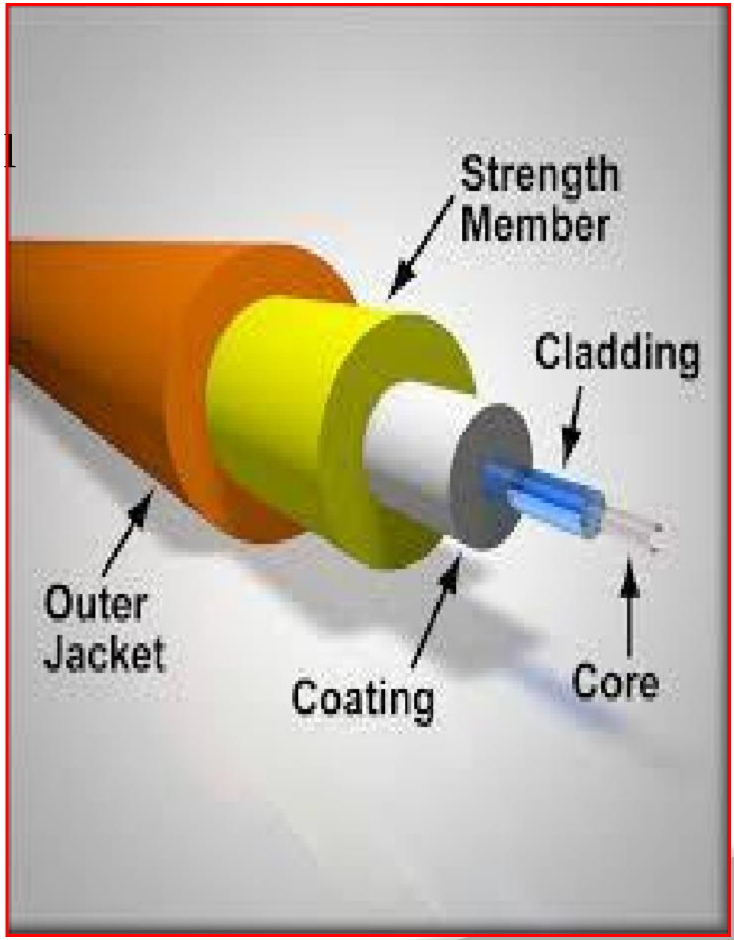
- The core is a cylindrical rod of dielectric material.
- Light propagates mainly along the core of the fiber.
- The core is generally made of glass.
- The core is described as having an index of refraction n_1.

The Cladding:

- The core is surrounded by a layer of material called the cladding, which is generally made of glass or plastic.
- The cladding layer is made of a dielectric material with an index of refraction n_2.
- The index of refraction of the cladding material is less than that of the core material.

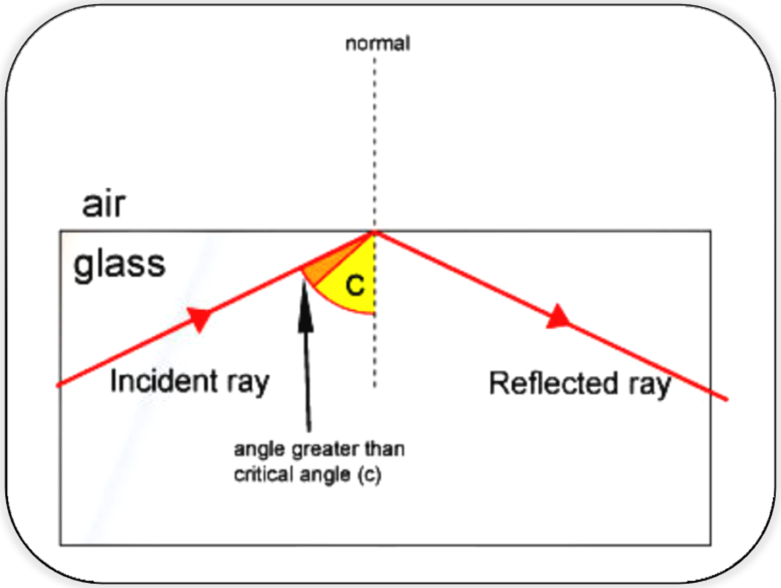
Buffer:

- The coating or buffer is a layer of material used to protect an optical fiber from physical damage.
- The material used for a buffer is a type of plastic.



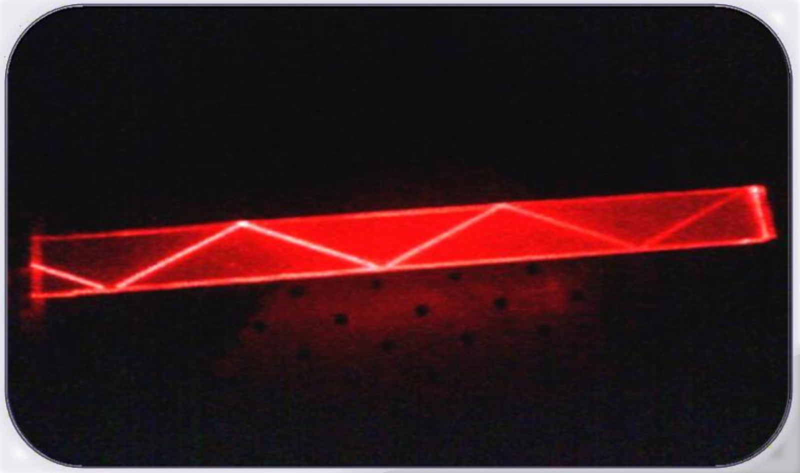
Principle of Optical Fibre

- Optical fibre carries light from one end of the fibre to the other by total internal reflection.
- When a ray of light passes from an optically denser medium into an optically rarer medium, the refracted ray bends away from the normal.



Total internal reflection: When a light ray, travelling from an optically denser medium into an optically rarer medium, is incident at an angle greater than the critical angle, then the ray is totally reflected back into the same medium by obeying the laws of reflection. This phenomenon is known as totally internal reflection.

Total Internal Reflection



Internally reflected light ray

Condition for Total Internal Reflection

Let the reflective indices of core and cladding materials be n_1 and n_2 respectively.

According to the law of refraction,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Here, θ_1=, θ_c and θ_2= 90

$$n_1 \sin\theta_c = n_2 \sin 90$$

$$\sin\theta_c = {}^{n}{}_{2}/n_1$$

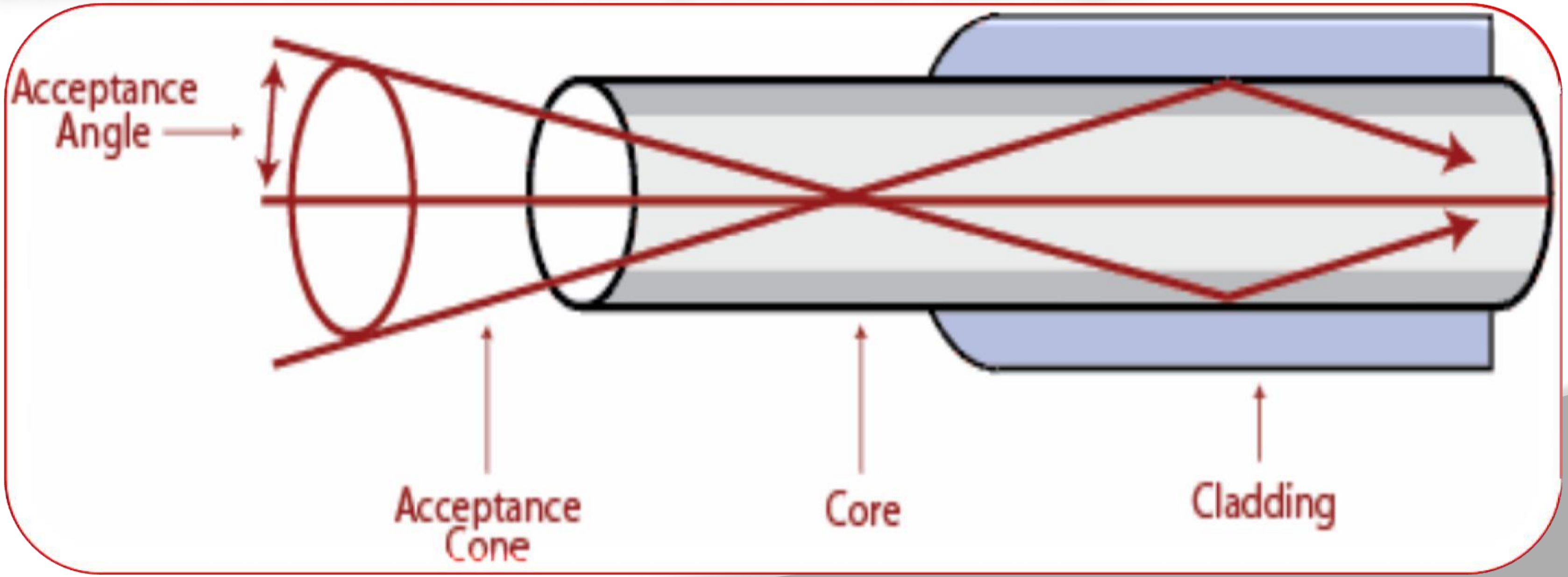
$$\theta c = \sin^{-1}({}^{n}{}_{2}/n_1) \qquad \rightarrow(1)$$

Equation (1) is the expression for condition for total internal reflection.

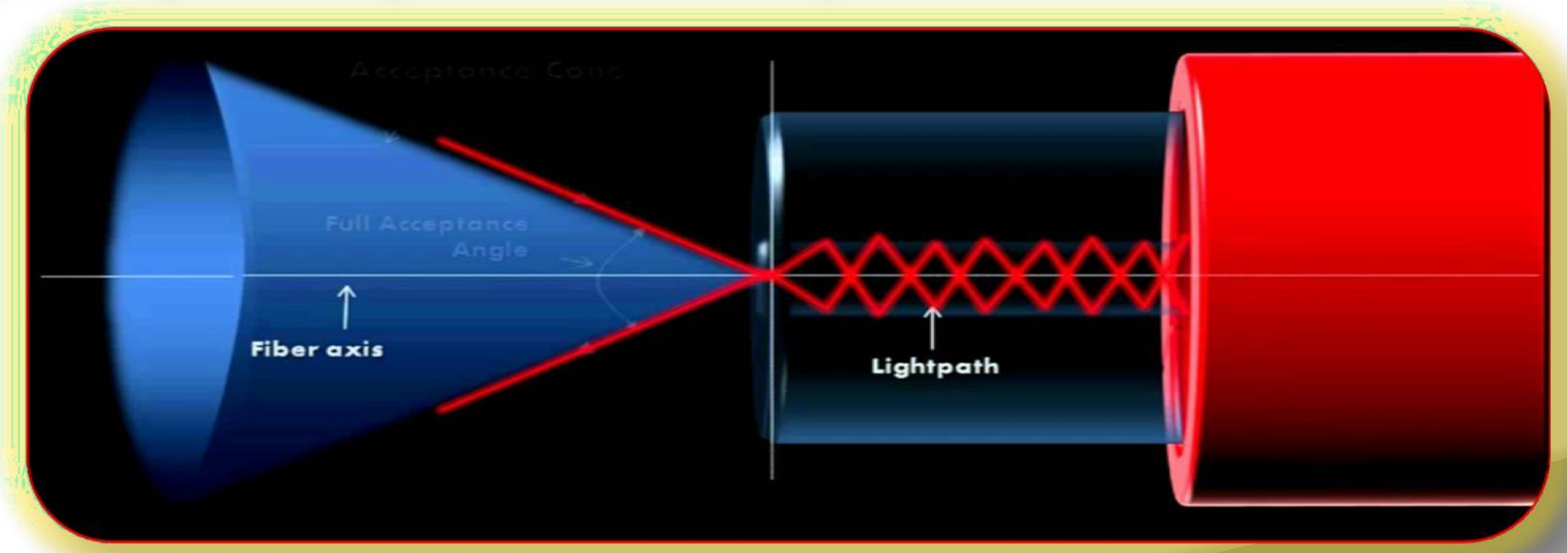
In case of total internal reflection, there is absolutely no absorption of light energy at the reflecting surface.

Acceptance angle and Acceptance cone

- Acceptance angle is the angle at which the beam has to be launched at one of its ends, in order to enable the entire light to propagate through the core.
- The acceptance angle is the maximum angle that a light ray can have with the axis of the fiber to propagate through the fiber.
- **Acceptance angle:** It is defined as the maximum angle of incidence at the end face of the optical fibre, for which the ray can be propagated through the core material. It is also called as Acceptance cone half angle.



- **Acceptance cone: The cone obtained by rotating a ray at the end face of an optical fibre, around the fibre axis with the acceptance angle, is known as acceptance cone.**
- **Light launched at the fiber end within this acceptance cone alone will be accepted and propagated to the other end of the fiber by total internal reflection.**
- **Larger acceptance angles make launching easier.**



Acceptance cone

Equation for Acceptance angle

- For light rays to propagate through the optical fibre, by total internal reflection, they must be incident on the fibre core within the angle θ_o , called the acceptance angle.
- Applying Snell‟s law at B,

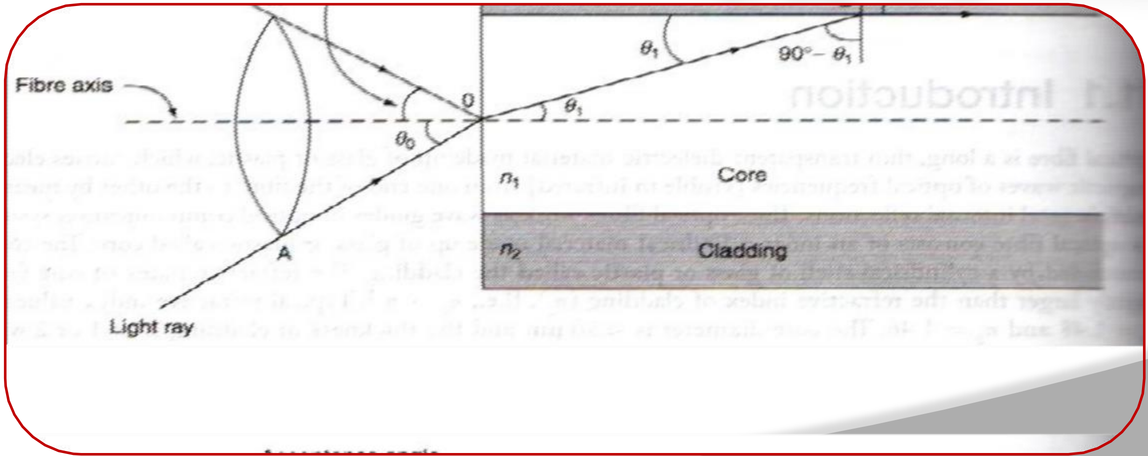
$$n_1 \sin(90^o-\theta_1) = n_2 \sin 90^o$$

$$n_1 \cos \theta_1 = n_2$$

$$\cos \theta_1 = n_2/n_1$$

$$\text{or } \sin\theta_1 = (1-\cos^2\theta_1)^{1/2}$$

$$=\{1-(n_2^2/n_1^2)\}^{1/2} \quad \text{........ (1)}$$



- Applying Snell"s law at O,

$$n_o \sin\theta_0 = n_1 \sin\theta_1$$

$$\text{or } \sin\theta_0 = (n_1/n_0)\sin\theta_1 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting eq. (1) in eq. (2),

$$\text{Sin } \theta_0 = (n_1/n_0)(1 - n_2^2/n_1^2)^{1/2}$$

$$= \frac{(n_1^2 - n_2^2)^{1/2}}{n_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

- As the fibre is in air, $n_0 = 1$

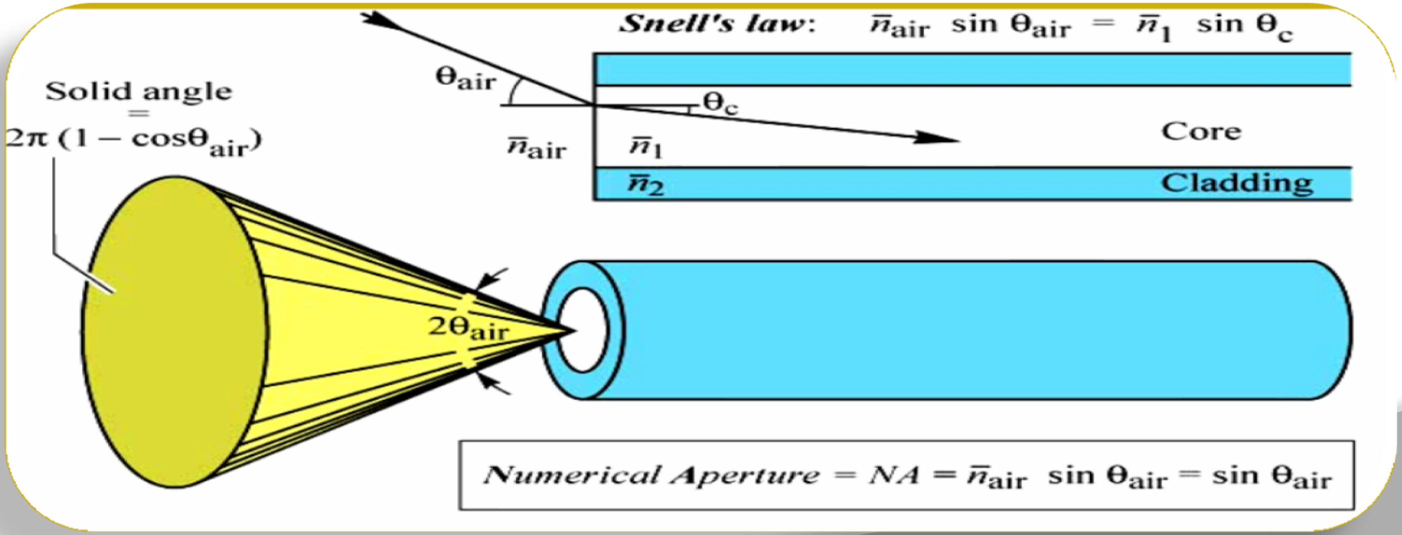
  Therefore, eq. (3) becomes

$$\text{Sin } \theta_0 = (n_1^2/n_2^2)^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Eq. (4) is the equation for Acceptance angle.

Numerical Aperture (NA)

- ✓ Light gathering capacity of the fiber is expressed in terms of maximum acceptance angle and is termed as "Numerical Aperture".
  - ✓ Light gathering capacity is proportional to the acceptance angle θ_{o}.
- ✓ So, numerical aperture can be represented by the sine of the acceptance angle of the fibre i.e., $\sin \theta_{o}$.
- ✓ For example, the light acceptance angle in air is $\theta_{air} = 11.5^{o}$ for a numerical aperture of NA=0.2.



Expression for Numerical aperture:

- According to the definition of Numerical aperture (NA),

$$NA= \sin\theta_0 = (n_1^2-n_2^2)^{1/2} \rightarrow (1)$$

- Let „Δ" the fractional change in the refractive index, be the ratio between the difference in the refractive indices of core and cladding material respectively.

$$\text{i.e., } \Delta = \frac{n_1-n_2}{n_1} \rightarrow (2)$$

$$\text{or } \Delta n_1 = n_1 - n_2 \rightarrow (3)$$

Eq. (1) can be written as,

$$NA= (n_1^2-n_2^2)^{1/2}$$

$$= \{(n_1-n_2)(n_1+ n_2)\}^{1/2} \rightarrow (4)$$

- Substituting eq. (3) in eq. (4),

$$NA = \{(\Delta n_1)(n_1+n_2)\}^{1/2}$$

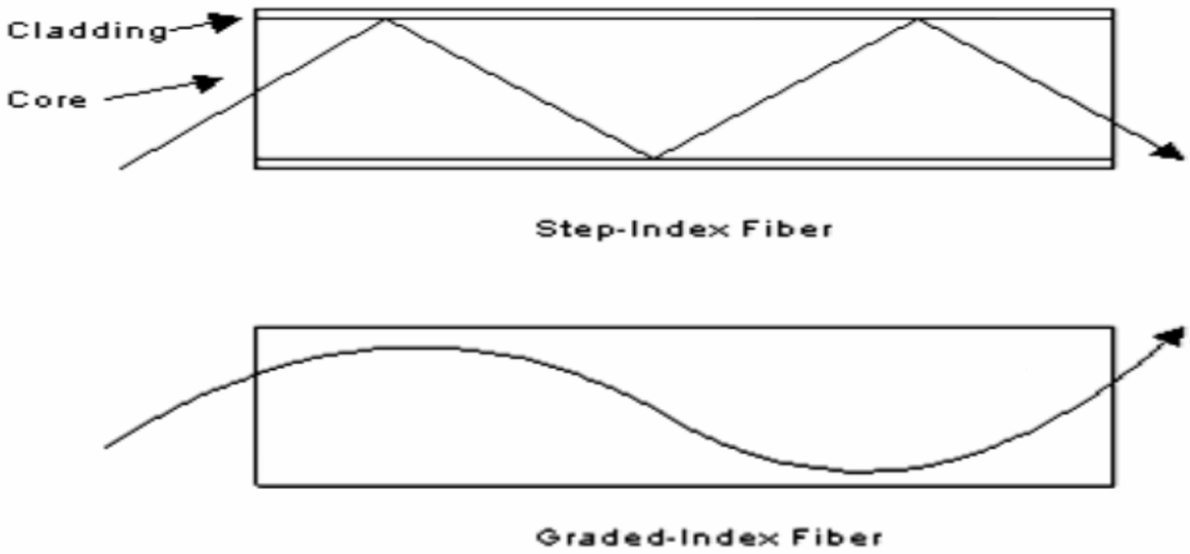
- As $n_1 \approx n_2$, $n_1+ n_2= 2n_1$

And therefore, Numerical Aperture = $(2n_1^2\Delta)^{1/2} = n_1(2\Delta)^{1/2}$ $\rightarrow (5)$

From equation (5) it is seen that numerical aperture depends only on the refractive indices of core and cladding materials and it is independent on the fiber dimensions.

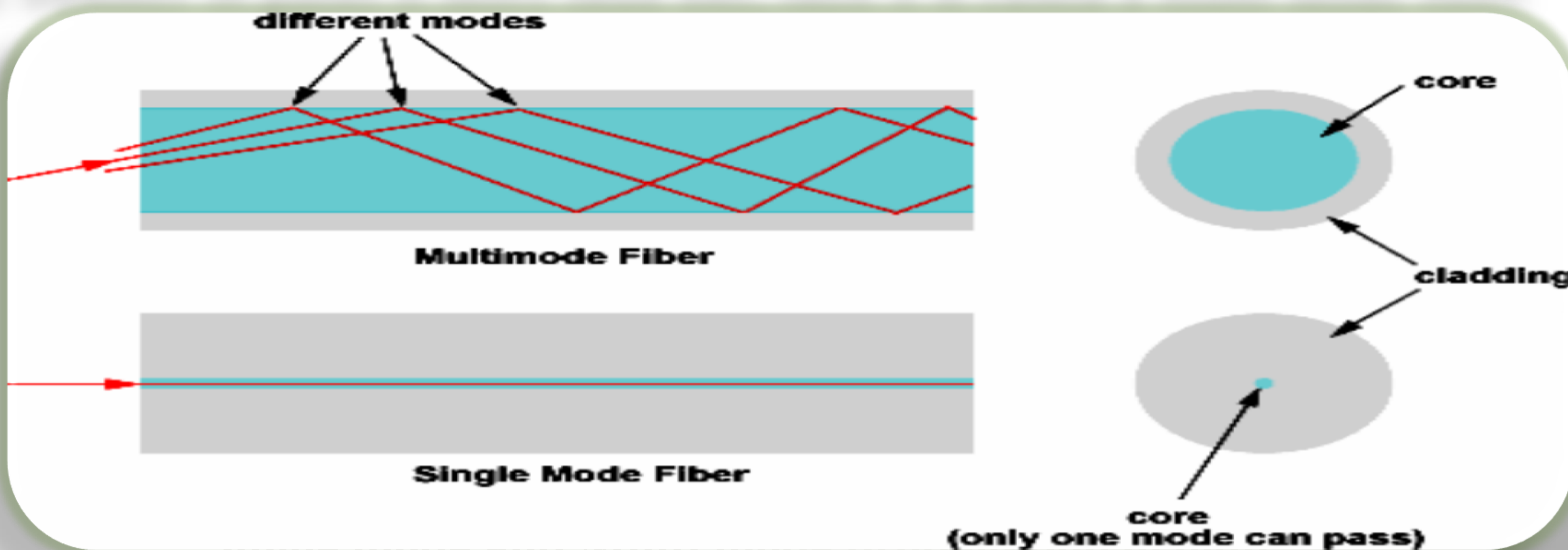
Types of Optical Fibres

- Based on the variation of refractive index of core, optical fibers are divided into: (1) step index and (2) graded index fibers.
- In all optical fibers, the refractive index of cladding material is uniform.



Light path through Step- index and Graded index Fibre

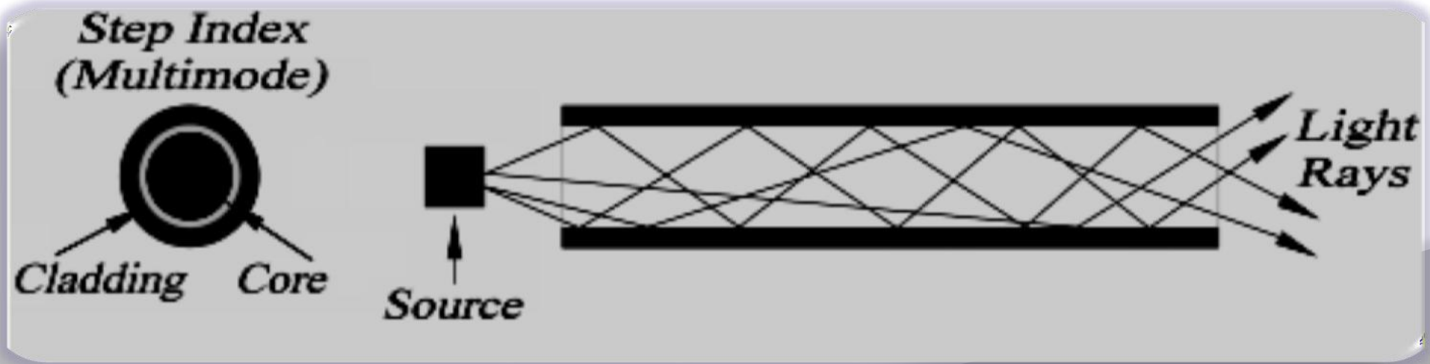
- **Based on the mode of propagation, all the fibers are divided into: (1) single mode and (2) multimode fibers.**
- **Mode means, the number of paths available for light propagation in the fiber.**
- **If there is only one path for the ray propagation, it is called a single mode fiber.**
- **If the number of paths is more than one, then it is called a multi mode fiber.**



Single mode and Multi mode propagation of light

Step index optical fibre

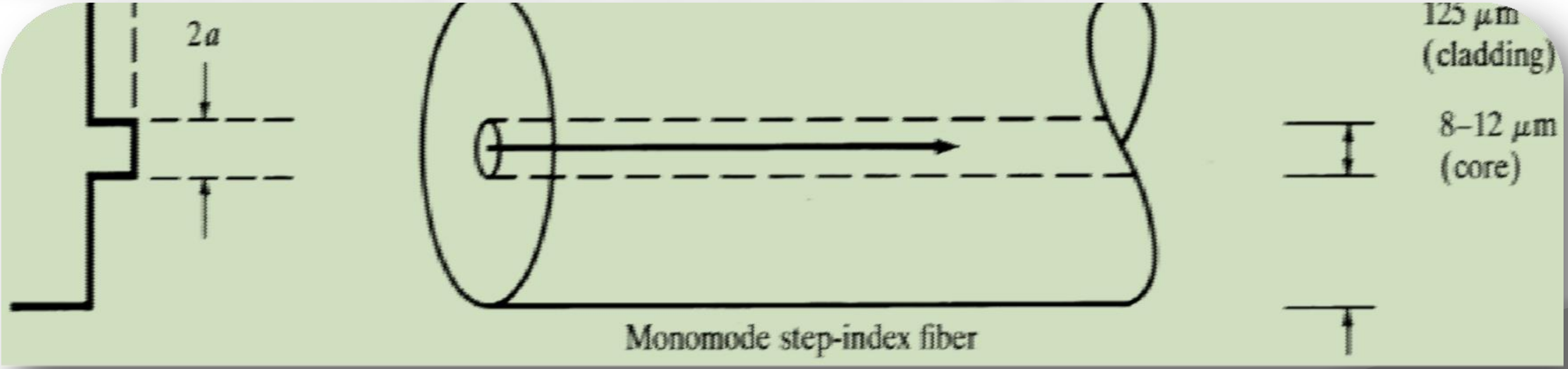
- Based on the mode of propagation of light rays, step index fibers are of 2 types: a) single mode step index fiber & b) multimode step index fibers.
- The light rays propagate in zigzag manner inside the core.



Multi mode Step index optical Fibre

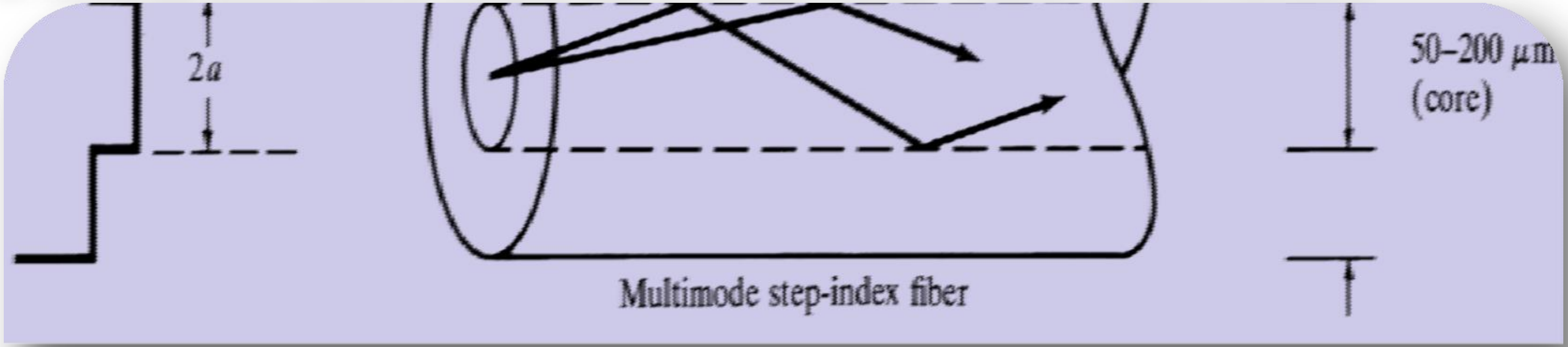
Refractive index profile in Single mode Step index fibre

- The refractive index is uniform throughout the core of this fibre.
- As we go radially in this fibre, the refractive index undergoes a step change at the core-cladding interface.
- The core diameter of this fibre is about 8 to 10 µm and outer diameter of cladding is 60 to 70 µm.
- In this fibre, the transmission of light is by successive total internal reflections i.e. it is a reflective type fiber.
- These fibres are mainly used in submarine cable system.



Refractive index profile in Multimode Step index fibre

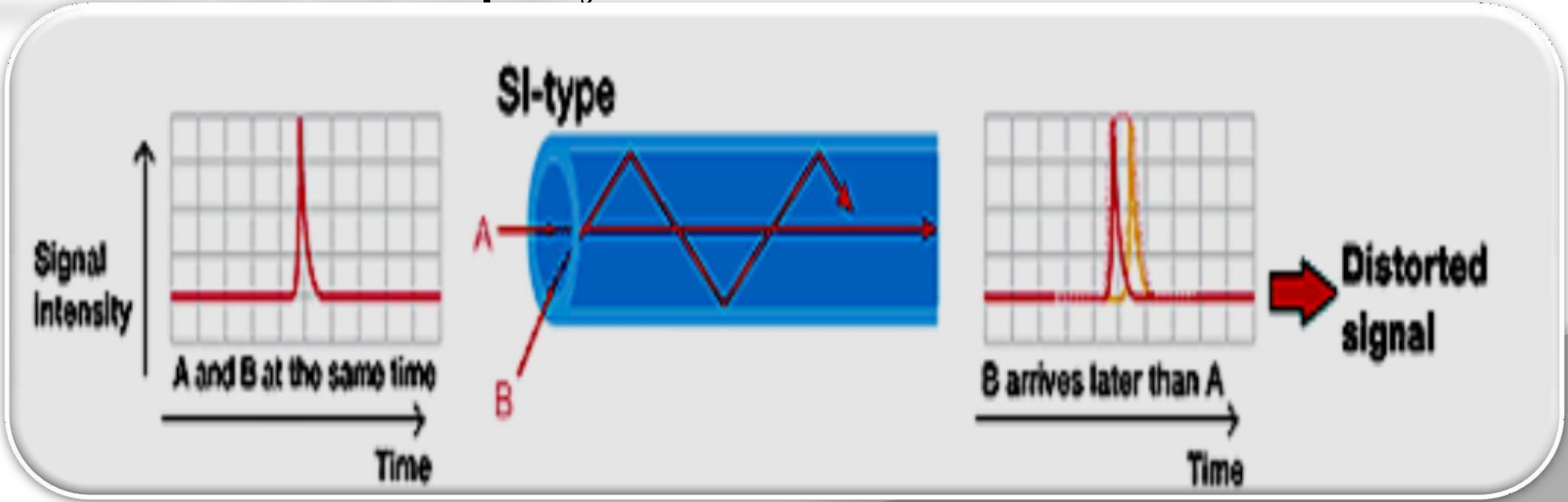
- Its core and cladding diameters are much larger to have many paths for light propagation.
- The core diameter of this fiber varies from 50 to 200 µm and the outer diameter of cladding varies from 100 to 250 µm.
- Light propagation in this fiber is by multiple total internal reflections i.e., it is a reflective type fiber.
- It is used in data links, which have lower band width requirements.



Multimode step-index fiber

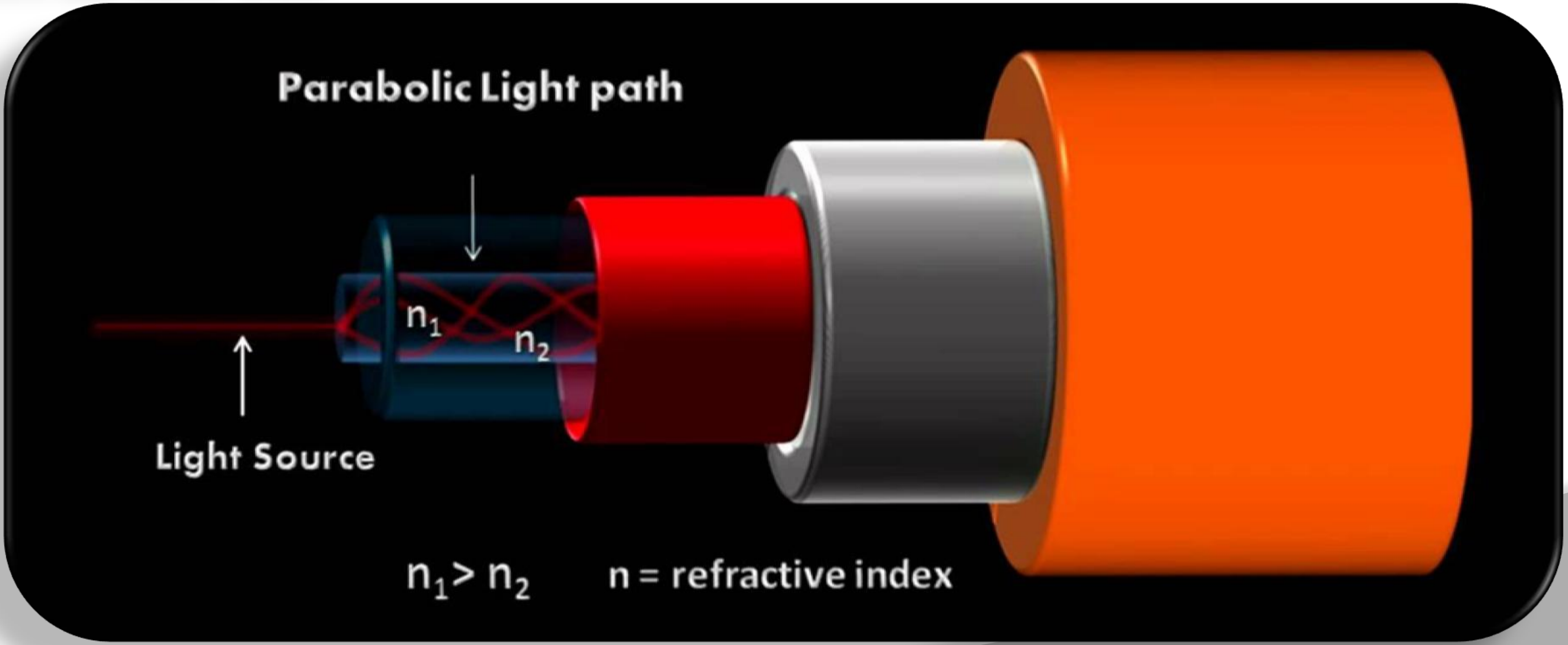
Transmission of signal in step index fibre

- Generally the signal is transmitted through the fiber in digital form i.e. in the form of 1‟s and 0‟s.
- In multimode fibre, the pulse which travels along path A (straight) will reach first at the other end of fiber. Next, the pulse that travels along with path B (zigzag) reaches the other end.
- Hence, the pulsed signal received at the other end is broadened. This is known as intermodal dispersion.
- This imposes limitation on the separation between pulses and reduces the transmission rate and capacity.



Graded index optical fibre

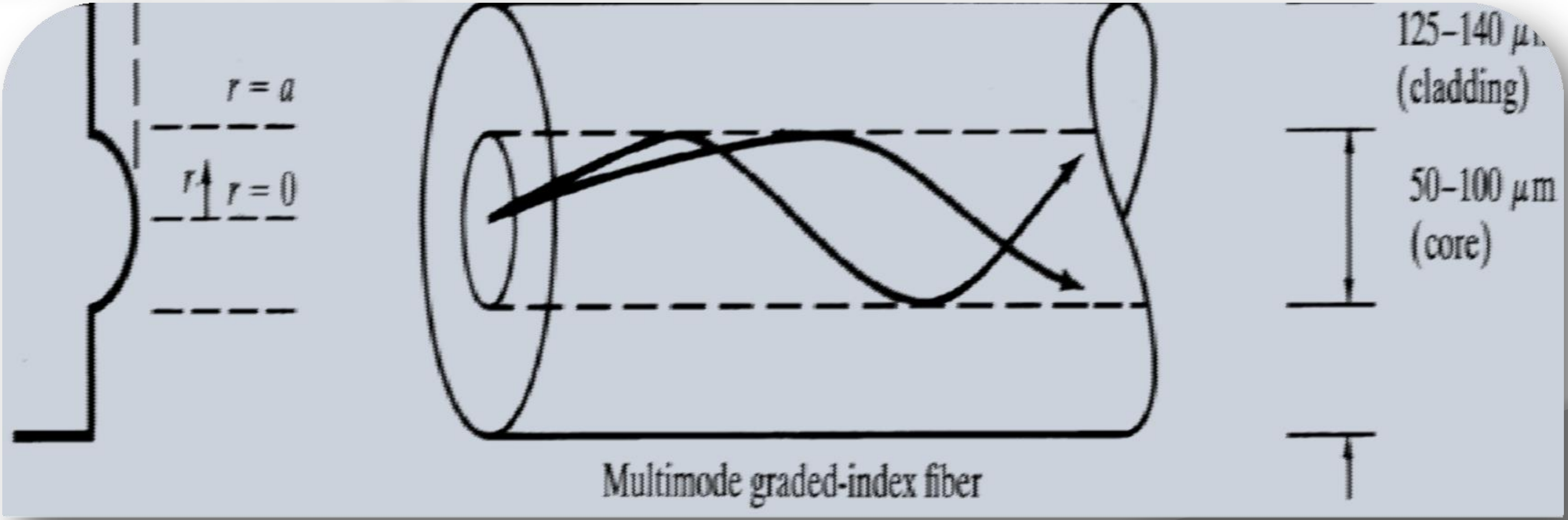
- To overcome the problem of inter modal dispersion caused due to step index optical fibres, graded index fibers are used.
- This fiber can be single mode or multimode fiber.
- Light rays propagate in the form of skew rays or helical rays. They will not cross the fiber axis.



Multimode Graded index optical fibre

Refractive index profile in Multimode graded index fibre

- In this fiber, the refractive index decreases continuously from center radially to the surface of the core.
- The refractive index is maximum at the center and minimum at the surface of core.
- The diameter of the core varies from 50 to 200µm and the outer diameter of the cladding varies from 100 to 250 µm.
- The refractive index profile is circularly symmetric.



Multimode graded-index fiber

Explanation:

- As refractive index changes continuously radially in core, light rays suffer continuous refraction in core.

- The propagation of light ray is not due to total internal reflection but by refraction.

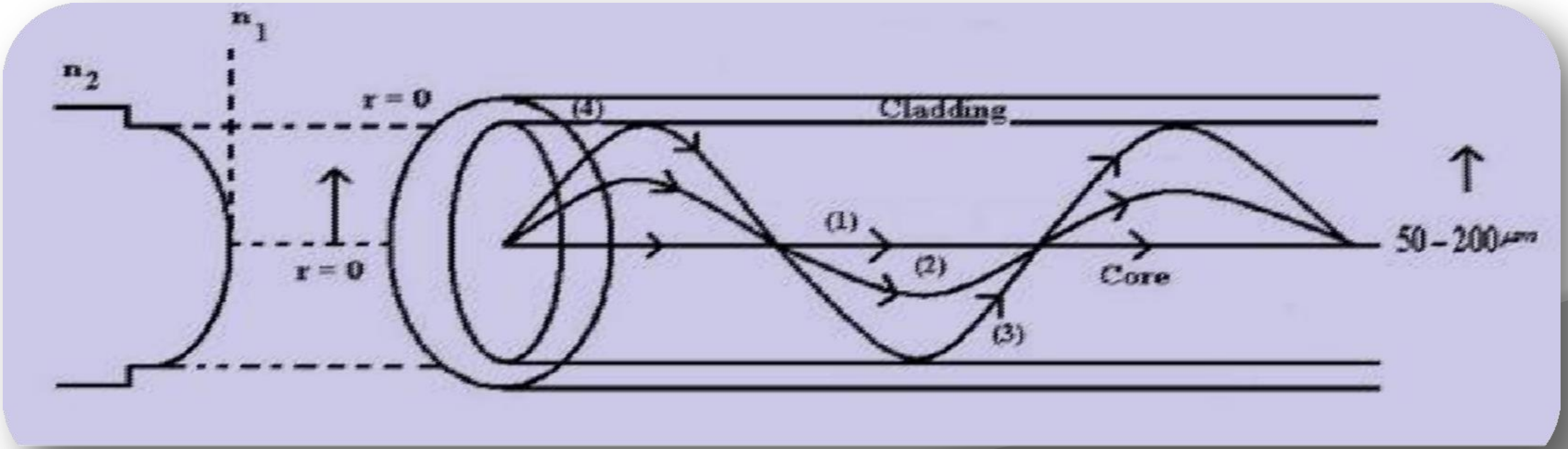
- In graded index fiber, light rays travel at different speed in different paths of the fiber.

- Near the surface of the core, the refractive index is lower, so rays near the outer surface travel faster than the rays travel at the center.

- Because of this, all the rays arrive approximately at the same time, at the receiving end of the fiber.

Transmission of signal in graded index fibre

- consider ray path 1 along the axis of fiber and another ray paths 2 and3.
- Along the axis of fiber, the refractive index of core is maximum, so the speed of ray along path 1 is less.
- Path 2 is sinusoidal and it is longer. This ray mostly travels in low refractive region and so the ray 2 moves slightly faster.
- Hence, the pulses of signals that travel along path 1, path 2 and path 3 reach the other end of the fiber simultaneously. Thus, the problem of intermodal dispersion can be reduced to a large extent using graded index fibers.



Differences between Single mode and Multimode fibers

Single mode Fibre	Multimode Fibre
✓In single mode fiber there is only one path for ray propagation.	✓In multimode fiber, large number of paths are available for light ray propagation.
✓A single mode step index fiber has less core diameter (<10 μm) and the difference between the refractive indices of core and cladding is very small.	✓Multi mode step index fibers have larger core diameter (50-200μm) and the difference between the refractive indices of core and cladding is large.
✓In single mode fibers, there is no dispersion.	✓Signal distortion and dispersion takes place in multimode fibers.
✓Signal transmission capacity is less but the single mode fibres are suitable for long distance communication.	✓Signal transmission capacity is more in multimode fibres. They are less suitable for long distance communication.
✓Launching of light into single mode fibers is difficult.	✓ Launching of light into multimode fibers is easy.
✓Fabrication cost is very high.	✓ Fabrication cost is less.

Attenuation in Optical Fibres

- Attenuation is the loss of power suffered by the optical signal as it propagates through the fiber.
- It is also called fiber loss.
- Signal attenuation is defined as "the ratio of the input optical power (P_i) into the fiber to the output optical power received (P_o) at the other end of the fiber".
- The attenuation coefficient of the signal per unit length is given as,

$$\alpha = 10/L \log (Pi/Po) \text{ dB/km}$$

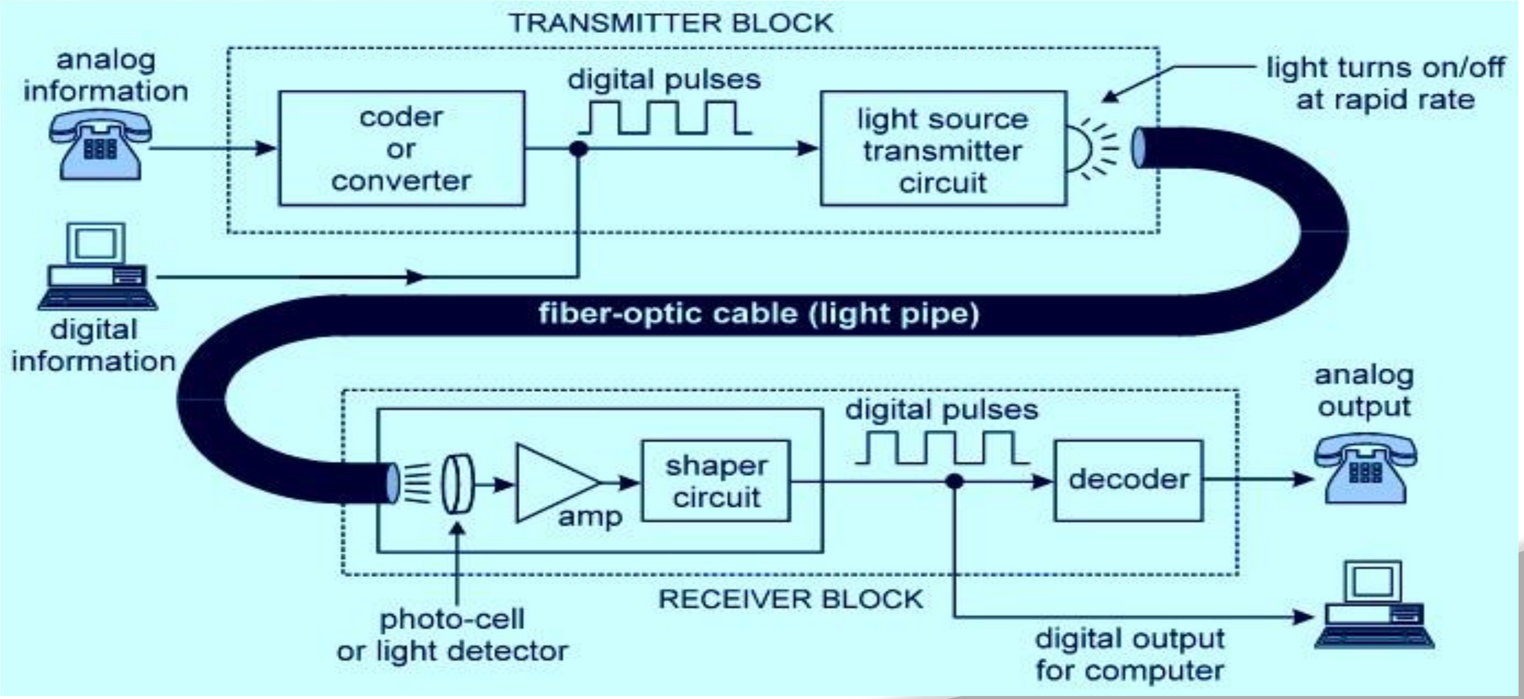
Where, L is the length of the fibre.

- The mechanisms through which attenuation takes place are

1. Absorption losses.
2. Scattering losses.
3. Bending losses.
4. Microbending and Wave guide losses.

Optical Fibres in Communication

- Optical fibre communication system essentially consists of three parts namely, (a) Transmitter (b) Optical fibre and (c) Receiver.
- The Transmitter includes modulator, encoder, light source, drive circuits and couplers.
- Basically, the fibre optic system simply converts an electrical signal to binary data by an encoder.



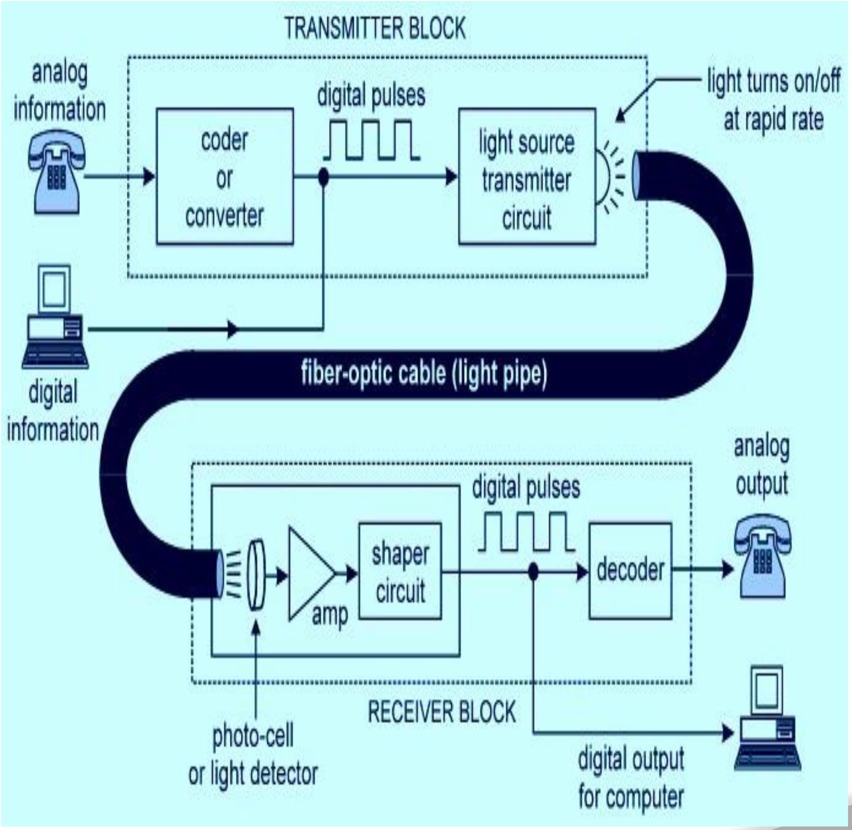
Block diagram of Optical Fibre communication system

- **This binary data comes out as a stream of electrical pulses and these pulses are converted into pulses of optical power, by modulating the light emitted by the light source.**

- **This means that the laser drive circuit directly modulates the intensity of the laser light with the encoded digital signal.**

- **This digital optical signal is launched into the optical fibre cable.**

- **The Couplers in the transmitter, couple the transmitted light signals**



- Finally, at the end of the optical fibre, the signal is fed to the receiver.

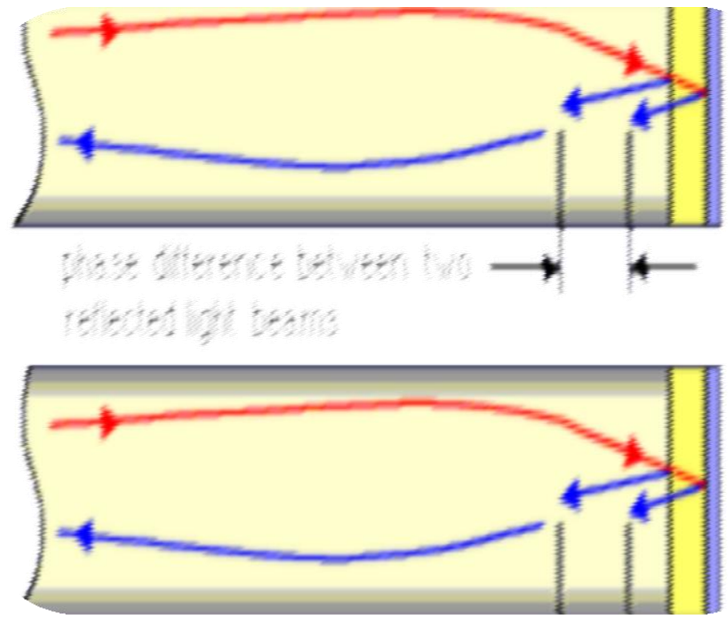
- The Receiver consists of a light detector, which can either be an Avalanche Photo Diode (ADP) or a Positive Intrinsic Negative(PIN) diode.

- In the photo detector, the signal is converted into pulses of electric current, which is then fed to the decoder, which converts the sequence of binary data stream into an analogue signal.

Applications of Optical Fibres

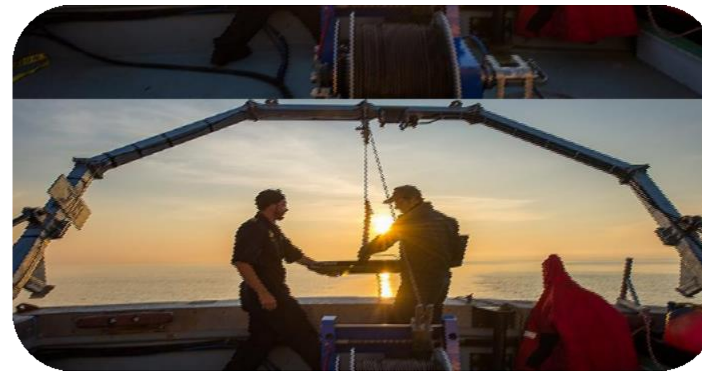
Sensors:

- **Fibers have many uses in remote sensing. In some applications, the sensor is itself an optical fiber.**
- **Optical fibers can be used as sensors to measure strain, temperature, pressure and other quantities.**
- **Extrinsic fiber optic sensors has the ability to reach places which are otherwise inaccessible. An example is the measurement of temperature inside aircraft jet engines.**
- **Extrinsic sensors can also be used in the same way to measure the internal temperature of electrical**



Fibre optic temperature sensor using Phase interference

- Optical fiber can be used to transmit power using a photovoltaic cell to convert the light into electricity.
- Fiber optics are used to connect users and servers in a variety of network settings and help increase the speed and accuracy of data transmission.
- They are also used in military as hydrophones for seismic and SONAR uses, as wiring in aircraft, submarines and other vehicles and also for field networking.
- Broadcast/cable companies are using fiber optic cables for wiring CATV, HDTV, internet, video on-demand



Fibre optics used in SONAR

Fibre optic cable system in Internet

Telecommunication:

- Optics fiber is used by many telecommunications companies to transmit telephone signals, Internet communication, and cable television signals.
- Unlike electrical cables, fiber optics transport information far distances with few repeaters.
- Fiber optic cables can carry a large number of different signals simultaneously through a technique called wavelength division multiplexing.
- Optical fibers are ideally suited

Optical fibres used in Telecommunication

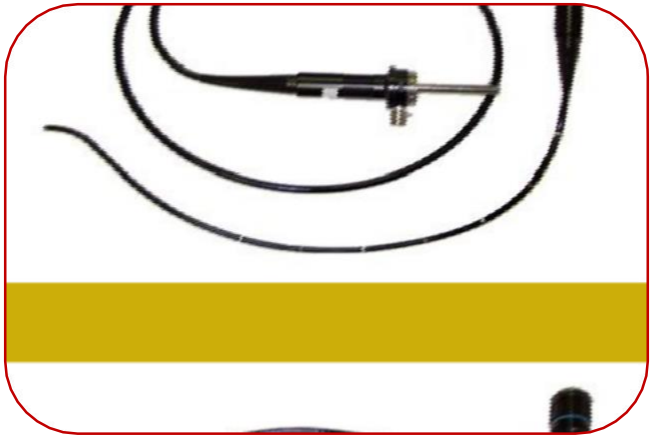
Medical Applications:

- Optical fiber is used in imaging optics. They are used as light guides in medical and other applications where bright light needs to be shone on a target without a clear line-of-sight path.
- A coherent bundle of fibers is used, along with lenses, for a long, thin imaging device called an endoscope, which is used to view objects through a small hole.
- Medical endoscopes are used for surgical procedures to view the internal parts of the human body.
- Industrial endoscopes are used for inspecting anything hard to reach, such

Optical fibre enabling the physician to look and work inside the body without performing surgery

Based on application, the endoscopes are classified into:

- Gastro scope is used to examine the stomach.
- Bronchoscope is used to see the upper passage of lungs.
- Ortho scope is used to see the small spaces within joints.
- Could scope is used to test female pelvic organs.
- Peritonea scope is used to test the abdominal cavity , lower parts of liver and gall bladder.
- In Ophthalmology, lasers guided by the fibres is used to reattach the detached retina.



A flexible Endoscope

Image of a Bronchoscope

MODULE-IV

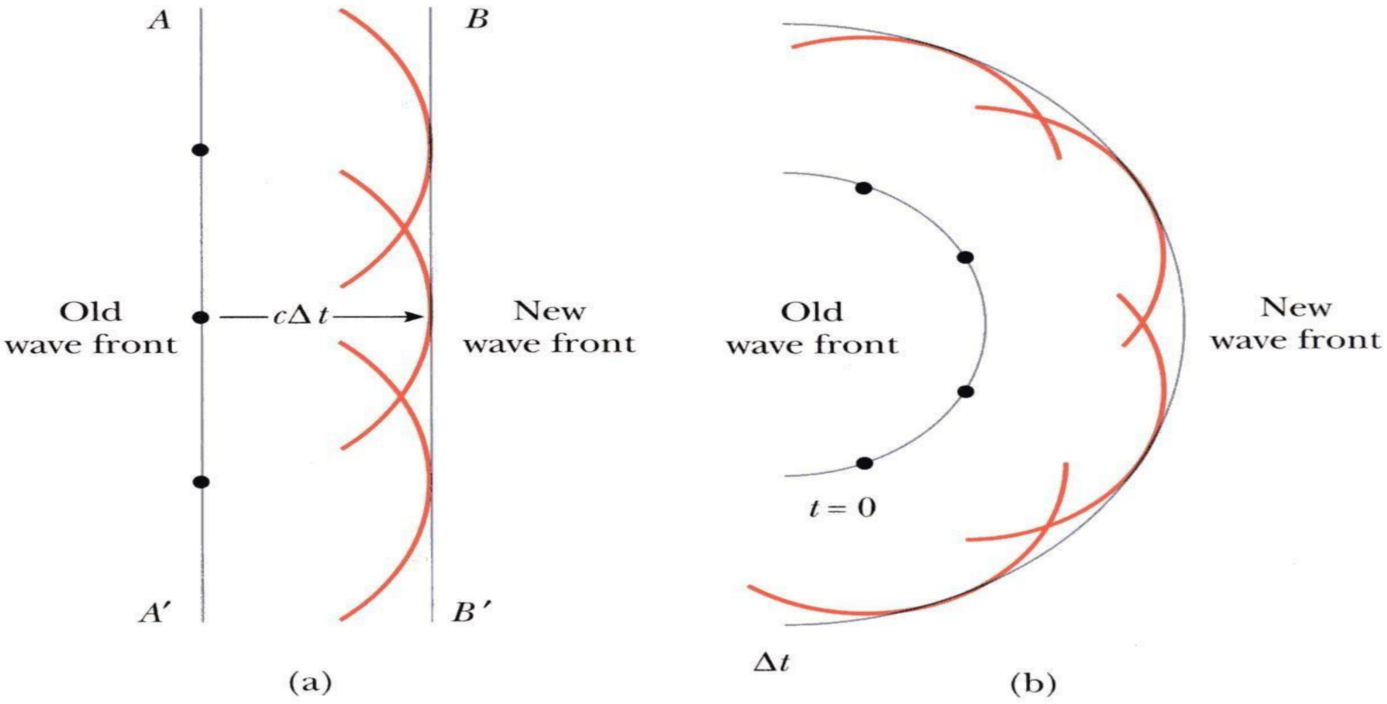
LIGHT AND OPTICS

MODULE-IV

CLOs	Course Learning Outcome
CLO 10	Apply different laws of radiation to understand the phenomenon behind production of light.
CLO 11	Explore the phenomenon of interference in thin films using Newton's rings experiment.
CLO 12	Identify diffraction phenomenon due to slits.

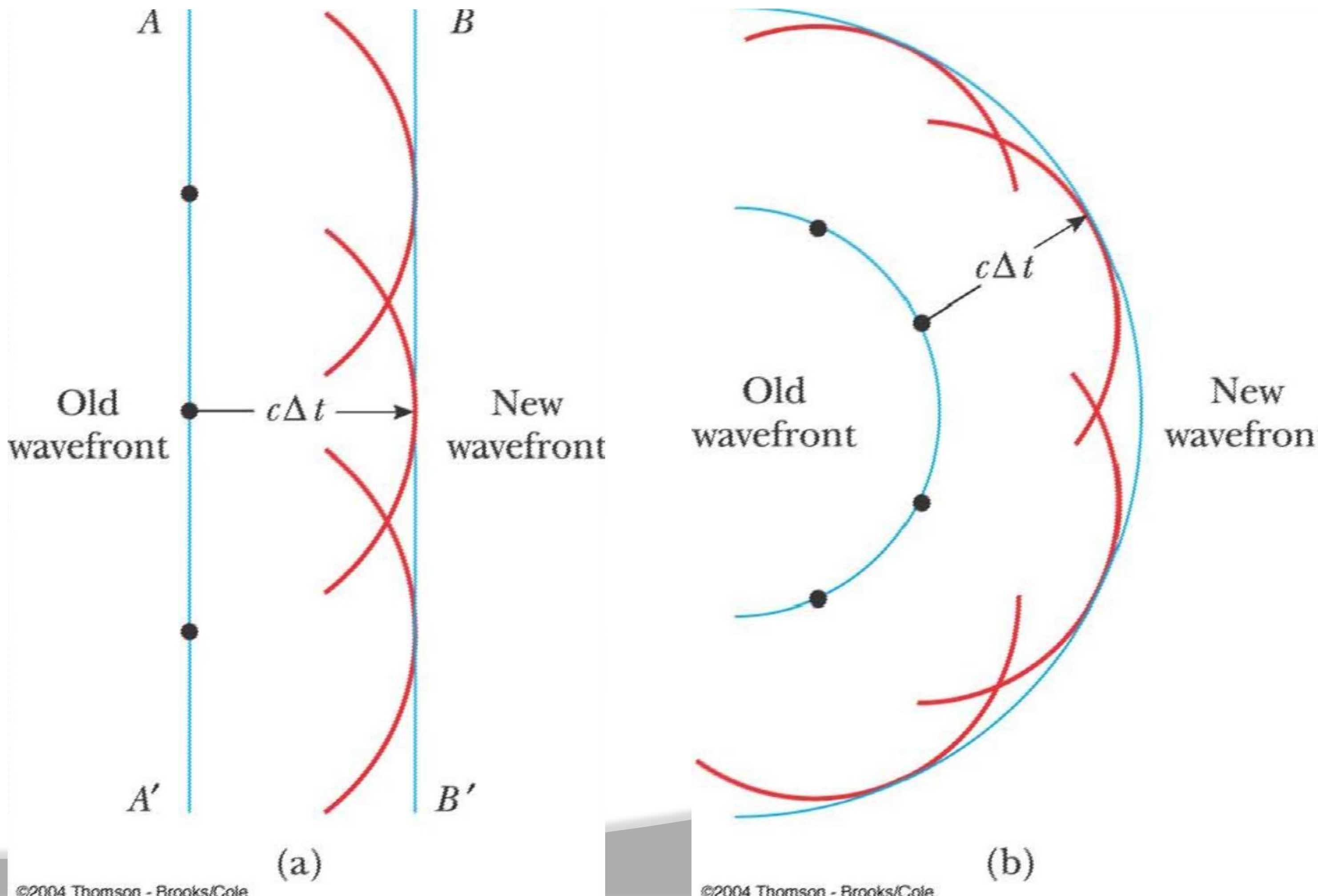
Huygens's Principle

All the points on a wave front can be considered as point sources for the production of spherical secondary Wavelets . After a time " t " the new position of a wave front is the surface tangent to these secondary wavelets



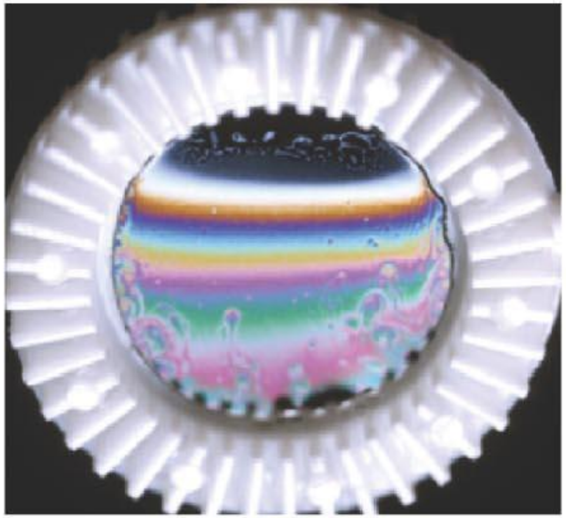
Huygens’s Principle

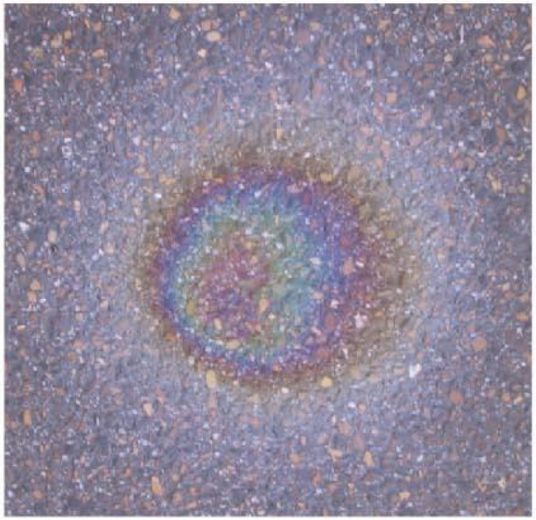
All points on a wave front act as new sources
for the production of spherical secondary waves



Interference in Thin Films

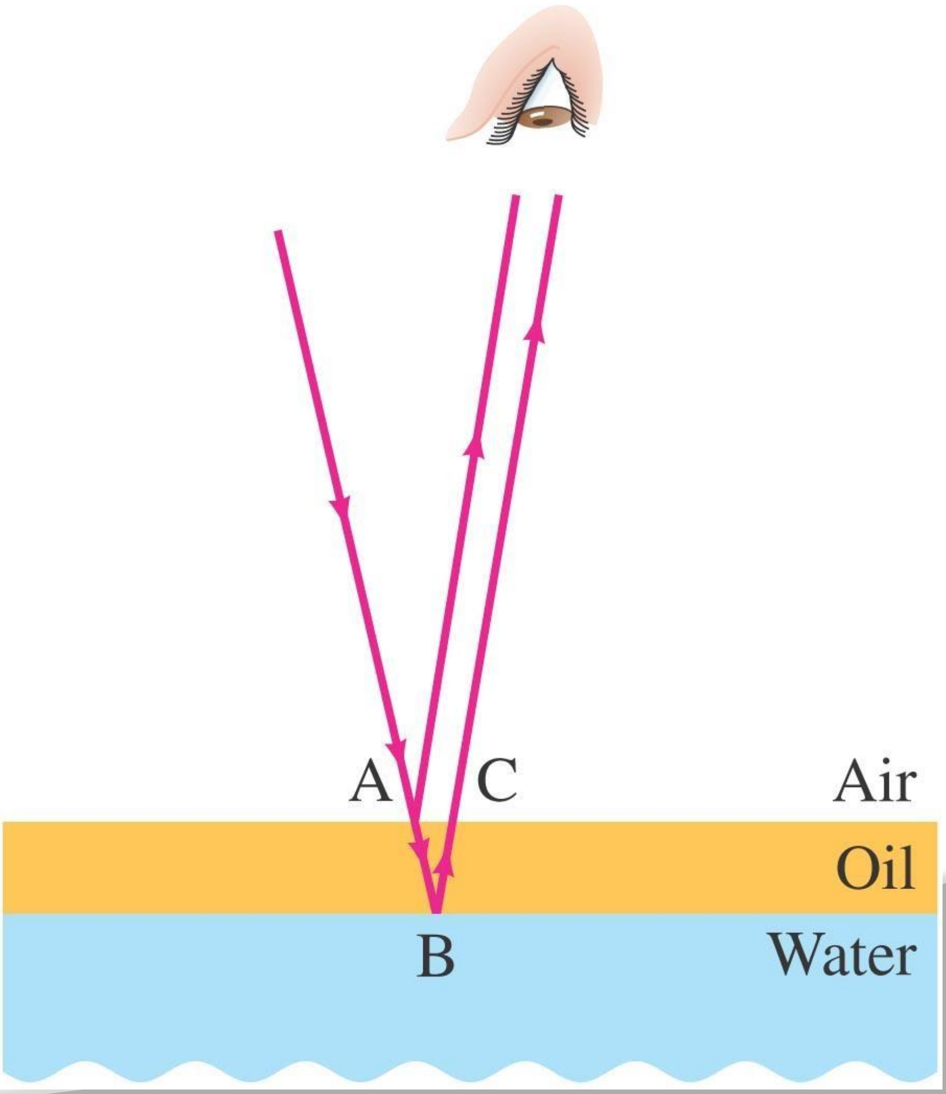
Another way path lengths can differ, and waves interfere, is if they travel through different media. If there is a very thin film of material – a few wavelengths thick – light will reflect from both the bottom and the top of the layer, causing interference. This can be seen in soap bubbles and oil slicks.





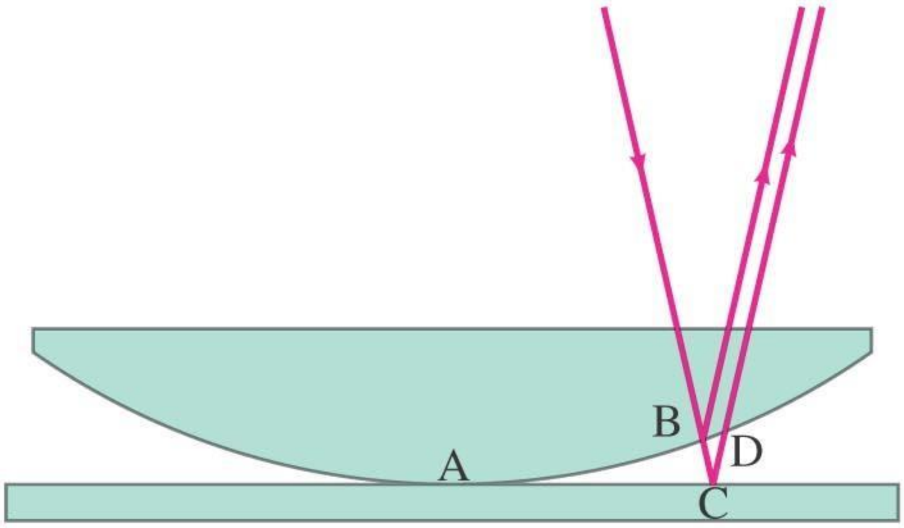
Interference in Thin Films

The wavelength of the light will be different in the oil and the air, and the reflections at points A and B may or may not involve phase changes.

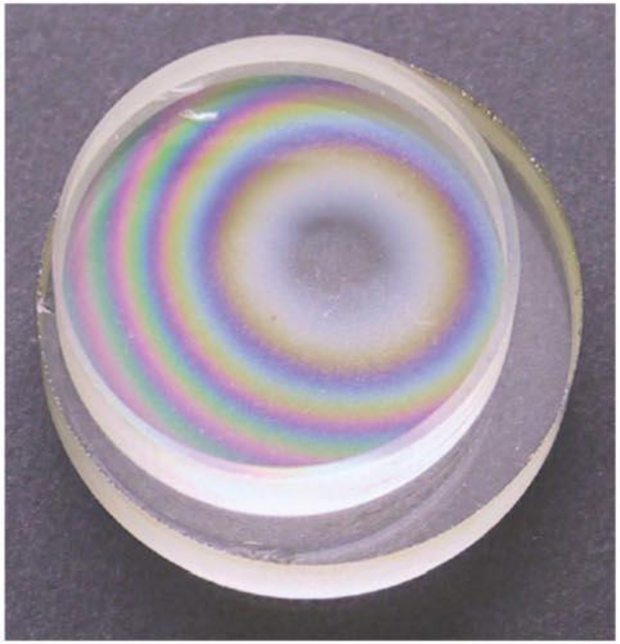


Interference in Thin Films

A similar effect takes place when a shallowly curved piece of glass is placed on a flat one. When viewed from above, concentric circles appear that are called Newton's rings.



(a)



(b)

- In the double-slit experiment, constructive interference occurs when

$$d \sin\theta = m\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \begin{bmatrix}\text{constructive}\\ \text{interference}\\ \text{(bright)}\end{bmatrix}$$

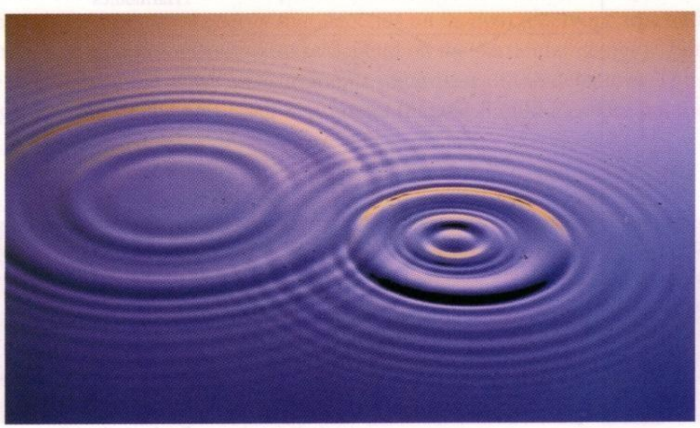
- and destructive interference when

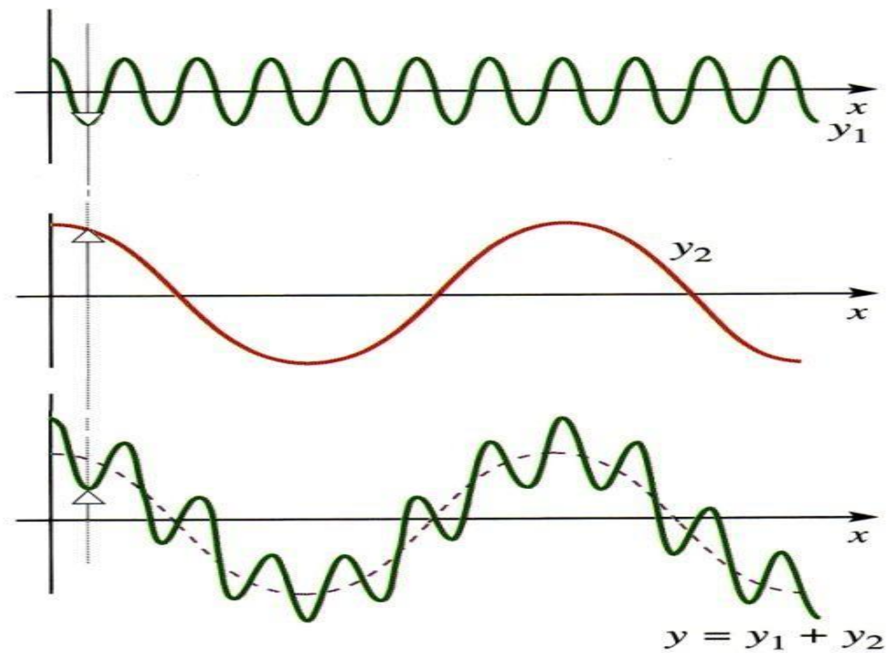
$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad m = 0, 1, 2, \cdots. \qquad \begin{bmatrix}\text{destructive}\\ \text{interference}\\ \text{(dark)}\end{bmatrix}$$

Two sources of light are coherent if they have the same frequency and maintain the same phase relationship.

SUPERPOSITION OF WAVES

- Two waves passing through the same region will superimpose - e.g. the displacements simply add
- Two pulses travelling in opposite directions will pass through each other unaffected
- While passing, the displacement is simply the sum of the individual displacements

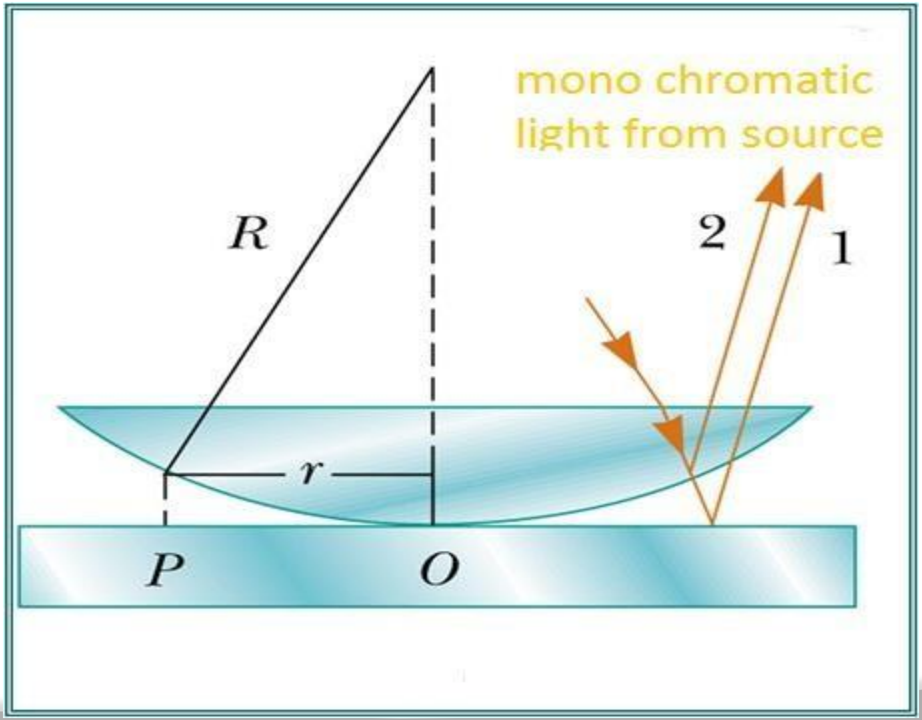




Newton’s Rings

Newton’s rings are one of the best examples for the interference in a non uniform thin film.

When a Plano-convex lens with its convex surface is placed on a plane glass plate, an air film of increasing thickness is formed between the two. The thickness of the film at the point of contact is zero.

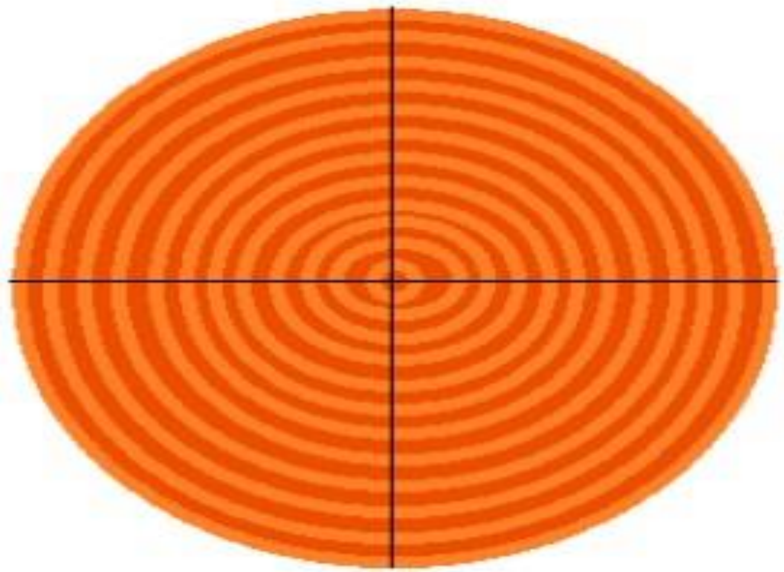


Ray 1
phase change of 180°

ray 2
no phase change

Newton’s Rings

If monochromatic light is allowed to fall normally and the film is viewed in the reflected light, alternate dark and bright rings concentric around the point of contact between the lens and glass plate are seen. These circular rings were discovered by Newton and are called Newton’s rings.



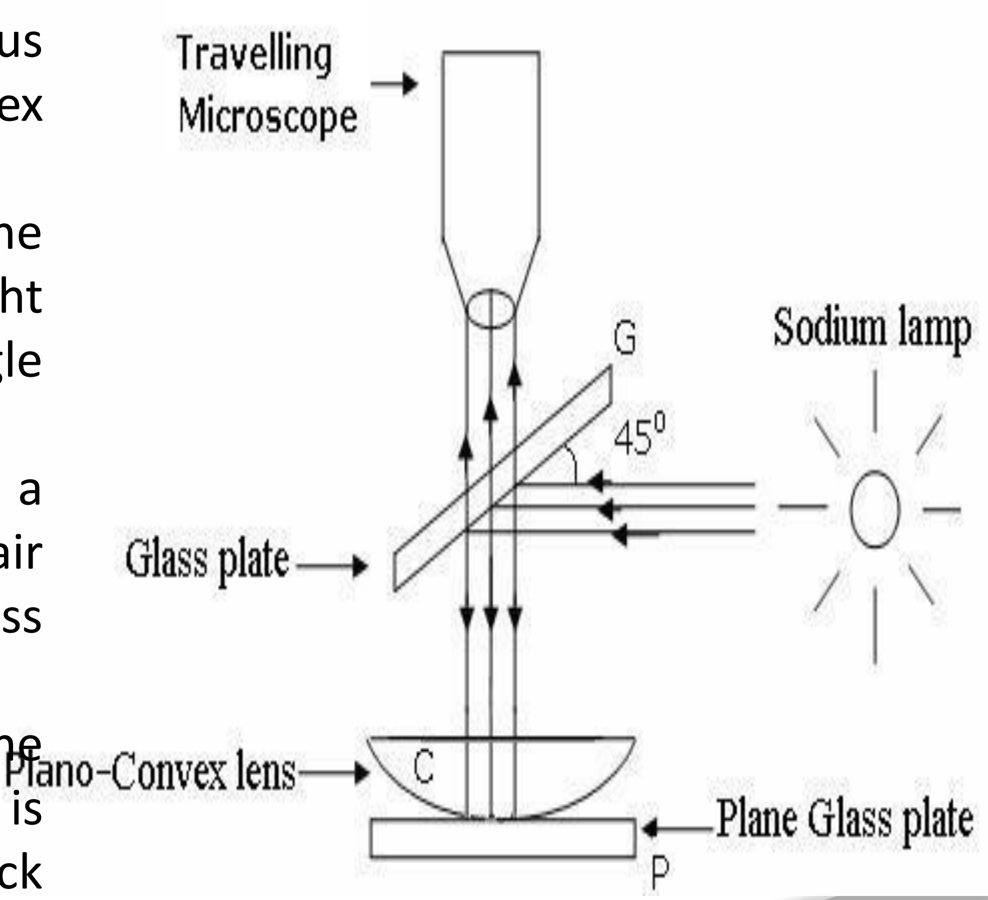
Experimental Arrangement

The Plano-convex lens (L) of large radius of curvature is placed with its convex surface on a plane glass plate (P).

The lens makes the contact with the plate at 'O'. The monochromatic light falls on a glass plate G held at an angle of 45° with the vertical.

The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P.

A part of the light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of the plate.



Explanation of Newton’s Rings

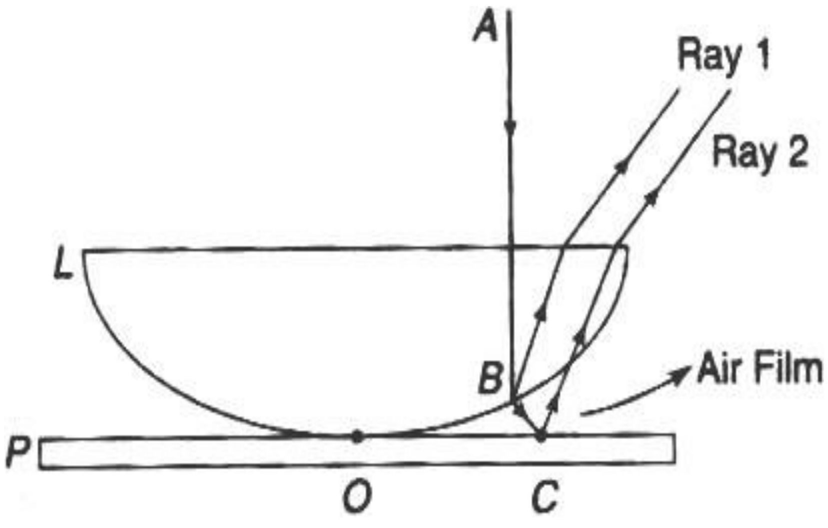


Fig.

Newton’s rings are formed due to interference between the light rays reflected from the top and bottom surfaces of air film between the plate and the lens.

The formation of Newton’s rings can be explained with the help of Fig.

A part of the incident monochromatic light AB is reflected at B (glass-air boundary) in the form of the ray (1) with any additional phase (or path) change. The other part of light is refracted along BC. Then at C (air-glass boundary), it is again reflected in the form of the ray (2)

with additional phase change of π or path change of $\lambda/2$

Explanation of Newton's Rings

The rings are observed in the reflected light, the path difference between them is $2\mu t \cos r + \lambda/2$

. For air film $\mu = 1$ and for normal incidence r=0, path difference is $2t + \lambda/2$

.

At the point of contact t=0, path difference is $\lambda/2$, i.e., the reflected light at the point of contact suffers phase change of π. Then the incident and reflected lights are out of phase and interfere destructively. Hence the central spot is dark.

The condition for bright ring is $2t + \lambda/2 = n\lambda$

$$2t = (2n-1)\lambda/2 \quad \text{where } n=1, 2, 3\ldots$$

The condition for dark ring is $2t + \lambda/2 = (2n+1)\lambda/2$

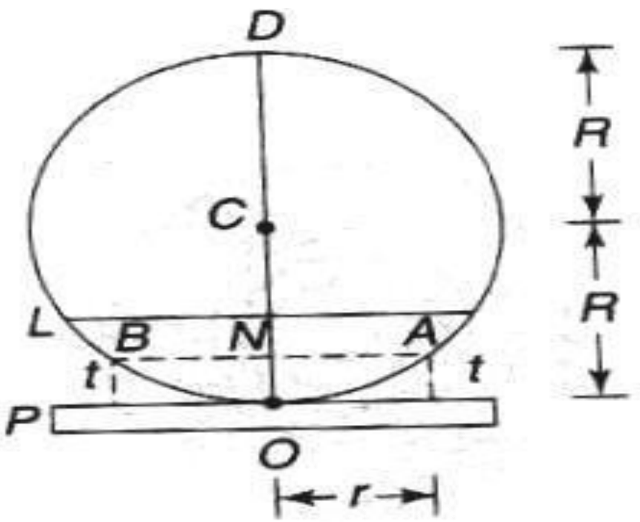
$$2t = n\lambda \quad \text{where } n = 0,1,2,3\ldots$$

For monochromatic light, the bright and dark rings depend on thickness of the air film. For a Newton's rings system, the focus of points having same thickness lie on a circle having its centre at the point of contact. Thus, we get bright and dark circular rings with the point of contact as the center.

Theory of Newton's Rings

To find the diameters of dark and bright rings,

let 'L' be a lens placed on a glass plate P.



The convex surface of the lens is the part of spherical surface (Fig.) with center at 'C'. Let R be the radius of curvature and r be the radius of Newton's ring corresponding to the film thickness 't'.

Substituting the values, r x r = t x (2R-t)

From the property of a circle,

NA x NB = NO x ND

$$r^2 = 2Rt - t^2$$

Derivation

$$t = \frac{r^2}{2R}$$

For bright ring, the condition is

$$2t = (2n-1)\frac{\lambda}{2}$$

$$2\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

Replacing r by $\frac{D}{2}$, the diameter of n^{th} bright ring will be

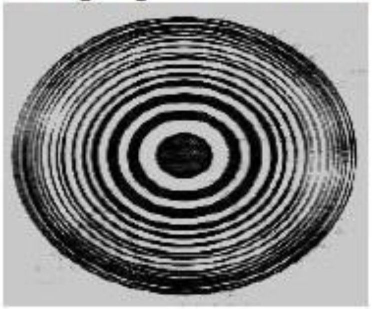
$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D = \sqrt{2n-1}\sqrt{2\lambda R}$$

$$D \,\alpha \sqrt{2n-1}$$

$$D\sqrt{odd\ natural\ number}$$

Thus, the diameters of the bright rings are proportional to the square root of odd natural numbers.



Derivation

$$2t = n\lambda$$

$$2\frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R$$

$$D^2 = 4n\lambda R$$

$$D = 2\sqrt{n\lambda R}$$

$$D \propto \sqrt{n}$$

$$D \propto \sqrt{natural\ number}$$

Thus, the diameters of dark rings are proportional to the square root of natural numbers.
With the increase in the order (n), the rings get closer and the fringe width decreases and are shown in Fig

Derivation

Determination of Wavelength of a Light Source

Let R be the radius of curvature of a Plano-convex lens, λ be the wavelength of light used. Let D_m and D_n are the diameters of m[th] and n[th] dark rings respectively. Then

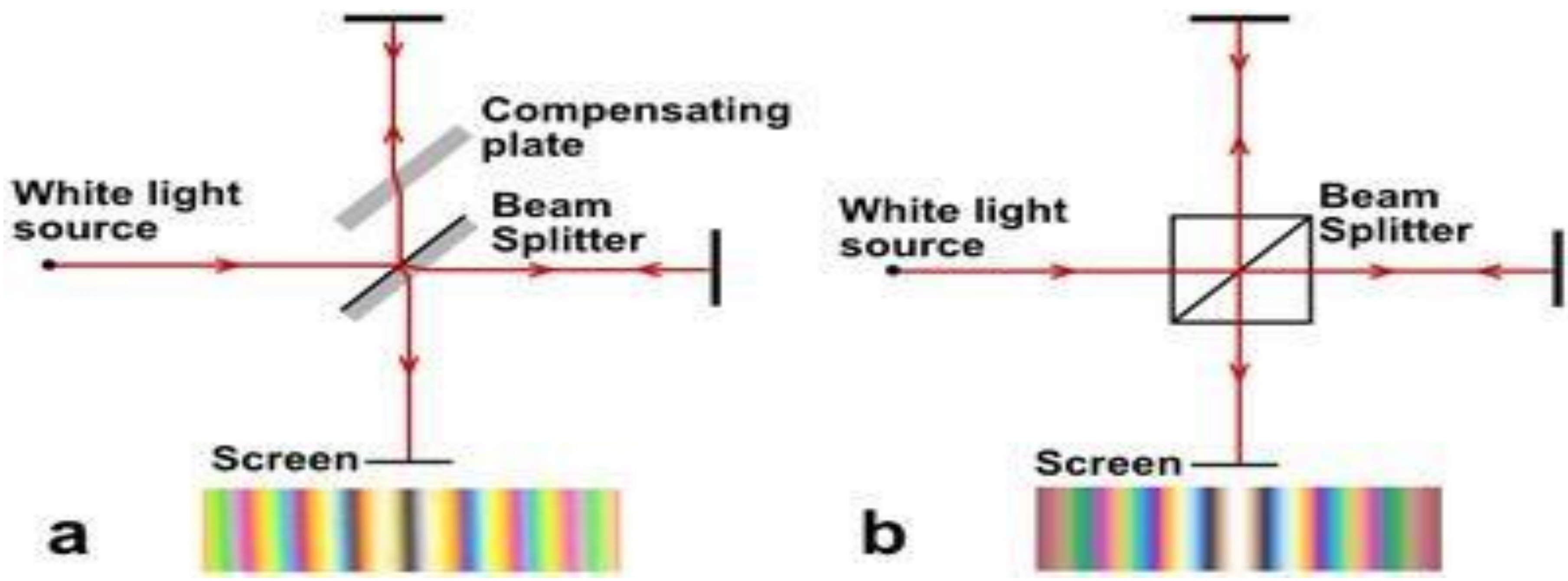
D_m^2=4mλR

$$\text{And } D_n^2=4(n)\ \lambda R$$

$$D_n^2-D_m^2=4(m\text{-}n)\ \lambda R$$

$$\lambda=\frac{D_n^2-D_m^2}{4(m-n)R}$$

Michelson Interferometer



Formation of Fringes

In Michelson interferometer the two coherent sources are derived from the principle of division of amplitude. The parallel light rays from a monochromatic source are incident on beams splitter (glass plate) *G1 which is semi silvered on its back surface and mounted at 45° to the axis.*

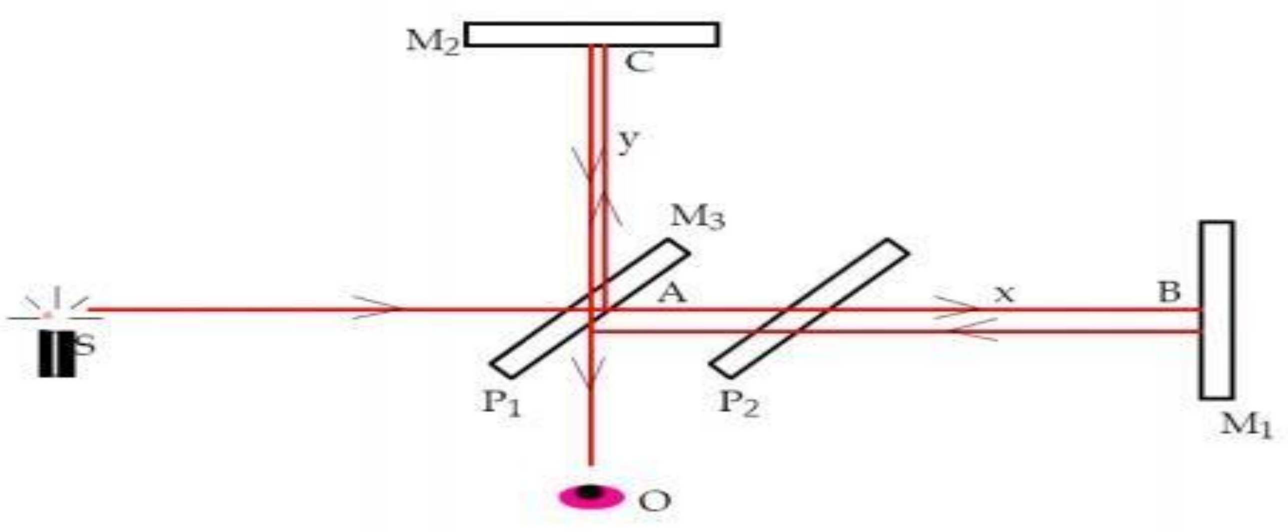
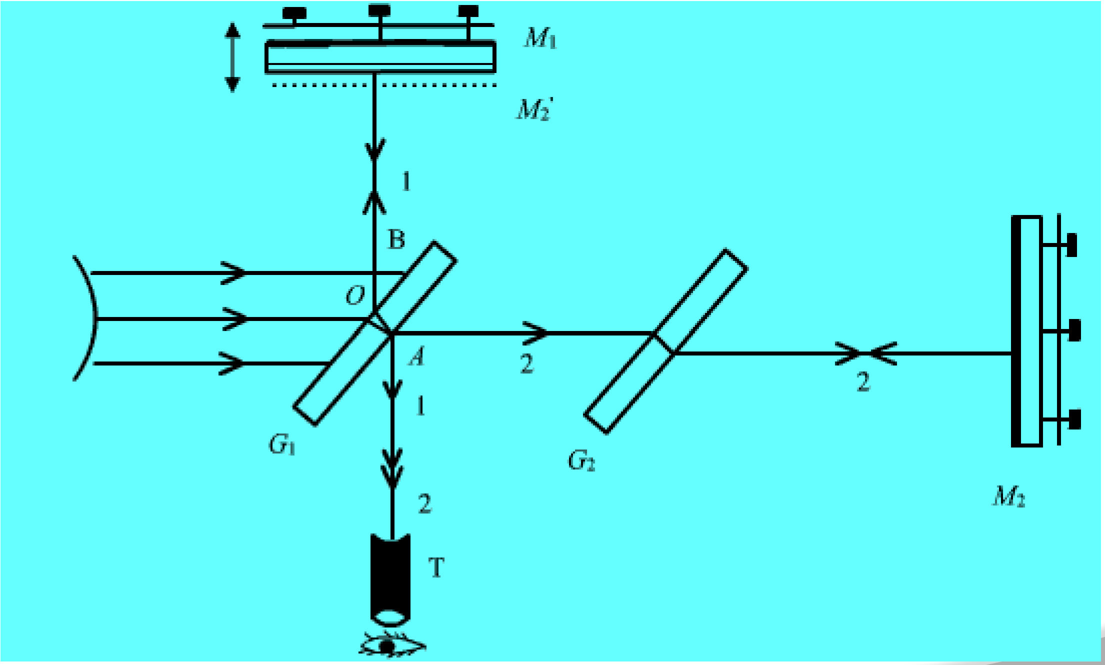


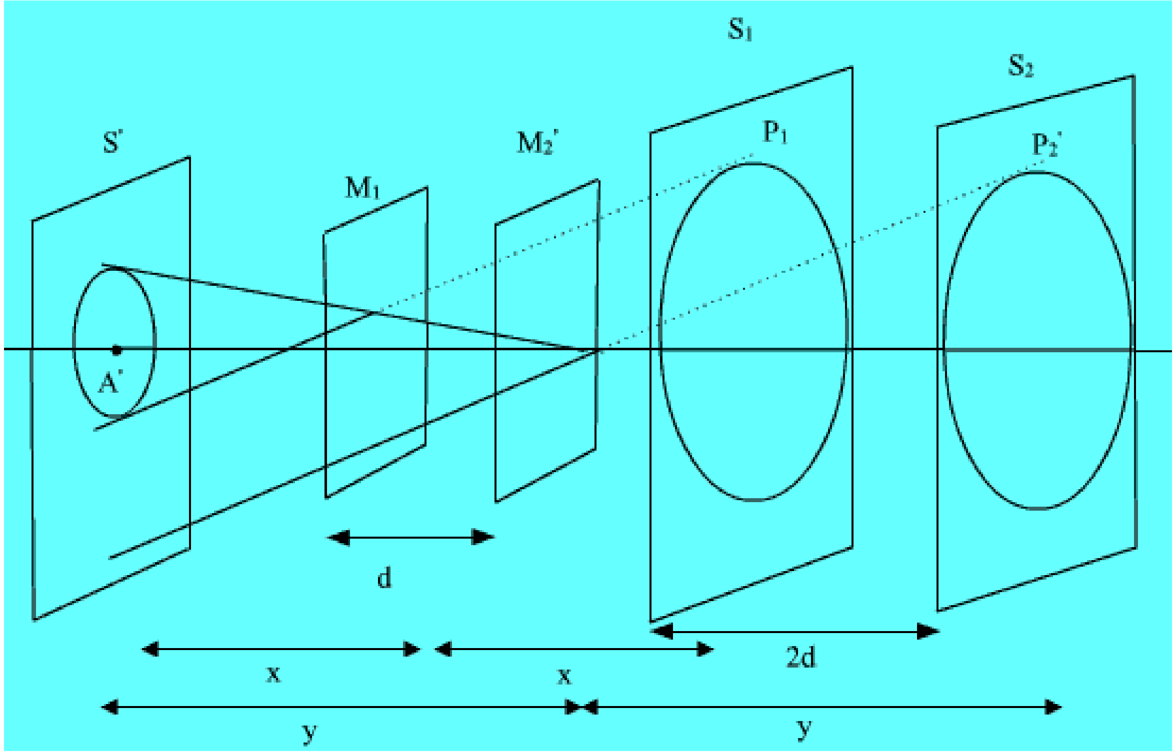
Figure 1: The Michelson interferometer.

Formation of Fringes

An observer at *'T' will see the images of* mirror *M2 and source S* *(M'2 and S' respectively)* *through beam splitter along with the* mirror *M1. S1 and S2 are the images of source in mirrors M1 and M2 respectively. The position of these* elements in figure depends upon their relative distances from point *A.*

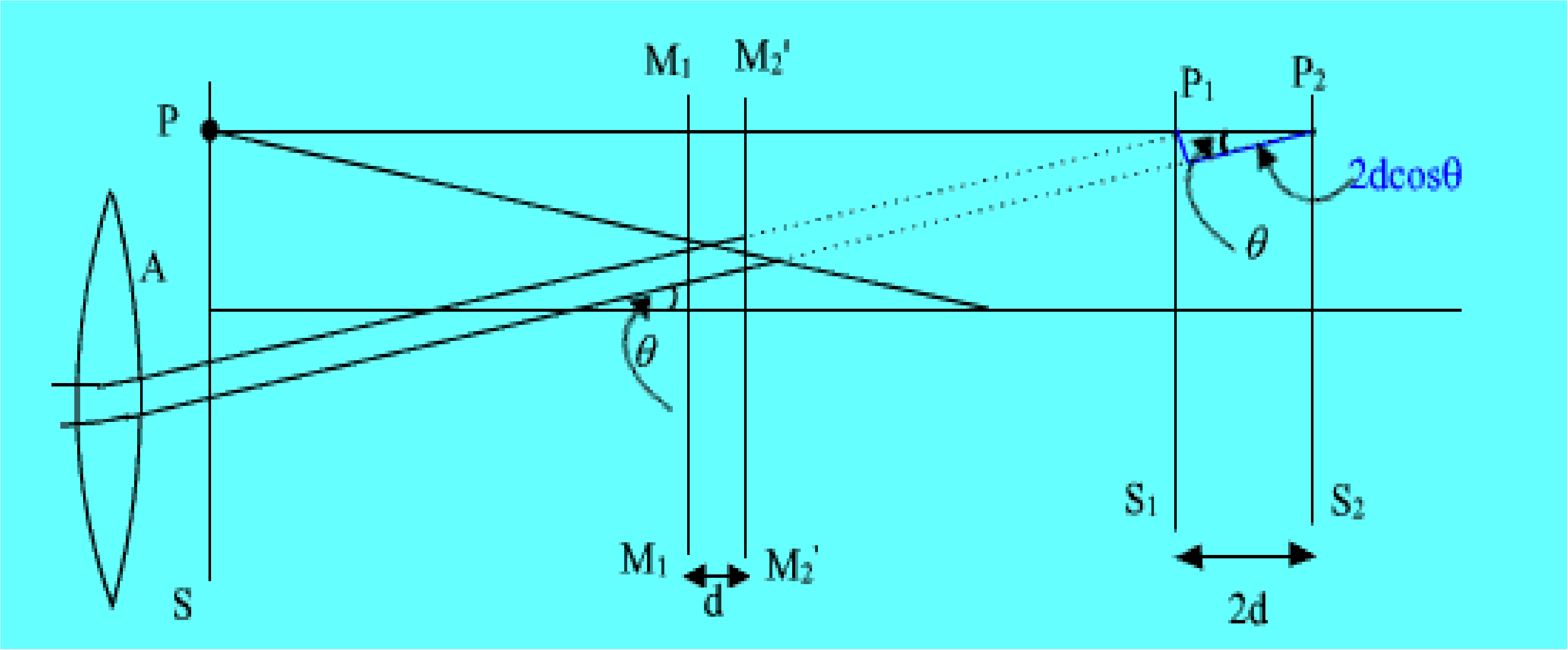


Formation of Fringes



Light from a point (say *P)* *from extended source appears to come from corresponding coherent* points *P1 and P2 on S1 and S2 .*

Formation of Fringes



Formation of Fringes

If *‘d ' is the separation between mirrors M1 and M2' then ‘2d' is the separation between virtual* sources *S1 and S2 The path difference between the two parallel rays coming from* point *P1 and P2 respectively and reaching the eyepiece is equal to 2d Cosϑ*

$$2d \cos\theta = n\lambda \text{ (Bright)}$$

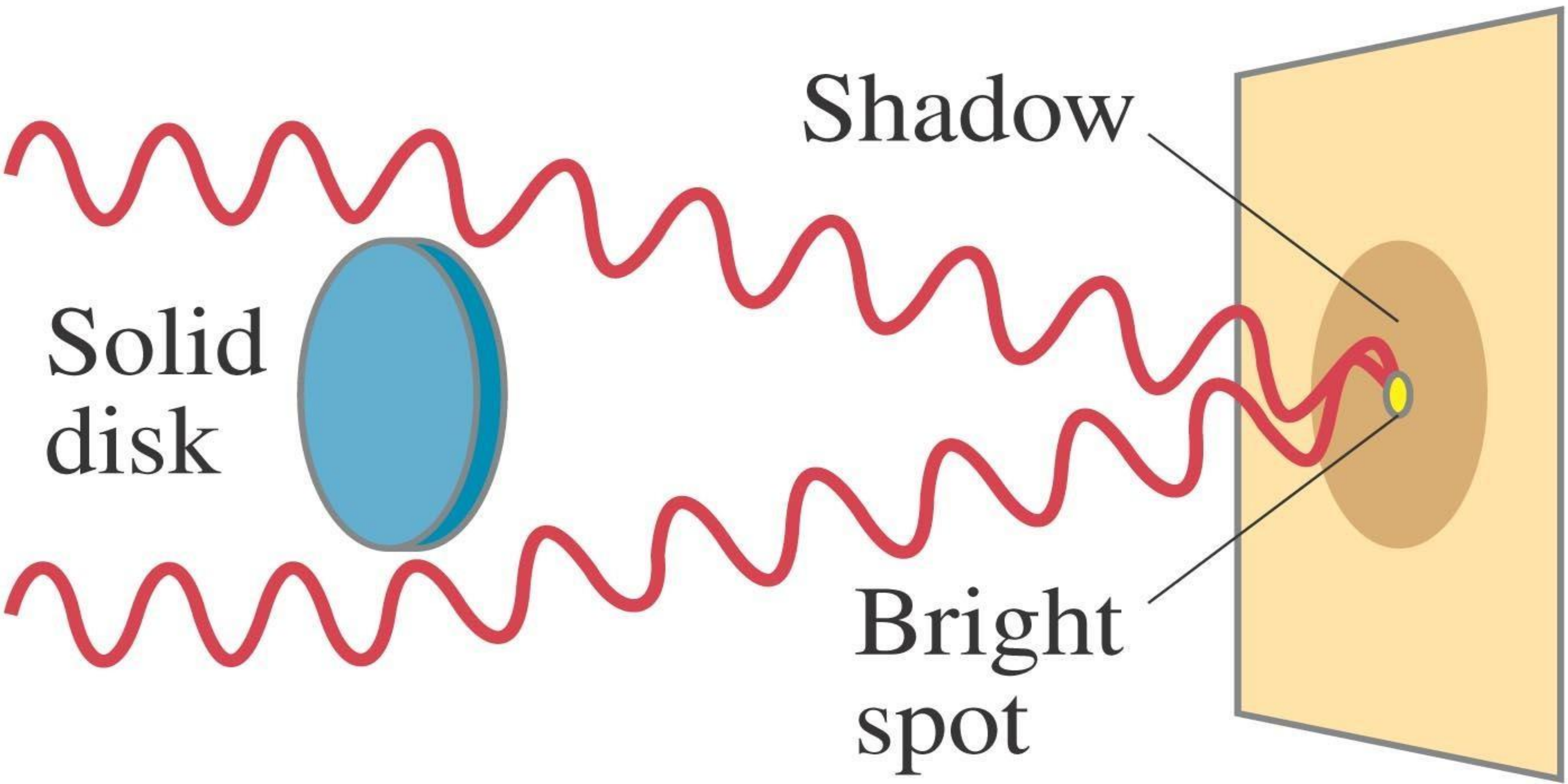
$$2d \cos\theta = (2n+1)\lambda/2 \text{ (dark)}$$

These fringes are concentric rings or straight line depending upon the mutual inclination of mirrors *M1 and M2 (M2’). If mirrors M1 and M2 are parallel to each other the case similar to the air film* between two parallel plate and fringes formed are concentric rings. Michelson interferometer is used to determine the wavelength of monochromatic source, the difference between two wavelengths, determination of thickness/refractive index of thin transparent sheet.

DIFFRACTION

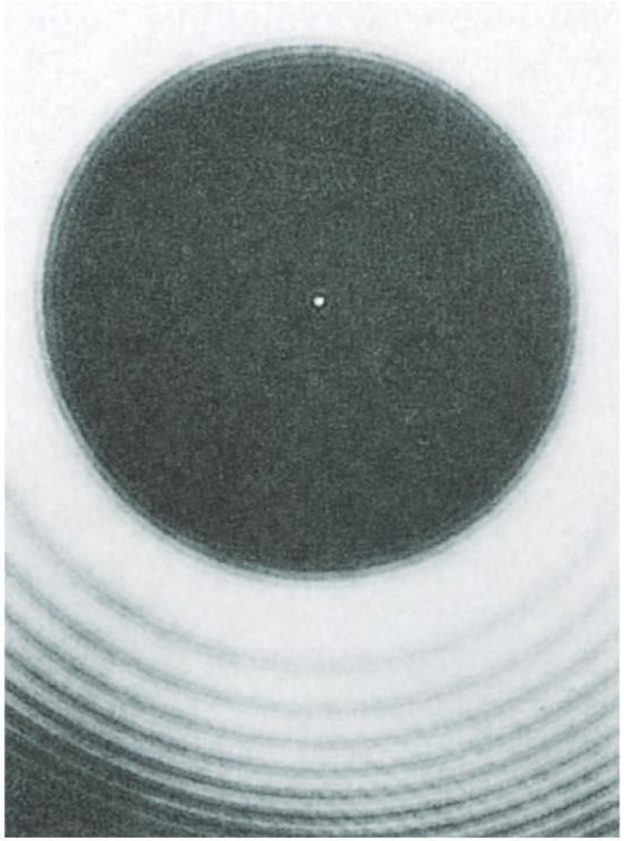
Diffraction by a Single Slit or Disk

If light is a wave, it will diffract around a single slit or obstacle



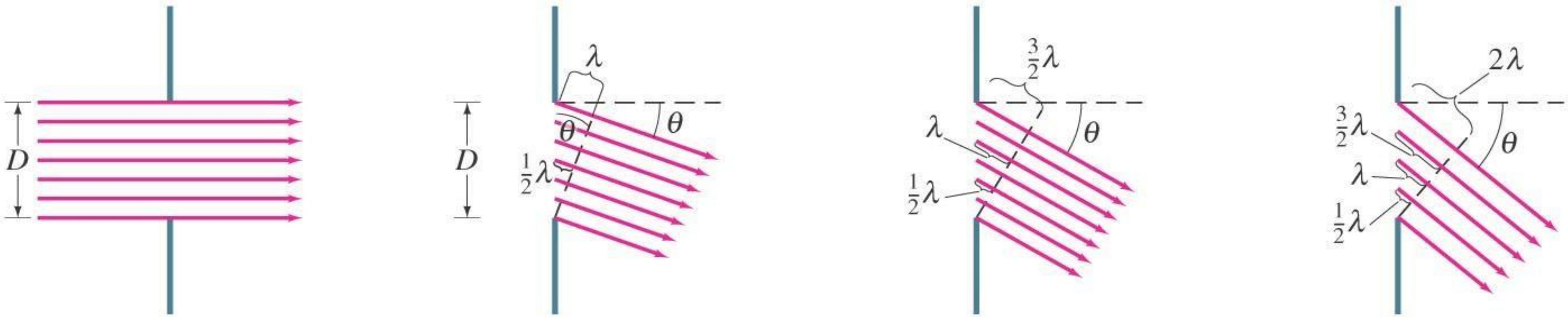
Diffraction by a Single Slit or Disk

The resulting pattern of light and dark stripes is called a diffraction pattern.



Diffraction by a Single Slit or Disk

This pattern arises because different points along a slit create wavelets that interfere with each other just as a double slit would.



(a) $\theta = 0$ Bright

(b) $\sin\theta = \frac{\lambda}{D}$ Dark

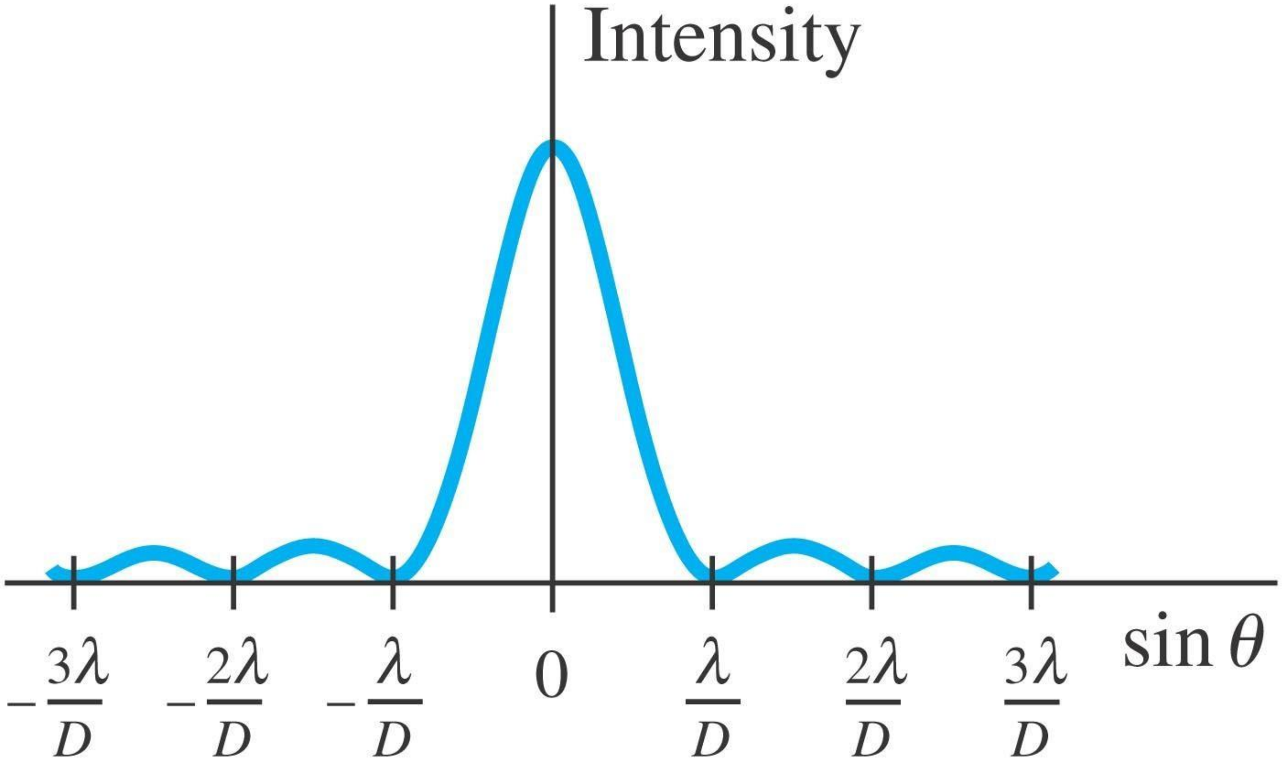
(c) $\sin\theta = \frac{3\lambda}{2D}$ Bright

(d) $\sin\theta = \frac{2\lambda}{D}$ Dark

Diffraction by a Single Slit or Disk

The minima of the single-slit diffraction pattern occur when

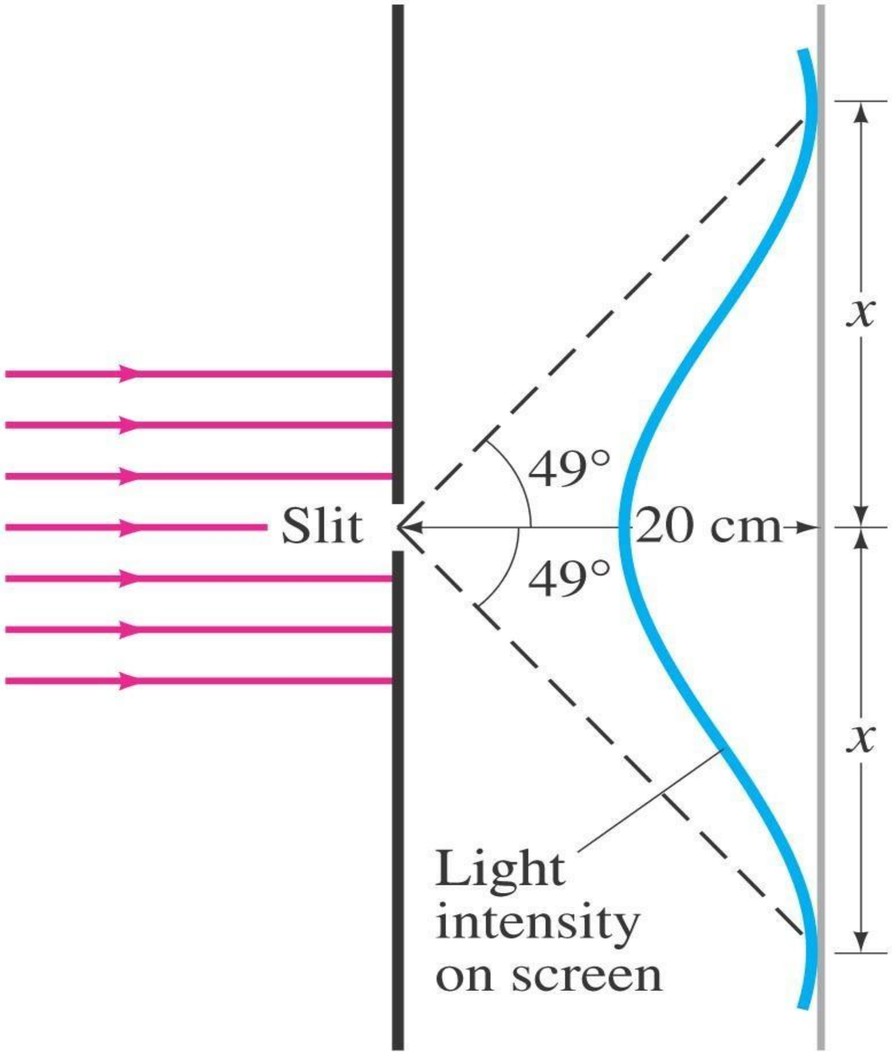
$$D \sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots. \qquad \text{[minima]}$$



Diffraction by a Single Slit or Disk

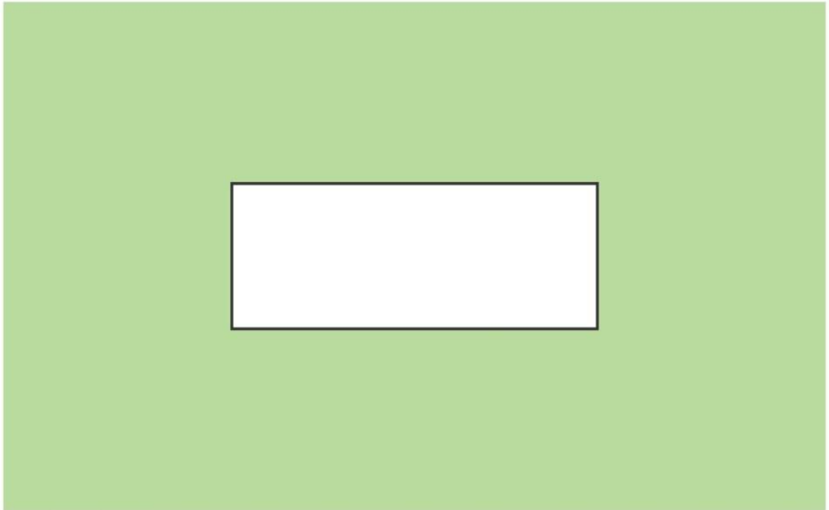
Example 35-1: Single-slit diffraction maximum.

Light of wavelength 750 nm passes through a slit 1.0×10^{-3} mm wide. How wide is the central maximum (a) in degrees, and (b) in centimeters, on a screen 20 cm away?



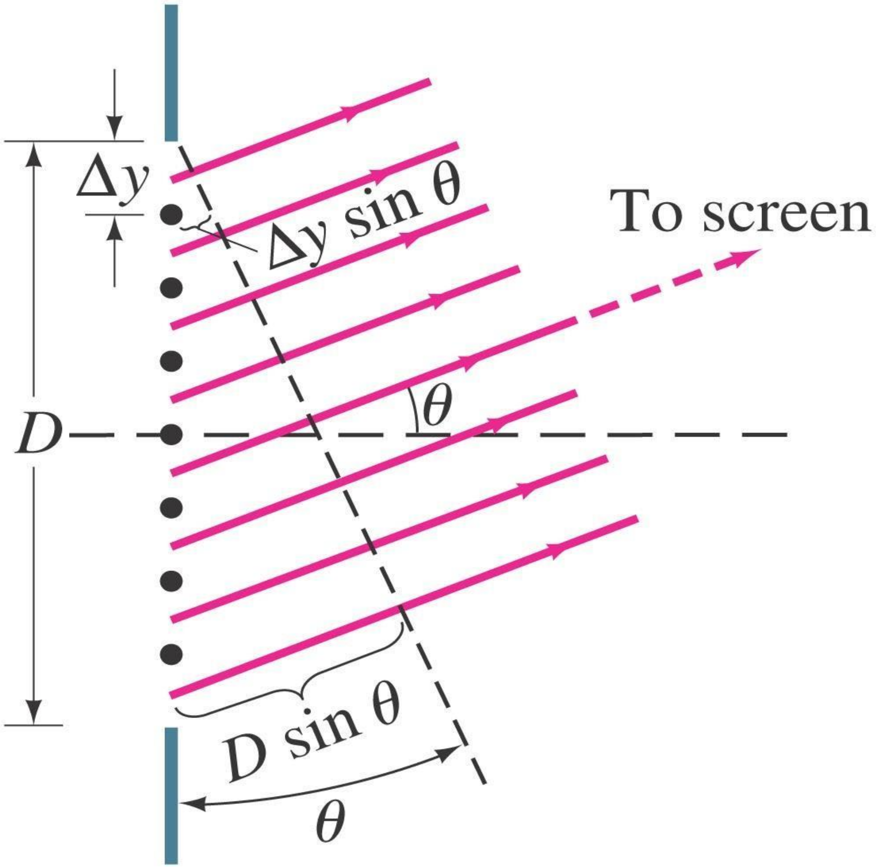
Conceptual Example 35-2: Diffraction spreads.

Light shines through a rectangular hole that is narrower in the vertical direction than the horizontal. (a) Would you expect the diffraction pattern to be more spread out in the vertical direction or in the horizontal direction? (b) Should a rectangular loudspeaker horn at a stadium be high and narrow, or wide and flat?



Intensity in Single-Slit Diffraction Pattern

Light passing through a single slit can be divided into a series of narrower strips; each contributes the same amplitude to the total intensity on the screen, but the phases differ due to the differing path lengths:

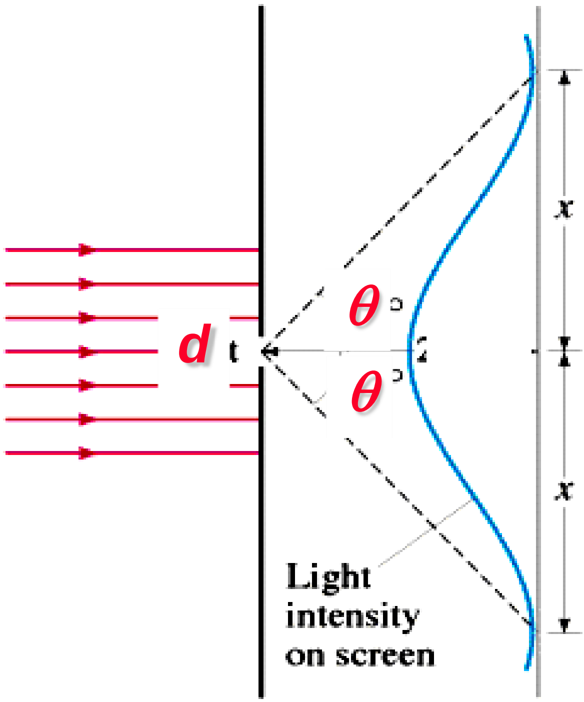


$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$$

Intensity in Single-Slit Diffraction Pattern

The angle at which one finds the first minimum is:

The central bright spot can be narrowed by having a smaller angle. This in turn is accomplished by widening the slit.



MODULE-V

HARMONIC OSCILLATIONS AND WAVES IN ONE DIMENSION

MODULE-V

CLOs	Course Learning Outcome
CLO 13	Acquire knowledge of basic harmonic oscillators and discuss in detail different types of harmonic oscillators.
CLO 14	Describe the steady state motion of forced damped harmonic oscillator.
CLO 15	Acquire knowledge of reflection and transmission of waves at a boundary of media.

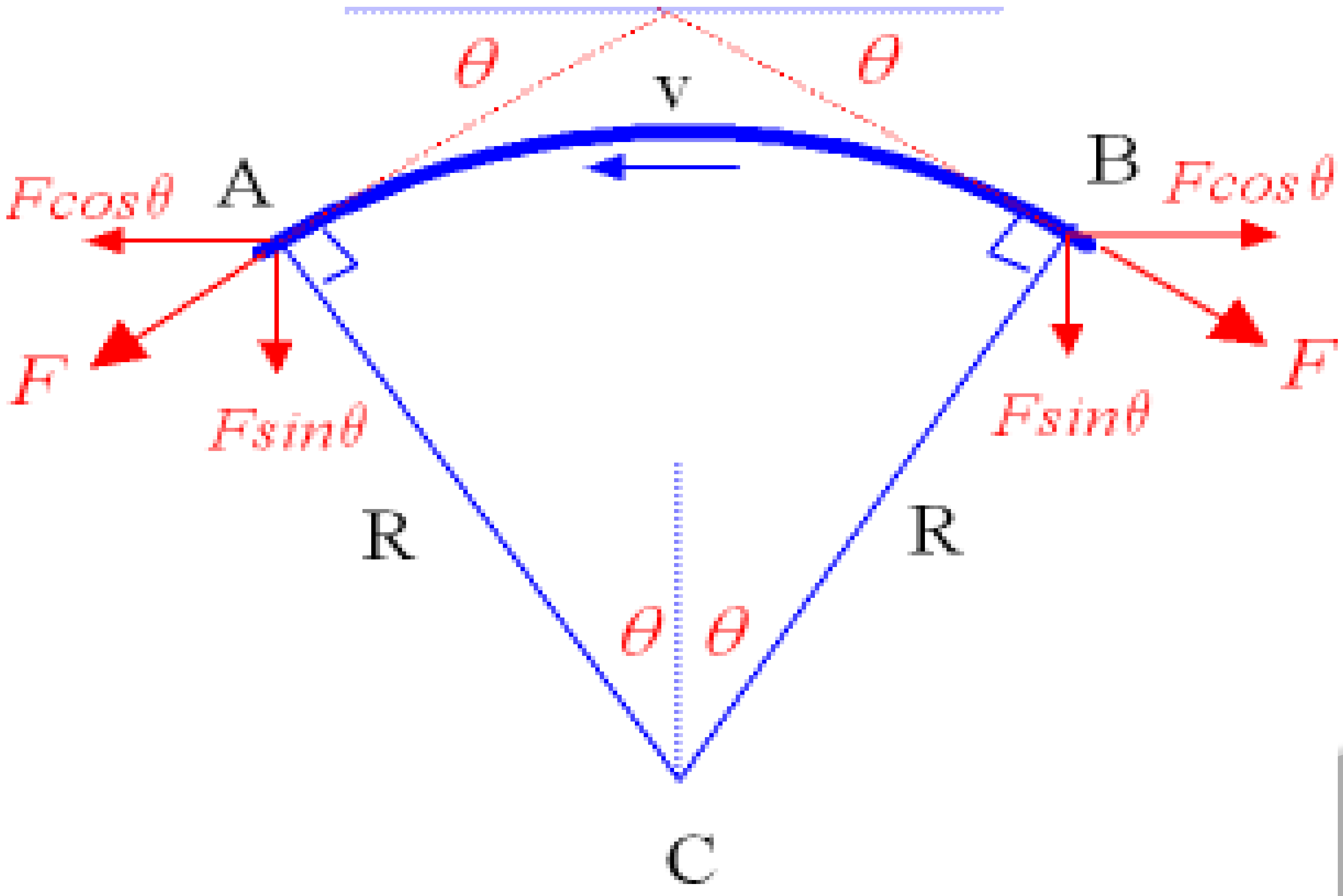
INTRODUCTION

- A wave is a disturbance of a medium which transfers energy from one place to another place without transport of matter. The medium for ocean waves is water, for example. When a string, fixed at both ends, is given a vertical hit by a stick, a dent appears in it that travels along the string.
  - **Mechanical Waves -** Those waves resulting from the physical displacement of part of the medium from equilibrium.
- **Electromagnetic Waves -** Those wave resulting from the exchange of energy between an electric and magnetic field.
  - **Matter Waves -** Those associated with the wave-like properties of elementary particles.

DEFINITIONS

- **Amplitude - (s_o) Maximum value of the displacement of a particle in a medium (radius of circular motion).**
- **Wavelength - (l) The spatial distance between any two points that behave identically, i.e. have the same amplitude, move in the same direction (spatial period)**
- **Wave Number - (k) Amount the phase changes per unit length of wave travel. (spatial frequency, angular wavenumber)**
- **Period - (T) Time for a particle/system to complete one cycle.**
- **Frequency - (f) The number of cycles or oscillations completed in a period of time.**
- **Angular Frequency - (w) Time rate of change of the phase.**

THE WAVE EQUATION ON A STRING



THE WAVE EQUATION ON A STRING

If we model the peak of a wave as it passes through the medium (the string) at speed v as shown in *above figure*.
consider that the peak segment is under a tensile force *F* that pulls it in opposite directions.
The hump can be looked at as a portion of a circle from *A* to *B* with its center at *C*.
The hump is being pulled down by a force of magnitude *2Fsinϑ*
This pulling down force passes through the center and therefore acts as a centripetal force for the segment that is equal to

therefore,

$$2F\,\text{Sin}\theta = \frac{Mv^2}{R} \quad \frac{Mv^2}{R}$$

. For small angles and in radians, $\sin\theta = 2\theta$. The formula becomes:

THE WAVE EQUATION ON A STRING

$$2F\,\theta = \frac{Mv^2}{R}$$

where M = the mass of the strin

If we calculate *mass M of the hump*, it results in $M = 2\mu R\theta$.

This is because the *length of the hump* is $2R\theta$ and μ is the mass per unit length of the string. In otherwords,

μ= *mass /length*.

$$2F\,\theta = \frac{2R\mu\theta\, v^2}{R}$$

$$v = \sqrt{\frac{F}{\mu}} \quad \text{where } \mu = \frac{M}{L}$$

is mass per unit length of the string.

Harmonics are group of frequencies to produce higher notes

Harmonic wave displacement *at any point x and at any instant t, y(x,t).* For harmonic waves, such equation has the general form:

$$y(x,t) = A \sin(kx - \omega t + \varphi)$$

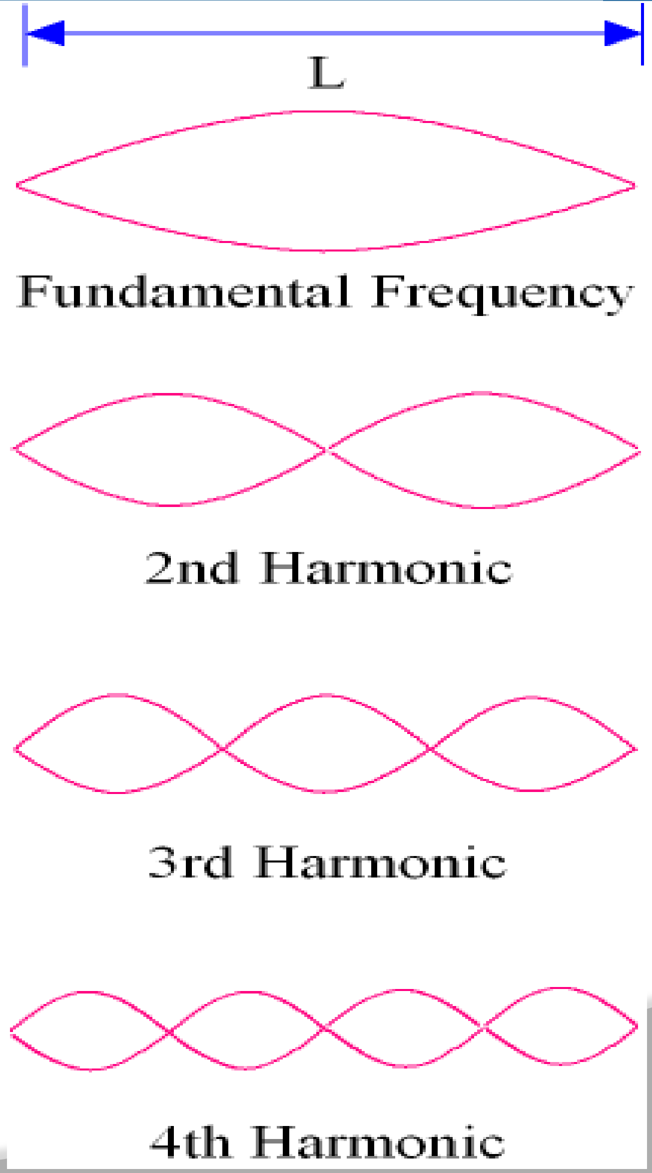
$$\text{where } \boxed{k = \frac{2\pi}{\lambda}}, \quad \boxed{\omega = \frac{2\pi}{T}},$$

$$\boxed{v = f\lambda}, \text{ and } \boxed{\phi = \text{phase angle}}.$$

RESONANT STANDING WAVES IN A STRING

It is also clear from *figure.* *That* the *each loop is one half of the wavelength* in each case. When the entire length of the rope accommodates *one loop only*, it is called *the fundamental frequency* and that is the lowest possible frequency.

The subsequent 2-loop, 3-loop, 4-loop, and ... cases are called the *2nd, 3rd, 4th, and ....* *harmonics* of that *fundamental frequency* as shown on the right



RESONANT STANDING WAVES IN A STRING

For 1 loop: $L = 1\,\frac{\lambda_1}{2}$

For 2 loops: $L = 2\,\frac{\lambda_2}{2}$

For 3 loops: $L = 3\,\frac{\lambda_3}{2}$

For 4 loops: $L = 4\,\frac{\lambda_4}{2}$

For n loops: $L = n\,\frac{\lambda_n}{2}$ or,

$$\lambda_n = \frac{2L}{n} \quad \text{or,} \quad \boxed{f_n = \frac{nv}{2L}}$$

Since $v = \sqrt{\frac{F}{\mu}}$; thus,

$$\boxed{f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}}$$

The speed C of waves in a rope under tension is

$$C = \frac{\sqrt{F}}{\mu}$$

where F is the tension, and μ is the mass per unit length, so the speed and the wavelength are less in the thicker rope.

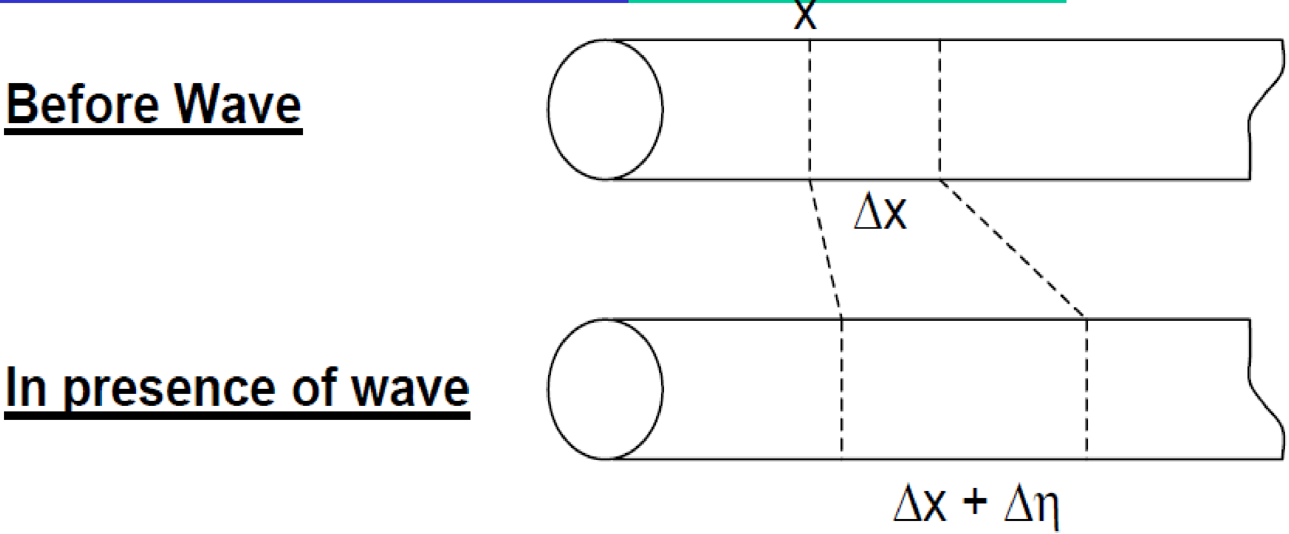
The speed in the left hand rope c1 and the speed in the right hand rope c2 . At the boundary (x=0) .

Some of the wave is transmitted, and some is reflected. To find how much is transmitted and how much is reflected. The amplitudes of the incident, transmitted and reflected waves 1, T and R respectively, and we consider that the wave is a sinusoidal wave of angular frequency ω . The equations to the incident, transmitted and reflected waves are as follows

To the right of the boundary, the displacement as a function of x and t is

$$T = \frac{2C2}{C1 + C2} \qquad R = \frac{C2 - C1}{C1 + C2}$$

Longitudinal Waves: consider a rod



For this case we must sum forces on Δx in the longitudinal (horizontal) direction

→ wave exists all through the rod but we just consider a small segment Δx

→ in the presence of the wave our segment Δx shifts and stretches by a longitudinal displacement $\Delta \eta$

LONGITUDINAL WAVES AND THE WAVE EQUATION FOR THEM

)

→ Our slice is shifted (due to cumulative strains) and also stretched by $\Delta\eta$

→ experiences forces:

F_1 (at front of slice)

F_2 (at back of slice)

Why? section Δx is stretched: so F_1 and F_2 depend on the interatomic separation at front and back of the slice. In string derivation we had different forces at each end of our segment also

→ ∴ average strain across slice $= \dfrac{\Delta \eta}{\Delta x}$

Young's modulus

→ ∴ average stress across slice $= Y \dfrac{\Delta \eta}{\Delta x} = \tau$

→ **stress at point x:**

(strain at any point x)

stress at x $= Y \frac{\partial \eta}{\partial x}$ (partial derivative because here strain is also time dependent)

→ **stress at point x + Δx**

how stress changes across any piece ∂x

stress at x + Δx = (stress at x) $+ \frac{\partial(stress)}{\partial x} \Delta x$

$$= Y \frac{\partial \eta}{\partial x} + \frac{\partial}{\partial x}\left(Y \frac{\partial \eta}{\partial x} \right) \Delta x$$

Now we have the stresses at each end, lets get the forces..

F = (cross sectional area)(stress)

$\therefore F_1$ = force at x $= \alpha Y \dfrac{\partial \eta}{\partial x}$ α = cross sectional area

F_2 = force at x+Δx $= \alpha Y \dfrac{\partial \eta}{\partial x} + \alpha Y \dfrac{\partial^2 \eta}{\partial x^2} \Delta x$

$\overset{+\rightarrow}{\sum F}$ gives $\boxed{F_2 - F_1 = \alpha Y \dfrac{\partial^2 \eta}{\partial x^2} \Delta x}$

also $F_2 - F_1 = ma$ (on our slice)

$m = \rho \alpha \Delta x$ $a = \dfrac{\partial^2 \eta}{\partial t^2}$

LONGITUDINAL WAVES AND THE WAVE EQUATION FOR THEM

)

becomes

$$\alpha Y \frac{\partial^2 \eta}{\partial x^2} \Delta x = (\rho \alpha \Delta x)\left(\frac{\partial^2 \eta}{\partial t^2}\right)$$

note

cancelling terms gives

$$\boxed{\frac{\partial^2 \eta}{\partial x^2} = \frac{\rho}{Y} \frac{\partial^2 \eta}{\partial t^2}}$$

WAVE EQUATION **for longitudinal waves!**

or, if

$$v = \sqrt{\frac{Y}{\rho}}$$

$$\boxed{\frac{\partial^2 \eta}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \eta}{\partial t^2}}$$

another equation for wave velocity

Note that η is just the longitudinal displacement, just like y was the transverse displacement for transverse waves

LONGITUDINAL WAVES AND THE WAVE EQUATION FOR THEM

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→ stretched string (transverse oscillations): $\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}$ (y: transverse displacement) $v = \sqrt{\frac{T}{\mu}}$ (T: tension; μ: mass/length)

→ longitudinal waves* in a solid: $\frac{\partial^2 \eta}{\partial t^2} = \frac{Y}{\rho}\frac{\partial^2 \eta}{\partial x^2}$ (η: longitudinal displacement) $v = \sqrt{\frac{Y}{\rho}}$ (Y: Young's modulus; ρ: density)

→ transverse (shear) waves in a solid: $\frac{\partial^2 y}{\partial t^2} = \frac{n}{\rho}\frac{\partial^2 y}{\partial x^2}$ $v = \sqrt{\frac{n}{\rho}}$ (n: shear modulus)

→ longitudinal waves in a gas or liquid: $\frac{\partial^2 \eta}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2 \eta}{\partial x^2}$ $v = \sqrt{\frac{B}{\rho}}$ (B: bulk modulus)

ACOUSTICS WAVES

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Acoustic waves (also known as sound waves): are a type of longitudinal waves that propagate by means of adiabatic compression and decompression.

Longitudinal waves are waves that have the same direction of vibration as their direction of travel. Important quantities for describing acoustic waves are sound pressure, particle velocity, particle displacement and sound intensity. Acoustic waves travel with the speed of sound which depends on the medium they're passing through.

$$C = \sqrt{P/\rho}$$

The propagation speed of acoustic waves is given by the speed of sound. In general, the speed of sound c is given by the Newton-Laplace equation:

P is a coefficient of stiffness, the bulk modulus (or the modulus of bulk elasticity for gas mediums), ρ is the density in kg/m^3


Thank you