ELECTRO-MAGNETIC FIELD THEORY

B.Tech III semester - R18

LECTURE NOTES

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Prepared By

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SYLLABUS

Unit-I	ELECTROSTATICS
Introduction to Cartesian, cylindrical and spherical co-ordinates. Conversion of one type of co-ordinates to another; Electrostatic fields: Coulomb's law, electric field intensity due to line and surface charges, work done in moving a point charge in an electrostatic field, electric potential, properties of potential function, potential gradient, Gauss's law, application of Gauss's law, Maxwell's first law, Laplace's and Poisson's equations, solution of Laplace's equation in one variable.	
Unit-II	**CONDUCTORS AND DIELECTRICS**
Dipole moment, potential and electric field intensity due to an electric dipole, torque on an electric dipole in an electric field, behavior of conductors in an electric field, electric field inside a dielectric material, polarization, conductor and dielectric, dielectric boundary conditions, capacitance of parallel plate and spherical and coaxial capacitors with composite dielectrics, energy stored and energy density in a static electric field, current density, conduction and convection current densities, Ohm's law in point form, equation of continuity.	
Unit-III	**MAGNETOSTATICS**
Biot-Savart's law, magnetic field intensity, magnetic field intensity due to a straight current carrying filament, magnetic field intensity due to circular, square and solenoid current carrying wire, relation between magnetic flux, magnetic flux density and magnetic field intensity, Maxwell's second equation, div(B)=0. Magnetic field intensity due to an infinite sheet of current and a long current carrying filament, point form of Ampere's circuital law, Maxwell's third equation, Curl (H)=Jc, field due to a circular loop, rectangular and square loops.	
Unit-IV	**FORCE IN MAGNETIC FIELD AND MAGNETIC POTENTIAL**
Moving charges in a magnetic field, Lorentz force equation, force on a current element in a magnetic field, force on a straight and a long current carrying conductor in a magnetic field, force between two straight long and parallel current carrying conductors, magnetic dipole and dipole moment, a differential current loop as a magnetic dipole, torque on a current loop placed in a magnetic field; Vector magnetic potential and its properties, vector magnetic potential due to simple configurations, Poisson's equations, self and mutual inductance, Neumann's formula, determination of self-inductance of a solenoid, toroid and determination of mutual inductance between a straight long wire and a square loop of wire in the same plane, energy stored and density in a magnetic field, characteristics and applications of permanent magnets.	
Unit-V	**TIME VARYING FIELDS AND FINITE ELEMENT METHOD**
Faraday's laws of electromagnetic induction, integral and point forms, Maxwell's fourth equation, curl (E)=∂B/∂t, statically and dynamically induced EMFs, modification of Maxwell's equations for time varying fields, displacement current. Derivation of Wave Equation, Uniform Plane Waves, Maxwell's equation in phasor form, Wave equation in Phasor form, Plane waves in free space and in a homogenous material. Wave equation for a conducting medium, Plane waves in loss dielectrics, Propagation in good conductors, Skin effect. Poynting theorem.	

UNIT-1
ELECTRO-STATICS AND VECTOR CALCULUS

1.1 Introduction

The most known particles are photons, electrons and neutrons with different masses. Their masses are

$m_e = 9.10x\ 10\text{-}^{31}$ kilograms

$m_p = 1.67x\ 10\text{-}^{27}$ kilograms

these masses leads to gravitational force between them, given as

$F = G\ m_e\ m_p / r^2$

The force between two opposite charges placed 1cm apart likely to be $5.5x10\text{-}^{67}$ and force between two like charges placed 1cm apart likely to be $2.3x10\text{-}^{24}$.this force between them is called as electric force .

Electric force is larger than gravitational force.

- ➔ Gravitational force is due to their masses.
- ➔ Electric force is due to their properties.
- ➔ Neutron has only mass but no electric force.

ELECTROSTATICS:

Electrostatics is the study of charge at rest. The study of electric and magnetic field can be done using MAXWELL'S equations. Electrostatic field is developed between static charges. Electrostatics got wide variety applications like X-rays, lightning protections etc.

Let us study the behavior of electric field using COLOUMB's and GAUSS laws.

1.2 Point Charge

A charge with smallest dimensions on the body compare to other charges is called as point charge. A group of charges concentrated on any pin head may be also called as point charge.

1.3 Columb's Law

Coloumb stated that the force between two point charges is

- ➔ Directly proportional to product of charges.
- ➔ Inversely proportional square of distance between the.

$F \ \alpha \ Q_1Q_2 / r^2$

$F = K Q_1Q_2 / r^2$, where K is the proportionality constant.

$K = 1/ 4\pi\varepsilon$, where ε is the permittivity of the medium.

$E = \varepsilon_0\varepsilon_r$, ε_0 = absolute permittivity = $8.854x10^{-12}$
ε_r = relative permittivity

most common medium is air or vacuum whose relative permittivity is 1, hence permittivity of air or vacuum is

$\varepsilon = 9x10^9$ m/F

Force between two point charges using vector analysis:

Let us consider two point charges separated by some distance given as $\vec{R}$.

Q_1 ←----------------$\vec{R}$-----------------→ Q_2

According to coloumb's law force between them is given as

$F = (K Q_1Q_2 / r^2) \times \hat{a}$, where $\hat{a}$ is the unit vector direction of force.

Let F_2 is the force experienced by Q_2 due to Q_1 and F_1 is force experienced by Q_1 due to Q_2. The direction of forces opposes each other , hence we can write in vector from forces as

$\overrightarrow{F1} = - \overrightarrow{F2}$

Hence unit vector can be $\hat{a}_{12}$ or $\hat{a}_{21}$, from the vector analysis we can write

$\hat{a}_{12} = \overrightarrow{R_{12}} / R_{12} = \overrightarrow{R} / R$ and

$\hat{a}_{21} = \overrightarrow{R_{21}} / R_{21} = \overrightarrow{R} / R$

Therefore the magnitude of force between them can be written as

$F_1 = F_2 = (K Q_1Q_2 / R^3) \times \widehat{\overrightarrow{R}}$

1.4 Electric Field

It is the region around the point and group charges in which another charge experiences force is called as electric field.

The force between two charges can be studied in terms of electric field as :

1) A charge can develop field surrounding it in space only.
2) The field of one charge leads to force on the other charge .

1.5 Electric Field Intensity

If an point charge q experiences the force F , then the electric field intensity of charge is defines as

$$E = F/q$$

Here charge q is called as test charge because the force experienced by it is due field of other charge.

The units of electric field intensity are N/C or V/mt.

q_1 ←----------------$\vec{r}$-----------------→ q_2

the force experienced by q_2 because of field of q_1 is

$$\text{vector, } F_2 = (K\, q_1 q_2 / r^2) \times \hat{a}$$

Therefore electric filed intensity on q2 charge is

$$\text{Vector,} E = F_2/q_2 = (K\, q_1 / r^2) \times \hat{a}$$

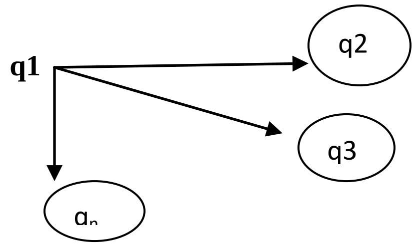
the force experienced by q_1 because of field of q_2 is

$$\text{vector, } F_1 = (K\, q_1 q_2 / r^2) \times \hat{a}$$

Therefore electric filed intensity on q2 charge is

$$\text{Vector,} E = F_1/_{q1} = (K\, q_2 / r^2) \times \hat{a}$$

1.5.1 Electric Field Intensity due to n point charges



let the point charges q_2,q_3--------------qn are placed at a distance of r_2,r_3---------------------------- r_n from q1. Hence total electric field intensity on q1 due remaining point charges is

force due to q2 on q1, $F2= (K\, q_1 q_2 / r^2) \times \hat{a}2$

force due to q3 on q1, $F3= (K\, q_1 q3 / r^2) \times \hat{a}3$

----

force due to qn on q_1, $Fn= (K\, q_1 qn / r^2) \times \hat{a}n$

therefore total electric field intensity is , $\vec{E} = (F_2+F_3$----------------$F_n) / q_1$

$= (K\, q_2 / r^2) \times \hat{a}2 + (K\, q3 / r^2) \times \hat{a}3$ ----+ $(K\, qn / r^2) \times \hat{a}n$

1.6 Charge Distribution

Charge distribution is of three types- line charge, surface and volume charge distribution.

Line charge: Here charge is distributed through out some length . the total charge distributed through a wire of length l is

$$Q = \int \rho_l \, dl$$

Where, ρ_l ----- line charge density

Hence electric field intensity due to line charge is ,

$$E = \int (K \int \rho_l dl / r^2) \times \hat{a}$$

Surface charge: Here charge is distributed through given area . the total charge distributed in an surface area is

$Q = \int \rho_s \, ds$

Where, ρ_s ----- surface charge density

Hence electric field intensity due to surface charge is ,

$E = \int (K \int \rho_s ds/ r^2) \times \hat{a}$

Volume charge: Here charge is distributed through given volume . the total charge distributed in an volume is

$Q = \int \rho_v \, dv$

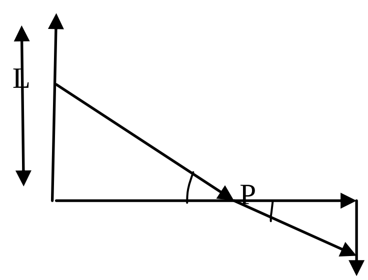
Where, ρ_v----- volume charge density

Hence electric field intensity due to volume charge is ,

$E = \int (K \int \rho_v dv/ r^2) \times \hat{a}$

1.7 Electric filed intensity due line charge

Let us consider a straight wire of length l is symmetrically placed in X-Y axis as shown in below figure



For a small length of dl on y-axis the charge is dq, the electric field intensity due dq at test point p is

$dE = Kdq/(y^2+a^2)$

then, $dEx = Kdq.\cos\Theta/(y^2+a^2)$-------------------0

$\cos\Theta = a/ \sqrt{y^2 + a^2}$------------------------1

we can write charge per unit length as dq/dl = Q/l,

dq =Q.dl/l (dl = dy)

$$dq = Q.dy/l \quad \text{---------------------}2$$

substituting equation 1 and 2 in 0

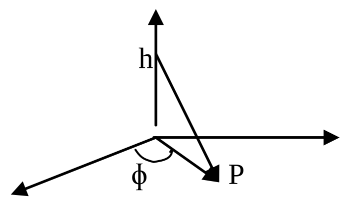
$$dEx = KQ\ dy.a\ /\ l.(y^2+a^2)^{3/2}$$

integrating on both sides with limits –l/2 and l/2, $Ex = \int KQ\ dy.a\ /\ l.(y^2+a^2)^{3/2}$

and when l tends to 0, $Ex = KQ\ /\ a^2$.

1.8 Electric filed intensity due surface charge

Let us consider an infinite sheet placed uniformly in xyz plane as shown in figure.



Let us consider a small area ds in xy plane , ds = ρ.dρ.dϕ.
Which is located at distance of ρ from origin making and angle of ϕ.
P be the point on z axis given as (0,0,h).
Distance from P to ds is $\vec{R}$.

$$\vec{R} = -\rho a_\rho + h a_z$$

$$R = \sqrt{(\rho^2+h^2)}$$

$$a_R = (-\rho a_\rho + h a_z)\ /\ \sqrt{(\rho^2+h^2)}$$

hence the electric field intensity at ds is given as ,

$$\vec{E} = \int \rho s\ ds.\ a_R\ /\ 4\pi\varepsilon.R^2$$

$$\vec{E} = \int \rho s\ \rho.d\rho.d\phi.\ (-\rho a_\rho + h a_z)\ /\ \sqrt{(\rho^2+h^2)}.\ 4\pi\varepsilon.R^2$$

The limits of ρ from 0 to ∞ and ϕ from 0 to 2π.

$$\vec{E} = \int\int \rho s\ \rho.d\rho.d\phi.\ (-\rho a_\rho + h a_z) / \sqrt{(\rho^2+h^2)}.\ 4\pi\varepsilon.R^2$$

By simplying above equation, the electric field intensity

$$\vec{E} = \rho s\ a z / 2\varepsilon.$$

1.9 Work done in moving point charge

Let us consider a charge q is placed in the existing electric field . The charge q experiences force. $\overrightarrow{F}$

$\vec{E}$

----

a -----------------------------b

Here charge q is made to move from a to b of length l through electric field intensity $\vec{E}$.

$$dw = -\vec{F}dl = -q\vec{E}dl$$

integrating on both sides, $w = -q\int \vec{E}dl$ with limits a to b.

1.10 Electric Potential

From the above discussion work done to move point charge through the existing electric field is

$$w = -q\int \vec{E}\ dl$$

but we know that electric potential is defined work done to move unit charge

$$V = w/q$$

Therefore, $V = w/q = -\int \vec{E}\ dl$ with limits a to b

$V = -\vec{E}\ l$ with limits a to b

Hence , electric potential $V = V_a - V_b$

1.11 Electric potential due to point charge

P (point charge)

$\vec{E}$

----

a -----------------------------b

Let the charge q is moved from a to b and at point charge is Q from r_a and r_b ,

We know that electric potential, V = q/(4πεr)

Electric field intensity at point p due to charge at a is, $V_a = Q/(4\pi\varepsilon r_a)$

Electric field intensity at point p due to charge at a is, $V_b = Q/(4\pi\varepsilon r_b)$

Hence potential difference or electric potential from a to b is,

$$V_{ab} = V_a - V_b$$

$$V_{ab} = Q/(4\pi\varepsilon r_a) - Q/(4\pi\varepsilon r_b)$$

$$V_{ab} = Q(r_b - r_a)/(4\pi\varepsilon r_a r_b).$$

1.12 Electric Flux

Macheal faraday has conducted experiment on two concentric spheres, inner layer is positively charges and outer layer negatively charge, then he observe that their some sort of displacement from inner layer to outer layer , this displacement is pronounced as electric flux between spheres.

1.13 Electric Flux Density

We know that electric field intensity is , $E = (K q / r^2)$

$$= q/ (4\pi\varepsilon r^2)$$

$$D=\varepsilon E = q/ (4\pi r^2)$$ —electric flux density

Electric flux density is defined as charge per unit area.

Potential gradient: potential gradient is defined as electric change in electric potential due to change in the distance or length.

$$E = - \nabla V$$

Properties of Potential:

- Potential is the energy acquired by the charge.
- When charge travel from one end to other end in any element there is potential change from high to low.
- Potential acquired by point charge leads to electric field.

1.14 Gauss Law

Gauss law states that the total flux in the given surface is equal to charge enclosed in it.

$$\phi = Q.$$

the total flux enlaced in given surface is

$$\phi = \int E \, ds$$

$$= \int Q / (4\pi\varepsilon\, r^2)\, ds$$

$$= Q / (4\pi\varepsilon\, r^2). \int ds$$

$$= Q4\pi\, r^2 / (4\pi\varepsilon\, r^2).$$

$$= Q / \varepsilon.$$

Applications of Gauss law

- To apply gauss law first assume Gaussian surface.
- The electric field intensity must be normal to the Gaussian surface.
- Gaussian surface must be symmetry.

1.15 Maxwell First Equation

We know that electric flux passing through the surface is equal to 1/ ε times the net charge enclosed.

$$\phi = \int_s E\, ds = Q/ \varepsilon$$

$$\phi = \int_s \varepsilon E\, ds = Q\, \varepsilon / \varepsilon$$

$$\phi = \int_s D\, ds = Q$$

from the strokes theorem we can say that surface integral function is volume integral of divergence of same function.

$$Q= \int_s D\, ds = \int_v (\nabla . D)\, dv \text{ ----------------------3}$$

from the gauss law we can write , $Q= \int_v \rho_v\, dv$ --------------------------- 4

by comparing equation 3 and 4

$$(\nabla . D) = \rho_v \text{---------------------------5}$$

Equation 3 and 5 are said to be Maxwell's first and second equation.

1.16 Poission and Laplace Equations

From the Maxwell's equation we know that,

$$(\nabla . D) = \rho_v \text{-----------------------------------6}$$

$$D = \varepsilon E \text{-------------------------- 7}$$

Equation 7 in 6,

$$\nabla \varepsilon E = \rho_v$$

but we know that, $E = - \nabla V$

$$\nabla \varepsilon(-\nabla V) = \rho_v$$

$$\nabla^2 V = - \rho_v / \varepsilon \text{ ------------------8}$$

Equation 8 is called poission's equation.

In the uniform Gaussian surface ,

$$\rho_v = 0$$

Then equation 8 can be rewrite as,

$$\nabla^2 V = 0 \text{ -------------------9}$$

Equation 9 is called as Laplace equation.

UNIT-II
ELECTRIC DIPOLE AND CAPACITANCE

2.1 Electric Dipole and Dipole Moment

Two opposite charges +q and –q separated by some distance d forms the electric dipole.

+q ----------------d--------------- -q

The distance travelled by the point charge is defined as dipole moment (or) the product of charge and distance travelled by it is called as electric dipole.

$$P = qd \quad \text{-- 1}$$

Here , P → electric dipole moment

d → distance between opposite charges

the line between two charges is called as axis of dipole. Potential

2.2 Electric Dipole Potential

Let us assume two charges separated by distance d as shown in the figure

+q ----------------d--------------- -q

Here, O → center of the axis between charges

P → be the test point where potential is required.

OP → with length of r.

AA^1 → perpendicular from A to OP.

BB^1 → perpendicular from A to OP.

∟POB = ϴ

r >>> d

the line $AP = A^1P = OP + OA^1$--------------------------------2

from the right angle triangle AA^1O, $OA^1 = OA \cos ϴ$

hence equation 2 can be written as, $AP = A^1P = r + OA \cos \Theta$

but, $OA = d/2$

$$AP = A^1P = r + d/2 \ \cos \Theta$$

Hence the potential at P due negative charge at A is ,

$$V_A = -Kq/ AP = -Kq/ r + d/2 \ \cos \Theta$$

Similarly from the right angle triangle BB^1O, $BP = B^1P = r - d/2 \ \cos \Theta$

Hence the potential at P due negative charge at A is ,

$$V_B = Kq/ BP = Kq/ r - d/2 \ \cos \Theta$$

Therefore the total potential acting on P is , $V = V_A + V_B$

$$V = Kq[(1/ r - d/2 \ \cos \Theta) - (1// r + d/2 \ \cos \Theta)]$$

$$= Kqd.\cos \Theta/ (r^2 – d^2/4 \ \cos^2 \Theta)$$

But we know that, $r >>> d$

$$V = Kqd.\cos \Theta/ r^2$$

$$V = KP.\cos \Theta/ r^2 \ , \ (P = qd) \text{-----------------} 3$$

2.3 Electric Field Intensity due to Dipole

We know that electric field intensity in terms of electric potential is given as ,

$$E = - \blacktriangledown V$$

From equation 3 we can say that potential due dipole is in spherical co-ordinates, therefore find electric field intensity we shall use spherical co-ordinates.

$$\blacktriangledown V = -[dv/dr \ + (1/r)dv/d\Theta]$$

Simplifying $\blacktriangledown V$, $dv/dr = -2KP.\cos \Theta/ r^3$

$$(1/r)dv/d\Theta = -KP.\sin\Theta / r^3$$

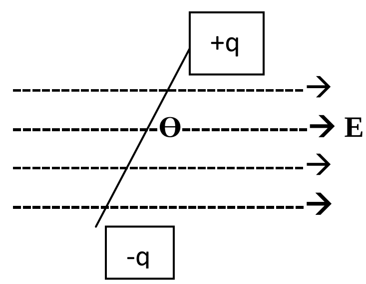
Substituting above two equations in E, $E = -[(-2KP.\cos\Theta/ r^3) + (- KP.\sin\Theta/ r^3)]$

$$= [(2KP.\cos\Theta/ r^3) + (KP.\sin\Theta/ r^3)]$$

$$= KP/ r^3 [(2\cos\Theta) + (\sin\Theta)] \text{ ------------------- } 4$$

2.4 Torque due to Electric Dipole

Let us consider two opposite charges are placed in the uniform electric field with their line of axis of 2r.



The experienced by +q is , $F_1 = E.q$

The experienced by -q is , $F_2= -E.q$

The total experienced by the dipole is , $F = F_1 + F_2$

$$F = 0$$

But the due to force experienced by +q it tends to oscillate in the direction of E and –q in the direction opposite to E, which leads torque of dipole.

T = magnitude of F x perpendicular distance
Between their line of action

$$T = E.q \times 2r \sin\Theta$$

$$T = PE.\sin\Theta. \quad (q.2r = P) \text{--------------} 5$$

2.5 Polarization

If an piece if dielectric or insulator placed between the charges plates of condenser, then center of gravity of negative charges is concentrated towards positive plate and center of gravity of positives charges concentrated towards negative plate, this process of separation opposite charges is called a polarization.

Polarization is also defined as electric dipole moment per unit volume.

Let A be the area of cross section of dielectric,

l be the distance by with opposite charges are separated,

q total charge in the volume of dielectric

then polarization, P = dipole moment / volume

= q.l / A.l

= q / A -- 6

i.e the polarization numerically equal to surface charge density.

2.6 Dielectric Constant and Electric Susceptibilty

Dielectric constant is defined as ratio capacitance of capacitor with dielectric to the capacitance of capacitor without dielectric .

Capacitance of capacitor with dielectric has low potential(V_d) than the capacitance of capacitor without dielectric(V) .

$$K = V / V_d$$ -- 7

The polarization is directly proportional to the electric field intensity created between charges.

$$P \alpha E$$

$$P = K_e E$$

K_e = P / E = electric susceptibility--------------------------- 8

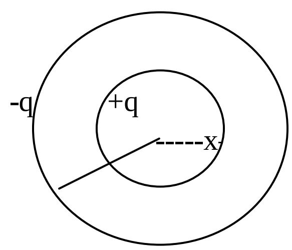
2.7 Capacitor and its Capacitance

The basic capacitor element is formed by separated two parallel plates with some dielectric medium. When some voltage is applied to such an element charge is formed between the plates, their by capacitance of capacitor is defined as charge Q developed between the plates when voltage V is applied.

$$C = Q / V \quad \text{---} \quad 9$$

The units of capacitance are Farads (F).

2.8 Capacitance of the Isolated Sphere



Let us consider an isolated sphere which is positively charges with radius x and negatively charges plate placed at infinite distance.

The electric flux density due to positive charge, $D = Kq / x^2$

Electric field intensity due to positive charge, $\varepsilon E = Kq / x^2$

$E = Kq /. x^2$

Work done, $w = - q \int E\, dl.$

$W = -q \int E\, dx$ with limits ∞ to x

$V = - \int E\, dx$ with limits ∞ to x

$V = - \int Kq /. x^2\, dx$ with limits ∞ to x

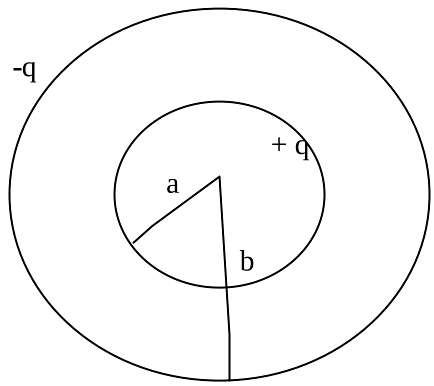
$= -K.q \; / \; (-. x)$ with limits ∞ to x

$= K.q \; / \; (. x)$

But the capacitance is given charge per voltage, $C = q / V$

$C = (x) / K$ ------------------------------- 10

2.9 Capacitance of the spherical Sphere



Let us consider an isolated sphere which is positively charges with radius a and negatively charges plate placed at b distance.

The electric flux density due to positive charge, $D = Kq / x^2$

Electric field intensity due to positive charge, $\varepsilon E = Kq / x^2$

$E = Kq /. x^2$

Work done, $w = - q \int E\, dl.$

$W = -q \int E\, dx$ with limits b to a

$V = - \int E\, dx$ with limits b to a

$V = - \int Kq /. x^2\, dx$ with limits b to a

$= -K.q / (-. x)$ with limits b to a

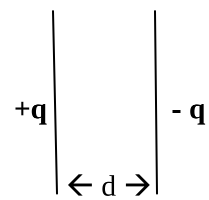
$= \frac{Kq}{}. [(1/a) – (1/b)]$

$$= \frac{Kq\ (b-a)}{ab}$$

But the capacitance is given charge per voltage, $C = q / V$

$C = ab / K(b-a)$ -------------------------------- 11

2.10 Capacitance of the Parallel Plates



Let potential applied to these parallel plates is V their by forming charge q between them.

Electric flux density between plates, $D = q / A$

$\varepsilon E = q / A$

$E = q / \varepsilon.A,$

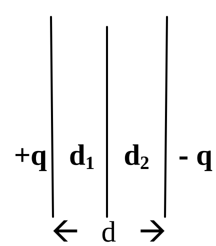
$V = E.d$

$V = q\, d / \varepsilon.A$

But the capacitance is given charge per voltage, $C = q / V$

$C = \varepsilon.A / d$ ------------------------------- 12

2.11 Capacitance of the Parallel Plates with two dielectric mediums



Let potential applied to first part is V_1 their by forming charge q between them.

Electric flux density between plates, $D = q / A$

$\varepsilon E_1 = q / A$

$E_1 = q / \varepsilon_1 . A,$

$V_1 = E . d_1$

$V_1 = q\, d_1 / \varepsilon_1 . A$

But the capacitance is given charge per voltage, $C = q / V$

$C_1 = \varepsilon_1 . A / d_1$ ------------------------------- 13

Let potential applied to first part is V_2 their by forming charge q between them.

Electric flux density between plates, $D = q / A$

$\varepsilon E_2 = q / A$

$E_2 = q / \varepsilon_2 . A,$

$V_2 = E . d_2$

$V_2 = q\, d_2 / \varepsilon 2_1 . A$

But the capacitance is given charge per voltage, $C = q / V$

$C_2 = \varepsilon_2 . A / d_2$ ------------------------------- 14

Hence total capacitance between plates with multiple dielectric mediums is ,

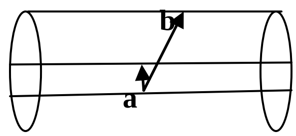
$C = C_1 + C_2$

$= (\varepsilon_1 . A / d_1) + (\varepsilon_2 . A / d_2)$

$= A / [(d_1 / \varepsilon_1) + (d_2 / \varepsilon_2)$ ---------- 15.

2.12 Capacitance of the Co-axial Cable

Let us consider co-axial cable two isolated sphere with radius a and b from center of axis



the length of cable is , then line charge distribution $\rho_l = q / l$

the electric flux density generally in cable is , $D = \rho_l / 2\pi r$

therefore electric filed intensity , $E = \rho_l / 2\pi r\varepsilon$

the electric potential of the cable is , $V = -\int Edr$, with limits b to a

$= -\int (\rho_l / 2\pi r\varepsilon)\, dr$

$= -(\rho_l / 2\pi\varepsilon) \int dr/r$

$= -(\rho_l / 2\pi\varepsilon)\ .\ln(r)$

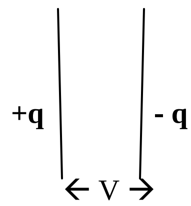
By applying limits, $V = -(\rho_l / 2\pi\varepsilon)\ .[\ln(a) - \ln(b)]$

$V = (\rho_l / 2\pi\varepsilon)\ .\ln(b/a)$

The capacitance of co-axial cable, $C = \rho_l / V$

$C = 2\pi\varepsilon / \ln(b/a)$ ----------------------------------- 21

2.13 Energy Stored in the Capacitor



By the definition capacitance between plates is , $C = q / V$

Electric potential, $V = dw / dq$

$dw = V\, dq$

$dw = (q / C)\, dq$

integrating on both sides, $w = \int (q / C)\, dq$

$w = q^2 / 2C$ (or) ---------------------------- 16

$w = (CV)^2 / 2C$

$w = CV^2 / 2$ (or)--------------------------- 17

$w = q^2 / 2C$

$w = Vq / 2$ ---------------------------------18

2.14 Energy Density in the Static Electric Field

Energy density of capacitor is defined energy stored per unit volume.

W_d = energy stored / volume

$W_d = CV^2 / 2 / Ad$

$W_d = \varepsilon A\, V^2/d \,/\, 2.\, Ad$

$W_d = \varepsilon\, V^2/2d^2$

$$W_d = \varepsilon E^2 / 2$$

$$W_d = DE/2 \text{ ------------------------------------ } 19$$

From equation 19 we can write,

$$dW / dV = DE/2$$

$$dW = (DE/2)\ dV$$

integrating on both sides, energy stored $W = \int_v (DE/2)\ dV$ -------------------- 20

2.15 current

The flow of electrons from one end to other end constitutes current. The rate of change of Charge is also defined as current.

$$i = q / t = dq / dt \text{ -- } 22$$

the units of current is ampere.

Current Density

If charge is distributed in the given area, then current density is defined as current constituted In given area.

$$J = i / A \quad (A/mt^2) \text{ --- } 23$$

$$J = di / ds$$

$$di = J\ .ds$$

integrating on both sides, $i = \int J\ .ds$

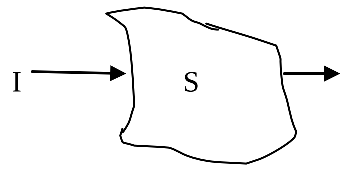
Convection Current Density

Let us consider a material with volume of charges (ρ_V) moving with drift velocity (V_d) **, then** Convection Current density is defined as product volume of charges moving with drift velocity.

$$\mathbf{J} = \rho_V \times V_d \text{ --- } 24.$$

2.16 Equation of Continuity

Let us an surface area through charges are moving in and out as shown in the figure



Let the charge q is moving through an area of S.

According law of conservation of charge, $[I]s = - dq / dt$

But current passing through area is , $[I]s = \int J \, ds$

Total charge in the given volume is, $q = \int_v \rho_v \, ds$

From above three equations we can write,

$$\int J \, ds = -(d/dt). \int_v \rho_v \, dv \text{------------------}25$$

from the stokes theorem we can write,

$$\int J \, ds = \int_v \nabla J \, dv \text{ ------------------------- } 26$$

by comparing equation 25and 26,

$$\int_v \nabla J \, dv = -(d/dt). \int_v \rho_v \, dv$$

$$\int_v \nabla J \, dv + (d/dt). \int_v \rho_v \, dv = 0$$

$$\int_v [\nabla J + d\rho_v / dt] \, dv = 0 \text{ ------------------ } 27$$

equation 27 is called as equation of continuity or maxwell's fifth equation.

UNIT-III
MAGNETO-STATICS

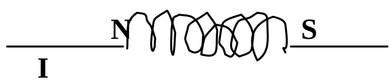
3.1 Introduction

Magneto-statics is the study of magnetic field developed by the constanr current through the coil Or due to permanent magnets.

The behavior of contant magnetic field is studied by using two basic laws, they are

- ➔ Bi-Savart's law
- ➔ Ampere's circutal law.

3.2 Magnetic Field



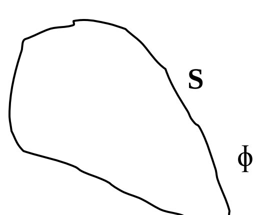
Let us consider a constant current I is passing through coil shown above which develops constant Flux surrounding the coil their by forming north and south poles. This formation magnetic from North pole to south pole ia called as magnetic field. The direction of magnetic flux in an coil is

Given by right hand thumb rule.

Right hand thumb rule says that if four fingers of hand folded such that they show direction of winding Then thumb indicates direction of flux.

3.3 Definitions of Magnetic Field

Magnetic flux density



magnetic flux density is defined as flux per unit area, $B = d\phi / ds$ (Wb/mt^2 or Tesla)

$$d\phi = B\, ds$$

by integrating on both sides we can determine total magnetic flux in area,

$$\phi = \int B\, ds \text{ -----------------------28}$$

Magnetic Field Intensity

The force experienced by coil when some current passes through it is magnetic field Intensity. Mathematically magnetic field intensity is givens as,

H = magnetic force / length

Magnetic force = NI

Length = l

Therefore magnetic field intensity, $H = NI / l$ (AT/mt) ---------------------------- 29

Magnetic Permeabilty

Permeability is the inherent property of core which helps in sustaining flux in the core.

Mathematically permeability is given as, $\mu = B / H$ ------------------------- 30

From equation 30 the relation between flux density and intensity is ,

$B = \mu H$ ------------------------ 31

Where $\mu = \mu_0 \mu_r$

μ_0 = absolute permeability = $4\pi x 10^{-7}$ H/mt

μ_r = relative permeability

varies from core to core

Intensity of Magnetization

When a magnetic substance is placed in a magnetic field it experiences magnetic momentum. The magnetic momentum per unit volume of substance is intensity of magnetization.

$I = M / V$

$M = m.l$ (m- pole strength of bar, l – length)

$$V = A.l$$

intensity of magnetization, $I = m.l / A.l$

$$I = m / A$$

Magnetic Susceptibility

The ratio intensity of magnetization to the magnetic field intensity is called as Magnetic Susceptibility.

$$K = I / H.$$

Total flux density, B = B due to magnetic field + B due to intensity of magnetization of bar

$$B = \mu_0 H + I$$

But we know that, $\mu = B / H$

$$= (\mu_0 H + I) / H$$

$$= \mu_0.+ (I/H)$$

$$\mu_0 \mu_r = \mu_0.+ K$$

$$\mu_r = 1 + K / \mu_0 \qquad \text{--} \; 31$$

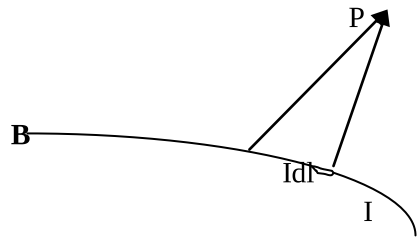
$\mu_r > 1$, paramagnetic materials

$\mu_r < 1$, diamagnetic materials

$\mu_r = 0$, non-magnetic materials

3.4 Biot-Savart's Law

Bio and savart are two scientists who conducted experiments on current carrying conductor To determine magnetic flux ddensity(B) at any point surrounding that conductor. Their Conclusion is named as "Biot-Savart's Law".



Let us consider an conductor carrying current I, which develops magnetic flux density B surrounding It. Here Idl is called as current element. To find total electric field intensity conductor is divided into Number of current elements.

The magnetic field intensity due to current element Idl is dH at point P. According Bio-Savart's law

$dH \propto Idl$ (current element)

$dH \propto \sin\Theta$ (angle between current element and length joining point)

$dH \propto 1 / r^2$ (square of distance between current element and point)

by combining above three,

$$dH \propto Idl \cdot \sin\Theta / r^2$$

by removing proportionality,

$$dH = Idl \cdot \sin\Theta / 4\pi r^2$$

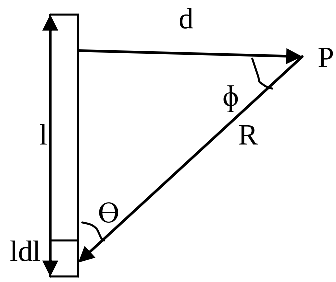
total magnetic field intensity at point P,

$$H = \int Idl \cdot \sin\Theta / 4\pi r^2$$

therefore total flux density at point P, $B = \mu H$

$$B = \mu \int Idl \cdot \sin\Theta / 4\pi r^2 \quad \text{--------------------------------------} \ 32$$

3.5 Magnetic Field Intensity due to a finite length of current carrying filament



Let us consider a straight conductor of length l, a test point P at which electric field intensity is to be Determined at a distance of d from conductor. Assume current element with a distance of R to P.

From Bio-Savart's law magnetic field intensity at test point P due to current element Idl is ,

$$dH = Idl \cdot \sin\Theta \, .\hat{a} / 4\pi R^2 \text{ ------------------------------- a}$$

from above right angle triangle, $\Theta + \phi = 90^0$ --- b

using equation a and b, $dH = Idl \cdot \cos\phi \, .\hat{a} / 4\pi R^2$ --------------------------------- c

the unit vector $.\hat{a}$, indicates the direction H at point P.

$$.\hat{a} = .\hat{R} / R \text{ -- d}$$

from above right angle triangle, $R = \sqrt{l^2 + d^2}$ -- e

$$\cos\phi = d / \sqrt{l^2 + d^2} \text{ ------------------------------------ f}$$

$$\tan\phi = l / d \text{ -- g}$$

$$l = d. \tan\phi$$
$$dl = d \sec^2\phi \, d\phi \text{ ------------------------------------ h}$$

substituting d,e,f in c,

$$dH = Idl \cdot \cos\phi \, .d \, .\hat{R} / 4\pi (l^2 + d^2)^2$$

integrating on both sides

$$H = \int Idl \, . \cos\phi \, .d \, .\hat{R} / 4\pi (l^2 + d^2)^{3/2}$$

$$H = \frac{I}{4\pi d^2} \int dl / (l^2 / d^2 + 1)^{3/2}$$

$$H = \frac{I}{4\pi d^2} \int dl / (\tan^2\phi + 1)^{3/2}$$

Substituting equation h in above equation is ,

$$H = \frac{I}{4\pi d^2} \int d \sec^2\phi \, d\phi / (\sec^2\phi)^{3/2}$$

$$H = \frac{I}{4\pi d^2} \int d \sec^2\phi \, d\phi / (\sec^3\phi)$$

$$H = \frac{I}{4\pi d} \int \cos\phi \, d\phi$$

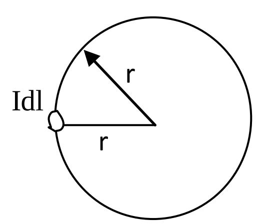
$$H = \frac{I}{4\pi d} \sin\phi \text{ --33}$$

For straight line of infinite length, ϕ varies between $-\pi/2$ to $\pi/2$

Substituting above limits in equation 33, $H = \frac{I}{2\pi d}$ --- 34

3.5 Magnetic Field Intensity due to a circular current carrying filament

Let us consider circular conductor with radius r,



magnetic field intensity at the center of circular conductor is,

from above figure we can say that idl and center are at 90^0

using Bio-Savart's law magnetic field intensity at center point P due to current element Idl is,

$$dH = idl \sin 90 / 4\pi r^2$$

$$dH = idl / 4\pi r^2$$

integrating on both sides, $$H = \int idl / 4\pi r^2$$

$$H = i \int dl / 4\pi r^2 \qquad (\int dl = 2\pi r)$$

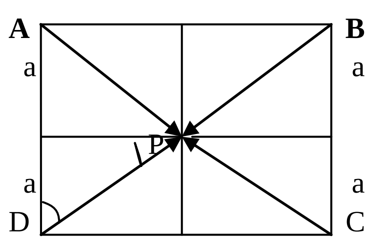
$$H = i\, 2\pi r / 4\pi r^2$$

$$H = i / 2r \text{ --- } 34$$

Magnetic field intensity at the center of circular conductor with N number of turns is,

$$H = Ni / 2r \text{ --- } 35$$

3.6 Magnetic Field Intensity due to a Square current carrying filament



From the above figure we can say that each side AB,BC,CD,DA has magnetic field intensity at the center Of square conductor.

In every right angle triangle angle between current element and center is 45^0.

The total magnetic field intensity at the center of square due to all corners using Bio-Savart's law

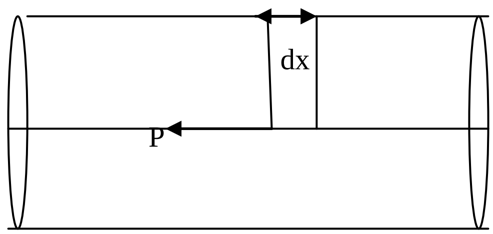
Because of any one side, $$H = (I / 4\pi a) \times [\sin 45^0 + \sin 45^0]$$
Using all sides, $$H = 4(I / 4\pi a) \times [\sin 45^0 + \sin 45^0]$$

$$H = (I / \pi a) \times [2 / \sqrt{2}]$$

$$H = (\sqrt{2}.I / \pi a) \text{ ------------------------------------- } 36$$

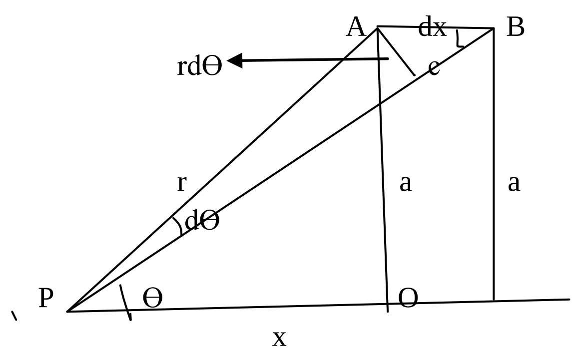
3.7 Magnetic Field Intensity due to a Solenoid current carrying filament

The construction of solenoid is same as coil wounded on a cylinder , let us take take cylinder As reference and derive expression for H due to solenoid. The solenoid with length l, number of turns N allowing an current of I is shown in below figure,



Assume a small length dx, with total turns ndx in it , let us derive what is the magnetic field intensity

Due to dx on P, their by total H at P.



total number of turns = N

total length = l

number of turns per unit length, $n = N / l$

x be the distance of the point,

the magnetic field intensity due to length dx on P is ,

$$dH = (Ia^2 / 2r^3)\, ndx$$

from figure , $r = \sqrt{a^2 + x^2}$, substituting r in dH.

$$dH = (Ia^2 / 2\,(a^2 + x^2)^{3/2})\, ndx$$

from above right angle triangles, $d\Theta <<< \Theta$, hence $\sin d\Theta = d\Theta$

$$\sin \Theta = r\, d\Theta / dx$$

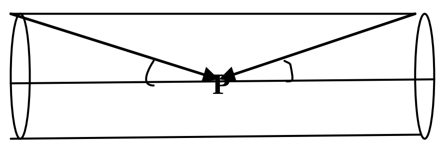
$$\sin \Theta = a / r$$

substituting above deduction in dH,

$$dH = (Ia^2\, r.d\Theta / \sin \Theta / 2r^3)\, n$$

$$dH = I.n.\ \sin \Theta.\ d\Theta / 2 \text{ --- a}$$

if seen from end points of solenoid the magnetic field intensity at P is



Here from one end to other end angle varies from 0 to 2π, substituting above and integrating equation a

$$\int dH = \int\ I.n.\ \sin \Theta.\ d\Theta / 2$$

$$H = -\ I.n.\cos \Theta.\ / 2, \text{ substituting above limits ----------------b}$$

$$H = -(I.n/2)\ [\cos 2\pi - \cos 0]$$

$$H = I.n = NI/l$$

if seen from end point of solenoid the magnetic field intensity at P at same end point,

then the limits varies between 0 to $\pi/2$
substituting above limits in b

$$H = -(I.n/2)\ [\cos\pi/2 - \cos 0]$$

$$H = n.I/2 = N.I/ 2l \text{ -- 37}$$

3.8 Maxwell's Second Equation

From the guass law we can write magnetic flux in the given surface is surface integral of

Magnetic flux density.

$$\Psi = \int B.ds$$

But total flux density in closed surface is always zero,

$$\Psi = \int B.ds = 0$$

By applying divergence theorem we can write,

$$\int B.ds = \int_v \nabla B.dv = 0$$

hence we can write , $\nabla B = 0$, is Maxwell's second equation------------------------ 38

3.9 Ampere Circuital Law

The ampere circuital law states line integral magnetic filed intensity around any closed path is equal to total current enclosed in that path.

$$\int H\, dl = I \text{ ---39}$$

Ampere's law is analogous to gauss law electro-statics.

Applications of Ampere's law :

➔ The magnetic field intensity in the surrounding closed path is always at tangential at Each and every point on it.

➔ At each every point on the closed path magnetic field intensity has the same value.

3.10 Maxwell's Third Equation

From the ampere circuital law we know that,

$$\int H\, dl = I$$

but current can be written as, $\int J\, ds = I$

equating above two equations, $\int H\, dl = \int J\, ds$ ---------------------------a

from stokes theorem, $\int H\, dl = \int \nabla \times H\, ds$ --------------------- b

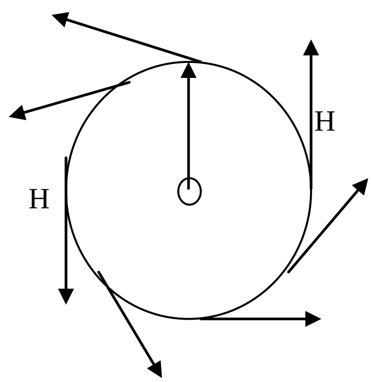
by combining equation a and b, $\int \nabla \times H\, ds = \int J\, ds$

by comparing on both sides, $\nabla \times H = J$, $\nabla \times H$ = curl of H ------40

equation 40 is called as differential, integral or point form of ampere's law and also called as **Maxwell's Third Equation**

3.11 Magnetic field intensity due to long straight conductor using ampere's law

Let us consider a straight conductor as shown in figure with closed path of magnetic field Intensity surrounding it with radius of r.



From ampere's circuital law we can write magnetic field intensity in closed path,

$$\int H\, dl = I$$ -------------------------------------a

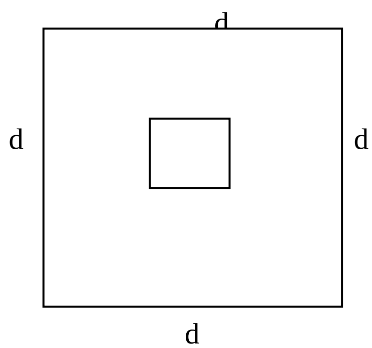
but we can write, $\int H\, dl = H \int dl$

$= H\, 2\pi r$ ----------------------------- b

Equating a and b, $H\,2\pi r = I$

$$H = I / 2\pi r \text{ ------------------------------------ } 41$$

3.12 Magnetic field intensity due to infinite sheet conductor using ampere's law



let us consider a square sheet as shown above with surrounding current path of side d.

according to Ampere's law ,

$$\int H\, dl = I$$

where $\int dl$ indicates the mean length closed path,

$$\int dl = 4d$$

their by , $H \int dl = I$

$$H.4d = I$$

$$H = I/4d. \text{--} 42$$

UNIT-IV
FORCE IN MAGNETIC FIELD AND MAGNETIC POTENTIAL

4.1 Force on moving charge.

When an charge Q is with velocity $\vec{V}$ is placed in the magnetic field of density $\vec{B}$, then it

Experiences force called as magnetic force.

$$\overrightarrow{Fm} = Q(\vec{V} X \vec{B}) \text{ -- } 43$$

$$=QVB \sin\Theta \, a_f$$

$\vec{V}$ is parallel to $\vec{B}$ then Θ= 0, therefore sinΘ = 0, hence always velocity direction and flux density

Direction must be normal to each other.

4.2 Moving charges in the magnetic field

The limitations of moving charge in the existing magnetic field,

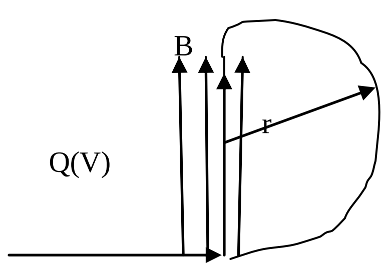
- ➔ If the velocity of charge in the magnetic field is zero then force experienced also zero.
- ➔ If the velocity direction and magnetic field direction are parallel to each other then force Experienced is zero.
- ➔ To say that moving charge in the magnetic field experiences force velocity and field must be normal to each other.

From the above discussion the force experienced by moving charge is ,

$$Fm = QVB.$$

Similarly we can also write force experienced by moving charge due to its mass is ,

$$Fm = ma$$



r is the radius made by path travlled by charge when it experiences force.

$$Fm = mV^2/r$$

By equating both forces, $QVB = mV^2/r$

$$r = mV / QB$$

time taken to complete one revolution in field is ,

$$T = 2\pi r / V$$

$$= 2\pi m / QB$$

Hence frequency of charge in field is ,

$$F = 1/ T$$

$= QB / 2\pi m$, as this expression of frequency is independent

Of velocity it is called as cyclotron.

4.3 Lorentz force equation

We know that the force acquire by point charge when kept in the static electric field is,

$$\overrightarrow{Fe} = Q\,\vec{E}$$

The force experienced by moving charge in the magnetic field is ,

$$\overrightarrow{Fm} = Q(\vec{V} X \vec{B})$$

The total force on the charge in the presence of both field is,

$$\vec{F} = \overrightarrow{Fe} + \overrightarrow{Fm}$$

$$= Q\,\vec{E} + Q(\vec{V} X \vec{B})$$

$= Q(\vec{E} + (\vec{V} X \vec{B}))$ --- 44

Equation 44 is called as **Lorentz force equation.**

4.4 Force on current element due to magnetic field

Let us a long conductor of length l which is partitioned into number parts allowing current

Of I. each part of conductor is of length dl, therefore individual part is represented with Idl called

As current element.

Force due to current element at any point

We know that convection current density is ,

$$\vec{J} = \rho_v \vec{V}$$

The current elements are ,

$$\vec{J}\, dv = K\, ds = \vec{I}\, dl$$

Using above two equations,

$$\vec{I}\, dl = \rho_v \vec{V}\, dv = Q\vec{V}$$

Also current element, $\vec{I}\, dl = (dQ/\, dt).dl$

$$= dQ.\vec{V}$$

The force experienced by moving charge we know as ,

$$\overrightarrow{dFm} = Q(\vec{V} X \vec{B})$$

$$= \vec{I}\, dl \; X\, \vec{B}$$

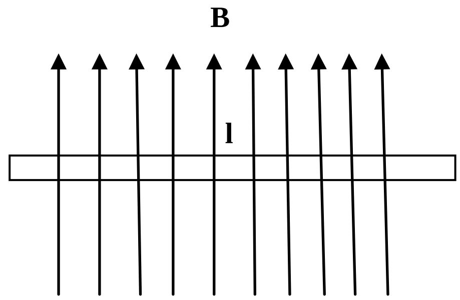
Integrating on both sides we can determine force due current element,

$$\overrightarrow{Fm} = \int \vec{I}\, dl \; X\, \vec{B} \text{ --- } 45$$

Similarly, $\overrightarrow{Fm} = \int_s \vec{K}\, ds \; X\, \vec{B}$

$$\overrightarrow{Fm} = \int_v \vec{J}\, dv \times \vec{B}$$

4.5 Force on a straight long current carrying conductor placed in the magnetic field



Let us consider a straight conductor placed in the magnetic field as shown in the figure,

Of length l, allowing current of I, hence current element if Idl,

The velocity of charges in the given length of conductor is $\vec{V}$.

The force experienced by current element is ,

$$\overrightarrow{dFm} = dQ(\vec{V} X \vec{B})$$

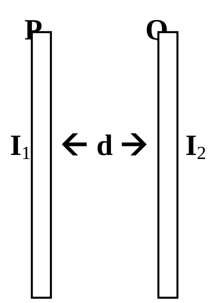
$$= dQ(dl/dt\ X\vec{B})$$

$$= I\ (\overrightarrow{dl} X \vec{B})$$

Their by integrating on both sides, $\overrightarrow{Fm} = I\ (\vec{l} X \vec{B})$

$$Fm = BIl \sin\Theta \text{ -------------------------------- } 46$$

4.6 Force on a straight parallel long current carrying conductors placed in the magnetic field



Let us consider two straight parallel current carrying conductors of length l separated by distance d

As shown above,

The magnetic field intensity due conductor P on Q is,

$$H = \mathbf{I}_1 / 2\Pi d$$

The magnetic flux density due conductor P on Q is,

$$B = \mu_0 \mathbf{I}_1 / 2\Pi d$$

Hence forced experienced by conductor Q due to field of P is,

$$F1 = B \mathbf{I}_2 l$$

$$= \mu_0 \mathbf{I}_1 \mathbf{I}_2 l / 2\Pi d$$

Similarly force experienced by P due to conductor Q is ,

$$F2 = \mu_0 \mathbf{I}_1 \mathbf{I}_2 l / 2\Pi d$$

Hence force per unit length of conductor is ,

$$(F / l) = \mu_0 \mathbf{I}_1 \mathbf{I}_2 / 2\Pi d \text{ ------------------------------------ } 47$$

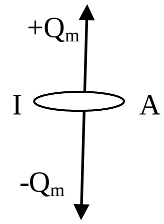
4.7 Magnetic dipole and dipole moment

Magnetic dipole is formed when two opposite magnetic charges are separated by distance l.

$-Q_m$ --------------l-------------- $+Q_m$

The line joining two charges is termed as axis of dipole. Direction magnetic dipole is from $-Q_m$ to $+Q_m$

In other words a bar magnet with pole strength Q_m and l has , magnetic dipole moment, $m = Q_m l$.



Let us consider a bar conductor allowing current I their forming loop of area A, magnet poles formed As shown in the figure.

Magnetic dipole moment , m= IA

Numerically both dipole moment must be same, Q_m l = IA

Magnetization

If their exist an conductor consisting of number of dipoles in its volume , then magnet dipole Moment per unit volume is called as magnetization.

$$M = m / V$$

$$= Q_m .l / A.l$$

$$= Q_m / A$$

Magnetic susceptibility

When the magnetic field is applied to an material the ,

Total magnetic field intensity is ,

$$\overrightarrow{B} = \mu_0 \overrightarrow{H} + \mu_0 \overrightarrow{M}$$

$$= \mu_0 \mu_r H$$

Therefore,

$$\mu_0 \mu_r \overrightarrow{H} = \mu_0 \overrightarrow{H} + \mu_0 \overrightarrow{M}$$

$$\overrightarrow{M} = (\mu_r - 1) \overrightarrow{H}$$

$$\overrightarrow{M} = X_m \overrightarrow{H}$$

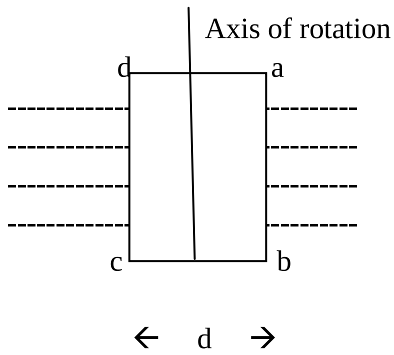
$X_m = (\mu_r - 1)$, is called as magnetic susceptibility

$$= \overrightarrow{M} / \overrightarrow{H} \text{ -------------------------------------- } 48$$

4.7 Torque due to Magnetic dipole

Let us a sheet of side abcd placed in the magnetic field , the side ab experiences the force into

The page and side cd out of the page. Angles made by sheet with magnetic field are α and β.



the total torque experienced by sheet due to dipole is ,

T = 2 x torque on each side

= 2 x force x distance from axis of rotation

= 2 x F x d/2

= 2 x BIl cos β x d/2

= BIA cos β

= mB cos β   or mB sin α

Therefore torque vector , $\vec{T} = \vec{m} \times \vec{B}$ -- 49

4.8 Scalar magnetic potential

Form the electro-statics we know that, $E = -\nabla V$

Similarly in the magneto-statics , $H = -\nabla V_m$

V_m – vector magnetic potential

Applying curl on both sides of H, $\nabla \times H = -\nabla \times (\nabla V_m)$

But curl of divergence of any vector is zero, $\nabla \times H = 0$

We can also write , $\nabla \times H = J$

From the above two equations we can write , J = 0.

This is possible only in the case constant magnetic field.

from the electro-statics we know that, $\int E\, dl = V$

Similarly in the magneto-statics , $\int H\, dl = V_m$

Ampere circuital law says that, $\int H\, dl = I$

Comparing last two equations, $V_m = I$ ---50

Hence the units of scalar magnetic potential is Amperes.

4.9 Vector magnetic potential

We know that divergence magnetic flux density over uniform closed surface is always zero.

$$\nabla B = 0$$

Also divergence of curl of vector is always zero.

$$\nabla .(\nabla \times A) = 0$$

By comparing above two equations,

$$B = \nabla \times A$$

$$\mu H = \nabla \times A$$

$$H = (\nabla \times A) / \mu$$

Applying curl on both sides,

$$\nabla \times H = \nabla \times (\nabla \times A) / \mu = J$$

But, $\nabla \times (\nabla \times A) = \nabla . (\nabla . A) - \nabla^2 A = \mu J$

For time invariant fields divergence of vector is zero, hence above can be written as

$$- \nabla^2 A = \mu J$$

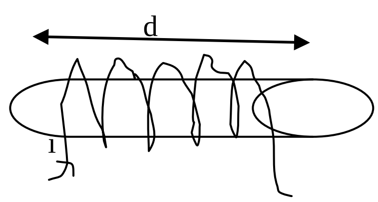
$$\nabla^2 A = - \mu J$$

Form the electro-statics we know that, $dv = dq/ 4\pi\varepsilon$

Similarly in the magneto-statics , $dA= \mu idl/ 4\pi r$

Integrating on both sides, $A = \int \mu idl/ 4\pi r$, A- vector magnetic potential ---------51

4.10 Self inductance of a solenoid



let us consider a solenoid as shown in figure with length l allowing an current of i A.

N – total turns of solenoid coil

N – number of turns per unit length

magnetic filed density inside solenoid is , $B = \mu_0 n.i.$

total flux linking with coil is $\phi = N B A$

$$= \mu_0 n\ l.i.A\ .n$$

$$= \mu_0 n^2.i.A\ .l$$

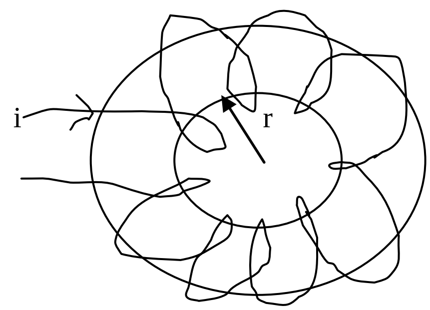
Self inductance is the property of coil which is responsible for emf induced in it,

$$L = N \phi / i$$

$$= \mu_0 \, n^2.i.A \, .l / i$$

$$= \mu_0 \, N^2A / l \;\; H \text{ ----------------------------52}$$

4.11 Self inductance of a Toroid



Let us a toroid on which a coil N turns is wounded allowing an current of i A.

Let r be the mean radius of the toroid.

Magnetic flux density in the toroid, $B = \mu_0 \, Ni / l$

Where , $l = 2\pi r$

$$B = \mu_0 \, Ni / 2\pi r$$

Total flux linkage with toroid is , $\phi = NBA$

$$= (N \mu_0 \, Ni / 2\pi r) . A$$

But, area $A = \pi R^2$

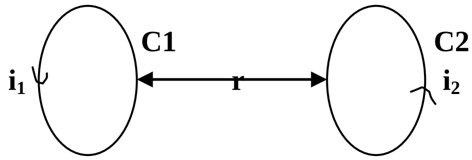
$$\phi = (N \mu_0 \, Ni / 2\pi r). \pi R^2$$

$$= (N^2 \mu_0 \, i \, R^2 / 2r).$$

Therefore self inductance of toroid is , $L = \phi / i$

$$= (N^2 \mu_0 \, R^2 / 2r). \, H \text{ ------------------------ 53}$$

4.11 Neumann's formulae



let us consider two circular coils brought as near as possible allowing i_1 and i_2 currents, with separation of r, of an areas S_1 and S_2 .

the magnetic flux density due to current i_1 is ,

$$B_1 = \nabla \times A_1.$$

Vector magnetic potential , $A_1 = \int \mu i_1 dl_1 / 4\pi r$

Hence flux with second coil due to i_1,

$$\phi_{21} = B_1\, dS_2$$

hence total flux linking with second coil is ,

$$\Psi_{21} = \int B_1\, dS_2$$

$$= \int (\nabla \times A_1)\, dS_2$$

From stokes theorem, $\int (\nabla \times A_1)\, dS_2 = \int A_1\, dl_2$

Substituting this inn above equation ,

$$\Psi_{21} = \int A_1\, dl_2$$

$$= \int \int \mu i_1 dl_1\, dl_2 / 4\pi r$$

Therefore mutual inductance between two coils is ,

$$M_{21} = \Psi_{21} / i_1$$

Mutual inductance is the imaginary concept which says that there is flux linkage with second Coil because of current flowing through first coil.

$$M_{21} = \int\int \mu i_1 dl_1\, dl_2 / 4\pi r \;/\; i_1$$

$$M_{21} = \int\int \mu dl_1\, dl_2 / 4\pi r \text{ ------------- } 54$$

This M_{21} is called as **Neumann's formulae.**

4.11 Energy stored in the magnetic field

Let the work done to increase the current by di is dw, by law of conservation of energy Work done is equal to energy stored .

$$dw = vi\, dt$$

$$= L.idi.\, dt/dt$$

$$dw = Lidi$$

integrating on both sides , $\int dw = \int Lidi$

$$w = Li^2 / 2$$

but we know that, $L = N\phi / i = \Psi / i$

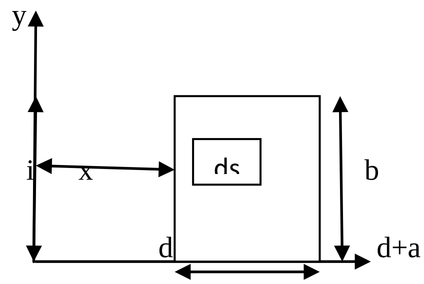
using above expressions we can write energy stored in the magnetic field also as,

$$w = \Psi\, i / 2$$

$$= \Psi^2 / 2\, L. \text{ ------------------------------ } 55$$

4.12 Mutual inductance between straight long and square conductors

Let us consider a straight and square conductor placed in the xy plane as shown.



here straight conductor is placed on y-axis and square loop as shown is xy plane.

The magnetic flux density due to straight wire o square loop is ,

$$B = \mu_0.i/ 2\pi x$$

The flux linking with square loop because current in straight wire is,

$$M = \Psi / I$$

From the gauss law we know that,

$$\Psi = \int_s B\ ds$$

$$= \int\int (\mu_0.i/ 2\pi x)\ dxdy, \text{ with limts}$$

$$x = d \text{ to } d+a,\ y = o \text{ to } b$$

$$= \int (\mu_0.i/ 2\pi x).y\ dx$$

Substituting limits of y, $= \int (\mu_0.i/ 2\pi x).b\ dx$

Then, $= (\mu_0.i\ b / 2\pi).\ln(x)$

Substituting limits of x, $= (\mu_0.i\ b\ \ln(d+a)/ 2\pi \ln(d))$.

Therefore mutual inductance between two conductors is ,

$$M = \Psi / i$$

$$= (\mu_0 . B. \ln(d+a)/ 2\pi \ln(d)) \text{.--------------- } 56$$

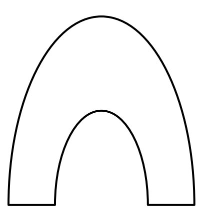
4.13 Characteristics and applications of permanent magnets

Characteristics:

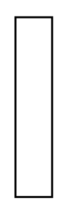
- ➔ Permanent magnets are the one which readily available in nature in the form of Bar and horse shoe shapes etc.
- ➔ Permanent magnets irrespective of supply always exhibits magnetic properties.
- ➔ Permanent magnets always develops a constant magnetic field.
- ➔ The strength of the permanent magnets measured in terms of their cohesive force.
- ➔ An permanent magnet with high cohesive force will have long life.
- ➔ Permanent magnet got the disadvantage of ageing effect i.e in long run they may get rusted.

Applications:

- ➔ Permanent magnets are used in the applications where ever it is required to develop Constant magnetic field . Eg- Dc generator, Dc motor.



Horse shoe magnet



Bar magnet

UNIT-V

TIME VARYING FIELDS AND WAVE EQUATIONS

5.1 introduction

Time varying fields are produced due to accelerated charges or time varying currents.

Here we shall study how time varying current affects electric and magnet fields.

5.2 Faraday's law of electro-magnetic induction

Micheal faraday has stated two laws

i) If any coil experiences change in flux or variable flux then emf is induced in it.

ii) The emf induced in the coil is directly proportional to rate of change of flux linking With the coil.

$$E \, \alpha \, - d\phi / dt$$

E- electro-motive force.

- Sign indicates that magnetic flux developed in coil opposes the current through it.

  From Lenz law.

For an coil with N turns emf induced in it ,

$$E = - N.d\phi / dt$$

5.3 Maxwell's Fourth equation or vector form of faraday's law

We know from the gauss law,

$$\phi = \int_s B \, ds$$

hence emf induced due to above flux is ,

$$e = \ - d\phi / dt$$

$$= -d(\int_s B\, ds)/dt \text{ --------------------------a}$$

Electric potential is given as ,

$$e = \int E\, dl \text{ ------------------------------------b}$$

equating above two equations,

$$\int E\, dl = -(\int_s dB\, ds)/dt \text{ -------------------------- c}$$

by applying stokes theorem,

$$\int E\, dl = \int_s (\nabla \times E)\, ds$$

substituting above equation in c,

$$\int_s (\nabla \times E)\, ds = -(\int_s dB\, ds)/dt$$

comparing on both sides,

$$\nabla \times E = -dB/dt \text{ -- 57}$$

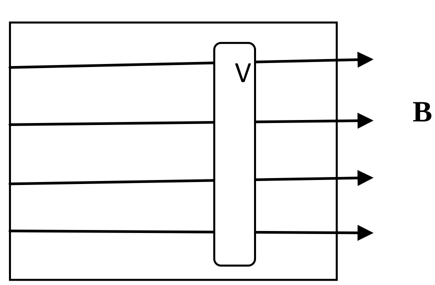
Equation 57 is called as Maxwell''s fourth equation of vector form of faraday's law.

5.4 Types of induced emf

The emf induced in the coil according faraday's law is mainly of two types. They are

i) Dynamically induced emf
ii) Statically induced emf.

Dynamically induced emf



Let us consider a straight conductor with charge velocity of v moving against the existing Magnetic field. Force experienced by conductor is ,

$$\vec{F} = Q\ ((\vec{V} X \vec{B})$$

$$\overrightarrow{F/Q} = ((\vec{V} X \vec{B})$$

$$\vec{E} = (\vec{V} X \vec{B})$$

Hence electric potential induced in the conductor is ,

$$e = \int \vec{E}\ dl$$

$$= \int (\vec{V} X \vec{B})\ dl$$

therefore potential induced can be written as, $e = BVl \sin\Theta$

the maximum value of potential induced is, $e = BVl$ ---------------------------------- 58

Statically induced emf

If an conductor experiences variable flux then emf induced in it is called as statically induced Emf.

$$e = \int \vec{E}\ dl = -Nd\phi / dt$$

since the flux is alternating,

$$\phi = \phi_m \sin wt$$

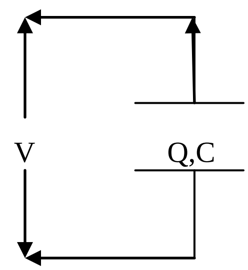
then the emf induced is ,

$$e = -Nd\ (\phi_m \sin wt\ ) / dt$$

$$= N \phi_m w \cos wt \text{ ------------------------------ } 59$$

5.5 Displacement current

Let us consider a capacitor is connected to Ac source as shown in figure



The current flowing through capacitor is ,

$$i_C = C\, dV / dt$$

the capacitance of capacitor,

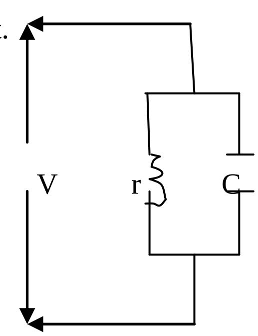
$$C = \varepsilon A / d$$

Then, $i_C = (\varepsilon A / d).\, dV / dt$

$$i_C / A = \varepsilon\, dE / dt$$

$$J_c = dD / dt$$

Jc is called as displacement current.



Above is the figure of actual capacitor with internal resistance,

Then the total current is ,

$$i = i_r + i_c$$

where , I – total current

i_r – current through resistance

i_c – current through capacitance

dividing above KCL on both sides by area A,

$$i / A = i_r / A + i_c / A$$

$$J = J_r + J_c$$

Jr – conducting current

Jc – displacement current

5.6 Maxwell's equations in time varying fields

In the time varying fields we can write,

$$E = Eo\ coswt$$

$$= Eo\ e^{jwt}$$

Similarly, $D = Do\ e^{jwt}$

$$d D / dt = Do\ wJ\ e^{jwt} = Jw\ Do$$

likely, $dB / dt = Jw\ B$

we know that,

$$\nabla xE = - dB / dt$$

$$= Jw\ B$$

Also, $\nabla \times E = -J w \mu H$

$\nabla \times H = J + dD/dt$

$= \sigma E + Jw\, Do$

$= \sigma E + Jw\, \varepsilon E$

$= E\,(\sigma + Jw\, \varepsilon)$

Integak form,,

$\int D\, ds = \int \rho_v dv$

$\int B\, ds = 0$

$\int E\, dl = -Jw \int B\, ds$

$\int H\, dl = (\sigma + Jw\, \varepsilon) \int E\, ds$

5.7 Wave Equation

After substituting the fields D and B in Maxwell's *curl* equations by the expressions and combining the two resulting equations we obtain the inhomogeneous wave equations.

$$\nabla \times \nabla \times \mathrm{E} + \frac{1}{c^2}\frac{\partial^2 \mathrm{E}}{\partial t^2} = -\mu_0 \frac{\partial}{\partial t}\left[\mathrm{j} + \frac{\partial \mathrm{P}}{\partial t} + \nabla \times \mathrm{M} \right]$$

$$\nabla \times \nabla \times \mathrm{H} + \frac{1}{c^2}\frac{\partial^2 \mathrm{H}}{\partial t^2} = \nabla \times \mathrm{j} + \nabla \times \frac{\partial \mathrm{P}}{\partial t} - \frac{1}{c^2}\frac{\partial^2 \mathrm{M}}{\partial t^2}$$

where we have skipped the arguments (r, *t*) for simplicity. The expression in the round brackets corresponds to the *total current density.*

$$\mathbf{j} = \mathbf{j} + \frac{\partial \mathrm{P}}{\partial t} + \nabla \times \mathrm{M} \;,$$

where j is the source and the conduction current density, $\partial P/\partial t$ the polarization current density, and $\nabla \times M$ the magnetization current density. The wave equations as stated in Eqations do not impose any conditions on the media and hence are generally valid.

Homogeneous Solution in Free Space

We first consider the solution of the wave equations in free space, in absence of matter and sources. For this case the right hand sides of the wave equations are zero. The operation ∇ × ∇× can be replaced by the identity, and since in free space ∇ · E = 0 the wave equation for E becomes

$$\nabla^2 \mathbf{E}(\mathbf{r},t) \;-\; \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}(\mathbf{r},t) \;=\; 0$$

with an identical equation for the H-field. Each equation defines three independent scalar equations, namely one for E_x, one for E_y, and one for E_z.

In the one-dimensional scalar case, that is $E(x, t)$, Equations. is readily solved by the ansatz of d'Alembert $E(x, t) = E(x - ct)$, which shows that the field propagates through space at the constant velocity c. To tackle three-dimensional vectorial
fields we proceed with standard separation of variables

$$\mathrm{E}(\mathrm{r}, t) = \mathrm{R}(\mathrm{r})\, T(t)\,.$$

$$c^2\,\frac{\nabla^2\mathbf{R}(\mathbf{r})}{\mathbf{R}(\mathbf{r})} \;-\; \frac{1}{T(t)}\frac{\partial^2 T(t)}{\partial t^2} \;=\; 0\,.$$

The first term depends only on spatial coordinates r whereas the second one depends only on time t. Both terms have to add to zero, independent of the values of r and t. This is only possible if each term is constant. We will denote this constant as $-\omega^2$. The equations for $T(t)$ and R(r) become

$$\frac{\partial^2}{\partial t^2}T(t) + \omega^2\, T(t) \;=\; 0$$

$$\nabla^2\mathbf{R}(\mathbf{r}) + \frac{\omega^2}{c^2}\,\mathbf{R}(\mathbf{r}) \;=\; 0\,.$$

Note that both R(r) and $T(t)$ are real functions of real variables.

Above is a harmonic differential equation with the solutions

$$T(t) \;=\; c'_\omega\cos[\omega t] + c''_\omega\sin[\omega t] \;=\; \mathrm{Re}\{c_\omega\exp[-i\omega t]\}\,,$$

ω

where $c_w{}'$ and c''_w are real constants and $c_\omega = c'_w + ic''_w$ is a complex constant. Thus,

according to ansatz (2.5) we find the solutions

$$\mathbf{E}(\mathbf{r},t) \;=\; \mathbf{R}(\mathbf{r})\,\mathrm{Re}\{c_\omega \exp[-i\omega t]\} \;=\; \mathrm{Re}\{c_\omega \mathbf{R}(\mathbf{r}) \exp[-i\omega t]\}\,.$$

In what follows, we will denote $c_\omega R$(r) as the *complex field amplitude* and abbreviate it by E(r). Thus,

$$\mathrm{E}(\mathbf{r},\, t) \,=\, \mathrm{Re}\{\mathbf{E}(\mathbf{r})\, \mathrm{e}^{-i\omega t}\}$$

Notice that E(r) is a *complex* field whereas the true field E(r, *t*) is real. The sym- bol E will be used for both, the real time-dependent field and the complex spatial part of the field. The introduction of a new symbol is avoided in order to keep the notation simple. Equation describes the solution of a *time-harmonic* electric field, a field that oscillates in time at the fixed angular frequency ω. Such a field is also referred to as *monochromatic* field.

$$\nabla^2\, \mathbf{E}(\mathbf{r}) \,+\, k^2\, \mathbf{E}(\mathbf{r}) \,=\, 0$$

with $k = |\mathrm{k}| = \omega / c$. This equation is referred to as *Helmholtz equation.*

Plane Waves

To solve for the solutions of the Helmholtz equation we use the ansatz

$$\mathbf{E}(\mathbf{r}) \;=\; \mathbf{E}_0\, \mathrm{e}^{\pm i\mathbf{k}\cdot\mathbf{r}} \;=\; \mathbf{E}_0\, \mathrm{e}^{\pm i(k_x x + k_y y + k_z z)}$$

which, after inserting

$$k_x^2 \;+\; k_y^2 \;+\; k_z^2 \;=\; \frac{\omega^2}{c^2}$$

The left hand side can also be represented by $\mathrm{k} \cdot bfk = k^2$. For the following we assume that k_x, k_y, and k_z are real. After inserting we find the solutions which are call $\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\{\mathbf{E}_0\, e^{\pm i\mathbf{k}\cdot\mathbf{r} - i\omega t}\}$ Solutions with the + sign in the exponent are waves tha ... $k_z]$. They are denoted *outgoing waves*. On the other hand, solutions with the − sign are incoming waves and propagate against the direction of k.

Although the field $E(r, t)$ fulfills the wave equation it is not yet a rigorous solu- tion of Maxwell's equations. We still have to require that the fields are divergence free, i.e. $\nabla\cdot E(r, t) = 0$. This condition restricts the k-vector to directions perpendicular to the electric field vector ($k\cdot E_0 = 0$). Fig. 2.1 illustrates the characteristic features of plane waves.

The corresponding magnetic field is readily found by using Maxwell's equation

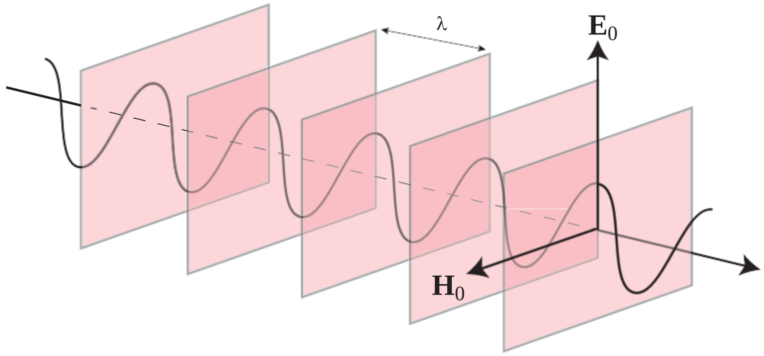
$\nabla \times E = i\omega\mu_0 H$. We find $H_0 = (\omega\mu_0)^{-1} (k \times E_0)$, that is, the magnetic field vector is perpendicular to the electric field vector and the wave vector k.

Let us consider a plane wave with real amplitude E_0 and propagating in direc- tion of the z axis. This plane wave is represented by $E(r, t) = E_0 \cos[kz - \omega t]$, where $k = |k| = \omega/c$. If we observe this field at a fixed position z then we'll mea- sure an electric field $E(t)$ that is oscillating with angular frequency $f = \omega / 2\pi$. On the other hand, if we take a snapshot of this plane wave at $t = 0$ then we'll ob- serve a field that spatially varies as $E(r, t = 0) = E_0 \cos[k z]$. It has a maximum at $z = 0$ and a next maximum at $k z = 2\pi$. The separation between maxima is $\lambda = 2\pi/k$ and is called the wavelength. After a time of $t = 2\pi/\omega$ the field reads $E(r, t = 2\pi/\omega) = E_0 \cos[k z - 2\pi] = E_0 \cos[k z]$, that is, the wave has prop- agated a distance of one wavelength in direction of z. Thus, the velocity of the wave is $v_0 = \lambda/(2\pi/\omega) = \omega/k = c$, the vacuum speed of light. For radio waves Figure 2.1: Illustration of a plane wave. In free space, the plane wave propagates with velocity c in direction of the wave vector $k = (k_x, k_y, k_z)$. The electric field vector E_0, the magnetic field vector H_0, and k are perpendicular to each other. $\lambda \sim 1$ km, for microwaves $\lambda \sim 1$ cm, for infrared radiation $\lambda \sim 10\ \mu$m, for visible light $\lambda \sim 500$ nm, and for X-rays $\lambda \sim 0.1$ nm, - the size range of atoms. Fig. 2.2 illustrates the length scales associated with the different frequency regions of the

electromagnetic spectrum.

A plane wave with a fixed direction of the electric field vector E_0 is termed *lin- early polarized*. We can form other polarizations states (e.g. circularly polarized waves) by allowing E_0 to rotate as the wave propagates. Such polarization states can be generated by superposition of linearly polarized plane waves.

Plane waves are mathematical constructs that do not exist in practice because their fields E and H are infinitely extended in space and therefore carry an infinite amount of energy. Thus, plane waves are mostly used to locally visualize or ap- proximate more complicated fields. They are the simplest form of waves and can be used as a basis to describe other wave fields (angular spectrum representation). For an illustration of plane waves go to http://en.wikipedia.org/wiki/Plane wave.


λ
E_0
H_0