MODERN POWER SYSTEM ANALYSIS
(BPSB01)

Dr. M.Pala Prasad Reddy
Associate Professor
Electrical and Electronics Engineering

UNIT-I

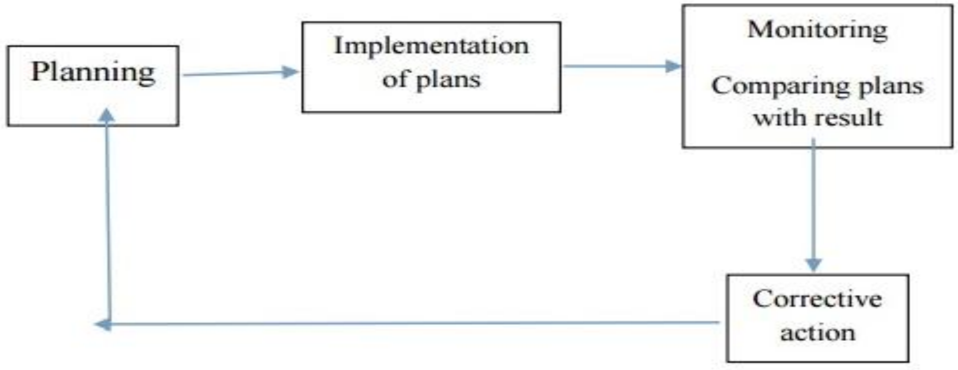
PLANNING AND OPERATIONAL STUDIES OF POWER SYSTEMS

Need for system planning and operational studies

Planning and operation of power system - Operational planning covers the whole period ranging from the incremental stage of system development

The system operation engineers at various points like area, space, regional & national load dispatch of power

Power system planning and operational analysis covers the maintenance of generation, transmission and distribution facilities



Planning and operation of power system the following analysis are very important

(a). Load flow analysis

(b). Short circuit analysis

(c). Transient analysis

Load flow analysis

Electrical power system operate - Steady state mode

Basic calculation required to determine the characteristics of this state is called as Load flow

Power flow studies - To determine the voltage current active and reactive power flows in given power system

A number of operating condition can be analyzed including contingencies. That operating conditions are

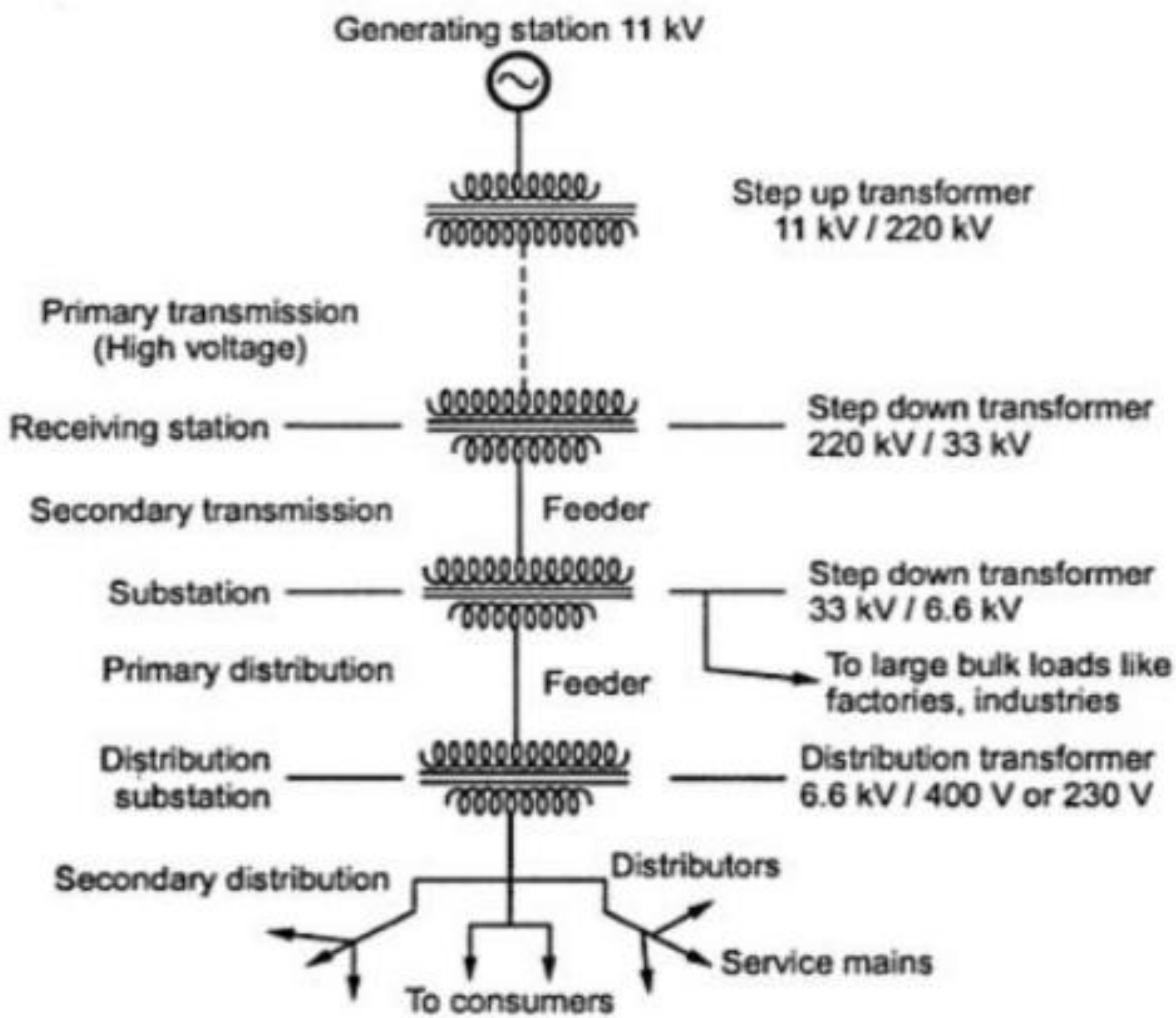
(a). Loss of generator

(b).Loss of a transmission line

(c).Loss of transformer (or) Load

(d). Equipment over load (or) unacceptable voltage levels

Basic components of a power system



Component # 1. Synchronous Generators:

The synchronous generators used in generating stations are revolving field type owing to its inherent advantages.

The synchronous generators, based on the type of prime movers to which they are mechanically coupled, may be classified as:

(i) Hydro-generators

(ii) Turbo-generators, and

(iii) Diesel engine driven generators.

Power Transformers:

Power transformers are used for stepping-up the voltage for transmission at generating stations and for stepping-down voltage for further distribution at main step-down transformer substations.

Usually naturally cooled, oil immersed, known as ON type, two winding, three-phase transformers, are used up to the rating of 10 MVA.

Component # 2. Switchgear:

Everyone is familiar with low voltage switches and rewirable fuses. A switch is used for opening and closing of an electric circuit while a fuse is used for over-current protection. Every electric circuit needs a switching device and protective device. Switching and protective devices have been developed in different forms. Switchgear is a general term covering a wide range of equipment concerned with switching and protection.

Circuit Breakers:

Circuit breakers are mechanical devices designed to close or open contact members, thus closing or opening of an electrical circuit under normal or abnormal conditions.t.

isolators:

Since isolators (or isolating switches) are employed only for isolating circuit when the current has already been interrupted, they are simple pieces of equipment. They ensure that the current is not switched into the circuit until everything is in order.

Earthing Switch:

Earthing switch is connected between the line conductor and earth. Normally it is open and it is closed to discharge the voltage trapped on the isolated or disconnected line. When the line is disconnected from the supply end, there is some voltage on the line to which the capacitance between the line and earth is charged.

Component # 3. Bus-Bars:

Bus-bar (or bus in short) term is used for a main bar or conductor carrying an electric current to which many connections may be made.

Component # 4. Lightning Arresters:

The lightning arrester is a surge diverter and is used for the protection of power system against the high voltage surges.

It is connected between the line and earth and so diverts the incoming high voltage wave to the earth.

Single line diagram

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

Symbols used for SLD

Motor (M) ; Generator (G)

Transformer : 2-Winding

: 3-Winding

Power Circuit braker

$3\text{-}\phi$ Delta : Δ , Star : Y

3ϕ Star-grounded neutral : Y

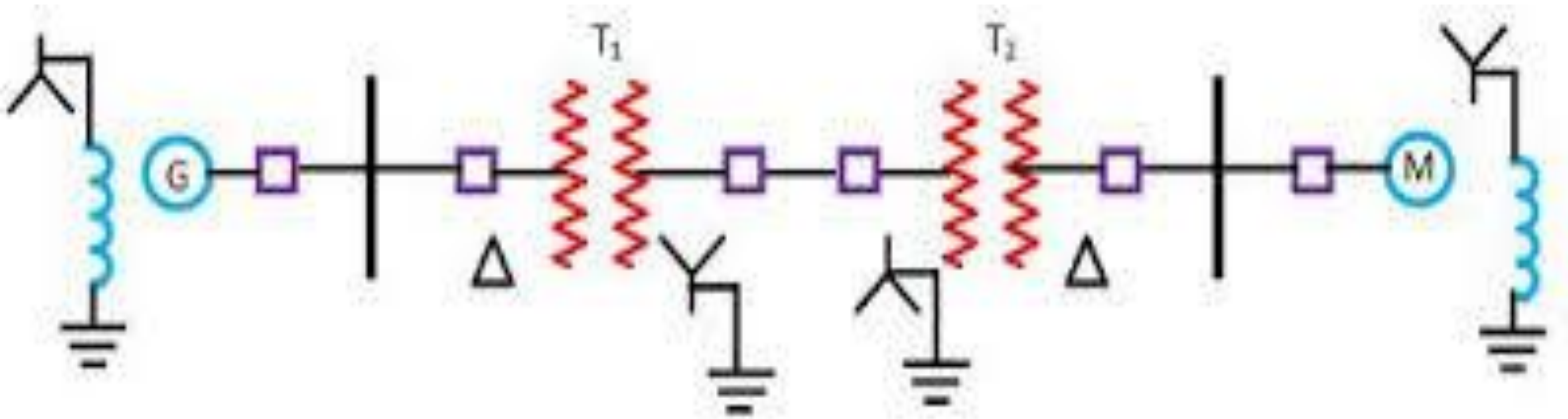
Grounded thro' X_n Y x_n

CT ; PT

SINGLE LINE DIAGRAM :

It gives diagrammatic & easy identification of power system network.

Components are represented in the form of symbols & they are interconnected in a straight line.



Single Line Representation of a Typical Power System

Impedance diagram

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

Assumptions:

1. The single phase transformer equivalents are shown as ideals with
2. impedances on appropriate side (LV/HV),

2. The magnetizing reactances of transformers are negligible,

3. The generators are represented as constant voltage sources with series resistance or reactance,

4. The transmission lines are approximated by their equivalent – Models,

5. The loads are assumed to be passive and are represented by a series branch of resistance or reactance and

6. Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram

Reactance Diagram:

With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- **The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible**
- **Loads are Omitted**
- **Transmission line capacitances are ineffective &**
- **Magnetizing currents of transformers are neglected.**

Per-unit system

•Voltages different level has to transformed to single voltage level in Per-Unit (P.U) form.

•In PS, impedance, current, voltage & power are expressed in P.U values.

•P.U systems are ideal for the computerized analysis & simulation of complex PS problems.

• Generally, manufacturers are preferring equipment impedance values in P.U rating.

For a single-phase system, the following formulas relate the various quantities.

$$\text{Base current, A} = \frac{\text{base VA}}{\text{base voltage, V}} = \frac{\text{base MVA} \times 10^6}{\text{base voltage, kV} \times 1000}$$

$$= \frac{\text{base MVA} \times 1000}{\text{base voltage, kV}} \qquad (1.1)$$

$$\text{Base impedance}, \Omega = \frac{\text{base voltage, V}}{\text{base current, A}} = \frac{\text{base voltage, kV} \times 1000}{\text{base current, A}} \qquad (1.2)$$

Substituting eq. (1.1) in the above

$$\text{Base impedance}, \Omega = \frac{(\text{base voltage, kV})^2 \times 1000}{\text{base MVA} \times 1000}$$ Thus

$$\boxed{\text{Base impedance}, \Omega = \frac{(\text{base voltage, kV})^2}{\text{base MVA}}} \qquad (1.3)$$

Power factor being a dimensionless quantity Base power, MW = base MVA

$$\text{Per-unit impedance} = \frac{\text{actual impedance}, \Omega}{\text{base impedance}, \Omega} \qquad (1.4)$$

$$\boxed{\text{Per-unit impedance = actual impedance x } \frac{\textit{Base MVA}}{(\textit{Base voltage,kV})^2}} \qquad (1.5)$$

$$\text{Base impedance}, \Omega = \frac{(\text{base voltage, kV})^2}{\text{base MVA}} \qquad (1.3)$$

$$\text{Per-unit impedance = actual impedance X } \frac{\textit{Base MVA}}{(\textit{Base voltage,kV})^2} \qquad (1.5)$$

For three phase system, when base voltage is specified it is line to line base voltage and the specified MVA is three phase MVA. Now let us consider a three phase system. Let *Base voltage, kV* and *Base MVA* be specified. Then single-phase base voltage, kV = *Base voltage, kV* $/\sqrt{3}$ and single-phase base MVA=*Base MVA / 3*. Substituting these in eq. (1.3)

$$\text{Base impedance}, \Omega = \frac{[Base\ voltage, kV/\sqrt{3}]^2}{Base\ MVA/3} = \frac{(Base\ voltage, kV)^2}{Base\ MVA} \quad (1.6)$$

It is to be noted that eq.(1.6) is much similar to eq.(1.3). Thus

$$\text{Per-unit impedance} = \frac{\text{actual impedance}}{\text{base impedance}}$$

$$= \text{actual impedance} \times \frac{Base\ MVA}{(Base\ voltage, kV)^2} \quad (1.7)$$

$$\text{Base impedance}, \Omega = \frac{(\text{base voltage, kV})^2}{\text{base MVA}} \qquad (1.6)$$

$$\text{Per-unit impedance} = \text{actual impedance} \times \frac{\text{Base MVA}}{(\text{Base voltage}, kV)^2} \qquad (1.7)$$

<u>EXAMPLE 1.4</u>

A three phase 500 MVA, 22 kV generator has winding reactance of 1.065 Ω. Find its per-unit reactance.

<u>Solution</u>

$$\text{Base impedance} = \frac{22^2}{500} = 0.968\ \Omega \ ; \quad \text{Per-unit reactance} = \frac{1.065}{0.968} = 1.1002$$

$$\text{Using eq.(1.7), per-unit reactance} = 1.065 \times \frac{500}{22^2} = 1.1002$$

$$\text{Per-unit impedance} = \text{actual impedance} \times \frac{\text{Base MVA}}{(\text{Base voltage,kV})^2} \qquad (1.7)$$

Per-unit quantities on a different base

Sometimes, knowing the per-unit impedance of a component based on a particular base values, we need to find the per-unit value of that component based on some other base values. From eq.(1.7) It is to be noted that the per-unit impedance is directly proportional to base MVA and inversely proportional to (base kV)^2. Therefore, to change from per-unit impedance on a given base to per-unit impedance on a new base, the following equation applies:

$$\text{Per-unit } Z_{new} = \text{per-unit } Z_{given} \; \frac{\text{base MVA}_{new}}{\text{base MVA}_{given}} \times \left(\frac{\text{base kV}_{given}}{\text{base kV}_{new}}\right)^2 \qquad (1.8)$$

$$\text{Per-unit } Z_{new} = \text{per-unit } Z_{given} \; \frac{\text{base MVA}_{new}}{\text{base MVA}_{given}} \times \left(\frac{\text{base kV}_{given}}{\text{base kV}_{new}}\right)^2 \qquad (1.8)$$

EXAMPLE 1.5

The reactance of a generator is given as 0.25 per-unit based on the generator's of 18 kV, 500 MVA. Find its per-unit reactance on a base of 20 kV, 100 MVA.

Solution

$$\text{New per-unit reactance} = 0.25 \times \frac{100}{500} \times \left(\frac{18}{20}\right)^2 = 0.0405$$

EXAMPLE 1.6

A single phase 9.6 kVA, 500 V / 1.5 kV transformer has an impedance of 1.302 Ω with respect to primary side. Find its per-unit impedance with respect to primary and secondary sides.

Solution

With respect to Primary

Per-unit impedance = $1.302 \times \dfrac{0.0096}{(0.5)^2} = 0.05$

With respect to Secondary

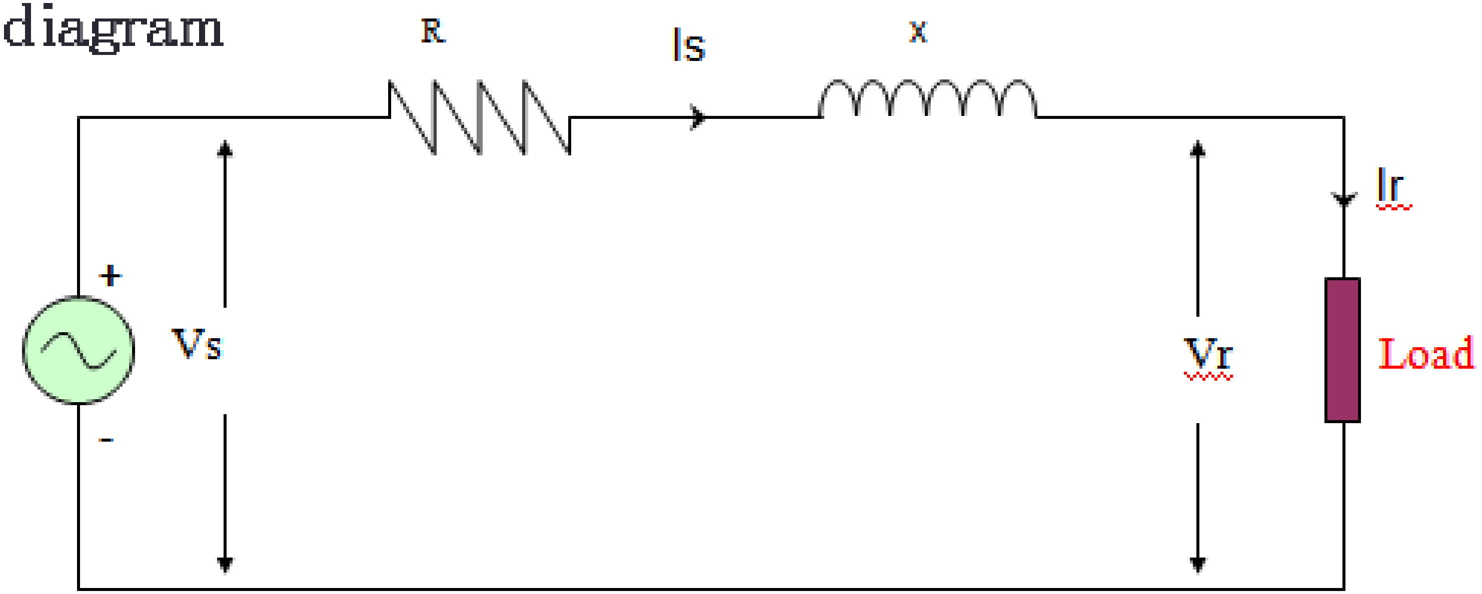
Impedance = $1.302 \times \left(\dfrac{1.5}{0.5}\right)^2 = 11.718\ \Omega$

Per-unit impedance = $11.718 \times \dfrac{0.0096}{(1.5)^2} = 0.05$

Advantages of per-unit calculation

1. Manufacturers usually specify the impedance of a piece of apparatus in percent or per-unit on the base of the name plate rating.
2. The per-unit impedances of machines of same type and widely different rating usually lie within narrow range although the ohmic values differ much.
3. For a transformer, when impedance in ohm is specified, it must be clearly mentioned whether it is with respect to primary or secondary. The per-unit impedance of the transformer, once expressed on proper base, is the same referred to either side.
4. The way in which the three-phase transformers are connected does not affect the per-unit impedances although the transformer connection does determine the relation between the voltage bases on the two sides of the transformer.

Circuit diagram



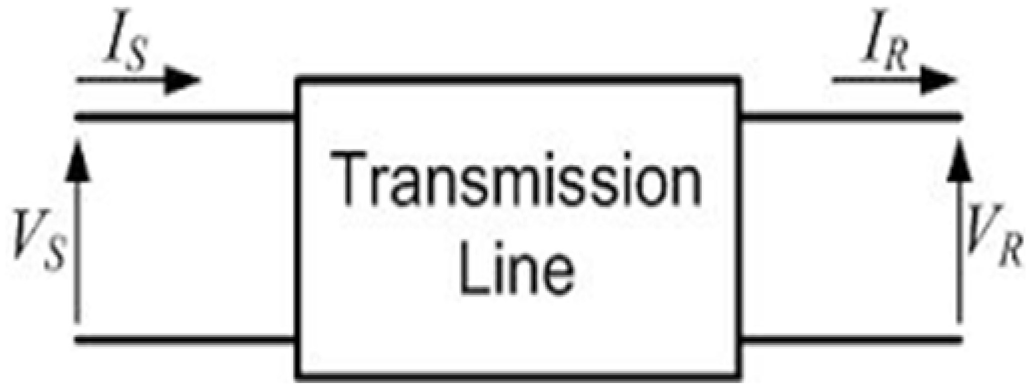
Vs – sending end voltage   Vr – receiving end voltage   I – load current

R – Loop resistance (Ω)   X – Loop inductance (Ω)

$$V_S = V_R + Z\,I_R \longrightarrow (1)$$

$$I_S = I_R \longrightarrow (2)$$

Generalized Circuit Constant of a TL



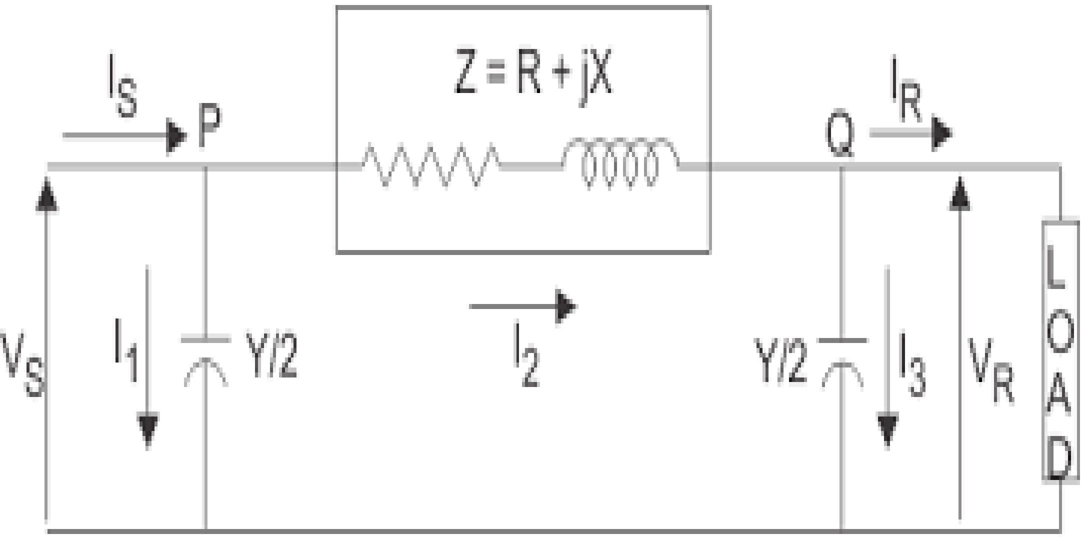
$$V_S = AV_r + BI_r \longrightarrow (3)$$

$$I_s = CV_r + DI_r \longrightarrow (4)$$

$$\begin{pmatrix} V_S \\ I_S \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_R \\ I_R \end{pmatrix}$$

A=1 ; B=Z ; C=0 ; D=1

- **When the length of the line is about 80km to 250km (50-150 miles) and the line voltage is moderately high between 20kV to 100kV.**
- **Due to sufficient length and line voltage, capacitance (C) is considered.**



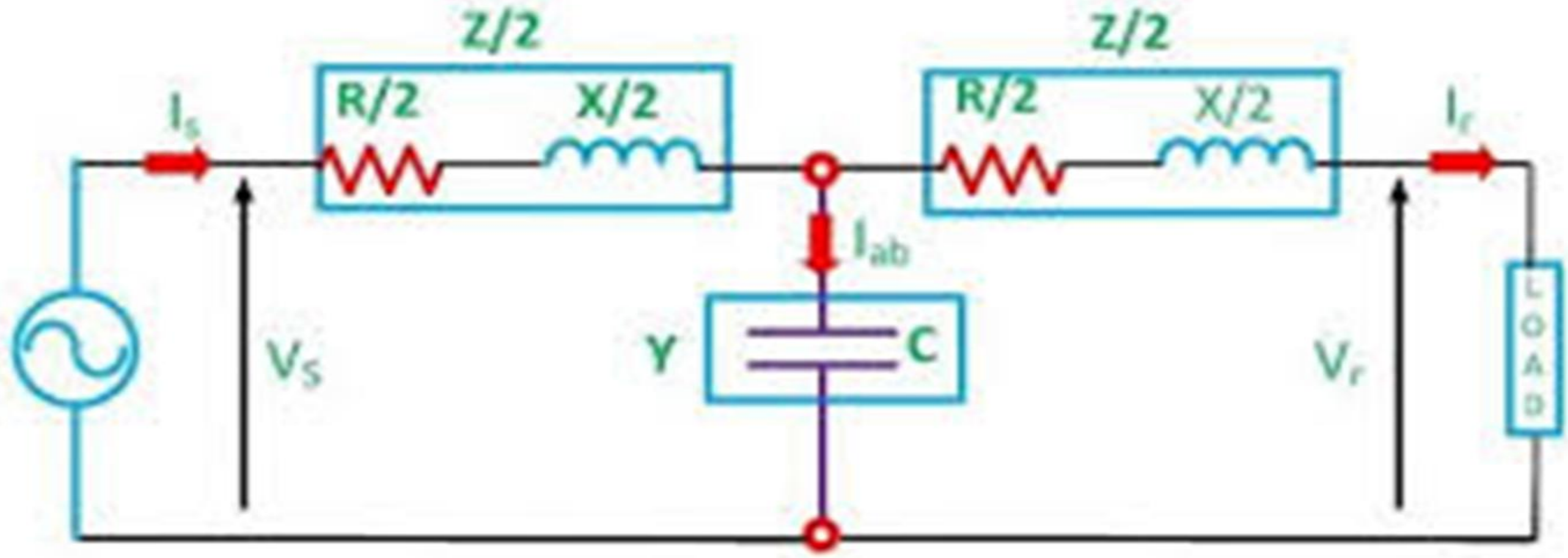
Nominal network of medium transmission line

$$A = (\frac{Y}{2}Z + 1)$$

$$B = Z\ \Omega$$

$$C = Y(\frac{Y}{4}Z + 1)$$

$$D = (\frac{Y}{2}Z + 1)$$



Nominal T model of a Medium Line

$$A = D = 1 + \frac{ZY}{2}$$

$$B = Z\left(1 + \frac{ZY}{4}\right)$$

$$C = Y$$

LONG TRANSMISSION LINE

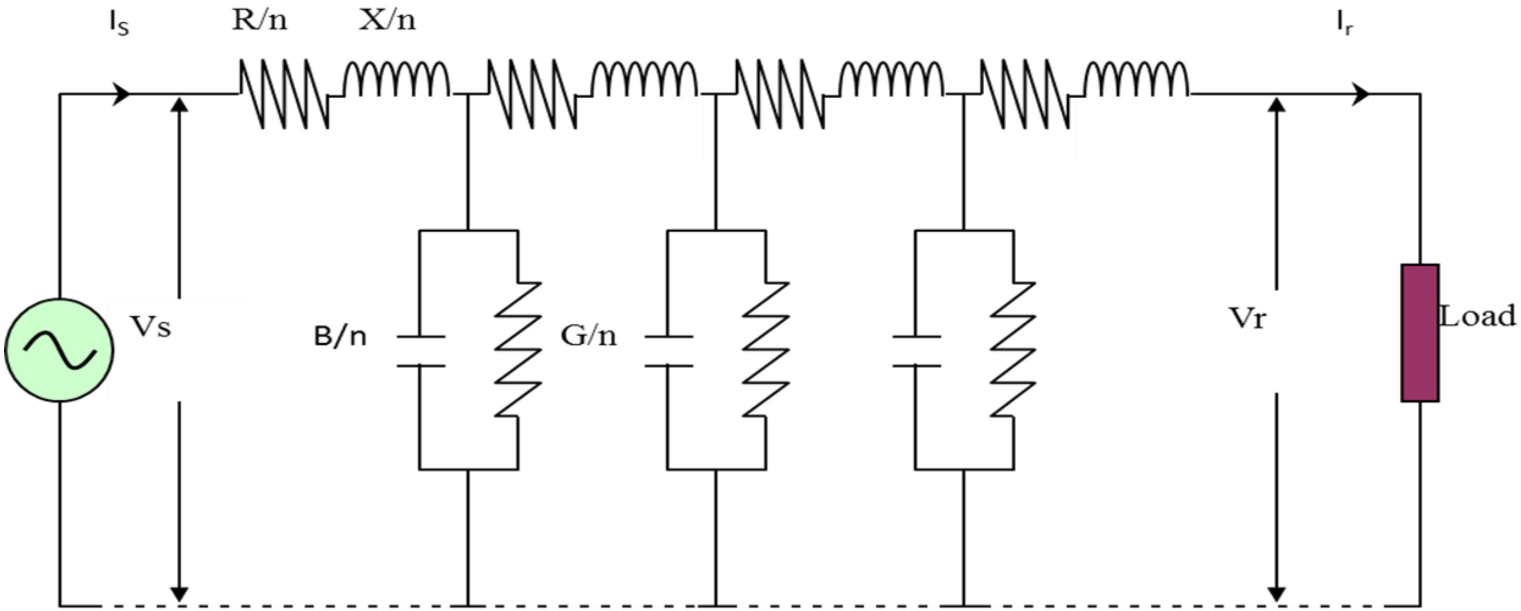
When the length of the line is more than 250km and line voltage is very high which is more than100kV.

The line constants (R,L,C,G) are uniformly distributed over the whole length of the line.

Resistance (R) and inductance (X) are serial elements of transmission line.

Capacitance (C) and conductance (G) are shunt elements of transmission line. It caused the power losses and corona effects.

LONG TRANSMISSION LINE CKT



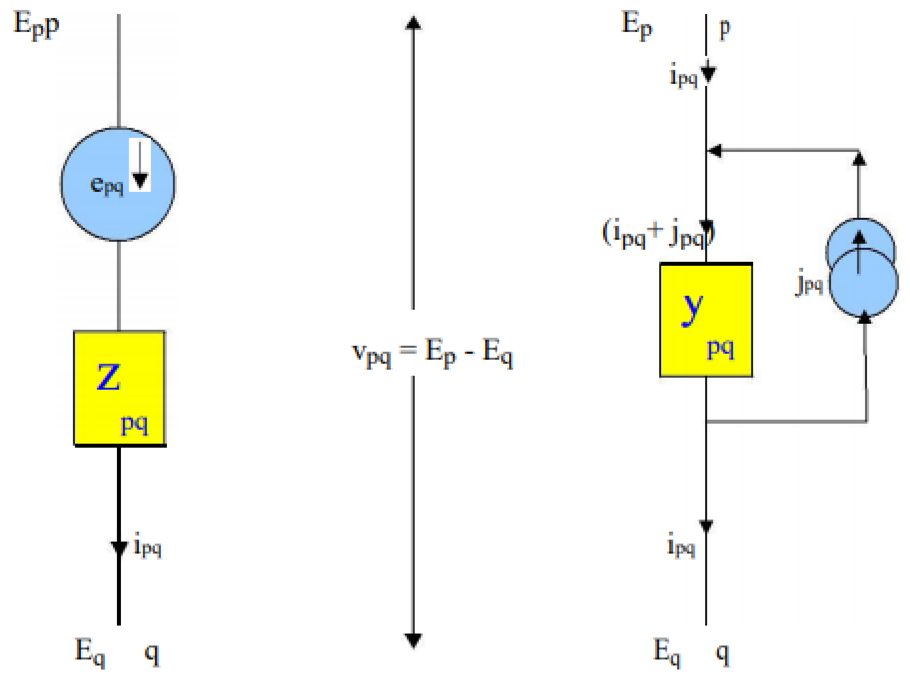
Vs- sending end voltage
Vr- receiving end voltage
Is- sending end current
Ir- receiving end current
Ic- capacitance current
R- loop Capacitance (Ω)
X- loop Inductance (Ω)
C- capacitance (F)
G - loop conductance

$$A = \cosh \delta l$$

$$B = Z_C \sinh \delta l$$

$$C = \frac{\sinh \delta l}{Z_C}$$

$$D = \cosh \delta$$



Representation of a primitive network element (a) Impedance form (b) Admittance form

The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices.

An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

v_{pq} = voltage across the element p-q,

e_{pq} = source voltage in series with the element p-q, i_{pq}= current through the element p-q,

j_{pq}= source current in shunt with the element p-q, z_{pq}= self impedance of the element p-q and

y_{pq}= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

$$v_{pq} + e_{pq} = z_{pq} i_{pq} \quad \text{(in its impedance form)}$$

$$i_{pq} + j_{pq} = y_{pq} v_{pq} \quad \text{(in its admittance form)} \qquad (6)$$

Rule of Inspection

Consider the 3-node admittance network as shown in figure5.

Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3(V_1 - V_3) + Y_6(V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5(V_2 - V_3) + Y_6(V_2 - V_1)$

At node 3: $0 = Y_3(V_3 - V_1) + Y_4V_3 + Y_5(V_3 - V_2)$ **(12)**

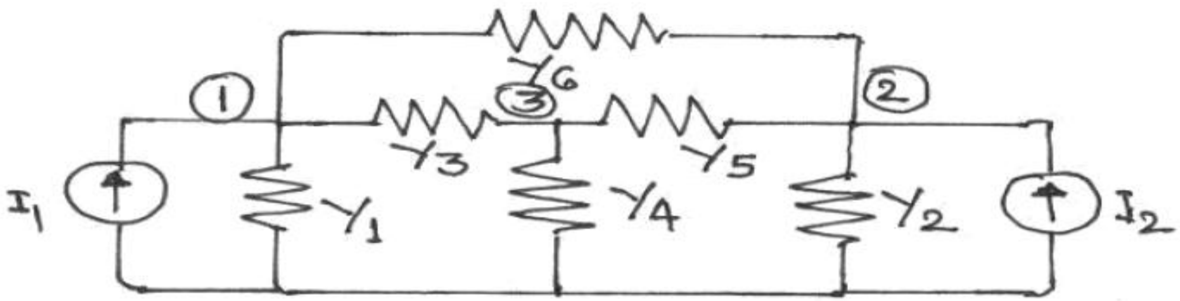


Fig. 3 Example System for finding Y_{BUS}

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad \mathbf{(13)}$$

In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS
In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus).

The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS}\, E_{BUS} \tag{17}$$

Where E_{BUS} = vector of bus voltages measured with respect to reference bus

I_{BUS} = Vector of currents injected into the bus

Y_{BUS} = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y]\, v$$

Pre-multiplying by A^t (transpose of A), we obtain

$$A^t i + A^t j = A^t [y] v \qquad (18)$$

However, as per equation (4),

$$A^t i = 0,$$

$$A^t j = I_{BUS} \qquad (19)$$

Thus from (18) we have, $I_{BUS} = A^t [y] v$ **(20)**

However, from (5), we have

$$v = A\, E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y] A\, E_{BUS} \qquad (21)$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y] A \qquad (22)$$

UNIT-II
POWER FLOW ANALYSIS

Importance of load flow analysis

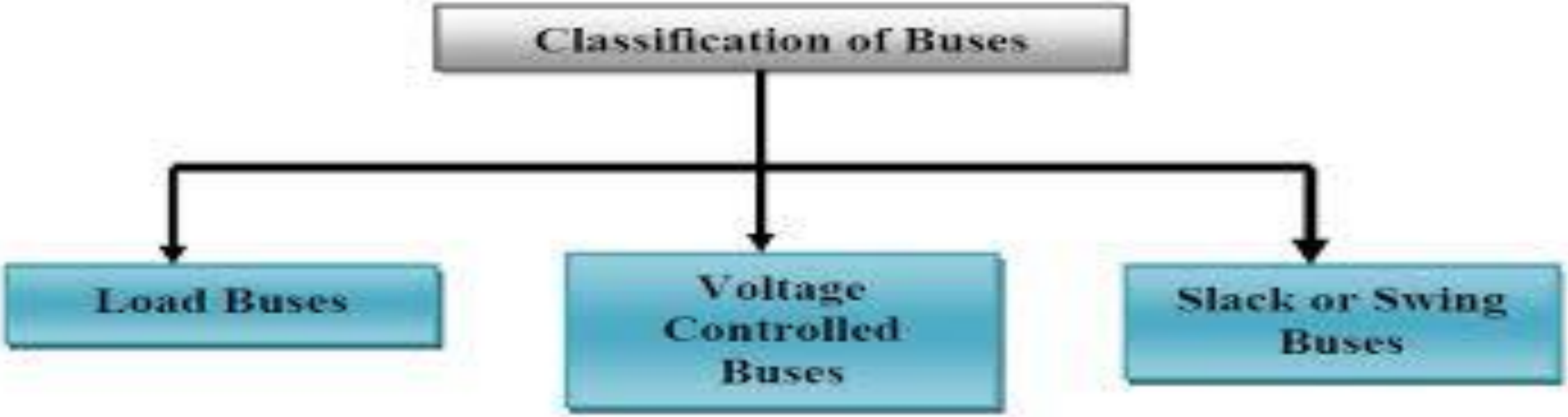
- Load flow analysis is the backbone of PSA.

- It is required for Planning, Operation, Economic Scheduling & Exchange of power b/w utilities . Expansion of system & also in design stage.

- Steady-state analysis, of an interconnected PS during normal operating conditions.

Compute steady-state voltage & voltage angle b/w all buses in n/w.

Real & Reactive power flow in every Tr. line and transformers under the assumption of known values of generation & load.



Types of bus	Known or specified quantities	Unknown quantities or quantities to be determined
Slack or Swing or Reference bus	V, δ	P,Q
Generator or Voltage control or PV bus	P, V	Q, δ
Load or PQ bus	P, Q	V, δ

Load Bus (or) PQ Bus

Bus where only load is connected & no generator exists in this load bus .i.e.,(PG & QG =0)

Real power demand (PD) & Reactive power demand (QD) are drawn for supply.

PD , QD are known values.

|V| , δ are unknown quantities.

In Power balance eqn ,

PD , QD → Negative Quantities

PG , QG Positive Quantities

$$P_i + j\,Q_i = (P_{Gi} - P_{Di}) + j\,(Q_{Gi} - Q_{Di})$$

Generator bus (or) Voltage control Bus (or) PV Bu

Bus Voltage magnitude can be controlled in this bus.

At each bus where alternators are connected, MW generation can be controlled by adjusting the prime mover.

Real power (PG) & Voltage Magnitude (|V|) is known quantities.

Phase angle (δ) & Reactive power (QD) to be find.
For good voltage profile , AVR can be used.

Slack Bus (or) Swing Bus (or) Reference Bus

In PS, load flows from the generator to load via Tr.Lines.
I^2R loss occurs due to losses in Tr. Line Conductors
Power balance relations:

$$P_L = \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{N} P_{Di}$$

$$Q_L = \sum_{i=1}^{N} Q_{Gi} - \sum_{i=1}^{N} Q_{Di}$$

P_L, Q_L are power losses in Tr. n/w
P_{Gi}, Q_{Gi} Generators Power, P_{Di}, Q_{Di} Demand Power
Known values are |V|, δ
Unknown values are P & Q

Solution for Load flow problems

- Gauss- Seidel Load Flow Method
- Newton- Raphson Load Flow Method
- Fast-Decouple Load Flow Method

In above mentioned load flow methods, voltage solution is assume & the iteration process can be proceeded until the reach of convergence.

Gauss-Seidel Algorithm

1. Form Y bus matrix
2. Assume, $V_k = V_{k(spec)} \angle 0^0$ at all generator buses.
3. Assume , $V_k = 1 \angle 0^0 = 1+j0$ at all load buses.
4. Iteration count setting (iter=1)
5. Let bus number i=1
6. If 'i' refer to generator bus go to step no.7,or else go to step 8
7. a. if 'i' refers to the slack bus go to step 9, or else go to step 7(b).

7. b. Compute Q_i using,

$$Q_i^{cal} = -\,\mathrm{Im}\left[\sum_{j=1}^{N} V_i^* Y_{ij} V_j\right]$$

$$Q_{Gi} = Q_i^{cal} + Q_{Li}$$

Contd…

Check for Q limit violation.

If $Q_{i(min)} < Q_{Gi} < Q_{i(max)}$, then $Q_{i(spec)} = Q_i^{cal}$

If $Q_{i(min)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(min)} - Q_{Li}$

If $Q_{i(max)} < Q_{Gi}$, then $Q_{i(spec)} = Q_{i(max)} - Q_{Li}$

If Q_{limit} is violated , then treat this bus as P-Q bus till convergence is obtained

8. Compute Vi using the equation,

$$V_i^{new} = \frac{1}{Y_{ij}} \left[\frac{P_{i(spec)} - Q_{i(spec)}}{V_i^{old\,*}} - \sum_{i=1}^{j-1} Y_{ij} V_j^{new} - \sum_{i=j+1}^{n} Y_{ij} V_i^{old} \right]$$

9. If ‘i’ is less than number of buses, increment i by 1 and go to step 6.

Contd…

10. Compare two successive iteration values for Vi

$$\text{If } V_i^{new} - V_i^{old} < \text{tolerance}, \text{ go to step 12}$$

11. Update new voltages as

$$V_i^{new} = V_i^{old} + \alpha\,(V_i^{new} - V_i^{old})$$

$$V_i^{old} = V_i^{new}$$

12. Compute relevant quantities

$$\text{Slack bus power, } S_1 = P_i - j\,Q_i = V^* I = V_i^* \left[\sum_{j=1}^{N} Y_{ij} V_j \right]$$

$$\text{Line flow, } \quad S_{ij} = P_{ij} + Q_{ij}$$

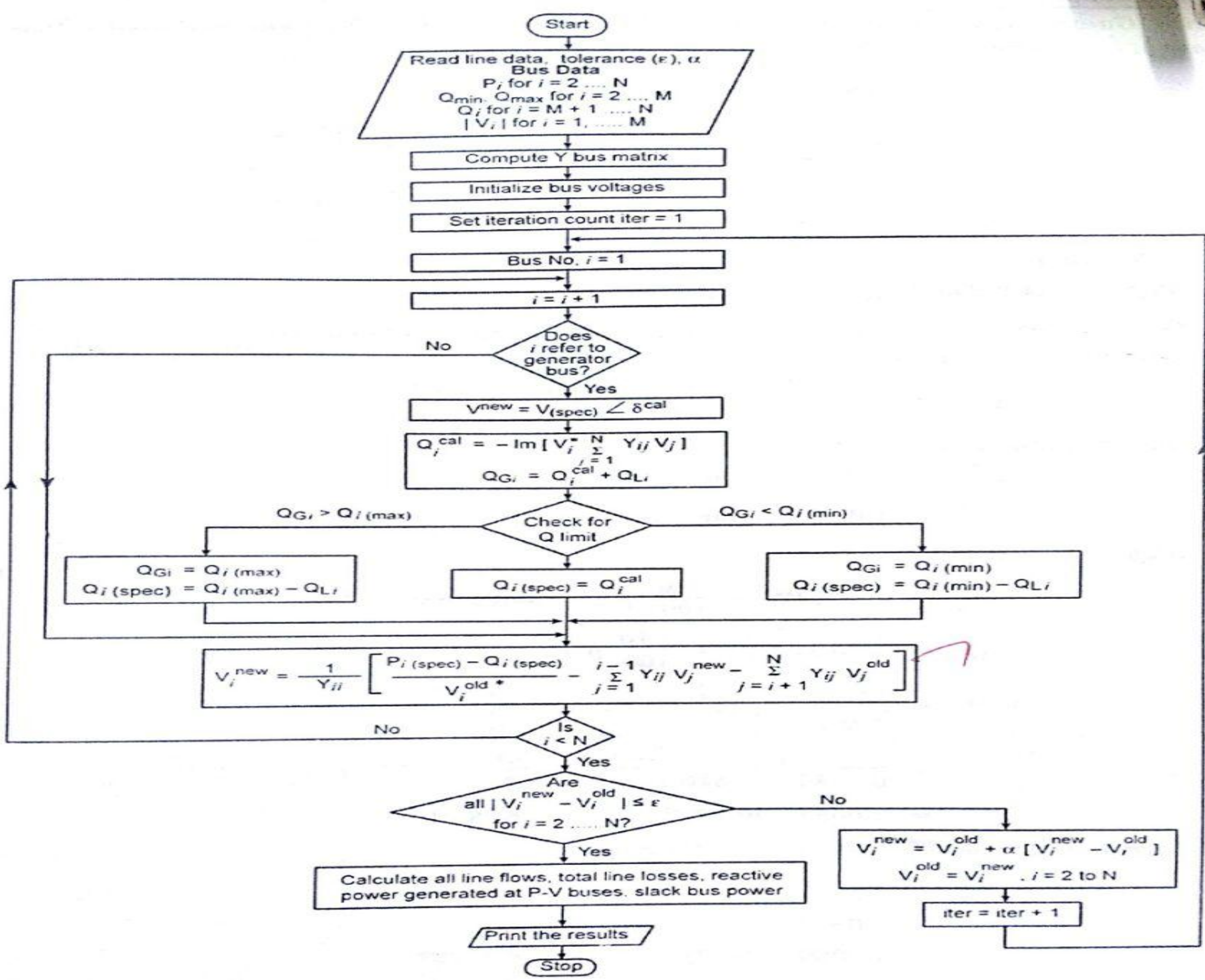
$$= V_i\,[\,V_i^* - V_j^*]\,Y_{ij\ series}^* + |V_i|^2\ Y_{ij}^*$$

$$P_{Loss} = P_{ij} + P_{ji}$$

$$Q_{Loss} = Q_{ij} + Q_{ji}$$

13. Stop the execution.

5.7.6. FLOW CHART FOR GAUSS-SEIDEL METHOD INCLUDING PV BUS ADJUS



Advantage of Gauss Seidel Method

i. Calculation are simple.

ii. Programming task is lesser.

iii. Used for small size system.

Disadvantage of Gauss Seidel Method

i. Not suitable for larger systems

ii. Required more no.of. iterations to reach convergence.

iii. Convergence time increases with size of the system.

1. Form Y-bus matrix
2. Assume flat start for starting voltage solution

$\delta i^{o} = 0,$ for i=1,2,…N for all buses except slack bus

$|Vi^{o}| = 1.0,$ for i=M+1,M+2,….N (for all PQ bus.

$|Vi| = |Vi|$ (Spec) , for all PV buses and Slack bus.

3. For load bus, calculate Pi^{cal} and Qi^{cal}
4. For PV buses, check Q-limit violation .

If $Qi(min) < Qi^{cal} < Qi(max)$, the bus acts as P-V bus.

If $Qi^{cal} > Qi(max)$, $Qi(spec)=Qi(min)$

If $Qi^{cal} < Qi(min)$, $Qi(spec) = Qi(min)$, the P-V bus will act as P-Q bus.

5. Compute mismatch vector using,

$$\Delta Pi = P_{i(spec)} - P_i^{cal} \qquad \Delta Q_i = Q_{i(spec)} - Q_i^{cal}$$

Contd….

6. Compute $\Delta Pi(max) = \max|\Delta Pi|$, $i=1,2,\ldots N$(except Slack bus)

$\Delta Q_i\,(max) = \max|\Delta Q_i|$, $i=M+1\ldots.N$

7. Compute Jacobian matrix using,

$$J = \begin{pmatrix} \dfrac{\partial Pi}{\partial \delta} & |V| \cdot \dfrac{\partial Pi}{\partial |V|} \\ \dfrac{\partial Qi}{\partial \delta} & |V| \cdot \dfrac{\partial Qi}{\partial |V|} \end{pmatrix}$$

8. Obtain static correction vector using

$$\begin{pmatrix} \Delta \delta \\ \dfrac{\Delta V}{|V|} \end{pmatrix} = [J]^{-1} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$$

9. Update state vector using,

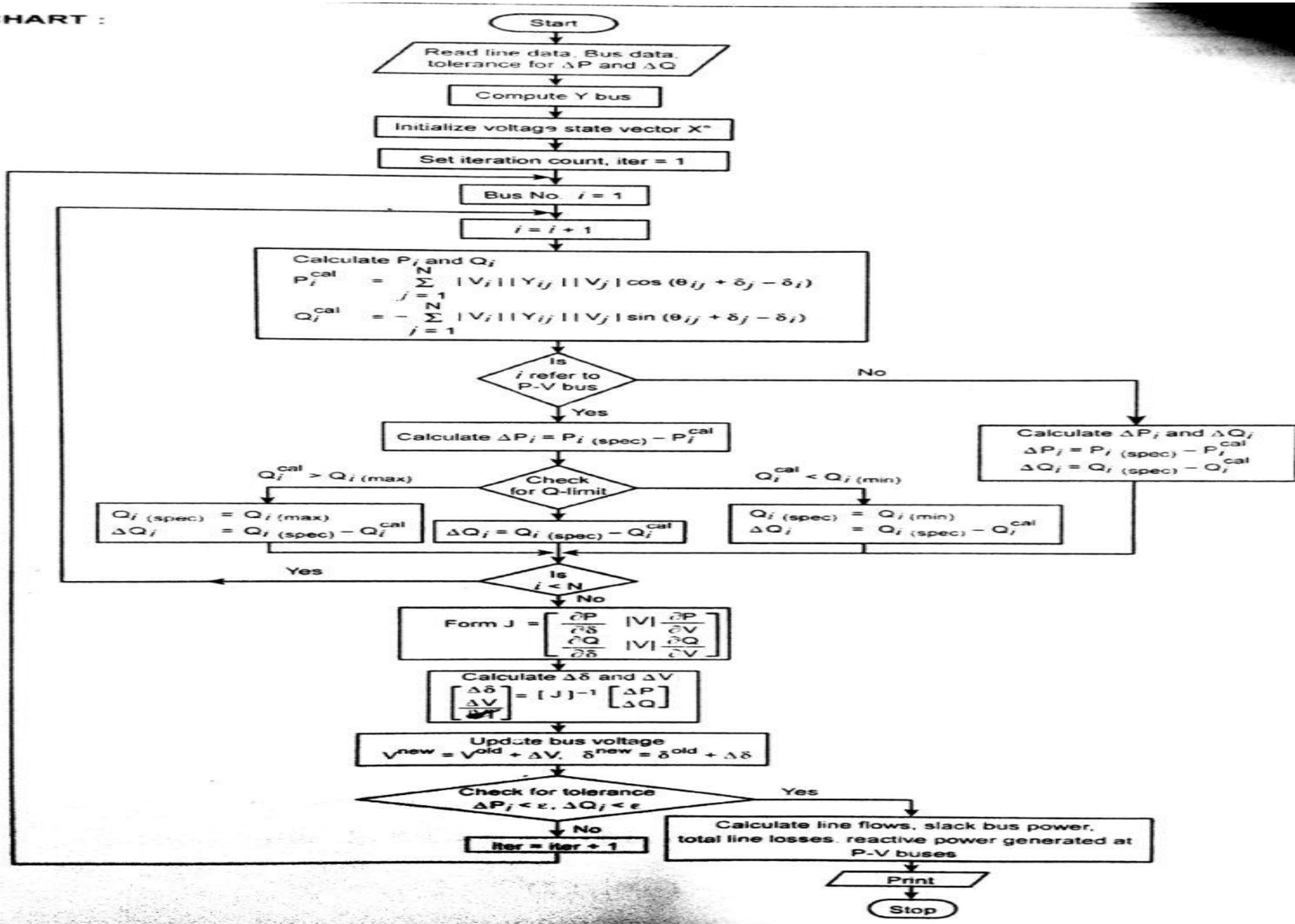
$$V^{new} = V^{old} + \Delta V$$
$$= V^{old} + [\, \Delta V / |V^{old}| \,]$$
$$= V^{old} + [1 + \{\Delta V / |V^{old}|\}]$$

$$\delta = \boldsymbol{\delta}^{old} + \Delta\boldsymbol{\delta}$$

10. This procedure is continued until,

$|\Delta Pi| < \varepsilon$ and $|\Delta Q_i| < \varepsilon$, otherwise go to step 3.

OW CHART :


Start
Read line data, Bus data, tolerance for ΔP and ΔQ
Compute Y bus
Initialize voltage state vector X°
Set iteration count, iter = 1
Bus No. i = 1
i = i + 1
Calculate P_i and Q_i
Is i refer to P-V bus
No
Yes
Calculate ΔP_i and ΔQ_i
Check for Q-limit
Is i < N
Yes
No
Form J
Calculate Δδ and ΔV
Update bus voltage
Check for tolerance ΔP_i < ε, ΔQ_i < ε
Yes
No
iter = iter + 1
Calculate line flows, slack bus power, total line losses, reactive power generated at P-V buses
Print
Stop

Advantage of Newton – Raphson Method

i. suitable for large size system.
ii. It is faster, reliable &the results are accurate.
iii. No.of. Iteration are less to reach convergence & also iterations are independent of the no.of.buses.

Disadvantage of Newton – Raphson Method

i. Programming logic is complex than GS Method
ii. Required more memory.
iii. No.of.calculation per iteration are higher than GS method

Decoupled Power Flow Formulation

General form of the power flow problem

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}^{(v)}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)} \\ \dfrac{\partial \mathbf{Q}^{(v)}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

where

$$\Delta \mathbf{P}(\mathbf{x}^{(v)}) = \begin{bmatrix} P_2(\mathbf{x}^{(v)}) + P_{D2} - P_{G2} \\ \vdots \\ P_n(\mathbf{x}^{(v)}) + P_{Dn} - P_{Gn} \end{bmatrix}$$

Decoupling Approximation

Usually the off-diagonal matrices, $\dfrac{\partial \mathbf{P}}{\partial |\mathbf{V}|}^{(v)}$ and $\dfrac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}}^{(v)}$

are small. Therefore we approximate them as zero:

$$-\begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} & \mathbf{0} \\ \mathbf{0} & \dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta}^{(v)} \\ \Delta |\mathbf{V}|^{(v)} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \\ \Delta \mathbf{Q}(\mathbf{x}^{(v)}) \end{bmatrix} = \mathbf{f}(\mathbf{x}^{(v)})$$

Then the problem can be decoupled

$$\Delta \boldsymbol{\theta}^{(v)} = -\left[\dfrac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}}^{(v)} \right]^{-1} \Delta \mathbf{P}(\mathbf{x}^{(v)}) \quad \Delta |\mathbf{V}|^{(v)} = -\left[\dfrac{\partial \mathbf{Q}}{\partial |\mathbf{V}|}^{(v)} \right]^{-1} \Delta \mathbf{Q}(\mathbf{x}^{(v)})$$

Off-diagonal Jacobian Terms

Justification for Jacobian approximations:

1. Usually r $\ll$ x, therefore $|G_{ij}| \ll |B_{ij}|$
2. Usually θ_{ij} is small so $\sin\theta_{ij} \approx 0$

Therefore

$$\frac{\partial \mathbf{P}_i}{\partial |\mathbf{V}_j|} = |V_i|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

$$\frac{\partial \mathbf{Q}_i}{\partial \boldsymbol{\theta}_j} = -|V_i||V_j|\left(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}\right) \approx 0$$

Fast Decoupled Power Flow

- By continuing with our Jacobian approximations we can actually obtain a reasonable approximation that is independent of the voltage magnitudes/angles.
- This means the Jacobian need only be built/inverted once.
- This approach is known as the fast decoupled power flow (FDPF)
- FDPF uses the same mismatch equations as standard power flow so it should have same solution
- The FDPF is widely used, particularly when we only need an approximate solution

FDPF Approximations

The FDPF makes the following approximations:

1. $\left|\mathrm{G}_{\mathrm{ij}}\right| = 0$
2. $\left|V_i\right| = 1$
3. $\sin\theta_{ij} = 0 \qquad \cos\theta_{ij} = 1$

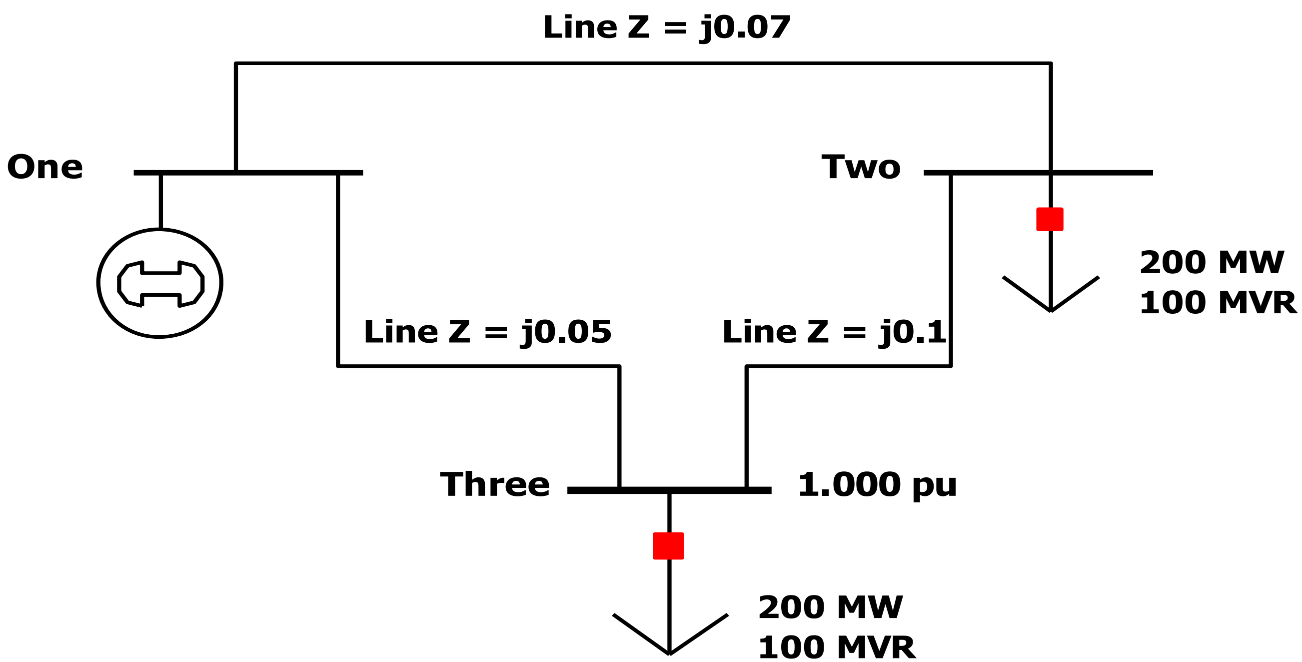
Then

$$\Delta\boldsymbol{\theta}^{(v)} = \mathbf{B}^{-1}\frac{\Delta\mathbf{P}(\mathbf{x}^{(v)})}{\mathbf{V}^{(v)}} \qquad \Delta\left|\mathbf{V}\right|^{(v)} = \mathbf{B}^{-1}\frac{\Delta\mathbf{Q}(\mathbf{x}^{(v)})}{\mathbf{V}^{(v)}}$$

Where $\mathbf{B}$ is just the imaginary part of the $\mathbf{Y}_{\mathrm{bus}} = \mathbf{G} + j\mathbf{B}$, except the slack bus row/column are omitted

FDPF Three Bus Example

Use the FDPF to solve the following three bus system



$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix}$$

FDPF Three Bus Example, cont'd

$$\mathbf{Y}_{bus} = j\begin{bmatrix} -34.3 & 14.3 & 20 \\ 14.3 & -24.3 & 10 \\ 20 & 10 & -30 \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} -24.3 & 10 \\ 10 & -30 \end{bmatrix}$$

$$\mathbf{B}^{-1} = \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix}$$

Iteratively solve, starting with an initial voltage guess

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix}$$

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9364 \\ 0.9455 \end{bmatrix}$$

$$\frac{\Delta \mathrm{P_i}(\mathbf{x}\ \ )}{|\mathrm{V_i}|} = \sum_{k=1}^{n} |V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) + \frac{P_{Di} - P_{Gi}}{|\mathrm{V_i}|}$$

$$\begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix}^{(2)} = \begin{bmatrix} -0.1272 \\ -0.1091 \end{bmatrix} + \begin{bmatrix} -0.0477 & -0.0159 \\ -0.0159 & -0.0389 \end{bmatrix} \begin{bmatrix} 0.151 \\ 0.107 \end{bmatrix} = \begin{bmatrix} -0.1361 \\ -0.1156 \end{bmatrix}$$

$$\begin{bmatrix} |V|_2 \\ |V|_3 \end{bmatrix}^{(2)} = \begin{bmatrix} 0.924 \\ 0.936 \end{bmatrix}$$

Actual solution: $\boldsymbol{\theta} = \begin{bmatrix} -0.1384 \\ -0.1171 \end{bmatrix}$ $\quad \mathbf{V} = \begin{bmatrix} 0.9224 \\ 0.9338 \end{bmatrix}$

“DC” Power Flow

- The “DC” power flow makes the most severe approximations: completely ignore reactive power, assume all the voltages are always 1.0 per unit, ignore line conductance
- This makes the power flow a linear set of equations, which can be solved directly

$$\boldsymbol{\theta} = \mathbf{B}^{-1}\,\mathbf{P}$$

DC Power Flow Example

EXAMPLE 6.17

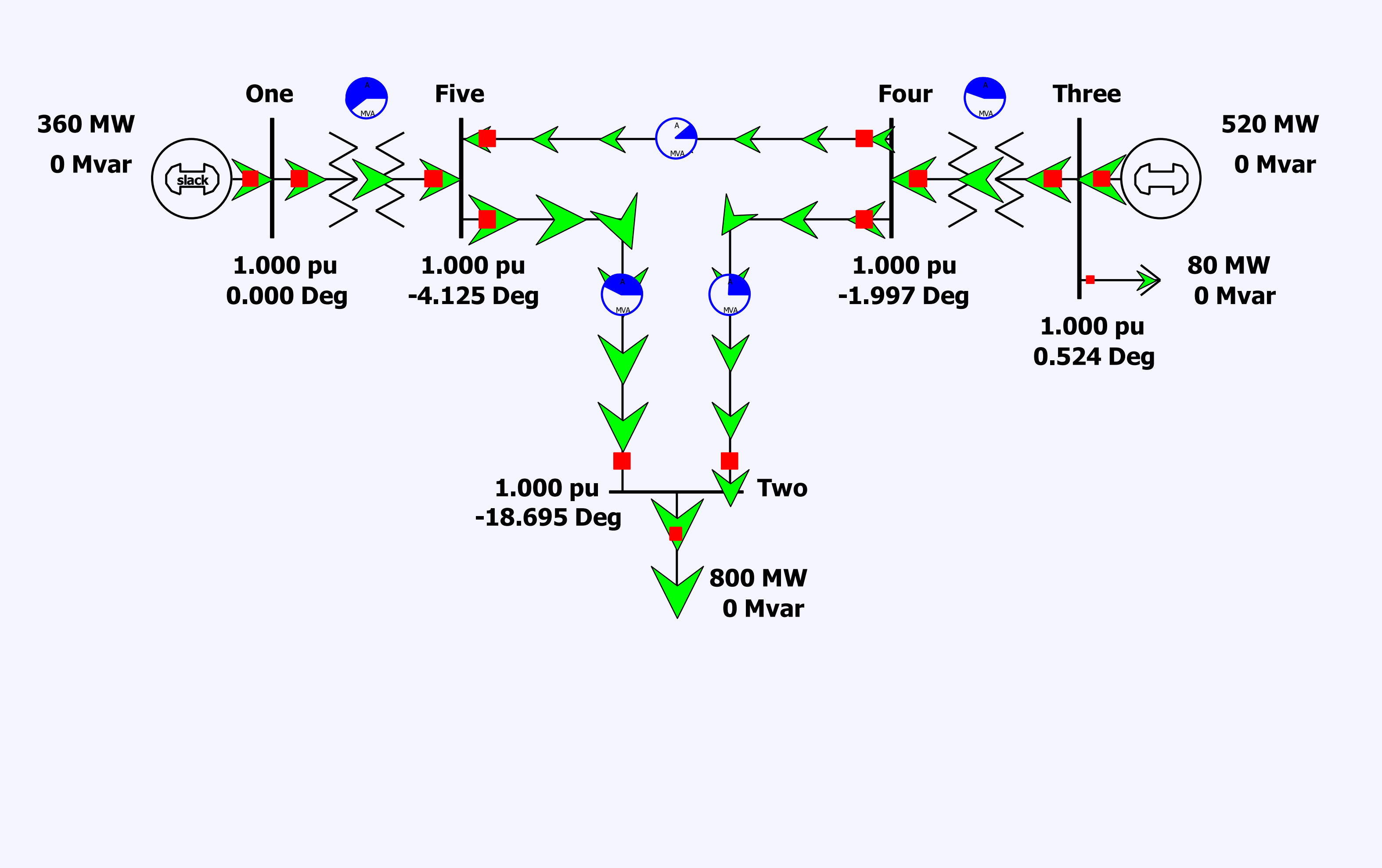
Determine the dc power flow solution for the five bus from Example 6.9.

SOLUTION With bus 1 as the system slack, the **B** matrix and **P** vector for this system are

$$\mathbf{B} = \begin{bmatrix} -30 & 0 & 10 & 20 \\ 0 & -100 & 100 & 0 \\ 10 & 100 & -150 & 40 \\ 20 & 0 & 40 & -110 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} -8.0 \\ 4.4 \\ 0 \\ 0 \end{bmatrix}$$

$$\delta = -\mathbf{B}^{-1}\mathbf{P} = \begin{bmatrix} -0.3263 \\ 0.0091 \\ -0.0349 \\ -0.0720 \end{bmatrix} \text{radians} = \begin{bmatrix} -18.70 \\ 0.5214 \\ -2.000 \\ -4.125 \end{bmatrix} \text{degrees}$$

DC Power Flow 5 Bus Example



UNIT-III
SHORT CIRCUIT ANALAYSIS

BALANCED FAULT

Analysis types

power flow - evaluate normal operating conditions

fault analysis - evaluate abnormal operating conditions

Fault types:

balanced faults

three-phase

unbalanced faults

single-line to ground and double-line to ground

line-to-line faults

Results used for:

Specifying ratings for circuit breakers and fuses

protective relay settings

specifying the impedance of transformers and generators

Introduction for Symmetrical Fault analysis

- A fault current that occurs in any normal circuit will tends to change the circuit to abnormal state.
- For example, a short circuit is a fault in which high current bypasses the normal load.
- An open-circuit fault occurs if a circuit is interrupted by some failure.

Fault usually occurs in a power system due to

- **Insulation failure of equipment.**
- **Flashover of lines initiated by a lighting stoke.**
- **Permanent damage of conductors and tower or accidental faulty operation.**
- **Wires blowing together in the wind.**
- **Animals or plants coming in contact with the wires**
- **Salt spray or pollution on insulators.**

Short-Circuit

"Whenever a fault occurs on a network such that a large current flows in one or more phases, a **short circuit** *is said to have occurred".*

Causes of short-circuit

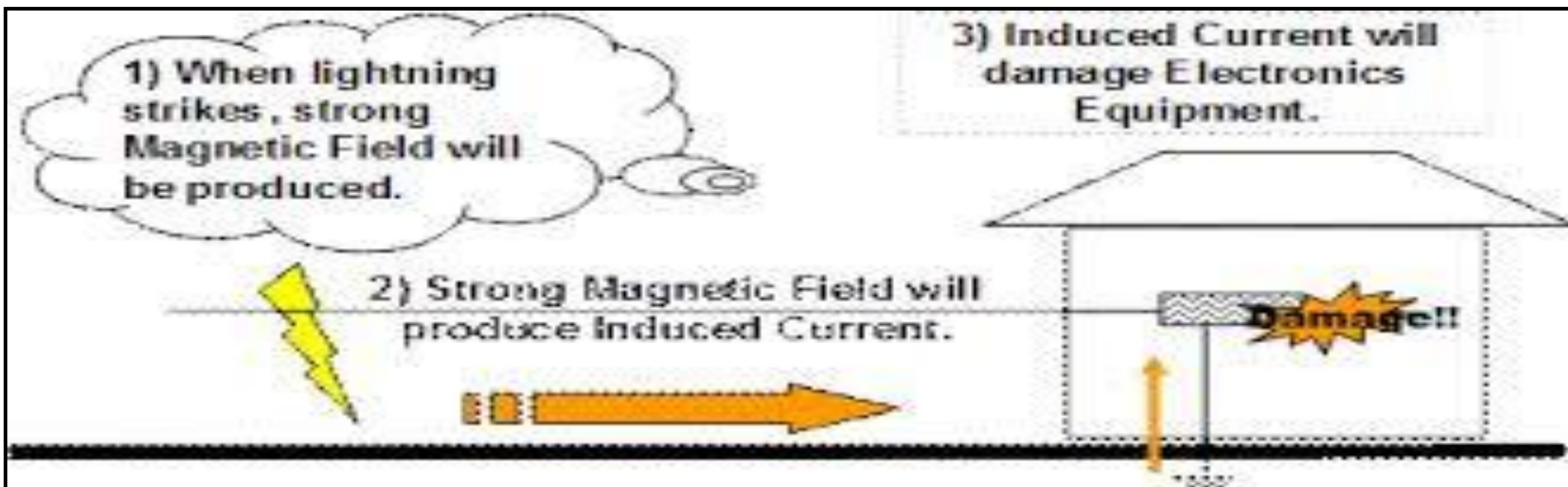
(*i*) Internal effects are caused by breakdown of equipment or transmission lines, from ***weakening of insulation*** in a generator, transformer *etc*. Such troubles may be due to ***ageing of insulation***, ***inadequate design*** or ***improper installation***.

(*ii*) External effects causing short circuit include insulation failure ***due to lightning surges***, ***overloading of equipment causing excessive heating***; ***mechanical damage by public*** *etc*.

Lightning

➢Lightning is a sudden electro static discharge between two clouds during storm.

➢When lightning strike transmission line then a very high voltage generated with in it.



Effects of short-circuit

When a short-circuit occurs, the current in the system increases to an abnormally high value while the system voltage decreases to a low value.

Short-circuit causes excessive heating which may result in fire or explosion.

Sometimes short-circuit takes the form of an arc and causes considerable damage to the system.

Low voltage created by the fault has a very harmful effect on the service rendered by the power system. If the voltage remains low for even a few seconds, the consumers' motors may be shut down and generators on the power system may become unstable.

Types of fault

- **Series fault (or) open circuit**

    i. One open conductor fault

    ii. Two open conductor fault

- **Shunt fault (or) short-circuit fault**

    Symmetrical fault (or) balanced fault

    ----Three-Phase fault

    Unsymmetrical fault (or) Unbalanced fault

    i. Line-to-ground (L-G) fault

    ii. Line-to-line (L-L) fault

    iii. Double line-to-ground (L-L-G) fault

Symmetrical fault

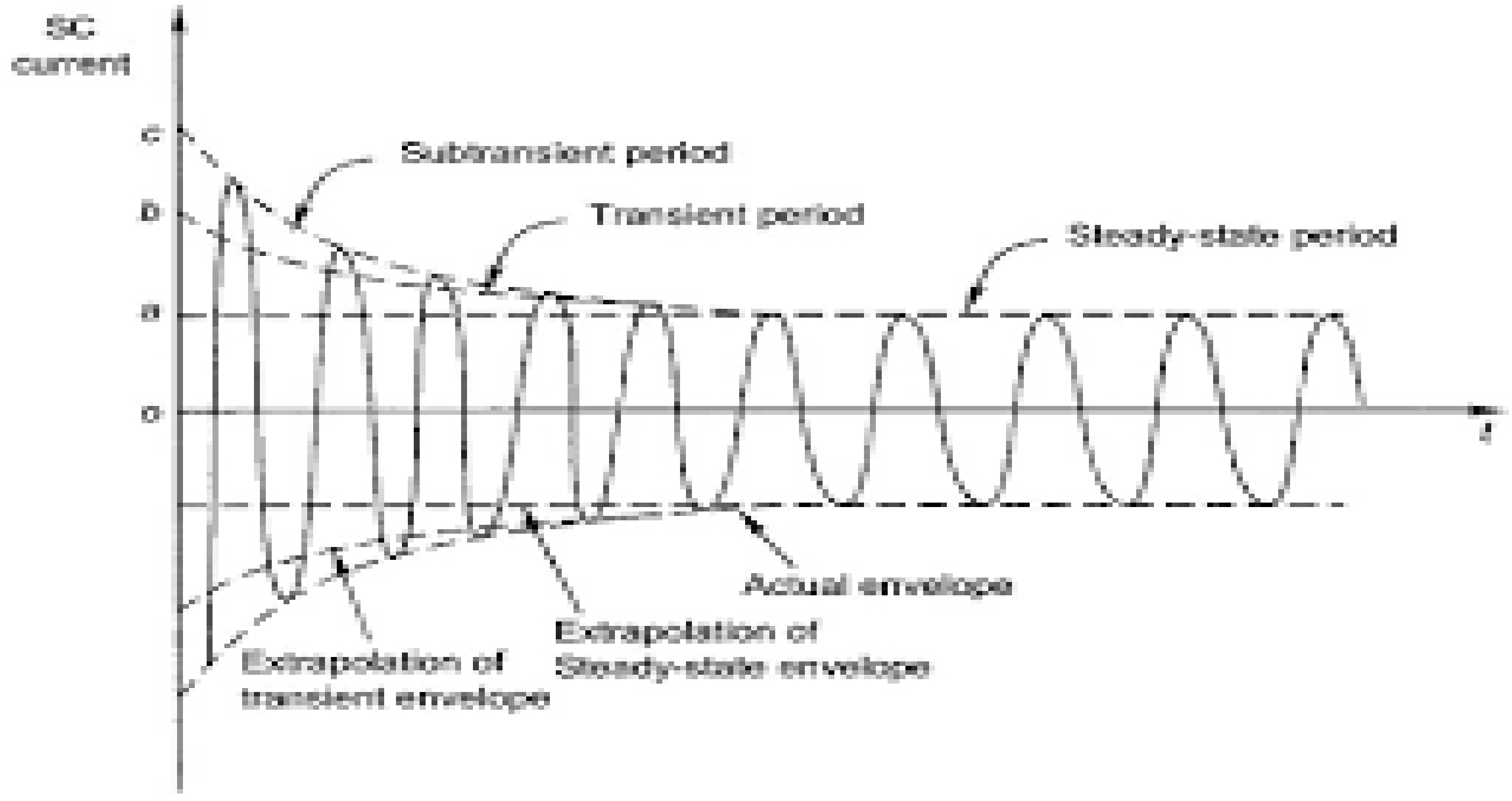


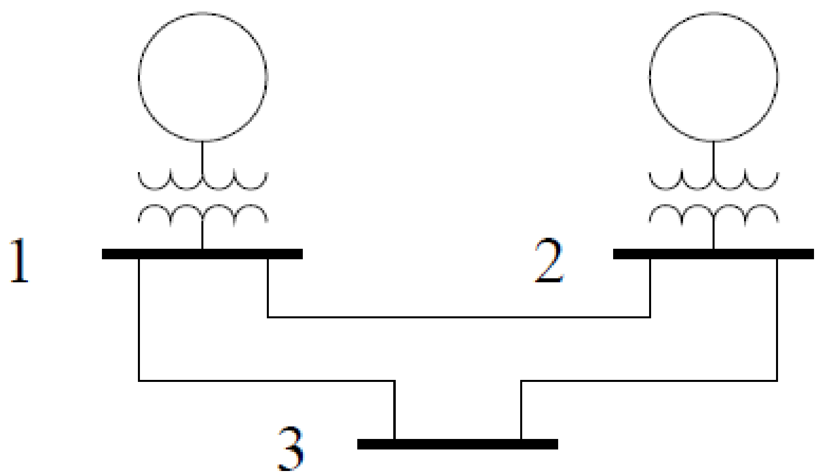
Fig. 8.71 *Symmetrical short circuit current in synchronous generator*

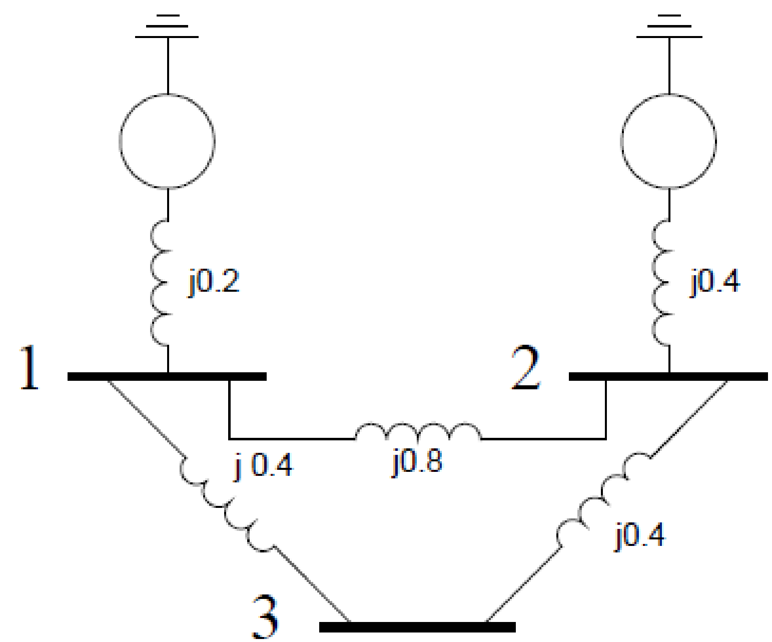
Application of Thevenin's Theorem:

- Fault current evaluation.
- Bus Voltage & Line current during the fault can be determined.
- Post fault voltage & current can be obtained using pre-fault voltages & current.

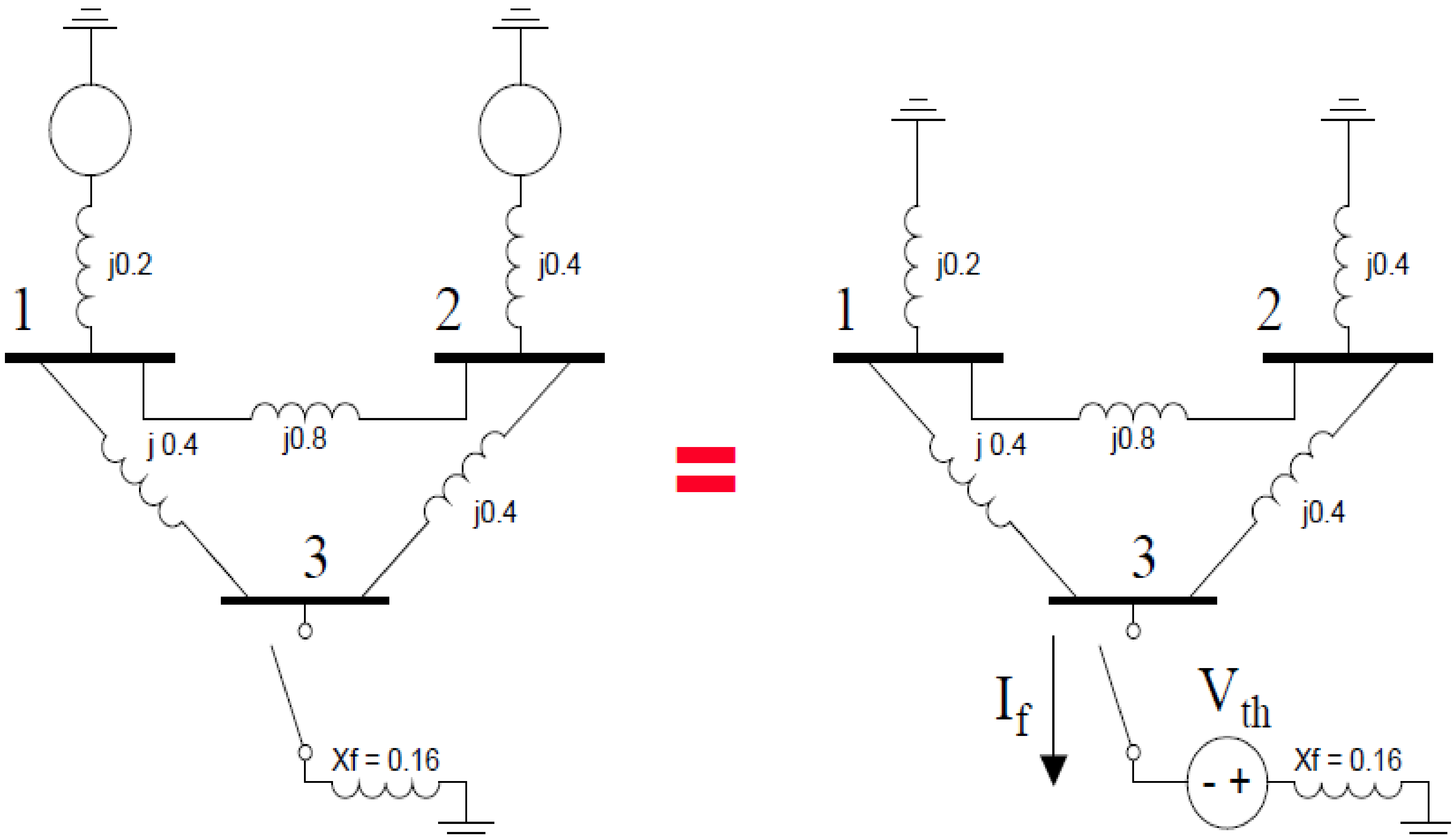
Thévenin’s Method

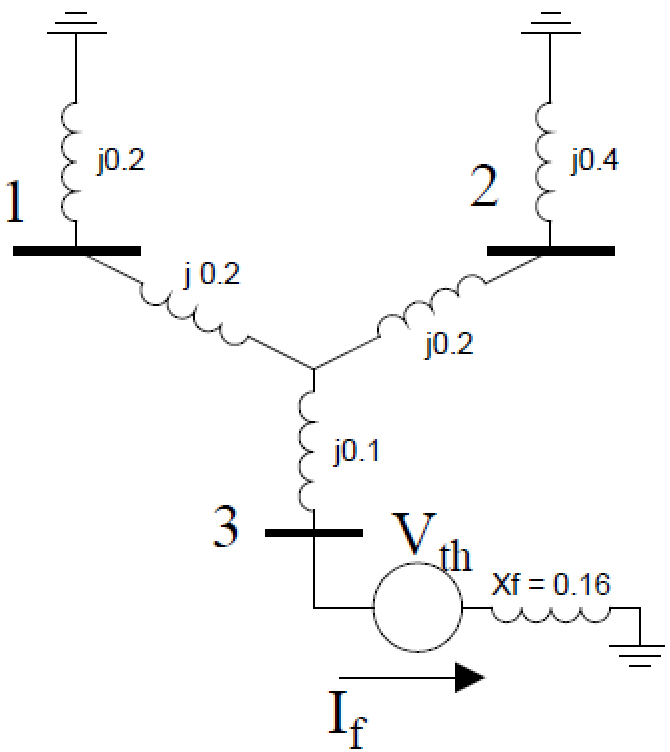
- The fault is simulated by switching a fault impedance at the faulted bus
- The change in the network voltages is equivalent to adding the prefault bus voltage with all other sources short curcuited





- 3-phase fault with $Z_f = j0.16$ on bus 3





$$I_3^{[f]} = \frac{V_3^{[0]}}{Z_{33} + Z_f}$$

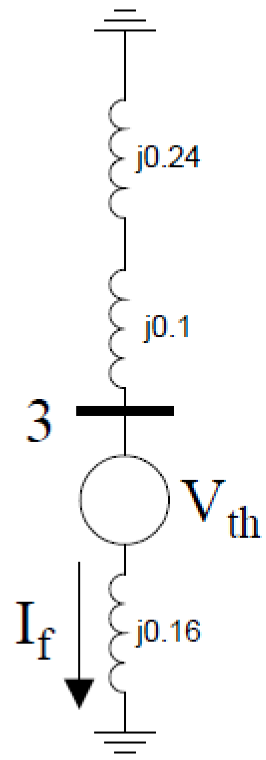
$$V_1^{[0]} = V_2^{[0]} = V_3^{[0]} = 1.0$$

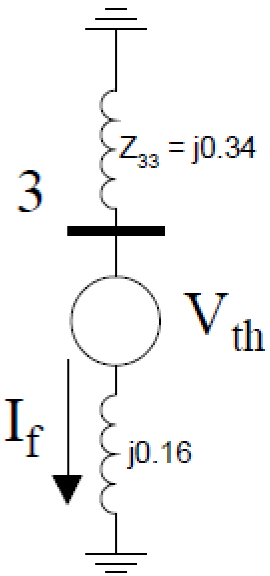
$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{(j1.6)} = j0.2$$

$$Z_{3s} = \frac{(j0.4)(j0.4)}{(j1.6)} = j0.1$$

$$Z_{33} = \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1$$

$$Z_{33} = j0.34$$





$$Z_{33} = j0.34$$

$$I_3^{[f]} = \frac{V_3^{[0]}}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0$$

Thévenin’s Method

- **For more accurate solutions**
  - use the pre-fault bus voltages which can be obtained from the results of a power flow solution
  - include loads - to preserve linearity, convert loads to constant impedance model
  - Thevenin’s theorem allows the changes in the bus voltages to be obtained
  - bus voltages are obtained by superposition of the pre-fault voltages and the changes in the bus voltages
  - current in each branch can be solved

Short Circuit Capacity (SCC)

- Measures the electrical strength of the bus
- Stated in MVA
- Determines the dimension of bus bars and the interrupting capacity of circuit breakers
- Definition:

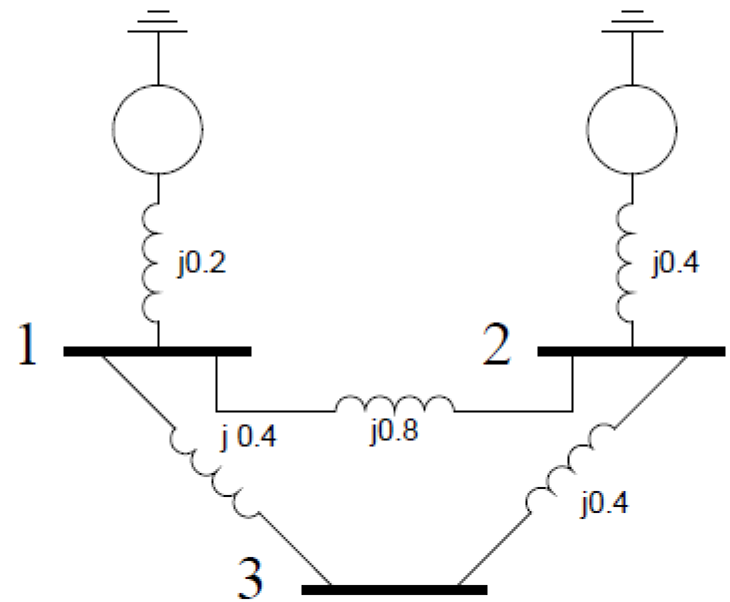
$$SCC = \sqrt{3}\, V_{L-L,k}^{[\text{pre-f}]}\, I_k^{[\text{f}]}$$

  - in per unit:

$$I_k^{[\text{f}]} = \frac{V_k^{[\text{pre-f}]}}{X_{kk}}$$

$$SCC = \frac{S_B}{X_{kk}}$$

- **Find the SCC for bus #3**



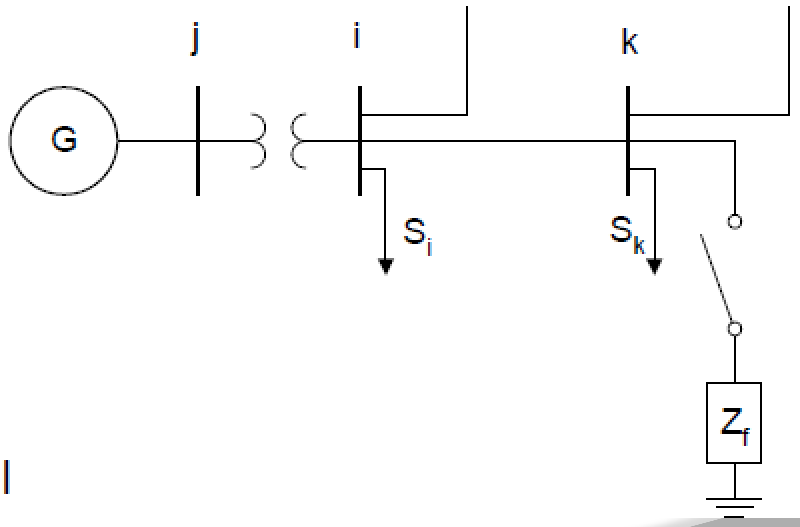
$$Z_{33} = j0.34$$

$$S_{base} = 100 \text{ MVA}$$

$$SCC_3 = \frac{S_{base}}{|Z_{33}|} = \frac{100 \text{ MVA}}{0.34} = 294 \text{ MVA}$$

Fault Analysis Using Impedance Matrix

- **Network reduction by Thévenin's method is not efficient**
  - difficult to apply to large networks
- **Matrix algebra formation**
  - seek a matrix where the diagonal elements represent the source impedance for the buses
  - consider the following system
    - operating under balanced conditions
    - each generator represented by a constant emf behind a proper reactance (X_d, X_d', or X_d'')
    - lines represented by their equivalent π model



- Using superpositioning, the fault voltages are calculated from the prefault voltages by adding the change in bus voltages due to the fault

$$\mathbf{V}_{bus}^{[\mathrm{f}]} = \mathbf{V}_{bus}^{[\mathrm{pre-f}]} + \Delta\mathbf{V}_{bus} \qquad \Delta\mathbf{V}_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

- The change in bus voltages can be calculated from the network matrix

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\ \mathbf{V}_{bus}$$

$$\mathbf{I}_{bus}^{[\mathrm{Fault}]} = \mathbf{Y}_{bus}\ \Delta\mathbf{V}_{bus}$$

$$\mathbf{I}_{bus}^{[\mathrm{Fault}]} = \begin{bmatrix} 0 \\ \vdots \\ -I_k^{[\mathrm{Fault}]} \\ \vdots \\ 0 \end{bmatrix}$$

Fault Analysis Using Impedance Matrix

$$\mathbf{I}_{bus}^{[\text{Fault}]} = \mathbf{Y}_{bus}\,\Delta\mathbf{V}_{bus}$$

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k^{[f]} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1k} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{k1} & \cdots & y_{kk} & \cdots & y_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nk} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

$$\Delta\mathbf{V}_{bus} = \mathbf{Y}_{bus}^{-1}\,\mathbf{I}_{bus}^{[\text{Fault}]}$$

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$$

$$\Delta\mathbf{V}_{bus} = \mathbf{Z}_{bus}\,\mathbf{I}_{bus}^{[\text{Fault}]}$$

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} z_{11} & \cdots & z_{1k} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ z_{k1} & \cdots & z_{kk} & \cdots & z_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nk} & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k^{[f]} \\ \vdots \\ 0 \end{bmatrix}$$

$$\mathbf{V}_{bus}^{[\mathrm{f}]} = \mathbf{V}_{bus}^{[\mathrm{pre-f}]} + \Delta\mathbf{V}_{bus}$$

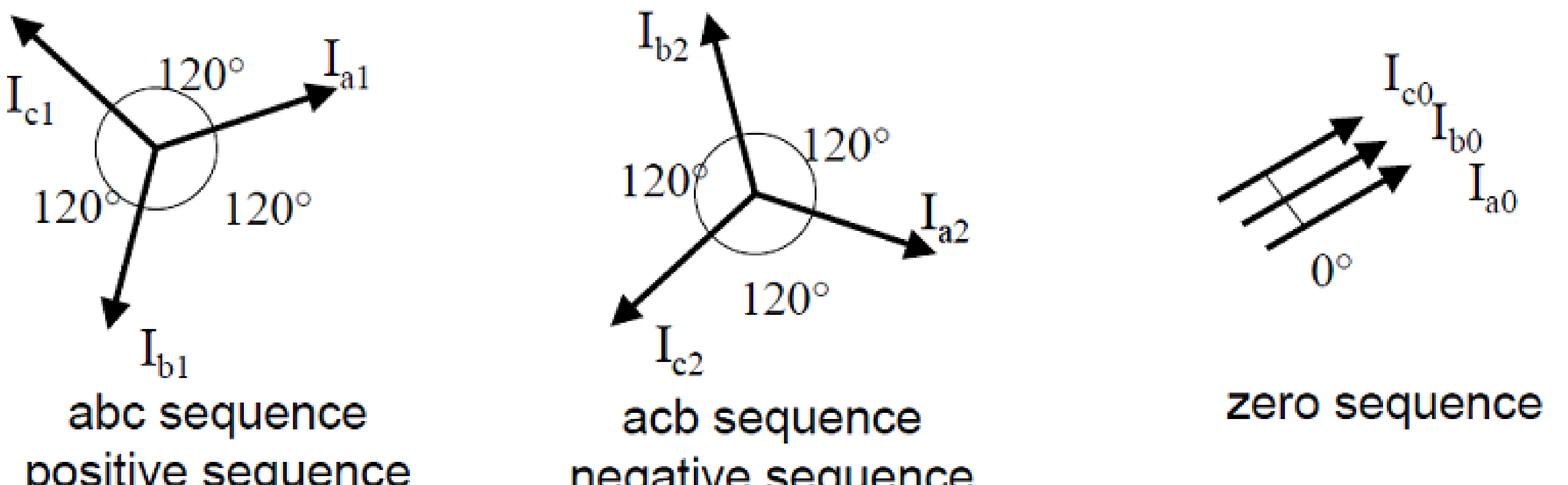
$$\mathbf{V}_{bus}^{[\mathrm{f}]} = \mathbf{V}_{bus}^{[\mathrm{pre-f}]} + \mathbf{Z}_{bus}\ \mathbf{I}_{bus}^{[\mathrm{f}]}$$

$$V_k^{[\mathrm{f}]} = V_k^{[\mathrm{pre-f}]} + Z_{kk}\ I_k^{[\mathrm{f}]} \quad \& \quad V_k^{[\mathrm{f}]} = Z_f\ I_k^{[\mathrm{f}]}$$

$$I_k^{[\mathrm{f}]} = \frac{V_k^{[\mathrm{pre-f}]}}{Z_{kk} + Z_f}$$

Symmetrical Components

- **Allow unbalanced three-phase phasor quantities to be replaced by the sum of three separate but balanced symmetrical components**
  - applicable to current and voltages
  - permits modeling of unbalanced systems and networks
- **Representative symmetrical components**



- **Positive sequence phasors**

$$I_{a1} = |I_{a1}| \angle(\delta + 0^\circ) \quad = I_{a1}$$

$$I_{b1} = |I_{a1}| \angle(\delta + 240^\circ) = a^2 I_{a1}$$

$$I_{c1} = |I_{a1}| \angle(\delta + 120^\circ) = a\, I_{a1}$$

- **Operator a identities**

$$a \; = 1\angle 120^\circ = -0.5 + j0.866$$

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$

$$a^3 = 1\angle 0^\circ \quad = 1 + j0$$

$$1 + a + a^2 = 0$$

- **Negative sequence phasors**

$$I_{a2} = |I_{a2}|\angle(\delta + 0^\circ) = I_{a2}$$

$$I_{b2} = |I_{a2}|\angle(\delta + 120^\circ) = a\, I_{a2}$$

$$I_{c2} = |I_{a2}|\angle(\delta + 240^\circ) = a^2 I_{a2}$$

- **Zero sequence phasors**

$$I_{a0} = |I_{a0}|\angle(\delta + 0^\circ) = I_{a0}$$

$$I_{b0} = |I_{a0}|\angle(\delta + 0^\circ) = I_{a0}$$

$$I_{c0} = |I_{a0}|\angle(\delta + 0^\circ) = I_{a0}$$

- **Relating unbalanced phasors to symmetrical components**

$$I_a = I_{a0} + I_{a1} + I_{a2} \quad = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{b0} + I_{b1} + I_{b2} \quad = I_{a0} + a^2 I_{a1} + a\, I_{a2}$$

$$I_c = I_{c0} + I_{c1} + I_{c2} \quad = I_{a0} + a\, I_{a1} + a^2 I_{a2}$$

- **In matrix notation**

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

- [A] is known as the symmetrical components transformation matrix

$$\mathbf{I}_{abc} = \mathbf{A}\,\mathbf{I}_{012} \qquad \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

- Solving for the symmetrical components leads to

$$\mathbf{I}_{012} = \mathbf{A}^{-1}\,\mathbf{I}_{abc}$$

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = \frac{1}{3}\mathbf{A}^*$$

- In component form, the calculation for symmetrical components are

$$I_{a0} = \tfrac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \tfrac{1}{3}\left(I_a + aI_b + a^2\, I_c\right)$$

$$I_{a2} = \tfrac{1}{3}\left(I_a + a^2\, I_b + aI_c\right)$$

- Similar expressions exist for voltages

$$\mathbf{V}_{abc} = \mathbf{A}\ \mathbf{V}_{012}$$

$$\mathbf{V}_{012} = \mathbf{A}^{-1}\ \mathbf{V}_{abc}$$

- The apparent power may also be expressed in terms of symmetrical components

$$S_{3\phi} = \mathbf{V}_{abc}^{\mathbf{T}}\ \mathbf{I}_{abc}^{*}$$

$$S_{3\phi} = \left(\mathbf{A}\mathbf{V}_{012}\right)^{\mathbf{T}}\left(\mathbf{A}\mathbf{I}_{012}\right)^{*}$$

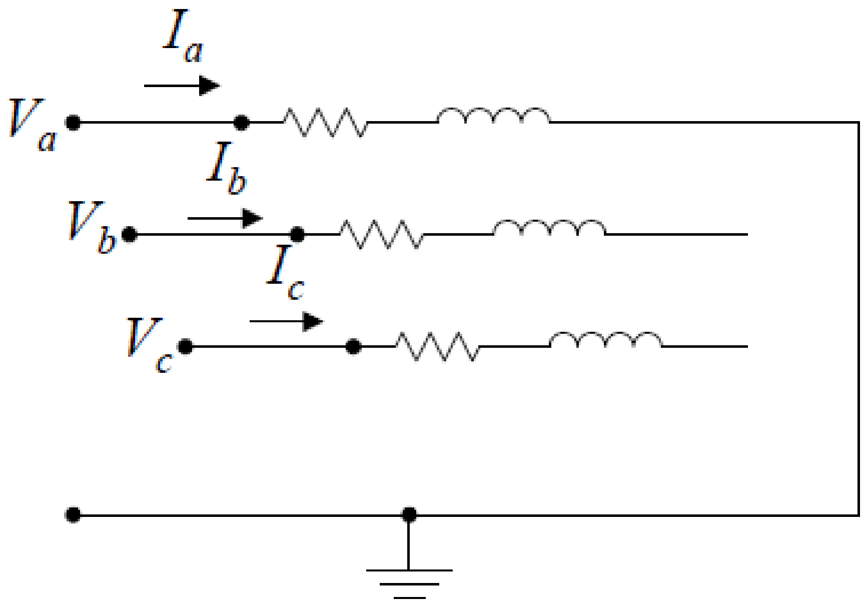
$$S_{3\phi} = \mathbf{V}_{012}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}\mathbf{A}^{*}\mathbf{I}_{012}^{*} \qquad \mathbf{A}^{\mathbf{T}}\mathbf{A}^{*} = 3$$

$$S_{3\phi} = 3\mathbf{V}_{012}^{\mathbf{T}}\mathbf{I}_{012}^{*} = 3\,V_{a0}I_{a0}^{*} + 3\,V_{a1}I_{a1}^{*} + 3\,V_{a2}I_{a2}^{*}$$

Sequence Impedances

- **The impedance offered to the flow of a sequence current creating sequence voltages**
  - positive, negative, and zero sequence impedances
- **Augmented network models**
  - wye-connected balanced loads
  - transmission line
  - 3-phase transformers
  - generators

Single Line to Ground Fault



$$V_a = 0$$

$$I_f = I_a$$

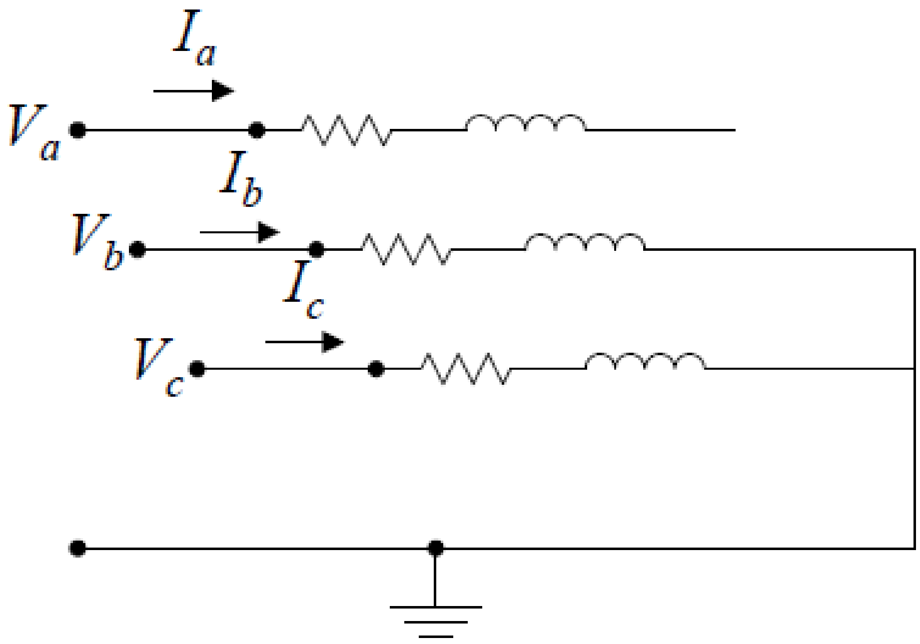
$$I_b = I_c = 0$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{a0} = I_{a1} = I_{a2} = \tfrac{1}{3} I_a$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

Double Line to Ground Fault



$$V_b = 0$$

$$V_c = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_f = I_b + I_c$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2}$$

$$I_{a0} + I_{a1} + I_{a2} = 0$$

$$\mathbf{I}_{abc} = \mathbf{A}\,\mathbf{I}_{012}$$

$$I_f = I_b + I_c$$

$$= I_{a0} + a^2 I_{a1} + a\,I_{a2} + I_{a0} + a\,I_{a1} + a^2 I_{a2}$$

$$= 2\,I_{a0} + a^2 \left(I_{a1} + I_{a2}\right) + a\,\left(I_{a1} + I_{a2}\right)$$

$$= 2\,I_{a0} + \left(a^2 + a\right)\left(I_{a1} + I_{a2}\right)$$

$$\left(a^2 + a\right) = -1$$

$$I_f = 2\,I_{a0} - \left(I_{a1} + I_{a2}\right)$$

Line-to-Line Fault

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

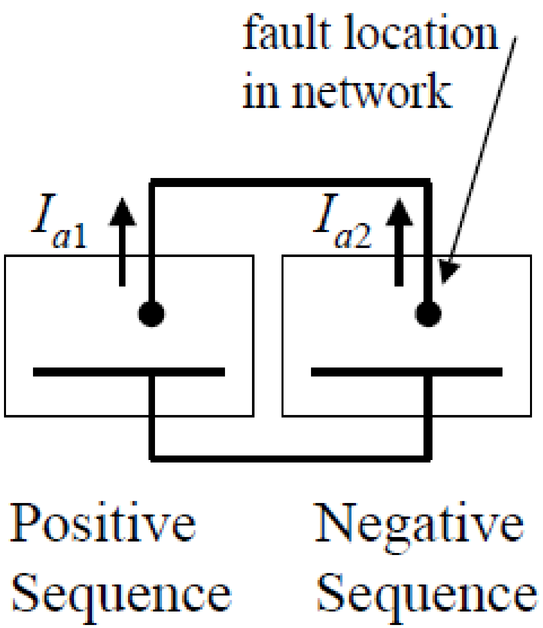
$$I_{a0} = 0$$

$$I_{a1} = \tfrac{1}{3}\left(a - a^2\right) I_b$$

$$I_{a2} = \tfrac{1}{3}\left(a^2 - a\right) I_b$$

$$I_{a1} = -I_{a2}$$

Network Diagram



Line-to-Line Fault

$$V_b - V_c = \left(a^2 - a\right)\left(V_k^{a1} - V_k^{a2}\right) = 0$$

$$V_k^{a1} = V_a - Z_k^1 I_{a1}$$

$$V_k^{a2} = -Z_k^2 I_k^{a2} = Z_k^2 I_k^{a1}$$

$$\left(a^2 - a\right)\left[V_a - \left(Z_k^1 + Z_k^2\right) I_k^{a1}\right] = 0$$

$$V_k^a - \left(Z_k^1 + Z_k^2\right) I_k^{a1} = 0$$

$$I_k^{a1} = \frac{V_{k,pre-f}^a}{Z_k^1 + Z_k^2}$$

Line-to-Line Fault

$$I_k^{a1} = -I_k^{a2}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_k^{a1} \\ -I_k^{a1} \end{bmatrix}$$

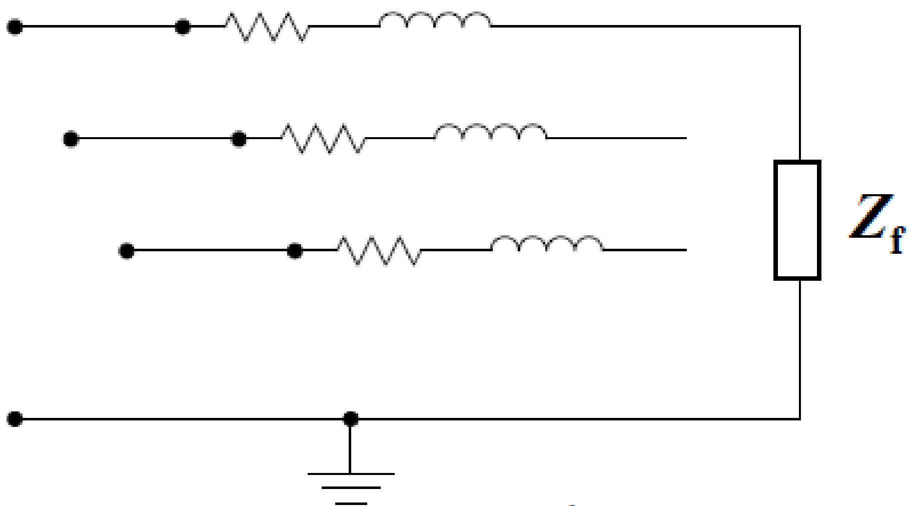
$$I_b = \left(a^2 - a\right) I_k^{a1} = -j\sqrt{3}\, I_k^{a1}$$

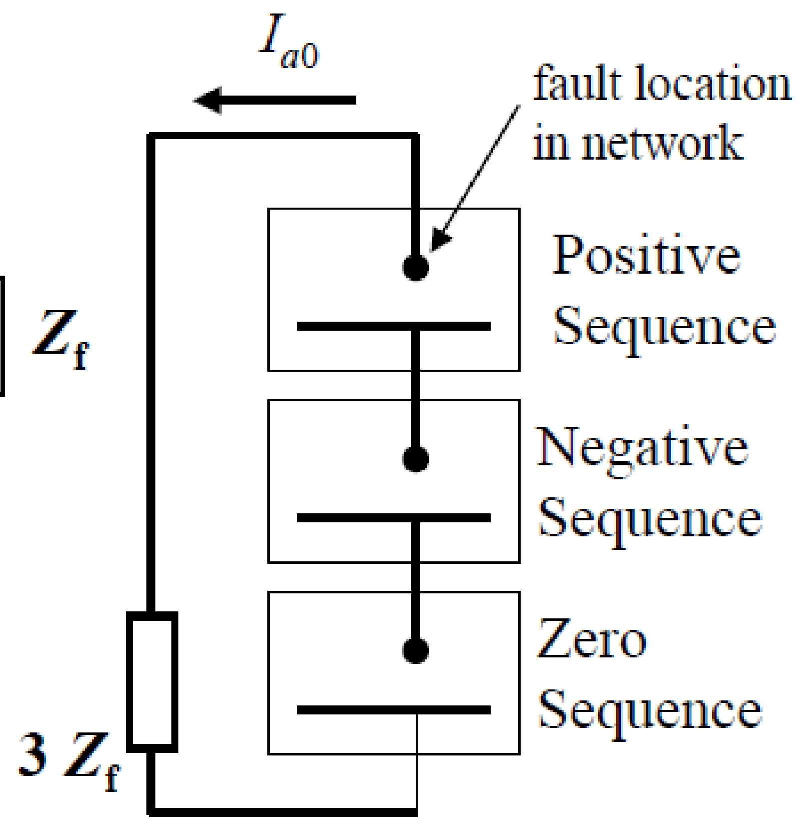
$$I_b = \frac{-j\sqrt{3}\, V_k^a}{Z_k^1 + Z_k^2}$$

$$I_c = -I_b$$

Fault Analysis with Fault Impedances

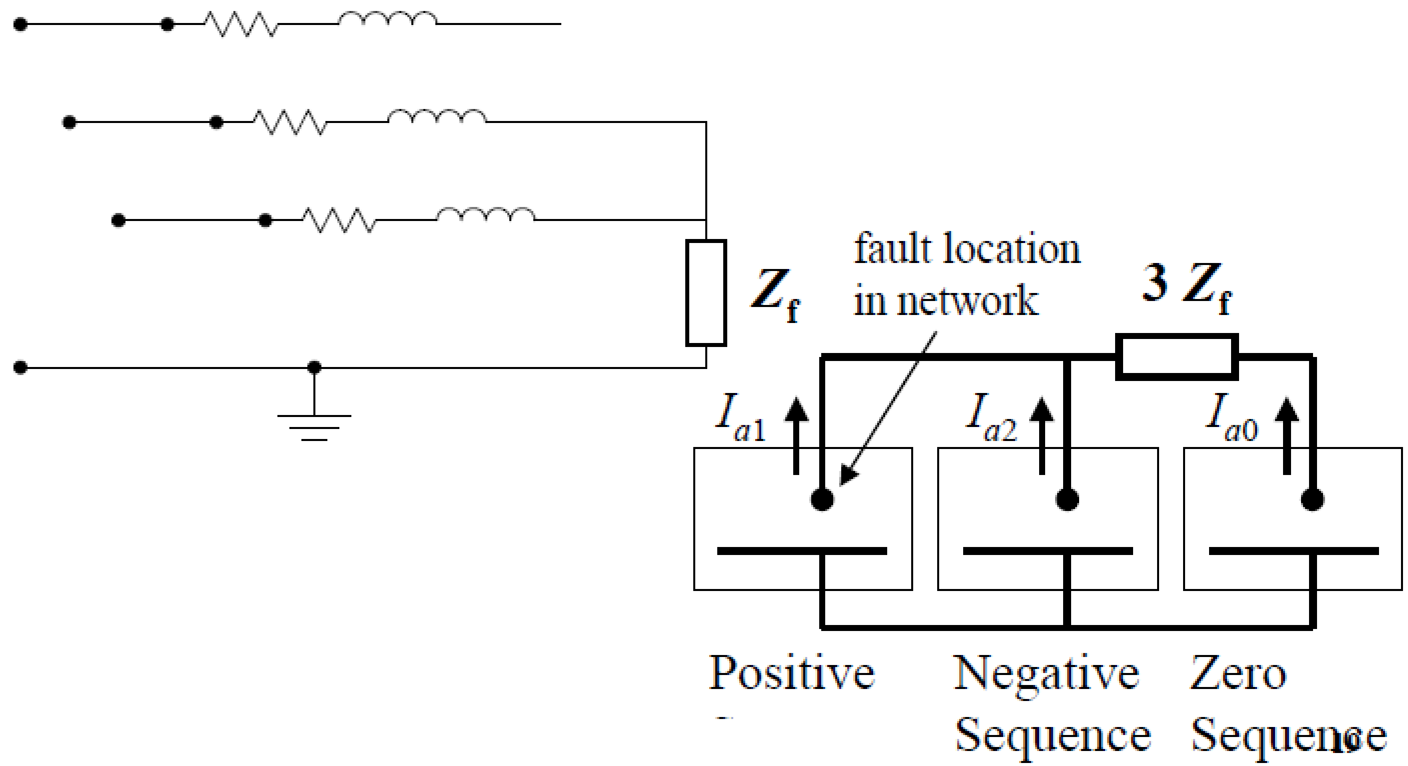
- **Single Line to Ground Fault**





$$I_k^{a0} = I_k^{a1} = I_k^{a2} = \frac{V_{k,pre-f}^{a1}}{Z_k^0 + Z_k^1 + Z_k^2 + 3Z_f}$$

- **Double Line to Ground Fault**



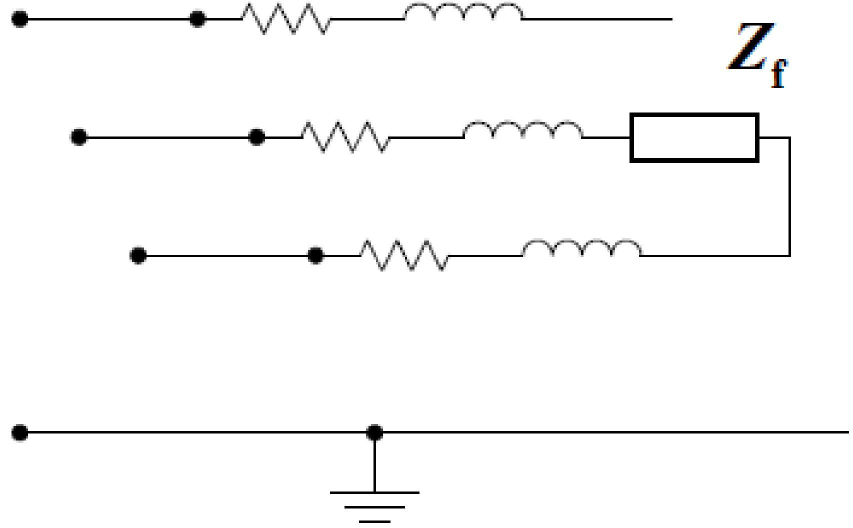
- **Double Line to Ground Fault**

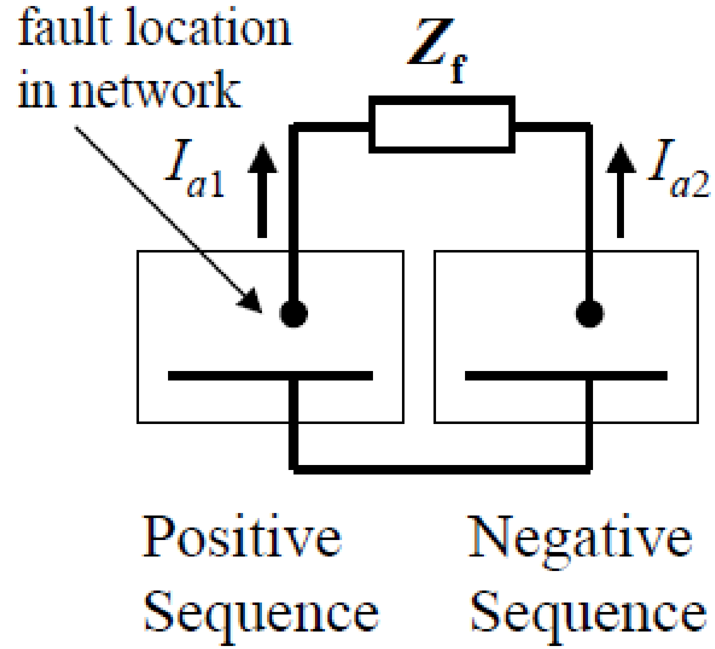
$$I_k^{a1} = \frac{V_k^{a1}}{Z_k^1 + \left(Z_k^0 + 3Z_f\right) \cdot Z_k^2 / \left(Z_k^0 + 3Z_f + Z_k^2\right)}$$

$$I_k^{a2} = -\frac{V_k^{a1} - Z_k^1 I_k^{a1}}{Z_k^2}$$

$$I_k^{a0} = -\frac{V_k^{a1} - Z_k^1 I_k^{a1}}{Z_k^0 + 3Z_f}$$

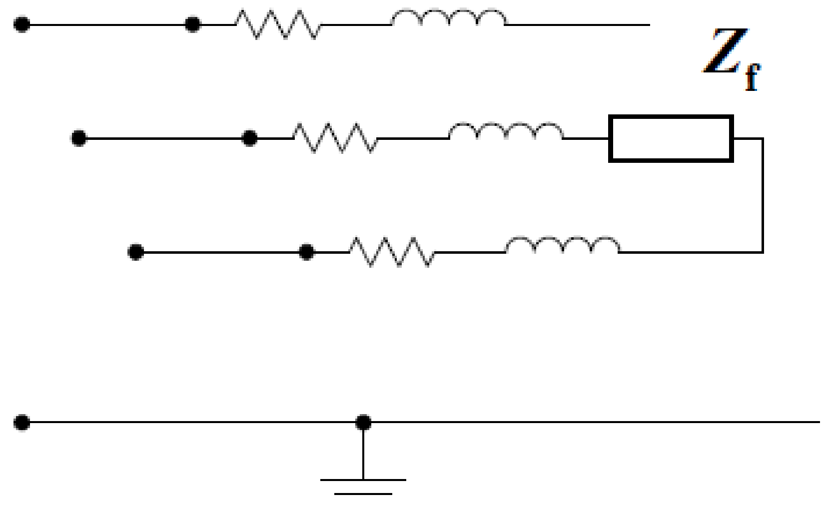
- **Line to Line Fault**

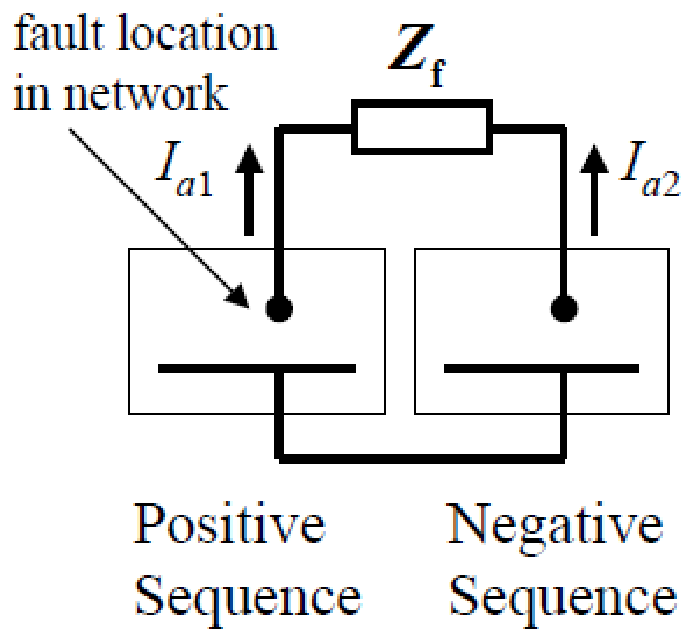




$$I_k^{a1} = -I_k^{a2} = \frac{V_k^a}{Z_k^1 + Z_k^2 + Z_f}$$

- **Line to Line Fault**





$$I_k^{a1} = -I_k^{a2} = \frac{V_k^a}{Z_k^1 + Z_k^2 + Z_f}$$

Fault Analysis using Z-Bus Matrix

- **Single Line to Ground Fault**

$$I_k^{a0} = I_k^{a1} = I_k^{a2} = \frac{V_{k,pre-f}^{a1}}{\mathbf{Z}_{kk}^0 + 2\cdot\mathbf{Z}_{kk}^1 + Z_f}$$

- **Line to Line Fault**

$$I_k^{a1} = -I_k^{a2} = \frac{V_k^a}{2\cdot\mathbf{Z}_{kk}^1 + Z_f}$$

- **Double Line to Ground Fault**

$$I_k^{a1} = \frac{V_k^{a1}}{\mathbf{Z}_{kk}^1 + \left(\mathbf{Z}_{kk}^0 + 3Z_f\right)\cdot\mathbf{Z}_{kk}^1 \Big/ \left(\mathbf{Z}_{kk}^0 + 3Z_f + \mathbf{Z}_{kk}^1\right)}$$

$$I_k^{a2} = -\frac{V_k^{a1} - \mathbf{Z}_{kk}^1 I_k^{a1}}{\mathbf{Z}_{kk}^1}$$

$$I_k^{a0} = -\frac{V_k^{a1} - \mathbf{Z}_{kk}^1 I_k^{a1}}{\mathbf{Z}_{kk}^0 + 3Z_f}$$

- **Bus voltages during fault**

$$V_i^{a0} = 0 - Z_{ik}^0 I_k^{a0}$$

$$V_i^{a1} = V_i^a - Z_{ik}^1 I_i^{a1}$$

$$V_i^{a2} = 0 - Z_{ik}^2 I_k^{a2}$$

- **Line currents during fault**

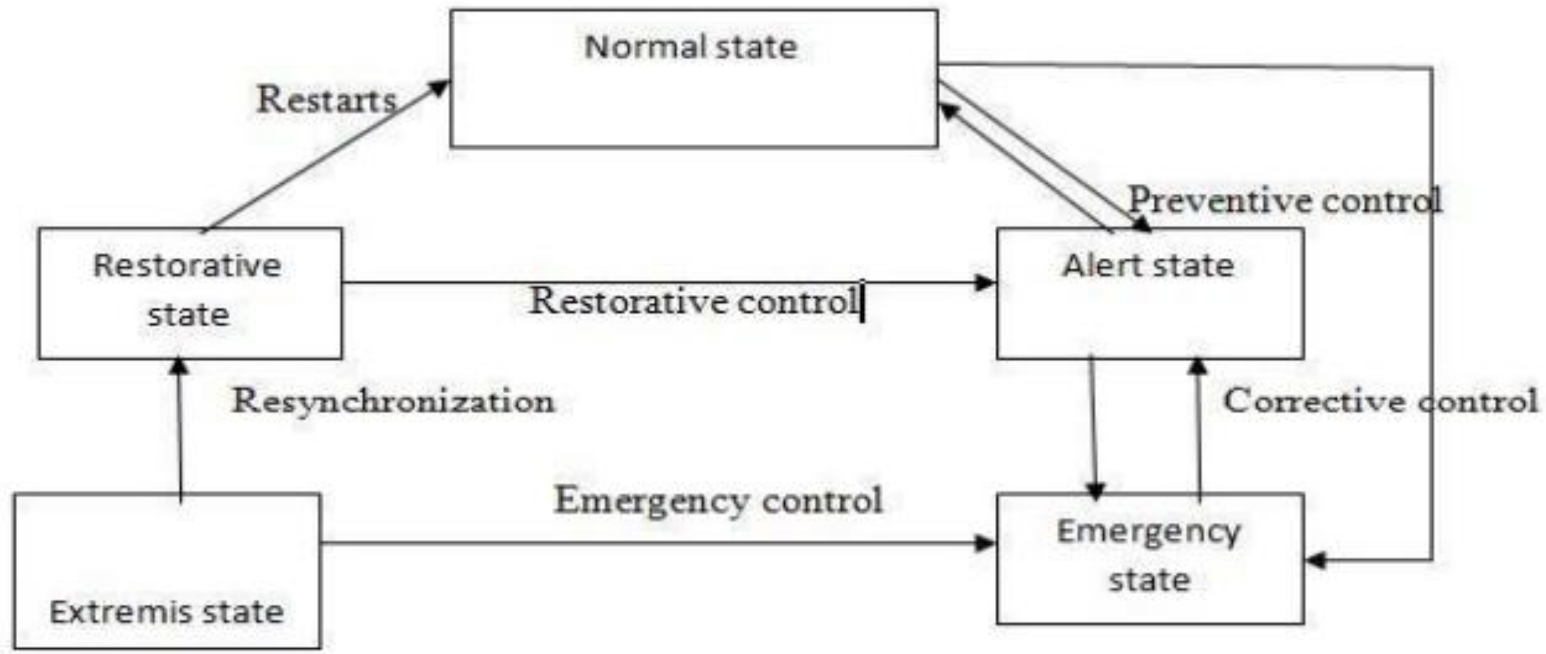
$$I_{ij}^{a0} = \left(V_i^{a0} - V_j^{a0}\right) / z_{ij}^0$$

$$I_{ij}^{a1} = \left(V_i^{a1} - V_j^{a1}\right) / z_{ij}^1$$

$$I_{ij}^{a2} = \left(V_i^{a2} - V_j^{a2}\right) / z_{ij}^2$$

UNIT-IV
CONTINGENCY ANALYSIS

Operating states of a power system



Operating states

Normal state:

A system is said to be in normal if both load and operating constraints are satisfied .It is one in which the total demand on the system is met by satisfying all the operating constraints.

Alert state:

- **A normal state of the system said to be in alert state if one or more of the postulated contingency states, consists of the constraint limits violated.**

Restorative state:

- From this state, the system may be brought back either to alert state or secure state .The latter is a slow process.
- Hence, in certain cases, first the system is brought back to alert state and then to the secure state .
- This is done using restorative control action.

Need for voltage regulation in power system

The transmission line and the distribution line need voltage control at various stages to maintain the voltage at the last consumers premises within permissible limits.

 Variations in supply voltage are determent in various aspects.

 Below normal voltage substantially reduced the light output from the incandescent lamps, above normal voltage reduces the life of the lamps.

 Motor operated at below normal voltage draw abnormal high currents and may overheat , even when carrying no more than the rated horse power load.

 If the voltage of the system deviates from the nominal value , the performance of the device suffers and its life expectance drop.

Need for frequency regulation in power system

In any power system, if the frequency changes there won't be required receiving end voltage. if we connected tow systems in parallel, it will spoil the system.

 Most of the AC motor runs at a speed that is directly related to the frequency.

 The overall operation of a power system can be much better controlled if the frequency error is kept within strict limit.

 A large number of electrical driven clocks are used they all driven by synchronous motor and the accuracy of these clocks is a function not only of a frequency error ,but actually of the integral of this error.

 When tow system working at different frequencies are to be tied together to make same frequency . frequency converting station or links are required.

What is power system security?

Power system security may be looked upon as the probability of the system's operating point remaining within acceptable ranges, given the probabilities of changes in the system (contingencies) and its environment.

Major components of security assessment

System monitoring

The prerequisite for security assessment of a power system is the knowledge of the system states. Monitoring the system is therefore the first step. I Measurement devices dispersed throughout the system help in getting a picture of the current operating state. The measurements can be in the form of power injections, power flows, voltage, current, status of circuit breakers, switches, transformer taps, generator output etc., which are telemetered to the control centre. I Usually a state estimator is used in the control centre to process these telemetered data and compute the best estimates of the system states. I Remote control of the circuit breakers, disconnector switches, transformer taps etc. is generally possible. The entire measurement and control system is commonly known as supervisory control and data acquisition (SCADA) system

Contingency analysis

Once the current operating state is known, the next task is the contingency analysis. Results of contingency analysis allow the system to be operated defensively. Major components of contingency analysis are: I Contingency definition I Contingency selection I Contingency evaluation.

Contingency definition involves preparing a list of probable contingencies. I Contingency selection process consists of selecting the set of most probable contingencies in preferred; they need to be evaluated in terms of potential risk to the system.
Usually, fast power flow solution techniques such as DC power flow are used to quickly evaluate the risks associated with each contingency.

On-line security assessment

In earlier days, security assessment in a power system was mainly offline in nature. Predefined set of rules or nomographs were used to assist the operators in the decision-making process. I However, due to the highly interconnected nature of modern power systems, and deregulated energy market scenarios, operating conditions and even the topology of a power system changes frequently. Off-line techniques for security assessment are therefore no- longer reliable in modern power systems. I On-line security assessment techniques use near-real-time measurements from different locations in a power system, and continuously update the security assessment of the system.

DC power flow

Active power flow between buses i and j is given by,

$$P_{ij} = \frac{V_i V_j}{X_{ij}} \sin(\theta_i - \theta_j) \tag{1}$$

Assuming $V_i = V_j = 1$ p.u., and $\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$, (since $(\theta_i - \theta_j)$ is usually very small),

$$P_{ij} \approx \frac{(\theta_i - \theta_j)}{X_{ij}} \tag{2}$$

Power injection at bus i therefore is given by,

$$P_i = \sum_{j=1}^{N} P_{ij} = \sum_{j=1}^{N} \frac{(\theta_i - \theta_j)}{X_{ij}} \tag{3}$$

Hence,

$$\frac{\partial P_i}{\partial \theta_i} = \sum_{j=1}^{N} \frac{1}{x_{ij}} \tag{4}$$

$$\frac{\partial P_i}{\partial \theta_j} = -\frac{1}{x_{ij}} \tag{5}$$

Now,

$$\Delta P_i = \frac{\partial P_i}{\partial \theta_1}\Delta\theta_1 + ... + \frac{\partial P_i}{\partial \theta_N}\Delta\theta_N \tag{6}$$

Iterative equations for power flow then become:

$$\begin{bmatrix} \Delta P_1 \\ . \\ . \\ . \\ \Delta P_N \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & . & . & . & B_{1N} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ B_{N1} & B_{N2} & . & . & . & B_{NN} \end{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ . \\ . \\ . \\ \Delta\theta_N \end{bmatrix} \tag{7}$$

Or,

$$\Delta P = [B][\Delta\theta] \tag{8}$$

Linear sensitivity factors

Analyzing in details a large number of contingencies is a difficult task.

I An easy (approximate) way to quickly compute any possible violation of operating limits is the one of linear sensitivity factors.

I Two such sensitivity factors for checking line flow violations are generation shift factors and line outage distribution factors.

Generation shift factors

They calculate the effect of change in generation on the line flows, as shown below:

$$a_{li} = \frac{\Delta f_l}{\Delta P_i} \tag{9}$$

where a_{li} is the linearized generation shift factor for the lth line for a change in output of ith generator; Δf_l is the MW change in power flow in the lth line; ΔP_i is the change in generation at the ith bus.

It is assumed here that the change in generation at the ith bus is picked up by the reference bus. The new values of power flows in each line can be found from:

$$f_l^{new} = f_l^{old} + a_{li}\Delta P_i; \ \forall l = 1, 2, \ldots, L \qquad (10)$$

where f_l^{old} is the power flow in the lth line before the ith generator went out. Assuming P_i^{old} to be the output of the ith generator before fault, above equation can be expressed as,

$$f_l^{new} = f_l^{old} - a_{li}P_i^{old}; \ \forall l = 1, 2, \ldots, L; [\because \Delta P_i = -P_i^{old}]. \qquad (11)$$

•Once the new values of flows are computed for all the lines, they are compared with corresponding line flow limits. Operators are ‘alarmed’ in case of any limit violations.

• In a practical power system, due to governor actions, the loss of generation at the bus i may be compensated by their generators throughout the system.

•A frequently used method is to assume that the loss of generation is distributed among participating generators in proportion to their maximum MW rating.

Therefore, the proportion of generation pickup by the jth generator is given by,

$$\gamma_{ji} = \frac{P_j^{\max}}{\sum_{\substack{k=1 \\ k \neq i}}^{N_G} P_k^{\max}} \tag{12}$$

where P_k^{max} is the maximum MW rating of the kth generator; N_G is the number of participating generators; γ_{ji} is the proportionality factor for pickup on generating unit j when unit i fails.

The new line flows are then given by,

$$f_l^{new} = f_l^{old} + a_{li}\Delta P_i - \sum_{\substack{j=1 \\ \neq i}}^{N_G} (a_{li}\gamma_{ji}\Delta P_i) \tag{13}$$

Line outage distribution factor

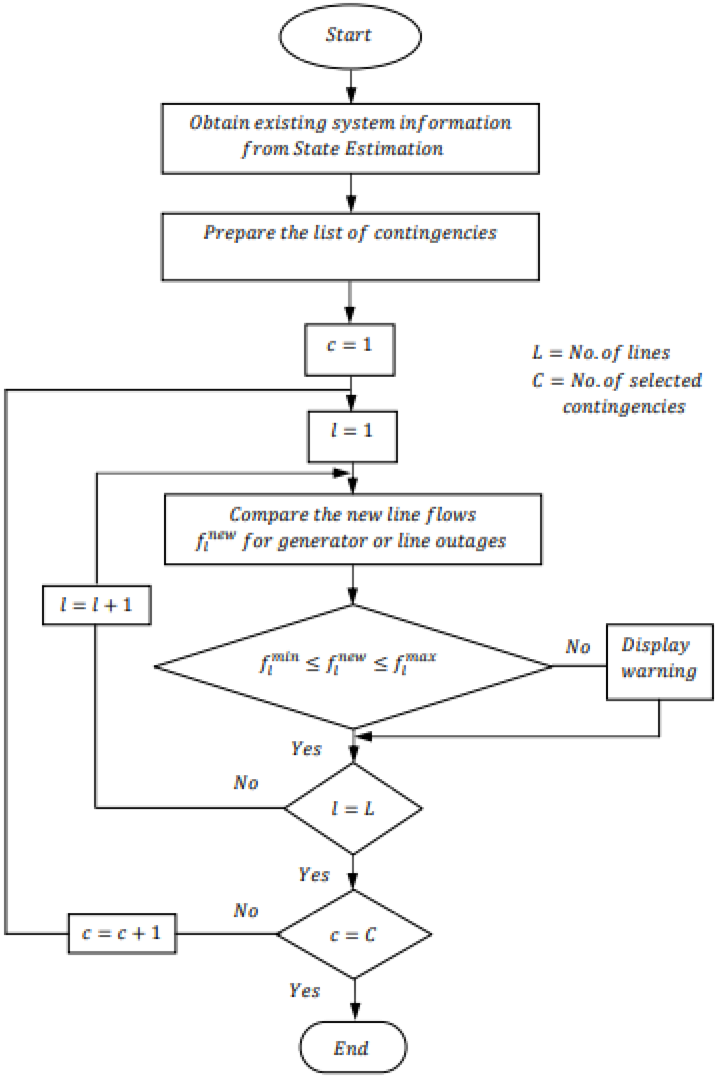
Line outage distribution factors are used to quickly check line overloading as a result of outage of any transmission line, and are defined as follows:

$$d_{lk} = \frac{\Delta f_l}{f_k^{old}} \tag{14}$$

where d_{lk} is the line outage distribution factor for line l after an outage of line k; Δf_l is the change in MW flow in line l due to the outage of line k; f_k^{old} is the flow in line k before its outage. The new value of line flow is given by,

$$f_l^{new} = f_l^{old} + d_{lk} f_k^{old} \tag{15}$$

Algorithm for contingency analysis



Z MATRIX METHOD

- Methods include inverting Y bus matrix and injecting a fictitious current into the bus
- Converting the MVA loads to impedance loads using

$$Z_{load\ i} = \frac{|V_i|^2}{S_i^*}$$

- Injecting a unit current, into the bus p which has to be removed

$$\begin{bmatrix} V_1 \\ . \\ . \\ . \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{1P} \\ . \\ . \\ . \\ Z_{NP} \end{bmatrix}$$

- I_{pq} can be calculated using the equation

$$I_{pq} = \frac{V_p - V_q}{Z_{line\,pq}}$$

- An adjustment parameter, *d*, has to be used

$$d \triangleq \frac{I_{pq}^{\;s}}{I_{pq}}$$

- Due to the injection $I_p = d$, the new current in other elements

$$I_{pq} = \frac{d(Z_{ip} - Z_{jp})}{Z_{line\;ij}}$$

where ij is not equal to mn

- The sought-after current flow changes due to removing line *pq* are

$$\Delta I_{ij} = \tilde{I}_{ij} - I_{ij} \qquad \text{for all } ij$$

- Calculating the current flow pattern in the modified network, in which line *pq* has been removed, requires only that we inject current $I_p = d$, as before, into the modified network.
- The voltages are

$$\begin{bmatrix} \tilde{V}_i \\ \cdot \\ \cdot \\ \cdot \\ \tilde{V}_N \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{11} & \tilde{Z}_{12} & \cdot & \cdot & \cdot & \tilde{Z}_{1N} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \tilde{Z}_{N1} & \tilde{Z}_{N2} & \cdot & \cdot & \cdot & \tilde{Z}_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \cdot \\ d \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} \tilde{Z}_{1p^d} \\ \cdot \\ \cdot \\ \cdot \\ \tilde{Z}_{Np^d} \end{bmatrix}$$

POST CONTINGENCY EQUILLIBRIUM

Methods to compute the equilibrium condition immediately following a disturbance to an electric power system

- Analysis by Integration
- Analysis by Simultaneous Iteration
- Analysis by partitioned iteration

UNIT-V
State Estimation

State Estimation Functions

Topology processor:

Creates one-line diagram of the system using the detailed circuit breaker status information.

Observability analysis:

Checks to make sure that state estimation can be performed with the available set of measurements.

State estimation:

Estimates the system state based on the available measurements.

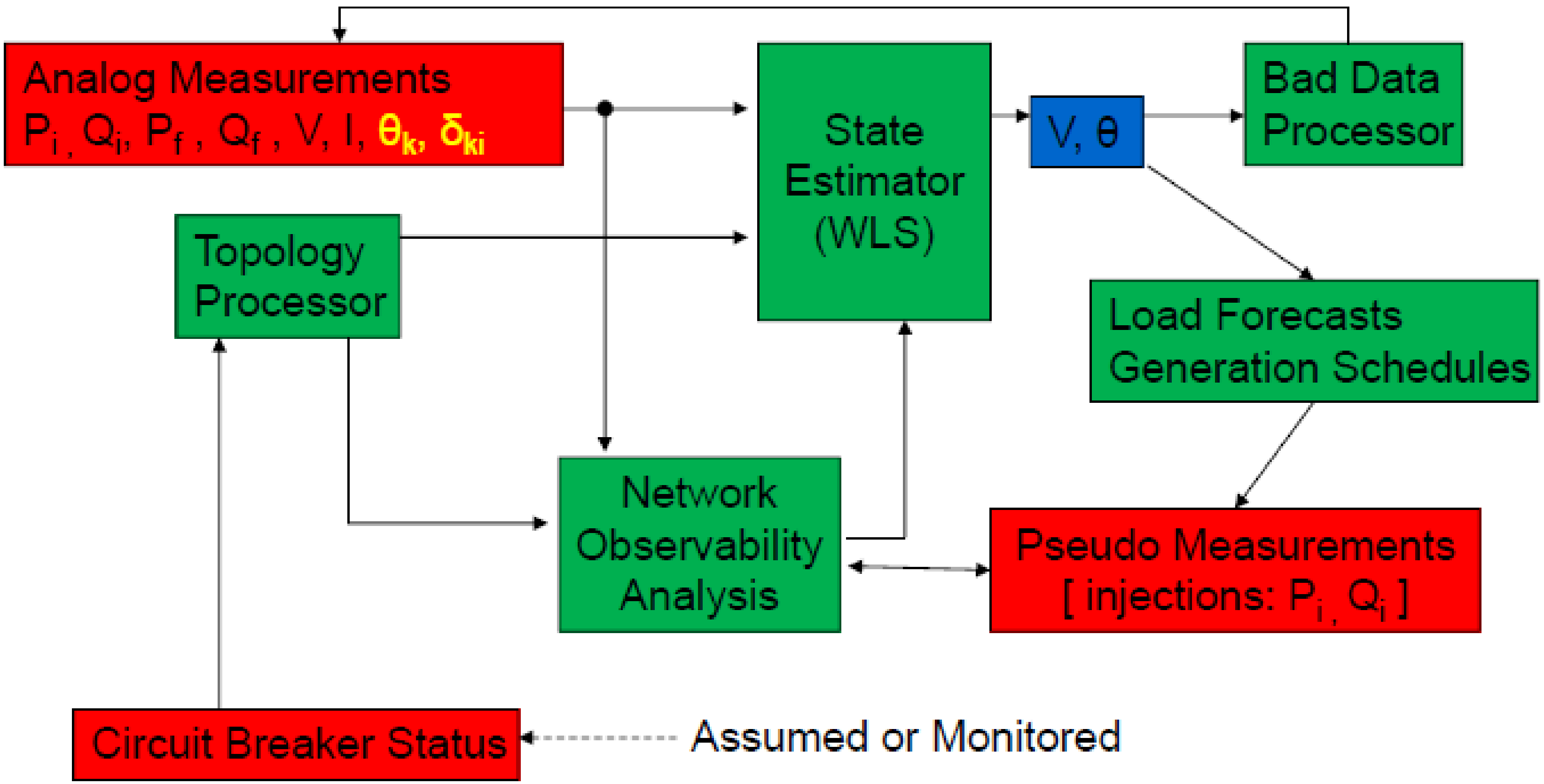
Bad data processing:

Checks for bad measurements. If detected, identifies and eliminates bad data.

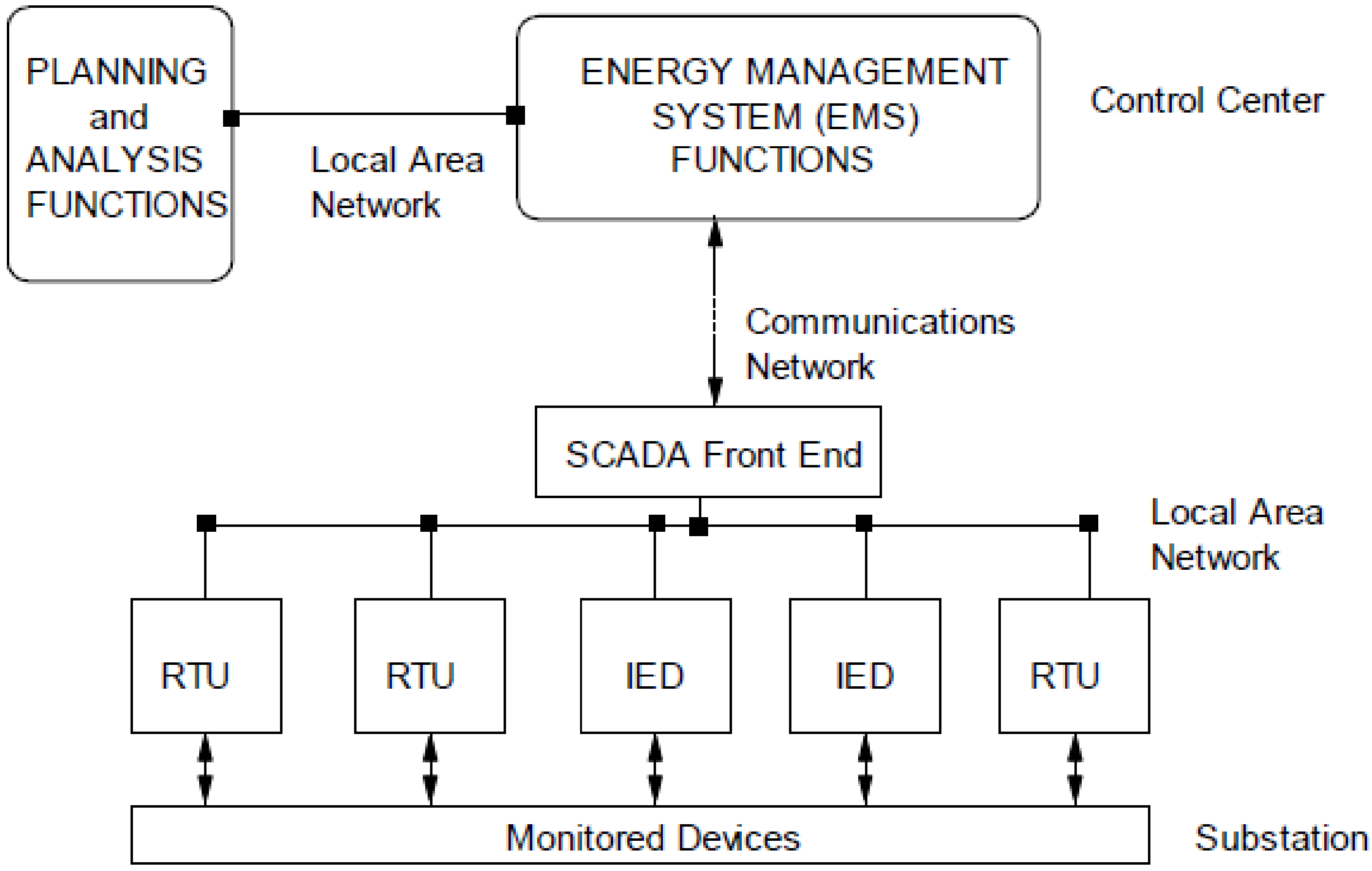
Parameter and structural error processing:

Estimates unknown network parameters, checks for errors in circuit breaker status.

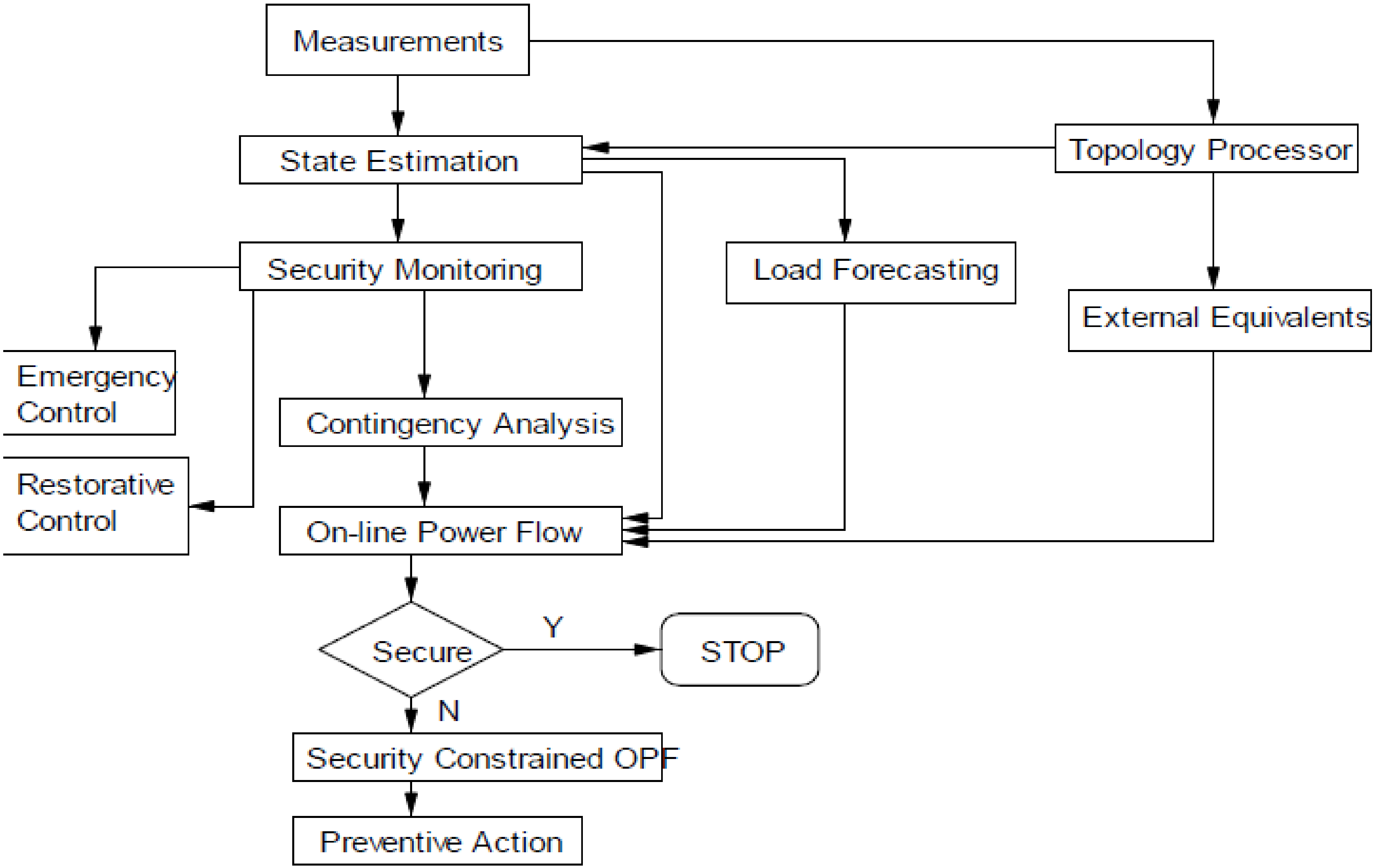
State Estimation and Related Functions



Communication Infrastructure



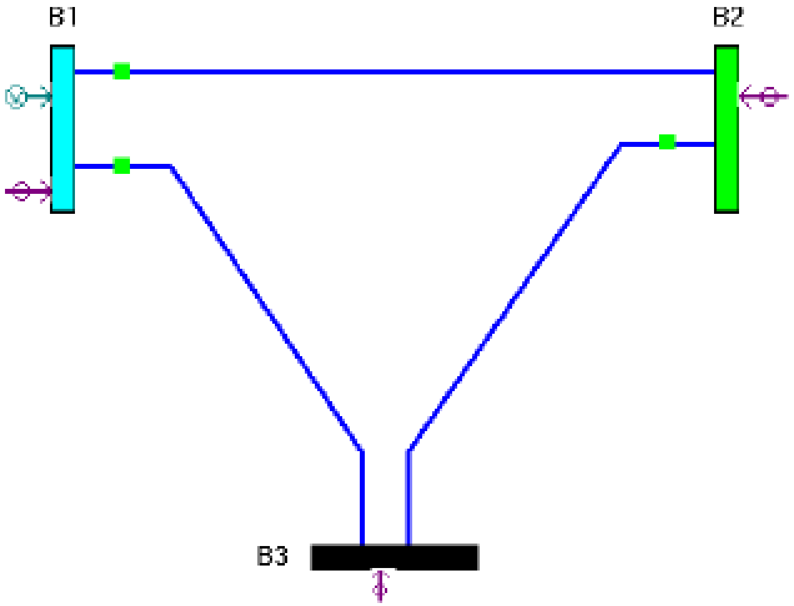
Energy Management System Applications



Problem Statement

- [z] : Measurements
  P-Q injections
  P-Q flows
  V magnitude, I magnitude

- [x] : States
  V, θ, Taps (parameters)

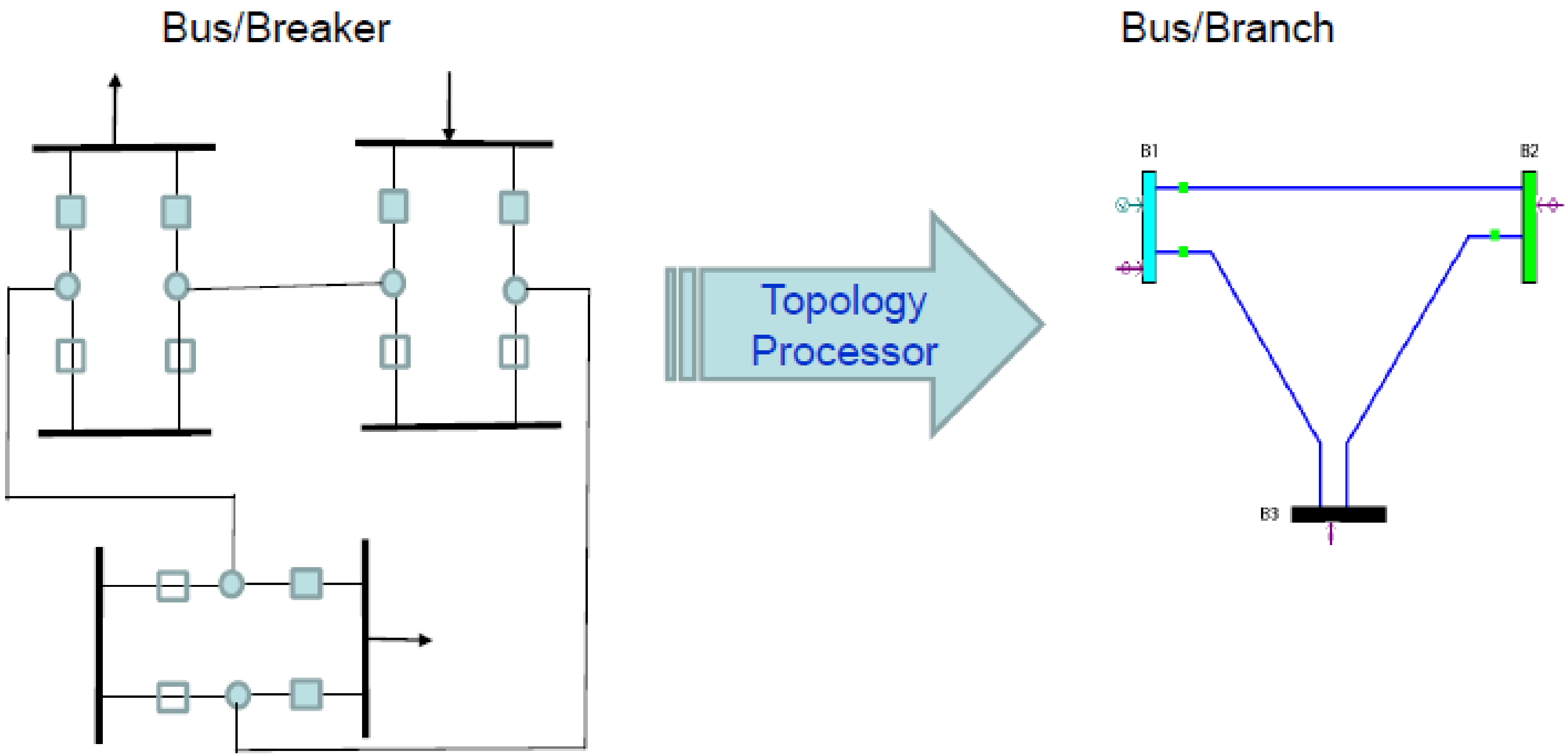


- **EXAMPLE:**

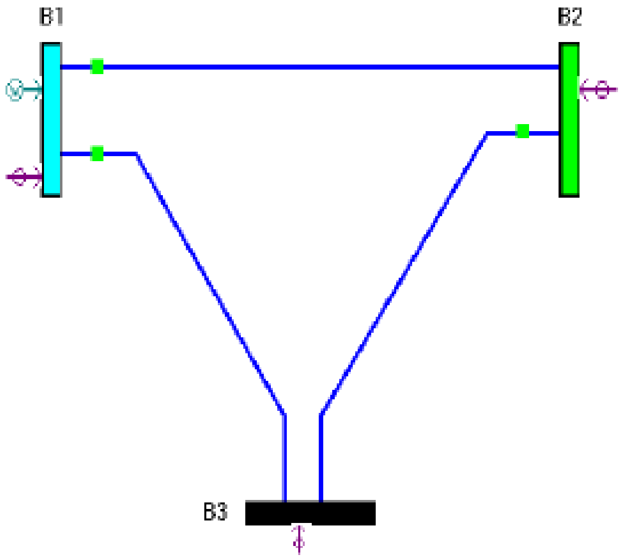
- [z] = [P12; P13; P23; P1; P2; P3; V1; Q12; Q13; Q23; Q1; Q2; Q3]
  m = 13 (no. of measurements)

- [x] = [V1; V2; V3; θ2; θ3]
  n = 5 (no. of states)

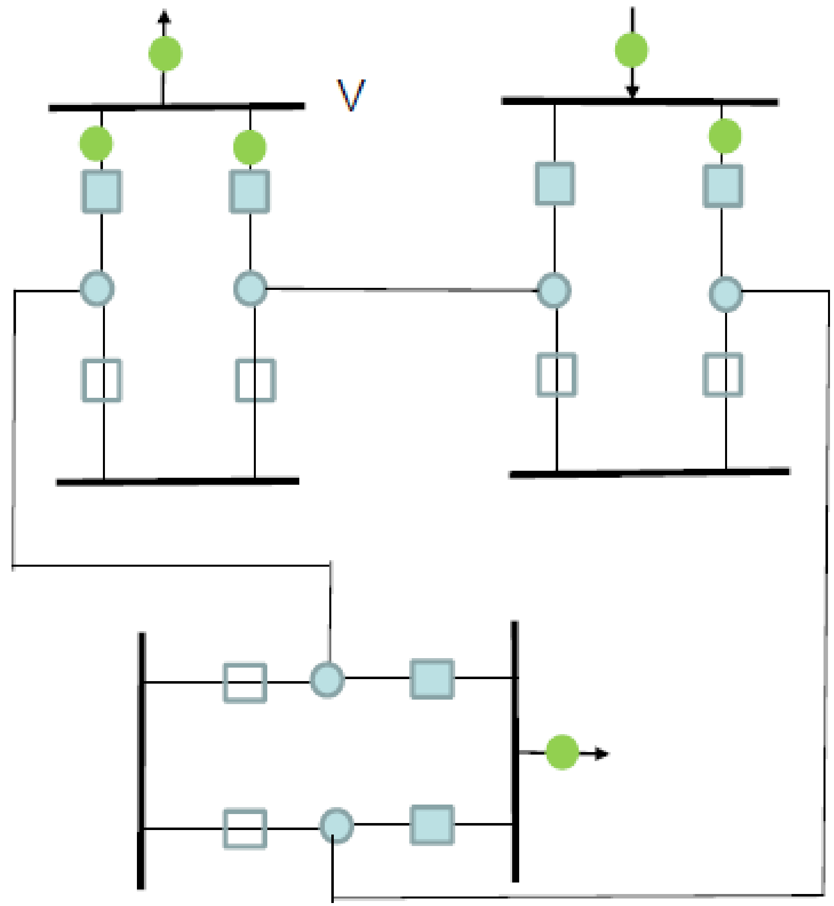
Bus/branch and bus/breaker Models



Bus/branch

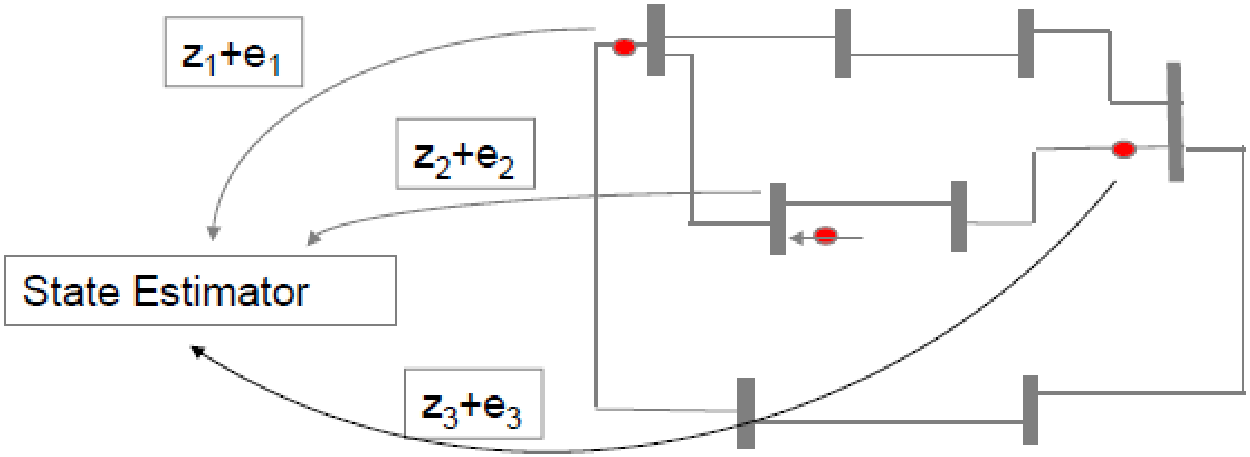


Bus/Breaker



Measurement Model

$[z_m] = [h([x])] + [e]$



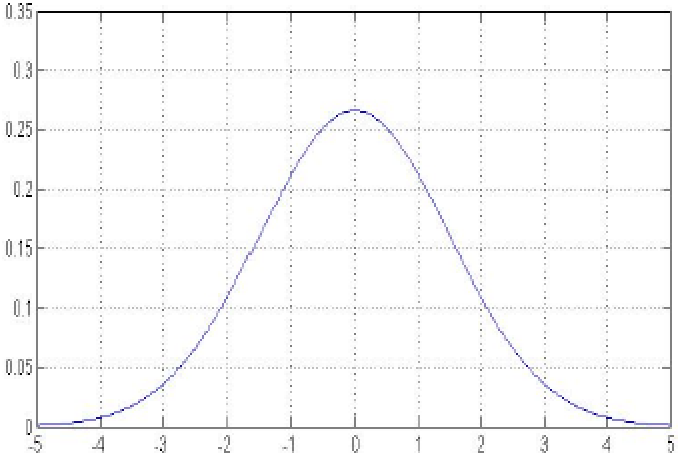
z_i : true measurement

e_i : measurement error

$e_i = e_s + e_r$

systematic random

Assumptions



- $e_i \sim N\ (\ 0,\ \sigma_i^2\ )$
- Holds true if:

  $e_s = 0,\ e_r \sim N\ (\ 0,\ \sigma_i^2\ )$

- If $e_s \neq 0$, then $E(e_i) \neq 0$,

  i.e. SE will be biased !

Maximum Likelihood Estimator (MLE)

Consider the random variables $X_1, X_2, \ldots, X_n$ with a p.d.f of $f(X \mid \theta)$, where θ is unknown.

The joint p.d.f of a set of random observations

$$x = \{ x_1, x_2, \ldots, x_n \}$$

will be expressed as:

$$f_n(x \mid \theta) = f(x_1 \mid \theta)\, f(x_2 \mid \theta) \ldots f(x_n \mid \theta)$$

This joint p.d.f is referred to as the ***Likelihood Function***.

The value of θ, which will maximize the function $fn(x \mid \theta)$ will be called the ***Maximum Likelihood Estimator (MLE)*** of θ.

Normal (Gaussian) Density Function, f(z)

$$f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\tfrac{1}{2}(\tfrac{z-\mu}{\sigma})^2\}$$

Likelihood Function, $f_m(z)$

$$f_m(z) = f_m(z_1) f_m(z_2) \cdots f_m(z_m)$$

Log-Likelihood Function, *L*

$$L = \log f_m(z) = \sum_{i=1}^{m} \log f(z_i)$$

$$= -\tfrac{1}{2} \sum_{i=1}^{m} (\tfrac{z_i - \mu_i}{\sigma_i})^2 - \tfrac{m}{2} \log 2\pi - \sum_{i=1}^{m} \log \sigma_i$$

Weighted Least Squares (WLS) Estimator

Given the set of observations $z_1, z_2, \ldots, z_n$ MLE will be the solution to the following:

$$\text{Maximize } f_m(z)$$

OR

$$\text{Minimize } \sum_{i=1}^{m} \left(\frac{z_i - \mu_i}{\sigma_i}\right)^2$$

Defining a new variable "r", measurement residual:

$$\text{Minimize } \sum_{i=1}^{m} W_{ii} r_i^{\,2} \qquad W_{ii} = \frac{1}{\sigma_i^2}$$

$$\text{Subject to } z_i = h_i(x) + r_i \quad i = 1, \ldots, m \qquad \mu_i = E(z_i) = h_i(x)$$

The solution of the above optimization problem is called the **weighted least squares (WLS)** estimator for **x**.

Linear case:

$$\text{Minimize} \sum_{i=1}^{m} W_{ii} r_i^{\,2}$$

$$\text{Subject to } [z] = [H] \cdot [x] + [r]$$

Solution is given by:

$$[\hat{x}] = [G^{-1}] \cdot [H^T] \cdot [W] \cdot [z]$$

$$[G] = [H^T] \cdot [W] \cdot [H]$$

$$W_{ii} = \frac{1}{\sigma_i^2} \qquad W = diag\{W_{ii}\}$$

Measurement Model

Given a set of measurements, [z]

and the correct network topology/parameters:

$$[z] = [h([x])] + [e]$$

Measurements:

Known !
They are measured
Contain errors

True System States:

Unknown !
Can be measured
or estimated

Measurement
Errors:

Unknown !
Can not be directly
measured
or computed

Following the state estimation, the estimated

state will be denoted by $[\hat{x}]$:

$$[z] = [h([\hat{x}])] + [r]$$

Measurements:
They are measured
Contain errors

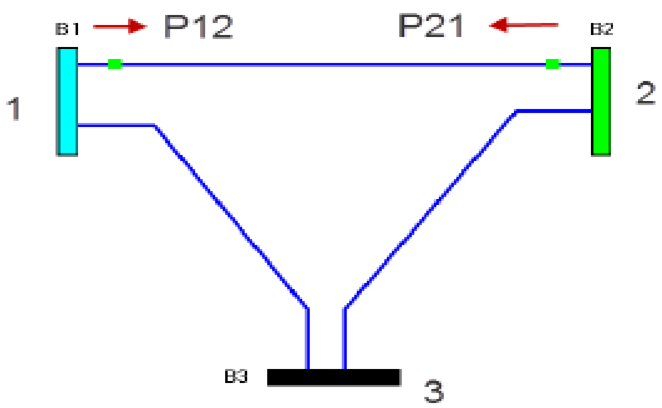
Estimated System
States

Measurement
Residuals:
Computed

Network Observability

Fully observable network:

A power system is said to be ***fully observable if voltage*** phasors at all system buses can be uniquely estimated using the available measurements.



$$Z_p = H \cdot \theta$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ H_{21} & \cdots & H_{2n} \\ H_{31} & \cdots & \vdots \\ \vdots & \cdots & \vdots \\ H_{m1} & \cdots & H_{mn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$\text{State Vector} = [\theta_2 \quad \theta_3]$$

$$\hat{\theta} = [G^{-1}] \cdot [H^T] \cdot [W] \cdot [Z_P]$$

Singular Matrix !
Cannot be inverted.

m ≥ n ➔ NECESSARY BUT "NOT" SUFFICIENT

EXAMPLE: m = 2, n = 2, UNOBSERVABLE SYSTEM

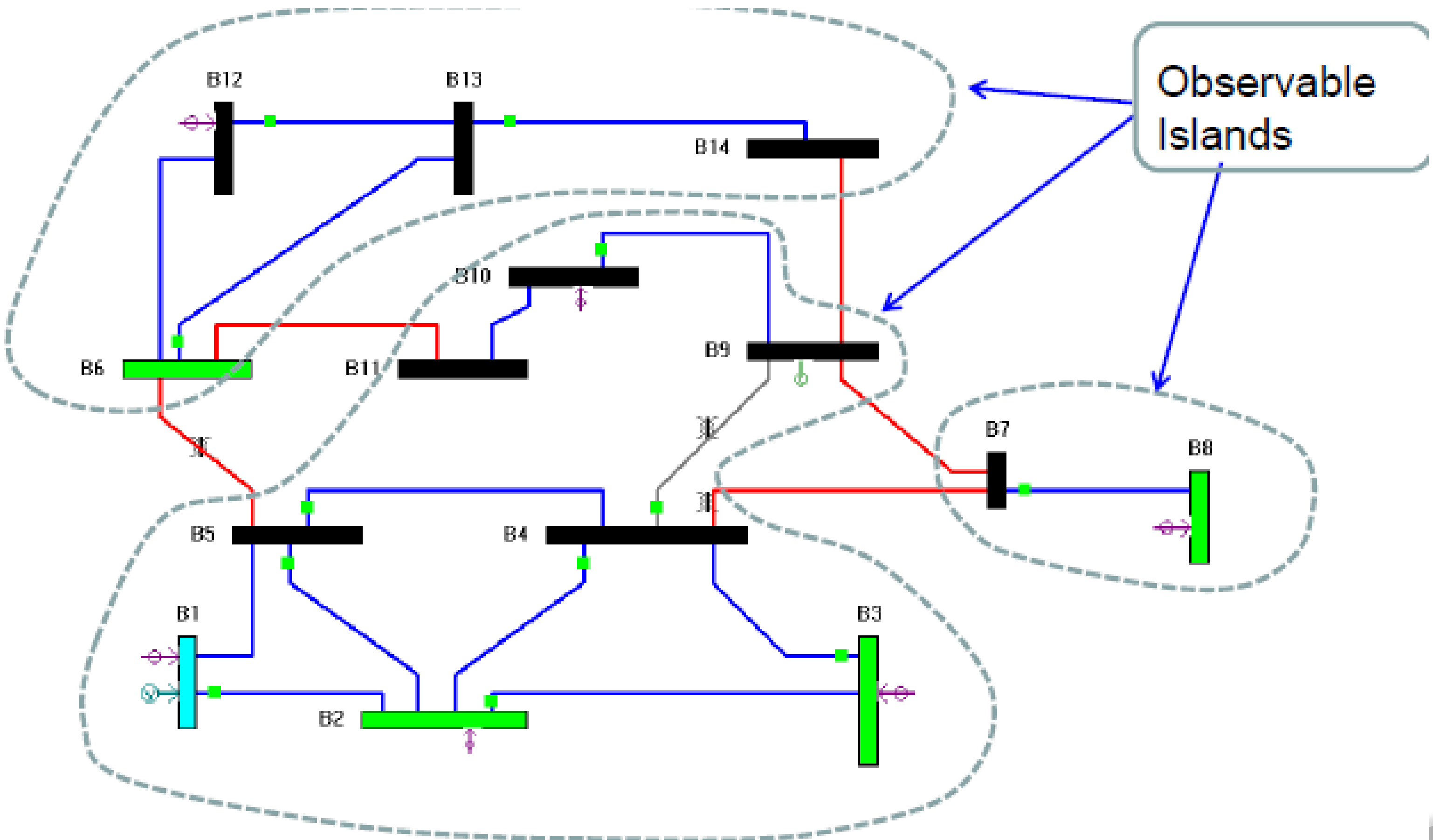
Network Observability

Unobservable branch:

- **If the system is found not to be observable, it will imply that there are unobservable branches whose power flows can not be determined.**

Observable island:

- **Unobservable branches connect observable islands of an unobservable system. State of each observable island can be estimated using any one of the buses in that island as the reference bus.**



RED LINES: Unobservable Branches

Bad Data Detection

Consider X_1, X_2, … X_N, a set of N independent random variables where:

$$X_i \sim N(0,1)$$

Then, a new random variable Y will have a χ^2 distribution with N degrees of freedom, i.e.:

$$\sum_{i=1}^{N} X_i^2 = Y \sim \chi_N^2$$

Now, consider the function

$$f(x) = \sum_{i=1}^{m} R_{ii}^{-1} e_i^2 = \sum_{i=1}^{m} \left(\frac{e_i^2}{R_{ii}} \right) = \sum_{i=1}^{m} \left(e_i^N \right)^2$$

and assuming:

$$e_i^N \sim N(0,1)$$

f(x) will have a χ^2 distribution with at most **(m-n)** degrees of freedom.

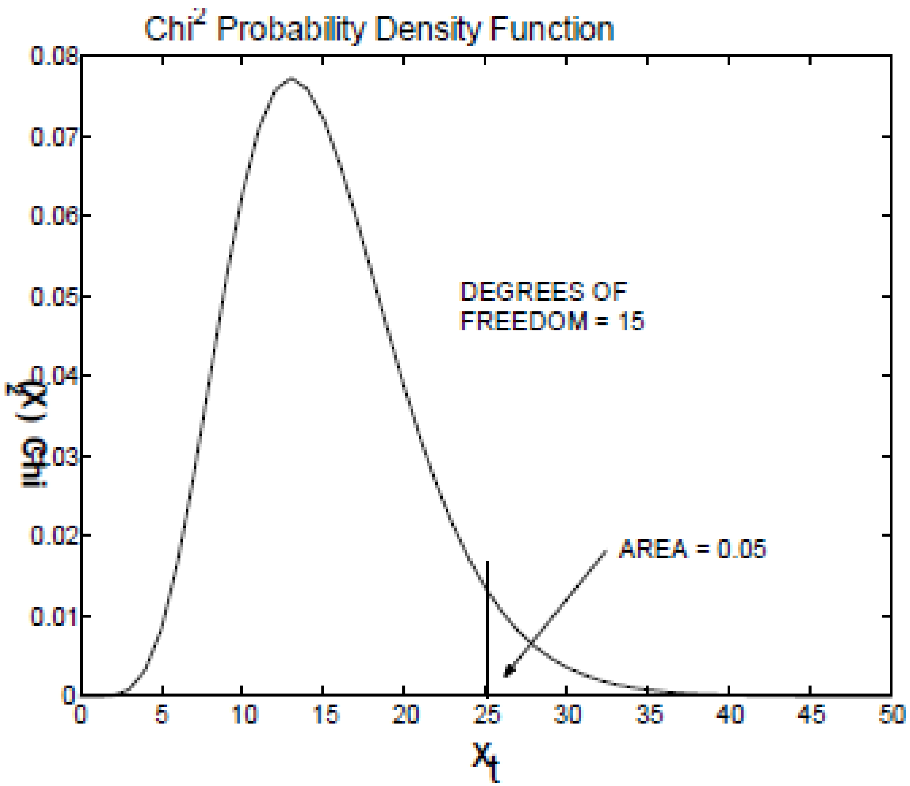
In a power system, since at least **n** measurements will have to satisfy the power balance equations, at most **(m-n)** of the measurement errors will be linearly independent.

Chi-squares Distribution:

$$\Pr\{X \geq x_t\} = \int_{x_t}^{\infty} \chi^2(u) \cdot du$$

Choose x_t such that:

$$\Pr\{X \geq x_t\} = \alpha = 0.05$$



Test:

If the measured $X \geq x_t$, then with 0.95 probability, bad data will be suspected.

Detection Algorithm

Solve the WLS estimation problem and compute the objective function:

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{\sigma_i^2}$$

Look up the value corresponding to ***p*** (e.g. 95 %) probability and ***(m-n)*** degrees of freedom, from the Chi-squares distribution table.

Let this value be $\chi^2_{(m-n),p}$ Here: $p = \Pr\{ J(x) \le \chi^2_{(m-n),p} \}$

Test if

$$J(x) \ge \chi^2_{(m-n),p}$$

If yes, then bad data are detected.

Else, the measurements are not suspected to contain bad data.

Properties of Measurement Residuals

Linear measurement model: $\Delta\hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z$

$$\Delta\hat{z} = H\Delta\hat{x} = K\Delta z, \qquad K = H(H^T R^{-1} H)^{-1} H^T R^{-1}$$

K *is called the hat matrix. Now, the measurement residuals can be expressed as follows:*

$$\begin{aligned} r &= \Delta z - \Delta\hat{z} \\ &= (I - K)\Delta z \\ &= (I - K)(H\Delta x + e) \\ &= (I - K)e \quad [\text{Note that KH} = \text{H}] \\ &= Se \end{aligned}$$

where ***S*** *is called the* ***residual sensitivity matrix.***

Distribution of Measurement Residuals

The residual covariance matrix Ω can be written as:

$$E[rr^T] = \Omega = S \cdot E[e \cdot e^T] \cdot S^T$$

$$= S \cdot R \cdot S^T = S \cdot R$$

Hence, the normalized value of the residual for measurement *i* will be given by:

$$r_i^N = \frac{r_i}{\sqrt{\Omega_{ii}}} = \frac{r_i}{\sqrt{R_{ii} S_{ii}}}$$

Classification of Measurements

Measurements can be classified as critical and redundant(or non-critical) with the following properties:

• A critical measurement is the one whose elimination from the measurement set will result in an unobservable system.

• The row/column of S corresponding to a critical measurement will be zero.

• The residuals of critical measurements will always be zero, and therefore errors in critical measurements can not be detected.

It can be shown that if there is a single bad data in the measurement set (provided that it is not a critical measurement) the largest normalized residual will correspond to bad datum.

Bad Data Identification / Elimination

Two commonly used approaches:

1. Post-processing of measurement residuals – Largest normalized residuals

2. Modifying measurement weights during iterative solution of WLS estimation

Largest Normalized Residual Test

Steps of the largest normalized residual test for identification of single and non-interacting multiple bad data:

Compute the elements of the measurement residual vector :

Compute the normalized residuals

Find k such that r_k^N is the largest among all $r_i^N, i = 1, \ldots, m$.

If r_k^N > c, then the k-th measurement will be suspected as bad data.

Else, stop, no bad data will be suspected. Here, c is a chosen identification threshold, e.g. 3.0.

Eliminate the k-th measurement from the measurement set and go to step 1.

Use of Synchrophasor Measurements

- Given enough phasor measurements, state estimation problem will become LINEAR, thus can be solved directly without iterations

$$Conventional\ Measurements$$

$$Z = h(X) + e$$

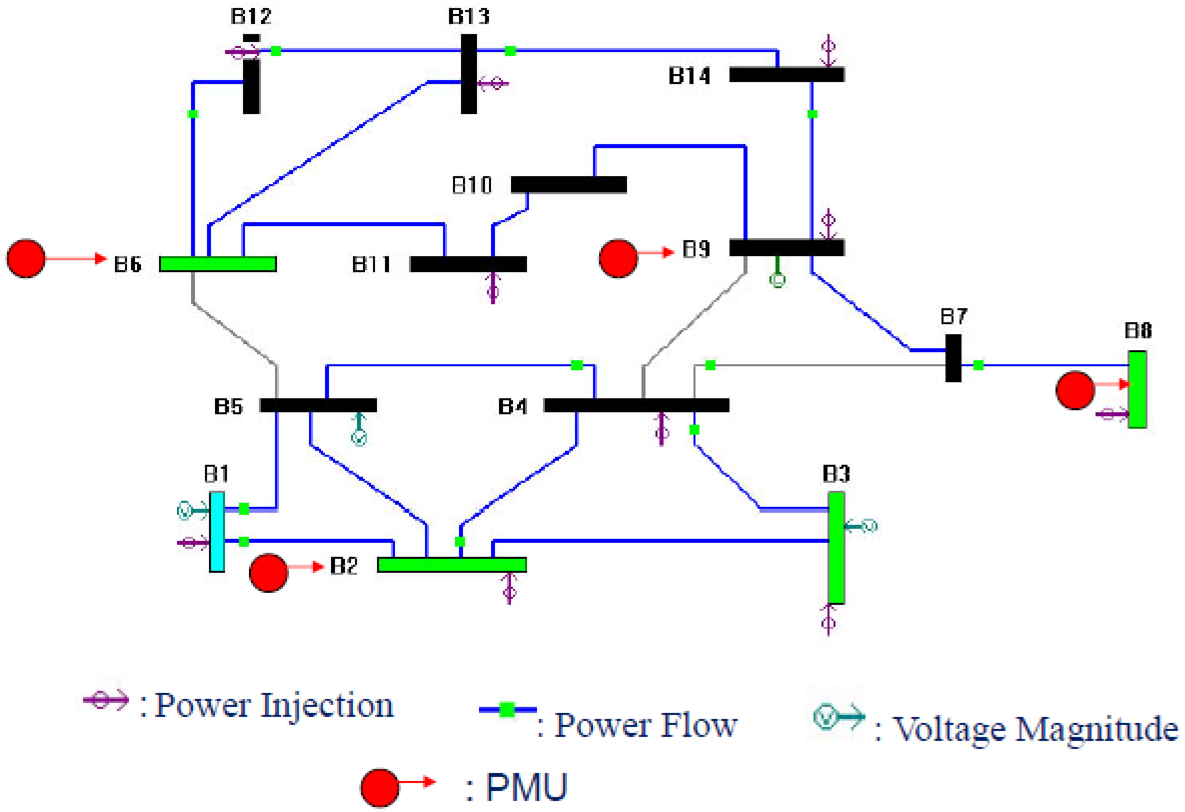
$$\Delta\hat{X} = (H^T R^{-1} H)^{-1} R^{-1} \Delta Z \quad Iterative$$

$$Phasor\ Measurements$$

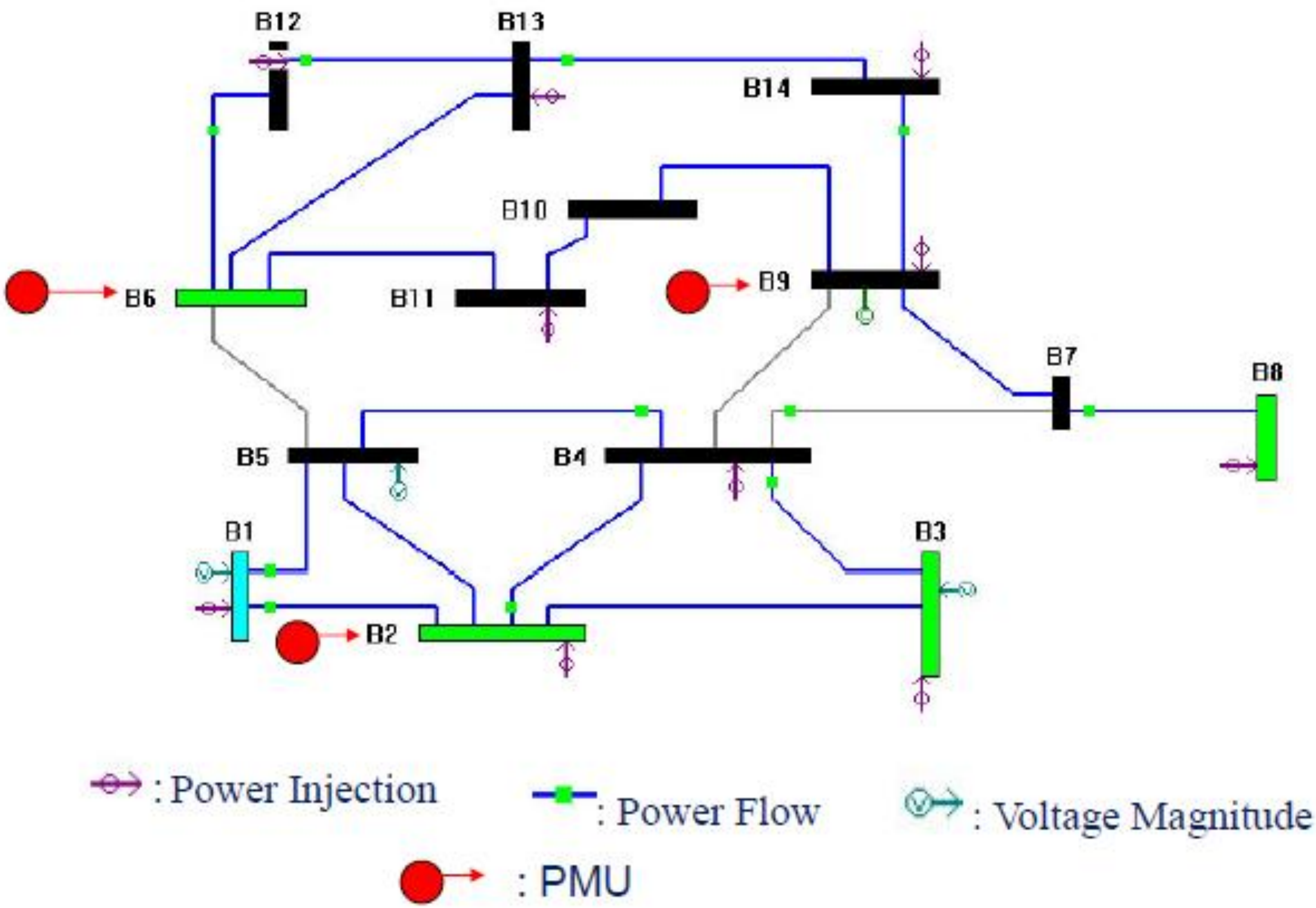
$$Z = H \cdot X + e$$

$$\hat{X} = (H^T R^{-1} H)^{-1} R^{-1} Z \quad Non-iterative$$

Placing PMUs:



Exploiting zero injections

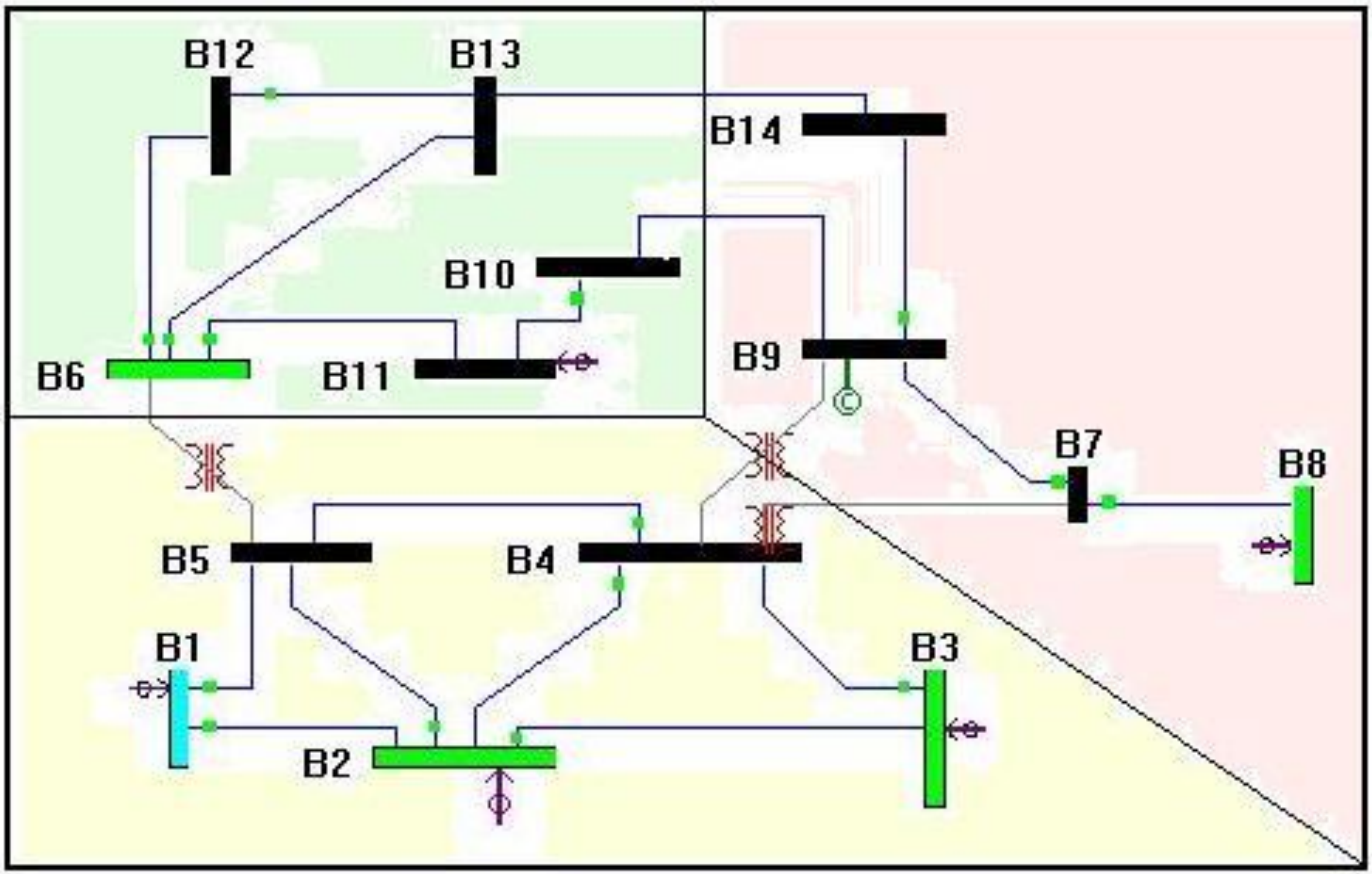


Use of Synchrophasor Measurements

- Given at least one phasor measurement, there will be no need to use a reference bus in the problem formulation
- Given unlimited number of available channels per PMU, it is sufficient to place PMUs at roughly 1/3rd of the system buses to make the entire system observable just by PMUs.

Systems	No. of zero injections	Number of PMUs	
		Ignoring zero Injections	Using zero injections
14-bus	1	4	3
57-bus	15	17	12
118-bus	10	32	29

Merging Observable Islands with PMUs



Performance Metrics

- State Estimation Solution
  - <u>Accuracy:</u>

    Variance of State = inverse of the gain matrix, $[G]^{-1}$

    $= E[\,(x - x^*)\,(x - x^*)'\,]$

  - <u>Convergence:</u>

    Condition Number = Ratio of the largest to smallest eigenvalue

    Large condition number implies an ill-conditioned problem.

- Measurement Quality

  - <u>Performance Index (WLS objective function):</u>

    Weighted sum of squares of residuals. Has a Chi-Squares distribution. Large numbers imply presence of bad data in the measurement set.

  - <u>Largest Absolute Normalized Residual:</u>

    If larger than 3.0, the measurement corresponding to the largest absolute value will be suspected of gross errors.

  - <u>Sample variance (Based on historical data):</u>

    Measurement weights are based on sample error variances calculated according to historical data and estimation results. They reflect the quality of individual measurements.