STRUCTURAL ANALYSIS (ACE008)
IARE-R16
B.Tech V Semester

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UNIT-I
ANALYSIS OF TRUSSES

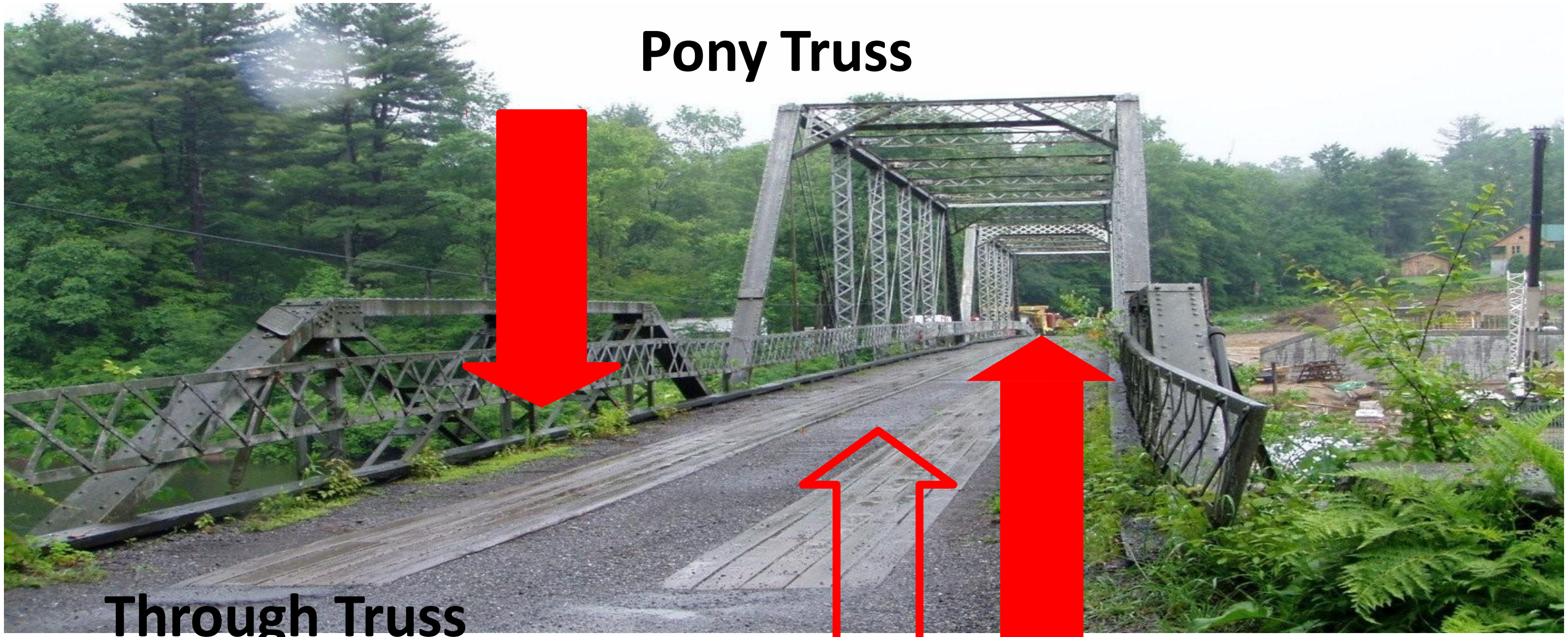
Truss Bridges

A metal truss bridge is a bridge whose main structure comes from a triangular framework of structural steel or iron.

Iron and Steel

metal truss

If the trusses run beside the deck, with no cross bracing above the deck, it is called a pony truss bridge



If cross-bracing is present above the deck of the bridge, then the bridge is referred to as a "through truss."

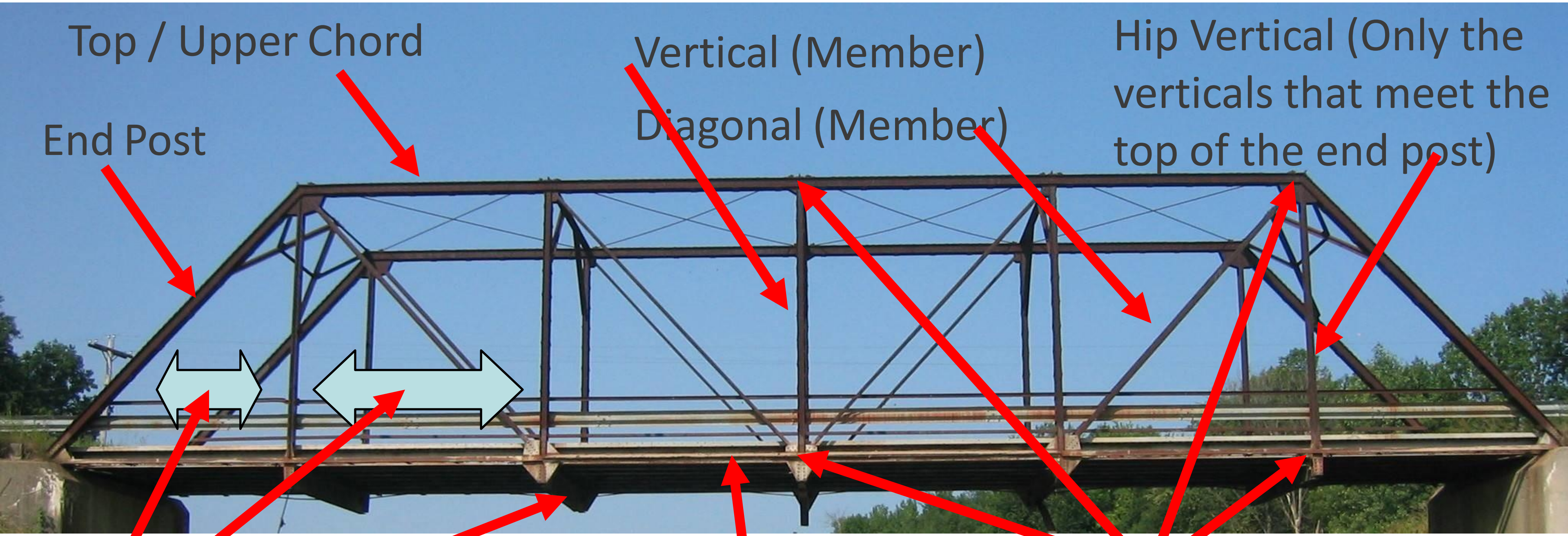
Truss Basics

Deck Truss

Trusses may run under the deck: these are called simply Deck truss bridges.

Truss Bridge Parts

The different parts of a truss bridge are all named. Some of the parts:



Floor beam

Bottom / Lower Chord

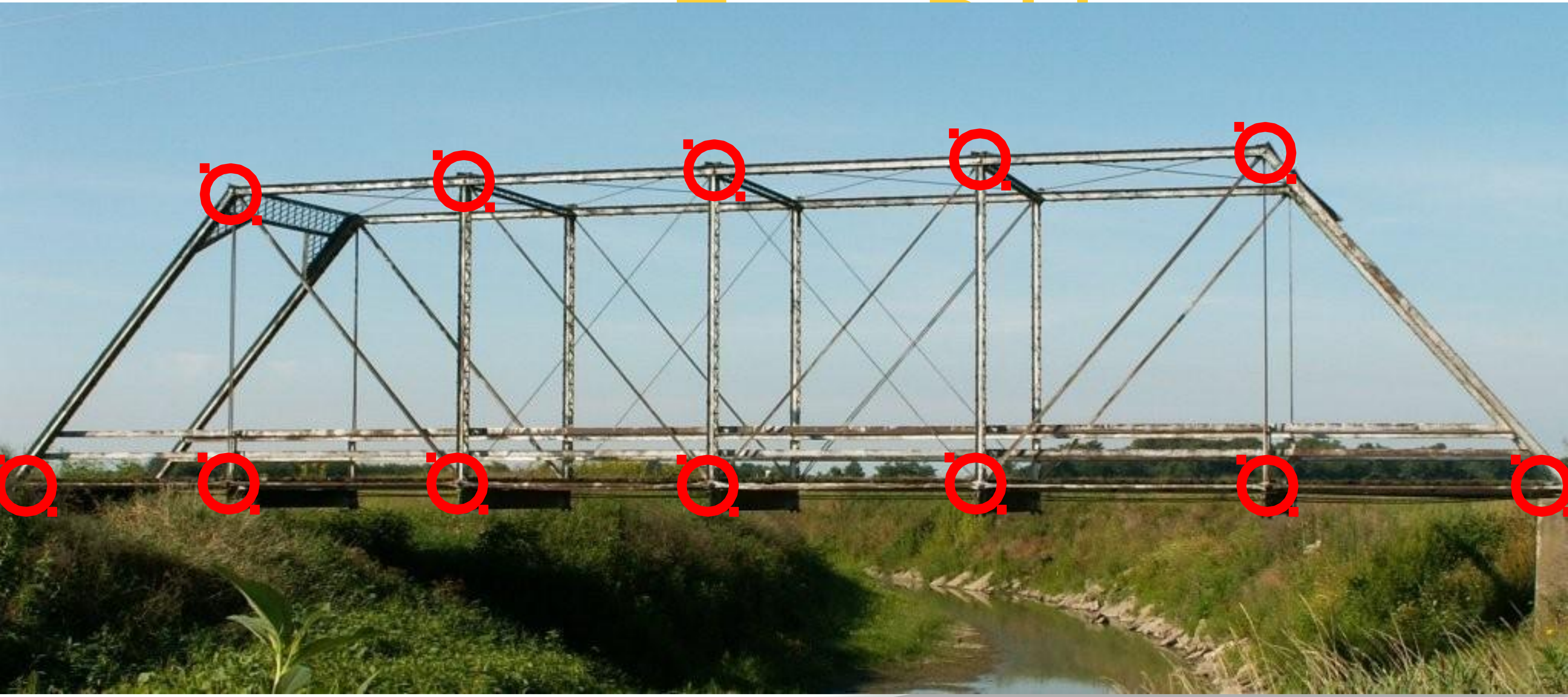
Each space between vertical members and end posts is one panel. This bridge has six

Portal Bracing

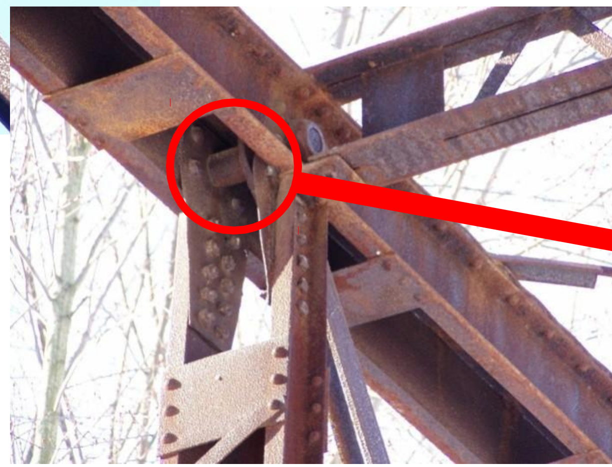
Sway Bracing

Lateral Bracing

The pieces of the framework of a truss bridge are held together by **connections**. Most connections on historic bridges are either riveted or pinned.



Pinned Connections

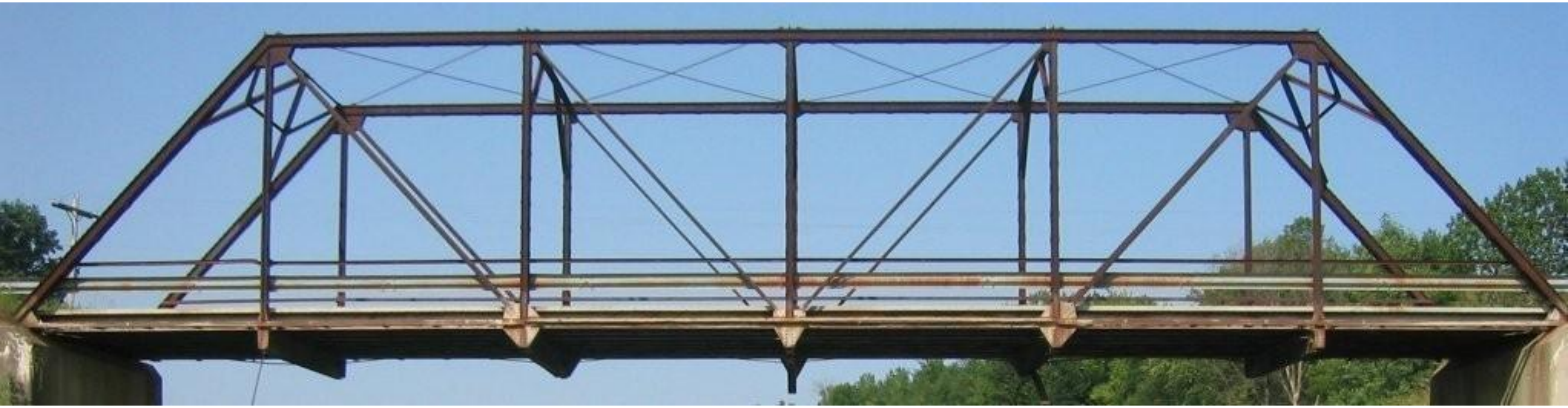


Pinned connections can be identified by the bolt-like object called a **pin** going through the loops of the members. They tend to show up on bridges from the first half of the truss bridge era.

Riveted Connections

Riveted connections are identified by a "**gusset plate**" which diagonals and vertical members are riveted to, and no pin is present. These connections tend to show up in the second half of the truss bridge era.

Truss Configurations



Overview: One of the two most common configurations, it tends to occupy the earlier half of the truss bridge era, but was used throughout. Originally developed by Thomas and Caleb Pratt in 1844.

Appearance: Diagonal members angle toward the center and bottom of bridge.

Pratt – Additional Notes

The Pratt may have additional diagonal members, sometimes of a smaller size, that do not follow the standard pattern to form an "X" shape on panels toward the center.

Truss Configurations

Whipple

Overview: The Whipple truss is also known as the double- intersection Pratt truss. It was patented by Squire Whipple in 1847 as a stronger version of the Pratt truss.

Appearance: Similar to the Pratt truss, but the diagonals pass through one vertical member before reaching the bottom chord. They tend to show up on longer spans built in the first half of the truss era, and with pinned connections.

Truss Configurations

Overview: The Baltimore railroad designed a truss configuration that eventually found use on both railroads and highways. It is a Pratt truss with additional members added for additional strength.

Appearance: Characterized by a Pratt configuration with extra smaller members branching off of the diagonals.

Truss Configurations

Overview: Charles H. Parker modified the Pratt design to create what became known as the Parker truss configuration. This design allowed one to use less materials to get the a similar load capacity. The downside was the more complex design.

Appearance: Characterized by an arch-shaped (polygonal) top chord, with diagonals that follow the Pratt configuration.

Pennsylvania

Overview: Sometimes called the Petit truss. Designed by the Pennsylvania railroad, this configuration combines the engineering ideas behind the Baltimore with those of the Parker or Camelback.

Appearance: Features an arch-shaped (polygonal) top chord with a diagonal arrangement like the Baltimore.

Truss Configurations

Overview: The other most common truss configuration, this design tended to be used in the second half of the truss bridge era, and with riveted connections. Originally developed in 1848 by James Warren and Willoughby Monzoni.

Appearance: Alternating diagonal members form a repeating "V" shape. A true Warren does not have vertical members.

Warren: With Verticals

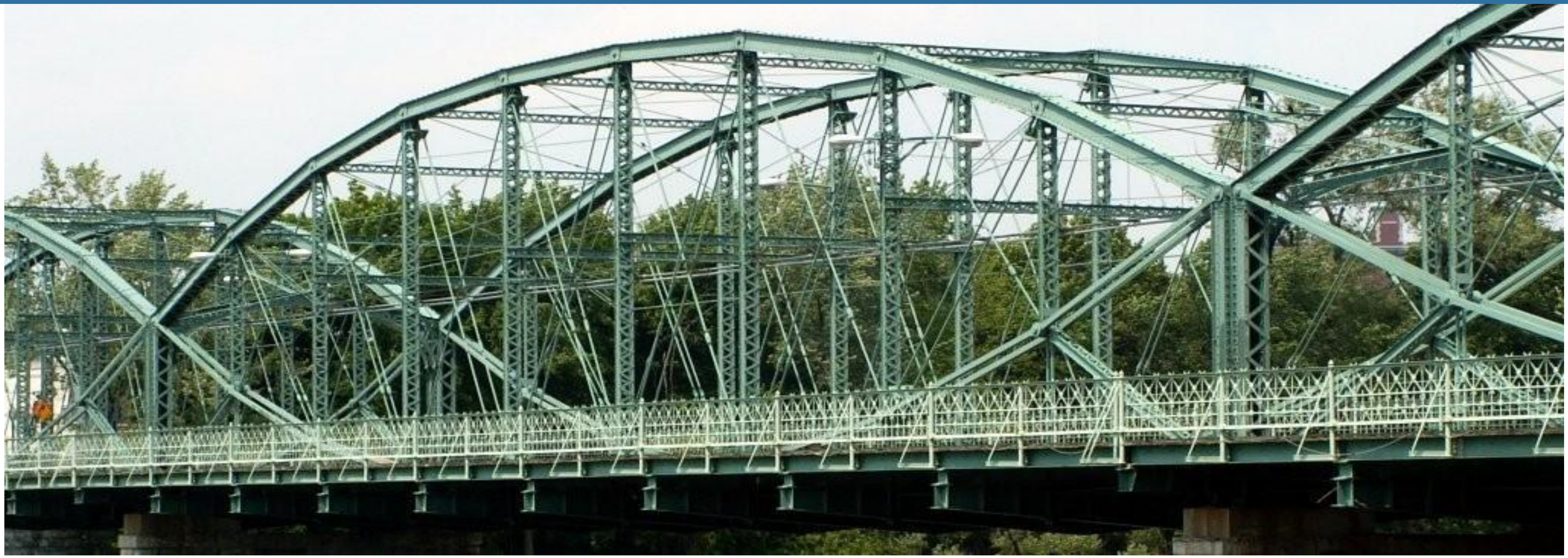
Most Warren truss bridges do in fact feature vertical members. They may be referenced simply as “**warren with verticals**” truss bridges. Vertical members may occur at each connection, or every other connection.

Double-Intersection Warren

Overview: Often called simply the Double Warren, this is an uncommon truss configuration. Bridges with this configuration often have riveted connections.

Appearance: Looks like two Warren trusses offset and superimposed on each other, forming a repeating “X” shape.

Lenticular



Overview: One of the rarest bridge designs in the country. Patented by the Berlin Iron Bridge Company of East Berlin, CT

Appearance: Both the top chord and bottom chord have an arched appearance, forming a distinctive oval or eye-like shape.

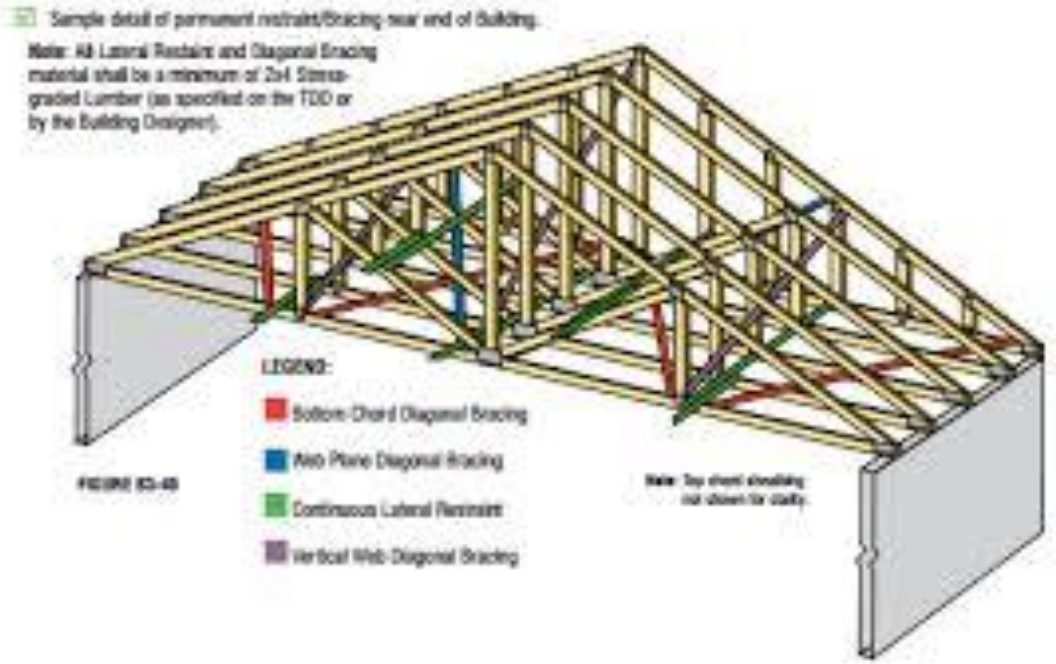
ANALYSIS OF PIN-JOINTED FRAMES (TRUSSES)

Truss/ Frame

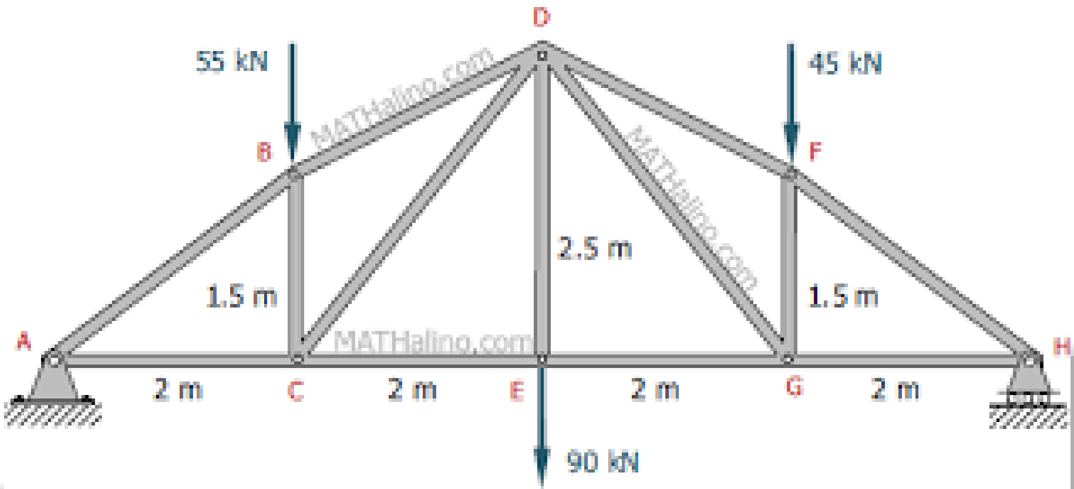
Truss/ Frame: A pin jointed frame is a structure made of slender **(cross-sectional** dimensions quite small compared to length) members pin connected at ends and capable of taking load at joints. Such frames are used as roof trusses to support sloping roofs and as bridge trusses to support deck.

Plane frame: A frame in which all members lie in a single plane is called plane frame. They are designed to resist the forces acting in the plane of frame. Roof trusses and bridge trusses are the example of plane frames.

Space frame: If all the members of frame do not lie in a single plane, they are called as space frame. Tripod, transmission towers are the examples of space frames.

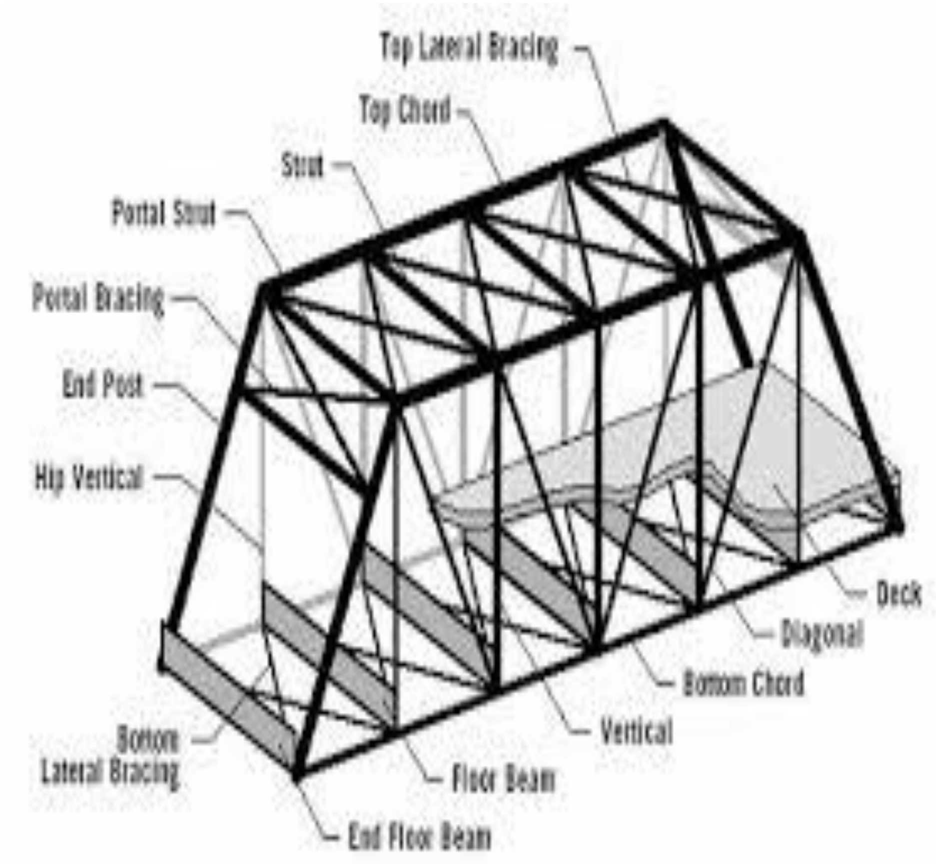


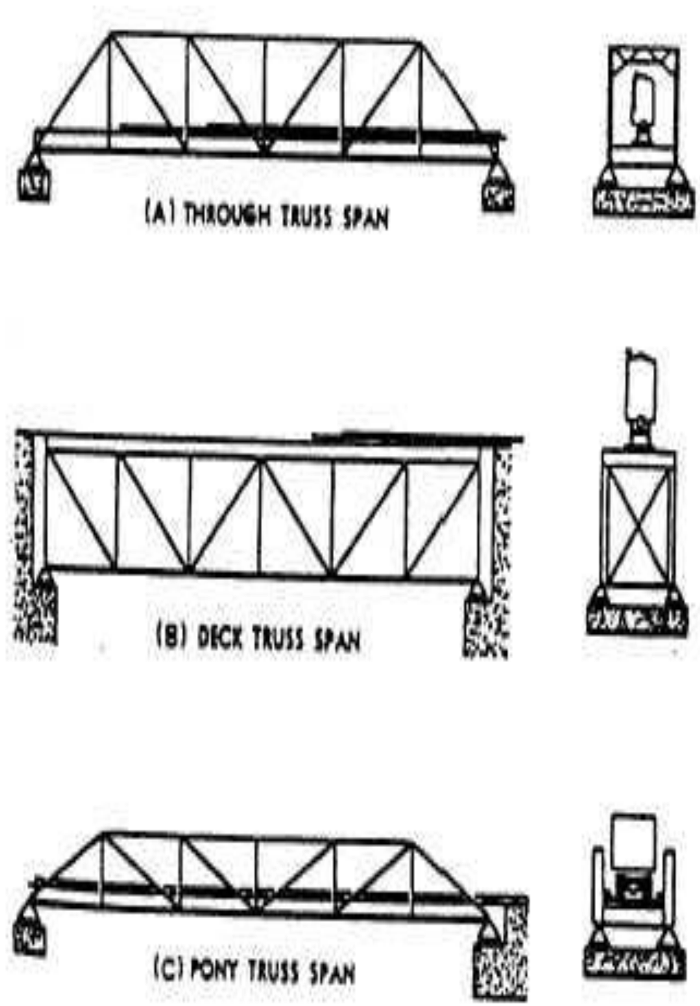
LEGEND:
Bottom Chord Diagonal Bracing
Web Plane Diagonal Bracing
Continuous Lateral Restraint
Vertical Web Diagonal Bracing



D
55 kN
45 kN
B
F
2.5 m
1.5 m
1.5 m
A
H
2 m
C
2 m
E
2 m
G
2 m
90 kN
MATHalino.com

Figure T-03


Top Lateral Bracing
Top Chord
Strut
Portal Strut
Portal Bracing
End Post
Hip Vertical
Deck
Diagonal
Bottom Chord
Vertical
Bottom
Lateral Bracing
Floor Beam
End Floor Beam


(A) THROUGH TRUSS SPAN
(B) DECK TRUSS SPAN
(C) PONY TRUSS SPAN

Perfect frame: A pin jointed frame which has got just sufficient number of members to resist the loads without undergoing appreciable deformation in shape is called a perfect frame. Triangular frame is the simplest perfect frame and it has 03 joints and 03 members.

It may be observed that to increase one joint in a perfect frame, two more members are required. Hence, the following expression may be written as the relationship between number of joint j, and the number of members m in a perfect frame.

$m = 2j - 3$

(a) When LHS = RHS, Perfect frame.

(b) When LHS<RHS, Deficient frame.

(c) When LHS>RHS, Redundant frame.

Assumptions

The following assumptions are made in the analysis of pin jointed trusses:

1. The ends of the members are pin jointed (hinged).

2. The loads act only at the joints.

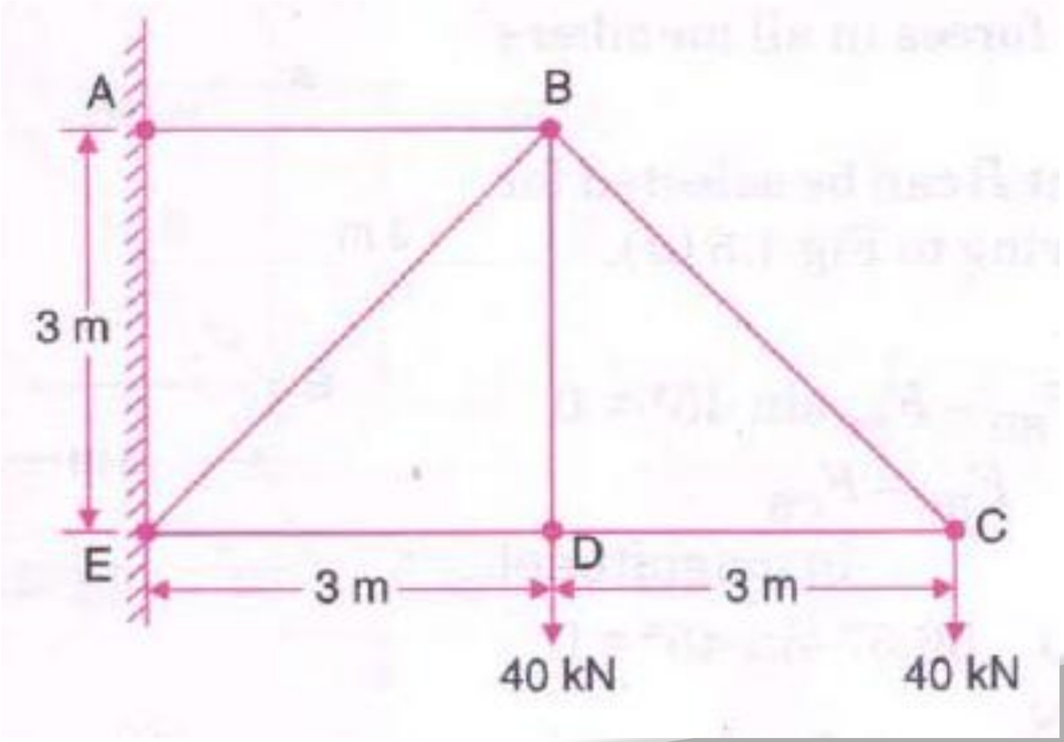
3. Self weight of the members is negligible.

Methods of analysis

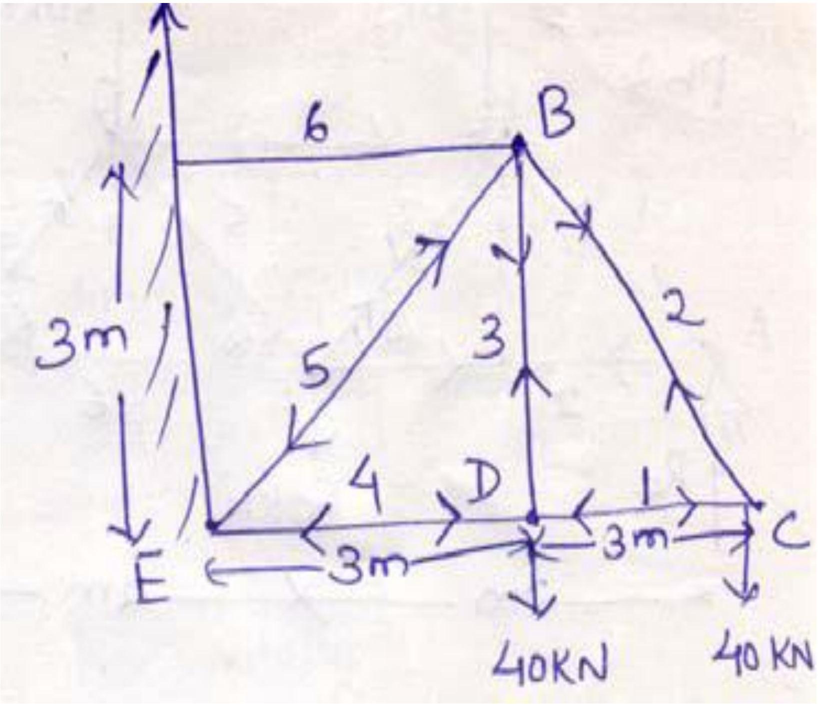
1. Method of joint
2. Method of section
3. Method of tension coefficient

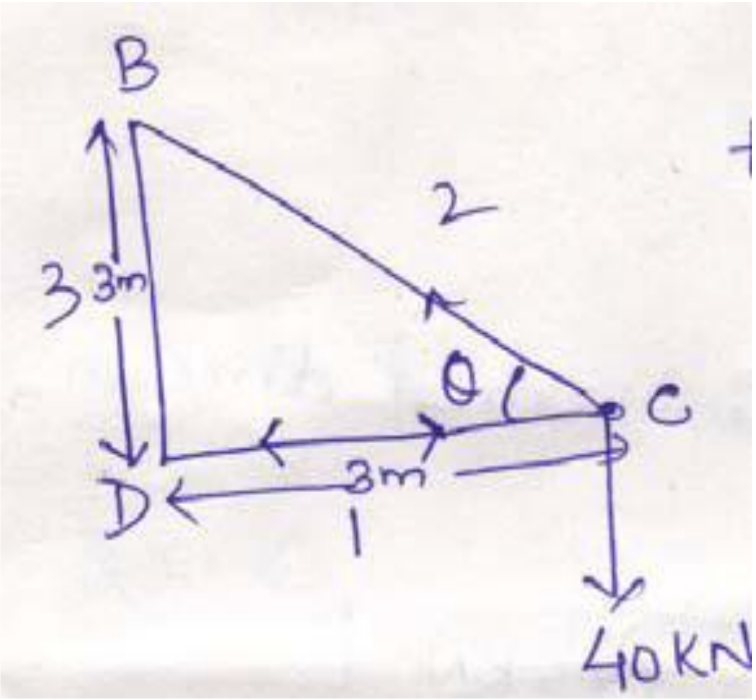
Problems on method of joints

Problem 1: Find the forces in all the members of the truss shown in figure.



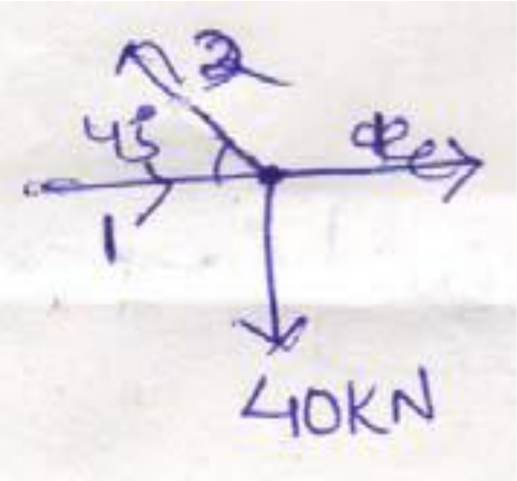
Solution





$\tan\theta = 1$

$\Rightarrow \theta = 45^{\circ}$



Joint C

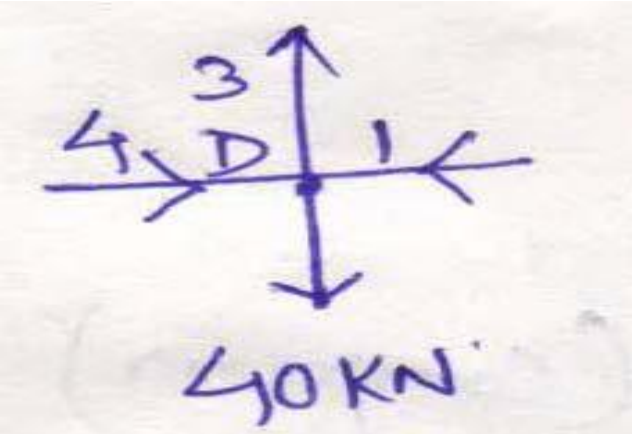
$S_1 = S_2 \cos 45$

$\Rightarrow S_1 = 40KN$ (Compression)

$S_2 \sin 45 = 40$

$\Rightarrow S_2 = 56.56KN$ (Tension)

Joint D



$S_3 = 40KN$ (Tension)

$S_1 = S_4 = 40KN$ (Compression)

Joint B

Resolving vertically,

$$\sum V = 0$$

$$S_5 \sin 45 = S_3 + S_2 \sin 45$$

$$\Rightarrow S_5 = 113.137 KN \text{ (Compression)}$$

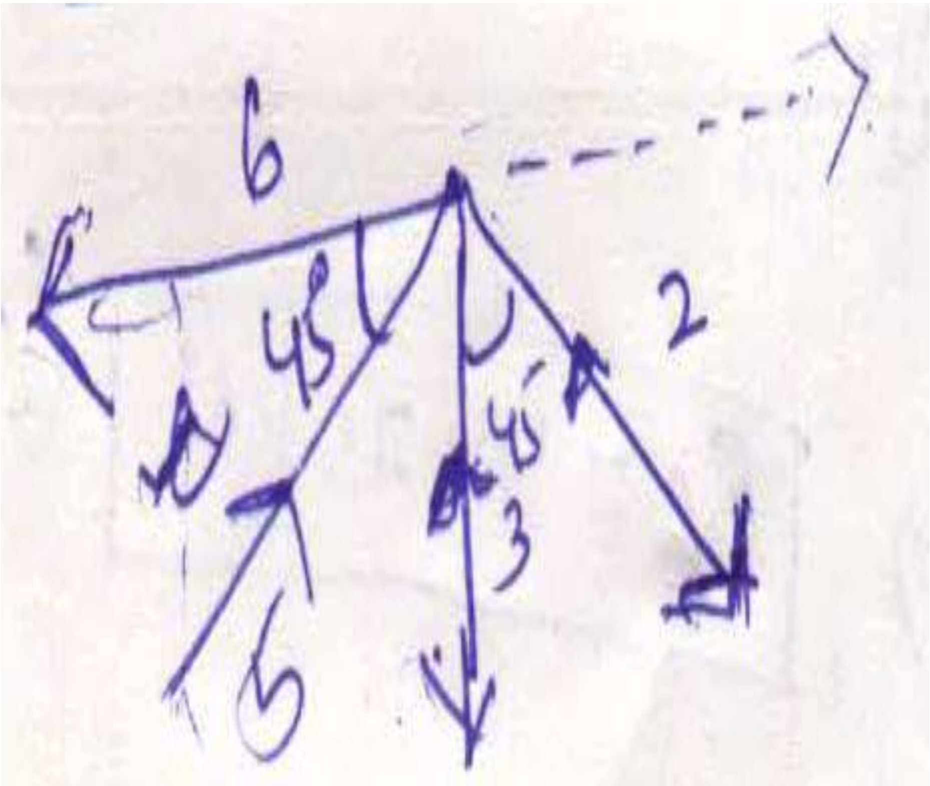
Resolving horizontally,

$$\sum H = 0$$

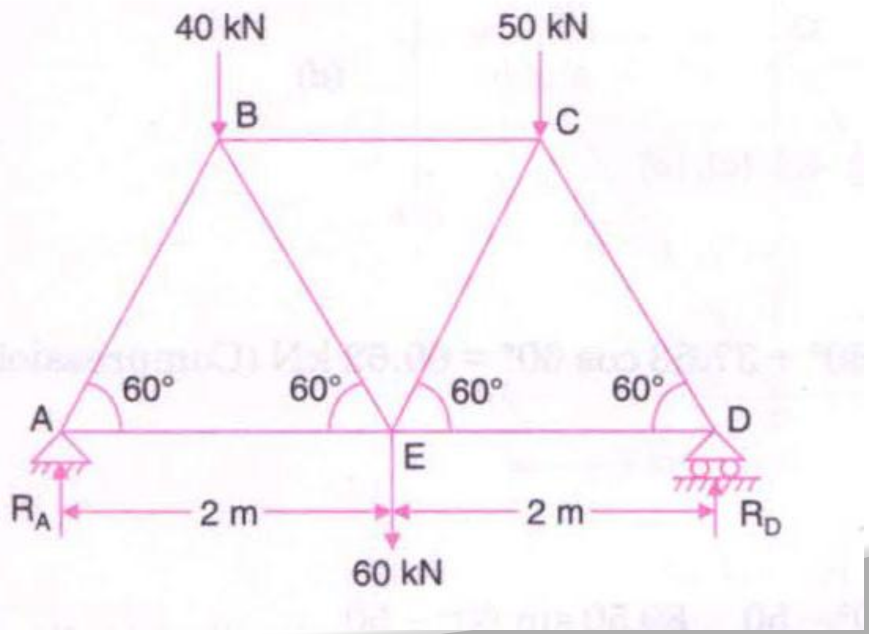
$$S_6 = S_5 \cos 45 + S_2 \cos 45$$

$$\Rightarrow S_6 = 113.137 \cos 45 + 56.56 \cos 45$$

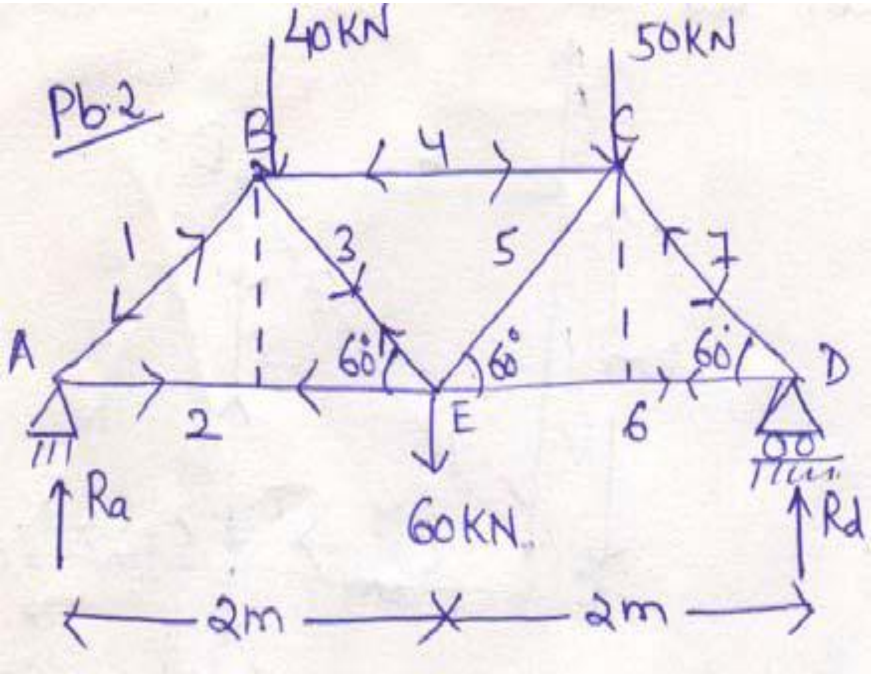
$$\Rightarrow S_6 = 120 KN \text{ (Tension)}$$



Problem 2: Determine the forces in all the members of the truss shown in figure and indicate the magnitude and nature of the forces on the diagram of the truss. All inclined members are at 60° to horizontal and length of each member is 2m.



Taking moment at point A,



$$\sum M_A = 0$$

$$R_d \times 4 = 40 \times 1 + 60 \times 2 + 50 \times 3$$

$$\Rightarrow R_d = 77.5KN$$

Now resolving all the forces in vertical direction,

$$\sum V = 0$$

$$R_a + R_d = 40 + 60 + 50$$

$$\Rightarrow R_a = 72.5KN$$

Joint A

$\sum V = 0$

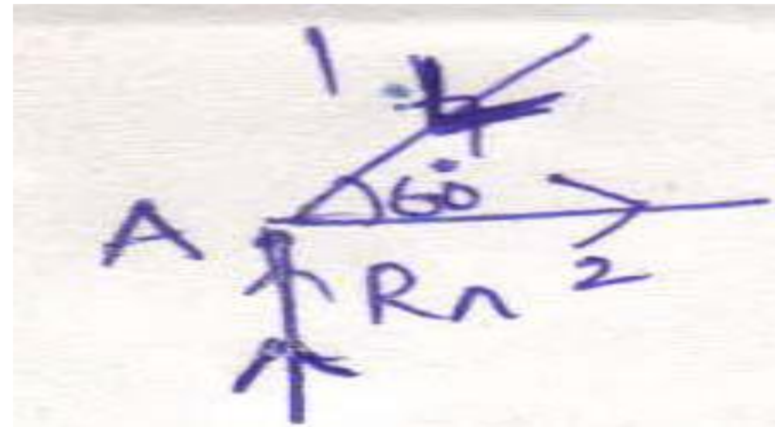
$\Rightarrow R_a = S_1 \sin 60$

$\Rightarrow S_1 = 83.72 KN$ (Compression)

$\sum H = 0$

$\Rightarrow S_2 = S_1 \cos 60$

$\Rightarrow S_1 = 41.86 KN$ (Tension)



Joint D

$\sum V = 0$

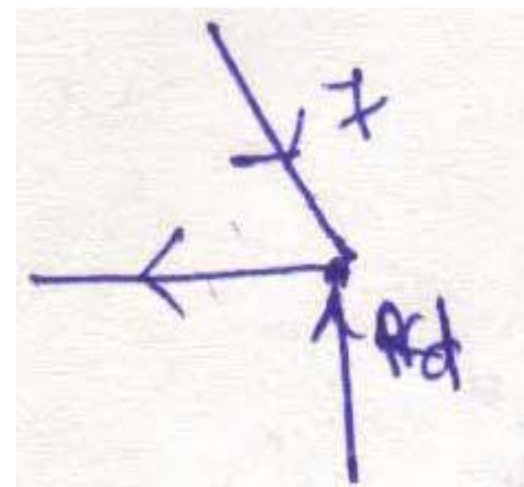
$S_7 \sin 60 = 77.5$

$\Rightarrow S_7 = 89.5 KN$ (Compression)

$\sum H = 0$

$S_6 = S_7 \cos 60$

$\Rightarrow S_6 = 44.75 KN$ (Tension)



Joint B

$$\sum V = 0$$

$$S_1 \sin 60 = S_3 \cos 60 + 40$$

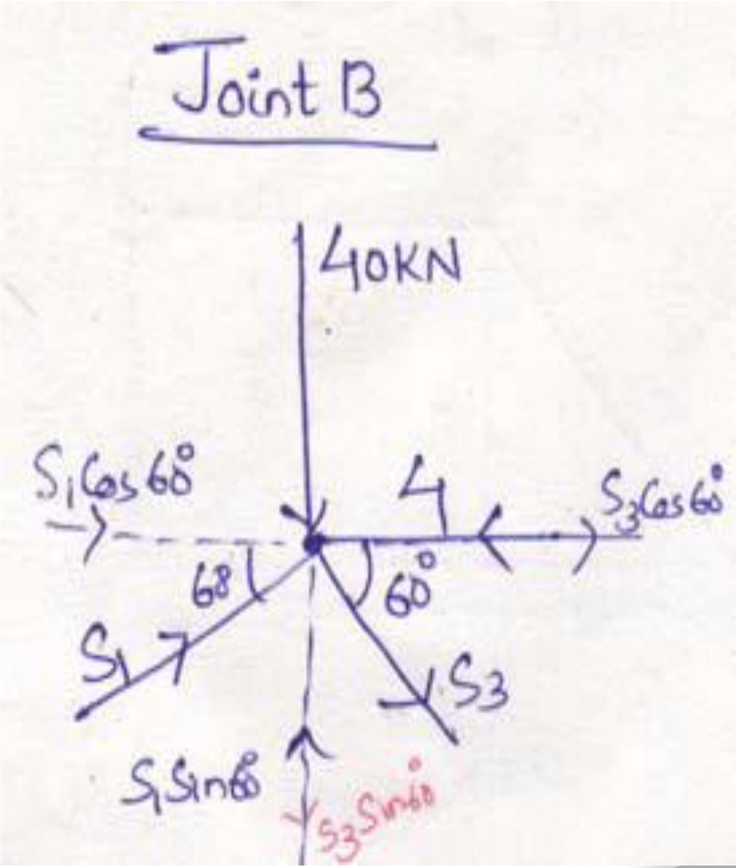
$$\Rightarrow S_3 = 37.532 KN \text{ (Tension)}$$

$$\sum H = 0$$

$$S_4 = S_1 \cos 60 + S_3 \cos 60$$

$$\Rightarrow S_4 = 37.532 \cos 60 + 83.72 \cos 60$$

$$\Rightarrow S_4 = 60.626 KN \text{ (Compression)}$$



Joint C

$$\sum V = 0$$

$$S_5 \sin 60 + 50 = S_7 \sin 60$$

$$\Rightarrow S_5 = 31.76KN \text{ (Tension)}$$

Plane Truss (Method of section)

The method of section is preferable for the following cases:

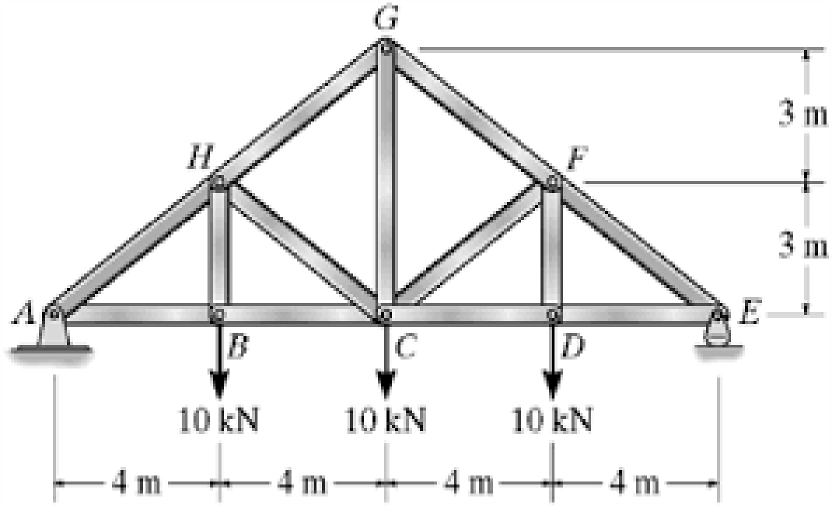
(i) analysis large truss in which forces in only few members are required.

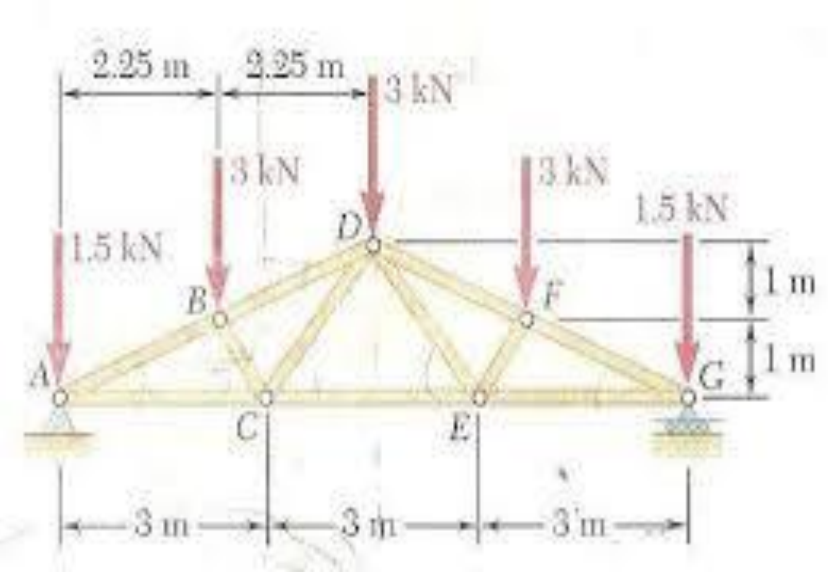
(ii) if method of joint fails to start or proceed with analysis

Steps of using Method of section

- First determine the support reactions
- draw a section line passing through not more than three members in which forces are unknown, such that the entire frame is cut into two separate parts

-each part should be in equilibrium under the action of loads, reaction and the force in the members


G
H
F
3 m
3 m
A
B
C
D
E
10 kN
10 kN
10 kN
4 m
4 m
4 m
4 m


2.25 m
2.25 m
3 kN
3 kN
3 kN
1.5 kN
1.5 kN
D
B
F
A
C
E
G
1 m
1 m
3 m
3 m
3 m

UNIT-II
INTRODUCTION TO ARCHES ANALYSIS

ARCHES

- An arch looks like curved girder , either a solid rib or braced supported at its ends and carrying transverse loads which are frequently vertical .

- The early Indian railway and highways bridges also use masonry arches. Arches are also used in buildings to carry loads over doorways, windows etc., as well as to add an aesthetic touch to building.

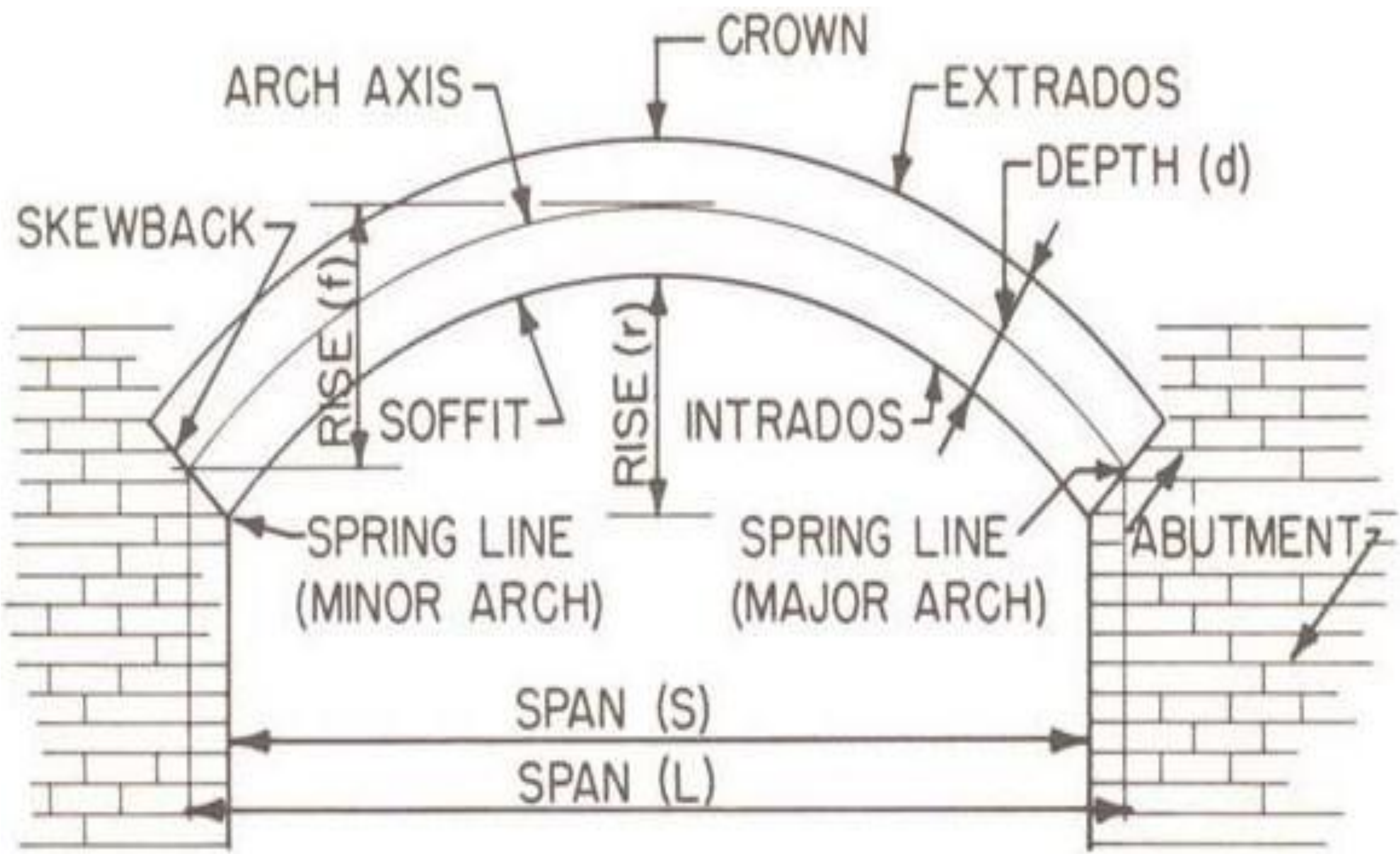
"An arch can be defined as a humped or curved beam subjected to transverse and other loads as well as the horizontal thrust at the supports."

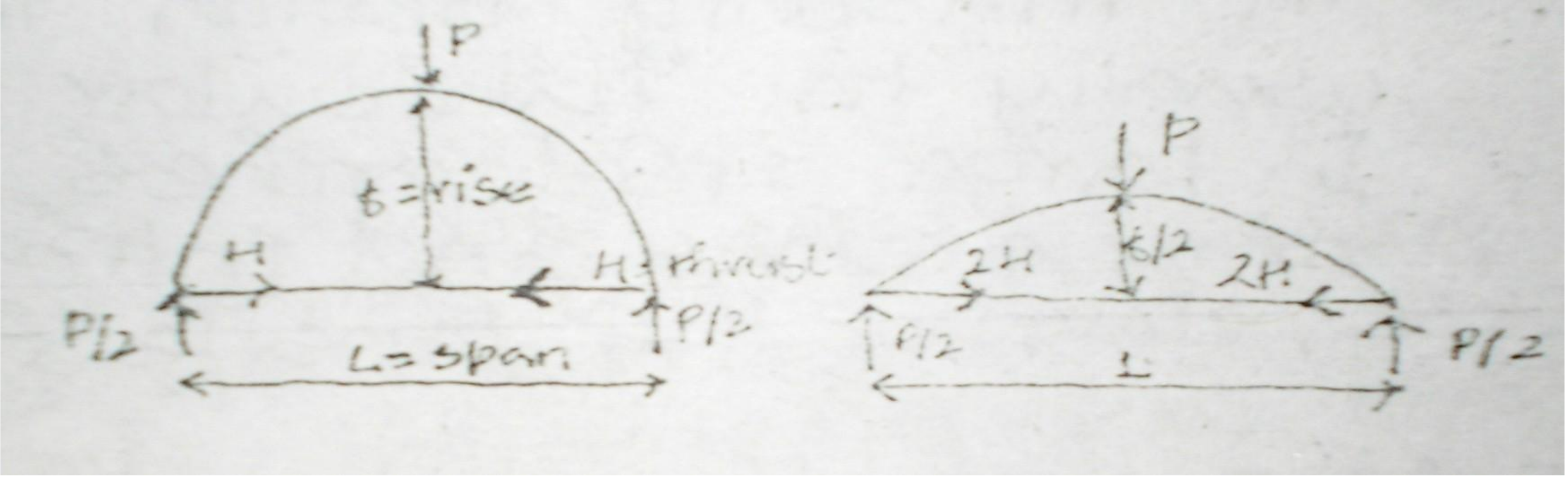
- It is simplest type of arch, consists of two section hinged at the crown and a hinge at support.
- The hinges at the support makes the ends of the arch to be fixed in position but not in direction.
- It is statically determinate structure.

- In case of beams supporting UDL, the maximum bending moment increases with the square of the span and hence they become uneconomical for long span structures. In such situations arches could be advantageously employed, as they would develop horizontal reactions, which reduces the design bending moment.

- Arches are mainly used in bridge construction and doorways. In earlier days arches were constructed using stones and bricks. In modern times they are being constructed of reinforced concrete and steel.

Arch Terminology





It is important to minimize the arch THRUST so as to reduce the dimensions of the tie rod, or to ensure that the soil will not move under the pressure of the abutments.

The THRUST is proportional to the total LOAD & to the SPAN, and inversely proportional to the RISE of the arch.

In arches rise to span ratio should not be less than 1/8 riser 2/3rd minimum should be 1/8 of the span & maximum.

Lesser rise takes compression but not tensile load

In masonry design the arch is heavy & loaded by the weight of walls, its shape is usually the funicular of the dead load, & some bending is introduced in it by live loads.

In large steel arches, the live load represents a greater share of the total load & introduces a large amount of bending but it is seldom in view of the tensile strength of steel.

The SHAPE of the arch may be chosen to be as close as possible to the FUNICULAR of the heaviest loads, so as to minimize BENDING.

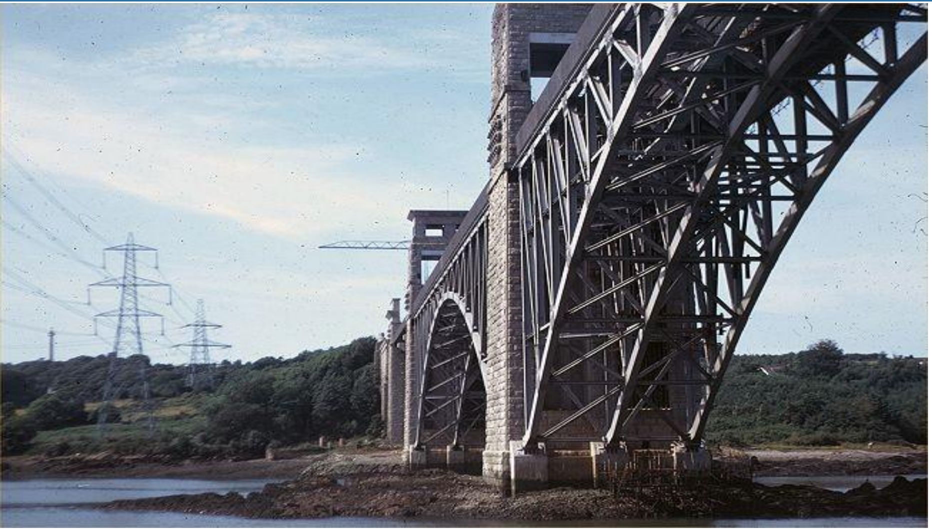
The arch thrust is absorbed by a tie-rod whenever the foundation material is not suitable to resist it.

When it must allow the free passage of traffic under it, its thrust is asorbed either by buttressesor by tie-rods buried under ground.

The stationary or moving loads carried by the arch are usually supported on a horizontal surface.

This surface may be above or below the arch, connected to it by compression struts or tension hangers.

MATERIALS USED



STEEL-takes more tension

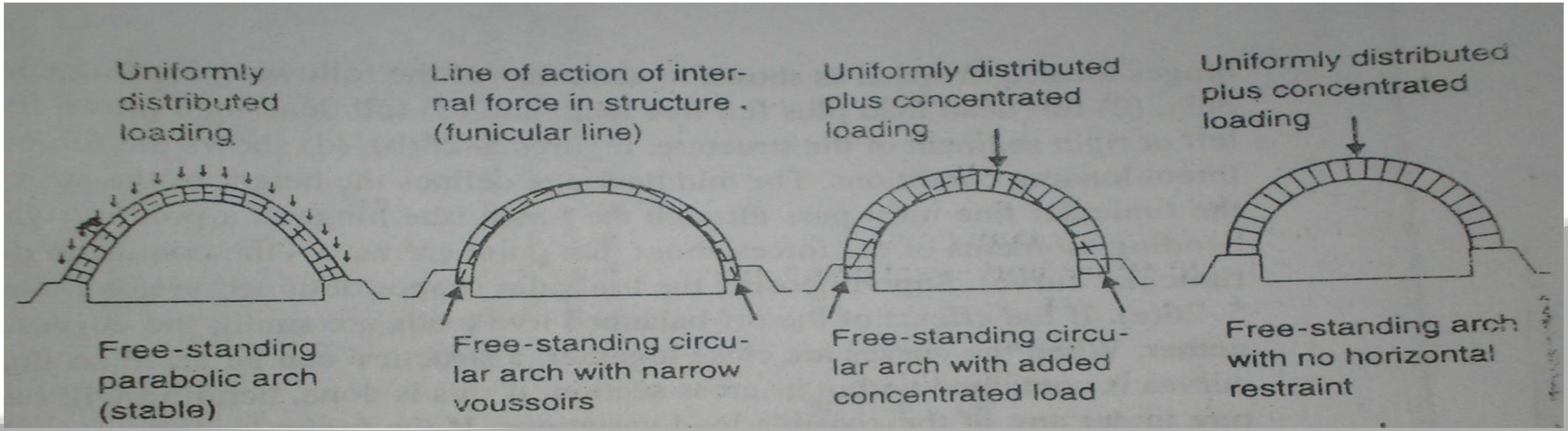
WOOD-both evenly

CONCRETE-takes more compression

FUNICULAR ARCHES – CONCENTRATED LOADS

The sum total of all rotational effects produced about any such location by the external and internal forces must be zero. In three hinged arch having a non-funicular shape, this observation is true only at three hinged conditions.

ϖThe external shear at a section is balanced by an internal resisting shear force that is provided by vertical component of the internal axial force.



DESIGN OF ARCH STRUCTURES

The first important consideration when designing a brick arch is whether the arch is structural or non-structural. That is, will the arch be required to transfer vertical loads to abutments or will it be fully supported by a steel angle.

While this may seem obvious, confusion often develops because of the many configurations of arch construction.

To answer this question, one must consider the two structural requirements necessary for a brick arch to adequately carry vertical loads.

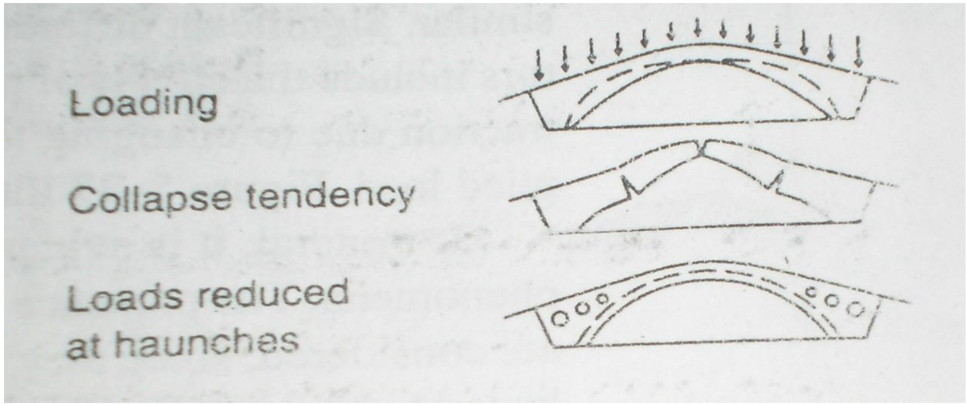
First, vertical loads must be carried by the arch and transferred to the abutments. Second, vertical load and lateral thrust from the arch must be resisted by the abutments.

If either the arch or the abutment is deficient, the arch must be considered as non-structural and the arch and its tributary load must be fully supported by a steel angle or plates.

Alternately, reinforcement may be used to increase the strength of either or both the arch and the abutments.

DESIGNING FOR LOAD VARIATIONS

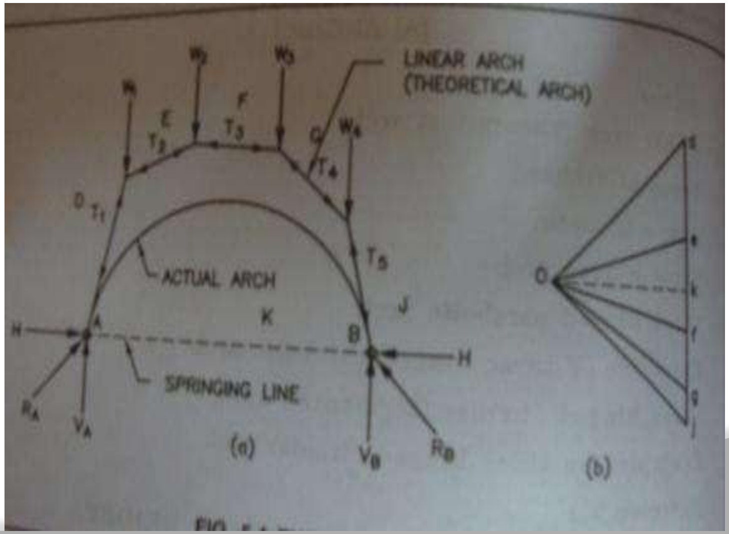
One of the most significant aspect of the modern arch is that it can be designed to sustain some amount of variation in load without either changing shape or experiencing damage.



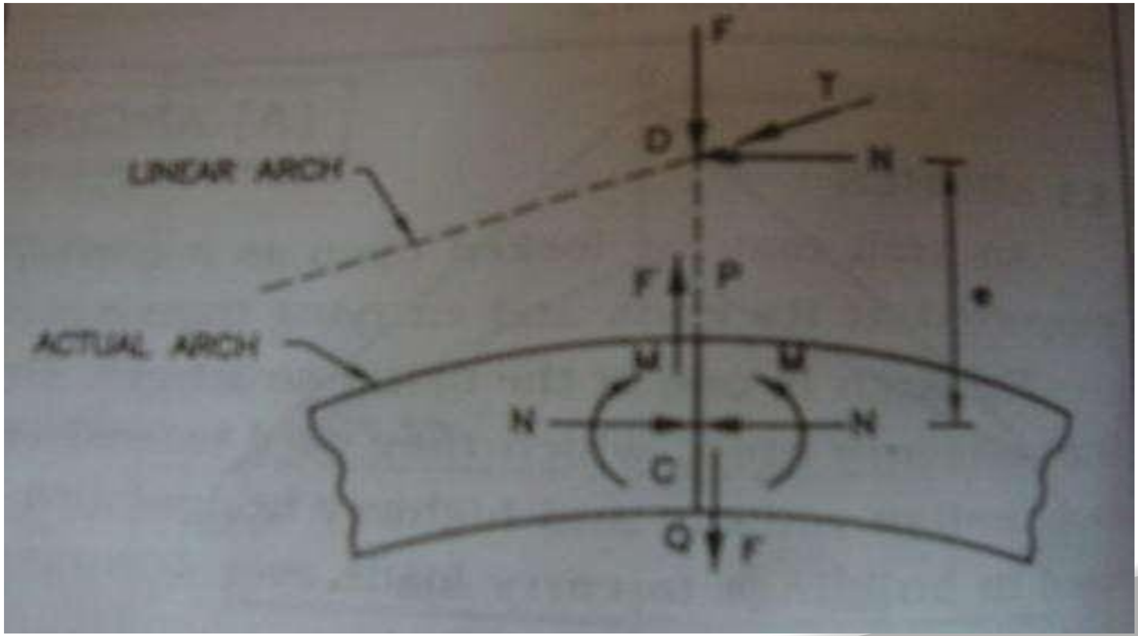
The shape of an arch is initially determined as a response to its primary loading condition (e.g.: parabolic for uniformly distributed loads)

LINEAR(THEORETICAL) ARCHES:

- Consider a system of jointed link work inverted about AB, with loads and shown in figure:
- Under a given system of loading , every link will be in a state of compression. The magnitudes of pushes T1,T2,T3,T4….. etc. can be known by the rays Od, Oe, Of, etc. in the force polygon. The actual lines of action of pushes T1,T2,T3….. etc. is known as the linear arch or **theoretical arch**.



- In practice the piston and magnitude of the loading over a structure goes on changing. It is therefore neither advisable nor possible to construct an arch according to its theoretical shape. In practice , the arch is made parabolic, circular or ellipse in shape for easy construction and aesthetic appearance. Such an arch is called **actual arch**.

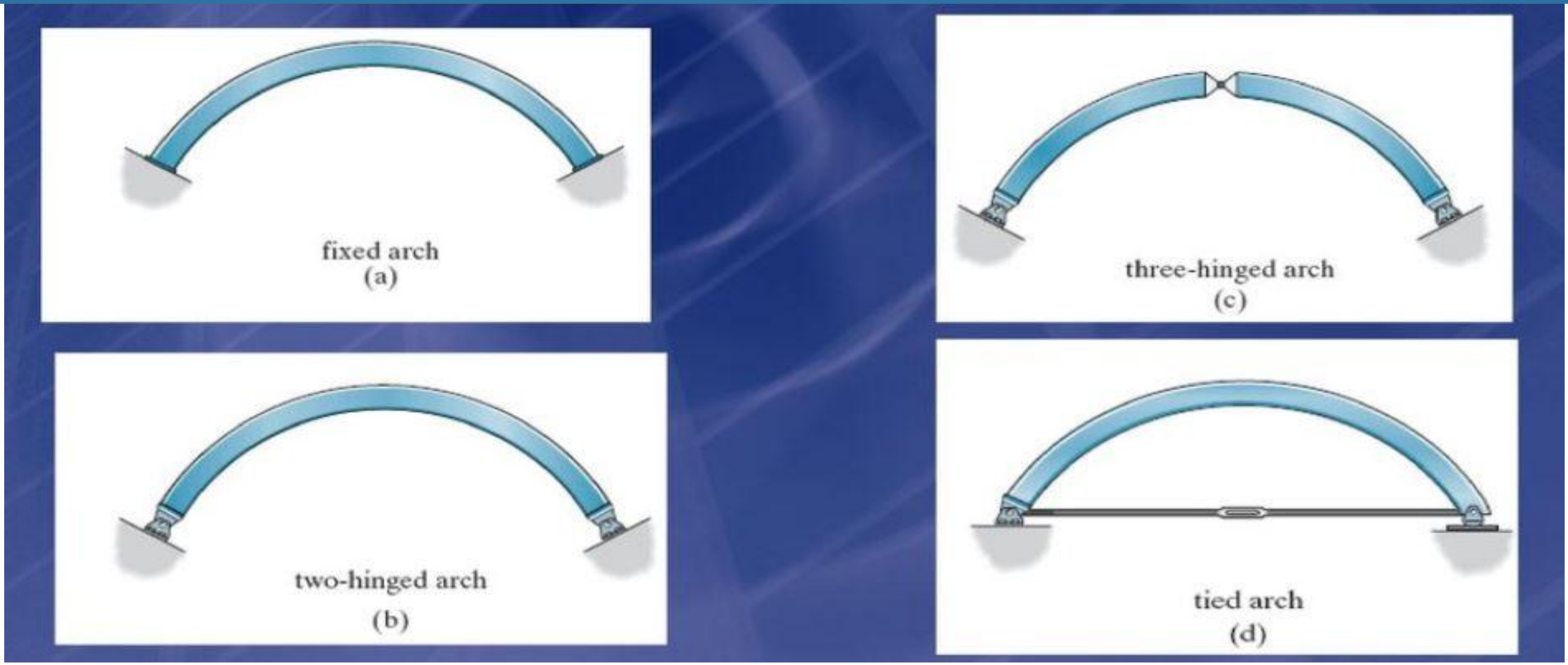


- Consider a cross-section PQ of the arches as shown in fig.
- Let T is neither normal to the cross-section nor does it act through centre C of the cross-section.
- The resultant thrust can be tangential to the section PQ.
- Let N is be the normal component and F be the tangential component F will cause shear force at the section PQ. The normal component N acts eccentrically, the eccentricity e being equal to CD. Thus the action of N acting at D is twofold (i) a normal thrust N at C and

  (ii) a bending moment , M = N.e at C.
- At any cross-section of arch is thus subjected to three stating actions:

1. Shear force / radial shear (F)
2. Bending moment (M)
3. Normal thrust (N)

TYPES OF ARCHES:



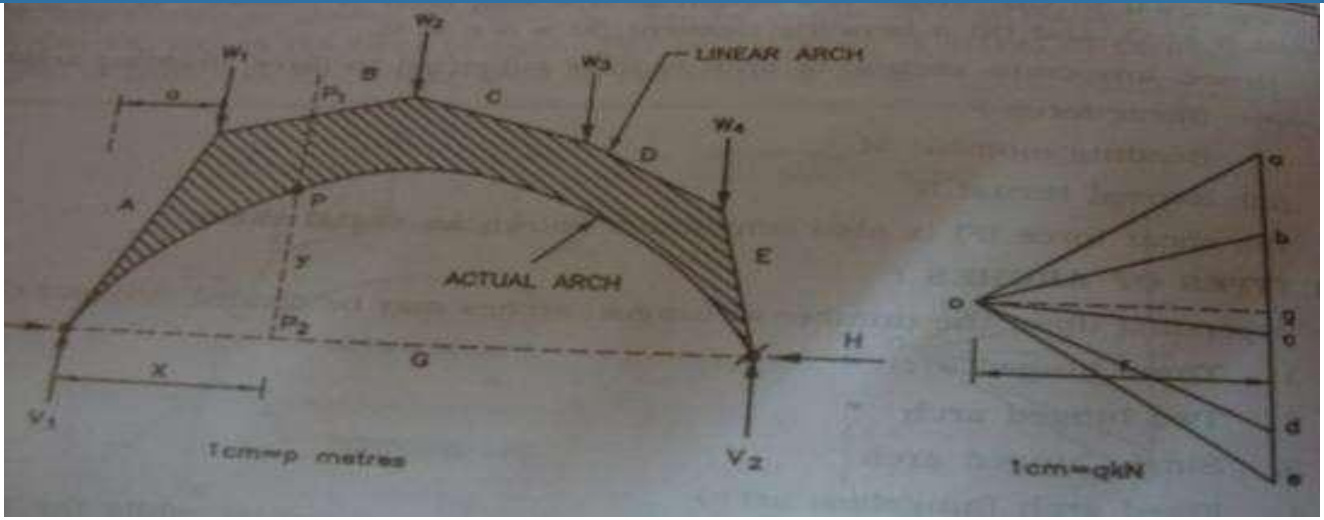
- Depending upon the number of hinges , arches may be divided into four classes:

1. Three hinged arch
2. Two hinged arch
3. Single hinged arch
4. Fixed(hinge less) arch

EDDY'S THEOREM:

"The bending moment at any section of an arch is equal to the vertical intercept between the linear arch and the centre line of the actual arch."

- consider a section at P distance at x from left hinge.
- Let the other co-ordinate of P be y.
- For the given system of loads the linear arch can be constructed, if H is known. Since funicular polygon represents the bending moment diagram to some scale, the vertical intercept P1 P2 at the section P will given the bending moment due to external load system.



Let the arch is drawn to a scale of 1 cm =p meters.

Let the plotted to a scale of 1 cm = q kN,

and if the distance of pole o from the load lines r, the scale of bending moment diagram will be,

1cm = p.q.r kN.m

Now theoretically, B.M. at P is given by,

$M_p = V_1.x - W_1(x-a) - H.y$

$= \mu x - Hy$

EDDY'S THEOREM:

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Where, $\mu x = V_1.x\text{-}W_1\,(x\text{-}a)$

= usual bending moment at a section due to load system on a simply supported beam.

From figure we having,

$\mu x = P_1P_2$* scale of B.M. dia.

$= P_1P_2(p.q.r)$

$Hy = PP_2$* scale of B.M. dia.

$= PP_2\,(p.q.r)$

Hence,

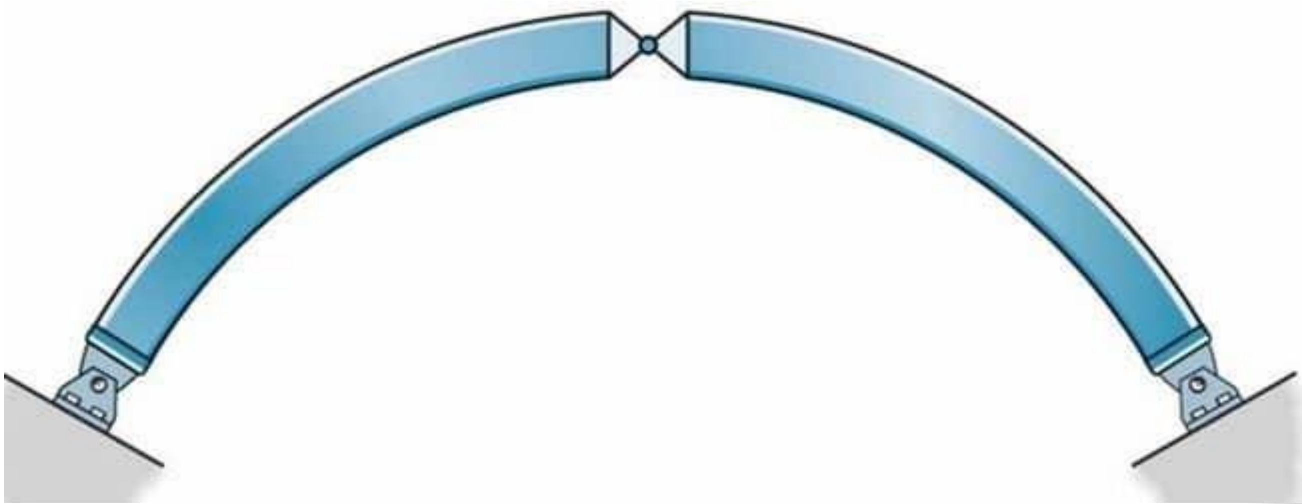
$Mp = \mu.x - H.y$

$= P_1P_2(p.q.r) – PP_2\,(p.q.r)$

$= PP_1\,(p.q.r)$

THREE HINGED ARCH:

- This is a statically determinate structure.
- A three hinged arch has two hinges at abutments and one hinge at the crown.



Let the arch is subjected to a number of loads W1,W2,W3…etc.

Since B.M. @ c is zero,

$M_c = \mu c - H.y = 0$

$H = \mu c / y$… horizontal thrust. Resolving forces along the section P,

$F = V\cos\theta - H\sin\theta$…..(1) radial shear

Similarly, resolving forces normal to the section, $N = V\cos\theta + H\sin\theta$…..(2) normal thrust

THREE HINGED PARABOLIC ARCH:

The equation of parabola, with origin at the left hand hinge A is given by,

$y = k \cdot x(L-x)$ ...(1) Where, k is constant .

When,

$x = L/2$ c, $y = r$ = central rise We

get from (1),

$r = k \cdot L/2\,(L - L/2)$

$= k \cdot L^2/4$

Therefore $k = 4r$

$/L^2$ So,

$y = 4r/L^2 \cdot x \cdot (L-x)$..... This is the equation of a parabolic arch

Where,

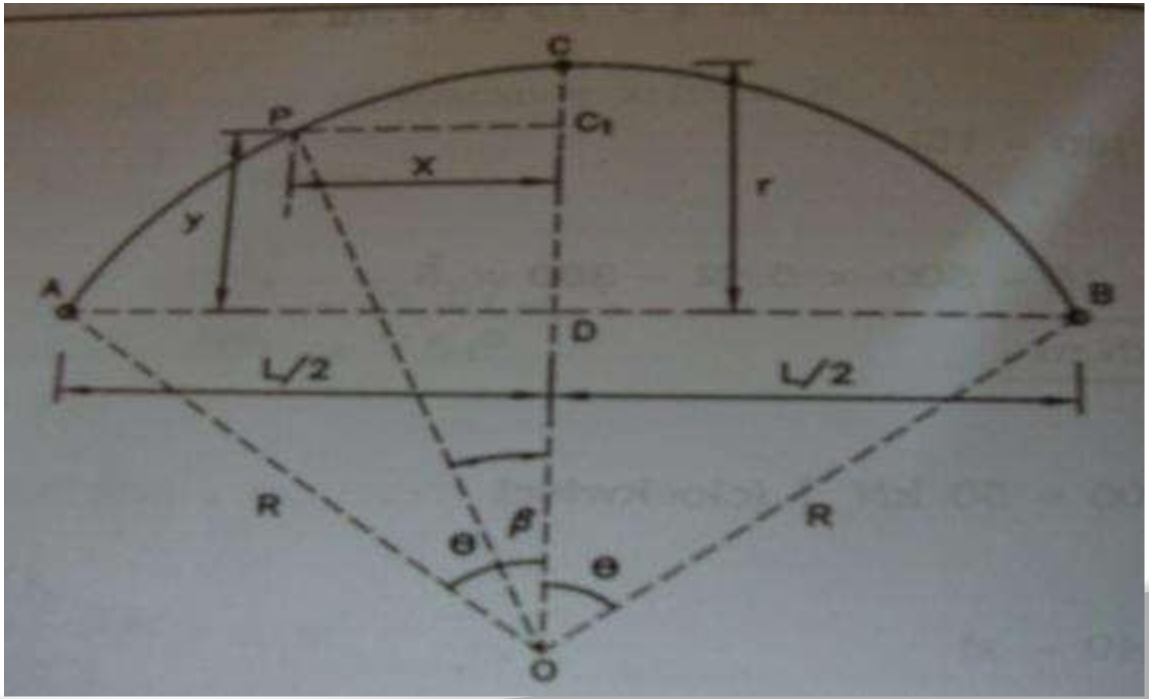
L = span of arch

r = central rise.

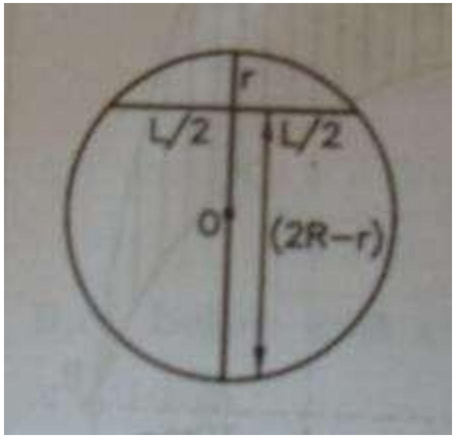
x = distance from support A and B.

THREE HINGED CIRCULAR ARCH:

- Consider the radius of arch is R, subtending an angle of 2θ at the centre. It is more convenient to have the origin at D the middle of span.
- Let (x,y) be the co-ordinates of the point P.



- From ΔOC1P, x = horizontal distance measured from C.
- $OP^2 = OC_1^2 + C_1P^2$
- So, $R^2 = \{(R-r)+y\}^2 + x^2$
- Also be the property of circle,
- $\{(2R-R).r\} = (L/2)(L/2) = L^2/4$



UNIT-III

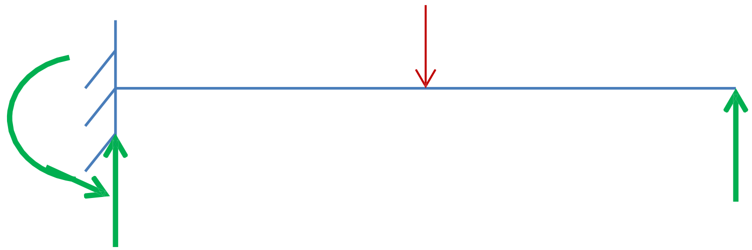
FORCE METHOD OF ANALYSIS OF INDETERMINATE BEAMS

INDETERMINATE BEAMS

- **Statically determinate beams:**
  - **Cantilever beams**
  - **Simple supported beams**
  - **Overhanging beams**

- **Statically indeterminate beams:**
  - **Propped cantilever beams**
  - **Fixed beams**
  - **Continuous beams**

Propped cantilever Beams:



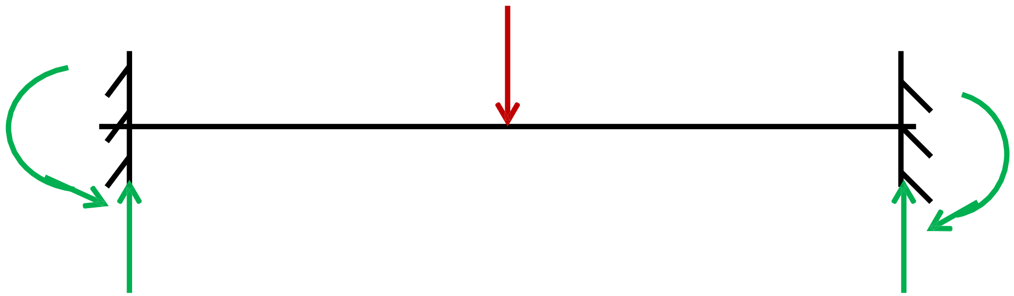
Degree of static indeterminacy

= N0. of unknown reactions – static equations

=3-2 = 1

- Fixed beam:

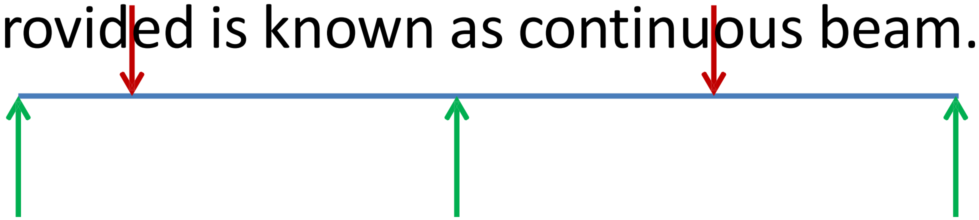
A fixed beam is a beam whose end supports are such that the end slopes remain zero (or unaltered) and is also called a built-in or encaster beam.



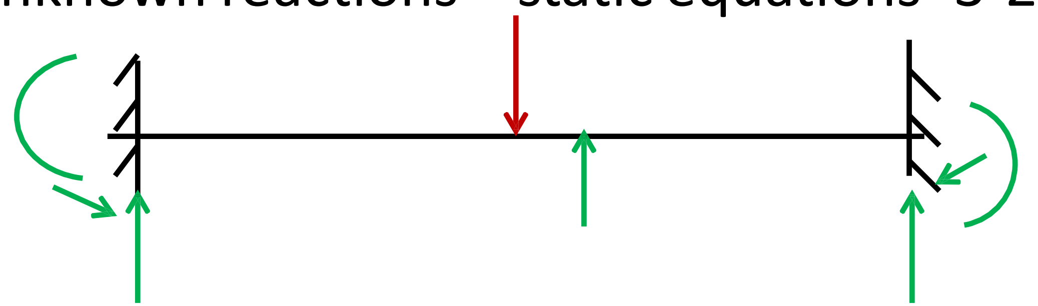
Degree of static indeterminacy=
N0. of unknown reactions – static equations=4-2 =2

Continuous beam:

Continuous beams are very common in the structural design. For the analysis, theorem of three moments is useful. A beam with more than 2 supports provided is known as continuous beam.



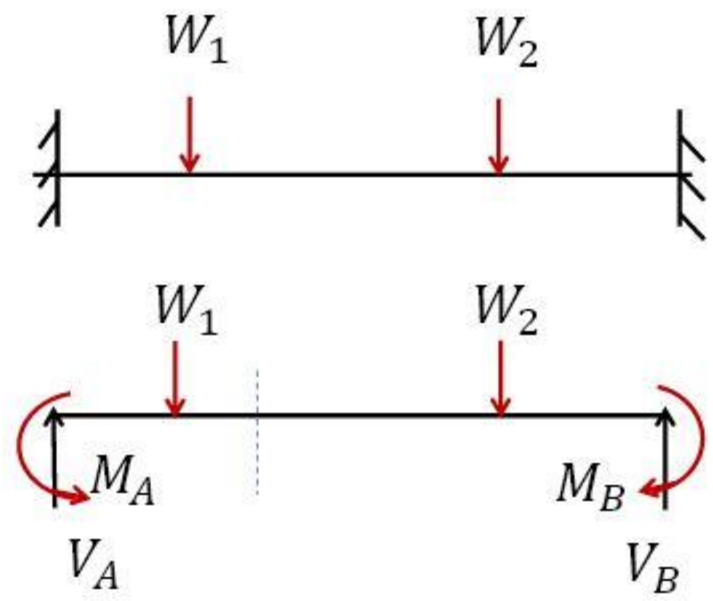
Degree of static indeterminacy=
N0. of unknown reactions – static equations=3-2 =1



Degree of static indeterminacy=
N0. of unknown reactions – static equations=5-2 =3

B.M. diagram for a fixed beam :

Figure shows a fixed beam AB carrying an external load system. Let V_A and V_B be the vertical reactions at the supports A and B.



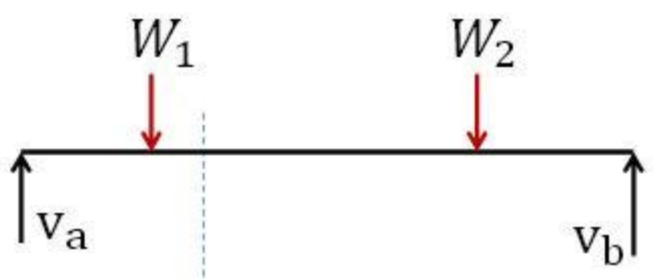
Let M_A and M_B be the fixed end Moments.

The beam may be analyzed in the following stages.

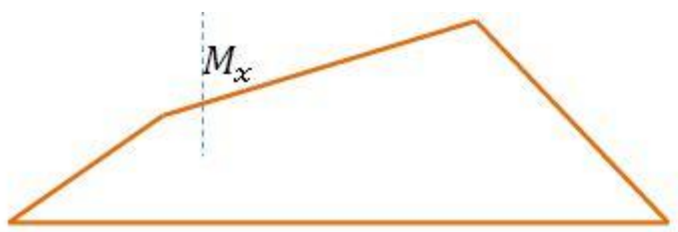
(i) Let us first consider the beam as Simply supported.

Let v_a and v_b be the vertical reactions at the supports A and B.

Figure (ib) shows the bending moment diagram for this condition. At any section the bending moment M_x is a sagging moment.



(ia) Freely supported condition



(ib) Free B.M.D.

(ii) Now let us consider the effect of end couples M_A and M_B

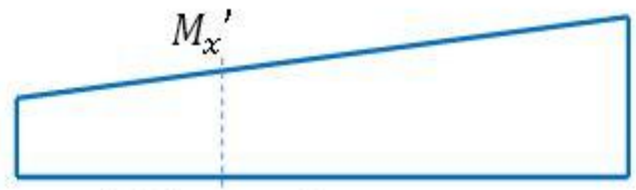
Let v be the reaction at each end due to this condition.

Suppose $M_B > M_A$.

Then $V = \frac{M_B - M_A}{L}$.

If $M_B > M_A$ the reaction V is upwards at B and downwards at A.

(iia) Effect of end couples

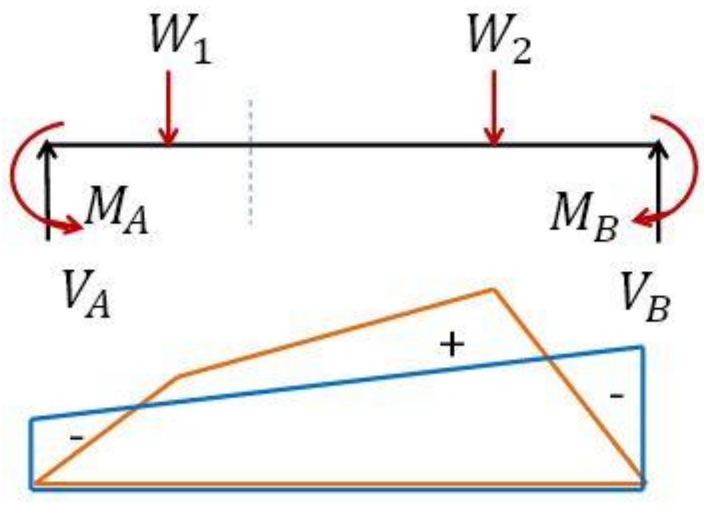


(iib) Fixed B.M.D.

Fig (iib). Shows the bending moment diagram for this condition.

At any section the bending moment M_x is hogging moment.

Now the final bending moment diagram can be drawn by combining the above two B.M. diagrams as shown in Fig. (iiib)

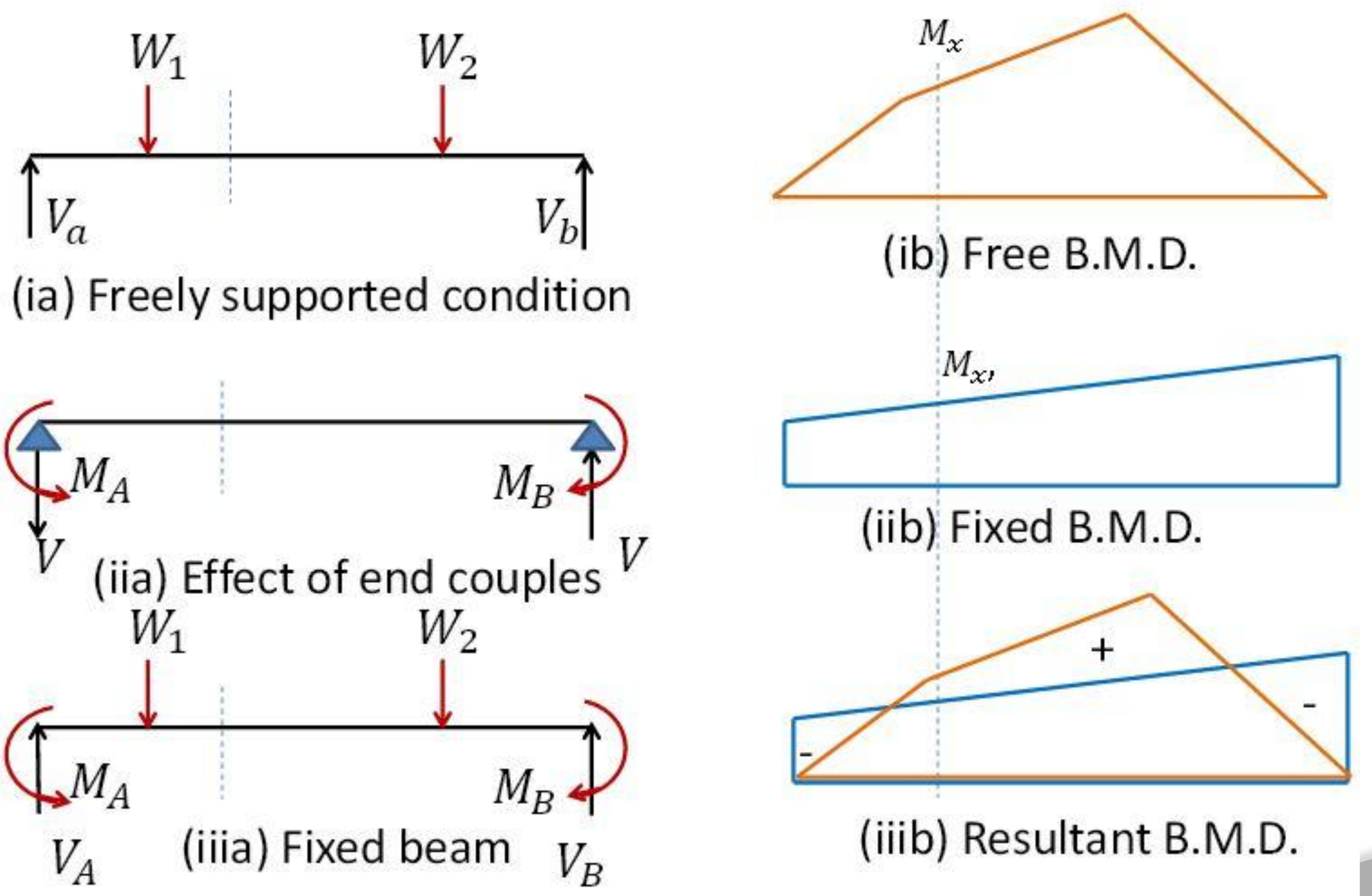


(iiib) Resultant B.M.D.

Now the final reaction $V_A = v_a - v_b$ and $V_B = v_b + v$

The actual bending moment at any section X, distance x from the end A is give by

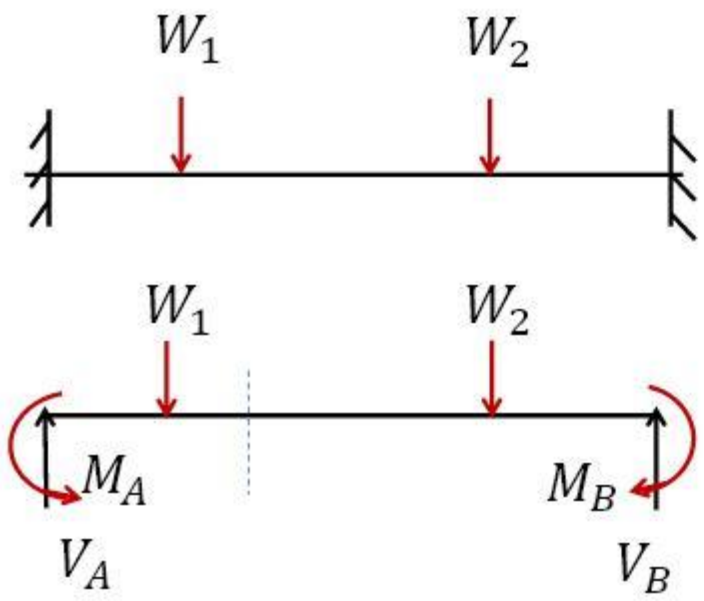
Fixed Beams


W1
W2
Va
Vb
(ia) Freely supported condition
Mx
(ib) Free B.M.D.
MA
MB
V
V
(iia) Effect of end couples
Mx'
(iib) Fixed B.M.D.
W1
W2
MA
MB
VA
(iiia) Fixed beam
VB
+
-
-
(iiib) Resultant B.M.D.

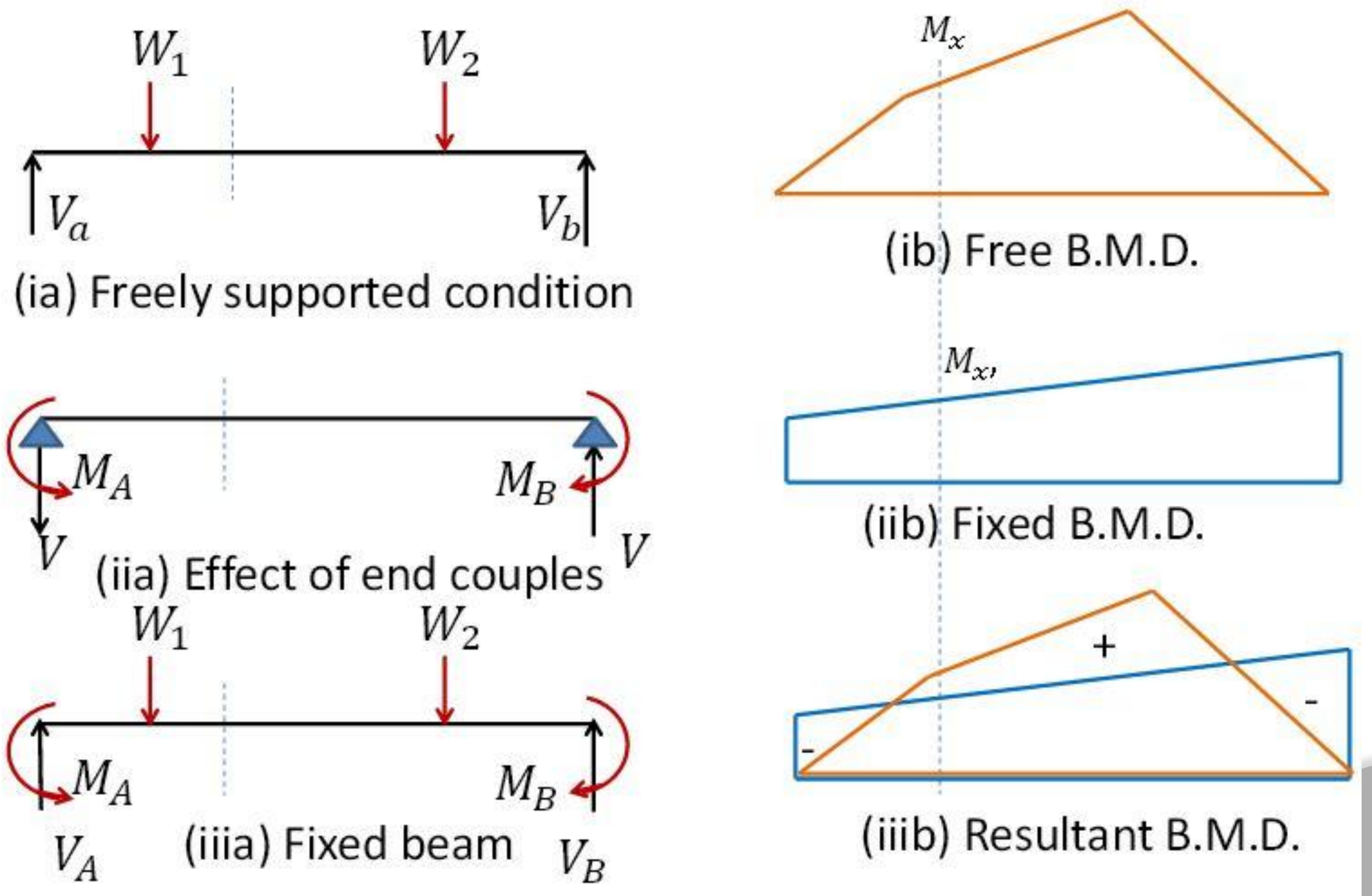
B.M. diagram for a fixed beam :

Figure shows a fixed beam AB carrying an external load system. Let V_A and V_B be the vertical reactions at the supports A and B.

Let M_A and M_B be the fixed end Moments.

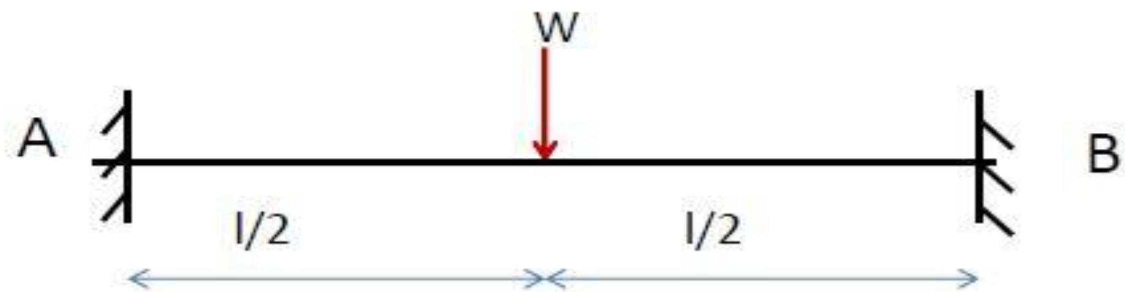


Fixed Beams


W_1
W_2
V_a
V_b
(ia) Freely supported condition
M_A
M_B
V
V
(iia) Effect of end couples
W_1
W_2
M_A
M_B
V_A
(iiia) Fixed beam
V_B
M_x
(ib) Free B.M.D.
$M_{x'}$
(iib) Fixed B.M.D.
+
-
-
(iiib) Resultant B.M.D.

Fixed beam problems

- Find the fixed end moments of a fixed beam subjected to a point load at the center.

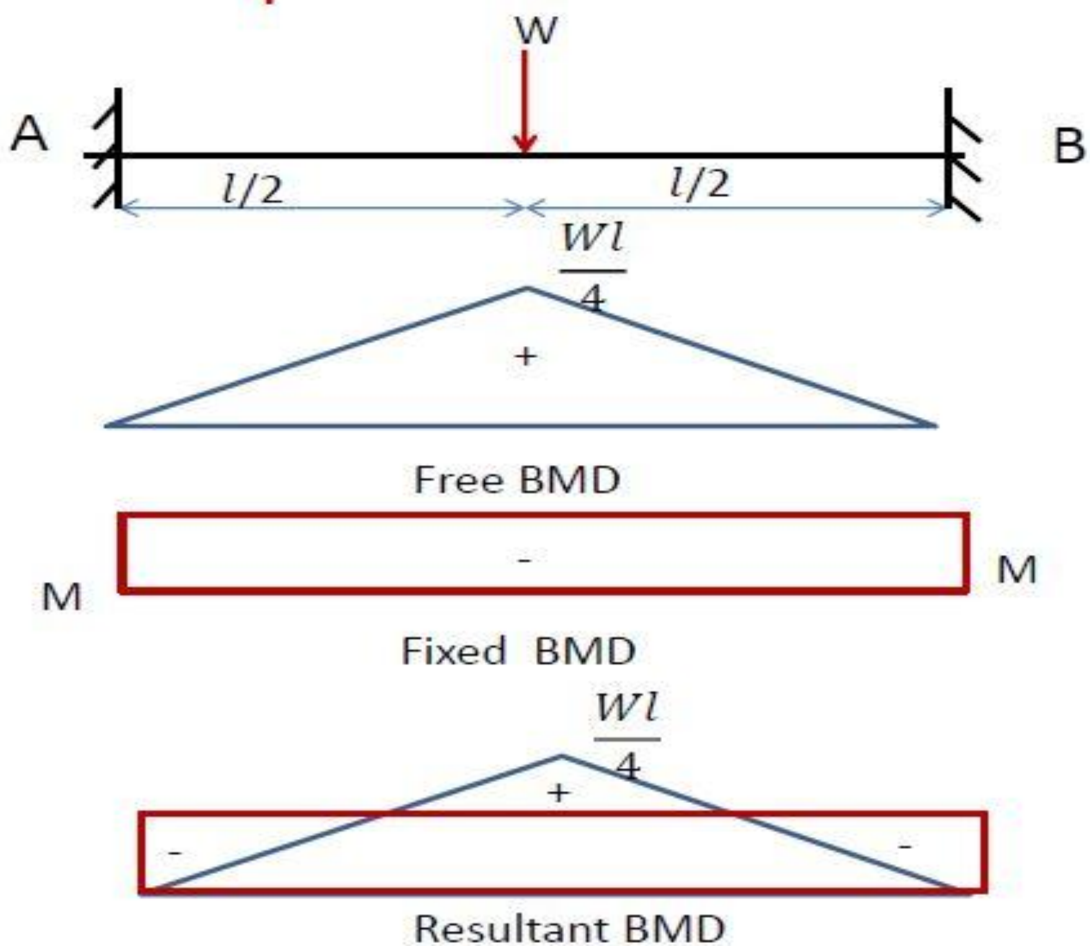


Fixed beam problems

- $A' = A$

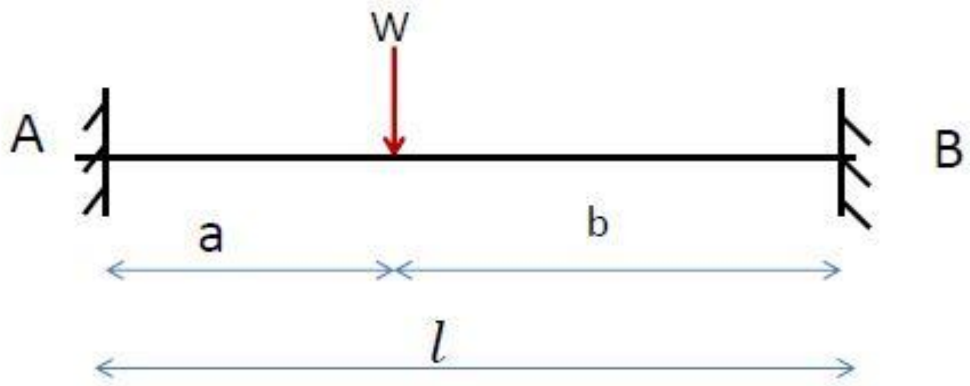
$$M \times l = \frac{1}{2} \times l \times \frac{Wl}{4}$$

$$M = \frac{Wl}{8} = M_A = M_B$$



Fixed beam problems

- Find the fixed end moments of a fixed beam subjected to a eccentric point load.



Fixed beam problems

- $A' = A$

$$\frac{M_A + M_B}{2} \times l = \frac{1}{2} \times l \times \frac{Wab}{l}$$

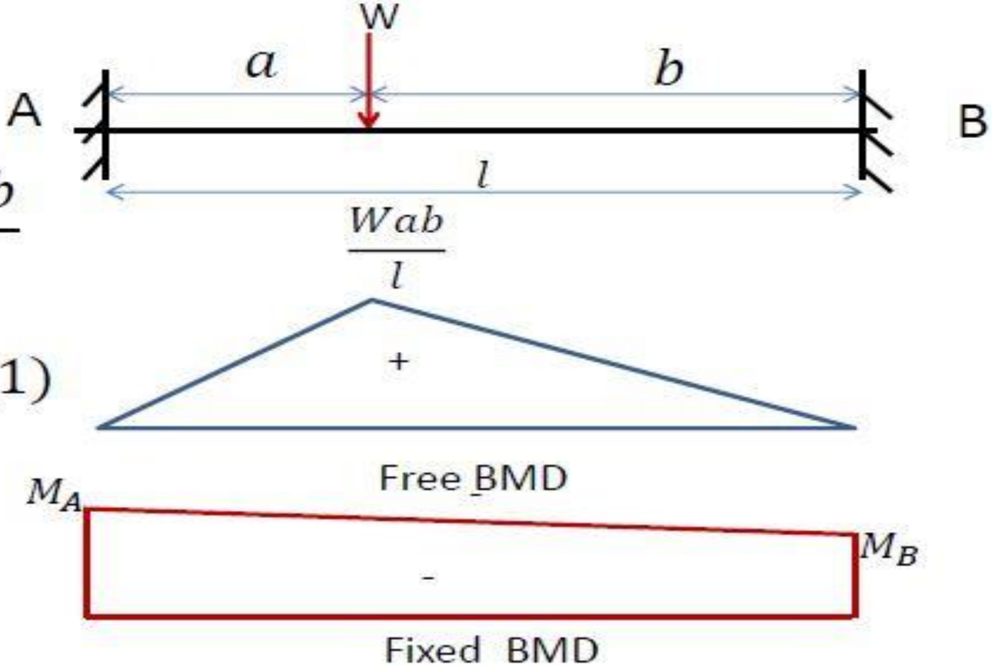
$$M_A + M_B = \frac{Wab}{l} ----(1)$$

- $x' = x$

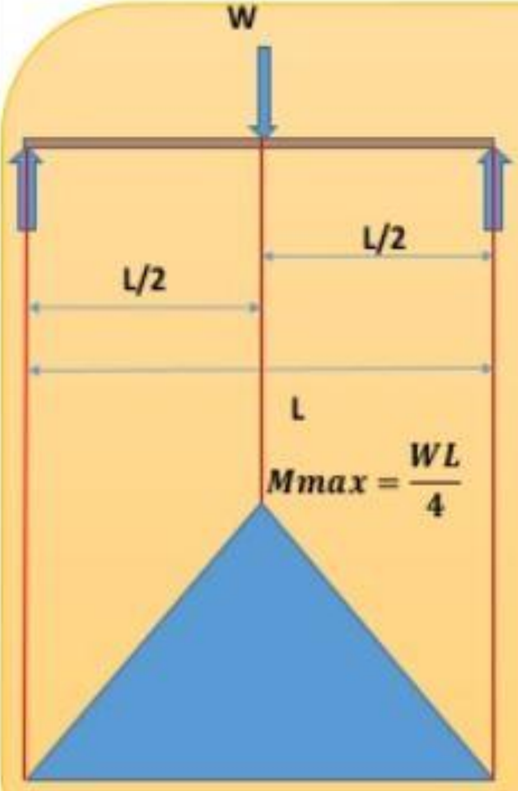
$$\frac{M_A + 2M_B}{M_A + M_B} \times \frac{l}{3} = \frac{l+a}{3}$$

$$M_B = M_A \times \frac{a}{l-a}$$

$$M_B = \frac{a}{b} ---(2)$$



STANDARD CASES:-



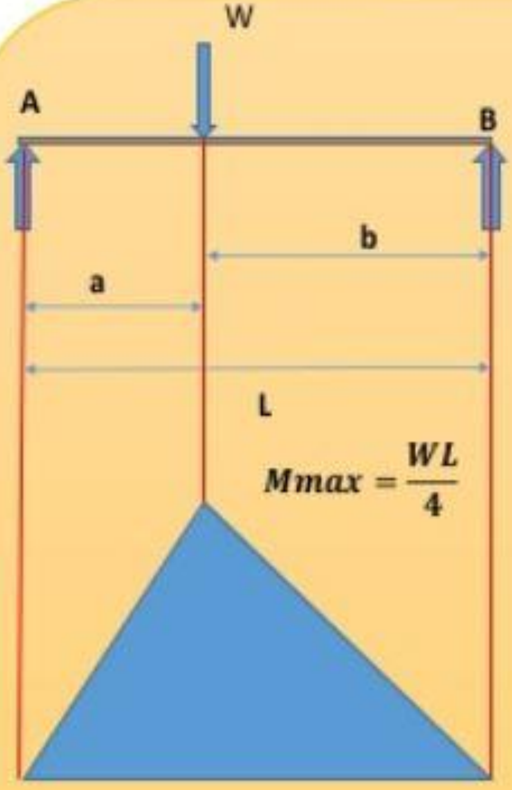
$$Mmax = \frac{WL}{4}$$

$$a = \frac{1}{2}.B.H$$

$$a = \frac{1}{2}.L.\frac{WL}{4}$$

$$X = \frac{L}{2}$$

CENTRAL POINTLOAD



$$Mmax = \frac{Wab}{L}$$

$$a = \frac{1}{2}.B.H$$

$$a = \frac{1}{2}.L.\frac{Wab}{L}$$

From A

$$X = \frac{L+a}{3}$$

From B

$$X = \frac{L+b}{3}$$

POINT LOAD ANYWHERE

UNIT-IV

DISPLACEMENT METHOD OF ANALYSIS

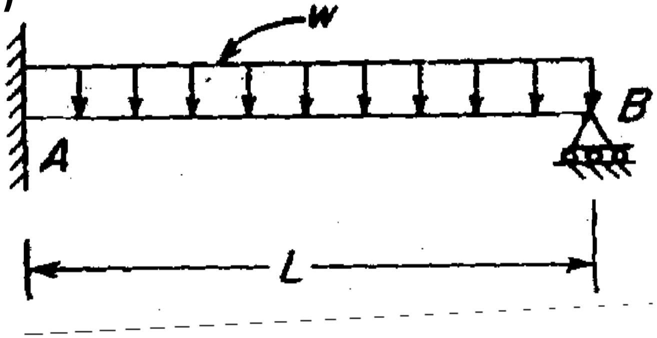
Displacement method of analysis

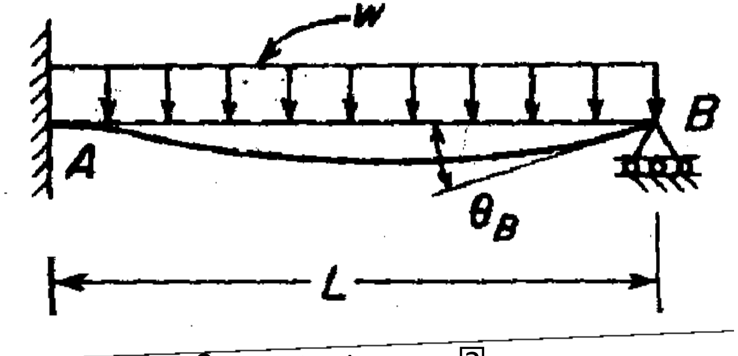
- Slope deflection method-Analysis of continuous beams and frames (with and without sway)

- Moment distribution method- Analysis of continuous beams and frames (with and without sway).

Displacement method

Example 1: **Propped cantilever** (Kinematically indeterminate to first degree)



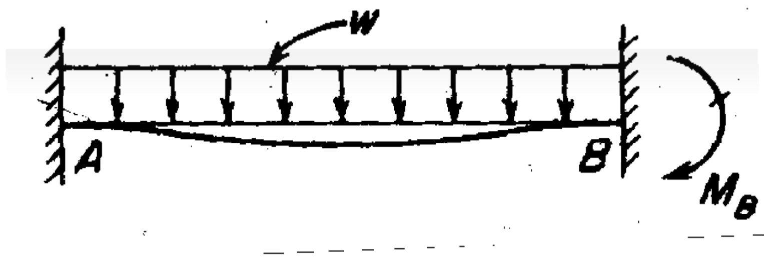


- Required to get ?$_B$

- degrees of freedom: one

- Kinematically determinate structure is obtained by restraining all displacements (all displacement components made zero - restrained structure)

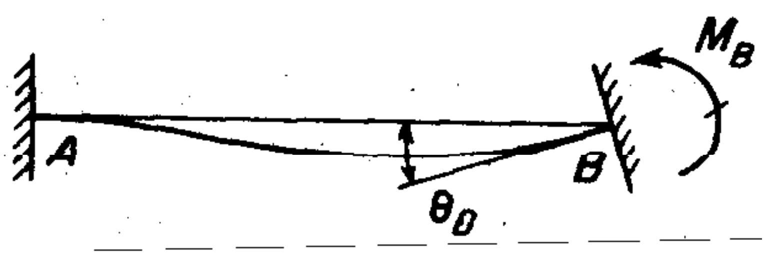
Restraint at *B* causes a reaction of M_B as shown.



$$M_B = \frac{wL^2}{12}$$

The actual rotation at ***B*** is θ_B

To induce a rotation of θ_B at *B,* it is required to apply a moment of M_B anticlockwise.



$$M_B = \frac{4EI}{L}\theta_B$$

$$\frac{wL^2}{12} = \frac{4EI}{L}\theta_B$$

Joint equilibrium equation
(or equation of action superposition)

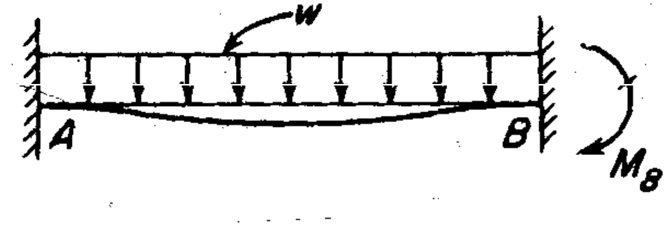
$$\therefore \theta_B = \frac{wL^3}{48EI}$$

- Restrained structure: Restraint at ***B*** causes a reaction of M_B.

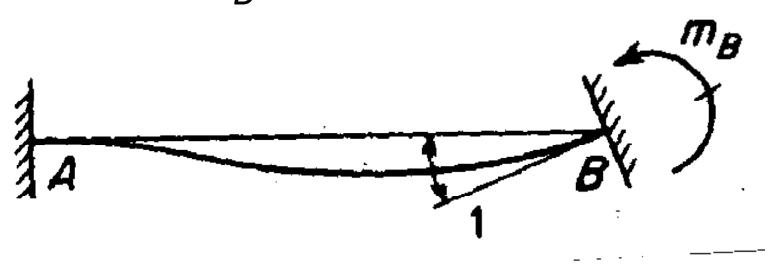
(Note the sign conventio clockwise positive)

M_B = wl²/12

r



Apply unit rotation corresponding to θ_B



Let the moment required for this unit rotation be

Moment required to induce a rotation of B

(Joint equilibrium equation)

$$\frac{wL^2}{12} - \frac{4EI}{L}\theta_B = 0 \qquad \therefore \theta_B = -\frac{M_B}{m_B} = \frac{wL^3}{48EI}$$

m_B (Moment required for unit rotation) is the stiffness coefficient here.

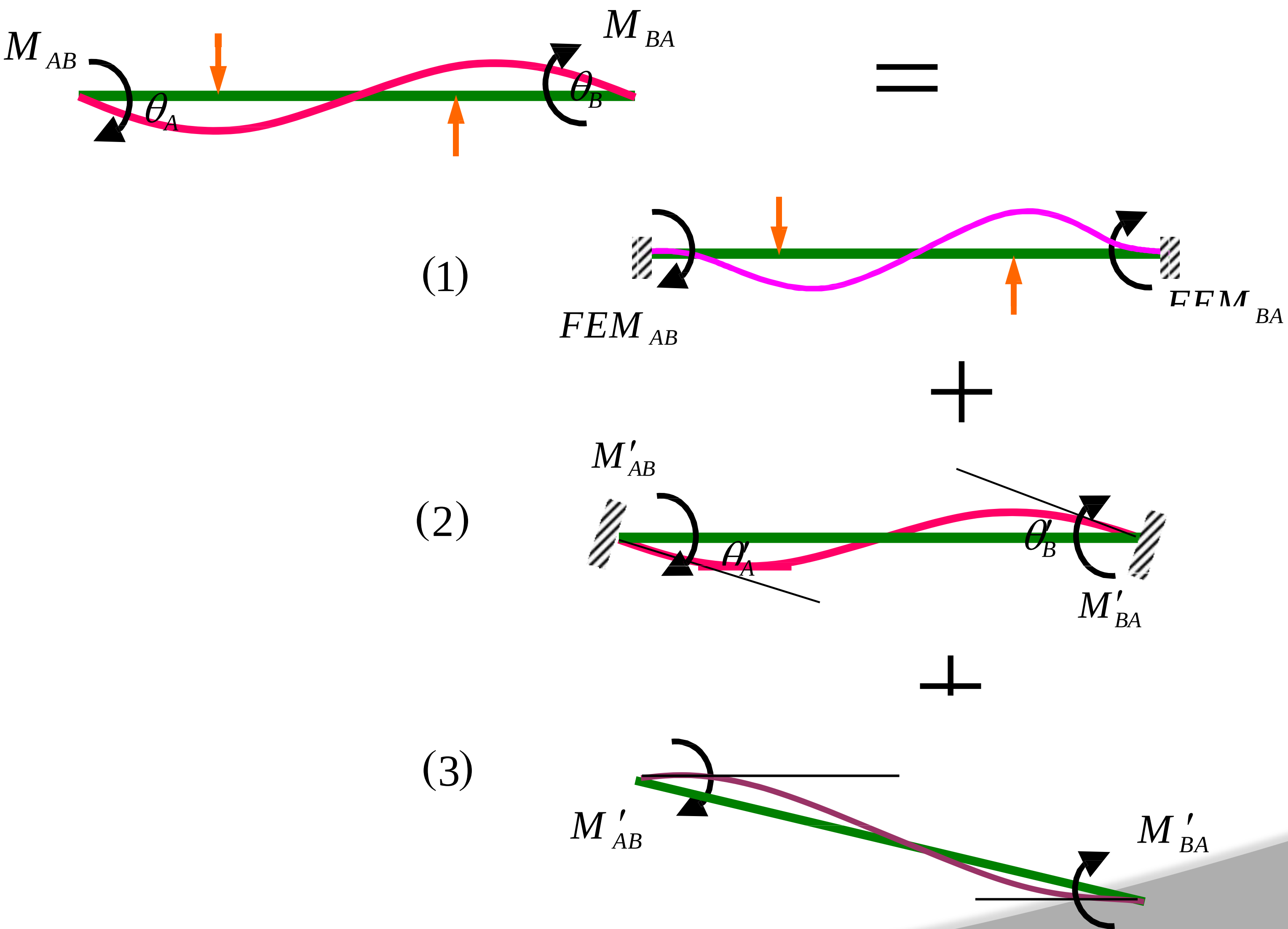
$$M_B + m_B\theta_B = 0$$

INTRODUCTION

- This method is based on the relationships of end moments with slopes and deflections (called slope-deflection equations) for each member.

Approach to solve problems

- The slope-deflection equations are written for each member.
- Joint equilibrium conditions are written.
- Solving the joint equilibrium conditions, unknown displacements are found out.
- Substituting these unknown displacements back in the slope-deflection equations, we get the unknown end moments.

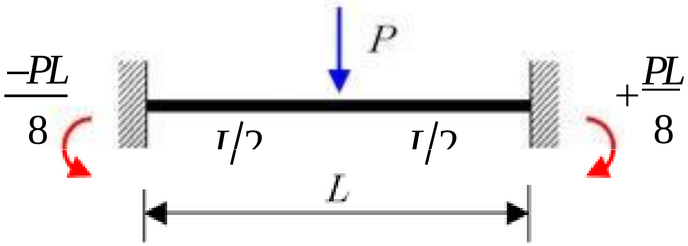


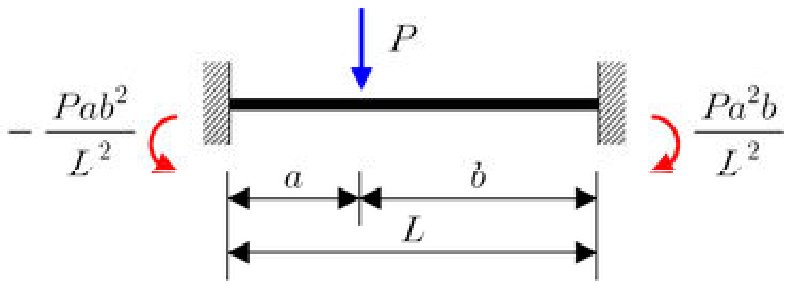
M_{AB}
M_{BA}
θ_A
θ_B
=
(1)
FEM_{AB}
FEM_{BA}
+
(2)
M'_{AB}
θ_A
θ_B
M'_{BA}
+
(3)
M'_{AB}
M'_{BA}

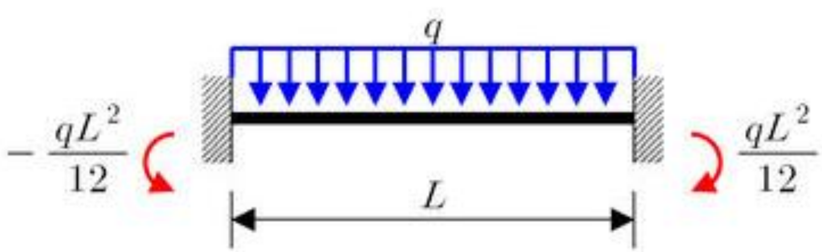
(1) Ends assumed as fixed (zero rotation). This requires restraining moments (fixed end moments) FEM_{AB} and FEM_{BA}. External loads are acting.

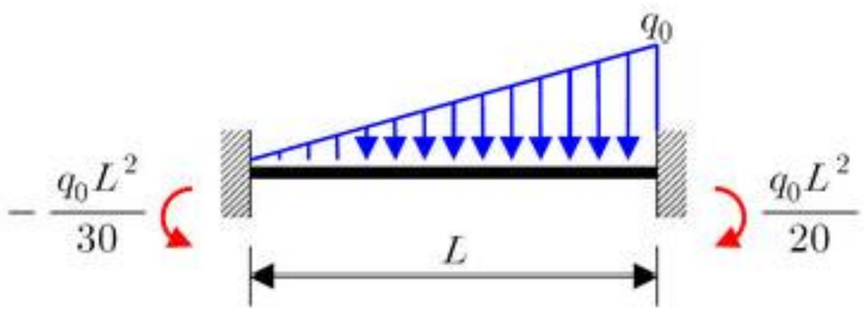
(2) Rotations are forced at ends. This requires moments M'_{AB} and M'_{BA}

(3) If there is a support settlement, moments M''_{AB} and M''_{BA} will be induced.

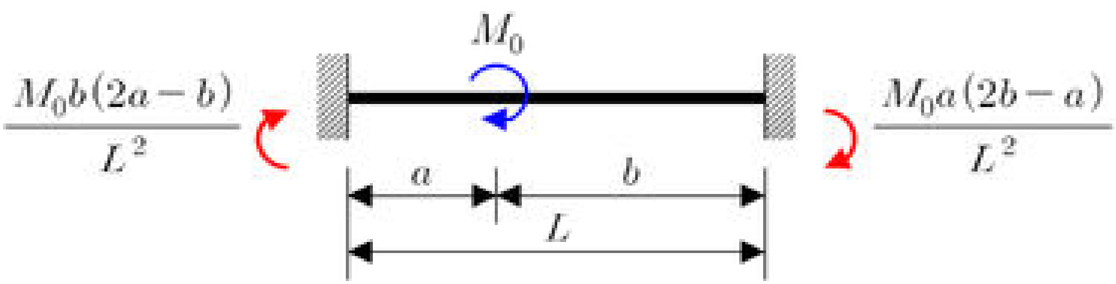

$-\frac{PL}{8}$
P
L
$+\frac{PL}{8}$


$-\frac{Pab^2}{L^2}$
P
a
b
L
$\frac{Pa^2b}{L^2}$

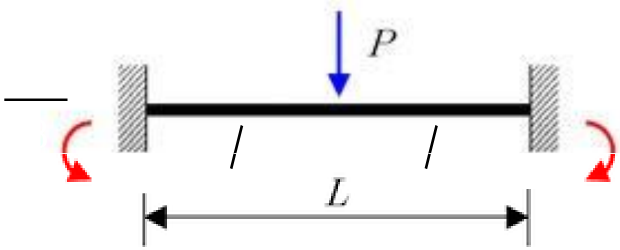

$-\frac{qL^2}{12}$
q
L
$\frac{qL^2}{12}$

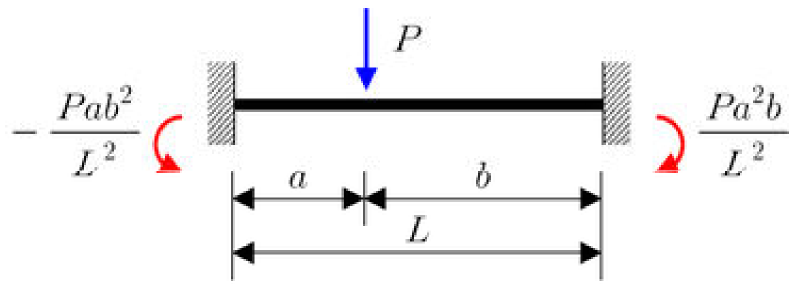


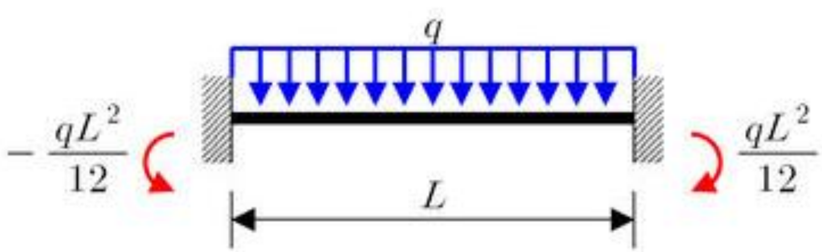
q_0
$-\frac{q_0 L^2}{30}$
$\frac{q_0 L^2}{20}$
L

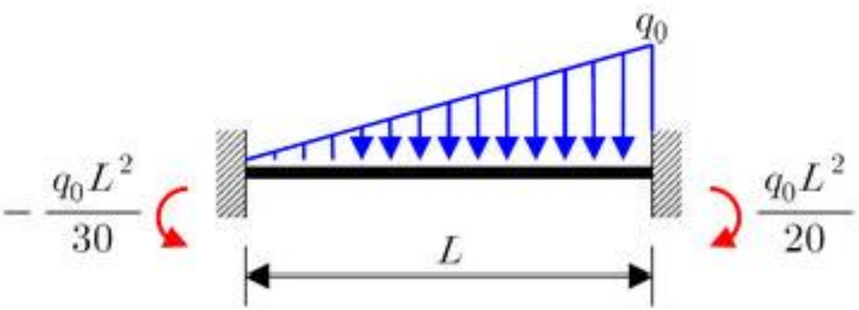


M_0
$\frac{M_0 b(2a-b)}{L^2}$
$\frac{M_0 a(2b-a)}{L^2}$
a
b
L

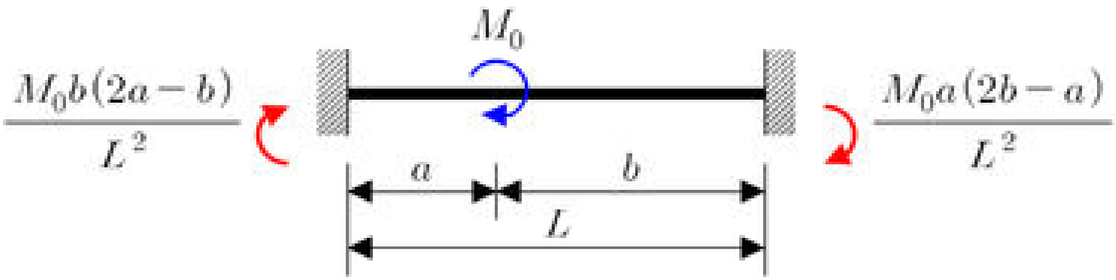

P
L


P
$-\frac{Pab^2}{L^2}$
$\frac{Pa^2b}{L^2}$
a
b
L


q
$-\frac{qL^2}{12}$
$\frac{qL^2}{12}$
L

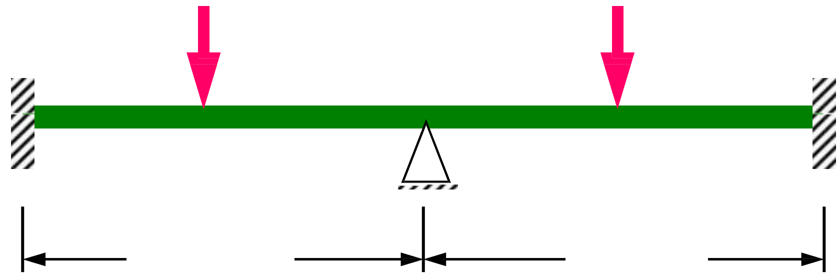


q_0
$-\frac{q_0 L^2}{30}$
$\frac{q_0 L^2}{20}$
L



M_0
$\frac{M_0 b(2a-b)}{L^2}$
$\frac{M_0 a(2b-a)}{L^2}$
a
b
L

Example



Problem structure

Fixed end moments (reactive)

$-Pab^2$

Fixed end moments

$$FEM_{AB} = \frac{}{l^2} = \frac{-5\times 3\times 2^2}{5^2} = -2.4\,kNm$$

$$FEM_{BA} = \frac{Pa^2b}{l^2} = \frac{5\times 3^2\times 2}{5^2} = 3.6\,kNm$$

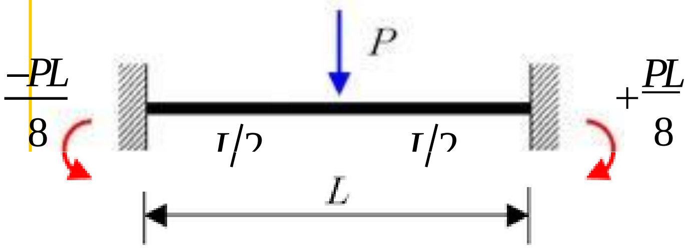
$$FEM_{BC} = \frac{-Pl}{8} = \frac{-8\times 5}{8} = -5\,kNm$$

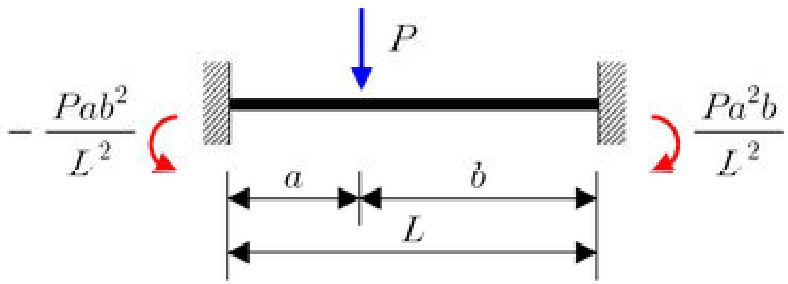
$$FEM_{CB} = \frac{Pl}{8} = \frac{8\times 5}{8} = 5\,kNm$$

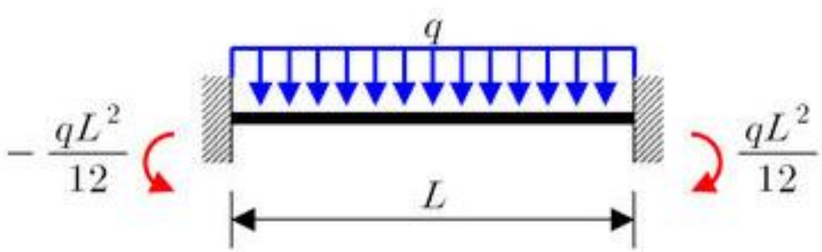
Known displacements

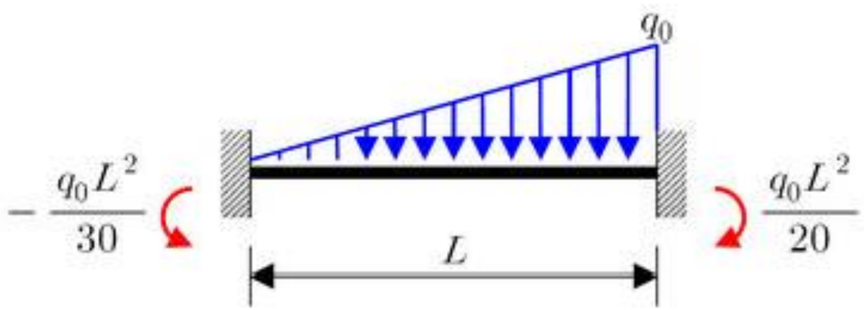
$$\theta_A = \theta_C = 0 \qquad \delta_A = \delta_B = \delta_C = 0$$

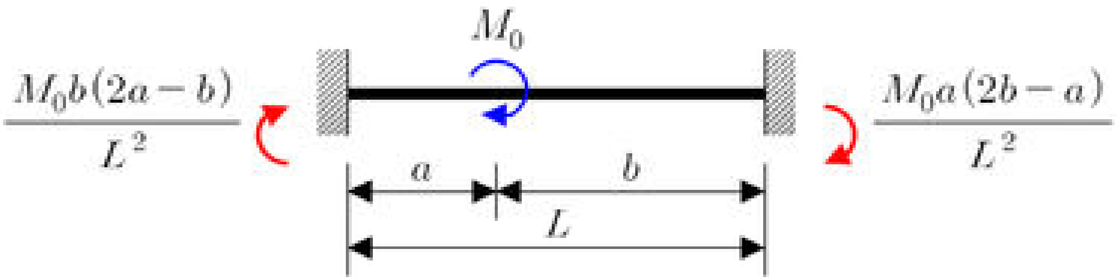
FIXED END MOMENTS



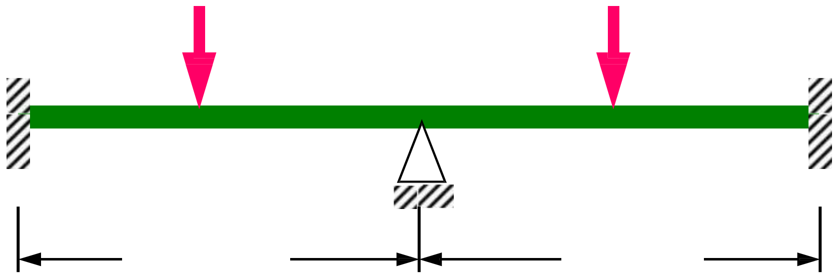





q_0
$-\frac{q_0 L^2}{30}$
$\frac{q_0 L^2}{20}$
L


M_0
$\frac{M_0 b(2a-b)}{L^2}$
$\frac{M_0 a(2b-a)}{L^2}$
a
b
L

Example



Problem structure

Fixed end moments (reactive)

$-Pab^2$

Fixed end moments

$$FEM_{AB} = \frac{\quad}{l^2} = \frac{-5\times3\times2^2}{5^2} = -2.4\,kNm$$

$$FEM_{BA} = \frac{Pa^2b}{l^2} = \frac{5\times3^2\times2}{5^2} = 3.6\,kNm$$

$$FEM_{BC} = \frac{-Pl}{8} = \frac{-8\times5}{8} = -5\,kNm$$

$$FEM_{CB} = \frac{Pl}{8} = \frac{8\times5}{8} = 5\,kNm$$

Known displacements

$$\theta_A = \theta_C = 0 \qquad \delta_A = \delta_B = \delta_C = 0$$

UNIT-V

MOVING LOADS AND INFLUENCE LINES

INTRODUCTION TO INFLUENCE LINE

Definition : An influence line is a plot of the magnitude of the resulting reaction/axial force/shear/moment generated in a beam or structure as a unit load travels across its length.

Influence lines can be generated for any of these actions (reactions, axial forces, shears, or moments) in a structure.

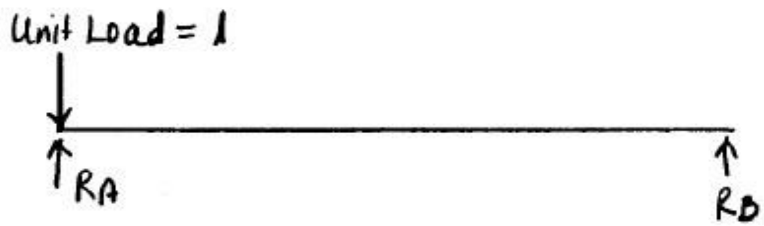
WHY WE USE INFLUENCE LINES

Influence lines are used

1. To determine where to place moving loads on a structure to obtain maximum results (reactions, shears, moments, axial forces).

1. To compute these reactions or other actions (shears/moments/axial forces) once the loads are placed in critical positions.

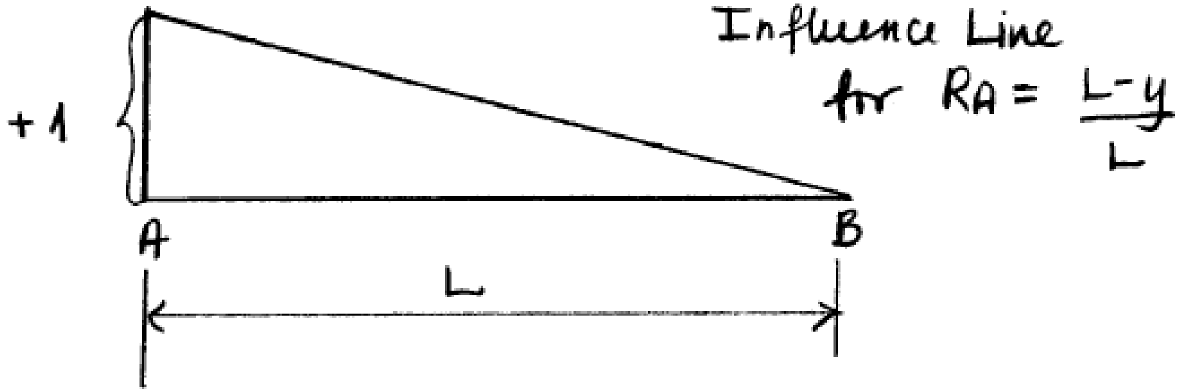
A TYPICAL INFLUENCE LINE



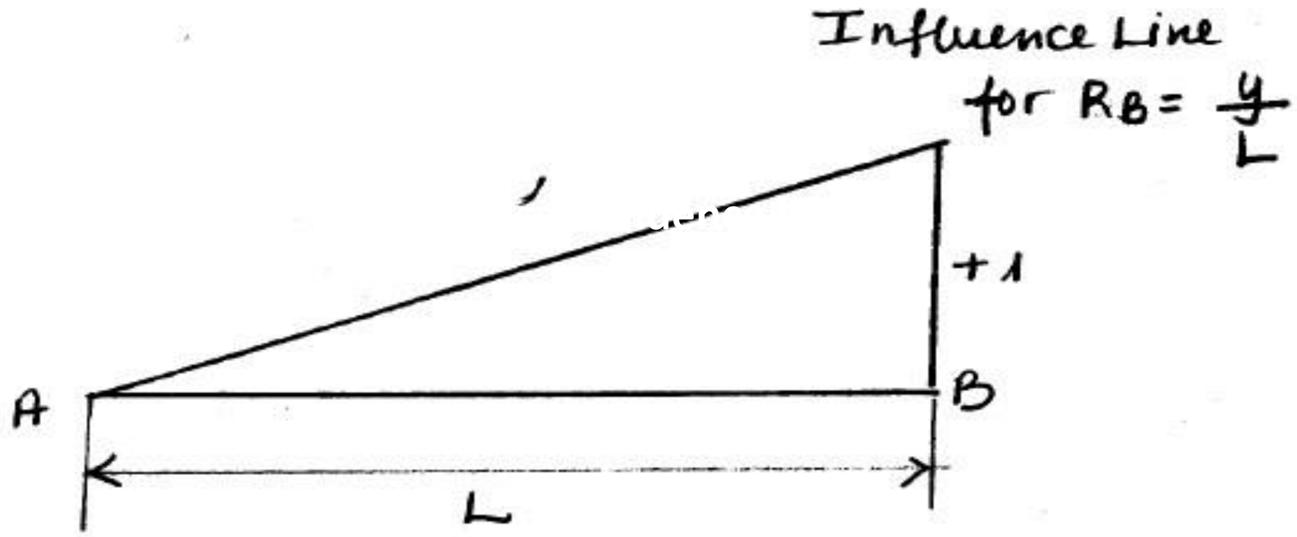
Here: $R_A + R_B = 1$

$R_B(L) = 0 \therefore R_A = 1$ when U.L. at A

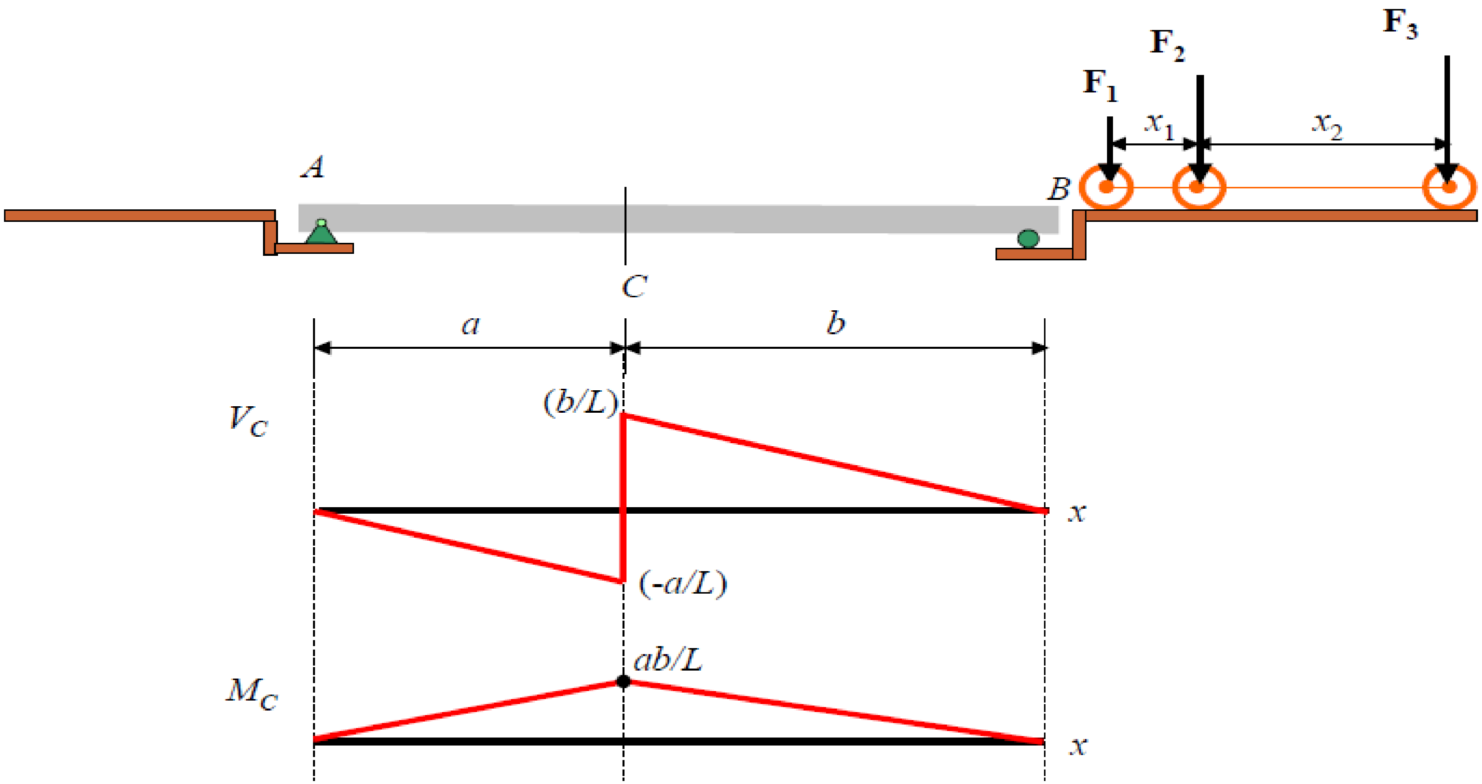
A TYPICAL INFLUENCE LINE



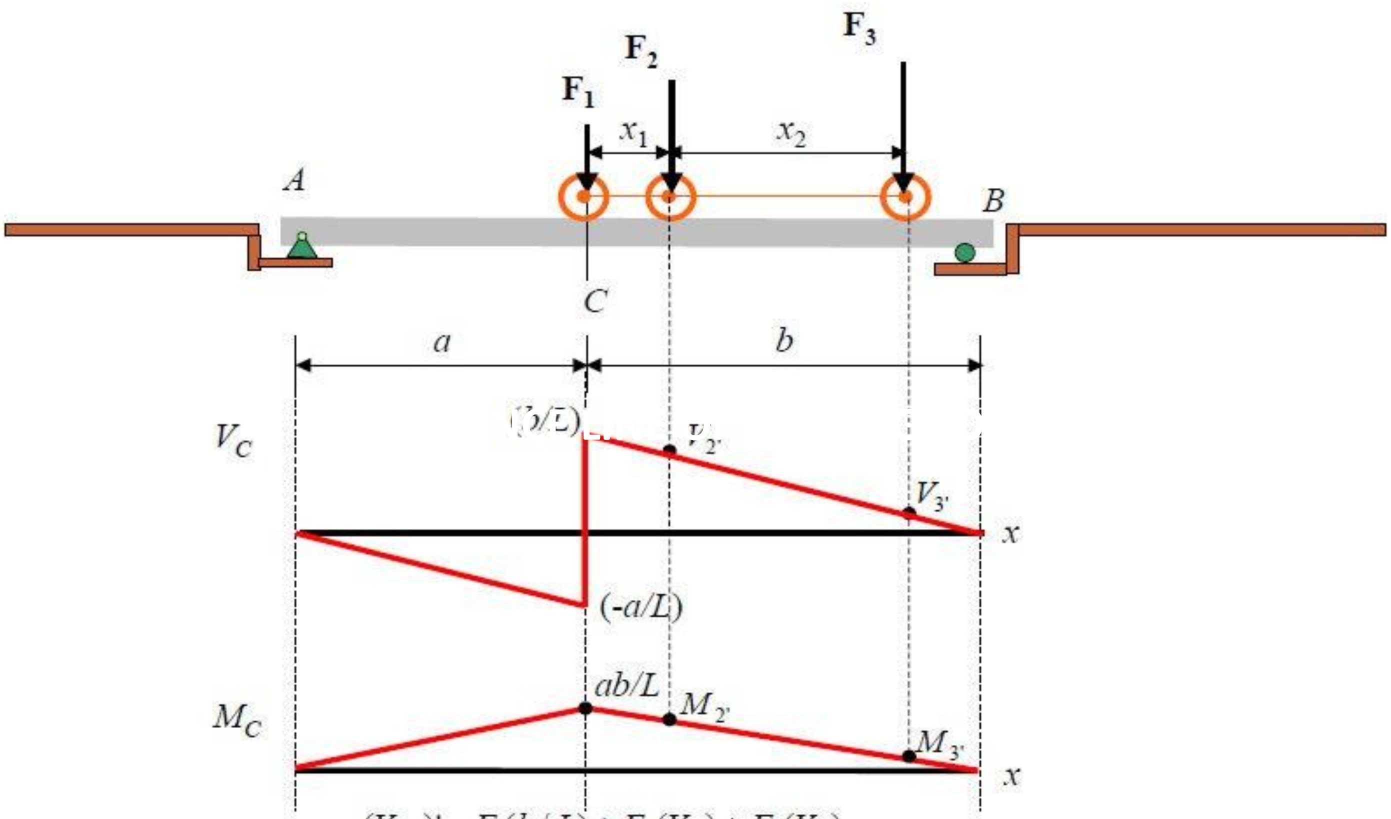
A TYPICAL INFLUENCE LINE



INFLUENCE LINE OF MOVING LOAD



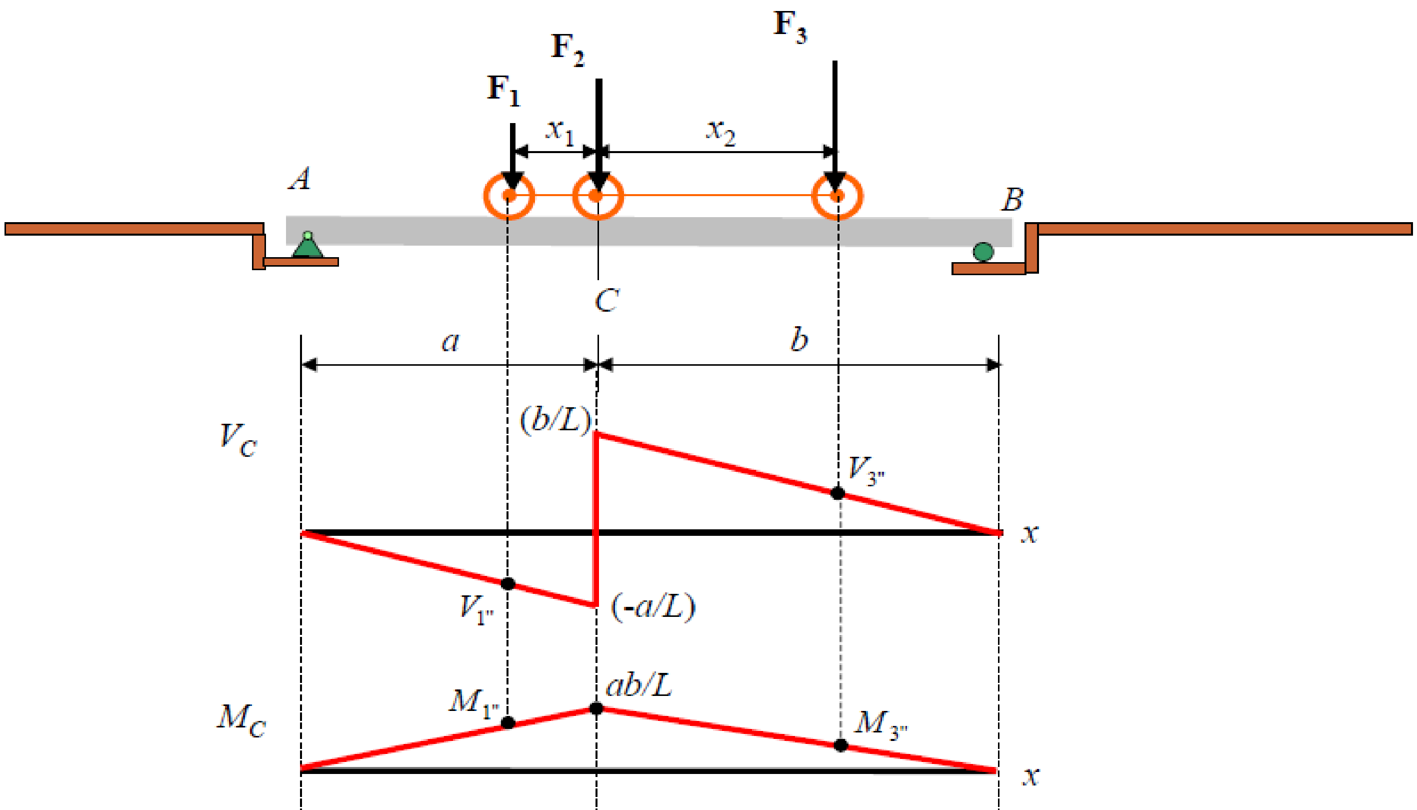
INFLUENCE LINE OF MOVING LOAD



$$(V_{CR})' = F_1(b/L) + F_2(V_{2'}) + F_3(V_{3'})$$

$$(V_{CL})' = F_1(-a/L) + F_2(V_{2'}) + F_3(V_{3'})$$

$$M' = F_1(ab/L) + F_2(M_{2'}) + F_3(M_{3'})$$

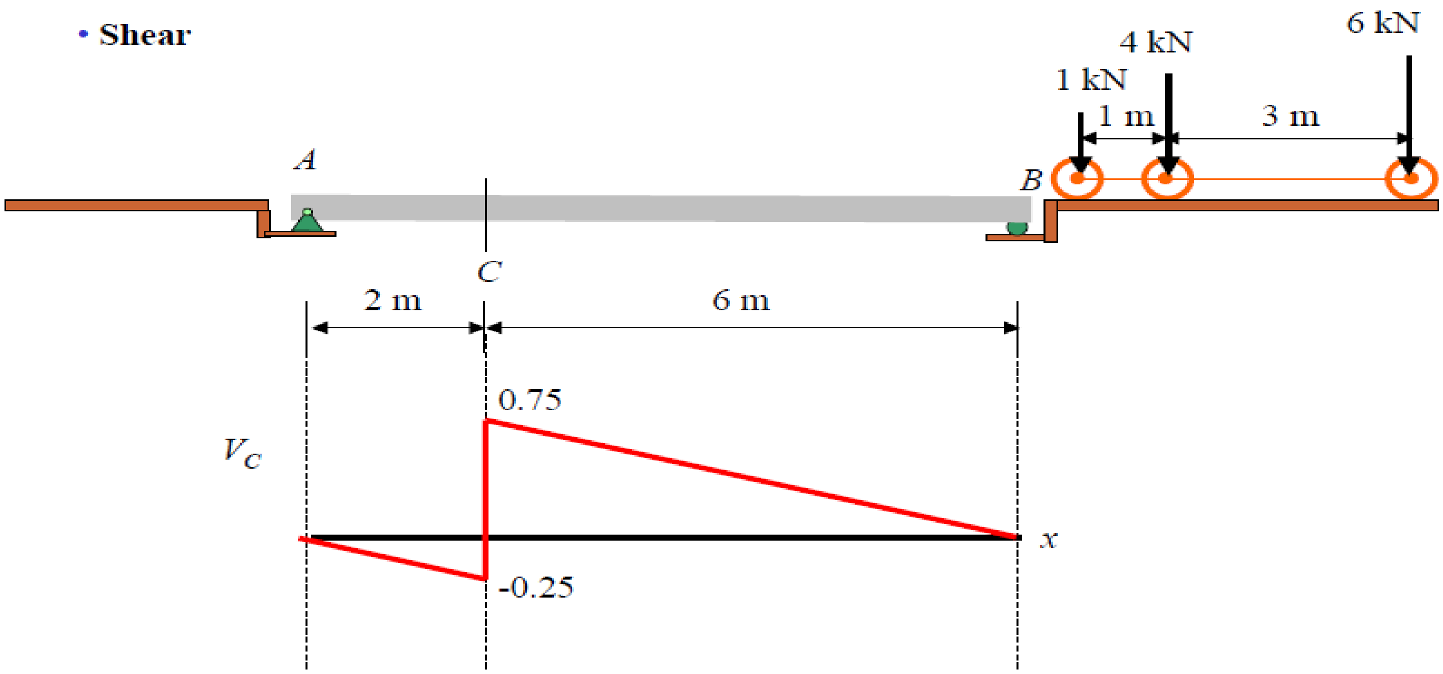


$$(V_{CR})'' = F_1 V_{1''} + F_2 (b/L) + F_3 (V_{3''})$$

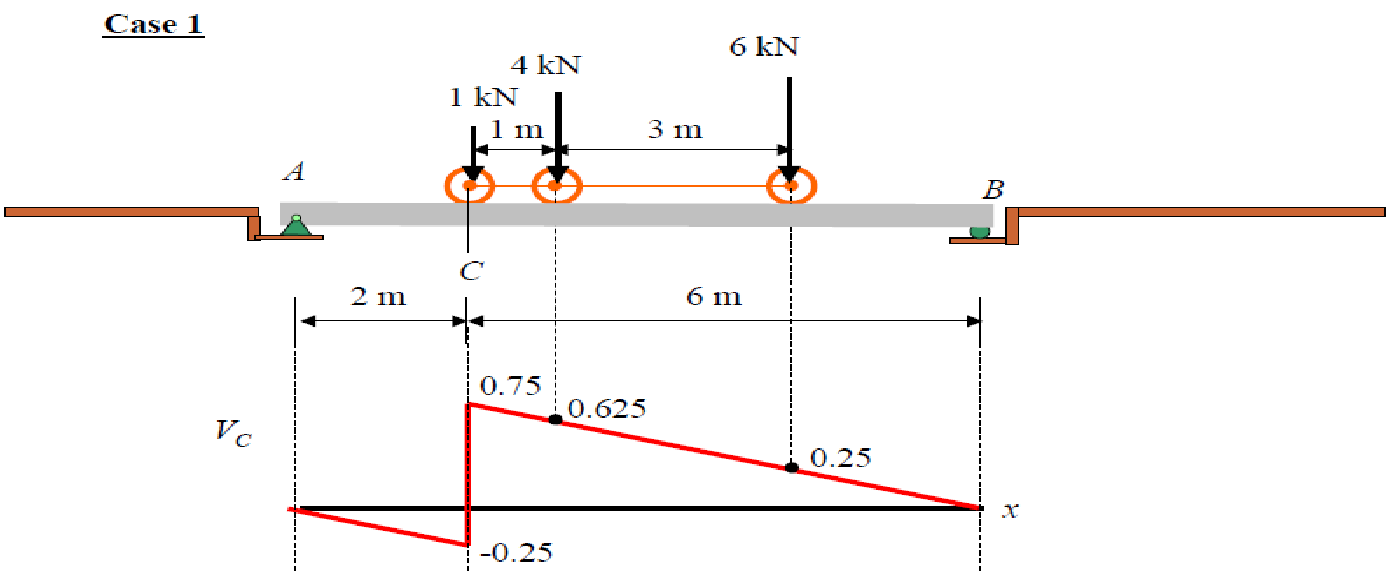
$$(V_{CL})'' = F_1 V_{1''} + F_2 (-a/L) + F_3 (V_{3''})$$

$$M'' = F_1 M_{1''} + F_2 (ab/L) + F_3 (M_{3''})$$

- **Shear**



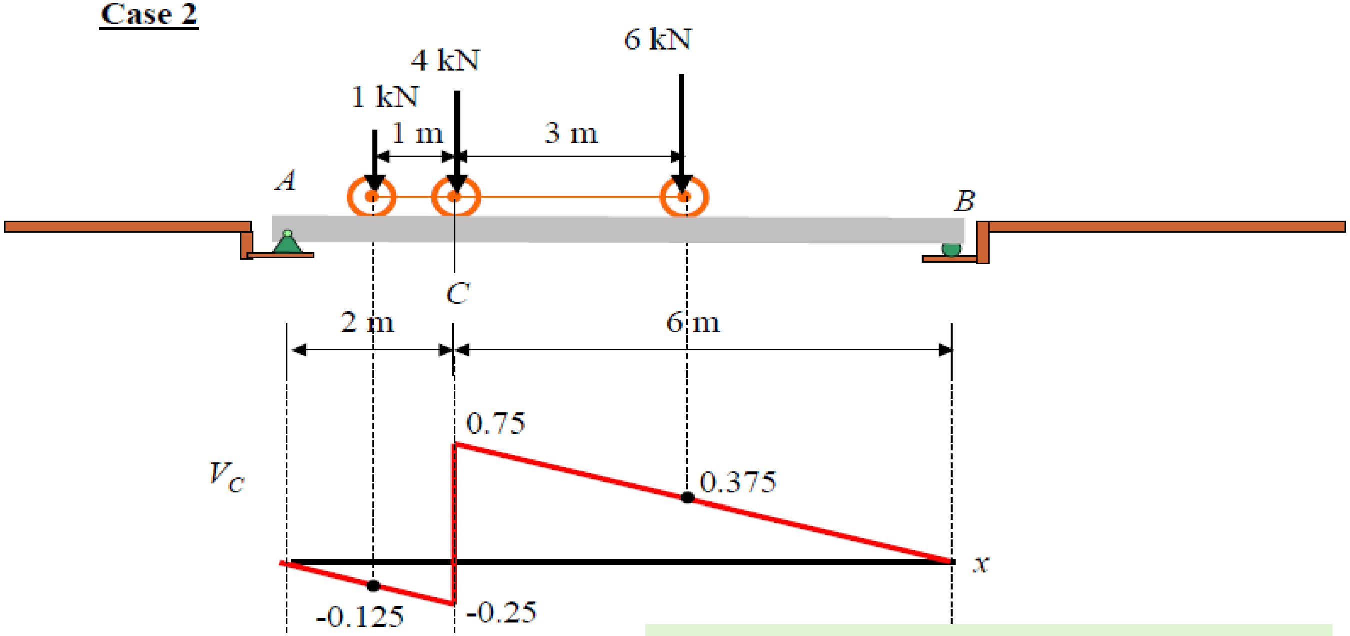
SHEAR DUE TO MOVING LOAD



$$(V_C)_1 = 1(0.75) + 4(0.625) + 6(0.25) = 4.75 \text{ kN}$$

SHEAR DUE TO MOVING LOAD

Case 2



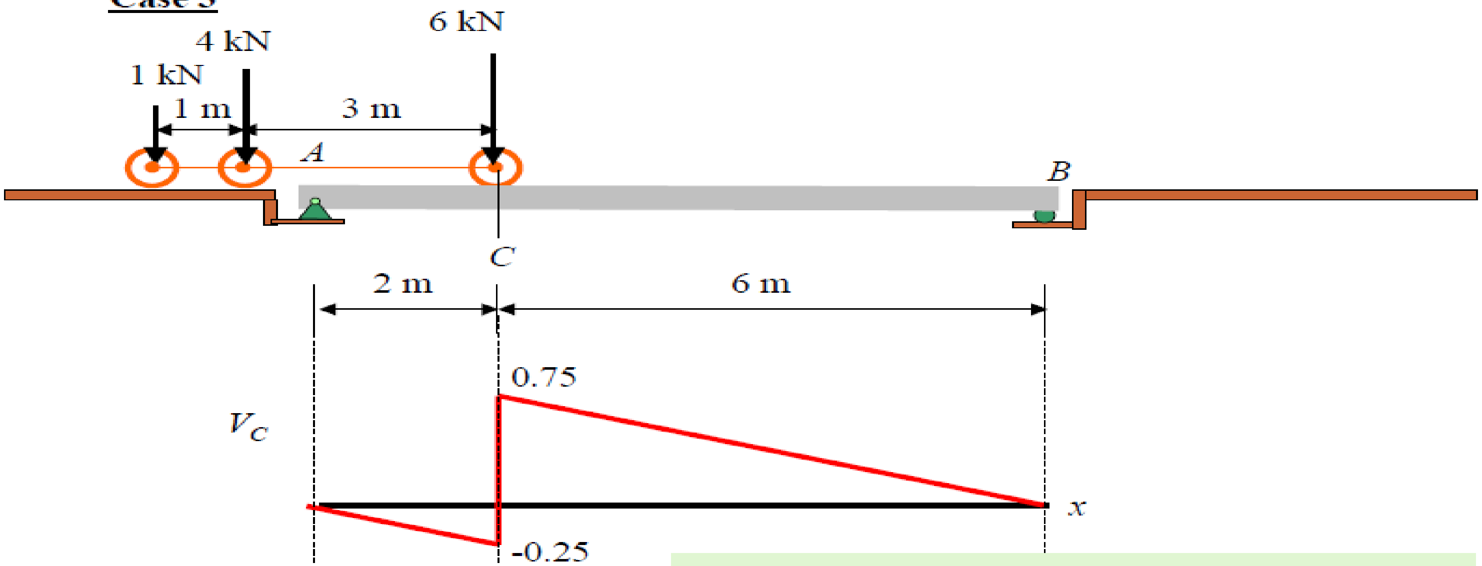
$(V_C)_1 = 1(0.75) + 4(0.625) + 6(0.25) = 4.75$ kN

$(V_C)_2 = 1(-0.125) + 4(0.75) + 6(0.375) = 5.125$ kN

$\Delta V_{1\text{-}2} = 5.125 - 4.75 = 0.375$ kN

SHEAR DUE TO MOVING LOAD

Case 3



$(V_C)_2 = 1(-0.125) + 4(0.75) + 6(0.375) = 5.125$ kN

$(V_C)_3 = 6(0.75) = 4.5$ kN

$\Delta V_{2\text{-}3} = 4.5 - 5.125 = -0.625$ kN

$(V_C)_{max} = (V_C)_2 = 5.125$ kN


Thank you