PPT ON
ELECTROMAGNETIC FIELDS(R18)

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Prepared
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UNIT-I
ELECTRO-STATICS AND VECTOR CALCULUS

INTRODUCTION

The most known particles are photons, electrons and neutrons with different masses. Their masses are

$$m_e = 9.10x\ 10\text{-}^{31} \text{ kilograms}$$

$$m_p = 1.67x\ 10\text{-}^{27} \text{ kilograms}$$

these masses leads to gravitational force between them, given as

$$F = G\ m_e\ m_p\ /\ r^2$$

The force between two opposite charges placed 1cm apart likely to be 5.5x10-^{67} and force between two like charges placed 1cm apart likely to be 2.3x10-^{24}.this force between them is called as electric force .

Electric force is larger than gravitational force. Gravitational force due to their masses. Electric force is due to their properties. Neutron has only mass but no electric force.

ELECTROSTATICS:

Electrostatics is the study of charge at rest. The study of electric and magnetic field can be done using MAXWELL'S equations. Electrostatic field is developed between static charges. Electrostatics got wide variety applications like X-rays, lightning protections etc.

Let us study the behavior of electric field using COLOUMB's and GAUSS laws.

Point Charge

A charge with smallest dimensions on the body compare to other charges is called as point charge.

A group of charges concentrated on any pin head may be also called as point charge.

COLOUMB'S LAW

Coloumb stated that the force between two point charges is directly proportional to product of charges and Inversely proportional square of distance between the. $F \alpha Q_1Q_2 / r^2$

$F = K Q_1Q_2 / r^2$, where K is the proportionality constant.

$K = 1/ 4\pi\varepsilon$, where ε is the permittivity of the medium.

$E = \varepsilon_0\varepsilon_{r,}$ ε_0 = absolute permittivity = $8.854x10^{-12}$

ε_r = relative permittivity

most common medium is air or vacuum whose relative permittivity is 1, hence permittivity of air or vacuum is

$\varepsilon = 9x10^9$ m/F

Force between two point charges using vector analysis

Let us consider two point charges separated by some distance given as .

$Q_1 \leftarrow$---------------------------------$\rightarrow Q_2$

According to coloumb's law force between them is given as

$F = (K\,Q_1Q_2 / r^2)$ x , where is the unit vector direction of force.

Let F_2 is the force experienced by Q_2 due to Q_1 and F_1 is force experienced by Q_1 due to Q_2. The direction of forces opposes each other , hence we can write in vector from forces as

$$F_1 = -F_2$$

Hence unit vector can be or , from the vector analysis we can write

$$a_{12} = R'_{12} / R_{12} = R' / R \text{ and}$$

$$a_{21} = R'_{21} / R_{21} = R' / R$$

Therefore the magnitude of force between them can be written as

$$F_1 = F_2 = (K\,Q_1Q_2 / R^3)\, xR'$$

Electric Field&Electric Field Intensity

Electric Field: It is the region around the point and group charges in which another charge experiences force is called as electric field.

The force between two charges can be studied in terms of electric field as : A charge can develop field surrounding it in space only,the field of one charge leads to force on the other charge .

Electric Field Intensity: If an point charge q experiences the force F , then the electric field intensity of charge is defines as

$$E = F/q$$

Here charge q is called as test charge because the force experienced by it is due field of other charge.

The units of electric field intensity are N/C or V/mt.

$$q_1 \leftarrow\text{---------------- ----------------}\rightarrow q_2$$

Electric Field&Electric Field Intensity

the force experienced by q_2 because of field of q_1 is

$$\text{vector, } F_2 = (K\, q_1 q_2 / r^2) \times a'$$

Therefore electric filed intensity on q2 charge is

$$\text{Vector,} E = F_2/q_2 = (K\, q_1 / r^2) \times a'$$

the force experienced by q_1 because of field of q_2 is

$$\text{vector, } F_1 = (K\, q_1 q_2 / r^2) \times a'$$

Therefore electric filed intensity on q2 charge is

$$\text{Vector, } E = F_1/_{q1} = (K\, q_2 / r^2) \times a'$$

Electric Field&Electric Field Intensity

let the point charges q_2,q_3--------------q_n are placed at a distance of r_2,r_3---------------------------r_n from q_1.

Hence total electric field intensity on q1 due to remaining point

Charges is , force due to q2 on q1, F2= $(K\, q_1 q_2 / r^2)$ x a'

force due to q3 on q1, F3= $(K\, q_1 q3 / r^2)$ x a'

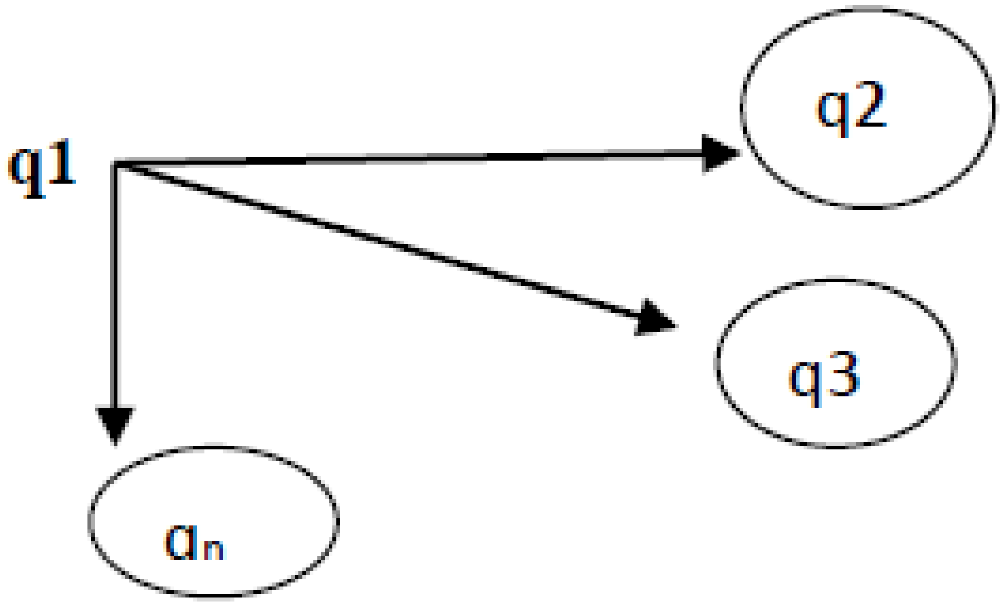
----

force due to qn on q_1, Fn= $(K\, q_1 qn / r^2)$ xa'

therefore total electric field intensity is , $= (F_2 + F_3$---------------$F_n) / q_1$

$= (K\, q_2 / r^2)$ x + $(K\, q_3 / r^2)$ x ----+ $(K\, q_n / r^2)$ x a'

Electric Field & Electric Field Intensity



CHARGE DISTRIBUTION

Line charge: Here charge is distributed through out some length . The total charge distributed through a wire of length l is $Q = \int \rho_l \ dl$

Where, ----- line charge density

Hence electric field intensity due to line charge is , $E = \int (K\, dl/ r^2)\, xa'$

Surface charge: Here charge is distributed through given area . The total charge distributed in an surface area is $Q = \int \rho_s\, dl$

Where, ----- surface charge density

Hence electric field intensity due to surface charge is , $E = \int (K\, ds/ r^2)\, x\, a'$, $E = \int (K\, ds/ r^2)\, x\, a'$

Volume charge: Here charge is distributed through given volume . the total charge distributed in an volume is $Q = \int \rho_V\, dl$

Where, ----- volume charge density

Hence electric field intensity due to volume charge is , $E = \int (K\, dv/ r^2)\, xa'$

LINE CHARGE DISTRIBUTION

Let us consider a straight wire of length l is symmetrically placed in X Y axis as shown in below figure

For a small length of dl on y-axis the charge is dq, the electric field intensity due dq at test point p is

$$dE = K.dq/(y^2+a^2)$$

Then,$dEx = K.dq.\cos\Theta/(y^2+a^2)$--------------------0

$$\cos\Theta = a\mathrm{Sqrt}(y^2+a^2) \text{ ------------------------1}$$

we can write charge per unit length as $dq/dl = Q/l$,

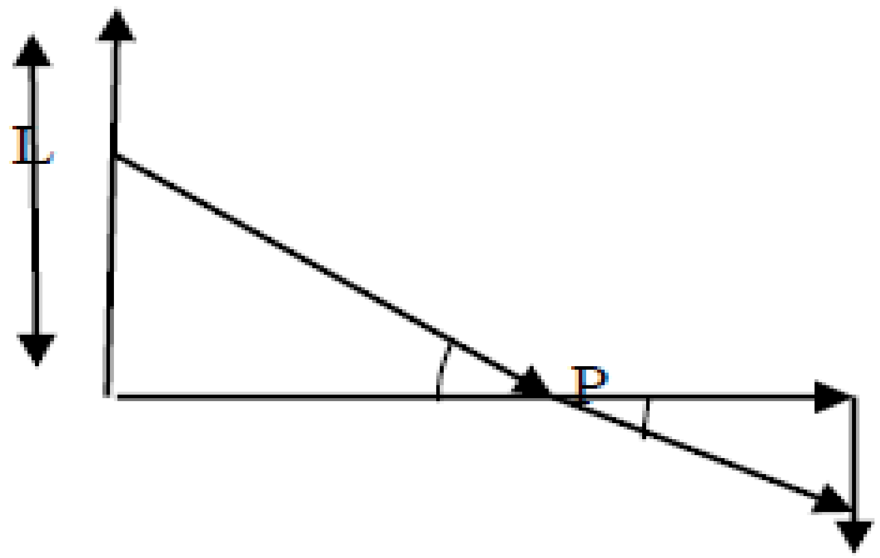
$$dq = Q.dl/l \quad (dl = dy)$$

$$dq = Q.dy/l \text{ ---------------------2}$$

Their $dEx = KQ\ dy.a / l.(y^2+a^2)^{3/2}$

integrating on both sides with limits –l/2 and l/2, $Ex = \int KQ\ dy.a /$ l.(and when l tends to 0, $Ex = KQ / a^2$.

CHARGE DISTRIBUTION



SURFACE CHARGE DISTRIBUTION

Let us consider an infinite sheet placed uniformly in xyz plane as shown in figure.t us consider a small area ds in xy plane , ds = ρ.dρ.dϕ.

Which is located at distance of ρ from origin making and angle of ϕ.

P be the point on z axis given as (0,0,h).

Distance from P to ds is .

$$R' = - \rho a_\rho + h a_{z,} R = \sqrt{(\rho^2+h^2)}, a_R = (- \rho a_\rho + h a_z) / \sqrt{(\rho^2+h^2)}$$

hence the electric field intensity at ds is given as ,

$$= \int \rho s \, ds. \, a_R / 4\pi\varepsilon.R^2$$

$$= \int \rho s \, \rho.d\rho.d\phi. \, (- \rho a_\rho + h a_z) / \sqrt{(\rho^2+h^2)}. \, 4\pi\varepsilon.R^2$$

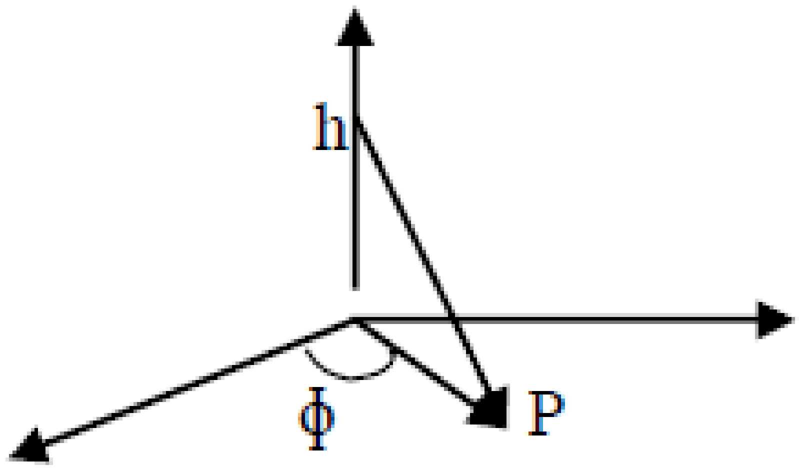
The limits of ρ from 0 to ∞ and ϕ from 0 to 2π.

$$= \int\int \rho s \, \rho.d\rho.d\phi. \, (- \rho a_\rho + h a_z) / \sqrt{(\rho^2+h^2)}. \, 4\pi\varepsilon.R^2$$

By simplying above equation, the electric field intensity

$$= \rho s \, az / 2\varepsilon.$$

SURFACE CHARGE DISTRIBUTION



WORK DONE ON POINT CHARGE

Charge q is placed in the existing electric field . The charge q experiences force F

----

a -------------E’---------------b

Here charge q is made to move from a to b of length l throug electric field intensity .E’

$$dw = - Fdl = -q.E\, dl$$

integrating on both sides, $w = -q\int E dl$ with limits a to b.

POTENTIAL DUE TO POINT CHARGE

From the above discussion work done to move point charge through the existing electric field is

$$w = -q\int E\,dl$$

but we know that electric potential is defined work done to move unit charge

$$V = w/q$$

Therefore, $V = w/q = -\int E\ dl$ **with limits a to b**

$V = -\ E.l$ **with limits a to b**

V- potential

w- work done

q- charge

Hence , **electric potential** $V = V_a - V_b$

POTENTIAL DUE TO POINT CHARGE

Point charge --------------------------------

a ------------------------------b

Let the charge q is moved from a to b and at point charge is Q from r_a and r_b ,

We know that electric potential, $V = q/(4\pi\varepsilon r)$

Electric field intensity at point p due to charge at a is, $V_a = Q/(4\pi\varepsilon r_a)$

Electric field intensity at point p due to charge at a is, $V_b = Q/(4\pi\varepsilon r_b)$

Hence potential difference or electric potential from a to b is,

$$V_{ab} = V_a - V_b$$

$$V_{ab} = Q/(4\pi\varepsilon r_a) - Q/(4\pi\varepsilon r_b)$$

$$V_{ab} = Q(r_b - r_a)/(4\pi\varepsilon r_a r_b).$$

Macheal faraday has conducted experiment on two concentric spheres, inner layer is positively charges and outerlayer negatively charge, then he observe that their some sort of displacement from inner layer to outer layer , thisdisplacement is pronounced as electric flux between spheres.We know that electric field intensity is ,

$E = (K q / r^2) = q/ (4\pi\varepsilon r^2)$, $D = \varepsilon E = q/ (4\pi r^2)$—electric flux density

Electric flux density is defined as charge per unit area.

Potential gradient: potential gradient is defined as electric change in electric potential due to change in thedistance or length.

$$E = - \nabla V$$

Properties of Potential: Potential is the energy acquired by the charge. When charge travel from one end to other end in any element there is potential change from high to low. Potential acquired by point charge leads to electric field.

GAUSS LAW

Gauss law states that the total flux in the given surface is equal to charge enclosed in it.

$$\phi = Q.$$

the total flux enlaced in given surface is $\phi = \int E \, ds$

$$= \int Q / (4\pi\varepsilon \, r^2) \, ds$$

$$= Q / (4\pi\varepsilon \, r^2). \int ds$$

$$= Q4\pi \, r^2 / (4\pi\varepsilon \, r^2).$$

$$= Q / \varepsilon.$$

Applications of Gauss law

→ **To apply gauss law first assume Gaussian surface.**

→ **The electric field intensity must be normal to the Gaussian surface.**

→ **Gaussian surface must be symmetry.**

POISSON'S AND LAPLACE EQUATIONS

We know that electric flux passing through the surface is equal to 1/ ε times the net charge enclosed.

$$\phi = \int_s E\, ds = Q/\varepsilon$$

$$\phi = \int_s \varepsilon E\, ds = Q\,\varepsilon/\varepsilon$$

$$\phi = \int_s D\, ds = Q$$

from the strokes theorem we can say that surface integral function is volume integral of divergence of same function.

$$Q = \int_s D\, ds = \int_v (\nabla \cdot D)\, dv \text{ ----------------------3}$$

from the gauss law we can write , $Q = \int_v \rho_v\, dv$ **-------------------------- 4**

by comparing equation 3 and 4 $(\nabla \cdot D) = \rho_v$**---------------------------5**

Equation 3 and 5 are said to be Maxwell's first and second equation.

From the Maxwell's equation we know that,

$(\nabla . D) = \rho_v$ -------------------------------- 6, $D = \varepsilon E$ ----------------------- 7

Equation 7 in 6, $\nabla \varepsilon E = \rho_v$

but we know that, $E = - \nabla V$

$$\nabla \varepsilon(-\nabla V) = \rho_v$$

$$\nabla^2 V = - \rho_v / \varepsilon \text{ -----------------} 8$$

Equation 8 is called poission's equation.

In the uniform Gaussian surface , $\rho_v = 0$

Then equation 8 can be rewrite as,

$$\nabla^2 V = 0 \text{ -----------------} 9$$

Equation 9 is called as Laplace equation.

UNIT-II
CONDUCTORS AND DIELECTRICS

ELECTRIC DIPOLE AND MOMENTUM

Two opposite charges +q and –q separated by some distance d forms the electric dipole.+q ----------------d--------------- -q

The distance travelled by the point charge is defined as dipole moment (or) the product of charge and distancetravelled by it is called as electric dipole. P = q.d ------------ 1

Here , P → electric dipole moment

d → distance between opposite charges

the line between two charges is called as axis of dipole.

Potential

ELECTRIC DIPOLE AND MOMENTUM

assume two charges separated by distance d as shown in the figure

+q ---------------d-------------- -q

Here, O → center of the axis between charges

P → be the test point where potential is required.

OP → with length of r.

AA^1 → perpendicular from A to OP

BB^1 → perpendicular from A to OP.

∟POB = ϴ

r >>> d

the line $AP = A^1P = OP + OA^1$---------------------------------2

from the right angle triangle AA^1O, $OA^1 = OA \cos \Theta$

hence equation 2 can be written as, $AP = A^1P = r + OA \cos \Theta$

but, $OA = d/2$

$AP = A^1P = r + d/2 \cos \Theta$

Hence the potential at P due negative charge at A is ,

$V_A = -Kq/ AP = -Kq/ r + d/2 \cos \Theta$

ELECTRIC DIPOLE AND POTENTIAL

Similarly from the right angle triangle BB^1O, $BP = B^1P = r - d/2 \cos \theta$

Hence the potential at P due negative charge at A is ,

$$V_B = Kq/ BP = Kq/ r - d/2 \cos \theta$$

Therefore the total potential acting on P is , $V = V_A + V_B$

$$V = Kq[(1/ r - d/2 \cos \theta) - (1// r + d/2 \cos \theta)]$$

$$= Kqd.\cos \theta / (r^2 – d^2/4 \cos^2 \theta)$$

But we know that, $r >>> d$

$$V = Kqd.\cos \theta / r^2$$

$$V = KP.\cos \theta / r^2 , (P = q.d) \text{----------------- } 3$$

know that electric field intensity in terms of electric potential is given as ,

$$E = - \nabla V$$

From equation 3 we can say that potential due dipole is in spherical co-ordinates, therefore find electric field

intensity we shall use spherical co-ordinates.

$$\nabla V = -[dv/dr + (1/r)dv/d\theta]$$

Simplifying ∇V, $dv/dr = -2KP.\cos \theta / r^3$

$$(1/r)dv/d\theta = -KP.\sin \theta / r^3$$

ELECTRIC DIPOLE AND TORQUE

Substituting above two equations in E, E = $-[\ (-2KP.\cos\Theta/ r^3) + (-KP.\sin\Theta/ r^3)\ ]$

$= [\ (2KP.\cos\Theta/ r^3) + (KP.\sin\Theta/ r^3)\ ]$

$= KP/ r^3\ [\ (2\cos\Theta) + (\sin\Theta)\ ]$ ----------- 4

Torque due to Electric Dipole

Let us consider two opposite charges are placed in the uniform electric field with their line of axis of 2r.

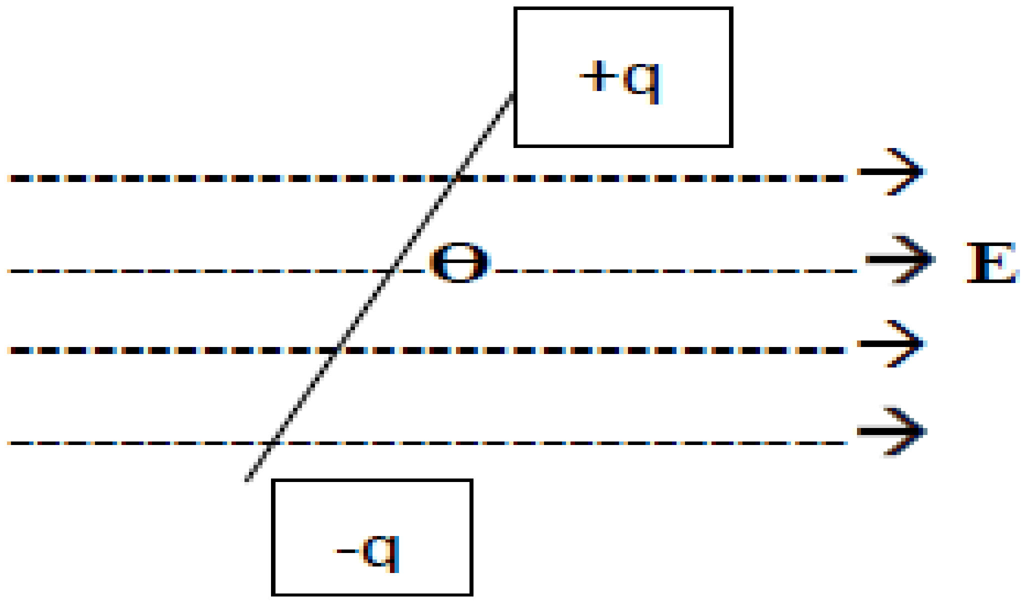
The experienced by +q is , $F_1 = E.q$

The experienced by -q is , $F_2 = -E.q$

The total experienced by the dipole is , $F = F_1 + F_2$

$F = 0$

ELECTRIC DIPOLE AND TORQUE



ELECTRIC DIPOLE AND TORQUE

due to force experienced by +q it tends to oscillate in th direction of E and –q in the direction opposite to E, which leads torque of dipole.

T = magnitude of F x perpendicular distance

Between their line of action

T = E.q x 2r sinϴ

T = PE.sinϴ.

POLARIZATION

If an piece if dielectric or insulator placed between the charges plates of condenser, then center of gravity of negative charges is concentrated towards positive plate and center of gravity of positives charges concentrated towards negative plate, this process of separation opposite charges is called a polarization.

Polarization is also defined as electric dipole moment per unit volume.

Let A be the area of cross section of dielectric,

l be the distance by with opposite charges are separated,

q total charge in the volume of dielectric

then polarization, P = dipole moment / volume

= q.l / A.l

= q / A ------------------------------------ 6

i.e the polarization numerically equal to surface charge density.

DIELECTRIC CONSTANT

Dielectric constant is defined as ratio capacitance of capacitor with dielectric to the capacitance of capacitor without dielectric .

Capacitance of capacitor with dielectric has low potential(V_d) than the capacitance of capacitor without dielectric(V) .

$$K = V / V_d \text{ -- } 7$$

The polarization is directly proportional to the electric field intensity created between charges.

$$P \alpha E$$

$$P = K_e E$$

$$K_e = P / E = \text{electric susceptibility} \text{ -------------------------- } 8$$

CAPACITOR

basic capacitor element is formed by separated two parallel plates with some dielectric medium.

When some voltage is applied to such an element charge is formed between the plates, their by

capacitance of capacitor is defined as charge Q developed between the plates when voltage V is applied.

$$C = Q / V \quad \text{---} \quad 9$$

The units of capacitance are Farads (F).

TORQUE DUE TO DIPOLE

But the due to force experienced by +q it tends to oscillate in the direction of E and –q in the direction opposite to E, which leads torque of dipole.

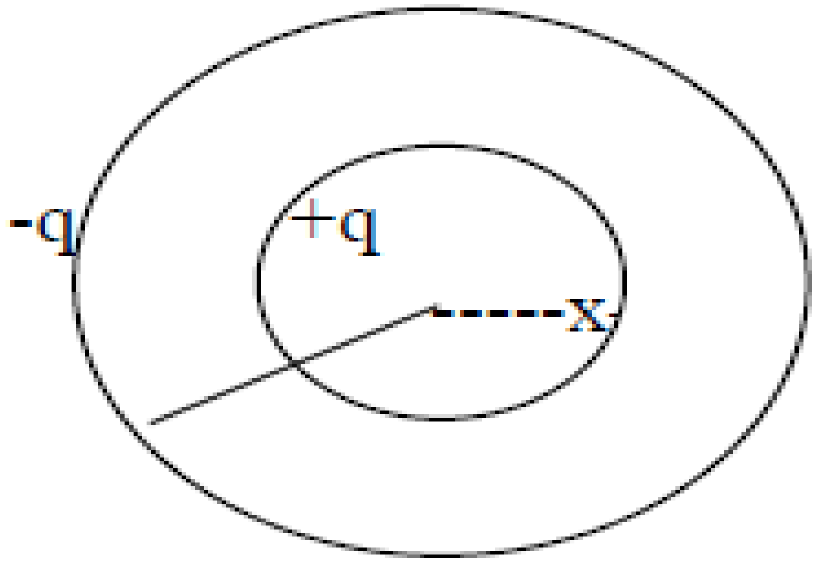
T = magnitude of F x perpendicular distance

Between their line of action

$$T = E.q \times 2r \sin\Theta$$

$$T = PE.\sin\Theta.$$

CAPACITANCE OF ISOLATED SPHERE



CAPACITANCE OF ISOLATED SPHERE

Let us consider an isolated sphere which is positively charges with radius x and negatively charges plate placed at infinite distance.

The electric flux density due to positive charge, $D = Kq / x^2$

Electric field intensity due to positive charge, $\varepsilon E = Kq / x^2$

$E = Kq /. x^2$

Work done, $w = - q \int E \, dl.$

$W = -q \int E \, dx$

with limits ∞ to x

$V = - \int E \, dx$

with limits ∞ to x

CAPACITANCE OF ISOLATED SPHERE

$V = - \int Kq /. x^2\, dx$ with limits ∞ to x

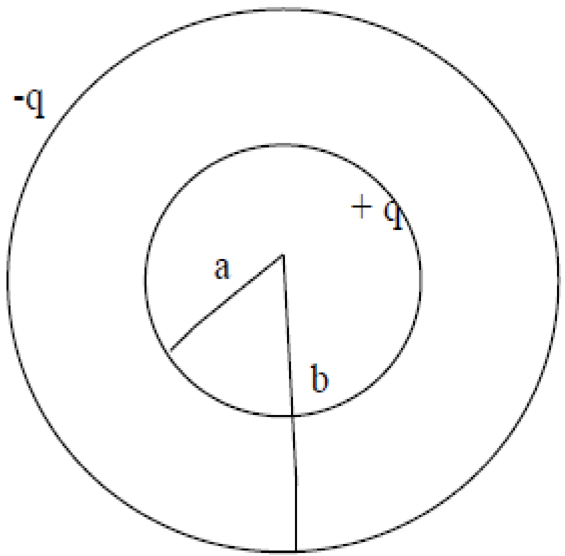
$= -K.q \;/\; (-. x)$ with limits ∞ t

$= K.q \;/\; (. x)$

But the capacitance is given charge per voltage, $C = q / V$

$C = (x) / K$ ------------------------------- 10

CAPACITANCE OF CONCENTRIC SPHERE



CAPACITANCE OF CONCENTRIC SPHERE

Let us consider an isolated sphere which is positively charges with radius a and negatively charges plate place at b distance.

The electric flux density due to positive charge, $D = Kq / x^2$

Electric field intensity due to positive charge, $\varepsilon E = Kq / x^2$

$E = Kq /. x^2$

Work done, $w = - q \int E\, dl$, $W = -q \int E\, dx$ with limits b to a

$V = - \int E\, dx$ with limits b to a

$V = - \int Kq /. x^2\, dx$ with limits b to a

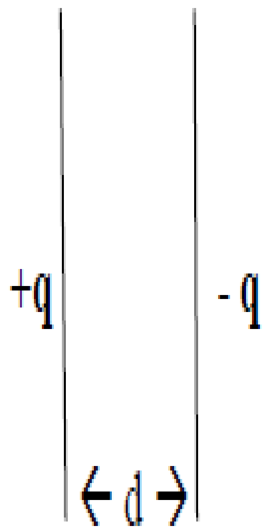
$= -K.q / (-. x)$ with limits b to a

$= . [(1/a) – (1/b)]$

But the capacitance is given charge per voltage, $C = q / V$

$C = ab / K(b-a)$ ------------- 11

CAPACITANCE OF PARALLEL PLATES



CAPACITANCE OF PARALLEL PLATES

Let potential applied to these parallel plates is V their by forming charge q between them.

Electric flux density between plates, $D = q / A$

$\varepsilon E = q / A$

$E = q / \varepsilon . A,$

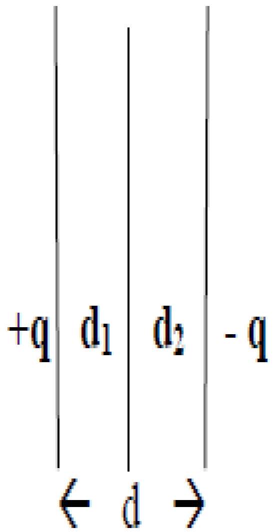
$V = E . d$

$V = q d / \varepsilon . A$

But the capacitance is given charge per voltage, $C = q / V$

$C = \varepsilon . A / d$ -------------------------------- 12

CAPACITANCE OF PARALLEL PLATES WTH MULTIPLE DIELECRICS



CAPACITANCE OF PARALLEL PLATES WTH MULTIPLE DIELECRICS

Let potential applied to first part is V_1 their by forming charge q between them.

Electric flux density between plates, $D = q / A$

$$\varepsilon E_1 = q / A$$

$$E_1 = q / \varepsilon_1.A,$$

$$V_1 = E.d_1$$

$$V_1 = q\, d_1 / \varepsilon_1.A$$

But the capacitance is given charge per voltage, $C = q / V$

$$C_1 = \varepsilon_1.A / d_1 \text{ ------------------------------ } 13$$

Let potential applied to first part is V_2 their by forming charge q between them.

CAPACITANCE OF PARALLEL PLATES WTH MULTIPLE DIELECRICS

Electric flux density between plates, $D = q / A$

$\varepsilon E_2 = q / A$

$E_2 = q / \varepsilon_2.A,$

$V_2 = E.d_2$

$V_2 = q\, d_2 / \varepsilon 2_1.A$

But the capacitance is given charge per voltage, $C = q / V$

$C_2 = \varepsilon_2.A / d_2$ ------------------------------ 14

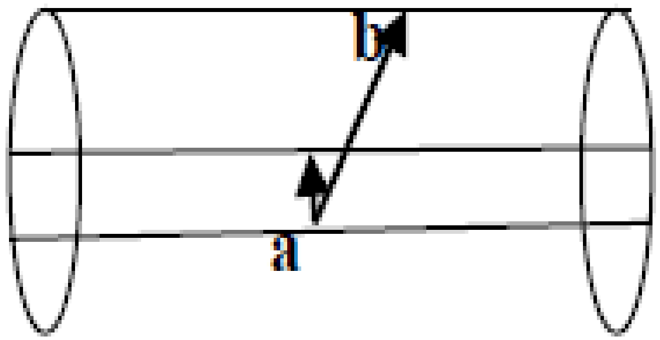
Hence total capacitance between plates with multiple dielectric mediums is ,

$C = C_1 + C_2$

$= (\varepsilon_1.A / d_1) + (\varepsilon_2.A / d_2)$

$= A / [(d_1/ \varepsilon_1) + (d_2/ \varepsilon_2)$ ---------- 15.

CAPACITANCE OF CO-AXIAL CABLE



ESTIMATE

Let us consider co-axial cable two isolated sphere with radius a and b from center of axis.The length of cable is , then line chargE distribution $\rho_l = q / l$

the electric flux density generally in cable is , $D = \rho_l / 2\pi r$

therefore electric filed intensity , $E = \rho_l / 2\pi r\varepsilon$

the electric potential of the cable is , $V = -\int E dr$, with limits b to a

$$= -\int (\rho_l / 2\pi r\varepsilon)\, dr$$

$$= -(\rho_l / 2\pi\varepsilon) \int dr/r$$

$$= -(\rho_l / 2\pi\varepsilon) .\ln(r)$$

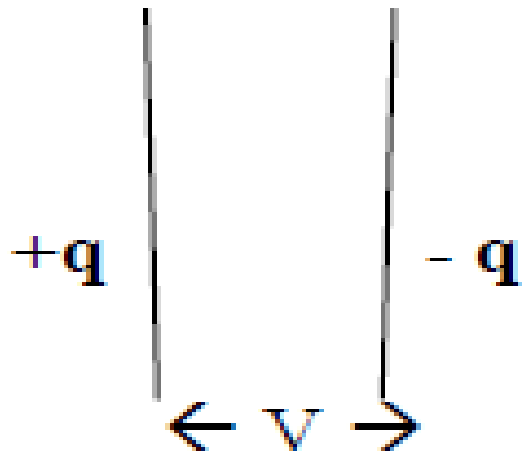
By applying limits, $V = -(\rho_l / 2\pi\varepsilon) .[\ln(a) – \ln(b)]$

$$V = (\rho_l / 2\pi\varepsilon) .\ln(b/a)$$

The capacitance of co-axial cable, $C = \rho_l / V$

$$C = 2\pi\varepsilon / \ln(b/a) \text{ -------- } 21$$

ENERGY STORED IN CAPACITOR



ENERGY STORED IN CAPACITOR

By the definition capacitance between plates is , $C = q / V$

Electric potential, $V = dw / dq$

$dw = V\, dq$

$dw = (q / C)\, dq$

integrating on both sides, $w = \int (q / C)\, dq$

$w = q^2 / 2C$ (or) ----------------------------- 16

$w = (CV)^2 / 2C$

$w = CV^2 / 2$ (or)----------------------------- 17

$w = q^2 / 2C$

$w = Vq / 2$ ---------------------------------18

ENERGY STORED IN CAPACITOR

W_d = energy stored / volume

$$W_d = CV^2 / 2/ Ad$$

$$W_d = \varepsilon A\, V^2/d \,/\, 2.\, Ad$$

$$W_d = \varepsilon\, V^2/2d^2$$

$$W_d = \varepsilon E^2/ 2$$

$$W_d = DE/ 2 \text{ -- } 19$$

From equation 19 we can write,

$$dW = (DE/ 2)\ dV$$

integrating on both sides, energy stored

$$W = \int_v (DE/ 2)\ dV \text{ -------------------- } 20$$

CURRENT DENSITY

The flow of electrons from one end to other end constitutes current.
The rate of change of Charge is also defined as current.

$$i = q / t = dq / dt \text{ ---------------------------------- } 22$$

the units of current is ampere.

Current Density

If charge is distributed in the given area, then current density is defined as current constituted

In given area.

$$J = i / A \quad (A/mt^2) \text{ ---------------------------------- } 23$$

$$J = di / ds$$

$$di = J .ds$$

integrating on both sides, $i = \int J .ds$

Convection Current Density

Let us consider a material with volume of charges (ρ_V) moving with drift velocity (V_d) , then

Convection Current density is defined as product volume of charges moving with drift velocity.

$$J = \rho_V \times V_d \text{ -- } 24.$$

Equation of Continuity

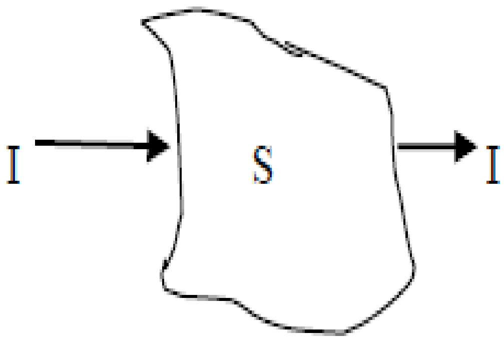
Let us an surface area through charges are moving in and out as shown in the figure

Let the charge q is moving through an area of S.

According law of conservation of charge, $[I]s = - dq / dt$

But current passing through area is , $[I]s = \int J \, d_s$

CURRENT DENSITY



EQUATION OF CONTINUITY

Total charge in the given volume is, $q = \int_v \rho_v \, ds$

From above three equations we can write,

$$\int J \, ds = -(d/dt). \int_v \rho_v \, dv \text{---------------------}25$$

from the stokes theorem we can write,

$$\int J \, ds = \int_v \nabla J \, dv \text{ --------------------------- } 26$$

by comparing equation 25and 26,

$$\int_v \nabla J \, dv = -(d/dt). \int_v \rho_v \, dv$$

$$\int_v \nabla J \, dv + (d/dt). \int_v \rho_v \, dv = 0$$

$$\int_v [\nabla J + d\rho_v / dt] \, dv = 0 \text{ ------------------ } 27$$

equation 27 is called as equation of continuity or maxwell's fifth equation.

CONDUCTORS

Conductors:
The substances having free charge carriers axe called the conductors. The examples of conductors are metallic substances e.g. copper, silver, gold, aluminum, iron, mercury etc.

Insulators:
The substances having no free charge carriers are called the *insulators* or *dielectrics*. The examples of insulators are glass, plastic, mica wood, cotton etc

The free and bound charges inside a conductor may be understood by the knowledge of structure of atom. Every substance is formed of atoms. Every atom is electrically neutral. It consists of a central, nucleus containing positive charge and negatively charged electrons revolving around the nucleus in various definite orbits. The electrons in orbits near the nucleus are tightly bound by Coulomb attractive forces; while the electrons in outermost orbit are very loosely bound.

DIELECTRICS

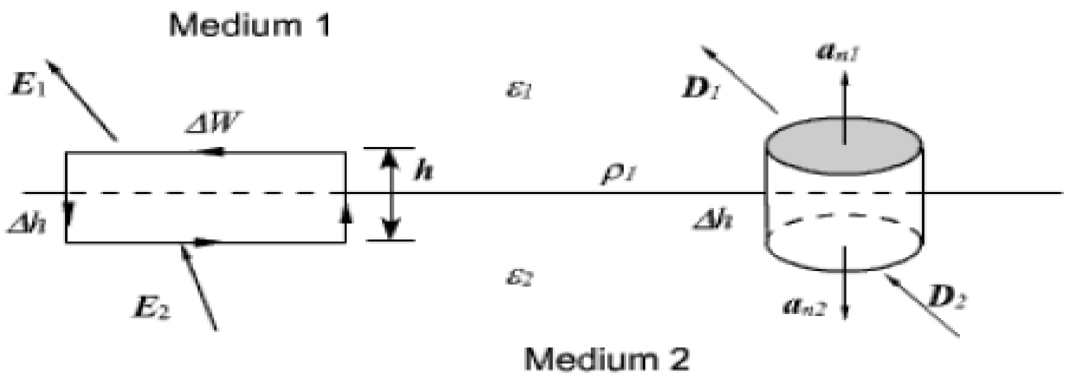
The absence of electron from a neutral atom makes it positively charged and the resulting atom is termed as positive ion. The positive ions are bound in the conductor in a regular pattern and are therefore termed as bound charges. Thus a conductor consists of free charges as well as bound charges. The free charges are free electrons and $H_2N_2O_2$ the bound charges are positive ions fixed in the lattice. Dielectrics are substances which do not contain free charge carriers. The examples of dielectrics are air, mica, rubber, wood, plastic etc. Each atom/molecule of a dielectric is neutral. The molecules of a dielectric may be of two types:

(i) **Non-polar Molecules**: If the centers of positive and negative charges in a molecule coincide; so that no electric dipole is formed, the molecule of the dielectric is said to be non-polar. The examples of non-polar molecules are H2N2O2 etc

ii) **Polar Molecules**: if the centers of positive and negative charges in molecules do not coincide, so that an electric dipole is formed, the molecule is said to be polar. The example of polar molecules are H2O, CO2 etc

Boundary Conditions for Electrostatic Fields:

Let us consider the relationship among the field components that exist at the interface between two dielectrics as shown in the figure . The permittivity of the medium 1 and medium 2 are ε_1 and ε_2 respectively and the interface may also have a net charge density ρ_s Coulomb/m.



Boundary Conditions at the interface between two dielectrics

We can express the electric field in terms of the tangential and normal components

$$\overrightarrow{E_1} = \overrightarrow{E_{1t}} + \overrightarrow{E_{1n}}$$

$$\overrightarrow{E_2} = \overrightarrow{E_{2t}} + \overrightarrow{E_{2n}} .$$

where E_t and E_n are the tangential and normal components of the electric field respectively.

Let us assume that the closed path is very small so that over the elemental path length the variation of E can be neglected. Moreover very near to the interface, $\Delta h \rightarrow 0$. Therefore

$$\oint \vec{E}.d\vec{l} = E_{1t}\Delta w - E_{2t}\Delta w + \frac{h}{2}(E_{1n} + E_{2n}) - \frac{h}{2}(E_{1n} + E_{2n}) = 0$$

Thus, we have,

$E_{1t} = E_{2t}$ or $\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ i.e. the tangential component of an electric field is continuous across the interface.

For relating the flux density vectors on two sides of the interface we apply Gauss's law to a small pillbox volume as shown in the figure. Once again as $\Delta h \rightarrow 0$, we can write

$$\oint \vec{D}\cdot d\vec{s} = (\vec{D_1}.\hat{a}_{n2} + \vec{D_2}\cdot\hat{a}_{n1})\Delta s = \rho_s \Delta s$$

i.e., $D_{1n} - D_{2n} = \rho_s$

.e., $\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$

Thus we find that the normal component of the flux density vector ***D*** is discontinuous across an interface by an amount of discontinuity equal to the surface charge density at the interface.

Two further illustrate these points; let us consider an example, which involves the refraction of D or E at a charge free dielectric interface as shown in the figure .

Using the relationships we have just derived, we can write

$$E_{1t} = E_1 \sin\theta_1 = \frac{D_1}{\varepsilon_1}\sin\theta_1 = E_{2t} = E_2 \sin\theta_2 = \frac{D_2}{\varepsilon_2}\sin\theta_2$$

$$D_{1n} = D_1 \cos\theta_1 = D_{2n} = D_2 Cos\theta_2$$

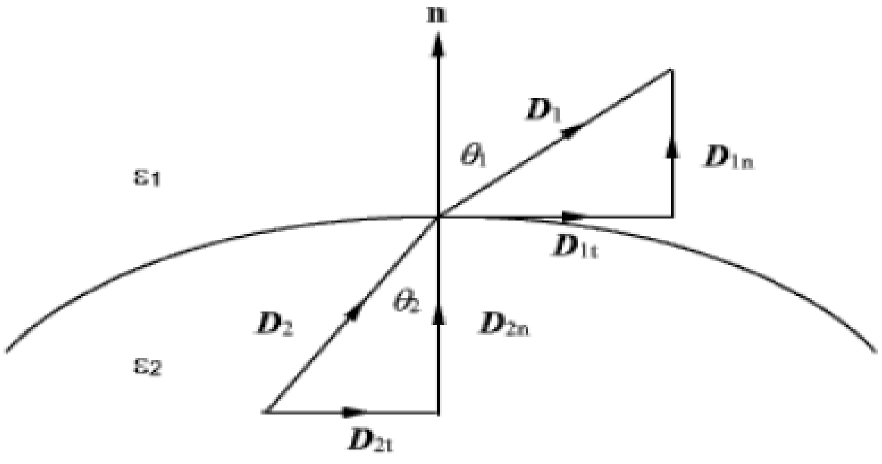
BOUNDARY CONDITIONS

In terms of flux density vectors,

$$\frac{D_1}{\varepsilon_1}\sin\theta_1 = \frac{D_2}{\varepsilon_2}\sin\theta_2$$

$$D_1\cos\theta_1 = D_2 Cos\theta_2$$

Therefore, $$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$



UNIT-III
MAGNETO-STATICS

INTRODUCTION

Magneto-statics is the study of magnetic field developed by the constant current through the coil Or due to permanent magnets.

The behavior of constant magnetic field is studied by using two basic laws, they are

→ Bi-Savart's law

→ Ampere's circutal law.

Magnetic Field

1. **Let us consider a constant current I is passing through coil shown above which develops constant Flux surrounding the coil their by forming north and south poles. This formation of magnetic from North pole to south pole is called as magnetic field. The direction of magnetic flux in an coil is Given by right hand thumb rule.**
2. **Right hand thumb rule says that if four fingers of hand folded such that they show direction of flux. Then thumb indicates direction of flux and other fingers how the coil is wounded (clock wise or anti-clock wise) .The means to develop the magnetic field is permanent magnets and above is said to be electro- magnets. Permanent magnetic posses the property of magnetism by nature, in order to develop strong magnetic one must choose permanent magnets with high cohesive force. Permanent magnet has disadvantage of ageing and getting rusted. This disadvantage of permanent is overcome by electro-magnets.**

DEFINITIONS

Magnetostatics is the study of magnetic fields in systems where the currents are steady (not changingwith time). It is the magnetic analogue of electrostatics, where the charges are stationary. Like in

electro-statics in magneto-statics we are going to deal with magnetic field intensity, magnetic flux density using Bio-Savart’s law and Amper’s circuital law.

Some of the important terms used to study characteristics of Magneto-statics are

→ Magnetic flux.

→ Magnetic flux density.

→ Magnetic field intensity.

→ Intensity of magnetization.

→ Magnetic susceptibility.

→ Permeability of core.

→ Reluctance of core.

Magnetic Flux Density

magnetic flux density is defined as flux per unit area,

$$B = d\phi / ds \quad (Wb/mt^2 \text{ or Tesla})$$

$$d\phi = B\, ds$$

by integrating on both sides we can determine total magnetic flux in area,

$$\phi = \int B\, ds \text{ ----------------------28}$$

MAGNETIC FIELD INTENSITY

The force experienced by coil when some current passes through it is magnetic field Intensity.

Mathematically magnetic field intensity is givens as,

H = magnetic force / length

Magnetic force = NI

Length = l

Therefore magnetic field intensity, H = NI / l (AT/mt) --------------- 29

PERMEABILITY

Magnetic Permeabilty

Permeability is the inherent property of core which helps in sustaining flux in the core.

Mathematically permeability is given as, $\mu = B/H$ --------------------- 30

From equation 30 the relation between flux density and intensity is ,

$B = \mu H$ ----------------------- 31

Where $\mu = \mu_0 \mu_r$

μ_0 = absolute permeability = $4\pi x 10^{-7}$ H/mt

μ_r = relative permeability

varies from core to core

INTENSITY OF MAGNETIZATION

When a magnetic substance is placed in a magnetic field it experiences magnetic momentum.

The magnetic momentum per unit volume of substance is intensity of magnetization.

$$I = M / V$$

$M = m.l$ (m- pole strength of bar, l – length)

$$V = A.l$$

intensity of magnetization, $I = m.l / A.l$

$$I = m / A$$

MAGNETIC SUSCEPTIBILITY

The ratio intensity of magnetization to the magnetic field intensity is called as Magnetic Susceptibility $K = I / H$.

Total flux density, B = B due to magnetic field + B due to intensity of magnetization of bar

$$B = \mu_0 H + I$$

But we know that, $\mu = B / H$

$$= (\mu_0 H + I) / H$$

$$= \mu_0 .+ (I/H$$

$$\mu_0 \mu_r = \mu_0 .+ K = 1 + K / \mu_0 \quad \text{--------} \; 31$$

$\mu_r > 1$, paramagnetic materials

$\mu_r < 1$, diamagnetic materials

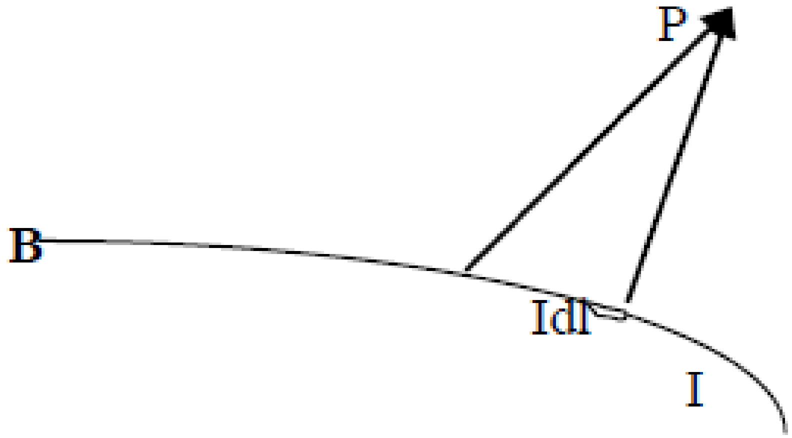
$\mu_r = 0$, non-magnetic materials

BIO-SAVART'S LAW

Bio and savart are two scientists who conducted experiments on current carrying conductor To determine magnetic flux density(B) at any point surrounding that conductor. Their Conclusion is named as "Biot-Savart's Law".

Let us consider an conductor carrying current I, which develops magnetic flux density B surrounding It. Here Idl is called as current element. To find total electric field intensity conductor is divided into Number of current elements.

BIO-SAVART'S LAW



BIO-SAVART'S LAW

The magnetic field intensity due to current element Idl is dH at point P. According Bio-Savart's law

dH α Idl (current element)

dH α sinϴ (angle between current element and length joining point)

dH α 1 / r^2 (square of distance between current element and point)

by combining above three,

dH α Idl . sinϴ / r^2

by removing proportionality,

dH = Idl . sinϴ / $4\pi r^2$

BIO-SAVART'S LAW

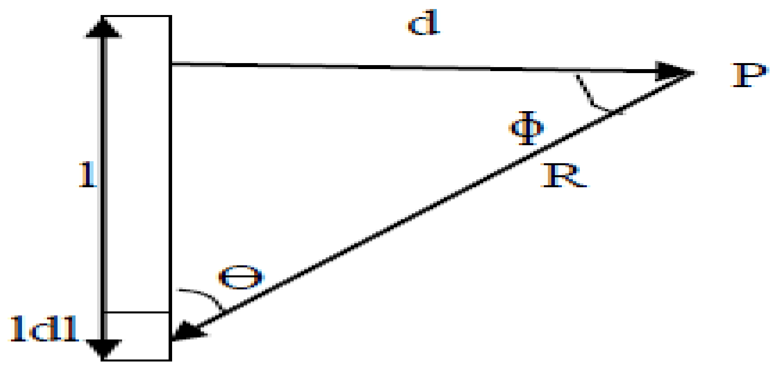
total magnetic field intensity at point P,

$$H = \int Idl \,.\, \sin\Theta / 4\pi r^2$$

therefore total flux density at point P, $B = \mu H$

$$B = \mu \int Idl \,.\, \sin\Theta / 4\pi r^2 \text{ ---------------- } 32$$

FIELD INTENSITY DUE TO STRAIGHT CONDUCTOR



FIELD INTENSITY DUE TO STRAIGHT CONDUCTOR

Let us consider a straight conductor of length l, a test point P at which electric field intensity is to be determined at a distance of d from conductor. Assume current element with a distance of R to

From Bio-Savart's law magnetic field intensity at test point P due to current element Idl is ,

$$dH = Idl \,.\, \sin\theta \,.\, / \, 4\pi R^2 \text{ ------------------------------- a}$$

from above right angle triangle, $\theta + \phi = 90^0$ ----------------- b

using equation a and b,

$$dH = Idl \,.\, \cos\phi \,.\, / \, 4\pi R^2 \text{ ---------------------------------- c}$$

the unit vector a' , indicates the direction H at point P.

$$a' = R' / R \text{ -- } d$$

from above right angle triangle, $R = \sqrt{l^2 + d^2}$ ------------------------------- e

$$\cos\phi = d / \sqrt{l^2 + d^2} \text{ ------------------------------------- } f$$

$$\tan\phi = l / d \text{ -- } g$$

$$l = d.\tan\phi$$

$$dl = d \sec^2\phi \, d\phi \text{ ------------------------------------- } h$$

substituting d,e,f in c,

$$dH = Idl \cdot \cos\phi \cdot d \cdot R' / 4\pi (l^2 + d^2)^2$$

FIELD INTENSITY DUE TO STRAUGHT CONDUCTOR

$$H = \int Idl \,.\, \cos\phi \,.d \,. / 4\pi (l^2 + d^2)^{3/2}$$

$$H = I/(4\Pi d^2\ ) \int dl / (l^2 / d^2 + 1)^{3/2}$$

$$H = I/(4\Pi d^2\ ) \int dl / (\tan^2\phi + 1)^{3/2}$$

Substituting equation h in above equation is ,

$$H = I/(4\Pi d^2\ ). \int d \sec^2\phi \, d\phi / (\sec^2\phi)^{3/2}$$

$$H = I/(4\Pi d^2\ ) .\int d \sec^2\phi \, d\phi / (\sec^3\phi)$$

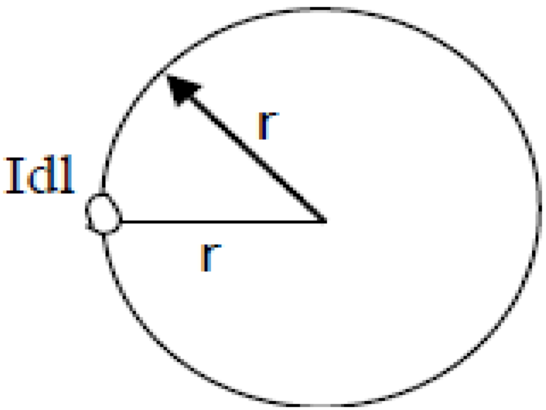
$$H = I/(4\Pi d\ ) .\int \cos\phi \, d\phi$$

$$H = I/(4\Pi d^2) .\sin\phi \text{ ----------------------------------33}$$

For straight line of infinite length, ϕ varies between $-\pi /2$ to $\pi /2$

Substituting above limits in equation 33, $H = I/(2\Pi d)$ ------------- 34

FIELD INTENSITY DUE TO CIRCULAR CONDUCTOR



FIELD INTENSITY DUE TO CIRCULAR CONDUCTOR

Let us consider circular conductor with radius r,

magnetic field intensity at the center of circular conductor is,

from above figure we can say that idl and center are at 90^0

using Bio-Savart's law magnetic field intensity at center point P due to current element Idl is,

$$dH = idl \sin 90 / 4\pi r^2$$

$$dH = idl / 4\pi r^2$$

FIELD INTENSITY DUE TO CIRCULAR CONDUCTOR

integrating on both sides, $H = \int i\,dl \;/\; 4\pi r^2$

$$H = i \int dl \;/\; 4\pi r^2 \qquad (\int dl = 2\pi r)$$

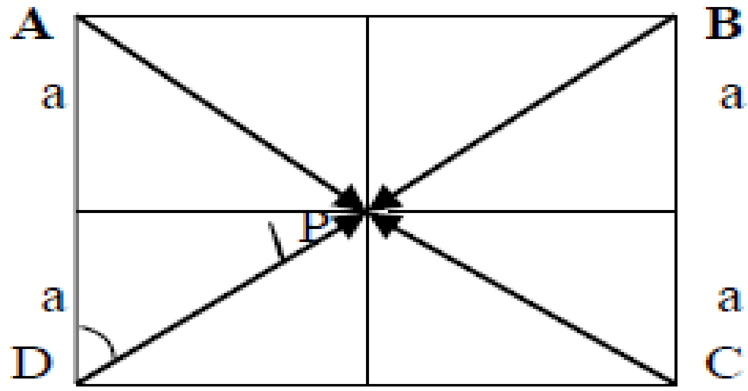
$$H = i\, 2\pi r \;/\; 4\pi r^2$$

$$H = i \;/\; 2r \text{ ------------------------------ } 34$$

Magnetic field intensity at the center of circular conductor with N number of turns is,

$$H = Ni \;/\; 2r \text{ ----------------------- } 35$$

FIELD INTENSITY DUE TO SQUARE CONDUCTOR



FIELD INTENSITY DUE TO SQUARE CONDUCTOR

From the above figure we can say that each side AB,BC,CD,DA has magnetic field intensity at the center Of square conductor.

In every right angle triangle angle between current element and center is 45^0.

The total magnetic field intensity at the center of square due to all corners using Bio-Savart's law

Because of any one side,

$$H = (I / 4\pi a) x[\sin45^0 + \sin45^0]$$

FIELD INTENSITY DUE TO SQUARE CONDUCTOR

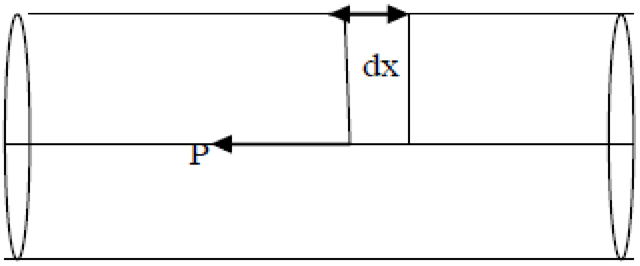
Using all sides,

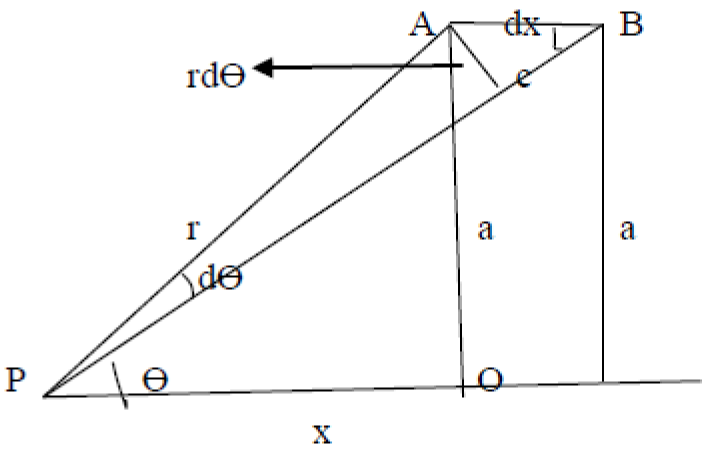
$$H = 4(I / 4\pi a) \times [\sin 45^0 + \sin 45^0]$$

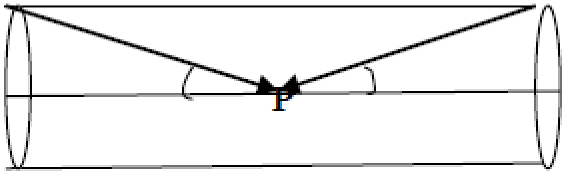
$$H = (I / \pi a) \times [2 / \sqrt{2}]$$

$$H = (\sqrt{2}.I / \pi a) \text{ ---------------------} 36$$

FIELD INTENSITY DUE TO SOLENOID CONDUCTOR







FIELD INTENSITY DUE TO SOLENOID CONDUCTOR

The construction of solenoid is same as coil wounded on a cylinder , let us take take cylinder As reference and derive expression for H due to solenoid. The solenoid with length l, number of turns N allowing an current of I is shown in below figure,

Assume a small length dx, with total turns ndx in it , let us derive what is the magnetic field intensity

Due to dx on P, their by total H at P.

total number of turns = N

total length = l

number of turns per unit length, n = N / l

x be the distance of the point,

the magnetic field intensity due to length dx on P is ,

FIELD INTENSITY DUE TO SOLENOID CONDUCTOR

$$dH = (Ia^2 / 2r^3)\, ndx$$

from figure , $r = \sqrt{a^2 + x^2}$, substituting r in dH.

$$dH = (Ia^2 / 2\,(a^2 + x^2)^{3/2})\, ndx$$

from above right angle triangles, $d\theta <<< \theta$, hence $\sin d\theta = d\theta$

$$\sin \theta = r\, d\theta / dx$$

$$\sin \theta = a / r$$

substituting above deduction in dH,

$$dH = (Ia^2\, r.d\theta / \sin \theta / 2r^3)\, n$$

$$dH = I.n.\, \sin \theta.\, d\theta / 2 \text{ --------------------------------- } a$$

FIELD INTENSITY DUE TO SOLENOID CONDUCTOR

if seen from end points of solenoid the magnetic field intensity at P is Here from one end to other end angle varies from 0 to 2π, substituting above and integrating equation a

$$\int dH = \int I.n. \sin \Theta . d\Theta / 2$$

$$H = - I.n.\cos \Theta . / 2$$

$$H = -(I.n/2)\ [\cos 2\pi – \cos 0]$$

$$H = I.n = NI/l$$

if seen from end point of solenoid the magnetic field intensity at P at same end point,

then the limits varies between 0 to π/2

substituting above limits in b

$$H = -(I.n/2)\ [\cos \pi/2 – \cos 0]$$

$$H = n.I/2 = N.I/ 2l \text{ ------------------ } 37$$

GAUSS LAW IN MAGNETIC FIELDS

From the guass law we can write magnetic flux in the given surface is surface integral of magnetic flux density.

$$\Psi = \int B.ds$$

But total flux density in closed surface is always zero,

$$\Psi = \int B.ds = 0$$

By applying divergence theorem we can write,

$$\int B.ds = \int_v \blacktriangledown\ B.dv = 0$$

hence we can write , $\blacktriangledown$ B = 0, is Maxwell's second equation------ 38

AMPERE CIRCUITAL LAW

The ampere circuital law states line integral magnetic filed intensity around any closed path is equal to total current enclosed in that path.

$$\int H\, dl = I \text{ --- } 39$$

Ampere's law is analogous to gauss law electro-statics.

Applications of Ampere's law :

→ **The magnetic field intensity in the surrounding closed path is always at tangential at Each and every point on it.**

→ **At each every point on the closed path magnetic field intensity has the same value.**

AMPERE CIRCUITAL LAW APPLICATION

From the ampere circuital law we know that,

$$\int H\, dl = I$$

but current can be written as,

$$\int J\, ds = I$$

equating above two equations,

$$\int H\, dl = \int J\, ds \text{ ---------------------------a}$$

from stokes theorem,

$$\int H\, dl = \int \nabla \times H\, ds \text{ --------------------- b}$$

by combining equation a and b,

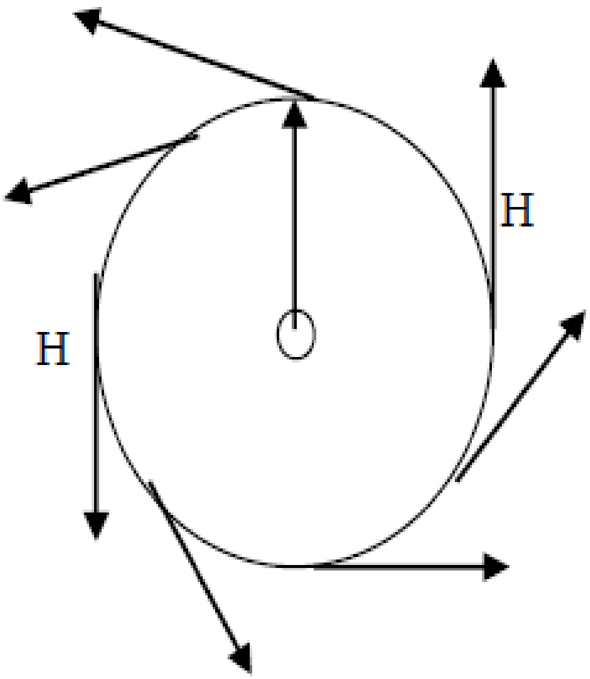
$$\int \nabla \times H\, ds = \int J\, ds$$

by comparing on both sides,

$\nabla \times H = J$, $\nabla \times H$ = curl of H ------40

Equation 40 is called as differential, integral or point form of ampere's law and also calledas Maxwell's Third Equation

AMPERE LAW FOR CIRCULAR CONDUCTOR



AMPERE LAW FOR CIRCULAR CONDUCTOR

Let us consider a straight conductor as shown in figure with closed path of magnetic field Intensity surrounding it with radius of r.

From ampere's circuital law we can write magnetic field intensity in closed path,

$$\int H\,dl = I \text{ -------------------------------------- } a$$

but we can write,

$$\int H\,dl = H \int dl$$

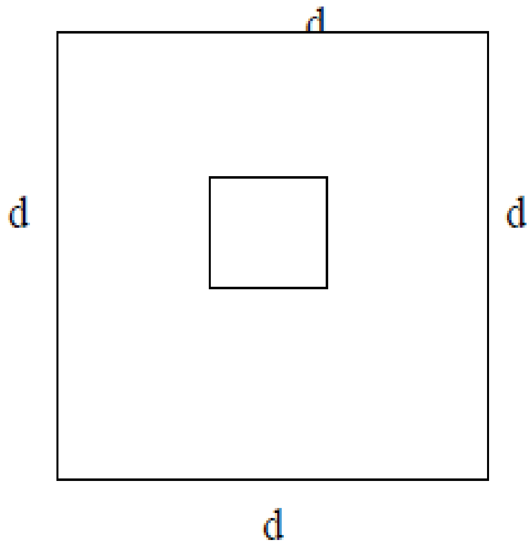
$$= H\,2\pi r \text{ ------------------------------ } b$$

Equating a and b,

$$H\,2\pi r = I$$

$$H = I / 2\pi r \text{ --------------------------------- } 41$$

AMPERE LAW FOR SQUARE CONDUCTOR



AMPERE LAW FOR SQUARE CONDUCTOR

let us consider a square sheet as shown above with surrounding current path of side d.

according to Ampere's law ,

$$\int H\, dl = I$$

where $\int dl$ indicates the mean length closed path,

$$\int dl = 4d$$

their by ,

$$H \int dl = I$$

$$H.4d = I$$

$$H = I/4d. \text{---------} 42$$

AMPERE LAW

The ampere circuital law states line integral magnetic filed intensity around any closed path is equal to total current enclosed in that path.

$$\int H\, dl = I \text{ -- } 39$$

Ampere's law is analogous to gauss law electro-statics.

UNIT-IV

MAGNETIC FORCE AND MAGNETIC POTENTIAL

FORCE ON POINT CHARGE

When an charge Q is with velocity is placed in the magnetic field of density , then it experiences force called as magnetic force.

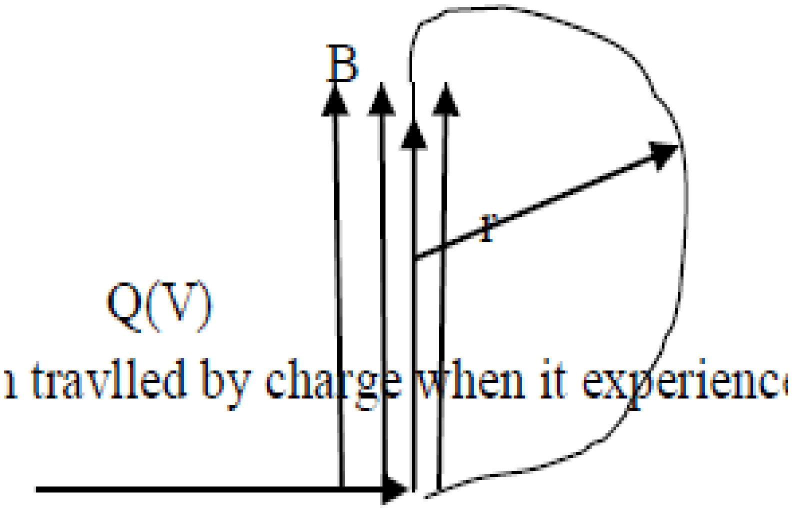
$$F_m = Q(V X B) \text{ ----------------------------------- } 43$$

$$= QVB \sin\theta \, a_f$$

V is parallel to B then Ɵ= 0, therefore sinƟ = 0, hence always velocity direction and flux density

Direction must be normal to each other.

FORCE ON POINT CHARGE



LIMITATIONS ON MOVING CHARGE

The limitations of moving charge in the existing magnetic field,

→ If the velocity of charge in the magnetic field is zero then force experienced also zero.

→ If the velocity direction and magnetic field direction are parallel to each other then force

experienced is zero.

To say that moving charge in the magnetic field experiences force velocity and field must be normal to each other.

From the above discussion the force experienced by moving charge is ,

$$Fm = QVB.$$

Similarly we can also write force experienced by moving charge due to its mass is ,

$$Fm = ma.$$

LIMITATIONS ON MOVING CHARGE

$$Fm = mV^2/r$$

By equating both forces, $QVB = mV^2/r$

$$r = mV / QB$$

time taken to complete one revolution in field is ,

$$T = 2\pi r / V$$

$$= 2\pi m / QB$$

Hence frequency of charge in field is ,

$$F = 1/ T$$

$= QB / 2\pi m$, as this expression of frequency is independent Of velocity it is called as cyclotron.

FORCE EQUATION

We know that the force acquire by point charge when kept in the static electric field is,

$$\overrightarrow{Fe} = \mathrm{Q}\,\vec{E}$$

The force experienced by moving charge in the magnetic field is ,

$$\overrightarrow{Fm} = \mathrm{Q}(\vec{V} X \vec{B})$$

The total force on the charge in the presence of both field is,

$$\vec{F} = \overrightarrow{Fe} + \overrightarrow{Fm}$$

$$= \mathrm{Q}\,\vec{E} + \mathrm{Q}(\vec{V} X \vec{B})$$

$$= Q(\vec{E} + (\vec{V} X \vec{B})) \text{ --- } 44$$

Equation 44 is called as **Lorentz force equation.**

Force on current element due to magnetic field

Let us a long conductor of length l which is partitioned into number parts allowing current

Of I. each part of conductor is of length dl, therefore individual part is represented with Idl called

As current element.

Force due to current element at any point

We know that convection current density is ,

$$\vec{J} = \rho_v \vec{V}$$

The current elements are ,

$$\vec{J}\, dv = K\, ds = \vec{I}\, dl$$

FORCE ON DIFFERENT CONFIGURATION

Using above two equations,

$$\vec{I}\,dl = \rho_v \vec{V}\,dv = Q\vec{V}$$

Also current element, $\vec{I}\,dl = (dQ/\,dt).dl$

$$= dQ.\,\vec{V}$$

The force experienced by moving charge we know as ,

$$\overrightarrow{dFm} = Q(\vec{V} X \vec{B})$$

$$= \vec{I}\,dl \,X\, \vec{B}$$

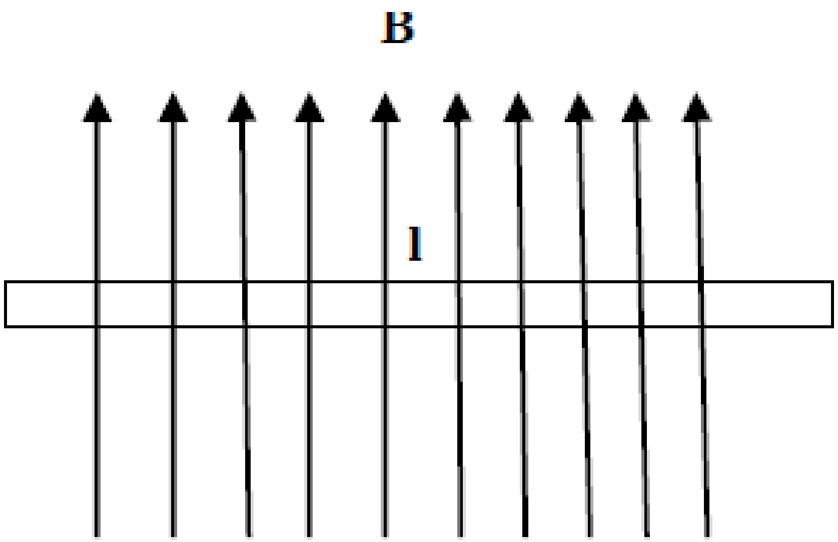
Integrating on both sides we can determine force due current element,

$$\overrightarrow{Fm} = \int \vec{I}\,dl \,X\, \vec{B} \text{ -- } 45$$

Similarly, $\overrightarrow{Fm} = \int_s \vec{K}\,ds \,X\, \vec{B}$

$$\overrightarrow{Fm} = \int_v \vec{J}\,dv \,X\, \vec{B}$$

FORCE ON STRAIGHT CONDUCTOR PLACED IN EXISTING MAGNETIC FIELD



FORCE ON STRAIGHT CONDUCTOR PLACED IN EXISTING MAGNETIC FIELD

Let us consider a straight conductor placed in the magnetic field as shown in the figure,

Of length l, allowing current of I, hence current element if Idl,

The velocity of charges in the given length of conductor is $\vec{V}$.

The force experienced by current element is ,

$$\overrightarrow{dFm} = dQ(\vec{V} X \vec{B})$$

$$= dQ(dl/dt\ X\vec{B})$$

$$= I\ (\overrightarrow{dl} X \vec{B})$$

Their by integrating on both sides, $\overrightarrow{Fm} = I\ (\vec{l} X \vec{B})$

$$Fm = BIl\ \sin\Theta \text{ ---------------------------------- } 46$$

MAGNETIC DIPOLE AND ITS MOMENTUM

Magnetic dipole is formed when two opposite magnetic charges are separated by distance l.

$-Q_m$ --------------l-------------- $+Q_m$

The line joining two charges is termed as axis of dipole. Direction magnetic dipole is from $-Q_m$ to $+Q_m$

In other words a bar magnet with pole strength Q_m and l has , magnetic dipole moment, $m = Q_m l$.

Let us consider a bar conductor allowing current I their forming loop of area A, magnet poles formed

As shown in the figure.

Magnetic dipole moment , $m = IA$

Numerically both dipole moment must be same, $Q_m l = IA$

Magnetization

If their exist an conductor consisting of number of dipoles in its volume , then magnet dipole Moment per unit volume is called as magnetization.

$$M = m / V$$
$$= Q_m .l / A.l$$
$$= Q_m / A$$

Magnetic susceptibility

When the magnetic field is applied to an material the ,
Total magnetic field intensity is ,

MAGNETIZATION AND SUSCEPTIBILITY

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$= \mu_0 \mu_r H$$

Therefore,

$$\mu_0 \mu_r \vec{H} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

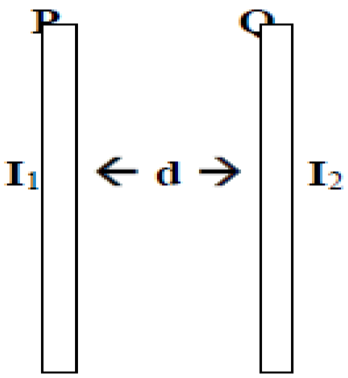
$$\vec{M} = (\mu_r - 1) \vec{H}$$

$$\vec{M} = X_m \vec{H}$$

$X_m = (\mu_r - 1)$ is called as magnetic susceptibility

$= \vec{M} / \vec{H}$ -------------------------------------- 48

MAGNETIC FORCE BETWEEN TWO CONDUCTORS



MAGNETIC FORCE BETWEEN TWO CONDUCTORS

As shown above,

The magnetic field intensity due conductor P on Q is,

$$H = I_1 / 2\Pi d$$

The magnetic flux density due conductor P on Q is,

$$B = \mu_0 I_1 / 2\Pi d$$

Hence forced experienced by conductor Q due to field of P is,

$$F1 = B I_2 l$$

$$= \mu_0 I_1 I_2 l / 2\Pi d$$

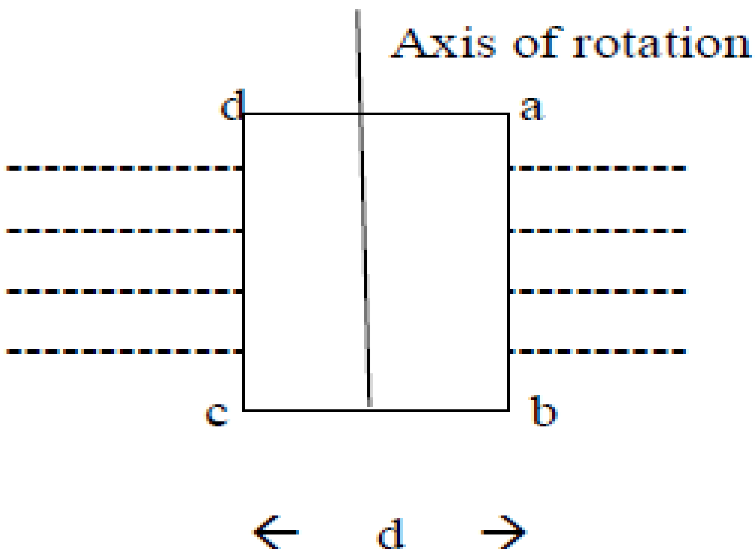
Similarly force experienced by P due to conductor Q is ,

$$F2 = \mu_0 I_1 I_2 l / 2\Pi d$$

Hence force per unit length of conductor is ,

$$(F / l) = \mu_0 I_1 I_2 / 2\Pi d \quad \text{------------------------------------} \quad 47$$

MAGNETIC FORCE ON SQUARECONDUCTORS



MAGNETIC FORCE ON SQUARECONDUCTORS

Let us a consider sheet of side abcd placed in the magnetic field , the side ab experiences the force into the page and side cd out of the page. Angles made by sheet with magnetic field are α and β.

the total torque experienced by sheet due to dipole is ,

T = 2 x torque on each side

= 2 x force x distance from axis of rotation

= 2 x F x d/2

= 2 x BIl cos β x d/2

= BIA cos β

= mB cos β or mB sin α

Therefore torque vector , $\vec{T} = \vec{m} \times \vec{B}$ -- 49

VECTOR AND SCALAR MAGNETIC POTENTIAL

Similarly in the magneto-statics , $H = -\nabla V_m$

V_m – vector magnetic potential

Applying curl on both sides of H, $\nabla \times H = -\nabla \times (\nabla V_m)$

But curl of divergence of any vector is zero, $\nabla \times H = 0$

We can also write , $\nabla \times H = J$

From the above two equations we can write , $J = 0.$

This is possible only in the case constant magnetic field.

from the electro-statics we know that, $\int E\, dl = V$

Similarly in the magneto-statics , $\int H\, dl = V_m$

Ampere circuital law says that, $\int H\, dl = I$

Comparing last two equations, $V_m = I$ ---------50

Hence the units of scalar magnetic potential is Amperes.

VECTOR AND SCALAR MAGNETIC POTENTIAL

We know that divergence magnetic flux density over uniform closed surface is always zero.

$$\nabla B = 0$$

Also divergence of curl of vector is always zero.

$$\nabla .(\nabla x A) = 0$$

By comparing above two equations,

$$B = \nabla x A$$

$$\mu H = \nabla x A$$

$$H = (\nabla x A) / \mu$$

VECTOR AND SCALAR MAGNETIC POTENTIAL

Applying curl on both sides, $\nabla \times H = \nabla \times (\nabla \times A) / \mu = J$

But, $\nabla \times (\nabla \times A) = \nabla . (\nabla . A) - \nabla^2 A = \mu J$

For time invariant fields divergence of vector is zero, hence above can be written as

$$- \nabla^2 A = \mu J$$

$$\nabla^2 A = - \mu J$$

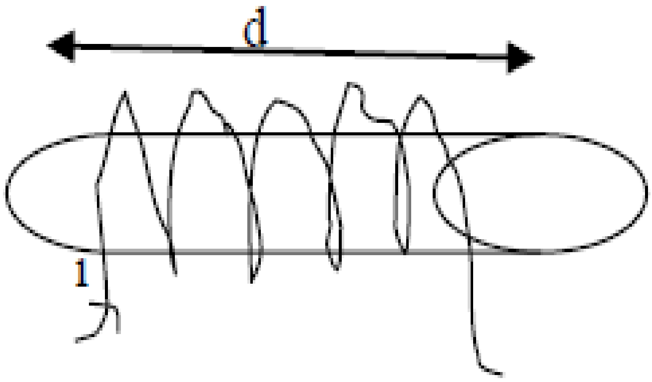
Form the electro-statics we know that, $dv = dq/ 4\pi\varepsilon$

Similarly in the magneto-statics , $dA= \mu idl/ 4\pi r$

Integrating on both sides,

$A = \int \mu idl/ 4\pi r$, **A- vector magnetic potential ---------51**

INDUCATNCE OF SOLENOID



INDUCATNCE OF SOLENOID

N – total turns of solenoid coil

n – number of turns per unit length

magnetic filed density inside solenoid is , $B = \mu_0 n.i.$

total flux linking with coil is $\phi = N B A$

$= \mu_0 n l.i.A .n$

$= \mu_0 n^2.i.A .l$

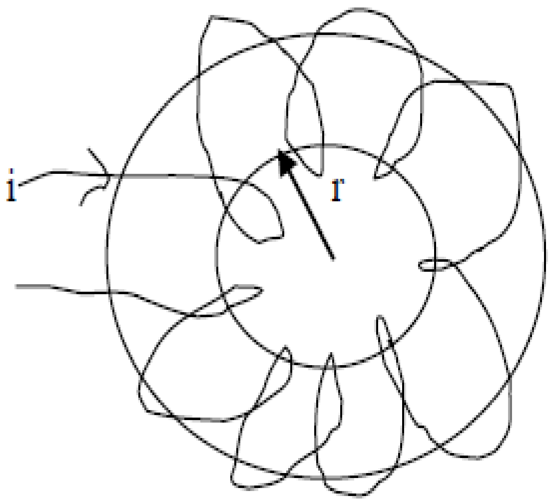
Self inductance is the property of coil which is responsible for emf induced in it,

$L = N \phi / i$

$= \mu_0 n^2.i.A .l / i$

$= \mu_0 N^2 A / l$ H ----------------------------52

INDUCATNCE OF TOROID



INDUCATNCE OF TOROID

Let us a toroid on which a coil N turns is wounded allowing an current of i A.

Let r be the mean radius of the toroid.

Magnetic flux density in the toroid, $B = \mu_0 Ni / l$

Where , $l = 2\pi r$

$B = \mu_0 Ni / 2\pi r$

Total flux linkage with toroid is , $\phi = NBA$

$= (N\ \mu_0\ Ni\ /\ 2\pi r\ )\ .\ A$

But, area $A = \pi R^2$

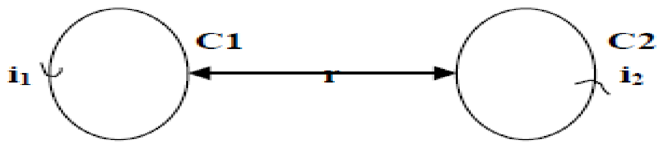
$\phi = (\ N\ \mu_0\ Ni\ /\ 2\pi r).\ \pi R^2$

$= (\ N^2\ \mu_0\ i\ R^2 /\ 2r).$

Therefore self inductance of toroid is , $L = \phi\ /\ i$

$= (\ N^2\ \mu_0\ R^2 /\ 2r).\ H$ ------------------------ 53

NEUMAN'S FORMULA



let us consider two circular coils brought as near as possible allowing i_1 and i_2 currents, with separation of r, of an areas S_1 and S_2 .

the magnetic flux density due to current i_1 is ,

$$B_1 = \nabla \times A_1.$$

Vector magnetic potential , $A_1 = \int \mu i_1 dl_1 / 4\pi r$

Hence flux with second coil due to i_1,

$$\phi_{21} = B_1 \, dS_2$$

hence total flux linking with second coil is ,

$$\Psi_{21} = \int B_1 \, dS_2$$

$$= \int (\nabla \times A_1) \, dS_2$$

From stokes theorem, $\int (\nabla \times A_1) \, dS_2 = \int A_1 \, dl_2$

NEUMAN’S FORMULA

Substituting this inn above equation ,

$$\Psi_{21} = \int A_1 \, dl_2$$

$$= \int \int \mu i_1 dl_1 \, dl_2 / 4\pi r$$

Therefore mutual inductance between two coils is ,

$$M_{21} = \Psi_{21} / i_1$$

Mutual inductance is the imaginary concept which says that there is flux linkage with second

Coil because of current flowing through first coil.

$$M_{21} = \int \int \mu i_1 dl_1 \, dl_2 / 4\pi r \; / \; i_1$$

$$M_{21} = \int \int \mu dl_1 \, dl_2 / 4\pi r \quad \text{-------------} \; 54$$

This M_{21} is called as **Neumann’s formulae.**

ENERGY STORED IN INDUCTOR

Let the work done to increase the current by di is dw, by law of conservation of energy

Work done is equal to energy stored .

$$dw = vi\ dt$$

$$= L.idi.\ dt/dt$$

$$dw = Lidi$$

integrating on both sides , $\int dw = \int Lidi$

$$w = Li^2 / 2$$

but we know that, $L = N\phi / i = \Psi / i$

ENERGY STORED IN INDUCTOR

using above expressions we can write energy stored in the magnetic field also as,

$$w = \Psi i / 2$$

$$= \Psi^2 / 2L. \text{ ----------------------------- } 55$$

MUTUAL INDUCTANCE

When two coils are brought together as close as possible then they form coupledcoils.

Here when current(i1) is allowed through first coil then magnetic flux Φ1 is developed in it, as other coil brought to close proximity some of Φ1 links with second coil called as Φm1 their by inducing voltage in it and when we close the second coil current flows in it (i2). This current i2 develops Φ2 in it and some of Φ2 links with 1^{st} coil called as Φm2. If the two coils are of same dimensions Φm1= Φm2 = Φm.

Here we define two inducatnces slef inductance of coils L1 and L2, mutal inductance between the coils M12=M21=M.

PERMANENT MAGNETS AND APPLICATION

Characteristics and applications of permanent magnets

Characteristics :

Permanent magnets are the one which readily available in nature in the form of Bar and horse shoe shapes etc. Permanent magnets irrespective of supply always exhibits magnetic properties. Permanent magnets always develops a constant magnetic field. The strength of the permanent magnets measured in terms of their cohesive force. An permanent magnet with high cohesive force will have long life. Permanent magnet got the disadvantage of ageing effect i.e in long run they may get rusted.

Applications:

Permanent magnets are used in the applications where ever it is required to develop Constant magnetic field . Eg- Dc generator, Dc motor.

Large industrial electromagnets, on the other hand, benefit greatly from the ability to control the magnetic flux. Electro lifting magnets can be positioned over materials to be moved before the magnetism is turned on, and the load can then be positioned before the magnet is de-energized.

On the negative side, electromagnets require a significant DC power source, create heat, and are vulnerable to power failures.

These problems are not insurmountable, however. Some electromagnets available today, for example, are up to 50% more energy efficient than any others previously available, have moreefficient cooling systems, and can be purchased with rectifiers and emergency generators (or other cut-in power source) to eliminate the vulnerability to power failure.

UNIT-V
TIME VARYING FIELDS AND WAVE PROPAGATION

INTRODUCTION

Time varying fields are produced due to accelerated charges or time varying currents.

Here we shall study how time varying current affects electric and magnet fields.

Faraday's law of electro-magnetic induction

Micheal faraday has stated two laws

If any coil experiences change in flux or variable flux then emf is induced in it.

The emf induced in the coil is directly proportional to rate of change of flux linking With the coil.

$$E \, \alpha \, - d\phi / dt$$

For an coil with N turns emf induced in it ,

$$E = - N . d\phi / dt$$

MAXWELL'S EQUATIONS

We know from the gauss law, $\phi = \int_s B\, ds$

hence emf induced due to above flux is ,

$$e = - d\phi / dt = -d(\int_s B\, ds) /dt$$

Electric potential is given as ,

$$e = \int E\, dl$$

equating above two equations,

$$\int E\, dl = - (\int_s dB\, ds) /dt$$

by applying stokes theorem,

$$\int E\, dl = \int_s (\nabla \times E)\, ds$$

TYPES OF EMF

substituting above equation in c,

$$\int_s (\nabla x E)\, ds = - (\int_s dB\, ds) /dt$$

comparing on both sides,

$$\nabla x E = -dB/dt$$

Equation is called as Maxwell''s fourth equation of vector form of faraday's law.

Types of induced emf

The emf induced in the coil according faraday's law is mainly of two types. They are

Dynamically induced emf

Statically induced emf.

Dynamically induced emf

Let us consider a straight conductor with charge velocity of moving against the existing magnetic field. Force experienced by conductor is , potential induced can be written as, e = BVl sinϴ

the maximum value of potential induced is, e = BVl

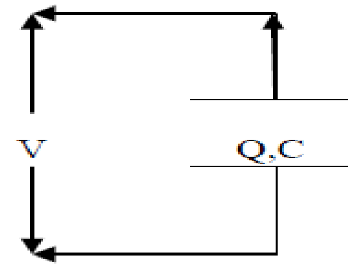
Statically induced emf

If an conductor experiences variable flux then emf induced in it is called as statically induced Emf.

$$e = -Nd\,(\phi_m \sin wt\,) / dt$$

DISPLACEMENT CURRENT DENSITY

Let us consider a capacitor is connected to Ac source as shown in figure



The current flowing through capacitor is ,

$$i_C = C\, dV / dt$$

the capacitance of capacitor,

$$C = \varepsilon A / d$$

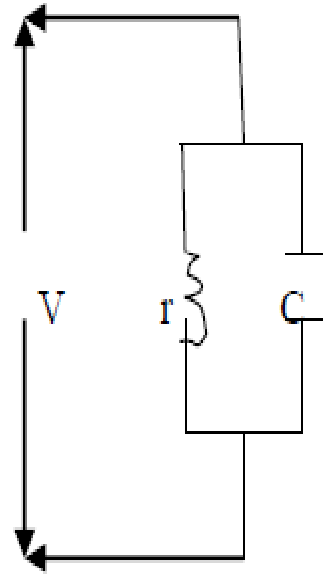
Then, $$i_C = (\varepsilon A / d).\ dV / dt$$

DISPLACEMENT CURRENT DENSITY

$$i_C / A = \varepsilon \, dE / dt$$

$$J_c = dD / dt$$

Jc is called as displacement current.



DISPLACEMENT CURRENT DENSITY

Above is the figure of actual capacitor with internal resistance,

Then the total current is ,

$$i = i_r + i_c$$

where , I – total current

i_r – current through resistance

i_c – current through capacitance

dividing above KCL on both sides by area A,

$$i / A = i_r / A + i_c / A$$

$$J = J_r + J_c$$

Jr – conducting current

Jc – displacement current

6 Maxwell's equations in time varying fields

In the time varying fields we can write,

$$E = Eo\ \cos wt$$

$$= Eo\ e^{jwt}$$

Similarly, $$D = Do\ e^{jwt}$$

$$d D / dt = Do\ wJ\ e^{jwt} = Jw\ Do$$

likely, $$dB / dt = Jw\ B$$

we know that,

$$\nabla x E = - dB / dt$$

MAXWELL'S EQUATION

$$= Jw\ B$$

Also,

$$\nabla x E = -Jw\mu H$$

$$\nabla x\ H = J + dD/dt$$

$$= \sigma E + Jw\ Do$$

$$= \sigma E + Jw\ \varepsilon E$$

$$= E\ (\sigma + Jw\ \varepsilon)$$

Integak form,,

$$\int D\ ds = \int \rho_v dv$$

$$\int B\ ds = 0$$

$$\int E\ dl = -\ Jw \int B\ ds$$

$$\int H\ dl = (\sigma + Jw\ \varepsilon) \int E\ ds$$

ADD ON INFORMATION

There are three ways that objects can be given a net charge. These are:

1. Charging by friction - this is useful for charging insulators. If you rub one material with another (say, a plastic ruler with a piece of paper towel), electrons have a tendency to be transferred from one material to the other. For example, rubbing glass with silk or saran wrap generally leaves the glass with a positive charge; rubbing PVC rod with fur generally gives the rod a negative charge.
2. Charging by conduction - useful for charging metals and other conductors. If a charged object touches a conductor, some charge will be transferred between the object and the conductor, charging the conductor with the same sign as the charge on the object.
3. Charging by induction - also useful for charging metals and other conductors. Again, a charged object is used, but this time it is only brought close to the conductor, and does not touch it. If the conductor is connected to ground (ground is basically anything neutral that can give up electrons to, or take electrons from, an object), electrons will either flow on to it or away from it. When the ground connection is removed , the conductor will have a charge opposite in sign to that of the charged object.