INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad -500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

TUTORIAL QUESTION BANK

Course Title	**COMPLEX ANALYSIS AND SPECIAL FUNCTIONS**				
Course Code	AHSB05				
Program	B. Tech				
Semester	THREE				
Course Type	Foundation				
Regulation	IARE - R18				
Course Structure	**Theory**			**Practical**	
	Lectures	**Tutorials**	**Credits**	**Laboratory**	**Credits**
	3	1	3	-	-
Course Coordinator	Ms. L Indira, Assistant Professor				

COURSE OBJECTIVES:

Students will try to learn:	
I	The applications of complex variable and conformal mapping in two dimensional complex potential theories.
II	The fundamental calculus theorems and criteria for the independent path on contour integral used in problems of engineering
III	The concepts of special functions and its application for solving the partial differential equation in mathematical physics and engineering.
IV	The Mathematics of combinatorial enumeration by using generating functions and Complex analysis for understanding the numerical growth rates.

COURSE OUTCOMES:

At the end of the course the students should be able to:

Course Outcomes		**Knowledge Level (Bloom's Taxonomy)**
CO 1	**Identify** the fundamental concepts of analyticity and differentiability for calculus of complex functions and their role in applied context.	Remember
CO 2	**Utilize** the concepts of analyticity for finding complex conjugates and their role in applied contexts.	Apply
CO 3	**Make use of the** conformal mapping technique for transferring	Apply

	geometric structure of complex functions with much more convenient geometry.	
CO 4	**Apply** integral theorems of complex analysis and its consequences for the analytic function having derivatives of all orders in simple connected region.	Apply
CO 5	**Extend** the Taylor and Laurent series for expressing the function in terms of complex power series.	Understand
CO 6	**Classify** Singularities and Poles of Complex functions for evaluating definite and indefinite Complex integrals.	Understand
CO 7	**Apply** Residue theorem for computing definite integrals of real and complex analytic functions over closed curves.	Apply
CO 8	**Relate** the concept of improper integral and second order differential equations of special functions for formulating real world problems with futuristic approach.	Understand
CO 9	**Determine** the characteristics of special functions generalization on elementary factorial function for the proper and improper integrals.	Apply
CO 10	**Choose** an appropriate special function on physical phenomena arising in engineering problems and quantum physics.	Apply
CO 11	**Analyze** the role of Bessel functions in the process of obtaining the series solutions for second order differential equation.	Analyze

MAPPING OF EACH CO WITH PO(s), PSO(s):

Course Outcomes	Program Outcomes												Program Specific Outcomes		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**
CO 1	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 2	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 4	3	1	-	1	-	-	-	-	-	-	-	-	-	-	-
CO 5	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 6	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 7	3	-	-	1	-	-	-	-	-	-	-	-	-	-	-
CO 8	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 9	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 10	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 11	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
TOTAL	33	6	-	2	-	-	-	-	-	-	-	-	-	-	-
AVERAGE	**3**	**1**	**-**	**1**	-	-	-	-	-	-	-	-	-	-	-

TUTORIAL QUESTION BANK

MODULE-I				
COMPLEX FUNCTIONS AND DIFFERENTIATION				
PART – A (SHORT ANSWER QUESTIONS)				
S No	**QUESTIONS**	**Blooms Taxonomy Level**	**How does this subsume the level below**	**Course Outcomes**
1	Define the term Analyticity of a complex variable function f (z).	Remember	----	CO 1,CO 2
2	Define the term Continuity of a complex variable function f (z).	Remember	----	CO 1,CO 2
3	Define the term Differentiability of a complex variable function f (z).	Remember	----	CO 1,CO 2
4	Show that complex variable function *f (z)* = z^3 to analyticity for all values of z in Cartesian form.	Understand	Learner to recall the concept of harmonic function and Understand how to prove that it is part of analytic function	CO 1,CO 2
5	Show that the function $v = x^3y - xy^3 + xy + x + y$ can be imaginary part of an analytic function *f (z)* where *z= x+ iy*.	Understand	Learner to recall the concept of harmonic function and Understand how to prove that it is part of analytic function.	CO 1,CO 2
6	Show that the function *f(z)* = $\|z\|^2$ does not satisfy Cauchy-Riemann equations in Cartesian form.	Understand	Learner to recall the Cauchy-Riemann equations and understand how to prove the analytic nature	CO 1,CO 2
7	Show that the complex variable function *f (z)* = $\frac{x-iy}{x^2+y^2}$ for analyticity in Cartesian form.	Understand	Learner to recall the Cauchy-Riemann equations and understand how to prove the analytic nature	CO 1,CO 2
8	Show that the function *f (z) = sinx siny – i cosx cosy* is not analytic function.	Understand	Learner to recall the Cauchy-Riemann equations and understand how to prove the analytic nature	CO 1,CO 2
9	Find the value of k such that *f(x, y)* = $x^3 + 3kxy^2$ may be harmonic function.	Remember	----	CO 1,CO 2
10	Find the analytic function *f (z)* whose real part of the analytic function is *u* = $x^2 - y^2 - x$.	Remember	----	CO 1,CO 2
11	Find the analytic function f (z) whose imaginary part of the analytic function is *v* = $e^x(xsiny + ycosy)$.	Remember	----	CO 1,CO 2
12	Show that the real part of an analytic function f(z) where *u* = $2log\,(x^2 + y^2)$ is harmonic.	Understand	Learner to recall the concept of harmonic function and then prove that it is analytic	CO 1,CO 2
13	Show that the function *f (z)* = $\|z\|^2$ is continuous at all points of z but not differentiable at any $z \neq 0$.	Understand	Learner to recall the concept of continuous function and understand the prove for differentiability.	CO 1,CO 2
14	List all the values of k such that *f (z)* = $e^x(cosky + isinky)$ is an analytic function.	Remember	----	CO 1,CO 2

15	Find the values of a, b, c such that $f(z) = x + ay - i(ax+by)$ is differentiable function at every point.	Remember	----	CO 1,CO 2
16	Show that every differentiable function is continuous or not. Give a valid example.	Understand	Learner to recall the concept of continuous function and understand the concept of differentiability with illustration.	CO 1,CO 2
17	Find the Bilinear transformation whose fixed points are *i*,-*i*.	Remember	----	CO 3
18	Find the Bilinear transformation which maps the points *(0,-i,-1)* into the points *(i,1, 0)*	Remember	----	CO 3
19	Find the points at which $w = \cosh z$ is not conformal.	Remember	----	CO 3
20	List the fixed points of the transformation $w = \frac{2i-6z}{iz-3}$	Understand	Learner to recall the definition of fixed points and understand the procedure to find them.	CO 3
	PART - B (LONG ANSWER QUESTIONS)			
1	Show that the real part of an analytic function f (z) where $u = e^{-2xy}\sin(x^2-y^2)$ is a harmonic function. Hence find its harmonic conjugate.	Understand	Learner to recall the concept of harmonic function and understand the procedure to find harmonic conjugate.	CO 1,CO 2
2	Show that the real part of analytic function f (z) where $u = \log\|z\|^2$ is harmonic function. If so find the analytic function by Milne Thompson method.	Understand	Learners recall Cauchy-Riemann equations and understand Milne Thompson's method of finding analytic functions.	CO 1,CO 2
3	Use Milne Thompson's method to find the imaginary part of an analytic function *f (z)* whose real part of an analytic function is $e^x(x\cos y - y\sin y)$.	Apply	Learners recall the Cauchy-Riemann equations and understanding the method to find imaginary part.	CO 1,CO 2
4	Show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|Real f(z)\|^2 = 2\|f'(z)\|^2$ where $w = f(z)$ is an analytic function.	Understand	Learner to recall Cauchy-Riemann equations and understand the concept of analytic functions.	CO 1,CO 2
5	Find an analytic function *f (z)* whose real part of an analytic function is $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$ by Milne-Thompson method.	Remember	----	CO 1,CO 2
6	Show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 4\|f'(z)\|^2$ If *f (z)* is a regular function of z	Understand	Learner to recall Cauchy-Riemann equations and understanding Milne Thompson's method.	CO 1,CO 2
7	Show that the function defined by $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic function even though Cauchy Riemann equations are satisfied at origin.	Understand	Learner to recall Cauchy-Riemann equations and understand the concept of regular functions through Cauchy Riemann equations.	CO 1,CO 2
8	Show that real part $u = x^3 - 3xy^2$ of an analytic function *f (z)* is harmonic. Hence find the conjugate harmonic function and the analytic function.	Understand	Learner to recall the concept of harmonic function and understand the concept of finding harmonic conjugate.	CO 1,CO 2

9	Find an analytic function *f (z) = u +iv* if the real part of an analytic function is $u = a(1+\cos\theta)$ using Cauchy-Riemann equations in polar form.	Remember	----	CO 1,CO 2
10	Construct the Cauchy-Riemann equations in polar form of an analytic function *f (z).*	Apply	Learner to recall polar form of complex function and understanding Cauchy's Riemann equations in Cartesian form and apply them to find Cauchy Riemann conditions in polar form.	CO 1,CO 2
11	Show that the real and imaginary parts of an analytic function *f (z)* are harmonic.	Understand	Learner to recall the concept of harmonic function and understand how to prove that analytic functions are harmonic.	CO 1,CO 2
12	Find the analytic function *f (z)* whose imaginary part of an analytic function is $r^2 cos2\theta + rsin\theta$ by Cauchy Riemann equations in polar form.	Remember	----	CO 1,CO 2
13	Show that the function $f(z) = \lvert z \rvert$ is continuous everywhere but nowhere differentiable.	Understand	Learner to recall the concept of continuous function and understand them for proving that it is not differentiable.	CO 1,CO 2
14	Show that the real part of an analytic function *f (z)* where $u = e^{-x}(xsiny - ycosy)$ is a harmonic function.	Understand	Learner to recall the concept of harmonic function and understand them for proving that it is analytic	CO 1,CO 2
15	Show that an analytic function *f (z)* with constant real part is always constant.	Understand	Learner to recall Cauchy-Riemann equations and understand the concept of analytic functions	CO 1,CO 2
16	Show that an analytic function *f (z)* with constant modulus is always constant.	understand	Learner to recall the concept of harmonic function and understand the concept of analytic functions	CO 1,CO 2
17	Show that uv is a harmonic functions if u and v is conjugate harmonic functions.	Remember	----	CO 1,CO 2
18	Construct the Bilinear transformation that maps the points *(1-2i, 2+i, 2+3i)* into the points *(2+i,1+3i,4).*	Apply	Learner to recall the cross ratio method to find transformation and understand the bilinear transformation between two points.	CO 3
19	Construct the Bilinear transformation that maps the points *(1, i,-1)* into the points *(2, i,-2).*	Apply	Learner to recall the cross ratio method to find transformation and understand the bilinear transformation between two points.	CO 3
20	Construct the Bilinear transformation that maps the points $(\infty, i, 0)$ into the points $(0, i, \infty)$.	Apply	Learner to recall the cross ratio method to find transformation and understand the bilinear transformation between two points.	CO 3

PART - C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)				
1	Construct the analytic function *f(z)* in terms of *z* if *f (z)* is an analytic function of *z* such that $u + v = \frac{sin2x}{cosh2y - cos2x}$.	Apply	Learner to recall Cauchy-Riemann equations and understand the concept of analytic functions.	CO 1,CO 2
2	Show that if $u = x^2 - y^2, v = -\frac{y}{x^2+y^2}$ both *u* and *v* satisfy Laplace's equation, but *u* + *iv* is not a regular (analytic) function of z.	Understand	Learner to recall condition for function to be Laplace equations and understand the concept of regular functions.	CO 1,CO 2
3	Construct the analytic function f(z) given $u - v = \frac{cosx + sinx - e^{-y}}{2cosx - e^{y} - e^{-y}}$ and *f(z)* is subjected to the condition $f(\frac{\pi}{2}) = 0$.	Apply	Learner to recall condition for function to be Laplace equations and understand the concept of regular functions and make use of Cauchy Riemann equations.	CO 1,CO 2
4	Construct the analytic function f(z) whose real part of it is $u = e^x[(x^2 - y^2)cosy - 2xysiny)]$.	Apply	Learner to recall condition for function to be Laplace equations and understand the concept of regular functions and make use of Cauchy Riemann equations.	CO 1,CO 2
5	Show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) log\lvert f'(z)\rvert = 0$ where $w = f(z)$ is an analytic function.	Understand	Learner to recall condition for function to be Laplace equations and understand the concept of analytic functions.	CO 1,CO 2
6	Construct the analytic function $f(z) = u(r,\theta) + iv(r,\theta)$ such that $v(r,\theta) = \left(r - \frac{1}{r}\right) sin\theta$, $r \neq 0$ using Cauchy-Riemann equations in polar form.	Apply	Learner to recall condition for function to be Laplace equations and understand the concept of regular functions and make use of Cauchy Riemann equations.	CO 1,CO 2
7	Construct an analytic function *f (z)* such that $Re[f'(z)] = 3x^2 - 4y - 3y^2$ and $f(1+i) = 0$.	Apply	Learner to recall condition for function to be Laplace equations and understand the concept of regular functions and make use of Cauchy Riemann equations.	CO 1,CO 2
8	Show that the function $f(z) = \sqrt{\lvert xy\rvert}$ is not analytic at the origin although Cauchy–Riemann equations are satisfied at origin.	Understand	Learner to recall condition for function to be Laplace equations and understand the concept of regular functions.	CO 1,CO 2
9	Utilize the complex potential for an electric field $w = \emptyset + i\varphi$ld where $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$ and determine the function $\emptyset$.	Apply	Learner to recall the concept of analytic function and understand the potential theory and apply it to find function.	CO 1,CO 2
10	Construct the Bilinear transformation that maps the points $(\infty, i, 0)$in the z-plane into the points $(0, i, \infty)$ in the w-plane.	Apply	Learners to recall the cross ratio method understand how to find transformation and apply the bilinear transformation between two points.	CO 3

MODULE-II				
COMPLEX INTEGRATION				
PART – A (SHORT ANSWER QUESTIONS)				
1	Define the Cauchy's integral formula.	Remember	----	CO1, CO 4
2	Define the Cauchy's General integral formula.	Remember	----	CO1, CO 4
3	Define the term Radius of convergence.	Remember	----	CO 5
4	Define the term Power series expansions of complex functions.	Remember	----	CO 5
5	Define the term Line Integral of complex variable function w = f (z).	Remember	----	CO 5
6	Define the term Contour Integration of a given curve in complex function.	Remember	----	CO 5
7	Define Cauchy's integral theorem for multiple connected region.	Remember	----	CO 1, CO 4
8	Find the value of $\int_0^{1+i} z^2 dz$.	Remember	----	CO1, CO 4
9	Find the value of $\int_C \frac{3z^2+7z+1}{(z+1)} dz$ with $C:\|z+i\| = 1$ by Cauchy integral formulae.	Remember	----	CO 1, CO 4
10	Find the value of line integral to $\int_0^{2+i} z^2 dz$ along the real axis to 2 and then vertically to *(2+i).*	Remember	----	CO 1, CO 4
11	Find the value of line integral to $\int_0^{3+i} z^2 dz$ along the straight line *y = x/3.*	Remember	----	CO 1, CO 4
12	Compare Cauchy integral formula with $\int_C e^{-z} dz$ and find the integral $C:\|z-1\| = 1$ by	Understand	Learner to recall Cauchy integral formula and understand how to find solution by comparison.	CO 1, CO 4
13	Solve line integral to $\int_0^{2+i} (x - y^2 + ix^3)\, dz$ along the real axis from *z=0* to *z=1.*	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
14	Solve the line integral $\int_C \bar{z} dz$ from *z =o* to *2i* and then from *2i to z = 4+2i.*	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
15	Find the radius of convergence of an infinite series *f (z) = sinz.*	Remember	----	CO 5
16	Find the radius of convergence of an infinite series *f (z)* $=\frac{1}{1-z}$.	Remember	----	CO 5
17	Find the radius of convergence of an infinite series $1+2^2 z+3^2 z^2+4^2 z^3+........$	Remember	----	CO 5

18	Solve the value of line integral $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x$.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
19	Solve the value of $\int_C \frac{1}{z-2}dz$ around the circle $\|z - 1\| = 5$ by Cauchy integral formulae.	Apply	Learner to recall Cauchy integral formula and understand how to find solution by comparison by applying integral concepts.	CO 1, CO 4
20	Show that by using line integral, $\int_C \frac{1}{(z-a)}dz = 2\pi i$ where c is the curve $\|z-a\| = r$.	Understand	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
	PART - B (LONG ANSWER QUESTIONS)			
1	Utilize the Cauchy's integral formula and find value of $\int_c \frac{z^3 - \sin 3z}{(z-\pi/2)^3}$ dz where c is the circle $\|z\| = 2$.	Apply	Learner to recall Cauchy integral formula and understand how to find solution by comparison by applying integral concepts.	CO 1, CO 4
2	Make use of vertices *-1 ,1,1+i ,-1+i* and verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle formed .	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
3	Solve the value of line integral to $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$ where c is the circle $\|z\| = 3$ using Cauchy's integral formula.	Apply	Learner to recall Cauchy integral formula and understand how to find solution by comparison by applying integral concepts.	CO 1, CO 4
4	Compare Cauchy's integral formula with line integral to $\int_c \frac{z^3 e^{-z}}{(z-1)^3} dz$ where c is $\|z-1\| = \frac{1}{2}$ and find the value of integral.	Understand	Learner to recall Cauchy integral formula and understand how to find solution by comparison.	CO 1, CO 4
5	Solve the value of line integral to $\int_c \frac{5z^2 - 3z + 2}{(z-1)^3}dz$ where c is any simple closed curve enclosing $\|z\| = 1$ using Cauchy's integral formula.	Apply	Learner to recall Cauchy integral formula and understand how to find solution by comparison by applying integral concepts.	CO 4
6	Solve the value of line integral to $\int_{z=0}^{z=1+i}[x^2 + 2xy + i(y^2 - x)]\,dz$ along the curve $y = x^2$.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 4
7	Solve the integral $\int_c (3z^2 + 2z - 4)dz$ around the square with vertices *at (0,0), (1,0), (1,1) and (0,1).*	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 4

8	Utilize the function $f(z) = 5\ sin2z$and verify Cauchy's theorem for c is the square with vertices at $1 \pm i and - 1 \pm i$.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 4
9	Compare Cauchy's integral formula with line integral to $\int_C \frac{(\sin z)^6}{\left(z-\frac{\pi}{6}\right)^3} dz$ around the unit circle and find the integral.	Understand	Learner to recall Cauchy integral formula and understand how to find solution by comparison.	CO 4
10	Solve the value of to $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c $is\ \lvert z-1 \rvert = 3$using Cauchy's general integral formulae.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO1, CO4
11	Make use of Cauchy's integral formula and evaluate $\int_c \frac{z+1}{z^2+2z+4} dz$ Where $c : \lvert z+1+i \rvert = 2$.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
12	Solve the value of line integral to $\int_C (y^2+z^2)dx+(z^2+x^2)dy+(x^2+y^2)dz$ from *(0, 0 ,0)* to *(1,1,1)* where C is the curve $x= t, y = t^2, z = t^3$ in the parametric form.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
13	Compare Cauchy general integral formula with $\int_c \frac{e^z}{z^2(z+1)^3} dz$ and estimate the value of integral with $C:\lvert z \rvert = 2$.	Understand	Learner to recall Cauchy integral formula and understand how to find solution by comparison.	CO 1, CO 4
14	Show that if *f(z)* is analytic function then $\int_A^B f(z)dz$ is independent of path followed.	Understand	Learner to recall condition for function to be analytic function and understand the concept of regular functions.	CO1 , CO 4
15	Solve the value of line integral to $\int_0^{3+i} z^2 dz$ along the parabola $x=3y^2$.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
16	Compare Cauchy general integral formula with $\int_C \frac{1}{e^z (z-1)^3} dz$ and estimate the value of integral with $C:\lvert z \rvert = 2$.	Understand	L learner to recall Cauchy general integral formula and understand how to find solution by comparison.	CO 1, CO 4
17	Solve the value of $\int_C \frac{e^z \sin 2z - 1}{z^2 (z+2)^2} dz$ where c *is* $\lvert z \rvert = \frac{1}{2}$ using Cauchy integral formulae.	Apply	Learner to recall Cauchy integral formula and understand how to find solution by comparison by applying integral concepts.	CO 1, CO 4
18	Compare Cauchy's integral formula with line integral to $\int_c \left[\frac{e^z}{z^3} + \frac{z^4}{(z-i)^2}\right] dz \quad ,c : \lvert z \rvert = 2$ and find the integral.	Understand	Learner to recall Cauchy general integral formula and understand how to find solution by comparison.	CO 1, CO 4

19	Find the value of line integral to $\int_C (z^2+3z)dz$ along the straight line from *(2,0) to (2,2)* and then from *(2,2) to (0,2).*	Remember	----	CO 1, CO 4
20	Solve the value of line integral to $\int_C \frac{\cos hz}{z^4}dz$ if C denote the boundary of a square whose sides lie along the lines $x=\pm 2$ *and* $y=\pm 2$ where C is described in positive sense.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
PART - C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)				
1	Solve the value of line integral to $\int_c \frac{z}{(z-1)(z-2)^2}dz$ where c is the circle $\lvert z-2\rvert = 1/2$ using Cauchy's integral formula.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
2	Solve the value of line integral to $\int_c \frac{z^4}{(z+1)(z-i)^2}dz$ where c is the ellipse $9x^2+4y^2=36$ using Cauchy's integral formula.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
3	Compare Cauchy's integral formula with line integral to $\int_c \frac{z^4-3z^2+6}{(z+1)^3}dz$ where c is the circle $\lvert z\rvert = 2$ and find the integral.	Understand	Learner to recall Cauchy general integral formula and understand how to find solution by comparison.	CO 1, CO 4
4	Compare Cauchy's integral formula with line integral to $\int_c \frac{z^2-2z-2}{(z^2+1)^2}dz$ where c is the circle $\lvert z-i\rvert = 1/2$ and find integral.	Understand	Learner to recall Cauchy general integral formula and understand how to find solution by comparison.	CO 1, CO 4
5	Solve the value of line integral to $\int_c \frac{e^z}{(z^2+\pi^2)^2}dz$ where *c is* $\lvert z\rvert = 4$ using Cauchy's integral formula.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
6	Solve the value of line integral to $\int_c \frac{\cos \pi z^2}{(z-1)(z-2)^3}dz$ where c is the circle $\lvert z\rvert = 3$ using Cauchy's integral formula.	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
7	Solve the value of line integral to $\int_0^{1+i}(x-y+ix^2)\,dz$ i) Along the straight line from *z = 0 to z = 1+i.* ii) Along the real axis from *z = 0 to z = 1* and then along a line parallel to imaginary axis from *z = 1 to z = 1+i* iii) Along the imaginary axis from *z = 0 to z = i* and then along a line parallel to real axis *z = i to z = 1+ i.*	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4

8	Make use of the vertices *-1+i ,-1-i,1+i ,-1-i* and verify Cauchy's theorem for the integral of $3z^2 + iz - 4$ taken over the boundary of the square .	Apply	Learner to recall the method of finding line integral in real analysis and understand the variable value along the line and apply integral concepts.	CO 1, CO 4
9	Derive the Cauchy general integral formulae of an analytic function *f(z)* within a closed contour c.	Remember	-	CO 1, CO 4
10	Solve the value of line integral to $\int_C (y^2+2xy)dx+(y^2-2xy)dy$ where C is the boundary of the region$y = x^2 and x = y^2$.	Understand	Learner to recall Cauchy general integral formula and understand how to find solution by comparison.	CO 1, CO 4
MODULE-III				
POWER SERIES EXPANSION OF COMPLEX FUNCTION				
PART – A (SHORT ANSWER QUESTIONS)				
1	What is Taylor's theorem of complex power series?	Remember	----	CO 5
2	What is Laurent's theorem of complex power series?	Remember	----	CO 5
3	Define the term pole of order m of an analytic function f(z).	Remember	----	CO 6
4	Define the terms Essential and Removable singularity of an analytic function *f(z).*	Remember	----	CO 6
5	Extend $f(z) = \frac{1}{z^2}$ in powers of $z+1$ as a Taylor's series.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
6	Extend $f(z) = e^z$ as Taylor's series about $z = 1$.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
7	Find the Poles of $\frac{1}{z^2-1}$.	Remember	----	CO 6
8	Extend the Taylor series expansion of $f(z) = e^z$ about the point $z = 1$.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
9	Find the Poles of the function $f(z) = \frac{ze^z}{(z+2)^4(z-1)}$.	Remember	----	CO 6
10	Define the Isolated singularity of an analytic function *f(z).*	Remember	----	CO 6
11	Define Cauchy's Residue theorem of an analytic function *f(z)* within and on the closed curve.	Remember	----	CO 6,CO 7
12	Find the Residue by Laurent's expansion to	Remember	----	CO 6,CO 7

	$f(z)=\frac{e^z}{(z-1)^2}$about $z=1$.			
13	Compare Laurent's expansion and find the Residues of the function $f(z)=\frac{1}{(z-\sin z)}$ about $z = 0$.	Understand	Learner to recall the Laurent's expansion formula and understanding how to residues of given analytic function.	CO 6,CO 7
14	Find the Residues of the function $f(z)=\frac{z}{(z+1)(z+2)}$ as a Laurent's series about $z = -2$.	Remember	----	CO 6,CO 7
15	Solve the value of $\oint_C \frac{1-2z}{z(z-1)(z-2)}dz$ where c is circle $\lvert z\rvert=\frac{1}{2}$ by Cauchy's Residue theorem.	Apply	Learner to recall Cauchy's Residue theorem and apply those formula to find integral.	CO 6,CO 7
16	State Residue formulae for simple pole.	Remember	----	CO 6,CO 7
17	Explain the types of evaluation of integrals by Cauchy's Residue theorem.	Apply	Learner to recall Cauchy's Residue theorem and classify the types to find integral.	CO 6,CO 7
18	Find the Residues of the function $f(z)=\frac{z}{(z-1)(z-2)}$ as a Laurent's series about $z = -1$.	Remember	----	CO 6,CO 7
19	Define the radius and region of convergence of a power series.	Remember	----	CO 5
20	Define the residue of a function by Laurent series expansion	Remember	----	CO 3
	PART - B (LONG ANSWER QUESTIONS)			
1	Extend $f(z)=\frac{z-1}{z+1}$ in Taylor's series about the point $z = 1$.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
2	Extend $f(z)=\frac{z-1}{z^2}$ in Taylor's series in powers of $z-1$. Also determine the region of convergence about the point $z = 1$.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
3	Extend Laurent's series expansion to the function $f(z)=\frac{z^2-4}{z^2+5z+4}$ *valid in* $1< z < 4$.	Understand	Learner to recall the Laurent's expansion formula and understanding how to residues of given analytic function.	CO 5
4	Extend $f(z)=\frac{e^{2z}}{(z-1)^3}$ *about* $z = 1$ as Laurent's series. Also find the region of convergence about $z = 1$.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point..	CO 5
5	Extend $f(z)=\frac{7z-2}{z(z+1)(z-2)}$ about $z=-1$ in the region $1<\lvert z+1\rvert<3$ as Laurent's series.	Understand	Learner to recall the Laurent's expansion formula and understanding how to residues of given analytic function.	CO 5

6	Extend $f(z)=\frac{2z^3+1}{z(z+1)}$ in Taylor's series about the point *z = 1*	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
7	Find Taylor's expansion of $f(z)=\frac{z+1}{(z-3)(z-4)}$ about the point *z=2*.Determine the region of convergence.	Remember	----	CO 5
8	Extend $f(z)=\cos z$ in Taylor's series about $z=\pi i$.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
9	Extend the Laurent's series expansion of $f(z)=\frac{e^z}{z(1-3z)}$ about *z = 1*.	Understand	Learner to recall the Laurent's expansion formula and understanding how to find residues of given analytic function.	CO 5
10	Develop $f(z)=\frac{1+2z}{z^2+z^3}$ in a series of positive and negative powers of z.	apply	Learner to recall the Taylor's and Laurent's expansion formula and understanding how to find residues of given analytic function.	CO 5
11	Solve the value of $\int_c \frac{2z-1}{z(2z+1)(z+2)}dz$ where c is the circle $\lvert z\rvert = 1$.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
12	Solve the integral $\oint_c \tan z dz$ where c is circle $\lvert z\rvert = 2$.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
13	Solve the integral $\oint_c \frac{dz}{(z^2+4)^2}$ where c is $\lvert z-i\rvert = 2$.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
14	Solve the integral $\oint_c \frac{\coth z}{z-i}dz$ where c is $\lvert z\rvert = 2$.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral..	CO 6,CO 7
15	Solve the integral $\int_{-\infty}^{\infty} \frac{x^2dx}{(x^2+1)(x^2+4)}$ using Residue theorem. .	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the	CO 6,CO 7

			formula to find integral.	
16	Solve the integral $\int_0^{\pi} \frac{d\theta}{(a+b\cos\theta)}$ using Residue theorem. .	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
17	Solve the integral of $\frac{\sin z}{z\cos z}dz$ where c is circle $\|z\| = \pi$.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral..	CO 6,CO 7
18	Find the value of $\oint_c \frac{1}{\sinh z}dz$ where c is circle $\|z\| = 4$ using Residue theorem.	Remember	----	CO 6,CO 7
19	Find the value of $\oint_c \frac{2e^z}{z(z-3)}dz$ where c is circle $\|z\| = 2$ using Residue theorem. .	Remember	----	CO 6,CO 7
20	Solve the integral $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)^2}$ using Residue theorem. .	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
PART - C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)				
1	Extend the Laurent expansion of *f(z)* = $\frac{1}{z^2-4z+3}$ for *1* <$\|z\| < 3$ *(ii)*$\|z\| < 1$ *(iii)* $\|z\| > 3$.	Understand	Learner to recall the Laurent's expansion formula and understanding how to residues of given analytic function.	CO 6,CO 7
2	Extend the Laurent expansion *f(z)* = $\frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region where $(i)\|z\| < 1$ *(ii)1*<$\|z\| < 4$.	Understand	Learner to recall the Laurent's expansion formula and understanding how to residues of given analytic function.	CO 6,CO 7
3	Extend $\frac{1}{z^2(z-3)^2}$ *as* Laurent's series in the region (i)$\|z\| < 1$ (ii)$\|z\| > 3$.	Understand	Learner to recall the Laurent's expansion formula and understanding how to residues of given analytic function.	CO 5
4	Extend *f(z)* = $\frac{2}{(2z+1)^3}$ in Taylor's series about z=0 and z=2.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
5	Extend *f(z)* = $\frac{e^z}{z(z+1)}$ in Taylor's series about z=2.	Understand	Learner to recall the Taylor series expansion formula and understand how to expand the given function about the given point.	CO 5
6	Solve the integral $\oint_c \frac{z-3}{(z^2+2z+5)}dz$ where c is	Apply	Learner to recall Cauchy's Residue theorem and understand where the	CO 6,CO 7

	circle $\|z\| = 1$.		integrand is valid over the given region and apply the formula to find integral.	
7	Solve the integral $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ using Residue theorem.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
8	Solve the integral $\int_0^{\infty} \frac{dx}{(x^6+1)}$ using Residue theorem.	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
9	Solve the integral $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ using Residue theorem (*a>0, b>0 and a* $\neq$ *b)*	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
10	Solve the integral $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5+4\cos\theta}$ using Residue theorem	Apply	Learner to recall Cauchy's Residue theorem and understand where the integrand is valid over the given region and apply the formula to find integral.	CO 6,CO 7
		MODULE-IV		
		SPECIAL FUNCTIONS-I		
		PART - A(SHORT ANSWER QUESTIONS)		
1	Show that the value of $\gamma(1/2) = \sqrt{\pi}$.	Understand	Leaner to recall the gamma function and understand the concept of solving improper integrals.	CO 8,CO 9
2	State the value of $\gamma(-7/2)$.	Understand	Leaner to recall the gamma function and understand the concept of solving improper integrals.	CO 8,CO 9
3	Find the value of $\gamma(11/2)$.	Remember	----	CO 8,CO 9
4	Define Gamma function	Remember	----	CO 8
5	Define Beta function	Remember	----	CO 8
6	State relation between Beta and Gamma function	Remember	----	CO 8
7	Find the value of $\int_0^{\infty} e^{x^2} dx$ using gamma function	Remember	----	CO 8,CO 9, CO10
7	Find the value of $\int_0^{\infty} e^{-x^2} dx$ using gamma function	Remember	----	CO 8,CO 9, CO10

8	Find the value of $\int_0^\infty e^{-x^3}\ dx$ using Gamma function	Remember	----	CO 8,CO 9, CO10
9	Find the value of $\int_{-\infty}^{\infty}$ □$^{-□^2}$ □ □using Gamma function	Remember	----	CO 8,CO 9, CO10
10	Solve the integral $\int_0^\infty x^2 e^{-x^2} dx$ using Gamma function	Apply	Leaner to recall the gamma function and understand the concept of solving improper integrals to apply for given integral.	CO 8,CO 9
11	What kind of Eulerian integral is Gamma function?	Remember	----	CO 8
12	What kind of Eulerian integral of Beta function?	Remember	----	CO 8
13	What are the convergent values of Gamma function?	Remember	----	CO 8
14	Find the value of integral $\int_0^\infty \frac{□^2}{\sqrt{1-□^5}}$ □ □ in terms of Beta function	Remember	----	CO 8,CO 9, CO10
15	Find the value of integral $\int_0^\infty$ □^{4}□$^{-□^2}$ □ □ using Gamma function	Remember	----	CO 8,CO 9, CO10
16	Find the value of integral $\int_0^\infty$ □$^4\left(□□□\frac{1}{□}\right)^3$ □ □ using Beta-Gamma function.	Remember	----	CO 8,CO 9, CO10
17	Find the value of integral $\int_0^{\frac{\pi}{2}} \sqrt{Tan\theta}\ d\theta$ in terms of Gamma function	Remember	----	CO 8,CO 9, CO10
18	Find the value of integral $\int_0^{\frac{\pi}{2}} \sqrt{Sec\theta}\ d\theta$ in terms of Gamma function	Remember	----	CO 8,CO 9, CO10
19	State any three properties of Beta function	Remember	----	CO 8
20	Show that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \beta(\frac{2}{5}, \frac{1}{2})$	Understand	Leaner to recall the gamma function and understand the concept of solving improper integrals.	CO 8,CO 9, CO10
PART - B (LONG ANSWER QUESTIONS)				
1	Show that $\frac{\beta(m+1,n)}{m} = \frac{\beta(m.n+1)}{n} = \frac{\beta(m,n)}{m+n}$	Understand	Leaner to recall the beta function and understand the different standard form of beta function.	CO 8,CO 9, CO10
2	Show that $\beta(m,n) = \beta(m+1.n) + \beta(m,n+1)$	Understand	Leaner to recall the beta function and understand the different standard form of beta function.	CO 8,CO 9, CO10
3	Prove that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{sin\theta}} \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\ d\theta = \pi$	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10

4	Solve the integral $\int_0^a x^4 \sqrt{a^2 - x^2}\, dx$ using Beta-Gamma functions	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
5	Solve the integral $\int_0^1 (xlogx)^4\, dx$ using Gamma function	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation..	CO 8,CO 9, CO10
6	Solve the integral $\int_0^1 x^{-3/2}(1 - e^{-x})\, dx$ using Gamma function	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
7	Solve the integral $\int_0^\infty \sqrt{x}\, e^{-x/3}\, dx$ using Gamma function	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
8	Show that $\Gamma n = \int_0^1 (log\frac{1}{x})^{n-1}\, dx$	Understand	Leaner to recall the beta-gamma relation and apply them for solving improper integrals	CO 8,CO 9, CO10
9	Show that $\beta(n,n) = \frac{\sqrt{\pi}\ \Gamma(n)}{2^{2n-1}\ \Gamma(n+\frac{1}{2})}$	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them.	CO 8,CO 9, CO10
10	Solve the integral $\int_0^1 \frac{x^8(1-x^6)}{(1+x)^{24}}\, dx$ using Beta-Gamma functions	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation..	CO 8,CO 9, CO10
11	Show that $\int_0^\infty x^m\, e^{-ax^n}\, dx = \frac{1}{na^{\frac{m+1}{n}}} \Gamma(\frac{1+m}{n})$ where m and n are positive constants	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
12	Show that $\Gamma(n).\Gamma(1-n) \ldots\ldots\ldots. = \frac{\pi}{sinn\pi}$	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them.	CO 8,CO 9, CO10
13	Solve the integral $\int_0^{\frac{\pi}{2}} (\sqrt{Tan\theta} + \sqrt{Sec\theta})d\theta$ using Beta-Gamma functions	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
14	Solve the integral $\int_0^\infty 3^{-4x^2}$ using Beta-Gamma functions	Apply	Leaner to recall the beta-gamma relation and understand how to find	CO 8,CO 9, CO10

			improper integrals by applying beta-gamma relation.	
15	Solve the integral $\int_0^1 \frac{dx}{\sqrt{(-\log x)}}$ using Gamma function	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
16	Show that $\int_0^\infty e^{-y^{1/m}}\, dy = m\Gamma(m)$	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
17	Solve the integral $\int_0^2 (8-x^3)^{1/3}\, dx$using Beta-Gamma functions	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
18	Solve the integral $\int_0^2 (8-x^3)^{-1/3}\, dx$using Beta-Gamma functions	Apply	Leaner to recall the beta-gamma relation and understand how to find improper integrals by applying beta-gamma relation.	CO 8,CO 9, CO10
19	State and prove the symmetry property of Beta function	Remember	----	CO 8
20	State and prove any two other forms of Beta function	Remember	----	CO 8
PART - C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)				
1	Show that $\beta(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\, dx$	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
2	Find the value of $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$ in terms of gamma function	Remember	----	CO 8,CO 9, CO10
3	Find the value of $\int_a^b (a-x)^m (x-b)^n\, dx$, $b > a$ using Beta-Gamma functions	Remember	----	CO 8,CO 9, CO10
4	Find the value of $\int_0^\infty \frac{dx}{1+x^4}$ using Beta-Gamma functions	Remember	----	CO 8,CO 9, CO10
5	Show that $\int_0^\infty \frac{x^4}{\sqrt{1-x^2}}\, dx = \frac{3\pi}{16}$ using Beta function	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
6	Prove that $4\int_0^\infty \frac{x^2}{\sqrt{1+x^4}}\, dx = \sqrt{2}\pi$ using Beta function	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
7	State and prove the relationship between Beta	Remember	----	CO 8,CO 9,

	and Gamma functions			CO10
8	State that $\beta\left(m+\frac{1}{2},m+\frac{1}{2}\right)=\frac{\pi}{m.2^{4m-1}}.\frac{1}{\beta(m,m)}$	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
9	Show that $\int_0^1 x^m(logx)^n\,dx=\frac{(-1)^m\,n!}{(m+1)^{n+1}}$ where n is a positive integer	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
10	Show that $\int_0^1 y^{q-1}(log\frac{1}{y})^{p-1}\,dx=\frac{\Gamma(p)}{(q)^p}$ where p,q are positive integers	Understand	Leaner to recall the beta-gamma relation and understand the concept to find different relation between them	CO 8,CO 9, CO10
MODULE-V				
SPECIAL FUNCTIONS-II				
PART - A(SHORT ANSWER QUESTIONS)				
1	State the expansion of $J_n(x)$.	Remember	----	CO 10,CO 11
2	State the expansion of $J_n(-x)$.	Remember	----	CO 10,CO 11
3	State Bessel differential equation.	Remember	----	CO 10,CO 11
4	State the most general solution of Bessel differential equation	Remember	----	CO 10,CO 11
5	State the expansion of $J_0(x)$	Remember	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO 10,CO 11
6	Relate $J_2(x)\ intermsof J_0(x)\ and J_1(x)$	Understand	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO 10,CO 11
7	Relate$J_3(x)\ intermsof J_0(x)\ and J_1(x)$	Understand	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO 5
8	Relate$J_4(x)\ intermsof J_0(x)\ and J_1(x)$	Understand	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO 10,CO 11
9	State the relationship between $J_n(x), J_{n-1}(x)\ and\ J_{n+1}\ (x)$	Remember	----	CO 10,CO 11
10	State the relationship between $J_n'(x), J_{n-1}(x)\ and\ J_{n+1}\ (x)$	Remember	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO 5
11	Show that $\left[J_{\frac{1}{2}}\right]^2+\left[J_{-\frac{1}{2}}\right]^2=\frac{2}{\pi x}$	Understand	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO10,CO 11

12	Show that $J_n(x)$=0 has no repeated roots except at x=0.	Understand	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO10,CO 11
13	Show that $\frac{d}{dx}(J_1(x))=-J_1(x)$ where $J_1(x)$ is the Bessel's function.	Understand	Leaner to recall the Bessel recurrence relation and understand them to prove different relation.	CO10,CO 11
14	State the trigonometric expansion of $\cos(xsin\theta)$	Remember	----	CO10,CO 11
15	State the trigonometric expansion of $\sin(xsin\theta)$	Remember	----	CO10,CO 11
16	State Orthogonality Property of Bessel's functions	Remember	----	CO 11
17	State the property of Generating function of Bessel's functions	Remember	Leaner to recall the Bessel recurrence relation and understand them finding trigonometric functions in term of Bessel functions	CO 11
18	Relate $J_{3/2}(x)$ in terms of sine and cosine trigonometric ratios	Understand	Leaner to recall the Bessel recurrence relation and understand them finding trigonometric functions in term of Bessel functions	CO10,CO 11
19	Relate $J_{5/2}(x)$ in terms of sine and cosine trigonometric ratios	Understand	Leaner to recall the Bessel recurrence relation and apply them finding trigonometric functions in term of Bessel functions	CO10,CO 11
20	Show that $J_{-1/2}(x) = J_{1/2}(x).cotx$	Apply	----	CO10,CO 11
PART - B (LONG ANSWER QUESTIONS)				
1	Show that $\int_0^x x^n J_{n-1}(x)dx = x^n J_n(x)$ where $J_n(x)$ is Bessel's function.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
2	Show that $\int_0^x x^{n+1} J_n(x)dx = x^{n+1} J_{n+1}(x)$ where $J_n(x)$ is Bessel's function.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
3	Show that $[J_n^2 + J_{n+1}^2] = \frac{2}{x}[nJ_n^2 - (n+1) J_{n+1}^2]$ where $J_n(x)$ is Bessel's function.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
4	Show that $\frac{d}{dx}[xJ_n(x)J_{n+1}(x)] = x[J_n^2(x) - J_{n+1}^2(x)]$ where $J_n(x)$ is Bessel's f unction.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
5	Show that $J_n(x)$ is an even function when n is even and odd function when n is odd function.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
6	Show that $\int J_3(x)dx = -J_2(x) - \frac{2}{x}J_1(x)$ using Bessel's Recurrence relation.	Apply	Leaner to recall the Bessel recurrence relation and apply	CO10,CO 11

			them to prove different relation	
7	Make use of generating function show that $\cos(x\sin\theta) = J_0 + 2(J_2\cos 2\theta + J_4\cos 4\theta + \ldots\ldots)$.	Analyse	Leaner to recall the Bessel recurrence relation and apply them finding trigonometric relations in term of Bessel functions	CO10,CO 11
8	Make use of generating function show that $\sin(x\sin\theta) = 2(J_1\sin\theta + J_3\sin 3\theta + J_5\sin 5\theta\ldots\ldots)$.	Analyse	Leaner to recall the Bessel recurrence relation and apply those finding trigonometric relations in term of Bessel functions.	CO10,CO 11
9	Show that $J_n(-x) = (-1)^n J_n(x)$ where n is a positive or negative integer.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
10	Show the Bessel's recurrence relation xJ_n'(x) = nJ_n(x) - x J_{n+1}(x).	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
11	Show the Bessel's recurrence relation xJ_n'(x) = -nJ_n(x) + x J_{n-1}(x).	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
12	Show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{1}{x}\sin x - \cos x\right)$	Apply	Leaner to recall the Bessel recurrence relation and apply those finding trigonometric functions in term of Bessel functions.	CO10,CO 11
13	Relate $J_5(x)\ intermsof J_0(x)\ and J_1(x.)$	Understand	Leaner to recall the Bessel recurrence relation and apply them find different functional values.	CO10,CO 11
14	Show that $\int xJ_0{}^2(x)dx = \frac{1}{2}x^2[J_0{}^2(x) + J_1{}^2(x)]$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
15	Show that $\frac{n}{x}J_n(x) + J_n'(x) = J_{n-1}(x)$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
16	Show that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \cdots) = 1$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
17	Show that $2nJ_n(x) = x[J_{n+1} + J_{n-1}]$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
18	Show the Bessel's recurrence relation J_n'(x) = $\frac{1}{2}[J_{n-1}$(x) $- J_{n+1}$(x)].	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
19	Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11

20	Show that $J_{-\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
PART - C (PROBLEM SOLVING AND CRITICAL THINKING QUESTIONS)				
1	Show that $J_{-n}(x)=(-1)^n J_n(x)$ where n is a positive integer.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
2	Show that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\theta - xsin\theta)d\theta$ where $J_n(x)$ is Bessel's function, n being a integer.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
3	Show that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)\,dx=\begin{cases}0, if\ \alpha=\beta\\ \frac{1}{2}[J_{n+1}(\alpha)]^2, if\ \alpha\neq\beta\end{cases}$	Analyse	Leaner to recall the Bessel recurrence relation and apply recurrence relations analyse them in different cases for finding orthogonal function to Bessel	CO 11
4	State and prove Generating function of Bessel's functions	Analyse	Leaner to recall the Bessel recurrence relation and apply recurrence relations analyse them in different cases that generates Bessel function.	CO 11
5	Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
6	Show that $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
7	Show that $\int J_3(x) + J_2(x) + \frac{2}{x}J_1(x) = 0$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
8	Show that $cosx = J_0 - 2J_2 + 2J_4 - \cdots$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
9	Show that $sinx = 2(J_1 - J_3 + J_5 - \cdots$	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11
10	Show that $J_{\frac{5}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x-\frac{3}{x}\cos x\right)$.	Apply	Leaner to recall the Bessel recurrence relation and apply them to prove different relation	CO10,CO 11

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