INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad - 500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

TUTORIAL QUESTION BANK

<table>
<tr><td>Course Title</td><td colspan="5">PROBABILITY THEORY AND STOCHASTIC PROCESSES</td></tr>
<tr><td>Course Code</td><td colspan="5">AECB08</td></tr>
<tr><td>Programme</td><td colspan="5">B.Tech</td></tr>
<tr><td>Semester</td><td colspan="5">THIRD</td></tr>
<tr><td>Course Type</td><td colspan="5">CORE</td></tr>
<tr><td>Regulation</td><td colspan="5">IARE - R18</td></tr>
<tr><td rowspan="3">Course Structure</td><td colspan="3">Theory</td><td colspan="2">Practical</td></tr>
<tr><td>Lectures</td><td>Tutorials</td><td>Credits</td><td>Laboratory</td><td>Credits</td></tr>
<tr><td>3</td><td>1</td><td>4</td><td>-</td><td>-</td></tr>
<tr><td>Course Faculty</td><td colspan="5">Dr. M V Krishna Rao, Professor</td></tr>
</table>

COURSE OBJECTIVES

The students will try to learn:	
I	The fundamental concepts of the 1-dimensional and 2-dimensional random variables and their characterization in probability space.
II	The stationary random process, its framework and application for analysing random signals and noises.
III	The characteristics of 1-dimensional stationary random signals in time and frequency domains.
IV	Analysis of the response of a linear time invariant (LTI) system driven by 1-dimensional stationary random signals useful for subsequent design and analysis of communication systems.

COURSE OUTCOMES

<table>
<tr><td colspan="3">After successful completion of the course, students will be able to:</td></tr>
<tr><td colspan="2">Course Outcomes</td><td>Knowledge Level (Bloom's Taxonomy)</td></tr>
<tr><td>CO 1</td><td>Infer the concepts of the random experiment, event probability, joint event probability, and conditional event probability for proving the Bayes theorem</td><td>Understand</td></tr>
</table>

	and for computing complex event probabilities and independence of multiple events.	
CO 2	**Explain** the concept of random variable, the probability distribution function (PDF), probability density function (pdf), joint and conditional probability density function (cpdf), and demonstrate the differences among various density functions such as Gaussian, Rayleigh, Poisson, Binomial etc.	Understand
CO 3	**Explain** the transformation of random variables, the Expectation operator on functions of random variables to formulate the definition of moments and demonstrate the use of the characteristic and moment generating functions to analytically derive the standard moments.	Understand
CO 4	**Interpret** the vector random variables as the extension of scalar random variables to characterize their joint, marginal and conditional density/ distribution functions.	Understand
CO 5	**Derive** the density function of sum of random variables for demonstrating the central limit theorem and its physical significance.	Apply
CO 6	**Explain** the Expectation operator on functions of vector random variables to formulate the definition of joint moments (e.g. Correlation and Covariance) and demonstrate the use of the joint characteristic and joint moment generating functions to alternatively derive the joint standard moments.	Understand
CO 7	**Develop** the framework for linear transformation of vector gaussian random variables using the properties of jointly gaussian variables.	Apply
CO 8	**Extend** the random variable concept to random process and its sample functions for demonstrating the time domain characteristics such as stationarity, independence, and ergodicity of a random process.	Understand
CO 9	**Relate** the correlation and covariance functions and their properties for the time domain classification of random processes.	Understand
CO 10	**Develop** analytically the auto-power and cross- power spectral densities to solve the related problems of random processes using correlation functions and the Fourier transform.	Apply
CO 11	**Analyze** the response of a linear time invariant (LTI) system driven by stationary random processes using the time domain description of random processes.	Analyze
CO 12	**Discover** the frequency domain characteristics of of a linear time invariant (LTI) system response driven by stationary random processes using the relationship between correlation functions and power density spectra.	Analyze

MAPPING OF EACH CO WITH PO(s), PSO(s):

Course Outcomes	Program Outcomes												Program Specific Outcomes		
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
CO 1	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 2	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 3	3	6	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 5	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 6	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 7	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-

CO 8	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 9	3	6	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 10	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 11	3	6	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 12	3	6	-	-	-	-	-	-	-	-	-	-	-	-	-

TUTORIAL QUESTION BANK

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
MODULE -I				
PROBABILITY AND RANDOM VARIABLES				
PART – A (SHORT ANSWER QUESTIONS)				
1	Define the probability of occurrence of an event.	Remember	---	CO 1
2	What are the axioms of probability?	Remember	---	CO 1
3	Define sample space and classify the types of sample space.	Remember	---	CO 1
4	How probability can be considered as relative frequency?	Remember	---	CO 1
5	Define conditional, joint and total probability.	Remember	---	CO 1
6	Explain Mutually Exclusive Events.	Understand	Recall the concept of mutual exclusiveness and explain the mutually exclusive events of a random experiment.	CO 1
7	Explain independent events.	Understand	Recall the concept of statistical independence and explain the independent events of a random experiment	CO 1
8	What is the concept of random variable?	Remember	---	CO 2
9	Give the classifications of Random variable	Remember	---	CO 2
10	Define Cumulative distribution function (CDF) and Probability density function (pdf).	Remember	---	CO 2
11	List any four properties of density function.	Remember	---	CO 1
12	List any four properties of distribution function.	Remember	---	CO 1
13	Define Bernoulli distribution function.	Remember	---	CO 2
14	Define Binomial Distribution function.	Remember	---	CO 2
15	Define Poisson Random Variable.	Remember	---	CO 2
16	Define Uniform Random Variable.	Remember	---	CO 2

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
17	Define gaussian random variable	Remember	---	CO 2
18	Define exponential random variable.	Remember	---	CO 2
19	Define Rayleigh random variable.	Remember	---	CO 2
PART – B (LONG ANSWER QUESTIONS)				
1	State and prove Bayes theorem.	Apply	Recall the statement of Bayes theorem and the definitions of conditional, joint probabilities, and apply them to prove the theorem.	CO 1
2	State and prove total probability theorem.	Apply	Recall the statement of total probability theorem and the definitions of conditional, joint probabilities, and apply them to prove the theorem.	CO 1
3	Derive expressions for mean and variance for uniform random variable.	Understand	Recall the probability density function of uniform random variable, and definitions of mean and variances and apply them to obtain the required expressions.	CO 2
4	Explain conditional probability theorem with properties.	Understand	Recall the definition of conditional probability and related theorem and explain its properties.	CO 1
5	Explain the properties the probability density function.	Understand	Recall the definition of probability, its axioms, and explain various properties of the probability density function of a random variable X.	CO 1
6	Derive the expression for mean and variance for Poisson random variable.	Apply	Recall the probability density function of Poisson random variable, and definitions of mean and variances and apply them to obtain the required expressions	CO 1
7	Derive expressions for mean and variance for binomial random variable.	Apply	Recall the probability density function of binomial random variable, and definitions of mean and variances and apply them to obtain the required expressions	CO 1
8	Explain the method of defining a conditioning event.	Understand	Recall the definition of at least two events and understand the interrelationship of the events and define the conditioning event.	CO 1
9	Derive expressions for mean and variance for exponential random variable.	Apply	Recall the probability density function of exponential random variable, and definitions of mean and variances and apply them to obtain the required expressions	CO 1
10	Derive the expressions for mean and variance for gaussian random variable.	Apply	Recall the probability density function of gaussian random variable, and definitions of mean and variances and apply them to obtain the required expressions.	CO 1
11	Compute the probability that exactly 5 of the items tested are defective if A batch of 50 items contains 10 defective items. Suppose 10 items are	Understand	Recall the permutation and combinations of choosing items from a sample, understand the definition of probability, and solve the problems of	CO 1

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
	selected at random and tested.		finding the required event probability.	
12	Assume that the height of clouds σ, above the ground at some location is a gaussian random variable X with mean value 2km and σ =0.25km. Find the probability of clouds higher than 2.5km.	Understand	Recall the gaussian density, understand its characteristics and solve the given problem to obtain the required probability.	CO 2
13	A production line manufactures 1KΩ resistors that must satisfy 10% tolerance. If a resistor is described by the gaussian random variable X, for which μ=1000Ω, σ = 40Ω, what fraction of the resistors is expected to be rejected?	Understand	Recall the gaussian density, understand its characteristics and solve the given problem to obtain the required fraction.	CO 2
14	The power reflected from an aircraft received by radar is described by an exponential density. The pdf is given by $f_X(x) = 0.1e^{-x/10}$, x>0. The average power is 10W. What is the probability that the received power is greater than the average power?	Understand	Recall the exponential density, understand its characteristics and solve the given problem to obtain the required fraction.	CO 2
15	Explain the Standard Moments and Central Moments.	Understand	Recall the definitions of Standard Moments and Central Moments, interpret their physical meaning from the mathematical expressions.	CO 2
PART – C (Problem Solving and Critical Thinking Questions)				
1	Assume that the number of automobiles arriving at a gasoline station is Poisson distributed and occur at an average rate of 50/hour. The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel, what is the probability that a waiting line will occur at the pump?	Understand	Recall the definition of Poisson density function and interpret its parameters and compute the required probability substituting the given parameters.	CO 1
2	Find the value of C, for A random variable X has a probability density function $f_X(x)= C(1-x^4)$; $-1<x<1$ = 0 ; elsewhere.	Remember	Recall definition of density function, understand the third axiom of probability and compute the required constant substituting the given parameters.	CO 2
3	Find the value of K and also find $P\{2 \le X \le 5\}$ Let X be a Continuous random variable with density function $f(x)=x/9+k$ for $0<x<6$; =0; otherwise	Understand	Recall definition of density function, understand the third axiom of probability and compute the required constant substituting the given parameters.	CO 2
4	A shipment of components consists of 3 identical boxes. one box contains 2000 components of which 25%are defective , the second box has 5000 components of which 20%are defective, third box contains 2000 components of which 55% are defective. A box is selected at random and a component is removed at	Understand	Recall the concept of the permutation and combinations of choosing items from a sample, understand probability definition , and compute the required probabilities.	CO 1

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
	random from the box what is the probability that this component is defective? What is the probability by that it came from the second box?			
5	The lifetime of IC chips manufactured by a semiconductor manufacturer is approximately normally distributed with a mean $=5 \times 10^6$ hours and standard deviation of 5×10^6 hours. A mainframe manufacturer requires that at least 95% of a batch should have a lifetime greater than 4×10^6 hours. Will the deal be made?	Understand	Recall the gaussian density, understand its characteristics and solve the given problem to obtain the required probability.	CO 2
6	Find the probability that at least one diode is defective. If a box contains 75 good diodes and 25 defective diodes and 12 diodes are selected at random?	Remember	Recall the concept of the permutation and combinations of choosing items from a sample, understand probability definition , and compute the required probabilities.	CO 1
7	A certain large city averages three murders per week and their occurrences follows a Poisson distribution 1. What is the probability that there will be five or more murders in a given week? 2. On the average, how many weeks a year can this city expect to have no murders? 3. How many weeks per year (average) can the city expect the number of murders per week to equal or exceed the average number per week?	Remember	Recall the concept of the permutation and combinations of choosing items from a sample, understand probability definition , and compute the required probabilities.	CO 1
8	Calculate the probabilities of system error and correct system transmission of symbols for an elementary binary communication system consisting of a transmitter that sends one of two possible symbols (a 1 or a 0) over a channel to a receiver. The channel occasionally causes errors to occur so that a "1" show up at the receiver as a "0" and vice versa. Assume the symbols **1**and **0** are selected for a transmission as 0.6 and 0.4 respectively. $p(A_1/B_1) = 0.95$ $p(A_1/B_2) = 0.05,$ $(A_2/B_1) = 0.05$ and $p(A_2/B_2) =$ $= 0.95$	Understand	Recall the definitions of joint and conditional probabilities, understand the communication system model, apply the related expressions and compute the required probabilities.	CO 1

<table>
<tr><th>S.No</th><th>Question</th><th>Blooms taxonomy level</th><th>How does this Subsume the levels?</th><th>Course Outcomes</th></tr>
<tr><td>9</td><td>Find the individual, joint and conditional probabilities. For a given problem as shown below.
In a box there are 100 resistors having resistance and tolerance values given in table. Let a resistor be selected from the box and assume that each resistor has the same likelihood of being chosen. Event A: Draw a 47Ω resistor, Event B: Draw a resistor with 5% tolerance, Event C: Draw a 100Ω resistor.
<table>
<tr><td rowspan="2">Resistance (Ω)</td><td colspan="2">Tolerance</td><td rowspan="2">Total</td></tr>
<tr><td>5%</td><td>10%</td></tr>
<tr><td>22</td><td>10</td><td>14</td><td>24</td></tr>
<tr><td>47</td><td>28</td><td>16</td><td>44</td></tr>
<tr><td>100</td><td>24</td><td>8</td><td>32</td></tr>
<tr><td>Total</td><td>62</td><td>38</td><td>100</td></tr>
</table></td><td>Understand</td><td>Recall the definitions of joint and conditional probabilities, understand the communication system model, apply the related expressions and compute the required probabilities.</td><td>CO 1</td></tr>
<tr><td>10</td><td>A man wins in a gambling game if he gets two heads in five flips of a biased coin. The probability of getting a head with the coin is 0.7. i) Find the probability the man will win. Should he play this game? ii) What is the probability of winning if he wins by getting at least four heads in five flips? Should he play this new game?</td><td>Understand</td><td>Recall the definitions of joint and conditional probabilities, understand the communication system model, apply the related expressions and compute the required probabilities.</td><td>CO 1</td></tr>
<tr><td colspan="5">MODULE -II</td></tr>
<tr><td colspan="5">OPERATIONS ON SINGLE & MULTIPLE RANDOM VARIABLES – EXPECTATIONS</td></tr>
<tr><td colspan="5">PART – A (SHORT ANSWER QUESTIONS)</td></tr>
<tr><td>1</td><td>What are the statistical mean and MS values?</td><td>Remember</td><td>---</td><td>CO 3</td></tr>
<tr><td>2</td><td>Give the expression for an arbitrary transformation of a single random variable, with a brief explanation.</td><td>Remember</td><td>---</td><td>CO 3</td></tr>
<tr><td>3</td><td>Define N-dimensional random vector.</td><td>Remember</td><td>---</td><td>CO 4</td></tr>
<tr><td>4</td><td>Define the joint distribution function of two continuous random variables X and Y.</td><td>Remember</td><td>---</td><td>CO 4</td></tr>
<tr><td>5</td><td>Define the joint distribution function of two discrete random variables X and Y.</td><td>Remember</td><td>---</td><td>CO 4</td></tr>
<tr><td>6</td><td>Define the joint distribution and density functions, in mathematical form, of N-dimensional continuous random vector X</td><td>Remember</td><td>---</td><td>CO 4</td></tr>
</table>

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
7	Define the joint distribution and density functions, in mathematical form, of N-dimensional discrete random vector X	Remember	---	CO 4
8	List any five properties of joint distribution function of two random variables.	Remember	---	CO 4
9	List any five properties of joint density function of two random variables.	Remember	---	CO 4
10	Define the relation between the marginal distribution functions and their joint density function of a 2-dimensional random vector	Remember	---	CO 4
11	Comment on the density function of a sum of two random variables.	Apply	Recall the definitions of density function and convolution integral and develop the density function of the new variable after addition.	CO 5
12	Explain joint probability distribution function properties of a random variable	Remember	---	CO 4
13	State the Central Limit Theorem.	Remember	---	CO 5
PART – B (LONG ANSWER QUESTIONS)				
1	Define and explain joint distribution function and joint density function of two random variables X and Y.	Apply	Recall the concept of a vector random variable, definitions of joint probabilities, and explain joint distribution and joint density functions of two variables X and Y.	CO 5
2	Distinguish between joint distribution and marginal distribution.	Understand	Recall the concept of a vector random variable, definitions of joint and marginal probabilities, and distinguish between joint distribution and marginal distributions.	CO 4
3	Define and explain conditional distribution and density function of two random variables X and *Y*.	Understand	Recall the concept of a vector random variable, definitions of joint and marginal probabilities, and distinguish between joint distribution and marginal distributions.	CO 4
4	What is the probability density function of the sum of two uniform random variables? Prove.	Apply	Recall the definitions of uniform density function and convolution integral and obtain the density function of the new variable after addition.	CO 5
5	The characteristic function for a Gaussian random variable X, having a mean value of 0, is $\Phi_X(\omega)$ =exp(-w ^{2}/ σ ^{2}). Find first three moments of X using $\Phi_X(w)$.	Understand	Recall the definition of characteristic function, understand the method of obtaining moments from characteristic function and compute the first three moments.	CO 3
6	State the properties of a joint distribution function.	Understand	Recall the definition of joint and marginal distribution functions, understand axioms of probabilities and state the properties of joint distribution function.	CO 4

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
7	Explain how a discrete random variable is transformed to a new discrete random variable.	Understand	Recall the definition of cumulative distribution of a discrete random variable and a transformation function and explain the mapping by equating the cumulative distributions of both input and output random variables.	CO 3
8	Find the moment generating function of Y, if X is a random variable with a Moment generating function of $M_X(v)$, Y=aX+b	Understand	Recall the definition of moment generating function, relate to the transformation of variables and obtain the moment generating function of the output variable.	CO 2
9	State and prove the central limit theorem.	Apply	Recall the concept of addition of two random variables and understand the convolution integral and obtain the density function of the new variable obtained after adding N-random variables to prove the central limit theorem.	CO 5
10	A random variable has a probability density function $f_X(x)=(5/4)(1-x^4)$ for $0<x<1$ = 0 other wise Find a) E[X] b) E[4X+2] and c). $E[X^2]$	Understand	Recall the concept of statistical expectation operator and understand its application on the function of random variables to compute the required expectations.	CO 3
11	The pdf of a random variable X is given by $fx(x)= x/20$; $2 \leq x \leq 5$, find the pdf of Y = 3X-5.	Understand	Recall the monotonic transformation and understand the relation for the monotonic transformation of a random variable and derive the required probability density function.	CO 3
12	The pdf of a random variable X is $fx(x)=(1/2)\cos(x)$; $-\pi/2<x<\pi/2$, find the mean of the function $g(X)=4X^2$.	Understand	Recall the nonmonotonic transformation and expectation operator, understand the relation for nonmonotonic transformation of a random variable and compute required mean.	CO 3
13	Explain the significance of the characteristic function of a random variable.	Understand	Recall the definition of characteristic function, understand its relationship with the density function, and explain its significance.	CO 3
14	Explain the significance of the moment generating function of a random variable.	Understand	Recall the definition of moment generating function, understand its relationship with the density function, and explain its significance.	CO 3
15	Obtain the probability of the random variable obtained by the monotonic transformation of a random variable.	Understand	Recall the definition of cumulative density function and the monotonic transformation and derive the required probability by equating the eq cumulative densities of both input and output random variables.	CO 3

<table>
<tr><th>S.No</th><th>Question</th><th>Blooms taxonomy level</th><th>How does this Subsume the levels?</th><th>Course Outcomes</th></tr>
<tr><td>16</td><td>Obtain the expression for a nonmonotonic transformation of a single random variable.</td><td>Understand</td><td>Recall the definition of cumulative density function and the nonmonotonic transformation and derive the required probability by equating the cumulative densities of both input and output random</td><td>CO 3</td></tr>
<tr><td colspan="5">PART-C (Problem Solving and Critical Thinking Questions)</td></tr>
<tr><td>1</td><td>Find $f_X(x)$ and $f_Y(y)$, the marginal pdfs of X and Y .The joint probability density function of random variables X and Y is
$f_{XY}(x, y) = 6(x + y^2)/5 \quad 0 \leq x \leq 1, 0 \leq y \leq 1$,
is zero otherwise.</td><td>Understand</td><td>Recall the definition of joint and marginal density functions, understand their relationship, and compute the marginal densities.</td><td>CO 3</td></tr>
<tr><td>2</td><td>Calculate the marginal pdfs $f_X(x)$ and $f_Y(y)$
The joint Pdf of X and Y is
$f_{X,Y}(x, y) = 5xy/4 \quad -1 \leq x \leq 1, -2 \leq y \leq 1$, =0 otherwise.</td><td>Understand</td><td>Recall the definition of joint and marginal density functions, understand their relationship, and compute the marginal densities.</td><td>CO 4</td></tr>
<tr><td>3</td><td>Find f(y/x) and f(x/y) for The joint density function of random variables X and Y is
$f_{X,Y}(x,y)=8xy; \quad 0<x<1, 0<y<1$
=0, otherwise</td><td>Understand</td><td>Recall the definition of joint and marginal density functions, understand their relationship with the conditional density functions, and compute the required conditional densities.</td><td>CO 4</td></tr>
<tr><td>4</td><td>Find the probability density function of the random variable Y obtained by the transformation
$Y = 3X^3 - 3X^2 + 2$ of the discrete random variable X whose density function is given below.
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(x,y$</td><td>0.2</td><td>0.15</td><td>0.3</td><td>0.15</td><td>0.2</td></tr></table></td><td>Understand</td><td>Recall the variable transformation of a discrete random variable and understand the relation for the monotonic transformation of a random variable and derive the required probability density.</td><td>CO 3</td></tr>
<tr><td>5</td><td>The probabilities of the random variables X and Yare given in Table . Find (a) value of K and (b) the joint distribution function and marginal distribution functions.
Joint probabilities of X and Y
<table><tr><td>X/Y</td><td>-1</td><td>0</td><td>1</td></tr><tr><td>0</td><td>$3/18$</td><td>$2/18$</td><td>$3/18$</td></tr><tr><td>1</td><td>$1/18$</td><td>$K/18$</td><td>$1/18$</td></tr><tr><td>2</td><td>$1/18$</td><td>$1/18$</td><td>$2/18$</td></tr></table></td><td>Understand</td><td>Recall the axioms of probability, use it to compute unknown K, understand the relationship between the joint and marginal distribution functions and compute the required probability functions.</td><td>CO 4</td></tr>
<tr><td>6</td><td>Find the probability density function of the random variable Y, obtained by a quadratic transformation a random variable X.</td><td>Understand</td><td>Recall the variable transformation of a random variable and understand the relation for the nonmonotonic transformation of a random variable and derive the required probability density.</td><td>CO 3</td></tr>
</table>

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
7	The joint pdf is given as $f_{X,Y}(x,y) = Ae^{-(2x+y)}$ for $x \geq 0$ and $y \geq 0$. Find (a) the value of A and (b) the marginal density functions.	Understand	Recall the axioms of probability, use it to compute unknown K, understand the relationship between the joint and marginal distribution functions and compute the required probability functions.	CO 4
8	Find the conditional density functions for the joint density function$f_{X,Y}(x,y) = 4xye^{-(x^2+y^2)}$u(x)u(y)	Understand	Recall and understand the relationship between the joint and conditional density functions and find the required joint density	CO 4
9	The joint pdf of two variables X and Y is (x) for $f_{X,Y}(x,y) = \frac{1}{18}e^{-\left(\frac{x}{6}+\frac{y}{3}\right)}$ for x $\geq$ 0 and y $\geq$ 0. Show that X and Y are independent random variables	Understand	Recall the definition of statistical independence, understand the conditions of independence, and show that variables are independent.	CO 4
10	Find the pdf of W = X+ Y, where X and Y have joint pdf $f_{XY}(x, y) = 1/15$ $0 \leq x \leq 3, 0 \leq y \leq 5$, =0 otherwise.	Understand	Recall the definition of joint and marginal density functions, understand their relationship and find the pdf of the output variable.	CO 4
MODULE –III				
OPERATIONS ON MULTIPLE RANDOM VARIABLES – EXPECTATIONS				
PART – A (SHORT ANSWER QUESTIONS				
1	Define the joint moments of order 3 about the origin of two random variables.	Remember	---	CO 6
2	Define the correlation between two random variables X and Y for both continuous and discrete cases.	Remember	---	CO 6
3	Define the joint central moments of order 3 of two continuous random variables X and Y.	Remember	---	CO 6
4	Define the covariance between two random variables X and Y for both continuous and discrete cases.	Remember	---	CO 6
5	Define the normalized second central moment of two continuous random variables X and Y.	Remember	---	CO 6
6	What are the conditions for the 2-dimentional random variable to be uncorrelated and orthogonal?	Remember	---	CO 6
7	State the properties of correlation coefficient.	Remember	---	CO 6

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
8	Define the expected value of a function of two random variables.	Remember	---	CO 6
9	Explain two joint central moments for two-dimensional random variable *(X, Y)*	Understand	Recall the definition of Central Moments, interpret their physical meaning from the mathematical expressions	CO 6
10	Describe joint characteristic function.	Understand	---	CO 6
11	How is the expected value of a conditional event defined? Explain.	Understand	---	CO 6
12	Define Joint Moments about the origin.	Remember	---	CO 6
13	Define the expected value of joint random variable.	Remember	---	CO 6
CIE-II				
1	Define the expected value of a joint random variable of 2-dimensions.	Remember	---	CO 6
2	Define the joint characteristic function of two random variables for both continuous and discrete cases.	Remember	---	CO 6
3	Define the joint moment generating function of two random variables for both continuous and discrete cases.	Remember	---	CO 6
4	List and explain any four properties of joint Gaussian variables.	Understand	Recall the characteristic parameters of gaussian variable, interpret the expression for the joint density function of vector gaussian variable and explain the properties.	CO 7
5	What are the elements of the covariance matrix of two random variables?	Remember	---	CO 6
6	What is Jacobian? Where is it used in probability theory?	Remember	---	CO 7
7	Define the joint characteristic function of a 2-dimensional random variable defined?	Remember	---	CO 6
8	Define the joint moment generating function of a 2-dimensional random variable defined?	Remember	---	CO 6
PART – B (LONG ANSWER QUESTIONS)				
1	State and prove the properties of correlation between two random variables.	Understand	Recall the definition of correlation, understand the properties of correlation between two random variables and prove the correlation properties.	CO 6

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
2	For two random variables X and Y, prove that $cov(X+a,Y+b)=C_{XY}$ and $cov(aX,bY)=abC_{XY}$.	Understand	Recall the definition of covariance, understand its relationship with correlation and prove the required properties.	CO 6
3	Prove that any two random variables X and Y, the inequality $\lvert\sigma_{XY}\rvert \le \sigma_X\sigma_Y$ is true.	Understand	Recall the definition of covariance, understand its relationship with correlation and prove the given inequality.	CO 6
4	Describe any four properties of the covariance between two random variables.	Understand	Recall the definition of covariance, understand its relationship with correlation and prove the required properties.	CO 6
5	Show that the variance of a weighted sum of uncorrelated random variables equals the weighted sum of the variances of the individual random variables.	Understand	Recall the definition of covariance, understand its relationship with correlation and prove the required properties.	CO 6
6	Show that the joint characteristic function of two independent random variables is equal to the product of the individual characteristic functions.	Understand	Recall the definition of joint characteristic function, condition for independence and prove the given property of joint characteristic function.	CO 6
7	Show that the characteristic function of sum of two independent random variables is equal to the product of the individual characteristic functions.	Understand	Recall the definition of joint characteristic function, condition for independence and prove the given property of joint characteristic function.	CO 6
8	Find variance and covariance of X-2Y. If E[X]=2, E[Y]=3, E[XY]=10, $E[X^2]$=9, and $E[Y^2]$=16.	Understand	Recall the definition of covariance and variance, understand the relationship and compute variance and covariance of sum of two variables.	CO 6
9	Prove that the moment generating function of sum of two independent random variables is equal to the product of the individual moment generating functions.	Understand	Recall the definition of joint moment generating function, condition for independence and prove the given property of joint moment generating function.	CO 6
CIE-II				
1	For random variables *X*, *Y* and *Z*, prove that $cov(X+Y,Z)=cov(X,Z)+cov(Y,Z)$.	Understand	Recall the definition of covariance, understand its relationship with correlation and prove the required property.	CO 7
2	Explain the Gaussian density function for *N* random variables.	Understand	Recall the Gaussian density function of scalar random variable, extend the same to N-dimensional vector random variable and describe the properties of N-dimensional Gaussian density function.	CO 7

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
3	Explain the linear transformations of Gaussian random variables.	Apply	Recall the transformation of scalar random variable, extend the same to the linear transformation of N-dimensional vector random variable using $N \times N$ transformation matrix and describe the properties of the output random variable.	CO 7
4	Explain the Gaussian density function for two random variables.	Understand	Recall the Gaussian density function of scalar random variable, extend the same to 2-dimensional vector random variable and describe the properties of 2-dimensional Gaussian density function.	CO 7
5	State the properties of jointly Gaussian random variables.	Understand	Recall the Gaussian density function of scalar random variable, extend the same to joint (vector) random variable and describe the properties of the jointly Gaussian density	CO 7
6	Show that the linear transformation of vector Gaussian random variable is another vector Gaussian random variable. What is the mean and variance of the resulting variable?	Apply	Recall the Gaussian density function of scalar random variable, understand the linear transformation of N-dimensional vector random variable and apply using $N \times N$ transformation matrix to show that the output Gaussian random variable.	CO 7
PART – C (Problem Solving and Critical Thinking Questions)				
1	The joint density function of two random variables X and Y is $f_{X,Y}(x,y) = \begin{cases} \frac{(x+y)^2}{40} & -1 \le x \le 1 ; -3 \le y \le 3 \\ 0 & elsewhere \end{cases}$ Find (a) the variances of X and Y, and (b) the correlation coefficient.	Understand	Recall the definition of covariance, understand its relationship with correlation, and solve the required variances and correlation coefficient.	CO 7
2	If the joint density function is $f_{X,Y}(x,y) = \begin{cases} x+y & 0 \le x \le 1, \quad 0 \le y \le \\ 0 & otherwise \end{cases}$ Find the correlation coefficient.	Understand	Recall the definition of covariance, understand its relationship with correlation, and solve the correlation coefficient.	CO 7
3	Find the coefficient of correlation between X and Y from the data given. Assume that X and Y are uniform random variables. X: 1, 2, 3, 4 Y: 2, 4, 8, 10	Understand	Recall the definition of correlation coefficient of discrete variable, understand the related expression, and solve for the correlation coefficient.	CO 6
4	For two random variables X and Y, the joint density function is $f_{X,Y}(x,y) = 0.15\delta(x+1)\delta(y) + 0.1\delta(x)\delta(y) + 0.1\delta(x)\delta(y-2) + 0.4\delta(x-1)\delta(y+2) + 0.2\delta(x-1)\delta(y-1) + 0.05(x-1)\delta(y-3)$	Understand	Recall the definitions of correlation coefficient of discrete variable, understand the correlation and orthogonality, and solve for the correlation coefficient.	CO 6

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
	Find (a) the correlation and (b) the correlation coefficient of X and Y. (c) Are X and Y either uncorrelated or orthogonal?			
5	Two random variables X and Y have the joint characteristic function $\emptyset_{X,Y}(w_1,w_2)=\exp(-2w_1^2-8w_2^2)$. Show that X and Y are both zero mean and that they are uncorrelated.	Understand	Recall the definition of joint characteristic function, understand the condition for correlation between two variables, solve and check for this condition and check the result.	CO 6
6	Calculate the following i) The variance of the sum of X and Y ii) The variance of the difference of X and Y for two random variables X and Y have zero mean and variance $\sigma_X^2 =$ 16 and σ_Y^2 = 36 and correlation coefficient is 0.5.	Understand	Recall the definition of covariance, understand its relationship with correlation and compute the required variances using this relationship.	CO 6
7	The joint density function of X and Y is $f(x,y) = \begin{cases} \frac{1}{100}, & 0 < x < 5, 0 < y < 20 \\ 0, & elsewhere \end{cases}$ Find the expected value of the functions (a). XY (b). X^2Y and (c) $(XY)^2$	Understand	Recall the definition of expectation operator, understand the joint moments, and their expressions, and compute the required expectations.	CO 6
CIE-II				
1	Let two random variables Y_1 and Y_2 be linear transformations of X_1 and X_2 given by $Y_1 = X_1 + X_2$, $Y_2 = 2X_1 + 3X_2$. If $f_{X_1 X_2}(x_1,x_2)$ is a joint density function of X_1 and X_2, then find the joint density function of Y_1 and Y_2.	Understand	Recall the transformation of scalar random variable, extend the same to the linear transformation of 2-dimensional vector random variable using 2×2 transformation matrix and solve for the output properties of the output joint density function.	CO 7
2	Consider two random variables X and Y such that $Y = -4X + 20$. The mean value and the variance of X are 4 and 2 respectively. Find the correlation between X and Y. comment on the result.	Apply	Recall the transformation of scalar random variable, and understand the covariance between two variables, and solve the correlation between these variables.	CO 7
3	Two random variables X and Y have mean values $\overline{X} = 1$ and $\overline{Y} = 1$, variances $\sigma_x^2 = 4$ and $\sigma_y^2 = 2$ and a correlation coefficient $\rho_{XY} = 0.2$. Define two new random variables $V = -X - Y$ and $W = 2X + Y$. Find (a) correlations of V and W and (b) correlation coefficient ρ_{VW}.	Apply	Recall the transformation of scalar random variable, extend the same to the linear transformation of 2-dimensional vector random variable using 2×2 transformation matrix and solve for the output properties of the output joint density function.	CO 7
4	Consider two correlated random variables X and Y with variances σ_x and σ_y respectively. These variables are to be transformed to uncorrelated random variables X_1 and Y_1 by	Apply	Recall the transformation of correlated 2-D vector random variable to uncorrelated ones, understand the axis rotation operation involved in this and derive expression	CO 7

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
	coordinate rotation as $X_1 = X\cos\theta + Y\sin\theta$ and $Y_1 = Y\cos\theta - X\sin\theta$, where θ is the rotation angle. Show that the angle of rotation is given by $\theta = \frac{1}{2}\tan^{-1}\left[\frac{2\rho\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}\right]$		for finding the angle of rotation required for this conversion.	
5	Consider two correlated random variables X and Y with variances 4 and 9 respectively, which are . transformed to uncorrelated variables X_1 and Y_1 by angle of rotation $\theta = \pi/8$. Compute the correlation coefficient between the variables.	Understand	Consider two correlated random variables X and Y with variances 4 and 9 respectively, which are . transformed to uncorrelated variables X_1 and Y_1 by angle of rotation $\theta = \pi/8$. Compute the correlation coefficient between the variables.	CO 7
6	Two Gaussian random variable X_1 and X_2 have zero mean and variances $\sigma^2{}_{X_1} = 4$ and $\sigma^2{}_{X_2} = 9$. Their covariance is $C_{X_1X_2} = 3$. Find the covariance $C_{Y_1Y_2}$ of new random variables Y_1 and Y_2, if the transformation is given as $Y_1 = X_1 - 2X_2$; $Y_2 = 3X_1 + 4X_2$	Understand	Recall the transformation of scalar random variable, extend the same to the linear transformation of 2-dimensional vector random variable using 2×2 transformation matrix and solve for the covariance between the output variables.	CO 7
MODULE -IV				
RANDOM PROCESSES – TEMPORAL CHARACTERISTICS				
PART – A (SHORT ANSWER QUESTIONS)				
1	Define random process.	Remember	---	CO 8
2	Define ergodicity.	Remember	---	CO 8
3	Explain the first order stationary process.	Understand	Recall the definition of the first order stationary process, and then explain the Ergodicity by stating that the two averages are same.	CO 8
4	Explain second order stationary	Understand	---	CO 8
5	State wide sense stationary random process.	Remember	---	CO 8
6	Define strict sense stationary random	Remember	---	CO 8
7	Explain briefly about the Ergodicity.	Understand	Recall the definitions of both the time average and the ensemble average of a random process, and then explain the Ergodicity by stating that the two averages are same.	CO 8
8	Define deterministic random process.	Remember	---	CO 8
9	Define distribution Function of a	Remember	---	CO 8
11.	Define mean ergodic process.	Remember	---	CO 8
12.	State correlation ergodic process.	Remember	---	CO 8
13.	Define auto correlation function of a	Remember	---	CO 8
14.	Explain cross correlation function of a	Remember	---	CO 8
15.	Define time mean square function.	Remember	---	CO 8
16.	Define Autocorrelation Ergodic	Remember	---	CO 8

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
17.	State Cross Correlation Ergodic	Remember	---	CO 8
18.	Define Auto Covariance function.	Remember	---	CO 8
19.	State Cross Covariance Function.	Remember	---	CO 8
	PART – B (LONG ANSWER QUESTIONS)			
1.	Explain classification of random processes with neat sketches.	Remember	---	CO 8
2.	Explain and write conditions for a wide sense stationary random process.	Remember	---	CO 8
3.	Briefly explain the distribution and density function in the context of stationary and independent random process	Understand	Recall the definition of 1-order and 2-order stationarity and the independence related to random processes and explain the required conditions.	CO 8
4.	Show that the process X(t)= A Cos (w_0t+θ) is wide sense stationary if it is assumed that A and w_0 are constants and θ is uniformly distributed random variable over the interval (0,2π).	Understand	Recall and understand the definition of wide sense stationarity (WSS) , compute the mean and ACF for the given process and show that the process is WSS.	CO 8, CO 9
5.	A random process Y(t)= X(t)Cos (wt+θ), where X(t) is wide sense stationery, w is constant and θ is uniformly distributed random variable over the interval (π, -π). Find E[Y(t)].	Remember	Recall the definition of wide sense stationarity (WSS), understand the mean and ACF condition for WSS process, and compute the expectation of the output random process.	CO 8, CO 9
6.	A random process consists of three sample functions X(t,s_1)=2, X(t,s_2)=2 cos(t), and X(t,s_3)=3 sin(t), each occurring with equal probability. Is the process stationary in any sense?	Understand	Recall the concept of sample functions of a random process, understand the stationarity conditions, compute mean and ACF to verify the stationarity.	CO 8
7.	Distinguish between stationary and nonstationary random processes.	Understand	Recall and understand the conditions for stationarity and contrast between the stationary and nonstationary processes.	CO 8
8.	A random process X(t)= a.cos (wt+θ), where w and θ are constants and a is uniformly distributed random variable over the interval (-A, A). Check X(t) for stationarity.	Understand	Recall the definition of stationarity, understand the mean and ACF condition for WSS process, and compute the expectation of the output random process.	CO 8, CO 9
9.	A random process X(t)= at+b, where b is constant and a is uniformly distributed random variable over the interval (-2, 2). Check X(t) for stationary.	Remember	Recall and understand the definition of stationarity, compute the mean and ACF and check whether stationarity conditions are satisfied or not.	CO 8, CO 9
10.	Obtain an expression for cross correlation function of input and output processes of a linear time invariant system.	Analyze	Recall and understand the definitions of cross correlation and the convolution operation, apply an input random process to LTI system, analyze the output and finally obtain cross correlation between the input and output processes.	CO 11
11.	State and prove any three properties of cross correlation function.	Understand	Recall and understand the definition of cross correlation, state the	CO 9

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
			properties of cross correlation of two random processes and derive the expressions to demonstrate the properties analytically.	
12.	Explain and prove any four properties of auto correlation function.	Understand	Recall and understand the definition of auto correlation, state the properties of auto correlation of two random processes and derive the expressions to demonstrate the properties analytically.	CO 9
13.	State and prove any four properties of cross covariance function.	Understand	Recall and understand the definition of cross covariance, state the properties of cross covariance of two random processes and derive the expressions to demonstrate the properties analytically.	CO 9
14.	State and prove any four properties of auto covariance function.	Remember	Recall and understand the definition of auto covariance, state the properties of auto covariance of two random processes and derive the expressions to demonstrate the properties analytically.	CO 9
15.	Analyze the output of an LTI system driven by a random process and develop the expression for the mean value of output process.	Analyze	Recall the definition of MS value of a random process and understand the convolution operation, apply an input random process to LTI system, analyze the output and finally obtain the mean of output process.	CO 11
16.	Analyze the output of an LTI system driven by a random process and develop the expression for the autocorrelation value of output process.	Analyze	Recall the definition of MS value of a random process and understand the convolution operation, apply an input random process to LTI system, analyze the output and finally obtain the autocorrelation of output process.	CO 11
17.	Analyze the output of an LTI system driven by a random process and develop the expression for the mean square value of output process.	Analyze	Recall the definition of MS value of a random process and understand the convolution operation, apply an input random process to LTI system, analyze the output and finally obtain the mean square value of output process.	CO 11
PART – C (Problem Solving and Critical Thinking Questions)				
1	Find whether X(t) is wide sense stationary or not, if A random process is given as X(t) = At, where A is a uniformly distributed random variable on (0,2).	Understand	Recall and understand the definition of wide sense stationarity (WSS) , compute the mean and ACF for the given process and check whether the process is WSS.	CO 8, CO 9
2	Let two random processes X(t) and Y(t) be defined by X(t) = A Cos(w_0t) + B Sin(w_0t) and Y(t) = B Cos(w_0t) - A Sin(w_0t). Where A and B are random variables and w_0 is constant. Show that X(t) and Y(t) are jointly wide sense stationery, assume A and B are	Understand	Recall the definition of wide sense stationarity (WSS) process, appreciate the linear transformation of 2-dimensional vector random variable using 2 $\times$ 2 transformation matrix, compute the mean and ACF of the output process and check for wide	CO 8, CO 9

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
	uncorrelated zero- mean random variables with same variance.		sense stationarity.	
3	Determine whether $X_1(t)$ and $X_2(t)$ are jointly wide sense stationary given that X(t) is a wide sense stationary random process. For each process $X_i(t)$ defined below (a) $X_1(t) = X(t + a)$ (b) $X_2(t) = X(at)$	Understand	Recall the definition of wide sense stationarity (WSS) process, appreciate the time translation and time scaling of a signal, compute the mean and ACF of the output processes and check for wide sense stationarity.	CO 8, CO 9
4	Define a random process by X(t)=A cos(πt), where A is a gaussian random variable with zero mean and variance σ^2_A a) find the density functions of X(0) and X(1) b) is X(t) is stationary in any sense?	Understand	Recall the definition of stationarity in strict sense or wide sense, understand and the stationarity conditions, compute the mean and ACF of the output processes and check for any kind of stationarity.	CO 4
5	X(t) is a stationary random process with a mean of 3 and an auto correlation function of $9 + 2e^{-\|\tau\|}$ Find the variance of the random process.	Understand	Recall the properties of auto correlation function (ACF), understand the method of obtaining MS value from ACF and then compute the variance.	CO 9
6	Given the random process X(t)=A sin(wt+ θ) where A and w are constants and θ is a random variable uniformly distributed on the interval (-π ,π) define a new random process $Y(t)= X^2(t)$. Find whether X(t) and Y(t) wide sense stationary? Are X(t) and Y(t) jointly wide sense stationary?	Understand	Recall the definition of (joint) wide sense stationarity, understand the stationarity conditions, compute the mean and ACF of the input processes and the crosscorrelation function of input and output processes.	CO 8, CO 9
7	A random process is defined as X(t)=A $\cos(w_c t+ \theta)$ where θ is a uniform random variable over(0,2π) .Verify the process is ergodic in the mean sense and auto correlation sense.	Understand	Recall the definition of stationarity, understand the ergodicity conditions, compute the time and ensemble statistics and check for ergodicity of the process.	CO 8, CO 9
8	Show that Z(t) is WSS but not strictly stationary, where $Z(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t)$ is a random process. If A and B are uncorrelated zero mean random process having different density functions but the same variance σ^2.	Understand	Recall the definition of (joint) wide sense stationarity, understand the stationarity conditions, compute the mean and ACF of the input processes and the crosscorrelation function of input and output processes	CO 4
9	Find E[Z], E[Z^2] and ${\sigma_z}^2$ of the random process $Z(t) = X_1 \cos\omega 0t - X_2 \sin\omega 0t$, and ω0 a constant, if X_1 and X_2 are independent Gaussian random variables, each with zero mean and variance σ^2.	Understand	Recall the definition of moments, understand transformation of scalar random variable by time functions, and compute the variance of the output process.	CO 9
10	Find mean, variance, and average power for a stationary ergodic random process having the Auto correlation function with the periodic components as $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}\cos 2\tau$	Understand	Recall the definition of moments and ergodicity, understand the conditions for stationarity and ergodicity on ACF, and finally compute the required parameters of the given process.	CO 9

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
11	Statistically independent zero mean random process X(t) and Y(t) have auto-correlation functions $R_{XX}(\tau)=e^{-\lvert\tau\rvert}$ and $R_{YY}(\tau)=\cos(2\pi\tau)$. Find the auto-correlation function of the difference w(t)=x(t)-y(t).	Understand	Recall and understand the definitions of ACF and statistical independence and compute the required parameters of the given process.	CO 9
12	Given E[X(t)]=6 and $R_{xx}(t,t+\tau)=$ 36+25 exp(-τ) for a random process X(t). Indicate which is true: (a). X(t) is first order stationary (b). X(t) is ergodic. (c). X(t) is wide sense stationary.	Understand	Recall and understand the definition of stationarity and ergodicity, understand the conditions for stationarity and ergodicity, compute the mean and ACF and finally check for the stationarity or ergodicity or both.	CO 8, CO 9
13	The random process X(t)= u(t) cos $(w_0t+\theta)$, where w_0 is constant and θ is uniformly distributed random variable over the interval (0,π). Find the power in X(t) if u(t) is a step function.	Understand	Recall the definition of power of a random process, understand the relation between ACF and power, compute the ACF and hence power of the given process.	CO 9
14	The random process X(t)= A_0 cos $(w_0t+\theta)$, where A_0 and w_0 are constants and θ is uniformly distributed random variable over the interval (-π ,π). Find the power in X(t).	Understand	Recall the definition of power of a random process, understand the relation between ACF and power, compute the ACF and hence power of the given process.	CO 9
15	An LTI system with an impulse response $h(t)$ was driven by a WSS process $X(t)$. Find the mean and MS values of the system output $Y(t)$.	Analyze	Recall and understand the moments and convolution integral, compute the output of the LTI system driven by WSS process, and analyze the output process to compute required moments.	CO 11
16	An LTI system with an impulse response $h(t)$ was driven by a WSS process $X(t)$ having an ACF $R_{xx}(\tau)$. Show that the ACF of the system output $Y(t)$ is given by the convolution of $R_{yy}(\tau)$ with $h(\tau)$ and $h(\tau)$	Analyze	Recall and understand the ACF of a process and impulse response of an LTI system, apply the convolution integral, compute the output of the LTI system driven by WSS process, and analyze the output process and find its ACF.	CO 11
MODULE -V				
RANDOM PROCESSES – SPECTRAL CHARACTERISTICS				
PART – A (SHORT ANSWER QUESTIONS)				
1	State wiener Khinchin theorem.	Remember	---	CO 10
2	Explain any two properties of cross-power density spectrum of two random processes.	Understand	Recall and understand the definition of cross-power density and state the properties of cross-power density spectrum of two random processes.	CO 10
3	Define cross –spectral density and its examples.	Remember	---	CO 10
4	Explain any two uses of spectral density spectrum.	Understand	Recall the definition of auto-power density, understand its relationship with ACF and state the uses of spectral density spectrum of a random	CO 10
5	Define power density spectrum.	Remember	---	CO 10

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
6	Explain any two properties of power density spectrum.	Understand	Recall the definition of auto-power density, understand its relationship with ACF and state the properties of spectral density spectrum of a random process.	CO 10
7	Define Narrow band random processes function.	Remember	---	CO 10
8	Prove that $S_{XX}(\omega) = S_{XX}(-\omega)$	Remember	---	CO 10
9	Define Average cross power.	Remember	---	CO 10
10	Explain the expression for the noise equivalent bandwidth of filter.	Understand	Recall the definition of filter frequency response and bandwidth, understand the noise equivalent bandwidth, and explain the meaning of each parameter in the expression.	CO 12
PART – B (LONG ANSWER QUESTIONS)				
1	What is significance of the power density spectrum of a random process? Derive the expression for it.	Apply	---	CO 10
2	Explain the cross-power density spectrum of two random processes and derive the expression for it.	Apply	Define the CCF and cross-power density spectrum, understand the relation between them, derive the cross power density spectrum of the two processes.	CO 10
3	Explain and derive the cross power spectral density of input and output random processes of a linear time invariant system.	Apply	Define the cross power density spectrum, understand the CCF and convolution integral, derive the cross power density spectrum from the CCF between input and output processes of an LTI system.	CO 12
4	State and Prove Wiener-Khinchin theorem.	Apply	State the Wiener-Khinchin theorem and understand the definitions of ACF, apply this in the definition of the power density spectrum of a random process, and prove the Wiener-Khinchin theorem.	CO 10
5	State and prove the properties of power density spectrum of a random process.	Apply	Define the power density spectrum, understand the ACF, apply this knowledge to prove the properties of power density spectrum	CO 10
6	Analyze any two properties of cross power density spectrum of input and output random processes of a LTI system.	Analyze	Define the cross power density spectrum, understand the convolution integral, derive the cross power density spectrum from the cross correlation of input and output of an LTI system, and analyze the system output properties.	CO 12
7	Derive the relation between power spectrum and auto correlation function of a random process.	Apply	Define the power density spectrum of a random process, understand the ACF and fourier transform, apply fourier transform on ACF to derive the power spectrum of the process.	CO 10

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
8	Derive the relation between cross power spectrum and cross correlation function of a random process.	Apply	Define the cross power density spectrum of a random process, understand the cross correlation function (CCF) and fourier transform, apply fourier transform on CCF to derive the cross-power spectrum of the process.	CO 10
9	Given that $X(t)=\sum_{i=1}^{N} \alpha_i X_i(t)$, where $\{\alpha_i\}$ is a set of real constants and the process $X_i(t)$ are stationary and orthogonal, show that $S_{XX}(w) = \sum_{i=1}^{N} (\alpha_i)^2 S_{XiXi}(w)$.	Apply	Recall the definition of power spectrum of the random, understand the orthogonality and the stationarity conditions and the linear transformation of processes, apply these concepts to obtain the power spectrum of the output processs.	CO 10
10	Find the power density spectrum of the random process for which $R_{XX}(\tau)=P\cos^4(\omega_0\tau)$, if P and ω_0 are constants. Determine the power in the process.	Apply	Recall and understand the ACF and power spectrum of a random process, apply this to develop the expression for the power density spectrum of process.	CO 10
11	Find the noise bandwidth of the power spectrum $S_{XX}(w)=1/[1+(\omega/W)^2]^N$, where W>0 is a constant and N>=2 is an integer.	Apply	Recall and understand the definition of power density spectrum and bandwidth, understand the noise equivalent bandwidth, and compute the noise bandwidth of the given power spectrum.	CO 12
12	A random process has the power density spectrum $S_{XX}(w)= 6\omega^2 /(1+\omega^2)^4$, find the average power.	Understand	Recall the definition of power density spectrum and bandwidth, understand and apply the method of obtaining process power from the power density to find the average power.	CO 10
13	Distinguish between white and colored noises. Where these noises are observed? Explain.	Analyze	Examine and bandwidth, apply this to find the power density spectrum of a noise process, analyze the shape of the power spectrum and Distinguish between white and colored noises	CO 12
14	An LTI system with an impulse response $h(t)$ was driven by white noise having a power density spectrum of $N_o/2$. Find the mean square value of the system output $Y(t)$.	Analyze	Recall the definition of MS value of a random process and understand the convolution operation, apply white random process to LTI system, analyze the output and finally obtain the mean square value of output process.	CO 12
15	Examine the autocorrelation and power density spectrum of white noise process.	Analyze	Recall the definition of ACF and power density spectrum of a random process, understand and compute the ACF of the white noise, apply this to compute the power density spectrum and examine the nature of ACF and power density.	CO 12

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
16	Examine the autocorrelation and power density spectrum of colored noise process and comment.	Analyze	Recall the definition of ACF and power density spectrum of a random process, understand and compute the ACF of the colored noise, apply this to compute the power density spectrum and examine the nature of ACF and power density, and comment.	CO 12
PART – C (Problem Solving and Critical Thinking Questions)				
1	Find an expression for its power spectral density $S_{XX}(\omega)$.Let the auto correlation function of a certain random process X(t) be given by $R_{XX}(\tau) = A^2/2 \cos(\omega\tau)$.	Apply	Recall and understand the ACF and power spectrum of a random process, apply this to find the power density spectrum of process.	CO 10
2	Describe the power spectral density function for a wide sense stationary process X(t) that has autocorrelation function $R_{XX}(\tau) = A e^{-b\|\tau}\|$ where b > 0. $S_{XX}(f)$ and calculate the average power $E[X^2(t)]$.	Apply	Recall and understand the ACF and power spectrum of a random process, understand the wide sense stationary process, apply this to find the power density spectrum and average power of given process	CO 10
3	Find the average power in a random process defined by $X(t)=A \cos(w_o t+\Theta)$ where A and w_o are constants and Θ is a random variable uniformly distributed on the interval $(0,\pi/2)$.	Apply	Recall and understand the ACF and power spectrum of a random process, understand the wide sense stationary process, apply this to find the power density spectrum and average power of given process.	CO 10
4	Find the autocorrelation function if the power Spectral density of X(t) is given by $S_{XX}(w)=1/(1+w^{2)}$ for w>0.	Apply	Recall and understand the ACF and power spectrum of a random process, understand the wide sense stationary process, apply this to find the ACF from the power density spectrum of given process.	CO 10
5	The auto correlation function of ana periodic random process is $R_{XX}(T)= e^{-\|\tau\|}$. Find the PSD and average power of the signal.	Apply	Recall and understand the ACF and power spectrum of a random process, understand the wide sense stationary process, apply this to find the power density spectrum and average power of given process.	CO 10
6	The cross spectral density of two random process X (t) and Y (t) is $S_{XY}(w)=1+(jw/k)$ for $–k<w<k$ and 0 elsewhere Where $k > 0$. Find the cross-correlation function between the processes?	Apply	Recall and understand the CCF and cross power spectrum of two random processes, apply this to find the CCF from the cross-power density spectrum of given process.	CO 10
7	A random process has the power density spectrum $S_{XX}(w)=w^2/(w^2+1)$.Find the average power in the random process.	Apply	Recall and understand the power spectrum of a random process, understand the wide sense stationary process, apply this to find the average power of given process from its power density spectrum.	CO 10
8	Estimate the power spectral density of a stationary random process for which	Apply	Recall and understand the ACF and power spectrum of a random process,	CO 10

S.No	Question	Blooms taxonomy level	How does this Subsume the levels?	Course Outcomes
	auto correlation function is $R_{XX}(\tau)=6.e^{-\alpha\|\tau\|}$.		understand the wide sense stationary process, apply this to find the power density spectrum and average power of given process.	
9	Find i) The Auto correlation function of a random process Y(t), if it has the power spectral density $S_{YY}(\omega)=9/(\omega^2+64)$. ii) The average power of the process.	Apply	Recall and understand the ACF and power spectrum of a random process, understand the wide sense stationary process, apply this to find the ACF from the power density spectrum and average power of given process	CO 10
10	Find the RMS bandwidth of the power spectrum $S_{XX}(w)= \omega^2 /[1+(\omega/W)^2]^4$, where W>0 is a constant.	Analyze	Recall and understand the definition of power density spectrum and bandwidth, understand the noise equivalent bandwidth, and compute the noise bandwidth of the given power spectrum.	CO 12
11	A random process has the power density spectrum $S_{XX}(w)= 6\omega^2 /1+\omega^4$, find the average power.	Apply	Recall and understand the power spectrum of a random process, understand the wide sense stationary process, apply this to find the average power of given process from its power density spectrum.	CO 10
12	Find the cross-correlation function of sin(wt) and cos(wt) and hence find its cross power spectral density.	Apply	Define the cross power density spectrum of a random process, understand the cross correlation function (CCF) and fourier transform, apply fourier transform on CCF to derive the cross-power spectrum of the process.	CO 10
14	A random process has a power spectrum $S_{XX}(w)= 4-(\omega^2/9)$; $\|\omega\|<=6$ = 0 ; elsewhere Find (a) the average power. (b) autocorrelation function of the process.	Apply	Recall and understand the ACF and power spectrum of a random process, understand the wide sense stationary process, apply this to find the ACF from the power density spectrum, and also average power of given process.	CO 10

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