INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad -500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

DEFINITIONS AND TERMINOLOGY QUESTION BANK

Course Title	**COMPLEX ANALYSIS AND SPECIAL FUNCTIONS**				
Course Code	AHSB05				
Program	B. Tech				
Semester	THREE				
Course Type	Foundation				
Regulation	IARE - R18				
Course Structure	**Theory**			**Practical**	
	Lectures	**Tutorials**	**Credits**	**Laboratory**	**Credits**
	3	1	3	-	-
Course Coordinator	Ms. L Indira, Assistant Professor				

COURSE OBJECTIVES:

Students will try to learn:	
I	The applications of complex variable and conformal mapping in two dimensional complex potential theories.
II	The fundamental calculus theorems and criteria for the independent path on contour integral used in problems of engineering
III	The concepts of special functions and its application for solving the partial differential equation in mathematical physics and engineering.
IV	The Mathematics of combinatorial enumeration by using generating functions and Complex analysis for understanding the numerical growth rates.

COURSE OUTCOMES:

At the end of the course the students should be able to:

Course Outcomes		**Knowledge Level (Bloom's Taxonomy)**
CO 1	**Identify** the fundamental concepts of analyticity and differentiability for calculus of complex functions and their role in applied context.	Remember
CO 2	**Utilize** the concepts of analyticity for finding complex conjugates and their role in applied contexts.	Apply

CO 3	**Make use of the** conformal mapping technique for transferring geometric structure of complex functions with much more convenient geometry.	Apply
CO 4	**Apply** integral theorems of complex analysis and its consequences for the analytic function having derivatives of all orders in simple connected region.	Apply
CO 5	**Extend** the Taylor and Laurent series for expressing the function in terms of complex power series.	Understand
CO 6	**Classify** Singularities and Poles of Complex functions for evaluating definite and indefinite Complex integrals.	Understand
CO 7	**Apply** Residue theorem for computing definite integrals of real and complex analytic functions over closed curves.	Apply
CO 8	**Relate** the concept of improper integral and second order differential equations of special functions for formulating real world problems with futuristic approach.	Understand
CO 9	**Determine** the characteristics of special functions generalization on elementary factorial function for the proper and improper integrals.	Apply
CO 10	**Choose** an appropriate special function on physical phenomena arising in engineering problems and quantum physics.	Apply
CO 11	**Analyze** the role of Bessel functions in the process of obtaining the series solutions for second order differential equation.	Analyze

DEFINITIONS AND TERMINOLOGY QUESTION BANK

S. No	QUESTION	ANSWER	Blooms Level	CO
MODULE – I				
1	What is complex number?	The number of the form z = x + iy is called a complex number.	Understand	CO1
2	What is complex conjugate?	The complex number z = x − iy is called the complex conjugate of z.	Understand	CO1
3	Define Functions of a Complex Variable.	Functions of a Complex Variable is a function that has a subset of the complex numbers as a domain and the complex numbers as a co domain	Remember	CO1
4	Explain Limits of complex Function.	Let $A\subseteq C$ and let $z0\in C$ be an accumulation point of A. The **Limit of** f **as** z **Approaches** $z0$ is L denoted $\lim_{z\to z0} f(z)=L$ if for all $\epsilon>0$ there exists a $\delta>0$ such that if $z\in A$ and $\lvert z-z0\rvert<\delta$ then $\lvert f(z)-L\rvert<\epsilon$.	Understand	CO1
5	Explain Differentiability of complex function.	Let w = f(z) be a given function defined for all z in a neighborhood of z_0.If $\lim_{\Delta z\to 0}\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}$ exists, the function f(z) is said to be derivable at z_0 and the limit is denoted by $f'(z_0)$ if exists is called the derivative of f(z) at z_0.	Understand	CO1

S. No	QUESTION	ANSWER	Blooms Level	CO
6	State polar form of Cauchy-Riemann equation.	If $f(z)=f(re^{i\theta})=u(r,\theta)+iv(r,\theta)$ and f(z) is derivable at $z_0=r_0e^{i\theta_0}$ then $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta},\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$	Understand	CO1
7	Define Analytic function.	A complex function is said to be analytic on a region R if it is complex differentiable at every point in R.	Remember	CO 2
8	Define Singularities.	The points where the function fails to be analytic.	Remember	CO 6
9	Explain the term Entire function.	A complex function that is analytic at all finite points of the complex plane is said to be entire function.	Understand	CO 1
10	State Cauchy–Riemann equations.	The Cauchy–Riemann equations on a pair of real-valued functions of two real variables *u*(*x*, *y*) and *v*(*x*, *y*) are the two equations: $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}$ $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$ Typically *u* and *v* are taken to be the real and imaginary parts respectively of a complex-valued function of a single complex variable z = x + iy, f(x + iy) = u(x,y) + iv(x,y)	Understand	CO2
11	Define harmonic function.	Analytic functions are intimately related to harmonic functions. We say that a real-valued function h(x, y) on the plane is harmonic if it obeys Laplace's equation: $\frac{\partial^2 h}{\partial^2 x}+\frac{\partial^2 h}{\partial^2 y}=0$.	Remember	CO2
12	Define Conjugate harmonic function.	If two harmonic functions u and v satisfy the Cauchy-Riemann equations in a domain D and they are real and imaginary parts of an analytic function f in D then v is said to be a conjugate harmonic function of u in D. If f(z)=u+iv is an analytic function and if u and v satisfy Laplace's equation ,then u and v are called conjugate harmonic functions.	Remember	CO2
13	State Milne Thomson method.	To express $f^{'}(z)$ completely in terms of z by replacing x by z and y by zero.	Understand	CO2
14	Define Harmonic Conjugate.	Given a function *u*(*x*, *y*) harmonic in an open disk, then we can find another harmonic function *v*(*x*, *y*) so that *u* + *iv* is an analytic function of *z* in the disk. Such a function *v* is called a *harmonic conjugate* of *u*.	Remember	CO2
15	What is the value of $f'(z)$.	$f'(z)=\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}$ or $\frac{\partial u}{\partial y}+i\frac{\partial v}{\partial y}$	Understand	CO1
16	What is simple curve?	A continuous curve which does not have self intersection	Understand	CO 1

S. No	QUESTION	ANSWER	Blooms Level	CO
17	When an integral of closed curve f(z) is equal to zero	an integral of closed curve f(z) is equal to zero if it is analytic and continuous at all point inside the closed curve.	Understand	CO 4
18	What is complex differentiation?	Given a complex-valued function f of a single complex variable, the derivative of f at a point z_0 in its domain is defined by the limit. This is the same as the definition of the derivative for real functions, except that all of the quantities are complex	Remember	CO 1
19	What is differentiation with example?	Differentiation allows us to find rates of change. For example, it allows us to find the rate of change of velocity with respect to time	Understand	CO 1
20	When a function is said to be continuous?	If limit of the function exists and is unique and is equal to its functional value then the function is said to be continuous.	Understand	CO 1
21	What are holomorphic functions?	Analytic functions are also called as holomorphic functions.	Understand	CO1
22	Is complex conjugate is differentiable?	If complex function is differentiable then its conjugate also differentiable.	Remember	CO1
23	What does it mean for a function to be analytic?	An analytic function is a function in which the derivative exists at all points in given region.	Remember	CO1
24	Why analytic functions are are important?	Analytic functions are so important because they come up in practical problems.	Understand	CO1
25	Is sin z is analytic everywhere?	A function is called analytic when Cauchy-Riemann equations hold in an open set. ... So sin z is not analytic anywhere. Similarly cos z = cosx cosh y + I sinx sinhy = u + iv, and the Cauchy-Riemann equations hold when z = n π for n ∈ Z.	Understand	CO1
26	Which functions are analytic everywhere?	The function is analytic throughout a region in the complex plane if f′ exists for every point in that region.	Remember	CO1
27	Are analytic functions are continuous?	Every analytic function has the property of being infinitely differentiable. Since the derivative is defined and continuous, the function is continuous everywhere.	Remember	CO1
28	Is the function f(z)=1/z analytic everywhere?	No, at z=0 it is not analytic.	Understand	CO2
29	What are conditions for functions to be analytic?	The functions z n, n a. The Cauchy-Riemann conditions are necessary and sufficient conditions for a function to be analytic at a point.	Remember	CO1
30	What are other names for analytic functions?	Regular function or holomorphic functions.	Understand	CO2
31	What is conjugate of '-i'	i	Understand	CO1

S. No	QUESTION	ANSWER	Blooms Level	CO
32	What are Cauchy-Riemann equations in polar form?	$\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}\Rightarrow u_r=\frac{1}{r}v_\theta$ $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}\Rightarrow v_r=-\frac{1}{r}u_\theta$	Understand	CO2
33	What is analytic complex function?	A complex function is said to be analytic on a region if it is complex differentiable at every point	Understand	CO2
34	Is constant function analytic?	Yes, a constant function is analytic on its domain of definition	Understand	CO1
35	Is cos z is analytic everywhere?	cosz is not analytic anywhere	Remember	CO1
36	Let f and g be Continuous functions at $z_0 \in C$. is f/g is continuous at z_0?	f/g is continuous at z_0 provided that $g(z_0)\neq 0$	Remember	CO1
37	If $z_1 = 2 + i$, $z_2 = 1 + 3i$, then ι Re ($z_1 - z_2$)	Answer is 'i'	Remember	CO1
38	What is polar form of complex number?	$z = r\,e^{i\theta}$	Remember	CO1
39	What is conjugate of 'i'	The conjugate of i is -i	Remember	CO1
40	What is a coefficient of a+ib?	The coefficients is b	Understand	CO1
MODULE -II				
1	Define line integral.	A line integral is an integral where the function to be integrated is evaluated along a curve. we define $\int_a^b F(t)\,dt=\int_a^b u(t)\,dt+i\int_a^b v(t)\,dt$	Remember	CO 4
2	What is real part of $\int_a^b F(t)dt$?	The real part of $\int_a^b F(t)dt$ is $\int_a^b u(t)dt$	Understand	CO 4
3	What is imaginary part of $\int_a^b F(t)dt$	The imaginary part of $\int_a^b F(t)dt$ is $\int_a^b v(t)dt$	Understand	CO 4
4	State Cauchy integral Theorem.	let F(z)=u(x, y)+iv(x, y) be analytic on and within a simple closed contour(or curve) 'c' and let f '(z) be continuous there, then $\int_c f(z)dz=0$	Understand	CO 4
5	State Cauchy integral formula.	Let f(z) be an analytic function everywhere on and within a closed contour c. If z=a is any point within c then $f(a)=\frac{1}{2\pi i}\int_c \frac{f(z)}{(z-a)}dz$ where the integral is taken in the positive sense around c.	Understand	CO 4

S. No	QUESTION	ANSWER	Blooms Level	CO
6	State generalization of Cauchy integral formula.	Let f(z) be an analytic function everywhere on and within a closed contour c. If z=a is any point within c then $f^{n}(a)=\frac{n!}{2\pi i}\int_{c}\frac{f(z)}{(z-a)^{n+1}}dz$	Understand	CO 4
7	Define indefinite integral.	If one the limit value is infinity or functional value is infinite at that limit then integral is called indefinite.	Remember	CO 4
8	What are simple connected regions?	The closed curve which does not intersect itself .	Understand	CO 4
9	Define singular point.	A point at which a function f(z) is not analytic is called a singular point .	Remember	CO 4
10	Define contour.	A continuous arc without multiple points is called contour.	Remember	CO 4
11	Define continuous function.	A function f(z) is said to be continuous at $z=z_0$,if $f(z_0)$ is defined and $\underset{z\to z_0}{Lt}\ f(z)=f(z_0)$	Remember	CO 4
12	Define Laplace equation.	If f(z) is analytic function in a domain D,then U and v satisfies the equation $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=0$	Remember	CO 4
13	Define orthogonality.	Two curves intersecting perpendicularly at a point are said to intersect orthogonally.	Remember	CO 4
14	Define Laplace operator.	The operator $\nabla=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is called laplacian operator.	Remember	CO 4
15	Define closed curve.	A curve in which initial and terminal points are equal.	Remember	CO 4
16	What is contour integration in complex analysis?	In the mathematical field of complex analysis, contour integration is a method of evaluating certain integrals along paths in the complex plane.	Remember	CO 4
17	What is complex analysis used for?	complex analysis used to solve abstract-looking equations	Understand	CO 4
18	What is Cauchy's theorem	the Cauchy integral theorem in complex analysis is called Cauchy's theorem.	Understand	CO 4
19	What is closed contour?	A contour line that forms a closed loop and does not intersect the edge of the map area on which it is drawn	Understand	CO 4
20	What is complex line integral?	a line integral is an integral where the function to be integrated is evaluated along a curve.	Understand	CO 4
21	Define complex integral.	Complex integration is integrals of complex functions.	Understand	CO 4
22	Define radius of convergence of power series.	A power series will converge only for certain values of. For instance, converges for. In general, there is always an interval in which a power series converges, and the number is called the radius of convergence	Remember	CO 4

S. No	QUESTION	ANSWER	Blooms Level	CO
23	How to find the radius of convergence	If the radius of convergence is R then the interval of convergence will include the open interval: (a − R, a + R). To find the radius of convergence, R, you use the Ratio Test.	Understand	CO 4
24	What is difference between radius of convergence and interval of convergence?	The convergence interval is the interval upon which the power series converges. The radius of convergence is the radius of this interval. converges iff $-1<x<1$, so the interval is $(-1,1)$ and the radius is 1.	Remember	CO 4
25	Is the radius of convergence always 1	Since 0<1 (in this example the limit does not depend on the value of x), the series converges for all x. Thus the interval of convergence is the interval.	Remember	CO 4
26	What happens when radius of convergence is 0	The distance between the center of a power series' interval of convergence and its endpoints. If the series only converges at a single point, the radius of convergence is 0. If the series converges over all real numbers, the radius of convergence is ∞.	Remember	CO 4
27	What is the center of convergence?	The Radius of Convergence of a Power Series. If a power series converges on some interval centered at the center of convergence, then the distance from the center of convergence to either endpoint of that interval is known as the radius of convergence	Remember	CO 4
28	Is Taylor series a power series	Yes	Remember	CO 5
29	Is zero convergent or divergent	A improper integral is convergent if the limit exists (and is finite). If an improper integral is not convergent, it is divergent. If the tail end of the function does not approach zero, $\lim x\to\infty f(x) = 0$, then the integral cannot converge and must diverge.	Remember	CO 5
30	What does a power series converge to	A power series converges absolutely in a symmetric interval about its expansion point, and diverges outside that symmetric interval. The distance from the expansion point to an endpoint is called the radius of convergence.	Remember	CO 5
31	Is Taylors and Maclourin's series the same	The Maclaurin series of the same function f is the particular case of the above Taylor series in case a=0, so that the series is in powers of x. ... The same goes with Maclaurin and Taylor series. A Maclaurin series is a Taylor series about x=0, where a Taylor series is about x=all real numbers.	Understand	CO 5
32	What is Taylor series method	Differential equations - Taylor's method. Taylor's Series method. Consider the one dimensional initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ where. f is a function	Remember	CO 5

S. No	QUESTION	ANSWER	Blooms Level	CO
		of two variables x and y and (x_0, y_0) is a known point on the solution curve.		
33	What is exactly line integral	A line integral (sometimes called a path integral) is the integral of some function along a curve.	Understand	CO 4
34	What does a circle on line integral mean	Basically it means you are integrating things over a loop.	Understand	CO 4
35	What does line and surface integral mean	In a line integral, the curve along which the integral is evaluated is not necessarily a x (or y) axis, or even a straight line. It can be any curve lying in higher dimensional space; though the curve itself is a 2 D entity	Understand	CO 4
36	Define complex number	A combination of a real and an imaginary number in the form a + bi. a and b are real numbers, and. i is the "unit imaginary number" $\sqrt{(-1)}$	Understand	CO 1
37	What is indefinite integral	A definite integral represents a number when the lower and upper limits are constants. The indefinite integral represents a family of functions whose derivatives are f. The difference between any two functions in the family is a constant.	Remember	CO 4
38	What is relationship between integrand function and integral function?	The symbol d x, called the differential of the variable x, indicates that the variable of integration is x. The function f(x) to be integrated is called the integrand. The symbol dx is separated from the integrand by a space (as shown). If a function has an integral, it is said to be integrable.	Remember	CO 4
39	What is definite integral of zero	A constant.	Remember	CO 4
40	What is meant by complex integration	Definition of complex integration. : the integration of a function of a complex variable along an open or closed curve in the plane of the complex variable.	Remember	CO 4
MODULE - III				
1	Define Power series.	A series of the form $\sum a_n z^n$ is called as power series. That is $\sum a_n z^n = a_1 z + a_2 z^2 + \ldots\ldots\ldots + a_n z^n + \ldots\ldots$	Remember	CO 5
2	State Taylor's series.	The Taylor series is an infinite series, whereas a Taylor polynomial is a polynomial of degree n and has a finite number of terms. The form of a Taylor polynomial of degree n for a function f (z) at x = a is $f(z) = f(a) + f'(a)(z-a) + f''(0)\frac{(z)^2}{2!} + \ldots\ldots$ $\ldots\ldots\ldots \lvert z-a \rvert < r$	Remember	CO 5

S. No	QUESTION	ANSWER	Blooms Level	CO
3	State Maclaurin series.	A Maclaurin series is a Taylor series expansion of a function about x=0, $f(z)=f(0)+f'(0)(z)+f''(0)\frac{(z)^2}{2!}+f'''(0)\frac{(z)^3}{3!}+........+f^n(0)\frac{(z)^n}{n!}+.......$ This series is called as Maclurins series expansion of f(z).	Remember	CO 5
4	State Laurent series.	The Laurent series for a complex function *f*(*z*) about a point *c* is given by: $f(z)=\sum_{n=-\infty}^{\infty}a_n(z-a)^n$ $f(z)=\sum_{n=0}^{\infty}a_n(z-a)^n+\sum_{n=1}^{\infty}b_n\frac{1}{(z-a)^n}$ where the a_n, *and a are* constants.	Remember	CO 5
5	Define Zero's of an analytic function.	A zero of an analytic function f (z) is a value of z such that f(z)=0 .Particularly a point a is called a zero of an analytic function f (z) if f(a) = 0.	Remember	CO 6
6	Define Zero's of m^{th} order.	If an analytic function f(z) can be expressed in the form $f(z)=(z-a)^m\Phi(z)$ where $\Phi(z)$ is analytic function and $\Phi(a)\neq 0$ then z=a is called zero of m^{th} order of the function f(z).	Remember	CO 6
7	Define Singular point of an analytic function.	A point at which an analytic function f (z) is not analytic, i.e. at which f '(z) fails to exist, is called a singular point or singularity of the function.	Remember	CO 6
8	Define Isolated singular points.	A singular point z_0 is called an isolated singular point of an analytic function f (z) if there exists a deleted ε-spherical neighborhood of z_0 that contains no singularity. If no such neighborhood can be found, z_0 is called a non-isolated singular point.	Remember	CO 6
9	Define non-isolated singular points.	A singular point z_0 is called an isolated singular point of an analytic function f(z) if there exists a deleted ε-spherical neighborhood of z_0 that contains no singularity. If no such neighborhood can be found, z_0 is called a non-isolated singular point.	Remember	CO 6
10	Define Simple pole.	A pole of order 1 is called a simple pole.	Remember	CO 6
11	Define Removable singular point.	An isolated singular point z_0 such that f can be defined, or redefined, at z_0 in such a way as to be analytic at z_0. A singular point z_0 is removable if $\lim_{z\to z_0} f(z)$ exist.	Remember	CO 6
12	Define Essential singular point.	A singular point that is not a pole or removable singularity is called an essential singular point.	Remember	CO 6

S. No	QUESTION	ANSWER	Blooms Level	CO
13	Define Residues at Poles.	If f(z) has a simple pole at z_0 , then $\operatorname{Re} s[f, z_0] = \lim_{Z \to Z_0} (z - z_0) f(z)$	Remember	CO 7
14	Stat Cauchy's Residue Theorem.	$\int_c f(z)dz = 2\pi i \sum_{a \in A} \operatorname{Re} s_{z=a_i} f(z)$ Where A is the set of poles contained inside the contour	Understand	CO 7
15	Define Residue at infinity.	The residue at infinity is given by: $\operatorname{Re} s[f(z)]_{Z=\infty} = -\frac{1}{2\pi i} \int_C f(z)dz$ Where f is an analytic function except at finite number of singular points and C is a closed contours so all singular points lie inside it.	Remember	CO 7
16	How to calculate residue	By finding limit at the poles.	Remember	CO 7
17	Can the residue of a pole be zero	If is analytic at , its residue is zero, but the converse is not always true	Remember	CO 6
18	What is pole of a function?	The basic example of a pole is , which has a single pole of order at . Plots of and are shown above in the complex plane.	Remember	CO 6
19	What is zero order	z is a pole or a zero of order n if the same is true for. If the curve is compact, and the function f is meromorphic on the whole curve, then the number of zeros and poles is finite, and the sum of the orders of the poles equals the sum of the orders of the zeros.	Remember	CO 6
20	Are branch point's singularities?	For complex functions, a singularity is where a function fails to be analytic. Being analytic at a point means having a derivative and being single valued in a neighborhood around that point. All branch points, by definition, are next to multiple valued points.	Remember	CO 6
21	What is pole singularity?	a singularity is a point at which an equation, surface, etc., blows up or becomes degenerate.	Remember	CO 6
22	What is point in complex analysis?	In the mathematical field of complex analysis, a branch point of a multi-valued function (usually referred to as a "multifunction" in the context of complex analysis) is a point such that the function is discontinuous when going around an arbitrarily small circuit around this point.	Remember	CO 5
23	What is isolated essential singularity?	n complex analysis, a branch of mathematics, an isolated singularity is one that has no other singularities close to it.	Remember	CO 5
24	What are the types of singularities?	There are three types of isolated singularities: removable singularities, poles and essential singularities.	Remember	CO 5

S. No	QUESTION	ANSWER	Blooms Level	CO
25	Is complex analysis useful?	The second application area is control theory, specifically in the analysis of stability of systems and controller design. engineering applications will only make use of parts of what is taught in a complex analysis	Remember	CO 5
26	What is complex variables math?	Complex variable, In mathematics, a variable that can take on the value of a complex number. The algebra of complex numbers (complex analysis) uses the complex variable z to represent a number of the form a + bi.	Remember	CO 5
27	What is analytic function in complex variables?	A complex function is said to be analytic on a region if it is complex differentiable at every point in.	Remember	CO 5
28	Are all smooth functions analytic?	Smooth functions (also called infinitely differentiable functions) and analytic functions are two very important types of functions. One can easily prove that any analytic function of a real argument is smooth.	Remember	CO 5
29	Is analytic continuation unique?	The analytic continuation is unique.	Understand	CO 5
30	What is analytic continuation?	Analytic continuation, starting from a representation of a function by any one power series, any number of other power series can be found which together define the value of the function at all points of the domain.	Remember	CO 5
31	What is power series in calculus?	Power series is a sum of terms of the general form $(x-a)^n$.	Understand	CO 5
32	Is a geometric series a power series?	Geometric series is a function.	Remember	CO 5
33	What makes series geometric?	Ratio of successive terms in the series is constant.	Understand	CO 5
34	What is series in math?	a description of the operation of adding infinitely many quantities, one after the other, to a given starting quantity. For a long time, the idea that such a potentially infinite summation could produce a finite result was considered paradoxical.	Remember	CO 5
35	What Are series used for	A Taylor series is an idea used in computer science, calculus, and other kinds of higher-level mathematics. It is a series that is used to create an estimate (guess) of what a function looks like. There is also a special kind of Taylor series called a Maclaurin series.	Understand	CO 5
36	What is Taylor's series approximation?	A Taylor series is a series expansion of a function about a point. A one dimensional Taylor series is an expansion of a real function about a point is given by. (1) If , the expansion is known as a Maclaurin series.	Understand	CO 5

S. No	QUESTION	ANSWER	Blooms Level	CO
37	What is the radius of convergence of a power series?	Radius of Convergence. A power series will converge only for certain values of. For instance, converges for. In general, there is always an interval in which a power series converges, and the number is called the radius of convergence (while the interval itself is called the interval of convergence)	Remember	CO 5
38	Are power series and taylor series the same?	If f has a power series expansion, then it is the Taylor series, so asking if f has a power series expansion is the same as asking if f is equal to its Taylor series. and this equality holds for any value of x.	Understand	CO 5
39	What does it mean for a function to be analytic?	an analytic function is a function that is locally given by a convergent power series. Functions of each type are infinitely differentiable, but complex analytic functions exhibit properties that do not hold generally for real analytic functions.	Remember	CO 5
40	What makes complex function analytic?	A complex function is said to be analytic on a region if it is complex differentiable at every point in. A complex function that is analytic at all finite points of the complex plane is said to be entire.	Understand	CO 5
MODULE – IV				
1	Define the gamma function?	The gamma function defined by the integral $\int_0^\infty e^{-x} x^{n-1} dx$ when n>0is an improper integral o third kind	Remember	CO 8, CO 9
2	Explain the improper integrals?	The integral $\int_a^b F(t)dt$ such an integrals,for which Either the interval of integration is not finite a= -∞ or b=∞ or both and the function f(t) is unbounded at one or more points in closed interval a and b	Remember	CO 8, CO 9
3	Define the Beta function?	The definite integral $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is called the beta function where m>0,n>0	Remember	CO 8, CO 9
4	State the relationship between beta and gamma function.	The relationship between beta and gamma function is $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$	Remember	CO 8, CO 9
5	Give the value of $\int_0^\infty e^{x^2}$	The value of $\int_0^\infty e^{x^2}\ is$ $[\Pi/2$	Remember	CO 8, CO 9
6	Give the value of $\int_0^\infty e^{-x^3}\ dx$	The value of $\int_0^\infty e^{-x^3}\ dx$ is (Γ(1/3)) /3	Remember	CO 8, CO 9
7	Define symmetric property of beta function.	The symmetric property of beta function $\beta(m,n)=\beta(n,m)$	Remember	CO 8, CO 9

S. No	QUESTION	ANSWER	Blooms Level	CO
8	If m is positive integral then write the value of $\Gamma(m+1)$	The value is m!	Remember	CO 8, CO 9
9	If m is positive fraction then write the value of $\Gamma(m+1)$	$m\Gamma(m)$	Remember	CO 8, CO 9
10	Express the value of Γ(n+(1/2))	The value of Γ(n+(1/2)) is 1.3.5………(2n-1) $\sqrt{\Pi}/2^{n}$-1	Remember	CO 8, CO 9
11	Write the value of $\int_0^\infty y^{3/2} e^{-y^2}\, dy$	the value of $\int_0^\infty y^{3/2} e^{-y^2}\, dy$ is Γ(5/4)/2	Understand	CO 8, CO 9
12	Define Eulerian integral of second kind	The Eulerian integral of second kind is called Gamma function.	Remember	CO 8, CO 9
13	Give the value of $\lceil n \lceil 1-n$.	The value of $\lceil n \lceil 1-n$ is $\frac{\pi}{sinn\pi}$	Remember	CO 8, CO 9
14	For what value of n does the Gamma function converges.	For n > 0 the gamma function converges.	Remember	CO 8, CO 9
15	Give the value of $\int_0^{\frac{\pi}{2}} sin^7 x\ dx$	The value of $\int_0^{\frac{\pi}{2}} sin^7 x\ dx$ is 16/35.	Remember	CO 8, CO 9
16	Is gamma function analytic?	This integral function is extended by analytic continuation to all complex numbers except the non-positive integers (where the function has simple poles), yielding the meromorphic function we call the gamma function.	Remember	CO 8, CO 9
17	What are beta and gamma functions?	The Gamma Function is defined as the single variable function. the Beta Function is defined as the two variable function	Remember	CO 8, CO 9
18	What does beta mean in math?	In mathematics, the beta function, also called the Euler integral of the first kind, is a special function defined by. for Re x > 0, Re y > 0.	Remember	CO 8, CO 9
19	What does beta mean in calculus?	The beta function (also known as Euler's integral of the first kind) is important in calculus and analysis due to its close connection to the gamma function, which is itself a generalization of the factorial function. Many complex integrals can be reduced to expressions involving the beta function.	Remember	CO 8, CO 9
20	What is beta integral?	the Eulerian integral of the first kind	Remember	CO 8, CO 9
21	What does the gamma symbol mean in math?	Gamma (uppercase/lowercase Γ γ), is the third letter of the Greek alphabet, used to represent the "g" sound in Ancient and Modern Greek. In the system of Greek numerals, it has a value of 3. ... The lowercase Gamma ("γ") is used in wave motion physics to represent the ratio of specific heat	Remember	CO 8, CO 9
22	What is meant by gamma function?	The gamma function (Γ(z)) is an extension of the factorial function to all complex numbers except negative integers. For positive integers, it is defined. The gamma	Remember	CO 8, CO 9

S. No	QUESTION	ANSWER	Blooms Level	CO
		function is defined for all complex numbers. But it is not defined for negative integers and zero.		
23	Is gamma function defined for negative integer.	Definition of the gamma functions for non-integer negative values. Through integration by parts, it can be shown that for, Γ (x) = 1 x Γ (x + 1).	Remember	CO 8, CO 9
24	Are complex numbers positive or negative?	A real number can be defined to be positive or negative or 0, as the real set lies on a straight line ie 1D. But a complex number lies in a complex plane called the Argand plane, i.e in 2 dimensions. It cannot be defined as positive or negative.	Remember	CO 8, CO 9
25	Can negative numbers have Factorials?	Factorial is defined only for non-negative integer numbers.	Remember	CO 8, CO 9
26	What is factorial number?	the factorial of a positive integer n, denoted by n!	Understand	CO 8, CO 9
27	Is the gamma function analytic?	This integral function is extended by analytic continuation to all complex numbers except the non-positive integers (where the function has simple poles), yielding the meromorphic function we call the gamma function. It has no zeroes, so the reciprocal gamma function 1/Γ(z) is a holomorphic function.	Remember	CO 8, CO 9
28	Is factorial function continuous?	The gamma function is a continuous extension to the factorial function, which is only defined for the nonnegative integers. While there are ther continuous extensions to the factorial function, the gamma function is the only one that is convex for positive real numbers.	Remember	CO 8, CO 9
29	Using the factorial representation of the gamma function, what is the solution for the gamma function $\Gamma(n)$ when $n = 8$?	5040	Remember	CO 8, CO 9
30	What is the value of $\frac{\Gamma(6)}{2\Gamma(3)}$	30	Remember	CO 8, CO 9
31	What is the value of $\int_0^\infty x^3 e^{-x} dx$	6	Remember	CO 8, CO 9
32	What is the value of $\int_0^\infty x^6 e^{-x} dx$	6!	Remember	CO 8, CO 9
33	What is the value of $\frac{6\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{2}{3}\right)}$	$\frac{4}{3}$	Remember	CO 8, CO 9
34	Give the value of $\beta(1,1)$	1	Remember	CO 8, CO 9

S. No	QUESTION	ANSWER	Blooms Level	CO
35	Using the factorial representation of the gamma function, what is the solution for the gamma function $\Gamma(n)$ when $n = 1$?	1	Remember	CO 8, CO 9
36	Give the value of $\beta(1,2)$	1	Remember	CO 8, CO 9
37	Give the value of $\Gamma\left(\frac{5}{2}\right)B\left(\frac{3}{2},1\right)$	$\frac{1}{2}\sqrt{\pi}$	Understand	CO 8, CO 9
38	Give the value of $\dfrac{B\left(\frac{7}{2},\frac{9}{2}\right)}{B\left(\frac{5}{2},\frac{11}{2}\right)}$	$\frac{5}{9}$	Remember	CO 8, CO 9
39	Give the value of $\beta(2, 2)$	$\frac{1}{6}$	Remember	CO 8, CO 9
40	Using the factorial representation of the gamma function, what is the solution for the gamma function $\Gamma(n)$ when $n = 0$?	Not defined	Remember	CO 8, CO 9
		MODULE –V		
1	Give Bessel differential equation.	The Bessel differential equation is $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$ where n is a non negative real constant or parameter	Understand	CO 9,CO 10
2	Define Orthogonality of Bessel.	The Orthogonality of Bessel is *If* α and β are the two distinct roots of $J_n(x) = 0$, then $\int_0^{\pi} xJ_n(\alpha x)J_n(\beta x)dx = \begin{cases} 0, & if\ \alpha \neq \beta \\ \frac{1}{2}[J_n'(\alpha)]^2 = \frac{1}{2}[J_{n+1}(\alpha)]^2, & if\ \alpha = \beta \end{cases}$	Remember	CO 9,CO 10
3	State generating function of Bessel	The generating function of Bessel is $e^{\frac{x}{2}(t-1/t)} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$	Understand	CO 9,CO 10
4	Write the first recurrence relation of Bessel.	The first recurrence relation of Bessel $J_{n+1}(x)=\frac{2n}{x}J_n(x)-J_{n-1}(x)$	Understand	CO 9,CO 10
5	Write the second recurrence relation of Bessel.	The second recurrence relation of Bessel. $J_n'(x)=J_{n-1}(x)-\frac{n}{x}J_n(x)$	Understand	CO 9,CO 10
6	Write the third recurrence relation of Bessel.	The third recurrence relation of Bessel is $J_n'(x)=\frac{n}{x}J_n(x)-J_{n+1}(x)$	Remember	CO 9,CO 10
7	Write the fourth recurrence relation of Bessel.	The fourth recurrence relation of Bessel is $J_n'(x)=\frac{1}{2}[J_{n-1}(x)-J_{n+1}(x)]$	Understand	CO 9,CO 10

S. No	QUESTION	ANSWER	Blooms Level	CO
8	Write the fifth recurrence relation of Bessel.	the fifth recurrence relation of Bessel $J_n(x) = \frac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$	Understand	CO 9,CO 10
9	If $m_1 \neq m_2$ are the roots of $J_n(m x) = 0$, then $\int_0^1 x\ J_n\ (m_1 x)\ J_n\ (m_2 x)\ dx$	The root is zero.	Understand	CO 9,CO 10
10	What is the value of $\int_0^{\pi/2} \sqrt{\pi x}\ J_{1/2}(x)dx$ is	The value of $\int_0^{\pi/2} \sqrt{\pi x}\ J_{1/2}(x)dx$ is one .	Understand	CO 9,CO 10
11	What is the value of $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2$	The value of $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2$ is $2/\Pi x$.	Understand	CO 9,CO 10
12	What is the value of $J_0^{\prime}(x)$	The value is $x\ J_1(x)$	Understand	CO 9,CO 10
13	Express the value of sin x in term of Bessel.	The value of sin x in term of Bessel is $2J_1 - 2J_3 + 2J_5 - 2J_7 + \ldots\ldots$	Understand	CO 9,CO 10
14	What is the value of $\frac{d}{dx}[x^3 J_8(x)$.	the value of $\frac{d}{dx}[x^3 J_8(x)$ is $x^3 J_2(x)$	Understand	CO 9,CO 10
15	What is the value of $\int_0^1 x\,[J_0(2x)]^2$	The value of $\int_0^1 x\,[J_0(2x)]^2$ is $\frac{1}{2}[J_1(2)]^2$	Understand	CO 9,CO 10
16	Express the standard value of $J_0(0)$	$J_0(0) = 1$	Understand	CO 9,CO 10
17	Express the standard value of $J_1(0)$	$J_1(0) = 0$	Remember	CO 9,CO 10
18	What is the standard value of $J_{-2}(x)$ in terms of $J_2(x)$	The value of $J_{-2}(x)$ is $J_2(x)$	Understand	CO 9,CO 10
19	What is the standard value of $\frac{d}{dx}[x^3 J_8(x)]$ in terms of $J_2(x)$	$\frac{d}{dx}[x^3 J_8(x)] = x^3 J_2(x)$	Understand	CO 9,CO 10
20	Express the coefficient of z^{-4} in the expansion of $e^{(x/2)}$ (z-(1/z))	The coefficient of z^{-4} in the expansion of $e^{(x/2)}$ (z-(1/z)) is $J_4(x)$	Understand	CO 9,CO 10
21	What is the value of $\int_0^1 x\,[J_0(2x)]^2$	$\int_0^1 x\,[J_0(2x)]^2 = \frac{1}{2}[J_1(2)]^2$	Remember	CO 9,CO 10
22	The equivalent value of the expression $\frac{2}{x} J_1(x)$ in terms of $J_0(x)$ and $J_2(x)$ is	$\frac{2}{x} J_1(x) = J_0(x) + J_2(x)$	Understand	CO 9,CO 10
23	For what value of n, $J_n(x)$ is even	$J_n(x)$ is even when n is Even	Understand	CO 9,CO 10
24	For what value of n, $J_n(x)$ is odd	$J_n(x)$ is odd when n is Odd	Understand	CO 9,CO 10
25	$J_n(x) = 0$ has repeated roots except at	x=0	Understand	CO 9,CO 10
26	For Bessel's function $J_n(x)$ the values of a and b where $\frac{d}{dx}[J_n(x)] = aJ_{n-1}(x) + bJ_{n+1}(x)$	The values of a and b are 1/2,1/2	Understand	CO 9,CO 10

S. No	QUESTION	ANSWER	Blooms Level	CO
27	The generating function of $J_n(x)$ is	The generating function is $e^{(x/2)(z-(1/z))}$	Understand	CO 9,CO 10
28	The value of $\int_0^{\pi/2} \sqrt{\pi x}\ J_{1/2}(x)dx$	$\int_0^{\pi/2} \sqrt{\pi x}\ J_{1/2}(x)dx = 1$	Understand	CO 9,CO 10
29	The value of $\frac{1}{\pi}\int_0^{\pi} \cos(xsin\theta)\, d\theta$ in terms of Bessels function is	$\frac{1}{\pi}\int_0^{\pi} \cos(xsin\theta)\, d\theta = J_0(x)$	Understand	CO 9,CO 10
30	Express the Jacobi series $\cos(xsin\theta)$	$\cos(xsin\theta) = J_0 - 2J_2cos2\theta + 2J_4cos4\theta \ldots..$	Understand	CO 9,CO 10
31	Express the Jacobi series $\text{Sin}(xsin\theta)$	$\text{Sin}(xsin\theta) = 2[J_1cos\theta - J_3cos3\theta + J_5cos5\theta - \cdots$	Understand	CO 9,CO 10
32	What is the standard value of $\frac{d}{dx}[xJ_1(x)]$ in terms of $J_0(x)$	$\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$	Remember	CO 9,CO 10
33	What is the standard value of $J_{3/2}(x)$	$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}[\frac{1}{x}\ sinx - cosx]$	Understand	CO 9,CO 10
34	What is the standard value of $J_{5/2}(x)$	$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}[\frac{3-x^2}{x^2}\ sinx - \frac{3}{x}cosx]$	Understand	CO 9,CO 10
35	What is the standard value of $J_{1/2}(x)$	$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\ sinx$	Understand	CO 9,CO 10
36	What is the standard value of $J_{-1/2}(x)$	$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\ cosx$	Remember	CO 9,CO 10
37	The relationship between $J_{1/2}(x)$ and $J_{-1/2}(x)$ is	$J_{-1/2}(x) = J_{1/2}(x)$ cotx	Understand	CO 9,CO 10
38	The condition on n for $J_n(-x) = (-1)^n J_n(x)$	n is a positive of negative integer	Understand	CO 9,CO 10
39	The most general solution of Bessel's differential equation is	$y = c_1J_n(x) + c_2J_{-n}(x)$ where c_1 and c_2 are arbitrary constants	Understand	CO 9,CO 10
40	Express the $J_0'(x)$ in terms of $J_1(x)$	$J_0'(x) = -J_1(x)$	Understand	CO 9,CO 10

Prepared by:
Ms. L Indira, Assistant Professor

HOD, FE