LECTURE NOTES

ON

MODERN POWER SYSTEM ANALYSIS

ELECTRICAL POWER SYSTEMS
(BPSB01)

Regulation-IARE-R18

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UNIT-I

PLANNING AND OPERATIONAL STUDIES OF POWER SYSTEMS

Development of Modern Power System - A Brief Historical Preview

The development of the modern day electrical energy system took a few centuries. Prior to 1800, scientists like William Gilbert, C. A. de Coulomb, Luigi Galvani, Benjamin Franklin, Alessandro Volta etc. worked on electric and magnetic field principles. However, none of them had any application in mind. They also probably did not realize that their work will lead to such an exciting engineering innovation. They were just motivated by the intellectual curiosity.

Between 1800 and 1810 commercial gas companies were formed - first in Europe and then in North America. Around the same time with the research efforts of scientists like Sir Humphrey Davy, Andre Ampere, George Ohm and Karl Gauss the exciting possibilities of the use of electrical energy started to dawn upon the scientific community.

In England, Michael Faraday worked on his induction principle between 1821 and 1831. The modern world owes a lot to this genius. Faraday subsequently used his induction principle to build a machine to generate voltage. Around the same time American engineer Joseph Henry also worked independently on the induction principle and applied his work on electromagnets and telegraphs.

For about three decades between 1840 and 1870 engineers like Charles Wheatstone, Alfred Varley, Siemens brothers Werner and Carl etc. built primitive generators using the induction principle. It was also observed around the same time that when current carrying carbon electrodes were drawn apart, brilliant electric arcs were formed. The commercialization of arc lighting took place in the decade of 1870s. The arc lamps were used in lighthouses and streets and rarely indoor due to high intensity of these lights. Gas was still used for domestic lighting. It was also used for street lighting in many cities.

From early 1800 it was noted that a current carrying conductor could be heated to the point of incandescent. Therefore the idea of using this principle was very tempting and attracted attention. However the incandescent materials burnt very quickly to be of any use. To prevent them from burning they were fitted inside either vacuum globes or globes filled with inert gas. In October 1879 Thomas Alva Edison lighted a glass bulb with a carbonized cotton thread filament in a vacuum enclosed space. This was the first electric bulb that glowed for 44 hours before burning out. Edison himself improved the design of the lamp later and also proposed a new generator design.

The Pearl Street power station in New York City was established in 1882 to sell electric energy for incandescent lighting. The system was direct current three-wire, 220/110 V and supplied Edison lamps for a total power requirement of 30 kW.

The only objective of the early power companies was illumination. However we can easily visualize that this would have resulted in the under utilization of resources. The lighting load peaks in the evening and by midnight it reduces drastically. It was then obvious to the power companies that an elaborate and expensive set up would lay idle for a major amount of time. This provided incentive enough to improve upon the design of electric motors to make them commercially viable. The motors became popular very quickly and were used in many applications. With this the electric energy era really and truly started.

However with the increase in load large voltage and unacceptable drops were experienced, especially at points that were located far away from the generating stations due to poor voltage regulation capabilities of the existing dc networks. One approach was to transmit power at higher voltages while consuming it at lower voltages. This led to the development of the alternating current.

In 1890s the newly formed Westinghouse Company experimented with the new form of electricity, the alternating current. This was called alternating current since the current changed direction in synchronism with the generator rotation. Westinghouse Company was lucky to have Serbian engineer Nicola Tesla with them. He not only invented polyphase induction motor but also conceived the entire polyphase electrical power system. He however had to face severe objection from Edison and his General Electric Company who were the proponents of dc. The ensuing battle between ac and dc was won by ac due to the following factors:

- Transformers could boost ac voltage for transmission and could step it down for distribution.
- The construction of ac generators was simpler.
- The construction of ac motors was simpler. Moreover they were more robust and cheaper than the dc motors even though not very sophisticated.

With the advent of ac technology the electric power could reach more and more people. Also size of the generators started increasing and transmission level voltages started increasing. The modern day system contains hundreds of generators and thousands of buses and is a large interconnected network.

Introduction of Modern Power System

Modern electric power systems have three separate components - generation, transmission and distribution. Electric power is generated at the power generating stations by synchronous alternators that are usually driven either by steam or hydro turbines. Most of the power generation takes place at generating stations that may contain more than one such alternator-turbine combination. Depending upon the type of fuel used, the generating stations are categorized as thermal, hydro, nuclear etc. Many of these generating stations are remotely located. Hence the electric power generated at any such station has to be transmitted over a long distance to load centers that are usually cities or towns. This is called the **power transmission**. In fact power transmission towers and transmission lines are very common sights in rural areas.

Modern day power systems are complicated networks with hundreds of generating stations and load centers being interconnected through power transmission lines. Electric power is generated at a frequency of either 50 Hz or 60 Hz.

In an interconnected ac power system, the rated generation frequency of all units must be the same. In India the frequency is 50 Hz.

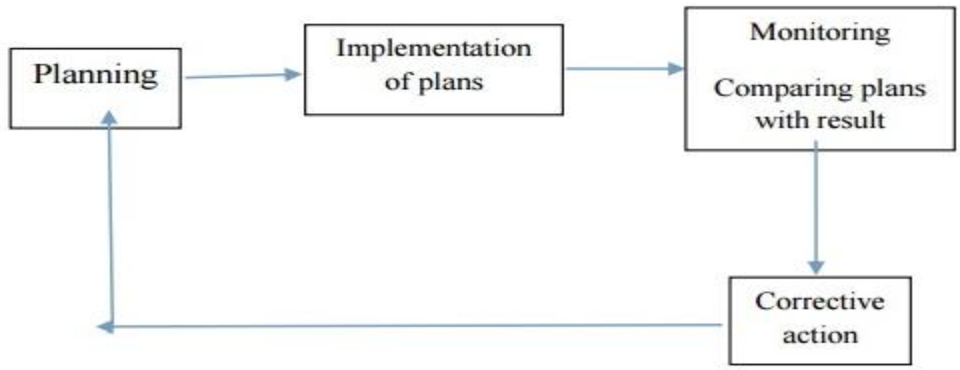
Need for system planning and operational studies:

Planning and operation of power system - Operational planning covers the whole period ranging from the incremental stage of system development

The system operation engineers at various points like area, space, regional & national load dispatch of power

- Power balance equation $P_D = \sum_{i=1}^{N} PGi$ This equation is satisfied it gives good economy and security

Power system planning and operational analysis covers the maintenance of generation, transmission and distribution facilities



Steps:

Planning of power system

Implementation of the plans

Monitoring system

Compare plans with the results

If no undesirable deviation occurs, then directly go to planning of system

If undesirable deviation occurs then take corrective action and then go to planning Of the system

Planning and operation of power system

Planning and operation of power system the following analysis are very important

(a). Load flow analysis

(b). Short circuit analysis

(c). Transient analysis

Load flow analysis

Electrical power system operate - Steady state mode

Basic calculation required to determine the characteristics of this state is called as Load flow

Power flow studies - To determine the voltage current active and reactive power flows in given power system

A number of operating condition can be analyzed including contingencies. That operating conditions are

(a). Loss of generator

(b).Loss of a transmission line

(c).Loss of transformer (or) Load

(d). Equipment over load (or) unacceptable voltage levels

The result of the power flow analysis is stating point for the stability analysis and power factor improvement. Load flow study is done during the planning of a new system or the extension of an existing one

Short circuit studies

To determine the magnitude of the current flowing through out the power system at various time intervals after fault. The objective of short circuit analysis - To determine the current and voltages at different location of the system corresponding to different types of faults

(a). Three phase to ground fault

(b). Line to ground fault

(c). Line to line fault

(d). Double line to ground fault

(e). Open conductor fault

Transient stability analysis

The ability of the power system consisting of two (or) more generators to continue to operate after change occur on the system is a measure of the stability. In power system the stability depends on the power flow pattern generator characteristics system loading level and the line parameters.

Basic Components Of A Power System

Major components of a power system are- synchronous generators, synchronising equipment, circuit breakers, isolators, earthing switches, bus-bars, transformers, transmission lines, current transformers, potential transformers, relay and protection equipment, lightning arresters, station transformer, motors for driving auxiliaries in power station. Some of the components will be discussed here as shown in Fig. 1.7 .

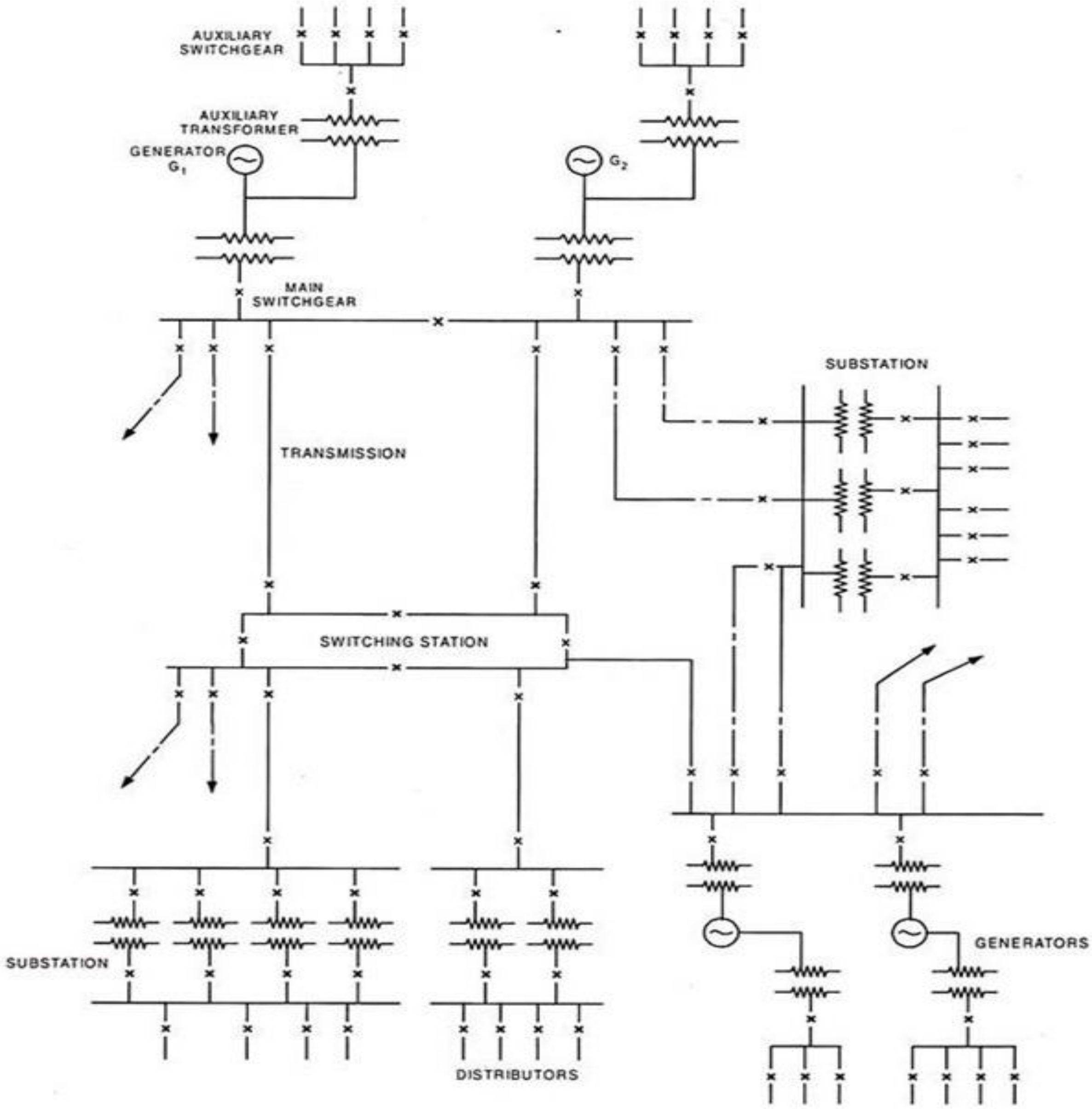


Fig. 1.7. *Simplified Single Line Diagram For a Typical Power System*

Component # 1. Synchronous Generators:

The synchronous generators used in generating stations are revolving field type owing to its inherent advantages.

The synchronous generators, based on the type of prime movers to which they are mechanically coupled, may be classified as:

(i) Hydro-generators

(ii) Turbo-generators, and

(iii) Diesel engine driven generators.

Power Transformers:

Power transformers are used for stepping-up the voltage for transmission at generating stations and for stepping-down voltage for further distribution at main step-down transformer substations. Usually naturally cooled, oil immersed, known as ON type, two winding, three-phase transformers, are used up to the rating of 10 MVA.

The transformers of rating higher than 10 MVA are usually air blast cooled. For very high rating, the forced oil, water cooling and air blast cooling may be used. For regulating the voltage the transformers used are provided with on load tap changer.

Component # 2. Switchgear:

Everyone is familiar with low voltage switches and rewirable fuses. A switch is used for opening and closing of an electric circuit while a fuse is used for over-current protection. Every electric circuit needs a switching device and protective device. Switching and protective devices have been developed in different forms. Switchgear is a general term covering a wide range of equipment concerned with switching and protection.

Circuit Breakers:

Circuit breakers are mechanical devices designed to close or open contact members, thus closing or opening of an electrical circuit under normal or abnormal conditions.

Automatic circuit breakers, which are usually employed for the protection of electrical circuits, are equipped with a trip coil connected to a relay or other means, designed to open the breaker automatically under abnormal conditions, such as over-current.

solators:

Since isolators (or isolating switches) are employed only for isolating circuit when the current has already been interrupted, they are simple pieces of equipment. They ensure that the current is not switched into the circuit until everything is in order.

Isolators or disconnect switches operate under no load condition. They are not equipped with arc-quenching devices. They do not have any specified current breaking capacity or current making capacity. The isolators in some cases are used for breaking charging current of transmission line.

Earthing Switch:

Earthing switch is connected between the line conductor and earth. Normally it is open and it is closed to discharge the voltage trapped on the isolated or disconnected line. When the line is disconnected from the supply end, there is some voltage on the line to which the capacitance between the line and earth is charged.

This voltage is significant in hv systems. Before commencement of maintenance work it is necessary that these voltages are discharged to earth by closing the earthing switch. Normally, the earthing switches are mounted on the frame of the isolator.

Component # 3. Bus-Bars:

Bus-bar (or bus in short) term is used for a main bar or conductor carrying an electric current to which many connections may be made.

Bus-bars are merely convenient means of connecting switches and other equipment into various arrangements. The usual arrangement of connections in most of the substations permits working on almost any piece of equipment without interruption to incoming or outgoing feeders.

In some arrangements two buses are provided to which the incoming or outgoing feeders and the principal equipment may be connected. One bus is usually called the “main” bus and the other “auxiliary” or “transfer” bus. The main bus may have a more elaborate system of measuring instruments, relays etc. associated with it. The switches used for connecting feeders or equipment to one bus or the other are called “selector” or “transfer” switches.

Bus-bars may be of copper, aluminium or steel. Copper has a comparatively low resistivity and also the advantage of relatively high mechanical strength; this makes it economical to use copper bus-bars in installations of very large capacity where the currents are particularly heavy.

During 1960’s the need for substituting the copper with aluminium became very urgent, particularly in countries like India where copper is imported. Now aluminium is being increasingly used for various switchgear installations due to its numerous advantages over copper such as higher conductivity on weight basis, lower cost for equal current carrying capacity, excellent corrosion resistance and ease of formability.

Component # 4. Lightning Arresters:

The lightning arrester is a surge diverter and is used for the protection of power system against the high voltage surges. It is connected between the line and earth and so diverts the incoming high voltage wave to the earth.

Lightning arresters act as safety valves designed to discharge electric surges resulting from lightning strokes, switching or other disturbances, which would otherwise flash-over insulators or puncture insulation, resulting in a line outage end possible failure of equipment.

They are designed to absorb enough transient energy to prevent dangerous reflections and to cut off the flow of power-frequency follow (or dynamic) current at the first current zero after the discharge of the transient. They include one or more sets of gaps to establish the breakdown voltage, aid in interrupting the power follow current, and prevent any flow of current under normal conditions (except that gap shunting resistors, when used to assure equal distribution of voltage across the gaps, permits a very small leakage current).

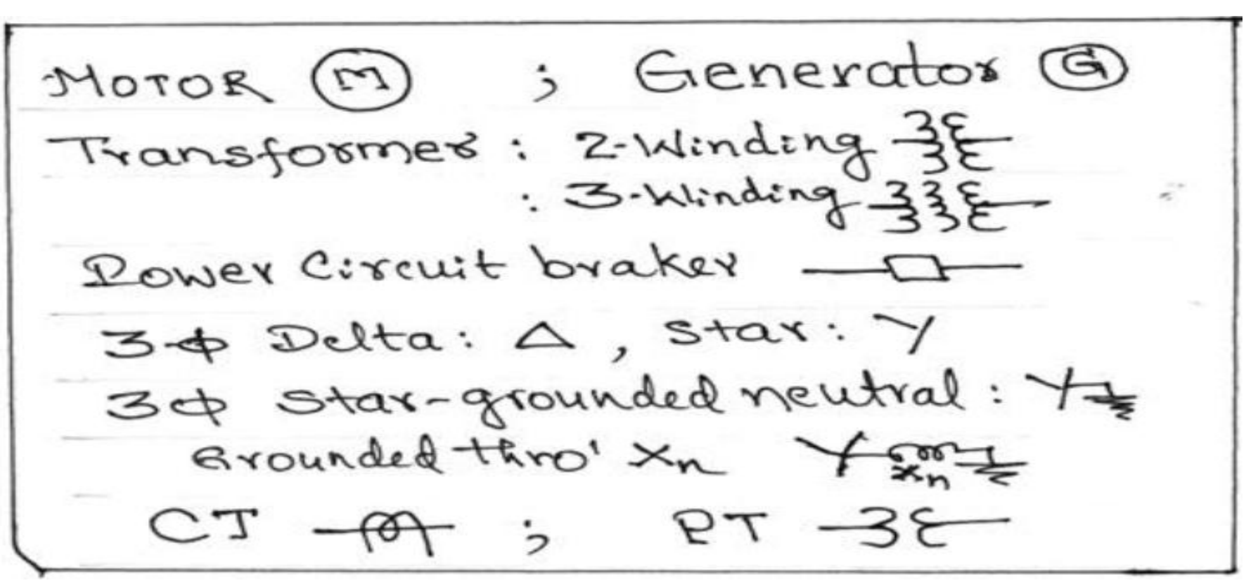
Either resistance (valve) elements to limit the power follow current to values the gaps can interrupt, or an additional arc extinguishing chamber to interrupt the power follow current are connected in series with gaps. Arresters have a short time lag of breakdown compared with the insulation of apparatus, the breakdown voltage being nearly independent of the steepness of the wave front.

Single line diagram:

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

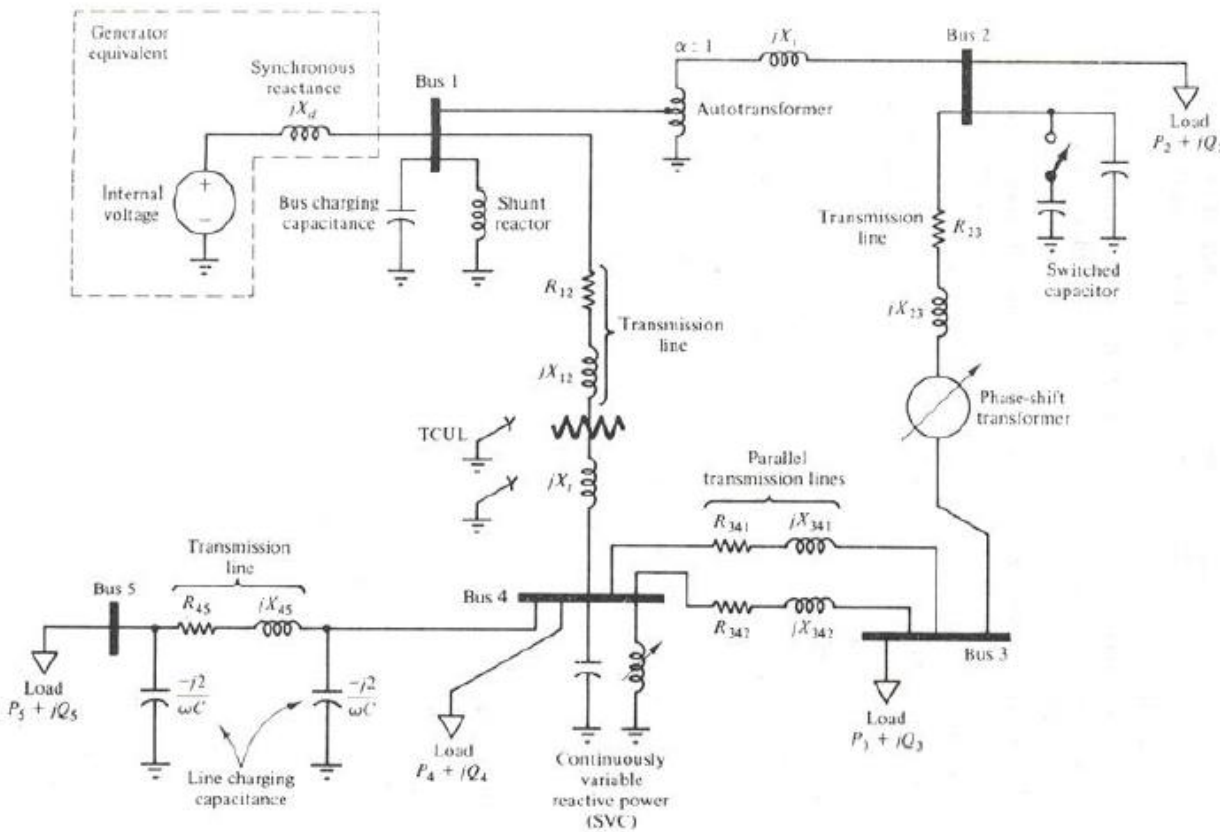
Symbols used for SLD

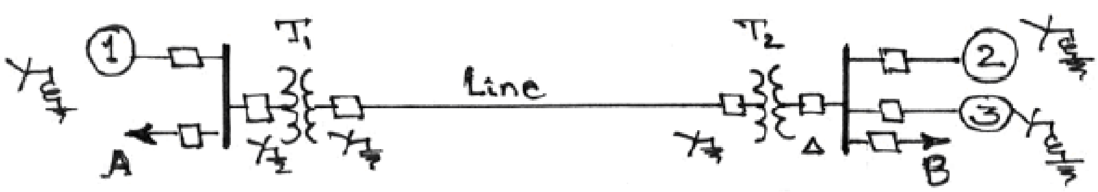
Various symbols are used to represent the different parameters and machines as single phase equivalents on the SLD,. Some of the important symbols used are as listed in the table of Figure 1.



Example system

Consider for illustration purpose, a sample example power system and data as under: Generator 1: 30 MVA, 10.5 KV, X”= 1.6 ohms, Generator 2: 15 MVA, 6.6 KV, X”= 1.2 ohms, Generator 3: 25 MVA, 6.6 KV, X”= 0.56 ohms, Transformer 1 (3-phase): 15 MVA, 33/11 KV, X=15.2 ohms/phase on HT side, Transformer 2 (3-phase): 15 MVA, 33/6.2 KV, X=16.0 ohms/phase on HT side, Transmission Line: 20.5 ohms per phase, Load A: 15 MW, 11 KV, 0.9 PF (lag); and Load B: 40 MW, 6.6 KV, 0.85 PF (lag). The corresponding SLD incorporating the standard symbols can be shown as in figure 2.





It is observed here, that the generators are specified in 3-phase MVA, L-L voltage and per phase Y-equivalent impedance, transformers are specified in 3-phase MVA, L-L voltage transformation ratio and per phase Y-equivalent impedance on any one side and the loads are specified in 3-phase MW, L-

L voltage and power factor.

Impedance diagram:

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

Assumptions:

1. The single phase transformer equivalents are shown as ideals with impedances on appropriate side (LV/HV),

2. The magnetizing reactances of transformers are negligible,

3. The generators are represented as constant voltage sources with series resistance or reactance,

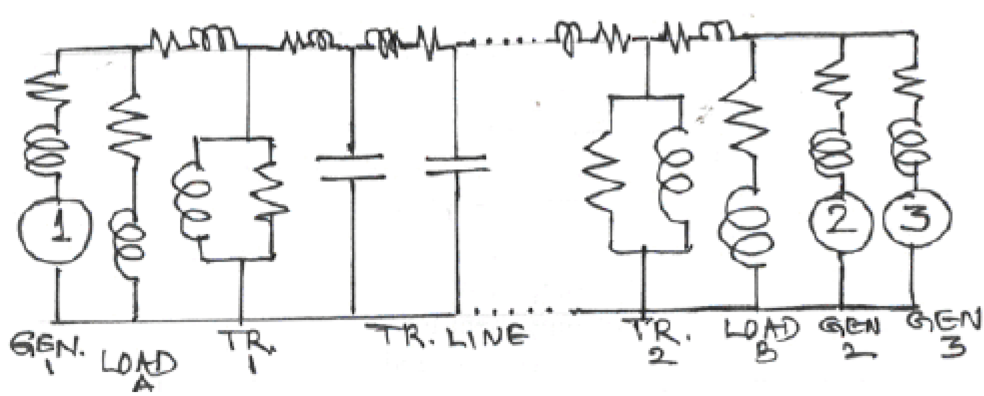
4. The transmission lines are approximated by their equivalent -Models,

5. The loads are assumed to be passive and are represented by a series branch of resistance or reactance and

6. Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram.

Example system

As per the list of assumptions as above and with reference to the system of figure 2, the impedance diagram can be obtained as shown in figure 3.



Reactance Diagram:

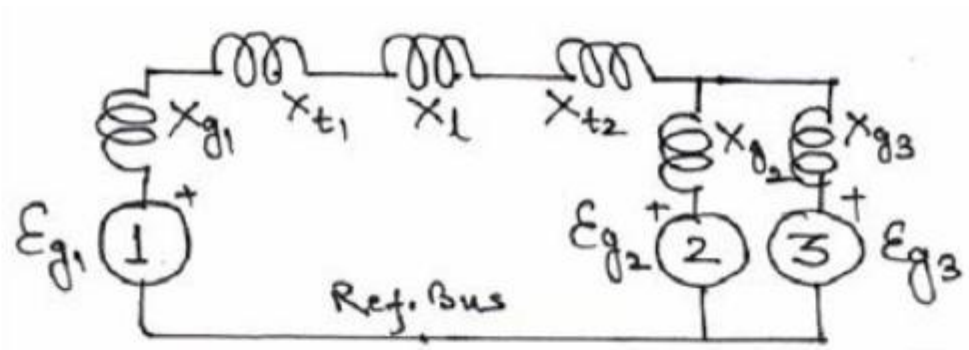
With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Loads are Omitted
- Transmission line capacitances are ineffective &
- Magnetizing currents of transformers are neglected.

Example system

as per the assumptions given above and with reference to the system of figure 2 and figure 3, the reactance diagram can be obtained as shown in figure



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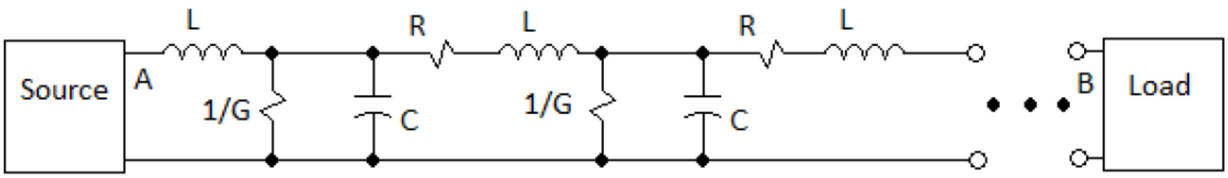
Note: These impedance & reactance diagrams are also referred as the Positive Sequence diagrams/ networks.

Transmission lines:

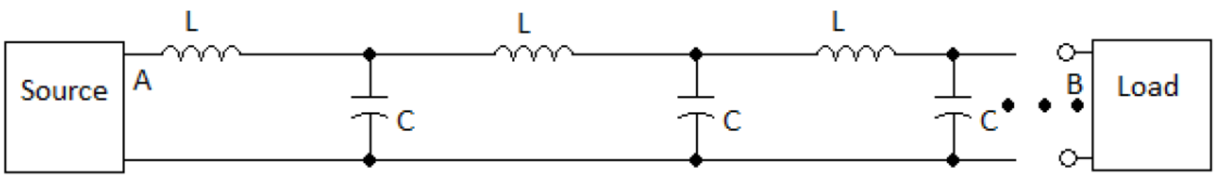
The Circuit Model:

A transmission line is a set of conductors used for transmitting electrical signals. In general, every connection in an electric circuit is a transmission line. Implicit in discussions of transmission line theory is the assumption that the lines are uniform. A uniform transmission line is one with uniform geometry and materials. This is, the conductor shape, size, and spacing or constant, and the electrical characteristics of the conductors and the material between them are uniform. Some examples of uniform transmission lines are coaxial cables, twisted-wire pairs, and parallel-wire pairs. For printed circuit boards, the common transmission lines are strip-line and microstrip.

In a simple transmission line circuit, a source provides a signal that is intended to reach a load. In basic circuit theory, you assume that the wires making up the transmission line are ideal and the voltage at all points on the wires is exactly the same. In reality, this situation is never quite true. Any wire has series resistance and inductance. Also, a capacitance exits between any pair of wires. You can model the transmission line using a basic circuit that consists of an infinite series of infinitesimal R, L, and C components. Because the elements are infinitesimal, the model parameters are usually specified in units per meter. Sometimes to simplify the discussion, we will ignore the resistances. A transmission line that is assumed to have no resistance is a *lossless* transmission line.



Lumped Circuit Model



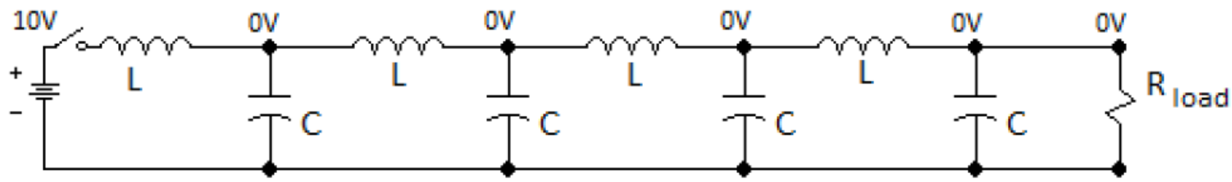
Lossless Lumped Circuit Model

Several important points

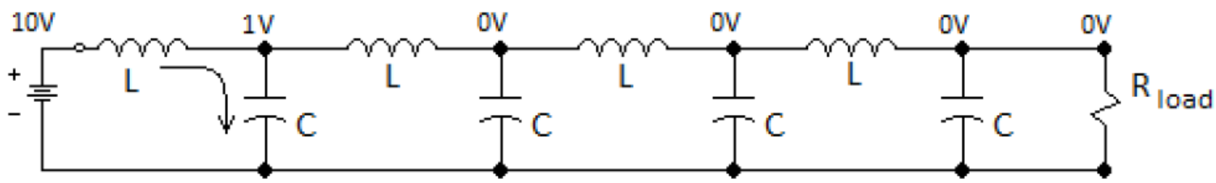
- With the LC model, points A and B may be at different potentials
- a signal transmitted from the source charges and discharges the line's inductance and capacitance. Therefore, the signal does not arrive instantly at point B but is delayed.
- The impedance at points A and B and each node in between depends not just on the source and load resistance, but also on the LC values of the transmission line
- At low frequencies, the LC pairs introduce negligible delay and impedance, reducing the model to a simple pair 136 of ideal wires.
- At higher frequencies, the LC effects dominate the behavior, and you cannot ignore them.

Characteristic Impedance:

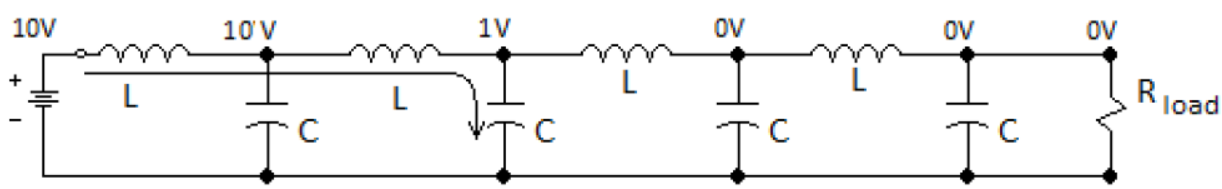
The circuit in the figures below demonstrates the behavior of a transmission line. In this circuit, a 10 volt battery is connected to a resistor through a transmission line that is modeled with a series of four L-C sections. In reality, a real transmission line is an infinite series of infinitesimal inductors and capacitors. This simplified model serves as a good learning tool. Between the battery and the transmission line is a mechanical switch, which is initially open. In the initial state there is no voltage on the line or the load, and no current flows. Immediately after the switch is closed, current flows from the battery into the transmission line. At this point, the current does not reach the load. instead the current is diverted by the first capacitor. The capacitor continues to sink charge until it reaches the 10 volts of the battery. During this process some energy is also transferred to the magnetic field of the inductor. As the voltage on the capacitor starts to climb, charge starts to trickle to the second stage.



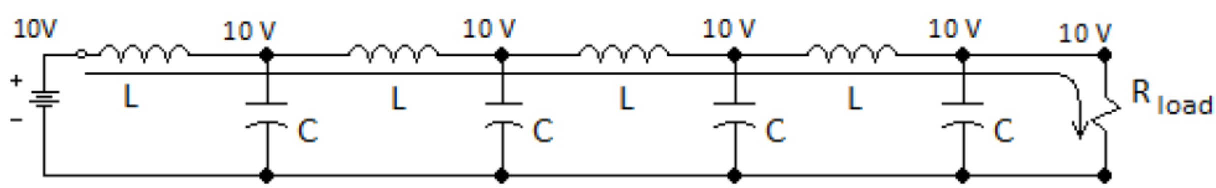
A) The switch is open. No current flows and the voltage of the transmission line is zero everywhere and I = 0.



B) The switch is closed. Current flows and starts charging the second stage. I = Vbattery/Z0, where Z0 = L/C.



C) The battery has charged up the first stage and is now charging the second stage. I = Vbattery/Z0.



D) The battery has charged the entire transmission line, and now current flows through the load resistor. I = Vbattery/Rload.

During the charging of the capacitors and inductors, no current reaches the load. Therefore the impedance that the battery "sees" is solely dependent on the value of the inductors and capacitors. This impedance is referred to as the *characteristic impedance* of the transmission line, and is easily calculated using the equation, $Z_{o} = \sqrt{\frac{L}{C}}$

For a lossy transmission line. $Z_{o} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

Synchronous Machine Model

The schematic diagram of a synchronous generator is shown in Fig. 1.15. This contains three stator windings that are spatially distributed. It is assumed that the windings are wye-connected. The winding currents are denoted by i_a , i_b and i_c. The rotor contains the field winding the current through which is denoted by i_f . The field winding is aligned with the so-called direct (d) axis. We also define a quadrature (q) axis that leads the d -axis by 90°. The angle between the d-axis and the a-phase of the stator winding is denoted by θ_d.

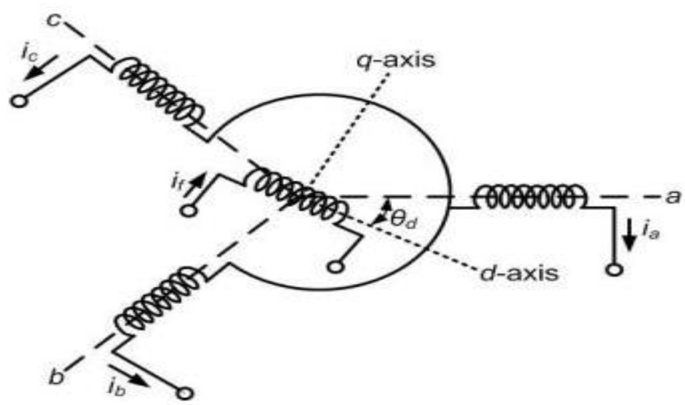


Fig. 1.15 Schematic diagram of a synchronous generator

Let the self-inductance of the stator windings be denoted by L_{aa}, L_{bb}, L_{cc} such that

$$L_s = L_{aa} = L_{bb} = L_{cc}$$

and the mutual inductance between the windings be denoted as

$$-M_s = L_{ab} = L_{bc} = L_{ca}$$

The mutual inductances between the field coil and the stator windings vary as a function of θ_d and are given by

$$L_{af} = M_f \cos\theta_d$$
$$L_{bf} = M_f \cos(\theta_d - 120°)$$
$$L_{cf} = M_f \cos(\theta_d + 120°)$$

The self-inductance of the field coil is denoted by L_{ff}.

The flux linkage equations are then given by

$$\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ca}i_c + L_{af}i_f = L_s i_a - M_s(i_b + i_c) + L_{af}i_f$$

$$\lambda_b = L_s i_b - M_s(i_a + i_c) + L_{bf}i_f$$

$$\lambda_c = L_s i_c - M_s(i_a + i_b) + L_{af}i_f$$

$$\lambda_f = L_{af}i_a + L_{bf}i_b + L_{cf}i_c + L_{ff}i_f$$

For balanced operation we have

$$i_a + i_b + i_c = 0$$

Hence the flux linkage equations for the stator windings (1.85) to (1.87) can be modified as

$$\lambda_a = (L_s + M_s) i_a + L_{af} i_f$$

$$\lambda_b = (L_s + M_s) i_b + L_{bf} i_f$$

$$\lambda_c = (L_s + M_s) i_c + L_{cf} i_f$$

For steady state operation we can assume

$$i_f = I_f = \text{constant}$$

Also assuming that the rotor rotates at synchronous speed ω_s we obtain the following two equations

$$\omega_s = \frac{d\theta_d}{dt}$$

$$\theta_d = \omega_s t + \theta_{d0}$$

where θ_{d0} is the initial position of the field winding with respect to the phase-a of the stator winding at time t = 0. The mutual inductance of the field winding with all the three stator windings will vary as a function of θ_d, i.e.,

$$L_{af} = M_f \cos(\omega_s t + \theta_{d0})$$

$$L_{bf} = M_f \cos(\omega_s t + \theta_{d0} - 120°)$$

$$L_{cf} = M_f \cos(\omega_s t + \theta_{d0} + 120°)$$

Substituting (1.92), (1.94), (1.95), (1.96) and (1.97) in (1.89) to (1.91) we get

$$\lambda_a = (L_s + M_s)i_a + M_f I_f \cos(\omega_s t + \theta_{d0})$$

$$\lambda_b = (L_s + M_s)i_b + M_f I_f \cos(\omega_s t + \theta_{d0} - 120°)$$

$$\lambda_c = (L_s + M_s)i_c + M_f I_f \cos(\omega_s t + \theta_{d0} + 120°)$$

Since we assume balanced operation, we need to treat only one phase. Let the armature resistance of the generator be R . The generator terminal voltage is given by

$$v_a = -Ri_a - \frac{d\lambda_a}{dt}$$

where the negative sign is used for generating mode of operation in which the current leaves the terminal. Substituting (1.98) in (1.101) we get

$$v_a = -Ri_a - (L_s + M_s)\frac{di_a}{dt} + M_f I_f \omega_s \sin(\omega_s t + \theta_{d0})$$

The last term of (1.102) is the internal emf e_a that is given by

$$e_a = \sqrt{2}\,|E_i|\sin(\omega_s t + \theta_{d0})$$

where the rms magnitude E_i is proportional to the field current

$$|E_i| = \frac{\omega_s M_f I_f}{\sqrt{2}}$$

Since θ_{d0} is the position of the d -axis at time t = 0, we define the position of the q -axis at that instant as

$$\delta = \theta_{d0} - 90°$$

Therefore (1.94) can be rewritten as

$$\theta_d = \omega_s t + \delta + 90°$$

Substituting (1.105) in (1.103) we get

$$e_a = \sqrt{2}\,|\,E_i\,|\cos(\omega_s t + \delta)$$

Hence (1.102) can be written as

$$v_a = -Ri_a - (L_s + M_s)\frac{di_a}{dt} + e_a$$

The equivalent circuit is shown in Fig. 1.16. Let the current i_a lag the internal emf e_a by θ_a . The stator currents are then

$$i_a = \sqrt{2}\,|\,I_a\,|\cos(\omega_s t + \delta - \theta_a)$$

$$i_b = \sqrt{2}\,|\,I_a\,|\cos(\omega_s t + \delta - \theta_a - 120°)$$

$$i_c = \sqrt{2}\,|\,I_a\,|\cos(\omega_s t + \delta - \theta_a + 120°)$$

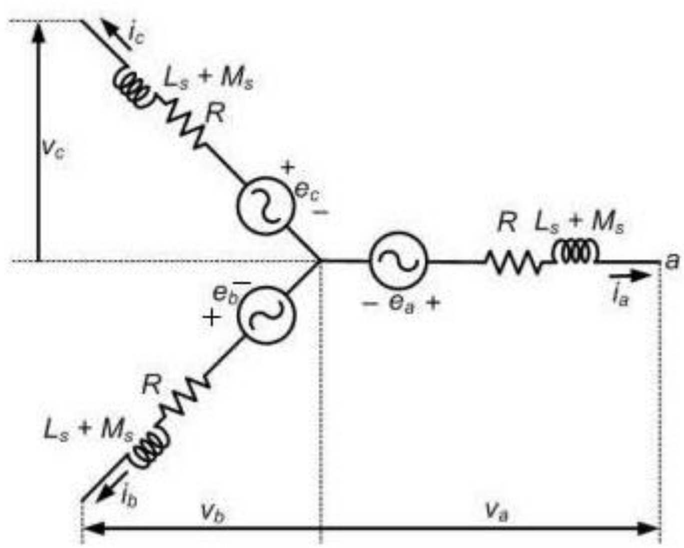


Fig. 1.16 Three-phase equivalent circuit of a synchronous generator.

The single-phase equivalent circuit is shown in Fig. 1.17. The phase angle θ_a between e_a and i_a is rather difficult to measure under load as e_a is the no load voltage. To avoid this, we define the phase angle between v_a and i_a to be θ . We assume that e_a leads v_a by δ . Therefore we can write

$$\theta = \theta_a - \delta$$

Then the voltages and currents shown in Fig. 1.17 are given as

$$v_a = \sqrt{2}|V_a|\cos \omega_s t$$

$$e_a = \sqrt{2}|E_i|\cos(\omega_s t + \delta)$$

$$i_a = \sqrt{2}|I_a|\cos(\omega_s t - \theta)$$

Equations (1.113) to (1.115) imply that

$$V_a = |V_a|\angle 0°, \quad E_a = |E_i|\angle \delta \text{ and } I_a = |I_a|\angle - \theta$$

The synchronous impedance is then defined as

$$Z_d = R + jX_d = R + j\omega_s(L_s + M_s)$$

he terminal voltage equation is then

$$V_a = E_a + (R + jX_d)I_a$$

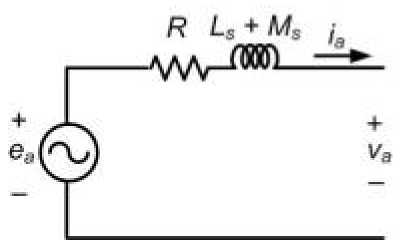


Fig. 1.17 Single-phase equivalent circuit of a synchronous generator.

Transformer Model

The equivalent circuit of a single-phase transformer is shown in Fig. 1.18. In this the primary voltage and currents are denoted by V_1 and V_2 respectively. The current entering the primary terminals is I_1. The core loss component is represented by R_c while the magnetizing reactance is denoted by X_m . The leakage inductance of the transformer is denoted by X_{eq} and R_{eq} is transformer winding resistance. It is to be noted that all the quantities are referred to the primary side. The turns ratio of the transformer is given by $N_1 : N_2$.

The impedance of the shunt branch is much larger compared to that of the series branch. Therefore we neglect R_c and X_m. Again of the series parameters, R_{eq} is much smaller than X_{eq}. We can therefore neglect the series impedance. Therefore the transformer can be represented by the leakage reactance X_{eq}. The single-phase transformer equivalent circuit, when referred to the primary side, is as shown is Fig. 1.19 (a). The equivalent circuit, when referred to the secondary side, is shown in Fig. 1.19 (b) where $a = N_1 / N_2$.

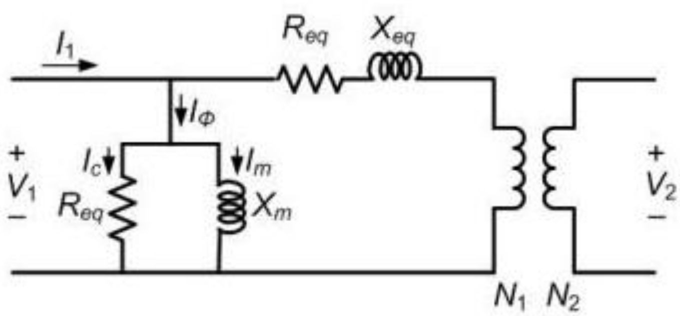


Fig. 1.18 Equivalent circuit of a single-phase transformer

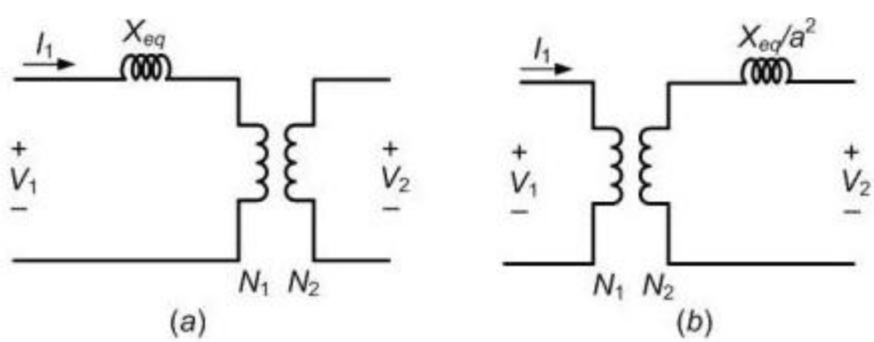


Fig. 1.19 Simplified equivalent circuit of a single-phase transformer: (a) when referred to the primary side and (b) when referred to the secondary side.

Balanced Operation Of a Three-Phase Citcuit

In the language of Power Systems, a three-phase circuit is said to be balanced if the following conditions are true.

- If all the sources and loads are y-connected.

- There is no mutual inductance between the phases.
- All neutrals are at the same potential.
- As a consequence of the points (2) and (3) above, all phases are decoupled.
- All network variables are balanced sets in the same sequence as the sources.

Consider the three-phase circuit shown in Fig. 1.20 that contains three balanced sources E_a , $E_b$and E_c along with three balanced source impedances, each of value Z_s. The sources supply two balanced loads - one wye-connected with impedance of Z_y and the other Δ-connected with impedance of Z_Δ . Since this is a balanced network, the sum of the currents at the neutrals N (or n) is zero. Therefore the neutral are at the same potential. Transforming the Δ-connected load to an equivalent y, we get the per phase equivalent circuit as shown in Fig. 1.21. In this fashion an entire power system can be converted into its per phase equivalent. The line diagram showing a per phase equivalent circuit is called a single-line diagram.

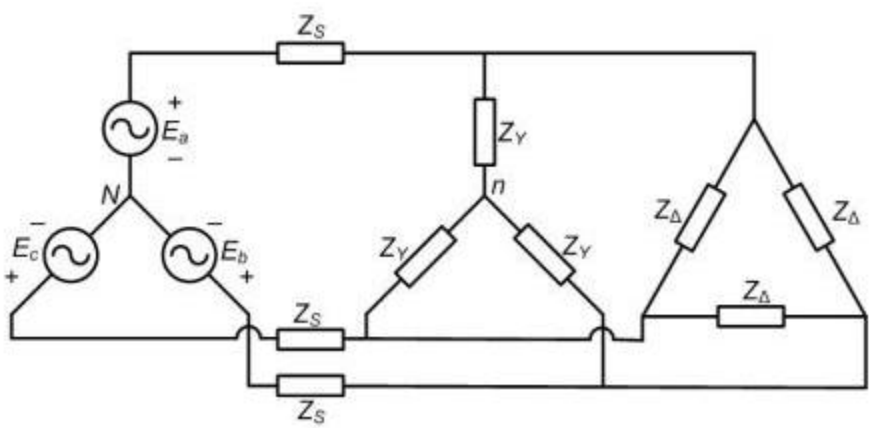


Fig. 1.20 Three balanced sources supplying two balanced load through balanced source impedances.

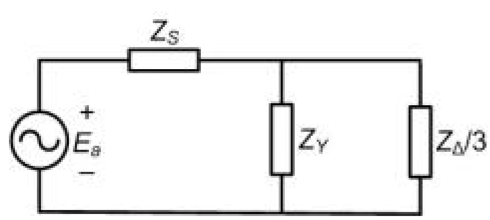


Fig. 1.21 Per phase equivalent circuit of the network of Fig. 1.20.

Per Unit Quantities:

During the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

Definition: Per Unit value of a given q*uantity is the ratio of the* ***actual value*** *in any given unit to the* ***base value*** *in the same unit.* The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in per unit itself.

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns-ratio of the connecting transformer.

If Ib is the base current in kilo amperes and Vb, the base voltage in kilovolts, then the base MVA is, Sb = (VbIb). Then the base values of current & impedance are given by

Base current (kA), Ib = MVAb/KVb

= Sb/Vb (1.1)

Base impedance, Zb = (Vb/Ib)

= (KVb2 / MVAb) (1.2)

Hence the per unit impedance is given by

Zpu = Zohms/Zb

= Zohms (MVAb/KVb2) (1.3)

In 3-phase systems, KVb is the line-to-line value & MVAb is the 3-phase MVA. [1-phase MVA = (1/3) 3-phase MVA].

Changing the base of a given pu value:

It is observed from equation (3) that the pu value of impedance is proportional directly to the base MVA and inversely to the square of the base KV. If Zpunew is the pu impedance required to be calculated on a new set of base values: MVAbnew & KVbnew from the already given per unit impedance Zpuold, specified on the old set of base values, MVAbold & KVbold , then we have

Zpunew = Zpuold (MVAbnew/MVAbold) (KVbold/KVbnew)2 (1.4)

On the other hand, the change of base can also be done by first converting the given pu impedance to its ohmic value and then calculating its pu value on the new set of base values.

Merits and Demerits of pu System

Following are the advantages and disadvantages of adopting the pu system of computations in electric power systems:

Merits:

- The pu value is the same for both 1-phase and & 3-phase systems
- The pu value once expressed on a proper base, will be the same when refereed to either side of the transformer. Thus the presence of transformer is totally eliminated
- The variation of values is in a smaller range 9nearby unity). Hence the errors involved in pu computations are very less.
- Usually the nameplate ratings will be marked in pu on the base of the name plate ratings, etc.

Demerits:

- If proper bases are not chosen, then the resulting pu values may be highly absurd (such as 5.8 pu, -18.9 pu, etc.). This may cause confusion to the user. However, this problem can be avoided by selecting the base MVA near the high-rated equipment and a convenient base KV in any section of the system.

P.U.Impedance / Reactance Diagram:

for a given power system with all its data with regard to the generators, transformers, transmission lines, loads, etc., it is possible to obtain the corresponding impedance or reactance diagram as explained above. If the parametric values are shown in pu on the properly selected base values of the system, then the diagram is refered as the per unit impedance or reactance diagram. In forming a pu diagram, the following are the procedural steps involved:

1. Obtain the one line diagram based on the given data

2. Choose a common base MVA for the system

3. Choose a base KV in any one section (Sections formed by the presence of transformers)

4. Find the base KV of all the sections present

5. Find pu values of all the parameters: R,X, Z, E, etc.

6. Draw the pu impedance/ reactance diagram.

Formation Of YBUS & ZBUS

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS} I_{BUS}$$
$$I_{BUS} = Y_{BUS} E_{BUS} \quad (1.5)$$

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR} I_{BR}$$

$$I_{BR} = Y_{BR} E_{BR} \quad (1.6)$$

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP} I_{LOOP}$$
$$I_{LOOP} = Y_{LOOP} E_{LOOP} \quad (1.7)$$

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

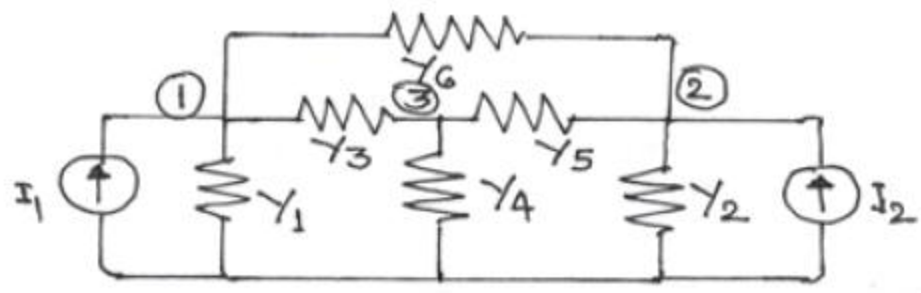
Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3 (V_1-V_3) + Y_6 (V_1 – V_2)$

At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2) (1.8)



These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

I1 = (Y1+Y3 +Y6) -Y6 -Y3 V1

I2 = -Y6 (Y2+Y5 +Y6) -Y5 V2

0 = -Y3 -Y5 (Y3 +Y4+Y5) V3 (1.9)

In other words, the relation of equation (9) can be represented in the form

IBUS = YBUS EBUS (1.10)

Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, YBUS of equation (9), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = yij (j = 1,2,…….n)

Yij = - yij (j = 1,2,…….n) (1.11)

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix

In cases where, the bus impedance matrix is also required, then it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible. **Note:** It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Per Unit Representation

n a power system different power equipment with different voltage and power levels are connected together through various step up or step down transformers. However the presence of various voltage and power levels causes problem in finding out the currents (or voltages) at different points in the network. To alleviate this problem, all the system quantities are converted into a uniform normalized platform. This is called the per unit system . In a per unit system each system variable or quantity is normalized with respect to its own base value. The units of these normalized values are per unit (abbreviated as pu) and not Volt, Ampere or Ohm. The base quantities chosen are:

- **VA base** (P_{base}): This is the three-phase apparent power (Volt-Ampere) base that is common to the entire circuit.
- **Voltage Base** (V_{base}): This is the line-to-line base voltage. This quantity is not uniform for the entire circuit but gets changed by the turns ratio of the transformer.

Based on the above two quantities the current and impedance bases can be defined as

$$I_{base} = \frac{P_{base}}{\sqrt{3} \times V_{base}}$$

$$Z_{base} = \frac{(V_{base})^2}{P_{base}}$$

Assume that an impedance Z is defined as Z_1 per unit in a base impedance of Z_{base_old} . Then we have

$$Z_1(\text{pu}) = \frac{Z}{Z_{base\_old}} \Rightarrow Z = Z_1 \times Z_{base\_old}$$

The impedance now has to be represented in a new base value denoted as Z_{base_new}. Therefore

$$Z_2(\text{pu}) = \frac{Z}{Z_{base\_new}} \Rightarrow Z_2 = \frac{Z_1 \times Z_{base\_old}}{Z_{base\_new}} \text{pu}$$

From (1.120) Z_2 can be defined in terms of old and new values of VA base and voltage base as

$$Z_2 = Z_1 \times \left(\frac{V_{base\_old}}{V_{base\_old}}\right)^2 \frac{P_{base\_new}}{P_{base\_old}}$$

Example 1.1:

Let us consider the circuit shown in Fig. 1.19 (a) which contains the equivalent circuit of a transformer. Let the transformer rating be

500 MVA, 220/22 kV with a leakage reactance of 10%.

The VA base of the transformer is 500 MVA and the voltage bases in the primary and secondary side are 200 kV and 22 kV respectively. Therefore the impedance bases of these two sides are

$$Z_{base1} = \frac{(220 \times 10^3)^2}{500 \times 10^6} = 96.8\Omega$$

$$Z_{base2} = \frac{(22 \times 10^3)^2}{500 \times 10^6} = 0.968\Omega$$

where the subscripts 1 and 2 refer to the primary (high tension) and secondary (low tension) sides respectively. Assume that the leakage reactance is referred to the primary side. Then for 10%, i.e., 0.1 per unit leakage reactance we have

$$X_{eq1} = 0.1 \times 96.8 = 9.68 \ \Omega$$

The above reactance when referred to the secondary side is

$$X_{eq2} = \frac{9.68}{a^2} = \frac{9.68}{10^2} = 0.0968 \quad \Omega$$

Hence the per unit impedance in the secondary side is 0.0968/0.968 = 0.1. Therefore we see that the per unit leakage reactance is the same for both sides of the transformer and, as a consequence, the transformer can be represented by only its leakage reactance. The equivalent circuit of the transformer is then as shown in Fig. 1.22. Since this diagram only shows the reactance (or impedance) of the circuit, this is called the reactance (or impedance) diagram .

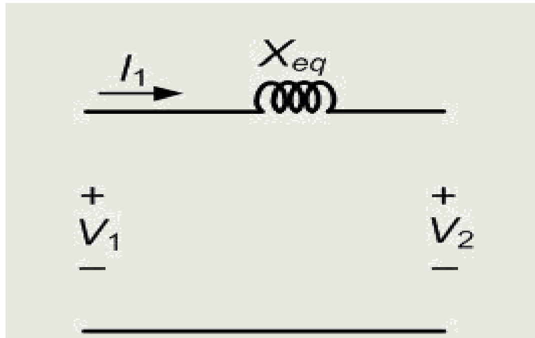


Fig. 1.22 Per unit equivalent circuit of a transformer.

Example 1.2:

Consider the 50 Hz power system the single-line diagram of which is shown in Fig. 1.23. The system contains three generators, three transformers and three transmission lines. The system ratings are

Generator G_1	200 MVA, 20 kV, X_d = 15%
Generator G_2	300 MVA, 18 kV, X_d = 20%
Generator G_3	300 MVA, 20 kV, X_d = 20%
Transformer T_1	300 MVA, 220Y/22 kV, X_d = 10%
Transformer T_2	Three single-phase units each rated 100 MVA, 130Y/25 kV, X = 10%
Transformer T_3	300 MVA, 220/22 kV, X = 10%

The transmission line reactances are as indicated in the figure. We have to draw the reactance diagram choosing the Generator 3 circuit as the base.

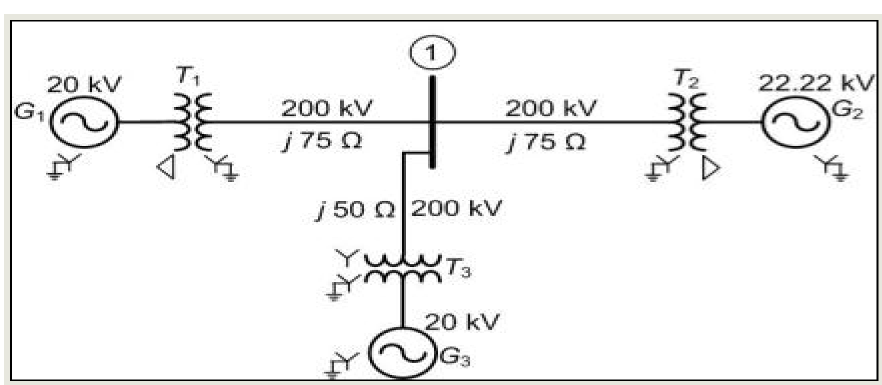


Fig. 1.23 Single-line diagram of the power system of Example 1.2.

s we have chosen the circuit of Generator3 as the base, the base MVA for the circuit is 300. The high voltage side of transformer T_2 is connected wye. Therefore its ratedline to line voltage is √3 X 130 = 225 kV. Since the low voltage side is connected in D , its line to line voltage is 25 kV. The base voltages are chosen as discussed below.

Since the base voltage of G_3 is 20 kV, the base voltage between T_3 and bus 1 will be 20 X 10 = 200 kV. Also as there is no transformer connected in bus 1, the base voltage of 200 kV must be chosen for both the lines that are connected to either side of bus 1. Then the base voltage for the circuit of G_1 will also be 20 kV. Finally since the turns ratio of T_2 is 9 (= 225 ÷ 25), the base voltage in the G 2 side is 200 ÷ 9 = 22.22 kV. The base voltages are also indicated in Fig. 1.23.

Once the base voltages for the various parts of the circuit are known, the per unit values for the various reactances of the circuit are calculated according to (1.123) for a base MVA of 300. These are listed below.

Generator G_1	$X_{G1} = 0.15 \times \dfrac{300}{200} = 0.225$
Generator G_2	$X_{G1} = 0.2 \times \left(\dfrac{18}{22.22}\right)^2 = 0.1312$
Generator G_3	$X_{G3} = 0.2$
Transformer T 1	$X_{T1} = 0.1 \times \left(\dfrac{220}{200}\right)^2 = 0.121$
Transformer T 2	$X_{T2} = 0.1 \times \left(\dfrac{25}{22.22}\right)^2 = 0.1266$
Transformer T 3	$X_{T3} = 0.1 \times \left(\dfrac{22}{20}\right)^2 = 0.121$

he base impedance of the transmission line is

$$Z_{base} = \frac{(200 \times 10^3)^2}{300 \times 10^6} = 133.33\ \Omega$$

Therefore the per unit values of the line impedances are

$$X_{j75} = \frac{75}{133.33} = 0.5625 \text{ pu} \quad \text{and} \quad X_{j50} = \frac{50}{133.33} = 0.375 \text{ pu}$$

The impedance diagram is shown in Fig. 1.24.

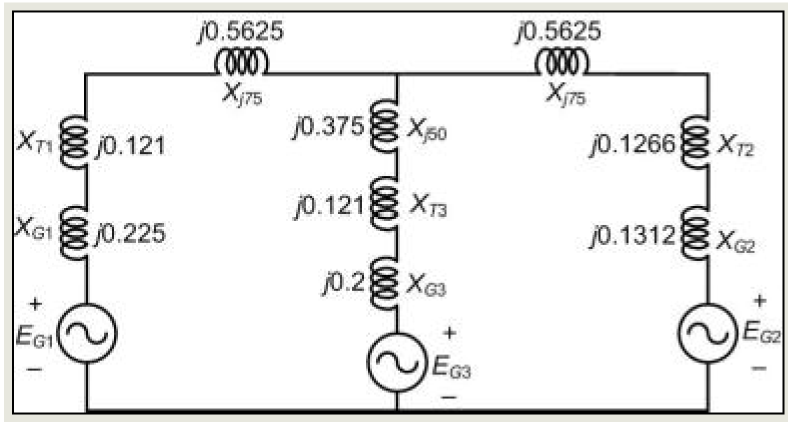


Fig. 1.24 The impedance diagram of the system of Fig. 1.23.

Closure

This completes our discussion on the modeling of power system components. In the subsequent portion of this course we shall use these models to construct a power system and use the per unit notation and the impedance diagram to represent the system.

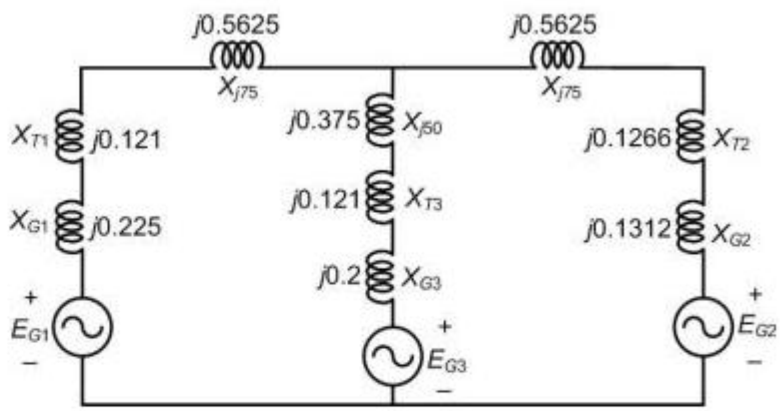


Fig. 1.24 The impedance diagram of the system of Fig. 1.23. Fig. 1.24 The impedance diagram of the system of Fig. 1.23.

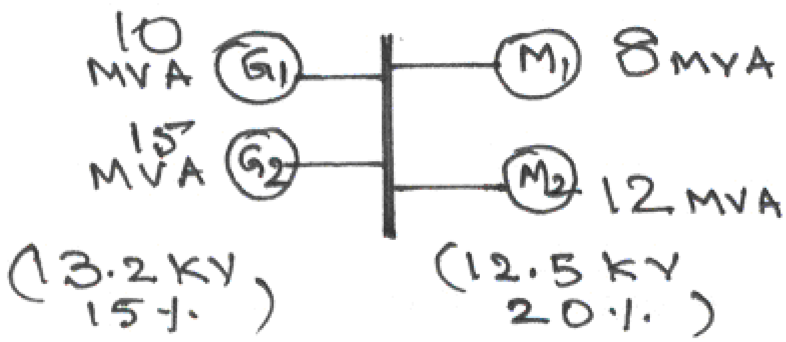
Problems :

Problem #1:

Two generators rated 10 MVA, 13.2 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to 2 motors of inputs 8 MVA and 12 MVA respectively. The operating voltage of motors is 12.5 KV. Assuming the base quantities as 50 MVA, 13.8 KV, draw the per unit reactance diagram. The percentage reactance for generators is 15% and that for motors is 20%.

Solution:

The one line diagram with the data is obtained as shown in figure P1(a).



Selection of base quantities: **50** MVA, **13.8** KV (Given)

Calculation of pu values:

XG1 = j 0.15 (50/10) (13.2/13.8)2 = j 0.6862 pu.

XG2 = j 0.15 (50/15) (13.2/13.8)2 = j 0.4574 pu.

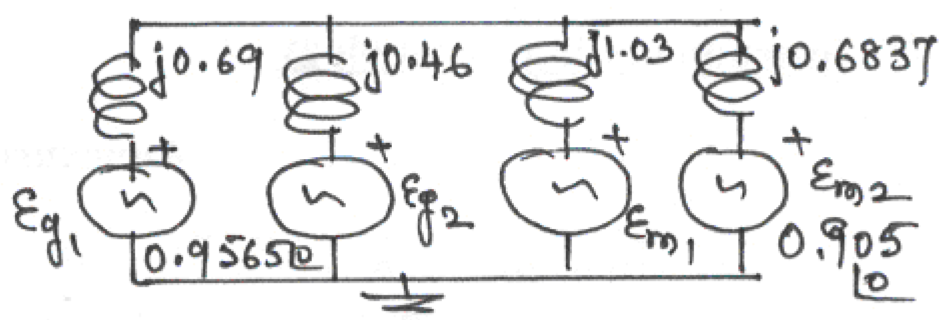
Xm1 = j 0.2 (50/8) (12.5/13.8)2 = j 1.0256 pu.

Xm2 = j 0.2 (50/12) (12.5/13.8)2 = j 0.6837 pu.

Eg1 = Eg2 = (13.2/13.8) = 0.9565 00 pu

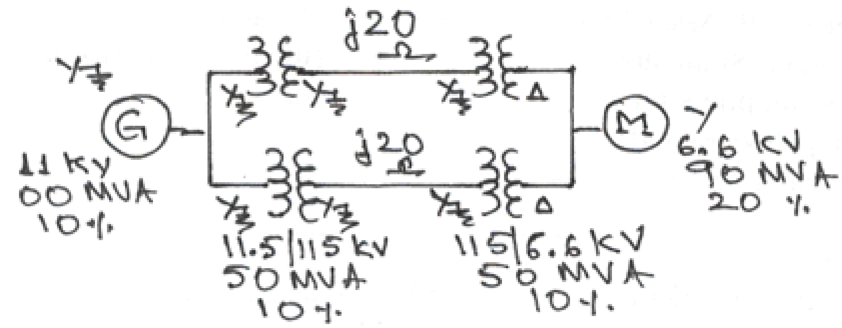
Em1 = Em2 = (12.5/13.8) = 0.9058 00 pu

Thus the pu reactance diagram can be drawn as shown in figure P1(b).



Problem #2:

Draw the per unit reactance diagram for the system shown in figure below. Choose a base of 11 KV, 100 MVA in the generator circuit.



Solution:

The one line diagram with the data is considered as shown in figure.

Selection of base quantities:

100 MVA, **11** KV in the generator circuit(Given); the voltage bases in other sections are: 11 (115/11.5) = **110** KV in the transmission line circuit and 110 (6.6/11.5) = **6.31** KV in the motor circuit.

Calculation of pu values:

XG = j 0.1 pu, Xm = j 0.2 (100/90) (6.6/6.31)2 = j 0.243 pu.

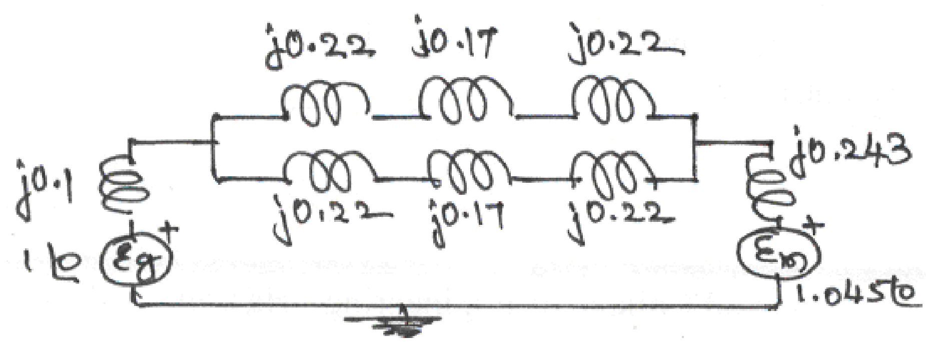
Xt1 =Xt2 = j 0.1 (100/50) (11.5/11)2 = j 0.2185 pu.

Xt3 =Xt4 = j 0.1 (100/50) (6.6/6.31)2 = j 0.219 pu.

Xlines = j 20 (100/1102) = j 0.1652 pu.

Eg = 1.000 pu, Em = (6.6/6.31) = 1.04500 pu

Thus the pu reactance diagram can be drawn as shown in figure P2(b).

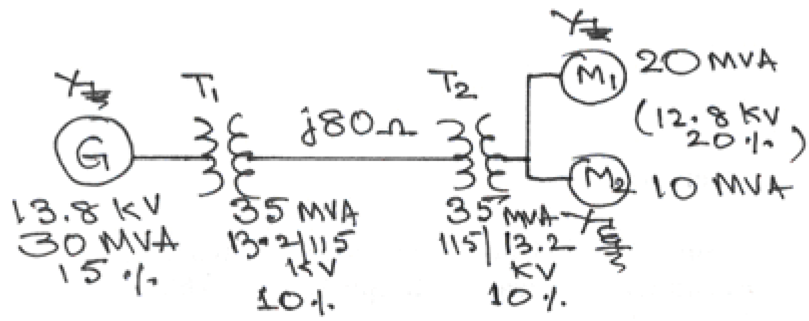


Problem #3:

A 30 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 15%. The generator supplies 2 motors through a step-up transformer - transmission line – step-down transformer arrangement. The motors have rated inputs of 20 MVA and 10 MVA at 12.8 KV with 20% sub transient reactance each. The 3-phase transformers are rated at 35 MVA, 13.2 KV- /115 KV-Y with 10 % leakage reactance. The line reactance is 80 ohms. Draw the equivalent per unit reactance diagram by selecting the generator ratings as base values in the generator circuit.

Solution:

The one line diagram with the data is obtained as shown in figure P3(a).



Selection of base quantities:

30 MVA, **13.8** KV in the generator circuit (Given);

The voltage bases in other sections are:

13.8 (115/13.2) = **120.23** KV in the transmission line circuit and

120.23 (13.26/115) = **13.8** KV in the motor circuit.

Calculation of pu values:

XG = j 0.15 pu.

Xm1 = j 0.2 (30/20) (12.8/13.8)2 = j 0.516 pu.

Xm2 = j 0.2 (30/10) (12.8/13.8)2 = j 0.2581 pu.

Xt1 =Xt2 = j 0.1 (30/35) (13.2/13.8)2 = j 0.0784 pu.

Xline = j 80 (30/120.232) = j 0.17 pu.

Eg = 1.000 pu; Em1 = Em2 = (6.6/6.31) = 0.9300 pu

Thus the pu reactance diagram can be drawn as shown in figure P3(b).

30 MVA, **13.8** KV in the generator circuit (Given);

The voltage bases in other sections are:

13.8 (115/13.2) = **120.23** KV in the transmission line circuit and

120.23 (13.26/115) = **13.8** KV in the motor circuit.

Calculation of pu values:

XG = j 0.15 pu.

Xm1 = j 0.2 (30/20) (12.8/13.8)2 = j 0.516 pu.

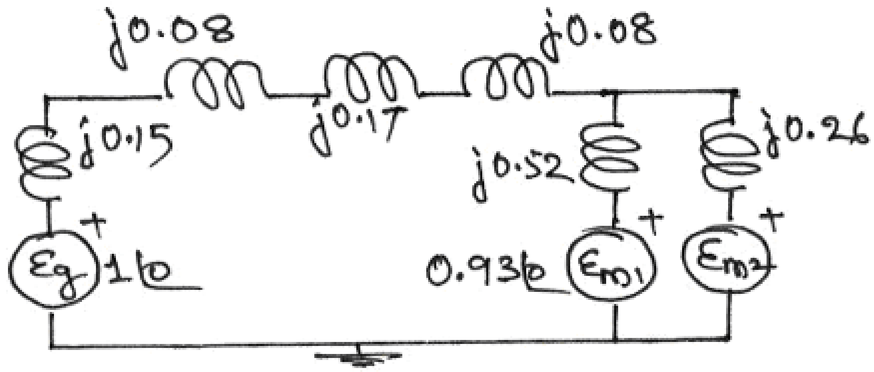
Xm2 = j 0.2 (30/10) (12.8/13.8)2 = j 0.2581 pu.

Xt1 =Xt2 = j 0.1 (30/35) (13.2/13.8)2 = j 0.0784 pu.

Xline = j 80 (30/120.232) = j 0.17 pu.

Eg = 1.000 pu; Em1 = Em2 = (6.6/6.31) = 0.9300 pu

Thus the pu reactance diagram can be drawn as shown in figure P3(b).



Problem #4:

A 33 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 0.5%. The generator supplies a motor through a step-up transformer - transmission line – step-down transformer arrangement. The motor has rated input of 25 MVA at 6.6 KV with 25% sub transient reactance. Draw the equivalent per unit impedance diagram by selecting 25 MVA (3), 6.6 KV (LL) as base values in the motor circuit, given the transformer and transmission line data as under:

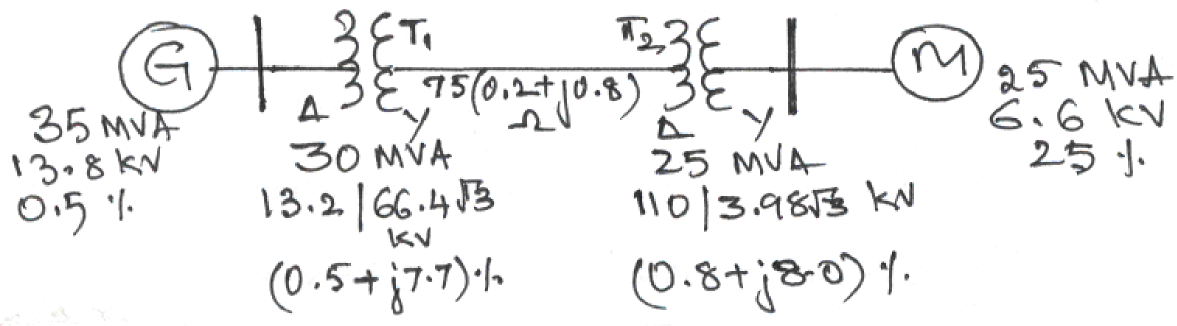
Step up transformer bank: three single phase units, connected Δ–Y, each rated 10 MVA, 13.2/6.6 KV with 7.7 % leakage reactance and 0.5 % leakage resistance;

Transmission line: 75 KM long with a positive sequence reactance of 0.8 ohm/ KM and a resistance of 0.2 ohm/ KM; and

Step down transformer bank: three single phase units, connected –Y, each rated 8.33 MVA, 110/3.98 KV with 8% leakage reactance and 0.8 % leakage resistance;

Solution:

The one line diagram with the data is obtained as shown in figure P4(a).



3-phase ratings of transformers:

T1: 3(10) = 30 MVA, 13.2/ 66.43 KV = 13.2/ 115 KV, X = 0.077, R = 0.005 pu.

T2: 3(8.33) = 25 MVA, 110/ 3.983 KV = 110/ 6.8936 KV, X = 0.08, R = 0.008 pu.

Selection of base quantities:

25 MVA, **6.6** KV in the motor circuit (Given); the voltage bases in other sections are: 6.6 (110/6.8936) = **105.316** KV in the transmission line circuit and 105.316 (13.2/115) = **12.09** KV in the generator circuit.

Calculation of pu values:

Xm = j 0.25 pu; Em = 1.000 pu.

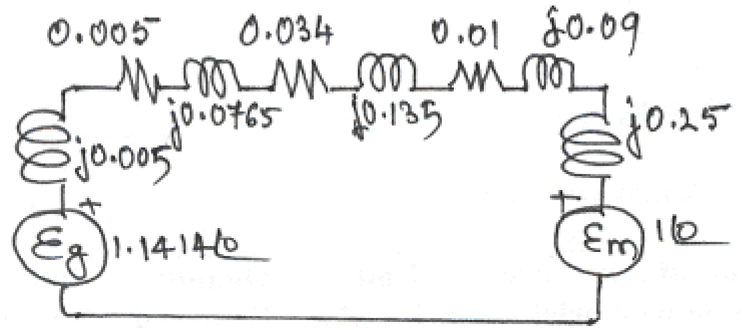
XG = j 0.005 (25/33) (13.8/12.09)2 = j 0.005 pu; Eg = 13.8/12.09 = 1.41400 pu.

Zt1 = 0.005 + j 0.077 (25/30) (13.2/12.09)2 = 0.005 + j 0.0765 pu. (ref. to LV side)

Zt2 = 0.008 + j 0.08 (25/25) (110/105.316)2 = 0.0087 + j 0.0873 pu. (ref. to HV side)

Zline = 75 (0.2+j 0.8) (25/ 105.3162) = 0.0338 + j 0.1351 pu.

Thus the pu reactance diagram can be drawn as shown in figure P4(b).



PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it. General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

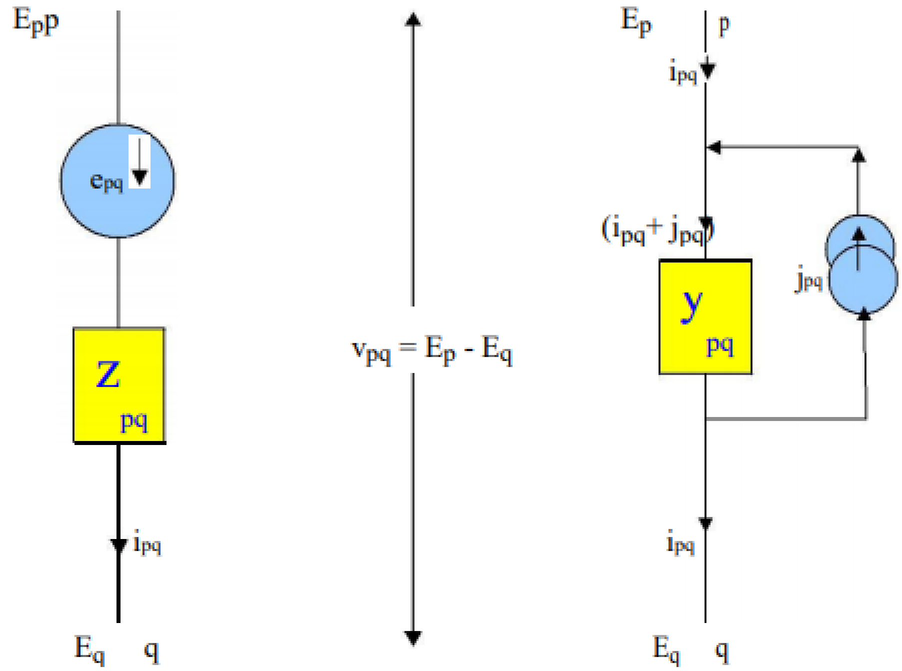


Fig.2 Representation of a primitive network element (a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

v_{pq} = voltage across the element p-q,

e_{pq} = source voltage in series with the element p-

q, i_{pq}= current through the element p-q,

j_{pq}= source current in shunt with the element p-

q, z_{pq}= self impedance of the element p-q and

y_{pq}= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

$v_{pq} + e_{pq} = z_{pq} i_{pq}$ (in its impedance form)

$i_{pq} + j_{pq} = y_{pq} v_{pq}$ (in its admittance form) **(6)**

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:

$j_{pq} = - y_{pq} e_{pq}$ **(7)**

A set of non-connected elements of a given system is defined as a primitive Network and an element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$$v + e = [z]\, i$$
$$i + j = [y]\, v \qquad \textbf{(8)}$$

Primitive network matrices: A diagonal element in the matrices, [z] or [y] is the self impedance zpq-pq or self admittance, ypq-pq. An off-diagonal element is the mutual impedance, zpq-rs or mutual admittance, ypq-rs, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by
inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Examples on Primitive Networks:

Example-4: Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: Z1=Z2=0.2, Z3=0.25, Z4=Z5=0.1 and Z6=0.4 units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Solution:

The element node incidence matrix, $\hat{A}$ can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$$\hat{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

Based on the conventional definitions of the elements of A , the oriented graph can be formed as under:

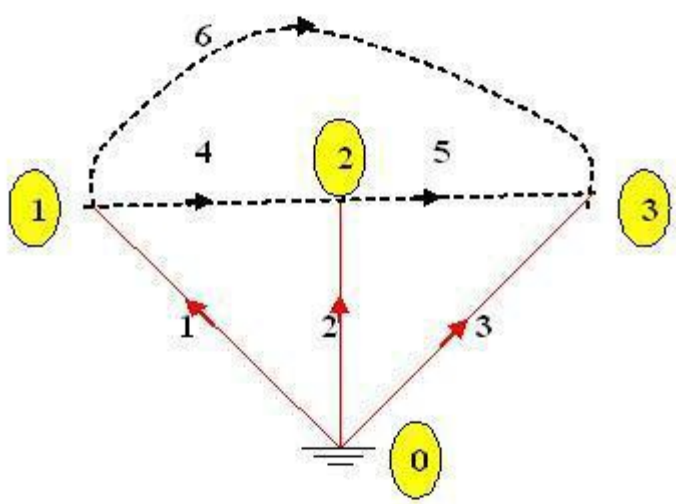


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

$$[z] = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix}$$

$$[y] = \begin{bmatrix} 5.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.5 \end{bmatrix}$$

FORMATION OF YBUS AND ZBUS

The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. ZBUS Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS}\, I_{BUS}$$

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad \textbf{(9)}$$

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR}\, I_{BR}$$

$$I_{BR} = Y_{BR}\, E_{BR} \qquad \textbf{(10)}$$

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP}\, I_{LOOP}$$

$$I_{LOOP} = Y_{LOOP}\, E_{LOOP} \qquad \textbf{(11)}$$

Of the various network matrices refered above, the bus admittance matrix (Y_{BUS}) and the bus impedance matrix (Z_{BUS}) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current

Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3 (V_1 - V_3) + Y_6 (V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5 (V_2 - V_3) + Y_6 (V_2 - V_1)$

At node 3: $0 = Y_3 (V_3 - V_1) + Y_4V_3 + Y_5 (V_3 - V_2)$ **(12)**

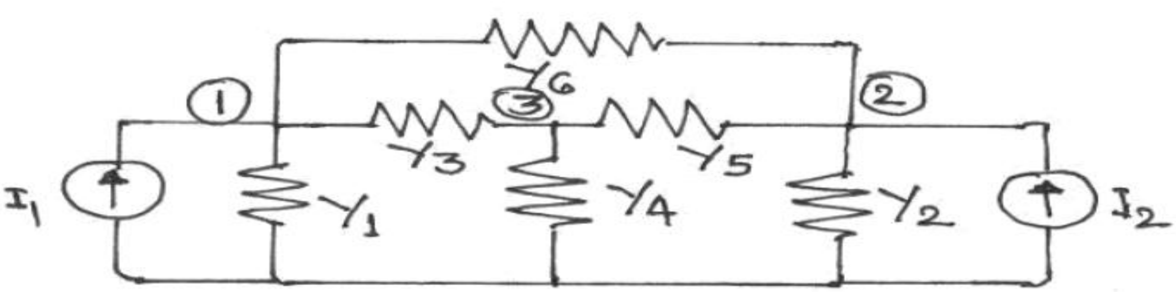


Fig. 3 Example System for finding Y_{BUS}

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad (13)$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad (14)$$

Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, YBUS of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots n)$$

$$Y_{ij} = -\,y_{ij} \quad (j = 1,2,\ldots\ldots n) \qquad (15)$$

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix

In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS} E_{BUS} \qquad \mathbf{(17)}$$

Where E_{BUS} = vector of bus voltages measured with respect to reference bus

I_{BUS} = Vector of currents injected into the bus

Y_{BUS} = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y]\ v$$

Pre-multiplying by A^t (transpose of A), we obtain

$$A^t i + A^t j = A^t [y]\ v \qquad \mathbf{(18)}$$

However, as per equation (4),

$$A^t i = 0,$$

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

$$A^t j = I_{BUS} \qquad \mathbf{(19)}$$

Thus from (18) we have, $\quad I_{BUS} = A^t [y]\ v \qquad \mathbf{(20)}$

However, from (5), we have

$$v = A\ E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y]\ A\ E_{BUS} \qquad \mathbf{(21)}$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y]\ A \qquad \mathbf{(22)}$$

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1} \quad\quad (23)$$

Example 8: *For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.*

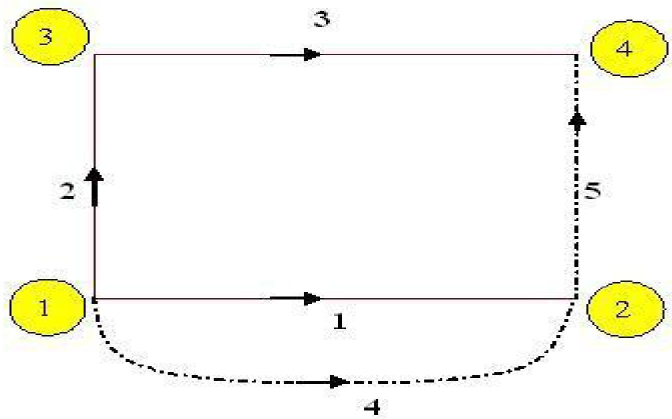


Fig E8 System for Example-8

Table E8: Data for Example-8

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by,

$$[z]=\begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix $[y] = [z]^{-1}$ and given by,

$$[y]=\begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by $(n-1)$ nodal equations, where n is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

UNIT-II
POWER FLOW ANALYSIS

Importance of power flow analysis in planning and operation of power systems:

Power flow studies are performed to determine voltages, active and reactive power etc. at various points in the network for different operating conditions subject to the constraints on generator capacities and specified net interchange between operating systems and several other restraints. Power flow or load flow solution is essential for continuous evaluation of the performance of the power systems so that suitable control measures can be taken in case of necessity. In practice it will be required to carry out numerous power flow solutions under a variety of conditions.

Power flow studies are undertaken for various reasons, some of which are the following:

I. The line flows

2. The bus voltages and system voltage profile

3. The effect of change in configuration and incorporating new circuits on system loading

4. The effect of temporary loss of transmission capacity and (or) generation on system loading and accompanied effects.

5. The effect of in-phase and quadrative boost voltages on system loading

6. Economic system operation

7. System loss minimization

8. Transformer taps setting for economic operation

9. Possible improvements to an existing system by change of conductor sizes and system voltages

For the purpose of power flow studies a single phase representation of the power network is used, since the system is generally balanced. When systems had not grown to the present size, networks were simulated on network analyzers for load flow solutions. These analyzers are of analogue type, scaled down miniature models of power systems with resistances, reactances, capacitances, autotransformers, transformers, loads and generators. The generators are just supply sources operating at a much higher frequency than 50 Hz to limit the size of the components. The loads are represented by constant impedances. Meters are provided on the panel board for measuring voltages, currents and powers. The power flow solution in obtained directly from measurements for any system simulated on theanalyzer.

With the advent of the modern digital computers possessing large storage and high speed the mode of power flow studies have changed from analog to digital simulation. A large number of algorithms are developed for digital power flow solutions. The methods basically distinguish between themselves in the rate of convergence, storage requirement and time of computation. The loads are generally represented by constant power.

Network equations can be solved in a variety of ways in a systematic manner. The most popular method is node voltage method. When nodal or bus admittances are used complex linear algebraic simultaneous equations will be obtained in terms of nodal or bus currents.

However, as in a power system since the nodal currents are not known, but powers are known at almost all the buses, the resulting mathematical equations become non-linear and are required to be solved by interactive methods. Load flow studies are required for power system planning, operation and control as well as for contingency analysis. The bus admittance matrix is invariably utilized in power flow solutions.

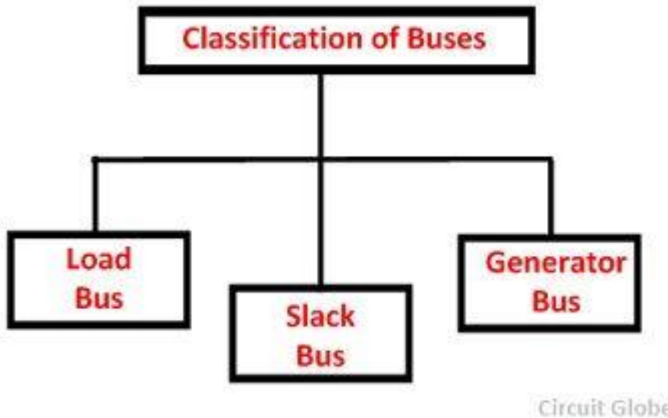
CONDITIONS FOR SUCCESSFUL OPERATION OF A POWER SYSTEM

There are the following:

1. There should the adequate real power generation to supply the power demand at various load buses and also the losses
2. The bus voltage magnitudes are maintained at values very close to the rated values.
3. Generators, transformers and transmission lines are not over loaded at any point of time or the load curve.

Classification of Power System Buses

A bus in a power system is defined as the vertical line at which the several components of the power system like generators, loads, and feeders, etc., are connected. The buses in a power system are associated with four quantities. These quantities are the magnitude of the voltage, the phase angle of the voltage, active or true power and the reactive power.In the load flow studies, two variable are known, and two are to be determined. Depends on the quantity to be specified the buses are classified into three categories generation bus, load bus and slack bus.



The table shown below shows the types of buses and the associated known and unknown value.

Type of Buses	Know or Specified Quantities	Unknown Quantities or Quantities to be determined.
Generation or P-V Bus	P, | V |	Q, δ
Load or P-Q Bus	P, Q	| V |, δ
Slack or Reference Bus	| V |, δ	P, Q

Generation Bus or Voltage control bus

This bus is also called the P-V bus, and on this bus, the voltage magnitude corresponding to generate voltage and true or active power P corresponding to its rating are specified. Voltage magnitude is maintained constant at a specified value by injection of reactive power. The reactive power generation Q and phase angle δ of the voltage are to be computed.

Load Bus

This is also called the P-Q bus and at this bus, the active and reactive power is injected into the network. Magnitude and phase angle of the voltage are to be computed. Here the active power P and reactive power Q are specified, and the load bus voltage can be permitted within a tolerable value, i.e., 5 %. The phase angle of the voltage , i.e.δ is not very important for the load.

Slack, Swing or Reference Bus

Slack bus in a power system absorb or emit the active or reactive power from the power system. The slack bus does not carry any load. At this bus, the magnitude and phase angle of the voltage are specified. The phase angle of the voltage is usually set equal to zero. The active and reactive power of this bus is usually determined through the solution of equations.

The slack bus is a fictional concept in load flow studies and arises because the I^2R losses of the system are not known accurately in advance for the load flow calculation. Therefore, the total injected power cannot be specified at every bus. The phase angle of the voltage at the slack bus is usually taken as reference or zero.

Development of power flow model in complex variables form

The *power flow problem* is a very well known problem in the field of power systems engineering, where voltage magnitudes and angles for one set of buses are desired, given that voltage magnitudes and power levels for another set of buses are known and that a model of the network configuration (unit commitment and circuit topology) is available. A *power flow solution procedure* is a numerical method that is employed to solve the power flow problem. A *power flow program* is a computer code that implements a power flow solution procedure. The *power flow solution* contains the voltages and angles at all buses, and from this information, we may compute the real and reactive generation and load levels at all buses and the real and reactive flows across all circuits. The above terminology is often used with the word "load" substituted for "power," i.e., load flow problem, load flow solution procedure, load flow program, and load flow solution. However, the former terminology is preferred as one normally does not think of "load" as something that "flows."

The power flow problem was originally motivated within planning environments where engineers considered different network configurations necessary to serve an expected future load. Later, it became an operational problem as operators and operating engineers were required to monitor the real-time status of the network in terms of voltage magnitudes and circuit flows. Today, the power flow problem is widely recognized as a fundamental problem for power system analysis, and there are many advanced, commercial power flow programs to address it. Most of these programs are capable of solving the power flow program for tens of thousands of interconnected buses. Engineers that understand the power flow problem, its formulation, and corresponding solution procedures are in high demand, particularly if they also have experience with commercial grade power flow programs.

Generator Reactive Limits

It is well known that generators have maximum and minimum real power capabilities. In addition, they also have maximum and minimum reactive power capabilities. The maximum reactive power capability corresponds to the maximum reactive power that the generator may produce when operating with a lagging power factor. The minimum reactive power capability corresponds to the maximum reactive power the generator may absorb when operating with a leading power factor. These limitations are a function of the real power output of the generator, that is, as the real power increases, the reactive power limitations move closer to zero. The solid curve in Figure T7.1 is a typical *generator capability* curve, which shows the lagging and leading reactive limitations (the ordinate) as real power is varied (the abscissa). Most power flow programs model the generator reactive capabilities by assuming a somewhat conservative value for P_{max} (perhaps 95% of the actual value), and then *fixing* the reactive limits Q_{max} (for the lagging limit) and Q_{ming} (for the leading limit) according to the dotted lines shown in Fig. T2.1.

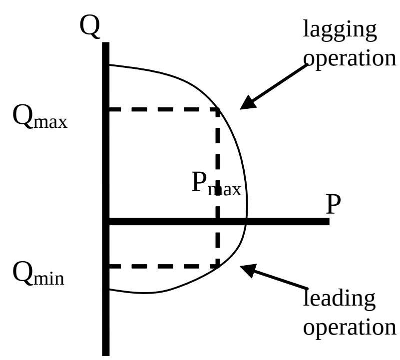


Fig T7.1: Generator Capability Curve and Approximate Reactive Limits

An *injection* is the power, either real or reactive, that is being injected into or withdrawn from a bus by an element having its other terminal (in the per-phase equivalent circuit) connected to ground. Such an element would be either a generator or a load. We define a positive injection as one where power is flowing from the element into the bus (i.e., into the network); a negative injection is then when power is flowing from the bus (i.e., from the network) into the element. Generators normally have positive real power injections, although they may also be assigned negative real power injections, in which case they are operating as a motor. Generators may have either positive or negative reactive power injections: positive if the generator is operating lagging and delivering reactive power to the bus, negative if the generator is operating leading and absorbing reactive power from the bus, and zero if the generator is operating at unity power factor. Loads normally have negative real and reactive power injections, although they may also be assigned positive real power injections in the case of very special modeling needs. Figure T7.3 (a) and (b) illustrate the two most common possibilities. Figure T.7.3 (c) illustrates that we must compute a net injection as the algebraic sum when a bus has both load and generation; in this case, the net injection for both real and reactive power is positive (into the bus). Thus, the net real power injection is $P_k=P_{gk}-P_{dk}$, and the net reactive power injection is $Q_k=Q_{gk}-Q_{dk}$. We may also refer to the net complex power injection as $S_k=S_{gk}-S_{dk}$, where $S_k=P_k+jQ_k$.

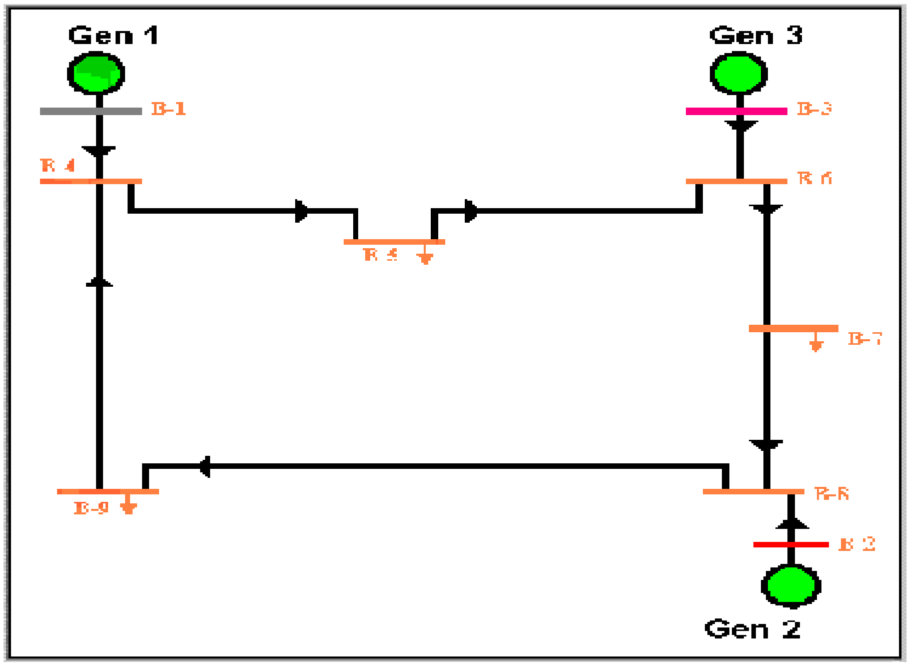


Figure T7.2: Single Line Diagram for Simple Power System

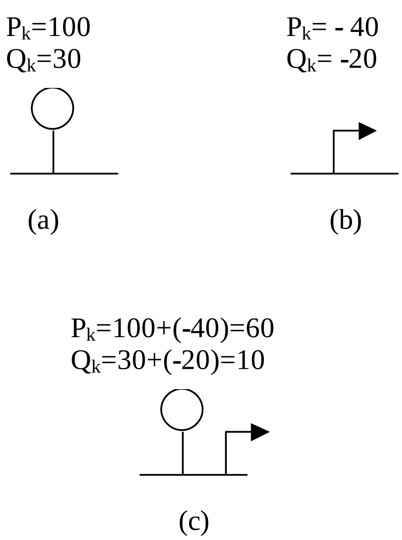


Fig T7.3: Illustration of (a) positive injection, (b) negative injection, and (c) net injection

Although it is physically appealing to categorize buses based on the generation/load mix connected to it, we need to be more precise in order to analytically formulate the power flow problem. For proper analytical formulation, it is appropriate to categorize the buses according to what information is known about them before we solve the power flow problem. For each bus, there are four possible variables that characterize the buses electrical condition. Let us consider an arbitrary bus numbered k. The four variables are real and reactive power injection, P_k and Q_k, respectively, and voltage magnitude and angle, $|V_k|$ and θ_k, respectively. From this perspective, there are three basic types of buses. We refer to the first two types using terminology that remind us of the known variables.

- *PV Buses*: For type PV buses, we know P_k and $|V_k|$ but not Q_k or θ_k. These buses fall under the category of *voltage-controlled buses* because of the ability to specify (and therefore to know) the voltage magnitude of this bus. Most generator buses fall into this category, independent of whether it also has load; exceptions are buses that have reactive power injection at either the generator's upper limit (Q_{max}) or its lower limit (Q_{min}), and (2) the system swing bus (we describe the swing bus below). There are also special cases where a non-generator bus (i.e., either a bus with load or a bus with neither generation or load) may be classified as type PV, and some examples of these special cases are buses having switched shunt capacitors or static var systems (SVCs). We will not address these special cases in this module. In Fig. T7.2, buses B2 and B3 are type PV. The real power injections of the type PV buses are chosen according to the system dispatch corresponding to the modeled loading conditions. The voltage magnitudes of the type PV buses are chosen according to the expected terminal voltage settings, sometimes called the generator "set points," of the units.
- *PQ Buses*: For type PQ buses, we know P_k and Q_k but not $|V_k|$ or θ_k. All load buses fall into this category, including buses that have not either load or generation. In Fig. T7.2, buses B4-B9 are all type PQ. The real power injections of the type PQ buses are chosen according to the loading conditions being modeled. The reactive power injections of the type PQ buses are chosen according to the expected power factor of the load.

The third type of bus is referred to as the *swing bus.* Two other common terms for this bus are *slack bus* and *reference bus*. There is only one swing bus, and it can be designated by the engineer to be any generator bus in the system. For the swing bus, we know $|V|$ and θ. The fact that we know θ is the reason why it is sometimes called the reference bus. Physically, there is nothing special about the swing bus; in fact, it is a mathematical artifact of the solution procedure. At this point in our treatment of the power flow problem, it is most appropriate to understand this last statement in the following way. The generation must supply both the load and the losses on the circuits. Before solving the power flow problem, we will know all injections at PQ buses, but we will not know what the losses will be as losses are a function of the flows on the circuits which are yet to be computed. So we may set the real power injections for, at most, all but one of the generators. The one generator for which we do not set the real power injection is the one modeled at the swing bus. Thus, this generator “swings” to compensate for the network losses, or, one may say that it “takes up the slack.” Therefore, rather than call this generator a |V|θ bus (as the above naming convention would have it), we choose the terminology “swing” or “slack” as it helps us to better remember its function. The voltage magnitude of the swing bus is chosen to correspond to the typical voltage setting of this generator. The voltage angle may be designated to be any angle, but normally it is designated as 0^o.

A word of caution about the swing bus is in order. Because the real power injection of the swing bus is not set by the engineer but rather is an output of the power flow solution, it can take on mathematically tractable but physically impossible values. Therefore, the engineer must always check the swing bus generation level following a solution to ensure that it is within the physical limitations of the generator.

The Admittance Matrix

Current injections at a bus are analogous to power injections. The student may have already been introduced to them in the form of current sources at a node. Current injections may be either positive (into the bus) or negative (out of the bus). Unlike current flowing through a branch (and thus is a branch quantity), a current injection is a nodal quantity. The admittance matrix, a fundamental network analysis tool that we shall use heavily, relates current injections at a bus to the bus voltages. Thus, the admittance matrix relates nodal quantities. We motivate these ideas by introducing a simple example.

Figure T7.4 shows a network represented in a hybrid fashion using one-line diagram representation for the nodes (buses 1-4) and circuit representation for the branches connecting the nodes and the branches to ground. The branches connecting the nodes represent lines. The branches to ground represent any shunt elements at the buses, including the charging capacitance at either end of the line. All branches are denoted with their admittance values y_{ij} for a branch connecting bus i to bus j and y_i for a shunt element at bus i. The current injections at each bus i are denoted by I_i.

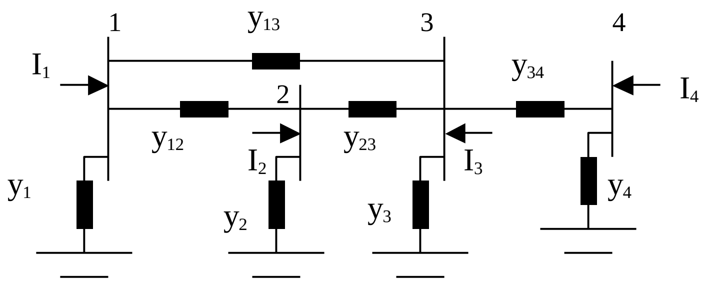


Fig. T7.4: Network for Motivating Admittance Matrix

Kirchoff's Current Law (KCL) requires that each of the current injections be equal to the sum of the currents flowing out of the bus and into the lines connecting the bus to other buses, or to the ground. Therefore, recalling Ohm's Law, I=V/Z=VY, the current injected into bus 1 may be written as:

$$I_1=(V_1-V_2)y_{12} + (V_1-V_3)y_{13} + V_1y_1 \qquad \text{(T7.1)}$$

To be complete, we may also consider that bus 1 is "connected" to bus 4 through an infinite impedance, which implies that the corresponding admittance y_{14} is zero. The advantage to doing this is that it allows us to consider that bus 1 *could be* connected to any bus in the network. Then, we have:

$$I_1=(V_1-V_2)y_{12} + (V_1-V_3)y_{13} + (V_1-V_4)y_{14} + V_1y_1 \qquad \text{(T7.2)}$$

Note that the current contribution of the term containing y_{14} is zero since y_{14} is zero. Rearranging eq. T7.2, we have:

$$I_1= V_1(y_1 + y_{12} + y_{13} + y_{14}) + V_2(-y_{12})+ V_3(-y_{13}) + V_4(-y_{14}) \qquad \text{(T7.3)}$$

Similarly, we may develop the current injections at buses 2, 3, and 4 as:

$$\begin{aligned} I_2&= V_1(-y_{21}) + V_2(y_2 + y_{21} + y_{23} + y_{24}) + V_3(-y_{23}) + V_4(-y_{24}) \qquad \text{(T7.4)}\\ I_3&= V_1(-y_{31})+ V_2(-y_{32}) + V_3(y_3 + y_{31} + y_{32} + y_{34}) + V_4(-y_{34})\\ I_4&= V_1(-y_{41})+ V_2(-y_{42}) + V_3(-y_{34})+ V_4(y_4 + y_{41} + y_{42} + y_{43}) \end{aligned}$$

where we recognize that the admittance of the circuit from bus k to bus i is the same as the admittance from bus i to bus k, i.e., $y_{ki}=y_{ik}$ From eqs. (T7.3) and (T7.4), we see that the current injections are linear functions of the nodal voltages. Therefore, we may write these equations in a more compact form using matrices according to:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad \text{(T7.5)}$$

The matrix containing the network admittances in eq. (T7.5) is the admittance matrix, also known as the Y-bus, and denoted as:

$$\underline{Y} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix} \quad \text{(T7.6)}$$

Denoting the element in row i, column j, as Y_{ij}, we rewrite eq. (T7.6) as:

$$\underline{Y} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \quad \text{(T7.7)}$$

where the terms Y_{ij} are not admittances but rather elements of the admittance matrix. Therefore, eq. (T7.6) becomes:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad \text{(T7.8)}$$

By using eq. (T7.7) and (T7.8), and defining the vectors $\underline{V}$ and $\underline{I}$, we may write eq. (T7.8) in compact form according to:

$$\underline{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}, \qquad \underline{I} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \qquad \Rightarrow \qquad \underline{I} = \underline{Y}\,\underline{V} \quad \text{(T7.9)}$$

We make several observations about the admittance matrix given in eqs. (T7.6) and (T7.7). These observations hold true for any linear network of any size.

1. The matrix is symmetric, i.e., $Y_{ij}=Y_{ji}$.
2. A diagonal element Y_{ii} is obtained as the sum of admittances for all branches connected to bus i, including the shunt branch, i.e., $Y_{ii} = y_i + \sum_{k=1, k \neq i}^{N} y_{ik}$, where we emphasize once again that y_{ik} is non-zero only when there exists a physical connection between buses i and k.
3. The off-diagonal elements are the negative of the admittances connecting buses i and j, i.e., $Y_{ij}=-y_{ji}$.

These observations enable us to formulate the admittance matrix very quickly from the network based on visual inspection. The following example will clarify.

Example T7.1

Consider the network given in Fig. T7.5, where the numbers indicate admittances.

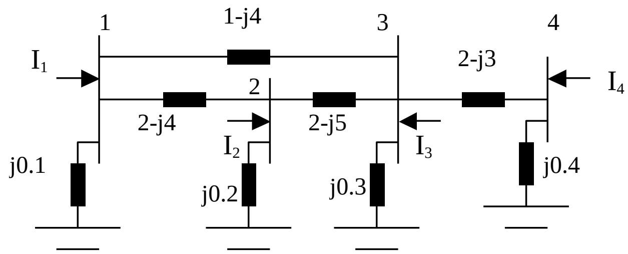


Fig. T7.5: Circuit for Example T7.1

The admittance matrix is given by inspection as:

$$\underline{Y} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} 2-j7.9 & -2+j4 & -1+j4 & 0 \\ -2+j4 & 4-j8.8 & -2+j5 & 0 \\ -1+j4 & -2+j5 & 5-j11.7 & -2+j3 \\ 0 & 0 & -2+j3 & 2-j2.6 \end{bmatrix}$$

The power flow equations

We have defined the net complex power injection into a bus, in Section T7.2, as $S_k=S_{gk}-S_{dk}$. In this section, we desire to derive an expression for this quantity in terms of network voltages and admittances. We begin by reminding the reader that all quantities are assumed to be in per unit, so we may utilize single-phase power relations. Drawing on the familiar relation for complex power, we may express S_k as:

$$S_k=V_kI_k^* \tag{T.7.10}$$

From eq. (T7.8), we see that the current injection into any bus k may be expressed as

$$I_k = \sum_{j=1}^{N} Y_{kj}V_j \tag{T7.11}$$

where, again, we emphasize that the Y_{kj} terms are admittance matrix elements and not admittances. Substitution of eq. (T7.11) into eq. (T7.10) yields:

$$S_k = V_k\left(\sum_{j=1}^{N} Y_{kj}V_j\right)^* = V_k\sum_{j=1}^{N} Y_{kj}{}^* V_j{}^* \qquad \text{(T7.12)}$$

Recall that V_k is a phasor, having magnitude and angle, so that $V_k=|V_k|\angle\theta_k$. Also, Y_{kj}, being a function of admittances, is therefore generally complex, and we define G_{kj} and B_{kj} as the real and imaginary parts of the admittance matrix element Y_{kj}, respectively, so that $Y_{kj}=G_{kj}+jB_{kj}$. Then we may rewrite eq. (T7.12) as

$$\begin{aligned} S_k &= V_k\sum_{j=1}^{N} Y_{kj}{}^* V_j{}^* = |V_k|\angle\theta_k\sum_{j=1}^{N}(G_{kj}+jB_{kj})^*\left(|V_j|\angle\theta_j\right)^* = |V_k|\angle\theta_k\sum_{j=1}^{N}(G_{kj}-jB_{kj})\left(|V_j|\angle-\theta_j\right) \\ &= \sum_{j=1}^{N}|V_k|\angle\theta_k\left(|V_j|\angle-\theta_j\right)(G_{kj}-jB_{kj}) = \sum_{j=1}^{N}\left(|V_k||V_j|\angle(\theta_k-\theta_j)\right)(G_{kj}-jB_{kj}) \end{aligned} \qquad \text{(T7.13)}$$

Recall, from the Euler relation, that a phasor may be expressed as complex function of sinusoids, i.e., $V=|V|\angle\theta=|V|\{\cos\theta+j\sin\theta\}$, we may rewrite eq. (T7.13) as

$$\begin{aligned} S_k &= \sum_{j=1}^{N}\left(|V_k||V_j|\angle(\theta_k-\theta_j)\right)(G_{kj}-jB_{kj}) \\ &= \sum_{j=1}^{N}|V_k||V_j|\left(\cos(\theta_k-\theta_j)+j\sin(\theta_k-\theta_j)\right)(G_{kj}-jB_{kj}) \end{aligned} \qquad \text{(T7.14)}$$

If we now perform the algebraic multiplication of the two terms inside the parentheses of eq. (T7.14), and then collect real and imaginary parts, and recall that $S_k=P_k+jQ_k$, we can express eq. (T7.14) as two equations, one for the real part, P_k, and one for the imaginary part, Q_k, according to:

$$\begin{aligned} P_k &= \sum_{j=1}^{N}|V_k||V_j|\left(G_{kj}\cos(\theta_k-\theta_j)+B_{kj}\sin(\theta_k-\theta_j)\right) \\ Q_k &= \sum_{j=1}^{N}|V_k||V_j|\left(G_{kj}\sin(\theta_k-\theta_j)-B_{kj}\cos(\theta_k-\theta_j)\right) \end{aligned} \qquad \text{(T7.15)}$$

The two equations of (T7.15) are called the power flow equations, and they form the fundamental building block from which we attack the power flow problem. It is interesting to consider the case of eqs. (T7.15) if bus k, relabeled as bus p, is only connected to one other bus, let's say bus q. Then the bus p injection is the same as the flow into the line pq. The situation is illustrated in Fig. T7.6.

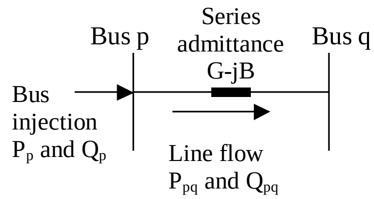


Fig. T7.6: Bus p Connected to Only Bus q

For the situation illustrated in Fig. T7.6, eqs. (T7.15) become:

$$
\begin{aligned}
P_p &= |V_p|^2 G_{pp} + |V_p||V_q| G_{pq}\cos(\theta_p - \theta_q) + |V_p||V_q| B_{pq}\sin(\theta_p - \theta_q) \\
Q_p &= -|V_p|^2 B_{pp} + |V_p||V_q| G_{pq}\sin(\theta_p - \theta_q) - |V_p||V_q| B_{pq}\cos(\theta_p - \theta_q)
\end{aligned}
\qquad \text{(T7.16)}
$$

If the line pq admittance is y=G-jB[1], as shown in Fig. T7.6, then G_{pq}=-G and B_{pq}=B (see eq. T7.6). If there is no bus p shunt reactance or line charging, then G_{pp}=G and B_{pp}=B. Under these conditions, eqs. (T7.16) become:

$$
\begin{aligned}
P_p &= |V_p|^2 G - |V_p||V_q| G\cos(\theta_p - \theta_q) + |V_p||V_q| B\sin(\theta_p - \theta_q) \\
Q_p &= |V_p|^2 B - |V_p||V_q| G\sin(\theta_p - \theta_q) - |V_p||V_q| B\cos(\theta_p - \theta_q)
\end{aligned}
\qquad \text{(T7.17)}
$$

If we simply rearrange the order of the terms in the reactive equation, then we have:

$$
\begin{aligned}
P_p &= |V_p|^2 G - |V_p||V_q| G\cos(\theta_p - \theta_q) + |V_p||V_q| B\sin(\theta_p - \theta_q) \\
Q_p &= |V_p|^2 B - |V_p||V_q| B\cos(\theta_p - \theta_q) - |V_p||V_q| G\sin(\theta_p - \theta_q)
\end{aligned}
\qquad \text{(T7.18)}
$$

Analytic statement of the power flow problem

Consider a power system network having N buses, N_G of which are voltage-regulating generators. One of these must be the swing bus. Thus there are N_G-1 type PV buses, and N-N_G type PQ buses. We assume that the swing bus is numbered bus 1, the type PV buses are numbered 2,…, N_G, and the type PQ buses are numbered N_G+1,…,N (this assumption on numbering is not necessary, but it makes the following development notationally convenient). It is typical that we know, in advance, the following information about the network (implying that it is input data to the problem):

1. The admittances of all series and shunt elements (implying that we can obtain the Y-bus),
2. The voltage magnitudes V_k, $k=1,\dots,N_G$, at all N_G generator buses,
3. The real power injection of all buses except the swing bus, P_k, $k=2,\dots,N$
4. The reactive power injection of all type PQ buses, Q_k, $k=N_G+1, \dots, N$

Statements 3 and 4 indicate power flow equations for which we know the injections, i.e., the values of the left-hand side of eqs. (T7.15). These equations are very valuable because they have one less unknown than equations for which we do not know the left-hand-side. The number of these equations for which we know the left-hand-side can be determined by adding the number of buses for which we know the real power injection (statement 3 above) to the number of buses for which we know the reactive power injection (statement 4 above). This is $(N-1)+(N-N_G)=2N-N_G-1$. We repeat the power flow equations here, but this time, we denote the appropriate number to the right.

$$
\begin{aligned}
P_k &= \sum_{j=1}^{N} |V_k||V_j| \left(G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j) \right), \qquad k = 2,\dots,N \\
Q_k &= \sum_{j=1}^{N} |V_k||V_j| \left(G_{kj} \sin(\theta_k - \theta_j) - B_{kj} \cos(\theta_k - \theta_j) \right), \qquad k = N_G + 1,\dots,N
\end{aligned}
\tag{T7.19}
$$

We are trying to find the following information about the network:

a. The angles for the voltage phasors at all buses except the swing bus (it is 0° at the swing bus), i.e., θ_k, $k=2,\dots,N$
b. The magnitudes for the voltage phasors at all type PQ buses, i.e., $|V_k|$, $k=N_G+1, \dots, N$

Statements a and b imply that we have N-1 angle unknowns and $N-N_G$ voltage magnitude unknowns, for a total number of unknowns of $(N-1)+(N-N_G)=2N-N_G-1$. Referring to the power flow equations, eq. (T7.19), we see that there are no other unknowns on the right-hand side besides voltage magnitudes and angles (the real and imaginary parts of the admittance values, G_{kj} and B_{kj}, are known, based on statement 1 above).

Thus we see that the number of equations having known left-hand side (injections) is the same as the number of unknown voltage magnitudes and angles. Therefore it is possible to solve the system of $2N-N_G-1$ equations for the $2N-N_G-1$ unknowns. However, we note from eq. (T7.19) that these equations are not linear, i.e., they are nonlinear equations. This nonlinearity comes from the fact that we have terms containing products of some of the unknowns and also terms containing trigonometric functions of some of the unknowns. Because of these nonlinearities, we are not able to put them directly into the familiar matrix form of "Ax=b" (where A is a matrix, x is the vector of unknowns, and b is a vector of constants) to obtain their solution. We must therefore resort to some other methods that are applicable for solving nonlinear equations. We describe such a method in Section T7.6. Before doing that, however, it may be helpful to more crisply formulate the exact problem that we want to solve.

Let's first define the vector of unknown variables. This we do in two steps. First, define the vector of unknown angles $\underline{\theta}$ (an underline beneath the variable means it is a vector or a matrix) and the vector of unknown voltage magnitudes $|\underline{V}|$.

$$\underline{\theta}=\begin{bmatrix}\theta_2\\ \theta_3\\ \vdots\\ \theta_N\end{bmatrix}, \qquad |\underline{V}|=\begin{bmatrix}|V_{N_G+1}|\\ |V_{N_G+2}|\\ \vdots\\ |V_N|\end{bmatrix} \tag{T7.20}$$

Second, define the vector $\underline{x}$ as the composite vector of unknown angles and voltage magnitudes.

$$\underline{x}=\begin{bmatrix}\underline{\theta}\\ |\underline{V}|\end{bmatrix}=\begin{bmatrix}\theta_2\\ \theta_3\\ \vdots\\ \theta_N\\ |V_{N_G+1}|\\ |V_{N_G+2}|\\ \vdots\\ |V_N|\end{bmatrix}=\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_{N-1}\\ x_N\\ x_{N+1}\\ \vdots\\ x_{2N-N_G-1}\end{bmatrix} \tag{T7.21}$$

With this notation, we see that the right-hand sides of eqs. (T7.19) depend on the elements of the unknown vector $\underline{x}$. Expressing this dependence more explicitly, we rewrite eqs. (T7.19) as

$$\begin{aligned} P_k &= P_k(\underline{x}), \qquad k=2,\ldots,N \\ Q_k &= Q_k(\underline{x}), \qquad k=N_G+1,\ldots,N \end{aligned} \tag{T7.22}$$

In eqs. (T7.22), P_k and Q_k are the specified injections (known constants) while the right-hand sides are functions of the elements in the unknown vector $\underline{x}$. Bringing the left-hand side over to the right-hand side, we have that

$$
\begin{aligned}
P_k(\underline{x}) - P_k &= 0, \qquad k = 2,\ldots,N \\
Q_k(\underline{x}) - Q_k &= 0, \qquad k = N_G + 1,\ldots,N
\end{aligned}
\tag{T7.23}
$$

We now define a vector-valued function f(x) as:

$$
\underline{f}(\underline{x}) = \left[\begin{array}{c} f_1(\underline{x}) \\ \vdots \\ f_{N-1}(\underline{x}) \\ \hdashline f_N(\underline{x}) \\ \vdots \\ f_{2N-N_G-1}(\underline{x}) \end{array}\right] = \left[\begin{array}{c} P_2(\underline{x}) - P_2 \\ \vdots \\ P_N(\underline{x}) - P_N \\ \hdashline Q_{N_G+1}(\underline{x}) - Q_{N_G+1} \\ \vdots \\ Q_N(\underline{x}) - Q_N \end{array}\right] = \left[\begin{array}{c} \Delta P_2 \\ \vdots \\ \Delta P_N \\ \hdashline \Delta Q_{N_G+1} \\ \vdots \\ \Delta Q_N \end{array}\right] = \left[\begin{array}{c} 0 \\ \vdots \\ 0 \\ \hdashline 0 \\ \vdots \\ 0 \end{array}\right] = \underline{0} \tag{T7.24}
$$

The mismatch vector.

Equation (T7.24) is in the form of f(x)=0, where f(x) is a vector-valued function and 0 is a vector of zeros; both f(x) and 0 are of dimension $(2N\text{-}N_G\text{-}1)\times 1$, which is also the dimension of the vector of unknowns, x. We have also introduced nom[illegible]nting the *mismatch vector* in eq. (T7.24), as the vector of ΔP_k's and ΔQ_k's. This vector is used during the solution algorithm, which is iterative, to identify how good the solution is corresponding to any particular iteration. In the next section, we introduce this solution algorithm, which can be used to solve this kind of system of equations. The method is called the Newton-Raphson method.

The Newton-Raphson Solution Procedure

There are two basic methods for solving the power flow problem: Gauss-Siedel (GS) and Newton-Raphson (NR). Both of these methods are *iterative root finding schemes*.

The GS and NR methods are often classified as root finding schemes because they are geared towards solving equations like f(x)=0 (or f(x)=0). The solution to such an equation, call it x* (or x*), is clearly a root of the function f(x) (or f(x)).

The methods are called iterative because they require a series of successive approximations to the solutions. The procedure is generally as follows. First, guess a solution. Unless we are very fortunate, the guess will be, of course, wrong. So we determine an update to the "old" solution that moves to a "new" solution with the intention that the "new" solution is closer to the correct solution than was the "old" solution. A key aspect to this type of procedure is the way we obtain the update. If we can guarantee that the update is always improving the solution, such that the "new" solution is in fact always closer to the correct solution than the "old" solution, then such a procedure can be guaranteed to work if only we are willing to compute enough updates, i.e., if only we are willing to iterate enough times.

Commercial grade power flow programs may make several different solutions procedures available, but almost all such programs will have available, minimally, the NR method. It is fair to say that the NR method has become the de-facto industry standard. The main reason for this is that the *convergence properties* of the NR scheme are very desirable when the initial, guessed solution is quite good, i.e., when it is chosen close to the correct solution. In the power flow problem, it is usually possible to make a good initial guess regarding the solution. One reason for this is that often, we may actually know the solution of a particular set of conditions because we have already gone through the solution procedure, and we want to resolve for a set of conditions that are almost the same as the previous ones, e.g., maybe remove one circuit or change the load level a little. In this case, we may utilize the previous solution as the initial guessed solution for the new conditions. This is sometimes referred to as a "hot" start. But even if we do not have a previous solution, we still may do very well with our guess. The reason for this is that the power flow problem is always solved with all quantities in per-unit. Because of the way we choose per-unit voltage bases, the per-unit voltages for all buses, under any reasonably normal condition, will be close to 1.0 per-unit. Of course, this tells us nothing about the angles, but it *is* something, and often it is enough to simply guess that all voltages are 1.0 per-unit and all angles are 0 degrees. This is sometimes called a "flat" start.

But what are "convergence properties" of a root finding method? There are basically two of them. One is *whether* the method will converge. The second one is *how fast* the method will converge. For NR, whether the method will converge depends on two things: how close the guessed solution is to the correct solution and the nature of the function close to the correct solution. If the guessed solution is close, and if the function is reasonably "smooth" close to the correct solution, then the NR will converge. Not only that, but it will converge *quadratically*. Quadratic convergence means that each iteration increases the accuracy of the solution by two decimal places. For example, if the correct solution for a particular problem is 0.123456789, and we guess 0.100000000, then the first iteration will yield 0.123xxxxxx, the second iteration will yield 0.12345xxxx, the third iteration will yield 0.1234567xx, and the correct solution will be obtained exactly on the fourth iteration.

In this module, we will not discuss the GS method, but the interested reader may find information about it in many texts on power systems analysis or in books on numerical methods. We will introduce the NR method with a simple illustration, obtained from [3].

Example T7.2

Consider the scalar function $f(x)=x^2-5x+4$. This function may be easily factored to find the roots as $x^*=4,1$.

Let us now illustrate how the NR method finds one of these roots. We first need the derivative: $f'(x)=2x-5$. Assume we are bad guessers, and try an initial guess of $x^{(0)}=6$. The following provides the first two iterations:

1. $f(x^{(0)})=f(6)=6^2-5(6)+4=10$
2. $f'(x^{(0)})=f'(6)=2(6)-5=7$
3. $\Delta x^{(0)}= -f(x^{(0)})/f'(x^{(0)})= -10/7=-1.429$
4. $x^{(1)}=x^{(0)}+\Delta x^{(0)}=6+(-1.429)=4.571$

1. $f(x^{(1)})=f(4.571)=2.03904$
2. $f'(x^{(1)})=f'(4.571)=4.142$
3. $\Delta x^{(1)}= -f(x^{(1)})/f'(x^{(1)})= -2.03904/4.142=-0.492284$
4. $x^{(2)}=x^{(1)}+\Delta x^{(1)}=4.571+(-0.492284)=4.0787$

One more iteration yields $x^{(3)}=4.002$. Note that by the third iteration, as it is getting very close to the correct solution, the algorithm has almost obtained quadratic convergence. Fig. T7.7 illustrates how the first solution $x^{(1)}$ is found from the initial guessed solution $x^{(0)}$ during the first iteration of this algorithm.

The NR algorithm is not smart enough to know which root you want, rather, it generally finds the closest root. This is another reason for making a good initial guess in regards to the solution. Fortunately, in the case of the power flow problem, alternative solutions are typically “far away” from initial guesses that have near-unity bus voltage magnitudes. On the other hand, it is possible for the solution to diverge, i.e., not to converge at all. This may occur if there is simply no solution, which is a case that engineers encounter frequently when studying highly stressed loading conditions served by weak transmission systems. It also might occur if the initial guessed solution is too far away from the correct solution. For this reason, “flat” starts encounter solution divergence more frequently than “hot” starts.

Graphical illustration of NR iteration 1 for a single variable root finding problem

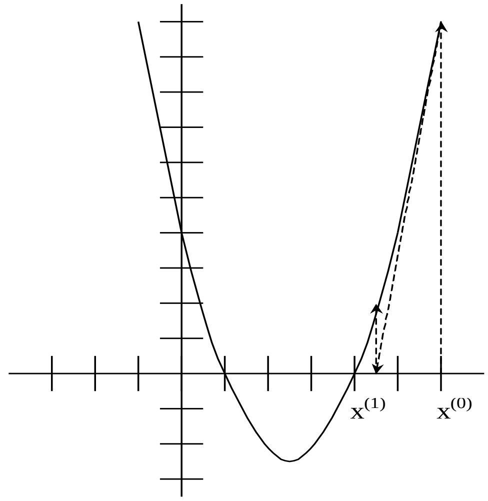


Fig. T7.7: Illustration of the first iteration of the Newton-Raphson algorithm

Next, we develop the NR update formula. We begin with the scalar case, where the update formula may be easily inferred from Example T7.2.

<u>Newton Raphson for the Scalar Case</u>:

Assume that we have guessed a solution $x^{(0)}$ to the problem f(x)=0. Then $f(x^{(0)})\neq 0$ because $x^{(0)}$ is just a guess. But there must be some $\Delta x^{(0)}$ which will make $f(x^{(0)} + \Delta x^{(0)})=0$. One way to study this problem is to expand the function f(x) in a Taylor series, as follows:

$$f(x^{(0)}+\Delta x^{(0)}) = f(x^{(0)}) + f'(x^{(0)})\Delta x^{(0)} + \frac{1}{2} f''(x^{(0)})(\Delta x^{(0)})^2 + ... = 0 \qquad \text{(T7.25)}$$

If the guess is a good one, then $\Delta x^{(0)}$ will be small, and if this is true, then $(\Delta x^{(0)})^2$ will be very small, and any higher order terms (h.o.t.) in eq. (T7.25), which will contain $\Delta x^{(0)}$ raised to even higher powers, will be infinitesimal. As a result, it is reasonable to approximate eq. (T7.25) as

$$f(x^{(0)}+\Delta x^{(0)}) \approx f(x^{(0)}) + f'(x^{(0)})\Delta x^{(0)} = 0 \qquad \text{(T7.26)}$$

Taking $f(x^{(0)})$ to the right hand side, we have

$$f'(x^{(0)})\Delta x^{(0)} = -f(x^{(0)}) \tag{T7.27}$$

We may easily solve eq. (T7.27) for $\Delta x^{(0)}$ according to:

$$\Delta x^{(0)} = -\left\{f'(x^{(0)})\right\}^{-1} f(x^{(0)}) \tag{T7.28}$$

Because f ’($x^{(0)}$) in eq. (T7.28) is scalar, it’s inverse is very easily evaluated using simple division so that:

$$\Delta x^{(0)} = \frac{-f(x^{(0)})}{f'(x^{(0)})} \tag{T.29}$$

Equation (T7.28) provides the basis for the update formula to be used in the first iteration of the scalar NR method. This update formula is:

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = x^{(0)} + \frac{-f(x^{(0)})}{f'(x^{(0)})} \tag{T7.30}$$

and from eq. (T7.28), we may infer the update formula for any particular iteration as:

$$x^{(j+1)} = x^{(j)} + \Delta x^{(j)} = x^{(j)} + \frac{-f(x^{(j)})}{f'(x^{(j)})} \tag{T7.31}$$

Next we develop the update formula for the case where we have n equations and n unknowns. We call this the multidimensional case.

<u>Newton Raphson for the Multidimensional Case</u>:

Assume we have *n* nonlinear algebraic equations and *n* unknowns characterized by $\underline{f}(\underline{x})=\underline{0}$, and that we have guessed a solution $\underline{x}^{(0)}$. Then $\underline{f}(\underline{x}^{(0)})\neq\underline{0}$ because $\underline{x}^{(0)}$ is just a guess. But there must be some $\Delta\underline{x}^{(0)}$ which will make $\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)})=\underline{0}$. Again, we expand the function $\underline{f}(\underline{x})$ in a Taylor series, as follows:

$$\begin{aligned}
f_1(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) &= f_1(\underline{x}^{(0)}) + f_1'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2} f_1''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = 0 \\
f_2(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) &= f_2(\underline{x}^{(0)}) + f_2'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2} f_2''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = 0 \qquad \text{(T7.32)} \\
&\vdots
\end{aligned}$$

$$f_n(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = f_n(\underline{x}^{(0)}) + f_n'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2} f_n''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + ... = 0$$

Equations (T7.32) may be written more compactly as

$$\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = \underline{f}(\underline{x}^{(0)}) + \underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2}\underline{f}''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + ... = \underline{0} \qquad \text{(T7.33)}$$

Assuming the guess is a good one such that $\Delta\underline{x}^{(0)}$ is small, then the higher order terms are also small and we can write

$$\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = \underline{f}(\underline{x}^{(0)}) + \underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} = \underline{0} \qquad \text{(T7.34)}$$

One reasonable question to ask at this point is: “Just what is $\underline{f}'(\underline{x}^{(0)})$?” That is, what is the derivative of a vector-valued function of a vector? Since we have *n* functions and *n* variables, we could compute a derivative for each individual function with respect to each individual unknown, like $\partial f_k(\underline{x})/\partial x_j$, which gives the derivative of the k^{th} function with respect to the j^{th} unknown. Thus, there will be a number of such derivatives equal to the product of the number of functions by the number of unknowns, in this case, $n \times n$. Thus, it is convenient to store all of these derivatives in a matrix. This matrix has become quite well-known as the *Jacobian* matrix, and it is often denoted using the letter $\underline{J}$. But how should the $n \times n$ derivatives be stored in this matrix $\underline{J}$?

The rows of $\underline{J}$ should be ordered in the same order as the functions, that is, the k^{th} row should contain the derivatives of the k^{th} functions. In eq. (T7.34), since the product $\underline{f}'(\underline{x}^{(0)})\ \Delta\underline{x}^{(0)}$ must provide a correction to the function $\underline{f}(\underline{x}^{(0)}+\Delta\underline{x}^{(0)})$, i.e., since $\Delta\underline{f}(\underline{x}^{(0)}) = \underline{f}'(\underline{x}^{(0)})\ \Delta\underline{x}^{(0)}$, it must be the case that any row of the matrix $\underline{J}$ must be ordered so that the term in the j^{th} column contains a derivative with respect to the j^{th} unknown of the vector $\underline{x}$.

The reasoning in the last paragraph suggests that we write the Jacobian matrix as:

$$\underline{J} = \begin{bmatrix} \dfrac{\partial f_1(\underline{x}^{(0)})}{\partial x_1} & \dfrac{\partial f_1(\underline{x}^{(0)})}{\partial x_2} & \cdots & \dfrac{\partial f_1(\underline{x}^{(0)})}{\partial x_n} \\ \dfrac{\partial f_2(\underline{x}^{(0)})}{\partial x_1} & \dfrac{\partial f_2(\underline{x}^{(0)})}{\partial x_2} & \cdots & \dfrac{\partial f_2(\underline{x}^{(0)})}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial f_n(\underline{x}^{(0)})}{\partial x_1} & \dfrac{\partial f_n(\underline{x}^{(0)})}{\partial x_2} & \cdots & \dfrac{\partial f_n(\underline{x}^{(0)})}{\partial x_n} \end{bmatrix} \tag{T7.35}$$

In eq. (T7.34), taking $\underline{f}(\underline{x}^{(0)})$ to the right hand side, we have

$$\underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} = -\underline{f}(\underline{x}^{(0)}) \tag{T7.36}$$

or, in terms of the Jacobian matrix $\underline{J}$, we have:

$$\underline{J}\Delta\underline{x}^{(0)} = -\underline{f}(\underline{x}^{(0)}) \tag{T7.37}$$

Solving eq. (T7.37) for $\Delta\underline{x}^{(0)}$, we have:

$$\Delta\underline{x}^{(0)} = -\left\{\underline{f}'(\underline{x}^{(0)})\right\}^{-1}\underline{f}(\underline{x}^{(0)}) = -\underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) \tag{T7.38}$$

Equation (T7.38) provides the basis for the update formula to be used in the first iteration of the multi-dimensional case. This update formula is:

$$\underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) \tag{T7.39}$$

and from eq. (T7.39), we may infer the update formula for any particular iteration as:

$$\underline{x}^{(i+1)} = \underline{x}^{(i)} + \Delta\underline{x}^{(i)} = \underline{x}^{(i)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(i)}) \tag{T7.40}$$

For problems of relatively small dimension, where the inverse of the Jacobian is easily obtainable, eq. (T7.40) is an appropriate update formula. In general, however, it is a good rule, in programming, to always avoid matrix inversion if at all possible, because for high-dimension problems, as is usually the case for large scale power networks, matrix inversion is *very* time consuming. We always want to avoid matrix inversion if possible, and it usually is.

To see how to avoid matrix inversion, we will state the update formula a little differently. To do this, we write eq. (T7.40) as

$$\underline{x}^{(i+1)} = \underline{x}^{(i)} + \Delta\underline{x}^{(i)} \qquad \text{(T7.41)}$$

where $\Delta\underline{x}^{(i)}$ is found from

$$-\underline{J}\Delta\underline{x}^{(i)} = \underline{f}(\underline{x}^{(i)}) \qquad \text{(T7.42a)}$$

Equation (T7.42a) is a very simple relation. Observing that J is just a constant n×n matrix, $\Delta\underline{x}^{(i)}$ is an n×1 vector of unknowns, and $\underline{f}(\underline{x}^{(i)})$ is an n×1 vector of knowns, we see that eq. (T7.42) is just the linear matrix equation

$$\underline{A}\ \underline{z}=\underline{b} \qquad \text{(T7.42b)}$$

There are a very many methods of solving (T7.42b). We will cover this topic later in these notes. First, however, let's illustrate the Newton-Raphson procedure for a multi-dimensional case. We will use the simplest multi-dimensional case we can, a two-variable problem.

Example T7.3

Solve the following two equations algebraically and using NR:

$$2x_1^2+x_1x_2-x_1-2=0,$$
$$x_1^2 -x_2=0$$

The steps for the algebraic solution are to first solve both equations for x_2, resulting in $x_2=(-2x_1^2+x_1+2)/x_1$ and $x_2=x_1^2$. Equating these two expressions for x_2, and manipulating, results in a cubic $x_1^3+2x_1^2-x_1-2=0$. This expression may be factored as: $(x_1-1)\ (x_1+1)(x_1+2)=0$, and we see that the solutions to the cubic in x_1 are 1, -1 and –2. Plugging these values for x_1 back into either expression for x_2 yields, respectively, 1, 1, and 4, and therefore there are three solutions to the original problem; they are: $(x_1, x_2)=(1,1), (-1,1), (-2,4)$.

Now let's solve this same problem using NR?

Define the functions $f_1(x_1,x_2)= 2x_1^2+x_1x_2-x_1-2$ and $f_2(x_1,x_2)= x_1^2 -x_2$. Then the Jacobian matrix is:

$$\underline{J} = \begin{bmatrix} \dfrac{\partial f_1(x_1,x_2)}{\partial x_1} & \dfrac{\partial f_1(x_1,x_2)}{\partial x_2} \\ \dfrac{\partial f_2(x_1,x_2)}{\partial x_1} & \dfrac{\partial f_2(x_1,x_2)}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 2x_1 + x_2 - 1 & x_1 \\ 2x_1 & -1 \end{bmatrix}$$

Let's act like we do not know the solution and guess at $(x_1^{(0)}, x_2^{(0)})=(0.9,1.1)$. Then the Jacobian $\underline{J}$, evaluated at this guessed solution, is

$$\underline{J}(x_1^{(0)}, x_2^{(0)}) = \begin{bmatrix} 2x_1 + x_2 - 1 & x_1 \\ 2x_1 & -1 \end{bmatrix}\Bigg|_{\underline{x}^{(0)}} = \begin{bmatrix} 1.9 & 0.9 \\ 1.8 & -1 \end{bmatrix}$$

Inverting the Jacobian results in:

$$\underline{J}^{-1} = \begin{bmatrix} 1.9 & 0.9 \\ 1.8 & -1 \end{bmatrix}^{-1} = \frac{1}{-3.52}\begin{bmatrix} -1 & -0.9 \\ -1.8 & 1.9 \end{bmatrix} = \begin{bmatrix} 0.28409 & 0.25568 \\ 0.51136 & -0.53977 \end{bmatrix}$$

We also need to evaluate:

$$\underline{f}(\underline{x}^{(0)}) = \begin{bmatrix} f_1(\underline{x}^{(0)}) \\ f_2(\underline{x}^{(0)}) \end{bmatrix} = \begin{bmatrix} 2x_1 + x_1x_2 - x_1 - 2 \\ x_1^{\,2} - x_2 \end{bmatrix}\Bigg|_{\underline{x}^{(0)}=(0.9,1.1)} = \begin{bmatrix} -0.11 \\ -0.29 \end{bmatrix}$$

We can now update the solution using eq. (T7.40), as

$$\underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)})$$

$$= \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.1 \end{bmatrix} - \begin{bmatrix} 0.28409 & 0.25568 \\ 0.51136 & -0.53977 \end{bmatrix}\begin{bmatrix} -0.11 \\ -0.29 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.1 \end{bmatrix} - \begin{bmatrix} -0.105397 \\ 0.100284 \end{bmatrix} = \begin{bmatrix} 1.005397 \\ 0.999716 \end{bmatrix}$$

We see that the first update results in a solution that is very close to the actual solution of (1,1). This good performance is due to the fact that we made a good initial guess. The student should repeat the above procedure, but try starting from other points, e.g., (-0.9,1.1), (-1.9,4.1), and (0,1.1), using two iterations each time. Writing a simple program will greatly reduce the effort.

In general, of course, we usually need to iterate several times in order to obtain a satisfactory solution. How many times is enough? The NR algorithm must employ a *stopping criterion* in order to determine when the solution is satisfactory. There are two ways to do this.

- Type 1 stopping criterion: Test the maximum change in the solution elements from one iteration to the next, and if this maximum change is smaller than a certain predefined tolerance, then stop. This means to compare the maximum absolute value of elements in $\Delta\underline{x}$ against a small number, call it ε_1. In example (T7.3), $\Delta\underline{x}$ = [-0.105397, 0.100284]^T, so the maximum absolute value of elements in $\Delta\underline{x}$ is 0.105397. If we had ε_1=0.15, we could stop. But if we had ε_1=0.05, we would need to continue to the next iteration.
- Type 2 stopping criterion: Test the maximum value in the function elements of the most current iteration $\underline{f}(\underline{x})$, and if this maximum value of elements in $\underline{f}(\underline{x})$ is smaller than a certain predefined tolerance, then stop. This means to compare the maximum absolute value of elements in $\underline{f}(\underline{x})$ against a small number, call it ε_1. In example (T7.3), $\underline{f}(\underline{x})$=[-0.11, -0.29]^T, so the maximum absolute value of elements in $\underline{f}(\underline{x})$ is 0.29. If we had ε_1=0.3, we could stop. But if we had ε_1=0.2, we would need to continue to the next iteration. This is the most common stopping criterion for power flow solutions, and the value of each element in the function is referred to as the "power mismatch" for the bus corresponding to the function. For type PQ buses, we test both real and reactive power mismatches. For type PV buses, we test only real power mismatches.

Application of NR to Power Flow Solution

Let's revisit the power flow problem outlined in Section T7.5, in light of the NR solution procedure described in Section T7.6. We desire to solve eq. (T7.24), with the vector of unknowns are given by eq. (T7.21) and the functions are in the form of eq. (T7.19). These equations are repeated here for convenience:

$$\underline{f}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ \vdots \\ f_{N-1}(\underline{x}) \\ ------- \\ f_N(\underline{x}) \\ \vdots \\ f_{2N-N_G-1}(\underline{x}) \end{bmatrix} = \begin{bmatrix} P_2(\underline{x}) - P_2 \\ \vdots \\ P_N(\underline{x}) - P_N \\ ------- \\ Q_{N_G+1}(\underline{x}) - Q_{N_G+1} \\ \vdots \\ Q_N(\underline{x}) - Q_N \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_N \\ ------- \\ \Delta Q_{N_G+1} \\ \vdots \\ \Delta Q_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ ------- \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta\underline{P} \\ \Delta\underline{Q} \end{bmatrix} = \underline{0} \quad \text{(T7.24)}$$

$$\underline{x} = \begin{bmatrix} \underline{\theta} \\ |\underline{V}| \end{bmatrix} = \begin{bmatrix} \theta_2 \\ \theta_3 \\ \vdots \\ \theta_N \\ |V_{N_G+1}| \\ |V_{N_G+2}| \\ \vdots \\ |V_N| \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N-1} \\ x_N \\ x_{N+1} \\ \vdots \\ x_{2N-N_G-1} \end{bmatrix} \qquad \text{(T7.21)}$$

$$\begin{aligned} P_k &= \sum_{j=1}^{N} |V_k||V_j|\left(G_{kj}\cos(\theta_k-\theta_j)+B_{kj}\sin(\theta_k-\theta_j)\right), \qquad k=2,\ldots,N \\ Q_k &= \sum_{j=1}^{N} |V_k||V_j|\left(G_{kj}\sin(\theta_k-\theta_j)-B_{kj}\cos(\theta_k-\theta_j)\right), \qquad k=N_G+1,\ldots,N \end{aligned} \qquad \text{(T7.19)}$$

The solution update formula is given by eq. (T7.40), repeated here for convenience:

$$\underline{x}^{(i+1)} = \underline{x}^{(i)} + \Delta\underline{x}^{(i)} = \underline{x}^{(i)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(i)}) \qquad \text{(T7.40)}$$

Clearly, an essential step in applying NR to the power flow problem is to enable calculation of the Jacobian elements, given for the general case by eq. (T7.35) as

$$\underline{J} = \begin{bmatrix} \frac{\partial f_1(\underline{x}^{(0)})}{\partial x_1} & \frac{\partial f_1(\underline{x}^{(0)})}{\partial x_2} & \cdots & \frac{\partial f_1(\underline{x}^{(0)})}{\partial x_n} \\ \frac{\partial f_2(\underline{x}^{(0)})}{\partial x_1} & \frac{\partial f_2(\underline{x}^{(0)})}{\partial x_2} & \cdots & \frac{\partial f_2(\underline{x}^{(0)})}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n(\underline{x}^{(0)})}{\partial x_1} & \frac{\partial f_n(\underline{x}^{(0)})}{\partial x_2} & \cdots & \frac{\partial f_n(\underline{x}^{(0)})}{\partial x_n} \end{bmatrix} \qquad \text{(T7.35)}$$

Evaluation of these elements is facilitated by the recognition, from eq. (T7.24), that there are only two kinds of equations (real power equations and reactive power equations), and from eq. (T7.21), that there are only two kinds of unknowns (voltage angle unknowns and voltage magnitude unknowns). Therefore, there are only four basic types of derivatives in the Jacobian. We denote four sub-matrices corresponding to these four basic types of derivatives as $\underline{J}^{P\theta}$, $\underline{J}^{Q\theta}$, $\underline{J}^{PV}$, $\underline{J}^{QV}$, where the first superscript indicates the type of equation we differentiate, and the second superscript indicates the unknown with respect to which we differentiate. Therefore,

$$\overbrace{\underline{J}}^{(2N-1-N_G)\times(2N-1-N_G)} = \begin{bmatrix} \overbrace{\underline{J}^{P\theta}}^{(N-1)\times(N-1)} & \overbrace{\underline{J}^{PV}}^{(N-1)\times(N-N_G)} \\ \overbrace{\underline{J}^{Q\theta}}^{(N-N_G)\times(N-1)} & \overbrace{\underline{J}^{QV}}^{(N-N_G)\times(N-N_G)} \end{bmatrix} \qquad \text{(T7.43)}$$

The numbers above each sub-matrix in eq. (T7.43) indicate its dimensions, which can be inferred by identifying the number of equations of that type (the number of rows of the sub-matrix) and the number of unknowns of that type (the number of columns of the sub-matrix). We may then identify an individual element of each sub-matrix as:

$$J_{jk}{}^{P\theta} = \frac{\partial P_j}{\partial \theta_k} \qquad J_{jk}{}^{Q\theta} = \frac{\partial Q_j}{\partial \theta_k} \qquad J_{jk}{}^{PV} = \frac{\partial P_j}{\partial |V_k|} \qquad J_{jk}{}^{QV} = \frac{\partial Q_j}{\partial |V_k|} \qquad \text{(T7.44)}$$

Notationally, observe that the element $J_{jk}{}^{P\theta}$ is *not* the element in row j, column k of the submatrix $\underline{J}^{P\theta}$, rather it is the derivative of the real power injection equation for bus j with respect to the angle of bus k. Since the swing bus is numbered 1, the Jacobian matrix will have $J_{22}{}^{P\theta}$ as the element in row 1, column 1. The situation is similar for the other submatrices.

The update equation (T7.42a) is repeated here for convenience:

$$-\underline{J}\Delta\underline{x}^{(i)} = \underline{f}(\underline{x}^{(i)}) \qquad \text{(T7.42a)}$$

Multiplying both sides by -1, we obtain

$$\underline{J}\Delta\underline{x}^{(i)} = -\underline{f}(\underline{x}^{(i)}) \qquad \text{(T7.42c)}$$

Using (T7.21), (T7.24), and (T7.43) we can write (T7.42c) as

$$\begin{bmatrix} \underline{J}^{P\theta} & \underline{J}^{PV} \\ \underline{J}^{Q\theta} & \underline{J}^{QV} \end{bmatrix} \begin{bmatrix} \Delta\underline{\theta} \\ \Delta|\underline{V}| \end{bmatrix} = -\begin{bmatrix} \Delta\underline{P} \\ \Delta\underline{Q} \end{bmatrix} \qquad \text{(T7.42d)}$$

From (T7.42d) we observe that

$$\begin{aligned} \underline{J}^{P\theta}\Delta\underline{\theta} + \underline{J}^{PV}\Delta|\underline{V}| &= -\Delta\underline{P} \\ \underline{J}^{Q\theta}\Delta\underline{\theta} + \underline{J}^{QV}\Delta|\underline{V}| &= -\Delta\underline{Q} \end{aligned} \qquad \text{(T7.42e)}$$

To get the needed derivatives, it is helpful to more explicitly write out the functions of eq. (T7.24). They are:

$$
\underline{f}(\underline{x}) = \begin{bmatrix} P_2(\underline{x}) - P_2 \\ \vdots \\ P_N(\underline{x}) - P_N \\ ------- \\ Q_{N_G+1}(\underline{x}) - Q_{N_G+1} \\ \vdots \\ Q_N(\underline{x}) - Q_N \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^{N} |V_2||V_j| \left(G_{2j}\cos(\theta_2 - \theta_j) + B_{2j}\sin(\theta_2 - \theta_j)\right) - P_2 \\ \vdots \\ \sum_{j=1}^{N} |V_N||V_j| \left(G_{Nj}\cos(\theta_N - \theta_j) + B_{Nj}\sin(\theta_N - \theta_j)\right) - P_N \\ ------- \\ \sum_{j=1}^{N} |V_{N_G+1}||V_j| \left(G_{N_G+1,j}\sin(\theta_{N_G+1} - \theta_j) + B_{N_G+1,j}\sin(\theta_{N_G+1} - \theta_j)\right) - P_{N_G+1} \\ \vdots \\ \sum_{j=1}^{N} |V_N||V_j| \left(G_{N,j}\sin(\theta_N - \theta_j) + B_{N,j}\sin(\theta_N - \theta_j)\right) - P_N \end{bmatrix} \qquad \text{(T7.45)}
$$

So each of the four sub-matrices of eq. (T7.43) has elements given by the expressions of eq. (T7.44), respectively. These expressions are evaluated by taking the appropriate derivatives of the functions in eq. (T7.45). One might think that this represents a formidable problem, since, based on eq. (T7.43), we have $(2N\text{-}1\text{-}N_G)\times(2N\text{-}1\text{-}N_G)$ elements in the Jacobian and therefore the same number of derivatives to evaluate. A typical power flow model for a US control area might have 5000 nodes (N=5000) and 1000 generators (N_G=1000), resulting in a 9898×9898 Jacobian matrix containing 97,970,404 elements, with each element requiring a differentiation of a function like those represented in eq. (T7.45). For a power flow model having 50000 nodes and 5000 generators, the dimension is 94998×94998, giving 9,024,600,000 elements.

Fortunately, all of the derivatives can be expressed by one of just a few differentiations. At first glance, one might think that there would be four differentiations, one for each sub-matrix. However, for each sub-matrix, the off-diagonal terms, with j≠k, are expressed differently than the diagonal terms, with j=k. Therefore, there are eight differentiations to perform. The student should attempt to obtain a few of these expressions. In doing so, the following tips are helpful.

- Before differentiating, it is helpful to pull out from the summation the term that corresponds to the bus injection being computed.
- When differentiating a sum of terms with respect to a particular unknown, the resulting derivative will be non-zero only for those terms in which the unknown appears.
- When differentiating with respect to the angles, the chain rule must be properly applied to account for the derivatives of the trigonometric functions and the arguments of those trigonometric functions.
- Each of the functions appear in the form of $f(\underline{x})=g(\underline{x})-A$. Because A is a constant (represented by $P_2,\ldots, P_N$ and $Q_{Ng+1},\ldots, Q_N$ in eq. (T7.45)), it has no effect on the resulting derivatives.

The resulting expressions are given below.

$$J_{jk}{}^{P\theta} = \frac{\partial P_j(\underline{x})}{\partial \theta_k} = |V_j||V_k|\left(G_{jk}\sin(\theta_j - \theta_k) - B_{jk}\cos(\theta_j - \theta_k)\right) \qquad \text{(T7.47)}$$

$$J_{jj}{}^{P\theta} = \frac{\partial P_j(\underline{x})}{\partial \theta_j} = -Q_j(\underline{x}) - B_{jj}|V_j|^2 \qquad \text{(T7.48)}$$

$$J_{jk}{}^{Q\theta} = \frac{\partial Q_j(\underline{x})}{\partial \theta_k} = -|V_j||V_k|\left(G_{jk}\cos(\theta_j - \theta_k) + B_{jk}\sin(\theta_j - \theta_k)\right) \qquad \text{(T7.49)}$$

$$J_{jj}{}^{Q\theta} = \frac{\partial Q_j(\underline{x})}{\partial \theta_k} = P_j(\underline{x}) - G_{jj}|V_j|^2 \qquad \text{(T7.50)}$$

$$J_{jk}{}^{PV} = \frac{\partial P_j(\underline{x})}{\partial |V_k|} = |V_j|\left(G_{jk}\cos(\theta_j - \theta_k) + B_{jk}\sin(\theta_j - \theta_k)\right) \qquad \text{(T7.51)}$$

$$J_{jj}{}^{PV} = \frac{\partial P_j(\underline{x})}{\partial |V_j|} = \frac{P_j(\underline{x})}{|V_j|} + G_{jj}|V_j| \qquad \text{(T7.52)}$$

$$J_{jk}{}^{QV} = \frac{\partial Q_j(\underline{x})}{\partial |V_k|} = |V_j|\left(G_{jk}\sin(\theta_j - \theta_k) - B_{jk}\cos(\theta_j - \theta_k)\right) \qquad \text{(T7.53)}$$

$$J_{jj}{}^{QV} = \frac{\partial Q_j(\underline{x})}{\partial |V_j|} = \frac{Q_j(\underline{x})}{|V_j|} - B_{jj}|V_j| \qquad \text{(T7.54)}$$

We are now in a position to provide the algorithm for using NR to solve the power flow problem. Before doing so, it is helpful to more explicitly define the mismatch vector, from eq. (T7.24) or (T7.45) as:

$$\underline{f}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ \vdots \\ f_{N-1}(\underline{x}) \\ -------- \\ f_N(\underline{x}) \\ \vdots \\ f_{2N-N_G-1}(\underline{x}) \end{bmatrix} = \begin{bmatrix} P_2(\underline{x}) - P_2 \\ \vdots \\ P_N(\underline{x}) - P_N \\ -------- \\ Q_{N_G+1}(\underline{x}) - Q_{N_G+1} \\ \vdots \\ Q_N(\underline{x}) - Q_N \end{bmatrix} = \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta P_N \\ -------- \\ \Delta Q_{N_G+1} \\ \vdots \\ \Delta Q_N \end{bmatrix} = \begin{bmatrix} \underline{\Delta P} \\ ------ \\ \underline{\Delta Q} \end{bmatrix} = \underline{0} \quad \text{(T7.53)}$$

The NR algorithm, for application to the power flow problem, is:

1. Specify:
    - All admittance data
    - P_d and Q_d for all buses
    - P_g and $|V|$ for all PV buses
    - $|V|$ for swing bus, with $\theta=0°$
2. Let the iteration counter j=1. Use one of the following to guess the initial solution.
    - Flat Start: $V_k=1.0 \angle 0°$ for all buses.
    - Hot Start: Use the solution to a previously solved case for this network.
3. Compute the mismatch vector for $\underline{x}^{(j)}$, denoted as $\underline{f}(\underline{x})$ in eq. (T7.24) and eq. (T7.45). In what follows, we denote elements of the mismatch vector as ΔP_k and ΔQ_k corresponding to the real and reactive power mismatch, respectively, for the k^{th} bus (which would not be the k^{th} element of the mismatch vector for two reasons: one reason pertains to the swing bus and the other reason to the fact that for type PQ buses, there are two equations per bus and not one – see boxed comments next to eq. T7.44). This computation will also result in all necessary calculated real and reactive power injections.
4. Perform the following stopping criterion tests:

    If $|\Delta P_k| < \varepsilon_P$ for all type PQ and PV buses and

    If $||\Delta Q_k| < \varepsilon_Q$ for all type PQ buses,

    Then go to step 6

    Otherwise, go to step 5.
5. Find an improved solution as follows:
    - Evaluate the Jacobian $\underline{J}$ at $\underline{x}^{(j)}$. Denote this Jacobian as $\underline{J^{(j)}}$
    - Solve for $\Delta\underline{x}^{(j)}$ from:

$$\underline{J}^{(j)}\Delta\underline{x}^{(j)} = -\begin{bmatrix} \underline{\Delta P} \\ ---- \\ \underline{\Delta Q} \end{bmatrix} \qquad \text{or} \qquad \Delta\underline{x}^{(j)} = -\left[\underline{J}^{(j)}\right]^{-1}\begin{bmatrix} \underline{\Delta P} \\ ---- \\ \underline{\Delta Q} \end{bmatrix}$$

    where we must use **factorization** with the left equation if the system is large, but if the system is not large, we may use the right hand equation.

    - Compute the updated solution vector as $\underline{x}^{(j+1)} = \underline{x}^{(j)} + \Delta\underline{x}^{(j)}$.
    - Return to step 3 with j=j+1.
6. Stop.

The above algorithm is applicable as long as all PV buses remain within their reactive limits. To account for generator reactive limits, we must modify the algorithm so that, **at each iteration**, we check to ensure PV bus reactive generation is within its limits (see Section T7.1 regarding modeling of reactive limits). In this case, steps 1-4 remain exactly as given above, but we need a new step 5 and 6, as follows:

5. Check reactive limits for all generator buses as follows:
   a. For all type PV buses, perform the following test:
      - If $Q_{gk}>Q_{gk,max}$, then
        ➔ $Q_{gk}=Q_{gk,max}$ and CHANGE bus k to a type PQ bus (see step 6a)
      - If $Q_{gk}< Q_{gk,min}$, then
        ➔ $Q_{gk}=Q_{gk,min}$ and CHANGE bus k to a type PQ bus (see step 6b)
   b. For all type PQ generator buses, perform the following test:
      - If $Q_{gk}=Q_{gk,max}$ and $|V_k|>|V_{k,set}|$ or if $Q_{gk}=Q_{gk,min}$ and $|V_k|<|V_{k,set}|$, then
        ➔ CHANGE this bus back to a type PV bus (see step 6b)
6. If there were no CHANGES in Step 5, then stop. If there were one or more CHANGES in step 5, then modify the solution vector and the mismatch vector as follows:
   a. For each CHANGE made in step 5-a (changing a PV bus to a PQ bus):
      - $N_G=N_G-1$
      - Include the variable ΔV_k to the vector $\Delta \underline{x}$ and the variable V_k to the vector $\underline{x}$.
      - Include the reactive equation corresponding to bus k to the vector $\underline{f}(\underline{x})$.
      - Modify the Jacobian by including a column to $\underline{J}^{PV}$ and including a row to $\underline{J}^{Q\theta}$ and $\underline{J}^{QV}$.
   b. For each CHANGE made in Step 5-b (changing a PQ gen bus back to a PV bus):
      - $N_G=N_G+1$
      - Remove the variable ΔV_k to the vector $\Delta \underline{x}$ and the variable V_k from the vector $\underline{x}$.
      - Remove the reactive equation corresponding to bus k from the vector $\underline{f}(\underline{x})$.
      - Modify the Jacobian by removing a column to $\underline{J}^{PV}$ and removing a row from $\underline{J}^{Q\theta}$ and $\underline{J}^{QV}$.

   After modifications have been made for all CHANGES, go back to Step 4.

When the algorithm stops, then all line flows may be computed using

$$S_{jk} = V_j I_{jk}^* = V_j [V_j - V_k]^* y_{jk}^*$$

Example T7.4 [5] (used with permission of V. Vittal)

Find θ_2, $|V_3|$, θ_3, S_{G1}, and Q_{G2} for the system shown in Fig. T7.8. In the transmission system all the shunt elements are capacitors with an admittance $y_c = j0.01$, while all the series elements are inductors with an impedance of $z_L = j0.1$.

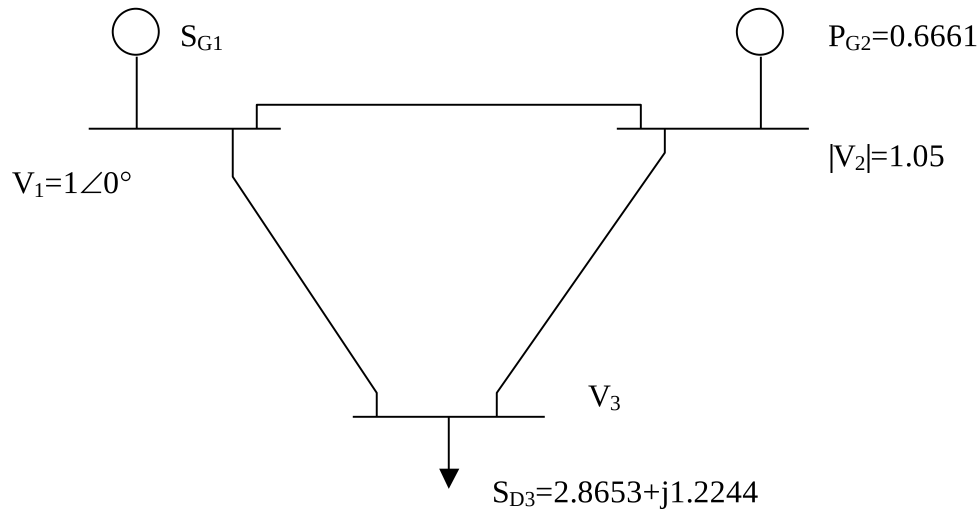


Fig. T7.8: Three Bus System for Example T7.4

Solution: The admittance matrix for the system shown in Fig. E10.6 is given by

$$\underline{Y} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

Bus 1 is the swing bus. Bus 2 is a PV bus. Bus 3 is a PQ bus. We use the NR method in the solution. The unknown variables are θ_2, θ_3, and $|V_3|$. Thus, we will need three equations, and the Jacobian is a 3 x 3 matrix.

We first write eq. (T7.45) for the case at hand, putting in the known values of $|V_1|$, $|V_2|$, θ_1, and the B_{ij}'s. Note that since we have neglected line resistance in the problem statement, all G_{ij}'s are zero.

$$P_2(\boldsymbol{x}) = |V_2||V_1|B_{21}\sin(\theta_2 - \theta_1) + |V_2||V_3|B_{23}\sin(\theta_2 - \theta_3)$$
$$= 10.5\sin\theta_2 + 10.5|V_3|\sin(\theta_2 - \theta_3) \quad \text{(T7.54a)}$$

$$P_3(\boldsymbol{x}) = |V_3||V_1|B_{31}\sin(\theta_3 - \theta_1) + |V_3||V_2|B_{32}\sin(\theta_3 - \theta_2)$$
$$= 10.0|V_3|\sin\theta_3 + 10.5|V_3|\sin(\theta_3 - \theta_2) \quad \text{(T7.54b)}$$

The equation for $Q_2(\underline{x})$ will not help since we do not know the reactive injection for bus 2, and its inclusion would bring in the reactive injection on the left-hand side as an additional unknown. But this loss of an equation is compensated by the fact that we know $|V_2|$ (and this will always be the case for a type PV bus). So we do not need to write the equation for $Q_2(\underline{x})$. Yet, because bus 3 is a type PQ bus, we do know its reactive injection, and so we will know the left hand side of the reactive power flow equation. This is fortunate, since we do not know $|V_3|$ (and this will always be the situation for a type PQ bus).

$$Q_3(\boldsymbol{x}) = -\Big[|V_3||V_1|B_{31}\cos(\theta_3 - \theta_1) + |V_3||V_2|B_{32}\cos(\theta_3 - \theta_2) + |V_3|^2 B_{33}\Big]$$
$$= -\Big[10|V_3|\cos\theta_3 + 10.5|V_3|\cos(\theta_3 - \theta_2) - 19.98|V_3|^2\Big] \quad \text{(T7.54c)}$$

The update vector and Jacobian matrix is:

$$\Delta\underline{x} = \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta|V_3| \end{bmatrix} \qquad \underline{J} = \begin{bmatrix} \frac{\partial P_2}{\partial\theta_2} & \frac{\partial P_2}{\partial\theta_3} & \frac{\partial P_2}{\partial|V_3|} \\ \frac{\partial P_3}{\partial\theta_2} & \frac{\partial P_3}{\partial\theta_3} & \frac{\partial P_3}{\partial|V_3|} \\ \frac{\partial Q_3}{\partial\theta_2} & \frac{\partial Q_3}{\partial\theta_3} & \frac{\partial Q_3}{\partial|V_3|} \end{bmatrix}$$

We obtain the various partial derivatives for the Jacobian from eqs. (T7.54a,b,c):

$$\frac{\partial P_2}{\partial\theta_2} = |V_2||V_1|B_{21}\cos(\theta_2 - \theta_1) + |V_2||V_3|B_{23}\cos(\theta_2 - \theta_3) = 10.5\cos\theta_2 + 10.5|V_3|\cos(\theta_2 - \theta_3)$$

$$\frac{\partial P_2}{\partial\theta_3} = -|V_2||V_3|B_{23}\cos(\theta_2 - \theta_3) = -10.5|V_3|\cos(\theta_2 - \theta_3)$$

$$\frac{\partial P_2}{\partial |V_3|} = |V_2|B_{23}\sin(\theta_2 - \theta_3) = 10.5\sin(\theta_2 - \theta_3)$$

$$\frac{\partial P_3}{\partial \theta_2} = -10.5|V_3|\cos(\theta_3 - \theta_2)$$

$$\frac{\partial P_3}{\partial \theta_3} = 10.0|V_3|\cos\theta_3 + 10.5|V_3|\cos(\theta_3 - \theta_2)$$

$$\frac{\partial P_3}{\partial |V_3|} = 10\sin\theta_3 + 10.5\sin(\theta_3 - \theta_2)$$

$$\frac{\partial Q_3}{\partial \theta_2} = -10|V_3||V_2|\sin(\theta_3 - \theta_2) = -10.5|V_3|\sin(\theta_3 - \theta_2)$$

$$\frac{\partial Q_3}{\partial \theta_3} = 10|V_3|\sin\theta_3 + 10|V_3||V_2|\sin(\theta_3 - \theta_2) = 10|V_3|\sin\theta_3 + 10.5|V_3|\sin(\theta_3 - \theta_2)$$

$$\begin{aligned}\frac{\partial Q_3}{\partial |V_3|} &= -\left[|V_1|B_{31}\cos(\theta_{31}) + |V_2|B_{32}\cos(\theta_{32}) + |V_3|B_{33}\right] - |V_3|B_{33} \\ &= -\left[10\cos\theta_3 + 10.5\cos(\theta_3 - \theta_2) - 39.96|V_3|\right]\end{aligned}$$

We are ready to start iterating using (T7.40). We note that the injections, to be used on the left hand side of eqs. (T7.54a,b,c) are $P_2 = P_{G2} = 0.6661$, $P_3 = -P_{D3} = -2.8653$, and $Q_3 = -Q_{D3} = -1.2244$; these quantities remain constant through the entire iterative process. We use a flat start; therefore our initial guess is $\theta_2=\theta_3=0°$ and $|V_3|=1.0$.Using eqs. (T7.53) and (T7.54a,b,c) we get:

$$\underline{f}(\underline{x}^{(0)}) = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}^{(0)} = \begin{bmatrix} P_2\left(\underline{x}^{(0)}\right) \\ P_3\left(\underline{x}^{(0)}\right) \\ Q_3\left(\underline{x}^{(0)}\right) \end{bmatrix} - \begin{bmatrix} P_2 \\ P_3 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.52 \end{bmatrix} - \begin{bmatrix} 0.6661 \\ -2.8653 \\ -1.2244 \end{bmatrix} = \begin{bmatrix} -0.6661 \\ 2.8653 \\ 0.7044 \end{bmatrix}$$

As expected for a flat start, the mismatch is large. Next we calculate the Jacobian matrix:

$$\underline{J} = \begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.46 \end{bmatrix} \qquad \text{(T7.55)}$$

Note that the Jacobian sub-matrices $\underline{J}^{PV}$ and $\underline{J}^{Q\theta}$ are both filled with zeros. This is because when resistance is neglected, these derivatives depend on sin terms, and because this is the first iteration of a flat start, all angles are zero and therefore the sin terms are all zero.

As mentioned, commercial power flow programs normally use LU factorization to obtain the update. In this case, however, because of the low dimensionality, we may invert the Jacobian. Taking advantage of the block diagonal structure, we have:

$$(\underline{J})^{-1} = \begin{bmatrix} \begin{bmatrix} 21 & -10.5 \\ -10.5 & 20.5 \end{bmatrix}^{-1} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & [19.46]^{-1} \end{bmatrix} = \begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0656 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix}$$

Now we compute:

$$\Delta \underline{x}^{(j)} = \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta|V_3| \end{bmatrix} = -[\underline{J}]^{-1} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ ---- \\ \Delta Q_3 \end{bmatrix} = -\begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0656 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix} \begin{bmatrix} -0.6661 \\ 2.8653 \\ 0.7044 \end{bmatrix} = \begin{bmatrix} -0.0513 \\ -0.1660 \\ -0.0362 \end{bmatrix}$$

The elements of the update vector corresponding to angles are in radians. We can easily convert them to degrees:

$$\begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \Delta|V_3| \end{bmatrix} = \begin{bmatrix} -0.0513 \text{ rad} \\ -0.1660 \text{ rad} \\ -0.0362 \end{bmatrix} = \begin{bmatrix} -2.9396^{o} \\ -9.5139^{o} \\ -0.0362 \end{bmatrix}$$

We now find $\underline{x}^{(1)}$ as follows:

$$\underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2.9396^{o} \\ -9.5139^{o} \\ -0.0362 \end{bmatrix} = \begin{bmatrix} -2.9396^{o} \\ -9.5139^{o} \\ 0.9638 \end{bmatrix}$$

We note that the exact solution is $\begin{bmatrix} -3.00^{o} \\ -10.01^{o} \\ 0.9499 \end{bmatrix}$, so this is pretty good progress for one iteration!

We proceed to the next iteration using the new values $\theta_2^{(1)}$=-2.9396°, $\theta_3^{(1)}$=-9.5139°, and $|V_3|^{(1)}$=0.9638.

Substituting in eq. (T7.54a), we get $P_2(\underline{x}^{(1)})$ = 0.6202, and thus $\Delta P_2^{(1)}$= 0.6202-0.6661=-0.0459. Similarly, using eqs. (T7.54b) and (T7.54c), we get the updated mismatch vector:

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} -0.0463 \\ 0.1145 \\ 0.2251 \end{bmatrix}$$

Note that, in just one iteration, the mismatch vector has been reduced by a factor of about 10. Calculating ***J*** using the updated values of the variables, we find that

$$\underline{J} = \begin{bmatrix} 20.5396 & -10.0534 & 1.2017 \\ -10.0534 & 19.5589 & -2.8541 \\ 1.1582 & -2.7508 & 18.2199 \end{bmatrix}$$

The matrix should be compared with $\underline{J}$ from the previous iteration. It has not changed much. The elements in the off-diagonal matrices $\underline{J}^{PV}$ and $\underline{J}^{Q\theta}$ are no longer zero, but their elements are small compared to the elements in the diagonal matrices $\underline{J}^{P\theta}$ and $\underline{J}^{QV}$. The diagonal matrices themselves have not changed much. It is also important to note that the upper left-hand Jacobian submatrix ($J^{P\theta}$) is symmetric. This fact allows for a significant savings in storage when dealing with large systems.

The updated inverse is

$$\left(\underline{J}\right)^{-1} = \begin{bmatrix} 0.0651 & 0.0336 & 0.0010 \\ 0.0336 & 0.0696 & 0.0087 \\ 0.0009 & 0.0084 & 0.0561 \end{bmatrix}$$

Comparing this inverted Jacobian with that of the last iteration, we do not see much change. Using the same procedure as before to calculate the update vector, we obtain

$$\underline{\mathrm{x}}^{(2)} = \begin{bmatrix} \theta_2 \\ \theta_3 \\ |V_3| \end{bmatrix} = \begin{bmatrix} -3.0023^{\mathrm{o}} \\ -9.9924^{\mathrm{o}} \\ 0.9502 \end{bmatrix}$$

This is very close to the correct answer. The largest error is only about 0.08%. Of course in the usual problem we do not know the answer and we would continue into the next iteration. We would stop the iterations when the mismatch vector satisfies the required tolerance. We would find:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix}^2 = \begin{bmatrix} -0.0019 \\ 0.0023 \\ -0.0031 \end{bmatrix}$$

The mismatch has been reduced from that of the last iteration by a factor of 100 and is small enough. On that basis we can stop here. So we stop with the values θ_2 = -3.0023$^{\mathrm{o}}$, θ_3 = -9.9924$^{\mathrm{o}}$, and $|V_3|$=0.9502. It remains to calculate the real and reactive power generation at the swing bus (bus 1) and the reactive power generation at the PV bus (bus 2) using the calculated values of θ_2, θ_3, and $|V_3|$.

$$\begin{aligned} P_{G1} = P_1 &= |V_1||V_2|B_{12}\sin(\theta_1 - \theta_2) + |V_1||V_3|B_{13}\sin(\theta_1 - \theta_3) \\ &= 10.5\sin 3.0023^{\mathrm{o}} + 9.502\sin 9.9924^{\mathrm{o}} = 2.1987 \end{aligned}$$

$$Q_{G1} = Q_1 = -\left[|V_1||V_2|B_{12}\cos(\theta_1 - \theta_2) + |V_1||V_3|B_{13}\cos(\theta_1 - \theta_3) + |V_1|^2 B_{11}\right]$$
$$= -\left[10.5\cos 3.0023^{o} + 9.502\cos 9.9924^{o} - 19.98\right]$$
$$= 0.1365$$

$$Q_{G2} = Q_2 = -\left[|V_2||V_1|B_{21}\cos(\theta_2 - \theta_1) + |V_2||V_3|B_{23}\cos(\theta_2 - \theta_3) + |V_2|^2 B_{22}\right]$$
$$= -\left[10.5\cos(-3.0023^{o}) + 9.977\cos(6.9901^{o}) - 22.028\right]$$
$$= 1.6395$$

This completes the example.

Advanced issues associated with power flow

Some more advanced issues in relation to the power flow problem are listed below.

- Jacobian elements as sensitivities and what they tell you about relations between real or reactive power injection and voltage magnitude or angle.
- Sparsity:
  - Sparse characteristic of Jacobian
  - Storage implications
  - Optimal ordering
- Different types of power flow formulations/algorithms:
  - Divide voltage magnitude part of update vector to reduce the Jacobian storage requirements
  - Fast decoupled power flow
  - Governor power flow
  - DC power flow
  - Gauss-Seidel
- Some advanced modeling issues:
  - Transformers: regulating transformers
  - Area interchange
  - Switched shunt capacitors
  -

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below. Case (a): Systems with PQ buses only: Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\, i = 2, 3, \ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.

2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.

3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be 1.0 0 0 . This is normally refered as the flat start solution.

4. Update the voltages. In any (k +1)st iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} \mathrm{V}_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} \mathrm{V}_j^{(k)}\right] \forall \text{ i=2,3,…n} \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current (k+1)st iteration. Hence these values are used. For buses (i+1)…..n, values from previous, kth iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall \text{ i = 2,3,…n} \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = \mathrm{P}_1 - \mathrm{jQ}_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.

8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present: At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\text{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij} V_j\right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\text{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij} V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)

2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$

3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Limitations of GS load flow analysis:

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under: • Systems having large number of radial lines • Systems with short and long lines terminating on the same bus • Systems having negative values of transfer admittances • Systems with heavily loaded lines, etc. GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

Decoupled Power Flow

- The completely Dishonest Newton-Raphson is not used for power flow analysis. However several approximations of the Jacobian matrix are used.
- One common method is the decoupled power flow. In this approach approximations are used to decouple the real and reactive power equations.

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1}\Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta\left|V^{(r)}\right|}{\left|V^{(r)}\right|} = \left[L^{(r)}\right]^{-1}\Delta Q^{(r)}$

(v)Update, $\left|V^{(r+1)}\right| = \left|V^{(r)}\right| + \left|\Delta V^{(r)}\right|$

(vi) Go to step (i) and iterate till convergence is reached.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = \left[|V_i||V_j|B'_{ij}\right][\Delta\delta]$$

$$[\Delta Q] = \left[|V_i||V_j|B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and B_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = \left[B'_{ij}\right]\left[\Delta\delta\right]$$

$$\left[\frac{\Delta Q}{|V|}\right] = \left[B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

UNIT-III

SHORT CIRCUIT ANALYSIS

Importance of short circuit analysis:

A Short circuit analysis is used to determine the magnitude of short circuit current, the system is capable of producing, and compares that magnitude with the interrupting rating of the overcurrent protective devices (OCPD). Since the interrupting ratings are based by the standards, the methods used in conducting a short circuit analysis must conform to the procedures which the standard making organizations specify for this purpose. The American National Standards Institute (ANSI) publishes both the standards for equipment and the application guides, which describes the calculation methods.

Short-Circuit Currents are currents that introduce large amounts of destructive energy in the forms of heat and magnetic force into a power system. A short circuit is sometimes called a fault. It is a specific kind of current that introduces a large amount of energy into a power system. It can be in the form of heat or in the form of magnetic force. Basically, it is a low-resistance path of energy that skips part of a circuit and causes the bypassed part of the circuit to stop working. The reliability and safety of electric power distribution systems depend on accurate and thorough knowledge of short-circuit fault currents that can be present, and on the ability of protective devices to satisfactorily interrupt these currents. Knowledge of the computational methods of power system analysis is essential to engineers responsible for planning, design, operation, and troubleshooting of distribution systems.

Short circuit currents impose the most serious general hazard to power distribution system components and are the prime concerns in developing and applying protection systems. Fortunately, short circuit currents are relatively easy to calculate. The application of three or four fundamental concepts of circuit analysis will derive the basic nature of short circuit currents. These concepts will be stated and utilized in a step-by step development.

The three phase bolted short circuit currents are the basic reference quantities in a system study. In all cases, knowledge of the three phase bolted fault value is wanted and needs to be singled out for independent treatment. This will set the pattern to be used in other cases.

A device that interrupts short circuit current, is a device connected into an electric circuit to provide protection against excessive damage when a short circuit occurs. It provides this protection by automatically interrupting the large value of current flow, so the device should be rated to interrupt and stop the flow of fault current without damage to the overcurrent protection device. The OCPD will also provide automatic interruption of overload currents.

Short-circuit calculations are required for the application and coordination of protective relays and the rating of equipment. All fault types can be simulated. Carelab's short-circuit study provides a detailed report identifying breaker ratings, breaker fault duties, discussions, and recommendations for any deficiencies found

Risks Associated With Short Circuit Currents

The building/facility may not be properly protected against short-circuit currents. These currents can damage or deteriorate equipment. Improperly protected short-circuit currents can injure or kill maintenance personnel. Recently, new initiatives have been taken to require facilities to properly identify these dangerous points within the power distribution of the facility.

Why Is A Short Circuit Dangerous?

A short circuit current can be very large. If unusually high currents exceed the capability of protective devices (fuses, circuit breakers, etc.) it can result in large, rapid releases of energy in the form of heat, intense magnetic fields, and even potentially as explosions known as an arc blast. The heat can damage or destroy wiring insulation and electrical components. An arc blast produces a shock wave that may carry vaporized or molten metal, and can be fatal to unprotected people who are close by.

Fault current calculations are necessary to properly select the type, interrupting rating, and tripping characteristics of power and lighting system circuit breakers and fuses. Results of the fault current calculations are also used to determine the required short-circuit ratings of power distribution system components including bus transfer switches, variable speed drives, switchboards, load centres, and panel boards. In calculating the maximum fault current, it is necessary to determine the total contribution from all generators that may be paralleled and the motor contribution from induction and synchronous motors.

Short Circuit Analysis is performed to determine the currents that flow in a power system under fault conditions. If the short circuit capacity of the system exceeds the capacity of the protective device, a dangerous situation exists. Since growth of a power system often results in increased available short-circuit current, the momentary and interrupting rating of new and existing equipment on the system must be checked to ensure the equipment can withstand the short-circuit energy (see Device Evaluation). Fault contributions for utility sources, motors and generators are taken into consideration.

A Short Circuit Analysis will help to ensure that personnel and equipment are protected by establishing proper interrupting ratings of protective devices (circuit breaker and fuses). If an electrical fault exceeds the interrupting rating of the protective device, the consequences can be devastating. It can be a serious threat to human life and is capable of causing injury, extensive equipment damage, and costly downtime.

On large systems, short circuit analysis is required to determine both the switchgear ratings and the relay settings. No substation equipment can be installed with knowledge of the complete short circuit values for the entire power distribution system. The short circuit calculations must be maintained and periodically updated to protect the equipment and CC the lives. It is not safe to assume that new equipment is properly rated.

The results of a Short Circuit Analysis are also used to selectively coordinate electrical protective devices.

What is Short Circuit Analysis?

Short circuit analysis essentially consists of determining the steady state solution of a linear network with balanced three phase excitation. Such an analysis provides currents and voltages in a power system during the faulted condition. This information is needed to determine the required interrupting capacity of the circuit breakers and to design proper relaying system. To get enough information, different types of faults are simulated at different locations and the study is repeated. Normally in the short circuit analysis, all the shunt parameters like loads, lime charging admittances are neglected* Then the linear network that has to be solved comprises of

- Transmission network
- Generator system and
- Fault. By properly combining the representations of these components we can solve the short circuit problem

Carelabs allows you to perform a per unit calculation on any system you are working with. We automatically converts the entire system (panel boards, transformers, generators, motorized items and cables) into a unique impedance unit from which you can obtain the rating of the short circuit current at any given point. The process is simple, efficient and will save you both money and time.
Carelabs provides short-circuit calculations for single and multiple faults, together with a number of reporting options. As short-circuit calculations are needed for a variety of purposes, the short-circuit

calculation in Carelabs supports different representations and calculation methods based on a range of international standards, as well as the superposition method (also known as the Complete Method),

What Are Bolted, Arcing and Ground Faults?

A bolted fault typically results from a manufacturing or assembly error that results in two conductors of different voltages being “bolted” together, or a source of power being directly connected (bolted) to ground. Since the connectors are solidly bolted there in no arc created and the high current quickly trips a protective device limiting the damage.

An arc fault is one in which the short circuit creates an arc. An arc is a flow of electricity between two conductors that are not in contact. The resulting intense heat can result in a fire, significant damage to the equipment, and possibly an arc flash or arc blast resulting in serious injuries.

A ground fault is when electricity finds an unintended, low resistance, path to ground. When that path goes through a human body the resulting heat can cause serious burns, and the electrical shock can disrupt the functioning of the human heart (fibrillation).

What Are Symmetrical and Asymmetrical Currents?

A polyphase system may experience either a symmetrical or an asymmetrical fault. A symmetrical fault current is one that affects all phases equally. If just some of the phases are affected, or the phases are affected unequally, then the fault current is asymmetrical.

Symmetrical faults are relatively simple to analyse, however they account for very few actual faults. Only about 5% of faults are symmetrical. Asymmetrical faults are more difficult to analyse, but they are the more common type of fault.

What Are Protective Devices for Short Circuit Analysis?

Protective devices are designed to detect a fault condition and shut off the electric current before there is significant damage. There are a number of different types of protective devices, the two most common are:

Fuses and Circuit Breakers

Fuses and circuit breakers are used to protect an electrical circuit from an over-current situation, usually resulting from a short circuit, by cutting off the power supply. Fuses can only be used once. Circuit breakers may be reset and used multiple times.

Ground Fault Interrupter (GFI)

This is a device that detects when the current flow in the energized conductor does not equal the return current in the neutral conductor. The GFI protects people by quickly cutting off the current flow preventing injuries resulting from shock. Ground Fault Interrupters are typically used in homes for bathroom, kitchen, and outdoor electrical sockets. The GFI will typically be built into the electrical socket.

A GFI does not provide over-current protection, and the circuit that includes a GFI will also include a fuse or circuit breaker.

In addition to fuses, circuit breakers, and GFIs, there are electrical protection devices that:

- detect changes in current or voltage levels
- monitor the ratio of voltage to current
- provide over-voltage protection
- provide under-voltage protection
- detect reverse-current flow
- detect phase reversal

When are Short Circuit Analysis Needed?

The first short-circuit analysis should be performed when a power system is originally designed, though this should not be the only time. These studies need to occur with any facility expansion or with the addition of any new electrical equipment such as circuit breakers or new transformers and cables. Without any new additions or changes, short circuit studies still need to occur on a regular basis of at least every 5-6 years.

How Is Short Circuit Current Calculated?

Short-circuit calculations are required to correctly apply equipment in accordance with NEC, and ANSI standards. Depending on the size and utility connection, the amount of detail required to perform these calculations can vary greatly. Carelabs short-circuit analysis will include calculations performed in accordance with the latest ANSI standards.

Switches, fuses, and breakers that need to interrupt or close into a fault are of special concern. Cables and buswork also have short-circuit withstand limitations, and a thorough study will examine non-interrupting equipment, as well as switches and breakers. Standards such as ANSI C37.010 and C37.13 outline the recognized calculation methods for these equipment-rating analyses.

These short circuit studies are performed using power system software as per IEEE standards. For larger systems, these short circuit calculations to be performed for both switch gear ratings and relay settings. Knowledge of the computational methods of power system analysis is essential to engineers responsible for planning, design, operation, and troubleshooting of distribution systems. A short-circuit study is an analysis of an electrical system that determines the magnitude of the currents that flow during an electrical fault. Comparing these calculated values against the equipment ratings is the first step to ensuring that the power system is safely protected. Once the expected short-circuit currents are known, a protection coordination study is performed to determine the optimum characteristics, ratings and settings of the power system protective devices.

NEC 110 requires that a short circuit analysis be done for all electrical equipment and panels. The two most common standards for short circuit current calculations are the ANSI/IEEE C37.010-1979 standard and the International Electro-technical Commission (IEC) 60909 standard.

The ANSI C37.010 standard was intended to be used for power circuit breaker selection, but it does provide the information needed for NEC 110 required labelling. The IEC 60909-3:2009 standard is more generic. It is intended to provide general guidelines for short-circuit analysis of any asymmetrical short circuit in a three-phase 50 Hz or 60Hz A.C. electrical system.

Either the ANSI or the IEC short circuit calculation method can be used. They have been compared and found to produce similar results. The ANSI method is commonly used in short circuit current calculation software.

Our short circuit analysis service:

- Is done with support of IEC 60909 (including 2016 edition), IEEE 141/ANSI C37, VDE 0102/0103, G74 and IEC 61363 norms and methods
- Is calculation of short-circuit currents in DC grids according to IEC 61660 and ANSI/IEEE 946
- We do complete superposition method, including dynamic voltage support of generators connected via power electronics
- Multiple fault analysis of any kind of fault incl. single-phase interruption, inter-circuit faults, fault sweep along lines, etc.

Diakoptic Model For The Short Circuit Analysis

In the short circuit analysis, it is customary to neglect the loads and other shunt parameters to the ground. Under this condition, impedance representation for the transmission network with ground as reference does not exist. However, connection to the ground is established at the generator buses, representing the generator as a constant voltage source behind appropriate reactant. Hence let us consider the combined transmission-generator network and while tearing the network, let us ensure that each sub-network has atleast one generator. In practice this should pose no difficulty since large power system 84 networks normally consist of different areas having generations in each area.

Neplan

Short circuit analysis is performed so that existing and new equipment ratings were sufficient to with stand the available short circuit current. This short circuit analysis can be done either through hand calculations or through known software like NEPLAN.

Using NEPLAN we can perform short circuit studies on electrical systems in a quick time and effective manner in four steps.

Data Collection and SLD Preparation

Short circuit calculations

Relay Coordination Studies

Load flow Analysis

Why chose Carelabs for Short Circuit Analysis?

At Carelabs we differ from competitors in our size and structure and this allows us to be more responsive to change. It also allows us to provide personalized and superior services to you. We follow NFPA-70E and IEEE 1584 guidelines in order to guarantee that we always meet the highest industry standards.

Benefits of Short Circuit Analysis

Conducting a short circuit analysis has the following benefits:

Helps avoid unplanned outages and downtime

Is critical for avoiding interruptions of essential services

Reduces the risk of equipment damage and fires

Increases safety and protects people from injuries

Determines the level and type of protective devices that are needed

Provides the information needed for NEC and NFPA required labels

Keeps you in compliance with NEC requirements

Reduces the risk a facility could face and help avoid catastrophic losses

Increases the safety and reliability of the power system and related equipment

Assumptions in Fault analysis:

In fault analysis study its necessary to make assumptions, because we cant predict 100% natural scenarios on this electrical energy. Following are some of the assumptions commonly made in three phase fault studies for the ease of calculations,

- Transformers are on nominal tap position. This will let us take nominal voltages of transformers in calculations.
- All sources are balanced and equal in magnitude and phase. We neglect the slight differences in magnitude and phase of the source voltages as it is nothing when compared with the fault.
- High Voltage Power Lines are assumed fully transposed and all 3 phase have same impedance. Transposed lines have more or less equal inductance's in all three phases.
- Loads currents are negligible compared to fault currents. Usually fault currents are about several kilo amperes, but load currents are mostly in ampere range. Therefore the effect of the load current on the final result is negligible.
- Sources represented by the Thevenin's voltage prior to fault at the fault point.
- Large systems may be represented by infinite bus bars. When comparing with large systems with a small one, effect from the small one will not make much effect on the larger system. Therefore there is not much different taking the large system as an infinite bus bar.
- Line charging currents can be completely neglected as line charging currents are smaller compared to load current.

- Resistances are negligible compared to the reluctance. Usually in power lines, the dominant component of the impedance is the reluctance as it is several times higher than the resistance

Fault Analysis using Thevenin's theorem

Short Circuit Current Computation through Thevenin Theorem – An alternate method of computing short circuit currents is through the application of the Thevenin theorem. This method is faster and easily adopted to systematic computation for large networks. While the method is perfectly general, it is illustrated here through a simple example.

Consider a synchronous generator feeding a synchronous motor over a line. Figure 9.13a shows the circuit model of the system under conditions of steady load. Fault computations are to be made for a fault at F, at the motor terminals. As a first step the circuit model is replaced by the one shown in Fig. 9.13b, wherein the synchronous machines are represented by their transient reactances (or subtransient reactances if subtransient currents are of interest) in series with voltages behind transient reactances. This change does not disturb the prefault current I° and prefault voltage V° (at F). As seen from FG the Thevenin equivalent circuit of Fig. 9.13b is drawn in Fig. 9.13c. It comprises prefault voltage V° in series with the passive Thevenin impedance network. It is noticed that the prefault current I° does not appear in the passive Thevenin impedance network. It is therefore to be remembered that this current must be accounted for by superposition after the SC solution is obtained through use of the Thevenin equivalent. Consider now a fault at F through an impedance Zf Figure 9.13d shows the Thevenin equivalent of the system feeding the fault impedance. We can immediately write

$$I^f = \frac{V^o}{jX_{\text{Th}} + Z^f} \qquad (9.12)$$

Current caused by fault in generator circuit

$$\Delta I_g = \frac{X'_{dm}}{(X'_{dg} + X + X'_{dm}} I^f \qquad (9.13)$$

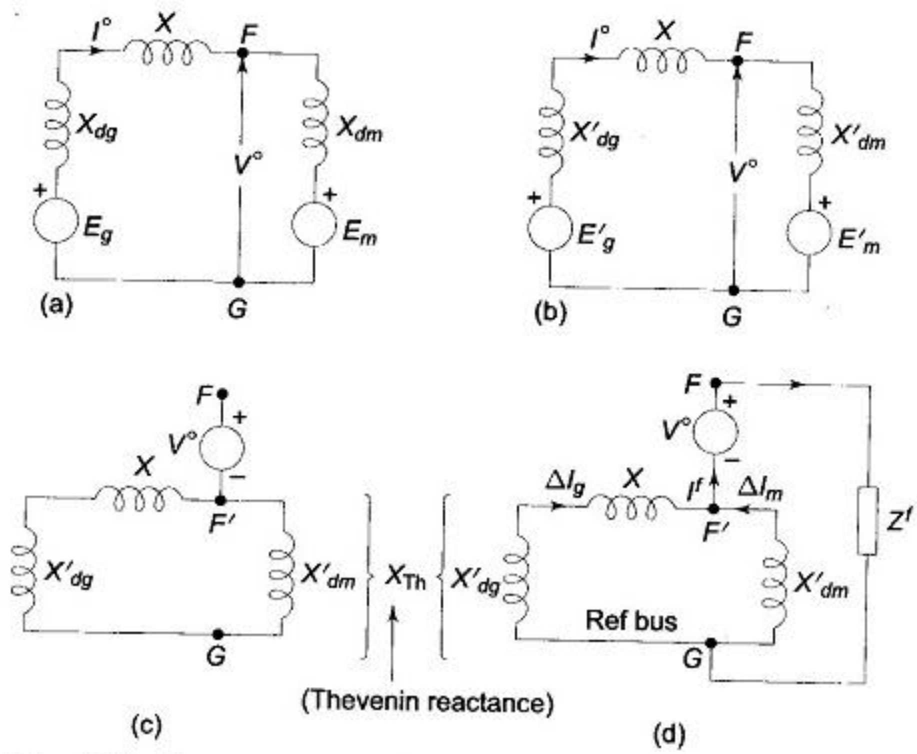


Fig. 9.13 Computation of SC current by the Thevenin equivalent

Current caused by fault in motor circuit

$$\Delta I_m = \frac{X'_{dg} + X}{(X'_{dm} + X + X'_{dg})} I^f \tag{9.14}$$

Postfault currents and voltages are obtained as follows by superposition:

$$I_g^f = I^o + \Delta I_g$$
$$I_m^f = -I^o + \Delta I_m \text{ (in the direction of } \Delta I_m) \tag{9.15}$$

Postfault voltage

$$V^f = V^o + (-jX_{\text{Th}} I^f) = V^o + \Delta V \tag{9.16}$$

where ΔV=-jXThIf is the voltage of the fault point F′ on the Thevenin passive network (with respect to the reference bus G) caused by the flow of fault current If. An observation can be made here. Since the prefault current flowing out of fault point F is always zero, the postfault current out of F is independent of load for a given prefault voltage at F. The above approach to SC computation is summarized in the following four steps: Step 1: Obtain steady state solution of loaded system (load flow study). Step 2: Replace reactances of synchronous machines by their subtransient/ transient values. Short circuit all emf sources. The result is the passive Thevenin network. Step 3: Excite the passive network of Step 2 at the fault point by negative of prefault voltage (see Fig. 9.13d) in series with the fault impedance. Compute voltages and currents at all points of interest. Step 4: Postfault currents and voltages are obtained by adding results of Steps 1 and 3. The following assumptions can be safely made in SC computations leading to considerable computational simplification: Assumption 1: All prefault Voltage magnitudes are 1 pu. Assumption 2: All prefault currents are zero. The first assumption is quite close to actual conditions as under normal operation all voltages (pu) are nearly unity. The changes in current caused by short circuit are quite large, of the order of 10-20 pu and are purely reactive; whereas the prefault load currents are almost purely real. Hence the total postfault current which is the result of the two currents can be taken in magnitude equal to the larger component (caused by the fault). This justifies assumption 2.

Z-Bus Building Algorithm

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\bar{E}_{bus} = [Z_{bus}]\bar{I}_{bus} \tag{9}$$

When expanded so as to refer to a n bus system, (9) will be of the form

$$E_1 = Z_{11} I_1 + Z_{12} I_2 + \ldots\ldots + Z_{1k} I_k \ldots + Z_{1n} I_n$$

$$E_k = Z_{k1} I_1 + Z_{k2} I_2 + \ldots\ldots + Z_{kk} I_k + \ldots + Z_{kn} I_n$$

$$E_n = Z_{n1} I_1 + Z_{n2} I_2 + \ldots\ldots + Z_{nk} I_k + \ldots + Z_{nn} I_n \tag{10}$$

Now assume that the bus impedance matrix Zbus is known for a partial network of *m* buses and a known reference bus. Thus, Zbus of the partial network is of dimension *m* by *m*. If now a new element is added between buses *p* and *q* we have the following two possibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p*-*q* is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p-q* is a **link** added to the p-network as shown in Fig 1b.

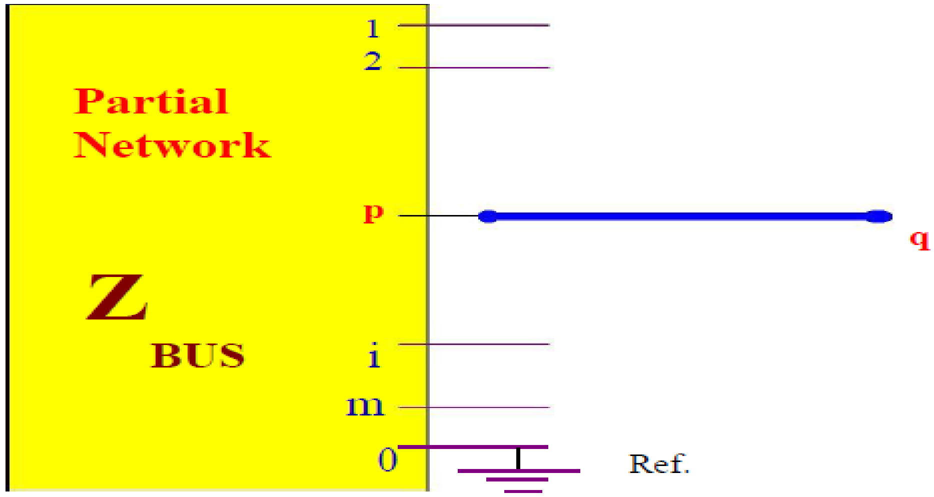


Fig 1a. Addition of branch p-q

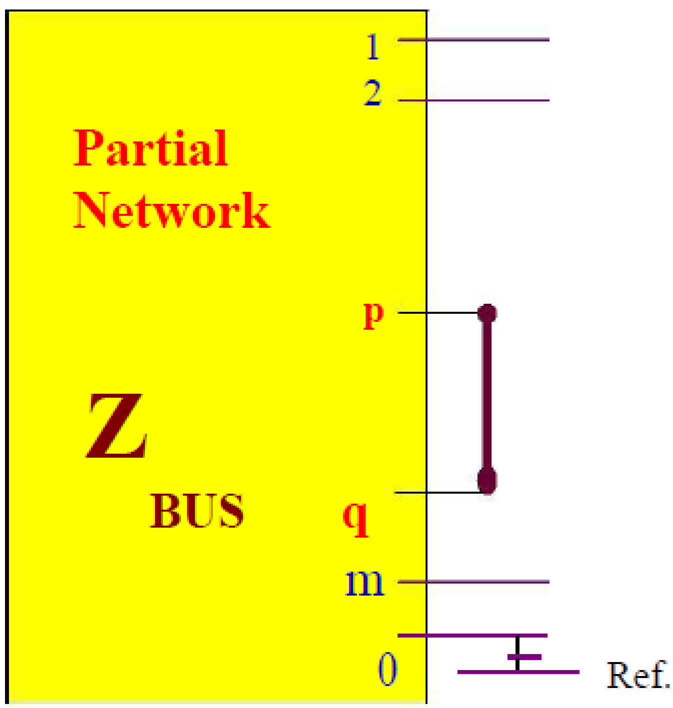


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix.

If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{1p} & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & Z_{2p} & Z_{2m} & Z_{2q} \\ Z_{p1} & Z_{p2} & Z_{pp} & Z_{pm} & Z_{pq} \\ Z_{m1} & Z_{m2} & Z_{mp} & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & Z_{qp} & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \qquad (11)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

$$\textbf{Vector } y_{pq\text{-}rs} \textbf{ is not equal to zero} \text{ and } \mathbf{Z_{ij}} = \mathbf{Z_{ji}} \quad \forall\ i,j = 1,2,\ldots m,q \qquad (12)$$

To find Z_{qi}:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

$$E_k = Z_{ki} I_i = Z_{ki} \qquad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots m, q \qquad (13)$$

$$\text{Hence, } E_q = Z_{qi}\ ;\ E_p = Z_{pi}\ \ldots\ldots\ldots$$

$$\text{Also, } E_q = E_p - v_{pq}\ ; \text{ so that } Z_{qi} = Z_{pi} - v_{pq} \quad \forall\ i = 1, 2, \ldots i \ldots\ldots p, \ldots m, \neq q \qquad (14)$$

To find v_{pq}:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \tag{15}$$

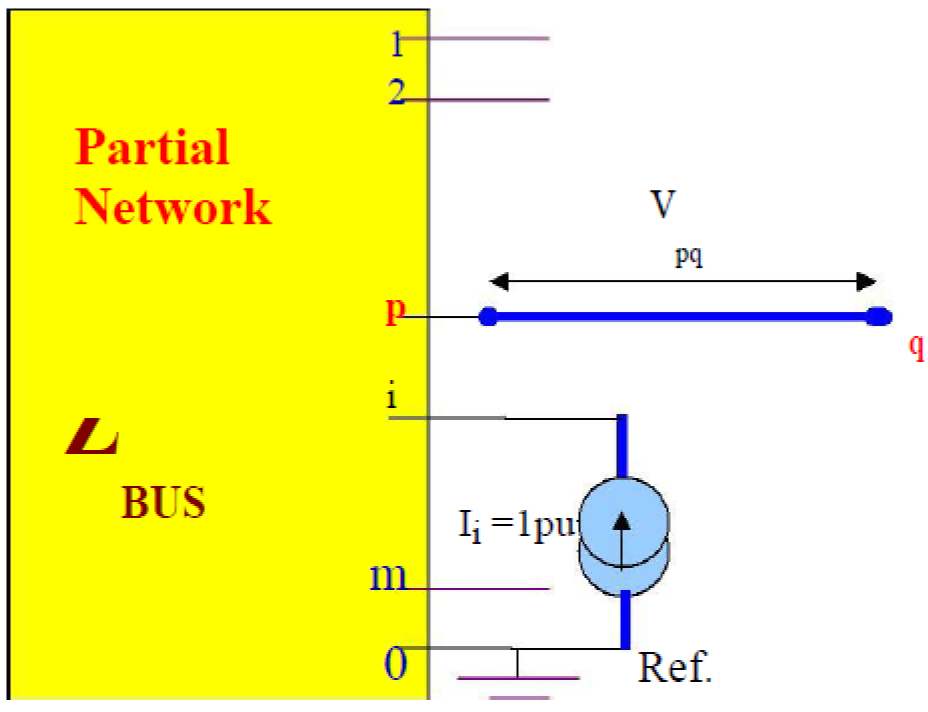


Fig.2 Calculation for Z_{qi}

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\dfrac{\bar{y}_{pq,rs} \bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs} \left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs} \left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2......m;\ i \neq q \tag{18}$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu.

As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq} \quad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots m, q$ (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \dfrac{\bar{y}_{pq,rs} \bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs} \left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs} \left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Short circuit capacity

Short circuit capacity calculation isused for many applications: sizing of transformers, selecting the interrupting capacity ratings of circuit breakers and fuses, determining if a line reactor is required for use with a variable frequency drive, etc.

The purpose of the presentation is to gain a basic understanding of short circuit capacity. The application example utilizes transformer sizing for motor loads.

Conductor impedances and their associated voltage drop are ignored not only to present a simplified illustration, but also to provide a method of approximation by which a plant engineer, electrician or production manager will be able to either evaluate a new application or review an existing application problem and resolve the matter quickly.

The following calculations will determine the extra kVA capacity required for a three phase transformer that is used to feed a single three phase motor that is started with full voltage applied to its terminals, or, "across-the-line."

Two transformers will be discussed, the first having an unlimited short circuit kVA capacity available at its primary terminals, and the second having a much lower input short circuit capacity available kVA of a single phase transformer = V x A kVA of a three phase transformer = V x A x 1.732, where 1.732 = the square root of 3.

The square root of 3 is introduced for the reason that, in a three phase system, the phases are 120 degrees apart and, therefore, can not be added arithmetically. They will add algebraically.

SYMMETRICAL COMPONENTS:
An unbalanced three-phase system can be resolved into three balanced systems in the sinusoidal steady state. This method of resolving an unbalanced system into three balanced phasor system has been proposed by C. L. Fortescue. This method is called **resolving symmetrical components** of the original phasors or simply **symmetrical components**.

In this chapter we shall discuss symmetrical components transformation and then will present how unbalanced components like Y- or Δ -connected loads, transformers, generators and transmission lines can be resolved into symmetrical components. We can then combine all these components together to form what are called **sequence networks**

Symmetrical Components

A system of three unbalanced phasors can be resolved in the following three symmetrical components:

- Positive Sequence: A balanced three-phase system with the same phase sequence as the original sequence.
- Negative sequence: A balanced three-phase system with the opposite phase sequence as the original sequence.
- Zero Sequence: Three phasors that are equal in magnitude and phase.

Fig. 7.1 depicts a set of three unbalanced phasors that are resolved into the three sequence components mentioned above. In this the original set of three phasors are denoted by V_a , V_b and V_c , while their positive, negative and zero sequence components are denoted by the subscripts 1, 2 and 0 respectively. This implies that the positive, negative and zero sequence components of phase-a are denoted by V_{a1} , V_{a2} and V_{a0} respectively. Note that just like the voltage phasors given in Fig. 7.1 we can also resolve three unbalanced current phasors into three symmetrical components.

Fig. 7.1 Representation of (a) an unbalanced network, its (b) positive sequence, (c) negative sequence and (d) zero sequence.

Symmetrical Component Transformation

Before we discuss the symmetrical component transformation, let us first define the α -operator. This has been given in (1.34) and is reproduced below

$$a = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

Note that for the above operator the following relations hold

$$a^2 = e^{j240^0} = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = a^*$$

$$a^3 = e^{j360^0} = 1$$

$$a^4 = e^{j480^0} = e^{j360^0}e^{j120^0} = a$$

$$a^5 = e^{j600^0} = e^{j360^0}e^{j240^0} = a^2 \text{ and so on}$$

Also note that we have

$$1 + a + a^2 = 1 - \frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} - j\frac{\sqrt{3}}{2} = 0$$

Using the a -operator we can write from Fig. 7.1 (b)

$$V_{b1} = a^2 V_{a1} \text{ and } V_{c1} = aV_{a1}$$

Finally from Fig. 7.1 (d) we get

$$V_{a0} = V_{b0} = V_{c0}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2} = V_{b0} + V_{b1} + V_{b2}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2} = V_{c0} + V_{c1} + V_{c2}$$

Therefore

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

he symmetrical component transformation matrix is then given by

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Defining the vectors V_{a012} and V_{abc} as

$$V_{a012} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}, \quad V_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

we can write (7.4) as

$$V_{a012} = CV_{abc}$$

where C is the symmetrical component transformation matrix and is given by

$$C = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

The original phasor components can be obtained from the inverse symmetrical component transformation, i.e.,

$$V_{abc} = C^{-1}V_{a012}$$

Finally, if we define a set of unbalanced current phasors as I_{abc} and their symmetrical components as I_{a012} , we can then define

$$I_{a012} = CI_{abc}$$
$$I_{abc} = C^{-1}I_{a012}$$

Real and Reactive Power

The three-phase power in the original unbalanced system is given by

$$P_{abc} + jQ_{abc} = V_a I_a^* + V_b I_b^* + V_c I_c^* = V_{abc}^T I_{abc}^*$$

where I^* is the complex conjugate of the vector I . Now from (7.10) and (7.15) we get

$$P_{abc} + jQ_{abc} = V_{a012}^T C^{-T} C^{-1*} I_{a012}^*$$

From (7.11) we get

$$C^{-T}C^{-1*} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore from (7.17) we get

$$P_{abc} + jQ_{abc} = 3\left(V_{a0}I_{a0}^* + V_{a1}I_{a1}^* + V_{a2}I_{a2}^*\right)$$

We then find that the complex power is three times the summation of the complex power of the three phase sequences

2.2.1 Symmetrical components

Let us consider a set of three unbalanced phasors having a phase sequence abc as depicted in fig. 2.6(a). The three phasors, in general, can be chosen arbitrarily. Each phasor has a magnitude and a phase angle. One, therefore, can have two degrees of freedom while defining a phasor. For defining the three unbalanced phasors in an unbalanced system there are (3x2) i.e. 6 degrees of freedom. But, while defining a balanced set one has only two degrees of freedom because choosing any one of the three phasors fixes the other two automatically. So, if we decide to resolve our three-phase unbalanced systems into fictitious symmetrical sets of phasors, we shall have to have three symmetrical sets each having a degree of freedom of two, so that the degrees of freedom of the original set are maintained. The fictitious phasors need to be defined in terms of the original phasors.

The original set of unbalanced phasors ($\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$) can be broken up into three symmetrical components:

(i) A set of balanced phasors $\mathbf{V}_{a1}$, $\mathbf{V}_{b1}$, $\mathbf{V}_{c1}$, having a phase sequence abc i.e. same as that of the original unbalanced set. The set is called a positive sequence set as shown in fig. 2.6(b).

(ii) A set of balanced phasors $\mathbf{V}_{a2}$, $\mathbf{V}_{b2}$, $\mathbf{V}_{c2}$ having a phase sequence acb i.e. opposite to that of the original unbalanced set. The set is called a negative sequence set as shown in fig. 2.6(c).

(iii) A set of three equal phasors $\mathbf{V}_{ao}$, $\mathbf{V}_{bo}$, $\mathbf{V}_{co}$ as shown in fig. 2.6 (d). This set is called a zero sequence set.

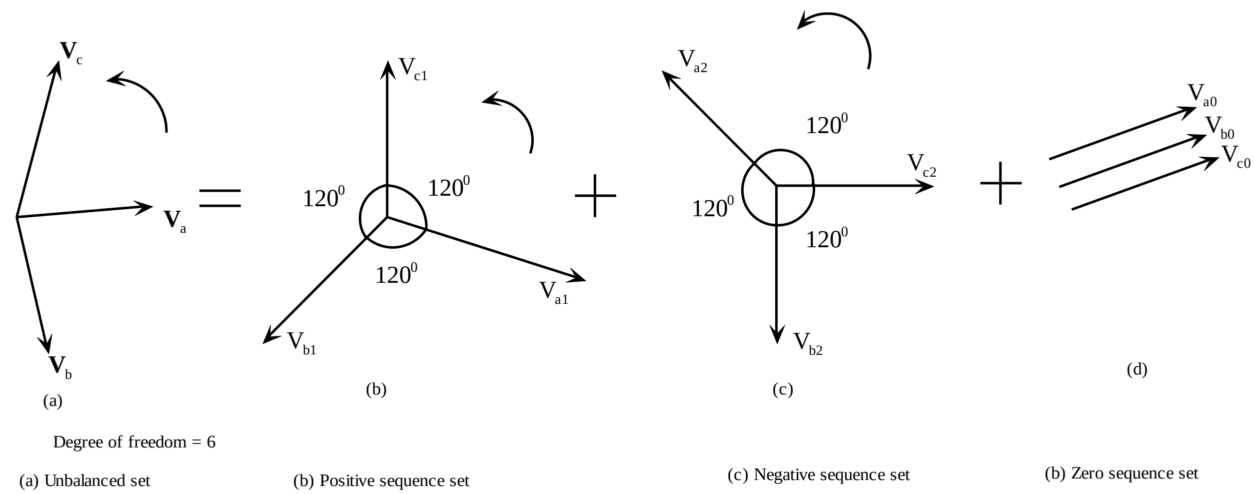


Fig 2.6 Resolution of a three phase unbalanced set of phasors into three balanced sets

Having decided to resolve our arbitrary three-phase unbalanced system into balanced systems one has to find the symmetrical components $\mathbf{V}_{a1}$, $\mathbf{V}_{a2}$, $\mathbf{V}_{ao}$ in terms of the original phasors $\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$.

Let us consider the positive sequence set of phasors $\mathbf{V}_{a1}$, $\mathbf{V}_{b1}$, $\mathbf{V}_{c1}$

$\mathbf{V}_{b1} = e^{j\,240}\ \mathbf{V}_{a1} = \mathbf{a}^2\,\mathbf{V}_{a1}$; $\mathbf{V}_{c1} = e^{j\,120}\ \mathbf{V}_{a1} = \mathbf{a}\,\mathbf{V}_{a1}$ … (2.3)

Where $\mathbf{a}$ is the unit phasor $e^{j\,120} = -0.5 + j0.866$

$\mathbf{a}^2$ is the unit phasor $e^{j\,240} = -0.5 - j0.866$ … (2.16)

Similarly, for negative sequence set:

$\mathbf{V}_{b2} = e^{j\,120}\,\mathbf{V}_{a2} = \mathbf{a}\,\mathbf{V}_{a2}$ … (2.17)

$\mathbf{V}_{c2} = e^{j\,240}\,\mathbf{V}_{a2} = \mathbf{a}^2\,\mathbf{V}_{a2}$ … (2.18)

For the zero sequence set,

$\mathbf{V}_{ao}, = \mathbf{V}_{bo}, = \mathbf{V}_{co}.$

The unit phasor, **a,** is an operator which, when operated on a phasor, shifts the phasor by 120^o in the anticlockwise direction (taken to be positive direction) without changing its magnitude. Some important properties of the phasor are:

$\mathbf{a}^3 = e^{j\,360} = 1.0 + j0$ (no change in phasor if operated by $\mathbf{a}^3$)

$\mathbf{a}^4 = \mathbf{a}^3\ \mathbf{a} = \mathbf{a}$;

$$\left.\begin{cases} \mathbf{a}^n = 1 + j0 & \text{if } n = 3m \\ \mathbf{a}^n = \mathbf{a} & \text{if } n = 3m+1 \\ \mathbf{a}^n = \mathbf{a}^2 & \text{if } n = 3m+2 \end{cases}\right\} m = \text{any positive integer}$$

$1+\mathbf{a}+\mathbf{a}^2 = 0$

$\mathbf{a} - \mathbf{a}^2 = j\sqrt{3}$

$1-\mathbf{a} = 1.5 - j\,0.866$

$1-\mathbf{a}^2 = 1.5 + j\,0.866$

$\mathbf{a}^* = \mathbf{a}^2$; $(\mathbf{a}^2)^* = \mathbf{a}$

2.2.1 Resolution of three phasors into their symmetrical components

In all three systems of symmetrical components, the subscripts denote the components in the different phases. The total voltage in any phase is then equal to the sum of the corresponding components of the different sequences in that phase. So,

$$\mathbf{V}_a = \mathbf{V}_{ao} + \mathbf{V}_{a1} + \mathbf{V}_{a2} \qquad \ldots(2.19)$$

$$\mathbf{V}_b = \mathbf{V}_{bo} + \mathbf{V}_{b1} + \mathbf{V}_{b2} = \mathbf{V}_{ao} + \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} \qquad \ldots(2.20)$$

$$\mathbf{V}_c = \mathbf{V}_{co} + \mathbf{V}_{c1} + \mathbf{V}_{c2} = \mathbf{V}_{ao} + \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2} \qquad \ldots(2.21)$$

The above equations (2.19) to (2.21) may be written in matrix form:

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \qquad \ldots(2.22)$$

The above equation may be written in compact form

$$[\mathbf{V}_{abc}] = [C][\mathbf{V}_{012}] \qquad \ldots(2.23)$$

[c] is the 3 x 3 connection matrix given by,

$$[c] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}$$

It can be easily found that the inverse of [c] is given by :

$$[c]^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \qquad \ldots(2.24)$$

From equation (2.22) we get,

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = [C]^{-1} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} \qquad \ldots(2.25)$$

From the matrix equation (2.25), we get

$$\mathbf{V}_{ao} = \frac{1}{3}(\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c) \qquad \ldots (2.26)$$

$$\mathbf{V}_{a1} = \frac{1}{3}(\mathbf{V}_a + \mathbf{a}\mathbf{V}_b + \mathbf{a}^2\mathbf{V}_c) \qquad \ldots (2.27)$$

$$\mathbf{V}_{a2} = \frac{1}{3}(\mathbf{V}_a + \mathbf{a}^2\mathbf{V}_b + \mathbf{a}\mathbf{V}_c) \qquad \ldots (2.28)$$

So, while equations (2.19) to (2.21) define the original phasors in terms of the symmetrical components, equations (2.26) to (2.28) help to calculate symmetrical components in terms of original unbalanced phasors.

It may be mentioned that the voltages and currents of a balanced system constitute positive sequence components only. Negative and zero sequence components are not present as the righthand side of each of the equations (2.26) and (2.28), is zero.

2.2.1 Some important interpretations

(i) Equation (2.26) shows that the zero sequence component will be present if and only if the sum of the original phasors is not zero. In a three phase systems, the sum of the currents $\mathbf{I}$a, $\mathbf{I}_b$ and $\mathbf{I}_c$ denotes the current returning to the source. For a star connected system flow of return current is possible if there is a return neutral path. So, we conclude that zero sequence current will be absent in a star connected system without neutral path or a ground connection that will carry the return current. In delta connected load, the line currents do not find return neutral path. Hence, line currents do not have zero sequence component. However, currents of zero sequence may circulate within the delta without getting out in the line. This circulating current cannot be determined from the line currents.

(ii) The line voltages or the delta voltages must by their very nature form a closed triangle i.e. $\mathbf{V}_{ab}$+$\mathbf{V}_{bc}$+$\mathbf{V}_{ca}$ must always be equal to zero. So, line voltages, howsoever unbalanced these may be, can never contain a zero-sequence component.

2.2.1 Sequence impedances in symmetrical systems

In symmetrical network systems the different sequences do not react upon each other. This means that currents of a particular sequence produce voltages of that sequence only.

We shall take up the static symmetric network shown in fig. 2.7 to justify the assertion made above.

Each of the lines has a series impedance of $\mathbf{z}_s$ and the mutual impedance between any two lines is $\mathbf{z}_m$. The phase voltages are $\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$ on one side and $\mathbf{V}_a'$, $\mathbf{V}_b'$, $\mathbf{V}_c'$ on the other side of the series impedances. An unbalanced set of currents $\mathbf{I}_a$, $\mathbf{I}_b$ and $\mathbf{I}_c$ are flowing through the lines. The drops $\Delta\mathbf{V}_a$, $\Delta\mathbf{V}_b$ and $\Delta\mathbf{V}_c$ constitute a set of unbalanced phasors and are given by :

$$\Delta\mathbf{V}_a = \mathbf{V}_a - \mathbf{V}_a' = \mathbf{I}_a\mathbf{z}_s + \mathbf{I}_b\mathbf{z}_m + \mathbf{I}_c\mathbf{z}_m \quad \ldots(2.29)$$

$$\Delta\mathbf{V}_b = \mathbf{V}_b - \mathbf{V}_b' = \mathbf{I}_a\mathbf{z}_m + \mathbf{I}_b\mathbf{z}_s + \mathbf{I}_c\mathbf{z}_m \quad \ldots(2.30)$$

$$\Delta\mathbf{V}_c = \mathbf{V}_c - \mathbf{V}_c' = \mathbf{I}_a\mathbf{z}_m + \mathbf{I}_b\mathbf{z}_m + \mathbf{I}_c\mathbf{z}_s \quad \ldots(2.31)$$

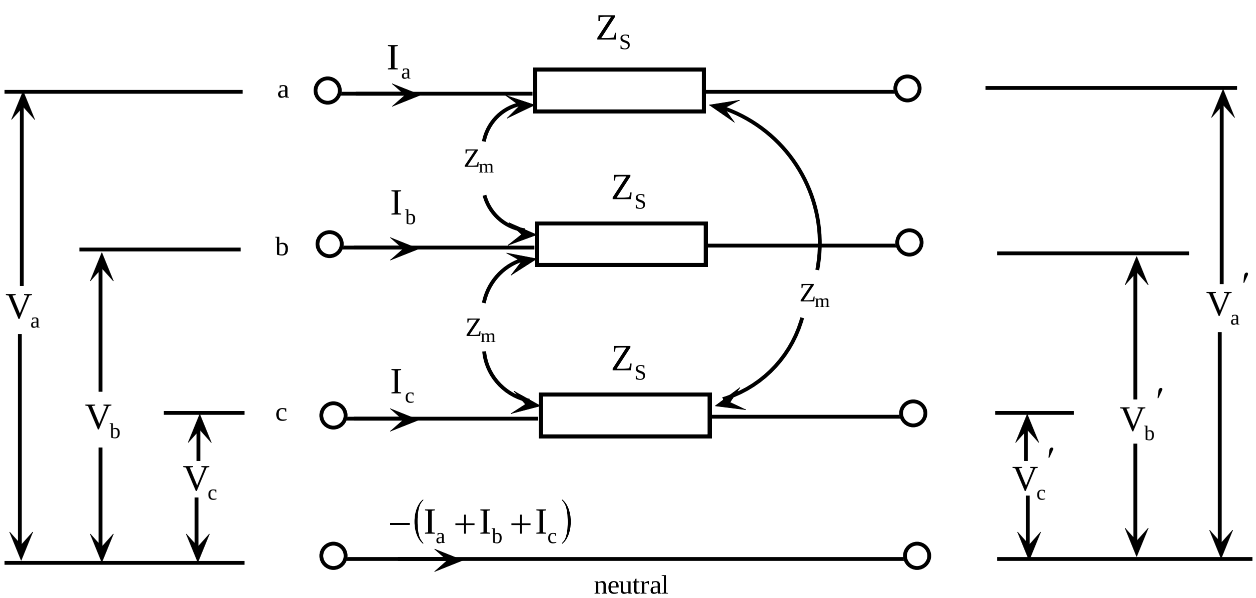


Fig. 2.7 A three-phase symmetric static circuit

The above equation can be expressed in matrix form as follows:

$$\begin{bmatrix} \Delta\mathbf{V}_a \\ \Delta\mathbf{V}_b \\ \Delta\mathbf{V}_c \end{bmatrix} = \begin{bmatrix} \mathbf{z}_s & \mathbf{z}_m & \mathbf{z}_m \\ \mathbf{z}_m & \mathbf{z}_s & \mathbf{z}_m \\ \mathbf{z}_m & \mathbf{z}_m & \mathbf{z}_s \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} \quad \ldots(2.32)$$

using equation (2.22) we can rewrite equation (2.32)

$$[c]\begin{bmatrix}\Delta\mathbf{V}_{a0}\\ \Delta\mathbf{V}_{a1}\\ \Delta\mathbf{V}_{a2}\end{bmatrix} = \begin{bmatrix}\mathbf{z}_s & \mathbf{z}_m & \mathbf{z}_m\\ \mathbf{z}_m & \mathbf{z}_s & \mathbf{z}_m\\ \mathbf{z}_m & \mathbf{z}_m & \mathbf{z}_s\end{bmatrix}[c]\begin{bmatrix}\mathbf{I}_{a0}\\ \mathbf{I}_{a1}\\ \mathbf{I}_{a2}\end{bmatrix} \quad \ldots(2.33)$$

$\Delta\mathbf{V}_{a0}$, $\Delta\mathbf{V}_{a1}$, $\Delta\mathbf{V}_{a2}$ are the zero sequence, positive sequence and negative sequence components of the unbalanced voltage drops $\Delta\mathbf{V}_a$, $\Delta\mathbf{V}_b$, $\Delta\mathbf{V}_c$.

So, from equation (2.33),

$$\begin{bmatrix}\Delta\mathbf{V}_{a0}\\ \Delta\mathbf{V}_{a1}\\ \Delta\mathbf{V}_{a2}\end{bmatrix}_c = [c]^{-1}\begin{bmatrix}\mathbf{z}_s & \mathbf{z}_m & \mathbf{z}_m\\ \mathbf{z}_m & \mathbf{z}_s & \mathbf{z}_m\\ \mathbf{z}_m & \mathbf{z}_m & \mathbf{z}_s\end{bmatrix}[c]\begin{bmatrix}\mathbf{I}_{a0}\\ I_{a1}\\ I_{a2}\end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \mathbf{a} & \mathbf{a}^2\\ 1 & \mathbf{a}^2 & \mathbf{a}\end{bmatrix}\begin{bmatrix}\mathbf{z}_s & \mathbf{z}_m & \mathbf{z}_m\\ \mathbf{z}_m & \mathbf{z}_s & \mathbf{z}_m\\ \mathbf{z}_m & \mathbf{z}_m & \mathbf{z}_s\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \mathbf{a}^2 & \mathbf{a}\\ 1 & \mathbf{a} & \mathbf{a}^2\end{bmatrix}\begin{bmatrix}\mathbf{I}_{a0}\\ \mathbf{I}_{a1}\\ \mathbf{I}_{a2}\end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix}(\mathbf{z}_s + 2\,\mathbf{z}_m) & (\mathbf{z}_s + 2\mathbf{z}_m) & (\mathbf{z}_s + 2\mathbf{z}_m)\\ (\mathbf{z}_s - \mathbf{z}_m) & (\mathbf{a}\mathbf{z}_s - \mathbf{a}\mathbf{z}_m) & (\mathbf{a}^2\mathbf{z}_s - \mathbf{a}^2\mathbf{z}_m)\\ (\mathbf{z}_s - \mathbf{z}_m) & (\mathbf{a}^2\mathbf{z}_s - \mathbf{a}^2\mathbf{z}_m) & (\mathbf{a}\mathbf{z}_s - \mathbf{a}\mathbf{z}_m)\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \mathbf{a}^2 & \mathbf{a}\\ 1 & \mathbf{a} & \mathbf{a}^2\end{bmatrix}\begin{bmatrix}\mathbf{I}_{a0}\\ \mathbf{I}_{a1}\\ \mathbf{I}_{a2}\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{z}_s + 2\mathbf{z}_m & 0 & 0\\ 0 & \mathbf{z}_s - \mathbf{z}_m & 0\\ 0 & 0 & \mathbf{z}_s - \mathbf{z}_m\end{bmatrix}\begin{bmatrix}\mathbf{I}_{a0}\\ \mathbf{I}_{a1}\\ \mathbf{I}_{a2}\end{bmatrix} \quad \ldots(2.34)$$

So, $\mathbf{V}_{a0} = (\mathbf{z}_s + 2\mathbf{z}_m)\mathbf{I}_{a0}$ …(2.35)

$\mathbf{V}_{a1} = (\mathbf{z}_s - \mathbf{z}_m)\,\mathbf{I}_{a1}$ …(2.36)

$\mathbf{V}_{a2} = (\mathbf{z}_s - \mathbf{z}_m)\,\mathbf{I}_{a2}$ …(2.37)

Equation (2.34) clearly shows that the elements of the impedance matrix are diagonal. We shall call this as the sequence impedance matrix of the static system and designate $\mathbf{Z}_o = \mathbf{z}_s + 2\,\mathbf{z}_m$; $\mathbf{Z}_1 = \mathbf{z}_s - \mathbf{z}_m$ and $\mathbf{Z}_2 = \mathbf{z}_s - \mathbf{z}_m$.

$\mathbf{Z}_o$, $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are the zero-sequence, positive sequence and negative sequence impedances of the static circuit.

The equation also shows that there is no mutual coupling between the current of one sequence with the voltage of another sequence. It can be shown that the same will be true for a symmetrical rotating machine. Hence, we make an important conclusion that the three fictitious sequence circuit equations (2.35), (2.36) and (2.37) are mutually exclusive i.e. have zero coupling. This gives us an opportunity to represent a symmetrical system under unbalanced operating condition, by three mutually independent sequence circuit on a per phase basis. We can also define the positive sequence impedance of a symmetrical circuit as the impedance offered when only positive sequence current flows in the circuit. Negative and zero sequence impedances are defined likewise.

A synchronous machine generates balanced positive sequence voltage only, under all conditions of loading and operation. Hence, only the positive sequence circuit can have voltage sources. Negative and zero sequence circuits cannot have any voltage sources.

So, the three mutually independent sequence networks on a per phase basis may be drawn as shown in fig. 2.8. The electrical connection of these fictitious networks will depend on the constraints imposed by the particular unbalanced operation.

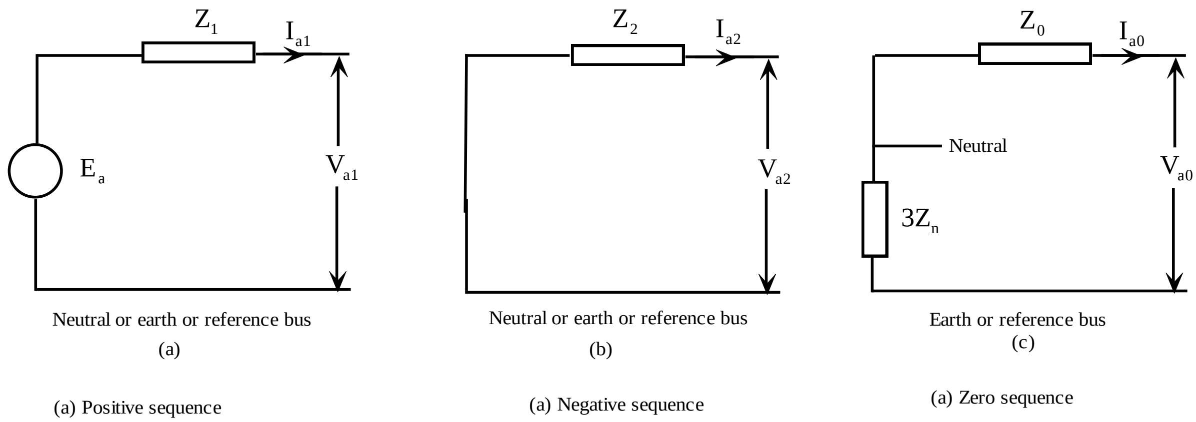


Fig 2.8 Sequence networks

In fig.2.8 the neutral bus is also the earth bus (i.e. both are at zero potential) for positive and negative sequence networks. This is because of the fact that the sum of the three positive sequence currents or three negative sequence currents at the neutral point is zero and no current flows to earth even if the neutral is earthed. However, zero-sequence currents in all the three phases are identical and sum up to 3 $\mathbf{I}_{ao}$ at the neutral point and flows to earth through any impedance $\mathbf{Z}_n$. The voltage drop between the neutral and earth bus will then be $3\boldsymbol{I}_{ao}\mathbf{Z}_{n}$ i.e. $\mathbf{I}_{ao}(3\,\mathbf{Z}_n)$. Hence in the zero-sequence circuit of fig. 2.8 (c) one has to connect an impedance of $3\mathbf{Z_n}$ between neutral and earth or reference bus. The terminal voltages are measured with respect to earth or reference bus.

So, from the above figure we get,

$$\mathbf{V}_{a1} = \mathbf{E}_a - \mathbf{I}_{a1}\,\mathbf{Z}_1$$

$$\mathbf{V}_{a2} = 0 - \mathbf{I}_{a2}\,\mathbf{Z}_2 = -\,\mathbf{I}_{a2}\,\mathbf{Z}_2$$

$$\mathbf{V}_{ao} = 0 - \mathbf{I}_{ao}\,(\mathbf{Z}_o + 3\,\mathbf{Z_n}) = -\,\mathbf{I}_{ao}\,(\mathbf{Z}_o + 3\,\mathbf{Z_n})$$

2.2.1 Power in terms of symmetrical components

The total complex power in a three-phase circuit is given by :

$$\mathbf{S = V_a\, I_a^* + V_b I_b^* + V_c I_c^*} \qquad \ldots(2.38)$$

$\mathbf{V}_a$, $\mathbf{V}_b$, $\mathbf{V}_c$ are phase voltages and $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$ are phase currents. In matrix form equation 92.38) will be :

$$\mathbf{S} = \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix}^* = \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix}^T [c]^T [c]^* \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \qquad \ldots(2.39)$$

$\mathbf{V}_{ao}$, $\mathbf{V}_{a1}$, $\mathbf{V}_{a2}$ are zero-sequence, positive sequence and negative sequence components of phase voltages; $\mathbf{I}_{ao}$, $\mathbf{I}_{a1}$, $\mathbf{I}_{a2}$ are the corresponding components of phase currents.

But, $$[c]^T [c]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a^2} & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a^2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a^2} \\ 1 & \mathbf{a^2} & \mathbf{a} \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \ldots(2.40)$$

Putting the result of equation (2.40) in equation (2.39) we get

$$\mathbf{S} = 3 \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix}^* = 3\,(\mathbf{V}_{a0} \quad \mathbf{I}_{a0}^{\,*} + \mathbf{V}_{a1} \; \mathbf{I}_{a1}^{\,*} + \mathbf{V}_{a2} \quad \mathbf{I}_{a2}^{\,*} \ldots(2.41)$$

We have two important observations on equation (2.41):-

(1) A voltage of a particular sequence interacts with the current of that sequence only to contribute to power.

(2) The total power is three times the sum of the symmetrical component powers per phase. In other words, the symmetrical component transformation is power invariant.

2.2.1 Sequence impedances of components

Before proceeding further let us examine the sequence impedances offered by various components of the power system. We have already defined the positive, negative and zero-sequence impedance of a component. Generally speaking the impedance of a particular sequence can be found by applying a voltage of that sequence only at the three terminals of the component and measuring the current. (We have considered all components to be symmetrical).

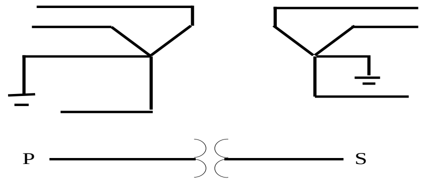
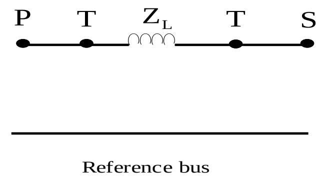
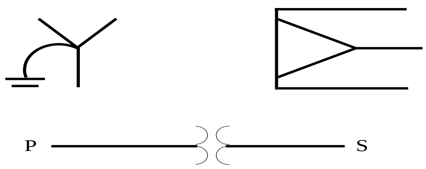
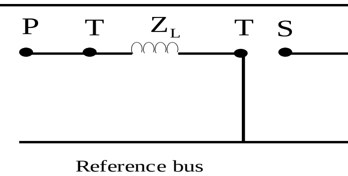
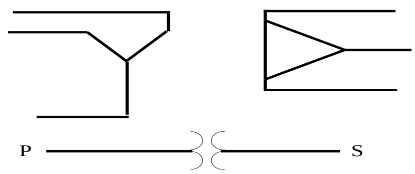
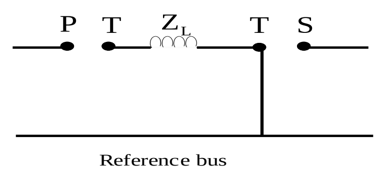
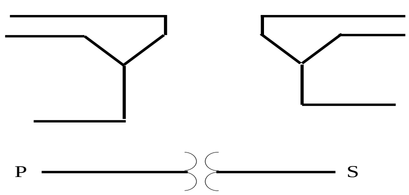
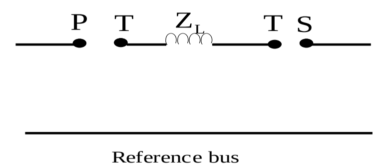
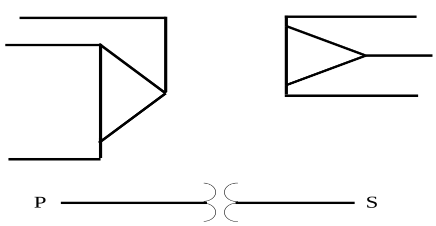
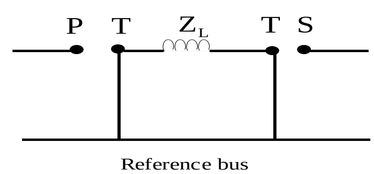
Connection: Primary / Secondary	Comment	Zero-sequence equivalent circuit
	As both primary and secondary sides are star connected with neutrals earthed, zero-sequence currents will flow in both primary and secondary windings.	
	Zero-sequence currents can flow in the line. Zero-sequence can also circulate in secondary delta, but cannot be present in secondary line.	
	Zero-sequence cannot flow in the primary because the star neutral is ungrounded. Zero-sequence current can circulate in secondary delta, but cannot be present in secondary line.	
	Zero-sequence current can neither flow in primary nor in secondary as both sides are star connected with ungrounded neutral.	
	Zero-sequence currents cannot be present in the lines of primary and secondary sides. But these can circulate in both the delta windings.	

Fig. 2.9 Zero-sequence networks of transformers

Fig. 2.9A summarizes the more usual cases of three winding transformers having delta winding tertiary. $\mathbf{Z}_p$, $\mathbf{Z}_s$, $\mathbf{Z}_T$ are p.u. leakage impedances of primary, secondary & tertiary windings.

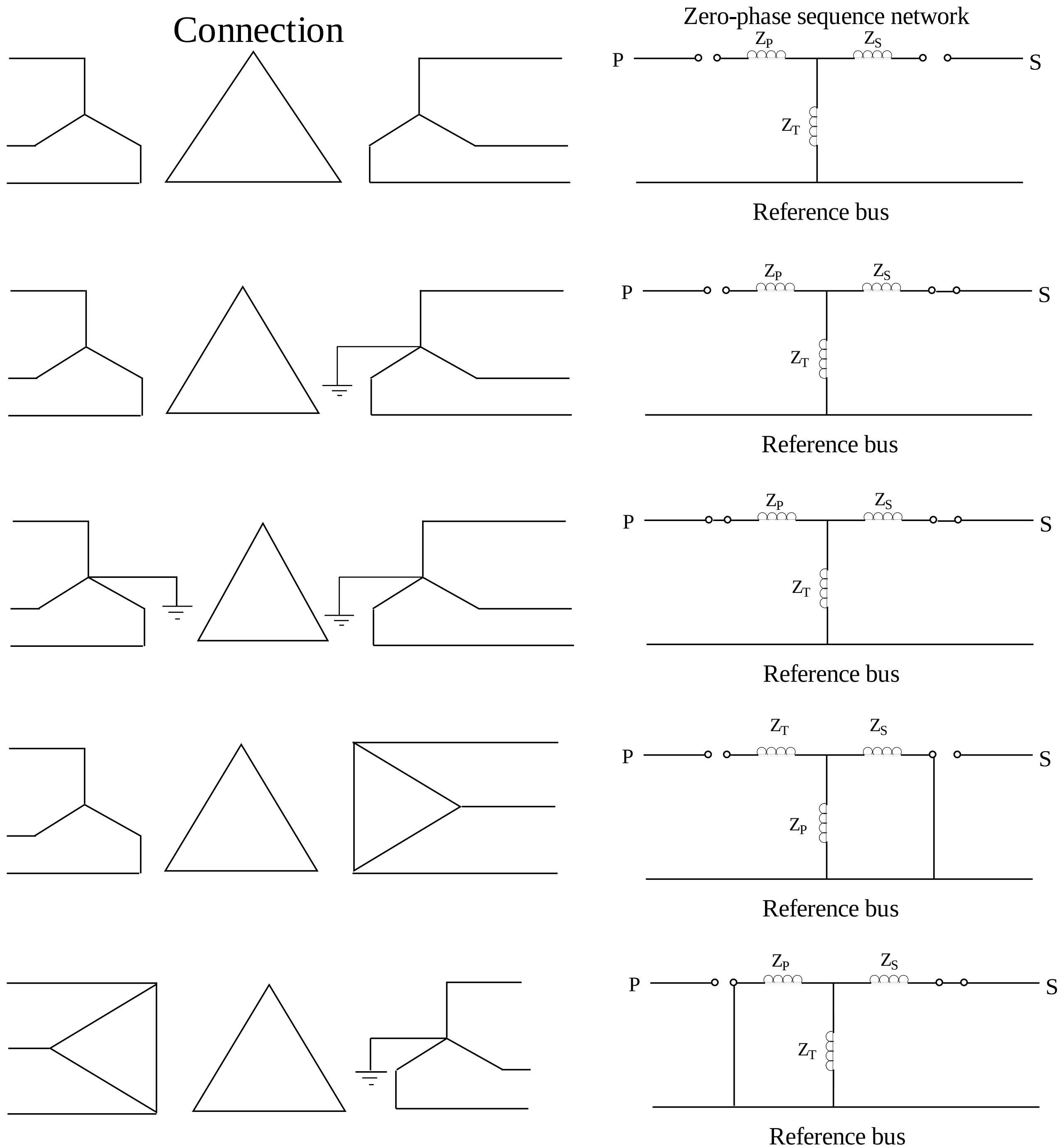


Fig. 2.9A Zero-sequence connections for three circuit transformers.

2.2.1 Phase shift in star-delta transformations

A three-phase transformer offers the same impedance $\mathbf{Z}_L$ to the flow of positive-sequence or negative-sequence current irrespective of the type of winding connection in the primary and secondary. However, the phase shift caused to the current during transformation differs in the two cases.

In general, if there is a phase-shift of γ degrees to the positive sequence set of voltages and currents, the corresponding shift for a negative-sequence set is $-\gamma$ degrees. Table 2.2 shows some common vector groupings of three-phase transformers for positive sequence and negative sequence applications.

Table 2.2 Vector grouping of transformers under positive and negative sequence applications

Sl. No.	Connection	Vector grouping positive sequence	Vector grouping negative sequence
1.	Star – Star	Y y 0	Y y 0
2.	Delta – Star	D y 0	D y 0
3.	Star – Star	Y y 6	Y y 6
4.	Star – Delta	Y d 1	Y d 11
5.	Star – Delta	Y d 11	Y d 1

Note: In the vector grouping nomenclature the first capital letter signifies the primary connection, the second letter denotes the secondary connection and the number denotes the phase angle of the secondary star phasor with respect to the corresponding primary star phasor. The primary phasor is assumed to be at 12 O'clock position of a clock. So, 1 means the secondary shift is 30^o lagging.

2.2.1 Assembling of system sequence networks

We have shown in article 2.2.4 that we can represent an unbalanced three-phase circuit by three de-coupled sequence networks. Only the positive sequence network will contain voltage sources. Further, the electrical connection of the three sequence networks will depend upon the constraints imposed by the type of unbalance. In other words, the electrical connection will be different for different type of unbalanced faults. Before we attempt to analyse the constraints for various faults we shall discuss the formation of sequence networks of simple systems with particular reference to the system, the single line diagram of which is shown in figure 2.10.

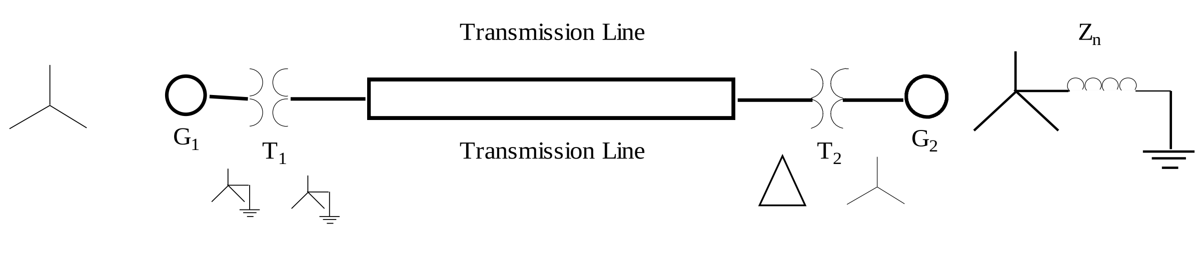
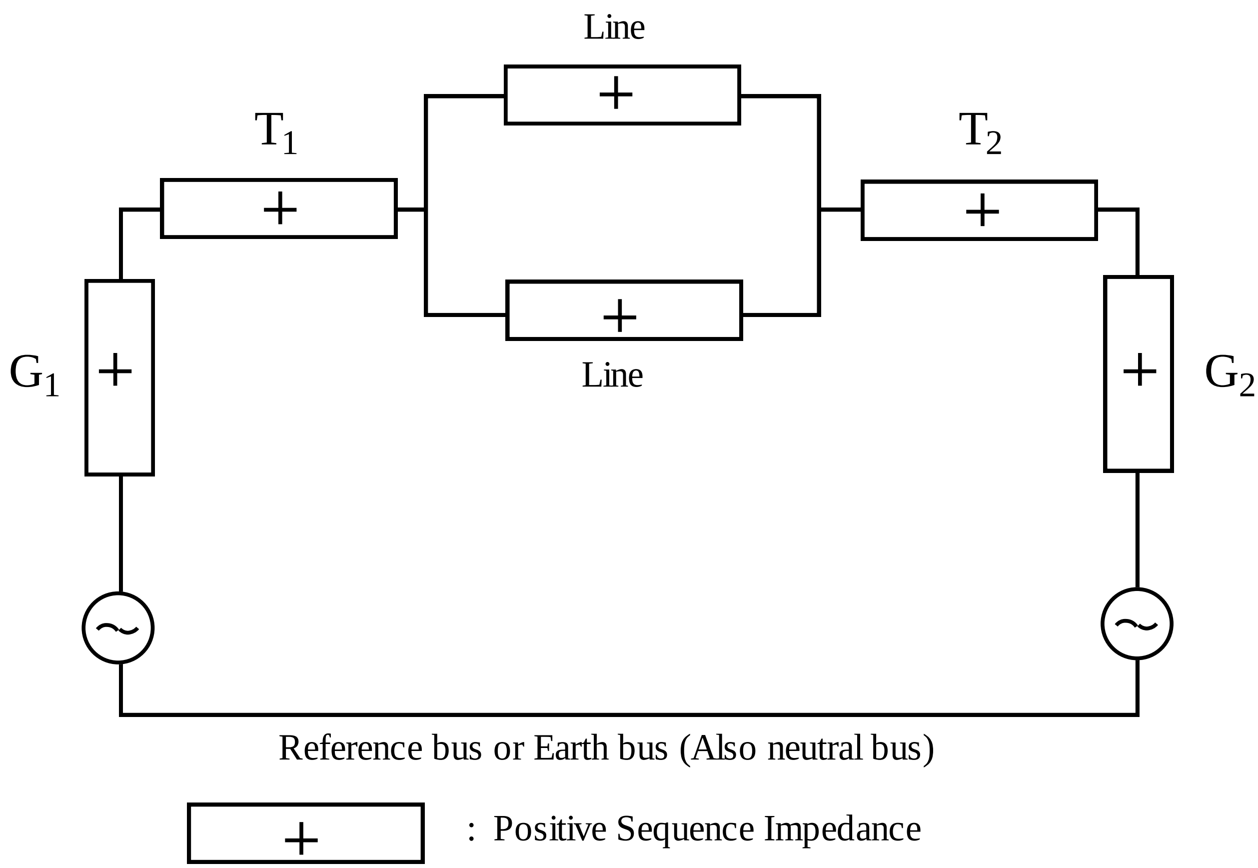
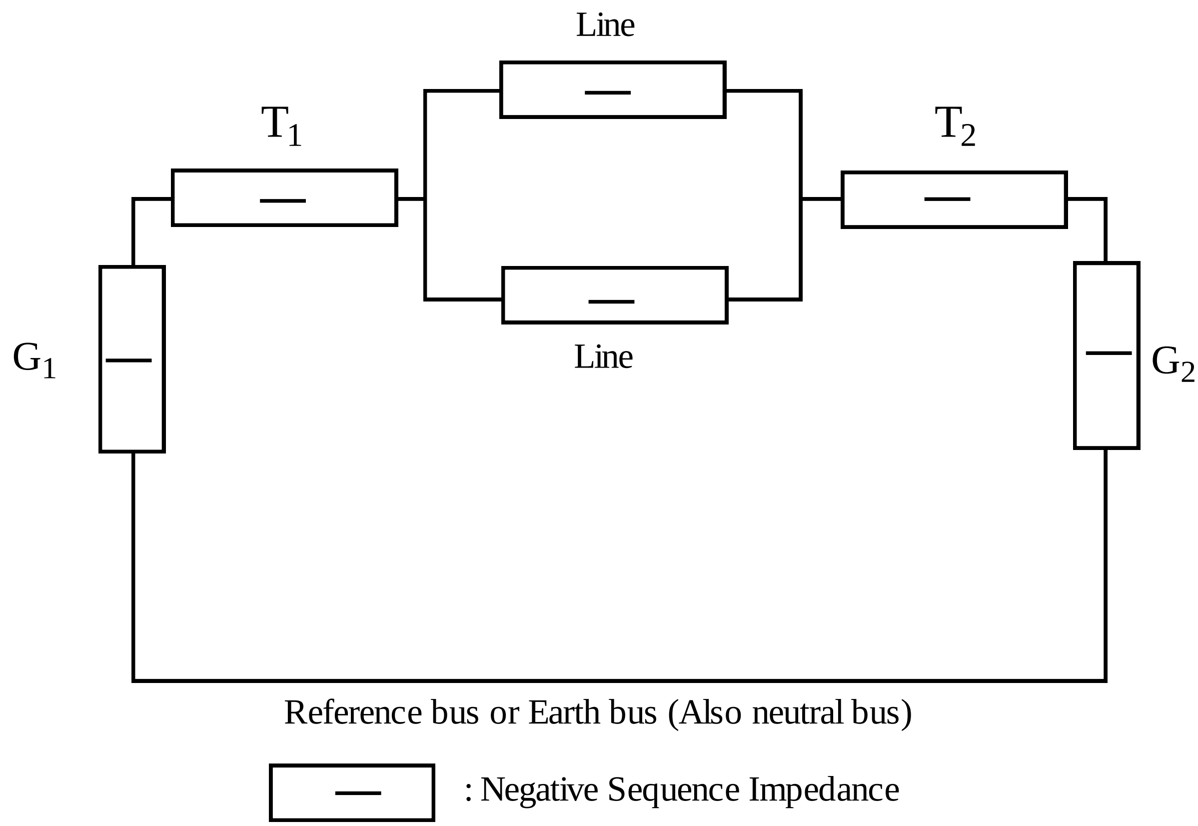


Fig 2.10 Single line diagram of a simple system

The positive sequence network is the same as is used for the calculation of symmetrical fault current. The negative sequence network will be same as the positive sequence network without the presence of the voltage sources and with the positive sequence impedances replaced by negative sequence impedances. A zero sequence network also does not contain any voltage source and special care is to be taken to represent the transformer depending upon its connection. Fig. 2.11 shows the three networks. The impedances in each sequence network represent the impedances of the components of that sequence only.

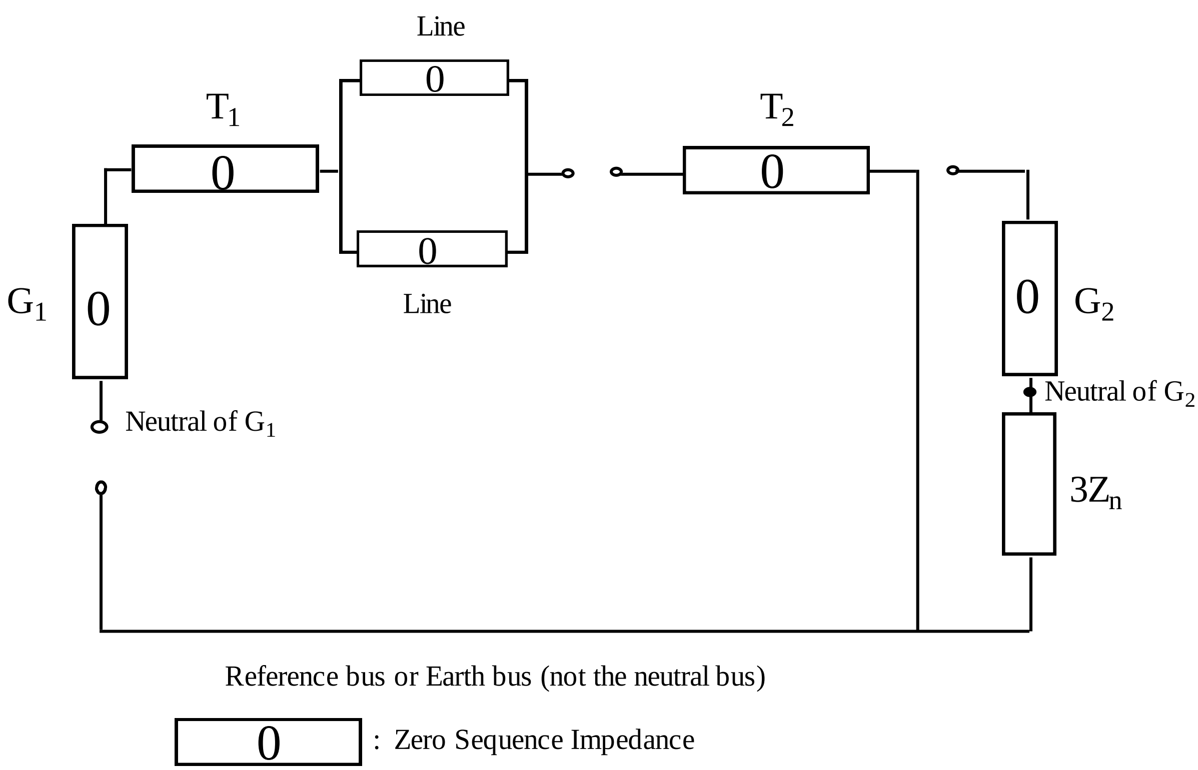


(a) Positive Sequence Network



(b)

Negative Sequence Network



(c) Zero Sequence Network

Fig 2.11 Sequence networks of the illustrative system

The zero-sequence network needs some explanation. The neutral of the generator G_1 is isolated. So, the neutral point of G_1 has not been connected to the reference or earth bus. The neutral of G_2 has been earthed through an impedance of $\mathbf{Z}_n$. So, its neutral is connected to the reference bus through the impedance 3 $\mathbf{Z}_n$. Both sides of transformer T_1 are earthed; hence zero sequence currents can flow through T_1 and its impedance is connected in the circuit. The star side of transformer T_2 cannot have zero-sequence current as the neutral is ungrounded. On the delta side zero sequence currents cannot be present in the line, but can circulate in the closed delta windings. So, the delta side terminal is shorted to the reference bus.

Computation of asymmetrical fault current

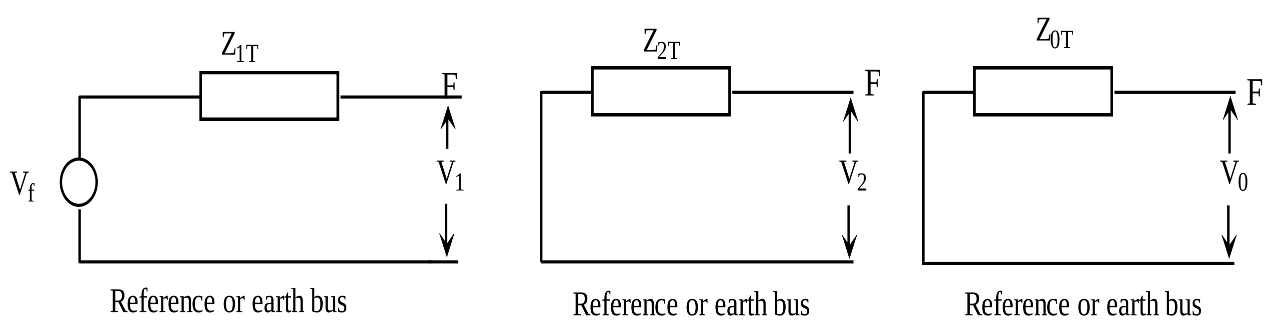
We are now in a position to list down the steps to be followed to compute asymmetrical fault current. There are:

(i) Draw the line diagram of the system

(ii) Assemble the three sequence networks individually and mark the fault point location F on each network.

(iii) Replace the network by their Thevenin equivalent at the point F. The Thevenin voltage source in the positive sequence network is V_f, the pre-fault voltage at F.

(iv) Connect the equivalent sequence networks at the points F according to the fault conditions and compute currents from the resulting network.



(a) Positive-sequence (b)Negative-sequence (c) Zero-sequence

$\mathbf{Z}_{1T}$ = Thevenin equivalent positive-sequence impedance as viewed from F

$\mathbf{Z}_{2T}$ =Thevenin equivalent negative-sequence impedance as viewed from F

$\mathbf{Z}_{0T}$ =Thevenin equivalent zero-sequence impedance as viewed from F

Single-line to ground faults (L-G fault)

Fig. 2.2 shows phase a shorted to earth through a fault impedance Z_f at the fault location F. We are neglecting capacitances at fault point.

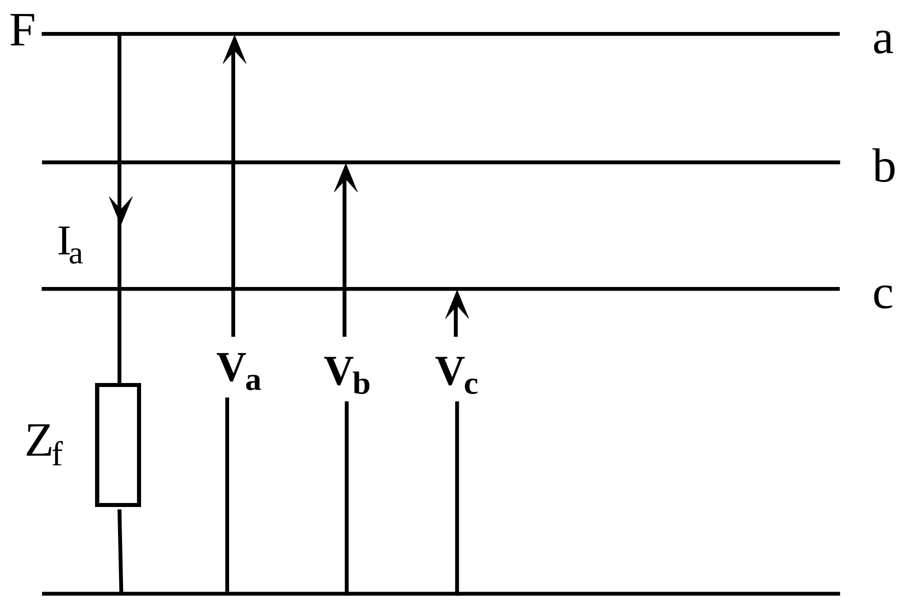


Fig. 2.2 Single line to ground fault at F.

The terminal conditions at the fault location F are :

$\mathbf{I}_b$ = current fed to the fault by phase b = 0 …(2.42)

$\mathbf{I}_c$ = current fed to the fault by phase c = 0 …(2.43)

$\mathbf{V}_a$ = voltage of phase a at the fault point F = $\mathbf{I}a\,\mathbf{Z}_f$ …(2.44)

$\mathbf{I}_a$ = current fed to the fault by phase a.

The sequence currents are then, according to equation (2.25):

$$\begin{bmatrix}\mathbf{I}_{a0}\\ \mathbf{I}_{a1}\\ \mathbf{I}_{a2}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \mathbf{a} & \mathbf{a}^2\\ 1 & \mathbf{a}^2 & \mathbf{a}\end{bmatrix}\begin{bmatrix}\mathbf{I}_a\\ \mathbf{I}_b\\ \mathbf{I}_c\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \mathbf{a} & \mathbf{a}^2\\ 1 & \mathbf{a}^2 & \mathbf{a}\end{bmatrix}\begin{bmatrix}\mathbf{I}_a\\ 0\\ 0\end{bmatrix} \qquad \text{…(2.45)}$$

Thus, $\mathbf{I}_{ao} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \frac{1}{3}\ \mathbf{I}_a$ …(2.46)

Also, from equation (2.44) and (2.46) we get:

$$\mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_{a0} = \frac{\mathbf{V}_a}{3\mathbf{Z}_f} = \frac{\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2}}{3\mathbf{Z}_f} \qquad \ldots(2.47)$$

The condition imposed by equation (2.47) can be met if the three sequence networks are connected in series through impedance 3 $\mathbf{Z}_f$, as shown in fig. 2.14. $\mathbf{V}_f$ is the pre-fault voltage at fault location F.

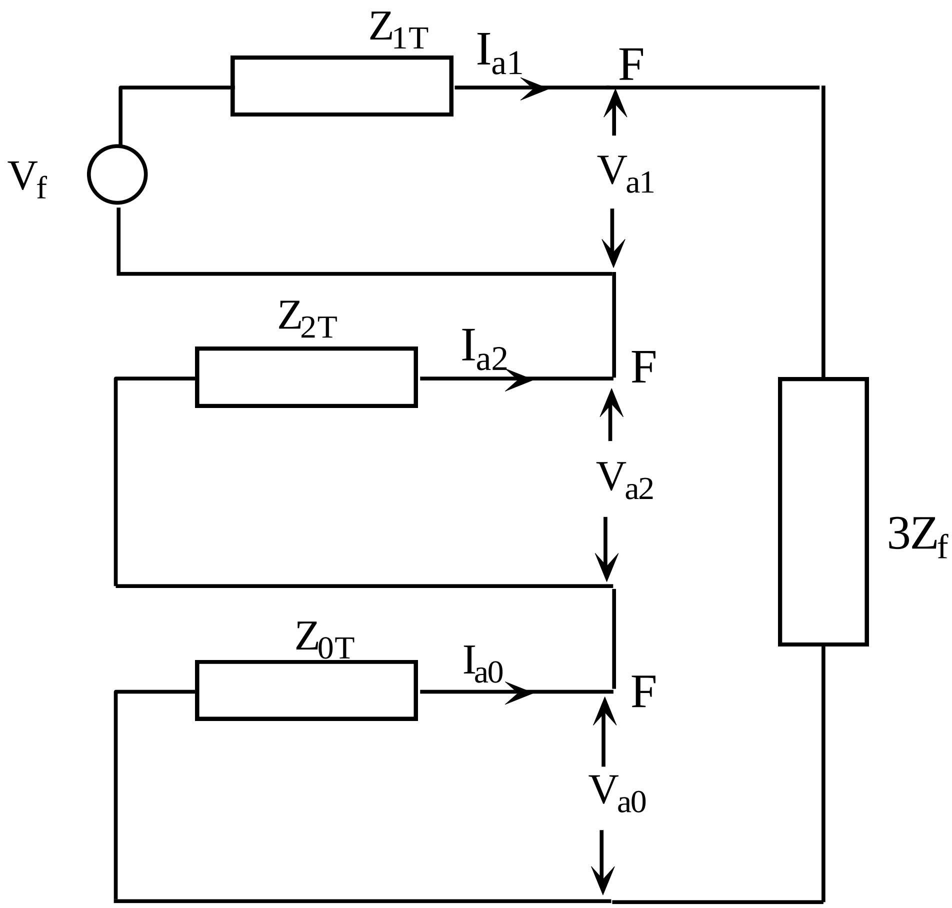


Fig. 2.14 Connection of sequence networks for a single line to ground fault.

From this resulting network, we get

$$\mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_{a0} = \frac{\mathbf{V}_f}{\mathbf{Z}_{1T} + \mathbf{Z}_{2T} + \mathbf{Z}_{0T} + 3\mathbf{Z}_f} \qquad \ldots(2.48)$$

So, $\mathbf{I}_a = 3\mathbf{I}_{a1} = \dfrac{3\mathbf{V}_f}{\mathbf{Z}_{1T} + \mathbf{Z}_{2T} + \mathbf{Z}_{0T} + 3\mathbf{Z}_f}$ $\qquad \ldots(2.48a)$

For computing voltages we follow these steps:

$$\mathbf{V}_{a1} = \mathbf{V}_f - \mathbf{I}_{a1}\,\mathbf{Z}_{1T} \qquad \ldots(2.49)$$

$$\mathbf{V}_{a2} = -\mathbf{I}_{a2}\,\mathbf{Z}_{2T} \qquad \ldots(2.50)$$

$$\mathbf{V}_{ao} = -\mathbf{I}_{ao}\,\mathbf{Z}_{0T} \qquad \ldots(2.51)$$

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \qquad \ldots(2.52)$$

Note: (i) Generally the fault impedance $\mathbf{Z}_f$ is taken as zero. So, $V_a = 0$.

(ii) For all unloaded system at normal system voltage $V_f = 1.0$ p.u.

(iii) If the system is ungrounded, then no zero sequence current can flow i.e. $Z_{0T} = \propto$ and hence, current due to single-line to ground fault is zero. However a small capacitance current will flow in the system through the fault point. This small capacitive current is capable to cause excessive overvoltage due to what is known as arcing ground, to be discussed in section 2.3.

Double line fault without involving ground (L-L fault)

Fig. 2.3 depicts the situation at the fault location F during L-L fault. Phases b and c are shorted through a fault impedance $\mathbf{Z}_f$.

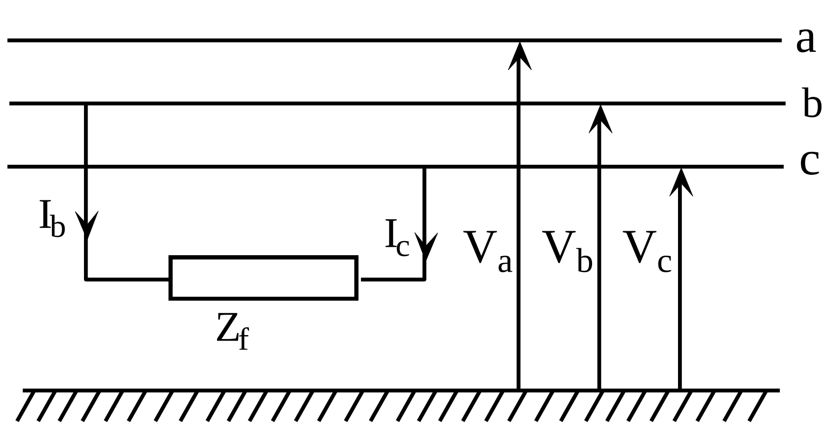


Fig. 2.3 Line to line fault

The fault conditions are:

$\mathbf{I}_a = 0$ …(2.53)

$\mathbf{I}_b = -I_c$ …(2.54)

$\mathbf{V}_b = \mathbf{V}_c + \mathbf{Z}_f I_b$ …(2.55)

The sequence currents at the fault are given by (following equation 2.25):

$$\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{0} \\ \mathbf{I}_b \\ -\mathbf{I}_b \end{bmatrix} \quad \text{…(2.56)}$$

The last equation yields :

$$\mathbf{I}_{a0} = \frac{1}{3}(\mathbf{I}_b - \mathbf{I}_b) = 0 \quad \text{…(2.57)}$$

This is indeed according to our expectation. Flow of zero-sequence current requires a return path and as the fault does not involve ground there is no return path and no zero-sequence current exists in the fault current.

$$\mathbf{I}_{a1} = \frac{1}{3}(\mathbf{a}\mathbf{I}_b - \mathbf{a}^2\mathbf{I}_b) = \frac{1}{3}\mathbf{I}_b(\mathbf{a} - \mathbf{a}^2) = \frac{1}{3}(j\sqrt{3})\,\mathbf{I}_b = \frac{j\mathbf{I}_b}{\sqrt{3}}$$

$$\mathbf{I}_{a2} = \frac{1}{3}(\mathbf{a}^2\mathbf{I}_b - \mathbf{a}\mathbf{I}_b) = \frac{1}{3}\mathbf{I}_b(\mathbf{a}^2 - \mathbf{a}) = \frac{-j\mathbf{I}_b}{\sqrt{3}} \quad \text{…(2.59)}$$

So, $\mathbf{I}_{a1} = -\mathbf{I}_{a2}$ …(2.60)

Moreover,

$\mathbf{I}_b = \mathbf{I}_{a0} + \mathbf{a}^2\,\mathbf{I}_{a1} + \mathbf{a}\,\mathbf{I}_{a2} = (\mathbf{a}^2\text{-}\mathbf{a})\,\mathbf{I}_{a1}$ …(2.61)

As, $\mathbf{I}_{a0} = 0$, $\mathbf{V}_{a0} = 0$ …(2.62)

Now, $\mathbf{V}_b = \mathbf{V}_{a0} + \mathbf{a}^2\,\mathbf{V}_{a1} + \mathbf{a}\,\mathbf{V}_{a2} = \mathbf{a}^2\,\mathbf{V}_{a1} + \mathbf{a}\,\mathbf{V}_{a2}$ …(2.63)

$\mathbf{V}_c = \mathbf{V}_{a0} + \mathbf{a}\,\mathbf{V}_{a1} + \mathbf{a}^2\,\mathbf{V}_{a2} = \mathbf{a}\,\mathbf{V}_{a1} + \mathbf{a}^2\,\mathbf{V}_{a2}$ …(2.64)

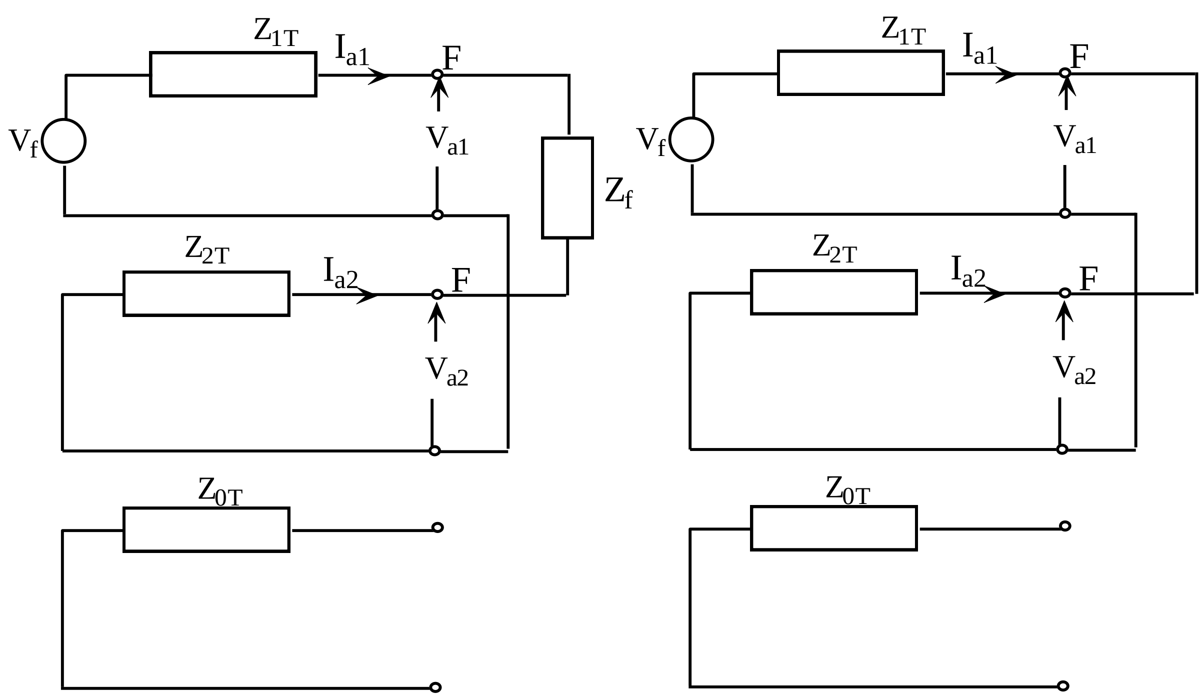
Putting values of $\mathbf{I}_b$, $\mathbf{V}_b$ and $\mathbf{V}_c$ from equations (2.61), (2.63) and (2.64) respectively into equation (2.55) one gets,

$$\mathbf{a}^2\,\mathbf{V}_{a1} + \mathbf{a}\,\mathbf{V}_{a2} = \mathbf{a}\,\mathbf{V}_{a1} + a^2\,\mathbf{V}_{a\,2} + \mathbf{Z}_f\,\mathbf{I}_{a1}\,(\mathbf{a}^2 - \mathbf{a})$$

i.e. $\mathbf{V}_{a1}\,(\mathbf{a}^2\text{-}\mathbf{a}) = \mathbf{V}_{a2}\,(\mathbf{a}^2\text{-}\mathbf{a}) + \mathbf{Z}_f\,\mathbf{I}_{a1}\,(\mathbf{a}^2\text{-}\mathbf{a})$

i.e. $\mathbf{V}_{a1} = \mathbf{V}_{a2} + \mathbf{Z}_f\,\mathbf{I}_{a1}$ …(2.65)

Equations (2.57), (2.60) and (2.65) are satisfied only if we connect the positive and negative sequence networks as shown in fig. 2.16. The zero-sequence network is kept isolated. If $Z_f = 0$, the positive and negative sequence networks are in parallel at the fault point as shown in fig. 2.16(b).



(a) Fault impedance ≠ 0 (b) Fault impedance = 0

Fig.2.16 Sequence network connections for line to line fault

From the equivalent circuit we get,

$$\mathbf{I}_{a1} = \frac{\mathbf{V}_f}{\mathbf{Z}_{1T} + \mathbf{Z}_{2T} + \mathbf{Z}_f}$$

$$\mathbf{I}_b = -I_c = (\mathbf{a}^2 - \mathbf{a})\,\mathbf{I}_{a1} = \text{-}\,j\,\sqrt{3}\,\mathbf{I}_{a1} = \frac{-\,j\sqrt{3}\,\mathbf{V_f}}{\mathbf{Z}_{1T} + \mathbf{Z}_{2T} + \mathbf{Z}_f} \qquad \text{…(2.66)}$$

Note: Current due to any fault that does not involve ground, cannot have zero-sequence component.

Double line fault involving ground (L-L-G fault):

The fault is shown in fig. 2.17. Phases b and c are shorted to the earth through the fault impedance Z_f.

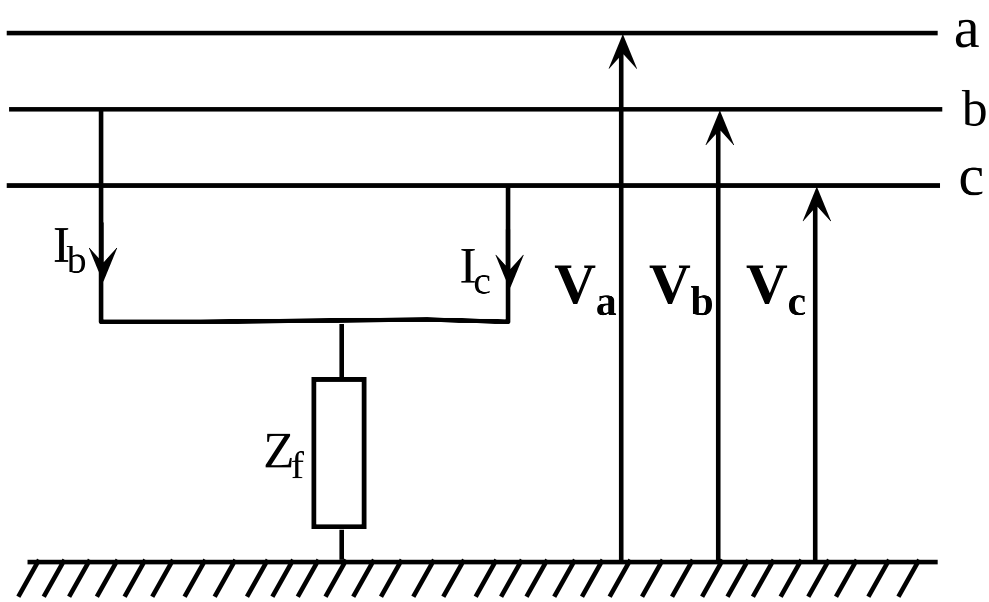


Fig. 2.17 Double line to ground fault

The fault conditions are :

$$\mathbf{I}_a = 0 \quad \ldots(2.67)$$

$$\mathbf{V}_b = \mathbf{V}_c \quad \ldots(2.68)$$

$$\mathbf{V}_b = \mathbf{Z}_f\,(\mathbf{I}_b + \mathbf{I}_c) \quad \ldots(2.69)$$

From equation (2.68) we get,

$$\mathbf{V}_{ao} + \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} = \mathbf{V}_{a0} + \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2}$$

i.e. $\mathbf{V}_{a1} = \mathbf{V}_{a2}$ …(2.70)

Also, $\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 3\,\mathbf{I}_{a0}$

Or, $\mathbf{I}_b + \mathbf{I}_c = 3\,\mathbf{I}_{a0}$ …(2.71)

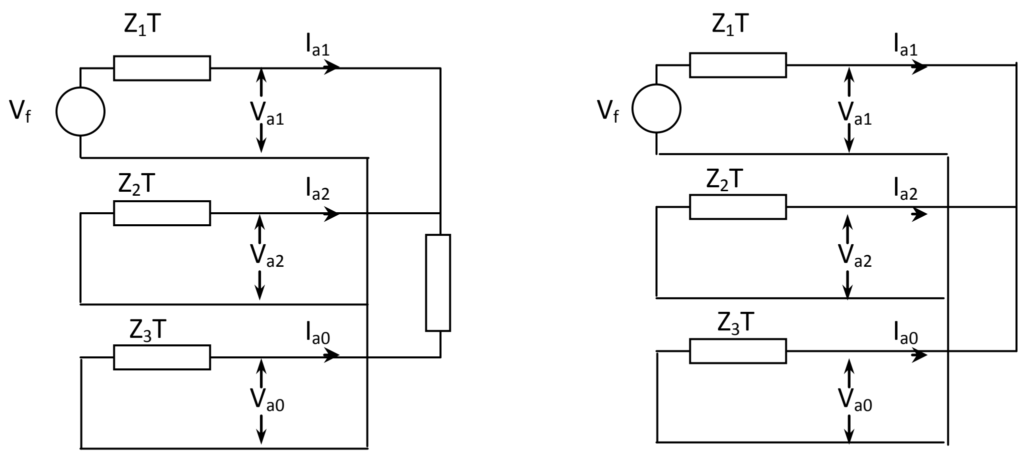
Expressing $\mathbf{V}_b$ in terms of symmetrical components and using equation (2.70) and (2.71) one gets from equation (2.69).

$$\mathbf{V}_{ao} - \mathbf{V}_{a1} = 3\,\mathbf{I}_{a0}\,\mathbf{Z}_f \qquad \ldots(2.72)$$

Equation (2.67) requires

$$\mathbf{I}_{a1} + \mathbf{I}_{a2} + \mathbf{I}_{a0} = \quad 0 \qquad \ldots(2.73)$$

Equations (2.70), (2.72) and (2.73) are satisfied by the interconnection of sequence networks shown in fig. 2.18. With $\mathbf{Z}_f = 0$, the sequence networks are connected in parallel at the fault point as shown in fig. 2.18b).



(a) Fault impedance>0 (b) Fault impedance=0

Fig. 2.18 Sequence network connections for L-L-G fault.

From the equivalent circuit of fig. 2.18, one gets

$$\mathbf{I}_{a1} = \frac{\mathbf{V}_f}{\mathbf{Z}_{1T} + \dfrac{\mathbf{Z}_{2T}(\mathbf{Z}_{0T} + 3\mathbf{Z}_f)}{\mathbf{Z}_{2T} + (\mathbf{Z}_{0T} + 3\mathbf{Z}_f)}} \qquad \ldots(2.74)$$

$$\mathbf{I}_{a2} = -\mathbf{I}_{a1}\cdot\frac{\mathbf{Z}_{0T} + 3\mathbf{Z}_f}{\mathbf{Z}_{2T} + \mathbf{Z}_{0T} + 3\mathbf{Z}_f} \qquad \ldots(2.75)$$

$$\mathbf{I}_{a0} = -\mathbf{I}_{a1}\cdot\frac{\mathbf{Z}_{2T}}{\mathbf{Z}_{2T} + \mathbf{Z}_{0T} + 3\mathbf{Z}_f} \qquad \ldots(2.76)$$

Example 2.7

The line currents in a three-phase system are:

$I_a = 10 \angle 90^o$ A, $I_b = 10 \angle -90^o$ A and $I_c = 10 \angle 0^o$ A.

Find the symmetrical components of the line currents.

$\mathbf{I}_{a0}$ = zero-sequence component

$$= \frac{1}{3}(\mathbf{I}_a + \mathbf{I_b} + \mathbf{I}_c) = \frac{1}{3}(j10 - j10 + 10) = 3.33\angle 0^0 \text{ A}.$$

I_{a1} = positive sequence component

$$= \frac{1}{3}(\mathbf{I_a} + \mathbf{aI_b} + \mathbf{a}^2\mathbf{I_c}) = \frac{1}{3}(\mathbf{j}10 + 10\angle 30^0 + 10\angle 240^0 = 2.437\angle 60^0 \textbf{ A}$$

I_{a2} = Negative sequence component

$$= \frac{1}{3}(\mathbf{I}_a + \mathbf{a}^2\mathbf{I}_b + \mathbf{aI}_c) = \frac{1}{3}(j10 + 10\angle 150^0 + 10\angle 120^0) = 9.1\angle 120^0 \text{ A}$$

Example 2.8

A 2.2 kV, 25 MVA three-phase synchronous generator with solidly earthed neutral, has a three-phase short circuit MVA of 170 MVA. Calculate the short circuit currents and the terminal voltages for (i) L-G fault, (ii) L-L fault, and (iii) L-L-G fault at the terminals of the generator. The negative and zero sequence reactances of the machine are 0.2 p.u. and 0.05 p.u. respectively. Neglect pre-fault current, and losses. Assume the pre-fault generated e.m.f. at the rated value. The faults are of dead short circuit ctype.

Let us assume base quantities to be the rating of the generator. So, base voltage = 2.2 kV (line to line) and base MVA = 25.

$$\text{Base current} = \frac{25 \text{ x } 10^6}{\sqrt{3} \text{ x } 13.2 \text{ x } 10^3} \text{A} = 1093.5\text{A}$$

$$\text{Fault MVA} = \frac{1}{x''} \text{ Base MVA}$$

x'' = sub-transient reactance in p.u.

x_1 = positive sequence reactance under sub-transient condition in p.u

$$= x'' = \frac{25}{170}\text{p.u} = 0.147\text{p.u}$$

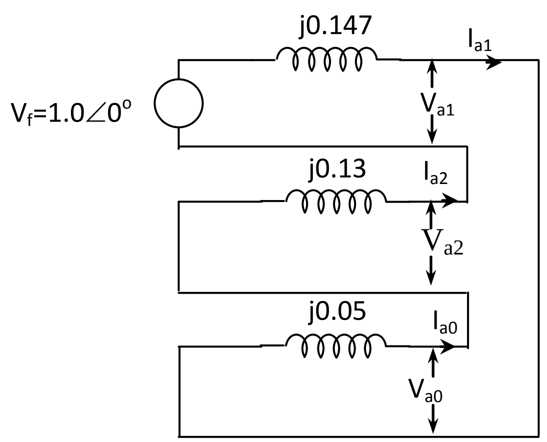
x_2 = 0.2 p.u.; x_0 = 0.05 p.u.

As the machine is unloaded before fault, the pre-fault terminal voltage = $\mathbf{V}_f = 1.0 \angle 0^o$ p.u.

The faults are dead short circuits i.e. $\mathbf{Z}_f$ = fault impedance = 0

<u>L-G fault</u>

The sequence diagram connection is shown below:



$$\mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_{a0} = \frac{\mathbf{V}_f}{jx_1 + jx_2 + jx_o}\text{p.u.} = \frac{1\angle 0^0}{j(0.147 + 0.13 + 0.05)} = -j3.058\text{p.u.}$$

Fault current at phase a = $\mathbf{I}_a$=3 $\mathbf{I}_{a1}$=-j9.174 p.u.=-j 9.174 x 1093.5 =-j 10031.8 A

$\mathbf{I}_b = \mathbf{I}_c = 0$.

The sequence terminal voltages are :

$\mathbf{V}_{a1} = \mathbf{V}_f - \mathbf{I}_{a1}\,(jx_1)$ = 1.0 – (-j3.058) (j0.147) = 0.550 p.u.

$\mathbf{V}_{a2} = -\mathbf{I}_{a2}\,(j\,x_2)$ = - (-j 3.058) (j 0.2) = - 0.397 p.u.

$\mathbf{V}_{a0} = -\mathbf{I}_{a0}\,(j\,x_0)$ = -(-j 3.058) (j0.05) = - 0.33 p.u.

So, line-to-neutral voltages at the terminal during fault are:

$\mathbf{V}_a = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2}$ = 0.55 – 0.397 – 0.33 = 0 (as expected)

$\mathbf{V}_b = \mathbf{V}_{a0} + \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} = -0.33 + 0.55\angle 240^0 + 0.397\angle 300^o$

$= -0.23 + j0.82 = 0.852\angle -105.67^0\,\text{p.u} = 0.852 \times \frac{13.2}{\sqrt{3}} \angle -105.67\,\text{kV} = 6.49\angle -105.67^0\ \text{kV}$

$\mathbf{V}_c = \mathbf{V_{a0}} + \mathbf{aV_{a1}} + \mathbf{a^2V_{a2}} = -0.153 + 0.55\angle 120^0 + 0.397\angle 420^0$

$= -0.23 + j0.82 = 0.852\angle 105.67\text{p.u} = 6.49\angle 105.67\text{kV}$

So, the line voltages are :

$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = -\mathbf{V}_b = -6.49 \angle -105.67^0$ kV $= 6.49 \angle 74.33^0$ kV

$\mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = 6.49 \angle -105.67^0 - 6.49 \angle 105.67^o = 12.49 \angle 270^o$ kV

$\mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = \mathbf{V}_c = 6.49 \angle 105.67^o$ kV

Note that $\mathbf{V}_{bc} > 11$ kV (normal system voltage).

<u>An important note</u>: If we would have calculated the line voltages from the p.u. phase voltages, then the base voltage value is to be taken as 11/√3 kV. For example:

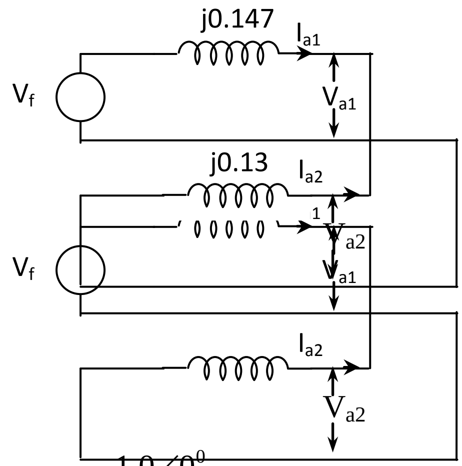
$\mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = 0.852 \angle -105.67^o - 0.852 \angle 105.67^0$ p.u. $= 1.64 \angle 270^0$ p.u.

$\mathbf{V}_{bc}$ is the subtraction of two p.u. phase voltages and hence in this case, $\mathbf{V}_{bc}$ in kV will be:

$$1.64\,\text{p.u.} \times \text{Base kV (phase)} = 1.64 \times \frac{13.2}{\sqrt{3}}\,\text{kV} = 12.49\,\text{kV}$$

<u>L-L fault</u>

The sequence connection diagram is shown below:



$$\mathbf{I}_{a1} = \frac{\mathbf{V}_f}{j(x_1 + x_2)} = \frac{1.0\angle 0^0}{j(0.147 + 0.13)} = -j3.61\,\text{p.u.}$$

$\mathbf{I}_{a2} = -\mathbf{I}_{a1} = j3.61$ p.u.

$\mathbf{I}_{a0} = 0$

$\mathbf{I}_a = \mathbf{I}_{a1} + \mathbf{I}_{a2} + \mathbf{I}_{a0} = 0$

$\mathbf{I}_b = \mathbf{a}^2\mathbf{I}_{a1}+\mathbf{a}\mathbf{I}_{a2}+\mathbf{I}_{a0} = (\mathbf{a}^2\text{-}\mathbf{a})\mathbf{I}_{a1}=-j\sqrt{3}\ \mathbf{I}_{a1} = -\sqrt{3} \text{ x } 3.61 \text{ p.u.} = 6.245 \text{ x } 1093.5 \angle 180^0 = 6829.44 \angle 180^o \text{ A.}$

$\mathbf{I}_c = -\mathbf{I}_b = 6.245 \angle 0^0 \text{ p.u.} = 6829.44 \angle 0^0 \text{ A.}$

$$\mathbf{V}_{a1}=\mathbf{V}_f-\mathbf{I}_{a1}\,(jx_1)=1.0-(-j3.61)(j0.147)=0.469\angle 0^0 \text{ p.u}$$

$$\mathbf{V}_{a2}=\mathbf{V}_{a1} = 0.469\angle 0^0 \text{ p.u;} \quad \mathbf{V}_{a0}=0$$

$$\mathbf{V}_a = \mathbf{V}_{a0}+\mathbf{V}_{a1}+\mathbf{V}_{a2} = 0.938\text{p.u}=0.938\times\frac{13.2}{\sqrt{3}}\angle 0^0=7.49\angle 0^0 \text{kV}$$

$$\mathbf{V}_b = \mathbf{V}_{a0}+\mathbf{a}^2\mathbf{V}_{a1}+\mathbf{a}\mathbf{V}_{a2} = (\mathbf{a}^2+\mathbf{a})\,\mathbf{V}_{a1}=-\mathbf{V}_{a1}=-0.469 \text{ p.u}= 3.574\angle 180^0 \text{kV}$$

$$\mathbf{V}_c = \mathbf{V}_{a0}+\mathbf{a}\,\mathbf{V}_{a1}+\mathbf{a}^2\mathbf{V}_{a2}=(\mathbf{a}+\mathbf{a}^2)\,\mathbf{V}_{a1}=3.574\angle 180^0 \text{ kV}$$

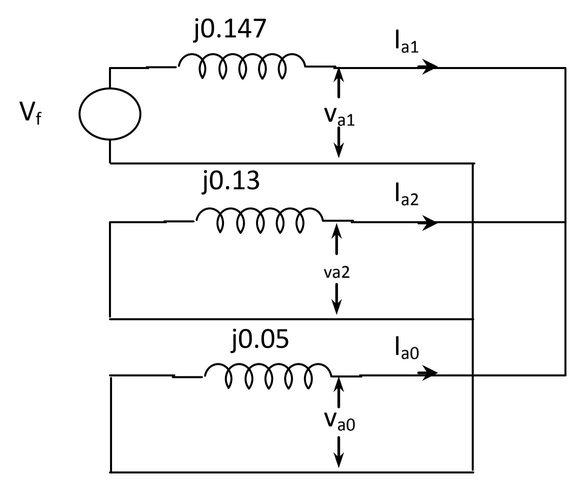
$$\mathbf{V}_{ab}=\mathbf{V}_a-\mathbf{V}_b=7.149\angle 0^0-3.574\angle 180^0=10.722\angle 0^0 \text{ kV}$$

$$\mathbf{V}_{bc}=\mathbf{V}_b-\mathbf{V}_c=0$$

$$\mathbf{V}_{ca}=\mathbf{V}_c-\mathbf{V}_a= 3.574\angle 180^0-7.149\angle 0^0=10.722\angle 180^0 \text{ kV}$$

<u>L-L-G fault</u>

The network connection is shown below:



$$\mathbf{I}_{a1}=\frac{\mathbf{V}_f}{j0.147+\dfrac{(j0.13)(j0.05)}{j0.13+j0.05}}=\frac{1.0\angle 0^0}{j0.147+j0.036}=-j5.46 \text{ p.u.}$$

$$\mathbf{I}_{a2}=-\mathbf{I}_{a1}\times\frac{j0.05}{j0.13+j0.05}=j5.46\times\frac{0.05}{0.18}=j1.517 \text{ p.u.}$$

$$\mathbf{I}_{a0}=-\mathbf{I}_{a1}\times\frac{j0.13}{j0.13+j0.05}=j5.46\times\frac{0.13}{0.18}=j3.943 \text{ p.u.}$$

$\mathbf{I}_a = \mathbf{I}_{a0} + \mathbf{I}_{a1} + \mathbf{I}_{a2} = 0$

$\mathbf{I}_b = \mathbf{I}_{a0} + \mathbf{a}^2\mathbf{I}_{a1} + \mathbf{a}\mathbf{I}_{a2}$

$$= j3.943 + (-j5.46)(1\angle 240^0) + (j1.517)(1\angle 120^0) = 8.45\angle 135.6^0 \text{ p.u}$$
$$= 8.45 \times 1093.5\angle 135.6^0 = 9240.54\angle 135.6^0 \text{ A}$$

$\mathbf{I}_c = \mathbf{I}_{a0} + \mathbf{a}\mathbf{I}_{a1} + \mathbf{a}^2\mathbf{I}_{a2}$

$$= j3.943 + 5.46\angle 30^0 + 1.517\angle 330^0 = 8.45\angle 44.4^0 \text{ p.u} = 9240.5\angle 44.4^0 \text{ A}$$

Fault current to ground $= \mathbf{I}_b + \mathbf{I}_c = 9240.54(1\angle 135.6^0 + 1\angle 44.4^0 = 13195\angle 90^0 \text{ A}$

$\mathbf{V}_{a1} = \mathbf{V}_f - \mathbf{I}_{a1}(jx_1)$

$$= 1\angle 0^0 - (-j5.46)(j0.147) = 0.197\angle 0^0 \text{ p.u}$$
$$\mathbf{V}_{a0} = \mathbf{V}_{a0} = \mathbf{V}_{a1} = 0.197\angle 0^0 \text{ p.u}$$
$$\mathbf{V}_a = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = 0.591\angle 0^0 \text{ p.u}$$
$$\mathbf{V}_b = \mathbf{V}_{a0} + \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} = 0$$
$$\mathbf{V}_c = \mathbf{V}_{a0} + \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2} = 0$$
$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = 4.513\angle 0^0 \text{ kV}$$
$$\mathbf{V}_{bc} = 0; \quad \mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = 4.513\angle 180^0 \text{ kV}$$

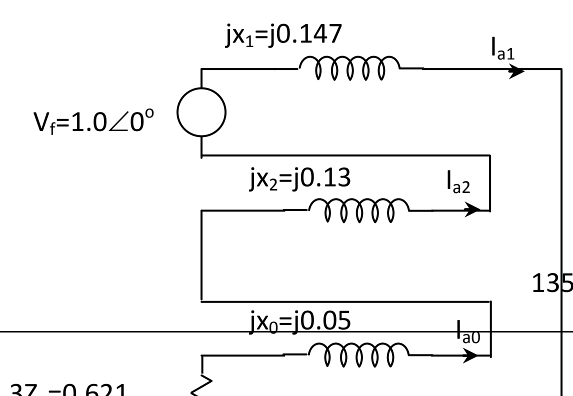
Example 2.9

In the previous problem the generator neutral is earthed through a 1-ohm resistance. Calculate the fault current for a L-G short circuit at its terminals. What is the neutral potential?

The p.u. value of the earthing resistance $= 1 \times \dfrac{\text{Base MVA}}{(\text{Base kV})^2} = 1 \times \dfrac{25}{(11)^2} = 0.207$

$\mathbf{Z}_n$ = neutral impedance = (0.207+j0) p.u

The interconnection of the sequence networks is:



$$\mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_{a0} = \frac{\mathbf{V}_f}{jx_1 + jx_2 + (3\mathbf{Z}_n + jx_0)}$$

$$= \frac{1\angle 0^0}{j0.147 + j0.13 + (0.621 + j0.05)} = \frac{1\angle 0^0}{0.621 + j0.327} = 1.425\angle -27.77^0$$

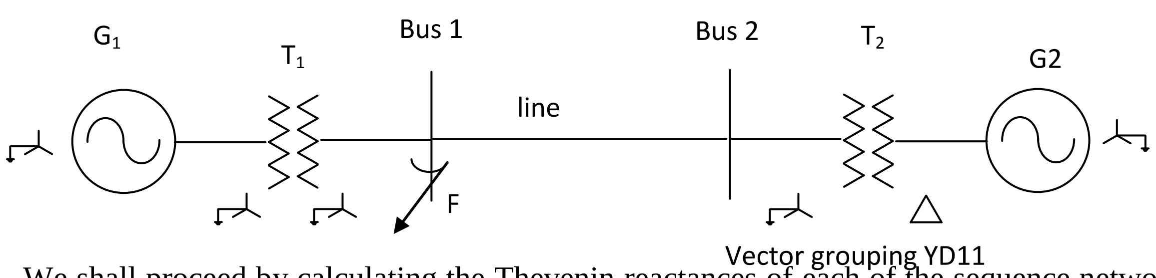
$\therefore \mathbf{I}_a = 3\mathbf{I}_{a1} = 4.275\angle -27.77^0$ p.u $= 4.275 \times 1093.5\angle -27.77^0$ A $= 4674.19\angle -27.77^0$ A

Potential of neutral $= 3\mathbf{I}_{a0}\,\mathbf{Z}_n = 4674.19 \times 1\angle -27.77$ V $= 4.674\angle -27.77$kV

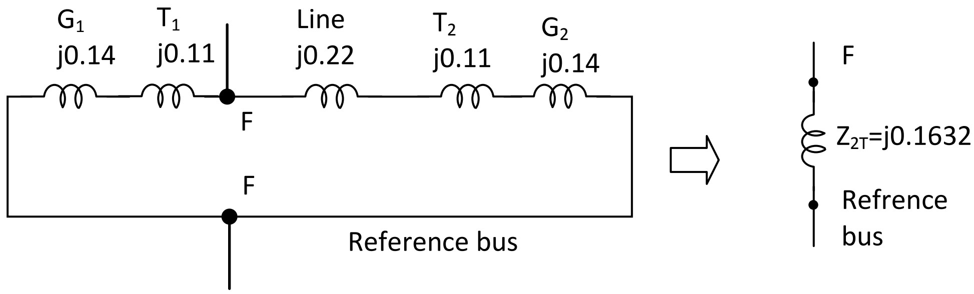
Example 2.10

A two bus system is show below. The generators G_1 and G_2 are identical. Neglecting pre-fault current and losses, calculate the fault current for a L-G fault at bus 1. Find out the currents contributed by G_1 and G_2. The pre-fault generated voltages were at rated values. The reactances of components are given below:

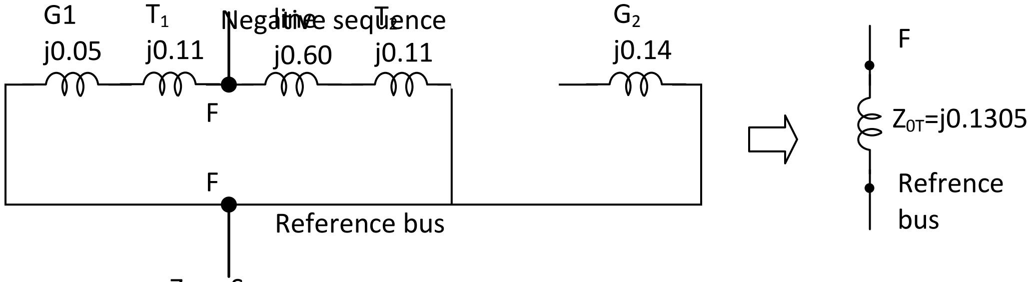
Equipment	Positive sequence reactance in p.u.	Negative sequence reactance in p.u.	Zero sequence reactance in p.u.
G_1	0.17	0.14	0.05
G_2	0.17	0.14	0.05
T_1	0.11	0.11	0.11
T_2	0.11	0.11	0.11
Line	0.22	0.22	0.60



We shall proceed by calculating the Thevenin reactances of each of the sequence networks as viewed from fault point F.



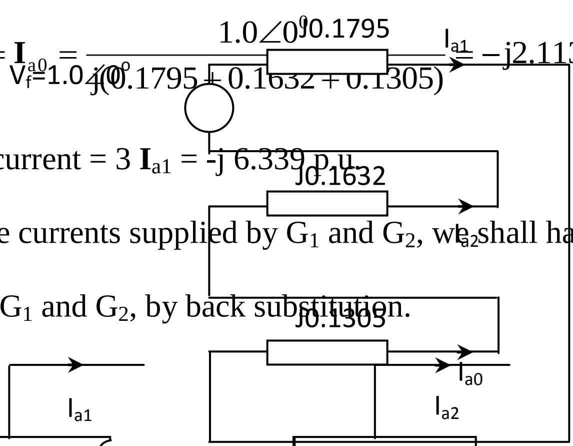
Negative sequence



Zero Sequence

The pre-fault voltage at F, V_f = 1.0 p.u

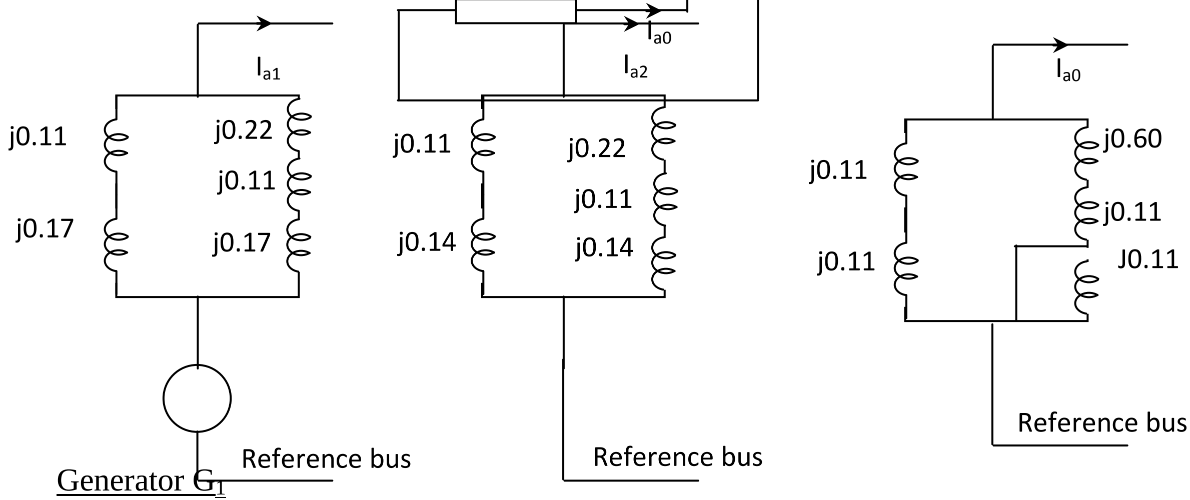
The connection of the sequence networks is shown below:



$$\mathbf{I}_{a1} = \mathbf{I}_{a2} = \mathbf{I}_{a0} = \frac{1.0\angle 0^0}{j(0.1795 + 0.1632 + 0.1305)} = -j2.113\text{p.u.}$$

$\mathbf{I}_a$ = fault current = 3 $\mathbf{I}_{a1}$ = -j 6.339 p.u.

To find the currents supplied by G_1 and G_2, we shall have to find the sequence components of currents shared by G_1 and G_2, by back substitution.



Positive sequence Negative sequence Zero sequence

<u>Generator G_1</u>

$\mathbf{I}_{a1}'$ = positive sequence current shared by G_1

$$= \mathbf{I}_{a1} \frac{j(0.22+0.11+0.17)}{j(0.28+0.5)} = -j1.354 \text{ p.u.}$$

$\mathbf{I}_{a2}'$ = negative sequence current shared by G_1

$$= \mathbf{I}_{a2} \frac{j(0.22+0.11+0.14)}{j(0.25+0.47)} = -j1.379 \text{ p.u.}$$

$\mathbf{I}_{a0}'$ = zero sequence current shared by G_1

$$= \mathbf{I}_{a0} \frac{j(0.60+0.11)}{j(0.71+0.16)} = -j1.724 \text{ p.u.}$$

So, the phase currents through G_1 are given by:

$$\begin{bmatrix} \mathbf{I}_a' \\ \mathbf{I}_b' \\ \mathbf{I}_c' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{a0}' \\ \mathbf{I}_{a1}' \\ \mathbf{I}_{a2}' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} -j1.724 \\ -j1.354 \\ -j1.379 \end{bmatrix} = \begin{bmatrix} -j\ 4.457 \\ 0.02 - j0.357 \\ -0.02 - j0.357 \end{bmatrix} \text{p.u.}$$

It is interesting to note that the b and c phase currents flowing through generator are not zero. They will be zero at the faulty point F.

The current flowing through each of the three neutrals of G_1, h.v. side of T_1 and H.V. side of T_1 is :

$$3\ \mathbf{I}_{ao}' = -j\ 5.172 \text{ p.u.}$$

<u>Generator G_2</u>

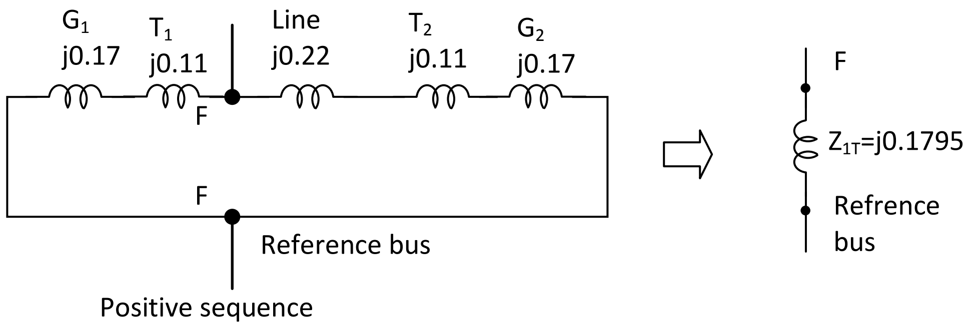
The vector grouping of the transformer T_2 is **Yd11** and so, the positive sequence currents will advance and negative sequence currents will retard by 30^0 as these currents cross T_2 and reach the generator.

$\mathbf{I}_{a1}''$ = Positive-sequence current shared by G_2

$$= \mathbf{I}_{a1}.\frac{j(0.11+0.17)}{j(0.28+0.5)}\angle 30^0 = 0.7585\angle -60^0 \text{p.u.} = 0.379 - j0.656$$

$\mathbf{I}_{a2}''$ = Negative-sequence current shared by G_2

$$= I_{a2}\frac{j(0.11+0.14)}{j(0.25+0.47)}\angle -30^0 = 0.7336\angle -120^0 \text{p.u.} = -0.367 - j0.653$$



$\mathbf{I}_{a0}''$ = Zero-sequence current shared by G_2 = 0

[Zero-sequence current cannot be present in the line connecting G_2 because of the delta connection of low voltage side of T_2].

The phase currents in G_2 are given by:

$$\begin{bmatrix} \mathbf{I}_a'' \\ \mathbf{I}_b'' \\ \mathbf{I}_c'' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{a0}'' \\ \mathbf{I}_{a1}'' \\ \mathbf{I}_{a2}'' \end{bmatrix} = \begin{bmatrix} 0.02 - j1.31 \\ -0.009 - j0.009 \\ -0.002 + j1.30 \end{bmatrix} \text{p.u.}$$

Example 2.11

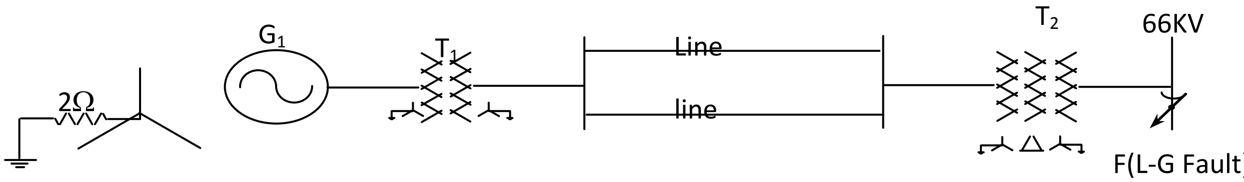
Consider the system whose one line diagram is shown below. The ratings and impedances of the various equipments are :

G_1 : 3-phase, 2.2 kV, 40 MVA; x_1 = 0.2 p.u., x_2 = 0.2 p.u., x_0 = 0.08 p.u. (resistance neglected); neutral point earthed through a resistance of 2 ohms.

T_1 : 2 winding transformer; 40 MVA, 2.2/22 kV, star / star connected with star points solidly earthed; x = 0.05 p.u. (resistance neglected).

Line: 22 kV, $x_1 = x_2$ = 40 ohms, x_0 = 100 ohms (resistance neglected).

T_2: 3 winding transformer; 40 MVA 22 kV/3.3 kV/66 kV, star/delta/star connection; 22 kV side neutral is solidly earthed, 66 kV side neutral is earthed through a 1Ω resistance; the tertiary delta is open circuited; x_p = 0.05 p.u. $x_{tertiary}$ = 0.06 pu x_s = 0.45 p.u.



Calculate the fault current and fault MVA for a L-G fault on one phase of the 66 kV bus.

Base MVA = 40

Base kV = 2.2 kV on generator side =22 kV on line side =3.3 kV on tertiary side of T_2

The neutral resistance of generator $R_{ng} = 2\times\dfrac{\text{Base MVA}}{(\text{Base kV})^2}\,\text{p.u.} = 2\times\dfrac{40}{(13.2)^2} = 0.459\,\text{p.u}$

The line reactances in p.u. are given by:

$$x_{1L} = x_{2L} = 40\ \text{x}\ \frac{\text{Base MVA}}{(\text{Base kV})^2} = 40\text{x}\frac{40}{(132)^2} = 0.092\ \text{p.u.}$$

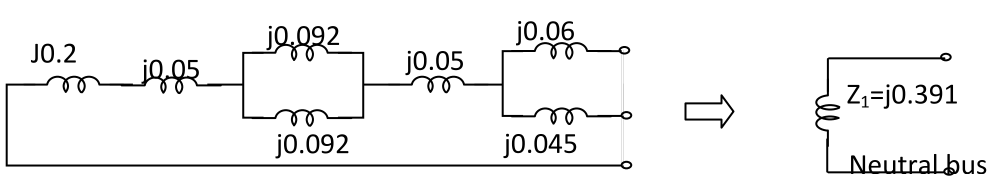
$$x_{0L} = 100\ \text{x}\ \frac{\text{Base MVA}}{(\text{Base kV})^2} = 100\text{x}\frac{40}{(132)^2} = 0.229\ \text{p.u.}$$

The neutral resistance of transformer T_2, in p.u

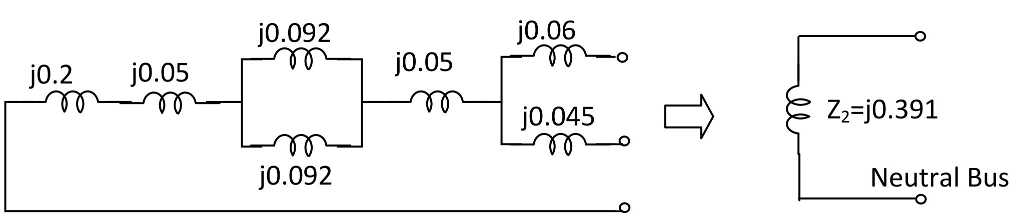
$$R_{nT} = 1\text{x}\frac{\text{Base MVA}}{(\text{Base kV})^2} = 1\text{x}\frac{40}{(66)^2} = 0.0092\ \text{p.u.}$$

As the pre-fault power is neglected, $\mathbf{V}_f$ = pre-fault voltage at the fault point = $1.0\angle 0^0$ p.u

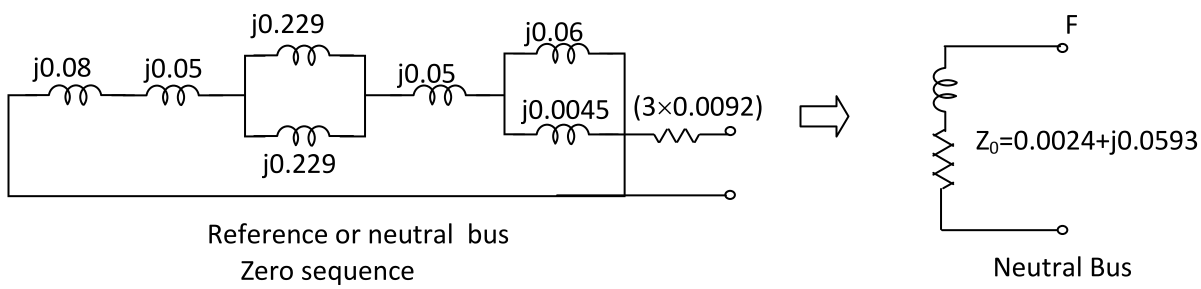
The Thevenin impedances of the sequence networks as viewed from the fault point, F will be computed as below:



Reference or neutral bus
Positive sequence



Reference or neutral bus
Negative sequence



The fault current is then given by:

$$\mathbf{I}_{\mathrm{f}} = \frac{3\ \mathbf{V}_{\mathrm{f}}}{\mathbf{Z}_{\mathbf{1}} + \mathbf{Z}_2 + \mathbf{Z}_0} = \frac{3 \times 1.0\angle 0^0}{\mathrm{j}0.391 + \mathrm{j}0.391 + 0.0024 + \mathrm{j}0.0593} = 3.566\angle -89.83$$

The magnitude of fault current in amperes

$$= \mathbf{I}_{\mathrm{f}}\,\mathrm{x}\,\frac{\text{Base MVA x } 10^3}{\sqrt{3\text{ x}}\ \text{Base kV}} = 3.566 \times \frac{40\text{ x }10^3}{\sqrt{3}\text{x}66}\ \mathrm{A} = 1247.78\mathrm{A}$$

.

Fault MVA = 3.566 x 40 = 142.64 MVA.

2.2.2 Series faults

Till now we have discussed faults which are short circuits. There may be instances where we come across a conductor /conductors snapping giving rise to open conductor faults. While short circuits are of shunt type faults, open conductors belong to series type of faults. Opening of conductor/conductors create unbalance in the system and therefore, are to be solved using symmetrical components.

One conductor open

Consider fig. 2.19 showing one conductor of a three-phase line opening due to snapping. This results in a potential difference between the two ends x and y of the snapped conductor. Let it be $\mathbf{v}_a$. Over the same length XY, voltage drops in

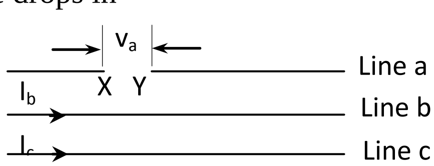


Fig. 2.19 Circuit for one open conductor

conductors b and c, $\mathbf{v}_b$ and $\mathbf{v}_c$, can be assumed to be zero, neglecting any voltage drop due to series impedance. Also, the current in line a, $\mathbf{I}_a$ is zero. So, these conditions boil down to the following equations:

$$\mathbf{I}_a = \mathbf{I}_{a1} + \mathbf{I}_{a2} + \mathbf{I}_{a0} = 0 \qquad \ldots(2.77)$$

The sequence components of the voltage drop are given by:

$$\begin{bmatrix} \mathbf{v}_{a0} \\ \mathbf{v}_{a1} \\ \mathbf{v}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{v}_a \\ \mathbf{v}_b \\ \mathbf{v}_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{v}_a \\ 0 \\ 0 \end{bmatrix}$$

So, $\mathbf{v}_{a0} = \mathbf{v}_{a1} = \mathbf{v}_{a2} = \dfrac{\mathbf{v}_a}{3}$ $\qquad \ldots(2.78)$

Equations (2.77) and (2.78) lead to the connection of sequence network as shown in fig. (2.20).

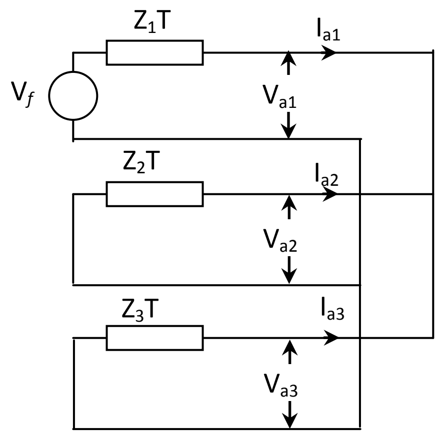
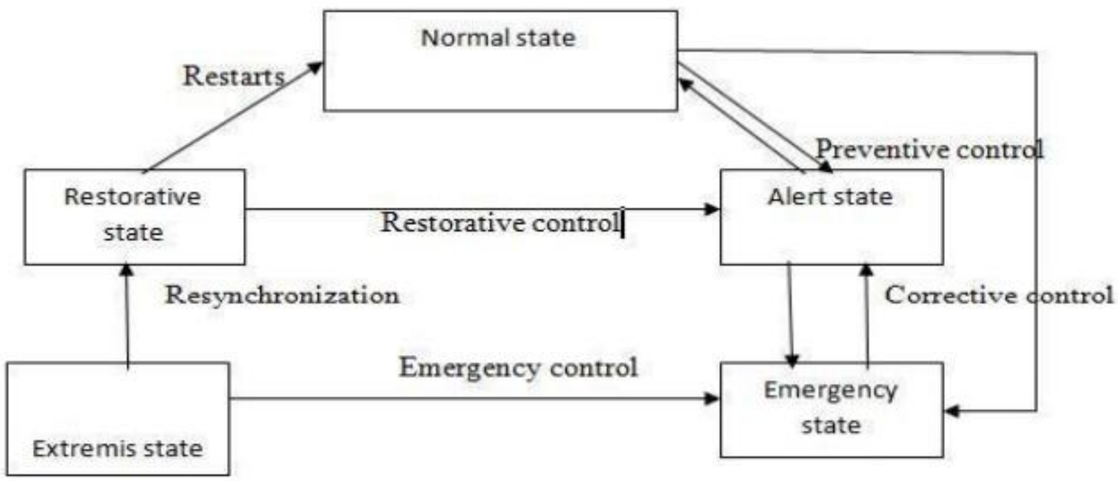


Fig. 2.20 Sequence Equivalent Circuit for one open conductor.

UNIT-IV

CONTINGENCY ANALYSIS

Operating states of a power system:



Operating states

Normal state:

A system is said to be in normal if both load and operating constraints are satisfied .It is one in which the total demand on the system is met by satisfying all the operating constraints.

Alert state**:**

- A normal state of the system said to be in alert state if one or more of the postulated contingency states, consists of the constraint limits violated.
- When the system security level falls below a certain level or the probability of disturbance increases, the system may be in alert state .
- All equalities and inequalities are satisfied, but on the event of a disturbance, the system may not have all the inequality constraints satisfied.
- If severe disturbance occurs, the system will push into emergency state. To bring back the system to secure state, preventive control action is carried out.

Emergency state:

Ø The system is said to be in emergency state if one or more operating constraints are violated, but the load constraint is satisfied .

Ø In this state, the equality constraints are unchanged.

Ø The system will return to the normal or alert state by means of corrective actions, disconnection of faulted section or load sharing.

Extremis state:

Ø When the system is in emergency, if no proper corrective action is taken in time, then it goes to either emergency state or extremis state.

Ø In this regard neither the load or nor the operating constraint is satisfied, this result is islanding.

Ø Also the generating units are strained beyond their capacity .

Ø So emergency control action is done to bring back the system state either to the emergency state or normal state.

Restorative state:

Ø From this state, the system may be brought back either to alert state or secure state .The latter is a slow process.

Ø Hence, in certain cases, first the system is brought back to alert state and then to the secure state .

Ø This is done using restorative control action.

Concept of security monitoring

Practically, the power system needs to be secured.We need to protect it from the black out or any internal or external damage.The operation of the power system is set to be normal only when the flow of power and the bus voltages are within the limits even though there is a change in the load or at the generation side. From this we can say that the security of the power system is an important aspect with respect to the continuation of its operation.

A very important aspect of the power system security is its ability to withstand the effect of contingency which is actually an output of either a generator, bus bars, transmission line, transformer etc. The contingency analysis technique is being widely used to predict the affect of the failures in the equipment used in power system. It is quite necessary task so as to keep the power system safe and secured. Though maintaining the security in power system is a challenging work for the engineers but it is even equally important to maintain the state of operation.

Security functionspower system security

1. Security control

2. Security assessment

Security control :- It determines the exact and proper security constraint scheduling which is required to obtain the maximized security level.

Security assessment :- It gives the security level of the system in the operating state.

The levels of power system security are classified into 5 states:-

1. **Normal**
2. **Alert**
3. **Emergency**
4. **Extreme emergency**
5. **Restorative**

- Usually the operation power system in the normal state where the voltages and the frequency of the system are within the range and no overloaded condition occurs.

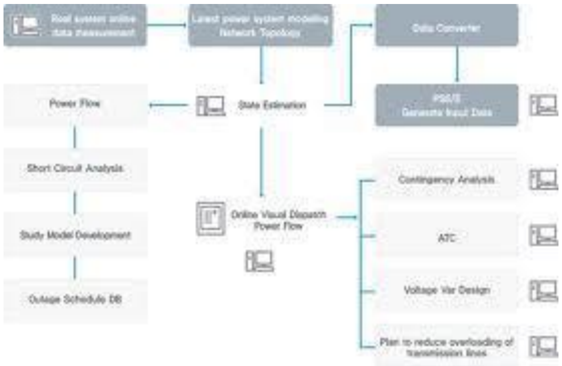
2.The system is transferred into the next state that is the emergency state if any sort of disturbance occurs. The system variables are not within the specified limits.

3.The control action which takes place during the emergency state includes generation tripping, generation run backup etc.

4.The system goes back to the further state when the extreme emergency comes into action that is occurrence of extreme disturbance. In this case the power system is in up stable state and may lead to shutting down of the major parts of the power system. Control action should be powerful such that the shedding of the load of the unimportant load are needs to be done.

Contingency

For the power system to be secured there must have continuity in the supply without any loses. Whenever the operating variable are out from the specified limits the power system comes into the emergency system. These violation of the operating variable result into the contingency occurring into the system. Thus an important of the security analysis moves around the power system to with stand affect of contingency.



contingency analysis

The contingency analysis is time consuming as it involves computation of load flow calculation followed by the outages from the transmission line, generator, transformer etc.

Contingency analysis

The contingency analysis basically involves the simulation of ever contingency of the power system. But this analysis involves three major difficulties

1. *Difficulty to develop the appropriate power system model.*

2. Confusion to choose contingency case.

3. Difficulty in computing the power flow and the bus voltages which leaves to high time consumption.

The contingency analysis is divided into three different stages

1**. Contingency definition** – It comprise of set of contingency that occur in the power system.

2. **Selection** – It is the process of selecting the most severe contingencies from the contingency list. Thus this process removes the unimportant contingencies and hence the contingency list is shortened.

3**. Evaluation** – In this process it involves the necessary security action or control to function in order to remove the affect of contingency.

contingency analysis using sensitivity factor

It is one of the easiest calculation way to provide quick calculation of the possible overloads. These factors show the changes in generation on the network configuration and are derived from dc load flow.

The system security assessment is carried out by calculating system operating limits in the **pre contingency and post contingency** operating states**.**

Pre contingency – It is the state of the power system before the contingency has occurred.

Post contingency– It is the state of the power system after the contingency has occurred.It is assumed that this type of the contingency has the security violations such as the line or transformer are beyond its flow limit or the bus voltage is within its limit.

Economic dispatch using linear programming formulation

Economic load dispatch means that generator's real and reactive power are allowed to vary within certain limits so as to meet particular load demand with minimum fuel cost. It is the process of finding out the maximum output from the generation facilities to meet the load demand while serving the power in reliable manner at lowest possible cost.

The economic dispatch problem is defined as the

Min $F_t = \sum F_n$ where n ranges from 1 to n

Subject to $P_d = \sum P_n$ where n ranges from 1 to n

Where F_t is the total fuel input to the system

F_n is the fuel input to the nth unit

P_d is the total load demand

P_n is the generation of the nth unit

By making the use of the Lagrangian multiplier the auxillary function is obtained as

$F = F_t + \boldsymbol{\lambda} (P_d - \sum P_n)$

Where $\boldsymbol{\lambda}$ is the Lagrangian multiplier

Differentiating the F with P_n and then equating it to zero we get,

$\delta F / \delta P_n = (\delta F_t / \delta P_n) + \boldsymbol{\lambda(0-1) = 0}$

since,

$F_t = F_1 + F_2 + F_3 —— + F_n$

$\boldsymbol{\Lambda =} \delta F_1 / \delta P_1 = \delta F_2 / \delta P_2 = ——— = \delta F_n / \delta P_n$

Hence dF_n/dP_n is the incremental production cost of plant n in Rs.per MWhr.

The incremental production cost of the given plant over the limited range is givan by

$\delta F / \delta P_n = F_{nn} P_n + \boldsymbol{f_n}$

where the F_{nn} slope of the incremental production cost curve

f_n is the intercept of the incremental production cost curve.

Corrective rescheduling

It is basically defined as the measures taken to avoid the the faults after it is occurred i.e.to isolate the defective fault from the non-faulty part of the power system and then further rectify the fault.After the rectification of the fault in the power system it is then restored in the power system from where it was isolated

System security can be said to comprise of three major functions that are carried out in energy control center: A. System monitoring B. Contingency analysis C. Corrective action analysis. System monitoring supplies the power system operations or dispatches with pertinent up-to-date information on the conditions of the power system on real time basis as load and generation change. Telemetry systems measure, monitor and transit the data, voltages, currents, current flows and the status of circuit breakers and switches in every substation in a transmission network. The second major security function is contingency analysis. Modern operation computers have contingency analysis programs stored in them. These foresee possible system troubles (outages) before they occur. They study outage events and alert the operators to any potential overloads or serious voltage violations. The third major security function, corrective action analysis, permits the operators to change the operation of the power system if a contingency analysis program predicts a serious problem in the event of the occurrence of a certain outage. Thus this provides preventive and post-contingency control .A simple example of corrective action is the shifting of generation from one station to another. This may result in change in power flows and causing a change in loading on overloaded lines.

POWER SYSTEM STATIC SECURITY LEVELS

In the diagram given below arrowed lines represent involuntary transitions between levels 1 to 5 due to contingencies. The removal of violations from level 4 normally requires corrective rescheduling or remedial action bringing the system to level 3, from where it can return to either level 1 or 2 by preventive rescheduling depending upon the desired operational security objectives. Levels 1 and 2 represent normal power system operation. Level 1 has the ideal security but is too conservative and costly. Level 2 is more economical, but depends on post contingency corrective rescheduling to alleviate violations without loss of load, within a specified period of time.

METHODS OF CONTINGENCY ANALYSIS

The different methods used for analyzing the contingencies are based on full AC load flow analysis or reduced load flow or sensitivity factors. But these methods need large computational time and are not suitable for on line applications in large power systems. It is difficult to implement on line contingency analysis using conventional methods because of the conflict between the faster solution and the accuracy of the solution. Some important methods are

1. AC load flow methods

2. DC load flow method.

3. Z-bus contingency analysis.

4. Performance Index method.

LOAD FLOW METHODS

The objective of power flow study is to determine the voltage and its angle at each bus, real and reactive power flow in each line and line losses in the power system for specified bus or terminal conditions. Power flow studies are conducted for the purpose of planning (viz. short, medium and long range planning), operation and control. The other purpose of the study is to compute steady state operating point of the power system, that is voltage magnitudes and phase angles at the buses. By knowing these quantities, the other quantities like line flow (MW and MVAr) real and reactive power supplied by the generators and loading of the transformers can also be calculated. The conditions of over loads and under or over voltages existing in the parts of the system can also be detected from this study.

The need of power flow study is summarized as follows:

- By performing this study over loaded as well as poor voltages existing in parts of the system can be detected.

- Load flow study is performed by the planning engineer for different configurations and load conditions before deciding on a final configuration.

- For accurate contingency evaluation purpose load flow analysis is an important tool to simulate various equipment outages.

- In a deregulated energy market this analysis is used to determine the available transfer capability.

- Another interesting application is in finding optimal location of capacitors and their size in a transmission line to improve voltage profile, compensate reactive power and to enhance transfer capability.

The different mathematical techniques [1, 2, 3] used for load flow study are

1. Gauss Seidel Method

2. Newton Raphson Method

3. Decoupled method.
4. Stott's fast decoupled method.

TRANSFER CAPABILITY CONCEPTS

For secured and economic supply of electric power, long distance bulk power transfers are essential, but the power transfer capability of a power system is limited. To operate the power systems safely and to gain the advantages of bulk power transfers, computations of transfer capability is essential. Transfer capability plays a vital role in liberalized electricity market. All the transmission lines are utilized significantly below their physical limits due to various constraints. By increasing the transfer capability the economic value of transmission lines can be improved and also there will be an increase in overall efficiency as more energy trading can take place between the competing regions with different price structures. The power system should be planned and operated such that these power transfers are within the limits of the system transfer capability. Transfer capability of a power system is defined as the maximum power that can be transferred from one area to another area.

NEED FOR TRANSFER CAPABILITY COMPUTATION

Transfer capability plays an important role in bi-lateral energy market. It indicates the amount of power that can be transferred on a transmission network between the two interconnected areas. Computation of transfer capability is essential and useful for several reasons. The need for transfer capability computation is summarized as follows:

1. A system is said to be more flexible and robust if it can accommodate large inter area power transfers compared to one with limited capacity. Thus transfer capability indicates the relative system security.

2. Transfer capability is useful in power system planning and designing. The relative merits of the planned improvements in transmission networks can be obtained from theses computations.

3. To appropriate the effects of multi area commerce or transactions and to furnish the details of the inexpensive power likely to be available to insufficient generation or high cost regions, transfer capability can be used as an alternate in specific circuit modeling.

4. In energy market applications it can be used to evaluate the transmission reservations.

TRANSFER CAPABILITY AND POWER SYSTEM SECURITY

Computation of Transfer Capability plays a vital role in power system planning and secured operation. To increase the reliability power systems are interconnected to form a grid. In such systems the loss of generation in one part can be substituted by the generation from the other part or area. This is an added advantage of the interconnected system compared to individual power system as it can survive such contingencies. In estimating the ability of the interconnected power system to remain secure during the unexpected contingencies like line outages and generator outages, computation of transfer capability is essential.

TRANSFER CAPABILITY AND ELECTRICITY MARKET

In the present deregulated environment with multiple power transactions computation of transfer capability emerges as the key issue to run the energy market smoothly. Total transfer capability forms the basis for the determination of Available Transfer Capability (ATC) which is the indication of the amount of inter area power transfer that can be increased without system security violations. The concept of deregulation rather than monopoly has become prominent to promote healthy competition between the sellers and to drive down the cost of energy. This has also initiated to accomplish reliable operation with better service at most competitive price.

The first country to initiate the deregulation of power industry is United Kingdom followed by Australia and Norway. The Federal Energy Regulatory Commission (FERC) in conjunction with North American Electric Reliability Council (NERC) approved the posting of ATC information through internet based Open Access Same Time Information System (OASIS) for the use of energy market participants. This information is important as it reflects the system realistic conditions such as demand levels of the customers, network paradigm, and generation dispatch and inter-area transfers.

1.3 FLEXIBLE AC TRANSMISSION SYSTEMS (FACTS)

The static and dynamic limits of transmission system restricted the power system transactions leading to underutilization of existing transmission lines. Previously traditional devices like fixed shunt, series reactors and capacitors were used to alleviate this problem however slow response; mechanical wear and tear confined their usage. The greater need for more efficient system has given rise to the development of alternative technology made of solid state, fast response devices. The other reasons like recent restructuring of power systems, difficulty in construction of new transmission lines and modified environmental and efficiency regulations have further fuelled the need for such devices. The invention of semiconductor devices like SCR opened the doors to the development of FACTS controllers.

Flexible Alternating Current Transmission Systems are used for control of voltage, phase angle and impedance of high voltage transmission lines. The strategic benefits of incorporating FACTS devices are improved reliability, better utilization of existing transmission system, improved availability, increased transient and dynamic stability and increased quality of supply. Due to dynamic nature of load and generation patterns, heavier line flows and higher losses are occurred causing security and stability problems. To overcome these problems in the present deregulated scenario more sophisticated control using FACTS devices is essential.

According to IEEE definition FACTS devices are power electronic base or other static controllers incorporated in AC transmission systems to enhance controllability and increase power transfer capability.

TYPES OF FACTS CONTROLLERS

FACTS controllers are classified as series controllers, shunt controllers, combined series–series controllers and combined series-shunt controllers.

i) SERIES CONTROLLERS

These devices are connected in series with the lines to control the reactive and capacitive impedance there by controlling or damping various oscillations in a power system. The effect of these controllers is equivalent to injecting voltage phasor in series with the line to produce or absorb reactive power. Examples are Static Synchronous Series Compensator (SSSC), Thyristor controlled Series Capacitor (TCSC), Thyristor-Controlled Series Reactor (TCSR). . They can be effectively used to control current and power flow in the system and to damp system's oscillations.

ii) SHUNT CONTROLLERS

Shunt controllers inject current in to the system at the point of connection. The reactive power injected can be varied by varying the 13 phase of the current. The examples are Static Synchronous Generator (SSG), Static VAR Compensator (SVC).

iii) COMBINED SERIES-SERIES CONTROLLERS

This controller may have two configurations consisting of series controllers in a coordinated manner in a transmission system with multi lines or an independent reactive power controller for each line of a multi line system. An example of this type of controller is the Interline Power Flow Controller (IPFC), which helps in balancing both the real and reactive power flows on the lines.

iv) COMBINED SERIES-SHUNT CONTROLLERS

In this type of controller there are two unified controllers a shunt controller to inject current in to the system and a series controller to inject series voltage. Examples of such controllers are UPFC and Thyristor- Controlled Phase-Shifting Transformer (TCPST).

OPTIMAL PLACEMENT OF FACTS DEVICES

The main considerations for incorporating the FACTS devices in power transmission system are improvement of system dynamic behavior, reliability and control of power. For the location of FACTS controller one of the following objectives may be chosen:

1. To reduce real power loss of a line.

2. To reduce Total real power loss of a system.

3. To reduce the total reactive power loss of the system.

4. To alleviate congestion by controlling power flow.

Sensitivity factors can be used for the first three objectives. To alleviate congestion and to improve transfer capability trial error methods can be used.

MODELLING CONTINGENCY ANALYSIS

Since contingency analysis involves the simulation of each contingency on the base case model of the power system, three major difficulties are involved in this analysis. First is the difficulty to develop the appropriate power system model. Second is the choice of which contingency case to consider and third is the difficulty in computing the power flow and bus voltages which leads to enormous time consumption in the Energy Management System.

It is therefore apt to separate the on-line contingency analysis into three different stages namely contingency definition, selection and evaluation. Contingency definition comprises of the set of possible contingencies that might occur in a power system, it involves the process of creating the contingency list. Contingency selection is a process of identifying the most severe contingencies from the contingency list that leads to limit violations in the power flow and bus voltage magnitude, thus this process eliminates the least severe contingencies and shortens the contingency list. It uses some sort of index calculations which indicates the severity of contingencies. On the basis of the results of these index calculations the contingency cases are ranked. Contingency evaluation is then done which involves the necessary security actions or necessary control to function in order to mitigate the effect of contingency.

Contingency Analysis using Sensitivity Factors

The problem of studying thousands of possible outages becomes very difficult to solve if it is desired to present the results quickly. One of the easiest ways to provide a quick calculation of possible overloads is to use sensitivity factors [1]. These factors show the approximate change in line flows for changes in generation on the network configuration and are derived from the DC load flow**.** These factors can be derived in a variety of ways and basically come down to two types:

The **generation shift factors** are designated a_{li} and have the following definition

$$a_{li} = \frac{\Delta f_l}{\Delta P_i} \tag{2.1}$$

where

l= line index

i=bus index

Δf_l = change in megawatt power flow on line l when a change in generation ΔP_i occurs at bus i

ΔP_i = change in generation at bus i

It is assumed that the change in generation ΔP_i is exactly compensated by an opposite change in generation at the reference bus, and that all other generators remain fixed. The a_{li} factor then represents the sensitivity of the flow on line *l* due to a change in generation at bus *i*. If the generator was generating P_{io} MW and it was lost, it is represented by ΔP_i, as the new

$$\Delta P_i = - P_i^o \tag{2.2}$$

power flow on each line in the network could be calculated using a pre calculated set of ***"a"*** factors as follows:

$$f_l = f_l^0 + a_{li} \Delta P_i \text{ for } l = 1 \ldots L \tag{2.3}$$

where,

f_l = flow on line *l* after the generator on bus *i* fails

f_l^0 = flow before the failure

The outage flow f_l on each line can be compared to its limit and those exceeding their limit are flagged for alarming. This would tell the operations personal that the loss of the generator on bus *i* would result in an overload on line *l*. The generation shift sensitivity factors are linear estimates of the change in flow with a change in power at a bus. Therefore, the effects of simultaneous changes on several generating buses can be calculated using superposition. The **line outage distribution factors** are used in a similar manner, only they apply to the testing for overloads when transmission circuits are lost. By definition, the line outage distribution factor has the following meaning:

$$d_{l,k} = \frac{\Delta f_l}{f_k^0} \tag{2.4}$$

where

$d_{l,k}$ = line outage distribution factor when monitoring line *l* after an outage on line *k*

Δf_l = change in MW flow on line *l*

f_k^0 = original flow on line *k* before it was outaged i.e., opened

If one knows the power on line *l* and line *k*, the flow on line *l* with line *k* out can be determined using ***"d"*** factors.

$$f_l = f_l^0 + d_{l,k} f_k^0 \tag{2.5}$$

where

f_l^0 and f_k^0 = pre outage flows on lines *l* and *k*, respectively

f_l = flow on line *l* with line *k* out

By pre calculating the line outage distribution factors, a very fast procedure can be set up to test all lines in the network for overload for the outage of a particular line. Furthermore, this procedure can be repeated for the outage of each line in turn, with overloads reported to the operations personnel in the form of alarm messages. The generator and line outage procedures can be used to program a digital computer to execute a contingency analysis study of the power system. It is to be noted that a line flow can be positive or negative so that we must check f_l against $-f_{lmax}$ as well as f_{lmax}. It is assumed that the generator output for each of the generators in the system is available and that the line flow for each transmission line in the network is also available and the sensitivity factors have been calculated and stored.

Contingency Analysis using AC Power Flow

The calculations made with the help of network sensitivity factors for contingency analysis are faster, but there are many power systems where voltage magnitudes are the critical factor in assessing contingencies. The method gives rapid analysis of the MW flows in the system, but cannot give information about MVAR flows and bus voltages. In systems where VAR flows predominate, such as underground cables, an analysis of only the MW flows will not be adequate to indicate overloads. Hence the method of contingency analysis using AC power flow is preferred as it gives the information about MVAR flows and bus voltages in the system. When AC power flow is to be used to study each contingency case, the speed of solution for estimating the MW and MVAR flows for the contingency cases are important, if the solution of post contingency state comes late, the purpose of contingency analysis fails. The method using AC power flow will determine the overloads and voltage limit violations accurately. It does suffer a drawback, that the time such a program takes to execute might be too long. If the list of outages has several thousand entries, then the total time to test for all of the outages can be too long. However, the AC power flow program for contingency analysis by the Fast Decoupled Power Flow (FDLF) [9] provides a fast solution to the contingency analysis since it has the advantage of matrix alteration formula that can be incorporated and can be used to simulate the problem of contingencies involving transmission line outages without re inverting the system Jacobian matrix for all iterations. Hence to model the contingency analysis problem the AC power flow method, using FDLF method has been extensively chosen.

CONTINGENCY SELECTION

Since contingency analysis process involves the prediction of the effect of individual contingency cases, the above process becomes very tedious and time consuming when the power system network is large. In order to alleviate the above problem contingency screening or contingency selection process is used. Practically it is found that all the possible outages does not cause the overloads or under voltage in the other power system equipments. The process of identifying the contingencies that actually leads to the violation of the operational limits is known as contingency selection. The contingencies are selected by calculating a kind of severity indices known as Performance Indices (PI) [1]. These indices are calculated using the conventional power flow algorithms for individual contingencies in an off line mode. Based on the values obtained the contingencies are ranked in a manner where the highest value of PI is ranked first. The analysis is then done starting from the contingency that is ranked one and is continued till no severe contingencies are found. There are two kind of performance index which are of great use, these are **active power performance index (PIP)** and **reactive power performance index (PIV)**. PIP reflects the violation of line active power flow and is given by eq.2.6.

$$PI_P = \sum_{i=1}^{L}\left(\frac{P_i}{P_{imax}}\right)^{2n} \qquad (2.6)$$

where,

P_i = Active Power flow in line *i*,

P_i^{max} = Maximum active power flow in line *i*,

n is the specified exponent,

L is the total number of transmission lines in the system.

If *n* is a large number, the PI will be a small number if all flows are within limit, and it will be large if one or more lines are overloaded. Here the value of n has been kept unity. The value of maximum power flow in each line is calculated using the formula

$$P_i^{max} = \frac{V_i * V_j}{X} \qquad (2.7)$$

where, V_i= Voltage at bus i obtained from FDLF solution

V_j= Voltage at bus j obtained from FDLF solution

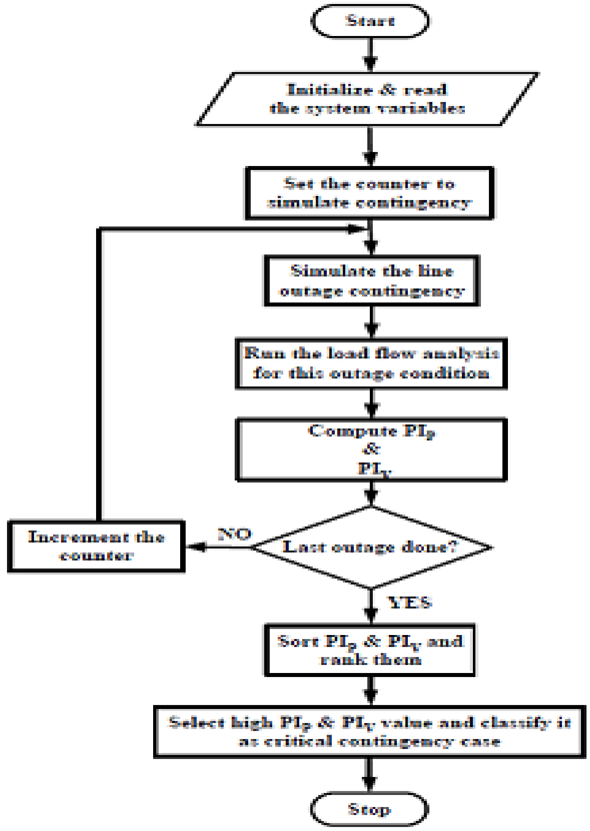
X = Reactance of the line connecting bus ‘i’ and bus ‘j’

Another performance index parameter which is used is reactive power performance index corresponding to bus voltage magnitude violations. It mathematically given by eq.2.8

$$PI_V = \sum_{i=1}^{Npq}\left[\frac{2(Vi - Vinom)}{Vimax - Vimin}\right]^2 \qquad (2.8)$$

where, V_i= Voltage of bus i

V_{imax} and V_{imin} are maximum and minimum voltage limits



For calculation of PIV it is required to know the maximum and minimum voltage limits, generally a margin of + 5% is kept for assigning the limits i.e, 1.05 P.U. for 14

maximum and 0.95 P.U. for minimum. It is to be noted that the above performance indices is useful for performing the contingency selection for line contingencies only. To obtain the value of PI for each contingency the lines in the bus system are being numbered as per convenience, then a particular transmission line at a time is simulated for outage condition and the individual power flows and the bus voltages are being calculated with the help of fast decoupled load flow solution.

ALGORITHM FOR CONTINGENCY ANALYSIS USING FAST DECOUPLED LOAD FLOW

The algorithm steps for contingency analysis using fast decoupled load flow solution are given as follows: ***Step 1*: **Read the given system line data and bus data. ***Step 2*:** Set the counter to zero before simulating a line contingency. ***Step 3*:** Simulate a line contingency. ***Step 4*:** Calculate the active power flow for in the remaining lines and the maximum power flow PMax using eq.2.7. ***Step 5*:** Calculate the active power performance index PIP which give the indication of active power limit violation using eq.2.6. ***Step 6*:** Calculate the voltages at all the load buses following the line contingency. ***Step 7*:** Calculate the reactive power performance index PIV which gives the voltage limit violation at all the load buses due to a line contingency using eq.2.8. ***Step 8*:** Check if this is the last line outage to be simulated; if not the step (3) to (7) is computed till last line of the bus system is reached. ***Step 9*:** The contingencies are ranked once the whole above process is computed as per the values of the performance indices obtained. ***Step 10*:** Do the power flow analysis of the most severe contingency case and print the results The flow chart of the algorithm is shown in Fig. 2.4.

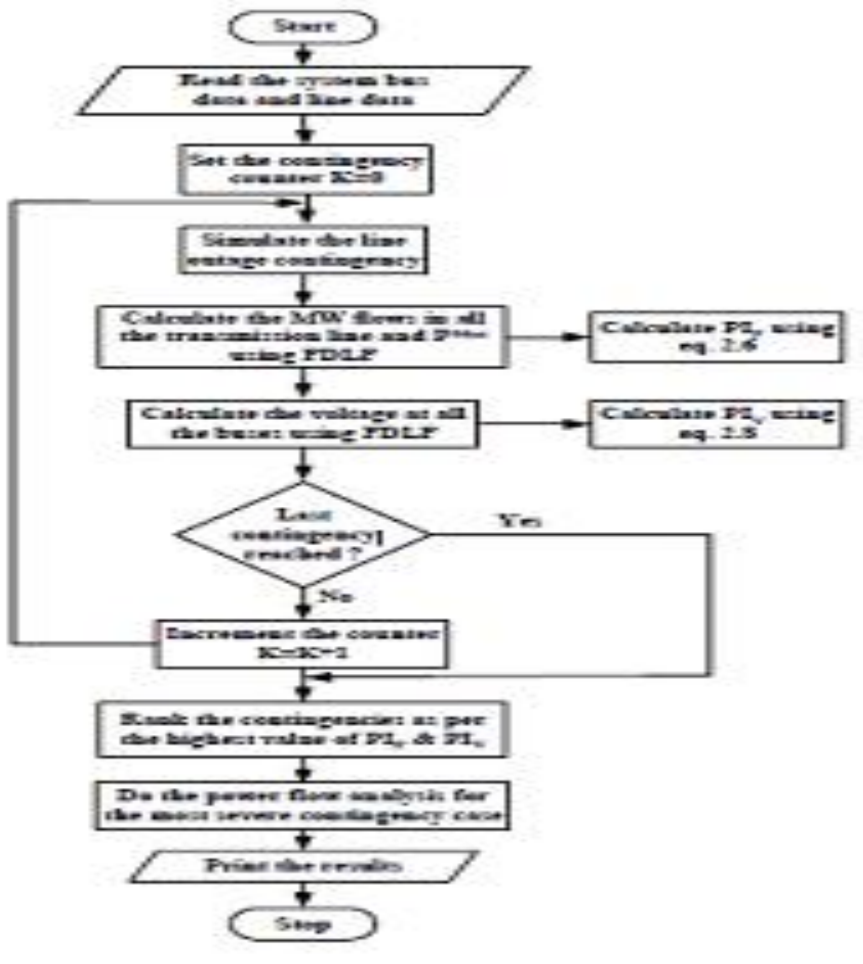


Fig. 2.4 Flow chart for Contingency Analysis using FDLF

SINGLE LINE OUTAGED

Single line removal can be performed using matrix compensation [3] or by modifying Zbus [5]. This paper presents an alternative method of line removal by creating a circulation current that completely self contains both an injection current and the original 'base case' line current. A test injection current of ($1\angle 0$) amp is injected in and out of line j to be removed as shown in Fig. 1. This creates a set of small $[\Delta V]j$ 'test' voltages throughout the network. Injecting both the in and out currents at the same time reduces the matrix computational error. Incremental voltages created on the from and to end of line j are $\Delta Vf\,j$ and $\Delta Vt\,j$ respectively

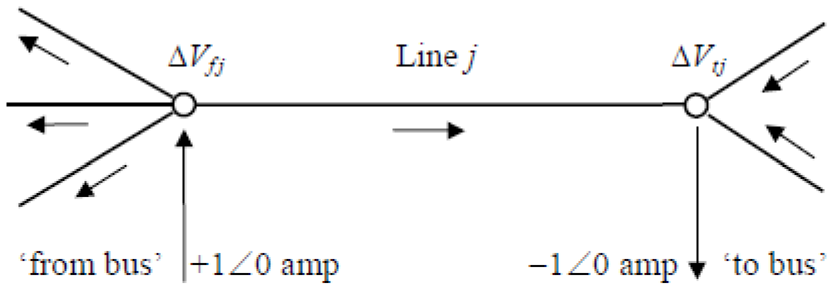


Fig. 1 Inject 1 Amp In And Out Of Line j

Fig. 2 shows the incremental line j current (_Vf j−_Vt j)Yj being scaled by a complex number Sj in order to create a circulation current that is completely self contained as a loop current within line j. This current includes the original base case load flow current as well as the portion of the injected current flowing in line j. Line j base case current is not canceled by this process. The purpose is to self-contain the base case current within the local circulation current set up by Sj so that no line currents from other adjacent lines from either the base case or from the injected currents flow across the gaps shown in Fig. 2. In practice the line is not removed from the matrix solution, but the equivalent delta voltages in the network are the same as though line j has been removed.

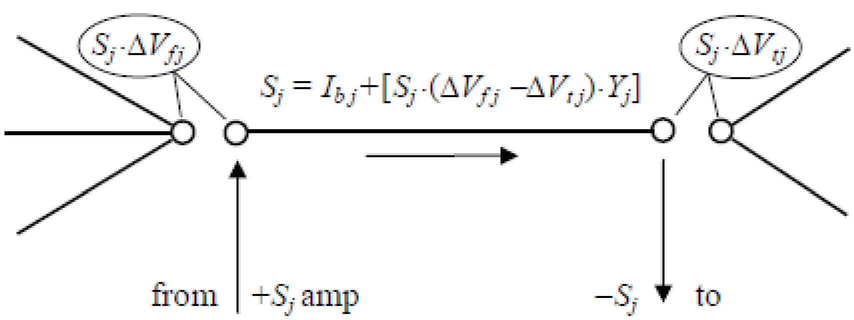


Fig. 2 Single Line Removal Using S_j Injection Current

The steps to calculate Sj are given below. The base case bus voltages are [V]b=[...Vfbj ...Vtbj ...]T and the base case complex current in line j to be removed is Ibj. The calculation of Ibj should not include shunt elements to ground such as line charging. Shunts are also excluded from the [Y] nodal admittance matrix to insure that incremental currents are contained within the transmission lines rather than being shorted to ground through shunt elements. The absence of shunts produces results more consistent with full AC load flow solutions of line outages.

Line currents are conveniently measured on the 'to' end of every line because the standard tapped transformer model normally has the series Z directly connected to the 'to' bus.

The transformer Z is used in the nodal admittance matrix [Y] as though it is a regular transmission line. Transformer tap and angle information is not included in the [Y] matrix. This simplification introduces error. However, the examples in section VI show this error is small for a tap ratio of .95

and a small phase shift angle of 3 degrees. The [Y] complex nodal admittance matrix of the network is constructed from real and reactive in-line series impedances. One bus in the network is grounded using a low impedance shunt element and remains at zero incremental volts at all times. While any bus may be the grounded bus, it

should be one that can regulate the voltage under severe line outage conditions in a full AC load flow. No other shunt elements are to be included in [Y]. The next step is to find the set of all [_V]j . _Vf j and _Vt j are incremental voltages resulting from the injection of ±1∠0

amp into line j as shown in Fig. 1. Eqn. (1) shows this is a standard nodal admittance matrix solution. The authors use the sparse matrix technique in [6] to efficiently solve (1). Other sparse matrix solution methods are presented in [7].

$$[\Delta \boldsymbol{V}]_j = [\ldots \Delta V_{fj} \ldots \Delta V_{tj} \ldots]^{\mathrm{T}} = [\boldsymbol{Y}]^{-1}[\ldots 1 \ldots -1 \ldots]^{\mathrm{T}} \qquad (1)$$

The [_V]j calculated from the ±1∠0 amp injections for line j are saved for use in other calculations such as the outaging of many lines. The complex scale factor Sj for scaling the incremental network bus voltages is given in (2).

$$S_j = \frac{I_{bj}}{1-(\Delta V_{fj}-\Delta V_{tj})Y_j} \qquad (2)$$

Sj is also the complex injection current that produces the totally self contained current in line j as shown in Fig. 2. If less than .00001 per unit amps injection current flows through the rest of the network, there effectively are no alternative paths for the injected current to flow other than the outaged line j. Then, the network will be broken into two islands by the outage of line j, if (3) is true.

$$\left|1-(\Delta V_{fj}-\Delta V_{tj})Y_j\right| \leq .00001 \qquad (3)$$

Eqn. (4) creates a temporary [V]new set of voltages for the outage of line j. Line currents including line shunt current.

$$[\boldsymbol{V}]_{\text{new}} = [\boldsymbol{V}]_b + S_j \cdot [\Delta \boldsymbol{V}]_j \qquad (4)$$

are calculated using [V]new to check for line overloads with line j outaged. This process is repeated for all single lines. outaged and all [_V]j are saved for use in other calculations.

MULTIPLE LINES OUTAGED

Multiple line removal is an extension of single line removal in which complex scalar Sj becomes complex vector [S] for n lines outaged simultaneously. Sj elements of [S] are injection currents into and out of each of the lines j=1...n. An example for n = 3 is shown in Fig. 3. Ib1, Ib2, Ib3 are the base case line complex currents for lines 1, 2, and 3, respectively.

_I11, _I22, _I33 are the line self currents from the ±1∠0 amp injections on each individual line. _I12, _I13, _I21, _I23, _I31, and _I32 are the line transfer coupling currents from the ±1∠0 amp injections. For example, _I12 is the current in line 1 from the ±1∠0 amp injection in line 2.

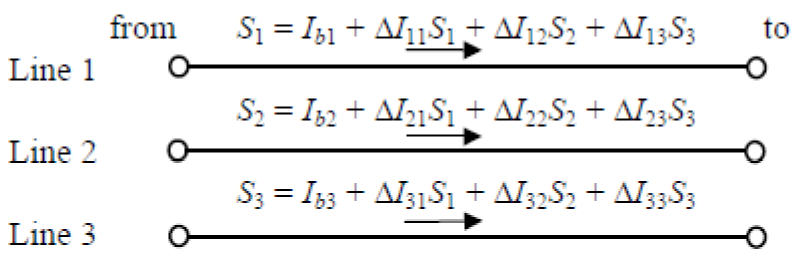


Fig. 3 Three Lines Outaged Example

Incremental _Iij currents on lines i for injections j are calculated as shown in (5) from the set of [_V]j calculated in Section III.

$$\Delta I_{ij} = (\Delta V_{fij} - \Delta V_{tij})\, Y_i \qquad (5)$$

Rearranging the equations shown in Fig. 3 for n = 3 produces a matrix equation for finding complex [S] vector.

$$\begin{bmatrix} 1-\Delta I_{11} & -\Delta I_{12} & -\Delta I_{13} \\ -\Delta I_{21} & 1-\Delta I_{22} & -\Delta I_{23} \\ -\Delta I_{31} & -\Delta I_{32} & 1-\Delta I_{33} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_{b1} \\ I_{b2} \\ I_{b3} \end{bmatrix}$$

[S] complex scale factors (bus injection currents) simultaneously disconnect all n lines from the network. Eqn. (6) is solved using Gauss elimination since the matrix is dense and small. Diagonal terms are used as pivot elements. A singularity of (6) occurs if a diagonal term becomes nearly zero. This condition indicates a system separation which means a part of the system is isolated. Skipping the outaging of lines that are electrically remote can be determined from the column elements of (6).
□ _I21 / (1−_I11)□ is the amount of current in outaged line 2
due to an incremental current of 1 A in outaged line 1. If this ratio is small (≤ .01), the two lines are remote from each other electrically. Being remote means the multiple line outage
case produces no new information over cases previously run. After (6) is solved, the new bus voltages [V]new for the case of multiple n lines simultaneously outaged can be calculated using (7). Line currents including line shunt

$$[V]_{\text{new}} = [V]_b + \sum_{j=1}^{n} S_j \cdot [\Delta V]_j \qquad (7)$$

currents are calculated using [V]new to check for line overloads with lines j=1...n outaged. The processes in sections III and IV are repeated for other sets of line outages.

Summary Of Steps For Outaging Multiple Lines:

1. Solve an initial load flow and store the complex line currents for this ‘base case’ with no lines outaged.

2. Outage each of the lines individually using (1)-(4), test the rest of the network for line overloads, and store in memory or disk the incremental line currents in all lines resulting from the 1 A injections for each line outaged.

3. Set up a procedure for stepping through each outage configuration for N-2, N-3, etc.

4. Calculate a probability of occurrence for each multiple line outage configuration and skip the simulation of configurations with too low a probability.

5. Construct matrix (6) from the currents in step 2.

6. Calculate the electrical ‘remoteness’ of lines being outaged by testing all the column elements of (6); example: □ _I21 / (1−_I11)□ , etc. If any of these ratios are below a small number (.01 for example), then skip the outage, because the same lines will have been outaged
individually at another point in the process of modeling all combinations of line outages.

7. Solve for new [S]. Matrix (6) is inverted using Gauss elimination and diagonal term pivoting. Singularity occurs if the lines outaged have isolated one or more buses from the main network.

8. Calculate new line currents for this contingency using the new bus voltages calculated in (7).

9. Overloaded lines are found and reported

10. Steps 3 - 9 are repeated for each multiple line outage.

UNIT-V

STATE ESTIMATION

Power System State Estimation:

In real-time environment the state estimator consists of different modules such as network topology processor, observability analysis, state estimation and bad data processing. The network topology processor is required for all power system analysis. A conventional network topology program uses circuit breaker status information and network connectivity data to determine the connectivity of the network.

Figure 14.6 is a schematic diagram showing the information flow between the various functions to be performed in an operations control centre computer system. The system gets information from remote terminal unit (RTU) that encode measurement trunsducer outputs and opened/closed status information into digital signals which are sent to the operation centre over communications circuits. Control centre can also transmit commands such as raise/lower to generators and open/close to circuit breakers and switches. The analog measurements of generator output would be directly used by the AGC program (Chapter 8). However, rest of the data will be processed by the state estimator before being used for other functions such as OLF (Optimal Load Flow) etc.

Before running the SE, we must know how the transmission lines are connected to the load and generator buses i.e. network topology. This keeps on changing and hence the current telemetered breaker/switch status must be used to restructure the electrical system model. This is called the *network topology program* or *system status processor* or *network configurator*.

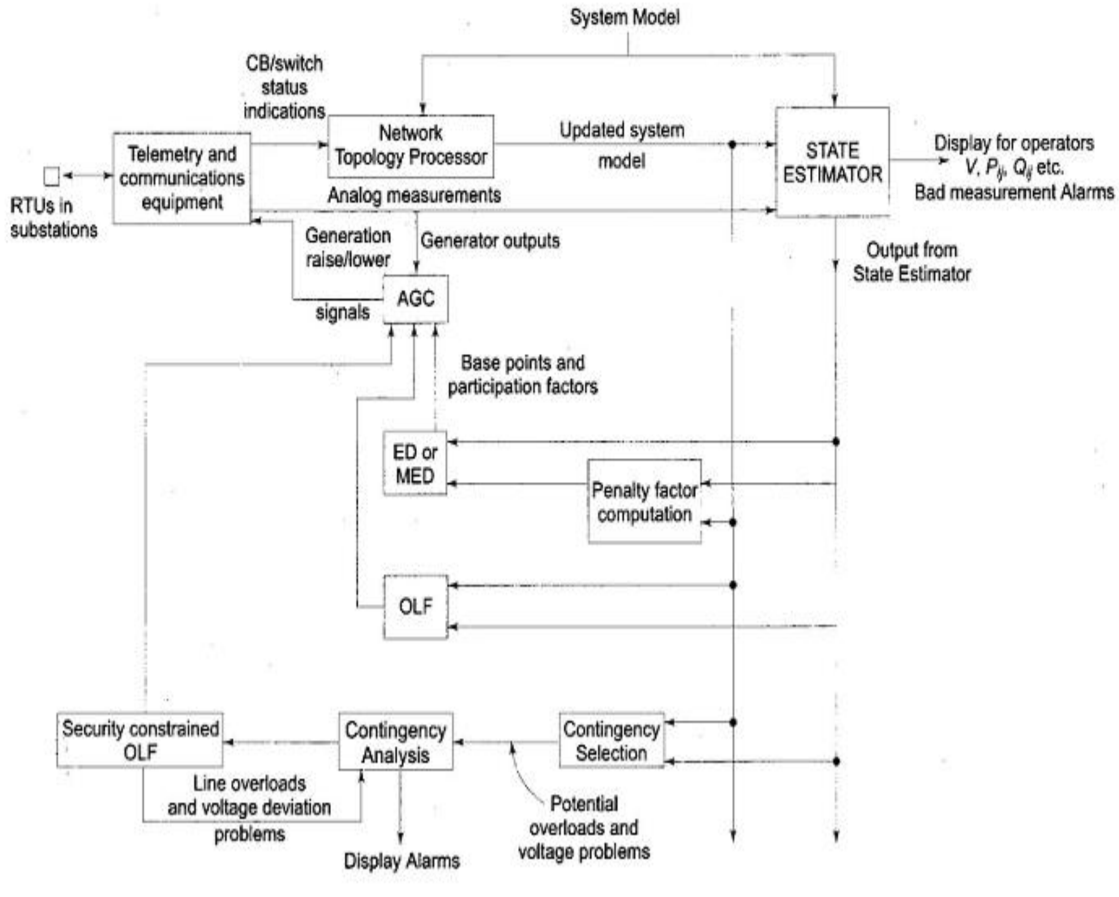


Fig. 14.6 A power system control centre

The output of the state estimator i.e. $|V|,\delta,P_{ij},Q_{ij}$ together with latest model form the basis for the economic dispatch (ED) or minimum emission dispatch (MED), contingency analysis program etc.

THE METHOD OF LEAST SQUARES

The electric power transmission system uses wattme ters, varmeters, voltmeters, and current meters to measure real power, reactive power, voltages, and currents, respectively. These continuous o r analog quantities are controllerd current and potential transformers (or other equivalent devices) installed on the l ines a nd on transformers and buses of t he power plants and substations of the system. The analog quantities p ass t hrough transdu.cers and analog-to-digital converters, and th e d igital outputs a re then telemetered to the energy control center over various communication links. The data rece ived at the e nergy control center i s processed by computer to inform the system operators of the present state o f the system . The acquired data always contains inaccuracies which are u n avoidable since physical measurements (as opposed to numerical cannot be entirely free of random errors or noise . These errors canbe quantified in a statistical sense a nd the estimated values of the quantities being m easured are then either accepted as reasonable or rejected if certain measures of accuracy are exceeded. Because of noise, the true values of physical quantities are never known and we have to consider how to calculate the best possible estima tes of the . u nknown quantities. The method of least squares is often used to " best fit" measured d a t a relating two or more qua ntities. Here we apply the method to a simple set of dc measurements w hich contain errors, and Sec. 1 5 .4 extends the estimation p rocedures to the ac power system. The best estimates are chosen as those which minimize the we igh ted s u m o f t h e squares o f the measurement errors.

Con sider the simple dc circu it of Fig. 15.1 with five resistances of 1 n each and two voltage sources VI and V2 of unknown values which are to be estimated. The measurement set consists of ammeter readings 2 1 and 22 and voltmeter readings Z3 and Z4' The symbol z is normally used for measurements regardless of the physical quantity being measured, and likewise, the symbol x applies to quant ities being esti mated. The system model based on elementary circuit analysis expresses the true values of the measured quantities in terms of

the n etwork parameters and the true (but unknown) source voltages X l = VI and x2 = V2 • Then , measurement equations characterizing the meter readings are found by adding error terms to the system model. For Fig. 15.1 we obtain

$$z_1 = \tfrac{5}{8}x_1 - \tfrac{1}{8}x_2 + e_1 \qquad (15.1)$$

$$z_2 = -\tfrac{1}{8}x_1 + \tfrac{5}{8}x_2 + e_2 \qquad (15.2)$$

$$z_3 = \tfrac{3}{8}x_1 + \tfrac{1}{8}x_2 + e_3 \qquad (15.3)$$

$$\underbrace{z_4}_{\text{Measurements}} = \underbrace{\tfrac{1}{8}x_1 + \tfrac{3}{8}x_2}_{\substack{\text{True values from}\\ \text{system model}}} + \underbrace{e_4}_{\text{Errors}} \qquad (15.4)$$

in wh ich the numerical coefficients are determined by the circuit resistances and the terms e l , e2 , e3, and c4 represent errors i n measuring the two currents Z l and Z 2 and the two voltages Z 3 and Z 4 ' Some a u thors use the term residua ls i nstead of errors, so we use both terms interchangeably. If e l , e2 , e:" and e4 were zero (the ideal case), then a ny two of the meter

readings would give exact and consistent readings from which t he true values Xl and X 2 of VI and V2 could be determined. But i n any measurement scheme there are unknown errors which generally fol low a statistical pattern, as w e shall disc uss in Sec. 15.2. Labeling the coefficients of Eqs. (1 5. 1) through (15.4) in an obvious way, we obtain

$$z_1 = h_{11}x_1 + h_{12}x_2 + e_1 = z_{1,\,\mathrm{true}} + e_1 \tag{15.5}$$

$$z_2 = h_{21}x_1 + h_{22}x_2 + e_2 = z_{2,\,\mathrm{true}} + e_2 \tag{15.6}$$

$$z_3 = h_{31}x_1 + h_{32}x_2 + e_3 = z_{3,\,\mathrm{true}} + e_3 \tag{15.7}$$

$$z_4 = h_{41}x_1 + h_{42}x_2 + e_4 = z_{4,\,\mathrm{true}} + e_4 \tag{15.8}$$

where $z_{j,\mathrm{true}}$ denotes the *true* value of the measured quantity z_j. We now rearrange Eqs. (15.5) through (15.8) into the vector-matrix form

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} - \begin{bmatrix} z_{1,\,\mathrm{true}} \\ z_{2,\,\mathrm{true}} \\ z_{3,\,\mathrm{true}} \\ z_{4,\,\mathrm{true}} \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ h_{41} & h_{42} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{15.9}$$

In more compact notation Eq. (1 5 .9) can b e written as

$$\mathbf{e} = \mathbf{z} - \mathbf{z}_{\mathrm{true}} = \mathbf{z} - \mathbf{Hx} \tag{15.10}$$

which represents the errors between the actual measu rements z and the true (but u nknown) values Z t rue g Hx of the measured quantities. The true values of X l and x 2 cannot be determined, but we can calculate estimates Xl and x 2 , as we shall soon see. Substituting these estimates in E q . (1 5 .9) gives estimated values of the errors in the form

$$\underbrace{\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix}}_{\text{Estimated errors}} = \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}}_{\text{Measurements}} - \underbrace{\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ h_{41} & h_{42} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}}_{\text{Estimates of } z_j} \tag{15.11}$$

Quantities with hats, I such as ej and Xi' are estimates of the corres ponding quantities without hats. In Eq. (15.11) the left-hand vector is e, which represents the differences between the actual measurements Z and their estim ated values Z A Hi so that we can write

$$\hat{\mathbf{e}} = \mathbf{z} - \hat{\mathbf{z}} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}} = \mathbf{e} - \mathbf{H}(\hat{\mathbf{x}} - \mathbf{x}) \qquad (15.12)$$

We must now decide upon a criterion for calculating the estimates x 1 and x2 from which e = [e1 e2 e3 e4Y and z = [21 22 23 24V are to be computed. It is not desirable to choose the algebraic sum of the errors to be minimized since posit ive and negative errors could then offset one another and the estimates would not necessarily be acceptable. It is preferable to minimize the direct sum of the squares of the errors. However, to ensure that measurements from m e ters of known greater accuracy are treated more favorably than l ess accurate measurements, each term i n the sum of squares is multip lied by a n appropriate weighting factor w to give the objective function

$$f = \sum_{j=1}^{4} w_j e_j^2 = w_1 e_1^2 + w_2 e_2^2 + w_3 e_3^2 + w_4 e_4^2$$

We select the best estimates o f the state variables a s those values x I and x 2 w h ich cause the objective function f to take on its minimum value. According t o the usual necessary cond itions for minimizing f, the estimates x I and x2 are those values of X I and X 2 which satisfy the equations

$$\left.\frac{\partial f}{\partial x_1}\right|_{\hat{\mathbf{x}}} = 2\left[w_1 e_1 \frac{\partial e_1}{\partial x_1} + w_2 e_2 \frac{\partial e_2}{\partial x_1} + w_3 e_3 \frac{\partial e_3}{\partial x_1} + w_4 e_4 \frac{\partial e_4}{\partial x_1}\right]\Bigg|_{\hat{\mathbf{x}}} = 0 \quad (15.14)$$

$$\left.\frac{\partial f}{\partial x_2}\right|_{\hat{\mathbf{x}}} = 2\left[w_1 e_1 \frac{\partial e_1}{\partial x_2} + w_2 e_2 \frac{\partial e_2}{\partial x_2} + w_3 e_3 \frac{\partial e_3}{\partial x_2} + w_4 e_4 \frac{\partial e_4}{\partial x_2}\right]\Bigg|_{\hat{\mathbf{x}}} = 0 \quad (15.15)$$

The nota tion I x indicates that t h e equations have t o b e evaluated from the state estimates x = [x I x2 V since the true values of t he states are not known. The u n kn own actual errors ej are then replaced by estimated errors ej, w h ich can be calculated once the state estimates Xi are known. Equations (1 5 . 14) and (1 5 . 1 5) i n vector-matrix form become

$$\left.\begin{bmatrix} \frac{\partial e_1}{\partial x_1} & \frac{\partial e_2}{\partial x_1} & \frac{\partial e_3}{\partial x_1} & \frac{\partial e_4}{\partial x_1} \\ \frac{\partial e_1}{\partial x_2} & \frac{\partial e_2}{\partial x_2} & \frac{\partial e_3}{\partial x_2} & \frac{\partial e_4}{\partial x_2} \end{bmatrix}\right|_{\hat{\mathbf{x}}} \underbrace{\begin{bmatrix} w_1 & \cdot & \cdot & \cdot \\ \cdot & w_2 & \cdot & \cdot \\ \cdot & \cdot & w_3 & \cdot \\ \cdot & \cdot & \cdot & w_4 \end{bmatrix}}_{\mathbf{W}} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (15.16)$$

where W is the diagonal matrix of weighting factors which have special significance, as shown i n Sec. 15.2. The partial derivatives for substitution i n Eq. (1 5 .16) are found from Eqs. (1 5.5) through (15.8) to be constants given by t h e elements of H, and so we obtain

$$\begin{bmatrix} h_{11} & h_{21} & h_{31} & h_{41} \\ h_{12} & h_{22} & h_{32} & h_{42} \end{bmatrix} \begin{bmatrix} w_1 & \cdot & \cdot & \cdot \\ \cdot & w_2 & \cdot & \cdot \\ \cdot & \cdot & w_3 & \cdot \\ \cdot & \cdot & \cdot & w_4 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15.17}$$

Using the compact notation of Eq. (1 5 . 1 2) in Eq. (1 5 . 17) yields

$$\mathbf{H}^T\mathbf{W}\hat{\mathbf{e}} = \mathbf{H}^T\mathbf{W}(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}) = 0 \tag{15.18}$$

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \Big(\underbrace{\mathbf{H}^T\mathbf{W}\mathbf{H}}_{\mathbf{G}}\Big)^{-1}\mathbf{H}^T\mathbf{W}\mathbf{z} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{z} \tag{15.19}$$

where i \ a n d i \ are t h e weighted least-squares estimates o f t h e state variab l es. Because H is rectangular, the symmetriCal matrix H T WH (often called t h e gain matrix G) must be inverted as a single entity to yield G - I = (H T WH) - l , w h i ch is also symme trical. Later in t h is chapter we discuss the case where G is not invertible due to the lack of sufficient m easurements. We expect the weighted least-squares procedure to yield estimates Xi' w hich a r e close t o t h e true values X i o f t h e state variables. An expression for the d ifferences (Xi - x) is found by substituting for z = Hx + e i n Eq. (1 5 . 1 9) to obtain

$$\hat{\mathbf{x}} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}(\mathbf{H}\mathbf{x} + \mathbf{e}) = \mathbf{G}^{-1}\underbrace{(\mathbf{H}^T\mathbf{W}\mathbf{H})}_{\mathbf{G}}\mathbf{x} + \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{e} \tag{15.20}$$

Canceling $\mathbf{G}$ with $\mathbf{G}^{-1}$ and rearranging the result lead to the equation

$$\hat{\mathbf{x}} - \mathbf{x} = \begin{bmatrix} \hat{x}_1 - x_1 \\ \hat{x}_2 - x_2 \end{bmatrix} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{e} \tag{15.21}$$

In a quite similar manner, we can compare the calculated values $\hat{\mathbf{z}} = \mathbf{H}\hat{\mathbf{x}}$ of the measured quantities with their actual measurements $\mathbf{z}$ by substituting for $\hat{\mathbf{x}} - \mathbf{x}$ from Eq. (15.21) into Eq. (15.12) to obtain

$$\hat{\mathbf{e}} = \mathbf{z} - \hat{\mathbf{z}} = \mathbf{e} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}\mathbf{e} = [\mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^T\mathbf{W}]\mathbf{e} \qquad (15.22)$$

where $\mathbf{I}$ is the *unit* or *identity matrix*. Immediate use of Eqs. (15.21) and (15.22) is not possible without knowing the actual errors e_i, which is never the practical case. But we can use those equations for analytical purposes after we have introduced the statistical properties of the errors in Sec. 15.2. For now let us numerically exercise some of the other formulas developed above in the following example.

Maximum Likelihood Estimation

The objective of state estimation is to determine the most likely state of the system based on the quantities that are measured. One way to accomplish this is by maximum likelihood estimation (MLE), a method widely used in statistics. The measurement errors are assumed to have a known probability distribution with unknown parameters. The joint probability density function for all the measurements can then be written in terms of these unknown parameters. This function is referred to as the likelihood function and will attain its peak value when the unknown parameters are chosen to be closest to their actual values. Hence, an optimization problem can be set up in order to maximize the likelihood function as a function of these unknown parameters. The solution will give the maximum likelihood estimates for the parameters of interest. The measurement errors are commonly assumed to have a Gaussian

(Normal) distribution and the parameters for such a distribution are its mean, ^ and its variance, o*^. The problem of maximum likelihood estimation is then solved for these two parameters. The Gaussian probability density function (p.d.f.) and the corresponding probability distribution function (d.f.) will be reviewed below briefly before describing the maximum likelihood estimation method.

Gaussian (Normal) probability density function

The Normal probability density function for a random variable 2 is defined as:

$$f(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left\{\frac{z-\mu}{\sigma}\right\}^2}$$

where z : random variable
μ : mean (or expected value) of $z = \mathrm{E}(z)$
σ : standard deviation of z

The function f(z) will change its shape depending on the parameters . However, its shape can be standardized by using the following change of variables:

$$u = \frac{z - \mu}{\sigma}$$

which yields:

$$\begin{aligned} E(u) &= \frac{1}{\sigma}(E(z) - \mu) = 0 \\ Var(u) &= \frac{1}{\sigma^2} Var(z - \mu) = \frac{\sigma^2}{\sigma^2} = 1.0 \end{aligned}$$

Hence, the new function becomes:

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$

The likelihood function

Consider the joint probability density function which represents the probability of measuring m independent measurements, each having the same Gaussian p.d.f. The joint p.d.f can simply be expressed as the product of individual p.d.f's if each measurement is assumed to be independent of the rest:

$$f_m(z) = f(z_1) f(z_2) \cdots f(z_m)$$

where z_i : ith measurement
z^T : $[z_1, z_2, \cdots, z_m]$

The function 7^(2) is called the likelihood function for 2. Essentially it is a measure of the probability of observing the particular set of measurements in the vector 2. The objective of maximum likelihood estimation is to maximize this likelihood function by varying the assumed parameters of the density function, namely its mean /^ and its standard deviation cr. In determining the optimum parameter values, the function is commonly replaced by its logarithm, in order to simplify the optimization procedure. The modified function is called the Log-Likelihood Function, Z! and is given by:

$$\begin{aligned}\mathcal{L} = \log f_m(z) &= \sum_{i=1}^{m} \log f(z_i) \\ &= -\frac{1}{2}\sum_{i=1}^{m}\left(\frac{z_i - \mu_i}{\sigma_i}\right)^2 - \frac{m}{2}\log 2\pi - \sum_{i=1}^{m}\log \sigma_i\end{aligned}$$

MLE will maximize the likelihood (or log-likelihood) function for a given set of observations 21,23,..., 2^,. Hence, it can be obtained by solving the following problem:

$$\begin{aligned}&\text{maximize} && \log f_m(z) \\ &\qquad\text{OR} && \\ &\text{minimize} && \sum_{i=1}^{m}\left(\frac{z_i - \mu_i}{\sigma_i}\right)^2 \qquad (2.6)\end{aligned}$$

This minimization problem can be re- written in terms of the res^&ta^ of measurement !, which is defined as:

$$r_i = z_i - \mu_i = z_i - E(z_i)$$

Hence, the minimization problem of Equation (2.6) will be equivalent to minimizing the weighted sum of squares of the residuals or solving the following optimization problem for the state vector 2:

$$\begin{aligned}&\text{minimize} && \sum_{i=1}^{m} W_{ii} r_i^2 && (2.7) \\ &\text{subject to} && z_i = h_i(x) + r_i, \quad i = 1, \ldots, m. && (2.8)\end{aligned}$$

The solution of the above optimization problem is called the weighted least gttares (WLS) estimator for 2. A review of the measurement model and the associated assumptions will be given next, before discussing the numerical solution methods.

WLS State Estimation Algorithm

WLS State Estimation involves the iterative solution of the Normal equations given by Equation (2.12). An initial guess has to be made for the state vector a:^. As in the case of the power How solution, this guess typically corresponds to the Hat voltage profile, where all bus voltages are assumed to be 1.0 per unit and in phase with each other. The iterative solution algorithm for WLS state estimation problem can be outlined as follows:

1. Start iterations, set the iteration index $k = 0$.
2. Initialize the state vector x^k, typically as a flat start.
3. Calculate the gain matrix, $G(x^k)$.
4. Calculate the right hand side $t^k = H(x^k)^T R^{-1}(z - h(x^k))$
5. Decompose $G(x^k)$ and solve for Δx^k.
6. Test for convergence, max $| \Delta x^k | \leq \epsilon$?
7. If no, update $x^{k+1} = x^k + \Delta x^k$, $k = k + 1$, and go to step 3. Else, stop.

The Measurement Jacobian, R

The structure of the measurement Jacobian J will be as follows:

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix}$$

The expressions for each partition are given below:

* Elements corresponding to real power injection measurements:

$$\frac{\partial P_i}{\partial \theta_i} = \sum_{j=1}^{N} V_i V_j(-G_{ij}\sin\theta_{ij} + B_{ij}\cos\theta_{ij}) - V_i^2 B_{ii}$$

$$\frac{\partial P_i}{\partial \theta_j} = V_i V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})$$

$$\frac{\partial P_i}{\partial V_i} = \sum_{j=1}^{N} V_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) + V_i G_{ii}$$

$$\frac{\partial P_i}{\partial V_j} = V_i(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})$$

Elements corresponding to reactive power injection measurements

$$\frac{\partial Q_i}{\partial \theta_i} = \sum_{j=1}^{N} V_i V_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) - V_i^2 G_{ii}$$

$$\frac{\partial Q_i}{\partial \theta_j} = V_i V_j(-G_{ij}\cos\theta_{ij} - B_{ij}\sin\theta_{ij})$$

$$\frac{\partial Q_i}{\partial V_i} = \sum_{j=1}^{N} V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) - V_i B_{ii}$$

$$\frac{\partial Q_i}{\partial V_j} = V_i(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})$$

Elements corresponding to real power flow measurements:

$$\frac{\partial P_{ij}}{\partial \theta_i} = V_i V_j(g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij})$$

$$\frac{\partial P_{ij}}{\partial \theta_j} = -V_i V_j(g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij})$$

$$\frac{\partial P_{ij}}{\partial V_i} = -V_j(g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij}) + 2\,(g_{ij} + g_{si})V_i$$

$$\frac{\partial P_{ij}}{\partial V_j} = -V_i(g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij})$$

Elements corresponding to reactive power flow measurements:

$$
\begin{aligned}
\frac{\partial Q_{ij}}{\partial \theta_i} &= -V_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}) \\
\frac{\partial Q_{ij}}{\partial \theta_j} &= V_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}) \\
\frac{\partial Q_{ij}}{\partial V_i} &= -V_j (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij}) - 2\,V_i (b_{ij} + b_{si}) \\
\frac{\partial Q_{ij}}{\partial V_j} &= -V_i (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij})
\end{aligned}
$$

Elements corresponding to voltage magnitude measurements:

$$
\frac{\partial V_i}{\partial V_i} = 1, \frac{\partial V_i}{\partial V_j} = 0, \frac{\partial V_i}{\partial \theta_i} = 0, \frac{\partial V_i}{\partial \theta_j} = 0
$$

Elements corresponding to current magnitude measurements (ignoring the shunt admittance of the branch) :

$$
\begin{aligned}
\frac{\partial I_{ij}}{\partial \theta_i} &= \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} V_i V_j \sin\theta_{ij} \\
\frac{\partial I_{ij}}{\partial \theta_j} &= -\frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} V_i V_j \sin\theta_{ij} \\
\frac{\partial I_{ij}}{\partial V_i} &= \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} (V_i - V_j \cos\theta_{ij}) \\
\frac{\partial I_{ij}}{\partial V_j} &= \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} (V_j - V_i \cos\theta_{ij})
\end{aligned}
$$

The Gain Matrix, G

Gain matrix is formed using the measurement Jacobian R and the measurement error covariance matrix, R. The covariance matrix is assumed to be diagonal having measurement variances as its diagonal entries. Since G is formed as:

$$G(x^k) = H^T R^{-1} H$$

it has the following properties:

1. It is structurally and numerically symmetric.

2. It is sparse, yet less sparse compared to R.

3. In general it is a non-negative definite matrix, i.e. all of its eigenvalues are non-negative. It is positive definite for fully observable networks.

G is built and stored as a sparse matrix for computational efficiency and memory considerations. It is built by processing one measurement at a time. Consider the measurement jacobian R and the covariance matrix for a set of m measurements, each one corresponding to one row, as shown below:

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_m \end{bmatrix}, R = \begin{bmatrix} R_{11} & 0 & \cdots & 0 \\ 0 & R_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & R_{mm} \end{bmatrix}$$

Then, the gain matrix can be re-written as follows:

$$G = \sum_{i=1}^{m} H_i^T R_{ii}^{-1} H_i$$

Since % arrays are very sparse row vectors, their product will also yield a sparse matrix. Nonzero terms in C can thus be calculated and stored in sparse form.

Bad Data Detection and Identification

One of the essential functions of a state estimator is to detect measurement errors, and to identify and eliminate them if possible. Measurements may contain errors due to various reasons. Random errors usually exist in measurements due to the finite accuracy of the meters and the telecommunication medium. Provided that there is sufficient redundancy among

measurements, such errors are expected to be filtered by the state estimator. The nature of this filtering action will depend on the specific method of estimation employed. Large measurement errors can also occur when the meters have biases, drifts or wrong connections. Telecommunication system failures or noise caused by unexpected interference also lead to large deviations in recorded measurements.

Some bad data are obvious and can be detected and eliminated apriori state estimation, by simple plausibility checks. Negative voltage magnitudes, measurements with several orders of magnitude larger or smaller than expected values, or large differences between incoming and leaving currents at a connection node within a substation are some examples of such bad data. Unfortunately, not all types of bad data are easily detectable by such means. Hence, state estimators have to be equipped wit more advanced features that will facilitate the detection and identification of any type of bad data.

Treatment of bad data depends on the method of state estimation used in the implementation. This chapter will focus on the bad data detection and identification techniques that are associated with the commonly used WLS method. Incorporate bad data processing as part of the state estimation procedure and hence their discussion will involve aspects of their treatment of bad data as well. When using the WLS estimation method, detection and identification of bad data are done only after the estimation process by processing the

measurement residuals. The analysis is essentially based on the properties of these residuals, including their expected probability distribution. Bad data may appear in several different ways depending upon the type, location and number of measurements that are in error. They can be broadly classified as:

1. Single bad data: Only one of the measurements in the entire system will have a large error.
2. Multiple bad data: More than one measurement will be in error.

Multiple bad data may appear in measurements whose residuals are strongly or weakly correlated. Strongly correlated measurements are those whose errors affect the estimated value of each other significantly, causing the good one to also appear in error when the other contains a large error. Estimates of measurements with weakly correlated residuals are not significantly affected by the errors of each other. When measurement residuals are strongly

correlated their errors may or may not be conforming. Conforming errors are those that appear consistent with each other. Multiple bad data can therefore be further classified into three groups:

1. Multiple non-interacting bad data: Bad data in measurements with weakly correlated measurement residuals.
2. Multiple interacting but non-conforming bad data: Non-conforming bad data in measurements with strongly correlated residuals.
3. Multiple interacting and conforming bad data: Consistent bad data in measurements with strongly correlated residuals.

Classification of Measurements

Power systems may contain various types of measurements spread out in the system with no apparent topological pattern. These measurements will exhibit different properties and affect the outcome of the state estimation accordingly, depending upon not only their values but also their location. Therefore, they may belong to one or more of the following categories [7]:

Critical measurement: A critical measurement is the one whose elimination from the measurement set will result in an unobservable system.The column of the residual covariance matrix H, corresponding to a critical measurement will be identically equal zero. Furthermore, the measurement residual of a critical measurement will always be zero.

Redundant measurement: A redundant measurement is a measurement which is not critical. Only redundant measurements may have nonzero measurement residuals.

Critical pair: Two redundant measurements whose simultaneous removal from the measurement set will make the system unobservable. Critical k-tuple: A critical k-tuple contains A; redundant measurements, where removal of all of them will cause the system to become unobservable. None of these /c measurements belong to a critical tuple of lower order. Those A; columns of the residual covariance matrix Q, corresponding to the members of a critical k-tuple, will be linearly dependent.

Bad Data Detection and IdentiRability

Detection refers to the determination of whether or not the measurement set contains any bad data. Identification is the procedure of finding out which specific measurements actually contain bad data. Detection and identifiabihty of bad data depends on the configuration of the overall measurement set in a given power system. Bad data can be detected if removal of the corresponding measurement does not render the system unobservable. In other words, bad data appearing in critical measurements can not be detected. A single measurement containing bad data can be identified if and only

if:

* it is not critical and

* it does not belong to a critical pair.

Bad data processing logic should be able to recognize the above inherent limitations of detection and single bad data identification. Provided that the above conditions are observed, single bad data can be detected and identified by the methods outlined next.

Bad Data Detection

One of the methods used for detecting bad data is the CM-s^wares test. Once bad data are detected, they need to be identiAed and eliminated or corrected, in order to obtain an unbiased state estimate. Test for Detecting Bad Data in WLS State Estimation

The WLS state estimation objective function J(a) can be used to approximate the above function /(a;) and a bad data detection test, referred to as the Chi-squares test for bad data, can be devised based on the properties of the x^ distribution.

The steps of the Chi-squares χ^2-test are given as follows:

- Solve the WLS estimation problem and compute the objective function:

$$J(\hat{x}) = \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x}))^2}{\sigma_i^2}$$

where:
$\hat{x}$: estimated state vector of dimension n.
$h_i(\hat{x})$: estimated measurement i.
z_i : measured value of the measurement i.
$\sigma_i^2 = R_{ii}$: variance of the error in measurement i.
m : number of measurements

- Look up the value from the Chi-squares distribution table corresponding to a detection confidence with probability p (e.g. 95%) and $(m - n)$ degrees of freedom. Let this value be $\chi^2_{(m-n),p}$.

  Here $p = \Pr\ (\mathrm{J}(\hat{\mathrm{x}}) \leq \chi^2_{(m-n),p})$.

- Test if $J(\hat{x}) \geq \chi^2_{(m-n),p}$.
  If yes, then bad data will be suspected.
  Else, the measurements will be assumed to be free of bad data.

Bad Data Identification

The properties of normalized residuals for a single bad data existing in the measurement set, can be used to devise a test for identifying and subsequently eliminating bad data.

1. Solve the WLS estimation and obtain the elements of the measurement residual vector:

$$r_i = z_i - h_i(\hat{x}), \qquad i = 1, \ldots, m$$

2. Compute the normalized residuals:

$$r_i^N = \frac{|\ r_i\ |}{\sqrt{\Omega_{ii}}} \qquad i = 1, \ldots, m$$

3. Find k such that r_k^N is the largest among all r_i^N, $i = 1, \ldots, m$.

4. If $r_k^N > c$, then the k-th measurement will be suspected as bad data. Else, stop, no bad data will be suspected. Here, c is a chosen identification threshold, for instance 3.0.

5. Eliminate the k-th measurement from the measurement set and go to step 1.