PPT ON
ELECTROMAGNETIC FIELDS(R18)

B.Tech III Semester (R18)
(2020-2021)

Prepared
By

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UNIT-I
ELECTRO-STATICS AND VECTOR CALCULUS

MODULE I - SYLLABUS

Introduction to Cartesian, cylindrical and spherical co-ordinates. Conversion of one type of co-ordinates to another; Electrostatic fields: Coulomb’s law, electric field intensity due to line and surface charges, work done in moving a point charge in an electrostatic field, electric potential, properties of potential function, potential gradient, Gauss’s law, application of Gauss’s law, Maxwell’s first law, Laplace’s and Poisson’s equations, solution of Laplace’s equation in one variable.

COURSE OUTCOMES MAPPED WITH MODULE I

CO	Course Outcomes	Blooms Taxonomy
CO 1	Make use of coloumb's law for obtaining force and electric filed intensity due to line, surface and volume charge distribution.	Apply
CO 2	Recognize the basic nomenclatures of point charge that helps in characterizing the behavior of electro-static fields .	Understand
CO 3	Make use of the Gauss law for obtaining electric field intensity, density and deduce Poisson's, Laplace equations.	Apply

PROGRAM OUTCOMES MAPPED WITH MODULE I

PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
PO 3	**Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

MAPPING OF COs AND POS FOR MODULE I

COs	PROGRAM OUTCOMES											
	1	2	3	4	5	6	7	8	9	10	11	12
CO1	√	√										
CO2	√											
CO3	√		√									

INTRODUCTION

The atom consists of protons(Nucleus), electrons(outer most region) and neutrons with different masses. Their masses are

$$m_e = 9.10x\ 10\text{-}^{31} \text{ kilograms}$$

$$m_p = 1.67x\ 10\text{-}^{27} \text{ kilograms}$$

these masses leads to gravitational force between them, given as

$$F = G\, m_e\, m_p / r^2$$

The force between two opposite charges placed 1cm(charge of particle = $1.6 * 10^{-19}$ C) (1C = $0.625 * 10^{19}$) apart likely to be 5.5x10-^{67} and force between two like charges placed 1cm apartlikely to be 2.3x10-^{24,} this force between them is called as electric force .

Electric force is larger than gravitational force. Gravitational force due to their masses. Electric force is due to their properties. Neutron has only mass but no electric force.

ELECTROSTATICS:

Electrostatics is the study of charge at rest. The study of electric and magnetic field can be done using MAXWELL'S equations. Electrostatic field is developed between static charges. Electrostatics got wide variety applications like X-rays, lightning protections etc.

Let us study the behavior of electric field using COLOUMB's and GAUSS laws.

POINT CHARGE

A charge with smallest dimensions on the body compare to other charges is called as point charge.

A group of charges concentrated on any pin head may be also called as point charge.

- An **electrostatic field** exists in the region surrounding an electrically charged object. This charged object, when brought in close proximity to an uncharged object, can induce a charge on the formerly neutral object. This is known as an induced charge.

- For **example**, a charged comb will attract small objects such as grains of sugar when brought close to them. If the grains of sugar come into contact with the comb then in a little while some of the grains will gain the same charge as the comb and will be rapidly repelled

- Based on the same types of experiments like the one you performed, scientists were able to establish **three laws** of electrical charges: Opposite charges attract each other. Like charges repel each other. Charged objects attract neutral objects

- **Static electricity** is **used in** pollution control by applying a static charge to dirt particles in the air and then collecting those charged particles on a plate or collector of the opposite electrical charge. Such devices are often called **electrostatic** precipitators.

COLOUMB'S LAW

Coloumb stated that the force between two point charges is directly proportional to product of charges and Inversely proportional square of distance between the. $F \; \alpha \; Q_1Q_2 / r^2$

$F = K\, Q_1Q_2 / r^2$, where K is the proportionality constant.

$K = 1/ 4\pi\varepsilon$, where ε is the permittivity of the medium.

$E = \varepsilon_0\varepsilon_{r,}$ ε_0 = absolute permittivity = $8.854x10^{-12}$

ε_r = relative permittivity

most common medium is air or vacuum whose relative permittivity is 1, hence permittivity of air or vacuum is

$\varepsilon = 9x10^9$ m/F

Force between two point charges using vector analysis

Let us consider two point charges separated by some distance given as

.

$Q_1 \leftarrow\text{--------------------------------}\rightarrow Q_2$

According to coloumb's law force between them is given as

$F = (K\ Q_1Q_2 / r^2)$ x , where is the unit vector direction of force.

Let F_2 is the force experienced by Q_2 due to Q_1 and F_1 is force experienced by Q_1 due to Q_2. The direction of forces opposes each other , hence we can write in vector from forces as

$$F_1 = -F_2$$

Hence unit vector can be or , from the vector analysis we can write

$$a_{12} = R'_{12}/ R_{12} = R' / R \text{ and}$$

$$a_{21} = R'_{21} / R_{21} = R' / R$$

Therefore the magnitude of force between them can be written as

$$F_1 = F_2 = (K\ Q_1Q_2 / R^3) \text{ x } R'$$

Electric Field & Electric Field Intensity

Electric Field: It is the region around the point and group charges in which another charge experiences force is called as electric field. The force between two charges can be studied in terms of electric field as : A charge can develop field surrounding It in space only, the field of one charge leads to force on the other charge .

Electric Field Intensity: If an point charge q experiences the force F , then the electric field intensity of charge is defines as

$$E = F/q$$

Here charge q is called as test charge because the force experienced by it is due field of other charge.

The units of electric field intensity are N/C or V/mt.

$$q_1 \leftarrow\text{---------------- ----------------}\rightarrow q_2$$

Electric Field & Electric Field Intensity

the force experienced by q_2 because of field of q_1 is

$$\text{vector, } F_2 = (K\, q_1 q_2 / r^2) \text{ x } a'$$

Therefore electric filed intensity on q2 charge is

$$\text{Vector,} E = F_2/q_2 = (K\, q_1 / r^2) \text{ x } a'$$

the force experienced by q_1 because of field of q_2 is

$$\text{vector, } F_1 = (K\, q_1 q_2 / r^2) \text{ x } a'$$

Therefore electric filed intensity on q2 charge is

$$\text{Vector, } E = F_1/_{q1} = (K\, q_2 / r^2) \text{ x } a'$$

Electric Field & Electric Field Intensity

let the point charges q_2, q_3--------------q_n are placed at a distance of r_2, r_3---------------------------r_n from q_1.

Hence total electric field intensity on q1 due to remaining point Charges is , force due to q2 on q1, F2= $(K\, q_1 q_2 / r^2)$ x a'

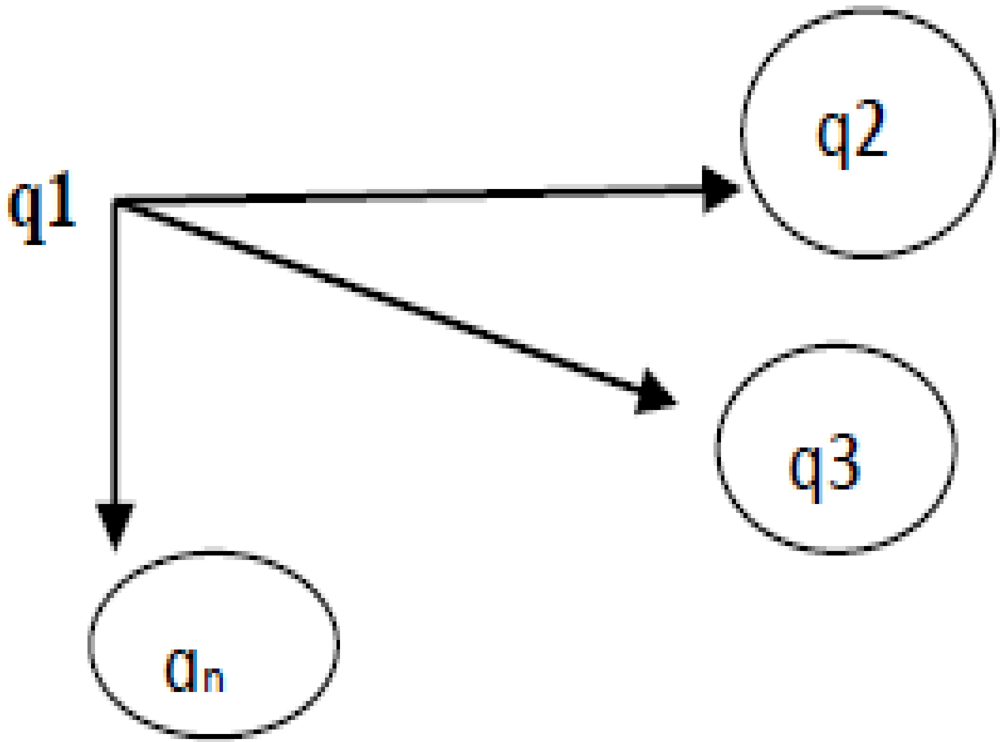
force due to q3 on q1, F3= $(K\, q_1 q3 / r^2)$ x a'

force due to qn on q_1, Fn= $(K\, q_1 qn / r^2)$ xa'

therefore total electric field intensity is , = $(F_2+F_3$---------------$F_n) / q_1$

= $(K\, q_2 / r^2)$ x + $(K\, q_3 / r^2)$ x ----+ $(K\, q_n / r^2)$ x a'

Electric Field & Electric Field Intensity



CHARGE DISTRIBUTION

Line charge: Here charge is distributed through out some length . The total charge distributed through a wire of length l is

$$\rho_l = dq / dl \text{ ----- line charge density}$$

$$\int \rho_l = \int dq / dl$$

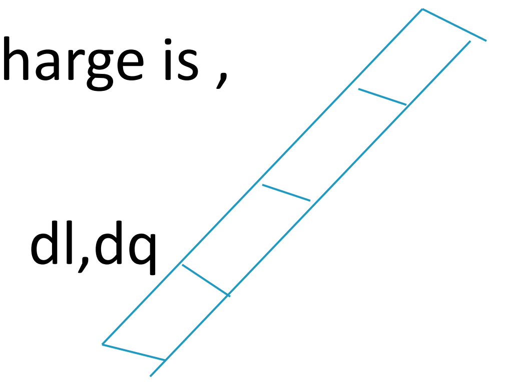
$$\int \rho_l \,.\, dl = \int dq$$

$$\int \rho_l \, dl = Q, \; Q - \text{total charge}$$

Hence electric field intensity due to line charge is ,

$$\mathbf{E = K\,Q / r^2 \times a}$$

$$E = \int (K \rho_l \, dl / r^2) \times a$$



CHARGE DISTRIBUTION

Surface charge: Here charge is distributed through given area . The total charge distributed in an surface area is

$$\rho_s = dq / ds \text{ , surface charge density}$$

$$\int \rho_s = \int dq / ds$$

$$\int \rho_s \, ds = \int dq$$

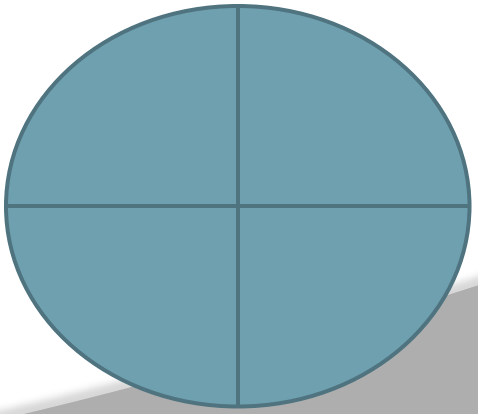
$$\int \rho_s \, ds = Q, \; Q - \text{Total Charge}$$

Hence electric field intensity due to surface charge is ,

$$E = K Q / r^2 \times a,$$

$$E = \int (K \rho_s \, ds / r^2) \times a,$$

ds, dq



CHARGE DISTRIBUTION

Volume charge: Here charge is distributed through given volume . The total charge distributed in an volume is

$$\rho_v = dq / dv \text{ , volume charge density}$$

$$\int \rho_v = \int dq / dv$$

$$\int \rho_v \, dv = \int dq$$

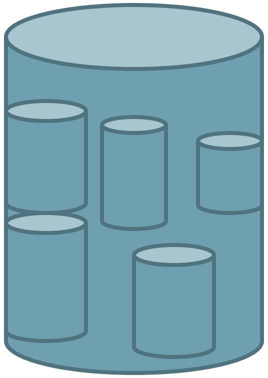
$$\int \rho_v \, dv = Q, \; Q - \text{Total Charge}$$

Hence electric field intensity due to volume charge is ,

$$E = K\,Q / r^2 \times a,$$

$$E = \int (K\,\rho_v\,dv / r^2) \times a,$$

dv, dq



Problems

Find the force on charge 100 μC due to charge -300 μC, where charges are placed at (0, 1,2) and (3, 0, 0) respectively.

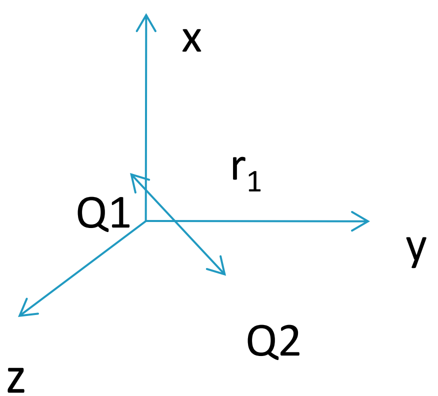
(-300 μC, Q1) ----------r_1'------------ (100 μC, Q2)

(3,0,0) $(x_1,y_1.z_1)$ (0,1,2) $(x_2,y_2.z_2)$

$r_1' = (x_2 - x_1)\, a_x + (y_2 - y_1)\, a_y + (z_2 - z_1)\, a_z$

$=(0-3)\, a_x + (1-0)\, a_y + (2-0)\, a_z$

$=-3\, a_x + 1.a_y + 2\, a_z$



Magnitude of r' is , $r_1 = \sqrt{(-3)^2 + (1)^2 + (2)^2} = \sqrt{14} = 3.74$

Problems

direction or unit vector is given as,

$$a = (r_1' / r_1)$$
$$= (-3\ a_x + 1.a_y + 2\ a_z\ ) / 3.74$$

therefore from coloumb's law force on 100 µC is given as ,

$$F_2 = \frac{(K.Q1.Q2) . a}{r_1^2} = \frac{(K.Q1.Q2) . r_1'}{r_1^2\ .\ r_1}$$
$$= \frac{(K.Q1.Q2) . r_1'}{r_1^3}$$

Problems

$K = (1/ 4\pi\varepsilon)$

$\varepsilon = \varepsilon_o\varepsilon_r = 8.854 \times 10^{-12} \times 1.$ (assuming air medium)

$K = (1/ 4\pi\ \varepsilon_o\varepsilon_r) = (1/ 4\pi \times 8.854 \times 10^{-12} \times 1)$

Therefore force on 100 μC,

$$F_2 = \frac{(1.\ Q1.Q2\ .\ r_1')}{4\pi \times 8.854 \times 10^{-12} \times 1 \times r_1^3}$$

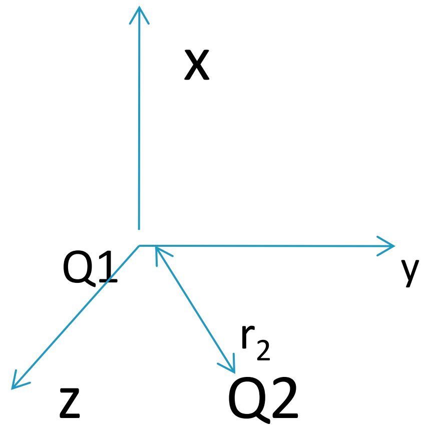
$$= \frac{(1 \times -300 \times 10^{-6} \times 100 \times 10^{-6} \times (-3\ a_x + 1.a_y + 2\ a_z\ )}{4\pi \times 8.854 \times 10^{-12} \times 1 \times 3.74^3}\ \text{N}$$

Problems

(100 μC, Q2) ----------r_2'------------ (-300 μC, Q1)

(0,1,2) (x_1,y_1,z_1) (3,0,0) (x_2,y_2,z_2)

$r_2' = (x_2 - x_1)\, a_x + (y_2 - y_1)\, a_y + (z_2 - z_1)\, a_z$



$=(3-0)\, a_x + (0-1)\, a_y + (0-2)\, a_z$

$=3\, a_x - 1.a_y - 2\, a_z$

Magnitude of r' is , $r_2 = \sqrt{(3)^2 + (-1)^2 + (-2)^2} = \sqrt{14} = 3.74$

Problems

direction or unit vector is given as,

$$a = (r_2' / r_2)$$
$$= (3\,a_x - 1.a_y - 2\,a_z\ ) / 3.74$$

therefore from coloumb's law force on -300 μC is given as ,

$$F_1 = \frac{(K.Q1.Q2)\ .\ a}{r_2^{\,2}} \qquad = \frac{(K.Q1.Q2)\ .\ r_2'}{r_2^{\,2}\ \ .\ \ r_2}$$

$$= \frac{(K.Q1.Q2)\ .\ r'}{r_2^{\,3}}$$

Problems

$K = (1/4\pi\varepsilon)$

$\varepsilon = \varepsilon_o\varepsilon_r = 8.854 \times 10^{-12} \times 1$. (assuming air medium)

$K = (1/4\pi\,\varepsilon_o\varepsilon_r) = (1/4\pi \times 8.854 \times 10^{-12} \times 1)$

Therefore force on 100 μC,

$$F_1 = \frac{(1.\ Q1.Q2\ .\ r_2')}{4\pi \times 8.854 \times 10^{-12} \times 1 \times r_2^3}$$

$$= \frac{(1\text{x}100\ \text{x}10^{-6}\text{x}{-300}\text{x}10^{-6} \times (3\ a_x - 1.a_y - 2\ a_z\ )}{4\pi \times 8.854 \times 10^{-12} \times 1 \times 3.74^3}\ \ N$$

Problems

Electric field intensity of -300 μC,

(-300 μC, Q1) ----------r_1'------------ (100 μC, Q2)

$$E1 = (F_2 / Q_2) = \frac{(K.Q1\ .a)}{r^2_1} = \frac{(K.Q1\ .r_1)}{r^3_1}$$

$$= \frac{(1x100\ x10^{-6}x\text{-}300x10^{-6}\ x\ (-3\ a_x + 1.a_y + 2\ a_z\ )}{4\pi\ x\ 8.854\ x\ 10^{-12}\ x\ 1\ x\ 3.74^3\ x\ (100x10^{-6}\ )}$$

Problems

Electric field intensity of 100 μC,

(100 μC, Q2) ---------r_2'------------ (-300 μC, Q1)

$$E_2 = (F_1 / Q_1) = \frac{(K.Q2 .a)}{r^2_2} = \frac{(K.Q2 .r_2)}{r^3_2}$$

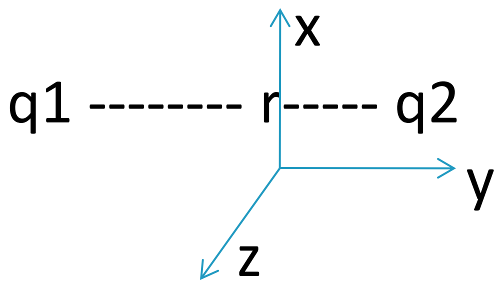
$$= \frac{(1x100 \times 10^{-6} x{-300}x10^{-6} \times (3\, a_x - 1.a_y - 2\, a_z)}{4\pi \times 8.854 \times 10^{-12} \times 1 \times 3.74^3 \times (-300x10^{-6})}$$

Problems

A point charge q1 = 200 μC, located at (2, -1, -4) experiences a force $F = 4a_x - 8a_y + 8a_z$ N due to point charge q2 at (3, -3, -2), Find the value of q2.

q2 -----------------r-----------------q1 (200 μC)

(3,-3,-2) (x_1,y_1,z_1) (2,-1,-4) (x_2,y_2,z_2)



From coloumb's law force on q1 can be written as,

$$F = \frac{(K.\ q2.q1.a)}{r^2} = \frac{(K.\ q2.q1.r')}{r^2.r} = \frac{(K.\ q2.q1.r')}{r^3} \quad \text{--------}1$$

Problems

The distance vector between q2 and q1,

$r' = (x_2 \ - x_1)\, a_x + (y_2 \ - y_1)\, a_y + (z_2 \ - z_1)\, a_z$

$= (2 \ - 3)\, a_x + (-1 \ -(- 3))\, a_y + (-4 \ -(- 2))\, a_z$

$= (-1.a_x + 2a_y - 2a_z)$

magnitude of r', $r = \sqrt{(-1)^2 + (2)^2 + (-2)^2} = \sqrt{9} = 3$

$K = (1/ 4\pi\varepsilon)$

$\varepsilon = \varepsilon_o \varepsilon_r = 8.854 \times 10^{-12} \times 1$. (assuming air medium)

$K = (1/ 4\pi\, \varepsilon_o \varepsilon_r) = (1/ 4\pi \times 8.854 \times 10^{-12} \times 1)$

Problems

Given force on q1 is

$$F = 4a_x - 8a_y + 8a_z \text{ ------------- } 2$$

Equating 1 and 2

$$\frac{(K.\ q2.q1.r')}{r^3} = 4a_x - 8a_y + 8a_z$$

Substituting values on LHS ,

$$\frac{(1 \times q2 \times 200 \times 10^{-6} \times (-1.a_x + 2a_y - 2a_z))}{(4 \times \pi \times 8.825 \times 10^{-12} \times 1 \times (3)^3} = 4a_x - 8a_y + 8a_z$$

Problems

Determine E at origin due to a point charge of 65 nC located at (-4, 3, 2) in Cartesian coordinates .

(65 nC, q1) ------------------r------------------ q2

(0,0,0) (x_1,y_1,z_1) (-4,3,2) (x_2,y_2,z_2)

(E_1, F_1) (E_2,F_2)nsity

As electric field intensity of q1 is required , it is given as

$$E1 = (F2 / q2) = (K.\ q1\ .a) / r^2$$

$$E1 = \frac{(K.\ q1\ .r')}{r^2.r} = \frac{(K.\ q1\ .r')}{r^3}$$

Problems

The distance vector between q1 and q2 is ,

$r' = (x_2 \ - x_1) a_x + (y_2 \ - y_1) a_y + (z_2 \ - z_1) a_z$

$= (-4 \ - 0) a_x + (3 \ - 0) a_y + (2 \ - 0) a_z$

$= (-4a_x + 3a_y + 2a_z)$

Magnitude of r', $r = \sqrt{(-4)^2 + 3^2 + 2^2} = \sqrt{29} = 5.38$

$K = (1/ 4\pi\varepsilon)$

$\varepsilon = \varepsilon_o\varepsilon_r = 8.854 \times 10^{-12} \times 1$. (assuming air medium)

$K = (1/ 4\pi\, \varepsilon_o\varepsilon_r) = (1/ 4\pi \times 8.854 \times 10^{-12} \times 1)$

Problems

$$E_1 = \frac{(1 \times 65 \times 10^{-9} \times (-4a_x + 3a_y + 2a_z)}{4 \times \pi \times 8.825 \times 10^{-12} \times 1 \times 5.38^3}$$

Vector Analysis

Scalar : Scalar is the quantity which has magnitude.

Eg: mass, length, temperature etc.

Vector : Vector is the quantity which has both magnitude and direction

Eg: force, velocity, electric field intensity etc.

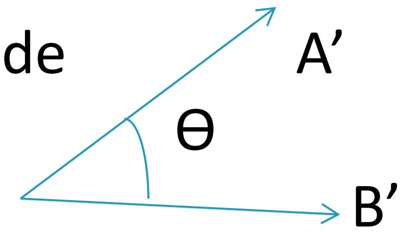
Representation of Vector : $\xrightarrow{\mathbf{A'}}$ **a**

A' = A . a, where A – magnitude of vector A'

a – direction of vector A'

Vector Analysis

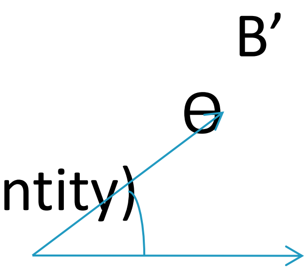
Dot Product: Let us consider two vector A' and B' where Θ rotation requires by B' to coincide with A' .



$A'.B' = A.B. \cos \Theta$ (Scalar quantity)

$\cos \Theta = (A'. B') / A.B$

Cross Product: Let us consider two vector A' and B' where Θ rotation requires by A' to coincide with B' .



$A' \times B' = A.B. \sin \Theta \, a$ (vector quantity)

$$\sin \Theta = \frac{(A' X B')}{A.B} .$$

Rules of vector analysis:

1) cumulative, $A' + B' = B' + A'$

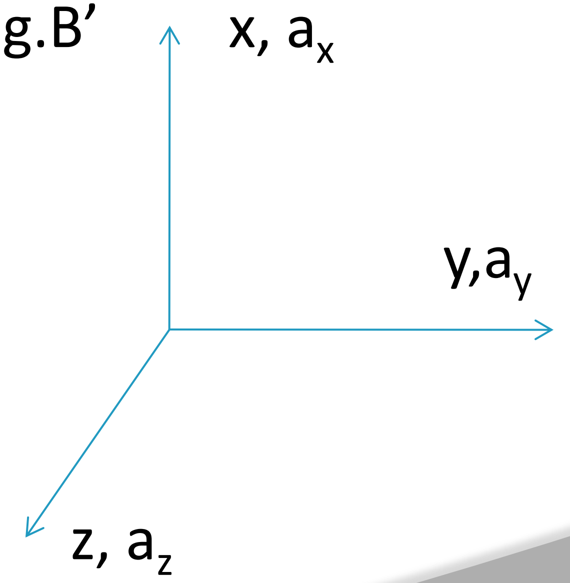
$$A' - B' = A' + (-B')$$

2) Multiplication of factor, $g\,(A' + B') = g.A' + g.B'$

3) Vector in Cartesian coordinates,

$$A' = A_x.a_x + A_y.a_y + A_z.a_z$$

$$B' = B_x.a_x + B_y.a_y + B_z.a_z$$



Problems

4) $A' + B' = (A_x.a_x + A_y.a_y + A_z.a_z) + (B_x.a_x + B_y.a_y + B_z.a_z)$

$= (A_x + B_x) a_x + (A_y + B_y) a_y + (A_z + B_z) a_z$

5) $a_x.a_x = a_y.a_y = a_z.a_z = 1 \quad (\Theta = 0^o)$

$a_x.a_y = a_y.a_z = a_z.a_x = 0 \quad (\Theta = 90^o)$

6) $a_x x\ a_x = a_y x a_y = a_z x a_z = 0$

$a_x\ x\ a_y = -a_y\ x\ a_x = a_z$

$a_y\ x\ a_z = -a_z\ x\ a_y = a_x$

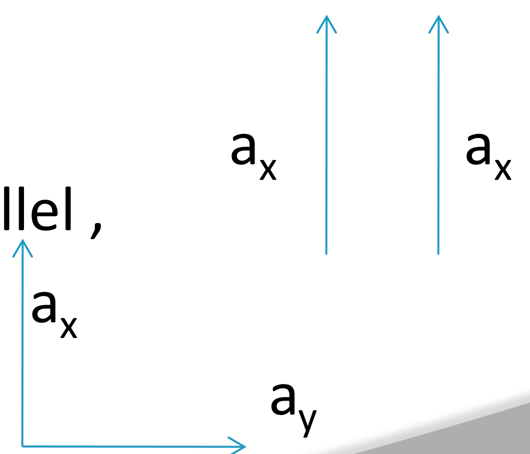
$a_z\ x\ a_x = -a_x\ x\ a_z = a_y$

Problems

7) $A' \cdot B' = (A_x.a_x + A_y.a_y + A_z.a_z) \cdot (B_x.a_x + B_y.a_y + B_z.a_z)$

$$= (A_x \cdot B_x)(a_x \cdot a_x) + (A_x \cdot B_y)(a_x \cdot a_y) + (A_x \cdot B_z)(a_x \cdot a_z) + (A_y \cdot B_x)(a_y \cdot a_x) + (A_y \cdot B_y)(a_y \cdot a_y) + (A_y \cdot B_z)(a_y \cdot a_z) + (A_z \cdot B_x)(a_z \cdot a_x) + (A_z \cdot B_y)(a_z \cdot a_y) + (A_z \cdot B_z)(a_z \cdot a_z)$$

$$= (A_x \cdot B_x) + (A_y \cdot B_y) + (A_z \cdot B_z)$$



Explanation, dot product of two unit in parallel ,

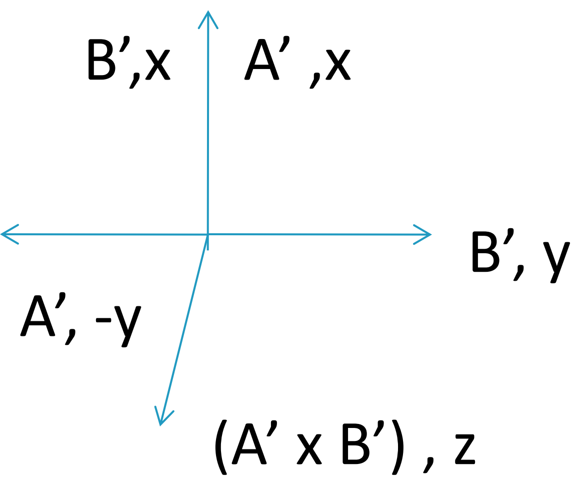
$$a_x \cdot a_x = 1 \times 1 \ . \cos o = 1$$

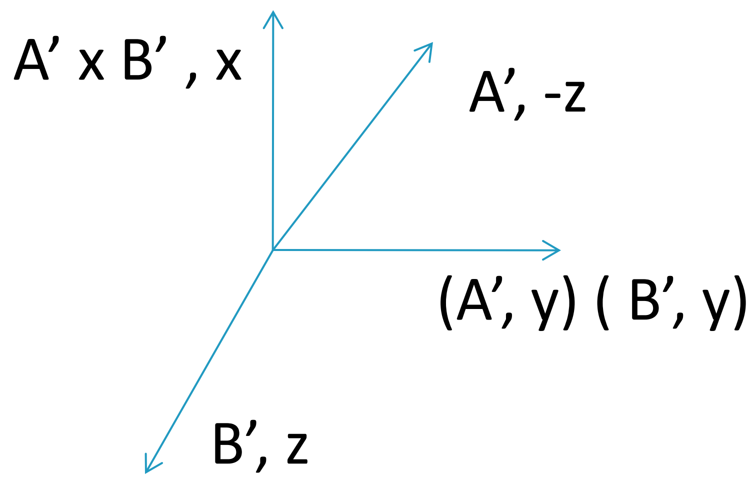
$$a_x \cdot a_y = 1 \times 1 \ . \cos 90^o = 0$$

Vector Analysis

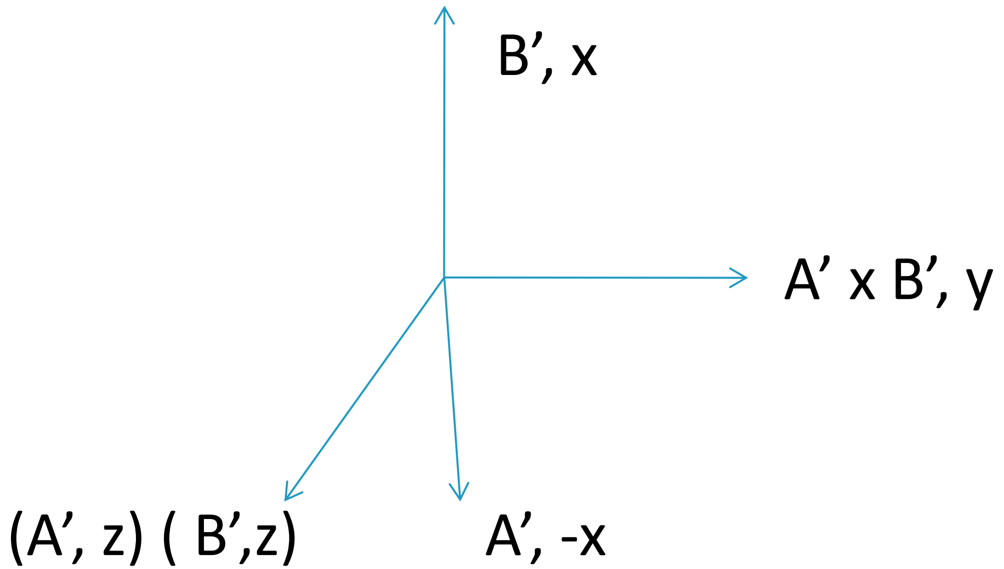
8) $A' \times B' = (A_x.a_x + A_y.a_y + A_z.a_z) \times (B_x.a_x + B_y.a_y + B_z.a_z)$

$$= (A_x . B_x)(a_x \times a_x) + (A_x . B_y)(a_x \times a_y) + (A_x . B_z)(a_x \times a_z) +$$
$$(A_y . B_x)(a_y \times a_x) + (A_y . B_y)(a_y \times a_y) + (A_y . B_z)(a_y \times a_z) +$$
$$(A_z . B_x)(a_z \times a_x) + (A_z . B_y)(a_z \times a_y) + (A_z . B_z)(a_z \times a_z)$$





Vector Analysis



Explanation cross product of two unit vectors or direction vectors,

$$a_x \times a_y = 1 \times 1 \sin(90^o) = 1$$

$$a_x \times a_x = 1 \times 1 \sin(0^o) = 0$$

Vector Analysis

$$= (A_x . B_y)(a_z) + (A_x . B_z)(-a_y) + (A_y . B_x)(-a_z) + (A_y . B_z)(a_x) + (A_z . B_x)(a_y) + (A_z . B_y)(-a_x)$$

$$= (A_y . B_z - A_z . B_y)\ a_x + (A_z . B_x - A_x . B_z)\ ay + (A_x . B_y - A_y . B_x)\ a_y$$

$$A' \times B' = \begin{vmatrix} ax & ay & az \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$$

Vector Analysis

9) $A' \cdot B' = B' \cdot A'$

$A' \times B' = -B' \times A'$

10) Scalar triple product, $A' . (B' \times C') = B' . (C' \times A') = C' . (A' \times B')$

$$A' . (B' \times C') = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$C' = C_x a_x + C_y a_y + C_z a_z$

11) $A' \times (B' \times C') = B' (A'.C') – C' (A' . B')$

Problems:

1) Find the magnitude of

$$A' = 3a_x + 2\, a_y – 6az \ , \ A = \sqrt{(3)^2 + (2)^2 + (-6)2} = \sqrt{49} = 7$$

2) $B'= 4.\cos\alpha\, a_x + 4.\sin\alpha\, a_y + 5\, a_z$

3) Given $A' = 3a_x - 2a_y + a_z$ and $B' = -a_x + 2.a_y + 7a_z$, find the angle between two vectors.

4) Given $A' = 2a_x + a_y - a_z$ and $B' = 6a_x - 3.a_y + 2a_z$, find cross product and resultant direction.

Vector Analysis

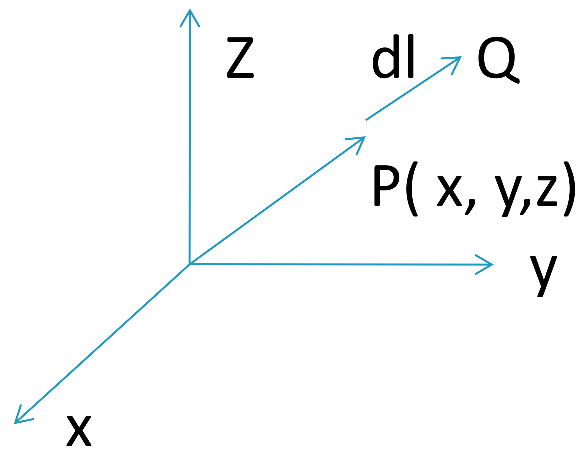
- Coordinate Systems are of three types :

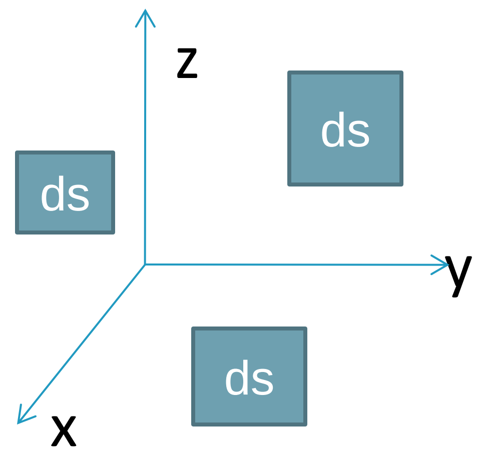
1) Cartesian coordinates

2) Cylindrical Coordinates

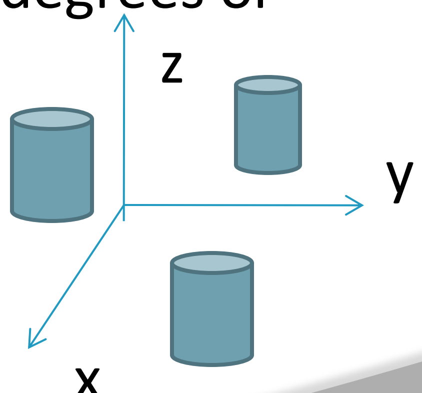
3) Spherical Coordintes.

Cartesian Coordinates





Above figure indicates 3-dimentional Cartesian coordinate system with x –axis, y-axis and z-axis displaced by 90 degrees or perpendicular to each other.



Any point in this is indicated as P(x, y, z)

Cartesian Coordinates

Point P(x,y,z) and Q(x+dx, y+dy, z+dz)

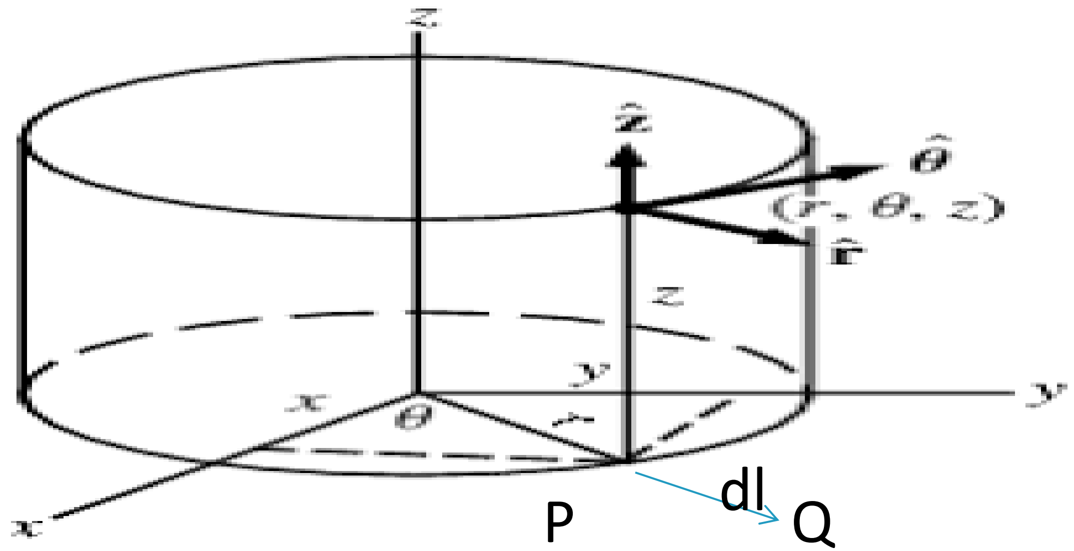
the change in length of P is dl,

differential length , $dl = dx.\ a_x + dy.a_y + dz.a_z$

differential area, $ds = dx.dy = dy.dz = dz.dx$

differential volume, $dv = dx.dy.dz$

Cylindrical Coordinates

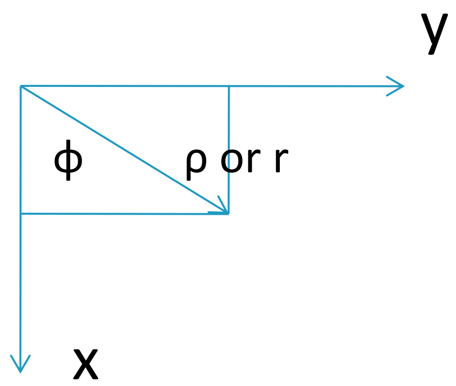


Above diagram represents cylindrical system in 3 dimensional

The point P(r, ϴ, z) or (ρ, ϕ, z)

where, r = ρ = radius of cylinder= is the resultant of xy plane.
ϴ = ϕ = if x-axis is reference , it is displacement to projectile.
z = height of cylinder.

Cylindrical Coordinates



$x = \rho \cos \phi$

$y = \rho \sin \phi$

$\rho = \sqrt{x^2 + y^2}$

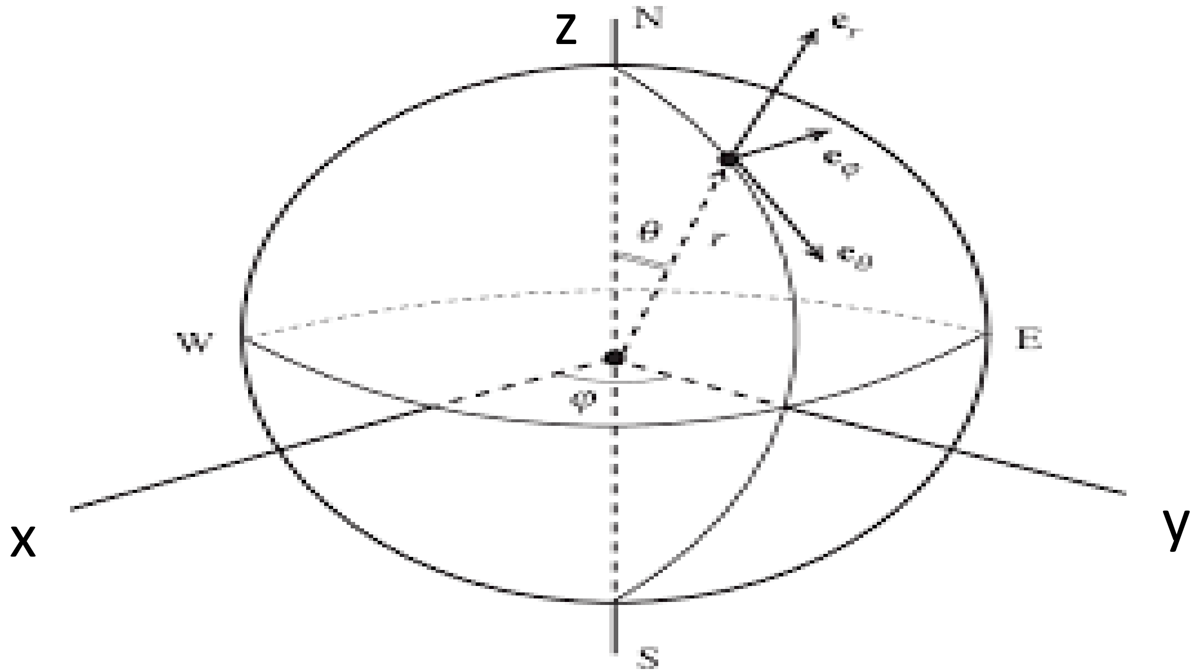
$\phi = \tan^{-1}(y / x)$

if P is the actual point let dl displacement takes place and reaches Q , therefore $dl = d\rho . a_\rho + \rho \, d\phi . \, a_\phi + dz. \, A_z$

Differential area = $ds = d\rho . dz = \rho d\phi . \, dz = d\rho . \, \rho d\phi$

Differential volume = $dv = d\rho . \, \rho d\phi . \, dz$

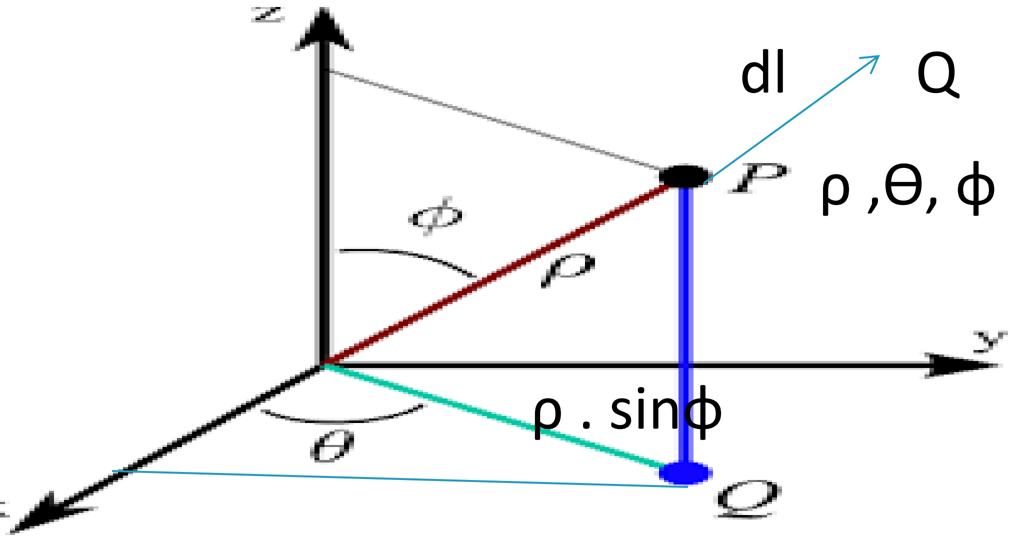
Spherical Coordinates



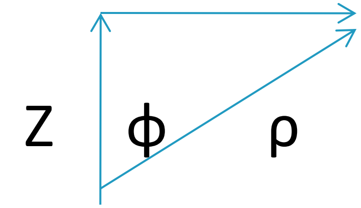
Above figure indicates spherical coordinate system with coordinates P (ρ, Ɵ, ϕ) .

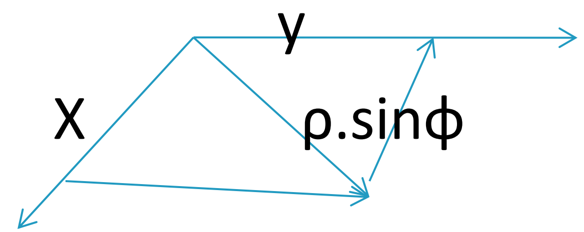
Where , ρ = is the radius of sphere in xyz plane
ϕ = displacement of radius with z axis
Ɵ = displacement of projectile from x –axis in xy plane.

Spherical Coordinates



x





Where, $x = \rho .\sin \phi . \cos \Theta$, $y = \rho .\sin \phi . \sin \Theta$, $z = \rho .\cos \phi$

$$x^2 + y^2 + z^2 = \rho^2$$

$$\sqrt{x^2 + y^2 + z^2} = \rho$$

Spherical Coordinates

$\text{Tan}\ \phi = (\rho / z)$

$\text{Tan}\ \Theta = (y / x)$

If P is actual point and Q is point after displacement of dl,

differential length $dl = d\rho.a_\rho + \rho.d\phi\ a_\phi + \rho.\sin\phi\ d\Theta\ a_\Theta$

differential area , $ds = d\rho\ .\ \rho.d\phi = \rho.d\phi\ .\ \rho.\sin\phi\ d\Theta$
$= \rho.\sin\phi\ d\Theta\ .\ D\rho$

Differential volume, $dv = d\rho.\ \rho.d\phi.\ \rho.\sin\phi$

Operators:

1) Operator del, ▼

$$▼ = (ɗ / ɗx\ ) a_x + (ɗ / ɗy\ ) a_y + (ɗ / ɗy\ ) a_y$$

if Ψ function is scalar, then

$$▼Ψ= (ɗΨ / ɗx\ ) ax + (ɗΨ / ɗy\ ) a_y + (ɗΨ / ɗy\ ) a_y$$

$$= \text{vector function}$$

If V is vector , $V= V_x a_x + V_y a_y + V_{z.} a_z$

$$▼V = \{(ɗ / ɗx\ ) a_x + (ɗ / ɗy\ ) a_y + (ɗ / ɗy\ ) a_y \}. \{V_x a_x + V_y a_y + V_{z.} a_z\}$$

$\blacktriangledown V = (dV_x / dx) + (d V_y / dy) + (dV_z / dy)$

$= \text{scalar value.}$

If V in cylindrical coordinates, $V = V_\rho . a_\rho + V_\phi . a_\phi + V_z . a_z$

$$\blacktriangledown V = \frac{d V_\rho}{d \rho} + \frac{d V\phi}{\rho \, d\phi} + \frac{d V_z}{d z}$$

If V in shperical coordinates, $V = V_\rho . a_\rho + V_\Theta . a_\Theta + V_\phi . a_\phi$

$$\blacktriangledown V = \frac{d V_\rho}{d \rho} + \frac{d V\Theta}{\rho \, d\Theta} + \frac{d V\phi}{\rho . \sin\Theta \, d\phi}$$

Vector Analysis

$$\nabla \times v = \begin{vmatrix} ax & ay & az \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \{(\partial V_z/\partial y) - (\partial V_y/\partial z)\}\, ax + \{(\partial V_x/\partial z) - (\partial V_z/\partial x)\}\, ay$$
$$+ \{(\partial V_y/\partial x) - (\partial V_x/\partial y)\}\, az$$

Vector Analysis

$$\nabla \times v = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \{(\partial V_z/\partial y) - (\partial V_y/\partial z)\}\, a_x + \{(\partial V_x/\partial z) - (\partial V_z/\partial x)\}\, a_y + \{(\partial V_y/\partial x) - (\partial V_x/\partial y)\}\, a_z$$

1) Convert given (2,5,-1) into cylindrical and spherical coordinates.

Cylindrical coordinates,

$$\rho = \surd\ x^2 + y^2 = \surd\ 2^2 + 5^2 = 5.38$$

$$\phi = \tan^{-1}(y / x) = \tan^{-1}(5/2) = 68.19^o$$

$$z = z = -1$$

Spherical coordinates,

$$\rho = \surd\ x^2 + y^2 + z^2 = \surd\ 2^2 + 5^2 + -1^2 = 5.47$$

$$\tan \Theta = (y / x)$$

$$\Theta = \tan^{-1}(y / x) = \tan^{-1}(5/2) = 68.19^o$$

Vector Analysis

$z = \rho .\cos \phi$

$\phi = \cos^{-1}(z / \rho) = \phi = \cos^{-1}(-1 / 5.47) = 100.53^o$

2) Given $A' = 3a_x - 2a_y + a_z$ and $B' = -a_x + 2.a_y + 7a_z$, find the angle between two vectors.

$$A'.B' = A.B.\cos\Theta$$

$$\cos\Theta = (A'.B') / (A.B)$$

2) $A = \sqrt{3^2 + -2^2 + 1^2} = \sqrt{14} = 3.74$

3) $B = \sqrt{-1^2 + 2^2 + 7^2} = \sqrt{52} = 7.21$

$$\cos\Theta = \frac{(3a_x - 2a_y + a_z)(-a_x + 2.a_y + 7a_z)}{3.74 \times 7.21} = \frac{(-3 - 4 + 7)}{3.74 \times 7.21} = 0$$

$\cos\Theta = 0$

$\Theta = 90^o$

Given $A' = 2a_x + 2a_y - a_z$ and $B' = 6a_x - 3.a_y + 2a_z$, find cross product and resultant direction.

$A' \times B' = A.B \sin\Theta.\ a$

$$A' \times B' = \begin{vmatrix} ax & ay & az \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix}$$

$$= (4 - 3)a_{x-}(4-(-6)a_y + (-6-12)a_z$$

$$= a_x - 10a_y - 18a_z$$

$$A = \sqrt{2^2 + 2^2 + -1^2} = \sqrt{9} = 3$$

$$B = \sqrt{6^2 + -3^2 + 2^2} = \sqrt{49} = 7$$

$$a = \frac{A' \times B'}{A.B \sin\Theta} = \frac{A' \times B'}{|A' \times B'|} = \frac{a_x + 10a_y - 18a_z}{\sqrt{1^2 + 10^2 + -18^2}}$$

Divergence Theorem:

$$\int_s A'. ds = \int_v \blacktriangledown A'. dv$$

Stokes Theorem:

$$\int_c .A'. dl = \int_s \blacktriangledown x\ A'. ds$$

2) Given $A' = 3a_x - 2a_y + a_z$ and $B' = -a_x + 2.a_y + 7a_z$, find the angle between two vectors.

$$A'.B' = A.B.\cos\Theta$$

$$\cos\Theta = (A'.B') / (A.B)$$

2) $A = \sqrt{3^2 + -2^2 + 1^2} = \sqrt{14} = 3.74$

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$$\cos\Theta = \frac{(3a_x - 2a_y + a_z)(-a_x + 2.a_y + 7a_z)}{3.74 \times 7.21} = \frac{(-3 - 4 + 7)}{3.74 \times 7.21} = 0$$

$\cos\Theta = 0$

$\Theta = 90^o$

Given $A' = 2a_x + 2a_y - a_z$ and $B' = 6a_x - 3.a_y + 2a_z$, find cross product and resultant direction.

$A' \times B' = A.B \sin\Theta.\, a$

$$A' \times B' = \begin{vmatrix} ax & ay & az \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix}$$

Vector Analysis

$= (4 - 3)ax - (4-(-6)a_y + (-6-12)a_z$

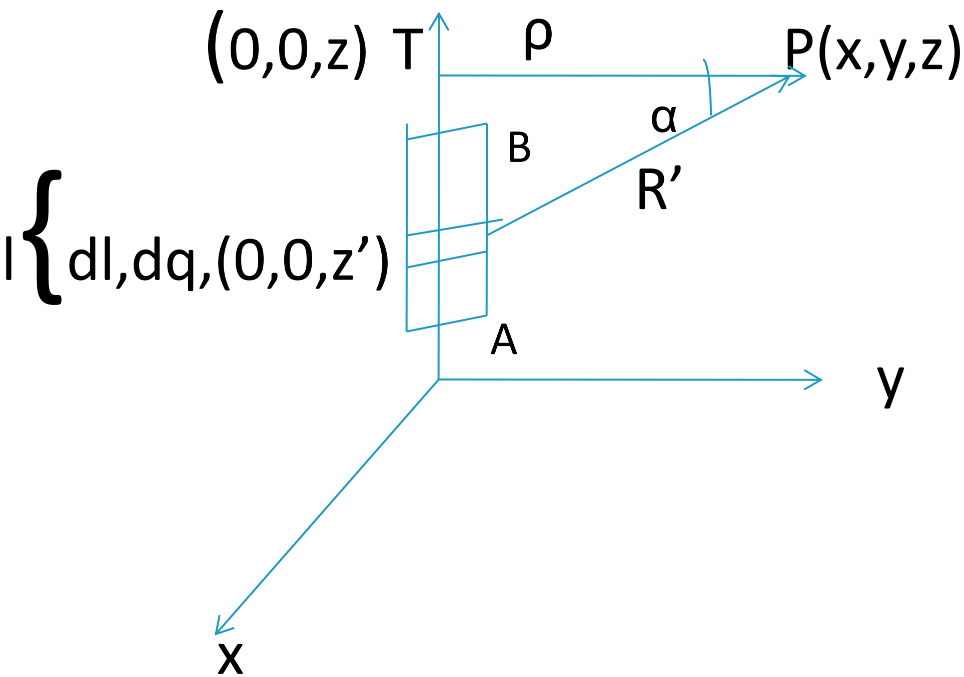
$= a_x - 10a_y - 18a_z$

$A = \sqrt{2^2 + 2^2 + -1^2} = \sqrt{9} = 3$

$B = \sqrt{6^2 + -3^2 + 2^2} = \sqrt{49} = 7$

$$a = \frac{A' \times B'}{A.B \sin\Theta} = \frac{A' \times B'}{|A' \times B'|} = \frac{a_x + 10a_y - 18a_z}{\sqrt{1^2 + 10^2 + -18^2}}$$

Electric Field Intensity Due To Line Charge



Let AB – straight conductor of length l

P - test point at which E is required due to straight conductor

(x,y,z) – location of P

dl – small length of l with charge dq located at (0,0,z’)

Q – total charge in straight conductor.

Electric Field Intensity Due To Line Charge

R' – is the distance vector between dl and point P.

Here, dl= dz , as conductor is located on z-axis.

$\rho_l = (dq/dl)$

$dq = \rho_l \, . \, dl = \rho_l \, . \, dz'$

$\int dq = \int \rho_l \, . \, dl$

$Q = \int \rho_l \, . \, dl = \int \rho_l \, . \, dz'$ ---------------------- 1

Therefore E due to Q at P is,

$$E = \frac{K.Q.\ a}{R^2} \qquad \text{-------------------}2$$

Electric Field Intensity Due To Line Charge

Let us find distance vector R', P(x, y, z) and dl(o ,o , z')

$$R' = (x_2 - x_1)a_x + (y_2 - y_1)a_y + (z_2 - z_1)a_z$$

$$R' = (x - 0)a_x + (y - 0)a_y + (z - z')a_z$$

$$R' = x\, a_x + y\, a_y + (z - z')\, a_z \qquad \text{or}$$

$R' = \rho\, a_\rho + (z - z')\, a_z$ (as ρ axis is result of x and y axis in cylindrical coordinates) ------------3

Magnitude of distance vector R,

$$R = \sqrt{x^2 + y^2 + (z-z')^2}$$

$$R^2 = x^2 + y^2 + (z-z')^2$$

Electric Field Intensity Due To Line Charge

$$R = \sqrt{\rho^2 + (z-z')^2}$$

$$R^2 = \rho^2 + (z-z')^2$$

But we know that , $R' = R\,.a$

$$a = (R' / R) \text{ ------------------ } 4$$

Where, $K = (1/4\pi\varepsilon)$

Substituting equation 1,3,4 in equation 2

$$E = \frac{K.\int \rho_l\,.\,dz'\,.\,R'}{R^2.R} = \frac{K.\int \rho_l\,.\,R'\,dz'}{R^3}$$

Electric Field Intensity Due To Line Charge

$$E = \frac{\int 1.\ \rho_l\ .\ (\rho\ a_\rho + (z - z')\ a_z\ )\ dz'}{4\pi\varepsilon\ .\ (\ \rho^2 + (z-z')^2\ )^{3/2}} \text{ ----------- } 5$$

ρ - distance between P and T

From right angle triangle PAT,

$$\cos\alpha = (\rho\ /\ R)$$

$$\sin\alpha = (z-z')\ /\ R$$

$$\sec\alpha = (\ R\ /\ \rho)$$

$$\tan\alpha = (z-z')\ /\ \rho$$

Where, $R = \rho\ .\ \sec\alpha$

Electric Field Intensity Due To Line Charge

$$(z-z') = \rho \tan \alpha$$

$$z' = (z- \rho \tan \alpha)$$

Differentiating on both sides,

$$dz' = - \rho \sec^2 \alpha \, .d\alpha$$

Substituting above values in equation 5,

$$E = \int 1. \frac{\rho_l . (R.\cos \alpha \, a_\rho + R.\sin\alpha \, a_z)(- \rho \sec^2 \alpha) \, d\alpha}{4\pi\varepsilon . (R^2)^{3/2}}$$

Electric Field Intensity Due To Line Charge

$$E = \frac{(-\rho \sec^2 \alpha) \,.\, \int 1.\, \rho_l \,.\, R(\cos\alpha\, a_\rho + \sin\alpha\, a_z)\, d\alpha}{4\pi\varepsilon \,.\, (R)^3}$$

$$= \frac{(-\rho \sec^2 \alpha) \,.\, \int 1.\, \rho_l \,.\, (\cos\alpha\, a_\rho + \sin\alpha\, a_z)\, d\alpha}{4\pi\varepsilon \,.\, (R)^2}$$

$$= \frac{(-\rho \sec^2 \alpha) \,.\, \int_{\alpha 1}^{\alpha 2} 1.\, \rho_l \,.\, (\cos\alpha\, a_\rho + \sin\alpha\, a_z)\, d\alpha}{4\pi\varepsilon \,.\, \rho^2 \sec^2 \alpha}$$

$$= \frac{-\rho_l \int_{\alpha 1}^{\alpha 2} (\cos\alpha\, a_\rho + \sin\alpha\, a_z)}{4\pi\varepsilon \,.\, \rho}\, d\alpha$$

$$= \frac{-\rho_l \;(\sin\alpha\, a_\rho - \cos\alpha\, a_z)}{4\pi\varepsilon \,.\, \rho} \quad \text{with limits } \alpha_1 \text{ and } \alpha_2$$

Electric Field Intensity Due To Line Charge

$$= - \frac{\rho_l \ [(\sin \alpha_2 - \sin \alpha_1).a_\rho - (\cos \alpha_2 - \cos \alpha_1)\ a_z \ ]}{4\pi\varepsilon \ . \ \rho}$$

For a straight conductor of infinite length,

i.e length of straight conductor on Z axis varies from $-\infty$ to ∞,

but we know that , $\tan \alpha = (z-z') / \rho$,

i.e when $z = -\infty$, $\tan \alpha_1 = -\infty$

$\alpha_1 = -90^o$

when $z = \infty$, $\tan \alpha_2 = \infty$

$\alpha_2 = 90^o$

Electric Field Intensity Due To Line Charge

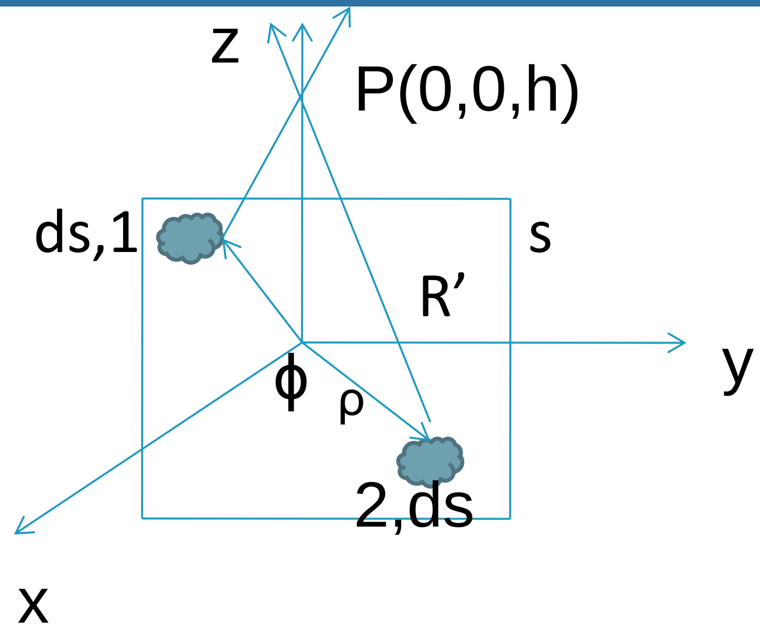
$$E = -\frac{\rho_l \, [\,(\sin 90^o - \sin -90^o).a_\rho - (\cos 90^o - \cos -90^o)\, a_z\,]}{4\pi\varepsilon \,.\, \rho}$$

$$E = -\frac{\rho_l .\ (1 - (-1)\, a_\rho}{4\pi\varepsilon \,.\, \rho}$$

$$= -\frac{\rho_l .\ 2.\ a_\rho}{4\pi\varepsilon \,.\, \rho}$$

$$= -\frac{\rho_l \,.\, a_\rho}{2\pi\varepsilon \,.\, \rho}$$ (if limits are reversed then the expression will be positive)

Electric Field Intensity Due To Surface Charge



Let us consider an square conductor with area **s** is placed in xy axis

With surface charge density $\rho_{s.}$

Q – total charge in square conductor

R’ – distance vector between ds (s) and P.

ρ – radius of square

Electric Field Intensity Due To Surface Charge

dq - charge in ds

Therefore surface charge density , $\rho_s = (dq / ds)$

$$dq = \rho_s .ds$$

$$\int dq = \int_s \rho_s .ds$$

$$Q = \int_s \rho_s .ds$$

The electric field intensity due to dq charge at P is

$$dE = \frac{K. dq .a}{R^2}$$

Electric Field Intensity Due To Surface Charge

Distance vector R' between ds,2(x,y,0) and P (0,0,h)

$(x_1, y_1, z_1) \quad (x_2, y_2, z_2)$

$R' = (x_2 - x_1)\, a_x + (y_2 - y_1)\, a_y + (z_2 - z_1)\, a_z$

$= (0 - x)\, a_x + (0 - y)\, a_y + (h - 0)\, a_z$

$= -x\, a_x - y\, a_y + h\, a_z$

$= -\rho\, a_\rho + h\, az$

Magnitude of R' , $R = \sqrt{(-\rho)^2 + h^2} = \sqrt{\rho^2 + h2} = (\rho^2 + h^2)^{1/2}$

unit vector, $a = (R' / R)$

Differential area , $ds = d\rho.\, \rho d\phi = \rho d\phi\, .dz = dz.\, d\rho$

$K = (1 / 4\pi\varepsilon)$

Electric Field Intensity Due To Surface Charge

Let us substitute above values in dE,

$$dE = \frac{K.\ dq\ .a}{R^2}$$

$$= \frac{1\ .\ \rho_s\ .ds\ .R'}{4\pi\varepsilon.\ R^2\quad .\ \ R}$$

$$= \frac{1\ .\ \rho_s\ .\ d\rho.\ \rho d\phi\ .(-\ \rho\ a_\rho + h\ az)}{4\pi\varepsilon\ .R^3}$$

As conductor symmetry over xy plane their will be counter ds on other side , $R' = \rho\ a_\rho + h\ az$

Electric Field Intensity Due To Surface Charge

Hence their will be no component of ρ in E as net electric field intensity due to ρ component will be zero.

$$dE = \frac{1 \,.\, \rho_s \,.\, d\rho.\ \rho d\phi \,.(\,h\ az)}{4\pi\varepsilon \,.R^3}$$

total field intensity due to square conductor at point P is

$$E = \int_s dE$$

$$= \frac{\int_s 1 \,.\, \rho_s \,.\, d\rho.\ \rho d\phi \,.h\ az}{4\pi\varepsilon \,.\, (\rho^2 + h^2)^{3/2}}$$

$$= \frac{\rho_s \,.h \quad \int d\phi \quad \int \rho\ d\rho\ az}{4\pi\varepsilon \quad . \qquad (\rho^2 + h^2)^{3/2}}$$

Electric Field Intensity Due To Surface Charge

$$= \frac{\rho_s\, .h \int_0^{2\pi} d\phi \int_0^{\infty} \rho\, d\rho\, az}{4\pi\varepsilon\ .\ (\rho^2 + h^2)^{3/2}}$$

$$= \frac{\rho_s\, .h\ [\phi]_0^{2\pi} \int_0^{\infty} \rho\, d\rho\, az}{4\pi\varepsilon\ .\ (\rho^2 + h^2)^{3/2}}$$

$$= \frac{\rho_s\, .h\ [2\pi - 0] \int_0^{\infty} \rho\, d\rho\, az}{4\pi\varepsilon\ .\ (\rho^2 + h^2)^{3/2}}$$

$$= \frac{\rho_s\, .h \int_0^{\infty} \rho\, d\rho\, az}{2\varepsilon\ .\ (\rho^2 + h^2)^{3/2}}$$

Electric Field Intensity Due To Surface Charge

Let us integrate second term,

$$= \frac{\int_0^\infty \rho \, d\rho \, az}{(\rho^2 + h^2)^{3/2}}$$

Let $t^2 = \rho^2 + h^2$, $t = (\rho^2 + h^2)^{1/2}$

Differentiating on both sides, $2t\,.dt = 2\,\rho.\,D\rho$

$$t\,.dt = \rho.\,d\rho$$

$$= \frac{\int t\,.dt\,az}{(t^2)^{3/2}} \qquad = \frac{\int t\,.dt\,az}{(t)^3}$$

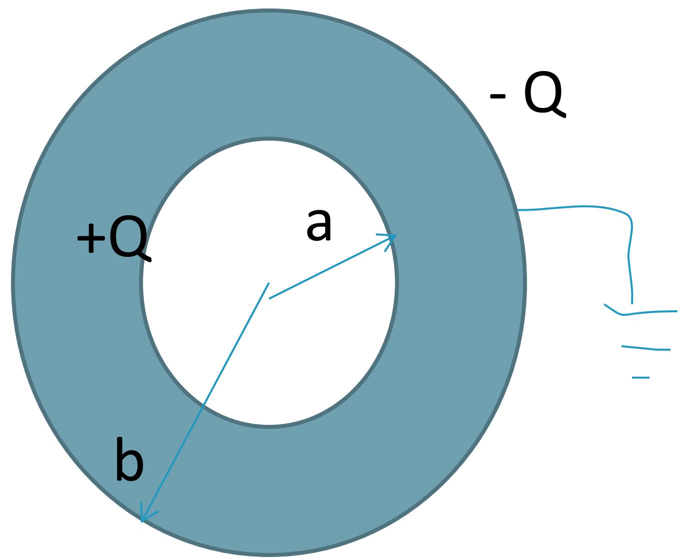
$$= \int (1/t^2)\,dt\,az \quad = \quad -t^{-1}\,az \quad = \quad -(\rho^2 + h^2)^{-1/2}\,az$$

Electric Field Intensity Due To Surface Charge

$$E = \frac{\rho_s \,.h\, [-(\rho^2 + h^2)^{-1/2}]_0^{\infty} a_z}{2\varepsilon\,.}$$

$$E = \frac{\rho_s \,. a_z}{2\varepsilon\,.}$$

Electric Displacement Or Flux



Micheal Faraday conducted an experiment on two concentric metalic spheres, where ineer sphere is positively charges (+Q) their by inducing negative charge on outer sphere(-Q).

Both spheres are isolated from each other and outer sphere is grounded.

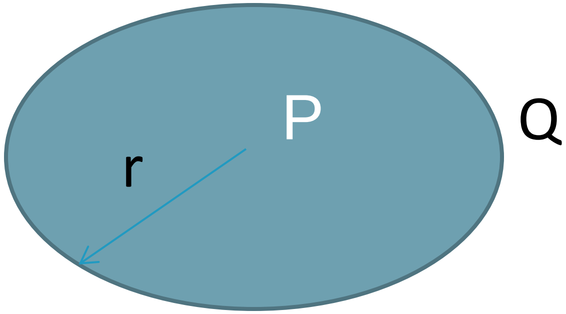
Electric Displacement Or Flux

Faraday then concluded that or observed that there is some displacement from inner sphere to outer sphere irrespective of medium between them. This displacement is called as electric displacement or electric flux (Ψ).

The total charge on inner sphere is Q, then Faraday's experiment

$$\Psi = Q \text{ (Coloumbs)}$$

Electric Flux Density:



The electric field intensity due to above sphere of charge Q at P

Electric Displacement Or Flux

$$E = \frac{(K.Q.)\,a}{r^2} \qquad = \frac{(Q)\,a}{4\pi\varepsilon.\,r^2}$$

Let us take only magnitude of E, $E = \dfrac{(Q)}{4\pi\varepsilon.\,r^2}$

further, $E.\,\varepsilon = (Q) / (4\pi r^2)$

$D = (Q) / (4\pi r^2)$

$D = (Q) / (A)$ ----- A area of sphere with radius r

Here D is referred as **electric flux density** or **displacement density** or **displacement flux density.**

Electric Displacement Or Flux

Electric flux density is defined as charge per unit area.

$$D = E.\varepsilon$$

If direction is also consider then , $D' = E'.\varepsilon$

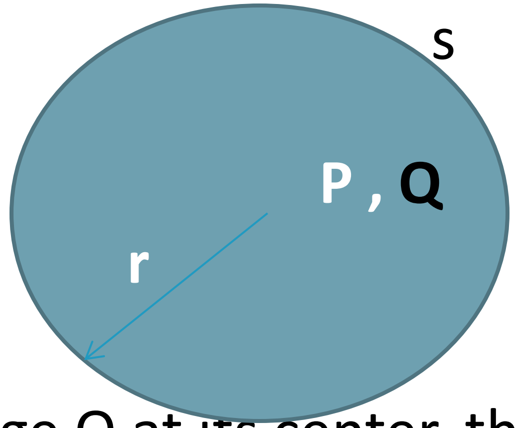
$$D' = (Q \,.\, a) / (4\pi r^2)$$

Gauss Law:

In the electrostatics Gauss law states that " The surface integral of the normal component of electric flux density over a closed surface equals to charge enclosed"

Electric Displacement Or Flux

“ The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.



$$\int D' \cdot ds = Q = \Psi$$

Let us consider a sphere of radius r with charge Q at its center, then Electric flux $d\Psi$ in ds of sphere.

$$d\Psi = D \cdot ds$$

$$\int d\Psi = \int\int D \cdot ds$$

$$\Psi = \int\int D \cdot ds$$

Electric Displacement Or Flux

$$\Psi = \int\int \frac{Q}{4\pi r^2} . ds$$

$$\Psi = \frac{Q}{4\pi r^2} \int\int ds$$

$$\Psi = \frac{Q . 4\pi r^2}{4\pi r^2}$$

$$\Psi = Q$$

$$\Psi = Q = \int\int D . ds = = \int_s D . ds$$

Maxwell's Equation From Gauss Law

We know that , $\Psi = \int\int D.\, ds$

$$\Psi = \int_s D.\, ds = Q \quad \text{--------------}1$$

As sphere has volume also total charge can be written as,

$$\Psi = \int_v \rho_v \,.\, dv = Q \text{ -------------- } 2$$

By comparing 1 and 2 ,

$$\int_s D.\, ds = \int_v \rho_v \,.\, dv = Q \text{ (applying divergence theorem) ----------- } 3$$

according divergence theorem, $\int_s A'.\, ds = \int_v \blacktriangledown .A'\, dv$

$$\int_s D.\, ds = \int_v \blacktriangledown .D\, dv \text{ ------------ } 4$$

Maxwell's Equation From Gauss Law

Substituting 4 in 3, $\int_v \nabla . D \, dv = \int_v \rho_v \, . \, dv$

Comparing both sides, $\nabla . D = \rho_v$ -------------- 5

Equation 5 called as → point form of Gauss law

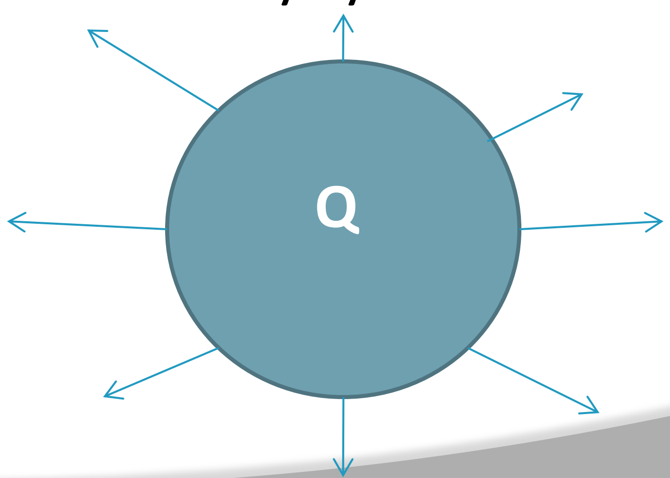
→ vector form of Gauss law

→ Differential form of Gauss Law

→ One of Maxwell Equation

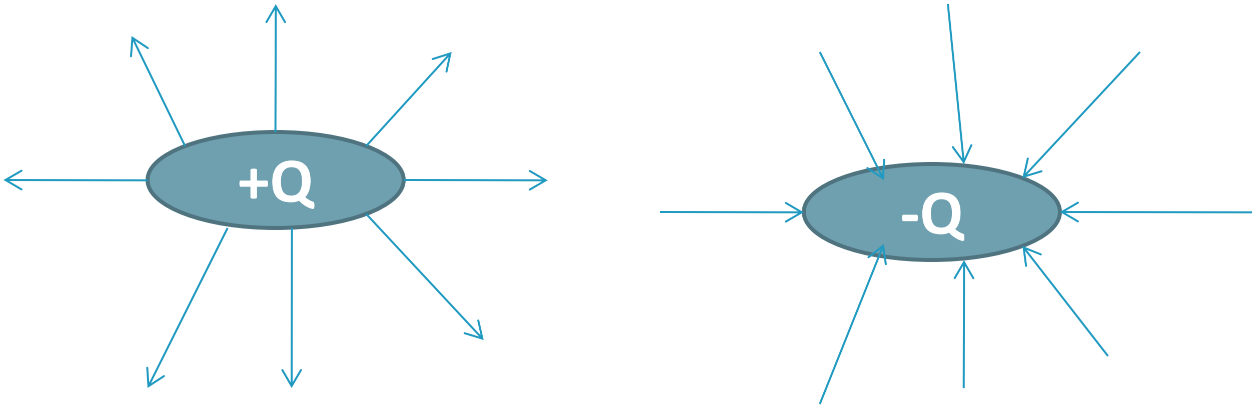
Applications Of Gauss Law

1) Gauss law is useful to determine the electric field intensity , to find E we assume surface as Gaussian Surface.
2) It is applicable only if Gaussian surface encloses total volume.
3) D’ must be normal to surface , therefore D’.ds = 0 as dot product of two vector perpendicular each other is zero.
4) At any point on Gaussian surface D’ magnitude must be same , otherwise Gaussian surface is not uniform shape.
5) Gauss law is applicable only symmetric or uniform surfaces.

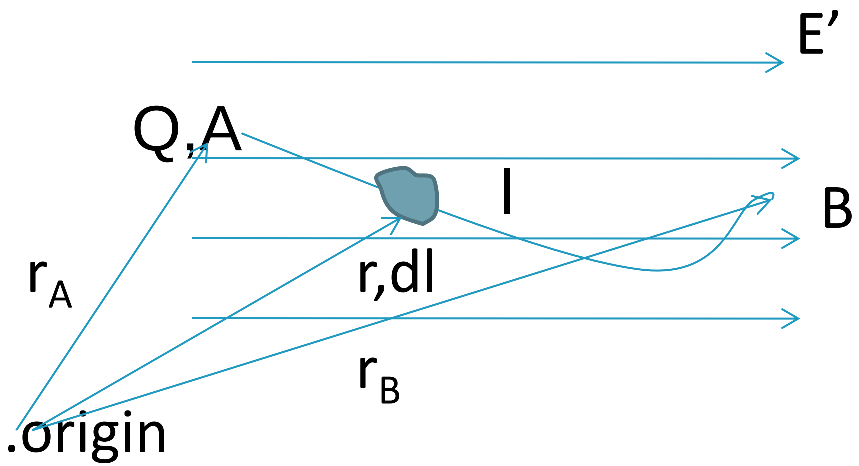


Direction Of Electric Field

Electric field is defined as the electric force per unit charge.
The direction of the field is taken to be the direction of the force it would exert on a positive test charge. The electric field is radially outward from a positive charge and radially in toward a negative point charge



Work Done



A charge Q placed in the existing electric field E’ with direction as shown , then Q experiences Force F’ in the Direction of E’.

The force on Q, F’ =Q. E’.

We suppose to move charge Q from point A to B, displacement length is l.

The work done in displacing charge Q by dl is,

Work Done

$$dw = -F'.\,Dl$$

$$dw = -Q.\,E'\,dl$$

$$\int dw = -\int Q.\,E'\,dl$$

Negative sign indicates that work is done due to external agent.

$$w = -\int_A^B Q.\,E'\,dl$$

Potential Difference And Electric Potential

Electric potential can be defined from work done expression,

$$w = -Q \int_A^B E' \, dl$$

The work done to move point charge from one point to another is called as electric potential.

$$(w / Q) = - \int_A^B E' \, dl$$

$$V = - \int_A^B E' \, dl$$

$$V_{AB} = - \int_A^B E' \, dl = V_A - V_B$$

Potential Difference And Electric Potential

1) To determine V_{AB} , A is initial and B is final point.
2) If V_{AB} is negative there is loss of potential from A to B.
3) If V_{AB} is positive there is gain of potential from A to B.
4) V_{AB} is independent of path travelled by charge.
5) V_{AB} measured in Joules per Coloumb or Volt.

The electric field intensity at a distance r fronm origin on charge Q is, $E' = (Q / 4\pi\varepsilon r^2). a_r$

The potential difference then computed as,

Differential length in spherical coordintes is,

$$\mathbf{dl = dr.a_r + \rho.d\phi\, a_\phi + \rho.\sin\phi\, d\Theta\, a_\Theta}$$

Potential Difference And Electric Potential

$$V_{AB} = - \int_{A}^{B} E' \, dl$$

$$V_{AB} = - \int_{A}^{B} \frac{Q}{4\pi\varepsilon r^2} . (dr.a_r + \rho.d\phi \, a_\phi + \rho.\sin\phi \, d\Theta \, a_\Theta). a_r$$

$$V_{AB} = - \frac{\int_{rA}^{rB} Q . dr}{4\pi\varepsilon r^2}$$

$$V_{AB} = - \frac{Q}{4\pi\varepsilon} \frac{1}{r} \quad \text{with limits } r_B \text{ and } r_A$$

$$V_{AB} = - \frac{Q}{4\pi\varepsilon} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

Potential Difference And Electric Potential

$$V_{AB} = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = V_A - V_B$$

Therefore , $V_A = (Q / 4\pi\varepsilon.r_A)$

$V_B = (Q / 4\pi\varepsilon.r_B)$

If point B is at infinite (∞), then $V_B = 0$

Therefore, $V_{AB} = V_A = (Q / 4\pi\varepsilon.r_A)$ = Absolute potential

General potential can be given as, $V = (Q / 4\pi\varepsilon.r)$

Problems

Find the work involved in moving a charge of 2C from (8,6,-4) to (2,3,-2) along straight line in the field $E' = x\,a_x + 2y\,a_y - 4z\,a_z$

The work done, $w = -Q \int E'\, dl$

$$dl = dx.\, a_x + dy.a_y + dz.a_z$$

$$E'\, dl = (x\, a_x + 2y\, a_y - 4z\, a_z)\,(dx.\, a_x + dy.a_y + dz.a_z)$$

$$= x.\, dx + 2y.\, dy - 4z.\, dz$$

therefore, $w = -2.\, {}_{(8,6,-4)}\int^{(2,3,-2)} (x.\, dx + 2y.\, dy - 4z.\, dz)$

$$= (-2)\left\{ {}_{8}[x^2/2]^{2} + {}_{6}[2y^2/2]^{3} - {}_{-4}[4Z^2/2]^{-2} \right\}$$

Problems

$W = (-2)\{[2-32] + [9-36] - [8-32]\}$

$= 66\ J$

Find the workdone in moving a point charge 5μC from (4,π,o) to (6,π,o) in the field $E' = (10^6 / \rho)\, a_\rho + 10^4 z\, a_z$

$E' = E_\rho a_\rho + E_\phi . a_\phi + E_z . a_z$

$E_\rho = (10^6 / \rho)$ and $E_z = 10^4 .z$

$dl = d\rho . a_\rho + \rho\, d\phi .\, a_\phi + dz .\, a_z$

$E' . dl = [(10^6 / \rho)\, a_\rho + 10^4 z\, a_z]\, [d\rho . a_\rho + \rho\, d\phi .\, a_\phi + dz .\, a_z]$

Problems

$E'. dl = (10^6 / \rho).d\rho$

Therefore work done, $w = - Q \int E' \, dl$

$= - 5 \times 10\text{-}6 \int_4^6 (10^6 / \rho).d\rho$

$= - 5 \times 10\text{-}6 \int_4^6 (10^6 / \rho).d\rho$

$= - 5 \times 10^{-6} .10^6 . [\ln \rho]_4^6$

$= - 5 \times 10^{-6} .10^6 . [\ln 6 - \ln 4]$

$= -2.027$ J

Potential gradient

we know that work done, $dw = - Q. E' dl$

$$(dw / Q) = - E' dl$$

$$dV = - E' dl \text{ -- -----------1}$$

We can write , $dV = (\partial V / \partial x) dx + (\partial V / \partial y) dy + (\partial V / \partial z) dz$

Further can be split as,

$$dV = [(\partial V / \partial x) a_x + (\partial V / \partial y) a_y + (\partial V / \partial z) a_z] \times [dx. a_x + dy.a_y + dz.a_z]$$

$$dV = \nabla .V\, dl \text{ ------------ 2}$$

Potential gradient

comparing 1 and 2 , $-E' = \nabla . V$

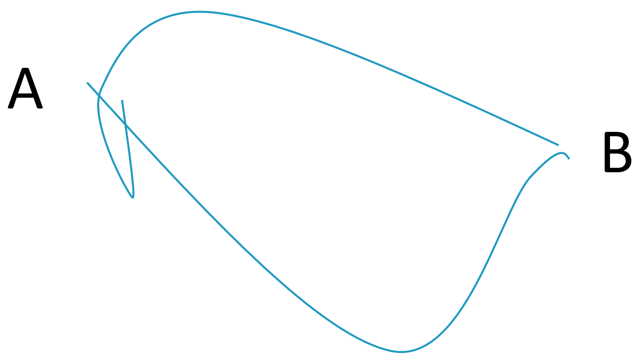
$E' = -\nabla . V$

Further simplifying,

$$E_x a_x + E_y a_y + E_z a_z = -[(\partial V / \partial x) a_x + (\partial V / \partial y) a_y + (\partial V / \partial z) a_z]$$

Where, $E_x = - \partial V / \partial x$ $E_y = - \partial V / \partial y$ $E_z = - \partial V / \partial z$

Maxwell's Curl Equation



From potential difference discussion we came to know that it is independent of path, $V_{BA} = - V_{AB}$

that is , $V_{BA} + V_{AB} = \int E' dl = 0$

$$\int E' dl = 0 \text{ --------------- } 1$$

Applying stokes theorem, $\int A' dl = \int_s \nabla \times A' ds$

$$\int E' dl = \int_s \nabla \times E' ds = 0$$

Therefore, $\nabla \times E' = 0$ (Maxwell Curl Equation) ------------- 2

Poisson's And Laplace equation

Equation 1 and 2 are proof for kirchoff's voltage law.

From point form of gauss law, $\nabla D = \rho_V$

But we know, $D = \varepsilon.E$

$E = -\nabla V$

By substitutions, $\nabla D = \nabla\ \varepsilon.E = \nabla\ \varepsilon.\ (-\nabla V) = \rho_V$

$\nabla^2 V = -\rho_V / \varepsilon$ ---------Poisson's equation

If $\rho_V = 0$ above equation becomes,

$\nabla^2 V = 0$ ----------- Laplace Equation

Poisson's And Laplace equation

$\rho_V = 0$ i.e volume charge density is zero is possible only during dielectric materials.

$$\blacktriangledown^2 = \text{Laplace operator}$$

$$= (d^2 \ / d^2 x) + (d^2 \ / d^2 y) + (d^2 \ / d^2 z)$$

$$= (d^2 \ / d^2 \rho) + (d^2 \ / \rho^2 d^2 \phi) + (d^2 \ / d^2 z)$$

$$= (d^2 \ / d^2 \rho) + (d^2 \ / \rho^2 d^2 \Theta) + (d^2 \ / \rho^2 .\sin \phi \, d^2 \phi)$$

Solution Of Laplace Equation

$V = 0, x = 0$ **dielectric** $V = V_0, x = d$

$-Q$ $+Q$

Let us consider two parallel kept one at x=0 and x=d with potential V=0 and $V=V_0$ respectively.

From Laplace equation , $\nabla^2 V = 0$

Solution Of Laplace Equation

$$(d^2V / d^2 x) + (d^2V / d^2 y) + (d^2V / d^2 z) = 0$$

As plates are placed in x axis terms of y and z axis in above equation can be eliminated, then

$$(d^2V / d^2 x) = 0$$

$$\int (d^2V / d^2 x) = \int 0$$

$$dV / dx = A$$

$$dV = A. dx$$

$$\int dV = \int A. dx$$

$$V = Ax + B$$

Solution Of Laplace Equation

At $x = 0, V = 0$ $\quad\quad$ $0 = A(0) + B$

$0 = B$

At $x = d, V = V_0$ $\quad\quad$ $V_0 = A(d) + 0$

$(V_0 / d) = A$

Therefore solution is , $\quad V = (V_0 / d)\, x$

UNIT-II
CONDUCTORS AND DIELECTRICS

MODULE II - SYLLABUS

Dipole moment, potential and electric field intensity due to an electric dipole, torque on an electric dipole in an electric field, behavior of conductors in an electric field, electric field inside a dielectric material, polarization, conductor and dielectric, dielectric boundary conditions, capacitance of parallel plate and spherical and coaxial capacitors with composite dielectrics, energy stored and energy density in a static electric field, current density, conduction and convection current densities, Ohm’s law in point form, equation of continuity.

COURSE OUTCOMES MAPPED WITH MODULE II

CO	Course Outcomes	Blooms Taxonomy
CO 4	Determine the potential and torque due to electric dipole used in structuring the principle of electrical equipments.	Understand
CO 5	Realize the behavior of conductors and dielectrics, their by compute the capacitance of different configured plates.	Understand

PROGRAM OUTCOMES MAPPED WITH MODULE II

PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
PO 3	**Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

MAPPING OF COs AND POS FOR MODULE II

COs	PROGRAM OUTCOMES											
	1	2	3	4	5	6	7	8	9	10	11	12
CO4	√	√										
CO5	√	√	√									

ELECTRIC DIPOLE AND MOMENTUM

Two opposite charges +q and –q separated by some distance d forms the electric dipole.

+q ---------------d-------------- -q

The distance travelled by the point charge Is defined as dipole moment (or) the product of charge and distance travelled by it is called as electric dipole.

P = q.d ----------- 1

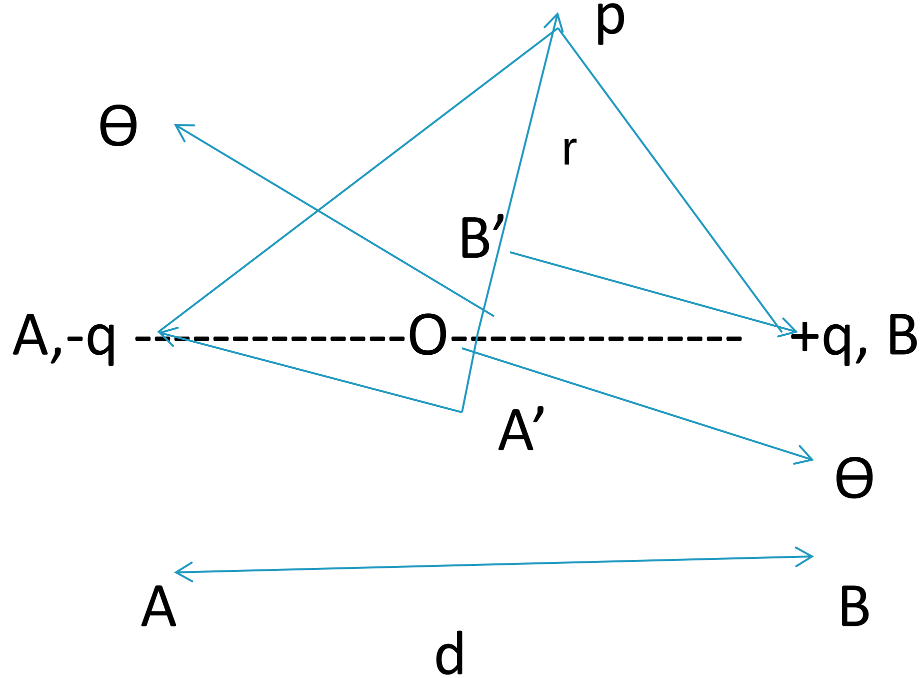
Here , P → electric dipole moment

d → distance between opposite charges

the line between two charges is called as axis of dipole.

POTENTIAL DUE TO ELECTRIC DIPOLE

Assume two charges separated by distance d as shown in the figure



POTENTIAL DUE TO ELECTRIC DIPOLE

Here, O → center of the axis between charges

P → be the test point where potential is required.

OP → with length of r.

AA′ → perpendicular from A to OP

BB′ → perpendicular from A to OP.

AO → d / 2

OB → d / 2

$\angle POB = \theta$

$r >>> d$

the line $AP = A'P = OP + OA'$-------------------------------2

from the right angle triangle $AA'O$, $OA' = OA \cos \Theta$

hence equation 2 can be written as, $AP = A'P = r + OA \cos \Theta$

but, $OA = d/2$

$AP = A'P = r + d/2 \cos \Theta$

Hence the potential at P due negative charge at A is ,

$V_A = -Kq/ AP = (-Kq) / (r + d/2 \cos \Theta)$

ELECTRIC DIPOLE AND POTENTIAL

Similarly from the right angle triangle BB′O, BP = B′P = OP – OB’

$= OP - OB.\cos\theta \quad = r - d/2 \cos\theta$

Hence the potential at P due positive charge at B is ,

$$V_B = Kq/ BP = (Kq) / (r - d/2 \cos\theta)$$

Therefore the total potential acting on P is , $V = V_A + V_B$

$$V = \frac{-Kq}{r + d/2 \cos\theta} + \frac{Kq}{r - d/2 \cos\theta}$$

$$= \frac{Kq [r + d/2 \cos\theta - (r - d/2 \cos\theta)]}{(r^2 - d^2/4 \cos^2\theta)} = \frac{Kq\, d. \cos\theta}{(r^2 - d^2/4 \cos^2\theta)}$$

But we know that, $r >>> d$

$$V = Kqd.\cos\theta/ r^2$$

$$V = KP.\cos\theta/ r^2 , (P = q.d) \text{ ---------------- } 3$$

know that electric field intensity in terms of electric potential is given as ,

$$E = -\nabla V$$

$$V = KP.\cos\Theta / r^2$$

From equation 3 we can say that potential due dipole is in spherical co-ordinates, therefore find electric field intensity we shall use spherical co-ordinates.

$$\nabla V = [\, dV/dr\ a_r + (1/r)dV/d\Theta\ .a_\Theta + (\,1/ r\sin\phi)\ dV/d\phi .a_\phi]$$

Simplifying ∇V, $dv/dr = -2KP.\cos\Theta / r^3$

$$(1/r)dv/d\Theta = (1/r)(-KP.\sin\Theta / r^2\,) = -KP.\sin\Theta / r^3$$

Substituting above two equations in E,

$$E = -[\ (-2KP.\cos \Theta / r^3) + (-\ KP.\sin \Theta / r^3)\ ]$$

$$= [\ (2KP.\cos \Theta / r^3) + (\ KP.\sin \Theta / r^3)\ ]$$

$$= (KP/ r^3) .[\ (2\cos \Theta)\ ar + (\sin \Theta)a\Theta\ ] \text{ ----------- } 4$$

Torque due to Electric Dipole

Let us consider two opposite charges are placed in the uniform electric field with their line of axis of 2r.

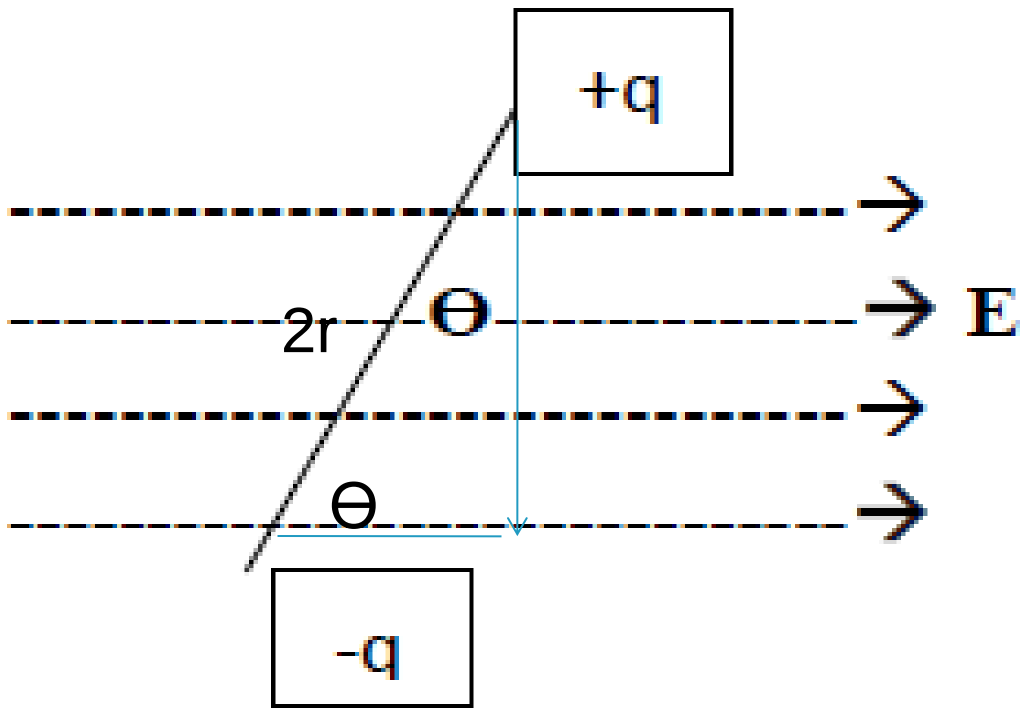
The force experienced by +q is , $F_1 = E.q$

The force experienced by -q is , $F_2 = -E.q$

The total experienced by the dipole is , $F = F_1 + F_2$

$$F = 0$$

ELECTRIC DIPOLE AND TORQUE



ELECTRIC DIPOLE AND TORQUE

due to force experienced by +q it tends to oscillate in the direction of E and –q in the direction opposite to E, which leads torque of dipole.

T = magnitude of F x perpendicular distance

Between their line of action

T = E.q x 2r sinϴ (2r = d)

T = PE.sinϴ. = P' x E'

BEHAVIOR OF CONDUCTOR

Key Features:

Conductor → charge easily carried from one point to another.

→ outer shell electrons are easily detachable.

→weak or small voltage is enough to detach electron.

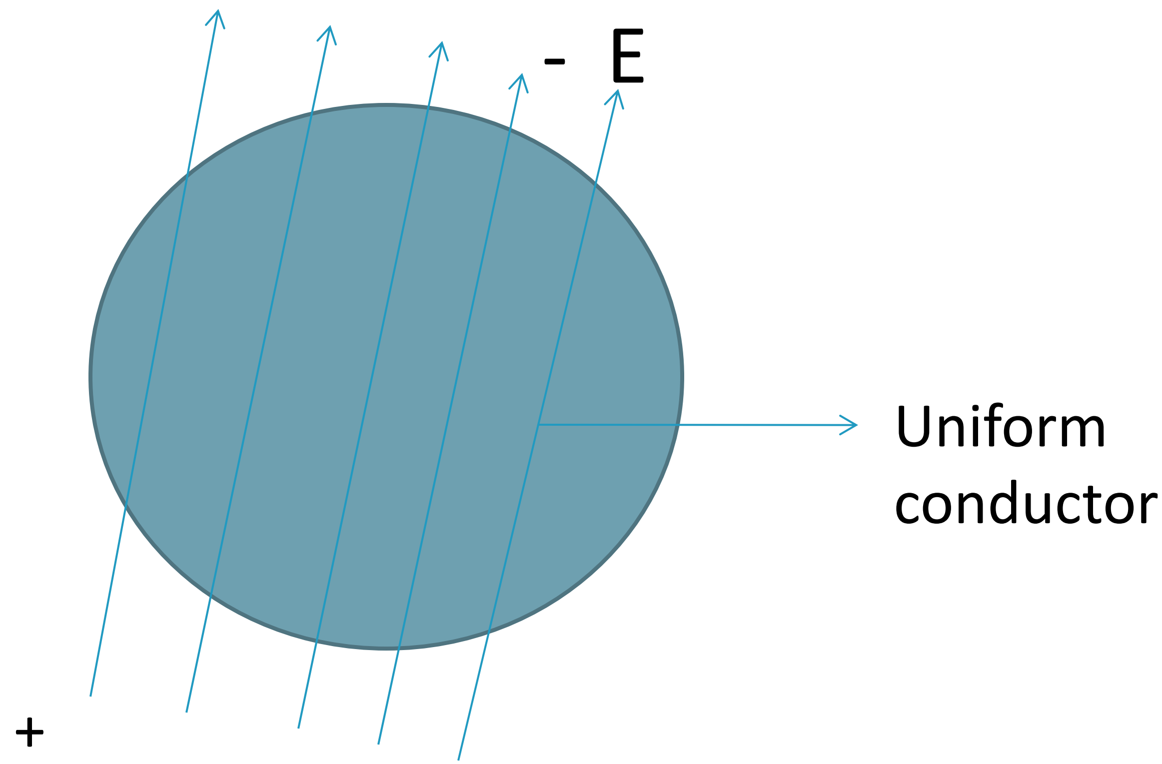
Dielectric → charge cant move from one point to another.

→ outer shell electrons are very difficult detachable.

→High Voltage is require to detach electron called as break down.

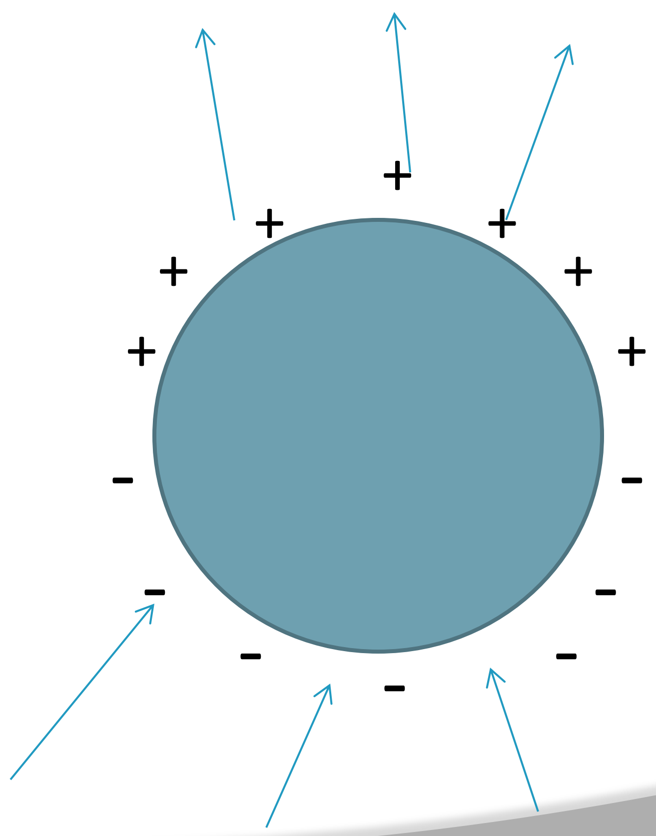
Semi-conductor → conduction occurs both by electron and holes.

BEHAVIOR OF CONDUCTOR



BEHAVIOR OF CONDUCTOR

- Uniform shaped conductor placed in existing static electric field.
- The free electrons move opposite to the direction of field.
- If by means of wire an closed path is provided the current will be constituted.



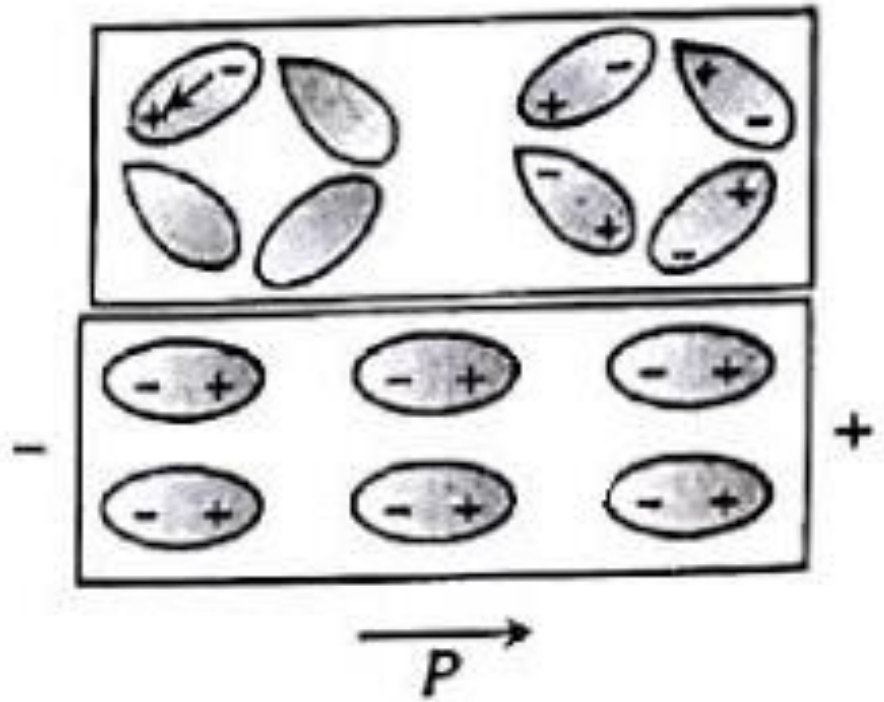
BEHAVIOR OF CONDUCTOR

- An conductor is bounded by free space.
- Therefore charge are restricted to move further due to atomic forces with in conductor and free space as insulator.
- i.e charge is always forms on the surface of conductor.
- Conductor should have uniform charge density.
- charge density inside conductor is zero.
- Electric field developed in conductor is exactly opposite to applied or existing field.
- electric field inside conductor is zero.
- Gauss law is applicable.
- Conductor is an equi potential surface.

BEHAVIOR OF DIELECTRIC

- Dielectric material doesn't have free electron therefore current is not constituted.
- The electrons in dielectric are tightly bounded to nucleus.
- Faraday first conducted experiment on parallel plates with dielectric and without dielectric and concluded that charge between plates with dielectric medium more than without dielectric medium.
- Therefore capacitance of capacitor increases with increase in charge.
- Dielectric helps in maintaining two large metal plates at very small seperation.
- Dielectric increases potential difference which capacitor can withstand without breakdown.
- Increases capacitance of capacitor.

BEHAVIOR OF DIELECTRIC



Dielectric are two types, they are polar and non-polar dielectrics.

BEHAVIOR OF DIELECTRIC

- In an atom positive and negative charge are equal in number.
- Let us assume all positive charges are concentrated at one point called as centre of gravity of positive charge.
- Let us assume all negative charges are concentrated at one point called as centre of gravity of negative charge.
- If centre of gravity of positive and negative charge are coincide they are called as non-polar dielectric otherwise polar dielectric.

POLARIZATION

If an piece if dielectric or insulator placed between the charges plates of condenser, then center of gravity of negative charges is concentrated towards positive plate and center of gravity of positives charges concentrated towards negative plate, this process of separation opposite charges is called a polarization.

Polarization is also defined as electric dipole moment per unit volume.

Let A be the area of cross section of dielectric,

l be the distance by with opposite charges are separated,

q total charge in the volume of dielectric

then polarization, P = dipole moment / volume

= q.l / A.l

= q / A ---------------------------------- 6

i.e the polarization numerically equal to surface charge density.

DIELECTRIC CONSTANT

Dielectric constant is defined as ratio of capacitance of capacitor with dielectric to the capacitance of capacitor without dielectric .

Capacitance of capacitor with dielectric has low potential(V_d) than the capacitance of capacitor without dielectric(V) .

$$K = V / V_d \text{ -- } 7$$

The polarization is directly proportional to the electric field intensity created between charges.

$$P \,\alpha\, E$$

$$P = K_e E$$

$$K_e = P / E = \text{electric susceptibility} \text{ -------------------------- } 8$$

Gauss Law In Dielectric

+q

No Dielectric

-q

Above figure indicates parallel plates arrangement with out dielectric medium.

From Gauss law we know that, $\phi = \int_s D' \, ds = q$

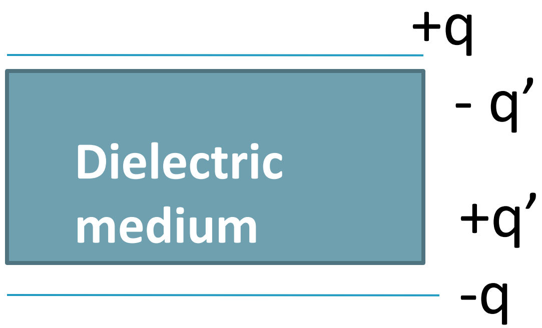
$$\phi = \int_s \varepsilon . E' \, ds = q$$

$$\phi = \int_s E' \, ds = q / \varepsilon$$

Applying Gauss law to above arrangement where charge on plate is q $\int E_0' \, ds = q / \varepsilon$

$$E_0' . \int_s ds = q / \varepsilon \rightarrow E_0' . \; A = q / \varepsilon \rightarrow E_0' = q / (\varepsilon A)$$

Gauss Law In Dielectric



Above figure indicates parallel plates with dielectric medium. The polarization takes place in the dielectric medium.

The charge on the +ve plate is +q and dielectric surface near to it is –q'. The total charge at that boundary is (q-q') .

Similarly other boundary side total charge is (q' – q).

Applying Gauss law to above figure, $\int_s E'\, ds = (q-q') / \varepsilon$

$E'. \int_s ds = (q-q') / \varepsilon \rightarrow E'. A = (q-q') / \varepsilon \rightarrow E' = (q-q') / (\varepsilon A)$

Gauss Law In Dielectric

$E' = (q / \varepsilon A) - (q' / \varepsilon A)$

But dielectric constant , $K = E_0 / E$

$E = E_0 / K$

Substituting E in above equation, $E_0 / K = (q / \varepsilon A) - (q' / \varepsilon A)$

But we know that , $E_0 = q / \varepsilon A$

Substituting E_0 in above equation, $(q / \varepsilon A K) = (q / \varepsilon A) - (q' / \varepsilon A)$

$$q / K = q - q'$$

$$q' = q - q / K = q\,(1 - (1 / K))$$

Substituting above equation in Gauss law expression,

Gauss Law In Dielectric

$$\int_s E' \, ds = (q - q') / \varepsilon$$

$$\int_s \varepsilon E' \, ds = (q - q')$$

$$\int_s \varepsilon E' \, ds = (q - q(1 - (1/K)))$$

$$= q - q + (q/K) = q/K$$

$$\int_s \varepsilon E' \, ds = q/K \quad \rightarrow \int D' \, ds = q/K$$

$$K. \int_s E' \, ds = q/\varepsilon \text{ (Gauss law in dielectric)}$$

$$K. E' \int_s ds = q/\varepsilon$$

$$K. E' A = q/\varepsilon \quad \rightarrow \quad E' = q/(K \varepsilon A)$$

Gauss Law In Dielectric

$$(q / \varepsilon A) = (q / \varepsilon A K) + (q' / \varepsilon A)$$

$$(q / A) = (q / A K) + (q' / A)$$

$$(q / A) = \varepsilon.(q / \varepsilon A K) + (q' / A)$$

$$(q / A) = \varepsilon.(E) + (P)$$

$$D = \varepsilon.(E) + (P)$$

From Gauss law, $\int D \, ds = q$

$$\int (\varepsilon.E + P) \, ds = q$$

Gauss Law In Dielectric

We know that, $E = (q / \varepsilon_0 K A)$ (for free space)

$$q / A = \varepsilon_0 K E$$

$$D = \varepsilon_0 K E \quad (D = q / A)$$

But we know that, $D = \varepsilon E$

Comparing above two equations, $\varepsilon = \varepsilon_0 K$

$$K = \varepsilon / \varepsilon_0 \text{ = relative permitivity.}$$

Substituting above D in Gauss law of dielectric,

$$D = \varepsilon_0.(E) + (P) \text{ (from Gauss Law)}$$

$$\varepsilon_0 K E = \varepsilon_0.(E) + (P)$$

$$P = \varepsilon_0 K E - \varepsilon_0.(E)$$

Gauss Law In Dielectric

$$P = \varepsilon_0 (k - 1) E$$

But we know polarization, $P = X_e . E$

Comparing above two equations, $X_e = \varepsilon_0 (k - 1)$

$$= \varepsilon_0 K - \varepsilon_0$$

$$= \varepsilon - \varepsilon_0$$

From which, $\varepsilon = \varepsilon_0 + X_e$

Now, $K = \varepsilon / \varepsilon_0 = (\varepsilon_0 + X_e) / \varepsilon_0 = 1 + (X_e / \varepsilon_0)$

CAPACITOR

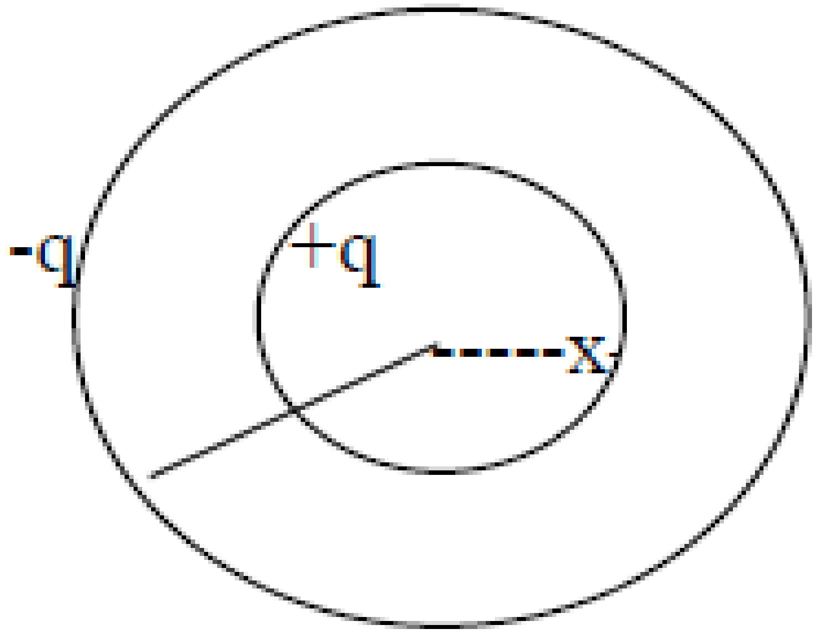
Basic capacitor element is formed by separated two parallel plates with some dielectric medium.

When some voltage is applied to such an element charge is formed between the plates, their by capacitance of capacitor is defined as charge Q developed between the plates when voltage V is applied.

$$C = Q / V \quad \text{-------------------------------------} \quad 9$$

The units of capacitance are Farads (F).

CAPACITANCE OF ISOLATED SPHERE



CAPACITANCE OF ISOLATED SPHERE

Let us consider an isolated sphere which is positively charges with radius x and negatively charges plate placed at infinite distance.

The electric flux density due to positive charge, $D = q / (4\pi x^2)$

Electric field intensity due to positive charge,

$E = Kq /. x^2$

Work done $w = - q \int E \, dl.$

$W = -q \int_{\infty}^{x} E \, dx$ with limits ∞ to x

$V = - \int_{\infty}^{x} E \, dx$ with limits ∞ to x

CAPACITANCE OF ISOLATED SPHERE

$$V = - \int_{\infty}^{x} Kq / x^2 \, dx$$

$$= -K.q \, . \, \frac{1}{-x} \quad \text{with limits } \infty \text{ and } x$$

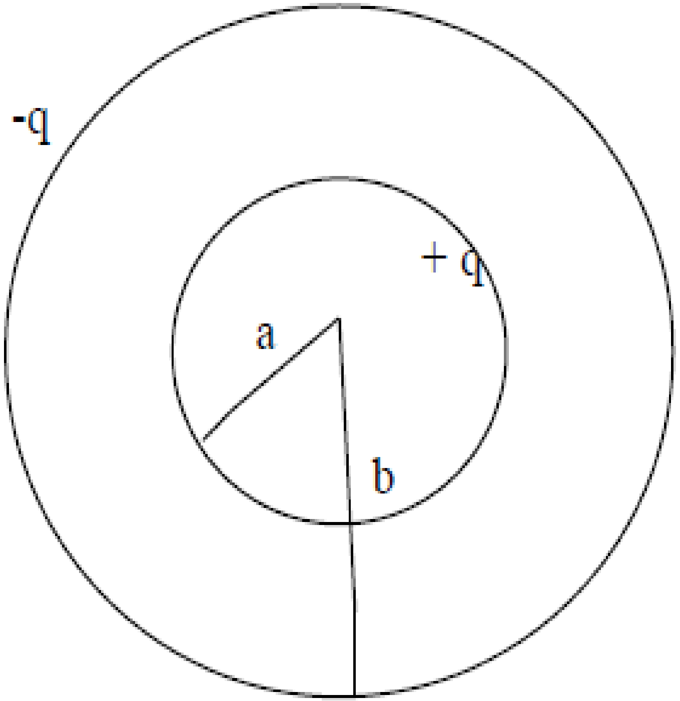
$$= - K.q \left[\frac{1}{-x} - \frac{1}{-\infty} \right] = K.q / x$$

But the capacitance is given charge per voltage, $C = q / V$

$$C = (x) / K = x / 1 / (4\pi\varepsilon)$$

$$C = 4\pi\varepsilon.x \ \ F$$

CAPACITANCE OF CONCENTRIC SPHERE



CAPACITANCE OF CONCENTRIC SPHERE

Let us consider two concentric spheres with radii 'a' and 'b' are separated by dielectric medium as sown in above figure .

The inner sphere is positively charged +q and outer sphere is negatively charged –q their it will go to earth.

Therefore the potential on inner sphere is , Va = K.q / a

$$V_a = q / (4\pi\varepsilon\, a)$$

Similarly the potential on outer sphere is , Vb = - K.q / b

$$V_b = - q / (4\pi\varepsilon\, b)$$

Hence total potential between plates, $V = V_a + V_b$

CAPACITANCE OF CONCENTRIC SPHERE

$$V = \frac{q}{4\pi\varepsilon\, a} - \frac{q}{4\pi\varepsilon\, b}$$

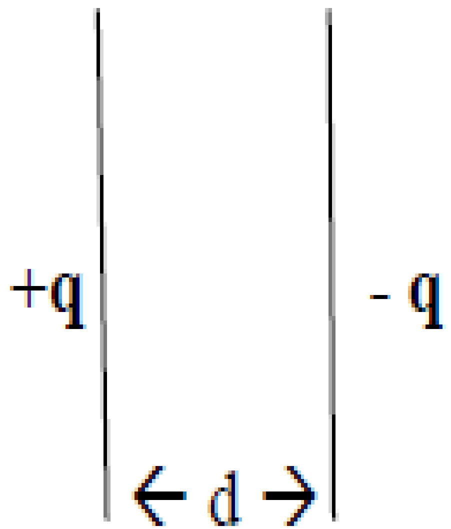
$$V = \frac{q}{4\pi\varepsilon} \left[(1/a) - (1/b) \right]$$

$$V = \frac{q.(b-a)}{4\pi\varepsilon.ab}$$

Therefore capacitance between plates , C = q / V

$$C = \frac{4\pi\varepsilon.ab}{(b-a)} \quad F$$

CAPACITANCE OF PARALLEL PLATES



CAPACITANCE OF PARALLEL PLATES

Let potential applied to these parallel plates is V their by forming charge q between them.

Electric flux density between plates, $D = q / A$

$\varepsilon E = q / A$

$E = q / \varepsilon.A,$

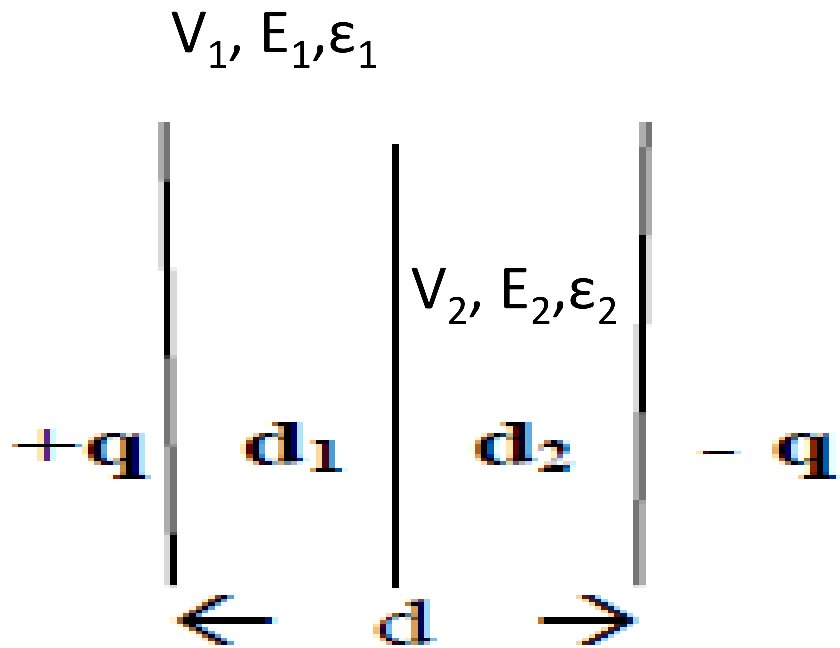
$V = E.d$

$V = q\,d / \varepsilon.A$

But the capacitance is given charge per voltage, $C = q / V_1$

$$C = \varepsilon.A / d \text{ ------------------------------- } 12$$

CAPACITANCE OF PARALLEL PLATES WTH MULTIPLE DIELECRICS



CAPACITANCE OF PARALLEL PLATES WTH MULTIPLE DIELECRICS

Let potential applied to first part is V_1 their by forming charge q between them.

Electric flux density between plates, $D_1 = q / A$

$$\varepsilon E_1 = q / A$$

$$E_1 = q / \varepsilon_1 .A,$$

$$V_1 = E_1 .d_1$$

$$V_1 = q\, d_1 / \varepsilon_1 .A$$

But the capacitance is given charge per voltage, $C = q / V$

$$C_1 = \varepsilon_1 .A / d_1 \quad \text{where } \varepsilon_1 = \varepsilon_0 \,.\, \varepsilon_{r1}$$

Let potential applied to first part is V_2 their by forming charge between them.

CAPACITANCE OF PARALLEL PLATES WTH MULTIPLE DIELECRICS

Electric flux density between plates, $D_2 = q / A$

$$\varepsilon E_2 = q / A$$

$$E_2 = q / \varepsilon_2 . A,$$

$$V_2 = E_2 . d_2$$

$$V_2 = q\, d_2 / \varepsilon_2 . A$$

But the capacitance is given charge per voltage, $C = q / V_2$

$$C_2 = \varepsilon_2 . A / d_2 \quad \text{where, } \varepsilon_2 = \varepsilon_0 . \varepsilon_{r2}$$

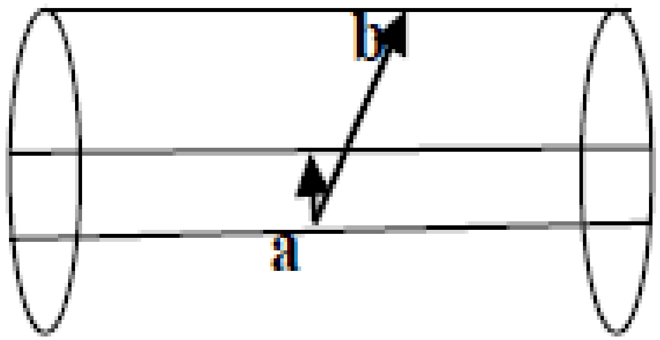
Hence total capacitance between plates with multiple dielectric mediums is ,

$$C = C_1 + C_2$$

$$= (\varepsilon_1 . A / d_1) + (\varepsilon_2 . A / d_2)$$

$$= \frac{A}{(d_1 / \varepsilon_1) + (d_2 / \varepsilon_2)} = \frac{\varepsilon_0 . A}{(d_1 / \varepsilon_{r1}) + (d_2 / \varepsilon_{r2})}$$

CAPACITANCE OF CO-AXIAL CABLE



ESTIMATE

Let us consider co-axial cable two isolated sphere with radius a and b from center of axis.The length of cable is , then line charge distribution $\rho_l = q / l$

the electric flux density generally in cable is , $D = \rho_l / 2\pi r$

therefore electric filed intensity , $E = \rho_l / 2\pi r\varepsilon$

the electric potential of the cable is , $V = -\int E dr$, with limits b to a

$$= -\int (\rho_l / 2\pi r\varepsilon)\, dr$$

$$= -(\rho_l / 2\pi\varepsilon) \int dr/r$$

$$= -(\rho_l / 2\pi\varepsilon) .\ln(r)$$

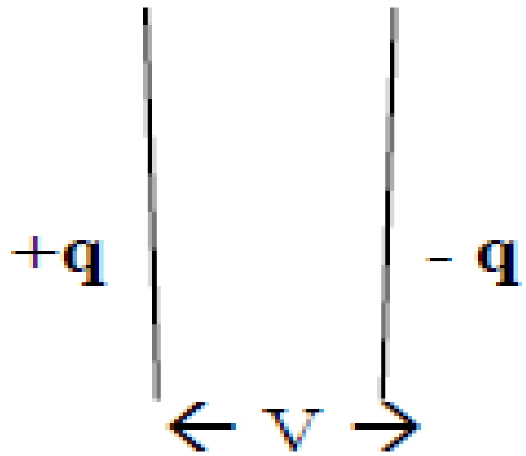
By applying limits, $V = -(\rho_l / 2\pi\varepsilon) .[\ln(a) - \ln(b)]$

$$V = (\rho_l / 2\pi\varepsilon) .\ln(b/a)$$

The capacitance of co-axial cable, $C = \rho_l / V$

$$C = 2\pi\varepsilon / \ln(b/a)$$

ENERGY STORED IN CAPACITOR



ENERGY STORED IN CAPACITOR

By the definition capacitance between plates is , $C = q / V$

Electric potential, $V = dw / dq$

$dw = V\, dq$

$dw = (q / C)\, dq$

integrating on both sides, $w = \int (q / C)\, dq$

$$w = q^2 / 2C$$
$$w = (CV)^2 / 2C$$
$$w = CV^2 / 2$$
$$w = q^2 / 2C$$
$$w = Vq / 2$$

ENERGY STORED IN CAPACITOR

W_d = energy stored / volume

$W_d = CV^2 / 2/ Ad$

$W_d = \varepsilon A\, V^2/d / 2.\, Ad$

$W_d = \varepsilon\, V^2/2d^2$

$W_d = \varepsilon E^2/ 2$

$W_d = DE/ 2$

From equation we can write,

$$dW = (DE/ 2)\ dV$$

integrating on both sides, energy stored

$$W = \int_v (DE/ 2)\ dV$$

CURRENT DENSITY

The flow of electrons from one end to other end constitutes current.
The rate of change of Charge is also defined as current.

$$i = q / t = dq / dt$$

the units of current is ampere.

CURRENT DENSITY:

If charge is distributed in the given area, then current density is defined as current constituted

In given area.

$$J = i / A \quad (A/mt^2)$$

$$J = di / ds$$

$$di = J .ds$$

integrating on both sides, $i = \int J .ds$

CONVECTION CURRENT DENSITY :

Let us consider a material with volume of charges (ρ_V) moving with drift velocity (V_d) , then

Convection Current density is defined as product volume of charges moving with drift velocity.

$$J = \rho_V x V_d$$

EQUATION OF CONTINUITY :

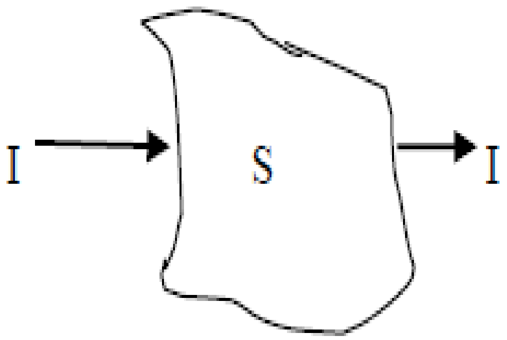
Let us an surface area through charges are moving in and out as shown in the figure

Let the charge q is moving through an area of S.

According law of conservation of charge, $[I]s = - dq / dt$

But current passing through area is , $[I]s = \int J d_s$

CURRENT DENSITY



EQUATION OF CONTINUITY

Total charge in the given volume is, $q = \int_v \rho_v \, ds$

From above three equations we can write,

$$\int J \, ds = -(d/dt). \int_v \rho_v \, dv$$

from the stokes theorem we can write,

$$\int J \, ds = \int_v \blacktriangledown J \, dv$$

by comparing equation 25and 26,

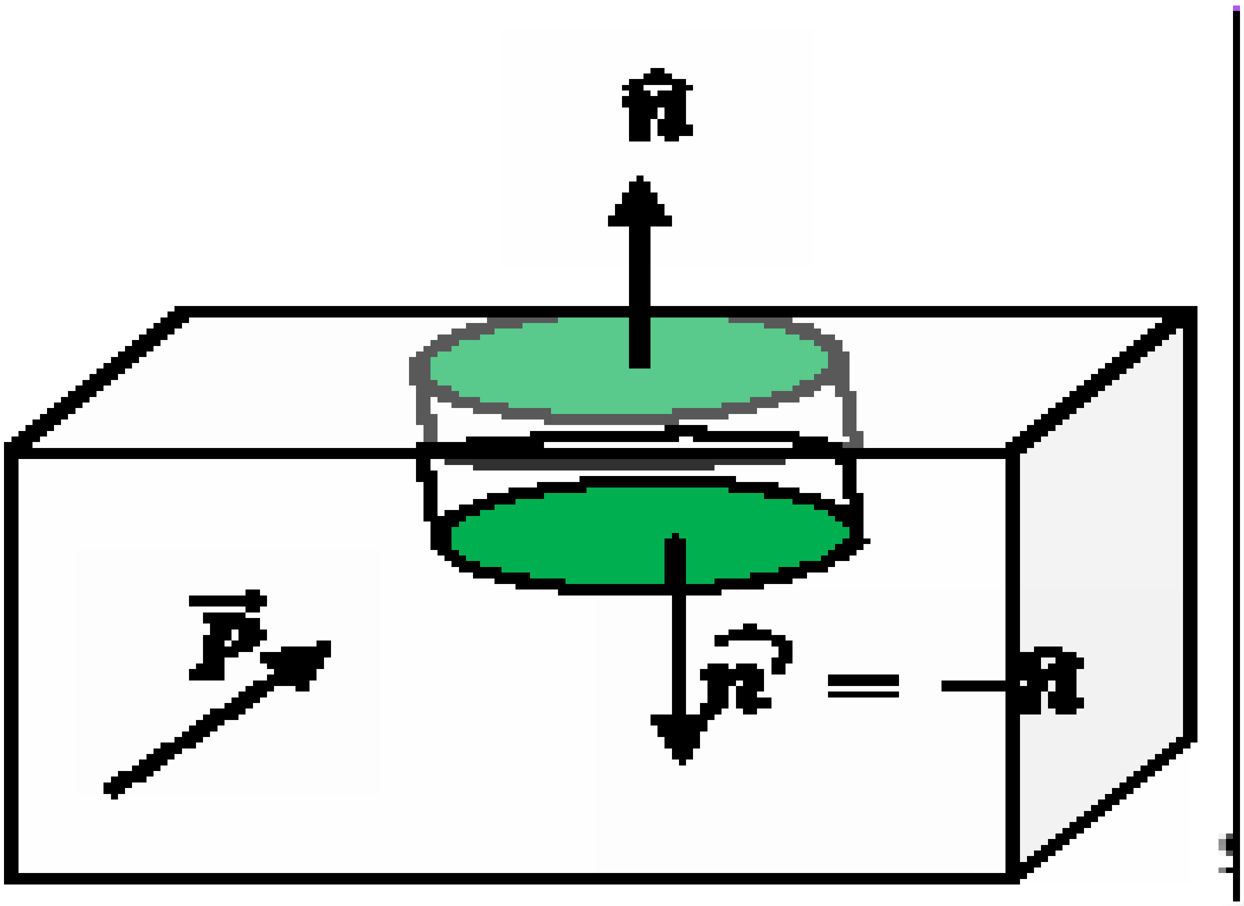
$$\int_v \blacktriangledown J \, dv = -(d/dt). \int_v \rho_v \, dv$$

$$\int_v \blacktriangledown J \, dv + (d/dt). \int_v \rho_v \, dv = 0$$

$$\int_v [\blacktriangledown J + d\rho_v \; / \; dt] \, dv = 0$$

equation called as equation is of continuity or maxwell's fifth equation.

Boundary Conditions Of Dielectrics



Boundary Conditions Of Dielectrics

We recall that in deriving the boundary condition On the surface of a conductor, we had taken a Gaussian Pillbox of infinitesimal height, half inside the conductor And half outside. The field inside the conductor being zero, the contribution to the flux came only from the top face of the pillbox, since the thickness of the pillbox being negligible the curved surface did not contribute. In the present case, there is a difference because the field inside is not zero. Further, one has to remember that at the interface, the normal is outward for the outside face and inward for the inside face.

Let us assume that there no free charges on the surface. The volume integral of the electric field over the pillbox is given by

Boundary Conditions Of Dielectrics

$$\int \nabla \cdot \vec{E} d^3r = \frac{1}{\epsilon_0} \int \rho_b d^3r$$
$$= -\frac{1}{\epsilon_0} \int \nabla \cdot \vec{P} d^3r$$

Using the divergence theorem on both sides,

$$\oint \vec{E} \cdot d\vec{S} = -\frac{1}{\epsilon_0} \oint \vec{P} \cdot d\vec{S}$$

The contribution to the surface integral on both sides, as mentioned before, are from the surfaces only with the directions of the normal being opposite, $\left(\vec{E}_{out} - \vec{E}_{in}\right) \cdot \hat{n} \Delta s = \frac{1}{\epsilon_0} \vec{P} \cdot \hat{n} \Delta S = \frac{\sigma_b \Delta S}{\epsilon_0}$

Where we have taken the direction of normal in this equation as going away from the surface (i.e. outward). Thus we have,

$$\left(\vec{E}_{out} - \vec{E}_{in}\right) \cdot \hat{n} = \frac{\sigma_b}{\epsilon_0}$$

Boundary Conditions Of Dielectrics

This is precisely the relationship that we had earlier for the boundary condition for a conductor except that the surface charge density was due to the free charges. Adding the free charge density, the discontinuity in the normal component of the electric field is given by

$$(\vec{E}_{out} - \vec{E}_{in}) \cdot \hat{n} = \frac{\sigma_f + \sigma_b}{\epsilon_0}$$

We can obtain the discontinuity in the normal component of the displacement vector from

$$D_n = \epsilon_0 E_n + P_n = \epsilon_0 E_n + \sigma_b$$

$$(\vec{D}_{out} - \vec{D}_{in}) \cdot \hat{n} = \epsilon_0 (\vec{E}_{out} - \vec{E}_{in}) \cdot \hat{n} - \sigma_b$$

the reason for the minus sign is that outside the material . Thus

$$(\vec{D}_{out} - \vec{D}_{in}) \cdot \hat{n} = \sigma_f$$

Boundary Conditions Of Dielectrics

Thus, if there are no free surface charges we have the normal component of to be continuous,

$$D_{1n} = D_{2n}$$

The tangential components can be shown to be continuous by use of Stoke's theorem. We take a rectangular contour across the surface.

$$E_{1t} = E_{2t}$$

UNIT-III
MAGNETO-STATICS

MODULE III - SYLLABUS

Biot-Savart's law, magnetic field intensity, magnetic field intensity due to a straight current carrying filament, magnetic field intensity due to circular, square and solenoid current carrying wire, relation between magnetic flux, magnetic flux density and magnetic field intensity, Maxwell's second equation, div(B)=0.

Magnetic field intensity due to an infinite sheet of current and a long current carrying filament, point form of Ampere's circuital law, Maxwell's third equation, Curl (H)=Jc, field due to a circular loop, rectangular and square loops.

COURSE OUTCOMES MAPPED WITH MODULE III

CO	Course Outcomes	Blooms Taxonomy
CO 6	Make use of Biot-Savart law and Ampere circuital law for obtaining magnetic field intensity due to circular, square, rectangular and solenoid current carrying wire.	Apply

PROGRAM OUTCOMES MAPPED WITH MODULE III

PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
PO 3	**Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

MAPPING OF COs AND POS FOR MODULE III

COs	PROGRAM OUTCOMES											
	1	2	3	4	5	6	7	8	9	10	11	12
CO6	√	√	√									

INTRODUCTION

Magneto-statics is the study of magnetic field developed by the constant current through the coil Or due to permanent magnets.

The behavior of constant magnetic field is studied by using two basic laws, they are

→ Bi-Savart's law

→ Ampere's circutal law.

MAGNETIC FIELD:

Let us consider a constant current I is passing through coil shown above which develops constant Flux surrounding the coil their by forming north and south poles. This formation of magnetic from North pole to south pole is called as magnetic field. The direction of magnetic flux in an coil is Given by right hand thumb rule.

Right hand thumb rule says that if four fingers of hand folded such that they show direction of flux. Then thumb indicates direction of flux and other fingers how the coil is wounded (clock wise or anti-

clock wise) .The means to develop the magnetic field is permanent magnets and above is said to be electro- magnets. Permanent magnetic posses the property of magnetism by nature, in order to develop strong magnetic one must choose permanent magnets with high cohesive force. Permanent magnet has disadvantage of ageing and getting rusted. This disadvantage of permanent is overcome by electro-magnets.

DEFINITIONS

Magnetostatics is the study of magnetic fields in systems where the currents are steady (not changing with time). It is the magnetic analogue of electrostatics, where the charges are stationary. Like in

electro-statics in magneto-statics we are going to deal with magnetic field intensity, magnetic flux density using Bio-Savart's law and Ampere's circuital law.

Some of the important terms used to study characteristics of

Magneto-statics are

→ Magnetic flux.

→ Magnetic flux density.

→ Magnetic field intensity.

→ Intensity of magnetization.

→ Magnetic susceptibility.

→ Permeability of core.

→ Reluctance of core.

MAGNETIC FLUX DENSITY

Magnetic Flux Density

magnetic flux density is defined as flux per unit area,

$$B = d\phi / ds \quad (\text{Wb/mt}^2 \text{ or Tesla})$$

$$d\phi = B\, ds$$

by integrating on both sides we can determine total magnetic flux in area,

$$\phi = \int B\, ds$$

MAGNETIC FIELD INTENSITY

The force experienced by coil when some current passes through it is magnetic field Intensity.

Mathematically magnetic field intensity is givens as,

$$H = \text{magnetic force} / \text{length}$$

$$\text{Magnetic force} = NI$$

$$\text{Length} = l$$

Therefore magnetic field intensity, $H = NI / l$ (AT/mt)

MAGNETIC PERMEABILTY:

Permeability is the inherent property of core which helps in sustaining flux in the core.

Mathematically permeability is given as, $\mu = B/H$

From equation 30 the relation between flux density and intensity is ,

$$B = \mu H$$

Where $\mu = \mu_0 \mu_r$

μ_0 = absolute permeability = $4\pi x 10^{-7}$ H/mt

μ_r = relative permeability

varies from core to core

INTENSITY OF MAGNETIZATION

When a magnetic substance is placed in a magnetic field it experiences magnetic momentum.

The magnetic momentum per unit volume of substance is intensity omagnetization.

$$I = M / V$$

$M = m.l$ (m- pole strength of bar, l – length)

$$V = A.l$$

intensity of magnetization, $I = m.l / A.l$

$$I = m / A$$

MAGNETIC SUSCEPTIBILITY

The ratio intensity of magnetization to the magnetic field intensity is called as Magnetic Susceptibility $K = I / H$.

Total flux density, B = B due to magnetic field + B due to intensity of magnetization of bar

$$B = \mu_0 H + I$$

But we know that,

$$\mu = B / H$$

$$= (\mu_0 H + I) / H$$

$$= \mu_0 .+ (I/H$$

$$\mu_0 \mu_r = \mu_0 .+ K = 1 + K / \mu_0$$

$\mu_r > 1$, paramagnetic materials

$\mu_r < 1$, diamagnetic materials

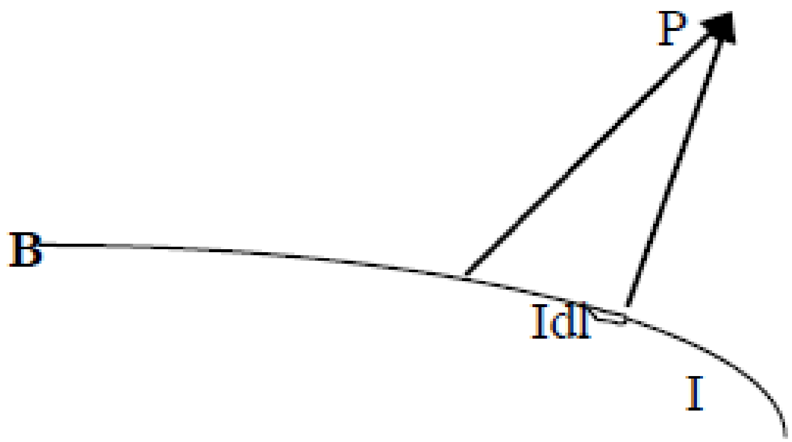
$\mu_r = 0$, non-magnetic materials

BIO-SAVART'S LAW

Bio and savart are two scientists who conducted experiments on current carrying conductor To determine magnetic flux density(B) at any point surrounding that conductor. Their Conclusion is named as "Biot-Savart's Law".

Let us consider an conductor carrying current I, which develops magnetic flux density B surrounding It. Here Idl is called as current element. To find total electric field intensity conductor is divided into Number of current elements.

BIO-SAVART'S LAW



BIO-SAVART'S LAW

The magnetic field intensity due to current element Idl is dH at point P.

According Bio-Savart's law

$$dH \propto Idl \text{ (current element)}$$

$dH \propto \sin\Theta$ (angle between current element and length joining point)

$dH \propto 1 / r^2$ (square of distance between current element and point)

by combining above three,

$$dH \propto Idl \,.\, \sin\Theta / r^2$$

by removing proportionality,

$$dH = Idl \,.\, \sin\Theta / 4\pi r^2$$

BIO-SAVART'S LAW

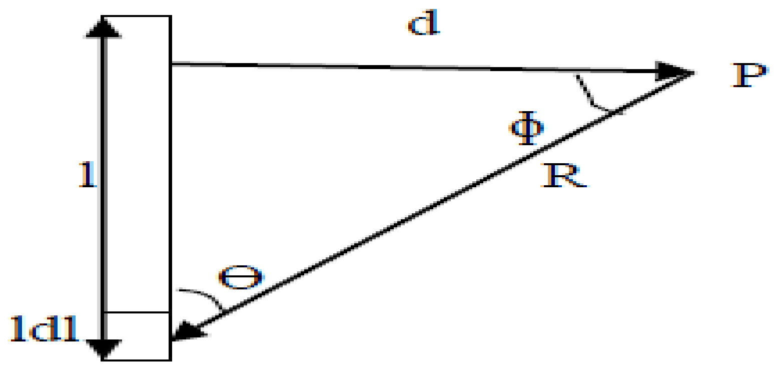
total magnetic field intensity at point P,

$$H = \int Idl \,.\, \sin\Theta / 4\pi r^2$$

therefore total flux density at point P, $B = \mu H$

$$B = \mu \int Idl \,.\, \sin\Theta / 4\pi r^2$$

FIELD INTENSITY DUE TO STRAIGHT CONDUCTOR



FIELD INTENSITY DUE TO STRAIGHT CONDUCTOR

Let us consider a straight conductor of length l, a test point P at which electric field intensity is to be determined at a distance of d from conductor. Assume current element with a distance of R to

From Bio-Savart's law magnetic field intensity at test point P due to current element Idl is ,

$$dH = Idl \,.\, \sin\Theta \,.\, / \, 4\pi R^2$$

from above right angle triangle, $\Theta + \phi = 90^0$

using equation a and b,

$$dH = Idl \,.\, \cos\phi \,.\, / \, 4\pi R^2$$

FIELD INTENSITY DUE TO STRAIGHT CONDUCTOR

the unit vector a' , indicates the direction H at point P.

$$a' = R' / R$$

from above right angle triangle, $R = \sqrt{l^2 + d^2}$

$$\cos\phi = d / \sqrt{l^2 + d^2}$$

$$\tan\phi = l / d$$

$$l = d.\tan\phi$$

$$dl = d\sec^2\phi\, d\phi$$

substituting ,

$$dH = Idl . \cos\phi .d . R' / 4\pi (l^2 + d^2)^2$$

FIELD INTENSITY DUE TO STRAUGHT CONDUCTOR

$$H = \int Idl \,.\, \cos\phi \,.d \,.\, / \, 4\pi \, (l^2 + d^2)^{3/2}$$

$$H = I/(4\Pi d^2\ ) \int dl \,/\, (l^2 / d^2 + 1)^{3/2}$$

$$H = I/(4\Pi d^2\ ) \int dl \,/\, (\tan^2\phi + 1)^{3/2}$$

Substituting equation h in above equation is ,

$$H = I/(4\Pi d^2\ ). \int d \sec^2\phi \, d\phi \,/\, (\sec^2\phi)^{3/2}$$

$$H = I/(4\Pi d^2\ ) .\int d \sec^2\phi \, d\phi \,/\, (\sec^3\phi)$$

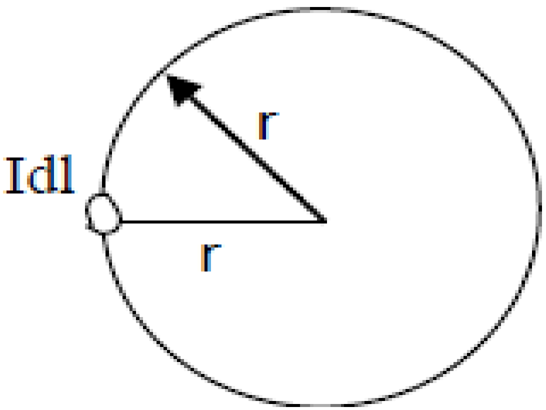
$$H = I/(4\Pi d\ ) .\int \cos\phi \, d\phi$$

$$H = I/(4\Pi d^2\ ) .\sin\phi$$

For straight line of infinite length, ϕ varies between $-\pi/2$ to $\pi/2$

Substituting above limits in equation , $H = I/(2\Pi d\ )$

FIELD INTENSITY DUE TO CIRCULAR CONDUCTOR



FIELD INTENSITY DUE TO CIRCULAR CONDUCTOR

Let us consider circular conductor with radius r,

magnetic field intensity at the center of circular conductor is,

from above figure we can say that idl and center are at 90^0

using Bio-Savart's law magnetic field intensity at center point P due to current element Idl is,

$$dH = idl \sin 90 / 4\pi r^2$$

$$dH = idl / 4\pi r^2$$

FIELD INTENSITY DUE TO CIRCULAR CONDUCTOR

integrating on both sides, $H = \int idl \ / \ 4\pi r^2$

$$H = i \int dl \ / \ 4\pi r^2 \qquad (\int dl = 2\pi r)$$

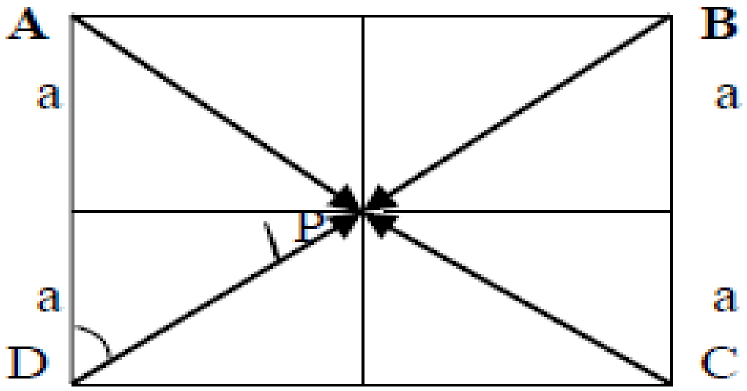
$$H = i \ 2\pi r \ / \ 4\pi r^2$$

$$H = i \ / \ 2r$$

Magnetic field intensity at the center of circular conductor with N number of turns is,

$$H = Ni \ / \ 2r$$

FIELD INTENSITY DUE TO SQUARE CONDUCTOR



FIELD INTENSITY DUE TO SQUARE CONDUCTOR

From the above figure we can say that each side AB,BC,CD,DA has magnetic field intensity at the center Of square conductor.

In every right angle triangle angle between current element and center is 45^0.

The total magnetic field intensity at the center of square due to all corners using Bio-Savart's law

Because of any one side,

$$H = (I / 4\pi a) \times [\sin 45^0 + \sin 45^0]$$

FIELD INTENSITY DUE TO SQUARE CONDUCTOR

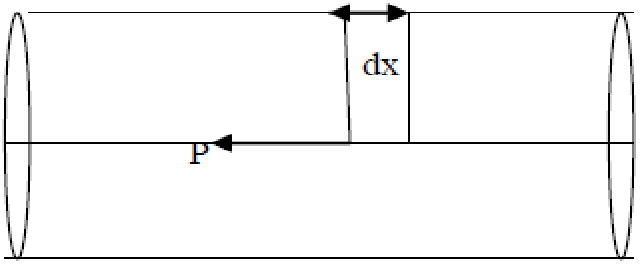
Using all sides,

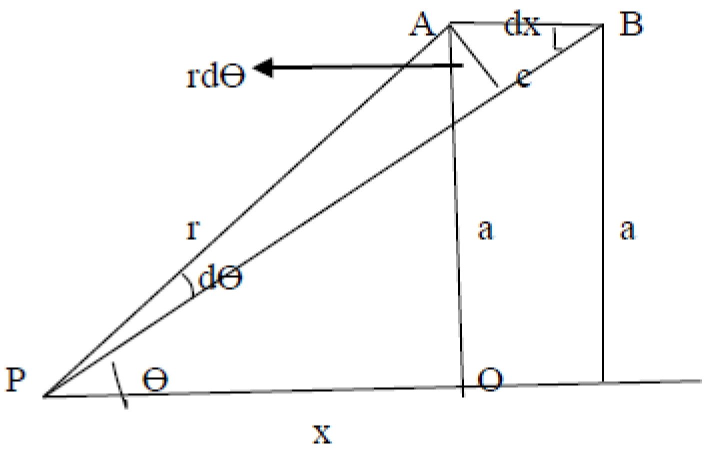
$$H = 4(I / 4\pi a) \times [\sin 45^0 + \sin 45^0]$$

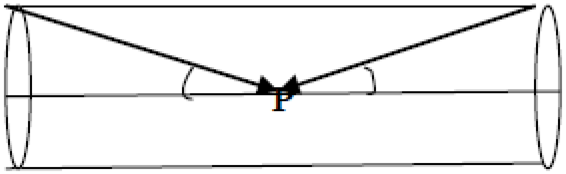
$$H = (I / \pi a) \times [2 / \sqrt{2}]$$

$$H = (\sqrt{2}.I / \pi a)$$

FIELD INTENSITY DUE TO SOLENOID CONDUCTOR







FIELD INTENSITY DUE TO SOLENOID CONDUCTOR

The construction of solenoid is same as coil wounded on a cylinder , let us take take cylinder As reference and derive expression for H due to solenoid. The solenoid with length l, number of turns N allowing an current of I is shown in below figure,

Assume a small length dx, with total turns ndx in it , let us derive what is the magnetic field intensity

Due to dx on P, their by total H at P.

total number of turns = N

total length = l

number of turns per unit length, $n = N / l$

x be the distance of the point,

the magnetic field intensity due to length dx on P is ,

FIELD INTENSITY DUE TO SOLENOID CONDUCTOR

$$dH = (Ia^2 / 2r^3)\, ndx$$

from figure , $r = \sqrt{a^2 + x^2}$, substituting r in dH.

$$dH = (Ia^2 / 2\,(a^2 + x^2)^{3/2})\, ndx$$

from above right angle triangles, $d\theta <<< \theta$,

hence $\sin d\theta = d\theta$

$$\sin \theta = r\, d\theta / dx$$

$$\sin \theta = a / r$$

substituting above deduction in dH,

$$dH = (Ia^2\, r.d\theta / \sin \theta / 2r^3)\, n$$

$$dH = I.n.\, \sin \theta.\, d\theta / 2$$

FIELD INTENSITY DUE TO SOLENOID CONDUCTOR

if seen from end points of solenoid the magnetic field intensity at P is Here from one end to other end angle varies from 0 to 2π, substituting above and integrating equation a

$$\int dH = \int I.n. \sin \Theta. d\Theta / 2$$

$$H = - I.n.\cos \Theta. / 2$$

$$H = -(I.n/2) [\cos 2\pi - \cos 0]$$

$$H = I.n = NI/l$$

if seen from end point of solenoid the magnetic field intensity at P at same end point,

then the limits varies between 0 to π/2

substituting above limits in b

$$H = -(I.n/2) [\cos\pi/2 - \cos 0]$$

$$H = n.I/2 = N.I/ 2l$$

GAUSS LAW IN MAGNETIC FIELDS

From the guass law we can write magnetic flux in the given surface is surface integral of magnetic flux density.

$$\Psi = \int B.ds$$

But total flux density in closed surface is always zero,

$$\Psi = \int B.ds = 0$$

By applying divergence theorem we can write,

$$\int B.ds = \int_v \blacktriangledown\ B.dv = 0$$

hence we can write , $\blacktriangledown$ B = 0, is Maxwell's second equation

AMPERE CIRCUITAL LAW

The ampere circuital law states line integral magnetic filed intensity around any closed path is equal to total current enclosed in that path.

$$\int H\,dl = I$$

Ampere's law is analogous to gauss law electro-statics.

AMPERE CIRCUITAL LAW

Applications of Ampere’s law :

→ The magnetic field intensity in the surrounding closed path is always at tangential at Each and every point on it.

→ At each every point on the closed path magnetic field intensity has the same value.

AMPERE CIRCUITAL LAW APPLICATION

From the ampere circuital law we know that,

$$\int H\, dl = I$$

but current can be written as,

$$\int J\, ds = I$$

equating above two equations,

$$\int H\, dl = \int J\, ds$$

from stokes theorem,

$$\int H\, dl = \int \nabla \times H\, ds$$

by combining equation a and b,

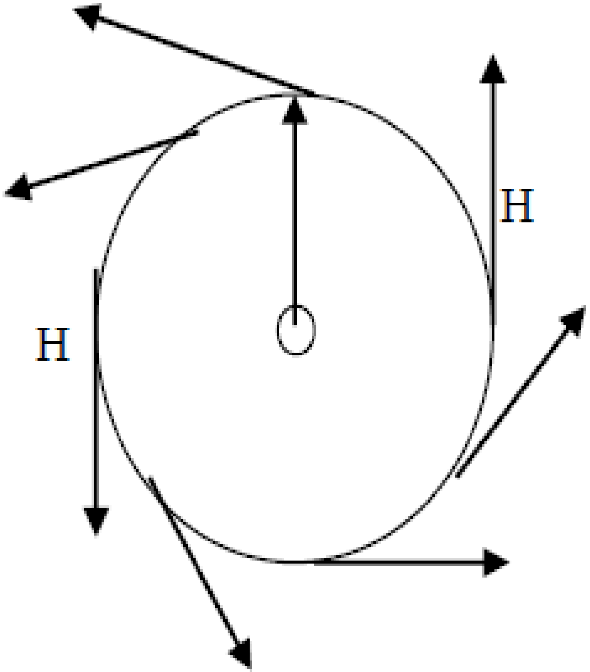
$$\int \nabla \times H\, ds = \int J\, ds$$

by comparing on both sides,

$\nabla \times H = J$, $\nabla \times H =$ curl of H

Equation 40 is called as differential, integral or point form of ampere's law and also calledas Maxwell's Third Equation

AMPERE LAW FOR CIRCULAR CONDUCTOR



AMPERE LAW FOR CIRCULAR CONDUCTOR

Let us consider a straight conductor as shown in figure with closed path of magnetic field Intensity surrounding it with radius of r.

From ampere's circuital law we can write magnetic field intensity in closed path,

$$\int H\, dl = I$$

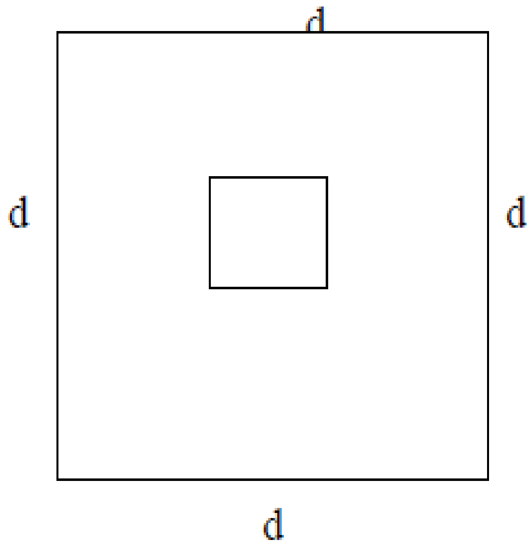
but we can write, $\int H\, dl = H \int dl$

$$= H\, 2\pi r$$

Equating a and b, $H\, 2\pi r = I$

$$H = I / 2\pi r$$

AMPERE LAW FOR SQUARE CONDUCTOR



AMPERE'S CIRCUITAL LAW

let us consider a square sheet as shown above with surrounding current path of side d.

according to Ampere's law ,

$$\int H \, dl = I$$

where $\int dl$ indicates the mean length closed path,

$$\int dl = 4d$$

their by ,

$$H \int dl = I$$

$$H.4d = I$$

$$H = I/4d.$$

AMPERE LAW

The ampere circuital law states line integral magnetic filed intensity around any closed path is equal to total current enclosed in that path.

$$\int H\, dl = I$$

Ampere’s law is analogous to gauss law electro-statics.

UNIT-IV
MAGNETIC FORCE AND MAGNETIC POTENTIAL

MODULE IV - SYLLABUS

Moving charges in a magnetic field, Lorentz force equation, force on a current element in a magnetic field, force on a straight and a long current carrying conductor in a magnetic field, force between two straight long and parallel current carrying conductors, magnetic dipole and dipole moment, a differential current loop as a magnetic dipole, torque on a current loop placed in a magnetic field;

Vector magnetic potential and its properties, vector magnetic potential due to simple configurations, Poisson's equations, self and mutual inductance, Neumann's formula, determination of self-inductance of a solenoid, toroid and determination of mutual inductance between a straight long wire and a square loop of wire in the same plane, energy stored and density in a magnetic field, characteristics and applications of permanent magnets.

COURSE OUTCOMES MAPPED WITH MODULE IV

CO	Course Outcomes	Blooms Taxonomy
CO 7	Predict the force due to moving charge in the magnetic field of various configuration for developing principles of electrical machines.	Understand
CO 8	Signify the magnetic dipole , dipole moment for obtaining torque due to magnetic dipole helps in structuring electrical devices..	Understand
CO 9	Calculate the self inductance and mutual inductance for different configurations of wires and energy stored in the coil.	Understand

PROGRAM OUTCOMES MAPPED WITH MODULE IV

PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
PO 3	**Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

MAPPING OF COs AND POS FOR MODULE IV

COs	**PROGRAM OUTCOMES**											
	1	2	3	4	5	6	7	8	9	10	11	12
CO7	√	√										
CO8	√	√										
CO9	√	√	√									

FORCE ON POINT CHARGE

When an charge Q is with velocity is placed in the magnetic field of density , then it experiences force called as magnetic force.

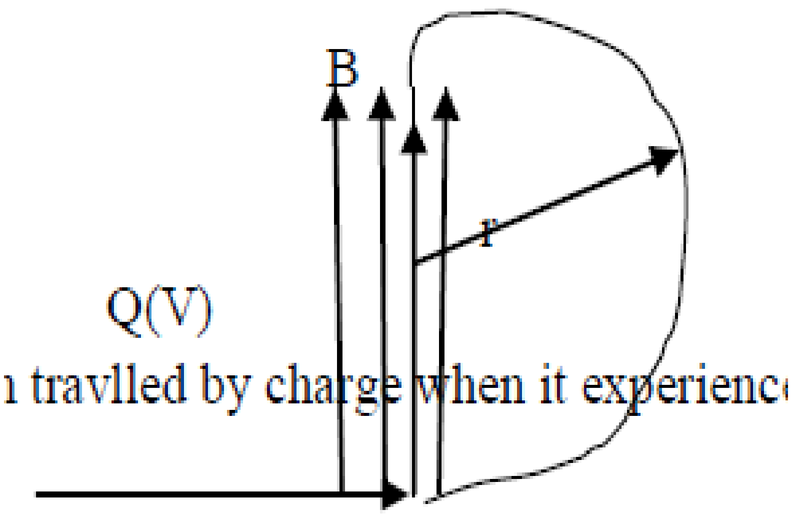
$$F_m = Q(VXB)$$

$$= QVB \sin\Theta \, a_f$$

V is parallel to B then Ө= 0, therefore sinӨ = 0, hence always velocity direction and flux density

Direction must be normal to each other.

FORCE ON POINT CHARGE



LIMITATIONS ON MOVING CHARGE

The limitations of moving charge in the existing magnetic field,

→ If the velocity of charge in the magnetic field is zero then force experienced also zero.

→ If the velocity direction and magnetic field direction are parallel to each other then force

experienced is zero.

To say that moving charge in the magnetic field experiences force velocity and field must be normal to each other.

From the above discussion the force experienced by moving charge is ,

$$Fm = QVB.$$

Similarly we can also write force experienced by moving charge due to its mass is ,

$$Fm = ma.$$

LIMITATIONS ON MOVING CHARGE

$$Fm = mV^2/r$$

By equating both forces, $QVB = mV^2/r$

$$r = mV / QB$$

time taken to complete one revolution in field is ,

$$T = 2\pi r / V$$

$$= 2\pi m / QB$$

Hence frequency of charge in field is ,

$$F = 1/ T$$

$= QB / 2\pi m$, as this expression of frequency is independent Of velocity it is called as cyclotron.

Lorentz Force equation

We know that the force acquire by point charge when kept in the static electric field is,

$$F_e = Q.E$$

The force experienced by moving charge in the magnetic field is ,

$$F_m = Qm\ (\ V \times B)$$

The total force on the charge in the presence of both field is,

$$F = F_e + F_m$$

$$= Q.E + Q\ (\ V \times B)$$

$$= Q\ (\ E + \ (\ V \times B)\ )$$

Equation is called as Lorentz force equation

FORCE ON CURRENT ELEMENT DUE TO MAGNETIC FIELD

Let us a long conductor of length l which is partitioned into number parts allowing current Of I. each part of conductor is of length dl, therefore individual part is represented with Idl called as current element.Force due to current element at any point We know that convection current density is , $J = \rho_v . V$

The current elements are ,

$$J\,dv = K\,ds = I\,dl$$

Using above two equations, $J\,dl = \rho_v\,dv = Q.V$

Also current element, $dl = (dQ/\,dt).dl$

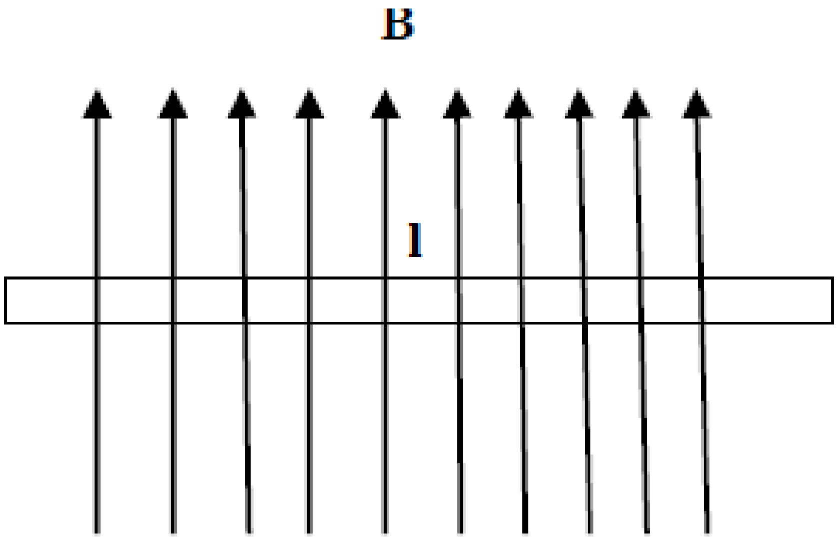
$$= dQ.\ V$$

The force experienced by moving charge we know as ,

$$dF_m = Q\,(V \times B)$$
$$= i.dl \times B$$

Integrating on both sides we can determine force due current element,

$$F_m = \int i.dl \times B$$
$$F_m = \int K.ds \times B$$
$$F_m = \int \rho_v.dv \times B$$



FORCE ON STRAIGHT CONDUCTOR PLACED IN EXISTING MAGNETIC FIELD

Let us consider a straight conductor placed in the magnetic field as shown in the figure, Of length l, allowing current of I, hence current element if Idl,

The velocity of charges in the given length of conductor is .

The force experienced by current element is ,

$$dF_m = dQ\,(V \times B)$$

$$= dQ\,(dl/dt \times B)$$

$$= (i.dl \times B)$$

$$F_m = I\,(l \times B)$$

$$= Bil \sin\theta$$

MAGNETIC DIPOLE AND ITS MOMENTUM

Magnetic dipole is formed when two opposite magnetic charges are separated by distance l.

$-Q_m$ -------------l------------- $+Q_m$

The line joining two charges is termed as axis of dipole. Direction magnetic dipole is from $-Q_m$ to $+Q_m$

In other words a bar magnet with pole strength Q_m and l has , magnetic dipole moment, $m = Q_m l$.

Let us consider a bar conductor allowing current I their forming loop of area A, magnet poles formed

As shown in the figure.

Magnetic dipole moment , $m = IA$

Numerically both dipole moment must be same, $Q_m l = IA$

Magnetization

If their exist an conductor consisting of number of dipoles in its volume , then magnet dipole Moment per unit volume is called as magnetization.

$$M = m / V$$
$$= Q_m .l / A.l$$
$$= Q_m / A$$

Magnetic susceptibility

When the magnetic field is applied to an material the , Total magnetic field intensity is ,

MAGNETIZATION AND SUSCEPTIBILITY

When the magnetic field is applied to an material the ,

Total magnetic field intensity is ,

$$B = \mu_0.H + \mu_0.M$$

$$= \mu_0 \mu_r H$$

Therefore,

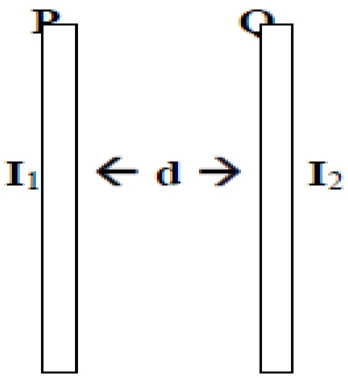
$$\mu_0 \mu_r .H = \mu_0 .H + \mu_0 .M$$

$$M = (\mu_r - 1) H$$

$$M = X_m .H$$

$X_m = (\mu_r - 1)$, is called as magnetic susceptibility, $X_m = m / H$

MAGNETIC FORCE BETWEEN TWO CONDUCTORS



MAGNETIC FORCE BETWEEN TWO CONDUCTORS

As shown above,

The magnetic field intensity due conductor P on Q is,

$$H = I_1 / 2\Pi d$$

The magnetic flux density due conductor P on Q is,

$$B = \mu_0 I_1 / 2\Pi d$$

Hence forced experienced by conductor Q due to field of P is,

$$F1 = B I_2 l$$

$$= \mu_0 I_1 I_2 l / 2\Pi d$$

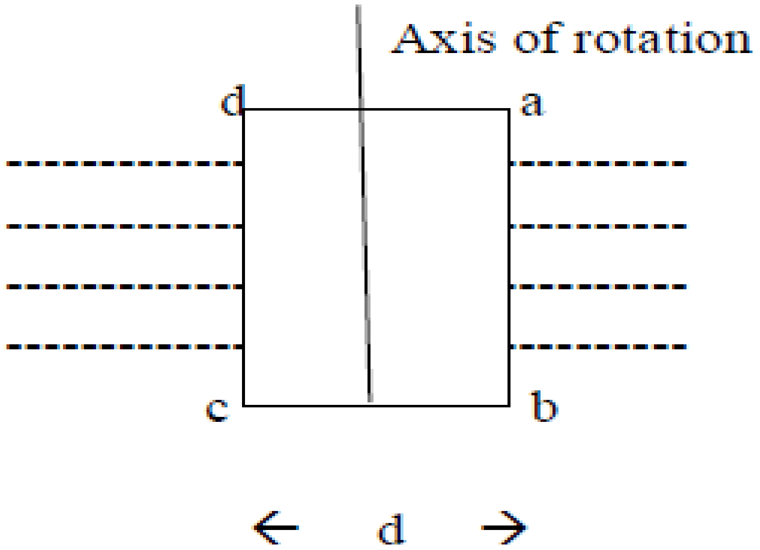
Similarly force experienced by P due to conductor Q is ,

$$F2 = \mu_0 I_1 I_2 l / 2\Pi d$$

Hence force per unit length of conductor is ,

$$(F / l) = \mu_0 I_1 I_2 / 2\Pi d$$

MAGNETIC FORCE ON SQUARECONDUCTORS



MAGNETIC FORCE ON SQUARECONDUCTORS

Let us a consider sheet of side abcd placed in the magnetic field , the side ab experiences the force into the page and side cd out of the page. Angles made by sheet with magnetic field are α and β.

the total torque experienced by sheet due to dipole is ,

$$
\begin{aligned}
T &= 2 \text{ x torque on each side} \\
&= 2 \text{ x force x distance from axis of rotation} \\
&= 2 \times F \times d/2 \\
&= 2 \times BIl \cos\beta \times d/2 \\
&= BIA \cos\beta \\
&= mB \cos\beta \quad \text{or} \quad mB \sin\alpha
\end{aligned}
$$

Therefore torque vector , $\vec{T} = \vec{m} \times \vec{B}$ -- 49

VECTOR AND SCALAR MAGNETIC POTENTIAL

Similarly in the magneto-statics , $H = -\nabla V_m$

V_m – vector magnetic potential

Applying curl on both sides of H, $\nabla \times H = -\nabla \times (\nabla V_m)$

But curl of divergence of any vector is zero, $\nabla \times H = 0$

We can also write , $\nabla \times H = J$

From the above two equations we can write , $J = 0.$

This is possible only in the case constant magnetic field.

VECTOR AND SCALAR MAGNETIC POTENTIAL

from the electro-statics we know that, $\int E \, dl = V$

Similarly in the magneto-statics , $\int H \, dl = V_m$

Ampere circuital law says that, $\int H \, dl = I$

Comparing last two equations, $V_m = I$

Hence the units of scalar magnetic potential is Amperes.

VECTOR AND SCALAR MAGNETIC POTENTIAL

We know that divergence magnetic flux density over uniform closed surface is always zero.

$$\nabla B = 0$$

Also divergence of curl of vector is always zero.

$$\nabla .(\nabla \times A) = 0$$

By comparing above two equations,

$$B = \nabla \times A$$

$$\mu H = \nabla \times A$$

$$H = (\nabla \times A) / \mu$$

VECTOR AND SCALAR MAGNETIC POTENTIAL

Applying curl on both sides, $\nabla \times H = \nabla \times (\nabla \times A) / \mu = J$

But, $\nabla \times (\nabla \times A) = \nabla . (\nabla . A) - \nabla^2 A = \mu J$

For time invariant fields divergence of vector is zero, hence above can be written as

$$- \nabla^2 A = \mu J$$

$$\nabla^2 A = - \mu J$$

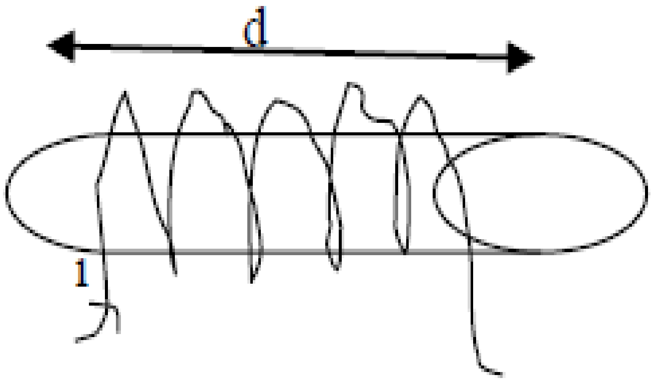
Form the electro-statics we know that, $dv = dq/ 4\pi\varepsilon$

Similarly in the magneto-statics , $dA= \mu idl/ 4\pi r$

Integrating on both sides,

$A = \int \mu idl/ 4\pi r$, A- vector magnetic potential

INDUCATNCE OF SOLENOID



INDUCATNCE OF SOLENOID

N – total turns of solenoid coil

n – number of turns per unit length

magnetic filed density inside solenoid is , $B = \mu_0\, n.i.$

total flux linking with coil is $\phi = N\,B\,A$

$= \mu_0\, n\, l.i.A\, .n$

$= \mu_0\, n^2.i.A\, .l$

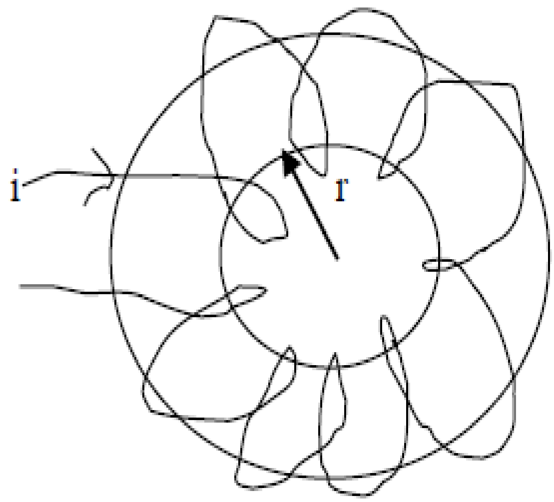
Self inductance is the property of coil which is responsible for emf induced in it,

$$L = N\,\phi / i$$

$$= \mu_0\, n^2.i.A\, .l / i$$

$$= \mu_0\, N^2 A / l \;\; H$$

INDUCATNCE OF TOROID



INDUCATNCE OF TOROID

Let us a toroid on which a coil N turns is wounded allowing an current of i A.

Let r be the mean radius of the toroid.

Magnetic flux density in the toroid, $B = \mu_0 \, Ni / l$

Where , $l = 2\pi r$

$B = \mu_0 \, Ni / 2\pi r$

Total flux linkage with toroid is , $\phi = NBA$

INDUCATNCE OF TOROID

$$= (N\, \mu_0\, Ni / 2\pi r)\,.\, A$$

But, area $A = \pi R^2$

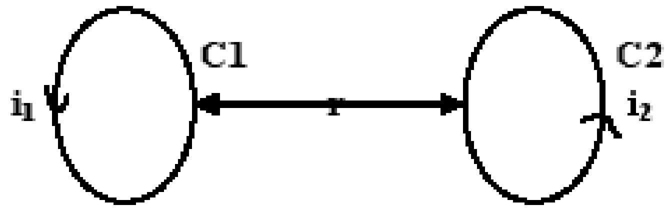
$$\phi = (N\, \mu_0\, Ni / 2\pi r).\, \pi R^2$$

$$= (N^2\, \mu_0\, i\, R^2 / 2r).$$

Therefore self inductance of toroid is , $L = \phi / i$

$$= (N^2\, \mu_0\, R^2 / 2r).\, H \text{ ------------------------ } 53$$

NEUMAN'S FORMULA



let us consider two circular coils brought as near as possible allowing i_1 and i_2 currents, with separation of r, of an areas S_1 and S_2 . the magnetic flux density due to current i_1 is ,

$$B_1 = \nabla \times A_1.$$

Vector magnetic potential , $A_1 = \int \mu i_1 dl_1 / 4\pi r$

Hence flux with second coil due to i_1,

$$\phi_{21} = B_1\, dS_2$$

hence total flux linking with second coil is , $\Psi_{21} = \int B_1\, dS_2$

$$= \int (\nabla \times A_1)\, dS_2$$

NEUMAN'S FORMULA

From stokes theorem, $\int (\blacktriangledown \times A_1)\, dS_2 = \int A_1\, dl_2$

Substituting this inn above equation ,

$$\Psi_{21} = \int A_1\, dl_2$$

$$= \int\int \mu i_1 dl_1\, dl_2 / 4\pi r$$

Therefore mutual inductance between two coils is ,

$$M_{21} = \Psi_{21} / i_1$$

Mutual inductance is the imaginary concept which says that there is flux linkage with second

Coil because of current flowing through first coil.

$$M_{21} = \int\int \mu i_1 dl_1\, dl_2 / 4\pi r \;/\; i_1$$

$$M_{21} = \int\int \mu dl_1\, dl_2 / 4\pi r$$

This M_{21} is called as Neumann's formulae.

ENERGY STORED

Let the work done to increase the current by di is dw, by law of conservation of energy

Work done is equal to energy stored

$$dw = vi\,dt$$

$$= L.idi.\ dt/dt$$

$$dw = Lidi$$

integrating on both sides , $\int dw = \int Lidi$

$$w = Li^2 / 2$$

but we know that, $L = N\phi / i = \Psi / i$

using above expressions we can write energy stored in the magnetic field also as, $w = \Psi^2 / 2.L$

MUTUAL INDUCTANCE

When two coils are brought together as close as possible then they form coupled coils.

Here when current(i1) is allowed through first coil then magnetic flux Φ1 is developed in it, as other coil brought to close proximity some of Φ1 links with second coil called as Φm1 their by inducing voltage in it and when we close the second coil current flows in it (i2). This current i2 develops Φ2 in it and some of Φ2 links with 1st coil called as Φm2. If the two coils are of same dimensions Φm1= Φm2 = Φm.

Here we define two inductances self inductance of coils L1 and L2, mutual inductance between the coils M12=M21=M.

PERMANENT MAGNETS AND APPLICATION

Characteristics and applications of permanent magnets

Characteristics :

Permanent magnets are the one which readily available in nature in the form of Bar and horse shoe shapes etc. Permanent magnets irrespective of supply always exhibits magnetic properties. Permanent magnets always develops a constant magnetic field. The strength of the permanent magnets measured in terms of their cohesive force. An permanent magnet with high cohesive force will have long life. Permanent magnet got the disadvantage of ageing effect i.e in long run they may get rusted.

Applications:

Permanent magnets are used in the applications where ever it is required to develop Constant magnetic field . Eg- Dc generator, Dc motor.

PERMANENT MAGNETS AND APPLICATION

Large industrial electromagnets, on the other hand, benefit greatly from the ability to control the magnetic flux. Electro lifting magnets can be positioned over materials to be moved before the magnetism is turned on, and the load can then be positioned before the magnet is de-energized.

On the negative side, electromagnets require a significant DC power source, create heat, and are vulnerable to power failures.

These problems are not insurmountable, however. Some electromagnets available today, for example, are up to 50% more energy efficient than any others previously available, have moreefficient cooling systems, and can be purchased with rectifiers and emergency generators (or other cut-in power source) to eliminate the vulnerability to power failure.

UNIT-V
TIME VARYING FIELDS AND WAVE PROPAGATION

MODULE V - SYLLABUS

Faraday's laws of electromagnetic induction, integral and point forms, Maxwell's fourth equation, curl (E)=$\partial B/\partial t$, statically and dynamically induced EMFs, modification of Maxwell's equations for time varying fields, displacement current.

Derivation of Wave Equation, Uniform Plane Waves, Maxwell's equation in phasor form, Wave equation in Phasor form, Plane waves in free space and in a homogenous material. Wave equation for a conducting medium, Plane waves in loss dielectrics, Propagation in good conductors, Skin effect. Poynting theorem.

COURSE OUTCOMES MAPPED WITH MODULE V

CO	Course Outcomes	Blooms Taxonomy
CO 10	State the Faraday's laws of electromagnetic induction and note the nature of emf induced in the coil for fixed and variable fields.	Remember
CO 11	List out the differential and integral forms of Maxwell's equation in time varying fields and fields varying harmonically with time for obtaining numerical solutions of complex engineering problems.	Understand
CO 12	Make use of the Maxwell Equations to produce a wave equation for the free- space, insulators and conductors for propagation of electromagnetic waves.	Apply

PROGRAM OUTCOMES MAPPED WITH MODULE V

PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
PO 3	**Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

MAPPING OF COs AND POS FOR MODULE V

COs	PROGRAM OUTCOMES											
	1	2	3	4	5	6	7	8	9	10	11	12
CO10	√		√									
CO11	√	√										
CO12	√	√										

INTRODUCTION

Time varying fields are produced due to accelerated charges or time varying currents.

Here we shall study how time varying current affects electric and magnet fields.

FARADAY'S LAW OF ELECTRO-MAGNETIC INDUCTION:

Micheal faraday has stated two laws

If any coil experiences change in flux or variable flux then emf is induced in it.

The emf induced in the coil is directly proportional to rate of change of flux linking With the coil.

$$E \propto - d\phi / dt$$

For an coil with N turns emf induced in it ,

$$E = - N.d\phi / dt$$

MAXWELL'S EQUATIONS

We know from the gauss law, $\phi = \int_s B\, ds$

hence emf induced due to above flux is ,

$$e = -d\phi / dt = -d(\int_s B\, ds) / dt$$

Electric potential is given as ,

$$e = \int E\, dl$$

equating above two equations,

$$\int E\, dl = -(\int_s dB\, ds) / dt$$

by applying stokes theorem,

$$\int E\, dl = \int_s (\nabla \times E)\, ds$$

substituting above equation in c,

$$\int_s (\blacktriangledown x E)\, ds = -\left(\int_s dB\, ds\right)/dt$$

comparing on both sides,

$$\blacktriangledown x E = -dB/dt$$

Equation is called as Maxwell''s fourth equation of vector form of faraday's law.

TYPES OF INDUCED EMF:

The emf induced in the coil according faraday's law is mainly of two types. They are

Dynamically induced emf
Statically induced emf.

TYPES OF EMF

DYNAMICALLY INDUCED EMF:

Let us consider a straight conductor with charge velocity of moving against the existing magnetic field. Force experienced by conductor is , potential induced can be written as, $e = BVl \sin\Theta$

the maximum value of potential induced is, $e = BVl$

STATICALLY INDUCED EMF:

If an conductor experiences variable flux then emf induced in it is called as statically induced Emf.

$$e = -Nd\,(\phi_m \sin wt\,)\,/\,dt$$

DISPLACEMENT CURRENT DENSITY

Let us consider a capacitor is connected to Ac source as shown in figure

The current flowing through capacitor is ,

$$i_C = C\, dV / dt$$

the capacitance of capacitor, $C = \varepsilon A / d$

Then, $i_C = (\varepsilon A / d).\, dV / dt$

$$i_C / A = \varepsilon\, dE / dt$$

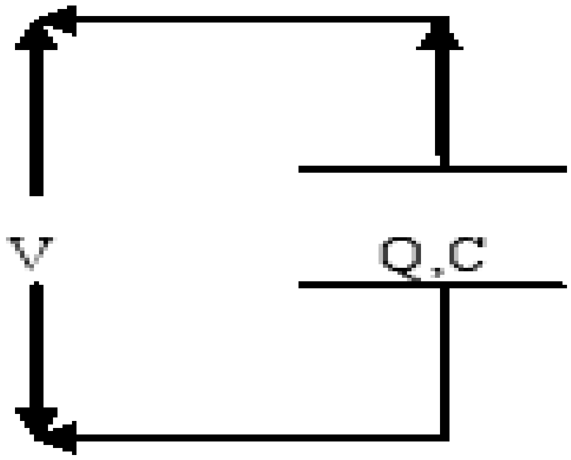
$$Jc = dD / dt$$

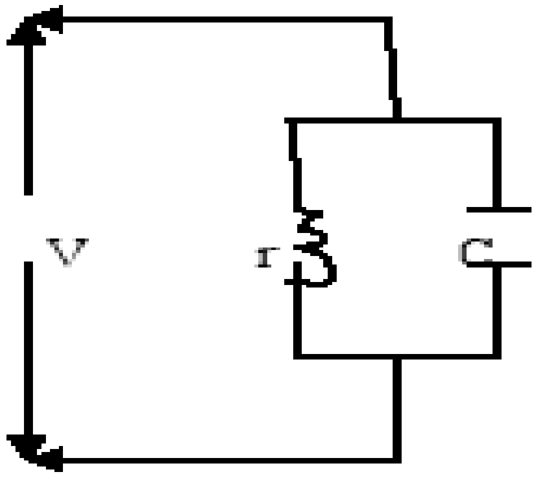
Jc is called as displacement current.

Above is the figure of actual capacitor with internal resistance,

Then the total current is , $i = i_r + i_c$

DISPLACEMENT CURRENT DENSITY





dividing above KCL on both sides by area A,

$$i / A = i_r / A + i_c / A$$

$$J = J_r + J_c$$

Maxwell's equations in time varying fields

In the time varying fields we can write,

$$E = Eo \cos wt$$

$$= Eo\, e^{jwt}$$

Similarly, $$D = Do\, e^{jwt}$$

$$d D / dt = Do\, wJ\, e^{jwt} = Jw\, Do$$

likely, $$dB / dt = Jw\, B$$

Maxwell's equations in time varying fields

we know that,

$$\nabla \times E = - dB / dt$$

$$= Jw\, B$$

Also,

$$\nabla \times E = -Jw\mu H$$

$$\nabla \times H = J + dD/dt$$

$$= \sigma E + Jw\, Do$$

$$= \sigma E + Jw\, \varepsilon E$$

$$= E\,(\sigma + Jw\, \varepsilon)$$

Integal form,

$$\int D\, ds = \int \rho_v dv\ ,\ \int D\, ds = \int \rho_v dv$$

$$\int B\, ds = 0$$

$$\int E\, dl = - Jw \int B\, ds$$

$$\int H\, dl = (\sigma + Jw\, \varepsilon) \int E\, ds$$

ADD ON INFORMATION

There are three ways that objects can be given a net charge. These are:

1. Charging by friction - this is useful for charging insulators. If you rub one material with another (say, a plastic ruler with a piece of paper towel), electrons have a tendency to be transferred from one material to the other. For example, rubbing glass with silk or saran wrap generally leaves the glass with a positive charge; rubbing PVC rod with fur generally gives the rod a negative charge.
2. Charging by conduction - useful for charging metals and other conductors. If a charged object touches a conductor, some charge will be transferred between the object and the conductor, charging the conductor with the same sign as the charge on the object.
3. Charging by induction - also useful for charging metals and other conductors. Again, a charged object is used, but this time it is only brought close to the conductor, and does not touch it. If the conductor is connected to ground (ground is basically anything neutral that can give up electrons to, or take electrons from, an object), electrons will either flow on to it or away from it. When the ground connection is removed , the conductor will have a charge opposite in sign to that of the charged object.

Wave Propagation

After substituting the fields D and B in Maxwell's *curl* equations by the expressions and combining the two resulting equations we obtain the inhomogeneous wave equations.

where we have skipped the arguments (r*, t*) for simplicity. The expression in the round brackets corresponds to the *total current density.*

$$j = j + \frac{\partial P}{\partial t} + \nabla \times M \, ,$$

where j is the source and the conduction current density, $\partial P/\partial t$ the polarization current density, and $\nabla \times M$ the magnetization current density. The wave equations as stated in Eqations do not impose any conditions on the media and hence are generally valid.

Homogeneous Solution in Free Space

We first consider the solution of the wave equations in free space, in absence of matter and sources. For this case the right hand sides of the wave equations are zero. The operation $\nabla \times \nabla \times$ can be replaced by the identity, and since in free space $\nabla \cdot E = 0$ the wave equation for E becomes

Wave Propagation

with an identical equation for the H-field. Each equation defines three independent scalar equations, namely one for E_x, one for E_y, and one for E_z.

In the one-dimensional scalar case, that is $E(x, t)$, Equations. is readily solved by the ansatz of d'Alembert $E(x, t) = E(x - ct)$, which shows that the field propagates through space at the constant velocity c. To tackle three-dimensional vectorial
fields we proceed with standard separation of variables

$$\mathrm{E}(\mathrm{r}, t) = \mathrm{R}(\mathrm{r})\, T(t)$$

$$c^2 \frac{\nabla^2 \mathrm{R}(\mathrm{r})}{\mathrm{R}(\mathrm{r})} - \frac{1}{T(t)} \frac{\partial^2 T(t)}{\partial t^2} = 0 .$$

The first term depends only on spatial coordinates r whereas the second one depends only on time *t*. Both terms have to add to zero, independent of the values of r and *t*. This is only possible if each term is constant. We will denote this constant as $-\omega^2$. The equations for $T(t)$ and R(r) become

$$\frac{\partial^2}{\partial t^2} T(t) + \omega^2 T(t) = 0$$

$$\nabla^2 \mathbf{R}(\mathbf{r}) + \frac{\omega^2}{c^2} \mathbf{R}(\mathbf{r}) = 0 .$$

Note that both R(r) and $T(t)$ are real functions of real variables.

Above is a harmonic differential equation with the solutions

Wave Propagation

$$T(t) = c'_{w} \cos[wt] + c''_{w} \sin[wt] = \operatorname{Re}\{c_{w} \exp[-iwt]\},$$

where $c_w{}'$ and c''_w are real constants and $c_\omega = c'_w + ic''_w$ is a complex constant. Thus,

according to ansatz (2.5) we find the solutions

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{R}(\mathbf{r}) \operatorname{Re}\{c_{w} \exp[-iwt]\} = \operatorname{Re}\{c_{w} \mathbf{R}(\mathbf{r}) \exp[-iwt]\}.$$

In what follows, we will denote $c_\omega R(\mathrm{r})$ as the *complex field amplitude* and abbrevi- ate it by E(r). Thus

Wave Propagation

Notice that E(r) is a *complex* field whereas the true field E(r, *t*) is real. The symbol E will be used for both, the real time-dependent field and the complex spatial part of the field. The introduction of a new symbol is avoided in order to keep the notation simple. Equation describes the solution of a *time-harmonic* electric field, a field that oscillates in time at the fixed angular frequency ω. Such a field is also referred to as *monochromatic* field.

$$\nabla \, E(r) + k \, E(r) = 0$$

Plane Waves

To solve for the solutions of the Helmholtz equation we use the ansatz

Wave Propagation

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \, e^{\pm i\mathbf{k}\cdot\mathbf{r}} = \mathbf{E}_0 \, e^{\pm i(k_x x + k_y y + k_z z)}$$

which, after inserting

$$k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

The left hand side can also be represented by k · *bfk* = k^2. For the following we assume that k_x, k_y, and k_z are real. After inserting we find the solutions