INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad -500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

COURSE DESCRIPTOR

Course Title	**COMPLEX ANALYSIS AND SPECIAL FUNCTIONS**				
Course Code	AHSB05				
Program	B. Tech				
Semester	THREE				
Course Type	Foundation				
Regulation	IARE - R18				
Course Structure	**Theory**			**Practical**	
	Lectures	**Tutorials**	**Credits**	**Laboratory**	**Credits**
	3	1	3	-	-
Course Coordinator	Ms. L Indira, Assistant Professor				

I. COURSE OVERVIEW:

This course Complex Analysis and Special Functions provides an introduction to complex analysis which is theory of complex functions with complex variable. The course includes complex functions and differentiation, complex integration, power series expansion of complex function and special functions. The mathematical skills derived from this course form a necessary base to analytical and design concepts encountered in the program.

II. COURSE PRE-REQUISITES:

Level	Course Code	Semester	Prerequisites
10 + 2	-	-	Basic Principles of complex functions

III. MARKS DISTRIBUTION:

Subject	SEE Examination	CIA Examination	Total Marks
Complex Analysis and Special Functions	70 Marks	30 Marks	100

IV. DELIVERY / INSTRUCTIONAL METHODOLOGIES:

✔	PPT	✔	Chalk & Talk	✔	Assignments	✗	MOOCs
✔	Open Ended Experiments	✔	Seminars	✗	Mini Project	✔	Videos
✔	Others						

V. EVALUATION METHODOLOGY:

The course will be evaluated for a total of 100 marks, with 30 marks for Continuous Internal Assessment (CIA) and 70 marks for Semester End Examination (SEE). Out of 30 marks allotted for CIA during the semester, marks are awarded by taking average of two CIA examinations or the marks scored in the make-up examination.

Semester End Examination (SEE): The SEE is conducted for 70 marks of 3 hours duration. The syllabus for the theory courses is divided into FIVE modules and each module carries equal weightage in terms of marks distribution. The question paper pattern is as follows. Two full questions with "either" or "choice" will be drawn from each module. Each question carries 14 marks. **There could be a maximum of two sub divisions in a question.**

The expected percentage of cognitive level of the questions is broadly based on the criteria given in Table: 1.

Table 1: The expected percentage of cognitive level of questions in SEE

Percentage of Cognitive Level	Blooms Taxonomy Level
10 %	Remember
25 %	Understand
50 %	Apply
15 %	Analyze
0 %	Evaluate
0 %	Create

Continuous Internal Assessment (CIA):

CIA is conducted for a total of 30 marks (Table 2), with 20 marks for Continuous Internal Examination (CIE), 05 marks for Quiz and 05 marks for Alternative Assessment Tool (Table 3).

Table 2: Assessment pattern for CIA

Component	Theory			Total Marks
Type of Assessment	CIE Exam	Quiz	AAT	
CIA Marks	20	05	05	30

Continuous Internal Examination (CIE):

Two CIE exams shall be conducted at the end of the 8th and 16th week of the semester respectively. The CIE exam is conducted for 20 marks of 2 hours duration consisting of five descriptive type questions out of which four questions have to be answered where, each question carries 5 marks. Marks are awarded by taking average of marks scored in two CIE exams

Quiz – Online Examination:

Two Quiz exams shall be online examination consisting of 25 multiple choice questions and are to be answered by choosing the correct answer from a given set of choices (commonly four). Such a question paper shall be useful in testing of knowledge, skills, application, analysis, evaluation and understanding of the students. Marks shall be awarded considering the average of two quiz examinations for every course.

Alternative Assessment Tool (AAT):

This AAT enables faculty to design own assessment patterns during the CIA. The AAT converts the classroom into an effective learning center. The AAT may include tutorial hours/classes, seminars, assignments, term paper, open ended experiments, METE (Modeling and Experimental Tools in Engineering), five minutes video, MOOCs etc. The AAT chosen for this course is given in table 3**.**

Table 3: Assessment pattern for AAT

5 Minutes Video	Assignment	Tech-talk	Seminar	Open Ended Experiment
20%	30%	30%	10%	10%

VI. COURSE OBJECTIVES:

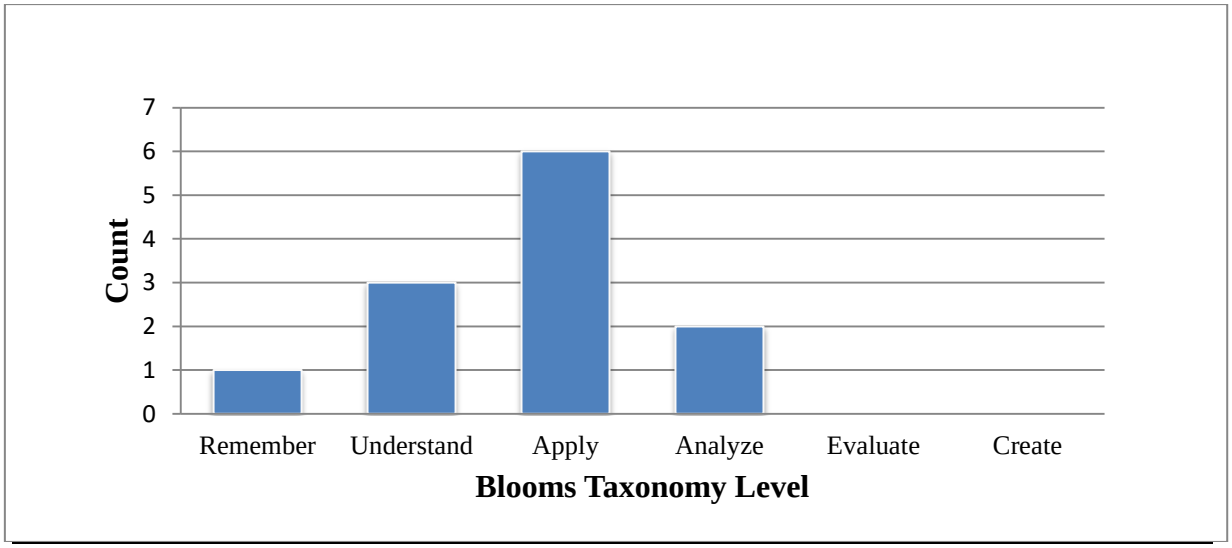
The students will try to learn:	
I	The applications of complex variable and conformal mapping in two dimensional complex potential theories.
II	The fundamental calculus theorems and criteria for the independent path on contour integral used in problems of engineering
III	The concepts of special functions and its application for solving the partial differential equations in physics and engineering.
IV	The mathematics of combinatorial enumeration by using generating functions and complex analysis for understanding the numerical growth rates.

VII. COURSE OUTCOMES:

After successful completion of the course, students will be able to:		
Course Outcomes		Knowledge Level (Bloom's Taxonomy)
CO 1	**Identify** the fundamental concepts of analyticity and differentiability for calculus of complex functions and their role in applied context.	Remember
CO 2	**Utilize the** concepts of analyticity for finding complex conjugates and their role in applied contexts.	Apply
CO 3	**Make use of the** conformal mapping technique for transferring geometric structure of complex functions with much more convenient	Apply

	geometry.	
CO 4	**Apply** integral theorems of complex analysis and its consequences for the analytic function with derivatives of all orders in simple connected region.	Apply
CO 5	**Extend** the Taylor and Laurent series for expressing the function in terms of complex power series.	Understand
CO 6	**Classify** Singularities and Poles of Complex functions for evaluating definite and indefinite Complex integrals.	Understand
CO 7	**Apply** Residue theorem for computing definite integrals of real and complex analytic functions over closed curves.	Apply
CO 8	**Relate** the concept of improper integral and second order differential equations of special functions for formulating real world problems with futuristic approach.	Understand
CO 9	**Determine** the characteristics of special functions generalization on elementary factorial function for the proper and improper integrals.	Apply
CO 10	**Choose** an appropriate special function on physical phenomena arising in engineering problems and quantum physics.	Apply
CO 11	**Analyze** the role of Bessel functions in the process of obtaining the series solutions for second order differential equation.	Analyze

COURSE KNOWLEDGE COMPETENCY LEVELS



VIII. HOW PROGRAM OUTCOMES ARE ASSESSED:

Program Outcomes		Strength	Proficiency Assessed by
PO 1	**Engineering knowledge**: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.	3	CIE/Quiz/AAT
PO 2	**Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences	3	CIE/Quiz/AAT

Program Outcomes		Strength	Proficiency Assessed by
PO 4	**Conduct Investigations of Complex Problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.	2	CIE/Quiz/AAT

3 = High; 2 = Medium; 1 = Low

IX. HOW PROGRAM SPECIFIC OUTCOMES ARE ASSESSED:

Program Specific Outcomes		Strength	Proficiency assessed by
PSO 1	Formulate and Evaluate the applications in the field of Intelligent Embedded and Semiconductor technologies.	-	-
PSO 2	Focus on the practical experience of ASIC prototype designs, Virtual instrumentation and SOC designs.	-	-
PSO 3	Build the Embedded hardware design and software programming skills for entry level job positions to meet the requirements of employers.	-	-

3 = High; 2 = Medium; 1 = Low

X. MAPPING OF EACH CO WITH PO(s), PSO(s):

Course Outcomes	Program Outcomes												Program Specific Outcomes		
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
CO 1	√	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 2	√	√	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 3	√	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 4	√	√	-	√	-	-	-	-	-	-	-	-	-	-	-
CO 5	√	√	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 6	√	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 7	√	-	-	√	-	-	-	-	-	-	-	-	-	-	-
CO 8	√	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 9	√	√	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 10	√	√	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 11	√	√	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 12	√	-	-	-	-	-	-	-	-	-	-	-	-	-	-

XI. JUSTIFICATIONS FOR CO – PO/PSO MAPPING – DIRECT

Course Outcomes	POs / PSOs	Justification for mapping (Students will be able to)	No. of key competencies
CO 1	**PO 1**	Identify the basic properties of analytic functions which are closed with respect to the fundamental operations of arithmetic (knowledge), algebra and applicability in solving majority of functions in various **engineering problems** by **applying Mathematical principles.**	**3**
CO 2	**PO 1**	Recognize system of two partial differential Equations (knowledge) which, together with certain continuity and differentiability criteria and connection of this system to the analytic functions and their application to construct analytic functions applied in **engineering disciplines** by **the principles of mathematics.**	**3**
	PO 2	**Identify** the Problem **formulation and abstraction** for obtaining the analytic function by Milne Thompson's method from the provided **information**.	**2**
CO 3	**PO 1**	Recognize the importance and application of Conformal mapping as an important technique (knowledge) in complex analysis used in different physical situations in various **engineering problems** by applying the **principles of mathematics and science.**	**3**
CO 4	**PO 1**	Apply the integral theorem of complex analysis (knowledge) and its consequences to the analytic function for solving complex **engineering** problems by applying the **principles of mathematics and science.**	**3**
	PO 2	**Identify the problem statement** to build extensions of Cauchy's theorem and application of necessary condition to vanish a contour integral around the simple connected regions from the provided **information and data** in reaching substantiated conclusions by using principles of mathematics.	**2**
	PO 4	Understand the given problem and **apply quantitative methods** to simplify the calculation of certain contour integrals (knowledge) on simply connected regions in order to solve engineering problems.	**1**
CO 5	**PO 1**	Apply the knowledge of geometric series that enable us to use Cauchy's integral formula for understanding power series representations of analytic functions in **engineering problems** by applying **the principles of mathematics.**	**3**
	PO 2	Identify the **problem formulation** and **abstraction** of rational complex functions for expressing in negative or positive terms of power series(knowledge) using Laurent's series and Taylor's series by applying the principles of mathematics.	**1**
CO 6	**PO 1**	Identify (knowledge) the singular points and poles of complex functions and their classification (understand) for applicability (apply) in solving definite and indefinite complex functions in **engineering problems** by applying **the principles of mathematics.**	**3**
CO 7	**PO 1**	Understand the method of finding residues of given real or complex integrand (knowledge) and applicability of Residue theorem to solve definite and indefinite complex integrals of specific interval in **engineering problems** by applying **the principles of mathematics.**	**3**

Course Outcomes	POs / PSOs	Justification for mapping (Students will be able to)	No. of key competencies
	PO 4	Make use of the **quantitative methods** of finding residues for evaluating line integrals (length of curve) of analytic functions over closed curves and applicability of Residue theorem by applying the principles of mathematics.	**1**
CO 8	**PO 1**	Identify the occurrence of special functions as solutions of differential equations or integrals of elementary functions(knowledge)and understand its generalization with factorial function for solving complex integrals of specific interval in **engineering problems** by applying the **principles of mathematics.**	**3**
	PO 2	**Identify the given problem** and **formulate** relationship between beta and gamma functions (knowledge) and their applicability for solving improper integrals by transforming by applying the principles of mathematics.	**2**
CO 10	**PO 1**	Identify special functions that have more or less established names and notations (knowledge) and Understand the major role of these functions which exists as solutions for integrals and differential equations of elementary functions in **engineering problems by principles of mathematics.**	**3**
	PO 2	**Identify the given problem** and **formulate** the special functions for solving and make use of mathematical methods to facilitate physical interpretation of the results obtained.	**2**
CO 11	**PO 1**	Recognize the Bessel functions as series solution of second order differential equation (knowledge) and find its generating function and use it to prove some useful standard results and recurrence relations applied in **engineering disciplines** by **principles of mathematics.**	**3**
	PO 2	**Identify the given problem** and **formulate** the recurrence relations to examine the behavior of the Bessel functions at some special values and their application to derive orthogonality of Bessel functions by applying the principles of mathematics**.**	**2**

XII. TOTAL COUNT OF KEY COMPETENCIES FOR CO – (PO, PSO) MAPPING

Course Outcomes	Program Outcomes / No. of Key Competencies Matched												PSOs/ No. of key competencies		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**
	3	**10**	**10**	**11**	**1**	**5**	**3**	**3**	**12**	**5**	**12**	**12**	**2**	**2**	**2**
CO 1	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 2	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 3	3	2	-	1	-	-	-	-	-	-	-	-	-	-	-
CO 4	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 5	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-

CO 6	3	-	-	1	-	-	-	-	-	-	-	-	-	-	-
CO 7	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 8	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 9	3	2	-	-	-	-	-	-	-	-	-	-	-		-
CO 10	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 11	3	2	-	-	-	-	-	-	-	-	-	-	-	-	-

XIII. PERCENTAGE OF KEY COMPETENCIES FOR CO – (PO/PSO):

Course Outcomes	**Program Outcomes / No. of key competencies**												**PSOs/ No. of key competencies**		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**1**	**2**	**3**
	3	**10**	**10**	**11**	**1**	**5**	**3**	**3**	**12**	**5**	**12**	**12**	**2**	**1**	**2**
CO 1	100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 2	100	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 3	100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 4	100	20.0	0.0	9.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 5	100	10.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 6	100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 7	100	0.0	0.0	9.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 8	100	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 9	100	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 10	100	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
CO 11	100	20.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

XIV. COURSE ARTICULATION MATRIX (PO – PSO MAPPING)

COs and POs and COs and PSOs on the scale of 0 to 3, **0** being **no correlation**, **1** being the **Low correlation**, **2** being **medium correlation** and **3** being **high correlation**.

0 – **0**≤ C≤ 5%–No correlation
1 – 5 <C≤ 40%– Low/ Slight
2 – 40 % <C< 60% –Moderate
3 – 60% ≤ C< 100% – Substantial /High

Course Outcomes	Program Outcomes												Program Specific Outcomes		
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3
CO 1	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 2	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 4	3	1	-	1	-	-	-	-	-	-	-	-	-	-	-
CO 5	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 6	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 7	3	-	-	1	-	-	-	-	-	-	-	-	-	-	-
CO 8	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 9	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 10	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
CO 11	3	1	-	-	-	-	-	-	-	-	-	-	-	-	-
TOTAL	33	6	-	2	-	-	-	-	-	-	-	-	-	-	-
AVERAGE	3	1	-	1	-	-	-	-	-	-	-	-	-	-	-

XV. ASSESSMENT METHODOLOGY - DIRECT

CIE Exams	PO 1,PO 2 PO 4	SEE Exams	PO 1,PO 2 PO 4	Assignments	PO 1,PO 2 PO 4	Seminars	PO 1,PO 2 PO 4
Laboratory Practices	-	Student Viva	-	Mini Project	-	Certification	-
Term Paper	PO 1,PO 2 PO 4						

XVI. ASSESSMENT METHODOLOGY - INDIRECT

✔	Early Semester Feedback	✔	End Semester OBE Feedback
✗	Assessment of Mini Projects by Experts		

XVII. SYLLABUS

Module-I	COMPLEX FUNCTIONS AND DIFFERENTIATION
Complex functions differentiation and integration: Complex functions and its representation on argand plane, concepts of limit, continuity, differentiability, analyticity, Cauchy-Riemann conditions	

and harmonic functions; Milne-Thomson method, Bilinear Transformation.	
Module-II	**COMPLEX INTEGRATION**
Line integral: Evaluation along a path and by indefinite integration; Cauchy's integral theorem; Cauchy's integral formula; Generalized integral formula; Power series expansions of complex functions And contour Integration: Radius of convergence.	
Module-III	**POWER SERIES EXPANSION OF COMPLEX FUNCTION**
Expansion in Taylor's series, Maclaurin's series and Laurent series. Singular point; Isolated singular point; Pole of order m; Essential singularity; Residue: Cauchy Residue Theorem. Evaluation of Residue by Laurent Series and Residue Theorem. Evaluation of integrals of the type $\int_0^{2\pi} f(\cos\theta, \sin\theta)d\theta$ and $\int_0^{\infty} f(x)dx$	
Module-IV	**SPECIAL FUNCTIONS-I**
Improper integrals; Beta and Gamma functions: Definitions; Properties of Beta and Gamma function; Standard forms of Beta functions; Relationship between Beta and Gamma functions	
Module-V	**SPECIAL FUNCTIONS-II**
Bessel's Differential equation: Bessel function, properties of Bessel function, Recurrence relations of Bessel function, Generating function and Orthogonality of Bessel function, Trigonometric expansions involving Bessel function..	
Textbooks:	
1. Erwin Kreyszig, "Advanced Engineering Mathematics", John Wiley & Sons Publishers, 10th Edition,2010 2. B. S. Grewal, "Higher Engineering Mathematics", Khanna Publishers, 43rd Edition, 2015.	
Reference Books:	
1. T.K.V Iyengar, B. Krishna Gandhi, "Engineering Mathematics - III", S. Chand & Co., 12th Edition, 2015. 2. Churchill, R.V. and Brown, J.W, "Complex Variables and Applications", Tata Mc Graw-Hill, 8th Edition, 2012.	

XVIII. COURSE PLAN:

The course plan is meant as a guideline. Probably there may be changes.

Lecture No	Topics to be covered	Course Outcomes	Text (T) book / Reference (R) book
1	Understanding the complex function in Argand plane	CO 1	T1:12.1, R1:4.2
2	Apply the limit of a complex function	CO 1,CO 2	T1:12.3, R1:4.4
3	Apply the continuity of a complex function	CO 1,CO 2	T1:12.3, R1:4.6
4	Apply the differentiability and analyticity of a complex function	CO 1,CO 2	T1:12.3, R1:4.7
5-6	Identify and Apply the of Cauchy-Riemann conditions in Cartesian and Polar forms	CO 1,CO 2	T1:12.4, R1:4.13
7	Evaluate the Harmonic Conjugates	CO 1,CO 2	T1:12.4, R1:4.15

Lecture No	Topics to be covered	Course Outcomes	Text (T) book / Reference (R) book
8-9	Apply the Milne-Thomson method to find the Analytic function	CO 1,CO 2	T1:12.4, R1:4.20
10	Apply the properties of Bilinear transformation for complex functions.	CO 3	T1:12.5, R1:8.8
11-12	Evaluate the Line Integral for a given path	CO 4	T1:13.1, R1:5.3
13	Apply the Cauchy's integral theorem in a given plane	CO 4	T1:13.2, R1:5.5
14-15	Apply the Cauchy's integral formula for evaluating contour integration.	CO 4	T1:13.3, R1:5.9
16-17	Apply the Cauchy's general integral formula for evaluating contour integration.	CO 4	T1:13.4, R1:5.10
18	Define the Power series expansions of complex functions and contour Integration	CO 5	T1:14.1, R1:6.1
19	Evaluate the Radius of convergence of power series complex function	CO 5	T1:14.2 , R1:6.1
20-21	Identify the types of power series expansions	CO 5	T1:14.4, R1:6.2
22	Define the types of Singularities and its nature	CO 6	T1:15.2 , R1:6.6
23	Define the concept of Residues	CO 6,CO 7	T1:15.1, R1:7.4,
24-25	Evaluate the Residues of complex functions.	CO 6, CO 7	T1:15.1, R1:6.5
26	Evaluate of contour integrals by Residue theorem.	CO 6, CO 7	T1:15.3, R1:7.9
27	Understand the basic concepts of Beta and Gamma functions	CO 8, CO 9	T2: 7.14, R1:1.6
28-29	Understand the properties of Beta and Gamma functions	CO 8, CO 9	T2: 7.15, R1:1.63
30-31	Evaluate the improper integrals by using the properties of Beta and Gamma functions.	CO 8, CO 9	T2: 7.15, R1:1.65
32	Define the Bessel differential equation and Bessel function	CO 8, CO 10	T2: 16.5, R1:7.32
33-34	Evaluate the Bessel functions using properties	CO 10	T2: 16.6, R1:7.36
35-36	Determine the recurrence relations of Bessel functions	CO 10	T2: 16.7, R1:7.36
37-39	Evaluate the Generating functions and Orthogonality of Bessel functions	CO 8,CO 11	T2: 16.8, R1:7.41
40-42	Determine the trigonometric expansions of Bessel functions	CO 11	T2: 16.9, R1:7.42
43-45	Understand and Apply the trigonometric expansions of Bessel functions	CO 11	T2: 16.9, R1:7.42

Prepared by:
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