

# **LECTURE NOTES**

**ON**

## **HYDRAULICS AND HYDRAULIC MACHINERY**

**B. Tech V Sem (IARE-R16)**

PREPARED BY

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# UNIT - I

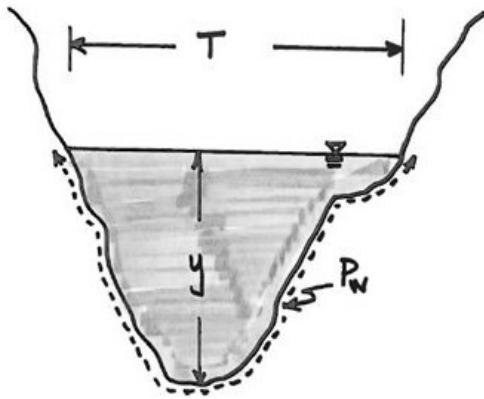
## OPEN CHANNEL FLOW

### INTRODUCTION:

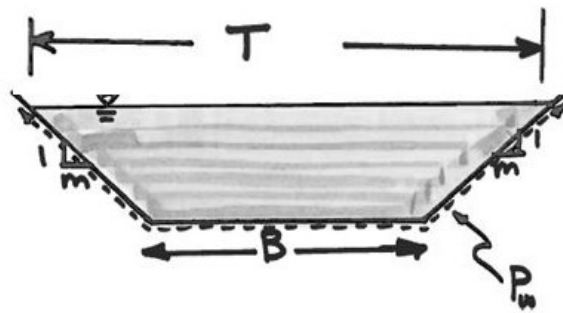
A channel flow is a flow which has a free surface and flows due to gravity, Pipes not flowing full also fall into the category of open channel flow. In open channels, the flow is driven by the slope of the channel rather than the pressure.

### Types of open channels:

1. Natural channel – River, hill slides, rivulets, tidal estuaries
2. Artificial channel – Drains, culverts, sewer, tunnels.



Natural Cross Section



Engineered Cross Section

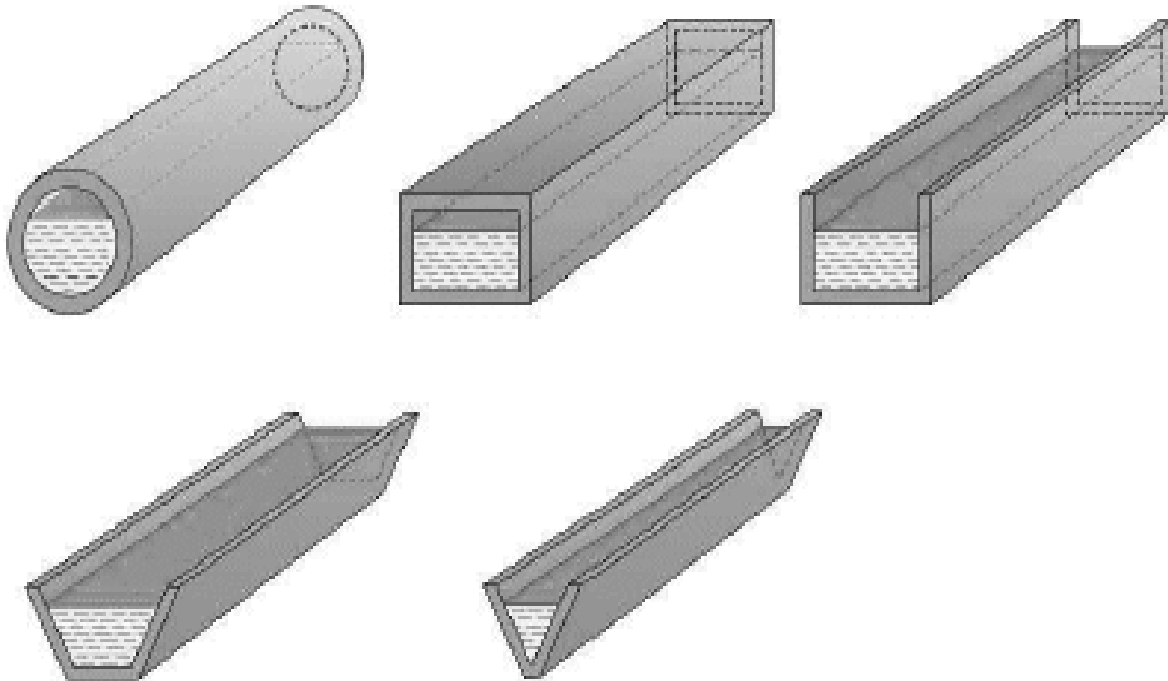
### Types of flow in open channels:

- **Steady flow:** Depth of water ( $y$ ) and velocity ( $v$ ) remain constant with respect to time.
- **Unsteady flow:** Depth of water ( $y$ ) and velocity ( $v$ ) does not remain constant with respect to time.
- **Uniform flow:** Depth of water ( $y$ ) and velocity ( $v$ ) remain constant along the length of the channel.
- **Non Uniform flow:** Depth of water ( $y$ ) and velocity ( $v$ ) varies along the length of the channel.
- **Laminar flow:** Depending on Reynolds number ( $R_e$ ), if  $R_e < 500$ , it is called as laminar flow.
- **Turbulent flow:** Depending on Reynolds number ( $R_e$ ), if  $R_e > 1000$ , it is called as turbulent flow.
- **Transition flow:** If  $R_e$  lies in between 500 and 1000, then the type of flow is called as transition flow.
- **Critical flow:** If the value of Froude number ( $F_r$ ) equals to unity, ( $F_r = 1$ ).
- **Super critical flow:** If the value of Froude number ( $F_r$ ) greater than unity, ( $F_r > 1$ ).
- **Sub critical flow:** If the value of Froude number ( $F_r$ ) less than unity, ( $F_r < 1$ ).
- **Gradually Varied Flow (GVF):** If the depth of flow changes gradually over the given length of the channel.
- **Rapidly Varied Flow (RVF):** If the depth of flow changes abruptly over the given length of the channel.
- **Rigid boundary channels:** Channels with non – changeable boundaries.
- **Non rigid boundary channels:** If the boundaries of channel changes due to scouring action, deposition of sediments etc.
- **Prismatic channels:** The channels in which shape, size of cross section and slope of the channel remain constant. These are also called as artificial channels.

**Types of open channels based on shape:**

Following are the types of channels based on shape:

- Rectangular channel
- Trapezoidal channel
- Triangular channel
- Circular channel



**Geometric Properties of Open Channels – Terminology:**

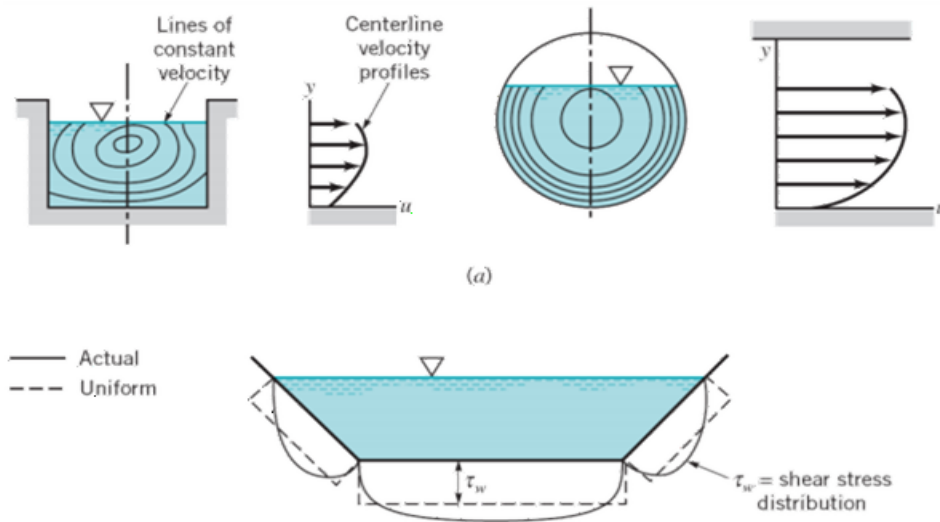
<b>y</b>	: depth of flow in the channel	<b>R</b>	: hydraulic radius at given cross section
<b>m</b>	: side slope of the channel	<b>S<sub>o</sub></b>	: channel bottom slope
<b>T</b>	: top width of the channel	<b>V</b>	: volume of flow
<b>b</b>	: bottom width of the channel	<b>E</b>	: specific energy
<b>L</b>	: length of channel	<b>q</b>	: discharge over width (m)
<b>A</b>	: area of the flow	<b>Q</b>	: flow rates / discharge
<b>P</b>	: wetted perimeter	<b>v</b>	: velocity of flow in channel
<b>D</b>	: hydraulics water depth	<b>Fr</b>	: Froude number

**Conditions for uniform flow in open channel:**

- Constant slope
- Constant cross section
- Constant roughness
- Depth, water area, velocity and discharge at every section of channel are constant.
- Channel bed, water surface and energy line are parallel,  $\Rightarrow S_o = S_w = S$
- $y_1 = y_2$  ,  $V_1 = V_2$

**Velocity distribution:**

- Velocity is always varying across channel because of friction along the boundary.
- The maximum velocity usually found just below the surface.

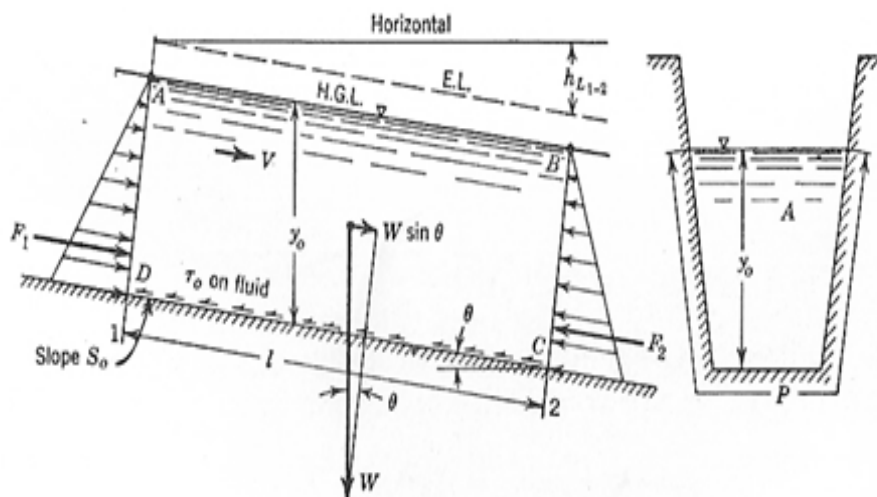


**Determination of discharge through open channels:**

Following are the basic methods to determine the discharge through open channels.

- Chezy's constant (C)
- Manning's value (N)
- Bazin's Formula and
- Kutter's Formula

**Derivation of Chezy's Formula:**



From the above figure,

- L = Length of the channel between section 1 – 1 & 2 – 2
- A = area of flow of water
- I = slope of the bed of the channel
- V = Mean velocity of flow of water
- P = Wetted perimeter of the cross section
- f = Frictional resistance per unit velocity per unit area

Considering forces acting on the water between sections 1-1 & 2-2

1. Component of weight of Water =  $W \sin i \rightarrow$
2. Friction Resistance =  $f P L V^2 \leftarrow$

Where:  $W = \text{Weight density} \times \text{Volume} = w \times (A \times L)$

Equating both forces  $f P L V^2 = w A L \sin i$

$$V = \sqrt{\frac{w}{f}} \sqrt{\frac{A}{P}} \sin i \rightarrow 1$$

$$\frac{A}{P} = m = \text{Hydraulic Radius} \rightarrow 2$$

$$\sqrt{\frac{w}{f}} = C = \text{Chezy's Constant} \rightarrow 3$$

substitute Eqn. 2 & 3 in Eqn. 1,

$$V = C \sqrt{m \sin i}$$

for small values of  $i$ ,  $\sin i = \tan i = i$

$$\therefore V = C \sqrt{m \cdot i}$$

#### Relation between Bazin's Formula and Chezy's formula:

$$C = \frac{157.6}{1.81 + \frac{k}{\sqrt{m}}}$$

Where,

$C = \text{Chezy's Constant}$

$k = \text{Bazin's constant}$

$m = \text{hydraulic radius}$

#### Relation between Manning's formula and Chezy's formula:

In applying the Manning equation, the greatest difficulty lies in the determination of the roughness coefficient,  $n$ ; there is no exact method of selecting the  $n$  value. Selecting a value of  $n$  actually means to estimate the resistance to flow in a given channel, which is really a matter of intangibles. (Chow, 1959). To experienced engineers, this means the exercise of engineering judgment and experience; for a new engineer, it can be no more than a guess and different individuals will obtain different results.

#### Factors Affecting Manning's Roughness Coefficient:

It is not uncommon for engineers to think of a channel as having a single value of  $n$  for all occasions. Actually, the value of  $n$  is highly variable and depends on a number of factors. The factors that exert the greatest influence upon the roughness coefficient in both artificial and natural channels are described below.

- **Surface Roughness:** The surface roughness is represented by the size and shape of the grains of the material forming the wetted perimeter. This is usually considered the only factor in selecting the roughness coefficient, but it is usually just one of the several factors. Generally, fine grains result in a relatively low value of  $n$  and coarse grains in a high value of  $n$ .
- **Vegetation:** Vegetation may be regarded as a kind of surface roughness, but it also reduces the capacity of the channel. This effect depends mainly on height, density, and type of vegetation.
- **Channel Irregularity:** Channel irregularity comprises irregularities in wetted perimeter and variations in cross-section, size, and shape along the channel length.

- **Channel Alignment:** Smooth curvature with large radius will give a relatively low value of n, whereas sharp curvature with severe meandering will increase n.
- **Silting and Scouring:** Generally speaking, silting may change a very irregular channel into a comparatively uniform one and decrease n, whereas scouring may do the reverse and increase n.
- **Obstruction:** The presence of logjams, bridge piers, and the like tends to increase n.
- **Size and Shape of the Channel:** There is no definite evidence about the size and shape of the channel as an important factor affecting the value of n.
- **Stage and Discharge:** The n value in most streams decreases with increase in stage and discharge.
- **Seasonal Change:** Owing to the seasonal growth of aquatic plants, the value of n may change from one season to another season.

The value of C according to Manning's formula as follows:

$$C = \frac{1}{n} m^{\frac{1}{6}}$$

**Where,**

N = Manning's Constant

C = Chezy's constant

m = hydraulic radius

#### **Most Economical Sections – Conditions**

1. Cost of construction should be minimum
2. Discharge should be maximum

Therefore,

$$Q = A V = A C \sqrt{m i}$$

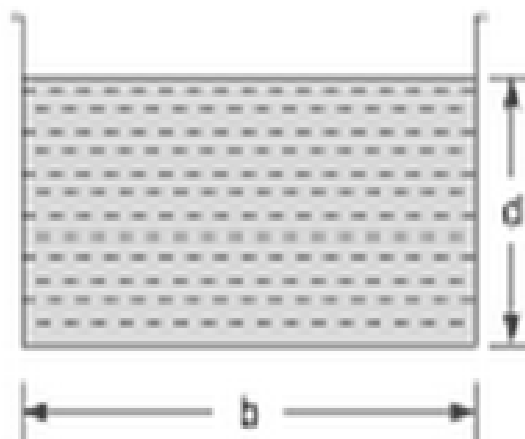
$$Q = K \frac{1}{\sqrt{P}} \text{ where } K = A C \sqrt{A i}$$

If P is minimum, Q will be maximum

**Geometrical properties of various sections:**

Shape	Section	Flow Area, $A$	Wetted Perimeter, $P$	Hydraulic Radius, $R_h$
Trapezoidal		$y(b + y \cot \alpha)$	$b + \frac{2y}{\sin \alpha}$	$\frac{y(b + y \cot \alpha)}{b + \frac{2y}{\sin \alpha}}$
Triangular		$y^2 \cot \alpha$	$\frac{2y}{\sin \alpha}$	$\frac{y \cot \alpha}{2}$
Rectangular		$by$	$b + 2y$	$\frac{by}{b + 2y}$
Wide Flat		$by$	$b$	$y$
Circular		$(\alpha - \sin \alpha) \frac{D^2}{8}$	$\frac{\alpha D}{2}$	$\frac{D}{4} \left( 1 - \frac{\sin \alpha}{\alpha} \right)$

**Condition for most economical section – Rectangular channel:**



for most economical section, P should be minimum

$$\frac{dP}{d(d)} = 0$$

$$A = bd \Rightarrow b = \frac{A}{d} \rightarrow 1$$

$$P = b + 2d = \frac{A}{d} + 2d \rightarrow 2$$

for most economical section, P should be minimum

$$\frac{dP}{d(d)} = 0 \Rightarrow \frac{d\left[\frac{A}{d} + 2d\right]}{d(d)} = 0 \Rightarrow \frac{-A}{d^2} + 2 = 0 \Rightarrow A = 2d^2 \Rightarrow bd = 2d^2$$

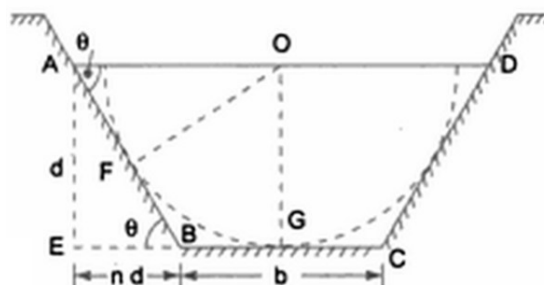
$$b = 2d \text{ or } d = b/2$$

$$m = \frac{A}{P} = \frac{bd}{b+2d} = \frac{2d^2}{2d+2d} = \frac{d}{2}$$

Hence, from the above equations, following points can be concluded:

1. Hydraulic mean depth (m) of the given rectangular channel is equal to half of the depth of flow in the channel.
2. Depth of the flow in the channel is equal to half the top width of the channel.

Condition for most economical section – Trapezoidal channel:



for most economical section,

P should be minimum

$$\frac{dP}{d(d)} = 0$$

$$A = (b + nd)d \Rightarrow b = \frac{A}{d} - nd \rightarrow 1$$

$$P = b + 2d\sqrt{n^2 + 1} = \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \rightarrow 2$$



for most economical section, P should be minimum

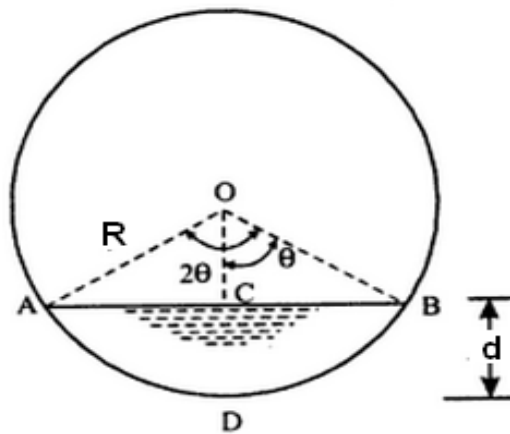
$$\frac{dP}{d(d)} = 0 \Rightarrow \frac{d \left[ \frac{A}{d} - nd + 2d\sqrt{n^2 + 1} \right]}{d(d)} = 0 \Rightarrow \frac{b + 2nd}{2} = d\sqrt{n^2 + 1}$$

$$m = \frac{d}{2} \text{ and } \theta = 60^\circ$$

Hence, following are the concluding statements for the most economical cross section of trapezoidal channel cross section:

1. The length of any sloping side is equal to half of top width of the channel.
2. Hydraulic mean depth of the channel is equal to half of the depth of flow in the channel.
3. The angle with respect to horizontal for the sloping sides is equal to  $60^\circ$ .
4. The portion of the semicircle drawn by taking the centre as the midpoint of top width should touch all the sides tangentially.

Condition for most economical section – Circular channel:



Circular channel.

$$A = R^2 \left( \theta - \frac{\sin 2\theta}{2} \right) \rightarrow 1$$

$$P = 2R\theta \rightarrow 2$$

$$m = \frac{A}{P} = \frac{R}{2\theta} \left( \theta - \frac{\sin 2\theta}{2} \right) \rightarrow 3$$

$$\text{for max. velocity, } \frac{dm}{d\theta} = 0 \Rightarrow \theta = 128^\circ 45', d = 0.81D, m = 0.3D$$

$$Q = AC\sqrt{m i} = AC\sqrt{\frac{A}{P} i} = C\sqrt{\frac{A^3}{P} i}, C \text{ and } i \text{ are constants}$$

$$\text{for max. discharge, } \frac{d \left[ \sqrt{\frac{A^3}{P}} \right]}{d\theta} = 0 \Rightarrow \theta = 154^\circ, d = 0.95D$$

### Non-uniform Flow:

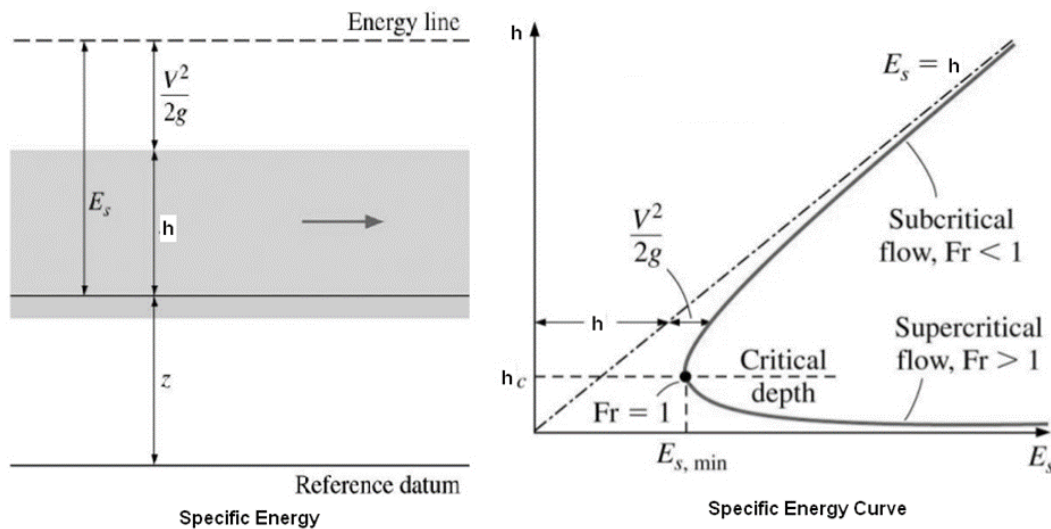
In Non-uniform flow, velocity varies at each section of the channel and the Energy Line is not parallel to the bed of the channel. This can be caused by

1. Differences in depth of channel and
2. Differences in width of channel.
3. Differences in the nature of bed
4. Differences in slope of channel and
5. Obstruction in the direction of flow

### Specific energy:

$$\text{Total Energy of flowing fluid, } E = z + h + \frac{v^2}{2g}$$

where  $z$  = Height of bottom of channel above datums,



For critical depth  $\frac{dE}{dh} = 0$ , Where:  $E = h + \frac{q^2}{2gh^2}$

$$h_c = \left[ \frac{q^2}{g} \right]^{\frac{1}{3}} \Rightarrow h_c^3 = \frac{q^2}{g} \Rightarrow h_c^3 \cdot g = q^2 \rightarrow 1$$

$$\text{but, } q = \frac{Q}{b} \Rightarrow q = \frac{bh \cdot v}{b} \Rightarrow h_c \cdot v$$

$$\text{Substitute in equation 1} \Rightarrow v_c = \sqrt{g \cdot h_c}$$

**Minimum Specific Energy in terms of Critical Depth;  $E = h + \frac{q^2}{2gh^2}$**

when specific energy is minimum, Depth of flow is critical

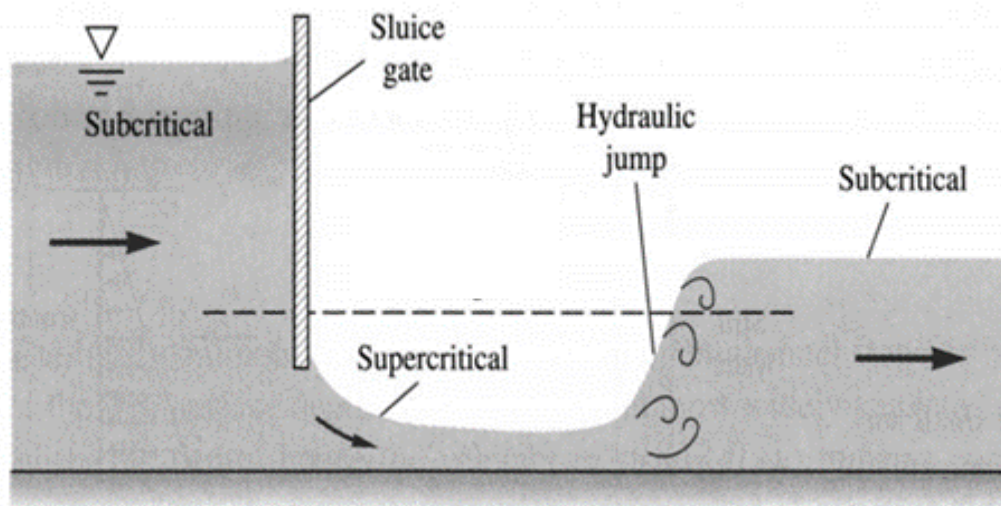
$$E = h_c + \frac{q^2}{2gh_c^2} \text{ substitute } h_c = \left[ \frac{q^2}{g} \right]^{\frac{1}{3}} \text{ or } h_c^3 = \frac{q^2}{g}$$

$$E_{\min} = h_c + \frac{h_c^3}{2gh_c^2} = h_c + \frac{h_c}{2} = \frac{3h_c}{2}$$

$$\text{or } h_c = \frac{2E_{\min}}{3}$$

### Hydraulic Jump:

It is defined as the place where the flow changes from supercritical flow to sub critical flow. It



can also define as the rise of water level, which takes place due to formation of the unstable shooting flow to stable streaming flow. Hence due to this transformation, loss of energy takes place. Following are the most typical locations for the occurrence of hydraulic jump are:

- Below control structures like weir, sluice of a channel.
- Due to the presence of any obstruction
- In the presence of a sharp change in the channel takes place due to slope or depth.
- At the toe of spillway dam.

The following formulas represent the values of  $d_2$  (Depth after the occurrence of jump) in terms of  $q$ ,  $V_1$  and  $F_1$  respectively.

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}} \rightarrow \text{interms of } q$$

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2v_1^2 d_1}{g_1}} \rightarrow \text{interms of } V_1$$

$$d_2 = \frac{d_1}{2} \left( \sqrt{1 + 8F_e^2} - 1 \right) \rightarrow \text{interms of } F_e$$

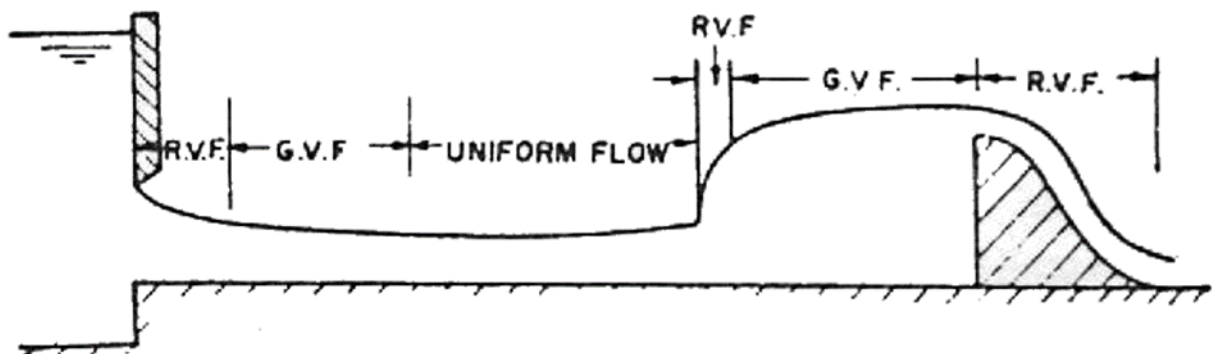
Loss of Energy:

$$h_L = E_1 - E_2 = \left[ \frac{[d_2 - d_1]^3}{4d_1 d_2} \right]$$

Length of jump = 5 to 7 times of  $(d_2 - d_1)$

Hydraulic Jump =  $d_2 - d_1$

### Gradually Varied Flow (GVF)



In GVF, depth and velocity vary slowly, and the free surface is stable

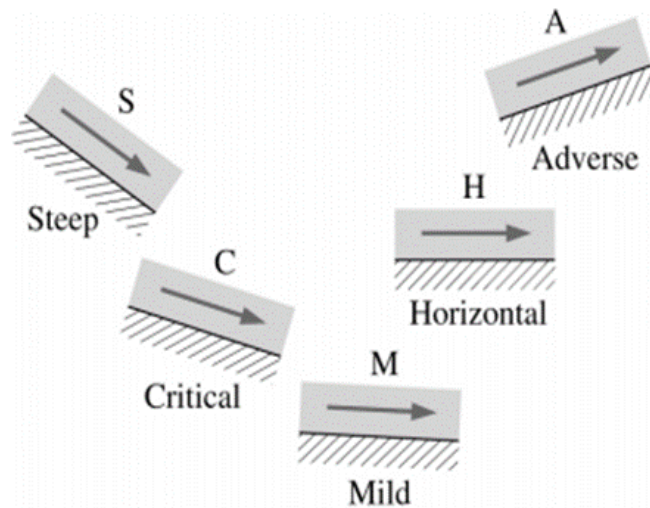
The GVF is classified based on the channel slope, and the magnitude of flow depth.

Steep Slope (S):	$S_o > S_c$ or $h < h_c$
Critical Slope (C):	$S_o = S_c$ or $h = h_c$
Mild Slope (M):	$S_o < S_c$ or $h > h_c$
Horizontal Slope (H):	$S_o = 0$
Adverse Slope(A):	$S_o = \text{Negative}$

where

$S_o$  : the slope of the channel bed,

$S_c$  : the critical slope that sustains a given discharge as uniform flow at the critical depth ( $h_c$ ).



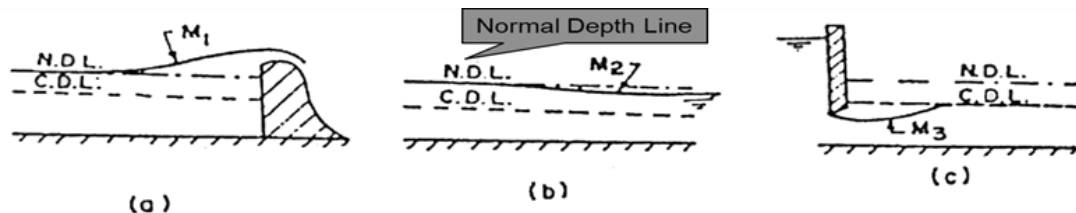
**Flow Profiles:**

The surface curves of water are called flow profiles (or water surface profiles).

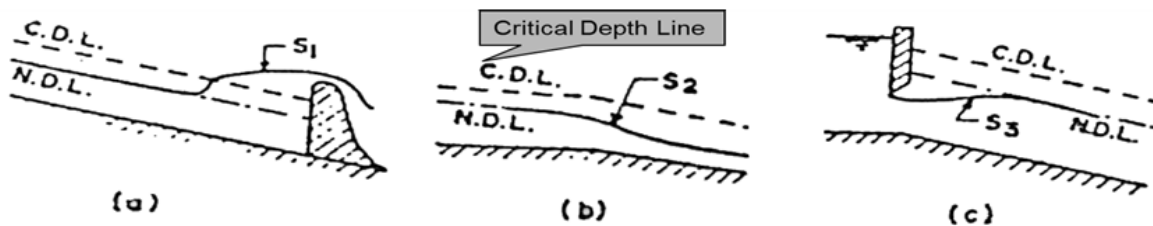
Depending upon the zone and the slope of the bed, the water profiles are classified into 13 types as follows:

- |                            |            |
|----------------------------|------------|
| 1. Mild slope curves       | M1, M2, M3 |
| 2. Steep slope curves      | S1, S2, S3 |
| 3. Critical slope curves   | C1, C2, C3 |
| 4. Horizontal slope curves | H2, H3     |
| 5. Adverse slope curves    | A2, A3     |

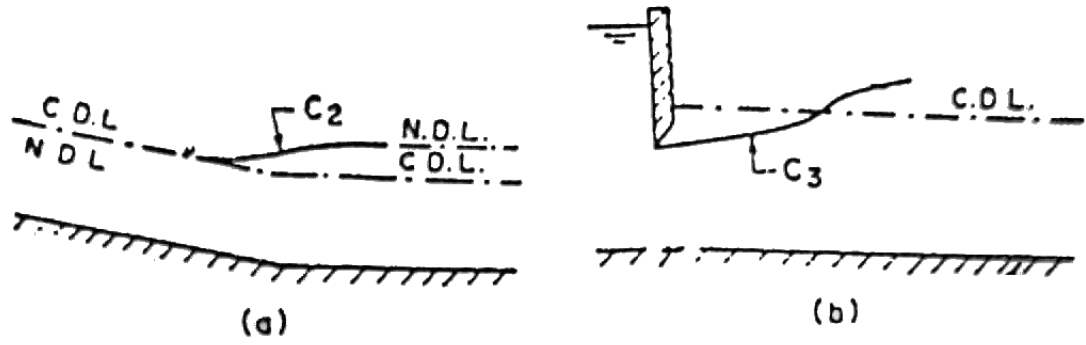
In all these curves, the letter indicates the slope type and the subscript indicates the zone. For example S2 curve occurs in the zone 2 of the steep slope.



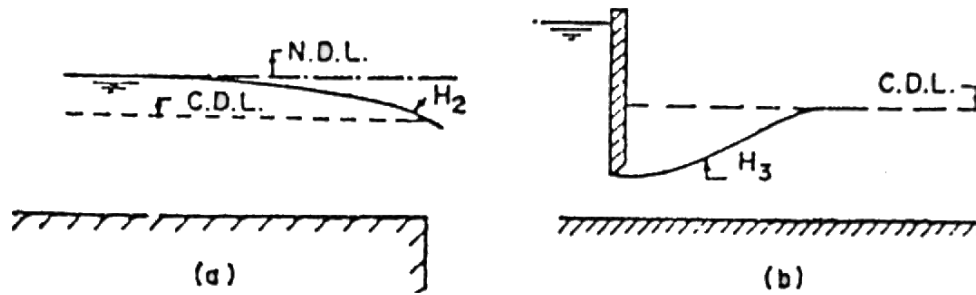
**Flow Profiles in Mild slope**



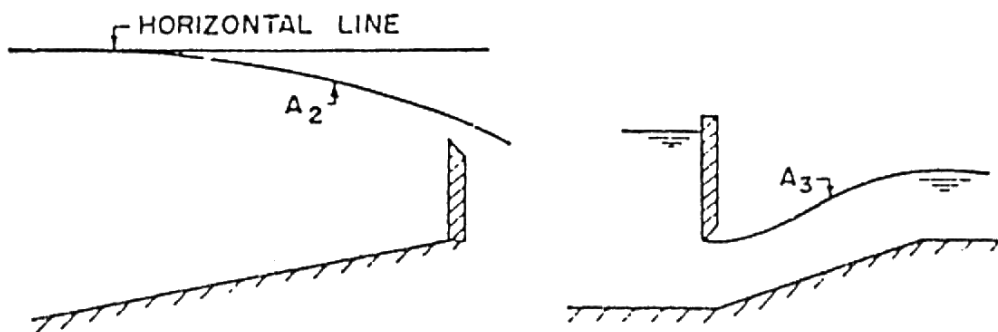
**Flow Profiles in Steep slope**



### Flow Profiles in Critical slope



### Flow Profiles in Horizontal slope



### Flow Profiles in Adverse slope

Equation of GVF:

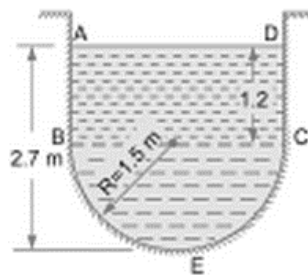
$$\frac{dh}{dx} = \frac{i_b - i_e}{\left[1 - \frac{V^2}{gh}\right]} \rightarrow \text{in terms of Velocity}$$

$$\frac{dh}{dx} = \frac{i_b - i_e}{\left[1 - (F_e)^2\right]} \rightarrow \text{in terms of } F_e$$

$$L = \frac{E_2 - E_1}{i_b - i_e}$$

### PROBLEMS:

- Find the velocity of flow and rate of flow of water through a rectangular channel of 6 m wide and 3 m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take Chezy's constant  $C = 55$
- Find slope of the bed of a rectangular channel of width 5m when depth of water is 2 m and rate of flow is given as 20 m<sup>3</sup>/s. Take Chezy's constant,  $C = 50$
- Find the discharge through a trapezoidal channel of 8 m wide and side slopes of 1 horizontal to 3 vertical. The depth of flow is 2.4 m and Chezy's constant  $C = 55$ . The slope of bed of the channel is 1 in 4000
- Find diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 litres/s when flowing half full. Take Manning's  $N = 0.020$
- Find the discharge through a channel show in fig. 16.5. Take the value of Chezy's constant  $C = 55$ . The slope of bed of the channel is 1 in 2000.



- A trapezoidal channel has side slopes of 1 horizontal and 2 vertical and the slope of the bed is 1 in 1500. The area of cross section is 40m<sup>2</sup>. Find dimensions of the most economical section. Determine discharge if  $C=50$ .
- A rectangular channel of width 4 m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take  $C=50$ .
- The rate of flow of water through a circular channel of diameter 0.6m is 150 litres/s. Find the slope of the bed of the channel for maximum velocity. Take  $C=50$ .
- The specific energy for a 3 m wide channel is to be 3 kg-m/kg. What would be the max. possible discharge.
- The discharge of water through a rectangular channel of width 6 m, is 18 m<sup>3</sup>/s when depth of flow of water is 2 m. Calculate:
  - Specific Energy
  - Critical Depth
  - Critical Velocity
  - Minimum Energy.
- The specific energy for a 5 m wide rectangular channel is to be 4 Nm/N. If the rate of flow of water through the channel is 20 m<sup>3</sup>/s, determine the alternate depths of flow.
- The depth of flow of water, at a certain section of a rectangular channel of 2 m wide is 0.3 m. The discharge through the channel is 1.5 m<sup>3</sup>/s. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water.
- A sluice gate discharges water into a horizontal rectangular channel with a velocity of 10 m/s and depth of flow of 1 m. Determine the depth of flow after jump and consequent loss in total head.
- Find the rate of change of depth of water in a rectangular channel of 10 m wide and 1.5 m deep, when water is flowing with a velocity of 1 m/s. The flow of water through the channel of bed slope in 1 in 4000, is regulated in such a way that energy line is having a slope of 0.00004.
- Find the slope of the free water surface in a rectangular channel of width 20 m, having depth of flow 5 m. The discharge through the channel is 50 m<sup>3</sup>/s. The bed of channel is having a slope of 1 in 4000. Take  $C=60$ .

## UNIT - II

### DIMENSIONAL ANALYSIS AND SIMILITUDE

#### INTRODUCTION:

Many practical real flow problems in fluid mechanics can be solved by using equations and analytical procedures. However, solutions of some real flow problems depend heavily on experimental data. Sometimes, the experimental work in the laboratory is not only time-consuming, but also expensive. So, the main goal is to extract maximum information from fewest experiments. In this regard, dimensional analysis is an important tool that helps in correlating analytical results with experimental data and to predict the prototype behavior from the measurements on the model.

#### Dimensions and Units:

Dimensional analysis is a mathematical technique which makes use of the study of dimensions as an aid to the solution of several engineering problems. It deals with the dimensions of the physical quantities are measured by comparison, which is made with respect to an arbitrarily fixed value.

- Length L, mass M and Time T are three fixed dimensions which are of importance in fluid mechanics. If in any problem of fluid mechanics, heat is involved then temperature is also taken as fixed dimension.
  - These fixed dimensions are called fundamental dimensions or fundamental quantity. The following are the Fundamental Dimensions (MLT)
  - Mass - kg - M
  - Length - m - L
  - Time - s - T
  - Secondary dimensions are those quantities which possess more than one fundamental dimension.
1. Geometric dimensions
    - Area -  $m^2$  -  $L^2$
    - Volume -  $m^3$  -  $L^3$
  2. Kinematic dimensions
    - Velocity -  $m/s$  -  $\frac{L}{T}$  or  $L.T^{-1}$
    - Acceleration -  $m/s^2$  -  $\frac{L}{T^2}$  or  $L.T^{-2}$
  3. Dynamic dimensions
    - Force - N -  $\frac{ML}{T}$  or  $M.L.T^{-1}$
    - Density -  $kg/m^3$  -  $M/L^3$  or  $M.L^{-3}$

#### Use of Dimensional Analysis:

1. Conversion from one dimensional unit to another
2. Checking units of equations (Dimensional Homogeneity)
3. Defining dimensionless relationship using
  - Rayleigh's Method
  - Buckingham's  $\pi$ -Theorem
4. Model Analysis



### Dimensional Homogeneity:

Dimensional Homogeneity means the dimensions in each equation on both sides equal.

Consider the following equation –  $V = \sqrt{2 X g X h}$

Dimensions of L. H. S =  $V = \frac{L}{T} = L X T^{-1}$

Dimensions of R. H. S =  $\sqrt{2 X g X h} = \sqrt{\frac{L}{T^{-2}} X L} = L X T^{-1}$

∴ Dimensions of L. H. S = Dimensions of R. H. S

Hence the above equation is said to be dimensionally homogenous as Dimensions of L. H. S and R. H. S are equal.

### Methods of dimensional analysis:

#### Rayleigh's Method:

To define relationship among variables. This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

#### Methodology:

1. Let X is a function of X<sub>1</sub>, X<sub>2</sub>, and X<sub>3</sub> and mathematically it can be written as  
 $X = f(X_1, X_2, X_3)$
2. This can be also written as  
 $X = K (X_1^a, X_2^b, X_3^c)$  where K is constant and a, b and c are arbitrarily powers.
3. The values of a, b and c are obtained by comparing the powers of the fundamental dimension on both sides.

#### Buckingham's π-Theorem:

This method of analysis is used when number of variables are more.

#### Theorem:

If there are **n** variables in a physical phenomenon and those **n** variables contain **m** dimensions, then variables can be arranged into (n – m) dimensionless groups called Φ terms.

#### Explanation:

1. If  $f(X_1, X_2, X_3, \dots, X_n) = 0$  and variables can be expressed using m dimensions then,  
 $f(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0$ , where,  $\pi_1, \pi_2, \pi_3, \dots$  are dimensionless groups.
2. Each π term contains (m + 1) variables out of which **m** are of repeating type and one is of non – repeating type.
3. Each π term being dimensionless, the dimensional homogeneity can be used to get each π term. π denotes a non-dimensional parameter.

#### Selecting Repeating Variables:

1. Avoid taking the quantity required as the repeating variable.
2. Repeating variables put together should not form dimensionless group.
3. No two repeating variables should have same dimensions.
4. Repeating variables can be selected from each of the following properties.
  - Geometric property → Length, height, width, area
  - Flow property → Velocity, Acceleration, Discharge
  - Fluid property → Mass density, Viscosity, Surface tension

### Model Analysis:

For predicting the performance of the hydraulic structures (such as dams, spillways etc.) or hydraulic machines (such as turbines, pumps etc.) before actually constructing or manufacturing, models of the structures or machines are made and tests are conducted on them to obtain the desired information.

**Model** is a small replica of the actual structure or machine. The actual structure or machine is called as **Prototype**. Models can be smaller or larger than the Prototype. Model Analysis is actually an experimental method of finding solutions of complex flow problems.

### Similitude or Similarities:

Similitude is defined as the similarity between the model and prototype in every aspect, which means that the model and prototype have similar properties.

Types of Similarities:

- Geometric Similarity → Length, Breadth, Depth, Diameter, Area, Volume etc.,
- Kinematic Similarity → Velocity, Acceleration etc.,
- Dynamic Similarity → Time, Discharge, Force, Pressure Intensity, Torque, Power

### Geometric Similarity:

The geometric similarity is said to exist between the model and prototype if the ratio of all corresponding linear dimensions in the model and prototype are equal. Mathematically,

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = L_r \text{ and } \frac{A_p}{A_m} = L_r^2 \text{ and } \frac{V_p}{V_m} = L_r^3$$

where  $L_r$  is Scale Ratio

### Kinematic Similarity

The kinematic similarity is said exist between model and prototype if the ratios of velocity and acceleration at corresponding points in the model and at the corresponding points in the prototype are the same. Also the directions of the velocities in the model and prototype should be same.

$$\frac{V_p}{V_m} = V_r \text{ and } \frac{a_p}{a_m} = a_r$$

where  $V_r$  is Velocity Ratio

where  $a_r$  is Acceleration Ratio

### Dynamic Similarity

The dynamic similarity is said exist between model and prototype if the ratios of corresponding forces acting at the corresponding points are equal.

$$\frac{F_p}{F_m} = F_r$$

where  $F_r$  is Force Ratio

It means for dynamic similarity between the model and prototype, the dimensionless numbers should be same for model and prototype.

### Forces acting on moving fluid:

Following are the different types of forces to be considered when the liquid is in motion.

- Inertia force,  $F_i$
- Viscous force,  $F_v$
- Gravity force,  $F_g$
- Pressure force,  $F_p$
- Surface tension force,  $F_s$
- Elastic force,  $F_e$

#### 1. Inertia Force, $F_i$ :

- It is the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.
- It always exists in the fluid flow problems.

#### 2. Viscous Force, $F_v$ :

- It is equal to the product of shear stress due to viscosity and surface area of the flow.
- It is important in fluid flow problems where viscosity is having an important role to play.

#### 3. Gravity Force, $F_g$ :

- It is equal to the product of mass and acceleration due to gravity of the flowing fluid.
- It is present in case of open surface flow.

#### 4. Pressure force, $F_p$ :

- It is equal to the product of pressure intensity and cross-sectional area of flowing fluid.
- It is present in case of pipe-flow.

#### 5. Surface Tension Force, $F_s$ :

- It is equal to the product of surface tension and length of surface of the flowing fluid.

#### 6. Elastic Force, $F_e$ :

- It is equal to the product of elastic stress and area of the flowing fluid

### Dimensionless Numbers:

Dimensionless numbers are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force. Following are some of the dimensionless numbers described below. The detailed applications of these dimensionless numbers shall be discussed detailed in further sections.

$$\text{Reynold's number, } R_e = \frac{\text{Inertia Force}}{\text{Viscous Force}} = \frac{\rho VL}{\mu} \text{ or } \frac{\rho VD}{\mu}$$

$$\text{Froude's number, } F_e = \sqrt{\frac{\text{Inertia Force}}{\text{Gravity Force}}} = \frac{V}{Lg}$$

$$\text{Euler's number, } E_u = \sqrt{\frac{\text{Inertia Force}}{\text{Pressure Force}}} = \frac{V}{\sqrt{p/\rho}}$$

$$\text{Weber's number, } W_e = \sqrt{\frac{\text{Inertia Force}}{\text{Surface Tension Force}}} = \frac{V}{\sqrt{\sigma/\rho L}}$$

$$\text{Mach's number, } M = \sqrt{\frac{\text{Inertia Force}}{\text{Elastic Force}}} = \frac{V}{C}$$

### Model Laws:

The laws on which the models are designed for dynamic similarity are called model laws or laws of similarity.

- Reynold's Model Law
- Froude Model Law.
- Euler Model Law.
- Weber Model Law
- Mach Model Law

#### 1. Reynold's Model Law:

If the viscous forces are predominant, the models are designed for dynamic similarity based on Reynold's number. Models based on Reynold's Number includes:

- Pipe Flow
- Resistance experienced by Sub-marines, airplanes, fully immersed bodies etc.
- Mathematically, it can be expressed as:

$$\begin{aligned} & \bullet \quad [Re]_m = [Re]_p \\ \Rightarrow & \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p} \end{aligned}$$

#### 2. Froude Model Law:

If the gravity force is predominant, the models are designed for dynamic similarity based on Froude number. Froude Model Law is applied in the following fluid flow problems:

- Free Surface Flows such as Flow over spillways, Weirs, Sluices, Channels etc.,
- Flow of jet from an orifice or nozzle
- Where waves are likely to formed on surface
- Where fluids of different densities flow over one another

$$\begin{aligned} & [F_r]_m = [F_r]_p \\ \Rightarrow & \frac{V_m}{\sqrt{g_m X L_m}} = \frac{V_p}{\sqrt{g_p X L_p}} \end{aligned}$$

#### 3. Euler Model Law:

Euler Model Law is applied in the following cases:

- Closed pipe in which case turbulence is fully developed so that viscous forces are negligible and gravity force and surface tension is absent where phenomenon of cavitation's takes place.

#### 4. Weber Model Law:

Weber Model Law is applied in the following cases:

- Capillary rise in narrow passages
- Capillary movement of water in soil
- Capillary waves in channels
- Flow over weirs for small heads

### 5. Mach Model Law:

Mach Model Law is applied in the following cases:

- Flow of aero plane and projectile through air at supersonic speed ie., velocity more than velocity of sound.
- Aero dynamic testing
- Underwater testing of torpedoes, and
- Water-hammer problems

### Problems:

1. Find Dimensions for the following:

- Stress / Pressure
- Work
- Power
- Kinetic Energy
- Dynamic Viscosity
- Kinematic Viscosity
- Surface Tension
- Angular Velocity
- Momentum
- Torque

2. Check Dimensional Homogeneity of the following:

- $Q = AV$
- $E_k = v^2/2g$

3. Water flowing through a pipe of diameter 30 cm at a velocity of 4 m/s. Find the velocity of oil flowing in another pipe of diameter 10cm, if the conditions of dynamic similarity is satisfied between two pipes. The viscosity of water and oil is given as 0.01 poise and 0.025 poise. The specific gravity of oil is 0.8.
4. In 1 in 40 model of a spillway, the velocity and discharge are 2 m/s and 2.5 m<sup>3</sup>/s. Find corresponding velocity and discharge in the prototype
5. In a 1 in 20 model of stilling basin, the height of the jump in the model is observed to be 0.20m. What is height of hydraulic jump in the prototype? If energy dissipated in the model is 0.1kW, what is the corresponding value in prototype?
6. A 7.2 m height and 15 m long spillway discharges 94 m<sup>3</sup>/s discharge under a head of 2m. If a 1:9 scale model of this spillway is to be constructed, determine the model dimensions, head over spillway model and the model discharge. If model is experiencing a force of 7500 N, determine force on the prototype.
7. A Dam of 15 m long is to discharge water at the rate of 120 cumecs under a head of 3 m. Design a model, if supply available in the laboratory is 50 lps
8. A 1:50 spillway model has a discharge of 1.5 cumecs. What is the corresponding discharge in prototype?. If a flood phenomenon takes 6 hour to occur in the prototype, how long it should take in the model.

## UNIT - III

# HYDRODYNAMIC FORCE ON JETS

### INTRODUCTION:

Analysis and Design of Hydraulic Machines (Turbines and Pumps) is essentially based on the knowledge of forces exerted on or by the moving fluids.

### Impact of Jets

The jet is a stream of liquid comes out from nozzle with a high velocity under constant pressure. When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted. Vane is a flat or curved plate fixed to the rim of the wheel.

#### 1. Force exerted by the jet on a stationary plate

- a) Plate is vertical to the jet
- b) Plate is inclined to the jet
- c) Plate is curved

#### 2. Force exerted by the jet on a moving plate

- a) Plate is vertical to the jet
- b) Plate is inclined to the jet
- c) Plate is curved

### Impulse-Momentum Principle

From Newton's 2<sup>nd</sup> Law, it can be written that:

$$F = m \times a = m \times \left( \frac{V_2 - V_1}{t} \right)$$

$$\Rightarrow F \times t = m \times (V_2 - V_1)$$

$$\Rightarrow F \times t = m V_2 - m V_1$$

$$\Rightarrow F \times t = \text{Final momentum} - \text{Initial momentum}$$

**Therefore,** Impulse of a force is given by the change in momentum caused by the force on the body.

Hence, based on the concept on of impulse, Force exerted by jet of water on the plate in the direction of jet can be calculated as:

$$F = \frac{m \times (V_2 - V_1)}{t}$$

$$F = \frac{m}{t} \times (V_2 - V_1)$$

$$F = \frac{m}{t} \times \text{Final velocity} - \text{Initial velocity}$$

$$F = \rho \times Q \times (V_2 - V_1)$$

$$F = \rho \times (a \times V) \times (V_2 - V_1)$$

Based on the above equation stated, the force exerted by the jet of water on different types of plates / vanes / buckets in both stationary and moving cases are discussed in the following sections. Following are different types of cases:

### Case: I

When the vanes / plates are stationary

- The plate / vane is flat and perpendicular to the direction of jet
- The plate / vane is flat and inclined to an angle  $\theta$  to jet of water
- The plate / vane is curved and the jet of water is striking at its centre exactly
- The plate / vane is curved and the jet of water is striking the vane at its one end of the tip.

### Case: II

When the vanes / plates are moving

- The plate / vane is flat and perpendicular to the direction of jet
- The plate / vane is flat and inclined to an angle  $\theta$  to jet of water
- The plate / vane is curved and the jet of water is striking at its centre exactly
- The plate / vane is curved and the jet of water is striking the vane at its one end of the tip.

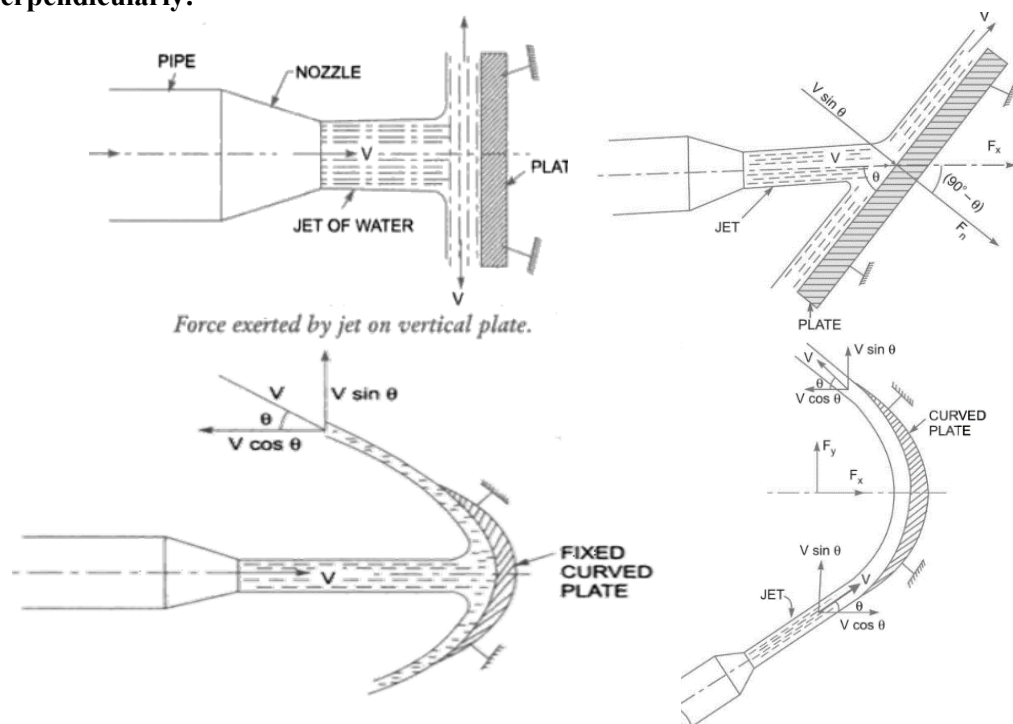
### Case: III

When the jet of water strikes on a series flat plate / vane mounted on periphery of wheel

### Case: IV

When the jet of water strikes on a series curved plate / vane mounted on periphery of wheel

**Force exerted by the jet on a stationary flat and curved plate and the jet is striking the plate perpendicularly:**



From the above figures, by the application of principle of impulse momentum, it can be seen that following equations can be derived:

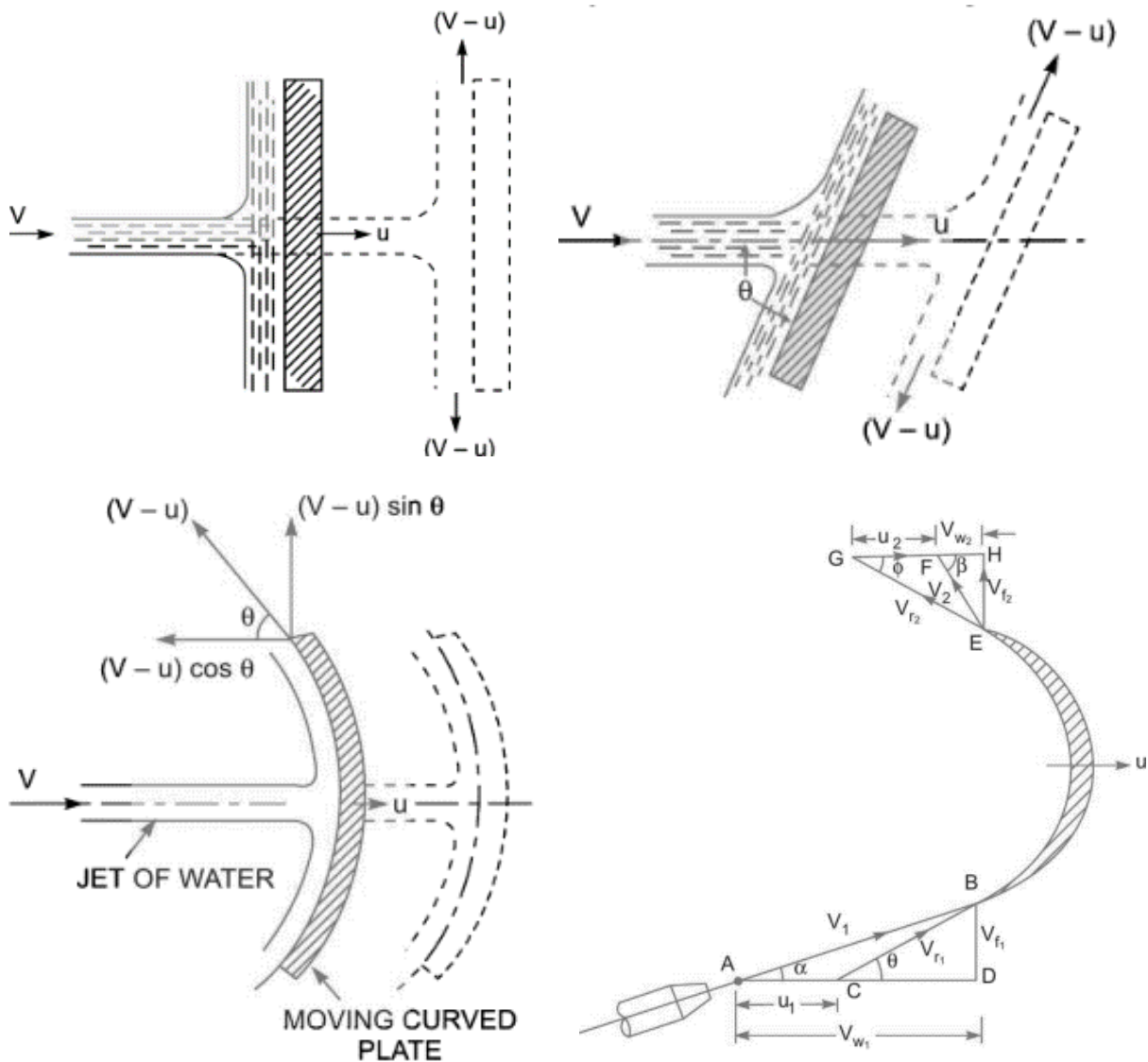
$$\text{When the plate is stationary, } F = \rho \times a \times V^2$$

$$\text{When the plate is inclined, } F = \rho \times a \times V^2 \times \sin \theta$$

$$\text{When the plate is curved, } F = \rho \times a \times V^2 \times (1 + \cos \theta)$$

$$\text{When the plate is curved, } F = 2 \times \rho \times a \times V^2 \times \cos \theta$$

Force exerted by the jet on a moving flat and curved plate and the jet is striking the plate perpendicularly:



From the above figures, by the application of principle of impulse momentum, it can be seen that following equations can be derived:

When the plate is moving and the jet strikes at its centre ,  $F = \rho X a X (V - U)^2$

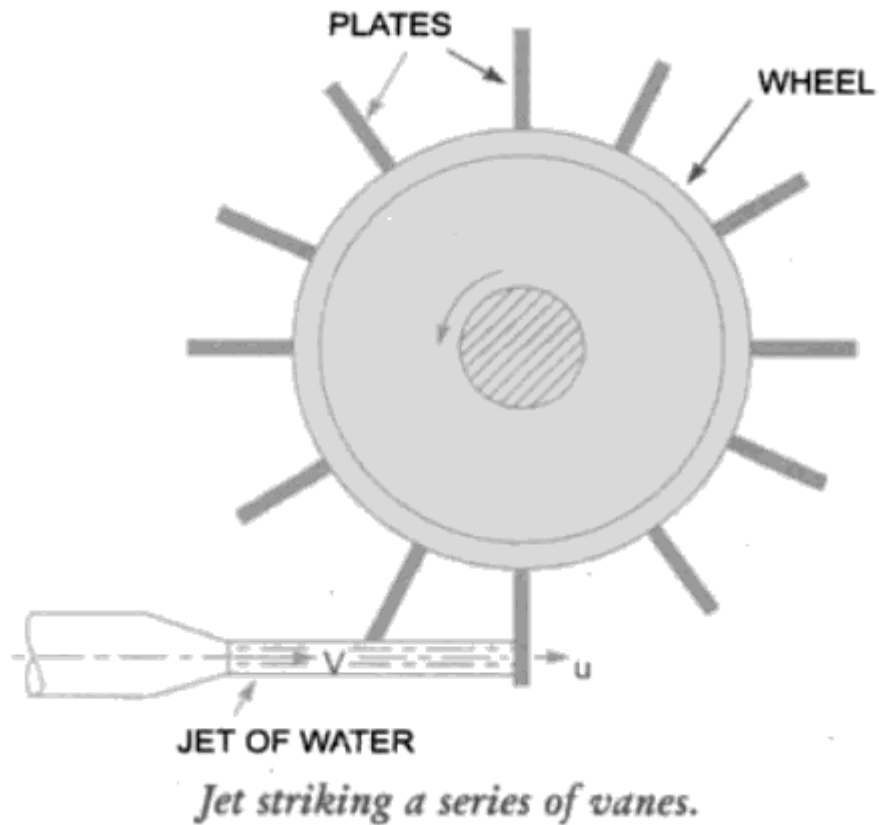
When the plate is moving and makes an angle  $\theta$  with jet of water,  $F_n = \rho X a X (V - U)^2 X \sin\theta$

When the plate is curved and moving, jet striking the plate at its centre,  $F = \rho X a X (V - U)^2 X (1 + \cos\theta)$

When the plate is curved and moving, jet striking the plate at its one tip,  $F = \rho X a X V_{r1} X (V_{w1} \pm V_{w2})$



**Impact of jet on a series of flat vanes mounted radially on the periphery of a circular wheel**



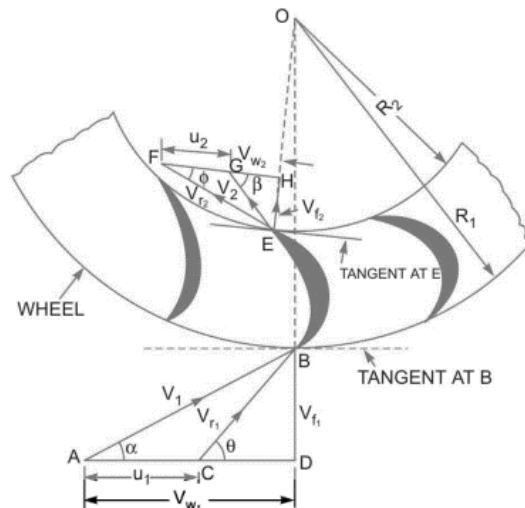
From the above figures, by the application of principle of impulse momentum, it can be seen that following equations can be derived:

The force exerted by the jet of water on the vanes is given by:  $F_x = \rho \times a \times V \times (V - U)$

Condition for maximum efficiency is  $U = \frac{V}{2}$

Maximum efficiency obtained through this wheel is  $\eta_{max} = 50\%$

**Impact of jet on a series of circular vanes mounted radially on the periphery of a circular wheel:**

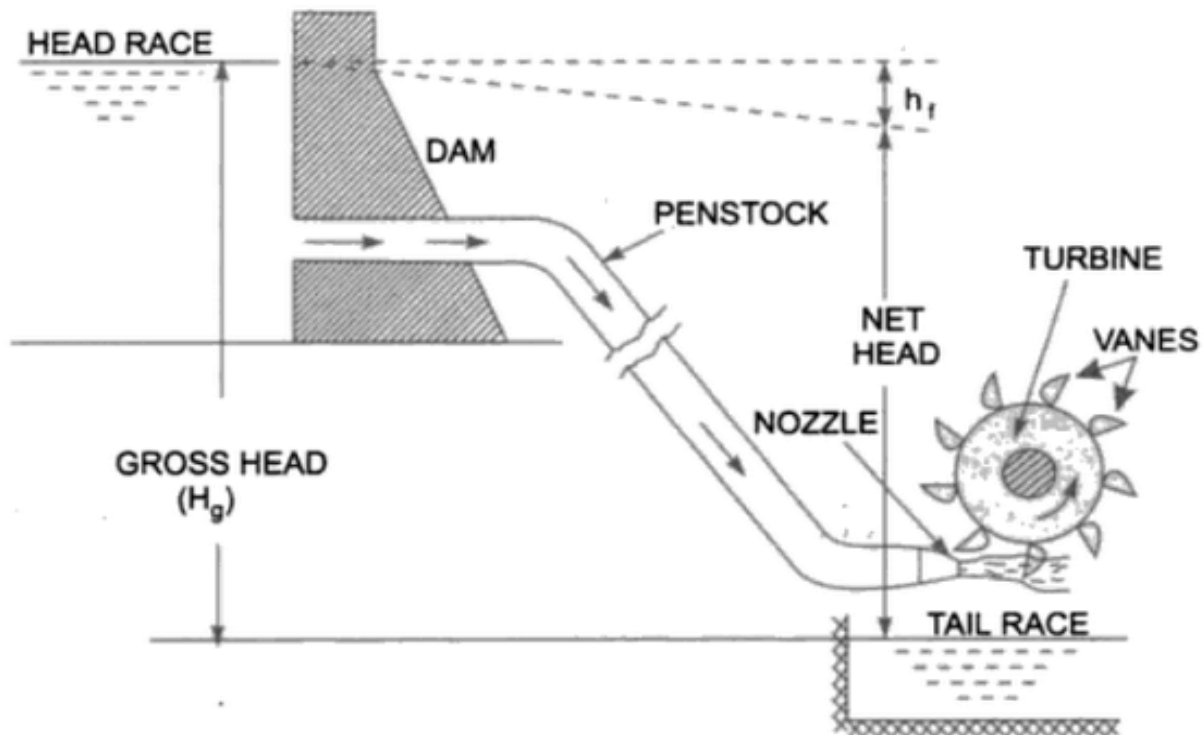


From the above figures, by the application of principle of impulse momentum, it can be seen that following equations can be derived:

Torque exerted by the jet of water on the wheel,  $T = \rho \times a \times V_1 \times (V_{w1} \times R_1 + V_{w2} \times R_2)$

Efficiency of radial vanes is given by:  $\eta = \frac{2 (V_{w1} \times U_1 \pm V_{w2} \times U_2)}{V_1^2}$

**Layout of hydroelectric power plant:**



*Layout of a hydro-eletric power plant.*

Some of the following terms mainly used in hydroelectric power plant are described below:

$H_g$  = Gross Head  
 $h_f$  = Head Loss due to Friction  
 $h_f = \frac{f \times L \times V^2}{2 \times g \times D}$

Where

$V$  = Velocity of Flow in Penstock  
 $L$  = Length of Penstock  
 $D$  = Dia. of Penstock  
 $H$  = Net Head  
 $H = H_g - h_f$

### Efficiencies of Turbine

1. Hydraulic efficiency =  $\frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$

$$\Rightarrow \eta_h = \frac{R.P}{W.P}$$

2. Mechanical efficiency =  $\frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}}$

$$\Rightarrow \eta_m = \frac{S.P}{R.P}$$

3. Volumetric efficiency =  $\frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$

4. Overall efficiency =  $\frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$

$$\Rightarrow \eta_o = \eta_m \times \eta_h$$

### Problems:

1. A jet of water 50 mm diameter strikes a flat plate held normal to the direction of jet. Estimate the force exerted and work done by the jet if, the plate is stationary and the plate is moving with a velocity of 1 m/s away from the jet along the line of jet.
2. The discharge through the nozzle is 76 lps. A jet of water 50 mm diameter exerts a force of 3 kN on a flat vane held perpendicular to the direction of jet. Find the mass flow rate.
3. A jet of water of diameter 50 mm strikes a stationary, symmetrical curved plate with a velocity of 40 m/s. Find the force extended by the jet at the centre of plate along its axis if the jet is deflected through  $120^\circ$  at the outlet of the curved plate.
4. A jet of water from a nozzle is deflected through  $60^\circ$  from its direction by a curved plate to which water enters tangentially without shock with a velocity of 30 m/s and leaves with a velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s, calculate the magnitude and direction of resultant force on the vane.
5. A jet of water strikes a stationary curved plate tangentially at one end at an angle of  $30^\circ$ . The jet of 75 mm diameter has a velocity of 30 m/s. The jet leaves at the other end at angle of  $20^\circ$  to the horizontal. Determine the magnitude of force exerted along 'x' and 'y' directions.
6. A jet of water of diameter 75 mm strikes a curved plate at its centre with a velocity of 25 m/s. The curved plate is moving with a velocity of 10 m/s along the direction of jet. If the jet gets deflected through  $165^\circ$  in the smooth vane, compute.
  - a) Force exerted by the jet.
  - b) Power of jet.
  - c) Efficiency of jet.
7. A jet of water impinges a curved plate with a velocity of 20 m/s making an angle of  $20^\circ$  with the direction of motion of vane at inlet and leaves at  $130^\circ$  to the direction of motion at outlet. The vane is moving with a velocity of 10 m/s. Compute.
  - a) Vane angles, so that water enters and leaves without shock.
  - b) Work done per unit mass flow rate
8. A jet of water having a velocity of 35 m/s strikes a series of radial curved vanes mounted on a wheel. The wheel has 200 rpm. The jet makes  $20^\circ$  with the tangent to wheel at inlet and leaves the wheel with a velocity of 5 m/s at  $130^\circ$  to tangent to the wheel at outlet. The diameters of wheel are 1 m and 0.5 m. Find
  - a) Vane angles at inlet and outlet for radially outward flow turbine.
  - b) Work done
  - c) Efficiency of the system

## UNIT - IV HYDRAULIC TURBINES

### INTRODUCTION:

Hydraulic turbines are those which convert hydraulic energy to Mechanical energy and then to electrical energy upon coupling to shafts or motors. At present hydro power generation is the cheapest as compared to other types of electrical energy. In this chapter the detailed design parameters of various types of turbines such as Pelton turbine, Francis turbine and Kaplan Turbine are discussed.

### Classification of Turbines:

1. According to type of energy at Inlet
  - a) Impulse Turbine: Requires High Head and Low Rate of Flow.  
Ex: Pelton Wheel
  - b) Reaction Turbine: Requires Low Head and High Rate of Flow.  
Ex: Francis, Kaplan
2. According to direction of flow through runner
 

a) Tangential Flow Turbine	-	Pelton Wheel
b) Radial Flow Turbine	-	Francis Turbine
c) Axial Flow Turbine	-	Kaplan Turbine
d) Mixed Flow Turbine	-	Modern Francis Turbine
3. According to Head at Inlet of turbine
 

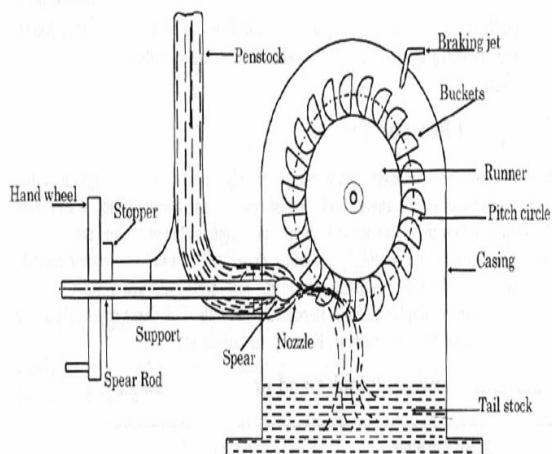
a) High Head Turbine	-	Pelton Wheel
b) Medium Head Turbine	-	Francis Turbine
c) Low Head Turbine	-	Kaplan Turbine
4. According to Specific Speed of Turbine
 

a) Low Specific Speed Turbine	-	Pelton Wheel
b) Medium Specific Speed Turbine	-	Francis Turbine
c) High Specific Speed Turbine	-	Kaplan Turbine
5. According to Disposition of Turbine Shaft
 

a) Horizontal Shaft	-	Pelton Wheel
b) Vertical Shaft	-	Francis & Kaplan Turbines
6. According to Disposition of Turbine Shaft
 

a) Low specific speed turbine	-	Pelton Wheel Turbine
b) Medium specific turbine	-	Francis Turbine
c) High specific turbine	-	Kaplan Turbines

### Pelton wheel turbine:

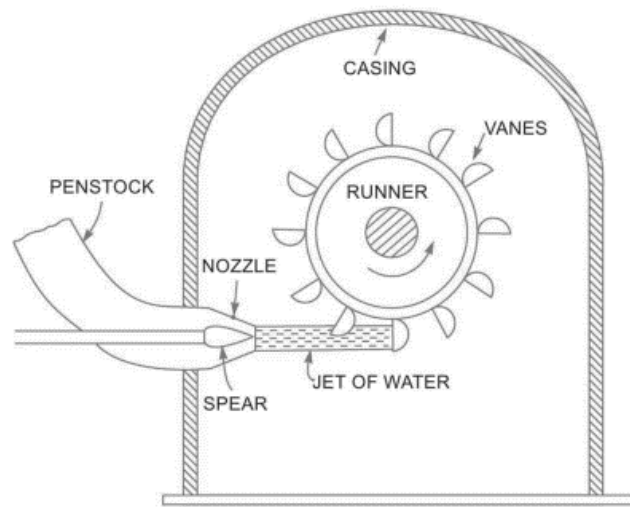


- Nozzle

### Main components of Pelton Wheel turbine:



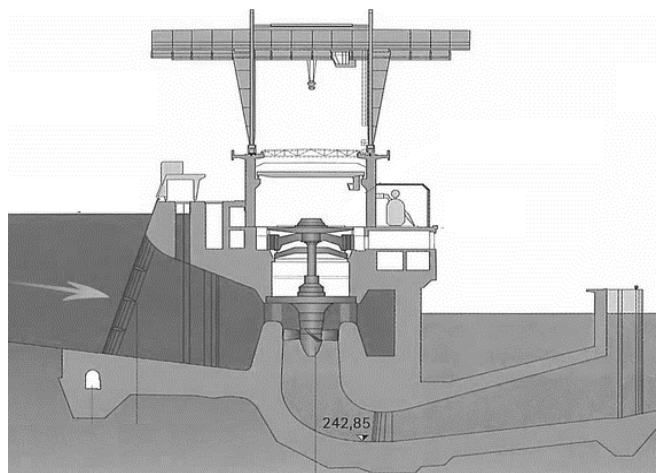
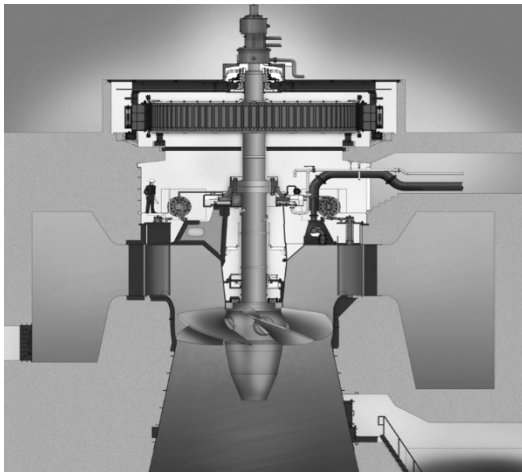
- Runner and buckets
- Casing
- Breaking jet



### Design parameters or guidelines for the design of Pelton Wheel

- Jet Ratio = Pitch Diameter of wheel / Dia. of Jet =  $D/d$
- Speed Ratio = Velocity of Wheel / Velocity of Jet =  $u/V$
- Water Power, =  $\frac{1}{2}mV^2 = \rho gQH$
- No. of Buckets =  $(0.5 \times \text{Jet Ratio}) + 15$

### Francis turbine:

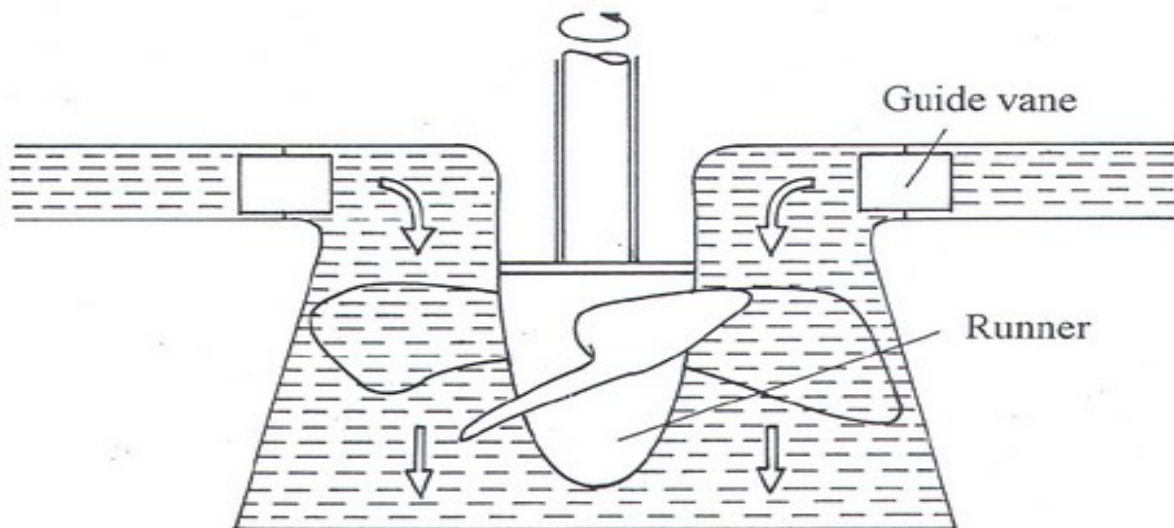


### Design parameters or guidelines for the design of Francis Wheel

- Velocity of Wheel  $U = U_1 = U_2 = \frac{\pi \times D \times N}{60}$
- Work done per second or Power  $\rho \times Q [V_{w1} \times U_1 \pm V_{w2} \times U_2]$
- Velocity of Wheel  $U_1 = \frac{\pi \times D_1 \times N}{60}$  and  $U_2 = \frac{\pi \times D_2 \times N}{60}$
- Discharge  $Q = \pi \times D_1 \times B_1 \times V_{f1} = \pi \times D_2 \times B_2 \times V_{f2}$

### Kaplan turbine:

Higher specific speed corresponds to a lower head. This requires that the runner should admit a comparatively large quantity of water. For a runner of given diameter, the maximum flow rate is achieved when the flow is parallel to the axis. Such a machine is known as axial flow reaction turbine. An Australian engineer, Vikton Kaplan first designed such a machine. The machines in this family are called Kaplan Turbines.

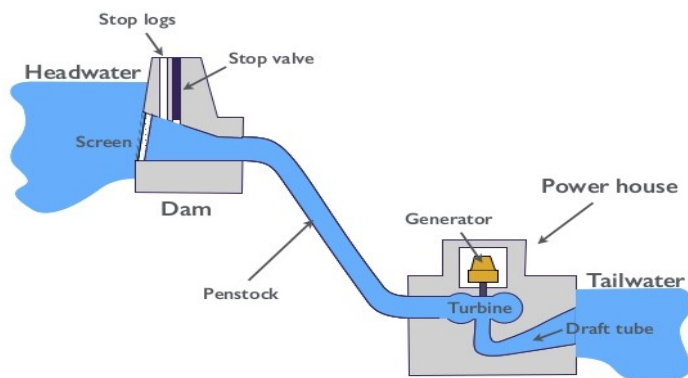
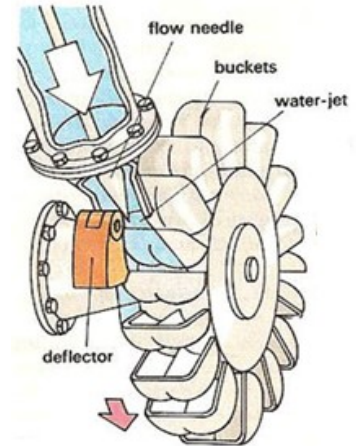
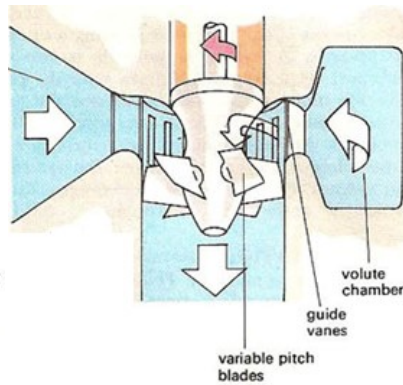
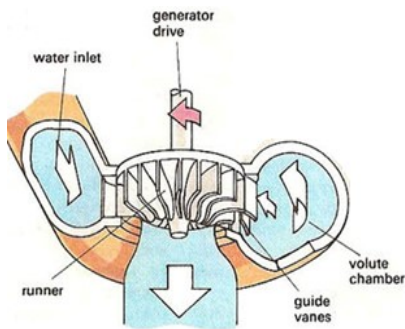


Guidelines for the designing of Kaplan turbine:

- Velocity of Wheel:  $U = U_1 = U_2 = \frac{\pi \times D_m \times N}{60}$   
Where,  $D_m = \text{Mean diameter, } D_m = \frac{D_0 + D_b}{2}$
- Work done per second:  $\rho \times Q [V_{w1} + V_{w2}] \times U$
- Velocity of Flow at Inlet and Outlet are equal  $V_{f1} = V_{f2}$
- Discharge  $Q = \frac{\pi}{4} \times (D_0^2 - D_b^2) \times V_{f1}$
- Flow Ratio  $\frac{V_{f1}}{\sqrt{2 \times g \times h}}$

## Draft Tube

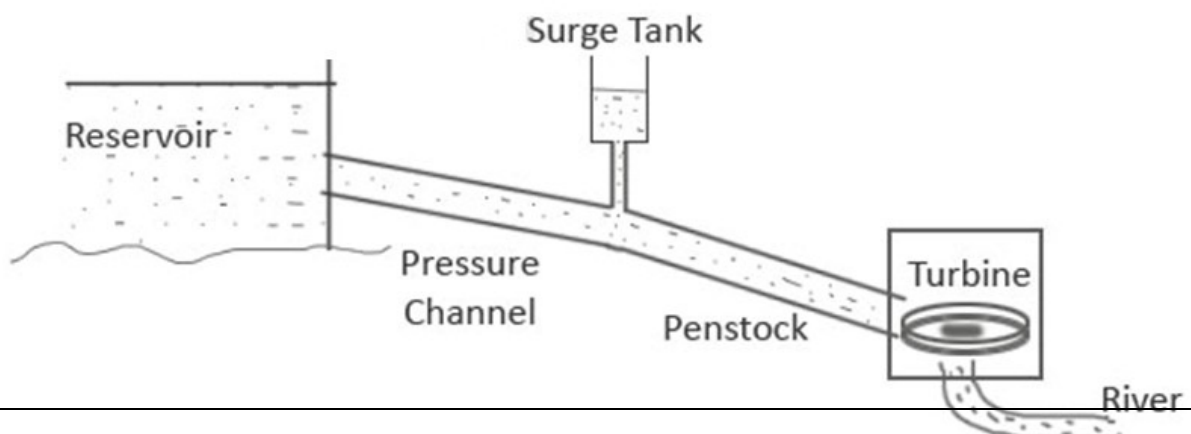
The water after working on the turbine, imparts its energy to the vanes and runner, thereby reducing its pressure less than that of atmospheric pressure. As the water flows from higher pressure to lower pressure, it cannot come out of the turbine and hence a divergent tube is connected to the end of the turbine. A draft tube is a divergent tube, one end of which is connected to the outlet of the turbine and the other end is immersed well below the tailrace (water level). The major function of the draft tube is to increase the pressure from the inlet to outlet of the draft tube as it flows through it and hence increase it more than atmospheric pressure. The other function is to safely discharge the water that has worked on the turbine to the tailrace.



## Types of Draft Tube

### Surge Tanks

Surge tank (or surge chamber) is a device introduced within a hydropower water conveyance system having a rather long pressure conduit to absorb the excess pressure rise in case of a sudden valve





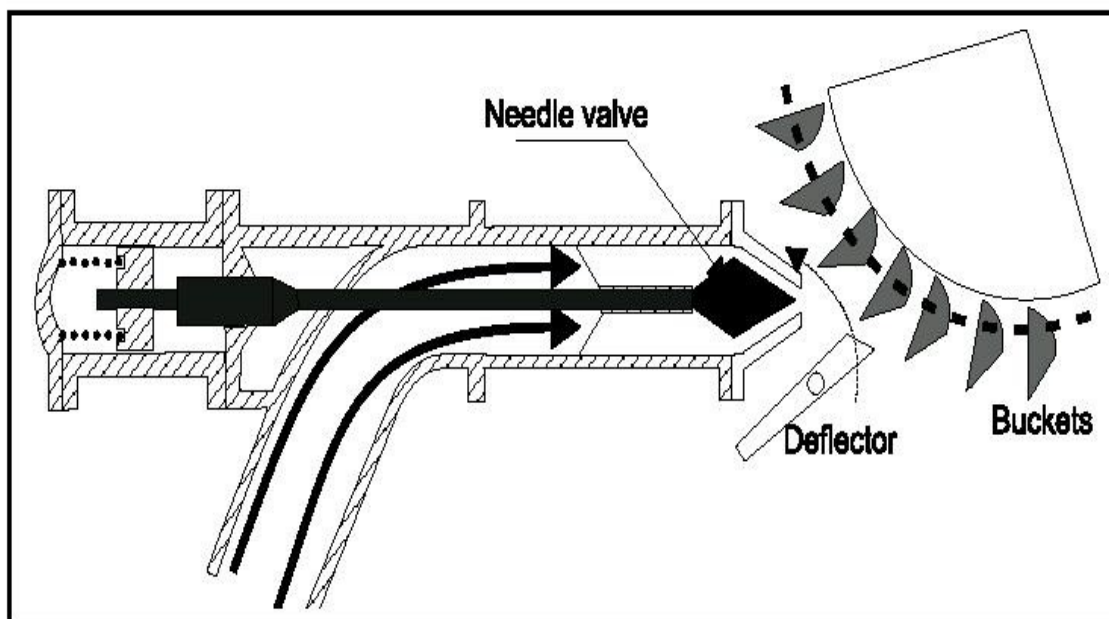
closure. The surge tank is located between the almost horizontal or slightly inclined conduit and steeply sloping penstock and is designed as a chamber excavated in the mountain. It also acts as a small storage from which water may be supplied in case of a sudden valve opening of the turbine. In case of a sudden opening of turbine valve, there are chances of penstock collapse due to a negative pressure generation, if there is no surge tank.

### Governing of Turbines:

Governing means Speed Regulation. Governing system or governor is the main controller of the hydraulic turbine. The governor varies the water flow through the turbine to control its speed or power output.

#### 1. Impulse Turbine

- a) Spear Regulation
- b) Deflector Regulation
- c) Combined



#### Performance of Turbines under unit quantities:

The unit quantities give the speed, discharge and power for a particular turbine under a head of 1m assuming the same efficiency. Unit quantities are used to predict the performance of turbine.

- Unit speed ( $N_u$ ) - Speed of the turbine, working under unit head
- Unit power ( $P_u$ ) - Power developed by a turbine, working under a unit head
- Unit discharge ( $Q_u$ ) - The discharge of the turbine working under a unit head

$$\text{Specific speed} = N_u = \frac{N}{\sqrt{H}}$$

$$\text{Unit power} = P_u = \frac{P}{\sqrt{H^3}}$$

$$\text{Unit discharge} = Q_u = \frac{Q}{\sqrt{H}}$$

#### Specific speed of the turbines ( $N_s$ ):

The specific speed of the turbine is the speed at which the turbine will run when developing unit power under a unit head. This is the type characteristics of a turbine. For a set of geometrically similar turbines, the specific speed will have the same value. Mathematically, it can be shown as:

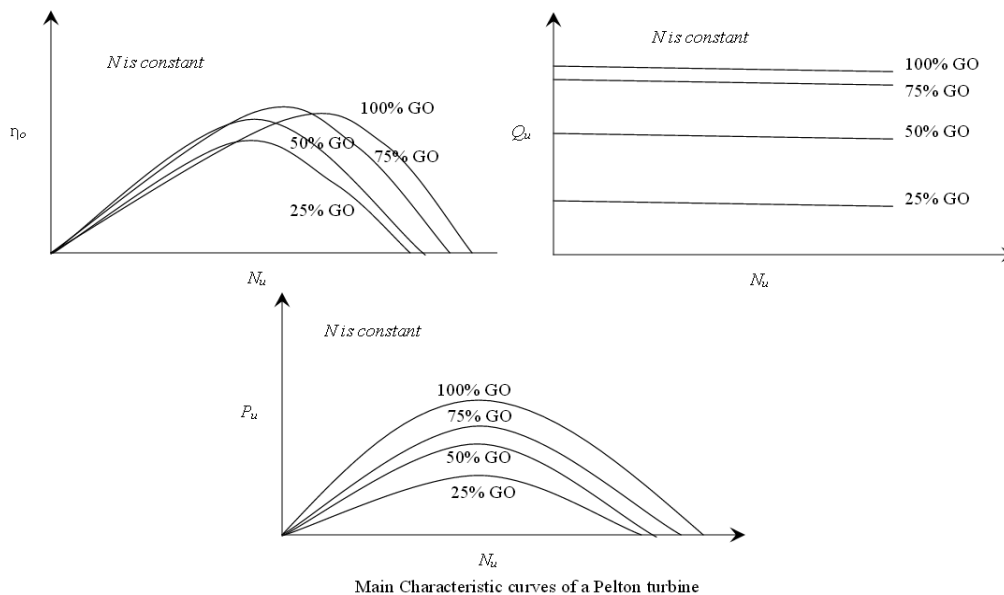
$$\text{Specific speed} = N_s = \frac{N \times \sqrt{P}}{H^{5/4}}$$

### Characteristics Curves of Turbine:

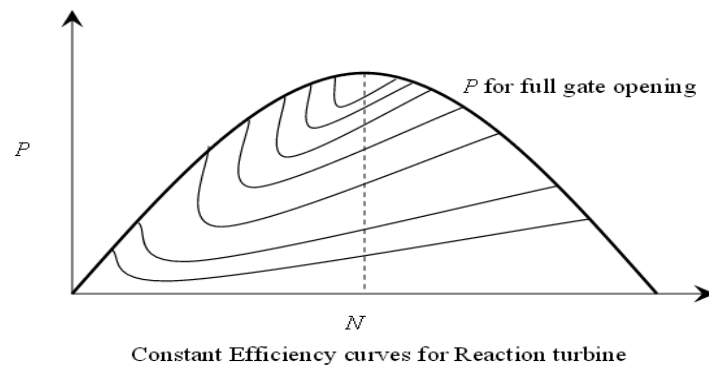
These are curves which are characteristic of a particular turbine which helps in studying the performance of the turbine under various conditions. These curves pertaining to any turbine are supplied by its manufacturers based on actual tests. The characteristic curves obtained are the following:

- Constant head curves or main characteristic curves
- Constant speed curves or operating characteristic curves.

### Constant efficiency curves or Muschel curves

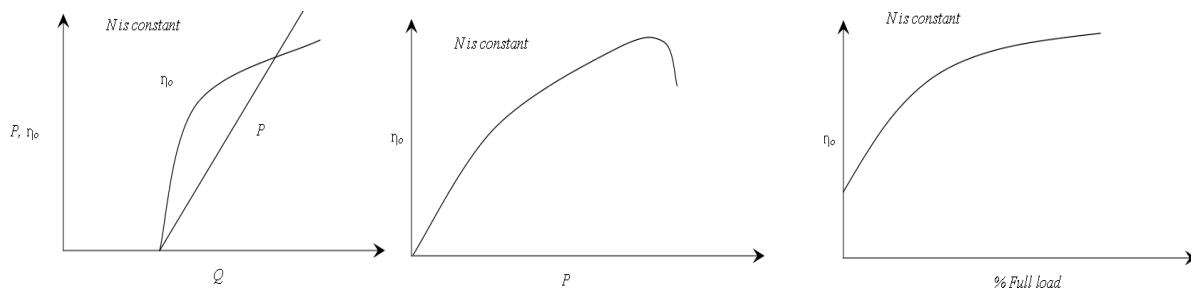


These curves are plotted from data which can be obtained from the constant head and constant speed curves. The object of obtaining this curve is to determine the zone of constant efficiency so that we can always run the turbine with maximum efficiency. This curve also gives a good idea about the performance of the turbine at various efficiencies.



**Constant speed curves or operating characteristic curves:**

In this case tests are conducted at a constant speed varying the head  $H$  and suitably adjusting the discharge  $Q$ . The power developed  $P$  is measured mechanically. The overall efficiency is aimed at its maximum value. The curves drawn are between:



**Cavitation:**

If the pressure of a liquid in course of its flow becomes equal to its vapour pressure at the existing temperature, then the liquid starts boiling and the pockets of vapour are formed which create vapour locks to the flow and the flow is stopped. The phenomenon is known as cavitation.

To avoid cavitation, the minimum pressure in the passage of a liquid flow, should always be more than the vapour pressure of the liquid at the working temperature. In a reaction turbine, the point of minimum pressure is usually at the outlet end of the runner blades, i.e., at the inlet to the draft tube.

**Methods to avoid Cavitation:**

- (i) Runner/turbine may be kept under water.
- (ii) Cavitation free runner may be designed.
- (iii) By selecting materials that can resist better the cavitation effect.
- (iv) By polishing the surfaces.
- (v) By selecting a runner of proper specific speed for given load.

### Problems:

1. A hydraulic turbine develops 120 KW under a head of 10 m at a speed of 1200 rpm and gives an efficiency of 92%. Find the water consumption and the specific speed. If a model of scale 1: 30 is constructed to operate under a head of 8m what must be its speed, power and water consumption to run under the conditions similar to prototype.
2. A model turbine 1m in diameter acting under a head of 2m runs at 150 rpm. Estimate the scale ratio if the prototype develops 20 KW under a head of 225 m with a specific speed of 100.
3. A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lps under a head of 30 m. The buckets deflect the jet through an angle of  $160^\circ$ . Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume the coefficient of nozzle as 0.98.
4. A Pelton wheel has to develop 13230 kW under a net head of 800 m while running at a speed of 600 rpm. If the coefficient of Jet  $C_y = 0.97$ , speed ratio is 0.46 and the ratio of the Jet diameter is  $1/16$  of wheel diameter. Calculate
  - Pitch circle diameter
  - the diameter of jet
  - the quantity of water supplied to the wheel
5. Design a Pelton wheel for a head of 80m. and speed of 300 RPM. The Pelton wheel develops 110 kW. Take co-efficient of velocity= 0.98, speed ratio= 0.48 and overall efficiency = 80%.
6. A double jet Pelton wheel develops 895 MKW with an overall efficiency of 82% under a head of 60m. The speed ratio = 0.46, jet ratio = 12 and the nozzle coefficient = 0.97. Find the jet diameter, wheel diameter and wheel speed in RPM.
7. A reaction turbine works at 450 rpm under a head of 120 m. Its diameter at inlet is 1.2 m and the flow area is  $0.4 \text{ m}^2$ . The angle made by the absolute and relative velocities at inlet are  $20^\circ$  and  $60^\circ$  respectively with the tangential velocity. Determine
  - the discharge through the turbine
  - power developed
  - efficiency.
8. A Francis turbine has inlet wheel diameter of 2 m and outlet diameter of 1.2 m. The runner runs at 250 rpm and water flows at 8 cumecs. The blades have a constant width of 200 mm. If the vanes are radial at inlet and the discharge is radially outwards at exit, make calculations for the angle of guide vane at inlet and blade angle at outlet.
9. A Kaplan turbine develops 9000 kW under a net head of 7.5 m. Overall efficiency of the wheel is 86% The speed ratio based on outer diameter is 2.2 and the flow ratio is 0.66. Diameter of the boss is 0.35 times the external diameter of the wheel. Determine the diameter of the runner and the specific speed of the runner.
10. A Kaplan turbine working under a head of 25 m develops 16,000 kW shaft power. The outer diameter of the runner is 4 m and hub diameter is 2 m. The guide blade angle is  $35^\circ$ . The hydraulic and overall efficiency are 90% and 85% respectively. If the velocity of whirl is zero at outlet, determine runner vane angles at inlet and outlet and speed of turbine.
11. Suggest a suitable type of turbine to develop 7000 kW power under a head of 20m while operating at 220 rpm. What are the considerations for your suggestion.

## UNIT - V CENTRIFUGAL PUMPS

### INTRODUCTION:

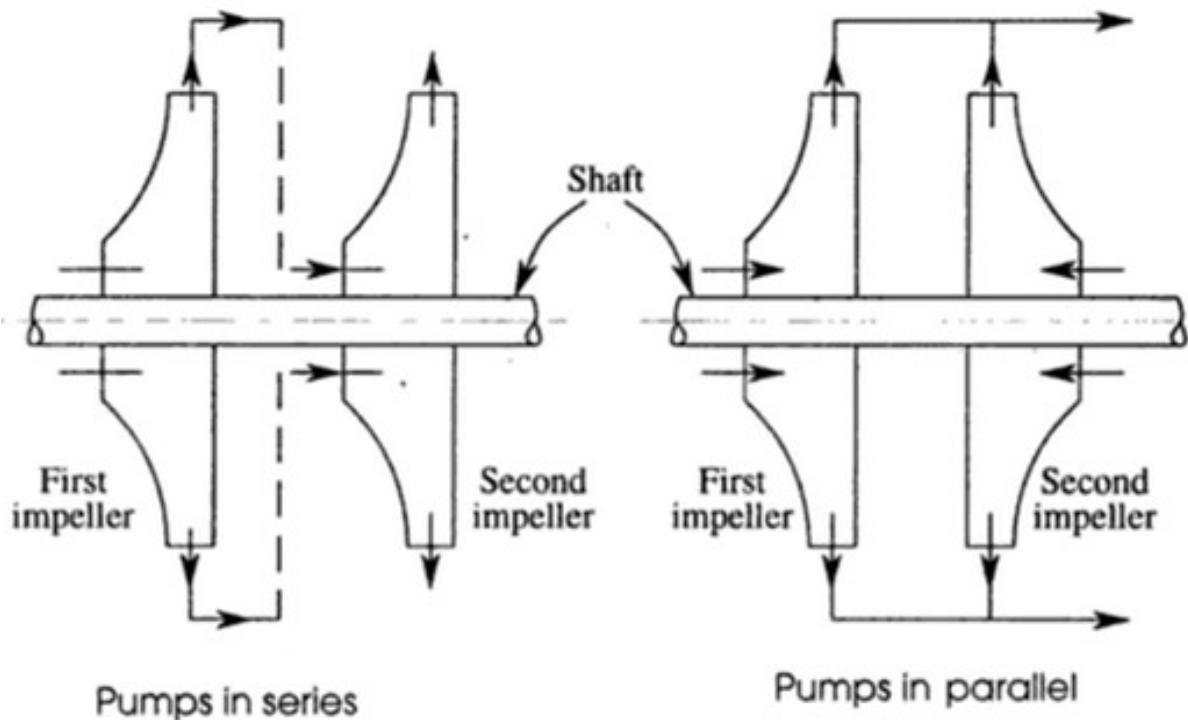
A pump is a hydraulic machine which converts mechanical energy into hydraulic energy or pressure energy. A centrifugal pump works on the principle of centrifugal force. In this type of pump, the liquid is subjected to whirling motion by the rotating impeller which is made of a number of backward curved vanes. The liquid enters this impeller at its centre or the eye and gets discharged into the casing enclosing the outer edge of the impeller. Generally centrifugal pumps are made of the **radial flow type only ( $\alpha = 90^\circ$ )**.

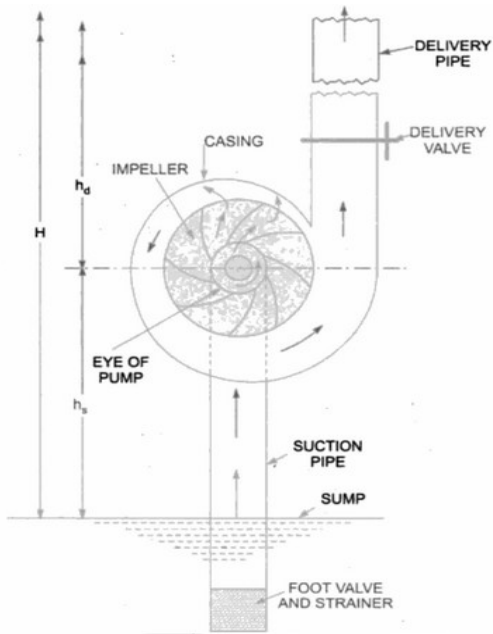
### Classification of Pumps

1. According to No. of Impellers
  - Single Stage Pump
  - Multistage Pump
2. According to Disposition of Shaft
  - Vertical Shaft Pump
  - Horizontal Pump
3. According to Head
  - Low Head Pump -  $H < 15\text{m}$
  - Medium Head Pump -  $15\text{m} < H < 40\text{m}$
  - High Specific Speed Turbine -  $H > 40\text{m}$

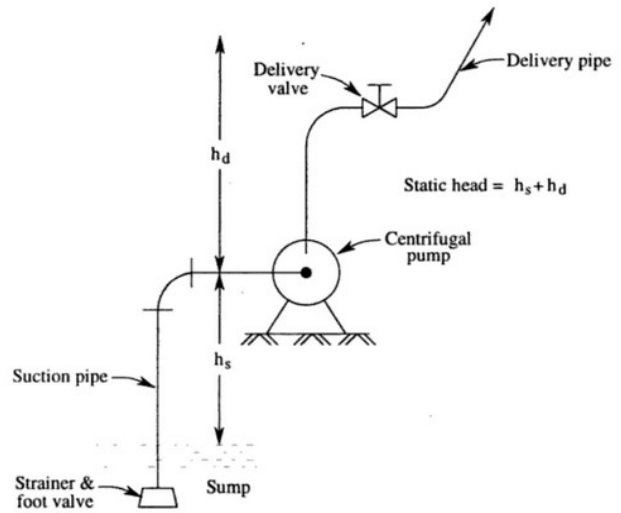
A centrifugal pump containing two or more impellers is called a multistage centrifugal pump.

- For higher pressures at the outlet, impellers can be connected in series.
- For higher flow output, impellers can be connected parallel.



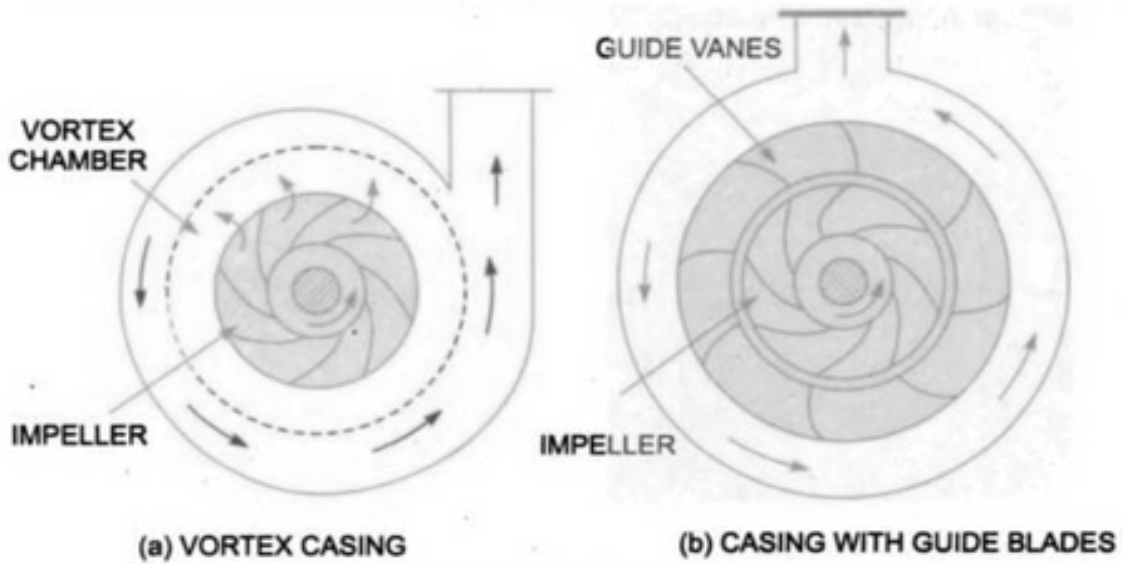


Main parts of a centrifugal pump.



### Components of Pump

1. Strainer and Foot Valve
2. Suction Pipe and its fittings
3. Pump
4. Delivery Valve
5. Delivery Pipe and its fittings



Different types of casing.

### Manometric Head:

It is the total head developed by the pump. This head is slightly less than the head generated by the impeller due to some losses in the pump.

$$H_m = \text{Suction Head} + \text{Delivery Head} + \text{Head Loss} + \text{Velocity Head in Delivery Pipe}$$
$$= h_s + h_d + h_f + V_d^2/2g$$

### Manometric Efficiency:

$$\eta_{\max} = \text{Manometric Head} / \text{Head Imparted by Impeller to Water}$$
$$= H_m / [(V_{w2} U_2) / g]$$
$$= g H_m / V_{w2} U_2$$

Head Imparted by Impeller to Water = Work done per Second

$$= \rho Q (V_{w2} U_2)$$

Head Imparted by Impeller to Unit Weight of Water

= Work done per Second per Unit Weight of Water

$$= \rho Q (V_{w2} U_2) / mg$$

$$= \rho Q (V_{w2} U_2) / (\rho Q) g$$

$$= V_{w2} U_2 / g$$

### Minimum Starting Speed of Pump:

A centrifugal pump will start delivering liquid only if the head developed by the impeller is more than the manometric head ( $H_m$ ). If the head developed is less than  $H_m$  no discharge takes place although the impeller is rotating. When the impeller is rotating, the liquid in contact with the impeller is also rotating. This is a forced vortex, in which the increase in head in the impeller is given by

$$\text{Head rise in impeller} = \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$$

Discharge takes place only when

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$$

substituting for  $u_1$ ,  $u_2$  and  $H_m$  in Equation (10.13), we obtain

$$N = \frac{120 \eta_m V_{w2} D_2}{\pi (D_2^2 - D_1^2)}$$

which is the minimum speed for the pump to discharge liquid.

### Specific Speed of Pump:

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver *one cubic metre* of liquid per second against a head of *one metre*. It is denoted by ' $N_s$ '.

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

### Model Analysis of Pump:

Before manufacturing the large sized pumps, their models which are in complete similarity with the actual pumps (also called prototypes) are made. Tests are conducted on the models and performance of the prototypes are predicted. The complete similarity between the model and actual pump (prototype) will exist if the following conditions are satisfied :

1. Specific speed of model = Specific speed of prototype

$$(N_s)_m = (N_s)_p \quad \text{or} \quad \left( \frac{N\sqrt{Q}}{H_m^{3/4}} \right)_m = \left( \frac{N\sqrt{Q}}{H_m^{3/4}} \right)_p$$

### Types of Reciprocating or displacement pumps:

- Piston pump
- Diaphragm pump
- Plunger pumps

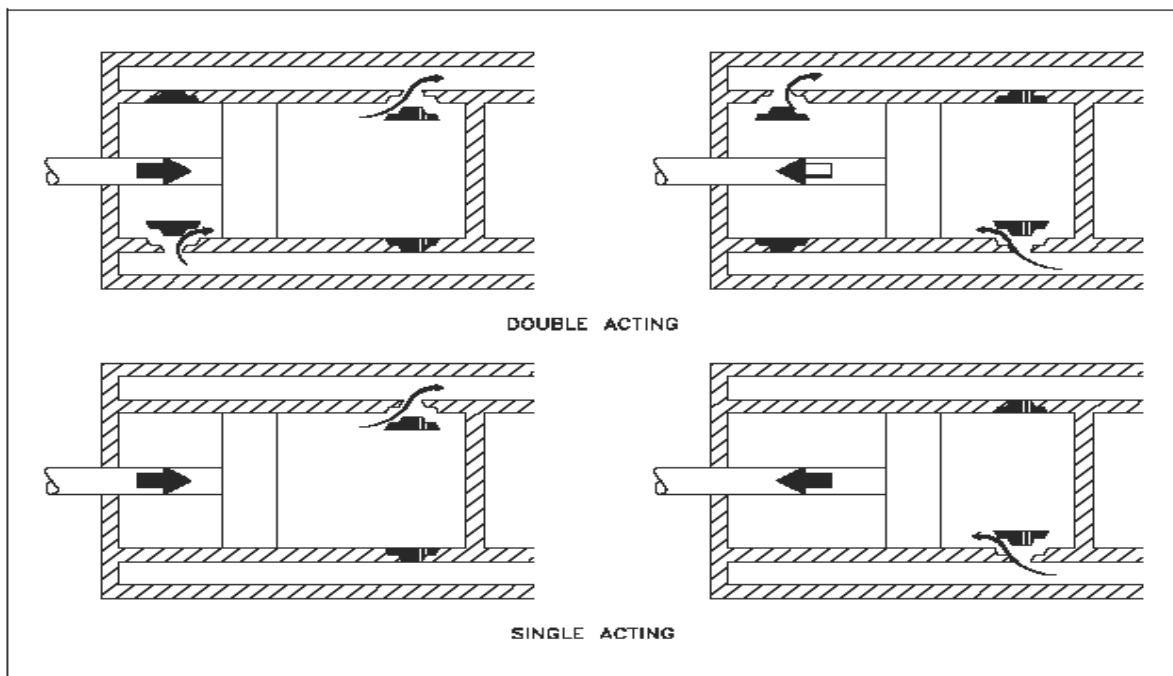
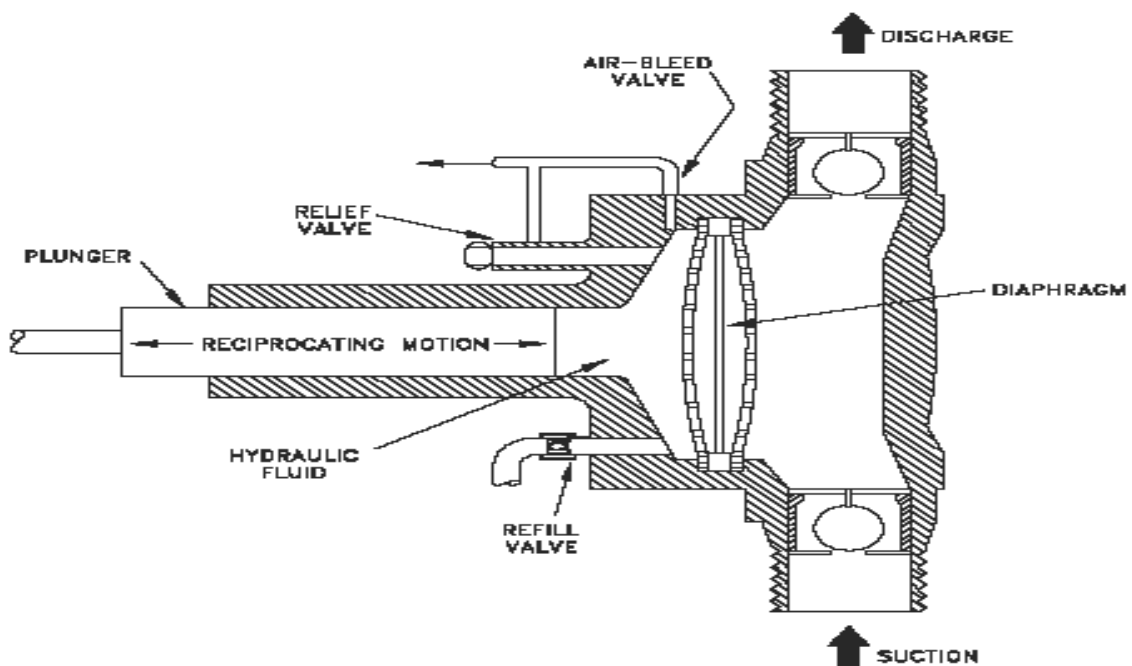
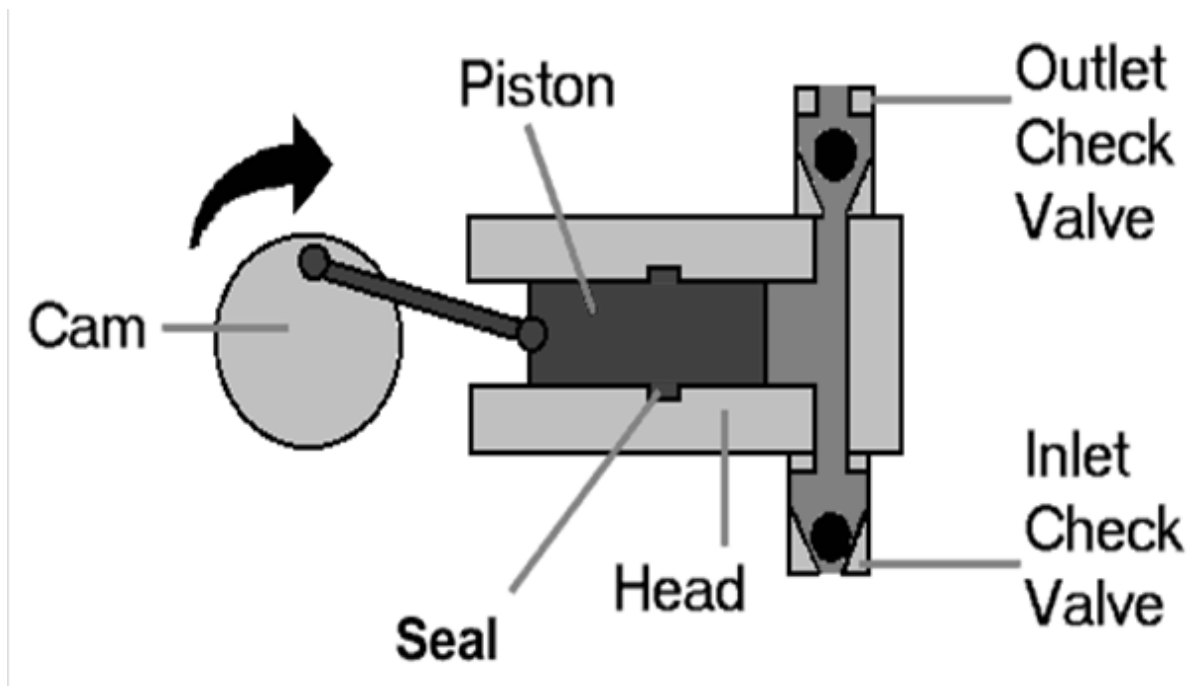


Figure 13 Single-Acting and Double-Acting Pumps





### Major components of a reciprocating pump or displacement pump

- Piston or plunger
- Crank and Connecting rod
- Suction pipe
- Delivery pipe
- Suction and Delivery valve

### Major terms used in reciprocating pump

- Brake Horsepower (B)
- Capacity(Q)
- Pressure ( $P_d$ )

- Mechanical efficiency
- Displacement (D)
- Slip (s)
- Valve Loss (VL)
- Speed (n)
- Pulsations
- Net Positive Suction Head Required (NPSHR)
- Net Positive Suction Head Available (NPSHA)

**Following are the important considerations in the selection of pump for any given application**

- Flow rate requirement
- Operating speed of pump
- Pressure rating
- Performance/application
- Reliability
- Cost
- Noise level of the pump
- Oil compatibility
- Type of pump control
- Pump contamination tolerance
- Availability of pump and parts

**Cavitation in Pump:**

Cavitation is the formation of bubbles or cavities in liquid, developed in areas of relatively low pressure around an impeller. The imploding or collapsing of these bubbles trigger intense shockwaves inside the pump, causing significant damage to the impeller and/or the pump housing. If left untreated, pump cavitations can cause:

- Failure of pump housing
- Destruction of impeller
- Excessive vibration leading to premature seal and bearing failure
- Higher than necessary power consumption

Precaution:  $NPSHA > NPSHR$

Where  $NPSHA =$  Net Positive Suction Head Available

$NPSHR =$  Net Positive Suction Head Required

- Cavitation: evaporation, followed by condensation (almost instantaneously)
- How to detect cavitation:
  - Change in performance curves
  - Visual observation of bubble formation
  - Noise and vibrations
- Cavitation effects:
  - Noise, vibration
  - Material erosion
  - Performance reduction (efficiency, etc.)
- Piston Head: Generally made grey cast iron.
- Valves: They are also made of steel, but a little improved form which has the stiffness and less wear and tear. Brass can also be used.
- Piston pin: Usually made of Case Hardened steel alloy
- containing nickel, chromium and molybdenum.
- Piston Rings: Made of grey cast iron or alloy cast iron because of their good wearing properties.

### **Occurrence of Cavitation in pumps**

- If  $p_1$  is less than the atmospheric pressure.
- If  $p_1 < p_{\text{vap}}$  (T) = Evaporation
- However, minimum pressure occurs inside the pump.
- $p_{\text{min}} < p_{\text{vap}}$  (T) = Evaporation = Cavitation.
- To avoid cavitation:  $H_s > H_{si}$