MATERIALS AND MECHANICS OF SOLIDS

B.Tech IV semester (Autonomous) IARE R-18

By

Dr K Viswanath Allamraju

Professor

Mr A. SOMAIAH

Assistant Professor

DEPARTMENT OF AERONAUTICAL ENGINEERING

INSTITUTE OF AERONAUTICAL ENGINEERING

(Autonomous)

Dundigal, Hyderabad- 500 043

S.No	COURSE OBJECTIVES
I	Determination of mechanical properties of different materials.
II	Establish the constitutive relations in metals using destructive methods.
III	Understand the behavior of members during twisting and transverse loading.
IV	Familiarize with standard test procedures.
V	Deriving slope and deflection for different types of beams.

COs	COURSE OUTCOMES
CO1	Describe the different types of crystal structures.
CO2	Discuss the phase transformations and equilibrium diagram.
CO3	Ability to apply the principles of elasticity, plasticity, stresses, strains and their relationships under various types of loads and to analyze the composite bars.
CO4	Able to draw shear force and bending moment diagrams for various loads.
CO5	Determination of slope and deflection of various types of beams.

MODULE- I

INTRODUCTION TO MATERIAL SCIENCE

CLOs	COURSE LEARNING OUTCOMES
CLO1	Describe the basic concepts of FEM and steps involved in it.
CLO2	Understand the difference between the FEM and Other methods.
CLO3	Understand the stress-strain relation for 2-D and their field problem.
CLO4	Understand the concepts of shape functions for one dimensional and quadratic elements, stiffness matrix and boundary conditions
CLO5	Apply numerical methods for solving one dimensional bar problems

- What is materials science?
- Why should we know about it?

- Materials drive our society
  - Stone Age
  - Bronze Age
  - Iron Age
  - Now?
    - Silicon Age?
    - Polymer Age?

Material -> something tangible that goes into the makeup of a physical object.

Material Science -> involves investigating the relationships that exist between the structures and properties of materials

Material Engineering -> is, on the basis of these *structure–property* correlations, designing or engineering the structure of a material to produce a predetermined set of properties

Structure -> The structure of a material usually relates to the arrangement of its internal components

Different levels of defining structure of a material

Property -> A property is a material trait (distinguishing feature) in terms of the kind and magnitude of response to a specific imposed stimulus

Six categories of properties -> mechanical, electrical, thermal, magnetic, optical, and deteriorative

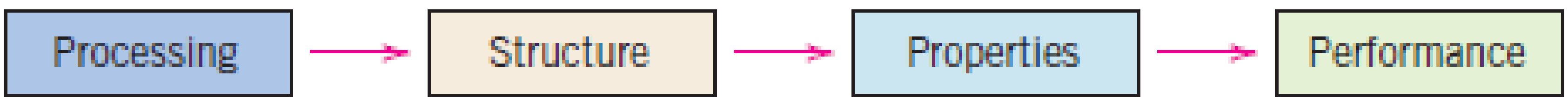
In addition to structure and properties, two other important components are involved in the science and engineering of materials—namely, “processing” and “performance.”

Processing -> preparing or putting through a prescribed procedure, e.g. the processing of ore to obtain material

Performance -> the accomplishment relative to stated goals or objectives

The *structure* of a material will depend on how it is *processed*.

Furthermore, a material's *performance* will be a function of its *properties*.



CLASSIFICATION OF MATERIALS

Three basic classifications of solid materials: metals, ceramics, and organic polymers (or just polymers).

In addition, there are the composites, combinations of two or more of the above three basic material classes

1. METALS

Materials in this group are composed of one or more metallic elements and often also nonmetallic elements in relatively small amounts

Atoms in metals and their alloys are arranged in a very orderly manner and in comparison to the ceramics and polymers, are relatively dense

Distinguishing characteristics -> stiff, strong, ductile, resistant to fracture

Metallic materials have large numbers of nonlocalized electrons

Some of the metals (Fe, Co, and Ni) have desirable magnetic properties

Metallic Objects

Crystal Structure

- Crystal Structure – matter assumes a periodic shape
  - Non-Crystalline or Amorphous “structures” exhibit

no long range periodic shapes

  - Xtal **Systems** – not structures but potentials
  - FCC, BCC and HCP – common Xtal Structures for metals
- Point, Direction and Planer ID�much‟ing in Xtals
- X-Ray Diffraction and Xtal Structure

Unit cell: smallest repetitive volume which contains the complete *lattice pattern* of a crystal.

ns of varying

y

meters:

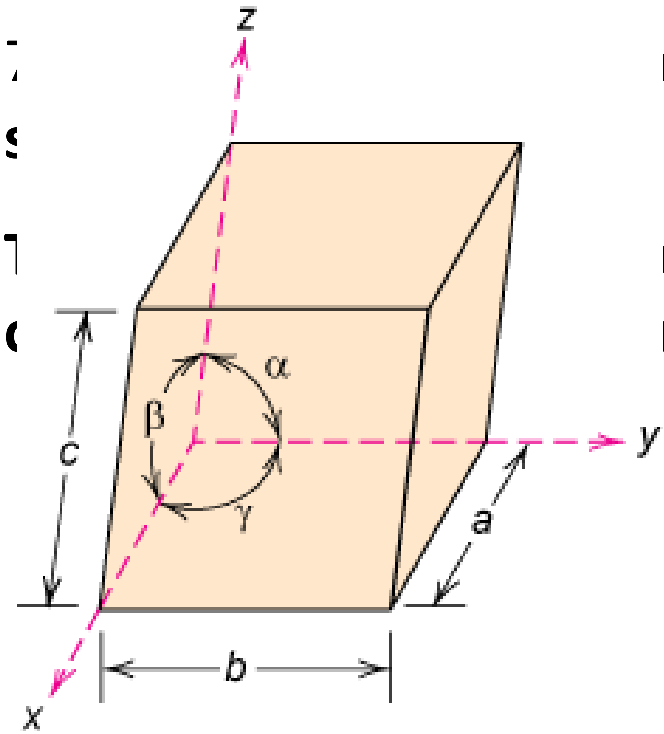
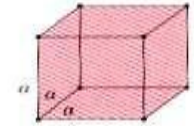
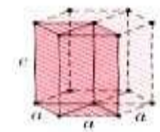
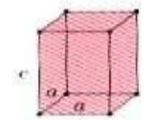
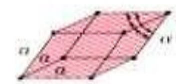
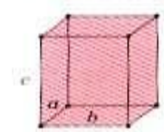
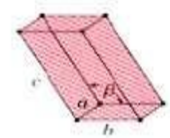
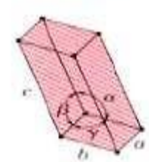
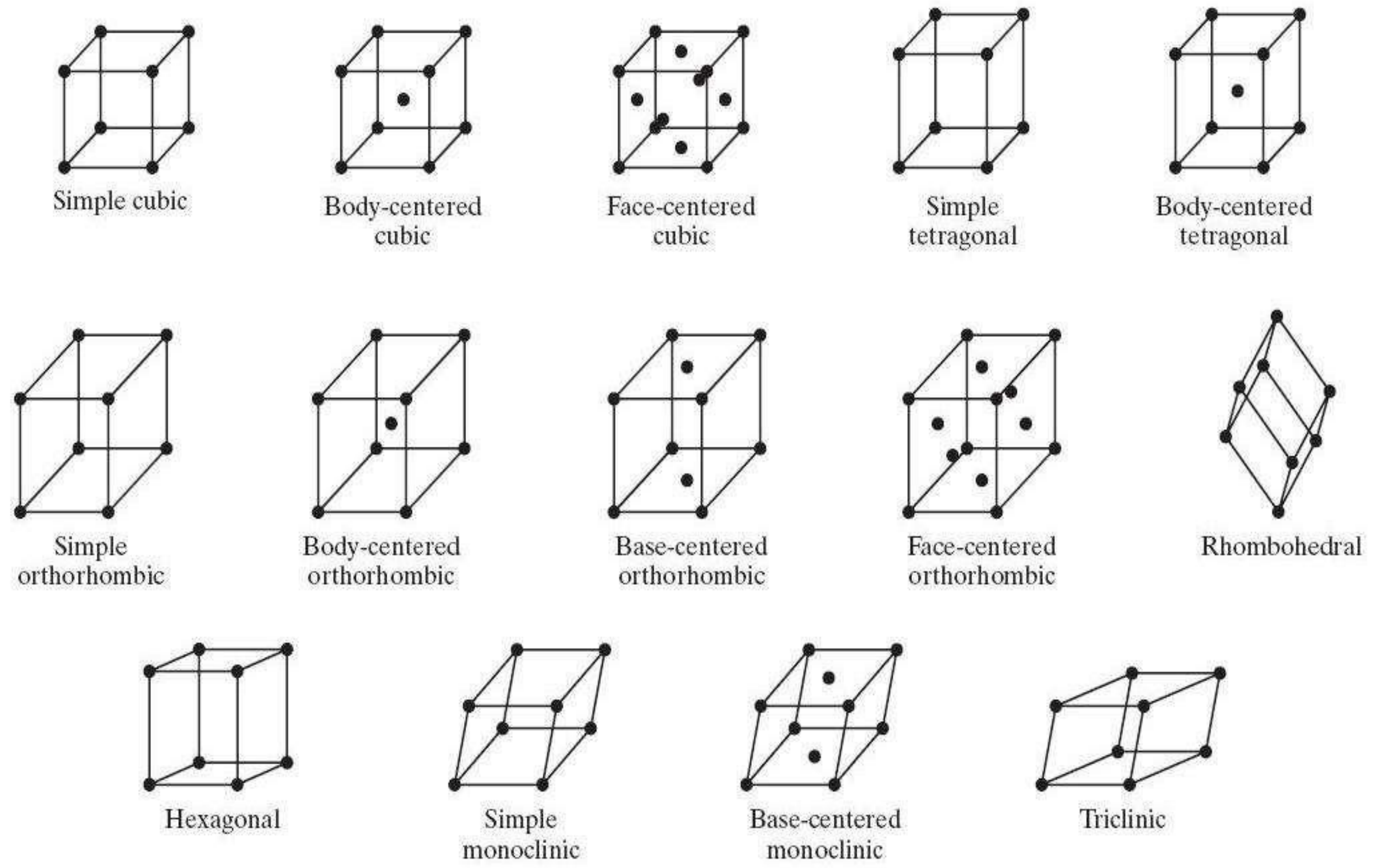


Table 3.2 Lattice Parameter Relationships and Figures Showing Unit Cell Geometries for the Seven Crystal Systems

Crystal System	*Axial Relationships*	*Interaxial Angles*	*Unit Cell Geometry*
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$	
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°, \gamma = 120°$	
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$	
Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90°$	
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$	
Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90° \neq \beta$	
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$	

The 14 Crystal (Bravais) Lattices



Metallic Crystal Structures

- Tend to be densely packed
- Reasons for dense packing:
- Typically, only one element is present, so all atomic radii are the same.

  -Metallic bonding is not directional.

  -Nearest neighbor distances tend to be small in order to lower bond energy.

  -Electron cloud shields cores from each other
- Have the simplest crystal structures

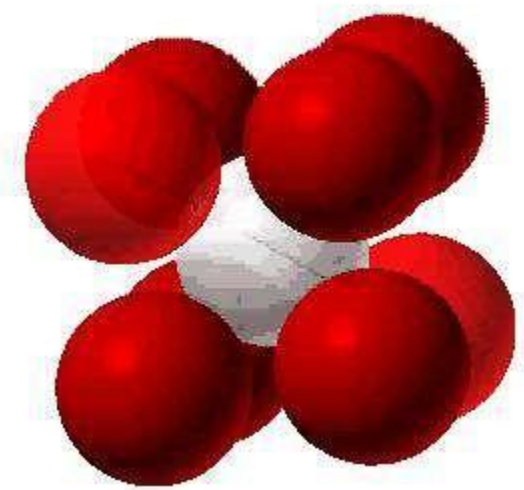
Atomic Radii and Crystal Structures for 16 Metals

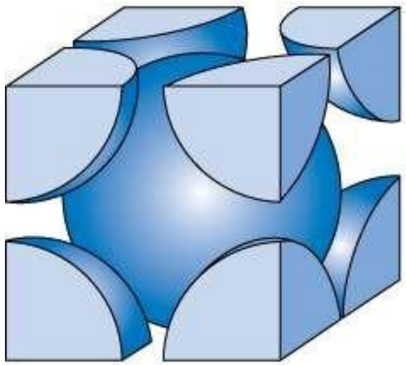
Metal	Crystal Structure[a]	Atomic Radius[b] (nm)	Metal	Crystal Structure	Atomic Radius (nm)
Aluminum	FCC	0.1431	Molybdenum	BCC	0.1363
Cadmium	HCP	0.1490	Nickel	FCC	0.1246
Chromium	BCC	0.1249	Platinum	FCC	0.1387
Cobalt	HCP	0.1253	Silver	FCC	0.1445
Copper	FCC	0.1278	Tantalum	BCC	0.1430
Gold	FCC	0.1442	Titanium (α)	HCP	0.1445
Iron (α)	BCC	0.1241	Tungsten	BCC	0.1371
Lead	FCC	0.1750	Zinc	HCP	0.1332

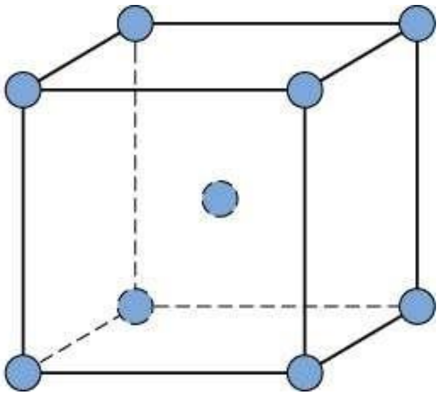
[a] FCC = face-centered cubic; HCP = hexagonal close-packed; BCC = body-centered cubic.

[b] A nanometer (nm) equals 10^{-9} m; to convert from nanometers to angstrom units (Å), multiply the nanometer value by 10.

Body Centered Cubic Structure (BCC)





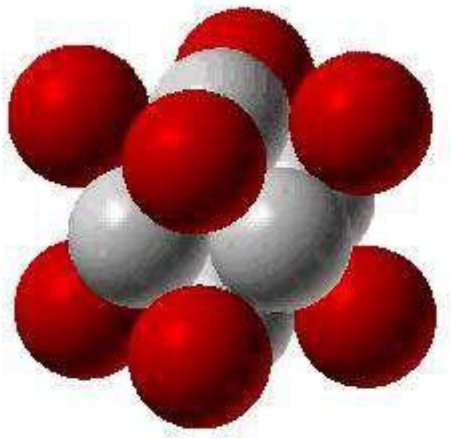


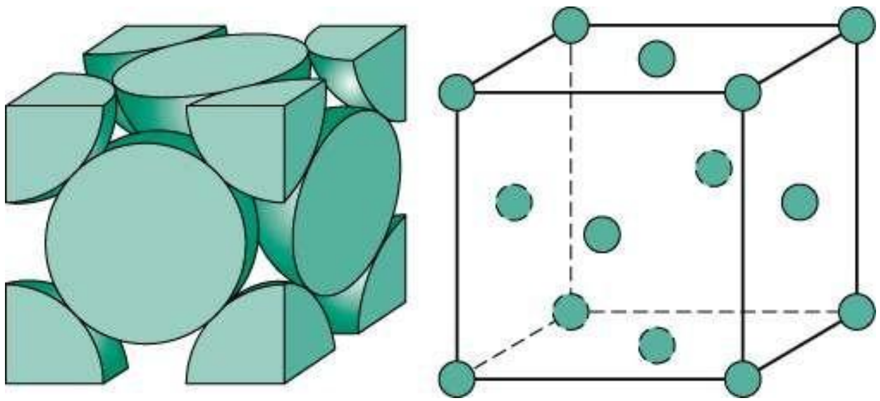
- Atoms touch each other along *cube diagonals within a unit cell*.

--Note: All atoms are identical; the center atom is shaded differently only for ease of viewing.

ex: Cr, W, Fe (?), Tantalum, Molybdenum

Face Centered Cubic Structure (FCC)





- Atoms touch each other along *face diagonals*.

  --Note: All atoms are identical; the face-centered atoms are shaded

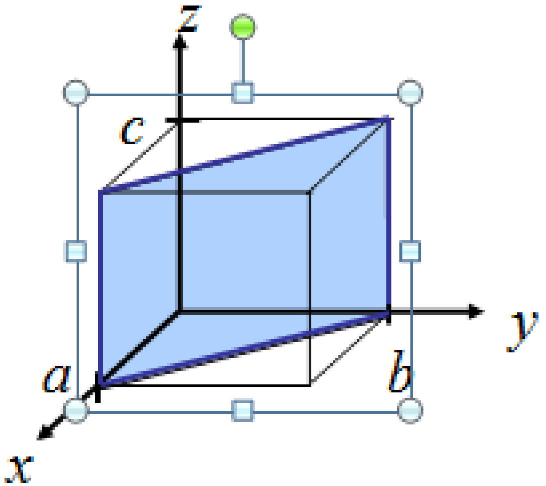
  differently only for ease of viewing.

ex: Al, Cu, Au, Pb, Ni, Pt, Ag

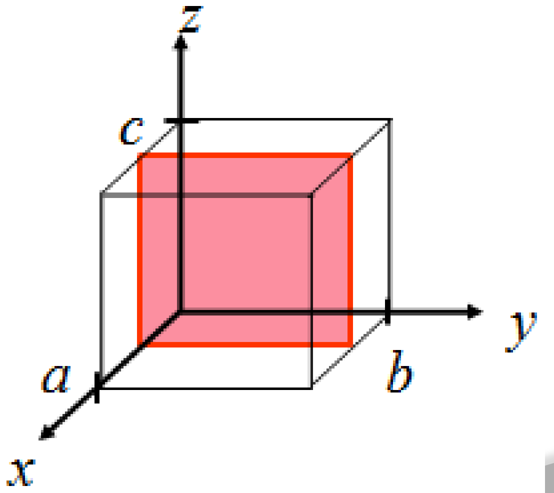
- Coordination # = 12

Crystallographic Planes

example		a	b	c
1.	Intercepts	1	1	∞
2.	Reciprocals	1/1	1/1	1/∞
		1	1	0
3.	Reduction	1	1	0
4.	Miller Indices	(110)		

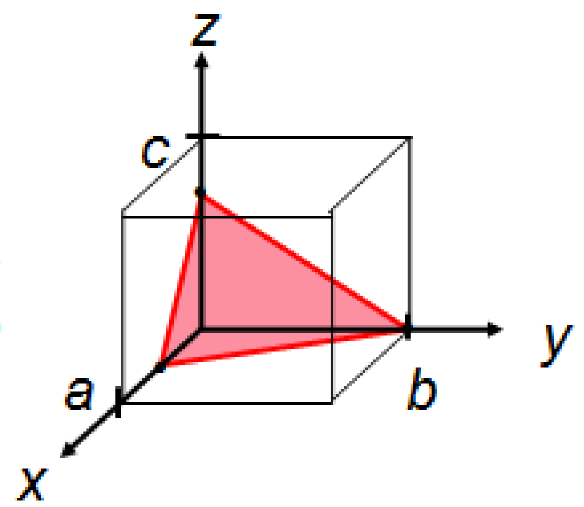


example		a	b	c
1.	Intercepts	1/2	∞	∞
2.	Reciprocals	1/½	1/∞	1/∞
		2	0	0
3.	Reduction	2	0	0
4.	Miller Indices	(100)		



Crystallographic Planes

example		a	b	c
1.	Intercepts	1/2	1	3/4
2.	Reciprocals	1/½	1/1	1/¾
		2	1	4/3
3.	Reduction	6	3	4
4.	Miller Indices	(634)		



Family of Planes $\{hkl\}$

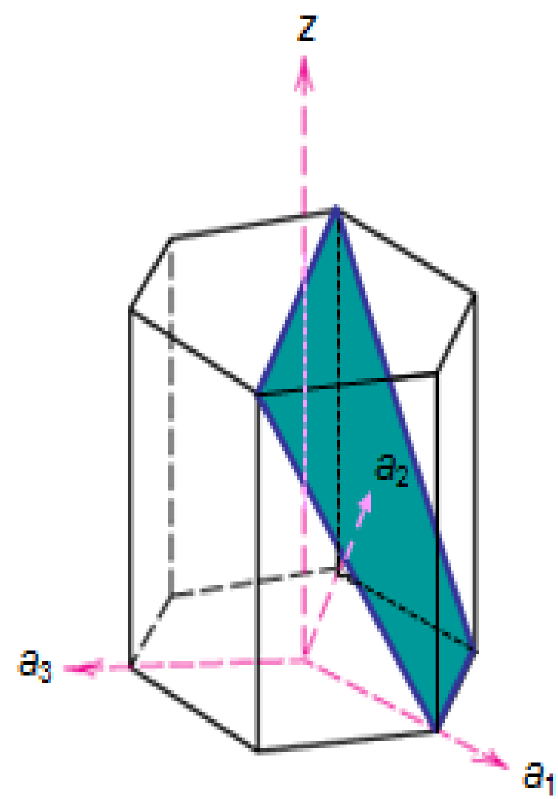
Ex: $\{100\} = (100),\ (010),\ (001),\ (\bar{1}00),\ (0\bar{1}0),\ (00\bar{1})$

Crystallographic Planes

- In hexagonal unit cells the same idea is used

example		a_1	a_2	a_3	c
1.	Intercepts	1	∞	-1	1
2.	Reciprocals	1	$1/\infty$	-1	1
		1	0	-1	1
3.	Reduction	1	0	-1	1

4. Miller-Bravais Indices (1011)



Adapted from Fig. 3.8(a), *Callister 7e.*

Bauschinger effect

The **Bauschinger effect** refers to a property of materials where the material's stress/strain characteristics change as a result of the microscopic stress distribution of the material. For example, an increase in tensile yield strength occurs at the expense of compressive yield strength. The effect is named after German engineer Johann Bauschinger.

Bauschinger effect

The Bauschinger effect is normally associated with conditions where the yield strength of a metal decreases when the direction of strain is changed. It is a general phenomenon found in most polycrystalline metals. The basic mechanism for the Bauschinger effect is related to the dislocation structure in the cold worked metal. As deformation occurs, the dislocations will accumulate at barriers and produce dislocation pile-ups and tangles. Based on the cold work structure, two types of mechanisms are generally used to explain the Bauschinger effect.

Cold Working and Hot working

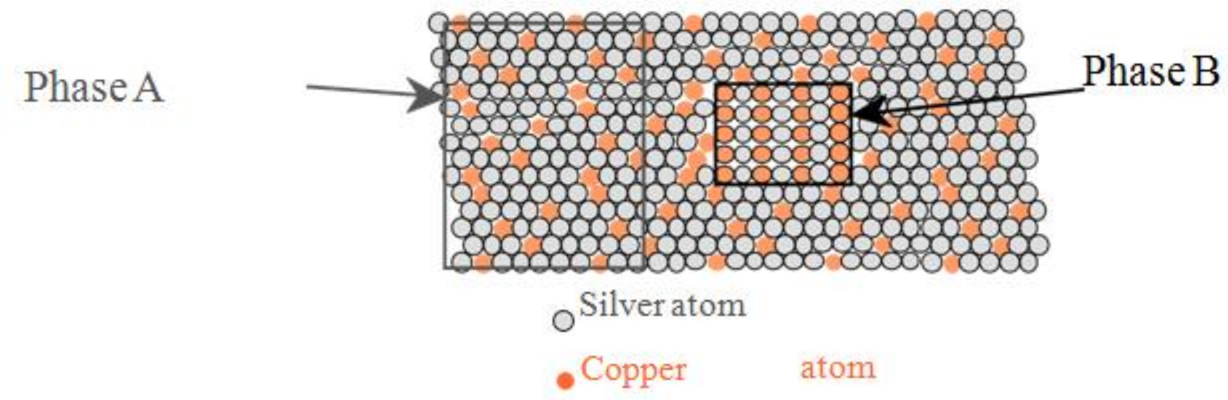
When the processing temperature of the mechanical deformation of steel is above the recrystallization temperature, the **process** is termed as **hot working process** otherwise it is termed as **cold working**. ... For **hot working processes**, large deformation can be successively repeated as the metal remain soft and ductile

Recrystallization is a process by which deformed grains are replaced by a new set of defect-free grains that nucleate and grow until the original grains have been entirely consumed. Recrystallization is usually accompanied by a reduction in the strength and hardness of a material and a simultaneous increase in the ductility. Thus, the process may be introduced as a deliberate step in metals processing or may be an undesirable byproduct of another processing step. The most important industrial uses are softening of metals previously hardened or rendered brittle by cold work, and control of the grain structure in the final product.

MODULE-II

PHASE DIAGRAMS

CLOs	COURSE LEARNING OUTCOMES
CLO1	Describe the basic concepts of FEM and steps involved in it.
CLO2	Understand the difference between the FEM and Other methods.
CLO3	Understand the stress-strain relation for 2-D and their field problem.
CLO4	Understand the concepts of shape functions for one dimensional and quadratic elements, stiffness matrix and boundary conditions
CLO5	Apply numerical methods for solving one dimensional bar problems



- When we combine two elements...
  what equilibrium state do we get?
- In particular, if we specify...
  --a composition (e.g., wt%Cu - wt%Ag), and
  --a temperature (T)

THE SOLUBILITY LIMIT

- Solubility Limit:

  Max concentration for which only a solution occurs.
  (No precipitate)

- Ex: Phase Diagram: Water-Sugar System

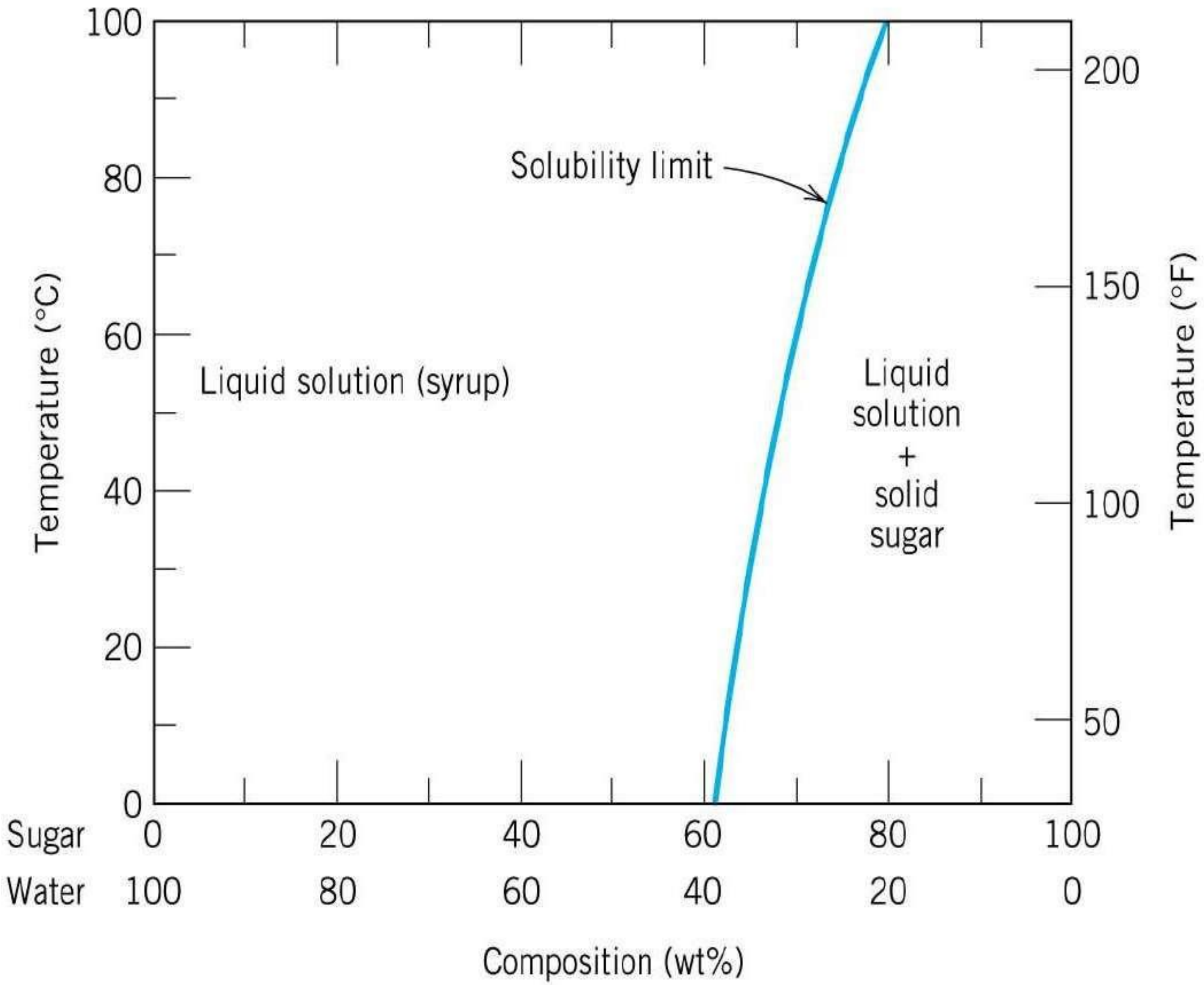
  Question: What is the solubility limit at 20C?

  Answer: 65wt% sugar.
  If Comp < 65wt% sugar: syrup
  If Comp > 65wt% sugar: syrup + sugar coexist

- Solubility limit increases with T:

  e.g., if T = 100C, solubility limit = 80wt% sugar.

THE SOLUBILITY LIMIT

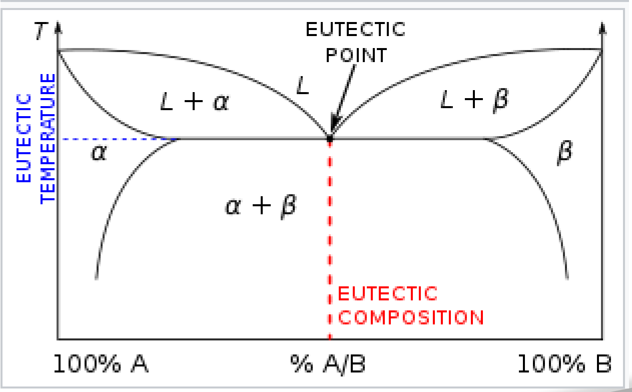


A **eutectic system**

A **eutectic system** : from the Greek "εύ" (eu = well) and "τήξις" (tēxis = melting) is a homogeneous mixture of substances that melts or solidifies at a single temperature that is lower than the melting point of either of the constituents.
The eutectic temperature is the lowest possible melting temperature over all of the mixing ratios for the involved component species.

A eutectic system

Upon heating any other mixture ratio, and reaching the eutectic temperature – see the adjacent phase diagram – one component's lattice will melt first, while the temperature of the mixture has to further increase for (all) the other component lattice(s) to melt. Conversely, as a non-eutectic mixture cools down, each mixture's component will solidify (form its lattice) at a distinct temperature, until all material is solid.



Peritectic

Peritectic transformations are also similar to eutectic reactions. Here, a liquid and solid phase of fixed proportions react at a fixed temperature to yield a single solid phase. Since the solid product forms at the interface between the two reactants, it can form a diffusion barrier and generally causes such reactions to proceed much more slowly than eutectic or eutectoid transformations. Because of this, when a peritectic composition solidifies it does not show the lamellar structure that is found with eutectic solidification.

IRON-CARBON (Fe-C) PHASE DIAGRAM

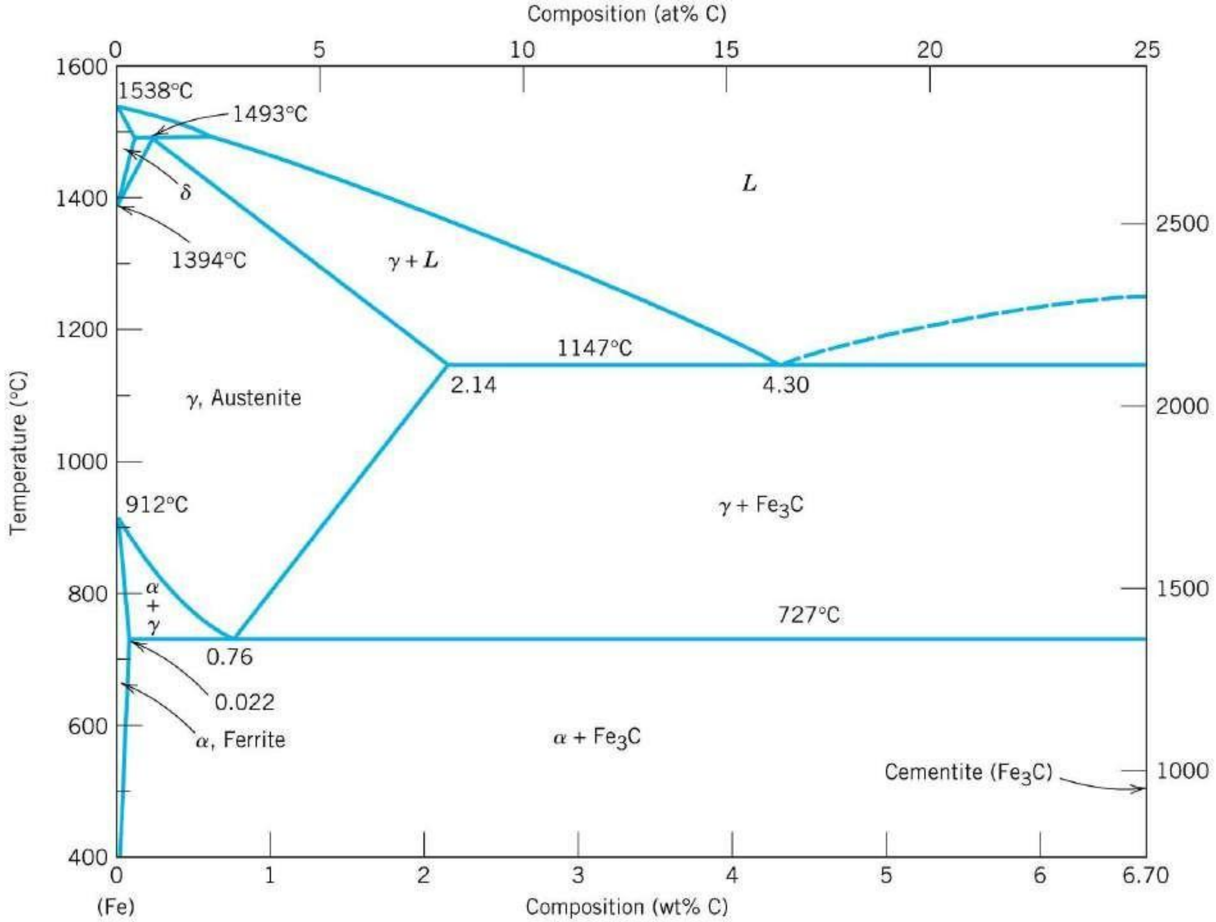


FIGURE 9.21 The iron–iron carbide phase diagram. [Adapted from *Binary Alloy Phase Diagrams*, 2nd edition, Vol. 1, T. B. Massalski (Editor-in-Chief), 1990. Reprinted by permission of ASM International, Materials Park, OH.]

Classification of steels

Low. Often called mild steels, low-carbon steels have less than 0.30 percent carbon and are the most commonly used grades. They machine and weld nicely and are more ductile than higher-carbon steels.

Medium. Medium-carbon steels have from 0.30 to 0.45 percent carbon. Increased carbon means increased hardness and tensile strength, decreased ductility, and more difficult machining.

High. With 0.45 to 0.75 percent carbon, these steels can be challenging to weld. Preheating, postheating (to control cooling rate), and sometimes even heating during welding become necessary to produce acceptable welds and to control the mechanical properties of the steel after welding.

Classification of steels

Very High. With up to 1.50 percent carbon content, very high-carbon steels are used for hard steel products such as metal cutting tools and truck springs. Like high-carbon steels, they require heat treating before, during, and after welding to maintain their mechanical properties.

MODULE-III

ELASTIC CONSTANTS AND PRINCIPAL STRESSES

CLOs	COURSE LEARNING OUTCOMES
CLO1	Describe the basic concepts of FEM and steps involved in it.
CLO2	Understand the difference between the FEM and Other methods.
CLO3	Understand the stress-strain relation for 2-D and their field problem.
CLO4	Understand the concepts of shape functions for one dimensional and quadratic elements, stiffness matrix and boundary conditions
CLO5	Apply numerical methods for solving one dimensional bar problems

- Strength and stiffness of structures is function of size and shape, certain physical properties of material.
- <u>Properties of Material:-</u>
- Elasticity
- Plasticity
- Ductility
- Malleability
- Brittleness
- Toughness
- Hardness

Stress

- Resistance offered by the material per unit cross-sectional area is called STRESS.

$\sigma = P/A$

Unit of Stress:

Pascal = 1 N/m^2

kN/m^2 , MN/m^2 , GN/m^2

1 MPa = 1 N/mm^2

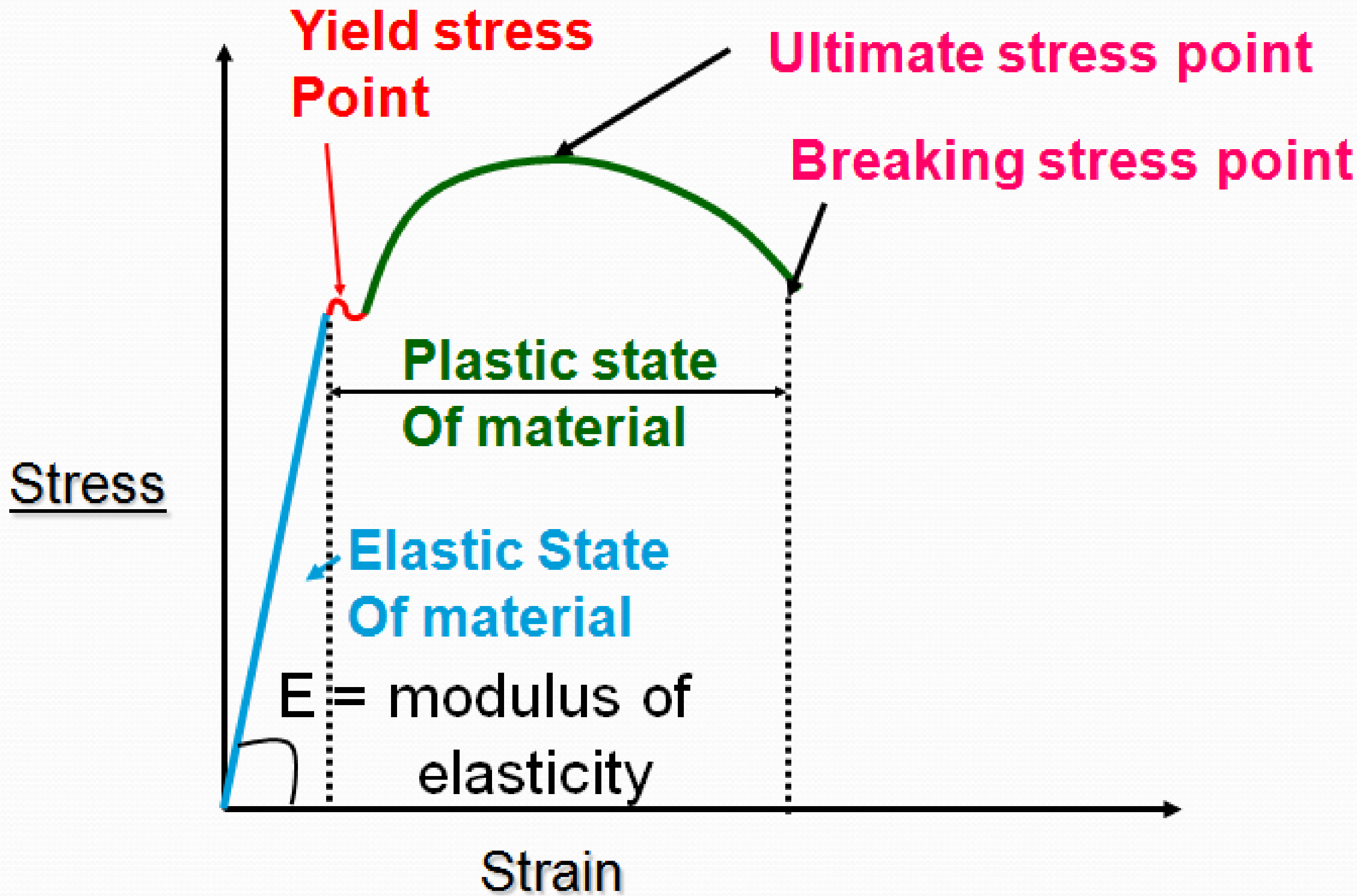
Example : 1

A short hollow, cast iron cylinder with wall thickness of 10 mm is to carry compressive load of 100 kN. Compute the required outside diameter `D' , if the working stress in compression is 80 N/mm^2. (D = 49.8 mm).

Example: 2

A Steel wire hangs vertically under its weight. What is the greatest length it can have if the allowable tensile stress σ_t =200 MPa? Density of steel γ=80 kN/m^3.(ans:-2500 m)

Stress- Strain Curve for Mild Steel (Ductile Material)



Modulus of Elasticity:

- Stress required to produce a strain of unity.
- i.e. the stress under which the bar would be stretched to twice its original length . If the material remains elastic throughout , such excessive strain.
- Represents slope of stress-strain line OA.

Example:4 An aluminium bar 1.8 meters long has a 25 mm square c/s over 0.6 meters of its length and 25 mm circular c/s over other 1.2 meters . How much will the bar elongate under a tensile load P=17500 N, if E = 75000 Mpa.

Example: 5 A prismatic steel bar having cross sectional area of A=300 mm^2 is subjected to axial load as shown in figure . Find the net increase δ in the length of the bar. Assume E = 2 x 10^5 MPa.(Ans δ = -0.17mm)

Example: 14

A steel bolt of length L passes through a copper tube of the same length, and the nut at the end is turned up just snug at room temp. Subsequently the nut is turned by 1/4 turn and the entire assembly is raised by temp 55^0C. Calculate the stress in bolt if L=500mm,pitch of nut is 2mm, area of copper tube =500sq.mm,area of steel bolt=400sq.mm

$Es=2 * 10^5\ N/mm^2 ;\alpha_s =12*10^{-6} /^0C$

$Ec=1 * 10^5\ N/mm^2 ;\alpha_c= 17.5*10^{-6} /^0C$

A circular section tapered bar is rigidly fixed as shown in figure. If the temperature is raised by 30^0 C, calculate the maximum stress in the bar. Take

$E=2*10^5$ N/mm^2 ;α $=12*10^{-6}$ /^{0}C

A composite bar made up of aluminum and steel is held between two supports.The bars are stress free at 40^0c. What will be the stresses in the bars when the temp. drops to 20^0C, if

(a) the supports are unyielding

(b)the supports come nearer to each other by 0.1 mm.

Take $E_{al} = 0.7*10^5$ N/mm^2 ; $\alpha_{al} = 23.4*10^{-6}$ /^{0}C

$E_S = 2.1*10^5$ N/mm^2 $\alpha_s = 11.7*10^{-6}$ /^{0}C

$A_{al} = 3$ cm^2 $A_s = 2$ cm^2

YOUNG'S MODULUS (E):--

Young's Modulus (E) is defined as the Ratio of Stress (σ) to strain (ε).

$$\boxed{E = \sigma / \varepsilon} \quad \text{-------------(5)}$$

BULK MODULUS (K):--

- When a body is subjected to the identical stress σ in three mutually perpendicular directions, the body undergoes uniform changes in three directions without the distortion of the shape.
- The ratio of change in volume to original volume has been defined as volumetric strain(ε_v)

- Then the bulk modulus, K is defined as $K = \sigma / \varepsilon_v$

ELASTIC CONSTANTS

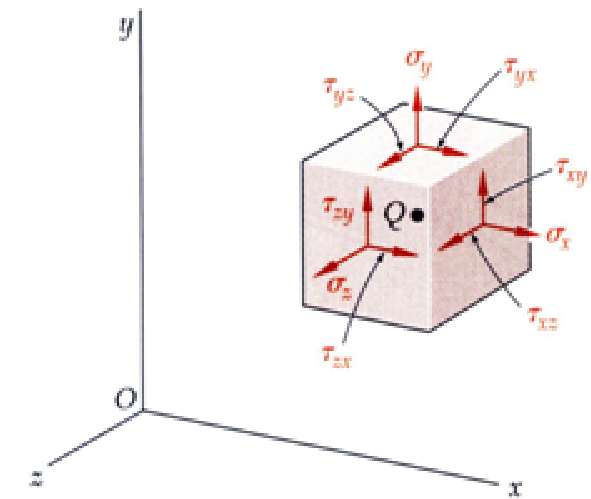
YOUNG'S MODULUS $E = \sigma / \varepsilon$ -------------(5)

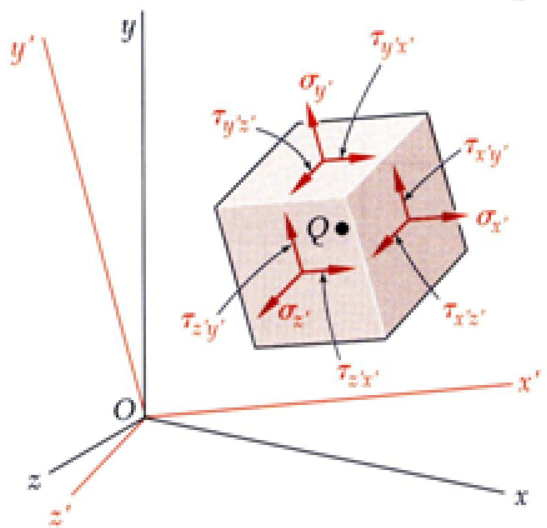
BULK MODULUS $K = \sigma / \varepsilon_v$ -------------(6)

MODULUS OF RIGIDITY $N = \tau / \phi$ -------------(7)

Principal plane

a **principal plane** as one on which there is no shearing stress; it is assumed that no shearing stress acts on a **plane** at an angle θ to Oy. A **principal** stress acting on an inclined **plane**; there is no shearing stress t associated with a **principal** stress σ.





- The most general state of stress at a point may be represented by 6 components,

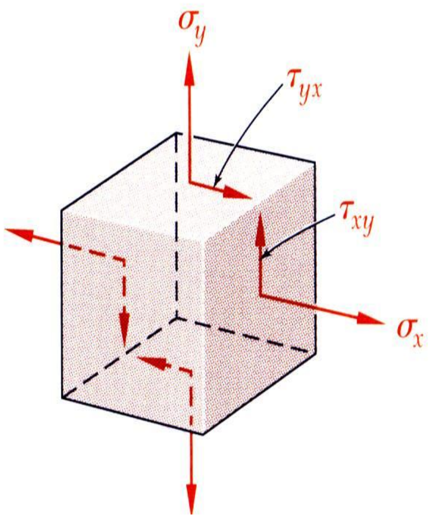
  $\sigma_x, \sigma_y, \sigma_z$ normal stresses

  $\tau_{xy}, \tau_{yz}, \tau_{zx}$ shearing stresses

  (Note: $\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz}$)

- Same state of stress is represented by a different set of components if axes are rotated.

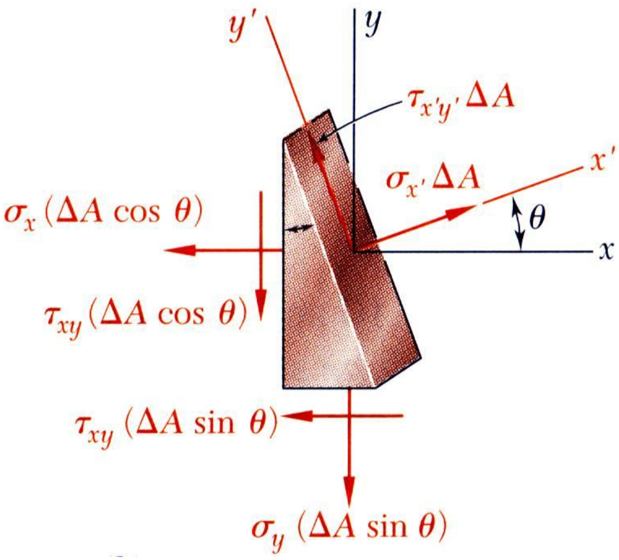
- The first part of the chapter is concerned with how the components of stress are transformed under a rotation of the coordinate axes. The second part of the chapter is devoted to a similar analysis of the transformation of the components of strain.



- *Plane Stress* - state of stress in which two faces of the cubic element are free of stress. For the illustrated example, the state of stress is defined by

$$\sigma_x, \sigma_y, \tau_{xy} \quad \text{and} \quad \sigma_z = \tau_{zx} = \tau_{zy} = 0.$$

Transformation of Plane Stress

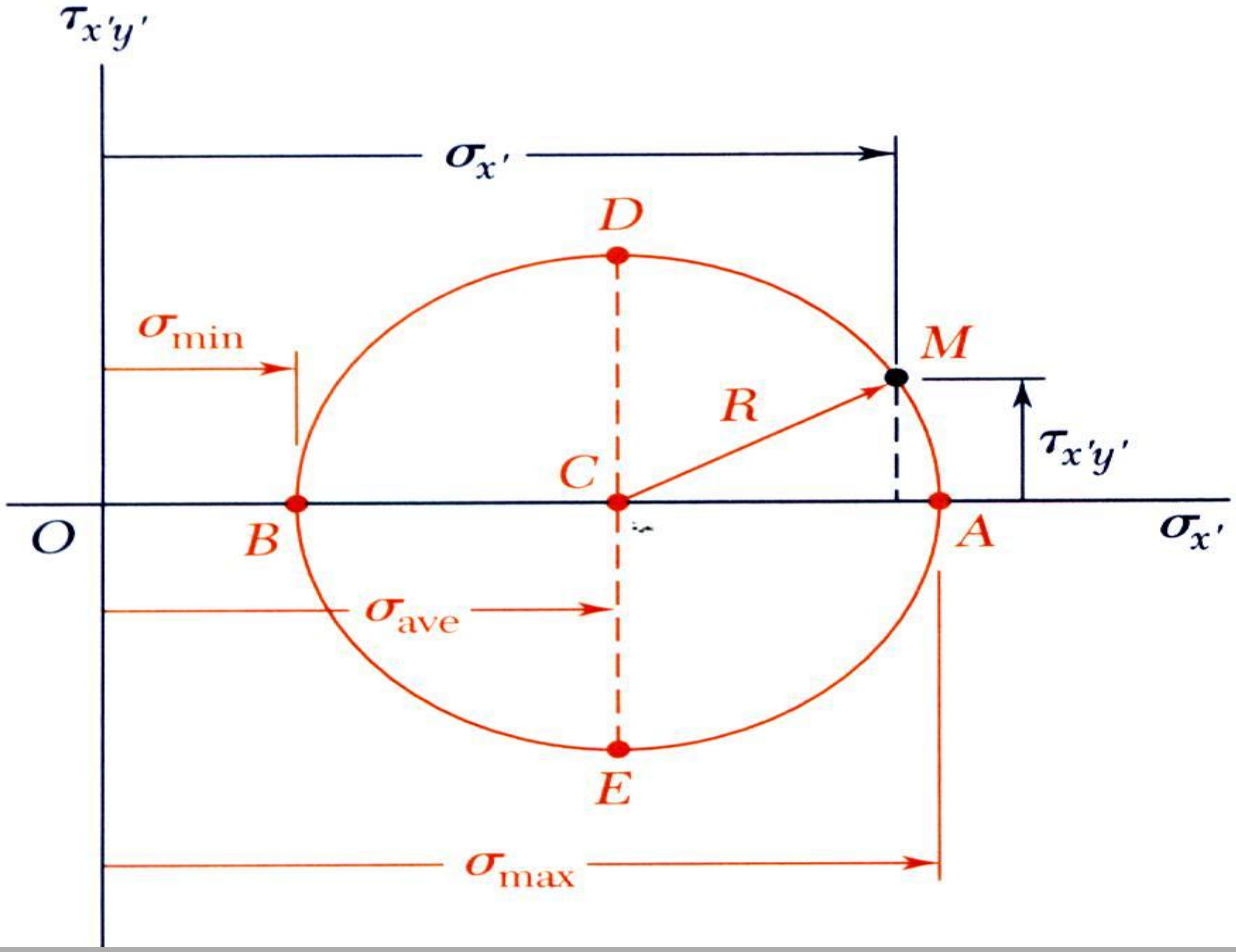


- Consider the conditions for equilibrium of a prismatic element with faces perpendicular to the *x*, *y*, and *x'* axes.

$$\sum F_{x'} = 0 = \sigma_{x'}\Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta$$
$$-\sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{xy}(\Delta A\sin\theta)\cos\theta$$

$$\sum F_{y'} = 0 = \tau_{x'y'}\Delta A + \sigma_x(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta$$
$$-\sigma_y(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\sin\theta)\sin\theta$$

Mohr's Circle



MODULE-IV

SHEAR FORCE AND BENDING MOMENT DIAGRAMS

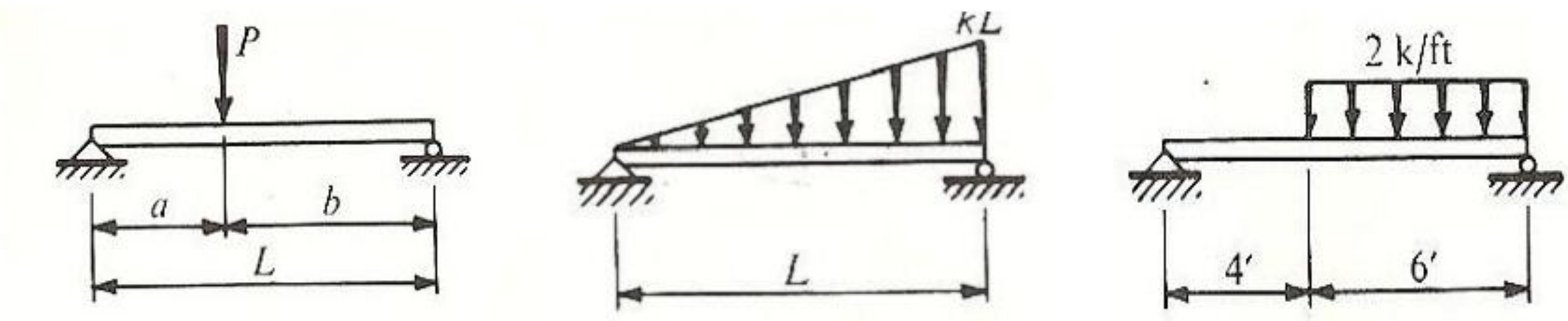
FLEXURAL STRESSES

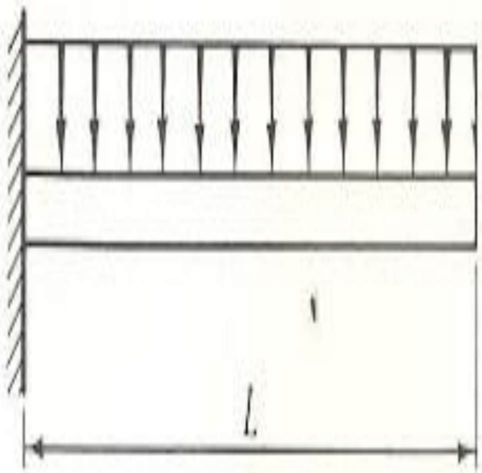
SHEAR STRESSES

CLOs	COURSE LEARNING OUTCOMES
CLO1	Describe the basic concepts of FEM and steps involved in it.
CLO2	Understand the difference between the FEM and Other methods.
CLO3	Understand the stress-strain relation for 2-D and their field problem.
CLO4	Understand the concepts of shape functions for one dimensional and quadratic elements, stiffness matrix and boundary conditions
CLO5	Apply numerical methods for solving one dimensional bar problems

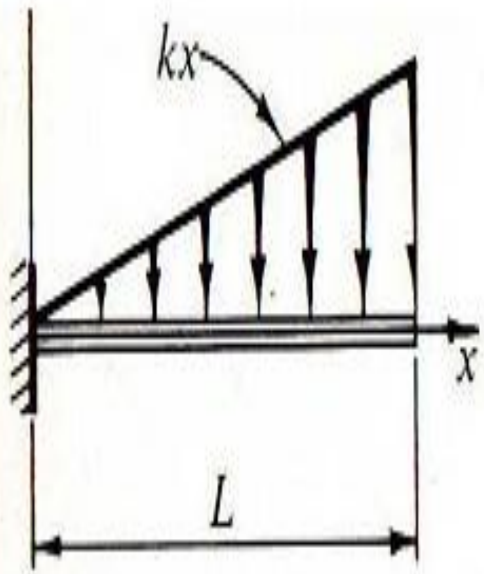
4-Classification of Beams:

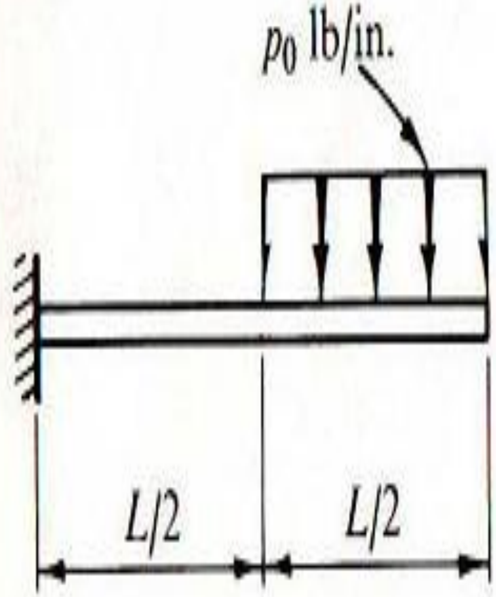
1) Simple Beam





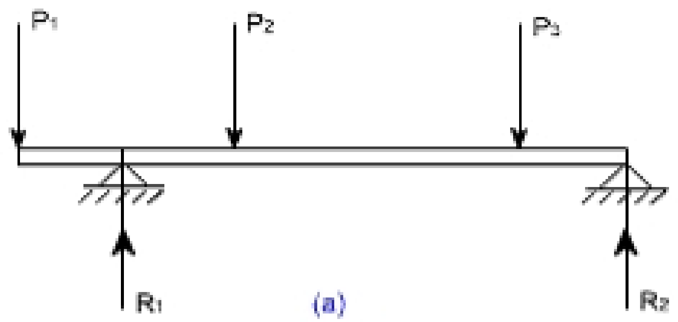
Cantilever

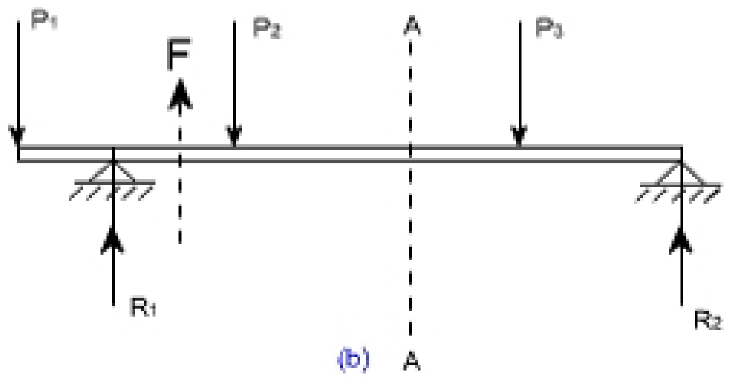




Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms





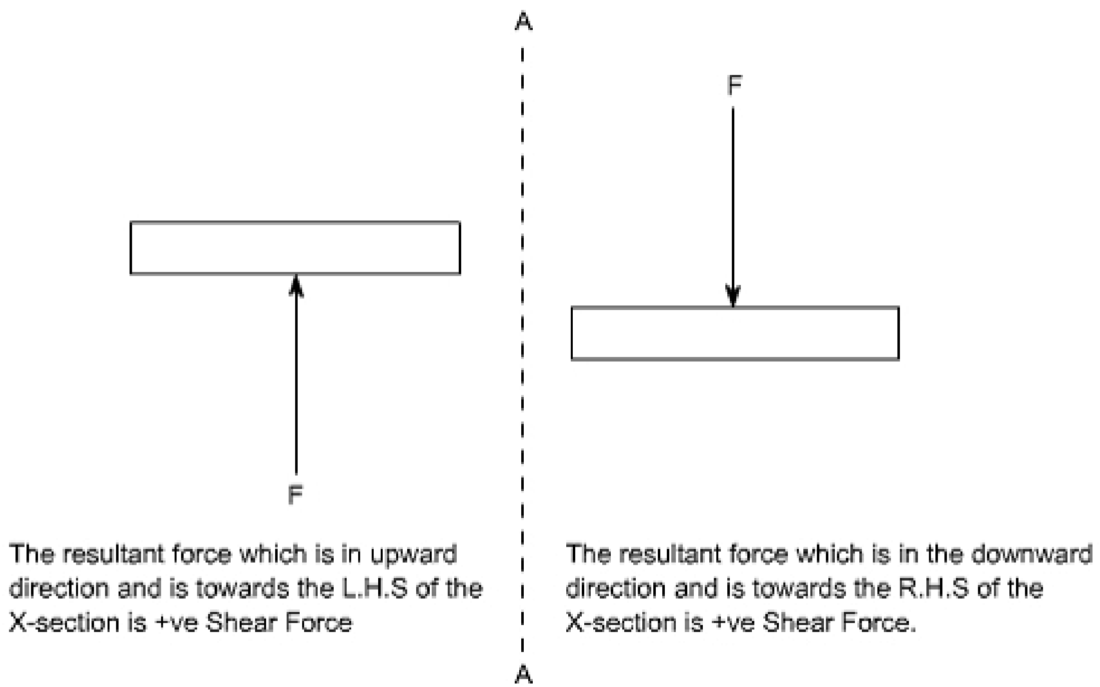
Now let us consider the beam as shown in fig 1(a) which is supporting the loads P_1, P_2, P_3 and is simply supported at two points creating the reactions R_1 and R_2 respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume that the resultant of loads and reactions to the left of AA is 'F' vertically upwards, and since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be F, acting downwards. This forces 'F' is as a shear force. The shearing force at any x-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

Therefore, now we are in a position to define the shear force 'F' to as follows:

At any x-section of a beam, the shear force 'F' is the algebraic sum of all the lateral components of the forces acting on either side of the x-section.

Sign Convention for Shear Force:

The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.



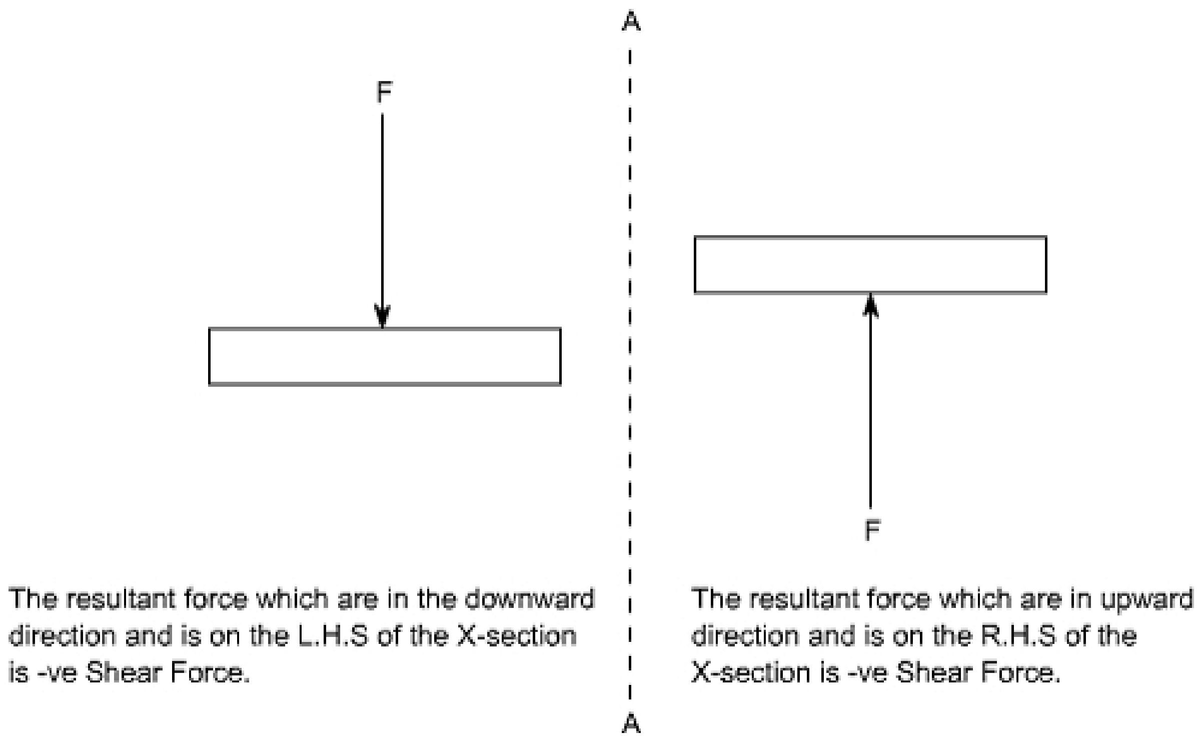


Fig 3: Negative Shear Force

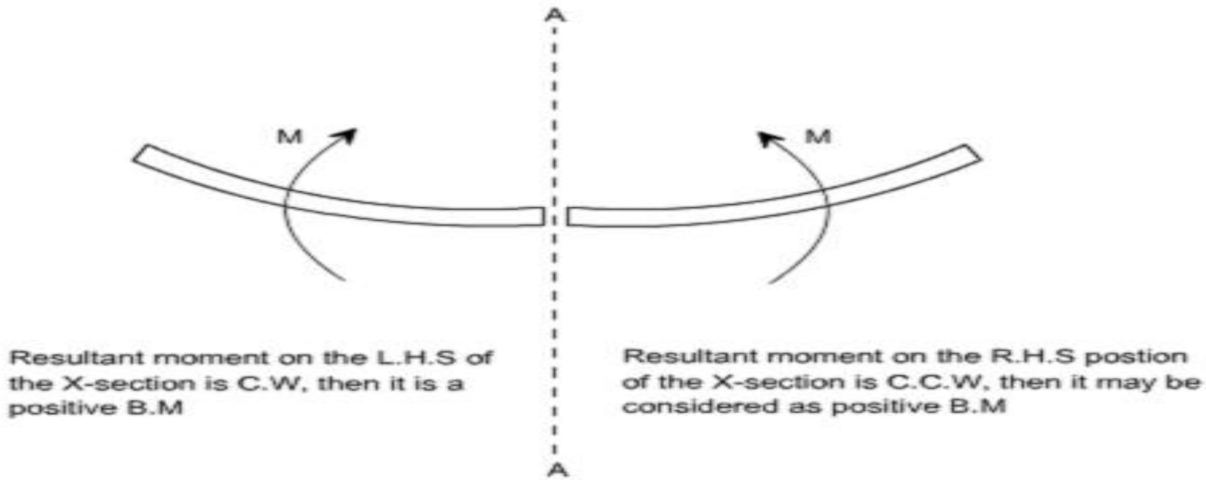
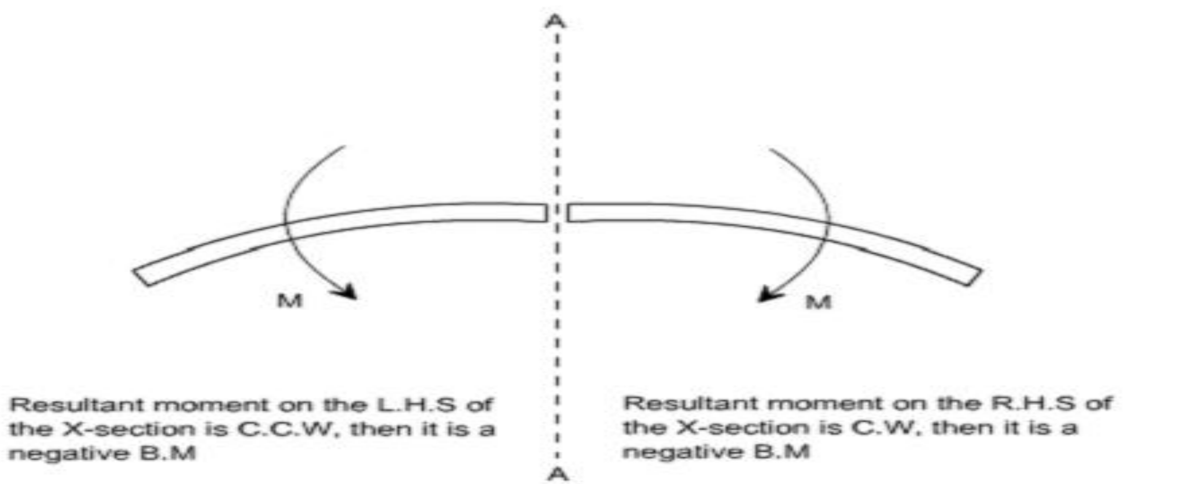
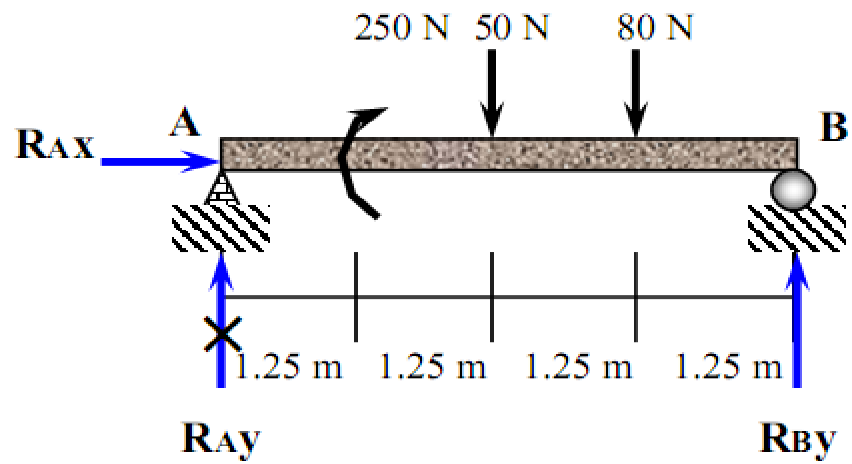


Fig 5: Positive Bending Moment



Calculations of Beam Reactions

Ex3:



$\rightarrow \sum F_x = 0$ --- (1)

$R_{Ax} = 0$

$\circlearrowleft + \sum M@A = 0$ --- (2)

$250 + 80 \times 2.5 + 80 \times 3.75 - R_B \times 5 = 0$

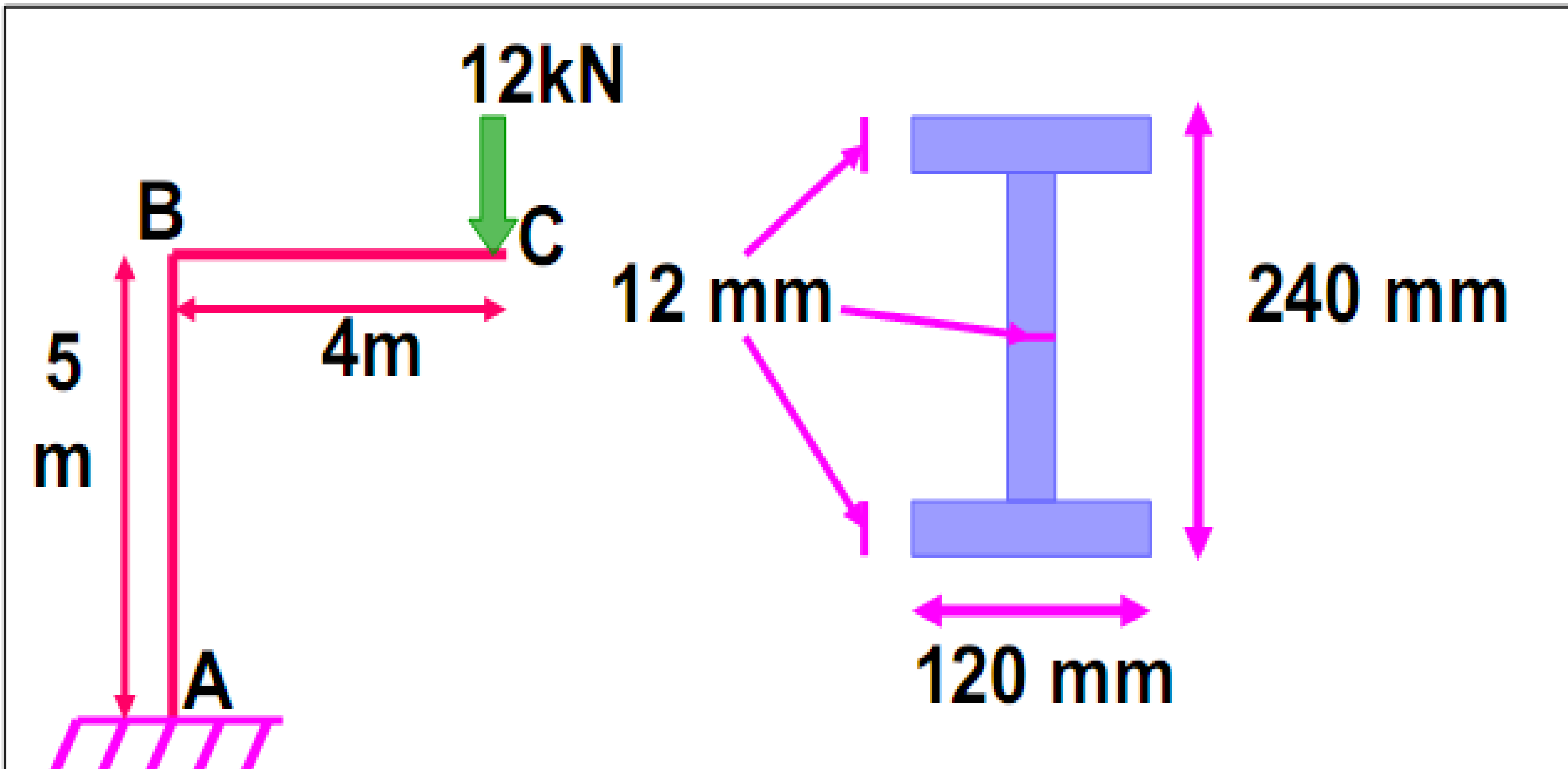
$\therefore R_{By} = +135 \text{ N} \uparrow$

R_{Ay} $\downarrow$

$\uparrow \sum F_y = 0$ --- (3)

$R_{Ay} = -5 \text{ N} \uparrow \Rightarrow R_{Ay} = 5 \text{ N} \downarrow$

2. Compare the strain energies due to three types of internal forces in the rectangular bent shown in Fig. having uniform cross section shown in the same Fig. Take E=2 x 105 MPa, G= 0.8 x 105 MPa, Ar= 2736 mm^2

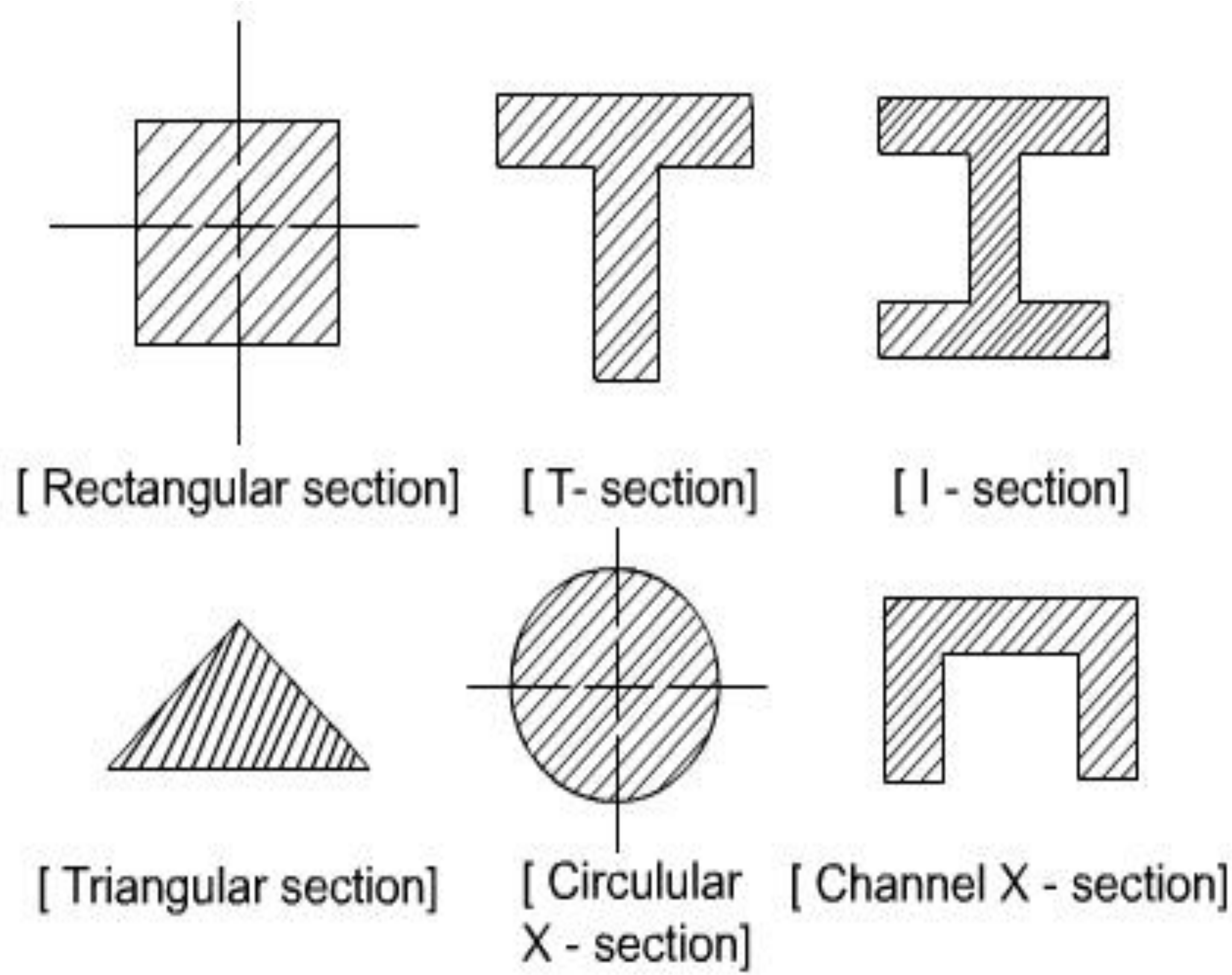


An I - section girder, 200mm wide by 300 mm depth flange and web of thickness is 20 mm is used as simply supported beam for a span of 7 m. The girder carries a distributed load of 5 KN /m and a concentrated load of 20 KN at mid-span.

Determine the

(i). The second moment of area of the cross-section of the girder

(ii). The maximum stress set up.


[Rectangular section]
[T- section]
[I - section]
[Triangular section]
[Circulular X - section]
[Channel X - section]

- **Loading restrictions:**
- **Concept of pure bending:**
- **As we are aware of the fact internal reactions developed on any cross-section of a beam may consists of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the bending effects alone are investigated, we shall put a constraint on the loading such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the member,**
- **That means F = 0**
- **since   or M = constant.**
- **Thus, the zero shear force means that the bending moment is constant or the bending is same at every cross-section of the beam. Such a situation may be visualized or envisaged when the beam or some portion of the beam, as been loaded only by pure couples at its ends. It must be recalled that the couples are assumed to be loaded in the plane of symmetry.**

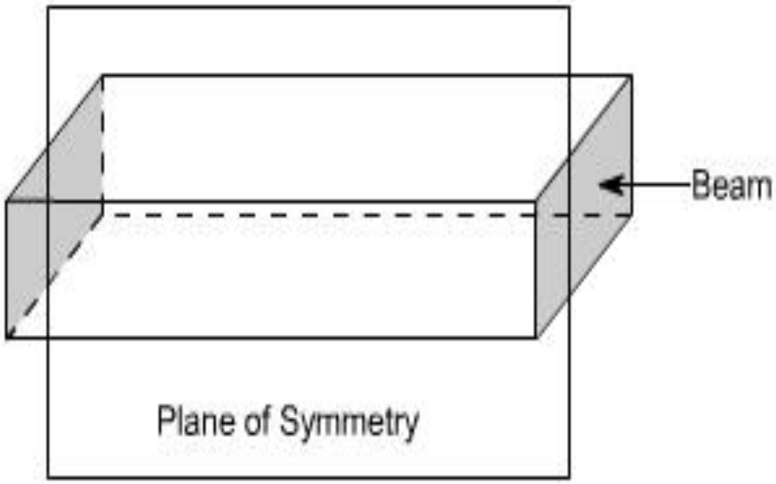


Fig (1)

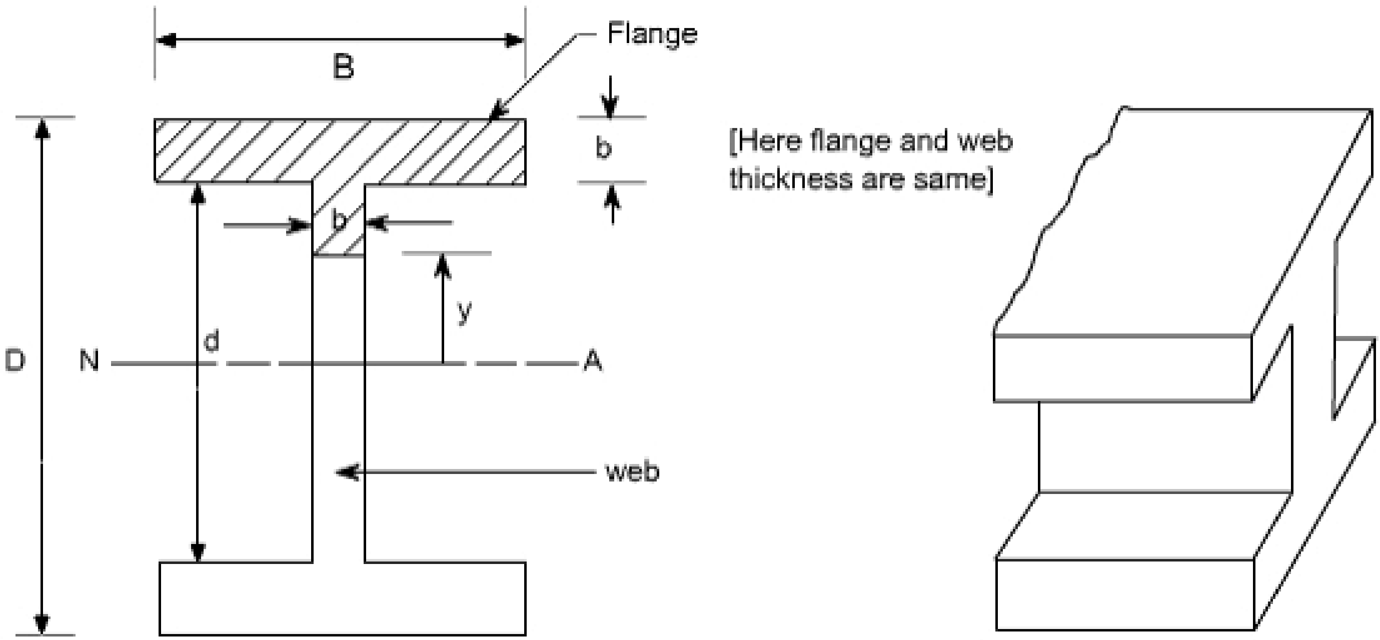
$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

However $\dfrac{\text{stress}}{\text{strain}} = E$ where E = Young's Modulus of elasticity

Therefore, equating the two strains as obtained from the two relations i.e,

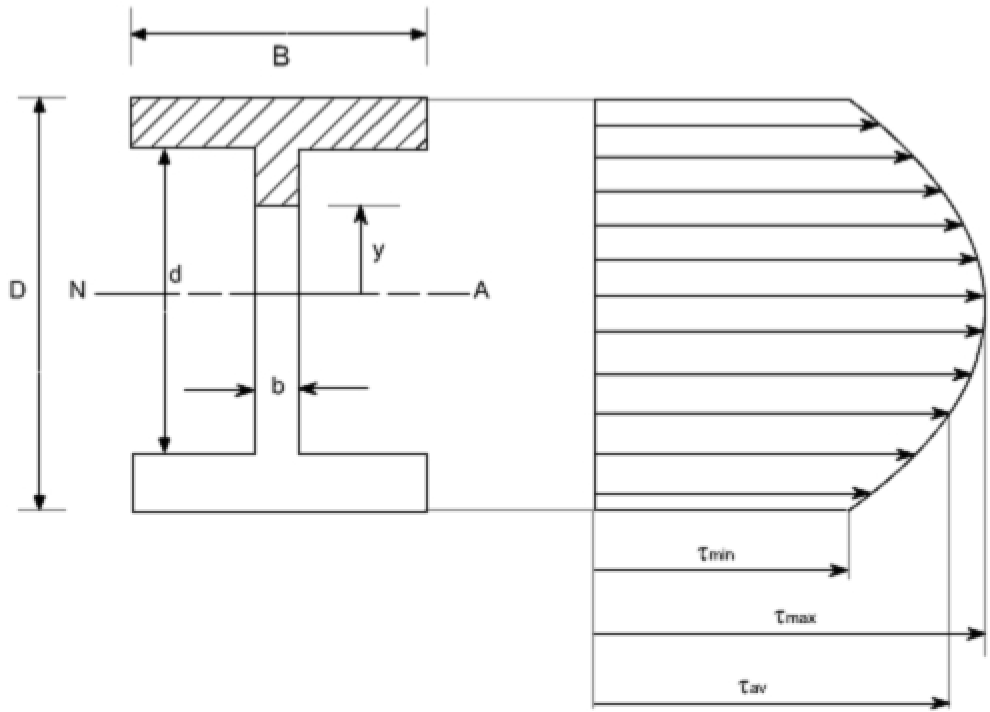
$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \qquad \text{.............(1)}$$

Consider an I - section of the dimension shown below.

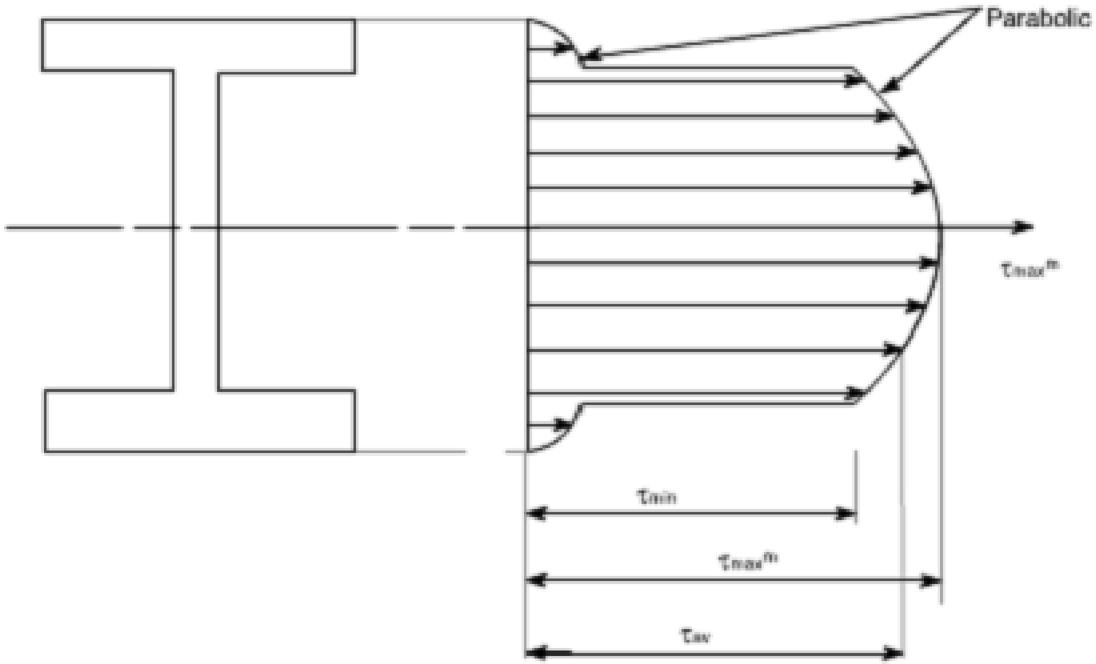


The shear stress distribution for any arbitrary shape is given as $\tau = \frac{F A \bar{y}}{Z I}$

Let us evaluate the quantity $A\bar{y}$, the $A\bar{y}$ quantity for this case comprise the contribution due to flange area and web area



$$\tau_{max^m} = \frac{F}{8bI}\left[B\left(D^2 - d^2\right) + bd^2\right]$$


Parabolic
τ_{max}^{m}
τ_{min}
τ_{max}^{m}
τ_{av}

MODULE-V

SLOPE AND DEFLECTION

CLOs	COURSE LEARNING OUTCOMES
CLO1	Describe the basic concepts of FEM and steps involved in it.
CLO2	Understand the difference between the FEM and Other methods.
CLO3	Understand the stress-strain relation for 2-D and their field problem.
CLO4	Understand the concepts of shape functions for one dimensional and quadratic elements, stiffness matrix and boundary conditions
CLO5	Apply numerical methods for solving one dimensional bar problems

Calculation of deflections is an important part of structural analysis

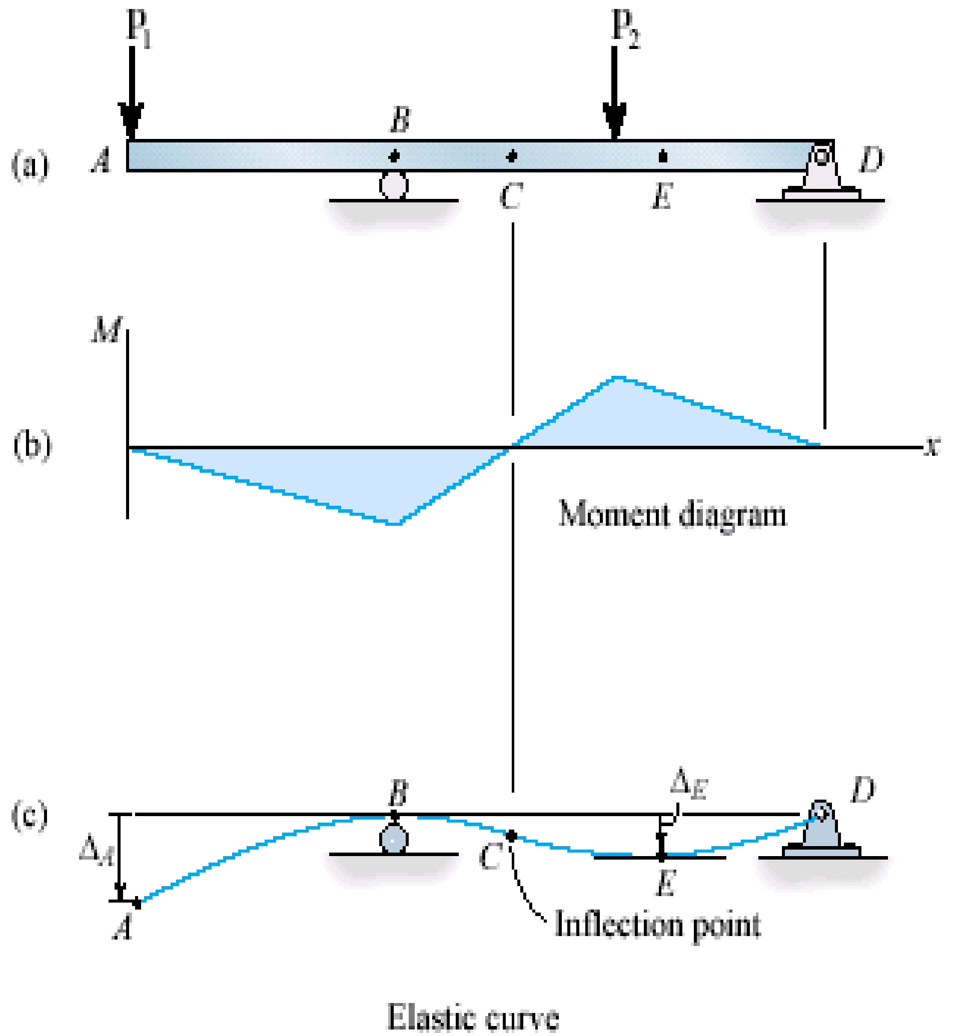
Excessive beam deflection can be seen as a mode of failure.

Extensive glass breakage in tall buildings can be attributed to excessive deflections

Large deflections in buildings are unsightly (and unnerving) and can cause cracks in ceilings and walls.

Deflections are limited to prevent undesirable vibrations

Beam Deflection



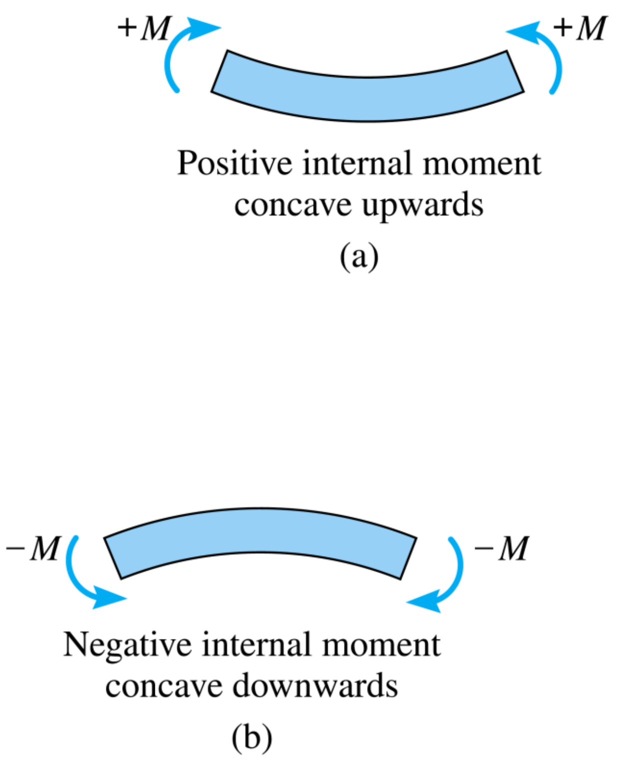
Beam Deflection

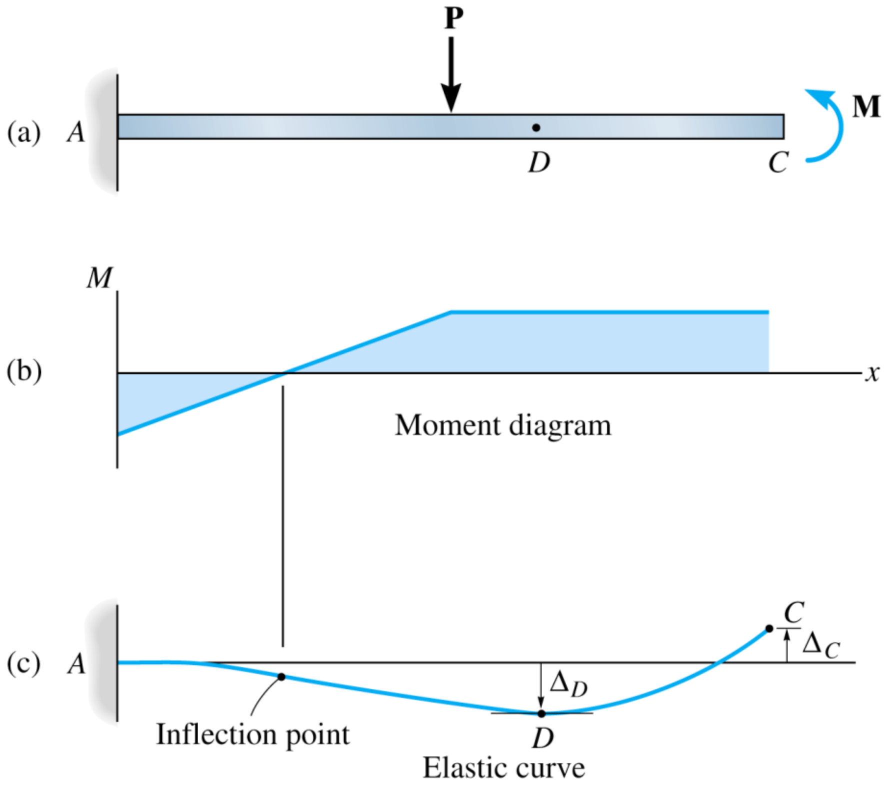
Consider a cantilever beam with a concentrated load acting upward at the free end.

Under the action of this load the axis of the beam deforms into a curve

The deflection Δ is the displacement in the y direction on any point on the axis of the beam

Beam Deflection





Elastic-Beam Theory

$$\varepsilon = \frac{\left(ds' - ds\right)}{ds} = \frac{\left(\rho - y\right)d\theta - \rho d\theta}{\rho d\theta} \quad \text{or}$$

$$\frac{1}{\rho} = -\frac{\varepsilon}{y}$$

Elastic-Beam Theory

Below the NA the strain is positive and above the NA the strain is negative for positive bending moments.

Applying Hooke's law and the Flexure formula, we obtain:

The Moment curvature equation

$$\frac{1}{\rho} = \frac{M}{EI}$$

The Double Integration Method

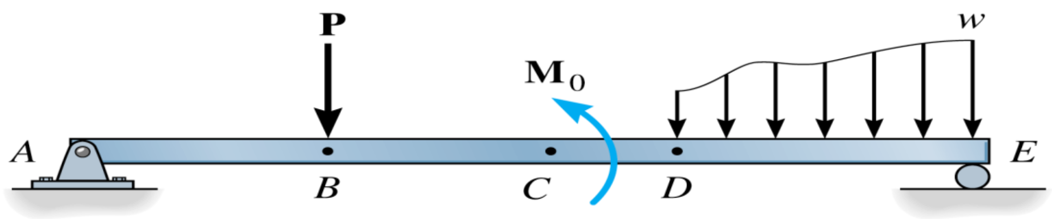
Once M is expressed as a function of position x, then successive integrations of the previous equations will yield the beams slope and the equation of the elastic curve, respectively.

Wherever there is a discontinuity in the loading on a beam or where there is a support, there will be a discontinuity.

Consider a beam with several applied loads.

- The beam has four intervals, AB, BC, CD, DE
- Four separate functions for Shear and Moment

The Double Integration Method



The Double Integration Method

Relate Moments to Deflections

$$\boxed{\frac{d^2v}{dx^2} = \frac{M}{EI}}$$

$$\theta(x) = \frac{dv}{dx} = \int \frac{M(x)}{EI(x)}\,dx$$

$$v(x) = \iint \frac{M(x)}{EI(x)}\,dx^2$$

Integration Constants

Use Boundary Conditions to Evaluate Integration Constants

Moment-Area Theorems

The moment-area theorems procedure can be summarized as:

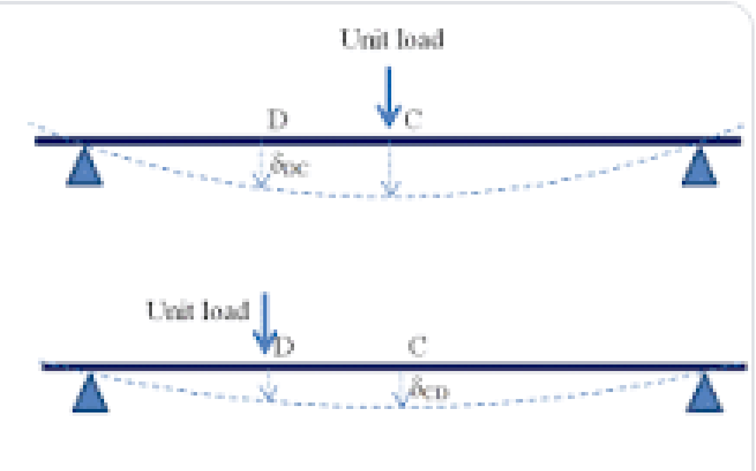
If A and B are two points on the deflection curve of a beam, EI is constant and B is a point of zero slope, then the Mohr's theorems state that:

(1) Slope at A = 1/EI x area of B.M. diagram between A and B

(2) Deflection at A relative to B = 1/EI x first moment of area of B.M diagram between A and B about A.

Maxwell's reciprocal theorem

Let the **deflection** at a point D be δ_{DC}. **Maxwell's reciprocal** theorem says that the **deflection** at D due to a unit load at C is the same as the **deflection** at C if a unit load were applied at D. In our notation, $\delta_{CD} = \delta_{DC}$. ... δ_{XY} is the **deflection** at point X due to unit load at point Y




Thank you