Presentation for
DESIGN OF MACHINE MEMBERS

DEPARTMENT OF MECHANICAL ENGINEERING

B.TECH : V SEM

by

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Fundamentals of Machine Design

What is Machine Design?

What is the importance of Machine Design for engineers?

Creation of new and better machines AND Improving existing ones So that it is economical in the cost of production and operation.

Definition of Machine Design

Machine design is defined as the use of scientific principles, technical information & imagination.

Machine design is defined as the use of scientific principles, technical information & imagination in the description of a machine or a mechanical system to perform specific functions with maximum economy & efficiency.

Machine Design is defined as the creation of new design or improving the exist
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Basic Requirement of Machine Elements

Strength and Rigidity

Wear Resistance

Minimum Dimensions & Weight

Manufacturability

Safety

Conformance to standards

Reliability

Maintainability

Minimum Life-cycle Cost

Engineering Materials and their Properties

- Selection of proper material for the machine components is one of the most important steps in process of machine design
- The best material is one which will serve the desired purpose at minimum costs
- Factors Considered while selecting the material:

i) Availability: Material should be readily available in market in large enough quantities to meet the requirement.

ii) Mechanical properties

Manufacturing Considerations:

- In some applications machinability of material is an important consideration in selection
- Where the product is of complex shape, castability or ability of the molten metal to flow into intricate passages is the criterion of material selection
- In fabricated assemblies of plates & rods, weldability becomes the governing factor

Toughness: Ability to absorb energy before fracture takes place

Malleability: Ability to deform to a greater extent before the sign of crack, when it is subjected to compressive force

Ductility: Ability to deform to a greater extent before the sign of crack, when subjected to tensile force

Brittleness: Property of the material which shows negligible plastic deformation fracture takes place

Hardness: Resistance to penetration or permanent deformation

General Procedure in Machine Design

Need or aim — Recognize and specify the problem

↓

Synthesis — Select the mechanism that would give the desired motion and form the basic model with a sketch etc

↓

Analysis of the FORCES

↓

Material selection

↓ ↑

Design of elements — Determine the stresses and thereby the sizes of components s.t. failure or deformation does not occur

↓

Modification — Modify sizes to ease construction & reduce overall cost

↓

Detailed drawing → Production

DESIGN CONSIDERATIONS

- Strength
- Rigidity
- Reliability
- Safety
- Cost
- Weight
- Ergonomics
- Aesthetics
- Manufacturing considerations
- Assembly considerations
- Conformance to standards
- Friction and wear
- Life
- Vibrations
- Thermal considerations
- Lubrication
- Maintenance
- Flexibility
- Size and shape
- Stiffness
- Corrosion
- Noise
- Environmental considerations

MANUFACTURING CONSIDERATIONS IN DESIGN

- Minimum total number of parts in a product
- Minimum variety of parts
- Use standard parts
- Use modular design
- Design parts to be multifunctional
- Design parts for multiple use
- Select least costly material
- Design parts for ease of manufacture
- Shape the parts for minimizing the operations

DESIGN CONSIDERATIONS FOR CASTINGS

- Design parts to be in compression then in tension
- Strengthen parts under tension by use of external devices
- Shape the casting for orderly solidification
- Avoid abrupt change in cross-section
- Provide more thickness at the boss
- Round off the corners
- Avoid concentration of metal at junctions
- Avoid thin sections
- Make provision for easy removal of pattern from the mould

DESIGN CONSIDERATIONS FOR FORGINGS

- Keep fibre lines parallel to tensile and compressive forces and perpendicular to shear forces
- Avoid deep machining cuts
- Keep vertical surfaces of forged parts tapered
- Keep the parting line in one plane
- Provide adequate fillet and corner radii
- Avoid thin sections

DESIGN CONSIDERATIONS IN WELDING

- Use the minimum possible number of welds
- Select the same thickness for the parts to be welded together
- Locate the welds at the areas in the design where stresses and/or deflections are not critical
- Effect of shrinkage and distortion should be minimized by post welding annealing and stress relief operations
- Decide proper welding sequence
- Design welding in the flat or horizontal position and not in the overhead position
- Use only the amount of weld metal that is absolutely required

STANDARDIZATION

- It is the process of establishing the set of norms to which a specified set of characteristics of a component or a product should conform
- **Example:** Standardizing the shaft consists of specifying the set of shaft diameters and material

➢ **Objectives of standardization**

- To make the interchangeability of the components possible
- To make the mass production of components easier

PREFERRED SERIES AND ITS SELECTION

- Preferred series are series of numbers obtained by geometric progression and rounded off
- The four basic preferred series are designated as:

R5 Series	$\sqrt[5]{10} = 1.58$
R10 Series	$\sqrt[10]{10} = 1.26$
R20 Series	$\sqrt[20]{10} = 1.12$
R40 Series	$\sqrt[40]{10} = 1.06$

- The other series are called derived series and are obtained by multiplying or dividing the basic sizes by 10, 100, etc.

MECHANICAL PROPERTIES OF MATERIALS

- Strength
- Stiffness/Rigidity
- Elasticity
- Plasticity
- Ductility
- Brittleness
- Malleability
- Toughness
- Machinability
- Resilience
- Creep
- Fatigue
- Hardness

EFFECT OF ALLOYING ELEMENTS

- Chromium
- Nickel
- Manganese
- Silicon
- Molybdenum
- Vanadium
- Tungsten

STEELS DESIGNATED ON THE BASIS OF MECHANICAL PROPERTIES

- These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress.
- According to Indian standard these steels are designated by a symbol ‘Fe’ or ‘Fe E’ depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm^2
- For example ‘Fe 290’ means a steel having minimum tensile strength of 290 N/mm^2 and ‘Fe E 220’ means a steel having yield strength of 220 N/mm^2.

STEELS DESIGNATED ON THE BASIS OF CHEMICAL COMPOSITION

- According to Indian standard, steels are designated in the following order :
- (***a***) Figure indicating 100 times the average percentage of carbon content,
- (***b***) Letter 'C', and
- (***c***) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.
- For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on an average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

INDIAN STANDARD DESIGNATION OF LOW AND MEDIUM ALLOY STEELS

- According to Indian standard, low and medium alloy steels shall be designated in the following order :

1. Figure indicating 100 times the average percentage carbon.

2. Chemical symbol for alloying elements each followed by the figure for its average percentage content multiplied by a factor as given below :

Element	*Multiplying factor*
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S and N	100

- For example 40 Cr 4 Mo 2 means alloy steel having average 0.4% carbon, 1% chromium and 0.25% molybdenum.

INDIAN STANDARD DESIGNATION OF HIGH SPEED TOOL STEEL

- According to Indian standard, the high speed tool steels are designated in the following order :

1. Letter 'XT'.

2. Figure indicating 100 times the percentage of carbon content.

3. Chemical symbol for alloying elements each followed by the figure for its average percentage content rounded off to the nearest integer, and

4. Chemical symbol to indicate specially added element to attain the desired properties.

- For example, XT 75 W 18 Cr 4 V 1 means a tool steel with average carbon content 0.75 percent, tungsten 18 per cent, chromium 4 per cent and vanadium 1 per cent.

Preferred Numbers

- The system is based on the use geometric progression to develop a set of numbers
- There are five basic series denoted as R5, R10, R20, R40, and R80 series which increases in steps of 56%, 26%, 12%, 6% and 3% respectively
- Each series has its own series factor as shown below

R5 Series	$\sqrt[5]{10} = 1.58$
R10 Series	$\sqrt[10]{10} = 1.26$
R20 Series	$\sqrt[20]{10} = 1.12$
R40 Series	$\sqrt[40]{10} = 1.06$
R80 Series	$\sqrt[80]{10} = 1.03$

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LIMITS & FITS

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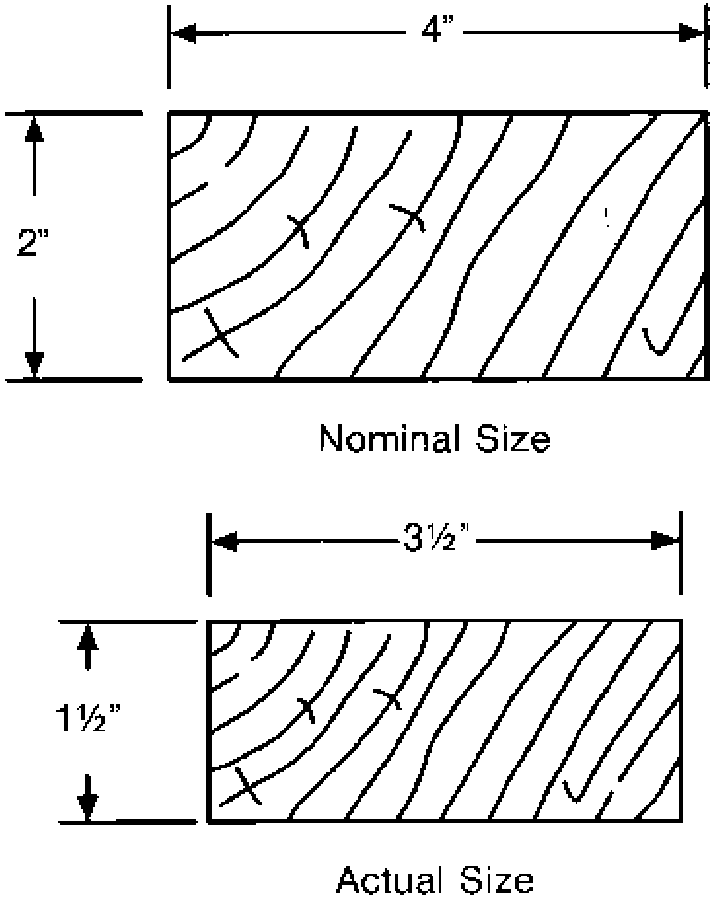
TERMINOLOGY

NOMINAL SIZE: It is the size of a part specified in the drawing.

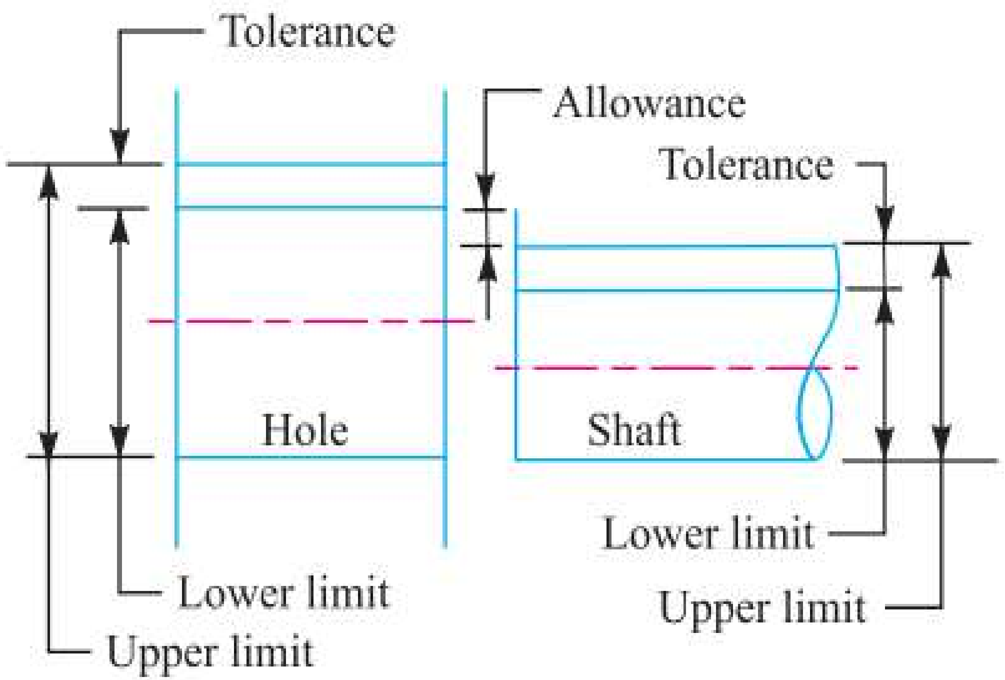
BASIC SIZE: It is the size of a part to which all limits of variation are determined.

ACTUAL SIZE: It is the actual measured dimension of a part. Nominal and basic size are often the same



LIMIT OF SIZES

There are two extreme possible sizes of a component. The largest permissible size for a component is called upper limit and smallest size is called lower limit.



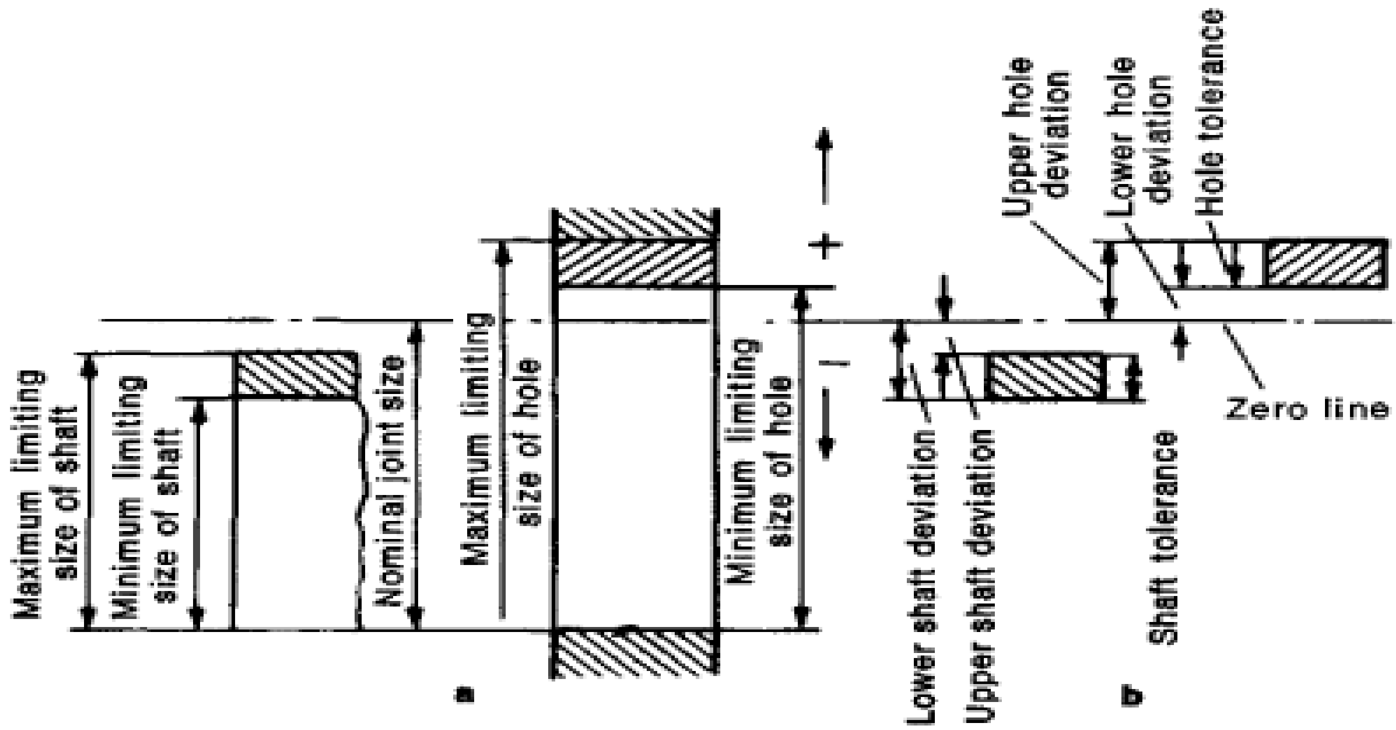
DEVIATION

LOWER DEVIATION: It is the algebraic difference between the minimum limit of size and the basic size.

UPPER DEVIATION: It is the algebraic difference between the maximum limit and the basic size.

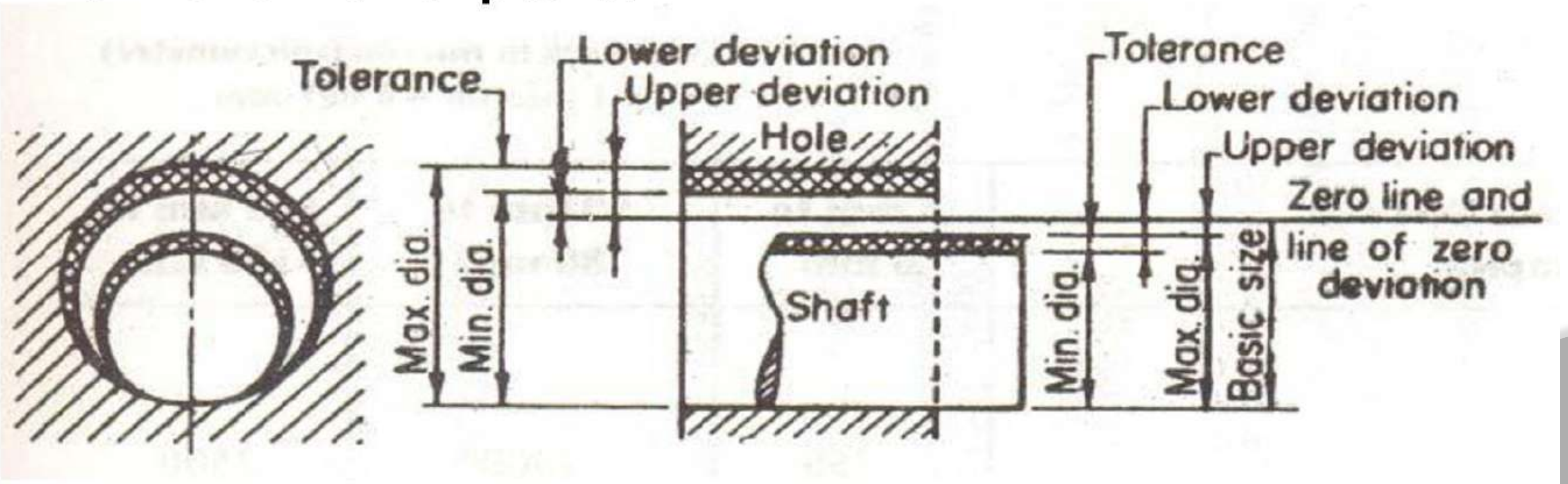
ZERO LINE

It is the straight line corresponding to the basic size. The deviations are measured from this line.



Tolerance

- A tolerance is the total permissible variation from the specified basic size of the part.



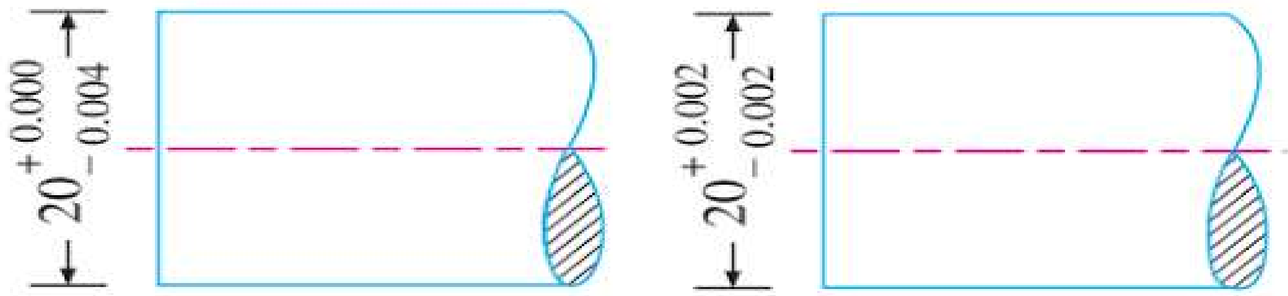
POSITIONAL TOLERANCES

Two types of positional tolerances are used:

Unilateral tolerances

Bilateral tolerances

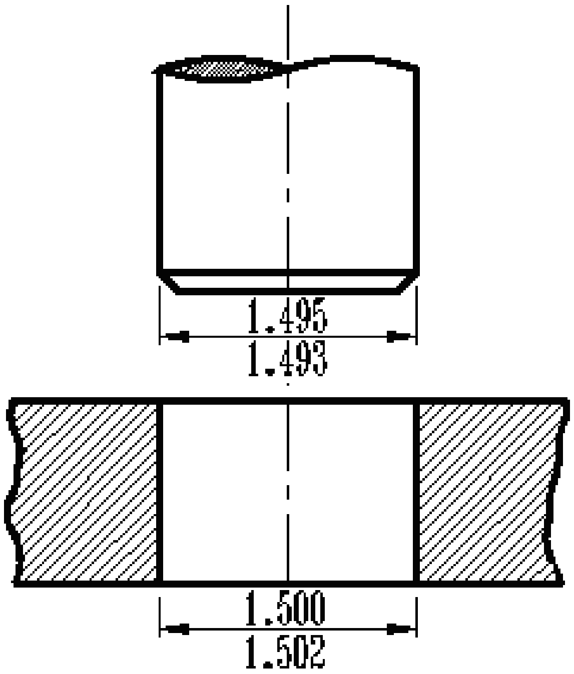
When tolerance is on one side of basic size, it is called unilateral and if it is both in plus and minus then it is known as bilateral tolerance.



(a) Unilateral tolerance (b) Bilateral tolerance.

FITS

The degree of tightness or looseness between two mating parts is called a fit.



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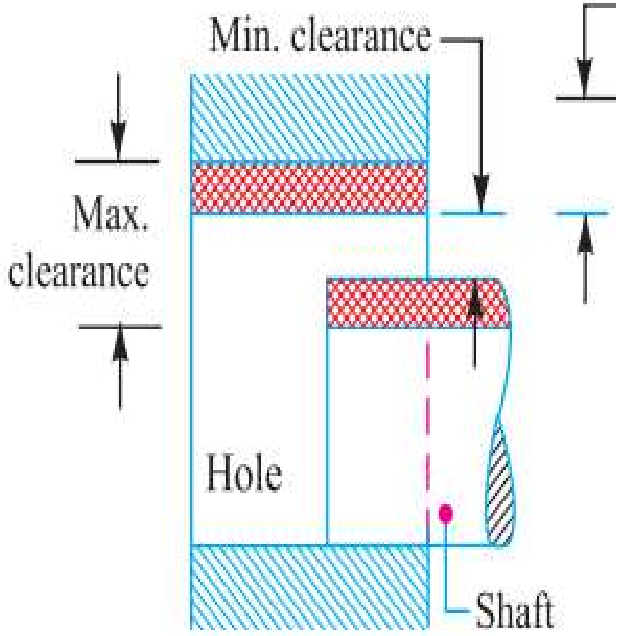
TYPES OF FITS

CLEARANCE FIT: There is a clearance or looseness in this type of fits. These fits maybe slide fit, easy sliding fit, running fit etc.



(*a*) Clearance fit.

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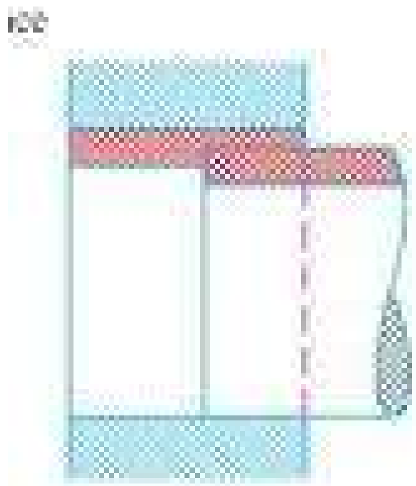
TYPES OF FITS

INTERFERENCE FIT: There is an interference or tightness in these type of fits. E.g. shrink fit, heavy drive fit etc.

TRANSITION FIT: In this type of fit, the limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts.



(c) Transition fit.

TYPES OF FITS

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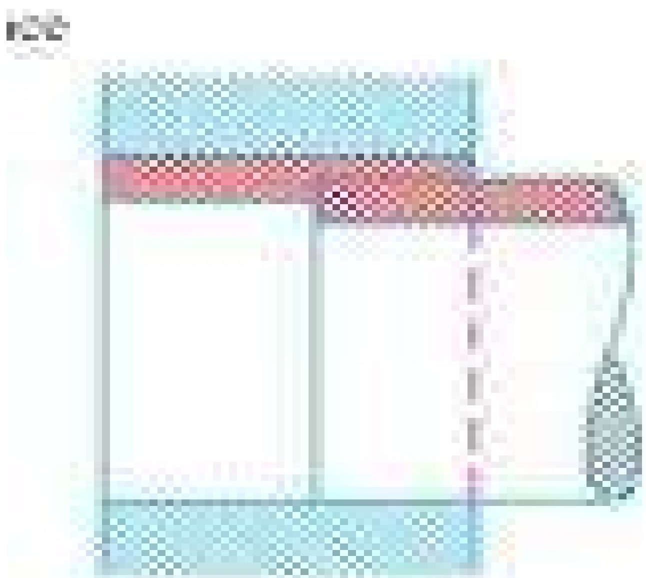


(c) Transition fit.

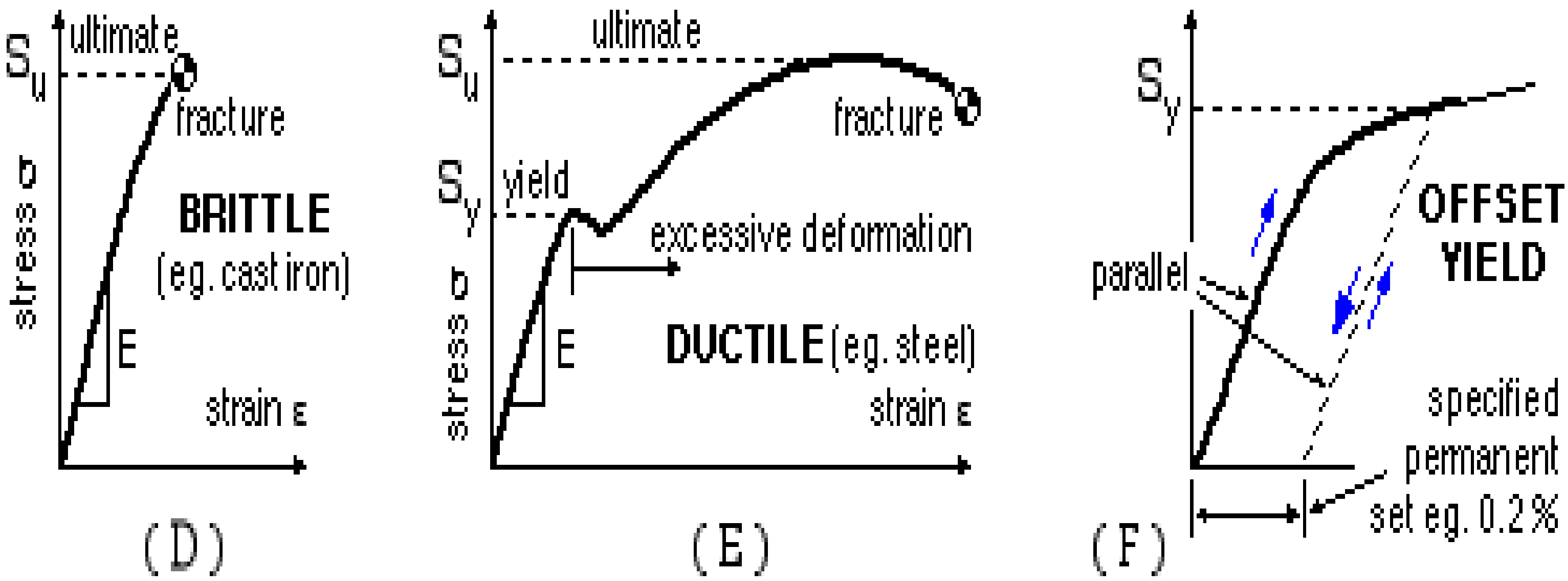
Factor of Safety (Safety Factor)

Eg: If a component needs to withstand a load of 100 N and a FoS of 3 is selected then it is designed with strength to support 300 N.

$$\text{FoS} = \frac{\text{Strength of the component (Max load)}}{\text{Load on the component (Actual load)}}$$

Factor of Safety (Safety Factor)

FoS (*Based on yeild strength*)	Application
1.25 – 1.5	Material properties known in detail Operating conditions known in detail Load and the resulting stresses and strains are known to a high degree of accuracy Low weight is important
2 – 3	For less tried materials or Brittle materials under average conditions of environment, load and stress
3 – 4	For untried materials under average conditions of environment, load and stress Better known materials under uncertain environment or uncertain stresses



S
u
ultimate
fracture
stress σ
BRITTLE
(eg. cast iron)
E
strain ε
(D)
ultimate
S
u
S
y
yield
fracture
excessive deformation
stress σ
E
DUCTILE (eg. steel)
strain ε
(E)
S
y
OFFSET
YIELD
parallel
specified
permanent
set eg. 0.2%
(F)

$$\text{FoS} = \frac{\text{Strength of the component } (S_u, S_y)}{\text{Stress in the component due to the actual load}}$$

- The cost factor (cost of material, manufacture)
- Whether failure could cause serious injury or death (a steam boiler or pressure vessel would use 8 – 10 FoS)
- Unknown stresses in the manufacturing process (casting would use 10 – 14 FoS)
- Environmental conditions (used in harsh environment or not)
- Knowledge of the environment
- Knowledge of the properties of the material used
- Knowledge of the loads (tension, compressive, shear, bending, cyclic loads, impact loads etc)
- Weight factor (aerospace 1.5 – 3 to reduce weight but strict quality control)
- Quality control, maintenance

Genesis of BIS

Government of India resolution on 3 September 1946 for establishment of Indian Standards Institution.

Indian Standards Institution (predecessor of Bureau of Indian Standards)set up on 6 January 1947.

Objectives : Promoting standardization ,quality control and simplification in industry & commerce.

Bureau of Indian Standards (BIS)Established on 1 April 1987 under BIS act, 1986 as statutory body

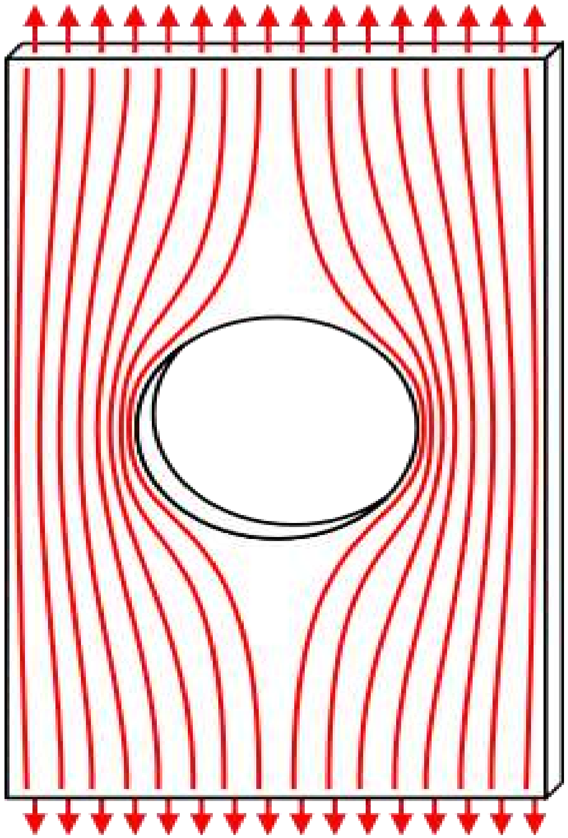
Bureau of Indian Standards

- Bureau of Indian Standards (BIS) took over work of ISI through enactment of ***BIS Act (1986)*** by the Indian Parliament.
- Formulates National standards and carried out conformity assessment by operating the Product and Management System Certification Scheme.
- Formulated over 18000 National Standards and also operating more than 22000 Certification licenses.
- Project India’s view in various committees of International Organization for Standardization (ISO) and International Electro technical Commission (IEC)

Stress Concentration

- Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.***
- A **stress concentration (stress raisers** or **stress risers)** is a location in an object where stress is concentrated. An object is strongest when force is evenly distributed over its area, so a reduction in area, e.g., caused by a crack, results in a localized increase in stress.
- A material can fail, via a propagating crack, when a concentrated stress exceeds the material's theoretical cohesive strength. The real fracture strength of a material is always lower than the theoretical value because most materials contain small cracks or contaminants (especially foreign particles) that concentrate stress.
- It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.

Internal Force lines are denser near the hole



Theoretical or Form Stress Concentration Factor

- The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area.
- Mathematically, theoretical or form stress concentration factor,
- The value of *Kt* depends upon the material and geometry of the part.

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

Fatigue Stress Concentration Factor

- When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.
- Mathematically, fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

- **Notch Sensitivity:** It may be defined as the degree to which the theoretical effect of stress concentration is actually reached.
- **Notch Sensitivity Factor "q":** Notch sensitivity factor is defined as the ratio of increase in the actual stress to the increase in the nominal stress near the discontinuity in the specimen.

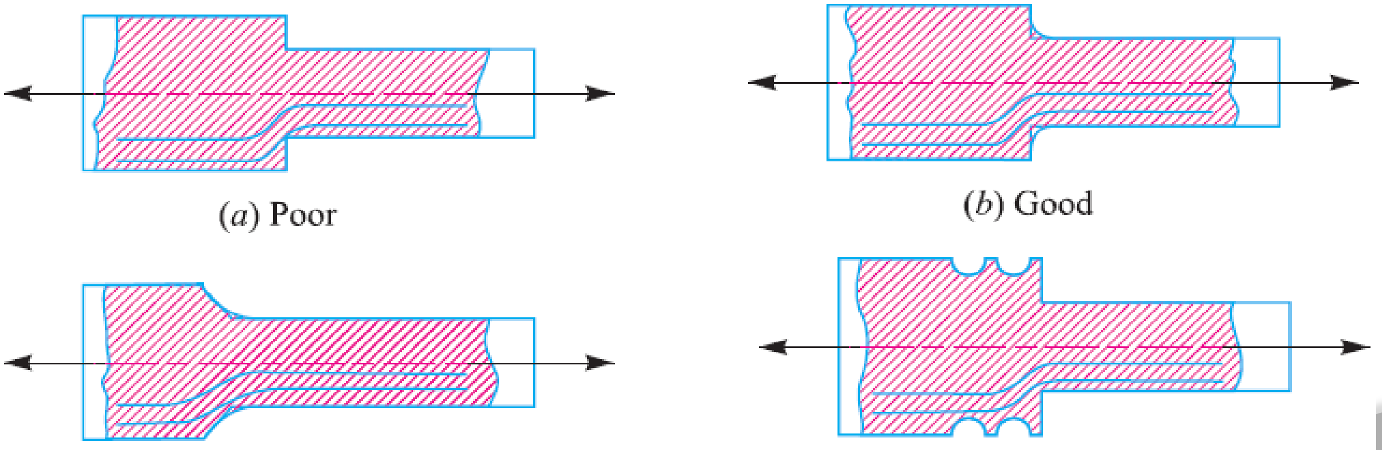
  Where, Kf and Kt are the fatigue stress concentration factor and theoretical stress concentration factor.
- The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

$$q = \frac{K_f - 1}{K_t - 1}$$

Methods to reduce stress concentration

- The presence of stress concentration can not be totally eliminated but it may be reduced to some extent.
- A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines.
- The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

- In Fig. (a), we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. (*b*) and (*c*) to give more equally spaced flow lines.
- It may be noted that it is not practicable to use large radius fillets as in case of ball and roller bearing mountings. In such cases, notches may be cut as shown in Fig. (*d*).



(*a*) Poor
(*b*) Good
(*c*) Preferred
(*d*) Preferred

Following figures show the several ways of reducing the stress concentration in shafts and other cylindrical members with shoulders

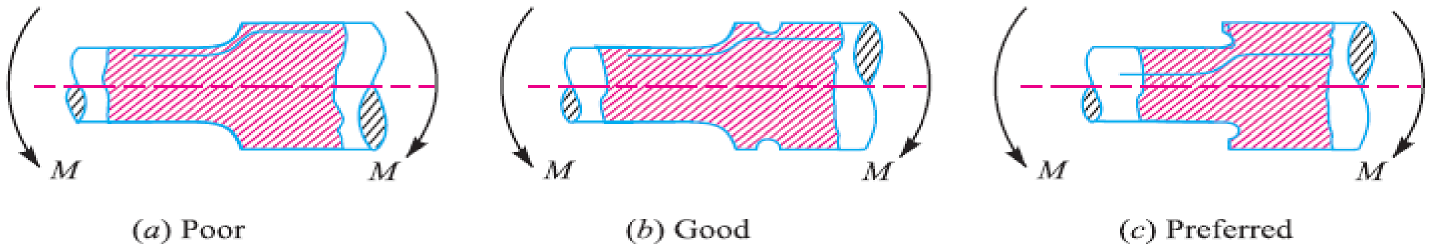


Fig. 6.9. Methods of reducing stress concentration in cylindrical members with shoulders.

Fig. 6.10. Methods of reducing stress concentration in cylindrical members with holes.

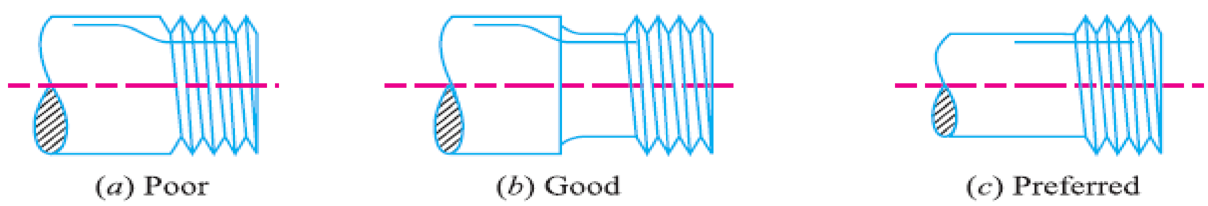


Fig. 6.11. Methods of reducing stress concentration in cylindrical members with holes.

Factors to be Considered while Designing Machine Parts Avoid Fatigue Failure

- The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.

2. The holes, notches and other stress raisers should be avoided.

3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.

4. The parts should be protected from corrosive atmosphere.

5. A smooth finish of outer surface of the component increases the fatigue life.

6. The material with high fatigue strength should be selected.

7. The residual compressive stresses over the parts surface increases its fatigue strength.

Endurance limit and Fatigue failure

- It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.**
- The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication.
- The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.

Factors affecting endurance limit

1) SIZE EFFECT:

- The strength of large members is lower than that of small specimens.
  This may be due to two reasons.
- The larger member will have a larger distribution of weak points than the smaller one and on an average, fails at a lower stress.
- Larger members have larger surface Ares. This is important because the imperfections that cause fatigue failure are usually at the surface.

2) SURFACE ROUGHNESS:

- Almost all fatigue cracks nucleate at the surface of the members.
- The conditions of the surface roughness and surface oxidation or corrosion are very important.
- Experiments have shown that different surface finishes of the same material will show different fatigue strength.
- Methods which Improve the surface finish and those which introduce compressive stresses on the surface will improve the fatigue strength.
- Smoothly polished specimens have higher fatigue strength.
- Surface treatments. Fatigue cracks initiate at free surface, treatments can be significant
- Plating, thermal or mechanical means to induce residual stress

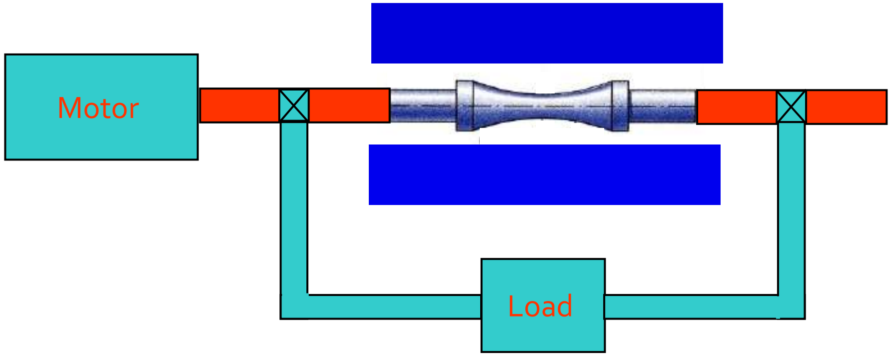
S-N Diagram

Fatigue strength of material is determined by R.R. Moore rotating beam machine. The surface is polished in the axial direction.

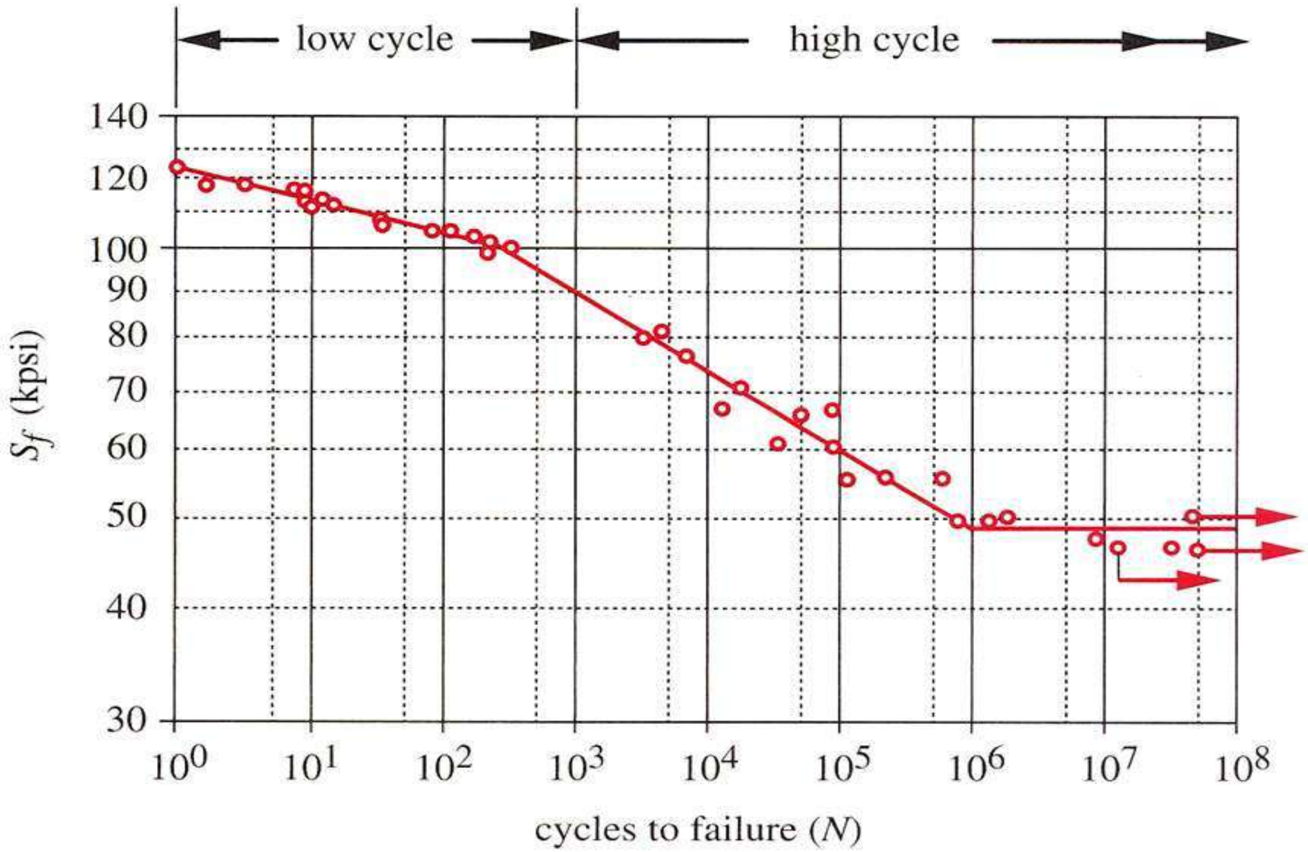
A constant bending load is applied.

The surface is polished in the axial direction. A constant bending load is applied.

Typical testing apparatus, pure bending



- A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in figure.
- A little consideration will show that if the stress is kept below a certain value the material will not fail whatever may be the number of cycles.
- This stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (σe).
- It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles).



low cycle
high cycle
140
120
100
90
80
70
60
50
40
30
S_f (kpsi)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
cycles to failure (N)

Correction Factors for Specimen's Endurance Limit

$S_e = k_a k_b k_c k_d k_e k_f S_e'$

Where S_e = endurance limit of component

S_e' = endurance limit experimental

k_a = surface finish factor (machined parts have different finish)

k_b = size factor (larger parts greater probability of finding defects)

k_c = reliability / statistical scatter factor (accounts for random variation)

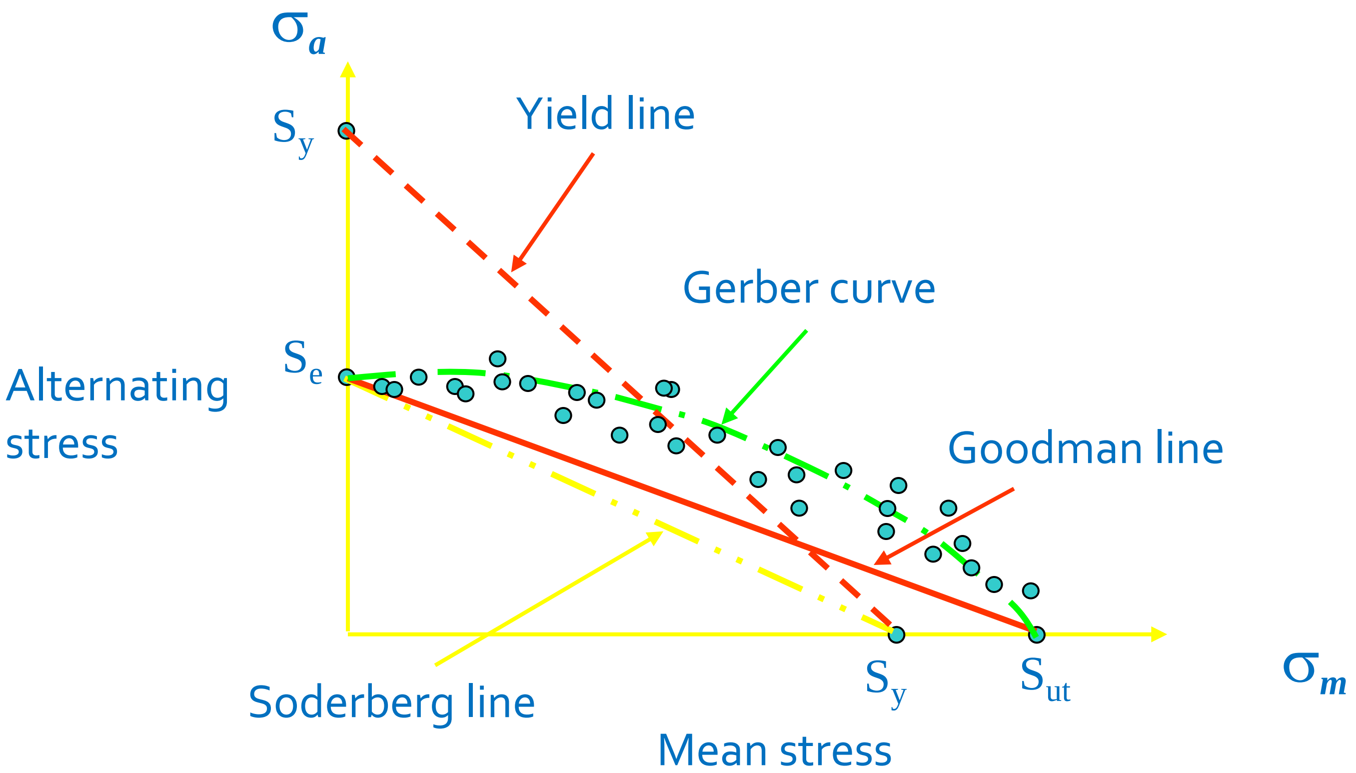
k_d = loading factor (differences in loading types)

k_e = operating T factor (accounts for diff. in working T & room T)

k_f = stress concentration factor

Fluctuating stresses

- The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in figure as functions of stress amplitude(σa) and mean stress (σm).
- The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile.
- In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

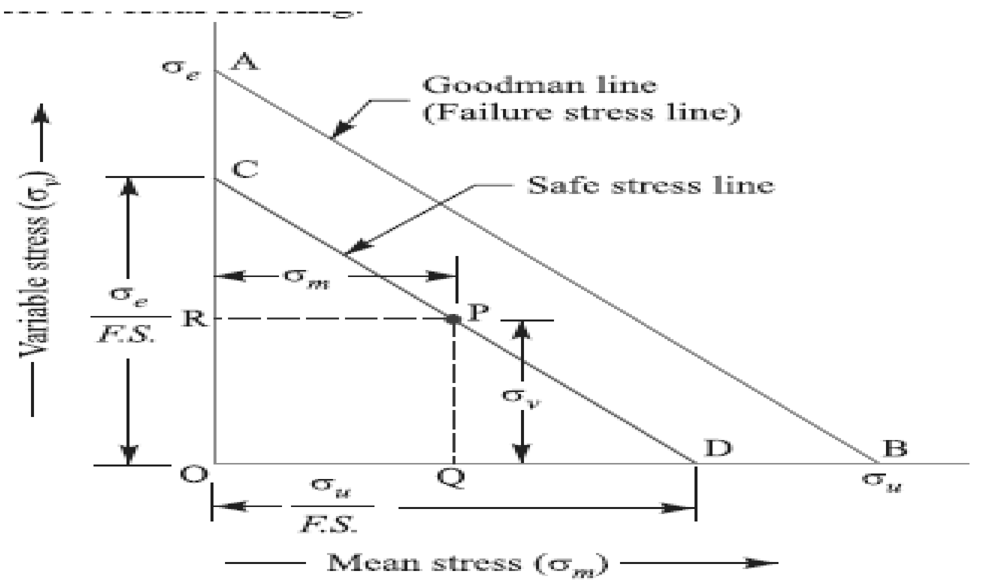


σa
Sy
Yield line
Gerber curve
Se
Alternating stress
Goodman line
Soderberg line
Sy
Sut
σm
Mean stress

Goodman Method for Combination of Stresses:

A straight line connecting the endurance limit (σ*e*) and the ultimate strength (σ*u*), as shown by line *AB* in figure given below follows the suggestion of Goodman.

A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.



Now from similar triangles COD and PQD,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \quad \ldots(\because\ QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \quad \ldots(i)$$

Soderberg Method for Combination of Stresses

- A straight line connecting the endurance limit (σe) and the yield strength (σy), as shown by the line *AB* in following figure, follows the suggestion of Soderberg line.
- This line is used when the design is based on yield strength. the line *AB* connecting σe and σy, as shown in following figure, is called ***Soderberg's failure stress line***.

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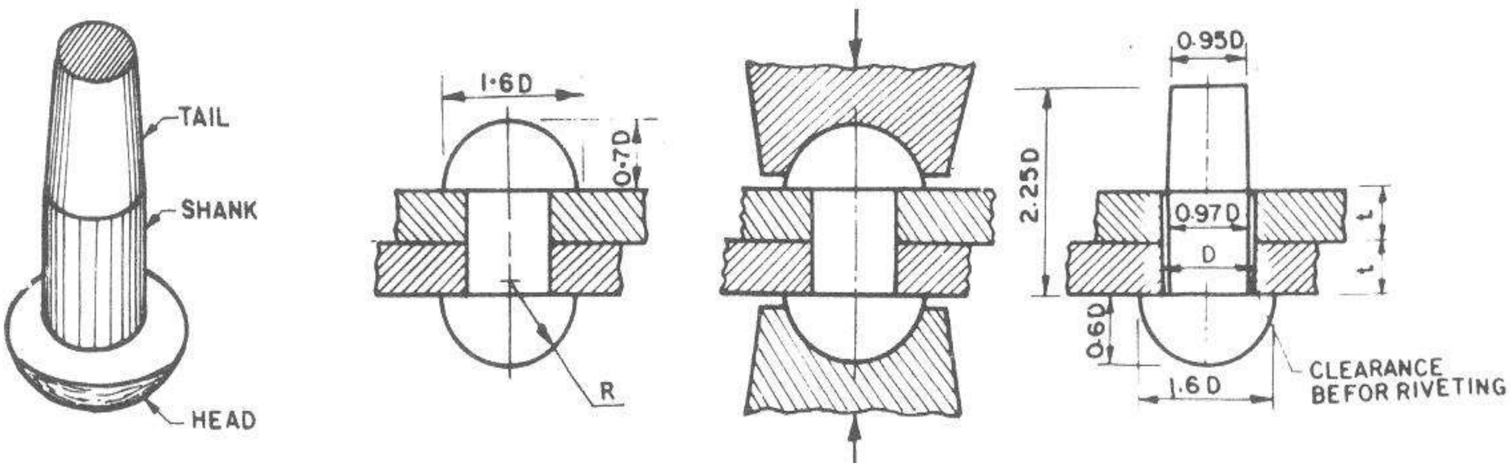
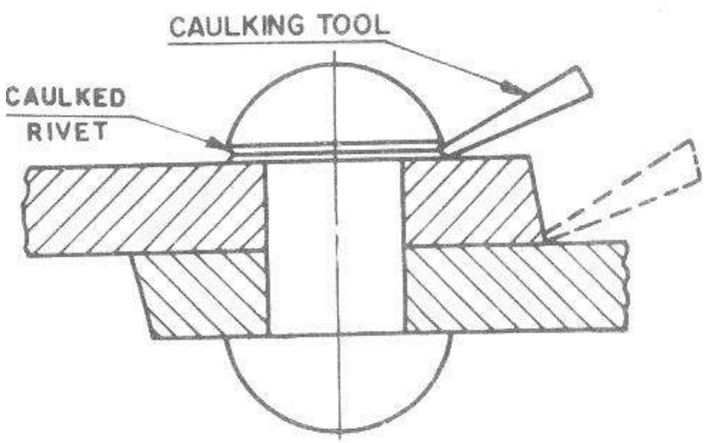
UNIT –II

Riveted Joints



Caulking and Fullering



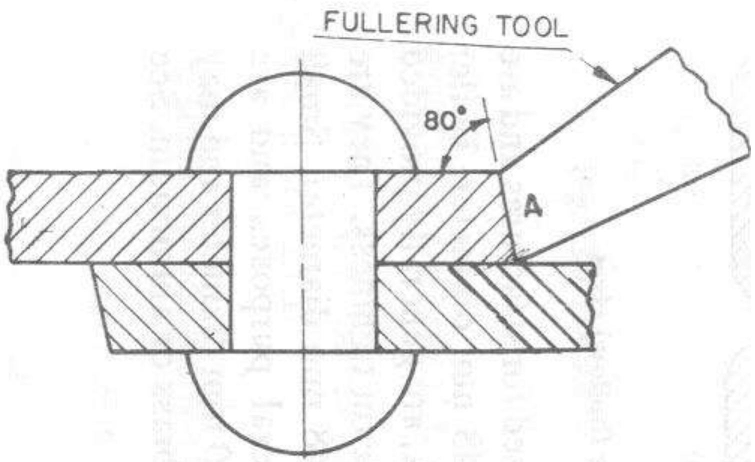


Fig. 13.4 Fullering

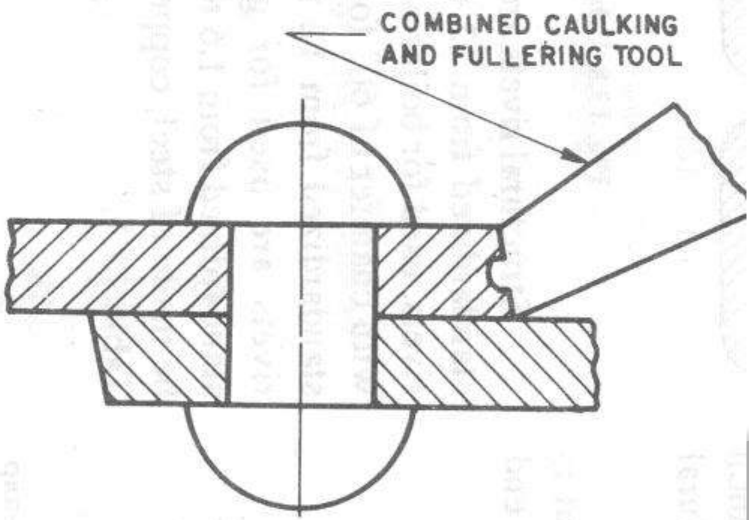


Fig. 13.5 Combined caulking and fullering

Caulking: Operation of burring down the edges of the plates and heads of the rivets to form a metal to metal joint. Fullering is a better option

65

Types of rivets

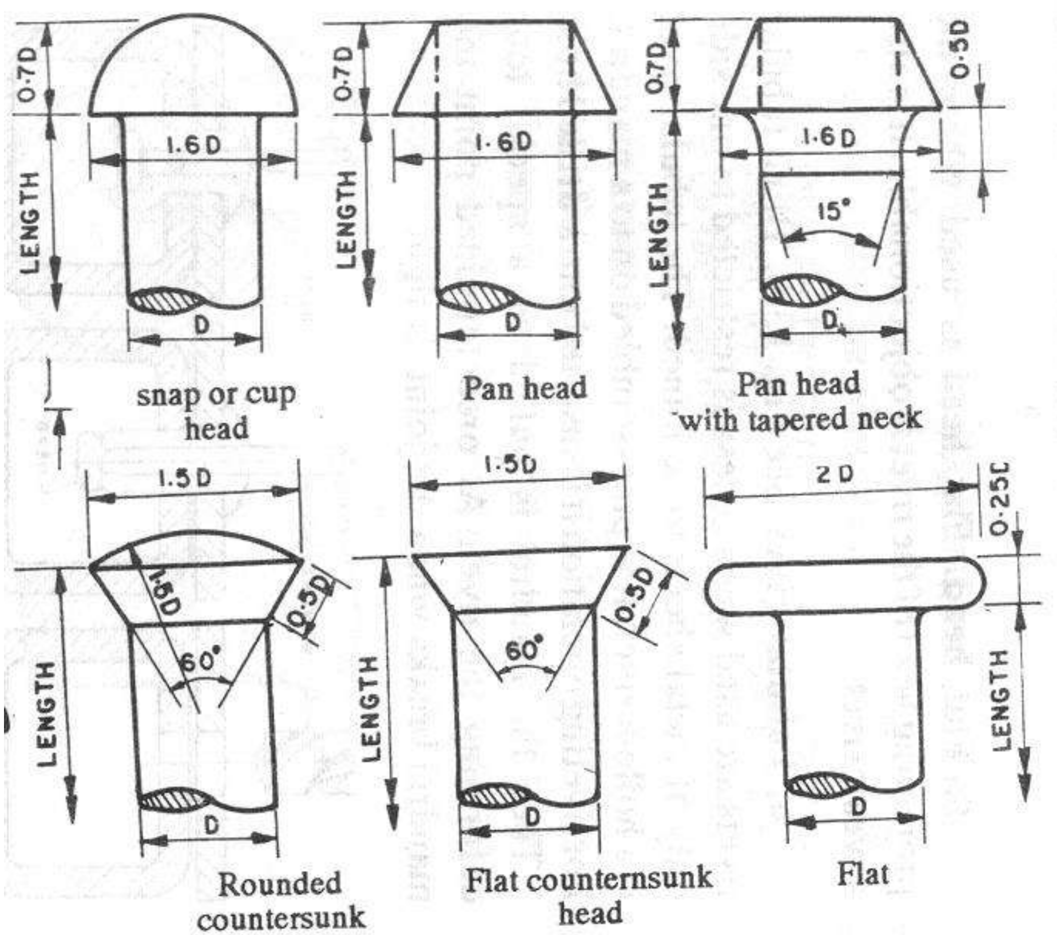
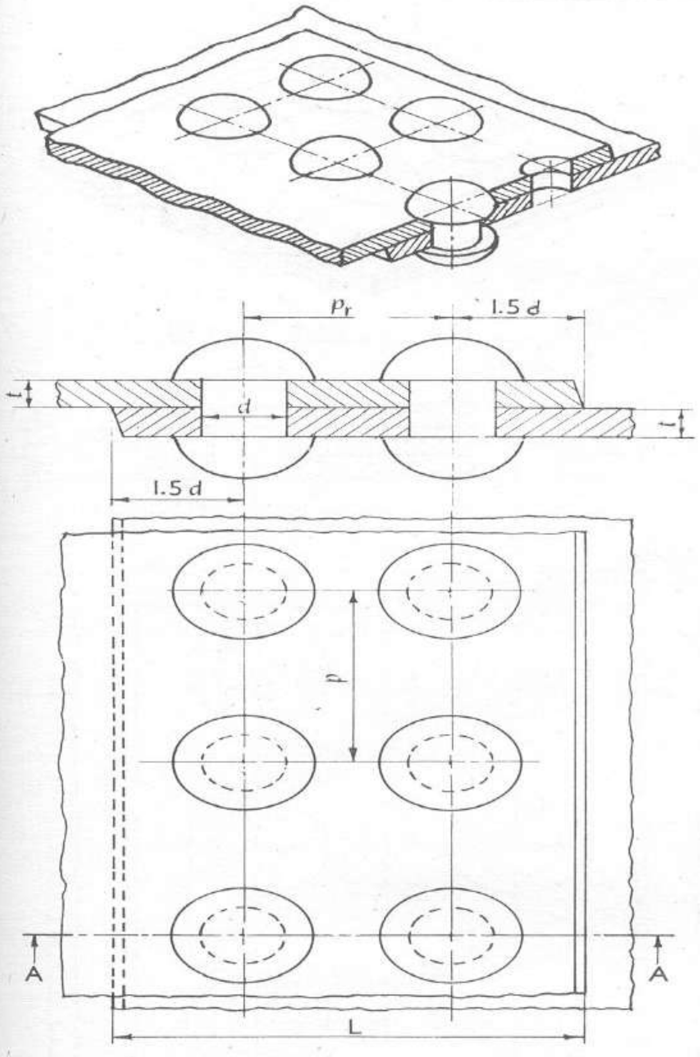
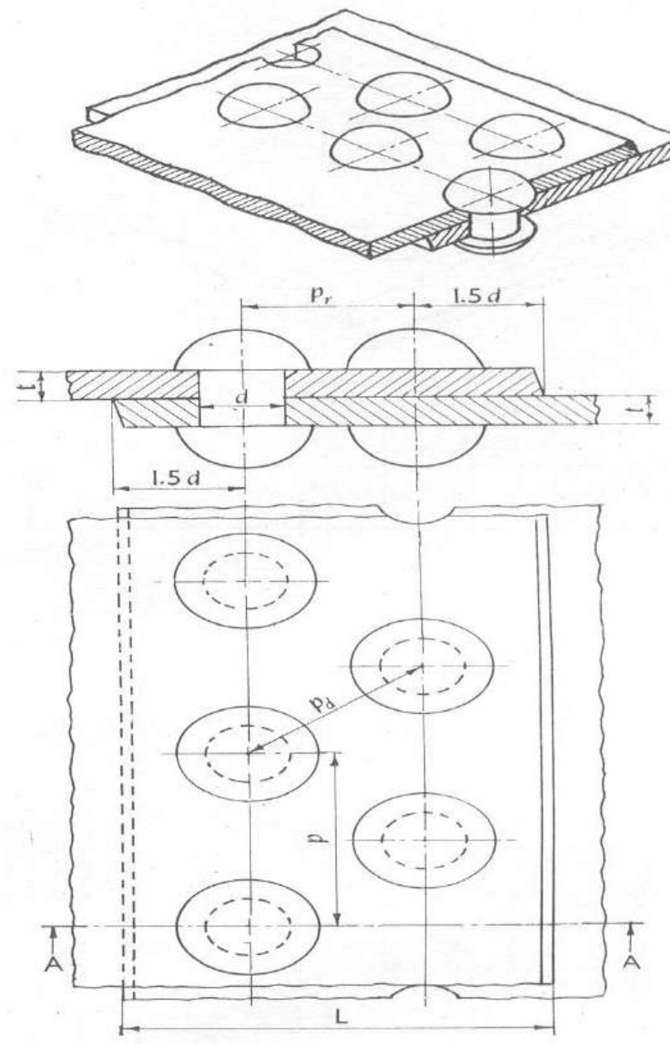


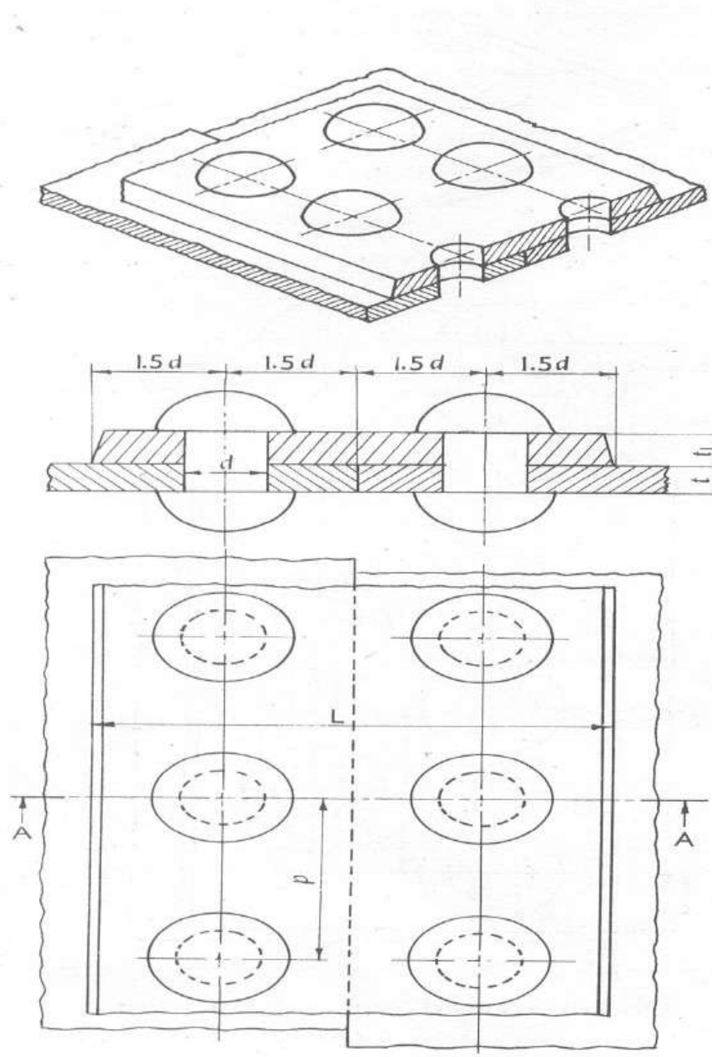
Fig.13.6 (a) Indian Standard rivet heads for general purposes (length of shank L = 2.5D to 10D) (see I.S. : 1929 and I.S. : 2155 for preferred length diameter combination)



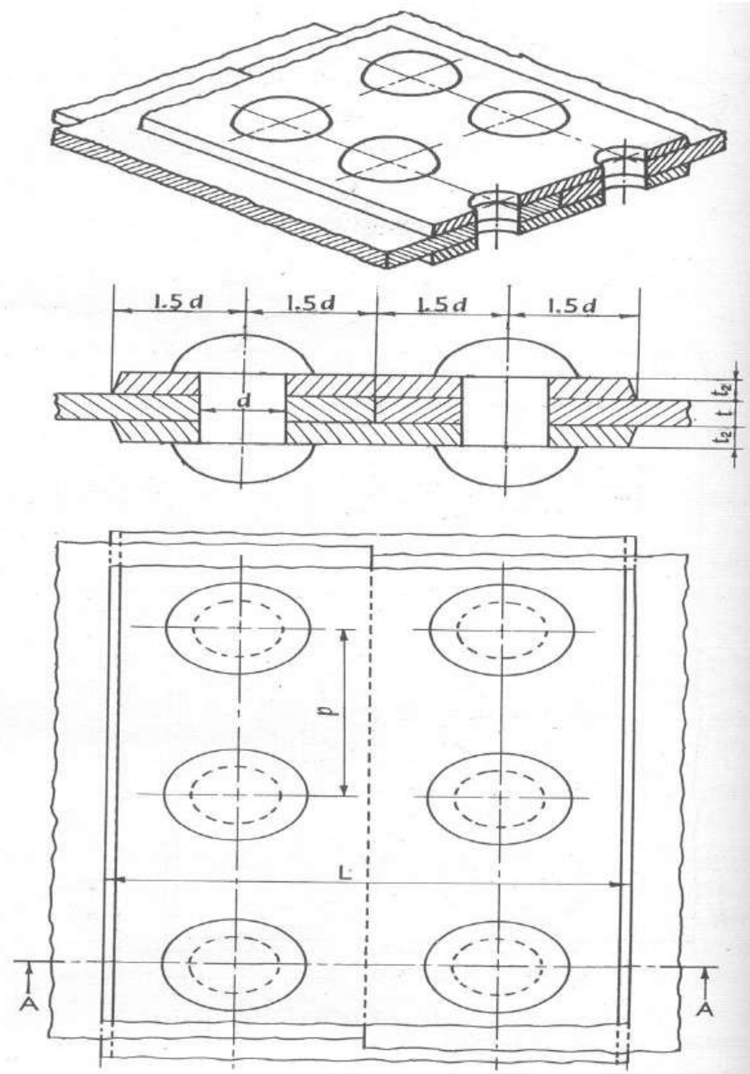
Double-riveted (chain) lap joint
FIG. 12-7



Double-riveted (zigzag) lap joint
FIG. 12-8



Single-riveted (single strap) butt joint
FIG. 12-9



Single-riveted (double straps) butt joint
FIG. 12-10

- Types of Riveted joints

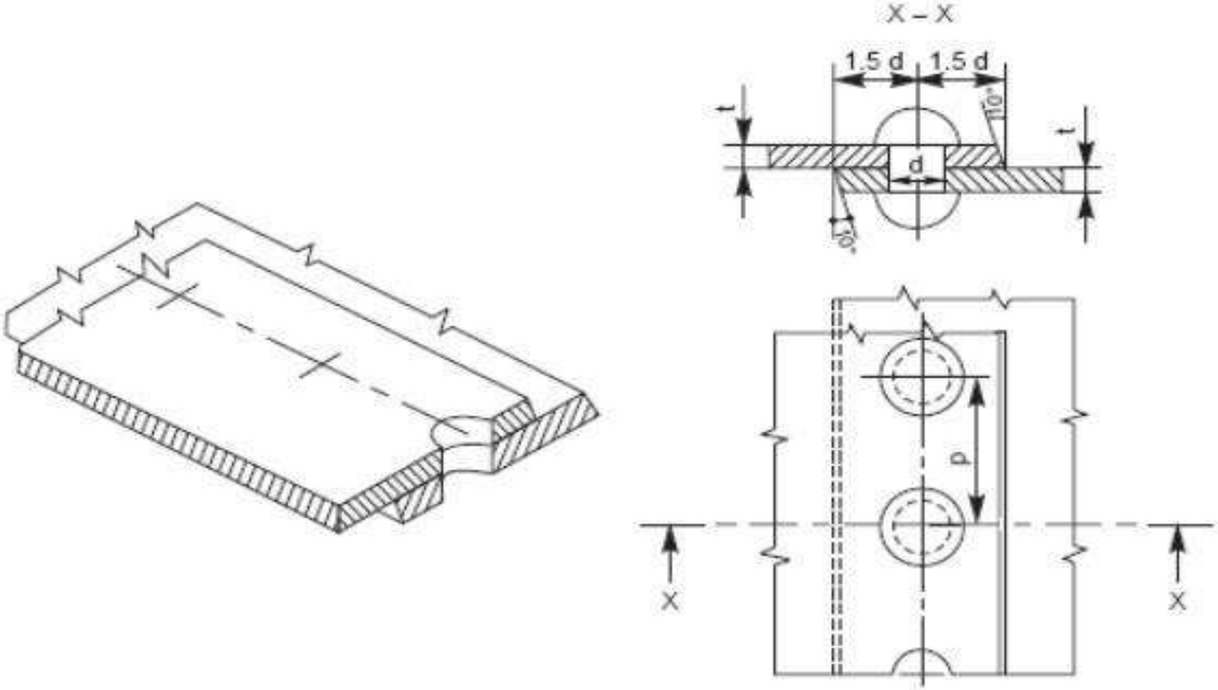


Fig. 10.9 Single riveted lap joint

- Double Riveted lap joints

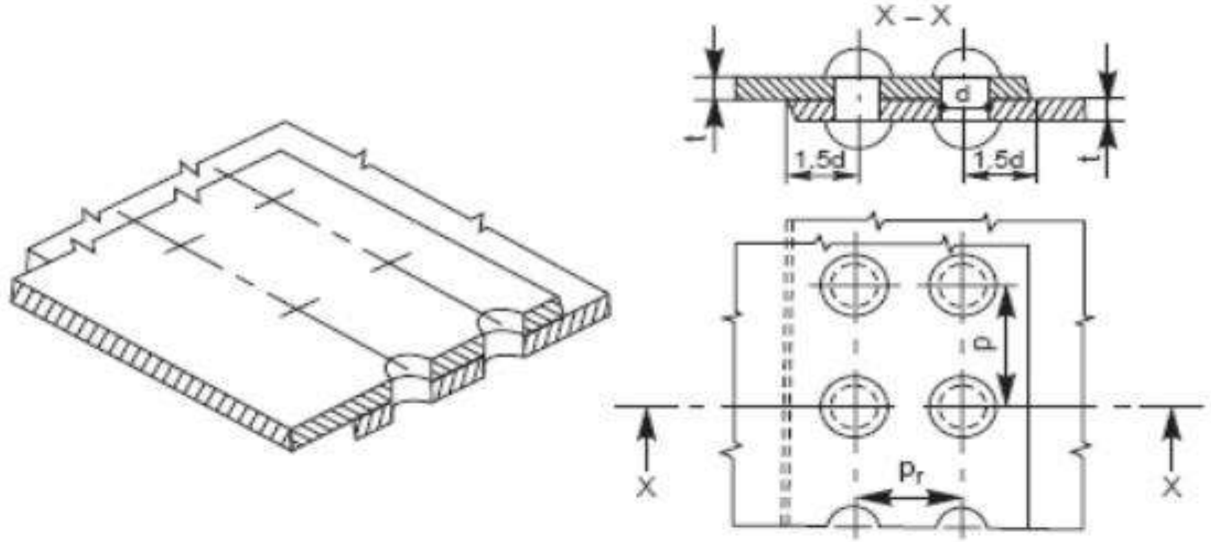


Fig. 10.10 Double riveted chain lap joint

* Double Riveted lap joints

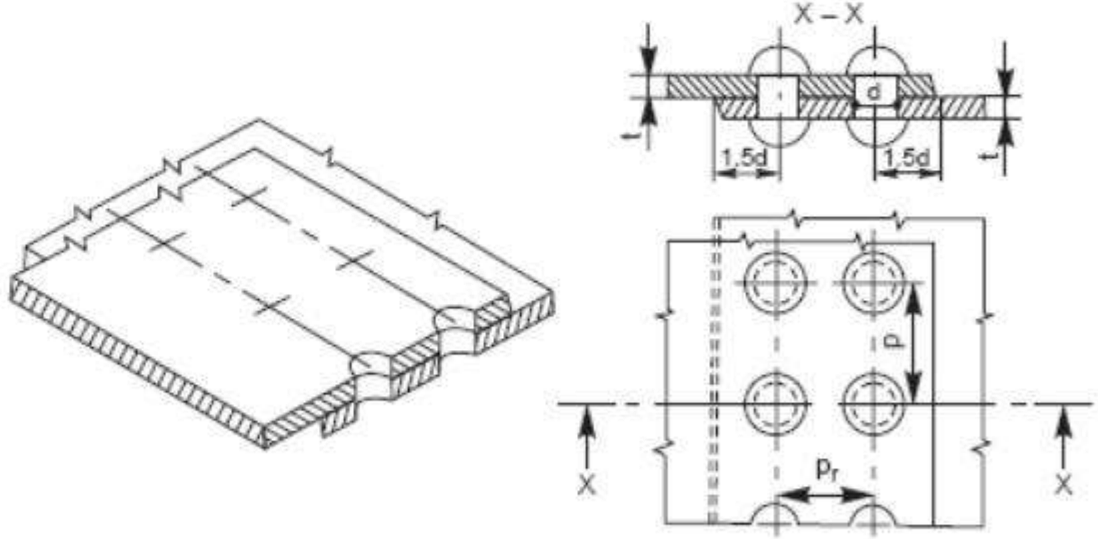


Fig. 10.10 Double riveted chain lap joint

- Double Riveted Zig zag Lap joint

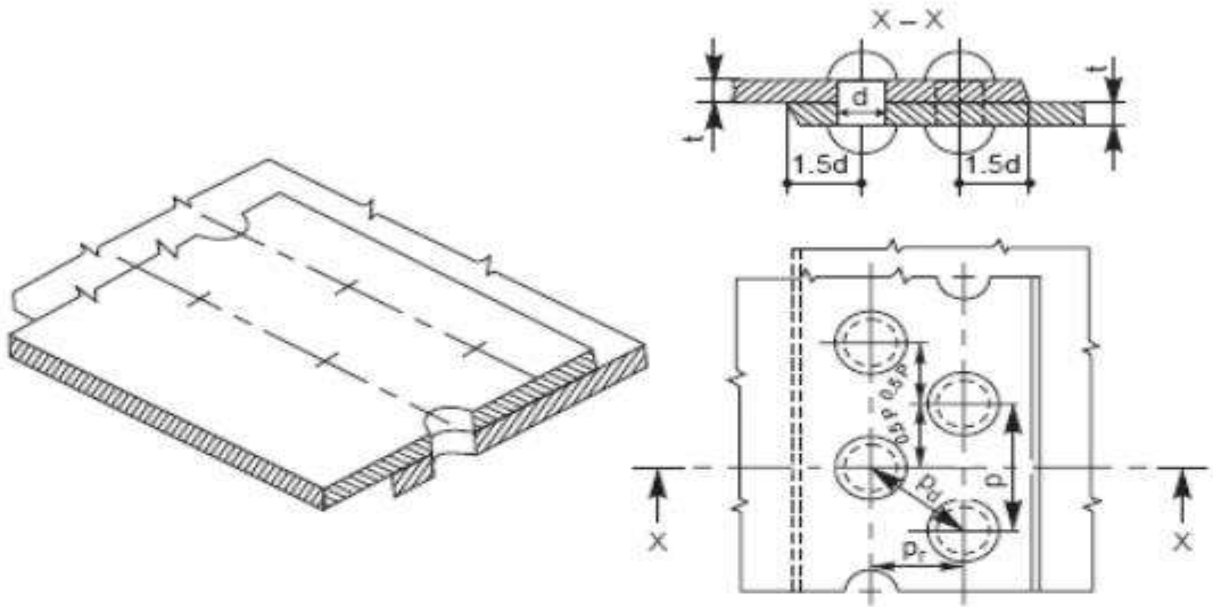


Fig. 10.11 Double riveted zig-zag lap joint

- Single riveted, single strap butt joint

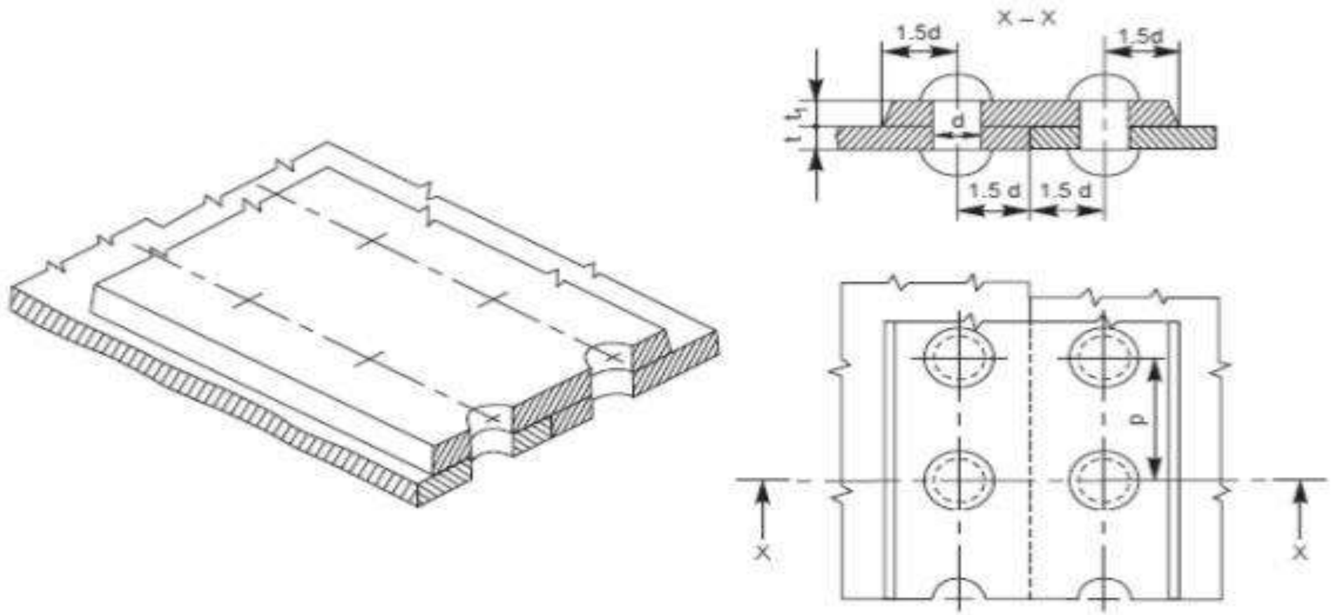


Fig. 10.12 Single riveted, single strap butt joint

- Single riveted, double strap butt joint

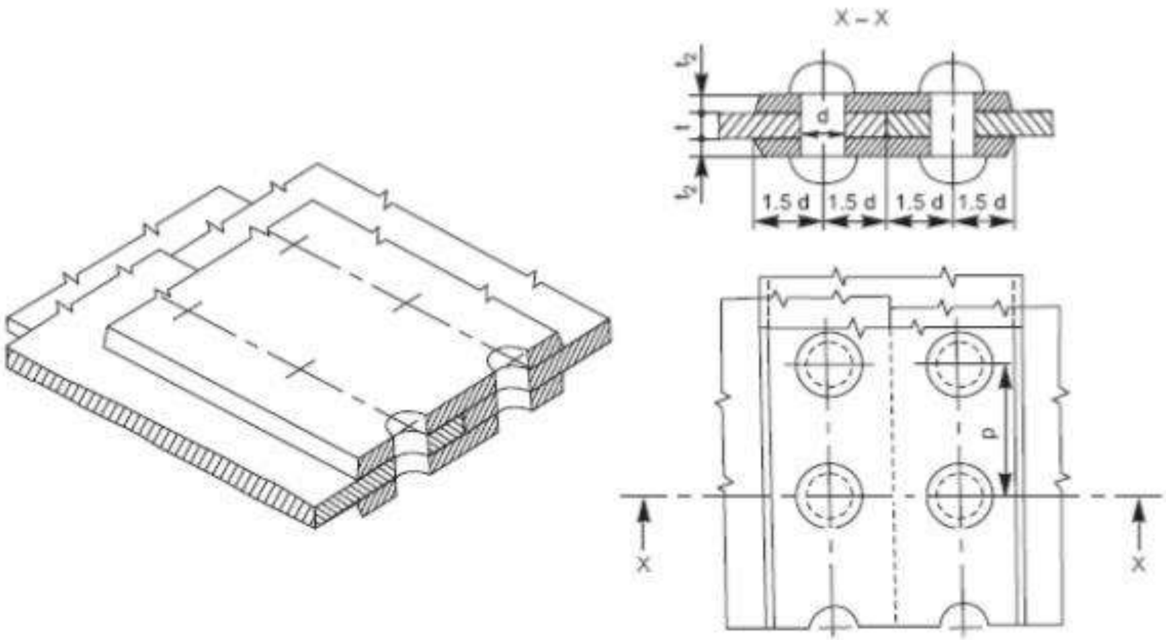


Fig. 10.13 Single riveted, double strap butt joint

- Double riveted, double strap chain butt joint

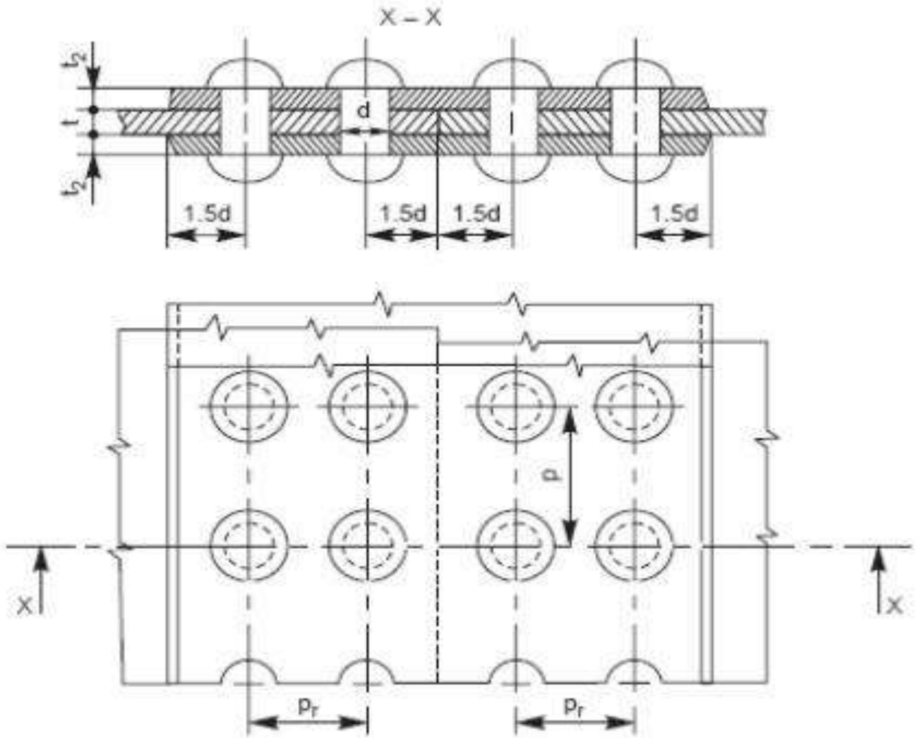


Fig. 10.14 Double riveted, double strap chain butt joint

- Double riveted, double strap zig zag butt joint

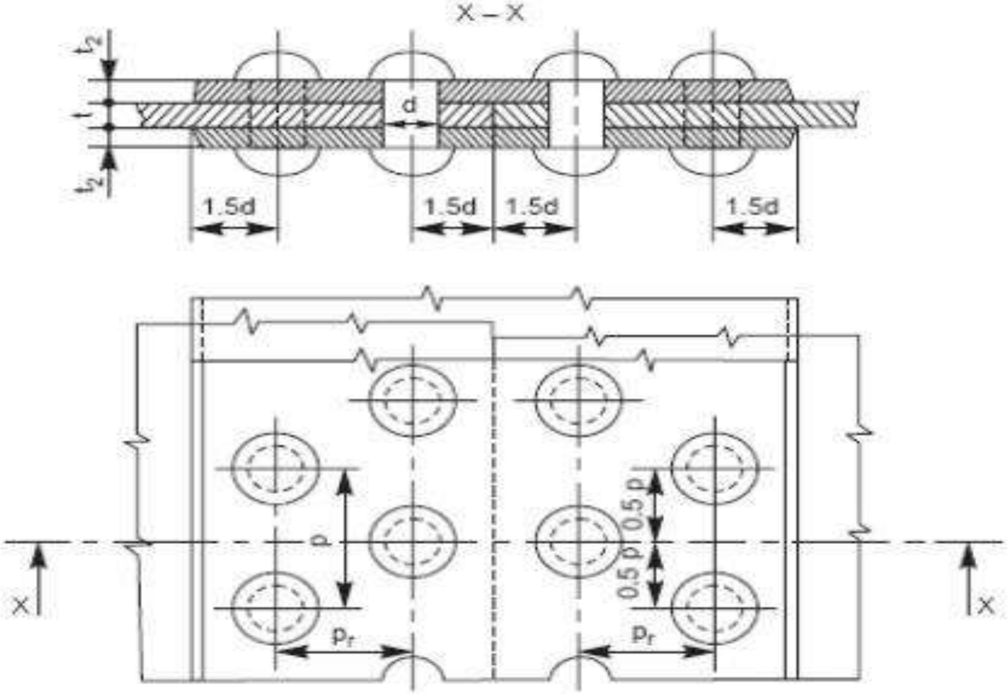
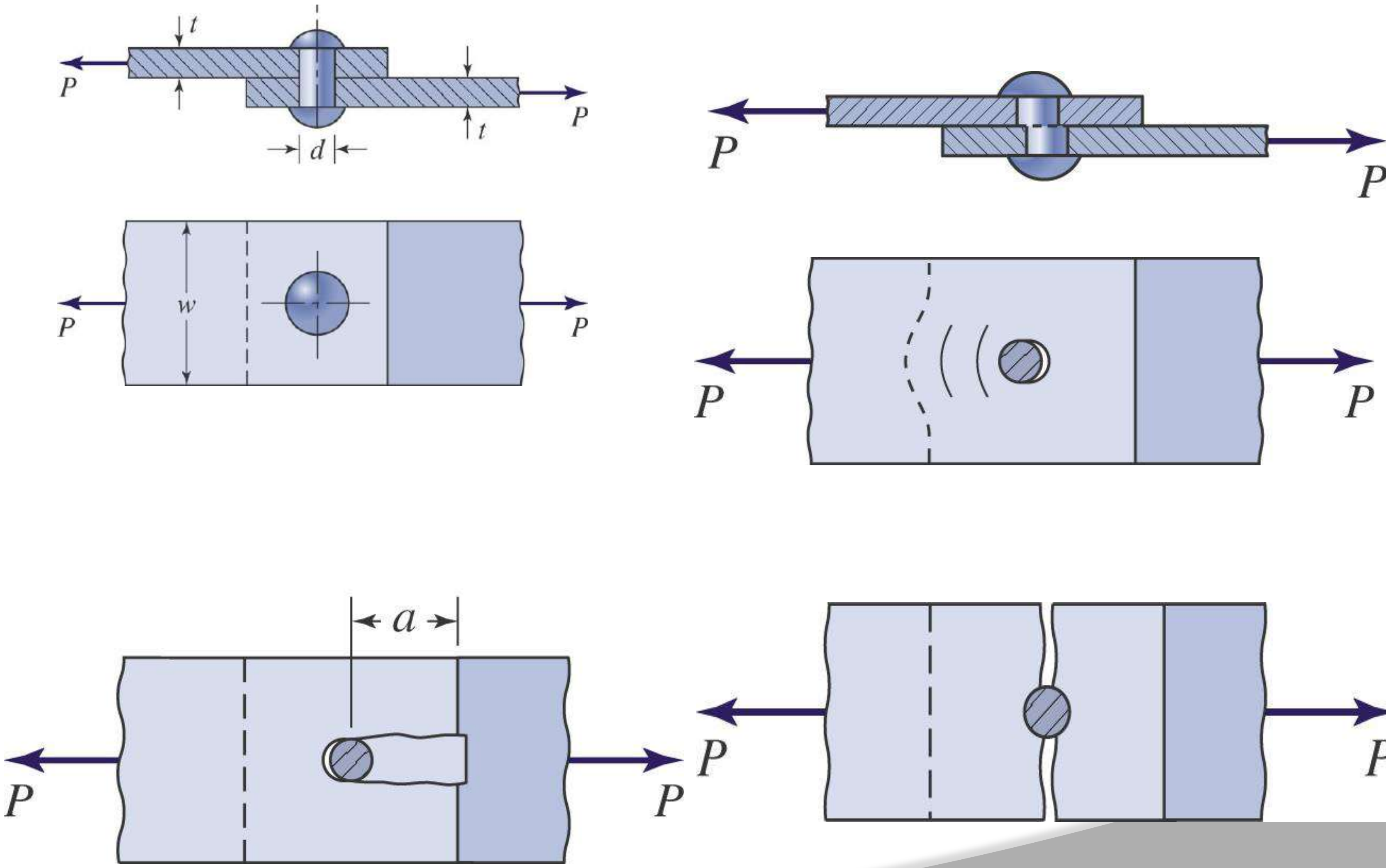
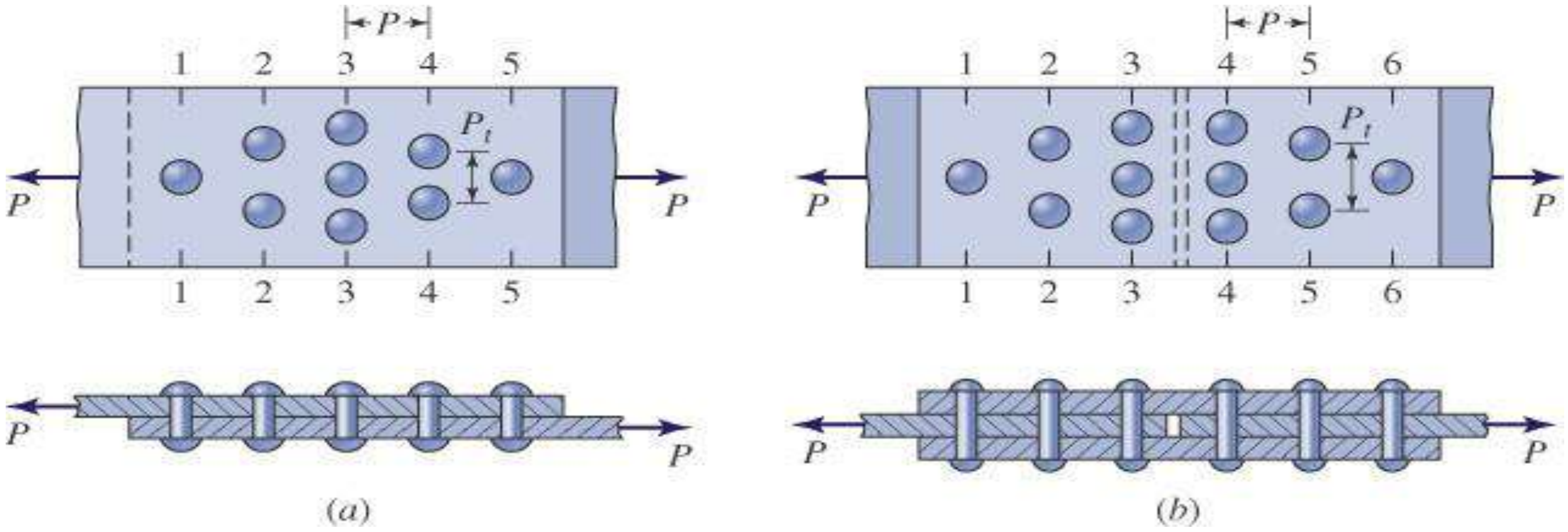


Fig. 10.15 Double riveted, double strap zig-zag butt joint

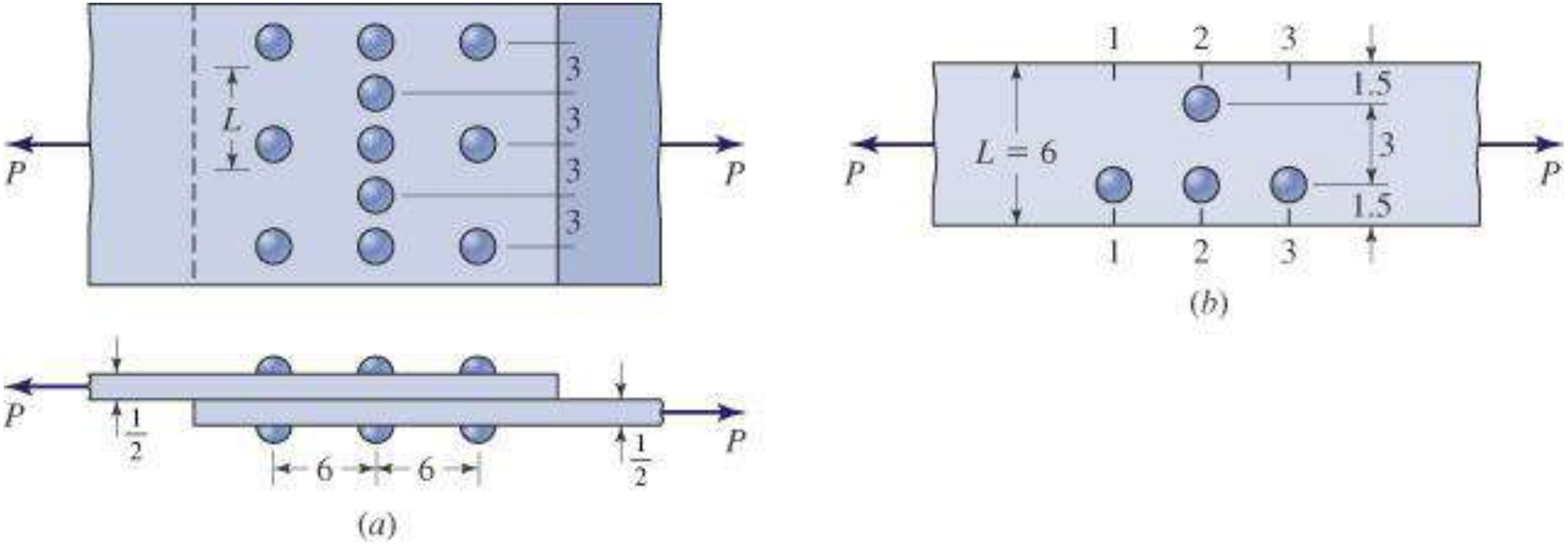
Failure of bolted or riveted joints



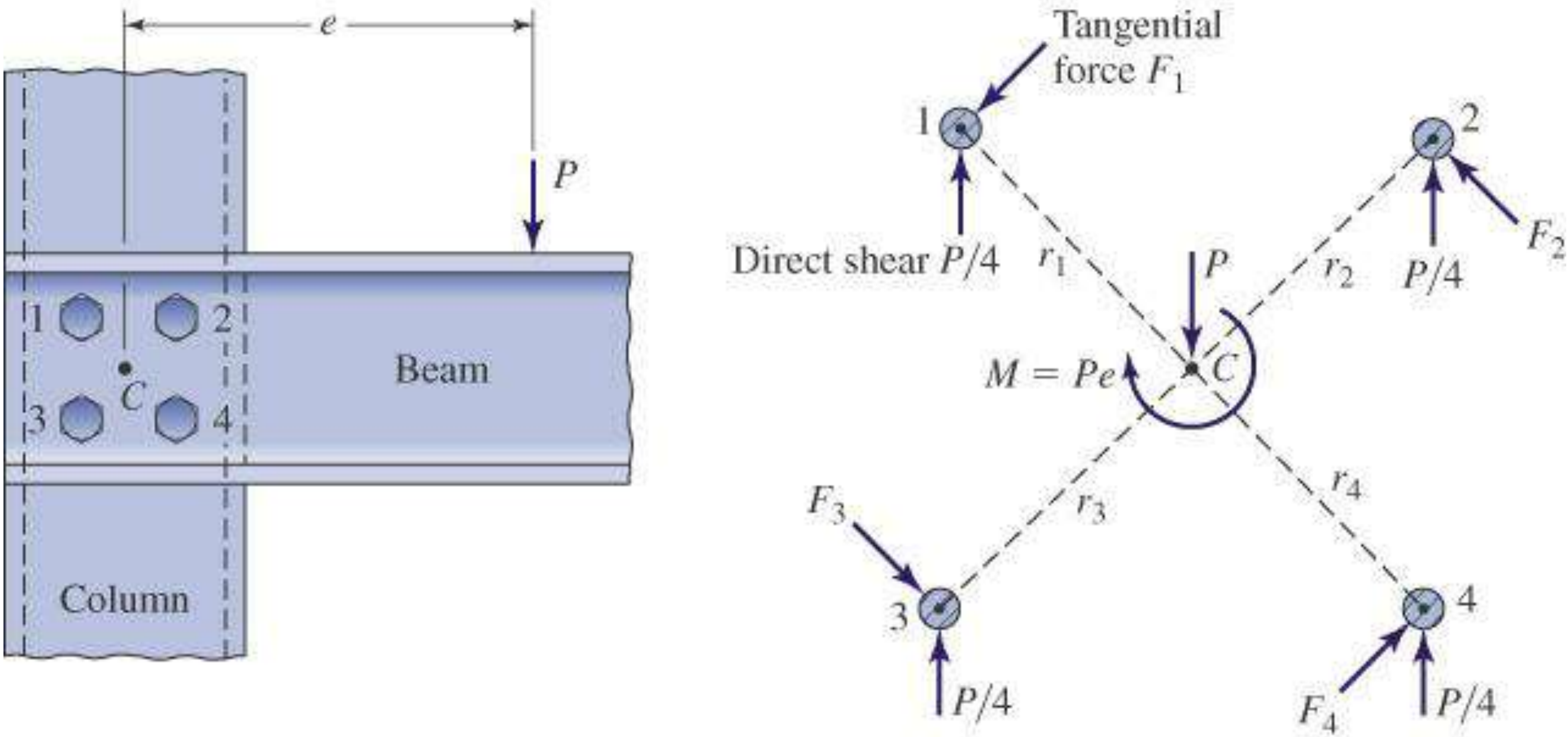
Lap Joint (single Joint) Butt Joint



EXAMPLE -1

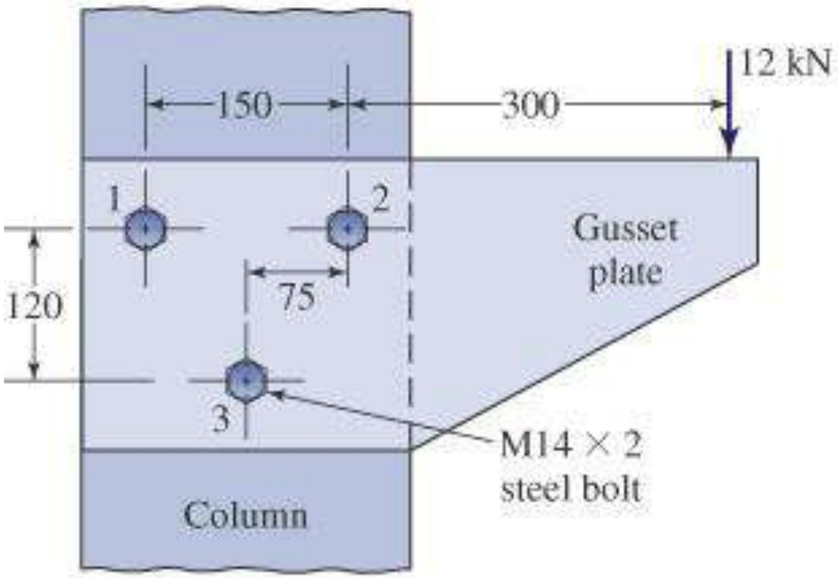


EXAMPLE - 2

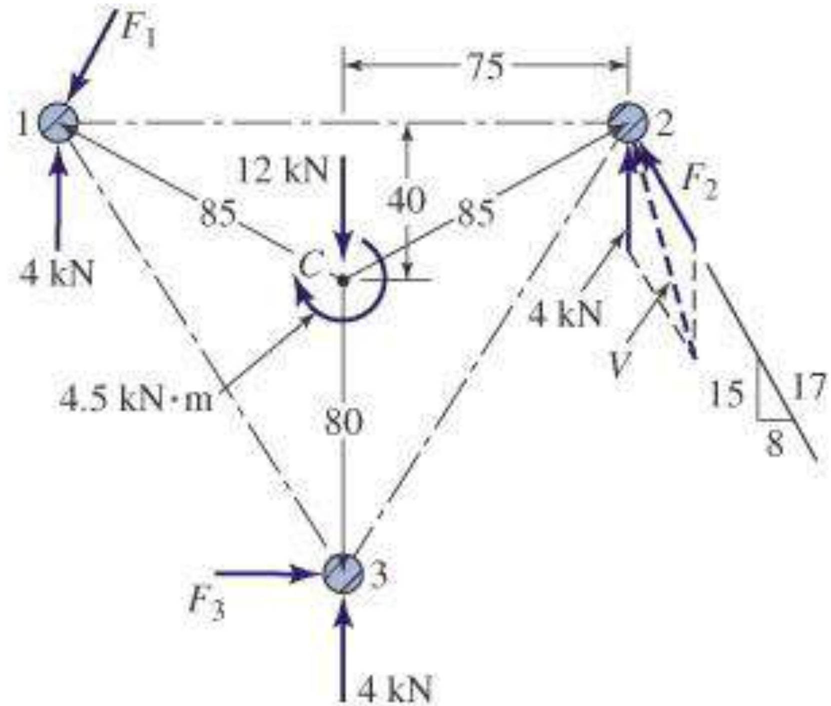


e
P
1
2
C
3
4
Beam
Column
Tangential
force F_1
Direct shear $P/4$
r_1
r_2
r_3
r_4
F_2
F_3
F_4
$P/4$
$M = Pe$

(a) (b)

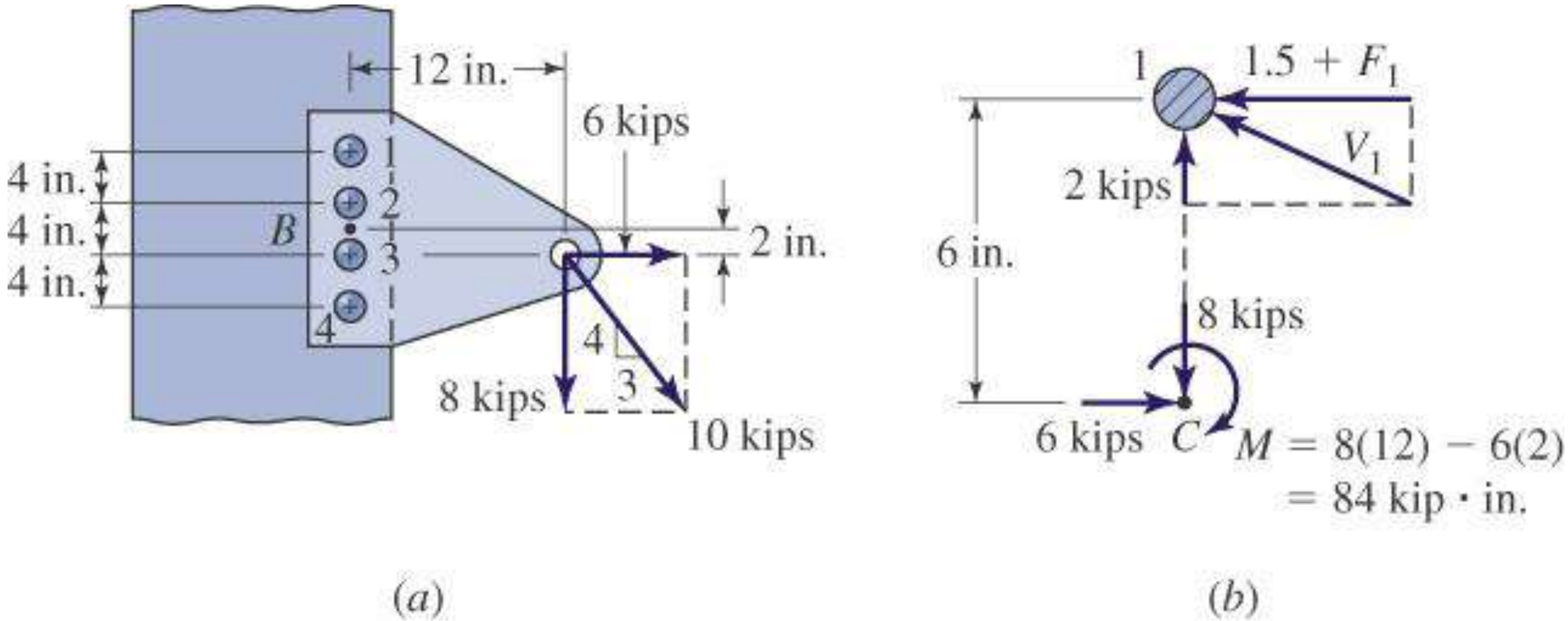

12 kN
150
300
1
2
120
75
3
Gusset
plate
M14 × 2
steel bolt
Column

(a)


F1
75
1
2
12 kN
40
85
85
F2
4 kN
C
4 kN
V
4.5 kN·m
15
17
8
80
F3
3
4 kN

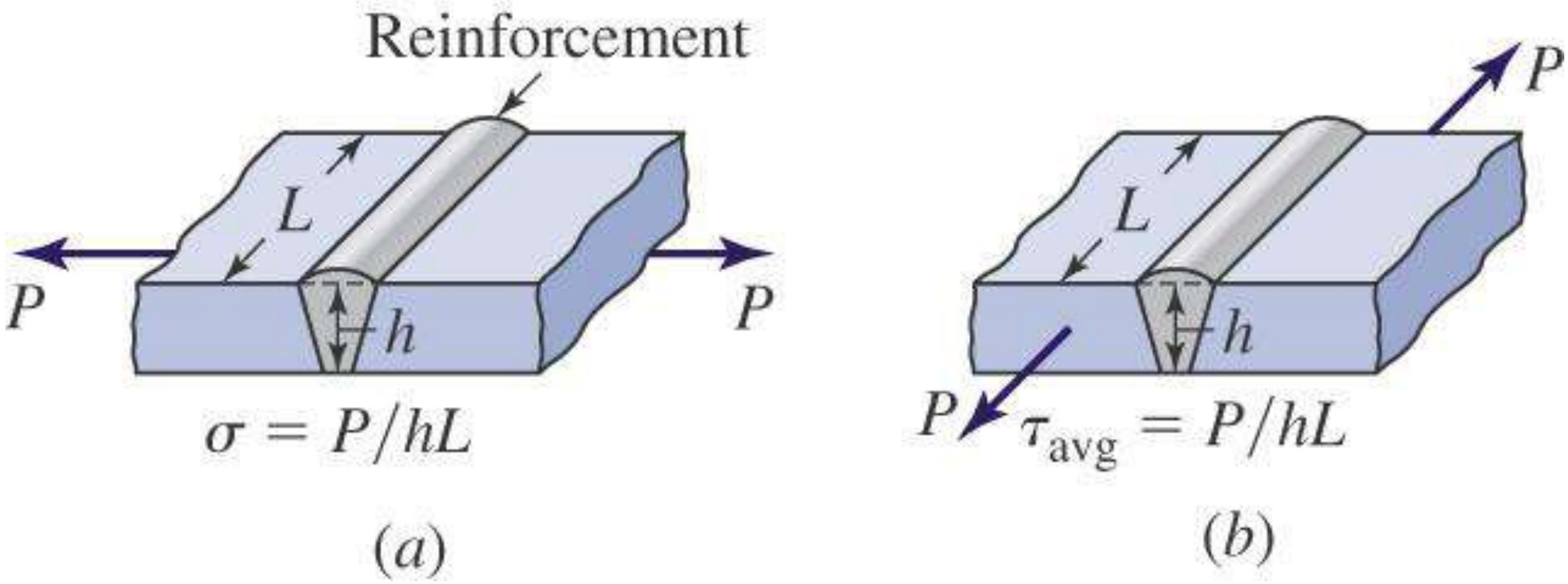
(b)

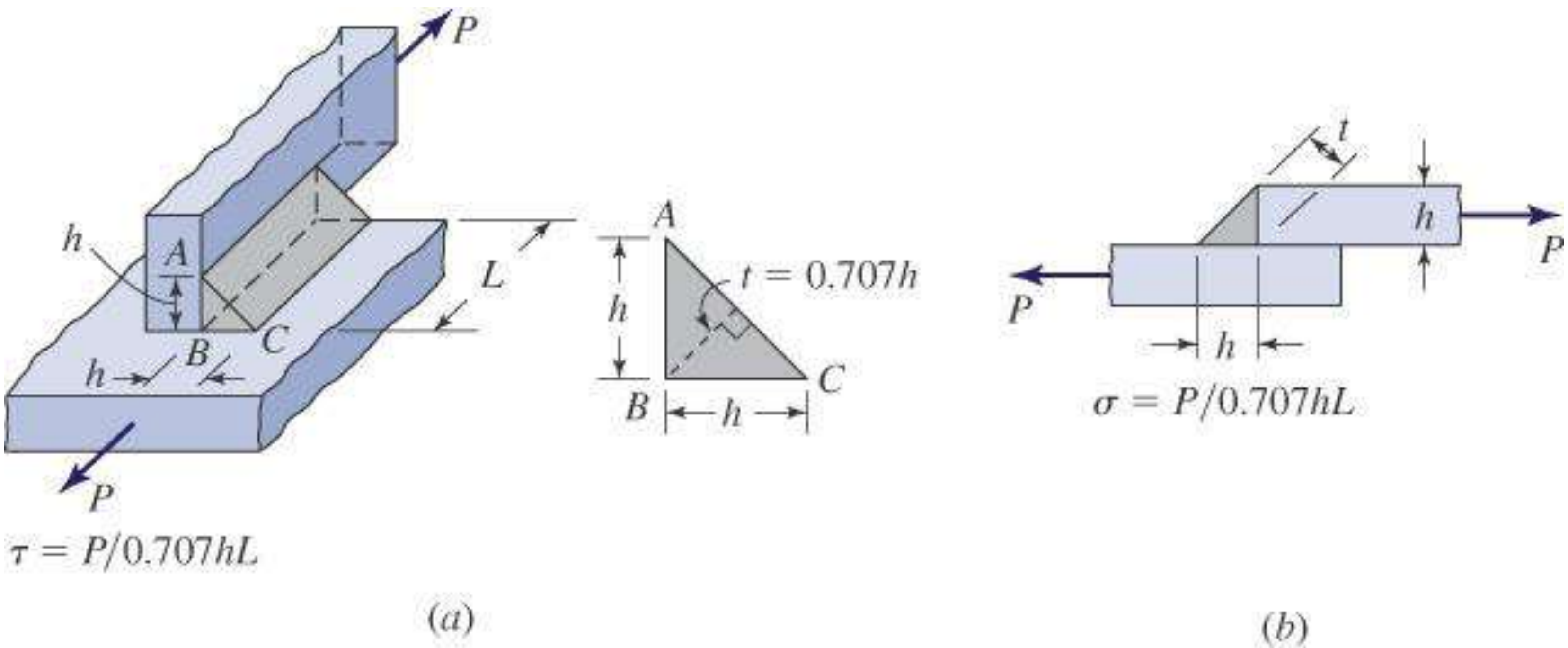
EXAMPLE -3



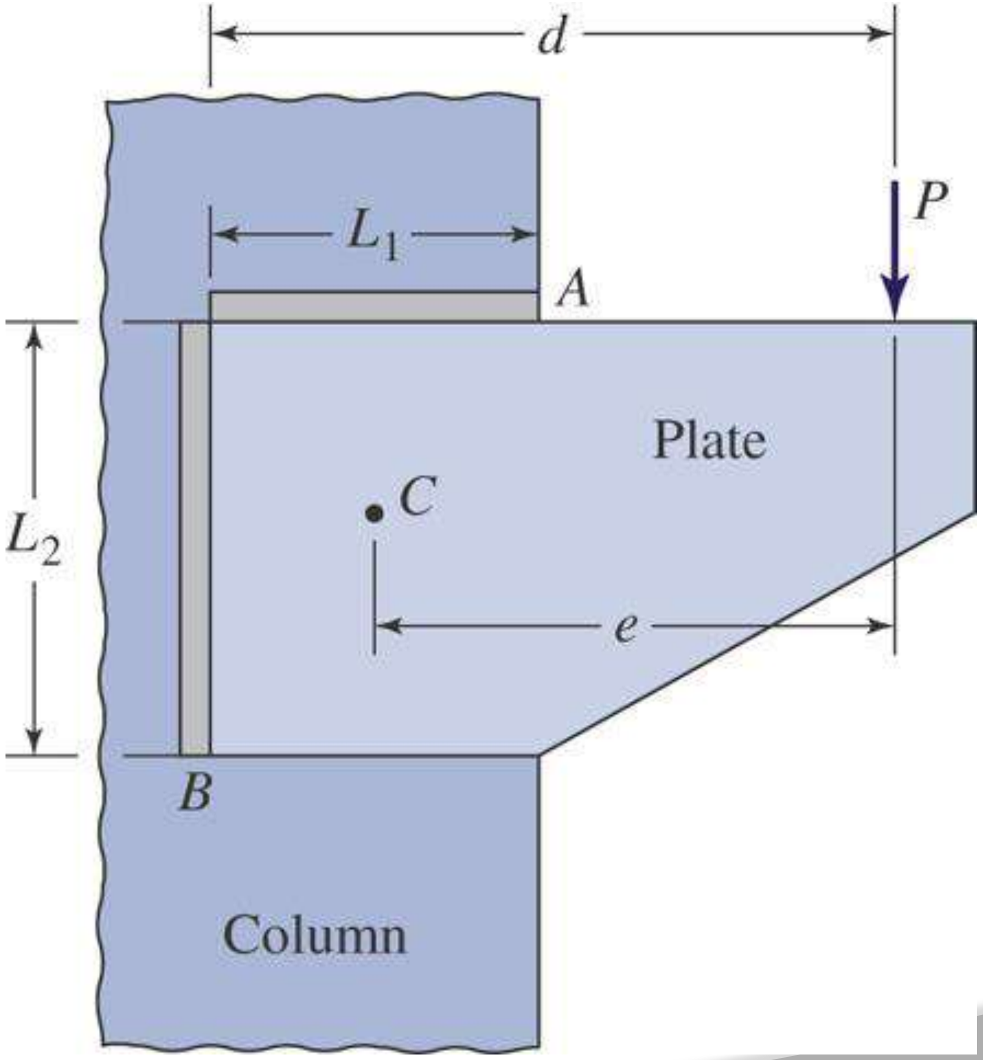
12 in.
6 kips
4 in.
4 in.
4 in.
B
1
2
3
4
2 in.
4
3
8 kips
10 kips
1
1.5 + F1
V1
2 kips
6 in.
8 kips
6 kips
C
M = 8(12) − 6(2)
= 84 kip · in.
(a)
(b)

WELDED JOINTS

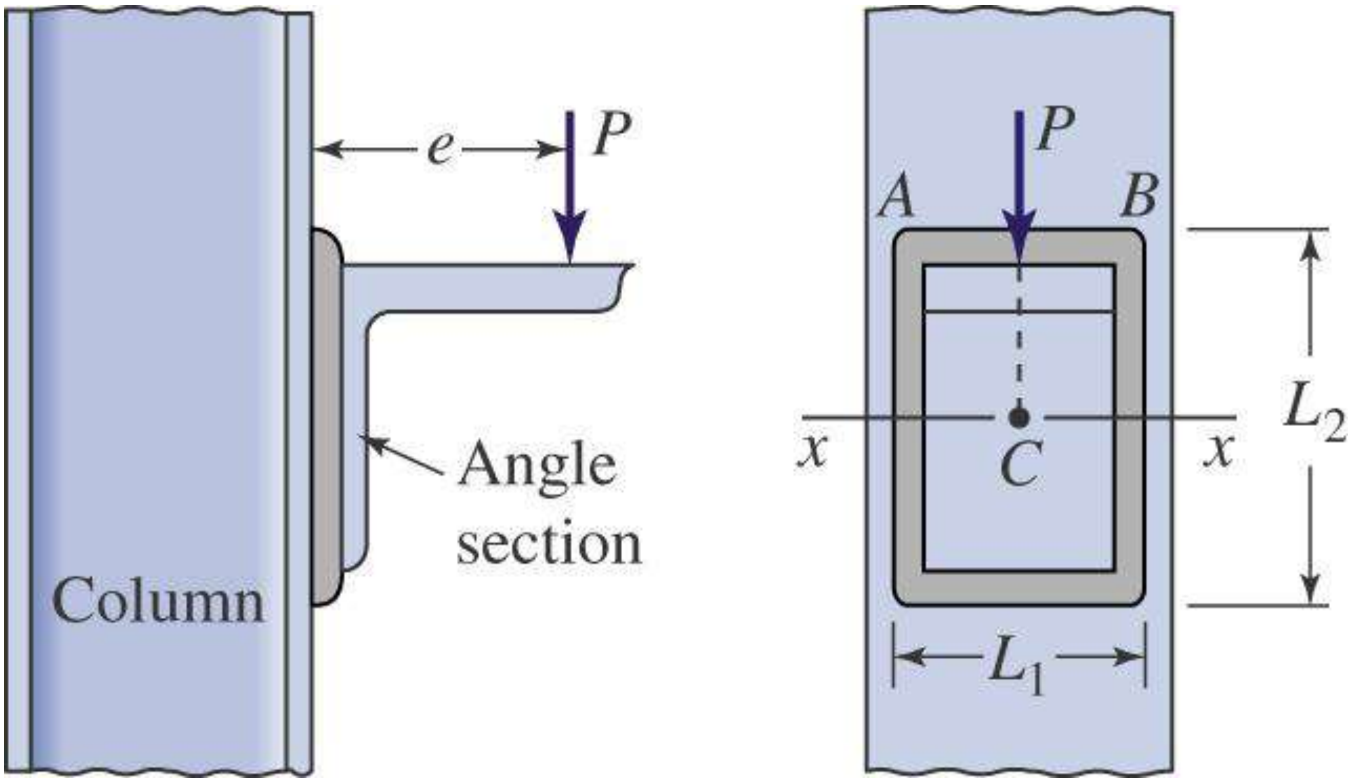


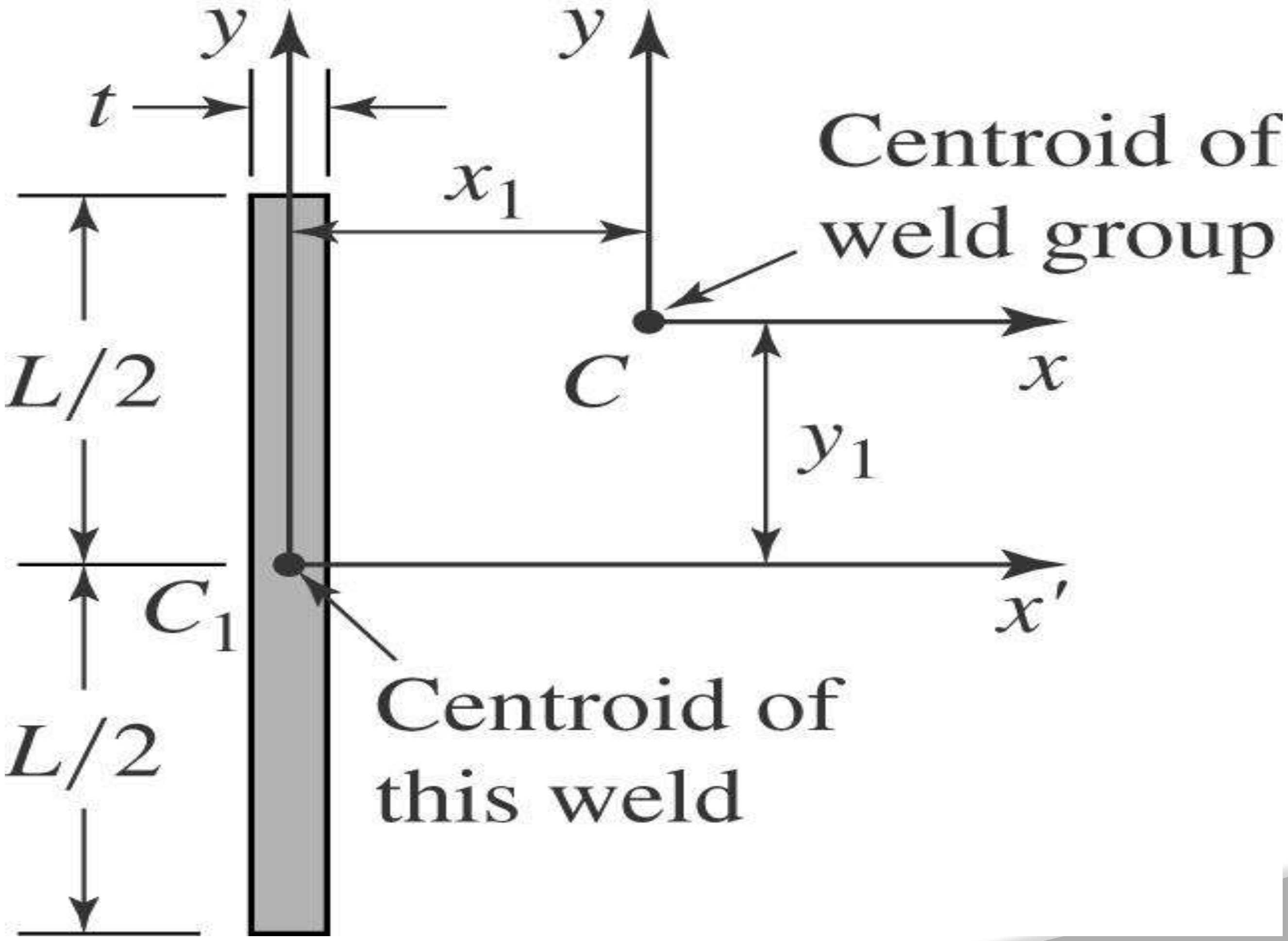

P
h
A
L
B
C
h
P
τ = P/0.707hL
A
t = 0.707h
h
B
h
C
(a)
t
h
P
P
h
σ = P/0.707hL
(b)

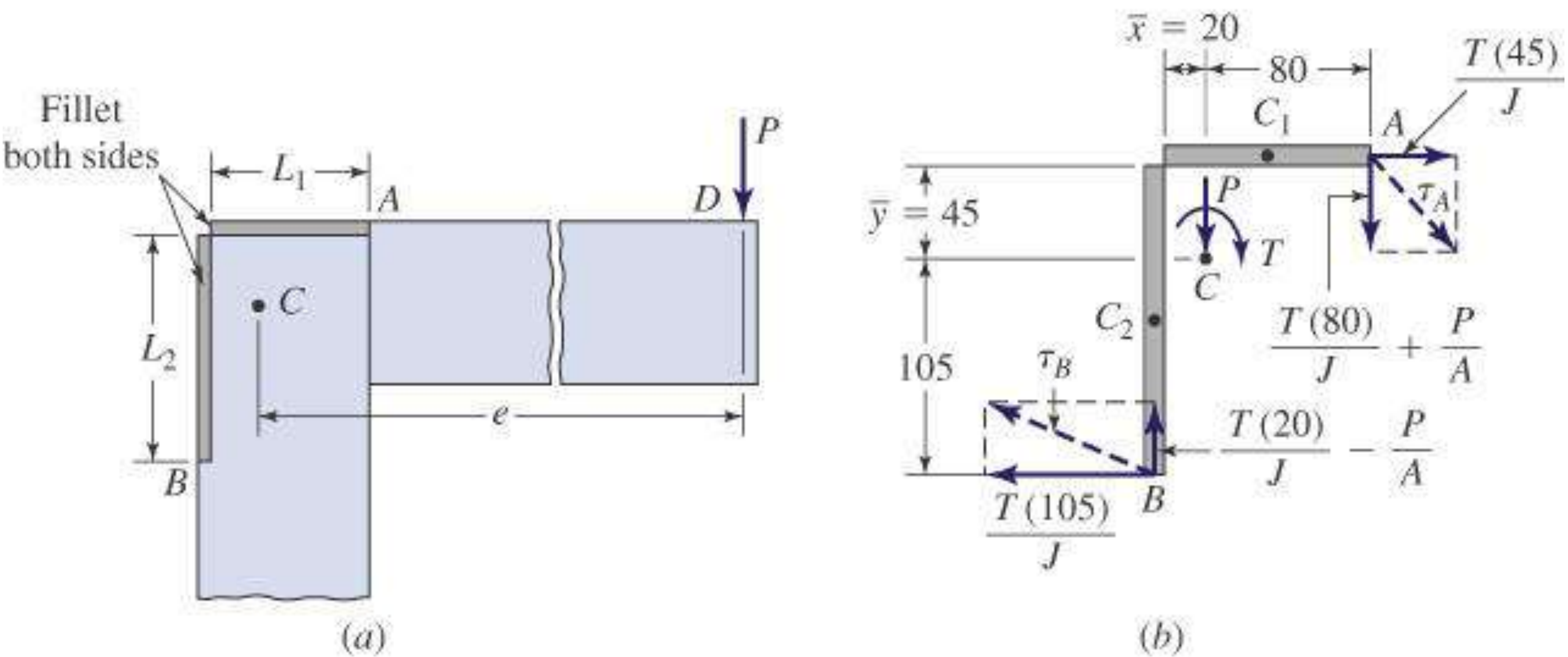
Weld under Bending



Weld under Bending




y
y
t
Centroid of
weld group
x1
L/2
C
x
y1
C1
x'
Centroid of
this weld
L/2


Fillet
both sides
L1
A
D
P
C
L2
B
e
(a)
x̄ = 20
80
T (45) / J
C1
A
ȳ = 45
P
T
C
τA
T (80) / J + P / A
C2
105
τB
T (20) / J − P / A
T (105) / J
B
(b)

Threaded Fasteners

Fastener Types

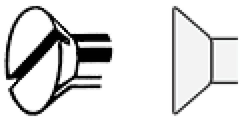
Bolt

Bolts are defined as headed fasteners having external threads that meet an exacting, uniform bolt thread specification such that they can accept a non-tapered nut.

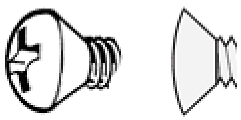
Screws are defined as headed, externally-threaded fasteners that do not meet the above definition of bolts. Screws always cut their own internal threads when initially installed, as there is generally no tool meeting the arbitrary specification of their threads to tap out the internal threads beforehand. Also, they are generally tapered to a sharp point.

Head Style

- **Flat**
  A countersunk head with a flat top.
  Abbreviated FH

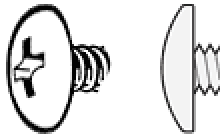


- **Oval**
  A countersunk head with a rounded top.
  Abbreviated OH or OV



- **Pan**
  A slightly rounded head with short vertical sides.
  Abbreviated PN

- **Truss**
  An extra wide head with a rounded top.



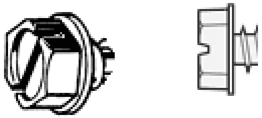
- **Round**
  A domed head.
  Abbreviated RH

90

- **Hex**
  A hexagonal Head.
  Abbreviated HH or HX

- **Hex Washer**
  A hex head with built in washer.

- **Slotted Hex Washer**
  A hex head with built in washer and a slot.



- **Socket Head Cap**
  A small cylindrical head using a socket drive.

- **Button**
  A low profile rounded head using a socket drive.

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Thread Types

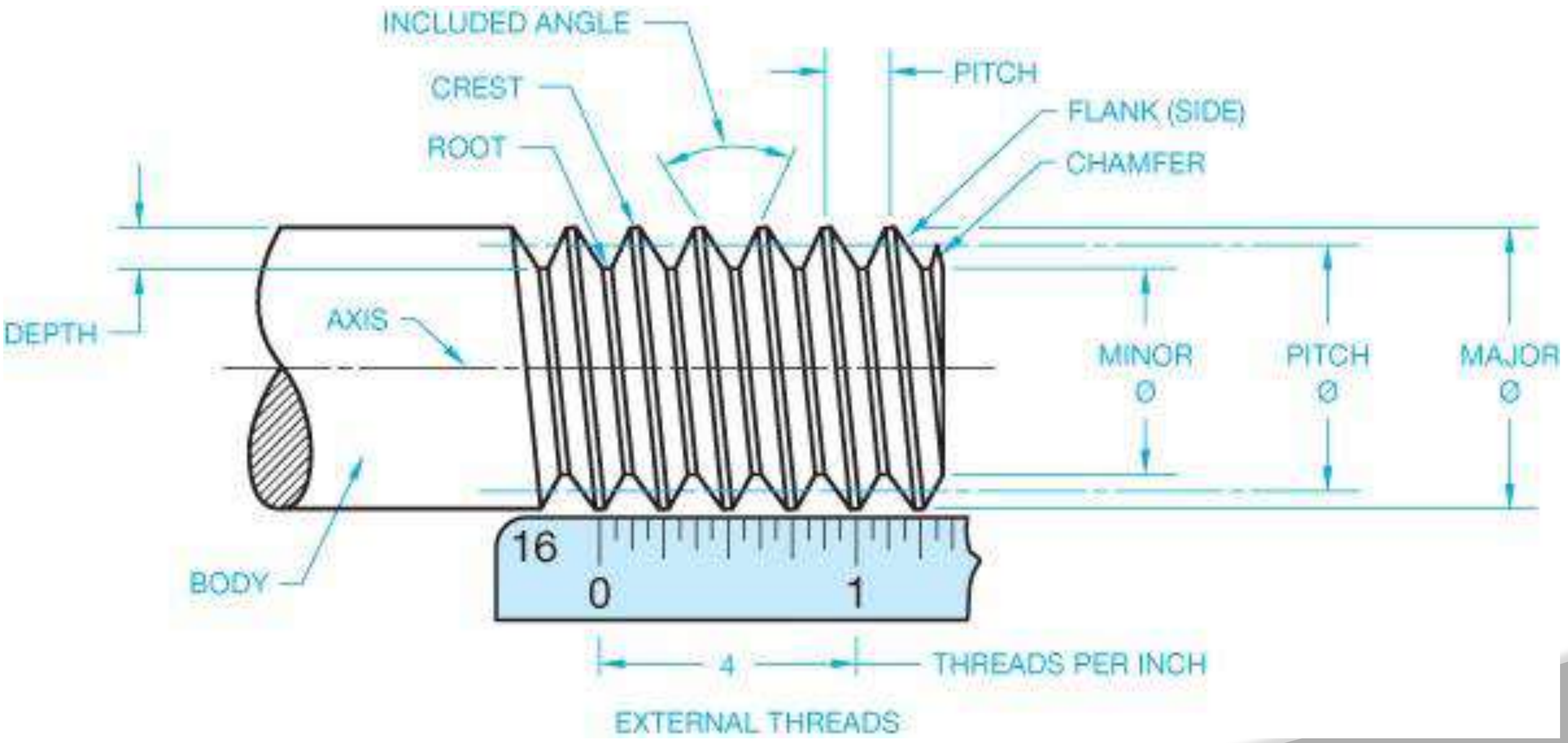
- Unified Thread
  - UNC – Unified National Coarse
  - UNF – Unified National Fine
- American National Thread
- Sharp-V Thread
- Metric Thread
- Whitworth Thread
- Square Thread
- Acme Thread
- Buttress Thread
- Rolled Thread
- Pipe Thread
  - NPT – National Pipe Taper
  - NPS – National Pipe Straight

Thread Detail

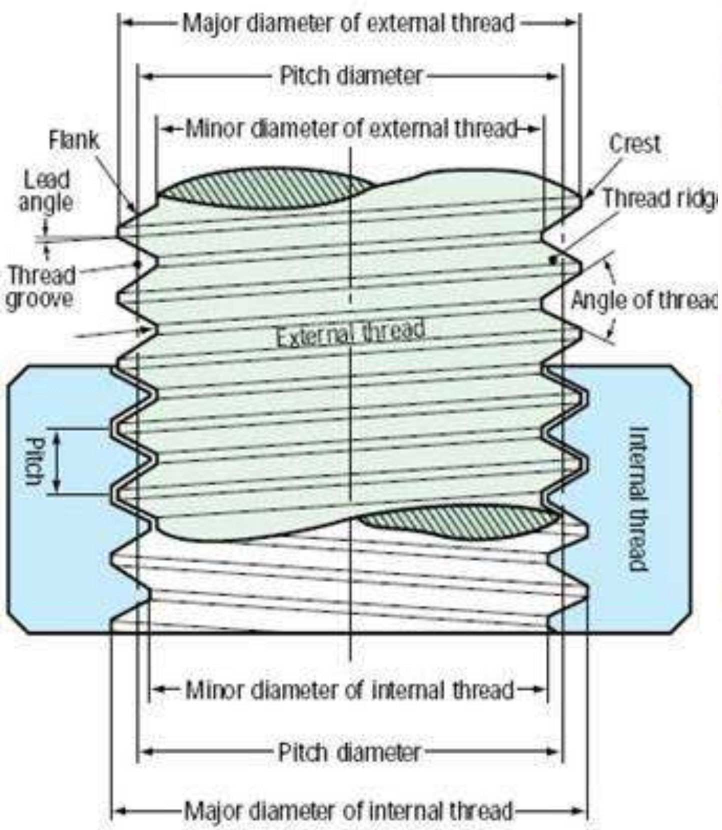
Crest Width = 1/8 Pitch

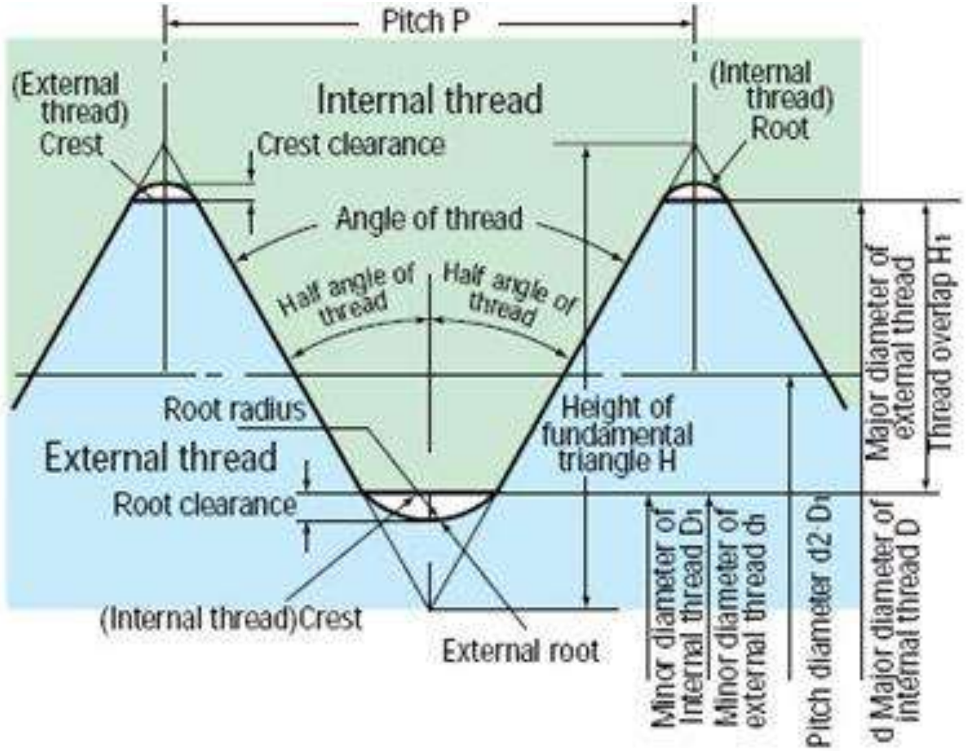
Root Width = ¼ Pitch

r = Radius at the Root = 0.1443 x P

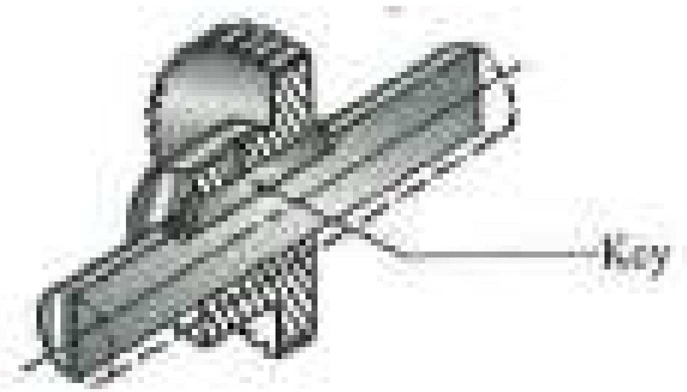


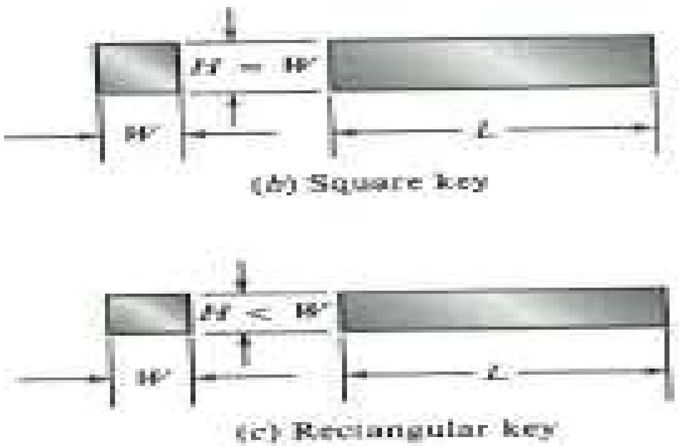
- Thread fit comparison



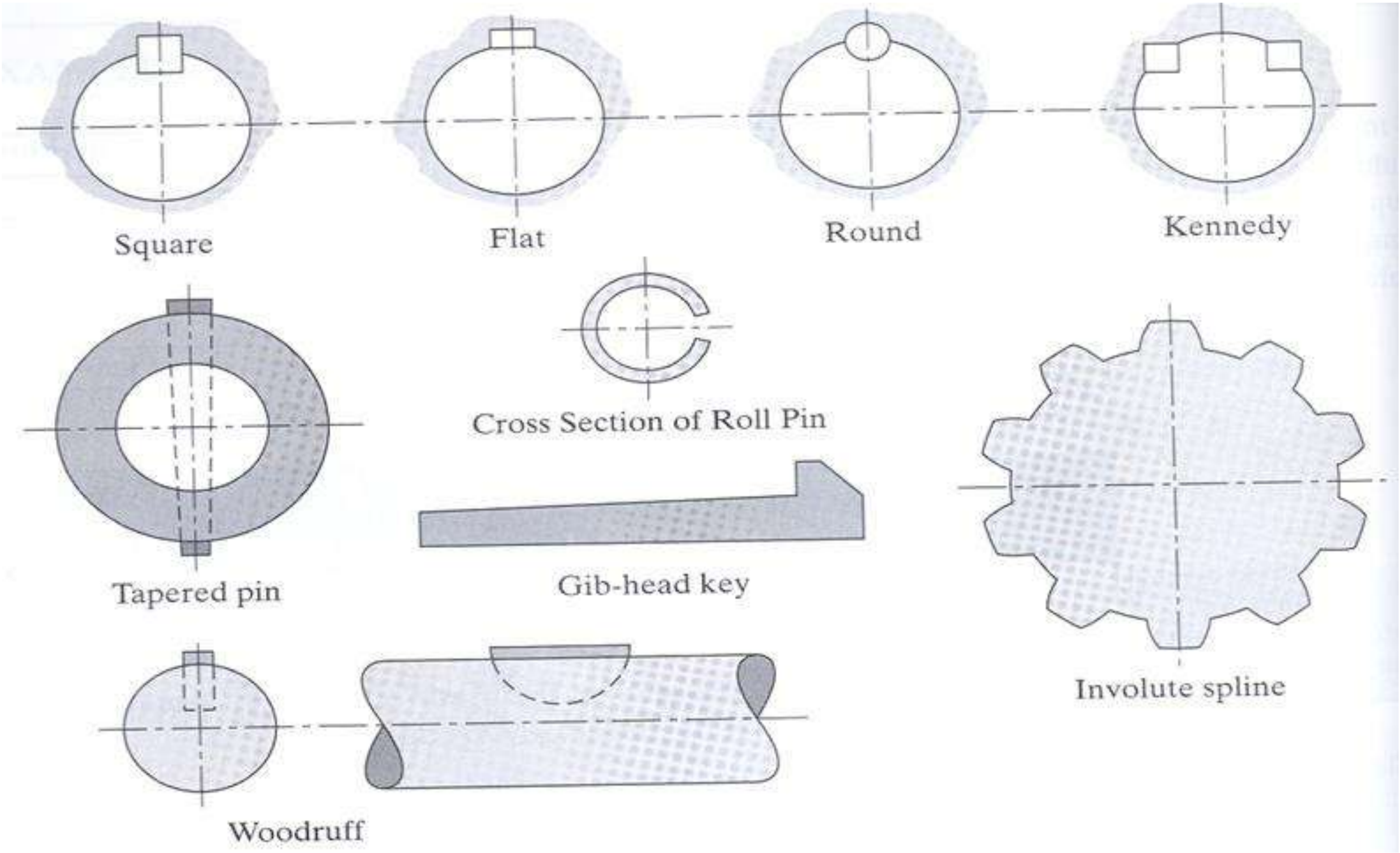


Key is used to transmit torque between shaft and the component mounted on the shaft.
Square and flat keys are most common.





TYPES OF KEYS



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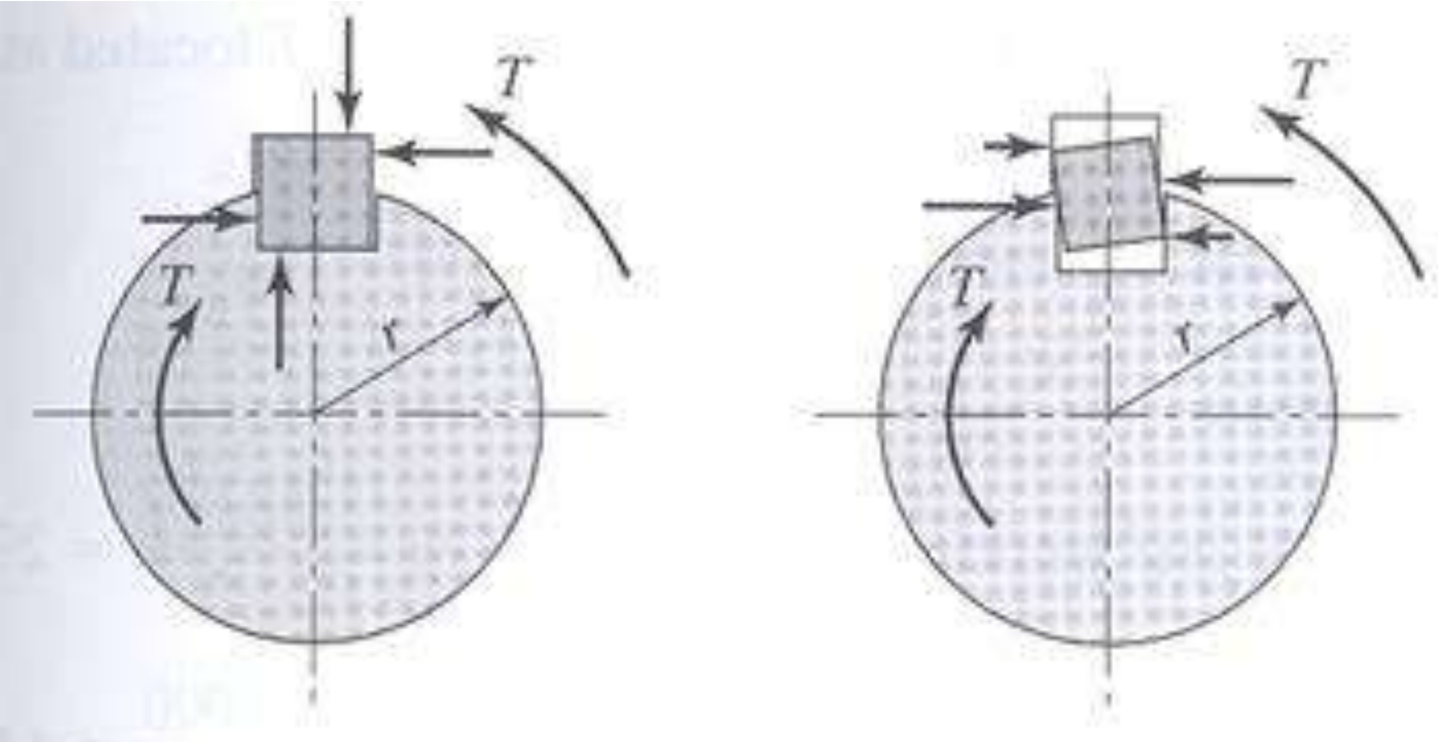
Stresses acting on keys



(a) Forces on key which fits tightly top and bottom.

(b) Forces acting on loosely fitted key.

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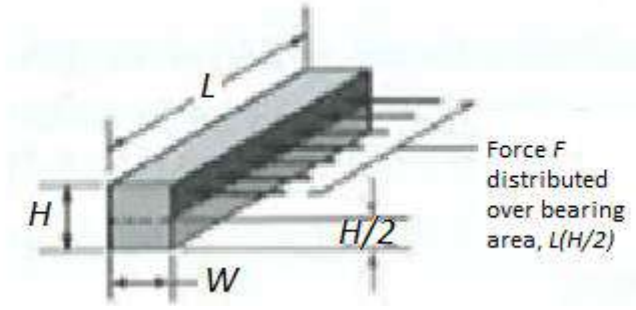
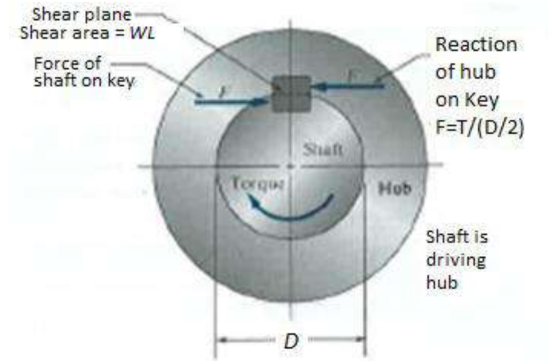
Simplified mechanical stresses equations for shear and compression





Tangential Force at shaft diameter: F= T/(D/2) = 2T/D

Average shear stress t in Key, should not exceed allowable shear stress $t_w = t_{yp}/N_{fs}$

$t = F/(W*L) < t_{yp}/N_{fs}$

or, $F/(W*L) < s_{yp}/(2N_{fs})$ ....(1)

Average bearing (compressive) stress s in Key, should not exceed compressive stress $s_w = s_{yp}/N_{fs}$

$s = F/[(H/2)*L] < s_{yp}/N_{fs}$

or, $s = F/(H*L) < s_{yp}/(2N_{fs})$........(2)

For square key, W=H, Equation (1) and (2) are essentially same, i.e., shear or bearing stress will produce the same key length.

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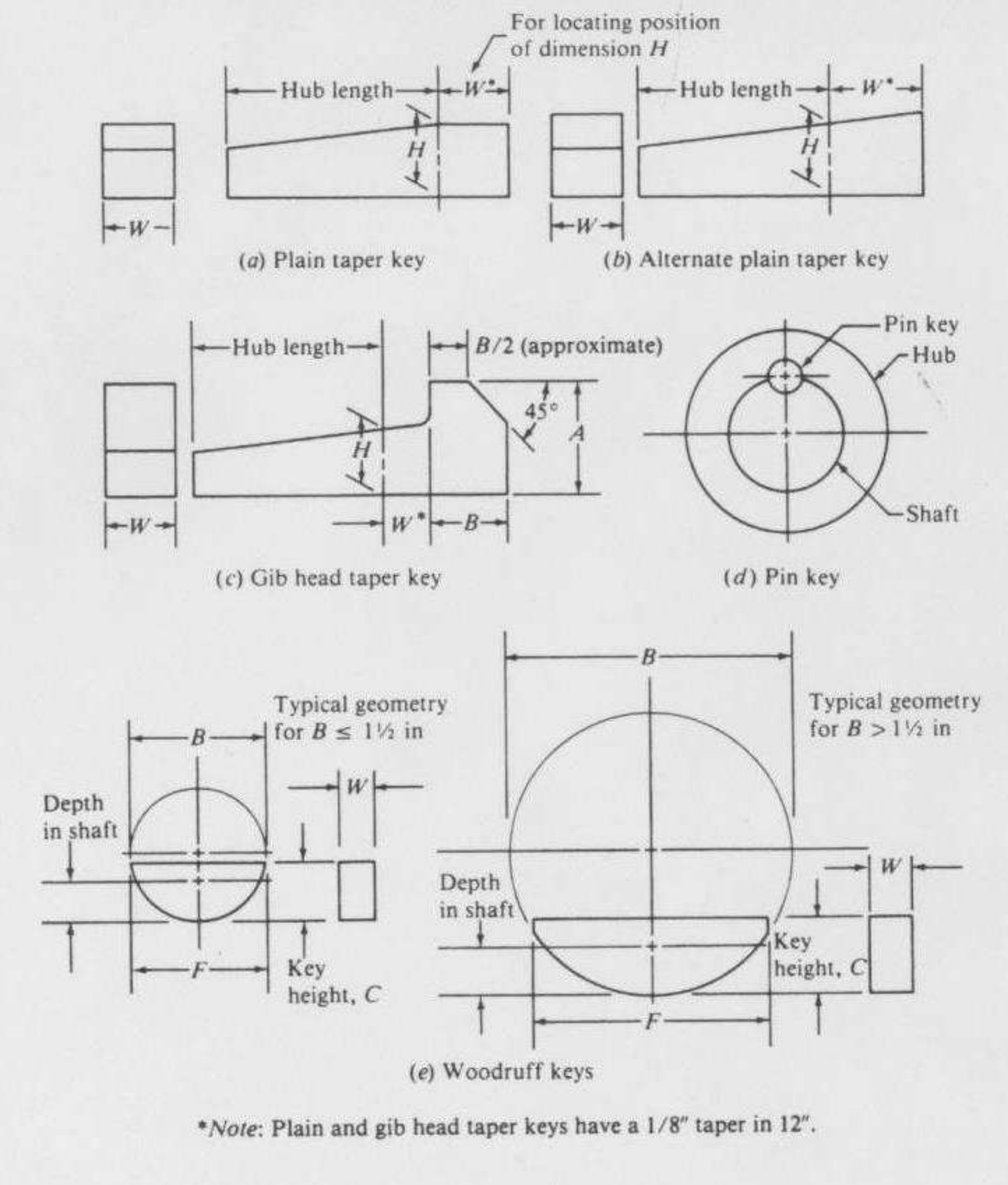
a. Tapered key – can install after hub (gear) is installed over shaft.

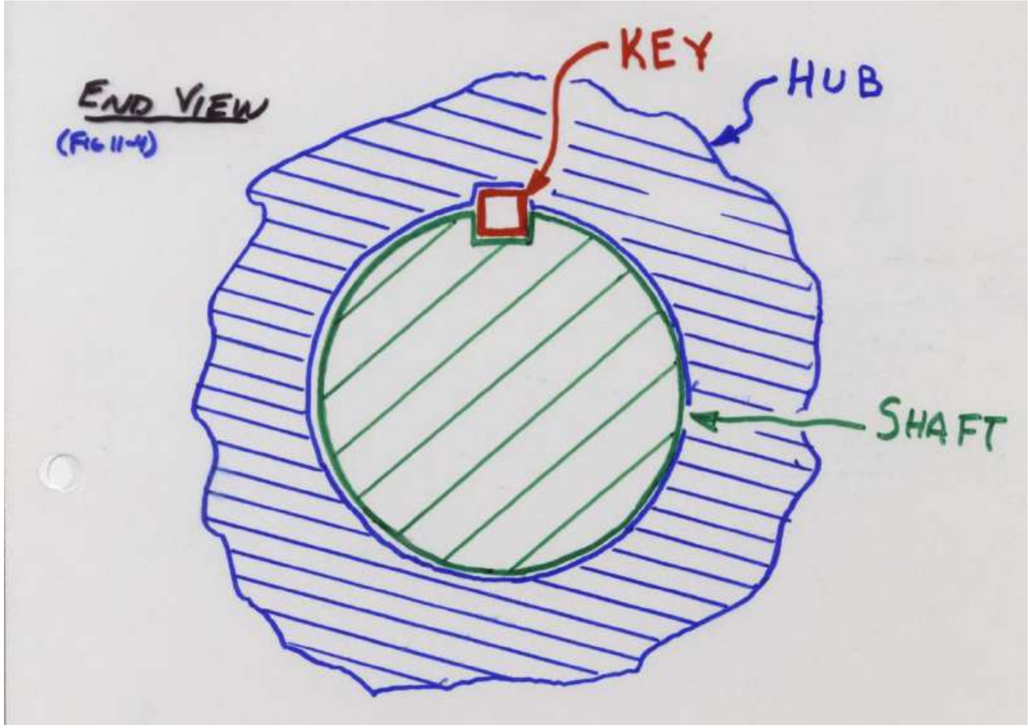
b. Gib head key – ease of extraction

c. Pin keys – low stress concentration

d. Woodruff key – light loading offers ease of assembly



Design of Keys – stress analysis



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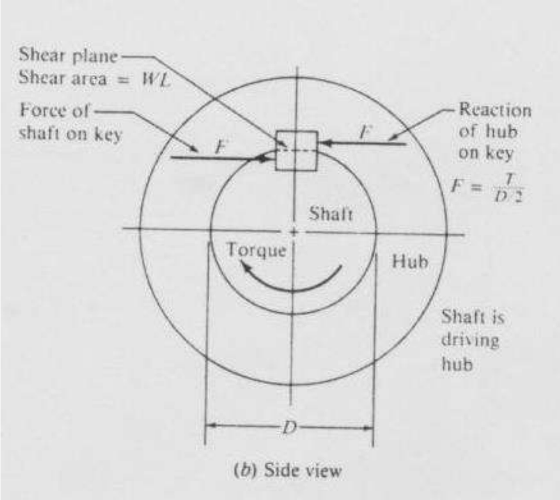
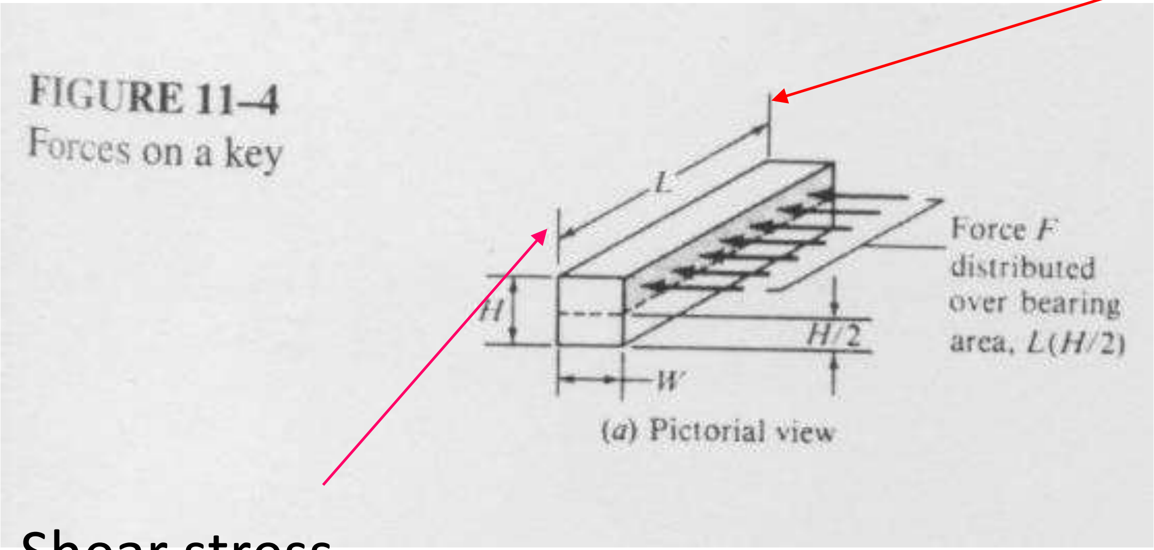


No load

Torque being transmitted

T = F/(D/2) or F = T/(D/2) this is the force the key must react!!!

Bearing stress



Shear stress

Required Length based on Shear Stress:

$$L = \frac{2T}{\tau_d DW} \quad where\ \tau_d = 0.5Sy/N$$

Required Length based on Bearing Stress:

$$L = \frac{4T}{\sigma_d DH} \quad where\ \sigma_d = Sy/N$$

Typical parameters for keys:

N = 3, material 1020 CD (Sy = 21,000 psi)

- A cotter joints is a flat wedge link piece of steel of rectangular cross section which is inserted through the rods at high angle to their axes .It is uniform in thickness but tapering in width , generally on one side only. Usually the taper is 1 in 30.

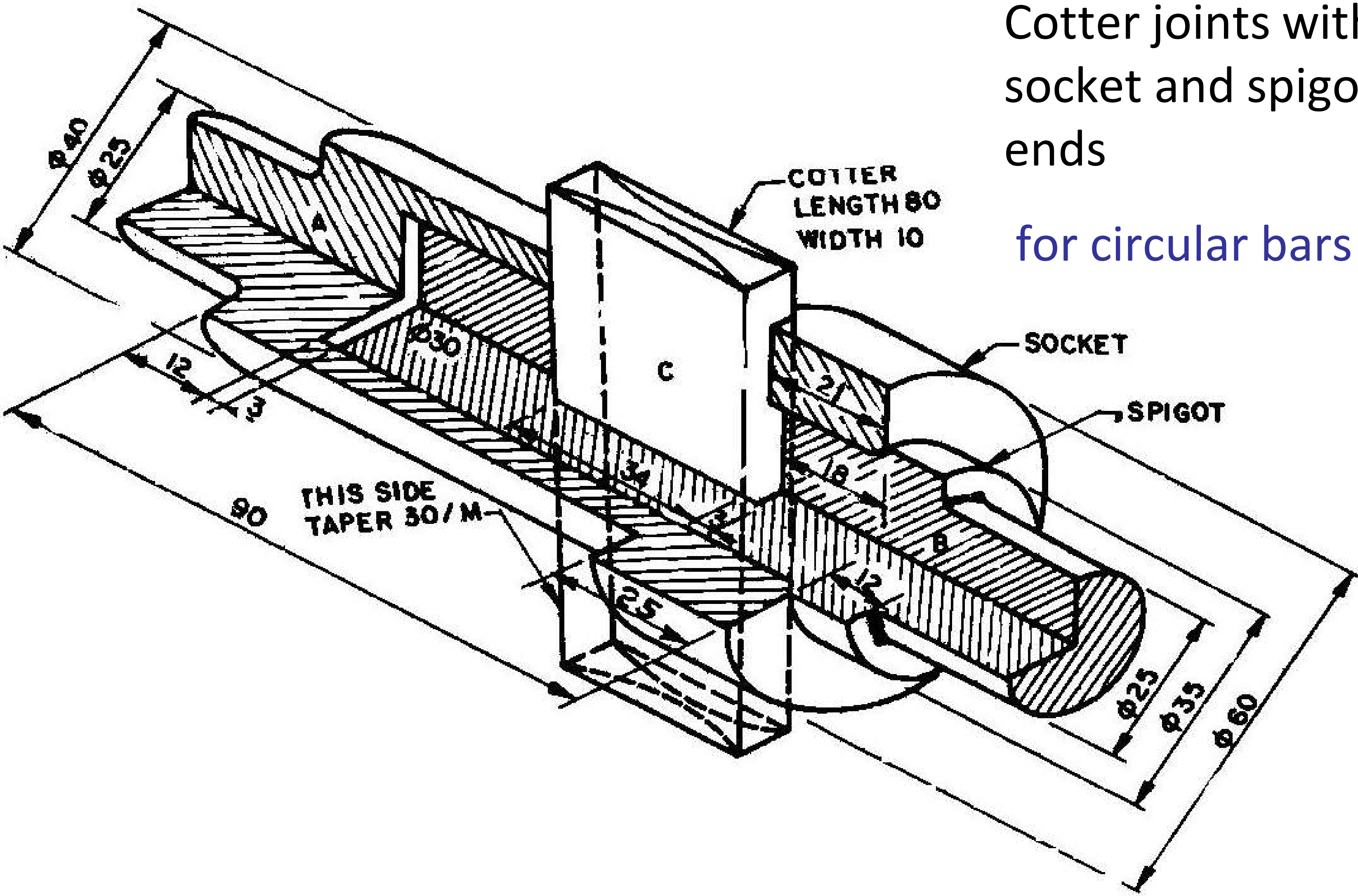
- when a special arrangement like a set-screw is provided for keeping the cotter from slackening ,its taper may be as large as 1 in 7. the end of the cotter are made narrow to facilitate the hammering for fixing and removing.

- cotter joins are generally use to fasten rigidly two rod s which is subjected to tensile or compressive stress along their axes. this joint is used to connect two circular rods.

- This joint in not suitable where the member are subjected under rotation.

- Thus they differ from key joints which are used to fasten shaft and hubs subjected to tensional stress:

Cotter joint



Cotter joints with socket and spigot ends

for circular bars

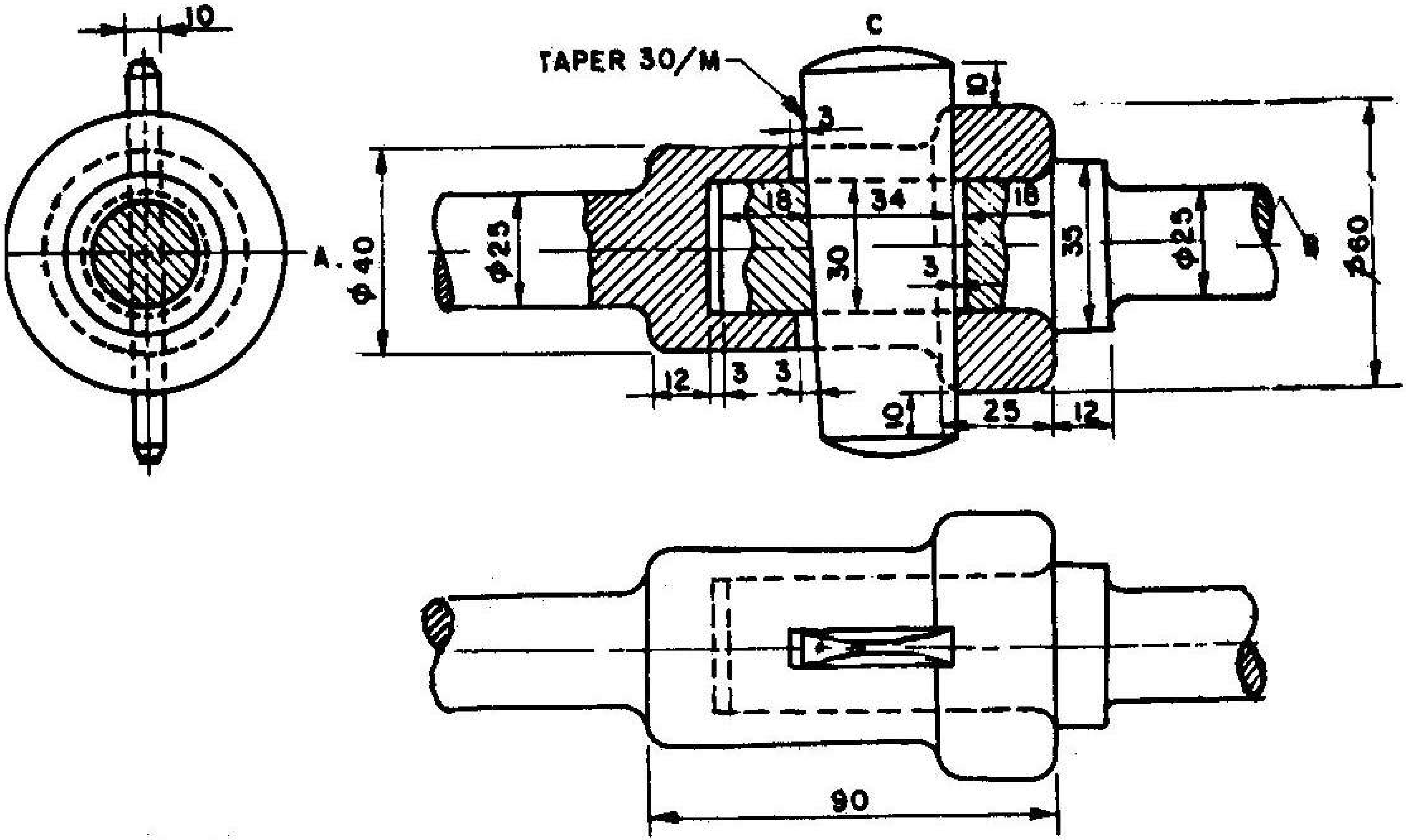
Slots are wider than the cotter.

Cotter pulls the rod and socket tightly together

Clearance: must be provided for adjustment.(2 to 3 mm)

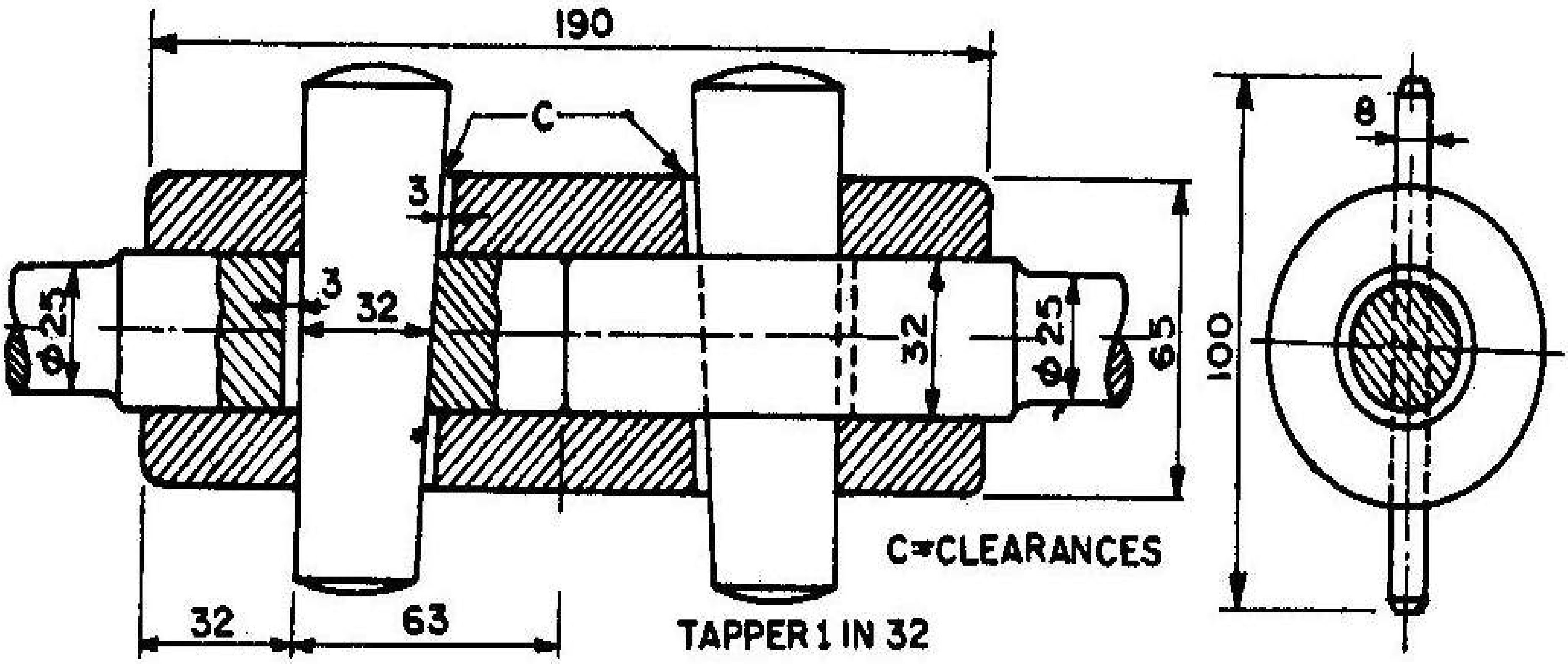
Proportions : cotter thickness = (1/3)diameter of rod

cotter width = rod diameter



10
TAPER 30/M
C
3
10
18
34
18
A.
Φ40
Φ25
30
3
35
Φ25
Φ60
12
3
3
10
25
12
90

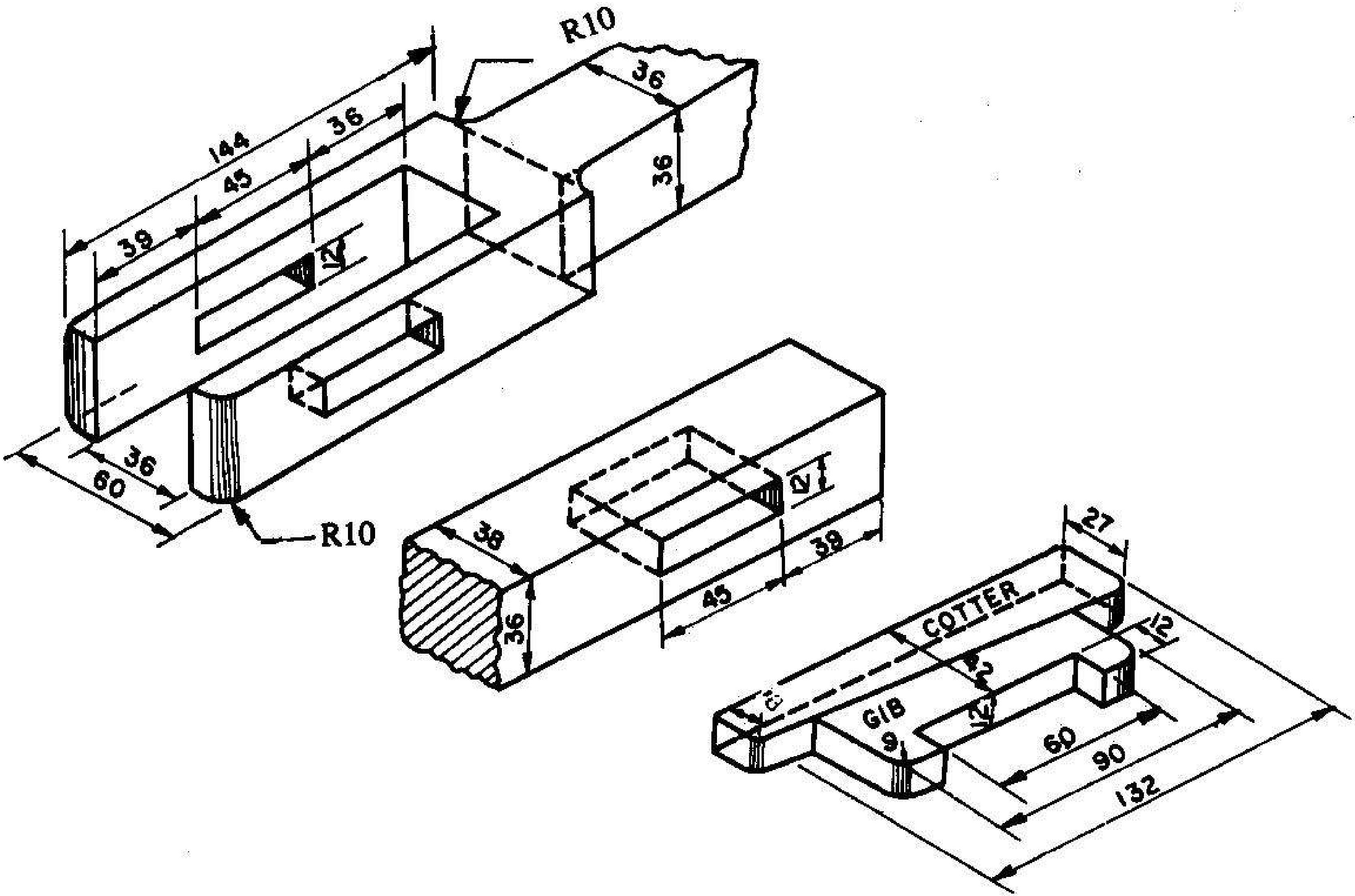
SLEEVE AND COTTER JOINT



COTTER WITH GIB

- Gib and cotter joints are used for rods of square or rectangular cross section .the end of one rod fits the end of the other rod which is made in the form of a strap. A gib is used along with the cotter to make this joint. Gib is likely a cotter but with two gib heads at its ends . The thickness of the gib and cotter are same.

Gib and cotter joint for rectangular rods

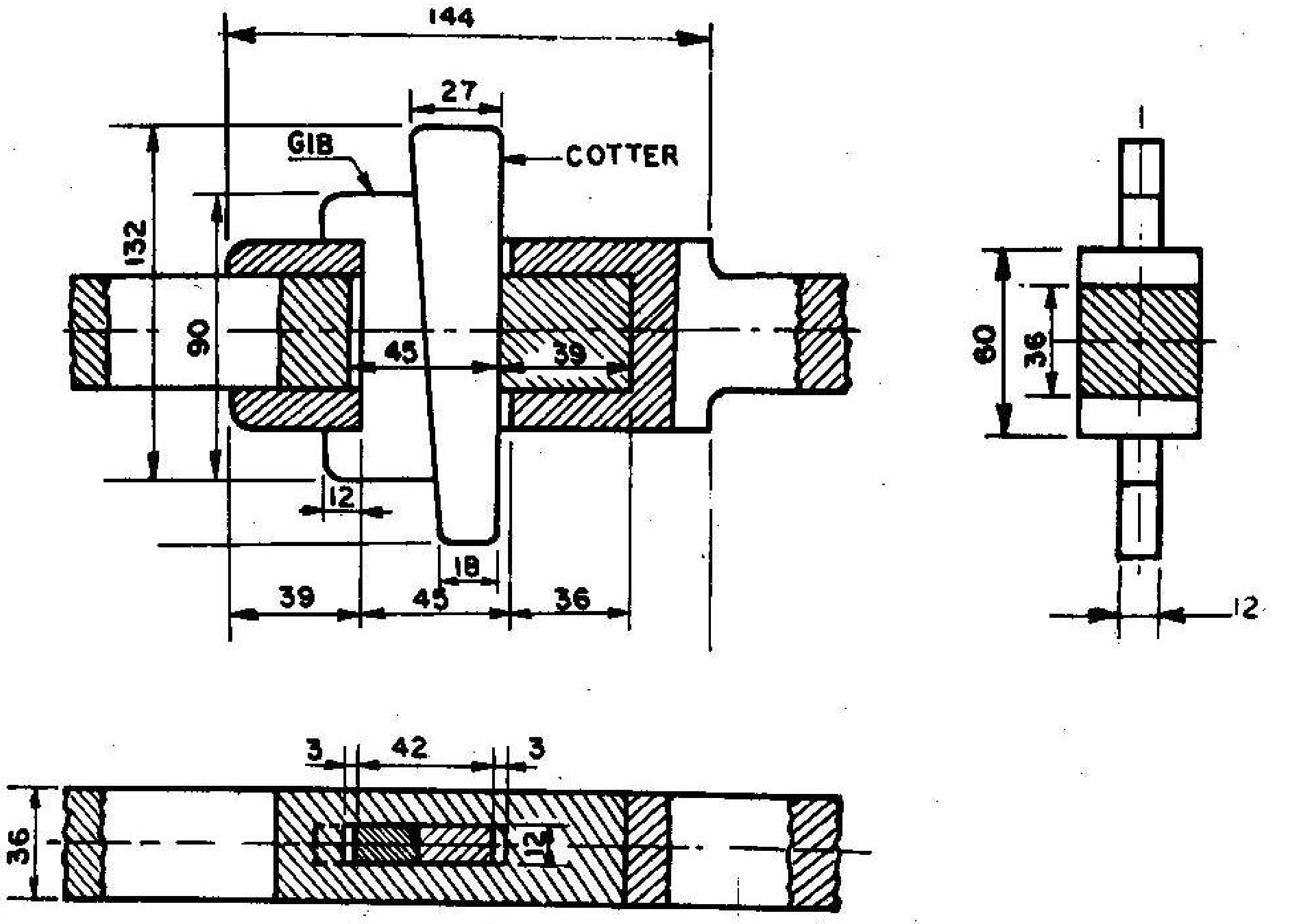


One bar end is made in the form of a strap

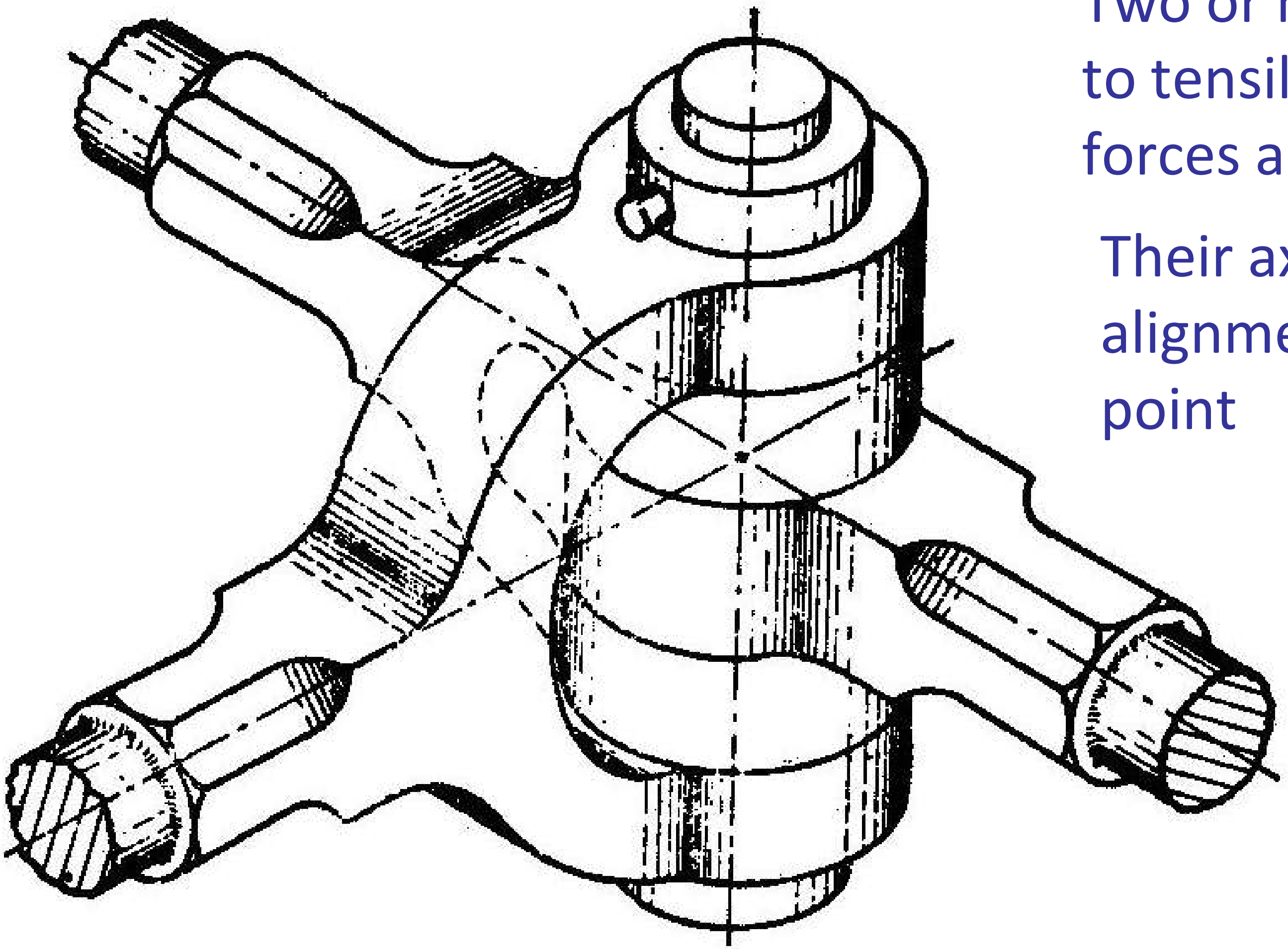
A Gib is used along with the cotter.

Gib is like a cotter but with two gib heads at its ends .

The thickness of the gib and cotter are same


144
27
GIB
COTTER
132
90
45
39
12
18
39
45
36
60
36
12
3
42
3
36
12

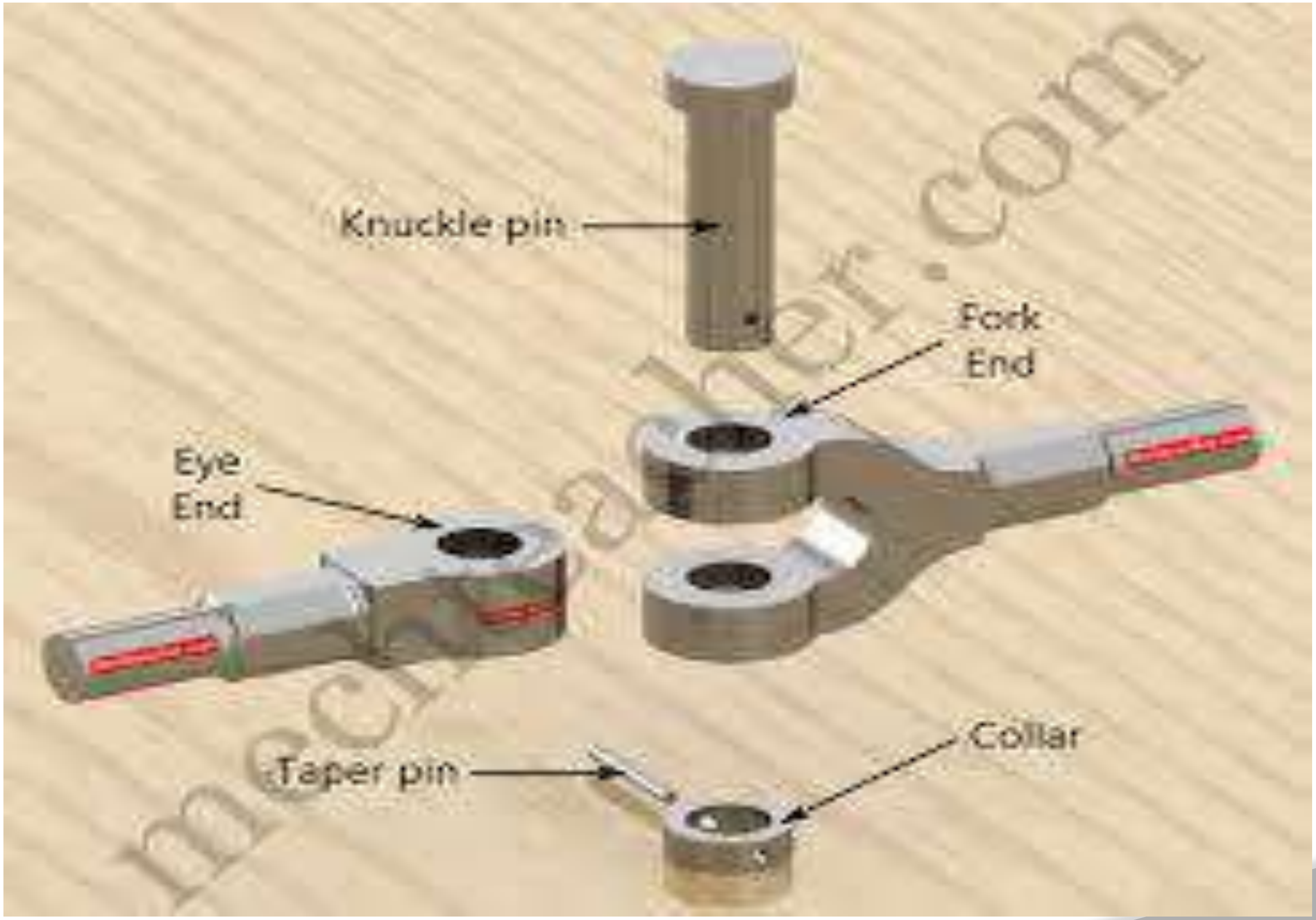
Knuckle joint

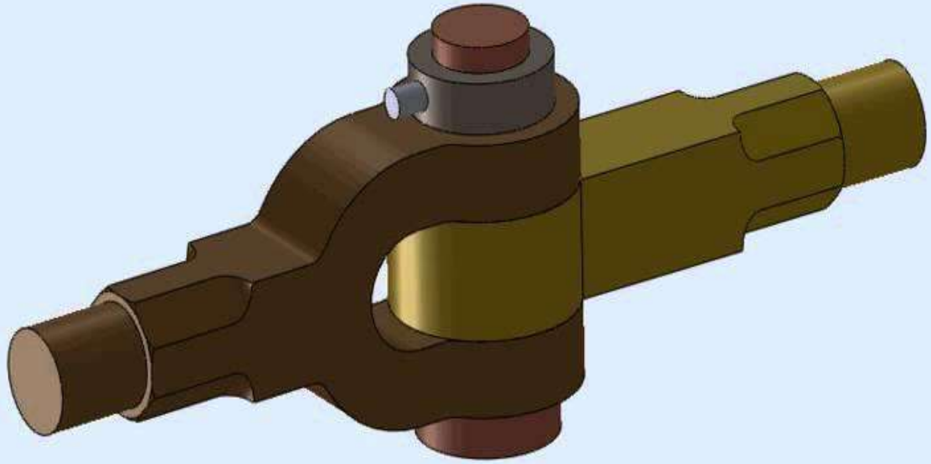


Two or more rods subjected to tensile and compressive forces are fastened together

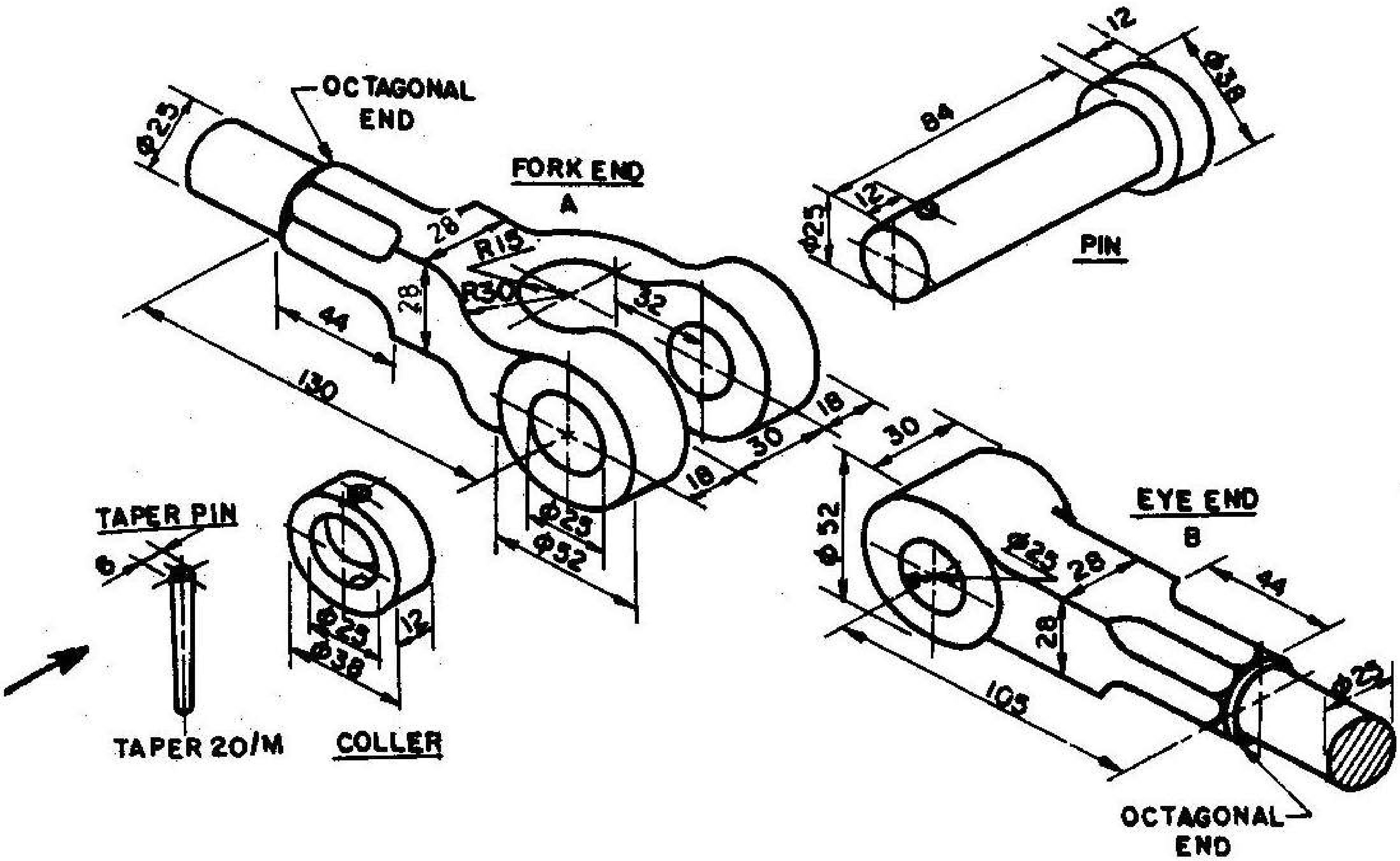
Their axes are not in alignments but meet in a point

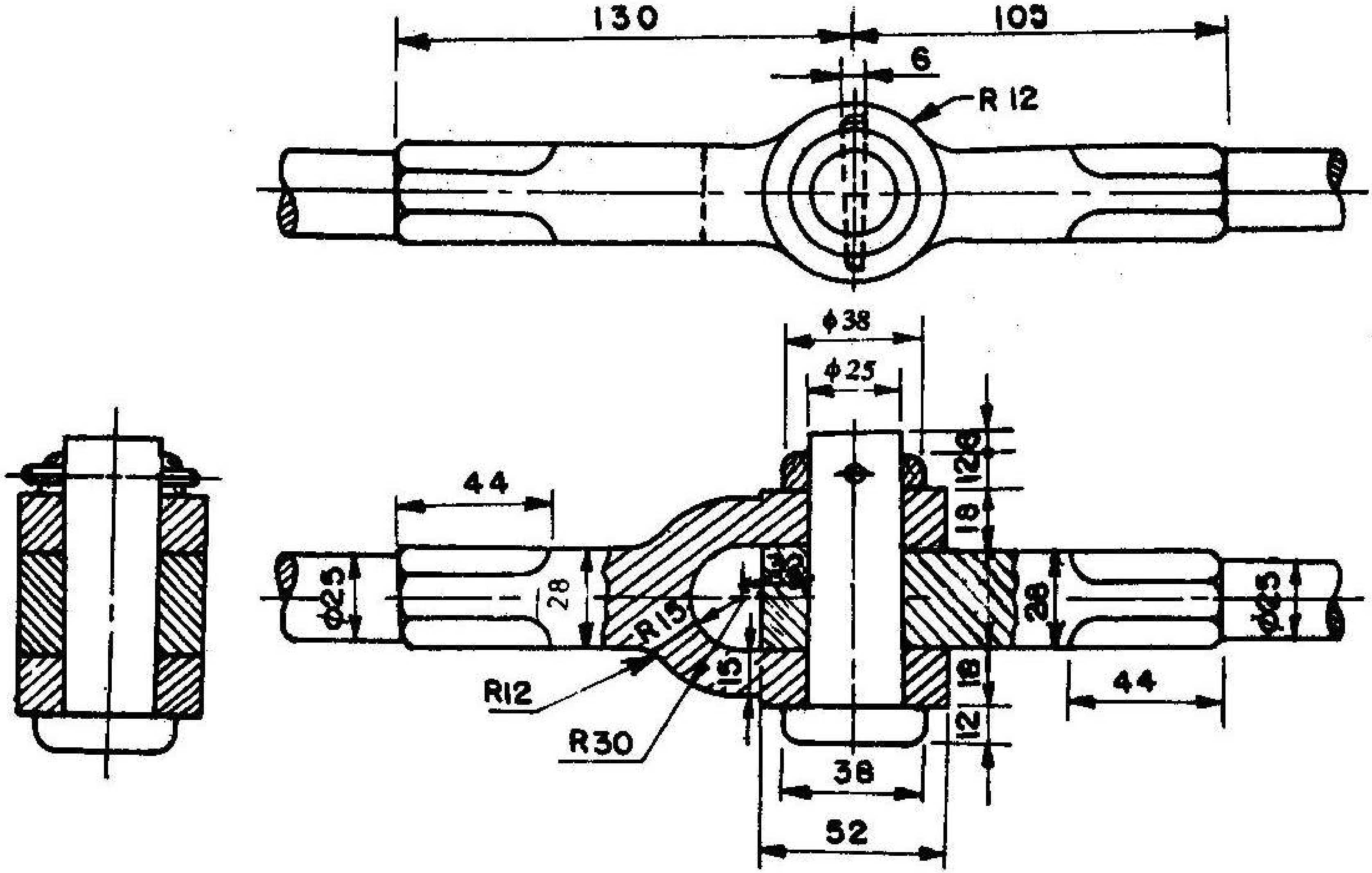
Applications: Elevator chains, valve rods, etc


Knuckle pin
Fork
End
Eye
End
Taper pin
Collar

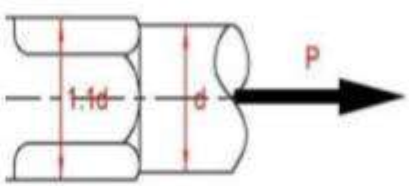


KUCKLE JOINT




130
6
R 12
ϕ38
ϕ25
44
28
ϕ25
R15
R12
R30
15
38
52
18
12
44

1. Failure of Rod In Tension



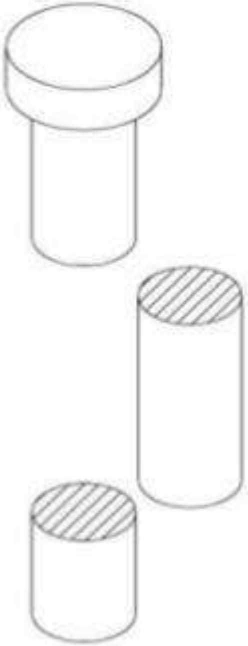
Area of Failure

$$A = \frac{\pi}{4} d^2$$

Stress Indused

$$\sigma_t = \frac{P}{\frac{\pi}{4} d^2}$$

2. Failure of Knuckle Pin Under Shear



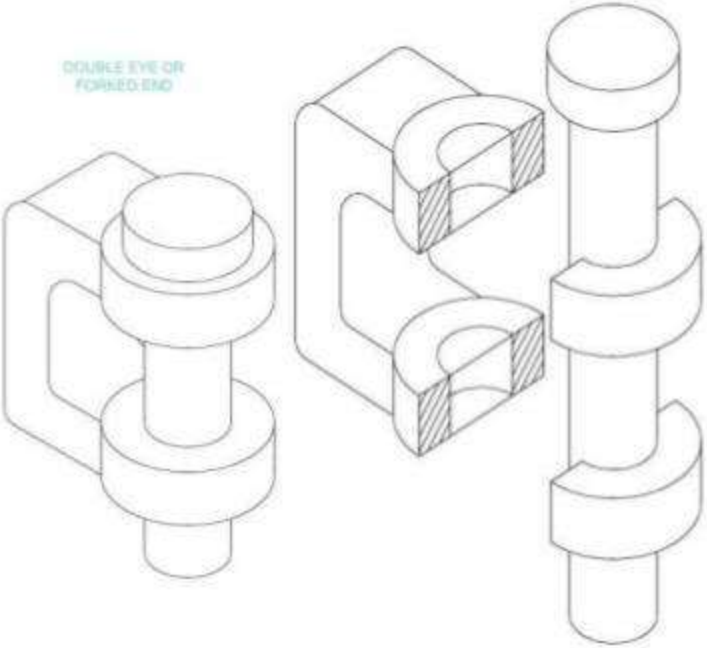
Area of Failure

$$A = 2 \times \frac{\pi}{4} d_1^2$$

Stress Indused

$$\tau = \frac{P}{2 \times \frac{\pi}{4} d_1^2}$$

3. Failure of Double Eye Under Tension



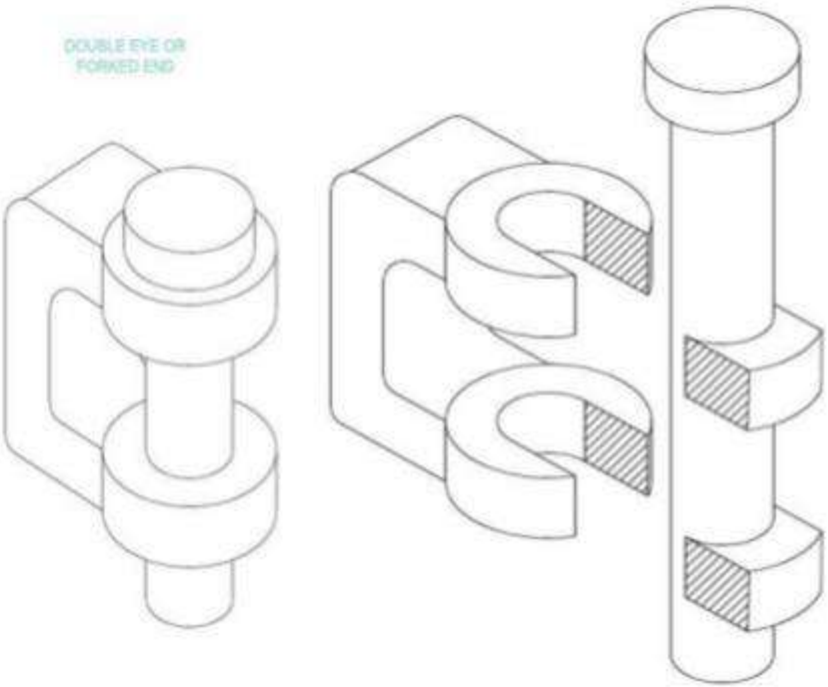
Area of Failure

$$A = 2 \times (d_2 - d_1) \times t_1$$

Stress Indused

$$\sigma_t = \frac{P}{2 \times (d_2 - d_1) \times t_1}$$

4. Failure of Double Eye Under Shear



Area of Failure

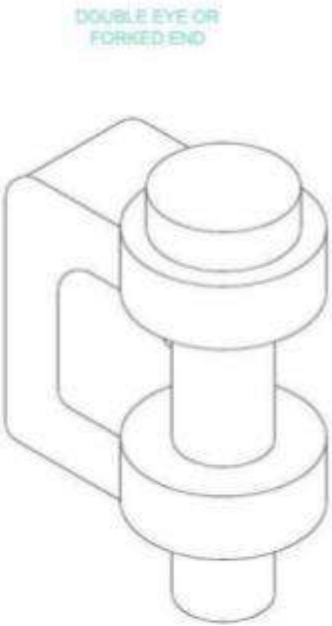
$$A = 4 \times \left(\frac{d_2 - d_1}{2}\right) \times t_1$$

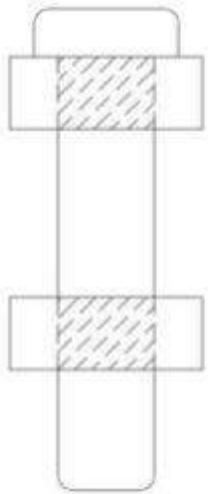
$$A = 2 \times (d_2 - d_1) \times t_1$$

Stress Indused

$$\tau = \frac{P}{2 \times (d_2 - d_1) \times t_1}$$

5. Failure of Double Eye Under Crushing





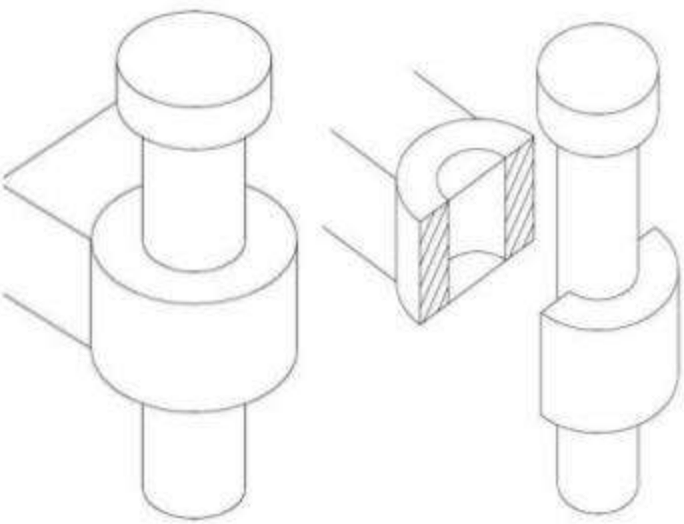
Area of Failure

$$A = 2 \times d_2 \times t_1$$

Stress Indused

$$\sigma_c = \frac{P}{2 \times d_2 \times t_1}$$

6. Failure of Single Eye Under Tension



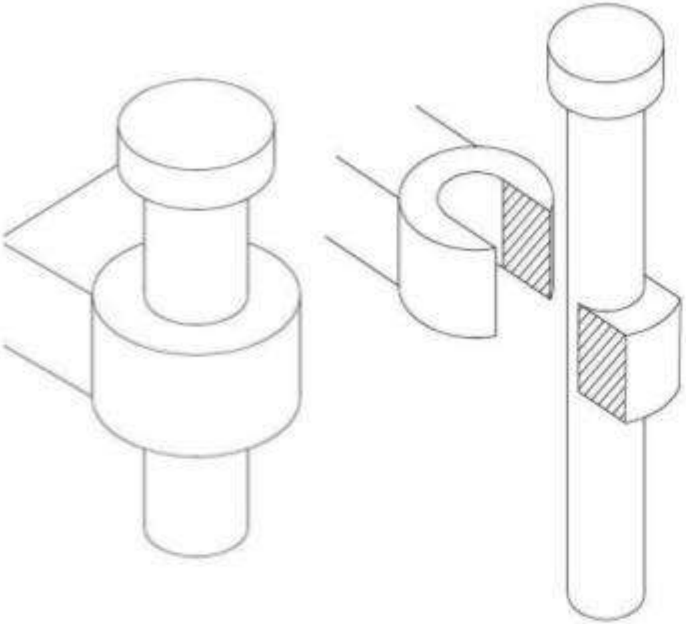
Area of Failure

$$A = (d_2 - d_1) \times t$$

Stress Indused

$$\sigma_t = \frac{P}{(d_2 - d_1) \times t}$$

7. Failure of Single Eye Under Shear



Area of Failure

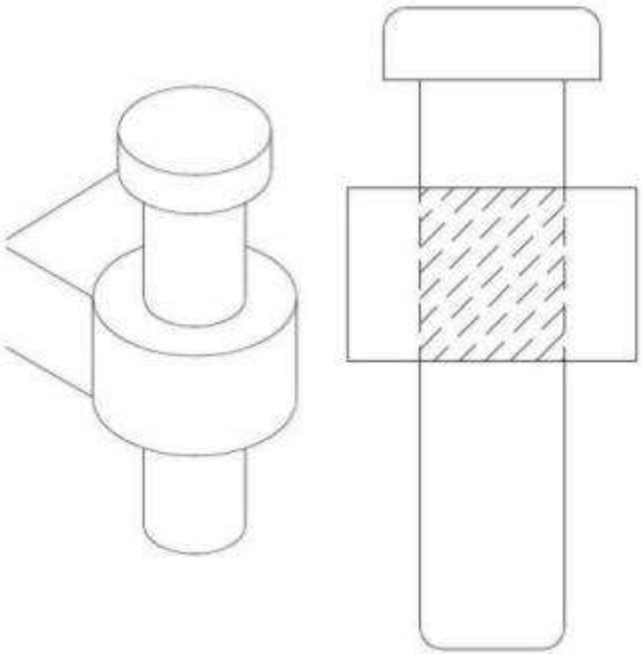
$$A = 2 \times \left(\frac{d_2 - d_1}{2}\right) \times t$$

$$A = (d_2 - d_1) \times t$$

Stress Indused

$$\tau = \frac{P}{(d_2 - d_1) \times t}$$

8. Failure of Single Eye Under Crushing



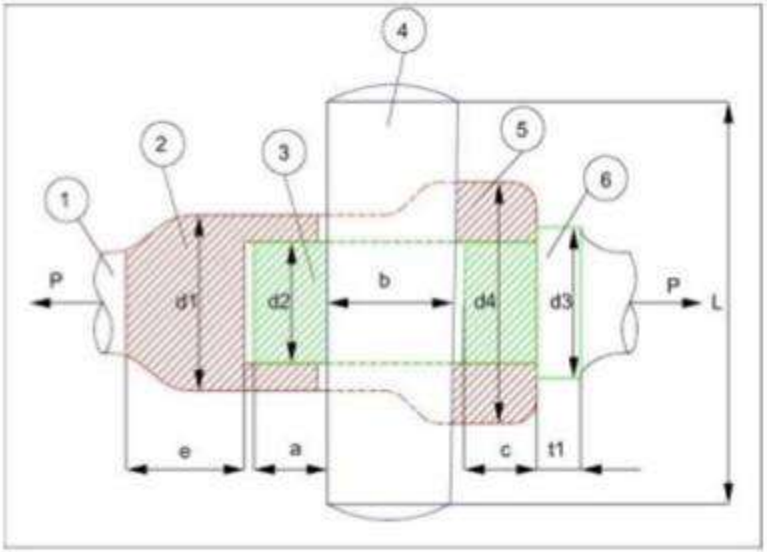
Area of Failure

$$A = d_1 \times t$$

Stress Indused

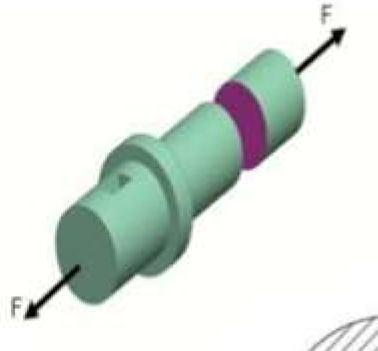
$$\sigma_c = \frac{P}{d_1 \times t}$$

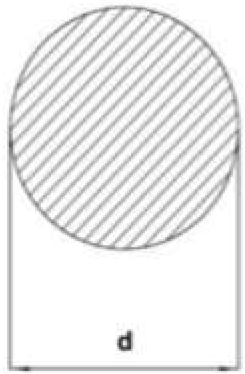
Design of Socket And Spigot Cotter Joint



Part Number	Name of Part
1	Shaft or Rod
2	Socket
3	Spigot
4	Cotter
5	Socket Collar
6	Spigot Collar

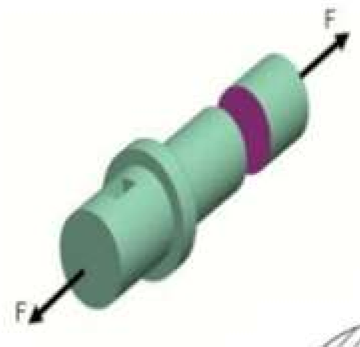
1. Failure of Shaft or Rod Under Tensile Stress

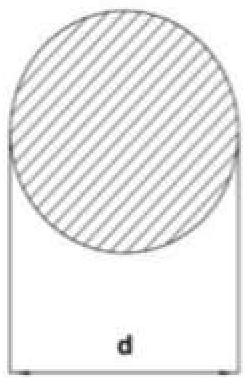




$$\sigma_t = \frac{P}{\frac{\pi}{4} d^2}$$

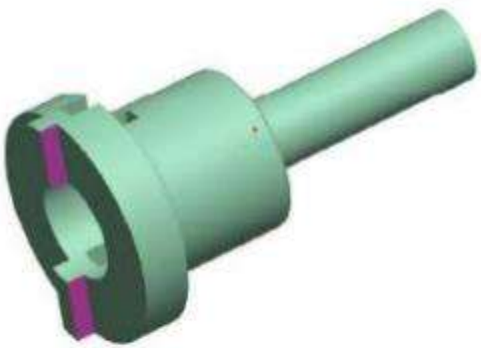
1. Failure of Shaft or Rod Under Tensile Stress





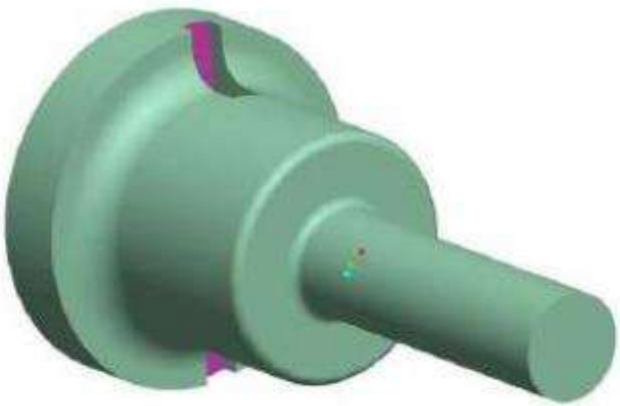
$$\sigma_t = \frac{P}{\frac{\pi}{4} d^2}$$

3. Failure of Socket Collar Under Shear Stress



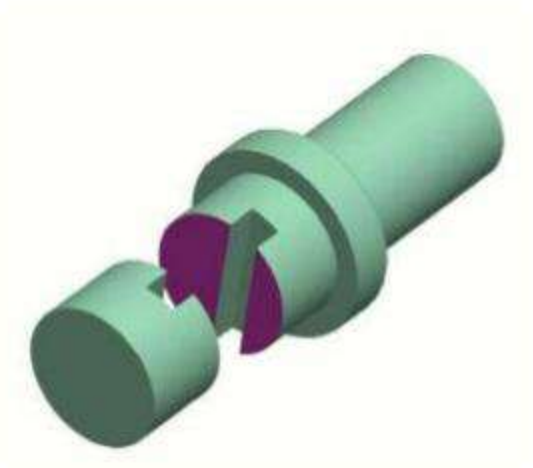
$$\tau = \frac{P}{2(d_4 - d_2) \times c}$$

4. Failure of Socket Coller Under Crushing



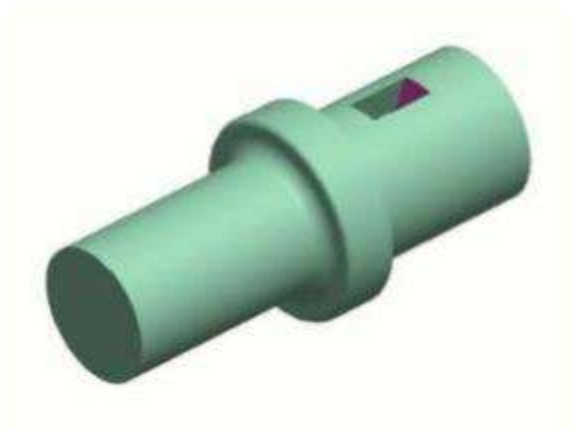
$$\sigma_c = \frac{P}{(d_4 - d_2) \times t}$$

5. Failure of Spigot Under Tension



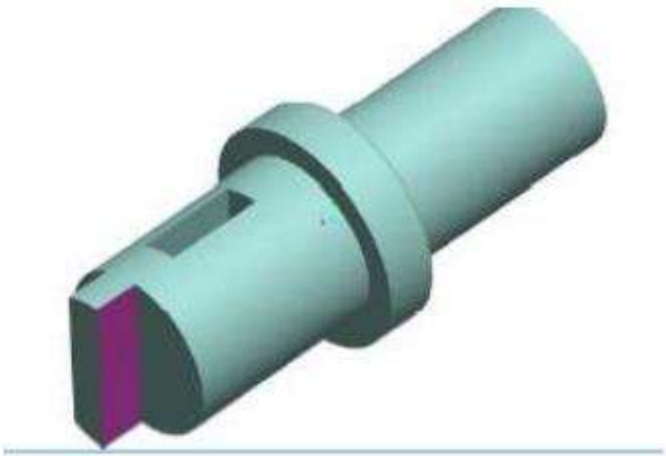
$$\sigma_t = \frac{P}{\left(\frac{\pi}{4} d_2^2\right) - (d_2 \times t)}$$

6. Failure of Spigot Under Crushing



$$\sigma_c = \frac{P}{(d_2) \times t}$$

7. Failure of Spigot Under Shear

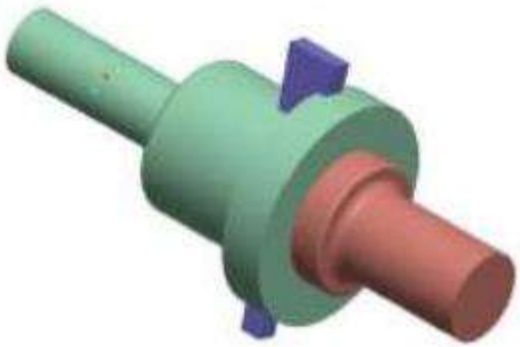


$$\tau = \frac{P}{2(d_2) \times a}$$

8. Failure of Cotter Under Shear

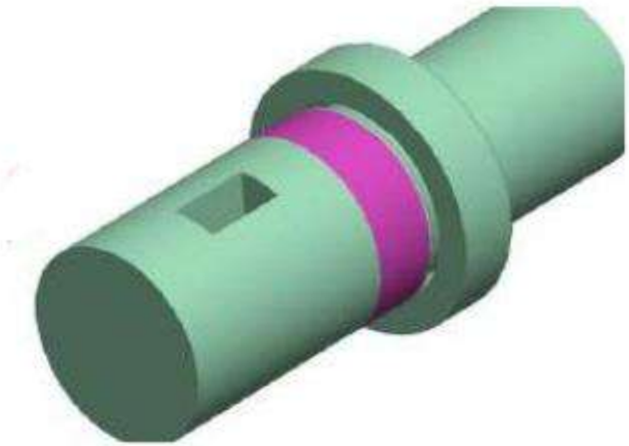
$$\tau = \frac{P}{2(b) \times t}$$

9. Failure of Spigot Collar Under Crushing



$$\sigma_c = \frac{P}{(d_3 - d_2) \times t_1}$$

10. Failure of Spigot Collar Under Shear



$$\tau = \frac{P}{\pi(d_3) \times t_1}$$

DESIGN OF SHAFTS ,SHAFT COUPLINGS

UNIT-IV : DESIGN OF SHAFTS ,SHAFT COUPLINGS

DESIGN OF SHAFTS

A shaft is a rotating member usually of circular cross-section (solid or hollow), which transmits power and rotational motion.

Machine elements such as gears, pulleys (sheaves), flywheels, clutches, and sprockets are mounted on the shaft and are used to transmit power from the driving device (motor or engine) through a machine

Press fit, keys, dowel, pins and splines are used to attach these machine elements on the shaft.

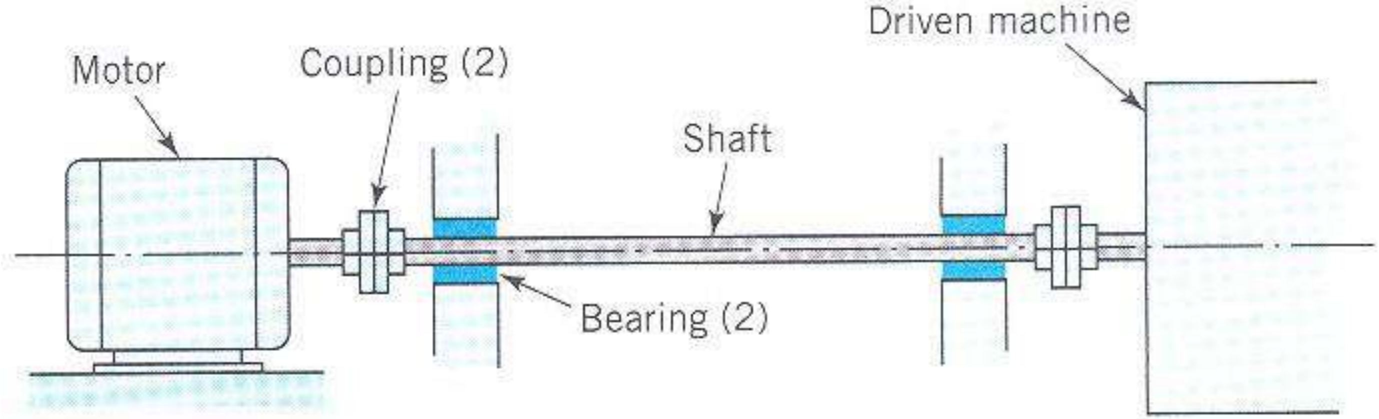
The shaft rotates on rolling contact bearings or bush bearings.

Various types of retaining rings, thrust bearings, grooves and steps in the shaft are used to take up axial loads and locate the rotating elements.

Couplings are used to transmit power from drive shaft (e.g., motor) to the driven shaft (e.g. gearbox, wheels).

- Shaft?
- Shaft Design
- ASME Shaft Equations
- Design of Shaft for Torsional Rigidity
- Standard Sizes of Shafts
- Bending and Torsional Moments

The connecting shaft is loaded primarily in torsion.



(*a*) Connecting shaft

Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that ***a shaft is used for the transmission of torque and bending moment***. The various members are mounted on the shaft by means of keys or splines.

1.The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2.An *axle,* though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3.A *spindle* is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (*e.g. lathe spindles).*

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

Manufacturing of Shafts

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.

The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

The following two types of shafts are important from the subject point of view

1.Transmission shafts. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2.Machine shafts. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ;

110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.

The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

The following stresses are induced in the shafts :

1.Shear stresses due to the transmission of torque (*i.e. due to torsional load).*

2.Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3.Stresses due to combined torsional and bending loads.

Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(a)112 MPa for shafts without allowance for keyways.

(b)84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σt) may be taken as 60 percent of the elastic limit in tension (σel), *but not more than 36 per cent of the* ultimate tensile strength (σu). *In other words, the permissible tensile stress,*

$$\sigma t = 0.6\ \sigma el \text{ or } 0.36\ \sigma u, \text{ whichever is less.}$$

- The maximum permissible shear stress may be taken as

(a)56 MPa for shafts without allowance for key ways.

(b)42 MPa for shafts with allowance for keyways.

- For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σel) *but not more than 18 percent of the ultimate* tensile strength (σu). *In other words, the permissible shear stress,*

  - $\tau = 0.3\ \sigma el$ *or* $0.18\ \sigma u$*, whichever is less.*

The shafts may be designed on the basis of
1. Strength, and   2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(a) Shafts subjected to twisting moment or torque only,
(b) Shafts subjected to bending moment only,
(c) Shafts subjected to combined twisting and bending moments, and
(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{...}(i)$$

where

T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

= d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i) may now be written as*

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this equation, we may determine the diameter of round solid shaft (*d).*

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where *do and di* = *Outside and inside diameter of the shaft, and r* = *do / 2.*

Substituting these values in equation (*i), we have*

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad ...(iii)$$

Let *k* = *Ratio of inside diameter and outside diameter of the shaft*

= *di / do*

Now the equation ***(iii) may be written as***

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \ldots(iv)$$

From the equations ***(iii) or (iv), the outside and inside diameter of a hollow shaft may be***
determined.
It may be noted that
1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous thanwould be possible for a solid shaft.
When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

2. The twisting moment (*T) may be obtained by using the following relation :*
We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (*T) is given by*

$$T = (T1 - T2)\, R$$

where
T1 and T2 = Tensions in the tight side and slack side of the belt respectively, and R = Radius of the pulley.

Example 1.

A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Solution.

Given : N = 200 r.p.m. ; P = 20 kW = 20 × 103 W; τ = 42 MPa = 42 N/mm2

Let d = Diameter of the shaft.

We k

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm},$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.25 = 115\,733 \text{ or } d = 48.7 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

Example 2.

A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Solution.

Given : *P = 1 MW = 1 × 106 W ; N = 240 r.p.m. ; Tmax = 1.2 Tmean ;*

$\tau = 60\ MPa = 60\ N/mm^2$

Let *d = Diameter of the shaft.*

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784 \text{ N-m} = 39\,784 \times 10^3 \text{ N-mm}$$

Maximum torque transmitted,

Tmax = 1.2 Tmean = 1.2 × 39 784 × 10 ^{3}= 47 741 × 10^3 N-mm

We know that maximum torque transmitted (Tmax),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

$$d = 159.4 \text{ say } 160 \text{ mm Ans.}$$

Example 3.

Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : P = 20 kW = 20×10^3 W ; N = 200 r.p.m. ; τu = 360 MPa = 360 N/mm^2 ; F.S. = 8 ; k = di / do = 0.5

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = *Diameter of the solid shaft.*

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$$d^3 = 955 \times 10^3 / 8.84 = 108\,032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

Diameter of hollow shaft

Let *di = Inside diameter, and*

do = Outside diameter.

We know that the torque transmitted by the hollow shaft (*T),*

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\, (d_o)^3\, [1 - (0.5)^4] = 8.3\, (d_o)^3$$

$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

$d_i = 0.5\, d_o = 0.5 \times 50 = 25$ mm **Ans.**

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \ldots(i)$$

where *M = Bending moment,*
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,
σb = Bending stress, and
y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4\,(1-k^4) \qquad \text{...(where } k = d_i/d_o)$$

$$y = d_o/2$$

Again substituting these values in equation ***(i), we have***

$$\frac{M}{\frac{\pi}{64}(d_o)^4\,(1-k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \qquad \text{or} \qquad M = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1-k^4)$$

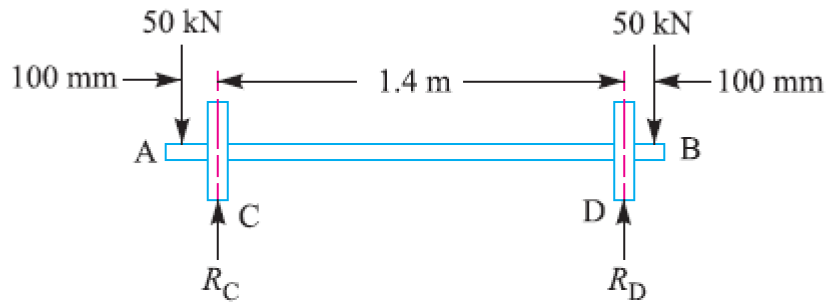
From this equation, the outside diameter of the shaft (do) may be obtained.

Example 4.

A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Solution.

Given : $W = 50\ kN = 50 \times 10^3\ N$; $L = 100\ mm$; $x = 1.4\ m$; $\sigma b = 100\ MPa = 100\ N/mm^2$



The axle with wheels is shown in Fig. 1.

A little consideration will show that the maximum bending moment acts on the wheels at C and D. Therefore maximum bending moment,

*$M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

The maximum B.M. may be obtained as follows :

$RC = RD = 50\ kN = 50 \times 10^3\ N$

B.M. at A, $MA = 0$

B.M. at C, $MC = 50 \times 10^3 \times 100 = 5 \times 10^6\ N\text{-}mm$

B.M. at D, $MD = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6\ N\text{-}mm$

B.M. at B, $MB = 0$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1.Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2.Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let

τ = Shear stress induced due to twisting moment, and

σb = *Bending stress (tensile or compressive) induced due to bending* moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16}{\pi\,d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...}(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad \text{...}(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

$$= \frac{32}{\pi\,d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad \text{...}(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress $[\sigma_{b(max)}]$ equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \quad \ldots(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4)$$

and

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b \, (d_o)^3 \, (1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 5.

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\,000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \quad \text{or} \quad d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\ d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

Example 6.

A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted?
The pressure angle of the gear may be taken as 20°.

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm^2 ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.

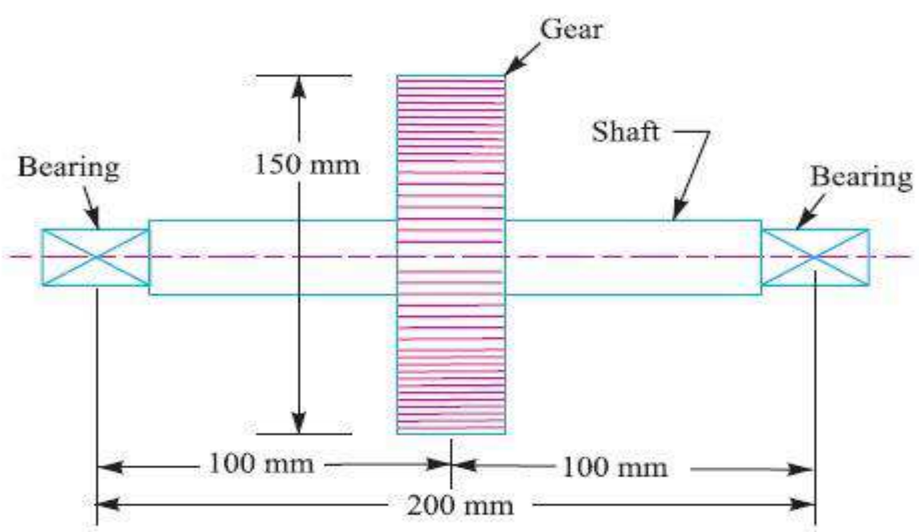


Fig. 14.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$

$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84 \ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

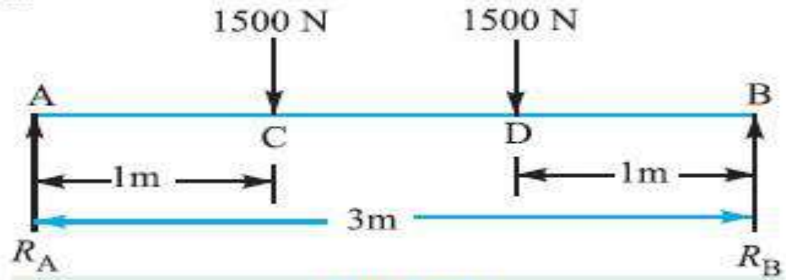


Fig. 14.3

∴ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$

$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

∴ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm **Ans.**

Example 8. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

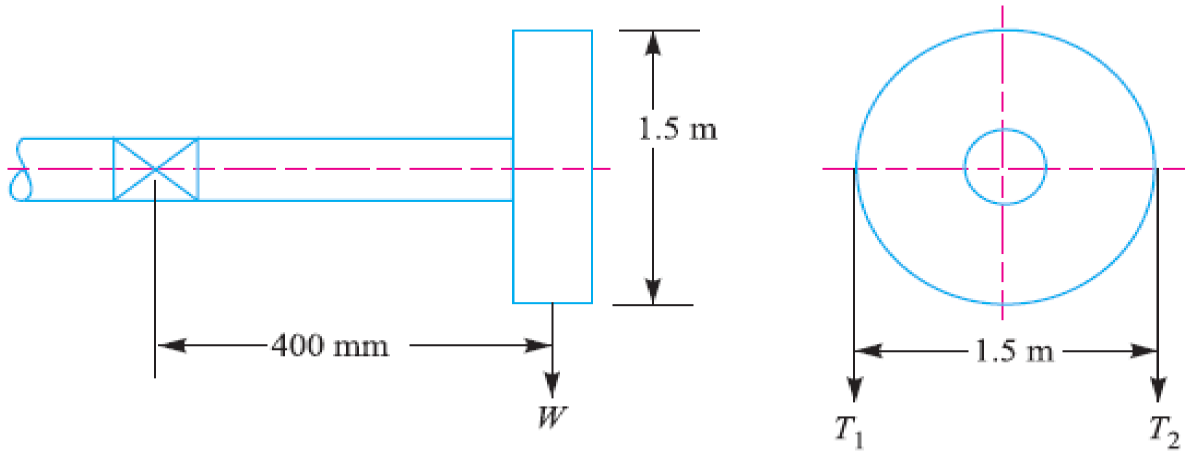
Solution .

Given : $D = 1.5$ m or $R = 0.75$ m; $T1 = 5.4$ kN $= 5400$ N ; $T2 = 1.8$ kN $= 1800$ N ;
$L = 400$ mm ; $\tau = 42$ MPa $= 42$ N/mm2

A line shaft with a pulley is shown in Fig 14.4.

We know that torque transmitted by the shaft,

$$T = (T1 – T2) R = (5400 – 1800)\ 0.75 = 2700 \text{ N-m}$$
$$= 2700 \times 103 \text{ N-mm}$$



1.5 m
400 mm
W
1.5 m
T_1
T_2

Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

$\therefore$ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$

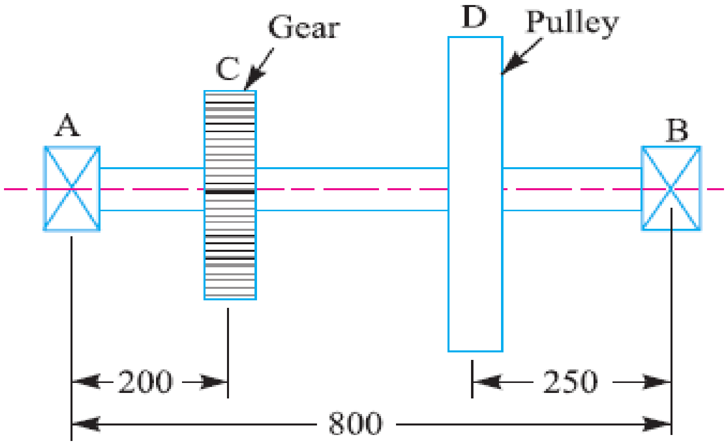
$$= 3950 \times 10^3 \text{ N-mm}$$

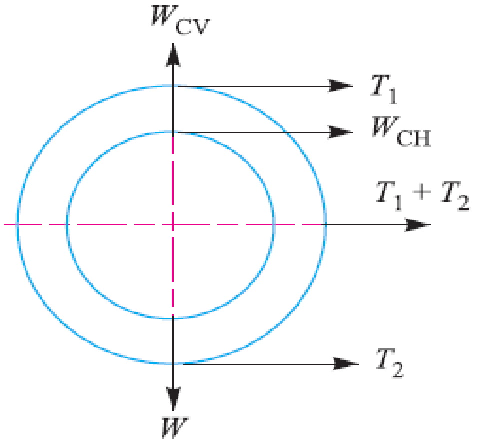
We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 3950 \times 10^3/8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

Example 14.9. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and* $\mu = 0.24$. *Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.*





Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm^2 ; $\tau = 42$ MPa $= 42$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.5 (*a*).

Let T_1 = Tension in the tight side of the belt on pulley $C = 2250$ N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \qquad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

$\therefore$ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley *C*,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley *D*, and

T_4 = Tension in the slack side of the belt on pulley *D*.

Since the torque on both the pulleys (*i.e.* *C* and *D*) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \ldots(i)$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

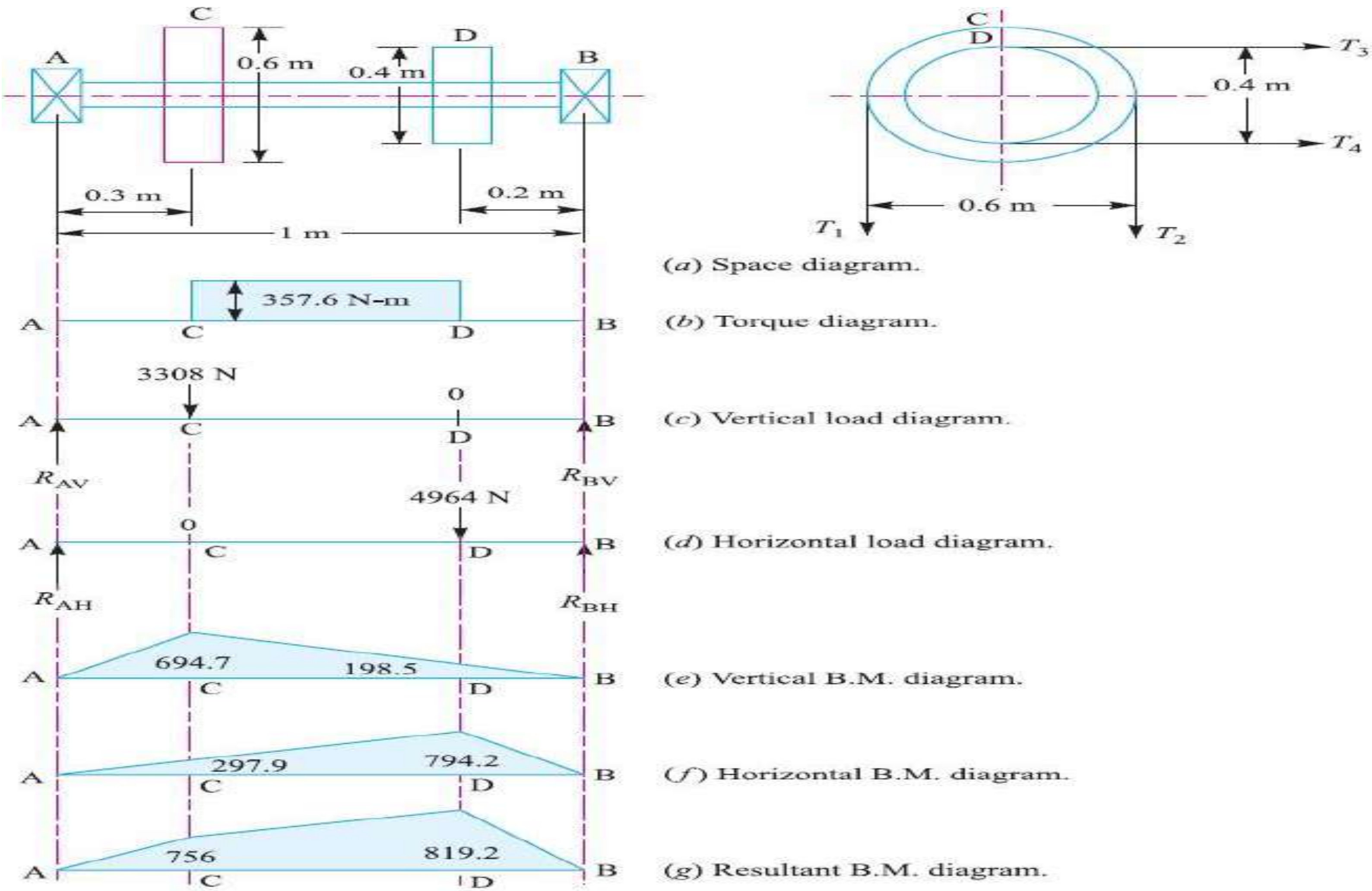
∴ Horizontal load acting on the shaft at *D*,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.



A
C
D
B
0.6 m
0.4 m
0.3 m
0.2 m
1 m
C
D
T3
0.4 m
T4
T1
0.6 m
T2
(a) Space diagram.
357.6 N-m
(b) Torque diagram.
3308 N
0
RAV
RBV
(c) Vertical load diagram.
4964 N
0
RAH
RBH
(d) Horizontal load diagram.
694.7
198.5
(e) Vertical B.M. diagram.
297.9
794.2
(f) Horizontal B.M. diagram.
756
819.2
(g) Resultant B.M. diagram.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about *A*,

$$R_{BV} \times 1 = 3308 \times 0.3 \quad \text{or} \quad R_{BV} = 992.4 \text{ N}$$

and

$$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7$ N-m

B.M. at *D*, $M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5$ N-m

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about *A*,

$$R_{BH} \times 1 = 4964 \times 0.8 \quad \text{or} \quad R_{BH} = 3971 \text{ N}$$

and

$$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9$ N-m

B.M. at *D*, $M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2$ N-m

The bending moment diagram for horizontal loading is shown in Fig. 14.5 (*f*).

Resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (g).

We see that bending moment is maximum at D.

∴ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$

$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\ d^3$$

$$\therefore \quad d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

d = 51.7 say 55 mm **Ans.**

Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed *twisting moment* (*T*) *and bending moment (M)*. Thus for a shaftsubjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where *Km = Combined shock and fatigue factor for bending, and*

Kt = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for *Km and Kt.*

Table 14.2. Recommended values for K_m and K_t

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 14.12. *A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?*

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $W = 900$ N ; $L = 2.5$ m ; $\tau = 42$ MPa $= 42$ N/mm² ; $\sigma_b = 56$ MPa $= 56$ N/mm²

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$
$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3 \text{ or } d = 51.2 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3 \text{ or } d = 53.4 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 53.4 \text{ say } 55 \text{ mm}$$ **Ans.**

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

From Table 14.2, for rotating shafts with gradually applied loads,

$$K_m = 1.5 \text{ and } K_t = 1$$

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3 \text{ or } d = 53.6 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \tfrac{1}{2}\left[1.5 \times 562.5 \times 10^3 + 1274 \times 10^3\right] = 1059 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 57.7 say 60 mm **Ans.**

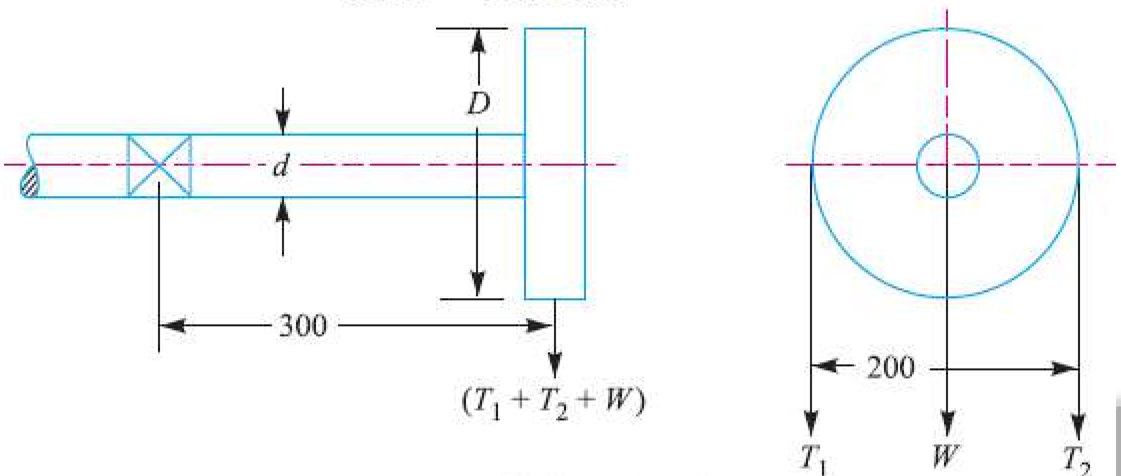
Example 14.13. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 MPa.*

Solution. Given : W = 200 N ; L = 300 mm ; D = 200 mm or R = 100 mm ; P = 1 kW = 1000 W ; N = 120 r.p.m. ; θ = 180° = π rad ; μ = 0.3 ; K_m = 1.5 ; K_t = 2 ; τ = 35 MPa = 35 N/mm^2

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

$$79.6 \times 10^3 = (T_1 - T_2)\,R = (T_1 - T_2)\,100$$

$$\therefore \quad T_1 - T_2 = 79.6 \times 10^3 / 100 = 796 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \quad \text{...}(ii)$$

...(Taking antilog of 0.4098)

From equations (*i*) and (*ii*), we get,

$$T_1 = 1303 \text{ N, and } T_2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

∴ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\,d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm}$$ **Ans.**

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M.y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi\,d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4\,F}{\pi\,d^2} \quad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4\,F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \quad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \quad \text{... } (\because k = d_i/d_o)$$

$\therefore$ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32\,M}{\pi\,d^3} + \frac{4\,F}{\pi\,d^2} = \frac{32}{\pi\,d^3}\left(M + \frac{F \times d}{8}\right) \quad \text{...(i)}$$

$$= \frac{32\,M_1}{\pi\,d^3} \quad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1-k^4)} + \frac{4\,F}{\pi\,(d_o)^2\,(1-k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1-k^4)}\left[M + \frac{F\,d_o\,(1+k^2)}{8}\right] = \frac{32\,M_1}{\pi(d_o)^3\,(1-k^4)}$$

$$\ldots\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1+k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

$\therefore$ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 (1 - k^2)}$$ ...(For hollow shaft)

The value of column factor (α) for compressive loads* may be obtained from the following relation :

Column factor, $$\alpha = \frac{1}{1 - 0.0044 (L/K)}$$

This expression is used when the slenderness ratio (L/K) is less than 115. When the slenderness ratio (L/K) is more than 115, then the value of column factor may be obtained from the following relation :

**Column factor, $$\alpha = \frac{\sigma_y (L/K)^2}{C \pi^2 E}$$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C = 1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha\, F\, d_o\,(1+k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1-k^4)$$

and

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o\,(1+k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o\,(1+k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1-k^4)$$

It may be noted that for a solid shaft, $k = 0$ and $d_0 = d$. When the shaft carries no axial load, then $F = 0$ and when the shaft carries axial tensile load, then $\alpha = 1$.

Example 14.18. *A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.*

Solution. Given : $T = 1.5$ kN-m $= 1.5 \times 10^3$ N-m ; $M = 3$ kN-m $= 3 \times 10^3$ N-m ; $F = 10$ kN $= 10 \times 10^3$ N ; $k = d_i / d_o = 0.5$; $d_o = 80$ mm $= 0.08$ m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ ; and } K_t = 1.0$$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1 + 0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

... ($\because \alpha = 1$, for axial tensile loading)

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) = \frac{\pi}{16} \times \tau (80)^3 (1 - 0.5^4) = 94\,260\, \tau$$

$\therefore$ $\tau = 4862 \times 10^3 / 94\,260 = 51.6$ N/mm^2 $= 51.6$ MPa **Ans.**

Example 14.19. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. *The maximum shear stress developed in the shaft, and*
2. *The angular twist between the bearings.*

Solution. Given : $d_o = 0.5$ m ; $d_i = 0.3$ m ; $P = 5600$ kW $= 5600 \times 10^3$ W ; $L = 6$ m ; $N = 150$ r.p.m. ; $F = 500$ kN $= 500 \times 10^3$ N ; $W = 70$ kN $= 70 \times 10^3$ N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\ 460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\ 500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

∴ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \quad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $$k = d_i / d_o = 0.3 / 0.5 = 0.6$$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\,500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\,(1 + 0.6^2)}{8}\right]^2 + (1 \times 356\,460)^2}$$

$$= \sqrt{(78\,750 + 51\,850)^2 + (356\,460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\,[1 - (0.6)^4] = 0.02\,\tau$$

$\therefore$ $$\tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19 \text{ MPa}$$ Ans.

2. *Angular twist between the bearings*

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m²)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \text{ Ans.}$$

Design of Shafts on the basis of Rigidity

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

where

θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity.* It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the

the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW $= 4000$ W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m $= 1000$ mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m² $= 84 \times 10^3$ N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$\therefore$ $d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\,740 / 8420 = 5.67$ N/mm² $= 5.67$ MPa **Ans.**

Example 14.22. *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \text{ **Ans.**}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(iv)$$

Dividing equation *(iii)* by equation *(iv)*, we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \qquad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \text{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \qquad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \qquad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

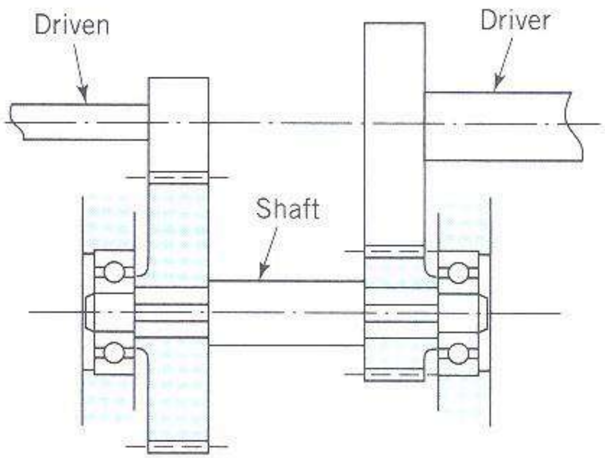
$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \qquad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \text{ Ans.}$$

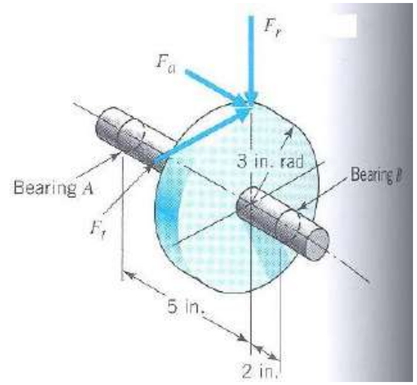
EXERCISES

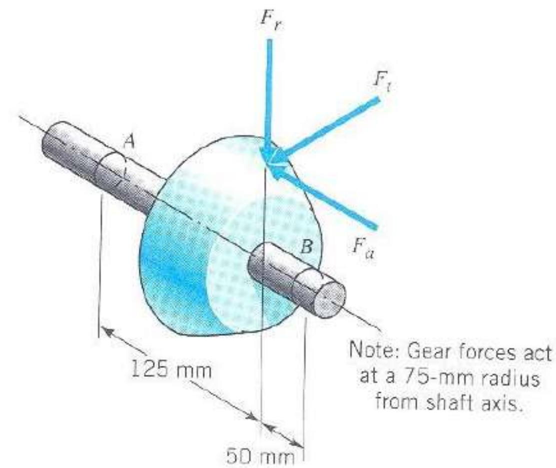
1. A shaft running at 400 r.p.m. transmits 10 kW. Assuming allowable shear stress in shaft as 40 MPa, find the diameter of the shaft. [Ans. 35 mm]
2. A hollow steel shaft transmits 600 kW at 500 r.p.m. The maximum shear stress is 62.4 MPa. Find the outside and inside diameter of the shaft, if the outer diameter is twice of inside diameter, assuming that the maximum torque is 20% greater than the mean torque. [Ans. 100 mm ; 50 mm]
3. A hollow shaft for a rotary compressor is to be designed to transmit a maximum torque of 4750 N-m. The shear stress in the shaft is limited to 50 MPa. Determine the inside and outside diameters of the shaft, if the ratio of the inside to the outside diameter is 0.4. [Ans. 35 mm ; 90 mm]
4. A motor car shaft consists of a steel tube 30 mm internal diameter and 4 mm thick. The engine develops 10 kW at 2000 r.p.m. Find the maximum shear stress in the tube when the power is transmitted through a 4 : 1 gearing. [Ans. 30 MPa]
5. A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of a bending moment of 10 kN-m and a torsional moment of 30 kN-m. Determine the diameter of the shaft using two different theories of failure and assuming a factor of safety of 2. [Ans. 100 mm]

Combined bending and torsion loads on shaft: Shaft carrying gears.



Gear countershaft





From power and rpm find the torque (T), which gives rise to shear stress.

From Torque (T) and diameter (d), find $F_t = 2T/d$. From F_t and pressure angles of gears you can find F_r and F_a.

F_r and F_t are orthogonal to each other and are both transverse forces to the shaft axis, which will give rise to normal bending stress in the shaft. When shaft rotates, bending stress changes from tensile to compressive and then compressive to tensile, ie, completely reversing state of stress.

F_a will give rise to normal axial stress in the shaft.

Loads on shaft due to pulleys

Pulley torque (T) = Difference in belt tensions in the tight (t_1) and slack (t_2) sides of a pulley times the radius (r), ie

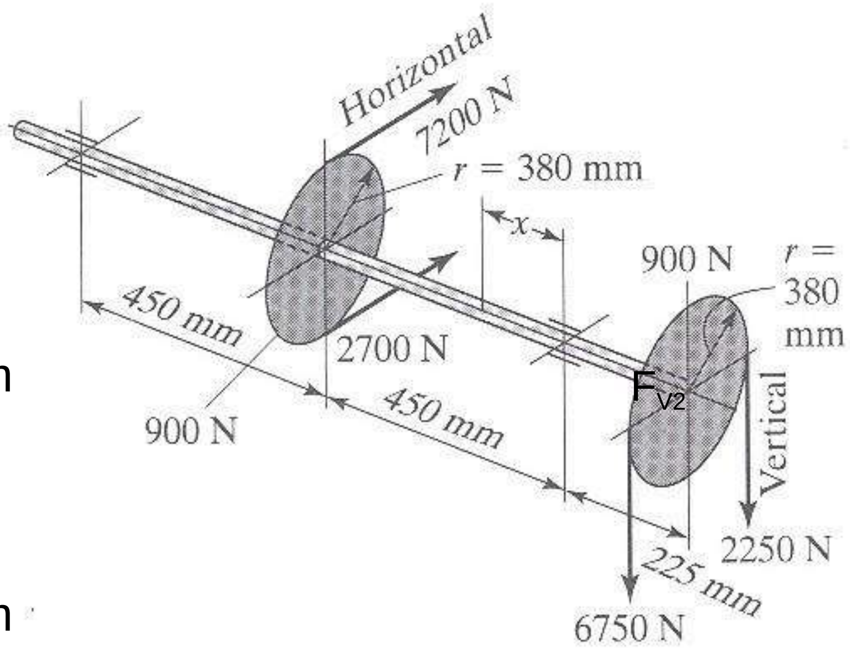
$T = (t_1 - t_2) x r$

Left pulley torque

$T_1 = (7200-2700)x380=1,710,000$ N-mm

Right pulley has exactly equal and opposite torque:

$T_2 = (6750-2250)x380=1,710,000$ N-mm



Bending forces in vertical (F_v) and horizontal (F_H) directions:

At the left pulley: F_{V1}=900N; F_{H1}=7200+2700 = 9900N

At the right pulley: F_{V2}=900+6750+2250=9900N; F_{H2}=0

Power, Toque & Speed

For linear motion:

Power = F.v (force x velocity)

For rotational motion

Power P = Torque x angular velocity

= T (in-lb).ω (rad/sec) in-lb/sec

= T.(2π n/60) in-lb/sec [n=rpm]

= T.(2π n/(60*12*550)) HP [HP=550 ft-lb/sec]

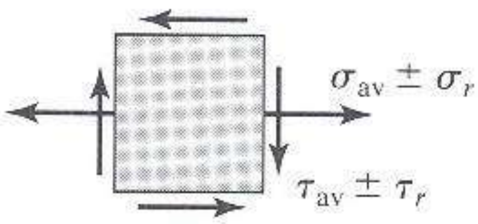
= T.n/63,025 HP

or, T= 63,025HP/n (in-lb), where n = rpm

Similarly, T= 9,550,000kW/n (N-mm), where n = rpm

Design of rotating shafts and fatigue consideration

The most frequently encountered stress situation for a rotating shaft is to have completely reversed bending and steady torsional stress. In other situations, a shaft may hav a reversed torsional stress along with reversed bending stress.



The most generalized situation the rotating shaft may have both steady and cyclic components of bending stress (σ_{av},σ_r) and torsional stress (τ_{av},τ_r).

From Soderberg's fatigue criterion, the equivalent static bending and torsional stresses are:

$$\sigma_{\text{equivalent}} = \sigma_{av} + \sigma_r K_{fb}\left(\frac{S_{yp}}{S_e}\right)$$

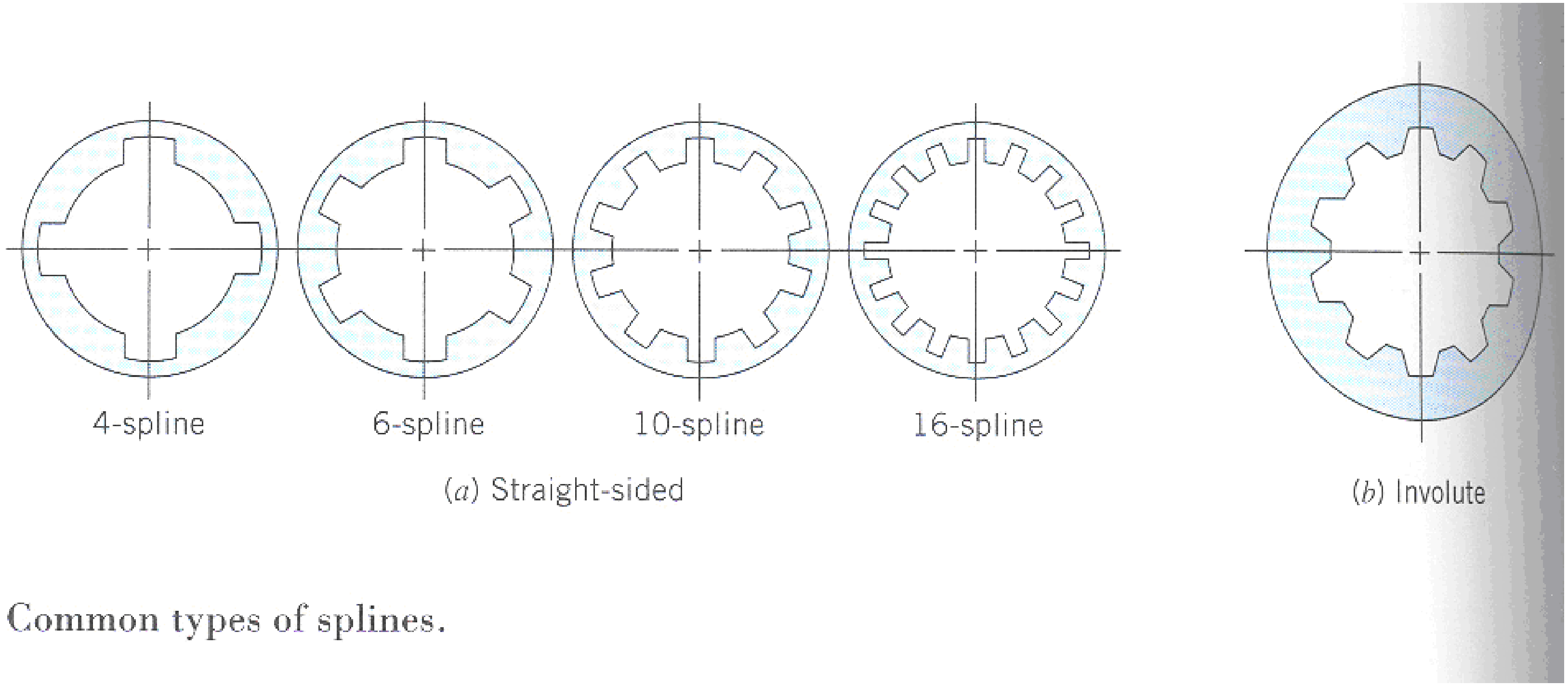
$$\tau_{\text{equivalent}} = \tau_{av} + \tau_r K_{ft}\left(\frac{S_{yp}}{S_e}\right)$$

Using these equivalent static stresses in our static

de

$$\left(\sigma_{av} + \sigma_r K_{fb}\left(\frac{S_{yp}}{S_e}\right)\right)^2 + 3\left(\tau_{av} + \tau_r K_{ft}\left(\frac{S_{yp}}{S_e}\right)\right)^2 \leq \left(\frac{S_{yp}}{N_{fs}}\right)^2$$

Integrated splines in hubs and shafts allow axial motion and transmits torque



Common types of splines.

All keys, pins and splines give rise to stress concentration in the hub and shaft

- Shaft couplings are used in machinery for several purposes,
- 1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
- 2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
- 3. To reduce the transmission of shock loads from one shaft to another.
- 4. To introduce protection against overloads.
- 5 .It should have no projecting parts

- Requirements of a Good Shaft Coupling
- 1.It should be easy to connect or disconnect.
- 2. It should transmit the full power from one shaft to the other shaft without losses.
- 3.It should hold the shafts in perfect alignment.
- 4.It should reduce the transmission of shock loads from one shaft to another shaft.
- 5.If should have no projecting parts.

TYPES OF SHAFTS COUPLINGS

- 1. Rigid coupling 2. Flexible coupling

1. Rigid coupling : It is used to connect two shafts which are perfectly aligned.

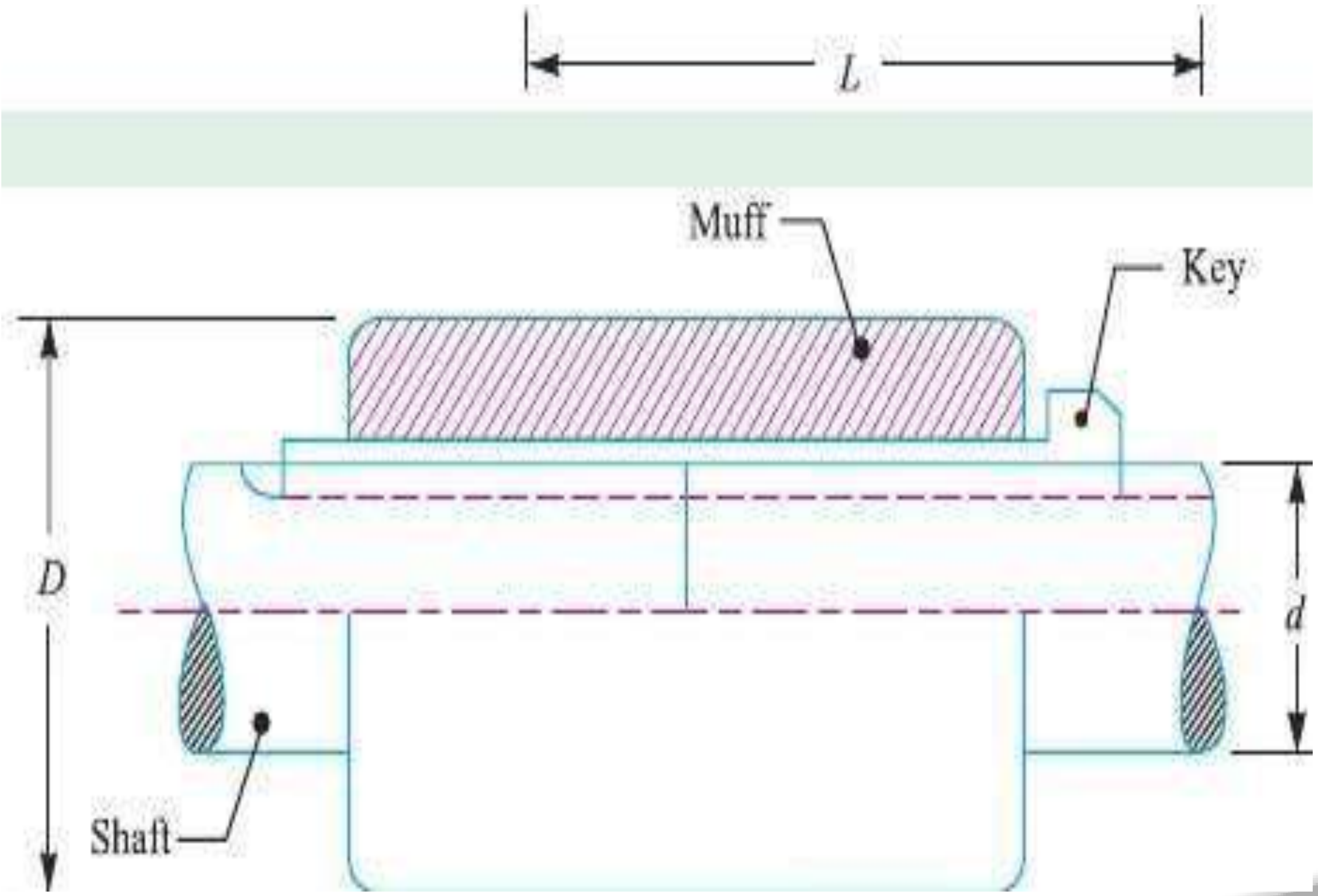
- types of rigid coupling are
- a)Sleeve or muff coupling.
- b)Clamp or split-muff or compression coupling,
- c)Flange coupling

- 2.Flexible coupling : It is used to connect two shafts having both lateral and angular misalignment.

- Types of flexible coupling are

- a)Bushed pin type coupling,
- b)Universal coupling, and
- c)Oldham coupling

SLEEVE OR MUFF-COUPLING



- It is the simplest type of rigid coupling, made of cast iron.
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft (sleeve).
- It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig.
- The power is transmitted from one shaft to the other shaft by means of a key and a sleeve.

- SHAFT - (d, T)

d = diameter of the shaft , T= torque

- SLEEVE – (D, L)

D= Outer diameter of the sleeve

- KEY- RED
- l= length, w= width, t=thickness

DESIGN FOR SLEEVE

- The usual proportions of a cast iron sleeve coupling

- Outer diameter of the sleeve, D =2d + 13 mm

  length of the sleeve, L=3.5d

  Where d = diameter of the shaft

- The sleeve is designed by considering it as a hollow shaft.

- T=Torque to be transmitted by the coupling
- τ_c=Permissible shear stress for the material of the sleeve which is cast rion.
- τ_c = 14 MPa.
- Torque transmitted by a hollow section

$$T = (\pi/16)\times\tau_c\times(D^4-d^4)/D$$

$$= (\pi/16)\times\tau_c\times D^3(1-K^4)$$

$$\ldots (\because k = d / D)$$

- From this expression, the induced shear stress in the sleeve may be checked

Design for key

- The length of the coupling key = sleeve

  (i.e. . 3.5d).

- The coupling key is usually made into two parts
- length of the key in each shaft

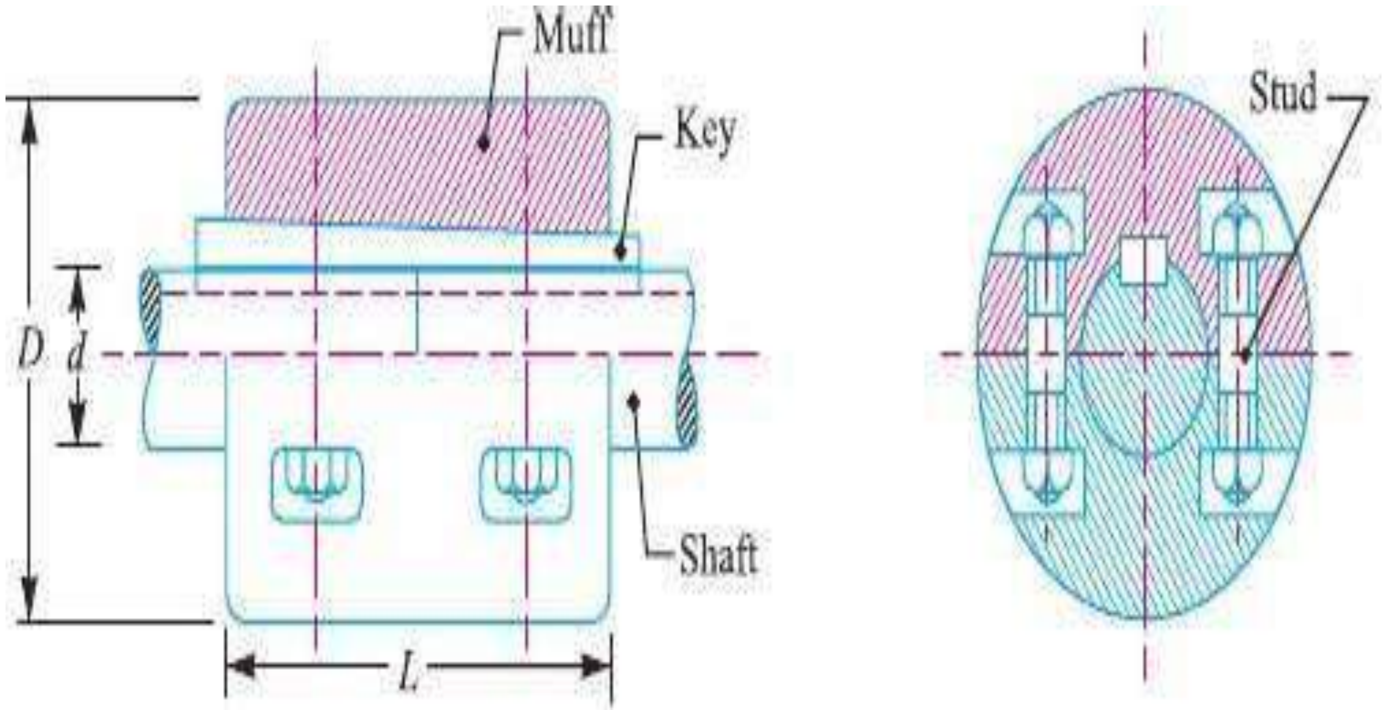
  $$l = L/2 = 3.5d/2$$

- After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

- $\mathbf{T = l \times w \times \tau \times (d/2)}$
  (Considering shearing of the key)

- $\mathbf{T = l \times t/2 \times \sigma_C \times (d/2)}$
  (Considering crushing of the key

Clamp or Compression Coupling



- the muff or sleeve is made into two halves and are bolted together.
- The halves of the muff are made of cast iron.
- The shaft ends are made to a butt each other
- a single key is fitted directly in the keyways of both the shafts.
- One-half of the muff is fixed from below and the other half is placed from above.
- Both the halves are held together by means of mild steel studs or bolts and nuts.
- The number of bolts may be two, four or six.
- The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the couplings

1. Design of muff

- The usual proportions of a cast iron sleeve coupling
- Outer diameter of the sleeve, D =2d + 13 mm
- length of the sleeve, L=3.5d

  Where d = diameter of the shaft
- The sleeve is designed by considering it as a hollow shaft.

- **T**=Torque to be transmitted by the coupling
- **τ_c**=Permissible shear stress for the material of the sleeve which is cast iron.
- **τ_c = <u>14 MPa.</u>**
- Torque transmitted by a hollow section

$$T = (\pi/16)\times\tau_c\times(D^4-d^4)/D$$

$$= (\pi/16)\times\tau_c\times D^3(1-K^4)$$

$$\ldots (\because k = d / D)$$

- From this expression, the induced **shear stress in the sleeve** may be checked

2. Design for key

- The length of the coupling key = length of the sleeve (**i.e. . 3.5d).**
- The coupling key is usually made into two parts
- length of the key in each shaft

l= L/2=3.5d/2

- After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted

- **$T = l \times w \times \tau \times (d/2)$**

  (Considering shearing of the key)

- **$T = l \times t/2 \times \sigma_C \times (d/2)$**

  (Considering crushing of the key

3. Design of clamping bolts

- T =Torque transmited by the shaft,
- d =Diameter of shaft,
- d_b=Root or effective diameter of bolt
- n=Number of bolts,
- σ_t =Permissible tensile stress for bolt material,
- μ=Coefficient of friction between the muff and shaft, and
- L=Length of muff.

- force exerted by each bolt **(F) =(π/4) (d** $_b$**²) σ**$_t$

- Force exerted by the bolts on each side of the shaft **(F)= (π/4) (d** $_b$**²) (σ**$_t$**)(n/2)**

- (P)be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface

- P=Force/Projected area

- **P= (π/4) (d** $_b$**²) (σ**$_t$**)(n/2)/(1/2)Ld**

- ∴Frictional force between each shaft and muff,

  F =μ× pressure × area

- **$F=(\mu \times (\pi/4)(d_b/2)(\sigma_t)(n/2)/(1/2)Ld) \times \pi (1/2) d L$**
- **$F= \mu \times (\pi^2/8)(d_b/2)(\sigma_t)(n)$**

- Torque that can be transmitted by the coupling

$$T=F \times d/2$$

$$T=\mu \times (\pi^2/8)(d_b^2)(\sigma_t)(n)\times d/2$$

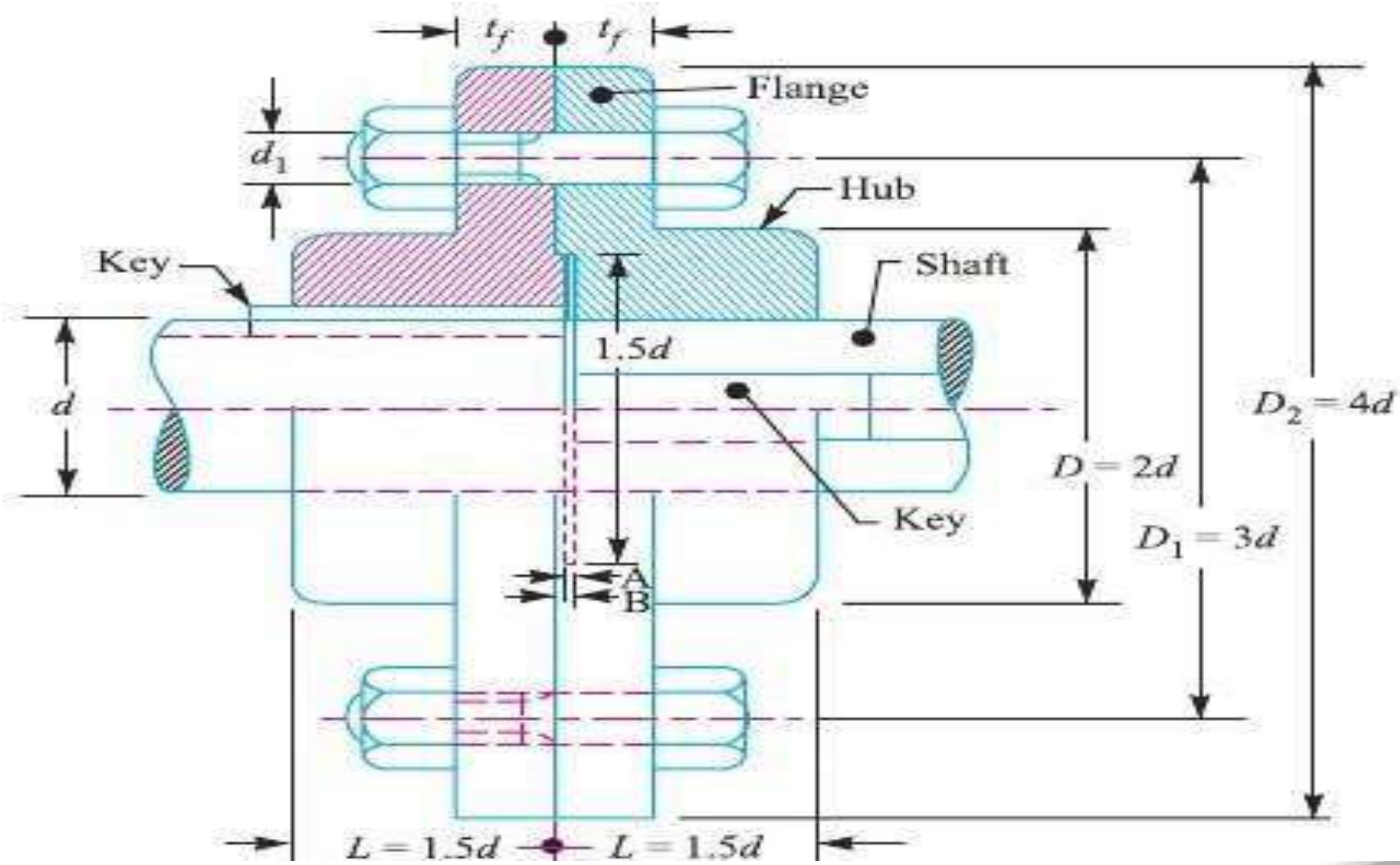
- From this relation, the root diameter of the bolt **(d_b)** may be evaluated.

μ=0.3

C. Flange coupling

- A flange coupling usually applies to a coupling having two separate cast iron flanges.
- Each flange is mounted on the shaft end and keyed to it.
- The faces are turned up at right angle to the axis of the shaft
- Flange coupling are
- 1.Unprotected type flange coupling
- 2. Protected type flange coupling
- 3. Marine type flange coupling

1.Unprotected type flange coupling



- In an unprotected type flange coupling each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.

- Generally, three, four or six bolts are used

Design of Unprotected type Flange Coupling

- The usual **proportions** for an unprotected type cast iron flange couplings
- d = diameter of the shaft or inner diameter of the hub
- D= Outside diameter of hub **D=2d**
- Length of hub, **L= 1.5d**
- Pitch circle diameter of bolts, **D_1=3d**
- Outside diameter of flange,

$$D_2 = D_1 + (D_1 – D) = 2D_1 – D = 4d$$

- Thickness of flange **t_f =0.5d**
- **Number of bolts =3, ford upto 40 mm**

  =4, for d upto 100 mm

  =6, for d upto 180 mm

- d =Diameter of shaft or inner diameter of hub,
- τ_s =Allowable shear stress for shaft,
- D=Outer diameter of hub,
- t_f =Thickness of flange
- τ_c=Allowable shear stress for the flange material

- d_1=Nominal or outside diameter of bolt,
- D_1 =Diameter of bolt circle,
- n=Number of bolts,
- $\tau_{b=}$ Allowable shear stress for bolt
- $\sigma_{cb,,}$ =Allowable crushing stress for bolt

- $\tau_{k=}$ Allowable shear stress for key material
- $\sigma_{ck=}$ key material

1. Design for hub

- The hub is designed by considering it as a hollow shaft,
- transmitting the same torque (T) as that of a solid shaft

$$T = \mathbf{T = (\pi/16)\times\tau_c\times(D^4-d^4)/D}$$

The outer diameter of hub is usually taken as twice the diameter of shaft.

- The length of hub **(L) = 1.5d**

2. Design for key

The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub **l=L**

- **$T = l \times w \times \tau \times (d/2)$**

  (Considering shearing of the key)

- **$T = l \times t/2 \times \sigma_C \times (d/2)$**

  (Considering crushing of the key

3. Design for flange

- T =Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

- $$T = \pi D \times t_f \times \tau_c \times D/2$$
  $$T = \pi \times t_f \times \tau_c \times D^2/2$$

  The thickness of flange is usually taken as half the diameter of shaft

4. Design for bolts

- Load on each bolt (F)= $(\pi/4)$ $(d_1/2)$ (τ_b)
- Total load on all the bolts (F) = $(\pi/4)$ $(d_1/2)$ $(\tau_b)(n)$
- The bolts are subjected to shear stress due to the torque transmitted

$$(T) = (\pi/4)\ (d_1/2)\ (\tau_b)(n)\ (D_1/2)$$

From this equation, the diameter of bolt (d_1) may be obtained.

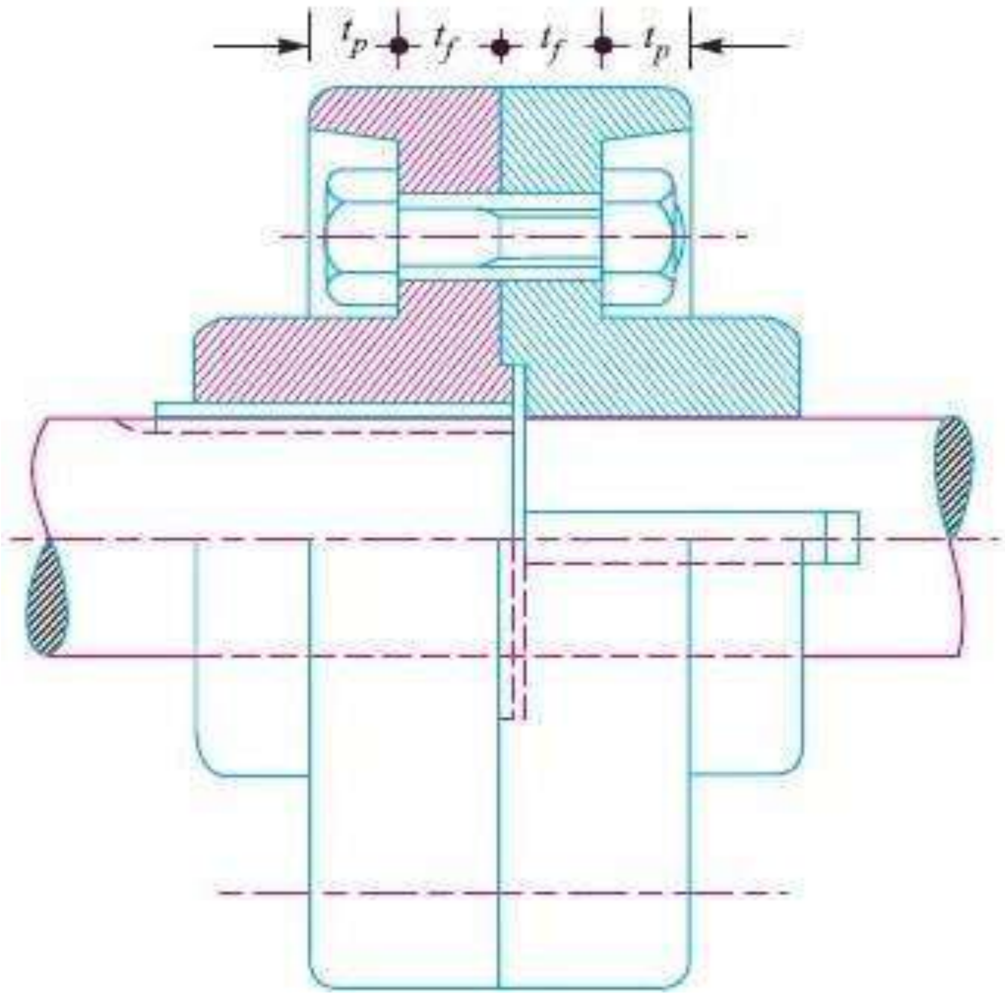
- We know that area resisting crushing of all the bolts = $n \times d_1 \times t_f$

- crushing strength of all the bolts = $n \times d_1 \times t_f \times \sigma_{Cb}$

  Torque = $n \times d_1 \times t_f \times \sigma_{Cb} \times (D_1/2)$

- From this equation, the induced crushing stress in the bolts may be checked

Protected type flange coupling

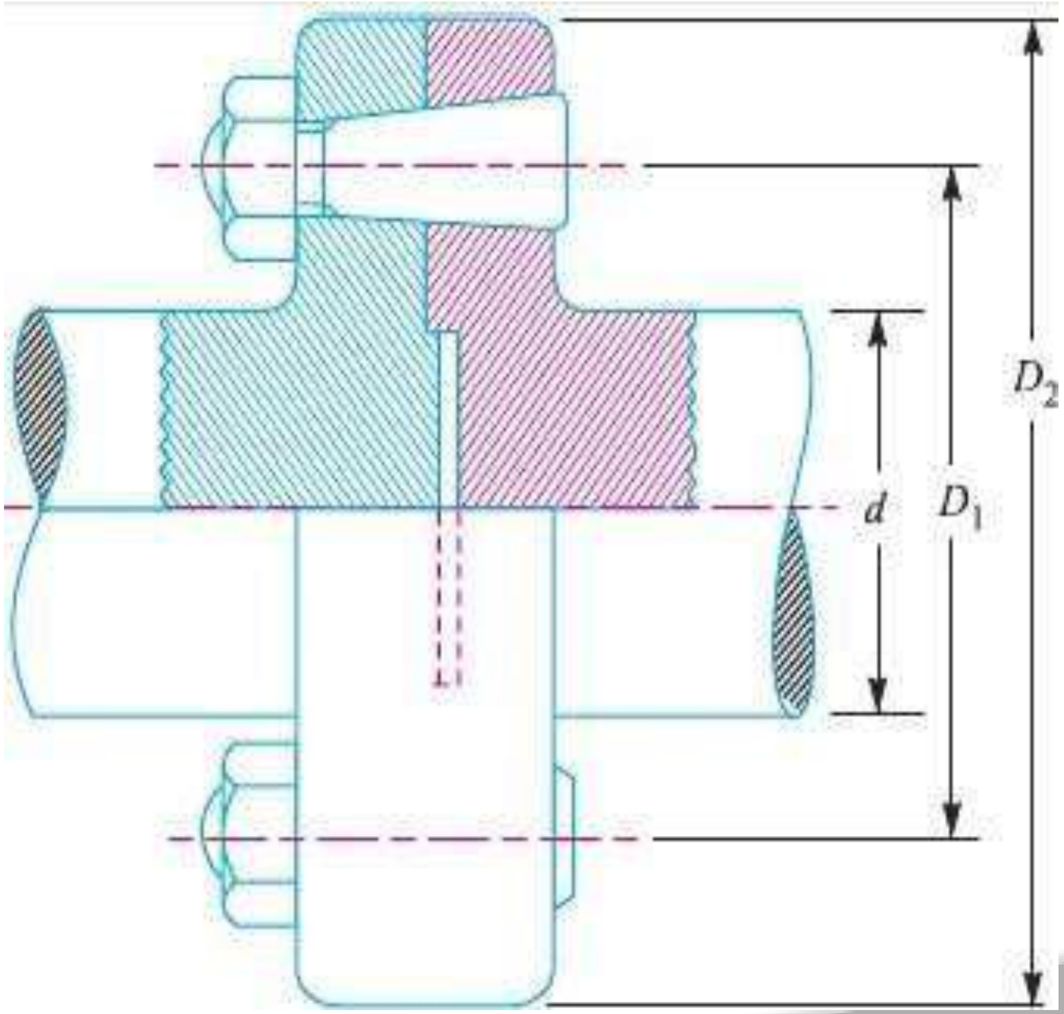


- the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman

$$(t_p) = 0.25d$$

The design of unprotective type is same process of protective type

Marine type flange coupling

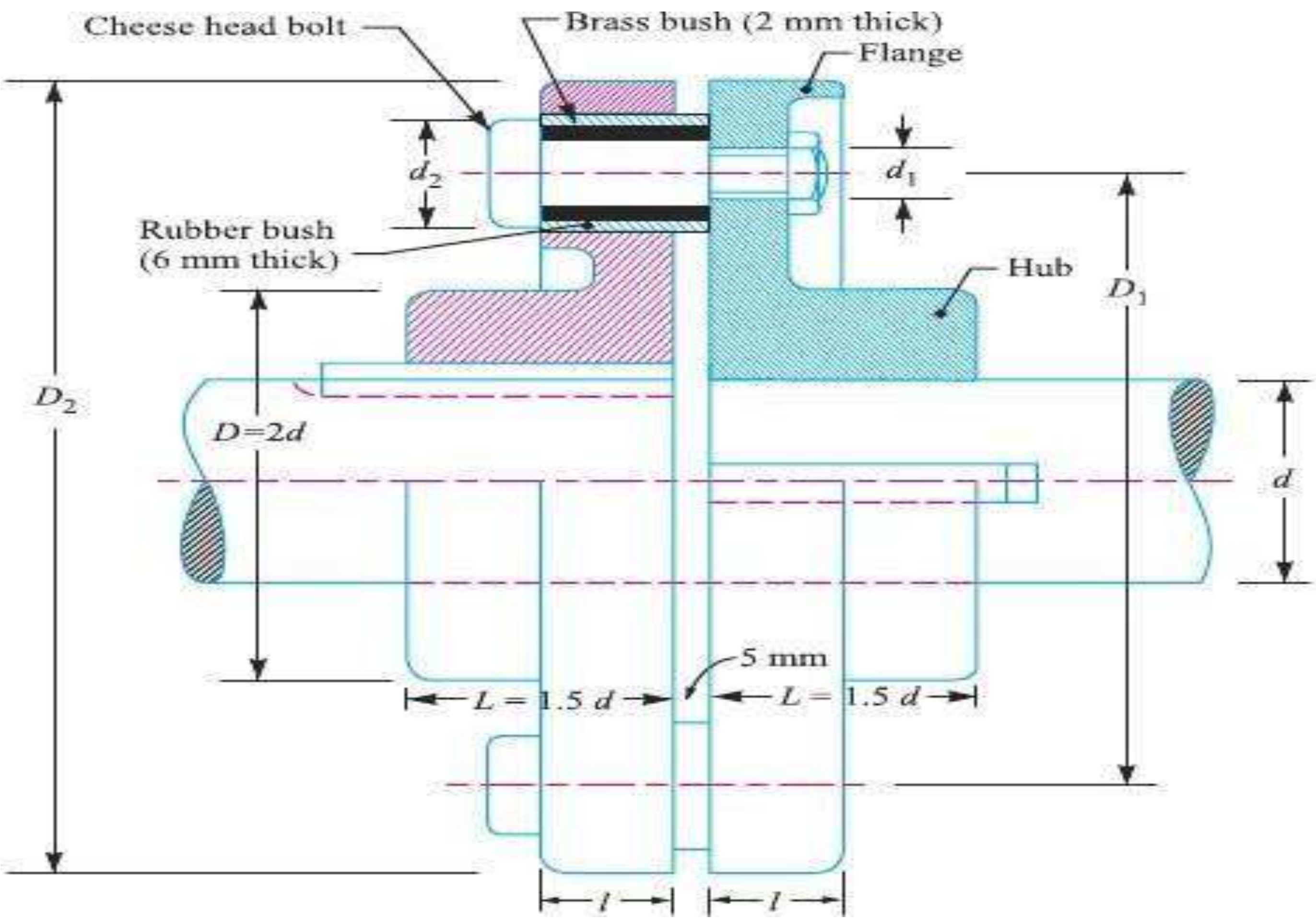


- In a marine type flange coupling, the flanges are forged integral with the shafts .
- The flanges are held together by means of tapered head less bolts.
- numbering from four to twelve depending upon the diameter of shaft.

Shaft diameter	No. of bolts
35 to 55	4
56 to 150	6
151 to 230	8
231 to 390	10
Above 390	12

- The other proportions for the marine type flange coupling
- Thickness of flange = d/ 3
- Taper of bolt= 1 in 20 to 1 in 40
- Pitch circle diameter of bolts, D_1= 1.6d
- Outside diameter of flange, D_2= 2.2d

Bushed-pin Flexible Coupling



A modification of the rigid type of flange coupling.

- The coupling bolts are known as pins. The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction.
- A clearance of 5 mm is left between the face of the two halves of the coupling.

- the **proportions** of the rigid type flange coupling
- the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm^2

Pin and bush design

- l=Length of bush in the flange,
- d_2=Diameter of bush,
- p_b=Bearing pressure on the bush or pin,
- n=Number of pins,
- D_1=Diameter of pitch circle of the pins

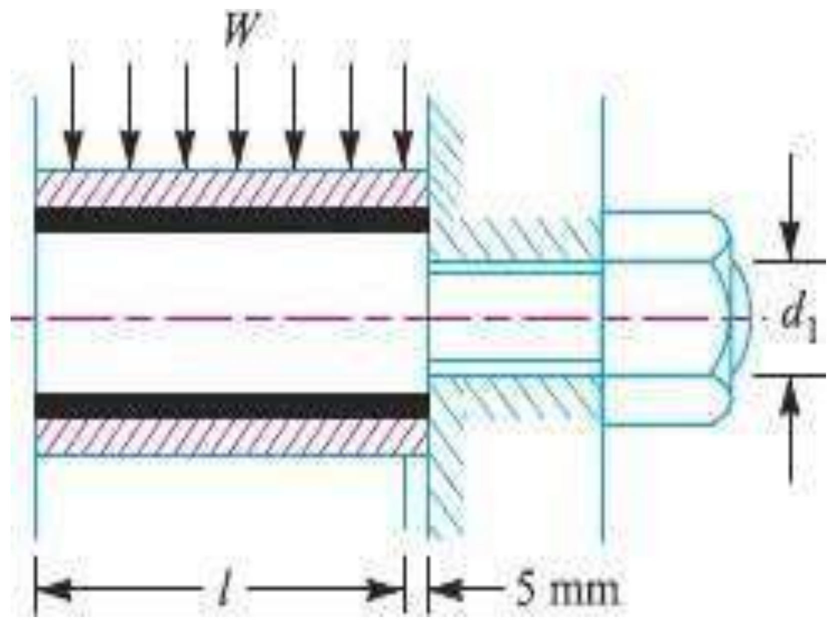
Pin and bush design

- Bearing load acting on each pin,
  $W = p_b \times d_2 \times l$

- ∴Total bearing load on the bush or pins,
- $W \times n = p_b \times d_2 \times l \times n$

- torque transmitted by the coupling,
- $T = W \times n \times (D_1/2)$

$$T = p_b \times d_2 \times l \times n \times (D_1/2)$$


W
d1
l
5 mm

- Direct shear stress due to pure torsion in the coupling halve
- $\tau = W / [\ \boldsymbol{(\pi/4)\ (d_1 /2)}\ ]$

- maximum bending moment on the pin
- $M = W\ (l/2 + 5mm)$
- bending stress

$$\sigma = M / Z$$

$$= W\ (l/2 + 5mm) / \boldsymbol{(\pi/32)\ (d_1^3)}$$

- Maximum principal stress, $= 1/2[\sigma + (\sigma + 4\tau^2)^{1/2}]$

- maximum shear stress on the pin, $= 1/2(\sigma + 4\tau^2)^{1/2}$

- The value of maximum principal stress varies from 28 to 42 MPa

UNIT-V : DESIGN OF SPRINGS

Introduction to Design of Helical Springs

Objectives of Spring

Cushioning , absorbing , or controlling of energy due to shock and vibration.

Car springs or railway buffers To control energy, springs-supports and vibration dampers.

2. Control of motion

- Maintaining contact between two elements (cam and its follower)

  In a cam and a follower arrangement, widely used in numerous applications, a spring maintains contact between the two elements. It primarily controls the motion.

- Creation of the necessary pressure in a friction device (a brake or a clutch) A person driving a car uses a brake or a clutch for controlling the car motion. A spring system keep the brake in disengaged position until applied to stop the car. The clutch has also got a spring system (single springs or multiple springs) which engages and disengages the engine with the transmission system.

- Restoration of a machine part to its normal position when the applied force is withdrawn (a governor or valve)

  A typical example is a governor for turbine speed control. A governor system uses a spring controlled valve to regulate flow of fluid through the turbine, thereby controlling the turbine speed.

3. Measuring forces

Spring balances, gages

4. Storing of energy

In clocks or starters

The clock has spiral type of spring which is wound to coil and then the stored energy helps gradual recoil of the spring when in operation. Nowadays we do not find much use of the winding clocks.

Commonly used spring materials

- One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are
- *Hard-drawn wire:*

This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 120ºC.

Oil-tempered wire:

It is a cold drawn, quenched, tempered, and general purpose spring steel. However, it is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 180ºC.

When we go for highly stressed conditions then alloy steels are useful.

Chrome Vanadium:

This alloy spring steel is used for high stress conditions and at high temperature up to 220ºC. It is good for fatigue resistance and long endurance for shock and impact loads.

Chrome Silicon:

This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 250ºC.

Music wire:

This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. However, it can not be used at subzero temperatures or at temperatures above 120ºC.

Normally when we talk about springs we will find that the music wire is a common choice for springs.

Stainless steel:

Widely used alloy spring materials.

Phosphor Bronze / Spring Brass:

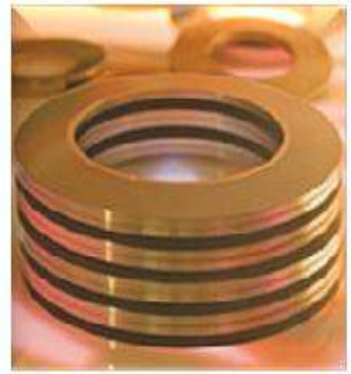
It has good corrosion resistance and electrical conductivity. That's the reason it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

Spring manufacturing processes

wire diameter is also small then the springs are normally manufactured by a cold drawn process through a mangle.

For very large springs having also large coil diameter and wire diameter one has to go for manufacture by hot processes.

Helical compression spring



Belleville spring

Torsion spring

Flat spring

leaf spring

Helical extension spring

Torsion spring

Flat spring`

Draw bar spring

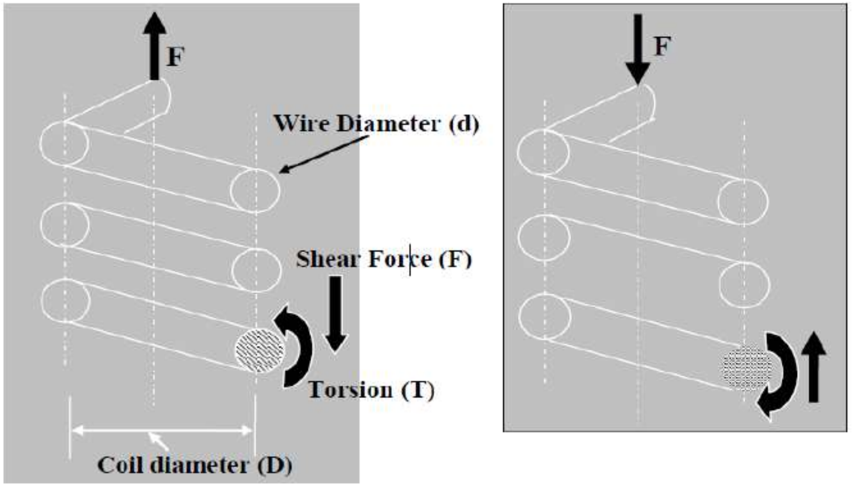
constant – force spring

Pull Types Springs

HELICAL SPRING

It is made of wire coiled in the form of helix having circular, square or rectangular cross section.

The figures below show the schematic representation of a helical spring acted upon by a tensile load F and compressive load F . The circles denote the cross section of the spring wire.



Terminology of helical spring

- They are as follows:

d = wire diameter of spring (mm)

Di = inside diameter of spring coil (mm)

Do =outside diameter of spring coil (mm)

D = mean coil diameter (mm)

There is an important parameter in spring design called spring index. It is denoted by letter C.

The spring index is defined as the ratio of mean coil diameter to wire diameter.

C= D/d

In design of helical springs, the designer should use good judgment in assuming the value of the spring index C. The spring index indicates the relative sharpness of the curvature of the coil.

A low spring index means high sharpness of curvature.

When the spring index is low ($C < 3$), the actual stresses in the wire are excessive due to curvature effect. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires. When the spring index is high ($C > 15$), it results in large variation in coil diameter.

Such a spring is prone to buckling and also tangles easily during handling. Spring index from 4 to 12 is considered better from manufacturing considerations.

Therefore, in practical applications, the spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.

Terms used in Helical springs

1. *Solid length*. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid***. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

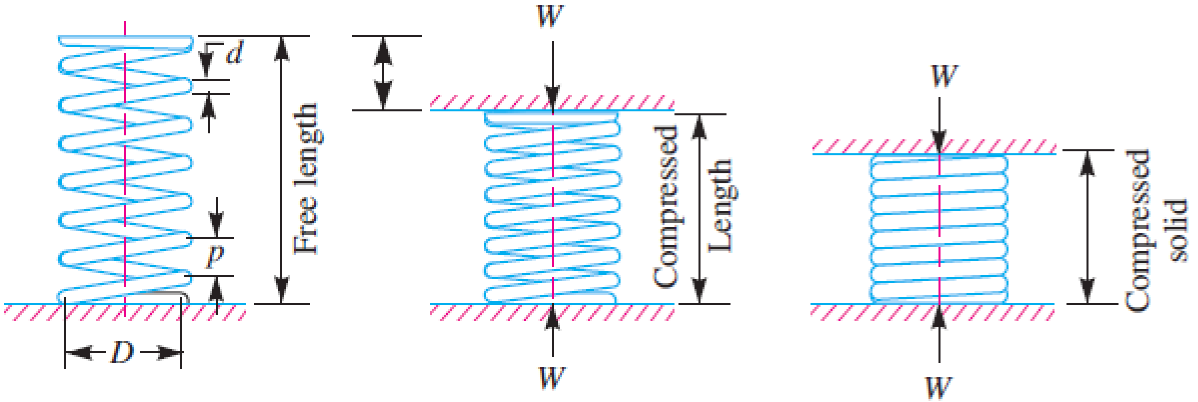
Solid length of the spring,

$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.

2. *Free length*. The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,



Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. ***Spring index.*** The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. ***Spring rate.*** The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. *Pitch.* The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $$p = \frac{L_F - L_S}{n'} + d$$

where L_F = Free length of the spring,

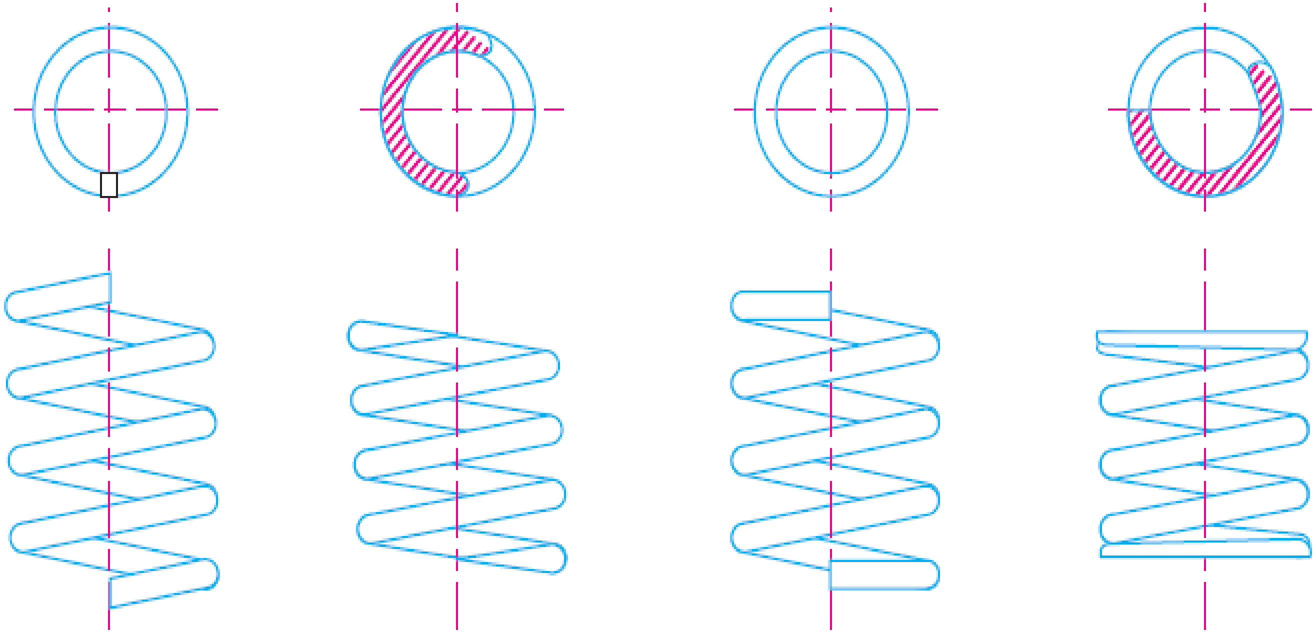
L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

End conditions for helical compression springs

- **The end condition are suitably formed to apply the loads on the springs.**
- **Various forms of end conditions are**



(*a*) Plain ends. (*b*) Ground ends. (*c*) Squared ends. (*d*) Squared and ground ends.

Stresses in the helical spring wire

Total number of turns, solid length and free length for different types of end connections.

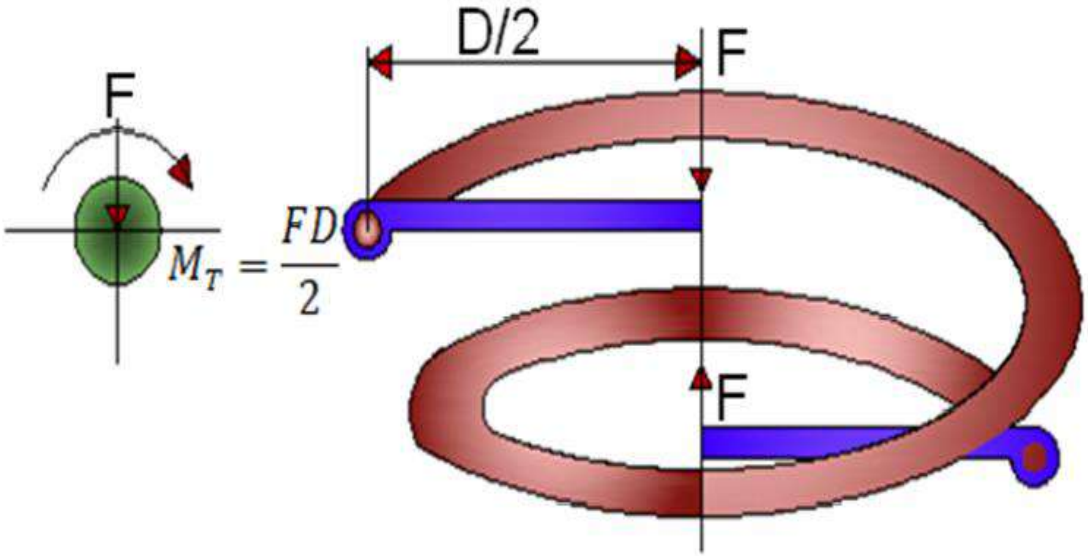
Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n+1)\,d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n+2$	$(n+3)\,d$	$p \times n + 3d$
4. Squared and ground ends	$n+2$	$(n+2)\,d$	$p \times n + 2d$

where

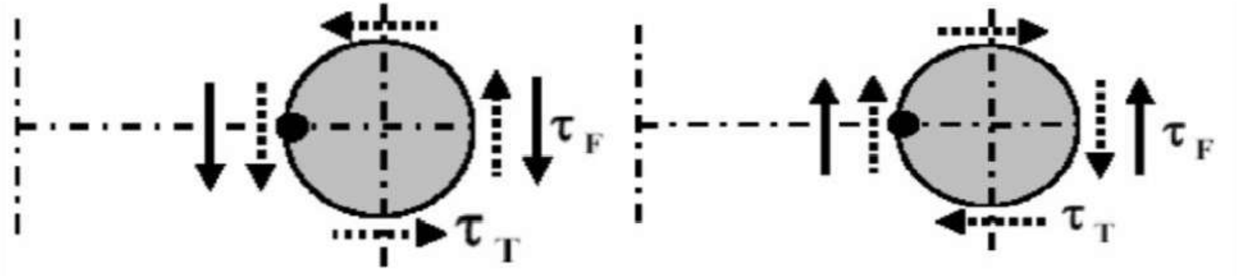
n = Number of active turns,
p = Pitch of the coils, and
d = Diameter of the spring wire.

Stresses in the helical spring wire

From the free body diagram, we have found out the direction of the internal torsion T and internal shear force F at the section due to the external load F acting at the centre of the coil



The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the figure.



The broken arrows show the shear stresses (τ_T) arising due to the torsion T and solid arrows show the shear stresses (F) due to the force (τ_F)

It is observed that for both tensile load as well as compressive load on the spring, maximum shear stress always occurs at the inner side of the spring. Hence, failure of the spring, in the form of crake, is always initiated from the inner radius of the spring. $(\tau_T + \tau_F)$

Consider a helical compression spring made of circular wire and subjected to an axial load W,

D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

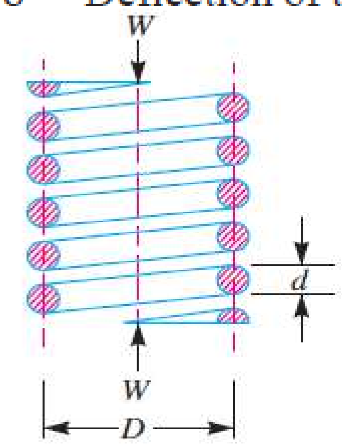
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

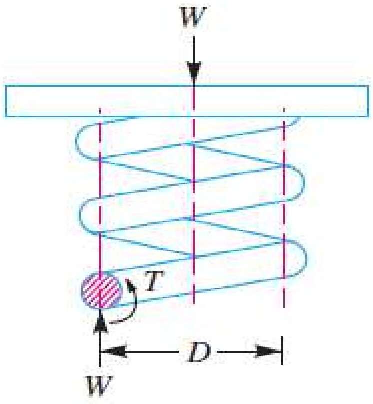
C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.



(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

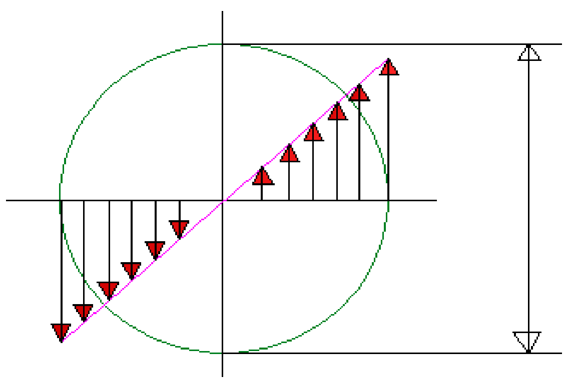
We know that the twisting moment,

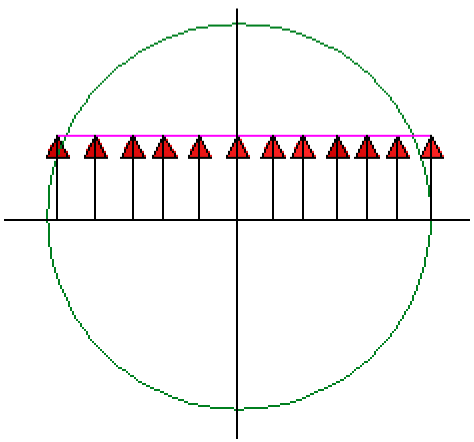
$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8W.D}{\pi d^3}$$

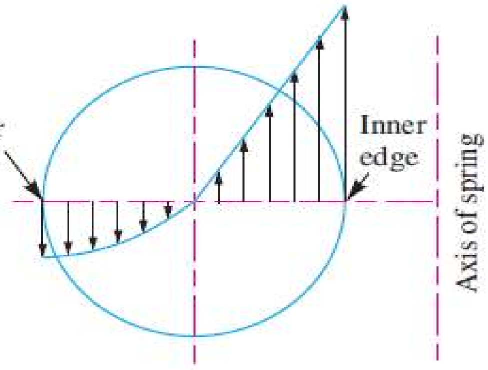
In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

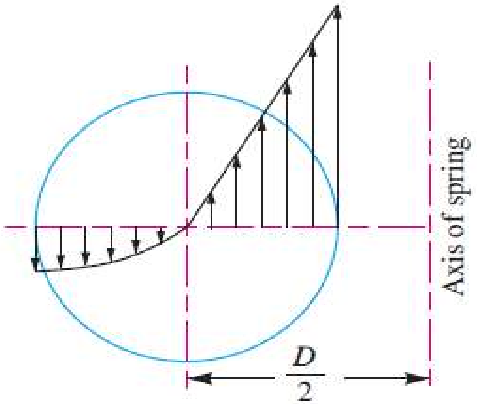




Average shear stress in the spring wire due to force F is



Resultant torsional shear and direct shear stress diagram.



Resultant torsional shear, direct shear and curvature shear stress diagram.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

πd πd

The *positive* sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8W.D}{\pi d^3} \qquad \text{...}(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \dfrac{1}{2C}$

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (*K*) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad ...(iv)$$

where

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

Deflection of helical spring of circular cross section wire

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \dots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \qquad \theta = \frac{T.l}{J.G} \qquad \dots\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

$$= \frac{\pi}{32} \times d^4$$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right) \pi D n}{\frac{\pi}{32} \times d^4 G} = \frac{16 W.D^2.n}{G.d^4}$$

Substituting this value of θ in equation (*i*), we have

Substituting this value of θ in equation (*i*), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \ldots (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

Energy stored in Helical Spring

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta \qquad \text{...}(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8\,K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Design against fluctuating load

Substituting the values of W and δ in equation (*i*), we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3 . \tau}{8\, K . D} \times \frac{\pi\, \tau . D^2 . n}{K . d . G}$$

$$= \frac{\tau^2}{4\, K^2 . G} (\pi\, D . n) \left(\frac{\pi}{4} \times d^2 \right) = \frac{\tau^2}{4\, K^2 . G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi\, D . n) \left(\frac{\pi}{4} \times d^2 \right)$$

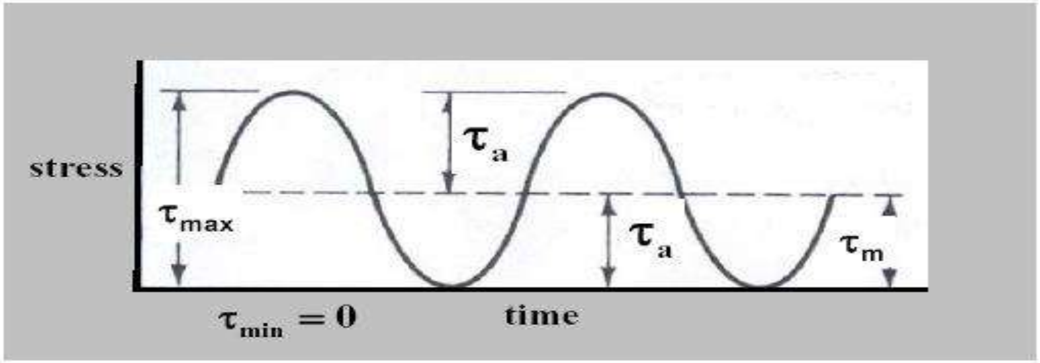
Note : When a load (say P) falls on a spring through a height h, then the energy absorbed in a spring is given by

$$U = P(h + \delta) = \frac{1}{2} W . \delta$$

where W = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load P, and

δ = Deflection produced in the spring.

- In many applications, the force acting on the spring is not constants but varies in magnitude with time. The valve springs of automotive engine subjected to millions of stress cycles during its life time.
- On the other hand, the springs in the linkages and mechanisms are subjected to comparatively less number of stress cycles.
- The spring subjected to fluctuating stresses are designed on the basis of two criteria- design for infinite life and design for finite life



$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

2. The value of mean shear stress (τ_m) is calculated by using the shear stress factor (K_S), while the variable shear stress is calculated by using the full value of the Wahl's factor (K). Thus

Mean shear stress,

$$\tau_m = K_s \times \frac{8 W_m \times D}{\pi d^3}$$

where $$K_S = 1 + \frac{1}{2C}; \text{ and } W_m = \frac{W_{max} + W_{min}}{2}$$

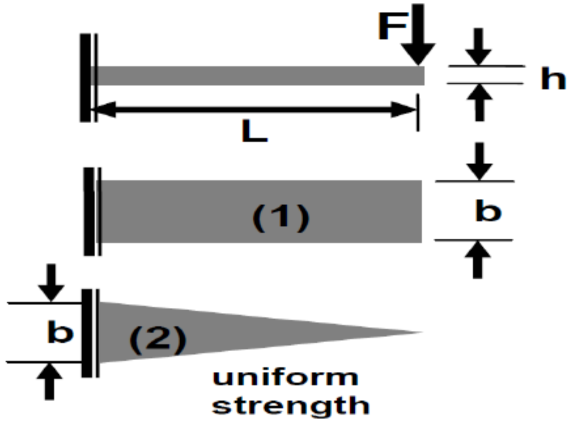
and variable shear stress, $$\tau_v = K \times \frac{8 W_v \times D}{\pi d^3}$$

where $$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}; \text{ and } W_v = \frac{W_{max} - W_{min}}{2}$$

Design of Leaf Spring

Characteristics

1. Sometimes it is also called as a semi-elliptical spring; as it takes the form of a slender arc shaped length of spring steel of rectangular cross section.
2. The center of the arc provides the location for the axle, while the tie holes are provided at either end for attaching to the vehicle body.
3. Supports the chassis weight
4. Controls chassis roll more efficiently-high rear moment center and wide spring base.
5. Controls rear end wrap-up
6. Controls axle damping
7. Controls braking forces
8. Regulates wheelbase lengths (rear steer) under acceleration and braking Leaf



For case 1(uniform width)

$$\sigma_{max} = \frac{6FL}{bh^2}$$

$$\delta_{max} = \frac{4FL^3}{Ebh^3}$$

For case 2(non uniform width)

$$\sigma_{max} = \frac{6FL}{bh^2}$$

$$\delta_{max} = \frac{6FL^3}{Ebh^3}$$

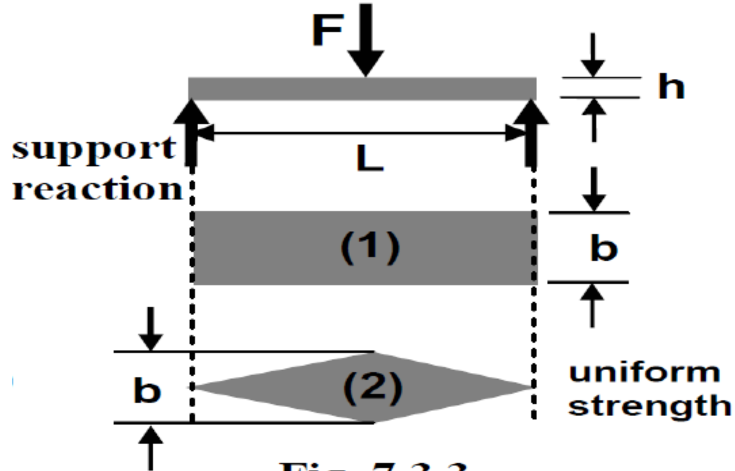


Fig. 7.3.3

For case 1(uniform width)

$$\sigma_{max} = \frac{3FL}{bh^2}$$

$$\delta_{max} = \frac{2FL^3}{Ebh^3}$$

$$\sigma_{max} = \frac{3FL}{bh^2}$$

$$\delta_{max} = \frac{3FL^3}{Ebh^3}$$

Laminated springs

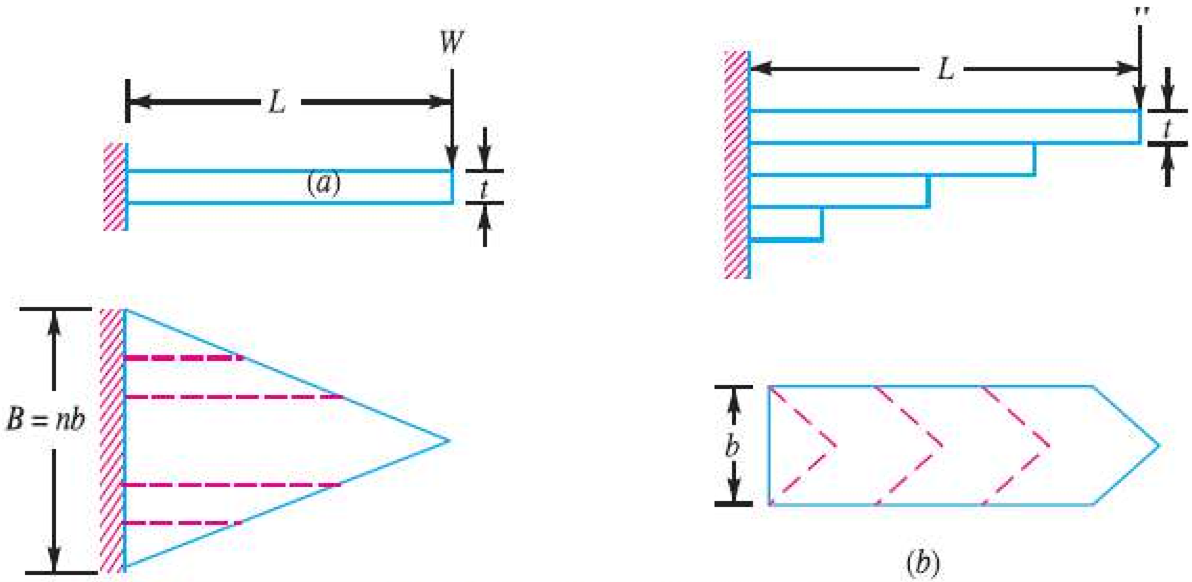


Fig. 23.29. Laminated leaf spring.

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(v)$$

and

$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad ...(vi)$$

where n = Number of graduated leaves.

∴ Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W = \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

Since $\sigma_F = \frac{3}{2}\,\sigma_G$, therefore

$$\sigma_G = \frac{2}{3}\,\sigma_F = \frac{2}{3} \times \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation (*iv*), *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$