Hall Ticket No | | | | | | | | | | Question Paper Code: AHSB05

INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)

Dundigal, Hyderabad - 500 043

MODEL QUESTION PAPER-II

B.Tech III Semester End Examinations, November 2020

Regulations: IARE - R18

COMPLEX ANALYSIS AND SPECIAL FUNCTIONS

ELECTRONICS AND COMMUNICATION ENGINEERING

Time: 3 hour Maximum Marks: 70

Answer ONE Question from each MODULE
All Questions Carry Equal Marks
All parts of the question must be answered in one place only

MODULE-I

1. (a) Define the term Analyticity of a complex variable function $f(z)$. Show that the real part of an analytic function $f(z)$ where $u = e^{-2xy}\sin(x^2 - y^2)$ is a harmonic function. Hence find its harmonic conjugate. **[7m]**

   (b) Define the term Continuity of a complex variable function $f(z)$. Show that the function $f(z) = \mid z \mid$ is continuous everywhere but nowhere differentiable. **[7m]**

2. (a) Construct the Bilinear transformation that maps the points $(\infty, i, 0)$ into the points $(0, i, \infty)$. **[7m]**

   (b) Find an analytic function $f(z)$ whose real part of an analytic function is $u = \dfrac{\sin 2x}{\cos 2y - \cos 2x}$ by Milne-Thompson method. **[7m]**

MODULE-II

3. (a) Utilize the Cauchys integral formula and find value of $\int_c \dfrac{z^3 - \sin 3z}{(z - \frac{\pi}{2})^3} dz$ where c is the circle $\mid z \mid = 2$. **[7m]**

   (b) Make use of vertices's -1 , 1, 1+i ,-1+i and verify Cauchys theorem for the integral of z taken over the boundary of the rectangle formed . **[7m]**

4. (a) Solve the value of line integral to $\int_{z=0}^{z=i+1} [x^2 + 2xy + i(y^2 - z)] dz$ along the curve$y = x^2$. **[7m]**

   (b) Make use of Cauchys integral formula and find the value of line integral $\int_c \dfrac{z^4 - 3z^2 + 6}{(z+1)^3}$ where c is the circle $\mid z \mid = 2$. **[7m]**

MODULE-III

5. (a) Extend Laurent's series expansion to the function $f(z) = \dfrac{1}{z^2 - 4z + 3}$ for $1 <| z |< 3$, $| z |< 1$ and $| z |> 3$ **[7m]**

   (b) Define Cauchys Residue theorem of an analytic function $f(z)$ within and on the closed curve. Find the value of $\oint \dfrac{1}{\sinh z} dz$ where c is circle $| z |= 4$ using Residue theorem. **[7m]**

6. (a) Define the following terms **[7m]**

   1. The Isolated singularity of an analytic function $f(z)$
   2. Pole of order m of an analytic function $f(z)$
   3. Essential and Removable singularities of an analytic function $f(z)$

   (b) Solve the value of $\int_0^\pi \dfrac{d\theta}{a + b\cos\theta}$ using Residue theorem. **[7m]**

MODULE-IV

7. (a) Define Gamma and Beta functions. Solve the integral $\int_0^\infty \sqrt{x}\, e^{-x/3} dx$ using Gamma functions **[7m]**

   (b) Solve the integral $\int_0^2 (8 - x^3)^{1/3}\, dx$ using Beta-Gamma functions **[7m]**

8. (a) State any three properties of Beta function. Show that $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$ using Beta-Gamma functions **[7m]**

   (b) Show that $\int_0^1 x^m (\log x)^n dx = \dfrac{(-1)^m\, n!}{(m+1)^{n+1}} dx$ where n is a positive integer **[7m]**

MODULE-V

9. (a) What are Bessel differential equation and most general solution of Bessel differential equation? Show the Bessels recurrence relation $J_n^{'}(x) = n\, J_n(x) - x\, J_{n+1}(x)]$ **[7m]**

   (b) Show that $J_{5/2}(x) = \sqrt{\dfrac{2}{\pi x}}(\dfrac{3 - x^2}{x^2} \sin x - \dfrac{3}{x} \cos x)$ **[7m]**

10. (a) Show that $\int_0^1 x\, J_n(\alpha, x)\, J_n(\beta, x) dx = \begin{cases} 0 & \alpha = \beta \\ 0.5[J_{n+1}(\alpha)]^2 & \alpha \neq \beta \end{cases}$ **[7m]**

    (b) Show that $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x\, \sin\theta)\, d\theta$ where $J_n(x)$ is Besel function and n being integer **[7m]**

****END OF EXAMINATION****

COURSE OBJECTIVES:

The course should enable the students to:

1	The applications of complex variable and conformal mapping in two dimensional complex potential theories.
2	The fundamental calculus theorems and criteria for the independent path on contour integral used in problems of engineering.
3	The concepts of special functions and its application for solving the partial differential equation in physics and engineering.
4	The mathematics of combinatorial enumeration by using generating functions and Complex analysis for understanding the numerical growth rates.

COURSE OUTCOMES:

After successful completion of the course, students should be able to:

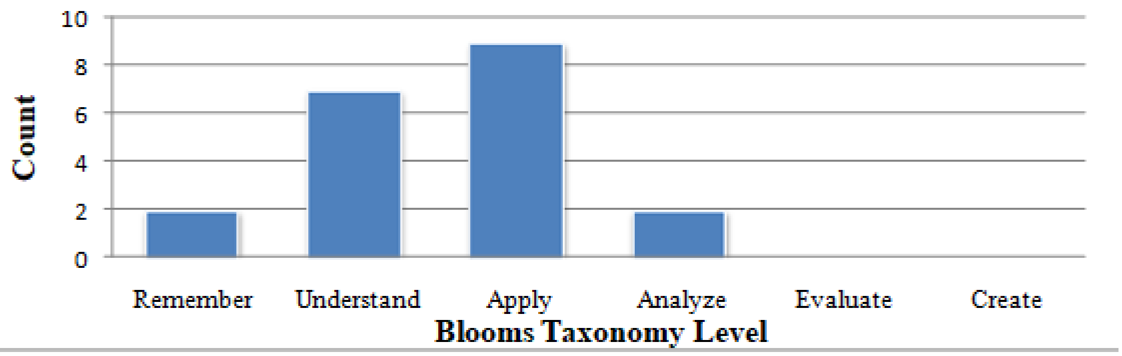
CO 1	Identify the fundamental concepts of analyticity and differentiability for calculus of complex functions and their role in applied context.
CO 2	Utilize the concepts of analyticity for finding complex conjugates and their role in applied contexts.
CO 3	Make use of the conformal mapping technique for transferring geometric structure of complex functions with much more convenient geometry.
CO 4	Apply integral theorems of complex analysis and its consequences for the analytic function having derivatives of all orders in simple connected region.
CO 5	Extend the Taylor and Laurent series for expressing the function in terms of complex power series.
CO 6	Classify Singularities and Poles of Complex functions for evaluating definite and indefinite Complex integrals.
CO 7	Apply Residue theorem for computing definite integrals of real and complex analytic functions over closed curves.
CO 8	Relate the concept of improper integral and second order differential equations of special functions for formulating real world problems with futuristic approach.
CO 9	Determine the characteristics of special functions generalization on elementary factorial function for the proper and improper integrals.
CO 10	Choose an appropriate special function on physical phenomena arising in engineering problems and quantum physics.
CO 11	Analyze the role of Bessel functions in the process of obtaining the series solutions for second order differential equation.

MAPPING OF SEMESTER END EXAMINATION QUESTIONS TO COURSE OUTCOMES

Q.No		All questions carry equal marks	Taxonomy	CO's	PO's
1	a	Define the term Analyticity of a complex variable function $f(z)$. Show that the real part of an analytic function $f(z)$ where $u = e^{-2xy}\sin(x^2 - y^2)$ is a harmonic function. Hence find its harmonic conjugate.	Remember	CO 1	PO 1
	b	Define the term Continuity of a complex variable function $f(z)$. Show that the function $f(z) = \mid z \mid$ is continuous everywhere but nowhere differentiable	Remember	CO 2	PO 1
2	a	Construct the Bilinear transformation that maps the points $(\infty, i, 0)$ into the points $(0, i, \infty)$.	Apply	CO 1	PO 1
	b	Find an analytic function $f(z)$ whose real part of an analytic function is $u = \dfrac{\sin 2x}{\cos 2y - \cos 2x}$ by Milne-Thompson method	Apply	CO 2	PO 1, 2
3	a	Utilize the Cauchys integral formula and find value of $\int_c \dfrac{z^3 - \sin 3z}{(z - \frac{\pi}{2})^3} dz$ where c is the circle $\mid z \mid = 2$.	Apply	CO 3	PO 1
	b	Make use of vertices's -1 , 1, 1+i ,-1+i and verify Cauchys theorem for the integral of z taken over the boundary of the rectangle formed.	Apply	CO 4	PO 1,2
4	a	Solve the value of line integral to $\int_{z=0}^{z=i+1} [x^2 + 2xy + i(y^2 - z)]dz$ along the curve$y = x^2$.	Apply	CO 3	PO 1
	b	Make use of Cauchys integral formula and find the value of line integral $\int_c \dfrac{z^4 - 3z^2 + 6}{(z+1)^3}$ where c is the circle $\mid z \mid = 2$.	Apply	CO 4	PO 1,2
5	a	Extend Laurent's series expansion to the function $f(z) = \dfrac{1}{z^2 - 4z + 3}$ for $1 < \mid z \mid < 3$, $\mid z \mid < 1$ and $\mid z \mid > 3$	Understand	CO 5	PO 1,2
	b	Define Cauchys Residue theorem of an analytic function $f(z)$ within and on the closed curve. Find the value of $\oint \dfrac{1}{\sinh z} dz$ where c is circle $\mid z \mid = 4$ using Residue theorem.	Understand	CO 6	PO 1

6	a	Define the following terms a. The Isolated singularity of an analytic function $f(z)$ b. Pole of order m of an analytic function $f(z)$ c. Essential and Removable singularities of an analytic function $f(z)$	Understand	CO 5	PO 1
	b	Solve the value of $\int_0^\pi \frac{d\theta}{a+b\cos\theta}$ using Residue theorem.	Understand	CO 7	PO 1,2
7	a	Define Gamma and Beta functions. Solve the integral $\int_0^\infty \sqrt{x}\, e^{-x/3} dx$ using Gamma functions.	Apply	CO 8	PO 1
	b	Solve the integral $\int_0^2 (8-x^3)^{1/3}\, dx$ using Beta-Gamma functions	Understand	CO 9	PO 1,2
8	a	State any three properties of Beta function. Show that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$ using Beta-Gamma functions	Understand	CO 8	
	b	Show that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^m\, n!}{(m+1)^{n+1}} dx$ where n is a positive integer	Understand	CO 9	PO 1,2
9	a	What are Bessel differential equation and most general solution of Bessel differential equation? Show the Bessels recurrence relation $J_n'(x) = n\, J_n(x) - x\, J_{n+1}(x)]$.	Apply	CO 10	PO 1,2
	b	Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x)$	Analyze	CO 11	PO 1,2
10	a	Show that $\int_0^1 x\, J_n(\alpha,x)\, J_n(\beta,x) dx = \begin{cases} 0 & \alpha = \beta \\ 0.5[J_{n+1}(\alpha)]^2 & \alpha \neq \beta \end{cases}$	Apply	CO 10	PO 1,2
	b	Show that $J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\theta - x\sin\theta)\, d\theta$ where $J_n(x)$ is Besel function and n being integer	Analyze	CO 11	PO 1,2

KNOWLEDGE COMPETENCY LEVELS OF MODEL QUESTION PAPER



Signature of Course Coordinator **HOD**
Mrs. L Indira, Assistant Professor