THEORY OF THIN PLATES AND SHELLS

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Outline

The concepts of Space Curves, surfaces, shell co-ordinates, boundary conditions.

The governing equation for a rectangular plate, Navier solution for simply- supported rectangular plate under various loadings.

Analyze , understand axi- symmetric loading, governing differential equation in polar co-ordinates.

The cylindrical and conical shells, application to pipes and pressure vessels. and understand the membrane theory of cylindrical.

UNIT-I

INTRODUCTION

SHELLS IN ENGINEERING STRUCTURES

Thin shells as structural elements occupy a leadership position in engineering and, in particular, in civil, mechanical, architectural, aeronautical, and marine engineering.

- Examples of shell structures in civil and architectural engineering are large-span roofs, liquid-retaining structures and water tanks, containment shells of nuclear power plants, and

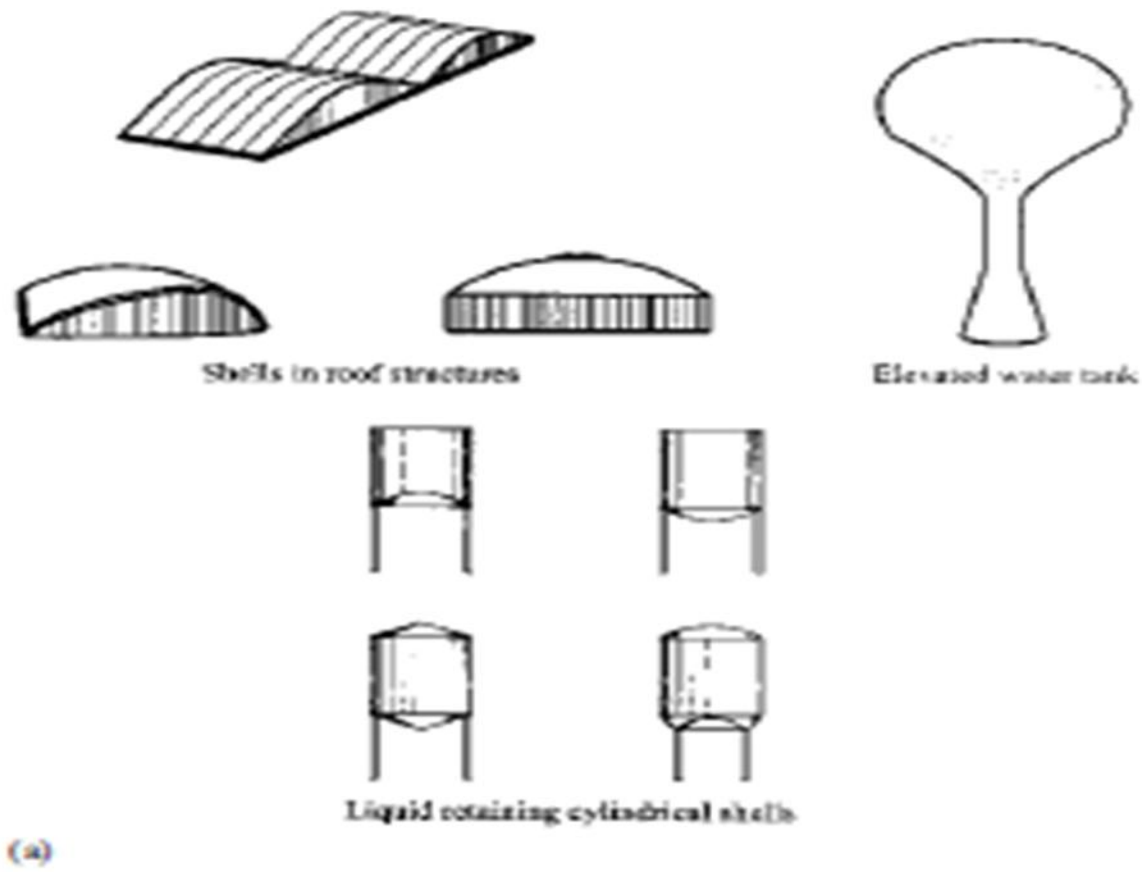
- concrete arch domes. In mechanical engineering, shell forms are used in piping systems, turbine disks, and pressure vessels technology. Aircrafts, missiles, rockets, ships, and submarines are examples of the use of shells in aeronautical and marine engineering.
- Another application of shell engineering is in the field of biomechanics: shells are found in various biological forms, such as the eye and the skull, and plant and animal shapes. This is only a small list of shell forms in engineering and nature.

ADVANTAGES

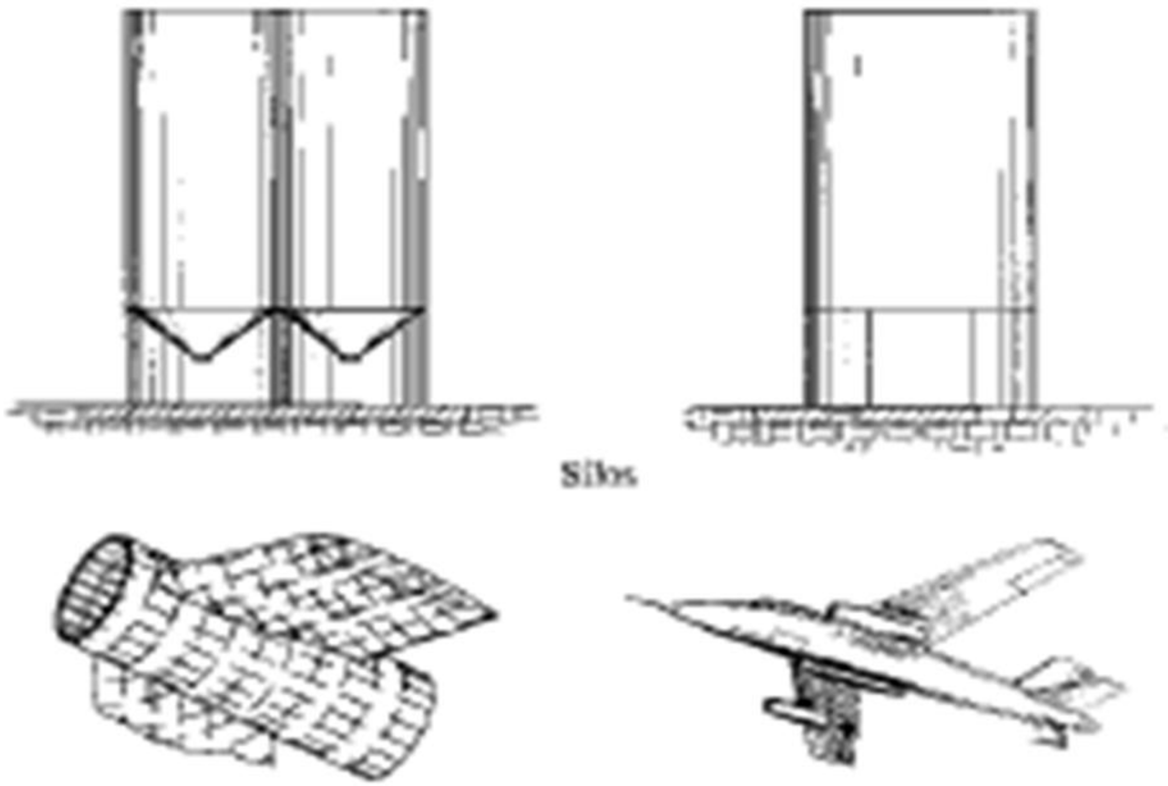
- Strategic cost management is understood in different ways in literature. Cooper and Slag Mulder argued that strategic cost management is “the application of cost management techniques so that they simultaneously improve the strategic position of a firm and reduce costs”.
- A hospital redesigns its patient admission procedure so it becomes more efficient and easier for patients. The hospital will become known for its easy admission procedure so more people will come to that hospital if the patient has a choice.

- In addition to these mechanical advantages, shell structures enjoy the unique position of having extremely high aesthetic value in various architectural designs.
- Shell structures support applied external forces efficiently by virtue of their geometrical form, i.e., spatial curvatures; as a result, shells are much stronger and stiffer than other structural forms

Examples



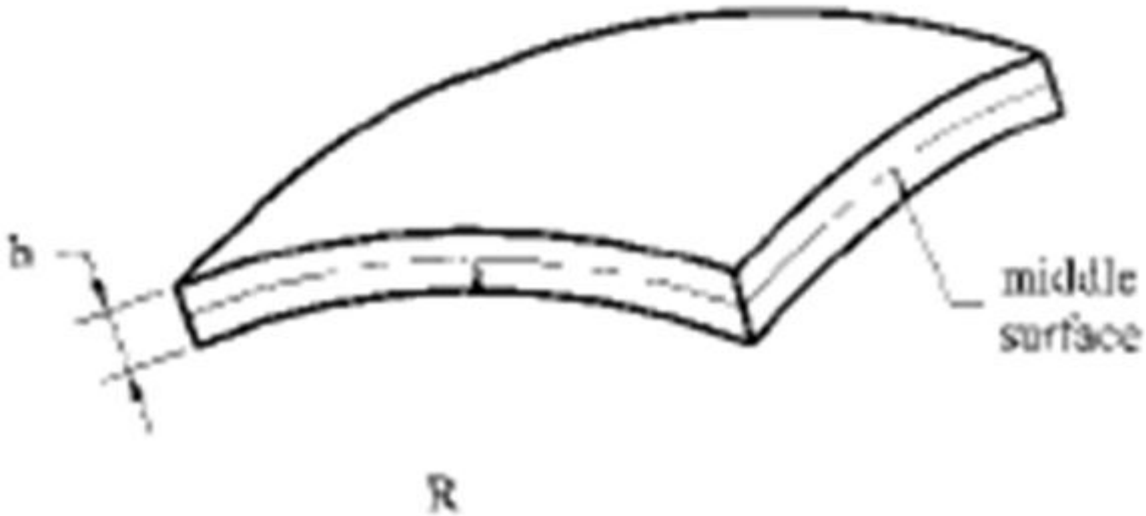
Shells in roof structures
Elevated water tank
Liquid retaining cylindrical shells
(a)



(b) Aircraft structures

GENERAL DEFINITIONS AND FUNDAMENTALS OF SHELLS

We now formulate some definitions and principles in shell theory. The term shell is applied to bodies bounded by two curved surfaces, where the distance between the surfaces is small in comparison with other body dimensions fig shown below



The locus of points that lie at equal distances from these two curved surfaces defines the middle surface of the shell. The length of the segment, which is perpendicular to the curved surfaces, is called the thickness of the shell and is denoted by h.

The geometry of a shell is entirely defined by specifying the form of the middle surface and thickness of the shell at each point. In this book we consider mainly shells of a constant thickness. Shells have all the characteristics of plates, along with an additional one – curvature.

The curvature could be chosen as the primary classifier of a shell because a shell’s behavior under an applied loading is primarily governed by curvature .

Depending on the curvature of the surface, shells are divided into cylindrical (noncircular and circular), conical, spherical, ellipsoidal, paraboloidal, toroidal, and hyperbolic paraboloidal shells. Owing to the curvature of the surface, shells are more complicated than flat plates because their bending cannot, in general, be separated from their stretching.

On the other hand, a plate may be considered as a special limiting case of a shell that has no curvature; consequently, shells are sometimes referred to as curved plates. This is the basis for the adoption of methods from the theory of plates, discussed in Part I, into the theory of shells.

There are two different classes of shells: thick shells and thin shells. A shell is called thin if the maximum value of the ratio h=R (where R is the radius of curvature of the middle surface) can be neglected in comparison with unity. For an engineering accuracy, a shell may be regarded as thin if [1] the following condition is satisfied:

$$\text{Max } (h/R) \leq 1/20$$

Hence, shells for which this inequality is violated are referred to as thick shells. For a large number of practical applications, the thickness of shells lies in the range

$$(1/1000) \leq (h/R) \leq 1/20$$

THE LINEAR SHELL THEORIES

The most common shell theories are those based on linear elasticity concepts. Linear shell theories predict adequately stresses and deformations for shells exhibiting small elastic deformations; i.e., deformations for which it is assumed that the equilibrium equation conditions for deformed shell surfaces are the same as if they were not deformed, and Hooke's law applies.

For the purpose of analysis, a shell may be considered as a three-dimensional body, and the methods of the theory of linear elasticity may then be applied.

However, a calculation based on these methods will generally be very difficult and complicated. In the theory of shells, an alternative simplified method is therefore employed. According to this method and adapting some hypotheses the 3D problem of shell equilibrium and straining may be reduced to the analysis of its middle surface only, i.e. the given shell, as discussed earlier as a thin plate, may be regarded as some 2D body.

In the development of thin shell theories, simplification is accomplished by reducing the shell problems to the study of deformations of the middle surface.

Shell theories of varying degrees of accuracy were derived, depending on the degree to which the elasticity equations were simplified. The approximations necessary for the development of an adequate theory of shells have been the subject of considerable discussions among investigators in the field. We present below a brief outline of elastic shell.

A second class of thin elastic shells, which is commonly referred to as higher order approximation, has also been developed. To this grouping it is possible to assign all linear shell theories in which one or another of the Kirchhoff–Love hypotheses are suspended. First, we consider some representative theories in which the thinness assumption is delayed in derivation while the rest of the postulates are

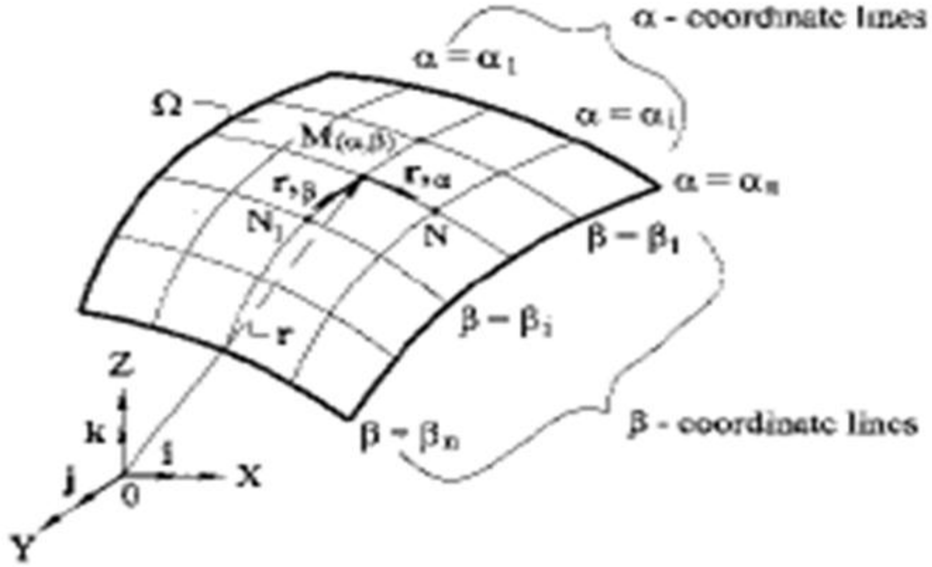
The membrane stress condition is an ideal state at which a designer should aim. It should be noted that structural materials are generally far more efficient in an extensional rather in a flexural mode because:

1. Strength properties of all materials can be used completely in tension (or compression), since all fibers over the cross section are equally strained and load-carrying capacity may simultaneously reach the limit for the whole section of the component.

2. The membrane stresses are always less than the corresponding bending

COORDINATE SYSTEM OF THE SURFACE

A surface can be defined as a locus of points whose position vector, r, directed from the origin 0 of the global coordinate system, OXYZ, is a function of two independent parameters α and β



If the α- and β -coordinate lines are mutually perpendicular at all points on a surface Ω _ (i.e., the angles between the tangents to these lines are equal to 90), the curvilinear coordinates are said to be orthogonal.

The derivatives of the position vector r with respect to the curvilinear coordinates and are given by the following:

$$\frac{\partial \mathbf{r}}{\partial \alpha} = \mathbf{r}_{,\alpha} \quad \text{and} \quad \frac{\partial \mathbf{r}}{\partial \beta} = \mathbf{r}_{,\beta},$$

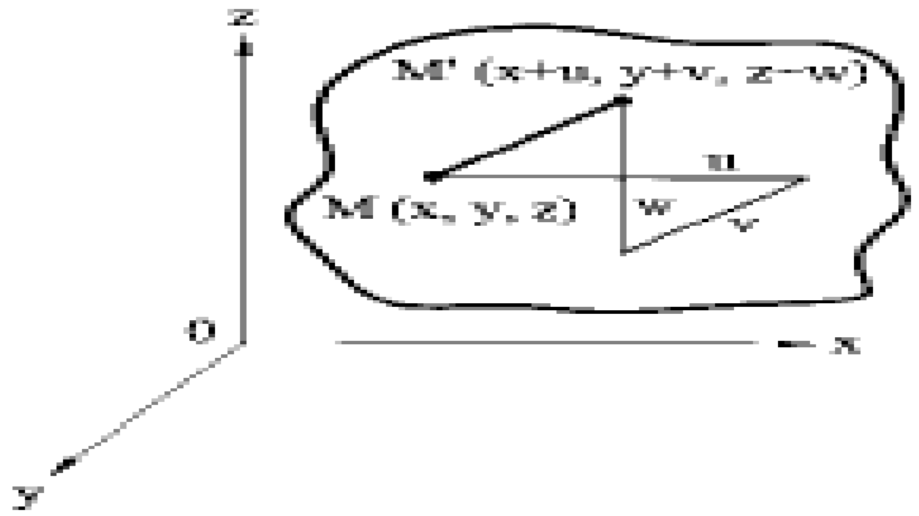
where we have introduced the comma notation to denote partial derivatives with respect to α and β

STRAINS AND DISPLACEMENTS

Assume that the elastic body shown in Fig. below is supported in such a way that rigid body displacements (translations and rotations) are prevented. Thus, this body deforms under the action of external forces and each of its points has small elastic displacements. For example, a point M had the coordinates x; y, and z in initial unreformed state. After deformation, this point moved into position M0and its coordinates became the following

$$x' = x + u,\quad y' = y + v,\quad z' = z' + w,$$

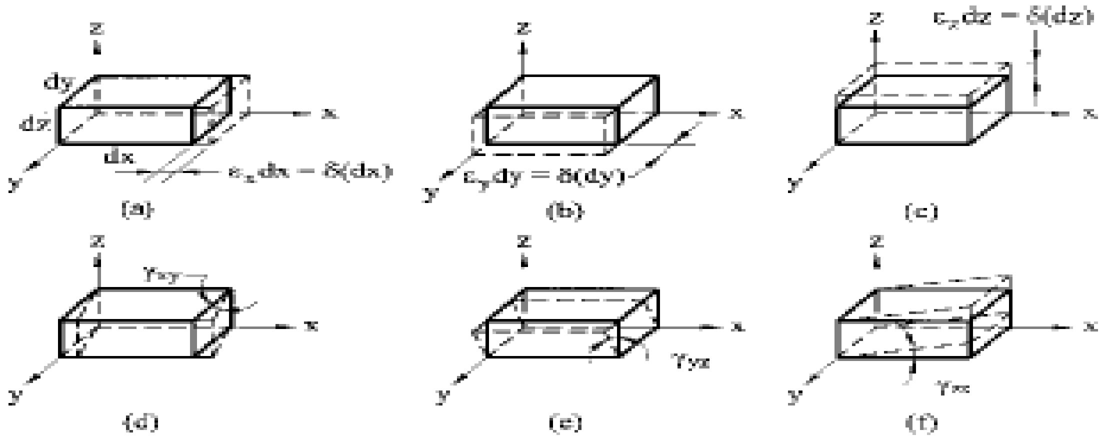
where u, v, and w are projections of the displacement vector of point M, vector MM0, on the coordinate axes x, y and z. In the general case, u, v, and w are functions of x, y, and z.



Again, consider an infinitesimal element in the form of parallelepiped enclosing point of interest M. Assuming that a deformation of this parallelepiped is small, we can represent it in the form of the six simplest deformations shown in Fig. a,b,c,d. define the elongation (or contraction) of edges of the parallelepiped in the direction of the coordinate axes and can be defined as

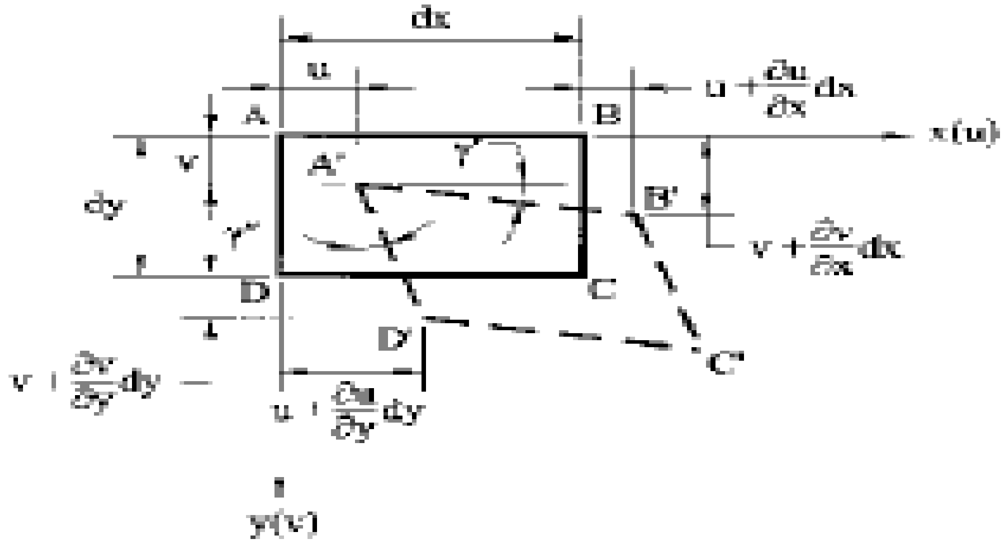
$$\varepsilon_x = \frac{\delta(dx)}{dx}, \varepsilon_y = \frac{\delta(dy)}{dy}, \varepsilon_z = \frac{\delta(dz)}{dz},$$

And they are called the normal or linear strains.



the increments delta dx can be expressed by the second term in the Taylor series, i.e., $\delta dx = (\partial u/\partial x)dx$, thus, we can write $\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \varepsilon_z = \frac{\partial w}{\partial z}.$

Since we have confined ourselves to the case of very small deformations, we may omit the quantities $\partial u/\partial x$ and $\partial v/\partial y$ in the denominator of the last expression, as being negligibly small compared with unity. Finally, we obtain



$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}.$$

Similarly, we can obtain _xz and _yz. Thus, the shear strains are given by

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.$$

Similar to the stress tensor at a given point, we can define a strain tensor a

$$T_D = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{pmatrix}$$

It is evident that the strain tensor is also symmetric because of

$$\gamma_{xy} = \gamma_{yx}, \quad \gamma_{xz} = \gamma_{zx}, \quad \gamma_{yz} = \gamma_{zy}$$

Constitutive equations

The constitutive equations relate the stress components to strain components. For the linear elastic range, these equations represent the generalized Hooke's law. In the case of a three-dimensional isotropic body, the constitutive equations are given by

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right], \quad \varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right], \quad \varepsilon_z = \frac{1}{2}\left[\sigma_z - \nu(\sigma_y + \sigma_x)\right],$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{xz} = \frac{1}{G}\tau_{xz}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz},$$

where E, and G are the modulus of elasticity, Poisson's ratio, and the shear modulus, respectively. The following relationship exists between E and G:

$$G = \frac{E}{2(1+\nu)}$$

EQUILIBRIUM EQUATIONS

The stress components introduced previously must satisfy the following differential equations of equilibrium:

where Fx; Fy; and Fz are the body forces (e.g., gravitational, magnetic forces). In deriving these equations, the reciprocity of the shear stresses, Eqs , has been used.

UNIT – II

STATIC ANALYSIS OF PLATES

THE ELEMENTARY CASES OF PLATE BENDING

Let us consider some elementary examples of plate bending of great importance for understanding how a plate resists the applied loads in bending. In addition, these elementary examples enable one to obtain closed-form solutions of the governing differential equation

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}$$

Cylindrical bending of a plate

Consider an infinitely long plate in the y axis direction. Assume that the plate is subjected to a transverse load which is a function of the variable x only, i.e., p ¼ pðxÞ

In this case all the strips of a unit width parallel to the x axis and isolated from the plate will bend identically. The plate as a whole is found to be bent over the cylindrical surface w ¼ wðxÞ. Setting all the derivatives with respect to y equal zero in Eq

we obtain the following equation for the deflection:

An integration of Eq. should present no problems. Let, for example, p ¼ p0xa, then the general solution of Eq. is of the following form:

$$w = w_h + w_p = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + \frac{p_0 x^5}{120 a D},$$

(b) Let $m_1 = m, m_2 = 0$

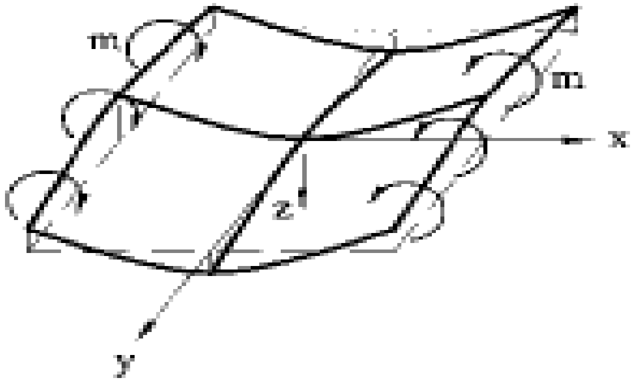
Then,

$$w = \frac{m}{2D(1 - \nu^2)}(-x^2 + \nu y^2).$$

A surface described by this equation has a saddle shape and is called the hyperbolic paraboloid of revolution .

(Horizontals of this surface are hyperbolas, asymptotes of which are given by the straight lines $\frac{x}{y} = \pm\sqrt{\nu}$

As is seen, due to the Poisson effect the plate bends not only in the plane of the applied bending moment Mx =m1 =m but it also has an opposite bending in the perpendicular plane



(c) Let $m_1 = m$, $m_2 = -m$

Then $$w = \frac{m}{2D(1-\nu)}(-x^2 + y^2).$$

Thus, a part of the plate isolated from the whole plate and equally inclined to the x and y axes will be loaded along its boundary by uniform twisting moments of intensity m. Hence, this part of the plate is subjected to pure twisting

Let us replace the twisting moments by the effective shear forces Vα rotating these moments through 90. Along the whole sides of the isolated part we obtain Vα=0, but at the corner points the concentrated forces S ¼ 2m are applied. Thus, for the model of Kirchhoff's plate, an application of self-balanced concentrated forces at corners of a rectangular plate produces a deformation of pure torsion because over the whole surface of the plate

$$\dot{M}_{nt} = m = \text{const.}$$

$$w = 0|_{x=0,a};\ \frac{\partial^2 w}{\partial x^2} = 0\bigg|_{x=0,a} \quad \text{and} \quad w = 0|_{y=0,b};\ \frac{\partial^2 w}{\partial y^2} = 0\bigg|_{y=0,b}. \tag{3.14}$$

In this case, the solution of the governing differential equation (2.24), i.e., the expressions of the deflection surface, wðx; yÞ, and the distributed surface load, pðx; yÞ, have to be sought in the form of an infinite Fourier series (see Appendix B), as follows:

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \tag{3.15a}$$

$$p(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} p_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}, \tag{3.15b}$$

where w_{mn} and P_{mn} represent coefficients to be determined. It can be easily verified that the expression for deflections (3.15a) automatically satisfies the prescribed boundary conditions (3.14).

Let us consider a general load configuration. To determine the Fourier coefficients pmn, each side of Eq. (3.15b) is multiplied by sin l_x a sin k_y b and integrated twice between the limits 0;a and 0;b, as follows (see Appendix B):

$$\int_0^a \int_0^b p(x, y) \sin \frac{l\pi x}{a} \sin \frac{k\pi y}{b} \mathrm{d}x\mathrm{d}y = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} p_{mn} \int_0^a \int_0^b \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{l\pi x}{a} \sin \frac{k\pi y}{b} \mathrm{d}x\mathrm{d}y. \quad \text{(a)}$$

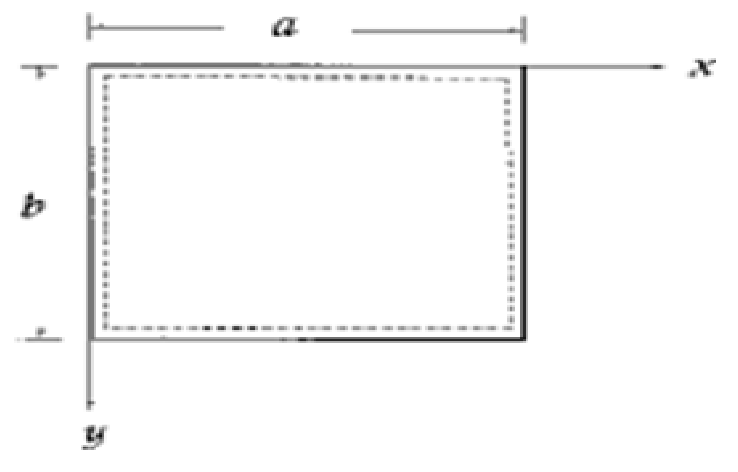


Fig. 3.5

It can be shown by direct integration that

$$\int_0^a \sin\frac{m\pi x}{a}\sin\frac{l\pi x}{a}\mathrm{d}x = \begin{cases} 0 \text{ if } m \neq l \\ a/2 \text{ if } m = l \end{cases}$$

$$\text{and } \int_0^b \sin\frac{n\pi y}{b}\sin\frac{k\pi y}{b}\mathrm{d}y = \begin{cases} 0 \text{ if } n \neq k \\ b/2 \text{ if } n = k. \end{cases} \quad (3.16)$$

The coefficients of the double Fourier expansion are therefore the following

$$p_{mn} = \frac{4}{ab}\int_0^a\int_0^b p(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\mathrm{d}x\mathrm{d}y. \quad (3.17)$$

Since the representation of the deflection (3.15a) satisfies the boundary conditions (3.14), then the coefficients wmn must satisfy Eq. (2.24). Substitution of Eqs (3.15) into Eq. (2.24) results in the following equation.

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{w_{mn}\left[\left(\frac{m\pi}{a}\right)^4+2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+\left(\frac{n\pi}{b}\right)^4\right]-\frac{p_{mn}}{D}\right\}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=0.$$

This equation must apply for all values of x and y.
We conclude therefore that

$$w_{mn}\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2-\frac{p_{mn}}{D}=0,$$

from which

$$w(x,y)=\frac{1}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{p_{mn}}{\left[(m/a)^2+(n/b)^2\right]^2}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \qquad (3.19)$$

where pmn is given by Eq. (3.17). It can be shown, by noting that $|\sin m\pi x/a|\leq 1$ and $|\sin n\pi y/b|\leq 1$ for every x and y and for every m and n, that the series (3.19) is convergent.

Since the stress resultants and couples are obtained from the second and third derivatives of the deflection wðx; yÞ, the convergence of the infinite series expressions of the internal forces and moments is less rapid, especially in the vicinity of the plate edges. This slow convergence is also accompanied by some loss of accuracy in the process of calculation. The accuracy of solutions and the convergence of series expressions of stress resultants and couples can be improved by considering more terms in the expansions and by using a special technique for an improvement of the convergence of Fourier's series.

RECTANGULAR PLATES SUBJECTED TO A CONCENTRATED LATERAL FORCE 'P'

Let us consider a rectangular plate simply supported on all edges of sides a and b and subjected to concentrated lateral force P applied at $x = \xi$ and $y = \eta$, ɔwn in Fig. 3.7.

Assume first that this force is uniformly distributed over the contact area of sides u and v (Fig. 3.6) i.e., its load intensity is defined as

$$p_0 = \frac{P}{uv}.$$

Substituting the above into Eq. (3.22), one obtains

$$p_{mn} = \frac{16P}{\pi^2 mnuv} \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b} \sin\frac{m\pi u}{2a} \sin\frac{n\pi v}{2b}. \qquad (3.24)$$

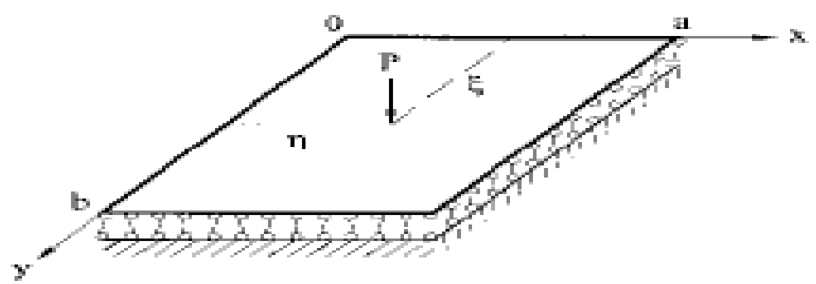


Fig. 3.7

Now we must let the contact area approach zero by permitting u ! 0 and v ! 0. In order to be able to use the limit approach first, Eq. (3.24) must be put in a more suitable form. For this purpose, the right-hand side is multiplied and divided by ab, giving the following:

$$p_{mn} = \lim_{u \to 0, v \to 0} \left[\frac{4P}{ab} \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b} \frac{\sin\frac{m\pi u}{2a} \sin\frac{n\pi v}{2b}}{(m\pi u/2a)(n\pi v/2b)} \right]. \tag{3.25}$$

Knowing that $\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$, Eq. (3.25) becomes

$$p_{mn} = \frac{4P}{ab} \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b}, \tag{3.26}$$

The deflected middle surface equation (3.27) in this case becomes

$$w(x, y) = \frac{4P}{\pi^4 Dab} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{\sin\frac{m\pi}{2} \sin\frac{n\pi}{2}}{\left[(m/a)^2 + (n/b)^2\right]} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}. \tag{3.28}$$

Furthermore, if the plate is square (a ¼ b), the maximum deflection, which occurs at the center, is obtained from Eq. (3.28), as follows

$$w_{max} = \frac{4Pa^2}{\pi^4 D} \sum_{m=1,3,\ldots}^{\infty} \sum_{n=1,3,\ldots} \frac{1}{(m^2+n^2)^2}.$$

Retaining the first nine terms of this series $(m = 1,\ n = 1, 3, 5;\ m = 3,\ n = 1, 3, 5;$ $m = 5,\ n = 1, 3, 5)$ we obtain

$$w_{max} = \frac{4Pa^2}{\pi^4 D}\left[\frac{1}{4}+\frac{2}{100}+\frac{1}{324}+\frac{2}{625}+\frac{2}{1156}+\frac{1}{2500}\right] = 0.01142\frac{Pa^2}{D}.$$

The ‘‘exact’’ value is $w_{max} = 0.01159\frac{Pa^2}{D}$ and the error is thus 1.5% [3].

This very simple Navier’s solution, Eq. (3.28), converges sufficiently rapidly for calculating the deflections. However, it is unsuitable for calculating the bending moments and stresses because the series for the second derivatives $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$ obtained by differentiating the series (3.28) converge extremely slowly. These series for bending moment, and consequently for stresses as well as for the shear forces, diverge directly at a point of application of a concentrated force, called a singular point.

where

$$\beta_m = \frac{1}{2}\sum_{n=1}^{\infty}\frac{\cos\frac{n\pi}{b}(y-\eta) - \cos\frac{n\pi}{b}(y+\eta)}{(m^2b^2/a^2+n^2)^2}. \tag{3.29b}$$

The series (3.29b) can be summed using the formula [4]:

$$\sum_{n=1}^{\infty}\frac{\cos nz}{(\alpha^2+n^2)^2} = -\frac{1}{2\alpha^4} + \frac{\pi}{4\alpha^3}\frac{\cosh\alpha(\pi-z)}{\sin\pi\alpha} - \frac{\pi(\pi-z)}{4\alpha^2} - \frac{\sinh\alpha(\pi-z)}{\sinh\pi\alpha} + \frac{\pi^2}{4\alpha^2}\frac{\cosh\alpha(\pi-z)\cosh\pi\alpha}{\sin h^2\pi\alpha}. \tag{a}$$

Using the above formula, we can represent the deflection surface (3.28) as

$$w(x,y) = \frac{Pa^2}{\pi^3 D}\sum_{m=1}^{\infty}\left(1+\beta_m\coth\beta_m - \frac{\beta_m y_1}{b}\coth\frac{\beta_m y_1}{b} - \frac{\beta_m\eta}{b}\coth\frac{\beta_m\eta}{b}\right) \times \frac{\sinh\frac{\beta_m\eta}{b}\sin h\frac{\beta_m y_1}{b}\sin\frac{m\pi\xi}{a}\sin\frac{m\pi x}{a}}{m^3\sinh\beta_m}, \tag{3.30}$$

where $\beta_m = \frac{m\pi b}{a},\ y_1 = b - y$, and $y \geq \eta$. If $y < \eta$, then the values y_1 and η must be replaced by y and $\eta_1 = b - \eta$, respectively.

Let us use an infinitely long (in the y-direction) plate loaded by a concentrated force P applied at $x = \xi$ and $y = 0$, as shown in Fig. 3.8, to illustrate the above-

LEVY'S SOLUTION (SINGLE SERIES SOLUTION)

In the preceding sections it was shown that the calculation of bending moments and shear forces using Navier's solution is not very satisfactory because of slow convergence of the series.

In 1900 Levy developed a method for solving rectangular plate bending problems with simply supported two opposite edges and with arbitrary conditions of supports on the two remaining opposite edges using single Fourier series [8]. This method is more practical because it is easier to perform numerical calculations for single series that for double series and it is also applicable to plates with various boundary conditions.

Levy suggested the solution of Eq. (2.24) be expressed in terms of complementary , wh ; and particular, wp , parts, each of which consists of a single Fourier series where unknown functions are determined from the prescribed boundary conditions. Thus, the solution is expressed as follows:

$$w = w_h + w_p. \qquad (3.40)$$

Consider a plate with opposite edges, x ¼ 0 and x ¼ a, simply supported, and two remaining opposite edges, y ¼ 0 and y ¼ b, which may have arbitrary supports.

The boundary conditions on the simply supported edges are

$$w = 0|_{x=0,x=a} \text{ and } M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) = 0\bigg|_{x=0,x=a}. \qquad (3.41a)$$

As mentioned earlier, the second boundary condition can be reduced to the following form:

$$\frac{\partial^2 w}{\partial x^2} = 0\bigg|_{x=0,x=a} \qquad (3.41b)$$

The complementary solution is taken to be

$$w_h = \sum_{m=1}^{\infty} f_m(y)\sin\frac{m\pi x}{a}, \qquad (3.42)$$

where fmðyÞ is a function of y only; wh also satisfies the simply supported boundary conditions (3.41). Substituting (3.42) into the following homogeneous differential equation

$$\nabla^2\nabla^2 w = 0$$

Gives

$$\left[\left(\frac{m\pi}{a}\right)^4 f_m(y) - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 f_m(y)}{dy^2} + \frac{d^4 f_m(y)}{dy^4}\right]\sin\frac{m\pi x}{a} = 0,$$

which is satisfied when the bracketed term is equal to zero. Thus,

$$\frac{d^4 f_m(y)}{dy^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 f_m(y)}{dy^2} + \left(\frac{m\pi}{a}\right)^4 f_m(y) = 0 \qquad (3.44)$$

The solution of this ordinary differential equation can be expressed as

$$f_m(y) = e^{\lambda y}.$$

Substituting the above into Eq. (3.44), gives the following characteristic equation

$$\lambda^4 - 2\frac{m^2\pi^2}{a^2}\lambda^2 + \frac{m^4\pi^4}{a^4} = 0, \qquad (3.46)$$

According to the obtained values of the characteristic exponents, the solution of the homogeneous equation can be expressed in terms of either exponential functions.

$$f_m(y) = A'_m e^{m\pi y/a} + B'_m e^{-m\pi y/a} + \frac{m\pi y}{a}\left(C'_m e^{m\pi y/a} + D'_m e^{-m\pi y/a}\right) \quad (3.48)$$

or hyperbolic functions

$$f_m(y) = A_m \sinh\frac{m\pi y}{a} + B_m \cosh\frac{m\pi y}{a} + \frac{m\pi y}{a}\left(C_m \sinh\frac{m\pi y}{a} + D_m \cosh\frac{m\pi y}{a}\right). \quad (3.49)$$

The second form, Eq. (3.49), is more convenient for calculations. The complementary solution given by Eq. (3.42) becomes

$$w_h = \sum_{m=1}^{\infty}\left[A_m \sinh\frac{m\pi y}{a} + B_m \cosh\frac{m\pi y}{a} + \frac{m\pi y}{a}\left(C_m \sinh\frac{m\pi y}{a} + D_m \cosh\frac{m\pi y}{a}\right)\right]\sin\frac{m\pi x}{a}, \quad (3.50)$$

where the constants Am; Bm;Cm; and Dm are obtained from the boundary conditions on the edges y =0 and y =b:

The particular solution, wp, in Eq. (3.40), can also be expressed in a single Fourier series as

$$w_p(x,y) = \sum_{m=1}^{\infty} g_m(y) \sin \frac{m\pi x}{a}. \tag{3.51}$$

The lateral distributed load pðx; yÞ is taken to be the following (see Appendix B):

$$p(x,y) = \sum_{m=1}^{\infty} p_m(y) \sin \frac{m\pi x}{a}, \tag{3.52}$$

where

$$p_m(y) = \frac{2}{a}\int_0^a p(x,y) \sin \frac{m\pi x}{a} \mathrm{d}x. \tag{3.53}$$

Substituting Eqs (3.51) and (3.52) into Eq. (3.44), gives

$$\frac{\mathrm{d}^4 g_m(y)}{\mathrm{d}y^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{\mathrm{d}^2 g_m(y)}{\mathrm{d}y^2} + \left(\frac{m\pi}{a}\right)^4 g_m(y) = \frac{p_m(y)}{D}. \tag{3.54}$$

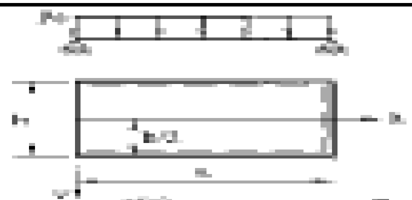
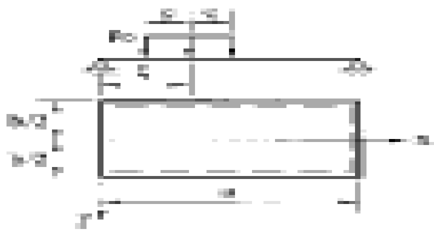
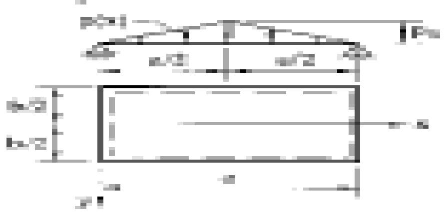
Solving this equation, we can determine gmðyÞ and, finally, find the particular solution, wpðx; yÞ.

CONTINUOUS PLATES

When a uniform plate extends over a support and has more than one span along its length or width, it is termed continuous. Such plates are of considerable practical interest. Continuous plates are externally statically indeterminate members (note that a plate itself is also internally statically indeterminate). So, the well-known methods developed in structural mechanics can be used for the analysis of continuous plates.

In this section, we consider the force method which is commonly used for the analysis of statically indeterminate systems. According to this method, the continuous plate is subdivided into individual, simple-span panels between intermediate supports by removing all redundant restraints. It can be established, for example, by introducing some fictitious hinges above the intermediate supports.

Table 3.2

No.	Load Geometry $p(x,y)=\sum_m p_m \sin\frac{m\pi x}{a}$	Expansion coefficient p_m [determined from Eq. (3.53)]
1		Uniform loading, $p_0 = \text{const}$ $p_m = \frac{4p_0}{m\pi}$ $(m = 1, 3, 5, \ldots)$
2		Hydrostatic loading, $p(x) = p_0 \frac{x}{a}$ $p_m = \frac{2p_0}{m\pi}(-1)^{m+1}$ $(m = 1, 2, \ldots)$
3		Line load p_0 at $x = \xi$ $p_m = \frac{2p_0}{a}\sin\frac{m\pi\xi}{a}$ $(m = 1, 2, 3, \ldots)$
4		Uniform load from $(\xi - c)$ to $(\xi + c)$ $p_m = \frac{4p_0}{m\pi}\sin\frac{m\pi\xi}{a}\sin\frac{m\pi c}{a}$ $(m = 1, 2, \ldots)$
5		Triangular load $p(x) = 2p_0\frac{x}{a}$ if $x \le a/2$ $p(x) = 2p_0\frac{a-x}{a}$ if $x \ge a/2$ $p_m = \frac{8p_0}{m^2\pi^2}(-1)^{\frac{m-1}{2}}$ $(m = 1, 3, \ldots)$

BASIC RELATIONS IN POLAR COORDINATES

As mentioned earlier, we use the polar coordinates r and ’ in solving the bending problems for circular plates. If the coordinate transformation technique is used, the following geometrical relations between the Cartesian and polar coordinates are applicable

$$x = r\cos\varphi,\ \ y = r\sin\varphi \text{ and } r^2 = x^2 + y^2,\ \varphi = \tan^{-1}\frac{y}{x}. \qquad (4.1)$$

Referring to the above

$$\frac{\partial r}{\partial x} = \frac{x}{r} = \cos\varphi, \qquad \frac{\partial r}{\partial y} = \frac{y}{r} = \sin\varphi,$$

$$\frac{\partial \varphi}{\partial x} = -\frac{y}{r^2} = -\frac{\sin\varphi}{r}, \qquad \frac{\partial \varphi}{\partial y} = \frac{x}{r^2} = \frac{\cos\varphi}{r}. \qquad (4.2)$$

Inasmuch as the deflection is a function of r and the chain rule together with the relations (4.2) lead to the following

$$\frac{\partial w}{\partial x}=\frac{\partial w}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial w}{\partial \varphi}\frac{\partial \varphi}{\partial x}=\frac{\partial w}{\partial r}\cos\varphi-\frac{1}{r}\frac{\partial w}{\partial \varphi}\sin\varphi. \qquad (4.3)$$

To evaluate the expression $\partial^2 w/\partial x^2$,

we can repeat the operation (4.3) twice. As a result, we obtain

$$\frac{\partial^2 w}{\partial x^2}=\cos\varphi\frac{\partial}{\partial r}\left(\frac{\partial w}{\partial x}\right)-\frac{1}{r}\sin\varphi\frac{\partial}{\partial \varphi}\left(\frac{\partial w}{\partial x}\right)$$

$$(4.4a)$$

$$=\frac{\partial^2 w}{\partial r^2}\cos^2\varphi-\frac{\partial^2 w}{\partial \varphi \partial r}\frac{\sin 2\varphi}{r}+\frac{\partial w}{\partial r}\frac{\sin^2\varphi}{r}+\frac{\partial w}{\partial \varphi}\frac{\sin 2\varphi}{r^2}+\frac{\partial^2 w}{\partial \varphi^2}\frac{\sin^2\varphi}{r^2}.$$

Similarly,

$$\frac{\partial^2 w}{\partial y^2}=\frac{\partial^2 w}{\partial r^2}\sin^2\varphi+\frac{\partial^2 w}{\partial r \partial \varphi}\frac{\sin 2\varphi}{r}+\frac{\partial w}{\partial r}\frac{\cos^2\varphi}{r}-\frac{\partial w}{\partial \varphi}\frac{\sin 2\varphi}{r^2}+\frac{\partial^2 w}{\partial \varphi^2}\frac{\cos^2\varphi}{r^2}, \qquad (4.4b)$$

$$\frac{\partial^2 w}{\partial x \partial y}=\frac{\partial^2 w}{\partial r^2}\frac{\sin 2\varphi}{2}+\frac{\partial^2 w}{\partial r \partial \varphi}\frac{\cos 2\varphi}{r}-\frac{\partial w}{\partial \varphi}\frac{\cos 2\varphi}{r^2}-\frac{\partial w}{\partial r}\frac{\sin 2\varphi}{2r}-\frac{\partial^2 w}{\partial \varphi^2}\frac{\sin 2\varphi}{2r^2}. \qquad (4.4c)$$

Adding term by term the relations (4.4a) and (4.4b), yields

$$\nabla_r^2 w \equiv \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}. \qquad (4.5)$$

After repeating twice the operation ∇_r^2,

the governing differential equation for the plate deflection (2.26) in polar coordinates becomes

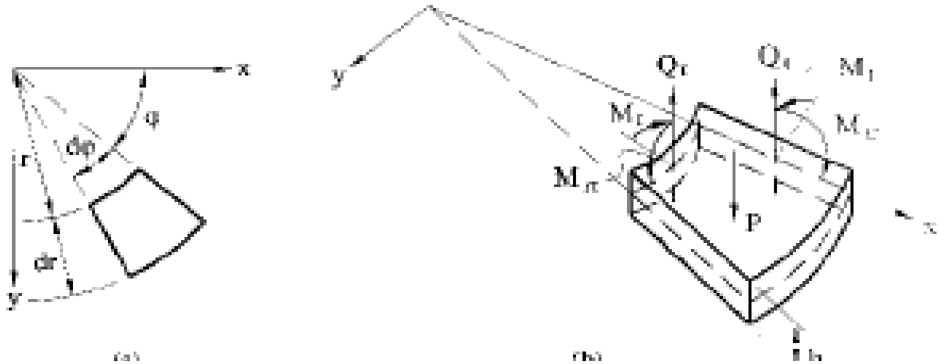
$$\nabla_r^4 w \equiv \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right) = \frac{p}{D}, \qquad (4.6a)$$

or in the expended form

$$\frac{\partial^4 w}{\partial r^4} + \frac{2}{r}\frac{\partial^3 w}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2 w}{\partial r^2} + \frac{1}{r^3}\frac{\partial w}{\partial r} + \frac{2}{r^2}\frac{\partial^4 w}{\partial r^2 \partial \varphi^2} - \frac{2}{r^3}\frac{\partial^3 w}{\partial \varphi^2 \partial r} + \frac{4}{r^4}\frac{\partial^2 w}{\partial \varphi^2} + \frac{1}{r^4}\frac{\partial^4 w}{\partial \varphi^4} = \frac{p}{D}.$$

(4.6b)

Let us set up the relationships between moments and curvatures. Consider now the state of moment and shear force on an infinitesimal element of thickness h, described in polar coordinates, as shown in Fig. 4.1b. Note that, to simplify the derivations, the x axis is taken in the direction of the radius r, at ’ ¼ 0 (Fig. 4.1b). Then, the radialMr, tangential Mt, twisting Mrt moments, and the vertical shear forces Qr;Qt will have the same values as the moments Mx;My; and Mxy, and shears Qx;Qy at the same point in the plate. Thus, transforming the expressions for moments (2.13) and shear forces (2.27) into polar coordinates, we can write the following:



$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right)\right]; \quad M_t = -D\left[\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2} + \nu\frac{\partial^2 w}{\partial r^2}\right];$$

$$M_{rt} = M_{tr} = -D(1-\nu)\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w}{\partial \varphi}\right); \qquad (4.7a)$$

$$Q_r = -D\frac{\partial}{\partial r}(\nabla_r^2 w); \; Q_t = -D\frac{1}{r}\frac{\partial}{\partial \varphi}(\nabla_r^2 w). \qquad (4.7b)$$

Similarly, the formulas for the plane stress components

$$\sigma_r = \frac{12M_r}{h^3}z, \quad \sigma_t = \frac{12M_t}{h^3}z, \quad \tau_{rt} = \tau_{tr} = \frac{12M_{tr}}{h^3}z, \qquad (4.8)$$

where Mr;Mt and Mtr are determined by Eqs (4.7). Clearly the maximum stresses take place on the surfaces z ¼ _h=2 of the plate.

Similarly, transforming Eqs (2.38) and (2.39) into polar coordinates gives the effective transverse shear forces. They may be written for an edge with outward normal in the r and ’ directions, as follows:

$$V_r = Q_r + \frac{1}{r}\frac{\partial M_{rt}}{\partial \varphi} = -D\left[\frac{\partial}{\partial r}(\nabla_r^2 w) + \frac{1-\nu}{r}\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w}{\partial \varphi}\right)\right],$$
$$V_t = Q_t + \frac{\partial M_{rt}}{\partial r} = -D\left[\frac{1}{r}\frac{\partial}{\partial \varphi}(\nabla_r^2 w) + (1-\nu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \varphi} - \frac{1}{r^2}\frac{\partial w}{\partial \varphi}\right)\right]. \quad (4.9)$$

UNIT – IV

STATIC ANALYSIS OF SHELLS: MEMBRANE THEORY OF SHELLS

CLASSIFICATION OF SHELL SURFACES

We define here some of the surfaces that are commonly used for shell structures in engineering practice. There are several possible classifications of these surfaces. One such classification, associated with the Gaussian curvature, was discussed in Sec. 11.6. Following Ref. [4], we now discuss other categories of shell surfaces associated with their shape and geometric develop ability.

Classification based on geometric form

(a) Surfaces of revolution
(b) Surfaces of translation
(c) Ruled surfaces

Classification based on geometric form

(a) Surfaces of revolution (Fig. 11.9)

As mentioned previously, surfaces of revolution are generated by rotating a plane curve, called the meridian, about an axis that is not necessarily intersecting the meridian. Circular cylinders, cones, spherical or elliptical domes, hyperboloids of revolution, and toroids (see Fig. 11.9) are some examples of surfaces of revolution. It can be seen that for the circular cylinder and cone (Fig. 11.9a and b), the meridian is a straight line, and hence, k1 ¼ 0, which gives _¼ 0. These are shells of zero

Gaussian curvature. For ellipsoids and paraboloids of revolution and spheres (Fig. 11.9c, d, and e), both the principal curvatures are in the same direction and, thus, these surfaces have a positive Gaussian curvature (_ > 0).

(b) Surfaces of translation (Fig. 11.10)

A surface of translation is defined as the surface generated by keeping a plane curve parallel to its initial plane as we move it along another plane curve. The two planes containing the two curves are at right angles to each other. An elliptic paraboloid is shown in Fig. 11.10 as an example of such a type of surfaces. It is obtained by translation of a parabola on another parabola; both parabolas have their curvatures in the same direction. Therefore, this shell has a positive Gaussian curvature. For this surface sections x ¼ constant and y ¼ constant are parabolas, whereas a section z ¼ constant represents an ellipse: hence its name, ‘‘elliptic paraboloid.’’

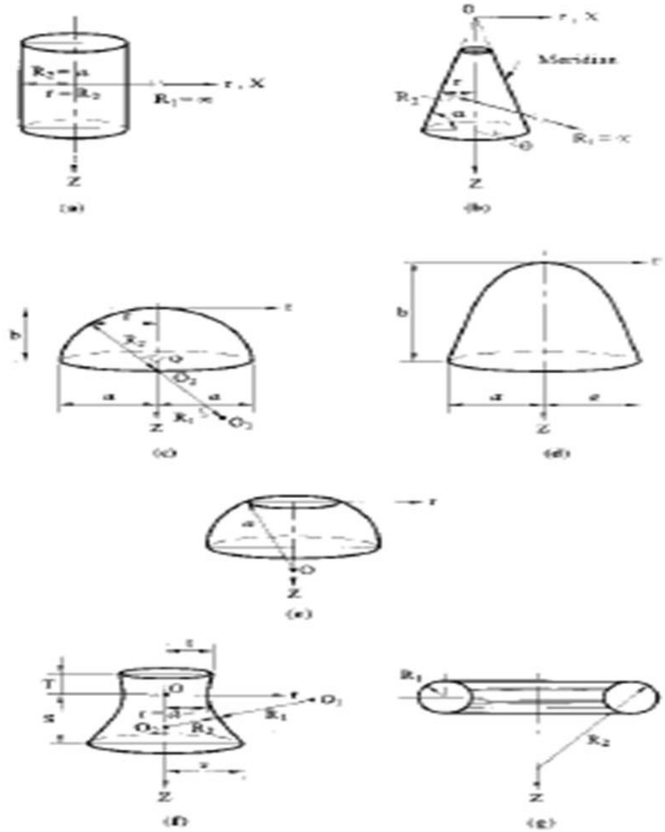


Fig. 11.9

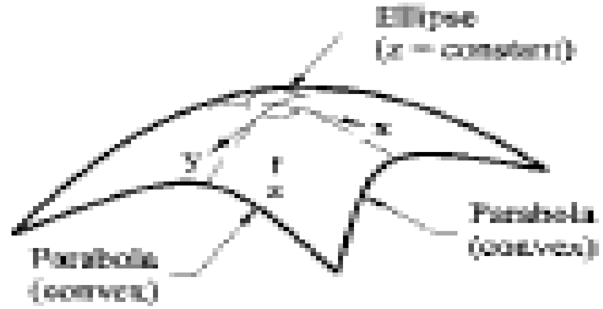


Fig. 11.10

(c) Ruled surfaces (Fig. 11.11)

Ruled surfaces are obtained by the translation of straight lines over two end curves (Fig. 11.11). The straight lines are not necessarily at right angles to the planes containing the end curves. The frustum of a cone can thus be considered as a ruled surface, since it can be generated by translation of a straight line (the generator) over two curves at its ends. It is also, of course, a shell of revolution. The hyperboloid of revolution of one sheet, shown in Fig. 11.11a, represents another example of ruled surfaces. It can be generated also by the translation of a straight line over two circles at its ends. Figure 11.11b shows a surface generated by a translation of a straight line on a circular curve at one end and on a straight line at the other end. Such surfaces are referred to as conoids. Both surfaces shown in Fig. 11.11 have negative Gaussian curvatures. 11.7.2 Classification based on shell curvature

Classification based on shell curvature

These shells have a zero Gaussian curvature. Some shells of revolution (circular cylinders, cones), shells of translation, or ruled surfaces (circular or noncircular cylinders and cones) are examples of singly curved shells.

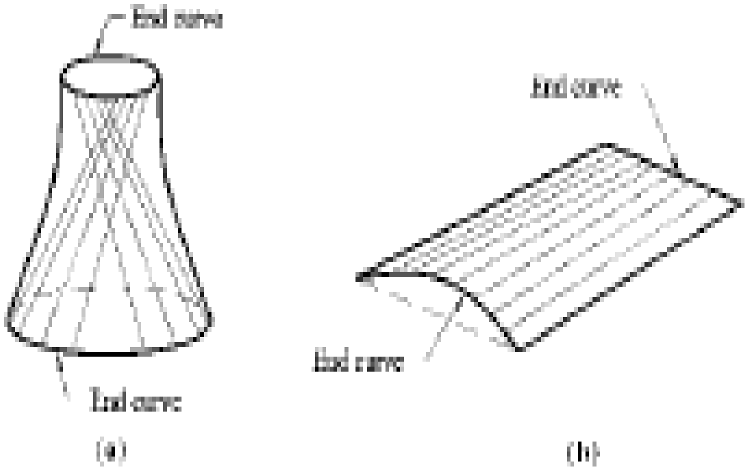


Fig. 11.11

(b) Doubly curved shells of positive Gaussian curvature

Some shells of revolution (circular domes, ellipsoids and paraboloids of revolution) and shells of translation and ruled surfaces (elliptic paraboloids, paraboloids of revolution) can be assigned to this category of surfaces.

(c) Doubly curved shells of negative Gaussian curvature

This category of surfaces consists of some shells of revolution (hyperboloids of revolution of one sheet) and shells of translation or ruled surfaces (paraboloids,conoids, hyperboloids of revolution of one sheet).It is seen from this classification that the same type of shell may appear in more than one category.

CLASSIFICATION BASED ON GEOMETRICAL DEVELOP ABILITY

(a) Developable surfaces

Developable surfaces are defined as surfaces that can be ‘‘developed’’ into a plane form without cutting and/or stretching their middle surface. All singly curved surfaces are examples of developable surfaces.

(b) Non-developable surfaces

A non-developable surface is a surface that has to be cut and/or stretched in order to be developed into a planar form. Surfaces with double curvature are usually non developable. The classification of shell surfaces into developable and non-developable has a certain mechanical meaning. From a physical point of view, shells with non-developable surfaces require more external energy to be deformed than do developable shells, i.e., to collapse into a plane form. Hence, one may conclude that non-developable shells are, in general, stronger and more stable than the corresponding developable.

SPECIALIZATION OF SHELL GEOMETRY

It is shown in the next chapter that the governing equations and relations of the general theory of thin shells are formulated in terms of the Lame´ parameters A and B as well as of the principal curvatures _1 ¼ 1=R1 and _2 ¼ 1=R2. In the general case of shells having an arbitrary geometry of the middle surface, the coefficients of the first and second quadratic forms and the principal curvatures are some functions of the curvilinear coordinates. We determine the Lame´ parameters for some shell geometries that are commonly encountered in engineering practice

Shells of revolution

The shells of revolution were discussed in Secs 11.2 and 11.7. As for the curvilinear coordinate lines and , the meridians and parallels may be chosen: they are the lines of principal curvatures and form an orthogonal mesh on the shell middle surface. Figure 11.12a shows a surface of revolution where R1 is the principal radius of the meridian, R2 is the principal radius of the parallel circle (as shown in Sec.11.2, R2 is the distance along a normal to the meridional curve drawn from a point of interest to the axis of revolution of the surface), and r is the radius of the parallel circle.

CLASSIFICATION BASED ON GEOMETRICAL DEVELOPABILITY

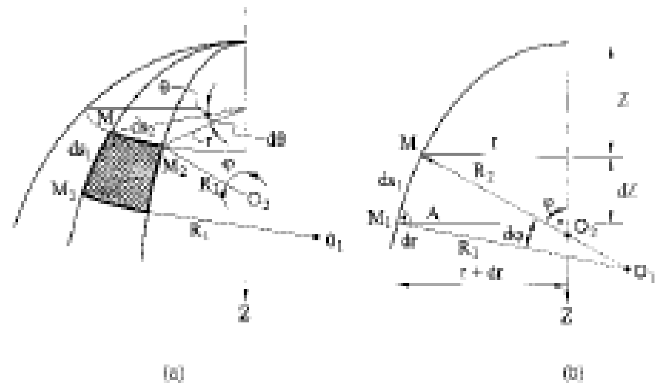


Fig. 11.12

There are several possibilities for a choice of the curvilinear coordinates. The overall goal is to be able to design reinforced concrete structures that are:

- Safe
- Economical
- Efficient

Reinforced concrete is one of the principal building materials used in engineered structures because:

- Low cost
- Weathering and fire resistance
- Good compressive strength
- Formability
- Identify the regions where the beam shall be designed as a flanged and where it will be rectangular in normal slab beam construction,
- Define the effective and actual widths of flanged beams,
- State the requirements so that the slab part is effectively coupled with the flanged beam,

- Write the expressions of effective widths of T and L-beams both for continuous and isolated cases,
- Derive the expressions of C, T and Mu for four different cases depending on the location of the neutral axis and depth of the flange.

Roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab. Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 5.10.1 a, b and c). Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e., between the supports of a continuous beam, the slab, up to a certain length.

Width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams - either T or L type depending on whether the slab is on both sides or on one side of the beam (Figs. 5.10.2 a to e) . Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression.

THE FUNDAMENTAL EQUATIONS OF THE MEMBRANE THEORY OF THIN SHELLS

The governing equations of the membrane theory can be obtained directly from the equations of the general theory of thin shells derived in Chapter 12. For this purpose, it is assumed that, in view of the smallness of the changes of curvature and twist, the moment terms in the equations of equilibrium for a shell element are unimportant, although in principle the shell may resist the external loads in bending. Note that neglecting the moments leads to neglecting the normal shear forces. Thus, for the membrane theory of thin shells, we can assume that

$$M1 = M2 = H = Q1 = Q2 = 0, \qquad (13:3)$$

Introducing Eq. (13.3) into Eqs below

$$N_1\frac{AB}{R_1}+N_2\frac{AB}{R_2}+\frac{\partial}{\partial\alpha}\left\{\frac{1}{A}\left[\frac{1}{A}\frac{\partial}{\partial\beta}(HA^2)+\frac{\partial}{\partial\alpha}(M_1B)-M_2\frac{\partial B}{\partial\alpha}\right]\right\}$$
$$+\frac{\partial}{\partial\beta}\left\{\frac{1}{B}\left[\frac{1}{B}\frac{\partial}{\partial\alpha}(HB^2)+\frac{\partial}{\partial\beta}(AM_2)-M_1\frac{\partial A}{\partial\beta}\right]\right\}+ABp_3=0.$$

and taking into account

$$M_{12}=M_{21}=H,\quad S=N_{12}+\frac{H}{R_2}=N_{21}+\frac{H}{R_1}.$$

One arrives at the following system of differential equations:

$$\begin{aligned}
&\frac{\partial}{\partial\alpha}(N_1B)+\frac{1}{A}\frac{\partial}{\partial\beta}(A^2S)-N_2\frac{\partial B}{\partial\alpha}+ABp_1=0,\\
&\frac{\partial}{\partial\beta}(N_2A)+\frac{1}{B}\frac{\partial}{\partial\alpha}(B^2S)-N_1\frac{\partial A}{\partial\beta}+ABp_2=0,\\
&N_1\frac{1}{R_1}+N_2\frac{1}{R_2}+p_3=0,
\end{aligned}\tag{13.4}$$

where $N_{12}=N_{21}=S$ (since $H=0$).

In this system, the number of unknowns is equal to the number of equations, so the problem of the membrane theory of shells is statically determinate (that is true for the equilibrium of an infinitely small shell element but is not always true for the equilibrium of the entire shell). This means that if the external load components, p1; p2; and p3, are known, then the membrane forces and stresses for such a shell are uniquely determined from Eqs (13.4).

Having determined the membrane forces, the shell displacements may be found. Solving the constitutive equations (12.45) for strains, and substituting them into Eqs (12.23), yields the following system of the three partial differential equations for the displacements:

$$\varepsilon_1 = \frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{v}{AB}\frac{\partial A}{\partial \beta} - \frac{w}{R_1} = \frac{1}{Eh}(N_1 - \nu N_2),$$

$$\varepsilon_2 = \frac{1}{B}\frac{\partial v}{\partial \beta} + \frac{u}{AB}\frac{\partial B}{\partial \alpha} - \frac{w}{R_2} = \frac{1}{Eh}(N_2 - \nu N_1), \quad (13.5)$$

$$\gamma_{12} = \frac{B}{A}\frac{\partial}{\partial \alpha}\left(\frac{v}{B}\right) + \frac{A}{B}\frac{\partial}{\partial \beta}\left(\frac{u}{A}\right) = \frac{2(1+\nu)}{Eh}S.$$

The system of the differential equations (13.4) in the membrane theory for determining the membrane internal forces (and stresses) is of the second order. Accordingly, the system (13.5) for the displacements is also of the second order. However, the stress resultants (N1;N2; and S) on the right-hand sides of Eqs (13.5) are themselves solutions of the second-order equations. Hence, the displacements in the membrane theory must satisfy a fourth-order system of differential equations. The latter can be obtained by substituting into Eqs (13.4) for the stress resultants from the corresponding expressions in terms of the strains.

The mathematical formulation of the theory of membrane shells is completed by adding appropriate boundary conditions. In the membrane theory, it follows from the above that only two boundary conditions may be specified on each edge of the shell. If the boundary conditions are given in terms of the stress resultants,

then only the membrane (or in-plane) forces (N1;N2; and S) are specified on edges of the shell. If the boundary conditions are formulated in terms of displacements, then only displacement components that are tangent to the middle surface, i.e., u and v,

must be prescribed on the shell boundary. In the membrane theory it is impossible to specify the edge displacements w and slopes #, since their assignment may result in the appearance of the corresponding boundary transverse shear forces and bending

moments. This is in a conflict with the general postulates of the membrane theory of thin shells introduced above.

THE MEMBRANE THEORY OF SHELLS OF REVOLUTION

Consider a particular case of a shell described by a surface of revolution (Fig. 11.12).The mid surface of such a shell of revolution, as mentioned in Sec. 11.8, is generated by rotating a meridian curve about an axis lying in the plane of this curve (the Z axis).

The geometry of shells of revolution is addressed in Sec. 11.8. There it is mentioned that meridians and parallel circles can be chosen as the curvilinear coordinate lines for such a shell because they are lines of curvature, and form an orthogonal mesh on its mid surface. Let us locate a point on the middle surface by the spherical coordinates and ’ (see Sec. 11.8), where is the circumferential angle characterizing a position of a point along the parallel circle, whereas the angle ’ is the meridional angle, defining a position of that point along the meridian. The latter represents the angle between the normal to the middle surface and the shell axis (Fig. 11.12a).

As before, R1;R2 are the principal radii of curvature of the meridian and parallel circle, respectively, and r is the radius of the parallel circle. The Lame´

parameters for shells of revolution in the above-mentioned spherical coordinate system are given by Eqs (11.39). Notice that, due to the symmetry of shells of revolution about the Z axis, these parameters are functions of ’ only and do not depend upon . Referring to Fig. 11.12a and b, we can obtain by inspection the following relations:

$$r = R_2 \sin\varphi;$$

$$AM_1 = \mathrm{d}r = \frac{\mathrm{d}r}{\mathrm{d}\varphi}\mathrm{d}\varphi \approx MM_1 \cos\varphi = R_1 \mathrm{d}\varphi \cos\varphi \qquad (13.6a)$$

or

$$\frac{\mathrm{d}r}{\mathrm{d}\varphi} = \frac{\mathrm{d}}{\mathrm{d}\varphi}(R_2 \sin\varphi) = R_1 \cos\varphi. \qquad (13.6b)$$

Finally from Eqs (13.6a) and (13.6b) we obtain

$$\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\varphi} = \frac{R_1}{R_2}\cot\varphi. \qquad (13.6c)$$

Substituting for A and B from Eqs (11.39) into the system of differential equations (13.4) and taking into account the relations (13.6), yields

$$R_1 \frac{\partial S}{\partial \theta} + \frac{\partial}{\partial \varphi}(rN_1) - N_2 R_1 \cos\varphi + rR_1 p_1 = 0,$$

$$R_1 \frac{\partial N_2}{\partial \theta} + \frac{1}{r}\frac{\partial}{\partial \varphi}(r^2 S) + rR_1 p_2 = 0, \qquad (13.7)$$

$$\kappa_1 N_1 + \kappa_2 N_2 + p_3 = 0,$$

where

$$\kappa_1 = \frac{1}{R_1} \quad \text{and} \quad \kappa_2 = \frac{1}{R_2}.$$

The last equation in the above system is known as the Laplace equation. Note that the membrane forces N1;N2; and S are, in a general case of loading, some functions of θ and φ.

Equations (13.7) can be reduced to one single, second-order differential equation for some function U: For this purpose, rewrite the above equations using the relations (13.6), as follows

$$\frac{1}{R_1}\frac{\partial N_1}{\partial \varphi} + \frac{N_1 - N_2}{R_2}\cot\varphi + \frac{1}{R_2 \sin\varphi}\frac{\partial S}{\partial \theta} + p_1 = 0, \tag{13.8a}$$

$$\frac{1}{R_2 \sin\varphi}\frac{\partial N_2}{\partial \theta} + \frac{1}{R_1}\frac{\partial S}{\partial \varphi} + \frac{2\cot\varphi}{R_2}S + p_2 = 0, \tag{13.8b}$$

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + p_3 = 0. \tag{13.8c}$$

Solving Eq. (13.8c) for N2 and substituting this into Eqs (13.8a) and (13.8b), one finds the following

$$\begin{aligned} \frac{1}{R_1}\frac{\partial N_1}{\partial \varphi} + N_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cot\varphi + \frac{1}{R_2 \sin\varphi}\frac{\partial S}{\partial \theta} &= -p_1 - p_3\cot\varphi \\ -\frac{1}{R_1 \sin\varphi}\frac{\partial N_1}{\partial \theta} + \frac{1}{R_1}\frac{\partial S}{\partial \varphi} + 2\frac{\cot\varphi}{R_2}S &= \frac{1}{\sin\varphi}\frac{\partial p_3}{\partial \theta} - p_2. \end{aligned} \tag{13.9}$$

We introduce new variables, U and V instead of N1 and S, as follows

$$N_1 = \frac{U}{R_2 \sin^2 \varphi}, \quad S = \frac{V}{R_2^2 \sin^2 \varphi}. \tag{13.10}$$

Substituting the above into Eqs (13.9), we obtain, after some simple transformations, the following system of equations:

$$\frac{R_2^2 \sin \varphi}{R_1} \frac{\partial U}{\partial \varphi} + \frac{\partial V}{\partial \theta} = -(p_3 \cos \varphi + p_1 \sin \varphi) R_2^3 \sin^2 \varphi,$$

$$-\frac{R_2}{\sin \varphi} \frac{\partial U}{\partial \theta} + \frac{\partial V}{\partial \varphi} = \left(\frac{\partial p_3}{\partial \theta} - p_2 \sin \varphi \right) R_1 R_2^2 \sin \varphi. \tag{13.11}$$

Differentiating then the first equation (13.11) with respect to ’ and the second one with respect to , and subtracting the second equation from the first, we obtain the following second-order differential equation for U:

$$\frac{1}{R_1 R_2 \sin\varphi}\frac{\partial}{\partial\varphi}\left(\frac{R_2^2 \sin\varphi}{R_1}\frac{\partial U}{\partial\varphi}\right)+\frac{1}{R_1 \sin^2\varphi}\frac{\partial^2 U}{\partial\theta^2}=F(\theta,\varphi), \tag{13.12}$$

where

$$F(\theta,\varphi)=-\frac{1|}{R_1 R_2 \sin\varphi}\frac{\partial}{\partial\varphi}\left[R_2^3 \sin^2\varphi(p_3\cos\varphi+p_1\sin\varphi)\right]+R_2\left(\frac{\partial p_2}{\partial\theta}\sin\varphi-\frac{\partial^2 p_3}{\partial\theta^2}\right). \tag{13.13}$$

Equation (13.12) may be written in the operator form

$$\mathbf{L}U=F(\theta,\varphi), \tag{13.14}$$

where the differential operator **L** is of the form

$$\mathbf{L}(\ldots)\equiv\frac{1}{R_1 R_2 \sin\varphi}\frac{\partial}{\partial\varphi}\left(\frac{R_2^2 \sin\varphi}{R_1}\frac{\partial(\ldots)}{\partial\varphi}\right)+\frac{1}{R_1 \sin^2\varphi}\frac{\partial^2}{\partial\theta^2}(\ldots). \tag{13.15}$$

The kinematic equations for displacements of shells of revolution in spherical coordinates are

$$\frac{1}{R_1}\frac{\partial u}{\partial \varphi} - \frac{w}{R_1} = \frac{1}{Eh}(N_1 - \nu N_2) = \varepsilon_1,$$

$$\frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\cos\varphi - \frac{w}{R_2} = \frac{1}{Eh}(N_2 - \nu N_1) = \varepsilon_2, \qquad (13.16)$$

$$\frac{r}{R_1}\frac{\partial}{\partial \varphi}\left(\frac{v}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \theta} = \frac{S}{Gh} = \gamma_{12}.$$

Now transform the above kinematic equations. Introducing the functions

$$\xi = \frac{u}{\sin\varphi}, \quad \eta = \frac{v}{R_2 \sin\varphi}, \qquad (13.17)$$

and making subsequent transformations associated with an elimination of deflection w and then , the system of equations

(Eqs., 13.16) may be reduced to one second order differential equation for the unknown function . In the operator form, this equation has the

Form

$$\mathbf{L}\eta = f(\theta, \varphi), \tag{13.18}$$

where

$$f(\theta, \varphi) = \frac{1}{R_1 R_2 \sin\varphi}\left\{\frac{1}{Gh}\frac{\partial(R_2 S)}{\partial\varphi} - \frac{1}{EhR_1}\left(R_1^2 + R_2^2 + 2\nu R_1 R_2\right)\frac{\partial N_1}{\partial\theta} - \frac{R_2}{Eh}(R_2 + \nu R_1)\frac{\partial p_3}{\partial\theta}\right\}, \tag{13.19}$$

and the operator L is given by Eq. (13.15). Thus, the governing differential equations for determining the membrane forces, Eq. (13.14), and displacements, Eq. (13.18), have an identical form. These equations can be solved by using the well-known method of separation of variables.

SYMMETRICALLY LOADED SHELLS OF REVOLUTION

Let us assume that the shell of revolution is subjected to loading that is symmetrical about the shell axis, i.e., the Z axis. A self-weight of a shell and a uniformly distributed snow load are examples of such a type of loading. In this case, the governing differential equations of the membrane theory of shells of revolution will be simplified considerably. All the derivatives with respect to will vanish because a given load, and hence all the membrane forces and displacements, does not change in the circumferential direction. The externally applied loads per unit area of the middle surface are represented at any point by the components p1 and p3 acting in the directions of the y and z axes of the local coordinate system at the above point respectively, where the y axis points in the tangent direction along the meridian and the z axis is a normal to the middle surface at that point (Fig. 13.1).

The load component p2 (acting along the x axis) is assumed to be absent. The presence of this component implies that the shell is twisted about its axis. If p2 = 0 and edge forces in the circumferential direction are also zero, then, as follows from the second Eq. (13.7), in the case of axisymmetrical loading,

S = N12 =N21 = 0. (13:31)

The nonzero membrane forces are shown in Fig. 13.1.

The first and third equations of the system (13.7) after some algebra transformations, eliminating N2, and taking into account Eqs (13.6), may be reduced to the following equation:

$$\frac{d}{d\varphi}(N_1 r \sin\varphi) + rR_1(p_1 \sin\varphi + p_3 \cos\varphi) = 0. \quad (13.32)$$

UNIT – V

SHELLS OF REVOLUTION: WITH BENDING RESISTANCEE

INTRODUCTION

As mentioned, shells of revolution belong to a highly general class of shells frequently used in engineering. One representative of this class, cylindrical shells, was considered in Chapter 15, and we will not dwell on these shells. The shell types analyzed in this chapter are subclasses of shells of revolution having non-zero Gaussian curvature. As mentioned in Sec. 11.7, such shells have non-developable surfaces. Hence, they are stronger, stiffer, and more stable than shells with zero Gaussian curvature. These shells are frequently used to cover the roofs of sport halls and large liquid storage tanks. The containment shield structures of nuclear power plants also have dome-like roofs. Various pressure vessels are either completely composed of a single rotational shell or have shells of revolution at their end caps. Conical shells with zero Gaussian curvature are also representative of this class of shells: they are used to cover liquid storage tanks and the nose cones of missiles and rockets.

In the membrane analysis of shells of revolution considered in earlier chapters, we saw that the membrane theory alone cannot accommodate all the loads, support conditions, and geometries in actual shells. Thus, in a general case, shells of revolution experience both stretching and bending to resist an applied loading, which distinguishes significantly the bending of shells from the elementary behavior of plates.

However, the character of bending deformation may be different. If a shell of revolution is subjected to a concentrated force (Fig. 16.1a), bending exerts a crucial effect on its strength, because, in this case, the bending deformation increases with a growth of the forces until the load-carrying capacity of the shell structure is exhausted. In places of junction of a shell with its supports (Fig. 16.1b) or other structural members (shell of another geometry, ring beam, etc.), or in places of jump change in the radii of curvature (Fig. 16.1c), the bending has another character; here, bending.

propagates only if it is needed to eliminate the discrepancies between the membrane displacements or to satisfy the conditions of statics. If a shell material is ductile, the bending deformations of the latter type are usually decreased and do not practically influence the load-carrying capacity of shell structures. If the material of the shell is brittle, the bending deformations remain proportional to the applied loads until failure and can result in a significant decrease in the strength of the shell structure. In this chapter we consider the bending theory of shells of revolution. It should be noted that the solutions of the governing differential equations involve many difficulties for a general shell of revolution, and therefore, we solve these equations for some particular shell geometries and load configurations that are frequently used in engineering practice.

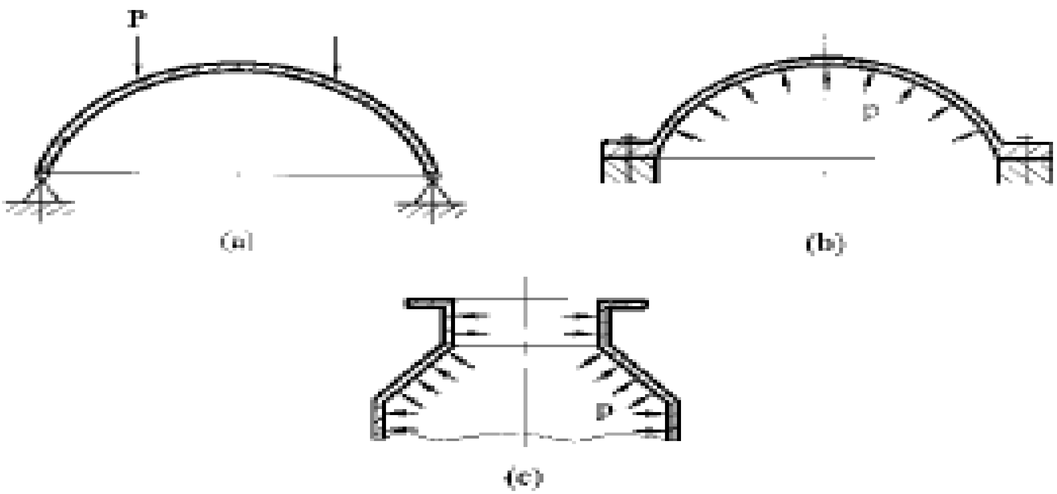


Fig. 16.1

Dead loads are those that are constant in magnitude and fixed in location throughout the lifetime of the structure such as: floor fill, finish floor, and plastered ceiling for buildings and wearing surface, sidewalks, and curbing for bridges

Live Loads

Live loads are those that are either fully or partially in place or not present at all, may also change in location; the minimum live loads for which the floors and roof of a building should be designed are usually specified in building code that governs at the site of construction (see Table1 - “Minimum Design Loads for Buildings and Other Structure.”)

Environmental Loads

Environmental Loads consist of wind, earthquake, and snow loads. such as wind, earthquake, and snow loads.

Serviceability

Serviceability requires that

• Deflections be adequately small. • Cracks if any be kept to a tolerable limits. • Vibrations be minimized

GOVERNING EQUATIONS

We present below the governing differential equations of the moment theory of shells of revolution of an arbitrary shape. As curvilinear coordinates and of a point on the shell middle surface, it is convenient to take the spherical coordinates, introducedin Sec. 11.8, and used in the membrane theory of shells of revolution in Chapters 13and 14. Thus, we take ¼ ’ and ¼ . As before, the angle ’ defines the location ofa point along the meridian, whereas characterizes the location of a point along the parallel circle (see Fig. 11.12). Let R1 and R2 be the principal radii of curvature of the meridian and parallel circle, respectively. Obviously, R1 and R2 will be functions of ’ only, i.e., R1 ¼ R1ð’Þ and R2 ¼ R2ð’Þ. The Lame´ parameters in this case are determined by the following formulas (see Sec.11.8):

$$A = R_1(\varphi), \quad B = R_2(\varphi)\sin\varphi, \quad r = R_2 \sin\varphi.$$

The Codazzi and Gauss conditions are given by Eqs (11.41).

Let us consider the kinematic relations of the moment theory of shells of revolution. Displacement components of the middle surface along the given coordinate axes are u (in the meridional direction), v (in the circumferential direction), and w (in the normal direction to the middle surface). The strain–displacement relations (12.23) and (12.24) of the general shell theory – taking into account Eqs (16.1) and (11.41) – take the following form for shells of revolution:

$$\varepsilon_1 = \frac{1}{R_1}\left(\frac{\partial u}{\partial \varphi} - w\right),$$

$$\varepsilon_2 = \frac{1}{R_2 \sin\varphi}\left(\frac{\partial v}{\partial \theta} + u\cos\varphi - w\sin\varphi\right),$$

Network models have three main advantages over linear programming:

1.They can be solved very quickly. Problems whose linear program would have 1000 rows and 30,000 columns can be solved in a matter of seconds. This allows network models to be used in many applications (such as real-time decision making) for which linear programming would be inappropriate.

2. They have naturally integer solutions. By recognizing that a problem can be formulated as a network program, it is possible to solve special types of integer programs without resorting to the ineffective and time consuming integer programming algorithms.

3.They are intuitive. Network models provide a language for talking about problems that is much more intuitive than the \variables, objective, and constraints" language of linear and integer programming.

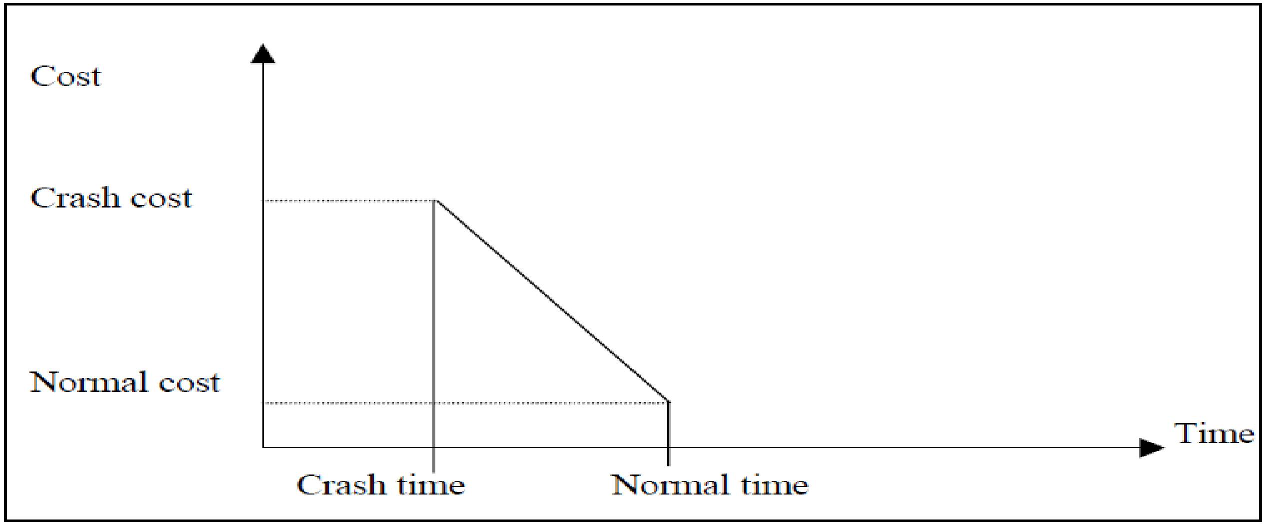
Production Planning

Quantitative analysis also helps individuals to make informed product-planning decisions. Let’s say a company is finding it challenging to estimate the size and location of a new production facility. Quantitative analysis can be employed to assess different proposals for costs, timing, and location. With effective product planning and scheduling, companies will be more able to meet their customers’ needs while also maximizing their profits.

Cost Slope

Cost slope is the increase in cost per unit of time saved by crashing. A linear cost curve is shown in Figure.

Linear Cost Curve



Cost slope=Crash cost C_c– Normal cost N_c/Normal time Ntt

Example:

The following Table gives the activities of a construction project and other data.

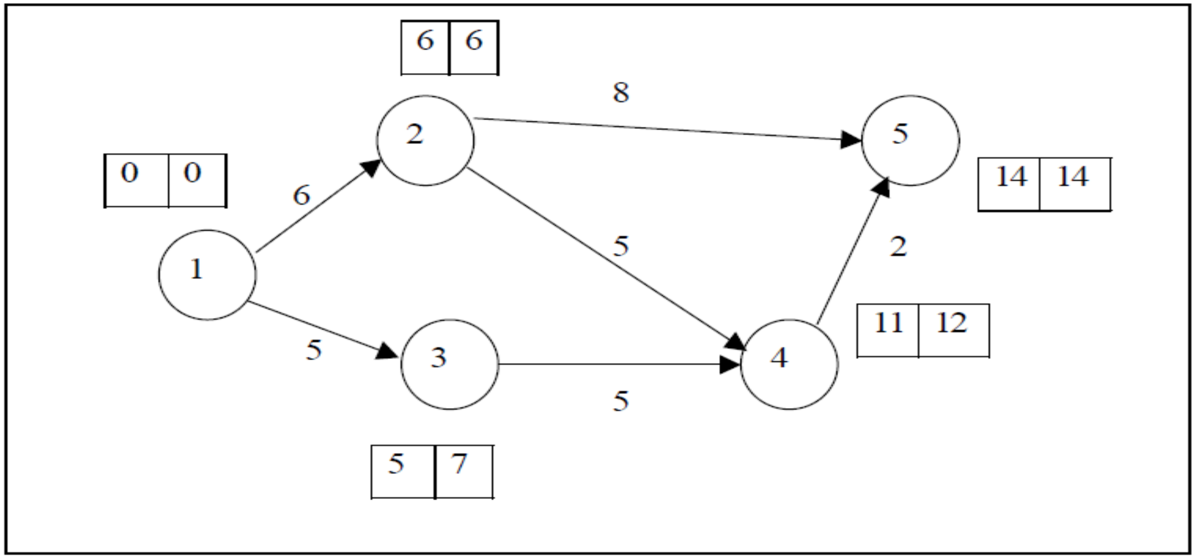
Construction Project Data

Activity	Normal		Crash	
	Time (days)	Cost (Rs)	Time (days)	Cost (Rs)
1-2	6	50	4	80
1-3	5	80	3	150
2-4	5	60	2	90
2-5	8	100	6	300
3-4	5	140	2	200
4-5	2	60	1	80

If the indirect cost is Rs. 20 per day, crash the activities to find the minimum duration of the project and the project cost associated.

Solution: From the data provided in the table, draw the network diagram and find the critical path.

Network Diagram



From the diagram, we observe that the critical path is 1-2-5 with project duration of 14 days The cost slope for all activities and their rank is calculated as shown in table below

Cost slope=Crash cost C_c– Normal cost N_c/Normal time Ntt

Cost Slope for activity 1– 2 = 80 – 50/6 – 4 = 30/2 = 15.

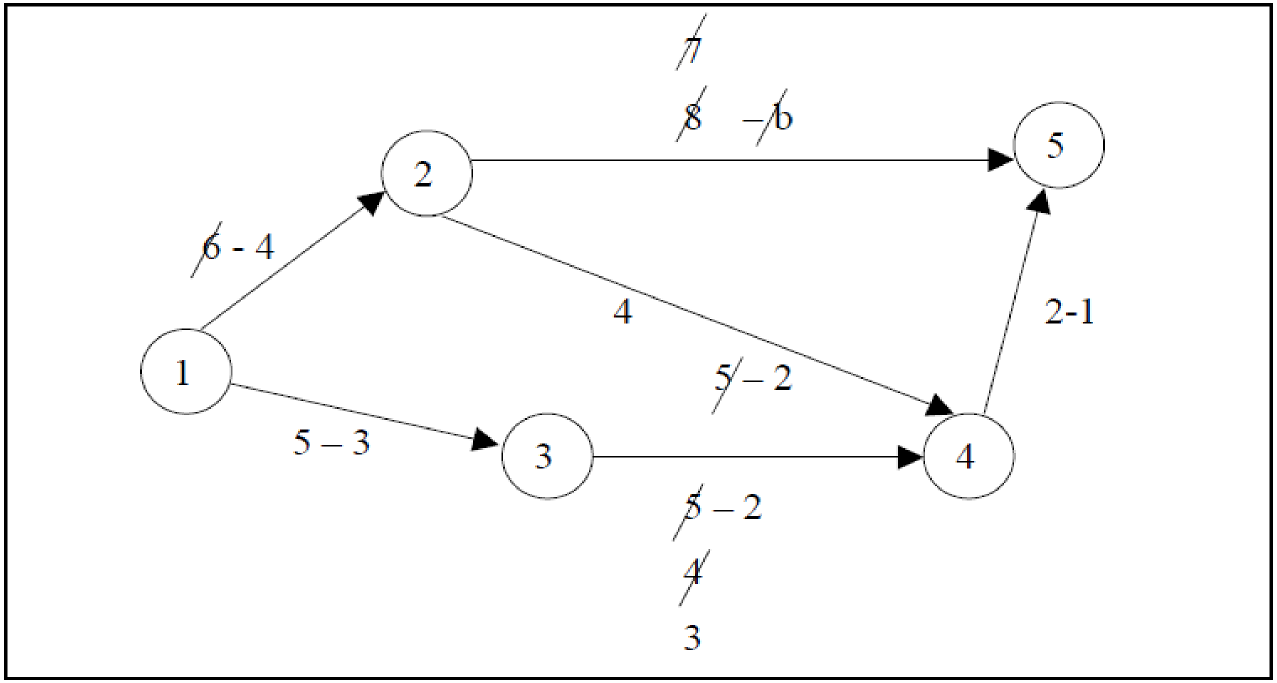
Cost Slope and Rank Calculated

Activity	Cost Slope	Rank
1-2	15	2
1-3	35	4
2-4	10	1
2-5	100	5
3-4	20	3
4-5	20	3

The available paths of the network are listed down in Table indicating the sequence of crashing.

Sequence of Crashing

Path	Number of days crashed
1-2-5	~~14~~ ~~12~~ ~~11~~ 10
1-2-4-5	~~13~~ ~~11~~ ~~11~~ 10
1-3-4-5	~~12~~ ~~12~~ ~~11~~ 10



The sequence of crashing and the total cost involved is given in the following table

Initial direct cost = sum of all normal costs given = Rs. 490.00

Sequence of Crashing & Total Cost

Activity Crashed	Project Duration	Critical Path	Direct Cost in (Rs.)	Indirect Cost (in Rs.)	Total Cost (in Rs)
–	14	1 – 2 – 5	490	$14 \times 20 = 280$	770
1 – 2(2)	12	1 – 2 – 5	$490 + (2 \times 15) = 520$	$12 \times 20 = 240$	760
2 – 5 (1) 3 – 4 (1)	11	1 – 2 – 5 1 – 3 – 4 – 5 1 – 2 – 4 – 5	$520 + (1 \times 100) + (1 \times 20) = 640$	$11 \times 20 = 220$	860
2 – 5 (1) 2 – 4 (1) 3 – 4 (1)	10	1 – 2 – 5 1 – 3 – 4 – 5 1 – 2 – 4 – 5	$640 + (1 \times 100) + (1 \times 10) + (1 \times 20) = 770$	$10 \times 20 = 200$	970

it is not possible to crash more than 10 days, as all the activities in the critical path are fully crashed. hence the project review techniques

Project review techniques

The project review techniques are

In the critical path method, the time estimates are assumed to be known with certainty. In certain projects like research and development, new product introductions, it is difficult to estimate the time of various activities. Hence PERT is used in such projects with a probabilistic method using three time estimates for an activity, rather than a single estimate, as shown in Figure. Minimum project duration is 10 days with the total cost of Rs. 970.00.

Example : A project schedule has the following characteristics as shown in the table

Project Schedule

Activity	Name	Time	Activity	Name	Time (days)
1-2	A	4	5-6	G	4
1-3	B	1	5-7	H	8
2-4	C	1	6-8	I	1
3-4	D	1	7-8	J	2
3-5	E	6	8-10	K	5
4-9	F	5	9-10	L	7

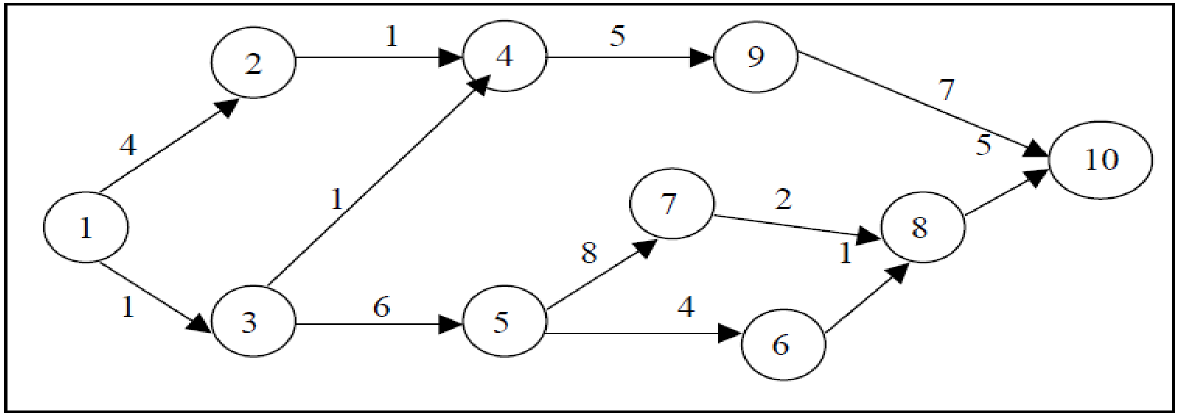
i. Construct PERT network.

ii. Compute T_E and T_L for each activity.

iii. Find the critical path.

(i) From the data given in the problem, the activity network is constructed as shown in the following figure.

Activity Network Diagram



(ii) To determine the critical path, compute the earliest, time T Network Model E and latest time T_L for each of the activity of the project. The calculations of TE and T_L are as follows:

To calculate T_E for all activities,

$T_{E1} = 0$

$T_{E2} = T_{E1} + t_1, 2 = 0 + 4 = 4$

$T_{E3} = T_{E1} + t_1, 3 = 0 + 1 = 1$

$T_{E4} = \max (T_{E2} + t_2, 4 \text{ and } T_{E3} + t_3, 4)$

$= \max (4 + 1 \text{ and } 1 + 1) = \max (5, 2)$

$= 5$ days

$T_{E5} = T_{E3} + t\ 3, 6 = 1 + 6 = 7$

$T_{E6} = T_{E5} + t\ 5, 6 = 7 + 4 = 11$

$T_{E7} = T_{E5} + t_5, 7 = 7 + 8 = 15$

$T_{E8} = \max (T_{E6} + t\ 6, 8 \text{ and } T_{E7} + t_7, 8)$

$= \max (11 + 1 \text{ and } 15 + 2) = \max (12, 17)$

$= 17$ days

$T_{E9} = T_{E4} + t_4, 9 = 5 + 5 = 10$

$T_{E10} = \max (T_{E9} + t_9, 10 \text{ and } T_{E8} + t_8, 10)$

$= \max (10 + 7 \text{ and } 17 + 5) = \max (17, 22)$

$= 22$ days

To calculate T_L for all activities

$T_{L10} = T_{E10} = 22$

$T_{L9} = T_{E10} – t_{9,10} = 22 – 7 = 15$

$T_{L8} = T_{E10} – t_{8, 10} = 22 – 5 = 17$

$T_{L7} = T_{E8} – t_{7, 8} = 17 – 2 = 15$

$T_{L6} = T_{E8} – t_{6, 8} = 17 – 1 = 16$

$T_{L5} = \min (T_{E6} – t_{5, 6} \text{ and } T_{E7} – t_{5, 7})$

$= \min (16 – 4 \text{ and } 15 –8) = \min (12, 7)$

$= 7$ days

$T_{L4} = T_{L9} – t_{4, 9} = 15 – 5 =10$

$T_{L3} = \min (T_{L4} – t_{3, 4} \text{ and } T_{L55} – t_{3, 5})$

$= \min (10 – 1 \text{ and } 7 – 6) = \min (9, 1)$

$= 1$ day

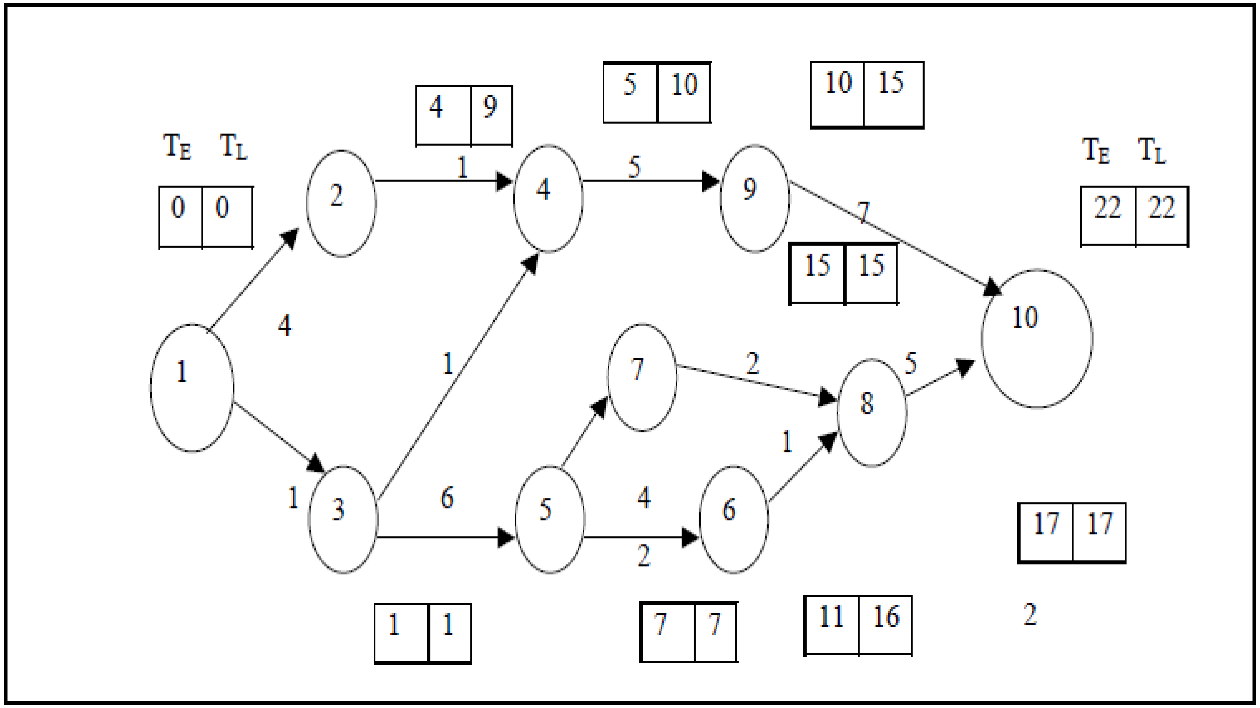
$T_{L2} = T_{L4} – t_2, 4 = 10 – 1 = 9$

$T_{L1} = \text{Min} (T_{L2} – t_{1, 2} \text{ and } T_{L3} – t_{1, 3})$

$= \text{Min} (9 – 4 \text{ and } 1 – 1) = 0$

Activity	Activity Name	Normal Time	Earliest Time		Latest Time		Total Float
			Start	Finish	Start	Finish	
1-2	A	4	0	4	5	9	5
1-3	B	1	0	1	0	1	0
2-4	C	1	4	5	9	10	5
3-4	D	1	1	2	9	10	8
3-5	E	6	1	7	1	7	0
4-9	F	5	5	10	10	15	5
5-6	G	4	7	11	12	16	5
5-7	H	8	7	15	7	15	0
6-8	I	1	11	12	16	17	5
7-8	J	2	15	17	15	17	0
8-10	K	5	17	22	19	22	0
9-10	L	7	10	17	15	22	5

(iii) From the table, we observe that the activities 1 – 3, 3 – 5, 5 – 7,7 – 8 and 8 – 10 are critical activities as their floats are zero.



The critical path is 1-3-5-7-8-10 (shown in double line in the above figure) with the project duration of 22 days.

Pert

PERT is an acronym for Program (Project) Evaluation and Review Technique, in which planning, scheduling, organizing, coordinating and controlling uncertain activities take place. The technique studies and represents the tasks undertaken to complete a project, to identify the least time for completing a task and the minimum time required to complete the whole project. It was developed in the late 1950s. It is aimed to reduce the time and cost of the project.

PERT uses time as a variable which represents the planned resource application along with performance specification. In this technique, first of all, the project is divided into activities and events. After that proper sequence is ascertained, and a network is constructed. After that time needed in each activity is calculated and the critical path (longest path connecting all the events) is determined.

Developed in the late 1950s, Critical Path Method or CPM is an algorithm used for planning, scheduling, coordination and control of activities in a project. Here, it is assumed that the activity duration is fixed and certain. CPM is used to compute the earliest and latest possible start time for each activity.

The process differentiates the critical and non-critical activities to reduce the time and avoid the queue generation in the process. The reason for the identification of critical activities is that, if any activity is delayed, it will cause the whole process to suffer. That is why it is named as Critical Path Method.

Differences between PERT and CPM

1.The most important differences between PERT and CPM are provided below:

2.PERT is a project management technique, whereby planning, scheduling, organizing, coordinating and controlling uncertain activities are done. CPM is a statistical technique of project management in which planning, scheduling, organizing, coordination and control of well-defined activities take place.

3.While PERT is evolved as a research and development project, CPM evolved as a construction project.

4.PERT is set according to events while CPM is aligned towards activities.

5.A deterministic model is used in CPM. Conversely, PERT uses a probabilistic model.

6.There are three times estimates in PERT, i.e. optimistic time (to), most likely time ™, pessimistic time (tp). On the other hand, there is only one estimate in CPM.

7.PERT technique is best suited for a high precision time estimate, whereas CPM is appropriate for a reasonable time estimate.

8.PERT deals with unpredictable activities, but CPM deals with predictable activities.

10.There is a demarcation between critical and non-critical activities in CPM, which is not in the case of PERT.

11.PERT is best for research and development projects, but CPM is for non-research projects like construction projects.

12.Crashing is a compression technique applied to CPM, to shorten the project duration, along with the least additional cost. The crashing concept is not applicable to PERT.


Thank you