INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad -500 043

ELECTRONICS AND COMMUNICATION ENGINEERING

COURSE LECTURE NOTES

Course Name	**ANTENNAS AND PROPAGATION**
Course Code	**AEC011**
Programme	B.Tech
Year	2019-2020
Semester	V
Course Coordinator	Ms. A Usharani, Assistant Professor,ECE
Course Faculty	Dr. V.Sivanagaraju,Professor, ECE Mrs.K.C.Koteswaramma,Assistant Professor,ECE

COURSE OBJECTIVES:

The course should enable the students to:

I. Be Proficient in the radiation phenomena associated with various types of antennas and understand basic terminology and concepts of antennas along with emphasis on their applications.
II. Analyze the electric and magnetic field emission from various basic antennas with mathematical formulation of the analysis.
III. Explain radiation mechanism of different types of antennas and their usage in real time field.
IV. Justify the propagation of the waves at different frequencies through different layers in the existing layered free space environment structure.

COURSE OUTCOMES:

CO 1	Describe the concept of probability, conditional probability, Baye's theorem and analyze the concepts of discrete, continuous random variables
CO 2	Determine the binomial, poisson and normal distribution to find mean, variance.
CO 3	Understand multiple random variables and enumerate correlation and regression to the given data.
CO 4	Explore the concept of sampling distribution and apply testing of hypothesis for sample means and proportions.
CO 5	Use t-test for means, F-test for variances and chi-square test for independence to determine whether there is a significant relationship between two categorical variables.

COURSE LEARNING OUTCOMES:

Students, who complete the course, will have demonstrated the ability to do the following:

S. No	Description
AEC011.01	Discuss about the radiation mechanism in single wire, double wire antennas and the current distribution of thin wire antenna.
AEC011.02	Discuss the different parameters of an antenna like radiation patterns, radiation intensity, beam efficiency, directivity and gain etc,.
AEC011.03	Analyze the concept of antenna properties based on reciprocity theorem, evaluate the field components of quarter wave monopole and half wave dipole.
AEC011.04	Understand the significance of loop antennas in high frequency range and its types; derive their radiation resistances and directivities.
AEC011.05	Discuss the uniform linear arrays such as broadside array and end fire array, derive their characteristics.
AEC011.06	Analyze the practical design considerations for monofilar helical antenna in axial and normal modes.
AEC011.07	Discuss the various types of Microwave antennas and analyze the design consideration of pyramidal horn.
AEC011.08	Analyze the concept of complementary in slot antennas using Babinet's principle and understand the impedance of slot antennas.
AEC011.09	Understand the significance, features and characteristics of micro strip patch antennas, analyze the impact of different parameters on characteristics.
AEC011.10	Understand and analyze the reflectors are widely used to modify the radiation pattern as a radiating element, its types.
AEC011.11	Discuss various concepts related to antennas such as feed methods like front feed, rear feed, offset feed and aperture blockage.
AEC011.12	Discuss various methods and techniques for experimental measurements of antennas such as pattern measurement, directivity measurement, gain measurement etc.
AEC011.13	Understand the wave propagation through the complete study of the wave by the nature and characteristics of media during the wave travels.
AEC011.14	Understand the space wave propagation focusing on field strength variation with distance and height, effect of earth's curvature, absorption and super refraction.
AEC011.15	Analyze the structure of ionosphere and understand the sky wave propagation through refraction and reflection by ionosphere.

SYLLABUS:

UNIT – I **ANTENNA BASICS AND THIN LINEAR WIRE ANTENNAS:** Antenna fundamentals: Introduction, radiation mechanism, single wire, 2 wires, dipoles, current distribution on a thin wire antenna; Antenna Parameters, radiation patterns, patterns in principal planes, main lobe and side lobes, beam widths, radiation intensity, beam efficiency, directivity, gain and resolution, antenna apertures, aperture efficiency, effective height; Antenna properties based on reciprocity theorem; Thin linear wire antennas: Retarded potentials; Radiation from small electric dipole, Quarter wave monopole and half wave dipole, current distributions, evaluation of field components; power radiated, radiation resistance, beam widths, directivity, effective area and effective height; Natural current distributions, fields and patterns of thin linear center-fed antennas of different lengths; Illustrated problems.
UNIT – II **LOOP ANTENNAS AND ANTENNA ARRAYS :** Loop Antennas: Introduction, small loop; Comparison of Far fields of small loop and short dipole; Radiation resistances and directivities of small and large loops. Antenna Arrays: Point sources, definition, patterns; Arrays of 2 isotropic sources, different cases; Principle of pattern multiplication; Uniform linear arrays - Broadside arrays; End-fire arrays; EFA with increased directivity; Derivation of their characteristics and comparison; BSAs with non-uniform amplitude distributions; General considerations and Binomial arrays; Folded Dipoles and their characteristics; Arrays with parasitic elements, Yagi-Uda array, Helical antennas-Helical geometry, Helix modes, Practical design considerations for monofilar Helical antenna in axial and normal modes.
UNIT – III **VHF,UHF AND MICROWAVE ANTENNAS** VHF, UHF and Microwave Antennas: Horn antennas- Types, Fermat‘s principle, optimum horns, design considerations of pyramidal horns; Illustrative problems; Lens antennas: Introduction, geometry of Non-metallic dielectric lenses zoning, tolerances, applications; Slot antenna, its pattern, Babinet‘s principle and complementary antennas, impedance of slot antennas. Microstrip Antennas: Introduction, features, advantages and limitations; Rectangular patch antennas-geometry and parameters, characteristics of micro strip antennas, Impact of different parameters on characteristics.
UNIT – IV **REFLECTOR ANTENNAS AND ANTENNA MEASUREMENTS** Reflector Antennas: Introduction, flat sheet and corner reflectors; Paraboloidal reflectors: Geometry, pattern characteristics, feed methods, reflector types- Related features; Illustrative problems. Antenna measurements: Introduction, concepts, reciprocity near and far fields; Coordinate system, sources of errors patterns to be measured; Pattern measurement arrangement directivity measurement; Gain measurements: Comparison method, absolute and 3-antenna methods.

UNIT – V **RADIO WAVE PROPAGATION** Wave Propagation - I: Introduction, definitions, categorizations , general classifications, different Modes of Wave Propagation; Ground wave propagation: Introduction, plane earth reflections, space and surface waves, wave tilt, curved earth reflections; Space wave propagation: Introduction, field strength variation with distance and height, effect of earth's curvature, absorption, super refraction, M-Curves, duct propagation, scattering phenomena, tropospheric propagation, fading and path loss calculations; Wave propagation – II: Sky wave propagation: Introduction, structure of ionosphere, refraction and reflection of sky waves by ionosphere; Ray path, critical frequency, MUF, LUF, OF, virtual height and skip distance; Relation between MUF and skip distance; Multi-hop propagation.

TEXT BOOKS:

1	John D. Kraus, Ronald J. Marhefka, Ahmad S. Khan, ―Antennas and Wave Propagation‖, TMH, 4th Edition, 2010.
2	C.A. Balanis, ―Antenna Theory, John Wiley and Sons, 2nd Edition, 2001.

REFERENCES:

1	E.C. Jordan, K.G. Balmain, ―Electromagnetic Waves and Radiating Systems‖, PHI, 2nd Edition, 2000.
2	E.V.D. Glazier, H.R.L. Lamont, ―Transmission and Propagation‖, Her Majesty's Stationery Office, 1958.
3	F.E. Terman, ―Electronic and Radio Engineering‖, McGraw-Hill, 4th Edition, 1955.
4	K.D. Prasad, Satya Prakashan, ―Antennas and Wave Propagation‖, Tech India Publications, 1st Edition, 2001.

UNIT – I
ANTENNA BASICS AND THIN LINEAR WIRE ANTENNAS:

Introduction:

Antennas are everywhere cat our homes & workplaces cars & aircrafts, ships, satellites). We are having infinite variety of antennas but all operate on the same basic participle of electromagnetism history- antennas are our electronic eyes & ears on the world. They are our links with space. Antennas were commonly called "aerials" (still used in some countries) ex: japan-middle sky wire. Antennas are the essential communication line for aircraft & ships

Antennas for all types of wireless devices & cellular phones line us to everyone and everything.By using the antennas and then arrays and probes we could visit the planets of the solar systems & beyond that also [ex: responding to our commands and reading back photograph & data at cm x &]

Electromagnetic spectrum and radio frequency band in accordance with us expectations, the third world was will be won by the side who will have a better command ever the electromagnetic spectrum CHF, VHF, UHF, SHF)- SHF further divided into a number of bounds and sub bounds like ELF, SLF, ULF, L,S,C,X, Ku, K, Ka.

Antenna Basics:

Before going to study the antennas, first we have to understand the –meaning of antenna

1) purpose (Application)
2) Parameters (what type of)
3) Types (how many-classifications)

We have to discuss the theoretical and practical aspects of antennas and their selection criterion for specific applications and range of applications. Here the words "specific" & range" are very important to analyze (where it is used, at what frequency depends on application). There is no rule for selecting an antenna for a particular frequency range or application while choosing an antenna many electrical, mechanical and structure aspects are to be taken into account. Application- everywhere we are using the same antenna may be used for $t_{\propto}$' ion & $R_{\propto}$' ion [ex: radars, mobiles]

Separate antennas required for televisions & radio but the principle is some and the parameters (called antenna parameters, \ selection factors) antennas may vary in size from order of a few millimeters to 1000's feet. Requirements for transmitting antennas- high gain, high efficiency and for Receiving antennas- low side lobes, large SNR.Antennas- it is a region of transition between a transmission line and (shape) space. Antennas radiate electromagnetic energy in the desired direction. An antenna may be isotropic\ (also called omni directional) or anisotropic (directional)

Radiation Mechanism:

The radiation from the antenna takes place when the Electromagnetic field generated by the source is transmitted to the antenna system through the Transmission line and separated from the Antenna into free space

Radiation from a Single Wire

Conducting wires are characterized by the motion of electric charges and the creation of current flow.

Assume that an electric volume charge density, q_v (coulombs/m^3), is distributed uniformly in a circular wire of cross-sectional area A and volume V.

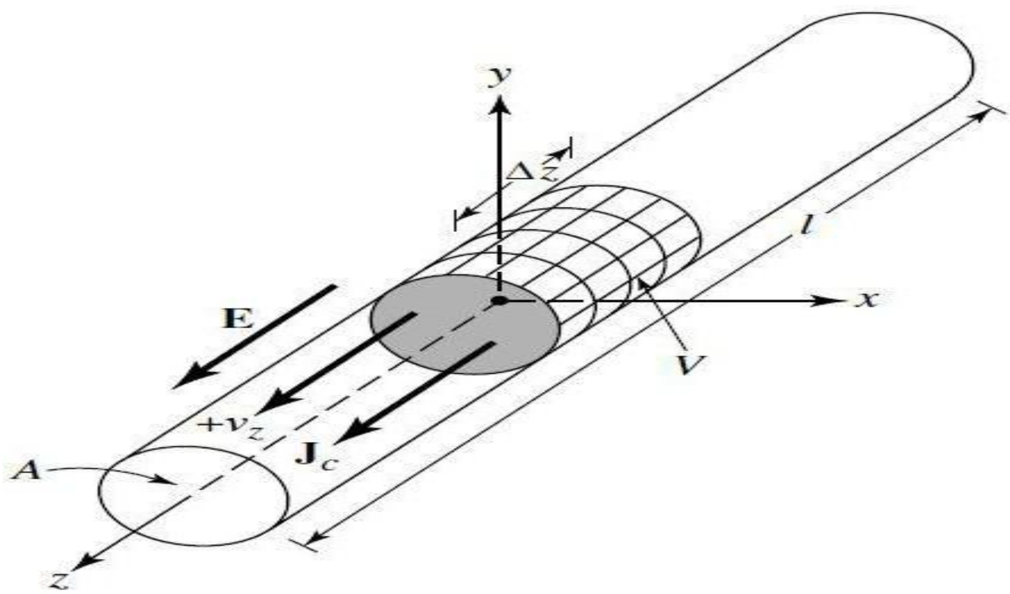


Figure: Charge uniformly distributed in a circular cross section cylinder wire

Current density in a volume with volume charge density q_v (C/m^3)

$J_z = q_v v_z$ (A/m^2) ---------------- (1)

Surface current density in a section with surface charge density qs (C/m^2)

$J_s = q_s v_z$ (A/m) ------------------ (2)

Current in a thin wire with a linear charge density q_l (C/m):

$I_z = q_l v_z$ (A) -------------------- (3)

To accelerate/decelerate charges, one needs sources of electromotive force and/or discontinuities of the medium in which the charges move. Such discontinuities can be bends or open ends of wires, change in the electrical properties of the region, etc.

In summary:

It is a fundamental single wire antenna. From the principle of radiation there must be some time varying current. For a single wire antenna,If a charge is not moving, current is not created and there is no radiation.If charge is moving with a uniform velocity:There is no radiation if the wire is straight, and infinite in extent.There is radiation if the wire is curved, bent, discontinuous, terminated, or truncated, as shown in Figure.
If charge is oscillating in a time-motion, it radiates even if the wire is straight

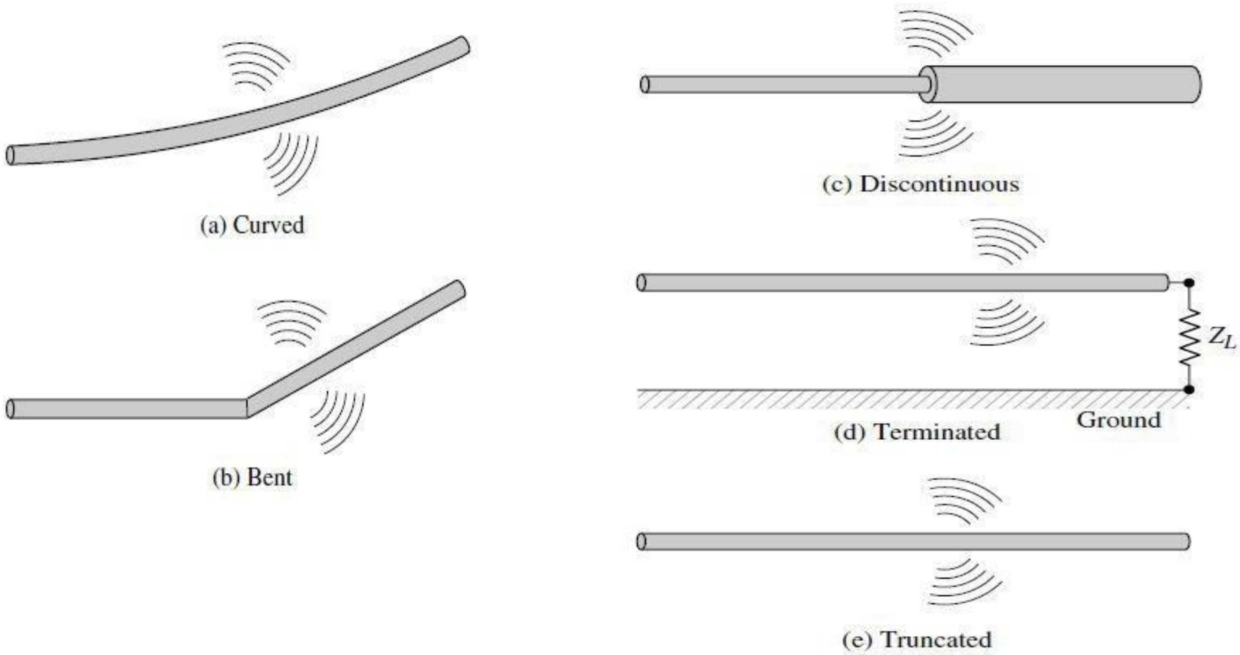


Figure : Wire Configurations for Radiation

Radiation from a Two Wire

Let us consider a voltage source connected to a two-conductor transmission line which is connected to an antenna. This is shown in Figure (a).

Applying a voltage across the two conductor transmission line creates an electric field between the conductors. The electric field has associated with it electric lines of force which are tangent to the electric field at each point and their strength is proportional to the electric field intensity.

The electric lines of force have a tendency to act on the free electrons (easily detachable from the atoms) associated with each conductor and force them to be displaced.

The movement of the charges creates a current that in turn creates magnetic field intensity.

Associated with the magnetic field intensity are magnetic lines of force which are tangent to the magnetic field. We have accepted that electric field lines start on positive charges and end on negative charges.

They also can start on a positive charge and end at infinity, start at infinity and end on a negative charge, or form closed loops neither starting nor ending on any charge.

Magnetic field lines always form closed loops encircling current-carrying conductors because physically there are no magnetic charges. In some mathematical formulations, it is often convenient to introduce equivalent magnetic charges and magnetic currents to draw a parallel between solutions involving electric and magnetic sources.The electric field lines drawn between the two conductors help to exhibit the Distribution of charge.

If we assume that the voltage source is sinusoidal, we expect the electric field between the conductors to also be sinusoidal with a period equal to that of the applied source. The relative magnitude of the electric field intensity is indicated by the density (bunching) of the lines of force with the arrows showing the relative direction (positive or negative). The creation of time- varying electric and magnetic fields between the conductors forms electromagnetic waves which travel along the transmission line, as shown in Figure 1.11(a).

The electromagnetic waves enter the antenna and have associated with them electric charges and corresponding currents. If we remove part of b the antenna structure, as shown in Figure (b), free-space waves can be formed by ―connecting‖ the open ends of the electric lines (shown dashed). The free-space waves are also periodic but a constant phase point P0 moves outwardly with the speed of light and travels a distance of λ/2 (to P1) in the time of one-half of a period.

It has been shown that close to the antenna the constant phase point P0 moves faster than the speed of light but approaches the speed of light at points far away from the antenna (analogous to phase velocity inside a rectangular waveguide).

Radiation from a Dipole:

Now let us attempt to explain the mechanism by which the electric lines of force are detached from the antenna to form the free-space waves. This will again be illustrated by an example of a small dipole antenna where the time of travel is negligible. This is only necessary to give a better physical interpretation of the detachment of the lines of force.

Although a somewhat simplified mechanism, it does allow one to visualize the creation of the free-space waves.

Figure(a) displays the lines of force created between the arms of a small center-fed dipole in the first quarter of the period during which time the charge has reached its maximum value (assuming a sinusoidal time variation) and the lines have traveled outwardly a radial distance λ/4.

For this example, let us assume that the number of lines formed is three. During the next quarter of the period, the original three lines travel an additional λ/4 (a total of λ/2 from the initial point) and the charge density on the conductors begins to diminish. This can be thought of as being accomplished by introducing opposite charges which at the end of the first half of the period have neutralized the charges on the conductors.

The lines of force created by the opposite charges are three and travel a distance λ/4 during the second quarter of the first half, and they are shown dashed in Figure (b).The end result is that there are three lines of force pointed upward in the first λ/4 distance and the same number of lines directed downward in the second λ/4. Since there is no net charge on the antenna, then the lines of force must have been forced to detach themselves from the conductors and to unite together to form closed loops.

This is shown in Figure(c). In the remaining second half of the period, the same procedure is followed but in the opposite direction.

After that, the process is repeated and continues indefinitely and electric field patterns are formed

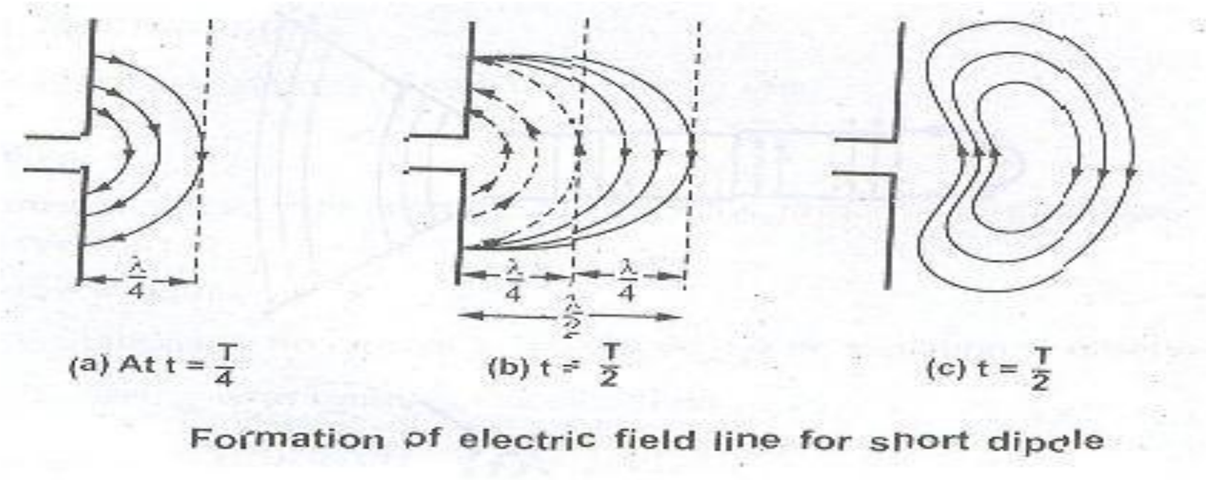


Fig. Formation of electric field line for short dipole

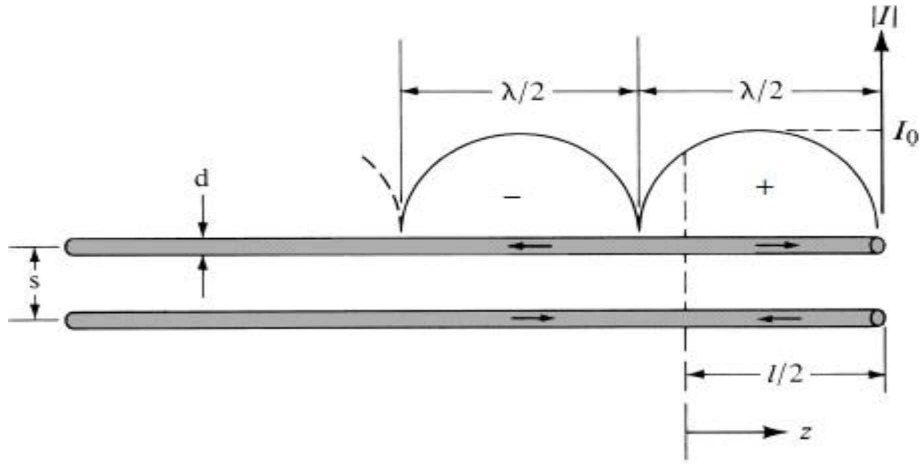
Current distribution on a thin wire antenna:

Let us consider a lossless two wire transmission line in which the movement of charges creates a current having value I with each wire. This current at the end of the transmission line is reflected back, when the transmission line has parallel end points resulting in formation of standing waves in combination with incident wave.

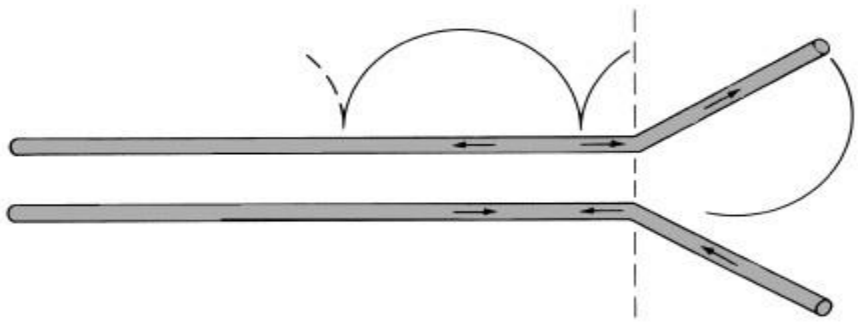
When the transmission line is flared out at 90^0 forming geometry of dipole antenna (linear wire antenna), the current distribution remains unaltered and the radiated fields not getting cancelled resulting in net radiation from the dipole. If the length of the dipole $l < \lambda/2$, the phase of current of the standing wave in each transmission line remains same.

If diameter of each line is small $d << \lambda/2$, the current distribution along the lines will be sinusoidal with null at end but overall distribution depends on the length of the dipole (flared out portion of the transmission line). The current distribution for dipole of length $l << \lambda$. When $l > \lambda$, the current goes phase reversal between adjoining half-cycles. Hence, current is not having same phase along all parts of transmission line.

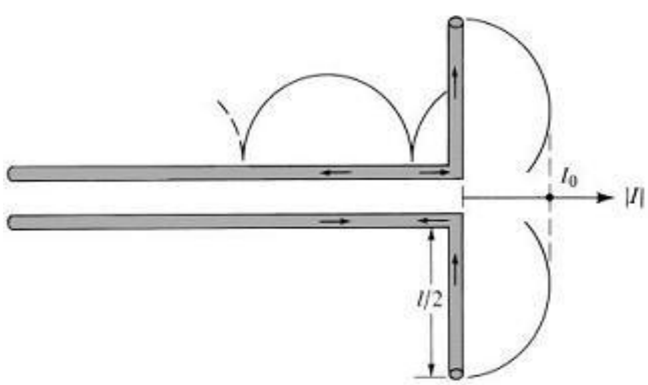
This will result into interference and canceling effects in the total radiation pattern. The current distributions we have seen represent the maximum current excitation for any time. The current varies as a function of time as well.



(a) Two-wire transmission line



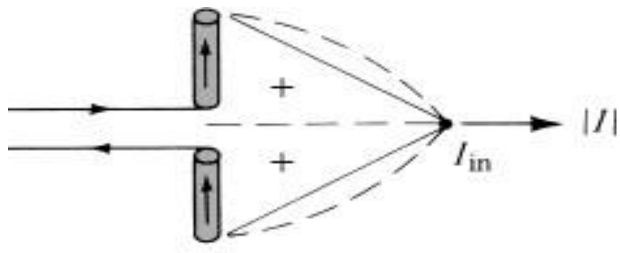
(b) Flared transmission line



(c) Linear dipole

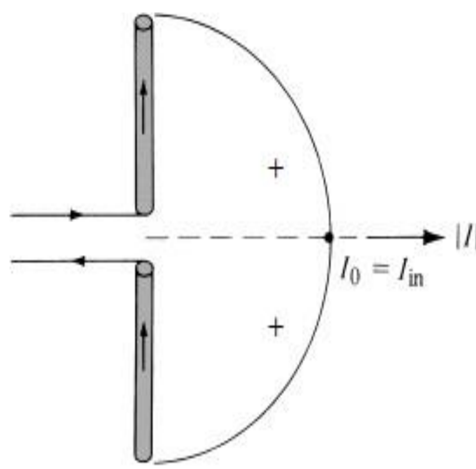
Fig. Current distribution on a lossless two-wire transmission line, flared transmission line

For $l = \lambda/2$



(a) $l << \lambda$

For λ /2<l< λ



(b) $l = \lambda/2$

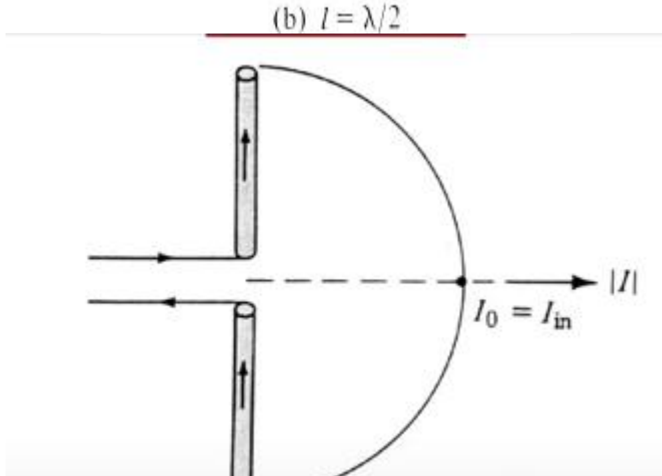
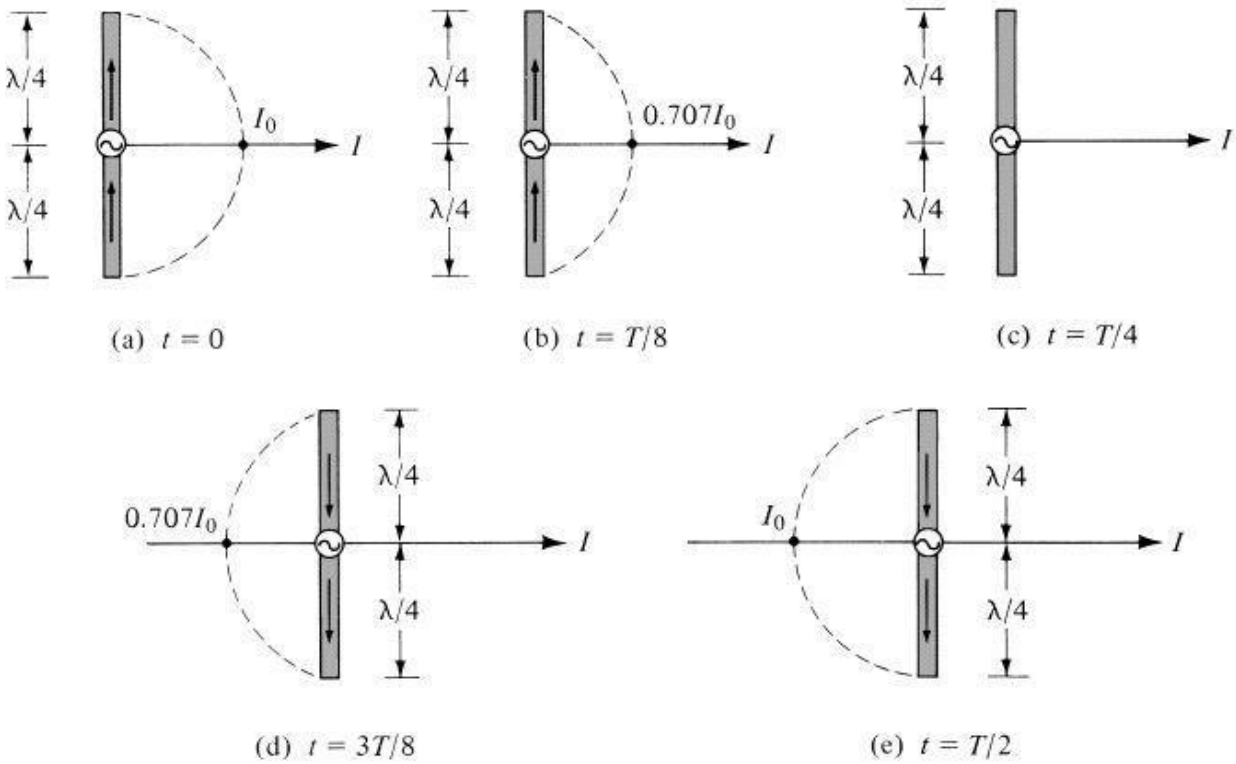


Fig. 1.16b



Basic antenna parameters:

Radiation pattern, gain, efficiency, impedance, frequency characteristics, shape, size, weight f appearance of antennas and cost is also one of the factor

Another definition: A radio antenna may be defined as the structure associated with the region of transition between a guided wave and a free space wave (Antennas convert electrons to photons)

[Photon is a quantum unit of electromagnetic energy] an antenna may be defined as [to radiate or receive electromagnetic waves as system of elevated conductors which couples or matches the t_x or R_x to antenna is a transition device or transducer free space between a guided wave and a free space wave

Basic radiation equation:

Motion of electric charges & creation of current flow.All antennas involve the same basic principle that radiation is produced by acceleration / decelerated charge. The basic equation of radiation may be expressed simply

$$D = \frac{4\pi(str)}{\Omega_A(str)} = \frac{41,253}{\theta_{HP}\phi_{HP}}$$

$$D \cong \frac{40,000}{(FNBW)^2} x4$$

$$HPBW \cong \left(\frac{FNBW}{2}\right)_\theta \left(\frac{FNBW}{2}\right)_\phi \frac{FNBW}{2})\Omega_A = \theta^0{}_{HP}\phi^0{}_{HP}$$

$$\varepsilon_p = \frac{A_e}{A_P} = SA_P$$

as

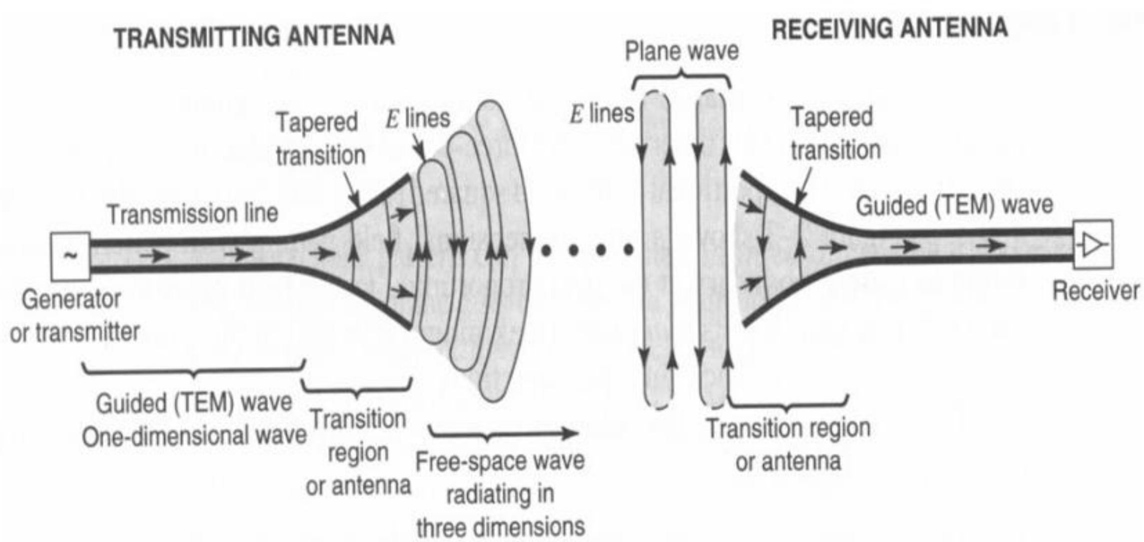
$$\overset{0}{I} L = Q\overset{0}{V} CAm5^{1}$$

$\overset{0}{I}$ = time charging current
L= length of current element
Q=charge c
$\overset{0}{V}$ = time change of velocity which equals the acceleration of charge ms^{-2}
Curved, bent, discontinues there is a radiation

IL & Qv radiates:

[if the charge is not moving current is not created & there is no radiation.Moving with uniform velocity there is no radiation if the wire is straight If the wire] ,For steady state harmonic variation, we focus on current ,For transients or pulses, we focus on charge

Radiation $\perp$ to the acceleration
Radiation power $\perp$ $(i(\ )^2/(QV)^2$



The two wire t_x' ion line is connected to the 't_x' (RF Generator). At first stage, the spacing between lines (wires) is assumed to be a small fraction of a wave length after that the $t_\propto$'ion line opens out in a tapered transition.
As the separation approaches the order of wave length or more, the wave tends to be radiated then the opened out line acts like an antenna which launches a free space wave [currents are flow out and end there but the fields associated with them keep on going]
Recall the transmission lines. There would be perfect reflection of a wave. If it is open circuited/short circuited.An equivalent circuit of a line with loss can be In terms of R, L, G, C (with loss) In terms of L&C (without loss)
The process of carrying energy of a propagating wave is shared by electric field (F) & magnetic field (H) or V/I energy share.

[Basic concept of radiation:-[Electromagnetic energy consist of E & M fields] at the OC end, the current becomes 'o' and pert of the energy shared by magnetic field becomes 'o' but it is mathematically. All of us know very well about the energy can neither be created nor be destroyed we have to survive the energy of magnetic field how it is possible.

So what is electric field carrying energy i.e $\frac{CV^2}{2}$ the line parameter cannot change $\left(C = \frac{EA}{d}\right)$ either ε,'A', d gets altered
So the possibility is only the change of voltage by which the additional energy can be carried by the electric filed.

Then the voltage rises at the C and to enable it to carry the total energy current wave following by a voltage wave at OC end voltage wave followed by a current wave at SC and Wave possess a moment of inertia like property, it will take some time to change its direction, this time may be small , some energy is likely to leak into the space this process of leakage can be termed as "Radiation"

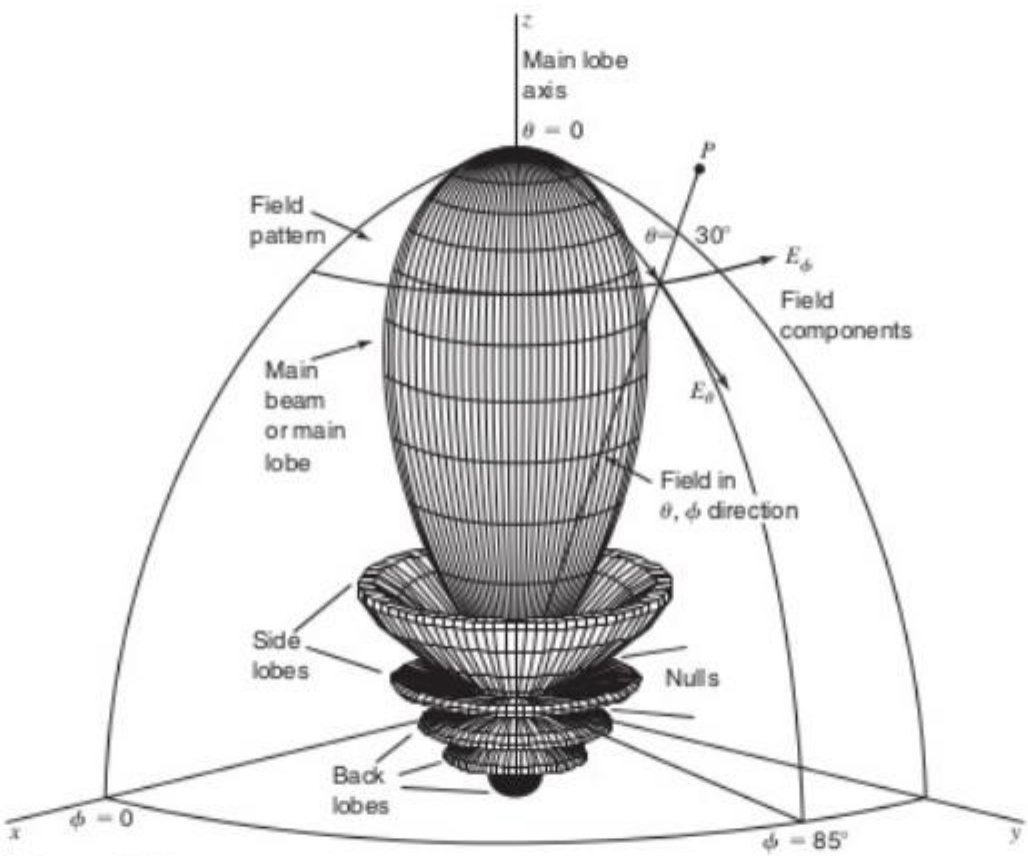
If the opening at the end is more, the more time will be taken by the wave to change its direction and thus more energy will leak into the space. So the maximum radiation will occur when the two wires at the end are flared to form a 180 angle. From the circuit point of view, the antennas appear to the t_x' ion lines as a resistance 'R_π' called "radiation resistance". In the t_x' ion case- Radiated power is desorbed by object at a distance tress, buildings, sky, gud, other R_x' ion case- passive radiation from distant objects active radiation other antennas

Patterns:- It is a SD quantity (quantities) involving the variation of field or power (proportional to the squared field) as 'E' (amplitude field pattern) a function E^2 (power pattern) of the spherical co-ordinate 2 and ϕ

Pattern with radians 'π' proportional to the field intensity in the direction 2 and ϕ

The pattern has its main lobe in the z-direction with (minimum radiation) minor lobes in other direction. To specify the radiation patters w.r to field intensity field pattern & polarization requires 3 patterns

1. The θ component of 'E' field as a function of the angles θ & ϕ $E^{\phi}(\theta, \phi)$
2. The ϕ component of 'E' field as a function of the angles θ &ϕ $E^{\phi}(\theta, \phi)$



The normalized field pattern for the electric field is given by (no directions)

$$E_\theta(\theta,\phi)_n = \frac{E_\phi(\theta,\phi)}{E_\theta(\theta,\phi)_{\max}}$$

[Patterns may be also be expressed in terms of the power per unit area]

Normalized power pattern= $\frac{s(\theta,\phi)}{s_n(\theta,\phi)_{\max}}$ (no directions)

$s(\theta,\phi)$ =pointing vector= $E^2{}_\theta(\theta,\phi)+E^2{}_\phi(\theta,\phi)]/z_0$ w/m^2

Half power beam width: the half power level occurs at those angles θ and Ø for which $E_\theta(\theta,\phi)_n$ (the angular beam width at the half = $\frac{1}{2}=0.907$ power level or half power beamwidth)

Pb1) an antenna has a field pattern given by $E(\theta)=\cos^2\theta \quad \forall 0^0 \le \theta \le 90^0$ find HPBW

Sol: $E(\theta)$ at half power = 0.707

0.707 = $\cos^2\theta$ $\cos\theta = \sqrt{0.707}$ & θ =33^0

HPBW=2θ=66^0

Pb2) an antenna has a field pattern given by $E(\theta)=\cos\theta\cos 2\theta \quad \forall 0^0 \le \theta \le 90^0$ find HPBW, FNBW

Sol: $E(\theta)$ at half power=0.707

$$0.707=\cos\theta\cos 2\theta=\frac{1}{\sqrt{2}}$$

$$\cos\theta=\frac{1}{\sqrt{2}\cos\theta}\rightarrow\theta=\frac{1}{2}\cos^{-1}(\frac{1}{\sqrt{2\cos\theta^{-1}}})$$

Beam Area/ Beam Solid angle (Ω_A):

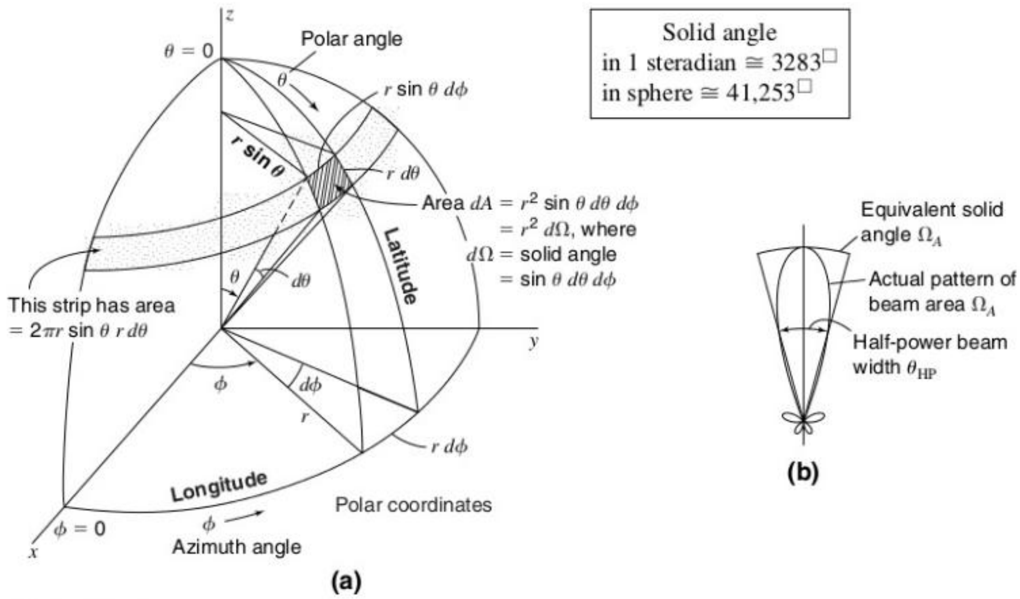
In spherical co-ordinate system, an incremental area dA on the surface of a sphere is the product of the length πda in the dθ(latitude) direction and $\pi\sin\theta d\phi$ (longitude) in the direction

$$dA=\pi d\theta.\pi\sin\theta d\phi=\pi^2\sin\theta d\theta d\phi$$

$$dA=\pi^2 d\Omega_A$$

$\phi\Omega$ =solid angle expressed in steradians or the area of the strip of width $\pi d\theta$ extending around the sphere at a constant angle θ is given by (2π r.sinθ) ($\pi d\theta$)

By integrating over for $\theta to\pi = \int_{\theta}^{\pi} 2\pi r \sin\theta \pi d\theta = 4\pi r^2$ subtended by a



sphere

The beam area or beam solid angle or Ω_A of an antenna is given by the integral of the normalized power pattern over a sphere (4π sr)

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} p_n(\theta,\phi)\sin\theta d\theta d\phi$$

$$d\Omega = \sin\theta d\theta d\phi$$

The beam area Ω_A is the solid angle through which all the power at radiated by the antenna the beam area of an antenna can often be described approximately in terms of the angles subtended by the half power points of the main lobe in the two principle planes.

Beam area= $\Omega_A \simeq \theta_{HP}\phi_{HP}$

Pb: An antenna has a field pattern given by $E(\theta) = \cos^2\theta \quad \forall 0^0 \le \theta \le 90^0$ Find the beam area Ω_A

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} p_n(\theta,\phi)\sin\theta d\theta d\phi$$

$$p_n(\theta,\phi) = E^2_\theta(\theta,\phi) = \int_0^{2\pi}\int_0^{\pi} \cos^4\theta \sin\theta d\theta d\phi$$

Radiation Intensity: the power radiated from an antenna per unit solid angle is called the radiation intensity 'U' watts/steraduan

$U = P_\pi.\pi^2 \quad P_\pi$ =radiated power

We know $d\Omega = \frac{ds}{\pi^2} \& \frac{ds}{d\Omega} = \pi^2$ →there are π^2 +meters surface area per unit solid angle

Power radiated per unit area in any direction is given by the pointing vector ⇒ p=EXH w/m^2

Where E & H are orthogonal to each other relation between E&H ⇒ $\frac{E}{H} = \eta_0$

η_0 =intrinsic impedance

P=E^1H ⇒ P=E $\frac{E}{\eta_0} = \frac{E^2}{\eta_0}$ w/m^2

Simple pointing vector $P = \frac{E^2}{\eta_0}$ watts/m^2

But this is average pointing vector $P_\pi = \frac{1}{2}\frac{E^2(\theta,\phi)}{\eta_0}$

P= $\frac{1}{2}[ExH^*]$

$E = a_e E_0 e^{jwt}$

$H^* = a_n H_0 e^{-(jw-8)}$

$H = a_n H_0 e^{j(wt-8)}$

8=phase difference but^4 E+H

Why we are taking P_{CV} E&H are changing with time then

P_q=real part of p $_{complex}$=R_e {P}

P_q= $\frac{1}{2}R_e\{ExH\}$ watts

In this E&H instantaneous values if runs values $\frac{1}{2}$ will be omitted are taken

So that $U = P_\pi.\pi^2$

$$U = \frac{1}{2}\frac{E^2(\theta,\phi)}{\eta_0}\pi^2$$

From $U(\theta,\phi)$ we can calculate normalized power

$$P_u(\theta,\phi) = \frac{U(\theta,\phi)}{U_m(\theta,\phi)} = \frac{s(\theta,\phi)}{s(\theta,\phi)_{mm}}$$

Beam efficiency:- The beam area or beam solid angle consists of the main beam area plus the minor lobe area

$$\Omega_{A=}\ \Omega_M + \Omega_m$$

Beam efficiency is defined as the ratio of the main beam area to the total beam area dimension less

The ratio of minor lobe area to the total beam area is called the stray factor

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A} \qquad \varepsilon_m = \frac{\Omega_m}{\Omega_A} \qquad \frac{\Omega_A}{\Omega_A} = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

Directivity (D) and Gain (G):- The directivity and gain are probably the most important parameters of an antenna

The directivity of an antenna is equal to the ratio of the maximum power density $P(\theta,\phi)$ max. w/m^2 to its average value over a sphere

$$D = \frac{P(\theta,\phi)_{\max}}{P(\theta,\phi)_{avg}} \quad D \geq 1 \text{ Dimension less}$$

This 'D' from pattern

The average power density over a sphere is given by

$$P(\theta,\phi)_{avg} = \frac{1}{4n}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{2\pi} P(\theta,\phi)\sin\theta d\theta d\phi$$

$$P_{avg} = \frac{1}{4n}\iint_{4\pi} P(\theta,\phi)d\Omega$$

$$D = \frac{P(\theta,\phi)_{\max}}{\frac{1}{4n}\iint_{4\pi} P(\theta,\phi)d\Omega} = \frac{1}{\frac{1}{4n}\iint_{4\pi} \frac{P(\theta,\phi)}{P(\theta,\phi)_{\max}} d\Omega}$$ [directivity from beam area]

$$D = \frac{1}{\frac{1}{4n}\iint P(\theta,\phi)d\Omega}$$

$$D = \frac{4\pi(str)}{\Omega_A(str)}$$ [Ratio of the area of the sphere to the beam area]

Gain:- the gain 'G' of an antenna is defined as the maximum power density of antenna / Test antenna with a reference antenna

$$G = \frac{P(\theta,\phi)_{\max} ofAUT}{P(\theta,\phi)_{\max} of \operatorname{Re}alanten}$$

Gain is always less then directivity (maximum directive gain) due to ohmic losses in the antenna the ratio of the gain to the directivity is the antenna efficiency factor G=KD K=0 to1

$$G = \frac{\max.powerR_X ledfromgivenantenna}{\max.powerR_X ledfromrefantenna} = \frac{p_1}{p_2}$$

Directive gain in a given direction is defined as the ratio of the radiation intensity in that direction to the average radiated power.

$$D = \frac{U(\theta,\phi)}{P_\pi/4\pi}$$

Gain=m.r.i from test/m.r.i from isotropic
with same power i/p in a gain direction

If the half power beam widths of an antenna are known, its directivity 'D'

$$D = \frac{4\pi(str)}{\Omega_A(str)} = \frac{41,253}{\theta_{HP}\phi_{HP}}$$ no. of rq degrees in sphere

Approximately $$D \cong \frac{40,000}{\theta_{HP}\phi_{HP}}$$

*the directivity –beam width product is 40,000(apper)

Directivity and Resolution:- The resolution of an antenna may be defined as equal to half the beam width between first nulls **FNBW/2_**To distinguish between t_x's on two adjacent satellites half power beam width then the *Directivity will half of the

$$HPBW \cong \frac{FNBW}{2} \quad \text{Become} \quad D \cong \frac{40,000}{(FNBW)^2} x4$$

$$\Omega_A = \theta^0_{HP} \phi^0_{HP}$$

Beam area $\Omega_A = \left(\frac{FNBW}{2}\right)_\theta \left(\frac{FNBW}{2}\right)_\phi$ (no.of T_X's or sources of radiation distribution over the slay uniformly

$$N = \frac{4\pi}{\Omega_A} \qquad D = \frac{4\pi}{\Omega_A}$$

But we know that ideally the number of point sources the antenna can resolve is numerically equal to the directivity of the antenna * when the antenna beam aligned with one satellite.

Principal Patterns:

For a linearly polarized antenna, performance is often described in terms of its principal E- and H-plane patterns. The E-plane is defined as ―the plane containing the electric field vector and the direction of maximum radiation,‖and the H-plane as ―the plane containing the magnetic-field vector and the direction of maximum radiation.

Although it is very difficult to illustrate the principal patterns without considering a specific example, it is the usual practice to orient most antennas so that at least one of the principal plane patterns coincide with one of the geometrical principal planes. An illustration is shown in Figure 2.5. For this example, the x-z plane (elevation plane; φ = 0) is the principal E-plane and the x-y plane (azimuthal plane; θ = π/2) is the principal H-plane. Other coordinate orientations can be selected.

The omni directional pattern of Figure2.6 has an infinite number of principal E-planes (elevation plan es; φ = φc) and one principal H- plane (azimuthal plane; θ = 90◦).

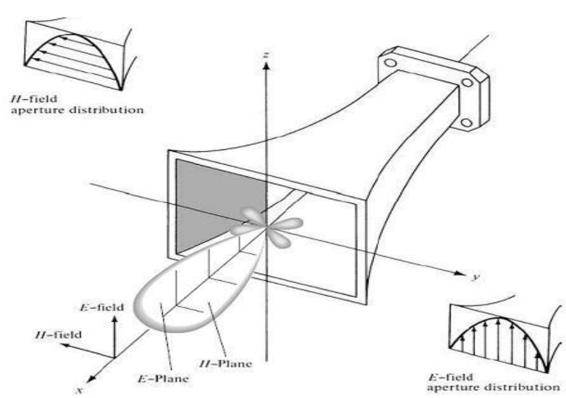


Fig. Principal E- and H-plane patterns for a pyramidal horn antenna

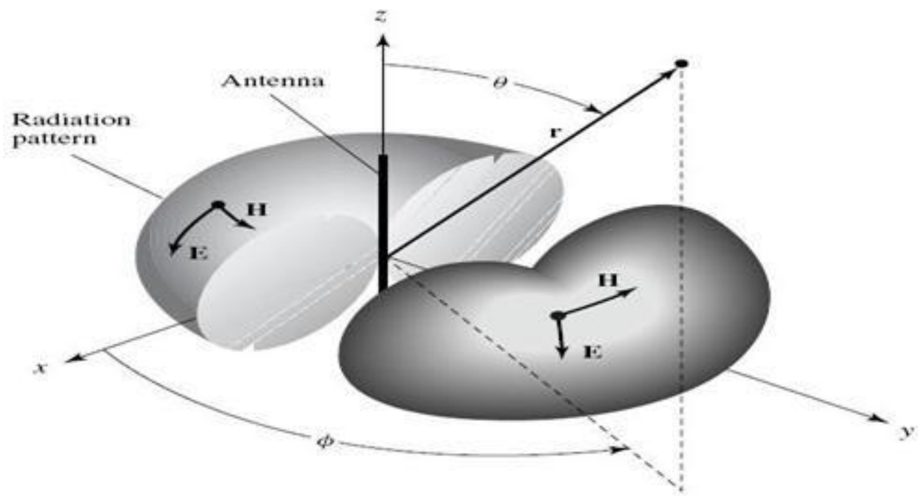
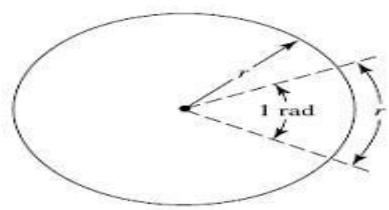


Fig. Omni directional antenna pattern

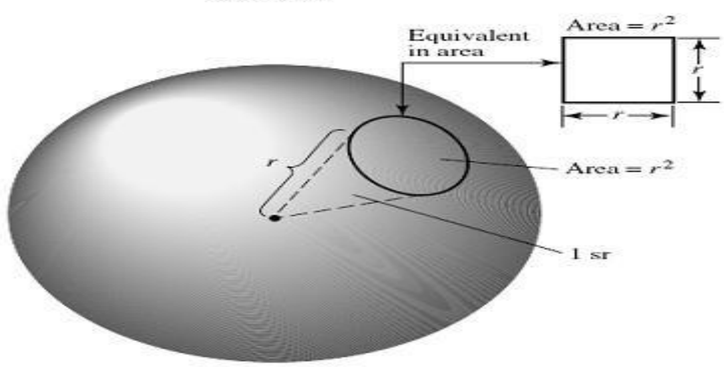
Radian and Steradian

The measure of a plane angle is a radian. One radian is defined as the plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is r. A graphical illustration is shown in Figure (a). Since the circumference of a circle of radius r is C = 2πr, there are 2π rad (2πr/r) in a full circle.

The measure of a solid angle is a steroidal. One steadied is defined as the solid angle with its vertex at the center of a sphere of radius r that is subtended by a spherical surface area equal to that of a square with each side of length r. A graphical illustration is shown in Figure (b). Since the area of a sphere of radius r is $A = 4\pi r^2$, there are 4π sr ($4\pi r^2/r^2$) in a closed sphere.



(a) Radian



(b) Steradian

Although the radiation pattern characteristics of an antenna involve three-dimensional vector fields for a full representation, several simple single-valued scalar quantities can provide the information required for many engineering applications.

Introduction:

Half power beam width: the half power level occurs at those angles θ and $\emptyset$ for which $E_\theta(\theta,\phi)_n$ (the angular beam width at the half $=\frac{1}{2}=0.907$ power level or half power beamwidth)

Pb1) an antenna has a field pattern given by $E(\theta)=\cos^2\theta \quad \forall 0^0 \le \theta \le 90^0$ find HPBW

Sol: $E(\theta)$ at half power = 0.707

$0.707 = \cos^2\theta \qquad \cos\theta=\sqrt{0.707} \quad \& \ \theta=33^0$

HPBW=2θ=66^0

Pb2) an antenna has a field pattern given by $E(\theta)=\cos\theta\cos 2\theta \quad \forall 0^0 \le \theta \le 90^0$ find HPBW, FNBW

Sol: $E(\theta)$ at half power=0.707

$$0.707=\cos\theta\cos 2\theta=\frac{1}{\sqrt{2}}$$

$$\cos\theta=\frac{1}{\sqrt{2}\cos\theta}\rightarrow\theta=\frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2\cos\theta^{-1}}}\right)$$

Beam Area/ Beam Solid angle (Ω_A):

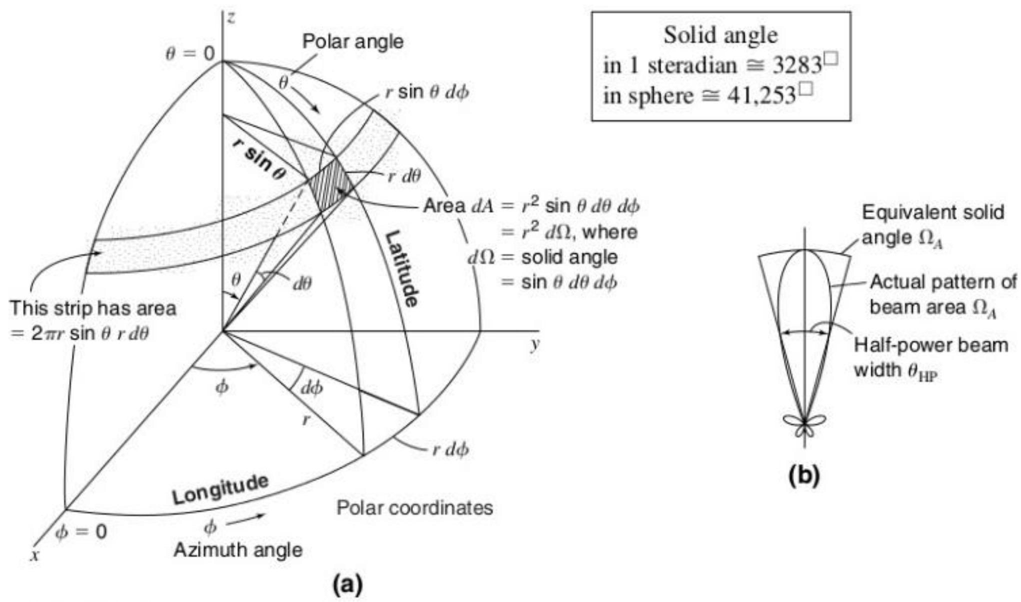
In spherical co-ordinate system, an incremental area dA on the surface of a sphere is the product of the length πda in the dθ(latitude) direction and $\pi\sin\theta d\phi$ (longitude) in the direction,

$$dA=\pi d\theta\,.\,\pi\sin\theta d\phi=\pi^2\sin\theta d\theta d\phi$$

$$dA=\pi^2 d\Omega_A$$

$\phi\Omega$=solid angle expressed in steradians or the area of the strip of width $\pi d\theta$ extending around the sphere at a constant angle θ is given by (2π r.sinθ) ($\pi d\theta$).

By integrating over for $\theta$to π=$\int_{\theta}^{\pi}2\pi r\sin\theta\pi d\theta=4\pi r^2$ angle subtended by a sphere.



The beam area or beam solid angle or Ω_A of an antenna is given by the integral of the normalized power pattern over a sphere (4π sr)

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} p_n(\theta,\phi)\sin\theta d\theta d\phi$$

$$d\Omega = \sin\theta d\theta d\phi$$

The beam area Ω_A is the solid angle through which all the power at radiated by the antenna the beam area of an antenna can often be described approximately in terms of the angles subtended by the half power points of the main lobe in the two principle planes.

$$\text{Beam area} = \Omega_A \simeq \theta_{HP}\phi_{HP}$$

Pb: An antenna has a field pattern given by $E(\theta) = \cos^2\theta$ $\forall 0^0 \le \theta \le 90^0$ Find the beam area Ω_A

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} p_n(\theta,\phi)\sin\theta d\theta d\phi$$

$$p_n(\theta,\phi) = E^2{}_\theta(\theta,\phi) = \int_0^{2\pi}\int_0^{\pi}\cos^4\theta\sin\theta d\theta d\phi$$

Radiation Intensity: the power radiated from an antenna per unit solid angle is called the radiation intensity 'U' watts/steraduan [Which does not depend upon the distance from the radiator?]

$U = P_\pi . \pi^2 P_\pi$ =radiated power

We know $d\Omega = \frac{ds}{\pi^2}$ & $\frac{ds}{d\Omega} = \pi^2$ →there are π^2 +meters surface area per unit solid angle

Power radiated per unit area in any direction is given by the pointing vector ⇒ p=EXH w/m^2. Where E & H are orthogonal to each other relation between E&H ⇒ $\frac{E}{H} = \eta_0$

η_0 =intrinsic impedance

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Simple pointing vector $P = \frac{E^2}{\eta_0}$ watts/m^2

But this is average pointing vector $P_\pi = \frac{1}{2}\frac{E^2(\theta,\phi)}{\eta_0}$

P=

$$E = a_e E_0 e^{jwt}$$

$$H^* = a_n H_0 e^{-(jw-8)}$$

$$H = a_n H_0 e^{j(wt-8)}$$

8=phase difference but^4 E+H , we are taking P_{CV} E&H are changing with time then

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$$D = \frac{1}{\frac{1}{4n}\iint P(\theta,\phi)d\Omega}$$

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Gain:-

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The resolution of an antenna may be defined as equal to half the beam width between first nulls **FNBW/2_**To distinguish between t_x's on two adjacent satellites half power beam width then the *Directivity will half of the

$$HPBW \cong \frac{FNBW}{2} \text{ Become } D \cong \frac{40,000}{(FNBW)^2} x4$$

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Beam area $\Omega_A = \left(\frac{FNBW}{2}\right)_\theta \left(\frac{FNBW}{2}\right)_\phi$ (no.of T_X's or sources of radiation distribution over the slay uniformly $N = \frac{4\pi}{\Omega_A}$)

But we know $D = \frac{4\pi}{\Omega_A}$

We conclude that ideally the number of point sources the antenna can resolve is numerically equal to the directivity of the antenna * when the antenna beam aligned with one satellite.

Signification:-

the directivity is equal to the number of point sources in the sky that the antenna can resolve (T&C) [under the ideal conditions of a uniform source distribution

Antenna apertures:-

the concept of aperture is most simply introduced by considering a receiving antenna,Let us consider pointing vector or power density of the plane wave be‘s’ watts/m^2 ,The area or physical aperture- APm^2

Power $P = \frac{E^2}{Z} A_P = SA_P$ (physical aperture)

The total power extracting by the horn from a passing wave being proportional to the aperture or area of its mouth
But the field response of horn is not uniform across the aperture ‘A’ because ‘E’ at the side walls must equal to ‘0’
Thus ‘A_e’ (the effective aperture) of the horn is less than the physical aperture A_p as given by

$$\varepsilon_{op} = \frac{A_e}{A_P} \quad \text{Aperture efficiency}$$

Horn parabolic} 50 to 80% ε_{op} 0.5 to 0.8, Consider an antenna with an effective aperture A_e which radiates its entire pole in a conical pattern

$$P = \frac{E^2}{Z_0} Ae \text{ Watts intrinsic impedance (377}\Omega \text{ for air)}$$

Assuming that uniform field ‘Eπ’ in the for field at a distance ‘r’ the power radiated is also given by

$$p = \frac{E_\pi^2 \pi^2}{Z_0} \Omega_A \text{ Watts}$$

By equating above two eq’s

$$E_\pi = \frac{E_a A_e}{\pi\lambda} \Rightarrow \lambda^2 = A_e \Omega_A$$

We get $D = 4\pi \frac{A_e}{\lambda^2}$ $\qquad \lambda^2 = A_e \Omega_A$ (m^2)

$$A_e = \frac{D\lambda^2}{4\pi}$$

If A_e is known, we can determine 'Ω_A' at a given wavelength

$$D = 4\pi \frac{A_e}{\lambda^2} \text{ (Directivity from aperture)}$$

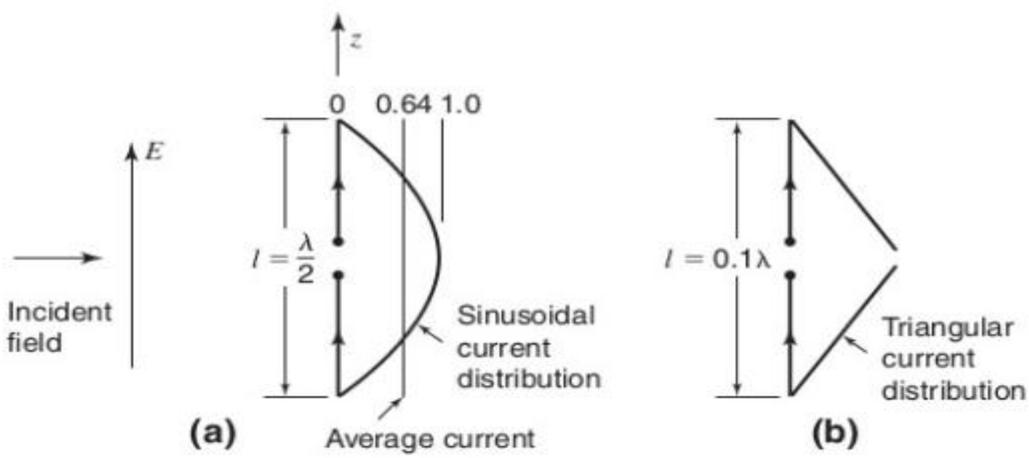
From that $A_e = \frac{D\lambda^2}{4\pi}$

Effective height:-

The effective height h(units) of an antenna is another parameter related to the aperture. By multiplying the effective height 'h' by the incident field β(v/m) of the same polarization given the voltage v induced.

V=hE

The effective height may be defined as the ratio of the induced voltage to the incident field h=v/E (m)



Consider the vertical of length l=$\lambda/2$ increased in an incident field E. if the current distribution is uniform, its 'G'=l.

but actual current distribution is nearly sinusoidal with an average value 2/π =0.64 so h=0.64lIf the same dipole is used at a longer wave length is only 0.1λ long then h=0.5lEffective height is a useful parameter for transmitting tower type antennas.

Antenna Theorems:

The well known network theorems including superposition theorem, thevenin's theorems, maximum power transfer theorem, compensation theorem and reciprocity theorem lead to very useful antenna theorems which relate the properties make it possible to derive/ correlate the properties of a receiving antenna from its properties as a transmitting antenna.Equality of directional patterns,Equality of transmitting and receiving antenna impedance and Equality of effective length

Field zones:

The fields surrounding the antenna are divided into 3 principle regionsReactive Near field, Radiating Near field or Fresnel Region,Far field or Fraunhofer Region

Reactive Near field:

The fields are predominately reactive fields which mean E&H fields are out of phase by 90^0 to each other boundery of this region is given as

$$R < 0.62\sqrt{\frac{0.3}{\lambda}}$$

Radiating Near field or Fresnel Region:
It is the region between near &far fields the reactive fields are not dominate. Here the shape of the radiation pattern may vary appreciably with distance & the radiating fields being to move.

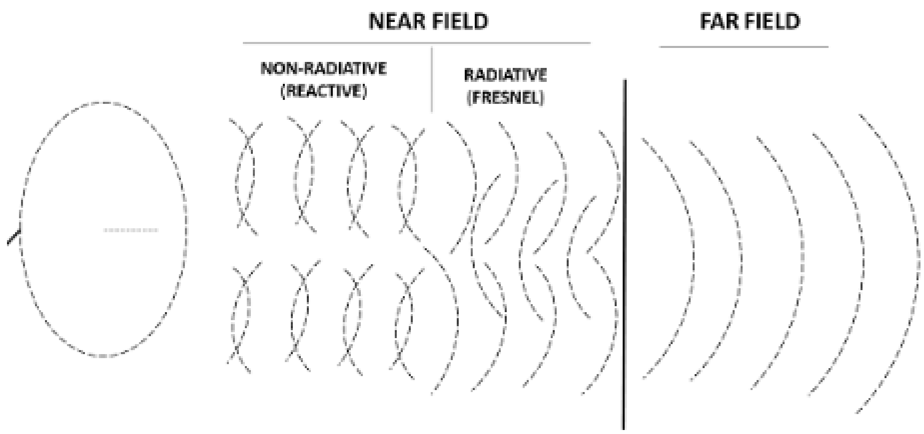
Radiating varies from $0.62\sqrt{\frac{0.3}{\lambda}}$ to $\frac{2D^2}{\lambda}$

This field may or may not exit depends on R&S

Equality of effective length:
In this region, the radiation pattern does not change shape with distance.
This region is dominated by radiated fields with E&H fields orthogonal to each other & the direction of propagation as with plane wave.

$$R > \frac{2D^2}{\lambda} \qquad R >> D \qquad R >> V$$

The powers radiated in a given direction from district parts of the antenna are approximately parallel. So the fields in the far field region behave like plane waves.



Antenna Efficiency:
The efficiency of the antenna is defined as the ratio of power radiated to the total i/p power to the antenna and is denoted by ɳ or k,

K=power radiated/total i/p power=G_P/D

In terms of resistances $K = \frac{R_r}{R_r + R_L} \times 100$

Loss resistance may consist-

- Ohmic loss in the anti conductor
- Dielectric loss
- I^2R loss in antenna & ground
- Loss in earth connections
- Leakage loss insulation

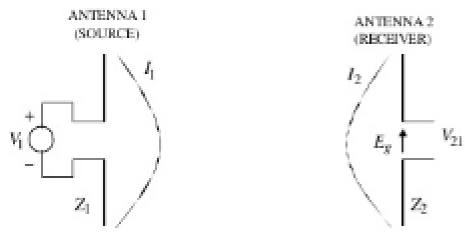
Radiation resistance:

$$R_r = \frac{P_r}{I^2}$$

It is defined as that fictitious resistance when substituted in series with the antenna will consume the some power as is actually radiated.

Reciprocity theorem:
Most powerful in circuit & field theories Rayleigh Helmholtz →Rayleigh Reciprocity Theorem

For antennas stat:
If an emf is applied to the terminals of an antenna no.1 & the current measured at the terminals of antenna no.2. then an equal current both in amplitude and phase will be obtained at the terminals of antenna no.of if the same e.m.f is applied to the terminals of antenna no.2



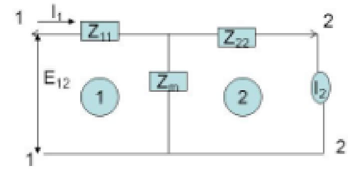
$$Z_m = Z_{12} = Z_{21} = \frac{E_{12}}{I_2} = \frac{E_{21}}{I_1} \Rightarrow \frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}$$

Application of Reciprocity theorem:
Equality of Directional patterns:
The directional patterns of transmitting and receiving antennas are identical if all the media are linear, passive, isotropic and the reciprocity holds good.

Consider the t_x antenna no.1 as test antenna which is placed at the centre of the observation circle. The receiving antenna no.2 is moved along along the surface the antenna no.2 is always kept perpendicular to the radius vector & parallel to the electric vector (if the polarization is linear), If a voltage 'E' is applied at no.1 & the resulting current 'I' at the terminals of $R_X$ing antenna 2 is measured- which will be the indication of electric field at the location of antenna no.2,If the process is reversed same will be occurred same voltage is applied to 2 & 'I' at no.1 this time the $R_x$ing petterns of 2

Accordingly to reciprocity theorem, for every position of test antenna no.1, the ratio E/I is the same.It is provided that radiation pattern of a test antenna no.1 observed by moving receiving antenna no.2 is identical with the positions inter changed.

Equality of Directivities:
If the radiation pattern of an antenna is same whether it is transmitting or receiving then the directivities will be same.
Proof of reciprocity theorem



$$0 = Z_m(I_2 - I_1) + Z_{22}I_2$$

$$I_2(Z_m + Z_{22}) = I_1 Z_m$$

$$I_2 = \frac{Z_m}{Z_m + Z_{22}} I_1 \text{ -------------------- (1)}$$

$$E_{12} = I_1 Z_{11} + Z_m(I_2 - I_1)$$

$$E_{12} = I_1(Z_{11} + Z_m) - I_2 Z_m$$

$$E_{12} = I_1(Z_{11} + Z_m) - \left(\frac{Z_m}{Z_m + Z_{22}}\right) I_1 Z_m$$

$$I_1 = \frac{(Z_m + Z_{22})E_{12}}{Z_{11}Z_m + Z_{11}Z_{22} + Z_m Z_{22}}, \; I_1 = \frac{(Z_m + Z_{22})}{(Z_{11} + Z_m)(Z_m + Z_{22}) - Z_m{}^2} \times E_{12}$$

Sub I_1 in I_2 (eq1)

$$I_2 = \frac{Z_m}{Z_m + Z_{22}} \times \frac{Z_m + Z_{22}}{(Z_{11} + Z_m)(Z_m + Z_{22}) - Z_m{}^2}$$

$$I_2 = \frac{E_{12}Z_m}{Z_{11}Z_m + Z_{11}Z_{22} + Z_m Z_{22}} = \frac{E_{12}Z_m}{Z_{11}Z_{22} + Z_m(Z_{11} + Z_{22})} \text{ ---------- (2)}$$

$$E_{12} = I_1(Z_{22} + Z_m) - I_1 Z_m = E_{21}$$

$$0 = Z_{11}I_1 + Z_m(I_1 - I_2)$$

$$I_1 = \frac{Z_m E_{21}}{Z_{11}Z_{22} + Z_m(Z_{11} + Z_{22})} = \frac{Z_m E_{12}}{Z_{11}Z_{22} + Z_m(Z_{11} + Z_{22})}$$

$$I_2(Z_{22} + Z_m) - \frac{Z_m Z_m}{Z_m + Z_{11}} I_2 = E_{21}$$

$$I_2 = \left[\frac{(Z_{11} + Z_m)}{Z_{11}Z_{22} + Z_m(Z_{11} + Z_{22})} E_{21}\right] =$$

$$E_{21} = Z_{22}I_2 + Z_m(I_2 - I_1)$$

$$= I_1(Z_{22} + Z_m) - I_1 Z_m = E_{21} \text{ ---------------- (3)}$$

$$0 = Z_{11}I_1 + Z_m(I_1 - I_2)$$

$$I_1 = \frac{Z_m I_2}{Z_m + Z_{11}} \text{ --------- (4)}$$

From (3)

$$I_2(Z_{22}+Z_m)-\frac{Z_mZ_m}{Z_m+Z_{11}}I_2=E_{21}$$

$$I_2\left[\frac{(Z_{22}+Z_m)(Z_{11}+Z_m)-Z_m{}^2}{Z_{11}+Z_m}\right]=E_{21}$$

$$I_2=\left[\frac{(Z_{11}+Z_m)}{Z_{11}Z_{22}+Z_m(Z_{11}+Z_{22})}E_{21}\right] \text{---------------- (5)}$$

From (4) I_2 in I_1

$$I_1=\frac{Z_mE_{21}}{Z_{11}Z_{22}+Z_m(Z_{11}+Z_{22})} \text{ ----------------- (6)}$$

Equation (2) & (6)

If $I_1 = I_2$

$$\frac{Z_mE_{21}}{Z_{11}Z_{22}+Z_m(Z_{11}+Z_{22})}=\frac{Z_mE_{12}}{Z_{11}Z_{22}+Z_m(Z_{11}+Z_{22})}$$

$E_{12}=E_{21}$

If $I_1 = I_2 \rightarrow E_{21} = E_{12}$

Retarded Potential (Time Varying Potential):

Application Maxwell's equations to the antenna radiation.Let us check & modify those eq's in different form radiation is a time varying phenomena.

Step (1)

We know $B=\nabla\times\ddot{\ddot{A}}$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t} \quad \text{Substitute 'B' in this}$$

$$\nabla\times\mathbf{E}=-\frac{\partial}{\partial t}(\nabla\times\ddot{\ddot{A}})$$

$$\nabla\times\mathbf{E}=-(\nabla\times\frac{\partial A}{\partial t})$$

$$\nabla\times\mathbf{E}=0 \qquad \nabla\times-\nabla V=0 \qquad \ddot{E}+\frac{\partial\dddot{A}}{\partial t}=-\nabla V$$

Then $E=-\nabla V-\frac{\partial A}{\partial t}$

Which satisfies both the static & the time varying conditions?

Step (2)

$$\nabla.D=P$$

$$\nabla.D=\nabla.\varepsilon E=\varepsilon(\nabla.E)$$

Subs (E)

$$=\varepsilon[\nabla(-\nabla V-\frac{\partial A}{\partial t})]$$

$$=\varepsilon(-\nabla.\nabla V-\frac{\partial}{\partial t}=\nabla.A)=P$$

$$= -\nabla^2 V - \frac{\partial}{\partial t} = \nabla . A = \frac{P}{\varepsilon}$$

$$\nabla^2 V - \frac{\partial}{\partial t} = \nabla . A = -\frac{P}{\varepsilon} \quad (\nabla^2 V = -\frac{P}{\varepsilon}\ forstatic)$$

$$\nabla^2 V = -\frac{P}{\varepsilon} - \frac{\partial}{\partial t} \nabla . A$$

For time varying condition

Step (3)

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad B = \mu H \rightarrow H = \frac{B}{\mu}$$

L.H.S

$$\nabla \times \frac{B}{\mu} = \frac{(\nabla \times \nabla \times A)}{\mu}$$

$$= \frac{[\nabla(\nabla . A) - \nabla^2 A]}{\mu} \quad \text{Vector identify}$$

R.H.S

$$J + \frac{\varepsilon \partial E}{\partial t} = J + \frac{\varepsilon \partial}{\partial t}(-\nabla V - \frac{\partial A}{\partial t})$$

$$= J + \varepsilon(-\nabla \frac{\partial V}{\partial t} - \frac{\partial^2 A}{\partial t^2})$$

$$= J - \varepsilon(\nabla \frac{\partial V}{\partial t} + \frac{\partial^2 A}{\partial t^2})$$

Equating L.H.S & R.H.S $\nabla^2 A$ is defined

$$\nabla(\nabla . A) - \nabla^2 A = \mu J - \mu\varepsilon[\nabla \frac{\partial V}{\partial t} + \frac{\partial^2 A}{\partial t^2}]$$

As per the Helmholtz theorem "A Vector field is completely define only when both its curt & divergence are known.There are some conditions which specify divergence of A. Two conditions we have to take Lorentz gauge condition & coulomb's gauge condition.

$$\nabla . A = -\mu\varepsilon \frac{\partial V}{\partial t}$$

$$\nabla . A = 0$$

Using (1) L.G $\quad \nabla^2 V = -\frac{P}{\varepsilon} - \mu\varepsilon \frac{\partial^2 V}{\partial t^2}$

$$\nabla^2 A = -\mu J - \mu\varepsilon \frac{\partial^2 A}{\partial t^2}$$

Step (4)

For sinusoidal time variations $V=V_0e^{jwt}$, $A=A_0e^{jwt}$

$$\nabla^2 V = -\frac{-P}{\varepsilon} - \mu\varepsilon \frac{\partial^2 V}{\partial t^2}$$

$$\nabla^2 A = -\mu J - W^2 \mu\varepsilon\pi$$

If P&J in the expressions of V&A are given in Maxwell's equations as above

They become functions of't' & this 't_1' is replaced by (t^1),

t^1=t-r/v

P&J can be [P] & [J]

$$V = \int_V \frac{[P]dv}{4\pi\varepsilon R}$$

$$A = \int_V \frac{\mu[J]dv}{4\pi R}$$

Suppose P=e^{-r}cosωt

e^{-r}cos(ωt-v)

'R' is distance between the elemental volume dv located in a current carrying conductor. So the equation of V&A are called as advanced potentials

$$V(r,t) = \frac{1}{4\pi\varepsilon}\int_V \frac{P(r,t)}{R}dv$$

$$A(r,t) = \frac{\mu}{4\pi}\int_V \frac{J(r,t)}{R}dv$$

$$V(r,t) = \frac{1}{4\pi\varepsilon}\int_V \frac{J(r^1, t - R/v)}{R}dv$$

$$A(r,t) = \frac{\mu}{4\pi}\int_V \frac{J(r^1, t - R/v)}{R}dv$$

Retarded Potential

The vector electric potential eq represents the super position of potentials due to various current elements at distant point (p) (r)

The effect of reaching a 'p' from a given element at an instant t due to a current which is followed at an earlier time.

Hence retardation time must be taken into according for sinusoidal time variation characterized by e^{jwt}

$$V = V_0 e^{jwt} \ \& \ A = A_0 e^{jwt}$$

$$\nabla^2 V = -\frac{-P}{\varepsilon} - \omega^2 \mu\varepsilon V$$

$$\nabla^2 A = -\mu J - \omega^2 \mu\varepsilon j$$

If P&J in the expressions of V and A given by they become functions of time and this time t is replaced by t^1

Such that t^1=t-r/v

'P' replaced by [P]

'J' replaced by [J]

$$V = \int \frac{[P]dv}{4\pi\varepsilon R} \qquad A = \int \frac{\mu[J]dv}{4\pi R}$$

The above two equations are called "retarded potentials"

If t^1=t+r/v then called "advanced potentials"

Radiation from small current element:

A small current element (short dipole) is considered as the basic source of radiation

For to find the E&H fields everywhere around in free space using the concept of retarded vector potential,Let us consider the small current element and the current flowing through it is 'I'. the current is constant along the length because of short length

Purpose:

This isolated current concept is unreal but still any physical circuit or antenna system carrying current may be considered to consist joined in series i.e end to end and hence if the Electromagnetic field of this building block is known the e.m field of any physical antenna having specified current distribution can be calculated.

Magnetic field components:

To find the electromagnetic field at any arbitrary point 'p'(r, Θ, ø), first calculate the vector magnetic potential 'A'

$$\mu H_{\phi} = \frac{1}{r}\left[\frac{\partial}{\partial r}(r(-A_z \sin\theta) - \frac{\partial}{\partial \phi} A_z \cos\theta\right]$$

$$(\Delta \times A)_{\phi} H_{\phi} = \frac{I_m \sin\theta dl}{4\pi}\left[-\frac{\omega \sin\omega(t-\frac{r}{v})}{v_r} - \frac{\cos\omega(t-\frac{r}{v})}{r^2}\right]$$

$$[A] = \frac{\mu}{4\pi}\int \frac{J(t-r/c)}{R}dv$$

As the current element is placed along the z-axis the vector potential will have only one component in +ve 'Z' direction & it is retarded by time 'r/v' sec

$$A_Z = \frac{\mu}{4\pi}\int \frac{J(t-r/v)}{R}dv$$

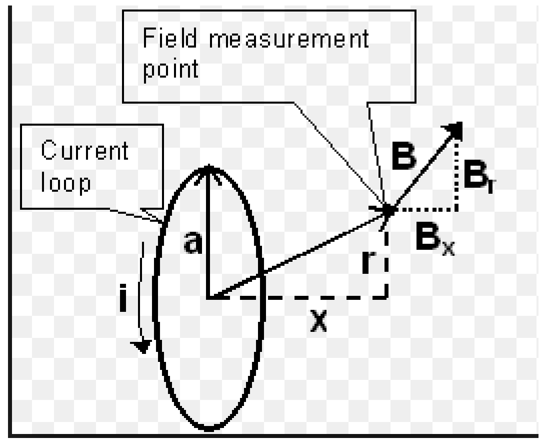
The current element $I_m \cos wt = I$

The integration of the current density J over a cross section area gives current I & this current is assumed to be constant along the length 'dl'

$$\int_v Jdv = I_m \cos wt$$

$$\int_v J(t-\frac{r}{v})dv = I_m dl \cos w(t-r)$$

$$A_Z = \frac{\mu}{4\pi}\frac{I_m dl \cos w(t-\frac{r}{v})}{R}$$



To find the curl A, we must find the component of A in r, Θ and ø directions

$$A_r = A_z \cos\theta$$

$$A_\theta = A_z \sin\theta$$

$$A_P = 0$$

$$\nabla \times A = (\nabla \times A)_r + (\nabla \times A)_\theta + (\nabla \times A)_\phi$$

$$\frac{\partial}{\partial \phi} = 0$$

$$(\nabla \times A)_r = \mu H_r = 0$$

$$(\nabla \times A)_\theta = \mu H_\theta = 0$$

$$(\nabla \times A)_\phi = \mu H_\phi$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right) - \frac{\partial A_r}{\partial \theta}\right] = B_\phi = \mu H_\phi$$

Cartesian to spherical

$$A_r = A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta$$

$$A_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$(\nabla \times A)_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] = 0$$

$$(\nabla \times A)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(A_\phi r)\right] = 0$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\phi r) - \frac{\partial A_r}{\partial \phi}\right] = \mu H_\phi$$

Thus the first two terms in the $\Delta \times \dddot{A}$ are equal to ‘o’

$$(\nabla \times \ddot{A})_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(A_\phi r) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\mu H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(r(-A_z \sin\theta) - \frac{\partial}{\partial \phi} A_z \cos\theta\right]$$

$$\nabla \times \ddot{A} = \frac{1}{r}\left[\frac{\partial}{\partial r}(-r\frac{\mu}{4\pi}\frac{I_m dl \cos w}{r}(t-\frac{r}{v})\sin\theta\right] - \frac{\partial}{\partial \theta}\left[\frac{\mu}{4\pi}\frac{I_m dl \cos w}{r}(t-\frac{r}{v})\cos\theta\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(-\frac{\mu I_m \sin\theta dl}{4\pi}\frac{\partial}{\partial r}\cos\theta(t-\frac{r}{v}) - \frac{\mu I_m dl \cos\omega}{4\pi}(t-\frac{r}{v})\frac{\partial}{\partial r}\cos\theta\right]$$

$$(\nabla \times A)_\phi = \frac{1}{r}\left[-\frac{\mu I_m \sin\theta dl \sin\omega(t-\frac{r}{v})}{4\pi}\left(-\frac{\omega}{v}\right) - \frac{\mu I_m \cos\omega}{4\pi r}(t-\frac{r}{v})\sin\theta\right]$$

$$\mu H_\phi = \frac{1}{r}\left[-\frac{\mu I_m \sin\theta dl}{4\pi}\left[\frac{\sin\omega(t-\frac{r}{v})}{v} - \frac{\cos\omega(t-\frac{r}{v})}{r}\right]\right]$$

$$H_\phi = \frac{I_m \sin\theta dl}{4\pi}\left[-\frac{\omega \sin\omega(t-\frac{r}{v})}{v_r} - \frac{\cos\omega(t-\frac{r}{v})}{r^2}\right]$$

Let us calculate the electric field.

$$\nabla \times \ddot{H} = \frac{\partial D}{\partial t} \qquad \text{(J is content)}$$

$$= \varepsilon \frac{\partial E}{\partial t} \qquad H_\theta=0, \quad H_r=0$$

$$\varepsilon \frac{\partial E_r}{\partial t} = (\nabla \times H)_r = \frac{1}{r\sin\theta}\left[\frac{r}{r\theta}(H_\phi \sin\theta - \frac{r}{r\theta}H_\theta\right]$$

$$\varepsilon \frac{\partial E_\theta}{\partial t} = (\nabla \times H)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}(\frac{rH_r}{r\theta} - \frac{\partial H_\phi r}{\partial \theta}\right]$$

$$\varepsilon \frac{\partial E_\theta}{\partial t} = (\nabla \times H)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(H_\theta r - \frac{\partial H_r}{\partial \theta}\right]$$

$$\varepsilon \frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{r}{r\theta}(\frac{I_m dl \sin^2\theta}{4\pi}\left(\frac{-w\sin wt_1}{vr} + \frac{\cos wt_1}{r^2}\right))\right]$$

$$= \frac{1}{r\sin\theta}\left[\frac{I_m dl}{4\pi}\left(\frac{-w\sin wt_1}{vr} + \frac{\cos wt_1}{r^2}\right)\frac{\partial}{\partial \theta}\sin^2\theta\right]$$

$$\varepsilon\frac{\partial E_r}{\partial t}=\frac{1}{r\sin\theta}\left[\frac{I_m dl}{4\pi}\left(\frac{-w\sin wt_1}{vr}+\frac{\cos wt_1}{r^2}\right)2\sin\theta\cos\theta\right]$$

$$\frac{\partial E_r}{\partial t}=\frac{I_m dl\cos\theta}{4\pi\varepsilon}\left(\frac{-w\sin wt_1}{vr^2}+\frac{\cos wt_1}{r^3}\right)$$

Apply integration on both sides

$$E_r=\frac{I_m dl\cos\theta}{4\pi\varepsilon}\int\frac{-w\sin wt_1}{vr^2}dt+\int\frac{\cos wt_1}{r^3}dt$$

$$E_r=\frac{I_m dl\cos\theta}{4\pi\varepsilon}\left[\frac{w\cos wt_1}{vr^2 w}+\frac{\sin wt_1}{wr^3}\right]$$

$$E_r=\frac{2I_m dl\cos\theta}{4\pi\varepsilon}\left[\frac{\cos wt_1}{vr^2}+\frac{\sin wt_1}{wr^3}\right]$$

$$\varepsilon\frac{\partial E_\theta}{\partial t}=(\nabla\times H)_\theta=\frac{1}{r}\left[-\frac{\partial H_\phi(r)}{\partial r}\right]$$

$$E_\theta=\frac{I_m dl\sin\theta}{4\pi\varepsilon}\left[-\frac{w\sin wt_1}{v^2 r}+\frac{\cos wt_1}{vr^2}+\frac{\sin wt_1}{wr^3}\right]$$

Therefore we have calculated all the 3 components of electric field intensity vector $E(E_r,E_\theta,E_\phi)\,\&\,H(H_r,H_\theta,H_\phi)$,We conclude here that out of six components of electromagnetic field, only three components E_r, E_θ &H_ϕ exist in the current element or Hertzian dipole & rest components E_θ, H_r, H_ϕ are everywhere ‘0’

$$\varepsilon\frac{\partial E_\theta}{\partial t}=(\nabla\times H)_\theta=\frac{1}{r}\left[-\frac{\partial H_\phi(r)}{\partial r}\right]$$

$$=\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{I_m dl\sin\theta}{4\pi}\left(\frac{-w\sin wt_1}{vr}+\frac{\cos wt_1}{r^2}\right)r\right]$$

$$=\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{I_m dl\sin\theta}{4\pi}\left(\frac{w\sin wt_1}{vr}-\frac{\cos wt_1}{r^2}\right)\right]$$

$$=\frac{I_m dl\sin\theta}{4\pi r}\left(\frac{\partial}{\partial r}\frac{w\sin wt_1}{v}-\frac{\partial}{\partial r}\frac{\cos wt_1}{r}\right)$$

$$\varepsilon\frac{\partial E_\theta}{\partial t}=\frac{I_m dl\sin\theta}{4\pi r}\left(\frac{w\cos wt_1}{v}\left(\frac{-w}{v}\right)-\frac{r\frac{\partial}{\partial r}\cos wt_1-\cos wt_1}{r^2}\right)$$

Apply Integration

$$\int\partial E_\theta=\int\frac{I_m dl\sin\theta}{4\pi r\varepsilon}\left(\frac{-w^2}{v^2}\cos wt_1-\frac{w}{vr}\sin wt_1+\frac{\cos wt_1}{r^2}\right)$$

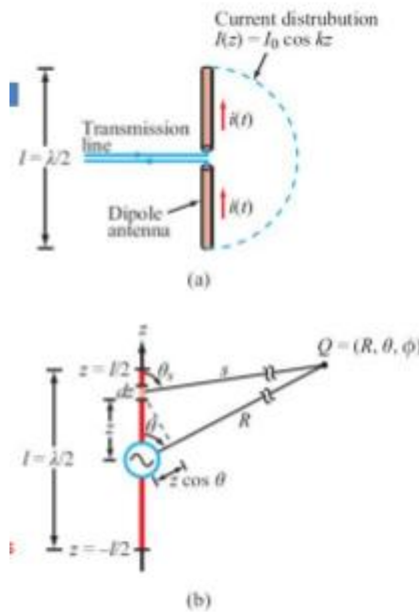
$$E_\theta=\frac{I_m dl\sin\theta}{4\pi r}\left(\frac{-w^2\sin wt_1}{v^2 wr}+\frac{w}{vr^2}\frac{\cos wt_1}{w}+\frac{\sin wt_1}{r^3 w}\right)$$

Half wave dipole and monopole:

The half wave dipole can be considered as a chain of Hertzian dipoles. For the uniform current distribution the +ve charges at the end of one Hertzian dipole gets cancelled with an equal negative charge at the opposite end of the adjacent dipole.But when the current distribution is not constant (non uniform suppose sinusoidal), So the successive dipoles of the chain have slightly different current amplitudes, where adjacent charges are not can called completely

Power radiated by the half wave dipole & the Monopole:

A dipole antenna is a vertical radiator fed in the centre,It produces maximum radiation in the plane normal to the axis In case of Hertzian dipole the expressions for E & H are derived assuming uniform current throughout the length At the ends antenna current is ‘o’ for the dipole, the current is not uniform throughout the length as it is maximum at centre & ‘o’ at ends Practically Hertzian dipole is not used but Half wave dipole (λ/2) & quarter wave monopole (λ/4)



The current element Idz is placed at a distance z from z=0 plane.

$$I = I_m \sin \beta(H - Z) \forall Z > 0$$

$$I = I_m \sin \beta(H + Z) Z > 0$$

‘A_Z’ at point ‘P’ due to current element Id_z is given by

$$dA_Z = \frac{\mu I}{4\pi r} e^{-j\beta R_{dz}}$$

The vector potential at point ‘P’ due to all suds currents can be obtained by integrating the vector potential Da_Z over the total length of the antenna

$$A_Z = \frac{\mu}{4\pi} \int_{-H}^{H} \frac{I}{r} e^{-j\beta R_{dz}}$$

The vector potential at point ‘P’ due to all suds currents can be obtained by integrating the vector potential Da_Z over the total length of the antenna

$$A_Z = \frac{\mu}{4\pi} \int_{-H}^{H} \frac{I}{r} e^{-j\beta R_{dz}}$$

$$A_Z = \frac{\mu}{4\pi}\int_{-H}^{H}\frac{I}{r}e^{-j\beta R_{dz}} + \frac{\mu}{4\pi}\int_{-H}^{H}\frac{I}{r}e^{-j\beta R_{dz}}$$

$$A_Z = \frac{\mu}{4\pi}\int_{-H}^{0}\frac{I_m \sin\beta(H+Z)}{r}e^{-j\beta R_{dz}} + \frac{\mu}{4\pi}\int_{0}^{H}\frac{I_m \sin\beta(H-Z)}{r}e^{-j\beta R_{dz}}$$

R=R-Z cosɵ

Substitute 'R' in the above equation

$$A_Z = \frac{\mu}{4\pi}\int_{-H}^{0}\frac{I_m \sin\beta(H+Z)}{r}e^{-j\beta(r-z\cos\theta)}dz + \frac{\mu}{4\pi}\int_{0}^{H}\frac{I_m \sin\beta(H-Z)}{r}e^{-j\beta(r-z\cos\theta)}dz$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{-H}^{0}\sin\beta(H+Z)e^{-j\beta z\cos\theta}dz + \int_{-H}^{0}\sin\beta(H+Z)e^{-j\beta z\cos\theta}dz$$

For quarter wave monopole H=λ/4 β=2Π/λ

$$\sin\beta(H+Z) = \sin(PH+\beta Z) = \sin(\frac{\pi}{2}+\beta z)$$

$$\sin\beta(H-Z) = \sin(\frac{\pi}{2}-\beta Z) = \cos\beta z$$

So $A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{-\pi}^{0}\cos\beta z e^{-j\beta z\cos\theta}dz + \int_{0}^{H}\cos\beta z e^{-j\beta z\cos\theta}dz$

$\int_{-\pi}^{0}e^{j\theta}d\theta = \int_{0}^{H}e^{-j\theta}d\theta$ Charging limits of integration

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H}\cos(-\beta z)e^{-j\beta z\cos\theta}dz + \int_{0}^{H}\cos\beta z e^{j\beta z\cos\theta}dz\right]$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H}\cos\beta z(e^{-j\beta z\cos\theta}+e^{j\beta z\cos\theta})dz\right]$$

$$e^{j\beta z\cos\theta}+e^{-j\beta z\cos\theta} = 2\cos(\beta z\cos\theta)$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{H}(\cos\beta z)2(\cos(\beta z\cos\theta))dz$$

$$2\cos A\cos B = \cos(A-B)+\cos(A+B)$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{H=\lambda/4}\cos\beta z(1+\cos\theta)+\cos\beta z(1-\cos\theta)dz$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin\beta z(1+\cos\theta)}{\beta(1+\cos\theta)}+\frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)}\right]_0^{\lambda/4}$$

By taking LCM

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin\beta z(1+\cos\theta)+\sin\beta z(1-\cos\theta)}{\beta(1+\cos\theta)(1-\cos\theta)}\right]_0^{\lambda/4}$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{(1-\cos\theta)\cos(\frac{\pi}{2}\cos\theta)+(1+\cos\theta)\cos\frac{\pi}{2}\cos\theta}{\sin^2\theta}\right]$$

$$A_Z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{2\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\right]$$

From Maxwell's equation $\nabla\times A = B$

We have to consider the radiation field only

$$(\nabla\times A)_\phi = \mu H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}A_\theta.r\right]$$

$$\mu H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(-A_z\sin\theta)r\right]$$

$$\mu H_\phi = \left[-\sin\theta\frac{\partial A_z}{\partial r}\right]$$

$$\mu H_\phi = -\frac{1}{r}\left[\frac{\partial}{\partial r}\left\{\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin^2\theta}\right]\right\}\sin\theta(r)\right]$$

$$H_\phi = \frac{-I_m}{4\pi\beta r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin\theta}\right]\frac{d}{dr}(e^{-j\beta r})(-j\beta)$$

$$H_\phi = \frac{jI_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin\theta}\right]$$

for half wave dipole & monopole

$$|H_\phi| = \frac{I_m}{2\pi r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

$$\frac{E_\theta}{H_\varphi} = \eta \qquad\qquad \eta = 120\pi H\phi$$

$$E_\theta = 120\pi H_\phi$$

$$\because E_\theta = 120\pi\,\frac{jI_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin\theta}\right]$$

$$E_\theta = \frac{60 I_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin\theta}\right]$$

$$\left|E_\phi\right| = \frac{60 I_m}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]$$

Power Radiated & Radiation resistance of λ/2 dipole & λ/4 monopole:

The field components Eɵ & HΘ are in time phase the maximum valve of the poyting vector can be obtained by P_{max}=| Eɵ || HΘ|

Multiplying the magnitudes= $\frac{60 I_m}{r}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]\times\frac{I_m}{2\pi r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin\theta}\right]$

$$P_{\max} = \frac{30 I^2{}_m}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

Average value of the power is help of the maximum value

$$P_{ave} = \frac{P_r}{2} = \frac{15 I^2{}_m}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = \sqrt{2} I_{rms}$$

$$P_{ave} = \frac{30 I^2{}_m}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2$$

$$P_r = \oint P_{ave}\, ds$$

$$= \int_0^{\pi}\frac{30 I^2{}_m}{\pi r^2}\left[\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right]^2 2\pi r^2 \sin\theta\, d\theta$$

$$= 60 I^2{}_{rms}\int_0^{\pi/2}\frac{\cos^2\frac{\pi}{2}\cos\theta}{\sin\theta}d\theta$$

Semispherical surface 0.609 by Trapezoidal rule

$$P_r = 60I^2{}_{rms}(0.609) = 36.54I^2{}_{rms}$$
$$P_r = 36.54\Omega[R_r = 36.5\Omega]$$
$$R_r = 73\Omega$$

Directivity of current element:

Directivity is defined as the maximum directive gain. Gain of an electric element obtained by comparing it with total radiated power. G=power radiated by the current element $P_{r(avg)}$ /total power radiated by the same current element

$$\frac{P_{max}}{p} = \frac{P_{r(avg)}}{P_{max}} = \frac{\dfrac{\eta(I_m dl \sin\theta)^2}{8\lambda^2 r^2}}{80\lambda^2\left(\dfrac{dl}{\lambda}\right)^2}$$

ɵ=90^0 →power will be maximum

$$P_{r(avg)} = \frac{\eta(I_m dl)^2}{8\lambda^2 r^2}$$
$$P_r = 80\lambda^2\left(\frac{dl}{\lambda}\right)^2 I^2{}_{rms}$$
$$G = 40\pi r^2 \times \frac{\dfrac{4(I_m dl)^2}{8\lambda^2 r^2}}{40\lambda^2\left(\dfrac{dl}{\lambda}\right)^2 I^2{}_{rms}}$$
$$G = \frac{\eta}{80\pi} = \frac{120\pi}{80\pi} = \frac{3}{2}$$
$$D = \frac{3}{2}\ G_{db} = 10\log G = 10\log {}^3\!/_2 = 10\times 0.1761(Gb = 1.761)$$

Beam Area:

$$\Omega_A = \frac{4\pi}{D} = \frac{4\pi(str)}{{}^3\!/_2} = \frac{8\pi}{3}(str)$$
$$\Omega_A = \frac{8\times 180}{3} = 240(str)$$
$$\Omega_A = 480(str)$$

Effective Area: $D = \dfrac{4\pi A_e}{\lambda^2}$

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{3\lambda^2}{2(4\pi)} = \frac{3\lambda^2}{8\pi}(m^2)$$

$$A_e = \frac{3\lambda^2}{8\pi}(m^2)$$

Power Radiated, Radiation Resistance, Beamwidths:

Power Radiated by current element and its radiation resistance:
For calculation of radiated power, pointing vector method is used (the current element is situated at the centre of a great spheres)

The power flow per unit area at any point 'P' on the sphere is given by pointing vector.

$$P = E \times H \qquad\qquad P_r = \int P_{r(cv)} ds$$

Out of 3 components $P(P_r, P_\theta, P_\phi)$ only the radial component contributes to power flow become.

$$P_\theta = E_r H_\phi \qquad P_\phi = E_\phi H_r \qquad P_r = E_\theta H_\phi$$

We $H_\theta = 0$ $E_\phi = 0$ P_θ & P_ϕ do not contribute to the net power flow.

Since the radiation field component from the current element, H_θ & H_ϕ are tangential to a spherical surface.

The poynting vector will be radial everywhere showing radial flow of power from the current element.

$$P_r = H_\phi E_\theta$$

$$P_r = \frac{I_m dl \sin\theta}{4\pi\varepsilon}\left(\frac{-w\sin wt_1}{v^2 r} + \frac{\cos wt_1}{vr^2} + \frac{\sin wt_1}{r^3 w}\right)$$

$$\frac{I_m dl \sin\theta}{4\pi}\left(\frac{-w\sin wt_1}{vr} + \frac{\cos wt_1}{r^2}\right)$$

$$P_r = \frac{(I_m dl \sin\theta)^2}{16\pi^2\varepsilon}\left(\frac{-w^2}{2v^3 r^2} + 0\right)$$

$$= \frac{(I_m dl \sin\theta)^2}{16\pi^2\varepsilon}\frac{4\pi^2 r^2}{2v^3 r^2}$$

$$P_{r(avg)} = \frac{\eta(I_m dl \sin\theta)^2}{8\pi^2\lambda^2} w\backslash m^2$$

Consider a sphere, and its elemental area on the spherical shell is

$$ds = 2\pi(r\sin\theta)(rd\theta)$$

$$= 2\pi r^2 \sin\theta d\theta$$

$$P = \oint P_{r(avg)} d.s$$

$$P = \int_0^\pi \frac{\eta(I_m dl \sin\theta)^2}{8\pi^2\lambda^2}(2\pi r^2 \sin\theta d\theta)$$

$$= \frac{\eta I^2{}_m dl^2 \pi}{4x^2}\int_0^\pi \sin^3\theta d\theta$$

$$= \frac{\eta I^2{}_m \pi}{4}\left(\frac{dl}{\lambda}\right)^2 \int_0^\pi \sin^3\theta d\theta$$

By willi's formula $\int_0^\pi f(t)dt = 2\int_0^{\pi/2} f(t)dt$

$$\int_0^{\pi/2} \sin^3 \theta d\theta = \frac{n-1}{n}\frac{n-3}{n-2}$$

$$2\int_0^{\pi/2} \sin^3 \theta d\theta = \frac{3-1}{3} = \frac{2}{3}$$

$$P = \frac{12\theta\pi I^2{}_m\pi}{4}\left(\frac{dl}{\lambda}\right)^2\left(\frac{2}{3}\right)$$

$$P = 4\theta\pi^2 I^2{}_m\left(\frac{dl}{\lambda}\right)^2$$

$$= 4\theta\pi^2(\sqrt{2}I_{rms})^2\left(\frac{dl}{\lambda}\right)^2 w \backslash m^2$$

$$P_r = I^2{}_{rms}R_r$$

$$R_r = \frac{P_r}{I^2{}_{rms}} = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \Omega$$

$$P_r = 80\pi\left(\frac{dl}{\lambda}\right)^2 \Omega$$

This is defined as the fictitious resistance which when inserted in series with the antenna will consume the same amount of power as is actually radiated.

Induction (near) field and Radiation (far) field:

Let us examine the expression for

$$H_\phi = \frac{I_m dl \sin\theta}{4\pi}\left(\frac{\cos wt_1}{r^2} - \frac{w\sin wt_1}{vr}\right)$$

Amplitude induction field radiation field

Induction field:

The first terms varies inversely as square of the distance ($1/r^2$) & this field will predominate at points close to the current element. It represents the energy shorted in the magnetic field surrounding the current element or conductor. This energy is alternatively stored in the field and returned to the source during each half cycle.

Radiation field:

The second term varies inversely as distance (1/r) which accounts for the radiation of electromagnetic waves from a conductor under suitable conditions.The radiation component of the magnetic field is produced by the alternating electric field and electric radiation component arises from the alternating magnetic field.

The flow of current or movement of the charge in the conductor establishes local induction fields.Whereas the radiation fields exist as a consequence of the charging induction fields. 1/w ∞ ($1/r^{3)}$ term varying-near the current element Indendepend of 'w'∞ ($1/r^2$) induction termThe two terms in 'E_r' belongs to the induction field and are negligible at a distance where r>>λ hence no radial component E_r is present in the radiation field.

In 'E_θ' also same

$$E_\theta = \frac{I_m dl \sin\theta}{4\pi\varepsilon}\left(-\frac{w\sin wt_1}{v^2 r}\right)$$

$$E_\theta = -\frac{2\pi f I_m dl \sin\theta \sin wt_1}{4\pi\varepsilon v^2 r}$$

$$= \frac{-I_m dl \sin\theta \sin wt_1}{2\lambda\varepsilon \frac{1}{\sqrt{\mu\varepsilon}} r} \qquad (\eta = \sqrt{\mu\varepsilon})$$

$$= \frac{-I_m dl \sin\theta \sin wt_1}{2\lambda r \sqrt{\frac{\varepsilon}{\mu}}} \qquad |E_\theta| = \frac{60\pi I_m dl}{\lambda r}$$

$$E_\theta = \frac{-\eta I_m dl \sin\theta \sin wt_1}{2\lambda r}$$

At a distance r>>λ

$$H_\phi = \frac{I_m dl \sin\theta}{2\pi}\left[\frac{-w \sin wt_1}{vr}\right]$$

$$= -\frac{2\pi f I_m dl \sin\theta \sin wt_1}{4\pi v r}$$

$$H_\phi = -\frac{I_m dl \sin\theta \sin wt_1}{2\lambda r}$$

$$|H_\phi| = -\frac{I_m dl}{2\lambda r}$$

Effective length or radiation height of linear antennas:

For linear current distribution expression for the radioactive field is given by

$$D = \frac{4\pi}{\lambda^2} A_e = \frac{D\lambda^2}{4\pi} = \frac{3\lambda^2}{2(4\pi)} A_e = \frac{3\lambda^2}{8\pi} (m^2)$$

$$|H_\phi| = \frac{I_m dl}{2\lambda r}$$

This radiation cannot be true for practical antennas due to non-uniform current distribution which are either linear or sinusoidal or co-sinusoidal, Due to this reason, there is a reduction of radiated power and makes the antenna equivalent to a shorter one with same current.So the length of antenna may be taken by finding the mean value of current over the length.

$$|H_\phi| = \frac{I_{av} l}{2\lambda r}$$

$$|H_\phi| = \frac{I_m l}{2\lambda r}$$

$l/2 = l_e$=effective length or height

Note: equivalent or radiation length l_e will be (l/2) as the mean value of current ($I_m/2$)

$$|H_\phi| = \frac{I_m l_e}{2\lambda r} \quad (l_e = l/2)$$

$$I_{ave} = \frac{I_m}{2}$$

$$l_e = \frac{2\lambda r |H_\phi|}{I_m}$$

$$\frac{l}{2} = \frac{2\lambda r |H_\phi|}{2I_{ave}}$$

$$l_e = \frac{\lambda |H_\phi|}{I_{ave}}$$

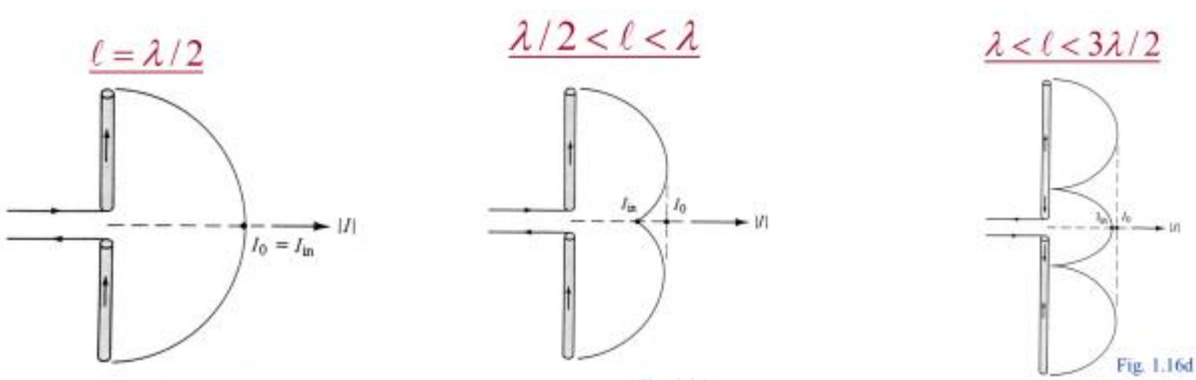
Natural current distributions; fields and patterns of Thin Linear Center-fed Antennas of different lengths:

Finite length dipole:

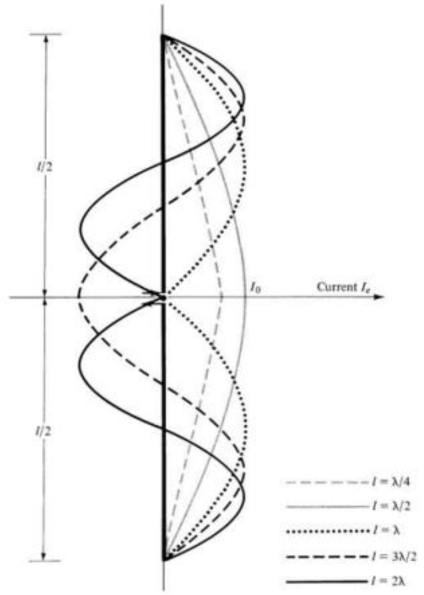
A finite length dipole is still in the order of a<<λ, However, the length l of the antenna is in the same order of magnitude as the operating wave length λ/10<l≤2λ,The current distribution is now approximated to a sinusoidal function:

$$I_e = \left\{ a_z I_0 \sin\left(k\left[\frac{l}{2} - z\right]\right), 0 \le z \le \frac{l}{2} \right.$$

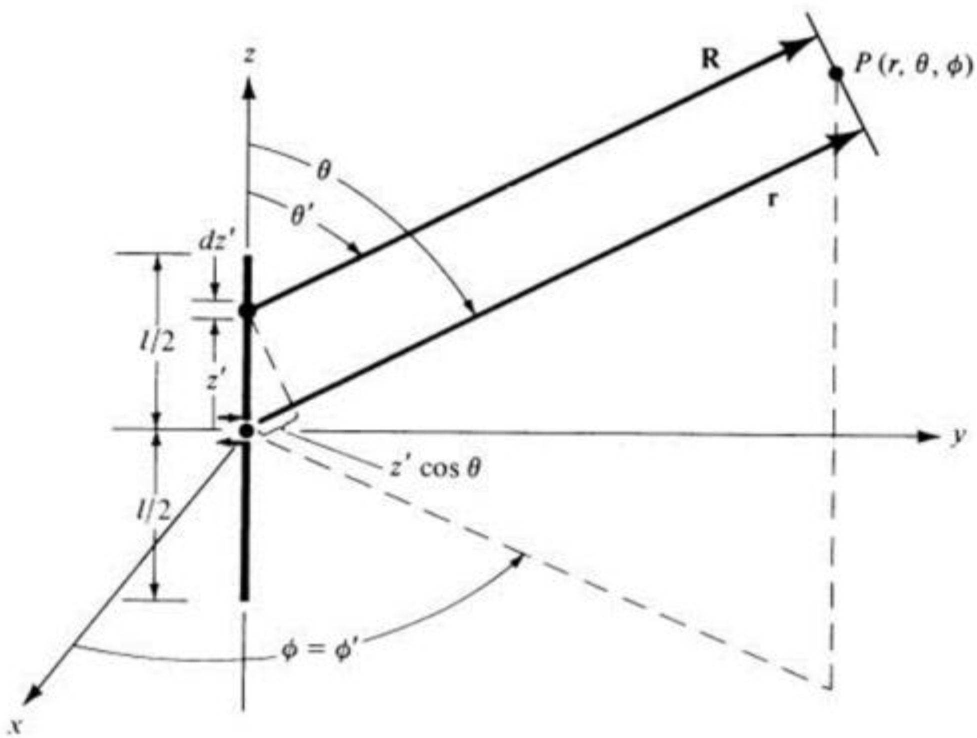
$$\left\{ a_z I_0 \sin\left(k\left[\frac{l}{2} - z\right]\right), -\frac{l}{2} \le z \le 0 \right.$$



Current Distributions along the Length of a LINEAR Wire Antenna:



Finite Dipole Geometry & Far-Field Approximations:



UNIT – II
LOOP ANTENNAS AND ANTENNA ARRAYS :

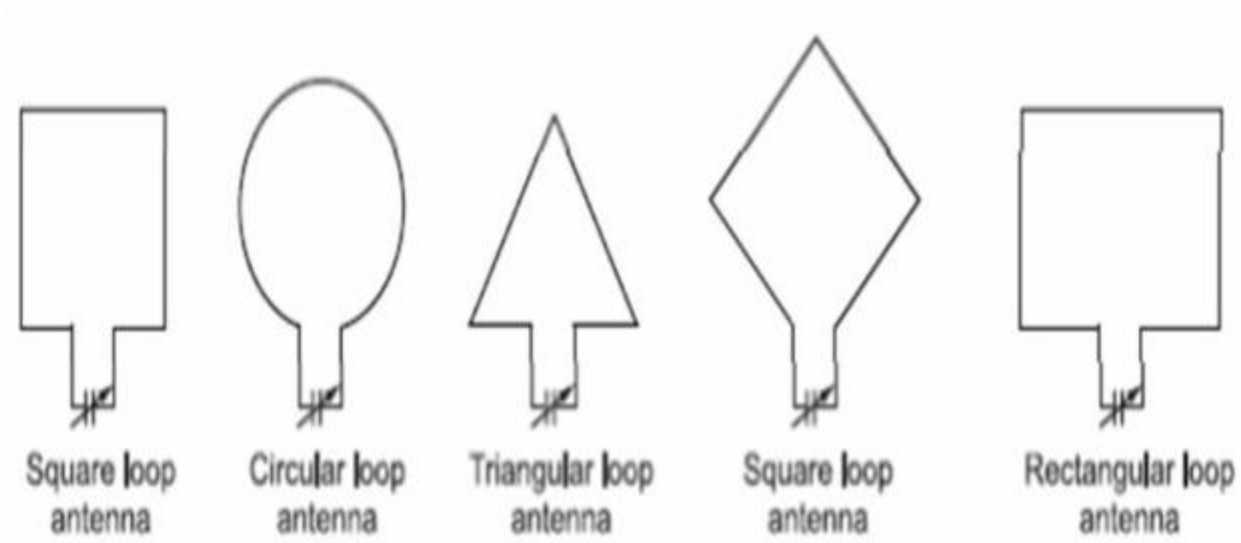
Loop antennas:

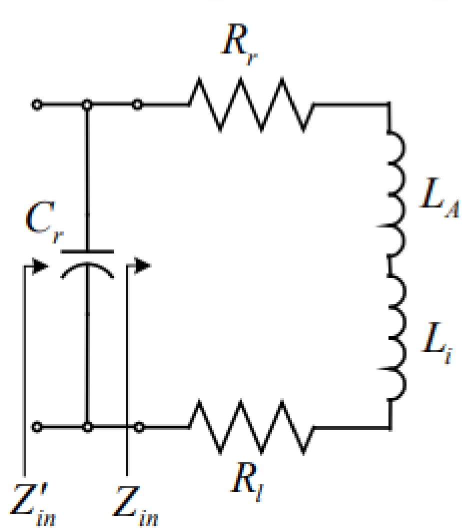
A radio direction finder (DE) is used to determine the direction of arrival of radial signal.Radio waves are propagating between T_X & R_X ,To find the direction of the unknown transmitter w.r.t Receiver & this process of finding the direction of incoming radio signal from a t_x & its position is known as Direction finding,Radio Direction finder (R.D.F)

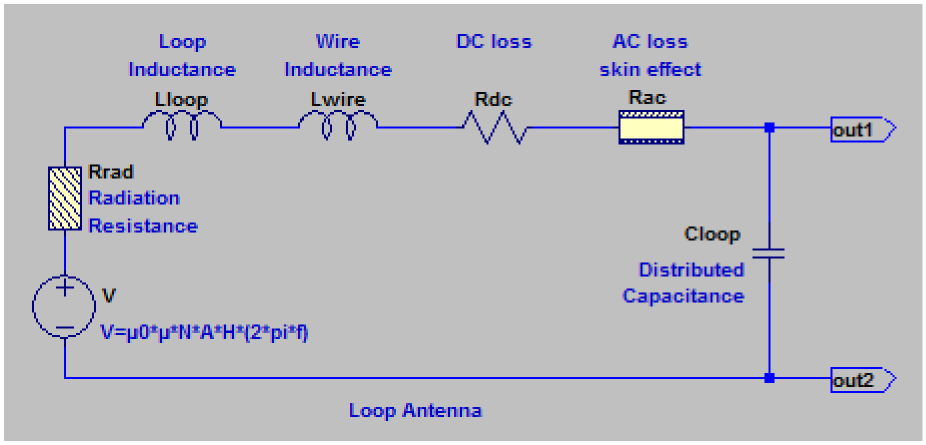
We can obtain the location by determining the direction of radio wave at two receiving points DF consists directional antenna, receiver direction indicator,For direction finding, we are using loop antennas.

Loop antennas:
The structure of this antenna is a radiating coil of any convenient cross section of one or more turns carrying radio frequency current.A loop of more than one turn is called frame.Dimensions of loop are small compared to wave length.

Different shapes of Loop:



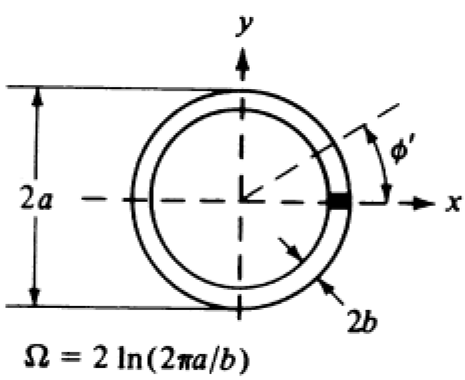


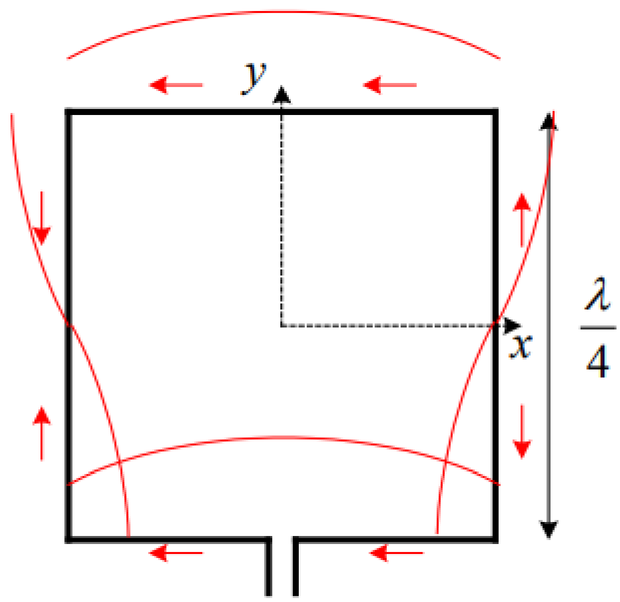


Small loop:
The radiation efficiency of closed loop antenna is low for t_x'ion purposes unless their dimensions are made comparable to the wave length employed.Let us consider/ take the small loop (circular) of low dimension compared to the wave length 'λ' so that the magnitude and phase remain the same throughout the loop.The field pattern of small circuit loop of radius 'r' may be determined by considering a square loop of the same area.

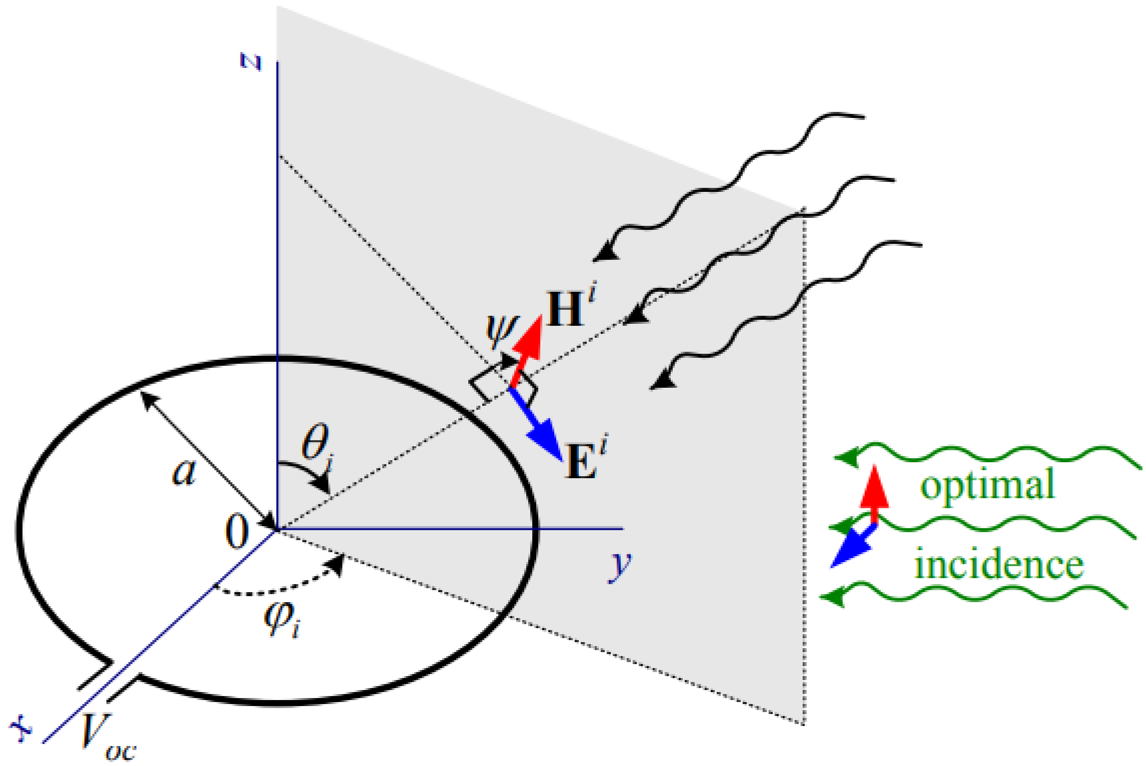
$$d^2=\Pi r^2$$

d=side length of the square loop





Its field pattern can be analyzed by treating loop as four short linear dipoles,To find the far-field pattern in the yz-plane, it is only necessary to consider two of the four small linear dipole, assuming that the loop is placed at the centre of the co-ordinate system; its far field component will have only 'Eθ'



Eɵ=field component due to +field component due to

$$d_{A\phi} = \frac{\mu d\pi}{4\pi r} A_\phi = \frac{\mu}{4\pi r}\int dM \quad E_\phi = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= -2jE_0\left[\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}\right]$$

$$E_\phi = -2Ej\sin\psi/2$$

E_0=Electric field amplitude of dipole AD+BC
Ψ=2π/λ dcos(90-θ)=phase difference
Ψ=βdsinθ
E_φ=-2jE_0sin (βdsin θ/2)
The term 'j' indicates that the total field E_φ is in phase quadrature with the individual dipole field 'E_0'

$$E_0 = \frac{j60\pi[I]\sin 90^0}{r}\frac{L}{\lambda}$$

$$E_0 = \frac{j60\pi[I]L}{r\lambda}$$

Here θ=90^0 because it is measured from x-axis instead of z-axis.
[I]=Retarded current

$$[I] = I_0 e^{jw(t-\frac{r}{c})}$$

$$E_\phi = -2j\left(\frac{j60A[I]}{r\lambda}\right)\frac{2\pi}{\lambda}\frac{d\sin\theta}{2}$$

If $\sin\frac{\psi}{2} = \frac{\psi}{2}$

$$E_\phi = \frac{120\pi^2[I]\sin\theta.A}{r\lambda^2}$$

For field pattern E_P field. Instantaneous value E_φ of loop area 'A'

The other component of far field is magnetic field component H_θ which is given by

$$E_\phi = 120\pi H_\theta$$

$$H_\theta = \frac{E_\phi}{120\pi}$$

$$H_\theta = \frac{\pi[I]\sin\theta}{r\lambda^2}A$$

Comparison of far fields of small loop and short dipole:

Electric dipole $E_\phi = \frac{120\pi^2[I]\sin\theta.A}{r\lambda^2}$

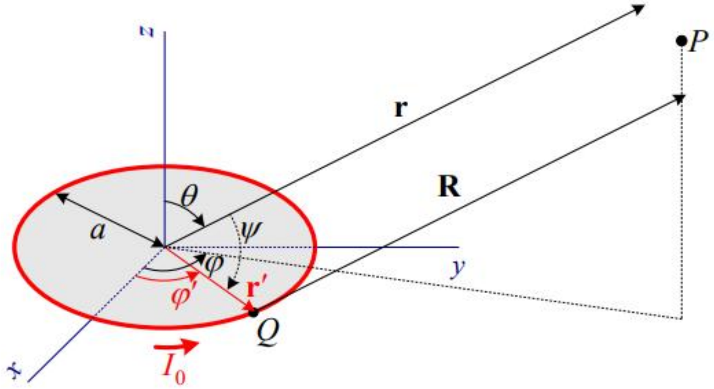
Electric $\frac{j60\pi[I]\sin 90^0 L}{r\lambda}$

Magnetic $H_\phi = \frac{j[I]\sin\theta L}{2r\lambda}$

$$H_\phi = \frac{\pi[I]\sin\theta}{r}\frac{(A)}{\lambda^2}$$

Loop antenna-General Case:

Consider a loop antenna with uniform, in phase current & any convenient size.Placed / located at the centre of the spherical co-ordinate system.Assuming the current 'I' uniform along the loop, the far field expressions can be obtained by finding the vector potential of the electric current.To find the vector potential consider first a pair of short dipoles placed diamenetrically opposite to each other loop of any radius a with length "adø"



To find total vector potential over the loop we can integrate vector potential obtained for above pair of short dipoles over the entire loop.

The for field component will have only a ø comp while the other components in r &ø direction will be zero.

$$d_{A\phi} = \frac{\mu d\pi}{4\pi r} \qquad A_\phi = \frac{\mu}{4\pi r}\int dM$$

d_M = current moment due to the pair of infinitesimal dipoles placed diametrically opposite,The ø-component of the retarded current moment due to one dipole is $F_Z = a_z \frac{\mu}{4\pi}\int_{-l/2}^{+l/2} I_m e^{jw(t-r/c)} dz = a_z \frac{\mu I_{m0}}{4\pi}\int_{-l/2}^{+l/2} e^{jw(t-r/c)} dz \;\; [I]ad\phi\cos\phi$

$$[I]I_0 e^{jw(t-\frac{r}{c})}$$

I_0=peak current or maximum current in time on the loop,The cross section on through the loop in x-z plane is as shown.

The resultant moment dM at a large distance due to a dipole pair

$$dM = 2j[I]ad\phi\cos\phi\sin\psi/2$$

$$\psi = 2\beta a\cos\phi\sin\theta \quad \text{Radiant}$$

$$A_\phi = \frac{\mu}{4\pi r}\int 2j[I]ad\phi\cos\phi\sin(\beta a\cos\phi\sin\theta)d\phi$$

$$= \frac{j\mu[I]a}{2\pi r}\int_0^\pi \cos\phi\sin(\beta a\cos\phi\sin\theta)d\phi$$

$$A_d = \frac{j\mu[I]a}{2Ar}\pi^{J_1}(\beta a\sin\theta)$$

J_1 (βasinθ)= Bessel's function of the first or der and of argument (βasinθ),

NOTE: Dipoles different orientation w.r.t ø & situated at the origin,

The far electric field of the loop has only a ø component,

$$E_\phi = -jwA_\phi = -jw\frac{j\mu[I]a}{2\pi r}J_1(\beta a\sin\theta)$$

$$E_\phi = \frac{w\mu[I]a}{2r}J_1(\beta a\sin\theta)$$

$$= \frac{2\pi f\times 4\pi\times 10^{-7}[I]a}{2r}J_1(\beta a\sin\theta)$$

$$= \frac{2\pi(f\lambda)4\pi\times 10^{-7}[I]a}{2r\lambda}J_1(\beta a\sin\theta)$$

$$= \frac{2\pi(3\times 10^8)4\pi\times 10^{-7}[I]a}{2r\lambda}J_1(\beta a\sin\theta)$$

$$E_\phi = \beta\frac{60\pi[I]a}{r}J_1(\beta a\sin\theta)$$

Instaneous electric field at a large distance 'r' from a loop of any radius 'a',

$$H_\theta = \frac{E_\phi}{\eta}$$

$$H_\theta = \frac{\beta[I]a}{2\pi r} J_1(\beta a \sin\theta)$$

Power radiated & Radiation Resistance of loop Antennas:

To calculate the radiation resistance of a loop antenna the poynting vector is integrated over a large sphere giving the total power 'p' radiated this power is then equated to the square of the current on the loop times the 'R_r'

$P = I_{rms} R_r$

I_m=max (or) peak current

$$P = \left(\frac{I_m}{\sqrt{2}}\right)^2 R_r \Rightarrow \frac{I^2{}_{rms}}{2} R_r (R_r = R.R)$$

The average poynting vector of a far field is given by

$$P = \frac{1}{2} R_e [E \times H^*]$$

The radial $P = \frac{1}{2} R_e [E \times H^*]$ (E=ηH)

$$P = \frac{1}{2} |H|^2 R_e(\eta)$$

$$|H_\phi| = \frac{\beta a[I]}{2r} J(\beta a \sin\theta)$$

$$P_r = \frac{1}{2}\left|\frac{\beta a[I]}{2r} J(\beta a \sin\theta)\right|^2 \times 120\pi$$

$$P_r = 15\pi \frac{(\beta a I_m)^2}{4r^2} J^2{}_1(\beta a \sin\theta)$$

The total power radiated 'P' is obtained by integrating the P_r over a large sphere.

$$P = \iint P_r ds$$

$$P = \int_0^{2\pi}\int_0^{\pi} \frac{15\pi(\beta a I_m)^2 J^2{}_1(\beta a \sin\theta)}{r^2} r^2 \sin\theta d\theta d\phi$$

$$ds = r^2 \sin\theta d\theta d\phi$$

$$P = 2\pi 15\pi (\beta a I_m)^2 \int_0^{\pi} J^2{}_1(\beta a \sin\theta) \sin\theta d\theta$$

For small arguments of the first order barrel function the following $J_1(x) = \frac{x}{2}$ $\quad J_1{}^2(x) = \left(\frac{x}{2}\right)^2$

$$P = 30\pi^2 (\beta a I_m)^2 \int_0^{\pi} J^2{}_1 \frac{(\beta a \sin\theta)^2}{2} \sin\theta d\theta$$

$$P = \frac{30\pi^2 (\beta a)^4 (I_m)^2}{4} \int_0^{\pi} \sin^3\theta d\theta$$

$$P = \frac{30\pi^2 (\beta a)^4 (I_m)^2}{4} \times \frac{4}{3} \quad \text{(Area of the circle=}\pi a^2\text{)}$$

$$P = 10\pi^2 (\beta a)^4 I_m{}^2$$

$$P = 10\pi^2 \beta^4 A^4 I_m{}^2$$

Assuming the loss less antenna, this power equals the power delivered to the loop terminals,

$$\frac{1}{2} R_r I_m{}^2 = 10\pi^2 \beta^4 A^4 I_m{}^2$$

$$\frac{1}{2} R_r I_m{}^2 = 10\pi^2 (\beta a)^4 I_m{}^2$$

$$R_r = 20\pi^2 (\beta a)^4$$

$$R_r = 20\pi^2 \beta^4 A^2 \Omega$$

$$R_r = 20\pi^2 \left(\frac{2\pi}{\lambda}\right)^4 \left(\pi a^2\right)^2 \Omega$$

$$= 20\pi^2 \frac{16\pi}{\lambda^4} \pi^2 a^4$$

$$R_r = 320\pi^8 \left(\frac{a}{\lambda}\right)^4$$

$$R_r \cong 31,200 \left(\frac{A}{\lambda^2}\right)^2$$

This is the equation of small single turn circular loop antenna or square with uniform in phase current.

Case:

If the loop antenna has N no.of turns so the magnetic field passes through all the loops the radiation resistance is equal to that of single turn multiplied by N^2

$$R_r = 31,200 \left(\frac{NA}{\lambda^2}\right)^2$$

The radiation resistance in terms of circumference can be written by the following equations (circumference) (C)=2π

$$R_r = 20\beta^4 A^2 = 20 \left(\frac{2\pi}{\lambda}\right)^4 \left(\pi a^2\right)^2$$

$$= 20 \frac{(2\pi)^4 \left(\pi^2 a^4\right)}{\lambda^4}$$

$$R_r = 20\pi^2 \frac{(2\pi a)^4}{\lambda^4}$$

$$R_r = 20\pi^2 \left(\frac{c}{\lambda}\right)^4 \Omega (c = 2\pi a)$$

$$R_r = 197 \left(\frac{c}{\lambda}\right)^4$$

Power radiated for any radius 'a':

$$P = 30\pi^2 (\beta a I_m)^2 \int_0^{\pi} J^2{}_1 \frac{(\beta a \sin\theta)^2}{2} \sin\theta d\theta$$

$$=\int_0^{\pi} J^2{}_1 \frac{(\beta a\sin\theta)^2}{2}\sin\theta d\theta = \frac{1}{x}\int_0^{2\pi} J_2(y)dy$$

$$\Rightarrow \frac{1}{\beta a}\int_0^{2\beta a} J_2(y)dy$$

c/λ >5 i.e is loop is large.

$$=\int_0^{\pi} J^2{}_1(\beta a\sin\theta)\sin\theta d\theta = \frac{1}{\beta a}\int_0^{2\beta a} J_2(y)dy = 1$$

$$P = 30\pi^2(\beta a I_m)^2 \frac{1}{\beta a}$$

$$P = 30\pi^2 I_m{}^2 \frac{c}{\lambda}\left[\beta a = \frac{2\pi}{\lambda} a = c \Rightarrow \beta a = \frac{c}{\lambda}\right]$$

$$\frac{1}{2} R_r I_m^2 = 30\pi^2 I_m{}^2 \frac{c}{\lambda}$$

$$R_r = 60\pi^2 \frac{c}{\lambda} = 592\frac{c}{\lambda}$$

$$R_r \cong 3720\frac{a}{\lambda}$$

Directivity of loop Antennas:

D=maximum radiation intensity/ average radiation intensity,Maximum radiation intensity for a loop antenna is given by multiplication of power with r2,Average radiation intensity is given by dividing the total power with 4π.

$$D = r^2 \frac{15\pi(\beta a I_m)^2 J^2{}_1(\beta a\sin\theta)}{r^2}$$

$$\frac{30\pi^2(\beta a I_m)^2\int_0^{\pi} J^2{}_1(\beta a\sin\theta)^2\sin\theta d\theta}{4\pi}$$

$$=\frac{2J^2{}_1(\beta a\sin\theta)}{\frac{1}{\beta a}\int_0^{2\beta a} J^2{}_1(\beta a\sin\theta)^2\sin\theta d\theta}$$

Substituting $\beta a = \frac{c}{\lambda}$

$$D = \frac{2\beta a\left[J^2{}_1(\beta a\sin\theta)\right]}{\int_0^{2\beta a} J^2{}_1(\beta a\sin\theta)\sin\theta d\theta}$$

$$D = \frac{2\frac{c}{\lambda}\left[J^2{}_1(\frac{c}{\lambda}\sin\theta)\right]}{\int_0^{2\beta a} J^2{}_1(\frac{c}{\lambda}\sin\theta)\sin\theta d\theta}$$

$$D = \frac{2\frac{c}{\lambda}\left[J^2{}_1(\frac{c}{\lambda}\sin\theta)\right]}{\int_0^{2c\lambda} J_2(y)dy}$$

For small loop: $\frac{c}{\lambda} \subset \frac{1}{3}$

$$D = \frac{2\left[J^2{}_1\beta a\sin\theta\right]_{\max}}{\int_0^{\pi} J^2{}_1(\beta a\sin\theta)^2 \sin\theta d\theta}$$

For small loop $J_1(x) = \frac{x}{2} \Rightarrow J^2{}_1(\beta a\sin\theta) = \left(\frac{\beta a\sin\theta}{2}\right)^2$

$$D = \frac{2\left(\frac{\beta a\sin\theta}{2}\right)^2}{\int_0^{\pi} J^2{}_1(\frac{\beta a\sin\theta}{2})^2 \sin\theta d\theta}$$

$$D = \frac{2\sin^2\theta}{\int_0^{\pi}\sin^3\theta d\theta} = \frac{2\sin^2\theta}{\frac{4}{3}} = 2\times\frac{3}{4}$$

D=3/2

For large loop: $\frac{c}{\lambda} \geq 5$

$$D = \frac{2\frac{c}{\lambda}\left[J^2{}_1(\frac{c}{\lambda}\sin\theta)\right]}{\int_0^{2c/\lambda} J_2(y)dy} \Rightarrow \int_0^{2c/\lambda} J_2(y)dy = 1$$

$J_1(\frac{c}{\lambda}\sin\theta) = 0.584$ When the loop size restricted to $\frac{c}{\lambda} \geq 1.84$

$$D = 2\frac{c}{\lambda}(0.584)^2$$

$$D = 0.682\frac{c}{\lambda}$$

$$A_e = \frac{\lambda^2 D}{4\pi} = \frac{\lambda^2 0.682\frac{c}{\lambda}}{4\pi} = 0.0543C\lambda$$

$$A_e = 0.0543C\lambda$$

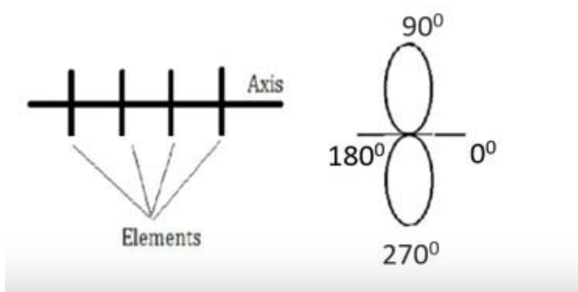
Antenna Arrays:

Antenna array is one of the common methods of combining the radiations from a group or array of similar antennas. The total field produced by an antenna array system at a great assistance from it & it is the vector sum of the fields produced by the individual antennas of the array system.The relative phases of individual field components depend on the relative distance of the individual antennas of the array.An antenna array is said to be linear, if the individual antennas of the array are equally spaced along a straight line.Individual antennas of the array are also termed as elements.

Various types of antenna array:

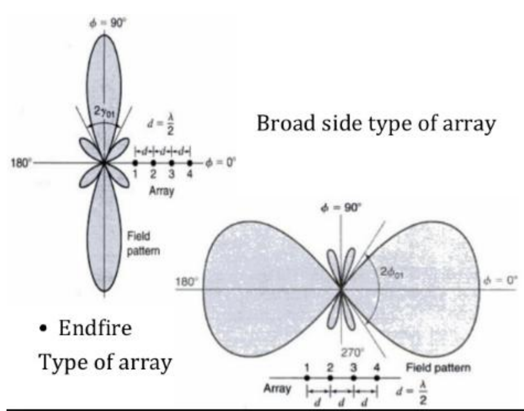
Broad side array:

In which a number of identical parallel antennas are set up along a line drawn '⊥' to their respective axes,

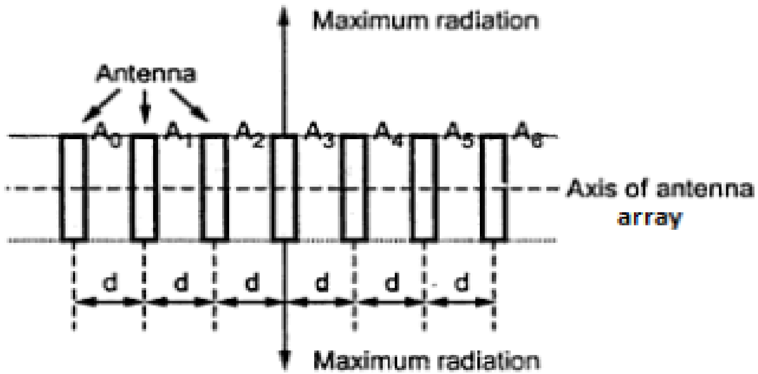


Broad side array

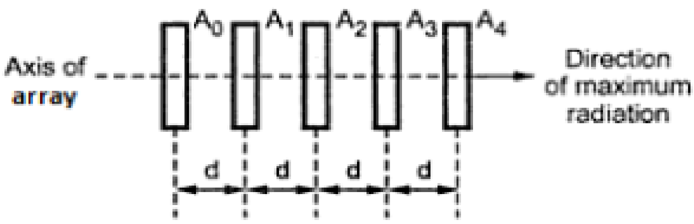
An arrangement in which the principal direction radiation is perpendicular to the array axis and also to the plane containing the array element,



End fire array:
The end fire array is nothing but broad side array except that individual elements are fed in, out of phase (180^0 usually)



Broadside array



(a) Top view

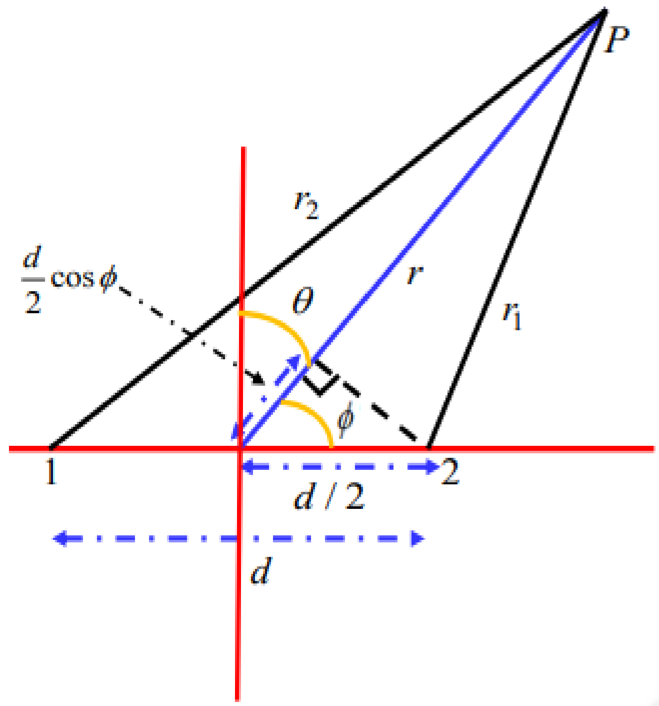
End-fire array

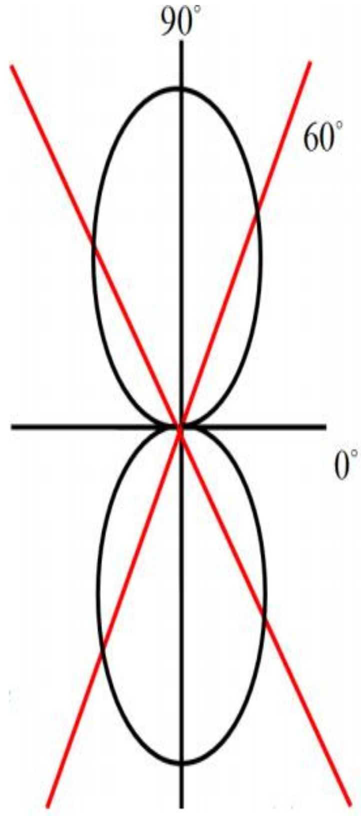
Thus in the end fire array, a number of identical antennas are spaced equally along a line and individual elements are fed with currents of equal magnitude but their phases varies.The arrangement in which the principal direction of radiation coincides with the direction of the array axis.

Arrays of 2 isotropic point sources:
Let us assume that the two point sources are separated by a distance & have the same polarization.

The superposition or addition of fields from the various sources at a great distance with due regard to phases
1) Equal amplitude & phase--------case (1)
2) Equal amplitude & opposite phase case(2)
3) Unequal amplitude any opposite phase case(3)

Case (1)
Arrays of two point sources with equal amplitude and phase:
Two isotropic point sources symmetrically situated w.r.t originTo calculate fields at a great distant point, at distance (R) from the origin ‘o’ which is taken as reference point for phase calculation





Path difference between the two waves in is given by
Path difference =dcosθ

$$\psi = \beta d \cos\theta radians$$

$$E = 2E_0 = \frac{1}{2}\cos\frac{\beta d\cos\theta}{2}\left(\frac{e^{-j\psi/2}+e^{j\psi/2}}{2}\right)$$

$$E = \cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1 = \cos\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta$$

$$\frac{\pi}{2}\cos\theta_{\max} = 90^0 \ \& \ 270^0$$

Thus the total far field at distant point ‘p’ in the direction of ‘θ’ is given by

$$E = E_1 e^{-jp/2} + E_2 e^{jp/2}$$

$E_1 e^{-jp/2}$ - Field component due to source

$E_2 e^{jp/2}$ - Source

In this case amplitudes are same

$$E_1 = E_2 = E_0$$

$$E = 2E_0\left(\frac{e^{-j\psi/2}+e^{j\psi/2}}{2}\right)$$

$$E = 2E_0\cos\frac{\psi}{2}$$

$$E = 2E_0 \cos\frac{\beta d \cos\theta}{2}$$

The above equation is the far field pattern of 2 isotropic point sources of same amplitude and phase.
The total amplitude is $2E_0$ maximum value may be ‘1’

$$2E_0 = 1$$

$$E_0 = \frac{1}{2}$$

The pattern is said to be normalized eq (1) becomes

$$E = \cos\left(\beta d \cos\theta\right) = \cos\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta$$

$$E = \cos\left(\frac{\pi}{2}\cos\theta\right) \text{(if d=}\lambda/2)$$

Maxima direction:

E is maximum when $\cos\left(\frac{\pi}{2}\cos\theta\right)$ is max

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta_{\max} = \pm n\pi \text{ Where n=0, 1, 2-----}$$

$$\frac{\pi}{2}\cos\theta_{\max} = 0 \quad \text{(if n=0)}$$

$$\cos\theta_{\max} = 0 \qquad \theta_{\max} = 90^0 \ \& \ 270^0$$

Minimum direction:

E is minimum when $\cos\left(\frac{\pi}{2}cos\theta\right)$ is minimum

$$\cos\left(\frac{\pi}{2}cos\theta\right) = 0$$

$$\left(\frac{\pi}{2}cos\theta_{\min}\right) = \pm\left(2n+1\right)\frac{\pi}{2} \text{ (n=0, 1, 2, …..)}$$

$$\frac{\pi}{2}cos\theta_{\min} = \frac{\pi}{2}$$

$$cos\theta_{\min} = \pm 1$$

$$\theta_{\min} = 0^0 \ \& 180^0$$

Half power point direction:

Power $V \ or \ I \ = \frac{1}{\sqrt{2}}$ ts times the max $V \ or \ I$

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4}$$
$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4}$$
$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$
$$\theta_{HPPD} = \pm\frac{1}{2}$$
$$\theta_{HPPD} = 60^0, 120^0$$

This is the simplest type of "Broad side array"

Case-(2)

Arrays of two point sources with equal amplitude and opposite phase:

This is exactly similar to above except that point source 1 is out of phase or opposite phase (180^0) to source 2 i.e when there is maximum in source 1 at one particular instant, then there is minimum in source 2 at that instant and vice versa

The total far field at distant point P_L is given by

$$(a+b)^{n-1} = a^{n-1}b^0 + \frac{n-1}{11}a^{n-2}b^1 + \frac{(n-1)(n-2)}{21}a^{n-3}b^2 + \frac{(n-1)(n-2)(n-3)}{31}a^{n-4}b^3 + ---$$

Because phase of source 1 and source 2 at distant point P is $-\frac{\psi}{2}$ and $\frac{\psi}{2}$, since the reference between midway

$$E_1 = E_2 = E_0$$
$$E = E_0 2j\left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j}\right)$$
$$E = 2jE_0 \sin\frac{\psi}{2}$$
$$E = 2jE_0 \sin\left(\frac{\beta d}{2}\cos\theta\right)$$

$2jE_0 = 1 \rightarrow$ (normalization)

$$E_{norm} = \sin\left(\frac{\beta d}{2}\cos\theta\right)$$
$$\beta = \frac{2\pi}{\lambda} \qquad d = \frac{\lambda}{2}$$
$$E_{norm} = \sin\left(\frac{\pi}{2}\cos\theta\right)$$

Maximum direction:

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$
$$\frac{\pi}{2}\cos\theta_{max} = \pm(2n+1)\frac{\pi}{2} \quad \text{(n=0, 1, 2, …..)}$$

$$\cos\theta_{\max} = \pm 1 \quad \text{if n=0}$$

$$\theta_{\max} = 0^0 \ \&180^0$$

Minimum direction:

$$\sin\left(\frac{\pi}{2} cos\theta\right) = 0$$

$$\left(\frac{\pi}{2} cos\theta_{\min}\right) = \pm n\pi \quad \text{(n=0, 1, 2,…..)}$$

$$cos\theta_{\min} = 0$$

$$\theta_{\min} = 90^0 \ \& -90^0$$

Half power point directions:

$$\sin\left(\frac{\pi}{2} cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

$$\theta_{HPPD} = 60^0, \pm 120^0$$

$$\theta_{HPPD} = \pm\frac{1}{2}$$

This is simplest type of “End fire array”

Arrays of two point sources with unequal amplitude and any phase:

Let us consider two point sources are not equal and any phase difference says α

E_1= Field due to source-(1)

E_2= Field due to source-(2)

$$\psi = \beta d\cos\theta + \alpha$$

α=phase angle by which the current ‘I_2’ of source (2)-leads the current I_1 of source (1)

If α=0 or 180^0 → $E_1 = E_2 = E_0$

The total field $E = E_1 e^{-j0} + E_2 e^{j\psi} = E_1\left(1+\frac{E_2}{E_1}e^{j\psi}\right)$

$$E = E_1\left(1+ke^{j\psi}\right)\left[\because k = \frac{E_2}{E_1}\right]$$

$$E_1 > E_2 k < 1$$

$$0 \le k \le 1$$

$$E = \left| E_1\left(1+k\left(\cos\psi + j\sin\psi\right)\right)\right|$$

$$E = E_1\sqrt{\left(1+k\cos\psi\right)^2 + \left(k\sin\psi\right)^2} < \phi$$

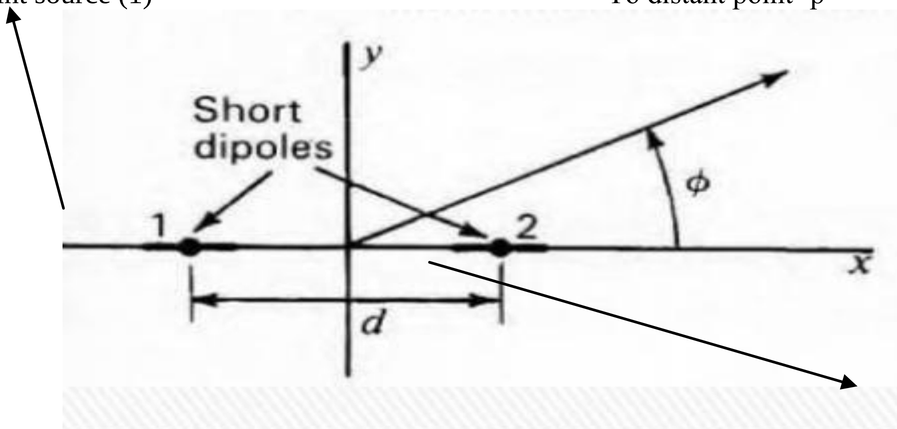
Ø=phase angle

$$\phi = tan^{-1}\frac{k\sin\psi}{1+k\cos\psi}$$

If $E_1 = E_2$, K=1

Non-isotropic but similar point sources:

Point source (1) To distant point ‘p’



Point source (2)

Array of two isotropic point source were up to considered, and this may be extended to the sources which are not isotropic and provided. Their field patterns are similar to that of isotropic point source.

Field patterns of non-isotropic must have the same shape & orientationAmplitudes are not necessarily equalSo there are called non-isotropic but similar point source.

Let us now consider 2 short dipoles which are superimposed over the two isotropic point sources & are separated by a distance.

$E_0 = E_1\sin\theta$ (for isotropic field pattern)------------- (1)

Field pattern for two identical isotropic sources is given by

$E_0 = 2E_0\cos\Psi/2$ ---------------------- (2)

Where $\Psi = \beta d\cos\theta + \alpha$

Combing equations (1) & (2)

$E = 2E_1\sin\theta\cos\Psi/2$

$E_{normal} = \sin\theta\cos\Psi/2$

$\sin\theta$ = pattern of individual isotropic source

$\cos\Psi/2$ = pattern of Array of two isotropic point sources

The above equation leads to the “principle of multiplication of pattern”

Multiplication of pattern (or) pattern multiplication:

It is being stated as follows:

“The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source patterns and the pattern of an array of isotropic point sources each located at the phase center of individual source and having the relative amplitude & phase. Whereas total phase pattern is the addition of the phase pattern of the individual sources & the array of isotropic point source”

$$E = \{E_i(\theta,\phi)\times E_a(\theta,\phi)\}\times\{E_{pi}(\theta,\phi)\times E_{pa}(\theta,\phi)\}$$

$\{E_i(\theta,\phi)\times E_a(\theta,\phi)\}$ =multiplication of field pattern

$\left\{E_{pi}(\theta,\phi)\times E_{pa}(\theta,\phi)\right\}$ =addition of phase pattern

$E_i(\theta,\varphi)$= field pattern of individual source

$E_a(\theta,\varphi)$= field pattern of array of isotropic point sources

$E_{pi}(\theta,\varphi)$= field pattern of individual source

$E_{pa}(\theta,\varphi)$= field pattern of array of isotropic point sources

Radiation pattern of 4 isotropic elements fed in phase, spaced λ/2 apart:-

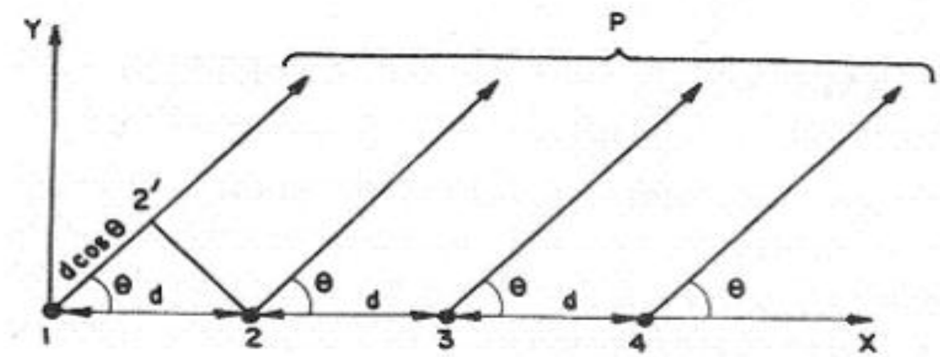
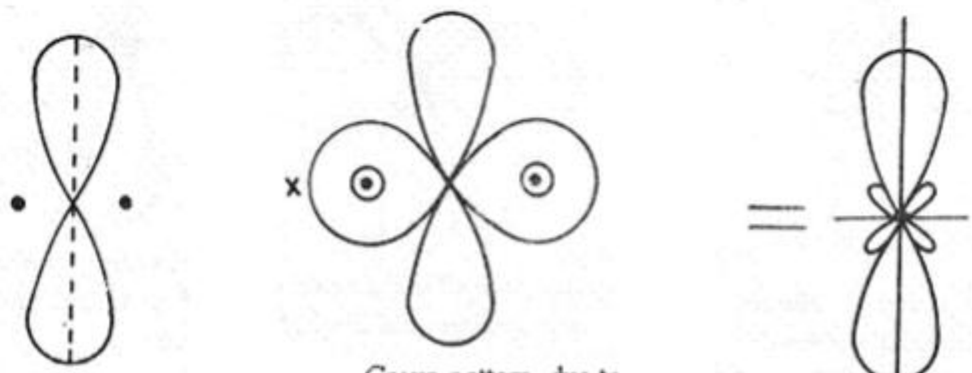


fig 4.1 Linear array of 4-isotrpic elements spaced λ/2 apart,fed in phase



Individual Pattern

Group pattern due to array of two(isotropic)

Resultant pattren of 4 isotropic elements

Figure:Radiation pattern of 8-isotropic elements fed in spaced λ/2 apart.

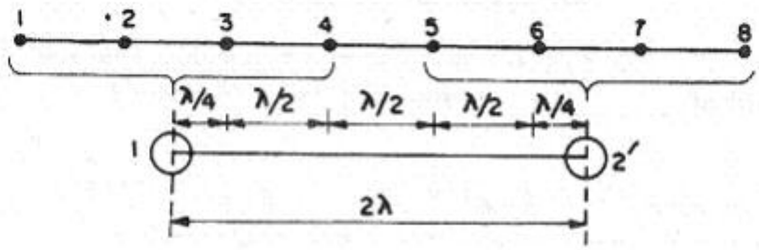


fig 4.5 (a)Linear array of 8 isotropic elements spaced λ/2.
(b) equivalent two units array spaced 2λ

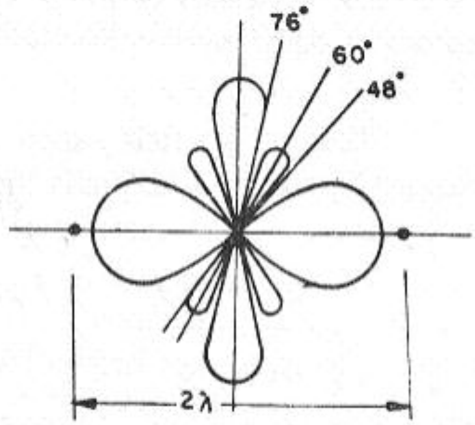
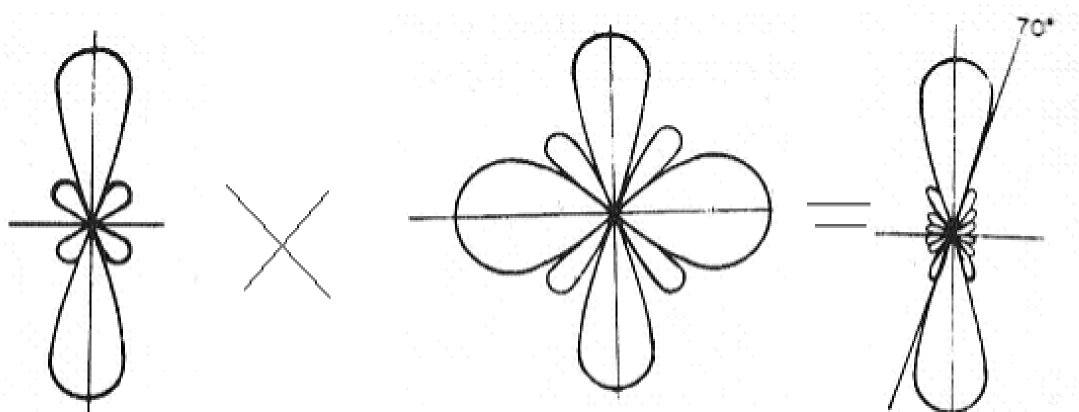


fig 4.6 Radiation Pattern of isotropic radiators spaced 2λ



"unit pattern" due to 4-individual elements

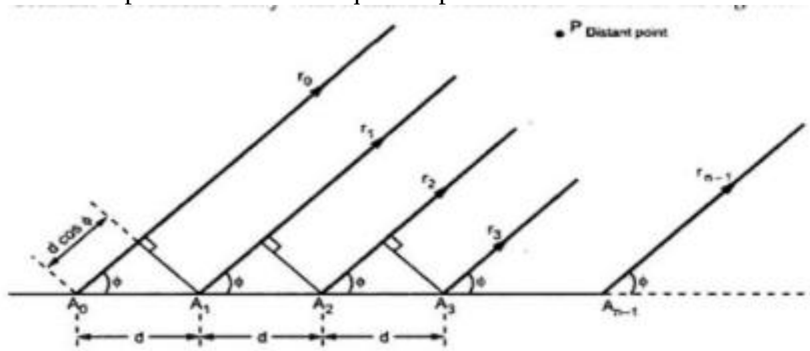
"group pattern" due to twomisotropic elements spaced 2λ apart

Resulting pattern of 8 isotropic elements

4.7 Resulatant radiation pattern of 8 isotropic elements by pattern multiplication.

Linear array with 'n' isotropic point sources of equal amplitude and spacing:

For point to point communication, at higher frequencies a single narrow beam of the radiation pattern is required which is usually obtained by multiunit linear arrays,Calculation of far field pattern for equally spaced 'n' isotropic point sources and are fed within phase currents of equal amplitudes



The total far fields patterns at a distant point 'P' is obtained by adding vector ally the fields of individual sources as

$$E_t = E_0 e^{(0)j\psi} + E_0 e^{(1)j\psi} + E_0 e^{(2)j\psi} + E_0 e^{(3)j\psi} + E_0 e^{(4)j\psi} + ------ + E_0 e^{(n-1)j\psi}$$

$$E_t = E_0\left(e^{j\psi} + e^{j2\psi} + e^{j3\psi} + e^{j4\psi} + ------ + e^{j(n-1)\psi}\right) \text{-------------- (1)}$$

$$\psi = \beta d \cos\theta + \alpha$$

α=phase difference in adjacent sources

Multiply the $e^{j\Psi}$ term to the eq (1) on both sides for computation

$$E_t e^{j\psi} = E_0\left(e^{j\psi} + e^{j2\psi} + e^{j3\psi} + e^{j4\psi} + ------ + e^{jn\psi}\right) \quad \text{--------- (2)}$$

Subtracting eq (2) from (1)

$$E_t\left(1 - e^{j\psi}\right) = E_0\left(1 - e^{jn\psi}\right)$$

$$E_t = \frac{E_0\left(1 - e^{jn\psi}\right)}{\left(1 - e^{j\psi}\right)} \quad \text{-------------- (3)}$$

Above equation is in the form of $\frac{1-a^n}{1-a}$ where 'a' is common ratio because it is geometric series if (a<1)

Then $a = \frac{e^{j\psi}}{1} = \frac{e^{j2\psi}}{e^{j\psi}} = \frac{e^{j3\psi}}{e^{j2\psi}} = ------ = e^{j\psi}$

Eq (3) may be rewritten as

$$E_t = \frac{E_0\left(1 - e^{jn\psi/2} + e^{jn\psi/2}\right)}{\left(1 - e^{j\psi/2} e^{j\psi/2}\right)}$$

$$E_t = E_0 \frac{e^{jn\psi/2}}{e^{j\psi/2}} \frac{\left(e^{-jn\psi/2} - e^{jn\psi/2}\right)}{\left(1 - e^{-j\psi/2} - e^{j\psi/2}\right)}$$

$$E_t = E_0 e^{j(n-1)\psi/2} \frac{\sin n\psi/2}{\sin \psi/2}$$

$$E_t = E_0 \left(\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right) e^{j\phi} \qquad \left[\because \varphi = \left(\frac{n-1}{2} \right) \psi \right]$$

$$E_t = E_0 \left(\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right) \cos\phi + j \sin\phi$$

The above equation is the total field pattern of linear array of n-isotropic point sources as reference point for phase.

Note: if the reference point is sifted to the centre of the co-ordinate then phase angle $\left(\frac{n-1}{2} \right) \psi = \phi$ is automatically eliminated and eq(4) reduces to $E_t = E_0 \left(\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right)$ array factor

$\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}$ =array factor or secondary pattern

E_0= individual source pattern or primary pattern

Array of 'n' isotropic sources of equal amplitude and spacing (broad side case):

Major lobe (maxima direction):

An array is said to be broad side array, if the phase angle is such that it makes maximum radiation perpendicular to the line of array i.e 90^0 and 270^0

In broad side array sources are in phase i.e α=0 & Ψ=0

For maximum must be satisfied

$$\psi = \beta d \cos\theta + \alpha = \beta d \cos\theta + 0$$

$$\beta d \cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90^0 or 270^0$$

The principal maximum occurs in these directions

Minor lobe (maxima direction):

$$E_t = E_0 \left(\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right)$$

This equation is maximum when numerator is maximum i.e $\sin \frac{\psi}{2}$ is maximum provided $\sin \frac{\psi}{2}$ not equal to '0'

$$\sin \frac{n\psi}{2} = 1$$

$$\frac{n\psi}{2} = \pm(2N+1)\frac{\pi}{2} \quad \text{(N=1, 2, 3,4)}$$

(N=0 corresponds to major lobe maxima)

$$n\psi/2 = \pm(2N+1)\frac{\pi}{2}\times\frac{1}{n}$$

$$\psi = \pm(2N+1)\frac{\pi}{n}$$

$$\beta d\cos(\theta_{max})_{min} = \pm(2N+1)\frac{\pi}{n}$$

$$\cos(\theta_{max})_{min} = \frac{1}{\beta d}\left[\pm(\frac{2N+1}{n})\pi - \alpha\right]$$

$$(\theta_{max})_{min} = \cos^{-1}\left[\frac{1}{\beta d}\left[\pm(\frac{2N+1}{n})\pi - \alpha\right]\right] \text{minor lobe maxima}$$

For broad side array α=0

$$(\theta_{max})_{min} = \cos^{-1}\left[\frac{1}{\beta d}\pm(\frac{2N+1}{n})\pi\right]$$

Minor lobe (minima direction):
Direction of pattern minima:

$$E_t = E_0\left(\frac{\sin n\psi/2}{\sin \psi/2}\right) = 0$$

$$\sin n\psi/2 = 0 \text{ [Provided } \left(\sin n\psi/2 \neq 0\right) \text{ where N=1, 2, 3]}$$

$$\frac{n\psi}{2} = \pm N\pi$$

$$\psi = \frac{\pm 2N\pi}{n}$$

$$\beta d\cos(\theta_{max})_{min} + \alpha = \pm\frac{2N\pi}{n}$$

$$\cos(\theta_{max})_{min} = \frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}$$

$$(\theta_{max})_{min} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}\right]$$

For broad side α=0

$$(\theta_{max})_{min} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n}\right\}\right]$$

Beam width of major lobe:
It is defined as the angle between first nulls or double the angle between first null and major lobe maximum directions BWFN=2r

R=angle between first null and maximum of major lobe=90^0-θ

$$(\theta_{\min})_{\min or} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n}\right\}\right]$$

$$\beta = \frac{2\pi}{\lambda}$$

$$(\theta_{\min})_{\min or} = \cos^{-1}\left(\pm\frac{N\lambda}{nd}\right)$$

$$90^0 - r = \pm\cos^{-1}\left(\pm\frac{N\lambda}{nd}\right)$$

$$\cos(90^0 - r) = \sin r = \pm\frac{N\lambda}{nd}$$

[r is very small0 $\sin r = r$

$$r = \pm\frac{N\lambda}{nd}$$

BWFN=2r

$$BWFN = \pm\frac{2N\lambda}{nd}$$ First null occurs when N=1

$$BWFN = \frac{2\lambda}{nd}$$

$$2r = \frac{2\lambda}{nd}$$

$$2r = \frac{2\lambda}{nd} \Rightarrow 2r = \frac{2\lambda}{L}$$

L =nd total length of array in meter

$$2r_1 = \frac{2\lambda}{L} = \frac{2}{L/\lambda}$$

$$2r_1 = \frac{2\times 57.3}{L/\lambda}\deg ree$$

$$2r_1 = \frac{114.6^0}{L/\lambda} BWFN$$

$$HPBW = \frac{BWFN}{2} = \frac{57.3^0}{L/\lambda}\deg ree$$

Array of n sources of equal amplitude and spacing (end-fire case):Major lobe (maxima):

For an array to be end fire, the phase angles is such that makes the maximum radiation in the line of array i.e θ=0^0 or 180^0 Thus for an array to be end fire Ψ=0 & θ=0^0 &180^0

$$\psi = \beta d\cos\theta + \alpha$$

$$0 = \beta d\cos 0^0 + \alpha$$

$$\alpha = -\beta d = \frac{-2\pi d}{\lambda}$$ [Phase difference depends on the spacing]

Minor lobe (maxima):
Direction of pattern maxima:

$$\sin\frac{n\psi}{2} = 1 if \sin\frac{n\psi}{2} \neq 0$$

$$\frac{n\psi}{2} = \pm(2N+1)\frac{\pi}{2}$$

$$n\psi = \pm(2N+1)\pi$$

$$\psi = \frac{\pm(2N+1)\pi}{n}$$

For end fire case

$$\alpha = -\beta d \psi = 0$$

$$\beta d \cos\left(\theta_{\max}\right)_{\min or} + \alpha = \pm\left(\frac{2N+1}{n}\right)\pi$$

$$\beta d \cos\left(\theta_{\max}\right) - \beta d = \pm\left(\frac{2N+1}{n}\right)\pi$$

$$\beta d\left(\cos\theta_{\max} - 1\right) = \pm\left(\frac{2N+1}{n}\right)\pi$$

$$\cos\theta_{\max} = \pm\frac{\left(2N+1\right)\pi}{\beta dn} + 1$$

$$\left(\theta_{\max}\right)_{\min or} = \cos^{-1}\left[\pm\frac{\left(2N+1\right)\pi}{\beta dn} + 1\right]$$

$$if n = 4, d = \frac{\lambda}{2}, \alpha = -\pi$$

$$\left(\theta_{\max}\right)_1 = \cos^{-1}\left[0.25\right] = 75.5^0$$

$$\left(\theta_{\max}\right)_1 = \cos^{-1}\left[\frac{-1}{4}\right] = -75.5^0$$

Minor lobe minima:
Direction of pattern minima: $\alpha = -\beta d$

$$\beta d \cos\left(\theta_{\min}\right)_{\min or} + \alpha(-\beta d) = \pm\left(\frac{2N\pi}{n}\right)$$

$$\beta d\left[\cos\left(\theta_{\min}\right)_{\min or} - 1\right] = \pm\frac{2N\pi}{n}$$

$$\left[\cos\theta_{\min} - 1\right] = \pm\frac{2N\pi}{\beta dn} = \pm\frac{2N\pi}{\frac{2\pi}{\lambda}dn} = \pm\frac{N\pi}{dn}$$

$$1 - 2\sin^2\frac{\theta_{\min}}{2} - 1 = \pm\frac{N\pi}{dn}$$

$$-2\sin^2\frac{\theta_{\min}}{2} = \pm\frac{N\pi}{dn}$$

$$\sin^2\frac{\theta_{\min}}{2}=\pm\frac{N\pi}{2dn}$$
$$\theta_{\min}=2\sin^{-1}\left(\pm\sqrt{\frac{N\pi}{2dn}}\right)=\pm60^0$$
$$(\theta_{\min})_2=\pm90^0 \quad (\theta_{\min})_3=\pm120^0 \quad (\theta_{\min})_4=\pm180^0$$

Beam width of major lobes:

Beam width=2θ_1

$$\theta_{\min}=2\sin^{-1}\left(\pm\frac{N\pi}{2dn}\right)$$
$$\sin\theta_{\min}\cong\theta_{\min}=2\sin^{-1}\left(\pm\sqrt{\frac{N\pi}{2dn}}\right)=\pm\sqrt{\frac{N\pi}{2dn}}$$
$$\theta_{\min}=\pm\sqrt{\frac{N\pi}{2dn}}$$

$$BWFN=\pm114.6\sqrt{\frac{2}{L/\lambda}}$$

Array of 'n' isotropic sources of equal amplitude and spacing end fire array with increased directivity:

The maximum radiation can be directed along the axis of the uniform array by allowing the phase shift α between elements equal to ±βd

$$\alpha=-\beta d,\ for\theta=0$$
$$\alpha=\beta d^0 \quad \theta=180$$

This produces a maximum field in the direction θ=0 but does not give the maximum directivity.

In order to improve the directivity of an EFA without destroying any other characteristics, required phase shift between closely spaced elements of a very long array should be

$$\alpha=-\left(\beta d+\frac{\pi}{n}\right)=-\left(\beta d+\frac{2.94}{n}\right)\text{ For max in }\theta=0\text{ ----- (1)}$$
$$\alpha=\left(\beta d+\frac{\pi}{n}\right)=+\left(\beta d+\frac{2.94}{n}\right)\text{ For max in }\theta=\pi\text{ ----- (2)}$$

These conditions are referred to as the "Hansen wood yard conditions for increased directivity".

These conditions do not necessarily yield the maximum possible directivity.

$$E_n=\frac{\sin\frac{n\psi}{2}\phi}{\sin\frac{\psi}{2}}$$

For maximum radiation along θ=0

$$|\psi|=|\beta d\cos\theta+\alpha|_{\theta=0}=\frac{\pi}{n}$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=180^0} \; \square \; \pi$$

For maximum radiation along θ=180^0

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=180^0} = \frac{\pi}{n}$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=0^0} \cong \pi$$

For array of ‘n’ elements the condition |Ψ|=π is satisfied by using (1) for θ=0 & (2) for θ=180^0 and choosing for each spacing of $d = \left(\frac{n-1}{n}\right)\frac{\lambda}{4}$,

If the element is large ,

$$E_n = (AF)_n = \frac{1}{n}\left|\frac{\sin\left[\frac{n}{2}\left(\beta d\cos\theta + \alpha\right)\right]}{\frac{\sin\left[\frac{n}{2}\left(\beta d\cos\theta + \alpha\right)\right]}{2}}\right|$$

$$\psi = \beta d\cos\theta + \alpha$$

To overcome the drawbacks a scientist named dolpls introduced an array with narrow beam width, high gain & minimum side lobes,

For this,Tchelys’chef’s polynormal is used & array is called. Dolph tcheby schef’s array

1) If beam width is specified then it is possible to reduce the side lobe level (with this we can over come the disadvantage of UAD (BSA).

2) If side lobe level is specified then it is possible to reduce the beam width & there by increase the directivity (NUAD) based on the above ‘2’ start’s the array was designed polynomial.

$T_N(x) = \cos(n\cos^{-1}x)$ for 1x1≤±1

$\cos^{-1}x=\delta$

$\cos\delta=x$

For n=0 $T_0(X) = \cos(0)=1$

For n=1 $T_1(X) = \cos(\cos^{-1}x)=\cos\delta=x$

For n=2 $T_2(X) = \cos(2\cos^{-1}x)=\cos(28)$

$= 2\cos^2\delta-1$

$= 2X^2-1$

For n=3 $T_3(X) = \cos(3\cos^{-1}x)=\cos(38)$

For higher values of n this can = $4\cos^3 8-3\cos 8$ be obtained by using = $4X^3-3X$

$T_{n+1}(x) = 2xT_n(x)- T_{n+1}(x)$

For n=3

$T_4(x) = 2xT_3(x)- T_2(x)=2x(4x^3-3x)-2x^2+1=8x^4-8x^2+1$

n=5 $T_5(x) = 16x^5-20x^3+5x$

$T_6(x) = 32x^6-48x^4+5x^2-1$

$T_7(x) = 64x^7-112x^5+56x^3.7x$

The degree of the polynomial is same the value of ‘n’ the value can be either even or odd

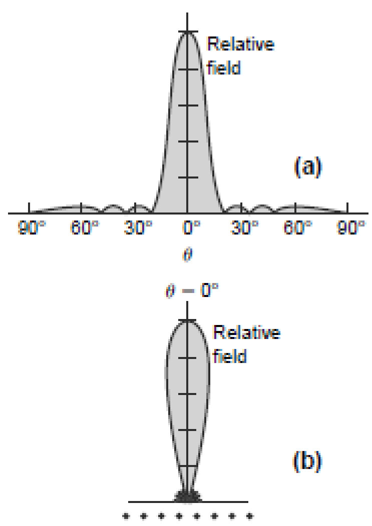
Binominal arrays:
in case of BSA as the no. of elements ↑'s D↑'s & also no. of side lobes ↑'s but in some applications it is desirable that side lobes should be eliminated totally &reduced to min desired level this can be achieved by binominal array.

Definition: it is an array in which all the elements are fed with current of non-uniform amplitudes & and the amp's arranged according to the co-efficient of binomial series

$$(a+b)^{n-1} = a^{n-1}b^0 + \frac{n-1}{11}a^{n-2}b^1 + \frac{(n-1)(n-2)}{21}a^{n-3}b^2 + \frac{(n-1)(n-2)(n-3)}{31}a^{n-4}b^3 + ---$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

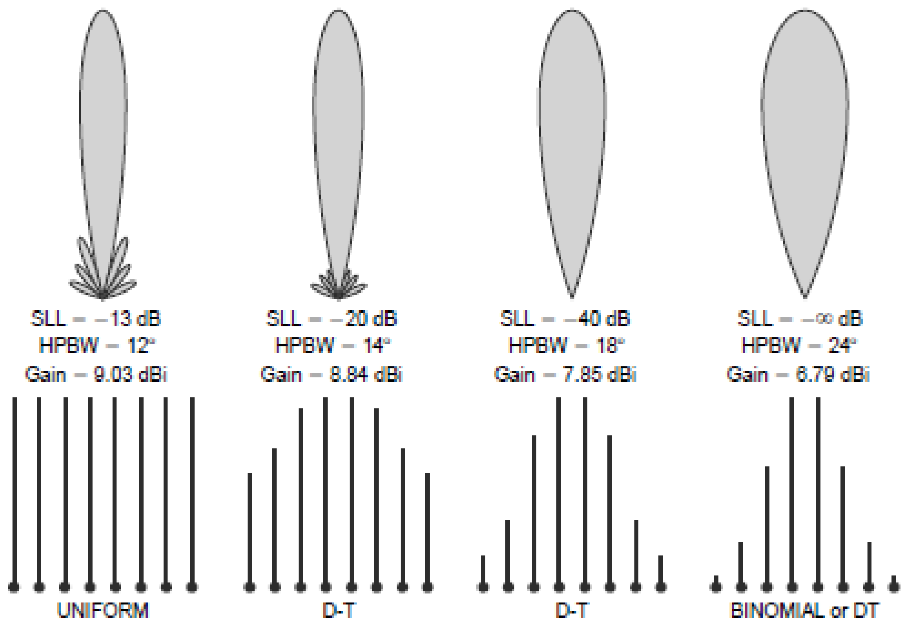
Comparison of Amplitude Distributions for Eight-Source Arrays:
In the problem worked in the preceding section, the side-lobe level was 26 dB below the maximum of the main beam (R = 20).
It is of interest to compare the amplitude for this case with the distributions for other side-lobe levels.
This is done in Fig. 5−48 for a uniform distribution and three optimum (D-T) distributions with side-lobe levels −20 dB, −40 dB, and −∞ dB below the main beam maximum. The infinite decibel 5–18 Comparison of Amplitude Distributions for Eight-Source Arrays,case corresponds to R=∞ (zero side-lobe level) and is identical with Stone's binomial distribution. The relative amplitudes for this case are 1, 7, 21, 35, 35, 21, 7, 1 (Riblet-1). The ratio of amplitudes of the center sources to the edge sources is 35 to 1.
Such a large ratio might be difficult to achieve in practice. Both the binomial and edge distributions are special cases of the
Dolph-Tchebyscheff (DT) distribution, but the uniform amplitude distribution is not.



The D-T optimum amplitude distribution, as discussed in the preceding sections, is optimum only if d ≥ λ/2,which covers the cases of most Relative field pattern of broadside array of eight isotropic sources spaced $\lambda/2$ apart.

The D-T amplitude distribution gives a minimum beam width for a side-lobe level 120 of the main lobe. The pattern is shown in rectangular coordinates at (*a*) and in polar at (*b*). Both diagrams show the pattern only from −90◦ to +90◦, the other half of the pattern being identical.Uniform and three optimum (D-T) source distributions for eight in-phase isotropic sources spaced $\lambda/2$ with field patterns. The distributions result in Side-Lobe Levels (SLLs) ranging from −13 dB for the uniform array to −∞ for the binomial array. Note that as the SLL is reduced, the distribution is more tapered, the HPBW is larger, and the gain is less.

Thus, if low SLL is required, the gain is reduced. Conversely, if maximum gain is desired, a larger SLL must be tolerated. This is the designer's dilemma interest for broadside arrays. By a generalization of the method, however, cases with smaller spacing can also be optimized.



In conclusion, it should be pointed out that the properties of the Tchebyscheff polynomials may be applied not only to antenna patterns as discussed above but also to other situations. It is necessary, however, that the function to be optimized be expressible as a polynomial.

The D-T (Dolph-Tchebyscheff) source amplitude distributions for linear arrays on N sources are given by the computer program ARRAYPATGAIN on the book's web site antennas3.com. The program also plots the pattern and gives the HPBW and gain as discussed .

Table 5–8

Variable	Range		
θ	$-\frac{\pi}{2}$	0	$+\frac{\pi}{2}$
x	0	1.15	0

Linear Broadside Arrays with No uniform Amplitude Distributions. General Considerations:
Let us begin by comparing the field patterns of four amplitude distributions, namely, uniform, binomial, edge,and optimum.To be specific, we will consider a linear array of five isotropic point sources with $\lambda/2$ spacing.If the sources are in phase and all equal in amplitude, we may calculate the pattern as discussed in Sec. 5−13 the result being as shown in Fig. 5−41 by the pattern designated uniform.

A uniform distribution yields the maximum directivity or gain. The pattern has a half-power beamwidth of 23◦, but the side lobes are relatively large.

n	Relative amplitudes (Pascal's triangle)										
3				1		2		1			
4			1		3		3		1		
5		1		4		6		4		1	
6	1		5		10		10		5		1

The amplitude of the first side lobe is 24 percent of the main-lobe maximum. In some applications this minor-lobe amplitude may be undesirably large.

To reduce the Side-Lobe Level (SLL) of linear in-phase broadside arrays, John Stone Stone (1) proposed that the sources have amplitudes proportional to the coefficients of a binomial series of the form where n is the number of sources.

$$(a+b)^{n-1} = a^{n-1} + (n-1)a^{n-2}b + \frac{(n-1)(n-2)}{2!}a^{n-3}b^{2}........$$

Thus, for arrays of three to six sources the relative amplitudes are given by Table 5−4, where the amplitudes are arranged as in Pascal's triangle (any inside number is equal to the sum of the adjacent numbers in the row above).
Applying the binomial distribution to the array of five sources spaced λ/2 apart, the sources have the relative amplitudes 1, 4, 6, 4, 1. The resulting pattern, designated binomial, is shown in Fig. 5−41.

Methods of calculating such patterns are discussed in the next section. The pattern has no minor lobes, but this has been achieved at the expense of an increased beam width (31◦). For spacing's of λ/2 or less between elements, the minor lobes are eliminated by Stone's binomial distribution. However, the increased beam width and the large ratio of current amplitudes required in large arrays are disadvantages.

Table 5–5

Type of distribution	Half-power beamwidth	Minor-lobe amplitude (% of major lobe)
Binomial	31◦	0
Edge	15◦	100

At the other extreme from the binomial distribution, we might try an edge distribution in which only the end sources of the array are supplied with power, the three central sources being either omitted or inactive. The relative amplitudes of the five-source array are, accordingly, 1, 0, 0, 0, 1.The array has, therefore, degenerated to two sources 2λ apart and has the field patter designated as edge . The beam width between half power points of the "main" lobe (normal to the array) is 15◦, but "minor" lobes are the same amplitude as the "main" lobe.

Comparing the binomial and edge distributions for the five-source array with λ/2 spacing, we have Table 5−5.Although for most applications it would be desirable to combine the 15◦ beam width of the edge distribution with the zero side-lobe level of the binomial distribution, this combination is not possible.

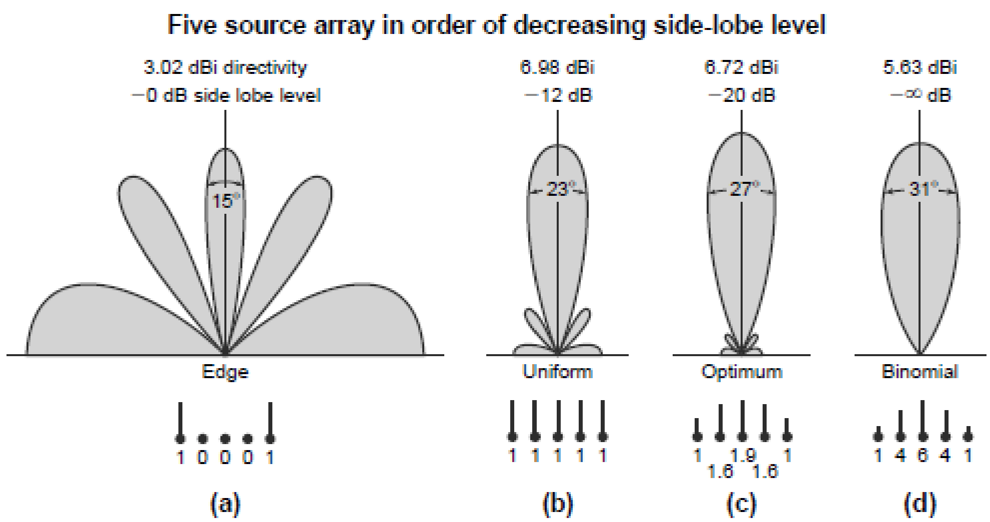
However, if the distribution is between the binomial and the edge type, a compromise between the beam width and the side lobe level can be made; i.e., the side-lobe level will not be zero, but the beam width will be less than for the binomial distribution.
An amplitude distribution of this nature for linear in-phase broadside arrays was proposed by Dolph (1) which has the further property of optimizing the relation between beam width and side-lobe level; i.e., if the side-lobe level is specified, the beam width between first nulls is minimized; or, conversely, if the beam width between first nulls is specified, the side-lobe level is minimized. Dolph’s distribution is based on the properties of the Tchebyscheff polynomials and accordingly will be referred to as the Dolph-Tchebyscheff or optimum distribution.

Applying the Dolph-Tchebyscheff distribution to our array of five sources with λ/2 spacing, let us specify a side-lobe level 20 dB below the main lobe, i.e., minor-lobe amplitude 10 percent of the main lobe. The relative amplitude distribution for this side-lobe level is 1, 1.6, 1.9, 1.6, 1 and yields the pattern designated optimum in Fig. 5−41. Methods of calculating the distribution and pattern are discussed in the next section.

The beam width between half-power points is 27◦, which is less than for the binomial distribution. Smaller beam widths can be obtained only by raising the side-lobe level. The Dolph-Tchebyscheff distribution includes all distributions between the binomial and the edge.

In fact, the binomial and edge distributions are special cases of the Dolph-Tchebyscheff distribution, the binomial distribution corresponding to an infinite ratio between main- and side-lobe levels and the edge distribution to a ratio of unity. The uniform distribution is, however, not a special case of the Dolph-Tchebyscheff distribution.

Referring to Fig. 5−41, we may draw a number of general conclusions regarding the relation between patterns and amplitude distributions. We note that if the amplitude tapers to a small value at the edge of the 5–15 Linear Broadside Arrays with No uniform Amplitude Distributions. General Considerations



Normalized field patterns of broadside arrays of five isotropic point sources spaced λ/2 apart. All sources are in the same phase, but the relative amplitudes have four different distributions: edge, uniform, optimum, and binomial. Only the upper half of the pattern is shown.

The relative amplitudes of the five sources are indicated in each case by the array below the pattern, the height of the line at each source being proportional to its amplitude. All patterns are adjusted to the same maximum amplitude.

Array (binomial distribution), minor lobes can be eliminated. On the other hand, if the distribution has an inverse taper with maximum amplitude at the edges and none at the center of the array (edge distribution), the minor lobes are accentuated, being in fact equal to the "main" lobe.

From this we may quite properly conclude that the side-lobe level is closely related to the abruptness with which the amplitude distribution ends at the edge of the array. An abrupt discontinuity in the distribution results in large minor lobes,while a gradually tapered distribution approaching zero at the edge minimizes the discontinuity and the minor-lobe amplitude.

In the next section, we shall see that the abrupt discontinuity produces large higher "harmonic" terms in the Fourier series representing the pattern. On the other hand, these higher harmonic terms are small when the distribution tapers gradually to a small value at the edge.

There is an analogy between this situation and the Fourier analysis of wave shapes. Thus, a square wave has relatively large higher harmonics, whereas a pure sine wave has none, the square wave being analogous to the uniform array distribution while the pure sine wave is analogous to the binomial distribution.

The preceding discussion has been concerned with arrays of discrete sources separated by finite distances. However, the general conclusions concerning amplitude distributions which we have drawn can be extended to large arrays of continuous distributions of an infinite number of point sources, such as might exist in the case of a continuous current distribution on a metal sheet or in the case of a continuous field distribution across the mouth of an electromagnetic horn or across a parabolic reflector antenna.

If the amplitude distribution follows a Gaussian error curve, which is similar to a binomial distribution for discrete sources, then minor lobes are absent but the beam width is relatively large.An increase of amplitude at the edge reduces the beam width but results in minor lobes, as we have seen.

Thus, in the case of a high-gain parabolic reflector type of antenna, the illumination of the reflector by the primary antenna is usually arranged to taper toward the edge of the parabola. However, a compromise is generally made between beam width and side-lobe level so that the illumination is not zero at the edge but has an appreciable value as in a Dolph-Tchebyscheff distribution.

General considerations:

Uniform amplitude arrays produce small half-power beam width and possess the largest directivity. But in certain instances the side lobe level of the radiation pattern has to be maintained at a desired level. The side lobe level can be reduced by varying the amplitude excitations of the array elements. Non uniform amplitude excitations of a linear antenna array produce a pattern with smaller side lobe level and a slightly increased half power beam width in comparison to the uniform linear antenna array.

In this section, we will discuss arrays with uniform spacing but non uniform amplitude distribution. Often, the broadside arrays are classified according to the type of their excitation amplitude.

The categories are:

(a) Uniform amplitude array: Relatively high directivity, but the side-lobe levels are high;

(b) Dolph–Chebyscheff array: For a given number of elements, its maximum directivity is next to that of the uniform array. Side-lobe levels are the lowest in comparison with the other two types of arrays for a given directivity;

(c) Binomial array: Does not have good directivity but has very l

Binomial Arrays:

The binomial array was investigated and proposed by J. S. Stone to synthesize patterns without side lobes. Let us first consider a 2–element array with equal current amplitudes and spacing, the array factor is given by

$$AF=1+e^{j\Psi}$$

For a board slid array (β=0)with element spacing d less than one-half wavelength, the array factor hasβFor a broadside array (no side lobe)

This can be proved in the following way:

$$|AF|^2 = (1+\cos\psi)^2 + \sin^2\psi = 2(1+\cos\psi) = 4\cos^2\left(\frac{\psi}{2}\right)$$

Where ψ = kd cosθ

The first null of this array factor can be obtained as:

$$\frac{1}{2}\frac{2\pi}{\lambda} d\cos\theta_n = \pm\frac{\pi}{2} \Rightarrow \theta_n = \pm\cos^{-1}\left(\frac{\lambda}{2d}\right)$$

As long as the $d < \lambda/2$, the first null does not exist. If $d = \lambda/2$, then null will be at $\theta=0^0$ and 180^0 . Thus, in "visible" the "visible" range of θ, All secondary lobes are eliminated.An array formed by taking the product of two arrays of this type gives:

$$AF = \left(1+e^{j\psi}\right)\left(1+e^{j\psi}\right) = 1+2e^{j\psi}+e^{j2\psi}$$

This array factor, being the square of an array factor with no side lobes, will also has no side lobes.Mathematically, the array factor above represents a 3-element equally-spaced array driven by current amplitudes with rations of 1:2:1.

In a similar fashion, equivalent arrays with more elements may be formed.

2-element $AF=1+e^{j\Psi}$

3-element $AF= (1+e^{j\Psi})^2 =1+2e^{j\Psi}+ e^{j2\Psi}$

4-element $AF= (1+e^{j\Psi})^3 =1+3e^{j\Psi}+ 3e^{j2\Psi}+ e^{j3\Psi}$

Similarly for N-element the array factor can be expressed as N-element $AF= (1+e^{j\Psi})^{N-1}$

if $d \le \lambda/2$, the above AF does not have side lobes regardless of the number of element N.

The excitation amplitude distribution can be obtained easily by the expansion of the binome in(6.50)Making use of Pascal's triangle, this can be given by

$$
\begin{array}{c}
1 \\
1 \quad 1 \\
1 \quad 2 \quad 1 \\
1 \quad 3 \quad 3 \quad 1 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
\ldots\ldots\ldots\ldots
\end{array}
$$

The relative excitation amplitudes at each element of an (N+1) element array can be determined from this triangle,An array with a binomial distribution of the excitation amplitudes is called a binomial array

$$AF = \left(1+e^{j\psi}\right)^{N-1} + \frac{(N-1)(N-2)}{2!}e^{j\psi} + \frac{(N-1)(N-2)(N-3)}{3!}e^{j3\psi} + \ldots\ldots\ldots\ldots$$

The excitation distribution as given by the binomial expansion gives the relative values of the amplitudes,It is immediately seen that there is too wide variation of the amplitude, which is a disadvantage of the binomial arrays

The overall efficiency of such an antenna would be low,Besides, the binomial array has a relatively wide beam,Its HPBW is the largest as compared to the uniform or the Dolph-Chebyshev array, An approximate closed-form expression for the HPBW of a binomial array with d=λ/2 is

$$HPBW = \frac{1.06}{\sqrt{N-1}} = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{1.75}{\sqrt{L/\lambda}}$$

Where L=(N-1)d is the array length

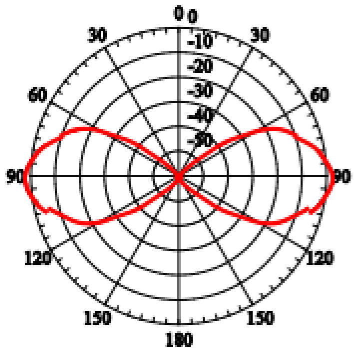
The directivity of a broadside binomial array with spacing d=λ/2 can be calculated as:

$$D_0 = \frac{2}{\int_0^{\pi}\left[\cos\left(\frac{\pi}{2}\cos\theta\right)\right]^{2(N-1)} d\theta}$$

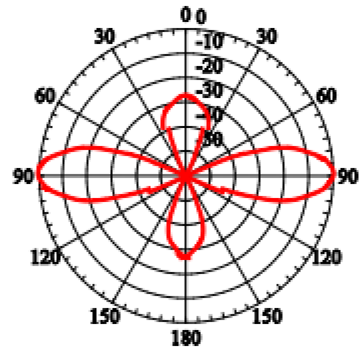
$$D_0 = \frac{(2N-2)(2N-4)\ldots\ldots\ldots 2}{(2N-3)(2N-5)\ldots\ldots\ldots 1}$$

$$D_0 \approx 1.77\sqrt{N} = 1.77\sqrt{1+2L/\lambda}$$

The array factor of a 10 element broad side binominal array (N=10)



(a) $d = \lambda/2$

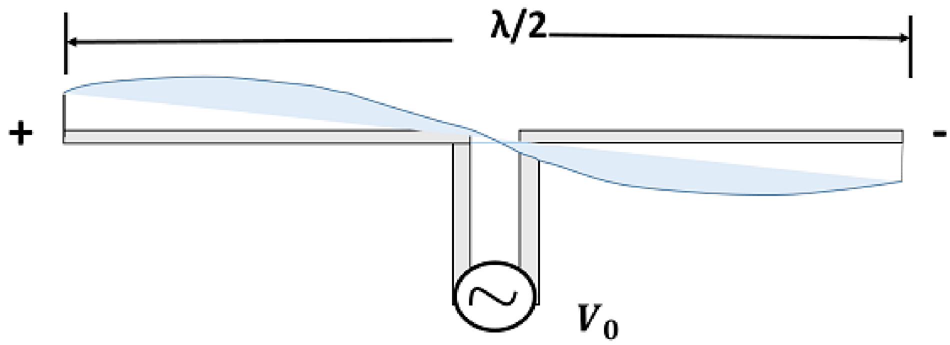


(b) $d = 3\lambda/4$

Radiation pattern for 10-element broad side binomial array

Folded Dipole Antenna:

A very important variation of conventional half wave dipole is the folded dipole in which two half wave dipoles-one continuous & the other split at the centre,Those two dipoles are folded & joined together in parallel at the ends



The split dipole is fed at the centre by a balanced transmission line,The two dipoles have the same voltages at their ends,Radiation pattern of a folded dipole and a conventional half wave dipole is same but the i/p impedance of the folded dipole is higher,Difference – Directivity, Band width (Broad for F.D),Directivity of Folded Dipole is bi-directional ,Due to the distribution of currents in the parts of the folded dipole the i/p impedance becomes higher,If the radii of the two conductors are equal, then equal currents flow in both the conductors, in the same direction i.e currents are equal in magnitude and phase in the two dipole

Total power is equal to the conventional dipole I/P impedance is higher,It can be proved that the i/p impedance at the terminals of a folded dipole antenna is equal to the square of no of conductors comprising the antenna times the impedance at the terminals of C.D

Equation of i/p impedance:

The equation for the i/p impedance or Terminal impedance or radiation resistance of F.D Antenna ,Let 'V' be the emf applied at the AA'Fig:- Equivalent diagram of two wire folded 1 /2 wave dipole By nodal analysis

$$\frac{v}{2} = I_1 Z_{11} + I_2 Z_{12}$$

I_1 & I_2 =Current flowing at the terminals of dipole no.1&2, Z_{11} & Z_{12} =self impedances of dipole no.1 & mutual impedance between 1&2,'V' divided equally in each dipole; hance voltage in each dipole is V/2 as shown by nodal analysis

$$\frac{v}{2} = I_1 \left(Z_{11} + Z_{12} \right)$$

The two dipoles in the system are very close to each other. The spacing 'a' between two dipoles is of the order of λ/100

$$Z_{11} \cong Z_{12}$$

$$\frac{v}{2} = I_1 \left(2Z_{11} \right) \qquad \frac{v}{I_1} = 4Z_{11}$$

$$z = 4Z_{11} \qquad z = 4 \times 73 = 292\Omega$$

Suppose for a folded dipole of 3 wires, it can be proved that termination impendence

$$\frac{v}{3} = I_1 \left(3Z_{11} \right) \qquad \frac{v}{I_1} = 3^2 Z_{11} \qquad Z = 9Z_{11} \qquad Z = 9 \times 73 = 657\Omega$$

For 'n' no.of wires:n=no.of Half wave dipoles

$$\frac{v}{n} = I_1 \left(nZ_{11} \right) \qquad \frac{v}{I_1} = n^2 Z_{11} \qquad z = n^2 Z_{11}$$

For unequal radii of 2 dipoles:

r_2&r_1=radii of elements

$$z = Z_{11} \left(1 + \frac{r_2}{r_1} \right)^2 \qquad z = 73 \left(1 + \frac{r_2}{r_1} \right)^2$$

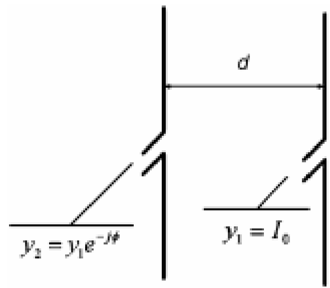
Impedance transformation depends on the relative spacing 'a' then according to Uda & Musleiave

$$z = z_{11} \left(1 + \frac{\log \frac{a}{r_1}}{\log \frac{a}{r_2}} \right)^2$$

$$z = z_{11} z_{ratio}$$

Voltage & current relations in parasitic Antennas (or) Dipole array with parasitic elements (next page):

Antennas can also be constructed with parasitic elements in which currents are induced by the fields from a driven element such elements have no transmission line connection,Dipole can be used as a parasitic element let us consider the case of an array in free space consisting of one driven element λ/2 dipole element & one parasitic element (2)



Both elements are vertical so that the azimuth angle ø is as indicated. The circuit relations for the elements are

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \text{ ---------- (1)}$$

$$0 = I_2 Z_{22} + I_1 Z_{12} \text{ ----------- (2)}$$

From eq (2)

$$I_2 = \frac{-I_1 Z_{12}}{Z_{22}} \text{ ------------- (3)}$$

$$= \frac{-I_1 |Z_{12}| \angle\theta_M}{|Z_{22}| \angle\theta_2} = -I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \angle\theta_M - \theta_2$$

$$\varepsilon = \pi + (\theta_M - \theta_2)$$

$$I_2 = -I_1 \left| \frac{Z_{12}}{Z_{22}} \right| \varepsilon$$

$$\theta_M = Tan^{-1} \frac{X_{12}}{R_{12}} \qquad \theta_2 = Tan^{-1} \frac{X_{22}}{R_{22}}$$

$Z_{12} = R_{12} + jX_{12} =$ Mutual impedance of (1) & (2)

$Z_{22} = R_{22} + jX_{22} =$ Self impedance of (1)

sub I_2 in I_1 (3) in (1)

$$V_1 = I_1 Z_{11} + \left(-I_1 \frac{Z_{12}}{Z_{22}} \right) Z_{12}$$

$$V_1 = I_1 \left(Z_{11} - \frac{Z^2{}_{12}}{Z_{22}} \right)$$

$$\frac{V_1}{I_1} = Z_{11} - \frac{Z^2{}_{12}}{Z_{22}} \Rightarrow Z_1 = Z_{11} - \frac{Z^2{}_{12}}{Z_{22}}$$

$$Z_1 = Z_{11} - \frac{|Z^2{}_{12}| \angle 2\theta_M}{|Z_{22}| \angle\theta_2}$$

Real part of Z_1

$$R_1 = R_{11} - \left|\frac{Z^2{}_{12}}{Z_{22}}\right| \cos(2\theta_M - \theta_2)$$

If any loss is present, so this equation will be charging & adding the term

$$R_1 = R_{11} - R_{1L} - \left|\frac{Z^2{}_{12}}{Z_{22}}\right| \cos(2\theta_M - \theta_2)$$

Power i/p to the driven element

$$P = R_1 I_1^2 \qquad I_1 = \sqrt{\frac{P}{R_1}}$$

$$I_1 = \sqrt{\frac{P}{R_{11} + R_{1L} - \left|\frac{Z^2{}_{12}}{Z_{22}}\right| \cos(2\theta_M - \theta_2)}} \quad \text{------------- (4)}$$

The electric field intensity $E_ø$ at a large distance from the array as a function of 'ø' is

$$\theta_E = \cos^{-1}\frac{h}{2L} = cos^{-1}\frac{10\lambda}{2\times 62.5\lambda} = 4.57^0$$

$$E_\phi = K\left(I_1 + I_2\angle\beta d\cos\phi\right)$$

$$\theta_E = \cos^{-1}\frac{L}{L+8} = \cos^{-1}\frac{33.33\lambda}{33.33+0.325\lambda} = 8.55^0$$

$$E_\phi = K\left(I_1 + I_1\left|\frac{Z_{12}}{Z_{22}}\right| \& \angle\beta d\cos\phi\right)$$

$$E_\phi = I_1 K\left(I_1\left|\frac{Z_{12}}{Z_{22}}\right| \& \angle\beta d\cos\phi\right) \quad \text{------------------ (5)}$$

eq (4) in (5)

$$E_\phi = K\sqrt{\frac{P}{R_{11} + R_{1L} - \left|\frac{Z^2{}_{12}}{Z_{22}}\right| \cos(2\theta_M - \theta_2)}} \left(1 + \left|\frac{Z_{12}}{Z_{22}}\right| (\& + \beta d\cos\phi)\right)$$

Yagi-Uda Antenna:

After the names prof S-Uda & H-Yagi, this antenna is called Yagi-Uda Antenna

[This antenna was invented and described in Japanese by the former sometime around 1928 & after wards it was described by H.Yagi in English]

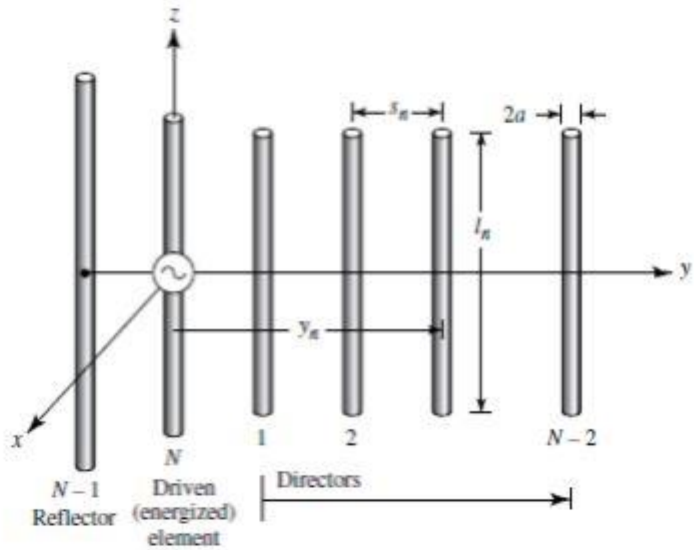
Arrangement of elements:

It consists of a driven element, a reflector and one or more director's i.e Yagi-Uda antenna is an array of a driven element (or) active element

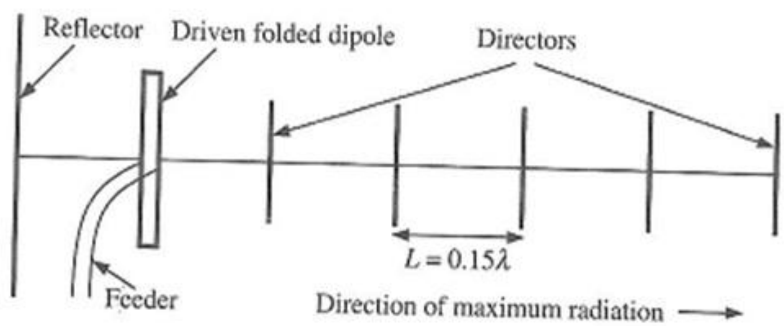
Driven element → where the power from the T_x is fed or which feeds received power to the R_X.

Parasitic element→ passive elements which are not connected directly to the transmission line but electrically coupled

The driven element is a resonant half-wave dipole usually of metallic rod at the frequency of operation,The parasitic elements of continuous metallic rods are arranged parallel to the driven element and at the same line of sight level,They are arranged co-linearly & close together with one reflector & one director



Yagi-Uda antenna configuration.



YAGI-UDA ANTENNA

The parasitic elements are excited from the voltage induced in them by the current flow in the driven elementThe phase & currents flowing due to the induced voltage depends on the spacing between the elements and upon the reactance of the elementsSpacing between element in front of driven element is known as director and its number may be more than one (1/5% more than D.E)Whereas the element in back of it is known as reflector (5% more than D.Ele)

- Reflector length=500/f (MHz) feet
- Driven element length= 475/f (MHz) feet
- Director length=455/f(MHz) feet

Action:

The spacing between the elements and the lengths of the parasitic elements determine the phases of currentsA parasitic element of equal or greater than λ/2 will be inductive while elements of lengths less than λ/2

- Inductive → Lag the induced voltage
- Capacitive→ Lead the induced voltage

Properly spaced dipoles shorter than λ/2 acts as director and the fields of driven element in the direction away from the driven element,If more than one director are employed then each director will excite the next,Additional gain is achieved by using additional directors in the beam direction,The distance between two elements may range from 0.1λ to 0.3λ, close spacing of elements are used in parasitic arrays to get a good excitation

Characteristics of Yagi-Uda Antenna:

If three elements array – one reflector, one driven & one director- beam Antenna,Unidirectional beam of moderate directivity with light weight, low cost & simplicity in feed system design-

- Gain-8db
- Front to back ratio-20db
- Super directive or Super gain Antenna

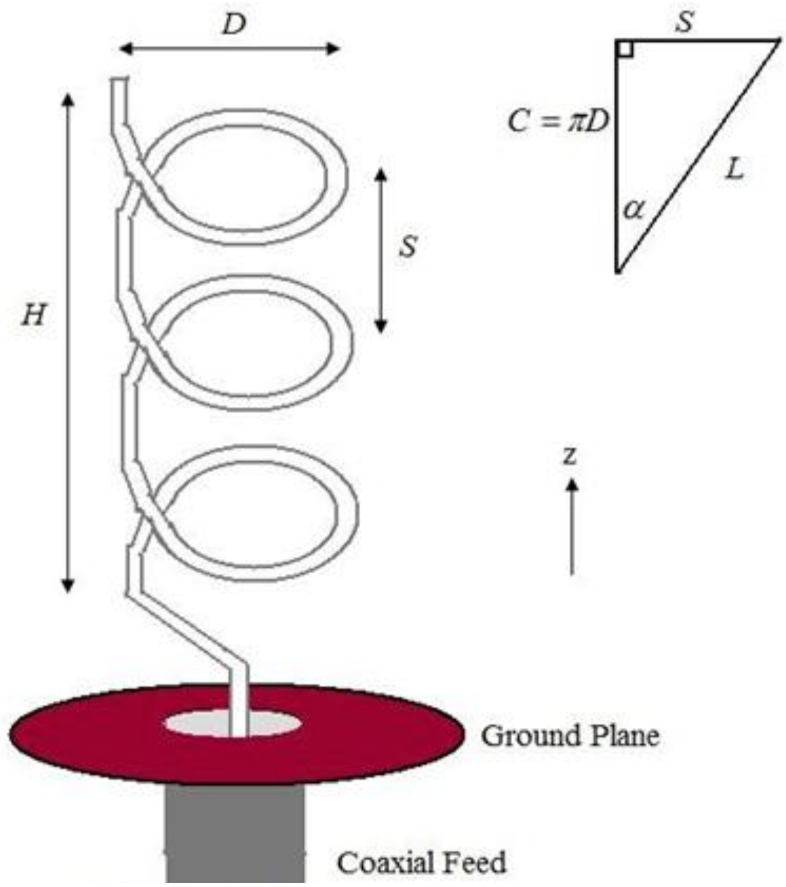
Helical Antenna:

It is another basic type of radiator,It is the simplest antenna to provide circularly polarized waves or nearly,Used in extra terrestrial communications in which satellite relays etc,Helical antenna to provide circular polarization characteristics

Helical Geometry:

It consists of a helix of thick copper code or tubing wound in the shape of screw thread and used as an antenna in conjunction with a flat metal plate called a ground plane,It is fed between one end and a ground plane ,The ground plane is simply made of sheet or of screen or of radial & Concentra conductor,The helix is fed by co-axial cable C=circumference of helix πD

- D=Diameter
- α=pitch angle=$\tan^{-1}$S/D
- d= diameter of helix conductor
- A=Axial length
- N=Number of one turns
- L=Length of one turn
- l=spacing of helix from the ground plane
- D=Diameter of one the helix turn
- S=Distance between turns (turn spacing)



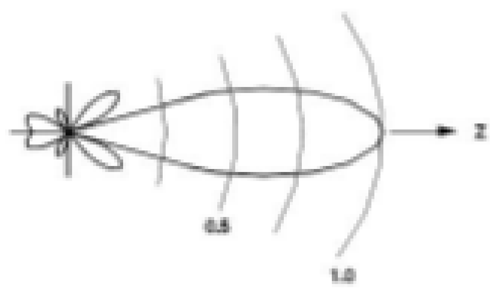


Fig:Radiation pattern of helical antenna

In general, the one end of the helix is connected to the centre conductor of the cable and the outer conductor is connected to the ground plane The parameters on which the mode of radiation depend are the diameter of helix ‘D’ & turn spacing For N turns of helix, the total length of the antenna is equal to NS & Circumference πd,If one turn of helix is unrolled on a plane surface, the circumference (πD), Spacing S, turn length ’l’ & pitch angle α are related by triangle

$$L = \sqrt{S^2 + C^2} = \sqrt{S^2 + (\pi D)^2}$$

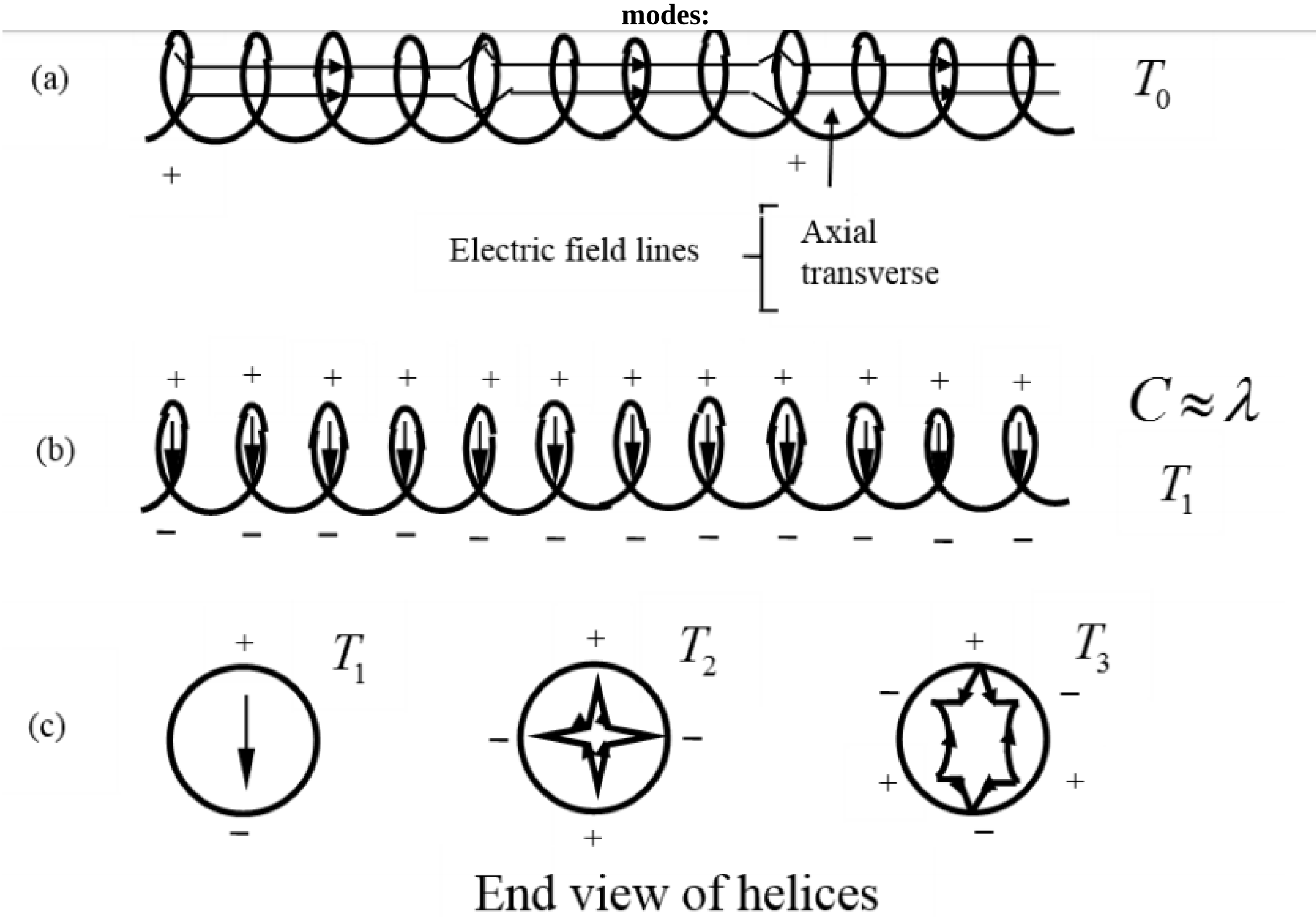
Pitch angle ‘α’ is the angle between a time tangent to the helix wire and the plane normal to the helix axis, it can be calculated from the triangle.

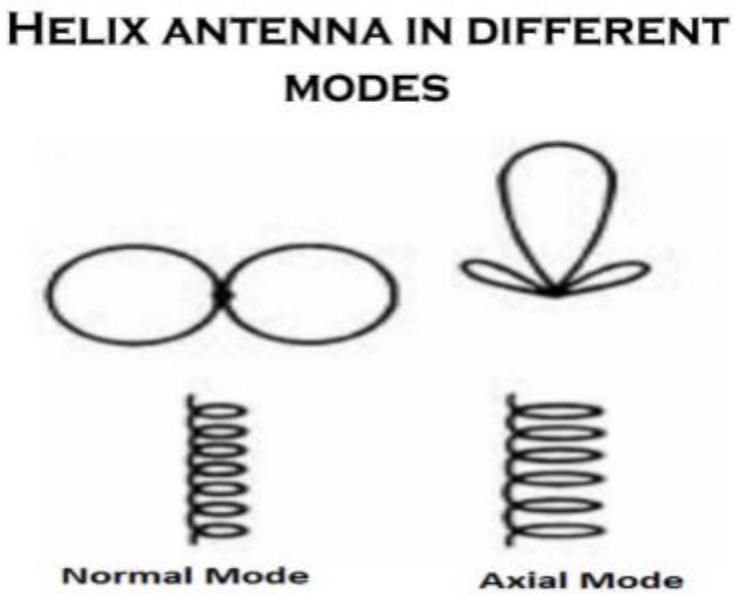
$$\tan\alpha = \frac{s}{c} = \frac{s}{\pi D}$$

$$\alpha = \tan^{-1}\left(\frac{s}{\pi D}\right)$$

So the properties of helical antenna can be described in terms of these geometric parameters,The different radiation characteristics are obtained by charging there parameters in relate on to wave length

Helix modes:$T_X$ion modes:





Normal mode of radiation: (perpendicular)

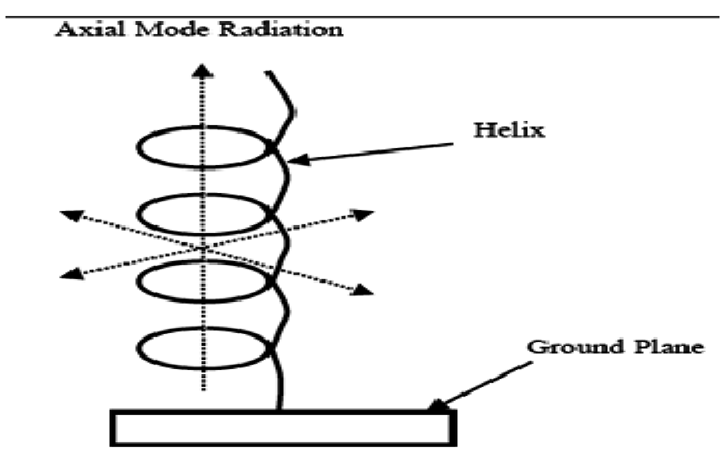
In this, the radiation field is maximum in broad way i.e in the direction normal to the helix axis (2) this mode of radiation is obtained if the dimensions of the helix is small compared with wave length i.e N<<<λ

Bandwidth of such a small helix is very narrow & radiation effiency is low

The between & rad effiency can be increased by increasing the size of helix & to have the current in phase along the helix axis

Phase shifted may be required to limit the practical situations

Radiation pattern is a combination of the equivalent radiation from a short dipole positioned on the same helix & a small loop which is also co-axial with the helix axis.



The helix become linear antenna suppose for a helix of fixed diameter

- If s→ 0 helix collapses to a loop
- If s→ constant D→0 linear conductor (S.D)
- Loop & linear antennas are limiting cases of the helix.

Radiation patterns of these two equivalent radiators are same.Polarizations are at right angles & the phase angle at any point in space is at 90^0 apart.Hence the resultant field is either circularly polarized or elliptically polarized depending upon the field strength ratio or the amplitudes of the two components (which in turn depends on the pitch angle α)

- α→ small loop radiation predominates
- α→ large → dipole radiation predominates

For the middle values of the 'α' the polarization is circular at one value of α. the polarization is elliptical at one value of α.A helical antenna may be considered of having a no.of small loops & short dipoles connected in series

- Loop diameter → helix diameter
- Dipole length → helix spacing

Then far field of the small loop is given by

$$E_\phi = \frac{120\pi^2[I]\sin\theta}{r}\frac{A}{\lambda^2} \qquad A = \frac{\pi D^2}{4}$$

Far field of a short dipole is given by

$$D = 2R \qquad A = \frac{\pi\left(2r\right)^2}{4} \qquad A = \pi r^2$$

$$E_\theta = \frac{j60\pi^2[I]\sin\theta}{r}\frac{L}{\lambda}$$

Length of the dipole there is 90^0 phase shift between them due to presence of j operator,The ration of magnitudes of these equations provides axial ratio (AR) of elliptical polarization

$$AR = \frac{E_\theta}{E_\phi} = \frac{\frac{j60\pi^2[I]\sin\theta}{r}S}{\frac{120\pi^2[I]\sin\theta}{r}\frac{A}{\lambda^2}}$$

$$AR = \frac{S\lambda}{2\pi A} = \frac{2S\lambda}{\pi^2 D^2}$$

AR=0→ Linear horizontal polarization
AR=α→ Linear vertical polarization
AR=1→ circular polarization

$$AR = 1\left|\frac{E_\theta}{E_\phi}\right| or \left|E_\theta\right| = \left|E_\phi\right|$$

$$2S\lambda = \pi^2 D^2$$

$$S = \frac{\pi^2 D^2}{2\lambda} = \frac{\left(\pi D\right)^2}{2\lambda} \rightarrow \frac{C^2}{2\lambda}$$

$$S = \frac{C^2}{2\lambda}$$

$$\alpha = \tan^{-1}\frac{S}{\pi D} = \tan^{-1}\frac{C^2}{2\lambda\pi D} = \tan^{-1}\frac{\pi^2 D^2}{2\lambda\pi D}$$

$$\alpha = \tan^{-1}\frac{\pi D}{2\lambda} = \tan^{-1}\frac{C}{2\lambda}$$

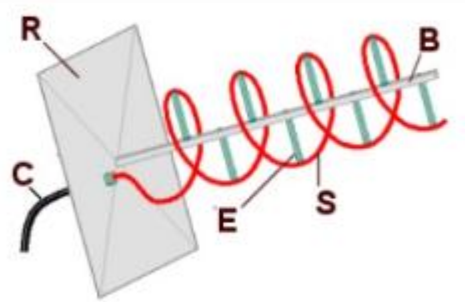
$$\alpha = \tan^{-1}\frac{C}{2\lambda}$$

This is the condition for pitch angle to get circular polarization

Very narrow Bandwidth Radiation efficiency is very small. Practically this made of operation is very rarely used Practical design considerations of monofilar helical

Axial or beam mode of radiation:

In this, the radiation field is maximum in the end fire direction i.e along the helix axis,Polarization is circular or nearly circular this mode occurs when the helix circumference and spacing‘s’ are of the order of one wave length (=λ) It produces a fairly directional beam in the axial direction with minor lobes at oblique angles,Mostly used in practical applications



The helix is operated in conjunction with a ground plane & is fed by a coaxial cableα varies from 12^0 to 18^0 & 14^0 is optimum pitch angle The antenna gain & beam width depends upon the helix length (NS),The terminal impedance is 100Ω at frequency C=λ R=140C/λ

Summary of Empirical Relation for Radiation properties of axial mode helix:

- $\alpha = 12^0 + 18^0$
- $c = \frac{3}{4}\lambda to \frac{4}{3}\lambda$
- $N = 3 to 15$
- Wire diameter‘d’ – negligible
- Ground plane diameter – 1/2λ

In 3-dimensuional spherical co-ordinate with θ=0 axis coincident with helix axis ,The pattern does not depend on angle ø

$$\therefore (HPBW)_{\theta 3db} = \frac{52}{C}\sqrt{\frac{\lambda^3}{NS}}$$

$$S = L\sin\alpha \qquad S = C\tan\alpha$$

$$BWFN = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}$$

$$D = \frac{15NSC^2}{\lambda^3}$$

$$AR = 1 + \frac{1}{2W}$$

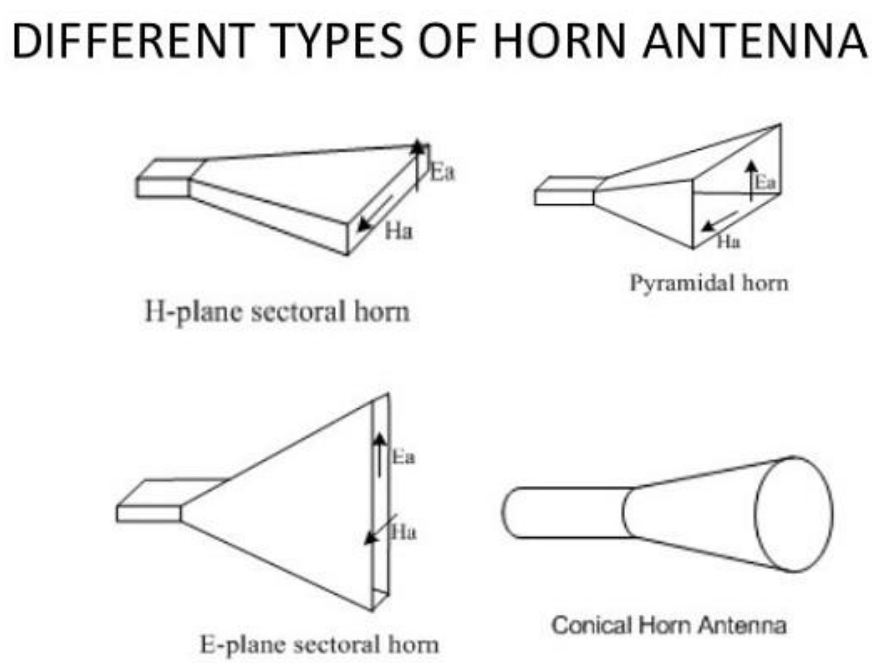
Horn Antenna:

A horn antenna may be regarded as a flared out or opened out waveguide,a wave guide is capable of radiating radiation into open space provided the same is excited at one end and opened at the other end The radiation is much greater through wave guide than the 2-wire t_x'ion lineIn wave guide, a small portion of the incident wave is radiated and the large portion is reflected back by the open cktThe open ckt is a discontinuity which matches the wave guide to space very poorly

To overcome these difficulties, the mouth of the waveguide is opened out which assume the shape of electromagnetic horn, just like a opened out t_x'ion line which gives a dipole,If the waveguide is terminated by any type of horn, the abrupt discontinuity existed is replaced by a gradual transformation

Then all the energy incident in forward direction in the waveguide will now be radiated (provided the impedance matching is proper),This improves directivity and reduces diffract a (diffraction around the edge will provide a poor radiation)

Types of horn antennas:



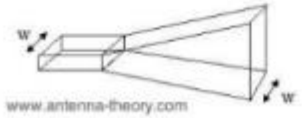


Figure 1. E-plane horn antenna.

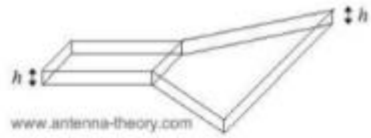


Figure 2. H-Plane horn antenna.

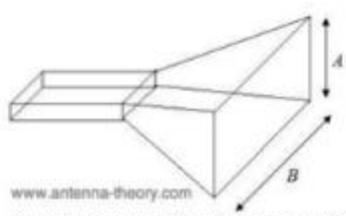
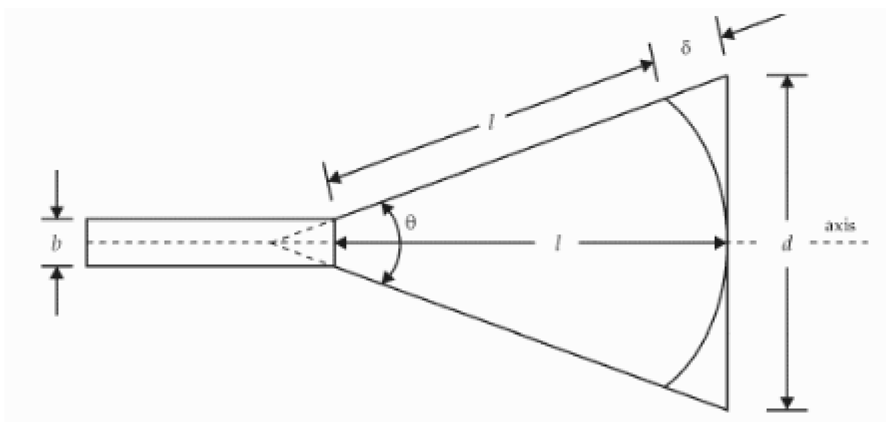


Figure 3. Pyramidal horn antenna

If flaring is done only in one direction, then sectional horn is produced, flaring in the direction of electric vector & the magnetic vector, the Sect oral E-plane horn & sectional 4-plane horn are obtained. If flaring is done along the both the walls (E&H) of the rectangular waveguide, then pyramidal horn is obtained.By flaring the walls of circular wave guide a conical horn is formed.

The principle of equality of path length (Fermat's principle):



This principle is applicable to horn design but with a different emphasis instead of requiring a constant phase across the horn mouth, the phase may deviate but by less than a specified amount 's' equal to path length difference between a ray travelling along the side and along the axis of the horn

$$\cos \theta/2 = \frac{L}{L+8}$$

$$\sin \theta/2 = \frac{a}{2(L+8)}$$

$$\tan \theta/_2 = \frac{a}{2L}$$

δ= path length difference
l= horn length (m)
a=aperture (m)
a_E for 'E' plane a_H for H-plane
θ= flare angle θ_E for 'E' plane
θ_H for H-plane
From the Geometry

Axial Length: $L=a^2/88$ (δ<<L)
Flare angle: $\theta=2\tan^{-1} a/2L = 2\cos^{-1}(L/L+8)$

$$(L+\delta)^2 = \left(\frac{a}{2}\right)^2 + L^2$$

$$L^2 + 2L8 + 8^2 = \frac{a^2}{4} + L^2 \quad \text{Neglected}$$

$$L = \frac{a^2}{88}$$

Optimum horn:
To obtain uniform horn, aperture distribution as possible, a very long horn with a small flere angle is required, But, for practical convenience the horn should be as short as possible,An optimum horn is between these extremes and has the minimum beam width, If flare angle is very high then a path

$$D = \frac{4\pi A_e}{\lambda^2} \quad \theta\uparrow \quad A\uparrow \quad D\uparrow$$

θ↓ A↓ D↓
'θ' value should be selected such that D is mix

Horn Antenna Design considerations (optimum pyramidal norm):
Horns are most widely used microwave antennas wireless communications primary antenna for electromagnetic sensing parabolic reflector R_F heating, Bio medicine.The horn antenna may be considered as an R_F transformer or impedance match between the wave guide feeder & free space

Impedance consideration:
Impedance matching is very desirable in com's standing waves increase the loss Suppose a wave guide without a horn in operation, the sudden interface of the conductive walls or free air, there may be abrupt change in impedance at the interface
This often results in reflections, losses and standing waves. When the flare angle becomes two large as it tends to 90^0, so there is no horn, resulting losses, reflections & standing wavesIn design there is an optimum flare angle for different horn types by using this horn we can overcome all difficulties,Such type of horn is considered as optimum horn

Aperture & Slant length considerations:
To realize design an optimum pyramidal horn, the width of the aperture is in either the E-field or it-field direction is dependent on the intended wavelength & the slant length of the aperture

$$\theta_E = \sqrt{2\lambda L_E}$$

$$a_H = \sqrt{2\lambda L_H}$$

a_E=width of the aperture in the E- field direction
L_E=slant length of the E-field direction
a_H = width
L_H=
λ=

To get an optimum conical horn, the diameter of the cylindrical horn aperture is dependent on the slant length of the cone from the approximate

$d = \sqrt{3}L$ d=diameter, L=slant

The between for practical horn antennas can be of the order of 20:1 for instance operating from 1GHZ-20 GHZ,The gain G of pyramidal horn antenna is the ratio of the power intensity along its beam axis to the intensity of an isotropic antenna with the same i/p power

G→ For pyramidal horn

$$G = \frac{4\pi A}{\lambda^2 a_e} \qquad G = \left(\frac{\pi d}{\lambda}\right)^2 a_e$$

$$a_e = 0.511 \qquad a_e = 0.522$$

Frequency considerations:

For any waveguide to be operational at intended frequency
The horn low cut off frequency

$$\lambda LC = 3412r \Rightarrow \lambda_{LC} = 3.412r$$

LC → low cut off wavelength
F=C/λ

λ_{LC} (mm)=1.706xbase length (mm)
λ_{HC}= 1.3065x base length (mm)

Directivity:

Assuming no loss D in terms of efficiency aperture is given by $D = \frac{4\pi A_e}{\lambda^2}$

Aperture efficiency $\varepsilon_{AP} = \frac{A_e}{A_p}$

$$A_e = \varepsilon_{AP} A_p$$

$$D = \frac{4\pi\varepsilon_{AP} A_p}{\lambda^e}$$

$$if\, \lambda \cong 1m \qquad \varepsilon_{AP} = 0.6$$

$$D = 7.5_{AP} \qquad \Rightarrow D = \frac{7.5A_p}{\lambda^e}$$

$$D_{d\beta} = 10\log_{10}(7.5A_P)$$

$$G = \frac{4.5A_P}{\lambda^2}$$

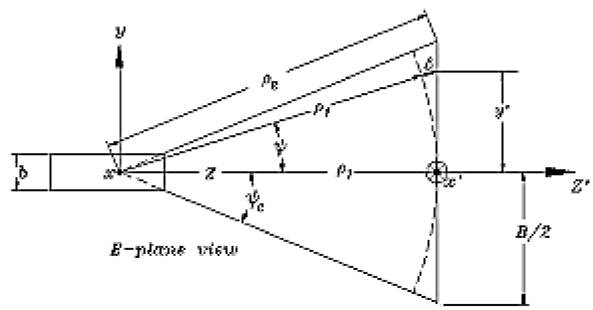
For rectangular w/G

$$D_{d\beta} = 10\log_{10}(7.5)(a_E \times a_H)$$

For circular w/G

$$D_{d\beta} = 10\log_{10}(7.5\pi r^2)$$

Operation of horn antenna:



The EM wave propagation in a waveguide is same as that in free space but it is restricted by the conducting walls of waveguide from being spherically spreading,On reaching the waveguide mouth these propagating fields spread laterally and wave front becomes spherical according to Huygens's principle, Near the mouth of the wave guide there exit transition region. It is the region where change of propagation takes place i.e from wave guide to free space

The flaring provides impedance matching, high radiation efficiency, high directivity, Horn produces a uniform phase spherical wave front with a larger aperture in comparison to a waveguide and thus the directivity is greater.

(1) calculate the directivity of 20 turn helix, having $\alpha=12^0$ circumference equal to one wave length
sol: The directivity of a helical antenna is given by

$$D = \frac{15NSC^2}{\lambda^3} \qquad N=20turns \quad C=\lambda \quad \alpha=12^0$$

$$\tan\alpha = \frac{s}{c} \qquad S=c\tan\alpha \quad S=\lambda\tan 12^0 \quad S=0.2126\lambda$$

$$D = \frac{15\times 20\times 0.2126\lambda\times\lambda^2}{\lambda^3}$$

$$D = 63.78$$

$$Dind\beta = 10\log D = 10\log 63.78$$

$$D = 18.046$$

(2) Find out the length L_L width W & half flare which the mouth height $h=10\lambda$. The horn is fed by a rectangular w.g with TE_{10} mode.

$$\theta = \tan^{-1}\frac{h}{2L} = \cos^{-1} = \frac{L}{L+8}$$

$$L = \frac{h^2}{88} \qquad h = bx$$

$L_1\, W_1\, \theta_E$ & θ_H --?

Typical values

$$\delta_E = 0.2\lambda in'E'plane$$

$$\delta_H = 0.375\lambda in'H'plane$$

$$L = \frac{10\lambda^2}{8\times 0.2\lambda} = \frac{100}{1.6} = 62.5\lambda$$

$$\theta_E = \tan^{-1}\frac{h}{2L} = \tan^{-1} = \frac{10\lambda}{2\times 62.5\lambda} = 4.6^0$$

$$\theta_H = \cos^{-1}\frac{L}{L+8} = \cos^{-1} = \frac{62.5}{62.5+0.375} = 6.3^0$$

$$\tan\theta_H = \frac{W}{2L} \qquad W = 2L\tan\theta_H$$

$$W = 2(62.5\lambda)\tan^{-1}6.3^0 = 13.8\lambda$$

(3) Find out the power gain in dB of paraboloidal reflector of open mouth aperture 10λ

$$G_P = 6\left(\frac{D}{X}\right)^2 = 600$$

$$= 10\log_w GP = 27.8db$$

(4) Find out the beam width between first nulls & power gain of a 2m paraboloidal reflector operating at 6000MHz

$$BWFN = \frac{140\lambda}{D} \,\&\, G_P = 6\left(\frac{D}{\lambda}\right)^2$$

$$D = 2m \qquad \lambda = \frac{300}{6000} = \frac{1}{20}m$$

$$BWFN = \frac{140\times 1}{2\times 20} = 3.5^0$$

$$G_P = 39.12dB = 9600$$

(5) A parabolic dish provides a gain of 75db at a frequency of 15GHz. Calculate capture area of the antenna its 3Db & null beam width

$G_P$db=7.5db f=15GHz

$$A_0 = KA = 0.65\left(\frac{\pi D^2}{4}\right)$$

$$G_{db} = 75 = 10\log_{10} GP$$

$$\log_{10} G_{db} = \frac{75}{10} = 7.5$$

$$G_p = 3162\times 10^4$$

$$G_p = 6\left(\frac{D}{\lambda}\right)^2 \qquad \frac{D}{\lambda} = \sqrt{\frac{3162}{6}\times 10^4}$$

$$\frac{D}{\lambda} = 2295 \qquad \lambda = \frac{300}{15000} = \frac{1}{50}m$$

$$D = 2295\times\frac{1}{50} = 45.9m$$

$$A = \frac{\pi D^2}{4} = 1653.8m^2$$

$$A_0 = E_A = 0.65\times 1653.8 = 1074.9 = 1075m^2$$

$$HPBW = \frac{78\lambda}{D}$$

(6) Design an optimum horn antenna with mouth height h=20λ & path difference δ=0.2λ, δ_E=0.2λ, δ_H=0.375λ Find L

Sol: given h=20λ

δ=0.2λ

$$l = \frac{h^2}{88} = \frac{(20\lambda)^2}{8\times 0.2\lambda} = 250\lambda$$

$$\theta_E = \cos^{-1}\frac{L}{L+8} = \cos^{-1}\frac{250\lambda}{250.2\lambda} = 2.29^0$$

$$\theta_H = \tan^{-1}\frac{h}{2L} = \tan^{-1}\frac{20\lambda}{2\times 25\phi\lambda}$$

Find out the length, width, & flare angles θ_E & θ_H of pyramidal horn antenna for which the mouth height is h=10λ The horn is fed by a rectangular wave guide with $T\beta_{10}$

h=10λ

$$L_E = \frac{h^2}{88_E} = \frac{100\lambda^2}{8\times 0.20\lambda} = 62.5\lambda$$

$$L_H = \frac{h^2}{88_H} = \frac{100\lambda^2}{8\times 0.375\lambda} = 33.33\lambda$$

$$\theta_H = \tan^{-1}\frac{h}{2L} = \tan^{-1}\frac{10\lambda}{2\times 62.5\lambda} = 4.57^0$$

UNIT – III
VHF,UHF AND MICROWAVE ANTENNAS

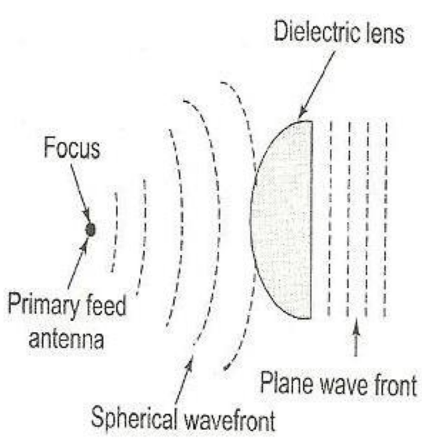
Introduction:

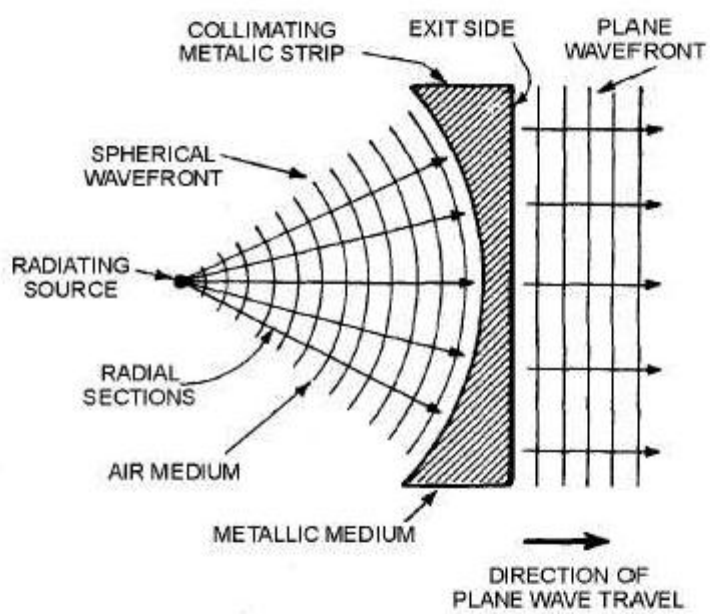
Lens Antennas:

Lens antennas may be divided into two types:

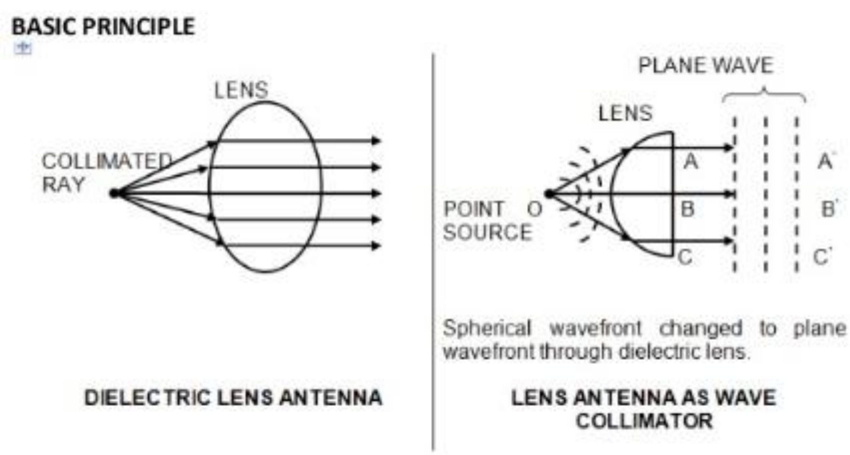
1) delay lenses – in which the electrical path length is increased by the lens medium
2) Fast lenses in which the electrical path length is decreased by the lens medium

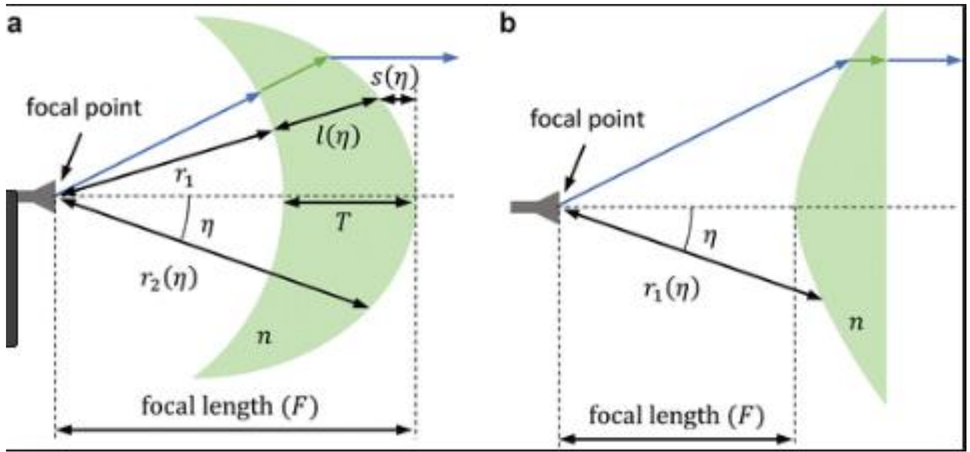
E-plane metal plate lenses are of the fast type.Dietetics lens sub divided intoLenses constructed of non-metallic, dielectrics, such as Lucite or polystyrene,Lenses constructed of metallic or artificial dielectrics





Geometry of Non-metallic Dielectric Lenses:





Let us determine the shape of the plane convex lens of fig shown above for transformation the spherical wave front from an isotropic point source or primary antenna into a plane wave frontA wave front is defined as a surface on all points of which the field is in the same phase,The field over the plane surface can be made everywhere in phase by shaping the lens.

So that all paths from the source to the plane are of equal electrical path length,This is the principle of equality of electrical path length (optical) (Fermat's principle),The electrical length of the path opposite must equal the electrical length of path $0QQ^1$

$$OPP^1=OQQ^{11}$$
$$OP^1= OQ^{11}$$
$$OQ=L \ \& \ OP=R$$
$$OP=OQ+QQ^1$$
$$\frac{R}{\lambda_0}=\frac{L}{\lambda_0}+\frac{x}{\lambda_d}$$
$$R=L+\left(\frac{\lambda_0}{\lambda_d}\right)x$$
$$R=L+\frac{\lambda_0}{\lambda_d}\left(R\cos\theta-L\right)$$
$$R=L+n\left(R\cos\theta-L\right)$$

In general $n=\frac{\lambda_0}{\lambda_d}=\frac{f\lambda_0}{f\lambda_d}=\frac{v_0}{v_d}=\frac{\sqrt{\mu\varepsilon}}{\mu_0\varepsilon_0}$

λ_0=Wave length in free space

λ_d= Wave length in the lens

$n=\frac{\lambda_0}{\lambda_d}$ =index of refraction

But $x=OQ^1-OQ$

$X=R\cos\theta-L$

f=frequency Hz

v_0=velocity in free space m/s

v_d =velocity in dielectric m/s

μ = permeability of the dielectric medium H/m

ε = permittivity of the dielectric medium

$$\mu_0 = 4\pi\times10^{-7} H/m \qquad \varepsilon_0 = 8.854\times10^{-12} F/m$$

$$R = L + nR\cos\theta - nL$$

$$R - nR\cos\theta = L(1-n)$$

$$R(1-n\cos\theta) = L(1-n)$$

$$R = \frac{L(1-n)}{(1-n\cos\theta)} \text{ Where n>1}$$

This equation gives the required shape of the lens. It is the equation of a hyperbola. Whose focal length is 'L' and radius of curvature (R)

R=L (n-1) provided 'θ' is small

The asymptote of the hyperbola is at '0' eq (1) implies that as long as θ is small the hyperbola lens can be replaced by a plano concave spherical lens of radius R=L (n-1) this is also the optical formula

Zoning:

The weight of the lens can be reduced by removing sections of Lens, which is called zoning of Lens,

Classification:

Curved surface zoning & plane surface zoning:In general the zoning of lens is carried out in such a way that particular design frequency the performance of lens antenna is not affected The zone step is denoted by 'z'so in the zoned lens antenna, the thickness 'z' of the lens antenna is such that the electrical length of the thickness Z in dielectric is an integral length of x longer that in air

That means 'z' in dielectric may be 3λd & that in air is $2\lambda_0$ where λ_d & λ_0 are the wave length in the dielectric & air respectively

For 1λ difference

$$\frac{z}{\lambda_d} = \frac{z}{\lambda_0} = 1$$

Refractive $n = \frac{z}{\lambda_0}$

$$\frac{z}{(\lambda_0/n)} - \frac{z}{\lambda_0} = 1$$

$$\frac{(n-1)z}{\lambda_0} = 1 \; z = \frac{\lambda_0}{(n-1)}$$

Curved surface zoning	**Plane surface zoning**
Zoning is done along the curved surface of lens	Zoning is done along the plane surface of lens
Mechanically stronger than plane surface zoning	Mechanically weaker than curved surface zoning
It has less weight	Comparatively bulker
Less power dissipation	power dissipation is more

Tolerances:

One can distinguish three different kinds of phase variation of the current distribution J(X) across the antenna aperture:

1) A linear phase variation along some direction across the aperture. That results only in a tilt of the main beam direction.

2) A quadratic phase variation across the aperture. Such phase error can be avoided by properly focusing the telescope.

3) The theoretical shape of the reflector can be approached up to a certain ε_{rms} limit (rms) of tolerance.

The phase error associated to the fabrication tolerances is: $\delta(x) = 4\pi\varepsilon(x)/\lambda$ and correspondingly the current grading can be written: $g(x) = g_0(x)e^{i\delta(x)}$ Where $g_0(x)$ is a real function, representing the ideal grading with no phase errors.

In terms of the current grading the directivity gain can be written:

$$G(n) = \frac{4\pi}{\lambda^2}\frac{\left|\int_A g_0(x)e^{-i[kn.x-\delta(X)]}dxdy\right|^2}{\int A g^2{}_0(x)dxdy}$$

Assuming:

δ (X) << 1 and δ (X) is randomly distributed

Then one can write: $e^{i\delta} \approx 1 + i\delta - \frac{1}{2}\delta^2$

$$\bar{\delta} = \frac{\int_A g_0(x)\delta(x)dxdy}{\int A g_0(x)dxdy} = 0$$

Using these approximations, the ratio between the gain with phase errors, G and the ideal gain, G_0, can be written as

$$\frac{G}{G_0} = 1 - \bar{\delta}^2 \approx 1 - 16\pi^2 \frac{\varepsilon_{rms}{}^2}{\lambda^2}$$

Where $$\bar{\delta}^2 = \frac{\int_A g_0(x)\delta^2(x)dxdy}{\int A g_0(x)dxdy}$$

In practical applications

1) $\delta \geq 1 \, for \lambda \rightarrow \varepsilon_{rms}$

2) δ (x) is not randomly distributed if at some point δ<0 ,it is likely that δ<0 in an area surrounding this point.

A correlation distance d for the phase error has to be introduced,Ruse (1952, 1966) has developed the theory of antenna gain for such a case

His results can be summarized with the following expression for the gain

$$G(u) = \eta A e^{-\bar{\delta}^2}\left(\frac{\pi D}{\lambda}\right)^2 A_1^2(\pi u D/\lambda) + \left(1 - e^{-\bar{\delta}^2}\right)\left(\frac{2\pi d}{\lambda}\right)^2 A_1^2(2\pi u d/\lambda)$$

Where η_A is the aperture efficiency

Applications:

They are used as feeders (called feed horn) for larger antenna,Structures such as parabolic antenna, as directive antenna for such device as radar guns, automatic doors openers, micro wave radiometer,A common element of phase array,satellite and microwave communications,Used in the calibration, other high gain antenna,Used for making electromagnetic interference measurement

Slot Antennas:

Slot antennas are useful in many applications, especially where low-profile or flush mountings are required as, for example, on high-speed aircraft. Any slot has its complementary form in wires or strips, so that pattern and impedance data of these forms can be used to predict the patterns and impedances of the corresponding slots.

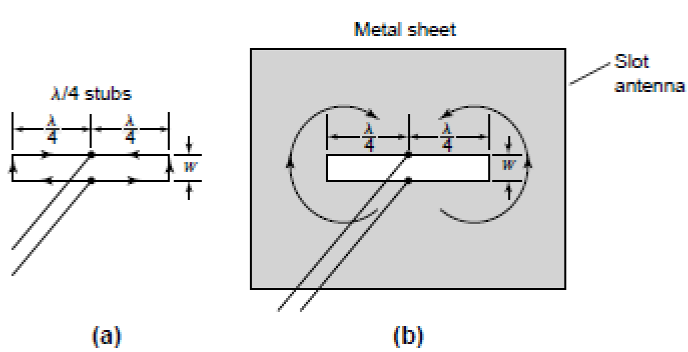
The discussion is based largely on a generalization and extension of Babinet's (Ba-bi-naý's) principle by Henry Booker (1). The antenna shown in Fig. 7–21a, consisting of two resonant λ/4 stubs connected to a 2-wire transmission line, is an inefficient radiator. The long wires are closely spaced (w<<λ) and carry currents of opposite phase so that their fields tend to cancel. The end wires carry currents in the same phase, but they are too short to radiate efficiently. Hence, enormous currents may be required to radiate appreciable amounts of power.

The antenna in Fig. 7–21b, on the other hand, is a very efficient radiator. In this arrangement a λ/2 slot is cut in a flat metal sheet.Although the width of the slot is small (w<<λ), the currents are not confined to the edges of the slot but spread out over the sheet.

This is a simple type of slot antenna. Radiation occurs equally from both sides of the sheet. If the slot is horizontal, as shown, the radiation normal to the sheet is vertically polarized.A slot antenna may be conveniently energized with a coaxial transmission line as in Fig. 7–22a. The outer conductor of the cable is bonded to the metal sheet.

Since the terminal resistance at the center of a resonant λ/2 slot in a large sheet is about 500Ω and the characteristic impedance of coaxial transmission lines is usually much less, an off-center feed such as shown in Fig. 7–22b may be used to provide a better impedance match.

For a 50Ω coaxial cable the distance s should be about λ/20. Slot antennas fed by a coaxial line in this manner are illustrated in Fig. 7–22c and d. The radiation normal to the sheet with the horizontal slot



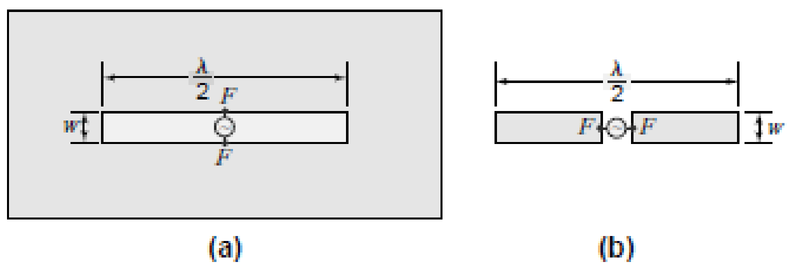
Patterns of Slot Antennas in Flat Sheets. Edge Diffraction:

Consider the horizontal λ/2 slot antenna of width w in a perfectly conducting flat sheet of infinite extent, as inFig.7–26a. The sheet is energized at the terminals FF. It has been postulated by Booker (1) that the radiation pattern of the slot is the same as that of the complementary horizontal λ/2 dipole consisting of a perfectly conducting flat strip of width w and energized at the terminals FF, as indicated in Fig. 7–26b, but with two differences.

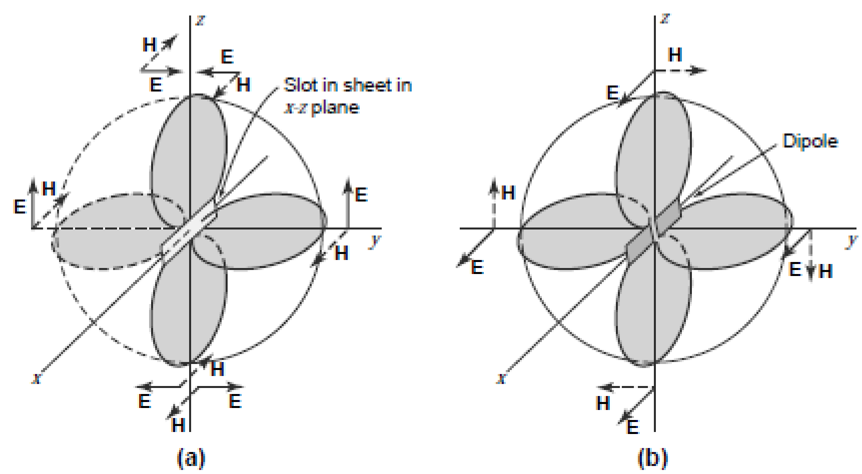
These are (1) that the electric and magnetic fields are interchanged and (2) that the component of the electric field of the slot normal to the sheet is discontinuous from one side of the sheet to the other, the direction of the field reversing.

The tangential component of the magnetic field is, likewise, discontinuous.The patterns of the λ/2 slot and the complementary dipole are compared in Fig.

The infinite flat sheet is coincident with the xz plane, and the long dimension of the slot is in the x direction (Fig. 7–27a).



A $\lambda/2$ slot in an infinite flat sheet (*a*) and a complementary $\lambda/2$ dipole antenna (*b*).



Radiation-field patterns of slot in an infinite sheet (*a*) and of complementary dipole antenna (*b*). The patterns have the same shape but with **E** and **H** interchanged.The complementary dipole is coincident with the *x* axis (Fig. 7–27*b*).

The radiation-field patterns have the same doughnut shape, as indicated, but the directions of **E** and **H** are interchanged. The solid arrows indicate the direction of the electric field **E** and the dashed arrows the direction of the magnetic field **H**.

If the *xy* plane is horizontal and the *z* axis vertical as in Fig. 7–27*a*, the radiation from the horizontal slotis vertically polarized everywhere in the *xy* plane. Turning the slot to a vertical position (coincident with the *z* axis) rotates the radiation pattern through 90◦ to the position shown in Fig. 7–28. The radiation in this case is everywhere horizontally polarized; i.e., the electric field has only an $E\varphi$ component.

If the slot is very thin
($w \ll \lambda$) and $\lambda/2$ long ($L = \lambda/2$), the variation of $E\varphi$ as a function of θ is given by

$$E_\phi(\theta) = \frac{\cos\left[(\pi/2)\cos\theta\right]}{\sin\theta}$$

Assuming that the sheet is perfectly conducting and infinite in extent, the magnitude of the field component $E\varphi$ remains constant as a function of φ for any value of θ.

Thus, $E\varphi\ (\varphi)$ = constant

Consider now the situation where the slot is cut in a sheet of finite extent as suggested by the dashed lines in Fig.This change produces relatively little effect on the $E\varphi(\theta)$ pattern given by (1). However there must be a drastic change in the $E\varphi(\varphi)$, Pattern since in the x direction, for example, the fields radiated from the two sides of the sheet are equal in magnitude but opposite in phase so that they cancel.

Hence, there is a null in all directions in the plane of the sheet. For a sheet of given length L in the x direction, the field pattern in the xy plane might then be as indicated by the solid curve in Fig. 7–29a. The dashed curve is for an infinite sheet ($L = \infty$). If one side of the slot is boxed in, there is radiation in the plane of the sheet as suggested by the pattern in Fig..1 With a finite sheet the pattern usually exhibits a scalloped or undulating characteristic, as suggested in Fig.

As the length L of the sheet is increased, the pattern undulations become more numerous but the magnitude of the undulations decreases, so that for a very large sheet the pattern conforms closely to a circular shapeMeasured patterns illustrating this effect are shown in Fig. 7–30 for 3 values of L.A method due to Andrew Alford for locating the angular positions of the maxima and minima is described by Dorne (1) and Lazarus.

In this method the assumption is made that the far field is produced by three sources (see Fig. 7–31), one (1) at the slot of strength 1 sin ωt and two (2 and 3) at the edges of the sheet (edge diffraction effect) with a strength k sin (ωt − δ), where k _ 1 and δ gives the phase difference of the edge sources with respect to the source (1) at the slot.

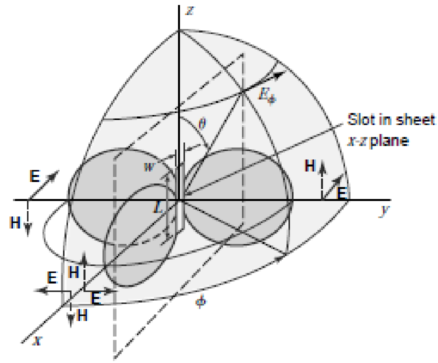
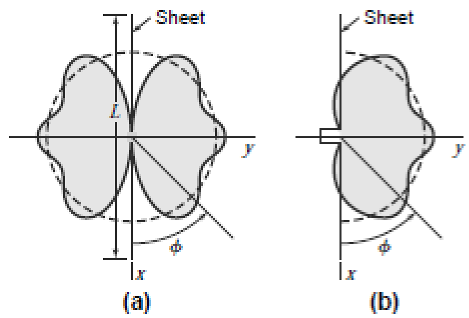


Figure 7–28 Radiation pattern of vertical slot in an infinite flat sheet.



Solid curves show patterns in xy plane of Fig. 7–26 for slot in finite sheet of length L. Slot is open on both sides in (a) and closed on left side in (b). Dashed curves show pattern for infinite sheet. All patterns idealized.

At the point P at a large distance in the direction φ, the relative field intensity is then

$$E = \sin \omega t + k \sin(\omega t - \delta - \varepsilon) + k \sin(\omega t - \delta + \varepsilon)$$

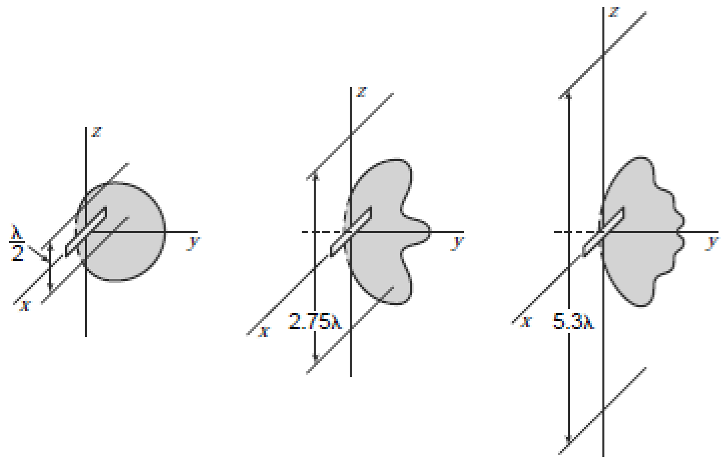
Where ε = (π/λ) L cos φ

By trigonometric expansion and rearrangement

$$E = (1 + 2k \cos\delta \cos\varepsilon) \sin(\omega t - \delta - \varepsilon) \sin \omega t - (2k \sin\delta \cos\varepsilon) \cos \omega t$$

and the modulus of E is

$$|E| = \sqrt{(1 + 2k \cos\delta \cos\varepsilon)^2 + (2k \sin\delta \cos\varepsilon)^2}$$



Babinet's Principle and Complementary Antennas:

By means of Babinet's (Ba-bi-naý's) principle many of the problems of slot antennas can be reduced to situations involving complementary linear antennas for which solutions have already been obtained. In optics Babinet's principle (Born-1) may be stated as follows:

The field at any point behind a plane having a screen, if added to the field at the same point when the complementary screen is substituted, is equal to the field when no screen is present.The principle may be illustrated by considering an example with 3 cases. Let a source and 2 imaginary planes, plane of screens *A* and plane of observation *B*, be arranged as in Fig.

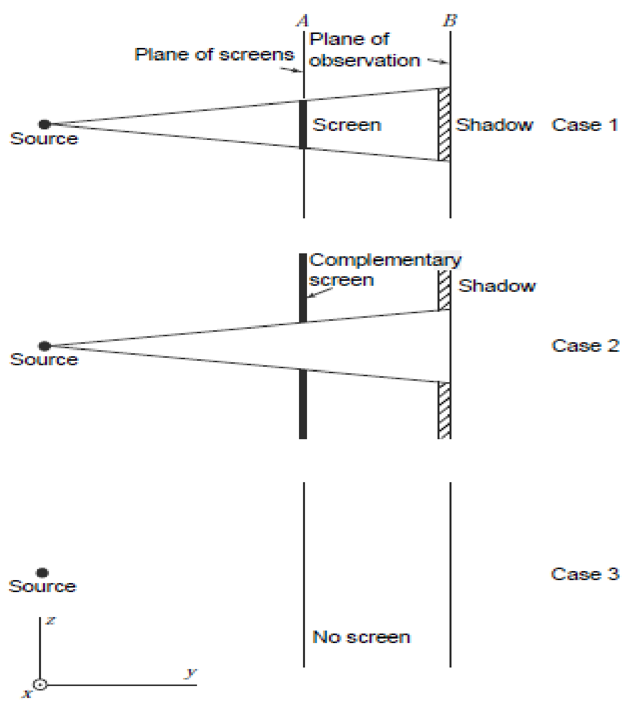


Figure 7–32 Illustration of Babinet's principle.

.

Thus,

$F_s = f_1(x, y, z)$ (1)

As Case 2 let the first screen be replaced by its complementary screen and the field behind it be given by

$F_{cs} = f_2(x, y, z)$ (2)

As Case 3 with no screen present the field is

$F_0 = f_3(x, y, z)$ (3)

Then Babinet's principle asserts that at the same point x_1, y_1, z_1,

$F_s + F_{cs} = F_0$

The source may be a point as in the above example or a distribution of sources. The principle applies not only to points in the plane of observation *B* as suggested in Fig. 7–32 but also to any point behind screen *A*.

Although the principle is obvious enough for the simple shadow case above, it also applies where diffraction is considered. Babinet's principle has been extended and generalized by Booker (1) to take into account the vector nature of the electromagnetic field.

In this extension it is assumed that the screen is plane, perfectly conducting and infinitesimally thin. Furthermore, if one screen is perfectly conducting ($\sigma = \infty$), the complementary screen must have infinite permeability ($\mu = \infty$).
Thus, if one screen is a perfect conductor of electricity, the complementary screen is a perfect "conductor" of magnetism.
No infinitely permeable material exists, but the equivalent effect may be obtained by making both the original and complementary screens of perfectly conducting material and interchanging electric and magnetic quantities everywhere.

The only perfect conductors are *superconductors* which soon may be available at ordinary temperatures for antenna applications.However, many metals, such as silver and copper, have such high conductivity that we may assume the conductivity is infinite with a negligible error in most applications.

As an illustration of Booker's extension of Babinet's principle, consider the cases in Fig. 7–33.

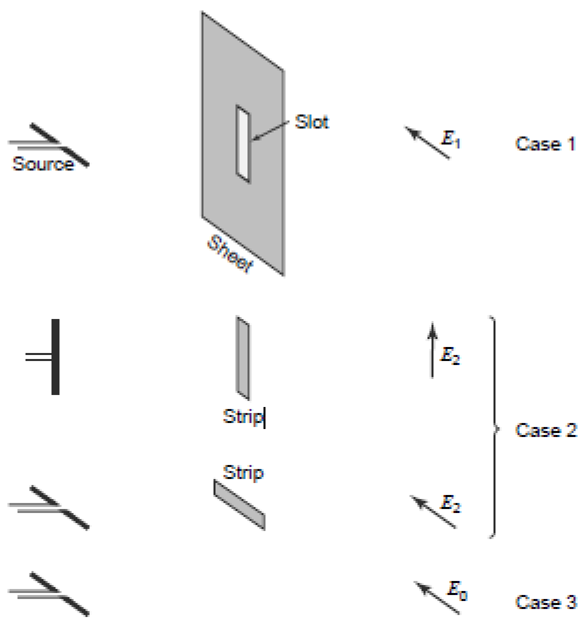


Illustration of Babinet's principle applied to a slot in an infinite metal sheet and the complementary metal strip. The source in all cases is a short dipole, theoretically an infinitesimal dipole.

In Case 1 the dipole is horizontal and the original screen is an infinite, perfectly conducting, plane, infinitesimally thin sheet with a vertical slot cut out as indicated. At a point *P* behind the screen the field is *E*1.

In Case 2 the original screen is replaced by the complementary screen consisting of a perfectly conducting, plane, infinitesimally thin strip of the same dimensions as the slot in the original screen. In addition, the dipole source is turned vertical so as to interchange **E** and **H**.

At the same point *P* behind the screen the field is *E*2. As an alternative situation for Case 2 the dipole source is horizontal and the strip is also turned horizontal. Finally, in Case 3 no screen is present and the field at point *P* is *E*0. Then, by Babinet's principle

E1 + E2 = E0 (5)

or

$$\frac{E_1}{E_0} + \frac{E_2}{E_0} = 1 \quad (6)$$

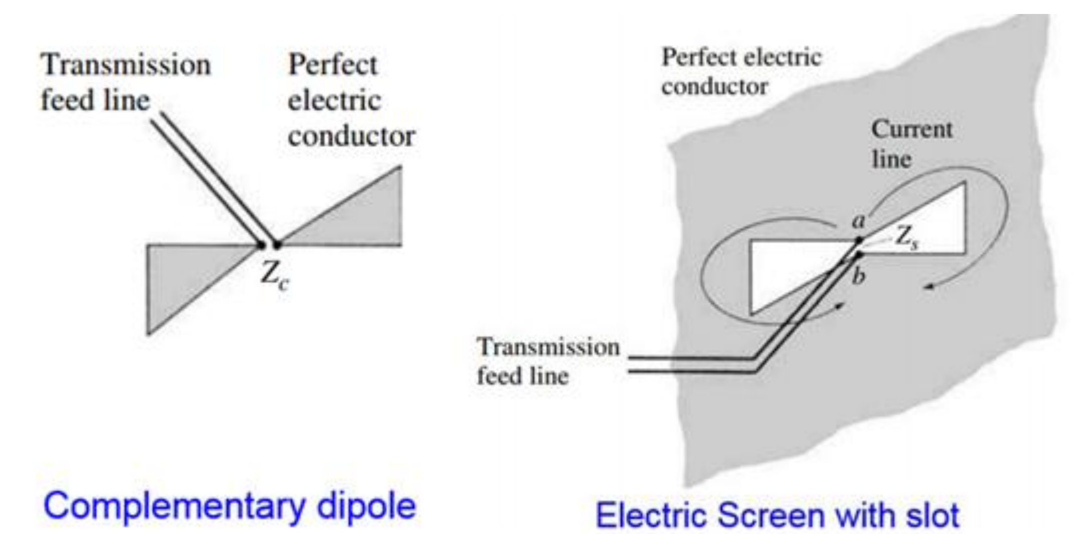
Babinet's principle may also be applied to points in front of the screens. In the situation of Case 1 (Fig. 7–33) a large amount of energy may be transmitted through the slot so that the field E1 may be about equal to the field E0 with no intermediate screen (Case 3).

In such a situation the complementary dipole acts like a reflector and E2 is very small. (See Chap. 13 on frequency-sensitive surfaces.) Since a metal sheet with a λ/2 slot or, in general, an orifice of at least 1λ perimeter may transmit considerable energy, slots or orifices of this size should be assiduously avoided in sheet reflectors such as described in Chap. 9 when **E** is not parallel to the slot.

Impedance of slot antennas:

If a electric screen (with slot) and its complement (strip dipole) are immersed in a medium with an intrinsic impedance η and have terminal impedance of Z_S and Z_C, respectively, the impedances are related by

$$Z_S Z_C = \frac{\eta^2}{4}$$



Far Field Electric and Magnetic Fields

$$E_{\theta S} = H_{\theta C}, E_{\phi S} = H_{\phi C}$$

$$H_{\theta S} = -\frac{E_{\theta C}}{\eta_0^{\ 2}}, H_{\phi S} = -\frac{E_{\phi C}}{\eta_0^{\ 2}}$$

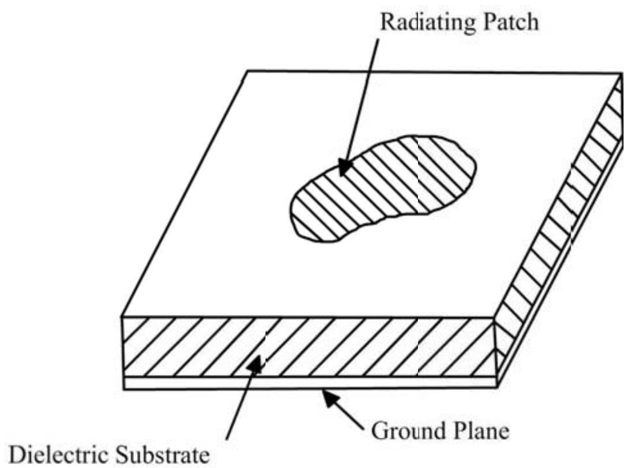
Radiation pattern of the slot is identical in shape to that of the dipole except that the E and H-fields are interchanged

Micro strip Antennas:- (patch antennas)

Is space craft or air craft applications, where size weight, cost, performance, ease of installation and Aerodynamic profile constraints, low profile antennas are required.In order to meet there specification micro strip or patch antennas are used.

There antennas can be mounted to metal or other existing surfaces & they require space for the feed line which is normally placed behind the ground planeMicro strip or patch antennas are popular for low profile applications at frequencies above 100 mHz also celled printed antennas.

Patch antennas can be directly printed on to a circuit board; there are becoming in creartygly popular within the mobile phone market.



Features of micro strip antennas:-

1) Basically a patch antenna is a metal patch suspended over a ground plane.
2) The assembly is usually contained in a plastic redone which protects

Radom: -

A dome or the structure protecting reder equipment & made from material transparent to radio waves, especially one on the outer surface of all aircraft .The structure form damage.Patch antennas are simple to fabricate & carry to modify

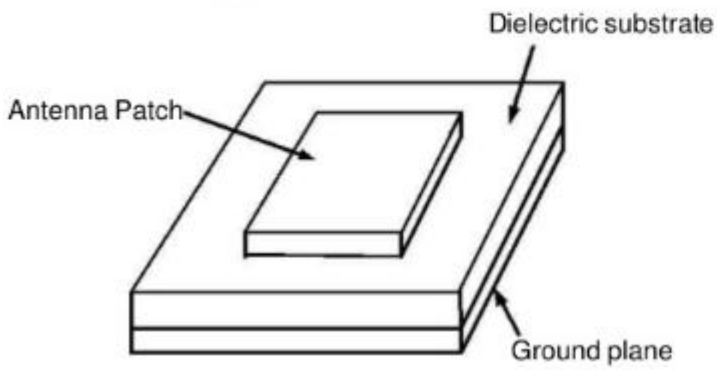
There are constructed on a dielectric substrate, by using to deprecate PCB'S,A micro strip patch antenna consists of radiating patch on one side of a dielectric sub state which has a ground plane on the other sideThe simplest patch antennas uses a $\lambda/2$ wave length long patch with a larger ground plane to give better performance. But at the cost of large antenna size,Ground plane > active patch

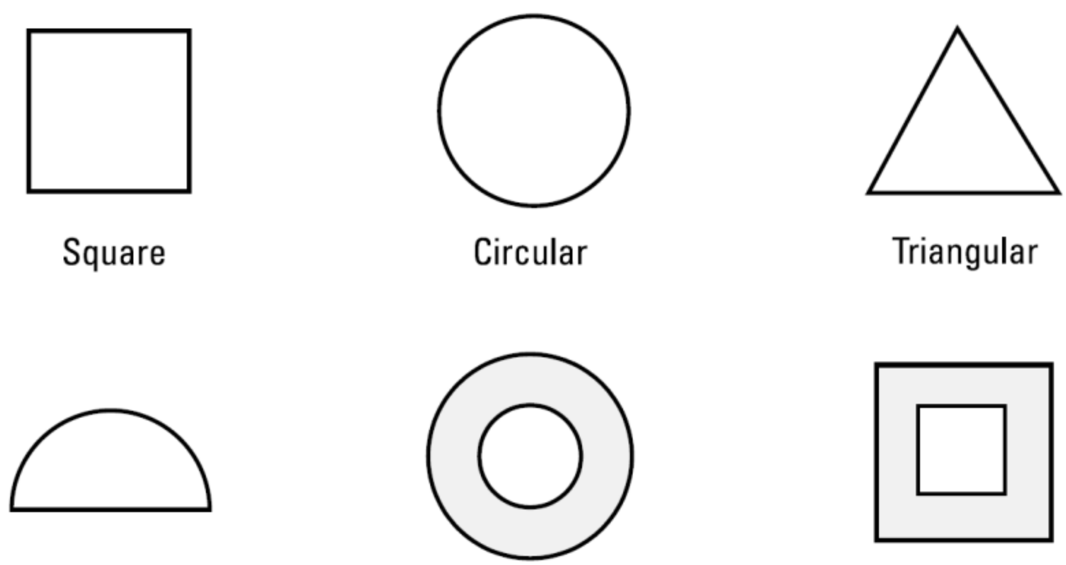
It is a narrow band, wide beam antenna fabricated by etching the antenna element pattern in metal trace bonded to the opposite side of the substrate which forms a ground plane,To get wide between, thick substrata is used

Types:

- Square → by using of regular shapes
- Rectangular → of well defined geometry
- Triangular → we can simplify the analysis
- elliptical→ &performance well be Circular rectangular & circular predicted we widely used.

Square patches are used to generate a pencil bean and rectangular patch for a fan beam,The size of micro strip antennas is inversely proportioned to its frequency,Ex:-for low freq tike am at 1 MHz filed patch size foot ball





The micro strip antennas is contracted on a this dielectric sheet using a printed circuit board & etching techniques, Alost common board is dual copper coated poly tietrafluoroethy lane

Advantages of micro strip antenna:

Micro strip antenna has several advantages compared to conventional microwave antenna ,These antennas are used in many applications over the broad frequency range from 100MHz to 50GHz.

Some of the principal advantages of these antennas are:

- Low weight, low cost, low profile and conformal
- Easy to fabricate and can be integrated with other micro strip components in monolithic application like RFIC and MMIC.
- The antenna can be easily mounted on missiles, rockets and satellite without major alterations.
- The antenna has low scattering cross section.
- Dual frequency antenna can be easily made.
- Micro strip antennas are compatible with modular designs (Solid state devices such as oscillators, amplifiers, variable attenuators, mixer, phase shifters etc. can be added directly to the antenna substrate board)

Disadvantages:

- Narrow bandwidth
- Radiation efficiency deteriorates as frequency and antenna array size increases due to an increase in the feeding network losses.
- Lower power handling capacity
- Poor isolation between the feed and the radiating elements

In recent years, with the advancement of technology, efforts have been made to minimize these effects dramatically.

Applications of MSA

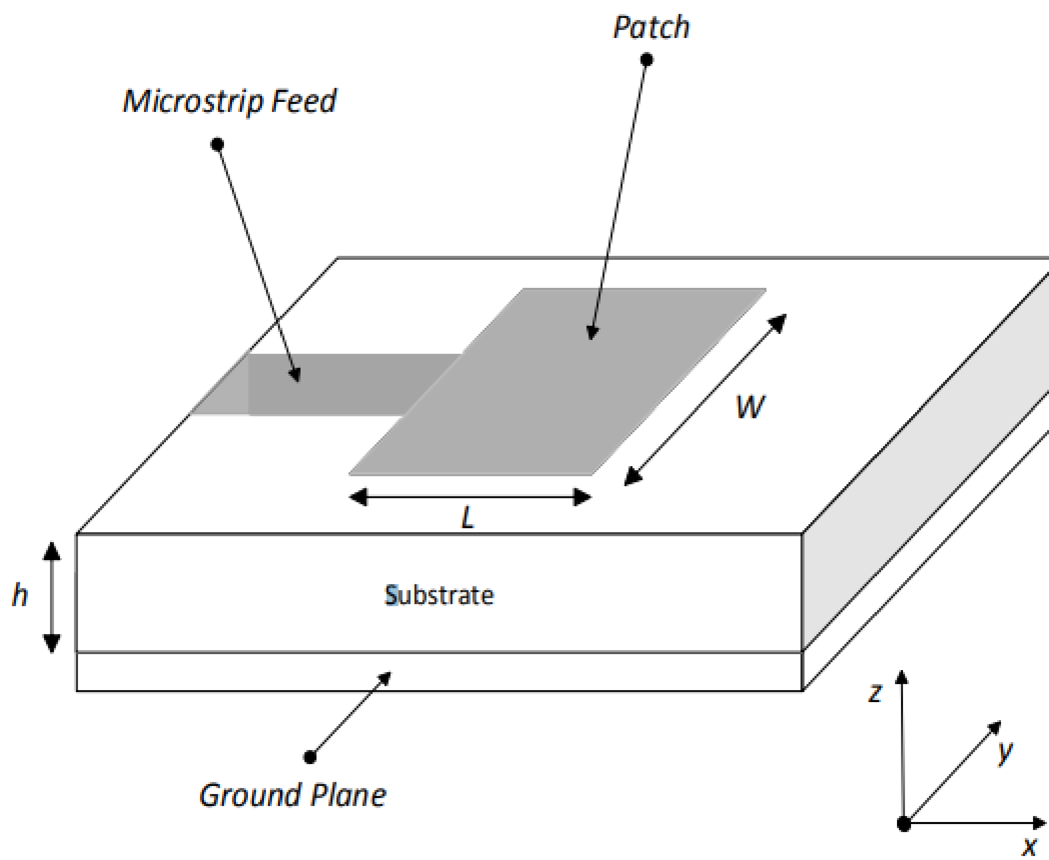
For many practical designs, the advantages of MSA far outweigh their disadvantages. With continuous research and development, the micro strip antennas have been applied in many different and successful applications. Now a days it is the most popular antenna in the wireless communication market. We can find applications of MSA in many various fields of high-tech technology which includes

- Satellite communication
- Mobile communication
- Missile telemetry
- Biomedical radiator
- Radar system
- Radio altimeter

Limitations

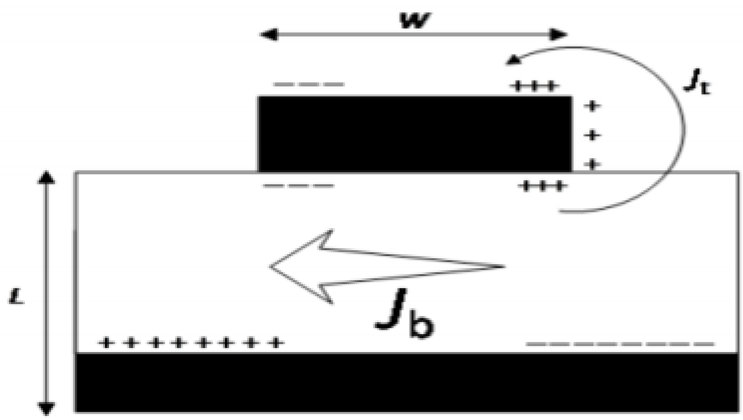
- Low between, low efficiency low gain antennae's with low power handling capacity
- The design complexity gets enhanced due to their smaller size
- There antennas also suffer from the effects of radiations from feeds and junctions
- Surface wave excitation is also the limitation

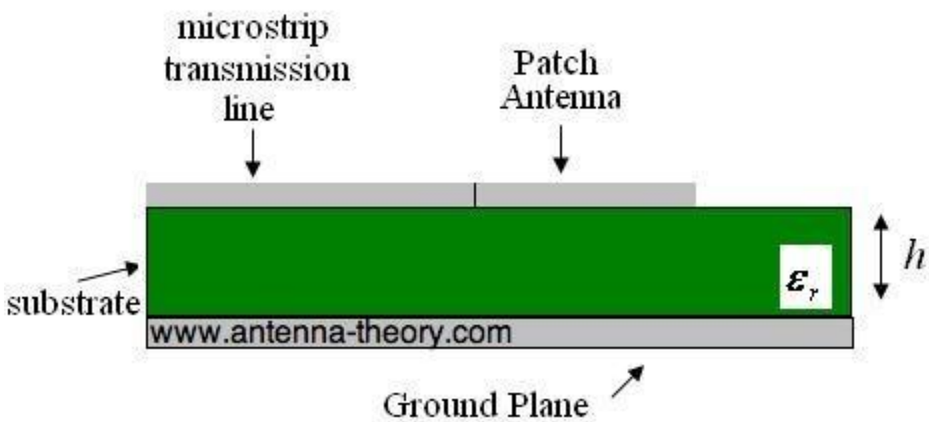
Rectangular patch / micro strip antennas:-
Geometry & parameters



Basic structure of rectangular micro strip antennas

The dimension L is universally taken to mean the long dimension, which causes resonance at its half wave. Length frequency,The radiating edges are at the ends of the L-dimension of the rectangle which sets the single polarization, If Radiation occurs at the ends of the W-dimensions is far leers & referred to as the cross polarization





The E-died distribution under the patch:

Due to the half wave nature of the patch, the fields under the L-edges of opposite polarity& when the field lines Crowe out and finally propagate out into the direction moaned. To the substrates they are now in the rime direction (both facing left)

In the far field perpendicular to the substrate the redaction from the two sides adds up because the fields are in phase, the radiation intensity drops as the fields of 2edges go farther &farther out of phase for effective radiation of micro strip antennas at two angles, the fields exactly cancel, MSA depends on directions

The structure has to be half wavelength resonator (L-λ/2),The dielectric (trident) sub striate should be sufficiently thicker & with low dielectric cuneal,The height of the substrate should be limited to a fraction of wave length,Let us consider a rectangular MSA. Fed by a micro strip t_x ion line as shown in fig the critical or centre frequency of operation of an antenna is a approximately given by

$$f_c = \frac{c}{2L\sqrt{\varepsilon_r}}$$ c=velocity of light

$$f_c = \frac{1}{2L\sqrt{\mu_0 \varepsilon_r \varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Characteristics of Micro strip Antennas:

All the characteristics (parameters) of an antenna are equally applicable to the micro strip antenna (MSA),This figure shows a cross section in a horizontal plane. The RP in the vertical plane is similar but not identical,Power radiated at 180^0 is about 15dB less than the power in the center of the beam. For linearly polarized MSA i.e 90^0

The beam width is about 65^0 and the gain is about 9dBi,An infinitely large ground plan would prevent any back radiation, but the real antenna has a fairly small ground plane & the power in the backward direction is only about 2dB down from that in the main beam

Beam width:-

there are different radiation patterns for an MSA from these we can be described that MSA'S generally have a very wide beam width, both in azimuth and elevation.

Directivity:-

- For TM_{10} $D = \frac{2h^2 F_{-0}^2 W^2 K_0^2}{P_r \pi \eta_0}$
- h=thickness of the substrate
- P_r=Radiated power
- $\eta_0 = 120\pi$
- k_0=wave number

Gain:-

Gain of a rectangular MSA with air dielectric is roughly estimated between 7-9dB in view of the following countsGain of the patch from the directivity relative to the vertical axis is normally about 2dB provided the length of the patch is half wavelength.

if the patch is of square shape, the pattern in the horizontal plane will be directional. such a patch is equivalent to a pair of dipoles repeated by half a wavelength thus counts to 3dB.if the addition of the ground plane cuts off most or all radiation behind the antenna the power averaged over all the directions is reduced by a factor of 2 & thus gain is increased by 3dB

Bandwidth:-

The impedance Bandwidth of a patch antenna is strongly influence by the spacing between the patch and the ground plane,As the patch is moved closer to the ground plane less energy is radiated & more energy is stored in the patch capacitance & inductance,The quality factor Q of the antenna increases & impedance between (↓) decreases,The feed structure also affects the bandwidth ,The voltage standing ratio 'S' is an important parameter to be accounted, particularly at the i/p and under resonance conditions

$$B = \frac{S-1}{Q_0\sqrt{S}}$$

Quality factor:-

MSA'S have a very high quality factor 'Q' represents the losses associated with the antenna , A large 'Q' leads to narrow bandwidth & low efficiency Q can be reduced by increasing the thickness of the dielectric substrate

Efficiency:-

The total loss factor for MSA is given by

$$L_T = L_c + L_d + L_r$$

L_r = loss in radiation

L_c = loss in conductor

L_d = loss in dielectric

This loss results in the reduction of radiation efficiency

$$\eta = \frac{P_r}{P_c + P_d + P_r}$$

Polarization:-

A very important advantage of patch antennas is their ability to have polarization diversity,Patch antennas can easily be designed to have vertical, horizontal, right-hand circular or left-hand circular polarizations,This unique property allows patch antennas to be used in many types of communication links L

Return loss:-

The return loss is defined as the ratio of the Fourier transforms of the incident pulse and the reflected signal,The between of a patch antenna is very small

RMSA----- order of 3%

Radar cross section:-

The GPS guidance system require low-radar cross section (RCS) plat form, the RCS of a conventional patch antenna is after too high to be acceptable, to reduce the RCS a standard technique is used to cover patch with a magnetic absorbing material (reduces antenna gain by several D (reduces antenna gain by several dB's)

Impact of Different Parameters on Characteristics:

The parameters (L, W, h, A and εr) shown in different illustrations of rectangular patch antennas control,

The antenna properties:

Therefore, the nature and quantum of impact of these parameters is to be properly,Accounted for an efficient design,It can be stated that the length L and the width W, or the aspect ratio of the Patch controls the resonant frequency. Earlier, it was also noted that the width w controls the input impedance and the radiation pattern.The wider the patch becomes, the lower will be the input impedance. Since the dimension helps in maximizing ,The efficiency, the best choice for the dimension W is given by

$$W = c/[2 f_0 \sqrt{\{(\varepsilon R + 1)/2\}}]$$

In this equation, the net dielectric constant used is the average of the dielectric constant of the substrate and that of air to obtain a half-wavelength.The permittivity εr of the substrate controls the fringing field. Lower the εr, wider more will be the fringing and better will be the radiation.

A decrease in εr also increases the antenna bandwidth. The efficiency of the antenna also increases with the lower value for permittivity. The impedance of the antenna increases with higher permittivities.

Higher values of permittivity result in 'shrinking' of the patch antenna. In cell phones, there is given very little space and the antenna needs to be half-wavelength long. One technique is to use a substrate with a very high permittivity.

Equation (1) of Sec. 14–4 can be manipulated to yield a relation for L which is given as below:

$$L = 1/2 f_c \sqrt{\varepsilon_0 \varepsilon_r \mu_0}$$

Thus, if the effective permittivity is increased by a factor of 4, the required length decreases by a factor of 2.Using higher values for permittivity is frequently exploited for miniaturization of antennas.As a general principle, ‘an antenna occupying more space in a spherical volume will have a wider bandwidth’.The impact of this principle is noticed when the increased thickness of a dipole antenna increases its bandwidth.

Since increase in height increases the volume, the bandwidth is bound to increase. Thus, the height h of the substrate controls the bandwidth. Besides, the increase in height also results in a more efficient antenna. Increase of height, however, induces surface waves that travel within the substrate.

This may result in undesired radiations which may couple to other components. Equation shows the dependence of bandwidth on various parameters discussed above.

$$B\alpha \frac{(\varepsilon_r - 1)}{\varepsilon_r{}^2} \frac{W}{L} h$$

Similarly, the bandwidth can also be written in terms of the proportionality relation, i.e., $B\alpha h / \sqrt{\varepsilon R}$.

UNIT – IV

REFLECTOR ANTENNAS AND ANTENNA MEASUREMENTS

Introduction:

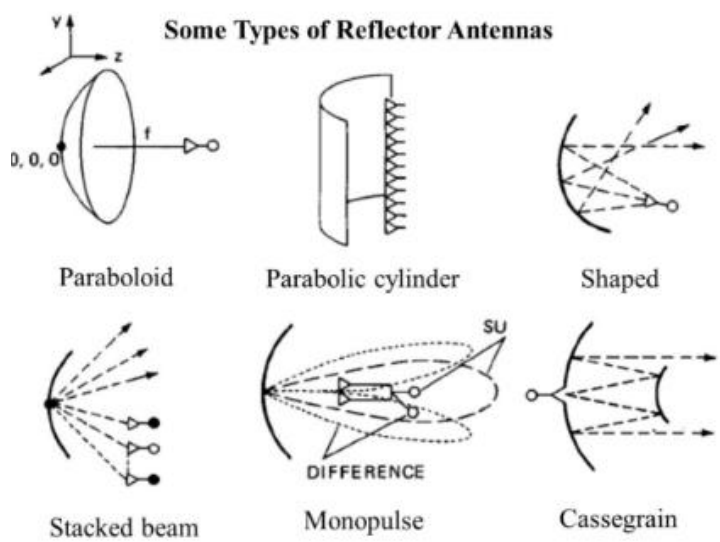
Reflector Antennas:

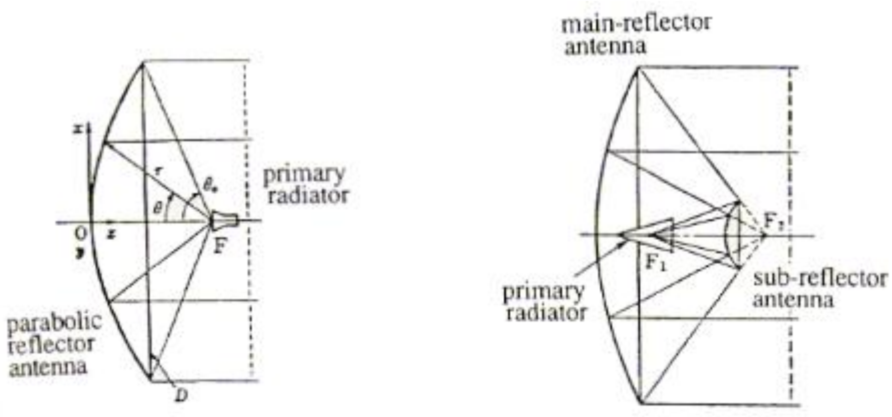
Reflectors are widely used to modify the radiation pattern of a radiating element

Ex: the backward radiation from an antenna may be eliminated with a plane sheet reflector of large enough dimensions

Types of Reflectors:

1) Flat sheet Reflectors
   - Small
   - Large $\alpha=180^0$
2) corner Reflectors
   - Active corner $\alpha<180^0$
   - Passive corner $\alpha=90^0$
3) Parabolic Reflectors
4) Elliptical Reflectors





(a) Parabolic reflector antenna (b) Cassegrain antenna

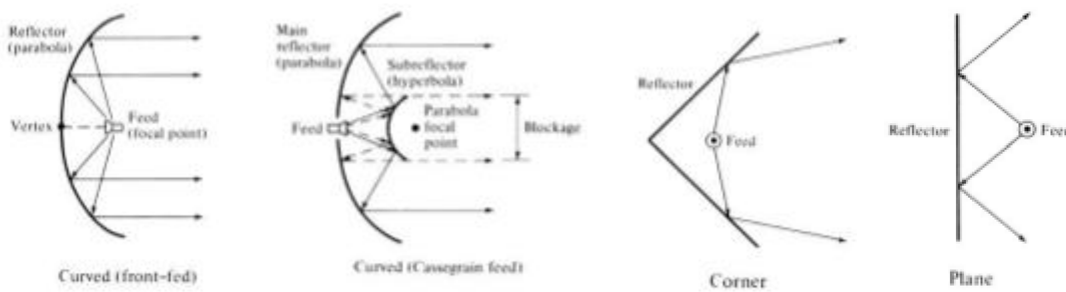
Flat sheet Reflectors:

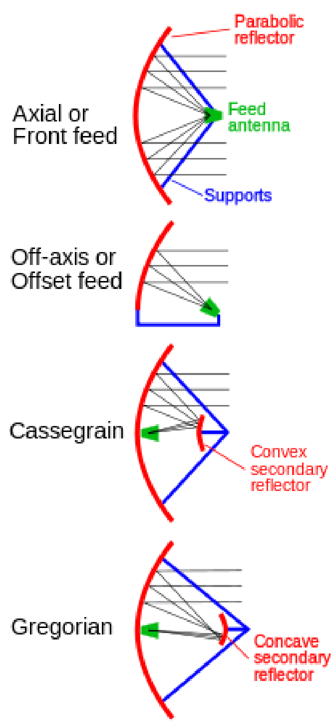
The problem of an antenna at a distance's' from a perfectly conducting plane sheet reflector of infinity extent is readily handled by the method of images (Brown- Scientist)

In this method the reflector is replaced by on image of the antenna at a distance 2s from the antenna,Assuming zero reflector losses, the gain in field intensity at a distance's' from an infinite plane is

$$G_f(\phi) = 2\sqrt{\frac{R_{11} + R_L}{R_{11} + R_L - R_{12}}}\left|\sin(s_r \cos\phi)\right|$$

The gain in eq(1) expressed relative to a λ/2 antenna in free space with same power input,The field patterns of λ/2 antennas at distances s= λ/4, λ/8 & λ/16 from the flat sheet reflector are shown

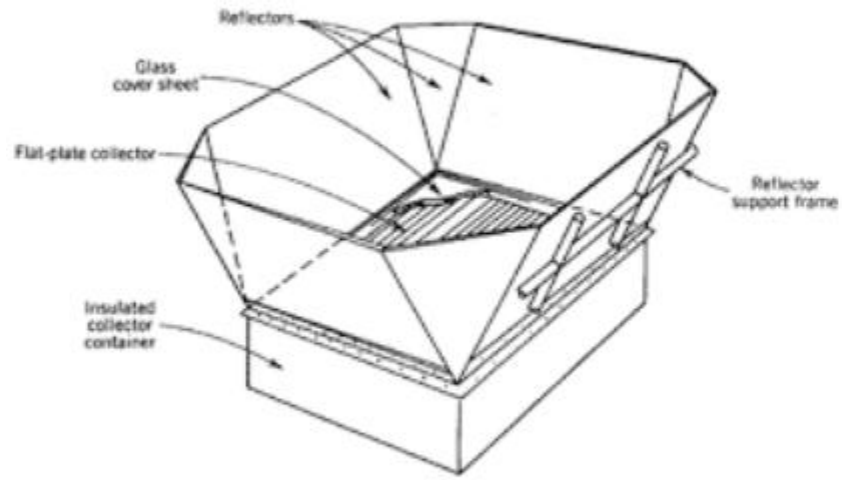




Field patterns of a λ/2 antenna at spacing's of ¼, 1/8 & 1/16 16 from infinite flat sheet reflector,When the reflecting sheet is reduced in size, the analysis is simple

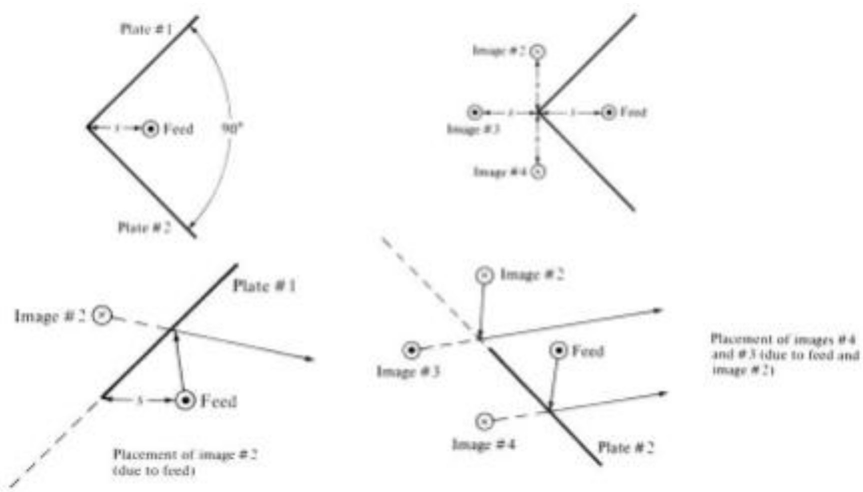
There are 3 principal angular regions Region-1 (air front of sheet) in this region above or the radiated field is given by the resultant of the direct field of the dipole and the reflected field from the sheet,Region-2: (above & below at the sides of sheet),In this there is only direct field from the dipole this region is in the shadow of the reflected field

Region-2: (below or behind the sheet),In this, the sheet acts as a shield, producing a full shadow (no direct or reflected fields, only diffracted fields)



Corner Reflectors:

Two flat reflecting sheets intersecting at an angle or corner form an effective directional antenna



When the corner angle $\alpha=90^0$, the sheets intersects at right angles, forming a square corner reflector,A corner with $\alpha=180^0$ is equivalent to a flat sheet reflector & limiting case of the corner reflector,There are practical disadvantages to angles much less than $90^{0,}$Assuming perfectly conducting reflecting sheets of infinite extent, the method of images can be applied to analyze the corner reflector antenna for angles $\alpha=180^0$ /n ,Where 'n' is any positive antigen n=1, 2, 3.

- n=1 α=180^0 or – flat sheet
- n=2 α=90^0 or π/2, radium - square corner reflector
- n=3 α=60^0 or π/3, radium
- n=4 α=45^0 or π/4, radium

Method of images can only be used for these angles – π, π/2, π/3, π/4 etc

Corner reflectors of intermediate angle cannot be determined by this method by applying interpolation we can estimate

Let us now analyze the method of images for square corner reflectorIn the analysis of 90^0 corner reflector, there are 3 images 2, 3, 4 corresponding to one driven antennaThe driven antenna 1 and the 3 images have currents of equal magnitude,The phase of the currents 1&2 are same and the image element 3&4 with 180^0 phase shift w.r.t 1&2 All the elements are assumed to be λ/2 long

The field pattern $E_ø(\theta)$ in the horizontal plane, at a large distance 'r' from the antenna is given by

$$E_ø(\theta) = 2KI_1\left[\cos(S_r\cos\phi) - \cos(S_r\sin\phi)\right]$$

$$S_r = \beta d$$

$$E_ø(\theta) = 2KI_1\left[\cos(\beta d\cos\phi) - \cos(\beta d\sin\phi)\right]$$

To obtain frequency of operation of a patch antenna accurately we should consider dimensional 'w' also so the expression for the frequency of operation of patch antenna considering L&W is given by

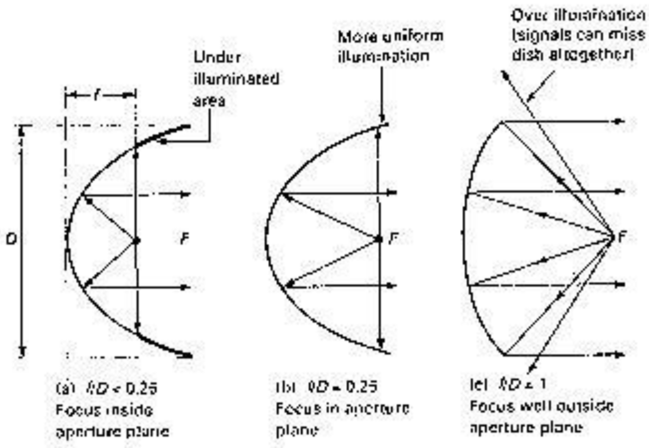
$$f = \frac{c}{2\sqrt{\varepsilon_r eff}}\left\{\left(\frac{n}{L+2\Delta L}\right)^2 + \left(\frac{m}{w+2\Delta w}\right)^2\right\}^{1/2}$$

for dominant mode with n=1, m=0

$$f_{r,nm} = \frac{c}{(L+2\Delta L)\sqrt{\varepsilon_r eff}}$$

Paraboloidal Reflectors: (Geometry & parameters)

"A parabola may be defined as the locus of a point which moves in such way that its distance from the fixed point (called focus) plus its distance from a straight line (called directrix) is constant"



Geometry of the 2D-plane curve parabolic Reflector

OF= Focal length=f

k= a constant (depends on the shape)

f= Focus

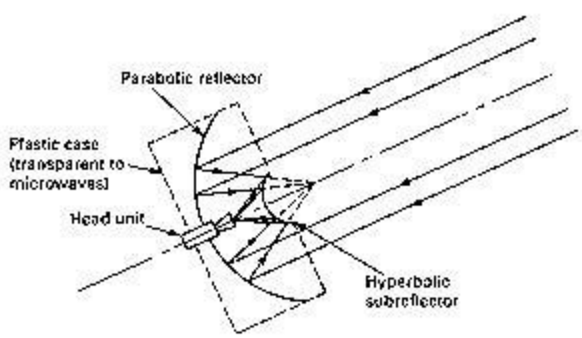
O=vertex

OO'= Axis of the parabola

$$FP_1 = P_1 P_1^1 = FP_2 = P_2 P_2^1 + FP_3 + P_3 P_3^1 = K$$

$$Y^2 = 4fx$$

The open mouth 'D' of the parabola is known as the Aperture,The ratio of focal length to Aperture size known as "fover D ratio" Range-0.25 to 0.5,A cylindrical parabola converts a cylindrical wave radiated by an in phase line source at focus into a plane wave at the aperture,All the wave originating from focus will be reflected parallel to the parabola axis,This implies that all the wave thus, reaching at the aperture plane are in phase so a very strong and concentrated beam of radiation is there along the cylindrical parabolic reflector parabola axis

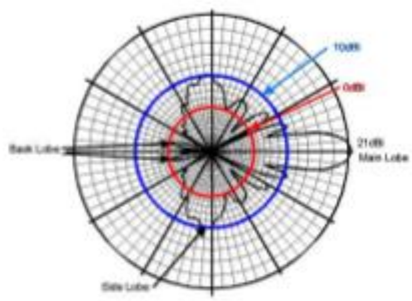


Pattern Characteristics:
Main Beam:
Main beam is the region around the direction of maximum radiation the main beam is cetered at 90 degrees

Side lobes:
Sides' lobes are smaller beams that are away from the main beam. Radiate in directions other than the main beam and can never be completely eliminated

Half Power Beam width (HPBW):
Half Power Beam width is the angular separation in which the magnitude of the radiation pattern decreases by 50% (or-3Db) from the peak of the main beam

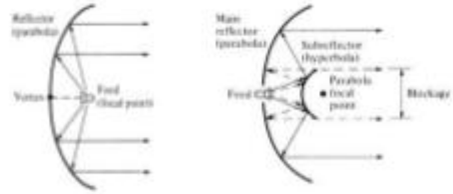
Null to Null Beam width:
Null to Null Beam width the angular separation from which the magnitude of the radiation pattern decreases to zero (negative infinity Db) away from the main beam

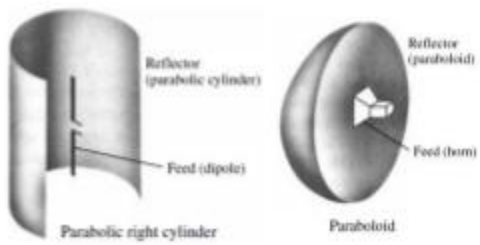


Feed Methods:
The parabolic reflector antenna has two basic components. They are,
1) Source a primary radiator or Feed radiator or Feed
2) Reflector a secondary radiator

A primary radiator or feed is said to be ideal if it radiates the energy towards the reflector such that it illuminates the entire surface of the reflector and if no enrgy is radiated in any other directions,But such an ideal radiator us not available in practice for a secondary radiator, paraboloid is the best choice to use

There are number of feeds available for a parabolic reflector
1) Dipole Feed
2) Horn Feed
3) cassegrain Feed
4) Offset Feed





Dipole Feed:

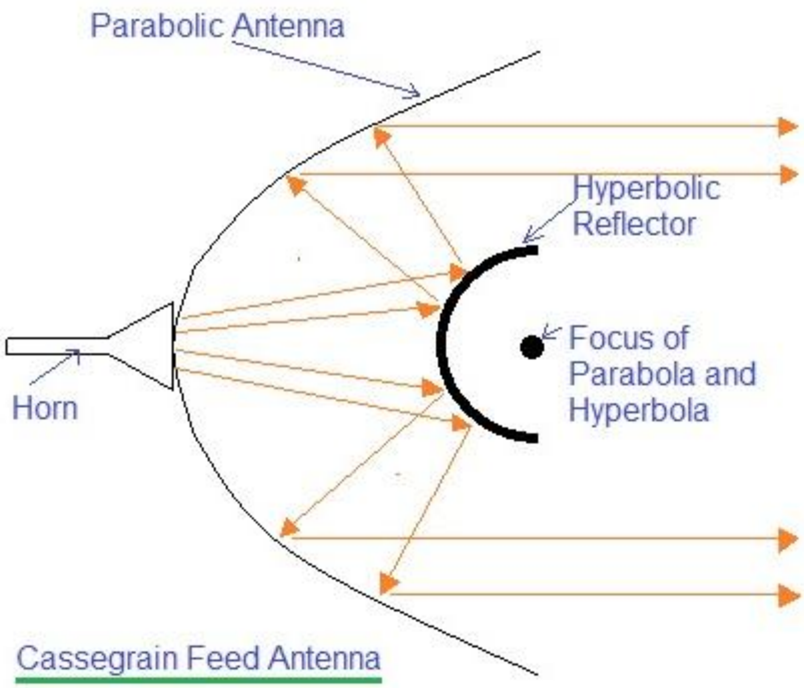
Feeding a parabolic reflector with dipole antenna is not very much suitable,It can also be fed by using a dipole with parasitic reflector or with a coaxial cable,End fire arrays of dipoles can also be used in front of the reflector,Which are spaced such that the pattern illuminates the entire paraboloid reflector,If a parabolic reflector is fed with a dipole, the system is changed from unbalanced to a balanced system

Horn Feed:

A paraboloid reflector antenna is fed with a waveguide horn ,If circlar polarization is required, then a helix or a conical horn antenna is used to feed the paraboloid,If the feed is placed at the moved along the axis, the pattern is broadened,But if the feed is moved laterally from the focus point, the pattern is narrowed or the beam is deteriorated

Cassegrain Feed:

The primary radiator or feed is placed at the vertex of the paraboloid instead of placing it at the focusThis system employes a hyperboloid secondary reflector,One of the hyperboloid's foci coincides with the focus point of the paraboloid ,The radiation emitted from the primary radiator are reflected from secondary reflector which illuminates the paraboloid reflector,The rays reflected from the secondary reflector will align in parallel to each other after reflecting at the paraboloid surface, A parabolid reflector which is fed by using a horn atenna as a primary radiator.



Offset Feed:

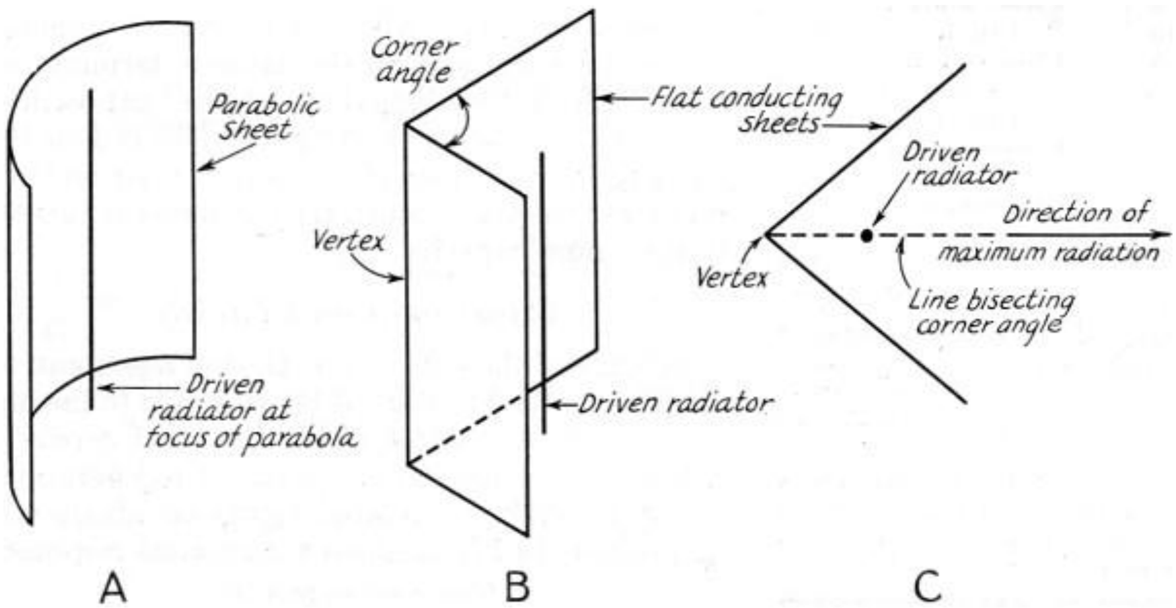
The offset feed system overcomes the problem of aperture blocking due to the secondary reflector dimensions.The dimensions depends on the distance between horn feed and hyperboloid reflector,The feed radiator is placed at the focus with this system all the rays are perfectly collimated without formation of the region of blocked rays

Reflector types- Related Features:

In addition to the paraboloid reflector, there are some reflectors in which properties of parabola are used

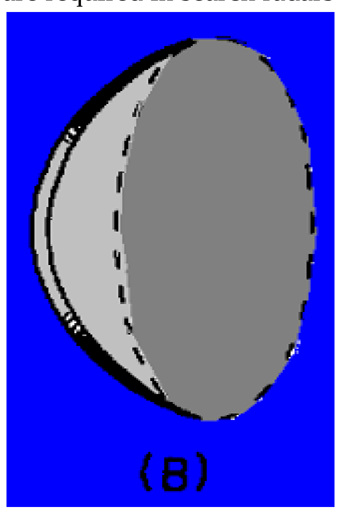
1) Cylindrical parabola:

Parabolic cylinder is formed by moving the parabolic contour parallel to itself, i.e, a plane sheet curved to parabolic shape in one dimension only forms the parabolic cylinder and It has a focal line instead of the focal point and a rectangular mouth.If a line source of radiation is directed along the focal line cylinder, then the parabolic cylinder curvature will be uniformly illuminated



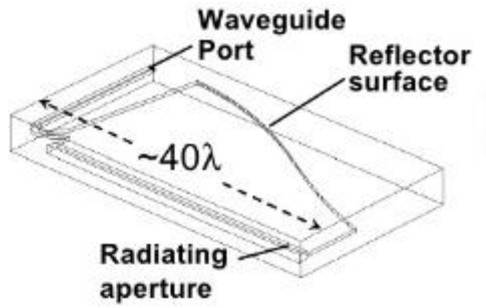
2) Truncated Paraboloid (or) cut paraboloid Reflector:

When the cut paraboloid reflectors are viewed from a point on the parabolic axis, their appearance is not circular These are used to generate fan beams which are required in search radars



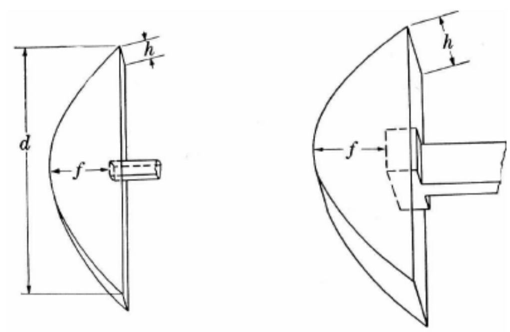
3) Pill Box Antenna (or) Short cylinder with plates:

Pill box antenna is a cylinder which is short in the axial directionIt has conducting end plates through which it is fed by using a coaxial cable or by a probe ,some times, the pill box antenna can also be fed by a wave guide horn,This antenna is used to generate a fan beam required in search.



4) Cheese Antenna:

when a pill box antenna and parabolic cylinder are combined the result structure resembles a cheese antenna



Introduction:

Antenna measurements:

Accurate measurements are necessary to establish the actual performance of antennas,Antennas having strict specifications are needed in many applications as in mobile and personal communications, satellite communications, remote sensing & Radar,In many situations, antenna properties can be calculated theoretically very accurately however for complex antennas this might not be possible.

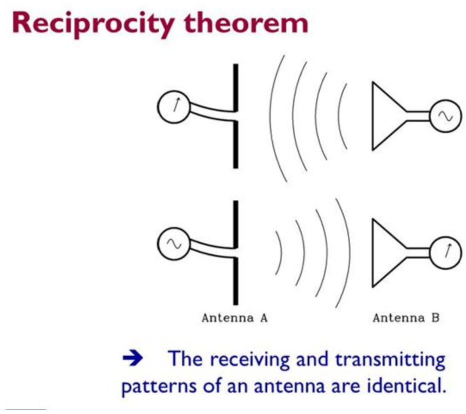
Modeling of usage environment is difficult Ex:-If the antenna is close to the human head or is installed on an air plane,Performance of real world antennas has to be checked by measurements, because due to fabrication tolerances & in some cases due to fabrication error, they may not work as well as predicated.The measurement results give valuable information for trouble shooting,

Basic concepts:

The most common antenna measurement is to measure its radiation properties like directional pattern, gain, phase pattern in the far field, The basic procedure is to place a transmitting or receiving source antenna at different locations w.r.t to the antenna under test (AUT) and get a no.of samples of the pattern by notating the AUT,To ensure the “sharpness” of the pattern sample only one direct signal path should exist between the AUT & Source antenna

Reciprocity in Antenna Measurements:

Two important consequences of the principle from the antenna measurement point of view The t_x‘ing & r_x‘ing patterns are same,Power flow is the same either way,All the radiation parameters of the AUT can be measured in either transmission or reception mode,In practical antenna measurements one has to be careful in applying the reciprocity principle,The emf’s in the terminals of the interchanged antennas are of same frequency,Linear, passive & isotropic medium,Power flow is equal for matched impedances only



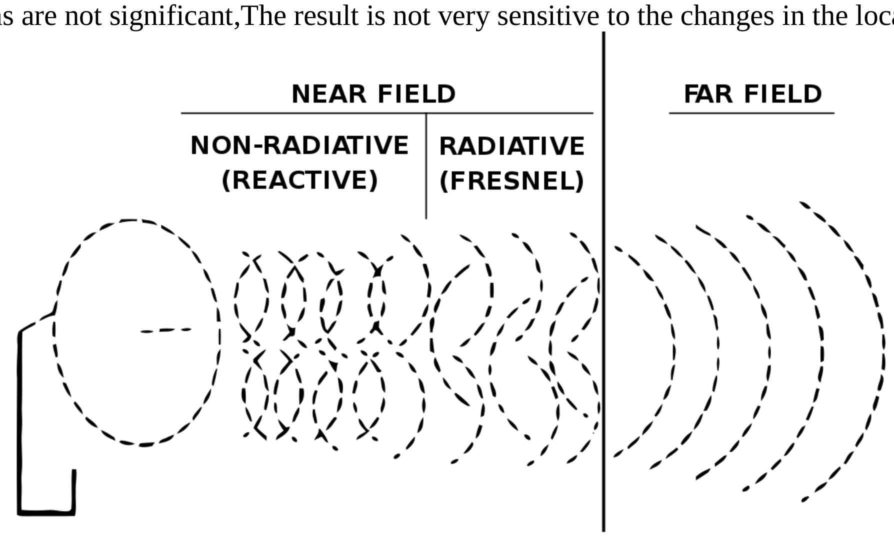
Conditions:
Should be met without problems
Should be considered always

$$\frac{V_R}{V_T} = \frac{e^{-r_1 l_1}}{1 - K_T K_{AS} e^{-2r_1 l_1}} .ts(1 - K_{AUT}). \frac{e^{-r_2 l_2}}{1 - K_{AUT} K_R e^{-2r_2 l_2}}$$

T_{FS}=voltage transmission co-efficient between the antenna terminals

Near field and far field:
The measurement usually takes place in the far field ,There are several advantages of the far field measurement
The measured field pattern is valid for any distance in the far field region: only simple transformation is required (1/r)

If a power pattern is required, only power (amplitude) measurement is needed,Coupling & multiple reflections between the antennas are not significant,The result is not very sensitive to the changes in the location of the antennas



The main disadvantages of the field measurements is the required large distance between the antennas leading to large antenna ranges, $r_{rmf} = \left(\frac{\lambda}{2\pi}\right)m$

Introduction:

Co-ordinate system:

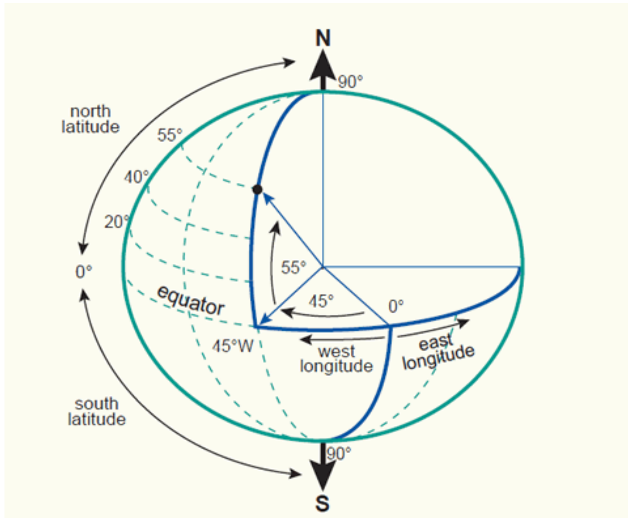


Fig: IEEE Standared Spherical co-ordinate System,AUT is at the origin,Thelivation angle ‘θ’ is measured from the Z-axis,The azimuth angle ‘φ’ is measured from the projection of the radius vector to the xy-plane with φ=0 increasing counter clock wise,Moving the source antenna along lines of constaint φ or constent θ results in conical cuts or φ cuts,When ‘θ’ is content, results in great circle cuts or θ cuts,When ‘φ’ is constant, the cut along the equator with θ=π/2 belongs to both the categories.Orthogonal great circle cuts through the axis of the main lobe of the antenna & the cuts are selected to coincide with the assumed direction of the E&ds fields in the main lobe & then they are called E&H plane cuts.

Sources of error in Antenna measurements:

Any measured quantity has a margin of error Thus the complete value for the gain of an antenna might be 15dB±0.5dB indicating half decibal uncertainityTo reduce the measurement uncertainity to an acceptible level, the critical sources of error have to be recognized.A pure plane wave is an ideal test field for the measurement of far field patterns,Insufficient distance between the antennas causes .Phase curvature amplitude taper reflections from surrounding} can have significant impact on the main beam,Let us assume that the AUT is a planer antenna which is receiving a wave coming from the direction of the main beam axis

If the measurement distance is too small, the fields received by different parts of AUT will not be in phase & there will be phase error,Doubling the measurement distance halves this phase error ,Due to the phase error, the measured gain is smaller & the side lobes are higher than in the ideal plane wave

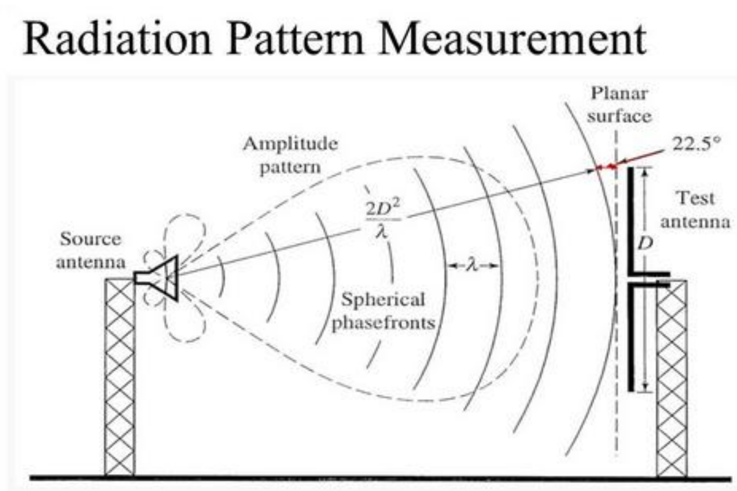
Reflections from surroundings produce field variations complitude & phase ripples) in the test zone as the direct wave & reflected waves interfere,Even small reflected waves causes large measurement errors, because the fields of the waves are added not the powers

Other sources of error:

Antenna measurements are 3-D vector field measurements. Therefore many kinds of alignment errors are possible, Careless alignment of the source antenna many increase the amplitude error.Incorrect use of cables many cause errors cables which have insufficient shielding may leak & act as antennas,Impedance mismatches between the interments & antennas may cause errors in gain measurements

Radiation pattern measurement:

The radiation pattern of an antenna is a 3-D figure & needs measurements of field intensity all over spatial anglesHence for radiation pattern of 'AUT' the various spatial angles must be specified ,Radiation pattern is function of direction (θ or φ)



For most antennas, it is generally necessary to take radiation pattern in XY plane (Horizontal plane) & XZ-plane (vert-plane)

For horizontal antennas:

two patterns are sufficient

The φ component of 'E' is measured as a function of φ in XY plane ($\theta=90^0$)

It is represented as $E_\varphi(\theta=90^0, \varphi)$ & is called E-plane pattern

The φ component of 'E' is measured as a function of 'θ' in xz-plane ($\varphi=0^0$)

It is represented as $E_\varphi(\theta, \varphi=0^0)$ & is called H-plane pattern

For vertical Antennas:

The θ component of b 'E' is measured as a function of φ in XY-plane ($\varphi=90^0$)

It is represented as E_θ ($\theta=90^0$, φ) & is called H-plane pattern

The θ component of the is measured as a function of 'φ' in xz-plane ($\varphi=0^0$)

It is represented as $E_\varphi(\theta, \varphi=0^0)$ & is called as E-plane pattern

Fundamental procedure:

Require two antennas 1) AUT (Antenna under test)

2) Source Antenna

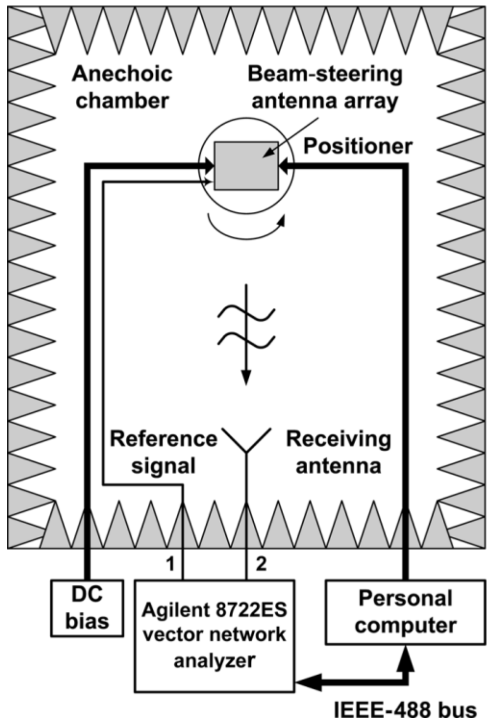
AUT→ primary antenna

Source→ secondary antenna } either t_x'ing or R_x'ing

Step1) →primary antenna kept stationary where as secondary antenna is transported around along a circular path at a constant distance

Step2) → the field strength reading & direction of the secondary antenna w.r.t primary antenna are recorded along the circle at different points
Step3) → from the readings, the post of radiation pattern of primary antenna is made either in rectangular from or in polar form

Arrangements for radiation patter measurement:



The equipment may be entirely automatic or point to point plot,It is usual to operate the antenna under test as a receives placing it under proper illumination by source antenna ,The source is fixed & the AUT is rotated on a vertical axis by antenna support shaft,For $E_\varphi(\theta=90^0, \varphi)$ pattern measurement, the antenna support shaft is rotated with both antennas horizontal and ,For $E_\varphi(\theta, \varphi=0^0)$ pattern the antenna support shaft is rotated with both antenna vertical ,Indication may be on a direct reading meter calibrated in field intensity,If large numbers of patterns are to be taken "automatic pattern recorder" may also be used.

Distance requirement:
In order to obtain accurate far-field radiation pattern, the distance between two antennas must be large otherwise near field pattern is obtained,Phase difference between centre and edges of receiving array for distance requirement

The distance should be $r^2+8^2+28r=\left(\frac{d}{2}\right)^2+r^2 or 28r=\frac{d^2}{88} \quad r\geq\frac{2d^2}{\lambda}$

d= max. Dimension of either antenna

From fig $(r+8)^2=\left(\frac{d}{2}\right)^2+r^2$

r≥8 8≤d

$$r^2 + 8^2 + 28r = \left(\frac{d}{2}\right)^2 + r^2 or 28r = \frac{d^2}{4}$$

8^2 can be neglected $r = \frac{d^2}{88}$

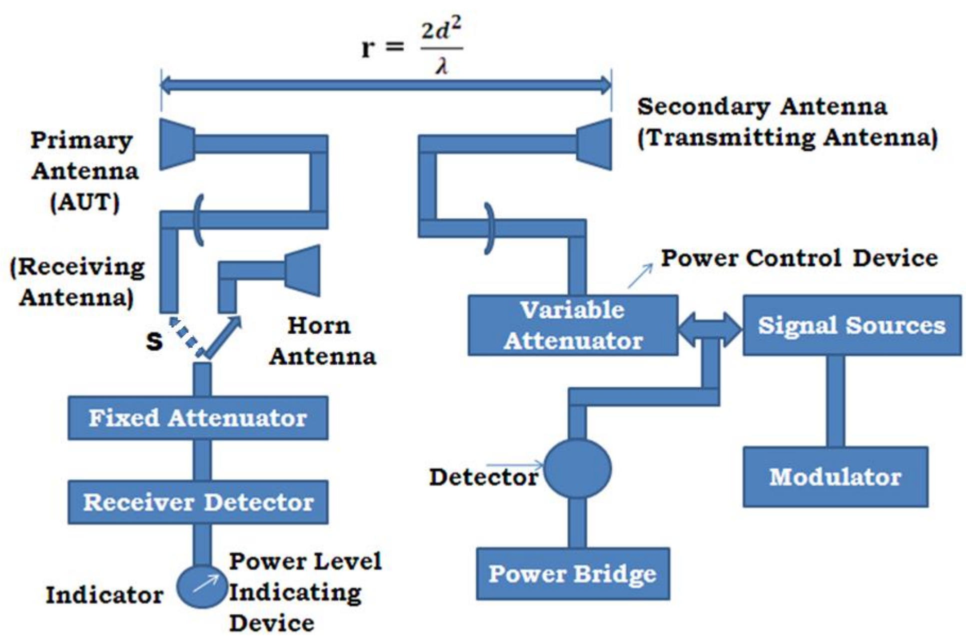
Uniform illumination requirement:

The requirement for an accurate field pattern is that primary antenna should produce a plane wave of uniform amplitude & phase over the distance at least equal to ‘r’.The interference between direct rays and indirect rays should be avoided as possible.Reflections from surrounding objects like building trees etc.Test should be conducted in open plain area and antenna should be directional and installed on higher

Measurement of Gain:

Definition:
Gain = maximum Radiation Intensity (Test or subject)/Maximum Radiation Intensity (reference)
G=KD
K= antenna efficiency factor/effectiveness ratio
D>G (Dis always greater than Gain)

Direct comparison Method:



As defined above, gain is a comparison of two antennas & gain measurement by comparison is done .At higher frequency the comparison method is done .In this measurement of gain is done by comparing the signal strengths t_x’ted or R_x’ted with the unknown gain antenna and standard gain antenna ,A standard gain antenna is that antenna where gain is accurately known so that it can be used in measurement of other antenna,Electromagnetic horn antenna at µ-wave frequency is mostly used as standard gain antenna,At AUT side, there will be two antennas one the subject antenna under test and the other a standard antenna

- Alternator pad – (R_x) To maintain the matched
- Power bridge (T_x)- to check on the stability of the transmittion

Step1) - standard antenna is connected to the receiver with the help of switch 's' and the antenna is aimed at secondary any antenna in the direction of max. Signal intensity, The i/p to the T_x'ing antenna is adjusted to a convenient level and corresponding reading at the receiver is recorded,Attenuator dial setting (w_1) & power bridge reading (p_1)

Step2) – now connect the subject antenna whose gain is to be measured in place of standard gain antenna ,The attenuator dial is adjusted such that receiver indicates the same previous reading as was with standard gain antenna

Let the attenuator dial setting be 'w_2' and power bridge reading p_2

Case-1) when $p_1 = p_2$ (no correction)

$$G_P = \frac{W_2}{W_1}$$

$$\log G_P = \log W_2 - \log W_1$$

$$G_P(dp) = W_2(db) - W_1(db)$$

Case 2) - when $p_1 \neq p_2$

The power level is changed during the measurements and $p_1 \neq p_2$ then actual power gain of the subject antenna can be obtained by multiplying G_p with ratio p_1/p_2

$$\frac{p_1}{p_2} = p$$

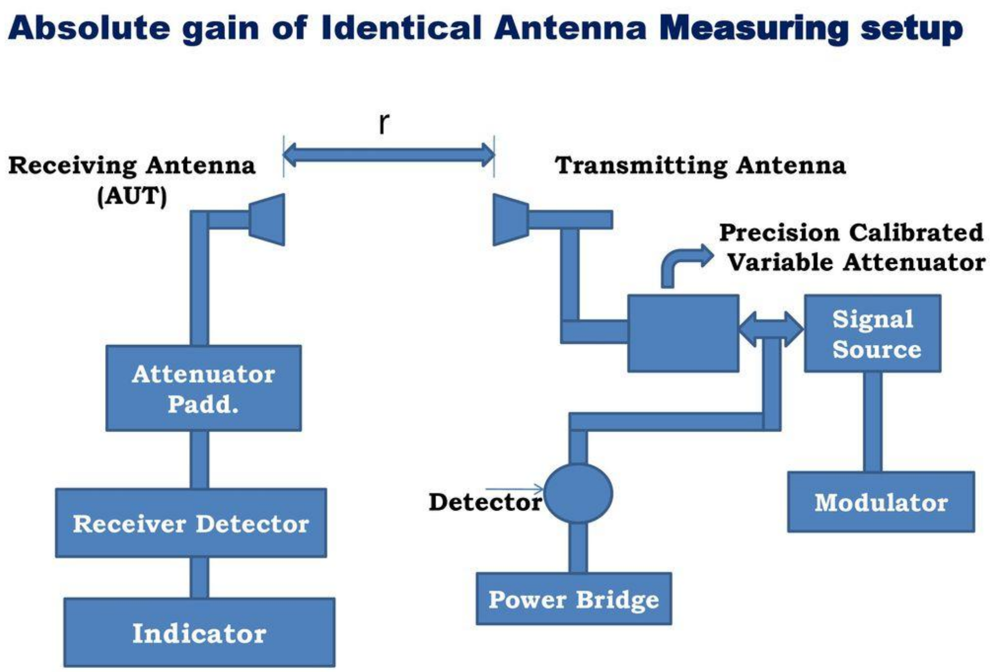
$$10 \log \frac{p_1}{p_2} = p(db)$$

$$G = G_p \frac{p_1}{p_2} = \frac{W_2}{W_1} . \frac{p_1}{p_2}$$

$$G(db) = G_p(db) + p(db)$$

$$G(db) = w_2(db) - w_1(db) + p(db)$$

Absolute gain of identical antennas:



The identical Antennas having distance 'r' is shown

P_t=t_x'ed power

P_r=R_x'ed power

A_{et}, A_{er}=Effective apertures of t_x'ing and R_x'ing antenna

Antennas are identical $A_{et} = A_{er} = \dfrac{G_0\lambda^2}{4\pi}$

From friis's t_x'ion eq

$$\frac{P_r}{P_t} = \frac{A_{er}A_{et}}{\lambda^2 r^2} = \frac{G_0\lambda^2}{4\pi}.\left(\frac{G_0\lambda^2}{4\pi}\right).\frac{1}{\lambda^2 r^2}$$

$$\frac{P_r}{P_t} = \left(\frac{G_0\lambda}{4\pi r}\right)^2$$

$$G_T G_R = \frac{|1-T_R T_L|^2 \, |1-T_G T_T|^2}{\left(1-|T_R|^2\right)\left(1-|T_T|^2\right)|1-T_G T_L|^2}\left(\frac{4\pi r}{\lambda}\right)^2\left(\frac{jP_L}{iP_L}\right)$$

$$G_0 = \frac{4\pi r}{\lambda}\sqrt{\frac{P_r}{P_t}}$$

If the effect of direct and indirect rays are considered

$$G_0 = \frac{4\pi r}{\lambda p}\sqrt{\frac{P_r}{P_t}}$$

F= propagation constant & raised due to the inter

The modified three-antenna method:

The three-antenna technique [6, 7] does not require a priori knowledge of the gain of any of the three antennas involved. On the other hand, in the modified three-antenna method, one of the antennas needs to be a reference antenna whose gains and error limits have been established through absolute calibration.

The reference antenna acts as a Reference Material (RM), which is defined as “a material or substance one or more of whose property values are sufficiently homogeneous and well established to be used for the calibration of an apparatus, the assessment of a measurement method, or for assigning values to materials”

In the modified three-antenna method, measurements are done just as in the case of the three-antenna method, with the RM also analyzed as presumed unknown.A comparison of the result obtained for the RM with its reference values essentially constitutes a “calibration of the whole measurement process against a traceable reference” and provides useful information on the combined effect of many of the potential sources of uncertainty.

The parameter used for this purpose is the so called bias, defined as the value obtained for the RM divided by the value expected.An overall uncertainty estimation from the above method requires that two contributions be taken into account at the minimum the uncertainty associated with the bias, and the precision of the measurement.

The bias uncertainty is estimated by combining the standard uncertainty on the RM values and the limiting error associated with the bias. Even when the bias is insignificant or is corrected for, the uncertainty associated with its determination needs to be considered in the overall uncertainty assessment.

A measure of the precision, defined as “the closeness of agreement between independent test results obtained under prescribed conditions is obtained by estimating the standard deviation associated with the measured data on the test antenna and by subsequently estimating the limiting error.

An actual measurement by using the modified three-antenna method would proceed as follows: Power measurements are done and repeated n times using all the pairs of the three antennas, with the RM also analyzed as presumed unknown.In this measurement effort, the antenna combinations were rotated n times, with a single reading taken each time.

The gains of the three antennas at a range r and wavelength λ are then determined by using simultaneous equations of the form

$$G_T G_R = \frac{|1 - T_R T_L|^2 \, |1 - T_G T_T|^2}{\left(1 - |T_R|^2\right)\left(1 - |T_T|^2\right)|1 - T_G T_L|^2}\left(\frac{4\pi r}{\lambda}\right)^2\left(\frac{jP_L}{iP_L}\right)$$

Where G_T and G_R are the transmitting and receiving antenna gains, iP_L is the power delivered to the power meter When the generator and the load are directly connected and fP_L is the power delivered when the antennas are connected.The variables Γ_T , Γ_R, Γ_G and Γ_L represent, respectively, the reflection coefficient of transmitting antenna, receiving antenna, generator and power meter.

In this work, the reflection coefficient values for the antennas were measured by using the HP 8510 B network analyzer, while for the generator and the power meter, the values given by the manufacturers were employed.

At each frequency, the "true" value G_r of the RM in dB (obtained through absolute calibration, or quoted by the manufacturer) is subtracted from its measured value G_m for each of the n trials.This gives the value of the bias β. The bias is used to correct the estimated (uncorrected) test antenna gains.

Thus, if G_u and G respectively represent the uncorrected and corrected gain values, then this operation proceeds as follows

$$\beta = G_m - G_r$$

$$G = G_u - \beta$$

Selvan The overall standard uncertainty U for the measurement is then estimated from the following equation

$$U = \sqrt{\left(\frac{\sigma^2{}_\beta + \sigma^2{}_G}{n}\right) + U^2{}_{RM}}$$

where σ_β is the standard deviation of the bias values, σ_G is the standard deviation of the bias-corrected test antenna gain values and U_{RM} is the standard uncertainty on the reference antenna gain values.The division by n in assumes normal distribution. The uncertainty with 99% confidence limits is, of course, given by 3U.

The measurement is repeated n times so as to be able to account for the random errors and to ensure the sustenance of the uncertainty estimate.As regards the effects of systematic error sources, the use of the same instrumentation throughout the measurement run will ensure their nearly identical effect on the gains of all the antennas.

It is desirable that the cables at the generator and load ends are not disturbed during the measurements; this is anyway quite practical, as only the antennas need to be replaced.

UNIT – V
RADIO WAVE PROPAGATION

Wave Propagation:

- Modern radio engg has penetrated all branches of national and international economy, science, industry culture and our everyday life
- One of its important applications involves long distance communications by means of electromagnetic waves
- In terms of wavelength, the lower limit of radio waves propogated in free space is ($8x10^{11}$ m) and the upper limit is $3x10^8$m
- Some frequencies are generated by fluctuations of the soler electron proton stream as it penetrates the earth's atmosphere these waves are closely related to magneto hydro dynamic waves (mechanical waves produced by the ion plasma of the atmosphere)
- [10^3Hz-10^{16}Hz presently extends Radio spectrum limits]
- The waves longer than 10^5m (sub audio & audio waves) have little or no commercial usage
- The waves having wave lengths between 10^3-10^5m, find applications in submarine & mine communications(can penetrate duper into water and earth)
- the waves having extremely high frequencies (short 'λ's) of terra range gaining ground in optical com's to their high b/w capability, very high speed.

Wave Definition and Broad categorization:
Mathematical relations of E&H are given by

$$\nabla^2 E = \mu\varepsilon \frac{r^2 E}{rt^2}$$

$$\nabla^2 H = \mu\varepsilon \frac{r^2 H}{rt^2}$$

This wave equation leads to the following equation
["It a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportion to the space reparation from the first location, the group of phenomena constitutes a wave"],Depending on the nature and location of space, some or all the characteristic of a propagating wave may get altered

Categorizations and General:

There are mainly due to variation of media parameters (σ, ε, μ) on the way or the shape & characteristics of obstacles (Reflection, diffraction, absorption, rotation of plane of polarization)

Electromagnetic waves can be classified in a no. of ways:

Guided waves:-
Waves guided by manmade structions such as parallel wire pairs, co-axial cables, wave guides, strip lines, optical fibers etc.

Unguided waves:-
Waves prorogating in the terrestrial atmosphere over and along the earth & in outer space

General classification:-
Plane wave- defined as which the equip has surface is a plane

Uniform plane wave:-
If the equiphase surface is also an equi amplitude surface then it is called uniform plane wave

Non-Uniform plane wave:-
The equiphase and equi-amplitude surfaces are neither same nor the parallel E & H need not necessarily be orthogonal

Slow wave:-
When the phase velocity normal to the equiphase surface is less than velocity of light 'c' referred to as a slow wave

Forward wave:-
A wave travelling in an assigned direction from the point of origin is called forward wave

Backward wave:-
A reflected wave, when a forward wave stribes a reflecting surface

Travelling wave:-
When a wave is progressing only in one direction and there is no reflected wave present it is called travelling wave

Standing wave:-
If both forward and reflected waves are simultaneously present, they combine to result in a wave called standing wave

Surface wave:-
If a wave is supported by some kind of surface between two media, it is called surface wave

Trapped wave:-
Sometimes a surface wave is also called a trapped wave because it carries energy within a small distance from the interface

Leaky wave:-
When discontinuities are densely placed along the line

Classification based on orientation of field vector:
Linearly, circularly, elliptically, vertically horizontally

Classification Based on the Presence of field components
TE (Transverse Electric Waves)
TM (Transverse Magnetic Waves)
TEM (Transverse Electromagnetic Waves)

Classification based on Modes of propagation:
Ground waves
Space waves
Sky waves

Different Modes of Wave Propagation:-

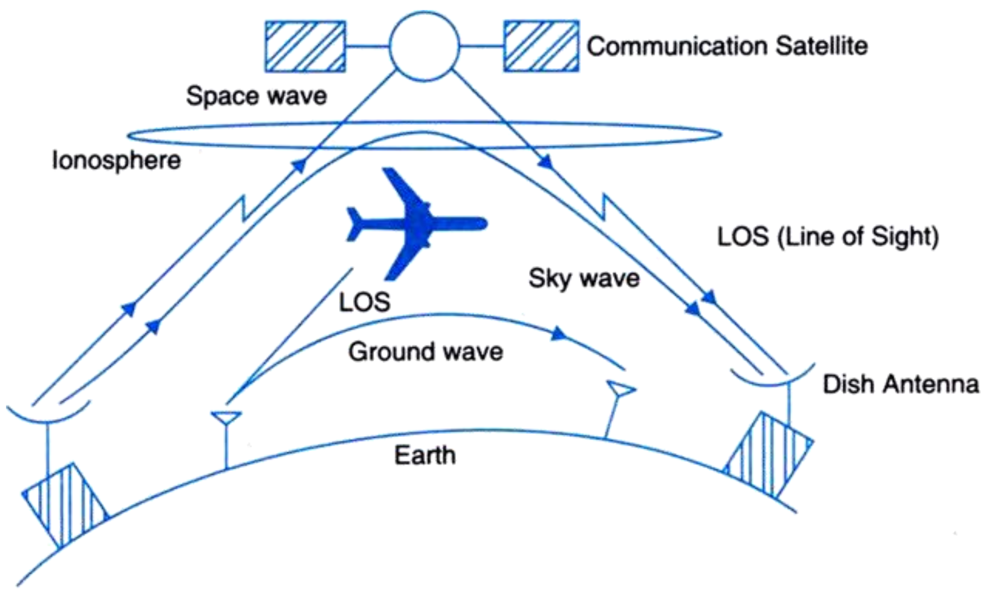
There are four major modes that the waves transmitted from a transmitter may follow to reach the destination and they are

(a) Surface wave propagation
(b) Space wave propagation
(c) Troposphere propagation
(d) Ionosphere propagation.

The first two propagation modes are grouped into ground wave propagation, but the behave differently enough for separate consideration.

Meaning of different terms in Figure:

T_X = Transmitting antenna
R_X = Receiving antenna
Path A = Ground wave propagation
Path B = Sky or ionosphere propagation
Path C = Space wave propagation



Ground Wave propagation:

Ground Wave propagation is a method of radio wave propagation that uses the area between the surface of the earth and the ionosphere for transmission. The ground wave can propagate a considerable distance over the earth's surface particularly in the low frequency and medium frequency portion of the radio spectrum.

Ground wave radio signal propagation is ideal for relatively short distance propagation on these frequencies during the daytime.Sky-wave ionospheric propagation is not possible during the day because of the attenuation of the signals on these frequencies caused by the D region in the ionosphere.

In view of this, lower frequency radio communications stations need to rely on the ground-wave propagation to achieve their coverage.Typically, what is referred to as a *ground wave* radio signal is made up of a number of constituent waves.If the antennas are in the line of sight then there will be a direct wave as well as a reflected signal.

As the names suggest the direct signal is one that travels directly between the two antennas and is not affected by the locality.There will also be a reflected signal as the transmission will be reflected by a number of objects including the earth's surface and any hills, or large buildings that may be present.

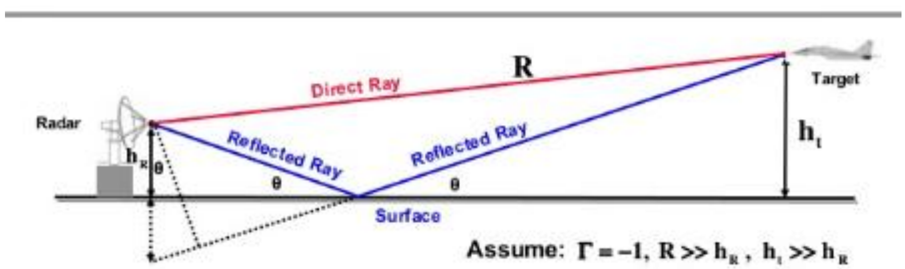
In addition to this there is a surface wave.This tends to follow the curvature of the Earth and enables coverage beyond the horizon. It is the sum of all these components that is known as the ground wave.

Beyond the horizon the direct and reflected waves are blocked by the curvature of the Earth, and the signal is purely made up of the diffracted surface wave.It is for this reason that surface wave is commonly called ground wave propagation.

Plane Earth Reflections:

For Elevated t_x'ing & R_X'ing antennas within the line of right of each other, the received signal reaching the receiver through a direct path and that reaching after being reflected by the ground

These two paths are shown in fig



For a smooth plane and finitely conducting earth the magnitude and phase of the reflected wave differ from that of the incident wave,When the earth is rough, the reflected wave tends to be scattered & may be much reduced in amplitude

The roughness is generally estimated by the Rayleigh criterion

$$R = 4\pi\sigma \sin\theta / \lambda$$

σ=standard deviate on of the surface irregularities relative to the mean surface height
θ=angle of incidence
if R<0.1-smooth (reflecting surface)
R>1.0 rough (reflecting surface)

When the incident wave is near grazing angle over a smooth earth, the reflection coefficient approaches (-1) for both polarizations (θ→0)The earth is not a good conductor like copper or silicon not a perfect dielectric, For a medium having dielectric constant 'ε' and conductivity σ, Maxwell's equations can be written as

$$\nabla \times H = \varepsilon^1 \frac{\partial E}{\partial t}$$

$$\varepsilon^1 = \left(\frac{\sigma}{jw} + \varepsilon \right)$$

The expression for reflection coefficient (E_r/ E_t) for (R_r) horizontal polarization (R_H) & for vertical

$$\frac{E_r}{E_i} = R_H = \frac{\sqrt{\varepsilon_1}\cos\theta - \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta}}{\sqrt{\varepsilon_1}\cos\theta + \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta}}$$

$$R_v = \frac{\frac{\varepsilon_2}{\varepsilon_1}\cos\theta - \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta}}{\frac{\varepsilon_2}{\varepsilon_1}\cos\theta + \sqrt{\frac{\varepsilon_2}{\varepsilon_1} - \sin^2\theta}}$$

If the medium 1 is free space $\varepsilon_1 = \varepsilon 0_1$

Medium 2 is flat earth surface $\varepsilon_2 = \ \varepsilon^1 = \left(\varepsilon + \frac{\sigma}{j\psi} \right)$

Then $R_H = \dfrac{\sqrt{\varepsilon_0}\cos\theta - \sqrt{\left(\varepsilon + \frac{\sigma}{jw} \right) - \varepsilon_0 sin^2\theta}}{\sqrt{\varepsilon_0}\cos\theta + \sqrt{\left(\varepsilon + \frac{\sigma}{jw} \right) - \varepsilon_0 sin^2\theta}}$

$$For = \psi_1 = \psi_2 = 90^0 - \theta$$

$$or \theta = 90^0 - \psi$$

$$\cos\theta = \sin\psi$$

$$\sin\theta = \cos\psi$$

$$R_H = \frac{\sqrt{\varepsilon_0}\cos\left(90^0 - \psi\right) - \sqrt{\left(\varepsilon + \frac{\sigma}{jw} \right) - \varepsilon_0 sin^2\left(90^0 - \psi\right)}}{\sqrt{\varepsilon_0}\cos\left(90^0 - \psi\right) - \sqrt{\left(\varepsilon + \frac{\sigma}{jw} \right) - \varepsilon_0 sin^2\left(90^0 - \psi\right)}}$$

$$R_H = \frac{\sqrt{\varepsilon_0}\sin\psi - \sqrt{\left(\varepsilon + \frac{\sigma}{jw} \right) - \varepsilon_0 \cos^2\psi}}{\sqrt{\varepsilon_0}\sin\psi + \sqrt{\left(\varepsilon + \frac{\sigma}{jw} \right) - \varepsilon_0 \cos^2\psi}}$$

$$\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = \frac{\left[\sin\psi - \sqrt{\left(\varepsilon_r - \frac{j\sigma}{w\varepsilon_0}\right) - \cos^2\psi}\right]}{\left[\sin\psi + \sqrt{\left(\frac{\varepsilon}{\varepsilon_0} + \frac{\sigma}{jw\varepsilon_0}\right) - \cos^2\psi}\right]}$$

Let $x = \frac{\sigma}{w\varepsilon_0}$

$$R_H = \frac{\left[\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}\right]}{\left[\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}\right]}$$

$$R_v = \frac{\left[(\varepsilon_r - jx)\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}\right]}{\left[(\varepsilon_r - jx)\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}\right]}$$

R_H& R_V are both complex quantities, and then can be written as

R_H= R_H< R_H amplitudes & R_V= $R_{V<}$ R_V phases

When the incident wave is horizontally polarized the phase of the reflected wave differs from that of the incident wave by nearly 180^0 for all angles of incidencesFor angles of incidence near grazing (Ψ=0), then reflected wave is equal in magnitude but 180^0 out of phase with the incident wave.

When the incident wave is vertically polarized At grazing incident E, the reflected wave is equal to that of the incident wave and has an 180^0 phase reversalWhen the angle of incidence is increased from ‘0’ both magnitude & phase angle of R_v↓ rapidly the magnitude reaches to its min value while the phase shift goes through -90^0 at an angle called Brewster angle.

Space wave and Surface waves:

In 1909, According to summer field the ground wave can be divided into two parts, a space wave and a surface wave. The space wave dominates at larger distances above the earth ,Surface wave is stronger nearer to the earth’s surface Norton reduced the complexity of the expressions developed by A-Somerfield and made suitable for the engineering workThe expressions for the electric field of an electric dipole above a finitely conducting plane earth surface & clearly shows

The distinction between surface wave and space wave

$$E_Z = j30\beta Idl\left[\cos^2\psi\left(\frac{e^{-j\beta R_1}}{R_1} + R_V\frac{e^{-j\beta R_2}}{R_2}\right) + (1-R_V)(1-u^2+u^4\cos^2\psi)F\frac{e^{-j\beta R_2}}{R_2}\right]$$

$$E_p = -j30\beta Idl\left[\sin\psi\cos\psi\left(\frac{e^{-j\beta R_1}}{R_1} + R_V\frac{e^{-j\beta R_2}}{R_2}\right) + \cos p(1-R_V)u\sqrt{1-u^2\cos^2\psi} - F\frac{e^{-j\beta R_2}}{R_2}(1-\sin^2)\right]$$

Space wave propagation:
It is used at frequencies greater than bands are VHF, UHF and other higher frequency bandsIt is also called troposphere propagation & also called line of sight propagation (receiver should be placed within the line of sight distance)

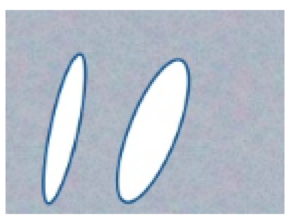
In ground wave propagation frequency ↑ 's alteration↑ 'sIn ionosphere propagation, the ionosphere does not reflect EM waves above 30Hz therefore we go for space wave propagation which is exhaustively used above 30Hz .Space wave energy is divided into direct wave and Reflected wave. The net field strength at the receiver is the vector sum of direct wave and reflected wave

In this field strength is inversely proportional to distance i.e this propagation covers only few kilometers,In this attenuation occurs due to rain fog, snow, clouds, absorption by gases present in the atmosphere.

Wave Tilt:
The wave front starts tilting in the forward direction as it progresses,The magnitude of the tilts will depend upon the conductivity & permittivity of the earth,Whenever there is even slight forward tilt in the electric field, the respective poynting vector drops vertically downwards

This supplies sufficient power to the earth over which the wave can be easily passed
Basically the electric field vector has two components
1) parallel to the earth surface
2) perpendicular to the earth surface

But due to a even alight forward tilt, these two components are not in phase & thus just above the surface of the earth, the electric field is found to be elliptically polarized



The components E_Z & E_P are respectively z-component & Radial component
R_1 R_2 & d are distances
R_v -Reflection coefficient
F – Attenuation constant depends on the earths constant

$$u^2 = \frac{1}{\varepsilon_r - j_x}$$

Equation 1 & 2 may be combined & separated into the following two parts
The field strengths for space and surfaces wave can be given as

$$E_{total}(space) = \sqrt{E_Z{}^2(sur)tE_p{}^2(sur)}$$

$$E_{total}(space) = j30\beta Idl\cos\psi\left(\frac{e^{-j\beta R_1}}{R_1} + RV\frac{e^{-j\beta R_2}}{R_2}\right)$$

$$E_{total}(space) = \sqrt{E_Z{}^2(space)tE_p{}^2(space)}$$

$$E_{total}(space) = j30\beta Idl(1-R_V)\frac{Fe^{-j\beta R_2}}{R_2}\left[1-2u^2\psi(\cos^2\psi)u^2\left(1+\frac{\sin^2\psi}{2}\right)\right]^{1/2}$$

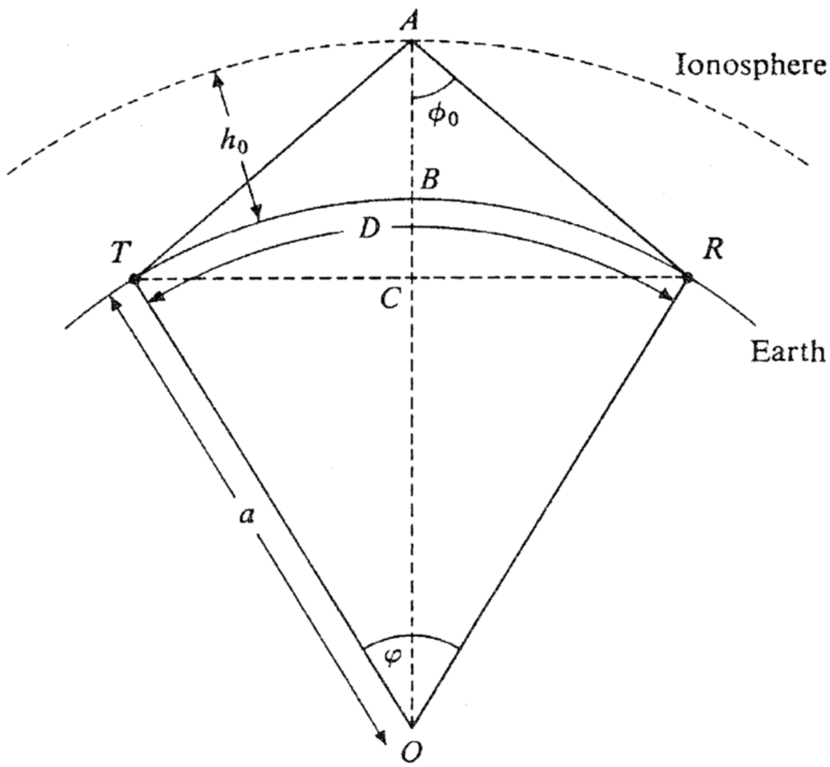
In the above relations u^4 & higher order terms are discarded

Curved Earth Reflections:

For greater distances, reduction in field strength below the free space value is much more; this enhanced reduction is mainly due to the curvature of the earth rather than due to losses in the ground,The effect of the curvature of the earth is entirely negligible up to certain distance and all the relations obtained are valid up

Surface waves- R_x'ed by diffraction

Space waves- - R_x'ed by refraction from lower atmosphere



The problem of curved earth can be easily tacked by the application of Maxwell's equations,One solution is in the form of an infinite series of spherical harmonics with co-efficient containing twelve barrel functions

For which $n = \frac{2\pi a}{\lambda}$

$\frac{a}{\lambda}$ = radius of the earth in wave lengths=$10^3$to 10^8

Whether t_x'ing and r_x'ing antennas are within line of sight range or not,the problem reduces to finding the distance to visible horizon (optical)

Earth radius 'a', antenna height h_1, α-angle
From OAC triangle

$$\cos\alpha = \frac{a}{a+h_1} \cong 1 - \frac{h_1}{a} \quad \text{----------1}$$

α is small in all practical problems

Then $\cos\alpha \cong 1 - \frac{r^2}{2}$ --------------2

Mathematical function

For eq 1&2 $\alpha = \frac{d_1}{a} = \sqrt{\frac{2h_1}{a}}$

The horizontal distance is

$$d_1 = \sqrt{2ah_1}m$$

$$d_2 = \sqrt{2ah_2}m$$

$$d = d_1 + d_2 = \sqrt{2a}\left(\sqrt{h_1} + \sqrt{h_2}\right)$$

$$d_0 = \sqrt{2\times 6.37\times 10^{-6}}\left(\sqrt{h_1} + \sqrt{h_2}\right)km$$

$$d_0 = 3.57\left(\sqrt{h_1} + \sqrt{h_2}\right)km$$

The curvature of the earth has the following effect on the wave propagation within the LOS Range:-For fixed antenna heights, the path length difference between DR and RR will be different from that of flat earth,The reflection at the convex surface will result in divergence of RR path and hence will reduce the power received via RR
[DR=Direct ray, RR=Reflected Ray]

To understand the process, consider the figure a tangent plane MN touching the earth at the point of reflection. the antenna heights can now be measured from this plane.

Practically there is little difference between h_1&h_2n && h_2+ h_2^1

h_1= h_1-Δ h_1

h_2^1= h_2-Δ h_2

Δ h_1=$A^{11}A^1$

Δ h_2=$B^{11}B^1$

d_1, d_2-Loss ranges at heights h_1&h_2

$$\Delta\, h_1 = \frac{{d_1}^2}{2a} \qquad \Delta\, h_2 = \frac{{d_2}^2}{2a}$$

$$h^1{}_1 = h_1 \frac{-{d_1}^2}{2a} \qquad h^1{}_2 = h_2 \frac{-{d_2}^2}{2a}$$

From triangles OAC & OBC with angles of luculent and reflection being the same

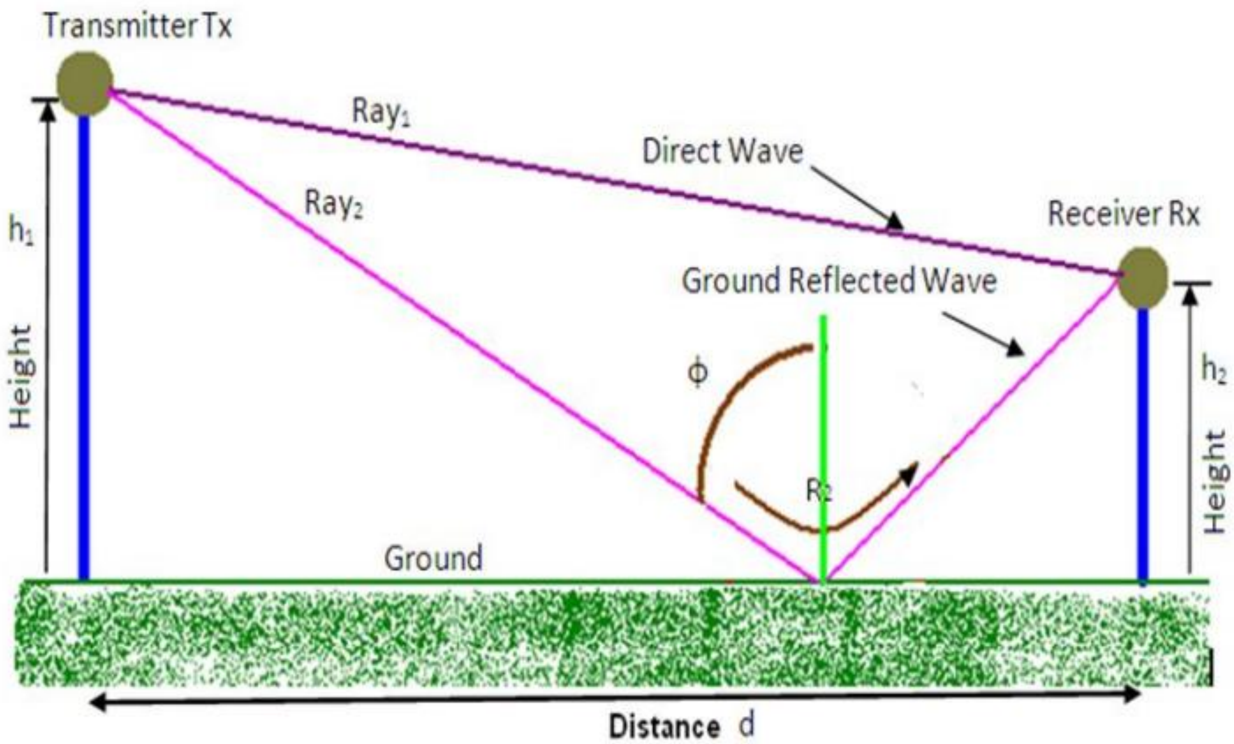
$$\left(a+h_1\right)\cos\left(\alpha+\psi_2\right) = a\cos\psi_2$$

$$\left(a+h_2\right)\cos\left(\alpha+\psi_2\right) = a\cos\psi_2$$

From the fig

$$\tan\psi = \frac{\cos\alpha - \frac{a}{a+h_1}}{\sin\alpha} = \frac{\cos\beta - \frac{a}{a+h_2}}{\sin\beta}$$

Field strength variation with distance:



When the distance (d) between t_x'ing & R_x'ing antennas is sufficiently large in comparison to antenna heights (h_t& h_r) the incidence angle 'Ψ' of the wave on earth is small

h_r = height of the Receiver

h_t = height of the Transmitter

R_1= distance travelled by the direct wave

R_2= distance travelled by the Reflected wave

Magnitudes of both R_1 & R_2 are same irrespective of polarization but with different phase

E_0= amplitude of DW& RW at a distance'd'

From the fig:

$$R_2 - R_1 = \frac{2h_t h_r}{d}\left(\frac{(h_t-h_r)^2-(h_t-h_r)^2}{2d}\right)$$

$$R^2{}_2 = (h_t-h_r)^2 + d^2$$

$$R_1^2 = (h_t-h_r)^2 + d^2$$

$$\Rightarrow d^2\left(1+\frac{(h_t-h_r)^2}{d^2}\right)$$

$$R_1 = d^2\left(1+\frac{(h_t-h_r)^2}{d^2}\right)^{+1/2} \text{-------------------1}$$

$$R^2{}_2 = (h_t-h_r)^2 + d^2$$

$$R_2 = d\left(1+\frac{(h_t-h_r)^2}{d^2}\right)^{1/2} \text{---------------------2}$$

Eq's 1&2 can be rewritten as

$$R_1 = d\left(1+\frac{(h_t-h_r)^2}{2d^2}\right)$$

$$R_1 = d+\frac{(h_t-h_r)^2}{2d^2}$$

$$R_2 = d+\left(1+\frac{(h_t-h_r)^2}{2d^2}\right)$$

The difference in path lengths 'R_2- R_1' is obtained to be

$$R_2 - R_1 = \left(\frac{(h_t-h_r)^2-(h_t-h_r)^2}{2d}\right)$$

$$R_2 - R_1 = \frac{2h_t h_r}{d}$$

The total amplitude due to DW & RW is given by

$$E_t = E_0 + E_0 e^{-j\psi}$$

Where

$$\psi = 180^0 + \alpha$$

α=phase difference due to path difference

$$E_t = E_0\left(1+e^{-j(180+\alpha)}\right)$$

$$E_t = E_0\left(1+\cos(180+\alpha) - j\sin(180+\alpha)\right)$$

$$E_t = E_0\left(1-\cos\alpha + j\sin\alpha\right)$$

$$|E_t| = E_0\sqrt{\left((1-\cos\alpha)^2+(\sin^2\alpha\right)}$$

$$E_t = E_0\sqrt{1-2\cos\alpha+\cos^2\alpha+\sin^2\alpha}$$

$$E_t = E_0\sqrt{2.2\sin^2 \alpha/2} = 2E_0 \sin\frac{\alpha}{2}$$

$$E_t = 2E_0 \sin\frac{4\pi h_t h_r}{2\lambda d} = 2E_0 \sin\left(\frac{2\pi h_t h_r}{\lambda d}\right)$$

Field strength variation with height: The impact of height on the distribution of field is shown

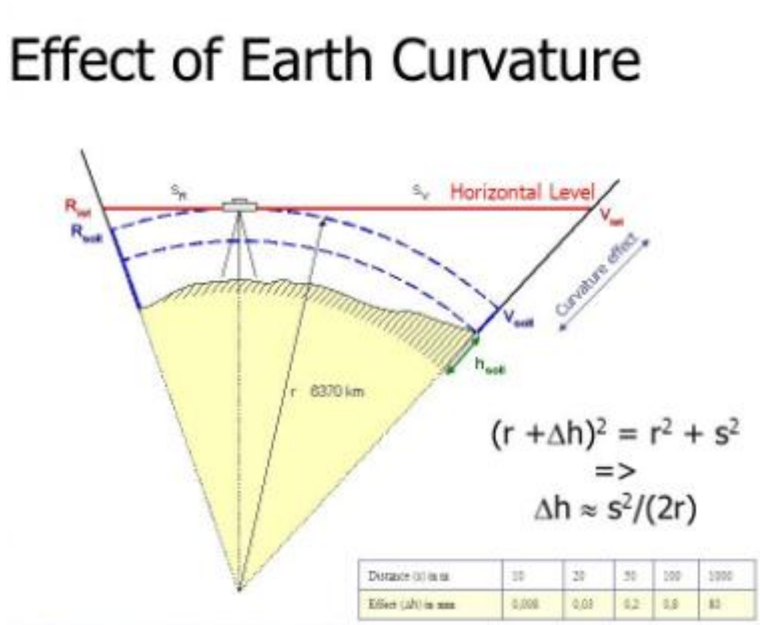
The location minima's & maxims depend on

1) h_t
2) h_r
3) Frequency
4) Distance between T_x & R_x

Effect of Earth's Curvature:

The curvature of earths creates shadow zones, also called differed zones

Due to curvature of earth



The effective antenna (h^1_t, h^1_r) & actual antenna (h_t, h_r) heights difference the quantum of difference will depend on the separation between T_x and R_x

Change in the number & location of maximas & minimas ,There is reduction in d^1, beyond which two waves tend to be out of phase,The effect of reflection is less when the angle of incidence is moderate or large,At larger distance, for small incidence angles, DW and RW in phase opposition

Absorption:

In VHF, the rain attenuates the wave partly due to absorption and partly by scattering,Attenuation is a function of wave length, ε,Drop diameter (water) & drop concentration and the losses due to scattering,For heavy rains-serious attenuation (λ=3cm)
Moderate rains- λ=1cm

Attenuation is proportional to the mass of water/unit volume and drop size,Losses in ice are considerably less than in liquid water,Due to molecular interaction, absorption of energy takes place at certain wavelengths due to water vapors and gases with peaks

Super Refraction:

The phenomenon of refraction in the troposphere is due to change in refractive index.

The reflective index 'n' for free space is given by $n = 1 + \frac{80}{T} 10^{-6} \left(P + \frac{4800w}{T} \right)$

T=absolute temp of air
P=air pressure in mille bars
w=partial pressure of water in mille bars

The gradient of refractive index n is not always uniform,Ray paths are dependent on variation of n with height, Variation of 'n' leads to the phenomena such as reflection, refraction, scattering, fading and ducting Duct can be assumed to be a wave guide with leakage

'n' is replaced by a modified index 'N'

$$N = n + \frac{h}{a} \qquad a=6.37\text{x}10^6\text{m (radius of the earth)}$$

'N' is always approx unity since h<<a

Let us introduce a new parameter called the refractive index 'M'

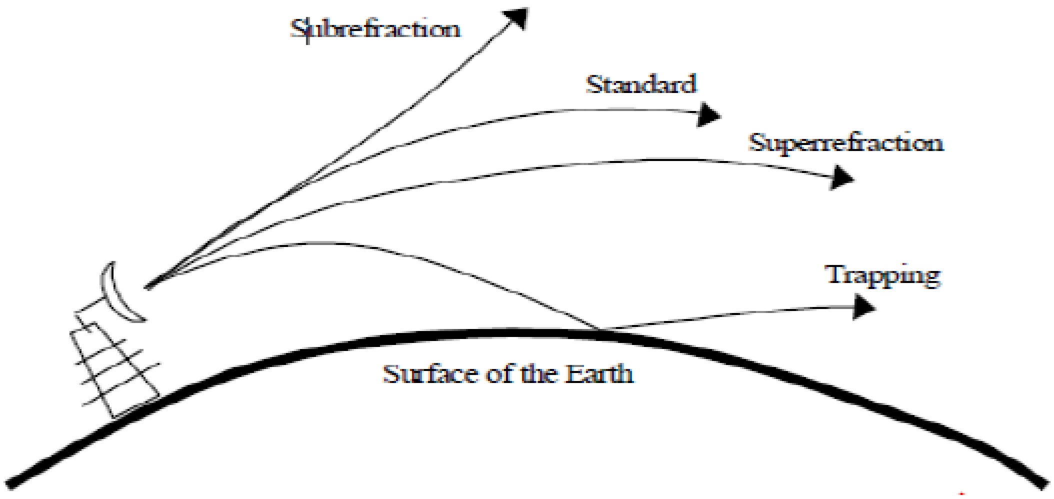
$$M = (N-1) + 10^6$$

The gradient of 'N' can be written as

$$\frac{dN}{dh} \times 10^6 = \frac{80}{T}\frac{dP}{dh} - \frac{80}{T^2}\left(p + \frac{9600W}{T} \right)\frac{dT}{du} + \frac{80 \times 4800}{T^2}\frac{dW}{dn} + \frac{10^6}{a}$$

Refractive n= $\sqrt{\varepsilon_r}$ height ↑ ε_r ↓

Super refraction occurs in areas where warm land air goes out over cool sea.



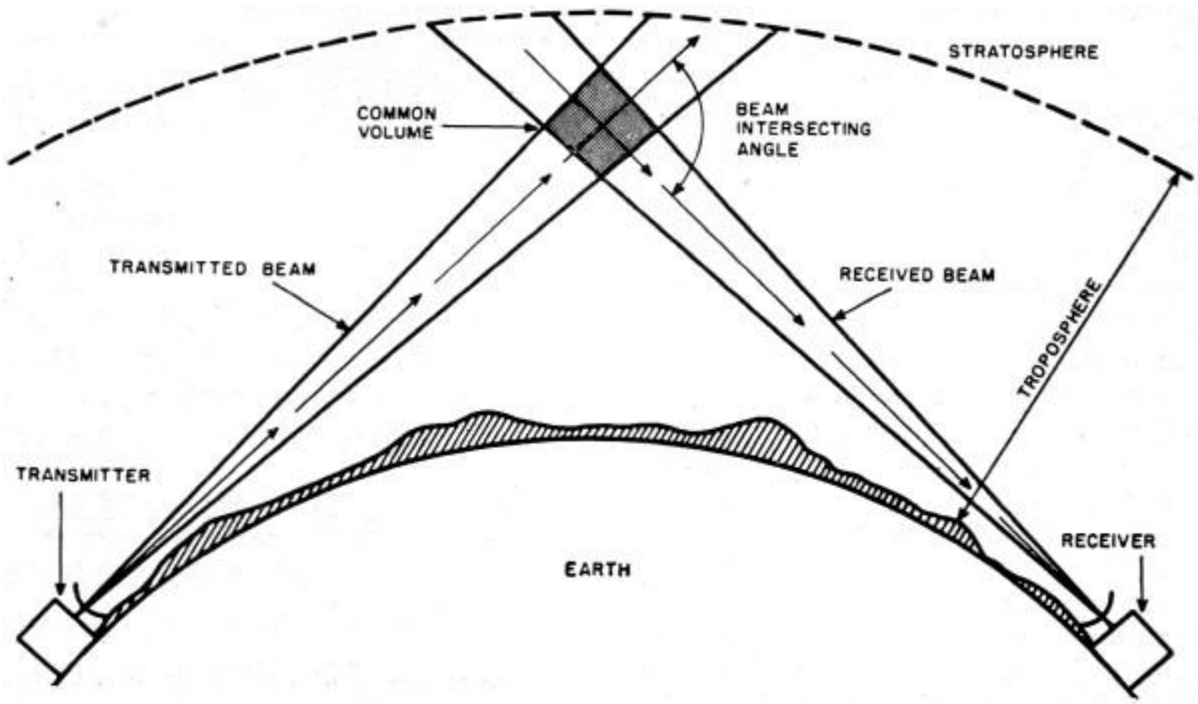
It is as shown the temperature inversion zone hot land area creates super refractor or duct phenomenon.

M-Curves:
'M' is the modified refractive index which was introduced to study the characteristics of troposphere,M-Curves are curves that shows the variation of modified index of refraction with height (dM/dn),M-Curve is useful in predicting the t_x'ion path

Duct propagation:
When the modified refractive index decreases with height change of tx'ion path occurs which is referred as duct propagation In this propagation the waves from transmitter reach the receiver after successive reflections from the ground and region beyond the line of sight

Features of duct propagation:
It happens when (dM/dn) is –ve i.e <0,When dielectric constant changes with height suddenly & rapidly,It takes place at VHF, UHF & microwave range & in areas which covers land & sea,It is a rare phenomenon & happens during monsoons,It occurs due to super reflection & temp inversion

Tropospheric Scattering:
It is the phenomenon in which we can receive signal beyond the optical horizon at VHF & UHF range ,In the upper part of tropospheric region'n' (refractive index) varies contimously when waves passing through such region gets scattered,Due to this scattered signal we can even receive the signal when the receiver is in shadow zone.The field strength of scattered signal is far better than the signal due to diffraction ,Both t_x'ing & R_x'ing antennas should be of high power & high gain



FADING:

The most troublesome and frustrating problem in receiving radio signals is variations in signal strength, most commonly known as FADING.

There are several conditions that can produce fading.

When a radio wave is refracted by the ionosphere or reflected from the Earth's surface, random changes in the polarization of the wave may occur. Vertically and horizontally mounted receiving antennas are designed to receive vertically and horizontally polarized waves, respectively. Therefore, changes in polarization cause changes in the received signal level because of the inability of the antenna to receive polarization changes Fading also results from absorption of the rf energy in the ionosphere.

Absorption fading occurs for a longer period than other types of fading, since absorption takes place slowly.Usually, however, fading on ionospheric circuits is mainly a result of multipath propagation

Path Loss:

Suppose s(t) of power P t is transmitted through a given channel   The received signal r(t) of power P r is averaged over any random variations due to shadowing.

We define the linear path loss of the channel as the ratio of transmit power to receiver power

$$P_L = \frac{P_t}{P_r}$$

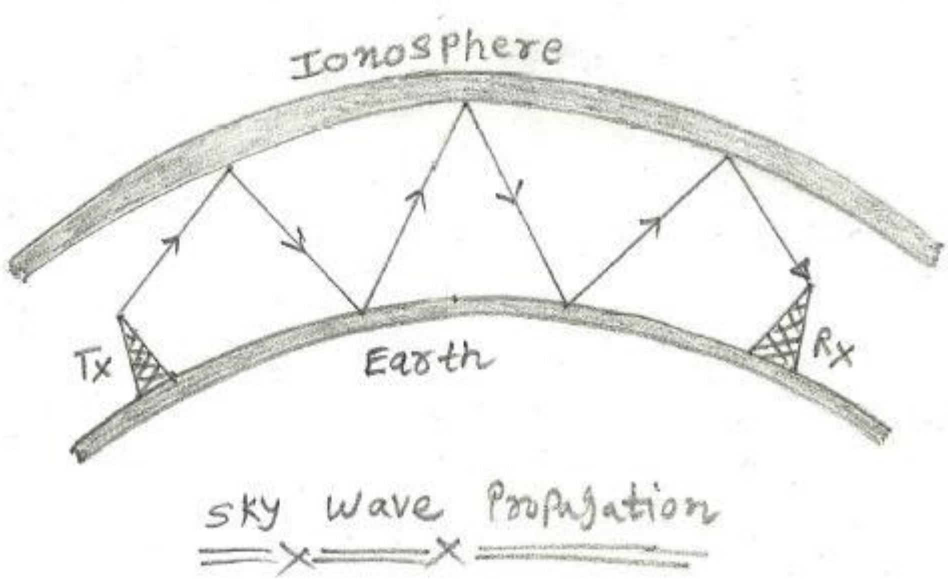
We define the path loss of the channel also in dB

$$P_L dB = 10\log_{10}\frac{P_t}{P_r} dB \text{ (Non negative number)}$$

Sky Wave:
The sky wave, often called the ionospheric wave, is radiated in an upward direction and returned to Earth at some distant location because of refraction from the ionosphere.

This form of propagation is relatively unaffected by the Earth's surface and can propagate signals over great distances. Usually the high frequency (hf) band is used for sky wave propagation.

The following in-depth study of the ionosphere and its effect on sky waves will help you to better understand the nature of sky wave propagation.



Structure of the ionosphere:
As we stated earlier, the ionosphere is the region of the atmosphere that extends from about 30 miles above the surface of the Earth to about 250 miles. It is appropriately named the ionosphere because it consists of several layers of electrically charged gas atoms called ions. The ions are formed by a process called ionization

Refraction in the ionosphere:
When a radio wave is transmitted into an ionized layer, refraction, or bending of the wave, occurs.

As we discussed earlier, refraction is caused by an abrupt change in the velocity of the upper part of a radio wave as it strikes or enters a new medium.

The amount of refraction that occurs depends on three main factors:
(1) the density of ionization of the layer,

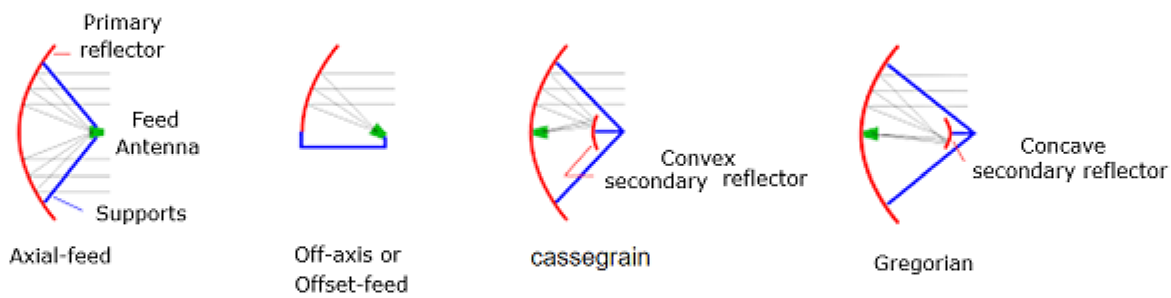
(2) the frequency of the radio wave, and
(3) the angle at which the wave enters the layer.

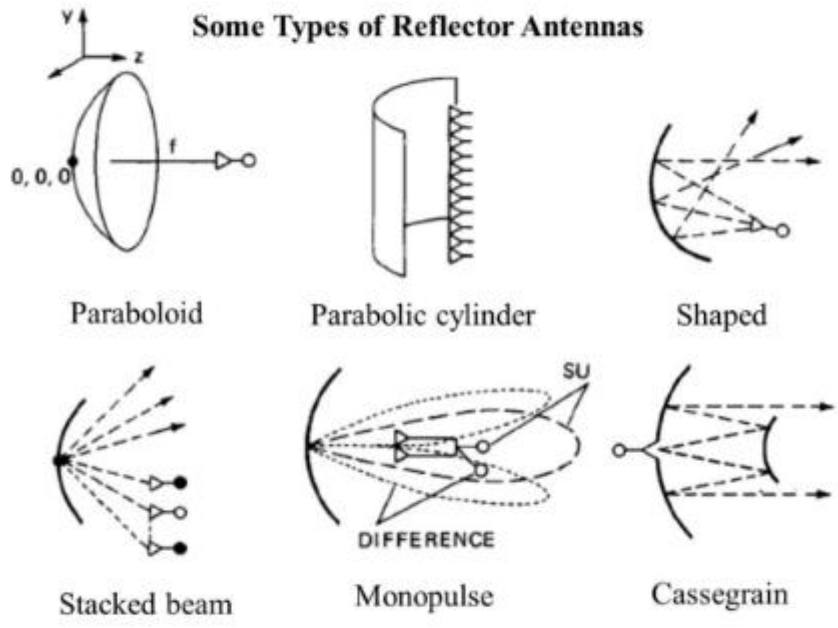
Density of Layer:
illustrates the relationship between radio waves and ionization density. Each ionized layer has a central region of relatively dense ionization, which tapers off in intensity both above and below the maximum region.As a radio wave enters a region of INCREASING ionization, the increase in velocity of the upper part of the wave causes it to be bent back TOWARD the Earth.

While the wave is in the highly dense center portion of the layer, however, refraction occurs more slowly because the density of ionization is almost uniform ,As the wave enters into the upper part of the layer of DECREASING ionization, the velocity of the upper part of the wave decreases, and the wave is bent AWAY from the Earth.

If a wave strikes a thin, very highly ionized layer, the wave may be bent back so rapidly that it will appear to have been reflected instead of refracted back to Earth. To reflect a radio wave, the highly ionized layer must be approximately no thicker than one wavelength of the radio wave.Since the ionized layers are often several miles thick, ionospheric reflection is more likely to occur at long wavelengths (low frequencies).

Types of reflectors:
1) Flat sheet reflectors
   a) small
   b) Large
2) Corner reflectors
   a) Active corner
   b) Passive corner
3) Parabolic reflectors
4) Elliptical reflectors
5) Hyper bolic reflectors
6) Circular reflectors





Reflection of Sky Waves by Ionosphere:

Sky Wave

The sky wave, often called the ionospheric wave, is radiated in an upward direction and returned to Earth at some distant location because of refraction from the ionosphere. This form of propagation is relatively unaffected by the Earth's surface and can propagate signals over great distances. Usually the high frequency (hf) band is used for sky wave propagation. The following in-depth study of the ionosphere and its effect on sky waves will help you to better understand the nature of sky wave propagation.

STRUCTURE OF THE IONOSPHERE:

As we stated earlier, the ionosphere is the region of the atmosphere that extends from about 30 miles above the surface of the Earth to about 250 miles. It is appropriately named the ionosphere because it consists of several layers of electrically charged gas atoms called ions. The ions are formed by a process called ionization.

Ionization:

Ionization occurs when high energy ultraviolet light waves from the sun enter the ionospheric region of the atmosphere, strike a gas atom, and literally knock an electron free from its parent atom. A normal atom is electrically neutral since it contains both a positive proton in its nucleus and a negative orbiting electron.

When the negative electron is knocked free from the atom, the atom becomes positively charged (called a positive ion) and remains in space along with the free electron, which is negatively charged. This process of upsetting electrical neutrality is known as IONIZATION.

The free negative electrons subsequently absorb part of the ultraviolet energy, which initially freed them from their atoms. As the ultraviolet light wave continues to produce positive ions and negative electrons, its intensity decreases because of the absorption of energy by the free electrons, and an ionized layer is formed.

The rate at which ionization occurs depends on the density of atoms in the atmosphere and the intensity of the ultraviolet light wave, which varies with the activity of the sun.Since the atmosphere is bombarded by ultraviolet light waves of different frequencies, several ionized layers are formed at different altitudes. Lower frequency ultraviolet waves penetrate the atmosphere the least; therefore, they produce ionized layers at the higher altitudes. Conversely, ultraviolet waves of higher frequencies penetrate deeper and produce layers at the lower altitudes

An important factor in determining the density of ionized layers is the elevation angle of the sun, which changes frequently. For this reason, the height and thickness of the ionized layers vary, depending on the time of day and even the season of the year. Recombination Recall that the process of ionization involves ultraviolet light waves knocking electrons free from their atoms.

A reverse process called RECOMBINATION occurs when the free electrons and positive ions collide with each other. Since these collisions are inevitable, the positive ions return to their original neutral atom state.The recombination process also depends on the time of day. Between the hours of early morning and late afternoon, the rate of ionization exceeds the rate of recombination.

During this period, the ionized layers reach their greatest density and exert maximum influence on radio waves. During the late afternoon and early evening hours, however, the rate of recombination exceeds the rate of ionization, and the density of the ionized layers begins to decrease. Throughout the night, density continues to decrease, reaching a low point just before sunrise.

Four Distinct Layers

The ionosphere is composed of three layers designated D, E, and F, from lowest level to highest level The F layer is further divided into two layers designated F1 (the lower layer) and F2 (the higher layer). The presence or absence of these layers in the ionosphere and their height above the Earth varies with the position of the sun. At high noon, radiation in the ionosphere directly above a given point is greatest. At night it is minimum.

When the radiation is removed, many of the particles that were ionized recombine. The time interval between these conditions finds the position and number of the ionized layers within the ionosphere changing. Since the position of the sun varies daily, monthly, and yearly, with respect to a specified point on Earth, the exact position and number of layers present are extremely difficult to determine.

However, the following general statements can be made:

The D layer ranges from about 30 to 55 miles. Ionization in the D layer is low because it is the lowest region of the ionosphere. This layer has the ability to refract signals of low frequencies. High frequencies pass right through it and are attenuated. After sunset, the D layer disappears because of the rapid recombination of ions.

The E layer limits are from about 55 to 90 miles. This layer is also known as the Kennelly- Heaviside layer, because these two men were the first to propose its existence. The rate of ionic recombination in this layer is rather rapid after sunset and the layer is almost gone by midnight. This layer has the ability to refract signals as high as 20 megahertz.

For this reason, it is valuable for communications in ranges up to about 1500 miles. The F layer exists from about 90 to 240 miles. During the daylight hours, the F layer separates into two layers, the F1 and F2 layers.

The ionization level in these layers is quite high and varies widely during the day. At noon, this portion of the atmosphere is closest to the sun and the degree of ionization is maximum. Since the atmosphere is rarefied at these heights, recombination occurs slowly after sunset. Therefore, a fairly constant ionized layer is always present. The F layers are responsible for high-frequency, long distance transmission.

REFRACTION IN THE IONOSPHERE:

When a radio wave is transmitted into an ionized layer, refraction, or bending of the wave, occurs. As we discussed earlier, refraction is caused by an abrupt change in the velocity of the upper part of a radio wave as it strikes or enters a new medium.

The amount of refraction that occurs depends on three main factors:
(1) The density of ionization of the layer
(2) The frequency of the radio wave, and
(3) The angle at which the wave enters the layer.

Ray Path:

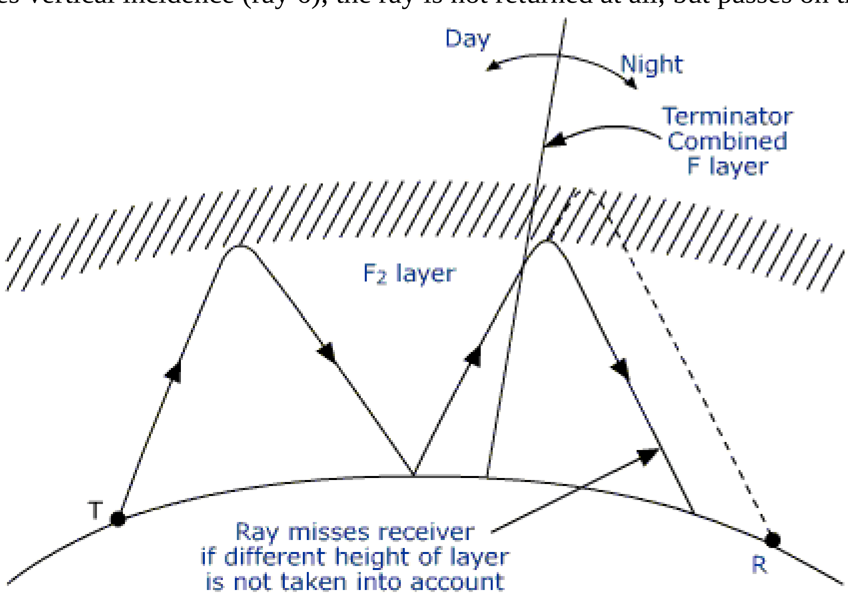
The path that a refracted wave follows to the receiver depends on the angle at which the wave strikes the ionosphere. You should remember, however, that the rf energy radiated by a transmitting antenna spreads out with distance. The energy therefore strikes the ionosphere at many different angles rather than a single angle. After the rf energy of a given frequency enters an ionospheric region, the paths that this energy might follow are many.

It may reach the receiving antenna via two or more paths through a single layer. It may also, reach the receiving antenna over a path involving more than one layer, by multiple hops between the ionosphere and Earth, or by any combination of these paths.

When the angle is relatively low with respect to the horizon (ray 1), there is only slight penetration of the layer and the propagation path is long. When the angle of incidence is increased (rays 2 and 3), the rays penetrate deeper into the layer but the range of these rays decreases.
When a certain angle is reached (ray 3), the penetration of the layer and rate of refraction are such that the ray is first returned to Earth at a minimal distance from the transmitter. Notice, however, that ray 3 still manages to reach the receiving site on its second refraction (called a hop) from the ionospheric layer.

As the angle is increased still more (rays 4 and 5), the rf energy penetrates the central area of maximum ionization of the layer. These rays are refracted rather slowly and are eventually returned to Earth at great distances.

As the angle approaches vertical incidence (ray 6), the ray is not returned at all, but passes on through the layer



Critical Frequency:

Critical Frequency is defined as the highest frequency which can be reflected by a particular ionospheric layer at vertical incidence.Each layer has different critical frequency. It is usually denoted by f_0 or f_c, For the regular layers it is proportional to the the square root of the maximum electron density in the layer

The refractive index 'μ' is given by

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81N}{f^2}}$$

Where,

N is the electron density

At vertical incidence, angle of incidence=0^0

$N = N_{max}$

$f = f_c$

As the angle of incidence goes on decreasing, the electron density goes on increasing and it reaches to maximum electron density (N_{max}) then,

$$\mu = \frac{\sin 0^0}{\sin r} = \sqrt{1 - \frac{81N_{max}}{f_c^2}} = 0$$

$$1 = \sqrt{1 - \frac{81N_{max}}{f_c^2}}$$

$$f_c = \sqrt{81(N_{max})}$$

$$f_c = 9\sqrt{(N_{max})}$$

f_c is expressed in MHz

N_{max} is expressed in per cubic meter

The critical frequency (wave) will get reflected only for vertical incidence not for any other angle of incidence,It is clear from the critical frequency that the radio waves of frequency equal to or less than the critical frequency ($f<f_c$) will certainly be reflected back by the ionosphere layer irrespective of the angle of incidence

Radio waves of frequency greater than critical frequency ($f<f_c$) will also be returned to earth only when the angle of incidence is glancing otherwise the wave will penetrate the layer concerned for a wave of frequency greater than critical frequency to be reflected, the condition is,

$\sin i > \mu$

$$> \sqrt{1 - \frac{81N_{max}}{f_c^2}}$$

$$sini > \sqrt{1 - \frac{f_c^2}{f^2}}$$

Where,

$$f_c = \frac{81N_{\max}}{f^2}$$

$$= \frac{N_m e^2}{m4\pi^2 f^2 k_0}$$

MUF: Maximum Usable Frequency:

As we discussed earlier, the higher the frequency of a radio wave, the lower the rate of refraction by an ionized layer. Therefore, for a given angle of incidence and time of day, there is a maximum frequency that can be used for communications between two given locations. This frequency is known as the MAXIMUM USABLE FREQUENCY (muf).

Waves at frequencies above the muf are normally refracted so slowly that they return to Earth beyond the desired location, or pass on through the ionosphere and are lost. You should understand, however, that use of an established muf certainly does not guarantee successful communications between a transmitting site and a receiving site. Variations in the ionosphere may occur at any time and consequently raise or lower the predetermined muf.

This is particularly true for radio waves being refracted by the highly variable F2 layer. The muf is highest around noon when ultraviolet light waves from the sun are the most intense.

It then drops rather sharply as recombination begins to take place.

MUF range=8-35 MHz

For a sky wave to return to earth, angle of reflection, i.e $<r=90^0$

$$\mu = \frac{\sin i}{\sin r}$$

$$= \sqrt{1 - \frac{81N_{\max}}{f_{muf}{}^2}}$$

$$\mu = \frac{\sin l}{\sin 90^0}$$

$$= \sqrt{1 - \frac{81N_{\max}}{f_{muf}{}^2}} \quad [r=90^0]$$

$$\mu = \sin l = \sqrt{1 - \frac{81N_{\max}}{f_{muf}{}^2}} \quad [\sin 90^0=1]$$

$$\sin^2 l = 1 - \frac{81N_{\max}}{f_{muf}{}^2}$$

But $f_c{}^2 = 81N_{\max}$

$$1 - \frac{f_c{}^2}{f_{muf}{}^2} = sin^2 l$$

$$\frac{f_c{}^2}{f_{muf}{}^2} = 1 - sin^2 l$$

$$\frac{f_c^{\ 2}}{f_{muf}^{\ 2}} = \cos^2 l$$

$$f_{muf}^{\ 2} = \frac{f_c^{\ 2}}{\cos^2 l}$$

$$= f_c^{\ 2} \sec^2 i$$

$$f_{muf} = f_c \sec i$$

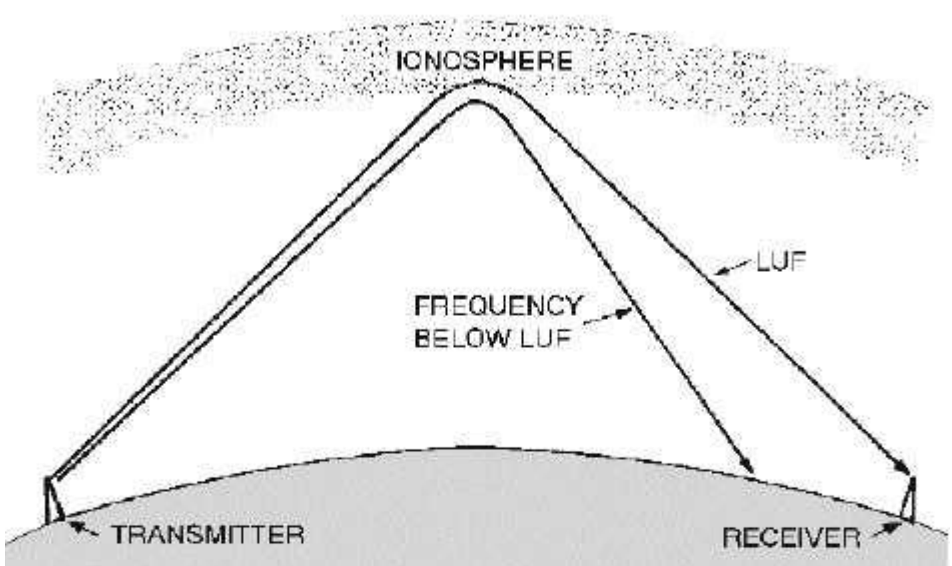
This means that f_{muf} is greater than f_c by a factor seci. this gives the maximum frequency which can be used for sky wave communication for a given angle of incidence (i) between two points on the earth

Lowest Usable Frequency:(LUF)

As there is a maximum operating frequency that can be used for communications between two points, there is also a minimum operating frequency. This is known as the LOWEST USABLE FREQUENCY (luf).

- As a frequency is lowered, absorption of the radio wave increases. A wave whose frequency is too low is absorbed to such an extent that it is too weak for reception.
- Likewise, atmospheric noise is greater at lower frequencies; thus, a low-frequency radio wave may have an unacceptable signal-to-noise ratio.

For a given angle of incidence and set of ionospheric conditions, the luf for successful communications between two locations depends on the refraction properties of the ionosphere, absorption considerations, and the amount of atmospheric noise present.



Optimum Working Frequency

Neither the muf nor the luf is a practical operating frequency.While radio waves at the luf can be refracted back to Earth at the desired location, the signal-to-noise ratio is still much lower than at the higher frequencies, and the probability of multipath propagation is much greater.

Operating at or near the muf can result in frequent signal fading and dropouts when ionospheric variations alter the length of the transmission path. The most practical operating frequency is one that you can rely on with the least amount of problems.

It should be high enough to avoid the problems of multipath, absorption, and noise encountered at the lower frequencies; but not so high as to result in the adverse effects of rapid changes in the ionosphere. A frequency that meets the above criteria has been established and is known as the OPTIMUM WORKING FREQUENCY.

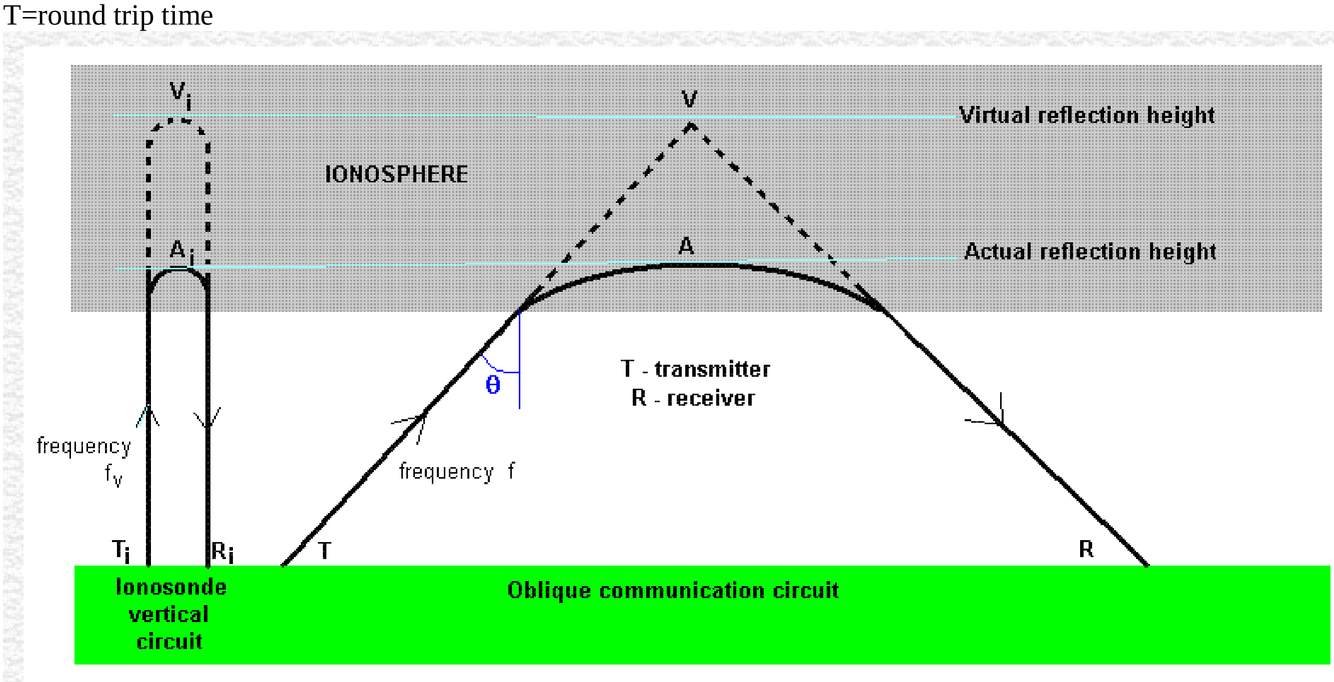
It is abbreviated "fot" from the initial letters of the French words for optimum working frequency,**Virtual Height:**

The virtual height of an ionosheric layer is the equivalent altitude of a reflection that would produce the same effect as the actual refraction

$$h_v = \frac{cT}{2}$$

c=speed of light=$3x10^8$

T=round trip time



APPLICATION OF SKY WAVE:

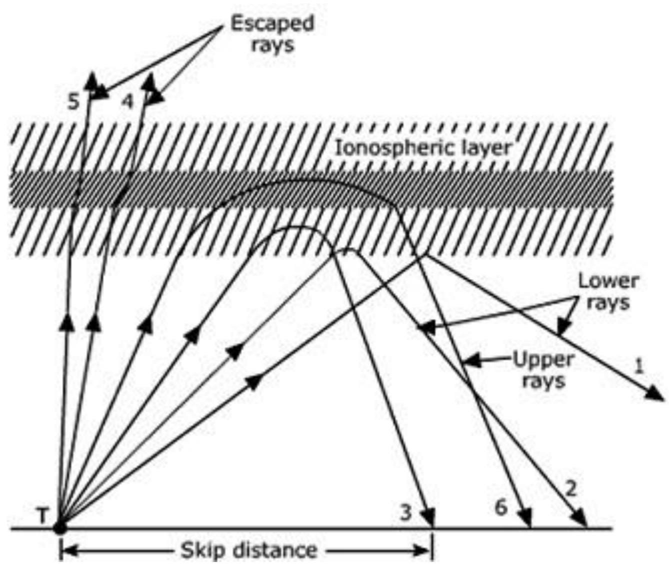
- Satellite communication
- Mobile communication

Skip Distance:

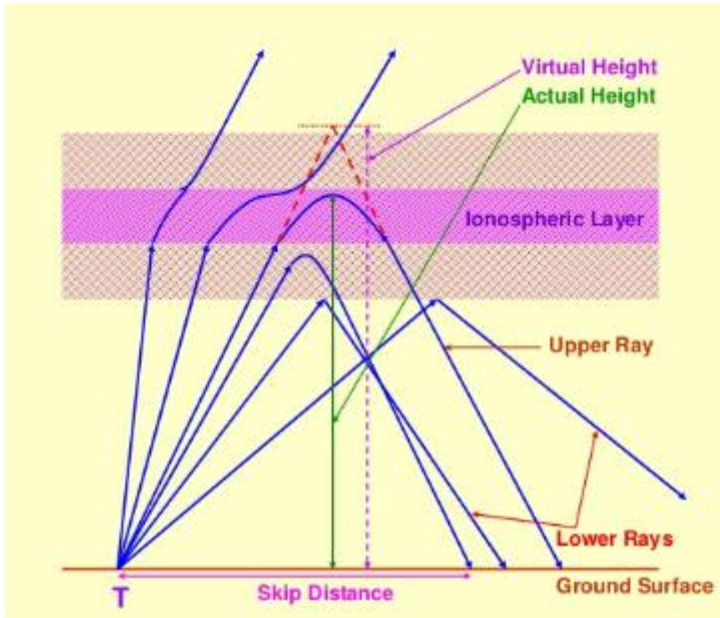
The minimum distance from the transmitter to the point on ground at which of a given frequency will return to the earth by the ionosphere is called skip distance

For flat earth $$D_{SKIP} = 2h\sqrt{\left(\frac{f_{MUF}}{f_{CR}}\right)^2 - 1}$$

1. Dskip=skip distance
2. h=height at which the reflection occurs
3. f_{MUF}=maximum usable frequency
4. fc=Critical frequency



Relation between MUF and Skip Distance:



The ionized layer which as assumed to be thin with sharp ionization density gradient so as to obtain mirror like reflections

d – Skip distance

h – Height of the ionospheric layer

θ_i – Angle of lucidness

θ_r – Angle of reflection

From the fig:

$$cos\ \theta_i = \frac{OB}{AB} = \frac{h}{\sqrt{h^2 + \frac{d^2}{4}}} = \frac{2h}{\sqrt{4h^2 + d^2}}$$

$$f_{MUF}{}^2 = f_c{}^2 \sec^2 \theta_i$$

$$f_{MUF}{}^2 = \frac{f_c{}^2}{\cos^2 \theta_i} \Rightarrow \cos^2 \theta_i = \frac{f_c{}^2}{f_{MUF}{}^2}$$

$$\left(\frac{2h}{\sqrt{4h^2 + d^2}}\right)^2 = \frac{f_c{}^2}{f_{MUF}{}^2}$$

$$f_{MUF}{}^2 = f_c{}^2 \left(\frac{\sqrt{4h^2 + d^2}}{2h}\right)^2$$

$$f_{MUF}{}^2 = f_c{}^2\left(\frac{4h^2 + d^2}{4h^2}\right)$$

$$f_{MUF} = f_c\sqrt{1+\left(\frac{d}{2u}\right)^2}$$

Above equation gives MUF in terms of skip distance

$$\frac{f^2{}_{MUF}}{f_c{}^2} = \sqrt{1+\frac{d^2}{4u^2}}$$

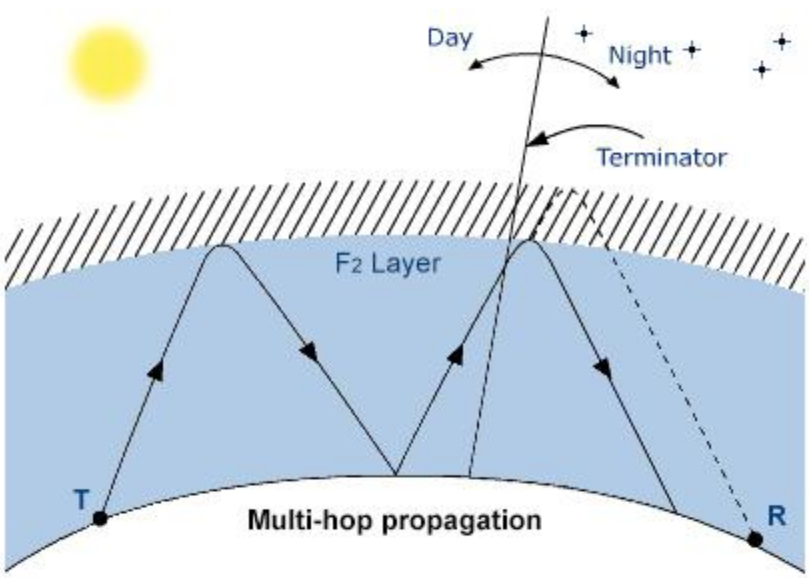
$$\frac{d^2}{(2h)^2} = \frac{f^2{}_{MUF}}{f_c{}^2} - 1$$

$$d^2 = (2h)^2\left[\frac{f^2{}_{MUF}}{f_c{}^2} - 1\right]$$

$$d = 2h\left[\frac{f^2{}_{MUF}}{f_c{}^2} - 1\right]$$

Multi-hop Propagation:

The transmission path is limited by the skip distance and the curvature of the earth,The longest single hop propagation is obtained when the transmitted ray is tangential at the earth surface



Skip distance decreases as the heights are different multiple hop in East-West

The maximum practical distance covered by a sky wave in single hop is 2000km for E-layer and 4000km for F_2 layer

The long distance short wave communication generally involves two to four transmission paths and each contribute appreciable energy to the receiver