HIGH SPEED AERODYNAMICS

III B. Tech V semester (Autonomous IARE R-16)

BY

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CO’s	Course outcomes
CO1	Explain a brief review of thermodynamics and fluid mechanics in relation to compressible flows
CO2	Demonstrate different types of shock waves and expansion waves and its properties across different situations.
CO3	Understand the importance of quasi one dimensional flow for obtaining supersonic speeds.
CO4	Illustrate the concepts of method of characteristics and its applications in nozzle designs.
CO5	Understand the experimental methods and their characteristics of various wind tunnels.

UNIT - I
INTRODUCTION TO COMPRESSIBLE FLOW

UNIT - I

CLOs	Course Learning Outcome
CLO1	Demonstrate the concept of supersonic flow, how it is different from incompressible flow.
CLO2	Understand governing equations of supersonic flow in various form and thermodynamics properties.
CLO3	Describe the governing equations required for compressible flows.

INTRODUCTION

Compressible flow is often called as variable density flow. For the flow of all liquids and for the flow of gases under certain conditions, the density changes are so small that assumption of constant density remains valid.

Let us consider a small element of fluid of volume. The pressure exerted on the element by the neighboring fluid is p. If the pressure is now increased by an amount dp, the volume of the element will correspondingly be reduced by the amount d .The compressibility of the fluid K is thus defined as

$$K=\frac{1}{\rho}.\frac{d\rho}{dp}$$

Brief Review of Thermodynamics and Fluid Mechanics

Thermodynamics Concepts:

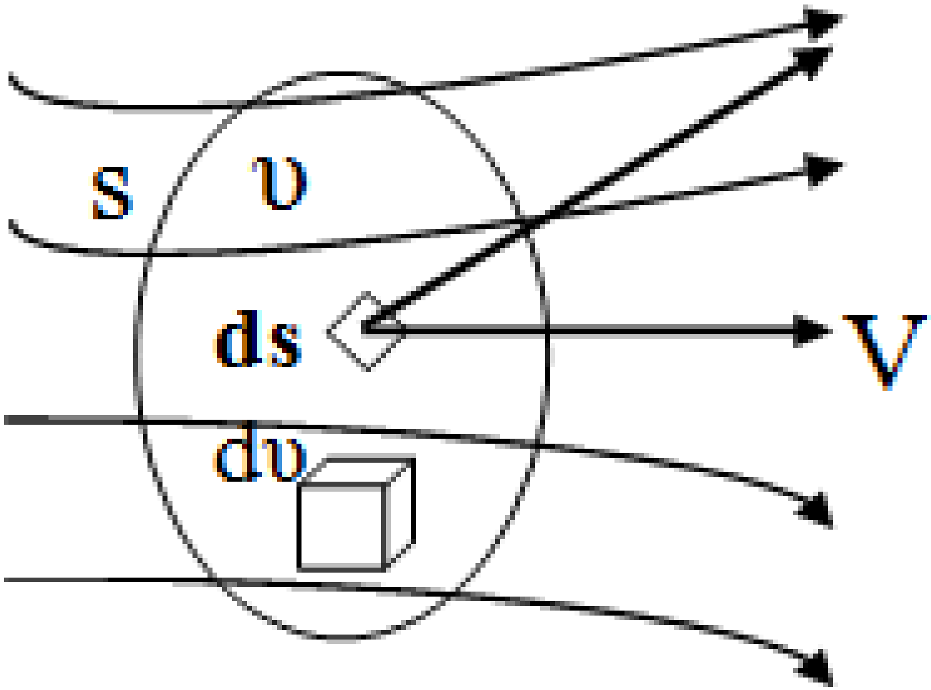
- System
- Surroundings
- Types of systems
- Laws of thermodynamics
    - Zeroth law of thermodynamics
    - First law of thermodynamics
    - Second law of thermodynamics
    - Third law of thermodynamics

Brief Review of Thermodynamics and Fluid Mechanics

- Important Fluid Properties
  - Continuum:
  - Transport phenomenon:
  - Compressibility of fluid and flow:
- Important Properties of Compressible Flows
  - Bulk modulus (Ev)
  - Coefficient of volume expansion (β)
  - Compressibility(k)
  - Viscosity (μ)

INTEGRAL FORMS OF CONSERVATION EQUATIONS

Conservation of Mass or Continuity Equation (Integral Form)



Finite Control Volume fixed in space

Conservation of Mass or Continuity Equation (Integral Form)

Total mass in the control volume (CV) =	$\oiiint_{v} \rho dv$
Rate of change of mass in CV=	$\frac{\partial}{\partial t} \oiiint_{v} \rho dv$
Thus the time rate of decrease of mass inside the CV is	$-\frac{\partial}{\partial t} \oiiint_{v} \rho dv$

Conservation of Mass or Continuity Equation (Integral Form)

$\rho V_n dS = \rho V.dS$

$$\oiint_s \rho V \,.\, ds$$

$$\oiint_s \rho V \,.\, ds = -\frac{\partial}{\partial t} \oiiint_v \rho dv$$

Or

$$\frac{\partial}{\partial t} \oiiint_v \rho dv + \oiint_s \rho V \,.\, ds = 0$$

Conservation of Momentum (Integral Form)

- Momentum equation is based on Newton's second law. This can be expressed as

$$\frac{d}{dt}(mV) = F$$
$$\oiiint_v \rho\, F_b dv$$
$$F = -\oiint_s P ds + \oiiint_v \rho\, F_b dv + F_{viscous}$$
$$\frac{\partial}{\partial t}\oiiint_v \rho V dv + \oiint_s (\rho V . ds)V = -\oiint_s P ds + \oiiint_v \rho F_b dv + F_{viscous}$$

Conservation of Energy (Integral Form)

- From the principle of energy conservation δq + δw = de

$$\frac{\delta q}{\delta t} = \oiiint_{v} q\,\rho\,dv + \dot{Q}_{viscous}$$
$$\frac{\delta w}{\delta t} = -\oiint_{s} PV.ds + \oiiint_{v} \rho(F_b.V)dv + \dot{W}_{viscous}$$
$$\frac{de}{dt} = \frac{\partial}{\partial t}\oiiint_{v} \rho(e + \frac{V^2}{2})dv + \oiint_{s} (\rho V.ds)\left(e + \frac{V^2}{2}\right)$$
$$\oiiint_{v} q\rho dv + \dot{Q}_{viscous} - \oiint_{s} PV.ds + \oiiint_{v} \rho(F_b.V)dv + \dot{W}_{viscous} = \frac{\partial}{\partial t}\oiiint_{v} \rho(e + \frac{V^2}{2})dv + \oiint_{s} \rho\left(e + \frac{V^2}{2}\right)V.ds$$

Differential Conservation Equations

- Conservation of Mass or Continuity Equation (Differential Form)
- Conservation of Momentum (Differential Form)
- Conservation of Energy (Differential Form)

Conservation of Mass or Continuity Equation (Differential Form)

- Using Gauss divergence theorem

$$\oiint_{s} (\rho V).\,ds = \oiiint_{v} \nabla.(\rho V)dv$$

$$\oiiint_{v} \frac{\partial \rho}{\partial t} dv + \oiiint_{v} \nabla.(\rho V)dv = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0$$

$$\frac{\partial \rho}{\partial t} + \rho\nabla.V + V.\nabla\rho = 0$$

$$\frac{D\rho}{Dt} + \rho\nabla.V = 0$$

Conservation of Momentum (Differential Form)

$$-\oiint_{s} P ds = -\oiiint_{v} \nabla P dv$$
$$\oiiint_{v} \frac{\partial(\rho V)}{\partial t} dv + \oiint_{s} (\rho V.ds)V = -\oiiint_{v} \nabla P dv + \oiiint_{v} \rho F_b dv + F_{viscous}$$
$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho F_{b,x} + F'_{viscous,x}$$
$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \rho F_{b,y} + F'_{viscous,y}$$
$$\rho \frac{Dw}{Dt} = -\frac{\partial P}{\partial z} + \rho F_{b,z} + F'_{viscous,z}$$
$$\rho \frac{DV}{Dt} = -\nabla P + \rho F_{b,z} + F'_{viscous}$$

Conservation of Energy (Differential Form)

$$\iiint\limits_{v} q\rho dv + \dot{Q}_{viscous} - \oiint\limits_{s} PV.ds + \iiint\limits_{v} \rho(F_b.V)dv + \dot{W}_{viscous} =$$

$$\frac{\partial}{\partial t}\iiint\limits_{v} \rho(e + \frac{V^2}{2})dv + \oiint\limits_{s} \rho\left(e + \frac{V^2}{2}\right)V.ds$$

$$\iiint\limits_{v} \frac{\partial}{\partial t}\left[\rho(e + \frac{V^2}{2}\right] dv + \oiint\limits_{s} \rho\left(e + \frac{V^2}{2}\right)V.ds$$

$$- \iiint\limits_{v} q\rho dv + \oiint\limits_{s} PV.ds - \iiint\limits_{v} \rho(F_b.V)dv - \dot{Q}_{viscous} - \dot{W}_{viscous} = 0$$

Conservation of Energy (Differential Form)

$$\frac{\partial}{\partial t}\left\{\rho\left(e+\frac{V^2}{2}\right)\right\}+\nabla.\left\{\rho\left(e+\frac{V^2}{2}\right)V\right\}$$
$$=q\rho-\nabla.(PV)+\rho(F_b.V)+\dot{Q}_{viscous}+\dot{W}_{viscous}$$

$$\rho\frac{D}{Dt}\left(e+\frac{V^2}{2}\right)=q\rho-\nabla.(PV)+\rho(F_b.V)+\dot{Q}_{viscous}+\dot{W}_{viscous}$$

CONTINUUM POSTULATES

- The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables.
- Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

$$\rho = \lim_{\Delta\forall \to 0} \left(\frac{m}{\Delta\forall} \right)$$

ACOUSTIC SPEED

- The **speed of sound** is the distance travelled per unit time by a sound wave as it propagates through an elastic medium.
- At 20 °C (68 °F), the speed of sound in air is about 343 metres per second (1,235 km/h; 1,125 ft/s; 767 mph; 667 kn), or a kilometre in 2.9 s or a mile in 4.7 s.
- It depends strongly on temperature, but also varies by several metres per second, depending on which gases exist in the medium through which a sound wave is propagating.
- The speed of sound in an ideal gas depends only on its temperature and composition. The speed has a weak dependence on frequency and pressure in ordinary air, deviating slightly from ideal behavior.

ACOUSTIC SPEED

In fluid dynamics, the speed of sound in a fluid medium (gas or liquid) is used as a relative measure for the speed of an object moving through the medium.

$$a^2 = \frac{dp}{d\rho} \; or \; a = \sqrt{\frac{dP}{d\rho}}$$

This is the general formula for acoustic speed or speed of sound.

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT}$$

MACH NUMBER

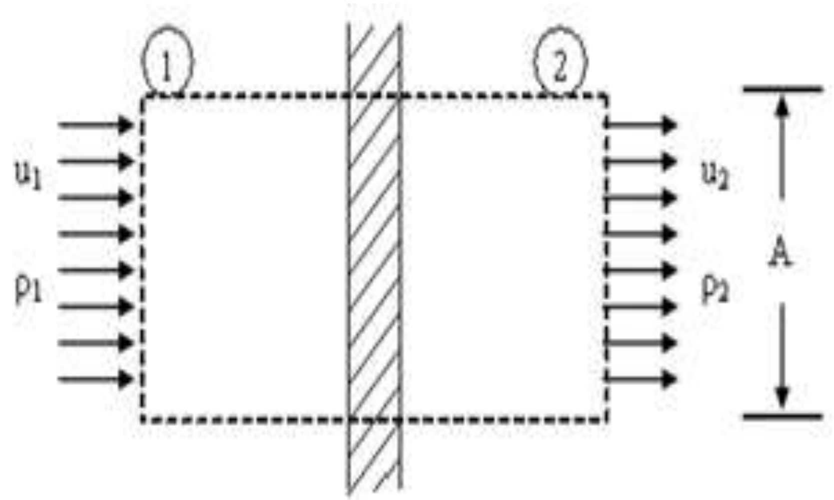
- Mach number is defined as the ratio of the particle (local) speed to the (local) speed of sound.

$$\text{Mach number} = \frac{\text{speed of the fluid particle(sound)}}{\text{acoustic apeed of that atmosphere}}$$

M < 0.3 (incompressible flow)
M < 1 (subsonic flow)
0.8 < M < 1.2 (transonic flow)
M > 1 (supersonic flow)
M > 5 an above (hypersonic flow)

GOVERNING EQUATIONS FOR COMPRESSIBLE FLOWS

One dimensional form of Mass conservation or Continuity Equation



One-dimensional flow

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

One dimensional form of Momentum conservation Equation

- For the steady and inviscid flow with no body forces

$$\oiint_S (\rho V.ds)V = \oiint_S Pds$$

$$\oiint_S (\rho V.ds)u = -\oiint_S (Pds)_x$$

$$\rho_1(-u_1A)u_1 + \rho_2(-u_2A)u_2 = -(-P_1A + P_2A)$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

One dimensional form of Conservation of Energy

- Consider the control volume for steady inviscid flow without body force, Then the equation reduces to,

$$\iiint_v q\rho dv - ds = \oiint_s \rho\left(e + \frac{V^2}{2}\right) V.ds$$

$$\dot{Q} - \oiint_s PV.ds = \oiint_s \rho\left(e + \frac{V^2}{2}\right) V.ds$$

$$\frac{\dot{Q}}{\rho_1 u_1 A} + \frac{P_1}{\rho_1} + e_1 + \frac{u_1^2}{2} = \frac{P_2}{\rho_2} + e_2 + \frac{u_2^2}{2}$$

$$\frac{\dot{Q}}{\rho_1 u_1 A} + P_1 v_1 + e_1 + \frac{u_1^2}{2} = P_2 v_2 + e_2 + \frac{u_2^2}{2}$$

Fundamental Equations for Compressible Flow

- **Conservation of Mass:**

$$\rho_1 u_1 + \rho_2 u_2$$

- **Conservation of Momentum:**

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

- **Steady Flow Energy Conservation:**

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

UNIT II

SHOCK AND EXPANSION WAVES

UNIT - II

CLOs	Course Learning Outcome
CLO4	Illustrate the impact of supersonic flow in the presence of compression and expansion corner
CLO5	Demonstrate supersonic aircraft design and applications to aircrafts, supersonic wind tunnel and shock tubes.
CLO6	Understand the concepts of shock wave boundary layer interaction.

Shock Waves

Consider a subsonic and supersonic flow past a body

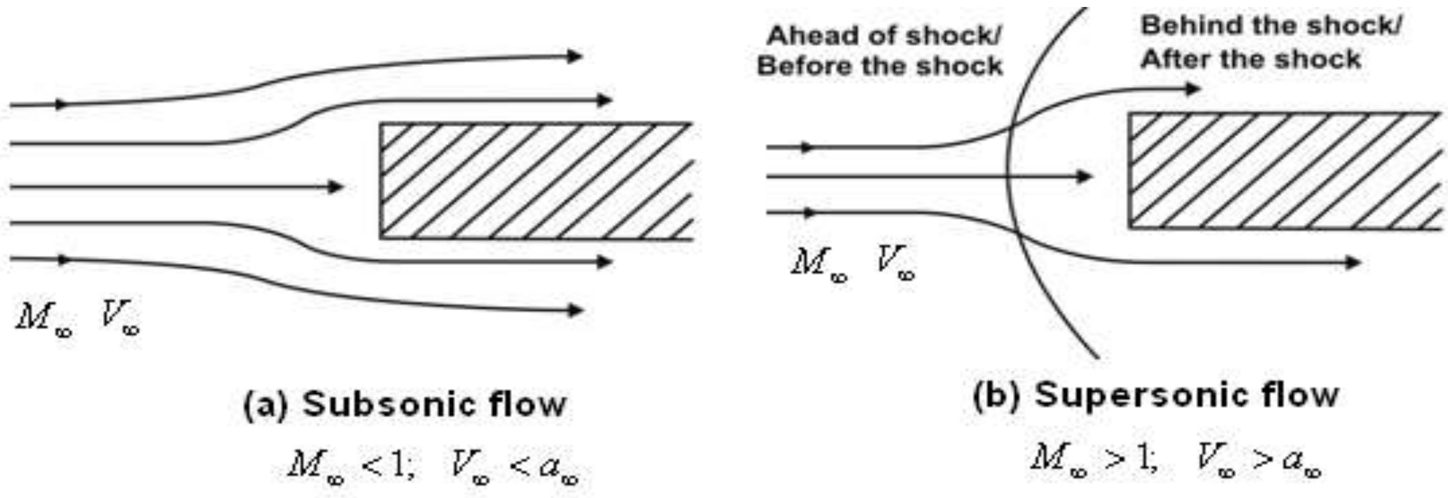
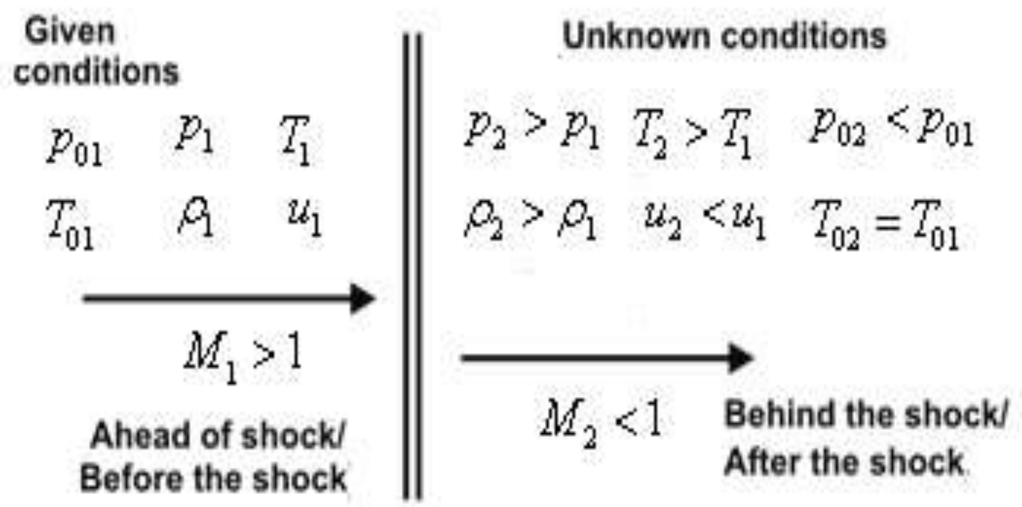


Illustration of shock wave phenomena

Normal Shock Waves

- A normal shock wave is one of the situations where the flow properties change drastically in one direction



Schematic diagram of a standing normal shock wave

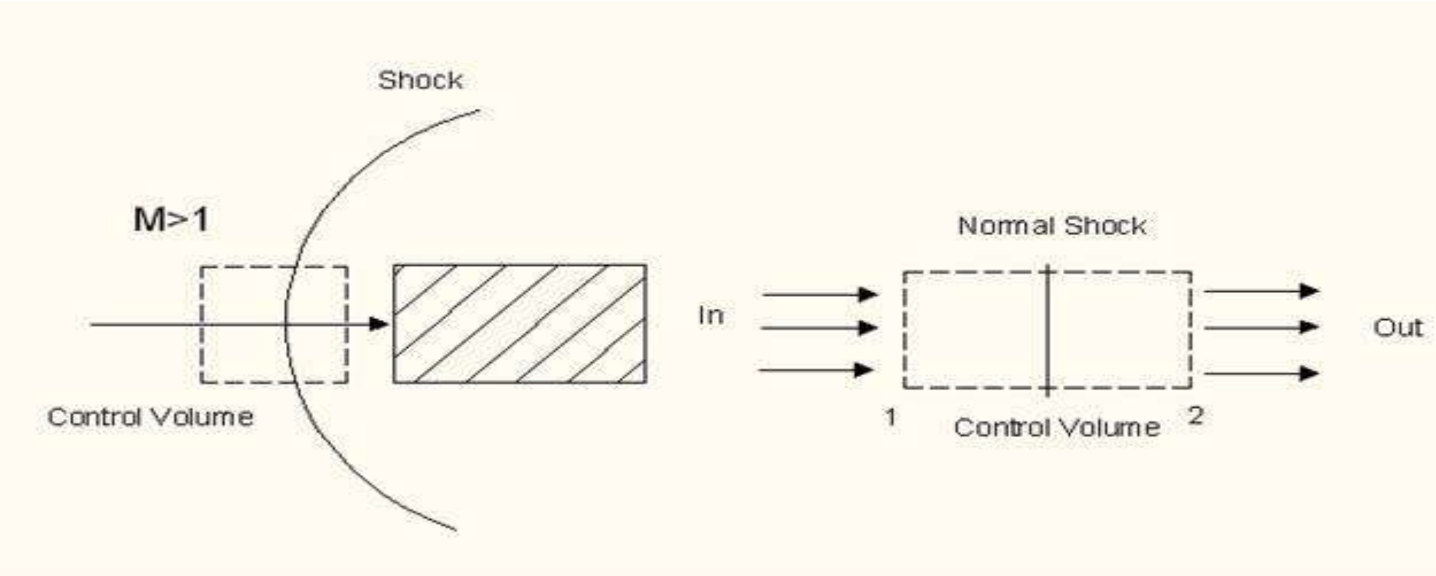
Entropy across a normal shock

- The compression through a shock wave is considered as irreversible process leading to an increase in entropy.
- The change in entropy can be written as a function of static pressure and static temperature ratios across the normal shock.

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

- $M_2 \leq 1; (p_2/p_1 \geq 1; (\rho_2/\rho_1) \geq 1; (T_2/T_1) \geq 1$

Normal shock relations



Shock pattern for a blunt or bluff obstacle

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

Normal shock relations

$$\left[\frac{a^{*2}(\gamma+1)}{u_1 u_2}\right]\frac{1}{2\gamma}=\frac{2\gamma-\gamma+1}{2\gamma}$$

$$a^{*2}=u_1 u_2$$

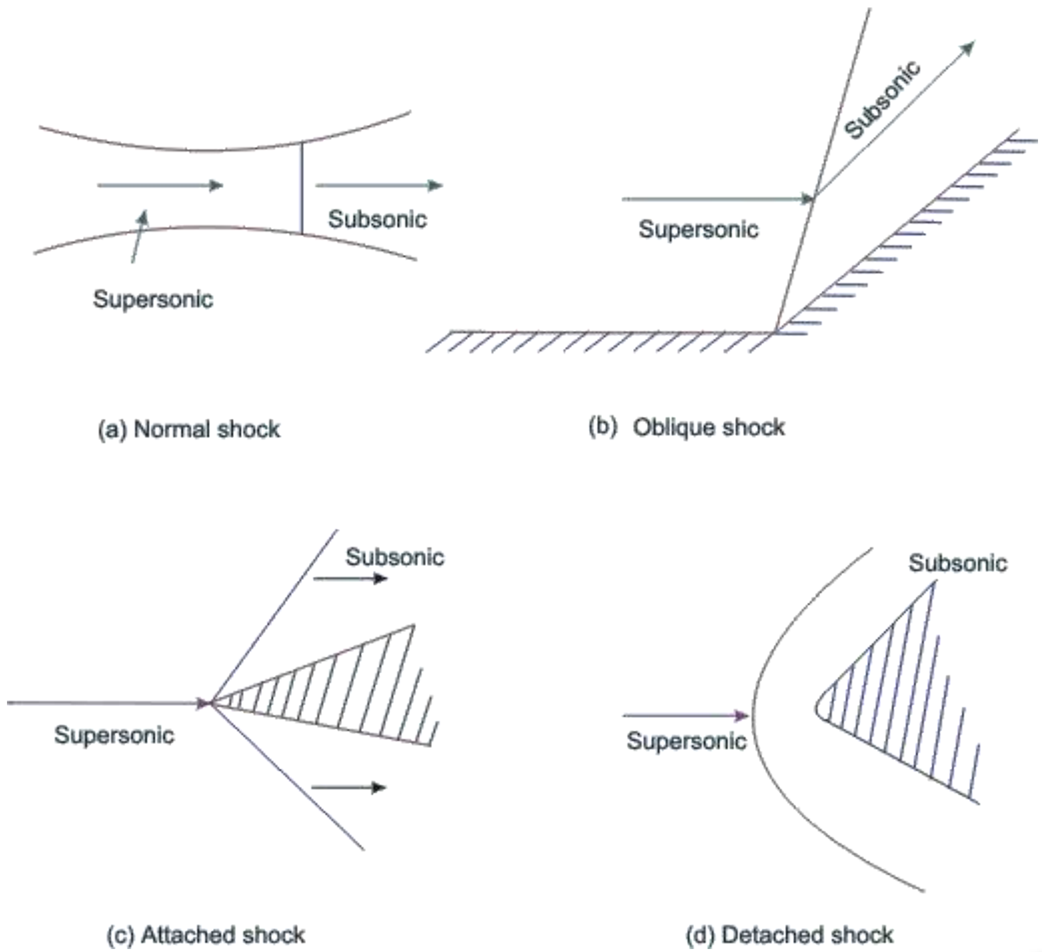
$$\frac{u_1}{a^{*2}}=\frac{u_2}{a^{*2}}\Rightarrow M_1^{*2}=\frac{1}{M_2^{*2}}$$

This expression shows that, M_1^{*2} and M_2^{*2} are reciprocal of each other for a normal shock. This equation is called as Prandtl's relation for normal shock which can be used to prove that Mach number becomes subsonic behind the normal shock

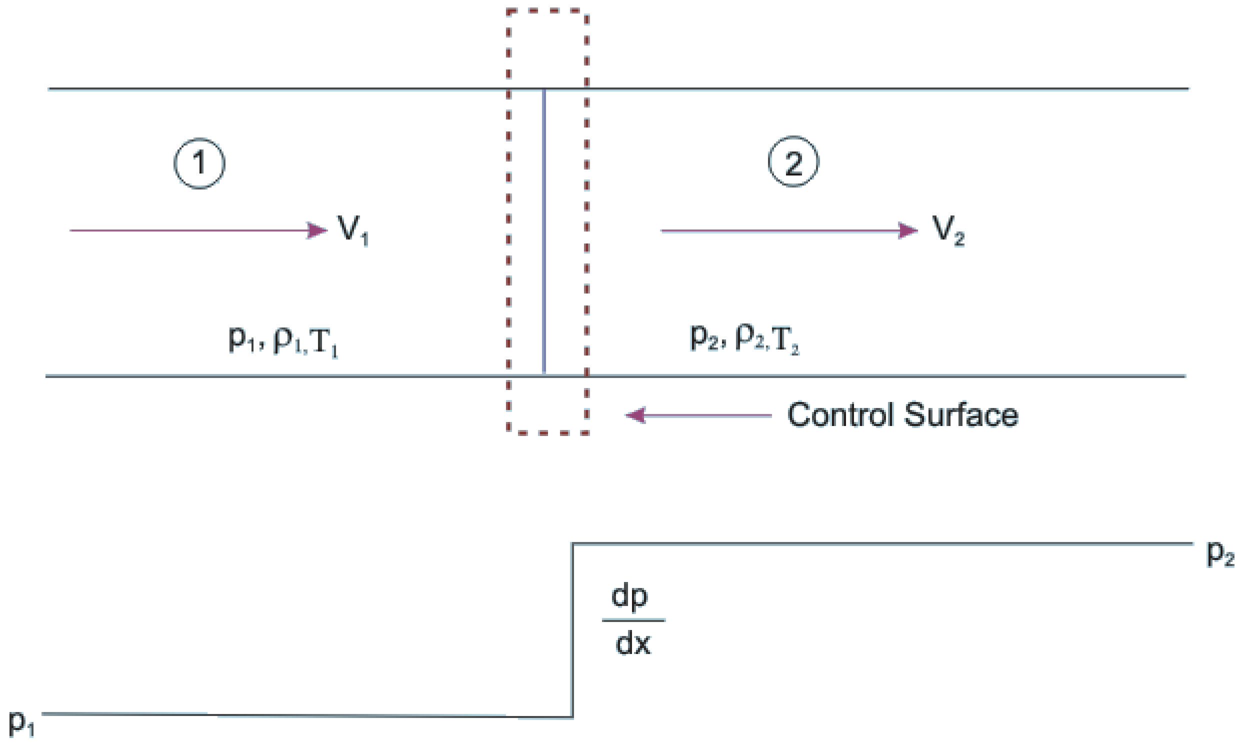
Normal shock relations

- Shock waves are highly localized irreversibilities in the flow .
- Within the distance of a mean free path, the flow passes from a supersonic to a subsonic state, the velocity decreases suddenly and the pressure rises sharply. A shock is said to have occurred if there is an abrupt reduction of velocity in the downstream in course of a supersonic flow in a passage or around a body.
- Normal shocks are substantially perpendicular to the flow and oblique shocks are inclined at any angle.
- Shock formation is possible for confined flows as well as for external flows.
- Normal shock and oblique shock may mutually interact to make another shock pattern.

Different type of Shocks



One Dimensional Normal Shock



One Dimensional Normal Shock

Stationary and Moving Normal Shock Waves

- The easiest way to analyze a normal shock is to consider a control surface around the wave.
- The energy equation can be simplified for an ideal gas as

$$T_{01}=T_{02}$$

By making use of the equation for the speed of sound equation and the equation of state for ideal gas equation, the continuity equation can be rewritten to include the influence of Mach number as:

$$\frac{p_1}{RT_1} Ma_1 \sqrt{\gamma RT_1} = \frac{p_2}{RT_2} Ma_2 \sqrt{\gamma RT_2}$$

APPLICATION TO AIRCRAFTS

- Air intakes for supersonic aircraft present an excellent example for the application of the theory of shock waves in steady flow. We know that the propulsion of aircraft is provided by the so-called propulsion nacelle, or more simply the nacelle, which brings together the various elements contributing to the propulsion , namely:

– the air intake, capturing air to feed the engine;

– the engine that can be a turbojet, a ramjet, or even a scramjet engine for air breathing hypersonic vehicles;

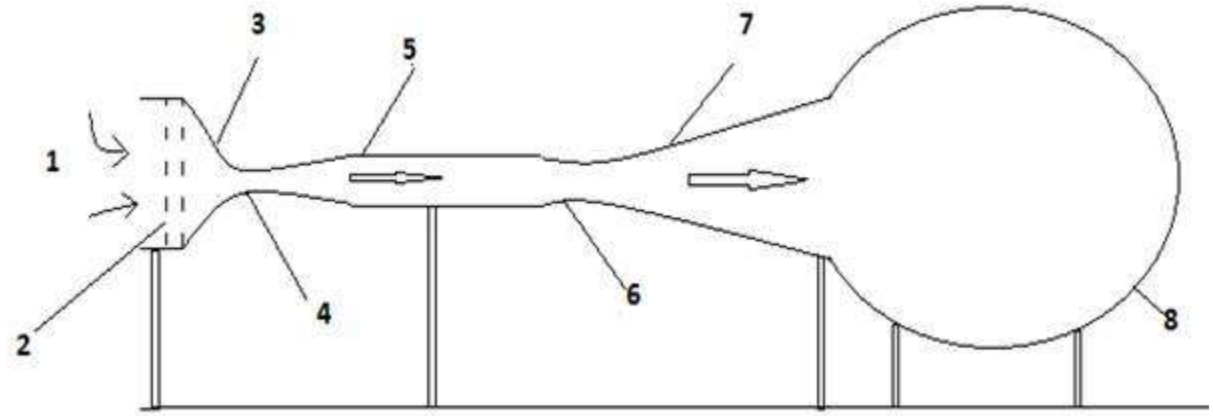
– a possible reheat duct or afterburner, where kerosene is burnt downstream of the engine in order to provide extra thrust; and

– the exhaust nozzle whose aerodynamics.

APPLICATION TO AIRCRAFTS

- The engines of current airliners additionally have a fan, arranged in front of the compressor and drawing an airflow that does not cross the compressor–turbine set.
- The operation of the fan is characterized by the bypass ratio, a ratio of the total drawn airflow and the airflow through the compressor.
- This bypass ratio is around 10 for modern engines.
- Engines for combat aircraft and missiles have no fan.

SUPERSONIC WIND TUNNEL

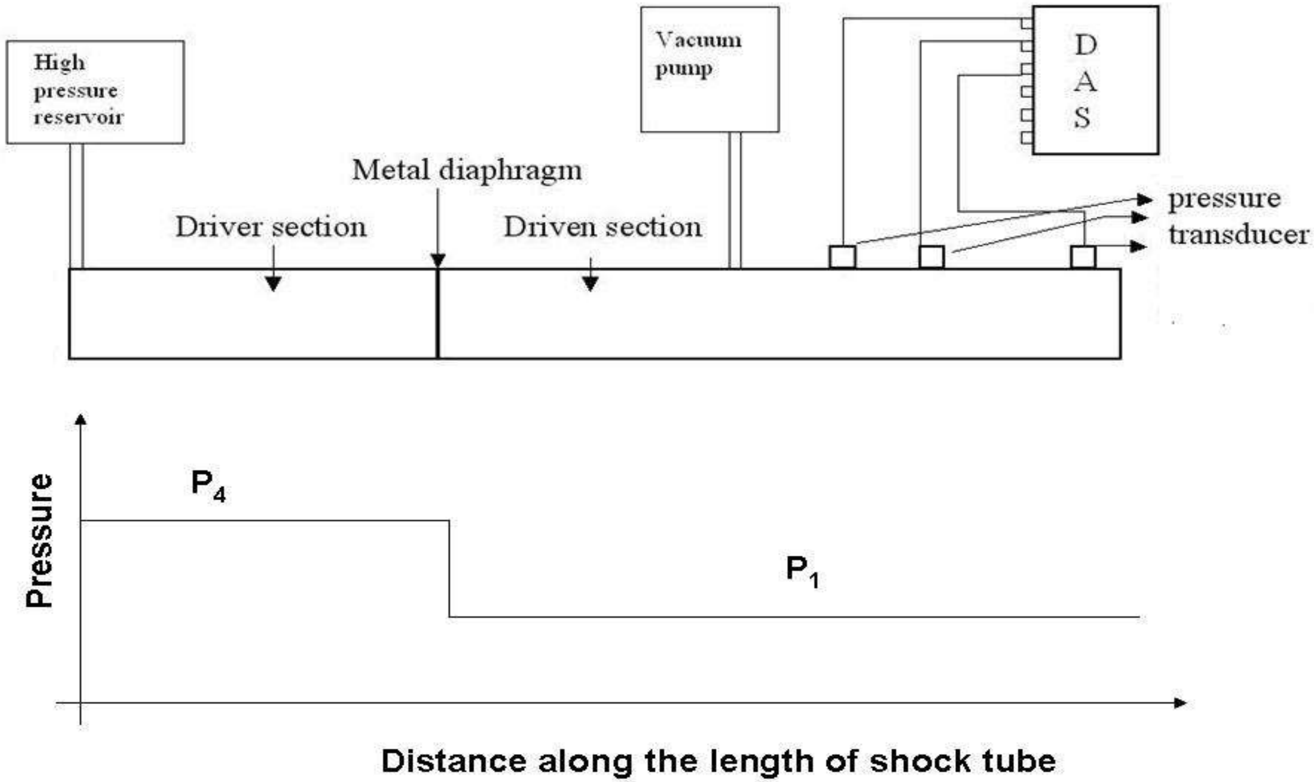


1. Air inlet
2. Settling chamber
3. Nozzle
4. Sonic throat
5. Test-section
6. Second throat
7. Diffuser
8. Vacuum chamber.

Shock Tube

- The shock tube is a simple duct closed at both ends. A diaphragm divides this duct into two compartments called as driver and driven sections.
- Driver section of the shock tube is the high pressure section which is supplied by the high pressure gas from the reservoir.
- Driven section of the shock tube is the low pressure section which contains the low pressure driven gas. These two sections are separated by a metal diaphragm.

Shock Tube



Schematic of the shock tube

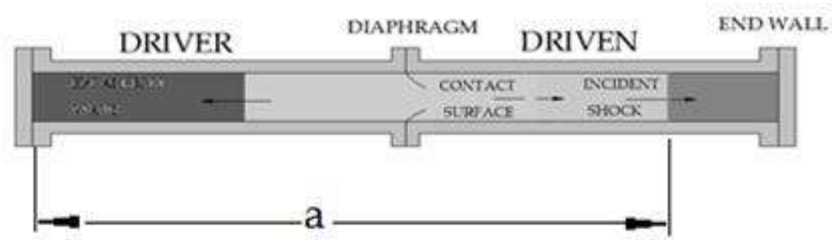
Shock Tube

Operating steps for this set up are as follows

1. Set the metal diaphragm between the driver and driven section.
2. Fill the driven gas in the driven section from the driven gas reservoir. This step is not required for case of air as the driven gas.
3. Set the required pressure in the driven section using vacuum pump.
4. Start filling the driver gas (say helium) in the driver section from the high pressure reservoir.

Motion of various waves inside the shock tube

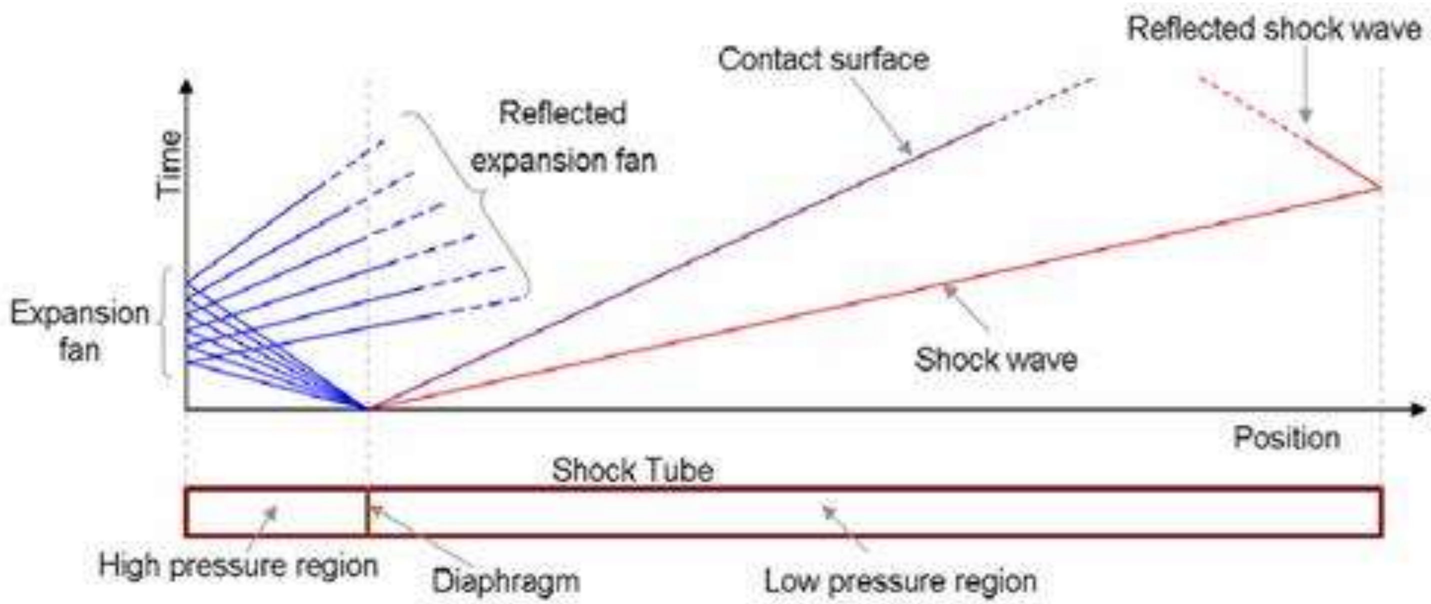
(a) On set of diaphragm burst.



(b) Propagation of shock, expansion and contact surface in the shock tube

(c) Reflection of shock and expansion from the ends of the shock tube

Motion of various waves inside the shock tube



Space time diagram for a typical shock tube

The notations of the regions between different waves in the shock tube

- Region 1: This is the region infront of the primary shock.
- Region 2: It is the region between contact surface and primary shock.
- Region 3: This is the region between tail of the expansion fan and contact surface.
- Region 4: It is the region ahead of the leading expansion wave.
- Region 5: It is the region behind the reflected

Applications of shock tube

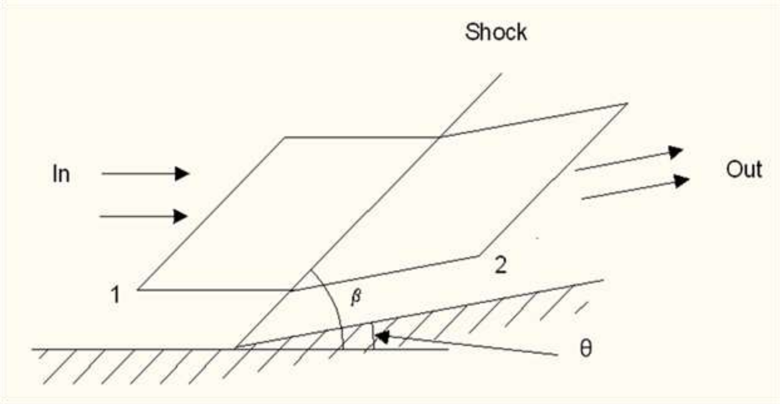
- Shock tube is very useful equipment and can be implemented for various applications in its various variants.
- Shock tube in its simplest format is used for the external flow aerodynamics studies in the higher subsonic, transonic and lower supersonic Mach numbers.
- Higher diameter tubes are advisable in these cases to avoid the domination of viscous effects.

Applications of shock tube

- For the experimentation in supersonic or hypersonic flow regimes, driven tube end is connected to a convergent divergent nozzle along with the test section and dump tank to comprise a shock tunnel. This formation helps to expand the stagnant gas behind the reflected shock through the nozzle to achieve high speed flow in the test section.
- Shock tube can also be used for chemical kinetic studies for measurement of reaction rates.
- Use of shock tube for contact less drug delivery has also been thought for its application in medical sciences.

Shock Polar

- Consider the supersonic flow of Mach number M_1 passing over a wedge of deflection angle θ



Supersonic flow over a wedge

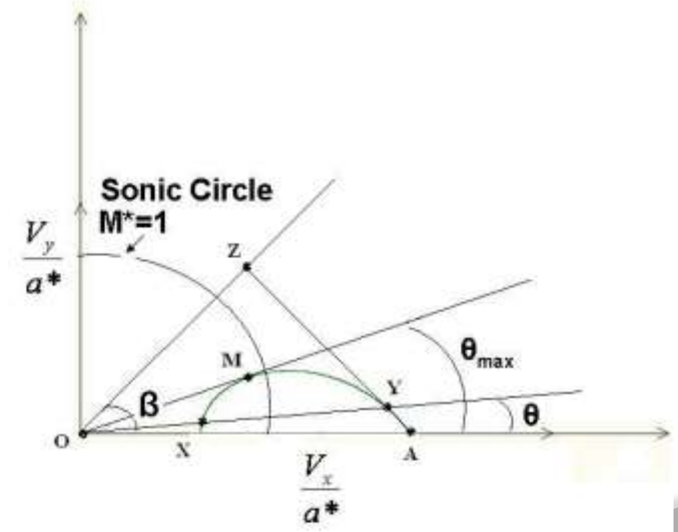
Shock Polar

- we can plot for various free stream Mach number for all the possible deflection angles.
- However, if we increase the freestream Mach number by increasing free stream velocity then it becomes impossible to plot such a graph when V_{x1} becomes ∞ for M_1 equal to ∞.
- To make such a plot possible, lets divide x axis and y axis by a^* which is the reference or stared quantity.
- This non-dimensionalisation makes it possible to represent V_{x1} since it will be represented by M_1^*.
- We can divide by a^* to post shock velocities as well, since a^* is constant in the flow field for an adiabatic flow.

Shock Polar

- Hence V_2 gets transformed to M_2^* which will make an angle equal to the flow deflection angle with the x-axis. This plot is called as shock polar. Such a shock polar can be plotted for very high Mach numbers M_1 also due to the fact that, if M_1 is equal to ∞, M_1^* is equal to 2.54.

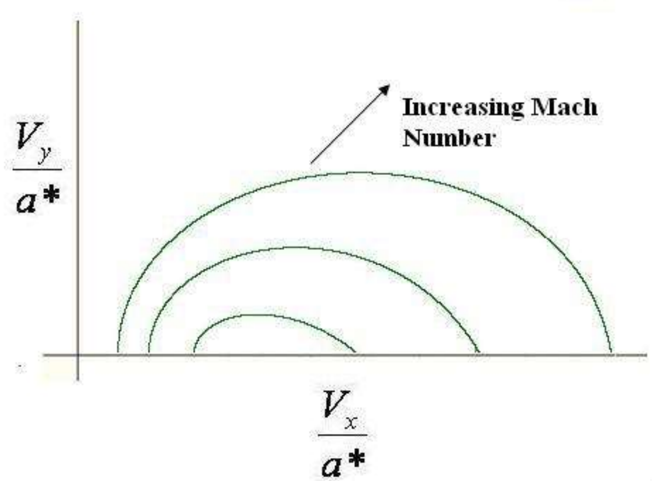
Therefore this plot is equally helpful for representing the pre and post shock velocities, flow deflection angle and the shock angle.



Typical Shock Polar

Analysis of Shock Polar

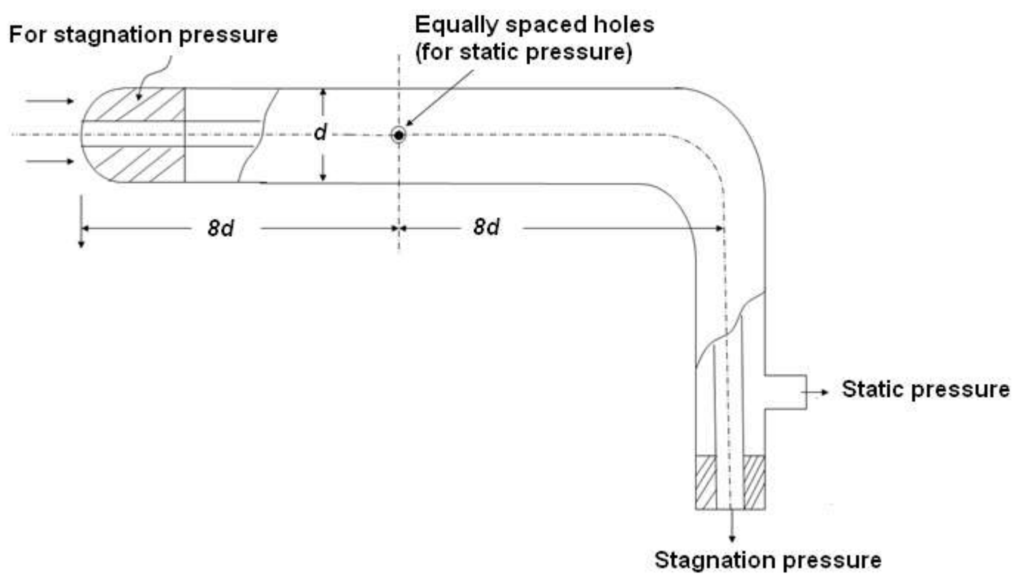
- Shock polar can be used as a tool to evaluate the post shock properties from known pre shock conditions.
- We can calculate a^* and hence M_1^* which is the required pre shock input condition from the known freestream conditions.
- Hence using flow deflection angle θ and M_1^*, we can evaluate the post shock condition as, M_2^* using a .given shock polar



Shock polar for various Mach numbers

Measurements of Flow Velocity

The flow velocity is obtained through simultaneous measurement of static and stagnation pressures using a Prandtl Pitot Static probe



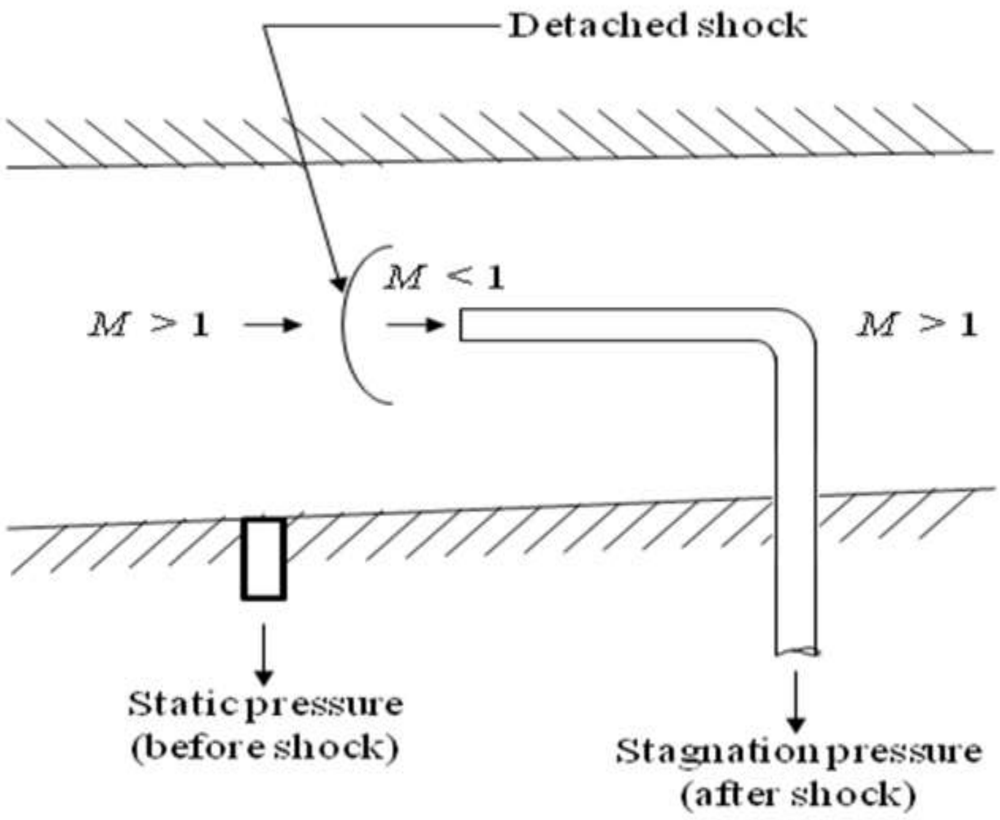
Prandtl Pitot static probe for simultaneous measurement

Measurements for Subsonic and Supersonic Flows

- The flow Mach number is one of the important parameter for subsonic and supersonic flows.
- All the flow parameters and their variations are the functions of local Mach number (M).
- The pressure measurements are one of the common practices to determine the Mach number.
- In subsonic flow, the simultaneous measurement of static(P) and stagnation pressures (p_0) using a *Prandtl Pitot Static tube* are made in a similar way.
- The isentropic relation is used to determine the flow Mach number.

$$\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$$

Measurements for Subsonic and Supersonic Flows



Detached shock ahead of the measuring pressure probe in a supersonic flow

Rayleigh-Pitot formula

The *Rayleigh-Pitot formula* with air as free stream is presented graphically in figure.

$$\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_{02}}{p_\infty} = \frac{\left(\frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{2\gamma}{\gamma+1} M_\infty^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}$$

$$\frac{p_d}{p_\infty} = \frac{\gamma M_\infty^2}{2}$$

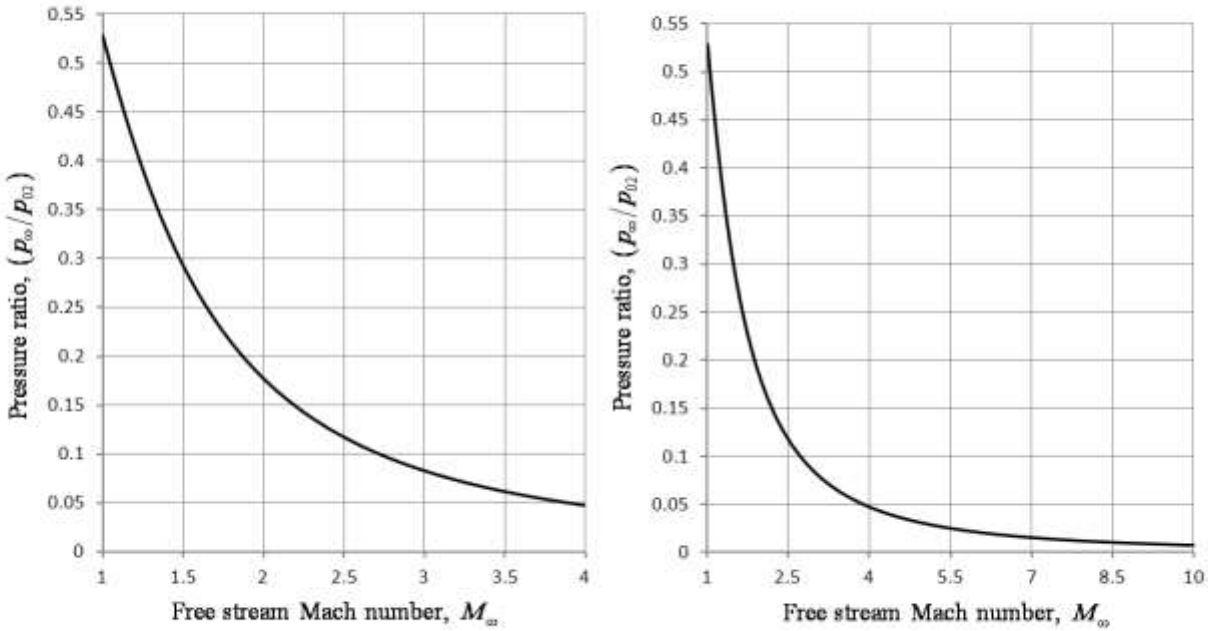
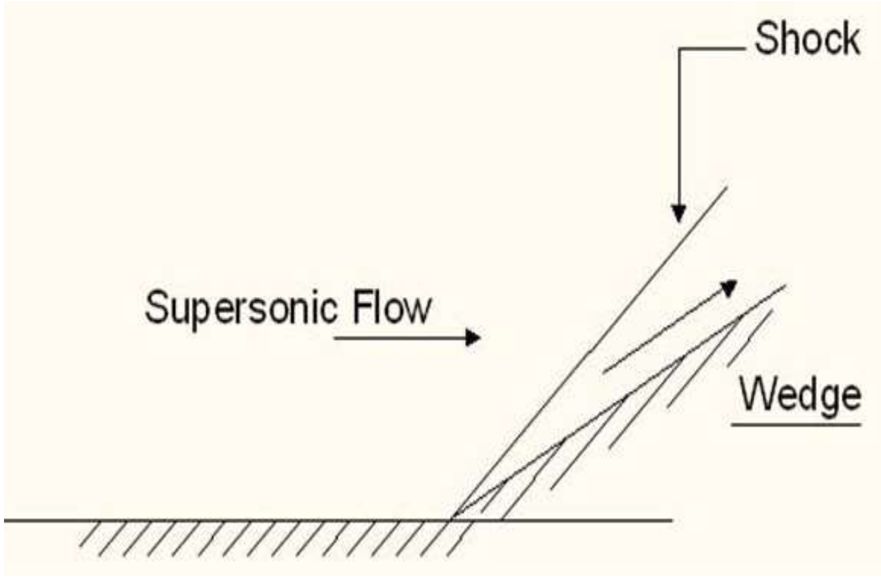


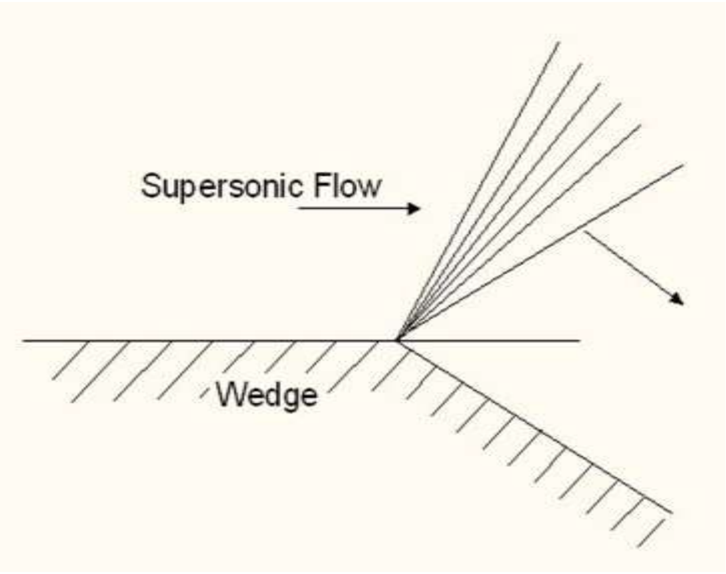
Fig. Mach number determination from Pitot tube measurement in a supersonic flow.

OBLIQUE SHOCKS

- The shock encountered by such flow is the normal shock. Herewith we are going to deal the compressible flows in two dimensions, 2D.
- There are two entities which are going to tackle the expansion and compression of the 2D compressible flow. Such situations are two situations which the flow might encounter.
- If the flow turns in to itself then the compression of flow takes place through shock. Since this shock makes certain angle with the flow, it is called as the oblique shock.

OBLIQUE SHOCKS

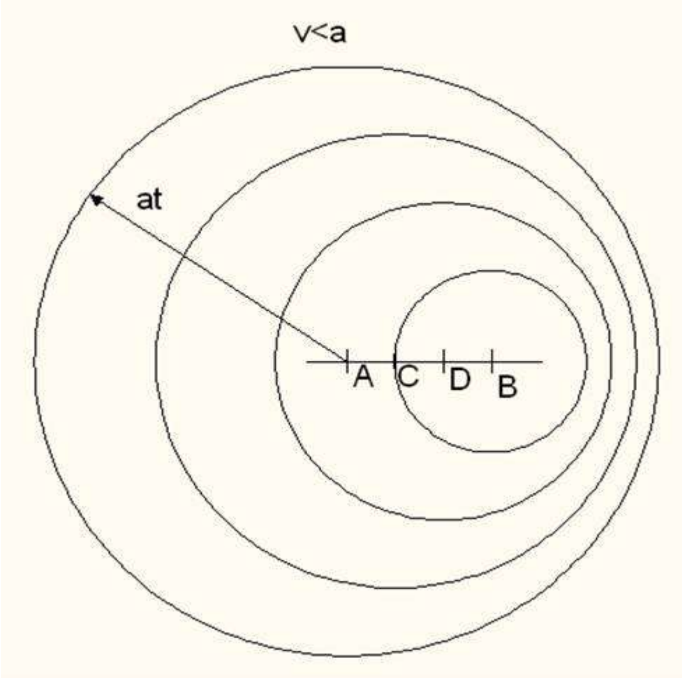




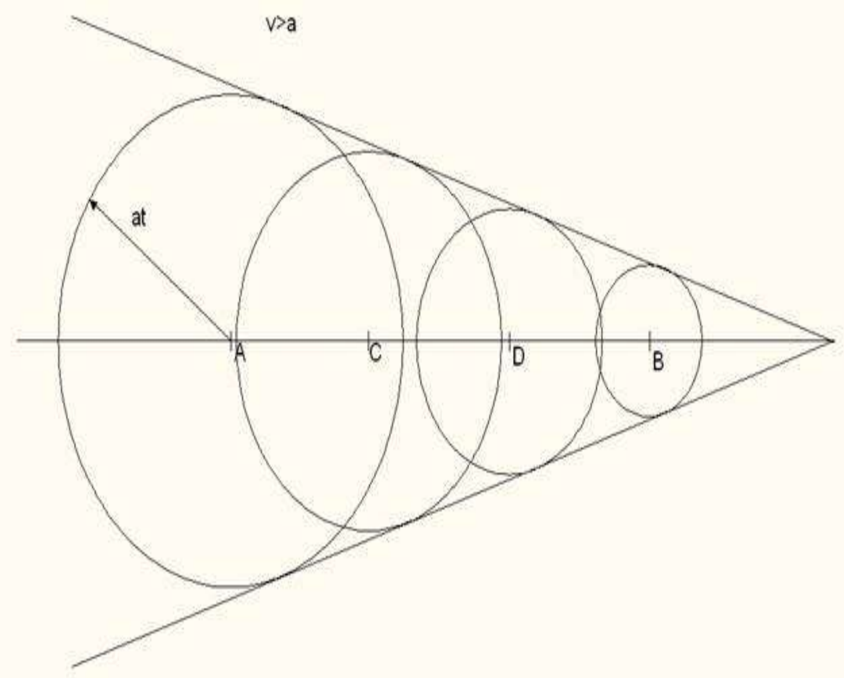
Supersonic flow turning into itself in the presence of shock

Supersonic flow turning away from itself in the presence of expansion fan

Mach Wave



Train travelling from A to B at subsonic speed

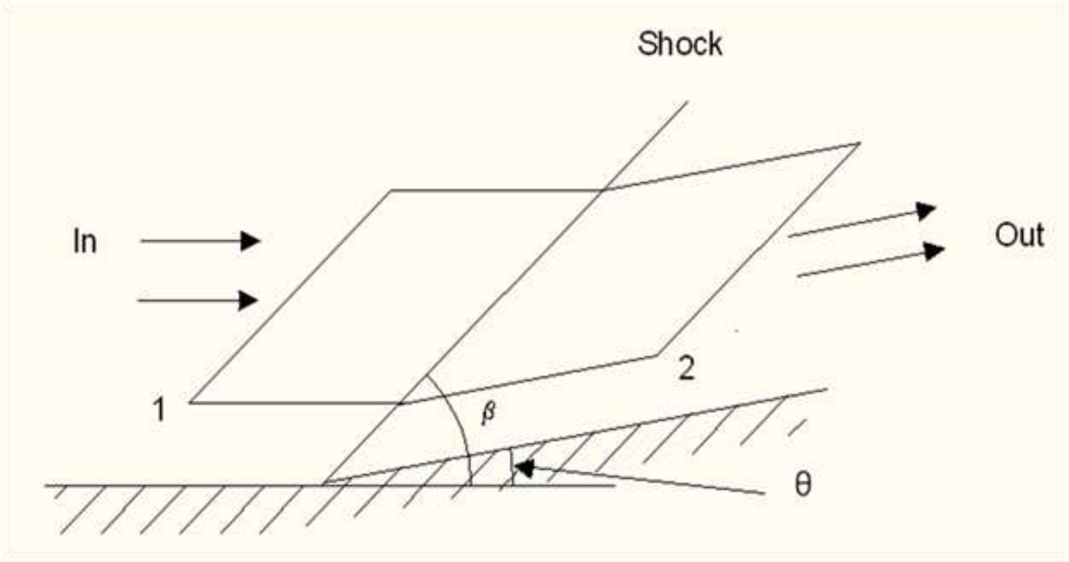


Train travelling from A to B at supersonic speed

Governing equations for Oblique Shock

- Consider the flow taking place along a wedge.
- Let θ be the wedge angle and β be the shock angle with the wall which is parallel to the approaching freestream.

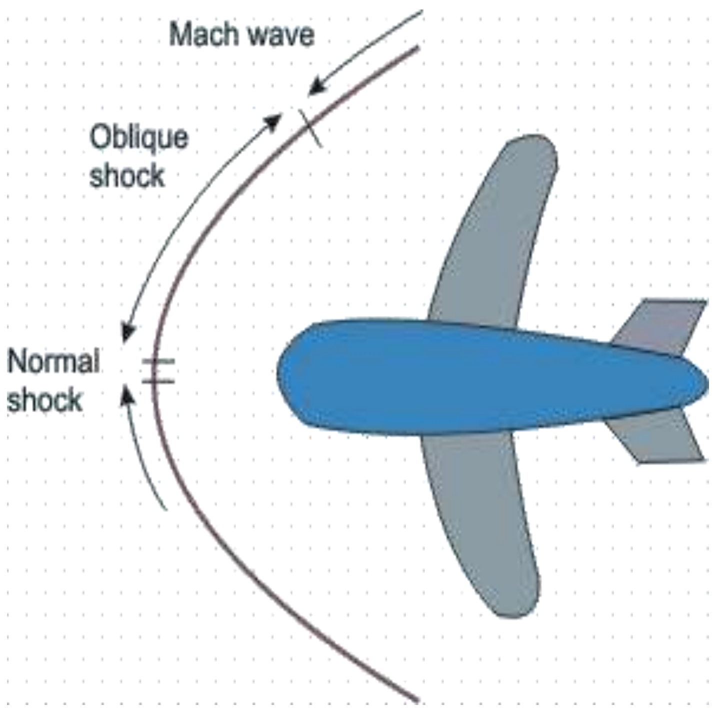
$$\iiint_V \frac{\partial \rho}{\partial t} dv + \oiint_S \rho V . ds = 0$$



An oblique shock for a supersonic flow over the wedge

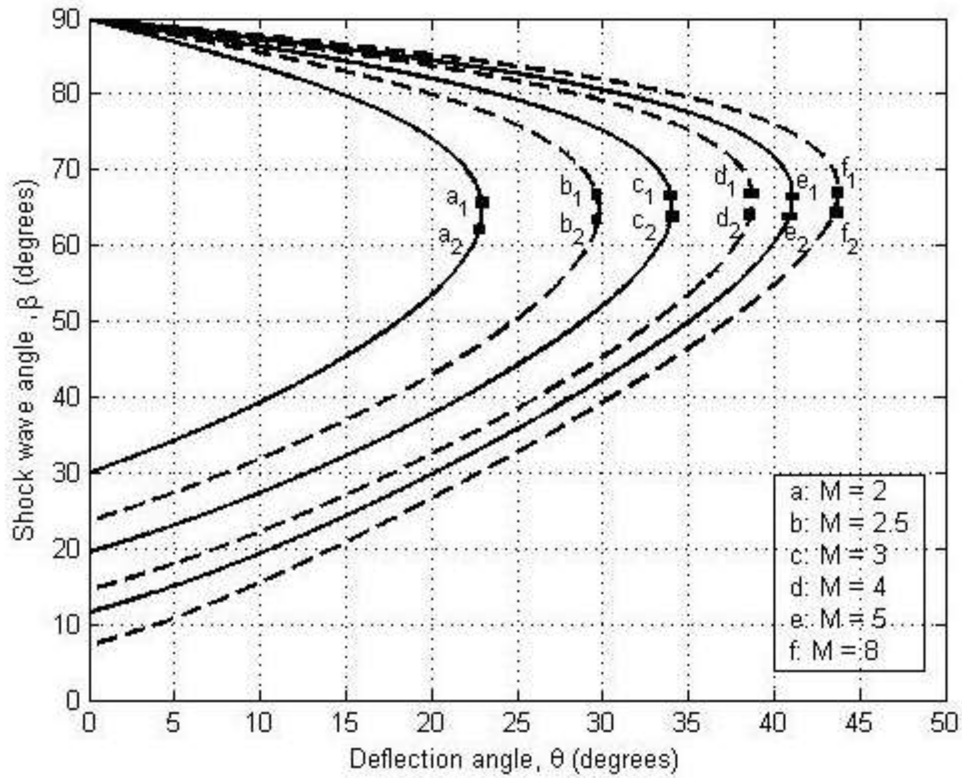
Oblique shock can be observed in following cases

- Oblique shock formed as a consequence of the bending of the shock in the free-stream direction.
- In a supersonic flow through a duct, viscous effects cause the shock to be oblique near the walls, the shock being normal only in the core region.
- The shock is also oblique when a supersonic flow is made to change direction near a sharp corner



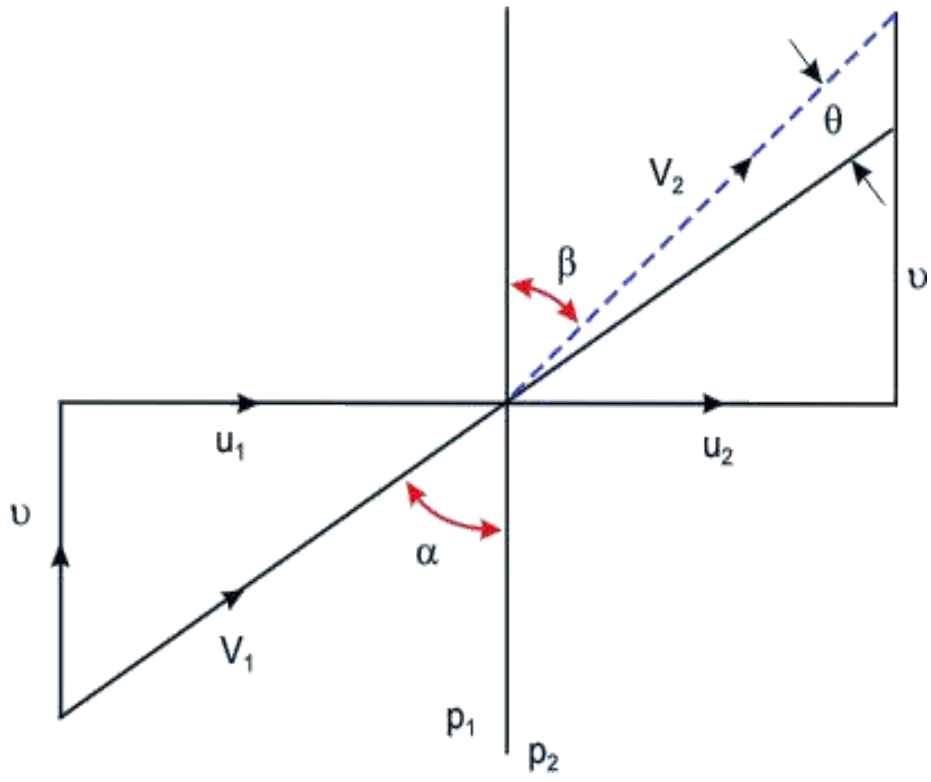
Normal and oblique Shock in front of an Obstacle

θ-β-M relation



θ-β-M relation

Two dimensional Oblique Shock



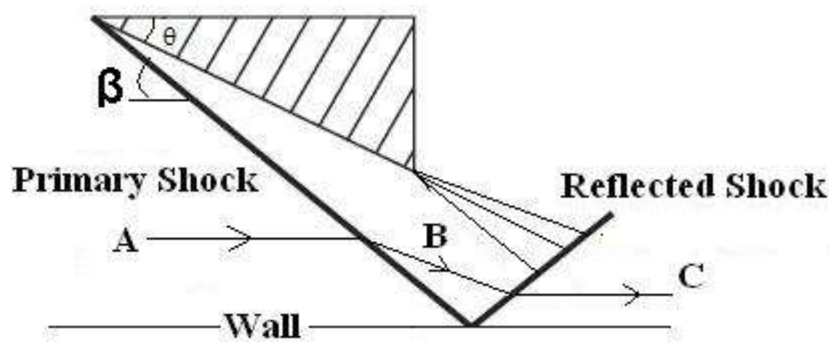
$$\tan\theta = \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\cdot\tan\beta}$$

$$= \frac{\tan\alpha - (\tan\beta/\tan\alpha)\tan\alpha}{1+\tan^2\alpha(\tan\beta/\tan\alpha)}$$

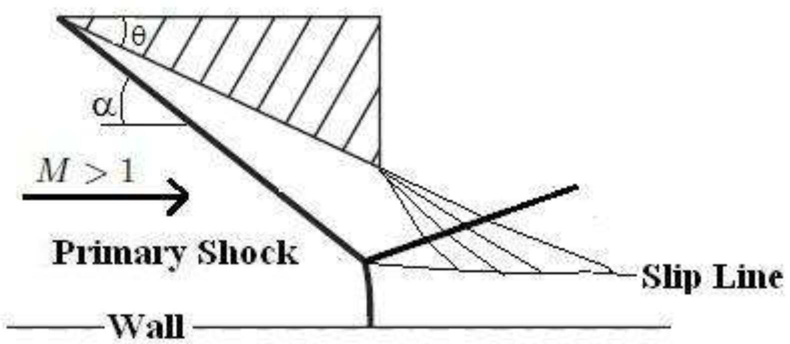
$$\tan\theta = \frac{\mathrm{Ma}_1^2\sin 2\alpha - 2\cot\alpha}{\mathrm{Ma}_1^2(\gamma+\cos 2\alpha)+2}$$

Two dimensional Oblique Shock

Shock Reflection

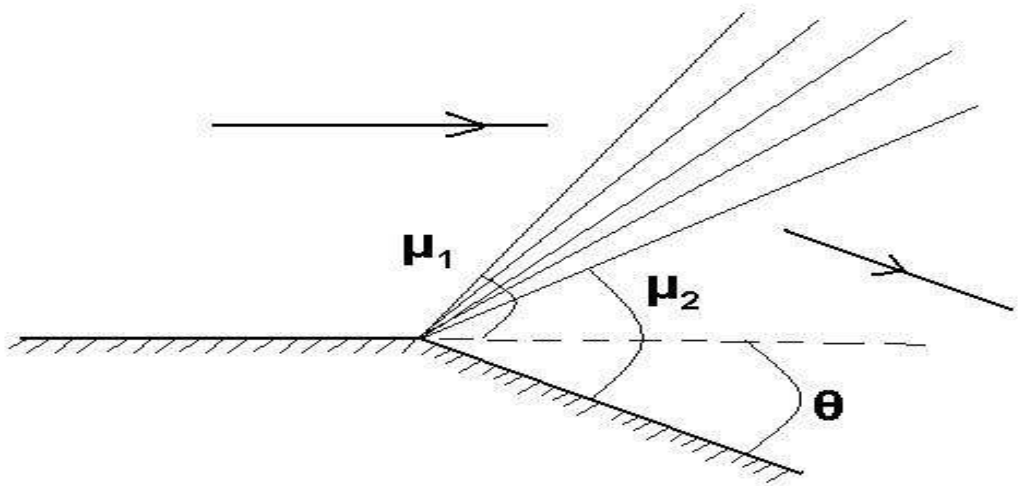


Typical regular reflection of shock wave

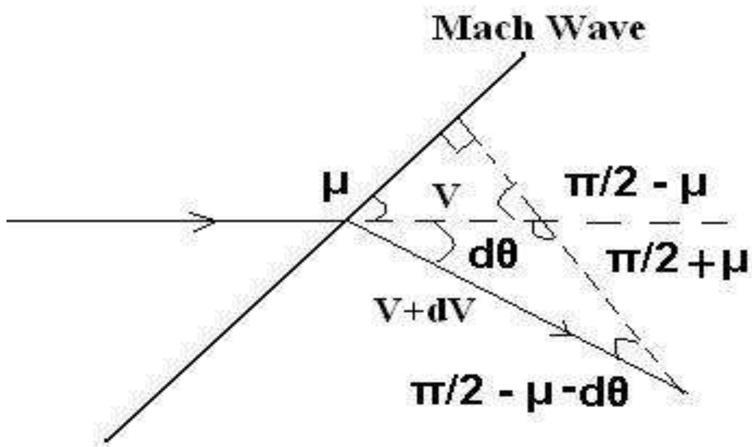


Typical Mach reflection of shock wave

Supersonic expansion



Expansion of supersonic flow



Velocity triangle across a typical Mach wave during supersonic expansion

Prandtl Meyer Function

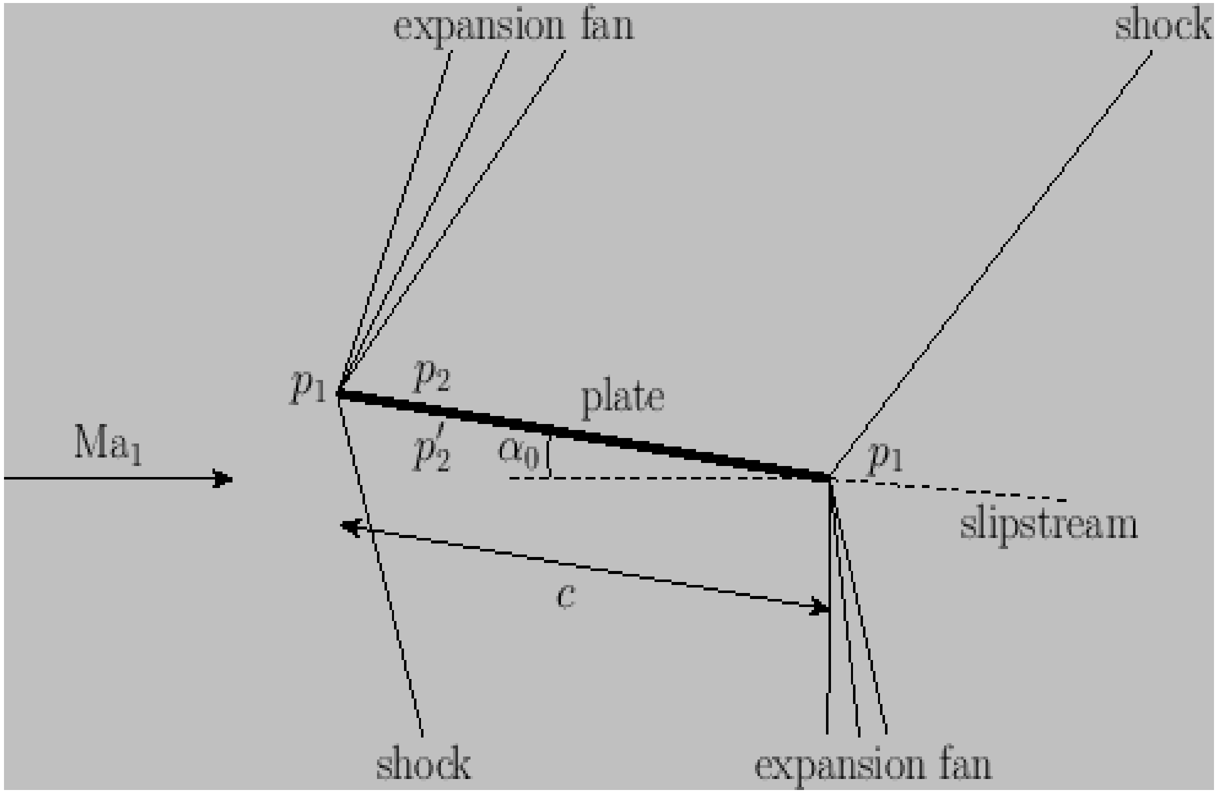
$$\int_{\theta_1}^{\theta_2} d\theta = \int_{M_1}^{M_2} \sqrt{M^2-1}\,\frac{dV}{V}$$

$$\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - 0 = \int_{M_1}^{M_2} \frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}M^2}\,\frac{dM}{M}$$

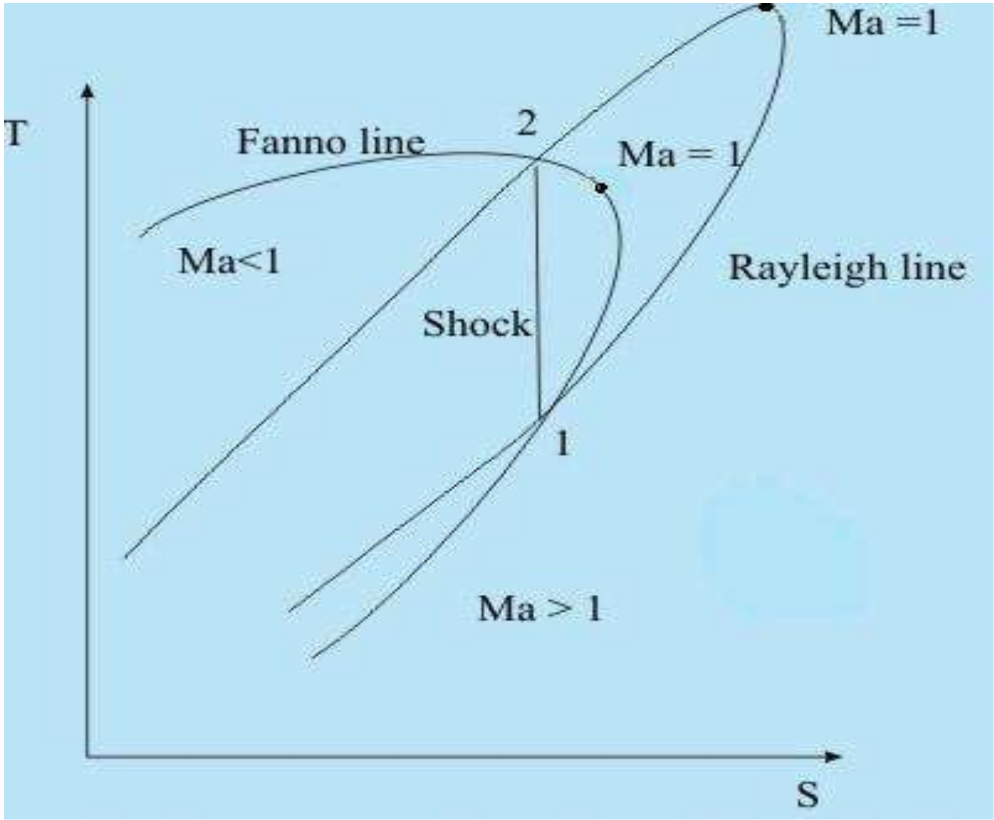
$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\left(M^2-1\right)} - \tan^{-1}\sqrt{M^2-1}$$

Here, v is called as the Prandtl-Meyer function.

Shock Expansion Method for Flow over Airfoil



Intersection of Fanno line and Rayleigh line



Intersection of Fanno line and Rayleigh line and the solution for normal shock condition

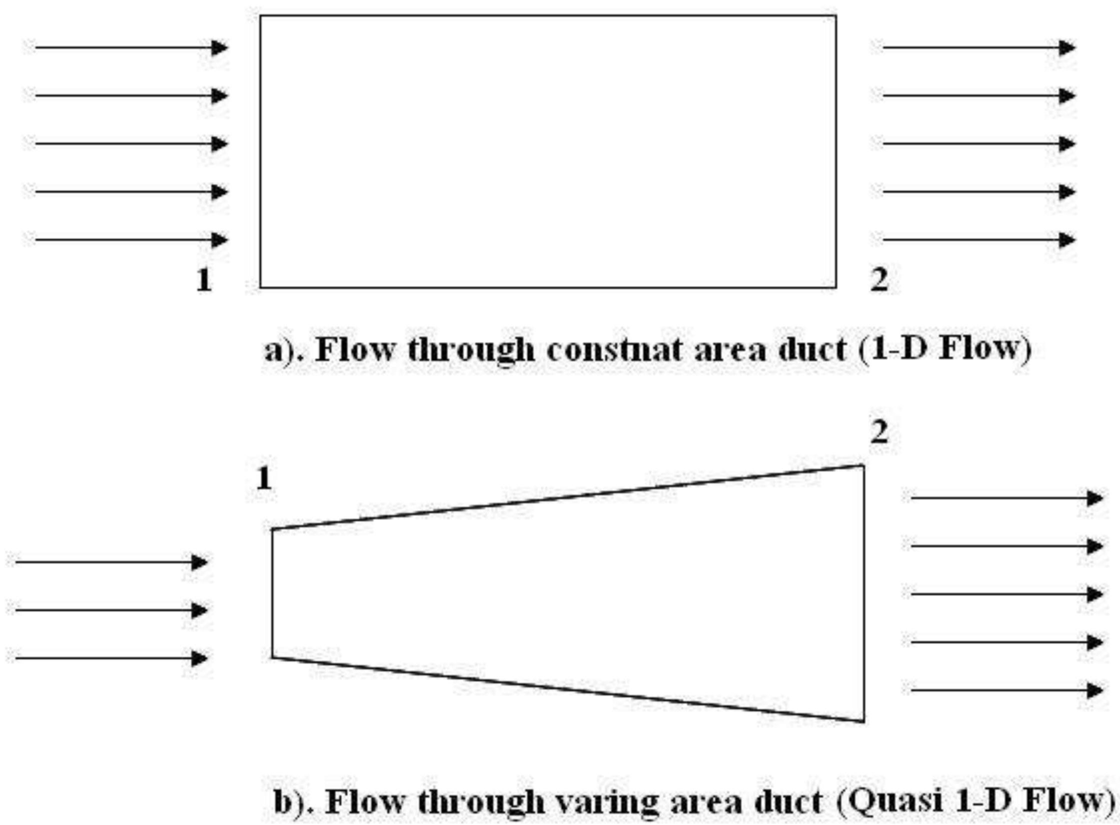
UNIT III

QUASI-ONE DIMENSIONAL FLOWS

UNIT - III

CLOs	Course Learning Outcome
CLO 7	Illustrate the concepts of quasi one dimensional flow for compressible flows
CLO 8	Describe isentropic flow in nozzles, area Mach relations, choked flow, under and over expanded nozzles, slipstream line.
CLO 9	Understand the impact of heat and Friction in duct flow and fanno flow

Isentropic flow in nozzles



Control volume for 1D and Quasi 1D flows

Mass conservation equation

$$\frac{\partial}{\partial t}\iiint^{v} \rho \, dv + \iint^{s} (\rho \vec{V} . \overrightarrow{ds}) = \mathbf{0}$$

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

$$\rho u A = const$$

$$d(\rho u A) = 0$$

Momentum conservation equation

$$\frac{\partial}{\partial t}\iiint\limits^{v} \rho\vec{V}\,dv + \iint\limits^{s} (\rho\vec{V}.\overrightarrow{ds})\vec{V} = -\iint\limits^{s} (p\overrightarrow{ds})$$

$$p_1A_1 + \rho_1u_1^2A_1 + \int_{A_1}^{A_2} p\,dA = p_2A_2 + \rho_2u_2^2A_2$$

$$pA + \rho u^2A + pdA = (p+dp)(A+dA) + (\rho + d\rho)(u+du)^2(A+dA)$$

$$\rho u^2 dA + \rho uAdu + Au^2 d\rho = 0$$

$$dp = -\rho udu$$

Energy conservation equation

$$\frac{\partial}{\partial t}\iiint^{v} \rho\left[e+\frac{V^2}{2}\right]dv + \iint^{s}(\rho\vec{V}.\overrightarrow{ds})\left[e+\frac{V^2}{2}\right] = -\iint^{s}(p\vec{V}.\overrightarrow{ds})$$

$$\left(\rho_1\left[e_1+\frac{u_1^2}{2}\right]\right)(-u_1A_1)+\left(\rho_2\left[e_2+\frac{u_2^2}{2}\right]\right)(u_2A_2) = -(-p_1u_1A_1+p_2u_2A_2)$$

$$p_1u_1A_1+\left(\rho_1u_1A_1\left[e_1+\frac{u_1^2}{2}\right]\right) = p_2u_2A_2+\left(\rho_2u_2A_2\left[e_2+\frac{u_2^2}{2}\right]\right)$$

$$h = e + \frac{p}{\rho}$$

$$dh + udu = Constant$$

Area-velocity relation for Quasi-One Dimensional flow

- Consider the differential mass conservation equation

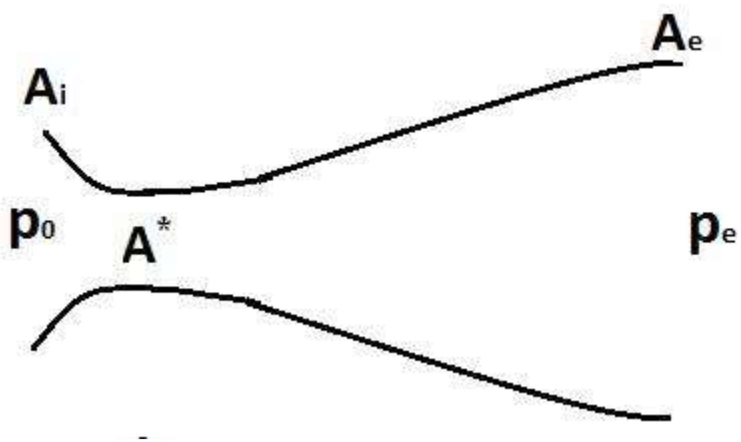
$$d(\rho uA) = 0$$

$$uA[d(\rho)] + \rho u[d(A)] + \rho A[d(u)] = 0$$

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$$

$$\frac{dA}{A} = \left(M^2 - 1\right)\frac{du}{u}$$

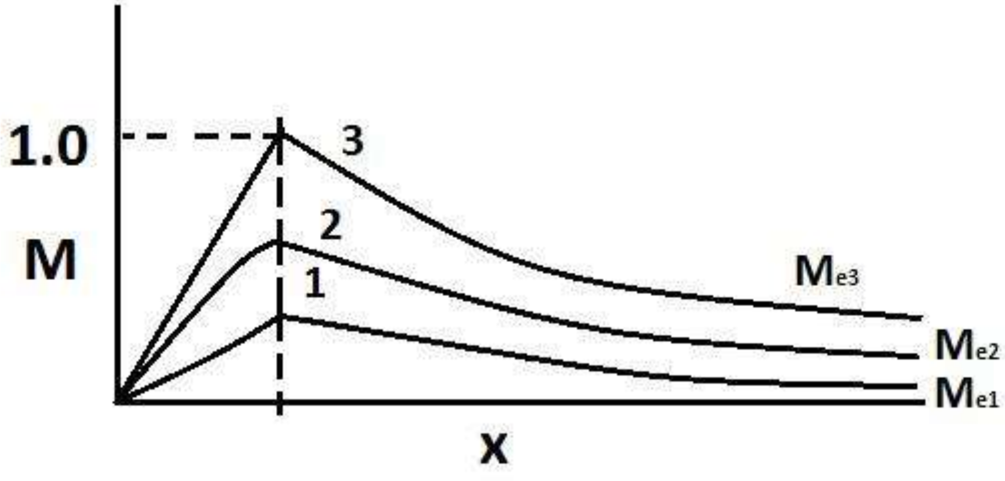
Isentropic flow through varying area duct



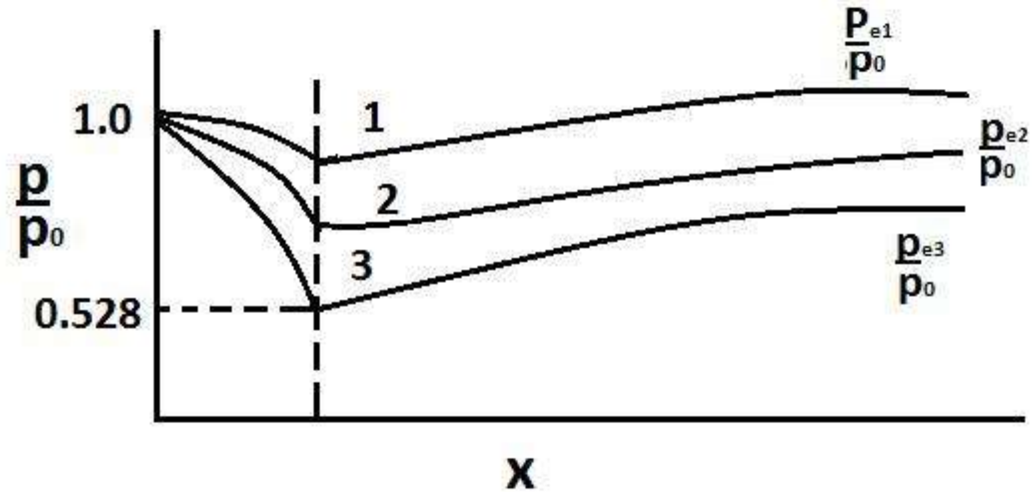
Flow through convergent divergent duct

$$\left[\frac{A}{A^*}\right]^2 = \frac{1}{M^2}\left[\frac{2}{(\gamma+1)}\left(1+\frac{(\gamma-1)}{2}M^2\right)\right]^{\frac{(\gamma+1)}{(\gamma-1)}}$$

Nozzle flow

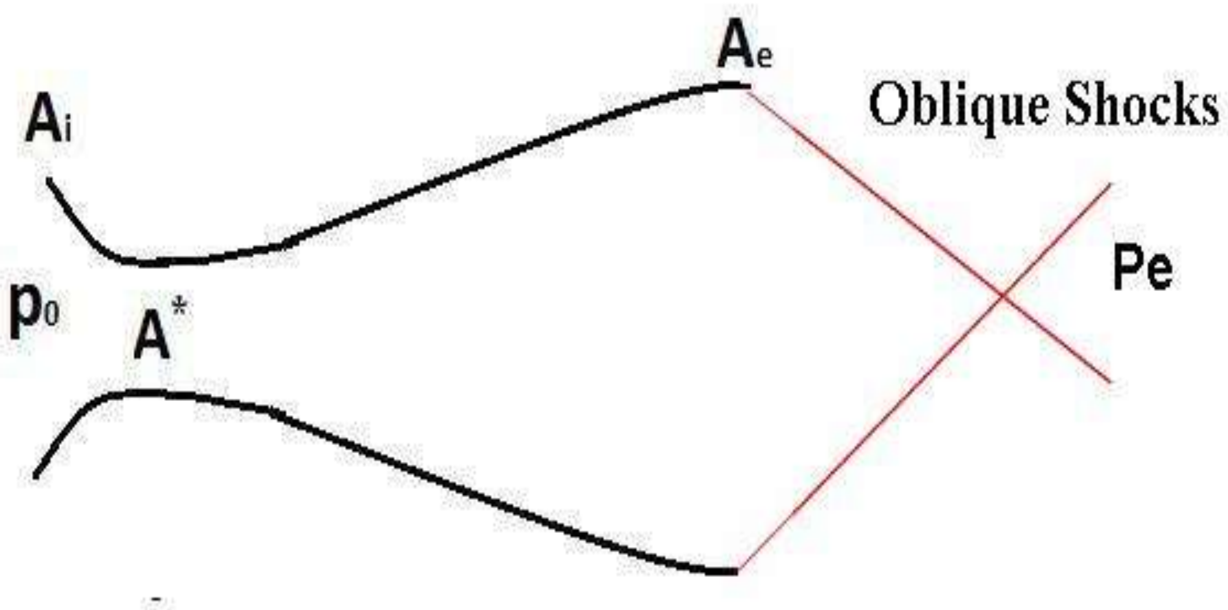


Mach number variation along the length of the duct for various exit pressure conditions



Pressure variation along the length of the duct for various exit pressure conditions

Oblique shock pattern for over expanded condition

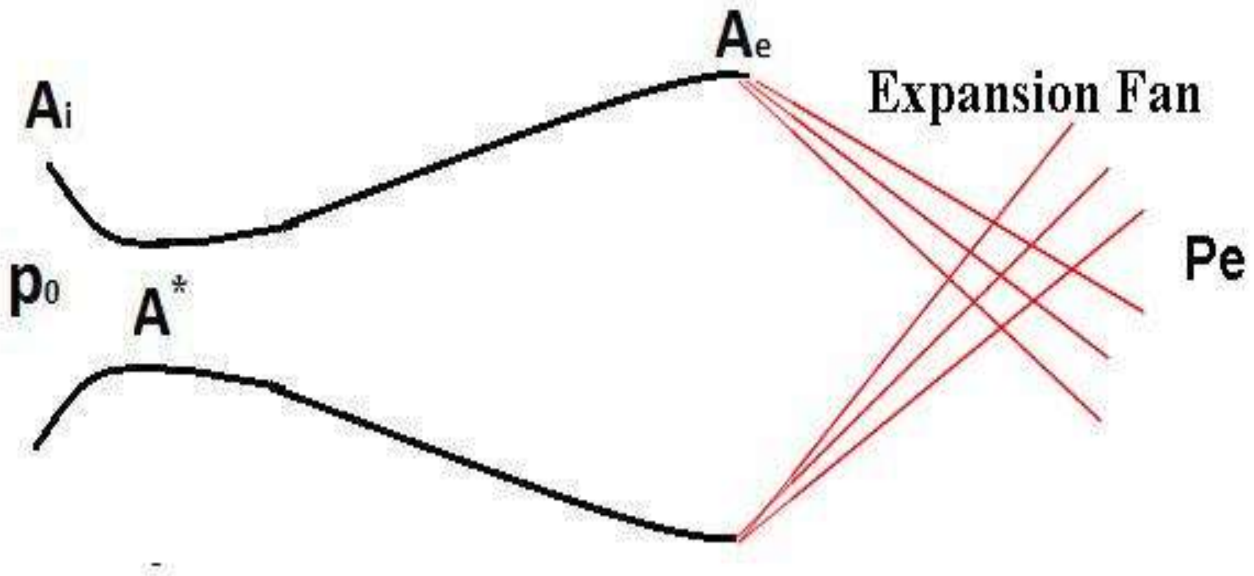


Oblique shock pattern for over expanded condition

Under expanded Nozzle

- Discharges fluid at an exit pressure greater than the external pressure
- This owes to the exit area being too small for an optimum area ratio
- The expansion of the fluid is incomplete
- Further expansion happens outside of the nozzle
- Nozzle exit pressure is greater than local atmospheric pressure

Expansion fan pattern for under expanded condition



Expansion fan pattern for under expanded condition

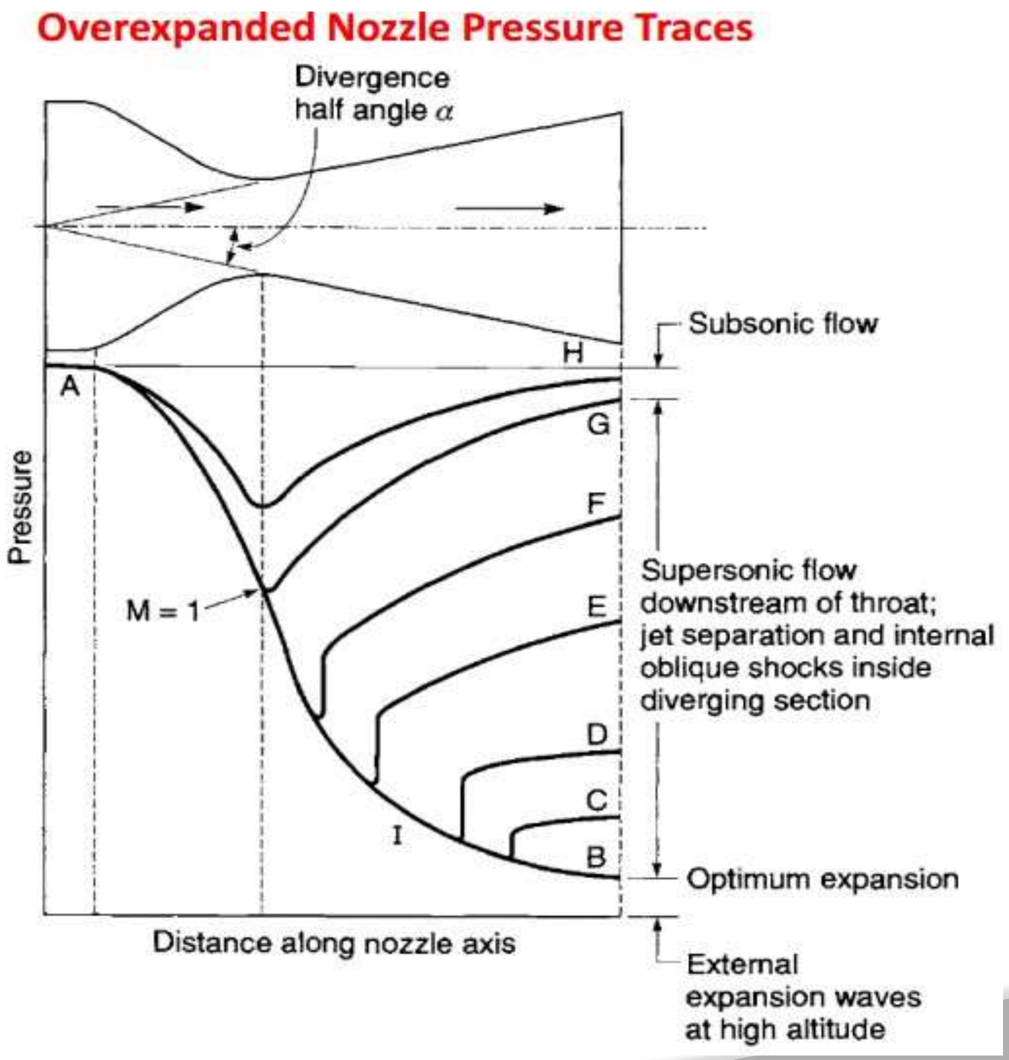
Over expanded Nozzle

- Fluid exits at lower pressure than the atmosphere
- This owes to an exit area too large for optimum Curve AB:
- variation of axial pressure with optimum back pressure for given nozzle area ratio Curves AC thru AF:
- Variation of axial pressure for increasingly higher external pressure. Sudden rise in pressure represents flow separation. This result in shock formation (sharp pressure rise) within the nozzle.
- This shock is pushed upstream toward the nozzle as ambient pressure increases Curve AG:

Over expanded Nozzle

- Significant back pressure has caused the nozzle to “unstart” meaning the nozzle is no longer choked.
- The diverging section now decelerates the flow Curve AH.
- Further back pressure increase drives reduces the pressure and resulting exit velocity

Over expanded Nozzle



Choked Flow

- Choked flow is a compressible flow effect.
- The parameter that becomes "choked" or "limited" is the fluid velocity.
- Choked flow is a fluid dynamic condition associated with the Venturi effect. When a flowing fluid at a given pressure and temperature passes through a restriction into a lower pressure environment the fluid velocity increases.

Choked Flow

- At initially subsonic upstream conditions, the conservation of mass principle requires the fluid velocity to increase as it flows through the smaller cross-sectional area of the restriction.
- At the same time, the Venturi effect causes the static pressure, and therefore the density, to decrease downstream beyond the restriction.
- Choked flow is a limiting condition where the mass flow rate will not increase with a further decrease in the downstream pressure environment while upstream pressure is fixed.

Chocked mass flow rate of the nozzle

$$\dot{m} = \rho^* u^* A^*$$

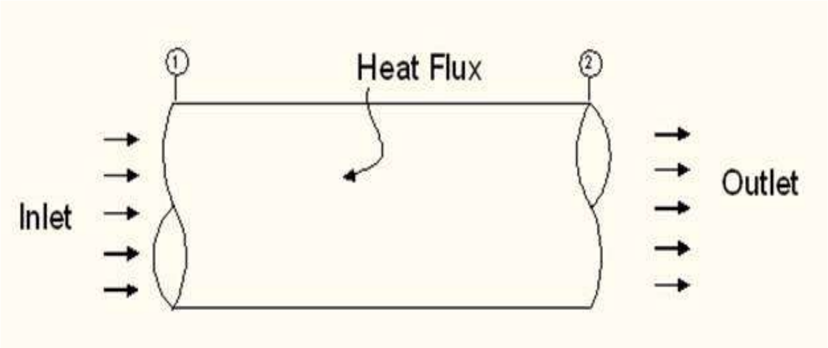
$$\frac{\rho_0}{\rho^*} = \left[\frac{(\gamma+1)}{2}\right]^{\frac{1}{(\gamma-1)}}$$

$$u^* = a^* = \sqrt{\gamma R T^*}$$

$$\dot{m} = \frac{p_0}{RT_0}\left[\frac{(\gamma+1)}{2}\right]^{\frac{-1}{(\gamma-1)}} \sqrt{\gamma R \frac{T^*}{T_0} T_0 A^*}$$

$$\dot{m} = p_0 \left[\frac{(\gamma+1)}{2}\right]^{\frac{-(\gamma+1)}{2(\gamma-1)}} \sqrt{\frac{\gamma}{RT_0}} A^*$$

- **Flow with friction**



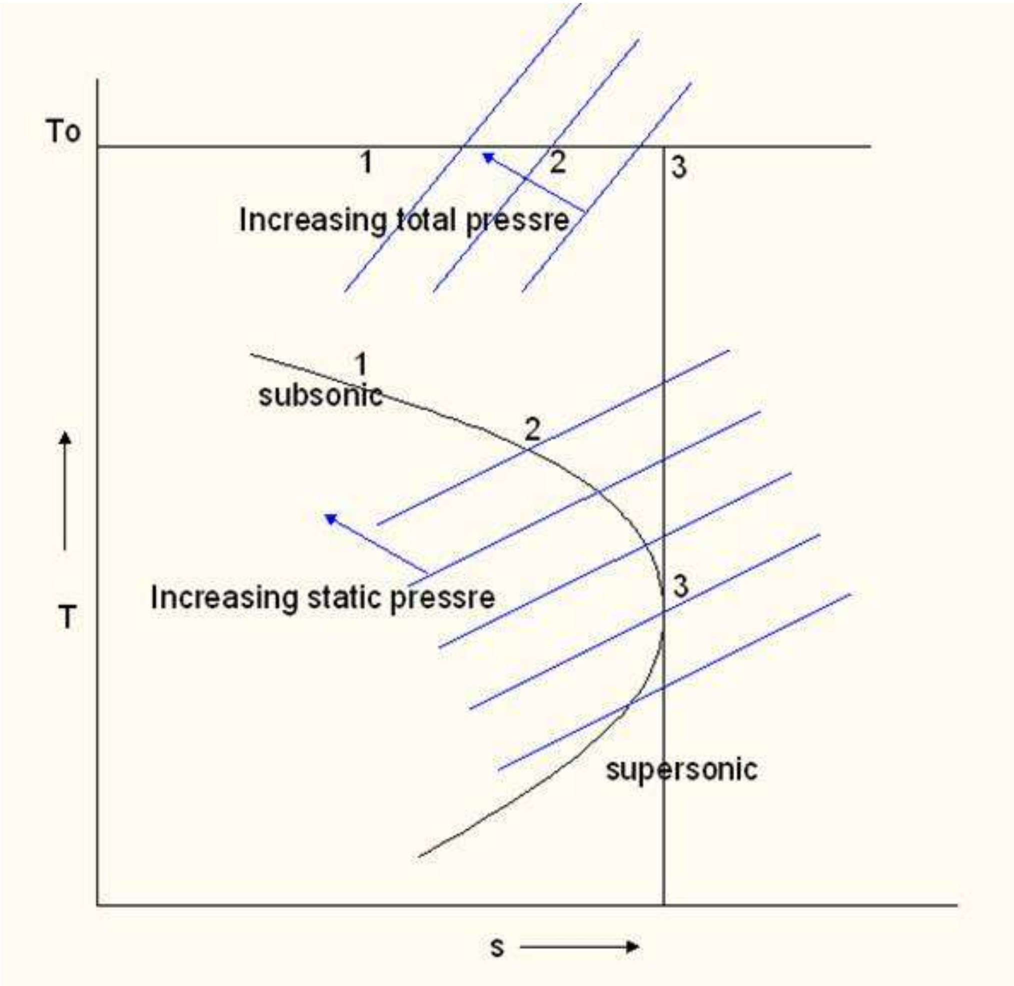
$$\frac{T_{0_2}}{T_{0_1}} = 1$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left(p_{0_2}/p_2\right)p_2}{\left(p_{0_1}/p_1\right)p_1} = \frac{\left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}}\frac{p_2}{p_1}$$

$$\frac{p_{0_2}}{p_{0_1}} = \left[\frac{2+(\gamma-1)M_2^2}{2+(\gamma-1)M_1^2}\right]^{\gamma+1/2(\gamma-1)}\frac{M_1}{M_2}$$

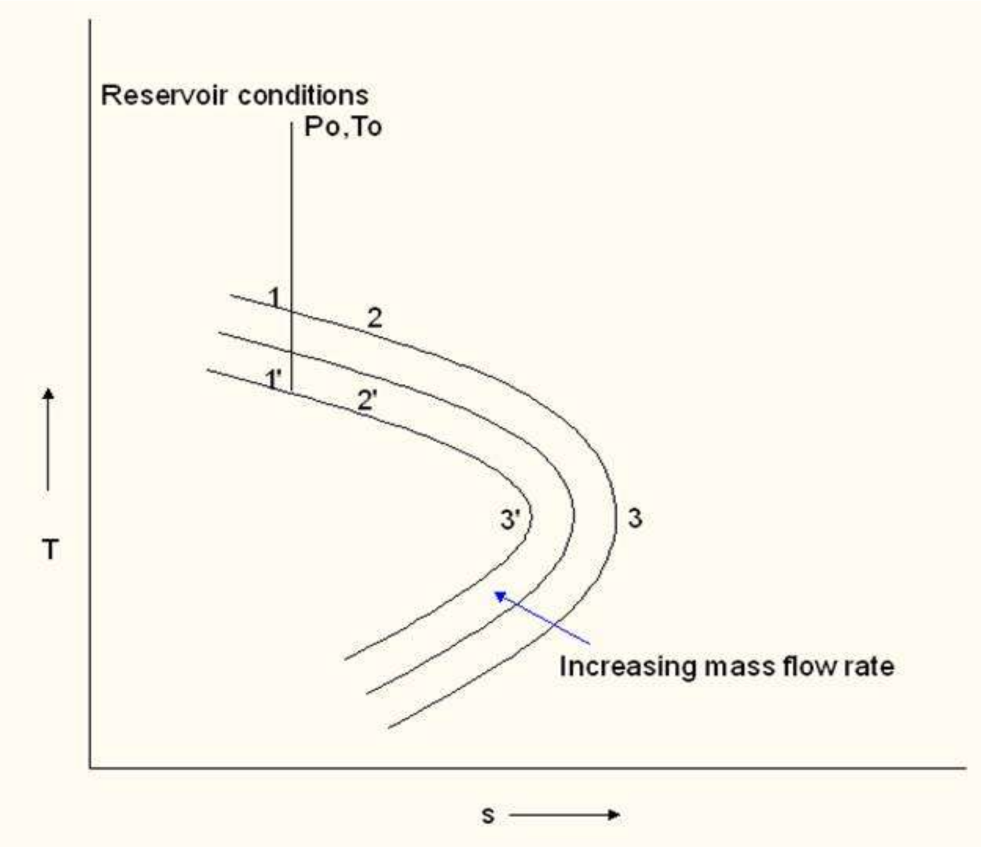
$$\frac{\rho_{0_2}}{\rho_{0_1}} = \frac{p_{0_2}}{p_{0_1}}\frac{T_{0_1}}{T_{0_2}} = \frac{p_{0_2}}{p_{0_1}}$$

Fanno line or curve



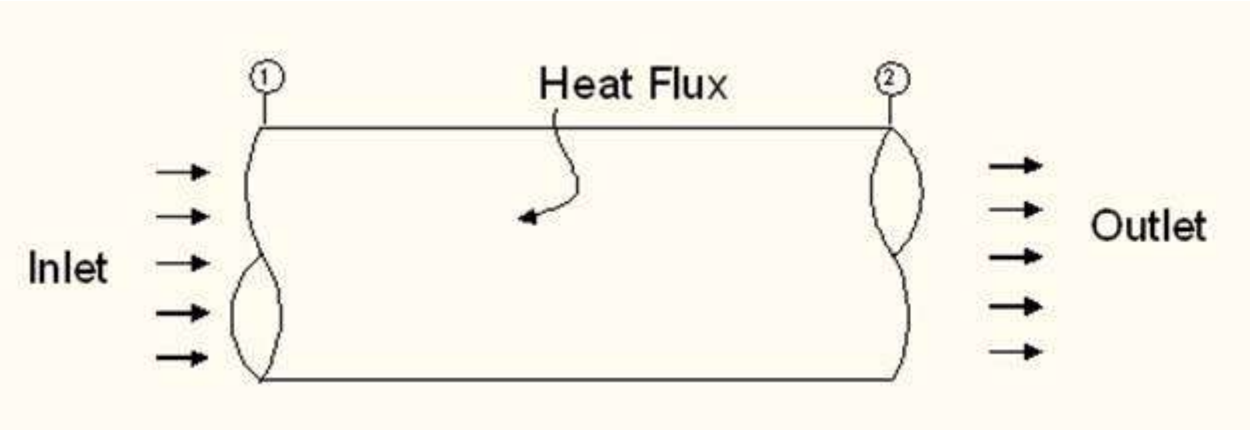
h-s diagram for Fanno flow

Fanno line or curve



h-s diagram for Fanno flow for various mass flow rates

One dimensional flow with heat addition



Typical Control volume for 1D flow with heat addition

$$\rho_1 u_1 = \rho_2 u_2$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

$$\frac{\rho_{0_2}}{\rho_{0_1}} = \frac{p_{0_2}}{p_{0_1}} \frac{T_{0_1}}{T_{0_2}}$$

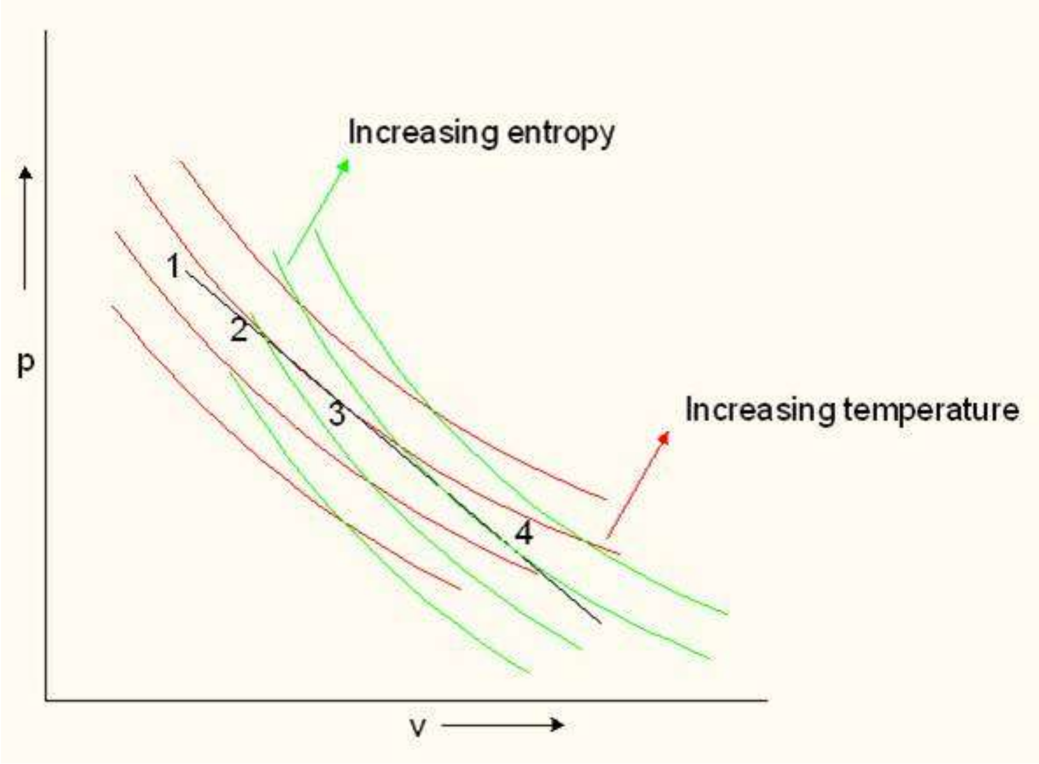
Addition of heat in supersonic flows

- Decreases Mach number
- Increases static pressure
- Increases static temperature
- Decreases total pressure
- Increases total temperature
- Decreases velocity

Addition of heat in subsonic flow

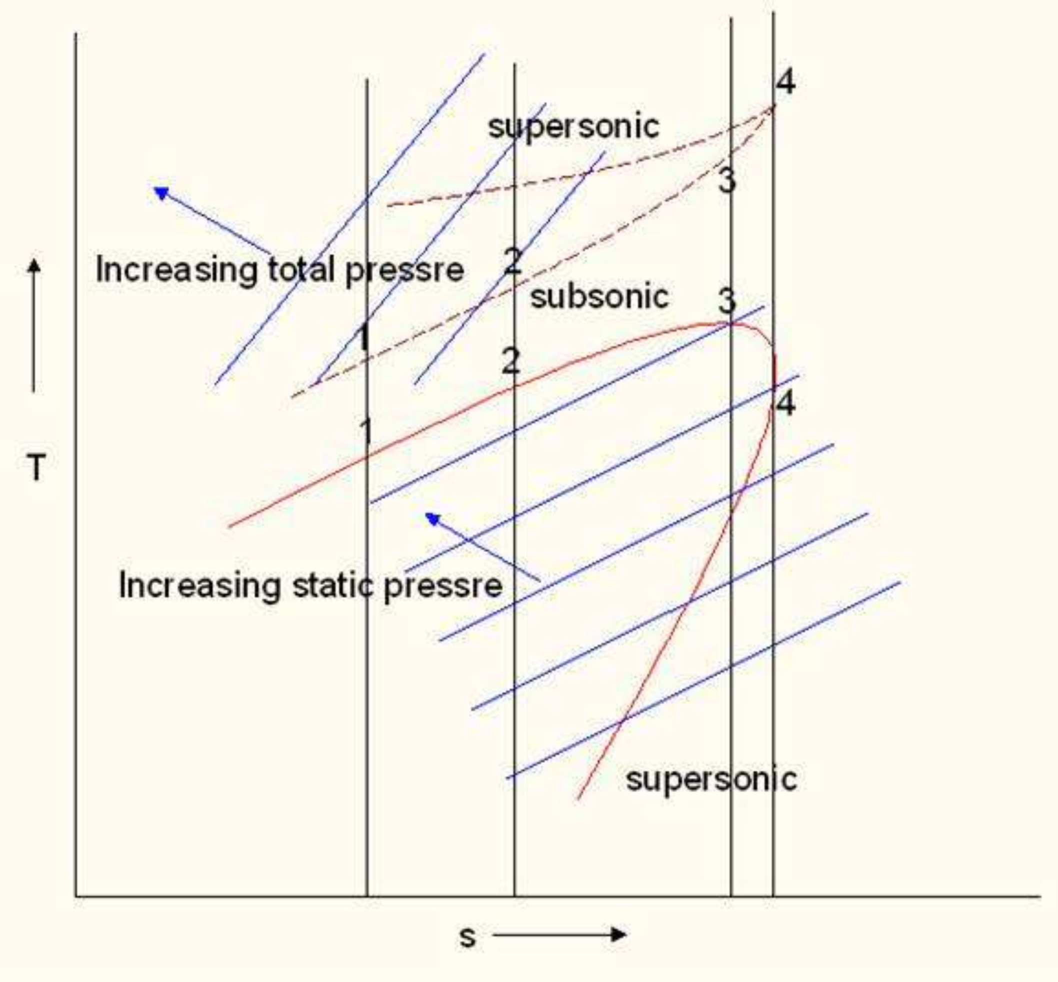
- Increases Mach number
- Decreases static pressure
- Increases static temperature if $M_\infty < \gamma^{-1/2}$ and decreases if $M_\infty > \gamma^{-1/2}$
- Decreases total pressure
- Increases total temperature
- Increases velocity

Rayleigh curve



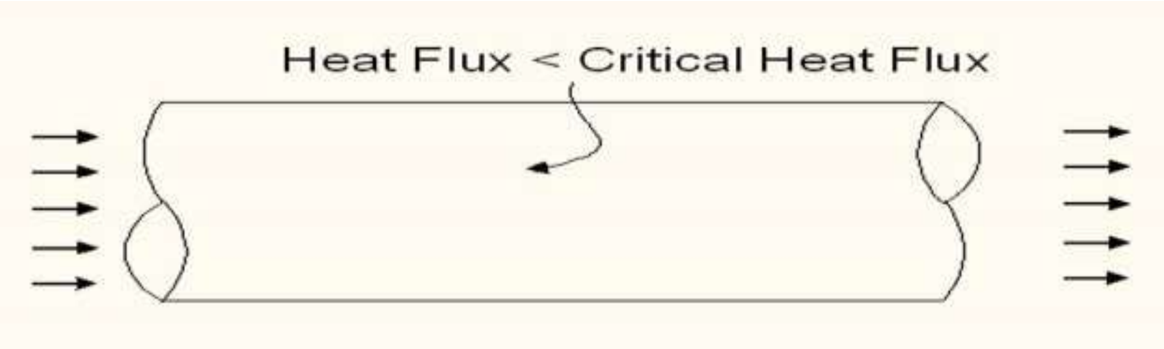
P-V diagram for the heat addition process

Rayleigh curve

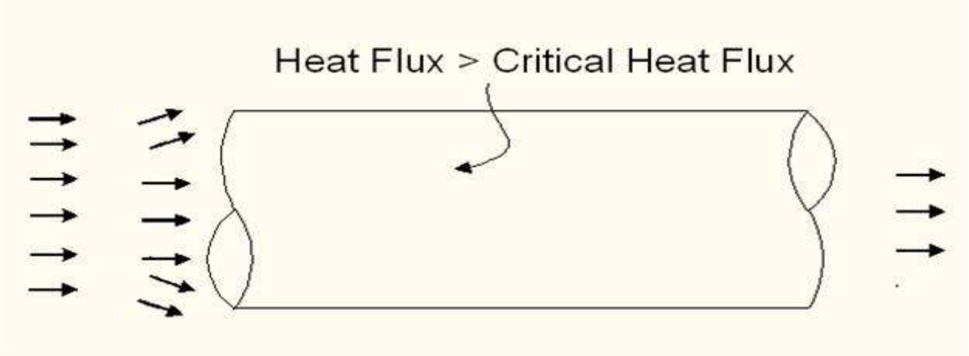


Rankine curve in h-s chart

Choking of the flow with Heat Addition



Streamline pattern for unchocked condition



Streamline pattern for chocked condition

Total Property Line of Rayleigh Curve

- Total enthalpy and entropy for 1D flow with heat addition
- Slope of this total enthalpy curve can be obtained from the above equation as,

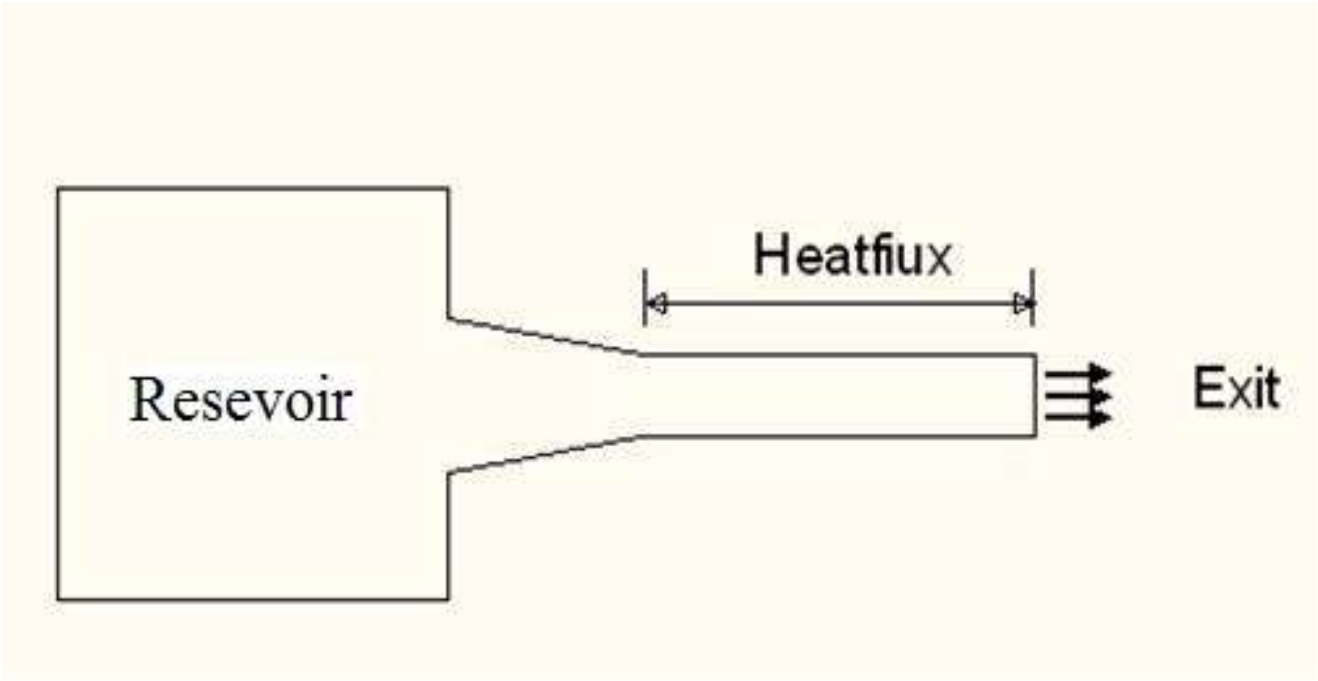
$$\frac{dh_0}{ds} = T$$

- Value of the local temperature on h_0-s diagram gives the slope of the curve.
- However both the lines should meet at highest total enthalpy point or highest entropy or sonic point for this mass flux condition.

Total Property Line of Rayleigh Curve

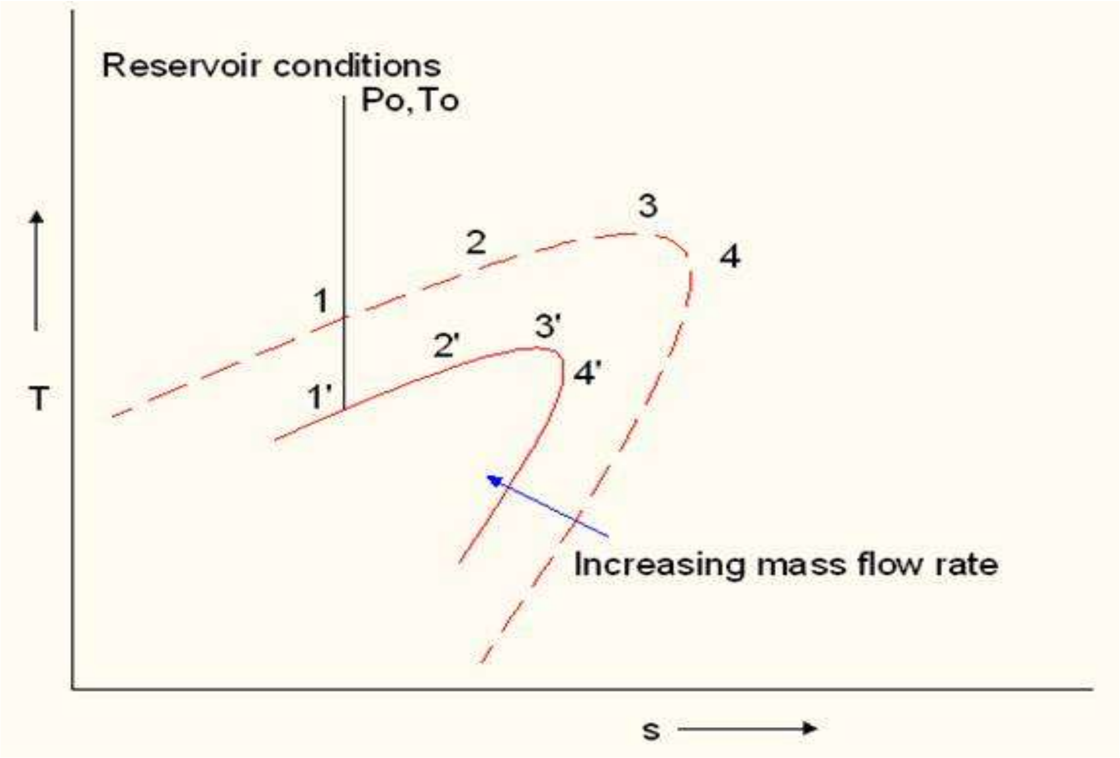
- Hence, supersonic flow total enthalpy line should be at the top of subsonic total enthalpy line which is unlike the static enthalpy entropy lines.
- This fact can also be interpreted from the definition of total enthalpy where velocity appears in square. Since, mass flow rate is same and velocity of supersonic flow is higher than the velocity of subsonic flow, total enthalpy of supersonic flow will be higher than that of subsonic flow.
- Both static and total enthalpy lines cut the constant pressure lines on the corresponding charts which represent static and total pressures respectively.

Effect of Change in Mass Flow Rate on T-S diagram for Rayleigh flow



A proposed experimental set up for 1D heat addition studies

Effect of Change in Mass Flow Rate on T-S diagram for Rayleigh flow



Rayleigh for increase in mass flow rate conditions

UNIT – IV

APPLICATIONS OF COMPRESSIBLE FLOWS AND NUMERICAL TECHNIQUES

UNIT - IV

CLOs	Course Learning Outcome
CLO 10	Describe small perturbation equations for subsonic, transonic, supersonic and hypersonic flow
CLO 11	Understand experimental characteristics of airfoils in compressible flow, supercritical airfoils and area rule.
CLO 12	Explain supersonic nozzle design using method of characteristics.

Small perturbation equations for subsonic, transonic, supersonic and hypersonic flow

- A great number of problems of interest in compressible fluid mechanics are concerned with the perturbation of a known flow pattern.
- The most common case is that of uniform, steady flow.
- Let U denote the uniform flow velocity, which is directed parallel to the x -axis.
- The density, pressure, and temperature are also assumed to be uniform, and are denoted ρ_∞, P_∞ and T_∞ respectively.
- The corresponding sound speed is c_∞ , and the Mach number is $Ma_\infty = U/c_\infty$

Small perturbation equations

- Finally the velocity field of the unperturbed flow pattern is
  - u = U,
  - v =0
- The body disturbs the flow pattern, and changes its velocity field, which is now written
  - u = U+u',
  - v = v'

where u' and v' are known as induced velocity components

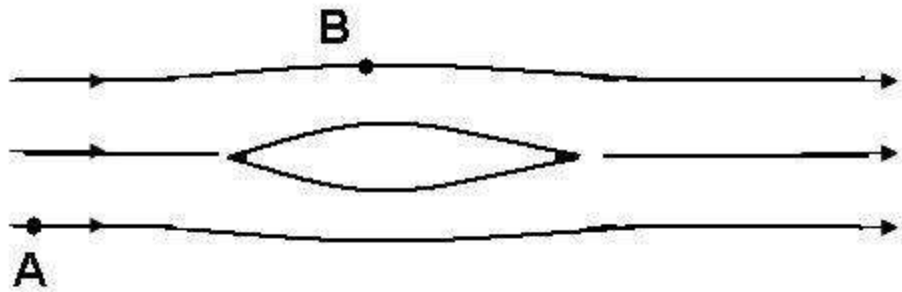
- Equation can be combined with the previous two equations to give

$$c^2 = c_\infty^2 - \frac{1}{2}(\gamma - 1)\left(2\,U\,u' + u'^2 + v'^2\right).$$

Experimental characteristics of airfoils in compressible

Linearization of Velocity Potential Equation

$$\vec{V} = (V_\infty + u')\vec{i} + v'\vec{j} + w'\vec{k}$$



$$\vec{V} = V_\infty \vec{i} + 0\vec{j} + 0\vec{k}$$

Schematic of the perturbed velocity field

Experimental characteristics of airfoils in compressible flow

At location A, velocity is only in x direction. However, presence of body perturbs the components of velocity at location B. Lets represent the general velocity field as,

$$\vec{V} = V_x\vec{i} + V_y\vec{j} + V_z\vec{k}$$

here, $V_x = V_\infty + u'$ and u', v', w' are the perturbed velocities in the x, y and z directions respectively such that.

- $V_y = v'$
- $V_w = w'$

Experimental characteristics of airfoils in compressible flow

We can use these expressions in the known velocity potential equation.

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}+\left(1-\frac{\Phi_z^2}{a^2}\right)\Phi_{zz}$$

$$-2\frac{\Phi_x\Phi_y}{a^2}\Phi_{xy}-2\frac{\Phi_x\Phi_z}{a^2}\Phi_{xz}-2\frac{\Phi_y\Phi_z}{a^2}\Phi_{yz}=0$$

This expression in the form of perturbed velocity potential can be written as,

$$\left[a^2-\left(V_\infty+\frac{\partial\phi}{\partial x}\right)^2\right]\frac{\partial^2\phi}{\partial x^2}+\left[a^2-\left(\frac{\partial\phi}{\partial y}\right)^2\right]\frac{\partial^2\phi}{\partial y^2}+\left[a^2-\left(\frac{\partial\phi}{\partial z}\right)^2\right]\frac{\partial^2\phi}{\partial z^2}$$

$$-2\left(V_\infty+\frac{\partial\phi}{\partial x}\right)\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y}-2\left(V_\infty+\frac{\partial\phi}{\partial x}\right)\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial x\partial z}-2\frac{\partial\phi}{\partial y}\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial y\partial z}=0$$

Pressure Coefficient for Small Perturbations

We can approximate the pressure coefficient under the assumption of small perturbation in the velocity field. We know that,

$$C_p = \frac{P - P_\infty}{\frac{1}{2} P_\infty V_\infty^2}$$

$$C_p = -\frac{2u'}{V_\infty}$$

This expression gives the local pressure coefficient based on perturbed velocity and freestream

Supercritical airfoil

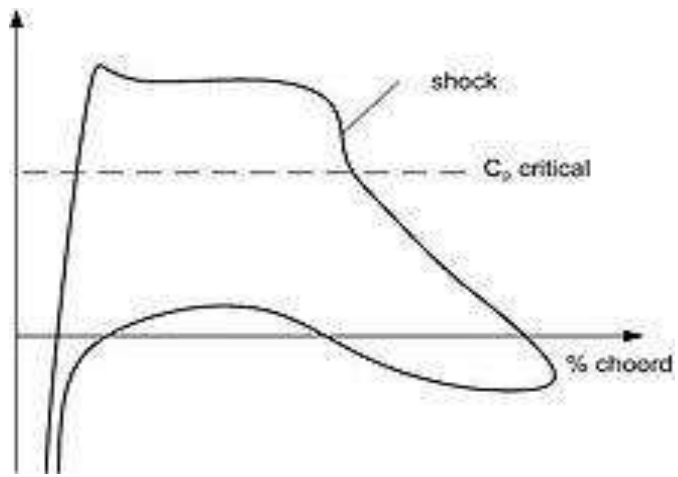
- A Supercritical airfoil is an airfoil designed, primarily, to delay the onset of wave drag in the transonic speed range.
- Supercritical airfoils are characterized by their flattened upper surface, highly cambered (curved) aft section, and smaller leading edge radius compared with traditional airfoil shapes.
- The supercritical airfoils were designed in the 1960s, by then NASA Engineer Richard Whitcomb, and were first tested on a modified North American T-2C Buckeye.
- After this first test, the airfoils were tested at higher speeds on the TF-8A Crusader While the design was initially developed as part of the supersonic transport (SST) project at NASA, it has since been mainly applied to increase the fuel efficiency of many high subsonic aircraft.

Supercritical airfoils feature four main benefits

The supercritical airfoil shape is incorporated into the design of a supercritical wing.

- they have a higher drag divergence Mach number,
- they develop shock waves further aft than traditional airfoils
- they greatly reduce shock-induced boundary layer separation,
- their geometry allows for more efficient wing design (e.g., a thicker wing and/or reduced wing sweep, each of which may allow for a lighter wing).

Supercritical airfoil



- Supercritical airfoil Mach Number/pressure coefficient diagram. The sudden increase in pressure coefficient at mid chord is due to the shock. (y-axis: Mach number (or pressure coefficient, negative up); x-axis: position along chord, leading edge left)

Second order equation for transonic flows

- The numerical solution of the compressible Euler and Navier - Stokes equations in primitive variables form requires the use of artificial viscosity or upwinding.
- Methods that are first - order - accurate are too dissipative and reduce the effective Reynolds number substantially unless a very fine grid is used.
- A first-order finite element method for the solution of the Euler and Navier - Stokes equations can be constructed by adding Laplacians of the primitive variables to the governing equations.

Second order equation for transonic flows

- Second-order schemes may require fourth-order dissipation and higher- order elements.
- A finite element approach is proposed in which the fourth-order dissipation is recast as the difference of two Laplacian operators, allowing the use of bilinear elements.
- The Laplacians of the primitive variables of the first-order scheme are thus balanced by additional terms obtained from the governing equations themselves, tensor identities or other forms of nodal averaging.

Area Rule

Whitcomb’s Area Rule

- The Whitcomb area rule, also called the transonic area rule, is a design technique used to reduce an aircraft's drag transonic and supersonic speeds, particularly between Mach 0.75 and 1.2.
- This is one of the most important operating speed ranges for commercial and military fixed-wing aircraft today, with transonic acceleration being considered an important performance metric for combat aircraft and necessarily dependent upon transonic drag.

Area Rule

- At high-subsonic flight speeds, the local speed of the airflow can reach the speed of sound where the flow accelerates around the aircraft body and wings.
- The speed at which this development occurs varies from aircraft to aircraft and is known as the critical Mach number.
- The resulting shock waves formed at these points of sonic flow can greatly reduce power, which is experienced by the aircraft as a sudden and very powerful drag, called wave drag.
- To reduce the number and power of these shock waves, an aerodynamic shape should change in cross sectional area as smoothly as possible.

Area Rule

- The area rule says that two airplanes with the same longitudinal cross-sectional area distribution have the same wave drag, independent of how the area is distributed laterally (i.e. in the fuselage or in the wing).
- Furthermore, to avoid the formation of strong shock waves, this total area distribution must be smooth.
- As a result, aircraft have to be carefully arranged so that at the location of the wing, the fuselage is narrowed or "waisted", so that the total area doesn't change much.
- Similar but less pronounced fuselage waisting is used at the location of a bubble canopy and perhaps the tail surfaces.

Area Rule

- The area rule also holds true at speeds exceeding the speed of sound, but in this case the body arrangement is in respect to the Mach line for the design speed.
- For example, consider that at Mach 1.3 the angle of the Mach cone formed off the body of the aircraft will be at about $\mu = \arcsin(1/M) = 50.3°$ (μ is the angle of the Mach cone, or simply Mach angle).
- In this case the "perfect shape" is biased rearward; therefore, aircraft designed for high speed cruise usually have wings towards the rear.

Area Rule

A classic example of such a design is the Concorde

- When applying the transonic area rule, the condition that the plane defining the cross-section meets the longitudinal axis at the Mach angle μ no longer prescribes a unique plane for μ other than the 90° given by $M = 1$.
- The correct procedure is to average over all possible orientations of the intersecting plane.

Theory of characteristics (method of characteristics)

- The method of characteristics has been used for many years to compute supersonic irrotational flows.
- Although the method has a strong analytical basis, its practical implementation is, essentially, always numerical and it is then used to compute the values of the flow variables at a series of distinct points in the flow rather than continuously throughout the flow field.

Theory of characteristics
(method of characteristics)

- Let's consider a general steady two-dimensional irrotational flowfield.
- The velocity potential (ϕ) equation for such flow field.

$$\left(1-\frac{u^2}{a^2}\right)\frac{\partial^2\phi}{\partial x^2}+\left(1-\frac{v^2}{a^2}\right)\frac{\partial^2\phi}{\partial y^2}-\frac{2uv}{a^2}\frac{\partial^2\phi}{\partial x\partial y}=0$$

Consider the change in any flow variable, *f*, *df* which can be determined by small changes in the coordinates *dx* and *dy* .

The change in the variable, *df*,

$$df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$$

Theory of characteristics (method of characteristics)

- We can introduce the local Mach angle, *α,* where (M=1/sinα, $\sqrt{M^2-1}$ =1/tan α) by replacing Mach number as,

$$\left(\frac{dy}{dx}\right)_{ch} = \frac{\cos\theta\sin\theta \pm \cos\alpha\sin\alpha}{\sin^2\alpha - \cos^2\alpha}$$

- After much manipulation and rearrangement, it can be shown that this equation gives:

$$\left(\frac{dy}{dx}\right)_{ch} = tan(\theta \pm \propto)$$

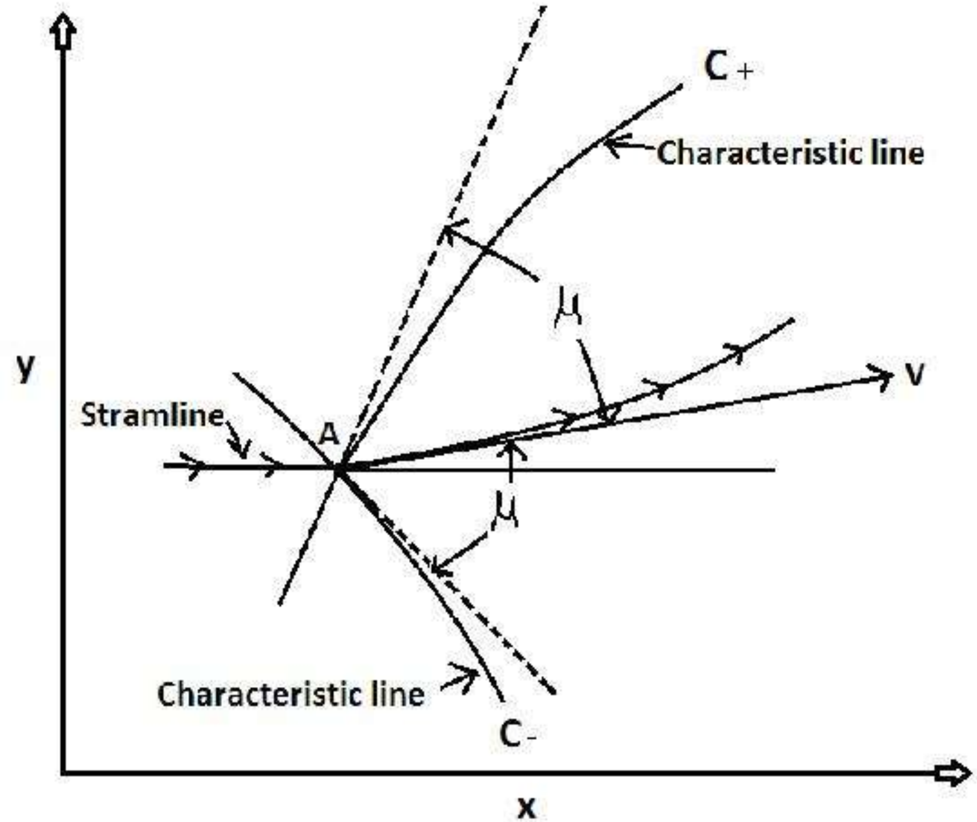
Governing Equation

- We know about the direction of the characteristic line obtained from the indeterminacy of the equation for zero denominators. However finiteness of the differential compels the zero value of the numerator.
- This condition evolves the equation to be solved along the characteristic lines.

$$(1 - u^2/a^2)dudy + (1 - v^2/a^2)dvdx = 0$$

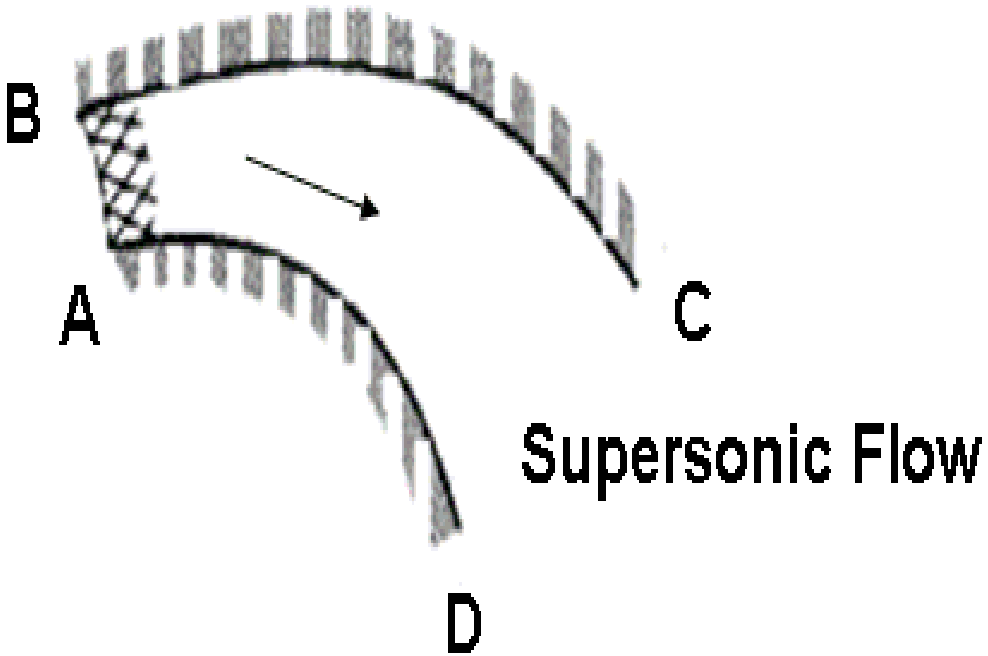
$$\frac{dv}{du} = \frac{(1 - u^2 / v^2)}{(1 - v^2 / a^2)(dy / dx)}$$

Governing Equation



Schematic representation of the details of the characteristic line

Flow through duct



Supersonic flow through duct

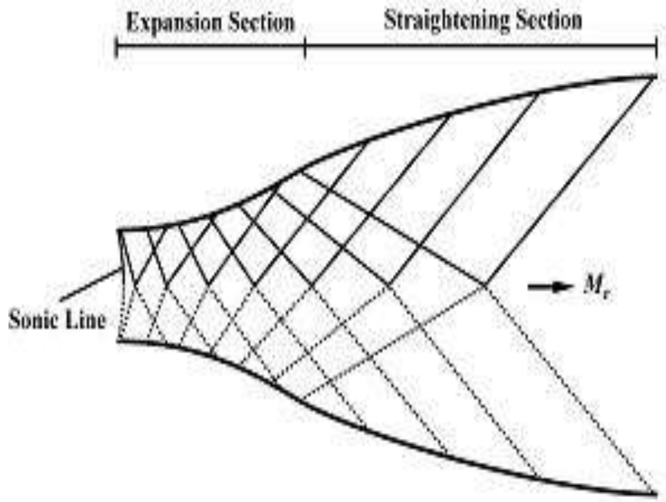
Procedure for using the method of characteristic lines

- The conditions on some initial line must be specified, e.g., conditions on the line AB must be specified.
- The shape of the walls, e.g., AD and BC , must be known.
- Using the initial values of the variables on line A, determine the stagnation pressure, temperature, etc.
- Starting with a series of chosen points on line AB, march the solution forward to the points defined by the intersection of characteristics with each other or with the wall as indicated.
- At each point, use the calculated values of ν and ϑ to get flow variables.

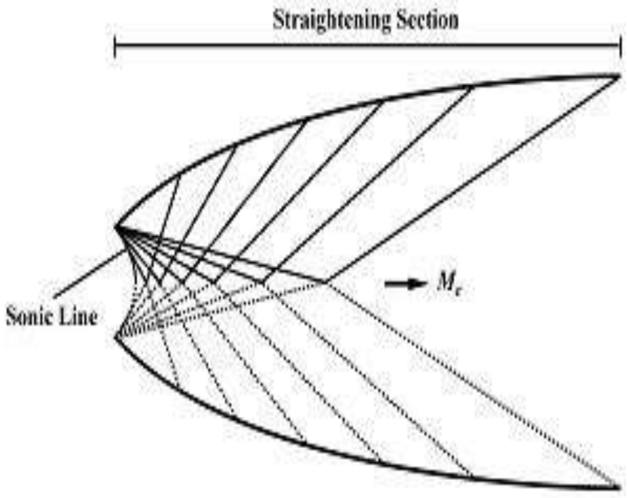
A computer program based on this procedure can be easily developed.

Nozzle Design

- Supersonic nozzles are used in a variety of engineering applications to expand a flow to desired supersonic conditions.
- Supersonic nozzles can be divided into two different types:



Gradual-expansion nozzles



Minimum-length nozzles

Nozzle Design

- Gradual-expansion nozzles are typically used in applications where maintaining a high-quality flow at the desired exit conditions is of importance (e.g., supersonic wind tunnels).
- For other types of applications (e.g., rocket nozzles), the large weight and length penalties associated with gradual-expansion nozzles make them unrealistic; therefore minimum-length nozzles, which utilize a sharp corner to provide the initial expansion, are commonly used.

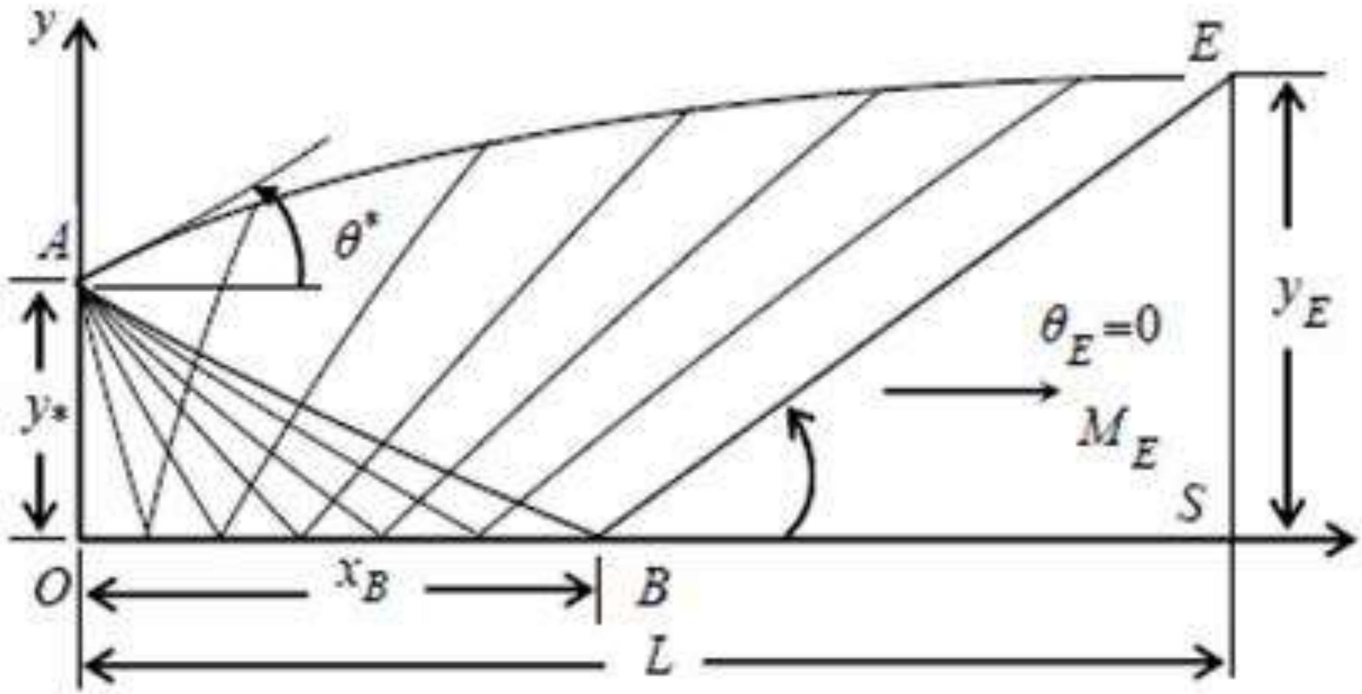
Nozzle Design

- For both gradual-expansion and minimum-length nozzles, the flow can be divided into simple and non-simple regions. A non-simple region is characterized by Mach wave reflections and intersections.
- In order to meet the requirement of uniform conditions at the nozzle exit, it is desirable to minimize the non-simple region as much as possible.
- This can be performed by designing the nozzle surface such that Mach waves (e.g., characteristics) are not produced or reflected while the flow is straightened.
- The Method of Characteristics is therefore applied to allow the design of a supersonic nozzle which meets these requirements.

Design of Minimum-Length Nozzle (MLN)

- It should be noted that for this two-dimensional nozzle configuration, flow symmetry implies that only half of the nozzle is physically required, assuming that the characteristic reflections in the non-simple region are maintained.
- Therefore, we can make the assumption of a half-symmetric minimum-length nozzle, in which a nozzle flap is extended from the symmetry plane such that it meets the length requirement for the last characteristic intersecting the nozzle surface

Design of Minimum-Length Nozzle (MLN)



Schematic of characteristic lines for MLN

Implementation of Method of Characteristics

- The two-dimensional Method of Characteristics is a relatively simple analytical model for analyzing supersonic two-dimensional flow.
- This analysis is performed by considering the characteristic lines in the flow.
- Points along each characteristic have five important properties: M (Mach number), θ (flow angle), ν (Prandtl-Meyer function), and x and y (position). For the assumption of steady, supersonic, we know that, $\theta \pm v = constant$
- The constant for summation can be said to be K+ and for subtraction to be K-.
- These constants are the Riemann invariants, which are constant along the characteristics C+ and C-.

UNIT-V

EXPERIMENTAL METHODS IN COMPRESSIBLE FLOWS

UNIT - V

CLOs	Course Learning Outcome
CLO 13	Illustrate working principle of subsonic wind tunnels, supersonic wind tunnels, shock tunnels
CLO 14	Explain free-piston shock tunnel, detonation-driven shock tunnels, and expansion tubes and characteristic features, their operation and performance.
CLO 15	Demonstrate flow visualization techniques for compressible flows.

INTRODUCTION

- With the advancement of aerospace vehicles, the human's dream is to fly faster and higher.
- Speed of manned and unmanned flight vehicles has increased by some orders of magnitude over the last few decades. As the speed of the vehicle is increased, the aerodynamic environment becomes increasingly hostile.
- At speeds around the local speed of sound (Transonic) and higher (Supersonic), the aerodynamic loads increase and their distributions change.
- When the speed of the vehicle becomes several times higher than the speed of sound (Hypersonic), additional problem of aerodynamic heating demands the change in design geometry and materials.

INTRODUCTION

- At still higher speeds (hypervelocity), the behavior of air begins to change significantly; both physically and chemically.
- There is a conventional 'rule-of-thumb' that defines the flow regimes based on the free stream Mach number (M_∞) i.e.
  - Subsonic: $0 < M_\infty < 0.8$;
  - Transonic: $0.8 < M_\infty > 1.2$
  - Supersonic: $1.2 > M_\infty > 5$;
  - Hypersonic: $M_\infty > 5$

Aerodynamic test facilities

- Low speed wind tunnel (continuous type; up to 40 m/s)
- High speed wind tunnel (intermittent/blow down type; Mach 3, 600m/s)
- Shock tunnel (impulse type; Mach 7, 2km/s)
- Free piston shock tunnel (impulse type, Mach 4-10, 5km/s)
- Expansion tube (impulse type, Mach 10, 10km/s)

SUBSONIC (Low Speed) Wind Tunnel

- In general, the wind tunnels are the devices which provide an airstream flowing under controlled conditions for external/internal flow simulations in the laboratory.

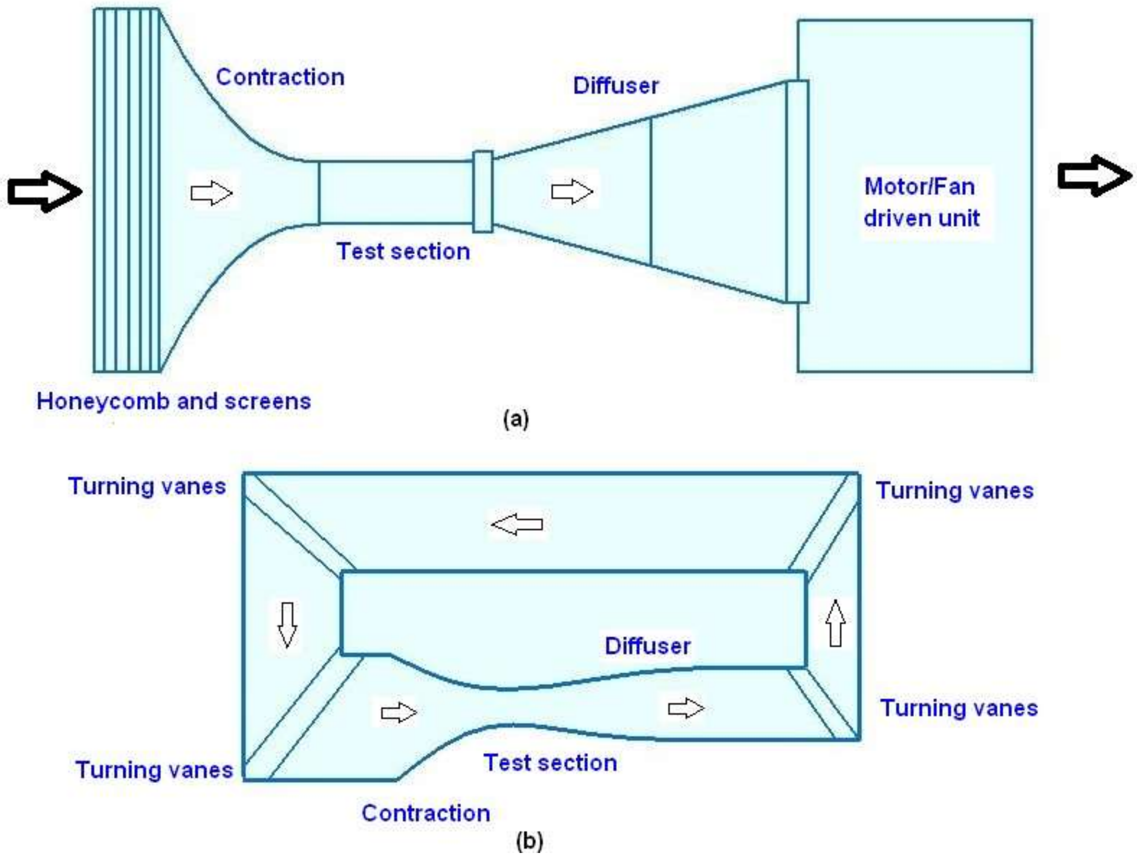
- The most fundamental experiments undertaken in the wind tunnel are the force/heat transfer measurement and flow visualization on aerodynamic models.

- The flows generated in the test section of the tunnel may be laminar/turbulent, steady/unsteady etc.

SUBSONIC (Low Speed) Wind Tunnel

- The other features may be study of boundary layer separation, vortex flow generation etc.
- The low speed wind tunnels limit their speed to 50-60 m/s and based on the need, the tunnel may be designed.
- Depending on the discharge of the air flow to atmosphere or recirculation of air, it is classified as open or closed circuit wind tunnels.

SUBSONIC (Low Speed) Wind Tunnel



Schematic representation of wind tunnel: (a) open circuit wind tunnel;(b) closed circuit wind tunnel.

Wind tunnel components

Motor/Fan Driven unit: This is the air supply unit that drives the air flow in the wind tunnel.

Typically, the fan is axial/centrifugal type and the axial fan is a better choice in the closed circuit tunnels since it produces a static pressure rise necessary to compensate for the total pressure loss in the rest of the circuit.

The fans with higher ratio of tip speed to axial velocity generally produce the required pressure rise in a small blade area. The wind tunnels fitted with blower are generally driven by a centrifugal impeller of squirrel-cage type.

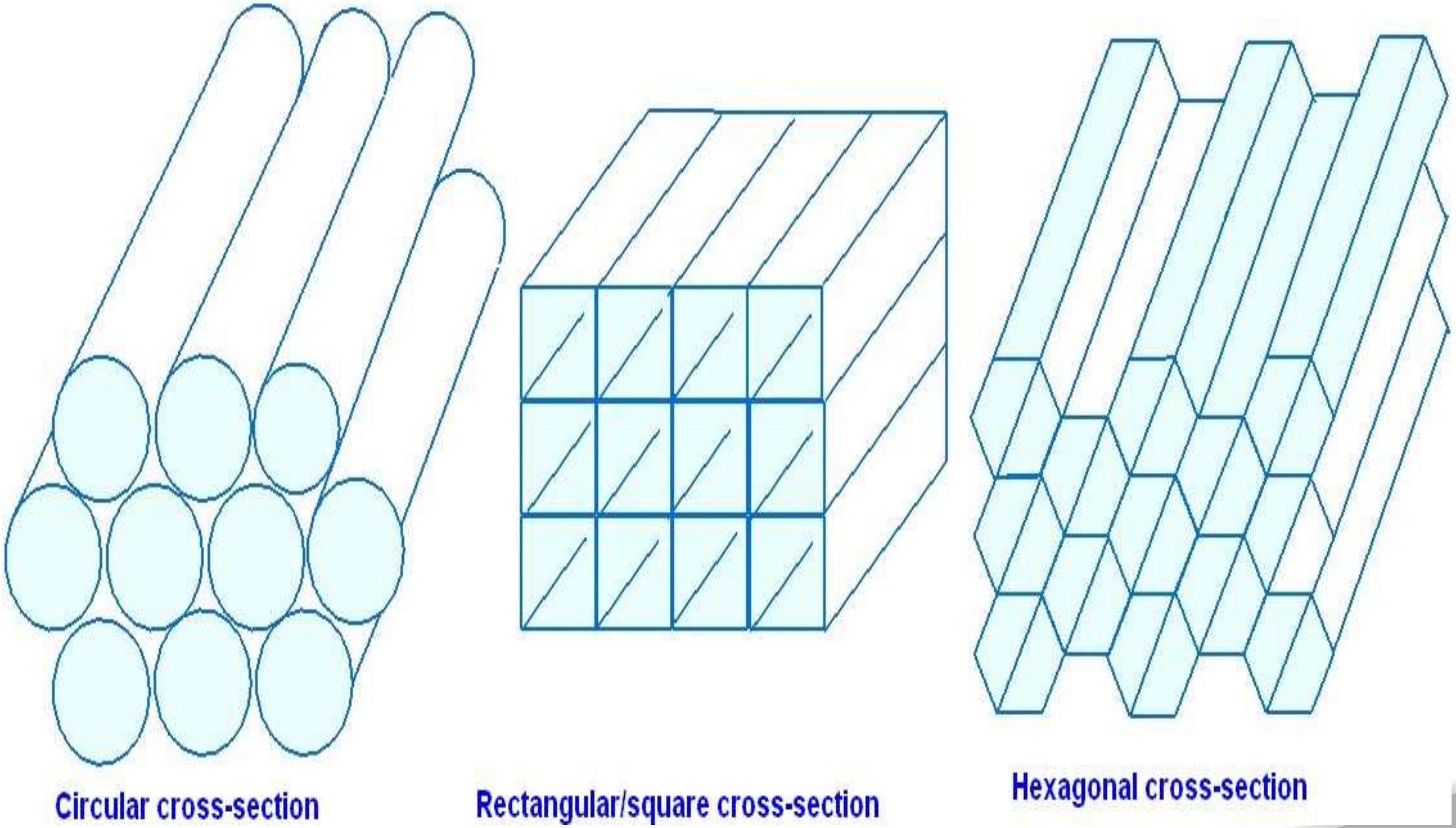
While in operation, the fan draws air from the atmosphere through the honeycomb/screen section.

Settling chamber and flow straightener

- It mainly comprises of honeycomb and screens as combination.
- The main function is to reduce the turbulence and straighten the flow only in the axial direction.
- In principle, the air can enter to the tunnel from any directions. But, only the axial flow is desired in the test section.
- The main purpose of the screen is to reduce the turbulent intensity in the flow and not to allow any unwanted objects to enter the tunnels. The honeycomb can be made with cells or various shapes.

Honeycomb structures for low speed wind tunnels

Honeycomb structures for low speed wind tunnels



Contraction

- The prime objective is to accelerate incoming flow from the settling chamber and supplies it to the test section at desired velocity.
- This section essentially reduces the cross-sectional velocity variation and maintains the flow uniformity.
- In general, small radius of curvature is used at the entry to this section and curvature of large radius is considered at the exit of the contraction section.
- However, the boundary-layer separation should be avoided at both the ends of this section.
- The contraction length is expected to be small so that large contraction area ratios are preferred.

Test Section

- It is the basic element of wind tunnel on which all other designs are generally made.
- All the aerodynamic models are mounted in the test section when the tunnel is operated with desired flow velocity.
- Various shapes for the test section are considered for constructing the wind tunnel viz. hexagonal, octagonal, rectangle etc.
- The test section is generally designed on the basis of utility and aerodynamic considerations since cost of construction depends on the test section area.

Test Section

- Length of the test section is mostly equal to major dimension of the cross-section of the same or twice of it.
- In addition, the test section should also be provided with facilities as per the testing requirement.
- The test section velocity is generally specified as percentage variation from the average of the cross-section.
- The ideal test section has steady uniform velocity at the inlet, no cross flow, less or no turbulence and less operating cost.

Diffuser

- It is basically a duct with increase in area attached downstream of the test section.
- After the test section, it is desired that the air must pass smoothly out of the test section.
- So, this geometry is made to decrease the flow velocity and increase in pressure.
- In order to avoid flow reversal, the exit pressure should be higher than the atmospheric in case of open circuit wind tunnel.

Diffuser

- This is a very critical section in design since the incurred pressure rise reduces the power requirement for the wind tunnel which is proportional to the cube of velocity.
- Hence maximum pressure recovery to be achieved at least possible distance is the main objective of diffuser design. In general practice, the cone angle of the diffuse is 7º or less so as to avoid boundary layer separation.

Turning vanes

- In a closed circuit wind tunnel, the air has to circulate in a controlled manner.
- Typically, the corners of the wind tunnel are of two bends aligned 90^0 each other.
- These corners are provided with turning vanes for smooth passage of the flow.
- Chambered aerofoils of bent planes are generally accepted as the turning vanes.
- These vanes are purposely made as adjustable for smooth operation thereby avoiding under/over tuning.

Special purpose low speed wind tunnels

- Low turbulence tunnels
- Two-dimensional tunnels
- Smoke tunnels
- Water tunnels

Low turbulence tunnels

- Low intensity turbulence is the prime requirement for many experimental situations.
- Hence, these tunnels are specifically designed with a wide- angle diffuser just ahead of the test section-settling chamber.
- The major task of reducing free stream turbulence is done by large settling chamber having honeycombs and the screens.
- Large contraction ratio of contraction section and several number of screens are some of the important components for this type of tunnel.

Two-dimensional tunnels

- These tunnels are intended for testing objects like cylinder, aerofoils etc.
- These tunnels can be of open or closed types but the main design feature of this tunnel is its height to width ratio of the test section which is more than or equal to 2.

Smoke tunnels

- Flow visualization is sole application of smoke tunnels.
- So, these tunnels are mostly three-dimensional open circuit type tunnels.
- The source of the smoke source is kept upstream of the test section so as to mix the smoke with free stream before passing over the object of interest.
- Very high contraction ratio and large number of screens are sometimes preferred for visualization of laminar flow.

Water tunnels

- These are the closed circuit type tunnels that are mostly preferred for hydrodynamics application like water flow over immersed bodies.
- These are preferred for flow visualization studies with particle image velocimetry (PIV) method because it is very simple to implement this technique in water tunnels.
- These tunnels are comprised of pumps for through circulation of water unlike fans in wind tunnels.

High Speed (supersonic) Wind Tunnel

- Experimental facilities for supersonic and hypersonic flow regimes are different from those used at subsonic speeds.
- In supersonic flows, the interest lies in simulating flow Reynolds number and the Mach numbers in the test section of the tunnel.
- In addition to these parameters, the total energy content (i.e. enthalpy) of the flow also becomes important at hypersonic speeds.
- The wind tunnels used in the Mach number range 1 to 5 are called as supersonic tunnels while the tunnels used for higher Mach numbers (>5) are called as hypersonic tunnels.

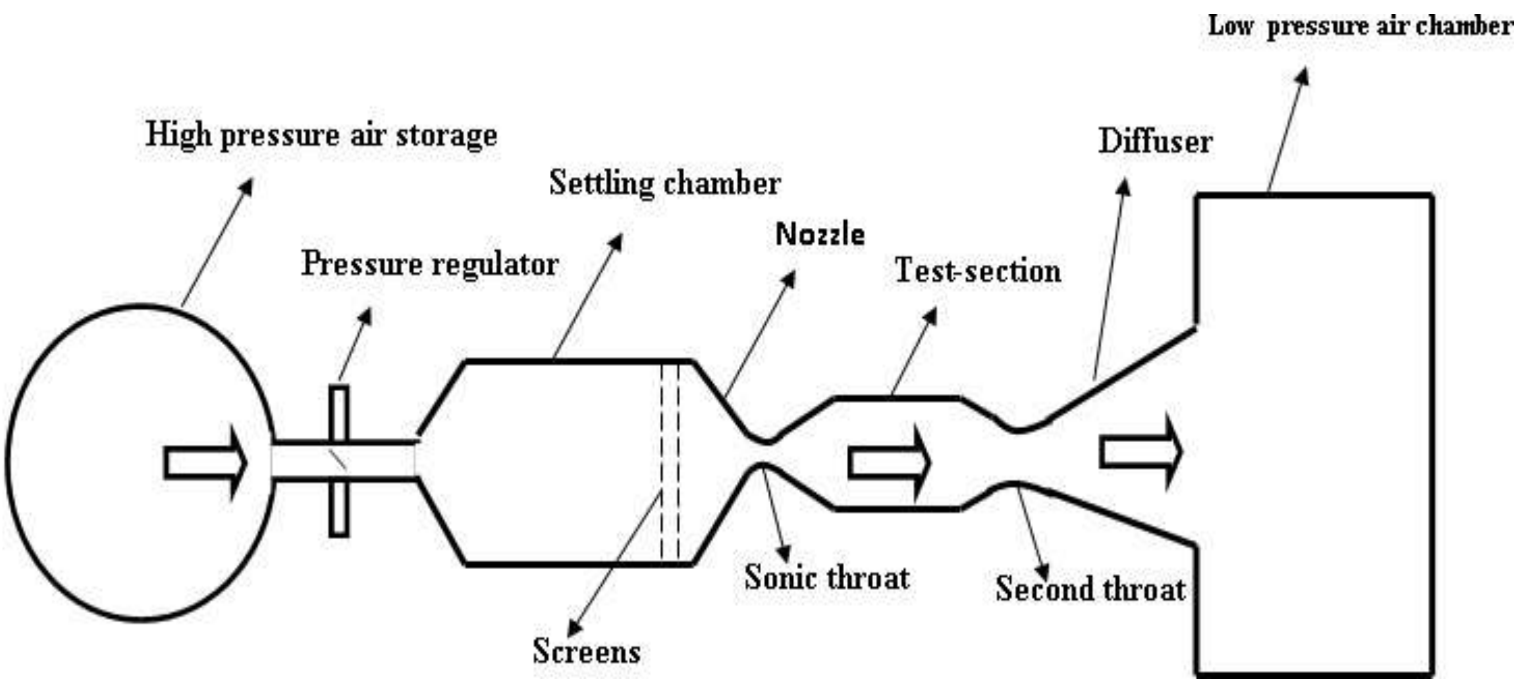
High Speed (supersonic) Wind Tunnel

- The high speed tunnels can be of open/closed circuit type.
- The open circuit wind tunnel takes the air from atmosphere and rejects them to a vacuum chamber.
- In contrast, the same air is re-circulated in a closed circuit wind tunnel.
- In the case of subsonic wind tunnels, experiments can be performed by running the tunnel continuously.
- But, when the velocity of air in the test section increases, the power requirement becomes very high because it is proportional to the cube of the velocity.

High Speed (supersonic) Wind Tunnel

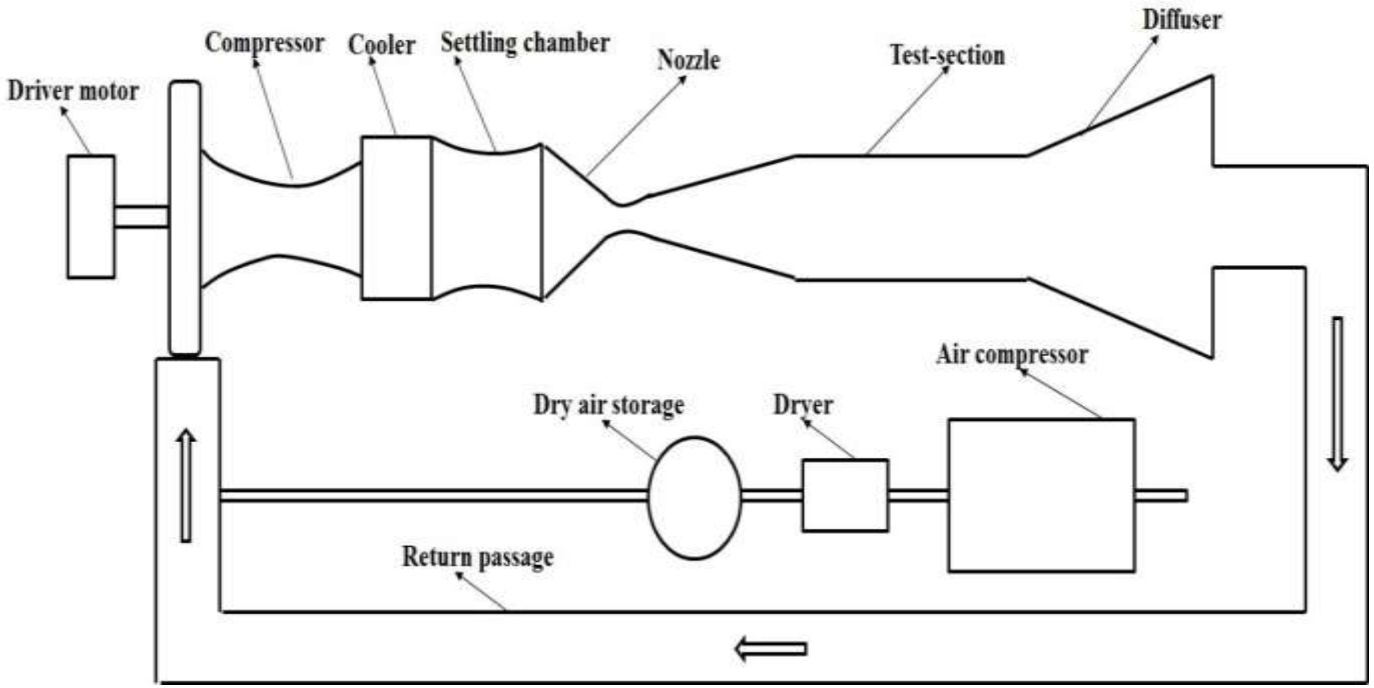
- Thus, in many cases, it is preferred to run high speed wind tunnels for a short duration and gather all the experimental data in this short time period (~1-5s).
- So, such types of tunnels are called as blow down tunnels.
- These tunnels operate intermittently using high pressure tanks and/or vacuum tanks.

Blow down wind Tunnel (Open circuit type)



Schematic diagram of a blow down wind tunnel (open circuit)

Continuous wind Tunnel (Closed circuit type)



Schematic diagram of a blow down wind tunnel (closed circuit)

Impulse-Type Tunnels

- The conventional hypersonic wind tunnels face the challenges of simulating re-entry flight phenomena, long range ballistic missile test conditions that occurs at very altitudes.
- Since, the temperature can rise up to 10000K, there will be dissociations, chemical reactions as well.
- Hence, such investigations need real gas simulation conditions and it is almost impossible to achieve them in conventional hypersonic wind tunnels.
- So, the concept of impulse type aerodynamic facilities is introduced where it is desired to produce low density and high enthalpy flows for a very short duration (~ few milliseconds).

Impulse-Type Tunnels

- Expansion and shock tunnels are the typical aerodynamic testing facilities with a specific interest in high speeds and high temperature testing.
- Shock tunnels use steady flow nozzle expansion whereas expansion tunnels use unsteady expansion with higher enthalpy/thermal energy.
- In both cases the gases are compressed and heated until the gases are released, expanding rapidly down the expansion chamber.
- The tunnels reach speeds from Mach numbers ranging 3 to 30, thus creating testing conditions similar to that of very high altitudes.

Shock Tunnel

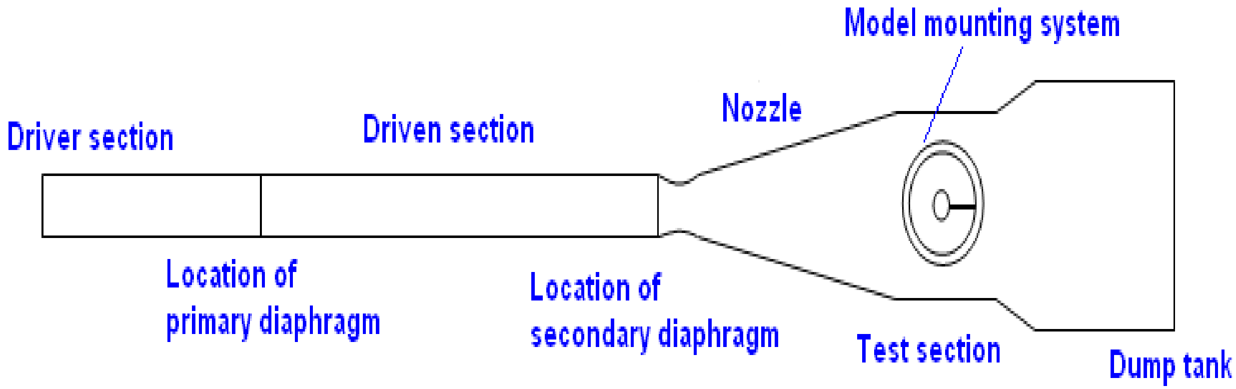
- It consists of two major parts, the shock tube and the wind tunnel portion.
- The general schematic layout of the shock tunnel, consisting of shock tube and wind tunnel section.
- The shock tube portion consists of a constant area tube separated by a diaphragm (generally, metal) into regions of high and low pressures.
- High and low pressure regions are called ‘driver tube’ and ‘driven tube’ respectively.
- The shock tube works on the principle of using a high-pressure gas in the driver tube to set up a shock wave, which propagates into the low pressure gas in the driven tube at the instant of diaphragm rupture.

Shock Tunnel

- The propagating shock wave compresses and heats the low-pressure test gas in the driven tube to a high pressure and temperature, and also imparts the test gas a high kinetic energy, with which it starts moving at a supersonic Mach number behind the propagating shock wave.
- This shock wave ideally travels through the driven section at a constant velocity, and there exists a region of steady supersonic flow of high temperature and pressure between the moving driver/driven gas interface and the shock wave.

Shock Tunnel

- The wind tunnel portion of the shock tunnel consists of a hypersonic nozzle that is attached to the driven end of the shock tube, a test section at the exit of the nozzle where the measurements are carried out, and a dump tank portion to accommodate the gas.

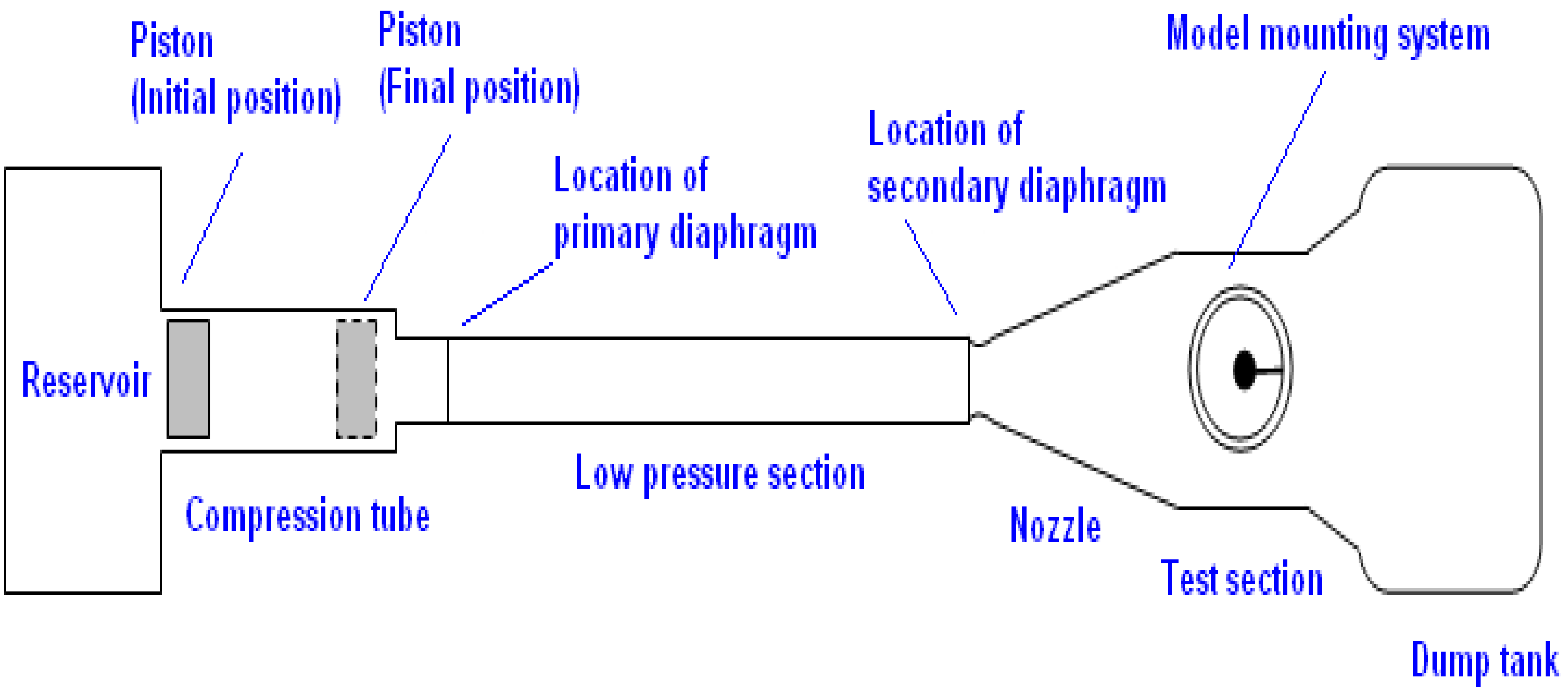


Schematic representation of a shock tunnel

Free Piston Shock Tunnel

- In a typical shock tunnel, the high pressure in the driver section is achieved by filling the tube from a high pressure gas cylinder. Thus, the driver gas is normally at room temperature.
- In the low density environment, it is likely that condensation may arise in the test section.
- So, many test facilities involve preheating the driver gas.
- One of them is to use a free piston driver to operate the shock tunnel.
- Here, the compression tube is filled with a driver gas (typically at atmospheric pressure) and is coupled to the driven section of low pressure region at one end and to a reservoir at very high pressure air (~100 bar).

Free Piston Shock Tunnel



Schematic diagram of a free piston shock tunnel

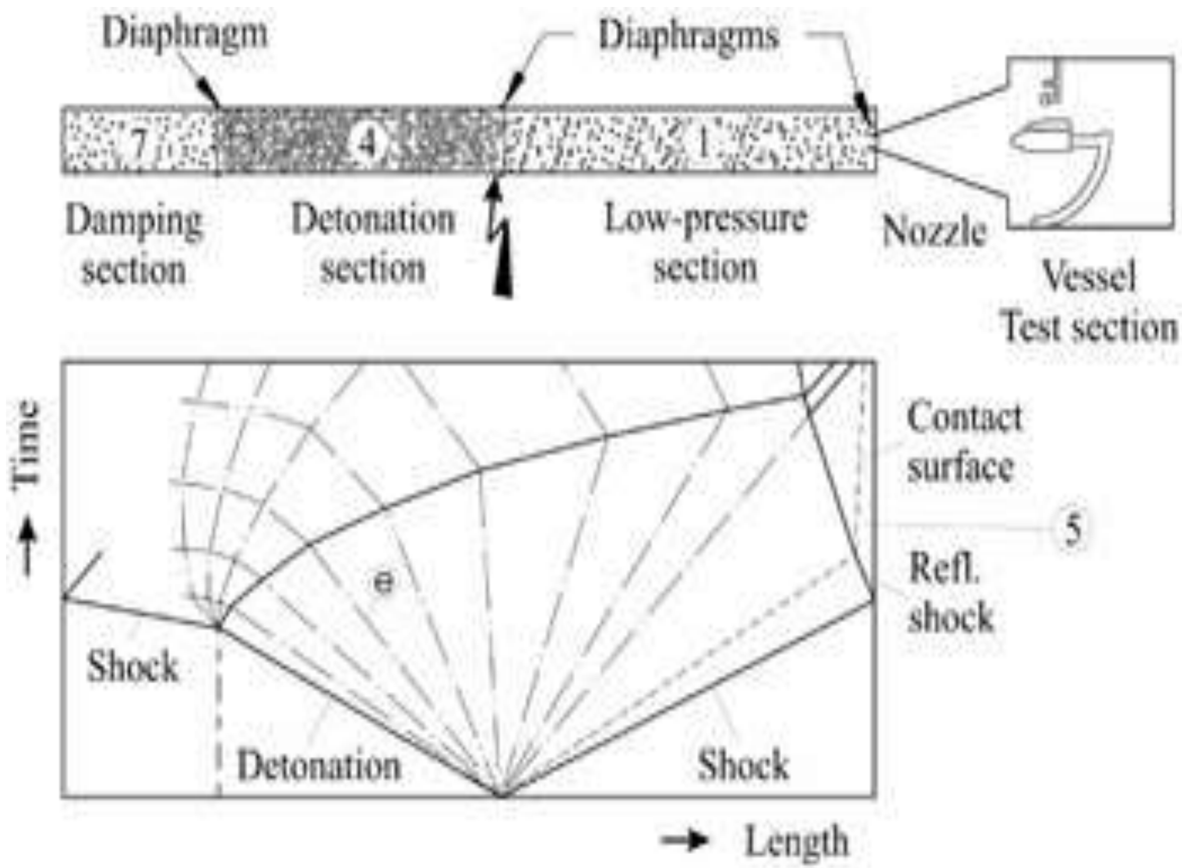
Free Piston Shock Tunnel

- In this way, the piston separates the reservoir from the compression tube.
- When the piston is released for firing, it accelerates in the compression tube by acquiring the energy of expanding reservoir gas.
- The piston then approaches down to the compression tube and transfers all its energy to the driver gas. In this way, both temperature and pressure can be increased due to adiabatic compression mechanism.
- In a typical facility, the pressure up 900 bar and temperature of 4500K can be achieved. In a well designed free piston shock tunnel, it is possible to achieve test flow duration up to 2ms.

Detonation-driven shock tunnels (tube)

- The detonation-driven shock tube, first proposed by Bird,3 has been studied by several investigators.
- In a detonation-driven shock tube or shock tunnel, the conventional driver is replaced by a detonation section filled with a detonable gas mixture.
- Typically, an oxyhydrogen mixture is used as the driver gas with helium or argon dilution.
- Helium dilution raises the sonic speed in the driver gas and also somewhat reduces the danger associated with premature detonation.
  - Upstream Mode
  - Downstream Mode

Upstream Mode



Principle of an upstream detonation-driven shock tunnel

Upstream Mode

- In the upstream-or backward propagation mode, the ignition source is just upstream of the primary diaphragm between the detonation and driven sections.
- In this case, the detonation wave propagates upstream.
- Direct initiation leads to a stable detonation wave where the products are at high temperature and pressure.
- The pressure rise following the detonation wave is constant, but the momentum imparted to the driver gas by the detonation wave is directed upstream, that is, opposite to the main flow.

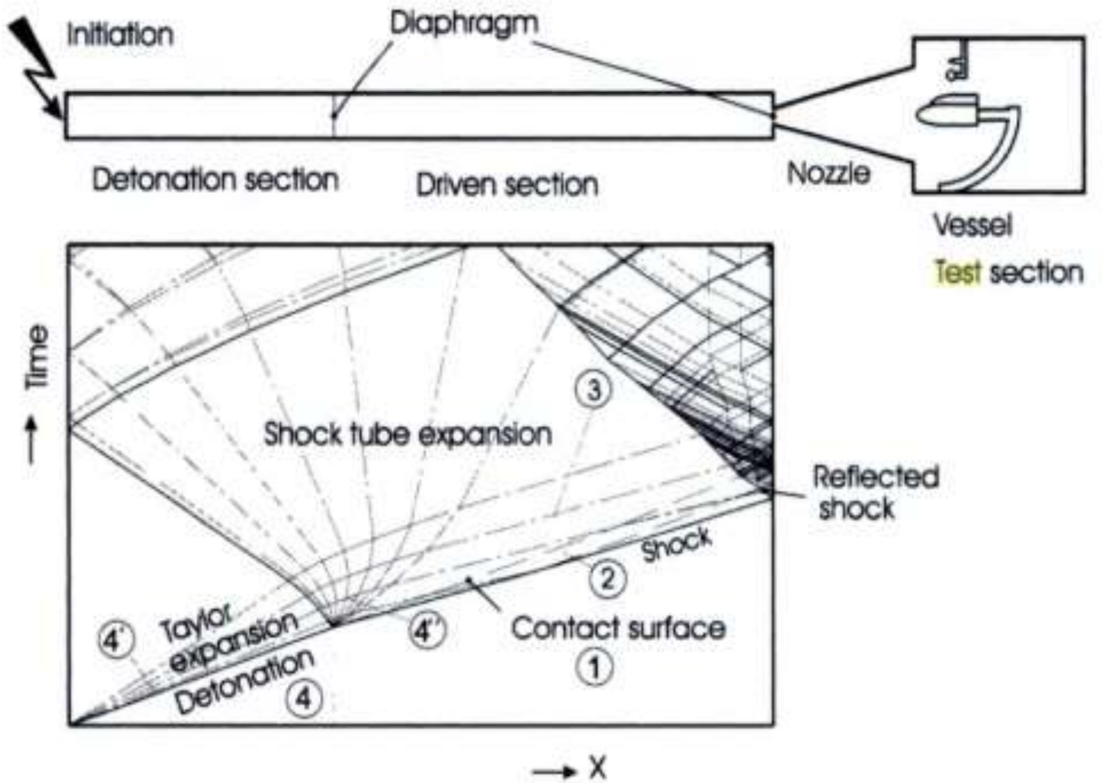
Upstream Mode

- This leads to a reduction of the effective driver performance.
- Shortly after initiation, the main diaphragm opens and the burnt products exhaust into the low-pressure section, driving the incident shock that compresses and heats the test gas.
- A Taylor expansion immediately follows the detonation wave, decelerating the burnt gas to zero velocity along the characteristic labeled *e* in Figure.

Downstream mode

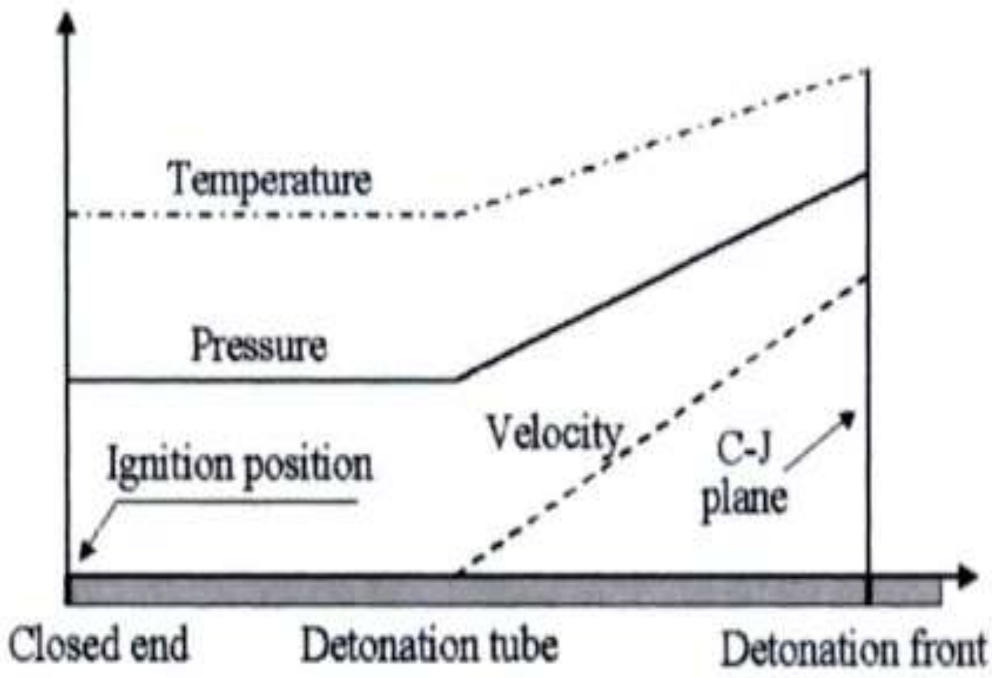
- In the downstream or forward propagation mode, the ignition source is located at the upstream end of the detonation section, producing a detonation wave that propagates downstream.
- The detonation wave, depicted as a solid line from the lower left- hand corner in the diagram, propagates downstream into region 4. The burnt gas following the detonation wave also flows downstream.
- The Taylor rarefaction decelerates the gas so that the flow velocity is zero at the end wall of the driver section.
- The detonation wave is reflected at the diaphragm to yield an effective, unsteady condition given by 4".

Principle of a downstream detonation-driven shock tunnel



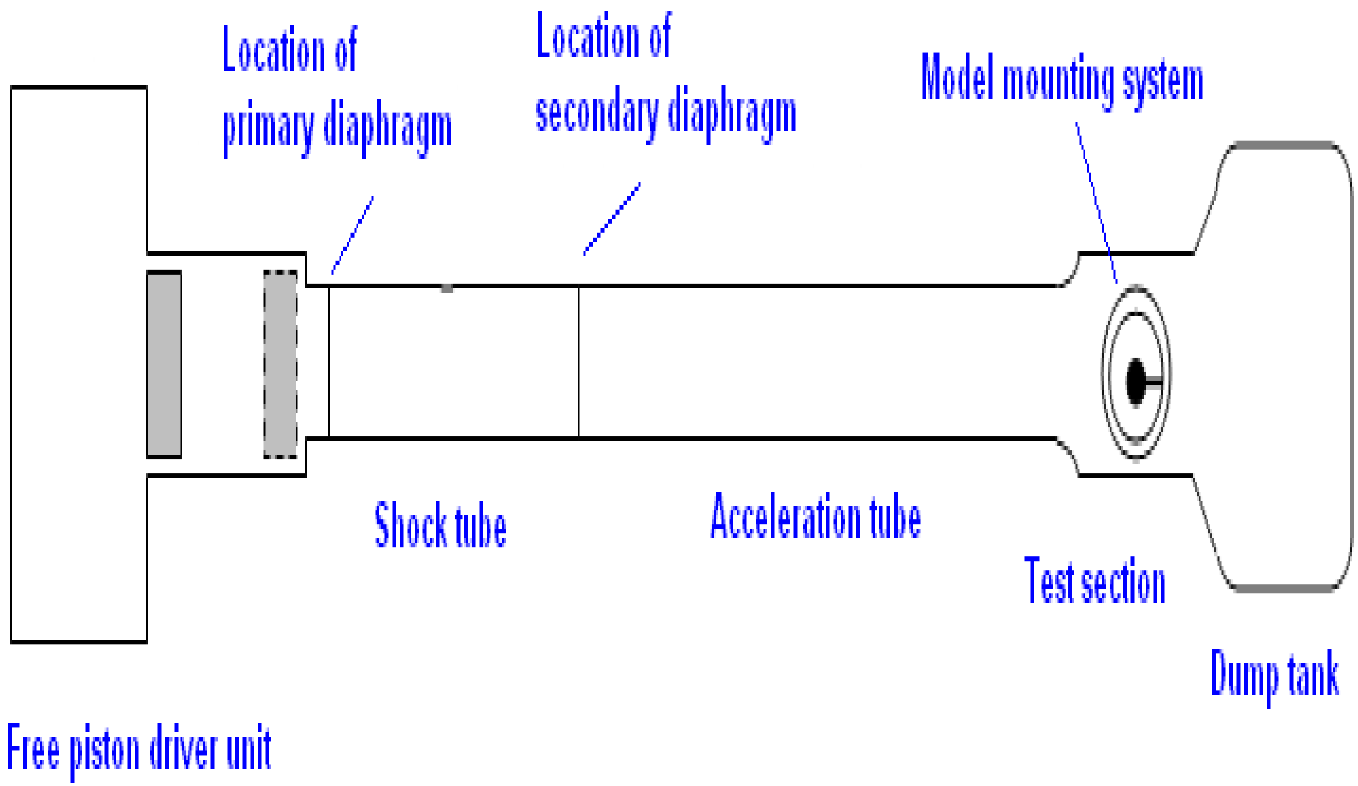
Principle of a downstream detonation-driven shock tunnel

Distribution of detonation wave along the length of a detonation tube



A simple model for the distribution of detonation wave along the length of a detonation tube.

Expansion Tunnel



Schematic diagram of an expansion tube

Expansion Tunnel

- It uses a dual-diaphragm system where the diaphragms act as rupture discs, or a pressure relief.
- The tunnel is separated into three sections: driver, driven, and acceleration.
- The driver section can be fired either by a high pressure gas or by a free piston driver.
- The driven section is filled with a lower pressure test gas.
- The acceleration section is filled with an even lower pressurized test gas.

Expansion Tunnel

- Each section is divided by a diaphragm, which is meant to be ruptured in sequence causing the first diaphragm to rupture, mixing and expanding the driver and the driven.
- When the shock wave hits the second diaphragm, it ruptures casing the two gases to mix with the acceleration and expand down the enclosed test section.
- In this way, the flow of the test flow can be increased for hypervelocity conditions.
- However, the operation time is approximately 250µs.

Flow Visualization Techniques for Compressible Flows

- Flow visualization is carried out to understand the physics of the flow in detail.
- Understanding of the flow provides excellent description which in turn helps to calculate flow properties for many problems of practical interests in both compressible subsonic and supersonic flow regimes.
- Flow visualization can be of interest for various problems.
- Since air is transparent, thus their flow patterns are invisible to us without incorporating special techniques to visualise the same. Therefore the present section deals with some important flow visualisation techniques.

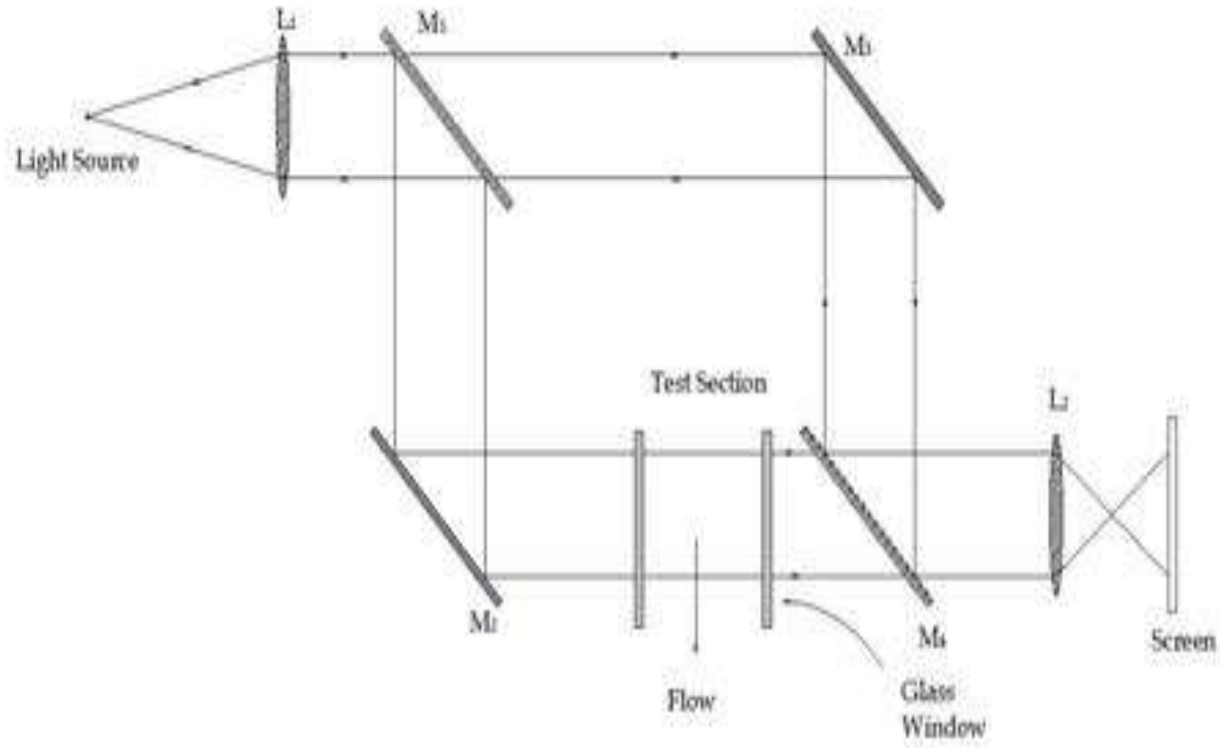
Flow Visualization Techniques

- Interferometer
- Schlieren Technique
- Planar Laser Induced Fluorescence (PLIF)

Interferometer

- The interferometer which is an optical method is specifically suited for qualitative determination of the density field of high speed flows.
- From the theory of light, we know that when light travels through a gas the velocity of propagation is affected by the physical properties of the gas.
- Light emitted from the source is allowed to pass through the lens L1 which makes the light rays parallel with each other.
- These parallel light beams are then passed through a monochromatic filter.
- The path of the light wave is then made by two ways as M1-M2-M4 and M1-M3-M4 in the way to fall on the screen

Interferometer

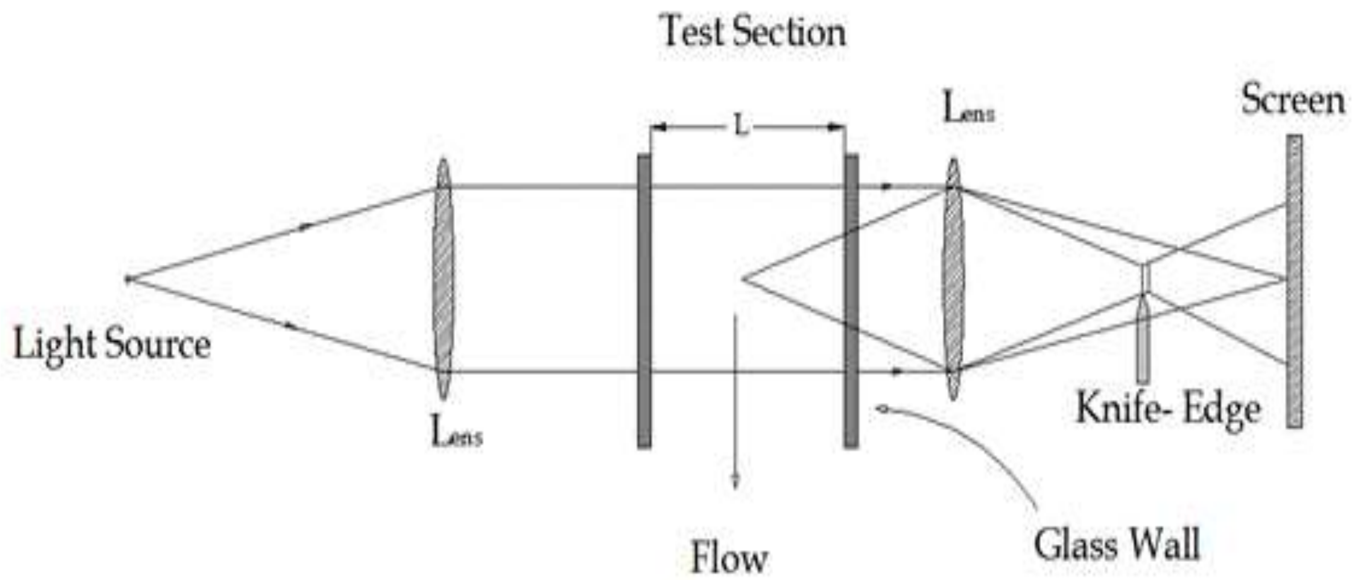


Mach-Zhender Interferometer

Schlieren Technique

- The Schlieren method is one more technique prevalently used for visualizing the compressible flow due to presence of large density gradients.
- This techniques also uses a light source and lens where Light emitted from the source is collimated by the lens before passing through the test section.
- These light beams are then passed through one more lens before getting on the screen.
- A knife edge is placed at the focal point of the second lens where the images of the source is formed.
- Kinfe edge can be any opaque object which can be placed at the same location.

Schlieren Technique



Schematic of Schlieren arrangement

Schlieren Technique

- This object or knife object is obstruct the light beam.
- If the beams of light escape the knife edge the screen gets uniformly illumination.
- This situation is seen for no flow case through the test section since both the beams pass though the medium of same density.
- However during the flow taking place in the test section with test model mounted in it, both the light beams encounter different densities therefore make different deflections one of the beams pass through uniform or freestream density region while other beam passes through the portion of shock placed ahead the test model.

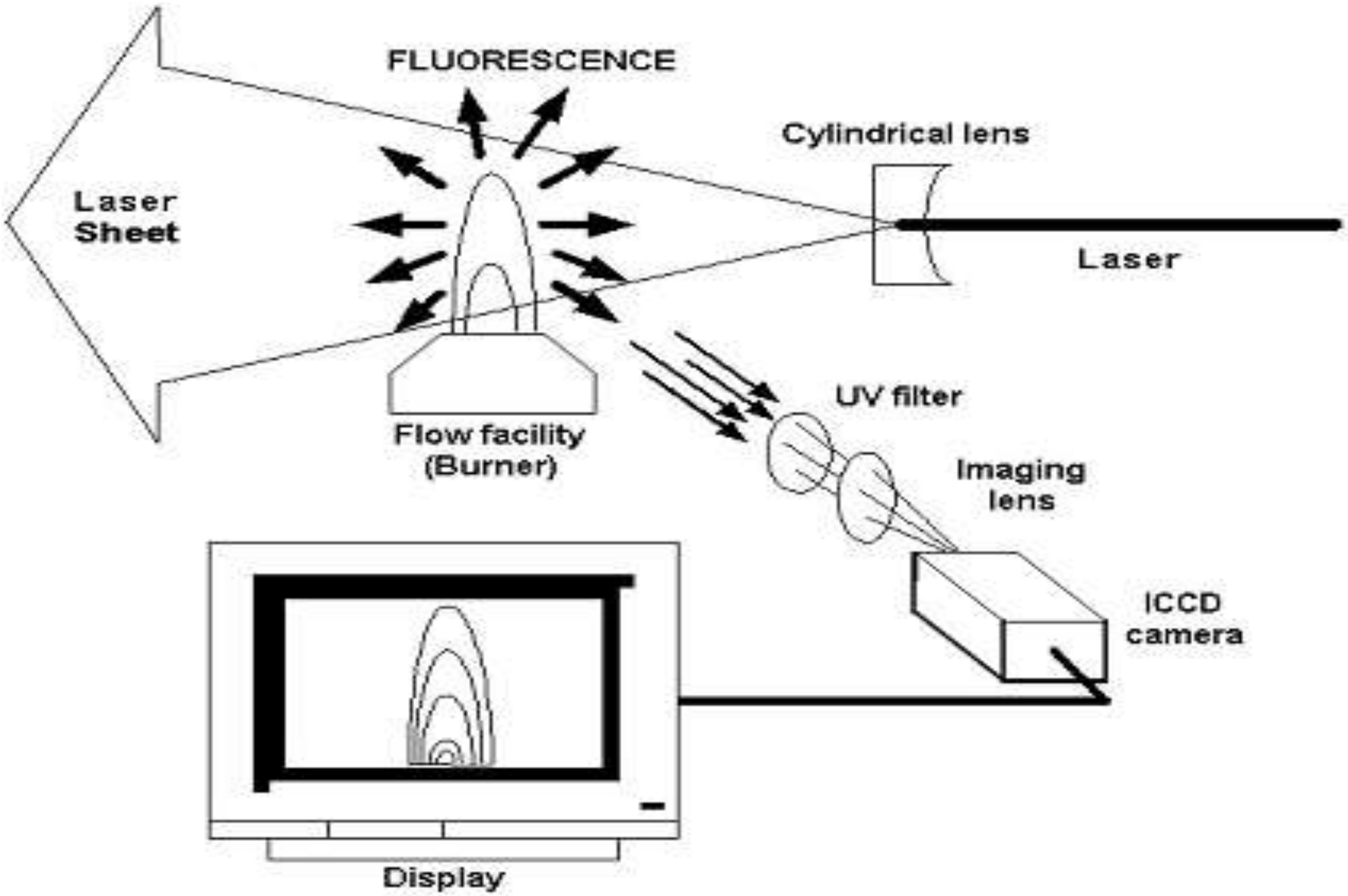
Planar Laser Induced Fluorescence (PLIF)

- Optical techniques are considered for flow visualization, Planar Laser Induced Fluorescence (PLIF) is among such methods.
- This lased based technique is popular due to its abilities like remote controlling, precise prediction and non-intrusive.
- In this technique, a laser sheet is used to illuminate the flow and captures the fluorescence.
- Radicals which are most active species on the flow are chosen to track in this technique using the laser sheet to excite the species and hence to emitted the fluorescence which is captured on charge-coupled device (ICCD) camera.

Planar Laser Induced Fluorescence (PLIF)

- Consider a burner whose fluorescence is expected to capture. Therefore a laser beam passes through a cylindrical lens to convert it into a thin laser sheet.
- This laser sheet passes through the flame produced by a flat flame burner and excites the OH radical which inturn causes fluorescence.
- This fluorescence signal is then acquired in the camera in the form of the images.
- The images captured in the ICCD can be further considered for processing.
- The major disadvantage is the quenching of the fluorescence at high pressure conditions due to high number of collisions of the molecules.

Planar Laser Induced Fluorescence (PLIF)



Basic Arrangement of PLIF