PPT ON
NETWORK ANALYSIS
III SEM (IARE-R18)

UNIT 1
NETWORK THEOREMS (DC AND AC)

TELLEGEN'S THEOREM

According to Tellegen theorem, the summation of instantaneous powers for the n number of branches in an electrical network is zero. Are you confused? Let's explain. Suppose n number of branches in an electrical network have i1, i2, i3, ... in respective instantaneous currents through them. These currents satisfy Kirchhoff's Current Law.

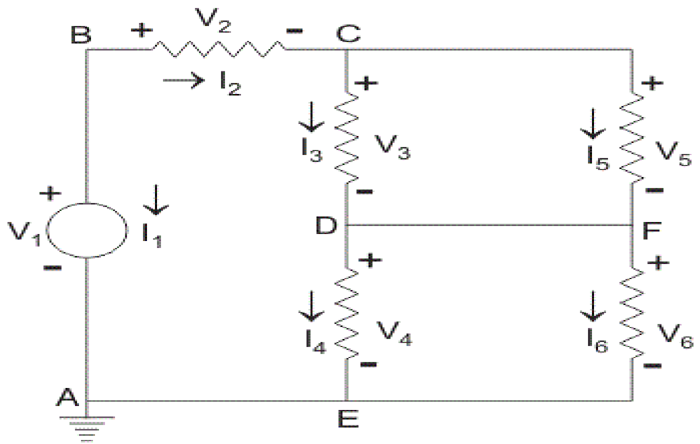
Again, suppose these branches have instantaneous voltages across them are v1, v2, v3, ... vn respectively. If these voltages across these elements satisfy Kirchhoff Voltage Law then,

Cont..

Vk is the instantaneous voltage across the kth branch and ik is the instantaneous current flowing through this branch.

Tellegen theorem is applicable mainly in general class of lumped networks that consist of linear, non-linear, active, passive, time variant and time variant elements.

Example



In the network shown, arbitrary reference directions have been selected for all of the branch currents, and the corresponding branch voltages have been indicated, with positive reference direction at the tail of the current arrow.

- For this network, we will assume a set of branch voltages satisfy the Kirchhoff voltage law and a set of branch current satisfy Kirchhoff current law at each node.

We will then show that these arbitrary assumed voltages and currents satisfy the equation.

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

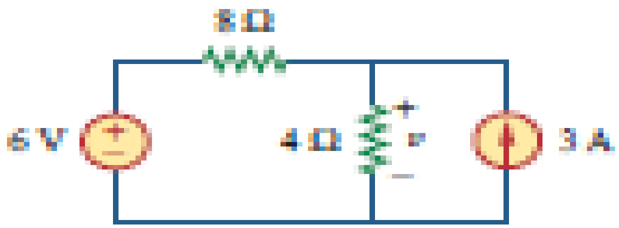
We get,

$$7 \times 5 + 2 \times (-5) + 3 \times 3 + 2 \times 4 + 3 \times (-8) + 2 \times (-9) = 0$$

Hence Tellegen theorem is verified

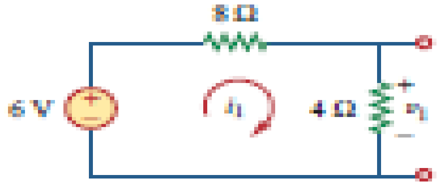
Super Position Theorem

- " In an any linear , bi-lateral network consisting number of sources , response in any element(resistor) is given as sum of the individual Reponses due to individual sources, while other sources are non-operative"
- **Example Problem**

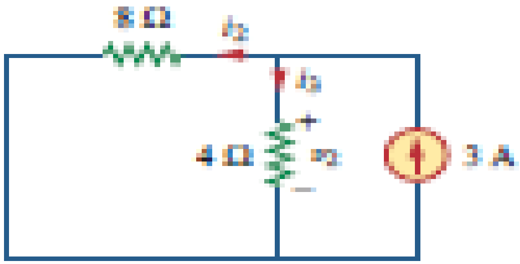


Let V = 6v, I = 3A, R1 = 8 ohms and R2 = 4 ohms

- Let us find current through 4 ohms using V source , while I is zero. then equivalent circuit is



Let i1 is the current through 4 ohms, i1 = V / (R1+R2)
Let us find current through 4 ohms using I source , while V is zero.
then equivalent circuit is



Let i2 is the current through 4 ohms, i2 = I. R1 / (R1+R2)
Hence total current through 4 ohms is = I1+I1(as both currents are in same direction or otherwise I1-I2)

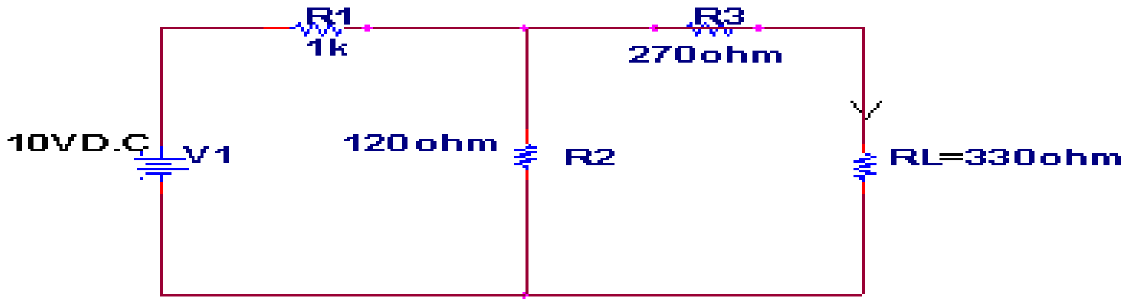
Problems

- Problem 01:
- In an network consisting three parallel branches, first across is defined as 20V in series with 5 ohms , second branch 7 ohms and third branch 10V in series with 4 ohms. Apply super-position theorem to Determine voltage drop across 7 ohms resistor.

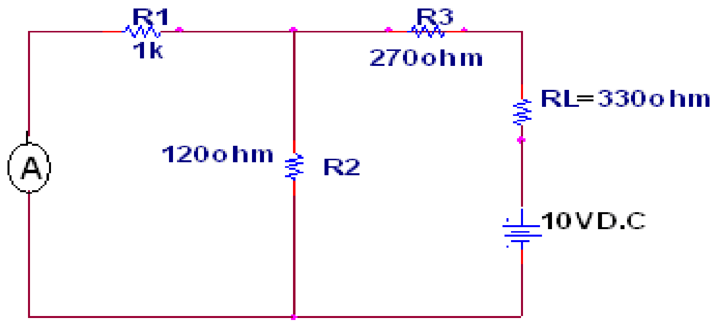
Reciprocity theorem

- In any linear bi-lateral network ratio of voltage in one mesh to current in other mesh is same even if their positions are inter-changed".

Example:



- Find the total resistance of the circuit, Rt = R1+ [R2(R3+Rl)] / R2+R3+RL.
- Hence source current, I = V1 / Rt.
- Current through RL is I1 = I. R2 / (R2+R3+RL)
- Take the ratio of , V1 / I1 ---1
- Draw the circuit by inter changing position of V1 and I1



Find the total resistance of the circuit, Rt = (R3+RL) + [R2(Rl)] / R2+R1.

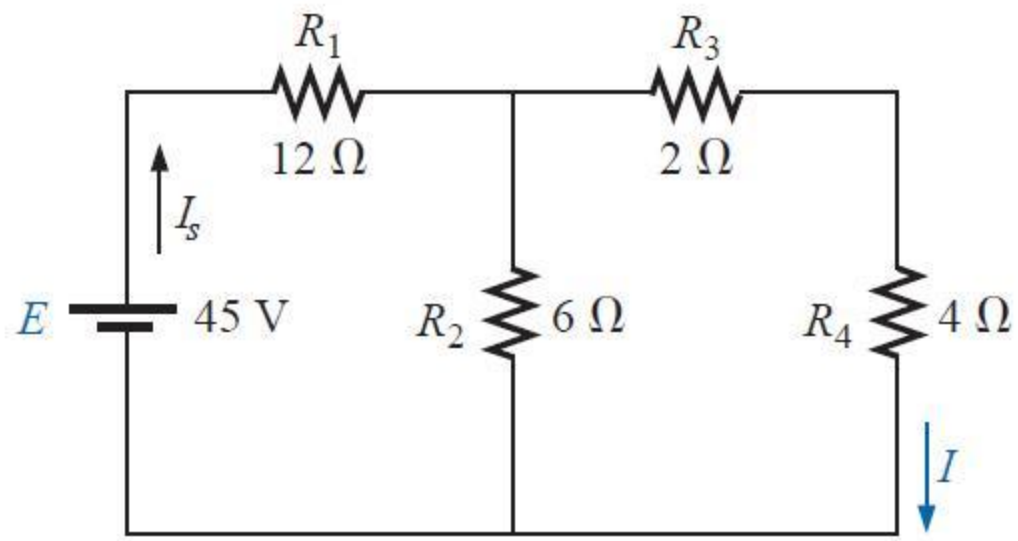
Hence source current, I = V1 / Rt.

Current through RL is I1 = I. R2 / (R2+R1)

Take the ratio of , V1 / I1 ---2

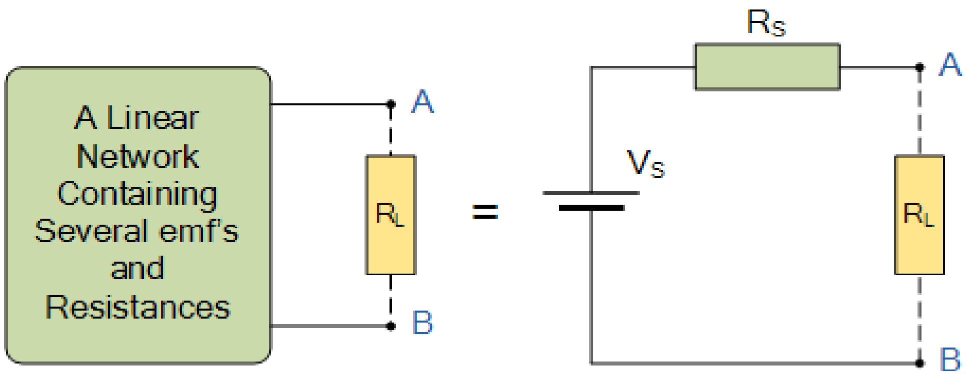
If ratio 1 = ratio 2, then circuit is said to be satisfy reciprocity.

Verify reciprocity theorem for given circuit



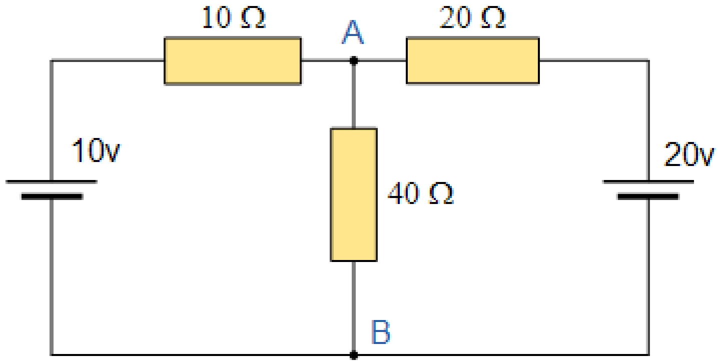
THEVENIN'S THEOREM

- DC: " An complex network consisting of number voltage and current sources can be replaced by simple series circuit consisting of equivalent voltage source in series with equivalent resistance, where equivalent voltage is called as open circuit voltage and equivalent resistance is called as Thevenin's resistance calculated across open circuit terminals while all energy sources are non-operative"
- Thevenin's equivalent circuit

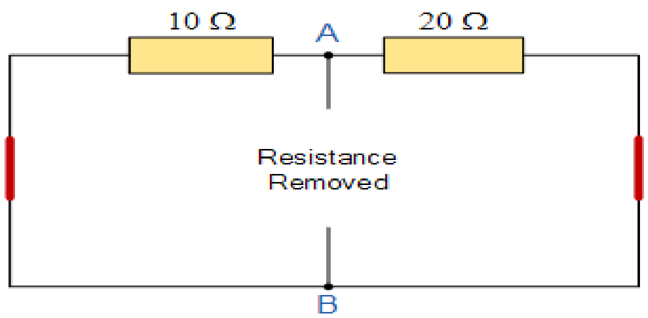


As far as the load resistor RL is concerned, any complex "one-port" network consisting of multiple resistive circuit elements and energy sources can be replaced by one single equivalent resistance Rs and one single equivalent voltage Vs. Rs is the source resistance value looking back into the circuit and Vs is the open circuit voltage at the terminals.

For example, consider the circuit from the previous section.



The value of the equivalent resistance, Rs is found by calculating the total resistance looking back from the terminals A and B with all the voltage sources shorted. We then get the following circuit.



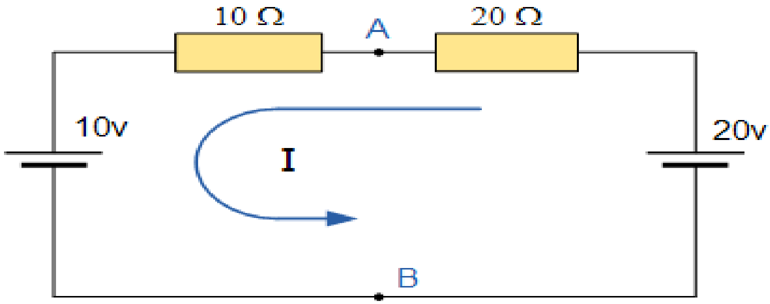
Find the Equivalent Resistance (Rs)

10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

The voltage Vs is defined as the total voltage across the terminals A and B when there is an open circuit between them. That is without the load resistor R_L connected.

Find the Equivalent Voltage (Vs)

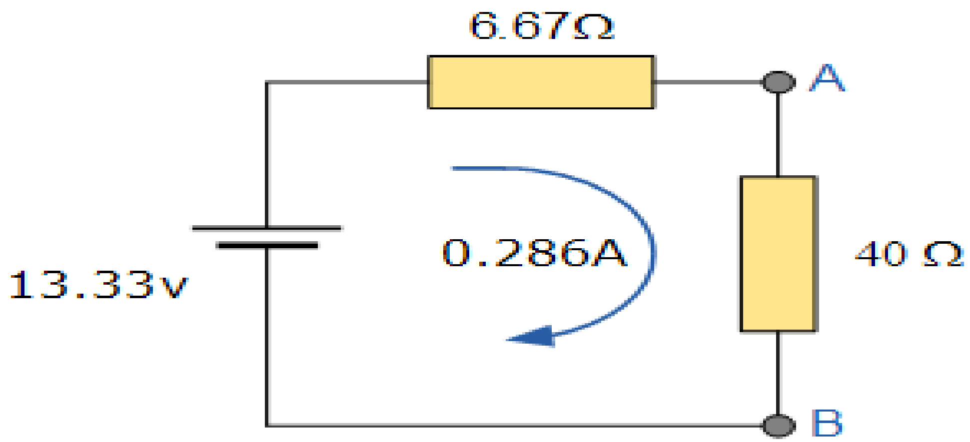


We now need to reconnect the two voltages back into the circuit, and as $V_S = V_{AB}$ the current flowing around the loop is calculated as:

$$I = \frac{V}{R} = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\ amps$$

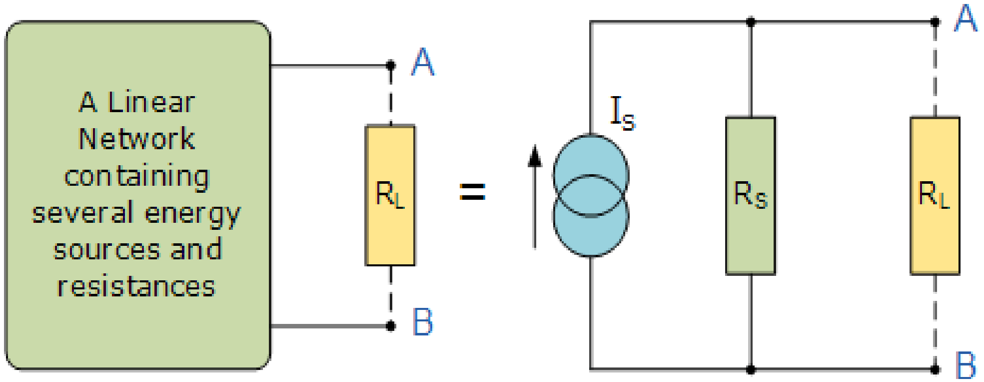
This current of 0.33 amperes (330mA) is common to both resistors so the voltage drop across the 20Ω resistor or the 10Ω resistor can be calculated as:

V_{AB} = 20 – (20Ω x 0.33amps) = 13.33 volts.
or
V_{AB} = 10 + (10Ω x 0.33amps) = 13.33 volts, the same



" An complex network consisting of number voltage and current sources can be replaced by simple parallel circuit consisting of equivalent current source in parallel with equivalent resistance, where equivalent current source is called as short circuit current and equivalent resistance is called as norton's resistance calculated across open circuit terminals while all energy sources are non-operative"

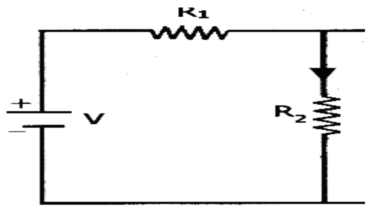
Norton's equivalent circuit



Maximum Power Transfer Theorem

In linear bi-lateral network maximum power can be transferred from source to load if load resistance is equal to source or Thevenin's or internal resistances".

For the below circuit explain maximum power transfer theorem



Let I be the source current, I = V / (R1+R2)

Power absorbed by load resistor is, PL = I^2 .R2

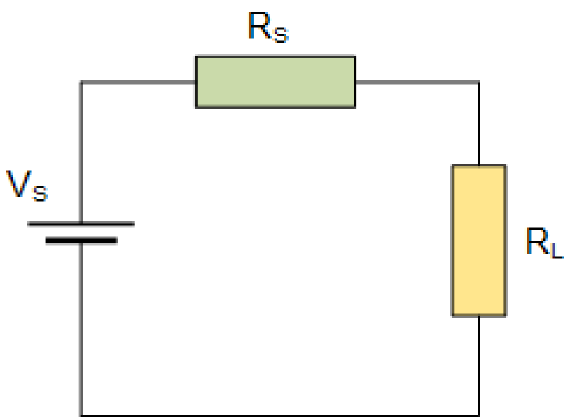
= [V / (R1+R2)]^2 .R2.

To say that load resistor absorbed maximum power, dPL / dR2 = 0.

When we solve above condition we get, R2 = R1.

Hence maximum power absorbed by load resistor is, PLmax = V^2 / 4R2

Example



Where:

R_S = 25Ω

R_L is variable between 0 – 100Ω

V_S = 100v

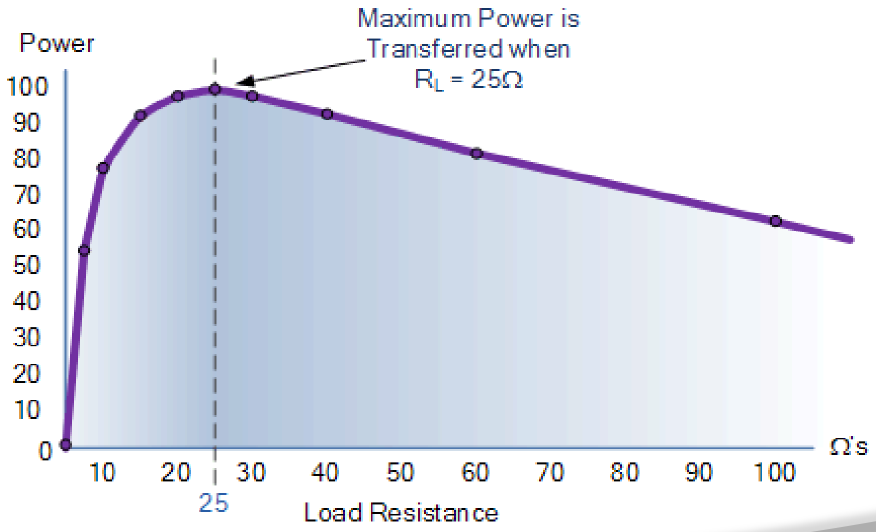
Then by using the following Ohm's Law equations:

$$I = \frac{V_S}{R_S + R_L} \quad \text{and} \quad P = I^2 R_L$$

We can now complete the following table to determine the current and power in the circuit for different values of load resistance.

R_L (Ω)	I (amps)	P (watts)
0	4.0	0
5	3.3	55
10	2.8	78
15	2.5	93
20	2.2	97
25	2.0	**100**
30	1.8	97
40	1.5	94
60	1.2	83
100	0.8	64

- Using the data from the table above, we can plot a graph of load resistance, RL against power, P for different values of load resistance. Also notice that power is zero for an open-circuit (zero current condition) and also for a short-circuit (zero voltage condition).
- **Graph of Power against Load Resistance**



From the above table and graph we can see that the Maximum Power Transfer occurs in the load when the load resistance, RL is equal in value to the source resistance, RS that is: RS = RL = 25Ω. This is called a “matched condition” and as a general rule, maximum power is transferred from an active device such as a power supply or battery to an external device when the impedance of the external device exactly matches the impedance of the source.

Milliman’s Theorem

An complex network consisting of number of parallel branches , where each parallel branch consists of voltage source with series resistance, can be replaced with equivalent circuit consisting of one voltage source in series with equivalent resistance

Millman's theorem is applicable to a circuit which may contain only voltage sources in parallel or a mixture of voltage and current sources connected in parallel. Let’s discuss these one by one.

Circuit consisting only Voltage Sources

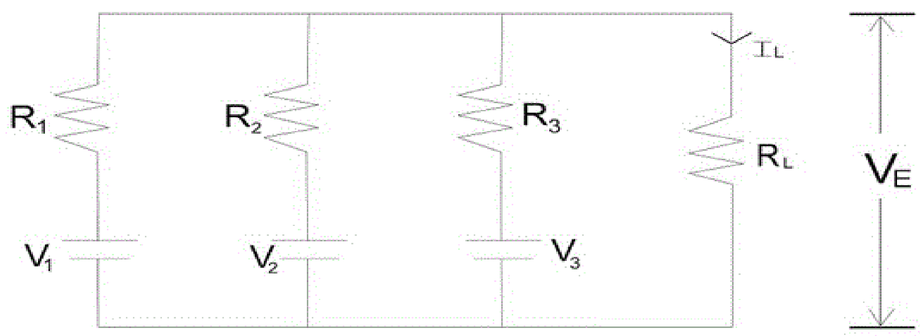


fig-a

Here V1, V2 and V3 are voltages of respectively 1st, 2nd and 3rd branch and R1, R2 and R3 are their respective resistances. IL, RL and VT are load current, load resistance and terminal voltage respectively.

Now this complex circuit can be reduced easily to a single equivalent voltage source with a series resistance with the help of Millman's Theorem as shown in figure b.

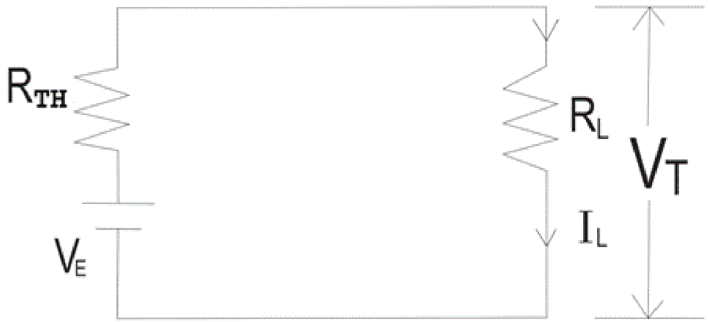


fig-b

The value of equivalent voltage V_E is specified as per Millman's theorem will be

Milliman's Theorem

This V_E is nothing but Thevenin voltage and Thevenin resistance R_{TH} can be determined as per convention by shorting the voltage source. So R_{TH} will be obtained as

$$V_E = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\sum \frac{V}{R}}{\sum \frac{1}{R}}$$

$$R_{TH} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Now load current and terminal voltage can be easily found by

$$I_L = \frac{V_{TH}}{R_L + R_{TH}} \quad \& \quad V_T = I_L \times R_L$$

- **Circuit is Consisting Mixture of Voltage and Current Source**
- Millman's Theorem is also helpful to reduce a mixture of voltage and current source connected in parallel to a single equivalent voltage or current source. Let's have a circuit as shown in below figure - f.

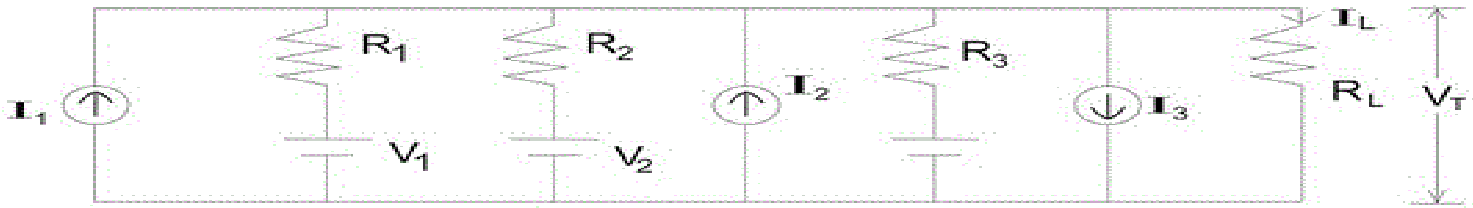


fig-f

- Here all letters are implying their conventional representation. This circuit can be reduced to a circuit as shown in figure - g.

Milliman's Theorem

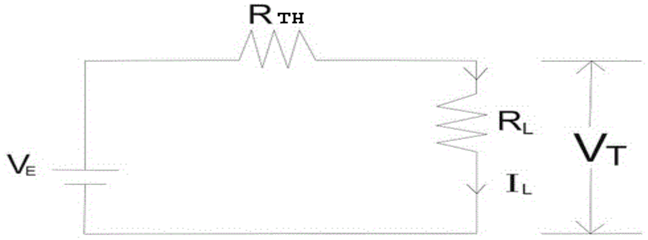


fig-g

Here V_E which is nothing but thevenin voltage which will be obtained as per Millman's theorem and that is

$$V_E = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + I_1 + I_2 - I_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\sum \frac{V}{R} + \sum I}{\sum \frac{1}{R}}$$

And R_{TH} will be obtained by replacing current sources with open circuits and voltage sources with short circuits.

$$R_{TH} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad I_L = \frac{V_E}{R_L + R_{TH}} \quad \& \quad V_T = I_L \times R_L$$

COMPENSATION THEOREM:

compensation theorem states that any element in the network can be replaced with Voltage source whose value is product of current through that element and its value” It is useful in finding change in current when sudden change in resistance value.

COMPENSATION THEOREM:

•For the above circuit source current is given as, I = V / (R1+R2)
•Element R2 can be replaced with voltage source of, V' = I.R2
•Let us assume there is change in R2 by ΔR, now source current is I'= V / (R1+R2+ ΔR)
•Hence actual change in current from original circuit to present circuit is = I – I'.
•This can be find using compensation theorem as, making voltage source non-operative and replacing ΔR with voltage source of I'. ΔR.
•Then change in current is given as = I'. ΔR / (R1+R2)

COMPENSATION THEOREM:

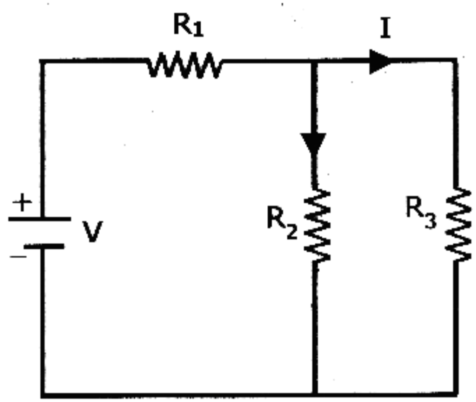


Fig 1

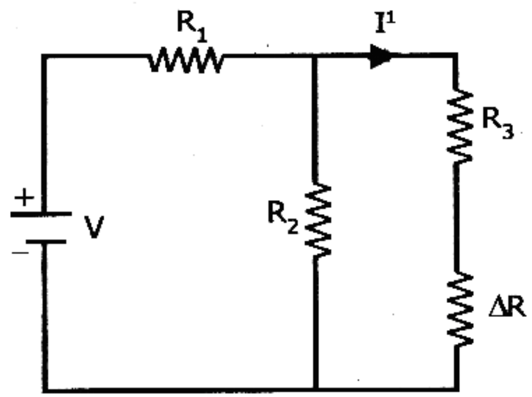


Fig 2

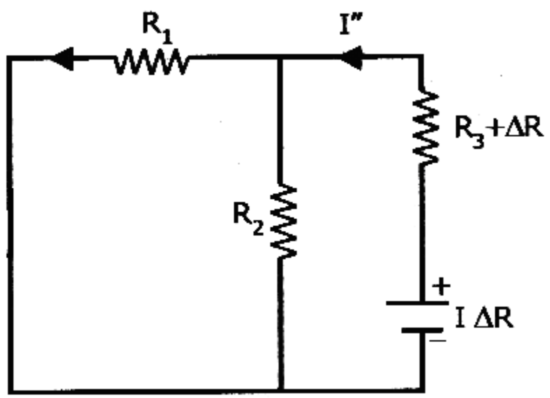
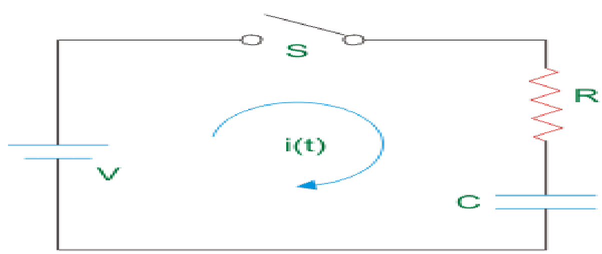


Fig 3

UNIT II
SOLUTION OF FIRST AND SECOND ORDER NETWORKS

Transient analysis of Capacitor

- The duration in which current changes in capacitor is known as transient period. The phenomenon of charging current or other electrical quantities like voltage, in capacitor is known as transient. To understand transient behavior of capacitor let us draw a RC circuit as shown below,



- Now, if the switch S is suddenly closed, the current starts flowing through the circuit. Let us current at any instant is i(t). Also consider the voltage developed at the capacitor at that instant is $V_c(t)$. Hence, by applying

DETAILED ESTIMATE

The problem of Economic operation of the power system involves two sub problems:

- Unit Commitment
- Economic Dispatch

Kirchhoff's Voltage Law, in that circuit we get,

$$V = Ri(t) + v_c(t)\ldots\ldots(i)$$

Now, if transfer of charge during this period (t) is q coulomb, then i(t) can be written as $\frac{dq(t)}{dt}$

Therefore,

$$i(t) = \frac{dq(t)}{dt} \Rightarrow dq(t) = i(t)dt \Rightarrow \int dq(t) = \int i(t)dt$$

$$\Rightarrow \int i(t)dt = q \;\; Again,\; q = Cv_c(+)$$

$$\therefore \int i(t)dt = Cv_c(+) \Rightarrow i(t) = C\frac{dv_c(t)}{dt}$$

Putting this expression of i(t) in equation (i) we get,

$$V = RC\frac{dv_c(+)}{dt} + v_c(+) \;\Rightarrow RC\frac{dv_c(+)}{dt} = V - v_c(+)$$

$$\Rightarrow \frac{dt}{RC} = \frac{dv_c(+)}{V - v_c(+)}$$

Now integrating both sides with respect to time we get,

$$\frac{t}{RC} = -log\left(V - v_c(t)\right) + K$$

Where, K is a constant can be determined from initial condition. Let us consider the time t = 0 at the instant of switching on the circuit putting t = 0 in above equation we get,

$$-log\left(V - v_c(0)\right) + K = 0 \Rightarrow \; K + logV$$

$$as,\; v_c(0) = 0$$

There will be no voltage developed across capacitor at t = 0 as it was previously unchanged. Therefore,

$$t/Rc = -log(V - v_c(t)) + log\,(V) \;\Rightarrow -t/Rc = log\,[V - v_c(t)] - logV$$

$$\Rightarrow -t/Rc = log\left[\frac{V - v_c(t)}{V}\right] \;\Rightarrow e^{-t/Rc} = \frac{V - v_c(t)}{V}$$

$$\Rightarrow v_c(t) = V - Ve^{-t/Rc} \;\Rightarrow v_c(t) = V[1 - e^{-t/Rc}].....(ii)$$

Now if we put RC = t at above equation, we get

$$V_c = 0.632V$$

Again, at the instant of switching on the circuit i.e. t = 0, there will be no voltage developed across the capacitor. This can also be proved from equation (ii).

$$v_c(0) = V[1 - e^o] = V[1 - 1] = 0$$

Cont..

So initial current through the circuit is, V/R and let us consider it as I_0. Now at any instant, current through the circuit will be,

$$i(t) = \frac{V - v_c(t)}{R} = \frac{V - V[1 - e^{-t/Rc}]}{R} = \frac{V}{R}e^{-t/Rc} = I_o e^{-t/Rc}$$

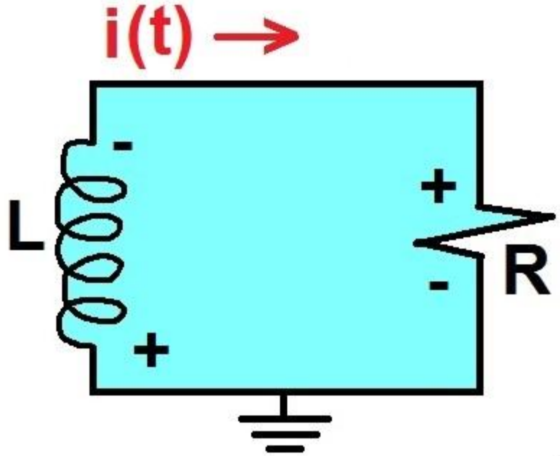
Now when, t = Rc the circuit current.

$$I = I_o e^{-1} = 0.367 I_o$$

So at the instant when, current through the capacitor is 36.7% of the initial current, is also known as time constant of the RC circuit. The time constant is normally denoted will τ (taw). Hence

$$\tau = Rc$$

The RL Circuit without a Source



$i(t=0) = I_0$ energy stored:

$$u(t=0) = \frac{1}{2} L I_0^2$$

$$v_L(t) = L\frac{di}{dt}$$

using KVL:

$$v_L + v_R = 0$$

$$L\frac{di}{dt} + iR = 0$$

$\vdots$

time constant: $\tau = \frac{L}{R}$

$$i(t) = I_0 e^{\frac{-t}{L/R}} \qquad i(t) = I_0 e^{\frac{-t}{\tau}}$$

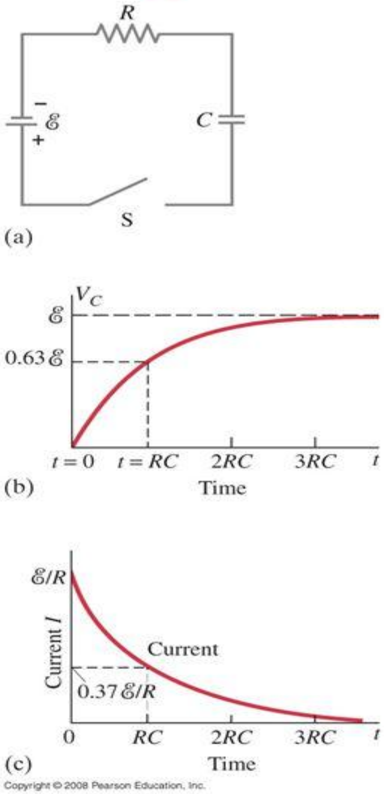
RC Circuit Transient Analysis

$$V_C = \xi\left(1 - e^{-t/RC}\right)$$

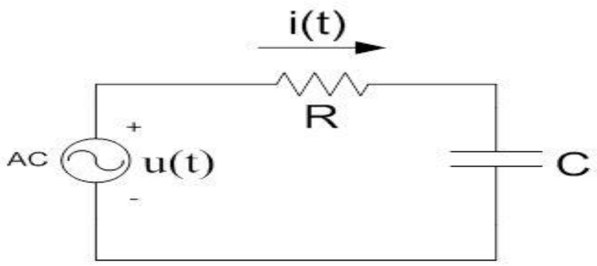
$$I = I_0 e^{-t/RC} \quad where \quad I_0 = \frac{\xi}{R}$$

$$and \quad let \quad \tau = RC$$

- τ is a time constant
- The voltage across the capacitor reaches 98% of the battery *EMF* in 4 τ
- The transient response of the circuit is over in approximately 4 - 5 τ



RC Circuits



The response of RC circuits can be categorized into two parts:

- Transient Response
- Forced Response

Transient response comes from the dynamic of R,C.
Forced response comes from the voltage source.

The complete response

- The combination of natural and step (or forced) responses
- For **RC** circuit, the complete response is:

$$v_c(t) = \underline{V_o e^{-\frac{t}{\tau}}} + \underline{V_s(1 - e^{-\frac{t}{\tau}})}$$

Natural response:

- Response due to initial energy stored in capacitor
- V_o is the <u>initial value</u>, i.e. $v_c(0)$

Forced response:

- Response due to the present of the source
- V_s is the <u>final value</u> i.e $v_c(\infty)$

Note: this is what we obtained when we solved the step response with initial energy (or initial voltage) at *t =0*

Step Response of A Series *RLC* Circuit

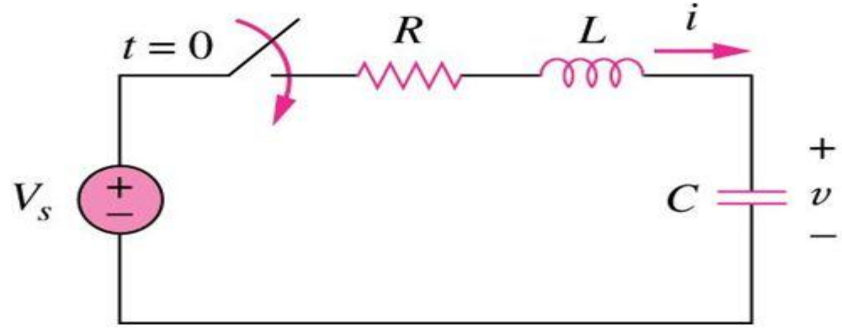
Applying KVL for $t > 0$,

$$Ri + L\frac{di}{dt} + v = V_S \qquad (1)$$

But $i = C\frac{dv}{dt}$

$$\Rightarrow \boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_S}{LC}} \quad (2)$$

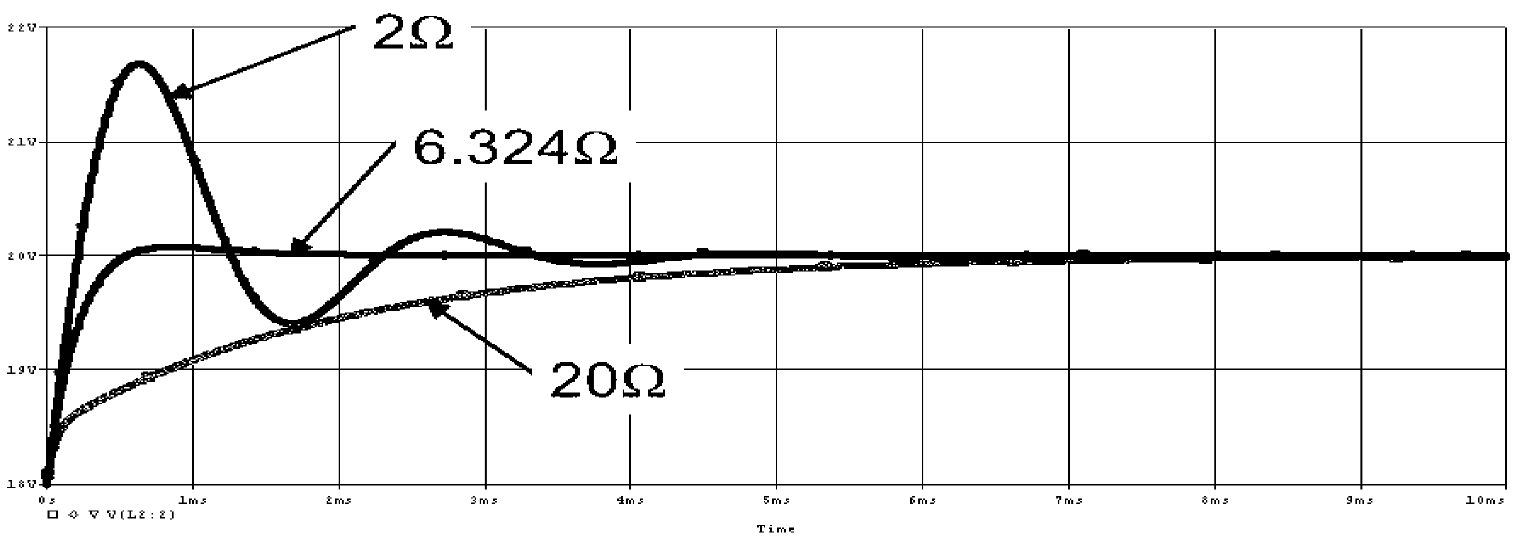
(2) has the same form as in the source-free case.



$$\boxed{\begin{aligned} &v(t) = v_t(t) + v_{ss}(t) \\ &\text{where} \\ &\begin{cases} v_t : \text{the transient response} \\ v_{ss} : \text{the steady-state response} \end{cases} \end{aligned}}$$

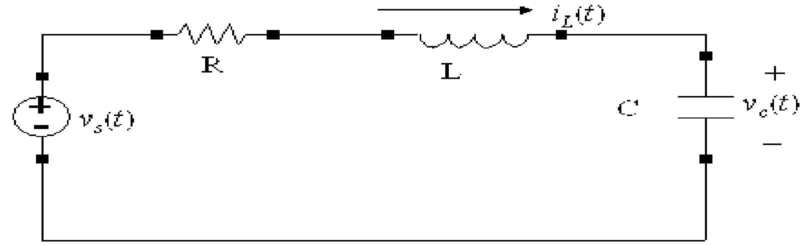
Step Response - Series RLC Circuit

- Comparison of the three responses with different R value.



Example

Find the differential equation for the circuit below in terms of v_c and also terms of i_L



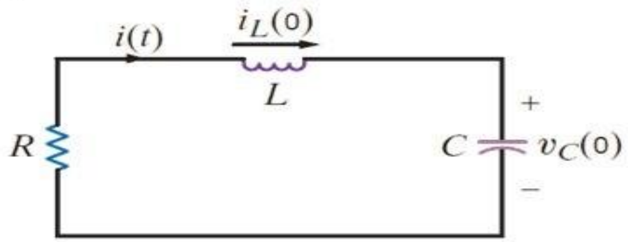
Show:

$$v_S(t) = LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C \quad \rightarrow \quad \frac{v_S(t)}{LC} = \frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C$$

$$v_S(t) = L\frac{di_L}{dt} + Ri_L + \frac{1}{C}\int_{-\infty}^{t} i_L(\tau)d\tau \quad \rightarrow \quad \frac{v_S(t)}{L} = \frac{di_L}{dt} + \frac{R}{L}i_L + \frac{1}{LC}\int_{-\infty}^{t} i_L(\tau)d\tau$$

Example

The series RLC circuit shown in Fig. 7.18 has the following parameters: C=0.04 F, L=1 H, R=6 , $i_L(0)$ = 4 A, and $v_C(0)$ = -4 V. Let us determine the expression for both the current and the capacitor voltage.



Applying Kirchhoff voltage law to the loop, $$iR+\frac{1}{C}\int_0^t i(x)dx+v_C(0)+L\frac{di}{dt}=0$$

$$\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0 \Rightarrow \frac{d^2i}{dt^2}+6\frac{di}{dt}+25i=0$$

Trial solution : $i(t)=Ke^{st}$

Characteristic equation : $s^2+6s+25=0$

$$s=\frac{-6\pm\sqrt{6^2-100}}{2}=\frac{-6\pm 8j}{2}=-3\pm 4j$$

First order transient circuits

Solution to 1st order differential equation :

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

$f(t) = 0 \longrightarrow$ homogeneous equation
$f(t) \neq 0 \longrightarrow$ inhomogeneous equation

$$\frac{dx_c(t)}{dt} + ax_c(t) = 0$$

$x_h(t)$ or $x_c(t) \longrightarrow$ homogeneous or complementary solution
$x_p(t) \longrightarrow$ inhomogeneous or particular solution

$$x(t) = x_p(t) + x_c(t)$$

Transform Pairs

The Laplace transforms pairs

$f(t)$	$\mathbf{F}(s)$
$\delta(t)$	1
$u(t)$ {a constant}	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$

Critically Damped Case (Series RLC)

- In the critically damped case, the two poles are real and equal. Assume that the poles are $s_1 = s_2 = -\alpha$, the forms for $I(s)$ and $V_C(s)$ can be expressed as

$$I(s) = \frac{\frac{V_i}{L}}{(s+\alpha)^2}$$

The damping factor: $\alpha = \dfrac{R}{2L}$

$$V_C(s) = \frac{\frac{V_i}{LC}}{s(s+\alpha)^2}$$

- The inverse transforms are of the forms

$$i(t) = C_0 t e^{-\alpha t} = \frac{V_i t}{L} e^{-Rt/2L}$$

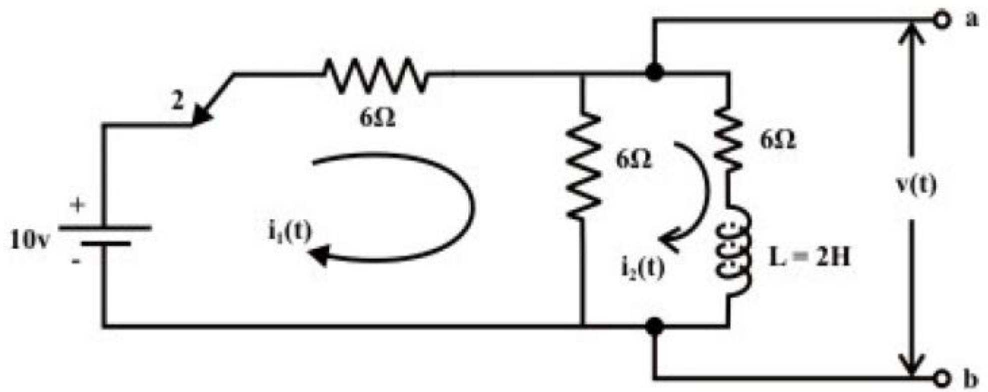
$C_0 = V_i / L$

$$v_C(t) = V_i + (C_1 t + C_2) e^{-Rt/2L}$$

The most significant aspect of the natural response function for the critically damped case is the $te^{-\alpha t}$ form. Although the t factor increases with increasing t, the $te^{-\alpha t}$ decreases at a faster rate, so the product eventually approaches zero.

DIFFERENTAL APPROACH CASE II

1 .Assign the loop currents in clockwise directions and redrawn the circuit as shown in . The voltage across the terminals " and " can be obtained by solving the following loop equations.



Solution

Loop-1:

$$10-6i_1(t)-6(i_1(t)-i_2(t))=0 \Rightarrow 10=12i_1(t)-6i_2(t) \Rightarrow i_1(t)=\frac{1}{12}(10+6i_2(t))$$

Loop-2:

$$-6i_2(t)-L\frac{di_2(t)}{dt}-6(i_2(t)-i_1(t))=0 \Rightarrow -6i_1(t)+12i_2(t)+2\frac{di_2(t)}{dt}=0$$

Using the value of $i_1(t)$ in equation , we get

$$9i_2(t)+2\times\frac{di_2(t)}{dt}=5$$

where, i2(t) and i1(t)can be obtained

To solve the above first order differential equation we must know inductor's initial and final conditions and their values are already known (see, $\Rightarrow i_2(0^-) = i_2(0^+) = 3\,A$ and $i_2(t=\infty) = \dfrac{5}{3+6} = 0.555\ amp.$). The solution of differential equation provides an expression of current $i_2(t)$ and this, in turn, will give us the expression of $i_1(t)$. The voltage across the terminals 'a' and 'b' is given by

$$v_{ab} = 10 - 6 \times i_1(t) = 6 i_2(t) + 2\frac{di_2(t)}{dt} = \left(3.339 - 7.335 \times e^{-\frac{9}{2}t}\right) V$$

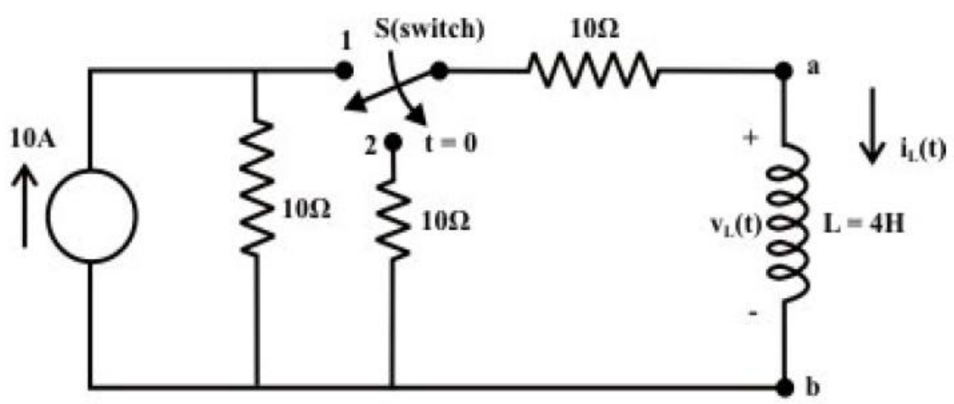
where, i2(t) and i1(t)can be obtained

$$i_2(t)=\left(2.445\times e^{-\frac{9}{2}t}+0.555\right) \text{ and } i_1(t)=\frac{1}{12}\left(10+6i_2(t)\right)=\left(1.11+1.2225e^{-\frac{9}{2}t}\right)$$

The switch '*S*' is closed in position '1' sufficiently

long time and then it is kept in position '2' as shown in fig. Compute the value of compute the value of V_L and I_L.

instant just prior to the switch changing; (ii) the instant just after the switch changes. Find also the rate of change of current through the inductor at time $t = 0^+$ $\left(i.e., \left. \frac{di_l(t)}{dt} \right|_{t=0^+} \right)$.



Solution: We assume that the circuit has reached at steady state condition when the switch was in position '1'. Note, at steady state the inductor acts as short circuit and voltage across the inductor is zero.

At $t=0^-$, the current through and the voltage across the inductor are $i_L(0^-)=\frac{10}{10+10}\times 10=5A$ and $v_L(0^-)=0V$ respectively. When the switch is kept in position '2', current through the inductor cannot change instantaneously but this is not true for the voltage across the inductor. At $t=0^+$, one can write the following expressions:

$i_L(0^+)=5A$ and $v_L(0^+)=-(10+10)\times 5=-100V$ ('b' is more + ve potential than 'a' terminal). Note that the stored energy in inductor is dissipated in the resistors. Now, the rate of change of current through inductor at time $t=0^+$ is obtained as

$$L\left.\frac{di_l(t)}{dt}\right|_{t=0^+}=-100V \Rightarrow \left.\frac{di_l(t)}{dt}\right|_{t=0^+}=\frac{-100}{4}=-25\,amp./\sec.$$

Step Response of A Series *RLC* Circuit

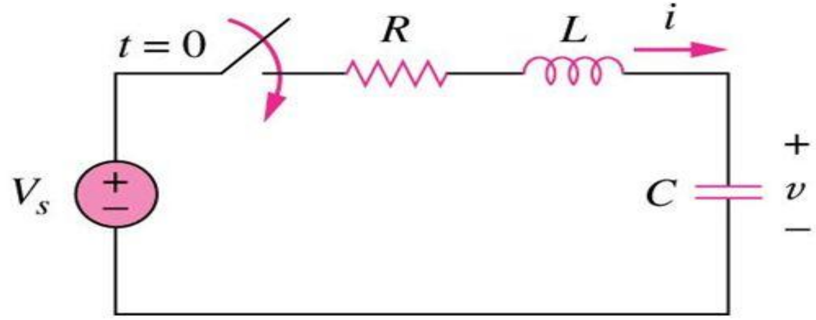
Applying KVL for $t > 0$,

$$Ri + L\frac{di}{dt} + v = V_S \qquad (1)$$

But $i = C\frac{dv}{dt}$

$$\Rightarrow \boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_S}{LC}} \quad (2)$$

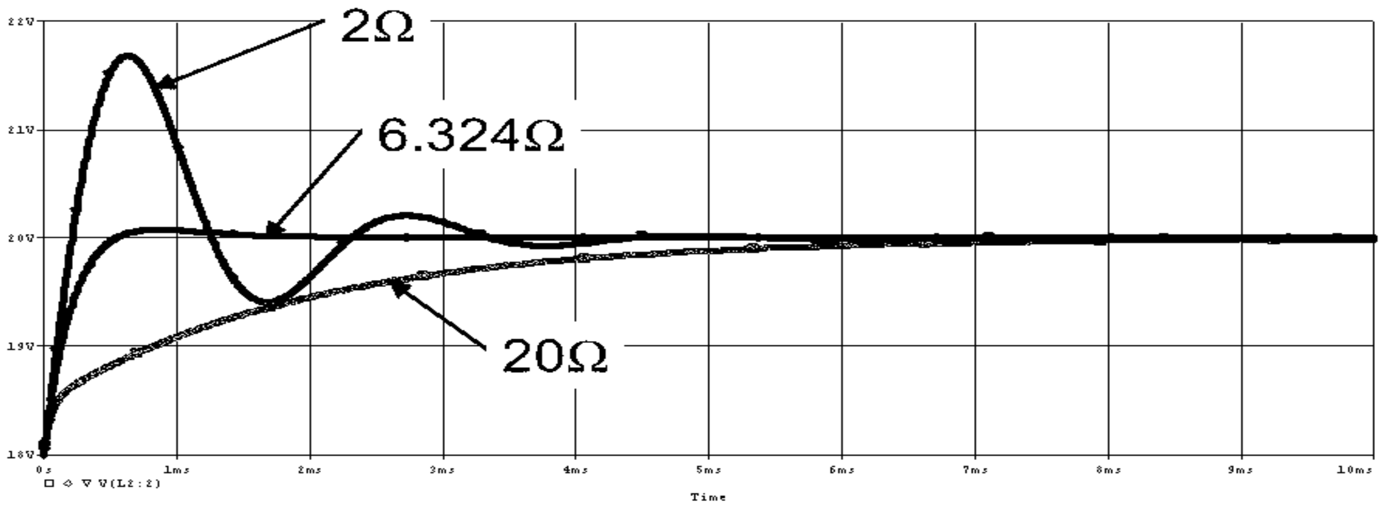
(2) has the same form as in the source - free case.



$$\boxed{\begin{aligned} & v(t) = v_t(t) + v_{ss}(t) \\ & \text{where} \\ & \begin{cases} v_t: \text{the transient response} \\ v_{ss}: \text{the steady - state response} \end{cases} \end{aligned}}$$

Step Response - Series RLC Circuit

- Comparison of the three responses with different R value.



Solution: We assume that the circuit has reached at steady state condition when the switch was in position '1'. Note, at steady state the inductor acts as short circuit and voltage across the inductor is zero.

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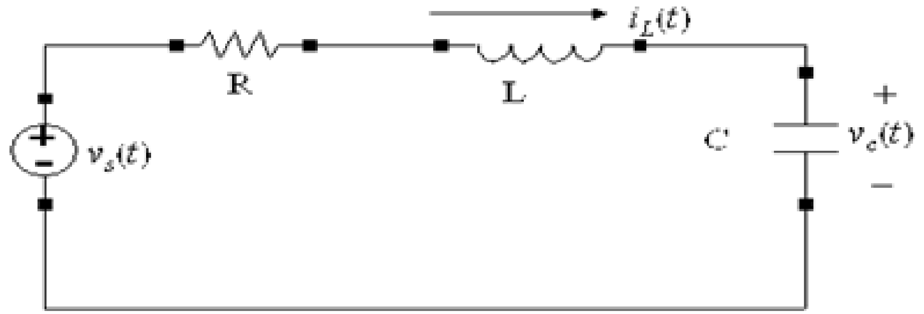
$i_L(0^+)=5A$ and $v_L(0^+)=-(10+10)\times 5=-100V$ ('b' is more + ve potential than 'a' terminal). Note that the stored energy in inductor is dissipated in the resistors. Now, the rate of change of current through inductor at time $t=0^+$ is obtained as

$$L\left.\frac{di_l(t)}{dt}\right|_{t=0^+}=-100V \Rightarrow \left.\frac{di_l(t)}{dt}\right|_{t=0^+}=\frac{-100}{4}=-25\,amp./\sec.$$

Find the differential equation for the circuit below in terms of v_c and also terms of i_L



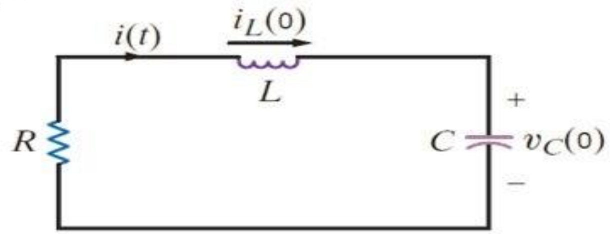
Show:

$$v_S(t) = LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C \quad \rightarrow \quad \frac{v_S(t)}{LC} = \frac{d^2v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{1}{LC}v_C$$

$$v_S(t) = L\frac{di_L}{dt} + Ri_L + \frac{1}{C}\int_{-\infty}^{t} i_L(\tau)d\tau \quad \rightarrow \quad \frac{v_S(t)}{L} = \frac{di_L}{dt} + \frac{R}{L}i_L + \frac{1}{LC}\int_{-\infty}^{t} i_L(\tau)d\tau$$

Example

The series RLC circuit shown in Fig. 7.18 has the following parameters: C=0.04 F, L=1 H, R=6 , $i_L(0)$ = 4 A, and $v_C(0)$ = -4 V. Let us determine the expression for both the current and the capacitor voltage.



Applying Kirchhoff voltage law to the loop, $$iR + \frac{1}{C}\int_0^t i(x)\,dx + v_C(0) + L\frac{di}{dt} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \quad \Rightarrow \quad \frac{d^2 i}{dt^2} + 6\frac{di}{dt} + 25i = 0$$

Trial solution : $$i(t) = Ke^{st}$$

Characteristic equation : $$s^2 + 6s + 25 = 0$$

$$s = \frac{-6 \pm \sqrt{6^2 - 100}}{2} = \frac{-6 \pm 8j}{2} = -3 \pm 4j$$

Co

First order transient circuits

Solution to 1st order differential equation :

$$\frac{dx(t)}{dt} + ax(t) = f(t)$$

$f(t) = 0 \longrightarrow$ homogeneous equation
$f(t) \neq 0 \longrightarrow$ inhomogeneous equation

$$\frac{dx_c(t)}{dt} + ax_c(t) = 0$$

$x_h(t)$ or $x_c(t) \longrightarrow$ homogeneous or complementary solution
$x_p(t) \longrightarrow$ inhomogeneous or particular solution

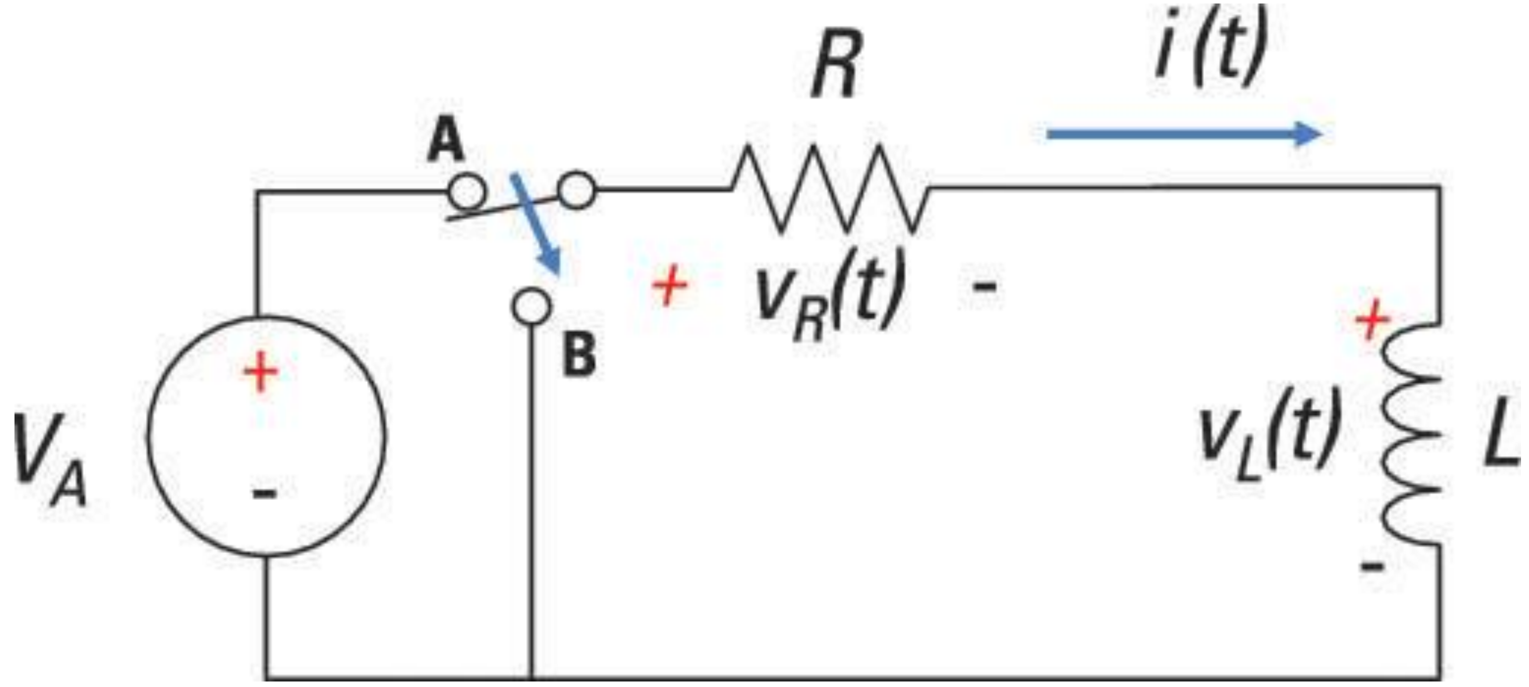
$$x(t) = x_p(t) + x_c(t)$$

Transient analysis of a series RL circuits

Follow these basic steps to analyze a circuit using Laplace techniques:

1. Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.
2. Apply the Laplace transformation of the differential equation to put the equation in the *s*-domain.
3. Algebraically solve for the solution, or response transform.
4. Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

- Here is an RL circuit that has a switch that's been in Position A for a long time. The switch moves to Position B at time t = 0.



For this circuit, you have the following KVL equation:

$$v_R(t) + v_L(t) = 0$$

Next, formulate the element equation (or *i-v* characteristic) for each device. Using Ohm's law to describe the voltage across the resistor, you have the following relationship:

$$v_R(t) = i_L(t)R$$

The inductor's element equation is

- **Substituting the element equations, $v_R(t)$ and $v_L(t)$, into the KVL equation gives you the desired first-order differential equation**

$$L\frac{di_L(t)}{dt}+i_L(t)R=0$$

- **On to Step 2: Apply the Laplace transform to the differential equation:**

$$\mathcal{L}\left[L\frac{di_L(t)}{dt}+i_L(t)R\right]=0$$
$$\mathcal{L}\left[L\frac{di_L(t)}{dt}\right]+\mathcal{L}\left[i_L(t)R\right]=0$$

- **The preceding equation uses the linearity property which says you can take the Laplace transform of each term. For the first term on the left side of the equation, you use the differentiation property:**

$$\mathcal{L}\left[L\frac{di_L(t)}{dt}\right]=L\left[sI_L(s)-I_0\right]$$

•This equation uses $I_L(s) = \mathcal{L}[i_L(t)]$, and I_0 is the initial current flowing through the inductor.

•The Laplace transform of the differential equation becomes

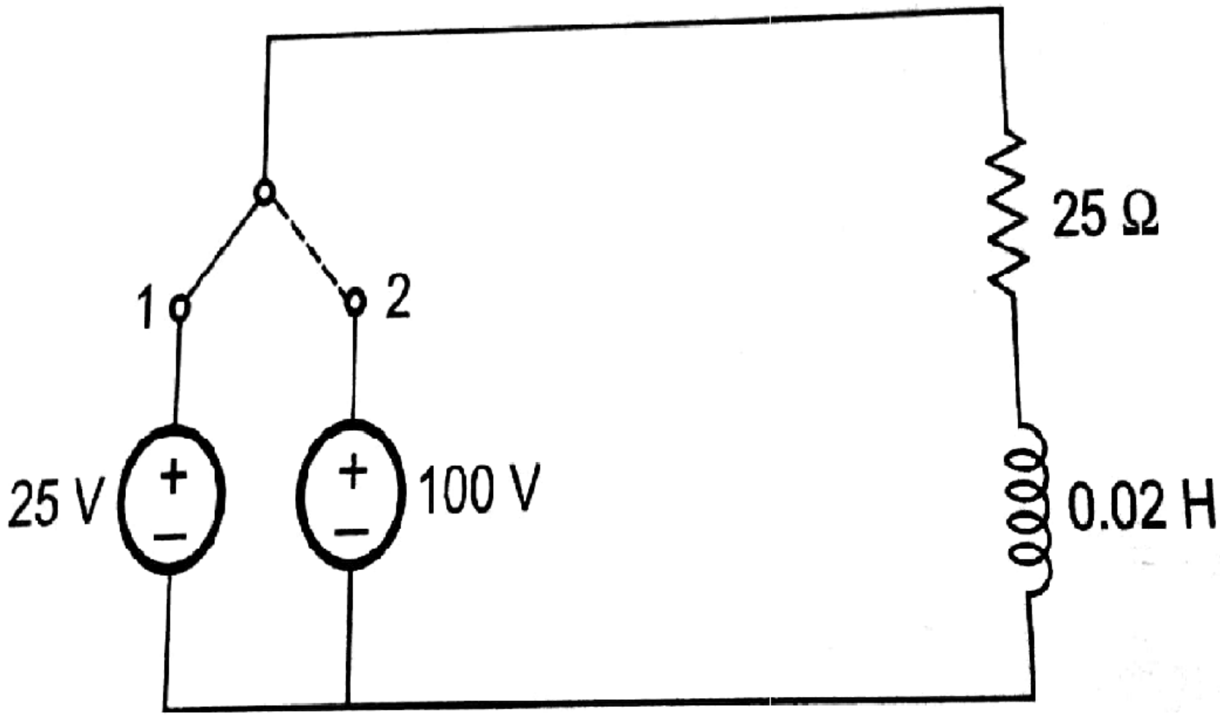
$I_L(s)R + L[sI_L(s) – I_0] = 0$

Solve for $I_L(s)$:

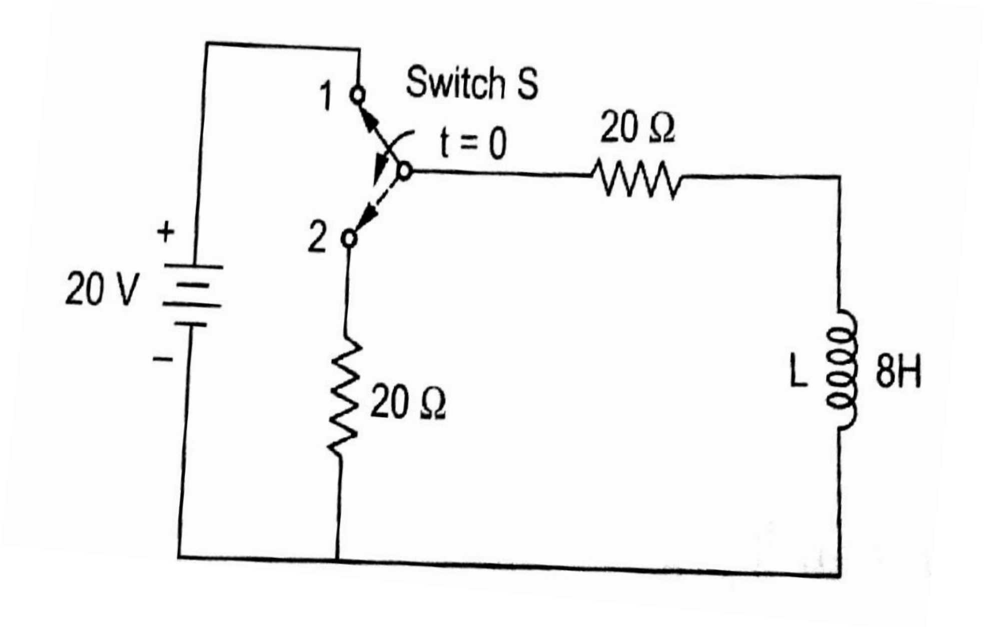
$$I_L(s)=\frac{I_0}{s+\frac{R}{L}}$$

Numerical Problems on series RL circuits

1. In the RL circuit shown in below figure the switch is in position 1 long enough to establish the steady state conditions. At t=0, the switch is thrown to position 2. Find the expression for the resulting current.

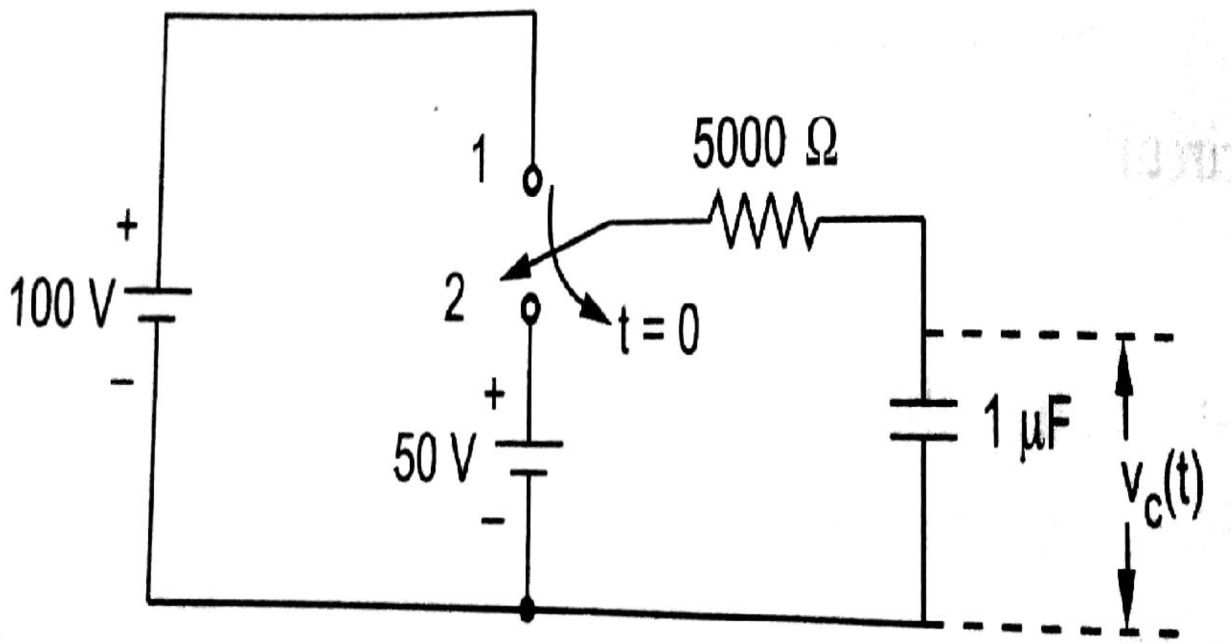


In the circuit shown in figure the switch S is kept in position 1 for long period to establish the steady state condition. The switch is then moved to position 2 at t=0. Find out the expression for the current after switching the switch to positi

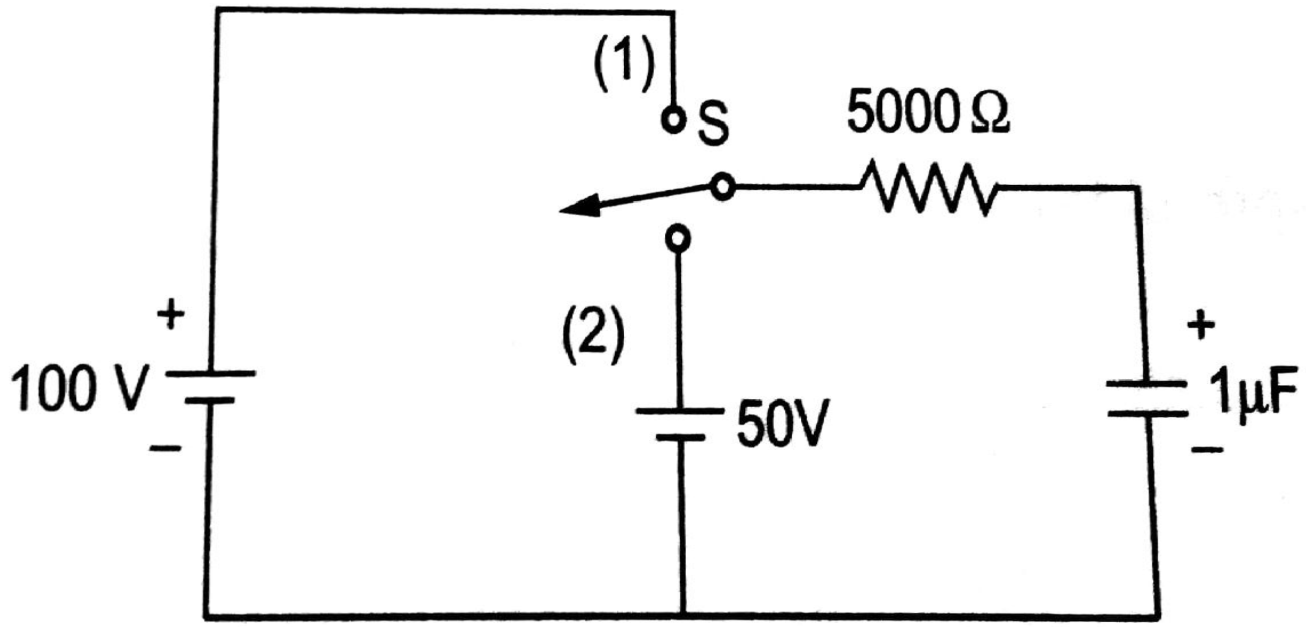


Numerical Problems on series RC circuits

The switch S is moved from position 1 to 2 at t=0. Find the voltages vR(t) and vC(t) for t>0.



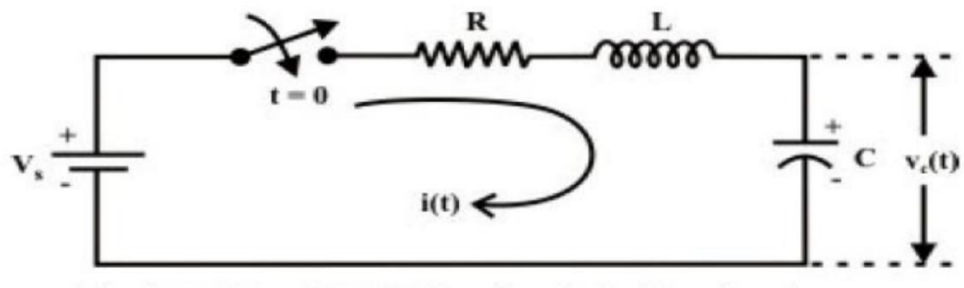
2. Switch is moved from position 1 to 2 at t=0. Find the voltages vR(t) and vC(t) for t>0.



Transient Response of a series R-L-C circuit

Consider a series RL circuit as shown in fig.11.1, and it is excited with a dc voltage source C−−sV.
Applying around the closed path for,

$$L\frac{di(t)}{dt}+Ri(t)+v_c(t)=V_s$$



The current through the capacitor can be written as Substituting the current ‘’expression in eq.(11.1) and rearranging the terms,

$$i(t) = C\frac{dv_c(t)}{dt}$$

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

The above equation is a 2nd-order linear differential equation and the parameters associated with the differential equation are constant with time. The complete solution of the above differential equation has two components; the transient response and the steady state response. Mathematically, one can write the complete solution as

$$v_c(t) = v_{cn}(t) + v_{cf}(t) = \left(A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}\right) + A$$

$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = 0 \Rightarrow \frac{d^2v_c(t)}{dt^2} + \frac{R}{L}\frac{dv_c(t)}{dt} + \frac{1}{LC}v_c(t) = 0$$

$$a\frac{d^2v_c(t)}{dt^2} + b\frac{dv_c(t)}{dt} + c\, v_c(t) = 0 \text{ (where } a = 1,\, b = \frac{R}{L} \text{ and } c = \frac{1}{LC})$$

The natural or transient response (see Appendix in Lesson-10) of second order differential equation can be obtained from the homogeneous equation (i.e., from force free system) that is expressed by

$$\alpha^2 + \frac{R}{L}\alpha + \frac{1}{LC} = 0 \quad \Rightarrow a\alpha^2 + b\alpha + c = 0 \ \text{(where } a = 1,\ b = \frac{R}{L} \text{ and } c = \frac{1}{LC}\text{)}$$

And solving the roots of this equation (11.5) on that associated with transient part of the complete solution (eq.11.3) and they are given below.

$$\alpha_1 = \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \right) = \left(-\frac{b}{2a} + \frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2 - ac} \right);$$

$$\alpha_2 = \left(-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \right) = \left(-\frac{b}{2a} - \frac{1}{a}\sqrt{\left(\frac{b}{2}\right)^2 - ac} \right)$$

where, $b = \frac{R}{L}$ and $c = \frac{1}{LC}$.

The roots of the characteristic equation are classified in three groups depending upon the values of the parameters, Rand of the circuit Case-A (over damped response): That the roots are distinct with negative real parts. Under this situation, the natural or transient part of the complete solution is written as

$$\dot{v}_{cn}(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

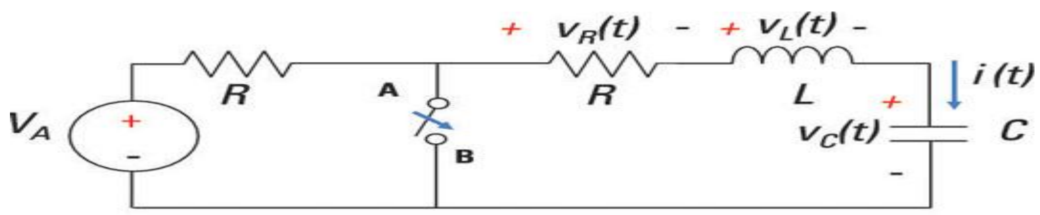
Transient analysis of a series RLC Circuit

Follow these basic steps to analyze a circuit using Laplace techniques:

1. Develop the differential equation in the time-domain using Kirchhoff's laws and element equations.
2. Apply the Laplace transformation of the differential equation to put the equation in the *s*-domain.
3. Algebraically solve for the solution, or response transform.
4. Apply the inverse Laplace transformation to produce the solution to the original differential equation described in the time-domain.

To get comfortable with this process, you simply need to practice applying it to different types of circuits such as an RC (resistor-capacitor) circuit, an RL (resistor-inductor) circuit, and an RLC (resistor-inductor-capacitor) circuit.

Here you can see an RLC circuit in which the switch has been open for a long time. The switch is closed at time *t = 0*.



$R = 800\ \Omega$ $\quad C = \frac{1}{4.1} \cdot 10^{-5}$ F $= 2.439\ \mu$F

$L = 1$ H $\quad V_A = 5$ V

Cont..

In this circuit, you have the following KVL equation:

$$v_R(t) + v_L(t) + v(t) = 0$$

Next, formulate the element equation (or i-v characteristic) for each device. Ohm's law describes the voltage across the resistor (noting that $i(t) = i_L(t)$ because the circuit is connected in series, where $I(s) = I_L(s)$ are the Laplace transforms):

$$v_R(t) = i(t)R$$

The inductor's element equation is given by

$$v_L(t) = L\frac{di_L(t)}{dt}$$

And the capacitor's element equation is

$$v_c(t) = \frac{1}{C}\int_0^t i(\tau) + v_c(0)$$

Cont..

Here, $v_C(0) = V_0$ is the initial condition, and it's equal to 5 volts.

Substituting the element equations, $v_R(t)$, $v_C(t)$, and $v_L(t)$, into the KVL equation gives you the following equation (with a fancy name: the *integro-differential equation*):

$$L\frac{di_L(t)}{dt} + i_L(t)R + \frac{1}{C}\int_0^t i(\tau) + v_c(0) = 0$$

The next step is to apply the Laplace transform to the preceding equation to find an $I(s)$ that satisfies the integral-differential equation for a given set of initial conditions:

$$\mathcal{L}\left[L\frac{di_L(t)}{dt} + i(t)R + \frac{1}{C}\int_0^t i(\tau)d\tau + V_0\right] = 0$$

$$\mathcal{L}\left[L\frac{di_L(t)}{dt}\right] + \mathcal{L}[i(t)R] + \mathcal{L}\left[\frac{1}{C}\int_0^t i(\tau)d\tau + V_0\right] = 0$$

The preceding equation uses the linearity property allowing you to take the Laplace transform of each term. For the first term on the left side of the equation, you use the differentiation property to get the following transform:

$$\mathcal{L}\left[L\frac{di(t)}{dt}\right] = L\left[sI(s) - I_0\right]$$

This equation uses $I_L(s) = \mathcal{L}[i(t)]$, and I_0 is the initial current flowing through the inductor. Because the switch is open for a long time, the initial condition I_0 is equal to zero.

For the second term of the KVL equation dealing with resistor *R*, the Laplace transform is simply

$$\mathcal{L}[i(t)R] = I(s)R$$

For the third term in the KVL expression dealing with capacitor *C*, you have

$$\mathcal{L}\left[\frac{1}{C}\int_0^t i(\tau)d\tau+V_0\right]=\frac{I(s)}{sC}+\frac{V_0}{s}$$

•The Laplace transform of the integro-differential equation becomes

$$L\left[sI(s)-I_0\right]+I(s)R+\frac{I(s)}{sC}+\frac{V_0}{s}=0$$

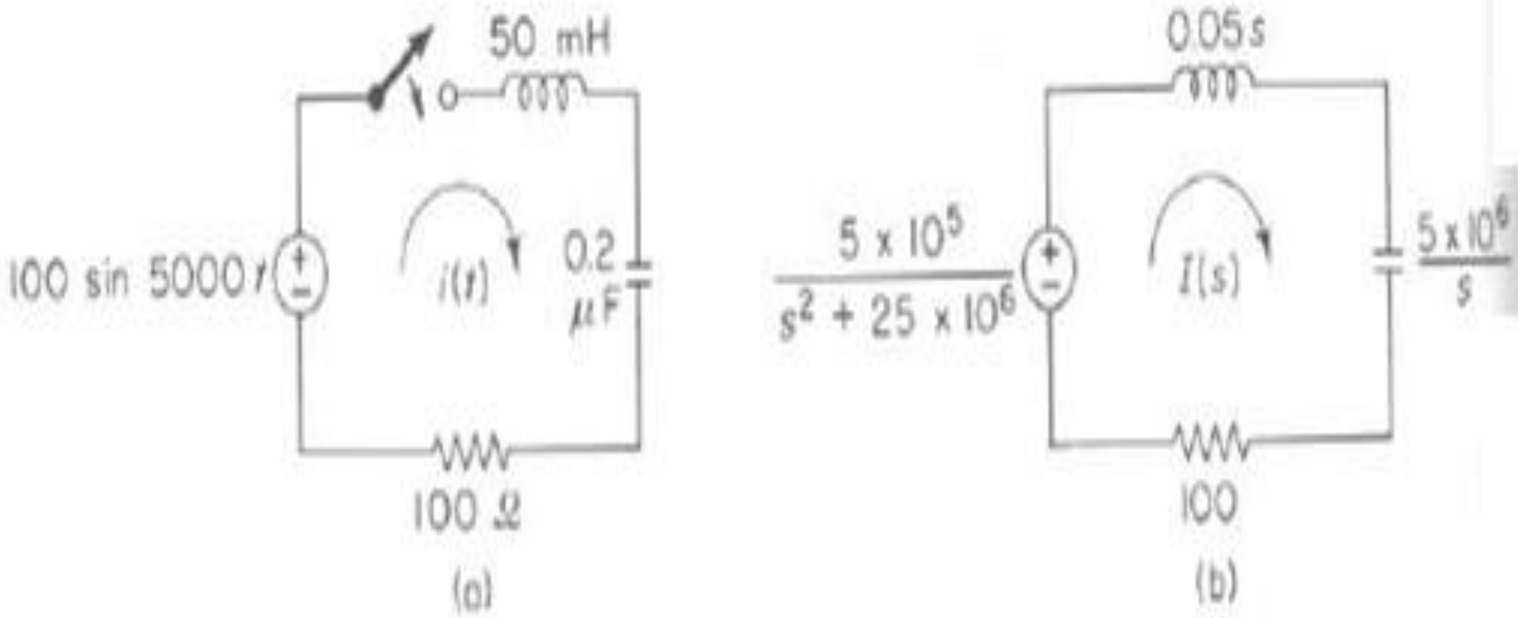
•Rearrange the equation and solve for I(s):

$$I(s)=\frac{sI_0-\frac{V_0}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

•To get the time-domain solution *i(t)*, use the following table, and notice that the preceding equation has the form of a damping sinusoid.

Numerical Problems on RLC Circuits

The relaxed series RLC circuit of Fig. 6-23a is excited at $t = 0$ by the sinusoidal source shown. Solve for the current $i(t)$ for $t > 0$.



Cont..

Solution Although the mathematics will eventually reveal the type of response, a preliminary calculation should prove interesting. We will first calculate $R/2L$ and $1/\sqrt{LC}$.

$$\frac{R}{2L} = \frac{100}{2 \times 0.05} = 10^3 \qquad (6\text{-}125a)$$

$$\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.05 \times 0.2 \times 10^{-6}}} = 10^4 \qquad (6\text{-}125b)$$

Since $R/2L < 1/\sqrt{LC}$, the circuit is underdamped and oscillatory. We have

$$\alpha = 10^3 \text{ nepers}^{\dagger} \qquad (6\text{-}126)$$

$$\omega_0 = 10^4 \text{ rad/s} \qquad (6\text{-}127)$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9.95 \times 10^3 \text{ rad/s} \qquad (6\text{-}128)$$

As a result of the relatively small amount of damping, the damped resonant frequency differs from the undamped resonant frequency by only 0.5%. As a matter of interest, the damped repetition frequency is $f_d = \omega_d/2\pi = 1548$ Hz. Notice that the natural damped frequency is about twice the frequency of the excitation. Again, we point out that these preliminary calculations are not absolutely necessary as the results will "fall out" of the math that follows.

The transformed circuit is shown in Fig. 6-23b. Using the impedance concept, we have

$$\begin{aligned} Z(s) &= 0.05s + 100 + \frac{5 \times 10^6}{s} \\ &= \frac{0.05s^2 + 100s + 5 \times 10^6}{s} \\ &= \frac{s^2 + 2000s + 10^8}{20s} \end{aligned} \qquad (6\text{-}129)$$

The current is

$$I(s) = \frac{E(s)}{Z(s)} = \frac{10^7 s}{(s^2 + 25 \times 10^6)(s^2 + 2000s + 10^8)} \qquad (6\text{-}130)$$

The poles due to the quadratic with three terms are

$$\begin{Bmatrix} s_1 \\ s_2 \end{Bmatrix} = -10^3 \pm j9.95 \times 10^3 \qquad (6\text{-}131)$$

which agrees with our preliminary calculations.

We obtain the final desired result by finding the inverse transform of $I(s)$. Since one quadratic has imaginary roots and the other has complex roots, we

Cont..

may invert the function by applying the special formula of Section 5-7 individually to the two quadratic factors. The reader is invited to show that the result is

$$i(t) = 0.133\epsilon^{-1000t} \sin(9.95 \times 10^3 t - 99.51^\circ) + 0.132 \sin(5000t + 82.41^\circ) \quad (6\text{-}132)$$

The response is seen to consist of a damped sinusoidal term whose frequency is the natural damped resonant frequency of the circuit, and an undamped sinusoid whose frequency is that of the excitation. The former term is transient in nature, whereas the latter term is the steady-state response. After the transient disappears, the steady-state or forced response is

$$i_{ss}(t) = 0.132 \sin(5000t + 82.41^\circ) \quad (6\text{-}133)$$

UNIT 3
LOCUS DIAGRAMS AND NEWORK FUNCTIONS

LOCUS DIAGRAMS

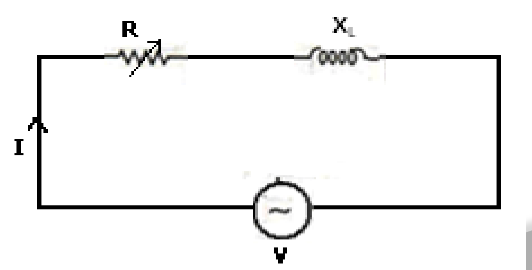
- Locus diagrams are the graphical representations of the way in which the response of electrical circuits vary, when one or more parameters are continuously changing. They help us to study the way in which current / power factor vary, when voltage is kept constant, Voltage / power factor vary, when current is kept constant, when one of the parameters of the circuit (whether series or parallel) is varied.

- The Locus diagrams yield such important information as I_{max}, I_{min} , V_{max} ,V_{min} & the power factor`s at which they occur. In some parallel circuits, they will also indicate whether or not, a condition for response is possible.

LOCUS OF SERIES RL CIRCUIT

- Consider an $R - X_L$ series circuit as shown below, across which a constant voltage is applied. By varying R or X_L, a wide range of currents and potential differences can be obtained.
- R can be varied by the rheostat adjustment and X_L can be varied by using a variable inductor or by applying a variable frequency source.
- When the variations are uniform and lie between 0 and infinity, the resulting locus diagrams are circles

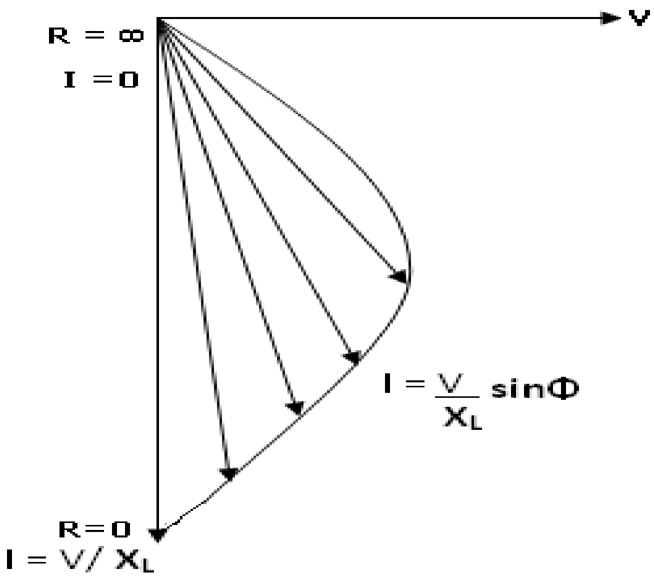
Case 1: when `R` is varied



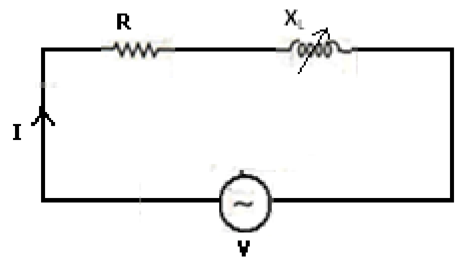
LOCUS OF SERIES RL CIRCUIT

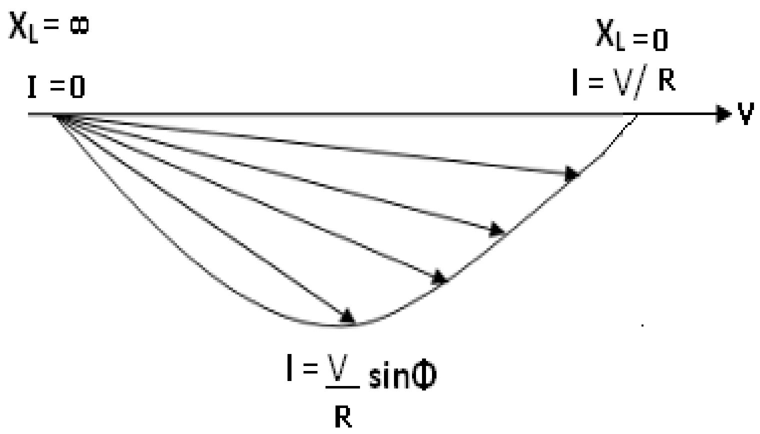
- When R = 0 , the current is maximum and is given by
- I_{max} = and lags V by 90^0
- Power factor is zero
- When R = infinity, the current is minimum and is given by I_{min} = 0, and power factor = 1
- For any other values of`R`, the current lags the voltage by an angle tan
- The general expression for current is
- The is the equation of a circle in the polar form, where is the diameter of the circle.
- The Locus diagram of current i.e the way in which the current varies in the circuit, as `R` is varied from zero to infinity is shown in below which is a semi -circle.

LOCUS OF SERIES RL CIRCUIT CURRENT WITH R VARIED



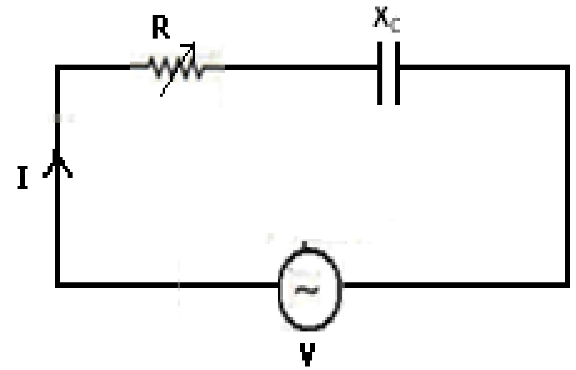
LOCUS OF SERIES RL CIRCUIT WITH X_L IS VARIED

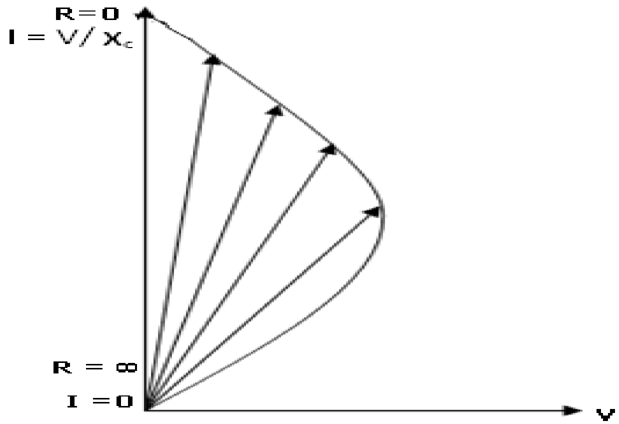




RC SERIES CIRCUIT

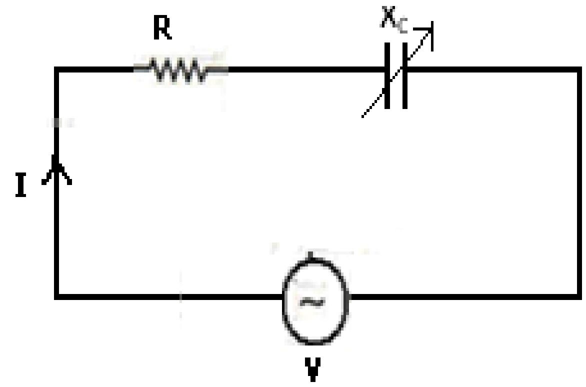
Case 1:when `R` is varied

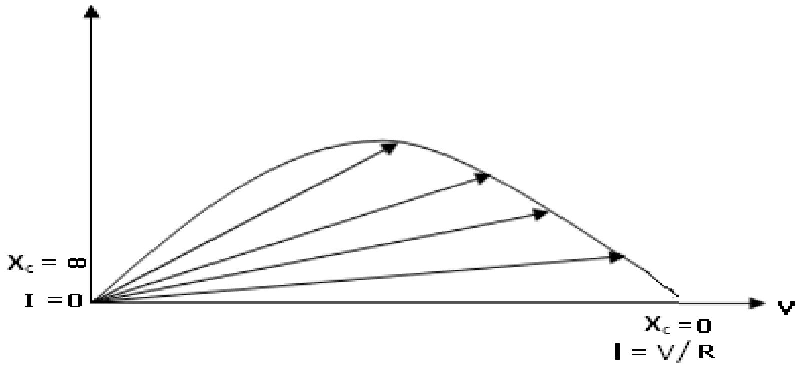




LOCUS OF SERIES RL CIRCUIT WITH X_C IS VARIED

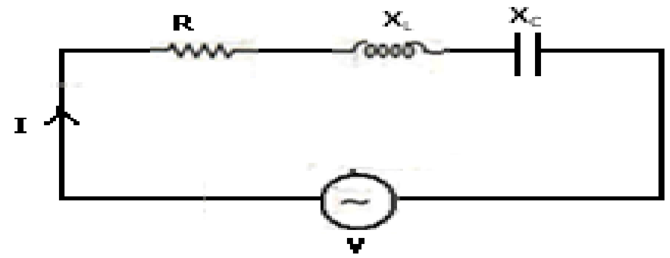
Case 2: Where X_c is varied

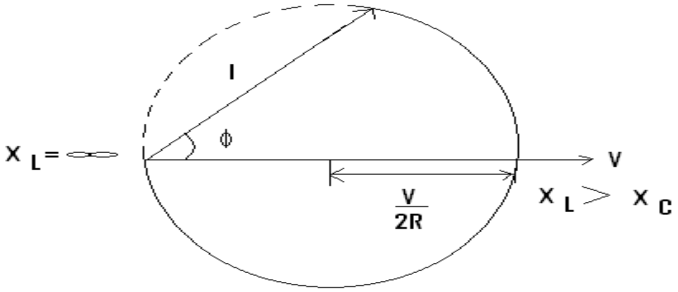




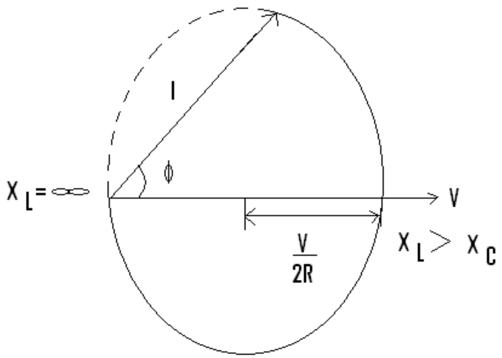
LOCUS OF CURRENT OF RLC SERIES CIRCUIT

- case1: when r is varied and the other three parameters are constant, the locus diagram of current shown below

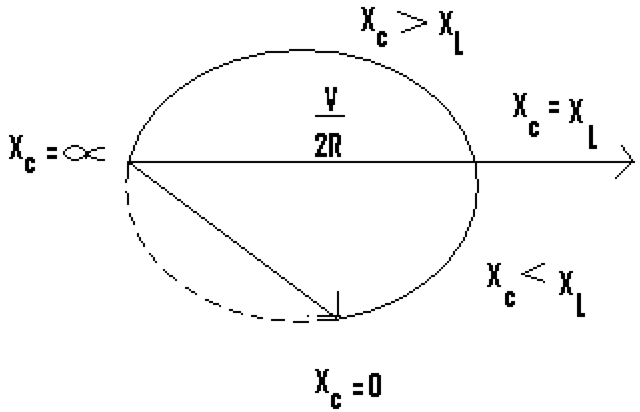




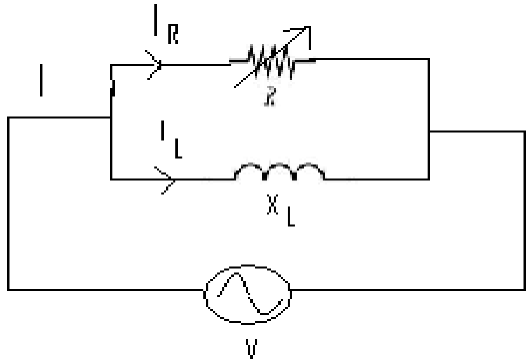
Case 2: When X_L is varied

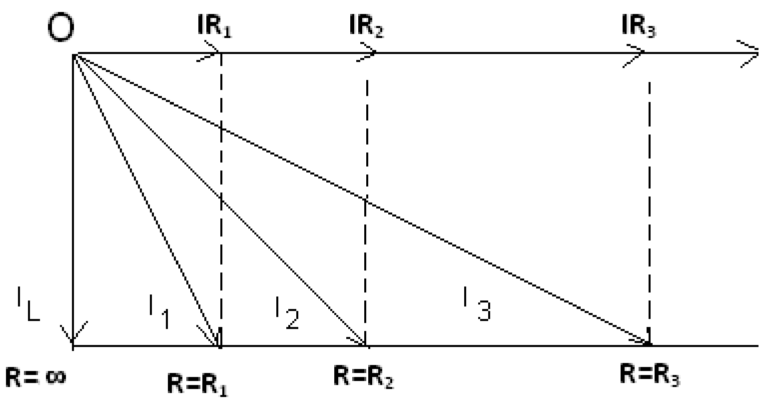


Case 3 : When X_C is varied

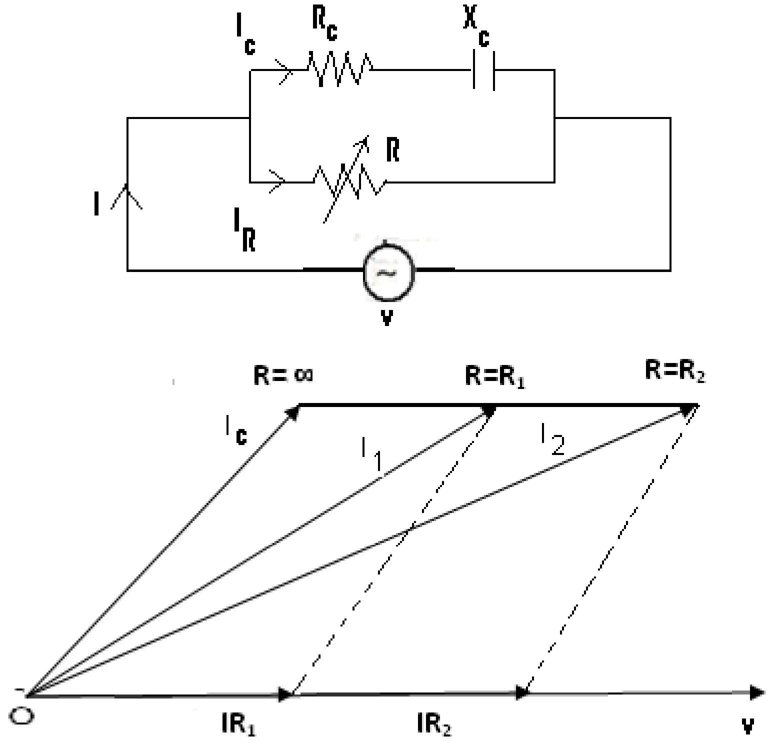


Case 1: R & X_L in parallel R Varying

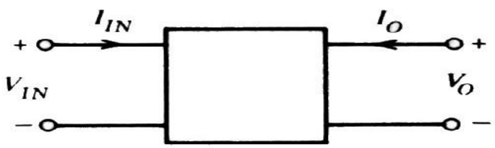




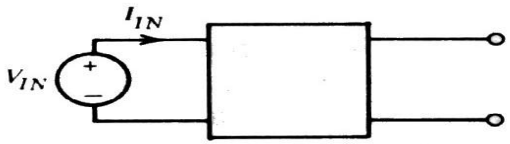
Case 2: R-X_C in parallel with R & 'R' varying.



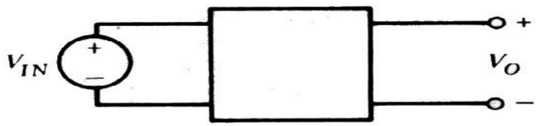
A **network function** is the Laplace transform of an impulse response. Its format is a ratio of two polynomials of the complex frequencies. Consider the general two-port network shown in Figure 2.2a. The terminal voltages and currents of the two-port can be related by two classes of network functions, namely, the driving point functions and the transfer functions.



(a) A two port network.



(b) Measuring input impedance.



(c) Measuring voltage gain.

POSSIBLE FORMS OF TRANSFER FUNCTIONS

- The voltage transfer function, which is a ratio of one voltage to another voltage.
- The current transfer function, which is a ratio of one current to another current.
- The transfer impedance function, which is the ratio of a voltage to a current.
- The transfer admittance function, which is the ratio of a current to a voltage

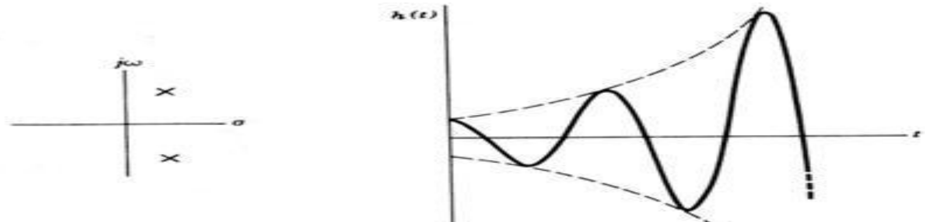
The voltage transfer functions are defined with the output

$$\text{voltage gain} = \frac{V_O(s)}{V_{IN}(s)}$$

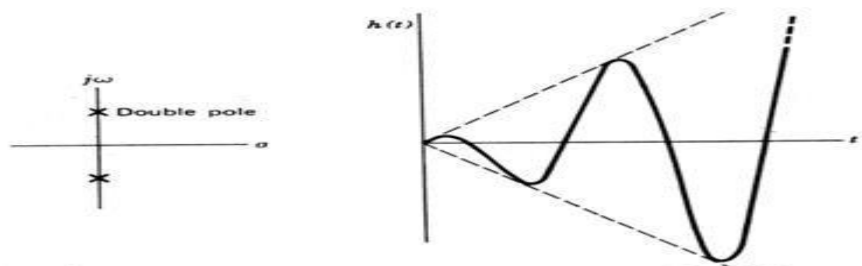
$$\text{voltage loss (attenuation)} = \frac{V_{IN}(s)}{V_O(s)}$$

The network functions of all passive networks and all stable active Must be rational functions in s with real coefficients.

- May not have poles in the right half s plane.
- May not have multiple poles on the jω axis.



Complex poles in the right half s plane.



Double poles on imaginary axis.

PROPERTIES OF DRIVING POINT FUNCTIONS

These conditions are required to satisfy to be positive realness

- Y(s) must be a rational function in s with real coefficients, i.e., the coefficients of the numerator and denominator polynomials is real and positive.
- The poles and zeros of Y(s) have either negative or zero real parts, i.e., Y(s) not have poles or zeros in the right half s plane.

Poles of Y(s) on the imaginary axis must be simple and their residues must be real and positive, i.e., Y(s) not has multiple poles or zeros on the jω axis. The same statement applies to the poles of l/Y(s).

PROPERTIES OF DRIVING POINT FUNCTIONS

- The degrees of the numerator and denominator polynomials in Y(s) differ at most by 1. Thus the number of finite poles and finite zeros of Y(s) differ at most by 1.

- The terms of lowest degree in the numerator and denominator polynomials of Y(s) differ in degree at most by 1. So Y(s) has neither multiple poles nor zeros at the origin.

- There be no missing terms in numerator and denominator polynomials unless all even or all odd terms are missing.

SYSTEM POLES AND ZEROS

- It is often convenient to factor the polynomials in the numerator and denominator, and to write the transfer function in terms of those factors

$$H(s) = \frac{a_n(s - z_1)(s - z_2)\cdots(s - z_n)}{b_m(s - p_1)(s - p_2)\cdots(s - p_m)}$$

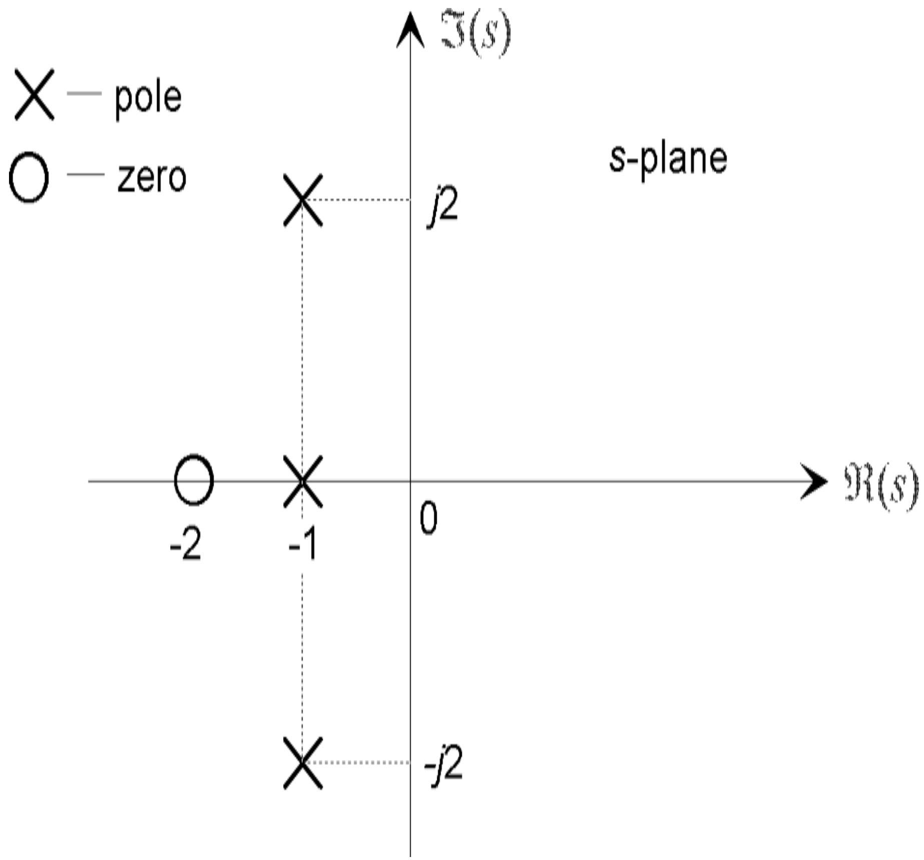
- Where the numerator and denominator polynomials, $N(s)$ and $D(s)$, have real coefficients defined by the system's differential equation and $K = b_m/a_n$. As written in Eq. (2) the z_i's are the roots of the equation N(s) =0
- and are defined to be the system zeros, and the p_i's are the roots of the equation D(s) = 0

POLE-ZERO PLOT

- The sense that they allow reconstruction of the input/output differential equation. In general, the poles and zeros of a transfer function may be complex, and the system dynamics may be represented graphically by plotting their locations on the complex s-plane, whose axes represent the real and imaginary parts of the complex variable s. Such plots are known as pole-zero plots.

- It is usual to mark a zero location by a circle (◦) and a pole location a cross (×). The location of the poles and zeros provide qualitative insights into the response characteristics of a system.

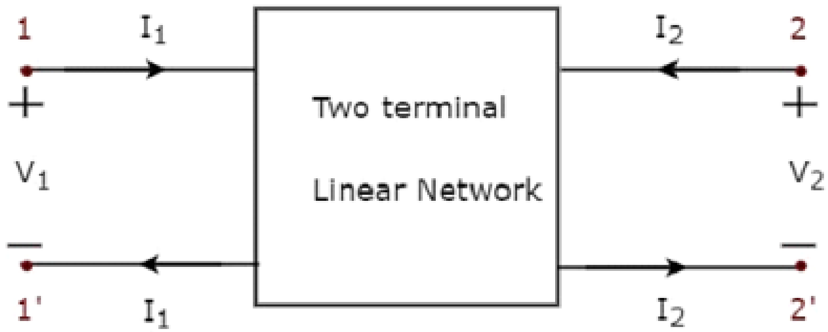
EXAMPLE OF POLE-ZERO PLOT



UNIT IV
TWO PORT NETWORK PARAMETERS

TWO PORT NETWORK

It is a pair of two terminal electrical network in which, current enters through one terminal and leaves through another terminal of each port. Two port network representation is shown in the following figure.



Here, one pair of terminals, 1 & 1' represents one port, which is called as port1 and the other pair of terminals, 2 & 2' represents another port, which is called as port2.

TWO PORT NETWORK

- There are four variables V_1, V_2, I_1 and I_2 in a two port network as shown in the figure.
- Out of which, we can choose two variables as independent and another two variables as dependent. So, we will get six possible pairs of equations.
- These equations represent the dependent variables in terms of independent variables.
- The coefficients of independent variables are called as parameters. So, each pair of equations will give a set of four parameters.

IMPEDANCE PARAMETERS (Z)

Z parameters are also known as impedance parameters. When we use Z parameter for analyzing two part network, the voltages are represented as the function of currents.

$$V_1 = f_1(I_1, I_2) \; and \; V_2 = f_2(I_1, I_2)$$

Two Port Networks

Z parameters:

$$z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

z_{11} is the impedance seen looking into port 1 when port 2 is open.

$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

z_{12} is a transfer impedance. It is the ratio of the voltage at port 1 to the current at port 2 when port 1 is open.

$$z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

z_{21} is a transfer impedance. It is the ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

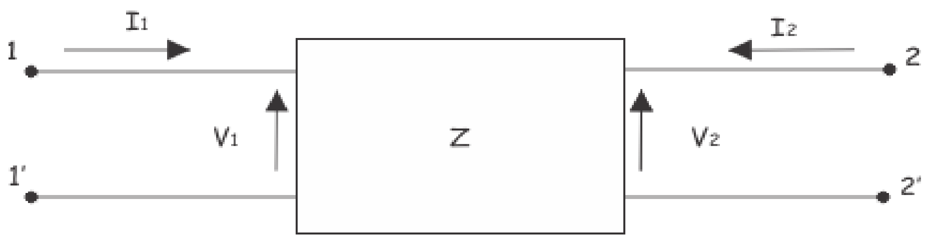
$$z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

z_{22} is the impedance seen looking into port 2 when port 1 is open.

IMPEDANCE PARAMETERS (Z)

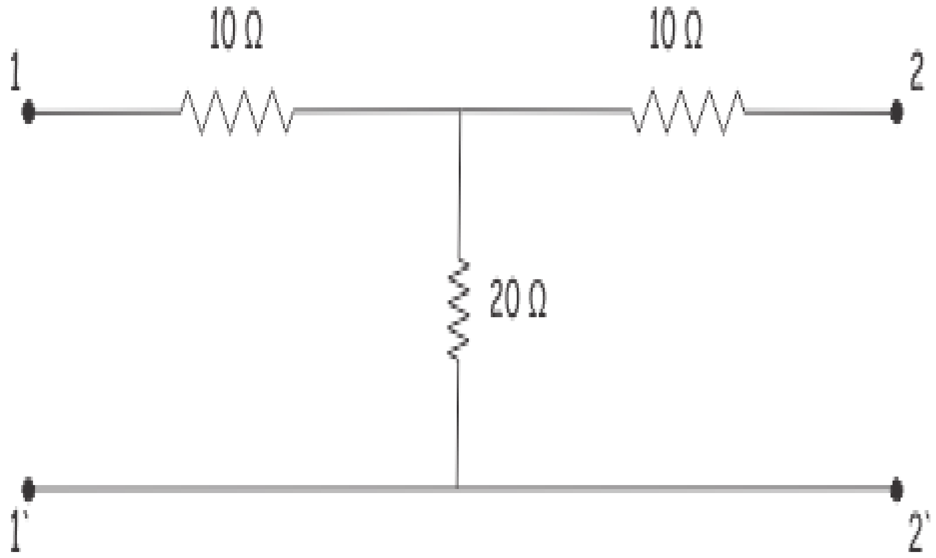
The voltages are represented as

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \text{ and } V_2 = Z_{21}I_1 + Z_{22}I_2$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Problem: Find the z parameters for network shown in figure



ADMITTANCE PARAMETERS (Y)

- We can represent current in terms of voltage for admittance parameters of a two port network. Then we will represent the current voltage relations as.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

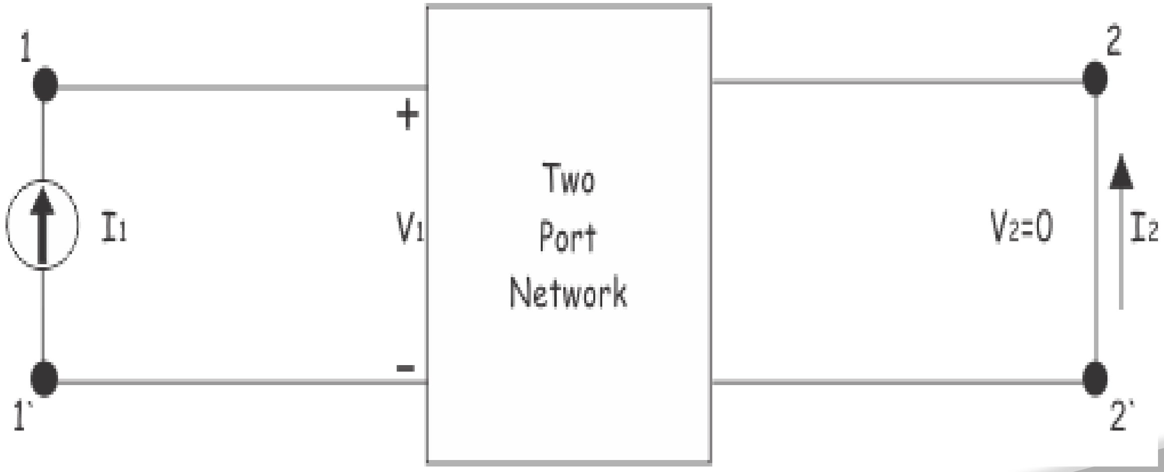
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

- This can also be represented in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

ADMITTANCE PARAMETERS (Y)

Here, Y11, Y12, Y21 and Y22 are admittance parameter. Sometimes these are called as Y parameters. We can determine the values of the parameters of a particular two port network by making short-circuited output port and input port alternatively as follows. First let us apply current source of I1 at input port keeping the output port short circuited as shown below.



ADMITTANCE PARAMETERS (Y)

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$

y_{11} is the admittance seen looking into port 1 when port 2 is shorted.

$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

y_{12} is a transfer admittance. It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

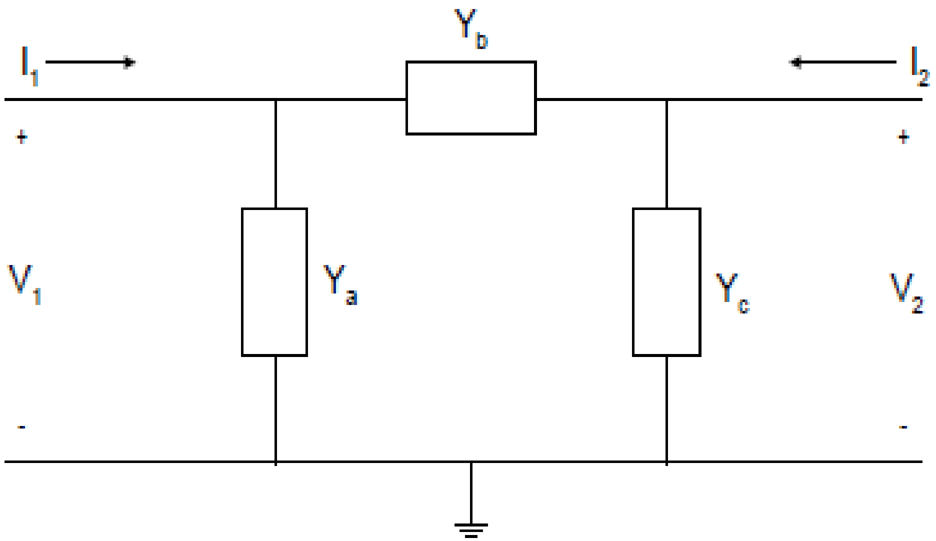
$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

y_{21} is a transfer impedance. It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

y_{22} is the admittance seen looking into port 2 when port 1 is shorted.

NUMERICAL PROBLEMS



HYBRID PARAMETERS OR h PARAMETERS

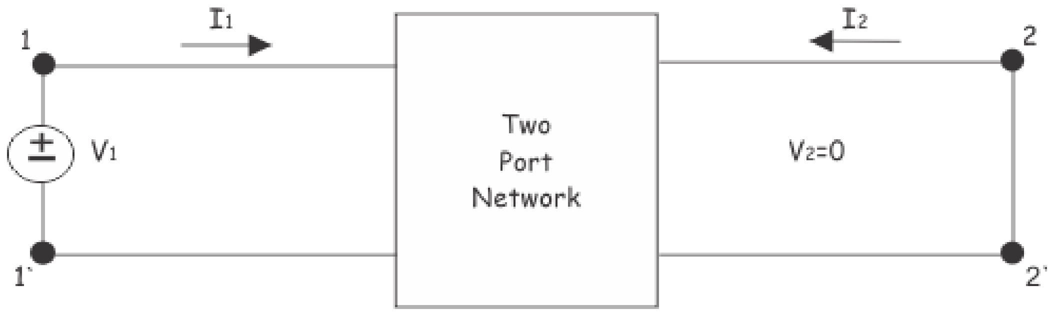
- Hybrid parameters are also referred as h parameters. These are referred as hybrid because, here Z parameters, Y parameters, voltage ratio, current ratio, all are used to represent the relation between voltage and current in a two port network.The relations of voltages and current in hybrid parameters are represented as

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

This can be represented in matrix form as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Let us short circuit the output port of a two port network as shown below



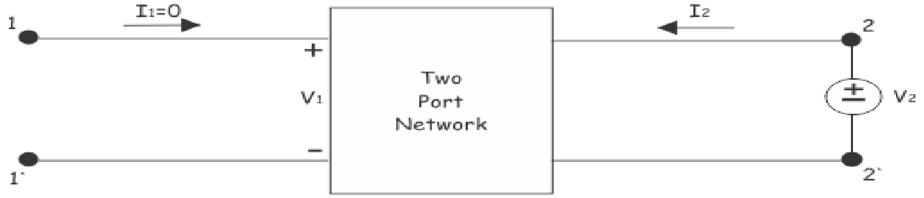
Now, ratio of input voltage to input current, at short circuited output port, is

$$\left.\frac{V_1}{I_1}\right|_{V_2=0} = h_{11}$$

This is referred as short circuit input impedance. Now, the ratio of the output current to input current at short circuited output port, is

$$\left.\frac{I_2}{I_1}\right|_{V_2=0} = h_{21}$$

- This is called short circuit current gain of the network. Now, let us open circuit the port 1. At that condition, there will be no input current (I1=0) but open circuit voltage V1 appears across the port 1, as shown below



$$\left.\frac{V_1}{V_2}\right|_{I_1=0} = h_{12} = open\ circuit\ reverse\ voltage\ gain$$

- This is referred as reverse voltage gain because, this is the ratio of input voltage to output voltage of the network, but voltage gain is defined as ratio of output voltage to input voltage of a network. Now

$$\left.\frac{I_2}{V_2}\right|_{I_1=0} = h_{21}$$

ABCD PARAMETERS

- Concerning the equivalent port representations of networks we've seen in this course:
- Z parameters are useful for series connected networks.
- Y parameters are useful for parallel connected networks.

parameters are useful for describing interaction of voltage and current waves with a network.

- There is another set of network parameters particularly suited for cascading two-port networks. This set is called the ABCD matrix or, equivalently, the transmission matrix

ABCD PARAMETERS

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2' \end{bmatrix}$$

$$I_2' = -I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2'=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2'=0}$$

$$B = \left.\frac{V_1}{I_2'}\right|_{V_2=0} \qquad D = \left.\frac{I_1}{I_2'}\right|_{V_2=0}$$

ABCD PARAMETERS

The ABCD parameters are defined as follows:

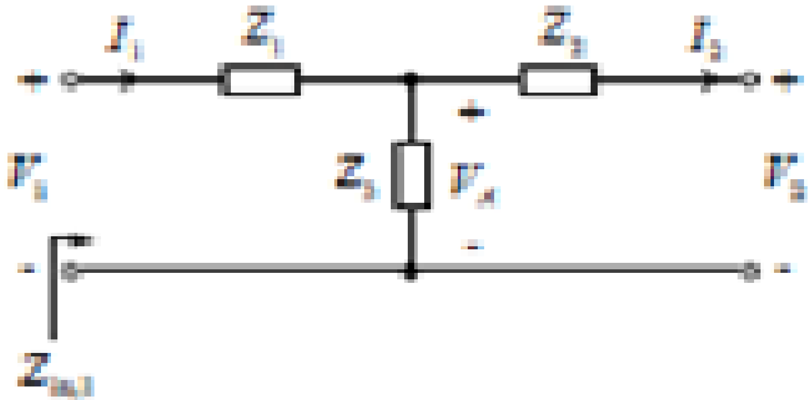
$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$ open circuit reverse voltage transfer ratio

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0}$$ = short circuit reverse transfer impedance

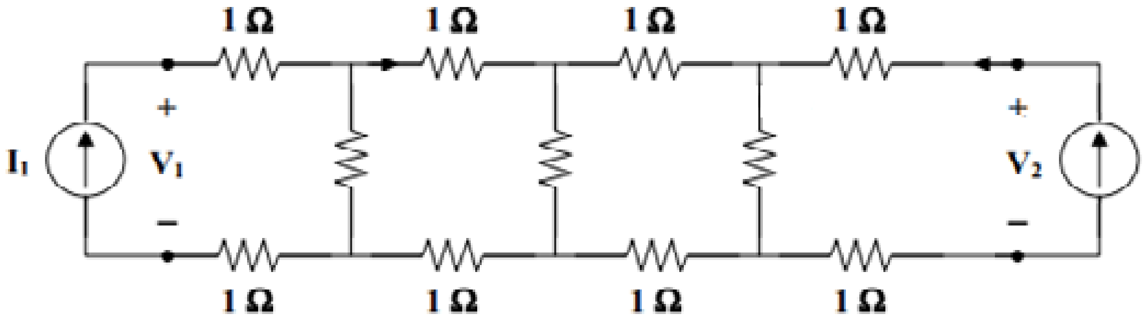
$$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$ open circuit reverse transfer admittance

$$D = \frac{I_1}{-I_2}\Big|_{V_2=0}$$ short circuit reverse current transfer ratio.

Derive the ABCD Parameters for the T network



- **Find the z parameters for network shown in figure**

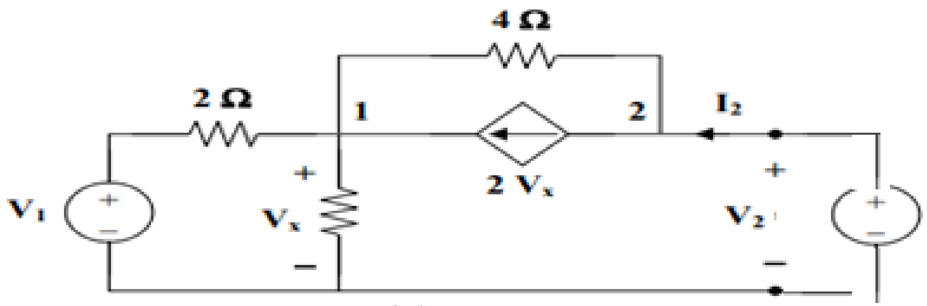


(a)

Thus,

$$[z] = \begin{bmatrix} 2.733 & 0.06667 \\ 0.06667 & 2.733 \end{bmatrix} \Omega$$

Find the Y parameters for network shown in figure

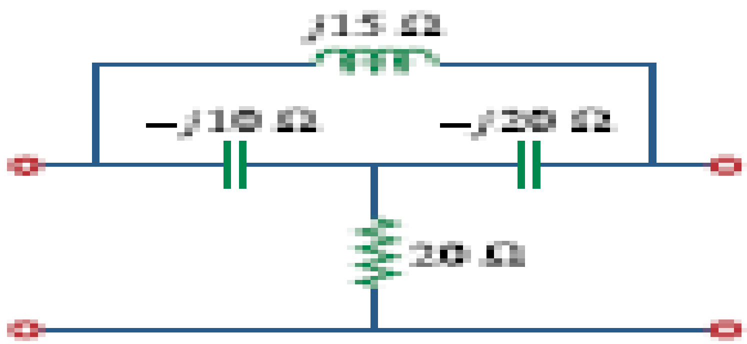


(a)

Thus,

$$[y] = \begin{bmatrix} 1.5 & 0.5 \\ -3.5 & -1.5 \end{bmatrix} S$$

Find the transmission parameters for network shown in figure



$$[T] = \begin{bmatrix} 0.7692 + j0.3461 & -6.923 + j25.385\ \Omega \\ 0.03461 + j0.023\ S & 0.5385 + j0.6923 \end{bmatrix}$$

CONDITION FOR RECIPROCITY

Properties:

I) Reciprocity

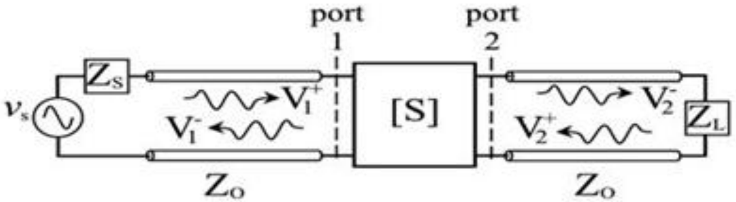
The two-port network is reciprocal if the transmission characteristics are the same in both directions (i.e. $S_{21} = S_{12}$).

It is a property of passive circuits (circuits with no active devices or ferrites) that they form reciprocal networks.

A network is reciprocal if it is equal to its transpose. Stated mathematically, for a reciprocal network

$$[S] = [S]^t,$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^t = \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix}.$$



By inspection If the network is

Symmetrical $\Longrightarrow$ Reciprocal

Condition for Reciprocity: $\mathbf{S_{12} = S_{21}}$

CONDITION FOR SYMMETRY

A network is symmetrical if its input impedance is equal to its output impedance. Most often, but not necessarily, symmetrical networks are also physically symmetrical

Symmetry . For the network to be symmetrical, the voltage–to–current ratio at one port should be the same as the voltage–to–current ratio at the other port with one of the port short circuited.

Condition for symmetry

$$\boxed{\begin{aligned} I_1 &= Y_{11} V_1 + Y_{12} V_2 \\ I_2 &= Y_{21} V_1 + Y_{22} V_2 \end{aligned}}$$

When the output port is short circuited, i.e., $V_2 = 0$. From the Y-parameter equation

$$I_1 = Y_{11} V_s$$

$$\frac{V_s}{I_1} = \frac{1}{Y_{11}}$$

When the input port is short circuited, i.e., $V_1 = 0$. From the Y-parameter equation,

$$I_2 = Y_{22} V_s$$

$$\frac{V_s}{I_2} = \frac{1}{Y_{22}}$$

Hence, for the network to be symmetrical,

$$\frac{V_s}{I_1} = \frac{V_s}{I_2}$$

$$\boxed{Y_{11} = Y_{22}}$$

TWO PORT PARAMETER CONVERSIONS

- convert one set of two-port network parameters into other set of two port network parameter inversion is known as two port network parameters conversion or simply, two-port parameters conversion.
- Step 1 − Write the equations of a two port network in terms of desired parameters
- Step 2 − Write the equations of a two port network in terms of given parameters.
- Step 3 − Re-arrange the equations of Step2 in such a way that they should be similar to the equations of Step1.
- Step 4 − By equating the similar equations of Step1 and Step3, we will get the desired parameters in terms of given parameters. We can represent these parameters in matrix form.

TWO PORT PARAMETER CONVERSIONS

Z parameters to T parameters

Here, we have to represent T parameters in terms of Z parameters. So, in this case T parameters are the desired parameters and Z parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations, which represents a two port network in terms of **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow V_2 - Z_{22}I_2 = Z_{21}I_1$$

$$\Rightarrow I_1 = \left(\frac{1}{Z_{21}}\right)V_2 - \left(\frac{Z_{22}}{Z_{21}}\right)I_2$$

TWO PORT PARAMETER CONVERSIONS

Step 4 − The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Step 5 − Substitute I_1 value of Step 3 in V_1 equation of Step 2.

$$V_1 = Z_{11}\{(\frac{1}{Z_{12}})V_2 - (\frac{Z_{22}}{Z_{21}})I_2\} + Z_{12}I_2$$

$$\Rightarrow V_1 = (\frac{Z_{11}}{Z_{21}})V_2 - (\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}})I_2$$

Step 6 − The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

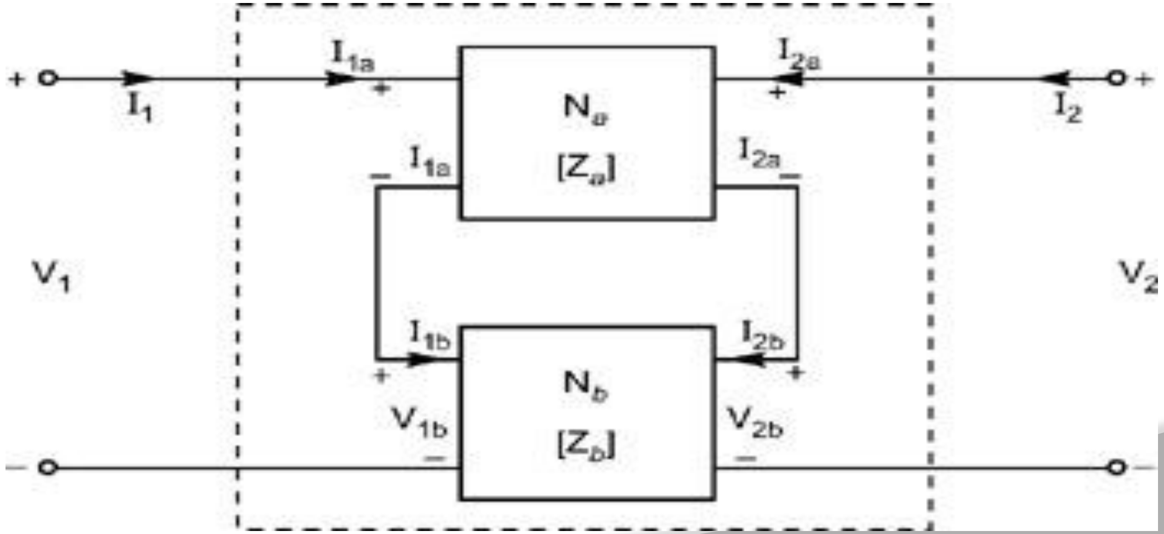
$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

- Interconnections of two-port networks

  Two-port networks may be interconnected in various configurations, such as series, parallel, cascade, series-parallel, and parallel-series connections. For each configuration a certain set of parameters may be more useful than others to describe the network.

- Series connection



- Series connection of two two-port networks for network N $_a$

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$(10.63) V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}.$$

$$(10.63) V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}.$$

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22}b \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$(10.65) V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$$

network can be written as

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2,$$

$$Z_11 = Z_{11a} + Z_{11b}$$

$$Z_12 = Z_{12a} + Z_{12b}$$

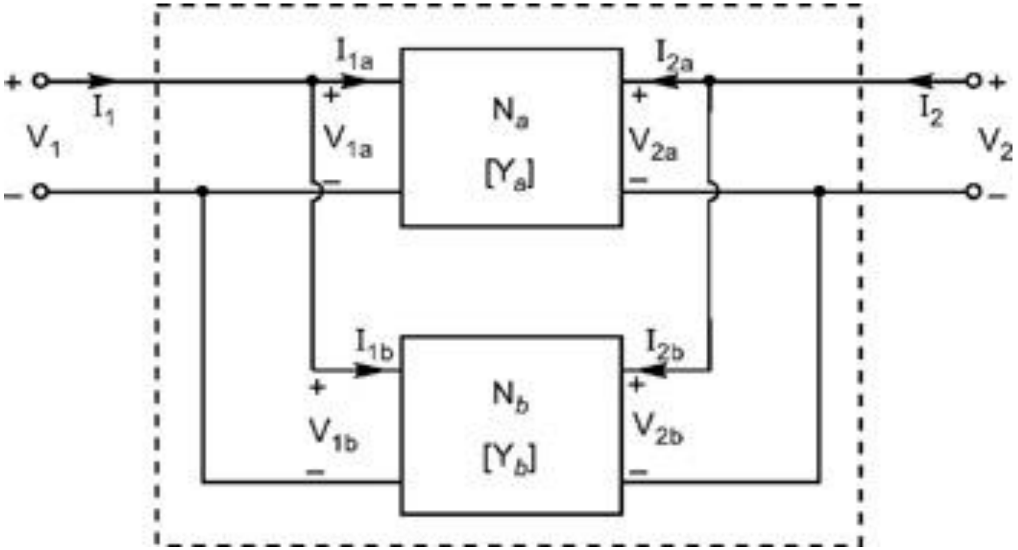
$$Z_21 = Z_{21a} + Z_{21b}$$

$$Z_22 = Z_{22a} + Z_{22b}$$

$$[Z] = [Z_a] + [Z_b].$$

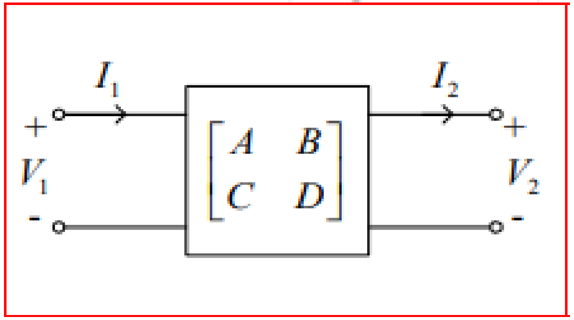
PARALLEL CONNECTION

Parallel connection of two two-port networks N a and N b. The resultant of two admittances connected in parallel is Y 1 + Y 2. So in parallel connection, the parameters are Y-parameters.



There is another set of network parameters particularly suited for cascading two-port networks. This set is called the *ABCD* matrix or, equivalently, the transmission matrix.

Consider this two-port network (Fig. 4.11a):



Unlike in the definition used for Z and Y parameters, notice that I_2 is directed away from the port. This is an important point and we'll discover the reason for it shortly.

CASCADE CONNECTION OF TWO PORT NETWORKS

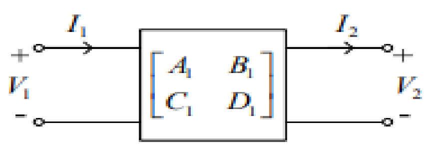
It is easy to show that

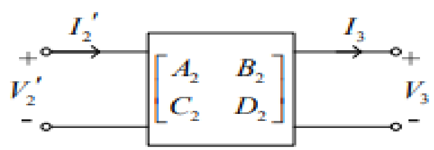
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \quad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}, \quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Note that not all of these parameters have the same units.

To see this, consider the following two-port networks:





In matrix form

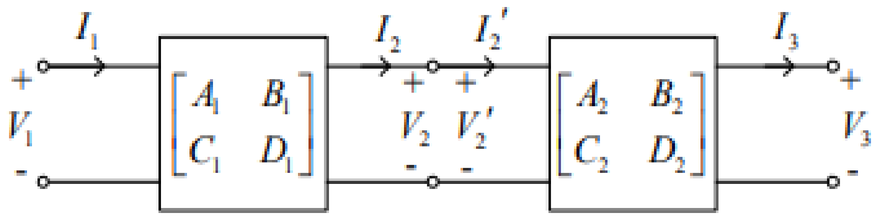
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad (2)$$

and

$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \qquad (3)$$

When these two-ports are cascaded,

CASCADE CONNECTION OF TWO PORT NETWORKS



it is apparent that $V_2' = V_2$ and $I_2' = I_2$. (The latter is the reason for assuming I_2 out of the port.) Consequently, substituting (3) into (2) yields

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \qquad (4)$$

We can consider the matrix-matrix product in this equation as describing the cascade of the two networks.

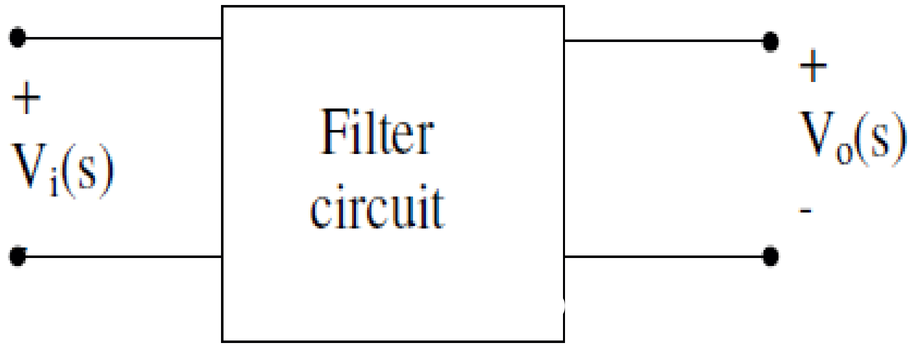
UNIT 5
FILTERS

INTRODUCTION TO FILTERS

- Filters are essential building blocks in many systems, particularly in communication and instrumentation systems. A filter passes one band of frequencies while rejecting another.
- Typically implemented in one of three technologies: passive RLC filters, active RC filters and switched capacitor filters. Crystal and SAW filters are normally used at very high frequencies.
- Passive filters work well at high frequencies, however, at low frequencies the required inductors are large, bulky and non-ideal.

INTRODUCTION TO FILTERS

- Furthermore, inductors are difficult to fabricate in monolithic from and are incompatible with many modern assembly systems. Active RC filters utilize op- amps together with resistors and capacitors and are fabricated using discrete, thick film and thin-film technologies.
- The performance of these filters is limited by the performance of the op-amps (e.g., frequency response, bandwidth, noise, offsets, etc.). Switched-capacitor filters are monolithic filters which typically offer the best performance in the term of cost.
- Fabricated using capacitors, switched and op-amps. Generally poorer performance compared to passive LC or active RC filters.



- The filter transfer function is given as follows: $T(j\omega) = T(s) = \frac{V_o(s)}{V_i(s)}$

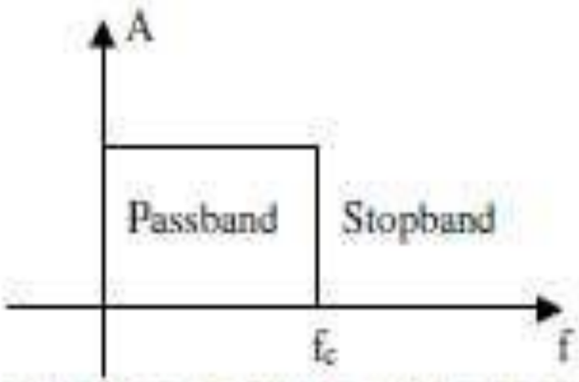
- The magnitude of the transmission is often expressed in dB in terms of gain function: G(w)dB=20log(|T(jw)|

Or, alternatively, in terms of the attenuation function:

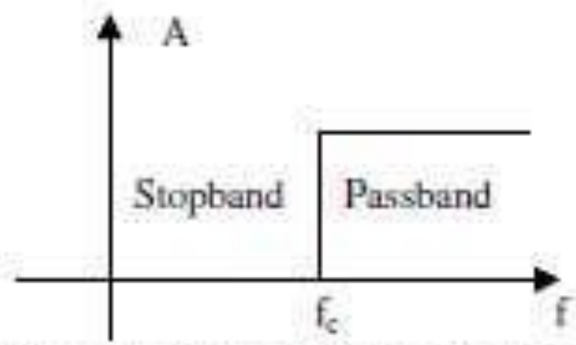
A(w)dB=-20log(|T(jw)|

CLASSIFICATION OF FILTERS

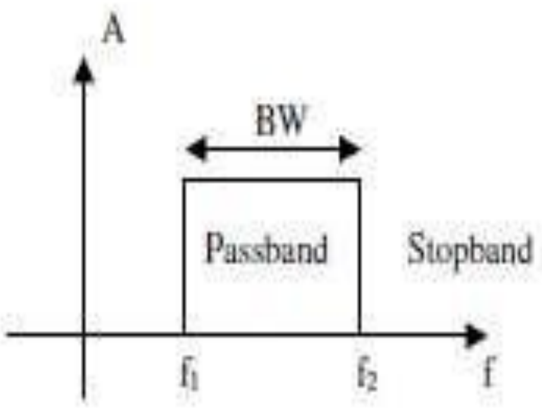
- A filter shapes the frequency spectrum of the input signal, according to the magnitude of the transfer function. The phase characteristics of the signal are also modified as it passes through the filter. Filters can be classified into a number of categories based on which frequency bands are passes through and which frequency bands are stopped. Figures below show ideal responses of various filters.



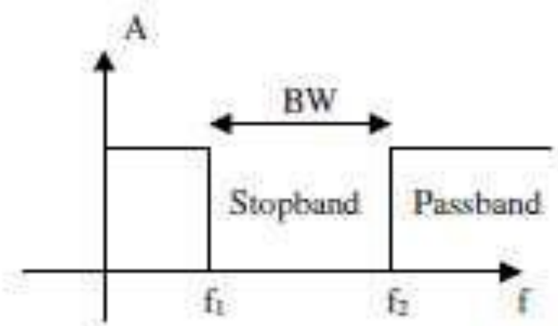
LOW-PASS FILTER RESPONSE



HIGH-PASS FILTER RESPONSE



BANDPASS FILTER RESPONSE



BANDSTOP FILTER RESPONSE

Classification of Pass band and Stop band:

Ideal filters could not be realized using electrical circuits, therefore the actual response of the filter is not a brick wall response as shown above but increases or decreases with a roll-off factor. Realistic transmission characteristics for a low pass filter are shown below

FILTERS

- Frequency-selective or filter circuits pass to the output only those input signals that are in a desired range of frequencies (called pass band).
- The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero).

- Typically in these circuits, the input and output currents are kept to a small valueand as such, the current transfer function is not an important parameter.
- The main parameter is the voltage transfer function in the frequency domain, Hv (jω) = Vo/Vi. As Hv (jω) is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals.
- To minimize the number of subscripts, hereafter, we will drop subscript v of Hv. Furthermore, we concentrate on the ‖open-loop‖ transfer functions, Hvo, and denote this simply by H(jω).

Low-Pass Filters:

- An ideal low-pass filter's transfer function is shown. The frequency between the
- pass- and-stop bands is called the cut-off frequency (ωc).
- All of the signals with frequencies below ωc are transmitted and all other signals are stopped.

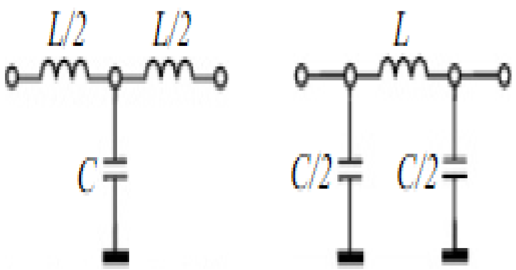
Band-pass filters:

- A band pass filter allows signals with a range of frequencies (pass band) to pass through and attenuates signals with frequencies outside this range.

THE LOW PASS CONSTANT-k FILTER

The constant-k LPF can have the configurations from *Figure*. The cutoff frequency is given by:

$$\omega_t = \frac{2}{\sqrt{LC}}$$

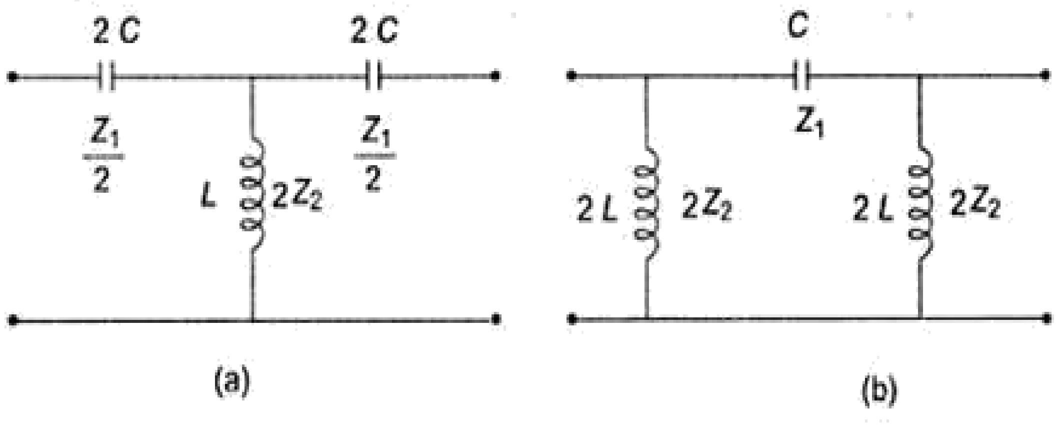


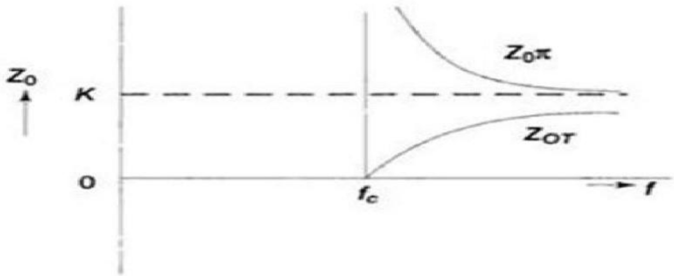
Figure

Generally the filter works on a constant load (R_s). To design the filter, R_s and ω_t are given. The matching can not be done at any frequency therefore we have to choose the frequency at which the filter will match. Most of the times, LPF matches in d.c. ($\omega = 0$). The elements of the filter are given by:

$$L = \frac{2R_s}{\omega_t} \qquad C = \frac{2}{\omega_t R_s}$$

CONSTANT K –HIGH PASS FILTER





DESIGN OF CONSTANT K-HIGH PASS FILTER

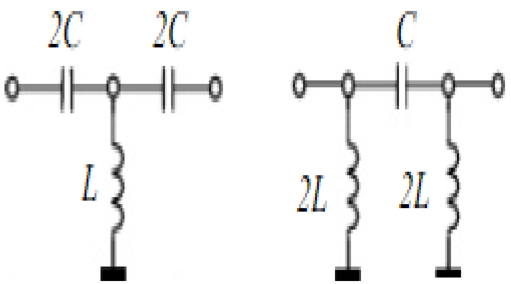
THE HIGH PASS CONSTANT-K FILTER

The possible configurations of the constant-k HPF are shown The cutoff frequency is given by:

$$\omega_t = \frac{1}{2\sqrt{LC}}$$

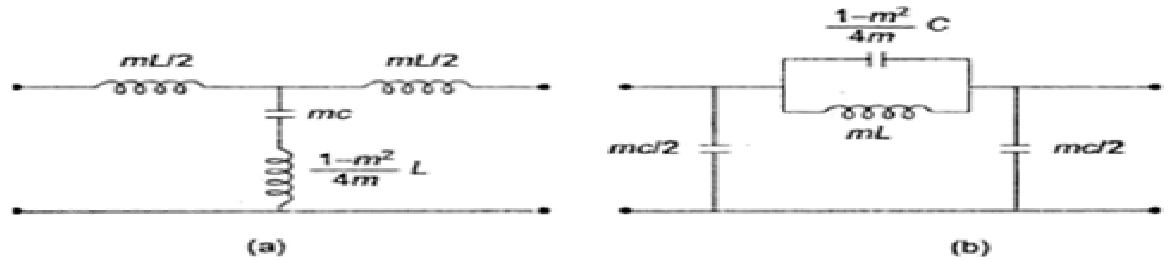
If we are interesting in matching at very high frequency $(\omega \to \infty)$, then L and C are given by:

$$L = \frac{R_s}{2\omega_t} \qquad C = \frac{1}{2R_s\omega_t}$$



Figure

m-DERIVED LOW PASS FILTER



$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4M}\right)\omega_r C}$$

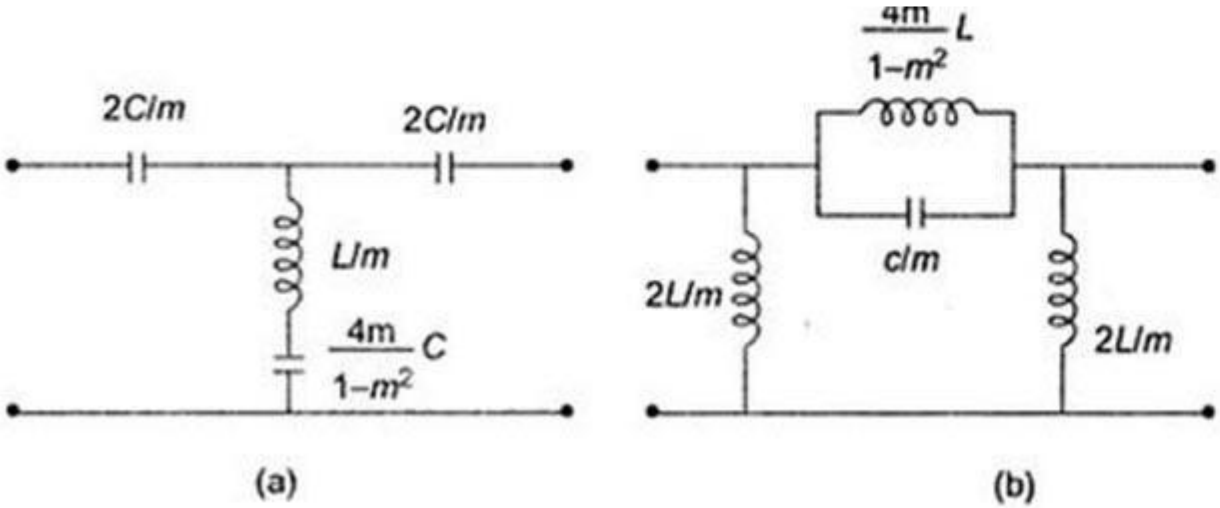
$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\alpha}\right)^2}}$$

And $$\beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1=m)^2}}$$

m-DERIVED HIGH PASS FILTER:

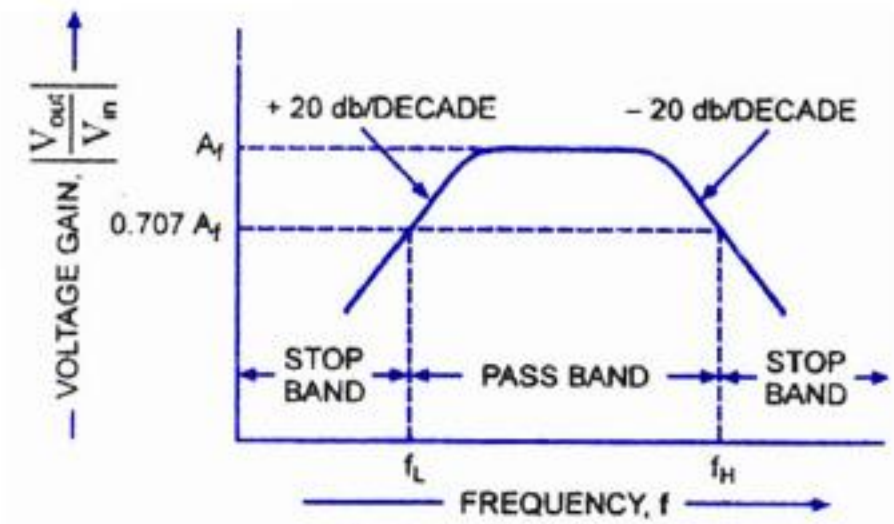


$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\alpha^2 = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \quad or \quad f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$

BAND PASS FILTER

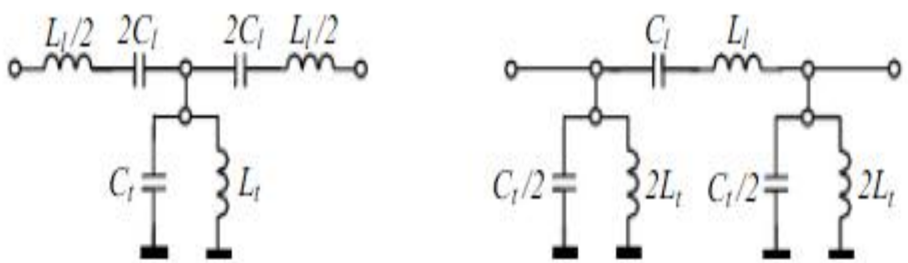


Frequency Response

Frequency Response wide bandpass filter

THE BAND PASS CONSTANT-K FILTER

The constant-*k BPF* configurations are shown in *Figure*



Figure

If the cutoff frequencies (ω_L and ω_H) and the load (R_s) are given, then the elements of the filter are given by:

$$\begin{cases} L_l = \dfrac{2R_s}{\omega_s - \omega_i} \\ C_l = \dfrac{\omega_s - \omega_i}{2R_s\omega_0^2} \end{cases} \qquad \begin{cases} L_t = \dfrac{(\omega_s - \omega_i)R}{2\omega_0^2} \\ C_t = \dfrac{2}{(\omega_s - \omega_i)R} \end{cases}$$

BAND STOP FILTER CHARACTERISTICS