Hall Ticket No | | | | | | | | | | Question Paper Code: AHSB05

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

MODEL QUESTION PAPER-I

B.Tech III Semester End Examinations, November 2020

Regulations: IARE - R18

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

ELECTRONICS AND COMMUNICATION ENGINEERING

Time: 3 hour Maximum Marks: 70

Answer ONE Question from each MODULE
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

MODULE-I

1. (a) Show that $(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}) \mid Real f(z) \mid^2 = 2. \mid f'(z) \mid^2$ where $w = f(z)$ is an analytical function. **[7m]**

   (b) Find the analytic function $f(z)$ whose imaginary part of an analytic function is $r^2 cos\theta + r\sin\theta$ by Cauchy Riemann equations in polar form. **[7m]**

2. (a) Construct the Bilinear transformation that maps the points $(1-2i, 2+i, 2+3i)$ into the points $(2+i, 1+3i, 4)$. **[7m]**

   (b) Use Milne Thompson's method to find the imaginary part of an analytic function $f(z)$ whose real part of an analytic function is $e^x(x\cos y - y\sin y)$. **[7m]**

MODULE-II

3. (a) Solve the value of line integral to $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$ where c is the circle $\mid z \mid = 3$ using Cauchy's integral formula. . **[7m]**

   (b) Compare Cauchy's integral formula with line integral to $\int_c \frac{z^3 e^{-z}}{(z-1)^3}dz$ where c is the circle $\mid z-1 \mid = \frac{1}{2}$ and find the value of integral. **[7m]**

4. (a) Solve the value of integral to $\int_c \frac{e^{2z}}{(z+1)^4}dz$ where c is the circle $\mid z-1 \mid = 3.$ and find the value of integral. **[7m]**

   (b) Make use of Cauchy's integral formula and evaluate $\int_c \frac{z+1}{z^2+2z+4}dz$ where $c : \mid z+1+i \mid = 2$ **[7m]**

MODULE-III

5. (a) Extend Laurent's series expansion to the function $f(z) = \frac{z^2 - 4}{z^2 + 5z + 4}$ valid in $1 < z < 4$ **[7m]**

   (b) Extend $f(z) = \frac{e^{2z}}{(z-1)^3}$ about $z = 1$ as Laurent's series. Also find the region of convergence about $z = 1$. **[7m]**

6. (a) Solve the value of $\int_c \frac{2z - 1}{z(2z+1)(z+2)} dz$ where c is the circle $\mid z \mid= 1$ **[7m]**

   (b) Solve the integral $\oint_c \tan z dz$ where c is the circle $\mid z \mid= 2$. **[7m]**

MODULE-IV

7. (a) Solve the integral $\int_0^a x^4 \sqrt{a^2 - x^2} dx$ using Beta-Gamma functions **[7m]**

   (b) Solve the integral $\int_0^1 (xlogx)^4 dx$ using Gamma functions **[7m]**

8. (a) Solve the integral $\int_0^\infty (3)^{-4x^2} dx$ using Beta-Gamma functions **[7m]**

   (b) Solve the integral $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ using Gamma functions **[7m]**

MODULE-V

9. (a) Show that $\int_0^1 x\, J_n(\alpha, x)\, J_n(\beta, x) dx = \begin{cases} 0 & \alpha = \beta \\ 0.5[J_{n+1}(\alpha)]^2 & \alpha \neq \beta \end{cases}$ **[7m]**

   (b) Make use of generating function show that $\cos(x \sin\theta) = J_0 + 2.(J_2 cos 2\theta + J_4 cos 4\theta + ...)$ **[7m]**

10. (a) Show the Bessels recurrence relation $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ **[7m]**

    (b) State and prove Generating function of Bessels functions. **[7m]**

****END OF EXAMINATION****

COURSE OBJECTIVES:

The course should enable the students to:

1	The applications of complex variable and conformal mapping in two dimensional complex potential theories.
2	The fundamental calculus theorems and criteria for the independent path on contour integral used in problems of engineering.
3	The concepts of special functions and its application for solving the partial differential equation in physics and engineering.
4	The mathematics of combinatorial enumeration by using generating functions and Complex analysis for understanding the numerical growth rates.

COURSE OUTCOMES:

After successful completion of the course, students should be able to:

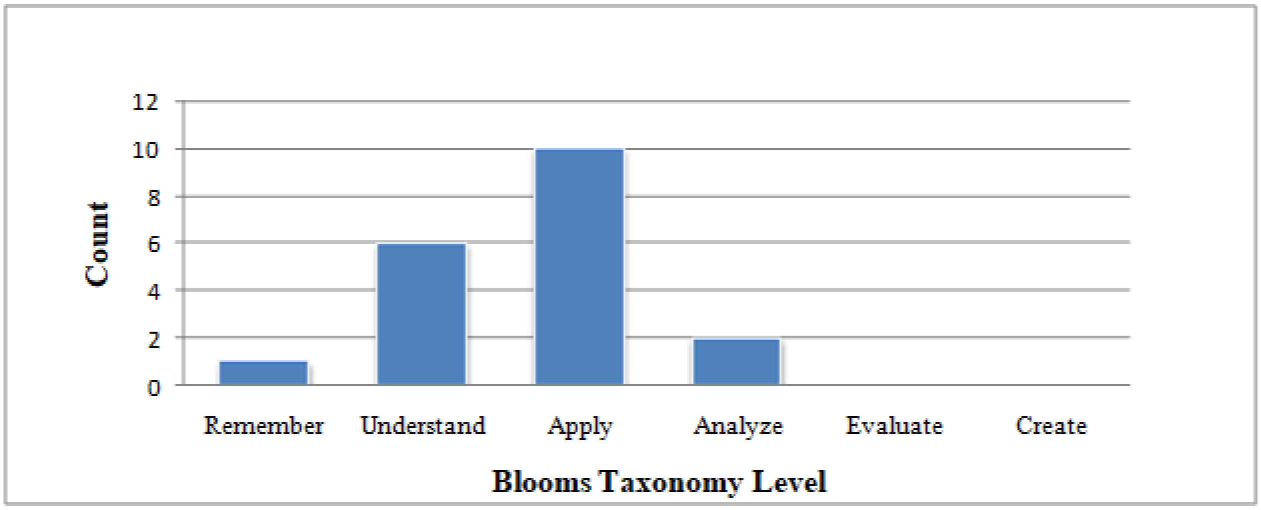
CO 1	Identify the fundamental concepts of analyticity and differentiability for calculus of complex functions and their role in applied context.
CO 2	Utilize the concepts of analyticity for finding complex conjugates and their role in applied contexts.
CO 3	Make use of the conformal mapping technique for transferring geometric structure of complex functions with much more convenient geometry.
CO 4	Apply integral theorems of complex analysis and its consequences for the analytic function having derivatives of all orders in simple connected region.
CO 5	Extend the Taylor and Laurent series for expressing the function in terms of complex power series.
CO 6	Classify Singularities and Poles of Complex functions for evaluating definite and indefinite Complex integrals.
CO 7	Apply Residue theorem for computing definite integrals of real and complex analytic functions over closed curves.
CO 8	Relate the concept of improper integral and second order differential equations of special functions for formulating real world problems with futuristic approach.
CO 9	Determine the characteristics of special functions generalization on elementary factorial function for the proper and improper integrals.
CO 10	Choose an appropriate special function on physical phenomena arising in engineering problems and quantum physics.
CO 11	Analyze the role of Bessel functions in the process of obtaining the series solutions for second order differential equation.

MAPPING OF SEMESTER END EXAMINATION QUESTIONS TO COURSE OUTCOMES

Q.No		All questions carry equal marks	Taxonomy	CO's	PO's
1	a	Show that $(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}) \mid Real f(z) \mid^2 = 2. \mid f'(z) \mid^2$ where $w = f(z)$ is an analytical function.	Apply	CO 1	PO 1
	b	Find the analytic function $f(z)$ whose imaginary part of an analytic function is $r^2 cos\theta + r\sin\theta$ by Cauchy Riemann equations in polar form.	Remember	CO 2	PO 1
2	a	Construct the Bilinear transformation that maps the points $(1-2i, 2+i, 2+3i)$ into the points $(2+i, 1+3i, 4)$.	Apply	CO 1	PO 1
	b	Use Milne Thompson's method to find the imaginary part of an analytic function $f(z)$ whose real part of an analytic function is $e^x(x\cos y - y\sin y)$.	Apply	CO 2	PO 1, 2
3	a	Solve the value of line integral to $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$ where c is the circle $\mid z \mid = 3$ using Cauchy's integral formula.	Apply	CO 3	PO 1
	b	Compare Cauchy's integral formula with line integral to $\int_c \frac{z^3e^{-z}}{(z-1)^3}dz$ where c is the circle $\mid z-1 \mid = \frac{1}{2}$ and find the value of integral.	Understand	CO 4	PO 1,2
4	a	Solve the value of integral to $\int_c \frac{e^{2z}}{(z+1)^4}dz$ where c is the circle $\mid z-1 \mid = 3.$ and find the value of integral.	Apply	CO 3	PO 1
	b	Make use of Cauchy's integral formula and evaluate $\int_c \frac{z+1}{z^2+2z+4}dz$ where $c :\mid z+1+i \mid = 2.$	Apply	CO 4	PO 1,2
5	a	Extend Laurent's series expansion to the function $f(z) = \frac{z^2-4}{z^2+5z+4}$ valid in $1 < z < 4$	Understand	CO 5	PO 1,2
	b	Extend $f(z) = \frac{e^{2z}}{(z-1)^3}$ about $z = 1$ as Laurent's series. Also find the region of convergence about $z = 1$.	Understand	CO 6	PO 1
6	a	Solve the value of $\int_c \frac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\mid z \mid = 1$	Apply	CO 5	PO 1

7	b	Solve the integral $\oint_c \tan z dz$ where c is the circle $\mid z \mid= 2$.	Apply	CO 7	PO 1,2
	a	Solve the integral $\int_0^a x^4 \sqrt{a^2 - x^2} dx$ using Beta-Gamma functions.	Apply	CO 8	PO 1
	b	Solve the integral $\int_0^1 (xlogx)^4 dx$ using Gamma functions	Understand	CO 9	PO 1,2
8	a	Solve the integral $\int_0^\infty (3)^{-4x^2} dx$ using Beta-Gamma functions	Understand	CO 8	
	b	Solve the integral $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ using Gamma functions	Understand	CO 9	PO 1,2
9	a	Show that $\int_0^1 x\, J_n(\alpha, x)\, J_n(\beta, x) dx = \begin{cases} 0 & \alpha = \beta \\ 0.5[J_{n+1}(\alpha)]^2 & \alpha \neq \beta \end{cases}$	Analyze	CO 10	PO 1,2
	b	Make use of generating function show that $\cos(x \sin\theta) = J_0 + 2.(J_2 cos2\theta + J_4 cos4\theta + ...)$	Analyze	CO 11	PO 1,2
10	a	Show the Bessels recurrence relation $J_n^{'}(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$	Apply	CO 10	PO 1,2
	b	State and prove Generating function of Bessels functions.	Apply	CO 11	PO 1,2

KNOWLEDGE COMPETENCY LEVELS OF MODEL QUESTION PAPER



Signature of Course Coordinator **HOD, AE**
Mrs. L Indira, Assistant Professor