COURSE: ENGINEERING MECHANICS
CODE: AMEB03

Institute of Aeronautical Engineering
(Autonomous)
Dundigal, Hyderabad- 500043

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COURSE OBJECTIVES

The course should enable the students to:

1. **Ability to work comfortably with basic engineering mechanics concepts required for analyzing static structures.**

2. **Identify an appropriate structural system to studying a given problem and isolate it from its environment, model the problem using good free body diagrams and accurate equilibrium equations.**

3. **Identify and model various types of loading and support conditions that act on structural systems, apply pertinent mathematical, physical and engineering mechanical principles to the system to solve and analyze the problem.**

4. **Understand the meaning of centre of gravity (mass)/ centroid and moment of inertia using integration methods and method of moments.**

COURSE LEARNING OUTCOMES (CLOs)

After completing this course the student must demonstrate the knowledge and ability to:

1. A basic understanding of the laws and principle of mechanics.
2. The ability to solve simple force system problems in mechanics.
3. Determine the resultant and apply conditions of static equilibrium.
4. Solve the problems of simple systems with the friction, calculate the linear moving bodies in general plane motion and applications.
5. Analyze planer and spatial systems to determine the force in the members of truss and frames of friction to a plane force system.
6. Solve the problems on different types of beams.
7. Obtain the centroid, center of gravity, first moment and second moment of area.
8. Understand the concept of virtual work and an ability to solve practical problems.
9. Understand the concepts of kinematics of the particles and rectilinear motion.

COURSE LEARNING OUTCOMES (CLOs)

10. Explore knowledge & ability to solve various particle motion problems.
11. Derive the D' Alembert's principle and apply it to various field problems of kinetic motion.
12. Determine the impact, impulse and impulsive forces occurring in the system and able to solve the problems.
13. Develop the work energy relations and apply to connected systems.
14. Understand the fixed axis rotation theory and solving the field problems by application of work energy method.
15. Introduction to concepts of vibration and explain the relation between simple harmonic motion and the equilibrium systems.
16. Derive the expressions for the concepts of simple, compound and torsional pendulums.
17. Explore the use of modern engineering tools, software and equipment to prepare for competitive exams, higher studies etc.

INTRODUCTION TO MECHANICS

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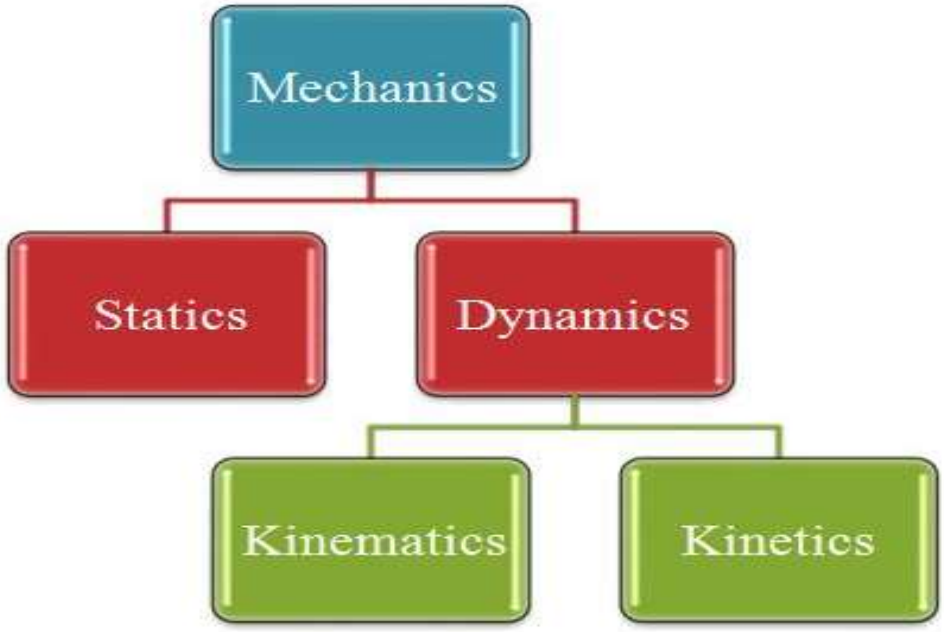
What is mechanics?

Physical science deals with the state of rest or motion of bodies under the action of force.

Why we study mechanics?

This science form the groundwork for further study in the design and analysis of structures.



BASIC TERMS

Essential basic terms to be understood

- Statics: dealing with the equilibrium of a rigid-body at rest
- Rigid body: the relative movement between its parts are negligible
- Dynamics: dealing with a rigid-body in motion
- Length: applied to the linear dimension of a straight line or curved line
- Area: the two dimensional size of shape or surface
- Volume: the three dimensional size of the space occupied by substance
- Force: the action of one body on another whether it's a push or a pull force
- Mass: the amount of matter in a body
- Weight: the force with which a body is attracted toward the centre of the Earth
- Particle: a body of negligible dimension

RIGID-BODY MECHANICS

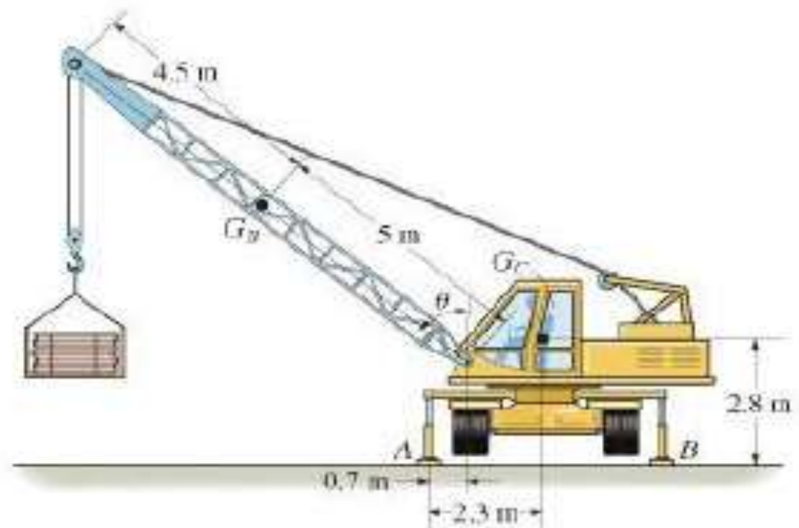
A basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids (advanced courses).

Essential for the design and analysis of many types of structural members, mechanical components, electrical devices etc, encountered in engineering.

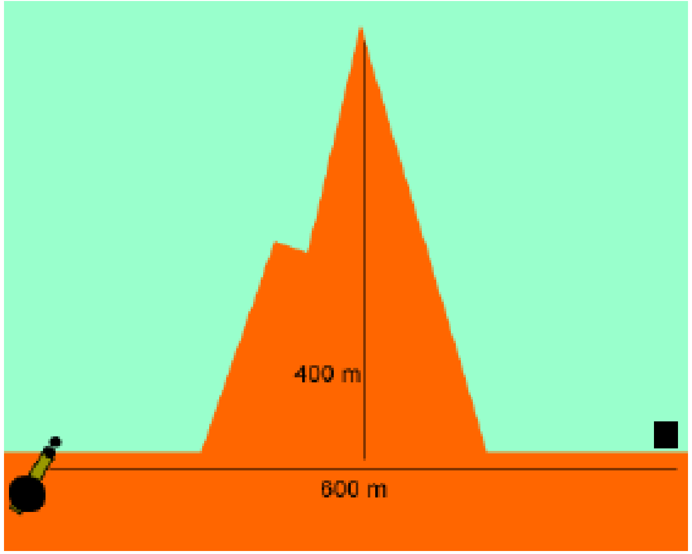
A rigid body does not deform under load!

RIGID-BODY MECHANICS

Statics: **Deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).**

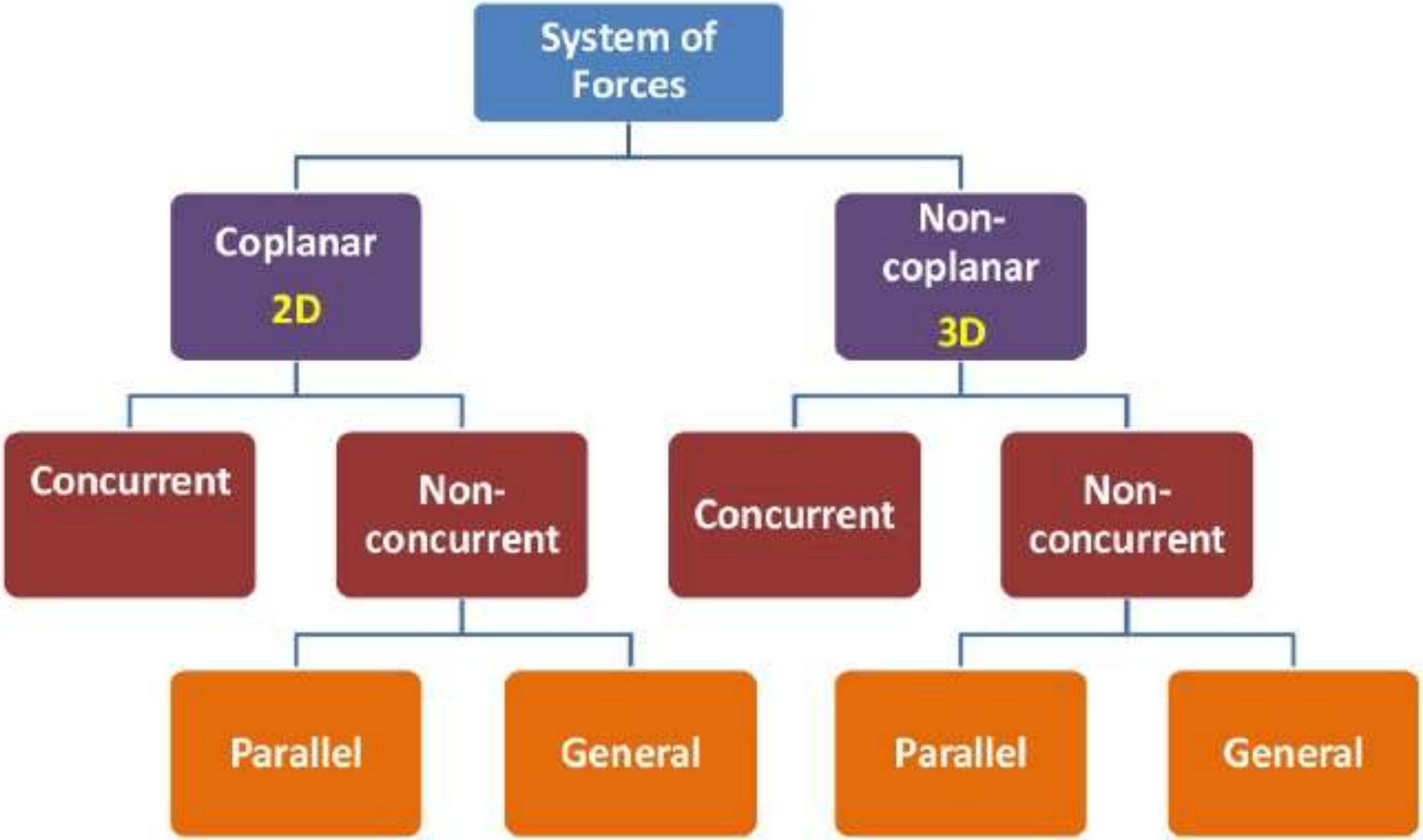


Dynamics: **Deals with motion of bodies (accelerated motion)**

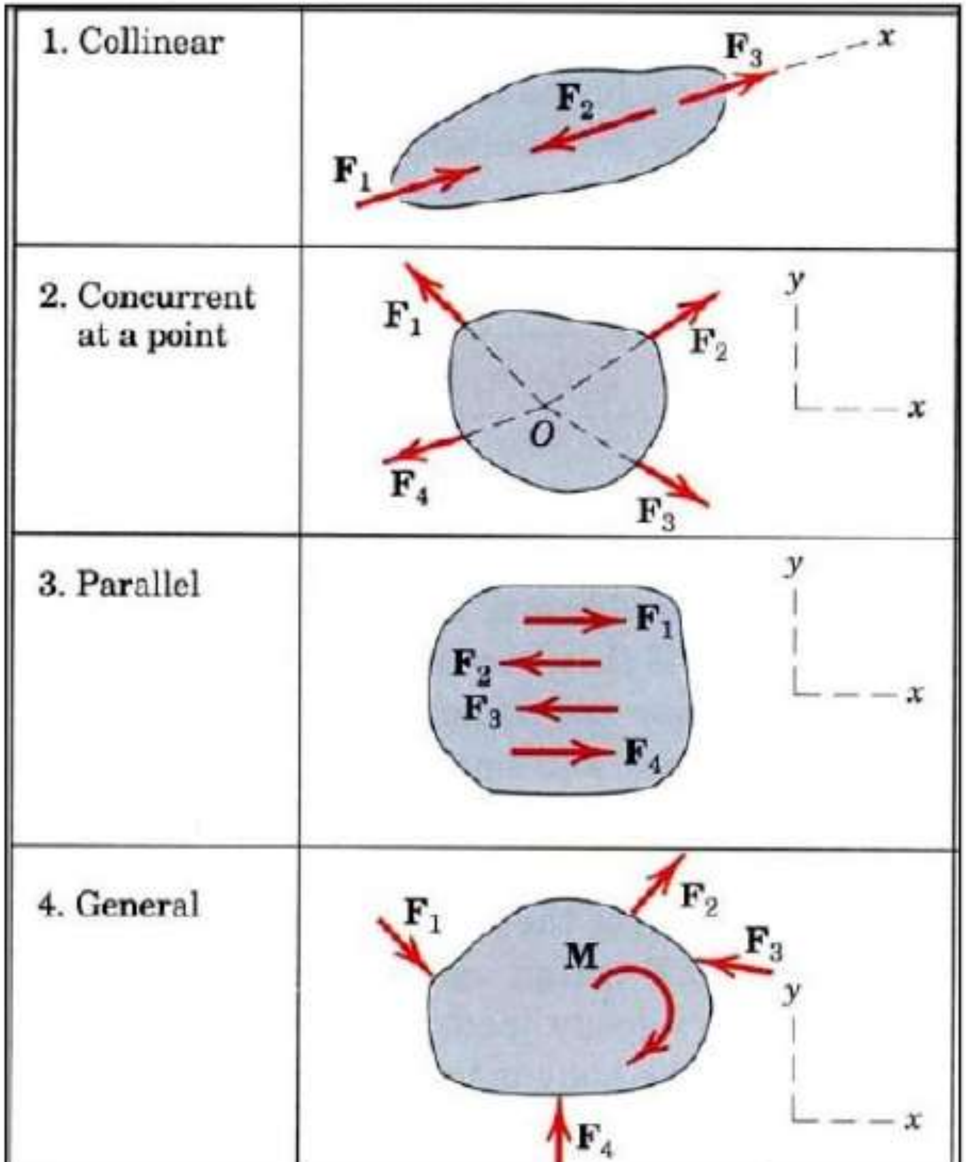


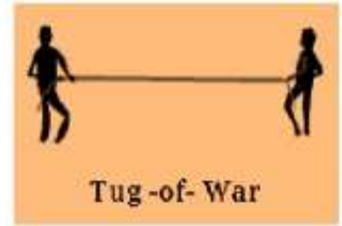
FORCE SYSTEMS BASIC CONCEPTS

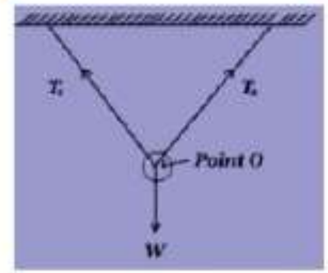
Systems of forces: Several forces acting simultaneously upon a body.

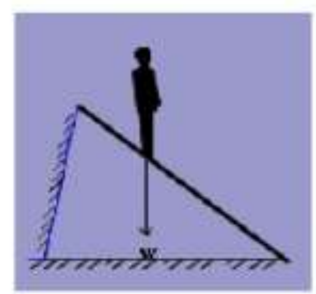


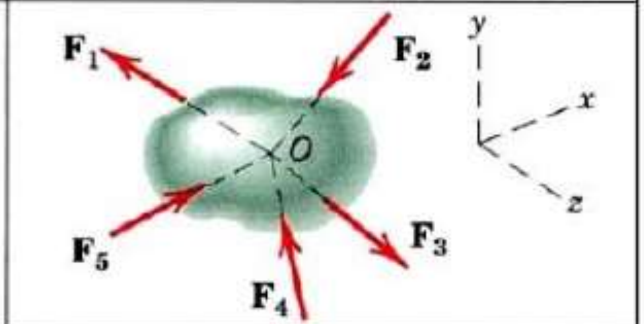
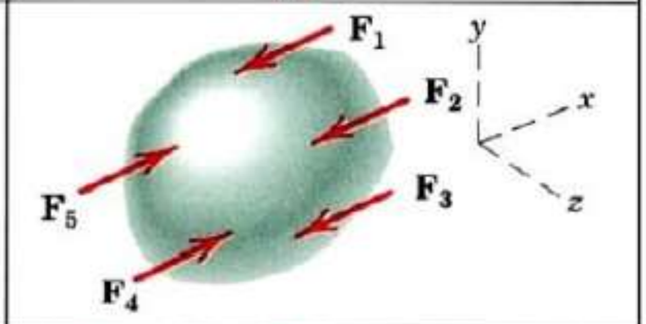
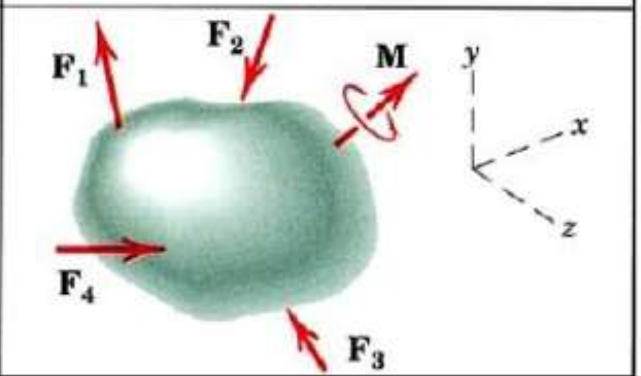
COPLANAR SYSTEM OF FORCES 2D

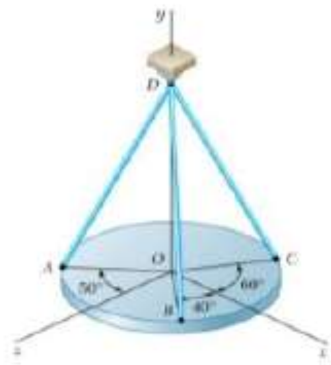


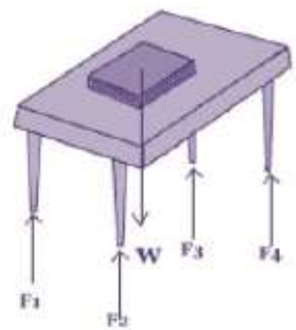




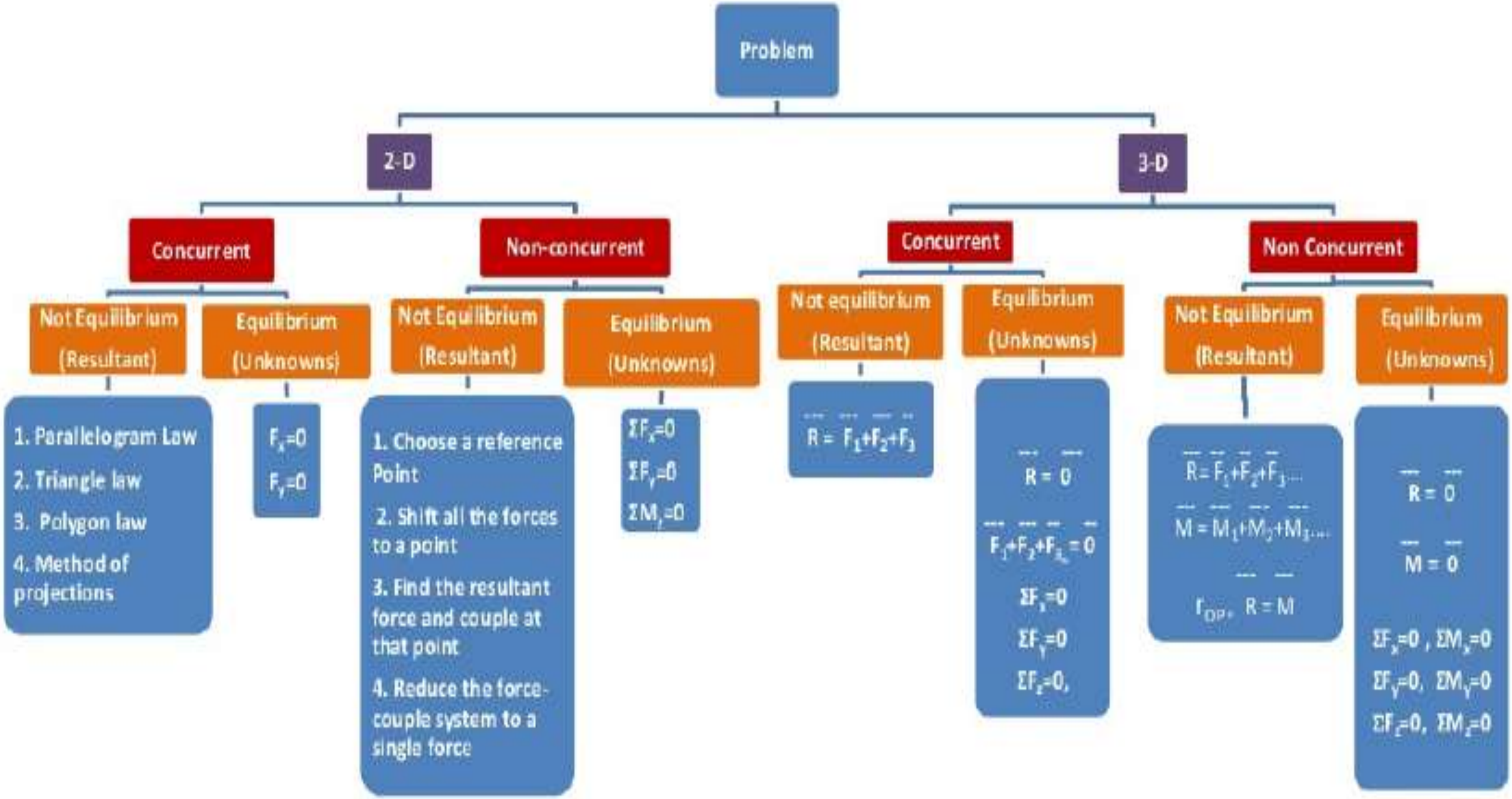


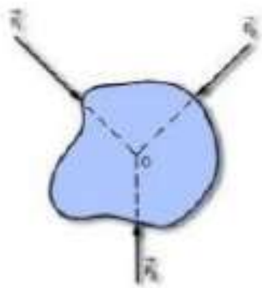
Force System	Free-Body Diagram
Concurrent at a point	
Parallel	
General	

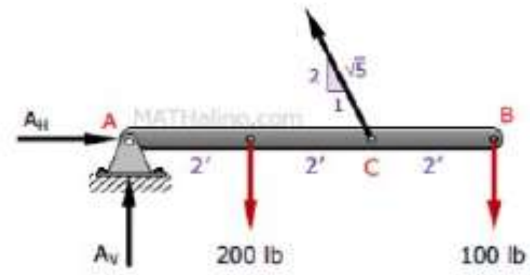




METHOD OF APPROACH TO SOLVE CONCURRENT AND NON-CONCURRENT FORCE SYSTEMS

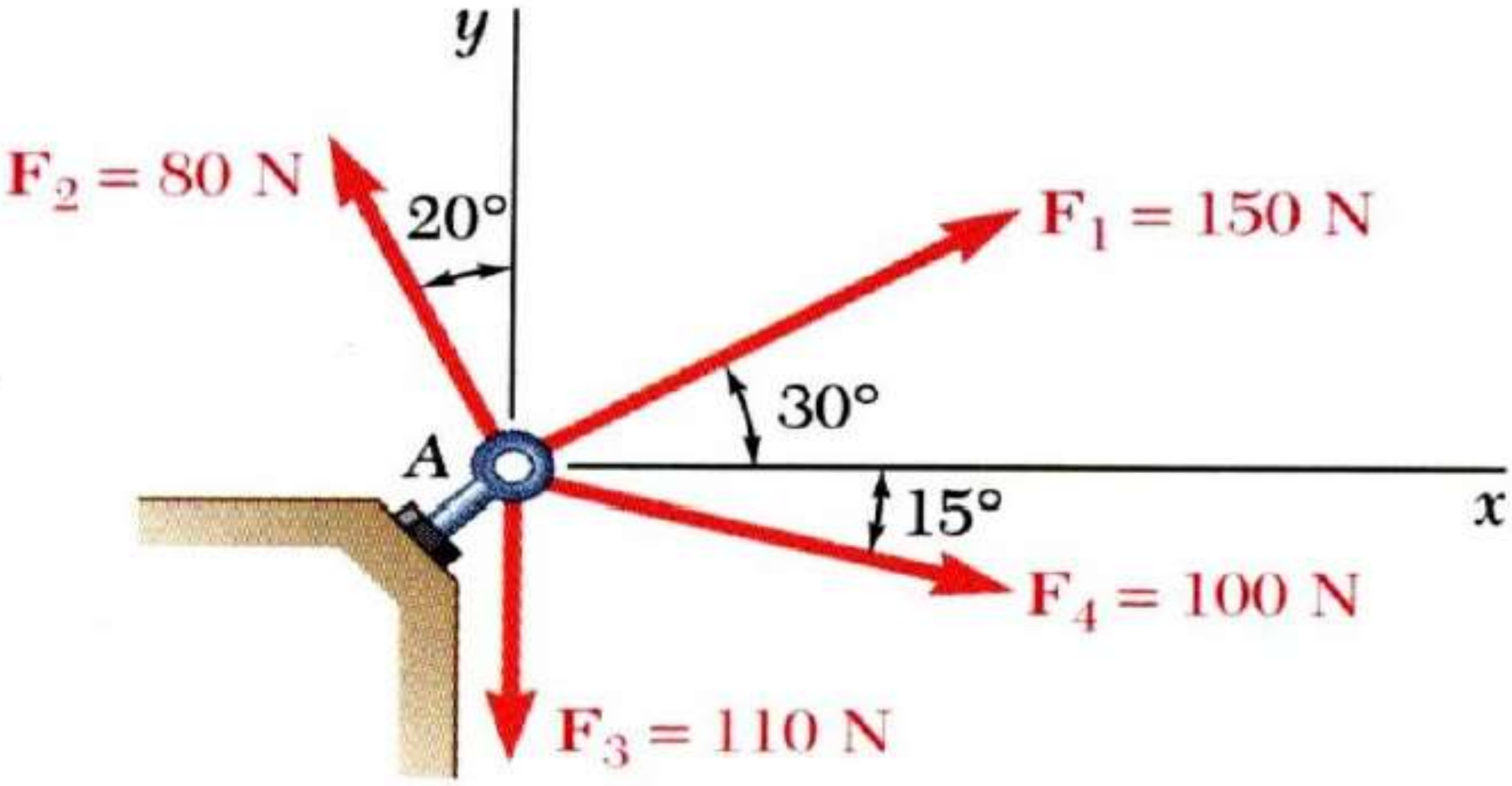






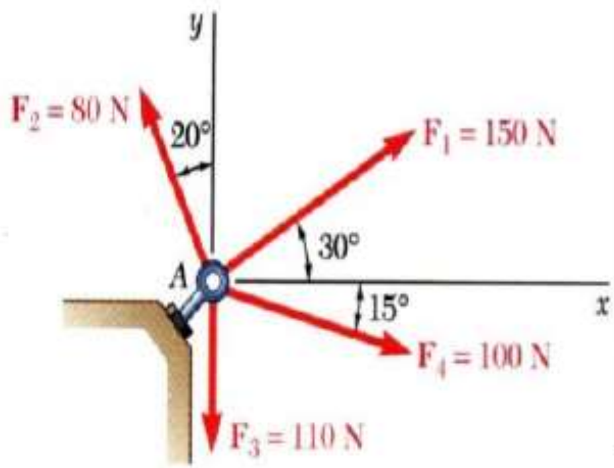
- Problem
  - 2-D
    - Concurrent
      - Not Equilibrium (Resultant)
        1. Parallelogram Law
        2. Triangle law
        3. Polygon law
        4. Method of projections
      - Equilibrium (Unknowns)
        - $F_x=0$
        - $F_y=0$
    - Non-concurrent
      - Not Equilibrium (Resultant)
        1. Choose a reference Point
        2. Shift all the forces to a point
        3. Find the resultant force and couple at that point
        4. Reduce the force-couple system to a single force
      - Equilibrium (Unknowns)
        - $\Sigma F_x=0$
        - $\Sigma F_y=0$
        - $\Sigma M_z=0$

Determine the resultant of the following figure

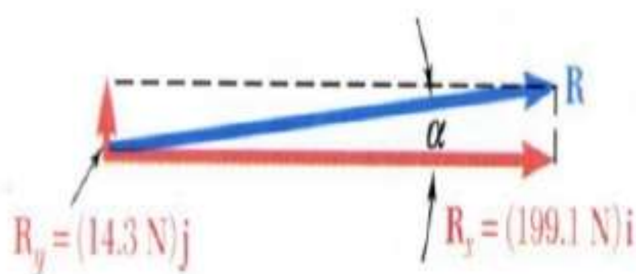


Problem – 2D- Concurrent - Resultant

Problem solution



Force	Mag	x − comp	y − comp
$\vec{F}_1$	150	$150\cos 30$	$+150\,Sin\,30$
$\vec{F}_2$	80	$-80\sin 20$	$+80\,Cos\,20$
$\vec{F}_3$	110	0	-110
$\vec{F}_4$	100	$+100\cos 15$	$-100\sin 15$
		$\sum F_x = +199.1$	$\sum F_y = +14.3$



Resultant is $R = \sqrt{\sum F_X{}^2 + \sum F_Y{}^2}$

$$R = \sqrt{199.1^2 + 14.3^2}$$

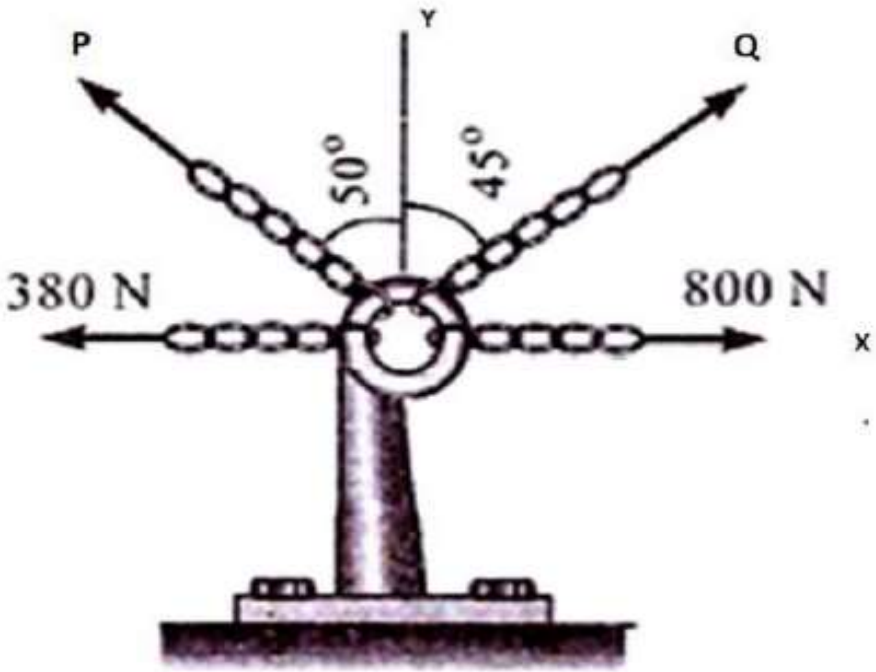
$$R = 199.6\text{N}$$

Direction is

$$\tan\alpha = \frac{14.3\text{ N}}{199.1\text{ N}}$$

$$\alpha = 4.1°$$

The resultant of the four concurrent forces as shown in Fig acts along Y-axis and is equal to 300N. Determine the forces P and Q.

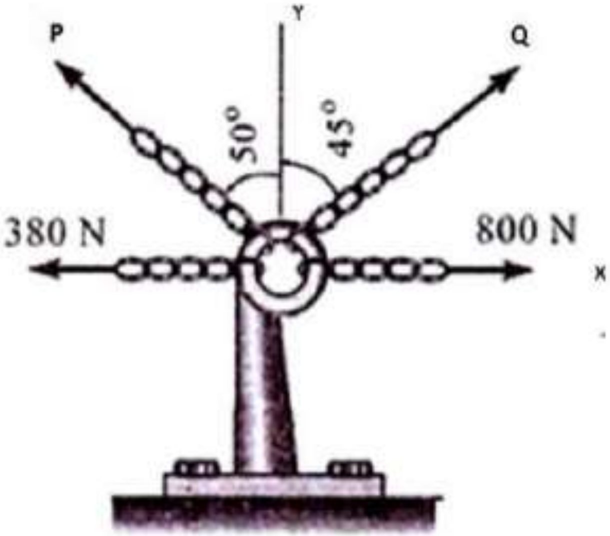


$$\sum F_x = 0$$

$$\sum F_y = R = 300N$$

Problem – 2D- Concurrent - Resultant

Problem solution



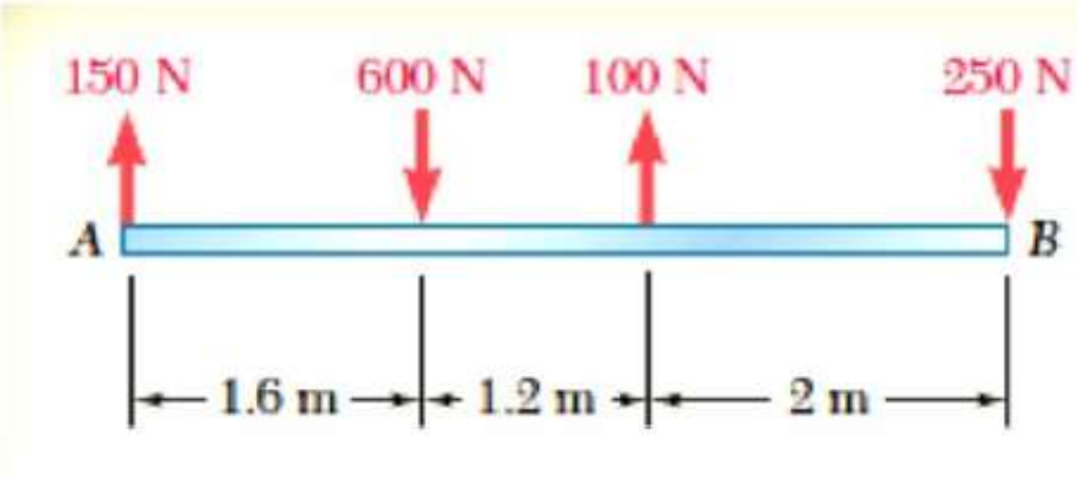
Force	Mag	x − comp	y − comp
$\vec{F}_1$	800	800	0
$\vec{F}_2$	380	−380	0
$\vec{F}_3$	Q	$+QSin45$	$+QCos45$
$\vec{F}_4$	P	$-PSin50$	$+PCos50$

$$\sum F_x = 0 \qquad \sum F_x = 800 - 380 + QSin45 - P\sin 50 = 0$$

$$\sum F_y = R = 300N \qquad \sum F_y = QCos45 + PCos50 = R = 300$$

P = 511 N
Q =- 40.3N

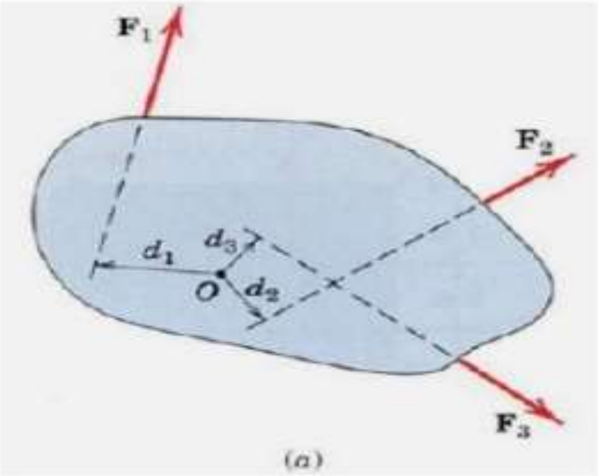
Determine the resultant of the following figure



Problem – 2D- Non Concurrent - Resultant

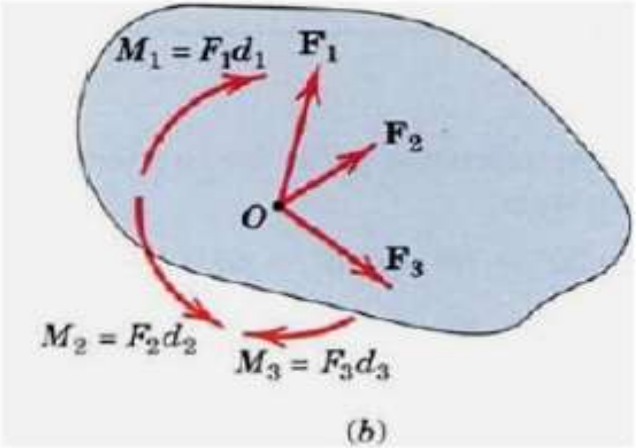
Resultant of General forces in a plane – Coplanar non-concurrent

Step 1: Choose a reference point

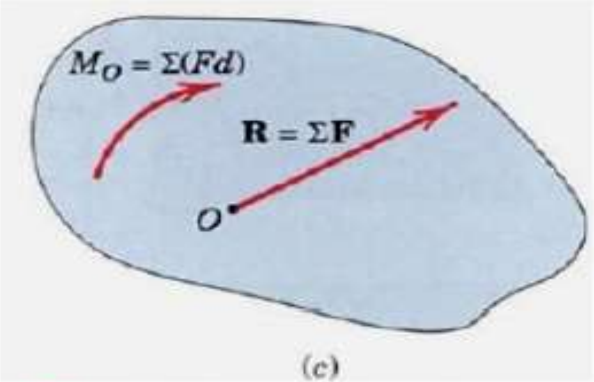


(a)

Step 2: Shift all the forces to a point

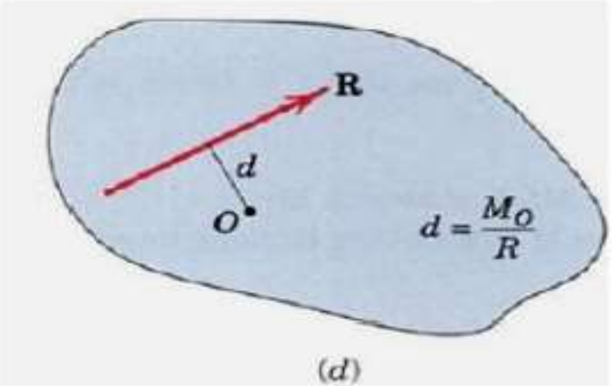


(b)



(c)

Step 3: Find the resultant force and moment of forces about O

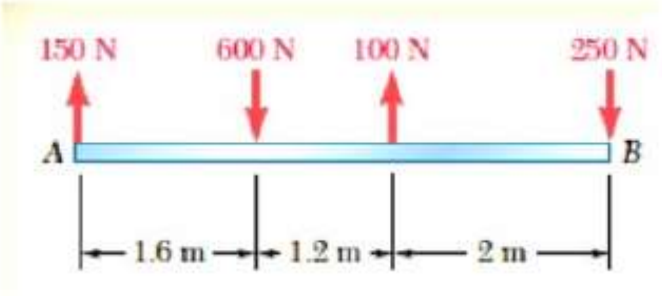


(d)

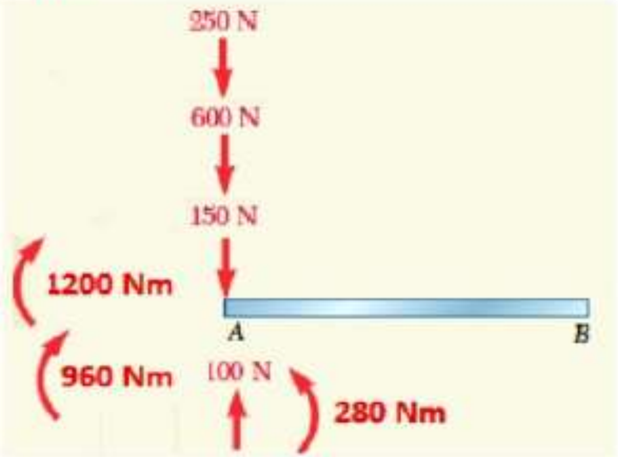
Step 4: Reduce resultant force and moment to a single force

Problem solution: **Resultant – Non-concurrent general forces in a plane**

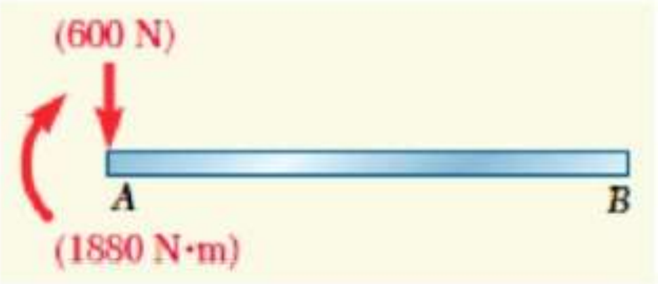
Step:1: Choose A as reference Point



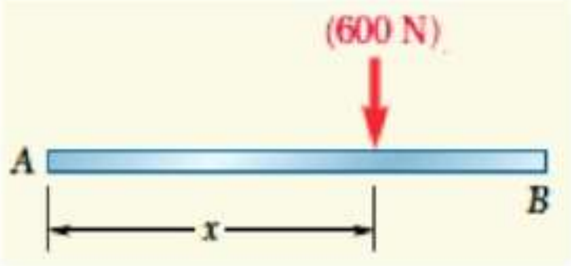
Step:2: Shift all forces to point A



Step 3: Find resultant force and couple



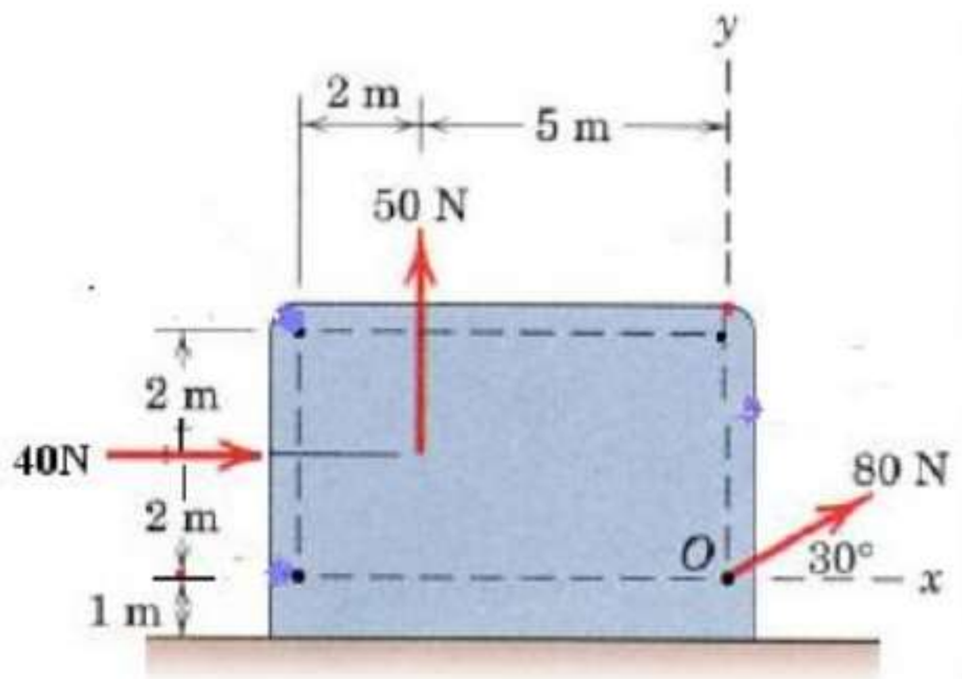
Step:4: Reduce it to a single force



x = 1880/600

x = 3.13m

Problem **Determine the resultant of the following figure**

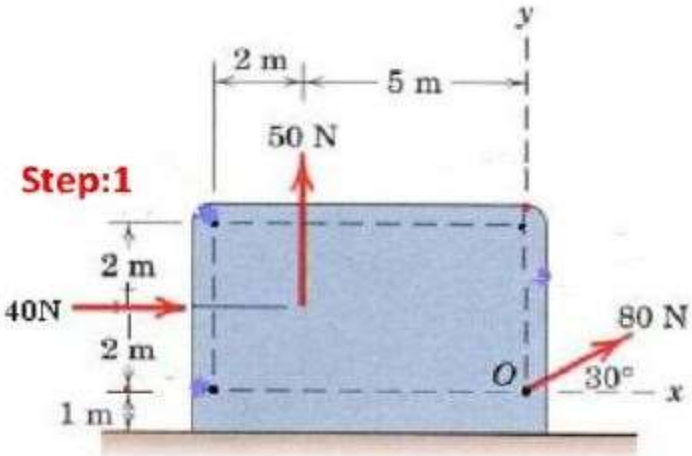


Problem – 2D- Non Concurrent - Resultant

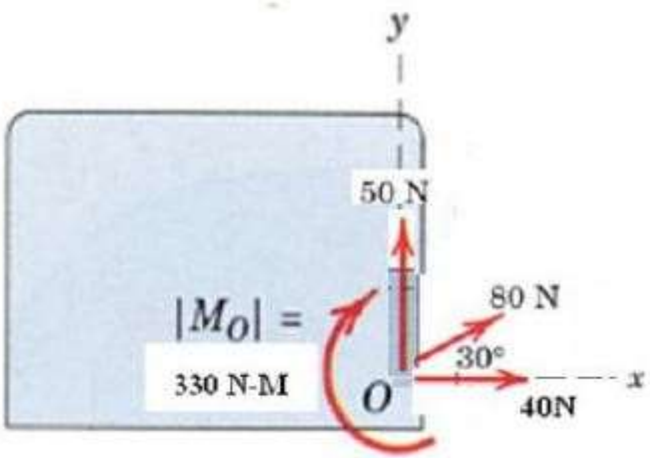
Problem solution **Resultant – Non-concurrent general forces in a plane**

Example: Determine the resultant force of the non-concurrent forces as shown in plate and distance of the resultant force from point 'O'.

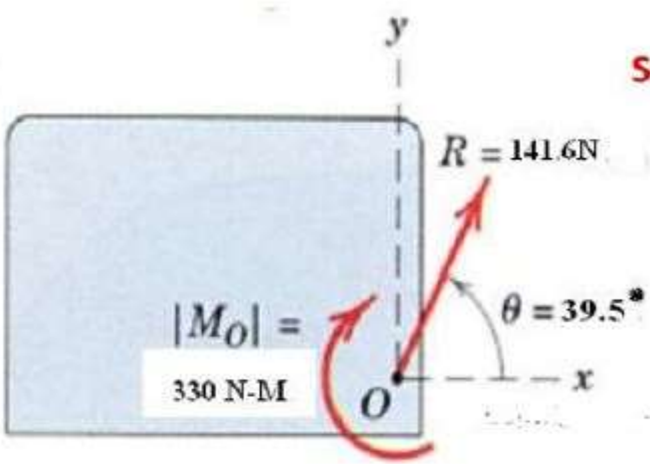
Step:1



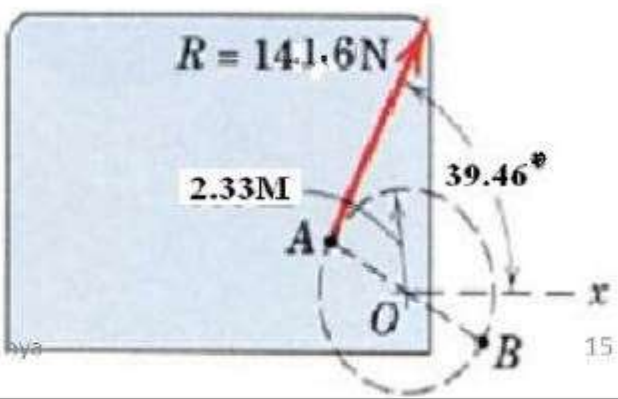
Step:2



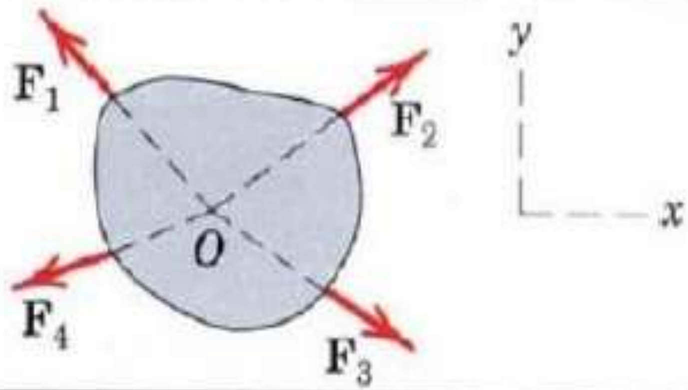
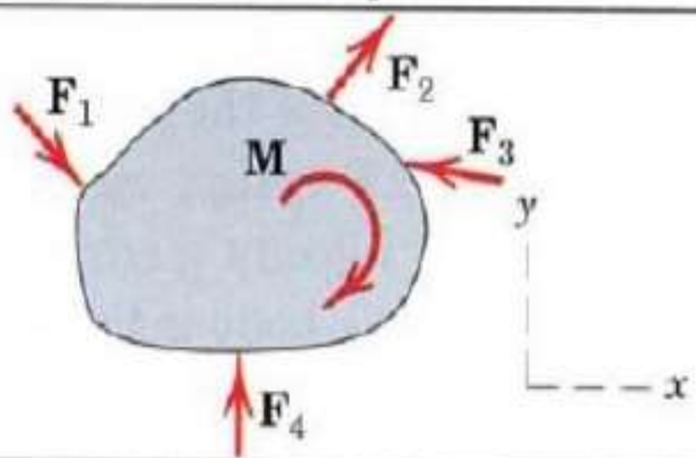
Step:3



Step:4



EQUATIONS OF EQUILIBRIUM

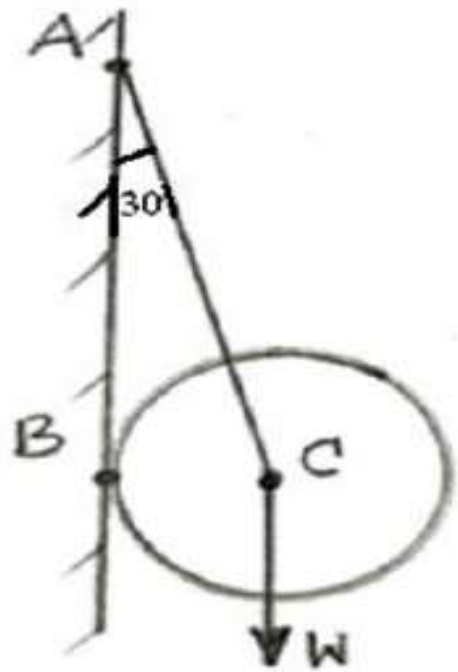
Force System	Free-Body Diagram	Independent Equations
Concurrent		$\Sigma F_x = 0$ $\Sigma F_y = 0$
Non Concurrent		$\Sigma F_x = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_y = 0$

TYPES OF SUPPORTS REACTION FORCES (2D)

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface

Problem

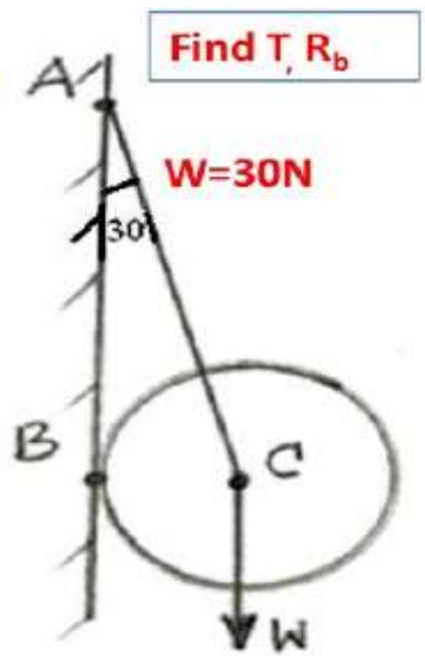
Find tension in the string and reaction at B

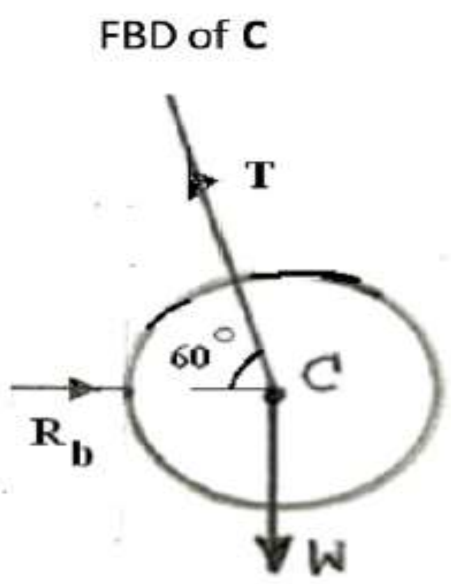


Problem – 2D - Concurrent - Equilibrium

PROBLEMS

Problem solution





Since the body is in equilibrium and the forces are concurrent ……

$\Sigma F_x = 0$; $R_b - T \cos 60^0 = 0$ ……………1

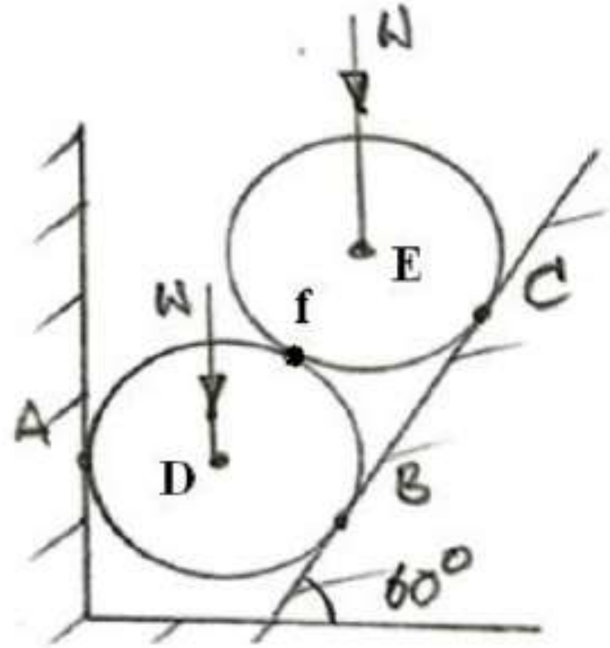
$\Sigma F_Y = 0$; $T \sin 60^0 - W = 0$ ………………2

T= 34.64N

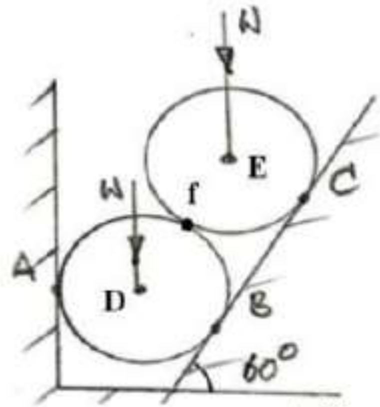
R_b = 17.32N

Problem

Find the reactions at A,B,C,D AND at F, Given W=100N



Problem – 2D - Concurrent - Equilibrium



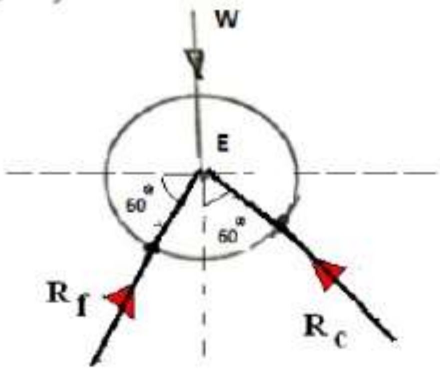
W=100N
Find R_a, R_b and R_c

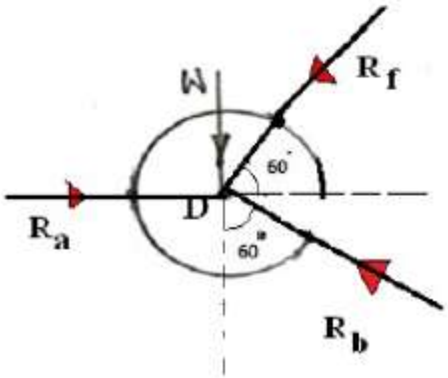
Since the body is in equilibrium and the forces are concurrent ......

$\Sigma F_X=0$; $R_f \cos 60^0 - R_c \sin 60^0 = 0$ 1

$\Sigma F_Y=0$; $R_f \sin 60^0 + R_c \cos 60^0 - W(100) = 0$......2

R_f=86.6N
R_c=50N





Since the body is in equilibrium and the forces are concurrent ......

$\Sigma F_X=0$; $-R_b \sin 60^0 - R_f \cos 60^0 + R_a = 0$ 1

$\Sigma F_Y=0$; $R_b \cos 60^0 - R_f \sin 60^0 - W = 0$..............2

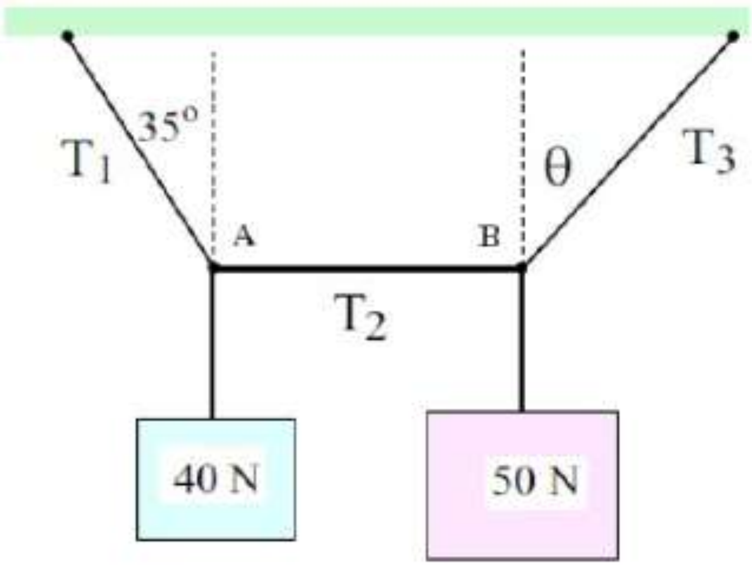
R_b = 350N
R_a = 346.4N

 21

PROBLEMS

Problem

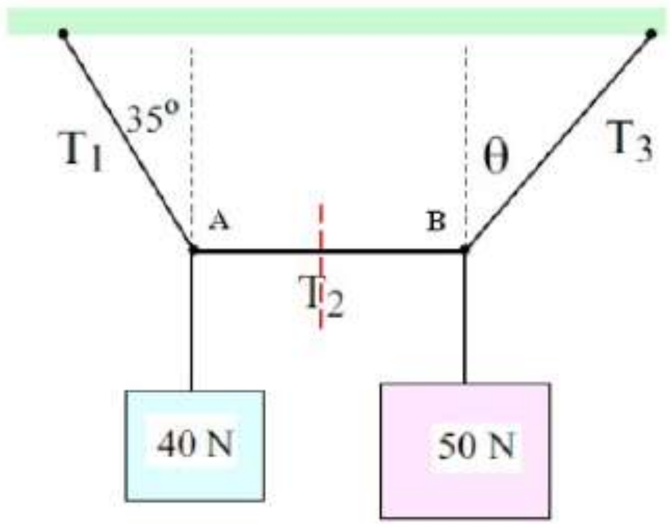
Find T_1, T_2, T_3 and θ



Problem – 2D - Concurrent - Equilibrium

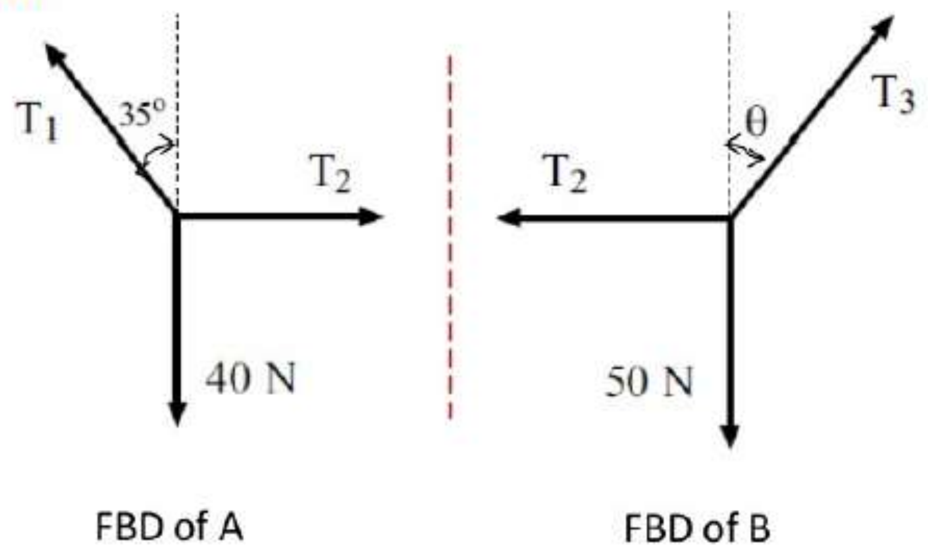
PROBLEMS

Problem

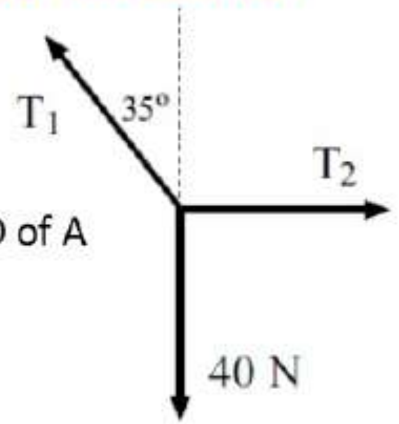


Find T_1, T_2, T_3 and θ

Problem solution:



Problem solution



FBD of A

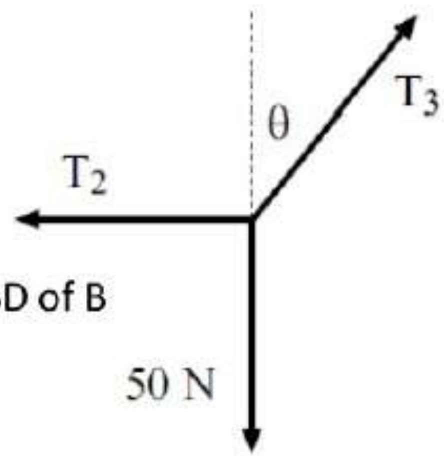
Since the body is in equilibrium and the forces are concurrent ......

$\Sigma F_X = 0$; $T_2 - T_1 \text{Sin } 35^0 = 0$ 1

$\Sigma F_Y = 0$; $T_1 \text{ Cos} 35^0$ - W(40N) = 0......2

T_1=48.8N

T_2=28.0N



FBD of B

Since the body is in equilibrium and the forces are concurrent ......

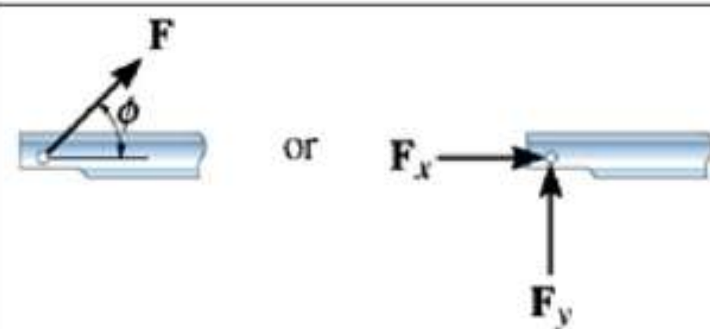
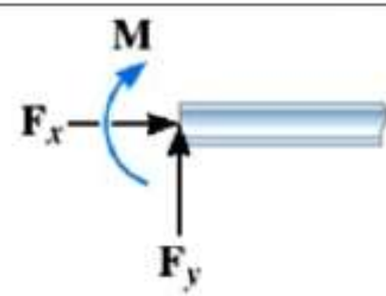
$\Sigma F_X = 0$; $-T_2 + T_3 \text{Sin } \theta^0 = 0$ 1

$\Sigma F_Y = 0$; $T_3 \text{ Cos } \theta^0$ - W(50N) = 0......2

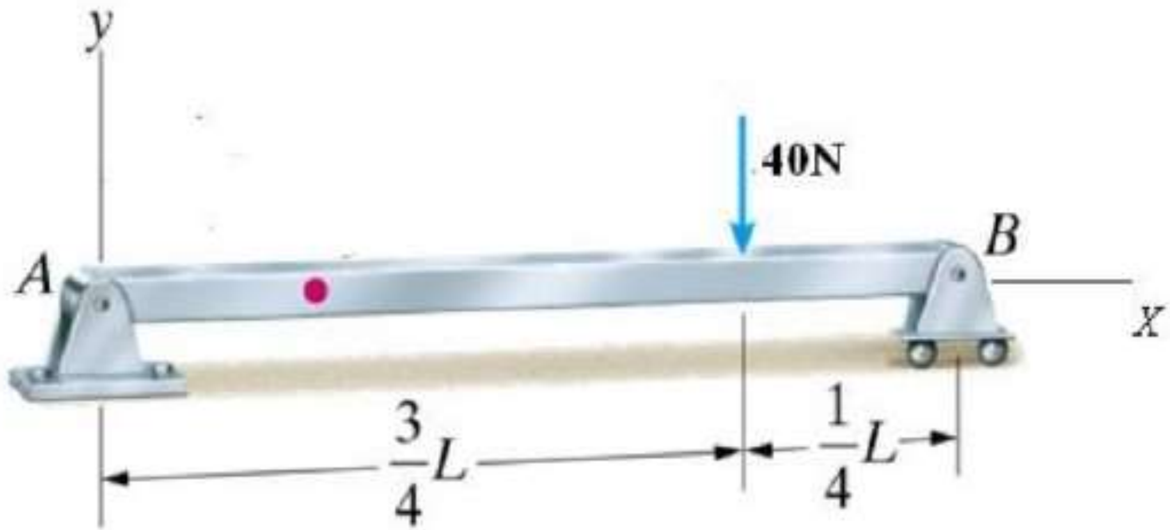
T_3=57.3N

θ=29.3N

TYPES OF SUPPORTS AND REACTION FORCES (2D)

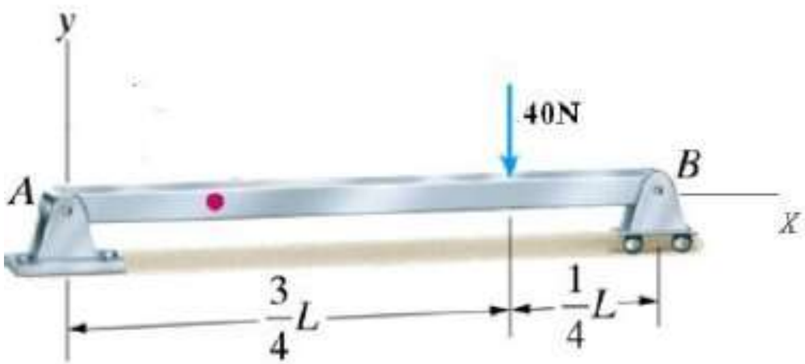
Constraints	Type and direction of forces produced
 roller The connection point on the bar can not move downward.	
 pin The joint can not move in vertical and horizontal directions.	
 fixed support The support prevents translation in vertical and horizontal directions and also rotation, Hence a couple moment is developed on the body in that direction as well.	

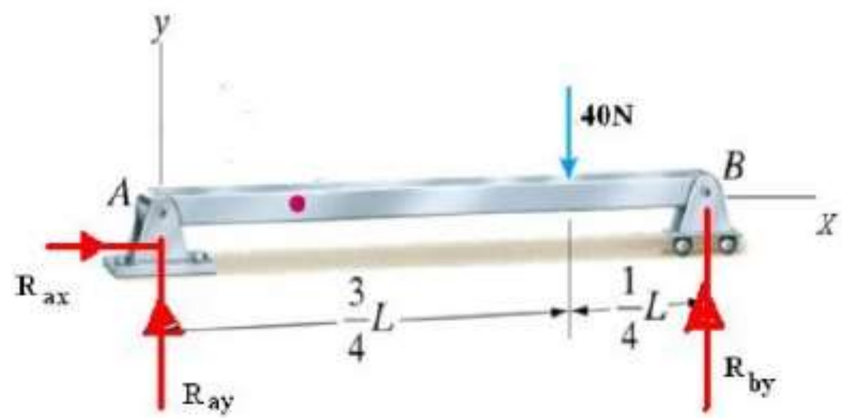
Problem Determine the reactions at A and B



Problem – 2D - Non Concurrent - Equilibrium

Problem solution





Since the body is in equilibrium and the forces are general forces then……

$\Sigma F_X=0$; $R_{ax} = 0$ ………………………………1

$\Sigma F_Y=0$; $R_{ay} + R_{by} - 40 = 0$……………2

$\Sigma M_A=0$; $(R_{by}*L) - 40*(3L/4) = 0$……3

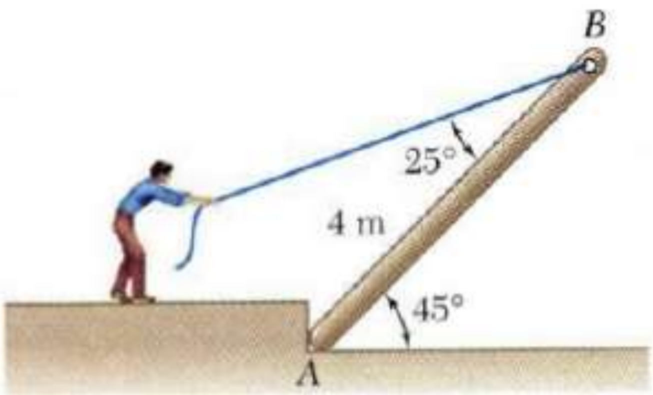
$R_{by}=30N$

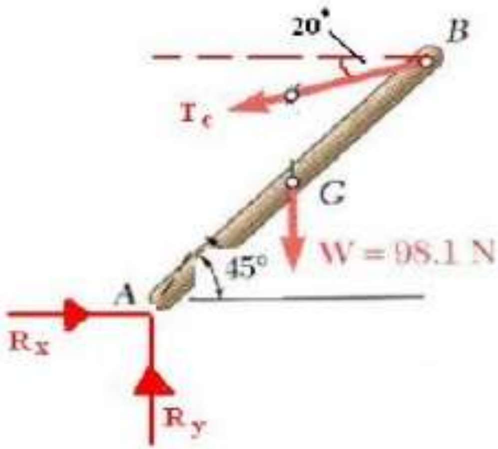
$R_{ay}=10N$

$R_{ax}=0N$

Problem

A man raises a 10 kg joist, of length 4 m, by pulling on a rope. Find the tension in the rope and the reaction at *A*.





Since the body is in equilibrium then…….

$\Sigma F_X=0$; $R_x - T_c*Cos20^0 = 0$ ………….1

$\Sigma F_Y=0$; $R_y - T_c* Sin20^0 - W(98.1) = 0$…………2

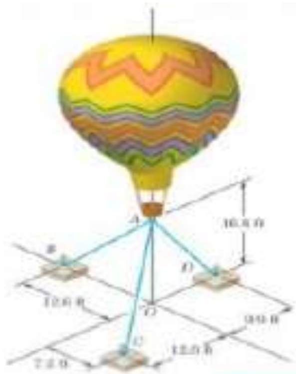
$\Sigma M_A=0$; $-(W*L/2) + (T_c*Cos20^0*4Sin45^0) - (Tc*Sin20^0*4*Cos45^0) = 0$……3

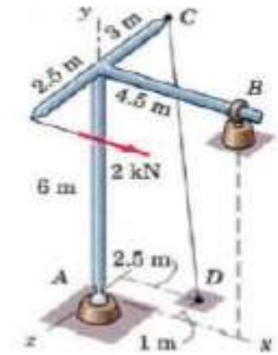
T_c=82N

R_x=77.1N

R_y=126.14N

METHOD OF APPRAOCH TO SLOVE NON COPLANAR (3D STATIC PROBLEMS)



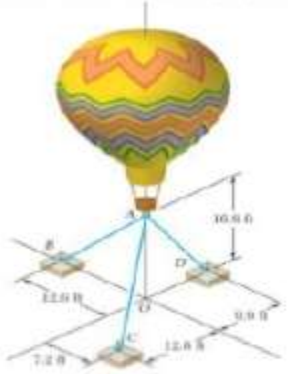


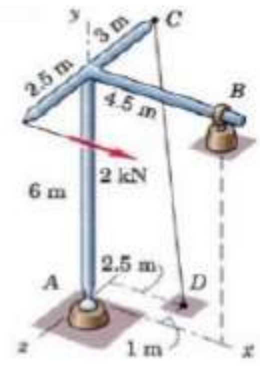
Problem

3-D

- **Concurrent**
  - Not equilibrium (Resultant)
    - $\overline{R} = \overline{F}_1+\overline{F}_2+\overline{F}_3.....$
  - Equilibrium (Unknowns)
    - $\overline{R} = \overline{0}$
    - $\overline{F}_1+\overline{F}_2+\overline{F}_3.... = \overline{0}$
    - $\Sigma F_x=0$
    - $\Sigma F_y=0$
    - $\Sigma F_z=0,$
- **Non Concurrent**
  - Not Equilibrium (Resultant)
    - $\overline{R}= \overline{F}_1+\overline{F}_2+\overline{F}_3....$
    - $\overline{M} = \overline{M}_1+\overline{M}_2+\overline{M}_3....$
    - $r_{OP} \times \overline{R} = \overline{M}$
  - Equilibrium (Unknowns)
    - $\overline{R} = \overline{0}$
    - $\overline{M} = \overline{0}$
    - $\Sigma F_x=0$, $\Sigma M_x=0$
    - $\Sigma F_y=0,$ $\Sigma M_y=0$
    - $\Sigma F_z=0,$ $\Sigma M_z=0$

METHOD OF APPRAOCH TO SLOVE NON COPLANAR (3D STATIC PROBLEMS)



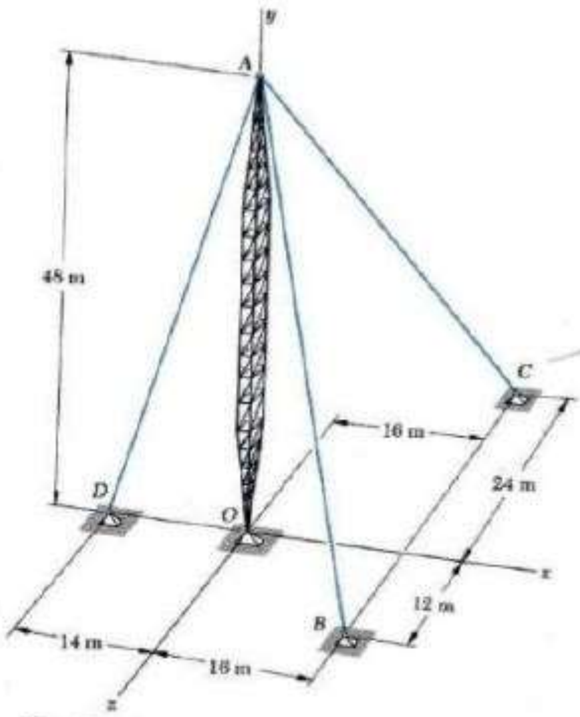


Problem

3-D

- **Concurrent**
  - **Not equilibrium (Resultant)**
    - $\overline{R} = \overline{F}_1+\overline{F}_2+\overline{F}_3.....$
  - **Equilibrium (Unknowns)**
    - $\overline{R} = \overline{0}$
    - $\overline{F}_1+\overline{F}_2+\overline{F}_3... = \overline{0}$
    - $\Sigma F_x=0$
    - $\Sigma F_y=0$
    - $\Sigma F_z=0,$
- **Non Concurrent**
  - **Not Equilibrium (Resultant)**
    - $\overline{R}= \overline{F}_1+\overline{F}_2+\overline{F}_3....$
    - $\overline{M} = \overline{M}_1+\overline{M}_2+\overline{M}_3....$
    - r_{OP}, $\overline{R} = \overline{M}$
  - **Equilibrium (Unknowns)**
    - $\overline{R} = \overline{0}$
    - $\overline{M} = \overline{0}$
    - $\Sigma F_x=0$, $\Sigma M_x=0$
    - $\Sigma F_y=0$, $\Sigma M_y=0$
    - $\Sigma F_z=0$, $\Sigma M_z=0$

RESULTANT OF CONCURRENT FORCES IN 3D



$$\overline{R} = \overline{F}_{AB} + \overline{F}_{AC} + \overline{F}_{AD}$$

$$\overline{R} = F_{AB} \cdot \hat{\lambda}_{AB} + F_{AC} \cdot \hat{\lambda}_{AC} + F_{AD} \cdot \hat{\lambda}_{AB}$$

$$\overline{R} = F_{AB} \cdot \frac{\overline{AB}}{AB} + F_{AC} \cdot \frac{\overline{AC}}{AC} + F_{AD} \cdot \frac{\overline{AD}}{AD}$$

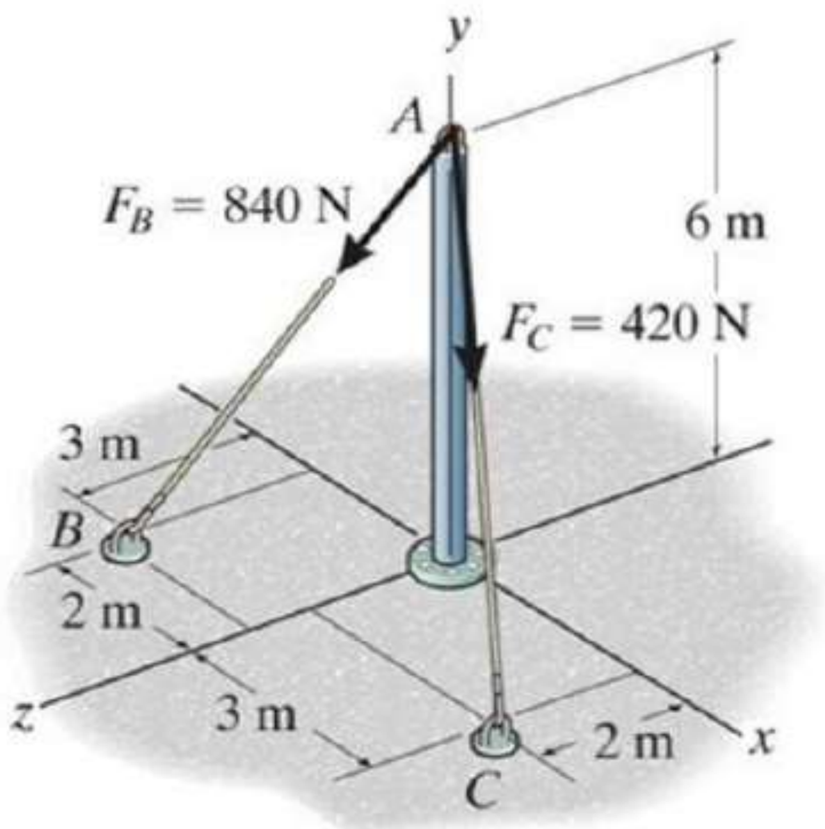
$$\overline{R} = \sum Fx \ i + \sum Fy \ j + \sum Fz \ k$$

$$\cos\theta_x = \frac{\sum Fx}{R}$$

$$\cos\theta_y = \frac{\sum Fy}{R}$$

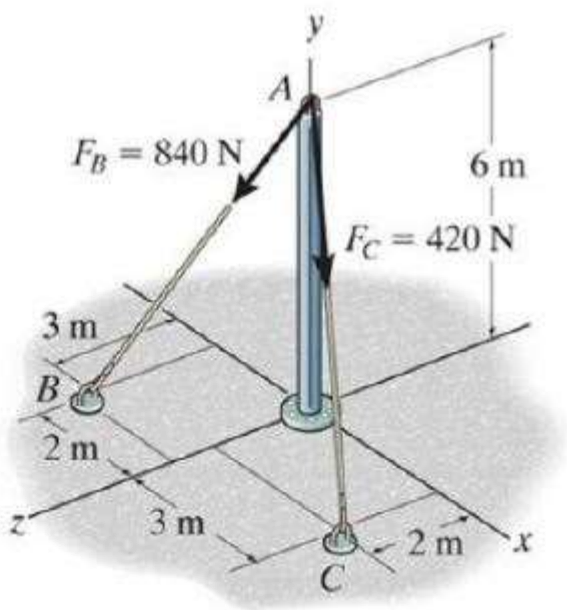
$$\cos\theta_z = \frac{\sum Fz}{R}$$

Problem Determine the Resultant acting at **A**



Problem – 3D- Concurrent - Resultant

Determine the Resultant acting at **A**



$$\overline{R} = \overline{F}_{ab} + \overline{F}_{ac}$$

$$\overline{F}_{ab} = F_{AB} \cdot \hat{\lambda}_{AB}$$

$$\overline{F}_{ab} = 840 * \frac{\{-2i - 6j + 3k\}}{\sqrt{(-2)^2 + (-6^2) + 3^2}}$$

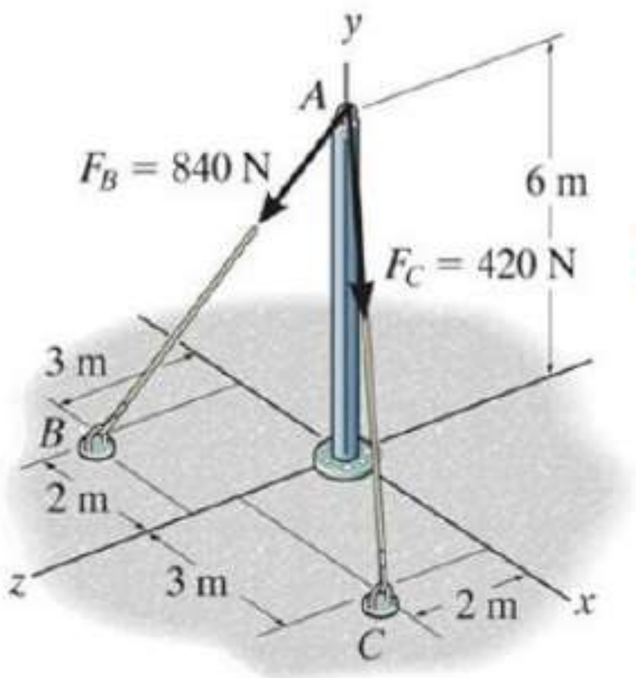
$$\overline{F}_{ac} = F_{AC} \cdot \hat{\lambda}_{AC}$$

$$\overline{F}_{ac} = 420 * \frac{\{3i - 6j + 2k\}}{\sqrt{(3)^2 + (-6^2) + 2^2}}$$

$$\overline{R} = \overline{F}_{ab} + \overline{F}_{ac} \qquad \overline{R} = -60i - 1080j + 480k$$

Magnitude of Resultant is $= \sqrt{(-60)^2 + (-1080^2) + 480^2} = 1183.3N$

RESULTANT OF CONCURRENT FORCES IN 3D



$$\overline{R} = -60i - 1080j + 480k$$

Magnitude of Resultant is

$$= \sqrt{(-60)^2 + (-1080^2) + 480^2} = 1183.3N$$

$$Cos\,\theta_x = \frac{-60}{1183.3}; \theta_x = 93^0$$

$$Cos\,\theta_y = \frac{-1080}{1183.3}; \theta_y = 155.8^0$$

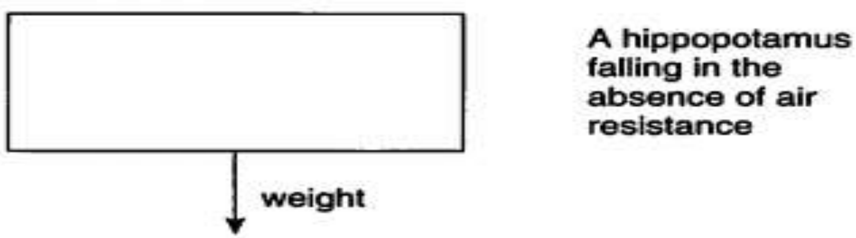
$$Cos\,\theta_z = \frac{480}{1183.3}; \theta_z = 66^0$$

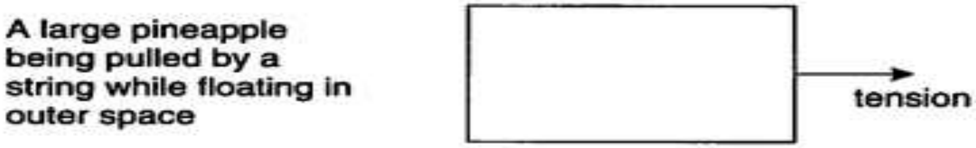
FREE BODY DAIGRAM

A free-body diagram is picture that represents one or more objects, along with the forces acting on those objects. The objects are almost always drawn as rectangles or circles, just for the sake of simplicity, and the forces are always shown as vectors. Below figures show few examples. Free-body diagrams are important because they help us to see forces as vectors. And if you can add vectors, you can analyze free-body diagram.

Let's look at the two examples in Figure 10.1. In the first, force is directed down. This force, which is the force of gravity, was labeled in the diagram as "weight." The force of gravity on the hippo (that is, the hippo's weight) pulls downward. In the second example, a force is directed to the right. The pineapple is being pulled by rope to the right.

Weight: The force due to gravity, equal to the mass of an object times *g*, the gravitational field (about 10 N/kg on Earth).

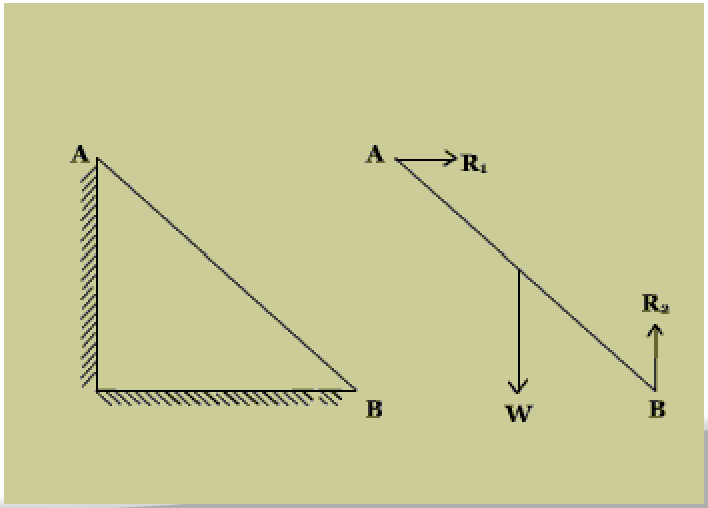




EQUATIONS OF EQUILIBRIUM OF COPLANAR SYSTEMS

A particle is in equilibrium if it is stationary or it moves uniformly relative to an inertial frame of reference. A body is in equilibrium if all the particles that may be considered to comprise the body are in equilibrium.

One can study the equilibrium of a part of the body by isolating the part for analysis. Such a body is called a free body. We make a free body diagram and show all the forces from the surrounding that act on the body. Such a diagram is called a free-body diagram. For example, consider a ladder resting against a smooth wall and floor. The free body diagram of ladder is shown in the right.



EQUATIONS OF EQUILIBRIUM OF COPLANAR SYSTEMS

Three forces are acting on the ladder. Gravitational pull of the earth (weight), W of the ladder, reaction of the floor R_2 and reaction of the wall R_1 .

When a body is in equilibrium, the resultant of all forces acting on it is zero. Thus that resultant force *R* and the resultant couple *M* $_R$ are both zero, and we have the equilibrium equations

$$\boldsymbol{R} = \sum \boldsymbol{F} = \mathbf{0} \qquad \text{and} \qquad M_R = \sum M = 0$$

EQUATIONS OF EQUILIBRIUM OF COPLANAR SYSTEMS

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When a body is in equilibrium, the resultant of all forces acting on it is zero. Thus that resultant force *R* and the resultant couple M_R are both zero, and we have the equilibrium equations

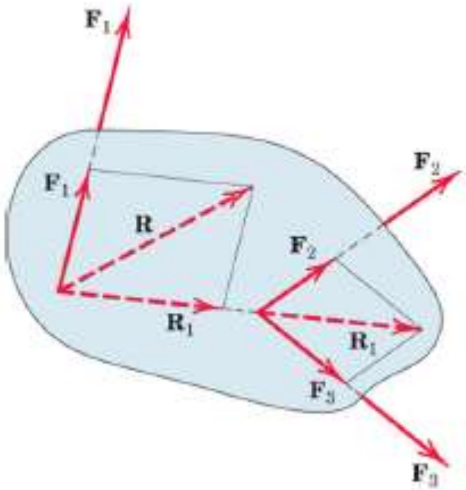
Equilibrium

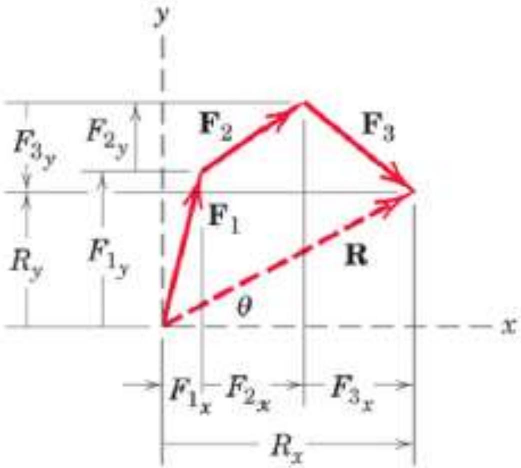
Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.

Condition studied in Statics

Equivalent Systems: Resultants

Vector Approach: Principle of Transmissibility can be used





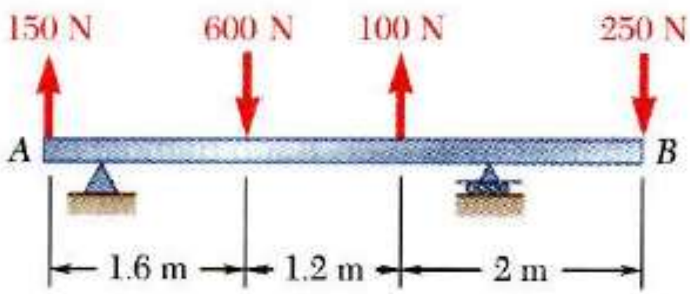
Magnitude and direction of the resultant force R is obtained by forming the force polygon where the forces are added head to tail in any sequence

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Equivalent Systems: Example



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

Solution:

a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

FRICTION AND BASICS STRUCTURAL ANALYSIS

Friction is the force distribution at the surface of contact between two bodies that prevents or impedes sliding motion of one body relative to the other. This force distribution is tangent to the contact surface and has, for the body under consideration, a direction at every point in the contact surface that is in opposition to the possible or existing slipping motion of the body at that point.

TYPES OF FRICTION

1. Dry friction:

Dry friction occurs when the unlubricated (rough) surfaces of two solids are in contact under a condition of sliding or a tendency to slide. A friction force tangential to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place. The direction of this friction force always opposes the motion or impending motion. This type of friction is also called Coulomb friction.

2. Fluid friction:

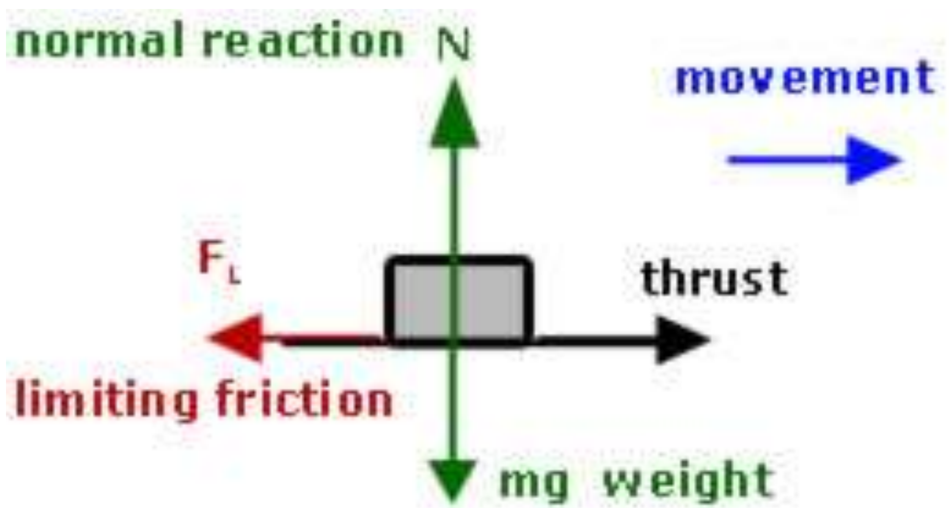
Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is relative velocity between layers, there is no fluid friction. Fluid friction depends on the velocity gradients within the fluid and on the viscosity of the fluid.

3. Internal friction:

Internal friction occurs in all solid materials subjected to cycle loading.

LIMITING FRICTION

The maximum friction that can be generated between two static surfaces in contact with each other. Once a force applied to the two surfaces exceeds the limiting friction, motion will occur. For two dry surfaces, the limiting friction is a product of the normal reaction force and the coefficient of limiting friction.



LAWS OF FRICTION

Laws of friction:

(1) The force of friction always acts in a direction opposite to that in which the body tends to move.

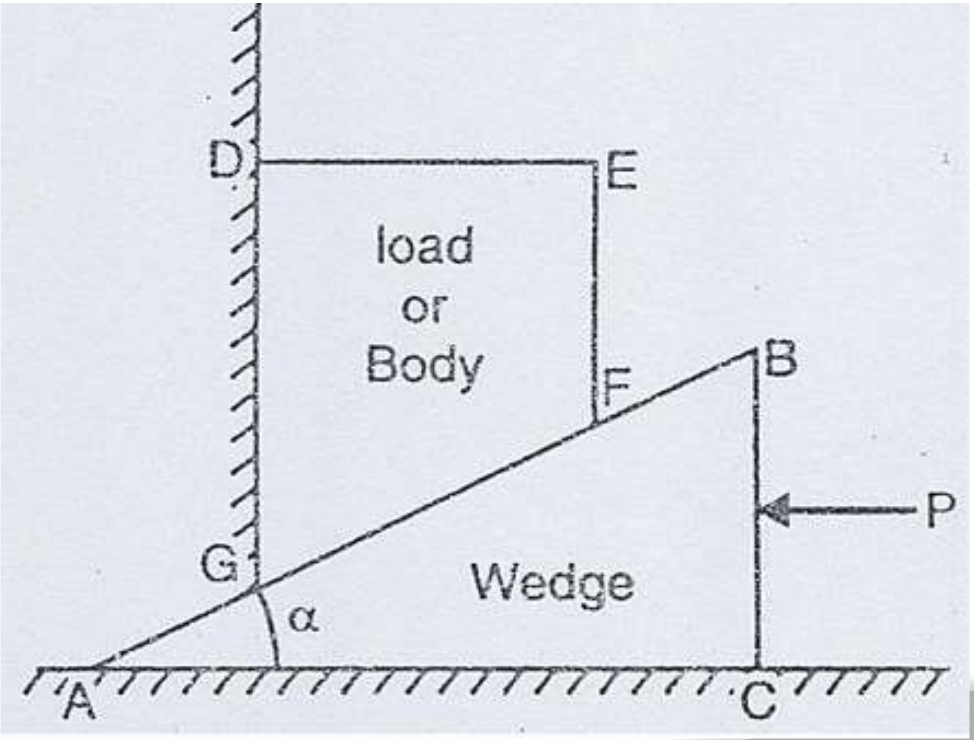
(2) Till the limiting value is reached, the magnitude of friction is exactly equal to the force which tends to move the body.

(3) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two surfaces.

(4) The force of friction depends upon the roughness/smoothness of the surfaces.

(5) The force of friction is independent of the area of contact between the two surfaces.

A wedge is a piece of metal or wood which is usually of a triangular or trapezoidal in cross-section. It is used for either lifting loads through small vertical distances or used for slight adjustments in the position of a body i.e., for tightening fits or keys for shafts.

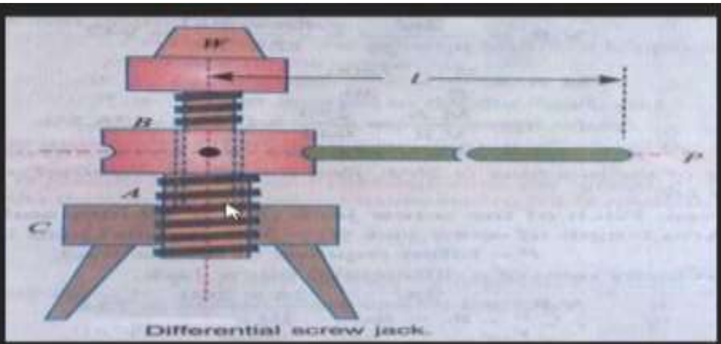


A jackscrew, or screw jack, is a type of jack that is operated by turning a lead screw. It is commonly used to lift moderately heavy weights, such as vehicles; to raise and lower the horizontal stabilizers of aircraft; and an adjustable supports for heavy loads, such as the foundations of houses

SCREW JACK & DIFFERENTIAL

A differential screw is a mechanism used for making small, precise adjustments to the spacing between two objects (such as in focusing a microscope moving the anvils of a micrometer or positioning optics). A differential screw uses a spindle with two screw threads of differing leads (aka thread pitch), and possibly opposite handedness, on which two nuts move. As the spindle rotates, the space between the nuts changes based on the difference between the threads.

1. It consists of two screws A and B.
2. Screw A is provided with external threads and internal thread to give differential movement.



Differential screw jack

3. The screw 'A' is mounted in nut 'C', which makes body of the screw jack.
4. Lever arrangement is connected to the screw 'A' through which screw 'B' can move.
5. As screw 'A' moves downwards, the screw 'B' moves upwards with a distance of difference of two pitches.

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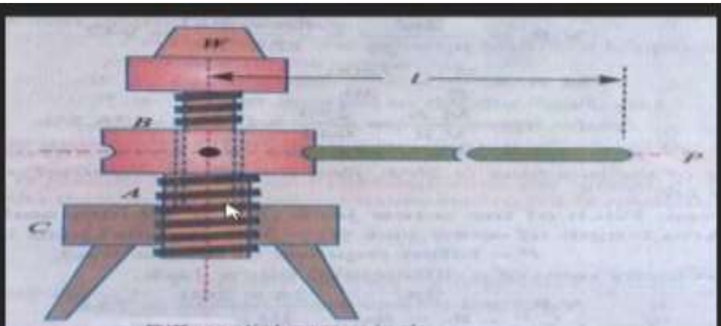

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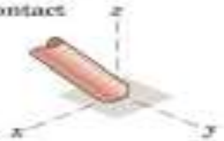
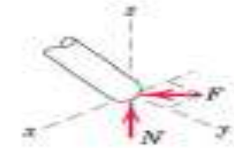
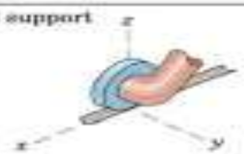
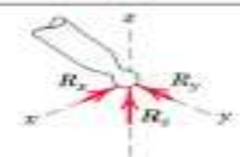
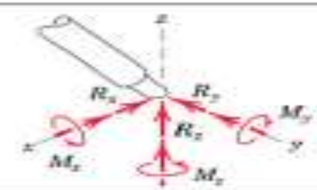
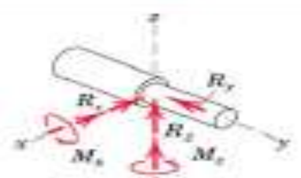
EQUILIBRIUM IN THREE DIMENSIONS

Various Supports
3-D Force
Systems

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint 	 A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded) 	 In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support 	 Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

METHODS OF SECTIONS

- The Method of Sections involves analytically *cutting* the truss into sections and solving for static equilibrium for each section.

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- The sections are obtained by *cutting* through some of the members of the truss to expose the force inside the members.

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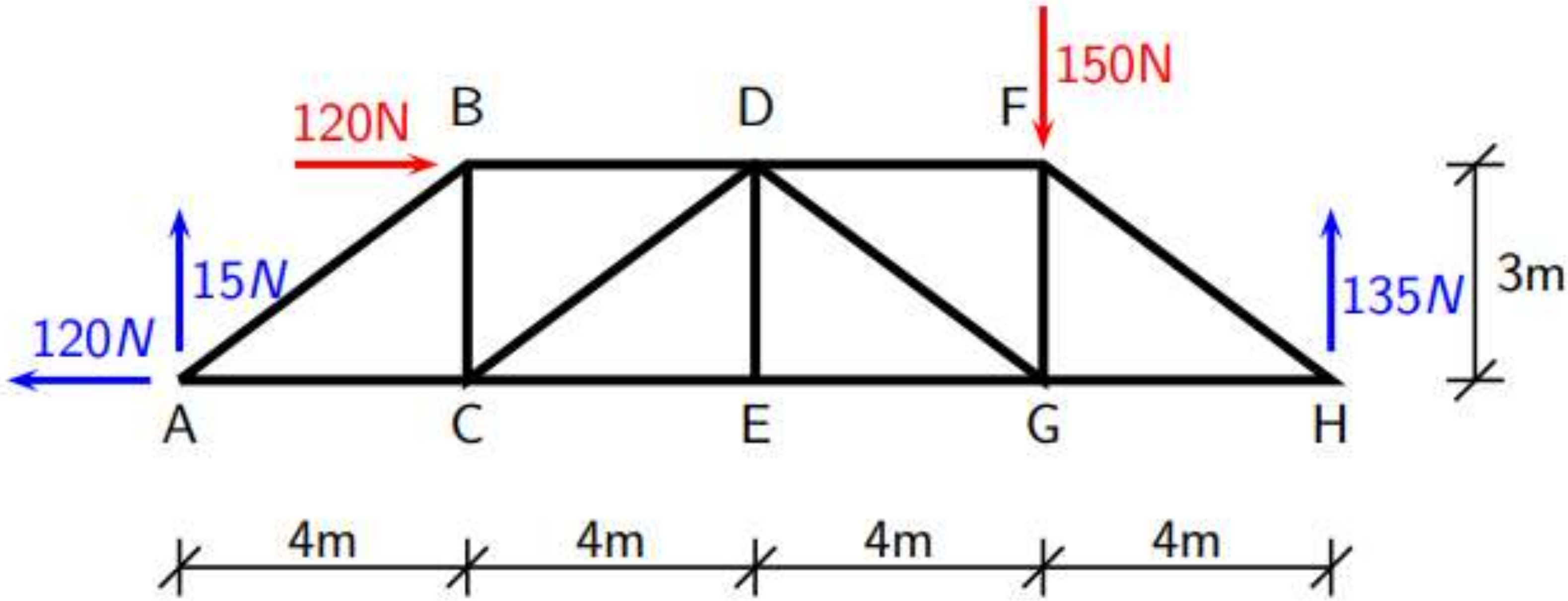
- The sections are obtained by *cutting* through some of the members of the truss to expose the force inside the members.

- In the Method of Joints, we are dealing with static equilibrium at a point. This limits the static equilibrium equations to just the two force equations. A section has finite size and this means you can also use moment equations to solve the problem. This allows solving for up to three unknown forces at a time.

Refer back to the end of the "truss-initial-analysis.pdf" file to see what has been solved so far for the truss. This is what has been solved for so far:

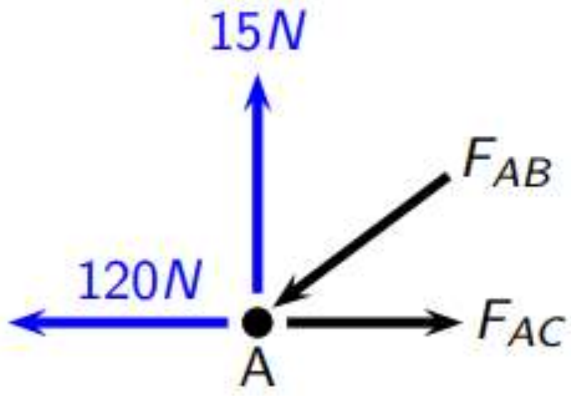


METHODS OF JOINTS

- The method of joints is one of the simplest methods for determining the force acting on the individual members of a truss because it only involves two force equilibrium equations.

- The method of joints is one of the simplest methods for determining the force acting on the individual members of a truss because it only involves two force equilibrium equations.
- Since only two equations are involved, only two unknowns can be solved for at a time. Therefore, you need to solve the joints in a certain order. That is, you need to work from the sides towards the center of the truss.

Let's start with joint A. We begin by drawing all the forces that act on the bolt at joint A.



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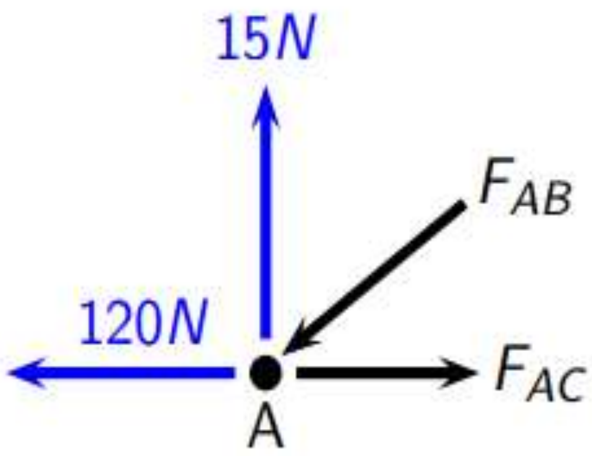


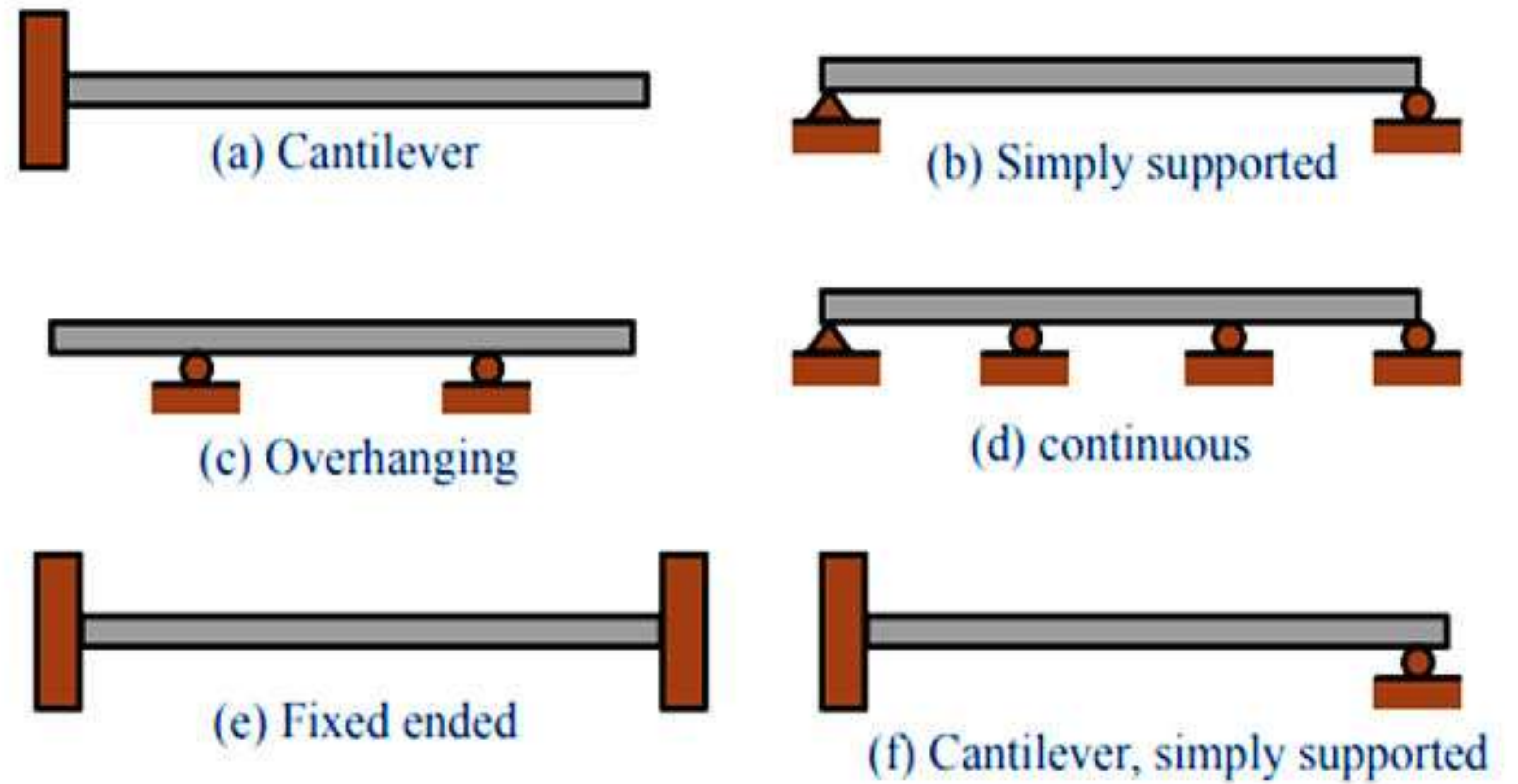
Note that F_{AB} points towards the joint. This is because F_{AB} and the 15 N force are the only vertical forces. Therefore F_{AB} must point downwards to balance the 15 N force pointing up.

TYPES OF BEAMS

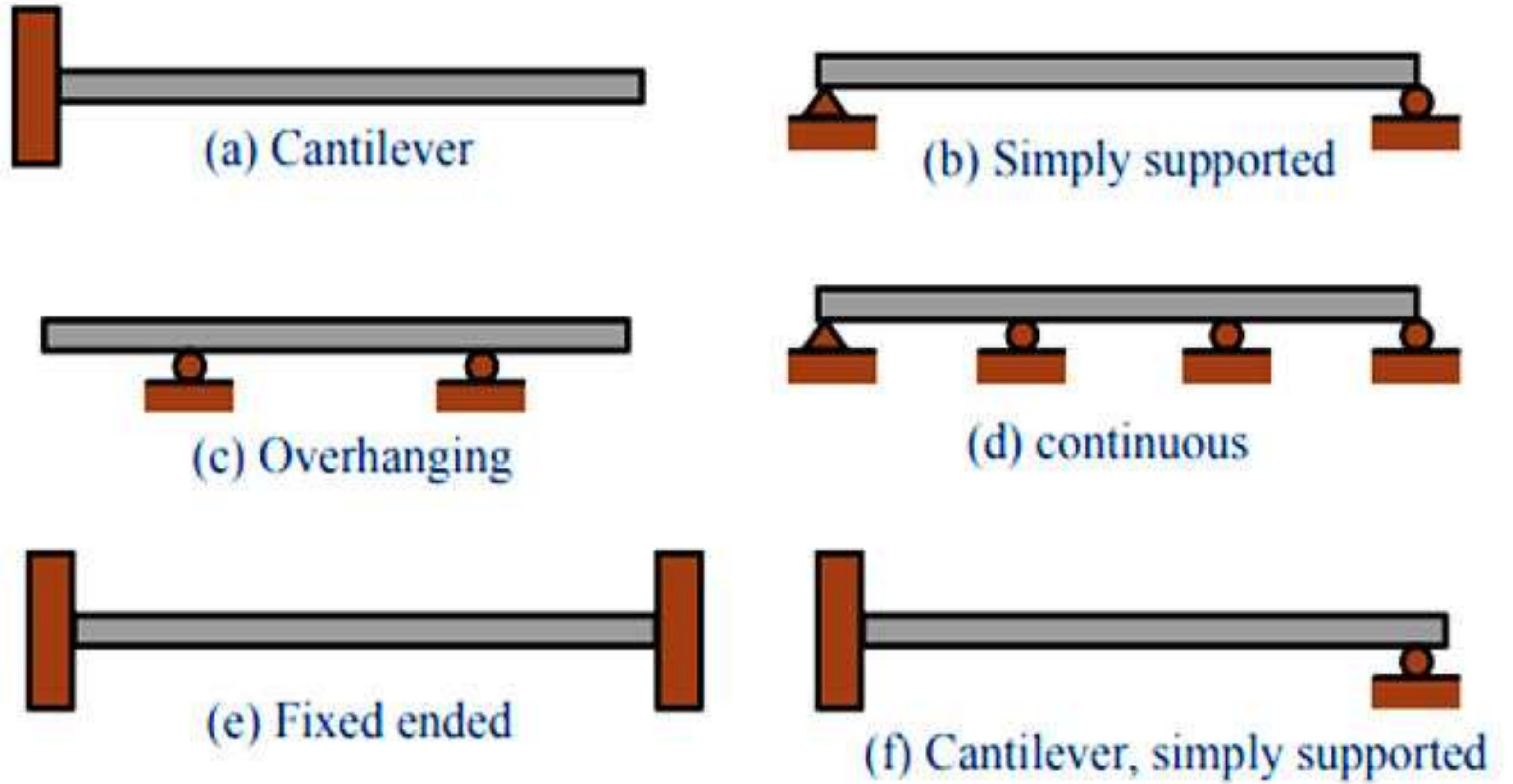
Different Types of Beams.

Beams are generally horizontal structural members which transfer loads horizontally along their length to the supports where the loads are usually resolved into vertical forces. Beams are used for resisting vertical loads, shear forces and bending moments

TYPES OF BEAMS

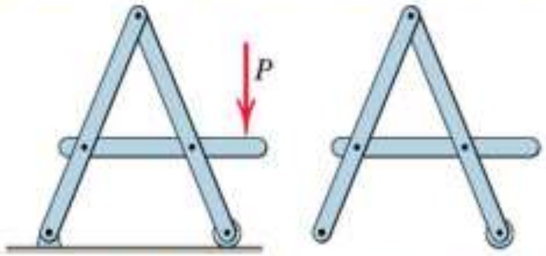


TYPES OF BEAMS

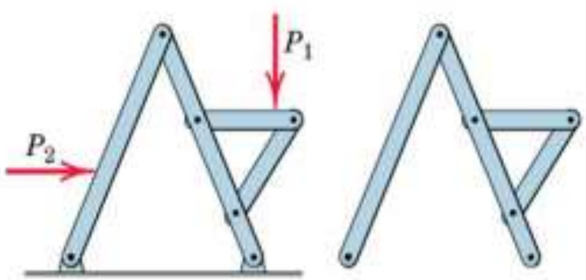


Interconnected Rigid Bodies with Multi-force Members

- Rigid Non-collapsible
  - –structure constitutes a rigid unit by itself when removed from its supports
  - –first find all forces external to the structure treated as a single rigid body
  - –then dismember the structure & consider equilibrium of each part

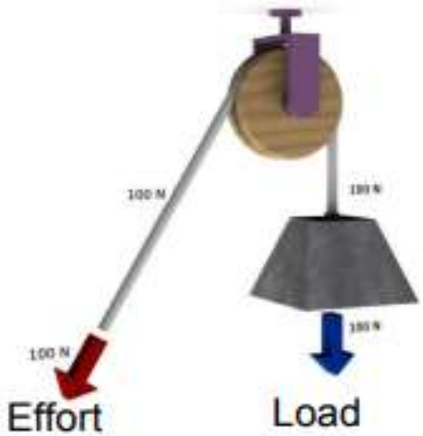


- Non-rigid Collapsible
  - –structure is not a rigid unit by itself but depends on its external supports for rigidity
  - –calculation of external support reactions cannot be completed until the structure is dismembered and individual parts are analysed.



RIGID FRAME

PULLEY SYSTEMS



Fixed Pulley

Effort = Load
→ Mechanical Advantage = 1

Distance moved by effort is equal to the distance moved by the load.
→ Velocity Ratio = 1

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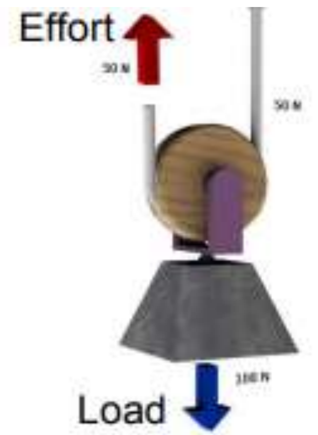
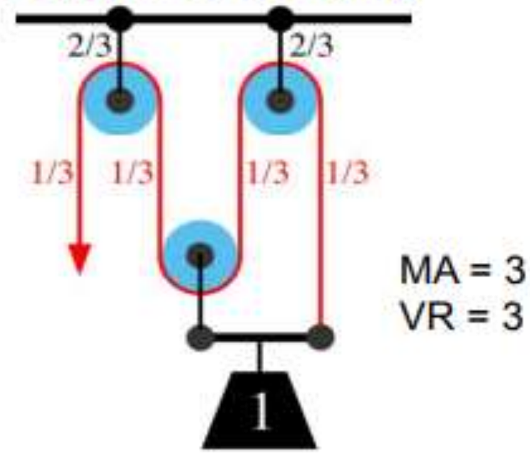


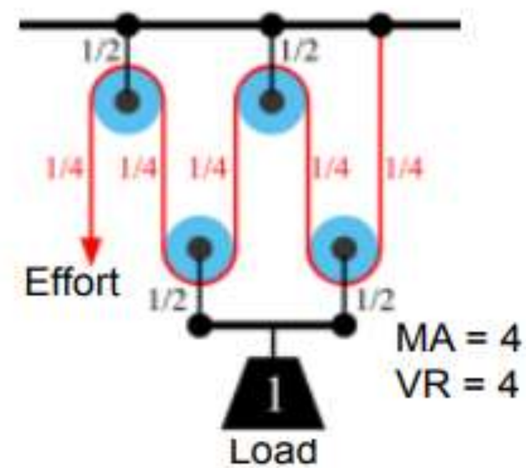
Movable Pulley

Effort = Load/2
→Mechanical Advantage = 2

Distance moved by effort is twice the distance moved by the load (both rope should also accommodate the same displacement by which the load is moved).
→ Velocity Ratio = 2

Compound Pulley

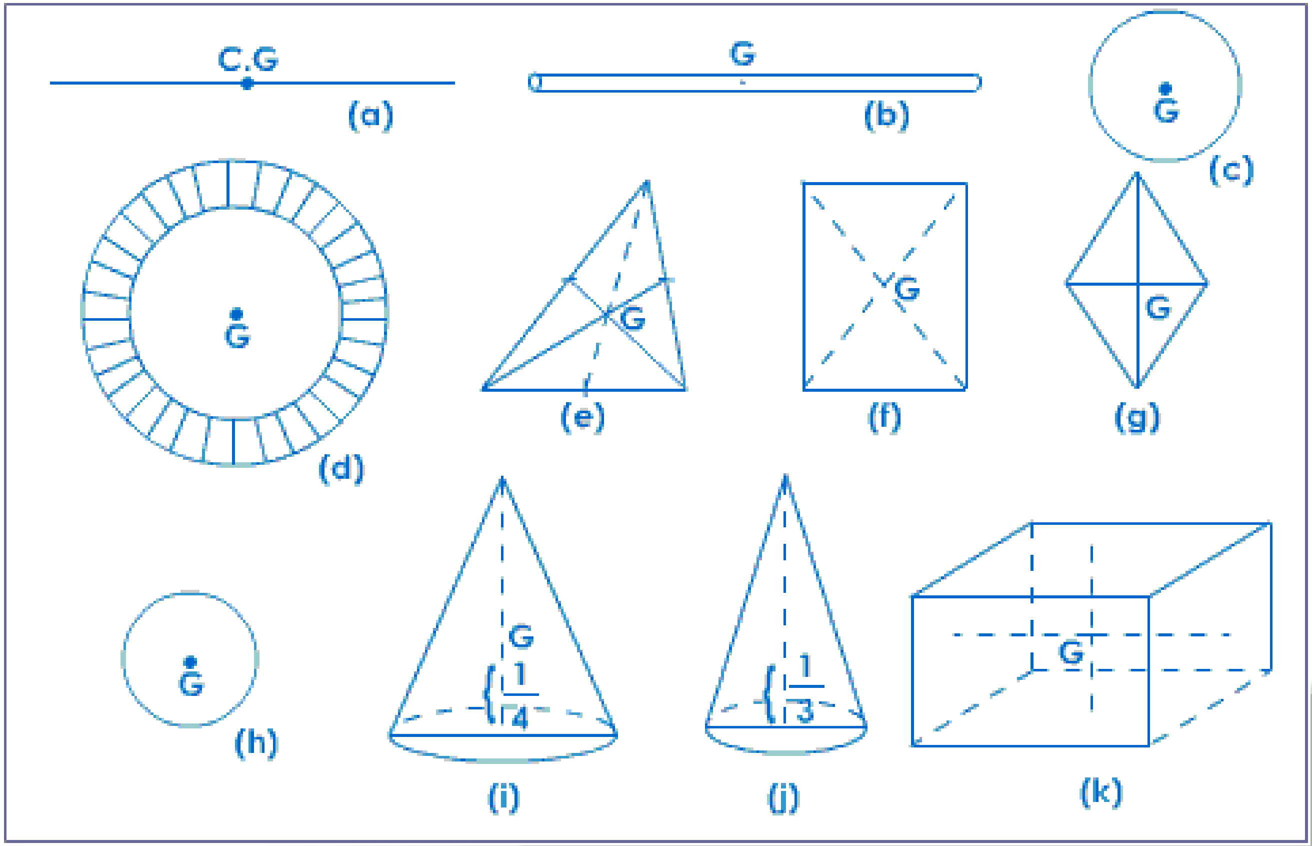




CENTROID AND CENTRE OF GRAVITY AND VIRTUAL WORK AND ENERGY METHOD

CENTROID OF SIMPLE FIGURE FROM FIRST PRINCIPLE

The centroid of an area is situated at its geometrical centre. In each of the following figures 'G' represents the centroid, and if each area was suspended from this point it would balance.

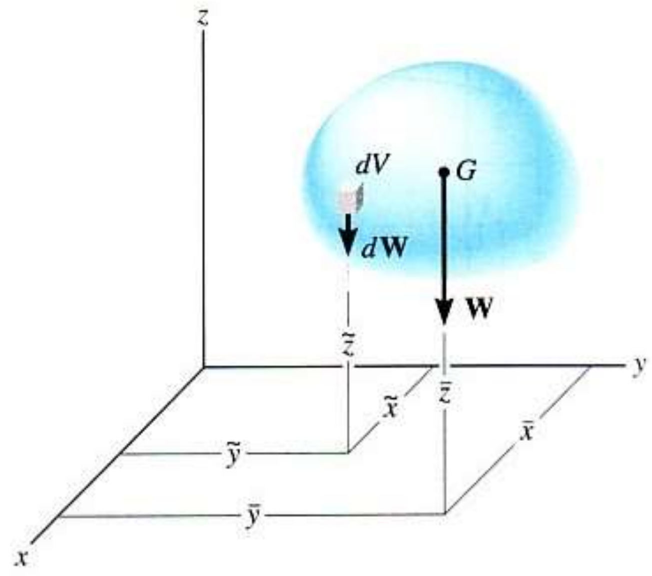


CENTER OF GRAVITY

The centre of gravity of a body is:

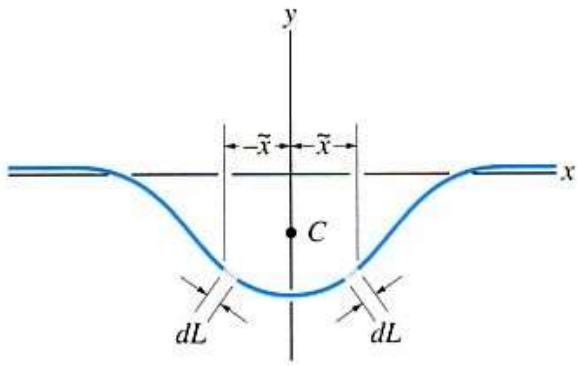
- **The point at which all the mass of the body may be assumed to be concentrated.**
- **The point through which the force of gravity is considered to act vertically downwards, with a force equal to the weight of the body.**
- **The point about which the body would balance.**
- **The centre of gravity of a homogeneous body is at its geometrical centre.**

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

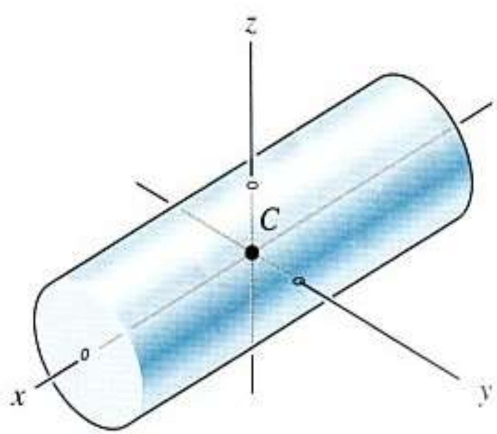


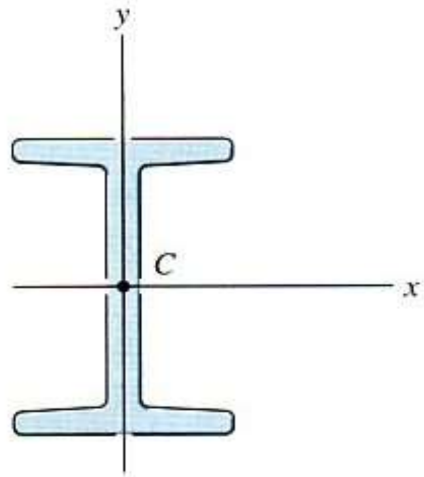
$\bar{x}, \bar{y}, \bar{z}$ represent the coordinates of the center of gravity G of the system of particles.

$\tilde{x}, \tilde{y}, \tilde{z}$ represent the coordinates of each particle in the system.



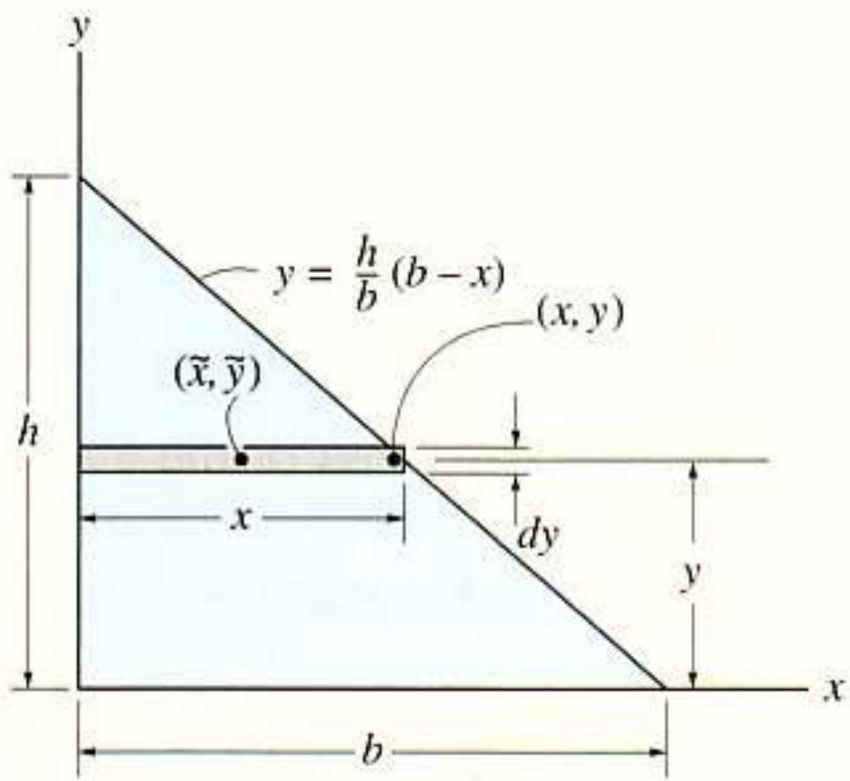
in cases where the shape has *an* axis of *symmetry*, the centroid of the shape will lie along the axis

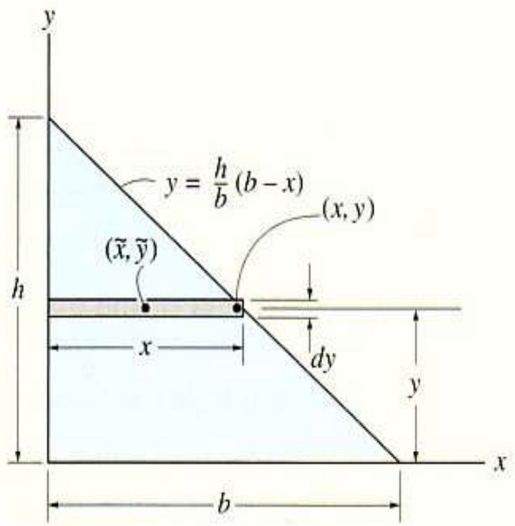




if the shape has *two* or *three* axes of *symmetry*,
the centroid lies at the intersection of these axes

Determine the distance $\overline{y}$ from the x axis to the centroid of the area of the triangle shown in Fig. 9–11.





Differential Element. Consider a rectangular element having thickness dy which intersects the boundary at (x, y), Fig. 9–11.

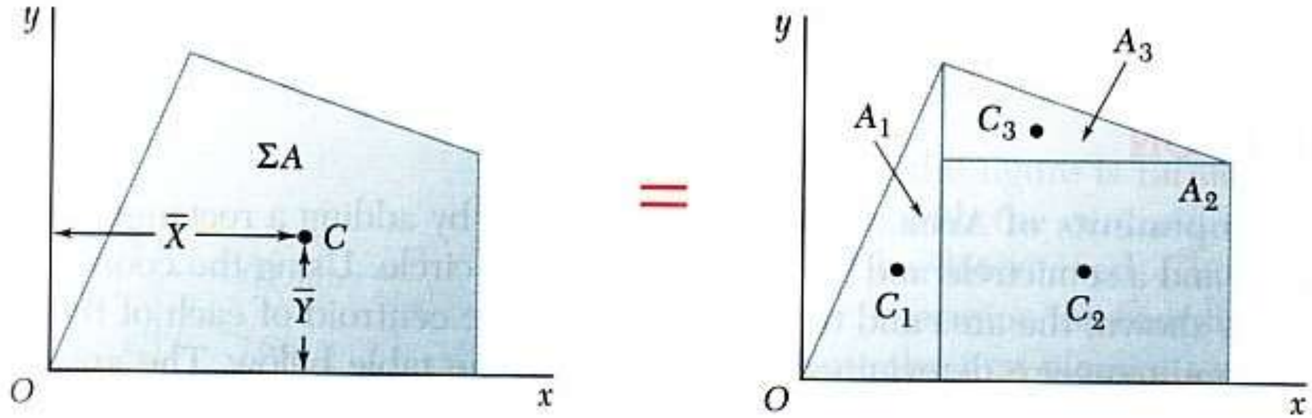
Area and Moment Arms. The area of the element is $dA = x\,dy = \frac{b}{h}(h - y)\,dy$, and its centroid is located a distance $\tilde{y} = y$ from the x axis.

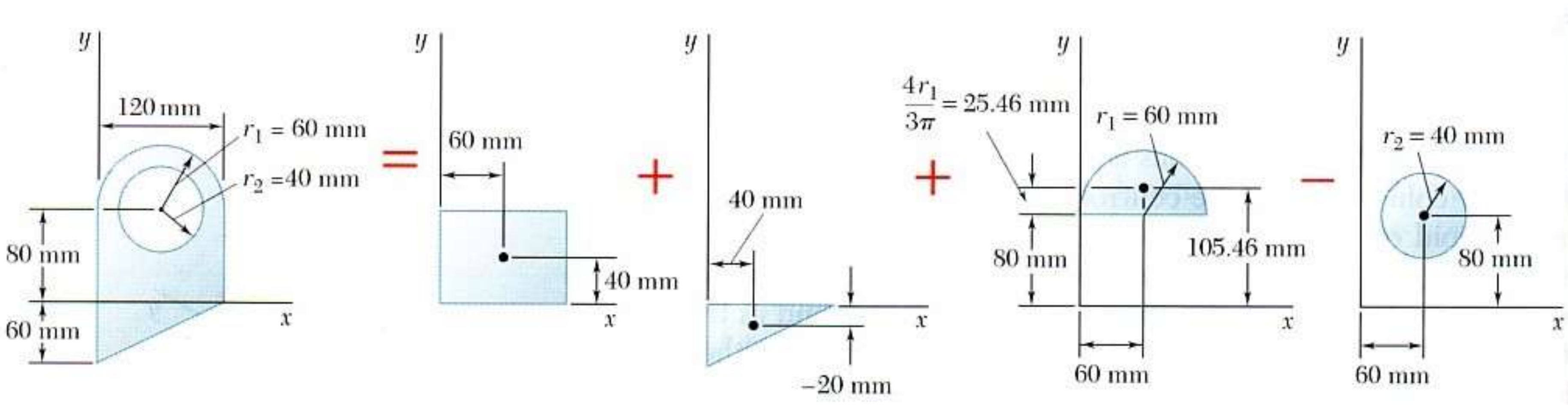
Integrations. Applying the second of Eqs. 9–6 and integrating with respect to y yields

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\frac{b}{h}(h - y)\,dy}{\int_0^h \frac{b}{h}(h - y)\,dy} = \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh}$$

$$= \frac{h}{3} \qquad \textit{Ans.}$$

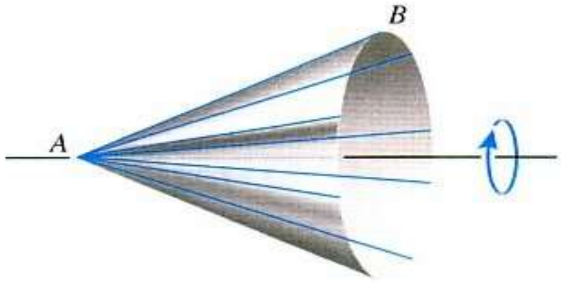
CENTROID OF COMPOSITE SECTIONS



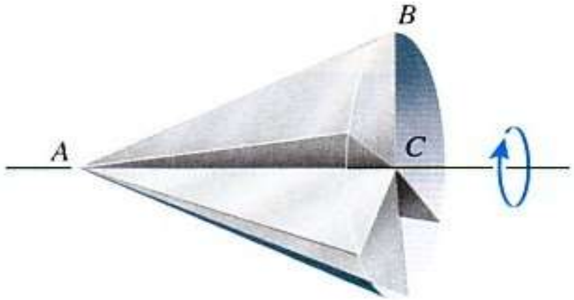


THEOREMS OF PAPPUS AND GULDINUS

surface area of revolution is generated by revolving a *plane curve* about a nonintersecting fixed axis in the plane of the curve

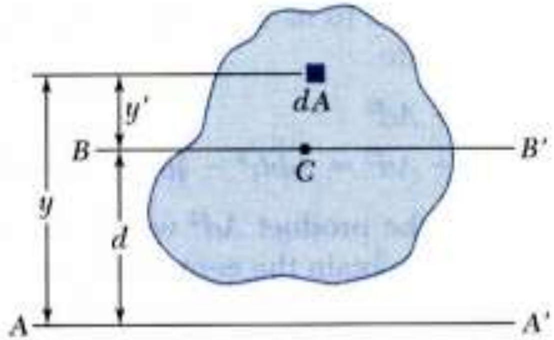


volume of revolution is generated by revolving a *plane area* about a nonintersecting fixed axis in the plane of the area



Area Moments of Inertia

Parallel Axis Theorem



- Consider moment of inertia *I* of an area *A* with respect to the axis *AA'*

$$I = \int y^2 dA$$

- The axis *BB'* passes through the area centroid and is called a *centroidal axis*.

$$I = \int y^2 dA = \int (y' + d)^2 dA$$
$$= \int y'^2 dA + 2d \int y' dA + d^2 \int dA$$

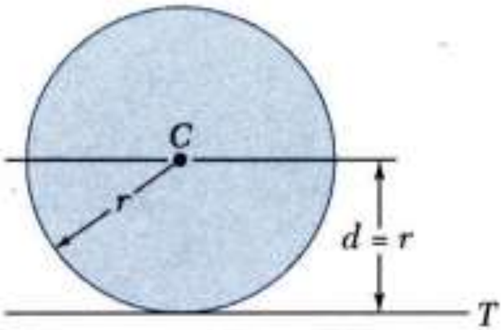
- Second term = 0 since centroid lies on BB' ($\int y' dA = y_c A$, and $y_c = 0$

$$I = \bar{I} + Ad^2 \quad \textit{Parallel Axis theorem}$$

Parallel Axis theorem:
MI @ any axis =
MI @ centroidal axis + Ad^2

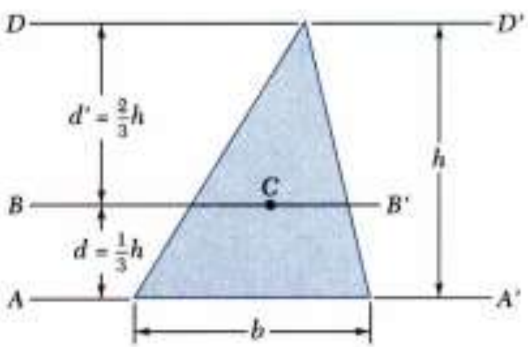
The two axes should be parallel to each other.

Parallel Axis Theorem



- Moment of inertia I_T of a circular area with respect to a tangent to the circle,

$$I_T = \bar{I} + Ad^2 = \tfrac{1}{4}\pi r^4 + \left(\pi r^2\right)r^2$$
$$= \tfrac{5}{4}\pi r^4$$

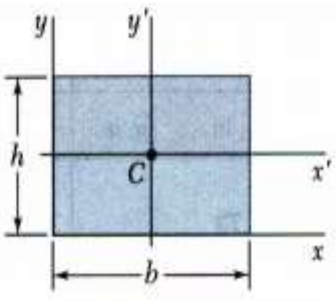


- Moment of inertia of a triangle with respect to a centroidal axis,

$$I_{AA'} = \bar{I}_{BB'} + Ad^2$$
$$I_{BB'} = I_{AA'} - Ad^2 = \tfrac{1}{12}bh^3 - \tfrac{1}{2}bh\left(\tfrac{1}{3}h\right)^2$$
$$= \tfrac{1}{36}bh^3$$

- The moment of inertia of a composite area A about a given axis is obtained by adding the moments of inertia of the component areas A_1, A_2, A_3, ... , with respect to the same axis.

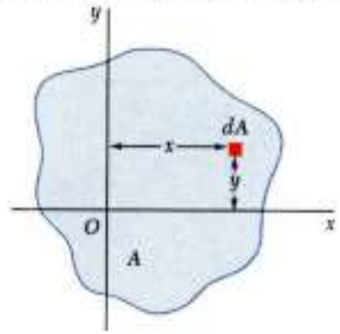
Area Moments of Inertia: Standard MIs



	Answer
Moment of inertia about x-axis	$I_x = \frac{1}{3}bh^3$
Moment of inertia about y-axis	$I_y = \frac{1}{3}b^3h$
Moment of inertia about x'-axis	$I_{x'} = \frac{1}{12}bh^3$
Moment of inertia about y'-axis	$I_{y'} = \frac{1}{12}b^3h$
Moment of inertia about z-axis passing through C	$I_C = \frac{1}{12}bh(b^2 + h^2)$

AREA MOMENT OF INERTIA

Products of Inertia: for problems involving unsymmetrical cross-sections and in calculation of MI about rotated axes. It may be +ve, -ve, or zero

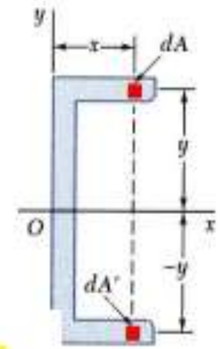


- Product of Inertia of area A w.r.t. x-y axes:

$$I_{xy} = \int xy\, dA$$

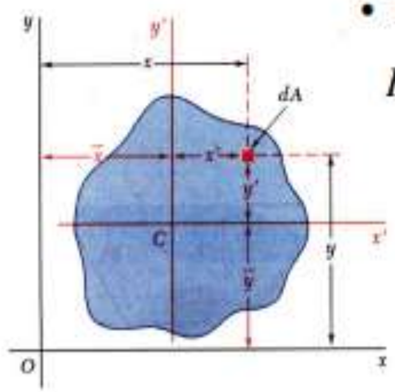
x and y are the coordinates of the element of area $dA = xy$

- When the x axis, the y axis, or both are an axis of symmetry, the product of inertia is zero.

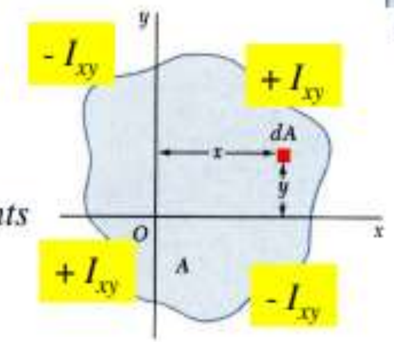


- Parallel axis theorem for products of inertia:

$$I_{xy} = \bar{I}_{xy} + \overline{x}\overline{y}A$$

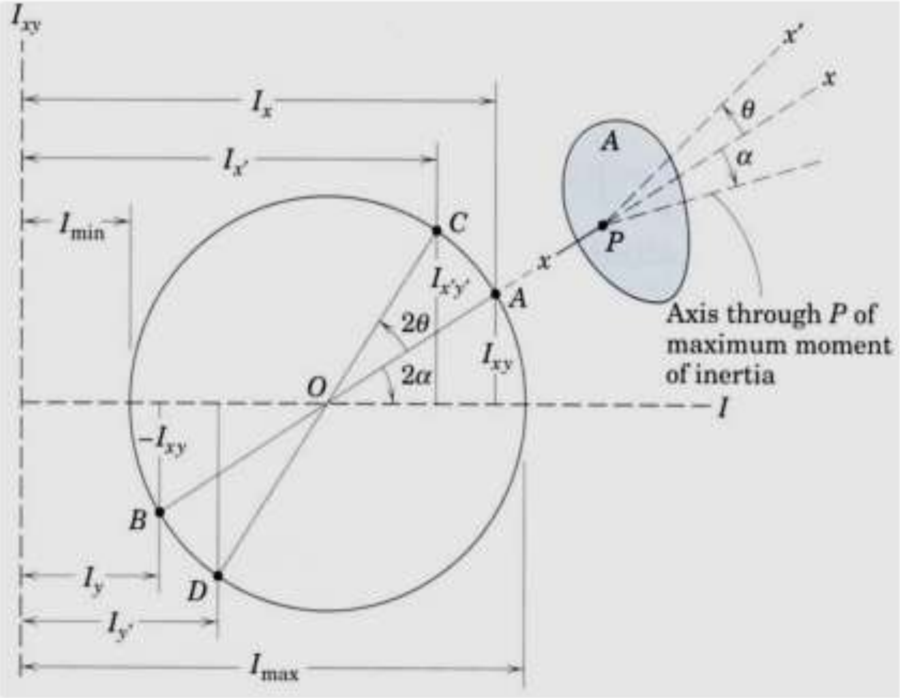


Quadrants



Area Moments of Inertia

Mohr's Circle of Inertia: **Construction**



$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{xy@\alpha} = 0$$

Choose horz axis → MI
Choose vert axis → PI
Point A – known $\{I_x, I_{xy}\}$
Point B – known $\{I_y, -I_{xy}\}$
Circle with dia AB
Angle α for Area
→ Angle 2α to horz (same sense) → I_{max}, I_{min}
Angle x to $x' = \theta$
→ Angle OA to OC = 2θ
→ Same sense
Point C → $I_{x'}$, $I_{x'y'}$
Point D → $I_{y'}$

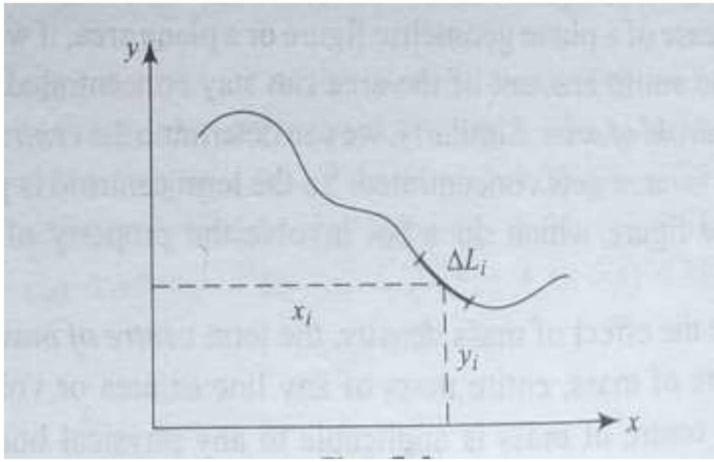
- Basic Definitions: The central point is defined as a point where the entire physical quantity (length/area/volume) can be assumed to be concentrated to give the same first moment of as that obtained by considering the elements of the body.

- The central points for a length, an area and a volume are called the centroids whereas the central points for the distribution of mass and gravitational force are termed as the centre of mass and centre of gravity respectively.

INTRODUCTION TO CENTROIDS & CENTRE OF GRAVITY

- **Basic Definitions:** The term centroid is purely related to any geometric shape which does not involve the property of mass or weight.
- **Centroid of Area:** It is a point where the entire area of a plane is supposed to concentrate.
- **Centroid of Volume:** It is a point where the entire volume of a plane is supposed to concentrate
- **Centre of Gravity**: It is a point where the entire weight of the body is assumed to concentrate.

- **Centroid of Line:**



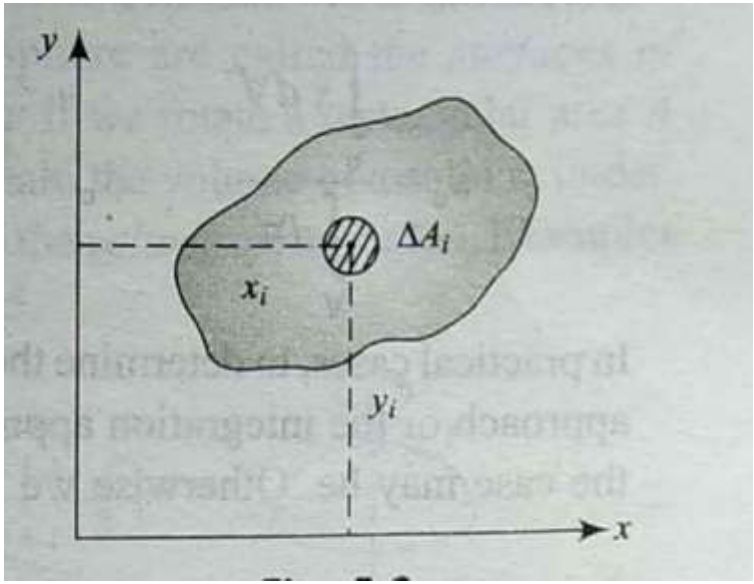
- $$x_c = \frac{\Sigma\, \Delta L_i\, x_i}{\Sigma\, \Delta L_i}$$

- $$y_c = \frac{\Sigma\, \Delta L_i\, y_i}{\Sigma\, \Delta L_i}$$

- **Considering the function as a continuous function, we can change discrete form to a continuous form as**

- $$x_c = \frac{\int x\, dL}{\int dL} \qquad y_c = \frac{\int y\, dL}{\int dL}$$

- **Centroid of Area:**



- $x_c = \dfrac{\Sigma\, \Delta A_i\, x_i}{\Sigma\, \Delta A_i}$

- $y_c = \dfrac{\Sigma\, \Delta A_i\, y_i}{\Sigma\, \Delta A_i}$

- **Considering the function as a continuous function, we can change discrete form to a continuous form as**

- $x_c = \dfrac{\int x\, dA}{\int dA}$ $\qquad y_c = \dfrac{\int y\, dA}{\int dA}$

INTRODUCTION TO CENTROIDS & CENTRE OF GRAVITY

- **Centroid of Volume:**

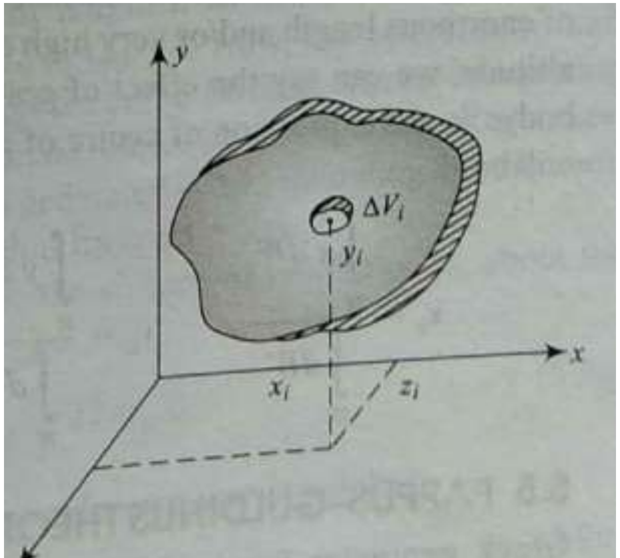


Fig. 5.3

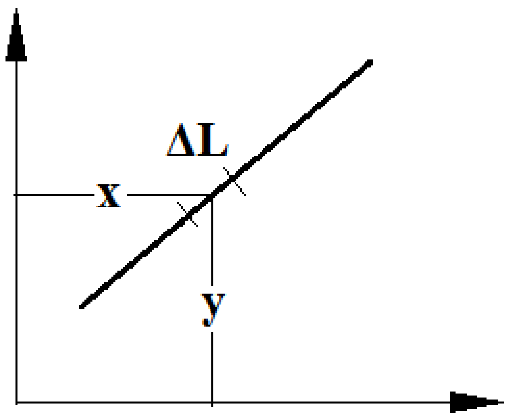
- $$x_c = \frac{\Sigma\, \Delta V_i\, x_i}{\Sigma\, \Delta V_i}$$

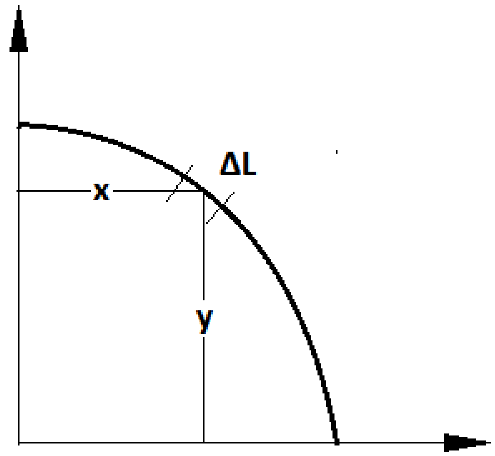
- $$y_c = \frac{\Sigma\, \Delta V_i\, y_i}{\Sigma\, \Delta V_i}$$

- **Considering the function as a continuous function, we can change discrete form to a continuous form as**

- $$x_c = \frac{\int x\, dV}{\int dV} \qquad y_c = \frac{\int y\, dV}{\int dV}$$

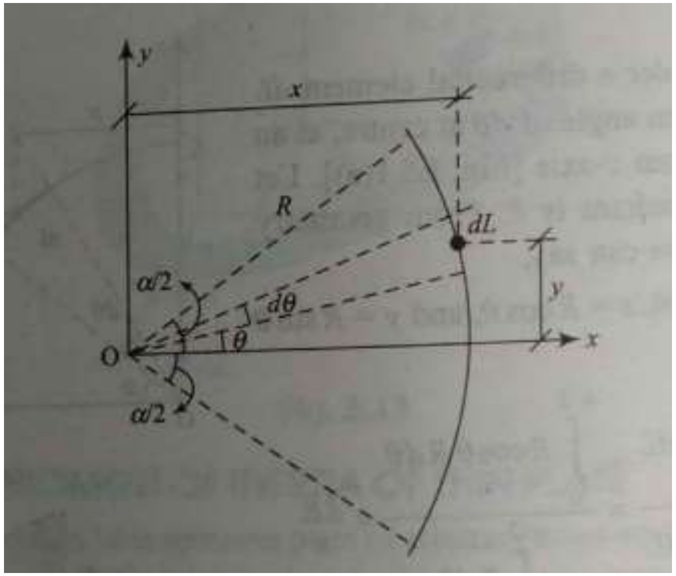
Problem 1: To locate the centroid of straight line.

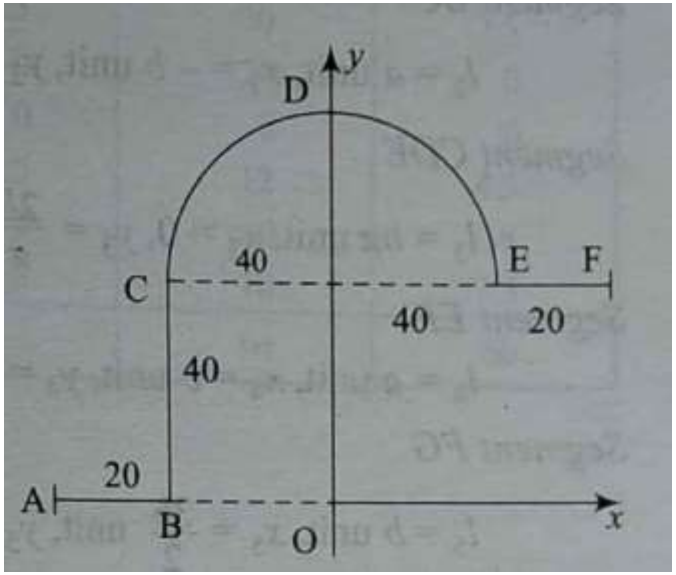




Problem 2: To locate the centroid of the curve AB bent in the shape of a quadrant.

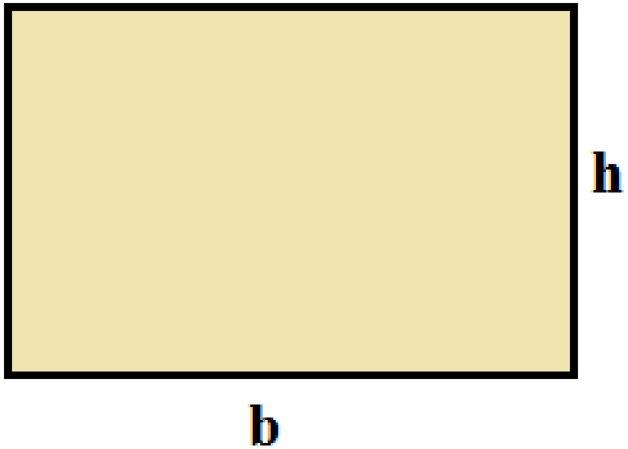
Problem 3: To locate the centroid of curve.

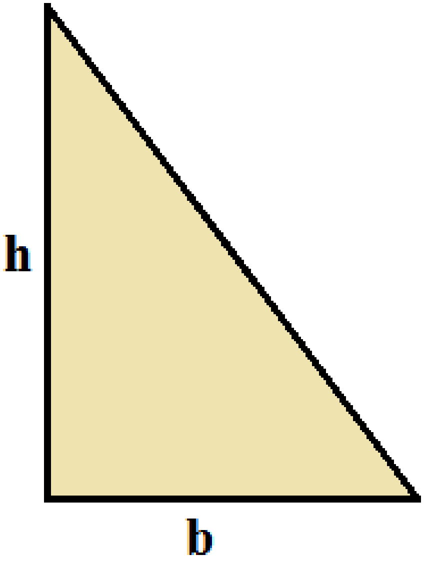




Problem 4: To locate the centroid of the curve

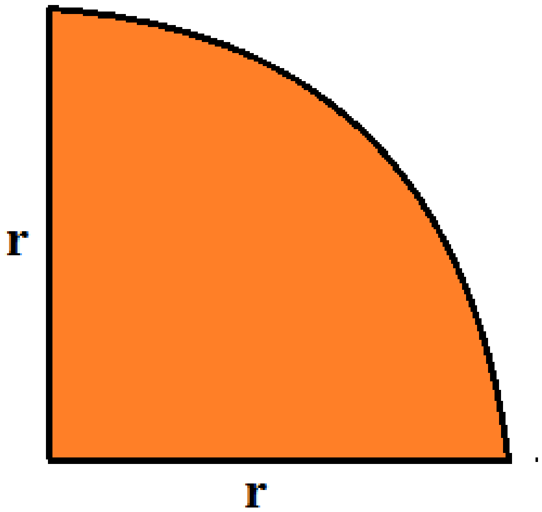
Problem 5: To locate the centroid of shaded area

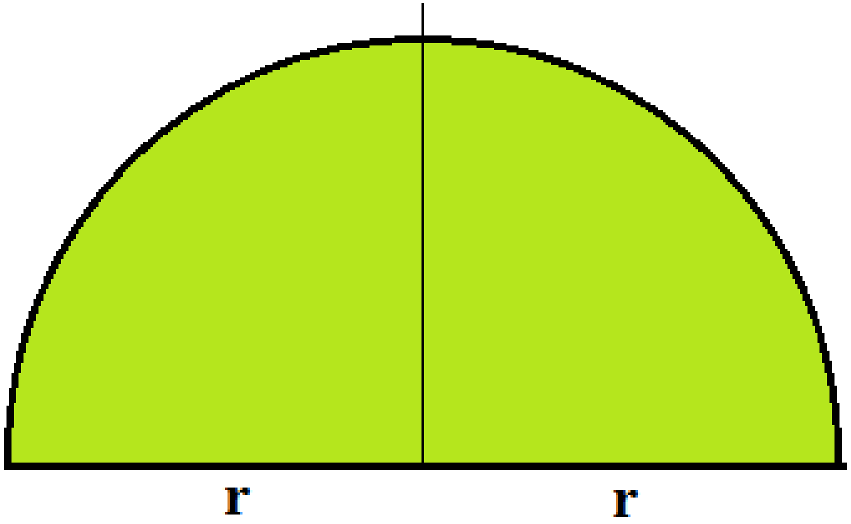




Problem 6: To locate the centroid of the shaded area

Problem 7: To locate the centroid of quadrant of a circle



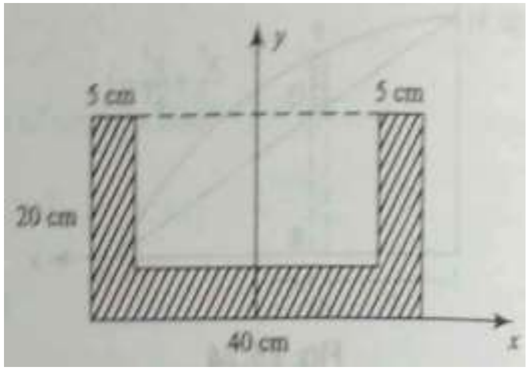


$x = \frac{4r}{3\pi}$ and $y = \frac{4r}{3\pi}$

$x = 0$ and $y = \frac{4r}{3\pi}$

Problem 8: To locate the centroid of the semi-circle.

Problem 1: To locate the centroid of shaded area

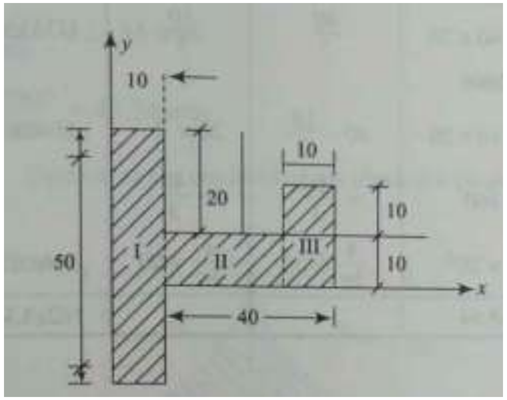


x = 0 (symmetrical about y-axis)

y = (20x5x10 + 20x5x10+30x5x2.5) /(20x5+20x5+30x5)

y = 6.786 cm

Problem 2: To locate the centroid of shaded area



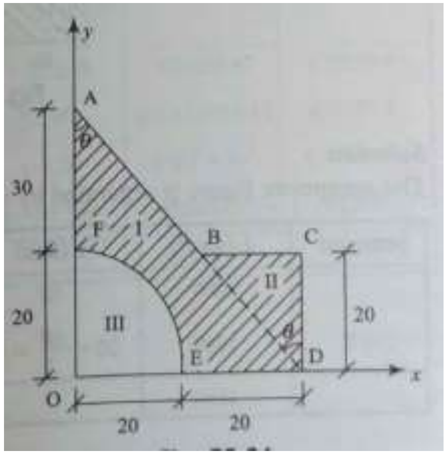
x = (50x10x5+30x10x25+20x10x45)/ (50x10+30x10+20x10)

x = 19 mm

y = (50x10x5+30x10x5+20x10x10)/ (50x10+30x10+20x10)

y = 6 mm

Problem 3: To locate the centroid of shaded area



AD=64.03MM

Θ = 38.66^{O}

BC =CD Tan Θ = 16mm

X_c =7.446 mm

Y_c = 7.446 mm

MOMENT OF INERTIA, POLAR MOMENT OF INERTIA AND RADIUS OF GYRATION

- It is a property of a shape of a plane geometric figure that is used to predict its resistance to bending and deflection.

- The concept of inertia is provided by Newton's First law of motion. The property of matter by virtue of which it resists ant change in its state of rest or of uniform motion is called inertia.

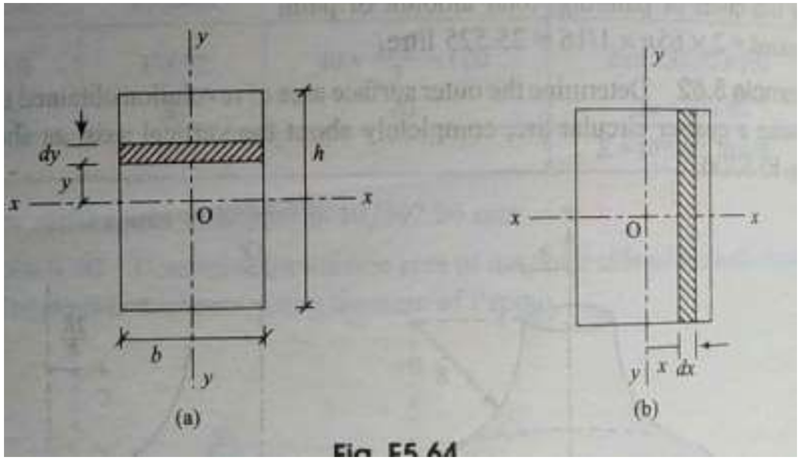
- The translatory inertia is identified as mass whereas the rotational inertia is termed as moment of inertia.

Moment of inertia, polar moment of inertia and radius of gyration

- In other words moment of inertia is the rotational analogue of mass, i.e., it plays the role of resisting a change in rotational motion in quite the same sense as mass plays in resisting a change in translatory motion.

- Moment of inertia is also defined as the second moment of area.

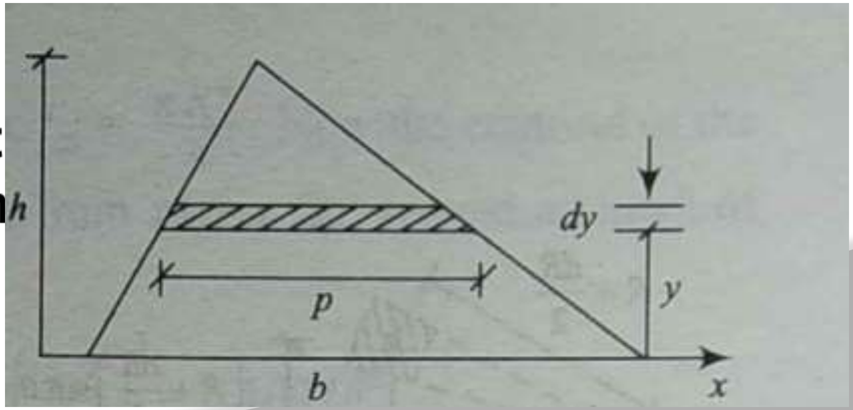
- **Centroid of Line:**



- $$x_c = \frac{\Sigma\, \Delta L_i\, x_i}{\Sigma\, \Delta L_i}$$

- $$y_c = \frac{\Sigma\, \Delta L_i\, y_i}{\Sigma\, \Delta L_i}$$

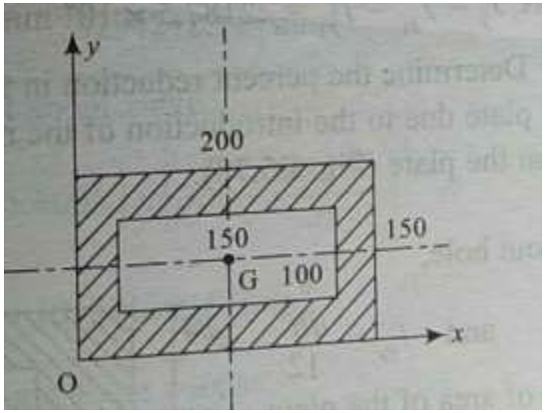
- **Considering the function as a cont discrete form to a continuous form**



$\int x\, dL$

Moment of inertia, polar moment of inertia and radius of gyration

- **Centroid of Volume:**



- $x_c = \dfrac{\Sigma\, \Delta V_i\, x_i}{\Sigma\, \Delta V_i}$

- $y_c = \dfrac{\Sigma\, \Delta V_i\, y_i}{\Sigma\, \Delta V_i}$

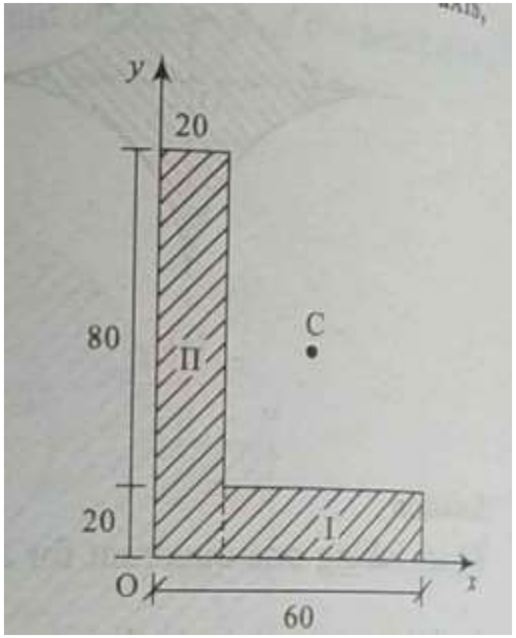
- **Considering the function as a continuous function, we can change discrete form to a continuous form as**

- $x_c = \dfrac{\int x\, dV}{\int dV}$ $\qquad$ $y_c = \dfrac{\int y\, dV}{\int dV}$

- Moment of Inertia of Composite Planes

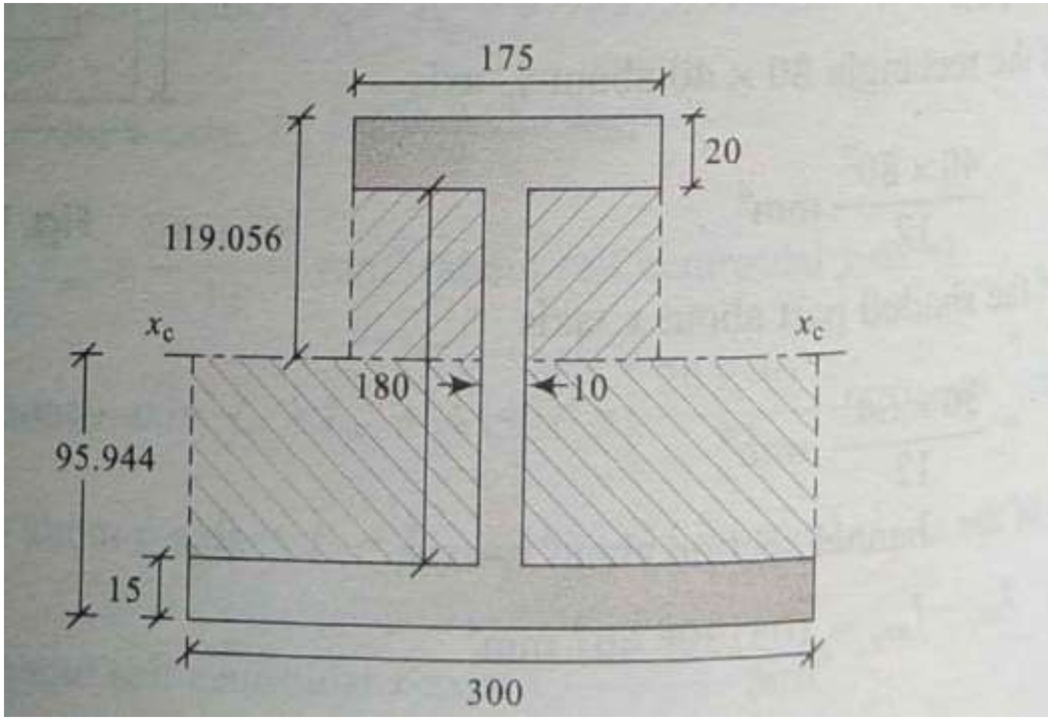
$I_x = 6773333.3 mm^4$

$I_y = 1653333.3\ mm^4$



Moment of inertia, polar moment of inertia and radius of gyration

- Moment of Inertia of I-section

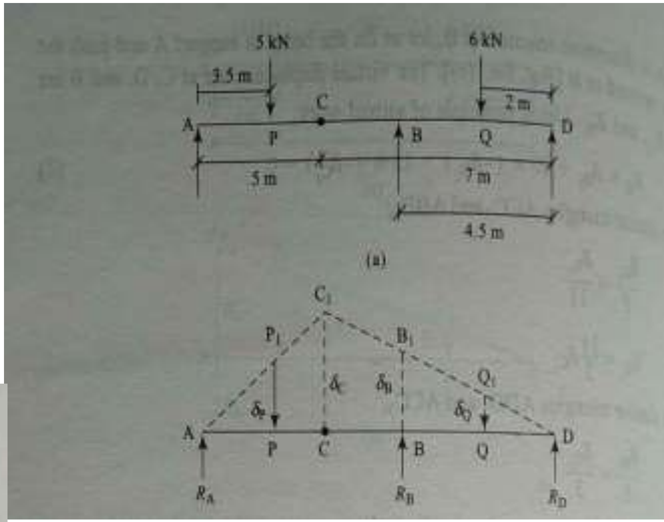


Principle of Virtual Work

The principle of virtual work states that for a system of initially stationary rigid bodies, the algebraic summation of virtual work done by all effective forces causing virtual displacement consistent with geometrical conditions, will be zero.

Introduction to concept of Virtual work. Principle of virtual wo

Problem: Using the method of virtual work, determine the reactions at the supports A and B of the transversely loaded beam as shown.



Putting the value of δ_C from Eq. (1), we get,

$$\delta_P = \frac{3.5}{5} \times \frac{7}{4.5}\delta_B$$

As the unknown is at point B, all the virtual displacements are expressed in terms of δ_B. Now using principle of virtual work, we obtain,

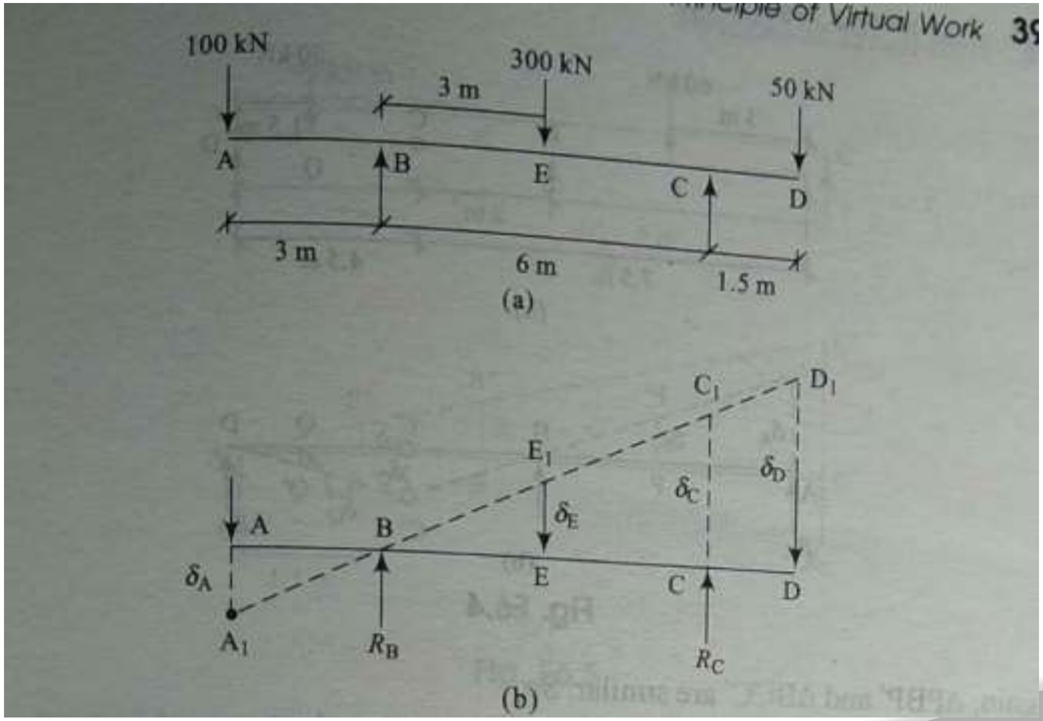
$$R_A \times 0 - 5 \times \delta_P + R_B \times \delta_B - 6 \times \delta_Q + R_D \times 0 = 0$$

Hence, $R_B\delta_B - 5\left(\frac{3.5}{5} \times \frac{7}{4.5}\right)\delta_B - 6\left(\frac{2}{4.5}\right)\delta_B = 0$

or $\quad R_B = 8.12$ kN

Problem: A simply supported beam with overhang on both sides is loaded as shown in figure. Using the method of virtual work, determine the reactions at the supports A and B.

.



INTRODUCTION TO CONCEPT OF VIRTUAL WORK. PRINCIPLE OF VIRTUAL WORK

We first determine the reaction at C. Keeping support B fixed, push support C upward, and virtual displacements at A, C, D, E are δ_A, δ_C, δ_D, and δ_E [Fig. E6.3(b)]. Here ΔBEE_1, ΔBCC_1, and ΔBDD_1, are similar. Thus,

$$\frac{\delta_E}{3} = \frac{\delta_C}{6} = \frac{\delta_D}{7.5}$$

or $\quad \delta_E = \frac{\delta_C}{2} \quad$ and $\quad \delta_D = \frac{7.5}{6}\delta_C$

Again ΔBAA_1 and ΔBEE_1 are similar. So,

$$\frac{\delta_A}{3} = \frac{\delta_E}{3}$$

Hence, $\delta_A = \delta_E = \frac{\delta_C}{2}$

Now, from the principle of virtual work, we can write,

$$100\delta_A - 300\delta_E + R_C\delta_C - 50\delta_D = 0$$

or $$100\frac{\delta_C}{2} - 300\frac{\delta_C}{2} + R_C\delta_C - 50\left(\frac{7.5}{6}\right)\delta_C = 0$$

Solving, we get

$$R_C = 162.5 \text{ kN}$$

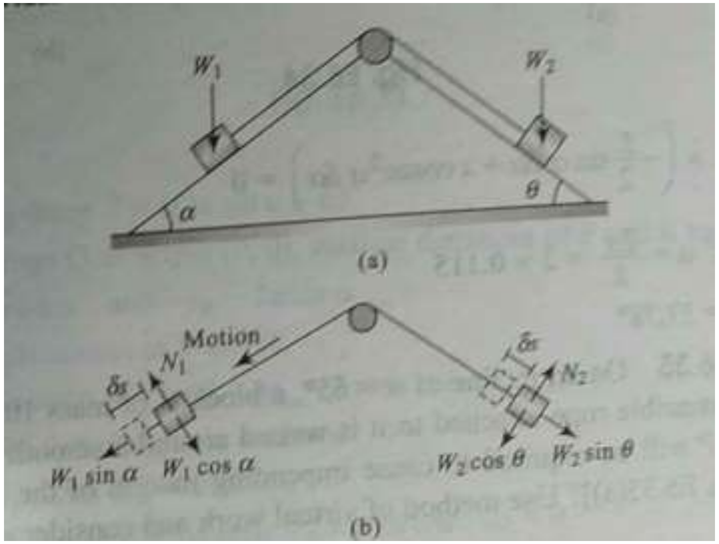
Using equation of equilibrium of vertical forces,

$$\overset{\oplus}{\uparrow}\sum F_y = 0: \quad R_B + R_C = 100 + 300 + 50$$

By solving,

$$R_B = 287.5 \text{ kN}$$

Two blocks are placed on two smooth inclined planes as shown in figure. The string connecting the two pulleys passing through a smooth pulley is inextensible. If W_1=75N and W_2=65N and α = 37°, Using the method of virtual work find θ.



In the pulley system shown in figure 1.35kN pull is applied. Considering the loss of work in friction as 17%, what should be the mass of the block. Use virtual work method.

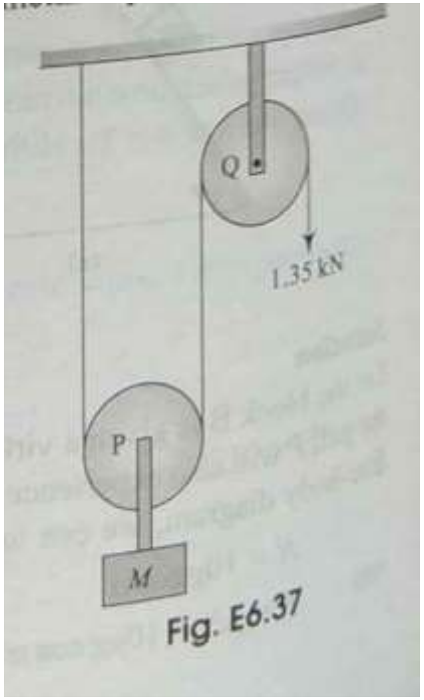


Fig. E6.37

PARTICLE DYNAMICS AND INTRODUCTION TO KINETICS

INTRODUCTION TO DYNAMICS

- Galileo and Newton (Galileo's experiments led to Newton's laws)
- Kinematics – study of motion
- Kinetics – the study of what causes changes in motion
- Dynamics is composed of kinematics and kinetics

The average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

The units of acceleration would be m/s^2, ft/s^2, etc.

The instantaneous acceleration is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad = \frac{dv}{dt} \quad = \frac{d}{dt}\frac{dx}{dt} \quad = \frac{d^2 x}{dt^2}$$

DETERMINATION OF THE MOTION OF A PARTICLE

$$\frac{dx}{dt} = v_0 + \int_0^t f(t)dt$$

$$\frac{dx}{dt} = v_0 + \int_0^t f(t)dt$$

$$dx = v_0 dt + \left[\int_0^t f(t)dt\right]dt$$

When independent particles move along the same line independent equations exist for each. Then one should use the same origin and time.

Relative motion of two particles

The relative position of B with respect to A

$$x_{B/A} = x_B - x_A$$

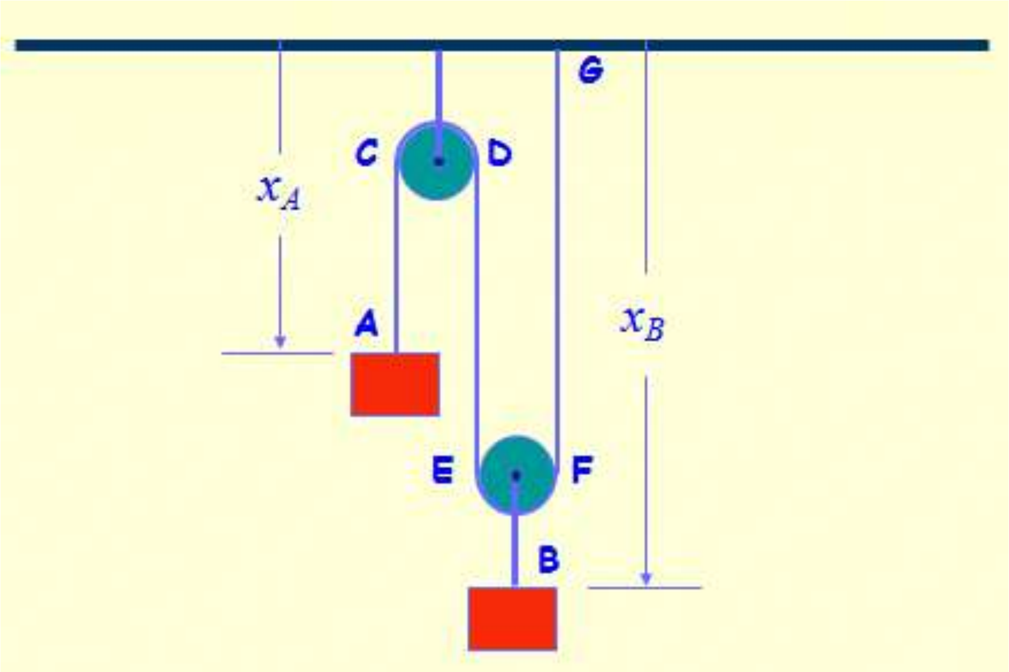
The relative velocity of B with respect to A

$$v_{B/A} = v_B - v_A$$

The relative acceleration of B with respect to A

$$a_{B/A} = a_B - a_A$$

DEPENDENT MOTIONS



Let's look at the relationships.

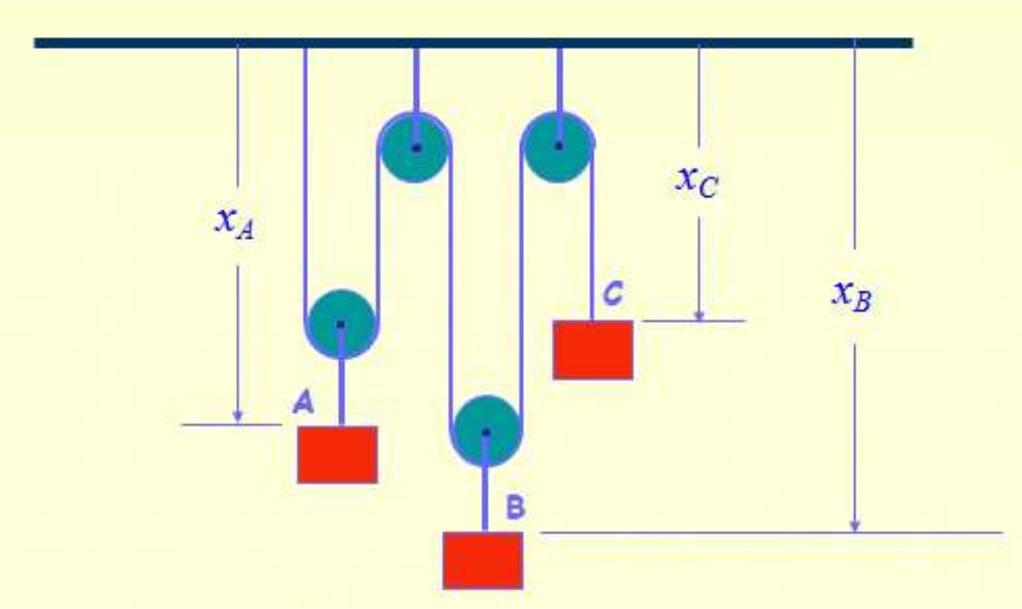
$$x_A + 2x_B = constant$$

$$v_A + 2v_B = 0$$

$$a_A + 2a_B = 0$$

System has one degree of freedom since only one coordinate can be chosen independently.

System has 2 degrees of freedom.



$$2x_A + 2x_B + x_C = constant$$

Let’s look at the relationships.

$$2v_A + 2v_B + v_C = 0$$

$$2a_A + 2a_B + a_C = 0$$

KINEMATICS OF A PARTICLE

- Kinematics is the area of mechanics concerned with the study of motion of particles and rigid bodies without consideration of what has caused the motion. When we take into consideration the factors causing the motion, the area of study is called dynamics.

- A particle has no size but has mass. This is a completely hypothetical concept. However, many times object can be approximated as particle. For example, a car is moving. Compared to distance, it travels, its size is very small and it can be treated as particle.

- Moreover, a rigid body can be considered as a combination of small particles. Thus, the concepts learned for a particle can be helpful in understanding the kinematics of rigid body.

Basic Definitions:

Dynamics : Dynamics is that branch of Engineering Mechanics which deals with the forces acting on the rigid bodies that are in motion.

Kinetics : Kinetics is the branch of classical mechanics that is concerned with the relationship between motion and its causes, specifically, forces and torques

Kinematics: Kinematics is that branch of classical mechanics which deals with motion of rigid bodies without considering the forces causing it.

Basic Definitions:

Motion: The continuous change in position of a body with respect to time and relative to the reference point or observer is called motion.

Rectilinear Motion : If the path followed by a point is a straight line then such motion is called rectilinear motion.

Curvilinear Motion: If the path followed by appoint is a curve then such motion is called a curvilinear motion

KINEMATICS OF A PARTICLE

Basic Definitions: Kinematics deals with displacement, velocity and acceleration of a point of interest at a particular time or with the passage of time.

At t_1 displacement S_1

At t_2 displacement S_2

During $(t_2 - t_1) = \Delta t$ displacement is $(S_2 - S_1) = \Delta S$

Displacement is dependent on time and thus $S = f(t)$.

This time dependant function may be of different types:

$S = c + dt$ ----(1)

At $t=0$, $S=c$ and rate of increment of variation is d which is constant throughout.

$S = c + dt^2$ ----(2)

Quadratic

$S = c + de^{-kt}$ ----(3)

Exponential or asymptotic relation of displacement with time.

For linear motion x marks the position of an object. Position units would be m, ft, etc.

Average velocity is,

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Velocity units would be in m/s, ft/s, etc.

The instantaneous velocity is,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

- The units of acceleration would be m/s^2, ft/s^2, etc.
- The instantaneous acceleration is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2}$$

Notice If *v* is a function of *x*, then

$$a = \frac{dv}{dt} \qquad = \frac{dv}{dx}\frac{dx}{dt} \qquad = v\frac{dv}{dx}$$

One more derivative

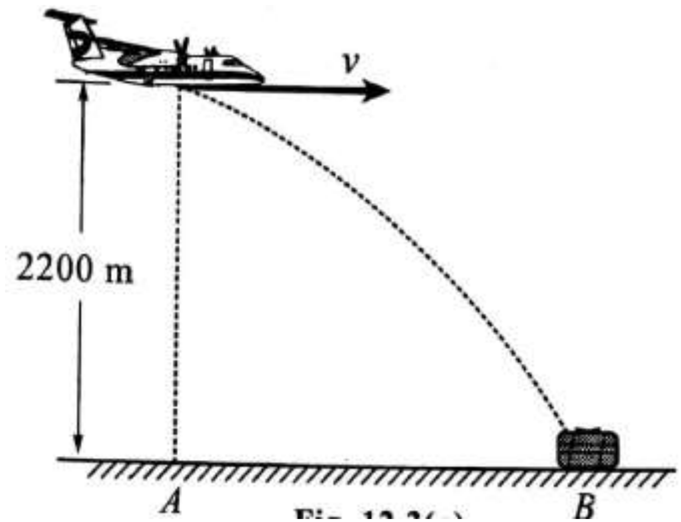
$$\frac{da}{dt} = \quad Jerk$$

PROBLEMS-RECTILINEAR MOTION

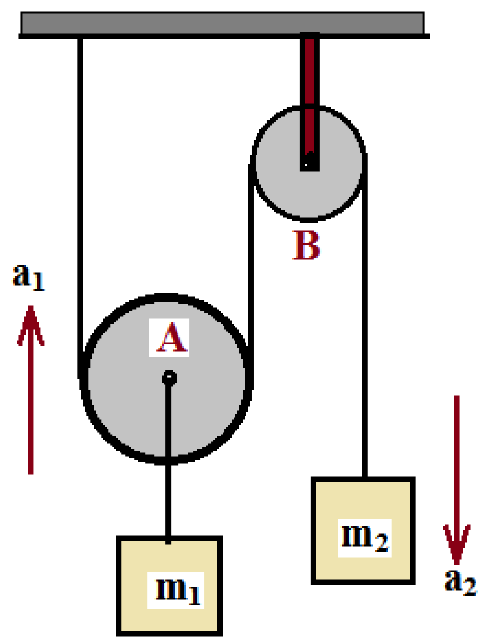
- The rectilinear motion of a particle is defined by the displacement time equation as $x=x_0+v_0t+(1/2)at^2$. Find the displacement and velocity at time t=2s while x_0=250mm, v_0=125mm/s and a=0.5mm/s^2.

- A particle starts from rest and moves along a straight line with constant acceleration a. If it acquires a velocity v=3 mm/s^2, after having travelled a distance S=7.5m, find the magnitude of the acceleration

- An aeroplane is flying in horizontal direction of 540 km/hr and at a height of 2200m as shown in figure. When it is vertically above the point *A* on the ground, a body is dropped from it. The body strike the ground at point *B*. Calculate the distance *AB* ignoring air resistance. Also find velocity at *B* and time taken to reach B



- Two masses are inter-connected with a pulley system, as shown in figure. Neglecting inertial and frictional effect of pulleys and cord, determine the acceleration of the mass m_2. Take m_1 = 40kg, m_2 = 30kg.



KINETICS OF A PARTICLE:

Method of work and energy

Involves relations between displacement, velocity, mass, and force.

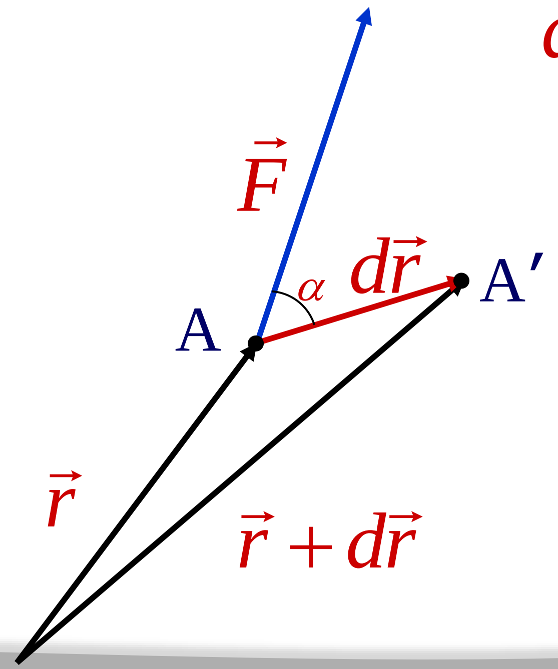
Method of impulse and momentum

Involves relations between mass, velocity, force, and time.

WORK OF A FORCE :

When a force is applied to a mass and the mass moves through an incremental distance, the work done by the force is

$$dU = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$



$$dU = \vec{F} \cdot d\vec{r} = F(\cos\alpha)ds$$

$$dU > 0 \quad if \quad \alpha < 90^0$$

$$dU = 0 \quad if \quad \alpha = 90^0$$

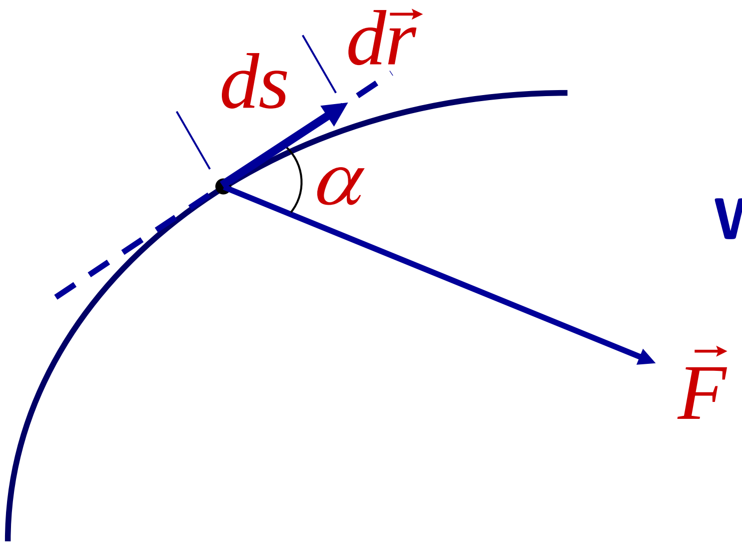
$$dU < 0 \quad if \quad \alpha > 90^0$$

$$Units \; - \; ft \cdot lb \;\; or \;\; N \cdot m = Joule\,(J)$$

To get the total work done along a path requires

$$U_{1\to 2} = \int_1^2 \vec{F} \cdot d\vec{r}$$

Notice that



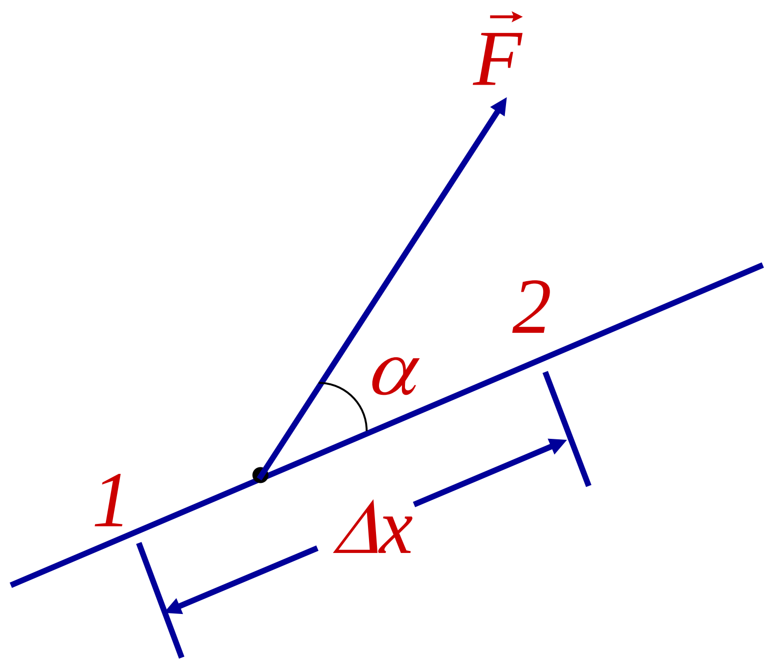
$$U_{1\to 2} = \int_1^2 F(\cos\alpha)ds = \int_1^2 F_t ds$$

When using rectangular coordinates

$$U_{1\to 2} = \int_1^2 (F_x dx + F_y dy + F_z dz)$$

Work of a Constant Force in Rectilinear Motion

$$U_{1\to 2} = F(\cos\alpha)\Delta x = F(\cos\alpha)(x_2 - x_1)$$



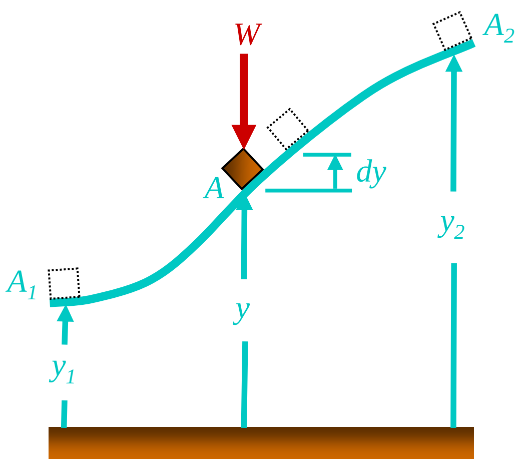
Work of the Force of Gravity

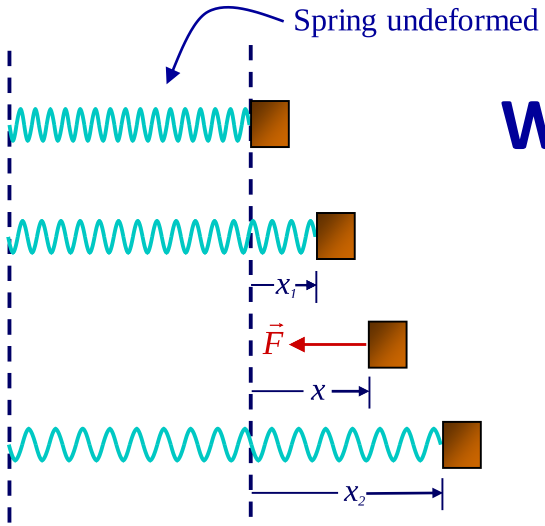
$$dU = -W\hat{j} \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$dU = -W dy$$

$$U_{1\rightarrow 2} = -\int_{1}^{2} W dy = W(y_1 - y_2)$$

$$U_{1\rightarrow 2} = -W\Delta y$$



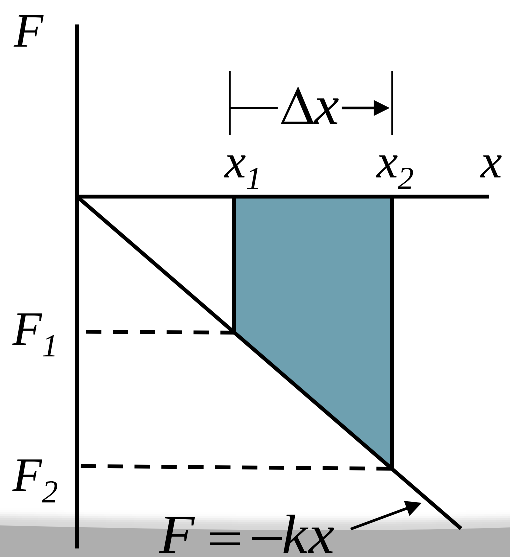


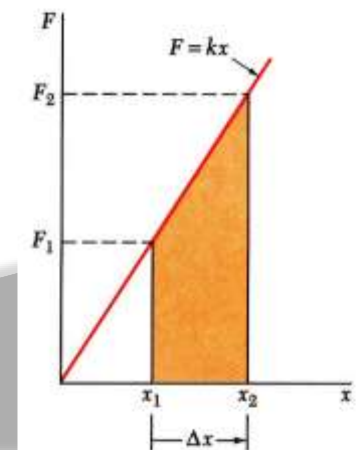
Work of the Force Exerted by a Spring

$$F = -kx$$

$$U_{1\to2} = -\int_1^2 kx\,dx = -\left(\tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2\right)$$

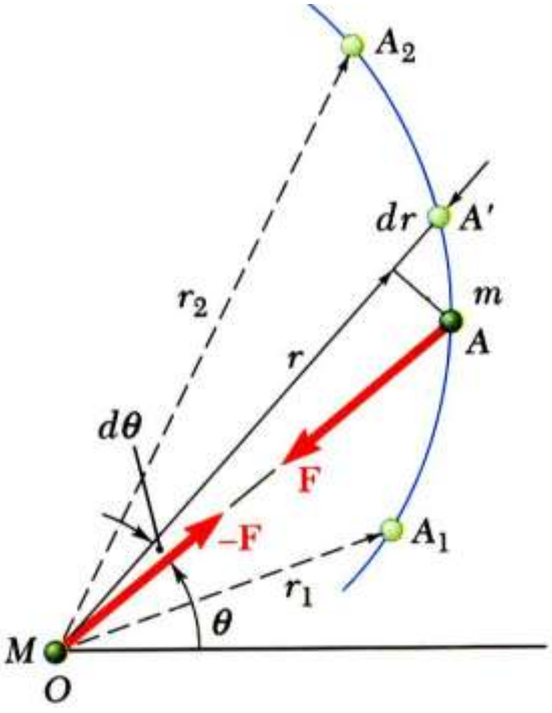
$$= -\tfrac{1}{2}k(x_2 + x_1)(x_2 - x_1)$$





Work of a Gravitational Force

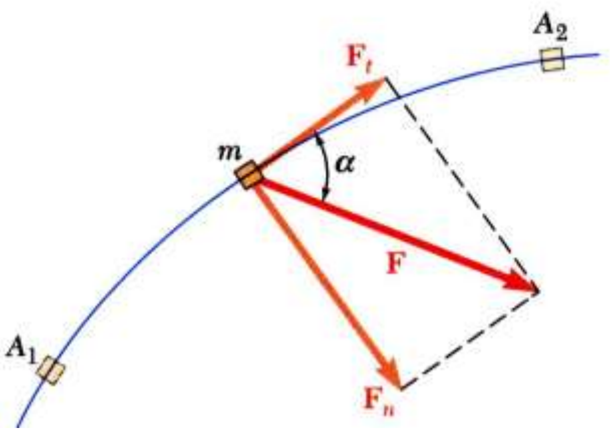
$$U_{1\to 2} = -\int_1^2 \frac{GMm}{r^2}\,dr = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$



WORK OF A GRAVITATIONAL FORCE

- Forces which *do not* do work (*ds* = 0 or cos α = 0):
- Reaction at frictionless pin supporting rotating body
- Reaction at frictionless surface when body in contact moves along surface.
- Reaction at a roller moving along its track, and
- Weight of a body when its center of gravity moves horizontally.

If the force doing work is the net force then



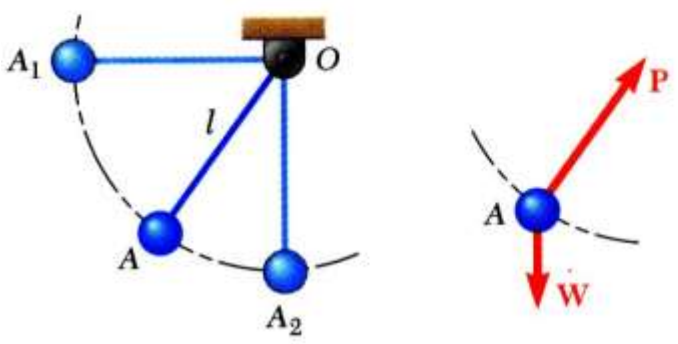
$$F_t = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$U_{1\to2} = \int_1^2 F_t ds = \int_1^2 mv\frac{dv}{ds}ds = \int_1^2 mv\,dv$$

$$U_{1\to2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$T = \tfrac{1}{2}mv^2 \qquad U_{1\to2} = T_2 - T_1 \qquad T_1 + U_{1\to2} = T_2$$

APPLICATIONS OF THE PRINCIPLE OF WORK AND ENERGY

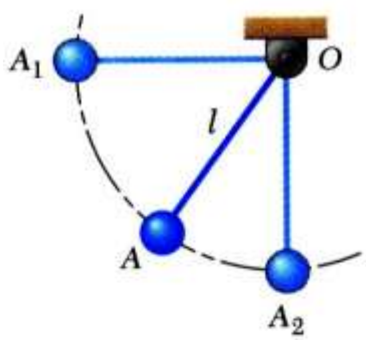


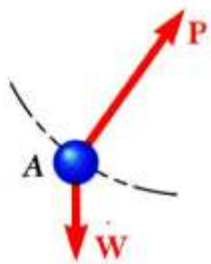
- **Wish to determine velocity of pendulum bob at A_2. Consider work & kinetic energy.**

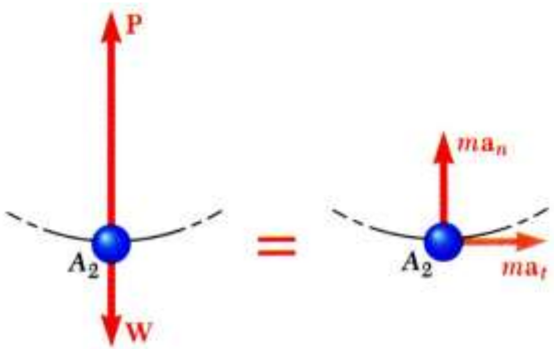
$$T_1 + U_{1\to 2} = T_2$$
$$0 + Wl = \frac{1}{2}\frac{W}{g}v_2^2$$
$$v_2 = \sqrt{2gl}$$

- Velocity found without determining expression for acceleration and integrating.
- All quantities are scalars and can be added directly.
- Forces which do no work are eliminated from the problem.

KINETIC ENERGY OF A PARTICLE, PRINCIPLE OF WORK AND ENERGY







$$v_2 = \sqrt{2gl}$$

- Principle of work and energy cannot be applied to directly determine the acceleration of the pendulum bob.
- Calculating the tension in the cord requires supplementing the method of work and energy with an application of Newton’s second law.
- As the bob passes through A_2

$$\sum F_n = m a_n$$

$$P - W = \frac{W}{g}\frac{v_2^2}{l}$$

$$P = W + \frac{W}{g}\frac{2gl}{l} = 3W$$

POWER AND EFFICIENCY

Power is the rate at which work is done.

$$P = \frac{dU}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

Units – 1 hp = 550 ft lb/s and 1 Watt = 1 J/s

Efficiency

$$\eta = \frac{\text{work out}}{\text{work in}} = \frac{\text{power out}}{\text{power in}}$$

POTENTIAL ENERGY

Close to the Earth

$$U_{1\to 2} = -\int_{1}^{2} W dy = W(y_1 - y_2)$$

Define $$V_g = Wy$$

Then $$U_{1\to 2} = (V_g)_1 - (V_g)_2$$

$$U_{1\to 2} = -\Delta V_g$$

Not So Close to the Earth

$$U_{1\to2} = -\int_1^2 \frac{GMm}{r^2}\,dr \qquad = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

Define $$V_g = -\frac{GMm}{r} \qquad = -\frac{WR^2}{r}$$

Then $$U_{1\to2} = (V_g)_1 - (V_g)_2$$

$$U_{1\to2} = -\Delta V_g$$

PRINCIPLE OF IMPULSE AND MOMENTUM

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

$$m\vec{v}_2 - m\vec{v}_1 = \int_{t_1}^{t_2} \vec{F}dt \quad = \boldsymbol{Imp}_{1\to 2}$$

$$m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}dt = m\vec{v}_2$$

- For a system of particles external impulses are considered only (remember Newton's third law)
- If no external forces act on the particle, then

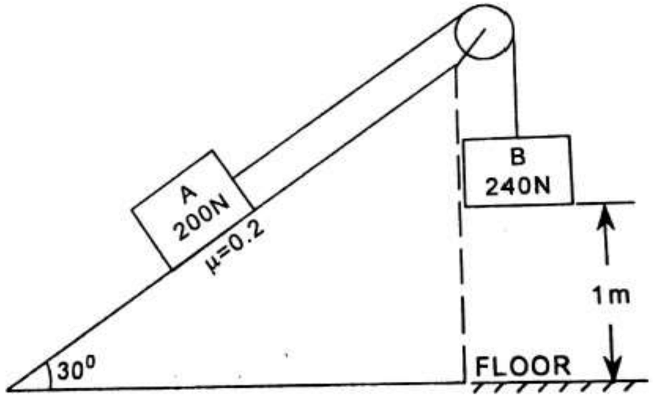
$$\Sigma m\vec{v}_2 = \Sigma m\vec{v}_1$$

PROBLEM

- Two blocks of A (200N) and B (240N) are connected as shown in figure given below. When the motion begins, the block B is 1m above the floor. Assuming the pulley to be frictionless and weightless, determine

(i) The velocity of block A when the block B touches the floor

(ii) How far the block A will move up the plane?



MECHANICAL VIBRATIONS

- Simple Harmonic Motion
- A pendulum, a mass on a spring, and many other kinds of oscillators exhibit a special kind of oscillatory motion called Simple Harmonic Motion (SHM).

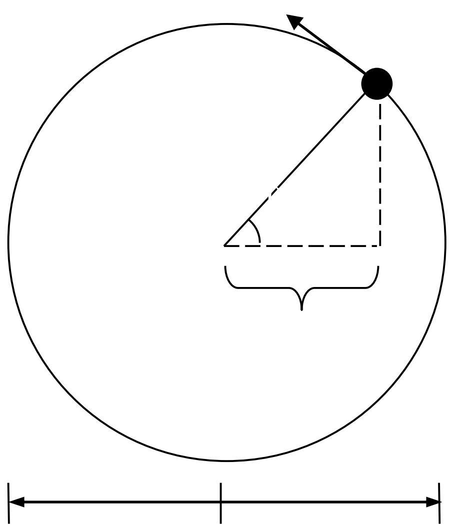
SHM occurs whenever :

- There is a restoring force proportional to the displacement from equilibrium: $F \propto -x$
- The potential energy is proportional to the square of the displacement: $PE \propto x^2$
- The period T or frequency f = 1 / T is independent of the amplitude of the motion.
- The position *x*, the velocity *v*, and the acceleration *a* are all sinusoidal in time.

SHM and circular motion

- There is an exact analogy between SHM and *circular motion*. Consider a particle moving with constant speed v around the rim of a circle of radius A.
- The x-component of the position of the particle has *exactly* the same mathematical form as the motion of a mass on a spring executing SHM with amplitude A.

- This same formula also describes the *sinusoidal* motion of a mass on a spring.

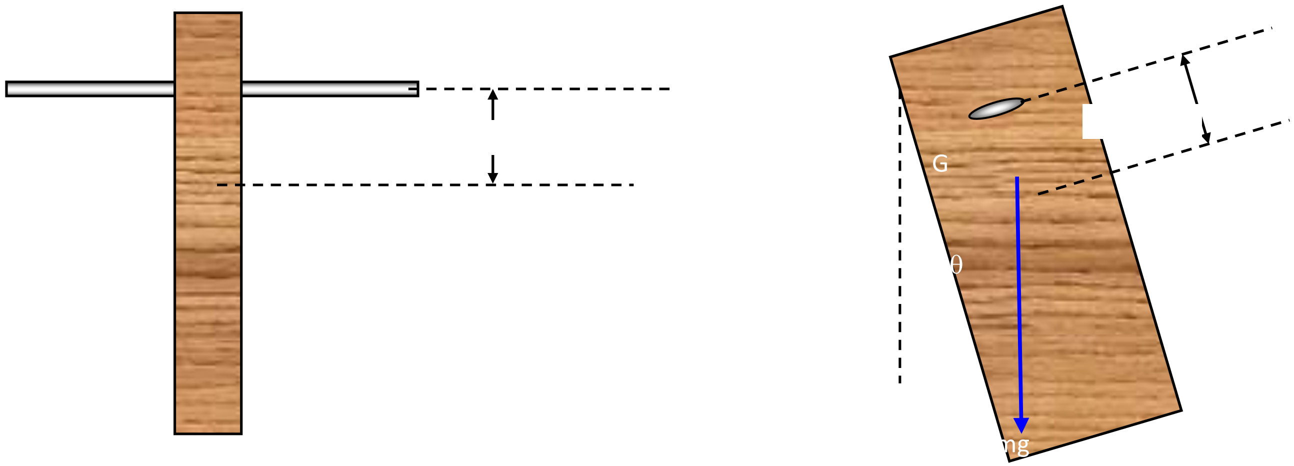


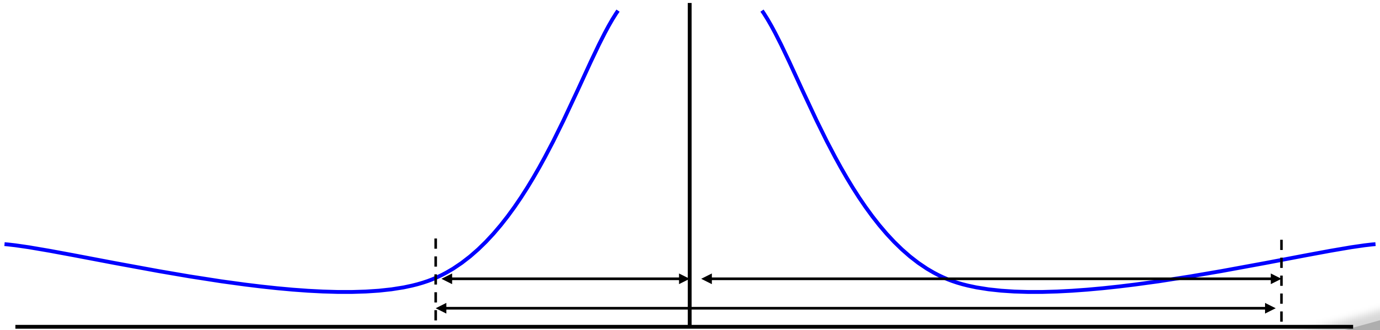
$$\omega = \frac{d\theta}{dt} = \text{const}$$

$$\theta = \omega t$$

$$x = A\cos\theta = A\cos\omega t$$

COMPOUND PENDULUM





ANALYSIS AND CALCULATIONS

- Plot a graph of T against d.
- From the graph record a series of values of the simple equivalent pendulum (L).
- Calculate the value of g from the graph or from the formula:

  $T^2 = 4(pi)^2L/g$

TORSIONAL PENDULUM

Torsion is a type of stress, which is easier to explain for a uniform wire or a rod when one end of the wire is fixed, and the other end is twisted about the axis of the wire by an external force.

TORSIONAL PENDULUM

- The external force causes deformation of the wire and appearance of counterforce in the material.
- If this end is released, the internal torsion force acts to restore the initial shape and size of the wire.
- This behavior is similar to the one of the released end of a linear spring with a mass attached.

TORSIONAL PENDULUM

- Attaching a mass to the twisting end of the wire, one can produce a torsion pendulum with circular oscillation of the mass in the plane perpendicular to the axis of the wire.

TORSIONAL PENDULUM

- To derive equations of rotational motion of the torsion pendulum, it would be useful to recall a resemblance of quantities in linear and rotational motion.
- We know that if initially a mass is motionless, its linear motion is caused by force F; correspondingly, if an extended body does not rotate initially, its rotation is caused by torque τ.

TORSIONAL PENDULUM

- The following relationship between the moment of inertia *I* of an oscillating object and the period of oscillation *T* as:

$$I = \left(\frac{T}{2\pi}\right)^2 \kappa \qquad (3)$$

- This relationship is true for oscillation where damping is negligible and can be ignored. Otherwise the relationship between *I* and κ is given by

$$I = \frac{\kappa}{\omega_0^2}$$

DEGREE OF FREEDOM

- The number of independent co ordinates required to complete the configuration of the system is called degree of freedom of the system.
- Single Spring and single mass system is comes under single degree of freedom system

Free vibration of One Degree of Freedom Systems

- Free vibration of a system is vibration due to its own *internal forces* (free of external impressive forces).
- It is initiated by an initial deviation (an energy input) of the system from its static equilibrium position.

DEGREE OF FREEDOM

- Once the initial deviation (a displacement or a velocity or both) is suddenly withdrawn, the strain energy stored in the system forces the system to return to its original, static equilibrium configuration.

- Due to the inertia of the system, the system will not return to the equilibrium configuration in a straight forward way.

- Instead it will oscillate about this position — free vibration.

Single degree of freedom system

- A system experiencing free vibration oscillates at one or more of its natural frequencies, which are properties of its mass and stiffness distribution.
- If there is no damping (an undamped system), the system vibrates at the (*undamped*) frequency (frequencies) forever. Otherwise, it vibrates at the (*damped*) frequency (frequencies) and dies out gradually.

Single degree of freedom system

- When damping is not large, as in most cases in engineering, undamped and damped frequencies are very close.
- Therefore usually no distinction is made between the two types of frequencies.
- The number of natural frequencies of a system equals to the number of its degrees-of-freedom. Normally, the low frequencies are more important.

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DEGREE OF FREEDOM

- Damping always exists in materials.
- This damping is called material damping, which is always positive (dissipating energy).
- However, air flow, friction and others may ‘present’ negative damping.


Thank you