LECTURE NOTES
ON
ENGINEERING MECHANICS

B. Tech III Semester (R-18)

Prepared By

Dr. BDY Sunil

Associate Professor

MECHANICAL ENGINEERING

INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad - 500 043

ENGINEERING MECHANICS

III Semester: ME									
Course Code	Category	Hours / Week			Credits	Maximum Marks		
AMEB03	Foundation	L	T	P	C	CIA	SEE	Total
		3	1	-	4	30	70	100
Contact Classes: 45	Tutorial Classes: 15	Practical Classes: Nil				Total Classes: 60		

COURSE OBJECTIVES:

The student will try to learn:

I. The application of mechanics laws to static and dynamic equilibrium conditions in a body for solving the field problems.

II. The importance of free body diagram for a given system and put in the knowledge of mathematics and science into the vast area of rigid body mechanics.

III. The effects of force and motion while carrying out the innovative design functions of engineering.

COURSE OUTCOMES (COs):

CO 1: **Determine** the reactions and resultants for the system of forces in engineering applications with principles of mechanics.

CO 2: **Analyze** the unknown forces with the help of free body diagrams to a given force system.

CO 3: **Identify** the equilibrium equations for a planar and spatial force systems from the rest or motion condition of the body

CO 4: **Apply** the static and dynamic friction laws for the equilibrium state of a wedge and ladder applications.

CO 5: **Apply** the friction laws to a standard and differential screw jack for conditions of self-locking and overhauling.

CO 6: **Demonstrate** the concepts of equilibrium for truss, beam, frames and machine applications.

CO 7: **Identify** the centroid, centre of gravity and moment of inertia for the simple plane sections from the first principles.

CO 8: **Explore** the theorems of moment and the mass moment of inertia of circular plate, cylinder, cone and sphere.

CO 9: **Apply** the concepts of virtual work and work-energy method for single and connected configured systems.

CO 10: **Determine** normal and tangential accelerations for a particle in rectilinear and curvilinear motion through kinematic equations.

CO 11: **Derive** the dynamic equilibrium of a body in motion by introducing inertia force through D' Alembert's principle.

CO 12: **Compute** the time period and frequencies of simple, compound and torsional pendulums using the basics of free and forced vibrations.

MODULE-I	**INTRODUCTION TO ENGINEERING MECHANICS**	**Classes: 10**
Force Systems Basic concepts, Particle equilibrium in 2-D & 3-D; Rigid Body equilibrium; System of Forces, Coplanar Concurrent Forces, Components in Space – Resultant- Moment of Forces and its Application; Couples and Resultant of Force System, Equilibrium of System of Forces, Free body diagrams, Equations of Equilibrium of Coplanar Systems and Spatial Systems; Static Indeterminacy.		
MODULE -II	**FRICTION AND BASICS STRUCTURAL ANALYSIS**	**Classes: 09**
Types of friction, Limiting friction, Laws of Friction, Static and Dynamic Friction; Motion of Bodies, wedge friction, screw jack & differential screw jack; Equilibrium in three dimensions; Method of Sections; Method of Joints; How to determine if a member is in tension or compression; Simple Trusses; Zero force members; Beams &types of beams; Frames &Machines.		
MODULE -III	**CENTROID AND CENTRE OF GRAVITY AND VIRTUAL WORK AND ENERGY METHOD**	**Classes: 10**
Centroid of simple figures from first principle, centroid of composite sections; Centre of Gravity and its implications; Area moment of inertia- Definition, Moment of inertia of plane sections from first principles, Theorems of moment of inertia, Moment of inertia of standard sections and composite sections; Mass moment inertia of circular plate, Cylinder, Cone, Sphere, Hook. Virtual displacements, principle of virtual work for particle and ideal system of rigid bodies, degrees of freedom. Active force diagram, systems with friction, mechanical efficiency. Conservative forces and potential energy (elastic and gravitational), energy equation for equilibrium. Applications of energy method for equilibrium. Stability of equilibrium.		
MODULE -IV	**PARTICLE DYNAMICS AND INTRODUCTION TO KINETICS**	**Classes: 08**
Particle dynamics- Rectilinear motion; Plane curvilinear motion (rectangular, path, and polar coordinates). 3-D curvilinear motion; Relative and constrained motion; Newton's 2nd law (rectangular, path, and polar coordinates). Work-kinetic energy, power, potential energy. Impulse-momentum (linear, angular); Impact (Direct and oblique). Introduction to Kinetics of Rigid Bodies covering, Basic terms, general principles in dynamics; Types of motion, Instantaneous centre of rotation in plane motion and simple problems.		
MODULE -V	**MECHANICAL VIBRATIONS**	**Classes: 08**
Basic terminology, free and forced vibrations, resonance and its effects; Degree of freedom; Derivation for frequency and amplitude of free vibrations without damping and single degree of freedom system, simple problems, types of pendulum, use of simple, compound and torsion pendulums.		
Text Books:		
1. Irving H. Shames (2006), "Engineering Mechanics", Prentice Hall, 4th Edition,2013 2. F. P. Beer and E. R. Johnston (2011), "Vector Mechanics for Engineers", Vol I - Statics, Vol II, – Dynamics, Tata McGraw Hill , 9th Edition,2013. 3. R. C. Hibbler (2006), "Engineering Mechanics: Principles of Statics and Dynamics", Pearson Press.		

Reference Books:
1. S.Bhavikatti,“ATextBookofEngineeringMechanics”,NewAgeInternational,1st Edition,2012 2. A.K.Tayal, “Engineering Mechanics”, Uma Publications, 14th Edition, 2013. 3. R. K. Bansal “Engineering Mechanics”, Laxmi Publication, 8thEdition, 2013. 4. Basudeb Bhattacharya, “Engineering Mechanics”, Oxford University Press, 2nd Edition, 2014. 5. K.Vijay Reddy, J. Suresh Kumar, “Singer’s Engineering Mechanics Statics and Dynamics”, B Publishers, 1st Edition, 2013.
Web References:
1. https://en.wikipedia.org/wiki/Dynamics_(mechanics) 2. https://www.youtube.com/playlist?list=PLUl4u3cNGP62esZEwffjMAsEMW_YArxYC
E-Text Books:
1. http://www.freeengineeringbooks.com/Civil/Engineering-Mechanics-Books.php 2. http://www.textbooksonline.tn.nic.in/books/11/stdxi-voc-ema-em-2.pdf 3. http://www.faadooengineers.com/threads/17024-Engineering-mechanics-pdf-Free-Download

MODULE – I

INTRODUCTION TO ENGINEERING MECHANICS

COURSE OUTCOMES (COs):

At the end of the course students are able to:		
	Course Outcomes	**Knowledge Level (Bloom's Taxonomy)**
CO 1	**Determine** the reactions and resultants for the system of forces in engineering applications with principles of mechanics.	Apply
CO 2	**Analyze** the unknown forces with the help of free body diagrams to a given force system.	Analyze
CO 3	**Identify** the equilibrium equations for a planar and spatial force systems from the rest or motion condition of the body.	Remember

PROGRAM OUTCOMES (POs):

Program Outcomes (POs)		Strength	Proficiency Assessed by
PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	3	CIE/Quiz/AAT
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences	1	CIE/Quiz/AAT
PO 4	**Conduct Investigations of Complex Problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.	1	Seminar/ conferences / Research papers

"The branch of physical science that deals with the state of rest or the state of motion of a body/particle is termed as MECHANICS. Application of laws of Mechanics, such as Newtonian Mechanics, Einstein's Relativistic Mechanics and Quantum Mechanics, to the field problem is termed as ENGINEERING MECHANICS."

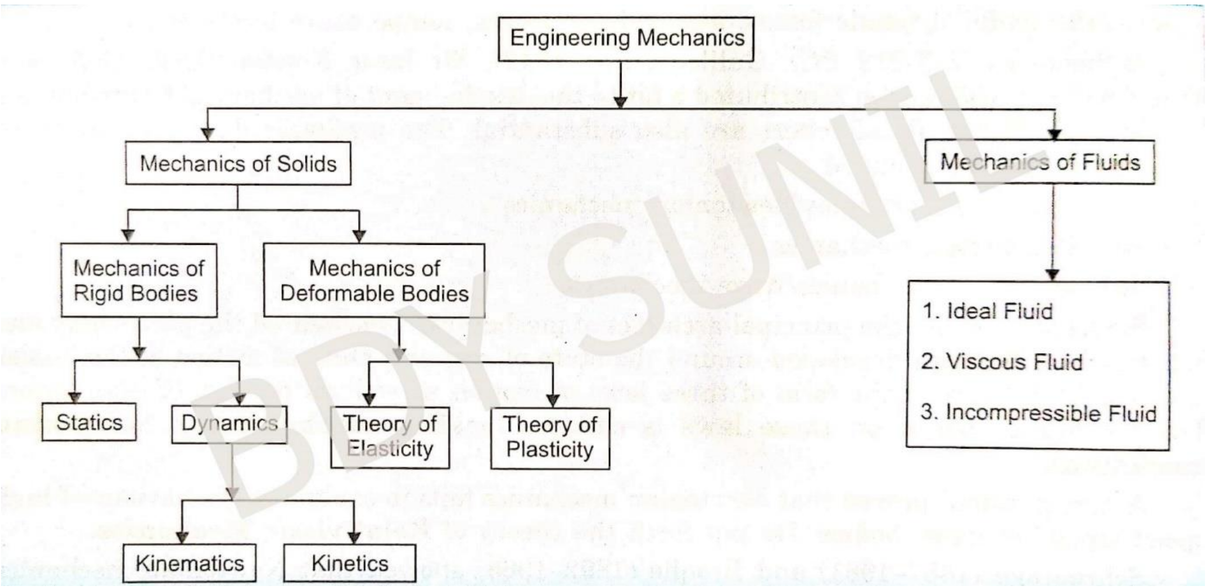
This can be treated as the definition of Engineering Mechanics. In this definition, if you observe carefully, the term particle or a body is the term used for the object which we consider in the problem. The body/particle may be as small as a molecule or as big as a building. It is a general term used for the object being considered in the problem.

Also, the term field problem refers to the problem in practical applications i.e., a real time application.

Now let us discuss about the **classification of Engineering Mechanics**. Depending upon the body to which Mechanics is applied, Engineering Mechanics is classified as:

(a) Mechanics of Solids and
(b) Mechanics of Fluids

The flow chart of classification can be seen here: [**classification figure**]



The broad classification of Engineering Mechanics can be given into Mechanics of Solids and Mechanics of Fluids.

The Solid Mechanics is classified into Mechanics of Rigid Bodies and Mechanics of Deformable Bodies. A rigid body is one in which the distance between two particles in the body will not change in any position or condition of the whole body, i.e., even when various forces are acting on the body. Whereas, a deformable body is one in which the distance varies with position and time.

In rigid body mechanics we have Statics and Dynamics. Static is the condition where the particle is at rest and we study the characteristics on that particle. Whereas, Dynamic is the condition where the particle is in motion and we study the characteristics. Studying the characteristics of a static particle is simple and easy but a particle in dynamic condition will be considered in two ways, Kinematics and Kinetics. Kinematics is the part of dynamics concerned with the study of motion of particles without considering the force which is the cause of motion. A particle will be in motion only when a force causes it to move or change its position. And if we study the characteristics of the body in motion without considering that force then it is called Kinematics. Whereas, if we consider the forces causing the motion of the body and study the characteristics then it is termed as Kinetics.

Mechanics of Deformable Bodies is classified into Theory of Elasticity and Theory of Plasticity.

In the Fluid Mechanics, the types of fluids seen are ideal, viscous and incompressible fluids.

For our present study we will learn the Static and Dynamic condition of a particle. We will see various laws configuring the systems, various methods to solve the field problems in Mechanics etc.

Now let us see what are the various **Laws of Mechanics** which are the basis for the problem solving in Engineering Mechanics.

Archemedes, Galileo, Sir Issac Newton, Einstein, Varignon, Euler, D' Alembert are some of the great scientists/inventors who have contributed a lot to the development of mechanics.

The fundamental laws of mechanics may be given as:

1. Newtons I law
2. Newtons II law
3. Newtons III law
4. Newtons law of gravitation
5. Law of transmissibility of forces and
6. Parallelogram law of forces

You have already studied about the Newton laws in your previous years. Let us briefly recollect them and clearly understand what the fundamental laws of mechanics are.

Newtons I law states that “every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it”. This gives us an idea that unless a force is acted upon, the body will continue to stay in its current state of movement or rest. This state of the body to continue in its current mode is termed as inertia. This can be easily understood by imagining as a body in outer space. When the body is moving in outer space, it continues to move in the same straight path until another body obstructs its path or an external agent in the form of force or pressure or torque is acted to change its direction or velocity.

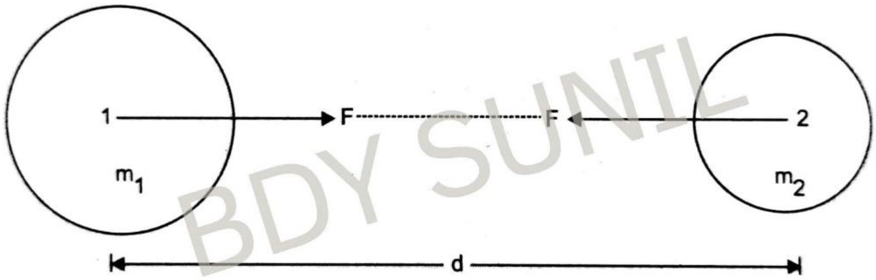
Newtons II law states that “the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it”. Thus, this law deduces the equation of force i.e., force is equal to mass X acceleration. i.e., F=ma.

Newtons III law, a very effective law, and we all remember it perfectly. It states that “for every action there is an equal and opposite reaction”. Let us see a small animation for understanding this.

Let us consider a tennis ball falling from a certain height. When the ball hits the ground, it executes a force on the ground which is called as an action. Because of this force acted on the ground, the response of the ground will also be in the form of force acted on the ball in reverse direction of the action which is called as reaction. The reaction force will be equal in magnitude of the action force and in the same line of the action force.

Let’s go to the next fundamental law, i.e., Newtons law of gravitation. This law states that, “every body attracts the other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them”.

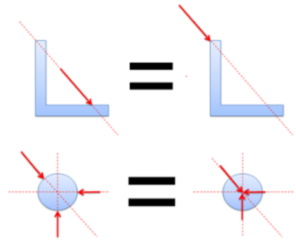
For understanding this, consider two bodies of different masses m1 and m2 which are at a distance d.



$$F = G\frac{m_1 m_2}{d^2}$$

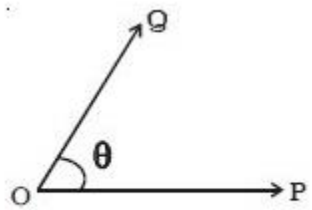
The force of attraction F between the bodies is given as F = G X m1m2/d^2. Where G is the constant of proportionality and is known as constant of gravitation.

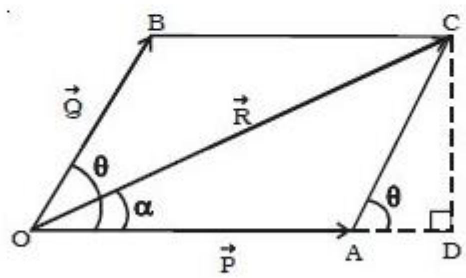
Now let us see the law of transmissibility of forces. This law states that "the state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force".



In the figure the dotted lines are the line of actions of the forces acting on the body. The equal symbol is given because when the same magnitude force is replaced anywhere on the line of action of the force, there won't be any change in the condition of the body, i.e., if the body is moving because of a force applied in a particular direction then there won't be any change in speed and direction of the motion of the body when that same magnitude force is replaced anywhere along the line of action of the applied force.

The next fundamental law which we will see is the Parallelogram law of forces. This law is useful in representing or identifying the resultant force of two concurrent forces. The law states that, "if two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces".

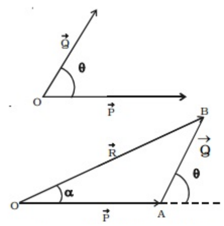




For understanding this law, consider two forces P and Q acting on a body at point O at an angle of θ. Now, if we represent these forces as two adjacent sides of a parallelogram and complete the parallelogram OACB then, the diagonal of this parallelogram i.e., OC will be the resultant of the two forces P and Q. In the figure, the resultant of the two forces P and Q is R acting at an angle α to the horizontal.

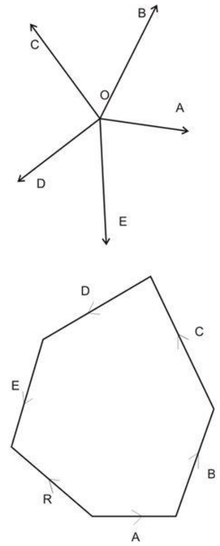
There are other ways to find the resultant force of the forces acting on a body using derived laws. The derived laws are the triangle law of forces and polygon law of forces.

Triangle law of forces states that, “if two forces acting on a body are represented by one after another by sides of a triangle, their resultant is represented by the closing side of the triangle taken from first point to the last”.



For understanding this law, consider two forces P and Q acting at point O on a body at an angle of θ. If we represent these forces as the sides of a triangle in same magnitude and direction one after the other, i.e., first if we represent force P in magnitude and direction then we have to represent the force Q starting from the end point of P, in magnitude and direction. Thus, the closing side of the triangle from O to B will be the resultant acting at an angle α.

The polygon law of forces is also similar to the triangle law of forces. This law is useful if there are more than two forces acting at a point on the body. The procedure for obtaining the resultant for all the forces acting is same as triangle law, where all the forces will be represented one after the other in continuous manner to make a polygon, and the final closing side of the polygon from starting point to the end point will be the resultant of all the forces.

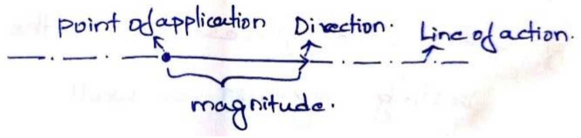


In the figure, the resultant of the forces A, B, C, D and E is R, which is the closing side of the polygon, formed by all the forces, and the direction of the resultant is from starting point of A to the ending point of E.

First let us discuss the characteristics of a force. A force is completely specified only when we provide the 4 characteristics. The characteristics are:

1. Magnitude
2. Point of application
3. Direction and

4. Line of action



The magnitude of the force should be given so that the quantity can be specified. For example, 100 N or 5000 N etc. Don’t mistake the size of the ray, which is used to represent the force, as the size that should be maintained for the magnitude i.e., for a smaller force small ray and for a bigger force big ray etc. It is not the physical representation of the force. It will be given only for an understanding that a force is being acted here. There is no relation between the size of the ray and the magnitude of the force. A smaller ray may be used for representing a 5000 N force and a bigger ray may be used for 100 N force. One should keep in mind that this size of the ray should only be maintained in calculations using graphical methods for a system of forces acting on a body.

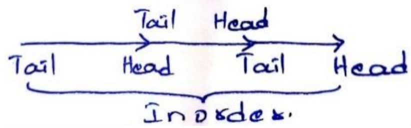
Then we have to specify the point where the force is applied. If a force is applied on certain area or the whole body, then it is termed as pressure, i.e., over the area, on each and every point, pressure magnitude is distributed in the form of force at all the points. So, the application of force implies that the force will be acted at a single point. That point should be specified.

Then we have to represent the direction in which force is acting. We already know that, force is a phenomenon where if it is applied on a certain mass, the mass will be acquiring an acceleration. Also, acceleration is a vector quantity and will be in a particular direction. Thus, it can be said that the direction in which the mass accelerates will be the direction of the force application. Representation of the direction of force is important because it decides the motion of the object, and the resultants, inertia force etc., will be calculated referring this direction. Thus, force is a vector quantity.

And finally, we have to represent the line of action of the force. The line of action is an imaginary infinite line drawn through the force applied along the direction. We have seen this line of action term in the Law of Transmissibility of Forces.

So, for completely representing a force these 4 characteristics should be specified.

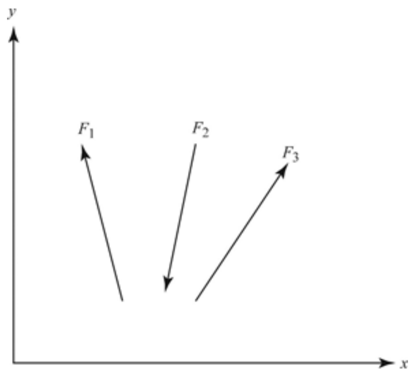
When we have discussed about the fundamental laws, we have seen the derived laws, which are, Triangle Law of Forces and Polygon Law of Forces. In those laws, it was stated that the forces are represented one after the other, i.e., the forces are represented in order. Then what does this in order mean?



The representation of the force is usually made using a ray and it can be given as the ray having a tail and a head, as shown in the figure. Now, when there are a number of forces acting, and if we have to connect them in order, then we should represent the first force in the direction acting with tail and head, then the second force will be connected to the first force, as shown in figure, as tail of the second force is connected to the head of the first force in the direction 2nd force is acting. Then, the tail of the 3rd force is connected to the head of the 2nd force in the direction 3rd force is acting, and so on. This systematic representation of connected forces is said to be the forces in order.

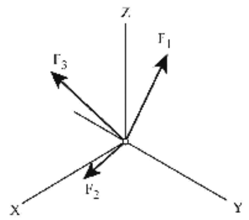
Next, let us learn about the system of forces. It is described as, when several forces act simultaneously on a body, they constitute a "system of forces." We can see several force systems depending upon the type of application of the forces.

If all the forces in a system lie in a single plane, it is called a "coplanar force system."



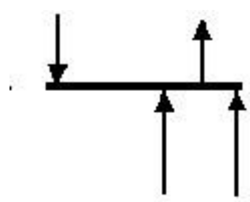
As given in the representation, number of forces that are acting will be in a 2-dimensional plane. The directions of the forces may be different, but these forces will lie in a single plane. Such force system is termed as a "coplanar force system."

Another type of force system is concurrent force system. It is described as, if the line of action of all the forces acting in a system pass through a single point, it is called as "concurrent force system." The forces may or may not lie in a single plane, but their line of action will be passing through a single point.



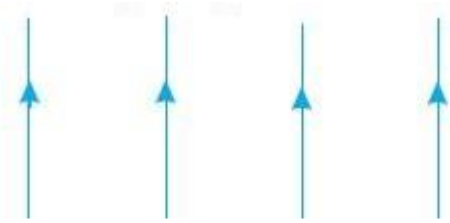
As given in the representation, forces F1, F2 and F3 are acting in different planes but their line of action is passing through a single point.

Another type of force system is parallel force system and it is described as, if all the forces are parallel to each other then it is called as "system of parallel forces."



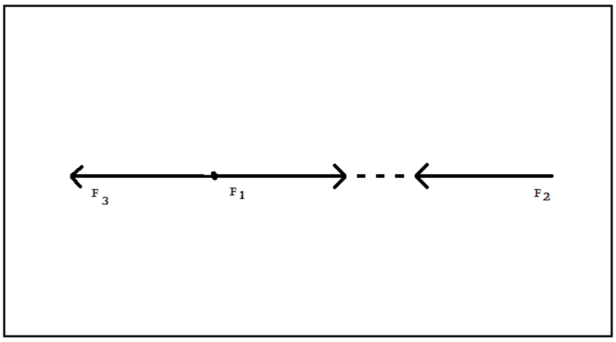
As given in the representation, all the forces will be parallel to each other, i.e., the lines of action are parallel to each other.

There are two types of parallel force systems. One is “like parallel force system” and other is “unlike parallel force system.” The representation given previously can be treated as unlike parallel force system, where, the forces acting are in different directions even though the line of actions are parallel. i.e., some of the forces are acting upwards and some of the forces are acting downwards. Such forces are called as unlike parallel forces.



As given in the representation, if all the forces are acting in one direction, i.e., either upwards or downwards, then that system of forces is termed as like parallel force system.

Another type of force system is collinear force system. It is described as, if the line of action of all the forces lie along a single line then it is called a “collinear force system.”

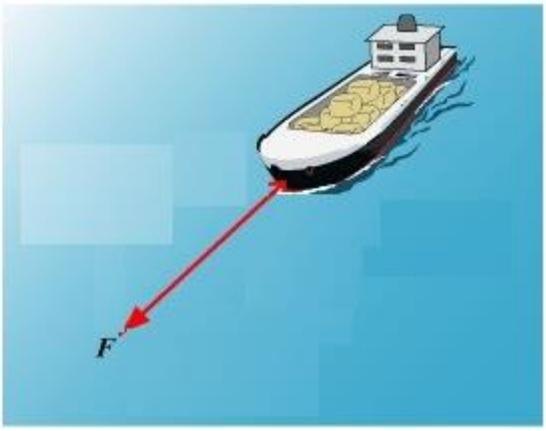


The forces F1, F2 and F3 are in different directions, but the line of actions lies in a single straight line. Hence, these forces constitute “collinear force system.”

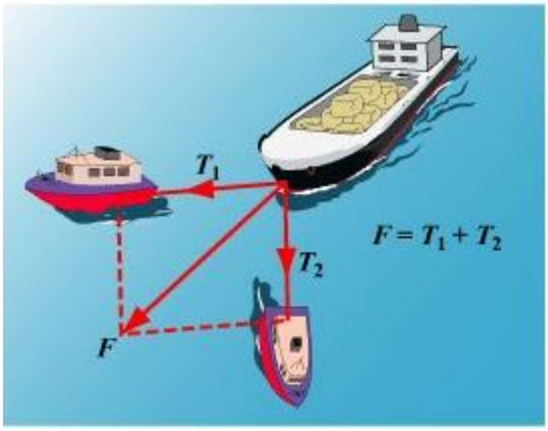
Now, let us see the resultant of system of forces. A resultant is described as a single force which will have the same effect that of a number of forces acting on a body. For understanding what a resultant is, let us refer an example.

Let a ship is being towed by two boats with a force of T1 and T2 in different directions as shown. Can you imagine in which direction the ship will move?

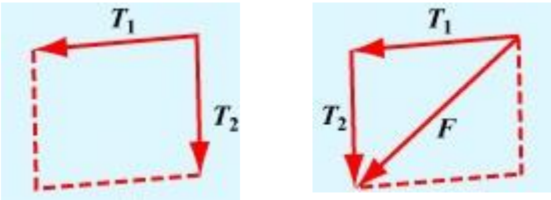
Yes, the ship will neither move in T1 direction nor in T2 direction, but it will move in a direction in between T1 and T2 with certain velocity let's say V, and let us say that the movement is in forward direction of the ship.



Now let's consider force F, being applied in the same direction as previous case which makes the ship to move with same velocity V. Then this force F can be treated as the resultant of the forces T1 and T2, because there is no change in the effect on the body i.e., direction and velocity, when the forces T1 and T2 are applied on the body and a single force F is applied on the body.



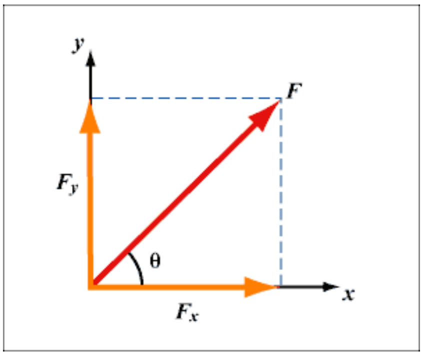
Hence, F is the resultant which is a composition of two forces T1 and T2.



From the graphical method, the resultant can be obtained using parallelogram law or the triangle law as shown.

After knowing what a resultant is, now let's discuss about the resultant of a coplanar concurrent force system. This is a force system in which the forces will be in a single plane and the line of actions of the forces will be passing through a single point. As discussed previously, the resultant for such a force system can be found by using parallelogram law, triangle law or polygonal law of forces. It can also be found by analytical method. The general method used for finding the resultant is the composition of forces by method of resolution.

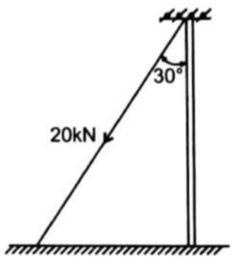
In previous example we have seen the composition of forces. Resolution of forces is exactly opposite process of composition of forces. It is the process of finding a number of component forces which will have the same effect as the given single force. The given single force will be resolved into its two components which are in mutually perpendicular directions.



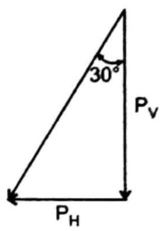
The process can be seen in the representation here. The force F is acting at an angle of θ to the horizontal. This force can be resolved into two mutually perpendicular component forces such as, Fx in x-direction and Fy in y-direction. It represents that the force F can be replaced with the two component forces Fx and Fy which will not change the effect on the body as when the single force F is acted upon. And the values of Fx and Fy can be obtained by trigonometric ratios as, Fx = F cosθ and Fy = F sinθ. With the help of these component forces of the system of forces, the resultant force can be calculated.

Let us solve a problem on resolution of forces for easy understanding.

The problem given is: the guy wire of an electric pole shown in figure makes an angle of 30o to the pole and is subjected to 20 kN force. Find the vertical and horizontal components of the force.



First let us represent the horizontal and vertical components of P=20 kN force.



The direction of the component forces should be such that they are to be reached to the final arrow of the main force. i.e., the vertical component will be moving towards the arrow of the main force, but in vertical direction upto the main force arrow, and the horizontal component also will be moving towards the arrow of the main force upto the main force arrow.

From the triangle law of forces, the vertical component Pv of force P is given by:

PV = Pcos30o = 20 cos30o = 17.321 kN (downward)

The horizontal component Ph is given by:

PH = Psin30o = 20 sin30o = 10 kN (left)

Thus, the component forces can be calculated.

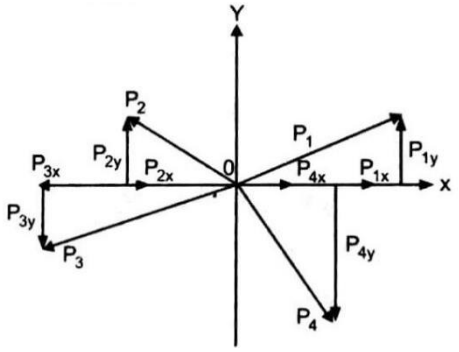
For finding the resultant of system of forces, now let us see the composition of concurrent forces by method of resolution. This is an analytical method of finding the resultant of multiple forces. It has to be done in three steps.

The first step is to find the components of each force in the system in two mutually perpendicular directions.

The second step is to add these components algebraically in each direction, i.e., in horizontal and vertical direction.

The third step is to combine the obtained components to get the resultant.

Let us see an example to understand this procedure.



Consider 4 concurrent forces P_1, P_2, P_3 and P_4 acting in various directions as represented in the figure. As step 1, let the horizontal and vertical components of force P_1 be P_{1x} and P_{1y} respectively. Similarly, for P_2, P_3 and P_4 the components are represented.

As step 2, the algebraic sum of the components should be given in x and y directions. So, the algebraic sum of horizontal component forces is given as

$$\sum P_x = P_{1x} + P_{2x} + P_{3x} + P_{4x}$$

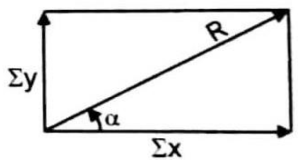
Similarly, the algebraic sum of vertical component forces is given as

$$\sum P_y = P_{1y} + P_{2y} + P_{3y} + P_{4y}$$

Now, the third step is to make the composition of two component forces from step 2 and deduce the resultant. The resultant is given as:

$$R = \sqrt{(\sum P_x)^2 + (\sum P_y)^2}$$

This will give the magnitude of the resultant force.



For obtaining the direction of the resultant, the inclination of the resultant force has to be deduced using the formula:

$$\alpha = \tan^{-1}\frac{\sum P_y}{\sum P_x}$$

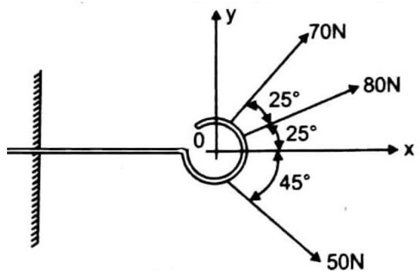
Using this formula, the inclination of the force to the horizontal can be known.

Please do remember that for representing a force, the magnitude and direction i.e., inclination should be compulsorily represented.

And as per the direction of the force that is, towards positive or negative directions, the sign convention should be given to the magnitude of the force. For example, if a force acts towards negative x-axis direction, then the horizontal component of that force should be negatively signed. Same will be the case with y-direction also. This system of sign convention is not mandatory, i.e., the sign convention may be followed in reverse also, but for solving a problem only one system should be followed. If positive x-axis and positive y-axis directions are considered as positive forces, then for the whole problem the same system of sign convention should be followed.

Let us do a problem for understanding this procedure.

The problem given is: Determine the resultant of the three forces acting on a hook as shown in figure.



In the problem there are 3 forces acting on the hook. So first let us resolve these forces into component forces.

For 70 N force, the x-component will be 70 cos50° = 45 and y-component will be 70 sin50° = 53.62

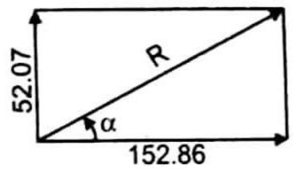
For 80 N force, the x-component will be 80 cos25° = 72.5 and y-component will be 80 sin25° = 33.81

For 50 N force, the x-component will be 50 cos45° = 35.36 and y-component will be -50 sin45° =-35.36, as the vertical component of the 50 N force is towards the negative y-direction, the magnitude is signed negatively.

Therefore, the sum of horizontal component forces is $\sum X = 152.86$ and $\sum Y = 52.07$

Now, the resultant $R = \sqrt{(152.86)^2 + (52.07)^2}$ = 161.48 N

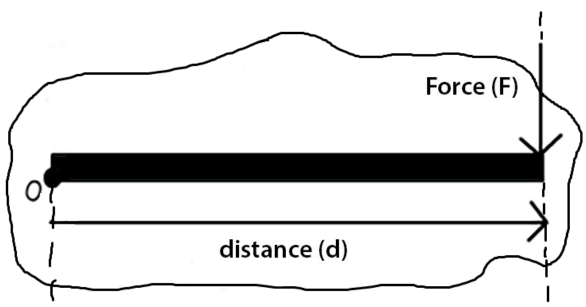
Angle $\alpha = \tan^{-1} \frac{52.07}{152.81}$ = 18.81°



So, the magnitude of the resultant is 161.48 N acting at an angle of 18.81° to the horizontal.

First let us learn about moment of a force. Moment is defined as "the product of magnitude of the force and the perpendicular distance of the point from the line of action of the force". Here, the point about which the moment is considered is termed as 'moment centre' and the perpendicular distance of the point from the line of action of the force is called 'moment arm'.

For calculations purpose, the clockwise moment is considered as positive moment and the anti-clockwise moment is considered as negative moment.

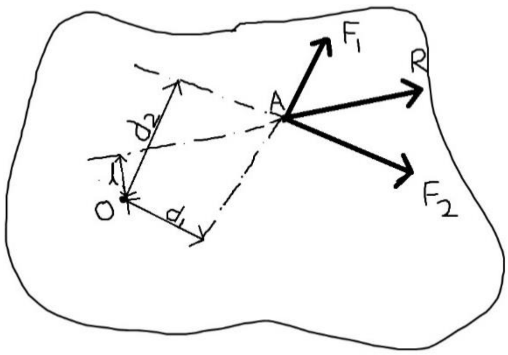


Consider a body on which a force F is acting at certain point. If we need to find the moment caused by the force on the body about the moment centre O, then the moment is given as F X d.

An example for moment on a body is the tightening of the nut using a wrench.

Here, a moment of F X l is being applied on the nut for tightening.

Now, let us learn about Varignon's Theorem. This theorem is also called as principle of moments. It states that, "the algebraic sum of the moments of a system of coplanar forces about a moment centre in their plane is equal to the moment of their resultant force about the same moment centre".

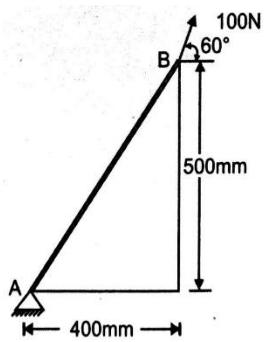


For understanding this let us see an example. Let there are two forces F1 and F2 acting on a body at a point A. The moments of F1 and F2 about the moment centre O are F1 X d1 and F2 X d2 respectively. Also, let R be the resultant of the forces F1 and F2. So, the moment caused by the resultant R on the body is given as R X l. Now, according to Varignon's theorem,

$$R \times l = (F_1 \times d_1) + (F_2 \times d_2)$$

The same procedure can be adopted for the resolution of forces also.

Let us solve a problem on moment principle. The problem given is: Find the moment of 100 N force acting at B about point A as shown in figure.



From the figure, the 100 N force can be resolved in to its horizontal component as 100 cos60° and the vertical component as 100 sin60°. From Varignon theorem, the moment of 100 N force is equal to the sum of the moments caused by its component forces. Therefore,

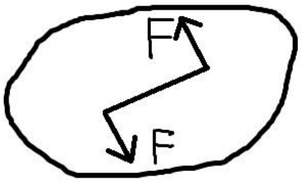
Moment, M = (100 cos60° × 500) + (-100 sin60° × 400); [negative sign is used because for the vertical component force, the moment will be anti-clockwise]

= 25000 – 34641.02

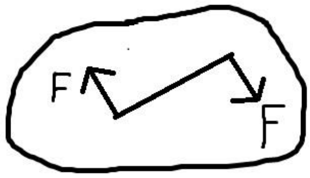
= **-9641.02 N-mm** or **9641.02 N-mm anti-clockwise**.

A bit extension for the moment is a couple. Two parallel forces equal in magnitude and opposite in direction, separated by a definite distance are said to form a couple.

Best example for a couple is, force applied on a tap by the fingers. Unknowingly, we use a couple force on the tap for opening and closing. We use our thumb and index finger and apply an equal force but opposite in direction, for opening and closing the tap.



Couple (Anti-clockwise, Negative Couple)



Couple (Clockwise, Positive Couple)

The force F is applied on the body at two positions in opposite directions which make the body to undergo a couple moment. In first image, because of the couple, the body will undergo an anti-clockwise couple which will be treated as negative couple, and in the second image, the body will undergo a clockwise couple which will be treated as a positive couple.

Now, let us see the characteristics of a couple.

First one, “a couple consists of a pair of equal and opposite parallel forces which are separated by a definite distance”.

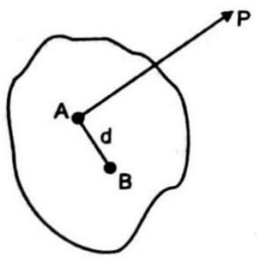
Second one, “the translator effect of a couple on the body is zero. i.e., the body will only rotate because of couple but no translator movement will be there”.

Third one, “the rotational effect of a couple about any point is a constant and it is equal to the product of the magnitude of the forces and the perpendicular distance between the two forces”.

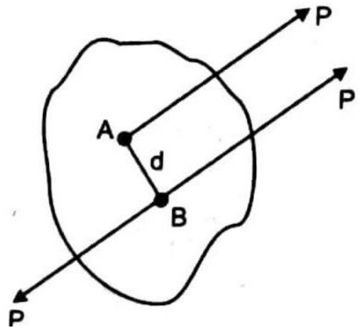
So, these are the characteristics of a couple acting on a body.

One of the applications of this couple is resolving a force into a force and a couple. This will be helpful in solving the complicated problems.

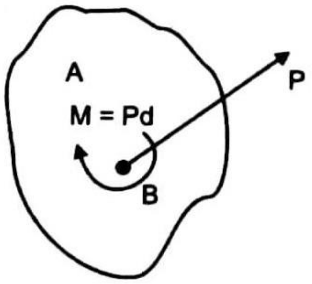
Let us see the procedure for doing this.



Let us consider a body on which a force P is acting at point A and need to find the effects on the body at point B. We can know the effect on the body by simply shifting the force P to point B and managing the equated effect by implementing the moment.



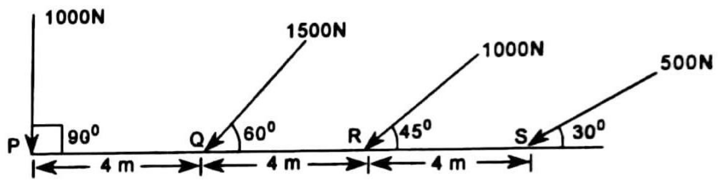
First, we can introduce two forces of equal magnitude P at point B in opposite directions. By applying equal and opposite forces, the system of forces is not disturbed. Hence, the original force P at point A and the opposite force P at point B form a couple of magnitude ‘Pd’.



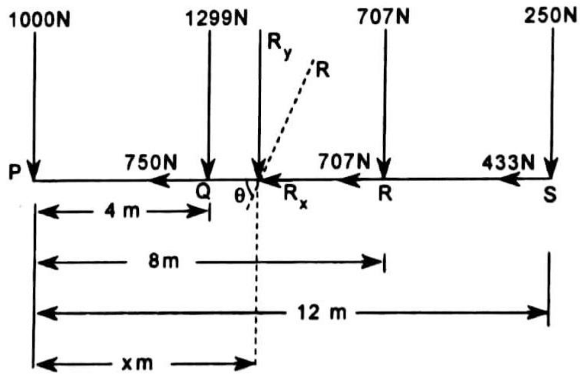
Thus, the force P at point A has been replaced with a force P at point B and a moment ‘Pd’.

Let us now solve some problems on resultant and moment.

The first problem given is: “4 forces are acting at various places and in various directions on a bar as shown in figure. Find the magnitude, direction and position of the resultant force”.

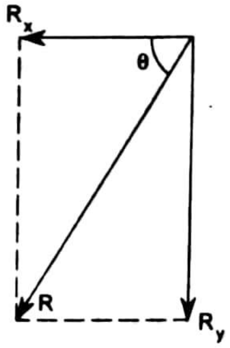


In the given problem, only the force acting at P is vertical and all other forces are inclined. Hence, we have to resolve the remaining forces into their horizontal and vertical components.



So, the forces acting at Q, R and S will be resolved into the component forces as represented. We have to be careful in representing the direction of the component forces also.

Let us assume that the resultant of these forces will act at a distance of Xm from P acting at an angle of θ to the horizontal as represented here.



Now, the algebraic sum of horizontal component forces is

$$\sum F_x = R_x = -750 - 707 - 433 = -1890N$$

Why we are taking negative sign here? Yes, as the forces are in negative direction.

And the algebraic sum of vertical component forces is

$$\sum F_y = R_y = -1000 - 1299 - 707 - 250 = -3256N$$

So, the magnitude of the resultant force is

$$R = \sqrt{(-1890)^2 + (-3256)^2} = \mathbf{3764.79N}$$

Also, $\theta = \tan^{-1}\frac{-3256}{-1890}$ = **59.86°**

Now, for finding the position of the resultant, we will use the Varignon theorem.

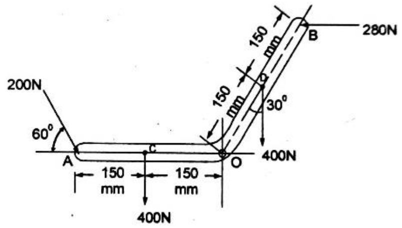
Taking moments about P,

Implies, $R_y \times x$ = (1299 × 4) + (707 × 8) + (250 × 12) [here we have taken positive sign because the moments are clockwise about point P]

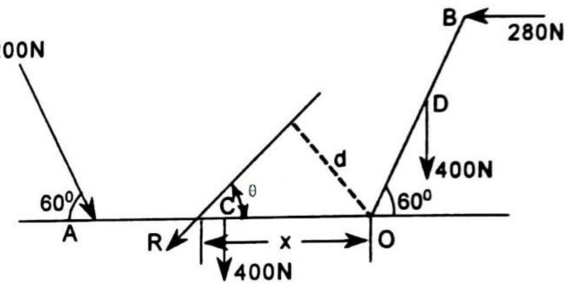
Therefore, x = 13852/3256 = **4.254 m**.

Hence, the resultant of the forces is 3764.79N, acting at an angle of 59.86° to the horizontal at 4.254m from the point P.

Now let us solve another problem. The problem given is: “determine the resultant of the forces acting on the bell crank shown in figure”.



In this problem there are 4 forces acting at various points and in various directions.



The force representation is made as given in the figure.

Now, resolving the forces horizontally,

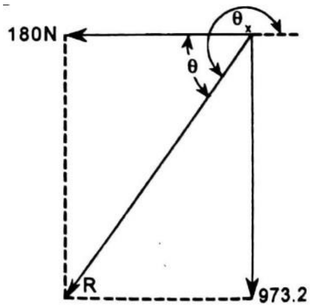
$$\sum F_x = R_x = -280 + 200\ \cos 60^o = -180\ N$$

Resolving the forces vertically,

$$\sum F_y = R_y = -400 - 400 + 200\ \sin 60^o = -973.2\ N$$

Now, the resultant, $R = \sqrt{(-180)^2 + (-973.2)^2}$ = **989.7 N**

The inclination, $\theta = \tan^{-1}\frac{-973.2}{-180}$ = **79.5°**



For identifying the position of the resultant let us apply Varignon's theorem.

Taking moments about O,

Implies, $R_y \times x$ = (-200 sin60° × 300) + (-400 × 150) + (400 × 150 cos60°) + (-280 × 300 sin60°)

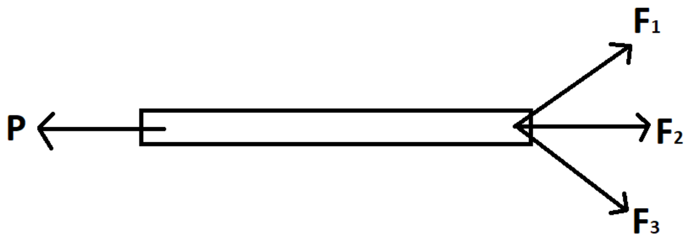
Solving this equation, we get x = **159 mm**.

Hence, the magnitude of resultant is 989.7 N acting at an angle of 79.5° and at a distance of 159 mm from the point O.

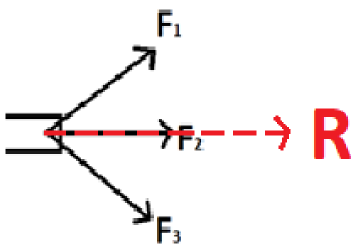
First let us learn about the equilibrium of forces. A body is said to be in equilibrium when there is no change in the state of rest under the effect of the forces acting on the body, i.e., the resultant is zero.

We have already learned that, resultant is a single force which can be replaced by the number of forces without changing the effect on the body. Let us say that a body is being applied by number of forces and even then, the body is at rest, i.e., the body is not moving by the effect of the forces, then we can say that the body is in equilibrium under the action of the forces. It represents that, all the forces acted on the body are making the body not to move in any direction, even when the forces are acting in many directions i.e., the forces are making themselves equivalent. So, the ultimate result is that, the sum of the forces is becoming zero. Hence, there will be no resultant i.e., the resultant of the forces acting is zero.

Let us consider an example of tug-of-war game. If the force applied by the person on left is greater than the forces applied by the persons on the right then the rope will move in left direction and vice versa. If, let us say that the force applied by the single person on the left and 3 persons on the right are equal, then there will be no movement of the rope on either side, i.e., the rope is in equilibrium. The force in the left is equating the 3 forces in the right and making the object/body into equilibrium state.



Schematically, this event is represented here. Let us say the force applied by the person on left is P and the forces applied by the persons on right be F1, F2 and F3. We can say that if P = F1+F2+F3 then the system will be in equilibrium.



Let us say that R is the resultant of the 3 forces F1, F2 and F3 acting on the right side. It implies that R is equal to P, which makes the body not to move in any direction. Let us imagine that there is no force P acting on left side. Then obviously, the body will move to the right. For making this body to come to rest, a force P which is equal to R should be applied exactly in opposite direction to R. Then the body will be in equilibrium. This force P which is making the body to be in equilibrium state is called as EQUILIBRANT.

Now, let us learn the conditions which will make the body into equilibrium state.

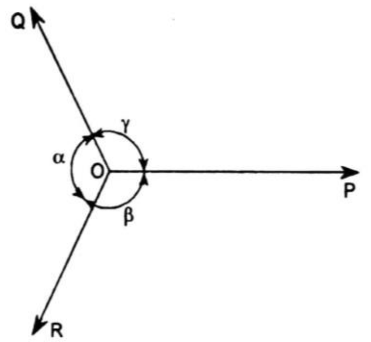
1. The algebraic sum of horizontal components of all the forces must be zero.
2. The algebraic sum of vertical components of all the forces must be zero.
3. The algebraic sum of moments of all the forces about any point in the plane must be zero.

So, it can be given as,

$$\sum F_x = 0, \sum F_y = 0, \text{ and } \sum M = 0$$

Thus, the body will be in equilibrium if the resultant force and resultant couple acting on the body are zero.

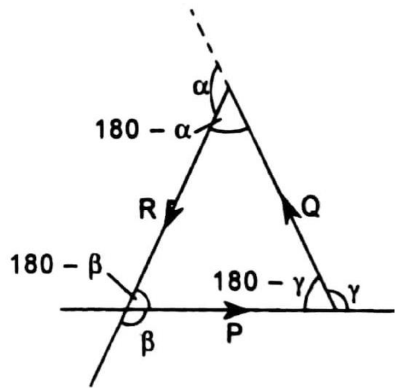
Now, let us see the LAMIs Theorem. It states that, "if three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces".



Let us consider the forces P, Q and R acting at point O and α, β and γ are the angles in between them as shown here. Then, Lamis Theorem can be expressed by the equation,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

The proof of the statement can be given using the in-order representation of the forces acting at point O.

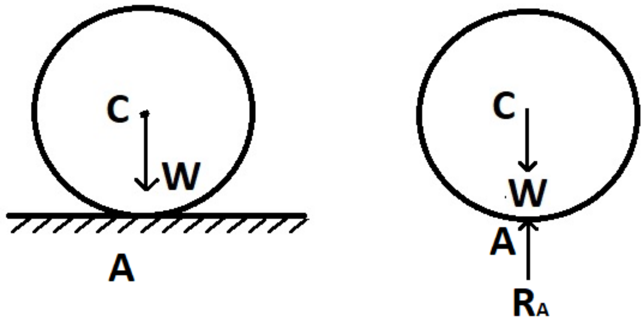


The three forces acting at a point are in equilibrium and can be represented by three sides of a triangle taken in the same order as shown here.

The angle between P and Q is γ, so, the inner angle will be 180-γ, similarly for the remaining forces. Now, from sine rule,

$$\frac{P}{\text{si}} = \frac{Q}{\ } = \frac{R}{\ }$$

si

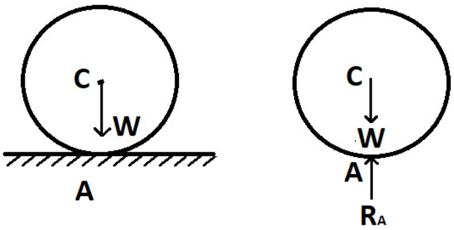


After this, let us learn about a constraint, action and reaction.

Consider a metal ball of weight W resting on a frictionless surface at point A. We can understand that, because of the surface prevailing, the ball cannot move downwards. The horizontal surface restricts the motion of the ball in downward direction and hence the surface is treated as a CONSTRAINT. Also, the ball applies a load of its own weight on to the surface at point A. This is called ACTION of a constrained body on its support. And according to Newtons III law, the surface exerts equal upward force on the ball, at the same point A where there is action. The upward force exerted by the support onto the body is called as REACTION, represented here as R_A.

Now, let us learn a very important aspect called FREE BODY DIAGRAM. This is a very needful tool for solving the problems in Engineering Mechanics. It is described as, “a sketch showing only the forces acting on a body by removing the support elements is called free-body diagram”.

For example, let us consider the figure from previous topic.

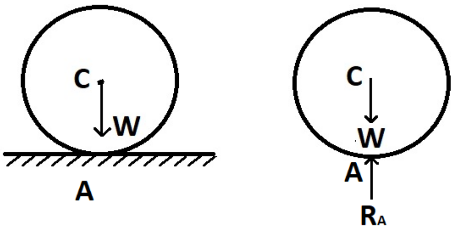


Here, the surface is the support for the ball. By removing this support, we have shown the reaction offered by the support at the point A as R_A, drawn normal to the body, i.e., the ray drawn here is perpendicular to the support. The actual forces acting in the system here are the weight of the object W alone. So, we have represented the force W at the same point C. It at all any other forces would have been acted on the ball, then we have to represent all the forces at the same point of application of individual forces. This is called as a free-body diagram of the system.

We will come to know in detail how to draw free body diagram while we solve the problems.

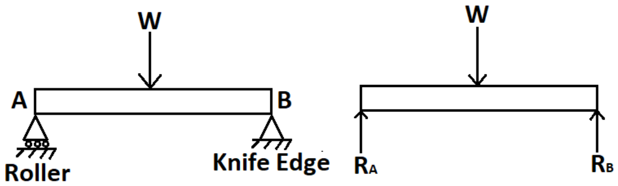
After knowing about the constraints, actions and reactions, now let us learn about various types of constraints/supports and support reactions.

Frist type of support is called as the frictionless support. This is a simple type of support which we have discussed in the previous topic.



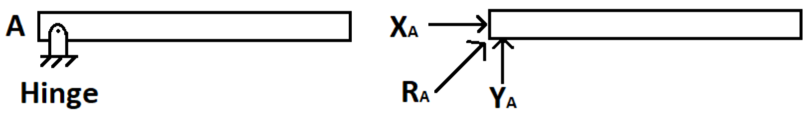
The flat surface acts as the support and the free body diagram should be represented as discussed previously.

The second type of supports are the roller and knife edge support.



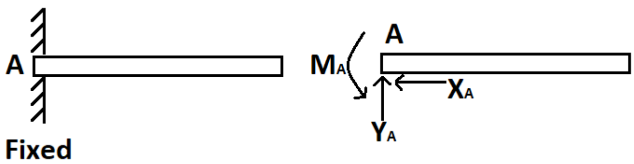
These are also the point load supports as shown here. The reactions at point A and B are shown in the free body diagram here towards and normal the body. These supports restrict the downward movement of the body.

And the third type of support is the hinged support.



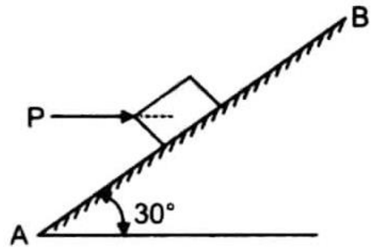
In this type of support, the body is restricted to move in x and y axis directions. So, there will be two support reactions at the hinge point. The final reaction will be the composition of the reactions XA and YA.

Next type of support is the built-in support, also called as fixed support.

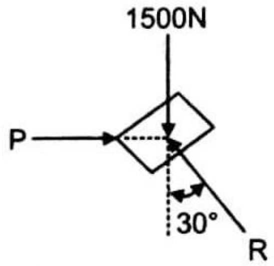


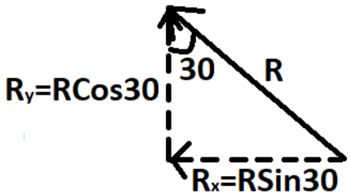
In this type of support, the body is fixed at the support point. There will be no movement in any direction at the support point. So, along with the x and y axes direction reaction, there will be a moment reaction at the support as shown here.

Let us solve a problem on state of equilibrium. The problem given is, “determine the horizontal force P to be applied to a block of weight 1500 N to hold it in position on a smooth inclined plane AB which makes an angle of 30 degrees with the horizontal”.



The very first thing that we have to do while solving a problem is to draw a free-body diagram of the given system. In the given system there is a force P that is being acted upon the body horizontally and the weight of the body acting vertically downwards, and there is a plane frictionless support at an angle of 30 degrees to the horizontal. Now, for drawing a free-body diagram, we have to remove the supports and mention the reactions on the body and in the direction towards the body. Also, we have to represent all the forces in the system as it is. So, the free-body diagram of the given system is as represented here.





So, in the free-body diagram of the given system, force P is represented at the point and direction as it is. And, the weight of the body 1500 N is shown vertically downwards which will act from the centre of the body. After removing the support, the reaction R is given normal to the support and towards the body centre. This 30-degree angle is obtained by simple triangle angles principle. Also, the inclined reaction force is divided into components Rx and Ry in mutual perpendicular directions and their values are Rsin30^o in horizontal direction and Rcos30^o in vertical direction as per trigonometric ratios. This is the free-body diagram of the given system.

Now we can solve the problem by using conditions of equilibrium.

First, we will do the sum of the horizontal components equal to zero.

i.e., $\sum F_x = 0$

which implies, P + (-R sin 30^o) = 0

We have taken negative symbol because the component is in negative axis direction.

So, P = R sin 30^o

Now, the sum of vertical components is equal to zero.

i.e., $\sum F_y = 0$

which implies, (-1500) + R cos 30^o = 0

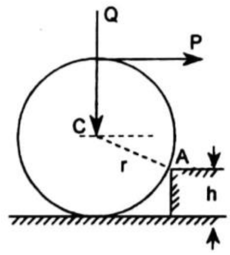
Solving this we get R = **1732.06 N**

Substituting this value in equation P we get,

P = 1732.06 × sin 30° = **866.03 N**.

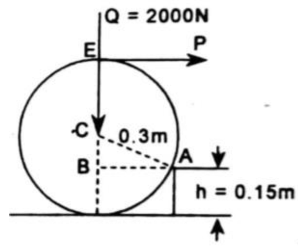
Thus, the force of 866.03 N is to be applied horizontally onto the block to hold it in position.

Let us solve another problem on equilibrium condition. The problem given is "A roller of radius 0.3 m and weight Q = 2000 N is to be pulled over a curb of height h = 0.15 m by the horizontal force P applied to the end of the string wound round the circumference of the roller. Find the magnitude of P required to start the roller over the curb".



In the figure we can see the forces acting. One of the forces is the weight of the object and the other force is P used to pull the roller over the curb. The roller is in contact with the curb at point A.

Now, as the first step, we will draw the free-body diagram of the given system.



In the system it is given that the force P is making the object to start the rolling. Hence, when the sufficient amount of force P is applied the roller starts to lose contact from the horizontal base support. So, there will not be any reaction from the support. And, at the point of contact of the roller to the curb, the roller tends to climb the curb by rolling, so we can apply the moment condition of equilibrium at moment centre A. That gives us the equation,

$$\sum M = -(Q \times AB) + (P \times EB) = 0$$

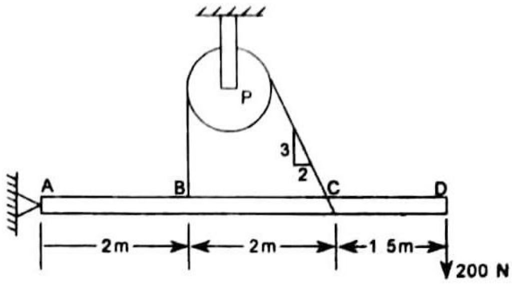
Here, negative moment is taken because the weight is causing anti-clockwise direction moment at the moment centre A.

For the required data of AB and EB, which are here, this is AB and this is EB, we can use simple triangle principles on triangle ABC.

Solving this we get, P = **1155.5 N**

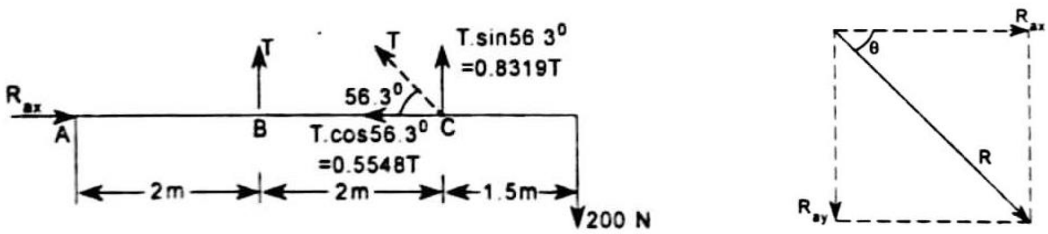
Hence, a force of 1155.5 N is to be applied so as to start the roller over the curb.

Let us now solve a problem involving string tension and a hinged support reaction. The problem given is, "determine tension in the cable and horizontal and vertical components of reaction at pin A as shown in the figure. The pully P is frictionless".



Here, the bar is supported by a string from the pulley at B and C, and a hinged support using a pin at point A. Also, there is a force of 200 N at the end of the bar acting vertically downwards at point D. As the bar is held by the string, there will be tension acting in the string because of the forces in the system. The tension will always be acting away from the body, which is considered during the solvation of the problem.

As the first step we have to draw the free-body diagram to the given system.



In this free body diagram, the hinge support reactions are given as Rax as horizontal reaction component and Ray as vertical reaction component. Also, the tension in the string is considered as T, which is acting away from the body at points B and C. The inclination of the string is given as $\tan^{-1}\frac{3}{2}$ = 56.3° at point C. Using this inclination angle, the components of tension at point C is deduced as 0.8319T in vertical direction and as 0.5548T in horizontal direction. The tension at point B is a vertical force.

Now let us first apply the moment condition for equilibrium of the system at moment centre A, so that the tension T can be found.

Hence, $\sum M =$ (-T × 2) + (-Tsin56.3° × 4) + (200 × 5.5) = 0

Solving this we will get the value of T = **206.47 N.**

Now, applying the $\sum F_x = 0$ condition, we get,

Rax – Tcos56.3° = 0

Which implies, Rax = **114.55 N**

And, applying the $\sum F_y = 0$ condition, we get,

Ray + T + Tsin56.3° + (- 200) = 0

Implies, Ray = **-178.23 N**, here negative sign indicates that the vertical component of the reaction is acting downwards.

Hence, reaction at A is given as, $R = \sqrt{(114.55)^2 + (178.23)^2}$ = **211.86 N.**

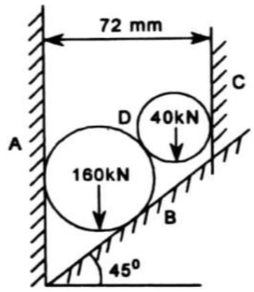
And the inclination of the reaction is given as, $\theta = \tan^{-1}\frac{178.23}{114.55}$ = **57.27°**.

Thus, the reaction at hinge support is 211.86 N acting at an angle of 57.27°, and the tension in the string is 206.47 N.

Let us solve a problem on connected bodies. If in a system two or more bodies are in contact with each other by any means, say, a string connected to two bodies or a bar connecting two bodies or even two bodies in direct contact with their surfaces, then these systems are termed as connected body systems.

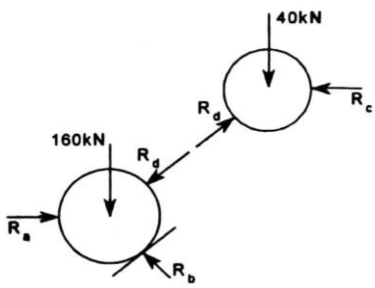
The procedure we use here to solve the problem on connected bodies systems is to apply the equilibrium conditions to individual bodies to find the reactions and unknown forces on the body and solve the equations we obtain.

The first problem given is, "two rollers of diameters 30 mm and 60 mm weighing 40 N and 160 N respectively are placed as shown in the figure. Assuming all contact surfaces are smooth, find the reactions at A, B and C".



In the system of connected bodies given, there are 2 rollers in contact with each other. Here, both the rollers will apply a reaction force on each other which is in equal magnitude, i.e., if Rd is the reaction force at the contact point of D, then at the contact point 160kN roller will apply Rd reaction force on 40kN roller and the 40kN roller will apply Rd reaction force on 160kN roller. And when we draw the free-body diagram we have to represent the reaction force Rd on both the rollers, in the direction towards their centre.

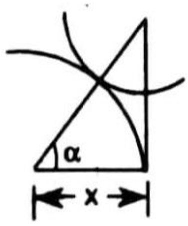
Now, let us draw the free-body diagram.



In this free-body diagram, we have removed the wall supports and represented the reactions at the contact points of A and C normal to the surface and towards the body centre, which are horizontal. We have also removed the inclined bottom surface support and represented the reaction at the contact point B, normal to the surface and towards the centre of the body. For 40kN roller, there is no contact of the roller to the inclined surface. Hence, we have not represented any reaction force here.

With the clue given by the inclination of the bottom inclined surface of 45°, we can represent the inclination of Rb as 45° to the horizontal. But for Rd, we have to find the inclination.

Let the horizontal distance of the centres of the rollers be 'x' and the inclination of the line joining the



centres be 'α'.

Now, for finding x, x=72-(30+15) =27mm, where 30 is this left side portion of the roller from the centre of big roller and 15 is the right-side portion of the small roller. So, if we remove these parts from the total horizontal distance of 72 mm, we will be getting the linear distance between the two centres of the rollers which is 27mm, as calculated.

Now, for finding the inclination, we can apply trigonometric ratio to the triangle formed here.

Therefore, Cos α = x/(30+15 which are the radii of the rollers) = 27/45

Solving this we get the inclination, α = 53.13°.

Now, let us apply equilibrium conditions to 40kN roller first.

So, applying $\sum F_y = 0$ implies,

-40 as the force is acting downwards + Rd Sin53.13° = 0. Solving this we get **Rd = 50kN**.

Also, $\sum F_x = 0$ implies,

-Rc as the force is in negative direction + Rdcos53.13° = 0. Solving this we get **Rc = 30kN**.

Now, let us apply equilibrium conditions to 160kN roller.

So, applying $\sum F_y = 0$ implies,

-160 as the force is acting downwards -Rdsin53.13° as the vertical component is acting downwards + Rbcos45° = 0

Solving this we get, **Rb = 282.89kN**.

Also, $\sum F_x = 0$ implies,

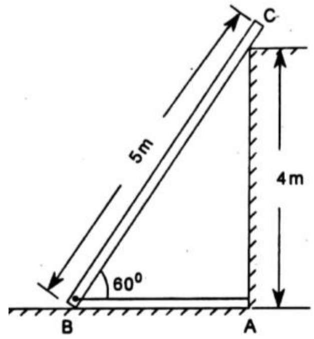
Ra -Rdcos53.13° as the component is acting in negative direction -Rbsin45° as the component is acting in negative direction = 0

Solving this we get, **Ra = 230kN**.

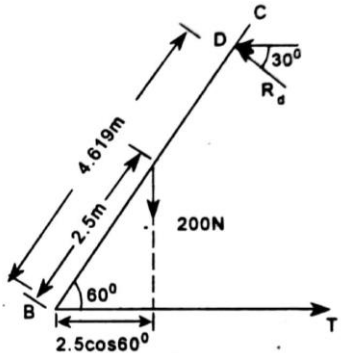
Thus, the reactions at A, B and C are 230kN, 282.89kN and 30kN respectively.

In this way, by drawing the free-body diagrams of individual bodies in a connected body system, we can obtain the reactions and other unknown forces.

Let us solve another problem. The problem given is, "determine the tension in the cable AB which holds a post BC from sliding as shown in figure. Weight of the post is 200N and assume all the surfaces as smooth".



In the problem given there are no extra forces acting in the system. The only forces that we able to see here are the weight of the post which is 200N and as this is the weight of the body, it will act vertically downwards. Then there will be reaction forces at the contact points of the surfaces, at the upper portion where the post is resting on the sharp edge and tension at the bottom of the post where it is tied using a cable. These things will be shown using the free-body diagram of the system.



As shown here, let the post is resting on the sharp edge at point D. Hence, at point D there will be a reaction force Rd acting normal to the surface and towards the body. By simplification we understand that the inclination of the reaction will be 30° from the horizontal. Also, for obtaining the BD length we can use simple triangle laws and deduce the length from triangle ABD as,

Sin60° in triangle ABD = AD/BD = 4/BD

Therefore, simplifying this we get, BD = 4.619m.

Also, the weight 200N acting vertically downwards, will be at a distance of 2.5 cos60° horizontally from point B, this is useful while we evaluate the moments.

The tension in the cable AB is given as T which is acting away from the body i.e., post.

All these data can be deduced by the help of free-body diagram of the system.

Now, let us apply the equilibrium condition to the system for finding the unknown forces acting in the system.

First let us apply $\sum F_x = 0$. This implies,

T + (-Rd cos30° as the horizontal component of the reaction force is in negative x-axis direction) = 0.

Simplifying this we get, T = 0.866Rd.

Now, let us take the moments about B, this gives,

$200 \times 2.5 \cos60^o$ + (-Rd × 4.619 as the perpendicular distance of this Rd reaction is directly known, there is no need to apply for individual component forces of Rd) =0

Simplifying this we get the value of Rd as **54.124N**.

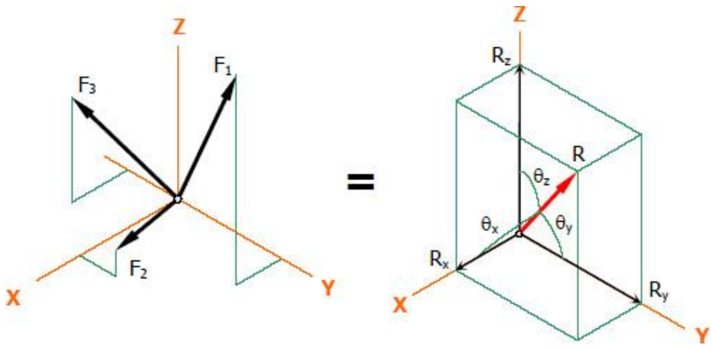
Now, substituting this Rd value in Tension equation, we get, T = **46.87N**.

Thus, the tension in the cable is 46.87N.

By this time, you may have understood that, there is no need of applying all the 3 equilibrium conditions in all the problems. We may use the equilibrium conditions according to the requirement of the problem for obtaining the unknown forces.

Coming to the introduction of the topic, we can say that "in spatial force system, the line of action of forces does not lie in the same plane".

For understanding this, let us consider an example of a tri-pod. Tri-pod is an equipment used to hold a body in position at certain height like a camera. We can adjust the position of the camera by changing the positions of the legs. In these various conditions, there are different forces acted through the legs of the tri-pod. If we consider the line of actions of the forces through the tri-pod, these lines will be concurrent but does not lie in the same plane. Thus, we can say that the forces acting in the system are spatial forces. Even the line of action of resultant of these spatial forces will be lying in a different plane.



Let us say that there are 3 forces F1, F2 and F3 acting at a point in various directions in different planes as shown here. These green lines are the intercepts of the forces to the concerned axes. Here, F1 and F2 are acting in this quadrant space and F3 is acting in this quadrant space. For F1 force, z intercept is this and y intercept is this. For F2 force, z intercept is this and x intercept is this. And for F3 force, z intercept is this and y intercept is this. If R is the resultant of these forces F1, F2 and F3, then in space, the resultant will be as represented here which has the inclinations of θx, θy and θz in x, y and z axes respectively. Here, Rx, Ry and Rz are the components of the R in mutually perpendicular directions in space. Therefore, the magnitude of Rx will be the compostion of x-direction components of all the 3 forces and Ry will be the composition of y-direction components of all the 3 forces and Rz will be the composition of z-direction components of all the 3 forces F1, F2 and F3.

Hence, we can write, Resulatnt, $R = \sqrt{(Rx)^2 + (Ry)^2 + (Rz)^2}$

Also, Rx = $\sum F_x$, Ry = $\sum F_y$, and Rz = $\sum F_z$

Where, $\sum F_x$ = $\sum(F_{1x} + F_{2x} + F_{3x})$ and $\sum F_y$ = $\sum(F_{1y} + F_{2y} + F_{3y})$ and $\sum F_z$ = $\sum(F_{1z} + F_{2z} + F_{3z})$

Here, 1x, 1y etc., represent the component forces of F1 and similarly for other component forces i.e., 2x, 2y, 2z for F2 force etc.

Also, for finding the inclination of the resultant, we can use the trigonometric ratios applied to the individual component as,

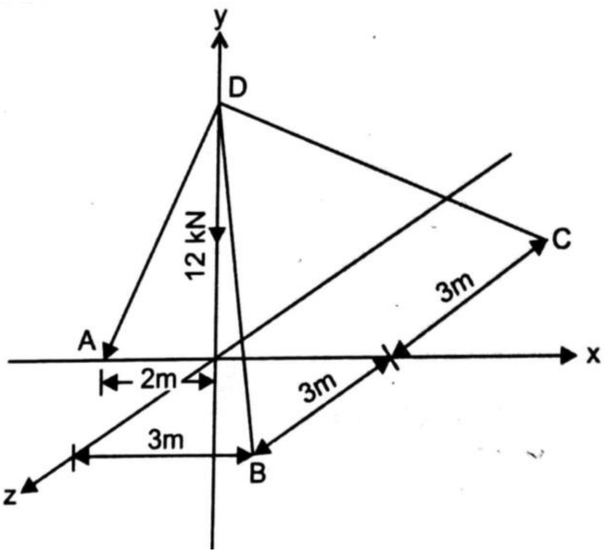
$$\theta_x = \cos^{-1}\frac{Rx}{R}$$

$$\theta_y = \cos^{-1}\frac{Ry}{R}$$

$$\theta_z = \cos^{-1}\frac{Rz}{R}$$

In this way we can find the resultant for a number of concurrent forces acting in space, also called as spatial concurrent force system.

Now, we will solve a problem on this spatial force system. The problem given is, "a tripod is resting with its legs on a horizontal plane at points A, B, C as shown in figure. Its apex point D is 4 m above the floor level and carries a vertically downward load of 12 kN. Determine the forces developed in the legs".



This problem can be treated as the reverse of the equilibrium problems which we have done so far i.e., the resultant force of 12 kN is provided and we have to find the forces along the DA, DB and DC directions. For getting this, first we have to identify the points A, B and C, then we have to find the lengths of DA, DB and DC, which will be useful for finding the inclinations of the forces through legs for each individual component. Using these inclinations, we will then find the component forces. Then, we apply the conditions for equilibrium and solve the equations for finding the forces through the legs of the tri-pod. So, this would be our procedure for solving the problem.

Now, let us solve the problem. First let us assume that the forces through DA, DB and DC be P1, P2 and P3 respectively. We see that the point D is in equilibrium under the action of the forces P1, P2, P3 and the 12 kN load. Let the projection of the point D be the origin of the 3-axis coordinate system as shown here. And by this we can identify the points A, B, C and D. Here, the point A is 2 m away from the origin in x-axis direction, and it is 0 distance from y-axis and z-axis. And, the point B is 3 m away from both x and z-axes but at a distance of 0 from the y-axis. Also, the point C is 3 m away from the x and z-axes but at a distance of 0 from the y-axis. Thus, the points are obtained as,

A at (-2,0,0)

B at (3,0,3)

C at (3,0,-3)

And D at (0,4,0)

Now, let us find the linear distances between these points.

Let L1 be the length of DA which is obtained using the length equation between the points D and A as, $L_1 = \sqrt{(-2-0)^2 + (0-4)^2 + (0-0)^2} = 4.472\ m$, and L2 be the length of DB which is obtained using the length equation between the points D and B as, $L_2 = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2} = 5.831\ m$, similarly the distance DC as, $L_3 = \sqrt{(3-0)^2 + (0-4)^2 + (-3-0)^2} = 5.831\ m.$

Now, we can find the inclination of the component forces using the trigonometric function as,

$cos\,\theta_{1x} = \frac{-2-0}{4.472}$, where, Cosθ1x is the inclination of the P1 force component in x-axis direction, similarly we get other component inclinations as,

$cos\,\theta_{1y} = \frac{0-4}{4.472}$, and $\cos\theta_{1z} = \frac{0-0}{4.472}$. From these, the component forces are obtained as $P_{1x} = P_1 \cos\theta_{1x} = -0.447P_1$, and $P_{1y} = P_1 \cos\theta_{1y} = -0.894P_1$ and $P_{1z} = P_1 \cos\theta_{1z} = 0$

Similarly, $\cos\theta_{2x} = \frac{3-0}{5.831}$, where, Cosθ2x is the inclination of the P2 force component in x-axis direction, similarly we get other component inclinations as,

$\cos\theta_{2y} = \frac{0-4}{5.831}$, and $\cos\theta_{2z} = \frac{3-0}{5.831}$. From these, the component forces are obtained as $P_{2x} = P_2 \cos\theta_{2x} = 0.514P_2$, and $P_{2y} = P_2 \cos\theta_{2y} = -0.686P_2$ and $P_{2z} = P_2 \cos\theta_{2z} = 0.514P_2$

Similarly, $cos\,\theta_{3x} = \frac{3-0}{5.831}$, where, Cosθ3x is the inclination of the P3 force component in x-axis direction, similarly we get other component inclinations as,

$cos\,\theta_{3y} = \frac{0-4}{5.831}$, and $\cos\theta_{3z} = \frac{-3-0}{5.831}$. From these, the component forces are obtained as $P_{3x} = P_3 \cos\theta_{3x} = 0.514P_3$, and $P_{3y} = P_3 \cos\theta_{3y} = -0.686P_3$ and $P_{3z} = P_3 \cos\theta_{3z} = -0.514P_3$

After obtaining the component forces, now let us apply the conditions for equilibrium of the system.

Equating the sum of the x-direction forces to zero, implies, $\sum P_x = 0\ i.e., (P_{1x} + P_{2x} + P_{3x}) = 0.$

Therefore, $-0.447P_1 + 0.514P_2 + 0.514P_3 = 0$

Now equating the sum of y-direction forces to zero, implies, $\sum P_y = 0\ i.e., (P_{1y} + P_{2y} + P_{3y}) = 0.$

Therefore, $-0.894P_1 + (-0.686P_2) + (-0.686P_3) = 0$

Now equating the sum of z-direction forces to zero, implies, $\sum P_z = 0$ i.e., $(P_{1z} + P_{2z} + P_{3z}) = 0$.

Therefore, $0P_1 + 0.514P_2 + (-0.514P_3) = 0$

Now solving these equations, we get the magnitudes of the forces through the legs of the tri-pod as, P1=-8.393 kN, P2=-3.649 kN and P3=-3.649 kN. The negative sign indicates that the forces are acting in negative direction.

MODULE – II

FRICTION AND BASICS STRUCTURAL ANALYSIS

COURSE OUTCOMES (COs):

At the end of the course students are able to:		
	Course Outcomes	**Knowledge Level (Bloom's Taxonomy)**
CO 4	**Apply** the static and dynamic friction laws for the equilibrium state of a wedge and ladder applications.	Apply
CO 5	**Apply** the friction laws to a standard and differential screw jack for conditions of self-locking and overhauling.	Apply
CO 6	**Demonstrate** the concepts of equilibrium for truss, beam, frames and machine applications.	Understand

PROGRAM OUTCOMES (POs):

Program Outcomes (POs)		Strength	Proficiency Assessed by
PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	3	CIE/Quiz/AAT
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences	1	CIE/Quiz/AAT
PO 4	**Conduct Investigations of Complex Problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.	1	Seminar/ conferences / Research papers
PSO 1	Formulate and evaluate engineering concepts of design, thermal and production to provide solutions for technology aspects in digital manufacturing.	3	Research papers / Group discussion / Short term courses

MODULE II

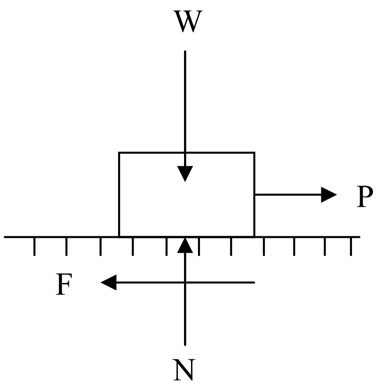
FRICTION AND BASICS STRUCTURAL ANALYSIS

Friction

- The force which opposes the movement or the tendency of movement is called **Frictional force or simply friction**. It is due to the resistance to motion offered by minutely projecting particles at the contact surfaces. However, there is a limit beyond which the magnitude of this force cannotincrease.
- If the applied force is more than this limit, there will be movement of one body over the other. This limiting value of frictional force when the motion is impending, it is known as **LimitingFriction**.
- When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called **Static Friction**, which will be having any value between zero and the limitingfriction.
- If the value of applied force exceeds the limiting friction, the body starts moving over the other body and the frictional resistance experienced by the body while moving is known as **Dynamic Friction**. Dynamic friction is less than limitingfriction.
- Dynamic friction is classified into following twotypes:

a) Slidingfriction
b) Rolling friction

- Sliding friction is the friction experienced by a body when it slides over the other body.
- Rolling friction is the friction experienced by a body when it rolls over a surface.
- It is experimentally found that the magnitude of limiting friction bears a constant ratio to the normal reaction between two surfaces and this ratio is called **Coefficient ofFriction**.



Coefficient of friction = $\frac{F}{N}$

where F is limiting friction and N is normal reaction between the contact surfaces.

Coefficient of friction is denoted by μ.

Thus, $\mu = \frac{F}{N}$

Laws of friction

1. The force of friction always acts in a direction opposite to that in which body tends tomove.
2. Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move thebody.
3. The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient offriction.
4. The force of friction depends upon the roughness/smoothness of thesurfaces.
5. The force of friction is independent of the area of contact between the two surfaces.
6. After the body starts moving, the dynamic friction comes into play, the magnitude of which is less than that of limiting friction and it bears a constant ratio with normal force. This ratio is called **coefficient of dynamicfriction**.

Angle of friction

Consider the block shown in figure resting on a horizontal surface and subjected to horizontal pull P. Let F be the frictional force developed and N the normal reaction. Thus, at contact surface the reactions are F and N. They can be graphically combined to get the reaction R which acts at angle θ to normal reaction. This angle θ called the angle of friction is givenby

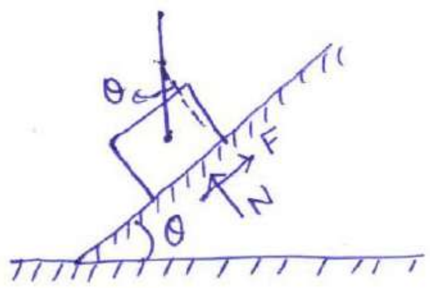
$$\tan\theta = \frac{F}{N}$$

As P increases, F increases and hence θ also increases. θ can reach the maximum value α when F reaches limiting value. At this stage,

$$\tan\alpha = \frac{F}{N} = \mu$$

This value of α is called Angle of Limiting Friction. Hence, the angle of limiting friction may be defined as the angle between the resultant reaction and the normal to the plane on which the motion of the body is impending.

Angle of repose



Consider the block of weight W resting on an inclined plane which makes an angle θ with the horizontal. When θ is small, the block will rest on the plane. If θ is gradually increased, a stage is reached at which the block start sliding down the plane. The angle θ for which the motion is impending, is called the angle of repose. Thus, the maximum inclination of the plane on which a body, free from external forces, can repose is called **Angle of Repose**.

Resolving vertically,
$N = W.\cos\theta$

Resolving horizontally,
$F = W.\sin\theta$

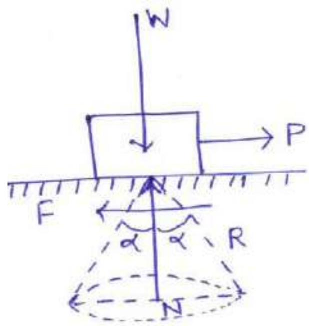
Thus, $\tan\theta = \frac{F}{N}$

If ϕ is the value of θ when the motion is impending, the frictional force will be limiting friction and hence,

$$\tan\phi = \frac{F}{N}$$
$$= \mu = \tan\alpha$$
$$\Rightarrow \phi = \alpha$$

Thus, the value of angle of repose is same as the value of limiting angle of repose.

Cone of friction

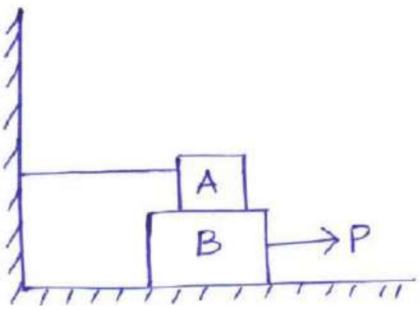


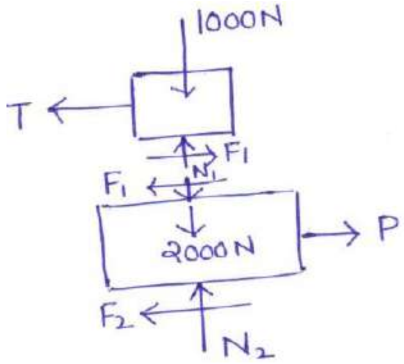
- When a body is having impending motion in the direction of force P, the frictional force will be limiting friction and the resultant reaction R will make limiting angle α with thenormal.
- If the body is having impending motion in some other direction, the resultant reaction makes limiting frictional angle α with the normal to that direction. Thus, when the direction of force P is gradually changed through 360˚, the resultant R generates a right circular cone with semi-central angle equal toα.

Problem 1: Block A weighing 1000N rests over block B which weighs 2000N as shown in figure. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is 1/3, what should be the value of P to move the block (B), if

(a) P ishorizontal.
(b) P acts at 30° upwards tohorizontal.

Solution: (a)





Considering block A,

$$\sum V = 0$$
$$N_1 = 1000N$$

Since F_1 is limiting friction,

$$\frac{F_1}{N_1} = \mu = 0.25$$
$$F_1 = 0.25N_1 = 0.25\times 1000 = 250N$$

$$\sum H = 0$$
$$F_1 - T = 0$$
$$T = F_1 = 250N$$

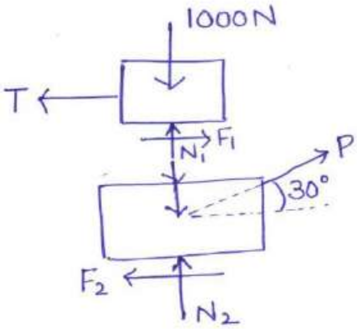
Considering equilibrium of block B,

$$\sum V = 0$$
$$N_2 - 2000 - N_1 = 0$$
$$N_2 = 2000 + N_1 = 2000 + 1000 = 3000N$$

$$\frac{F_2}{N_2} = \mu = \frac{1}{3}$$
$$F_2 = 0.3N_2 = 0.3\times 1000 = 1000N$$

$$\sum H = 0$$
$$P = F_1 + F_2 = 250 + 1000 = 1250N$$

(b) When P is inclined:



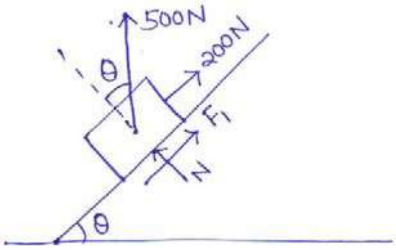
$$\sum V = 0$$
$$N_2 - 2000 - N_1 + P.\sin 30 = 0$$
$$\Rightarrow N_2 + 0.5P = 2000 + 1000$$
$$\Rightarrow N_2 = 3000 - 0.5P$$

From law of friction,

$$F_2 = \frac{1}{3}N_2 = \frac{1}{3}(3000 - 0.5P) = 1000 - \frac{0.5}{3}P$$

$$\sum H = 0$$
$$P\cos 30 = F_1 + F_2$$
$$\Rightarrow P\cos 30 = 250 + \left(1000 - \frac{0.5}{3}P\right)$$
$$\Rightarrow P\left(\cos 30 + \frac{0.5}{3}P\right) = 1250$$
$$\Rightarrow P = 1210.43N$$

Problem 2: A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. The same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coefficient of friction between the inclined plane and theblock.

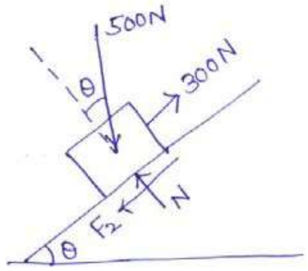


$$\sum V = 0$$
$$N = 500.\cos\theta$$
$$F_1 = \mu N = \mu.500\cos\theta$$

$\sum H = 0$

$200 + F_1 = 500.\sin\theta$ (1)

$\Rightarrow 200 + \mu.500\cos\theta = 500.\sin\theta$



$\sum V = 0$

$N = 500.\cos\theta$

$F_2 = \mu N = \mu.500.\cos\theta$

$\sum H = 0$

$500\sin\theta + F_2 = 300$ (2)

$\Rightarrow 500\sin\theta + \mu.500\cos\theta = 300$

Adding Eqs. (1) and (2), we get

500 = 1000. sinθ

sin θ = 0.5

θ = 30°

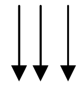
Substituting the value of θ in Eq. 2,

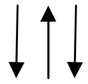
$500\sin 30 + \mu.500\cos 30 = 300$

$$\mu = \frac{50}{500\cos 30} = 0.11547$$

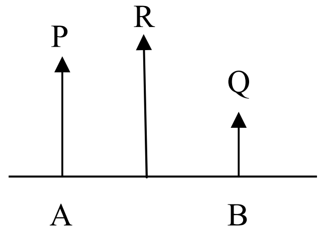
<u>Parallel forces on a plane</u>

Like parallel forces: Coplanar parallel forces when act in the same direction. **Unlike parallel forces:** Coplanar parallel forces when act in different direction. **Resultant of like parallel forces:**



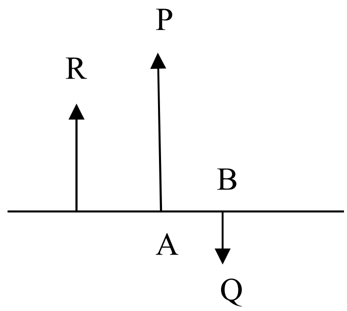


Let P and Q are two like parallel forces act at points A and B. R = P + Q



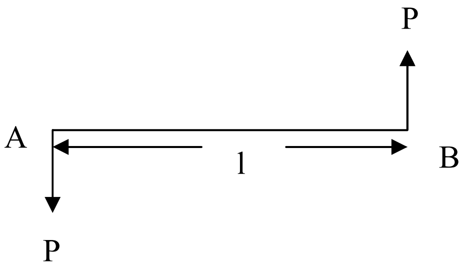
Resultant of unlikeparallelforces:
R = P-Q

R is in the direction of the force havinggreatermagnitude.



Couple:

Two unlike equal parallel forces form a couple.

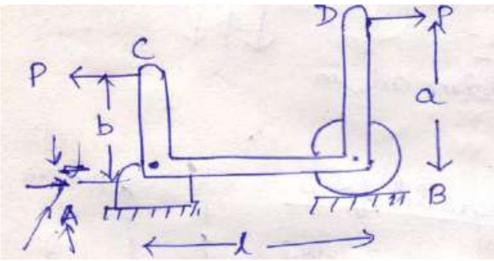


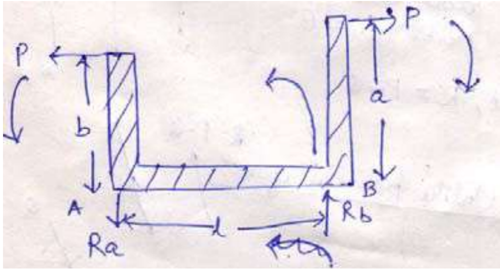
The rotational effect of a couple is measured by its moment.

Moment = P × l

Sign convention: Anticlockwise couple (Positive)
Clockwise couple (Negative)

Problem 1 :A rigid bar CABD supported as shown in figure is acted upon by two equal horizontal forces P applied at C and D. Calculate the reactions that will be induced at the points of support. Assume l = 1.2 m, a = 0.9 m, b =0.6 m.





$$\sum V = 0$$

$$R_a = R_b$$

Taking moment about A,

$$R_a = R_b$$

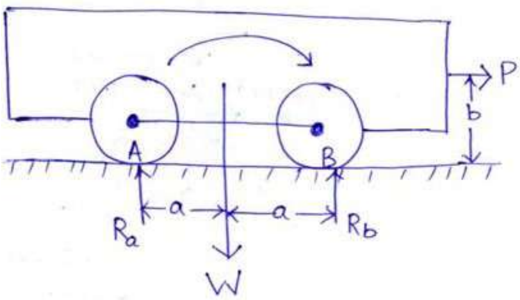
$$R_b \times l + P \times b = P \times a$$

$$\Rightarrow R_b = \frac{P(0.9 - 0.6)}{1.2}$$

$$\Rightarrow R_b = 0.25P(\uparrow)$$

$$\Rightarrow R_a = 0.25P(\downarrow)$$

Problem 2: Owing to weight W of the locomotive shown in figure, the reactions at the two points of support A and B will each be equal to W/2. When the locomotive is pulling the train and the drawbar pull P is just equal to the total friction at the points of contact A and B, determine the magnitudes of the vertical reactions R_a and R_b.



$$\sum V = 0$$

$$R_a + R_b = W$$

Taking moment about B,

$$\sum M_B = 0$$

$$R_a \times 2a + P \times b = W \times a$$

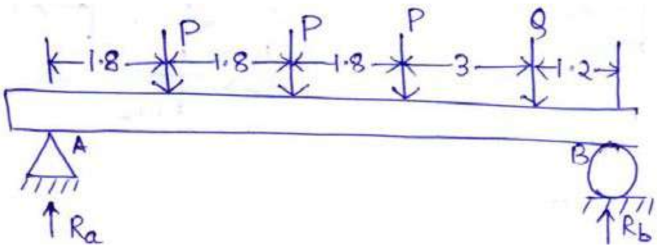
$$\Rightarrow R_a = \frac{W.a - P.b}{2a}$$

$$\therefore R_b = W - R_a$$

$$\Rightarrow R_b = W - \left(\frac{W.a - P.b}{2a}\right)$$

$$\Rightarrow R_b = \frac{W.a + P.b}{2a}$$

Problem 3: The four wheels of a locomotive produce vertical forces on the horizontal girder AB. Determine the reactions R_a and R_b at the supports if the loads P = 90 KN each and Q = 72 KN (All dimensions are in m).



$$\sum V = 0$$

$$R_a + R_b = 3P + Q$$

$$\Rightarrow R_a + R_b = 3 \times 90 + 72$$

$$\Rightarrow R_a + R_b = 342 KN$$

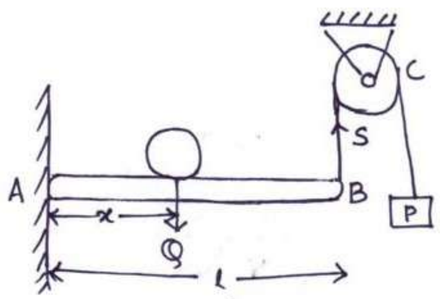
$$\sum M_A = 0$$

$$R_b \times 9.6 = 90 \times 1.8 + 90 \times 3.6 + 90 \times 5.4 + 72 \times 8.4$$

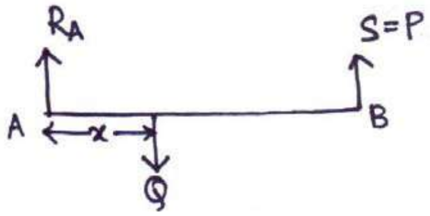
$$\Rightarrow R_b = 164.25 KN$$

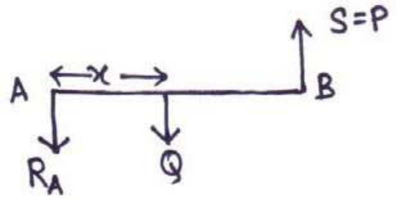
$$\therefore R_a = 177.75 KN$$

Problem 4: The beam AB in figure is hinged at A and supported at B by a vertical cord which passes over a frictionless pulley at C and carries at its end a load P. Determine the distance x from A at which a load Q must be placed on the beam if it is to remain in equilibrium in a horizontal position. Neglect the weight of thebeam.



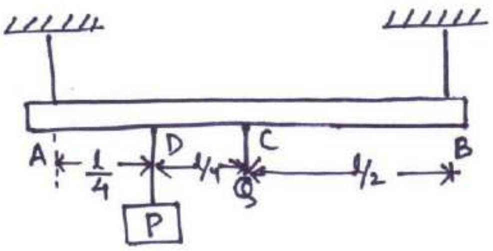
FBD





$$\sum M_A = 0$$
$$S \times l = Q \times x$$
$$\Rightarrow x = \frac{P.l}{Q}$$

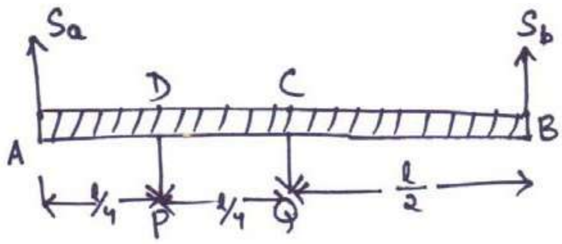
Problem 5: A prismatic bar AB of weight Q = 44.5 N is supported by two vertical wires at its ends and carries at D a load P = 89 N as shown in figure. Determine the forces S_a and S_b in the two wires.



Q = 44.5 N
P = 89 N

Resolving vertically,

$$\sum V = 0$$
$$S_a + S_b = P + Q$$
$$\Rightarrow S_a + S_b = 89 + 44.5$$
$$\Rightarrow S_a + S_b = 133.5N$$



$$\sum M_A = 0$$

$$S_b \times l = P \times \frac{l}{4} + Q \times \frac{l}{2}$$

$$\Rightarrow S_b = \frac{P}{4} + \frac{Q}{2}$$

$$\Rightarrow S_b = \frac{89}{4} + \frac{44.5}{2}$$

$$\Rightarrow S_b = 44.5$$

$$\therefore S_a = 133.5 - 44.5$$

$$\Rightarrow S_a = 89N$$

MODULE – III

CENTROID AND CENTRE OF GRAVITY AND VIRTUAL WORK AND ENERGY METHOD

COURSE OUTCOMES (COs):

At the end of the course students are able to:		
Course Outcomes		Knowledge Level (Bloom's Taxonomy)
CO 7	**Identify** the centroid, centre of gravity and moment of inertia for the simple plane sections from the first principles.	Apply
CO 8	**Explore** the theorems of moment and the mass moment of inertia of circular plate, cylinder, cone and sphere.	Apply
CO 9	**Apply** the concepts of virtual work and work-energy method for single and connected configured systems.	Apply

PROGRAM OUTCOMES (POs):

Program Outcomes (POs)		Strength	Proficiency Assessed by
PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	3	CIE/Quiz/AAT
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences	1	CIE/Quiz/AAT
PO 4	**Conduct Investigations of Complex Problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.	1	Seminar/ conferences / Research papers
PSO 1	Formulate and evaluate engineering concepts of design, thermal and production to provide solutions for technology aspects in digital manufacturing.	3	Research papers / Group discussion / Short term courses

MODULE III

CENTROID AND CENTRE OF GRAVITY AND VIRTUAL WORK AND ENERGY METHOD

Centre of gravity

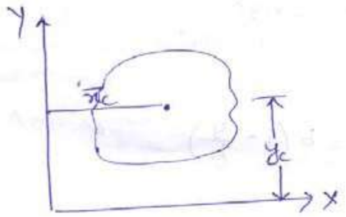
Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

As the point through which resultant of force of gravity (weight) of the bodyacts.

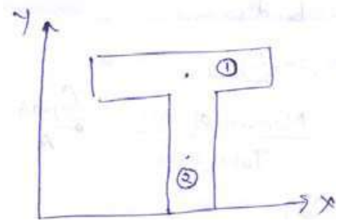
Centroid: Centrroid of an area lies on the axis of symmetry if it exits.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

$$x_c = \sum A_i x_i y_c$$
$$= \sum A_i y_i$$



$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$
$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

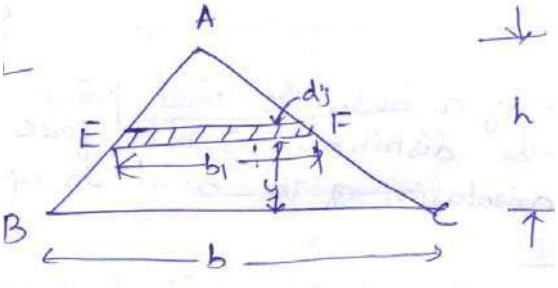


$$x_c = y_c = \frac{\text{Moment of area}}{\text{Totalarea}}$$

$$x_c = \frac{\int x.dA}{A}$$

$$y_c = \frac{\int y.dA}{A}$$

Problem 1: Consider the triangle ABC of base 'b' and height 'h'. Determine the distance of centroid from the base.



Let us consider an elemental strip of width 'b_1' and thickness 'dy'.

$$\Delta AEF \sim \Delta ABC$$

$$\therefore \frac{b_1}{b} = \frac{h-y}{h}$$

$$\Rightarrow b_1 = b\left(\frac{h-y}{h}\right)$$

$$\Rightarrow b_1 = b\left(1-\frac{y}{h}\right)$$

Area of element EF (dA) = $b_1 \times dy$

$$= b\left(1-\frac{y}{h}\right)dy$$

$$\bar{y} = \frac{\int y.dA}{A}$$

$$= \frac{\int_0^h y.b\left(1-\frac{y}{h}\right)dy}{\frac{1}{2}b.h}$$

$$= \frac{b\left[\frac{y^2}{2}-\frac{y^3}{3h}\right]_0^h}{\frac{1}{2}b.h}$$

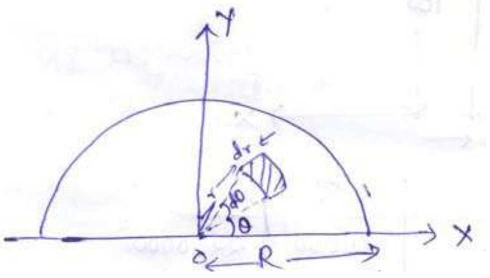
$$= \frac{2}{h}\left[\frac{h^2}{2}-\frac{h^3}{3}\right]$$

$$= \frac{2}{h}\times\frac{h^2}{6}$$

$$= \frac{h}{3}$$

Therefore, y_c is at a distance of h/3 from base.

Problem 2: Consider a semi-circle of radius R. Determine its distance from diametral axis.



Due to symmetry, centroid 'y_c' must lie on Y-axis.

Consider an element at a distance 'r' from centre 'o' of the semicircle with radial width dr.

Area of element = (r.dθ)×dr

Moment of area about x = $\int y.dA$

$$=\int_0^{\pi}\int_0^{R}(r.d\theta).dr\times(r.\sin\theta)$$

$$=\int_0^{\pi}\int_0^{R} r^2\sin\theta.dr.d\theta$$

$$=\int_0^{\pi}\int_0^{R}\left(r^2.dr\right).\sin\theta.d\theta$$

$$=\int_0^{\pi}\left[\frac{r^3}{3}\right]_0^R.\sin\theta.d\theta$$

$$=\int_0^{\pi}\frac{R^3}{3}.\sin\theta.d\theta$$

$$=\frac{R^3}{3}\left[-\cos\theta\right]_0^{\pi}$$

$$=\frac{R^3}{3}\left[1+1\right]$$

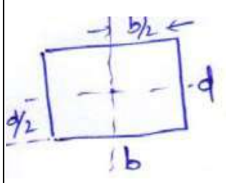
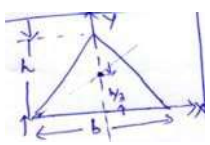
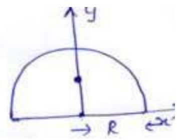
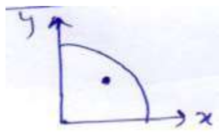
$$=\frac{2}{3}R^3$$

$$y_c=\frac{\text{Moment of area}}{\text{Totalarea}}$$

$$= \frac{\frac{2}{3}R^3}{\frac{\pi R^2}{2}}$$

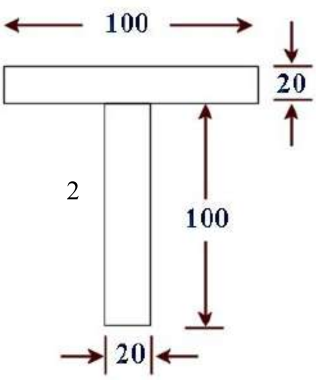
$$= \frac{4R}{3\pi}$$

Centroids of different figures

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

Problem 3: Find the centroid of the T-section as shown in figure from the bottom.

1

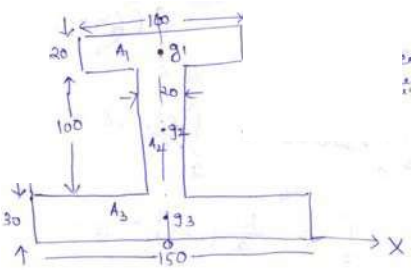


Area (A_i)	x_i	y_i	A_i x_i	A_iy_i
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Problem 4: Locate the centroid of the I-section.



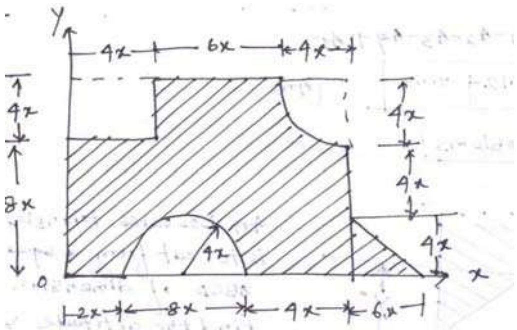
As the figure is symmetric, centroid lies on y-axis. Therefore, $\bar{x} = 0$

Area (A_i)	x_i	y_i	A_i x_i	A_iy_i
2000	0	140	0	280000
2000	0	80	0	160000
4500	0	15	0	67500

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.71\,mm$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

Problem 5: Determine the centroid of the composite figure about x-y coordinate. Take x = 40 mm.



A_1 = Area of rectangle = $12x.14x = 168x^2$

A_2 = Area of rectangle to be subtracted = $4x.4x = 16\,x^2$

A_3 = Area of semicircle to be subtracted = $\frac{\pi R^2}{2} = \frac{\pi(4x)^2}{2} = 25.13x^2$

A_4 = Area of quatercircle to be subtracted = $\frac{\pi R^2}{} = \frac{\pi(4x)^2}{} = 12.56x^2$

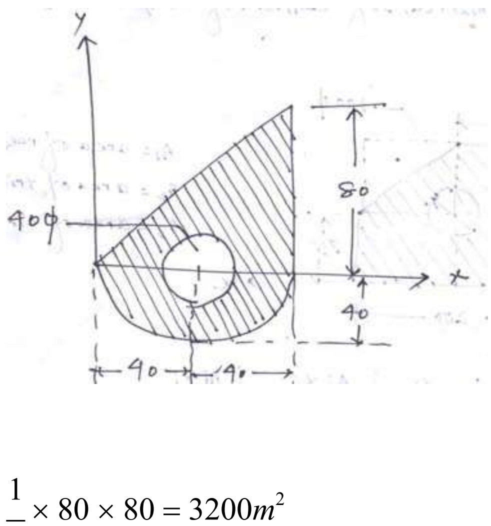
A_5 = Area of triangle =44 $\frac{1}{2} \times 6x \times 4x = 12x^2$

Area (A_i)	x_i	y_i	$A_i\, x_i$	$A_i y_i$
A_1 = 268800	7x = 280	6x =240	75264000	64512000
A_2 = 25600	2x = 80	10x=400	2048000	10240000
A_3 = 40208	6x =240	$\frac{4 \times 4x}{3\pi}$ =67.906	9649920	2730364.448
A_4 = 20096	$10x + \left(4x - \frac{4\times 4x}{3\pi}\right)$ = 492.09	$8x + \left(4x - \frac{4\times 4x}{3\pi}\right)$ = 412.093	9889040.64	8281420.926
A_5 = 19200	$14x + \frac{6x}{3} = 16x$ = 640	$\frac{4x}{3}$ = 53.33	12288000	1023936

$$\bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4 + A_5x_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 326.404mm$$

$$\bar{y} = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4 + A_5y_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 219.124mm$$

Problem 6: Determine the centroid of the following figure.



A_1 = Area of triangle = $\frac{1}{2} \times 80 \times 80 = 3200m^2$

A_2 = Area of semicircle= $\frac{\pi d^2}{8} - \frac{\pi R^2}{2}$ 2513.274m

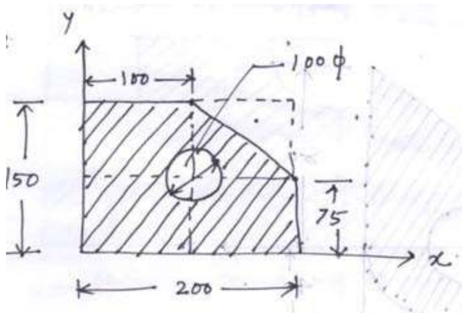
A_3 = Area of semicircle $\frac{\pi D^2}{} = 1256.64m^2$

Area (A_i)	x_i	y_i	$A_i\ x_i$	A_iy_i
3200	2×(80/3)=53.33	80/3 = 26.67	170656	85344
2513.274	40	$\frac{-4\times 40}{3\pi}=-16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c=\frac{A_1x_1+A_2x_2-A_3x_3}{A_1+A_2+A_3}=49.57mm$$

$$y_c=\frac{A_1y_1+A_2y_2-A_3y_3}{A_1+A_2-A_3}=9.58mm$$

Problem 7: Determine the centroid of the following figure.



A_1 = Area of the rectangle
A_2 = Area of triangle
A_3 = Area of circle

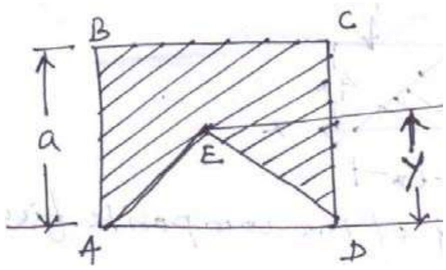
Area (A_i)	x_i	y_i	$A_i\ x_i$	A_iy_i
30,000	100	75	3000000	2250000
3750	100+200/3 = 166.67	75+150/3 =125	625012.5	468750
7853.98	100	75	785398	589048.5

$$x_c=\frac{\sum A_ix_i}{\sum A_i}=\frac{A_1x_1-A_2x_2-A_3x_3}{A_1-A_2-A_3}=86.4mm$$

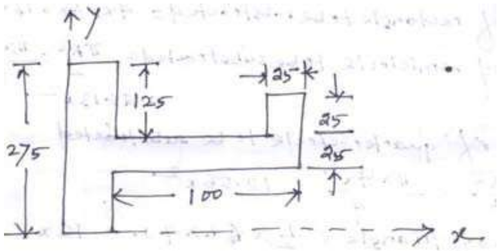
$$y_c=\frac{\sum A_iy_i}{\sum A_i}=\frac{A_1y_1-A_2y_2-A_3y_3}{A_1-A_2-A_3}=64.8mm$$

Numerical Problems (Assignment)

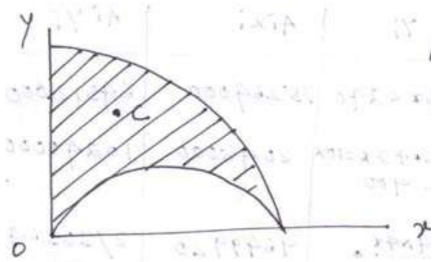
1. An isosceles triangle ADE is to cut from a square ABCD of dimension ‘a’. Find the altitude ‘y’ of the triangle so that vertex E will be centroid of remaining shadedarea.



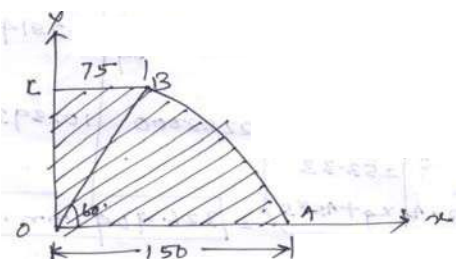
2. Find the centroid of the followingfigure.



3. Locate the centroid C of the shaded area obtained by cutting a semi-circle of diameter ‘a’ from the quadrant of a circle of radius‘a’.



4. Locate the centroid of the compositefigure.



Truss/ Frame: A pin jointed frame is a structure made of slender (cross-sectional dimensions quite small compared to length) members pin connected at ends and capable of taking load at joints.

Such frames are used as roof trusses to support sloping roofs and as bridge trusses to support deck.

Plane frame: A frame in which all members lie in a single plane is called plane frame. They are designed to resist the forces acting in the plane of frame. Roof trusses and bridge trusses are the example of plane frames.

Space frame: If all the members of frame do not lie in a single plane, they are called as space frame. Tripod, transmission towers are the examples of spaceframes.

Perfect frame: A pin jointed frame which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a perfect frame. Triangular frame is the simplest perfect frame and it has 03 joints and 03members.

It may be observed that to increase one joint in a perfect frame, two more members are required. Hence, the following expression may be written as the relationship between number of joint j, and the number of members m in a perfect frame.

$$m = 2j - 3$$

(a) When LHS = RHS, Perfectframe.
(b) When LHS<RHS, Deficientframe.
(c) When LHS>RHS, Redundantframe.

Assumptions

The following assumptions are made in the analysis of pin jointed trusses:

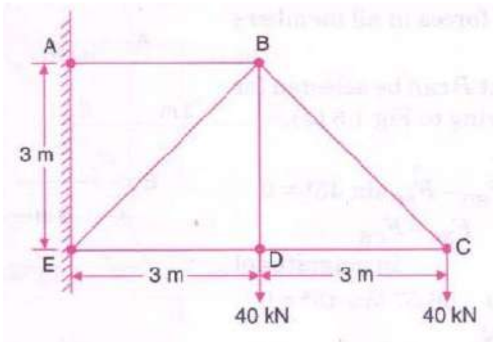
1. The ends of the members are pin jointed(hinged).
2. The loads act only at thejoints.
3. Self weight of the members isnegligible.

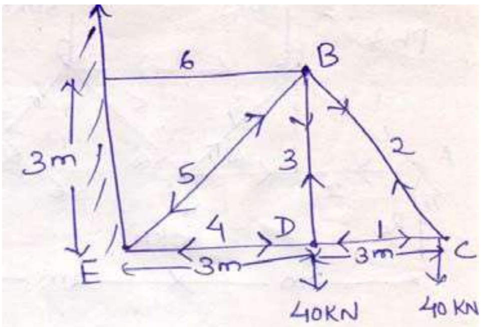
Methods of analysis

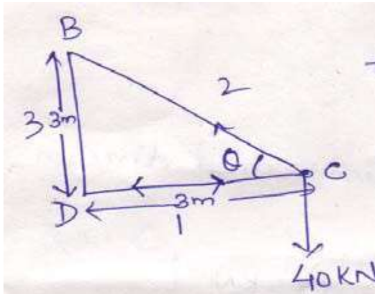
1. Method ofjoint
2. Method ofsection

Problems on method of joints

Problem 1: Find the forces in all the members of the truss shown in figure.



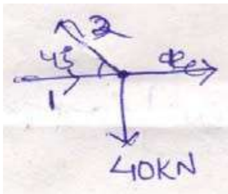




$\tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

<u>Joint C</u>



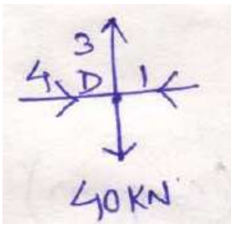
$S_1 = S_2 \cos 45$

$\Rightarrow S_1 = 40KN$ (Compression)

$S_2 \sin 45 = 40$

$\Rightarrow S_2 = 56.56KN$ (Tension)

<u>Joint D</u>

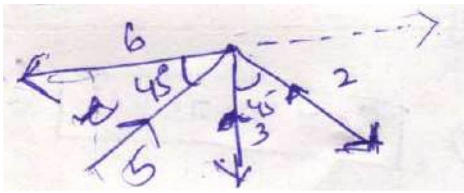


$S_3 = 40KN$ (Tension)

$S_1 = S_4 = 40KN$ (Compression) <u>Joint</u>

<u>B</u>

Resolving vertically,



$\sum V = 0$

$S_5 \sin 45 = S_3 + S_2 \sin 45$

$\Rightarrow S_5 = 113.137 KN$ (Compression)

Resolving horizontally,

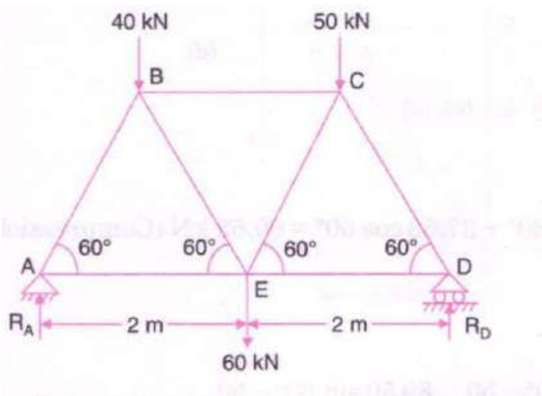
$\sum H = 0$

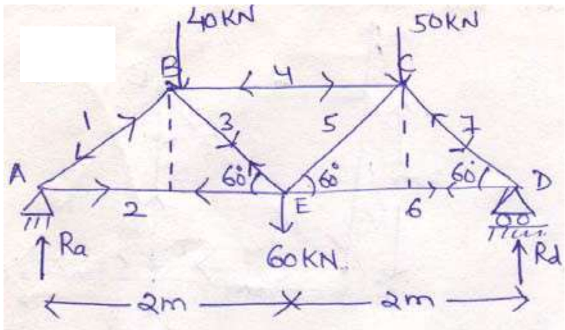
$S_6 = S_5 \cos 45 + S_2 \cos 45$

$\Rightarrow S_6 = 113.137 \cos 45 + 56.56 \cos 45$

$\Rightarrow S_6 = 120 KN$ (Tension)

Problem 2: Determine the forces in all the members of the truss shown in figure and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is2m.





Taking moment at point A,

$\sum M_A = 0$

$R_d \times 4 = 40 \times 1 + 60 \times 2 + 50 \times 3$

$\Rightarrow R_d = 77.5 KN$

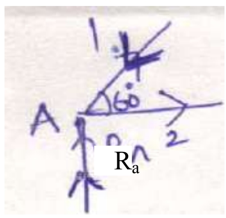
Now resolving all the forces in vertical direction,

$\sum V = 0$

$R_a + R_d = 40 + 60 + 50$

$\Rightarrow R_a = 72.5 KN$

<u>Joint A</u>



$\sum V = 0$

$\Rightarrow R_a = S_1 \sin 60$

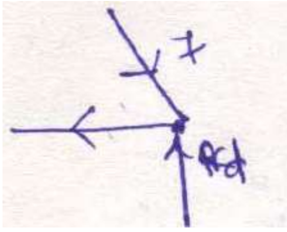
$\Rightarrow S_1 = 83.72 KN$ (Compression)

$\sum H = 0$

$\Rightarrow S_2 = S_1 \cos 60$

$\Rightarrow S_1 = 41.86 KN$ (Tension)

Joint D



$\sum V = 0$

$S_7 \sin 60 = 77.5$

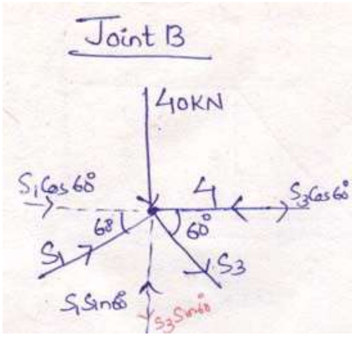
$\Rightarrow S_7 = 89.5 KN$ (Compression)

$\sum H = 0$

$S_6 = S_7 \cos 60$

$\Rightarrow S_6 = 44.75 KN$ (Tension)

Joint B



$\sum V = 0$

$S_1 \sin 60 = S_3 \cos 60 + 40$

$\Rightarrow S_3 = 37.532 KN$ (Tension)

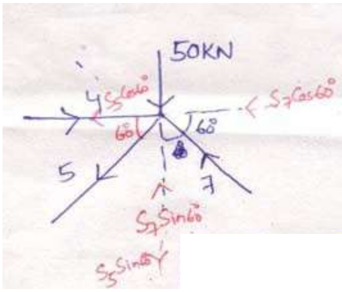
$\sum H = 0$

$S_4 = S_1 \cos 60 + S_3 \cos 60$

$\Rightarrow S_4 = 37.532 \cos 60 + 83.72 \cos 60$

$\Rightarrow S_4 = 60.626 KN$ (Compression)

Joint C



$\sum V = 0$

$S_5 \sin 60 + 50 = S_7 \sin 60$

$\Rightarrow S_5 = 31.76 KN$ (Tension)

Plane Truss (Method of Section)

In case of analysing a plane truss, using method of section, after determining the support reactions a section line is drawn passing through not more than three members in which forces are unknown, such that the entire frame is cut into two separate parts.

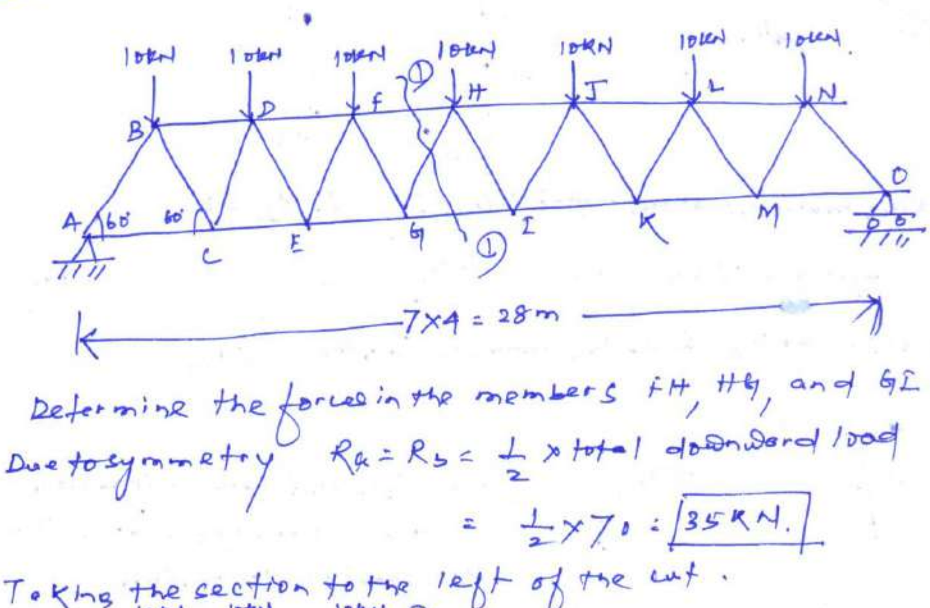
Each part should be in equilibrium under the action of loads, reactions and the forces in the members.

Method of section is preferred for the following cases:

(i) analysis of large truss in which forces in only few members are required

(ii) If method of joint fails to start or proceed with analysis for not getting a joint with only two unknown forces.

Example 1.

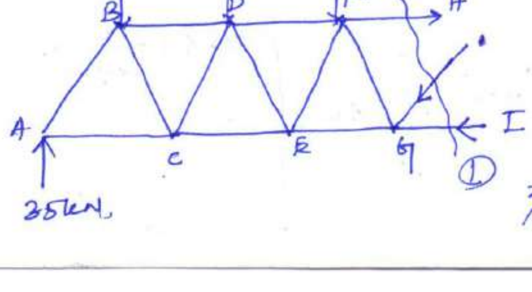


Determine the forces in the members FH, HG, and GI in the truss

Due to symmetry $R_A = R_O = \frac{1}{2} \times$ total downward load

$= \frac{1}{2} \times 70 = \boxed{35\,KN}$

Taking the section to the left of the cut.



Taking moment about G

$\Sigma M_G = 0.$

$F_{FH} \times 4\sin 60 + 35 \times 12$

$= 10 \times 2 + 10 \times 6 + 10 \times 10$

$\Rightarrow F_{FH} = \frac{(20+60+100) - 420}{4\sin 60^\circ}$

$= -69.28\ kN.$

Negative sign indicates that direction should have opposite i.e it is compressive in nature.

Now Resolving all the forces vertically $\Sigma Y = 0$

$$10 + 10 + 10 + F_{GH} \sin 60 = 35$$

$$\Rightarrow F_{GH} = \frac{35 - 30}{\sin 60^\circ}$$

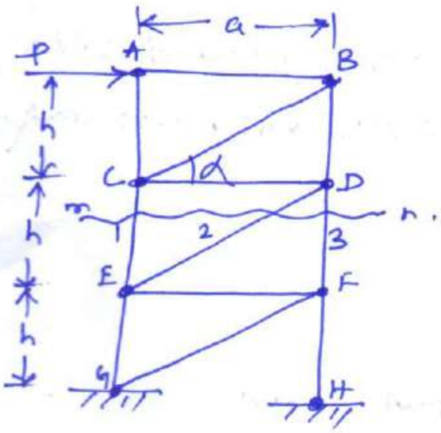
$$\Rightarrow \boxed{F_{GH} = 5.78 \text{ kN.}} \text{ (compressive)}$$

Resolving all the forces horizontally $\Sigma X = 0$.

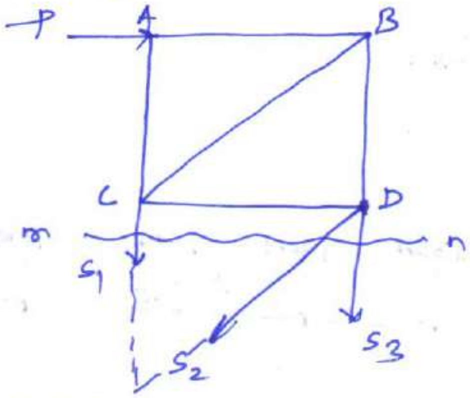
$$F_{FH} + F_{GH} \cos 60 = F_{GI}$$

$$\Rightarrow F_{GI} = 69.28 + 5.78 \cos 60^\circ = \boxed{72.17 \text{ kN.}} \text{ (tension)}$$

Q.2



Using method of sections determine the axial forces in bars 1, 2 and 3.



Taking moment about ~~poi~~ joint D $\Sigma M_D = 0$.

$$S_1 \times a = P \times h \Rightarrow \boxed{S_1 = \frac{Ph}{a}} \quad \text{— (1)} \quad \text{(tension)}$$

Similarly taking E as the moment centre $\Sigma M_E = 0$

$$S_3 \times a + P \times 2h = 0$$

$$\Rightarrow \boxed{S_3 = \frac{-2Ph}{a}}$$

(−ve sign indicates direction of force will be opposite and it will be compressive in nature)

Resolving all the forces horizontally $\Sigma X = 0$.

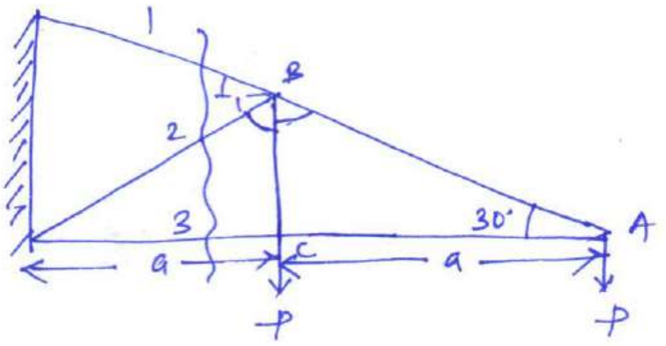
$$S_2 \cos\alpha = P$$

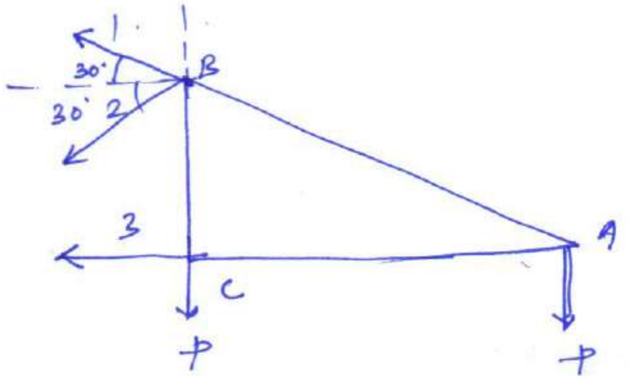
$$\Rightarrow S_2 = \frac{P}{\cos\alpha} = \boxed{\frac{P\sqrt{a^2 + h^2}}{a}} \text{ (Ans)}$$

$$\cos\alpha = \frac{a}{\sqrt{a^2 + h^2}}$$

$\frac{BC}{AC} = \tan 30°$

$\Rightarrow BC = a \tan 30 = \boxed{0.578a}$





$\Sigma M_B = 0.$

$S_3 \times 0.578a + P \times a = 0$

$\Rightarrow S_3 = \frac{-Pa}{0.578a} = -1.73P$

(-ve sign indicates direction is opposite and it is compressive in nature)

Resolving vertically $\Sigma Y = 0$

$S_1 \sin 30 = 2P + S_2 \sin 30$

$\Rightarrow S_1 = \frac{2P + S_2/2}{\sin 30} = (4P + S_2)$ — (2)

Now resolving horizontally $\Sigma X = 0.$

$S_1 \cos 30 + S_2 \cos 30 = 1.73P$

$\Rightarrow (4P + S_2) \times \frac{\sqrt{3}}{2} + S_2 \frac{\sqrt{3}}{2} = 1.73P$

$\Rightarrow 2\sqrt{3}P + \frac{\sqrt{3}}{2} S_2 + \frac{\sqrt{3}}{2} S_2 = 1.73P$

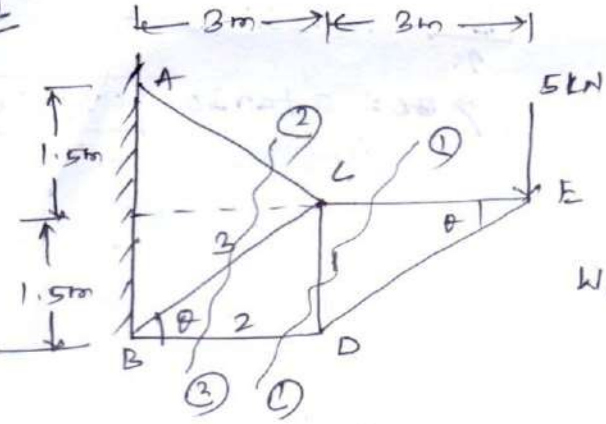
$\Rightarrow \frac{\sqrt{3}}{\cancel{2}} S_2 = 1.73P - 2\sqrt{3}P$

$= -1.73P$

$\Rightarrow S_2 = \frac{-1.73P}{\sqrt{3}} = \boxed{-P}$ (-ve sign indicates the direction is opposite and it is compressive)

Now $S_1 = 4P - P = \boxed{3P}$ (tension)

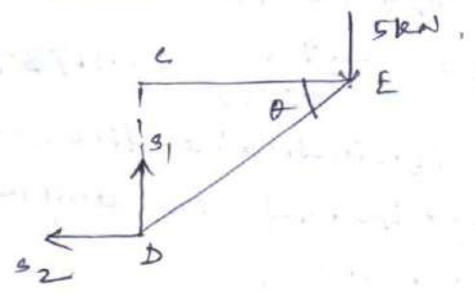
Q.4



Using method of sections, find axial forces in each bar 1, 2 and 3 of the plane truss.

We have $\tan\theta = \left(\frac{1.5}{3}\right) \Rightarrow \boxed{\theta = 26.56^\circ}$

considering section 1-1



Resolving vertically, $\Sigma Y = 0$

$\boxed{S_1 = 5kN}$ (tension)

Now taking moment about C

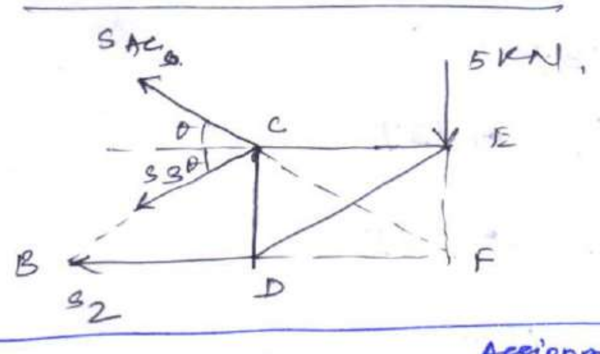
$S_2 \times 1.5 + 5 \times 3 = 0$

$\Rightarrow S_2 = -10kN$

-ve sign indicates direction should have been opposite

$\boxed{S_2 = 10kN}$ (compression)

Considering section 2-2

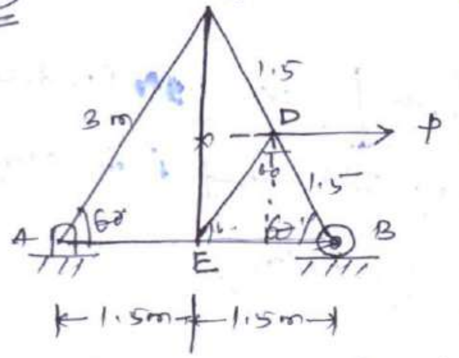


Taking moment about F

$\Sigma M_F = 0$

$\Rightarrow \boxed{S_3 = 0}$

Q.5



Assignment

Using method of joint and method of section find the axial force in the bar x.

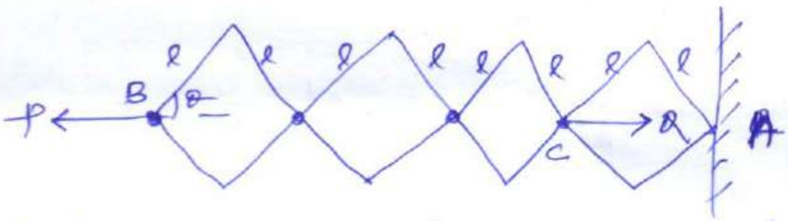
Method of Joint

considering the whole structure and taking moment about A $\Sigma M_A = 0$.

$R_B \times 3 = P \times 1.5 \sin 60$

$\Rightarrow \boxed{R_B = \frac{\sqrt{3}}{4} P.}$

Q.1 (6.3) Calculate the relation betn active forces P and Q for equilibrium of system of bars. The bars are so arranged that they form identical rhombuses.



Let l = length of each side of bar.

θ = angle made by each side of the rhombus

Distance of P from fixed point A = $6l\cos\theta$

" Q " = $2l\cos\theta$

Let the virtual displacement of P is B-B'

$B\text{-}B' = dx_1 = \frac{d}{d\theta}(6l\cos\theta)\,d\theta = -6l\sin\theta\, d\theta$

Similarly the virtual displacement of Q is C-C'

$= dx_2 = -2l\sin\theta\, d\theta$

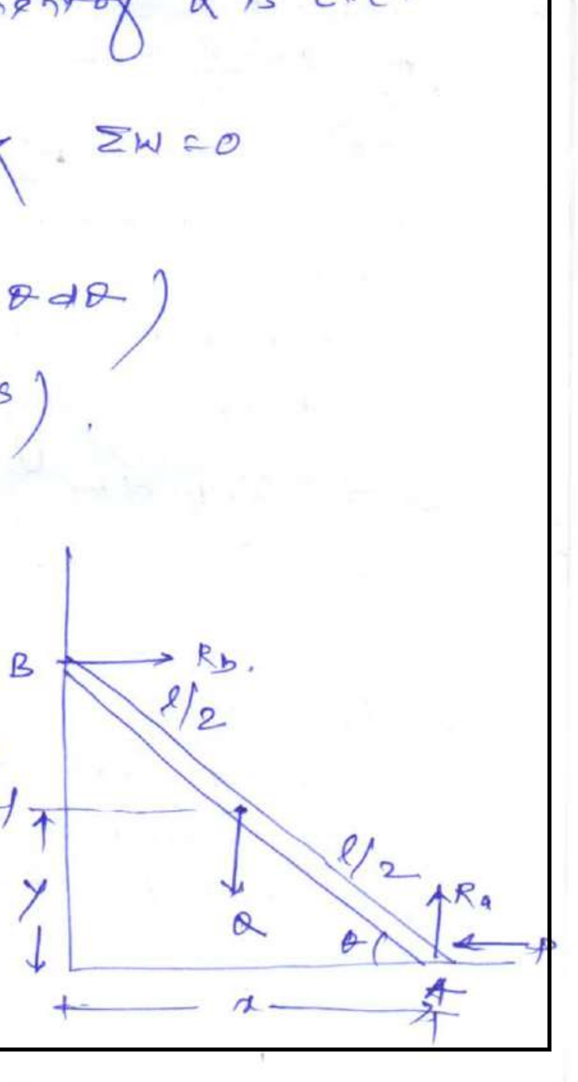
Applying principle of virtual work $\Sigma W = 0$

$P.\,dx_1 = Q.\,dx_2$

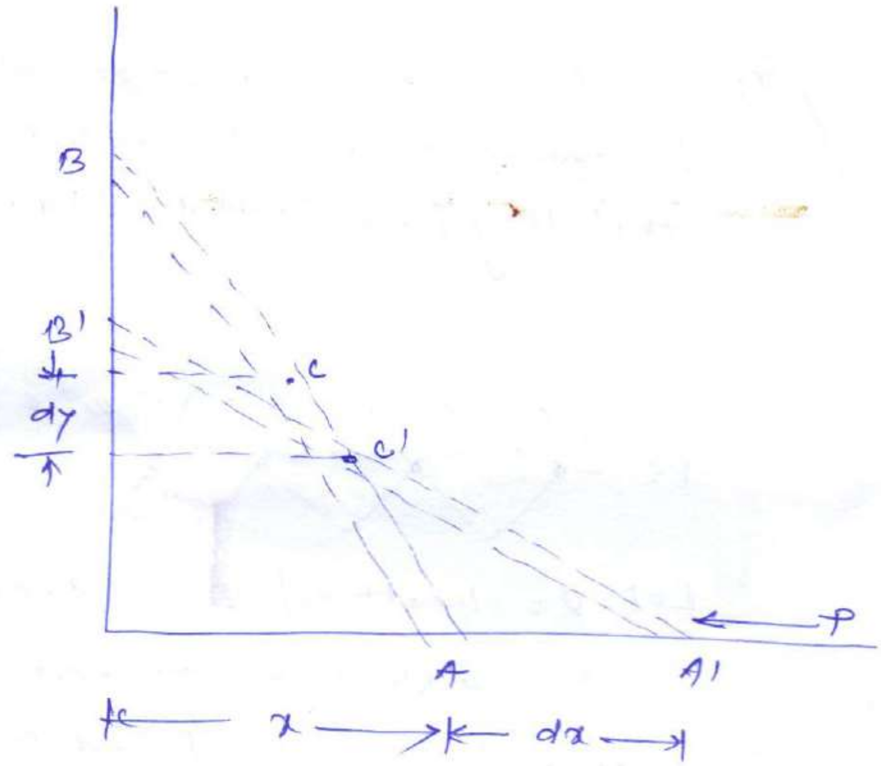
$\Rightarrow P.(6l\sin\theta\, d\theta) = Q(2l\sin\theta\, d\theta)$

$\Rightarrow \boxed{P = \frac{Q}{3}}$ (Ans).

Q.2 A prismatic bar AB of length l and wt. Q stands in a vertical plane and is supported by smooth surfaces at A and B. Using principle of virtual work find the magnitude of horizontal force P applied at A if the bar is in equilibrium.



5



Let the horizontal distance of P from D is x

$x = l\cos\theta$

$AA' = dx = -l\sin\theta\, d\theta$

vertical distance of Q from D is y

$y = \frac{l}{2}\sin\theta$

$CC' = dy = \frac{l}{2}\cos\theta\, d\theta$

Normal reactions Ra and Rb have no work along the planes.

Applying principle of virtual work $\Sigma W = 0$

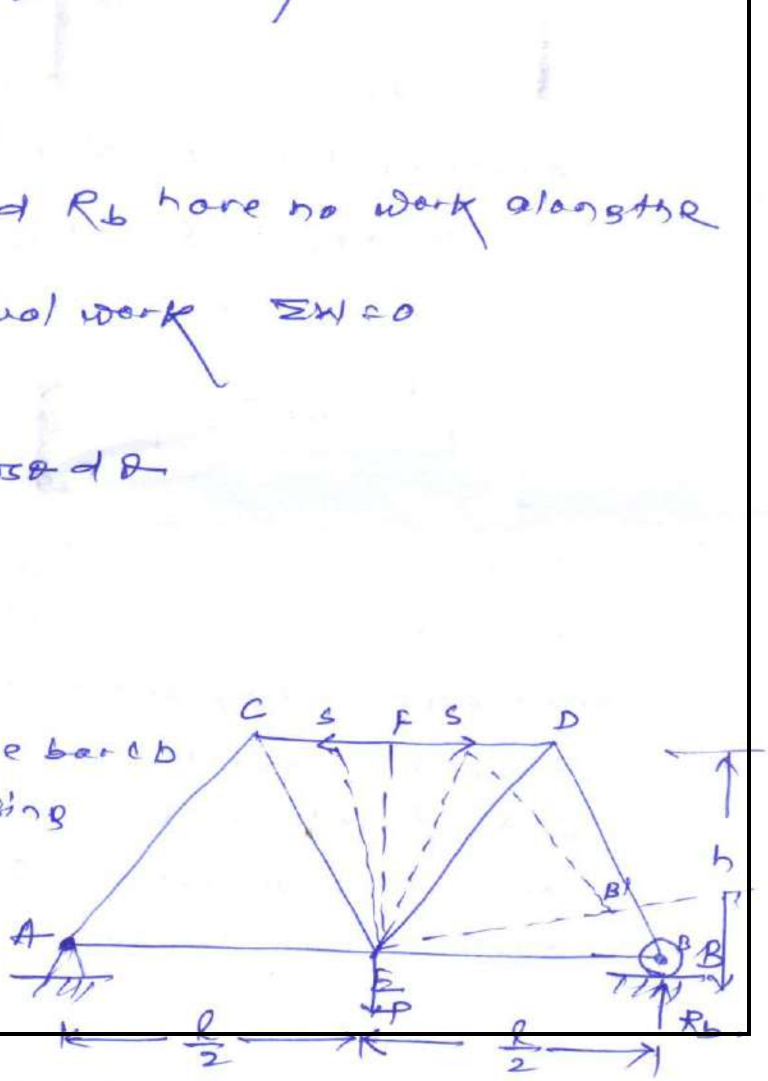
$P\,dx = Q\,dy$

$Pl\sin\theta\, d\theta = Q\frac{l}{2}\cos\theta\, d\theta$

$\Rightarrow \boxed{P = \frac{Q}{2}\cdot\cot\theta}$

Q.3 (6.14)

Find axial forces in the bar CD of the simple truss by using method of virtual work.



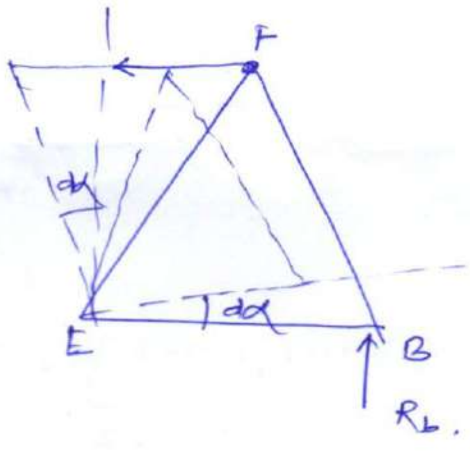
13/11/14 (2)

Let S be the compressive force in bar CD.

consider the part EBDF of the truss under the action of force R_b, P and S

Keeping E fixed and giving EB an angular displacement $d\alpha$

$\Sigma W = 0$.



$R_b \times BB' = S \times FF'$

$BB' = \frac{l}{2} d\alpha$

$FF' = h\, d\alpha$

$R_b \times \frac{l}{2} d\alpha = S \times h\, d\alpha$

$\Rightarrow \boxed{S = \frac{R_b l}{2h}}$ — (1)

Now considering whole frame as equilibrium body $\Sigma y = 0$.

$R_a + R_b = P$.

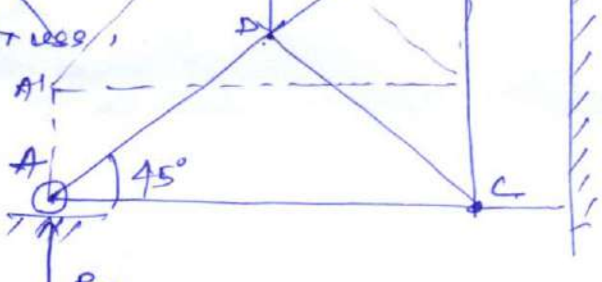
$R_b \cdot l = P \cdot \frac{l}{2} \Rightarrow \boxed{R_b = \frac{P}{2}}$ — (2)

Substituting the value of R_b in eq. (1)

$\boxed{S = \frac{Pl}{4h}}$ — (3) (Ans)

Q.4 (6.15)

Using principle of virtual work find reactions R_a for the truss.



Let the truss is virtual displaced by an amount dy

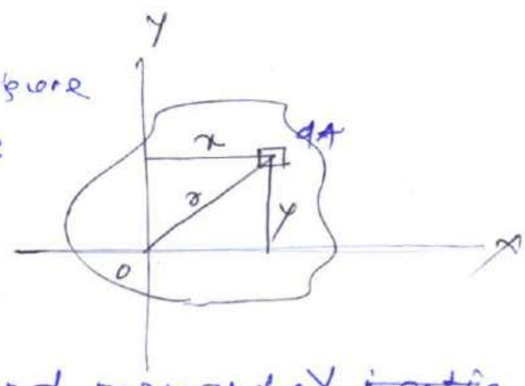
$\Sigma W = 0$.

$R_a \times AA' = P \times DD'$

where $AA' = DD' = dy$

$\Rightarrow \boxed{R_a = P}$

modipada to bigbazar
near jagnnath mandir
righthandside



The moment of inertia of any plane figure with respect to x and y axes in its plane are expressed as

$I_x = \int y^2 \, dA \quad I_y = \int x^2 \, dA$

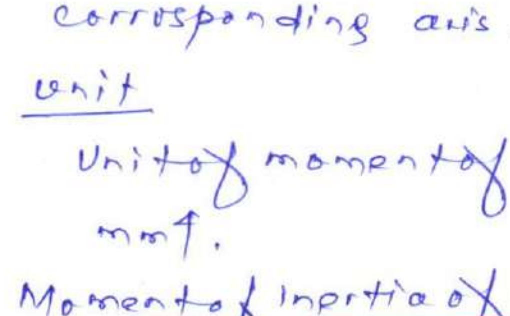
- I_{xx} and I_{yy} are also known as second moment of ~~inertia~~ area about the axes as it is distance is squared from corresponding axis.

unit

Unit of moment of inertia of area is expressed as m^4 or mm^4.

Moment of inertia of Plane figures:-

(i) Rectangle



Considering a rectangle of width b and depth d. Moment of inertia about centroidal axis x-x parallel to the short side i.e b

Now considering an elementary strip of width dy

Moment of inertia of the elemental strip about centroidal axis xx is

$I_{xx} = y^2 \, dA$

$= y^2 \, b\,dy$

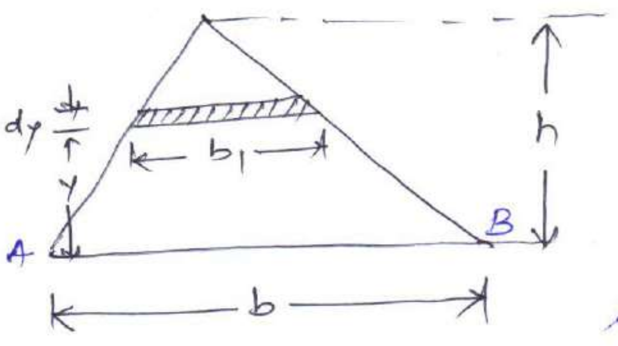
So moment of inertia of entire ~~figure~~ area

$$I_{xx} = \int_{-d/2}^{d/2} y^2 b\,dy = b \left| \frac{y^3}{3} \right|_{-\frac{d}{2}}^{d/2} = b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

$$\Rightarrow \boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly ~~moment at~~ $\boxed{I_{yy} = \frac{db^3}{12}}$

(ii) Triangle :- (Moment of inertia of a triangle about it's b



Consider a small elementary strip at a distance y from the base of thickness dy. Let dA is the area of strip

$dA = b_1 dy$

And $b_1 = \frac{(h-y)}{h} \times b$.

Moment of inertia of strip about base AB

$= y^2 dA = y^2 b_1 dy$

$= y^2 \frac{(h-y)}{h} . b\, dy$

$\therefore$ Moment of inertia of the triangle about AB

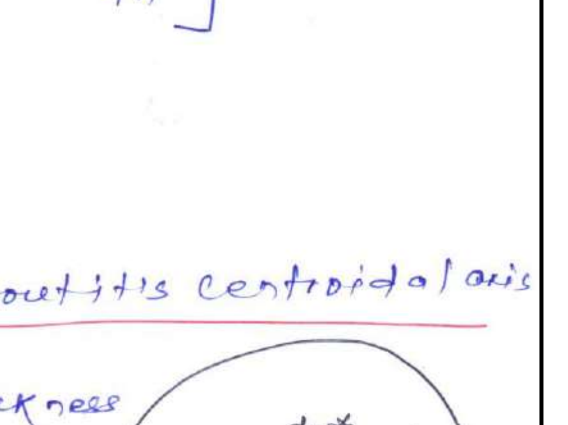
$$I_{AB} = \int_0^h \frac{y^2(h-y)\, b\,dy}{h} = \int_0^h \left(y^2 - \frac{y^3}{h}\right) b\,dy$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h = b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$= b\left[\frac{h^3}{3} - \frac{h^3}{4}\right] = \frac{bh^3}{12}$$

$$\Rightarrow \boxed{I_{AB} = \frac{bh^3}{12}}$$

(iii) Moment of inertia of a circle about it's centroidal axis



Considering an elementary strip of thickness dr, the side of strip $rd\theta$.

moment of inertia of strip about xx

$= y^2 dA$

$= (r\sin\theta)^2\, r\,d\theta\, dr$

$= r^3 \sin^2\theta\, d\theta\, dr$

$\therefore$ Moment of inertia of circle about xx axis

$$I_{xx} = \int_0^R \int_0^{2\pi} r^3 \sin^2\theta\, d\theta\, dr$$

$$= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1-\cos 2\theta}{2}\right) d\theta\, dr$$

$$= \int_0^R \frac{r^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr$$

$$= \int_0^R \frac{r^3}{2}\left(2\pi - \frac{\sin 4\pi}{2}\right) dr$$

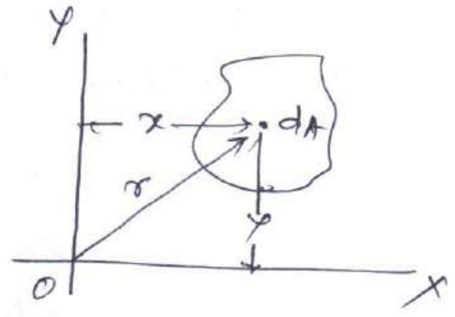
$$= \left[\frac{r^4}{8}\right]_0^R \left[2\pi - 0\right]$$

$$= \frac{R^4}{8} 2\pi = \frac{\pi R^4}{4}$$

$$\Rightarrow \boxed{I_{xx} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}} \qquad \left(\because R = \frac{D}{2}\right)$$

Polar moment of inertia:-

Moment of inertia about an axis perpendicular to the plane of area is called polar moment of inertia it may denoted as J or I_{zz}



$$\boxed{I_{zz} = \Sigma r^2 dA}$$

Radius of Gyration:-

Radiaus of gyration may be defined by a relation

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

where K = radius of gyration

I = moment of inertia

A = cross-sectional area

so, we can have the following relations

$$\boxed{\begin{aligned} K_{xx} &= \sqrt{\frac{I_{xx}}{A}} \\ K_{yy} &= \sqrt{\frac{I_{yy}}{A}} \\ K_{AB} &= \sqrt{\frac{I_{AB}}{A}} \end{aligned}}$$

Theorems of Moment of inertia

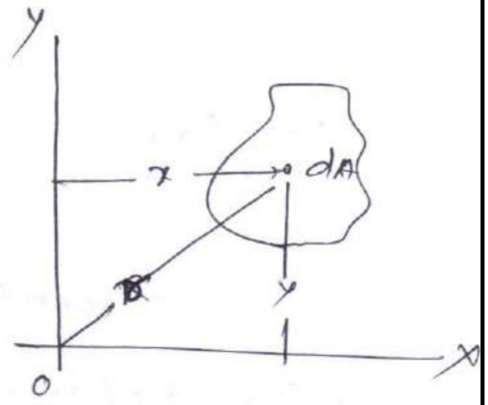
There are two theorems of moment of inertia

(a) perpendicular axis theorem

(b) parallel axis theorem.

Perpendicular axis theorem:-

Moment of inertia of an area about an axis ⊥r to it's plane at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of area.



$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = \Sigma r^2 dA$$

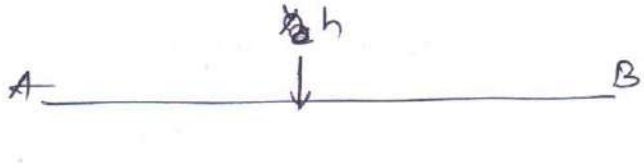
$$= \Sigma (x^2 + y^2)\, dA$$

$$= \Sigma x^2 dA + \Sigma y^2 dA$$

$$\Rightarrow \boxed{I_{zz} = I_{xx} + I_{yy}}$$

Parallel axis theorem:-

Moment of inertia about an axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance betn the two parallel axes.



$$\boxed{I_{AB} = \cancel{I_{xx}}\ I_{GG} + Ah^2}$$

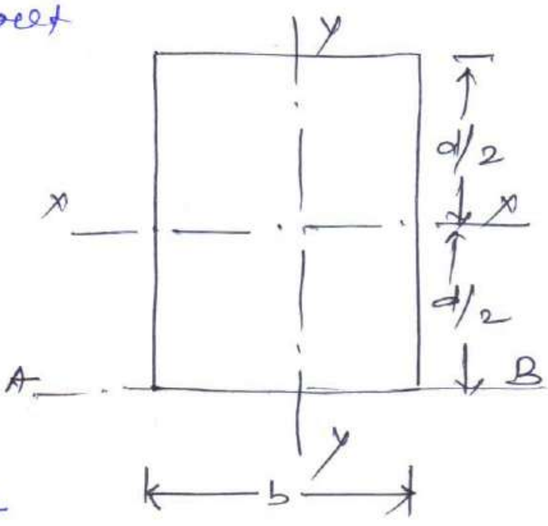
Moment of inertia of standard sections:-

i) Moment of inertia of a rectangle about it's centroidal axis xx

$$I_{xx} = \frac{bd^3}{12}$$

Similarly moment of inertia about it's centroidal axis yy

$$I_{yy} = \frac{db^3}{12}$$



Now moment of inertia of rectangle about it's base AB can be obtained by applying parallel axis theorem

$$I_{AB} = I_{xx} + Ah^2$$

$$= \frac{bd^3}{12} + (bd)\left(\frac{d}{2}\right)^2$$

$$= \frac{bd^3}{12} + \frac{bd^3}{4}$$

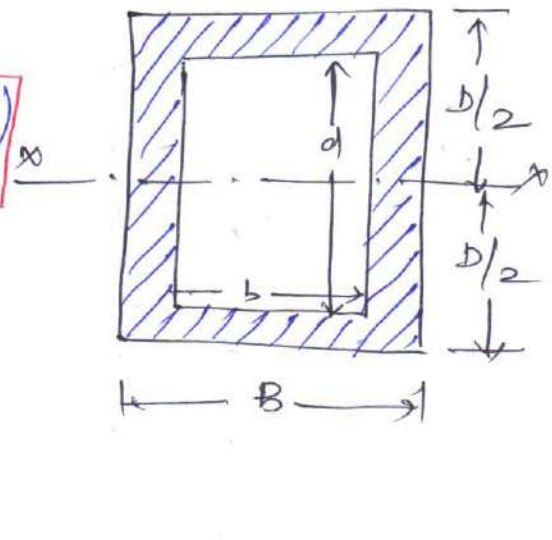
$$= \frac{3bd^3 + bd^3}{12} = \frac{bd^3}{3}$$

$$\Rightarrow \boxed{I_{AB} = \frac{bd^3}{3}}$$

(ii) Moment of inertia of a hollow rectangular section:-

Moment of inertia of hollow rectangular section

$$\boxed{I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}\left(BD^3 - bd^3\right)}$$



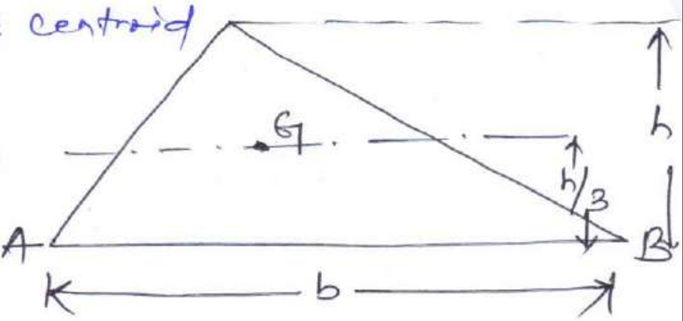
(iii) Moment of inertia of ~~triangle about it's base~~ centroidal

Moment of inertia of triangle about it's base

= moment of inertia about its centroid

+ Ah^2

(using parallel axis theorem)



$\Rightarrow I_{AB} = I_{xx} + Ah^2$

$$\Rightarrow \frac{bh^3}{12} = I_{xx} + \frac{1}{2} b \times h \times \left(\frac{h}{3}\right)^2$$

$$= I_{xx} + \frac{bh^3}{18}$$

$$\Rightarrow I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$= \frac{3bh^3 - 2bh^3}{36} = \frac{bh^3}{36}$$

$$\Rightarrow \boxed{I_{xx} = \frac{bh^3}{36}}$$

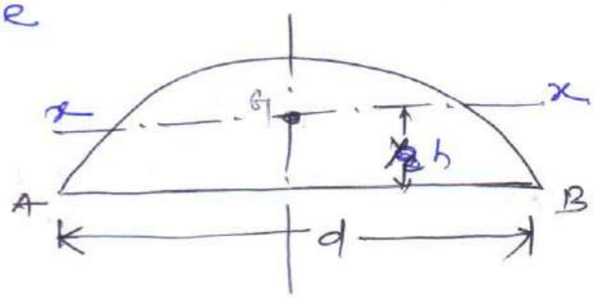
(iv) Moment of inertia of semicircle

(a) about diametral axis

Moment of inertia of semicircle

$$\text{about } AB = \frac{1}{2}\frac{\pi d^4}{64}$$

$$= \boxed{\frac{\pi d^4}{128}}$$



(b) about centroidal axis xx

$$\boxed{h = \frac{4R}{3\pi} = \frac{2d}{3\pi}}$$

$$\text{area } A = \frac{1}{2}\frac{\pi d^2}{4} = \frac{\pi d^2}{8}$$

Using parallel axis theorem

$$I_{AB} = I_{xx} + Ah^2$$

$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \left(\frac{2d}{3\pi}\right)^2 =$$

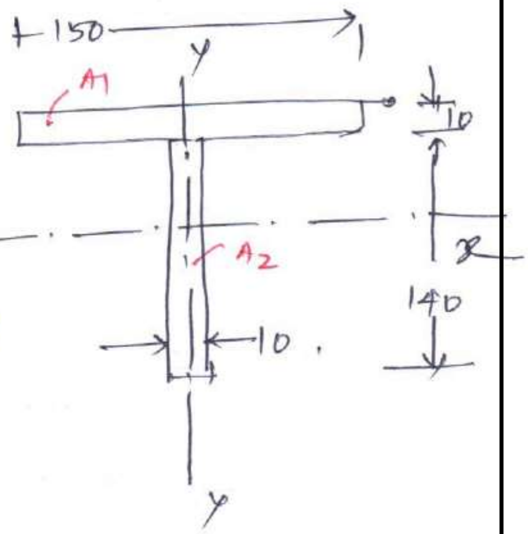
$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \frac{4d^2}{9\pi^2}$$

$$= I_{xx} + \frac{\cancel{\pi} d^4}{18\pi}$$

$$\Rightarrow \boxed{I_{xx} = \left(\frac{\pi d^4}{128} - \frac{d^4}{18\pi}\right)}$$

Moment of inertia of composite figures :-

Q.1 Determine the moment of inertia of the composite section about an axis passing through the centroidal axis. Also determine MI about axis of symmetry and radius of gyration.



Soln Dividing the composite area into A_1 and A_2

$A_1 = 150 \times 10 = 1500\ mm^2$

$A_2 = 140 \times 10 = 1400\ mm^2$

Distance of centroid from base of the composite figure

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{(A_1 + A_2)} = \frac{1500 \times 145 + 1400 \times 70}{2900}$$

$$= 108.79\ mm$$

Moment of inertia of the area about xx axis

$$I_{xx} = \left\{\frac{150 \times 10^3}{12} + 1500 \times (145 - 108.79)^2\right\} + \left\{\frac{10 \times 140^3}{12} + 1400 \times (108.79 - 70)^2\right\}$$

$$= (12500 + 1966746.15) + (2286666.667 + 2106529.74)$$

$$= 6372442.557\ mm^4$$

Similarly

$$I_{yy} = \frac{10 \times 150^3}{12} + \frac{140 \times 10^3}{12} = 2812500 + 11666.66667$$

$$= 2824166.667\ mm^4$$

Radius of gyration $k = \sqrt{\frac{I}{A}}$

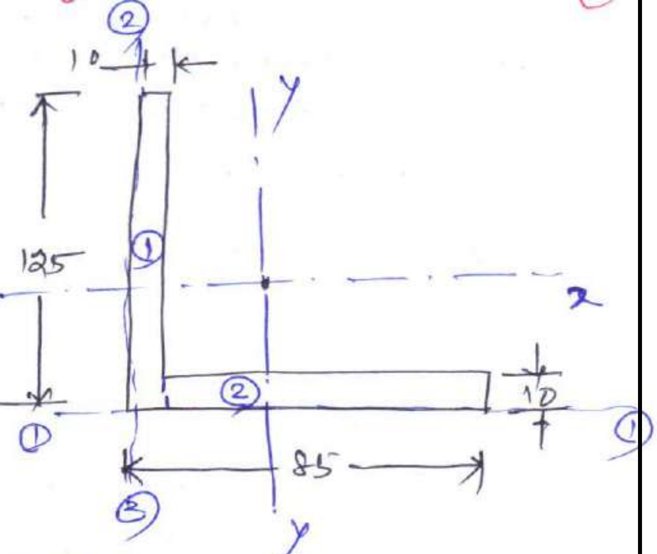
so $K_{xx} = \sqrt{\frac{I_{xx}}{A}}$

$= \sqrt{\frac{6372442.5}{2900}} = 46.87$ mm

Similarly $k_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{2824166.667}{2900}}$

$= 31.206$ mm (Ans)

Q.2 Determine the MI of L-section about its centroidal axes parallel to the legs. Also find the polar moment of inertia.



We have $A_1 = 125 \times 10 = 1250$ mm^2

$A_2 = 75 \times 10 = 750$ mm^2

Total area $A_1 + A_2 = 2000$ mm^2

Distance of centroid from 1-1 axis

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{1250 \times 62.5 + 750 \times 5}{2000} = 40.9375$ mm

Distance of centroidal axis yy from 2-2 axis

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$

$= \frac{1250 \times 5 + 750 \times \left(\frac{75}{2} + 10\right)}{2000}$

$= \frac{1250 \times 5 + 750 \times 47.5}{2000} = 20.93$ mm

Moment of inertia about xx axis

$I_{xx} = \left\{\frac{10 \times 125^3}{12} + 1250 \times (62.5 - 40.9375)^2\right\}$

$+ \left\{\frac{75 \times 10^3}{12} + 750 \times (40.9375 - 5)^2\right\}$

$= (1627604.167 + 581176.7578) + (6250 + 968627.9297)$

$= \boxed{3183658.854 \text{ mm}^4}$

Similarly MI about yy centroidal axis

$$I_{yy} = \left\{\frac{125\times10^3}{12} + 1250\times(20.93-5)^2\right\} + \left\{\frac{10\times75^3}{12} + 750\times(47.5-20.93)^2\right\}$$

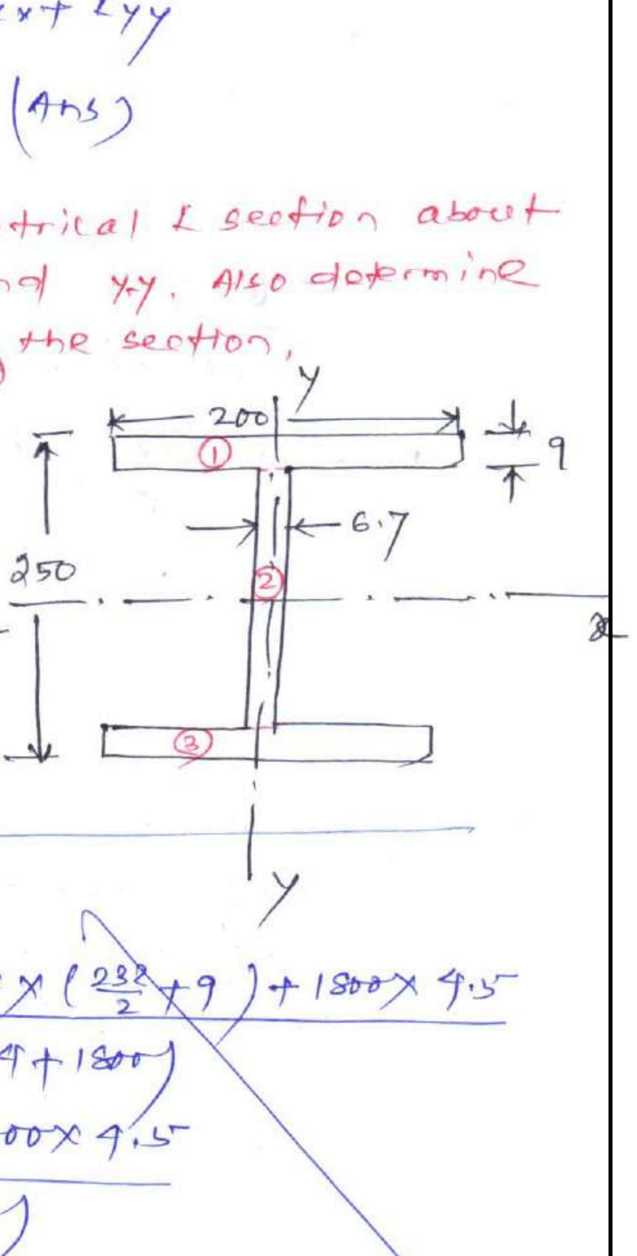
$$= (10416.66667+317206.125)+(351562.5+529473.675)$$

$$= \boxed{1208658.967\ mm^4}$$

Polar moment of inertia $I_{zz} = I_{xx}+I_{yy}$

$$= \boxed{4392317.821\ mm^4} \quad (Ans)$$

Q.3 Determine the MI of the symmetrical I section about its centroidal axes x-x and y-y. Also determine the polar moment of inertia of the section.



We have from the figure

$A_1 = 200\times9 = 1800\ mm^2$

$A_2 = 232\times6.7 = 1554.4\ mm^2$

$A_3 = 200\times9 = 1800\ mm^2$

~~Position of centroidal axis xx from base~~

$$\bar{y} = \frac{A_1y_1+A_2y_2+A_3y_3}{(A_1+A_2+A_3)}$$

$$= \frac{1800\times(4.5+232+9)+1554.4\times(\frac{232}{2}+9)+1800\times4.5}{(1800+1554.4+1800)}$$

$$= \frac{1800\times245.5+1554.4\times125+1800\times4.5}{(1800+1554.4+1800)}$$

$$= 125\ mm.$$

$$\bar{x} = \frac{A_1x_1+A_2x_2+A_3x_3}{(A_1+A_2+A_3)}$$

$$= \frac{1800\times100+1554.4\times96.65+1800\times100}{(1800+1554.4+1800)} = 98.98$$

MI about xx axis

$$I_{xx} = \left\{\frac{200\times 9^3}{12} + 1800\times(125-4.5)^2\right\} + \left\{\frac{6.7\times 232^3}{12} + 1554.4\times(\right.$$

$$+ \left\{\frac{200\times 9^3}{12} + 1800\times(125-4.5)^2\right\}$$

$$= (12150 + 26136450) + (6972002.133 + 0)$$

$$+ (12150 + 26136450)$$

$$= 26148600 + 6972002.133 + 26148600$$

$$= \boxed{59269202.13 \text{ mm}^4}$$

MI about yy axis

$$I_{yy} = \frac{9\times 200^3}{12} + \frac{232\times 6.7^3}{12} + \frac{9\times 200^3}{12}$$

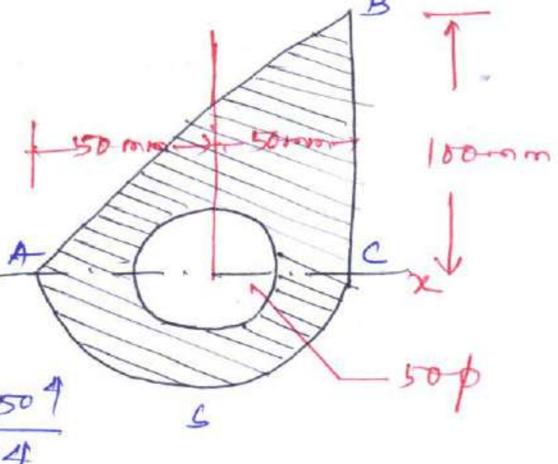
$$= 6000000 + 5814.751 + 6000000$$

$$= \boxed{12005814.75 \text{ mm}^4}$$

Polar moment of inertia $I_{zz} = I_{xx} + I_{yy}$

$$= \boxed{71275016.88 \text{ mm}^4}$$

Q.A Calculate the moment of inertia of the shaded area about xx axis.



MI of the shaded section about xx = MI of triangle ABC about xx + MI of semicircle ACS about xx − MI of circle

$$= \frac{100\times 100^3}{12} + \frac{\pi\times 100^4}{128} - \frac{\pi\times 50^4}{64}$$

$$= 8333333.333 + 2454369.261 - 306796.1576$$

$$= 10480906.44 \text{ mm}^4$$

$$= \boxed{1.048\times 10^7 \text{ mm}^4}$$

MODULE – IV

PARTICLE DYNAMICS AND INTRODUCTION TO KINETICS

COURSE OUTCOMES (COs):

At the end of the course students are able to:		
Course Outcomes		Knowledge Level (Bloom's Taxonomy)
CO 10	**Determine** normal and tangential accelerations for a particle in rectilinear and curvilinear motion through kinematic equations.	Apply
CO 11	**Derive** the dynamic equilibrium of a body in motion by introducing inertia force through D'Alembert's principle.	Apply

PROGRAM OUTCOMES (POs):

Program Outcomes (POs)		Strength	Proficiency Assessed by
PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	3	CIE/Quiz/AAT
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences	1	CIE/Quiz/AAT
PSO 1	Formulate and evaluate engineering concepts of design, thermal and production to provide solutions for technology aspects in digital manufacturing.	3	Research papers / Group discussion / Short term courses

MODULE IV

PARTICLE DYNAMICS AND INTRODUCTION TO KINETICS

-: Rectilinear Translation :-

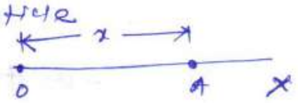
In statics, it was considered that the rigid bodies are at rest. In dynamics, it's considered that they are in motion. Dynamics is commonly divided into two branches. Kinematics and kinetics.

In, kinematics we are concerned with space time relationship of a given motion of a body and not at all with the forces that cause the motion.

- In kinetics we are concerned with finding the kind of motion that a given body or system of bodies will have under the action of given forces or with what forces must be applied to produce a desired motion.

Displacement

From the fig. displacement of a particle can be defined by its x-coordinate, measured from the fixed reference point O.



- When the particle is to the right of fixed point O, this displacement can be considered positive and when it's towards the ~~sig~~ left hand side it's considered as negative.

General displacement time equation

$$x = f(t) \quad -(1)$$

where $f(t)$ = function of time.

for example $\boxed{x = c + bt}$

In the above equation c, represents the initial displacement at $t=0$, while the constant b shows the rate at which displacement increases. It is called uniform rectilinear motion.

Second example is $\boxed{x = \frac{1}{2} g t^2}$

where x is proportional to the square of time.

Velocity

Acceleration

Example The rectilinear motion of a particle is defined by the displacement - time equation $x = x_0 - v_0 t + \frac{1}{2} a t^2$ construct displacement - time and velocity diagram for this motion and find the displacement and velocity at time $t_2 = 2s$. $x_0 = 750$ mm, $v_0 = 500$ mm/s

$a = 0.125$ m/s^2

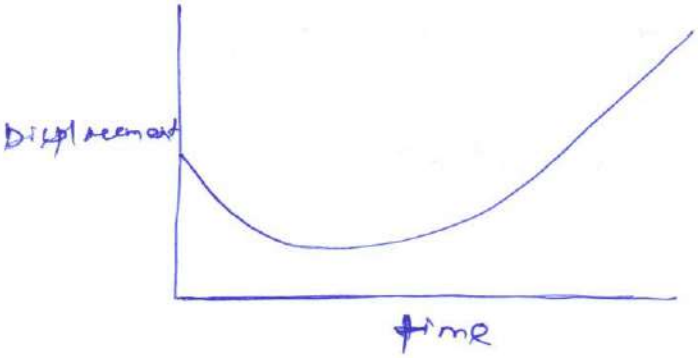
The equation of motion is

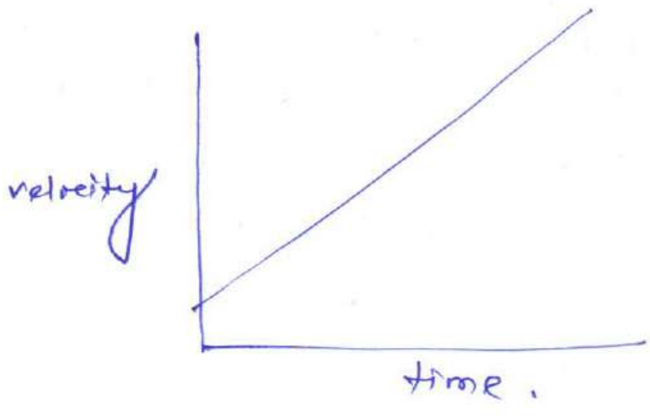
$$x = x_0 - v_0 t + \frac{1}{2} a t^2 \quad -(1)$$

$$v = \frac{dx}{dt} = -v_0 + at \quad -(2)$$

substiting x_0, v_0 and a in equation (1)

~~$x = 75 - 500$~~





Q-1 A bullet leaves the muzzle of gun with velocity $v = 750$ m/s. Assuming constant acceleration from breech to muzzle find time t occupied by the bullet in travelling through gun barrel which is 750 mm long.

initial velocity of bullet $u = 0$
final velocity of bullet $v = 750$ m/s.
total distance $s = 0.75$ m.
$t = ?$

We have $v^2 - u^2 = 2as$,

$\Rightarrow v^2 = 2as \Rightarrow a = \frac{v^2}{2s} = \frac{750^2}{2 \times 0.75}$

$= 375000$ m/sec^2

Again $v = u + at$

$\Rightarrow 750 = 375000 \times t$

$\Rightarrow t = \frac{750}{375000} = \boxed{0.002 \text{ sec.}}$

Q-2 A stone is dropped into well and falls vertically with constant acceleration $g = 9.81$ m/sec^2. The sound of impact of stone on the bottom of well is heared after 6.5 sec. If velocity of sound is 336 m/s. How deep is the well?

$V = 336$ m/sec
let s = depth of well
t_1 = time taken by the stone into the well
t_2 = time taken by the sound to be heared.
total time $t = (t_1 + t_2) = 6.5$ sec.

Now $s = ut + \frac{1}{2} gt^2$

$\Rightarrow s = 0 + \frac{1}{2} gt^2$

$\Rightarrow t_1 = \sqrt{\frac{2s}{g}}$

When the sound travels with uniform velocity

$s = Vt_2$ or $t_2 = \frac{s}{V}$

$$\sqrt{\frac{2s}{g}} + \frac{s}{V} = 6.5$$

$$\Rightarrow \frac{2s}{g} = \left(6.5 - \frac{s}{336}\right)^2$$

$$\Rightarrow 2s = 9.81\left(6.5 - \frac{s}{336}\right)^2$$

$$= 9.81\left(\frac{2184 - s}{336}\right)^2$$

$$s = 0.0291\,(2184 - s)^2$$

$$= 0.0291\,(4769856 + s^2 - 4368s)$$

$$= 138802.809 + 0.0291s^2 - 127.1088s$$

$$\Rightarrow 0.0291s^2 - 129.1088s + 138802.809 = 0$$

$$\Rightarrow s =$$

$$0.2038s = 42.25 + 0.0000088s^2 - 0.0386s$$

~~s = 174~~

$$0.0000088s^2 - 0.1658s + 42.25 = 0$$

$$\boxed{s = 17.31\,m.}$$

A2 A rope AB is attached at B to a small block of negligible dimensions and passes over a pulley C so that it's free end A hangs 1.5 m above ground when the block rests on the floor. The end A of the rope is moved horizontally in a str. line by a man walking with a uniform velocity $v_0 = 3\,m/s$. Plot the velocity-time diagram

(b) find the time t required for the block to reach the pulley if h = 4.5 m, pully dimension are negligible.

A3 A particle starts from rest and moves along a str. line with constant acceleration a. If it acquires a velocity $v = 3\,m/s$. after having travelled a distance s = 7.5 m. find magnitude of acceleration.

20/11/2014 ①

Principles of Dynamics:

Newton's law of motion:

First law: Everybody continues in it's state of rest or of uniform motion in a straight line except in so far as it may be compelled by force to change that state.

Second Law :-

The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.

Third law To every action there is always an equal and contrary reaction or the mutual action's of any two bodies are always equal and oppositely directed.

General Equation of Motion of a Particle:

$$\boxed{ma = f}$$

Differential equation of Rectilinear motion:

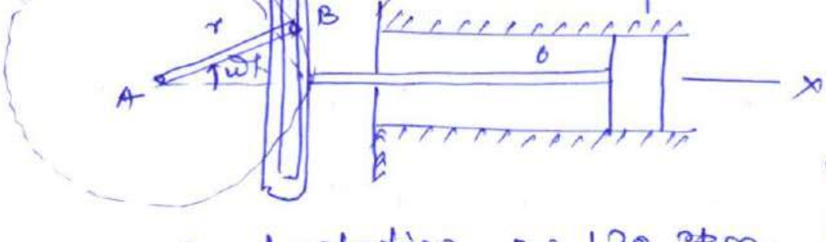
Differential form of equation for rectilinear motion can be expressed as

$$\boxed{\frac{W}{g}\ddot{x} = X}$$

where $\ddot{x}$ = acceleration

X = Resultant acting force.

Example



For the engine shown in fig., the combined wt. of piston and piston rod $W = 450N$, crank radius $r = 250mm$ and uniform speed of rotation $n = 120$ rpm. Determine the magnitude of resultant force acting in piston (a) at extreme position and (b) at the middle position piston has a simple harmonic motion represented by displacement-time equation

$$x = r\cos\omega t \quad \text{---(1)}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi\times 120}{60} = 4\pi \text{ rad/s.}$$

$$\dot{x} = -r\omega\sin\omega t$$

$$\ddot{x} = -r\omega^2\cos\omega t \quad \text{---(2)}$$

Differential equation of motion

$$\frac{W}{g}\ddot{x} = X$$

$$\Rightarrow -\frac{W}{g}r\omega^2\cos\omega t = X$$

$$\Rightarrow X = -\frac{450}{9.81}\times 0.25\,(4\pi)^2\cos(4\pi t)$$

for extreme position

$$\cos\omega t = -1$$

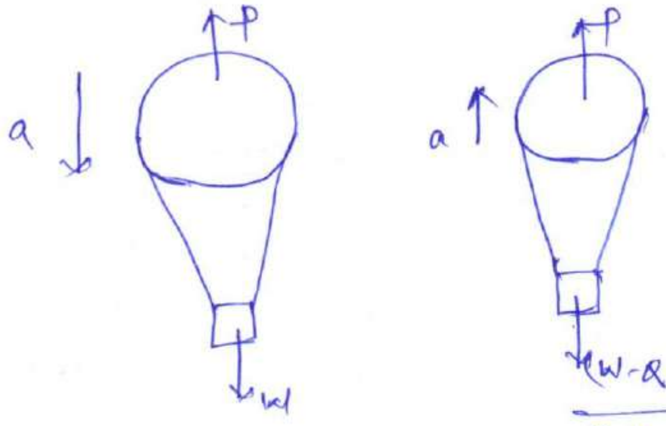
so $\boxed{X = 1810\,N.}$

~~for extre~~ middle position $\cos\omega t = 0$.

so Resultant force = 0.

E-2 A ballon of gross wt W is falling vertically downward with constant acceleration a. What amount of ballast Q must be thrown out in order to give balloon an equal upward acceleration a.

P = buoyant force.



(i) (ii)

(i) considering 1st case when ballon is falling,

$$\frac{W}{g}a = W - P \quad \text{---(1)}$$

(ii) $$\frac{W-Q}{g}a = P - (W-Q) \quad \text{---(2)}$$

Eq(1) + Eq(2)

$$\frac{Q}{g}\cdot a = \cancel{W+W-Q} = 2W + Q$$

$$\Rightarrow Q\left(\frac{a}{g} - 1\right) = 2W$$

$$\Rightarrow Q = \frac{2Wg}{(a-g)}$$

$$\frac{Wa}{g} = (W - P)$$

$$\frac{(W-Q)a}{g} = P - (W - Q)$$

$$\frac{Wa + (W-Q)a}{g} = W - \cancel{P} + \cancel{P} - (W - Q) = Q$$

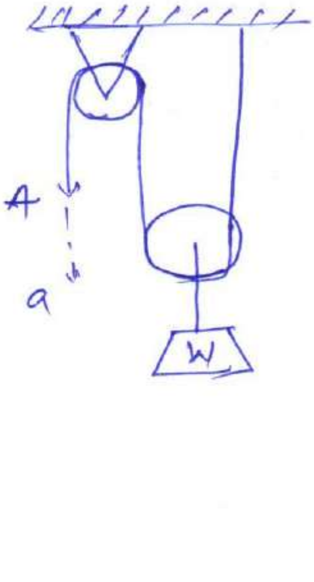
$$\Rightarrow \frac{Wa + Wa - Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg + Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. W = 4450N is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~sth~~ the string is pulled vertically downward with constant acceleration a = 18 m/s2 find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

$$= \cancel{\frac{W}{2}\left(2 + \frac{a}{2g}\right)}$$

$$= W\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1 + \frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1 + \frac{18}{2 \times 9.81}\right) = \boxed{4266.28\ N.}$$

$$\frac{Wa}{g} = (W-P)$$

$$\frac{(W-Q)a}{g} = P-(W-Q)$$

$$\frac{Wa+(W-Q)a}{g} = W-\cancel{P}+\cancel{P}-(W-Q) = Q$$

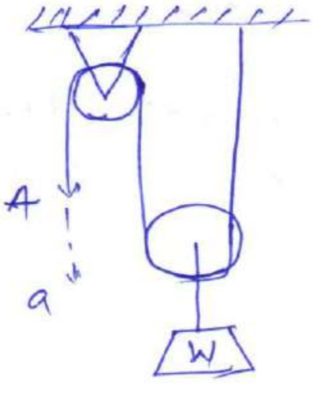
$$\Rightarrow \frac{Wa+Wa-Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg+Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. W = 4450N is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~ath~~ the string is pulled vertically downward with constant acceleration $a = 18\ m/s^2$ find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

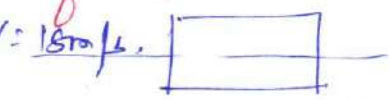
$$= \cancel{\frac{W}{2}\left(2+\frac{a}{2g}\right)}$$

$$= W\left(1+\frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1+\frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1+\frac{18}{2\times 9.81}\right) = \boxed{4266.28\ N.}$$

Q.2 An elevator of gross wt W = 4450N starts to move upward direction with a constant acceleration and acquires a velocity v = 18m/s, after travelling a distance = 1.8 m. find tensile force S in the cable during it's motion.

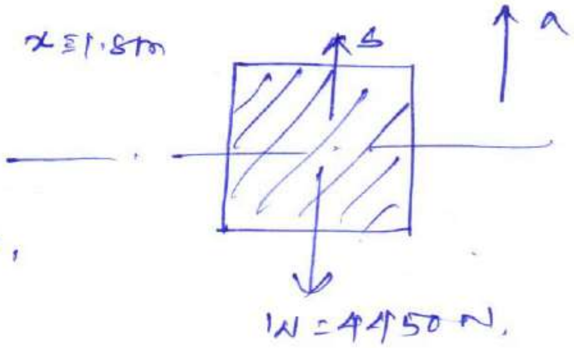


W = 4450N.

V = 18 m/s.

initial velocity u = 0

distance travelled x = 1.8 m,



$S - W = \frac{W}{g} \cdot a$

$\Rightarrow S = W + \frac{W}{g} a = W\left(1 + \frac{a}{g}\right)$ ——(1)

Now applying equation of kinematics

$v^2 - u^2 = 2as$

$\Rightarrow 18^2 - 0 = 2a \times 1.8$

$\Rightarrow a = \frac{18^2}{2 \times 1.8} = \boxed{90\ m/s^2}$

substituting the values of a in eq. (1)

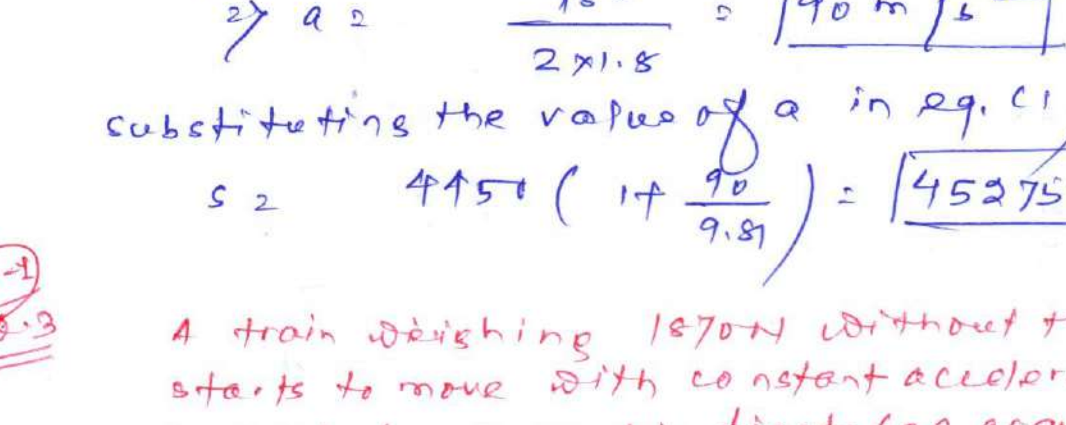
$S = 4450\left(1 + \frac{90}{9.81}\right) = \boxed{45275.7\ N.}$

A-1

Q.3

A train weighing 1870N without the locomotive starts to move with constant acceleration along a straight track and in first 60s acquires a velocity of 56 Kmph. Determine the tension S in draw bar both locomotive and train if the air resistance is 0.005 times the wt. of the train.

V = 56 Kmph = 15.56 m/s.



$S - F = \frac{W}{g} \cdot a$

$\Rightarrow S = 0.005W + \frac{Wa}{g}$ —(1)

from eq. of kinematics.

$v = u + at$

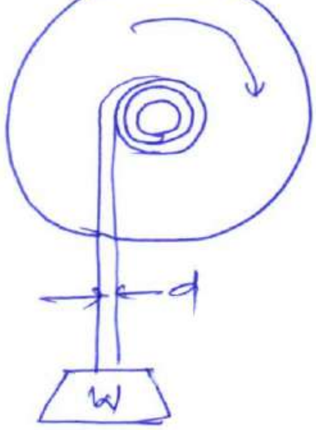
$\Rightarrow a = \left(\frac{15.56 - 0}{60}\right) = 0.26 \text{ m/sec}^2$

substituting the value of a in eq. (1)

$S = W\left(0.005 + \frac{a}{g}\right)$

$= 1870\left(0.005 + \frac{0.26}{9.81}\right) = \boxed{58.9 \text{ kN.}}$

A-2
Q. 4 A wt. W is attached to the end of a small flexible rope of dia. d = 6.25mm. and is raised vertically by winding the rope on a reel. if the reel is turned uniformly at a rate of 2 rps. what will be the tension in rope.



dia of rope d = 6.25mm = 0.00625m.

No of revolutions N = 2rps.

let x = initial radius of reel.

t = time taken for N revolutions.

Net radius after t sec.

$R = [x + (Nt d)]$

Now mean velocity $v = R\omega$

$\omega = 2\pi N$.

$\therefore v = (x + Ntd)\,2\pi N$

acceleration of rope $= a = \frac{dv}{dt}$

$a = \frac{d}{dt}\left[2\pi N x + 2\pi N^2 t d\right] = 2\pi N^2 d$

$S - W = \frac{W}{g} \cdot a \Rightarrow S = W + \frac{Wa}{g} = W\left(1 + \frac{a}{g}\right)$

$\Rightarrow S = W\left(1 + \frac{2\pi N^2 d}{g}\right)$ ~~W(1+~~

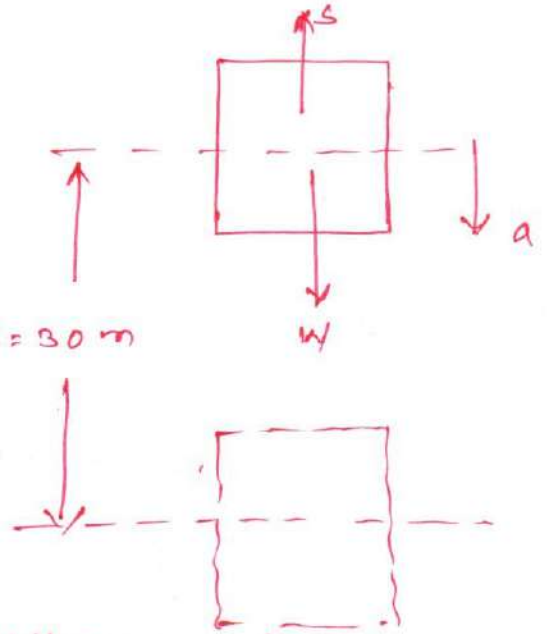
$\Rightarrow s = W\left(1 + \frac{2\pi \times 2^2 \times 0.00625}{9.81}\right)$

$=$

Ass-3

Q.5

A mine cage of wt $W = 8.9$ kN starts from rest and moves downward with constant acceleration travelling a distance $s = 30$ m in 10 sec. find the tensile force in the cable.



Wt. of cage $W = 8.9$ kN.

initial velocity $u = 0$.

distance travelled $s = 30$ m

time $t = 10$ sec.

$s = \cancel{ut}^{0} + \frac{1}{2} a t^2$

$\Rightarrow 30 = \frac{1}{2} a \times 10^2$

$\Rightarrow t = \frac{60}{10^2} = \boxed{0.6 \text{ m/sec}^2}$

Differential equation of rectilinear motion

$W - S = \frac{W}{g} \cdot a$

$\Rightarrow S = W - \frac{W}{g} a = W\left(1 - \frac{a}{g}\right)$

$= 8.9\left(1 - \frac{0.6}{9.81}\right)$

$\Rightarrow \boxed{S = 8.35 \text{ kN.}}$ (Ans)

D'Alembert's Principle

Differential equation of motion (rectilinear) can be written as

$$X - m\ddot{x} = 0 \quad \text{——— (1)}$$

Where x = Resultant of all applied force in the direction of motion

m = mass of the particle

The above equation may be treated as equation of dynamic equilibrium. To express this equation, in addition to the real forces acting on the particle a fictitious force $m\ddot{x}$ is required to be considered. This force is equal to the product of mass of the particle and it's acceleration and directed in opposit direction, and is called the inertia force of the particle.

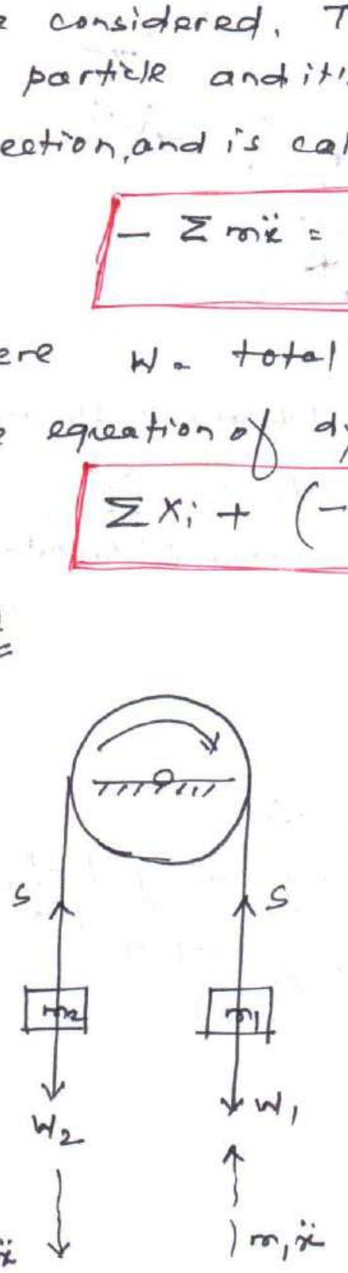
$$\boxed{-\Sigma m\ddot{x} = -\ddot{x}\,\Sigma m = -\frac{W}{g}\ddot{x}}$$

Where W = total weight of the body

so the equation of dynamic equilibrium can be expressed as:

$$\boxed{\Sigma X_i + \left(-\frac{W}{g}\ddot{x}\right) = 0} \quad \text{——— (2)}$$

Example 1



for the example shown considering the motion of pulley as shown by the arrow mark. we have upward acceleration $\ddot{x}_2$ for W_2 and downward acceleration $\ddot{x}_1$ for W_1

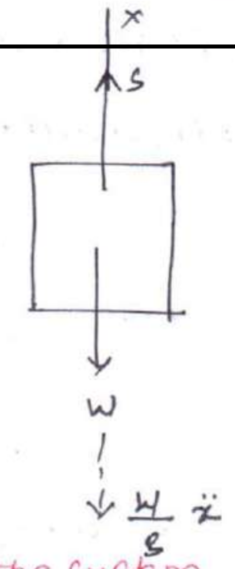
- corresponding inertia forces and their direction are indicated by dotted line.
- By adding inertia forces to the real forces (such as W_1, W_2 and tension in strings) we obtain, for each particle, a system of forces in equilibrium.

The equilibrium equation for the entire system without S

$$W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

$$\Rightarrow (m_1+m_2)\ddot{x} = (W_1 - W_2) \Rightarrow \boxed{\ddot{x} = \frac{W_1 - W_2}{(W_1+W_2)}\cdot g}$$

Example 2 A body is moving in upward direction by a rope.

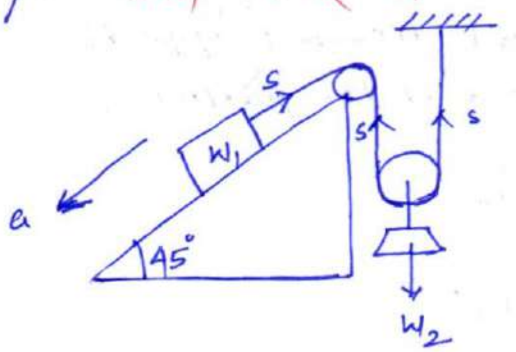


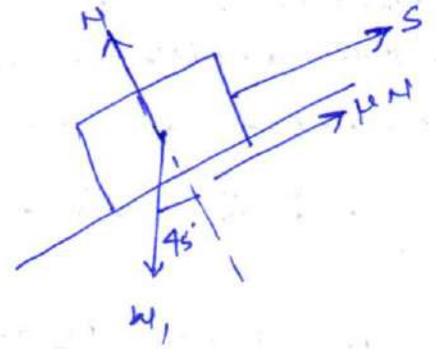
so the equation of dynamic equilibrium considering the real and inertia forces.

$S - W - \frac{W}{g} a = 0$, so tensile force in rope

$$\Rightarrow \boxed{S = W\left(1 + \frac{a}{g}\right)}$$

Q.1 Find tensions in the string during motion of the system if $W_1 = 900N$, $W_2 = 450N$. The μ betn the inclined plane and block $W_1 = 0.2$





When W_1 moves downward in the inclined plane with an acceleration a, then acceleration of $W_2 = \frac{a}{2}$

Considering dynamic equilibrium of W_1, from D'Alembert's principle

$$\left(W_1 \sin 45° - \mu N - S\right) - \frac{W_1}{g} a = 0$$

$$\Rightarrow \frac{W_1}{g} a = W_1 \sin 45° - \mu N - S$$

$$= W_1 \sin 45 - \mu W_1 \cos 45 - S$$

$$\Rightarrow a = \left(900 \times \frac{1}{\sqrt{2}} - 0.2 \times 900 \times \frac{1}{\sqrt{2}} - S\right)\frac{9.81}{900}$$

$$= \left(636.4 - 127.28 - S\right) 0.0109$$

$$\Rightarrow a = \cancel{5.549}\ 6.93676 - 1.387352 - 0.0109 S \quad \text{--- (1)}$$

Similarly ~~from~~ for weight W_2

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow \cancel{\frac{W_2 a}{2g}} = W_2\left(1 + \frac{a}{2g}\right) = 2S$$

$$\Rightarrow S = \frac{450}{2}\left(1 + \frac{a}{19.62}\right) = 225 + 11.46\, a \quad \text{--- (2)}$$

substituting the value of S in eq. (1)

$\cancel{a = 5.549 - 2.4525 - 0.125a} \Rightarrow a =$

$a = 6.93676 - 1.387352 - 0.0109\left(225 + 11.46a\right)$

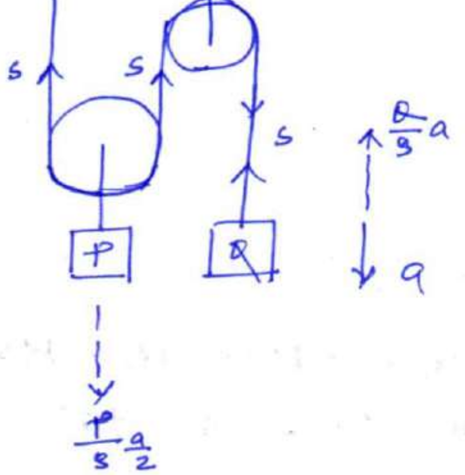
$= \cancel{6.93}\ 5.549408 - 2.4525 - 0.124914a$

$= 3.096908 - 0.124914a$

$\Rightarrow \boxed{a = 2.75\ m/s^2}$

Q.2 Two weights P and Q are connected by the arrangement shown in fig. Neglecting friction and inertia of pulley and cord find the acceleration a of wt - Q

Assume $P = 178\ N$, $Q = 133.5\ N$.



Applying D'Alembert's principle to Q

$Q - s - \frac{Q}{g}a = 0$

$\Rightarrow s = Q\left(1 - \frac{a}{g}\right)$ — (1)

$= 133.5\left(1 - \frac{a}{9.81}\right)$

Applying D'Alembert's principle to P

~~P - 2s = P~~

$2s - P - \frac{Pa}{2g} = 0$

$\Rightarrow 2s = P\left(1 + \frac{a}{2g}\right)$

$\Rightarrow s = \frac{P}{2}\left(1 + \frac{a}{2g}\right)$ — (2)

$= \frac{178}{2}\left(1 + \frac{a}{19.62}\right)$

$133.5\left(1 - \frac{a}{9.81}\right) = 89\left(1 + \frac{a}{19.62}\right)$

$\Rightarrow 133.5 - 13.608a = 89 + 4.536a$

$\Rightarrow 18.144a = 44.5$

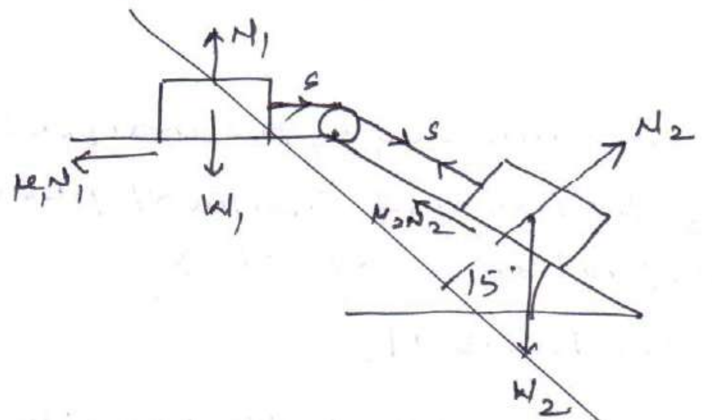
$\Rightarrow \boxed{a = 2.45\ m/s^2}$ (Ans)

Q.3 Assuming the car in the fig. to have a velocity of 6 m/s find shortest distance s in which it can be stopped with constant deceleration without disturbing the block. Data: $c = 0.6\ m$, $h = 0.9\ m$

$\mu = 0.5$

31

Q.3 Two blocks of wt $W_1 = 150N$ and $W_2 = 500N$ are connected by an inextensible string, Find the acceler of the blocks and tension in the string. $\mu_1 = 0.1$, $\mu_2 = 0.$



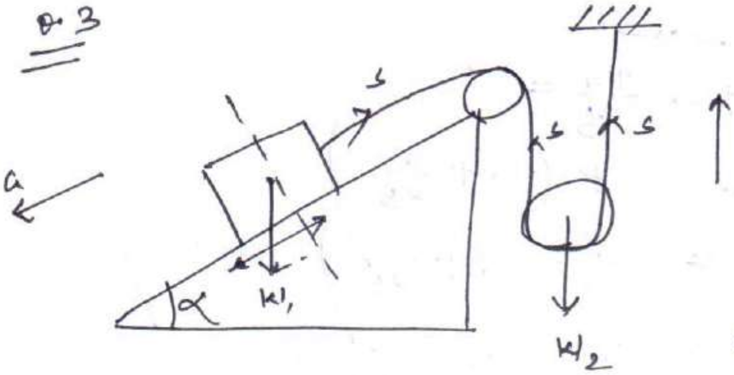
for block 1

$s - \mu_1 N_1 = 0$

$\Rightarrow s = \mu_1 W_1 = 0.1 \times 150 = 15N.$

for block 2

Q.3



$W_1 = 890N$ $W_2 = 445N$.

$\mu = 0.2$ $\alpha = 45°$

find s.

considering equilibrium of W_1 and applying D'Alembert's principle

$$W_1 \sin 45° - \mu N_1 - S - \frac{W_1}{g} a = 0$$

$$\Rightarrow S = W_1 \sin 45 - \mu N - \frac{W_1}{g} a$$

$$= \frac{890}{\sqrt{2}} - 0.2 \times 890 \times \frac{1}{\sqrt{2}} - \frac{890}{9.81} a$$

$$= 629.32 - 125.865 - 90.72a$$

$$\boxed{S = 503.455 - 90.72a} \quad \text{——(1)}$$

Applying D'Alembert's principle for W_2

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow 2S = W_2\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W_2}{2}\left(1 + \frac{a}{2g}\right) = \frac{445}{2}\left(1 + \frac{a}{19.62}\right) = 222.5 + 11.34a \quad \text{——(2)}$$

equating (1) and (2)

$503.455 - 90.72a = 222.5 + 11.34a$

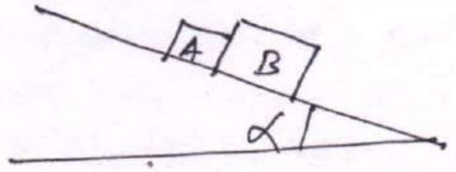
$\Rightarrow 102.0604\,a = 280.955$

$\Rightarrow \boxed{a = 2.75\ m/s^2}$

so $S = 222.5 + 11.34 \times 2.75$

$= \boxed{253.71\ N.}$

Q4

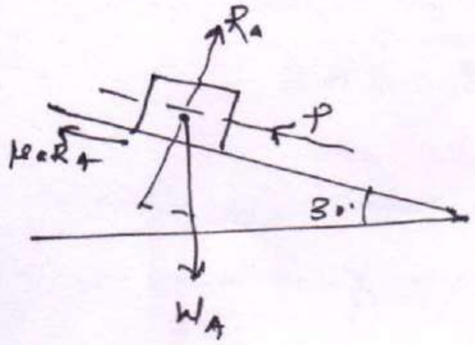


$W_A = 44.5N \quad W_B = 89N$

$\alpha = 30^\circ \quad \mu_a = 0.15$

$\mu_B = 0.3$

find pressure P betn blocks.



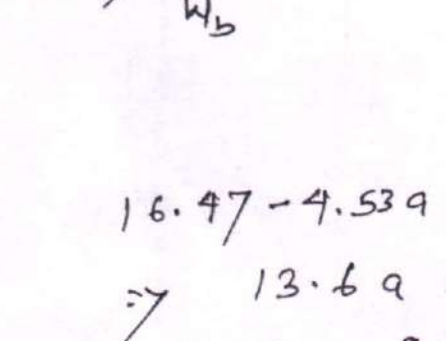
$W_a \sin 30 - P - \mu_a R_a - \frac{W_a}{g} a = 0$

$\Rightarrow P = W_a \sin 30 - \mu_a R_a - \frac{W_a}{g} a$

$= 44.5 \times \frac{1}{2} - 0.15 \times 44.5 \times \cos 30 - \frac{44.5}{9.81} a$

$= 22.25 - 5.78 - 4.53a$

$= 16.47 - 4.53a \quad \text{—(1)}$



$P + W_b \sin 30 - \mu_b R_b - \frac{W_b}{g} a = 0$

$\Rightarrow P = -\frac{W_b}{2} + 0.3 \times 89 \cos 30 + \frac{89}{9.81} a$

$= -\frac{89}{2} + 23.122 + 9.07a$

$= -21.378 + 9.07a \quad \text{—(2)}$

$16.47 - 4.53a = -21.378 + 9.07a$

$\Rightarrow 13.6a = 37.848$

$\Rightarrow a = 2.78\ m/s^2$

$P = 3.87\ N.$

Momentum and Impulse

We have the differential equation of rectilinear motion of a particle

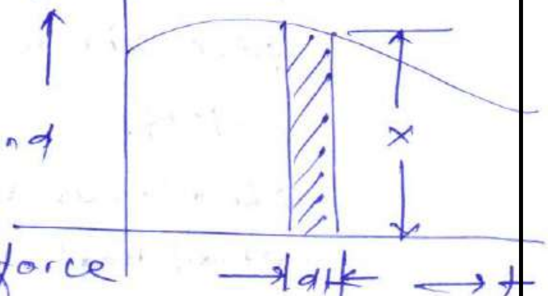
$$\frac{W}{g}\ddot{x} = X$$

Above equation may be written as

$$\frac{W}{g}\frac{d\dot{x}}{dt} = X$$

or $$\boxed{d\left(\frac{W}{g}\dot{x}\right) = X\,dt} \quad \text{---(1)}$$

In the above equation we will assume force X as a function of time represented by a force time diagram.



The right hand side of eq. (1) is then represented by the area of shaded elemental strip of ht X and width dt. This quantity i.e $(X\,dt)$ is called impulse of the force X in time dt. The expression on the left hand side of the expression $\left(\frac{W}{g}\dot{x}\right)$ is called momentum of particle.

so the eq. (1) represents the differential change in momentum of a particle in time dt.

Integrating eq. (1) we have

$$\boxed{\frac{W}{g}\dot{x} + C = \int_0^t X\,dt} \quad \text{---(2)}$$

where C is a constant of integration

Now assuming an initial moment, $t=0$, the particle has an initial velocity $\dot{x}_0$

so $$\boxed{C = -\frac{W}{g}\dot{x}_0} \quad \text{---(3)}$$

So equation (2) becomes

$$\boxed{\frac{W}{g}\dot{x} - \frac{W}{g}\dot{x}_0 = \int_0^t X\,dt} \quad \text{---(4)}$$

From equation (4) it is clear that the total change of momentum of a particle during a finite interval of time is equal to the impulse of acting force.

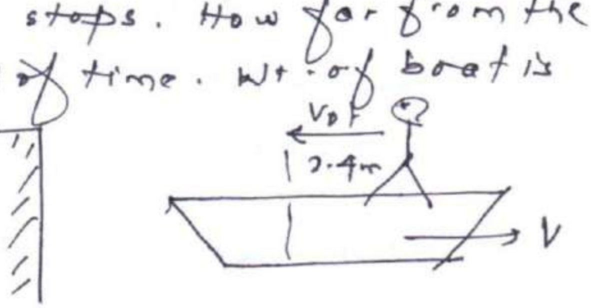
~~Example~~ in other words

$$\boxed{F \cdot dt = d(mv)}$$

where $m \times v =$ momentum

~~Recoil of gun~~

Q-1 A man of wt 712 N stands in a boat so that he is 4.5 m from a pier on the shore. He walks 2.4 m in the boat towards the pier and then stops. How far from the pier will he be at the end of time. Wt of boat is 890 N.



wt of man $W_1 = 712\,N$

wt of boat $W_2 = 890\,N$

Let v_0 is the initial velocity of man and t is time

then $v_0 t = x$

$\Rightarrow v_0 t = 2.4\,m$

$\Rightarrow v_0 = \left(\frac{2.4}{t}\right)$ m/s.

let V = velocity of boat towards right

according to conservation of momentum

$$W_1 v_0 = (W_1 + W_2) V$$

$$\Rightarrow V = \frac{W_1 v_0}{(W_1 + W_2)}$$

distance covered by boat

$$s = V \cdot t = \frac{W_1 v_0}{(W_1 + W_2)} \cdot t$$

$$\Rightarrow s = \frac{712 \times 2.4}{\cancel{t}\,(712 + 890)} \cdot \cancel{t} = \boxed{1.067\,m}$$

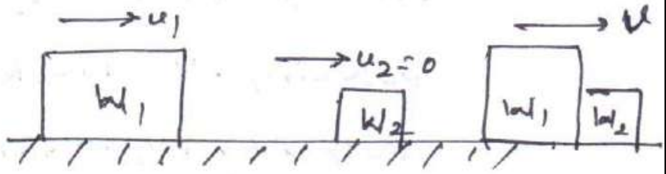
position of man from pier

$= 4.5 + s - x$

$= 4.5 + 1.667 - 2.4 = \boxed{3.167\,m}$ (Ans).

Q.2 A locomotive wt 534 kN has a velocity of 16 kmph and backs into a frieght car of wt 86 kN that is at rest on a track. after coupling at what velocity v the entire system continues to move. Neglect friction.

conservation of momentume



$W_1 u_1 + W_2 u_2 = (W_1 + W_2) V$

$\Rightarrow V = \dfrac{534 \times 4.45}{(534 + 86)} = \boxed{3.82\ m/s.}$

Q.3 A 667.5 man sits in a 333.75 N canoe and fire a rifle bullet horizontally. ~~directed over~~ find velocity v with which the canoe will move after the shot. the rifle has a muzzle velocity 660 m/s and wt. of bullet is 0.28 N.

Wt. of man $W_1 = 667.5\,N$.

Wt. of canoe $W_2 = 333.75\,N$.

Wt. of bullet $W_2 = 0.28\,N$.

velocity of muzzle $u = 660\,m/s$.

V = final velocity of canoe.

According to conservation of momentum

$\cancel{W_3 u}\ W_3 u = (W_1 + W_2) V$

$\Rightarrow V = \dfrac{0.28 \times 660}{(667.5 + 333.75)} = \boxed{0.182\ m/s.}$

Q.4 A wood block wt 22.25 N rests on a smooth horizontal surface. A revolver bullet weighing 0.14 N is shot horizontally into the side of block. If the block attains a velocity of 3 m/s what is muzzle velocity.

Wt. of wood block $W_1 = 22.25$ N.

Wt. of bullet $W_2 = 0.14$ N.

velocity of block $V = 3$ m/s.

velocity of muzzle $= u$

According to conservation of momentum

$$\cancel{W_1 V} = W_2 u = (W_1 + W_2) V$$

$$\Rightarrow u = \frac{(22.25 + 0.14)\,3}{0.14}$$

$$= \boxed{479.98 \text{ m/s.}}$$

Conservation of momentum

When the sum of impulses due to external force is zero the momentum of the system remain conserved

When $\Sigma \int_0^t X\, dt = 0$

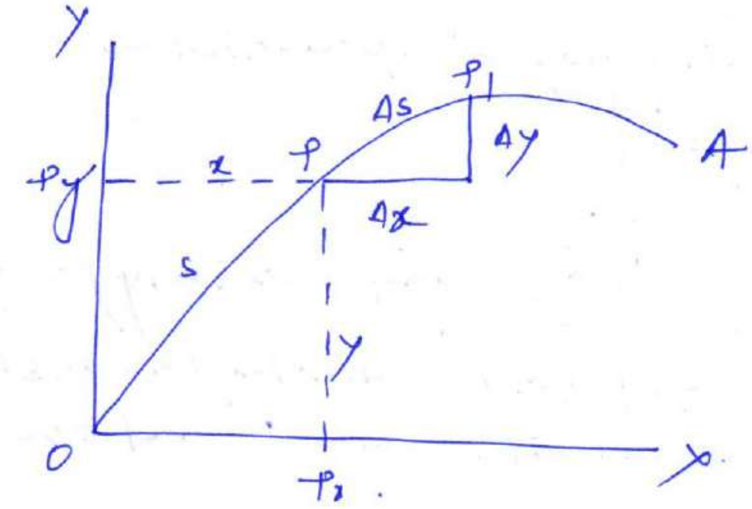
$$\boxed{\Sigma \left(\frac{W}{g}\right) \dot{x}_2 = \Sigma \left(\frac{W}{g}\right) \dot{x}_1}$$

$\therefore$ final momentum = initial momentum.

Curvilinear Translation

When moving particle describes a curved path it is said to have curvilinear motion.

Displacement



consider a particle P in a plane on a curved path.
To define the particle we need two coordinates x and y.
as the particle moves, these coordinates ~~move~~ change with time and the displacement time equations are

$$x = f_1(t) \qquad y = f_2(t) \qquad \text{—(1)}$$

The motion of particle can also be expressed as

$$y = f(x) \qquad s = f_1(t)$$

where $y = f(x)$ represents the equation of path OA and $s = f_1(t)$ gives displacement s measured along the path as a function of time.

velocity :-

Considering an infinitesimal time difference from t to t + Δt during which the particle moves from P to P_1 along it's path.
then velocity of particle may be expressed as

$$\boxed{\bar{v}_{av} = \frac{\Delta s}{\Delta t}}$$

$$\boxed{\begin{aligned}(v_{av})_x &= \frac{\Delta x}{\Delta t} \\ (v_{av})_y &= \frac{\Delta y}{\Delta t}\end{aligned}}$$

(average velocity along x and y coordinates)

It can also be expressed as

$$v_x = \frac{dx}{dt} = \dot{x}$$

$$v_y = \frac{dy}{dt} = \dot{y}$$

so the total velocity may be represented by

$$\boxed{v = \sqrt{\dot{x}^2 + \dot{y}^2}}$$

and $\cos(v, x) = \frac{\dot{x}}{v}$ and $\cos(v, y) = \frac{\dot{y}}{v}$

where (v, x) and (v, y) denotes the angles betn the direction of velocity vector $\overline{v}$ and the coordinate axes.

Acceleration :—

The acceleration particles may be described as

$$\boxed{\begin{aligned} a_x &= \frac{d\dot{x}}{dt} = \ddot{x} \\ a_y &= \frac{d\dot{y}}{dt} = \ddot{y} \end{aligned}}$$

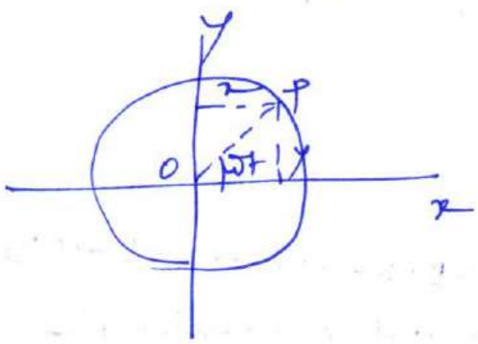
It is also known as instantaneous acceleration

Total acceleration $a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$

Considering particular path for above case.

$x = r\cos\omega t \quad y = r\sin\omega t.$

$x^2 + y^2 = r^2$



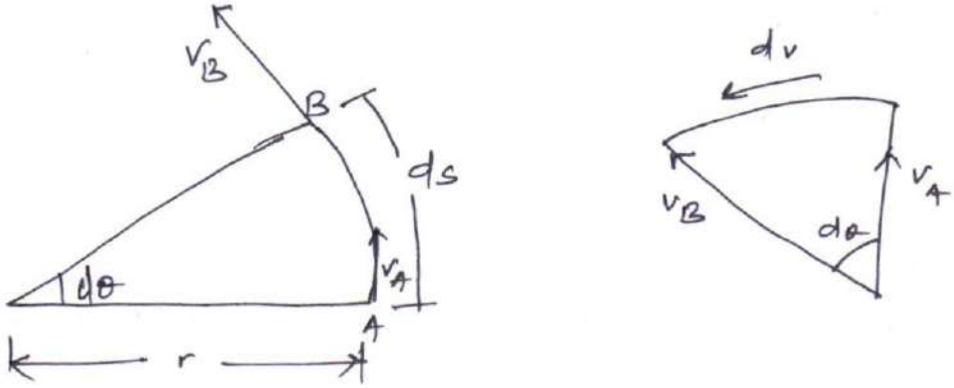
$\dot{x} = -r\omega\sin\omega t \quad \dot{y} = r\omega\cos\omega t$

$v = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\ddot{x} = -r\omega^2\cos\omega t \quad \ddot{y} = -r\omega^2\sin\omega t$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$

Acceleration during circular motion



V_A = tangential velocity at A
= tangential velocity at B
$= V_B = V$

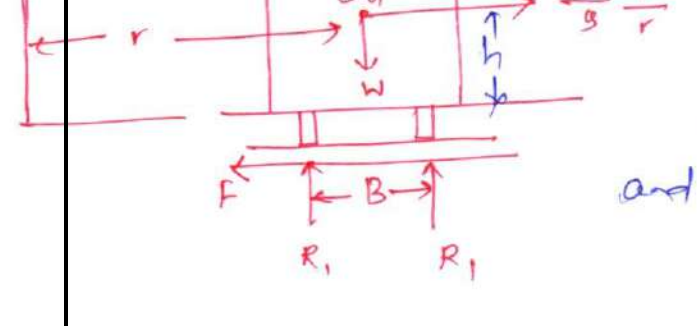
Now $dv = v\,d\theta = v\frac{ds}{r} = \frac{v}{r}ds$

acceleration $= \frac{dv}{dt} = \boxed{\frac{v^2}{r}}$

so when a body moves with uniform velocity v along a curved path of radius r, it has a radial inward acceleration of magnitude $\frac{v^2}{r}$

Applying D'Alembert's principle to get equilibrium condition an inertia force of magnitude $\frac{W}{g}a$ $= \frac{W}{g}\frac{v^2}{r}$ must be applied in outward direction it is known as centrifugal force.

Motion on a level road



Consider a body is moving with uniform velocity on a curvilinear curve of radius r. Let the road is flat.

Let W = wt. of the body

and inertia force is given by

$\frac{W}{g}a = \frac{W}{g}\frac{v^2}{r}$

Condition for skidding :-

Let W = wt. of vehicle

R_1, R_2 = reactions at wheel

F = frictional force.

$\frac{W}{g} \cdot \frac{v^2}{r}$ = inertia force

Skidding takes place when the frictional forces reaches limiting value i.e

$F = \mu W$

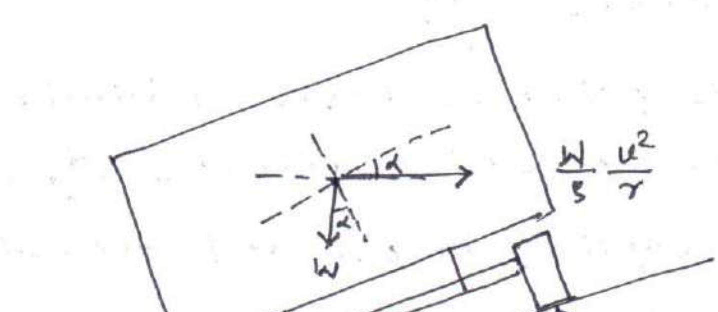
Then maxm permissible speed to avoid skidding

$$\boxed{v = \sqrt{\frac{gr}{2} \frac{B}{h}}}$$

The distance betn inner and outer wheel is equal to the gauge of railway track and expressed as G.

so $$\boxed{v = \sqrt{\frac{gr}{2} \frac{G}{h}}}$$

Designed speed and angle of Braking



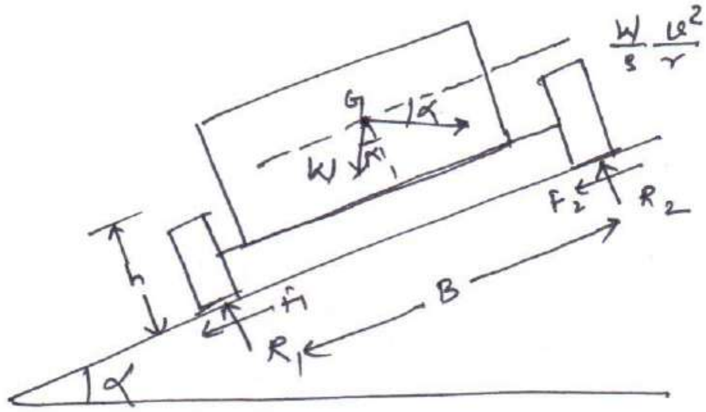
Σ of all the forces in the inclined plane

$\frac{W}{g} \frac{v^2}{r} \cos\alpha - W \sin\alpha = 0$

$\Rightarrow \boxed{\tan\alpha = \frac{v^2}{gr}}$

Relation betn the angle of braking and designed speed is $\boxed{\tan\alpha = \frac{v^2}{gr}}$

condition for skidding and overturning:-



(a) condition for skidding

$$\boxed{v = \sqrt{\tan(\alpha + \phi) \times g r}}$$

where α = angle of inclination

$\tan\phi = \mu$

g = ~~coeffic~~ gravitational acceleration

r = radius of curve

~~then the~~ vehicle will skid if the velocity is more than this value.

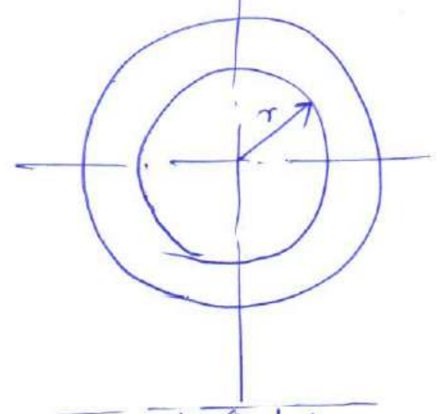
(b) condition for overturning:

Limiting speed from consideration of overturning

$$\boxed{v = \sqrt{g r \frac{G + (2he/G)}{2h - e}}}$$

Q.1 A circular ring has a mean radius $r = 500$ mm and is made of steel for which $w = 77.12\ kN/m^3$ and for which ultimate strength in tension is 413.85 MPa. Find the uniform speed of rotation about its geometrical axis perpendicular to the plane of the ring at which it will burst?

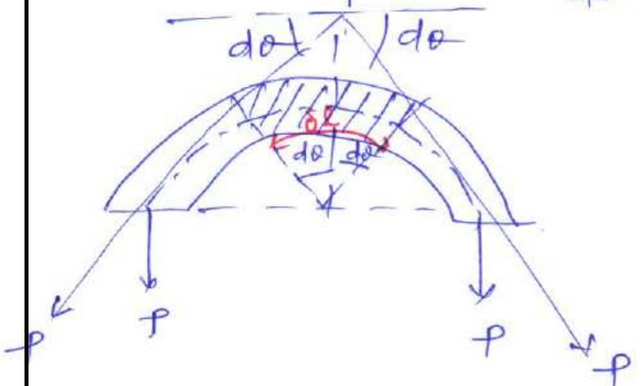
mean radius $r = 500\,mm = 0.5\,m$,



density of the wheel $w = 77.12\,kN/m$

σ_t = ultimate strength $= 413.85 \times 10^6 Pa$

Now considering an infinitesimal small elementary ring extruded at an angle of $2d\theta$



Centrifugal force acting

$$\delta F_c = \frac{dW}{g} \cdot \frac{v^2}{r}$$

Let P = tension on the ring

A = cross-sectional area of ring.

dW = wt. of the element

$= w \times \text{volume}$

$= w \times A \times \delta l$

$= w \times A \times r\,2d\theta$

Now centrifugal force

$$\frac{w}{g}(A\,dl) \times \frac{v^2}{r} = \frac{w}{g} \times A \cancel{r}\, 2d\theta \times \frac{v^2}{\cancel{r}} = \frac{2wA\,d\theta\, v^2}{g}$$

Balancing forces along the radius $= 2P \sin d\theta$

$$= \frac{2wA\,d\theta\, v^2}{g} \quad \text{--- (1)}$$

as $d\theta$ is very small $\sin d\theta \simeq d\theta$

Eq. (1) may be written as

$$\cancel{2}P\,\cancel{d\theta} = \frac{\cancel{2}wA\,\cancel{d\theta} \cdot v^2}{g}$$

$$\Rightarrow \boxed{P = \frac{wAv^2}{g}} \quad \text{--- (2)}$$

Tensile stress on the ring $\sigma_t = \frac{P}{A} = \frac{wv^2}{g}$

Now substituting the values

$$413.85 \times 10^6 = \frac{77.12 \times 10^3 \times v^2}{9.81} \Rightarrow v = 229.45\,m/s.$$

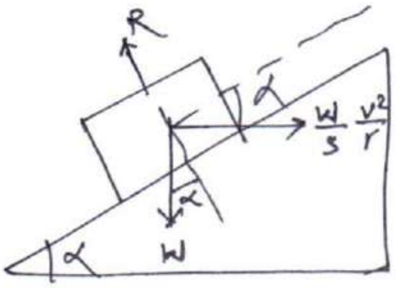
Now $v = \frac{\pi D N}{60} \Rightarrow N = \frac{60 \times 229.45}{\pi \times 1} = \boxed{4382.\ rpm.}$

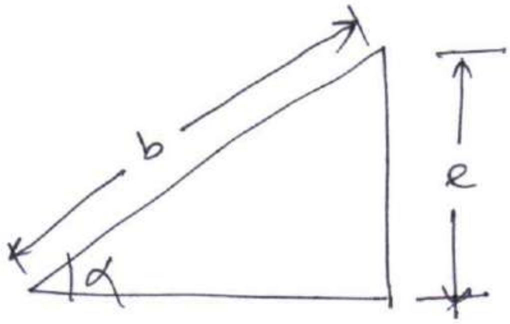
D' Alembert's Principle in Curvilinear Motion

Equation of motion of a particle may be written as

$$\left.\begin{aligned} X - m\ddot{x} &= 0 \\ Y - m\ddot{y} &= 0 \end{aligned}\right\} \quad \text{---} (1)$$

Q.2 Find the proper super elevation 'e' for a 7.2 m highway curve of radius r = 600 m in order that a car travelling with a speed of 80 Kmph will have no tendency to skid sidewise.





$b = 7.2\,m \qquad r = 600\,m \qquad v = 80\,Kmph = 22.23\,m/s.$

Resolving along the inclined plane

$$W \sin\alpha = \frac{W}{g} \cdot \frac{v^2}{r} \cos\alpha$$

$$\Rightarrow \boxed{\tan\alpha = \frac{v^2}{rg}}$$

from the geometry $\sin\alpha = \frac{e}{b}$, since α is very small

let $\sin\alpha \simeq \tan\alpha$

$$\frac{v^2}{rg} = \frac{e}{b} \Rightarrow e = \frac{bv^2}{rg} = \frac{7.2 \times 22.23^2}{600 \times 9.81}$$

$$= 0.604\,m \quad (\text{Ans})$$

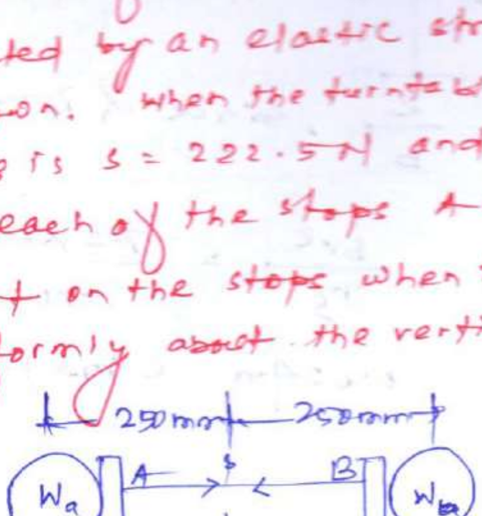
Q.3 A racing car travels around a circular track of 300m radius with a speed of 384 kmph. What angle α should the floor of the track make with horizontal in order to safeguard against skidding.

Velocity $v = 384$ kmph $\quad r = 300$m
$= 106.67$ m/s.

We have angle of banking $\tan\alpha = \dfrac{v^2}{rg}$

$\Rightarrow \alpha = \tan^{-1}\left(\dfrac{106.67^2}{300 \times 9.81}\right) = \boxed{75.5^\circ}$ (Ans)

Q.4 Two balls of wt $W_a = 44.5N$ and $W_b = 66.75N$ are connected by an elastic string and supported on a turntable as shown. When the turntable is at rest, the tension in the string is $S = 222.5N$ and the balls exert this same force on each of the stops A and B. What forces will they exert on the stops when the turntable is rotating uniformly about the vertical axis CD at 60 rpm?



We have;

$W_a = 44.5N \quad W_b = 66.75N$

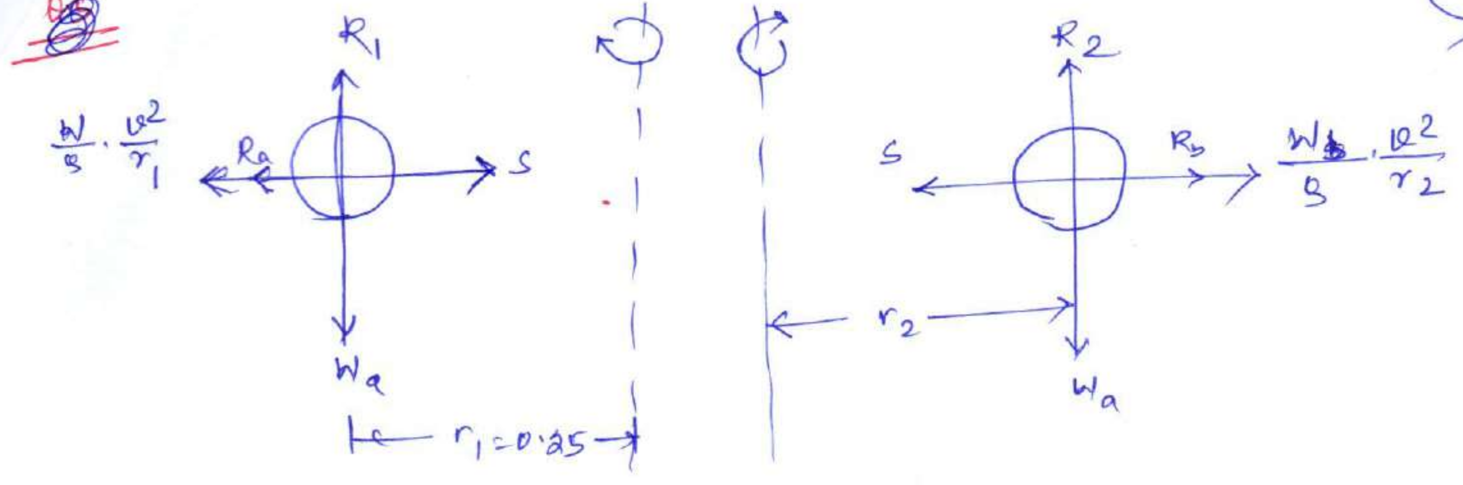
$S = 222.5N$

$N = 60$rpm,

radius of rotation $r_1, r_2 = 0.25$ m

Now angular velocity

$\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 60}{60} = 2\pi$ rad/s.



Considering the left hand side ball

$$R_a + \frac{W_a}{g} \cdot r_1 \omega^2 = S$$

$$\Rightarrow R_a = 222.5 - \frac{44.5}{9.81} \times 0.25 \times (2\pi)^2$$

$$= \boxed{177.72\ N.}$$

Considering the ball on right hand side

$$R_b + \frac{W_b}{g} \times r_2 \times \omega^2 = S$$

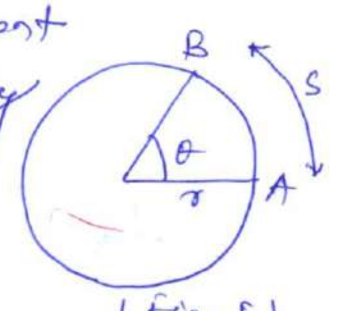
$$\Rightarrow R_b = 222.5 - \frac{66.75}{9.81} \times 0.25 \times (2\pi)^2$$

$$= \boxed{155.34\ N.}$$

Rotation of Rigid Bodies:-

Angular motion:-

The rate of change of angular displacement with time is called angular velocity and denoted by ω.



(fig-1)

$$\boxed{\omega = \frac{d\theta}{dt}} \quad \text{—(1)}$$

- The rate of change of angular velocity with time is called angular acceleration and denoted by α

$$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}} \quad \text{—(2)}$$

Angular acceleration may also be expressed as:

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{\alpha = \omega \cdot \frac{d\omega}{d\theta}} \quad \text{—(3)} \quad \left(\because \frac{d\theta}{dt} = \omega\right)$$

Relationship between angular motion and linear motion

from fig-1 $\quad s = r\theta$

tangential velocity (linear) of the particle p.

$$\boxed{v = \frac{ds}{dt} = r \cdot \frac{d\theta}{dt}} \quad \text{—(4)}$$

linear acceleration $\boxed{a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}}$ —(5)

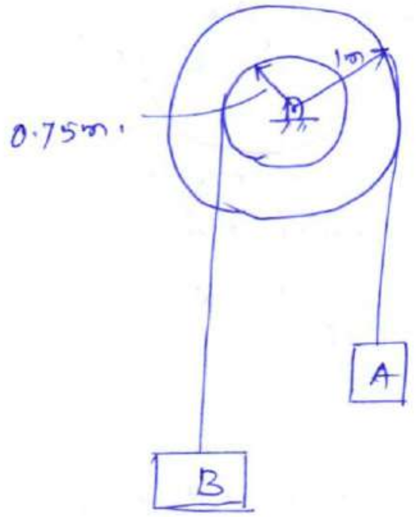
If $\frac{v^2}{r}$ = radial acceleration

Then $\boxed{a_n = \frac{v^2}{r} = r\omega^2}$ —(6) (where a_n = radial acceleration)

uniform angular velocity (ω)

$$\boxed{\omega = \frac{2\pi N}{60} \text{ rad/sec}} \quad \text{—(7)}$$

Q. The step pulley starts from rest and accelerates at 2 rad/s^2. How much time is required for block A to move 20m. Find also the velocity of A and B at that time.



When A moves by 20m, the angular displacement of pulley θ is given by

$$r\theta = s$$

$$\Rightarrow 1 \times \theta = 20$$

$$\Rightarrow \boxed{\theta = 20 \text{ rad}}$$

$\alpha = 2$ rad/s^2 and $\omega_0 = 0$

from kinematic relation

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow 20 = 0 \times t + \frac{1}{\cancel{2}} \times \cancel{2} t^2$$

$$\Rightarrow \boxed{t = 4.472 \text{ sec.}}$$

Velocity of pulley at this time

$$\omega = \omega_0 + \alpha t$$

$$= 0 + 2 \times 4.472$$

$$= \boxed{8.944 \text{ rad/s}}$$

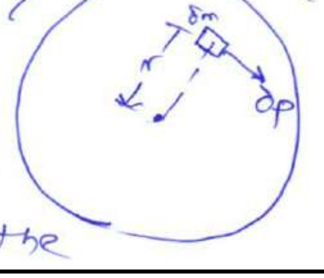
Velocity of block A $v_A = 1 \times 8.944$

$$= \boxed{8.944 \text{ m/s}}$$

Velocity of block B $v_B = 0.75 \times 8.944$

$$= \boxed{6.708 \text{ m/s.}}$$

Kinematics of rigid body for rotation:-



Consider a wheel rotating about it's axis in clockwise direction with an acceleration α. Let δm be mass of an element at a distance r from the axis of rotation, δp be the resulting force on this element

$\delta p = \delta m \times a$ (a = tangential acceleration)

but $a = r \times \alpha$ (α = angular acceleration)

$\therefore \boxed{\delta p = \delta m \, r \, \alpha}$

Rotational moment $\delta M_t = \delta p \times r$

$= \delta m \, r^2 \alpha$

$M_t = \Sigma \, \delta M_t = \Sigma \, \delta m \, r^2 \alpha$

$= \alpha \, \Sigma \, \delta m \, r^2$

$= \alpha I$

$\Rightarrow \boxed{M_t = \alpha I}$ (I = mass moment of inertia)

Product of mass moment of inertia and angular velocity of rotating body is called angular momentum

so $\boxed{\text{Angular momentum} = I\omega}$

Kinetic energy of rotating bodies

$\boxed{K.E = \frac{1}{2} I \omega^2}$

Q.2 A flywheel weighing 50 kN and having radius of gyration 1m losses its speed from 400 rpm to 280 rpm in 2 min. Calculate (a) retarding torque, (b) change in KE during the period, (c) change in angular momentum.

we have $\omega_0 = 400 \text{rpm} = \frac{2\pi \times 400}{60} = 41.89$ rad/s

$\omega = 280 \text{rpm} = \frac{2\pi \times 280}{60} = 29.32$ rad/s.

$t = 2 \text{min} = 120$ sec,

$\omega = \omega_0 + \alpha t$

$\Rightarrow \alpha = \frac{\omega - \omega_0}{t} = \boxed{-.1047 \text{ rad/s}^2}$

Wt of flywheel = 50000 N

mass of " $= \frac{50000}{9.81} = 5096.84$ kg.

Radius of gyration $k = 1$ m.

$I = mk^2$

$= 5096.84 \times 1 = 5096.84$

(a) Retarding torque

$I\alpha = 5096.84 \times 0.1047$

$= \boxed{533.64\ Nm}$

(b) change in KE

= initial KE − final KE

$= \frac{1}{2} I\omega_0^2 - \frac{1}{2} I\omega^2$

$= \frac{1}{2} \times 5096.84 \left(41.89^2 - 29.32^2\right)$

$= \boxed{\cancel{2280442.9}\ Nm}$ $\boxed{2281115.462\ Nm}$

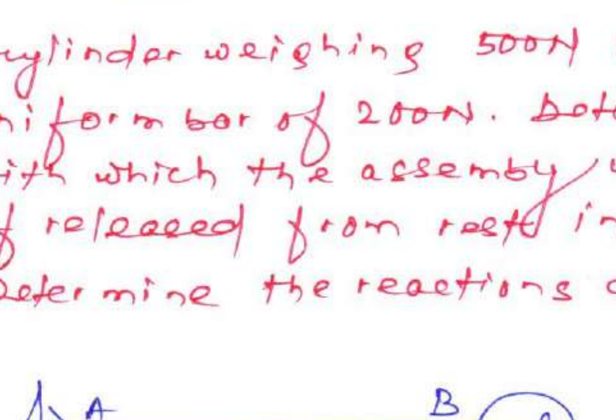
(c) change in angular momentum

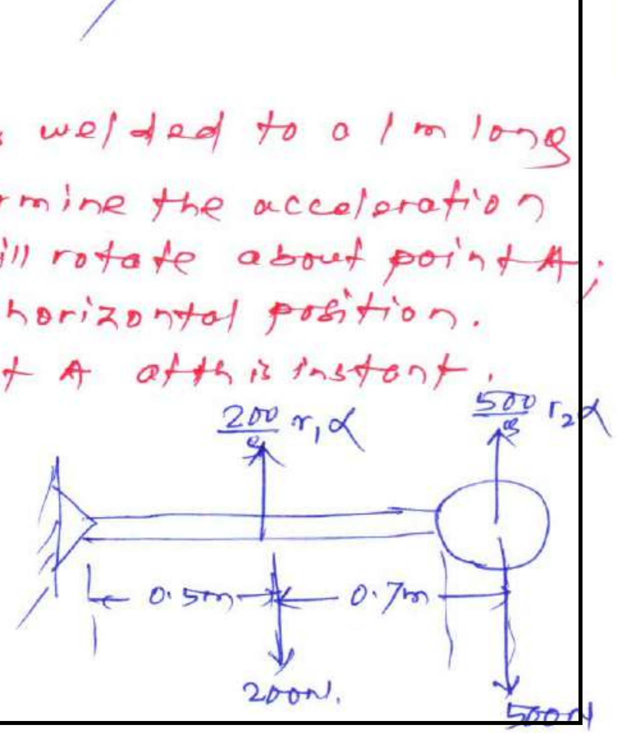
$I\omega_0 - I\omega$

$= 5096.84 \left(41.89 - 29.32\right)$

$= \boxed{64067.298\ Nm}$

Q.3 A cylinder weighing 500N is welded to a 1m long uniform bar of 200N. Determine the acceleration with which the assemby will rotate about point A, if released from rest in horizontal position. Determine the reactions at A at this instant.





Let α = angular acceleration of the assembly (3)

I = mass moment of inertia of the assembly

$I = I_G + Md^2$ (transfer formula)

mass MI of bar about A $= \frac{1}{2} \times \frac{200}{9.81} \times 1^2 + \frac{200}{9.81} \times (0.5)^2$

$= 6.7968$

mass MI of cylinder about A

$= \frac{1}{2} \frac{500}{9.81} \times 0.2^2 + \frac{500}{9.81} \times 1.2^2$

$= 74.4$

MI of the system $= 6.7968 + 74.4 = 81.2097$

Rotational moment about A

$M_t = 200 \times 0.5 + 500 \times 1.2 = 700$ Nm.

$M_t = I\alpha$

$\Rightarrow \alpha = \dfrac{700}{81.2097} = \boxed{8.6197}$ rad/sec.

Instantaneous acceleration of rod AB is vertical and $= r_1\alpha = 0.5 \times 8.6197$

$= 4.31$ m/s.

Similarly instantaneous acceleration of cylinder

$= r_2\alpha = 1.2 \times 8.6197$

$= 10.34$ m/s.

Applying D'Alembert's dynamic equilibrium

$R_A = 200 + 500 - \frac{200}{9.81} \times 4.31 - \frac{500}{9.81} \times 10.34$

$\Rightarrow \boxed{R_A = 84.93\text{ N.}}$ (Ans)

MODULE –V

MECHANICAL VIBRATIONS

COURSE OUTCOMES (COs):

At the end of the course students are able to:		
	Course Outcomes	**Knowledge Level (Bloom's Taxonomy)**
CO 12	**Compute** the time period and frequencies of simple, compound and torsional pendulums using the basics of free and forced vibrations.	Understand

PROGRAM OUTCOMES (POs):

	Program Outcomes (POs)	**Strength**	**Proficiency Assessed by**
PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	3	CIE/Quiz/AAT
PO 4	**Conduct Investigations of Complex Problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.	1	Seminar/ conferences / Research papers
PSO 1	Formulate and evaluate engineering concepts of design, thermal and production to provide solutions for technology aspects in digital manufacturing.	3	Research papers / Group discussion / Short term courses

MODULE V
MECHANICAL VIBRATIONS

Definitions and Concepts

Amplitude : Maximum displacement from equilibrium position; the distance from the midpoint of a wave to its crest or trough.

Equilibrium position: The position about which an object in harmonic motion oscillates; the center of vibration.

Frequency: The number of vibrations per unit of time.

Hooke's law: Law that states that the restoring force applied by a spring is proportional to the displacement of the spring and opposite in direction.

Ideal spring: Any spring that obeys Hooke's law and does not dissipate energy within the spring.

Mechanical resonance: Condition in which natural oscillation frequency equals frequency of a driving force.

Period: The time for one complete cycle of oscillation.

Periodic motion: Motion that repeats itself at regular intervals of time.

Restoring force: The force acting on an oscillating object which is proportional to the displacement and always points toward the equilibrium position.

Simple harmonic motion: Regular, repeated, friction-free motion in which the restoring force has the mathematical form F = - kx.

Simple Harmonic Motion

A pendulum, a mass on a spring, and many other kinds of oscillators exhibit a special kind of oscillatory motion called Simple Harmonic Motion (SHM).

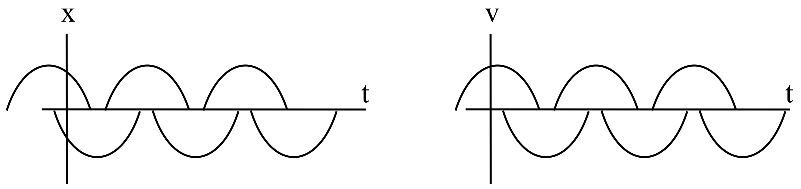
SHM occurs whenever :

i. here is a restoring force proportional to the displacement from equilibrium: $F \propto -x$

ii. the potential energy is proportional to the square of the displacement: $PE \propto x^2$

iii. the period T or frequency f = 1 / T is independent of the amplitude of the motion.

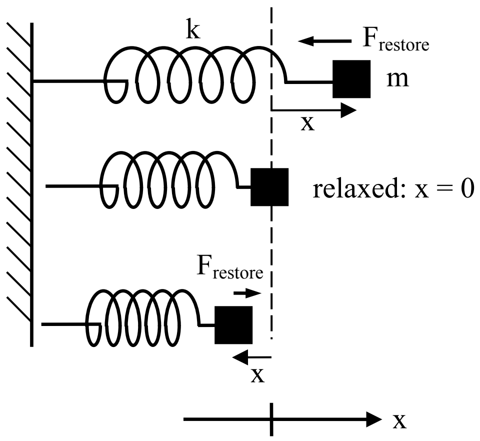
iv. the position *x*, the velocity *v*, and the acceleration *a* are all sinusoidal in time.



(*Sinusoidal* means sine, cosine, or anything in between.)
As we will see, any one of these four properties guarantees the other three. If one of these 4 things is true, then the oscillator is a simple harmonic oscillator and all 4 things must be true.

Not every kind of oscillation is SHM. For instance, a perfectly elastic ball bouncing up and down on a floor: the ball's position (height) is oscillating up and down, but none of the 4 conditions above is satisfied, so this is not an example of SHM.

A mass on a spring is the simplest kind of Simple Harmonic Oscillator.



Hooke's Law: $\mathbf{F}_{spring} = -k\,\mathbf{x}$

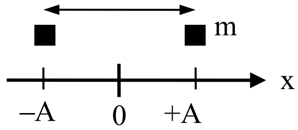
(–) sign because direction of $\mathbf{F}_{spring}$ is opposite to the direction of displacement vector **x**(**bold** font indicates vector)

k = spring constant = stiffness,
units [k] = N / m

Big k = stiff spring

Definition: *amplitude*A = $|x_{max}| = |x_{min}|$.

Mass oscillates between extreme positions x = +A and x = –A



Notice that Hooke's Law (F = –kx) is condition i : restoring force proportional to the displacement from equilibrium. We showed previously (Work and Energy Chapter) that for a spring obeying Hooke's Law, the potential energy is $U = (1/2)kx^2$, which is condition ii. Also, in the chapter on Conservation of Energy, we showed that F = –dU/dx, from which it follows that condition ii implies condition i. Thus, Hooke's Law and quadratic PE ($U \propto x^2$) are equivalent.

We now show that Hooke's Law guarantees conditions iii (period independent of amplitude) and iv (sinusoidal motion).

We begin by deriving the *differential equation* for SHM. A differential equation is simply an equation containing a derivative. Since the motion is 1D, we can drop the vector arrows and use sign to indicate direction.

$$F_{net} = m\,a \quad \text{and} \quad F_{net} = -k\,x \quad \Rightarrow \quad m\,a = -k\,x$$

$$a = dv/dt = d^2x/dt^2 \qquad \Rightarrow \qquad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

The constants k and m and both positive, so the k/m is always positive, always. For notational convenience, we write $k/m = \omega^2$. (The square on the ω reminds us that ω^2 is always positive.) The differential equation becomes

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \text{(equation of SHM)}$$

This is the *differential equation* for SHM. We seek a solution x = x(t) to this equation, a function x = x(t) whose second time derivative is the function x(t) multiplied by a negative constant ($-\omega^2 = -k/m$). The way you solve differential equations is the same way you solve integrals: you *guess* the solution and then check that the solution works.

Based on observation, sinusoidal solution: $x(t) = A\cos(\omega t + \varphi)$,

where A, φ are <u>any</u> constants and (as we'll show) $\omega = \sqrt{\frac{k}{m}}$.

A = amplitude: x oscillates between +A and –A

φ = phase constant (more on this later)

Danger: ωt and φ have units of radians (not degrees). So set your calculators to radians when using this formula.

Just as with circular motion, the angular frequency ω for SHM is related to the period by

$\omega = 2\pi f = \frac{2\pi}{T}$, T = period.

(What does SHM have to do with circular motion? We'll see later.)

Let's check that $x(t) = A\cos(\omega t + \varphi)$ is a solution of the SHM equation.

Taking the first derivative dx/dt , we get $v(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$.

Here, we've used the Chain Rule: $\frac{d}{dt}\cos(\omega t + \varphi) = \frac{d\cos(\theta)}{d\theta}\frac{d\theta}{dt}$, $(\theta = \omega t + \varphi)$

$$= -\sin\theta \cdot \omega = -\omega\sin(\omega t + \varphi)$$

Taking a second derivative, we get

$$a(t) = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{d}{dt}\left(-A\omega\sin(\omega t + \varphi)\right) = -A\omega^2\cos(\omega t + \varphi)$$

$$\frac{d^2x}{dt^2} = -\omega^2\left[A\cos(\omega t + \varphi)\right]$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

This is the SHM equation, with $\omega^2 = \frac{k}{m}$, $\omega = \sqrt{\frac{k}{m}}$

We have shown that our assumed solution is indeed a solution of the SHM equation. (I leave to the mathematicians to show that this solution is unique. Physicists seldom worry about that kind of thing, since we know that nature usually provides only one solution for physical systems, such as masses on springs.)

We have also shown condition iv: x, v, and a are all sinusoidal functions of time:

$$x(t) = A\cos(\omega t + \varphi)$$
$$v(t) = -A\omega \sin(\omega t + \varphi)$$
$$a(t) = -A\omega^2 \cos(\omega t + \varphi)$$

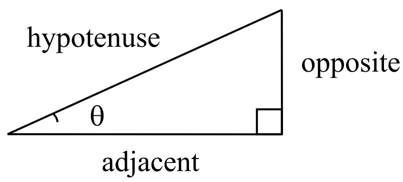
The period T is given by $\omega = \sqrt{\dfrac{k}{m}} = \dfrac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\dfrac{m}{k}}$. We see that T does not depend on the amplitude A (condition iii).

Let's first try to make sense of $\omega = \sqrt{k/m}$: big ω means small T which means rapid oscillations. According to the formula, we get a big ω when k is big and m is small. This makes sense: a big k (stiff spring) and a small mass m will indeed produce very rapid oscillations and a big ω.

A closer look atx(t) = A cos(ωt+φ)

Let's review the sine and cosine functions and their relation to the *unit* circle. We often define the sine and cosine functions this way:

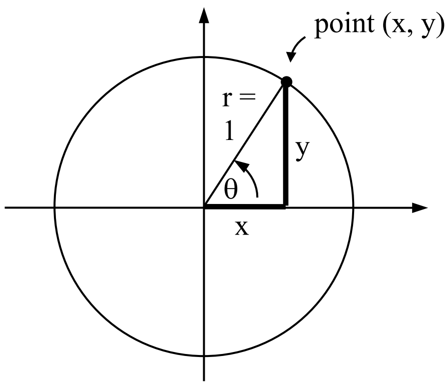
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

This way of defining sine and cosine is correct but incomplete. It is hard to see from this definition how to get the sine or cosine of an angle greater than 90°.

A more complete way of defining sine and cosine, a way that gives the value of the sine and cosine for *any* angle, is this: Draw a *unit* circle (a circle of radius r = 1) centered on the origin of the x-y axes as shown:



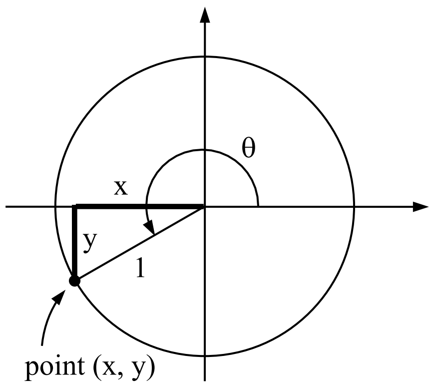
Define sine and cosine as

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$$

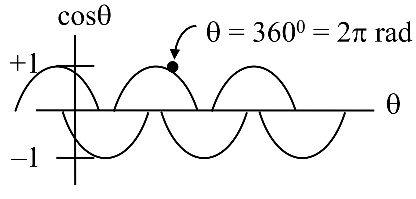
This way of defining sin and cos allows us to compute the sin or cos of *any* angle at all.

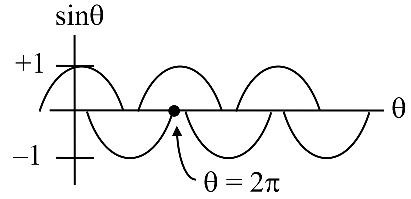
For instance, suppose the angle is θ = 210°. Then the diagram looks like this:



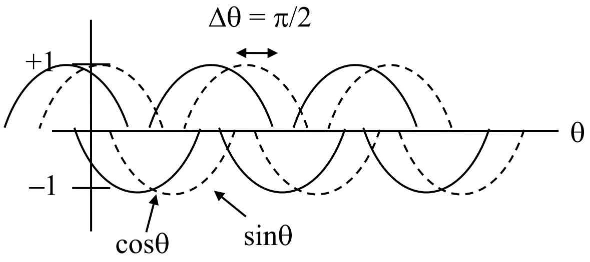
The point on the unit circle is in the third quadrant, where both x and y are negative. So both cosθ = x and sinθ = y are negative

For any angle θ, even angles bigger than 360° (more than once around the circle), we can always compute sin and cos. When we plot sin and cos vs angle θ, we get functions that oscillate between +1 and –1 like so:





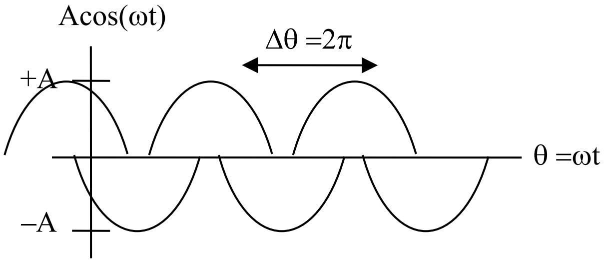
We will almost always measure angle θ in radians. Once around the circle is 2π radians, so sine and cosine functions are periodic and repeat every time θ increases by 2π rad. The sine and cosine functions have exactly the same shape, except that sin is shifted to the right compared to cos by $\Delta\theta = \pi/2$. Both these functions are called *sinusoidal* functions.



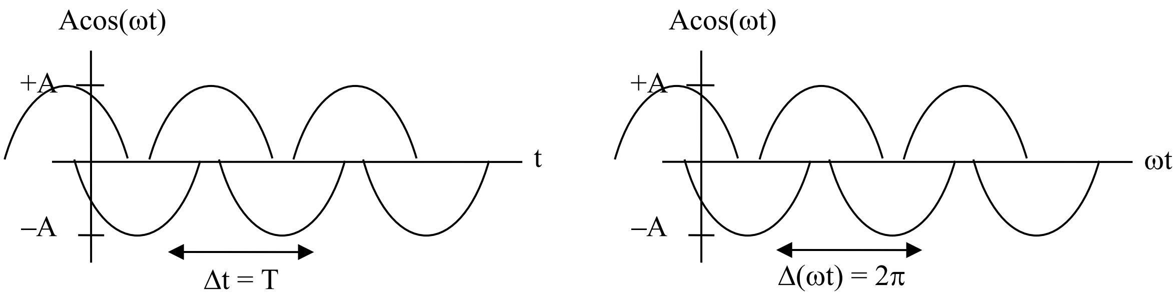
The function cos(θ + φ) can be made to be anything in between cos(θ) and sin(θ) by adjusting the size of the *phase*φ between 0 and –2π.

$$\cos\theta,\ (\varphi = 0)\ \rightarrow\ \sin\theta = \cos\left(\theta - \frac{\pi}{2}\right),\quad (\varphi = -\pi/2)$$

The function cos(ωt + φ) oscillates between +1 and –1, so the function Acos(ωt + φ) oscillates between +A and –A.



Why $\omega = \dfrac{2\pi}{T}$? The function f(θ) = cosθ is periodic with period Δθ = 2π. Since θ =ωt+φ, and φ is some constant, we have Δθ =ω Δt. One complete cycle of the cosine function corresponds to Δθ = 2π and Δt = T, (T is the period). So we have 2π = ω T or $\omega = \dfrac{2\pi}{T}$. Here is another way to see it: $\cos(\omega t) = \cos\left(2\pi\dfrac{t}{T}\right)$ is periodic with period Δt = T. To see this, notice that when t increases by T, the fraction t/T increases by 1 and the fraction 2πt/T increases by 2π.



Now back to simple harmonic motion. Instead of a circle of radius 1, we have a circle of radius A (where A is the amplitude of the Simple Harmonic Motion).

SHM and Conservation of Energy:

Recall $PE_{elastic} = (1/2)\ k\ x^2$ = work done to compress or stretch a spring by distance x.

If there is no friction, then the total energy $E_{tot} = KE + PE$ = constant during oscillation. The value of E_{tot} depends on initial conditions – where the mass is and how fast it is moving initially. But once the mass is set in motion, E_{tot} stays constant (assuming no dissipation.)

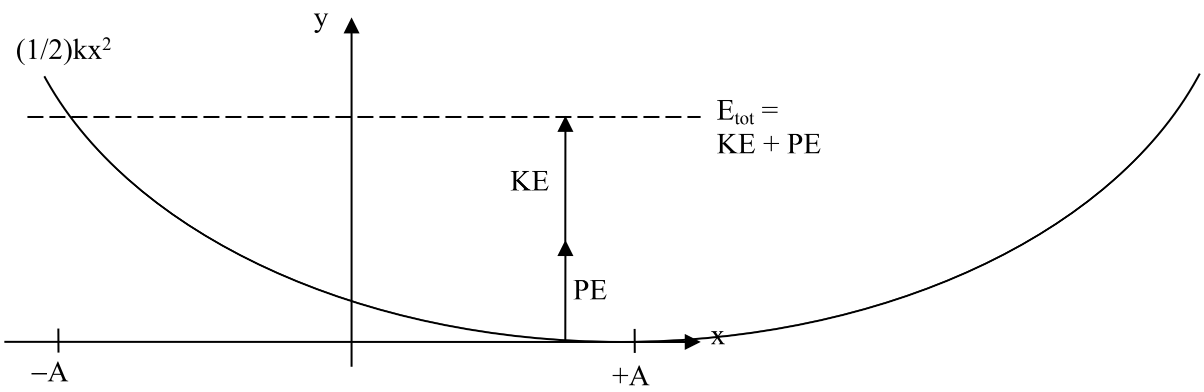
At any position x, speed v is such that $\boxed{\tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2} = E_{tot}$.

When |x| = A, then v = 0, and all the energy is PE: $\underbrace{KE}_{0} + \underbrace{PE}_{(1/2)kA^2} = E_{tot}$

So total energy $E_{tot} = \tfrac{1}{2} k A^2$

When x = 0, v = v_{max}, and all the energy is KE: $\underbrace{KE}_{(1/2)m v_{max}^2} + \underbrace{PE}_{0} = E_{tot}$

So, total energy $E_{tot} = \tfrac{1}{2} m v_{max}^2$.



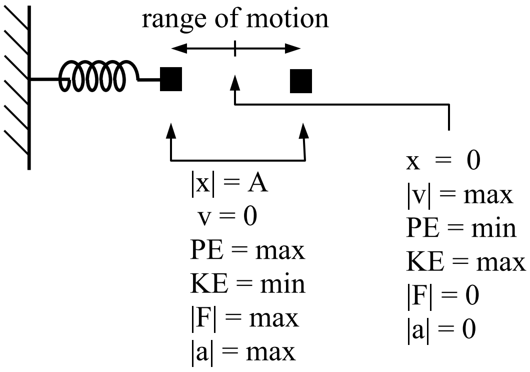
So, we can relate v_{max} to amplitude A :$PE_{max} = KE_{max} = E_{tot} \Rightarrow \tfrac{1}{2} k A^2 = \tfrac{1}{2} m v_{max}^2 \Rightarrow$

$$v_{max} = \sqrt{\frac{k}{m}}\ A$$

Example Problem: A mass m on a spring with spring constant k is oscillating with amplitude A. Derive a general formula for the speed v of the mass when its position is x.

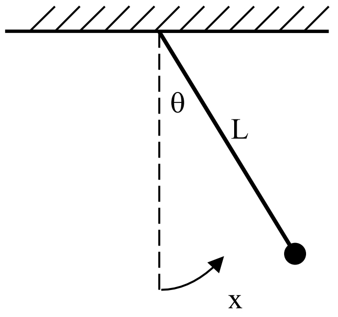
Answer: $v(x) \;=\; A\sqrt{\frac{k}{m}}\;\sqrt{1-\left(\frac{x}{A}\right)^2}$

Be sure you understand these things:

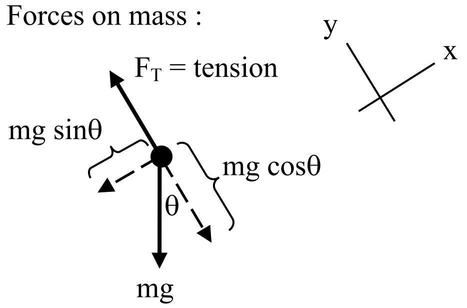


Pendulum Motion

A simple pendulum consists of a small mass m suspended at the end of a massless string of length L. A pendulum executes SHM, if the amplitude is not too large.



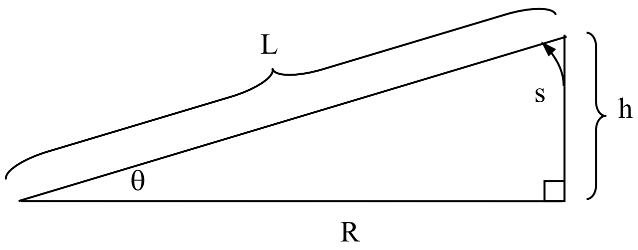
$\theta = x / L$ (rads)



The restoring force is the component of the force along the direction of motion:

$$\text{restoring force} = -mg\sin\theta \cong -mg\,\theta = -mg\frac{x}{L}$$

Claim: $\sin\theta \cong \theta\,(\text{rads})$ when θ is small. $\sin\theta = \dfrac{h}{L}$



If θ small, then $h \approx s$, and $L \approx R$, so $\sin\theta \approx \theta$.

Try it on your calculator: $\theta = 5^{o} = 0.087266..\text{rad}$, $\sin\theta = 0.087156..$

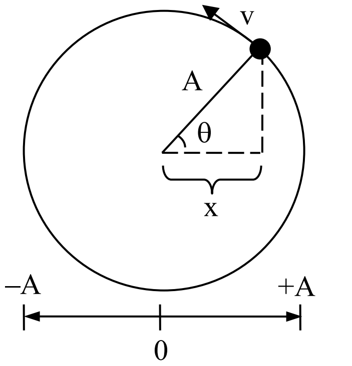
$F_{restore} = -\left(\dfrac{mg}{L}\right)x$ is exactly like Hooke's Law $F_{restore} = -k\,x$, except we have replaced the constant k with another constant (mg / L). The math is exactly the same as with a mass on a spring; all results are the same, except we replace k with (mg/L).

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}} \;\Rightarrow\; T_{pend} = 2\pi\sqrt{\frac{m}{(mg/L)}} = 2\pi\sqrt{\frac{L}{g}}$$

Notice that the period is independent of the amplitude; the period depends only on length L and acceleration of gravity. (But this is true only if θ is not too large.)

SHM and circular motion

There is an exact analogy between SHM and *circular motion*. Consider a particle moving with constant speed v around the rim of a circle of radius A.
The x-component of the position of the particle has *exactly* the same mathematical form as the motion of a mass on a spring executing SHM with amplitude A.



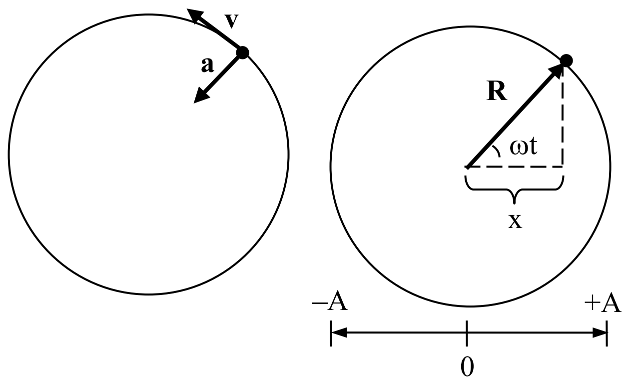
Angular velocity $\omega = \dfrac{d\theta}{dt} = \text{const} \Rightarrow$

$\theta = \omega t$ so

This same formula also describes the *sinusoidal* motion of a mass on a spring.

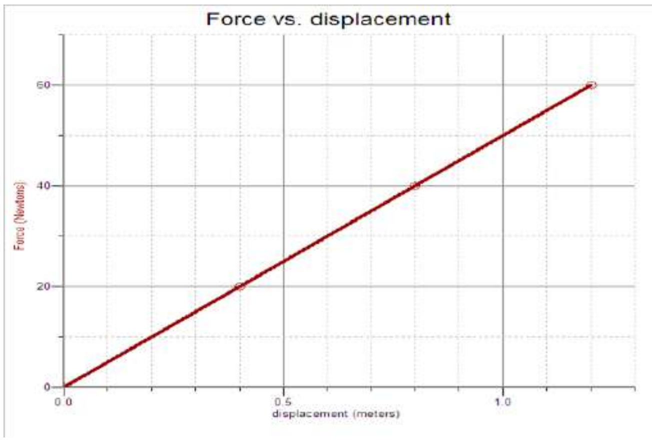
That the same formula applies for two different situations (mass on a spring & circular motion) is no accident. The two situations have the same solution because they both obey the same equation. As Feynman said, "The same equations have the same solutions". The equation of SHM is $\dfrac{d^2x}{dt^2} = -\omega^2 x$. We now show that a particle in circular motion obeys this same SHM equation.

Recall that for circular motion with angular speed ω, the acceleration of a the particle is toward the center and has magnitude $|\vec{a}| = \dfrac{v^2}{R}$. Since v = ω R , we can rewrite this as $|\vec{a}| = \dfrac{(\omega R)^2}{R} = \omega^2 R$



Let's set the origin at the center of the circle so the position vector **R** is along the radius. Notice that the acceleration vector **a** is always in the direction opposite the position vector **R** . Since $|\vec{a}| = \omega^2 |\vec{R}|$, the vectors **a** and **R** are related by $\vec{a} = -\omega^2 \vec{R}$. The x-component of this vector equation is: $a_x = -\omega^2 R_x$. If we write $R_x = x$, then we have $\dfrac{d^2x}{dt^2} = -\omega^2 x$, which is the SHM equation. Done.

Example
A mass of 0.5 kg oscillates on the end of a spring on a horizontal surface with negligible friction according to the equation $x = A\cos(\omega t)$. The graph of *F vs. x* for this motion is shown below.



The last data point corresponds to the maximum displacement of the mass.
Determine the
(a) angular frequency ω of the oscillation,
(b) frequency f of oscillation,
(c) amplitude of oscillation,
(d) displacement from equilibrium position ($x = 0$) at a time of 2 s.

Solution:
(a) We know that the spring constant $k = 50$ N/m from when we looked at this graph earlier. So,

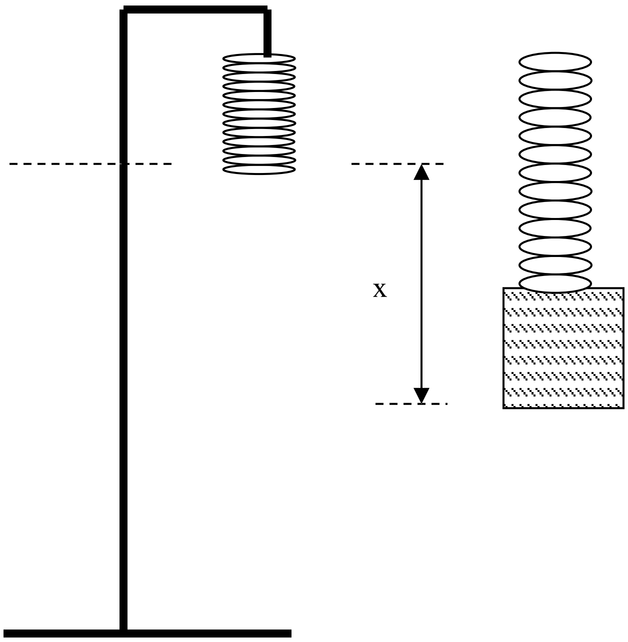
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\,N/m}{0.5\,kg}} = 10\frac{rad}{s}$$

(b) $f = \dfrac{\omega}{2\pi} = \dfrac{10\,rad/s}{2\pi} = 1.6\,Hz$

(c) The amplitude corresponds to the last displacement on the graph, $A = 1.2$ m.

(d) $x = A\cos(\omega t) = (1.2\,m)\cos[(10\,rad/s)(2\,s)] = 0.5\,m$

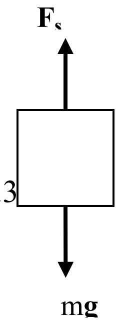
Example
A spring of constant $k = 100$ N/m hangs at its natural length from a fixed stand. A mass of 3 kg is hung on the end of the spring, and slowly let down until the spring and mass hang at their new equilibrium position.



(a) Find the value of the quantity *x* in the figure above.The spring is now pulled down an additional distance *x* and released from rest.
(b) What is the potential energy in the spring at this distance?
(c) What is the speed of the mass as it passes the equilibrium position?
(d) How high above the point of release will the mass rise?
(e) What is the period of oscillation for the mass?

Solution:
(a) As it hangs in equilibrium, the upward spring force must be equal and opposite to the downward weight of the block.

$$F_s = mg$$
$$kx = mg$$
$$x = \frac{mg}{k} = \frac{(3\,kg)(10\,m/s^2)}{100\ N/m} = 0.3$$



(b) The potential energy in the spring is related to the displacement from equilibrium position by the equation
$$U = \frac{1}{2}kx^2 = \frac{1}{2}(100\,N/m)(0.3m)^2 = 4.5\,J$$
(c) Since energy is conserved during the oscillation of the mass, the kinetic energy of the mass as it passes through the equilibrium position is equal to the potential energy at the amplitude. Thus,

$$K = U = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(4.5\,J)}{3\,kg}} = 1.7\,m/s$$
(d) Since the amplitude of the oscillation is 0.3 m, it will rise to 0.3 m above the equilibrium position.
(e) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3\,kg}{100\,N/m}} = 1.1\,s$

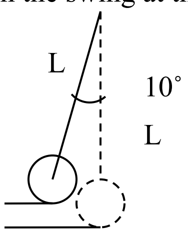
Example
A pendulum of mass 0.4 kg and length 0.6 m is pulled back and released from and angle of 10˚ to the vertical.

(a) What is the potential energy of the mass at the instant it is released. Choose potential energy to be zero at the bottom of the swing.
(b) What is the speed of the mass as it passes its lowest point?

This same pendulum is taken to another planet where its period is 1.0 second.
(c) What is the acceleration due to gravity on this planet?

Solution
(a) First we must find the height above the lowest point in the swing at the instant the pendulum is released.

Recall from chapter 1 of this study guide that $h = L - L\cos\theta$.
Then
$$U = mg(L - L\cos\theta)$$
$$U = (0.4\,kg)(10\,m/s^2)(0.6\,m - 0.6\,m\cos 10°) = 0.$$



(b) Conservation of energy:

$$U_{\max} = K_{\max} = \frac{1}{2}mv^2$$

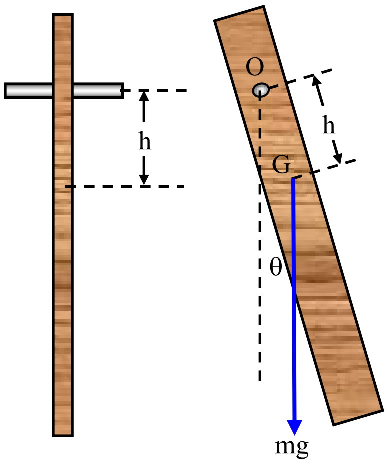
$$v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(0.4\,J)}{0.4\,kg}} = 1.4\,m/s$$

(c)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.6\,m)}{(1.0\,s)^2} = 23.7\,\frac{m}{s^2}$$

COMPOUND PENDULUM



AIM:
The aim of this experiment is to measure g using a compound pendulum.

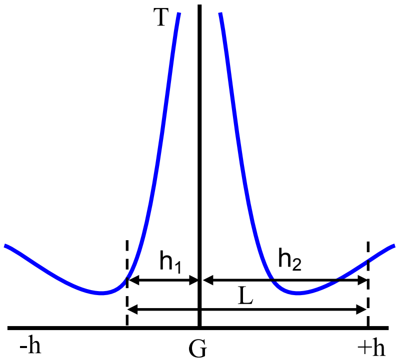
YOU WILL NEED:

WHAT TO DO:

First put the knife edge through the hole in the metre rule nearest end A, and record the time for 10 oscillations. Hence work out the time for one oscillation (T).
Repeat this for each hole in the ruler for a series of different distances (d) from end A.

ANALYSIS AND CALCULATIONS:
Plot a graph of T against d.
From the graph record a series of values of the simple equivalent pendulum (L).
Calculate the value of g from the graph or from the formula:

$$T^2 = 4\pi^2 L/g$$



Torsion Pendulum:

1. Introduction
Torsion is a type of stress, which is easier to explain for a uniform wire or a rod when one end of the wire is fixed, and the other end is twisted about the axis of the wire by an external force. The external force causes deformation of the wire and appearance of counterforce in the material. If this end is released, the internal torsion force acts to restore the initial shape and size of the wire. This behavior is similar to the one of the released end of a linear spring with a mass attached.

Attaching a mass to the twisting end of the wire, one can produce a torsion pendulum with circular oscillation of the mass in the plane perpendicular to the axis of the wire.

To derive equations of rotational motion of the torsion pendulum, it would be useful to recall a resemblance of quantities in linear and rotational motion. We know that if initially a mass is motionless, its linear motion is caused by force F; correspondingly, if an extended body does not rotate initially, its rotation is caused by torque τ. The measure of inertia in linear motion is mass, m, while the measure of inertia in rotational motion is the moment of inertia about an axis of rotation, I. For linear and angular displacement in a one-dimensional problem, we use either x or θ. Thus, the two equations of motion are:

$$F_x = ma_x \text{and} \quad \tau = I\alpha \qquad (1)$$

where a_x and α are the linear and the angular acceleration.
If the linear motion is caused by elastic, or spring, force, the Hooke's law gives $F_x = -kx$, where k is the spring constant. If the rotation is caused by torsion, the Hooke's law must result in

$\tau = -\kappa\theta$

where κ is the torsion constant, or torsional stiffness, that depends on properties of the wire. It is essentially a measure of the amount of torque required to rotate the free end of the wire 1 radian.

Your answer to the Preparatory Question 2 gives the following relationship between the moment of inertia I of an oscillating object and the period of oscillation T as:

This relationship is true for oscillation where damping is negligible and can be ignored. Otherwise the relationship between I and κ is given by

$$I = \frac{\kappa}{\omega_0^2} \qquad (3^*)$$

where ω_0 can be found from $\omega = \sqrt{\omega_0^2 - \left(\frac{c}{2I}\right)^2} \qquad (3^{**})$

$\omega = \frac{2\pi}{T} = 2\pi f$; f is the frequency of damped oscillation; and c is the *damping coefficient*.
The relationship between the torsion constant κ and the diameter of the wire d is given in [3] (check your answer to the Preparatory Question 1) as

$$\kappa = \frac{\pi G d^4}{32 l} \qquad (4)$$

where l is the length of the wire and G is the shear modulus for the material of the wire.

As any mechanical motion, the torsional oscillation is damped by resistive force originating from excitation of thermal modes of oscillation of atoms inside the crystal lattice of the wire and air resistance to the motion of the oscillating object. We can estimate the torque of the resistive force as being directly proportional to the angular speed of the twisting wire, i.e. the torque $\tau_R = -c\,d\theta/dt$ (recall the drag force on mass on spring in viscose medium as $R = -bv$). Combining Eq.(1), (2) and the expression for τ_R, we obtain the equation of motion of a torsional pendulum as follows:

$$I\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + \kappa\theta = 0 \qquad (5)$$

The solution of Eq.(5) is similar to the solution of the equation for damped oscillation of a mass on spring and is given by:

$$\theta = Ae^{-\alpha t}\cos(\omega t + \varphi) \qquad (6)$$

where $\alpha = c/2I \qquad (7)$

and $\alpha = \beta^{-1}$ with β being the time constant of the damped oscillation; c is the damping coefficient; ω is the angular frequency of torsional oscillation measured in the experiment; and φ can be made zero by releasing the object on the wire at a position of the greatest deviation from equilibrium.

Equation (6) can be used to calculate c (damping coefficient) and β (time constant = amount of time to decay e times) with DataStudio interface and software.

Another important formula is $\alpha = \omega_0/2Q$, where Q is the *quality factor* and $\omega_0^2 = \kappa / I$ (see Eq.3'). The ratio

$$\zeta = \alpha/\omega_0 = (2Q)^{-1} \qquad (8)$$

is called the *damping ratio*.

Free vibration of One Degree of Freedom Systems

Free vibration of a system is vibration due to its own ***internal forces*** (free of external impressive forces). It is initiated by an initial deviation (an energy input) of the system from its static equilibrium position. Once the initial deviation (a displacement or a velocity or both) is suddenly withdrawn, the strain energy stored in the system forces the system to return to its original, static equilibrium configuration. Due to the inertia of the system, the system will not return to the equilibrium configuration in a straightforward way. Instead it will oscillate about this position — free vibration.

A system experiencing free vibration oscillates at one or more of its natural frequencies, which are properties of its mass and stiffness distribution. If there is no damping (an undamped system), the system vibrates at the (*undamped*) frequency (frequencies) forever. Otherwise, it vibrates at the (*damped*) frequency (frequencies) and dies out gradually. When damping is not large, as in most cases in engineering, undamped and damped frequencies are very close. Therefore usually no distinction is made between the two types of frequencies.

The number of natural frequencies of a system equals to the number of its degrees-of-freedom. Normally, the low frequencies are more important.

Damping always exists in materials. This damping is called material damping, which is always positive (dissipating energy). However, air flow, friction and others may 'present' negative damping.

Undamped Free Vibration

Equation of motion based on the free-body diagram

$$m\ddot{x} + kx = 0$$

$$\Big\downarrow$$

$$\ddot{x} + \omega_n^2 x = 0 \qquad \omega_n = \sqrt{\frac{k}{m}}$$

natural frequency

$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

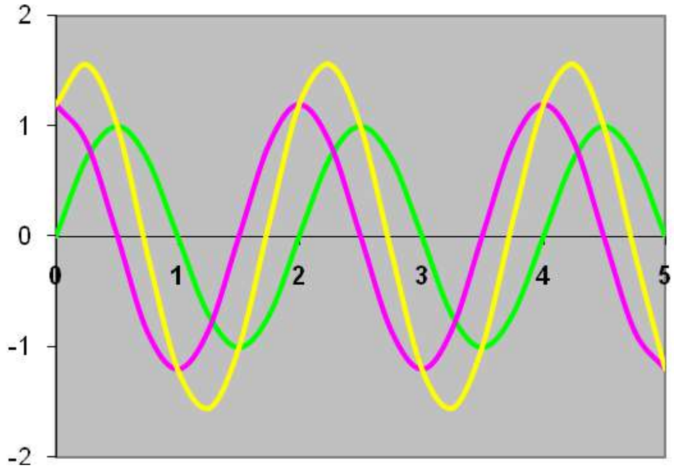
period

$$x(t) = A\sin\omega_n t + B\cos\omega_n t$$

A and B are determined by the *initial conditions.*

$\boxed{\text{Sin or Cos}}$

$$\boxed{\begin{array}{ll} \tau = ? & \omega_n = ? \\ x(0) = ? & \dot{x}(0) = ? \end{array}}$$



$$x(t) = \frac{\dot{x}(0)}{\omega_n} \sin \omega_n t + x(0) \cos \omega_n t$$

$$= \sqrt{\left(\frac{\dot{x}(0)}{\omega_n}\right)^2 + [x(0)]^2} \sin(\omega_n t + \varphi) \text{ where } \varphi = \arctan\left(\frac{x(0)\omega_n}{\dot{x}(0)}\right)$$

Vibration of a pendulum

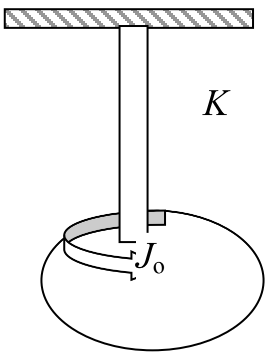
How to establish the equation of motion?

What is its natural frequency?

$$ml^2\ddot{\theta} = -mgl\sin\theta \rightarrow l\ddot{\theta} + g\sin\theta = 0$$

$$l\ddot{\theta} + g\theta = 0 \quad \rightarrow \quad \omega_n = \sqrt{\frac{g}{l}}$$

Systems with Rotational Degrees-of-Freedom



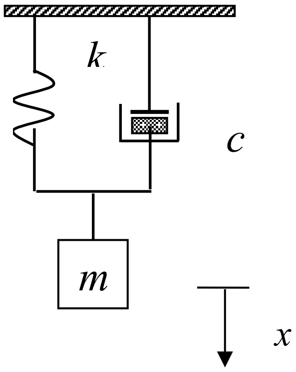
Equation of Motion

$$J_o\ddot{\theta} + K\theta = 0$$

natural frequency $$\omega_n = \sqrt{\frac{K}{J_o}}$$

Systems involving rotational degrees-of-freedom are always more difficult to deal with, in particular when translational degrees-of-freedom are also present. Gear care is needed to identify both degrees-of-freedom and construct suitable equations of motion.

Damped Free Vibration (first hurdle in studying vibration)



$$m\ddot{x} = -kx - c\dot{x} \qquad m\ddot{x} + c\dot{x} + kx = 0$$

standard equation $$\ddot{x} + 2\zeta\omega_{\mathrm{n}}\dot{x} + \omega_{\mathrm{n}}^2 x = 0$$

damping factor $$\zeta = \frac{c}{2m\omega_{\mathrm{n}}} = \frac{c}{2\sqrt{km}}$$

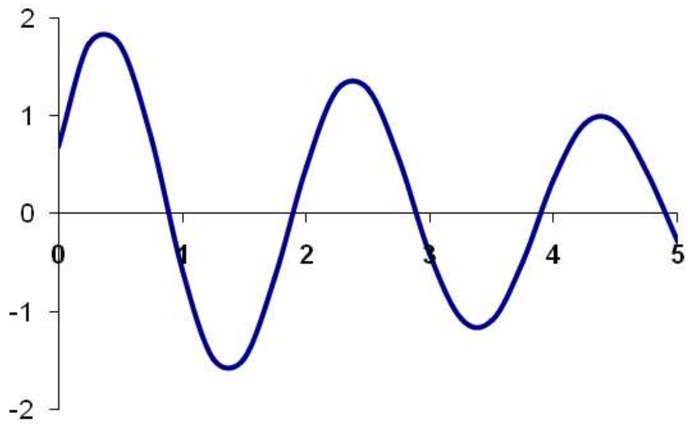
1. **oscillatory motion** (under-damped $\zeta < 1$)

$$x(t) = \exp(-\zeta\omega_{\mathrm{n}}t)[C_1\exp(\sqrt{\zeta^2 - 1}\,\omega_{\mathrm{n}}t) + C_2\exp(-\sqrt{\zeta^2 - 1}\,\omega_{\mathrm{n}}t)]$$

$$x(t) = \exp(-\zeta\omega_{\mathrm{n}}t)(A\sin\omega_{\mathrm{d}}t + B\cos\omega_{\mathrm{d}}t) = X\exp(-\zeta\omega_{\mathrm{n}}t)\sin(\omega_{\mathrm{d}}t + \varphi)$$

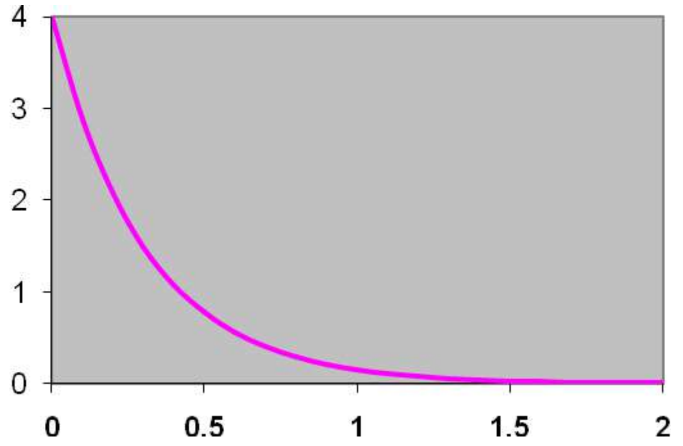
$$x(t) = \exp(-\zeta\omega_{\mathrm{n}}t)[\frac{\dot{x}(0) + \zeta\omega_{\mathrm{n}}x(0)}{\omega_{\mathrm{d}}}\sin\omega_{\mathrm{d}}t + x(0)\cos\omega_{\mathrm{d}}t] \qquad \omega_{\mathrm{d}} = \omega_{\mathrm{n}}\sqrt{1-\zeta^2}$$

damped natural frequency

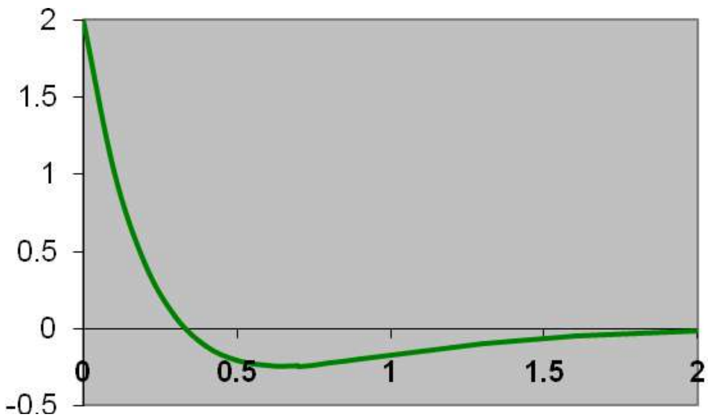


2. **nonoscillatory motion** (over-damped $\zeta > 1$)

$$x(t) = \exp(-\zeta\omega_{\mathrm{n}}t)[A\exp(\sqrt{\zeta^2 - 1}\,\omega_{\mathrm{n}}t) + B\exp(-\sqrt{\zeta^2 - 1}\,\omega_{\mathrm{n}}t)]$$



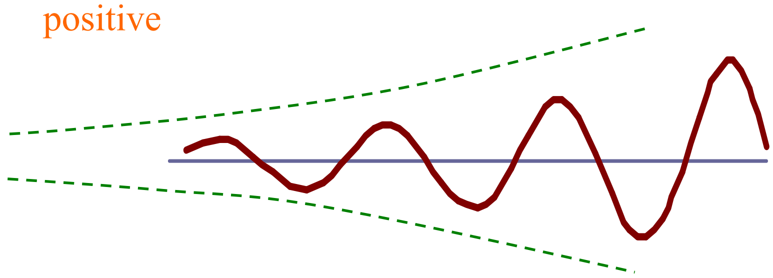
3. **critically damped motion** ($\zeta = 1$)



$$x(t) = (A + Bt)\exp(-\omega_n t)$$

4. negative damping of $\zeta < 0$ as a special case of $\zeta < 1$:

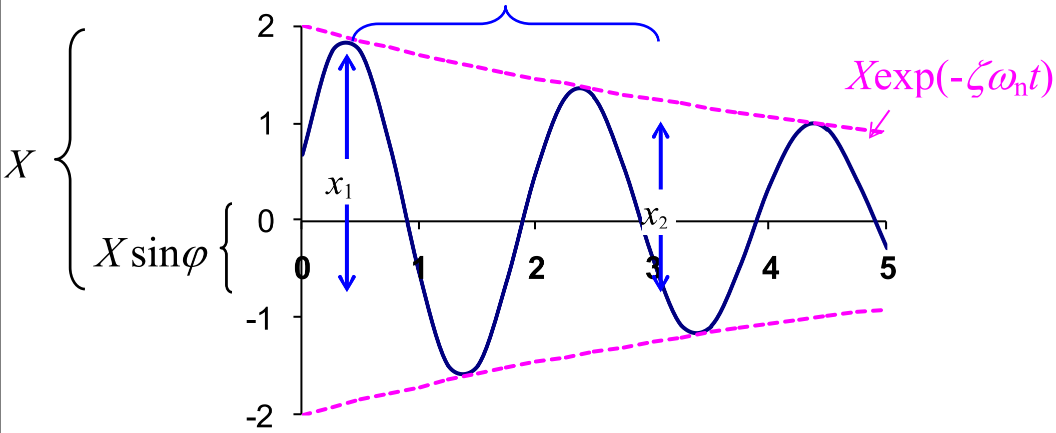
$$x(t) = \exp(\underbrace{-\zeta\omega_n}_{\text{positive}} t)[C_1 \exp(\sqrt{\zeta^2 - 1}\,\omega_n t) + C_2 \exp(-\sqrt{\zeta^2 - 1}\,\omega_n t)]$$



Divergent oscillatory motion (flutter) due to negative damping

Determination of Damping

$$x(t) = X\exp(-\zeta\omega_{\mathrm{n}}t)\sin(\omega_{\mathrm{d}}t + \varphi)$$



$$2\exp(-0.05\pi t)\sin(0.9988\,\pi t + \varphi)$$

two consecutive peaks:

$$x_1 = X\exp(-\zeta\omega_{\mathrm{n}}t_1)\sin(\omega_{\mathrm{d}}t_1 + \varphi)$$

$$x_2 = X\exp(-\zeta\omega_{\mathrm{n}}t_2)\sin(\omega_{\mathrm{d}}t_2 + \varphi) = X\exp(-\zeta\omega_{\mathrm{n}}t_2)\sin(\omega_{\mathrm{d}}t_1 + \varphi)$$

logarithm decrement $\qquad \delta = \ln\dfrac{x_1}{x_2} = \zeta\omega_{\mathrm{n}}\tau_{\mathrm{d}} \Rightarrow \zeta = \dfrac{\delta}{\omega_{\mathrm{n}}\tau_{\mathrm{d}}}$

Example:

The 2nd and 4th peaks of a damped free vibration measured are respectively 0.021 and 0.013. What is damping factor?

Solution:

$$\frac{x(t_2)}{x(t_4)} = \exp(\zeta\omega_n 2\tau_{\mathrm{d}}) \quad \rightarrow \quad 2\zeta\omega_n\tau_{\mathrm{d}} = \ln\left(\frac{x(t_2)}{x(t_4)}\right)$$

$$2\zeta\omega_n\tau_{\mathrm{d}} = 2\zeta\omega_n\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{4\pi\zeta}{\sqrt{1-\zeta^2}} = \ln\left(\frac{x(t_2)}{x(t_4)}\right)$$

If a small damping is assumed, $2\zeta\omega_n\tau_{\mathrm{d}} = 4\pi\zeta = \ln\left(\dfrac{x(t_2)}{x(t_4)}\right)$. This leads to

$$\zeta = \frac{1}{4\pi}\ln\left(\frac{x(t_2)}{x(t_4)}\right) = 0.0382 = 3.82\%.$$

If such an assumption is not made, then $\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{4\pi}\ln\left(\frac{x(t_2)}{x(t_4)}\right)$ and hence $\frac{\zeta^2}{1-\zeta^2} = \left[\frac{1}{4\pi}\ln\left(\frac{x(t_2)}{x(t_4)}\right)\right]^2$. This leads to

$$\zeta = \frac{\frac{1}{4\pi}\ln\left(\frac{x(t_2)}{x(t_4)}\right)}{\sqrt{1+\left[\frac{1}{4\pi}\ln\left(\frac{x(t_2)}{x(t_4)}\right)\right]^2}} = 0.0381 = 3.81\%$$

. So virtually the same value.

General differential equations

$$a_n \frac{\mathrm{d}^n x}{\mathrm{d}t^n} + a_{n-1}\frac{\mathrm{d}^{n-1} x}{\mathrm{d}t^{n-1}} + \ldots\ldots + a_1 \frac{\mathrm{d}x}{\mathrm{d}t^1} + a_0 = 0$$

first solve the characteristic equation

$$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots\ldots + a_1\lambda + a_0 = 0$$

If all roots $\lambda_j$aredistinct, then the general solution is

$$x(t) = \sum_{j=1}^{n} b_j \exp(\lambda_j t)$$

whereb_j are constants to be determined.

If there are repeated roots, t^m (integer $m > 1$) appears in a solution. These are not interesting cases for mechanical vibration.

λ in response to the change of a parameter reveal stability properties