Mathematical foundation in computer Science

Proposition: A proposition is a declarative Sentence which in a given contest can be Said to be either true or false but not both. (It should be noted that not all Sentences are propositions.)

Ex: NewDelhi is the capital of India (T)

Every rectangle is a Square (F)

Δ^{le} ABC (not a complete Sentence)

$xy = yx$ (not a Proposition)

If we Know the Values of x and y then only it is a complete Proposition. Otherwise

→ it is not a complete proposition. Generally propositions are indicated by Small alphabets p, q, r, s etc

→ Truth or falcity of a proposition is called its truth Value.

→ If the proposition is true, then Proposition is indicated by '1'

→ If the proposition is false, then Proposition is indicated by '0'

Negation : A proposition obtained by inserting the word not at an appropriate place in a given proposition is called the negation of a given proposition. It is indicated by "~" or "¬".

Ex: P : NewDelhi is capital of India → Truth Value 1

~P : NewDelhi is not capital of India → 0

Truth table of a given proposition P
"3" is a Prime number

P	~P
1	0
0	1

conjunction : A compound proposition obtained by combining two given propositions by inserting the word 'and' in b/w them is called a conjunction of given proposition. It is indicated by the symbol '$\wedge$'.

The conjunction of $p \wedge q$ is true only when both p and q are true.
In all other cases it is false

p	q	$p \wedge q$
0	1	0
1	0	0
1	1	1
0	0	0

<u>Disjunction</u>: It is indicated by the Symbol "V"

p	q	$p \vee q$
0	0	0
1	0	1
0	1	1
1	1	1

The Disjunction of p or q is false only when p is false and q is false. In all other cases it is true

<u>Exclusive Disjunction</u>: It is indicated by the Symbol $\overline{\vee}$ or $\underline{\vee}$. The rule of this exclusive disjunction is

p	q	$p \underline{\vee} q$
0	0	0
1	1	0
0	1	1
1	0	1

~~[illegible]~~ q	~q	p	~p
0	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0

In the conjunction p or q of two propositions p and q the symbol $\vee$ is used in the inclusive sense i.e $p \vee q$ is taken to be true when p is true or q is true or both p and q are true. Sometimes we require the use of the word 'or' in the exclusive sense i.e we require that the compound proposition p or q to be true only when p is true or q is true but not both. It is denoted by $\bar{\vee}$ or $\underline{\vee}$. This is called as Exclusive or or Exclusive disjunction

Conditional: It is denoted by the symbol $(\longrightarrow)$. The truth table of p and q is shown in the below table $(p \rightarrow q)$

p	q	$p \rightarrow q$	$q \rightarrow p$
0	0	1	1
1	0	0	1
0	1	1	0
1	1	1	1

A compound propositional obtained by combining two given propositions by using the words "if and then" at appropriate places is called a conditional.

The conditional, if p then q is denoted by '$p \to q$'. The conditional, if q then p is denoted by '$q \to p$'

Rule : The conditional $p \to q$ is false only when p is true & q is false in all other cases it is true.

Biconditional : It is denoted by the symbol '$\leftrightarrow$' or "$\rightleftarrows$" or "$\rightleftharpoons$"

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1
1	1	1	1	1

Let p and q be two Propositions, then conjunction of the conditionals $(p \to q) \wedge (q \to p)$ is called the biconditional of p and q

and $p \Leftrightarrow q$ is same as $(p \to q) \wedge (q \to p)$

→ Combined Truthtable :

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$\neg p \wedge \neg q$	$\neg p \vee \neg q$	$p \underline{\vee} q$	$p \to q$	$q \to p$	$p \leftrightarrow q$
1	0	0	1	0	1	0	1	1	0	1	0
0	1	1	0	0	1	0	1	1	1	0	0
1	1	0	0	1	1	0	0	0	1	1	1
0	0	1	1	0	0	1	1	0	1	0	1

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<u>NAND condition</u> ($\uparrow$) : It is negation of a conjuntion

i.e $\neg(p \wedge q)$

Truth table of NAND condition.

p	q	$p \wedge q$	$\neg(p \wedge q) = p \uparrow q$
0	1	0	1
1	0	0	1
1	1	1	0
0	0	0	1

<u>NOR condition</u> ($\downarrow$) : It is negation of disjunction (OR)

i.e $\neg(p \vee q)$ (or) $(p \downarrow q)$

Truth table of ~~AND~~ NOR

p	q	$p \vee q$	$(p \downarrow q) = \neg(p \vee q)$
0	1	1	0
1	0	1	0
0	0	0	1
1	1	1	0

→ Construct truth table for following compound propositions

i) $p \wedge \sim q$

$\sim p$	p	q	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$
1	0	1	0	0	1
0	1	0	1	1	0
1	0	0	1	0	1
0	1	1	0	0	1

$\sim p \veebar (r-q)$	$p \to \sim q$
1	~~0~~ 1
1	[illegible]
0	1
0	0

11) Let p and q be Primitive Statements for which the implication $p \to q$ is false determine the following truth Values

$p \wedge q$

$\sim p \vee q$

$q \to p$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee q$	$q \to p$	$\sim q \to \sim p$
0	1	1	0	0	1	0	1
1	0	0	1	0	0	1	0
0	0	1	1	0	1	1	1
1	1	0	0	1	1	1	1

→ Construct the truth tables of the following compound Proposition

i) $(p \vee q) \wedge r$

$p \vee (q \wedge r)$

$(p \wedge q) \to (\sim r)$

$q \wedge ((\sim r) \to p)$

p	q	r	$\sim r$	$(p \vee q)$	1 $(p \vee q) \wedge r$	2 $p \vee (q \wedge r)$		3 $(p \wedge q) \to (\sim r)$
1	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	1
1	1	0	1	1	0	1	1	1
0	0	0	1	0	0	0	0	1
1	0	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1	0
1	1	1	0	1	1	1	1	0
0	0	1	0	0	0	1	1	1

$(p \wedge q)$	4 $q \wedge ((\sim r) \to p)$	$(\sim r) \to p$
1	0	1
1	0	0
1	1	1
0	0	0
1	0	1
1	1	1
1	1	1
0	0	0

→ $(p \vee q) \wedge r \neq p \vee (q \wedge r)$

→ **Well formed formulae:**

i) Statements represented in symbolic forms which cannot be integrated in more than one way are called well formed formulae.

While representing a statement involving

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Connectives in Symbolic representation conveys the intended meaning of the statement without any ambiguity. Appropriate Paranthesis are to be used at appropriate places to achieve this objective.

→ In a compound Preposition P, q, r, s are the Primitives ∴ The number of possible combinations of truth values of these components that we have to consider is 2^4. Hence 16 rows are needed for the construction of a truth table. Give

→ Given truth values of P and q as 1 & r and s as 0. find truth values of following

p	q	r	s
1	1	0	0

i) $p \vee (q \wedge r)$ $1 \vee 0 = 1$

$(r \wedge (q \wedge r))$ $(1 \wedge 0 = 0)$

ii) $\sim (p \wedge q)$ $\sim 1 = 0$

iii) $(p \wedge (q \wedge r)) \vee \sim ((p \vee q) \wedge (r \wedge s))$

$0 \vee \sim (1 \wedge 0)$

$0 \vee \sim (0)$

$0 \vee 1$

1

iv) $(p \vee (q \rightarrow (q \wedge \sim p))) \leftrightarrow (q \vee \sim s)$

1

$p \vee (q \rightarrow (0)$

$p \vee (0)$

$1 \vee 0 \qquad 1 \leftrightarrow 1$

$1 \qquad = 1$

iv) $(\sim (p \wedge q) \vee \sim r) \vee ((\sim p \wedge q) \vee \sim r) \wedge s)$

$(0 \vee 1) \vee ((1 \vee 0) \wedge 0)$

$1 \vee (1 \wedge 0)$

$1 \vee 0$

1

ii) $(p \leftrightarrow r) \wedge (\sim q \rightarrow s)$

$0 \wedge 1 \quad \wedge 1$

$0 \wedge 1$

$= \underline{\underline{0}}$

Tautology : A compound Proposition which is true for all possible truth Values of its components is called Tautology.

→ A Tautology is generally denoted by "T_0"

p	$\sim p$	$p \vee \sim p$
1	0	1
0	1	1

} Tautology

Contradiction : (Contradictory)

A compound Proposition which is false for all possible truth Values of its components is called a contradiction. It is denoted by "F_0"

p	$\sim p$	$p \wedge \sim p$
1	0	0
0	1	0

} Contradiction

Contigency : A Statement formula is said to be contigency if it is neither tautology nor contradiction is contigency. In other word of any truth table contains all truth values are true or false.

→ Determine whether following statements are Tautology or not.

i) $[\sim P \wedge (P \to q)] \to \sim q$

P	q	$\sim P$	$\wedge$ $P \to q$	$\sim P \wedge (P \to q)$	$\to \sim q$	Ans
0	1	1	1	1	0	0
1	0	0	0	0	1	1
0	0	1	1	1	1	1
1	1	0	1	0	0	1

ii) $[\sim q \wedge (P \to q)] \to \sim P$

$\sim q$	$P \to q$	$\sim P$	Ans	$\to$ $\sim P$		Ans
0	1	1	0	0	1	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1
0	1	0	0	0	0	1

iii) $[[(P \vee q) \wedge (P \to r)] \wedge (q \to r)] \to r$

$P \vee q$	$P \to$
1	
1	
0	
1	

P	q	r	$(P \vee q) \wedge (P \to r)$		$(q \to r)$
1	0	0	1	0	1
0	1	0	1	1	0
1	1	0	0	0	0
0	0	0	1	1	1
1	0	1	1	1	1
0	1	1	1	1	1
1	1	1	1	1	1
0	0	1	0	1	1

	$\wedge$	
0	1	0
1	0	0
0	0	0
1	1	1
1	1	1
1	1	1
1	1	1
0	1	0

	$\to$	Ans
0	0	0
0	0	0
0	0	0
1	0	0
1	1	1
1	1	1
1	1	1
0	1	1

iv) $(p \wedge q) \rightarrow (p \rightarrow q)$

0	$\rightarrow$	1		1
0		0		1
0		1		1
1		1		1

Tautology

v) $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$

0	$\wedge$	1	$\rightarrow$	0
1		0		1
1		0		0
1		1		1
0		1		1
1		1		1
1		1		1
1		1		1

0	$\rightarrow$	0	1
0		1	1
0		0	1
1		1	1
0		1	1
1		1	1
1		1	1
1		1	1

Tautology

→ find the following situations in which the following are contradictions.

1) $[(p \wedge q) \wedge r] \rightarrow [(s \vee t)]$

2) $[p \wedge (q \wedge r)] \rightarrow [(s \veebar t)]$

$(0 \wedge 0) \wedge r$

$0 \wedge 1 \rightarrow 0$

$1 \rightarrow 0$

$(p \wedge q) \wedge r$

$(p \wedge q) = 1 \quad r = 0$

p	q	r	s	t
			1	0
			0	1
			1	1
1	1	1	0	0

contradictory

2) $p, q, r = 1 \qquad s \veebar t = 0$

$s, t = 0, 0$
$s, t = 1, 1$

1) Here, on the given compound propositions, if primitives p,q,r are true and s,t are false then the given compound proposition will become contradictory

2) Here on the given compound propositions of p,q are true & s,t are false or true. on that case it will become contradictory.

Tautology Implication : If P and q are any two compound statements P is tautologically implied by q If P and q is a tautology it is represented by Symbol "$P \Rightarrow q$"

Ex:

$$\underbrace{P \rightarrow (q \rightarrow r)}_{} \Rightarrow \underbrace{[(P \rightarrow q) \rightarrow (P \rightarrow r)]}_{}$$

P	q	r	$(q \rightarrow r)$	$P \rightarrow (q \rightarrow r)$ ①	$(P \rightarrow q)$	$(P \rightarrow r)$
0	1	0	0	1	1	1
1	0	0	1	1	0	0
0	0	0	1	1	1	[illegible]
1	1	0	0	0	1	0
0	1	1	1	1	1	1
1	0	1	1	1	0	1
0	0	1	1	1	[illegible]	1
1	1	1	1	1	1	1

$(P \rightarrow q) \rightarrow (P \rightarrow r)$ ②	Tautological implication ① → ②
1	1
1	1
[illegible]	1
0	1
1	1
1	1
1	1
1	1

② $p \wedge q \Rightarrow p$

Tautological implication

p	q	$p \wedge q$	$(p \wedge q) \to p$
0	1	0	1
1	0	0	1
0	0	0	1
1	1	1	1

Tautology

Logical Equilance (or) Equilance implication :

The truth values of one compound Proposition is equal to the truth value of another compound proposition then those two compound Propositions are in logical equilance. Denoted by $\Leftrightarrow$

Ex: Determine $\sim(p \vee (\sim p \wedge q))$ & $(\sim p \wedge \sim q)$ are $\Leftrightarrow$ or not

p	q	$\sim p$	$(\sim p \wedge q)$	$p \vee (\sim p \wedge q)$	$\sim(p \vee (\sim p \wedge q))$
0	1	1	1	1	0
1	0	0	0	1	0
1	1	0	0	1	0
0	0	1	0	0	1

$\sim p$	$\sim q$	$\sim p \wedge \sim q$
1	0	0
0	1	0
0	0	0
1	1	1

logically equivalent

$\therefore \sim(p \vee (\sim p \wedge q)) \Leftrightarrow (\sim p \wedge \sim q)$

→ Determine following statements are logically equivalent or not.

1. $\sim p \leftrightarrow q$ & $p \leftrightarrow \sim q$

p	q	$\sim p$	$\sim p \leftrightarrow q$	$\sim q$	$p \leftrightarrow \sim q$
0	1	1	1	0	1
0	0	1	0	1	0
1	1	0	0	0	0
1	0	0	1	1	1

logically eq

2. $p \leftrightarrow q$ & $(p \wedge q) \vee (\sim p \wedge \sim q)$

p	q	$p \leftrightarrow q$	$(p \wedge q)$	$\sim p$	$\sim q$	$(\sim p \wedge \sim q)$	Ans
0	1	0	0	1	0	0	0
1	0	0	0	0	1	0	0
0	0	1	0	1	1	1	1
1	1	1	1	0	0	0	1

logically equivalent

3) $p \to q$ & $(\sim p \to \sim q)$

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim p \to \sim q$
0	1	1	1	0	0
1	0	0	0	1	1
0	0	1	1	1	1
1	1	1	0	0	1

The laws of logic or Properties of logical equilance

The following results Known as the laws of logic. In this laws 'T_0' denotes Tautology and 'F_0' denotes contradiction

① The law of Double negation:

for any given Proposition P

$$\sim(\sim P) \Leftrightarrow P$$

P	~P	~(~P)
1	0	1
0	1	0

$$\therefore \sim(\sim P) \Leftrightarrow P$$

② Idempotent laws for any Proposition:

Here, P is a given compound Proposition then,

$$P \vee P \Leftrightarrow P$$

$$P \wedge P \Leftrightarrow P$$

P	P	P∨P	P∧P
0	0	0	0
1	1	1	1

③ Identity law for a given Proposition P:

$$\boxed{\begin{array}{l} P \vee F_0 \Leftrightarrow P \\ P \wedge T_0 \Leftrightarrow P \end{array}}$$

P	F_0	$P \vee F_0$	T_0	$P \wedge T_0$
1	0	1	1	1
0	0	0	1	0

④ **Inverse laws** : Here for a given compound Proposition

$P \vee \sim P$ (or) $P \wedge \sim P \Leftrightarrow$

P	$\sim P$	$P \vee \sim P$
0	1	1
1	0	1

$\} T_0$

P	$\sim P$	$P \wedge \sim P$
0	1	0
1	0	0

$\} F_0$

⑤ **Domination laws**. Here, for a Compound Proposition

$P \vee T_0 \Leftrightarrow T_0$

$P \wedge F_0 \Leftrightarrow F_0$

⑥ **Commutative laws** :

$$P \vee q \Leftrightarrow q \vee P$$
$$P \wedge q \Leftrightarrow q \wedge P$$

P	q	$P \vee q$	$q \vee P$	$P \wedge q$	$q \wedge P$
0	1	1	1	0	0
1	0	1	1	0	0
1	1	1	1	1	1
0	0	0	0	0	0

Logically equivalent

⑦ **Absorption laws** :

$$(P \vee (P \wedge q)) \Leftrightarrow P$$
$$(P \wedge (P \vee q)) \Leftrightarrow P$$

p	q	p∧q	p∨(p∧q)	p∨q	p∧(p∨q)
1	0	0	1	1	1
0	1	0	0	1	0
1	1	1	1	1	1
0	0	0	0	0	0

8) <u>De-Morgan's law</u> :

$$\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$$

$$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$$

~(p∨q) =	~(p∧q)	~p	~q	~p∧~q
0	1	0	1	0
0	1	1	0	0
0	0	0	0	0
1	1	1	1	1

9) <u>Associative laws</u> :

$$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$$

$$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$$

~~p∧q~~	p	q	r	p∧q	(p∧q)∧r	q∧r	p∧(q∧r)
~~0~~	0	0	0	0	0	0	0
~~0~~	1	0	0	0	0	0	0
~~1~~	0	1	0	0	0	0	0
~~0~~	1	1	0	1	0	0	0
	0	0	1	0	0	0	0
	1	0	1	0	0	0	0
	0	1	1	0	0	1	0
	1	1	1	1	1	1	1

logically equilent

10) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$q \wedge r$	$p \vee (q \wedge r)$
0	1	0	1	0	0	0	0
1	0	0	1	1	1	0	1
0	0	0	0	0	0	0	0
1	1	0	1	1	1	0	1
0	1	1	1	1	1	1	1
1	0	1	1	1	1	0	1
0	0	1	0	1	0	0	0
1	1	1	1	1	1	1	1

→ show that

$[p \vee (p \wedge (p \vee q))]$ & p are logically equivalent

$p \vee ([(p \wedge p) \vee (p \wedge q)]$

$p \vee [p \vee (p \wedge q)]$

$p \vee p = p$ $\quad \left[\because p \vee (p \wedge q) \Leftrightarrow p \right.$

The given compound Propositions

$\sim(p \vee (\sim p \wedge q))$

$\sim p \wedge \sim(\sim p \wedge q)$

$\sim p \wedge (p \vee \sim q)$

$(\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$F_0 \vee (\sim p \wedge \sim q)$

$\sim p \wedge \sim q$

→ Prove the following logical c.p logically equivalent to each other without using truth tables.

$$[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$$

$$[(p \vee q) \wedge (p \vee \sim q)] \vee q$$

$$(p \wedge q) \vee r$$

$$\cancel{p \vee q}$$

$$\cancel{(p \vee (q \wedge q)) \vee (p \vee q)}$$

$$p \vee (q \wedge p)$$

$$p$$

$$[p \vee (q \wedge \sim q)] \vee q$$

$$(p \vee F_0) \vee q$$

$$p \vee q$$

Law of Negation of conditional :

Given a conditional Statement $p \to q$ its negation is obtained by using following laws

$$\boxed{\begin{array}{c} p \to q \Leftrightarrow \sim p \vee q \\ \sim(p \to q) \Leftrightarrow p \wedge \sim q \end{array}}$$

p	q	p→q	~p	~p∨q	~(p→q)	p∧~q	~q
0	1	1	1	1	0	0	0
1	0	0	0	0	1	1	1
0	0	1	1	1	0	0	1
1	1	1	1	1	0	0	0

logically equal

→ Prove following

$p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$

$p \to (\sim q \vee r)$

$p \to q$

$\sim p \vee (\sim q \vee r)$

$(\sim p \vee \sim q)$

$\sim p \vee (q \to r)$

$\sim p \vee (\sim q \vee r)$

$(\sim p \vee \sim q) \vee r$

$\sim (p \vee q) \vee r$

$\cancel{(p \vee q) \to r}$

$(p \wedge q) \to r.$

→ $(p \to q) \wedge (p \to r) \Leftrightarrow p \to (q \wedge r)$

$(\sim p \vee q) \wedge (\sim p \vee r)$

$\sim p \vee (q \wedge r)$

$p \to (q \wedge r)$

→ $(p \to q) \wedge (r \to q) \Leftrightarrow (p \vee r) \to q$

$(\underbrace{\sim p \vee r}_{p}) \wedge (\underset{q}{\sim r} \vee \underset{r}{q})$

$\sim p \vee (r \wedge \sim r) \vee \sim p \vee (r \wedge q)$

$(\sim p \vee F_0) \vee \sim p \vee (r \wedge q)$

$\sim p \vee \sim p \vee (r \wedge q)$

$\sim p \vee (r \wedge q)$

$p \rightarrow (r \wedge q)$

2) $(p \rightarrow q) \wedge (p \rightarrow \sim q) \Leftrightarrow \sim p$

$(\sim p \vee q) \wedge (\sim p \vee \sim q)$

$\sim p \vee (q \wedge \sim q)$

$\sim p \vee F_0$

$\sim p$

1) $(\sim p \vee q) \wedge (\sim r \vee q)$

$q \vee (\sim p \wedge \sim r)$

$q \vee \sim(p \vee r)$

$\sim(q \rightarrow (p \vee r))$

$(p \vee r) \rightarrow q$

→ Prove the followings

① $(\sim p \vee q) \wedge (\sim p \vee r) \Leftrightarrow (p \to q \wedge r)$

$(p \to q) \wedge (p \to r)$

$p \to (q \wedge r)$

②

$(p \to q) \wedge (r \to q) \Leftrightarrow (p \vee r) \to q$

$(\sim p \vee q) \wedge (\sim r \vee q)$

$(q \vee \sim p) \wedge (q \vee \sim r)$

$q \vee (\sim p \wedge \sim r)$

$q \vee \sim (p \vee r)$

$\sim (p \vee r) \vee q$

$(p \vee r) \to q$

③ $(p \to q) \wedge (p \to \sim q) \Leftrightarrow \sim p$

$(\sim p \vee q) \wedge (\sim p \vee \sim q)$

$\sim p \vee (q \wedge \sim q)$

$\sim p \vee (F_0)$

$\sim p$

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$\rightarrow$ 4) $[P \wedge (\sim r \vee q \vee \sim q)] \vee [(r \vee T_0 \vee \sim r) \wedge \sim q]$

$\Leftrightarrow P \vee \sim q$

$\Rightarrow [P \wedge (\sim r \vee T_0) \vee (r \vee T_0) \wedge \sim q]$

$(P \wedge T_0) \vee (T_0 \wedge \sim q)$

$P \vee \sim q$

5) i) $\underbrace{P \vee (q \wedge r)}_{P} \vee \sim \underbrace{[P \vee (q \wedge r)]}_{\sim P} \Rightarrow T_0$

first expression $a = P \vee (q \wedge r)$

$b = \sim (P \vee (q \wedge r))$

$a \vee b \rightarrow b = \sim a$

$\boxed{a \vee \sim a = T_0}$

ii) $(P \wedge (P \rightarrow q)] \rightarrow q$

$(P \wedge (\sim P \vee q)) \rightarrow q$

$((P \wedge \sim P) \vee (P \wedge q)) \rightarrow q$

$F_0 \vee (P \wedge q) \rightarrow q$

$(P \wedge q) \rightarrow q$

$\sim (P \wedge q) \vee q \Rightarrow \sim P \vee (\sim q \vee q)$

$\sim P \vee T_0 = T_0$

iii) $[p \wedge (p \to q)] \to q$

$$p \to (q \to (p \wedge q))$$

$$p \to (\sim q \vee (p \wedge q))$$

$$p \to ((\sim q \vee p) \wedge (\sim q \vee q))$$

$$p \to ((\sim q \vee p) \wedge T_0)$$

$$p \to (\sim p \vee p)$$

$$\sim p \vee (\sim p \vee p)$$

$$(\sim p \vee p) \vee \sim p$$

$$T_0 \vee \sim p$$

$$T_0$$

iv) $[(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r)) \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$

$[\cancel{(p \vee q)}$.

X $(\sim[(p \vee q) \to (\sim p \wedge (\sim q \vee \sim r)] \vee \sim(p \vee q)$

$\vee \sim(p \vee r)$

$\sim[(p \vee q) \to (\sim p \wedge (\sim q \vee \sim r)) \vee \sim p \wedge (\sim q \vee \sim r)$

$\sim(p \vee q) \to \;)$ X

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$(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r)) \vee \sim p \wedge (\sim q \vee \sim r)$

$\sim p \vee p$

$(p \vee q) \wedge T_0$.

$p \vee q$

Not a Tautology

Principle of Duality :

If U is a compound Propisition that contains the connections or connectives and ($\wedge$) or ($\vee$). Suppose we replace each occurence of and ($\vee$) with or ($\wedge$), or ($\vee$) with and ($\wedge$) respectively. Also, if U compound proposition contains Tautology (T_0) & Contradictory (F_0) as components. Suppose we replace each occurence of Tautology with contradictory and contradictory with Tautology respectively. then the resulting compound Proposition is called the dual of the compound proposition U. And it is denoted by "u^d". ex:

$u = [(p \vee q) \wedge \sim(F_0 \wedge (\sim q \vee T_0))$.

$u^d = [(p \wedge q) \vee \sim(T_0 \wedge (\sim q \vee F_0))$

The following two results are very important

1: $(u^d)^d \Leftrightarrow u$

The dual of dual form of u (C.P) is logically equal to compound proposition (u).

2. For any two compound propositions u and v, if $u \Leftrightarrow v$ i.e $u^d \Leftrightarrow v^d$. This is known as Principle of duality

→ The following c.p's

1) $q \to p$ $\quad \sim q \vee p \Rightarrow \boxed{\sim q \wedge p}$

2) $(p \vee q) \wedge r$ $\quad \boxed{(p \wedge q) \vee r}$

3) $(p \wedge q) \vee T_0$ $\quad \boxed{(p \vee q) \wedge F_0}$

4) $p \to (q \wedge r)$ $\quad \sim p \vee (q \wedge r)$

$\boxed{\sim p \wedge (q \vee r)}$

5) $(p \to q) \wedge (q \to p)$

$(\sim p \vee q) \wedge (\sim q \vee p)$

$\boxed{\text{Ans}: (\sim p \wedge q) \vee (\sim q \wedge p)}$

6) $p \veebar q$ =

p	q	$p \veebar q$
1	0	0
0	1	1
1	1	0
0	0	0

p	q	$\boxed{p \veebar q}$	~p	$\sim p \leftrightarrow q$	p	~q	$p \leftrightarrow \sim q$
0	0	0	1	1	0	1	0
1	1	0	0	1	1	0	0
0	1	1	1	0	0	0	1
1	0	1	0	0	1	1	1

$\boxed{p \leftrightarrow \sim q}$

$\cancel{q \leftrightarrow \sim p}$

$p \veebar q \Leftrightarrow \sim(\sim p \leftrightarrow q)$

$(\sim p \to \sim q) \wedge (\sim q \to \sim p)$

$(p \vee q) \wedge (\sim q \vee \sim p)$

$\sim((p \vee q) \wedge (q \wedge p))$

dual form $\leftarrow \sim((p \wedge q) \vee (q \vee p))$

$\sim(p \leftrightarrow \sim q)$

$(\sim p \wedge q) \vee (\cancel{\sim q}\; p \wedge \sim q)$

					0	0	0	
					0	0	0	
					1	0	1	
p	q	~p	~q			1	1	
0	0	1	1	0	0	0		
0	1	1	0	1	0			
1	0	0	1	0	1	0		
1	1	0	0	0	0	1		
						1		
						0		

→ write dual forms of following given CP.

→ $(p \leftrightarrow q) \veebar (q \leftrightarrow p)$

$((p \to q) \wedge (q \to p)) \veebar ((q \to p) \wedge (p \to q))$

$((\sim p \vee q) \wedge (\sim q \vee p)) \veebar ((\sim q \vee p) \wedge (\sim p \vee q))$

$\sim((\sim p \vee q) \wedge (\sim q \vee p) \Leftrightarrow (\sim q \vee p) \wedge (\sim p \vee q))$

→ $\sim(p \vee q) \wedge (p \vee \sim(q \wedge \sim s))$

$\sim(p \wedge q) \vee (p \wedge \sim(q \vee \sim s)) \Rightarrow u^d$

→ $(p \wedge q) \vee ((\sim p \vee q) \wedge (\sim r \vee s)) \vee (r \wedge s)$

$(p \vee q) \wedge ((\sim p \wedge q) \vee (\sim r \wedge s)) \wedge (r \vee s)$

write down duals

i) $p \to q \Rightarrow \sim p \vee q \Rightarrow \sim p \wedge q$

ii) $(p \to q) \to r$

$(\sim p \vee q) \to r$

$\sim(\sim p \vee q) \vee r$

$(p \wedge \sim q) \vee r$

$u^d = (p \vee \sim q) \wedge r$

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$\sim\big((\sim p \vee q) \wedge (\sim q \vee p) \rightarrow (\sim q \vee p) \wedge (\sim p \vee q)\big) \wedge$

$\big((\sim q \vee p) \wedge (\sim p \vee q) \rightarrow (\sim p \vee q) \wedge (\sim q \vee p)\big)$

$\sim\big((\sim(\sim p \vee q) \wedge (\sim q \vee p)) \vee ((\sim q \vee p) \wedge (\sim p \vee q))\big) \wedge$

iii)

$p \rightarrow (q \rightarrow r)$

$p \rightarrow (\sim q \vee r)$

$\sim p \vee (\sim q \vee r)$

$u^d = \sim p \wedge (\sim q \wedge r)$

$[\cancel{(\sim p \vee q) \wedge (p \wedge (p \wedge q))}]$

$[(\cancel{\sim p \vee q}) \wedge (\cancel{p \wedge q})]$

$[\cancel{(\sim p \vee q) \wedge (p \wedge q)}]$

$[\cancel{\sim p \vee q \wedge R}]$

IV

p	q	r
0	0	0
1	1	1

$[(\sim p \vee q) \wedge (p \wedge q)]$ $[\cancel{F_0 \vee q}]$

$\sim p \wedge (p \wedge q) \vee (q \wedge (p \wedge q))$

$(F_0 \wedge q) \vee (q \wedge p)$

$q \vee q \wedge p$

$q \wedge p$

q	q	
0	0	0
1	1	1

→ Verify the Principle of duality to the following logical equilance.

1)

$$\left[\sim(p\wedge q) \longrightarrow \sim p \vee (\sim p \vee q)\right]$$

$$\sim(p\wedge q) \rightarrow (\sim p \vee q)$$

$$\cancel{(\sim p \vee \sim q) \rightarrow (\sim p \vee q)}$$

$$(p\wedge q) \vee (\sim p \vee q)$$

$$\cancel{p\wedge (q \vee \sim p \vee q)}$$

$$(p \vee \sim p \vee q) \wedge (q \vee \sim p \vee q)$$

$$(T_0 \vee q) \wedge (\sim p \vee q)$$

$$T_0 \wedge (\sim p \vee q)$$

$$\sim p \vee q$$

→ $\text{ud} = \left(\sim(p\vee q) \longrightarrow \sim p \wedge (\sim p \wedge q)\right] \Leftrightarrow (\sim p \wedge q)$

$$\sim(p\vee q) \Rightarrow (\sim p \wedge q)$$

$$(p\vee q) \vee (\sim p \wedge q)$$

$$\cancel{(p \vee \sim p \wedge q) \vee (q \vee \sim p \wedge q)}$$

$(T_0 \wedge q) \vee (q \wedge \sim p)$

$\cancel{q \vee q \vee \sim p}$

$\cancel{q \vee \sim p}$

p	~p	
0	1	1
1	0	p
1	0	1
1	1	1

$(\sim p \wedge q) \vee (p \vee q)$

$(\sim p \vee p \vee q) \wedge (q \vee p \vee q)$

$(T_0 \vee q) \wedge (q \vee p)$

1 0 1
0 1 1

$(p \vee (\sim p \wedge q)) \vee (q \vee (\sim p \wedge q))$

$((p \vee \sim p) \wedge (p \vee q)) \vee ((q \vee \sim p) \wedge (q \vee q))$

$(T_0 \wedge (p \vee q)) \vee ((q \vee \sim p) \wedge q)$

$(p \vee q) \vee (q \wedge (q \vee \sim p))$

$(p \vee q) \vee ((q \wedge q) \vee (q \wedge \sim p))$

$(p \vee q) \vee (q \vee (q \wedge \sim p))$

$(p \vee q) \vee q \vee (\sim p \wedge q)$

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→ Express the following Propositions in the form of NAND and NOR connections

$\sim(p \wedge q) = p \uparrow q$

$\sim(p \vee q) = p \downarrow q$

①

$\sim p$	$\sim p$
$\sim p \vee \sim p$	$\sim p \wedge \sim p$
$\sim(p \wedge p)$	$\sim(p \vee p)$
$p \uparrow p$	$p \downarrow p$

② $p \wedge q$

$\sim(\sim(p \wedge q))$

$\sim(\sim p \vee \sim q)$

$\sim((\sim p \vee \sim q) \vee (\sim p \vee \sim q))$

$(\sim p \vee \sim q) \downarrow (\sim p \vee \sim q)$

$\sim(p \wedge q) \downarrow \sim(p \wedge q)$

$(p \uparrow q) \downarrow (p \uparrow q)$

③ $p \vee q$

$\sim(\sim(p \vee q))$

$\sim(\sim p \wedge \sim q)$

$\sim((\sim p \wedge \sim q) \vee (\sim p \wedge \sim q))$

$(\sim p \wedge \sim q) \downarrow (\sim p \wedge \sim q)$

$\sim(p \vee q) \downarrow \sim(p \vee q)$

$(p \downarrow q) \downarrow (p \downarrow q)$

④ $p \to q$

$\sim p \vee q$

$\sim(\sim(\sim p \vee q))$

$\sim(p \wedge \sim q)$

~~$\sim((p \wedge \sim q) \vee (p \wedge \sim q))$~~

~~$(p \wedge \sim q) \downarrow (p \wedge \sim q)$~~

$(p \uparrow \sim q)$

$p \uparrow (\sim q \vee \sim q)$

$p \uparrow \sim(q \wedge q)$

$p \uparrow q \uparrow q$

① $(p\vee q)\wedge(\sim p\wedge(\sim p\wedge q)) \Leftrightarrow \sim p\wedge q$

LHS: $(p\vee q)\wedge(\sim p\wedge q)$

$(p\wedge\sim p\wedge q)\vee(q\wedge\sim p\wedge q)$

$(F_0\wedge q)\vee(q\wedge\sim p)$

$(q\wedge F_0)\vee(q\wedge\sim p)$

$q\wedge(F_0\vee\sim p)$

$q\wedge\sim p$

$\sim p\wedge q$

duality

~~RHS~~

$(p\wedge q)\vee(\sim p\vee(\sim p\vee q)) \Leftrightarrow \sim p\vee q$

$(p\wedge q)\vee(\sim p\vee\sim p\vee q)$

$(p\wedge q)\vee(\sim p\vee q)$

$(p\vee\sim p\vee q)\wedge(q\vee\sim p\vee q)$

$(T_0\vee q)\wedge(q\vee\sim p)$

$T_0\wedge(\sim p\vee q)$

$\sim p\vee q$

Hence I verify Principle of duality

→ for any Proposition Prove following :

i) $p\uparrow(q\uparrow r) \Leftrightarrow \sim p\vee(q\wedge r)$

$\sim p(p\wedge(q\uparrow r))$

$\sim p(p\wedge\sim(q\wedge r))$

$\sim p\vee(q\wedge r)$

ii) $(p \uparrow q) \uparrow r \Leftrightarrow (p \wedge q) \vee \sim r$

$\sim(p \wedge q) \uparrow r$

$\sim(\sim(p \wedge q) \wedge r)$

$(p \wedge q) \vee \sim r$

I) $p \downarrow (q \downarrow r)$	$(p \downarrow q) \downarrow r$
$\sim(p \vee (q \downarrow r))$	$\sim((p \downarrow q) \vee r)$
$\sim(p \vee \sim(q \vee r))$	$\sim(\sim(p \vee q) \vee r)$
$\sim p \wedge (q \vee r)$	$(p \vee q) \wedge \sim r$

Hence it is observed that both a and b, c and d connections are not associative

$\rightarrow (p \leftrightarrow q)$

$(p \rightarrow q) \wedge (q \rightarrow p)$

$\sim(\sim(\cancel{(p \vee \sim q) \wedge (q \vee \sim q)}))$

$(p \uparrow q \uparrow q) \wedge (q \uparrow p \uparrow p)$

$\sim(\sim(p \uparrow q \uparrow q) \wedge (q \uparrow p \uparrow p))$

$\sim((p \downarrow q \downarrow q) \vee (q \downarrow p \downarrow p)$

$(p \downarrow q \downarrow q) \downarrow (q \downarrow p \downarrow p)$

Normal forms :

1) Disjunction Normal form

2) Conjunctive Normal form

① Disjunction NF : Given a compound proposition u. Suppose we obtain a compound proposition v such that $u \Leftrightarrow v$. ~~Here~~ 'v' is a disjunction of two or more compound proposition each of which is a conjunction involving the compounds of 'u' or their negations. Then 'v' is called disjunction normal form of u.

Ex: $u \Leftrightarrow (a \vee b \vee \sim c) \wedge (a \vee \sim b \vee c)$

→ 2) Conjuctive NF: A compound Proposition 'u', if we obtain an equivalent C.P 'v' which is a conjunction of two or more compound Proposition each of which a disjunction is involving in the components of 'u' or their negations. Then V is called a conjuctive NF of u.

Ex: $u \Leftrightarrow (a \wedge b \wedge \sim c) \vee (a \wedge \sim b \wedge c)$

→ find conjuctive NF's of a disjective NF of a compound proposition.

1) $p \wedge p \rightarrow q$

$p \wedge (\sim p \vee q)$

$(p \vee p) \wedge (\sim p \vee q) \rightarrow$ ~~Conjuctive~~ disjuctive NF

$(p \wedge p) \vee (\sim p \wedge q) \rightarrow$ ~~disjuctive~~ conjuctive NF

2) Convert the following compound propositions into disjuctive NF & conjuctive NF

① $\sim(p \vee q) \leftrightarrow (p \wedge q)$

$(\sim(p \vee q) \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow \sim(p \vee q))$

$((p \vee q) \vee (p \wedge q)) \wedge ((p \wedge q) \rightarrow \sim(p \vee q))$

$((p\vee q)\vee(p\wedge q))\wedge(\sim(p\wedge q)\vee\sim(p\vee q))$

$((p\vee q)\vee(p\wedge q))\wedge((\sim p\vee q)\vee\sim(p\vee q))$

$((p\vee q)\vee\sim(\sim(p\wedge q))\wedge((\sim p\vee q)\vee\sim(p\vee q))$

$((p\vee q)\vee\sim(\sim p\vee\sim q))\wedge((\sim p\vee q)\vee\sim(p\vee q))$

$\cancel{((p\vee p\wedge q)\vee(q\vee p\wedge q))\wedge((\sim p\vee\sim p\wedge\sim q)\vee(q\vee\sim p\wedge\sim q))}$

$\cancel{((p\wedge q)\vee(q\vee p))\wedge((\sim p\wedge\sim q)\vee(}$

$\longrightarrow$

$\cancel{(p\vee q)\wedge\sim(p\wedge q)}$

$(p\vee q)\wedge(\sim p\vee\sim q)$ b :

$\cancel{(p\wedge\sim p\vee\sim q)}\vee(q\wedge\sim p\vee\sim q)$

$\cancel{(t_0\vee\sim q)\vee(}$

$((p\vee q)\vee(p\wedge q))\wedge((\sim p\vee q)\vee\sim(p\vee q))$

$(((p\vee q\vee p)\wedge(p\vee q\vee q))\wedge((\sim p\vee q\vee\sim p)\wedge(\sim p\vee q\vee\sim q))$

$((p\vee q)\wedge(p\vee q))\wedge((\sim p\vee q)\wedge(T_0))$

$(p\vee q)\wedge(\sim p\vee q)$. $\longrightarrow$ disjunction

$(p\wedge q)\vee(\sim p\wedge q)$ $\longrightarrow$ conjuction

3) Q2. $p \to \{(p \to q) \wedge \sim(\sim q \vee \sim p)\}$

$p \to$

(i) $= (\_\_\_\_\_) \to (\_\_\_)$

$\equiv \cancel{(\ \vee\ ) \wedge (\ \ )}$

$\equiv (\ \wedge\ ) \vee (\ \wedge\ )$

$\to$

$p \to \{(\sim p \vee q) \wedge (q \wedge p)\}$

$\sim p \vee ((\sim p \vee q) \wedge (q \wedge p))$

$\cancel{\sim p \vee ((\sim p \vee q \wedge q) \wedge (\sim p \vee q \wedge p))}$

$\cancel{\sim p \vee ((\sim p \vee q) \wedge}$

$\cancel{\sim p \vee ((\sim p \vee q \wedge q) \wedge (\sim p \vee q}$

$\sim p \vee ((\sim p \wedge q \wedge p) \vee (q \wedge q \wedge p))$

$\sim p \vee ((F_0 \wedge q) \vee (q \wedge p))$

$\sim p \vee (F_0 \vee (q \wedge p))$

$\sim p \vee (q \wedge p)$

$(\sim p \vee q) \wedge (\sim p \vee p) \rightarrow$ disjuctive

$\rightarrow (\sim p \wedge \sim p) \vee (q \wedge p) \rightarrow$ conjuctive

Principal of Normal Forms:

Given two simple propositions p and q, the compound propositions are

$$\left.\begin{array}{l} p \wedge q \\ p \wedge \sim q \\ \sim p \wedge q \\ \sim p \wedge \sim q \end{array}\right\} \text{are called min terms involving } p \,\&\, q$$

The duals of these min terms are

$$\left.\begin{array}{rl} (p \wedge q)^d & = p \vee q \\ & = p \vee \sim q \\ & \sim p \vee q \\ & \sim p \vee \sim q \end{array}\right\} \text{are called max terms}$$

Given a compound proposition 'u' involving two simple proposition's p and q an equivalent compound proposition v, consisting of disjunctions of min terms, involving p and q. It is known as principle of disjunctive normal form or Sum of Products canonical form.

Ex :

$$u \equiv v$$

$$(p \wedge q) \vee (\sim p \wedge \sim q)$$

min terms   min terms combined with OR

Similarly given compound Proposition 'u' involving two simple proposition p and q and an equal compound proposition v which is logically equal to u consisting of conjuctions of the max terms involving p and q. It is known as Principle of conjunctive NF or Product of sums canonical form.

Ex :

$$U \equiv V$$

$$(p \vee q) \wedge (\sim p \vee \sim q)$$

max terms   max terms combined with conjuction

Q) obtain the principle of disjunctive NF of the given compound proposition.

i) $p \vee (p \wedge q)$

$(p \wedge T_0) \vee (p \wedge q)$

$p \wedge (\sim q \vee q) \vee (p \wedge q)$

$(p \wedge \sim q) \vee (p \wedge q) \vee (p \wedge q)$

$$\underbrace{(p \wedge q)}_{\text{min terms}} \vee \underbrace{(p \wedge \sim q)}_{\text{min term}} \qquad \sim p \wedge (p \vee q)$$

Principle of disjunctive NF

we observed that the right hand side is a disjunction min terms involving p & q

$\therefore$ This is a principle of disjunctive NF

b) $(\sim p) \vee q \quad = (\sim p \wedge T_0) \vee (q \wedge T_0)$

$$(\sim p \wedge (\sim q \wedge q)) \vee (q \wedge (\sim p \wedge p))$$

$$\underbrace{[(\sim p \wedge \sim q) \wedge (\sim p \wedge q)]}_{\text{min terms}} \vee \underbrace{[(q \wedge \sim p) \wedge (q \wedge p)]}_{\text{min terms}}$$

$\cancel{(\sim p \wedge \sim q) \vee (\sim p \wedge q)}$

disjunctive NF

c). $p \rightarrow \{(p \rightarrow q) \wedge \sim(\sim q \vee \sim p)\}$

$$p \rightarrow \{(\sim p \vee q) \wedge (p \wedge q)\}$$

$$\sim p \vee \{(\sim p \vee q) \wedge (p \wedge q)\}$$

$$\sim p \vee (\sim p \vee q) \wedge \sim p \vee (p \wedge q)$$

$$(\sim p \vee q) \wedge (\sim p \vee p) \wedge (\sim p \vee q)$$

$\sim p$

$$\sim p \vee \{(\sim p) \wedge (q \wedge p)) \vee (q \wedge (q \wedge p)\}$$

$$\sim p \vee \{(F_0 \wedge q) \vee (q \wedge p)\}$$

$$\sim p \vee \{f_0 \vee (p \wedge q)\}$$

$$\sim p \vee \{p \wedge q\}$$

$$\sim p \wedge \{\sim q \vee q\} \vee \{p \wedge q\}$$

$$(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge q)$$

→ $p \rightarrow q$

$$\sim p \vee q$$

$$\sim p \wedge (\sim q \vee q) \quad \vee q \wedge (\sim p \vee p)$$

$$(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee \cancel{(q \wedge q)}\, (q \wedge \sim p) \vee (q \wedge p)$$

→ $p \leftrightarrow q$

Obtain the Principle of conjuctive NF's of the following C.P

i) $(\sim p \to q) \wedge (q \leftrightarrow p)$

$$(p \vee q) \wedge (q \to p) \wedge (p \to q)$$

$$(\underline{p \vee q}) \wedge (\underline{\sim q \vee p}) \wedge (\underline{\sim p \vee q})$$

Max terms

Principle Conjuctive NF

2) $(p \wedge q) \vee (\sim p \wedge q)$

$$((p \wedge q) \vee \sim p) \wedge ((p \wedge q) \vee q)$$

$$(\sim p \vee (p \wedge q)) \wedge (q \vee (p \wedge q))$$

$$((\sim p \vee p) \wedge (\sim p \vee q)) \wedge ((q \vee p) \wedge (q \vee q))$$

$$(\sim p \vee q) \wedge \cancel{(p \vee q)} ((q \vee p) \wedge (q \vee F_0))$$

$$(\sim p \vee q) \wedge ((q \vee p) \wedge (q \vee (\sim p \wedge p)))$$

$$(\sim p \vee q) \wedge ((q \vee p) \wedge ((q \vee \sim p) \wedge (q \vee p)))$$

$$(\sim p \vee q) \wedge (q \vee p) \wedge (q \vee \sim p) \wedge (q \vee p)$$

$$(\sim p \vee q) \wedge (q \vee p)$$

→ we observe that the right hand side is a conjuction of maximum terms involving p & q. This is a principle of conjuctive Normal form. of a given compound proposition.

Converse, Inverse and Contrapositive :

① converse :

$q \rightarrow p$ is the converse of $p \rightarrow q$

$p \rightarrow q$ is converse of $q \rightarrow p$

② Inverse :

Inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$

③ Contrapositive :

It means combination of converse and inverse

→ contrapositive of $q \rightarrow p$ is $\sim p \rightarrow \sim q$

Q. write down contra positive of $p \rightarrow (q \rightarrow r)$

a) only one occurence of the "$\rightarrow$" symbol

b) No occurence of "$\rightarrow$"

a) $p \rightarrow (q \rightarrow r)$

$\sim(q \rightarrow r) \rightarrow \sim p$

$(q \rightarrow r) \vee \sim p$

$\boxed{(\sim q \vee r) \vee \sim p}$

→ Truth table for converse, inverse, contrapositive

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	0	0	1	1	1	1

<u>Logically Implication:</u>

* Difference between logical equilance and logically implication?

→ When a hypothetical statement $p \to q$ is such that q is true whenever p is true we say that $p \Rightarrow q$ this is logical implication. Symbolically written as $\boxed{p \Rightarrow q}$

Denoting the word implies.

→ When a hypothetical statement $p \to q$ is such that q is not necessarily true whenever p is true i;e $p \not\Rightarrow q$. we say that p doesn't necessarily implies q. This is symbolically written as $\boxed{p \not\Rightarrow q}$.

Prove the following logically implication :-

i) $p \wedge (p \to q) \Rightarrow q$

$p \wedge (\sim p \vee q)$

$(p \wedge \sim p) \vee (p \wedge q)$

$f_0 \vee (p \wedge q)$

$p \wedge q$

p	q	$p \to q$	$p \wedge (p \to q)$
0	0	1	0
1	0	0	0
0	1	1	0
1	1	1	1

logically implies

ii) $p \vee (p \to q) \Rightarrow p$

p	q	$p \to q$	$p \vee (p \to q)$
0	0	1	1
1	0	0	1
0	1	1	1
1	1	1	1

logically implies

iii) $((p \to q) \wedge \sim q) \Rightarrow \sim p$

p	q	$p \to q$	$\sim q$	$(p \to q) \wedge \sim q$	$\sim p$
0	0	1	1	1	1
1	0	0	1	0	0
0	1	1	0	0	1
1	1	1	0	0	0

logically not implies

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Prove the following

→ $(p \wedge (p \to q) \wedge r) \Rightarrow [(p \vee q) \to r]$

p	q	r	$p \to q$	$p \wedge (p \to q)$	$p \wedge (p \to q) \wedge r$	$(p \vee q)$	$(p \vee q) \to r$
1	0	0	0	0	0	1	0
0	1	0	1	0	0	1	0
1	1	0	1	1	0	1	0
0	0	0	1	0	0	0	1
1	0	1	0	0	0	1	1
0	1	1	1	0	0	1	1
1	1	1	1	1	1	1	1
0	0	1	1	0	0	0	1

logically implies.

→ $(p \vee q) \wedge \sim p \Rightarrow q$

p	q	$(p \vee q)$	$\sim p$	$(p \vee q) \wedge \sim p$
1	0	1	0	0
0	1	1	1	1
1	1	1	0	0
0	0	0	1	0

logically not implies

→ Prove the following are logical implication

① $(p \to q) \wedge ($

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$(p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s) \longrightarrow (\sim p \vee \sim r)$

p	q	r	s	$p\to q$	$r \to s$	$\sim q \vee \sim s$	Ans	Ans	$\sim p \vee \sim r$
1	0	0	0	0	1	1	0	0	1
0	1	0	0	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1
1	0	1	0	0	0	1	0	0	1
0	1	1	0	1	0	1	0	0	0
1	1	1	0	1	0	1	0	0	1
0	0	1	0	1	0	1	0	0	0
1	0	0	1	0	1	1	1	1	1
0	1	0	1	1	1	0	1	0	1
1	1	0	1	1	1	0	1	0	1
0	0	0	1	1	1	1	1	1	1
1	0	1	1	0	1	1	0	0	1
0	1	1	1	1	1	0	1	0	0
1	1	1	1	1	1	$\boxed{0}$	1	0	0
0	0	1	1	1	1	1	1	1	1

~~$(p\to q) \wedge (r \to s)$~~

$\therefore$ By using above truth table the given compound expression is logically not implies

Necessary and Sufficient Condition :

Consider two propositions p and q whose values are interrelated. Suppose $p \Rightarrow q$ then in order that q may be true it is sufficient that p is true. Also if p is true then it is necessary that q is true.

In view of this interpretation all of the following statements are taken to carry the same meaning.

i) $p \Rightarrow q$ ii) p is sufficient for q

iii) q is necessary for p

These statements are also written in following forms: ii) q if p

iii) p only if q

→ For two propositions p and q the following situations are possible

i) $p \Rightarrow q$ but $q \not\Rightarrow p$

ii) $q \not\Rightarrow p$. but $p \Rightarrow q$

iii) $p \Rightarrow q$ but $q \Rightarrow p$

→ Then, in the first of the above cases p is sufficient but not a necessary condition for q

→ In the second case p is a necessary but not a sufficient condition for q Thus a condition may be necessary but not sufficient.

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→ In the last case p is necessary & sufficient condition for q and vice-versa

→ Write down necessary & sufficient conditions for p & q

p : ABC is a equilateral triangle

q : ABC is a equiangle triangle

(write the suffi)

$p \Rightarrow q$ but $q \Rightarrow p$

Rules of inference for propositional logic :

Consider a set of propositions $p_1, p_2, ---, p_n$ and a proposition c. Then a compound proposition of the form $p_1 \wedge p_2 \wedge p_3 ---- p_n \rightarrow c$ is called an argument. Here, $p_1, p_2, ---, p_n$ are called the premises of the argument and c is called conclusion of the argument.

→ "Inference means deriving conclusions from evidences"

Rules of inferences :

1. Rule of conjunctive simplification : This rule states that for any two propositions p and q if $p \wedge q$ is true then p is true then

if $p \wedge q \Rightarrow p$

This rule follows from the definition of conjuction

2. Rule of disjuctive Amplification : This rule states that for any two propositions p and q if p is true then $p \vee q$ also true

if $\boxed{p \Rightarrow p \vee q}$

true then true

3.) Rule of Syllogism : This rule states that any three propositions p, q, r if $p \rightarrow q$ is true and $q \rightarrow r$ is true then $p \rightarrow r$ is true

if $p \rightarrow q$ is true &
$q \rightarrow r$ is true

then $p \rightarrow r$ is true

This rule follows from the tautology.

4) Modus pones : This rule states that if p is true and $p \rightarrow q$ is true then it is logically implies to q is also true

if p is true
$p \rightarrow q$ is true

then $p \wedge (p \rightarrow q) \Rightarrow q$

5) Modus Tollens : This rule states if $p \rightarrow q$ is true and q is false then the result p is also false.

$p \rightarrow q$ is true
q is false
then $\boxed{(p \rightarrow q) \wedge \sim q \Rightarrow \sim p}$

6) Rule of disjuctive Syllogism : This rule states that if $p \vee q$ is true and p is false then result q is true

i.e $\boxed{(p \vee q) \wedge \sim p \Rightarrow q}$

7) Test Validity of following arguments

$$\begin{array}{l} (p \to r) \\ q \to r \\ \hline (p \vee q) \to r \end{array}$$

$(p \to r) \wedge (q \to r) \Leftrightarrow (p \vee q) \to r$

$\Big((p \to r) \wedge (\sim q \vee r)$

$(p \to r) \wedge \sim q \vee (p \to r) \wedge r$

$\sim p \vee p \Big)$

$$\begin{array}{l} (p \to r) \wedge (r \to q) \\ \hline p \to q \end{array}$$

$(\sim p \vee r) \wedge (\sim q \vee r)$

$r \vee (\sim p \wedge \sim q)$

$\sim (p \vee q) \vee r$

$(p \vee q) \to r$

→ Test whether following arguments are Valid or not

$$\begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline q \vee s \end{array}$$

$\overset{T}{(p \to q)} \wedge \underline{\overset{T}{(r \to s)}} \wedge (p \vee r) \Leftrightarrow q \vee s$

$(\sim p \vee q) \wedge (\sim r \vee s) \wedge (p \vee r)$

$(\sim p \vee q) \wedge (p \vee r) \wedge (\sim r \vee s)$

$(\sim p \to q) \wedge (\sim s \to \sim r) \wedge (r \vee p)$

$(\sim p \to q) \wedge (\sim s \to \underline{\underline{\sim r}}) \wedge (\underline{\underline{\sim r}} \to p)$

$p \to q$

$(\sim p \to q) \wedge (\sim s \to \sim r) \wedge (\sim r \to p)$

rule of syllogism

$\sim p$

$(\sim p \to q) \wedge (\sim s \to p)$

$(\sim p \to q) \wedge (\sim s \to p)$

$(\sim q \to \underline{\sim p}) \wedge (\underline{\sim p} \to s)$

$(\sim q \to s)$

$(q \vee s)$

$\Rightarrow \quad (p \to q) \wedge (r \to s) \wedge (\sim q \vee \sim s) \Leftrightarrow \sim (p \wedge r)$

$(p \to q) \wedge (\overset{\sim s \to \sim r}{\cancel{\sim r \vee s}}) \wedge (\sim q \vee \sim s)$

$(p \to q) \wedge (\cancel{\ldots}\ r \to s) \wedge (\cancel{\ldots})$

$(p \to q) \wedge ($

$\rightarrow$ ~~$(p\rightarrow q)\wedge(r\rightarrow s)\wedge(p\vee r)\Leftrightarrow$~~

$(p\rightarrow q)\wedge(r\rightarrow s)\wedge(\sim q\vee\sim s)\Leftrightarrow\sim(p\wedge r)$

$\underbrace{(p\rightarrow q)\wedge(r\rightarrow s)\wedge(q\rightarrow\sim s)}$

$(p\rightarrow\sim s)\wedge(r\rightarrow s)$

$(p\rightarrow\sim s)\wedge(\sim s\rightarrow\sim r)$

$(p\rightarrow\sim r)$

$\sim p\vee\sim r$

$\sim(p\wedge r)$

$\rightarrow$ $(p\rightarrow r)\wedge(\sim p\rightarrow q)\wedge(q\rightarrow s)\Leftrightarrow\sim r\rightarrow s$

$(p\rightarrow r)\wedge(\sim p\rightarrow s)$

$(\sim r\rightarrow\sim p)\wedge(\sim p\rightarrow s)$

$(\sim r\rightarrow s)$

$\rightarrow$ $((\sim p\vee\sim q)\rightarrow(r\wedge s))\wedge\boxed{(r\rightarrow t)\wedge\sim t}$ $\rightarrow$ modus tollens

~~$\sim(p\rightarrow\sim q)\vee(r\wedge s)\wedge(r\rightarrow t)\wedge\sim t$~~

~~$\sim(p\rightarrow\sim q)\vee($~~

$\rightarrow$ $(\sim p\vee\sim q)\rightarrow(r\wedge s)\wedge\sim r$

$\sim(\sim p \vee \sim q) \vee (r \wedge s) \wedge \sim r$ $\quad (p \to q) \wedge r$

$((p \wedge q) \vee (r \wedge s)) \wedge \sim r$ $\quad p \to (q \wedge r)$

$((p \wedge q) \wedge \sim r) \vee ((r \wedge s) \wedge \sim r)$

$((p \wedge q) \wedge \sim r) \vee s$

$\cancel{(\sim(p \wedge q) \to \sim r) \vee s}$ $\quad r \wedge s$

$\to ((p \to \sim q) \to (r \wedge s)) \wedge \sim r$

$(\sim(p \to \sim q) \vee (r \wedge s)) \wedge \sim r$

$(\sim(p \to \sim q) \wedge \sim r) \vee s$

$\to ((\sim p \vee \sim q) \to (r \wedge s)) \wedge (r \to t) \wedge \sim t$

$((\sim p \vee \sim q) \to (r \wedge s)) \wedge \underline{\sim r}$

by applying rule of disjunction amplification

$((\sim p \vee \sim q) \to (r \wedge s)) \wedge \underline{(\sim r \vee \sim s)}$

$\boxed{(\sim(p \wedge q) \to (r \wedge s))} \wedge (\sim r \vee \sim s)$

$\cancel{(p \wedge q) \vee (r \wedge s)}$

$\sim(\sim p \vee \sim q) \vee (r \wedge s) \wedge (\sim r \vee \sim s)$

$(p \wedge q) \Rightarrow p$

Quantifiers :

consider the following Propositions

① All squares are rectangles

② For every integer x, x^2 is a non-negetive integer

③ Some determinants are equal to '0'

④ There exist a real number whose Square is equal to itself.

In this Propositions the words all, Every, Some, there exist are associated with the idea of a quantity such words are called as Quantifiers.

→ The Propositions indicated above may be re-writtened in alternative forms as explained below. Let 'S' denote the set of all Squares Then the Proposition 1 may be rewrittened as for $\forall x \in S$, x is a rectangle

Symbollically writtened as $\forall x \in S, P(x)$

→ $\forall x \in A, q(x)$

→ $\exists x \in d, s(x)$

→ $\exists x \in b, r(x)$

~~Similarly the Proposit~~

It is observed that '∀' has been used to denote the phrases "for all" and "for Every" in logic these phrases are regarded as equivalent phrases.

→ The phrases "for each", "for any" are also taken to be Equivalent. The symbol '∀' is used to denote all those phrases. Each of these equivalent phrases is called the universal Quantifiers.

→ for phrases ③ & ④ representing by using "there exist" symbol.

There exist :- ∃

where the symbol denotes phrase for "some one" "atleast one".

It is observed that the '∃' symbol has been used to denote the phrase "for someone", "atleast one", "there exist one".

→ The symbol "∃" is used to denote all of these equivalent phrases. Each of these equivalent phrases is called "Existential Quantifiers".

$\forall(\,) = $ ("for all" / for any / for each / for every)

$\exists(\,) = $ ("there exist / there is / for some / atleast one)

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Quantified Statement : A Proposition involving the universal or the existential Quantifier is called a Quantified statement.

→ A Quantified Statement is a Proposition of the form "$\forall x \in S, P(x)$" (or) "$\exists x \in S, P(x)$" where $P(x)$ is an open statement and S is the universe for x in $P(x)$

→ 1) Write down following Quantified Statements in the form of symbolic representation -

① Atleast one integer is even

$$\exists\ x \in I,\ P(x)$$

② There exist a positive integer i.e even

$$\exists\ x \in Z+,\ (P(x) \wedge q(x))$$

$P(x)$: x is even
$q(x)$: x is odd

③ Some even integers are divisible by 3

$$\exists\ x \in Z,\ (P(x) \wedge q(x))$$

$P(x)$: x is even
$q(x)$; x is divisible by 3

④ Every integer is either even or odd

$$\forall\ x \in Z\ (P(x) \veebar q(x))$$

$P(x)$: x is odd
$\sim P(x)$: x is even

⑤. If x is even and a Perfect Square then x is not divisible by 3.

$$\boxed{\cancel{\forall x,\ P(x) \Rightarrow \sim q(x)}}$$

$P(x)$ is even
$q(x)$ is P.S
$r(x)$ is not divisible by 3

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Write each of the following in Symbolic forms:

① All men are good

$P(x)$ is a men

$q(x)$: x is good

$\forall x$, if x is men then x is good

$$\boxed{\forall x, (P(x) \rightarrow q(x))}$$

② No men are good

$P(x)$ is a men

$\sim q(x)$: x is not good

$\forall x$, if x is men then x is not good

$$\boxed{\forall x, (P(x) \rightarrow \sim q(x))}$$

③ Some men are good

$P(x)$: x is a men

$q(x)$: some x are good.

$\exists x$, such that x is a men and x is good

$$\boxed{\exists x, (P(x) \wedge q(x))}$$

④ Some men are not good

$$\boxed{\exists x, (P(x) \wedge \sim q(x))}$$

→ If x is odd or is not divisible by 7 then x is divisible by 3

$$\boxed{(P(x) \vee q(x)) \rightarrow r(x)}$$

→ $(P(x) \wedge q(x)) \rightarrow \sim r(x)$

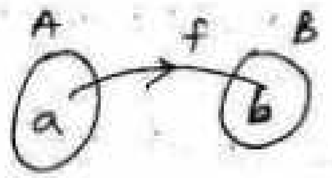
UNIT-II

<u>Functions</u>: Let A and B are two sets, then a function $f: A \to B$ is a relation, such that for each element a in set A there is a unique b (element) in set B such that $(a,b) \in f$.

→ We note $b = f(a)$, b is called image of a. 'a' is called Pre-image of 'b'.

→ 'a' is also called as argument of the function f.

→ $b = f(a)$ is called the value of the function f for the argument 'a'



$f: A \to B$

<u>Note</u>:

→ Every function is a relation, but a relation need not be a function

→ In function $f: A \to B$ 'A' is called the domain of the function f and 'B' is called the co-domain of function 'f'.

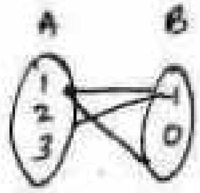
→ The subset of set B consists of images of all elements of A under function f.

is called the range of function f. and it is denoted by $f(A) = B$.

→ Let $A = \{1,2,3\}$ $B = \{-1,0\}$ and R be a relation from $A \to B$. Defined by

$$R = \{(1,-1),(1,0),(2,-1)\}$$ is R a

function from $A \to B$

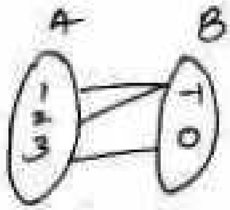


under R, the element '1' of set A is related to two different elements -1 and 0 of set B. ∴ R is not a function.

2) Let $A = \{1,2,3\}$ $B = \{-1,0\}$ and S be the relation $S: A \to B$ defined by

$$S = \{(1,-1),(2,-1),(3,0)\}$$

Is S a function from $A \to B$



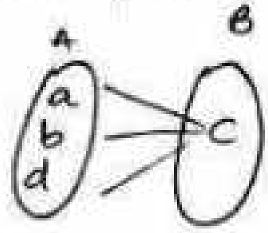
Is a function

Types of functions :

1) Identity function : A function $f: A \to A$ such that $f(A) = A$ for every element $a \in A$ is called the identity function on set A.

2) constant function : If a function $f: A \to B$ such that for every $a \in A$ where c is a

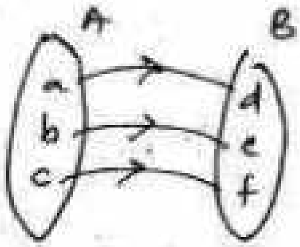
fixed element of B is called constant function.



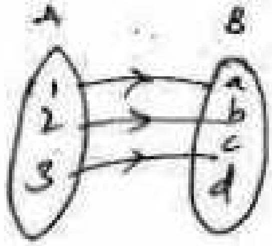
$$f(a) = f(b) = f(d) = c$$

3) <u>Onto function</u> : (<u>Surjuctive function</u>) :

A function $f: A \to B$ is said to be a onto function if every element of B has a pre-image in A

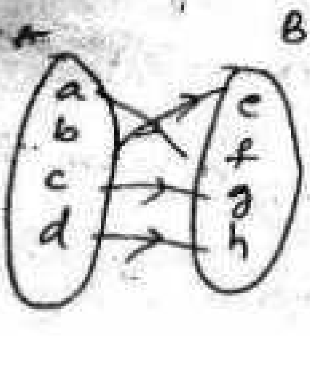


Onto



not Onto

4) <u>One to One function</u> : A function $f: A \to B$ is said to be a one to one function if different elements of set A have different images in set B :



5) <u>inverse function:</u> A function $f: A \to B$ is said to be a inverse function if $\exists$ function $g: B \to A$ such that

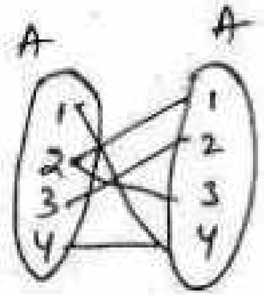
$$f^{-1}(b) = g \begin{cases} g \circ f = I_A \ \& \\ f \circ g = I_B \end{cases}$$

where I_A and I_B are identity function on set A & B then g is called inverse function of f then

$$f = g^{-1} \text{ or } f = g^{-1}$$

<u>Ex</u>: Let $A = \{1, 2, 3, 4\}$ determine whether or not the following relations on A are functions

1) $f = \{(2,3), (1,4) (2,1) (3,2) (4,4)\}$



2) $g = \{(3,1), (4,2), (1,1)\}$

'2' has no image
not a function

3) $h = \{(2,1), (3,4), (1,4), (2,1), (4,4)\}$

function

→ Let $A = \{0, \pm 1, \pm 2, \pm 3\}$ consider $f: A \to R$ defined by $f(x) = x^3 - 2x^2 + 3x + 1$ for $x \in A$

$27 - 18 + 9 + 1$

$f(2) = 8 - 8 + 6 + 1 = 7$ $\quad f(0) = 1$

$f(-2) = -8 - 8 - 6 + 1$ $\quad f(1) = 1 - 2 + 3 + 1 = 3$

$f(-1) = -1 - 2 - 3 + 1 = -5$

$\frac{\begin{matrix}37 \\ 18\end{matrix}}{19}$

$f = \{1, -5, 3, 7, -21, 19, -63\}$

Of $f : R \to R$ be defined by $f(x) = x^2 + 1$ determine the images of following subsets R

i) $A_1 = \{2, 3\}$ $\quad f(2) = 5$

$f(3) = 10$

ii) $A_2 = \{-2, 0, 3\}$

$f(x) = x^2 + 1$

$= 5$
1
10

iii) $A_3 = \{0, 1\} = 1$
2

iv) $A_4 = \{-6, 3\} = 37$
10

→ find the inverse of following functions.

$f(x) = 3x^2 + 4$

$f(x) = y$

$\cancel{f^{-1}(x) \neq \frac{y}{3x^2+4}}$ $\quad x = f^{-1}(y)$

let. $y = 3x^2 + 4$

$3x^2 = y - 4$

$x = \sqrt{\frac{y-4}{3}}$

$f^{-1}(y) = \sqrt{\frac{y-4}{3}}$

$f^{-1}(x) = \sqrt{\frac{x-4}{\ldots}}$

$f(x) = \frac{10}{\sqrt[5]{7-3x}}$

$f(x) = y$

$x = f^{-1}(y)$

$y = \frac{10}{\sqrt[5]{7-3x}}$

$\sqrt[5]{7-3x} = \frac{10}{y}$

$7 - 3x = \left(\frac{10}{y}\right)^5$

$\frac{7 - \left(\frac{10}{y}\right)^5}{3} = x$

$$\frac{7y^5-(10)^5}{3y^5}=x$$

$$f^{-1}(x)=\frac{7x^5-10^5}{3x^5}$$

$\rightarrow$ $f(x)=4e^{6x+2}$

$\log a+\log b=\log ab$

$f(x)=y$

$x=f^{-1}(y)$

$y=4(e^{6x+2})$

$\log y=\log 4+\log(e^{6x+2})$

$\log y=2\log 2+6x+2$

$$\frac{\log y-2\log 2-2}{6}=f^{-1}(y)\quad\forall y\in R$$

$$f^{-1}(x)=\frac{\log x-2\log 2-2}{6}$$

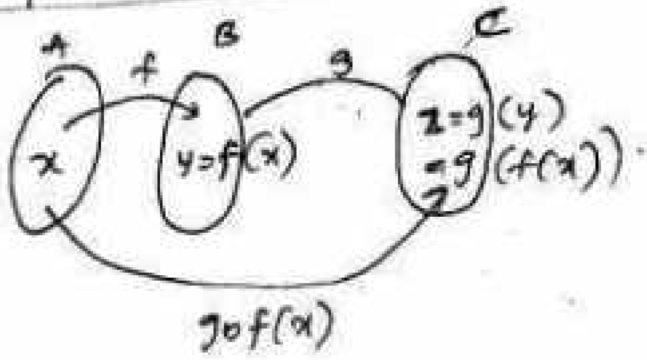
$$f^{-1}(x)=\frac{\log x-0.602-2}{6}$$

$$f^{-1}(x)=\frac{\log x-2.602}{6}\quad\forall x\in R$$

<u>composition function</u> : Let a function $f:A\rightarrow B$ and $g:B\rightarrow C$ are two functions then the composition function can be written as $gof(x)$ $:A\rightarrow C$

such that $gof(x)=g(f(x))$

Pictorial representation of composition function :



→ Let $f(x) = x^2+3 \quad g(x) = 2x^2+4x+3 \quad h(x) = 4x^3$

find $(fog)oh(x)$

$f(g(h(x))$

$f(g(4x^3)$

$f(2(4x^3)^2+4(4x^3)+3)$

$f(32x^6+16x^3+3)$

$(32x^6+16x^3+3)^2+3$

$(32x^6)^2+256x^6+9+1024x^9+96x^3+192x^6+3$

$1024x^{12}+256x^6+1024x^9+96x^3+192x^6+12$

2) $g(f(h(x))$

$g(f(4x^3))$

$g(16x^6+3)$

$2(16x^6+3)^2+4(16x^6+3)+3$

$2(256x^{12}+9+96x^6)+64x^6+12+3$ 18 15 33

$512x^{12}+192x^6+64x^6+33$

3) $h(f(g(x))$

$h(f(2x^2+4x+3))$

$h((2x^2+4x+3)^2+3)$

$h(4x^4+16x^2+9+16x^3+24x+12x^2+3)$

$h(4x^4+16x^3+28x^2+24x+12)$

$4(4x^4+16x^3+28x^2+24x+12)^3$

4) $h(g(f(x))$

$h(g(x^2+3)$

$h(2(x^2+3)^2+4(x^2+3)+3)$

$h(2(x^4+9+6x^2)+4x^2+12+3)$

$h(2x^4+16x^2+18+12+3)$

$h(2x^4+16x^2+33)$

$4(2x^4+16x^2+33)^3$

$\begin{array}{r} 15 \\ 18 \\ \hline 3 \end{array}$

$\begin{array}{r} 121 \\ 4 \\ \hline 484 \\ 11 \\ \hline 473 \end{array}$

$\Rightarrow$ Let $f(x) = 6x^2+4x+2$

$g(x) = 4x^2+x$

find values of $g(f(-9))$ $f(g(-11)$

$g(6(-9)^2+4(-9)+2)$

817668

$f(4(-11)^2-11)$

$f(473)$

1344268

for Ex1 of a composite function $f(g(h(1))$

$f(g(4)$

$f(32+16+3)$

$f(51)$

$(51)^2+3$

2604

$$\begin{array}{r} 32 \\ 19 \\ \hline 51 \end{array} \qquad \begin{array}{r} 51 \\ 51 \\ \hline 151 \\ 2554 \\ \hline 2601 \end{array}$$

Recursive function : A function called itself.

→ consider the factorial function $f(n) = n!$ which is defined for every natural number 'n'. The explicity method of describing this function is $f(0) = 1$ i.e $f(0) = 0! = 1$ and $f(n) = 1 \cdot 2 \cdot 3 \cdots n$ for $n \in Z^+$ this recursive method of describing this function is $f(0) = 1$

$f(n) = n f(n-1)$

$f(n-1) = (n-1) f(n-2)\, n$

bijective function : if $f: A \to B$ is a bijective function. Such that f is a onto function and one - one function

→ Obtain recursive defination for following function, $f(n) = a_n$ in each of following cases

i) $a_n = 6^n$

$a_0 = 6^0 = 1$

$a_1 = 6^1 = 6 \cdot 1 = 6 \cdot a_0$

$a_2 = 6^2 = 6 \cdot 6^1 = 6a_1$

$a_3 = 6^3 = 6 \cdot 36 = 6a_2$

$|$

$a_n = 6^n = 6a_{n-1}$

$a_{n+1} = 6^{n-1} = 6a_n$

Thus a recursive function of a given function $a_n = 6^n$ is $a_0 = 1$ & $a_{n+1} = 6a_n \ \forall n \in Z^+$

ii) $a_n = 5n$

$a_0 = 5 \times 0 = 0$

$a_1 = 5 \times 1 = 5 + a_0$

$a_2 = 5 \times 2 = a_1 + 5$

$a_3 = 5 \times 3 = a_2 + 5$

$|$

$\boxed{a_n = 5n = a_{n-1} + 5}$ recursive function

$a_{n+1} = 5(n+1) = a_n + 5$

The recursive function of given function $a_n = 5n$ is $a_0 = 0$ & $a_n = a_{n-1} + 5 \ \forall n \in Z^+$

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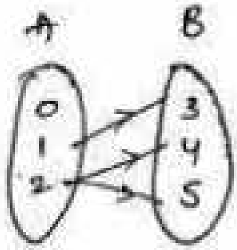
Relations :

Let A & B are two sets. Then a subset of $A \times B$ is called a relation from A to B. Here, R is a relation from A to B then R is a set of ordered pairs a, b where $a \in A$, $b \in B$ if $(a, b) \in R$ we say that a is related to b by a relation R. This is also denoted by aRb

Ex :



its a relation but not a function

$R = \{(1,3)\ (2,4), (2,5)\}$

$\Rightarrow 1R3, 2R4, 2R5$

Matrix of a relation :

consider the sets $A = \{a_1, a_2, --, a_n\}$ and $B = \{b_1, b_2, --, b_n\}$ of orders $m \times n$ respectively. Then $A \times B$ consists of all ordered pairs of the form (a_i, b_j), $1 \le i \le m$, $1 \le j \le n$ which are in number. Let R be a relation from A to B so that R in a subset of $A \times B$. Now let us put $m_{ij} = (a_i, b_j)$ & assign values 1 (or 0 to m_{ij} according to following

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i b) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

then $m \times n$ matrix formed by these m_{ij} is called the adjacency matrix, or the matrix of the relation.

R to the relation matrix for R & denoted by M_R

Ex: As Another ex, consider $A = \{1, 2, 3, 4\}$ & a relation R defined on A by $R = \{(1,2), (1,3), (2,4), (3,2)\}$

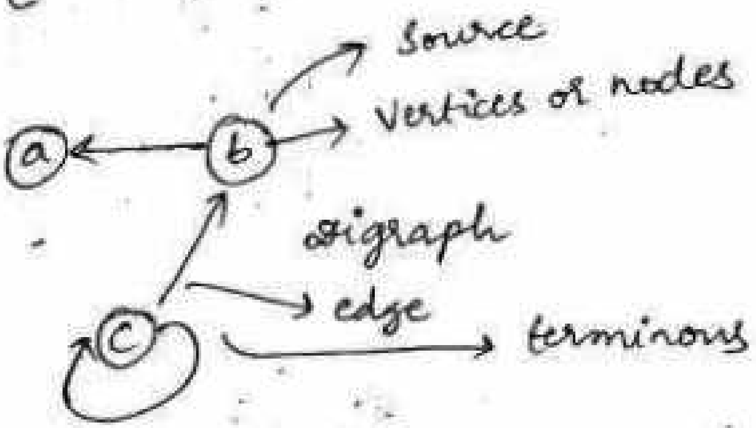
Thus here $A = \{a_1, a_2, a_3, a_4\} = B$ where $a_1 = 1$, $a_2 = 2$, $a_3 = 3$, $a_4 = 4$

$$M_R = (m_{ij}) = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

→ <u>Directed graph or digraph</u> :

If matrix of a relation is

$$\begin{array}{c c c} & & \begin{array}{ccc} a & b & c \\ 3 & 4 & 5 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$



→ Draw a small circle or bullet for each element of A and label the circle with the corresponding element of A. These circles are called <u>Vertices or nodes</u>.

→ Draw an ~~array~~ arrow called an edge from one vertex to another vertex based on the relation. The resulting pictorial representation is called a digraph or directed graph of a given relation.

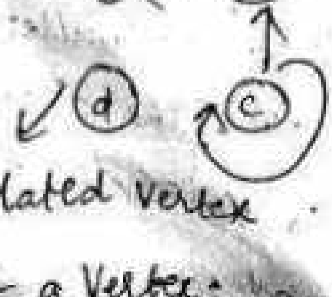
→ <u>Origin / Source</u> : A vertex from which an edge leaves is called the origin or the source for that edge.

→ A vertex where an edge ends is called the <u>terminous</u>

→ <u>Isolated Vertex</u> : A vertex which is neither a source nor a terminous of any edge (no connection

ex:

$$
\begin{array}{cc|cccc}
 & & a & b & c & d \\
 & & 3 & 4 & 5 & 6 \\
\hline
a & 0 & 0 & 0 & 0 & 0 \\
b & 1 & 1 & 0 & 0 & 0 \\
c & 2 & 0 & 1 & 1 & 0 \\
d & 3 & 0 & 0 & 0 & 0
\end{array}
$$



Isolated Vertex

→ <u>Indegree of a Vertex</u> : a b c d

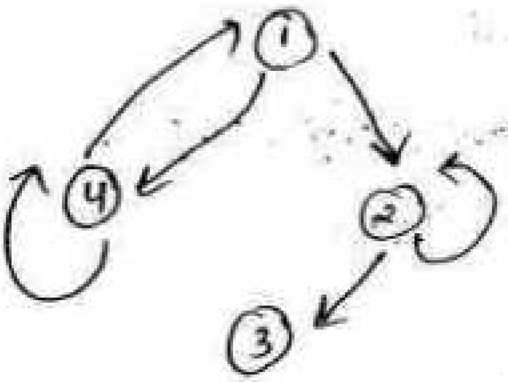
indegree

outdegree

→ The number of edges terminating at the vertex is called indegree of that vertex.

→ The number of edges leaving a vertex is outdegree

→ Find the relation, indegree, outdegree of the vertex represented by diagraph given below. Also write down its matrix.



Matrix =
$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(1,2), (2,2), (2,3), (1,4), (4,1), (4,4)\}$

	1	2	3	4
indegree ↳	1	2	1	2
outdegree →	2	2	0	2

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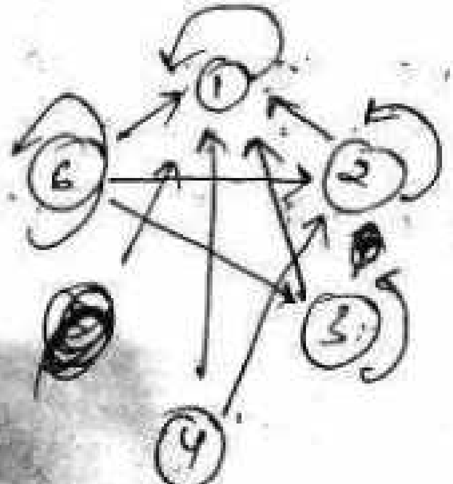
→ Let $R = \{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if a is multiple of b. represent the relation R as a matrix and draw its digraph. Also find indegree & out degree

$\boxed{aRb}$

$R = \{1,2,3,4,6\}$

$R = \{(1,1), (2,1), (3,1), (4,1), (6,1),$
$(2,2)\ (4,2)\ (6,2)$
$(3,3)\ (6,3)$
$(6,6)\}$

$$M = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$



	1	2	3	4	6
indegree	5	3	2	0	1
outdegree	1	2	2	2	4

Operations on Relations :

① **Union** : $R_1 \cup R_2 = \{(a,b) \in (A \times B) / (a,b) \in R_1$ (or) $(a,b) \in R_2\}$

② **intersection** : $R_1 \cap R_2 = \{(a,b) \in (A \times B) / (a,b) \in R_1$ and $(a,b) \in R_2\}$

③ $R_1 - R_2 = \{(a,b) \in (A \times B) / (a,b) \in R_1$ and $(a,b) \notin R_2\}$

4) $\overline{R_1} = A \times B - R_1$ → compliment of a relation

5) Converse relation : $(R^c)^c = R$

ex : $R = \{(a,b), (c,d)\}$

$R^c = \{(b,a), (d,c)\}$

→ Consider the sets $A = \{a,b,c\}$ $B = \{1,2,3\}$ here

$R = \{(a,1), (b,1), (c,2), (c,3)\}$ &

$S = \{(a,1), (a,2), (b,1)\ (b,2)\}$

from A to B

determine $R \cup S$, $R \cap S$, $\overline{R}$, $\overline{S}$, R^c, S^c $[S^{-1}]$

$R \cup S = \{(a,1), (b,1), (c,2), (c,3), (a,2), (b,2)\}$

$R \cap S = \{(a,1), (b,1)\}$

$R^c = \{(1,a), (1,b), (2,c), (3,c)$

$S^c = \{(1,a), (2,a), (1,b), (2,b)\}$

$A \times B = \{(a,1), (a,2), (a,3), (b,1)\ (b,2)\ (b,3)\ (c,1)\ (c,2)\ (c,3)\}$

$\overline{R} = A \times B - R = \{(a,2), (a,3), (b,2)\ (b,3)\ (c,1)\}$

$\overline{S} = A \times B - S = \{(a,3), (b,3), (c,1), (c,2), (c,3)\}$

→ Let $A = \{1,2,3\}$ $B = \{1,2,3,4\}$ the relations R and S from A to B by the following matrices

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \quad \& \quad M_S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Determine i) $\bar{R}$ ii) $\bar{S}$ iii) $R \cup S$ iv) $R \cap S$ v) R^c vi) S^c vii) Matrix representation of all the above

→ $\bar{R} = A \times B - R = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$.

$R = \{(1,1), (1,3), (2,4), (3,1), (3,2), (3,3)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,4), (3,2), (3,4)\}$

i) $\bar{R} = \{(1,2), (1,4), (2,1), (2,2), (2,3), (3,4)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

ii) $\bar{S} = \{(2,1), (2,2), (2,3), (3,1), (3,3)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

iii) $R \cup S = \{(1,1), (1,3), (2,4), (3,2), (3,1), (3,3), (1,2), (1,4), (3,4)\}$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 1 & 1 \end{array}$$

iv) $R \cap S = \{(1,1), (1,3)\ (2,4)\ (3,2)\}$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \end{array}$$

v) $R^c = \{(1,1), (3,1), (4,2), (1,3)\ (2,3)\ (3,3)\}$

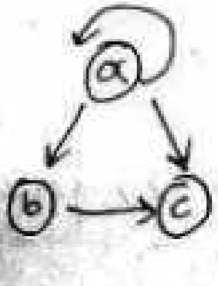
$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 4 & 0 & 1 & 0 \end{array}$$

vi) $S^c = \{(1,1), (2,1), (3,1), (4,1), (4,2), (2,3), (4,3)\}$

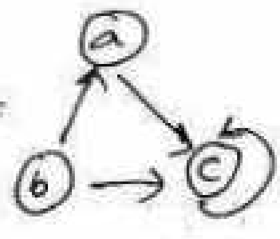
$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 \end{array}$$

→ The diagraph of relation R & S on the set A = $\{a, b, c\}$ -

R =



S =



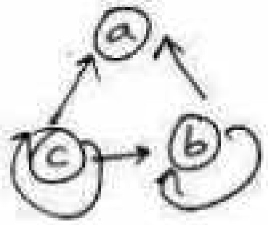
$R = \{(a,a), (a,c), (a,b), (b,c)\}$

$S = \{(a,c), (b,a), (c,c), (b,c)\}$

i) $\overline{R} = A \times B - R$

$\overline{R} = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\} - \{(a,a),(a,c),(a,b),(b,c)\}$

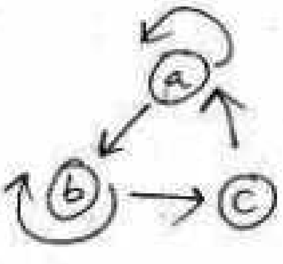
$= \{(b,a),(b,b),(c,a),(c,b),(c,c)\}$



$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 0 & 0 \\ b & 1 & 1 & 0 \\ c & 1 & 1 & 1 \end{array}$$

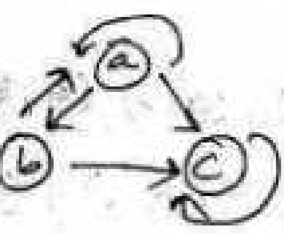
ii) $\overline{S} = A \times B - S =$

$\{(a,a),(a,b),(b,b),(b,c),(c,a)\}$



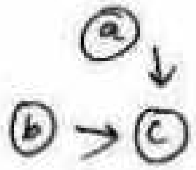
$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 1 & 0 \\ b & 0 & 1 & 1 \\ c & 1 & 0 & 0 \end{array}$$

iii) $R \cup S = \{(a,c),(b,c),(a,a),(a,b),(b,a),(c,c)\}$



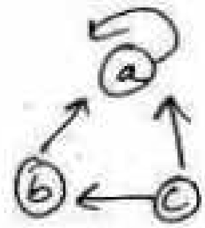
$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 1 & 1 \\ b & 1 & 0 & 1 \\ c & 0 & 0 & 1 \end{array}$$

iv) $R \cap S = \{(a,c),(b,c)\}$



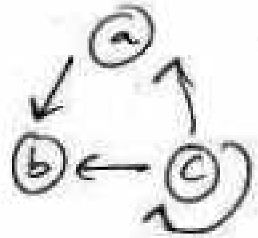
$$\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

v) $R^c = \{(a,a),(c,a)\ (b,a)\ (c,b)\}$



$$\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

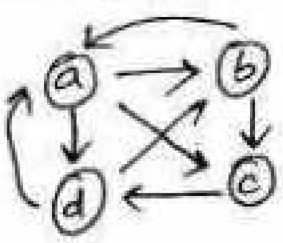
vi) $S^c = \{(c,a),(a,b),(c,c),(c,b)\}$



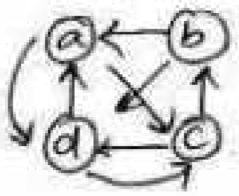
$$\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

→ Let $A = \{a,b,c,d\}$ and relations R and S

R =



S =



find $\overline{R}$, $\overline{S}$, $\overline{R\cap S}$, & their diagraphs

$$\boxed{\overline{R\cap S} = (A\times B) - R\cap S}$$

$R = \{(a,b),(a,d),(a,c),(b,a),(b,c),(c,d),$
$(d,b),(d,a)\}$

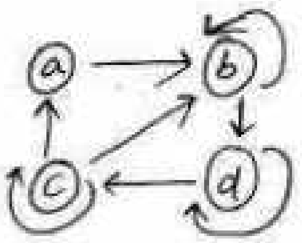
$S = \{(a,c),(a,d),(b,a),(b,d),(c,b),(c,d),$
$(d,c),(d,a)\}$

$\{a,b,c,d\} \quad \{a,b,c,d\}$

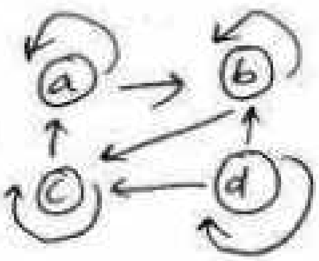
$\bar{R} = (A \times B) - R = \{(a,a), (a,b), (a,c), (a,d), (b,a),$
$(b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d),$
$(d,a), (d,b), (d,c), (d,d)\}$

$- \{(a,b), (a,d), (a,c), (b,a), (b,c), (c,d),$
$(d,b), (d,a)\}$

$\bar{R} = \{(a,b), (b,b), (b,d), (c,a), (c,b), (c,c),$
$(d,c), (d,d)\}$



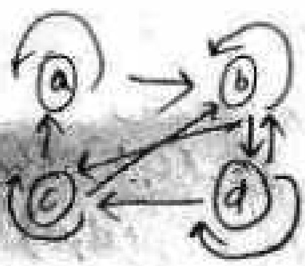
$\bar{S} = \{(a,a), (a,b), (b,b), (b,c), (c,a), (c,c),$
$(d,b), (d,c), (d,d)\}$



$\overline{R \cap S} = (A \times B) - (R \cap S)$

$R \cap S = \{(a,d), (a,c), (b,a), (c,d), (d,a)\}$

$\overline{R \cap S} = A \times B - (R \cap S) = \{(a,a), (a,b), (b,b), (b,c), (b,d),$
$(c,a), (c,b), (c,c), (d,b), (d,c),$
$(d,d)\}$



* Properties of binary relation :

* Reflexive relation : A Relation R on set A is said to be a reflexive relation if $(a,a) \in R$ $\forall a \in A$.

Ex : consider Set $A = \{1,2,3\}$ is the relation $R = \{(1,1), (2,2), (3,3)\}$ is Reflexive or not ?

The given relation R is a reflexive relation because $(1,1), (2,2), (3,3) \in R \; \forall \{1,2,3\} \in A$

consider $A = \{1,2,3,4\}$. then the relation $R = \{(1,1), (2,2), (3,3)\}$ is reflexive or not ? →

The given relation R is not a reflexive $(4,4) \notin R \; \forall \{1,2,3\} \in A$ matrix relation of reflexive relation is for a given set

$$A = \{1,2,3\} \text{ is } \quad \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

The reflexive relation must have 1's on its diagonals.

② **Irreflexive relation** : A relation R on set A is said to be a irreflexive relation if $(a,a) \notin R$ for all $a \in A$

ex: consider $A = \{1,2,3\}$ & $R = \{(2,3),(1,2),(1,3)\}$

Here, R is irreflexive relation because $(2,3),(1,2),(1,3) \in R \ \forall \ \{1,2,3\} \in A$

③ **Symmetric relations** : A relation R on set A is said to be symmetric relation $(b,a) \in R$, whenever $(a,b) \in R$, $\forall \ (a,b) \in A$

Ex: Consider a set $A = \{1,2,3\}$ & $R = \{(1,1),(1,2),(2,1)\}$

The given relation is a symmetric relation because $(1,1),(1,2),(2,1) \in R$, $\forall \ \{1,2,3\} \in A$

→ $R = \{(1,1),(1,2),(2,1),(2,3)\}$ is not symmetric

④ **Compatibility relation** : A relation R on set A is said to be a compatibility relation which contains reflexive relation and symmetric relation

ex: i.e

Reflexive relation

if $(a,a) \in R$, $\forall \ (a,a) \in R$

Symmetric relation

if $(a,b) \in R$, $\forall (b,a) \in R$

ex: let $A = \{1,2,3\}$ &

$R = \{(1,1)\ (2,2), (3,3), (1,3), (3,1)\}$

Here R is compatibility relation

The given relation R is a compatibility relation because $\{(1,1), (2,2), (3,3)\} \in R, \forall$ $\{(1,2,3)\} \in A$ having Reflexive Property.

2. $\{(1,3), (3,1)\} \in R, \forall \{(1,2,3)\} \in A$ having Symmetric relation

(5) Anti-Symmetric Property: A relation R on set A is said to be a anti symmetric relation whenever $(a,b) \in R$ & $(b,a) \in R$ then $a=b$

for ex: $A = \{1,2,3\}$ $R = \{(1,1), (2,2), (3,3)\}$ Here for all the relation $\forall a=b$ then it is called Anti - symmetric Property.

(6) Transitive relation: A relation R on set A is said to be a Transitive relation if

$(a,b) \in R$
$(b,c) \in R$ $\quad \forall \{a,b,c\} \in A$
$(a,c) \in R$

If R is not a Transitive relation

$(a,b) \in R$
$(b,c) \in R$ but
$(a,c) \notin R, \forall \{a,b,c\} \in A$

→ Consider the set $A = \{1,2,3\}$ & $R = \{(1,1)\ (1,2), (2,3)$
$(1,3), (3,1), (3,3)\}$

Transitive relation

(7) <u>Equivalence relation</u>: If the relation is having reflexive, Symmetric and Transitive Properties (RST) then the relation is Equivalence relation.

<u>ex</u>: R is reflexive if $(a,a) \in R, \forall a \in A$

R is Symmetric if $(a,b) \in R$ whenever $(b,a) \in R$

R is Transitive if $(a,b) \in R, (b,c) \in R$ then $\forall (a,b) \in R$
$(a,c) \in R, \forall \{a,b,c\} \in A$

Let $A = \{1,2,3,4\}$ $R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,4), (2,4), (3,3), (4,4)\}$

R is reflexive, Symmetric, Transitive

∴ It is Equivalence relation.

* <u>Partial Order relation</u> (po-set): (RAT): If the given relation R is having reflexive, Anti-Symmetric and transitive properties (RAT) then the relation is partial order relation.

<u>Ex</u>: If $\subseteq$ (inclusion) is Partial order relation on set P then the ordered pair is $(P, \subseteq)$ is called a Partial order relation. <u>PO-SET or Ordered Set</u>

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→ Let set $A = \{1, 2, 3, 4, 6, 12\}$ define relation $A \to B$ if a divides b. Verify that R is a equivalence relation or not.

$R = \{(1,2), (1,3), (1,4), (1,6), (1,12), \underline{(1,1)},$
$\underline{(2,2)}, (2,4), (2,6), (2,12)$
$\underline{(3,3)}, (3,6), (3,12)$
$\underline{(4,4)}, (4,12), \underline{(6,6)}, (6,12)$
$\underline{(12,12)}\}$

R is reflexive relation
not equivalence relation

→ ① reflexive : if $(a,a) \in R \ \forall a \in A$
② Antisymmetric : if $(a,b) \in R$, $(b,a) \in R$
$\forall a = b$
③ Transitive : if $(a,b) \in R$, $(b,c) \in R$ then
$(a,c) \in R \ \forall \{a,b,c\} \in A$

HASSE diagram or PO-set :

A Partial Order $\subseteq$ on set P can be represented by means of a diagram is known as HASSE diagram of $(P, \subseteq)$

→ Some rules of Hasse diagram.

① Eliminate or remove the reflexive relations
② Eliminate or remove the transitive relations
③ We represent the vertices using small circles or dots.

④ Draw the diagram in such a way that all edges point upward direction.

⑤ We need not put arrows to edges

→ Let $A = \{1,2,3,4,6,12\}$ on set A define the relation $A \to B$ if a divides b. Prove that R is a Partial order relation & draw the Hasse diagram from this relation.

$R = \{(1,1),(1,2),(1,3),(1,4),(1,6),(1,12),$
$(2,2),(2,4),(2,6),(2,12),(3,3),$
$(3,6),(3,12),(4,4),(4,12),(6,6),$
$(6,12),(12,12)\}$

R is reflexive ⇒ $\{(1,1),(2,2),(3,3),(4,4),(6,6),(12,12)\}$

Anti-Symmetric ⇒ $\{(1,1),(2,2),(3,3),(4,4),(6,6),(12,12)\}$

a divides b and b divides a
⇒ $a=b$

Transitive ⇒ $\{(1,2),(2,4),(1,4)\}, \{(1,6),(6,12),(1,12)\},$
$\{(1,3),(3,6),(1,6)\},$
$\{(2,6),(6,12),(2,12)\}$
$\{(1,3),(3,12),(1,12)\},$
$\{(3,6),(6,12),(3,12)\}$

Hence R Satisfies Partial Order relation

Hasse diagram :

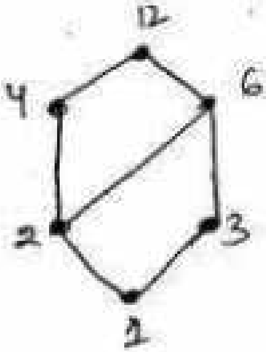
Step 1 : eliminate reflexive relations

$R = \{ (1,2), (1,3), (1,4), (1,6), (1,12),$
$(2,4), (2,6), (2,12), (3,6), (3,12),$
$(4,12), (6,12) \}$

Step 2 : eliminate transitive relations.

$R = \{ (1,2), (1,3), (2,4), (2,6), (3,6), (4,12)$
$(6,12) \}$

Step 3 :



→ Draw the hasse diagram of the relation R

on set $A = \{1, 2, 3, 4, 5\}$ of the matrix

$$M_R = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{ (1,1), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4)$
$(2,5), (3,3), (3,4), (3,5), (4,4), (5,5) \}$

Step 1 : ~~eliminate reflexive~~ reflexive : $\{(1,1), (2,2),$
~~$\{(1,3), (1,4),$~~ $(3,3), (4,4),$
$(5,5)\}$

Anti-Symmetric : $\{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

Transitive : $\{(1,3), (3,4), (1,4)\}$

$\{(1,3), (3,5), (1,5)\}$

$\{(2,3), (3,4), (2,4)\}$

$\{(2,3), (3,5), (2,5)\}$

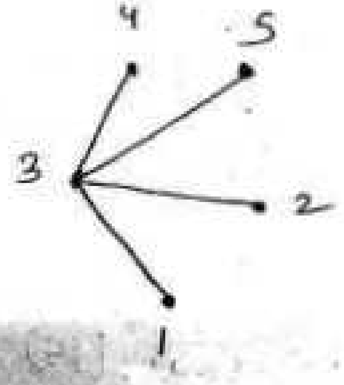
Hasse diagram :

① Step : 1 : remove reflexive

$\{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5)\}$

Step 2 : Transitive

$\{(1,3), (2,3), (3,4), (3,5)\}$



→ Draw the hasse diagram positive division of 45

A = {1, 3, 5, 9, 15, 45}

R = { (1,1), (1,3), (1,5), (1,9), (1,15), (1,45),
(3,3), (3,5), (3,9) (3,15) (3,45),
(5,5), (5,9), (5,15), (5,45),
(9,9), (9,15), (9,45),
(15,15), (15,45), (45,45) }

To Prove that is Partial order relation:

Reflexive : (1,1), (3,3), (5,5), (9,9), (15,15), (45,45)

Anti Symmetric : (1,1) (3,3), (5,5), (9,9), (15,15), (45,45)

Transitive : (1,3) (1,5) (1,3) (1,3)
(3,5) (3,9) (3,15) (3,45)
(1,5) (1,9) (1,15) (1,45)

(3,5) (3,5)
(5,9) (5,15)
(3,9) (3,15)

(3,15) (3,9) (3,9)
(5,45) (9,15) (9,45)
(3,45) (3,15) (3,45)

(3,15) (5,9) (5,9)
(15,45) (9,15) (9,45)
(3,45) (5,15) (5,45)

(1,5) (1,5) (1,5)
(5,9) (5,15) (5,45)
(1,9) (1,15) (1,45)

(1,9) (1,9) (1,15)
(9,15) (9,45) (15,45)
(1,15) (1,45) (1,45)

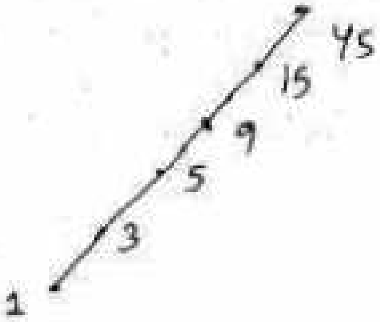
(5,15)
(15,45)
(5,45)

∴ It is Partial order relation.

Step 1: Reflexive

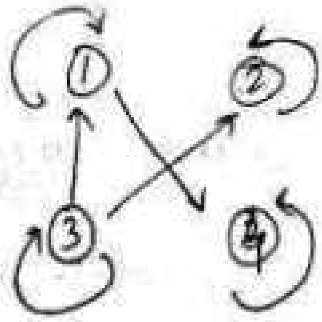
$\{(1,3), \cancel{(1,5)}, \cancel{(1,9)}, \cancel{(1,15)}, \cancel{(1,45)}, (3,5),$
$\cancel{(3,9)}, \cancel{(3,15)}, \cancel{(3,45)}, (5,9), \cancel{(5,15)},$
$\cancel{(5,45)}, (9,15), (9,45), \cancel{(15,45)}\}$

$R = \{(1,3), (3,5), (5,9), (9,15), (15,45)\}$



→ The diagraph of Relation R on set $A = \{1,2,3,4$ is shown in below. Verify that (A,R) is a Po-set and draw corresponding Hasse diagram

$A = \{(1,1), (1,2), (1,3), (1,4)$
$(2,2), (2,3)\ (2,4)$
$(3,3), (3,4)$
$(4,4)$



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

$R = \{(1,1), (1,4), (2,2), (3,1), (3,2), (3,3), (4,4)\}$

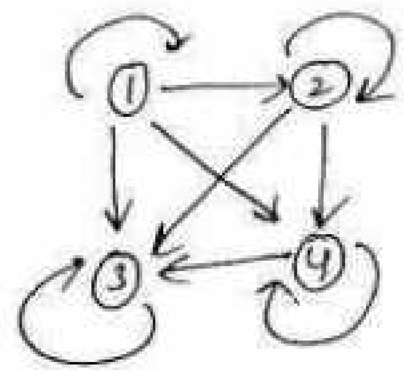
Reflexive : (1,1), (2,2), (3,3), (4,4)

Anti-Symmetric : (1,1) (2,2) (3,3) (4,4)

Transitive : No transitive elements

∴ Given relation is not Partial order

→



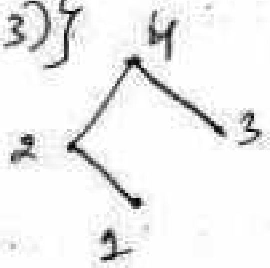
$R = \{(1,1), (1,2), (1,3), (1,4),$
$(2,2), (2,3), (2,4),$
$(3,3), (4,4), (4,3)\}$

Reflexive : {(1,1) (2,2) (3,3) (4,4)}

Anti-Symmetric {(1,1) (2,2) (3,3) (4,4)

Transitive : {(1,2), (2,3), (1,3)}
{(1,2), (2,4), (1,4)}
{(1,4), (4,3), (1,3)}
{(2,4), (4,3), (2,3)}

→ remove reflexive, Transitive
{(1,2), (2,4), (4,3)}



Lattice : A Lattice is introduced as Po-set $(P, \subseteq)$ As $(P, \subseteq)$ in each Pair of relation has

① Greatest Lower Bound (GLB)

② Least upper Bound (LUB) is called a Lattice.

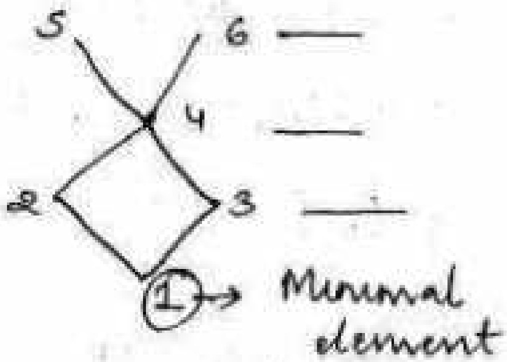
→ The greatest Lower bound of (a,b) is

$a \times b$
$a \cdot b$
$a \wedge b$
~~$a*b$~~ $a \cap b$
GCD

→ Least upper bound of (a,b) is

$a + b$
$a \vee b$
$a \cup b$
LCM

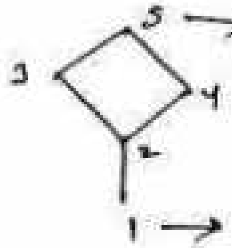
Maximal elements :



For the given example (hasse diagram) 5 and 6 are maximal elements and '1' is minimal element. Here '1' is also called as

Least element and there is no greatest element in the given diagram

Q2:

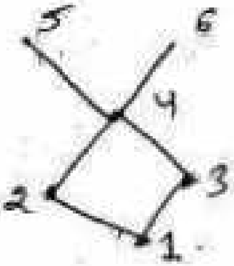


5 → Greatest (or) Maximal ele

1 → Least (or) Minimal ele

Greatest element is an element $a \in A$ if $xRa, \forall x \in A$. Least element is an element $a \in A$ if $aRx, \forall x \in A$.

→ GLB (Greatest Lower bound)



Take order Pair (2,3)

Take upper bound values of element 2 = {2,4,5,6}

Take upper bound values of element 3 = {3,4,5,6}

common elements = {4,5,6}

'4' is the Least upper bound for given ordered Pair (2,3)

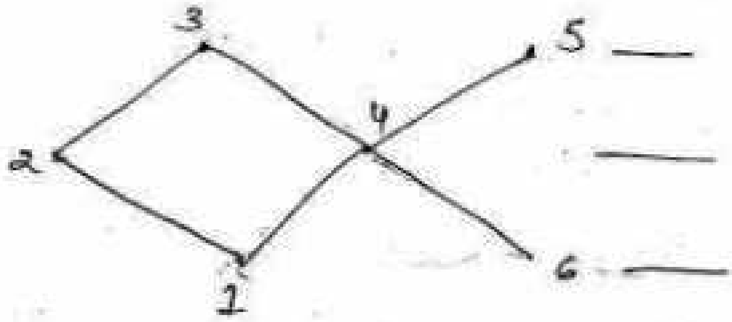
Greatest Lower bound :

for element 2. {2,1}.

element 3. {3,1}

1 → Greatest Lower bound

→ find maximal and minimal elements and GLB and LUB elements for each Pair for a following figures.



(3,5) are Maximal elements

(1,6) are Minimal elements

GLB :

(1,2) ⇒ for ele 1 : {1}

for ele 2 : {2, 1, 6}.

common ele = 1 = GLB

LUB : (1,2) : for ele 1 : {1, 2, 3, 4, 5}

for ele 2 : {2, 3, 4, 5}

common = {2, 3, 4, 5}

LUB = 2

GLB :

(2,3) ⇒ for ele 2 : {2, 1, 6}

for ele 3 : {3, 2, 4, 1, 6}

c = {1, 2, 6}

GLB = 1

LUB

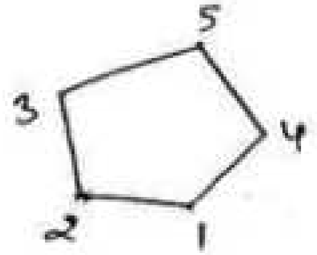
for ele 2 = {2, 3, 5}

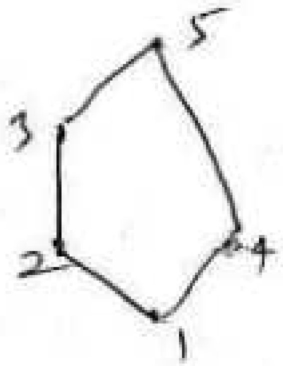
ele 3 = {3}

c = {3} = LUB

(1,4) ⇒ GLB : ele 1

→





(1,2) , (2,3) , (3,5) , (5,4) , (1,4) , (2,4)

→ (1,2)

GLB :
for ele 1 : {1}
ele 2 : {2,1}
GLB = 1

LUB :
for ele 1 : {1, 2, 4, 3, 5}
for ele 2 : {2, 3, 5}
~~LUB~~ = comon = {2,3,5}
LUB = 2

→ (2,3) GLB :
for ele 2 : {2,1}
ele 3 : {3, 2, 1, 4}
common = {1,2}
GLB = 2

LUB :
for ele 2 : {2,3,5}
ele 3 : {3,5}
c = {3,5}
LUB = 3

(3,5)

GLB =

ele 3 : {3,2,4,1}

ele 5 : {3,2,1,4,5}

c = {1,2,3,4}   GLB = 1

LUB : ele3 : {3,5}

ele 5 : {5}

c = 5 = LUB   Lattice

→ Supremum : The Least upper bound Value of Hasse diagram is Known as Supremum

→ Infinum : The greatest Lower bound Value of a Hasse diagram (any Order Pair) is Known as Infinum

UNIT-3 ALGEBRAIC STRUCTURES

Properties of Binary operations on Algebraic Structures

1) Commutative Operation : A binary operation star on set A is said to be commutative operation if $a*b = b*a \; \forall \; a,b \in A$

Additive : $\cancel{a+b}$ $(a+b) = (b+a)$

Multiplicative : $(a*b) = (b*a)$

Similarly for union and intersection

$\cancel{A\cap B = A\cup B}$ $A\cup B = B\cup A$

$\cancel{B\cap A = B\cup A}$ $A\cap B = B\cap A$

→ It doesn't hold subtraction and division

2) Association Operation : A binary operation star is said to be an association operation if

Addition: $(a*b)*c = a*(b*c) \; \forall \; a,b,c \in A$

Multiplication $(a+b)+c = a+(b+c) \; \forall \; a,b,c \in A$

3) Identity Operation :

$1*a = a*1 = a$ → multiplicative identity

$a+0 = 0+a = a$ → additive identity

4) Inverse Operation : Let E an identity element in set A for operation '*'

$a*b = c = 1 = b*a \; \forall \; a,b \in A$

Here 'b' is inverse of element 'a' & Vice-Versa

⑤ Distributive :

$$a+(b\times c) = (a+b)*(a+c)$$
$$a*(b+c) = (a\times b)+(a\times c)$$

⑥ closure operation : of any relation $a,b \in G$ then $(a*b)\in G$. This Property is closure operation

→ Find whether following algebraic systems satisfies the Properties under binary operations + & ×

① odd integers ⇒ {1, 3, 5, 7, - - -}

commutative : $1+3 = 3+1 = 4$
$1\times3 = 3\times1 = 3$

Associative : $1+(3+5) = (1+3)+5 = 9$
$1\times(3\times5) = (3\times1)\times5 = 15$

identity : Odd integer + 0 = Odd integer
Odd integer × 1 = odd integer

inverse : $3\times\frac{1}{3} = 1$
$9-9=0$

distributive : $1+(3\times5) = (1+3)\times(1+5)$
$16 \neq 24$
$1\times(3+5) = (1\times3)+(1\times5)$
$8 = 3+5$
8

Not distributive

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→ closure Property under addition doesn't satisfy.

⇒ positive integers = {0, 1, 2, 3, 4, ... }

[illegible]

$1 + (4+5)$ [illegible]

$2 + (3 \times 4) = 14$

$(2+3) \times (2+4)$

5×6

→ Structure (CAII)

Group : A group $(G, *)$ is said to be a group which satisfies closure, associative, identity & inverse operations

Note : ① A group with addition binary operations is known as additive group.

② A group with multiplication B.O is multiplicative group

③ The no. of diff elements in a group is Order of group. it is represented by symbol $O(G)$

Abelian Group : A group $(G, *)$ is said to be an abelian group which satisfies commutative Property (CCAII)

→ If $A = \{1, -1, i, -i\}$ are 4th root of unity. Show that $(A, *)$ is a group

$\times$	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

C A I I

closure : $1 - 1 = 0 \notin A$
Not a closure

Associative : $1, -1, -i$

$1 + (-1 + i) = (1 + (-1))$

→ accepts multiplication closure

→ associative $1, -1, i$

$$1 \times (-1 \times i) = (1 \times (-1)) \times i$$

$$-i = -i$$

obeys associative

→ Identity : $1 * 1 = 1 * 1 = 1$

obeys identity law

→ Inverse : $i, -i$ $\quad i \times \left(\frac{1}{i}\right) = 1$

∴ it is not a group

finite and infinite group:

→ A group $(G, *)$ is said to be a finite group of the set G contains finite number of diff elements

→ of G contains infinite number of different ele then the set G is called infinite group

semi group : An algebraic structure $(S, *)$ is said to be a semi group which satisfies the following properties

i) Closure

ii) Associative

→ Monoid : The algebraic structure $(M, *)$ is said to a monoid which satisfies the following properties closure, Associative, Identity (C, A, I)

→ Ring : An algebraic structure "G" with two binary operation '+' & 'x' $(G, +, \cdot)$ is a ring which satis the following properties ① Abelion grp
② Semi-group
③ Distributive

→ Show the set of integers is a group

→ Prove that $(\{1,-1\}, \times)$ where $*$ is the multiplication operation is an abelion grp or not

Commutative : $1-1 = -1+1 = 0$

obeys commutative law

$1\times(-1) = (-1)\times 1 = -1$

Closure : $1\times(-1) = -1 \in A$ obeys

Associative : $1, -1, 1 \Rightarrow 1\times(-1\times 1) = (1\times -1)\times 1$

$= -1$

obeys

Inverse : $1\times\frac{1}{1} = 1$ $1 \in A$ obeys

Identity : $1\times 1 = 1\times 1 = 1$ obeys

∴ It is Abelion grp

Determine whether the given table with respect to operation $(*)$ on set $A = \{a, b\}$ is a semi grp & monoid

$\times$	a	b
a	a	b
b	b	b

<u>Commutative</u> : $a \times b = b \times a$

$b = b$

commutative

<u>closure</u> : $a \times b = b \in A$

<u>Associative</u> : $(a \times b) \times a = a \times (b \times a)$

$b \times a = a \times b$

$b = b$

obeys

<u>Identity</u> : ~~$a \times \left(\frac{1}{a}\right) = 1$~~ $\Rightarrow$ $a \times 1 = 1 \times a = a$

identity

~~doesn't hold~~

$\therefore$ It is a semigroup and monoid

→ Prove that $S = \{0, 1, 2, 3, 4\}$ is a ring w.r.t to operation of Multiplication & addition using $|5|$

<u>under addition</u> ; <u>closure</u> ;

$\times$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

<u>Multiplication</u> : $2 \times 3 = 1$

$\in$

Obeys closure Property

<u>Commutative</u> $\Rightarrow$ $2 \times 3 = 3 \times 2 = 6 \% 5$

obeys $= 1$

<u>Associative</u> $\Rightarrow$ $(2 \times 3) \times 4 = 2 \times (3 \times 4)$

~~doesn't~~ obeys

$a \times \frac{1}{a} = 1$

$a \times 1 = 1 \times a = a$

$a + 0 = 0 + a = a$

+	0	1	2	3	4
0	0	1	2	3	4
1	0	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

identity : $2 \times 1 = 1 \times 2 = 2$ (obeys)

→ it doesn't hold inverse Property

under ~~(Multiplication)~~ Addition :

closure : $1+2 = 3 \in A$

Commutative : $2+3 = 3+2$ obeys

Associative : $(2+3)+4 = 2+(3+4)$

identity : $2+0 = 0+2 = 2$

inverse : $1 - 1 = 0$ doesn't hold

$2 - 2 = 0$

distributive : $a \times (b+c) = (a \times b) + (a \times c)$

$1 \times (2+3) = 2+3$

$5 = 5$

$(1+2) \times (4+3)$ ✓ $a + (b \times c) = (a+b) \times (a+c)$

$3 \times 4 = 12$ $\quad 1 + (2 \times 3) = 3 \times 4$

$2 = \cancel{4}$ $\quad 7 = 12$ $\quad (1 \times 2) + 4$

$5 \times 6 = 30$ $\quad (1+4) \times (2+4)$

Semi groups Homo-morphism and Iso-morphism :

Consider two Semi groups $(S_1, *_1)$ and $(S_2, *_2)$. Let f be a function from S_1 to S_2 then f is called homomorphism from S_1 and S_2 if $\forall$ $(a,b) \in S_1$. Symbollically

$$f(a *_1 b) = f(a) *_2 f(b)$$

composition

Isomorphism: If f is called iso morphism from $S_1 \to S_2$ if ① f is homomorphism

② f is one-one & onto function

→ consider the semi groups $(Z,+)$ $(E,+)$ then define $f: Z \to E$ by $f(x) = 2x, \forall x \in Z$ show that we find that for any $(a,b) \in Z$ f is homomorphism and isomorphism

Sol

$$f(a+b) = f(a) + f(b)$$

$$2(a+b) = 2a + 2b$$

Here operation is addition.

$$\therefore f(x) = f(a) + f(b)$$

$\therefore$ It is homo-morphism

one-one : $f(x) = 2x$ $f(a) = f(b)$

$f(a) = 2a$

$f(b) = 2b$ $2a = 2b$

$a = b$ / one-one

Onto: $f(x) = 2x = y$

$y = 2x$

$y \in E$ thus every y in E has Preimage in Z under f. then it is onto function

Then the semigroups $(Z,+)$, $(E,+)$ function $f: Z \to E$ is isomorphism

→ consistency, inconsistency, Proof of contradiction, Automatic theorem proving*

→ Predicate is a main factor or Part of a given statement.

Ex: He is a $\underbrace{\text{good boy}}_{\text{Predicate}}$

Predicate Logic: symbolic representation of a statement

Types of Predicates:

① Oneplace: If a Predicate is associative with only one individual name then it is a one place Predicate. $P(x)$

② two place: If a Predicate is associative with more than one (two) names $P(x,y)$

③ three place: If a Predicate is associative with more than two names. $P(x,y,z)$

Inputs

Free and Bound Variables: If a given formulae containing the form of $\forall x, P(x)$ then this formulae is called as x bound Part of the formulae. It is also known as bound Variable.

Ex: $\forall x \underbrace{(P(x) \rightarrow Q(x))}_{\text{scope of Variable}}$

bound occurrence / x Variable

free occurrence / y Variable

→ Show that $S \vee R$ is a Valid inference from the premesis

$$\begin{array}{c} (P \vee Q) \\ (P \rightarrow R) \\ (Q \rightarrow S) \\ \hline S \vee R \end{array}$$

$(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

$(\sim P \rightarrow Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

$(\sim Q \rightarrow P) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

$(\sim Q \rightarrow R) \wedge (Q \rightarrow S)$

$(\sim R \rightarrow Q) \wedge (Q \rightarrow S)$

$\sim R \rightarrow S$

$S \vee R$

consistency & inconsistency:

consistency: Let H_1 and H_2 are any two statement formulaes then conjunction of H_1 and H_2 has the truth value is true, then ~~H_1 and~~

$H_1 \wedge H_2$ has consistency formulaes.

denoted by $H_1 \wedge H_2 \Rightarrow T$

Ex: H_1: All humans are mortal

H_2: Simon is immortal

Let $H_1 : p \vee q$

$H_2 : \sim p \vee \sim q$

p	q	H_1: $p \vee q$	$\sim p$	$\sim q$	H_2: $\sim p \vee \sim q$	$H_1 \wedge H_2$
1	0	1	0	1	1	1
0	1	1	1	0	1	1
0	0	0	1	1	1	0
1	1	1	0	0	0	0

Inconsistent: in above truth table the conjunction of H_1 and H_2 has truth values in 2 and 3 inputs. At this time $H_1 \wedge H_2$ are called as consistency formulaes

→ Let H_1 and H_2 are two statement formulaes then the conjunction of $H_1 \wedge H_2$ has truth values false then $H_1 \wedge H_2$ are inconsistency formulaes

Denoted by $H_1 \wedge H_2 \Rightarrow F$

In the above Truth table the conjunction of H_1, H_2 has truth value false in 1 & 4 inputs. At this time H_1 and H_2 are called inconsistency formulas.

Proof of contradiction:

Hypothesis: Assume that $P \rightarrow q$ is false that is P is true, q is false

Analysis: Analyse the given Statement and Solve it.

Conclusion: Because of contradiction arrived in the analysis that $P \rightarrow q$ is true

① Provide a Proof by contradiction

for every integer 'n', if n^2 is odd then n is odd

P: n^2 is odd
q: n is odd

$P \rightarrow q$

P is true
q is false

$\sim q$: n is not odd (even)

$n = 2k$

$n^2 = (2k)^2$

$n^2 = 4k^2$

$\therefore n^2$ is even

P is false, q is true

n is odd

$n = 2k+1$

$n^2 = (2k+1)^2$

$n^2 = 4k^2 + 1 + 4k$

$n^2 = 4k(k+1) + 1$

n^2 is odd

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② Assume that $p \to q$ is false that is p is true & q is false

Now q is false means 'n' is even so that

③ Prove that if m is an even integer

p: m is even integer

q: m+7 is odd integer 1 → 0

~q assume m+7 is even integer

m is even

$m = 2k$

$m+7 = (2k)+7 \Rightarrow (2k+6)+1$

↓ even + 7

$m+7 =$ odd — false

Hence Proved

Elementary combinatorics :

① Counting rule : we have two types of counting

rules : ① Sum rule

② Product rule

Sum rule :

→ if t_1 task can be Performed in 'm' diff ways

t_2 task can be Performed in 'n' diff ways

then t_1 or t_2 can be Performed in $(m+n)$ ways

→ Product rule : if t_1 task performed in 'm' diff ways & 't_2' task Performed in 'n' diff ways then two tasks t_1, t_2 Performed in $m \times n$ ways

→ find the number of ways we can select the counting rule from the class which is having 6 boys and 5 girls.

$$6+5 = 11 \text{ ways} \qquad \begin{matrix} t_1 = 6 \\ t_2 = 5 \end{matrix}$$

→ find the number of ways of selecting a book from the library which having 10 mathematics, 5 physics, 6 cse, 7 electronic books.

$T_1 = 10$
$T_2 = 5$
$T_3 = 6$
$T_4 = 7$

$\underline{28 \text{ ways}}$

→ If a Person having three shirts, 4 trousers then find no. of ways of selecting a Pair.

$T_1 = 3$
$T_2 = 4$

$\underline{3 \times 4 = 12 \text{ ways}}$

→ A Person has 4 transport modems for travelling from Hyd to chennai and three transport modems for travelling from chennai to Banglore find the no. of ways of the Person travelling from Hyd to Bang Via chennai.

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T_1 = (Hyd to che) 4

T_2 = (ch to Bang) 3

$4 \times 3 = 12$ ways

<u>Permutations and combinations</u> :

The number of Permutations of 'n' different elements by selecting 'r' elements at a time is denoted by $^nP_r = \frac{n!}{(n-r)!}$

choosing 'r' number of objects from 'n' number of objects

→ find the number of ways of forming three digit number from 5 elements

$^5P_3 = \frac{5 \times 4 \times \cancel{3} \times \cancel{2}}{\cancel{2!}} = 60$ ways

→ find the number of ~~ways~~ of letters the word Permutation success

$\frac{7 \times \cancel{6} \times 5 \times \cancel{4} \times 3}{\cancel{2!} \cdot \cancel{3!} \cdot 3!}$

30×42

1260

without repition $\frac{7!}{3!\,2!\,2!} = 420$

3q

with repition is 7!

→ How many different strings of length '4' can be performed using letters of word FLOWER

$^6P_4 = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3$

30×12

360 ways

Combinations :

The no. of ways of combining of 'n' different objects selecting 'r' ~~all~~ objects at a time. It is denoted by $\boxed{{}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}}$

→ find the number of ways of selecting 9 commi[ttee] with 7 persons.

$${}^{9}C_{7} = \frac{9 \times \cancel{8}^{4}}{\cancel{2!} \times \cancel{7!}} = 36 \text{ ways}$$

→ find the no. of ways of arranging 5 boys and 4 girls in a line and the line can start with boy and end with a Boy

B								B

${}^{7}P_{3} \times 4!$

$24 \times$ $\frac{7\times6\times5}{\cancel{3!}}$

$\cancel{3! \times 4! \times 2}$ $\cancel{5} \times \cancel{...}$

$\cancel{6 \times 24}$

$$\begin{array}{r} 24 \\ 6 \\ \hline 144 \\ \hline \end{array}$$

${}^{5}P_{2} \times 7!$ $\quad {}^{7}P_{3} \times \cancel{...}$

$= 1,00,050$

→ find no. of ways of selecting three boys, 4 girls from a group of 5 boys and 7 girl

$${}^{5}C_{3} \times {}^{7}C_{4}$$

$= \cancel{...}\ 350$ ways

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→ find the no. of ways forming a comitee of 5 Persons from a group of 5 Indians, 4 russians such that there are atleast 3 Indians in commitee

$$^5C_3 \times {}^4C_2 + {}^5C_4 \times {}^4C_1 + {}^5C_5 \times {}^4C_0$$

$$= 81$$

Permutations with repititions:

The number of Permutations of 'n' different objects, select r objects at a time with repititions are allowed then the number of Permutations are n^r. Here n is total available objects

r is selected objects at a time (repetitions will consider)

find the no. of ways forming 4 digit number from the digits {1, 2, 3, 4, 5, 6} when repititions are allowed.

without repititions $= {}^nP_r = {}^6P_4$

with repititions $= n^r = 6^4$

The number of Permutations without repititions is ^nP_r and with repitition is n^r

→ find the no. of ways forming a four letter word from the word MIXTURE in which atleast one letter is repeated

$$7^4 - {}^7P_4 = 1561$$

$n = 7$

$r = 4$

The no. of Permutations are ways to get a four word letter from a given word MIXTURE with out repetion is ${}^7P_4 = 840$

→ and with repetition is $7^4 = 2401$

⇒ The no. of ways in which atleast one letter is repeated is $7^4 - {}^7P_4 = 2401 - 840$
$= 1561$

Combination with repititions :

The no. of combinations of n different objects selecting 'r' objects at a time when repetitions are allowed is ${}^{(n+r-1)}C_r$

$8-14-1_{C_4}$

→ In how many ways we can distribute 12 ~~identical~~ Pencils to 4 children such that every children get atleast 1 Pencil.

$${}^{15}C_4 - {}^{12}C_4$$

$n+4-1$ $\quad$ $1 \cdot r$

11_{C_4} $\quad$ $12+$

$^{8}C_4$

No. of children = 4 = r

No. of objects = 12 = n (Pencils)

n = 8

$4 \times {}^{(8+4-1)}C_4$

$4 \times {}^{11}C_4$ = 1320 ways

8	0	0	0
7	1	0	0
6	1	1	0
6	2	0	0
5	1	1	1
5	2	1	0
5	3	0	0
4	4	0	0
4	3	1	0
4	2	2	0

1430

1820

* How many Permutations can be made with letters of the word CONSTITUTION and

① In how many ways vowels occur together — 99,79,200

$\boxed{OIUIO}$ CNSTTTN

$$\frac{8!}{3! \times 2!} \times \frac{5!}{2! \times 2!}$$

1,00,800

② consonants occur together

OIUIO $\boxed{CNSTTTN}$

$$\frac{6!}{2! \times 2!} \times \frac{7!}{3! \times 2!}$$

75,600

③ Vowels & consonants occur alternatively

_ C _ N _ S _ _ _ _ _ _ _ _

C _ N _ S _ T _ T _ T _ N

$$\frac{7!}{3!\,2!} \times \frac{5!}{2! \times 2!}$$

④ Letter N will occur at beginning & ending

CONSTITUTION

N _ _ _ _ _ _ _ _ _ _ N

$$\frac{10!}{2!\,2!\,3!} = 1{,}51{,}200$$

→ How many three digit numbers can be formed by using 6 digits to

$\{2, 3, 4, 5, 6, 8\}$

find

a) If repititions of digits are allowed

$\Rightarrow 6^3$

b) If repititions are not allowed

${}^6P_3$

c) The number is to be odd and repititions are not allowed

_ _ 3 $\Rightarrow 1 \times {}^5P_2$

_ _ 5

$1 \times {}^5P_2 \Rightarrow 2 \times {}^5P_2$

d) No. is to be even and repititions are not allowed

$\_\ \_\ \underline{2}$ $\quad 4 \times {}^5P_2$

$\_\ \_\ \underline{4}$ $\quad 4 \times {}^5P_2$

$\_\ \_\ \underline{6}$ $\quad 4 \times {}^5P_2$

$\_\ \_\ \underline{8}$ $\quad 4 \times {}^5P_2$

$\Rightarrow 4 \times {}^5P_2$

$= 80$ ways

e) The number is to be multiple of 5 and repetitions are not allowed.

$\_\ \_\ \underline{5}$. $\quad 1 \times {}^5P_2 = 20$

f) The number must contain digit 5 and repititions are not allowed

$\_\ \_\ \_ \quad 3 \times {}^5P_2 = 60$

g) The number must contain digit 5 and repititions are allowed.

$\_\ \_\ \_$

$6^3 - 5^3$

91

$3 \times \frac{6 \times 5}{4!}$

$\begin{array}{r} 30 \\ 3 \\ \hline 90 \end{array}$

$\begin{array}{r} 60 \\ +6 \\ \hline 66 \end{array}$

${}^6P_2$

③ How many Permutations can be made to the of word MISSISSIPPI .

1 1 1 1

$$\frac{11!}{4!\,4!\,2!}$$

→ In how many of these vowels occupying the even places

_ _ _ I _ I _ I O I _

$1 \times \frac{7!}{4!\,2!}$ $\frac{5P_4}{4!} \times \frac{7!}{4!\,2!}$

→ $^nP_r = 684800$ & $^nC_r = 120$

n, r, P

$\frac{n!}{(n-r)!} = 684800$ $\frac{n!}{(n-r)!\,r!} = 120$

$120 \times r! = 684800$

$\boxed{r! = 5706}$

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Find the value of n if ${}^{n}P_5 : {}^{(n-1)}P_4 = 8:1$

$$\frac{\dfrac{n!}{\cancel{(n-5)!}}}{\dfrac{(n-1)!}{\cancel{(n-5)!}}} = \frac{8}{1}$$

$$\frac{n!}{(n-1)!} = \frac{8}{1}$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = \frac{8}{1}$$

$$\boxed{n=8}$$

→ If ${}^{n}C_3 = {}^{n}C_5$ ${}^{2n}C_2$

$$\frac{\cancel{n!}}{(n-3)!\,3!} = \frac{\cancel{n!}}{(n-5)!\,5!}$$

$$\frac{2n!}{(2n-2)!\,2!}$$

$$\frac{(2n)(2n-1)}{\cancel{2!}}$$

$$\frac{1}{\cancel{(n-5)!\,3!}} = \frac{1}{(n-5)(n-4)\times 5\times 4}$$

$4n^2-$

$2'$

$$20(n-5)(n-4) = 1$$

$$20(n^2-9n+20) = 1$$

$\cancel{n^2-9}$

$$20n^2 - 180n + 399 = 0$$

$$\frac{1}{(n-3)(n-4)\times \cancel{3!}} = \frac{1}{\cancel{(n-5)!}\times 5\times 4}$$

$^8C_3 = {}^8C_5$

$$20 = n^2 - 7n + 12$$

$$n^2 - 7n - 8 = 0$$

$$\boxed{n = 8}$$

$$^{2n}C_n = n(2n-1)$$
$$= 8(15)$$
$$= 120$$

→ find value of n in $^nP_4 = 12\,^nP_2$

$$\frac{n!}{\cancel{(n-4)!}} = \frac{12\,n!}{(n-2)!} \quad 12 \times \frac{n!}{(n-2)(n-3)}$$

$$1 = \frac{12}{(n-2)(n-3)}$$

$$n^2 - 5n + 6 = 12$$

$$n^2 - 5n - 6 = 0$$

$$n^2 \cancel{-3n} - 6n + n - 6$$

$$n(n-6) + (n-6)$$

$$(n-6)(n+1) = 0$$

$$\boxed{n = 6}$$

Binomial theorem (Coefficients)

$1 + nx +$

$$(1+x)^n = \sum_{r=0}^{n} {}^nC_r \cdot x^r$$

$$(1+x)^{-n} = \sum_{r=0}^{n} (-1)^r \, {}^{(n+r-1)}C_r \cdot x^r$$

$$(1-x)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r \, x^r$$

$3+3-1C_3$ $\frac{5\times4\times2}{2\times1}$

$3+2-1C_2 \, x^2$

$$(1-x)^{-n} = \sum_{r=0}^{n} {}^{(n+r-1)}C_r \cdot x^r$$

$3+2-1$ $4C_2$ $\frac{4\times3}{2\times1}$

$4C_2$ $\frac{4\times3}{2\times1}$ $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3$

$\frac{5\times4}{2\times1}$ $5C_3$

Problems:

Find the coefficient of x^{12} of the exp $x^3(1-2x)^{10}$

x^3

$$(1-x)^n = 1 - nx + {}^nC_2 x^2 - {}^nC_3 x^3$$

$-{}^nC_9$

$-{}^nC_9$

$x^{r+3} = x^{12}$

$-{}^{10}C_9$ $\frac{10\times9}{1\times2}$

-10

$r = 9$.

${}^{10}C_9 \cdot (-1)^9 \cdot 2^9$

$= -5120$

$$x^3\left[\sum_{r=0}^{10} (-1)^r \, {}^{10}C_r (2x)^r\right]$$

$$x^3\left[{}^{10}C_r (-1)^r \cdot (2x)^r\right]$$

$$x^3\left({}^{10}C_r \, (-1)^r \cdot 2^r \cdot x^r\right)$$

$$= {}^{10}C_r \cdot (-1)^r \cdot 2^r \cdot x^{r+3}$$

→ find coeff of x^5 of the exp $(1-2x)^{-7}$

$$\sum_{r=0}^{7} {}^{(7+r-1)}C_r \cdot (2x)^r$$

$${}^{(7+r-1)}C_r \; 2^r \cdot x^r$$

$${}^{11}C_5 \cdot (2)^5$$

coeff of x^5 is $\underline{14784}$

→ find coeff of x^{12} of the exp $\dfrac{x^2}{(1-x)^{10}}$

$$x^2(1-x)^{-10}$$

$$\sum_{r=0}^{10} {}^{(9+r)}C_r \cdot x^{2+r}$$

$2+r = 12$

$r = 10$

$\cancel{{}^{(9+12)}C_{12}}$ $\quad {}^{19}C_{10}$

$\cancel{2},93,1$ $\quad$ 92,378

$${}^{11}C_7$$

→ find coefficient of x^{12} of $(1+x)^{-20}$

$$\sum_{r=0}^{20} (-1)^r \, {}^{(20+r-1)}C_r \, x^r$$

$$(-1)^r \, {}^{(19+r)}C_r \, x^{12}$$

$$^{(19+12)}C_{12}$$

$$^{31}C_{12} = 14,11,20,525$$

– find coeff of x^{27} of exp

$$\left(x^4 + x^5 + x^6 + \text{---}\right)^5$$

$$\left[x^4\left[1 + x + x^2 + \text{---}\right]\right]^5$$

$$x^{20}\left[\underline{1 + x + x^2 + \text{---}}\right]^5$$

$x^{20} \cdot {}^{(n+r-1)}C_r \cdot x^r$ $\qquad$ $x^{20}\left[(1-x)\right]^{-5}$

$$x^{20}\left[(1-x)^{-1}\right]^5$$

~~5~~

${}^{(4+r)}C_r \cdot x^{r+20}$ $\qquad$ $x^{2}\left[(1-x)^{-5}\right]$ $\qquad$ $20+r = 27$, $r = 7$

$r + 20 = 27$

$r = 7$

$$x^{20}\left[\sum_{r=0}^{5} {}^{(4+r)}C_r \, x^r\right] \qquad {}^{11}C_7$$

$$330 = \sum_{r=0}^{5} {}^{(4+r)}C_r \, x^{20+r}$$

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→ → find coeff of x^{27} of

$$\left[x^4 + 2x^5 + 3x^6 + \cdots\right]^5 \qquad (1-x)^{-2}$$

$$\left[x^4\left[1 + 2x + 3x^2 + \cdots\right]\right]^5$$

$$\left[x^4\left[(1-x)^{-2}\right]\right]^5$$

$$x^{20}\left[(1-x)^{-10}\right]$$

$$x^{20}\left[\sum_{r=0}^{\infty} {}^{(9+r)}C_r \, x^r\right]$$

$$20 + r = 27$$

$$\boxed{r = 7}$$

$${}^{(9+7)}C_7$$

$${}^{16}C_7$$

$$11{,}440$$

Multinomial Theorem :

The coefficient of $x_1^{n_1} \cdot x_2^{n_2} \cdot x_3^{n_3}$ in the exp of $(ax + by + cz)^n$ is $\dfrac{n!}{n_1!\, n_2!\, n_3!} \cdot (ax)^{n_1} \cdot (by)^{n_2} (cz)^{n_3}$

Find the coeff of $x^3y^3z^2$ of $(2x-3y+5z)^8$

$$\frac{8\times \cancel{6}\times 7\times 5\times \cancel{4}^{2}}{\cancel{3!\,3!\,2!}}\times (2)^3\cdot(-3)^3(5)^2$$

$$-\ 8\times 7\times 10\times 8\times 27\times 25$$

$$-64\times 70\times 27\times 25$$

$$-30{,}24{,}000$$

→ find coeff of xyz^2 in exp $(2x-y-z)^4$

$$\frac{4\times 3}{1!\,1!\,\cancel{2!}}(2x)^1(-y)^1(-z)^2$$

$$12\times 2\times(-1)\times(1)$$

$$-24$$

→ find coeff of $a^2b^3c^3d^5$ in exp $(a+2b-3c+2d+5)^{16}$

$$\frac{16!}{2!\,3!\,3!\,5!\,3!}(1)^2\times(2)^3\cdot(-3)^3\cdot(2)^5\,(5)^3$$

$$-\frac{16!\times 50}{3}\qquad \frac{16!\times 1\times \cancel{8}^{4}\times(-\cancel{27}^{\cancel{9}^{3}})\times \cancel{32}^{\cancel{16}^{\cancel{8}\,\cancel{4}\,2}}\times \cancel{125}^{25}}{\cancel{2}\times\cancel{6}^{\cancel{3}}\times\cancel{6}^{\cancel{3}}\times\cancel{120}^{\cancel{30}^{\cancel{15}\,3}}\times\cancel{6}\,\cancel{3}}$$

$$-3\cdot 487\times 10^{14}$$

→ find coeff of x^{18} in the following exp

$$(x+x^2+x^3+x^4+x^5)(x^2+x^3+x^4+\cdots)^5$$

$$x[1+x+x^2+x^3+x^4]\,[x^2[1+x+x^2+\cdots]]$$

$$\cancel{x}\,[\cancel{(1-x)}^{-1}]\,[x^2\,[1-x]^{-1}]^5$$

$$x^{11}[1+x+x^2+x^3+x^4]\,[1-x]^{-5}$$

$$x^{11}(1+x+x^2+x^3+x^4)\left[{}^{5+r-1}C_r\,x^r\right]$$

$$(1+x+x^2+x^3+x^4)\left[{}^{5+r-1}C_r\,x^{11+r}\right]$$

$11+r=18$

$r=7$

$$(1+x+x^2+x^3+x^4)\left[{}^{11}C_7\,x^{18}\right]$$

$$^{11}C_7\,x^{18}+{}^{10}C_6\,x^{18}+{}^{9}C_5\,x^{18}+{}^{8}C_4\,x^{18}+{}^{7}C_3\,x^{18}$$

$$x^{18}\left[{}^{11}C_7+{}^{10}C_6+{}^{9}C_5+{}^{8}C_4+{}^{7}C_3\right]$$

$$=771$$

→ find coefficient of x^{18} in exp

$$(x+x^3+x^5+x^7+x^9)(x^3+2x^4+3x^5+\cdots)^3$$

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$$x(1+x^2+x^4+x^6+x^8)\left[x^9[1+2x+3x^2+\cdots]^3\right]$$

$$(1-x)^{-2}$$

$$x\left[1+x^2+x^4+x^6+x^8\right]\left[x^9\left[1-x\right]^{-6}\right]$$

$$x^{10}\left[1+x^2+x^4+x^6+x^8\right]\left[1-x\right]^{-6}$$

$$^{6+r-1}C_r\, x^{10+r}$$

$$\left[1+x^2+x^4+x^6+x^8\right]\left[{}^{13}C_8\, x^{18}\right]$$

$10+r=18$

$r=8$

$$^{13}C_8 x^{18}+{}^{12}C_6 x^{18}+{}^{9}C_4 x^{18}+{}^{7}C_2 x^{18}+{}^{5}C_0 x^{18}$$

$$= \underline{\underline{1897}}$$

→ find coeff of x^n in the following

$$(x^2+x^3+x^4+\cdots)^4$$

$$\left[x^2[1+x+x^2+\cdots]\right]^4$$

$$x^8\left[(1-x)^{-1}\right]^4$$

$$x^8(1-x)^{-4}$$

$$\left[{}^{4+r-1}C_r\, x^{r+8}\right]$$

$r=n$

$r+8=n$

$\boxed{r=n-8}$

$$^{4+n-8-1}C_{n-8}\, x^8 \qquad {}^{n-5}C_{n-8}$$

→ find coeff of x^n

$$(1+x^2+x^4+\cdots)^7$$

$$(1-x^2)^{-7}$$

$$1+x^2$$

$$\cancel{(1+x^2[1+x^2+}$$

$$(1-x^2)^{-7}$$

$$(1+x)^{-7}(1-x)^{-7}$$

$$\left[(-1)^7\ {}^{7+r-1}C_r\, x^r\right]\left[{}^{7+r-1}C_r\, x^r\right]$$

$$\left[\sum_{r=0}^{n} -\ {}^{7+r-1}C_r\, x^r\right]\left[\sum_{r=0}^{n} {}^{7+r-1}C_r\, x^r\right]$$

$$-\left(\sum_{r=0}^{n} {}^{7+r-1}C_r\, x^r\right]^2$$

$$x^n = {}^{(6+\frac{n}{2})}C_6$$

$(1-x^2)^{-7}$

$1 + x^2 + (x^2)^2 + (x^2)^3$

$\sum_{r=0}^{n} {}^{n+r-1}C_{2r} \cdot x^{2r}$

$2r = n$

$r = n/2$

$$\boxed{{}^{n+n/2-1}C_{n/2}}$$

${}^{7+2r-1}C_{2r} \cdot x^{2r}$

$2r = n$

$r = n/2$

${}^{6+2(n/4)}C_{7\cdot(n/4)}$

${}^{(6+4)}C_{n}$

pigeon hole Principle :

If 'n' Pigeons are assigned to 'm' pigeon holes then atleast one pigeon hole contains two or more pigeons i.e $m<n$

$$\left[\frac{n-1}{m}+1\right]$$

11 P

10 h

$\frac{10}{10}+1$

Here , n = no. of pigeons

m = no. of holes

→ on a group of 13 children there must be atleast two children born in same month

$$\frac{13-1}{12}+1 \Rightarrow \frac{12}{12}+1 = \underline{\underline{2}}$$

→ Prove that eight cars carry 26 Passengers atleast one car has four or more passengers

$$\frac{26-1}{8}+1 = \frac{25}{8}+1 = 4.125$$
$$= 4$$

Principle of Inclusion and exclusion :

1. $|A \cup B|$ (or) $n(A \cup B) = n(A)+n(B)-n(A \cap B)$
2. $|\overline{A \cup B}|$ or $n(\overline{A \cup B}) = n(U)-n(A \cup B)$
3. $n(A \cup B \cup C) = n(A)+n(B)+n(C)-n(A \cap B)-n(B \cap C)-n(C \cap A)+n(A \cap B \cap C)$

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4. $|\overline{A \cup B \cup C}| = n(U) - n(A \cup B \cup C)$

$n(U) - n(A) - n(B) - n(C) + n(A \cap B) + n(B \cap C) + n(C \cap A) - n(A \cap B \cap C)$

5. $n(\overline{A} \cap \overline{B}) = n(\overline{A \cup B}) = n(U) - n(A \cup B)$

$= n(U) - n(A) - n(B) + n(A \cap B)$

Problems :

→ how many integers between 1 and 300

$\boxed{\quad 300}$

i) divisible by atleast one of 5, 6, 8

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 60 + 50 + 37 - 10 - 12 - 225 + 2 \cdot 5$

$\underline{\underline{125}}$

ii) divisible none of 5, 6, 8

$300 - (105)$

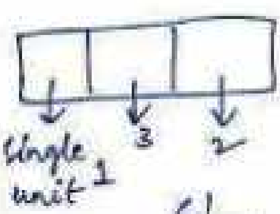
$n(B \cap C) = \dfrac{m(U)}{n(B) \times n(C)}$

$\underline{\underline{195}}$

→ how many ways can the letters {4a's, 3b's, 2c's} be arranged so that so that all the letters of same kind are not in single block

$\dfrac{9!}{4!\,3!\,2!} = S = 1,260$

$S_1 =$



$S_1 = 4$

$S_2 = 3$

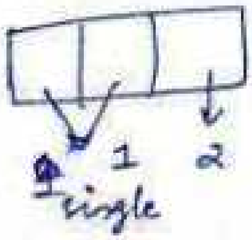
$S_3 = 2$

$n(S_1) = 60 = {}_3 \dfrac{6!}{3!\,2!}$

$n(S_2) =$ [illegible] $= 21 \times 5 = 105$

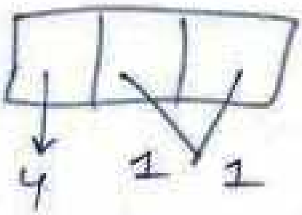
$n(S_3) = 280$

$n(S_1 \cap S_2) =$



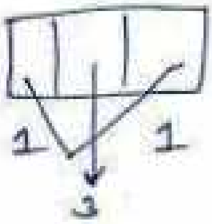
$= \dfrac{4!}{2!} = 4 \times 3 = 12$

$n(S_2 \cap S_3)$



$\dfrac{6!}{4!} = 6 \times 5 = 30$

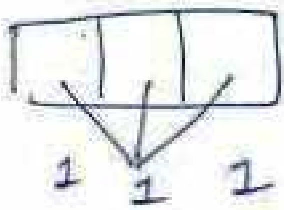
$n(S_3 \cap S_1) =$



$\dfrac{5!}{3!} = 5 \times 4 = 20$

280
105
60
448
62
386

$n(S_1 \cap S_2 \cap S_3) =$



$= 3! = 6$

$n(S_1 \cup S_2 \cup S_3) = 60 + 105 + 280 - 12 - 30 - 20 + 6$

$448 - 62$

$386 + 3 = 389$

$n(\overline{S_1 \cup S_2 \cup S_3}) = 1260 - 389 = 871$

How many integers b/w 1 and 10^4 contain one '8' and one '9'

$n(S) = 10{,}000$

$\boxed{2\times1\times7 + 2\times1\times8 + 8\times1\times8}$

8 9   9 8

$= 14+16+16$

$\Rightarrow 2$

_ 8 9 → 7

_ 9 8 → 7

9 _ 8 → 8

8 _ 9 → 8

9 8 _ → 8

8 9 _ → 8

$3\times2\times7 +$ 42

$= 42$

~~7x2x1 = 7~~

$3\times2\times8 +$

$3\times2\times8$

8 9

$= \cancel{4\times3\times8\times7}$

$=$

$4C$

Total ways $= 2\times1\times7\times8 + 2\times1\times7\times8 + 2\times1\times8\times8 + 2\times1\times8\times8$

_ _ 9 8

8 9 _ _

$2[56+56+64+64]$

$2[112+128]$

_ _ 9 8

$720 + 46 + 2$

$= 768$

_ _ _ _

9 8

UNIT-IV

Recurrence relations:

$$(1-x)^{-1} = 1+x+x^2+x^3+ \cdots = \sum_{r=0}^{n} {}^{n+r-1}C_r x^r$$

A sequence may be defined by indicating a relation connecting its general term with one or more its preceding terms is called as recurrence relation. Here a general term is connected with its preceding terms a_{n-1}, a_{n-2}. Such a relation is called recurrence relation and the process of determining the value of a_n is called solution of recurrence relation.

→ Find coeff of x^{12} in $\frac{x^2}{(1-x)^{10}}$

$$x^2(1-x)^{-10}$$

$$x^2\left[\sum_{r=0}^{n} {}^{10+r-1}C_r\, x^r\right]$$

$${}^{10+r-1}C_r\; x^{2+r}$$

$2+r=12$
$r=10$

$${}^{19}C_{10}\; x^{12}$$

92,378

→ Find coeff of x^{10} in given exp $\frac{x^3-5x}{(1-x)^3}$

$$x^3-5x\,(1-x)^{-3}$$

$$(x^3-5x)\sum_{r=0}^{n} {}^{3+r-1}C_r\, x^r$$

$$(x^3-5x)\sum_{r=0}^{n} {}^{2+r}C_r\, x^r$$

$1+3+6+10+15$

$${}^{2+r}C_r\, x^{3+r} - 5x\, {}^{2+r}C_r\, x^{1+r}$$

$r=7$ $\qquad$ $r=9$

$${}^{9}C_7 - 5\, {}^{11}C_9$$

$$-239$$

Generating Function :

Consider a sequence of real numbers $a_0, a_1, a_2, \ldots, a_n$. Let us denote this by a_r where $r = 0,1,2,3,\ldots$. Suppose there exist a function $f(x)$ whose expansion is a series of powers of x

$$f(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + \cdots$$

$$\sum_{r=0}^{\infty} a_r x^r$$

Here $f(x)$ is generating function for given sequ..

→ find the generating function for Sequence

$1, 1, 1, 1, ----$

$\Rightarrow 1 \cdot x^0 + 1 \cdot x + 1 \cdot x^2 + 1 \cdot x^3 + ---$

$1 + x + x^2 + x^3 + --$

$(1-x)^{-1}$

→ find generating func for Sequence $1, -1, 1, -1, -$

$1 - x + x^2 - x^3 + ---$

$(1+x)^{-1}$

→ $1, 2, 3, 4, 5, 6, --$

$1 + 2x + 3x^2 + 4x^3 + --$

$(1-x)^{-2}$

→ find G.F of Sequence $1, 1, 0, 1, 1, 1, --$

$1 + x + x^3 + x^4 + x^5 + --$

$(1 + x + 1 \cdot x^2 + x^3 + x^4 + --) - x^2$

$f(x) = \boxed{(1-x)^{-1} - x^2}$

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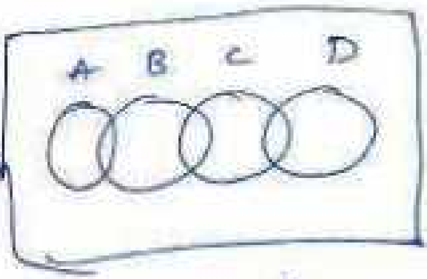
→ 0, 1, 2, 3, 4

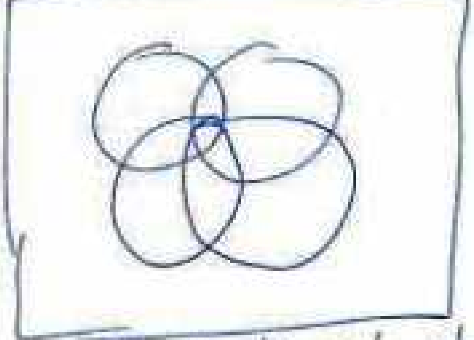
$$x + 2x^2 + 3x^3 + 4x^4 + \cdots$$

$$x\left[1 + 2x + 3x^2 + 4x^3 + \cdots\right]$$

$$x\left[(1-x)^{-2}\right]$$

→ Among the students in a hostel, 12 students study mathematics (denoted by A). Slly 20 students study physics (B), 20 students study chemistry (C) and 8 students student biology (D). There are 5 students for A & B, 7 students for A & C, 4 students for A & D 16 for B & C, 4 students for B & D 3 students for C & D. There are 3 students for A, B & C. two students for A, B & D, two (2) students for B, C & D. 3 students for A, C & D. Finally there are two students who study all these subjects (A, B, C, D). Further more there are 71 students who donot study any of these subjects. Find total no. of students in hostel.





$n(A) = 12$

$n(B) = 20$

$n(C) = 20$

$n(D) = 8$

$n(A \cap B) = 5$

$n(A \cap C) = 7$

$n(A \cap D) = 4$

$n(B \cap C) = 16$

$n(B \cap D) = 4$

$n(C \cap D) = 3$

$n(A \cap B \cap C) = 2$

$n(A \cap B \cap D) = 2$

$n(B \cap C \cap D) = 2$

$n(A \cap C \cap D) = 3$

$n(A \cap B \cap C \cap D) = 2$

$n(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}) \cancel{n(\overline{A \cap B \cap C \cap D})} = 71$

$n(S) = ?$

$n(S) = n(A \cup B \cup C \cup D) + 71$

$n(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}) = n(U) - (n(A) + n$

$+ n(C) + n(D) - n(A \cap B) - n(A \cap$

$- n(A \cap D) - n(B \cap C) - n(B \cap D) - n(C \cap D) +$

$n(A \cap B \cap C) + n(A \cap B \cap D) + n(B \cap C \cap D$

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$n(A\cap C\cap D) + n(A\cap B\cap C\cap D))$

$n(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}) = n(U)$

$71 + (12+20+20+8+5+7+4+16+4+3+2+2+2+3+2)$

$71 + (29)$

$= \underline{\underline{100}}$

→ A Survey of 500 televisions Viewers of a Sports channel Produced the following information. 285 People are watching cricket, 195 people are watching hockey. 115 People are watching football, 45 are watching both C & F 70 watching both C & H. 50 People H & F 50 People donot watch three games

i) How many Viewers watch all three kinds of games

$(\overline{A\cup B\cup C}) = 50$

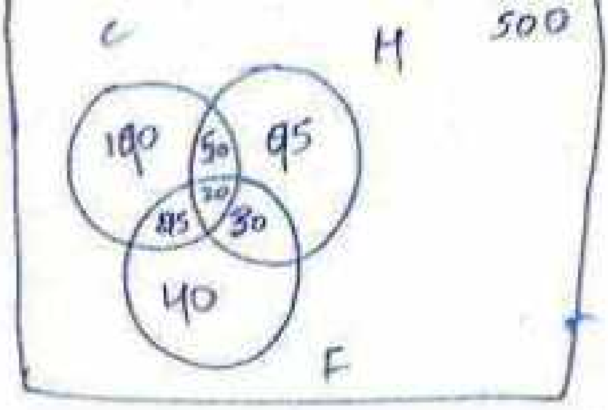
$500 - 50 = 450$

$115 + x$

$450 = 285+195+115 - 70 - 45 - 50 + n(A\cap B\cap C)$

$n(A\cap B\cap C) = n(A)$

ii) How many watch exactly one of Sports



$n(C) = 285$

$n(H) = 195$

$n(F) = 115$

$n(C \cap H) = 70$

$n(C \cap F) = 45$

$n(H \cap F) = 50$

$n(C \cap F \cap H) = n(C \cup F \cup H) - n(C) - n(F) - n(H) + n(C \cap F) + n(F \cap H) + n(C \cap H)$

$n(C \cap F \cap H) = 20$

→ No. of People watch all three games = 20

→ No. of People watch cricket = 190

No. of People watch Hockey = 95

No. of People watch football = 40

325

→ find generating function for a Sequence

$1^2, 2^2, 3^2, 4^2, --$

$1^2 x^0 + 2^2 x + 3^2 \cdot x^2 + 4^2 \cdot x^3 + --$

$1 + 4x + 9x^2 + 16x^3 + 25x^4 + --$

$\cancel{(1-x)^{-4}}$

$\sum_{r=0}^{n} 4+r-1_{C_r} x^r$

$\sqrt{1} + 4x + 10x^2 + 20x^3 + \cdots) \cancel{-(x^2+4x^3}$

$f(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + \cdots$

assume $(1-x)^{-2} = 1+2x+3x^2+4x^3+\cdots$

$\Rightarrow x(1-x)^{-2} = x+2x^2+3x^3+4x^4+\cdots$

$\Rightarrow$ diff on both sides

$$\underline{LHS} = \frac{(x)}{(1-x)^{+2}} \Rightarrow \frac{(1-x)^2 \cdot 1 + x \cdot 2(1-x)}{(1-x)^4}$$

$$= \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4}$$

$$\Rightarrow \frac{(1-x)+2x}{(1-x)^3}$$

$$\Rightarrow \frac{1+x}{(1-x)^3}$$

$\underline{RHS}$: $1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots$

$1^2x^0 + 2^2x^1 + 3^2x^2 + 4^2x^3 + \cdots$

$\rightarrow$ Coeff of x^{20} in sequence $(x^2+x^3+x^4+x^5+x^6)^5$

$\Rightarrow x^{10}[1+x+x^2+x^3+x^4+\cdots]^5$ $\quad (10+r=20$

$x^{10}\left[(1-x)^{-1}\right]^5 \Rightarrow x^{10}\left[(1-x)^{-5}\right]$

$x^{10}\left[{}^{5+r-1}C_r\, x^r\right]^{10}$

$\Rightarrow {}^{14}C_{10}$

$\rightarrow x^{10}[1+x+x^2+x^3+x^4]^5$

$x^{10}\left[\sum_{r=0}^{4} {}^{n+r-1}C_r\, x^r\right]^5$

$\left[\sum_{r=0}^{4} {}^{3+r}C_r\, x^{10+r}\right]^5$

$\left[{}^{3}C_0 x^{10} + {}^{4}C_1 x^{11} + {}^{5}C_2 x^{12} + {}^{6}C_3 x^{13} + {}^{7}C_4 \right.$

$\rightarrow 1^3+2^3+3^3+ \cdots$

$\rightarrow$ find G.F for sequence $1^2, 2^2, 3^2, 4^2, \cdots$

$1^2 \cdot x^0 + 2^2 \cdot x + 3^2 x^2 + 4^2 x^3 + \cdots$

$a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \cdots$

$(1-x)^{-2} = 1+2x+3x^2+4x^3+\cdots$

$1^3 x^0 + 2^3 x + 3^3 x^2 + \cdots$

$(1-x)^{-3} = 1+3x+6x^2+10x^3+\cdots$

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→ find G.F for sequence $0^2, 1^2, 2^2, 3^2, 4^2, \ldots$

$$0^2 \cdot x^0 + 1^2 \cdot x^1 + 2^2 x + 3^2 \cdot x^2 + 4^2 x^3 + \ldots$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$$

$$x(1-x)^{-2} = x + 2x^2 + 3x^3 + 4x^4 + \ldots$$

$$\frac{d}{dx}\left(x(1-x)^{-2}\right) = 1 + 2^2 x + 3^3 x + 4^4 x + \ldots$$

$$\frac{d}{dx}\left(\frac{x}{(1-x)^2}\right) = 1 + 2^2 x + 3^2 x + 4^4 x + \ldots$$

$$\frac{(1-x)^2 + x \cdot 2(1-x)}{(1-x^0)^4}$$

<u>Counting technique</u>: In this technique we find the generating function or general soln for the equation of the form $x_1 + x_2 + x_3 + \ldots = \text{Value}$

The GF for this equation is

$$f(x) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot \ldots$$

and we find coeff of $\boxed{x^{\text{value}}}$

3/5

find the generating function for eq

$x_1 + x_2 + x_3 + x_4 = 25$ for

a) Non-negative integers $0, 1, 2, 3, \ldots$

b) Positive integers $1, 2, 3, 4, \ldots$

$$f(x_1) = x^0 + x^1 + x^2 + x^3 + \ldots = (1-x)^{-1}$$
$$f(x_2) = x^0 + x^1 + x^2 + x^3 + \ldots = (1-x)^{-1}$$
$$f(x_3) = (1-x)^{-1}$$
$$f(x_4) = (1-x)^{-1}$$

$$f(x) = f(x_1)\, f(x_2)\, f(x_3)\, f(x_4)$$
$$= (1-x)^{-4}$$

$\cancel{\rightarrow 1 + 4x +}$

$$\sum_{r=0}^{\infty} {}^{n+r-1}C_r \, x^r$$

$$ {}^{3+r}C_r \, x^r$$

$r = 25$, because $x^{\text{value}} = x^r = x^{25}$

As per counting technique

$$= {}^{28}C_{25} \, x^{25}$$

$$= 3,276$$

$4+2$ ... $5c_2$ ...

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→ Positive integers:

$f(x_1) = x + x^2 + x^3 + x^4 + \dots$

$= x(1 + x + x^2 + x^3 + \dots)$

$f(x_2) = f(x_3) = x\left[(1-x)^{-1}\right] = f(x_4)$

$f(x) = f(x_1)\, f(x_2)\, f(x_3)\, f(x_4)$

$x^4\,[1-x]^{-4}$

$x^4 . \sum_{r=0}^{\infty} {}^{4+r-1}C_r\, x^r$

$\sum_{r=0}^{\infty} {}^{3+r}C_r\, x^{4+r}$ $\qquad 4 + r = 25$

$r = 21$

${}^{3+21}C_{21}$

${}^{24}C_{21} = 2{,}024$

→ find no. of integral solutions for the equation

$x_1 + x_2 + x_3 + x_4 + x_5 = 30$ for $x_i \geq 0$, $i = 1,2,3,4,5$,

$\boxed{x_2 \text{ is even}}$ and x_3 is odd

31 31

$f(x) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot f(x_4) \cdot f(x_5) = x^{30}$

$f(x_1) = x^0 + x^1 + x^2 + x^3 + x^4 + \dots = (1-x)^{-1}$

$f(x_2) = x^0 + x^2 + x^4 + x^6 + x^8 + \cdots$

$1 + x^2 + x^4 + \cdots$

$\cancel{1 + x^2 [1 + x^2 + x^3 + \cdots]}$

$\cancel{1 + x^2 [1 - x}$

$(1 - x^2)^{-1}$

$f(x_3) = x^1 + x^3 + x^5 + x^7 + \cdots$

$= x [1 + (x^2)^1 + (x^2)^2 + \cdots]$

$x [(1 - x^2)^{-1}]$

$f(x_4) = (1 - x)^{-1}$

$f(x_5) = (1 - x)^{-1}$

$f(x) = (1 - x)^{-3} (1 - x^2)^{-2} x \cdot (1-x^2) = x^{30}$

$= x \cdot \sum_{r=0}^{3} {}^{2+r}C_r \, x^r \cdot \sum_{r=0}^{2} {}^{2+r-1}C_2 \, x^{2r} = x^{30}$

$= \left[{}^{2+r}C_r \; x^{1+r} \right] \left[{}^{1+s}C_s \cdot x^{2s} \right] = x^{30}$

9,10
1,14
3,13

$= (1+r) + 2s = 30$

$\boxed{r + 2s = 29}$

15

$2s + r = 29$

No. of integral

$r = 29 - 2s$

s	r
1	27
2	25
13	3
14	1

Total no of integral solutions are 14

In how many ways 12 oranges, be distributed among three children A, B, C so that a gets 4, B and C gets atleast '2' but C gets not more than 5

$(4)A \quad \overset{(2)}{B} \quad \overset{(2)}{C}$

$12-4-2-2$

$f(x) = f(x_1)\, f(x_2)\, f(x_3) = (4)$ remaining

$x_1 \geq 4 \qquad x_1 + x_2 + x_3 = 12$

$\downarrow$

$f(x_1) = x^4 + x^5 + x^6 + \cdots$

$= x^4 [1-x]^{-1}$

$x_2 \geq 2$

$f(x_2) = x^2 + x^3 + x^4 + \cdots$

$x^2 [1-x]^{-1}$

$2 \leq x_3 \leq 5$

$f(x_3) = x^2 + x^3 + x^4 + x^5$

$=$

$f(x) = x^6 (1-x)^{-2} \left[x^2 + x^3 + x^4 + x^5 \right]$

$(x^8 + x^9 + x^{10} + x^{11}) \left[\sum_{r=0}^{2} {}^{2+r-1}C_r \, x^r \right]$

$x^8 + x^9 + x^{10} + x^{11} \left[{}^{1+r}C_r \, x^r \right]$

0	2	3	→	4
[illegible]	2	2	⇒	3
[illegible]	2	2	⇒	3
[illegible]	2	2	⇒	3
0	3	2	⇒	3
				2
4	0	0	⇒	2
8				14

Recurrence relation using Substitution Method :

① Solve recurrence relation

$a_n = a_{n-1} + n$ where $n \geq 1$ $a_0 = 2$

by using Substitution method

If $a = 1 \Rightarrow a_1 = a_0 + 1$

If $a = 2 \Rightarrow a_2 = a_1 + 2$

$a_0 + 1 + 2$

If $n = 3 \Rightarrow a_3 = a_0 + 1 + 2 + 3$

⋮

If $n = n \Rightarrow a_n = a_0 + 1 + 2 + 3 + \cdots + n$

$$a_n = a_0 + \frac{n(n+1)}{2}$$

$$a_n = 2 + \frac{n^2+n}{2}$$

$$\boxed{a_n = \frac{4+n^2+n}{2}}$$

2) Solve RR

$a_n = a_{n-1} + n^3$ $n \geq 1$ $a_0 = 5$

If $a = 1 \Rightarrow a_1 = a_0 + 1^3$

$a = 2 \Rightarrow a_2 = a_1 + 2^3$

|

$a_n = a_0 + 1^3 + 2^3 + 3^3 + \cdots + n^3$

$$a_n = 5 + \left(\frac{n(n+1)}{2}\right)^2$$

$$5 + \frac{n^2(n+1)^2}{4}$$

$$\frac{20 + n^2(n^2+1+2n)}{4}$$

$$\frac{20 + n^4 + 2n^3 + n^2}{4}$$

Solve RR $\quad a_n = a_{n-1} + 3n^2 + 3n + 1, \quad n \geq 1$

$a_1 = a_0 + 3 + 3 + 1 = a_0 + 7$

$a_2 = a_1 + 12 + 6 + 1$

$a_2 = a_1 + 19 \quad = a_1 + 3(2)^2 + 3(2) + 1$

$a_2 = a_0 + 7 + 19$ $\qquad \frac{n(n+1)(2n+1)}{6}$

$a_3 = a_2 + 27$

$n=3 \quad a_3 = a_2 + 3(3)^2 + 3(3) + 1$

$= a_1 + 3(2)^2 + 3(2) + 1 + 3(3)^2 + 3(3) + 1$

$a_3 = a_0 + \underline{3(1)^2} + 3(1) + 1 + \underline{3(2)^2} + 3(2) + 1 + 3(3)^2 + 3(3) + 1 + \cdots$

$\underline{3(n)^2} + 3(n) + 1$

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$$a_0 + \cancel{2}\left[\frac{n(n+1)(2n+1)}{\cancel{6}\,2}\right] + 3\left[\frac{n(n+1)}{2}\right] + n$$

$$1 + \frac{n(n+1)(2n+1)}{2} + \frac{3n(n+1)}{2} + n$$

$$\frac{2 + (n^2+n)(2n+1) + 3n^2 + 3n + 2n}{2}$$

$$\frac{2 + 2n^3 + n^2 + 2n^2 + n + 3n^2 + 3n + 2n}{2}$$

$$\frac{2n^3 + 6n^2 + 6n + 2}{2}$$

$$= n^3 + 3n^2 + 3n + 1$$

→ Solve RR $a_n = a_{n-1} + 3^n \quad n \geq 1 \quad a_0 = 1$

$a_1 = a_0 + 3^1$

$a_2 = a_1 + 3^2 \Rightarrow a_0 + 3^1 + 3^2$

$a_n = a_0 + 3^1 + 3^2 + 3^3 + \cdots + 3^n$

$= 3^0 + 3^1 + 3^2 + 3^3 + \cdots + 3^n$

$\frac{3}{1}$ $a(a^{n-1})$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1}$$

$\frac{a(a^n-1)}{n}$

$\frac{1(1^n-1)}{3}$

$$= 1 + \frac{3(3^n - 1)}{3 - 1} = \frac{3^n - 1}{2}$$

$$\frac{2+3^{n+1}-3}{2} = \frac{3^{n+1}-1}{2}$$

$\rightarrow$ $a_n = a_{n-1} + n^2$, when $n \geq 1$ & $a_0 = 7$

$$= 7 + \left[\frac{n(n+1)(2n+1)}{6}\right.$$

$$= \frac{42 + (n^2+n)(2n+1)}{6}$$

$$= \frac{42 + 2n^3 + n^2 + 2n^2 + n}{6}$$

$$\frac{2n^3 + 2n^2 + n + 42}{6}$$

Solution for first order (or) direct Method of recurrance relation

Let the recurrance relation $a_n = c \cdot a_{n-1} + f(n)$ where $n \geq 1$ the gen soln of above R·R is $a_n = c^n \cdot a_0 + f(n)$

Note: If $f(n) = 0$ then the R·R is homogeneous, then $a_n = c^n \cdot a_0$. If RR is Non-homogeneous then

$$a_n = c^n a_0 + \sum_{k=1}^{n} c^{n-k} \cdot f(k) \quad \text{for } n \geq 1$$

Here $an = c^n a_0 + c^{n-1} f(1) + c^{n-2} f(2) + \cdots + c f(n-1) + f(n)$

$\rightarrow$ Solve RR for $4a_n - 5a_{n-1} = 0$ where $n \geq 1$ & $a_0 = 1$

$$a_n = \frac{5a_{n-1}}{4} \qquad c = 5/4, \; f(n) = 0$$

$$a_n = \left(\frac{5}{4}\right)^n \cdot 1$$

Solve RR $a_{n+1} = 8a_n$ where $n \geq 0$, $a_0 = 4$

$a_n = 8a_{n-1}$

$a_n = 2^3 a_{n-1}$

$a_n = (2^3)^n \cdot 4 = 2^{3n+2}$

→ Solve RR $a_n = 7 \cdot a_{n-1}$ where $n \geq 1$ given $a_2 = 98$

$a_n = 7 \cdot a_{n-1}$

$c = 7 \quad f(n) = 0$

$\boxed{a_n = (7)^n \cdot 2}$

$a_1 = 7 \cdot a_0$

$a_2 = 7 \cdot a_1$

$\boxed{\frac{98}{7} = a_1}$

$\boxed{\frac{98}{2 \times 7} = a_0}$

Q) Solve RR. $a_n = n \cdot a_{n-1}$ for $n \geq 1$, $a_0 = 1$

$a_n = n \cdot a_{n-1}$

$a_1 = 1 \cdot a_0$

$a_2 = 2 \cdot 1 \cdot a_0$

$(n)^n \cdot a_0$

$a_3 = 3 \cdot 2 \cdot 1 \cdot a_0$

$a_n = n!\, a_0$

Q) Solve RR $2a_n - 3a_{n-1} = 0$ $n \geq 1$, $a_4 = 81$

$a_n = \frac{3a_{n-1}}{2}$

$\left(\frac{3}{2}\right)^n \cdot 2^4$

$\frac{3^n}{2^{n-4}}$

$2a_n = 3a_{n-1}$

$2a_1 = 3a_0$

$2a_2 = 3a_1$

$2a_3 = 3a_2$

$2a_4 = 3a_3$

$\boxed{a_2 = 36}$

$\boxed{a_1 = 24}$

$\boxed{a_0 = 16}$

$\boxed{a_3 = 54}$

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$\rightarrow$ $3a_{n+1} - 4a_n = 0$ where $n \geq 0$ & $a_1 = 5$

$3a_{n+1} = 4a_n$

$3a_n = 4a_{n-1}$

$3a_1 = 4a_0$

$\frac{15}{4} = a_0$

$3a_n = 4a_{n-1}$

$a_n = \left(\frac{4}{3}\right) a_{n-1}$

$\left(\frac{4}{3}\right)^n \cdot \frac{15}{4} = \frac{4^{n-1}}{3^{n-1}} \cdot 5$

$\rightarrow$ $a_{n+1} = k \cdot a_n$ for $n \geq 1$ & $a_3 = \frac{153}{49}$ $a_5 = \frac{1377}{2401}$

$a_4 = k \cdot a_3$ $\qquad a_5 = k \cdot a_4$

$a_4 = k \cdot \frac{153}{49}$ $\qquad \frac{1377}{2401} = k^2 \cdot \frac{153}{49}$ $\qquad \boxed{k = \pm 3/7}$

$\rightarrow$ $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$

non-homogeneous

$a_n = 3a_{n-1} + 5 \times 3^n$

$a_n = c^n \cdot a_0 + f(n)$

$= (3)^n \cdot 2 + [f(n)]$

$a_n = c^n a_0 + c^{n-1} f(1) + c^{n-2} f(2) + \cdots + c f(n-1) + f(n)$ — (1)

Here comparing with $a_n = c a_{n-1} + f(n)$.

$a_n = 3a_{n-1} + 5 \times 3^n$

$f(n) = 5 \times 3^n$, $\boxed{c = 3}$ $\rightarrow$ sub in (1)

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$a_n = (3)^n a_0 + (3)^{n-1}(5\times 3^1) + (3)^{n-2}(5\times 3^2) + \cdots + 3\times(5\times 3^{n-1}) + 5\times 3^n$

$(\because a_0 = 2)$

$= 3^n \times 2 + 3^{n-1}(5\times 3) + 3^{n-2}(5\times 3^2) + \cdots + 3(5\times 3^{n-1}) + 5\times 3^n$

$= 3^n [2 + (5+5+5+ \cdots + 5)\, n \text{ times}]$

$= 3^n [2 + 5n]$

→ Solution of second & higher order RR

A relation of form $c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} = f(n)$

where $n \geq 2$

where c_0, c_1, c_2 are constants such type of relations are called second order linear RR. with const coefficients. If we substitute $a_n = \alpha^n$ then relation form will become

$c_0\alpha^n + c_1\alpha^{n-1} + c_2\alpha^{n-2} = f(n)$. This is called auxilary eq or characteristic equation.

If $f(n) = 0$ then given relation becomes

$\boxed{c_0\alpha^n + c_1\alpha^{n-1} + c_2\alpha^{n-2} = 0}$ then it is known as second order homogeneous equation

$c_0\alpha^n + c_1\alpha^{n-1} + c_2\alpha^{n-2} = f(n)$ is called second order non-homogeneous equation

Note/ If different roots are formed the equation then solution is

$$\alpha_n \text{ (or) } a_n = c_1\alpha^n + c_2\alpha^n + c_3\alpha^n + \cdots$$

If two roots are same then homogeneous soln is

$$\alpha_n^{h} = (c_1 + c_2 n)\alpha_1^n + c_3\alpha_3^n + \cdots$$

If 3 roots are same then homogenous sol is

$$\alpha_n^{h} = (c_1 + c_2 n + c_3 n^2)\alpha_1^n + c_4\alpha^n + \cdots$$

1) Solve RR

$$a_n - 7a_{n-1} + 10a_{n-2} = 0 \quad \text{where } n \geq 2 \ (a_0 = 10, a_1 = 41)$$

Given $a_n - 7a_{n-1} + 10a_{n-2} = 0$

Given is 2nd order homogeneous

Auxilary eq is

$$\alpha^2 - 7\alpha + 10 = 0$$

$$\alpha^2 - 5\alpha - 2\alpha + 10 = 0$$

$$\alpha = 2, 5$$

roots are different

Homogeneous equation is

$$a_n \text{ (or) } \alpha_n = c_1\alpha^n + c_2\alpha^n$$

$$a_n = c_1 2^n + c_2 5^n \quad \text{—— (1)}$$

given $a_0 = 10$

$$a_n = c_1 \cdot 2^n + c_2 5^n$$

$$\boxed{n=0}$$

$a_0 = c_1 2^0 + c_2 5^0$

$10 = c_1 + c_2$ — (2)

S^{ly} $a_1 = 41$ keep $n = 1$

$a_1 = c_1 \cdot 2^1 + c_2 5^1$

$41 = 2c_1 + 5c_2$ — (3)

Solve (2) (3) $c_1 = 3, \; c_2 = 7$

sub in eq (1)

$$\boxed{a_n = 3 \cdot 2^n + 7 \cdot 5^n}$$

2) $a_n - 6a_{n-1} + 8a_{n-2} = 0$ where $n \geq 2$, $a_0 = 3, a_1$

auxilary eq is

$\alpha^2 - 6\alpha + 8 = 0$

$\alpha^2 - 4\alpha - 2\alpha + 8 = 0$

$\alpha = 2, 4$ (roots are diff)

$a_n = c_1 \alpha^n + c_2 \alpha^n$

$a_n = c_1 2^n + c_2 \cdot 4^n$ — (1)

given $a_0 = 3$

if $n = 0$

$a_0 = c_1 2^0 + c_2 4^0$

$3 = c_1 + c_2$ — (2)

$\boxed{a_1 = 7}$ if $n = 1$

$a_1 = c_1 \cdot 2^1 + c_2 \cdot 4^1$

$7 = 2c_1 + 4c_2$ — (3)

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solving $c_1 = 5/2$, $c_2 = 1/2$

sub in ①

$$\boxed{a_n = \left(\frac{5}{2}\right)2^n + \left(\frac{1}{2}\right)4^n}$$

3) solve:

i) $a_n - 6a_{n-1} + 9a_{n-2} = 0$ & $a_0 = 5$, $a_1 = 12$

ii) $a_n + 3a_{n-1} - 10a_{n-2} = 0$ & $a_0 = 4$, $a_1 = 3$

iii) $2a_{n+2} - 11a_{n+1} + 5a_n = 0$ & $a_0 = 2$, $a_1 = -8$, $n \geq 0$

i) auxilary Eq:

$$\alpha^2 - 6\alpha + 9 = 0$$

$$\alpha^2 - 3\alpha - 3\alpha + 9 = 0$$

$$\alpha(\alpha - 3) - 3(\alpha - 3) = 0$$

$$\alpha = 3, 3$$

a_n (or) $\alpha_n^4 = (c_1 + c_2 n)\alpha_1^n$ —— ①

$$a_n = (c_1 + c_2 n)3^n$$

$$a_0 = (c_1 + c_2 \cdot 0) \cdot 3^0$$

$$\boxed{5 = c_1}$$

$$a_1 = (c_1 + c_2)3^1$$

$$\boxed{4 = c_1 + c_2} \Rightarrow \boxed{c_2 = -1}$$

$$\boxed{a_n = (5 - n)3^n}$$

11) auxilary equation:

$\alpha^2+3\alpha-10=0$

$\alpha^2+5\alpha-2\alpha-10=0$

$\alpha(\alpha+5)-2(\alpha+5)=0$

$(\alpha+5)(\alpha-2)=0$

$\boxed{\alpha=2,-5}$

$a_0=4, a_1=3$

roots are different.

α_n (or) $a_n = c_1\alpha^n + c_2\alpha^n$ —— ①

$a_n = c_1\alpha^n + c_2\alpha^n$

$a_n = c_1(2)^n + c_2(-5)^n$

$a_0 = c_1(2)^0 + c_2(-5)^0$

$\boxed{4 = c_1 + c_2}$ —— ②

$a_1 = c_1(2)^1 + c_2(-5)^1$

$\boxed{3 = 2c_1 - 5c_2}$ —— ③

$\boxed{c_1 = 4 - c_2}$

$3 = 2(4-c_2) - 5c_2$

$3 = 8 - 2c_2 - 5c_2$

$3 = 8 - 7c_2$

$\cancel{-}5 = \cancel{-}7c_2$

$\boxed{c_2 = 5/7}$

$c_1 = 4 - \frac{5}{7}$

$\boxed{c_1 = \frac{23}{7}}$

$a_n = \frac{23}{7}(2)^n + \frac{5}{7}(-5)^n$

auxilary equation :

$$2a_{n+2} - 11a_{n+1} + 5a_n = 0 \quad , a_0 = 2, a_1 = -8$$

$$2a_n - 11a_{n-1} + 5a_{n-2} = 0$$

auxilary Equation :

$$2\alpha^2 - 11\alpha + 5 = 0$$

10

$$2\alpha^2 - 10\alpha - \alpha + 5 = 0$$

$$2\alpha(\alpha - 5) - (\alpha - 5) = 0$$

$$(\alpha - 5)(2\alpha - 1) = 0$$

$$\alpha = 1/2, 5$$

roots are diff

$$\alpha_n (\text{or})\ a_n = c_1\alpha^n + c_2\alpha^n$$

$$a_n = c_1\left(\tfrac{1}{2}\right)^n + c_2(5)^n \quad \text{--- (1)}$$

$$a_0 = c_1\left(\tfrac{1}{2}\right)^0 + c_2(5)^0$$

$$\boxed{2 = c_1 + c_2} \quad \text{--- (2)}$$

$$a_1 = c_1\left(\tfrac{1}{2}\right) + c_2(5)$$

$$2a_1 = c_1 + 10c_2$$

$$-16 = c_1 + 10c_2 \quad \text{--- (3)}$$

$$c_1 + c_2 = 2$$

$$c_1 + 10c_2 = -16$$

$$- \quad - \quad +$$

$$-9c_2 = 18 \Rightarrow \boxed{c_2 = -2}$$

$$\boxed{c_1 = 4}$$

$$\boxed{a_n = 4\left(\frac{1}{2}\right)^n + (-2)(5)^n}$$

$$= \frac{2^2}{2^n} + (-2)5^n$$

$$\boxed{a_n = 2^{2-n} - 2 \cdot 5^n}$$

Solve the RR $a_n + a_{n-1} - 8a_{n-2} - 12a_{n-3} = 0$ where $n \geq 3$

$a_0 = 1,\ a_1 = 5$

$a_2 = 1$

Step 1 :

auxilary Equation of given equation is

$$\alpha^3 + \alpha^2 - 8\alpha - 12 = 0$$

$1 + 1 - 8 - 12$

$8 + 4 - 16 - 12$

$$(\alpha+2)^2(\alpha-3) = 0$$

$$(\alpha+2)(\alpha+2)(\alpha-3) = 0$$

$$\alpha = -2, -2, 3$$

$$a_n \text{ (or) } \alpha_n = (c_1 + c_2 n)\alpha_1^n + c_3\alpha_2^n \quad \text{—— (1)}$$

$$a_n = (c_1 + c_2 n)(-2)^n + c_3(3)^n$$

$\boxed{n=0}$

$$a_0 = (c_1 + c_2(0))(-2)^0 + c_3(3)^0$$

$$\boxed{1 = c_1 + c_3}$$

$\boxed{n=1}$

$$a_1 = (c_1 + c_2)(-2) + c_3(3)$$

$$5 = -2c_1 - 2c_2 + 3c_3$$

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$\boxed{n=2}$

$$1 = (c_1 + 2c_2)4 + c_3(9)$$

$$4c_1 + 8c_2 + 9c_3 = 1$$

solving ①, ②, ③

-2 0 1

-8 +9

4 +5

$$c_1 + 0 + c_3 = 1$$
$$-2c_1 - 2c_2 + 3c_3 = 5$$
$$4c_1 + 8c_2 + 9c_3 = 1$$

$c_1 = 0, \; c_2 = -1 \quad c_3 = 1$ sub in ①

R.R is

$$\boxed{a_n \text{ (or) } \alpha_n = -n(-2)^n + (3)^n}$$

→ solve RR for a given homogeneous equation

$$a_n - 7a_{n-2} + 10a_{n-4} = 0 \text{ where } n \geq 4$$

$$\alpha^4 - 7\alpha^2 + 10 = 0$$

Put $\alpha^2 = t$

$$t^2 - 7t + 10 = 0$$
$$t^2 - 5t - 2t + 10 = 0$$
$$t(t-5) - 2(t-5) = 0$$
$$(t-5)(t-2) = 0$$
$$t = 2, 5$$

$2 = \alpha^2 \qquad 5 = \alpha^2$

$\alpha = \pm\sqrt{2} \qquad \alpha = \pm\sqrt{5}$

$$\therefore \alpha = \pm\sqrt{2}, \pm\sqrt{5}$$

$\cancel{a_n = (c_1 + c_2 n)}$

$$\boxed{a_n = c_1(\sqrt{2})^n + c_2(-\sqrt{2})^n + c_3(\sqrt{5})^n + c_4(-\sqrt{5})^n}$$

→ Solve RR for given homogeneous Equation

$$2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n \quad \text{for } n \geq 0$$

$a_0 = 0, a_1 = 1$

$a_2 = 2$

$$2a_{n+3} - a_{n+2} - 2a_{n+1} + a_n = 0$$

$$2a_n - a_{n-1} - 2a_{n-2} + a_{n-3} = 0$$

$$2\alpha^3 - \alpha^2 - 2\alpha + 1 = 0$$

$$\alpha = -1, 1, 1/2$$

$2 - 1 - 2 + 1$

$-2 - 1 + 2 + 1$

$$a_n = c_1\alpha_1^n + c_2\alpha_2^n + c_3\alpha_3^n$$

$$a_n = c_1(-1)^n + c_2(1)^n + c_3\left(\frac{1}{2}\right)^n \text{ ——— } (1)$$

$\boxed{n=0}$ $a_0 = c_1 + c_2 + c_3 = 0$

$\boxed{n=1}$ $a_1 = -c_1 + c_2 + \frac{c_3}{2} = 1$

$\boxed{n=2}$ $a_2 = c_1 + c_2 + \frac{c_3}{4} = 2$

$\frac{15}{6} + \frac{1}{6} - \frac{16}{6}$

$$c_1 = 1/6 \quad c_2 = \frac{5}{2} \quad c_3 = \frac{-8}{3}$$

sub in (1)

$$a_n = \frac{1}{6}(-1)^n + \frac{5}{2}(1)^n - \frac{8}{3}\left(\frac{1}{2}\right)^n$$

→ Non-homogeneous (higher Order) / first RR :

Method for finding Solution for given non-homogeneous equation.

(i) If $f(n) = c$, where c is constant then the solution for a given homogeneous equation is $a_n = \alpha$

(ii) If $f(n) = a + bn$ (or) n then the Solution is $a_n = c + dn$

(iii) If $f(n) = a + bn + cn^2$ (or) n^2 then solution is

$a_n = c_1 + c_2 n + c_3 n^2$

(iv) If $f(n) = a^n$ then the Solution is $a_n = d^n$

Note: 1) If we have one root then the Particular solution is $a_n = n \cdot (d \cdot a^n)$

2) If we have repeated roots then solution is

$a_n = n^k (d \cdot a^n)$

→ Solve the solution of Non-homogeneous recursive relns

$a_n = 6a_{n-1} - 9a_{n-2} + f(n)$ where $f(n)$ is

i) 3^n

ii) $n \cdot 3^n$

iii) $n^2 \cdot 2^n$

Here $f(n) = 3^n$ solution is $a_n = d^n$

$$f(n) = \underbrace{a_n - 6a_{n-1} + 9a_{n-2}}$$

<u>auxilary equation</u> :

$$\alpha^2 - 6\alpha + 9 = 0$$

$$\alpha^2 - 3\alpha - 3\alpha + 9 = 0$$

$$\alpha(\alpha - 3) - 3(\alpha - 3)$$

$$(\alpha - 3)^2 = 0$$

$$\alpha = 3, 3$$

$$a_n = (c_1 + c_2 n)(3)^n$$

Here , $f(n) = 3^n$

roots are repeated for two times

$$k = 2$$

$$a_n = n^2 \cdot d \cdot a^n$$

ii) $f(x) = n \cdot 3^n$

$$(c + dn)\, n^2 \cdot d \cdot 3^n$$

iii) $f(n) = n^2 \cdot 2^n$

$$(c_1 + c_2 n + c_3 n^2) \cdot n^2 \cdot d \cdot 9^n$$

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→ Solve the recurrence relation $a_n = 3a_{n-1} + 2n$, $a_1 = 3$

$f(n) = 2n$

$$2n = a_n - 3a_{n-1}$$

auxiliary Eq :

$$\alpha - 3 = 0$$

$$\boxed{\alpha = 3}$$

The homogeneous solution is

$$\boxed{a_n = c_1 \alpha^n = c_1 3^n}$$

Here we assume that y_p
$f(n) = 2n$
then
$a_n = a_n = c + dn$

let $\boxed{n = 1}$

$$a_1 = c + 3$$

$$3 = c + 3$$

$$\boxed{c = 0}$$

$\boxed{a_n = 3n}$ recurrence relation

$$a_n = c + dn$$
$$a_{n-1} = c + d(n-1)$$

$$(c + dn) - 3(c + d(n-1)) = 2n$$

$$c + dn - 3(c + dn - d)$$

$$c + dn - 3c - 3dn - 3d = 2n$$

$$-2c - 2dn - 3d = 2n$$

Comparing n coefficients

$$-2dn - 3d = 2n + 2c$$

$$d = 2(n + c)$$

$$dn - 3dn = 2n$$
$$-2d = 2$$
$$\boxed{d = -1}$$

→ Compare constants.

$$c - 3c + 3d = 0$$

$$-2c + 3d = 0$$

$$\boxed{2a = 3d}$$

$$2c = -3$$

$$\boxed{c = -3/2}$$

$$a_n = c - n$$

$$\boxed{a_n = \frac{-3}{2} - n}$$

$$\boxed{Y_{NH} = Y_H + Y_p}$$

$$Y_{NH} (a) \; a_n = c\,3^n - \frac{3}{2} - n$$

Put $\boxed{n = 1}$

$$a_1 = c_1 \cdot 3 - \frac{3}{2} - 1$$

$$3 = 3c_1 - \frac{5}{2}$$

$$3 + \frac{5}{2}$$

$$\frac{11}{2} = 3c_1$$

$$\boxed{c_1 = \frac{11}{6}}$$

$$\therefore \boxed{a_n = \frac{11}{6} 3^n - \frac{3}{2} - n}$$

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Solve RR of Non homogeneous Equation

$$a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 3^n$$

auxilary eq :

$$\alpha^3 + 6\alpha^2 + 12\alpha + 8 = 0 \qquad (\alpha+2)^3 = 0$$

$$\alpha = -2, -2, -2$$

$$\boxed{a_n^H = (c_1 + c_2 n + c_3 n^2)(-2)^n}$$

$$f(n) = 3^n$$

$$\boxed{k = 3}$$

$$a_n^P = n^3(d \cdot 3^n)$$

$$a_{n-1} = (n-1)^3 \, d \cdot 3^{n-1}$$

$n^3+1 - 3n(3+n)$

$$a_{n-2} = (n-2)^3 \, d \cdot 3^{n-2}$$

$$a_{n-3} = (n-3)^3 \cdot d \cdot 3^{n-3}$$

$$n^3 \cdot d \cdot 3^n + (n-1)^3 \cdot d \cdot 3^{n-1} + (n-2)^3 \cdot d \cdot 3^{n-2} + (n-3)^3 \cdot d \cdot 3^{n-3} = 3^n$$

$$\left[n^3 + (n-1)^3 \cdot 1/3 + (n-2)^3 \cdot \frac{1}{9} + (n-3)^3 \cdot \frac{1}{27}\right] 3^n \cdot d = 3^n$$

$\longrightarrow$

Generating functions :

Solution of recurrence relation by Method of GF with first order recurrence relation

$$a_n = ca_{n-1} + f(n) \quad n \geq 1$$

$$a_{n+1} = ca_n + \phi(n) \quad n \geq 0$$

then GF of above RR is

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad \text{where } a_n \text{ is Soln of RR}$$

formula for finding GF is $f(x) = \dfrac{a_0 + xg(x)}{1-cx}$

where $g(x) = \sum \phi(x) x^n$

if $\phi(x) = 0$ then $g(x) = 0$ then GF of given recurrence relation is $f(x) = \dfrac{a_0}{1-cx}$

① Solve RR $a_{n+1} - a_n = 3^n$, $n \geq 0$, $a_0 = 1$ using GF

The given non-homogeneous relation is

$$\boxed{a_{n+1} = a_n + 3^n}$$

Compare with Non-homogeneous relation with general term

$$a_{n+1} = ca_n + \phi(n) \quad , n \geq 0$$

$$\boxed{c = 1}$$

$$\phi(n) = 3^n$$

Now, we are gng to find $g(x)$ value

$$g(x) = \sum \phi(x) \cdot x^n$$

$$\sum 3^n \cdot x^n$$

$\sum (3x)^n$

$(1-3x)^{-1} = \frac{1}{1-3x}$

$$f(x) = \frac{a_0 + x g(x)}{1-cx}$$

$$= \frac{1+x\left(\frac{1}{1-3x}\right)}{1-x}$$

$$= \frac{1+\frac{x}{1-3x}}{1-x} = \frac{1-3x+x}{(1-3x)(1-x)} = \frac{1-2x}{(1-x)(1-3x)}$$

By using Partial fractions

$$\frac{1-2x}{(1-3x)(1-x)} = \frac{A}{(1-3x)} + \frac{B}{(1-x)} = \frac{A-Ax+B-3Bx}{(1-3x)(1-x)}$$

$$A+B=1$$
$$A+3B=2$$
$$B=1/2 \quad A=1/2$$

$$f(x) = \frac{1}{2(1-3x)} + \frac{1}{2(1-x)}$$

$$= \frac{1}{2}(1-3x)^{-1} + \frac{1}{2}(1-x)^{-1}$$

$$= \frac{1}{2}\sum (3x)^n + \frac{1}{2}\sum x^n$$

Comparing

$$f(x) = \sum \left(\frac{1}{2} + \frac{1}{2}3^n\right) \cdot x^n$$

$$f(x) = \sum a_n \cdot x^n$$

$$\boxed{a_n = \left(\frac{1}{2} + \frac{1}{2}3^n\right)}$$

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2) $a_n - 3a_{n-1} = n \quad n \geq 1 \quad a_0 = 1$

$a_{n+1} - 3a_n = n+1$

$a_{n+1} = 3a_n + (n+1)$

on comparing with

$$\boxed{a_{n+1} = ca_n + \phi(n)}$$

$\boxed{c = 3}$ $\boxed{\phi(n) = n+1}$

$nx^n + x^n$

$g(x) = \sum \phi(n) \cdot x^n$

$= \sum (n+1) \cdot x^n$ $\qquad \frac{1}{(1-x)^2}$

$= (1-x)^{-2}$ $\qquad 3n \cdot x^n$

$6x^2$ $\qquad 1+2x+3x^2$

$g(x) = \frac{1}{(1-x)^2}$ $\qquad 9x^3$ $\qquad (1-x)^{-2}$

$f(x) = \frac{a_0 + x\, g(x)}{1-cx}$

$f(x) = \frac{1 + x\left[\frac{1}{(1-x)^2}\right]}{1-3x}$

$= \frac{1 + \frac{x}{1+x^2-2x}}{1-3x} = \frac{1+x^2-2x+x}{(1-3x)(1-x)^2}$

$= \frac{x^2-x+1}{(1-3x)(1-x)^2} = \frac{A}{1-3x} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$

$\cancel{A} + Ax^2 - 2Ax + \cancel{B} - 3Bx + Cx - 3\cancel{C}x^2$

$A + B + (A-3C)x^2 - (2A + 3B - C)x$

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$A+B=1$
$A-3C=1$ add
$2A+3B-C=1$

$2A+B-3C=2$
$2A+3B-C=1$
$-2B-2C=1$

1

~~$1=B-3C=1$~~

$A=7/4 \quad B=-3/4, \; C=1/4$

$\frac{7}{4}-\frac{3}{4}$

$f(x) \Rightarrow \frac{7/4}{1-3x}+\frac{-\frac{3}{4}+\frac{x}{4}}{(1-x)^2}$

$2A+3B-C$

$\frac{14}{4}-\frac{9}{4}-\frac{1}{4}$

$\frac{7}{4}(1-3x)^{-1}-\frac{3}{4}(1-x)^{-2}+\frac{1}{4}x(1-x)^{-2}$

$\frac{4}{4}=2$

$\frac{7}{4}\sum(3x)^n-\frac{3}{4}\sum(n+1)x^n+\frac{1}{4}\sum$

$A=7/4 \quad B=-1/2 \quad C=-1/4$

$1+x^2$

$1+2x+4x^2$

$(1-x)^{-2}$

+

3) $a_n = a_{n-1} + 2n \quad n \geq 1 \quad a_0 = 1 \quad \boxed{n^2+n+1}$

$a_{n+1} = a_n + 2(n+1)$

on comparing

$$\boxed{a_{n+1} = ca_n + \phi(n)}$$

$\boxed{c=1} \quad \boxed{\phi(n) = 2(n+1)}$

$g(x) = \sum \phi(n) \cdot x^n$

$= \sum 2(n+1) \cdot x^n$

$= 2[1-x]^{-2}$

$= 2 + 4x + 6x^2$

$= 2[1+2x+3x^2]$

$= 2[1-x]^{-2}$

$$f(x) = \frac{1 + x[2[1-x]^{-2}]}{1-x}$$

$$= \frac{1 + \frac{2x}{(1-x)^2}}{1-x} = \frac{1+x^2-\cancel{2x}+\cancel{2x}}{(1-x)(1-x)^2}$$

$$= \frac{1-x^2+\cancel{2x}}{\cancel{(1-x^2)}(1-x)} = \frac{A}{1-x} + \frac{B+Cx}{1-x^2}$$

$$\frac{A}{(1-x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2} = \frac{\cancel{A(1-x)^2 + B(1}}{(1-x)^2}$$

$$\frac{A(1-x) + B(1-x) + C}{(1-x)^2}$$

$A - Ax + B - Bx + C = 1 + x^2$

$A + B + C = 1$

$-A - B = 0$

$-A = B$

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$$\frac{1+x^2}{(1-x)^3} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2} + \frac{C}{(1-x)^3}$$

$$= \frac{A(1-x)^2 + B(1-x) + C}{(1-x)^3}$$

$$\frac{A(1+x^2-2x) + B - Bx + C}{(-x)^3}$$

$$\frac{A + Ax^2 - 2Ax + B - Bx + C}{(1-x)^3}$$

$$1+x^2 = A+B+C + Ax^2 + (-2A-B)x$$

$A+B+C = 1$

$\boxed{A = 1}$

$-2A - B = 0$

$2A = -B$

$\boxed{B = -2}$

$\boxed{C = 1+1 = 2}$

$1+2x+4x^2$

$1-2x+3x$

$1+2x+3x^2$

$1+x^2$

$(1-x)^{-3} =$

$1+x+x^2$

$\sum \frac{(n+1)(n+\ldots}{2}$

$1+x+x^2$

$$f(x) = \frac{1}{(1-x)} + \frac{-2}{(1-x)^2} + \frac{2}{(1-x)^3}$$

$$= (1-x)^{-1} - 2(1-x)^{-2} + 2(1-x)^{-3}$$

$$\sum x^n - 2\sum \frac{(n+1)x^n}{1!} + 2 \cdot$$

$n(n+1)\ 2-x)$

$$(1-x)^{-1}\left[1 - 2(1-x)^{-1} + 2(1-x)^{-2}\right]$$

$(1-x)^{-2}(1-x)^{-1}$

$(n+1)x^{n+1}$

$(n+1)x^n$

$(n+1)\ x^n x^n$

$x(n+1)x^n$

Note

$1+x+x^2+x^3+\cdots = (1-x)^{-1} = \sum x^n$

$1-x+x^2-x^3+\cdots \quad (1+x)^{-1} = \sum (-1)^n x^n$

$1+2x+3x^2+\cdots = (1-x)^{-2} = \sum (n+1)x^n$

$1-2x+3x^2-4x^3+\cdots = \sum (-1)^n (n+1)x^n$

Solve recurrence relation

1) $a_n - 4a_{n-1} = 0 \quad n \geq 1 \quad a_0 = 1$

$a_{n+1} = 4a_n + 0$

$\boxed{c=4} \quad \cancel{f(\phi)=0} \quad \phi(x) = 0$

$g(x) = \sum \phi(x) \cdot x^n$

$\boxed{g(x) = 0}$

$f(x) = \dfrac{1+x(0)}{1-4x}$

$= \dfrac{1}{1-4x}$

$(1-4x)^{-1}$

$f(n) = \sum (4x)^n$

$f(n) = \sum a_n \cdot x^n$

comparing

$\boxed{a_n = 4^n}$

Solution of 2nd Order or higher Order R.R by the method of

generating functions.

Let $a_n + A_1 a_{n-1} + A_2 a_{n-2} = f(n)$

can be written as

$$a_{n+2} + A_1 a_{n+1} + A_2 a_n = \phi(n) \qquad \phi(n) = f(n+2)$$

then $$f(x) = \frac{a_0 + (a_1 + a_0 A_1)x + x^2 g(x)}{1 + A_1 x + A_2 x^2}$$

where $g(x) = \sum \phi(x) x^n$

If $\phi(x) = 0$ then $g(x) = 0$ at that time the general function is homogeneous

$$f(x) = \frac{a_0 + (a_1 + a_0 A_1)x}{1 + A_1 x + A_2 x^2}$$

Solve the RR

$$a_{n+2} - 2a_{n+1} + a_n = 2^n \qquad n \geq 0 \qquad a_0 = 1, a_1 = 2$$

compare with General term

$A_1 = -2 \qquad a_0 = 1$

$A_2 = 1 \qquad a_1 = 2$

$$\phi(n) = 2^n$$

$$g(x) = \sum \phi(x) \cdot x^n = \sum 2^n \cdot x^n = \sum (2x)^n$$
$$= (1-2x)^{-1}$$

$$f(x) = \frac{1 + (2 + 1(-2))x + x^2 \cdot \frac{1}{1-2x}}{1 - 2x + x^2}$$

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$$\frac{1+\frac{x^2}{1-2x}}{1-2x+x^2}$$

$$\frac{\cancel{1-2x+x^2}}{(1-2x)\cancel{(1-2x+x^2)}}$$

$$f(x) = \frac{1}{1-2x}$$

$$= (1-2x)^{-1}$$

$$= \sum (2x)^n$$

$$= 2^n \sum x^n$$

$$\Rightarrow \boxed{a_n = 2^n}$$

→ $a_n - 7a_{n-1} + 10a_{n-2} = 4^n$

$a_0 = 8, \quad a_1 = 36$

$a_{n+2} - 7a_{n+1} + 10a_n = 4^{n+2}$

$A_1 = -7, \quad A_2 = 10 \qquad \phi(n) = 4^{n+2}$

$$g(x) = \sum 4^{n+2} \cdot x^n = \sum 16 \cdot (4x)^n = 16 \cdot (1-4x)^{-1}$$

$$f(x) = \frac{8 + (36 - 56)x + 16x^2(1-4x)^{-1}}{1-7x+10x^2}$$

$$= \frac{8 - 20x + \frac{16x^2}{1-4x}}{1-7x+10x^2} = \frac{8 - 32x - 20x + 80x^2 + 1}{(1-4x)(1-7x+10x^2)}$$

rap

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$$\frac{96x^2-52x+8}{(1-4x)(10x^2-7x+1)}$$

$$\frac{8\left[12x^2-7x+1\right]}{(1-4x)\left[10x^2-7x+1\right]}$$

$$\frac{96x^2-52x+8}{(1-4x)(1-2x)(1-5x)}$$

$$\Rightarrow \frac{A}{1-2x}+\frac{B}{1-5x}+\frac{C}{(1-4x)} = \frac{A(1-5x)(1-4x)+B(1-2x)(1-4x)+C(1-2x)(1-5x)}{(1-2x)(1-5x)(1-4x)}$$

$\cancel{A}-9xA+20x^2A+\cancel{B}-6Bx+8Bx^2$

$\cancel{C}-7Cx+10Cx^2$

$A+B+C=8$

$-9A-6B-7C=52$

$20A+8B+10C=96$

$$A=4,\ B=12,\ C=-8$$

$$\frac{4}{1-2x}+\frac{12}{1-5x}-\frac{8}{1-4x}$$

$$4(1-2x)^{-1}+12(1-5x)^{-1}-8(1-4x)^{-1}$$

$$4\sum(2x)^n+12\sum(5x)^n-8\sum(4x)^n$$

$$\sum x^n\cdot\left[4\cdot2^n+12\cdot5^n-8\cdot2^{2n}\right]$$

$$\sum x^n\left[2^{2+n}+12\cdot5^n-2^{3+2n}\right]$$

$$\therefore\ \boxed{a_n=2^{n+2}+12\cdot5^n-2^{2n+3}}$$

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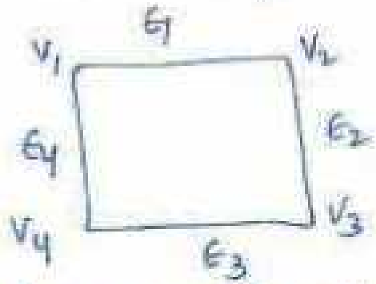
UNIT-V

Graph is a collection of Vertices and Edges.

→ represent by "G". G[V, E] where V is Vertices and E is a Edge

ex:



The above graph contains 4 Vertices (V_1, V_2, V_3, V_4) and 4 Edges (E_1, E_2, E_3, E_4)

Representation of Graphs :

graphs can be classified into two types

① Matrix representation

② Linked list representation

① Matrix representation again classified into

a) adjacency Matrix representation (vert to vert)

b) Incidence Matrix representation (vert to edges)

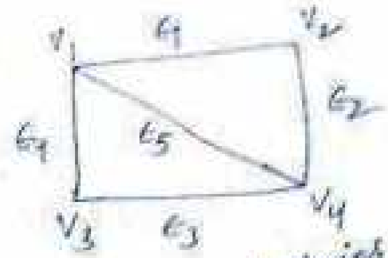
The Matrices are commonly used to represent the graph for computer processing.

Representation of undirected graphs (adjancency)

Let $A = \{a_{ij}\}_{n \times n}$ where $a_{ij} = 1$, if there is an edge b/w i^{th} Vertices to the j^{th} Vertices.

$a_{ij} = 0$, if there is no Edge between i^{th} to j^{th} Vertices

Example:



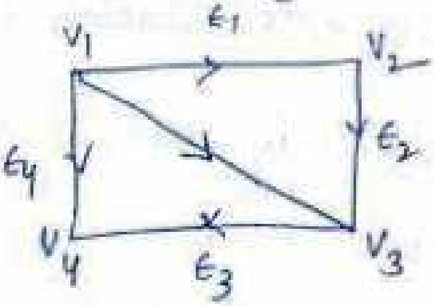
In the above graph contains four vertices & five edges

$$\begin{array}{c|c} V & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

<u>Representation of a directed graph of adjacency Matrix</u>

The adjacency matrix of directed graph G with n vertices is

$$A = \{a_{ij}\}_{n \times n} \begin{cases} \text{where } a_{ij} = 1 \text{ (if arrow directs to the vertex } (V_i, V_j) \text{ is in graph } G) \\ \text{otherwise } 0 \end{cases}$$



$$\begin{array}{c|c} & \begin{matrix} V_1 & V_2 & V_3 & V_4 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

<u>Incidence Matrix representation of undirected graph</u>

Consider an undirected graph $G = (V, E)$ which has 'n' vertices and 'm' edges then the matrix representation is $B = \{b_{ij}\}_{n \times m}$

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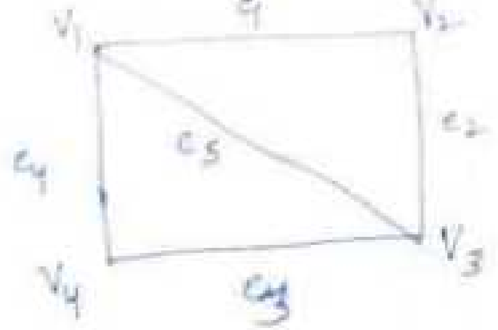
'n' is no of vertices

'm' is no of Edges

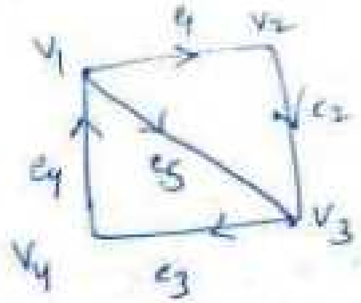
where $\begin{cases} b_{ij} = 1, \text{ when edge } e_j \text{ is incident with vertices } v_i \\ b_{ij} = 0, \text{ otherwise} \end{cases}$



$$\begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline v_1 & 1 & 0 & 0 & 1 & 1 \\ v_2 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 1 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 1 & 0 \end{array}$$

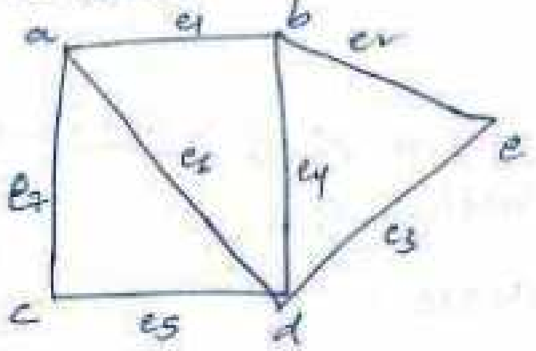
Incidence Matrix representation of directed graph

let $B = \{b_{ij}\}_{m \times n}$ $\begin{cases} \text{where } b_{ij} = 1 \text{ (if arrow is directed away from the vertex V)} \\ b_{ij} = -1 \text{ (if arrow is directed towards vertex V)} \\ \text{otherwise } 0 \end{cases}$



$$\begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline v_1 & 1 & 0 & 0 & -1 & 1 \\ v_2 & -1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & -1 & 1 & 0 & -1 \\ v_4 & 0 & 0 & -1 & 1 & 0 \end{array}$$

1) undirected



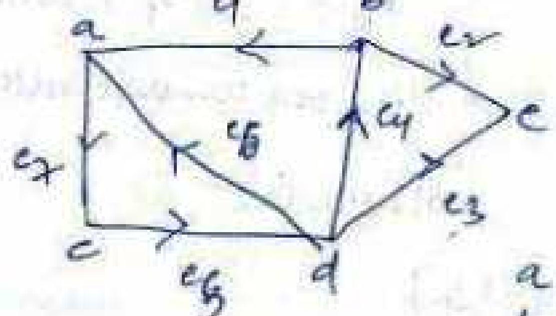
adjacency :

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 1 & 1 & 1 & 0 \\ b & 1 & 0 & 0 & 1 & 1 \\ c & 1 & 0 & 0 & 1 & 0 \\ d & 1 & 1 & 1 & 0 & 1 \\ e & 0 & 1 & 0 & 1 & 0 \end{array}$$

incidence :

$$\begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline a & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ b & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ d & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ e & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}$$

2)

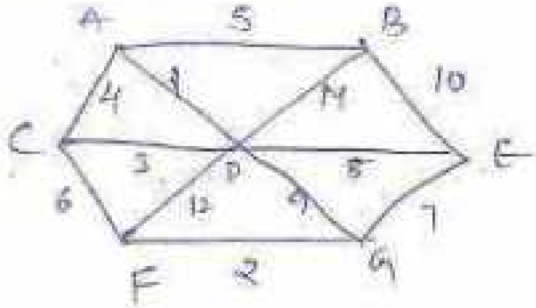


adjacency :

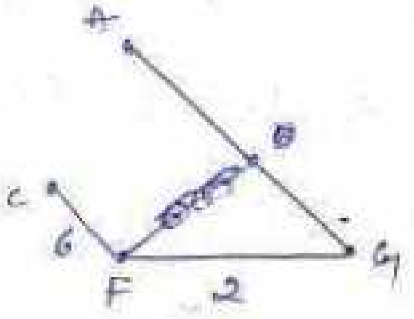
$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 1 & 0 \\ d & 1 & 1 & 0 & 0 & 1 \\ e & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$
\begin{array}{c|ccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\
\hline
a & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\
b & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\
c & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
d & 0 & 0 & 1 & 1 & -1 & 1 & 0 \\
e & 0 & -1 & -1 & 0 & 0 & 0 & 0
\end{array}
$$

→ find minimum cost of spanning using prim's

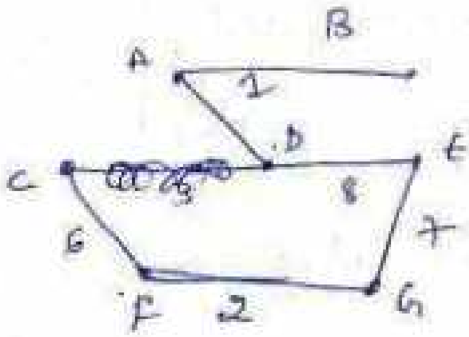


$w(C,A) = 4$
$w(C,D)\ 3$



$w(A,B) = 5$
$w(A,C) = 4$
$w(A,D) = 1$

$w(G,E) = 7$
$w(G,F) = 2$



$w(F,C) = 6$
$w(F,D) = 12$

$w(E,B) = 10$
$w(E,G) = 7$

$w(G,E)$

$1+8+5+7+6+2$

$w(F,D) = 12$
$w(F,C) = 6$
$w(F,$

$$
\begin{array}{r}
18 \\
11 \\
\hline
29
\end{array}
$$

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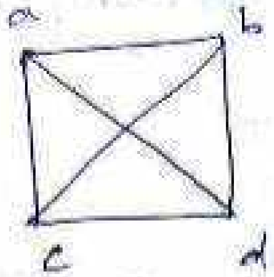
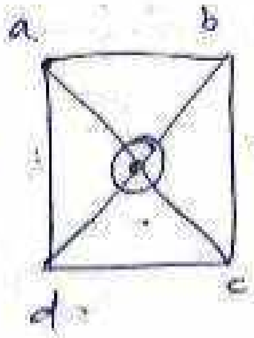
→ Planar Graph : A Graph 'G' is said to be a Planar Graph if it is in one plane without any cross overs.

→ Cross overs means any two edges shouldn't Intersect to each other.

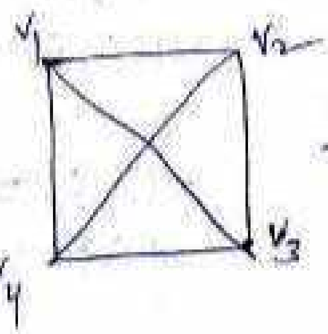


Planar

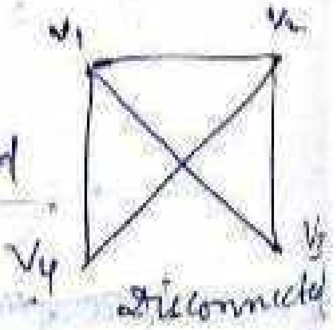


not a Planar

→ Connected graphs : A graph 'G' is said to be a connected graph if there is atleast one Path inbetween every Pair of Vertices. Otherwise It is called as Disconnected graph

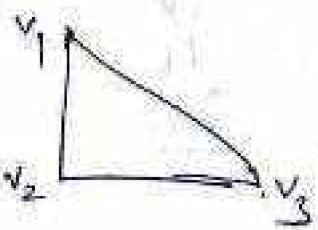


→ Connected



Disconnected

→ Simple graph : A graph 'G' is said to be a simple graph if it is not containing Parallel Edges or self loops.



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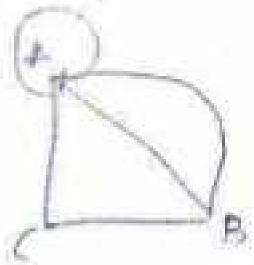
General Graph: A General Graph 'G' is said to be a General graph, if it contains either a Parallel edge or self loop or both

ex:



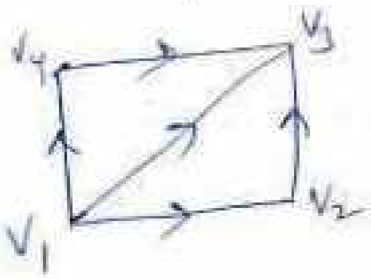
$D(V_i) = 10$

Degree of a vertex: The no. of edges on vertex is Known as Degree of vertex.

if it is self loop (count twice)

indicated by the Symbol $D(V_i)$

Indegree Vertex: The no. of Edges coming to Vertex is called Indegree Vertex. denoted by $D^+(V_i)$



$D^+(V_i) = D^+(V_1) + D^+(V_2) + D^+(V_3) + D^+(V_4)$

$0+1+3+1 = 5$

Outdegree: The no. of edges Leaving from the vertex is called as Outdegree of Vertex

$D^-(V_i) = 3+1+0+1 = 5$

Order of Graph: No. of vertices in a graph is called the Order of graph

$O(G) = 4$

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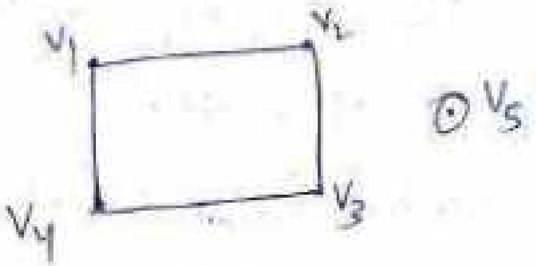
Size of graph : No. of edges in a graph.

$s(G) = 5$

Isolated Vertex : A Vertex is said to be isolated if there are no connections with remaining vertices (or) of degree = 0

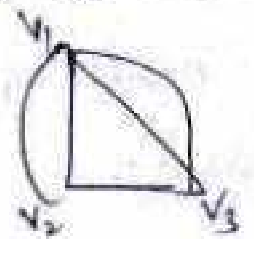


Pendent Vertex : A Vertex is said to be a Pendent vertex if it is having the degree '1'.

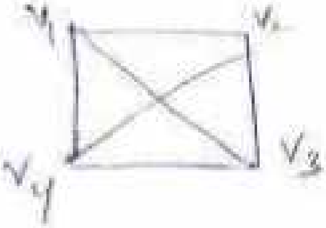
Self loop : An edge between a Vertex and itself is called as self loop.

loop free : A Graph which doesn't containing self loop is called loop free

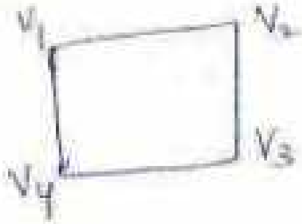
Multigraph: A Graph 'G' is said to be a Multigraph which containing multiple edges but no loops is called Multigraph.



complete graph: Every vertex of a graph is connected to remaining all vertices indicated by k_n
↓
no. of vertices

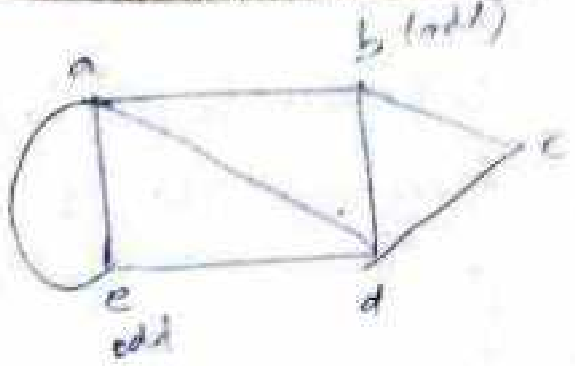


Regular graph: A graph 'G' is said to be a regular graph, it contains all vertices having same degree.



Euler Graph:

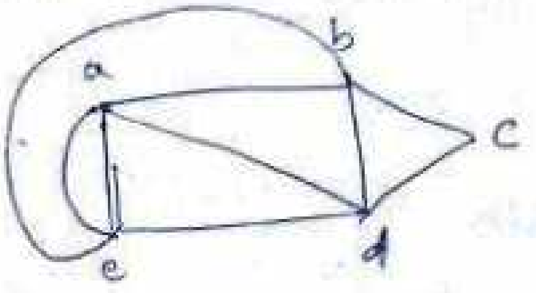
* ① Euler's Path: It is defined as visit all the edges exactly once and visit all the vertices atleast once.
* In Euler's Path starting vertex and ending vertex shouldn't be same
* To Obtain the Euler's Path for given graph all the vertices degree should be even, except two vertices, whose degree is odd.
* To Obtain Euler's Path we can start from one odd degree and End with another odd degree vertex.



b - c - d - b - a - d - e - a - e

Euler's Circuit: It is defined as start from one vertex and visit all the edges once and visit all the vertices atleast once and come back to the starting vertex.

To obtain Euler's Circuit in the given graph. All the vertices degree's should be even



Hamiltation graph.

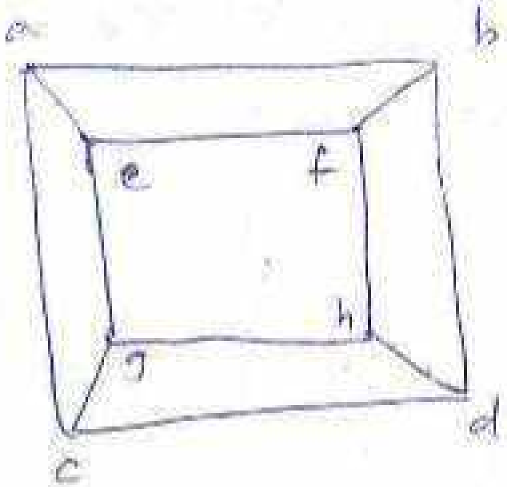
① Hamiltation Path is defined visit all the vertices exactly once

→ In hamiltanian Path starting & ending vertex shouldn't be same

→ If the given graph having 'n' vertices then the hamiltanian path consists of (n-1) Edges.

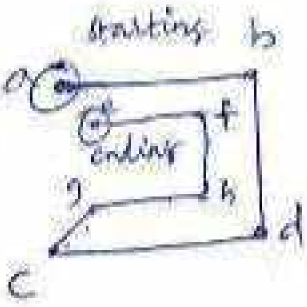
→ In hamiltanian path terminal vertices degree (starting & ending) should be '1' and remaining vertices degree should be '2'.

Problem: find the hamiltanian path for a Graph G

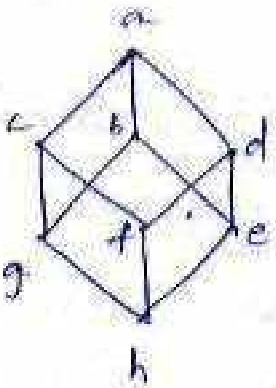


Step 1: a.

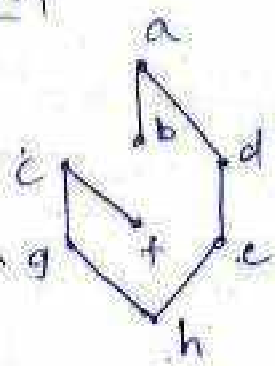
Step 2:



Prob:



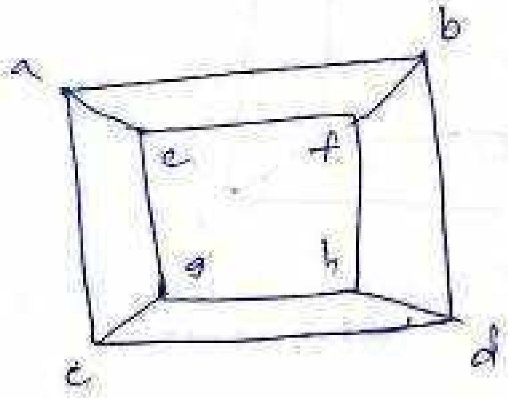
Step: 1



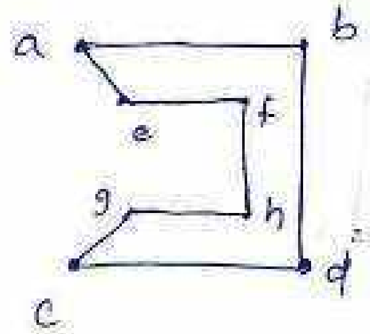
① Hamiltatian cycle : is defined as start from one vertex and visit all the vertices exactly and come back to starting vertex.

→ In hamiltanian cycle starting vertex and ending vertex should be same.

→ In hamiltatian cycle the degree of all the vertices should be 'two'.



Step 1:



hamiltanian cycle

Directed Graph: A Directed graph is a pair (V,E) where V is a non-empty set and E is a set of ordered pairs of elements taken from the set V. Here 'E' is called directed graph.

Directed acyclic graph (DAG): A directed acyclic graph is an acyclic graph that has a directions as well as lack of proper cycles. A "DAG" has a topological ordering. This means that the starting node has a lower or higher degree and ending node has lower or higher degree.

Ex: 

Graph Colouring:

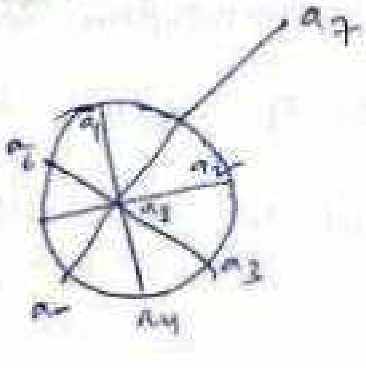
→ A Graph can have more than one proper colouring

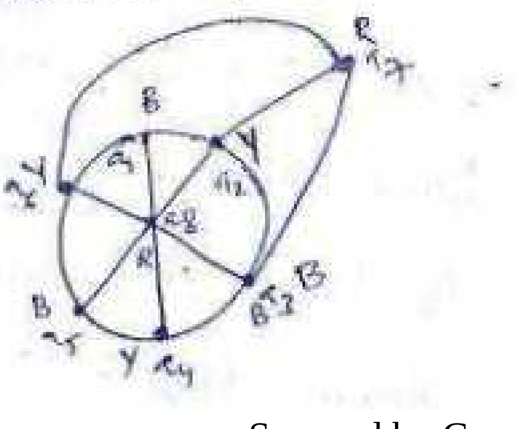
→ Two non-adjacent vertices in the graph should get the same colour.

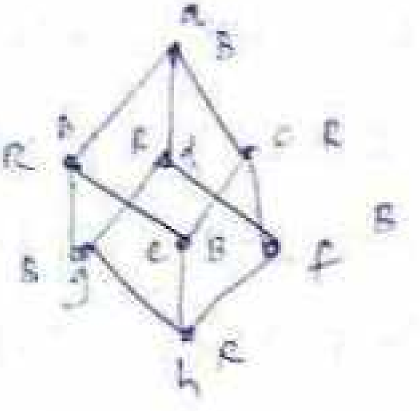
→ A graph is said to 'k' colourable if we can properly colour with 'k' colours

→ A graph 'G' which is k colourable but not 'k-1' colourable is called chromatic Graph

→ If a Graph 'G' is 'k' chromatic then 'k' is called chromatic number of 'G'.







Chromatic number = 2

Isomorphism in Graphs :

Consider two graph $G = (V, E)$, $G' = (V', E')$

Suppose there exists a $f: V \to V'$ such that f is one to one and onto functions and $\forall$ vertices of G, the edges $(AB, BC, CD, DA) \in E$

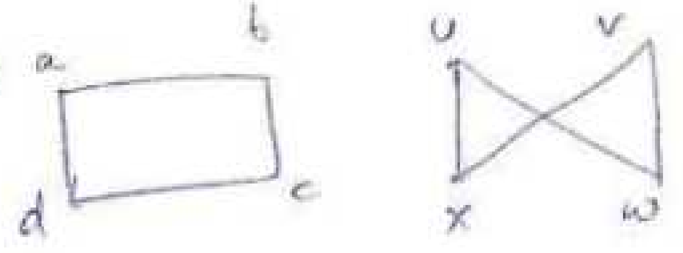
(A,B,C,D)

if and only if the edge $\{f(a), f(b), f(c), f(d)\} \in E'$

then f is called an Isomorphism ~~of~~ b/w G and G' and we say that G & G' are isomorphic Graphs.

→ If two graphs are isomorphic then they must have the same no. of vertices, the same no. of edges and equal no. of vertices with a given edge.

ex:



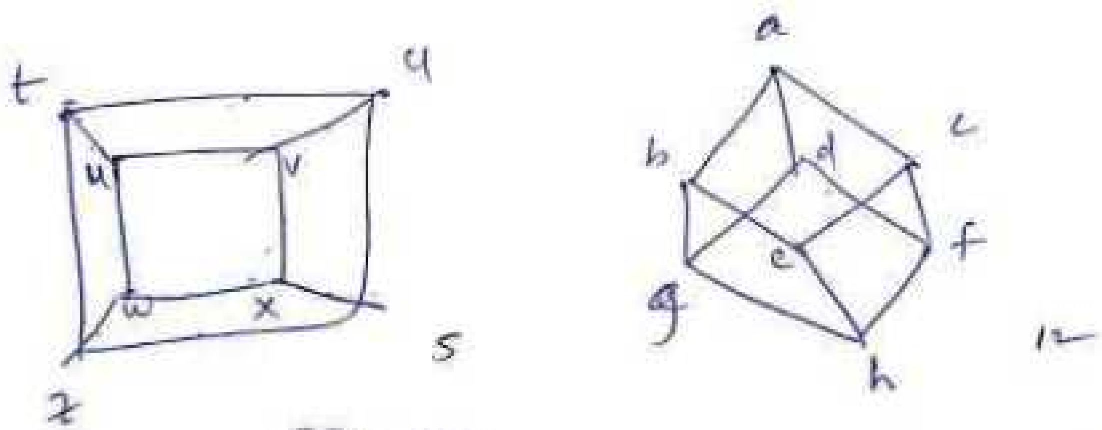
a $\longleftrightarrow$ u

b $\longleftrightarrow$ v

d $\longleftrightarrow$ x

c $\longleftrightarrow$ w

→ Check whether the following graphs are Isomorphic or not.

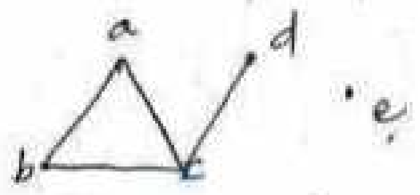


12 8 same no. of edges, vertices and degree's

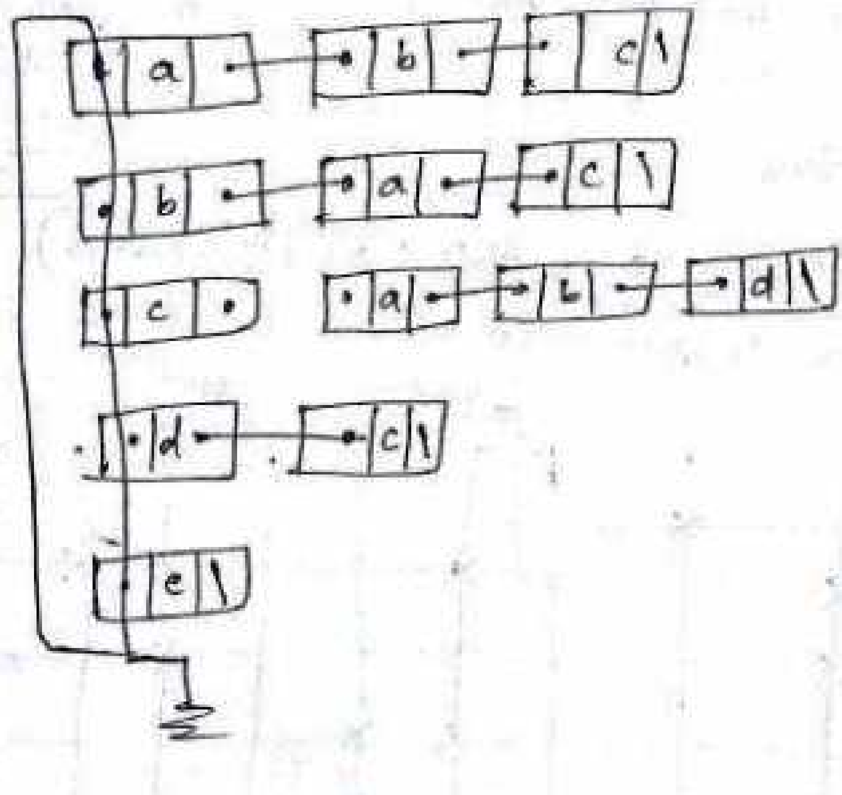
∴ Isomorphic

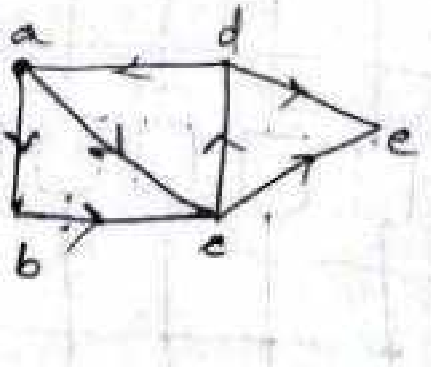
Linked list representation of a graph :

undirected graph

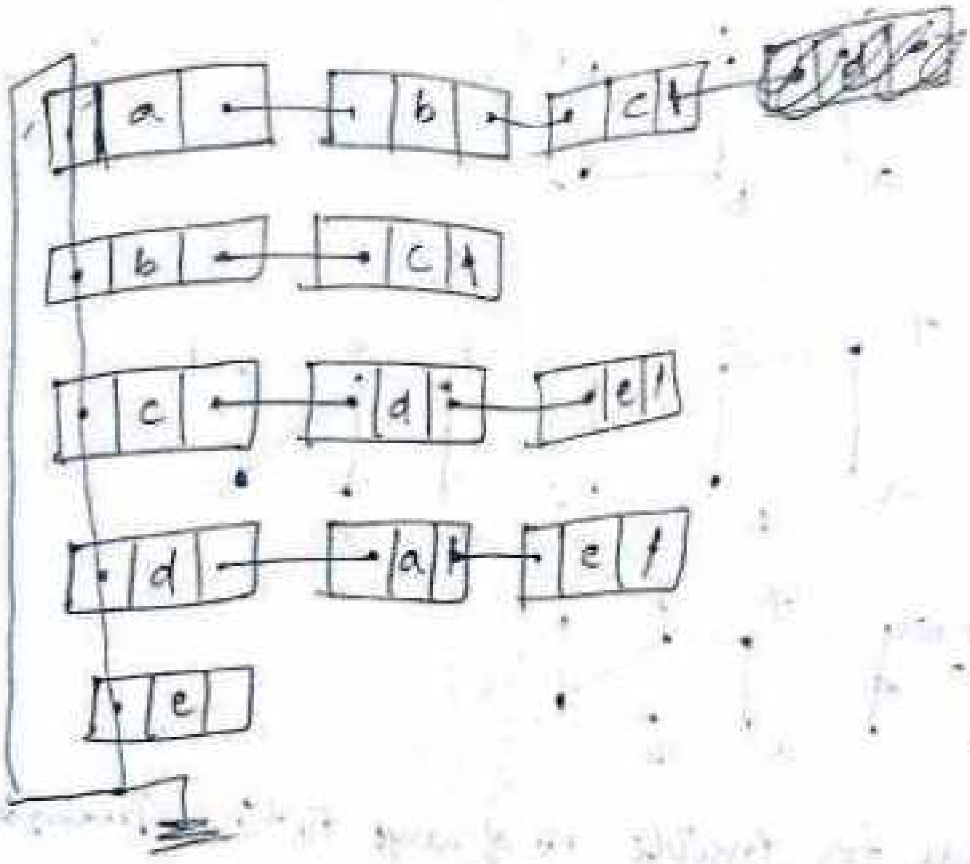


Vertices	connecting Vertices
a	b,c
b	a,c
c	a,b,d
d	c
e	ϕ



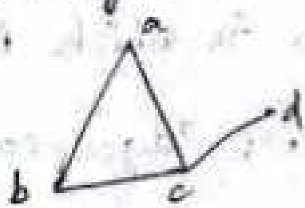


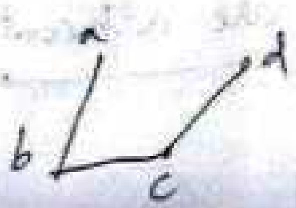
a	b, c, d	bc
b	c, a	c
c	d, e, b, a	de
d	a, c, e	ae
e	~~b~~ c, d	φ



→ What is a spanning tree

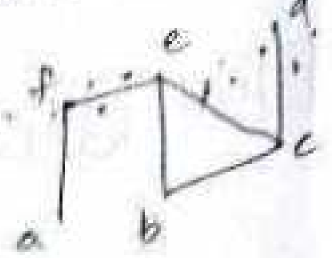
A Sub graph 'H' of graph 'G' is said to be a spanning tree. It contain 'n' vertices & it doesn't form any loop or cycle.

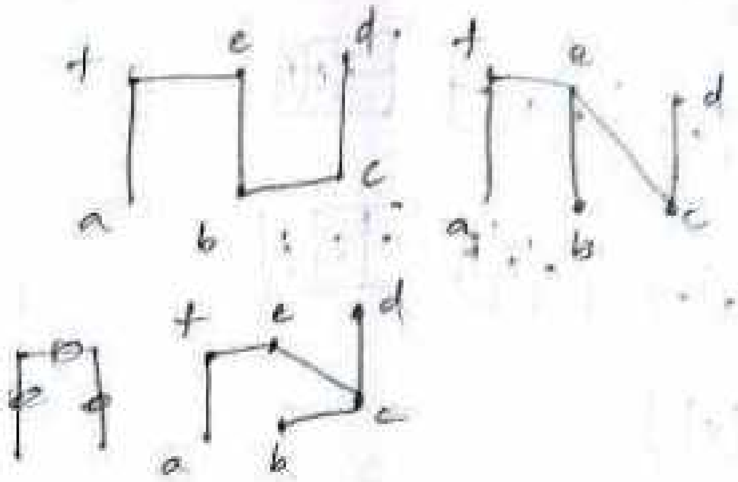




Note: ① In the given graph 'G' we have to delete (m-n+1) edges. Here m is no. of edges and 'n' is no. of vertices.

Prob: Find all spanning trees for the given graph G.





Procedure for possible no. of ways to find spanning trees for a given graph:

Let 'G' be a connected graph, then $V(G) = \{v_1, v_2, v_3, \ldots v_p\}$ no. of vertices. Here matrix $M = (M_{ij})_{p \times p}$ is matrix in which $M_{ii} = \text{Degree}(V_i)$ where degree is no. of edges comming to vertex

$m_{ij} = -1$, when v_i & v_j are adjacent, otherwise

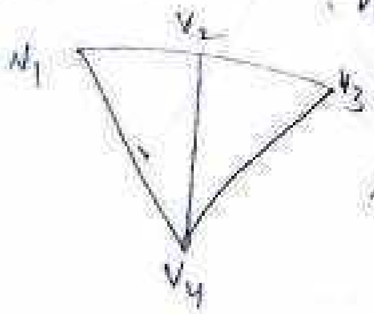
$m_{ij} = 0$

(m-n+1)

o. of

and we have to calculate matrix of given graph and find determinant of that matrix - we get possible no. of spanning trees for a given graph.

ph G_1



find the all possible spanning trees of given graph G_1

$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}
\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \end{array}
$$

$$\begin{matrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{matrix}$$

$2(6-1)+1(-2) = 10-2 = 8$

$$\Rightarrow 2\begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix} + 1\begin{bmatrix} -1 & -1 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix} + 1\begin{bmatrix} -1 & 3 & -1 \\ 0 & -1 & 2 \\ -1 & -1 & -1 \end{bmatrix}$$

$$= 2\left[3(6-1)+1(-3-1)-1(1+2)\right] + \left[(-1)(6-1)+1(0-1)-1(+2)\right]$$

$$+ \left[-1.(1+2)-3(+2)-1(0-1)\right]$$

$$= 2\left[15-4-3\right] + \left[-5-1-2\right] + \left[-3-6+1\right]$$

$$2\left[8\right] + \left[-8\right] + \left[-8\right]$$

$20-16$

8

$$\begin{matrix} -1 & 0 & -1 \\ 3 & -1 & -1 \\ -1 & 2 & -1 \end{matrix}$$

$-1(1+2)-1(6-1)$

$-3-5$

$$\begin{matrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{matrix}$$

$3(6-1)+1(-3-1)$

$-1(1+2)$

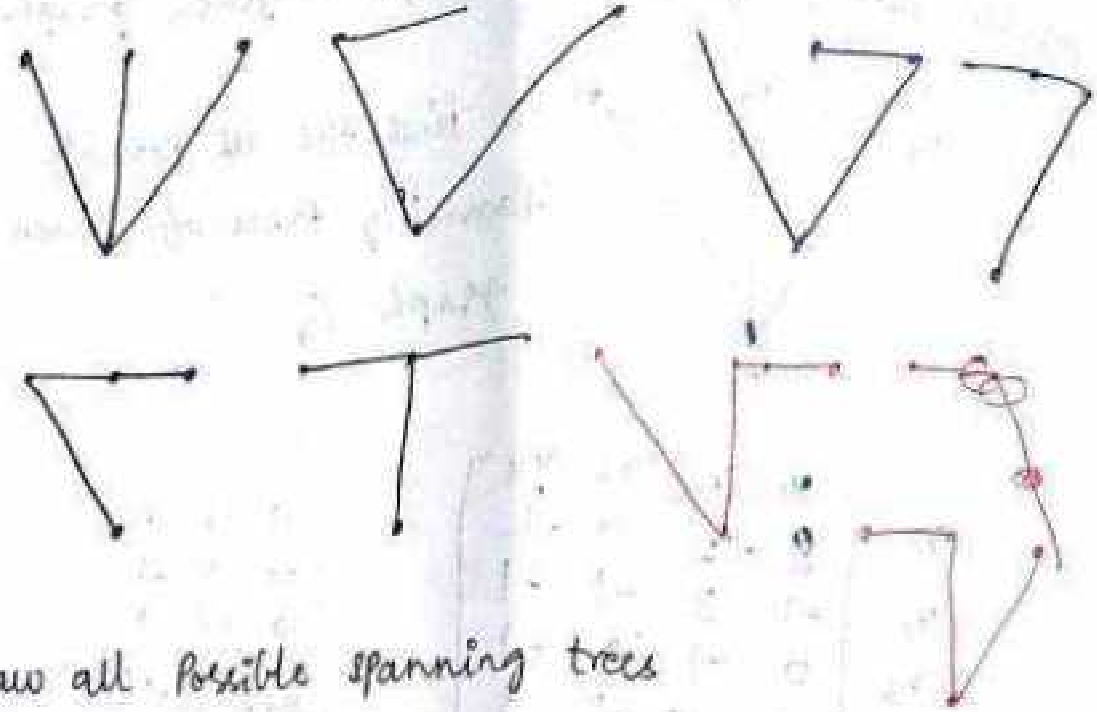
15-

s

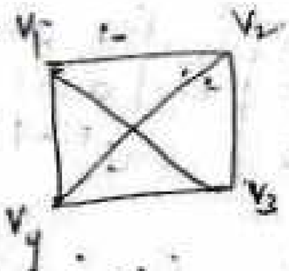
es

)

Remove m-n+1 edges = 5-4+1 = 2



Draw all Possible spanning trees



6-4+1 = 3 remove edges

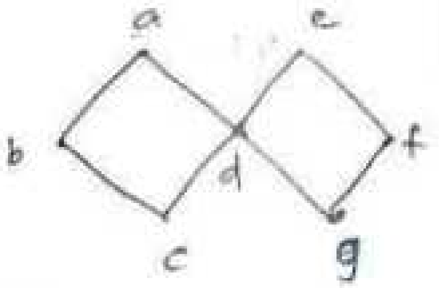
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \end{array}$$

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} = 3(9-1)+1(-3-1)-1(1+3)$$

$$= 24-4-4 = 16$$

Draw Spanning tree of graph G by using DFS algorithm

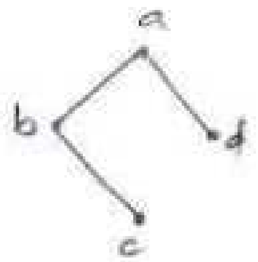
$8-7+1 = 2$



① Step 1: Take any arbitary vertex

Step 2:

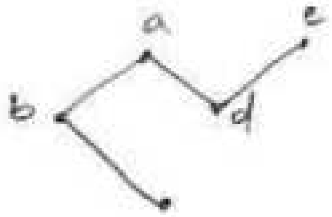
Step 3:



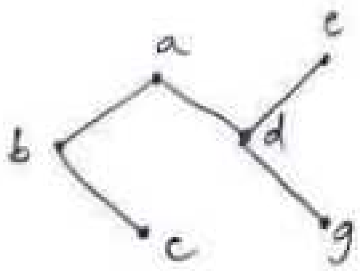
If we consider d to c loop forms so, eliminate.

→ consider either b to c or d to c

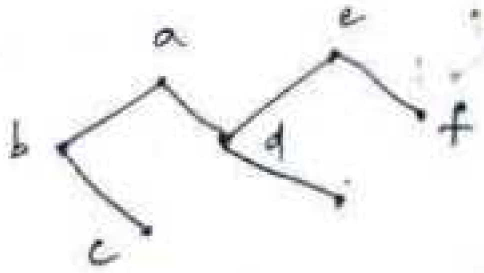
Step 4:



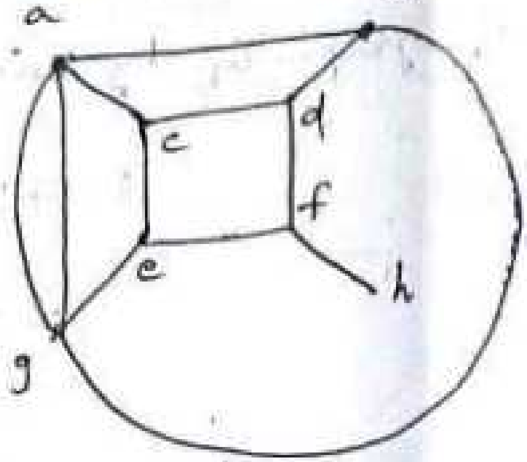
Step 5:



Step 6:



2)

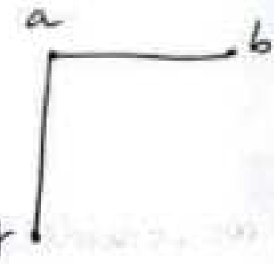


12-8+1

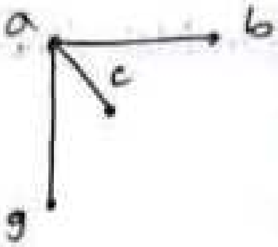
4+1=5

Step 1 :

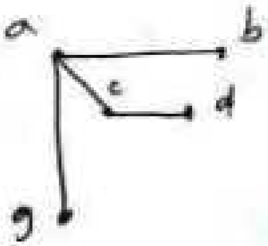
Step 2 :



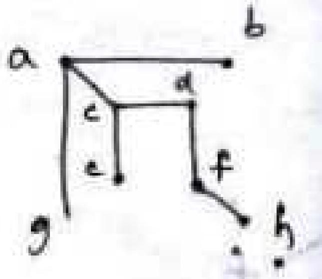
Step 3 :



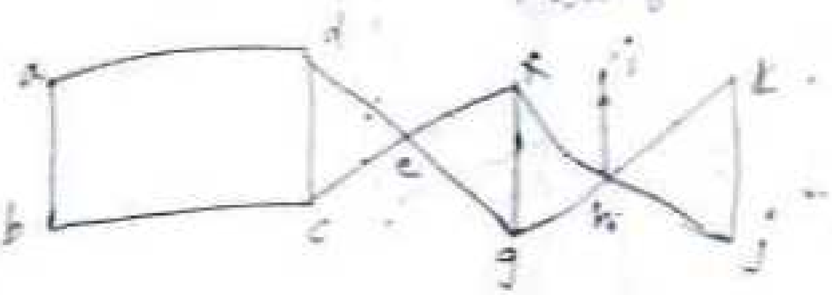
Step 4 :



Step 5 :

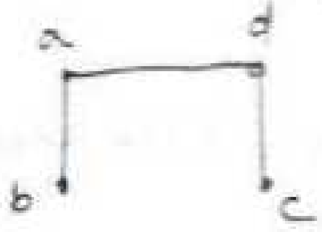


Spanning tree of 'G' by using BFS

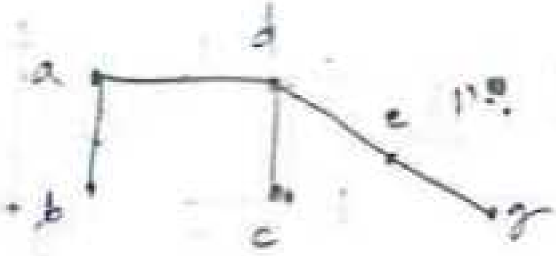


15 − 11 + 1
4 + 1 = 5

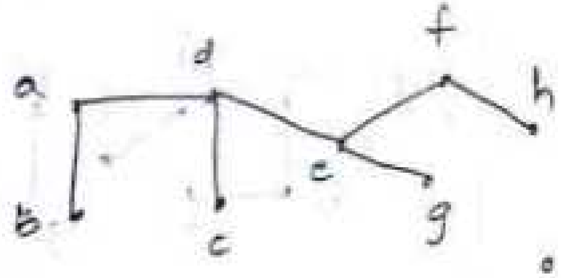
Step 1:



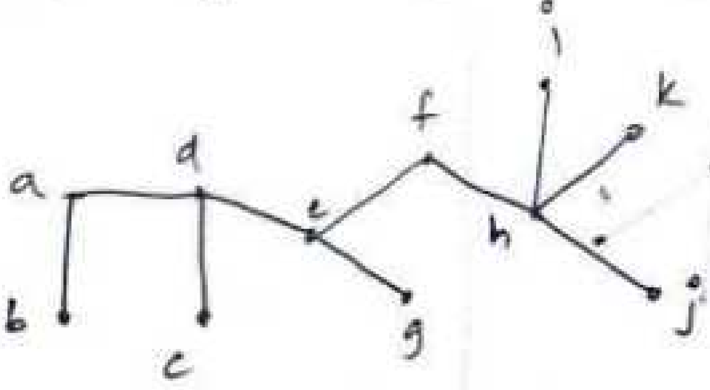
Step 2:



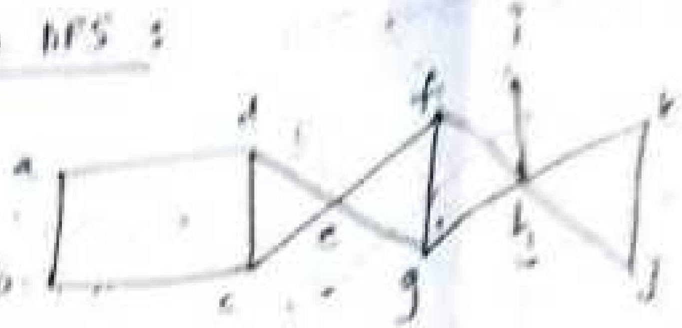
Step 3:



Step 4:

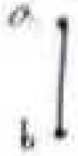


The above graph is a spanning tree because 'n' vertices and n-1 edges and doesn't form a cycle or loop using DFS :



Step1: consider arbitrary vertex a

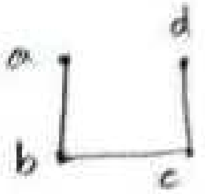
Step2 :



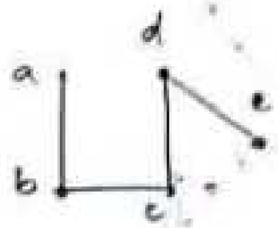
consider and either a to b edge or a to d edge

Step3:

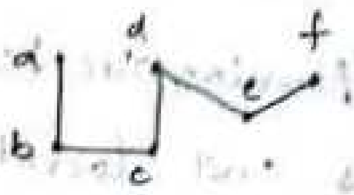
Step4 :



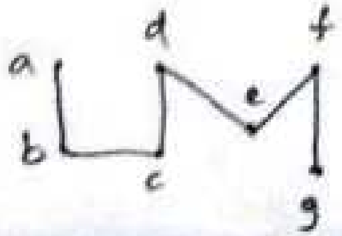
Step5 :



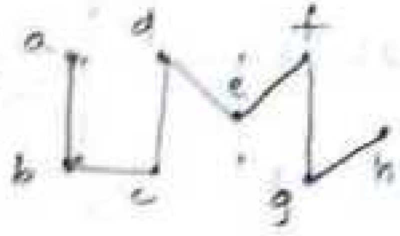
Step6 :



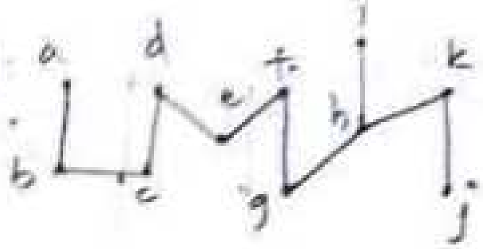
Step7 :



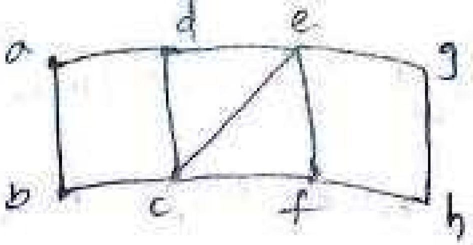
Step8 :



Step9



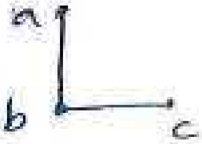
a)



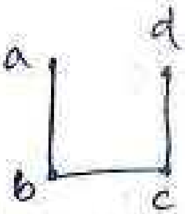
Step1 :

Step2 :

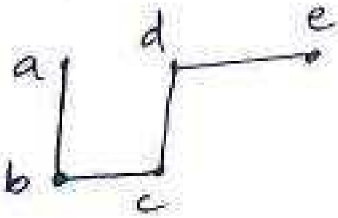
Step3 :



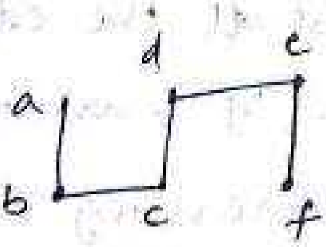
Step4 :



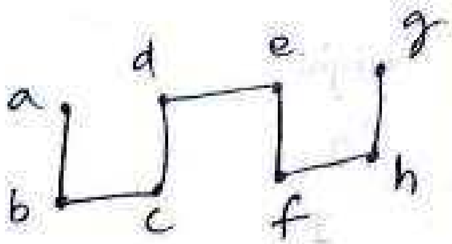
Step5 :

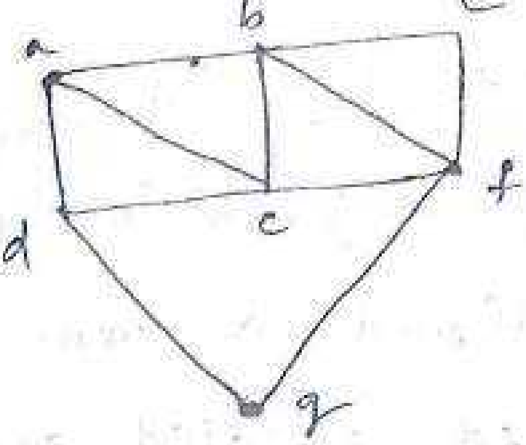


Step6 :



Step7 :



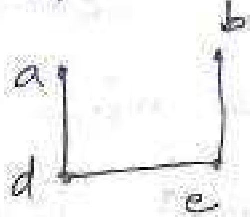


i)

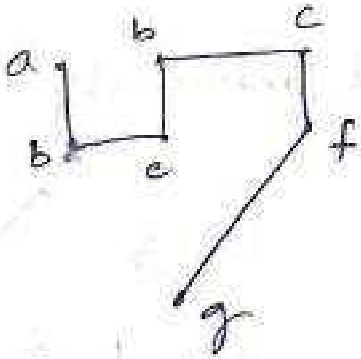
Step1 :

Step2 :

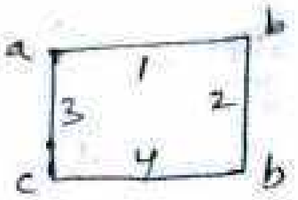
Step3 :



Step4 :

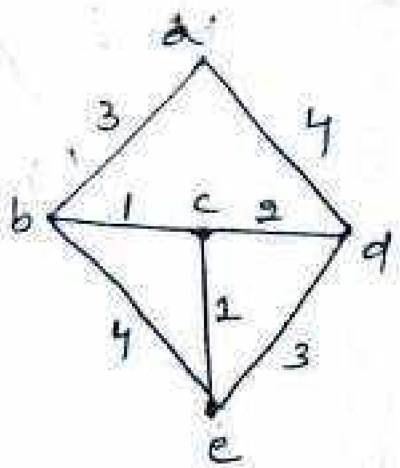


Weighted Graph: A Graph 'G' is said to be a weighted graph in which each edge has been assigned a non-negetive number is called weighted graph. It is represented by the symbol W(e). Ex:



In these minimum spanning trees we have to implement i) Krushkal's ii) Prim's

→ find minimal spanning tree by using Krushkal's

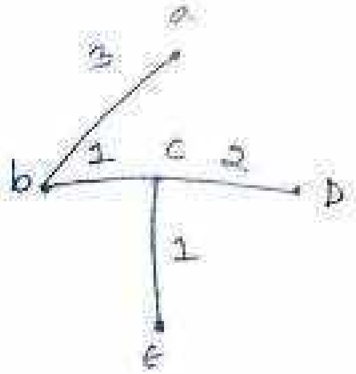


List all the edges of Graph 'g' & arrange them in increasing order

~~edeb~~ :

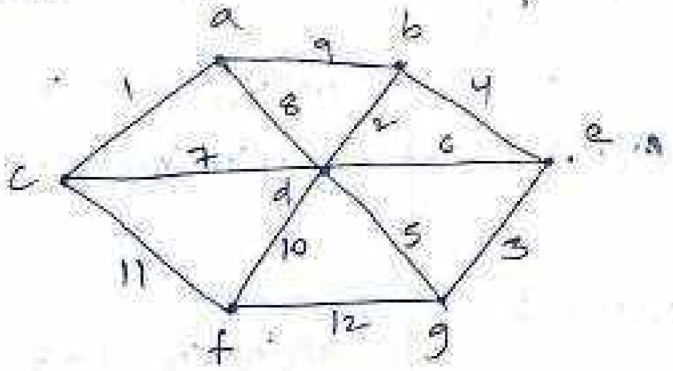
Edge	Weight
B – C	1
C – E	1
C – D	2
A – B	3
D – E	3
B – E	4
A – D	4

final ans:



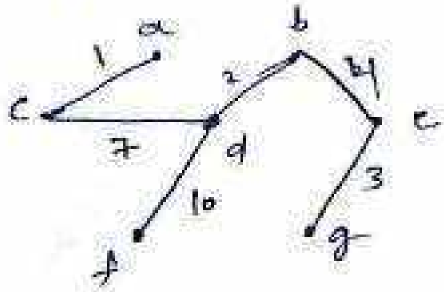
Stop the Procedure and the above graph contains 'n' vertices and n-1 Edges without forming a loop or cycle. The total minimal cost of Spanning tree is '7'.

→ find minimal cost of a graph 'G' by using Krushkal's Algorithm.

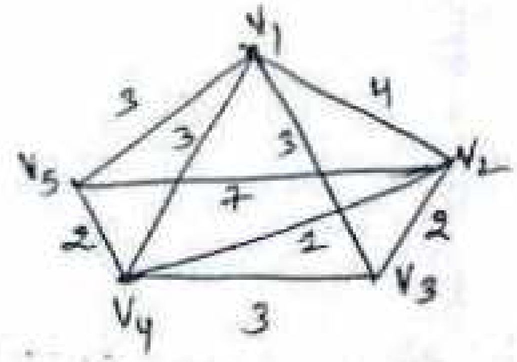


Edge	Weight
a-c	1
d-b	2
e-g	3
b-e	4
d-g	5
d-e	6
d-c	7
d-a	8
a-b	9
d-f	10
c-f	11
f-g	12

10+4+

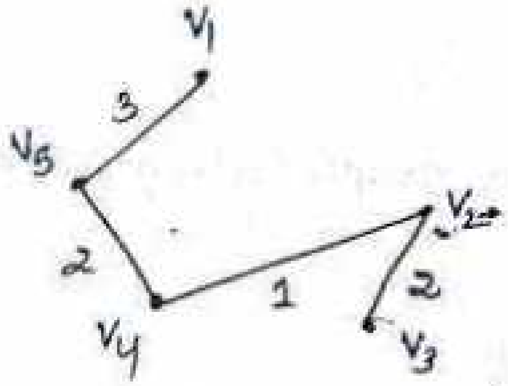


→ find the minimal spanning tree of weighted graph 'G' by using Prim's



Steps:

we choose the vertex V_1 and find the weights of edges from vertex V_1 to other connected vertices



$\omega(V_1, V_5) = 3$

$\omega(V_1, V_4) = 3$

$\omega(V_1, V_3) = 3$

$\omega(V_1, V_2) = 4$

$\omega(V_5, V_2) = 7$

$\omega(V_5, V_4) = 2$

$\omega(V_2, V_3) = 2$

Minimum cost = 8

~~$\omega(V_4, V_1) = 3$~~

$\omega(V_4, V_2) = 1$

$\omega(V_4, V_3) = 3$