OPERATIONS RESEARCH

Course code:AME021

IV. B. Tech II semester

Regulation: IARE R-16)

BY

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CO’s	Course outcomes
CO1	Formulate the mathematical model of real time problem for optimization, using Linear programming.
CO2	Establish the problem formulation by using transportation, assignment models
CO3	Apply sequencing for flow and replacement for maintenance of machines programming, game theory and queuing models.
CO4	Formulate game theory model and apply stochastic models for discrete and continuous variables to control inventory.
CO5	Formulate queuing models and visualize dynamic programming and simulation models

CLOs	Course Learning Outcome
CLO 1	Understand the characteristics, phases, types of operation research models and its applications.
CLO 2	Visualize modeling principles scope, decision making, general methods for solving OR models.
CLO 3	Understand linear programming concepts problem formulation and graphical models.
CLO 4	Understand simplex method and artificial variable techniques.
CLO 5	Comprehend two-phase method and Big-M method of linear programming.
CLO 6	Apply to build and solve transportation models of balanced .
CLO 7	Understand the degeneracy model problem of transportation, unbalanced type-maximization.
CLO 8	Apply to build assignment models for optimal solution.
CLO 9	Understand variants of assignment model and travelling salesman model.

CLO 10	Understand the flow shop sequencing model of 'n' jobs through two machines and three machines.
CLO 11	Comprehend job shop sequencing of two jobs through 'm' machines.
CLO 12	Understand the concept of replacement of items that deteriorate with time when money value is not counted
CLO 13	Understand the concept of replacement of items that deteriorate with time when money value is n counted .
CLO 14	Visualize the replacement of items that fail completely and group replacement.
CLO 15	Understand minmax (maximini) criterion, optimal strategy , solution od games with saddle point
CLO 16	Visualize dominance principle while solving game theory problem.

CLO 17	Apply to solve m * 2 , 2 *n model of games and graphical method.
CLO 18	Understand the concepts of deterministic inventory model and purchase inventory model with one price break and multiple price breaks.
CLO 19	Visualize stochastic inventory models – demand may be discrete variable or continuous variable.
CLO 20	Understand the concepts of waiting line model of single channel and multi server model.
CLO 21	Visualize dynamic programming concepts and models
CLO 22	Comprehend the simulation models, phases of simulation, application of simulation
CLO 23	Visualize the application of simulation for inventory and queuing problems.

SYLLABUS OF UNIT-I

- Development, definition,
- Characteristics and phases,
- Types of operation research models
- Applications: Allocation: linear programming, problem formulation, graphical solution, simplex method, artificial variables techniques, two–phase method, big-M method

OPTIMIZATION TECHNIQUES

- Operations Research is the science of rational decision-making and the study, design and integration of complex situations and systems with the goal of predicting system behavior and improving or optimizing system performance

HISTORICAL DEVELOPMENT OF OR /OT (EVOLUTION OF OR)

- The main origin of Origin of Operations Research was during second world -war .
- During that time , military management in England called upon a team of sientists to study the strategic and tactical problems related to air and land defense of the country.
- Since they were having very limited military recourses, it was necessary to decide upon most effective utilization of them, eg. Efficient ocean transport, effective bombing etc.

HISTORICAL DEVELOPMENT OF OR /OT (EVOLUTION OF OR)

- During world-war-II, the military commands of UK and USA engaged several inter-disciplinary teams of scientists to undertake, scientific research into strategic and tactical militarary operations.
- Their mission was to formulate specific proposals ans plans for aiding the Military Commands to arrive t the decisions on optimal utilization of scarce military recourses and efforts , and also to implement the decisions effectively.
- Hence OR can be associated with ‘ an art of winning the war with out actually fighting it”.

NATURE & MEANING OF OR

- A few opinions about the definitions of OR which has been changed according to the development of the subject.

1. OR is a scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control. ------ Morse & Kimbol 1940
2. OR is a scientific method of providing excecutive with an analytical and objective basis for decisions.

(P M S Blackett 1950)

NATURE & MEANING OF OR

4. OR is the application of scientific methods , techniques and tools to problems involving the operations of systems so as to provide these in control of the operations with optimum solutions to the problem. (Churchman , Acoff, Anoff 1948)

5. OR is the art of giving bad answers to problems to which otherwise worse answers are given ---(TL Saaty 1958)

6. OR is the scientific method oriented study of the basic structure, characteristics, functions and relationships of an organization to provide the executive wit a sound , scientific and quantitative basis for decision making. (EL Arnoff & MJ

NATURE & MEANING OF OR

7 OR is an experimental and applied science devoted to observing, understanding and predicting the behavior of purposeful man-machine systems and OR workers are actively engaged in applying this knowledge to practical problems in business, government and society.

(OR Society of America)

8. OR is the application of scientific method by inter-disciplinary teams to problems involving the control of organized (man-machine) systems so as to provide solutions which best serve the purpose of the organization as a whole. (Ackoff & Sasieni 1968)

CHARECTERISTIS (FEATURES) OPERATIONS RESEARCH

1. **Inter-disciplinary team approach:** In OR, the optimum solution is found by a team of scientists selected from various disciplines such as mathematics, statistics, economics, engineering, physics, etc.
2. **Wholistic approach to the system:** The most of the problems tackled by OR hav the characteristic that OR have the characteristic that OR tries to find the best (optimum) deisions relative to larger possible portion of the total organization.
3. **Imperfectness of solutions:** by OR techniques, we cannot obtain perfect answers to our problems but, only the quality of the solution is improved from worse to bad answers.
4. Use of scientific research: OR uses techniques of scientific research to reach the optimum solutions.
5. **To optimize the total output:** OR tries to optimize total return by maximizing the profit and minimizing the cost or loss.

APPLICATIONS OF OPEARATIONS RESEARCH

1) **In Agriculture:**

- Optimum allocation of land to various crops in accordance with climatic conditions.
- Optimum distribution of water from various resources like canal for irrigation purposes.

2) In Finance:

- To maximize the per capita income with minimum recourses.
- To find out the profit plan for the company.
- To determine the best replacement policies.

APPLICATIONS OF OPEARATIONS RESEARCH

3) **In Industry:** allocation of various limited resources such as men, machines, material, money, time etc,

4) In marketing: with the help of OR techniques a marketing manager can decide

- Where to distribute the products for sales so that the total cost of transportation is minimum.
- The minimum per unit of sale price.
- Thee size of stock to meet the future demand.
- How to select the best advertising media with respect to time, cost etc.
- How, when , and where to purchase at the minimum possible cost?

APPLICATIONS OF OPEARATIONS RESEARCH

6)Personnel Management: A Personnel manager can use OR techniques

- To appoint the most suitable persons on minimum salary.
- To determine the best age of retirement for employees.
- To find out the number of persons to be appointed on full time basis when the work load is seasonal.

APPLICATIONS OF OPEARATIONS RESEARCH

7**) In Production management**. A production manager can use OR techniques

- To find out the number and size of the items to be replaced.
- In scheduling and sequencing the production run by proper allocation of machines.
- In calculating the optimum product mix.
- To select , locate and design the sites for production plants.

8) In LIC:

- What should be the premium rates for various modes of policies.
- How best the profits could be distributed in he case of with profit policies.

QUANTITATIVE TECHNIQUES OF OR

1. **Distribution (Allocation) Models. Distribution** models are concerned with allotment of available resources so as to minimize cost or maximize profits subject to prescribed conditions.
2. **Production/Inventory Models:** These models are concerned with determination of the optimal (economic) order quantity and ordering (production) intervals considering the factors such s demand per unit t ime, cost of placing orders, costs associated with goods held up in he inventory and the cost due to shortage of goods etc.
3. **Waiting Line (Queueing) Models> In queueing models an attempt is made to predict**

5 Competitive Strategy Models(Game Theory): these models are used to determine the behavior of decision making under competition or conflict.

6.Network Models: These models are applicable in large projects involving complexities and inter-dependencies activities. PERT and CPM are used for planning , scheduling and controlling complex project which can be characterized as net-work.

QUANTITATIVE TECHNIQUES OF OR

7. **Job Sequencing Models:** These models involve the selection of such a sequence of performing a series of jobs too be done on service facilities(machines) that optimize the efficiency measure of performance of the system.

8. **Replacement Models:** These models deal with determination of optimum replacement policy in situation that arise when some items or machinery need replacement by a new one.

9. **Simulation Models**: Simulation is very powerful technique for solving such complex models which cannot be solved otherwise and thus it is being extensively applied to solve a variety of problems.

MODELLING IN OPERATIONS RESEARCH

Definition: A model in the sense used in 0 R is defined as a representation of an actual object or situation. It shows the relationships (direct or indirect) and inter-relationships of action and reaction in terms of cause and effect.

- Since a model is an abstraction of reality, it thus appears to be less complete than reality itself. For a model to be complete, it must be a representative of those aspects of reality that are being investigated.

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MAIN OBJECTIVES OF MODELLING

- The main objective of a model is to provide means for analyzing the behavior of the system for the purpose of improving its performance.
- If a system is not in existence, then a model defines the ideal structure of this future system indicating the functional relationships among its elements.
- The reliability of the solution obtained from a model depends on the validity of the model in representing the real systems.
- A model permits to ‘examine the behavior of a system without interfering with ongoing operations.

CLASSIFICATION OF MODELS
1. Classification by Structure

1A.Iconic models. Iconic models represent the system as it is by scaling it up or down (i.e., by enlarging or reducing the size). In other words, it is an image.

- For example, a toy airplane is an iconic model of a real one. Other common examples of it are: photographs, drawings, maps etc. In a globe, the diameter of the earth is scaled down, but the globe has approximately the same shape as the earth, and the relative sizes of continents, seas, etc., are approximately correct.

1B.Analogue models. The models, in which one set of properties is used to represent another set of properties, are called analogue models. After the problem is solved, the solution is reinterpreted in terms of the original system.

- For example, graphs are very simple analogues because distance is used to represent the properties such as: time, number, percent, age, weight, and many other properties. Contour lines on a map represent the rise and fall of the heights. In general, analogues are less specific, less concrete but easier to manipulate than are iconic models.

1C. Symbolic (Mathematical) models. The symbolic or mathematical model is one which employs a set of mathematical symbols (i.e., letters, numbers, etc.) to represent the decision variables of the system. These variables are related together by means of a mathematical equation or a set of equations to describe the behavior (or properties) of the system. The solution of the problem is then obtained by applying well-developed mathematical techniques to the model.

CLASSIFICATION OF MODELS

2. **Classification by Purpose**

Models can also be classified by purpose of its utility. The purpose of a model may be descriptive predictive or prescriptive.

2A.Descriptive models. A descriptive model simply describes some aspects of a situation based on observations, survey. Questionnaire results or other available data. The result of an opinion poll represents a descriptive model.

2B.Predictive models. Such models can answer 'what if' type of questions, i.e. they can make predictions regarding certain events. For example, based on the survey results, television networks such models attempt to explain and predict the election results before all the votes are actually counted.

2C.Prescriptive models. Finally, when a predictive model has been repeatedly successful, it can be used to prescribe a source of action. For example, linear programming is a prescriptive (or normative) model because it prescribes what the managers ought to do.

3. Classification by Nature of Environment

3A.Deterministic models. Such models assume conditions of complete certainty and perfect knowledge.

For example, linear programming, transportation and assignment models are deterministic type of models.

3B.Probabilistic (or Stochastic) models. These types of models usually handle such situations in which the consequences or payoff of managerial actions cannot be predicted with certainty. However, it is possible to forecast a pattern of events, based on which managerial decisions can be made. For example, insurance companies are willing to insure against risk of fire, accidents, sickness and so on, because the pattern of events have been compiled in the form of probability distributions.

CLASSIFICATION OF MODELS

4. **Classification by Behavior**

4a).**Static models:** These models do not consider the impact of changes that takes place during the planning horizon, i.e. they are independent of time. Also, in a static model only one decision is needed for the duration of a given time period.

4b).**Dynamic models:** In these models, time is considered as one of the important variables and admits the impact of changes generated by time. Also, in dynamic models, not only one but a series of interdependent' decisions is required during the planning horizon.

CLASSIFICATION OF MODELS

5.. **Classification by Method of Solution**

5a)**Analytical models**. These models have a specific mathematical structure-and thus can be solved by known analytical or mathematical techniques. For example, general linear programming models as well as the specially structured transportation and assignment models are analytical models. .

5b). **Simulation models.** They also have a mathematical structure but they cannot be solved by purely using the ‘tools’ and ‘techniques’ of mathematics. A simulation model is essentially computer-assisted experimentation on a mathematical structure of a real time structure in order to study the system under a variety of assumptions.

PRINCIPLES OF MODELING

Modeling Principles are useful in guiding to formulate the models within the context of OR.

1. **Do not build up a complicated model when simple one will suffice.** Building the strongest possible model is a common guiding principle for mathematicians who are attempting to extend the theory or to develop techniques that have wide applications. However, in the actual practice of building models for specific purposes, the best advice is to "keep it simple".
2. **Beware of molding the problem to fit the technique.** For example, an expert on linear programming techniques may tend to view every problem he encounters as required in a linear programming solutions. In fact, not all optimization problems involve only linear functions. Also, not all OR problems involve optimization. As a matter of fact, not all real-world problems call for operations research! Of course, everyone search reality in his own terms, so the field of OR is not unique in this regard. Being human we rely on the methods we are most comfortable in using and have been most successful within the past. We are certainly not able to use techniques in which we have no competence, and we cannot hope to be competent in all techniques. We must divide OR experts into three main categories:

PRINCIPLES OF MODELING

3.**The deduction phase of modeling must be conducted rigorously**. The reason for requiring rigorous deduction is that one wants to be sure that if model conclusions are inconsistent with reality, then the defect lie in the assumptions. One application of this principle is that one must be extremely careful when programming computers. Hidden "bugs" are especially dangerous when they do not prevent the program from running but simply produce results, which are not consistent with the intention of the model.

4.**Models should be validated prior to implementation.** For example, if a model is constructed to forecast the monthly sales of a particular commodity, it could be tested using historical data to compare the forecasts it would have produced to the actual sales. In case, if the model cannot be validated prior to its implementation, then it can be implemented in phases for validation. For example a new model for inventory control may be implemented for a certain selected

MODELLIG PRINCIPLES

5. **A model should never be taken too literally.** For example, suppose that one has to construct an elaborate computer model of Indian economy with many competent researchers spending a great deal of time and money in getting all kinds of complicated interactions and relationships. Under such circumstances, it can be easily believed as if the model duplicates itself the real system. This danger continues to increase as the models become larger and more sophisticated, as they must deal with increasingly complicated problems.

6. **A model should neither be pressed to do, nor criticized for failing to do that for which it was never intended.** One example of this error would be the use of forecasting model to predict so far into the future that the data on which the forecasts are based have no relevance. Another example is the use of certain network methods to describe the activities involved in a complex project. A model should not be stretched beyond its capabilities.

MODELLIG PRINCIPLES

7. **Beware of over-selling a model**. This principle is of particular importance for the OR professional because most non- technical benefactors of an operations researcher's work are not likely to understand his methods. The increased technicality of one's methods also increases the burden of responsibility on the OR professional to distinguish clearly between his role as model manipulator and model interpreter. In those cases where the assumptions can be challenged, it would be dishonest to use the model.
8. **Some of the primary benefits of modeling are associated with the process of developing the model.** It is true in general that a model is never as useful to anyone else as it is to those who are involved in building it up. The model itself never contains the full knowledge and understanding of the real system that the builder must acquire in order to

MODELLIG PRINCIPLES

9. **A model cannot be any better than the information that goes into it.** Like a computer program, a model can only manipulate the data provided to it; it cannot recognize and correct for deficiencies in input. Models may condense data or convert it to more useful forms, but they do not have the capacity to generate it. In some situations it is always better to gather more information about the system instead of exerting more efforts on modern constructions.

10. **Models cannot replace decision makers.** The purpose of OR models should not be supposed to provide "Optimal solutions" free from human subjectivity and error. OR models can aid decision makers and thereby permit better decisions to be made. However they do not make the job of decision making easier. Definitely, the role of experience, intuition and judgment in decision- making is undiminished.

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GENERAL METHODS FOR SOLVING 'OR' MODLES

Generally the methods are used for solving OR models. Are as follows:

1.Analytic Method: If the OR model is solved by using all the tools of classical mathematics such as: differential calculus and finite differences available for this task, then such type of solutions are called analytic solutions. Solutions of various inventory models are obtained by adopting the so-called analytic procedure.

2. Iterative Method: If classical methods fail because of complexity of the constraints or of the number of variables, then we are usually forced to adopt an iterative method. Such a procedure starts with a trial solution and a set of rules for improving it. The trial solution is then replaced by the improved solution, and the process is repeated until either no further improvement is possible or the cost of further calculation cannot be justified.

- Iterative method can be divided into three groups:
- After a finite number of repetitions, no further improvement will be possible.
- Although successive iterations improve the solutions, we are only guaranteed the solution as a limit of an infinite process.
- Finally we include the trial and error method, which, however, is likely to be lengthy, tedious, and costly even if electronic computers are used.
- The Monte-Carlo Method: The basis of so-called Monte- Carlo technique is random sampling of variable's values from a Distribution of that variable. Monte-Carlo refers to the use of sampling methods to estimate the value of non-stochastic variables

ROLE OF OPERATIONS RESEARCH IN DECISSION MAKING (Advantages of OR)

1. **Better control.**
2. **Better Co-ordination.**
3. **Better System.**
4. **Better Decisions**

MAIN PHASES OF OPERATION RESEARCH STUDY (procedure for for or study involves following phases)

Phase-I: **Formulating the Problem in appropriate model.** To do this following information is required.

i. Who has to take the decision?
ii. What are the objectives?
iii. What are the ranges of controlled variables?
iv. What are the restrictions or constraints on the variables?

(wrong formulation cannot yield a right decision/solution one should be careful while executing this phase.

MAIN PHASES OF OPERATION RESEARCH STUDY/methodology of OR (procedure for for or study involves following phases)

Phase-II: Constructing a mathematical model: the second phase of investigations in a appropriate model which is convenient for analysis. A mathematical model should include the following three important basic factors

Decision variables and parameters, Constraints or Restrictions, objective function.

Phase-III.: Deriving Solutions from the model: This phase devoted to computation of decision variables that optimize the objective function.

Phase-IV. Testing the model and its solution: after completing the model it is once again tested as a whole for the errors if any. A model may be said to be valid if it can provide a reliable prediction of the system's performance.

MAIN PHASES OF OPERATION RESEARCH STUDY (procedure for OR study involves following phases)

Phase-V: Controlling the solution: This phase establishes controls over the solution with any degree of satisfaction. The model requires immediate modification as soon as the controlled variables (one or more) change significantly, otherwise the model goes out of control.

Phase-VI: Implementing the solution: Finally , the tested results of the model are implemented to work . This phase primarily executed with the cooperation of Operations Research experts and those who are responsible for managing and operating the systems.

LP FORMULATION – EXAMPLE 1
PRODUCTION ALLOCATION PROBLEM

A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs 3 on type B. Each product is processed on two machines G and H. Type A requires one minute of processing time on G and two minutes on H; type B requires one minute on G and one minute on H. the machine G is available for not more than 6 hour 40 minutes while machine H is available for 10 hours during any of working day. Formulate the problem as a linear programming problem.

LP FORMULATION – EXAMPLE 1
PRODUCTION ALLOCATION PROBLEM

- **let X_1 be the no. of products of type A , X_2 the number products of type B.**

process Time of Products (minutes)

MACHINE	**Type A (X_1 units**	**Type B (X_2 units)**	**Available time (minutes)**
G	1	1	400
H	2	1	600
Profit /unit	Rs 2	Rs 3	

SOLUTION: Find X_1 and X_2 such that the profit $Z = 2X_1 + 3X_2$ is maximum

$1 X_1 + 1 X_2 <= 400$,

$1 X_1 + 1 X_2 <= 600$

$X_1 , >=0 , X_1 >=0$

The total profit for X_1 units of typew A, and X_2 units of type B . **Z = 2X1 + 3X2** (objective function)

Considering Processing time on Machine G -> $1 X_1 + 1 X_2$ 400 ---- eqn-1 (first constraint)

Considering processing time on machine H -> $1 X_1 + 1 X_2$ 600 ---- eqn-2 (second constraint)

Since it is not possible to produce negative quantities

$X_1 , >=0 , X_1 >=0$ (non negativity restrictions

Hence the allocation problem of the firm ca be finally put in the form

LP FORMULATION – EXAMPLE-2

A company produces two types of Hats. Each hat of first type requires twice as much labour than the second type. If all the hats are of the second type only, the company can produce a total of 500 hats a day. The material limits the daily sales of the first and second type to 150 and 250 hats . Assuming that the profits per hat are Rs8 for type A and Rs 5 for type B, formulate the problem as a linear programming model in order to determine the number of hats of each type so as to maximize the profit

LP FORMULATION – EXAMPLE 2
PRODUCTION ALLOCATION PROBLEM

- let X_1 be the no. of hats of type A , X_2 the no. of hats of type B.

process Time of Products (minutes)

product	Hat- A (X_1 units	hat B (X_2 units)	Available
Proces time	2t	1t	500t minutes
Material- A hat B	X1 ----	--- x2	150 nos 250 nos
Profit /unit	Rs 8	Rs 5	

SOLUTION: Find X_1 and X_2 such that the profit $Z = 8X_1 + 5X_2$ is maximum

$2X_1 + 1\,X_2 <= 500$,

$X_1 <= 150$

$X_2 <= 250$

$X_1 > 0$, $X_1 > 0$

The total profit for X_1 units of HAT A, and X_2 units of HAT B . $Z = 8X1 + 5X2$ (objective fn)

Considering Processing time on HAT A -> $2t^* X_1 + 1t^* X_2 <= 500t$ ---(first constraint)

=➔ $2X_1 + 1\,X_2 <= 500$

Considering material limit HAT-A -> $1X_1 <= 150$ ------ eqn-2 (2nd constraint)

Considering material limis on HAT-B $X_2 <= 250$ ------- eqn---3

Since it is not possible to produce negative quantities

LP FORMULATION – EXAMPLE 3
PRODUCTION ALLOCATION PROBLEM

A manufacturer of patent medicines is proposed to prepare a production plan for medicines A and B. There are sufficient ingredients available to make 20,000 bottles of medicine A and 40,000 bottles of medicine B, but there are only 45,000 bottles into which either of the medicines can be filled. Further, it takes three hours to prepare enough material to fill1000 bottles of medicine A and one hour to prepare enough material to fill 1 000 bottles of medicine B, and there are 66hours available for this operation. The profit is Rs. 8 per bottle for medicine A and Rs. 7 per bottle for medicine B.

- Formulate this problem as a L.P.P. ,
- How the manufacturer schedule his production in order to maximize profit.

LP FORMULATION – EXAMPLE 3
PRODUCTION ALLOCATION PROBLEM

- **let X_1 , and X_2 be the thousands of bottles medicine A & B.**

	Type A (X_1 thousand units	Type B (X_2 thousand units)	Availability
Processtime	3 hrs	1hr	66 hours
Bottle	*	*	45 thousand
Ingredient A Ingredient B	*	 *	20 40
Profit /unit	Rs 8	Rs 7	

SOLUTION: Find X_1 and X_2 such that the profit Z = $8000X_1 + 7000X_2$ is maximum

$3X_1 + 1X_2 <= 66$,

$X_1 + X_2 <= 45$

$X_1 <= 20$

$X_2 <= 40$

$X_1 > 0$, $X_1 > 0$

The **total profit for X_1 thousands units of type A, and X_2 thousands of type B .**

Z = 8000X1 + 7000 X2 (objective function)

Considering processing time $3X_1 + 1X_2 <= 66$ ---- eqn-1 (1st constraint)

Considering limitations on bottles for A & B → $X_1 + X_2 <= 45$ --- eqn-2 (2nd constraint)

Considering ingredient availability → $X_1 <= 20$, $X_2 <= 40$

Since it is not possible to produce negative quantities

X_1 , X_2 >=0 (non negativity restrictions

The total pr

LP FORMULATION – EXAMPLE 4
PRODUCTION ALLOCATION PROBLEM

- A toy company manufactures two types of doll. A basic version doll-A and a delux version doll-B. Each type of doll of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 per day. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The delux version requires a fancy dress of which there are only 600 per day available. If the company makes a profit of Rs 3 and Rs 5 per doll respectively on doll A and B, then how many of each one has to be produced so as to maximize the profit.

LP FORMULATION – EXAMPLE 4
DOLLS PRODUCTION ALLOCATION PROBLEM

- let X_1 , and X_2 be the thousands of bottles medicine A & B.

	DOLL A (X_1 units	DOLL B (X_2 units)	Availability
Processtime	1 t	2 t	2000 t
plastics	X1	X2	1500
Fancy matl I	-	X2	600
Profit /unit	Rs 3	Rs 5	

SOLUTION: Find X_1 and X_2 such that the profit
$Z = 3X_1 + 5 X_2$ is maximum
$X_1 + 2 X_2 <= 2000$
$X_1 + X_2 <= 1500$
$X_2 <= 600$
$X_1 > 0$, $X_1 > 0$

The **total profit for X_1 thousands units of type A, and X_2 thousands of type B .**
maximize Profit = max $Z = 3X1 + 5 X2$ (objective function)
Considering processing time $t X_1 + 2 t X_2 <= 2000 t = X_1 + 2 X_2 <= 1500$ (2nd constraint)
Considering plastic availability → $X_1 + 2 X_2 <= 1500$,
Considering fancy material $X_2 <= 600$
Since it is not possible to produce negative quantities X... X... > =o n...

The total pr

LP FORMULATION – EXAMPLE 5
PRODUCTION ALLOCATION PROBLEM

- **A firm an produce three types of cloths, say A, B, and C and three kinds are wools are required for it say red. Green and blue wool. One unit length of type A cloth needs 2 meters of red wool and 3 meters of blue wool. One unit length of type B cloth needs 3 meters of red wool , 2 meters of green wool and 2 meters of blue wool. One unit of type C cloth needs 5 meters of green wool and 4 meters of blue wool. The firm has only a stock of 8 meters of red wool, 10 meters of green wool and 15 meters of blue wool. The profit expected Rs 3, 5, 8 respectively on A, B and C products. Formulate the LPP problem.**

LP FORMULATION – EXAMPLE 5
DOLLS PRODUCTION ALLOCATION PROBLEM

- let X_1 , and X_2 be the thousands of bottles medicine A & B.

	Type- A (X_1 units	Type- B (X_2 units)	Type-CX_3 units)	Availability
Red wool	2 mtrs	3 mtrs		8 mtrs
Green wool		2 mtrs	5 mtrs	10 mtrs
Blue wool	3 mtrs	2 mtrs	4 mtrs	15 mtrs
Profit /unit	Rs 3	Rs 5	Rs 8	

SOLUTION: Find X_1 and X_2 such that the profit

$Z = 3X_1 + 5X_2 + 8X_3$ is maximum

$2X_1 + 3X_2 <= 8$

$2X_2 + 5X_3 <= 10$

$3X_1 + 2X_2 + 4X_3 <= 15$

$X_1 > 0, \quad X_1 > 0$

The **total profit for X_1 units of type A, X_2 units of type B and X_3 units of type C**
maximize Profit = max Z = 3X1 + 5 X2 + 8 X3 (objective function)
Considering Red wool availability $2X_1 + 3X_2 <= 8$ (1st constraint)
Consideringgreen wool availability $2X_2 + 5X_3 <= 10$ ----(2nd constraint)
Considering Blue wool availability 3X1 + 2 X2 + 4 X3 < = 15 (3rd constraint)
Since it is not possible to produce negative quantities X_1 , X_2 > =o n

GRAPHICAL METHOD

Practical Steps Involved in Solving LPP By Graphical Method

- Simple linear programming problems of two decision variables can be easily solved by graphical method. The outlines of graphical procedure are as follows:
- Step 1: Consider each inequality constraint as equation.
- Step 2: plot each equation on he graph, as each will geometrically represent a srraight line.
- step-3. Shade the feasible region. Every point on the line will satisfy the equation of the line. If the inequality- constraint corresponding to that line is ‘<=’, then the region below the line lying in the first quardrant (due to non-negativity of variables) is shaded. For the equality –constraint with ‘ >=’ sign, the region above he line in the first quardrant is shaded. The poins lying in the common region will satisfy all the constraints simultaneously. The common region thus obtained is called the feasible region.

GRAPHICAL METHOD
Practical Steps Involved in Solving LPP By Graphical Method

- Step-4. Choose the convenient value of Z (say =0) and plot the objective function line.
- Step-5. Pull the objective function line until the exreme points of the feasible region. In the maximization case, this line will stop farthest from the origin and passing through at least one carner of he feasible region. In the minimization case, , the line , this line will stop nearest to the origin and passing through at least one corner of the easible region.
- Step-6. Read the coordinates of the exteme point (s) selected in step-5, and find the maximum or minimum (as thec case may be) value od Z.

Example-1. Find a geometrical interpretation and a solution for the following problem.(Graphical method)

- **Maximize $Z = 3X_1 + 5X_2$, subjective to restrictions:**

  $X_1 + 2X_2 \leq 2000$, $X_1 + X_2 \leq 1500$, ,

  $X_2 \leq 600$ and X_1 0, , X_2 0,
- **Graphical Solution.**
- **Step-1. To graph the inequality constraints, consider two mutually perpendicular lines OX_1 and OX_2 as axis of coordinates. Since X_1, X_2 0 we have to draw in first quadrant only.**
- **To plot the $X_1 + 2X_2 \leq 2000$,**

  consider the equation $X_1 + 2X_2 = 2000$,

  for $x_1 = 0$, $X_2 = 1000$ and for $X_2 = 0$, $X_1 = 2000$
- **Take appropriate scale and represent the points and draw the line of $X_1 + 2X_2 = 2000$, since constraint is of type, consider region toward the origin as feasible region.**

Example-1. Find a geometrical interpretation and a solution for the following problem.(Graphical method)

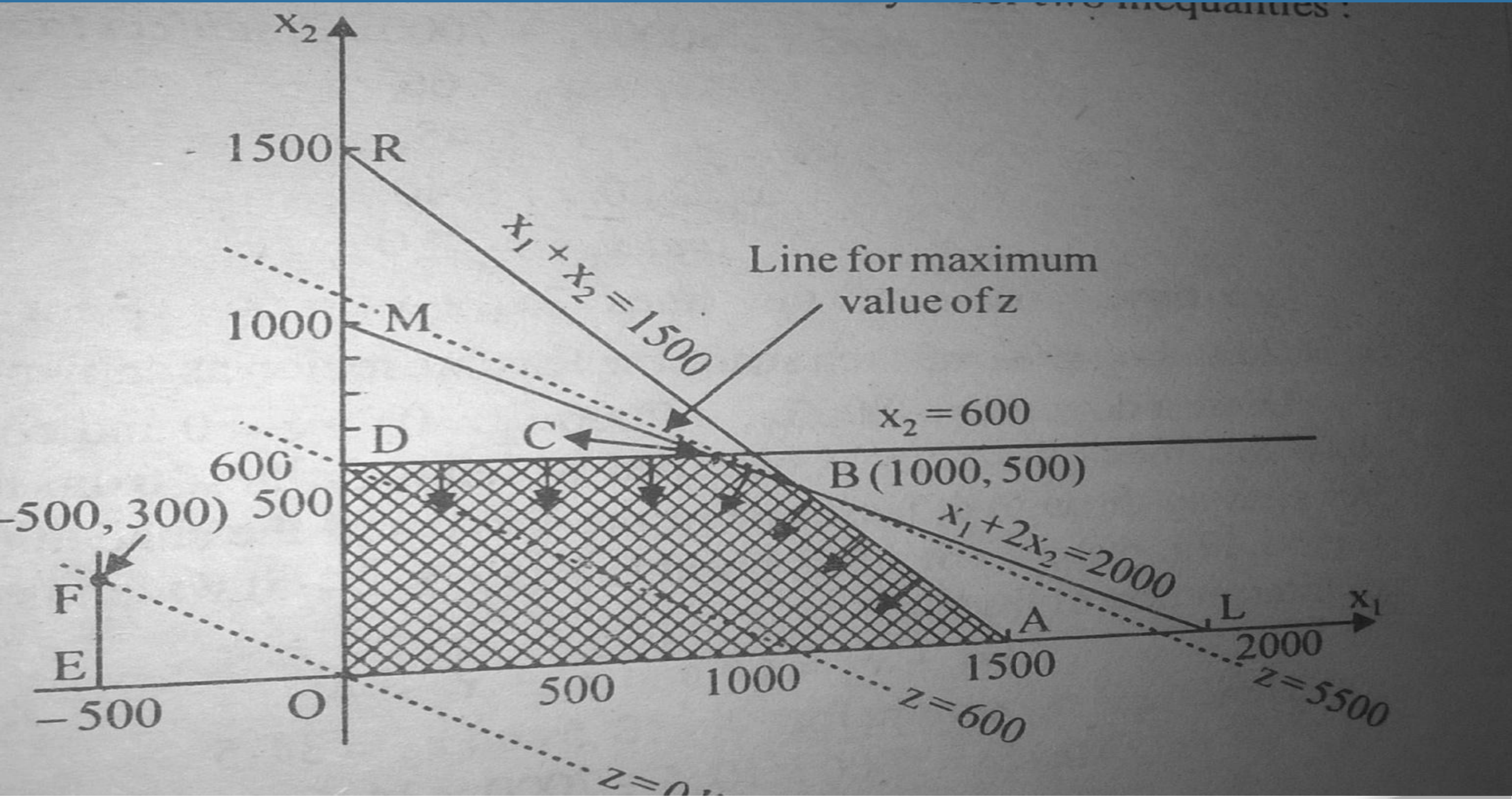
- Consider second constraint $X_1 + X_2 \leq 1500$,

  when $X_1 = 0$, $X_2 = 1500$

  when $X_2 = 0$ $X_1 = 1500$
- Consider third constraint $X_2 \leq 600$,

  $X_2 = 600$, when $X_1 = 0$, $X_2 = 600$
- Similarly plot the all the straight lines and consider common n region towards origin satisfying all the all the constraints as feasible region.
- Step-2. Find the feasible region or solution space by combining all constraints. A common shaded area of OABCD is obtained which is feasible solution to the given LPP. Which satisfies all the constraints.
- Step-3. Find the coordinates of the corner points of feasible region O, A, B, C, D
- Step-4. Locate the corner point of optimal solution either by calculating the value of Z for each corner point o, A, B, C and D

GRAPHICAL METHOD



GRAPHICAL METHOD

@ Point A (1500, 0) $Z = 3x_1 + 5x_2 = 4500 + 0 = 4500$

@ point B (1000, 500), $Z = (3*1000 + 5*500) = 5500$ maximum

@ point C (800,600), $Z = (3*800 + 5*600) = 5400$

@ point D (0, 600), $Z = (3*0 + 5*600) = 3000$

@ point O (0, 0) $Z = (3*0 + 5*0) = 0$

- In this problem, maximum value of Z is attained at the corner point B (1000, 500), which is point of intersection of lines
- $X_1 + 2X_2 = 2000$, and $X_1 + X_2 \quad 1500$,
- Hence the required solution is $X_1 = 1000$, $X_2 = 500$ and maximum value Z = Rs 5500

SECOND METHOD TO FIND OPTIMUM POINT FOR GRAPHICAL METHOD

- **To find the point or points in the feasible region . for some fixed value of Z, Z** $= 3X_1 + 5X_2$ is a straight line and any point on it gives the same value of Z
- Considering Z=3000, $3X_1 + 5X_2 = 3000$
- when x1 = 0, X2= 600
- When x2=0 X1 = 1000
- Represent the objective function line also on the feasible region graph. Move the objective function straight line parallel such that it touches the extreme point on the feasible region.. it touches point C, where $X_1 = 1000$, $X_2 = 500$, maximum objective function value
- $Z_{max} = 5500$.

Example-2. Solve following LPP(maximization type) by Graphical method

Solve by graphical method Max $Z = X_1 + X_2$ Subject to

$X_1 + 2X_2 \leq 2000$, , $X_1 + X_2 \leq 1500$, $X_2 \leq 600$ and $X_{1,}$ $X_2 \geq 0$

- Considering First constraint as equation $X_1 + 2X_2 = 2000$,

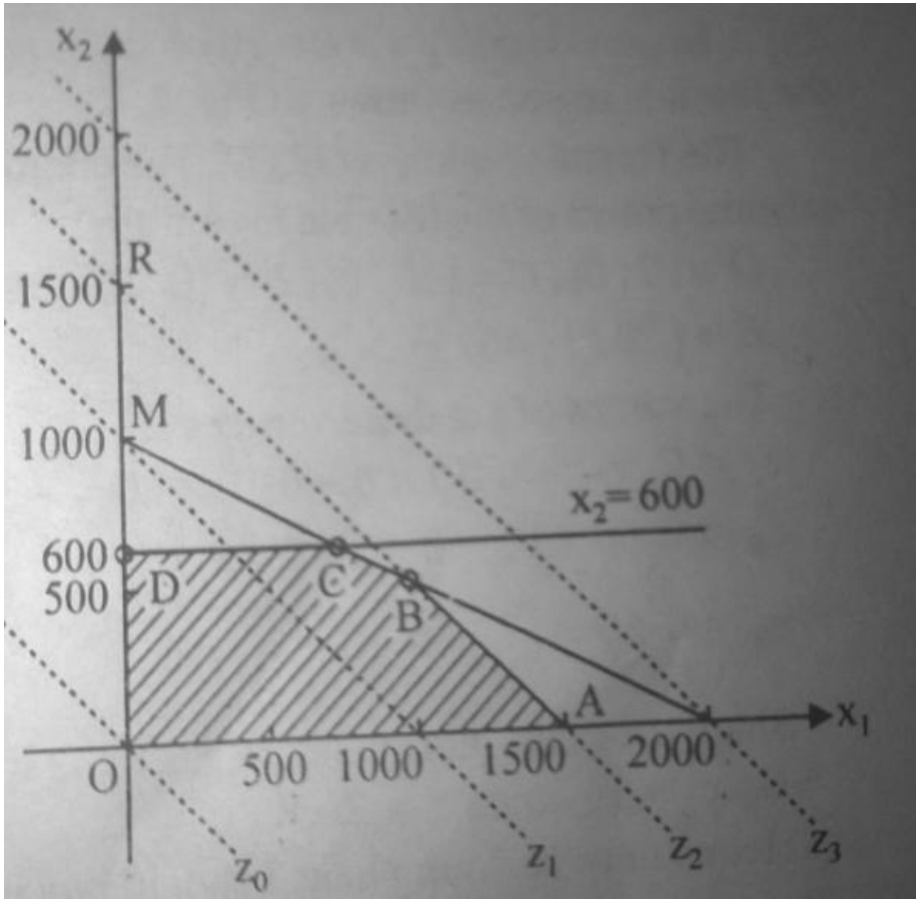
When [$X_1 = 0$, $X_2 = 1000$] and When [$X_2 = 0$, $X_1 = 2000$]

- Considering the second constraint as equation , $X_1 + X_2 = 1500$
- when [$X_1 = 0$, $X_2 = 1500$] & [When $X_2 = 0$, $X_1 = 1500$]
- The feasible region is similar to earlier problem.

It is clear from figure that Z is maximum and it lies along the edge AB of the polygon of the feasible solutions. This indicates that the values of X_1, X_2, which maximize Z are not unique., but any point on the edge AB of OABCD polygon will give the optimum value of Z. The maximum value of Z is always unique, but there will be an infinite number of feasible solutions which give unique value of 'Z'.

Example-2. Solve by Graphical method

- thus , two corners A and B as well as any other point on the line AB (segment) will give optimal solution of this problem.
- It should be noted that if a linear programming problem has more than one optimum solution, there exists alternative optimum solutions.



- **one of the optimum solutions will be corresponding to corner point B. ie X_1= 1000, X_2= 500 with max. profit Z = 1000+500= 1500**

Example-3. Solve following LP problem Graphically(maximization type)

- Max Z = 8000 X_1 + 7000$X_{2,}$ subject to

$X_1 + X_2 \leq 66$, $X_1 + 2X_2 \leq 45$,

$X_1 \leq 20$, $X_2 \leq 40$

and $X_1 \geq 0$, , $X_2 \geq 0$

- Solution: First plot the lines 3 $X_1 + X_2 = 66$, $X_1 + 2X_2 = 45$, $X_1 = 20$ and $X_2 = 40$ and shade the fesible region as shown in fig-3.
- Draw a dotted line 8000 X_1 + 7000X_2= 0 for z-0 and continue to draw the lines till a point is obtained which is farthest from the origin but passing through at least one of the corners of the shaded (feasible) region.

Example-3. Solve mqximization - LPP by Graphical method

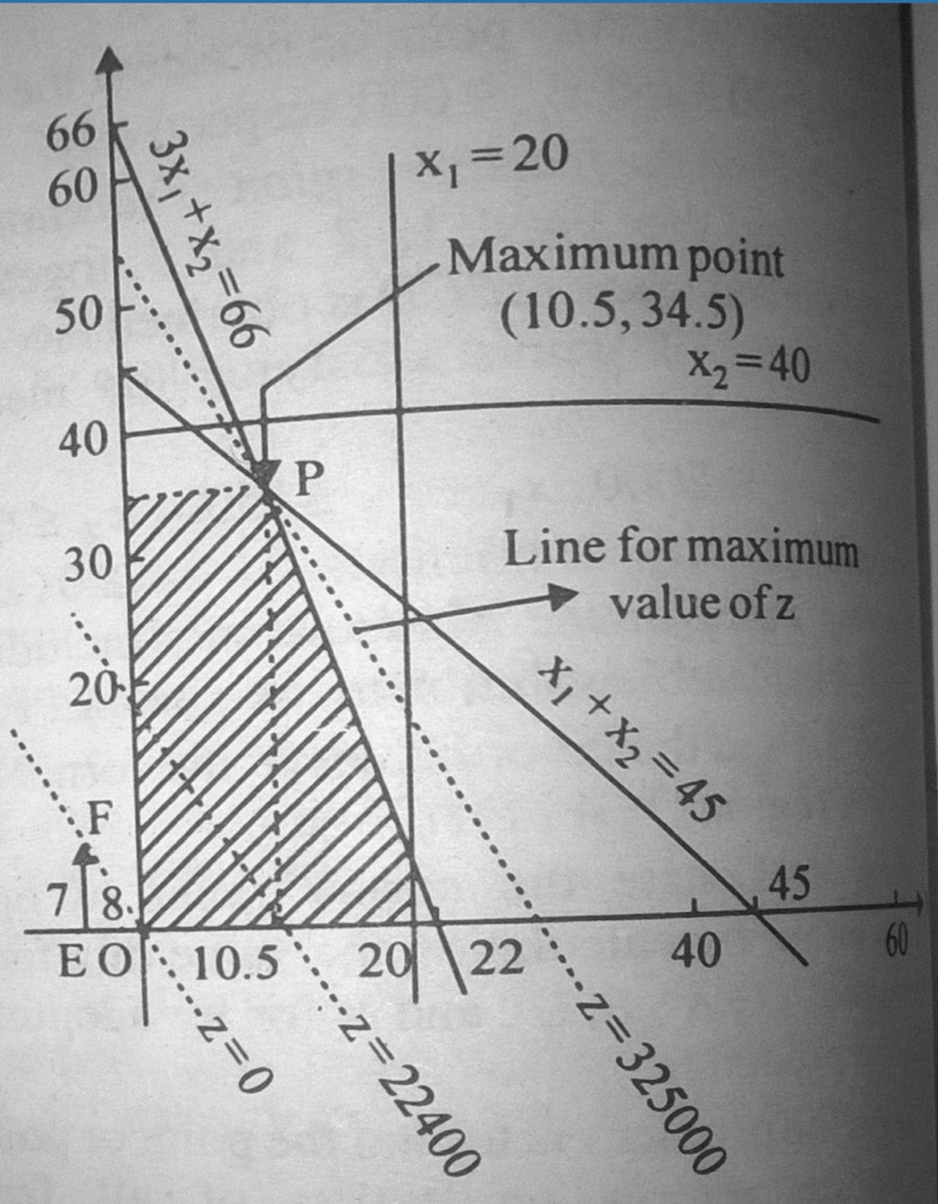


Fig. 6

The figure shows that this point **P** (10.5, 34.5) which is the intersection of lines

$3\ X_1 + X_2 = 66$,

$X_1 + 2X_2 = 45$,

Hence , Z is maximum for $X_1 = 10.5$ and $X_2 = 34.5$

Max Z = 8000 X_1 + 7000X_2

Max Z = 8000* 10.5 + 7000*34.5 = **Rs 3,25,000.**

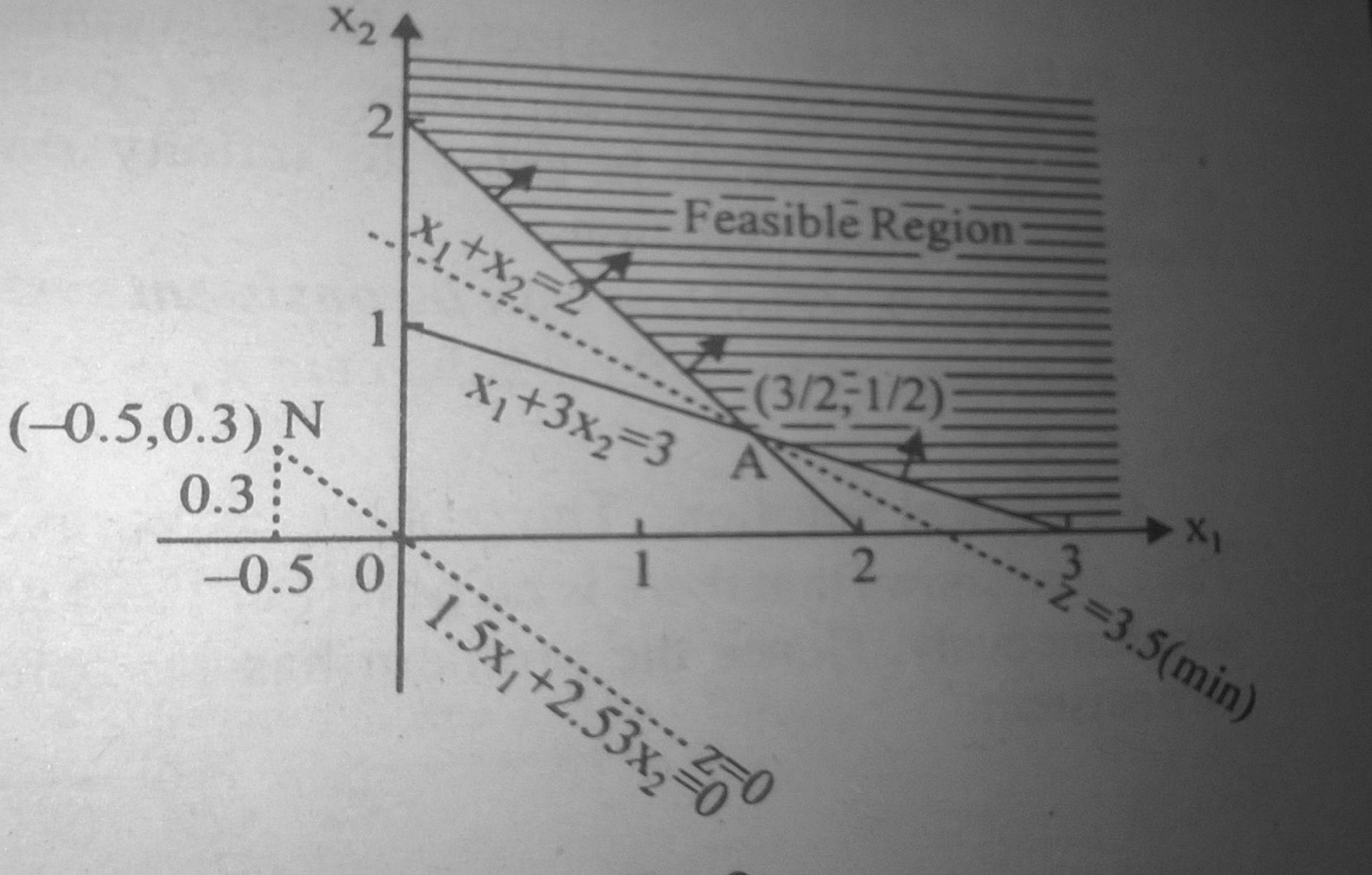
Example-4. Solve by Graphical method (minimization type problem)

- **Example-4. Min $Z = 1.5\ X_1 + 2.5\ X_2$ subject to**
- $X_1 + 3X_2 \leq 3 \quad X_1 + X_2 \leq 2$ and $X_1 \quad 0, \quad , X_2 \quad 0$
- Graphical Solution: The geometrical interpretation of the problem is given in fig-4. The minimum is attained at the point of intersection A of the lines
- $X_1 + 3X_2 = 3$ and $X_1 + X_2 = , 2$. this is

the unique point to give the minimum value of Z. now , solving these two equations

- $X_1 = 1.5 \quad , X_2 = 0.5$ and min $Z = 3.5$

Example-4. Solve minimization type problem by Graphical method

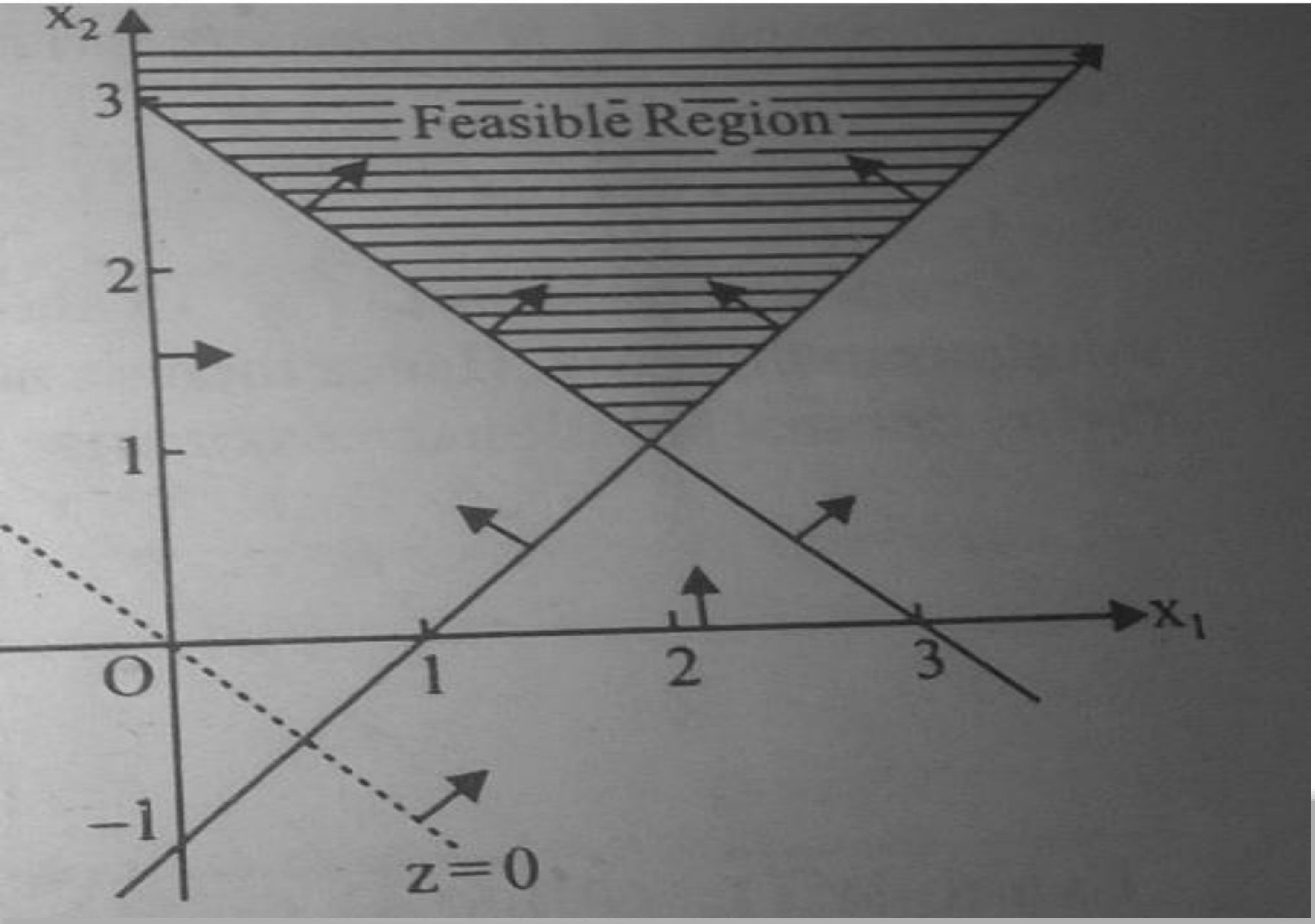


Example-7. Problem having unbounded solution Graphical method

- Max $Z = 3X_1 + 2X_2$ subject to $X_1 - X_2 \leq_{,} 1$

  $X_1 + X_2 \leq 3$, and $X_1 \geq 0$, $,X_2 \geq 0$
- Graphical Solution: the region of feasible solution is the shaded area as shown in figure-7.
- It is clear from the figure that the line representing the objective function can be moved far even parallel to itself in the direction of increasing Z, and still have some points in the region of feasible solutions.
- Hence Z can be made arbitrarily large, and the problem has no finite maximum value of Z. such problems are said to have unbounded solutions.

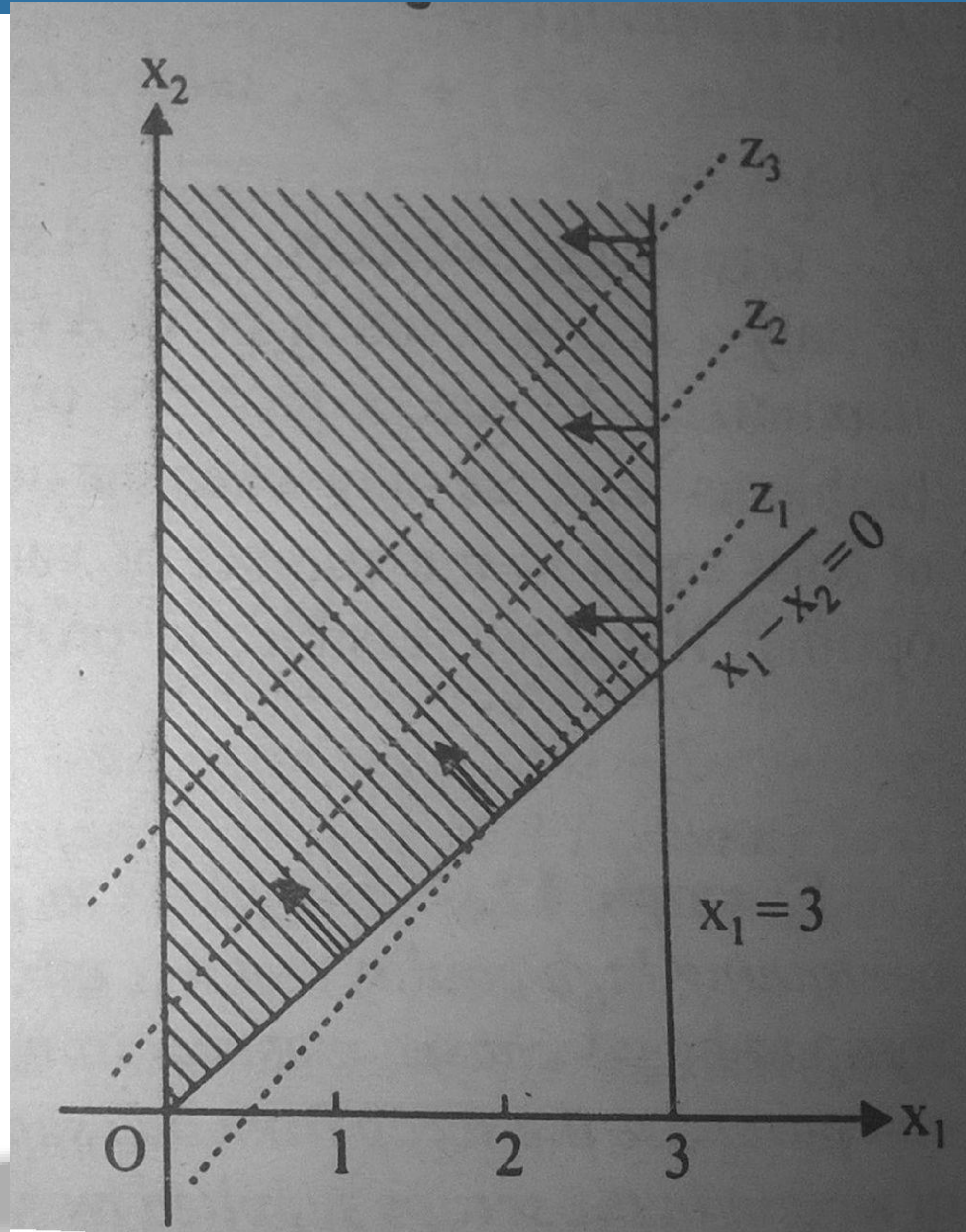
Example-7. Problem having unbounded solution Graphical method



Example-8 Problem having unbounded solution Graphical method

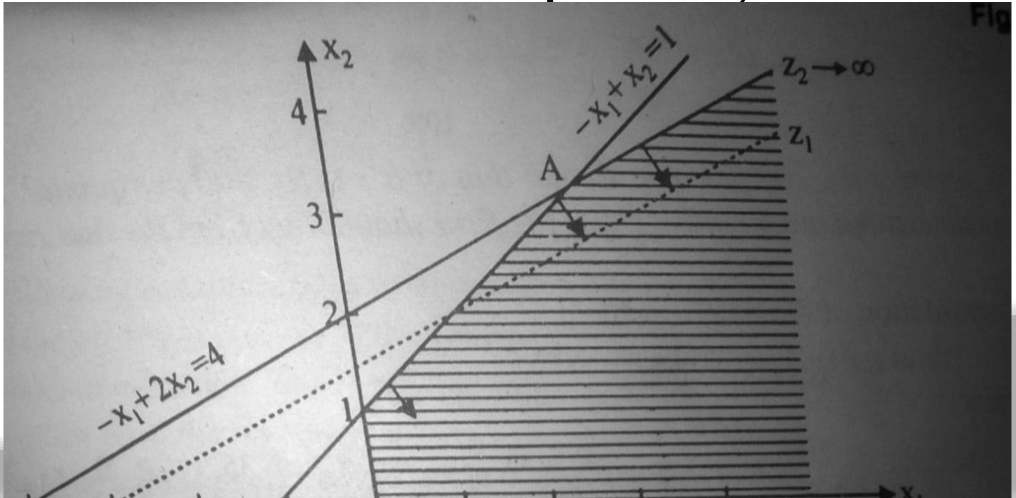
- . Max $Z = -3X_1 + 2X_2$
- s/t $X_1 \leq 3$, $X_1 - X_2 \leq 0$ and X_1 , X_2 0

In previous example-7, it has been seen that the both the variables can be made arbitrarily large as Z is increased. In this problem, an unbounded solution does not necessarily imply that all the variables can be made arbitrarily large as Z approaches infinity. Here the variable X_1 remains constant as shown in fig.



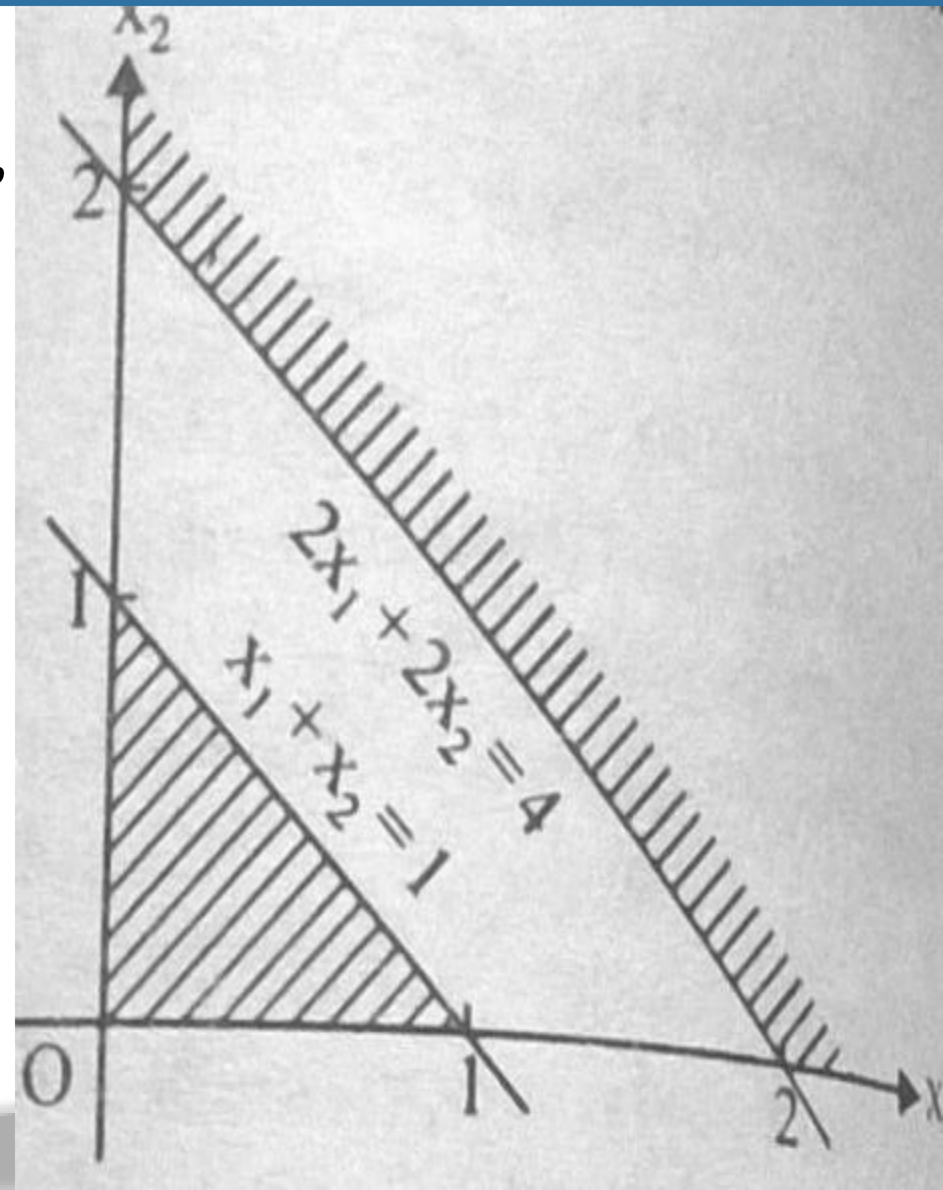
Example-9.Problem which not completely normal

- Maximize $Z= -X_1 + 2 X_2$ subject to $- X_1 +X_2 \leq 1$,
  $- X_1 + 2X_2 \geq 4$ and $X_1 \geq 0$, X_2 0
- Graphical solution: the problem is solved graphically in fig.
- Observing graph figure the line of objective function coincides with the edge of equation-1 of the region of the feasible solutions. Thus every point (X_1, X_2) lying on this edg ($- X_1 + 2X_2 = 4$), which is going infinity on the right gives Z=4 and is therefore an optimal; solution.



Example-10. (Problem with inconsistent system of constraints)

- Maximize $Z = 3X_1 - 2X_2$ subject to $X_1 + X_2 \leq 1$, $2X_1 + 2X_2 \geq 4$ and $X_1 \geq 0$, $X_2 \geq 0$
- Graphical solution: the problem is represented graphically in fig-12. The figure shows that there is no point (X_1, X_2) which satisfies both the constraints simultaneously. Hence the problem has no solution because the constraints are

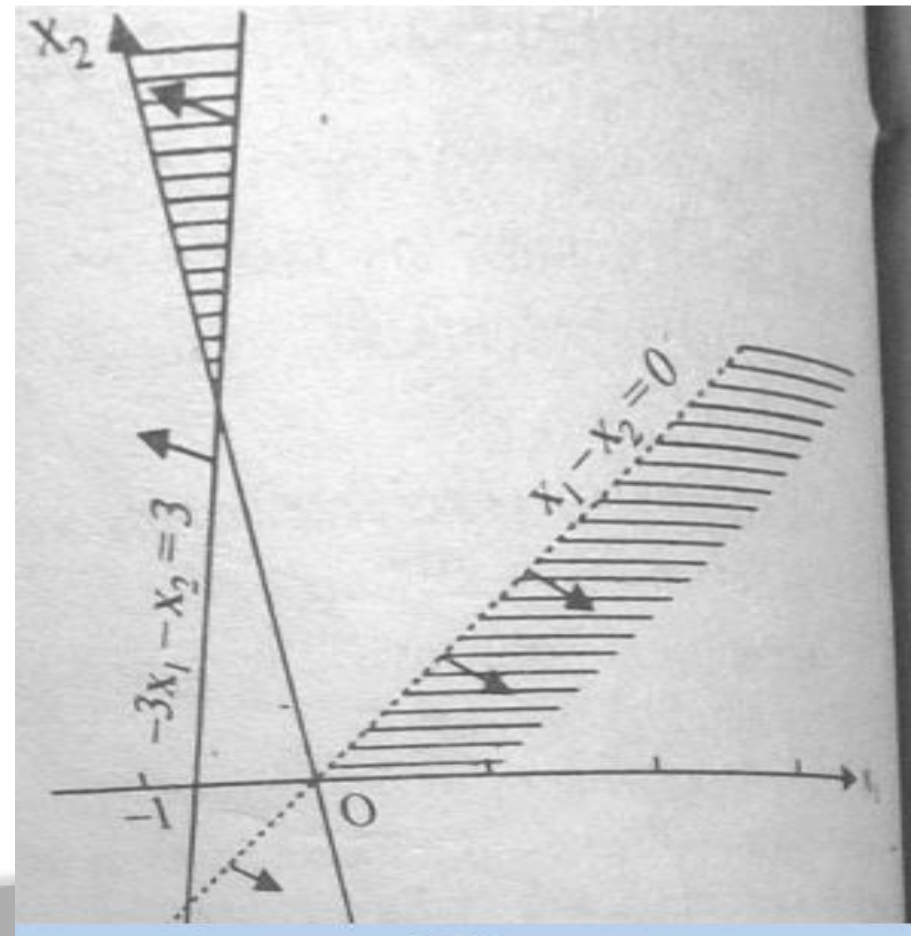


Graphical solution: the figure shows that there is no region of feasible solutions in this case. Hence there is no feasible solution. So the question of having optimal s

Example-11. Solve by Graphical method

Maximize $Z = X_1 + X_2$ subject to $X_1 - X_2 \geq 0$, $-3X_1 + X_2 \geq 3$, and X_1 , $X_2 \geq 0$

Graphical solution: the figure shows that there is no region of feasible solutions in this case. Hence there is no feasible solution. So the question of having optimal solution does not arise.



Example-12. (Problem in which constraints are equations rather than in equalities

- Maximize $Z = 5X_1 + 3X_2$ subject to $3X_1 + 5X_2 = 15$, , $5X_1 + 2X_2 = 10$, and X_1 , X_2 0
- Graphical solution: fig-14 shows the graphical solution. Since there is only a single solution point A

(20/19, 45/19), thee is nothing to be maximized. Hence , a problem of this kind is of no importance. Such problems can arise only when the number of equations in the constraints is at least equal to the number of variables. If the

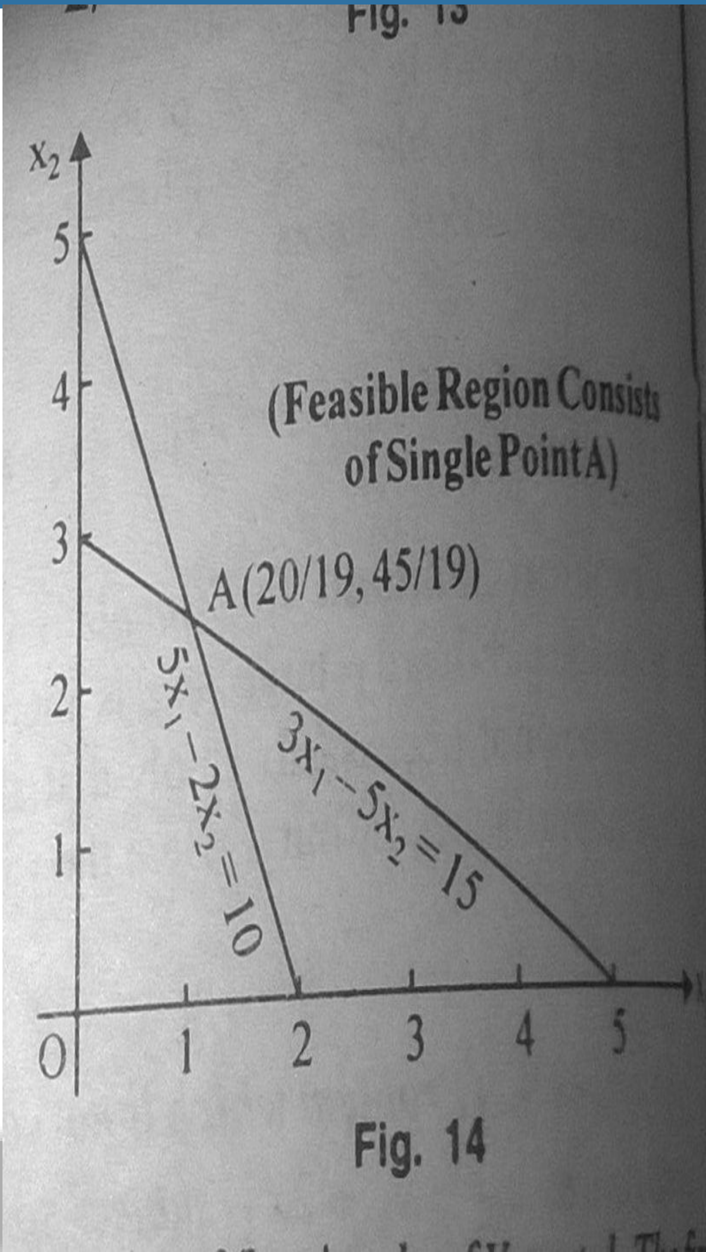


Fig. 14

SIMPLEX METHOD

- *Simplex*: a linear-programming algorithm that can solve problems having more than two decision variables.
- The simplex technique involves generating a series of solutions in tabular form, called tableaus. By inspecting the bottom row of each tableau, one can immediately tell if it represents the optimal solution. Each table corresponds to a corner point of the feasible solution space. The first tableau corresponds to the origin. Subsequent tableaus are developed by shifting to an adjacent corner point in the direction that yields the highest (smallest) rate of profit (cost). This process continues as long as a positive (negative) rate of profit (cost) exists.

SIMPLEX METHOD

- Steps:

1. Initialization:

Transform all the constraints to equality by introducing slack, surplus, and artificial variables as follows:

Constraint type	Variable to be added
$\leq$	+ slack (s)
$\geq$	- Surplus (s) + artificial (A)
=	+ Artificial (A)

SIMPLEX METHODOLOGY

2. Test for optimality:

Case 1: Maximization problem

the current BF solution is optimal if every coefficient in the objective function row is nonnegative

Case 2: Minimization problem

the current BF solution is optimal if every coefficient in the objective function row is nonpositive

Example-1: COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- Example-1

Solve the following problem using the simplex method

- Maximize

$$Z = 3X_1 + 2X_2$$

Subject to

$$X_1 + X_2 \leq 4$$

$$X_1 - X_2 \leq 4$$

$$X_1, X_2 \geq 0$$

COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- Step-1: First observe whether all the right side constants of the constraints are non-negative. If not , it can be changed into positive value on multiplying both sides of the constraints by -1. In rthis example all 'b_i' (right side constants) are already positive.
- Step-2. Next convert the inequality constraints to equation by introducing the non-negative slack or surplus variable. The coefficients of slack or surplus variables are always taken as zero in the objective function.
- In this problem, all inequality constraints being ≤ type. only slack variables S_1, S_2 are needed. Adding slack variables S_1, S_2 the given problem now becomes
-

Example-1: COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- Maximize $Z = 3X_1 + 2X_2 + 0\,S_1 + 0\,S_2$

Subject to

$$X_1 + X_2 + 0\,S_1 = 4$$

$$X_1 - X_2 + 0\,S_2 = 2$$

and $X_1, X_2, S_1, S_2 \geq 0$

Step-3. Construct the starting simplex table

It should be remembered that the value of non-basic variables are always taken as zero at each iteration. So $X_{1=0}$, $X_{2=0}$.

X_B gives the value of basic variables as indicated in the first column.

Example-1 ITERATION-1 SIMPLEX TABLE-1

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	3	2	0	0	
Basic variable	C_B	X_B	$\rightarrow X_1$	X_2	S_1	S_2	Minimum Ratio=X_B / X_B For $X_k > 0$
S_1	0	4	1	1	1	0	4/1 =4
S_2	0	2	1	-1	0	1	2/1 = 2 min
S_1= 4, S_2= 4, X_1 = 0, X_1 = 0	$Z=\sum C_B X_B$		Δ_1 = -3 Most -ve	Δ_2= - 2	Δ_3 = 0	Δ_4 = 0	

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Entering Variable

$Z= \ 0 * 4 + 0 * 2 = 0,$ $\Delta_j = \sum C_B X_B - C_j$

$\Delta_1 = 0 * 1 + 0 * 1 - 3 = -3$, $\Delta_2 = 0 * 1 + 0 * (-1) - 2 = -2$

$\Delta_3 = 0 * 1 + 0 * 0 - 0 = 0,$ $\Delta_4 = 0 * 0 + 0 * 1 - 0 = 0$

At this stage the solution i: basic variables are S_1 = 4, S_2 = 2; the nonbasic variables are X_1 = 0, X_2 = 0 and Z = 0

Example-1: COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- STEP-5. Now proceed to tse the basic feasible solution for optimality by the rules given below. This is done by computing the 'net evaluation' $\boldsymbol{\Delta_j}$ **for each variable** X_{j} (column vector X_j) by the formula $\boldsymbol{\Delta_J} = Z_j - C_{ij} = \sum \mathbf{C_B X_B} - \mathbf{C_j}$

OPTIMALITY TEST OF SIMPLEX METHOD

1. If all $\Delta_j \geq 0$, the solution under test will be optimal

Alternative optimal solution will exist if any no-basic Δ_j is also zero.

2. If at least one Δ_j is negative, the solution under test is not optimal, then proceed to improve the solution in the next step.

3. If corresponding to to any negative Δ_j , all the elements of the column X_j are negative or zero ($\leq$), the solution under test will be unbounded.

- Decide entering vector(most –ve Δ_j) and leaving vector (ratio is minimum)
- Identify the pivot row, pivot column, pivot element.
- Perform optimality test and repeat same process for second iteration and so on till you get all $\Delta_j \geq 0$,

ITERATION-2 SIMPLEX TABLE-2

Apply the same rules we will obtain this solution

		C_j →	3	2	0	0	
Basic variable	C_B	X_B	X_1	X_2	S_1	S_2	Minimum Ratio$=X_B / X_B$ For $X_k > 0$
S_1	0	2	0	2	1	-1	2/1=1min
x_1	3	2	1	-1	0	1	2/1 = 2 min →
$S_{1}= 2$, $S_{2}= 0$, $X_{1} = 2$, $X_{2} = 0$	$Z=\sum C_B X_B$ $Z = 6$		$\Delta_1 = 0$	$\Delta_{2} = -5$ Most -ve	$\Delta_3 = 0$	$\Delta_4 = 3$	

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Entering Variable

$Z= 0 * 4 + 0 * 2 =0$, $\Delta_j =\sum C_B X_B - C_j$

$\Delta_1 = 0 * 1 + 0 * 1 – 3 = - 3$, $\Delta_2 = 0 * 1 + 0 * (-1) – 2 = -2$

$\Delta_3 = 0 * 1 + 0 * 0 – 0 = 0$, $\Delta_4 = 0 * (-1) + 3 * 1 – 0 = 3$

At this stage the solution i: basic variables are $S_1 = 4$, $S_2 = 2$; the nonbasic variables are $X_1 = 0$, $X_2 = 0$ and $Z = 6$

ITERATION-3 SIMPLEX TABLE-3

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	3	2	0	0	
Basic variable	C_B	X_B	X_1	X_2	S_1	S_2	Minimum Ratio=X_B / X_B For $X_k > 0$
x_2	2	1	0	1	1/2	-1/2	2/1=1min
x_1	3	3	1	0	1/2	1/2	2/1 = 2 min →
$X_1 = 3, X_2 = 1;$ $S_1 = 0, S_2 = 0,$	$Z=\sum C_B X_B$ **Z = 11**	$\Delta_1 = 0$	$\Delta_2 = 0$ Most -ve	$\Delta_3 = 5/2$	$\Delta_4 = 1/2$	36	

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Entering Variable

$Z = 2 * 1 + 3 * 3 = 11$, $\Delta_j = \sum C_B X_B - C_j$

$\Delta_1 = 2 * 1 + 3 * 0 - 3 = 0$, $\Delta_2 = 2 * 1 + 3 * 0 * - 2 = 0$

$\Delta_3 = 2 * 1/2 + 3 * 1/2 - 0 = 5/2$, $\Delta_4 = 2 * (-1/2) + 3 * 1/2 - 0 = 1/2$

Since Δ_j ≥ 0, the optimal solution is attained. **Solution: basic variables are $X_1 = 3$, $X_2 = 2$; the nonbasic variables are $S_1 = 0$, $s_2 = 0$ and $Z = 0$**

COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- Example-2: Solve the following problem using the simplex method

Mini $Z = X_1 - 3X_2 + 3X_3$ Subject to

$3X_1 - X_2 + 3X_3 \leq 7$

$-2X_1 + 4X_2 \leq 12$,

$-4X_1 + 3X_2 + 8X_3 \leq 10$

and $X_1, X_2, X_3 \quad 0$

This is problem of minimization. Converting the objective function from minimization to maximization, we have

$$\text{Min } Z = \text{Max } (-Z) = \max (Z^1) \quad \text{where } -Z = Z^1$$

COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- Step-1: First observe whether all the right side constants of the constraints are non-negative. If not , it can be changed into positive value on multiplying both sides of the constraints by -1. In rthis example all ‘b_i’ (right side constants) are already positive.
- Step-2. Next convert the inequality constraints to equation by introducing the non-negative slack or surplus variable. The coefficients of slack or surplus variables are always taken as zero in the objective function.
- In this problem, all inequality constraints being ≤ type. only slack variables S_1, S_2 are needed. Adding slack variables S_1, S_2 the given problem now becomes
-

Example-2. COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- **Max $Z^1 = -X_1 + 3X_2 - 2X_3$ Subject to**
- **$3X_1 - X_2 + 3X_3 \leq 7$**
- **$-2X_1 + 4X_2 \leq 12$,**
- **$-4X_1 + 3X_2 + 8X_3 \leq 10$ and X_1, X_2, X_3**

adding slack variables S1, S2 and S3

- $Z^1 = -X_1 + 3X_2 - 2X_3 + 0S_1 + 0S_2 + 0S_3$
- Subject to $3X_1 - X_2 + 3X_3 + S_1 = 7$
- $-2X_1 + 4X_2 + S_2 = 12$
- $-4X_1 + 3X_2 + 8X_3 + S_3 = 10$
- X_1, X_2, X_3, **S1, S2 and S3** ≥ 0

OPTIMALITY TEST OF SIMPLEX METHOD

1. If all $\Delta_j \geq 0$, the solution under test will be optimal

Alternative optimal solution will exist if any no-basic Δ_j is also zero.

2. If at least one Δ_j is negative, the solution under test is not optimal, then proceed to improve the solution in the next step.

3. If corresponding to to any negative Δ_j , all the elements of the column X_j are negative or zero ($\leq$), the solution under test will be unbounded.

- Decide entering vector(most –ve Δ_j) and leaving vector (ratio is minimum)
- Identify the pivot row, pivot column, pivot element.
- Perform optimality test and repeat same process for second iteration and so on till you get all $\Delta_j \geq 0$,

example-2 ITERATION-1 SIMPLEX TABLE-1

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	-1	3	-2	0	0		
Basic variable	C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Minimum Ratio= X_B / X_B For $X_k > 0$
S_1	0	7	3	-1	3	1	1/4	0	7/-1 = - →
x_2	0	12	-2	4	0	0	1	0	**12/4=3 min →**
S_3	0	10	-4	3	8	0	0	1	**10/3=3.3**
S_{1}= 4, S_{2}= 4, **X_{1} = 0, X_1 = 0**	**$Z=\sum C_B X_B$** **$Z = 0$**		**Δ_1 = 1**	**Δ_{2}= - 3 Most -ve ↑**	**Δ_{3}= - 2**	**Δ_4 = 0**	**Δ_5 = 0**	**Δ_5= 0**	

example-2 ITERATION-2 SIMPLEX TABLE-1

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	-1	3	-2	0	0		
Basic variable	C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Minimum Ratio=X_B/ X_B For $X_k > 0$
S_1	0	10	5/2	0	3	1	¼	0	**10/(5/2) = 04 min**
X_2	3	3	-1/2	1	0	0	1/4	0	**3/ -= -**
S_3	0	1	-5/2	0	8	0	-3/4	1	**-**
$S_{1=}$ 4, $S_{2=}$ 4, **$X_{1=}$ 0, $X_{1=}$ 0**	**$Z=\sum C_B X_B$ $Z = 0$**		**Δ_1 = -1/2 Most -ve**	**$\Delta_{2=}$ 0**	**$\Delta_{3=}$ 2**	**Δ_4 = 0**	**Δ_5 = 3/4**	**Δ_6= 0**	

example-2 ITERATION-3 SIMPLEX TABLE-3

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	**-1**	**3**	**-2**	**0**	**0**		
Basic variable	C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Minimum Ratio=X_B/ X_B For $X_k > 0$
x_1	-1	4	5/2 1	0	6/5	2/5	¼	0	**10/(5/2) = 04 min**
X_2	3	1	0	1	6/10	1/5	3/8	0	**3/ -= -**
S_3	0	11	0	0	5	1	11/8	1	**-**
$X_{1=}$ 4, $X_{2=}$ 41, **$X_{3=}$ 0, $s_{1=}$ 0, S2 = 0, s3 =11**	**$Z^*=\sum C_B X_B$ Z^{1*} = -1 Z = 1**	**Δ_1 = 0**	**$\Delta_{2=}$ 0**	**$\Delta_{3=}$ 5**	**Δ_4 =** 1/5	**Δ_5 = 7/8**	**Δ_6= 0**		

Iteration

- Step 1: Determine the entering variable by selecting the variable with the most negative in the last row.
- From the initial tableau, in the last row (Z row), the coefficient of X_1 is -3 and the coefficient of X_2 is -5; therefore, the most negative is -5. consequently, X_2 is the entering variable.
- X_2 is surrounded by a box and it is called the pivot column

Example-3 COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

Solve the following problem using the simplex method

Max $Z = 3X_1 + 2X_2 + 5X_3$ Subject to

$X_1 + 2X_2 + X_3 \leq 430$

$3X_1 + 2X_3 \leq 460$

$X_1 + 4X_2 + \quad \leq 420$

and $X_1, X_2, X_3 \geq 0$

COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

- Step-1: First observe whether all the right side constants of the constraints are non-negative. If not , it can be changed into positive value on multiplying both sides of the constraints by -1. In rthis example all ‘b_i’ (right side constants) are already positive.
- Step-2. Next convert the inequality constraints to equation by introducing the non-negative slack or surplus variable. The coefficients of slack or surplus variables are always taken as zero in the objective function.
- In this problem, all inequality constraints being ≤ type. only slack variables S_1, S_2 are needed. Adding slack variables S_1, S_2 the given problem now becomes
-

COMPUTATIONAL PROCEDURE OF SIMPLEX METHOD

adding slack variables S1, S2 and S3

- **Max Z = $3X_1 + 2X_2 + 5X_3 + 0\,S_1 + 0\,S_2 + 0\,S_3$**
- **Subject to**
- $X_1 + 2X_2 + X_3 + S_1 = 430$
- $3X_1 + 2X_3 + S_2 = 460$
- $X_1 + 4X_2 + S_3 = 420$
- **and X_1, X_2, X_3, S1, S2 and S3** ≥ 0

example-3 ITERATION-1 SIMPLEX TABLE-1

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	3	2	5	0	0		
Basic variable	C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Minimum Ratio=X_B/X_B For $X_k > 0$
S_1	0	430	1	2	1	1	0	0	**430/1=430**
S_2	0	460	3	0	2	0	1	0	**460/2=230 min** →
S_3	0	420	1	4	0	0	0	1	**420/0 =---**
S_1 = 430 **S_2 = 460** **S_3 = 420** **X1 =0 X2=0** **X3 = 0**	**$Z=\sum C_B X_B$** **Z = 0**	**Δ_1 = -3**	**Δ_2= - 2**	**Δ_3 = -5 Most -ve** ↑	**Δ_4 = 0**	**Δ_5 = 0**	**Δ_6 = 0**		

example-3 ITERATION-2 SIMPLEX TABLE-2

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	3	2	5	0	0		
Basic variable	C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Minimum Ratio=X_B/ X_B For $X_k > 0$
S_1	0	200	-1/2	2	0	1	1/2	0	200/2 =100 MIN
X_3	5	230	3/2	0	1	0	½	0	---
S_3	0	420	1	4	0	0	0	1	420/4
$X_3 = 230$ $X_1 =$ $X_2 = 0$ $S_2 = 0$, $S_1 = -1/2$, $S_3 = 420$	$Z=\sum C_B X_B$ Z = 0	$\Delta_1 =$ 7/2	$\Delta_2 =$ - 2 Most -ve ↑	$\Delta_3 =$ 0	$\Delta_4 = 0$	$\Delta_5 =$ 5/2	$\Delta_6 = 0$		

example-3 ITERATION-3 SIMPLEX TABLE-3

Apply the same rules we will obtain this solution

		$C_j \rightarrow$	3	2	5	0	0		
Basic variable	C B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Minimum Ratio=X_B/ X_B For $X_k > 0$
S_1	0	200	-1/2	2	0	1	1/4	0	200/2
X_3	5	230	3/2	1	1	0	-1/2	0	460/2min→
S_3	0	420	1	0	0	0	0	1	2/1 = 2 min
X_1 = X_2 = 0 X_3 = 230 X_3 = 230 S1 = 200 S3 = 420	$Z=\sum C_B X_B$ Z = 0	Δ_1 = 9/2	Δ_2= 3	Δ_3 = -5	Δ_4 = 0	Δ_5 = 0	Δ_6 = 0		

A) TWO PHASE SIMPLEX METHOD

ARTIFICIAL VARIABLE MTHOD
A) TWO PHASE SIMPLEX METHOD- problem-1

Auxiliary Table-2

		C_j	- 2	- 1	0	- 1	- 1	
Basic variable	C_B	X_B	X_1	X_2	X_3	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0$
a_1	- 1	5/4	-7/2	-5/4	0	1	1	
X_3	0	1/4	1/2	3/4	1	0	1/4	
	Z^{1*} =	-- 5/4	7/2	5/4	0	0	-1/4	

since all $\Delta_j \geq 0$, an optimum basic feasible solution to the auxiliary LPP has been attained. But at the same time max Z^{1*} is negative and the artificial variable a_1 apperars in the basic solution at a +ve level (ie +5/4). Hence the original problem does not possess any feasible solution. Here there is no need to enter phase-II.

ARTIFICIAL VARIABLE MTHOD
A) TWO PHASE SIMPLEX METHOD

- Used to, solve a problem in which some artificial variables are introduced. The solution is solved in two in two phases.
- Phase-I : In this phase , the simplex method is applied to a specially constructed auxiliary linear programming problem leading to a final simplex table containing a basic feasible solution to the original problem.
-

ARTIFICIAL VARIABLE MTHOD
A) TWO PHASE SIMPLEX METHOD

- Step-1: Assign a cost -1 to each artificial variable and a cost '0' to all other variables (in place of their original cost) in the objective function.
- Step-2. Construct auxiliary linear programming problem in which the new objective function Z^* to be maximum subject to the given set of constraints.
- Step-3. Solve the auxiliary problem by simlex method until either of the following three possibilities do arise.

ARTIFICIAL VARIABLE MTHOD
A) TWO PHASE SIMPLEX METHOD

- Max $Z^* < 0$ and atleast one artificial vector appear in the optimum basis at a +ve level. In this case given problem does not possess any feasible solution.
- Max $Z^* = 0$ and at least one artificial vector appears in the basis at zero level. In this case proceed to phase-II.
- Max $Z^* = 0$ and no artificial vector appears in the optimum basis. In this case also proceed to

Phase-II

ARTIFICIAL VARIABLE MTHOD
A) TWO PHASE SIMPLEX METHOD

- Phase-II: Now assign the actual costs to the variables in the objective function and a a zero cost to every artificial variable that appears in the basis at the zero level. This new objective function is now maximized by simplex method subject to he given constraints. That is simplex method is applied to the modified simplex table obtaoined at the end of phase-I, until an optimum basic feasible solution has been attained (if exists). The artificial variables which are non-basic at the end of phase-I are removed.

Example-1. Solve by TWO PHASE SIMPLEX METHOD

Minimize $Z = X_1 - 2X_2 - 3X_3$ Subject to

$-2X_1 + X_2 + 3X_3 = 2$

$2X_1 + 3X_2 + 4X_3 = 1$ and $x_1, x_2, x_3 >= 0$

Step-1. First convert the objective function into maximization form

minz = max (-z)= Max $Z^1 = -X_1 + 2X_2 + 3X_3$, where $Z^1 = -Z$

Introducing the artificial variables a_1, a_2

The constraints of the given problem become

$-2X_1 + X_2 + 3X_3 + a_1, = 2$

$2X_1 + 3X_2 + 4X_3 + a_2, = 1$

and $x_1, x_2, x_3, a_1, a_2 >= 0$

A) Solve by TWO PHASE SIMPLEX METHOD

- Phase-I: auxiliary LP problem is

  $\text{Max } Z^1 = \quad 0\,X_1 + 0\,X_2 + 0\,X_3\ ,- 1\,a_1 - a_2$

  Subject to the above given

  $-2X_1 + X_2 + 3X_3 + a_1\, , = 2$

  $2X_1 + 3X_2 + 4X_+ \quad a_2\, , = 1$

Then construct the auxiliary simplex table

A) TWO PHASE SIMPLEX METHOD – Example-1

Auxiliary Table-1								
		C_j	- 2	- 1	0	- 1	- 1	
Basic variable	C_B	X_B	X_1	X_2	X_3	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0$
a_1	- 1	2	-2	1	3	1	0	2/3
a_2	- 1	1	2	3	4	0	1	1/4 mini
	$Z^{1*} =$	-- 3	0	-4	-7	0	0	

Example-1. Solve by TWO PHASE SIMPLEX METHOD

Auxiliary simplex table--2								
		$C_j \rightarrow$	0	0	0	- 1	-- 1	
Basic variable	C_B	X_B	X_1	X_2	X_3	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
a_1	- 1	5/4	-7/2	-5/4	0	1	-3/4	2/3 =1
X_2	0	¼	½	¾	1	0	1/4	¼ mini
	Z^{1*} = -5/4		7/4	5/4	0 ↑	0	3/4	

since all $\Delta_j \geq 0$, an optimum basic feasible solution to the auxiliary LPP has been attained. But at the same time max Z^{1*} is negative and the artificial variable a_1 appears in the basic solution at a +ve level (ie +5/4). Hence the original problem does not possess any feasible solution. Here there is no need to enter phase-II.

Example-2. Solve by TWO PHASE SIMPLEX METHOD

- Minimize $Z = 15/2\ X_1 - 3X_2$ subject to the constraints.

$$3X_1 - X_2 - X_3 \geq 3$$

$$X_1 - X_2 \geq 2 \quad \text{and } x_1, x_2, x_3 >= 0$$

- Step-1. First convert the objective function into maximization form

Max $Z^1 = -15/2\ X_1 + 3X_3$, where $Z^1 = -Z$

Introducing the surplus variables $X_4 \geq 0$, and $X_5 \geq 0$

Introducing the artificial variables $a_1 \geq 0$, $a_2 \geq 0$

The constraints of the given problem become

$$3X_1 - X_2 - X_3 - X_4 + a_1 = 3$$

$$X_1 - X_2 - X_5 + a_2 = 2$$

and $x_1, x_2, x_3, a_1, a_2 \geq 0$

Example-2. Phase-I simplex table-1

Phase-I: in first phase consider zero costs associated with original variables and -1 as coefficients of artificial variables a1 and a2., the objective function thus becomes

Max $Z1^* = -o\,X_1 + 0\,X_2 + 0\,X_3 + 0\,X_4 + 0\,X_5$ and apply simplex

Auxiliary simplex table--1

		$C_j \rightarrow$	0	0	0	0	0	- 1	-- 1	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	X_5	A1	A2	Mini Ratio X_B/X_K for $X_K > 0_0$
a_1	- 1	3	3	-1	-1	-1	0	1	0	3/3 =1 mini →
a_2	- 1	2	1	-1	1	0	-1	0	1	2/1=2
	Z^{1*} =	- 5	-4 min ↑	2	0	1	1	0	0	

Since all Δ_2 is most negative, ≥ 0, X_1 will enter into basis

Example-2. Phase-I Auxiliary simplex table-2

Phase-I: in first phase consider zero costs associated with original variables and -1 as coefficients of artificial variables a1 and a2., the objective function thus becomes

Max $Z1^* = - o X_1 + 0 X_2 + 0 X_3 + 0 X_4 + 0 X_5$ and apply simplex

Auxiliary simplex table--2										
	$C_j \rightarrow$		0	0	0	0	0	- 1	-- 1	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	X_5	A1	A2	Mini RatioX_B/X_K for $X_K > 0_0$
X_1	0	1	1	-1/3	-1/3	-1/3	0	1/3	0	--
a_2	- 1	1	0	0	4/3	1/3	-1	1/3	1	¾ mini
	Z^{1*} =	- 1	0	0	- 4/3 min	-1/3	1	2/3	0	

Since all Δ_3 - 4/3 is most negative, X_3 will enter into basis

Example-2. Phase-I Auxiliary simplex table-2

Auxiliary simplex table--3										
		$C_j \rightarrow$	0	0	0	0	0	- 1	-- 1	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	X_5	A1	A2	Mini RatioX_B/X_K for $X_K > 0_0$
X_1	0	5/4	1	-1/2	0	-1/4	-1/4	¼	1/4	--
X_3	0	¾	0	-1/2	1	-1/4	-3/4	¼	3/4	
	Z^{1*} = -	0	0	0	0	1	2	1	1	

Since all Δ_j >=0 , and no artificial variable appears in the basis, an optimal solution to the auxiliary problem has been attained. No we can proceed to phase-ii.

Example-2. Phase-II simplex table-3

Phase-II: in second phase consider the actual costs associated with original variables, the objective function thus becomes

Max Z1* = -15/2 X1 + 3 X2 + 0 X3 + 0 X4 + 0 X45 and apply simplex

Phase-II. simplex table--1								
		$C_j \rightarrow$	-15/2	3	0	0	0	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	X_4	Mini RatioX_B/X_K for $X_K > 0_0$
X_1	-15/4	5/4	1	-1/2	0	-1/4	-1/4	--
X_3	0	¾	0	-1/2	1	-1/4	-3/4	
	Z^{1*} = - 75/8		0	¾	0	15/8	15/8	

Since all $\Delta_j \geq 0$, an optimum basic feasible solution has been attained.

Hence optimum solution is $X_{1=}$ 5/4, $X_2 = 0$, $X_3 = 3/4$

Minimum Z = 75/8

ARTIFICIAL VARIABLE MTHOD
A) BIG –M METHOD (Charne's Penalty Method)

- The Computational Steps of Big-M- Method are as follows:
- Step-1: express the problem in the standard form.
- Step-2: Add non-negative artificial variables to the LHS of each of the equations of type **(≥) and '='**
- When artificial variables are added, it causes violation of corresponding constraints.
- In this assigning a very large price **(per unit penalty -M)** to these artificial variables in the objective function.

-M for Maximization problem, +M for minimization .

Spep-3: use the artificial variables for starting the solution and proceed with usual simplex routine until optimal solution is obtained.

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

$$\text{Max } Z = -2x_1 - X_2 \quad \text{subjective to}$$

$$3X_1 + x_2 = 3$$

$$4X_1 + 3x_2 \geq 6$$

$$X_1 + 2x_2 \leq 4 \text{ and } x_1, x_2 \geq 0$$

Step-1. Introducing slack variables and artificial variables, the system of constraint equation becomes

$3X_1 + X_2 \quad + a_1 \quad = 3$ → adding AV a_1

$4X_1 + 3X_2 - X_3 \quad + a_2 \quad = 6$ → adding AV a_1

$X_1 + 2X_2 + \quad X_4 = 4$ → adding Slack variable X_4

and $x_1, x_1, x_{2,}\ a_1, a_2 \geq =0$

Step-2. Assigning large negative price -M to the artificial variables a_1, a_2, the objective function becomes

- $\text{Max } Z = -2x_1 - X_2 + 0X_3 + 0X_4 - MA_1 - MA_2$

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Step-3: Construct the Starting simplex table-1 as follows:

Simplex table-2							Iteration-2		
		C_j	-2	-1	0	0	- M	-M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
a_1	- 2	1	1	1/3	0	1	1	0	1/(1/3)= 3
a_2	-M	2	0	5/3	1	0	0	1	2/(5/3)= 1.2 mini
X_4	0	3	0	5/3	0	0	0	0	4/1 = 4
	Z = -2-2M		0 min	(1-5M)/3	M	0	(-2+7M)/3	0	

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Step-3: Construct the iteraration-3, simplex table-3 as follows:

Simplex table-3							Iteration-3		
		C_j	-2	-1	0	0	- M	-M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
X_1	- 2	3/5	1	0	1/5	0	3/5	- 1/5	
X_2	- 1	26/5	0	1	-3/5	0	- 4/5	3/5	
X_4	0	1	0	0	1	1	1	- 1	
	Z = - 12/5		0	0	1/5	0	M- 2/5	M-1/5	

Since M is as large as possible, all $\Delta_j \geq 0$, an optimum solution to LPP has been attained. Solution is $X_1 = 3/5$, $X_2 = 6/5$, Max Z = - 12/5

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Step-3: Construct the Starting simplex table-1 as follows:

Simplex table-3									Iteration-3
		C_j	-2	-1	0	0	- M	-M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
X_1	- 2	3/5	1	0	1/5	0	3/5	- 1/5	1/(1/3)= 3
X_2	- 1	26/5	0	1	-3/5	0	- 4/5	3/5	2/(5/3)= 1.2 mini
X_4	0	1	0	0	1	1	1	- 1	4/1 = 4
	Z = - 12/5		0	0	1/5	0	M- 2/5	M-1/5	

Since M is as large as possible, all $\Delta_j \geq 0$, an optimum solution to LPP has been attained. Solution is X_1= 3/5, X_2= 6/5, Max Z = - 12/5

Example-2. Solve LPP with BIG –M Method (Charne's Penalty Method)

Max Z = $X_1 + 2X_2 + 3X_3 - X_4$

subject to

$X_1 + 2X_2 + 3X_3 = 15$

$2X_1 + X_2 + 5X_3 = 20$

$X_1 + 2X_2 + X_3 + X_4 = 10$ and $x_1, x_2, x_{3,} X_4 \geq 0$.

Since the constraints of the given problem are equations, introduce the artificial variables $a_1, a_2 \geq 0$

Max Z = $X_1 + 2X_2 + 3X_3 - X_4 - Ma_1 - Ma_2$

subject to

$X_1 + 2X_2 + 3X_3 + a_1 = 15$

$2X_1 + X_2 + 5X_3 + a_2 = 20$

$X_1 + 2X_2 + X_3 + X_4 = 10$ and $x_1, x_2, x_{3,} X_4 \geq 0$.

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Simplex Table-1						Iteration - 1			
	$C_j \rightarrow$		1	2	3	- 1	-M	- M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0$
a_1	**-M**	**15**	**1**	**2**	**3**	**0**	**1**	**0**	15/3
a_2	**-M**	**20**	**2**	**1**	**5**	**0**	**0**	**1**	20/5 =4 mini →
X_4	**-1**	**10**	**1**	**2**	**1**	**1**	**0**	**0**	10/1 = 10
	Z= (35M-10)		**(-3M-2)**	**(-3M-2)**	**(-8M-4)** ↑	**0**	**0**	**0**	

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Simplex Table-2 Iteration-2

	$C_j \rightarrow$		1	2	3	- 1	- M	- M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
a_1	-M	3	-1/5	7/5	0	0	1	X	3/(7/5)=15/7 →
X_3	3	4	2/5	1/5	1	0	0	X	= 20
X_4	-1	6	3/5	9/5	0	1	0	X	= 30/9
	Z= (-3M+6)		(M-2)/5	(-7M+16)/5	0	0	0	X	

Most – ve

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Simplex Table-3 Iteration-3

		$C_j \rightarrow$	1	2	3	- 1	- M	- M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
X_1	2	15/7	-1/7	1	0	0	X	X	-
X_3	3	25/7	3/7	0	1	0	X	X	(25/7)/3/7= 25/3
X_4	-1	15/7	6/7	0	0	1	X	X	15/6 $\longrightarrow$
	Z= 90/7		- 6/7	0	0	0	X	X	

Example-1. Solve LPP with BIG –M Method (Charne's Penalty Method)

Simplex Table-4 **Iteration-4**

		$C_j \rightarrow$	1	2	3	- 1	- M	- M	
Basic variable	C_B	X_B	X_1	X_2	X_3	X_4	A_1	A_2	Mini Ratio X_B/X_K for $X_K > 0_0$
X_2	2	5/2	0	1	0	1/6	X	X	-
X_3	3	5/2	0	0	1	1/2	X	X	(25/7)/3/7= 25/3
X_4	-1	5/2	1	0	0	7/6	X	X	15/6 $\longrightarrow$
	Z= 15		0 $\uparrow$	0	0	75/36	X	X	

. **Since all $\Delta_j \geq 0$, an optimum basic feasible solution to LPP has been attained. Solution is $X_1=X_2= X_3= 5/2$, Max Z = 15**

UNIT-II
TRANSPORTATION PROBLEM

- Transportation Modeling
- Finding an Initial Basic Feasible Solution
  - The Northwest-Corner Rule
  - The Lowest-Cost Method
  - Vogel's Approximation Method
- Finding Optimal Solution
- The Stepping-Stone Method
- MODI (Modified Distribution Method
- Special Issues in Modeling
  - Demand Not Equal to Supply (Unbalnced TP problem
  - Degeneracy

Transportation Modeling

- An interactive procedure that finds the least costly means of moving products from a series of sources to a series of destinations

- Can be used to help resolve distribution and location decisions

Transportation Modeling

- It is a special class of linear programming
- Need to know
  1. The origin points and the capacity or supply per period at each
  2. The destination points and the demand per period at each
  3. The cost of shipping one unit from each origin to each destination

TRANSPORTATION EXAMPLE

- The following example was used to demonstrate the formulation of the transportation model.
- **Bath tubes are produced in three Factories namely Factory-D, Factory-E and Factory-F. The product is shipped to the distributers namely WH-A, WH-B and WH-C in different cities in railroad. Each Factory is able to supply the following number of units , Factory-D 100 units, Factory-E 300 units and Factory-F 300 units .**
- **The distribution requires the following unites ; WH-A 300 units , WH-B 200 units and WH-C 200 units**
- The transportation cost matrix is shown in below table.

SHIPPING COST PER UNIT

From \ To	WH-A	WH-B	WH-C
Factory-D	5	4	3
Factory-E	8	4	3
Factory-F	9	7	5

TRANSPORTATION PROBLEM

Transportation Problem

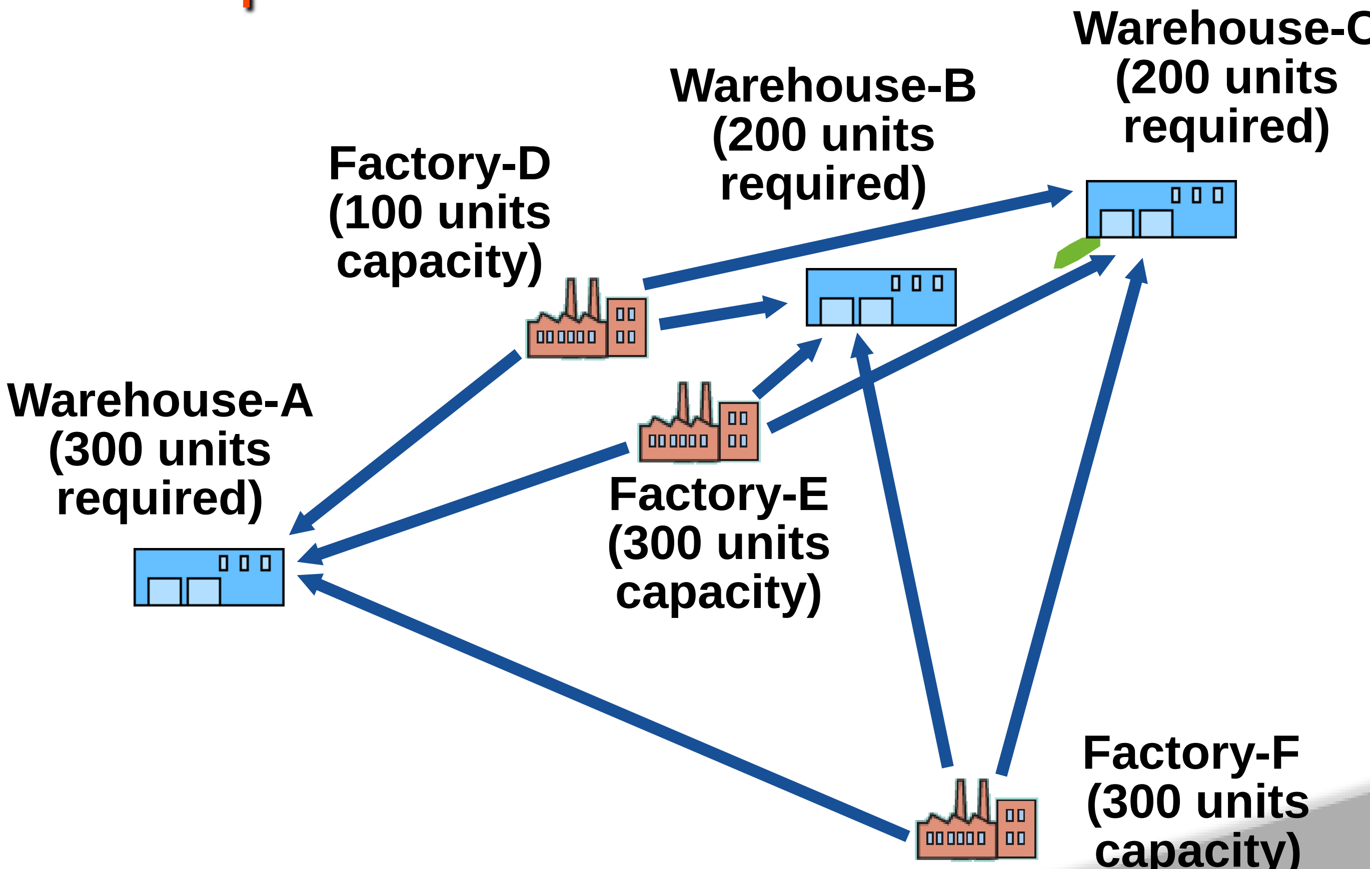


Figure C.1

TRANSPORTATION MATRIX

From \ To	WH-A	WH-B	WH-C	Factory capacity
FACTORY-D	5	4	3	100
FACTORY-E	8	4	3	300
FACTOFY-C	9	7	5	300
Warehouse requirement	300	200	200	700

Factory-A capacity constraint

Cell representing a possible source-to-destination shipping assignment (factory-B to WH-2)

Cost of shipping 1 unit from Factory-C to warehouse-2

warehouse-3 demand

Total demand and total supply

STEP BY STEP PROCEDURE FO SOLVING TRANSPORTATION P PROBLEM

STEP-2: Establish the Initial Basic Feasible Solution using any one of the following methods

i. The Northwest corner rule
ii. Least Cost Method
iii. Vogel's Approximation Model

STEP-3. Testing the Initial Basic Feasible solution for optimality and find Optimal Solution using MODI method or Stepping Stone method

STEP BY STEP PROCEDURE FO SOLVING TRANSPORTATION P PROBLEM

STEP-1: Understand the given problem and formulate in Standard form.

STEP-2: Check the equality of the total supply & demand (if not, we need to balance it creating dummy row or dsummy column according to the problem. assign Zero transportation cost for the dummy cells

- how many ship of unit should be transported from each demand to each distribution center can be expressed as (X_{ij})
- i ……..for Factory and j …….for distribution center

STEP BY STEP PROCEDURE FO SOLVING TRANSPORTATION PROBLEM

STEP-3: Establish the Initial Basic Feasible Solution using any one of the following methods

i. The Northwest corner rule
ii. Least Cost Method
iii. Vogel's Approximation Model

STEP-4. Testing the Initial Basic Feasible solution for optimality and find Optimal Solution using MODI method or Stepping Stone method

STEP-5. Finding Optimal solution and Optimal transportation cost.

i. Northwest-Corner Rule for finding Initial Basic Feasible Solution.

- **Start in the upper left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows:**
  1. **Exhaust the supply (factory capacity) of each row before moving down to the next row**
  2. **Exhaust the (warehouse) requirements of each column before moving to the next column**
  3. **Check to ensure that all supplies and demands are met**

NORTH WEST –CORNER RULE

- The steps of the northwest corner method are summarized here:
- 1. Allocate as much as possible to the cell in the upper left-hand corner, subject to the supply and demand constraints.
- 2. Allocate as much as possible to the next adjacent feasible cell.
- 3. Repeat step 1,2 until all units all supply and demand have been satisfied

Northwest-Corner Rule

INITIAL BASIC FEASIBLE SOLUTION USING NORTHWEST-CORNER RULE

To / From	WH-A	WH-B	WH-C	Factory capacity
FACTORY-D	र 5 100	र 4	र 3	100
FACTORY-E	र 8 100	र 4 100	र3	300
FACTORY-F	र 9	र7 100	र 5 200	300
Warehouse requirement	300	200	200	700

Means that the Factory-F is shipping 100 bathtubs from Factory-F to WH-B

INITIAL BASIC FEASIBLE SOLUTION TABLE USING NORTHWEST-CORNER RULE

From \ To	WH-A	WH-B	WH-C	Factory capacity
FACTORY-D	100 ₹ 5	₹ 4	₹3	100
FACTORY-E	200 ₹8	100 ₹4	₹3	300
FACTORY-F	₹9	100 ₹ 7	200 ₹ 5	300
Warehouse requirement	300	200	200	700

INITIAL BASIC FEASIBLE SOLUTION:

(TRANSPORT) FORM FACORY-D TO WAREHOUE-- A 100 UNITS

ASSIGN (TRANSPORT) FORM FACORY-E TO WAREHOUE--A 200 UNITS

ASSIGN (TRANSPORT) FORM FACORY-E TO WAREHOUE--B 100 UNITS

TRANSPORT FORM FACORY-F TO WAREHOUE-B 100 UNITS UNITS

TRANSPORT FORM FACORY-F TO WAREHOUE-C 200 UNITS UNITS

TOTAL TRANSPORT COST = 100*5 + 200 * 8 + 100*4 + 100 * 7 + 200 * 5 = Rs 4200

INITIAL BASIC FEASIBLE SOLUTION USING NORTHWEST-CORNER RULE

Computed Shipping Cost

	Route				
	From	To	Qty.Shipped	Cost per Unit	Total Cost
Transport	D	A	100 units	Rs 5	Rs 500
Transport	E	A	200 units	8	1,600
Transport	E	B	100 units	4	400
Transport	F	B	100 units	7	700
transport	F	C	200 units	5	1,000
					Total: Rs 4,200

Table C.2

This is a Initial feasible solution but not necessarily the lowest cost alternative (not the optimal solution)

ii. SECOND METHOD OF IBFS
LOWEST-COST METHOD

1. **Evaluate the transportation cost and select the cell with the lowest cost (in case a Tie make an arbitrary selection).**
2. **Depending upon the supply & demand condition , allocate the maximum possible units (min of capacity, demand)to lowest cost cell.**
3. **Delete the satisfied allocated row or the column (or both).**
4. **Repeat steps 1 and 3 until all units have been allocated.**

THE LOWEST- COST METHOD

To / From	WH- A	WH-B	WH-C	Factory capacity
FACTORY-D	र 5	र4	□3	100
FACTORY-E	□ 8	□	□ 3 100	300
FACTORY-F	□ 9	□ 7	□ 5	300
Warehouse requirement	300	200	200	700

STEP-1: **First, □ 3 is the lowest cost cell ie cell(D,A). so ship 100 units from Factory-D to Warehouse –C and cross off the first row as Factory-D is is satisfied**

THE LOWEST-COST METHOD …Contd.

To From	WH- A	WH - B	WH - C	Factory capacity
FACTORY- D	□ 5	□ 4	□ 3 100	100
FACTORY - E	□8	□4	□3 100	300
FACTORY - F	□9	□ 7	□5	300
Warehouse requirement	300	200	200	700

STEP-2: **Again search next lowest cost cell, □ 3 is again the lowest cost pertaining to cell (E,C). So ship 100 units from FACTOTY- E to WH-C and cross off column C, as WHARE-HOUSE-C is satisfied**

The Lowest-Cost Method

L-5 L-2

From \ To	WH -- A	WH- B	WH - C	Factory capacity
FACTORY- D	$5	$4	$3 100	100
FACTORY - E	$8	$4 200	$3 100	300
FACTORY- F	$9	$7	$5	300
Warehouse requirement	300	200	200	700

L-1

L-3

STEP-3: **Again search next lowest cost cell, □4 is the lowest cost cell , pertains to cell(EB), so ship 200 units from FACTORY-E to WH-B and cross off column B and row E as FACTORY-E and WH-B are satisfied**

Figure C.4

The Lowest-Cost Method

L-6 L-3 L-2

To / From	WH-A	WH-B	WH-C	Factory capacity
FACTORY-D	5	4	100 3	100
FACTORY-E	8	200 $4	100 $3	300
FACTORY-F	300 9	7	5	300
Warehouse requirement	300	200	200	700

L-1

L-4

L-5

STEP-4: **Finally, select left out cell(F,A) and ship 300 units from FACTORY-F to WAREHOUSE-A as this is the only remaining cell to complete the allocations**

Figure C.4

The Lowest-Cost Method

To / From	WH-A	WH-B	WH-C	Factory capacity
FACTORY -D	□ 5	□ 4	3 □ 100	100
FACTORY –E	□ 8	□ 4 200	□ 3 100	300
FACTORY -F	□ 9 300	□ 7	□ 5	300
Warehouse requirement	300	200	200	700

initial basic feasible solution:

Transport form facory-D to warehoue-- A 100 units @ cost of rs 5/-
Transport form facory-E to warehoue--A 200 units @ cost of rs 8/-
Transport) form facory-E to warehoue-B 100 units @ cost of rs 4/-

Transport form facory-F to warehoue-B 100 units units @ cost rs 7/-
Transport form facory-F to warehoue-C 200 units units @ cost rs 5/-

Total Cost = □ 3(100) + □ 4(200) + □ 3(100) + □ 9 (300)
= □ Rs4,100

Initial Basic Feasible Solution using Lowest-Cost Method

Computed Shipping Cost

Route				
From	To	Tubs Shipped	Cost per Unit	Total Cost
D	C	100	□ 3	₹ 300
E	B	200	4	800
E	C	100	3	300
F	A	300	9	2700
			Total: □	₹ 4,100

Table C.2

This is a feasible solution but not necessarily the lowest cost alternative
(not the optimal solution)

Third Method of IBFS
Vogel's Approximation Method (VAM)

1. Calculate penalties for each row and column by taking the difference between the smallest and next smallest unit transportation cost .
2. observe the penalty computed row and colum , select the cell with lagest penalty and allocate maximum possible quanty. min of capacity, demand).
3. Adjust the supply and demand and cross out the satisfied row or column (or both).
4. Repeat steps 1 and 3 until all units have been allocated.

Vogel's Approximation Method

L-2

From \ To	WH- A	WH-B	WH-C	Factory capacity
FACTORY-D	100 (5)	4	3	100
FACTORY-E	8	200 (4)	100 (3)	300
FACTORY-F	200 (9)	7	100 (5)	300
Warehouse requirement	300 200 ,0	200	200	700

	P-1	P-2	P-3	
FACTORY-D	1			L-1
FACTORY-E	1	1	1	L-3
FACTORY-F	2	2	2	2 L-4

	WH-A	WH-B	WH-C
	3 max	0	0
	1	3 MAX	2
	1		2MAX
	9MAX	L-2	5

D – A =100
E- B = 200
E-C = 100
F-A = 200
F-C =100

Figure C.4

TOTAL COST = 100*5 + 200*4 + 100*3 + 200* 9 + 100* 5 = Rs 3900

Initial Basic Feasible Solution using Vogel's Approximation Method

Computed Shipping Cost

Route				
From	To	Tubs Shipped	Cost per Unit	Total Cost
D	A	100	5	Rs 500
E	B	200	4	800
E	C	100	3	300
F	A	200	9	1800
F	C	100	5	500

Total: 3900

Testing the initial feasible solution

- Test for optimality
- Optimal solution is achieved when there is no other alternative solution give lower cost.
- Two method for the optimal solution :-
- Stepping stone method
- Modified Distribution method (MODI)

MODI Method for optimal Solution

- Consider earlier TP problem and find the initial Basic feasible solution using Vogel's Approximation Method

USING VOGEL'S APPROXIMATION METHOD TO FIND IBFS

From \ To	WH- A	WH-B	WH-C	Factory capacity
FACTORY-D	100 (cost 5)	(cost 4)	(cost 3)	100
FACTORY-E	(cost 8)	200 (cost 4)	100 (cost 4) 3	300
FACTORY-F	200 (cost 9)	(cost 7)	100 (cost 5)	300
Warehouse requirement	300 ,, 200 ,0	200	200	700

	P-1	P-2	P-3	
FACTORY-D	1			L-1
FACTORY-E	1	1	1	L-3
FACTORY-F	2	2	2	L-4 2

	WH- A	WH-B	WH-C
	3 max	0	0
	1	3 MAX	2
	1		2MAX
	9MAX	L-2	5

L-2

D – A =100
E- B = 200
E-C = 100
F-A = 200
F-C =100

TOTAL COST = 100*5 + 200*4 + 100*3 + 200* 9 + 100* 5 = Rs 3900

Figure C.4

From \ To	WH--A	WH-B	WH-C	Factory capacity	
Factory-D	100 (5)	4	3	100	$U_{1} = -2$
Factory-E	8	200 (4)	100 (3)	300	$U_{2} = 0$
Factory-F	200 (9)	$7	100 ($5)	300	$U_{3} = 2$
Warehouse requirement	300	200	200	700	
	$V_{1} = 7$	$V_{2} = 4\ 8$	$V_{3} = 3$		

Total Cost = 5(100) + 4(200) + 3(100) + 9(200) + 5(100)

= Rs 39,000 is this the optimal solution

No. ocupied cells = m + n - 1 = 3+3-1 = 5

The actual cells = 5

Since no. of acupied cells = no. of actual cells, the problem is said to be non-degenerate. Thus Optimal Solution can be attained using MODI method

Figure C.8

Find Optimal Solution using MODI Method

1. **Check IBFS for non-denerate**

no of actulas cells = (m +n-1) = 5 satisfies

2. To calculate set of U_i, V_j values consider row or column whoch has more no of ocupied cells (Ui /Vj) as zero.

3. For ocupied cells $C_{ij} = U_i + V_j$

considering U2 = 0 , $U_2+V_3 = C_{23}$, U2 + V3 = 3, 0+ V3 = 3-→ V3= 3

$U_2 + V_2 = C_{22}$, 0 + V_2 = 4 -→ V2 = 4

Cons$_{idering}$ C33 = U3+V3 5 = U3 + 3→ U3 = 2

Considering C31 = U3 +V1 , 9= 2 + v1 -→ V1 = 7

Considerinf C11 = U1 +V1 , 5 = U1 + 7 -→ U1 = -2

.

3.

Computation of Cell Evaluation for un-ocupied cells dij = $C_{ij} – u_i - v_j$

To / From	WH-A	WH-B	WH-C	Factory capacity	
Factory-D	100 (5)	(4)	(3)	100	$U_{1} = -2$
Factory-E	(8)	200 (4)	100 (3)	300	$U_{2} = 0$
Factory-F	200 (9)	($7)	100 ($5)	300	$U_{3} = 2$
Warehouse requirement	300	200	200	700	
	$V_{1} = 7$	$V_{2} = 4$	$V_{3} = 3$		

d_{12} = C22 – U2 –v2 , = 4 – (-2 + 4) = 2

d_{23} = C23 – U2 –V3 = 3 – 0 – 3 = 0

d_{21} = C21 – U2 – V1 = 8 – 0 -7 = 1

d_{32} = C32 – U3 – V2 = 7 – 2 – 4 = 1

Since all d_{ij} are >=0, the optimal solution has been achieved

OPTIMAL SOLUTION TABLE

From \ To	WH-A	(WH-B	WH-C	Factory capacity
Factory-D	5 100	4 +2	3 +2	100
Factory-E	8 +1	4 200	3 100	300
Factory-F	9 200	$7 +1	$5 100	300
Warehouse requirement	300	200	200	700

Transport from Factory –D to WH-A = 100 UNITS
Transport from Factory-E to WH-B – 200 units
Transport from Factory –E to WH-C = 100 units
Transport from Factory-F to WH-A = 200 units
Transport from Factory-F to WH-C = 100 units

Since all d_{ij} are >
solution has bee

Total transportation cost = 100*5 + 200*4 + 100*3 + 200*9 + 100* 5
= 500+800+300+1800+500 = Rs 3900

TRANSPORTATION MODEL Example-2

- The has three production facilities S1, S2 and S3 with Production Capacity0f 7, 9 and 18 units (in ooo's) per week f a product, respectively. These units are to be shipped to four warehouses D1, D2, D3 and D4 with requirement of 5,6, 7 and 14 units (in 000^{s} per week , respectively. The transportation costs (in rupees) per unit between factories to ware houses are given in the table below.

SHIPPING COST PER UNIT

To / From	D1	D2	D3	D4
S1	19	30	50	10
S2	70	30	40	60
S3	40	8	70	20

Transportation Problem

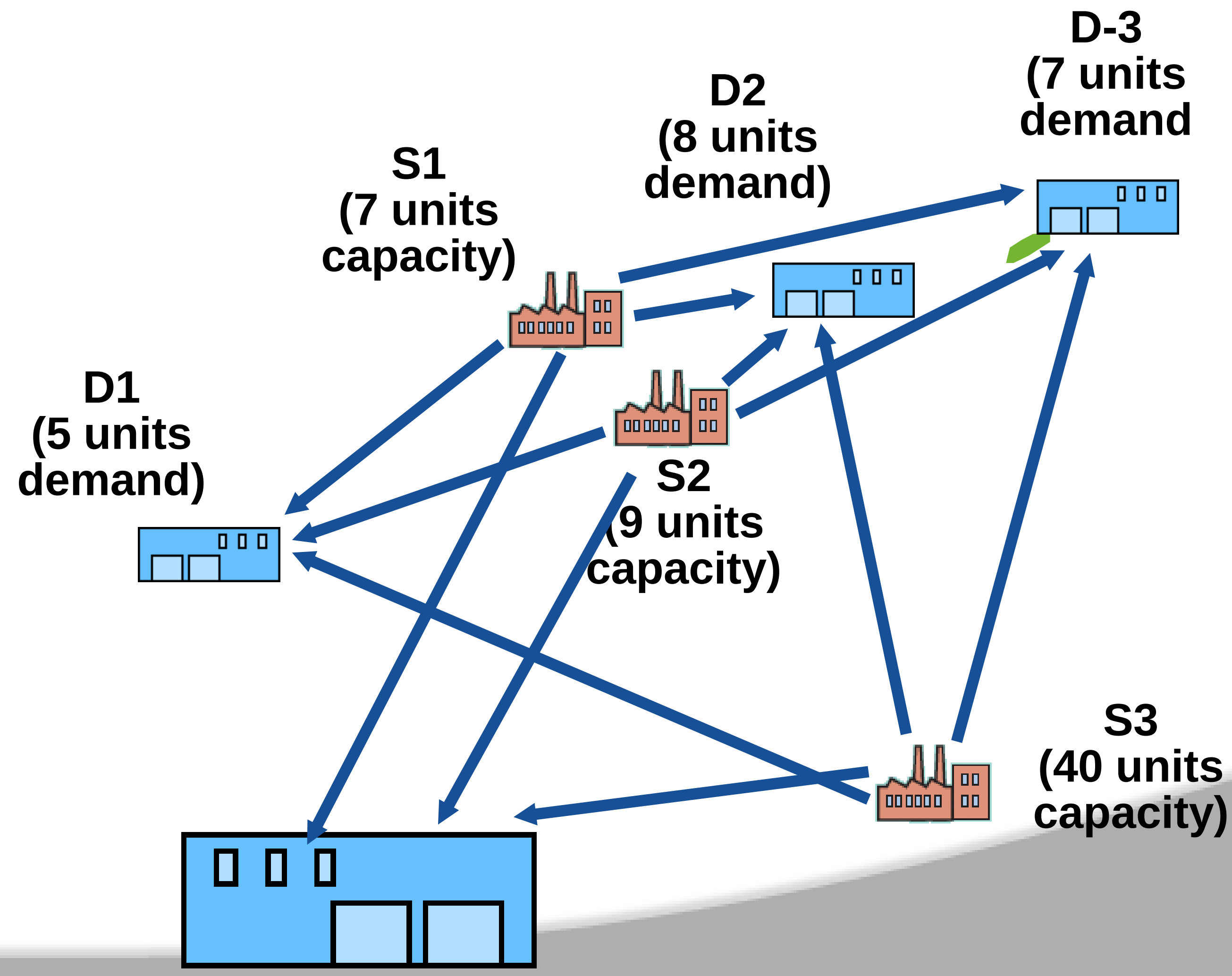


Figure C.1

INITIAL TRANSPORTATION TABLE

	D1	D2	D3	D4	CAPACITY
S1	19र	30र	50र	10र	7
S2	70र	30र	40र	60र	9
S3	र40	8र	70र	20र	18
DEMAND	5	8	7	14	34

IBFS- using North West Corner Method

L-1 L-2 L-3 L-4 L-5

	D1	D2	D3	D4	CAPACITY
s1	₹19 5	30₹ 2	50₹	10	7,2
S2	₹70	30₹ 6	₹40 3	60₹	,3,9
s3	40₹	8₹	70₹ 4	20₹ 14	18
	5	,,6,8	,4,7	14	34

IBFSOLUTION: TRANSPORT FROM S1 TO D1 5 UNITS
TRANSPORT FROM S1 TO D2 2 UNITS
TRANSPORT FROM S2 TO D2 6 UNITS
TRANSPORT FROM S2 TO D3 3 UNITS
TRANSPORT FROM S3 TO D3 4 UNITS
TRANSPORT FROM S3 TI D4 14 UNITS

TITAL TRANSPORTATION COST = 5 *19 + 2 * 30 + 6 * 30 + 3 * 40 + 4* 70+ 14*20=
= 95 + 60 + 180 + 120 + 280 + 280 = Rs 1015/-

Least
8,10,20,40,40

IBFS- using Least Cost Method

L-1 L-4 L-3

L-2

L-5

	D1	D2	D3	D4	CAPACITY	
			7			7,0
s1	19₹	₹30	7 ₹50	10₹ 7	7	9,2
S2	70₹ 3	30₹ 8	40₹ 7 7	60₹	9	18,10,3
s3	40₹ 3 5,2	8₹ 8	₹70 7,0	20₹ 14,7,0 7	,10,18	
	5	8	7	14	34	

IBFSOLUTION: TRANSPORT FROM S1 TO D1 5 UNITS
TRANSPORT FROM S1 TO D2 2 UNITS
TRANSPORT FROM S2 TO D2 6 UNITS
TRANSPORT FROM S2 TO D3 3 UNITS
TRANSPORT FROM S3 TO D3 4 UNITS
TRANSPORT FROM S3 TI D4 14 UNITS

ToTAL TRANSPORTATION COST = 5 *19 + 2 * 30 + 6 * 30 + 3 * 40 + 4* 70+ 14*20=Rs 1015

IBFS- using Vogel's approximation

L-2 L-1 L-3

	D1	D2	D3	D4	CAPACITY	PENALTY P-1	PENALTY -P2	P3	P4	
S1	19₹ 5	30₹	50₹	10₹ 2	7,2,0 7	9	9	9	10	10
S2	70₹	30₹	40₹ 7	60₹ 2	9,2,0 9	10	20	20 MAX	MAX 60	
S3	3 40₹	8₹ 8	₹70	20₹ 10	18	12	12	20	20	20
	5	8	7	14	34					
P1	21	MAX 22	14,12,2 10	10						
P2	MAX21		10	10						
P3				10						

IBFS- using Vogel's approximation

L-2 L-1 L-3

	D1	D2	D3	D4	CAPACITY
s1	19₹ 5	30₹	50₹	10₹ 2	7
S2	70₹	30₹	40₹ 7	60₹ 2	9
s3	40₹ 3	8₹ 8	₹70 10	20₹ 10	18
	5	8	7	14	34

S1-D1 = 5 UNITS
S1-D4 = 2 UNITS
S2-D3 = 7 UNITS
S2-D4 = 2 UNITS
S3- D2 = 8 UNITS
S3 –D4= 10 UNITS

TOTAL TRANSPORT COST = 5*19 + 2 * 10 + 7*40 + 2 * 60 + 8* 8+ 10* 20 = Rs779

APPLYING OPTIMALITY TEST (MODI -METHOD)

	D1	D2	D3	D4	CAPACITY	U_i
s1	19₹ 5	30₹	50₹	10₹ 2	7	$U_1 =$ 10
S2	70₹	30₹ add	40₹ 7	60₹ 2 sub	9	U_2 =60
s3	40₹ 3	8₹ 8 sub	₹70	20₹ add 10	18	$U_3 =$ 20
DEMAND	5	8	7	14	34	
V_j	V_1 = 9	V_2 = -12	V_3 - 20	$V_4 =$ 0		

$d_{12} =$ C12 – U1 –v2 , = 19 -10 - (-12)= 21

$d_{13} =$ C13 – U1 –v3 , = 50 – 10 – (-20) = 60 $_{d23}$ = C23 – U2 –V3 = 3 – 0 – 3 =

d_{21} = C21 – U2 – V1 = 70 – 60 – 9 = 1

d_{22} = C22 – U2 – V2 = 30 – 60 – (-12) = - 18

$d_{31} =$ C31 – U3 – V1 = 40 – 20 – 9 = 11

$d_{33} =$ C33 – U3 – V3 = 70 – 20 – (-20) = 70

Since D_{22} < 0 , ie = -18 , solution is not optimal, form loop starting from most – ve cell is cell (2, 2)

APPLYING OPTIMALITY TEST (MODI -METHOD) Revised Table , test for optimality computing net evaluation of unoccupied cell

	D1	D2	D3	D4	CAPACITY	U_i
S1	19₹ 5	30₹	50₹	10₹ 2	7	U_1 =10
S2	70₹	30₹ 2	40₹ 7	60₹	9	U_2 = 42
S3	40₹ 3	8₹ 6 sub	₹70	20₹ 12	18	U_3 =20
DEMAND	5	8	7	14	34	
V_j	V_1 =9	V_2 = - 12	V_3 =- 2	V_4 = 0		

d_{12} = C12 – U1 –v2 , = 19 – 10 - (-12)= 21
d_{13} = C13 – U1 –v3 , = 50 – 10 – (-2) = 38
d_{21} = C21 – U2 – V1 = 70 – 42 – 9 = 19
d_{24} = C24 – U2 – V4 = 60 –42 – 0 = 18
d_{31} = C31 – U3 – V1 = 40 – 20 – 9 = 11
d_{33} = C33 – U3 – V3 = 70 – 20 – (- 2) = 52
Since all Dij >= an optimal solution has been attained

Total Transport cost = 5*19 + 2 * 10 + 2 * 30 + 7 * 40 + 6 * 8 + 12 * 20 = 743

THE OPTIMUM SOLTUTION FOR TRANSPORTATION PROBLEM

	D1	D2	D3	D4	CAPACITY	U_i
S1	19₹ 5	30₹	50₹	10₹ 2 2	7	U_1 =10
S2	70₹	30₹ 2	40₹ 7	60₹	9	U_2 = 42
S3	40₹ 3	8₹ 6 sub	₹70	20₹ 12	18	U_3 =20
DEMAND	5	8	7	14	34	
V_j	V_1 =9	V_2 = - 12	V_3 =- 2	V_4 = 0		

Transport from S1 to D1 = 5 UNITS
Transport from S1 TO D4 = 2 UNITS
Transport from S2 TO D2 = 2 UNITS
Transport from S2 TO D3 = 7 UNITS
Transport from S3 TO D2 = 6 UNITS
Transport from S3 TO D4 = 12 UNITS

Total Transport cost = 5*19 + 2 * 10 + 2 * 30 + 7 * 40 + 6 * 8 + 12 * 20 = 743

STEPPING-STONE METHOD

1. Proceed row by row and Select a water square (a square without any allocation) to evaluate
2. Beginning at this square, trace/forming a closed path back to the original square via squares that are currently being used.
3. Beginning with a plus (+) sign at the unused corner (water square), place alternate minus and plus signs at each corner of the path just traced.

STEPPING-STONE METHOD … contd.

4. Calculate an improvement index (the net cost change for the path)by first adding the unit-cost figures found in each square containing a plus sign and subtracting the unit costs in each square containing a minus sign.

STEPPING-STONE METHOD

5. Repeat steps 1 though 4 until you have calculated an improvement index for all unused squares.

EVALUATE THE SOLUTION FROM OPTIMALITY TEST.

- Evaluate the solution from optimality test by observing the sign of the net cost change

I. The negative sign (-) indicates that a cost reduction can be made by making the change.

II. Zero result indicates that there will be no change in cost.

III. The positive sign (+) indicates an increase in cost if the change is made.

IV. If all the signs are positive , it means that the optimal solution has been reached .

V. If more than one squares have a negative signs then the water squared with the largest negative net cost change is selected for quicker solution , in case of tie; choose one of them randomly.

EVALUATE THE SOLUTION FROM OPTIMALITY TEST.

6. If an improvement is possible, choose the route (unused square) with the largest negative improvement index

7. On the closed path for that route, select the smallest number found in the squares containing minus signs, adding this number to all squares on the closed path with plus signs and subtract it from all squares with a minus sign .Repeat these process until you have evaluate all unused squares

8. Prepare the new transportation table and check for the optimality.

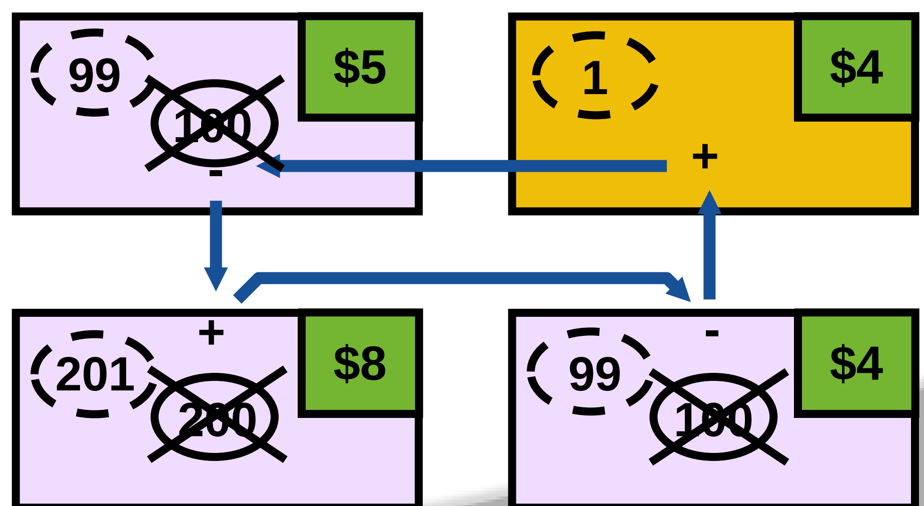
STEPPING-STONE METHOD ... contd.

From \ To	WH-A	WH-B	WH-C	Factory capacity
FACTORY-D	100 [5]	– [4] +	[3]	100
(FACTORY-E	200 [8] +	100 [4]	[3]	300
[illegible]	[9]	100 [7]	200 [5]	300
Warehouse requirement	300	200	200	700

Factory-D to WH-B index

= ₹4 - ₹5 + ₹8 -₹ 4

= +₹ 3



STEPPING-STONE METHOD ..Contd.

From \ To	WH-A	WH-B	WH-C	Factory capacity
Factory-D	₹ 5 100 -	₹4	Start ₹3 +	100
Factory-E	8 ₹ 200 +	₹4 ₹100 -	₹3	300
Factory-F	₹9	₹7 100 +	₹5 200 -	300
Warehouse requirement	300	200	200	700

Factory-D – WH-C index

= ₹3 – ₹5 + ₹7 –₹4 + ₹8 – ₹5 = + ₹4

STEPPING-STONE METHOD .... Contd

From \ To	WH-A	WH-B	WH -C	Facto capac
FACTORY -D	₹5 100	₹4	₹3	100
FACTORY- E	₹8 200 S A	₹4 100 sub	₹3 Add	300
FACTORY -F	₹ ₹9 A S	₹ 7 100 Add	₹5 200	300
Warehouse requirement	300	200	sub 200	700

Factory-E to WH- C index

= ₹3 - ₹4 + ₹7 - ₹5 = + ₹1

(Closed path = EC - EB + FB - FC)

Factory-F to WH-A index

= ₹9 - ₹7 + ₹4 - ₹8 = ₹ -2

(Closed path = FA - FB + EB - EA)

STEPPING-STONE METHOD

From \ To	WH-A	WH-B	WH-C	Factory capacity
FACTORY- D	100 (र 5)	(र 4)	(र 3)	100
FACTORY - E	200 (र8) -	100 (र 4) +	(र 3)	300
FACTORY - F	(र 9) +	100 (र 7) -	200 (र5)	300
Warehouse requirement	300	200	200	700

1. Add 100 units on route FA
2. Subtract 100 from routes FB
3. Add 100 to route EB
4. Subtract 100 from route EA

THE FIRST MODIFIED TRANSPORTATION TABLE BY STEPPING-STONE METHOD ..contd

To / From	WH-A	WH-B	WH-C	Factory capacity
Factory-D	100 (5)	(4)	(3)	100
Factory-E	100 (8)	200 (4)	(3)	300
Factory-F	100 (9)	(7)	200 (5)	300
Warehouse requirement	300	200	200	700

Total Cost = ₹ 5(100) + ₹8(100) + ₹4(200) + ₹9(100) + ₹5(200)
= ₹ 4,000 is this the optimal solution

FIRST MODIFIED TRANSPORTATION TABLE BY STEPPING-STONE METHOD ..contd

- Since the condition for the acceptability is met (m+n -1 = the used cells).3+3 -1 =5
- Repeat steps 1 though 4 until you have calculated an improvement index for all unused squares.
- Evaluate the unused cells as follow :-
-

MODIFIED TRANSPORTATION TABLE BY STEPPING-STONE METHOD ..contd

- DB …… DB, DA,EA,EB= +4 -5+8 – 4 = +3
- DC……..DC,DA,FA,FC= +3-5+9-5 = +2
- EC ……..EC,FC,FA,EA = +3 -5 +9-8 = -1
- FB…….FB,FA,EA,EB= +7-9+8-4 = +2
- Since there is a negative sign appear, then the pervious solution is not the optimal solution and there is chance to modify the solution

The second modified transportation table by Stepping-Stone Method

To / From	WH-A	WH-B	WH-C	Factory capacity
Factory-D	100 (5)	(4)	(3)	100
Factory-E	(8)	200 (4)	(3)	300
Factory-F	200 (9)	(7)	100 (5)	300
Warehouse requirement	300	200	200	700

Total Cost = 5(100) + 4(200) + 3(100) + 9(200) + 5(100)
= Rs 39,00 is this the optimal solution ?

RE-EVALUATE THE UNUSED CELLS

- AD; DB,EB,EC,FC,FA,DB= +4-4+3-5+9-5 = +2
- DC; DC,FC,FA,DA= +3 -5+9-5 = +2
- EA; EA,EC,FC,FA =+8 -3+5-9 = +1
- FB; FB,EB,EC,FC= +7 – 4 +3 – 5 = +1
- Since there is no negative sign appear, then the pervious solution is the optimal solution and there is no chance to modify the solution

The second modified transportation table by Stepping-Stone Method

To / From	WH-(A)	WH-B	WH-C	Factory capacity
Factory-D	100 (5)	(4)	(3)	100
Factory-E	(8)	200 (4)	100 (3)	300
Factory-F	200 (9)	($7)	100 ($5)	300
Warehouse requirement	300	200	200	700

Total Cost = 5(100) + 4(200) + 3(100) + 9(200) + 5(100)
= Rs 39,000 is this the optimal solution ?

- The initial cost with NWCR method = ₹ 4,200
- The first modified cost = ₹ 4,000
- The second modified cost = ₹ 3,900

- Optimal solution

SPECIAL ISSUES IN MODELING

- **Demand not equal to supply**
  - **Called an unbalanced problem**
  - **Common situation in the real world**
  - **Resolved by introducing dummy sources or dummy destinations as necessary with cost coefficients of zero**

UNBALANCED TRANSPORTATION PROBLEM

From \ To	WH-A	WH-B	WH-C	Factory capacity
FACTORY-D	र 5	र 4	र 3	250
FACTORY-E	र 8	र 4	र3	300
FACTORY-F	र 9	र7	र 5	300
Warehouse requirement	300	200	200	850

TOTAL FACTORY CAPACITY = 850 UNITS
TOTAL WAREHOUSE REQUIREMENT = 700 UNITS
TOTALS ARE NOT EQUAL WE HAV E INSERT DUMMY –WAREHOUSE OF REQUIREMENT = 150 UNITS WITH TRANSPORT COST = 0

UNBLANCED TP PROBLEM
IBFS BY USING NOH WEST CORNER RULE

From \ To	WH-A	WH-B	WH-C	WH-Dummy	Factory capacity
FACTORY-D	250 (5र)	(र4)	(र3)	(0)	250
FACTORY-E	50 (8र)	200 (र4)	50 (र3)	(0)	300
FACTORY-F	(र9)	(र7)	150 (र5)	150 (0)	300
Warehouse requirement	300	200	200	150	850

Total Cost = 250(5) + 50(8) + 200(4) + 50(3) + 150(5) + 150(0)
= Rs 3,350

New Factory-E capacity

SPECIAL ISSUES IN MODELING UNBLANCED TP PROBLEM

From \ To	WH-A	WH-B	WH-C	WH-Dummy	Factory capacity
FACTORY-D	$5	$4	$3	0	250
FACTORY-E	50 $8	$4	$3	0	300
FACTORY-F	$9	$7	$5	0	300
Warehouse requirement	300	200	200	150	850

New Factory-E capacity

Total Cost = 250(5) + 50(8) + 200(4) + 50(3) + 150(5) + 150(0)

= Rs 3,350

CHECK FOR OPTIMALITY TEST

1). no of occupied cell required = m + n -1 = 3+4-1 = 6

no of available occupied cells = 6

if both are equal, this problem is not degenerate, it has optimal solution

2).find set of Ui, Vj for all row and columns such that

$$C_{ij} = U_i + V_j$$

3) then compute net cell evaluation for unoccupied cells (d_{ij})

4) If all (d_{ij}) >=0 , then solution table is optimal

5) If not select most –ve cell and form loop for modified solution and repeat the steps 2 to 4

SPECIAL ISSUES IN MODELING

- **Degeneracy**

We must make the test for acceptability to use the stepping-stone methodology, that mean the feasible solution must met the condition of the number of occupied squares in any solution must be equal to the number of rows in the table plus the number of columns minus 1

M (number of rows) + N (number of columns) = allocated cells

If a solution does not satisfy this rule it is called degenerate

SPECIAL ISSUES IN MODELING: WHEN THE PROBLEM IS IN DEGENERATE

To / From	Customer 1	Customer 2	Customer 3	Warehouse supply
Warehouse 1	8	2	6	100
Warehouse 2	100 10	9	20 9	120
Warehouse 3	ε 7	100 10	80 8	80
Customer demand	100	100	100	300

Ini
Pla
un
(qt
eva

No. of ocupied cells required = m+n-1 = 3 +3-1= 5
No. of ocupied cells available = 5

No. of occupied cells =4
5 > 4
So this problem is said to be DEGENERATE

SPECIAL ISSUES IN MODELING: WHEN THE PROBLEM IS IN DEGENERATE

From \ To	Customer 1	Customer 2	Customer 3	Warehouse supply
Warehouse 1	8	2 100	6	100
Warehouse 2	10 20	9	9 100	120
Warehouse 3	7 80	10	8	80
Customer demand	100	100	100	300

Ini
Pla
un
(qt
eva

No. of ocupied cells required = m+n-1 = 3 +3-1= 5

No. of ocupied cells available = 4

5 > 4

So this problem is said to be DEGENERATE

select least cost cell as ocupied cell with wit very small quantity ε

Optimal solution for degeneracy transportation problem

From \ To	Customer 1	Customer 2	Customer 3	Warehouse supply	
Warehouse 1	8	100 / 2	ε / 6	100	U1= - 3
Warehouse 2	20 / 10	9	100 / 9	120	U2 = 0
Warehouse 3	80 / 7	10	8	80	U3 = - 3
Customer demand	100	100	100	300	
	V1 =10	V2 = 5	V3=9		

Transport warehouse-1 customer -2 = 100 units
Transport warehouse-1 customer -3 = 0 units
Transport warehouse-2 customer -1 = 20 units
Transport warehouse-3 customer -3 = 100 units
Transport warehouse-3 customer -1 = 100 units

ε

Total Transportation cost = 100 * 2 + 0 * 6 + 20 * 10 + 100 *9 + 80 *7 = Rs 1860

THE ASSIGNMENT PROBLEM

- In many business situations, management needsto assign - personnel to jobs, - jobs to machines, - machines to job locations, or - salespersons to territories.
- Consider the situation of assigning *n* jobs to *n* machines.
- When a job i (=1,2,....,n) is assigned to machine j (=1,2, ...n) that incurs a cost Cij.
- The objective is to assign the jobs to machines at the least possible total cost.

ASSIGNMENT PROBLEM

- This situation is a special case of the Transportation Model and it is known as the *assignment problem*.
- Here jobs represent "sources" and machines represent "destinations."
- The supply available at each source is 1 unit and demand at each destination is 1unit.

ASSIGNMENT PROBLEM

- Assignment Problem is a special type of linear programming problem where the objective is to minimize the cost or time of completing a number of jobs by a number of persons.
- The assignment problem in the general form can be stated as follows:
- "Given *n* facilities, *n* jobs and the effectiveness of each facility for each job, the problem is to assign each facility to one and only one job in such a way that the measure of effectiveness is optimized (Maximized or Minimized)."Several problems of management have a structure identical with the assignment problem.

EXAMPLES OF ASSIGNMENT PROBLEM:

- Example-1: A manager has four persons (i.e. facilities) available for four separate jobs (i.e. jobs) and the cost of assigning (i.e. effectiveness) each job to each person is given. His objective is to assign each person to one and only one job in such a way that the total cost of assignment is minimized.
- Example-2: A manager has four operators for four separate jobs and the time of completion of each job by each operator is given. His objective is to assign each operator to one and only one job in such a way that the total time of completion is minimized.

Example 1.*(Maximization Problem).*

A Department head has four subordinates, and four tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrensic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hours.

		SUBORDINATES			
		I	II	III	IV
TASKS	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	3

Example 2 .*(Minimization Problem).*

- **Find the minimum cost assignment for the following problem, explaining each step.**

Worker	Job				
	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

EXAMPLE-3 (MINIMIZATION)

The assignment cost of assigning any one operator to any one machine is given in the following table. Solve the optimal assignment by Hungarian method

	Operators				
		I	II	III	IV
Machine	A	10	5	13	15
	B	3	9	18	3
	C	10	7	3	2
	D	5	11	9	7

EXAMPLE-4 ASSIGNMENT MODEL (minimization)

- Example-4:A tourist car operator has four cars in each of the four cities and four customers in four different cities. The distance between different cities is given. His objective is to assign each car to one and only one customer in such a way that the total distance covered is minimized

HUNGARIAN METHOD FOR ASSIGNMENT PROBLEM

- Although an assignment problem can be formulated as a linear programming problem, it is solved by a special method known as Hungarian Method because of its special structure.. The Hungarian Method is based on the principle that if a constant is added to every element of a row and/or a column of cost matrix, the optimum solution of the resulting assignment problem is the same as the original problem and vice versa. The original cost matrix can be reduced to another cost matrix by adding constants to the elements of rows and columns where the total cost or the total completion time of an assignment is zero. Since the optimum solution remains unchanged after this reduction, this assignment is also the optimum solution of the original problem

STEPS FOR SOLVING MINIMIZATION ASSIGNMENT PROBLEM

- Step 1: See whether Number of Rows are equal to Number of Column. If yes, problem is balanced one; if not, then add a Dummy Row or Column to make the problem a balanced one by allotting zero value or specific value (if any given) to each cell of the Dummy Row or Column, as the case may be.
- Step 2: Row Subtraction: Subtract the minimum element of each row from all elements of that row.
- Note: If there is zero in each row, there is no need for row subtraction.

STEPS FOR SOLVING MINIMIZATION ASSIGNMENT PROBLEM

- Step 3: Column Subtraction: Subtract the minimum element of each column from all elements of that column.

Note: If there is zero in each column, there is no need for column subtraction.

- Step 4: Draw minimum number of Horizontal and/or Vertical Lines to cover all zeros.

LINE DRAWING PROCEDURE

when it is not possible to assign one cell on each of the row and column (ie any row or column found with out assigment), we have to draw minimum no of lines covering all zeros

step-1. tick (✔) row that do not have any assinment

Step-2. tick (✔) column having crossed zero ($\cancel{0}$)

Step-3. observe the ticked column, mark rows having assignment. $\boxed{0}$ (assigned zero)

Step-4. Repeat the steps-2 and 3 until the chain of ticking complete.

Step-5. draw lines through all ticked column and un-ticked rows.

This will give minimum no. of lines. Then go for Assignment

STEPS FOR SOLVING MINIMIZATION ASSIGNMENT PROBLEM ….. Contd.

Step 5: If the total lines covering all zeros are equal to the size of the matrix of the Table, we have got the optimal solution; if not, subtract the minimum uncovered element from all uncovered elements and add this element to all elements at the intersection point of the lines covering zeros.

Step 6: Repeat Steps 4 and 5 till minimum number of lines covering all zeros is equal to the size of the matrix of the Table.

STEPS FOR SOLVING MINIMIZATION OF ASSIGNMENT PROBLEM ... Contd..

Step 7: Assignment: Select a row containing exactly one unmarked zero and surround it by ,‘and draw a vertical line through the column containing this zero. Repeat this process till no such row is left; then select a column containing exactly one unmarked zero and surround it by, and draw a horizontal line through the row containing this zero and repeat this process till no such column is left.

Note: If there is more than one unmarked zero in any row or column, it indicates that an alternative solution exists. In this case, select anyone arbitrarily and pass two lines horizontally and vertically.

LINE DRAWING PROCEDURE ....CONTD..

- Step 8: Add up the value attributable to the allocation, which shall be the minimum value.
- Step 9 : Alternate Solution: If there are more than one unmarked zero in any row or column, select the other one (i.e., other than the one selected in Step 7) and pass two lines horizontally and vertically.
- Step 10:Add up the value attributable to the allocation, which shall be the minimum value.

ASSIGNMENT PROBLEM -1 (MINIMIZATION)

ABC Corporation has four plants each of which can manufacture any one of the four products. Product costs differ from one plant to another as follow:

COST MATRIX

PLANT	PRODUCT			
	1	2	3	4
A	33	40	43	32
B	45	28	31	23
C	42	29	36	29
D	27	42	44	38

You are required to obtain which product each plant should produce to minimize cost,

SOLUTION

PLANT	PRODUCT			
	1	2	3	4
A	33	40	43	32
B	45	28	31	23
C	42	29	36	29
D	27	42	44	38

Initial Table

Step 1 : Row Deduction: Subtracting the minimum element of each row from all the elements of that row

PLANT	PRODUCT			
	1	2	3	4
A	1	8	11	0
B	22	5	8	0
C	13	0	7	0
D	0	15	17	11

After row operation

SOLUTION

PLANT	PRODUCT			
	1	2	3	4
A	1	8	11	0
B	22	5	8	0
C	13	0	7	0
D	0	15	17	11

Step 2 : Column Deduction: Subtracting the minimum element of each column from all the elements of that column

PLANT	PRODUCT			
	1	2	3	4
A	1	8	4	0
B	22	5	1	0
C	13	0	0	0
D	0	15	17	11

After column operation

SOLUTION

Step-3: perform assignment considering row / Columns having single zero

✔

PLANT	PRODUCT				
	1	2	3	4	
A	1	8	4	$\boxed{0}$	
B	22	5	1	~~0~~	✔ 1
C	13	~~0~~	$\boxed{0}$	~~0~~	
D	$\boxed{0}$	15	17	11	

Here row-c does not have any assignment. To proceed further apply line drawing procedure to revise the table.

Example 1.*(Minimization Problem).*

STEP-3 now test whether it is possible to make an assignment using only zeros. If it is possible , the assignment must be optimal

Assign Machine A to Producto-IV =Rs 32
Assign Machine B to Product- III = 31
Assign Machine C to Product –II = 29
Assign Mchine –D to product -I = 27
total cost = Rs 119 hours

product

	1	2	3	4	
A	1	8	4	$\boxed{0}$	3
B	22	5	1	~~0~~	1
C	13	$\boxed{0}$	~~0~~	~~0~~	
D	$\boxed{0}$	15	17	11	

,minimum unlined element = 1

- **Subtract mini element from all unlined elements**
- **add min element to elements of line of intersection**

Solution Table

PRODUCT

MACHINE	I	II	III	IV
A	0	7	3	$\boxed{0}$
B	21	4	$\boxed{0}$	0
C	13	$\boxed{0}$	0	1
D	$\boxed{0}$	15	17	12

Example -1 *OPTIMAL SOLLUTION*

STEP-6 now test whether it is possible to make an assignment using only zeros. If it is possible , the assignment must be optimal

PRODUCT

MACHINE	I	II	III	IV
A	33	40	43	32
B	45	28	31	23
C	42	29	36	29
D	27	42	44	38

Assign Machine A to Product-IV = Rs 32
Assign Machine B to Product- III = 31
Assign Machine C to Product –II = 29
Assign Mchine –D to product -I = 27
total cost = 119

OPTIMUM ASSIGNMENT

Step-5: the optimal assignment is as follows:
Assign Plant –a to product -4
Plant B to product -3

plant	product	Cost (Rs)
Assign Machine-A	4	32
Assign Machine -B	3	31
Assign Machine-C	2	29
Assign Machine-D	1	27
Total cost		Rs 119

STEPS INVOLVING MAXIMIZATION TYPE OF ASSIGNMENT PROBLEM

- Step 1: See whether Number of Rows is equal to Number of Columns. If yes, problem is a balanced one; if not, then adds a Dummy Row or Column to make the problem a balanced one by allotting zero value or specific value (if any given) to each cell of the Dummy Row or Column, as the case may be.
- Step 2: Derive Profit Matrix by deducting cost from revenue.
- Step 3: Derive Loss Matrix by deducting all elements from the largest element.
- Step 4: Follow the same Steps 2 to 9 as involved in solving Minimization Problems.

Example 2.*(MiNization Problem).*

A Department head has four subordinates, and four tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrensic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hours.

TASKS	SUBORDINATES I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	3

Example 2.*(Minimization Problem).*

STEP-1. Subtract the smallest element in each row from every element of that row, we will get the reduced matrix

SUBORDINATES

SKS

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	3

Reduced matrix

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	16	23	21	0

Example 2.*(Manimization Problem).*

STEP-2. next Subtract the smallest element in each rcolumn from every element of that column, we will get the reduced matrix

SUBORDINATES

TASKS		I	II	III	IV
	A	0	18	9	3
	B	9	24	0	22
	C	23	4	3	0
	D	16	23	21	0

Reduced matrix

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	16	19	21	0

Example 2. *(Minimization Problem)*
Optimum Solution

STEP-3 now test whether it is possible to make an assignment using only zeros. If it is possible , the assignment must be optimal

	I	II	III	IV
A	$\boxed{0}$	14	9	3
B	9	20	$\boxed{0}$	22
C	23	$\boxed{0}$	3	0
D	16	19	21	$\boxed{0}$

Assign Task A to Subordinate-I = 8 hours
Assign Task B to Subordinate – III = 4
Assign Task C to Subordinate –II = 19
Assign Task D to subordinate –IV = 10
total man-hours = = 42 man hours

Solution Table

TASKS	SUBORDINATES I	II	III	IV
A	$\boxed{8}$	26	17	11
B	13	28	$\boxed{4}$	26
C	38	$\boxed{19}$	18	15
D	19	26	24	$\boxed{10}$

Example 2.*(Manimization Problem).*

STEP-2. next Subtract the smallest element in each rcolumn from every element of that column, we will get the reduced matrix

SUBORDINATES

SKS

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	16	23	21	0

Reduced matrix

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	16	19	21	0

Example-3: ASSIGNMENT (minimization)

- Five men are available to do five different jobs. From the past records , the time (in hours) that each man takes to do each job is known and given in the following table.

		Job				
		I	II	III	IV	V
Man	A	2	9	2	7	1
	B	6	8	7	6	1
	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find the assignment of men to jobs that will minimize the total time taken.

Example-3: ASSIGNMENT

- Step-1. Subtracting the smallest element of each row from every element of the corresponding row, we get the reduced matrix

Job

Man	I	II	III	IV	V
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Reduced Matrix

	I	II	III	IV	V
A	1	8	1	6	0
B	5	7	6	5	0
C	3	5	4	2	0
D	3	1	6	2	0
E	4	2	8	4	0

Example-3: Assignment ...contd.

- Step-2. Subtracting the smallest element of each column from every element of the corresponding column , to get the adjoining reduced matrix reduced matrix

Job

	I	II	III	IV	V
A	1	8	1	6	0
B	5	7	6	5	0
C	3	5	4	2	0
D	3	1	6	2	0
E	4	2	8	4	0

Reduced Matrix

	I	II	III	IV	V
A	0	7	0	4	0
B	4	6	5	3	0
C	2	4	3	0	0
D	2	0	5	0	0
E	3	2	7	2	0

Example-3: Assignment .. Contd.

- Step-3 Perform assignment for the rows/ columns where there is only one zero only. Mini = 2
- Unlined .

	I	II	IV	V
A	0	7	0	0
B	4	6	3	0
C	2	4	0	0
D	2	0	0	0
E	3	2	2	0

Reduced Matrix

	I	II	III	IV	V
A	0	7	0	4	2
B	2	4	3	2	0
C	2	4	3	0	2
D	2	0	5	0	2
E	1	0	5	0	0

Example-3: Assignment .. Contd.

- Step-3 Perfom assignment for the rows/ columns where there is only one zero only. Mini unlined=1
- Unlined .

	I	II		IV	V
A	$\boxed{0}$	7	~~0~~	4	2
B	2	4	3	2	$\boxed{0}$
C	2	4	3	$\boxed{0}$	2
D	2	~~0~~	5	~~0~~	2
E	1	$\boxed{0}$	5	~~0~~	~~0~~

Reduced Matrix

	I	II	III	IV	V
A	~~0~~	8	$\boxed{0}$	5	2
B	2	5	3	3	$\boxed{0}$
C	1	4	2	$\boxed{0}$	1
D	1	$\boxed{0}$	4	~~0~~	1
E	$\boxed{0}$	0	4	~~0~~	~~0~~

Example-3: Solution

Assign man –A to job-III = 2 hours

Assign Man-B to job-V = 1 hour

Assign Man-C to Job –IV = 3 hours

Assign Man –D to Job –II = 2 hours

Assign Man –E to Job-I = 5 hours Total Time = 13 hours

Man	I	II	III	IV	V
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

	I	II	III	IV	V
A	0	8	0	5	2
B	2	5	3	3	0
C	1	4	2	0	1
D	1	0	4	0	1
E	0	0	4	0	0

Example 4. ASSIGNMEMT *(Maximization Problem).*

A company has 5 jobs to be done. The following matrix shows the return in rupees on assigning i th (i = 1,2,3,4,5) machine to the jth job (j =A, B, C, D, E). Assign the five jobs to the five machines so as to maximize the total expected profit

Jobs

Machines		**A**	**B**	**C**	**D**	**E**
	1	**5**	**11**	**10**	**12**	**4**
	2	**2**	**4**	**6**	**3**	**5**
	3	**3**	**12**	**5**	**14**	**6**
	4	**6**	**14**	**4**	**11**	**7**
	7	**9**	**8**	**12**	**5**	**5**

SOLUTION OF EXAMPLE-4 (MAXIMIZATION)..contd.

Solution.*Step* 1.*Converting from Maximization to Minimization:* Since the highest element is 14, so subtracting all the elements from 14, the following reduced cost (opportunity loss of maximum profit) matrix is obtained.

	A	B	C	D	E
1	5	11	10	12	4
2	2	4	6	3	5
3	3	12	5	14	6
4	6	14	4	11	7
7	9	8	12	5	1

COST MATRIX

	A	B	C	D	E
1	9	3	4	2	10
2	12	10	8	11	9
3	11	2	9	0	8
4	8	0	10	3	7
5	7	5	6	2	9

Then perform standard, steps of minimization.

SOLUTION OF EXAMPLE-4 (MAXIMIZATION)..contd.

Step 2. Perform Row operation : consider each of the row and subtract smallest element from other elements of that row.

JOB

NES

	A	B	C	D	E
1	9	3	4	2	10
2	12	10	8	11	9
3	11	2	9	0	8
4	8	0	10	3	7
5	7	5	6	2	9

After row operation

	A	B	C	D	E
1	7	1	2	0	8
2	4	2	0	3	1
3	11	2	9	0	8
4	8	0	10	3	7
5	5	3	4	0	7

SOLUTION OF EXAMPLE-4 Contd..

Step 3.. Performing column operation : consider each of the column and subtract smallest element from other elements of that column

	A	B	C	D	E
1	7	1	2	0	8
2	4	2	0	3	1
3	11	2	9	0	8
4	8	0	10	3	7
5	5	3	4	0	7

After column operation

	A	B	C	D	E
1	3	1	2	0	7
2	0	2	0	3	0
3	7	2	9	0	7
4	4	0	10	3	6
5	1	3	4	0	7

SOLUTION OF EXAMPLE-4 (maximization) ..contd.

Step-4. after assigning, row-3 and row-5 does not contain any assignment. At this stage no optimal solution is attained. Draw minimum no. of lines passing through all zeros. Select minimum among unlined element.

3* No change of elemen on the line other than li intersection

- **1.Subtract this element from all other unlined elements.**
- 2. **add this element to intersection elements.**

JOB

	A	B	C	D (3 ✔)	E	
1	3	1	2	$\boxed{0}$	7	✔ 4
2	$\boxed{0}$	2	~~0~~	3	~~0~~	
3	7	2	9	~~0~~	7	✔ 1
4	4	~~0~~	10	3	$\boxed{0}$	
5	1	3	4	~~0~~	7	✔ 2

Optimal solution is attained

	A	B	C	D	E
1	2	$\boxed{0}$	1	~~0~~	6
2	~~0~~	2	$\boxed{0}$	4	~~0~~
3	6	1	8	$\boxed{0}$	6
4	5	~~0~~	10	4	$\boxed{0}$
5	$\boxed{0}$	2	3	~~0~~	6

OPTIMAL SOLUTION OF EXAMPLE-4

Optimal solution is attained

	A	B	C	D	E
1	2	$\boxed{0}$	1	~~0~~	6
2	~~0~~	2	$\boxed{0}$	4	~~0~~
3	6	1	8	$\boxed{0}$	6
4	5	~~0~~	10	4	$\boxed{0}$
5	$\boxed{0}$	2	3	~~0~~	6

MACHINE

JOB

	A	B	C	D	E
1	5	11	10	12	4
2	2	4	6	3	5
3	3	12	5	14	6
4	6	14	4	11	7
7	7	8	12	5	1

plant	JOB	PROFIT(Rs)
Assign Machine-1	B	11
Assign Machine -2	C	6
Assign Machine-3	D	14
Assign Machine-4	E	7
Assign Machine-5	A	7
Maximum Profit		**Rs 45**

UNBALANCED ASSIGNMENT PROBLEM

- If the cost matrix of an assignment problem is not a square matrix (number of sources not equal to number of destinations), then the assignment problem is called an Unbalanced problem,
- In such cases , fictitious rows / columns with ‘0’ costs are added in the matrix so as to form a square matrix. Then the usual assignment algorithm can be applied to the resulting problem.

Example : UNBALANCED ASSIGNMENT PROBLEM

- A company is faced with the problem of assigning six different machines to five different jobs. The profits in rupees are estimated as follows.

JOBS

MACHINES

	A	B	C	D	E
1	25	50	10	60	10
2	20	50	15	70	30
3	30	65	20	80	30
4	35	70	20	90	45
5	40	70	30	90	60
6	60	90	50	100	60

Solve the problem assuming that the objective to maximize totlal profit

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION ...... Contd.

JOBS

Profit matrix

IINES

	A	B	C	D	E	Dummy
1	25	50	10	60	10	0
2	20	50	15	70	30	0
3	30	65	20	80	30	0
4	35	70	20	90	45	0
5	40	70	30	90	60	0
6	60	90	50	100	60	0

Step-1. introducing one dummy column with profit elements as zeo
Since this problem is maximization type . Convert the problem into cost matrix by subtracting all elements from the highest element (100)

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION …… Contd.

Step-2. . Convert the problem into cost matrix by subtracting all elements from the highest element (100)

cost matrix

IINES

	A	B	C	D	E	Dummy
1	75	50	90	40	90	100
2	80	50	85	30	70	100
3	70	35	80	20	70	100
4	65	30	80	20	55	100
5	60	30	70	10	40	100
6	40	10	50	0	40	100

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION ...... Contd.

Step-3: perform column operation

	A	B	C	D	E	Dummy
1	75	50	90	40	90	100
2	80	50	85	30	70	100
3	70	35	80	20	70	100
4	65	30	80	20	55	100
5	60	30	70	10	40	100
6	40	10	50	0	40	100

1.After row operation

	A	B	C	D	E	Dummy
1	35	10	50	0	50	60
2	50	20	55	0	40	70
3	50	15	60	0	50	80
4	45	10	60	0	45	80
5	50	20	60	0	30	90
6	40	10	50	0	40	100

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION …… Contd.

Step-2: perform column operation

	A	B	C	D	E	Dummy
1	35	10	50	0	50	60
2	50	20	55	0	40	70
3	50	15	60	0	50	80
4	45	10	60	0	45	80
5	50	20	60	0	30	90
6	40	10	50	0	40	100

1.After column operation

	A	B	C	D	E	Dummy
1	0	0	0	0	20	0
2	15	10	5	0	10	10
3	15	5	10	0	20	20
4	10	0	10	0	15	20
5	15	10	10	0	0	30
6	5	0	0	0	10	40

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION ...... Contd.

Step-3: Perform assignment with single zeros

	A	B	C	D	E	Dummy	
1	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	20	$\cancel{0}$	
2	15	10	5	$\boxed{0}$	10	10	✔ 3
3	15	5	10	$\cancel{0}$	20	20	✔ 1
4	10	$\boxed{0}$	10	$\cancel{0}$	15	20	
5	15	10	10	$\cancel{0}$	$\boxed{0}$	30	
6	5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	10	40	
				✔ 2			

→

Aftter correcting with unlined small element = 5

	A	B	C	D	E	Dummy
1	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	5	20	0
2	10	5	$\cancel{0}$	$\boxed{0}$	5	5
3	10	$\cancel{0}$	5	$\cancel{0}$	15	15
4	10	$\boxed{0}$	10	5	15	20
5	15	10	10	5	$\boxed{0}$	30
6	5	$\cancel{0}$	$\boxed{0}$	5	10	40

✔ ✔ ✔

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION …… Contd.

	A	B	C	D	E	Dummy
1	0	0	0	5	15	0
2	5	5	5	0	10	10
3	15	5	10	0	20	20
4	10	0	10	0	15	20
5	15	10	10	~~0~~	0	30
6	5	0	0	~~0~~	10	40

Step-5: perform line drawing operation

	A	B ✔3	C	D ✔2	E	Dummy	
1	$\boxed{0}$	~~0~~	~~0~~	5	20	0	✔
2	10	5	~~0~~	$\boxed{0}$	5	5	✔4
3	10	~~0~~	5	~~0~~	15	15	1✔
4	10	$\boxed{0}$	10	5	15	20	✔
5	15	10	10	5	$\boxed{0}$	30	5
6	5	~~0~~	$\boxed{0}$	5	10	40	

RESTRICTION S ON ASSIGNMENT

- Some times technical, legal or other restrictions do not permit the assignment of a particular facity to a particular job. Such a difficulty can be overcome by assigning a very high cost (say , infinite cost) to the corresponding cell, so that the activity will be automatically excluded from the optimal solution.

Example - RESTRICTION S ON ASSIGNMENT

A job shop has purchased 5 new machines of different type. They are 5 available locations in the shop where a machine could be installed. Some of these locations are more desirable than others for particular machines because of their proximity to work centres which would have a heavy work flow to and from these machines. Therefore , the objective is to assign the new machines to the available locations in order to minimize the total cost of material handling. The estimated cost per unit time of materials handling involving each of the machines is given in table for respective locations. Locations 1,2,3, 4, and 5 ae not considered suitable for machines A, B, C, D and E respectively.

Example - RESTRICTION S ON ASSIGNMENT

Since locations 1, 2, 3, 4 and 5 are not suitable for machines A, B, C, D and E respectively, an ‘ X’ is shown in cost matrix some times denoted by “---”

LOCATIONS

MACHINES

	1	2	3	4	5
A	X	10	25	25	10
B	1	X	10	15	2
C	8	9	X	20	10
D	14	10	24	X	15
E	10	8	25	27	X

Example - RESTRICTION S ON ASSIGNMENT

Since locations 1, 2, 3, 4 and 5 are not suitable for machines A, B, C, D and E respectively, an extremely large cost (say ∞) should be attached to these locations. The cost matrix is as follows:

LOCATIONS

MACHINES

	1	2	3	4	5
A	∞	10	25	25	10
B	1	∞	10	15	2
C	8	9	∞	20	10
D	14	10	24	∞	15
E	10	8	25	27	∞

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION ...... Contd.

Step-1: perform row operation

	1	2	3	4	5
A	∞	10	25	25	10
B	1	∞	10	15	2
C	8	9	∞	20	10
D	14	10	24	∞	15
E	10	8	25	27	∞

1.After row operation

	1	2	3	4	5
A	∞	0	15	15	0
B	0	∞	9	14	1
C	0	1	∞	12	2
D	4	0	14	∞	5
E	2	0	15	17	∞

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION ...... Contd.

1.After column peration

	1	2	3	4	5
A	∞	0	6	3	0
B	0	∞	0	2	1
C	0	1	∞	0	2
D	4	0	5	∞	5
E	2	0	6	5	∞

Step-2: perform column operation

	1	2	3	4	5
A	∞	0	15	15	0
B	0	∞	9	14	1
C	0	1	∞	12	2
D	4	0	14	∞	5
E	2	0	15	17	∞

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION …… Contd.

Step-3. perform assignment operation

	1	2 ✔ 2	3	4	5	
A	∞	$\cancel{0}$	6	3	$\boxed{0}$	
B	$\cancel{0}$	∞	$\boxed{0}$	2	1	
C	$\cancel{0}$	1	∞	$\boxed{0}$	2	
D	4	$\boxed{0}$	5	∞	5	✔ 3
E	2	$\cancel{0}$	6	5	∞	✔ 1

Step-4. unlined mini element = 2

OPTIMUM SOLUTION

LOCATION

M/C	1	2	3	4	5
A	∞	2	6	3	$\boxed{0}$
B	$\cancel{0}$	∞	$\boxed{0}$	2	1
C	$\cancel{0}$	3	∞	$\boxed{0}$	2
D	2	$\boxed{0}$	3	∞	3
E	$\boxed{0}$	$\cancel{0}$	4	3	∞

Example : UNBALANCED ASSIGNMENT PROBLEM SOLUTION …… Contd.

Step-5. OPTIMUM SOLUTION

	1	2 ✔ 2	3	4	5	
A	∞	10	25	25	$\boxed{10}$	
B	1	∞	$\boxed{10}$	15	2	
C	8	9	∞	$\boxed{20}$	10	
D	14	$\boxed{10}$	24	∞	15	✔ 3
E	$\boxed{10}$	8	25	27	∞	✔ 1

Step-4. unlined mini element = 2 OPTIMUM SOLUTION

M/C

LOCATION

	1	2	3	4	5
A	∞	2	6	3	$\boxed{0}$
B	~~0~~	∞	$\boxed{0}$	2	1
C	~~0~~	3	∞	$\boxed{0}$	2
D	2	$\boxed{0}$	3	∞	3
E	$\boxed{0}$	~~0~~	4	3	∞

Assign Machine A to Location-5 = Rs 10
Assign Machine- B to Location -3 = 10
Assign Machine –C to Location-4 = 20
Assign Machine-D to Location-2 = 10
Assign Machine-E to location -1 = 10
Total cost **= Rs 60**

Introduction

Suppose there are n jobs to perform, each of which requires processing on some or all of m different machines. The effectiveness (i.e. cost, time or mileage, etc.) can be measured for any given sequence of jobs at each machine, and the most suitable sequence is to be selected (which optimizes the effectiveness measure) among all (n!)m theoretically possible sequences. Although, theoretically, it is a1ways possible to select the best sequence by testing each one, but it is practically impossible because of large number of computations.

DEFINITION OF SEQUENCING

DEFINITION:Suppose there are n jobs (1, 2, 3,..., n), each of which has to be processed one at a time at each of m machines A, B, C, ... The order of processing each job through machines is given (for example, job I is processed through machines A, C, B-in this order). The time that each job must require on each machine is known. The problem is to find a sequence among (n!)m number of all possible sequences (or combinations) (or order) for processing the jobs so that the total elapsed time for all the jobs will be minimum.

Mathematically, let
Ai = time for job i on machine A,
Bi = time for job i on machine B, etc.
T = time from start of first job to completion of the last job.
Then, the problem is to determine for each machine a sequence of jobs i1, i2, i3, ..., in- where
(i1,i2, i3, ..., in) is the permutation of the integers which will minimize T.

TERMINOLOGY AND NOTATIONS

- Number of Machines. It means the service facilities through which. a job must

pass before it is completed. .

- For example, a book to be published has to be processed through composing, printing, binding, etc. In this example, the book constitutes the job and the different processes constitute the number of machines.

- Processing Order. It refers to the order in which various machines are required for completing the job.

- Processing Time. It means the time required by each job on each machine. The notation Tij will denote the processing time required for Ith job b yjth machine (i = 1,2,..., n;j = 1, 2, ...,m).

TERMINOLOGY AND NOTATIONS

- Idle Time on a Machine. This is the time for which a machine remains idle during the total elapsed time. The notation Xij shall be (used to denote the idle time of machine j between the end of the (i - 1)th job and the start of the ith job.
- Total Elapsed Time. This is the time between starting the first job and completing the last job. This also includes idle time, if exists. It will be denoted by the symbol T.
- No Passing Rule. This rule means that P1lssing is not allowed, i.e. the same order of jobs is maintained over each machine. If each of the n-jobs is to be processed through two machines A and B in the order AB, then this rule means

PRINCIPAL ASSUMPTIONS

- Each job will go to machine A first and then to B.

- No machine can process more than one operation at a time.
- Each operation, once started, must be performed till completion.
- A job is an entity, i.e. even though the job represents a lot of individual parts, no lot may be processed by more than one machine at a time.
- Each operation must be completed before any other operation, which it must precede, can begin.
- Time intervals for processing are independent of the order in which operations are performed.
- There is only one of each type of machine.
- A job is processed as soon as possible subject to ordering requirements.
- All jobs are known and are ready to start processing before the period under consideration begins.
- The time required to transfer jobs between machines is negligible.

SEQUENCING PROBLEM MODELS

1. n jobs and two machines A and B, all jobs processed in the order AB.
2. n jobs and three machines A, Band C, all jobs processed in the order ABC.
3. Two jobs and m machines. Each job is to be processed through the machines in a prescribed order (which is not necessarily the same for both the jobs)
4. Problems with n jobs and m-machines.

Our syllabus focusses on above models

Processing n Jobs Through Two Machines

Processing n Jobs Through Two Machines

The problem can be described as: (i) only two machines A and B are involved, (ii) each job is processed in the order AB, and (iii) the exact or expected processing times A), Az, A3, ..., An; B), B2, B3,..., Bn are known

Processing Times	Job(i)				
	1	2	3	...	n
A_i	A_1	A_2	A_3	...	A_n
B_i	B_1	B_2	B_3	...	B_n

The problem is to sequence (order) the jobs so as to minimize the total elapsed time T

Solution Procedure

Step 1. Select, the least processing time occurring in the list A I, Az, A3,..., Ar and Bt, B2, B3' ..., BII' If there is a tie, either of the smallest processing time should be selected.

Step 2. If the least processing time is Ar select r th job first. If it is Bs, do the s th job last (as the given order is AB).

Step 3. There are now n - I jobs left to be ordered. Again repeat steps I and n for the reduced set of processing times obtained by deleting processing times for both the machines corresponding to the job already assigned.

Continue till all jobs have been ordered. The resulting ordering will minimize the elapsed time T.

Proof. Since passing is not allowed, all n jobs must be processed on machine A without any idle time for it. On the other hand, machine B is subject to its remaining idle time at various stages. Let Yj be the

ft n

I £0 I Y,

JOHNSON'S ALGORITHM FOR N JOBS 2 MACHINES

The Johnson's iterative procedure for determining the optimal sequence for an n-job 2-machine sequencing problem can be outlined as follows:

Step 1. Examine the A;'s and B;'s for i = 1,2, ..., n and find out min [Ai,Bi]

Step2.

i. If this minimum be A_k for some i =k, do (process) the kth job first of all.
ii. If this minimum be Br for some i =r, do (process) the rth job last of all.

Step 3.
i. If there is a tie for minima $A_k = B_n$ process the k^{th} job first of all and r^{th} job in the last.
ii. If the tie for the minimum occurs among the A;'s, select the job corresponding to the minimum of B;'s and process it first of all.

iii. If the tie for minimum occurs among the B;'s, select the job corresponding to the minimum of A_i's and process it in the last. Go to next step.

Step4. Cross-out the jobs already assigned and repeat steps 1 to 3 arranging the jobs next to first or next to last, until all the jobs have been assigned.

Example 1. There are five jobs each of which must go through the two machines A and B in the order AB. Processing times are given below:

Processing time (hours)					
Job	1	2	3	4	5
Time for A	5	1	9	3	10
Time for B	2	6	7	8	4

Determine a sequence for five jobs that will minimize the elapsed time T. Calculate the total idle time for the machines In this period.

Solution. Apply steps I and II of solution procedure. It is seen that the smallest processing time is one hour for job 2 on the machine A. So list the job 2 at first place as shown below.

Now, the reduced list of processing times becomes

Job	*A*	*B*
1	5	0
3	9	7
4	3	8
5	10	4

Again, the smallest processing time in the reduced list is 2 for job I on the machine *B*. So place job I last.

Continuing in the like manner, the next reduced list is obtained

continuing in the like manner, the next reduced list is obta

I.'h	A	A
3	9	7
4	S	8
5	10	4

leading to sequence

2	4			1

and the list

Job	A	B
3	9	$\boxed{7}$
5	10	4

gives rise to sequence

2	4		5	1

Finally, the optimal sequence is obtained,

2	4	3	5	1

2				

Now, the reduced list of processing times becomes

Job	*A*	*B*
1	5	0
3	9	7
4	3	8
5	10	4

Again, the smallest processing time in the reduced list is 2 for job I on the machine *B.*

Continuing in the like manner, the next reduced list is obtained

I.'h	*A*	A
3	9	7
4	S	8
5	10	4

leading to sequence

2	4			1

and the list

Job	*A*	*B*
3	9	$\boxed{7}$
5	10	4

gives rise to sequence

2	4		5	1

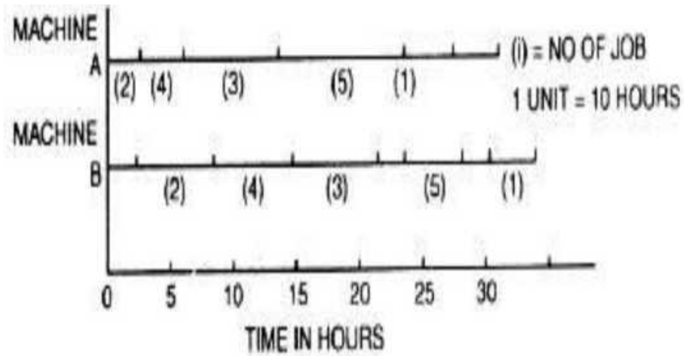
Finally, the optimal sequence is obtained,

2	4	3	5	1

Further, it is also possible to calculate the minimum elapsed time corresponding to the optimal sequencing, using the individual processing time given in the statement of the problem. The details are given in Table

Job sequence	Machine *A*		Machine *B*	
	Time in	Time out	Time in	Timeout
2	0	1	1	7
4	1	4	7	15
3	*4*	13	13	22
5	*n*	33	23	27
1	23	28	28	ran

Thus, the minimum time, i.e. the time for starting of job 2 to completion of the last job 1, is 30 hrs only. During this time, the machine A remains idle for 2 hrs (from 28 to 30 hrs) and the machine B remains idle for 3 hrs only (from 0-1,22-23, and 27-28 hrs). The total elapsed time can also be calculated by using Gantt chart as follows:



From the Fig it can be seen that the total elapsed time is 30 hrs, and the idle time of the machine B is 3 hrs. In this problem, it is observed that job may be held in inventory before going to the machine. For example, 4th job will be free on machine A after 4th hour and will start on machine B after 7th hr. Therefore, it will be kept in inventory for 3 hrs.

PROCESSING N JOBS THROUGH THREE MACHINES

The problem can be described as: (i) Only three machines A, B and C are involved, (ii) each job is processed in the prescribed order ABC,

(iii) transfer of jobs is not permitted, i.e. adhere strictly the order over each machine, and

(iv) exact or expected processing times are given in Table

Job	Machine k	Machine H	Machine C
1	M	ii	C\
!		ii	c,
J	A*	B)	Cj
	_ii_	_L_	_4_

However,the earlie rmethod adopted by Johnson(1954) can be extended to cover the special cases where either one or both of the following conditions hold:

, The minimum time on machine A the maximum time on machine B.

* The minimum time on machine C the maximum time on machine B.

The procedure explained here (without proof) is to

$C^{*}j — A_{r}4- Hf = Bi +$

replace the problem with an equivalent problem, involving n jobs and two fictitious machines denoted by G and H, and corresponding time Gj and Hj are defined by

If this problem with prescribed ordering *GH* is solved, the resulting optimal sequence will also be optimal

for the original problem,

Rub for deleting the programs which cannot be optimal,

Rule no,	Momordcnn**for		WctcPfojnfni cmninp
	m		
1	*u*	*t*	j*f*
II	*u...*	..XV	1
III	J...V	JL	JR
IV	_X V .	*H...*	Jff'
V	*i*|,*L*	*.HI.*	n;
i	*..HI..*	*1U*	*

Example. There are five jobs, each of which must go through machines A, Band C in the order ABC.

Jobt	ProccssingTimcs		
	*	i	Ci
1	s	S	i
1	10	i	9
1	6	2	S
4	1		i
f	ll	4	9

Processing times are given in Table

determine a sequence for five jobs that will minimize the elapsed time T.

Solution. Here min Ai = 6, max Bi = 6, min Ci = 4. Since one of two conditions is satisfied by min Ai =max Bi, so the procedure adopted in Example 1 can be followed.

The equivalent problem, involving five jobs and two fictitious machine G and H, becomes:

Job i	Processing Times	
	$G_i (= A_i + B_i)$	$H_i (= B_i + C_i)$
1	13	9
2	16	15
3	8	10
4	10	9
5	15	9

This new problem can be solved by the procedure described earlier. Because of ties, possible optimal sequences are :

(i) | 3 | 2 | 1 | 4 | 5 |

(ii) | 3 | 2 | 4 | 1 | 5 |

(iii) | 3 | 2 | 4 | 5 | 1 |

(iv) | 3 | 2 | 5 | 4 | 1 |

(v) | 3 | 2 | 1 | 5 | 4 |

(vi) | 3 | 2 | 5 | 1 | 4 |

It is possible to calculate the minimum elapsed time for first sequence as shown in Table

Jub	M#cltd		Mithiuefl		MacicC	
	Timeia	Timeout	Time in	Tine out	Timein	Timeout
	0	6	6	8	1	10
I	!'	16	It	22	22	31
1		24	24	24	31	35
■1	24	31	11	34	33	41
s;	31	42	42	40	40	51

Thus, any of the sequences from (i) to (vi) may be used to order the jobs through machines A, B and C. and they all will give a minimum elapsed time of 51 hrs. Idle time for machine A is 9 hrs, for B 31 hrs, for C 19hrs.

Graphical Method

In the two job m-machine problem, there is a graphical procedure, which is rather simple to apply and usually provides good (though not necessarily optimal) results. The following example will make the graphical procedure clear.

Example 3

Example 3. Use graphical method to minimize the time needed to process the following jobs on the machines shown below, i.e. for each machine find the job, which should be done first. Also calculate the total time needed to complete both the jobs.

Job 1	Sequence of Machines	:	A	B	C	D	E
	Time	:	2	3	4	6	2
Job 2	Sequence of Machines	:	C	A	D	E	B
	Time	:	4	5	3	2	6

Example Solution.

Solution.

Step 1. First, draw a set of axes, where the horizontal axis represents processing time on job 1 and the vertical axis represents processing time on job 2

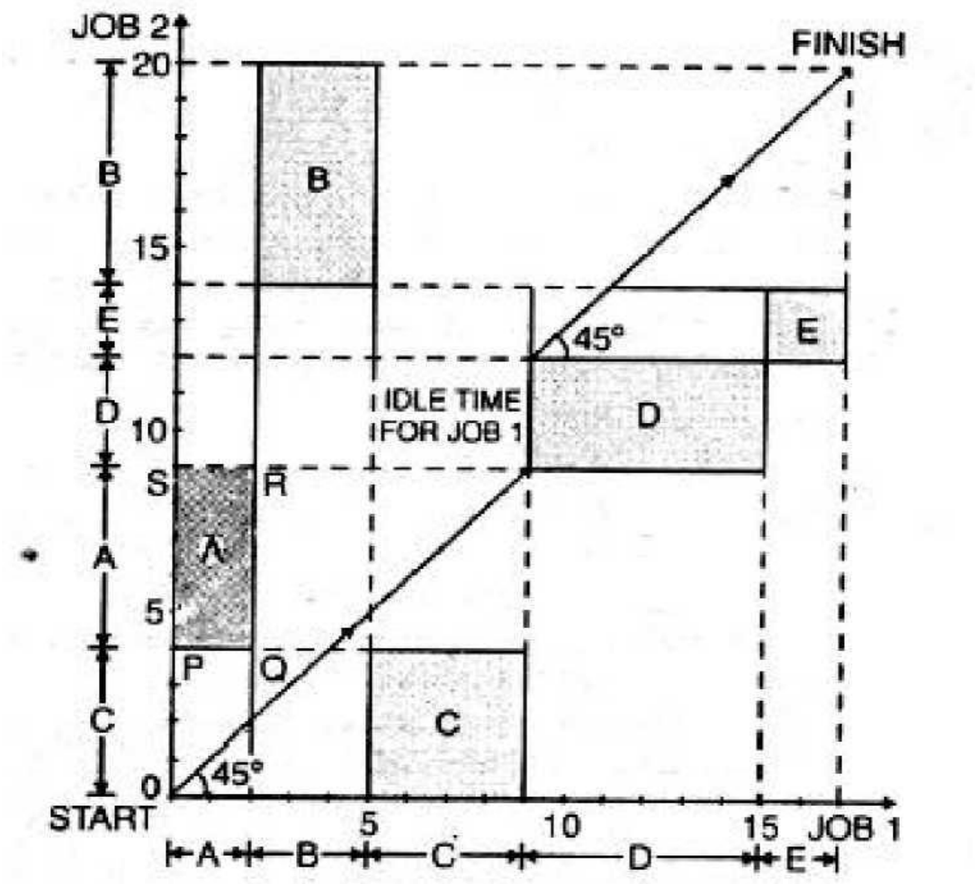
Step 2. Layout the machine time for two jobs on the corresponding axes in the given technologicalorderMachineAtakes2hrsforjob1and5hrsforjob2. Constr uct the rectangle PQRS for the machine A. Similarly, other rectangles for machines B, C, D and E are constructed as shown.

Step 3. Make programme by starting from the origin 0 and moving through various states of completion (points) until the point marked Graphical solution for the 2-job 5-machine sequencing problem. ‘finish’ is obtained. Physical interpretation of the path thus chosen involves the series of segments, which are horizontal or vertical or diagonal making an angle of 45° with the horizontal. Moving to the right means that job 1is proceeding while job 2 is idle, and moving upward means that job 2 is proceeding while job 1 is idle, and moving diagonally means that both the jobs are proceeding simultaneously.

Further, both the jobs cannot be processed simultaneously on the same machine.Graphically, diagonal movement through the blocked-out (shaded) area is not allowed, and similarly for other machines too.

Step4.To find an optimal path. An optimal path (programme) is one that minimizes idle time for job (horizontal movement). Similarly, an optimal path is one that minimizes idle time for job 2 (vertical movement). Choose such a path on which diagonal movement is as much as possible. According to this, choose a good path by inspection as shown by arrows.

Step5. To find the elapsed time. The elapsed time is obtained by adding the idle time for either of the job to the processing time for that job. In this problem, the idle time for the chosen path is seen to be 3 hrs. for the job I, and zero for the job 2. Thus, the total elapsed time, 17 + 3 = 20 hrs is obtained.



Game is defined as an activity between two or more persons involving activities by each person according to a set of rule at the end of which each person receives some benefit or satisfaction or suffers loss (negative benefit). The set of rules defines the game. Going through the set of rules once by the participants defines a play.

Games. They can be classified on the basis of the following characteristics. ,

1. Chance of strategy:
2. Number of persons:
3. Number of activities:
4. These may be finite or infinite.

. Number of alternatives (choices) .. .

5. Information to the players about the past activities of other .

Payoff: A quantitative measure of satisfaction a person gets at the end of each play is called a payoff.

Competitive Game. A competitive situation is called acompetitive game if it has the following four properties

2. Zero-sum and Non-zero-sum Games. Competitive games are classified according to the number of players involved, i.e. as a two person game.. three person game, etc. Another important distinction is between zero-sum games and nonzero-sum games. If the players make payments only to each other, i.e. the loss of one is the gain of others, and nothing comes from outside, the competitive game is said to be zero-sum.

STRATEGIES

Strategy .A strategy of a player has been loosely defined as a rule for decision-making in advance of all the plays by which he decides the activities he should adopt. In other words, a strategy for a given player is a set of rules (programmes) that specifies which of the available course of action he should make at each play.

i. **Pure Strategy**. : If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action. A pure strategy is usually represented by a number with which the course of action is associated.

STRATEGIES .. CONTD.

ii. Mixed Strategy: If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus, the mixed strategy is a selection among pure strategies with fixed probabilities.

TWO-PERSON, ZERO-SUM (RECTANGULAR) GAMES:

A game with only two players (say, Player A and Player B) is called a 'two- person, zero-sum game' if the losses of one player are equivalent to the gains of the other, so that the sum of their net gains is zero. Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in rectangular form.

Pay-off Matrix. Suppose the player A has m activities and the player B has n activities. Then a payoff matrix can be formed by adopting the following rules:

- Row designations for each matrix are activities available to player A.
- Column designations for each matrix are activities available to player B.
- Cell entry 'v_{ij} , is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity.

- With a ‘zero-sum, two person game’, the cell entry in the player B’ s payoff matrix will be negative of the corresponding cell entry ‘Vi}, in the player A’s payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

MINIMAX (MAXIMIN) CRITERION AND OPTIMAL STRATEGY

The ‘Minimax criterion of optimality’ states that if a player lists the worst possible outcomes of all his potential strategies, he will choose that strategy to be most suitable for him which corresponds to the best of these worst outcomes. Such a strategy is called an optimal strategy.

- Example 1. Consider (two-person, zero-sum) game matrix, which represents payoff to the player A. Find, the optimal strategy for the payoff matrix given below.

B

	i \ j	1	2	3
	1	3	−4	8
A	2	−8	5	−6
	3	6	−7	6

SADDLE POINT

- A saddle point of a payoff matrix is the position of such an element in the payoff matrix, which is minimum in its row and maximum in its column.

RULES FOR DETERMINING A SADDLE POINT:

- Select the minimum element of each row of the payoff matrix and mark them by ‘O’.
- Select the greatest element of each column of the payoff matrix and mark them by ‘O ‘.
- there appears an element in the payoff matrix marked by ‘0’ and ‘D’ both, the position of that element is a saddle point of the payoff matrix.

SOLUTION OF GAMES WITH SADDLE POINTS

To obtain a solution of a rectangular game, it is feasible to find out:

- the best strategy for player A
- the best strategy for player B
- the value of the game (V).

Player A can choose his strategies from {AI, A2, A3} only, while B can choose from the set (B1, B2) only. The rules of the game state that the payments should be made in accordance with the selection of strategies:

PRINCIPLE OF DOMINANCE:

- A given strategy can also be said to be dominated if it is inferior to some convex linear combination of two or more strategies

RULES OF DOMINANCE:

- Delete the minimum row: if all the elements of one row is less than or equal to the corresponding element in the other row
- Delete the maximum Column : if all the if all the elements of one column is greater than or equal to the corresponding element in the other column)
- A given strategy can be dominated if it is inferior to an average of two or more other pure strategies.

PRINCIPLE OF DOMINANCE …contd..

- If the ith row dominates the convex linear combination of some other rows, then one of the rows involving in the combination may be deleted. Similar arguments follow for columns also.
- The optimal strategy for player A is reduced
- The optimal strategy for player B also is reduced
- Finally find the strategy of player-A and player-B

GRAPHICAL METHOD FOR (2 X N) AND (MX2) GAMES

The optimal strategies for a (2 x n) or (m x 2) matrix game can be located easily by a simple graphical method. This method enables us to reduce the 2 x n or m x 2 matrix game to 2 x 2 game that could be easily solved by the earlier methods.

Step 1. Construct two vertical axes, axis I at the point x) =0

and axis 2 at the point x) :;: 1.

Step 2. Represent the payoffs V2j'j = 1. 2, ..., n on axis I and payoff line vlj ,j =I, 2, ..., n on axis 2. Step 3. Join the point representing vij on Axis 2 to the point representing V2jon axis I. The resulting straight-line is the expected payoff line

STEPS INVOLVED

Step 4. Mark the lowest boundary of the lines E.i(x) so plotted, by thick line segments. The highest point

- on this lowest boundary gives the maximin point P and identifies the two critical. moves of player B.
- If there are more than two lines passing through the maximum point P. there are ties for the optimum mixed strategies for player B. Thus any two such lines with opposite sign slopes will define an alternative optimum .

Why Inventory is maintained?

- Inventory helps in smooth and efficient running of business.
- Inventory provides service to the customers immediately or at a short notice.
- Due to absence of stock, the company may have to pay high prices because of piece-wise purchasing.
- Maintainingof inventory may earn price discount because of bulk- purchasing.
- Inventory also acts as a buffer stock when raw materials are received late and so many sale-orders are likely to be rejected.
- Inventory also reduces product costs because there is an

INVENTORY:

- In broad sense, inventory may be defined as the stock of goods, commodities or other economic resources that are stored or reserved in order to ensure smooth and efficient running of business affairs.

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FORMS OF INVENTORY:

- The inventory or stock of goods may be kept in any of the following forms:
- Raw material inventory, i.e. raw materials which are kept in stock for using in the production of\ goods.
- Work-in-process inventory, i.e. semi finished goods or goods in process, which are stored during the production process.
- Finished goods inventory, i.e. finished goods awaiting shipment from the factory.
- Inventory also include: furniture, machinery, fixtures, etc. The term inventory may be classified in two main categories

I. DIRECT INVENTORY:

The items which playa direct role in the manufacture and become an integral part of finished goods are included in the category of direct inventories

CLASSIFICATION OF INVENTORIES:

A).Raw material inventories are provided:

- for economical bulk purchasing,
- to enable production rate changes
- to provide production buffer against delays in transportation,
- for seasonal fluctuations.

B) Work-in-process inventories are provided:

- to enable economical lot production,
- to cater to the variety of products
- for replacement of wastages,
- to maintain uniform production even if amount of sales may vary.

C) Finished-goods inventories are provided:

- for maintaining off-self delivery,
- to allow stabilization of the production level
- for sales promotion.

D)Spare parts.

II. IN-DIRECT INVENTORIES

Indirect inventories include those items, which are necessarily required for manufacturing but do not become the component of finished production, like: oil, grease, lubricants, petrol, and office-material maintenance material, etc.

INVENTORY DECISIONS:

The managers must take two basic decisions in order to accomplish the functions of inventory. The decisions made for every item in the inventory are:

- How much amount of an item should be ordered when the inventory of that item is to be replenished?
- When to replenish the inventory of that item

INVENTORY COSTS:

I. Holding Cost (C or Ch):

The cost associated with carrying or holding the goods in stock is known as holding or carrying cost which is usually denoted by C. or Ch per unit of goods for a unit of time components of holding cost:,

- Invested Capital Cost. This is the interest charge over the capital investment.
- Record Keeping and Administrative Cost.
- Handling Costs. These include all costs associated with movement of stock such as: cost of labour overhead cranes, gantries and other machinery required for this purpose.
- Storage Costs. These involve the rent of storage space or depreciation and interest even if the own space is used.
- Depreciation, Deterioration and Obsolescence Costs.

- Taxes and Insurance Costs. All these costs require careful study and generally amounts to I% to 2% of the invested capital.
- Purchase Price or Production Costs. Purchase price per- unit item is affected by the quantity

II. Shortage Costs or Stock-out Costs (C2 or C,).

The penalty costs that are incurred as a result of running out of stock (i.e., shortage) are known as shortage or stock-out costs. These are denoted by C2 or Cs per unit of goods (or a specified period.

These costs arise due to shortage of goods, sales may be lost, and good will may be lost either by a delay in meeting the demand or being quite unable to meet the demand at all.

III. SET-UP COSTS (C3 or Co):

these include the fixed cost associated with obtaining goods through placing of an order or purchasing or manufacturing or setting up a machinery before starting production. So they include costs of purchase, requisition, follow-up, receiving the goods, quality control, etc. These are also called order costs or replenishment costs, usually denoted by C3 or Co per production run (cycle).

If P is the purchase price of an item

and I is the stock holding cost per unit item expressed as a fraction of stock value (in rupees),

then the holding cost C. = IP.

ECONOMIC ORDERING QUANTITY (EOQ)

Economic Ordering Quantity (EOQ) is that size of order which minimizes total annual (or other time period as determined by individual firms) cost of carrying inventory and cost of ordering under the assumed conditions of certainty and that annual demands are known

CONDITION TO APPLY EOQ MODEL:

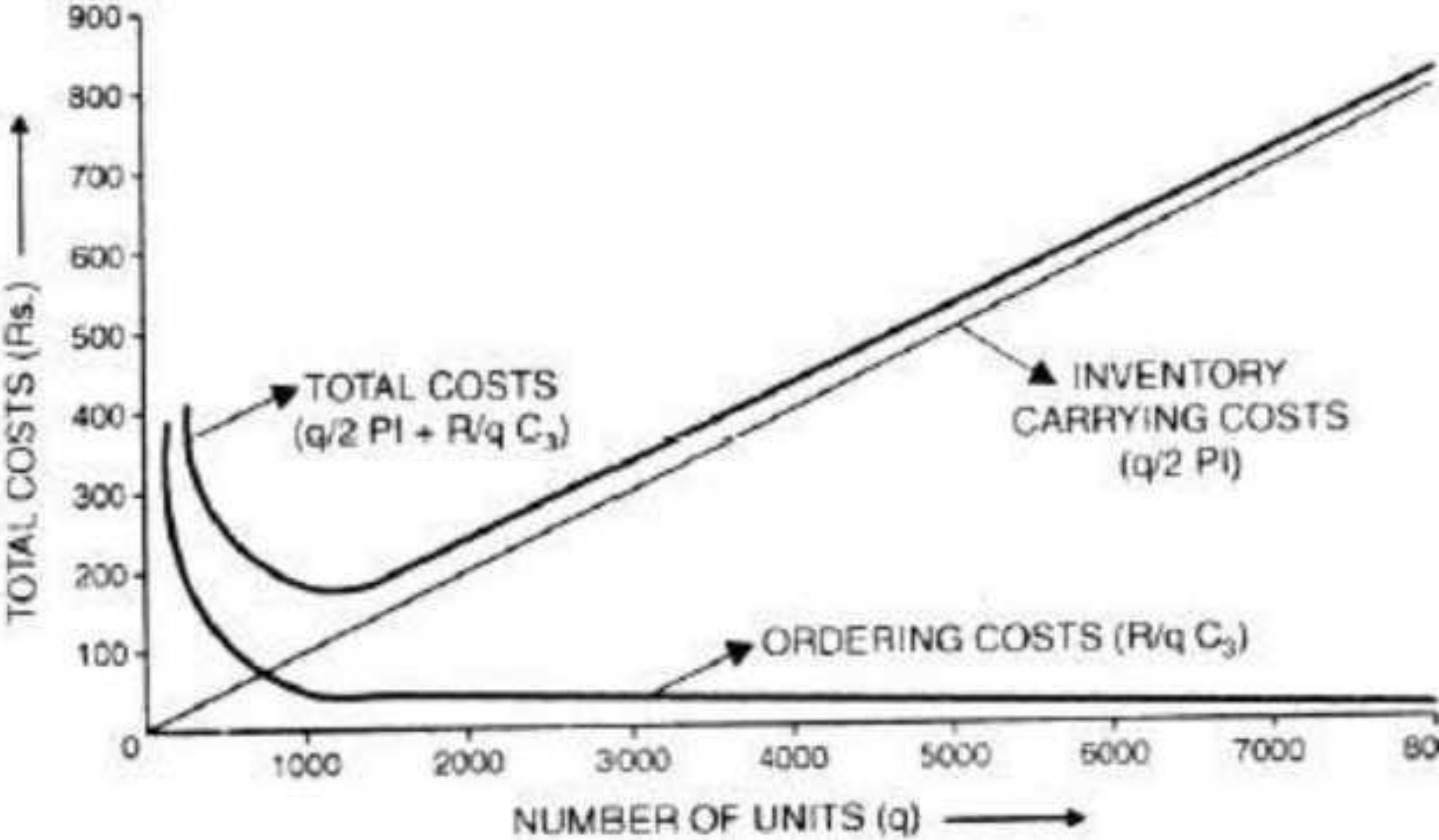
a. The item is replenished in lots or batches, either by purchasing or by manufacturing.

b. Consumption of items (or sales or usage rate) is uniform and continuous.

EOQ MODEL IS DESCRIBED UNDER THE FOLLOWING SITUATIONS:

- Planning period is one year.
- Demand is deterministic and indicated by parameter D units per year.
- Cost of purchases, or of one unit is C.
- Cost of ordering (or procurement cost of replenishment cost) is C3or Co ‘For manufacturing goods, it is known as set-up cost.
- Cost of holding stock (also known as inventory carrying cost) is C1or Ch per unit per year expressed either in items of cost per unit per period or in terms of percentage charge of the purchase price.
- Shortage cost (mostly it is back order cost) is C2or Cs per unit per year.
- Lead time is L, expressed in unit of time.
- Cycle period in replenishment is t.
- Order size is Q.

RELATION NUMBER OF UNITS WITH ORDERING COSTS, INVENRORY CARRYING COSTS AND TOTAL COSTS.



UNIT-V

WAITING LINES , DYNAMIC PROGRAMMING AND SIMULATION

Examples of waiting lines

, Example-1: waiting of customer at cinema ticket counter .

- The arriving people are called the customers
- The person issuing the tickets is called a server.

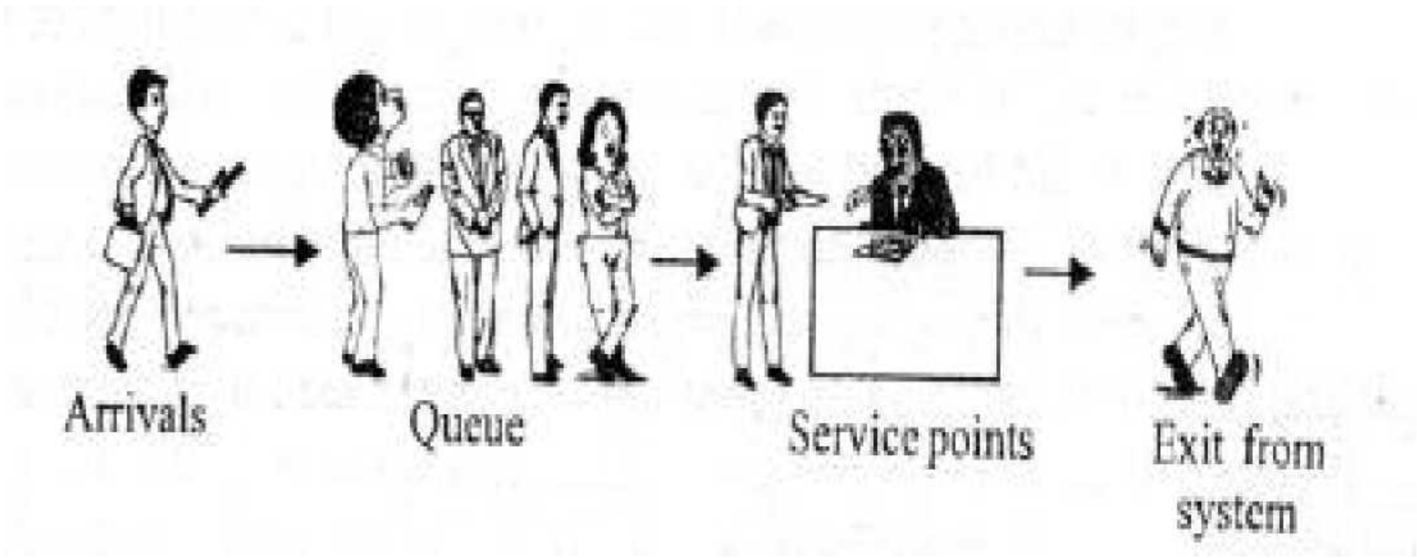
Example-2 : In a office letters arriving at a typist's desk.

- the letters represent the customers
- the typist represents the server.

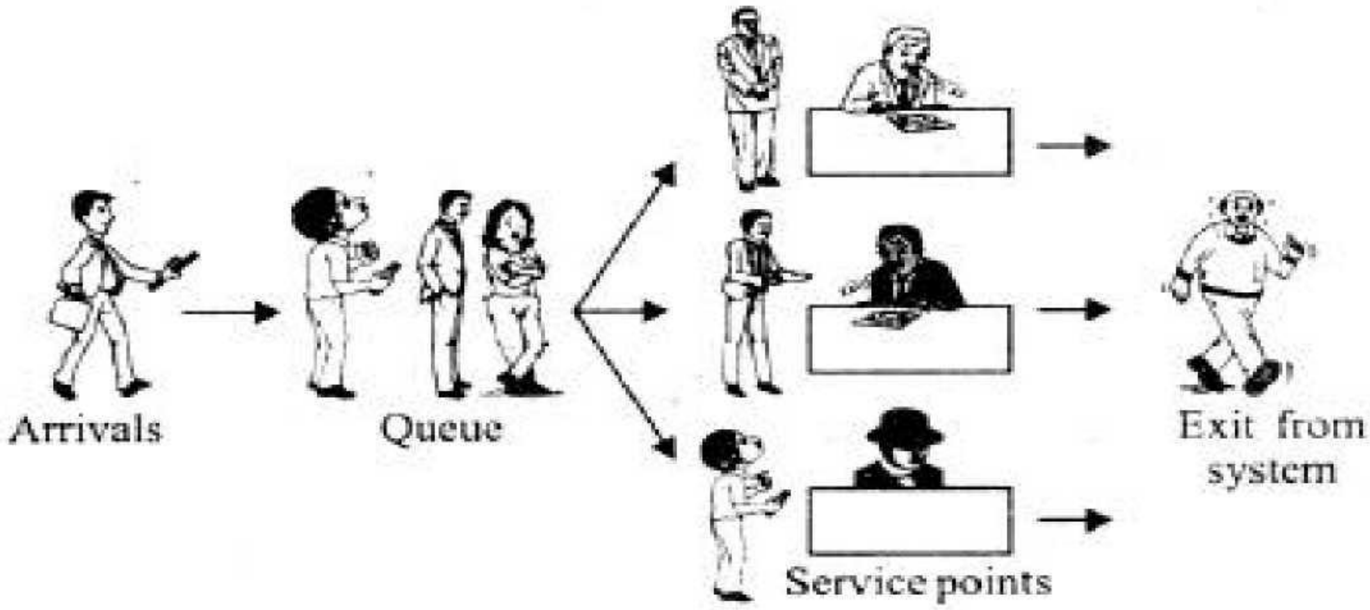
Example-3: machine breakdown situation.

- A broken machine represents a customer calling for the service of a repairman
- Service mechanic/Engineer is the server

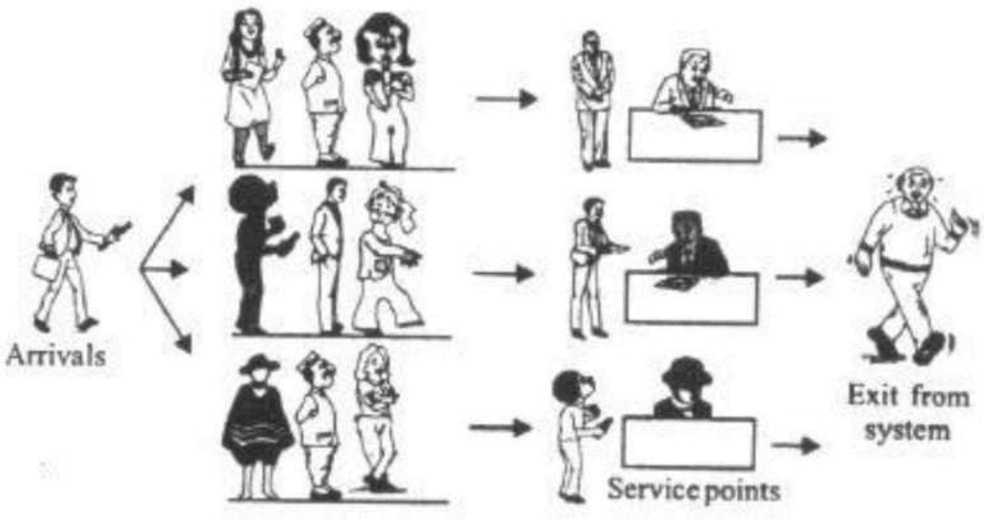
1. QUEUING SYSTEM WITH SINGLE QUEUE AND SINGLE SERVICE STATION.



2. QUEUING SYSTEM WITH SINGLE QUEUE AND SEVERAL SERVICE STATIONS.



3. QUEUING SYSTEM WITH SEVERAL QUEUES AND SEVERAL QUEUES



QUEUING THEORY:

Queuing theory is concerned with the statistical description of the behavior of queues with finding, *e.g.*, the probability distribution of the number in the queue from which the mean and variance of queue length and the probability distribution of waiting time for a customer, or the distribution of a server's busy periods can be found.

Meaning of a Queuing Model

A Queuing Model is a suitable model to represent a service- oriented problem where customers arrive randomly to receive some service, the service time being also a random var

Objective of a Queuing Model

The objective of a queuing model is to find out the optimum service rate and the number of servers so that the average cost of being in queuing system and the cost of service are minimized

PARAMETERS INVOLVED IN QUEUING MODEL

- Customer
- Server
- Queue Discipline
- Time Spent in the Queuing System

KENDALL'S NOTATION

Kendall's Notation is a system of notation according to which the various characteristics of a queuing model are identified. Kendall (Kendall, 1951) has introduced a set of notations, which have become standard in the literature of queuing models.

A general queuing system is denoted by (a/b/c) :(d/e)
where a = probability distribution of the inter arrival time.
b = probability distribution of the service time. c = number of servers in the system. d = maximum number of customers allowed in the system. e = queue discipline

. Traditionally, the exponential distribution in queuing problems is denoted by M.

Model-1: Thus (MIMI!): (00/ FIFO)

Model-1: Thus (MIMI!): (00/ FIFO) indicates a queuing system when the inter arrival times and service times are exponentially distributed having one server in the system with first in first out discipline and the number of customers allowed in the system can be infinite

Model-1: Thus (MIMI!): (00/ FIFO)

State of Queuing System

The transient state of a queuing system is the state where the probability of the number of customers in the system depends upon time. The steady state of a queuing system is the state where the probability of the number of customers in the system is independent *of* *t*.

Let Pn (t) indicate the probability of having n customers in the system at time *t*.
Then if Pn(t) depends upon *t*, the queuing system is said to be in the transient state.
After the queuing system has become operative *for* a considerable period of time, the probability Pn(t) may become independent of t

Poisson Process

When the number of arrivals in a time interval of length t follows a Poisson distribution with parameter (λt), which is the product of the arrival rate (A-)and the length of the interval t, the arrivals are said to follow a poison process

M/M/1 Queueing Model

The M/M/1 queuing model is a queuing model where the arrivals follow a Poisson process, service times are exponentially distributed and there is one server.

ASSUMPTION OF M/M/1 QUEUING MODEL:

1. The number of customers arriving in a time interval/ follows a Poisson Process with parameter A.
2. The interval between any two successive arrivals is exponentially distributed with parameter A.
3. The time taken to complete a single service is exponentially distributed with parameter
4. The number of server is one.
5. Although not explicitly stated both the population and the queue size can be infinity.
6. The order of service is assumed to be FIFO.

$$L_s = E(n) = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} n\left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

$$= \frac{\lambda}{\mu-\lambda} = \frac{\rho}{1-\rho}$$

. The expected number of customers in the queue is given by

l* = V (n-])P$_n$ = V nP„ =yp,

ii-/ tl- l ■I = I

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho}$$

Average waiting Lime of a customer in the system W_r ~

ju -A.

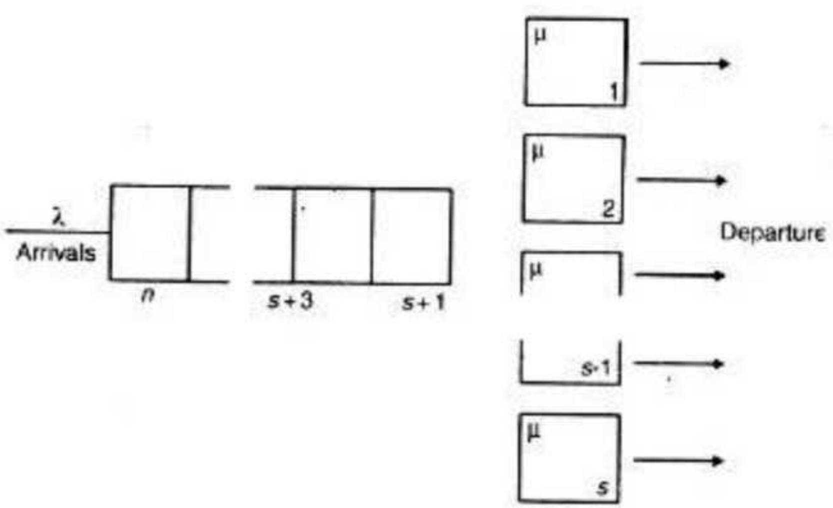
Average wailmg time of a customer m the queue W

$$_q = \frac{}{\mu(\mu}$$

PRACTICAL FORMULAE INVOLVED IN QUEUEING THEORY

1.	Arrival Rate per hour	$= \lambda$
2.	Service Rate per hour	$= \mu$
3.	Average Utilisation Rate (*or* Utilisation Factor), ρ	$= \dfrac{\lambda}{\mu}$
4.	Average Waiting Time in the System, (waiting and servicing Time) W_s	$= \dfrac{1}{(\mu - \lambda)}$
5.	Average Waiting Time in the Queue, W_q	$= \dfrac{\lambda}{\mu(\mu - \lambda)}$
6.	Average Number of Customers (including the one who is being served) in the System, L_s	$= \dfrac{\lambda}{(\mu - \lambda)}$

MODEL IV (A): (M | *M* \ *S*): (~ | *FCFS*)



SIMULATION

SIMULATION: simulation is the representative model for real situations.

Example: the testing of an air craft model in a wind tunnel from which the performance of the real aircraft is determined for being under fit under real operating conditions.

In the laboratories, we often perform a number of experiments on simulated models to predict the behavior of the real system under true environments.

SIMULATION

Definition: Simulation is a representation of reality through the use of a model or other device which will react in the same manner as reality under a given set of conditions

Simulation is the use of system model that has designed the characteristics of reality in order to produce the essence of actual operation

According to Donald G. Malcolm, a simulated model may be defined as one which depicts the working of a large scale system of men, machines, materials and information operating over period of time in a simulated environment of the actual real world conditions.

TYPES OF SIMULATION

- Analog simulation
- Computer simulation (System Simulation)

i Deterministic models

i. Stochastic model ***a*** Static models

b. Dynamic models

NEED FOR SIMULATION

- Simulation techniques allow experimentation with a model of real-life system rather than the actual operating system.
- Sometimes there is no sufficient time to allow the actual system to operate extensively.
- The non-technical manager can comprehend simulation more easily than a complex mathematical model.

The use of simulation enables a manager to provide insights into certain managerial problems.

SIMULATION PROCESS

Step-1. First define and identify the problem clearly.

- Step-2.Secondly, list the decision variable and decision rules of the problem.
- Step-3. Formulate the suitable model for the given problem.
- Step-4. Test the model and compare its behavior with the behavior of real problem situation.
- Step-5. Collect and identify the data required to test the model.

SIMULATION PROCESS

•Step-6. Execute (run) the simulation model.

•Step-7' the results of simulation run are then analyzed. If the simulation run is complete, then choose the best course of action, other wise , required changes are done in model decision variables, design or parameters and go to step-4.

•Step-8. Run the simulation again to find the new solution. Step-9. Validate the simulation.

LIMITATION OF SIMULATION

- Optimum results can not be attained
- The another difficulty lies in the quantification of the variables.
- In very large and complex problems, it becomes difficult to make the computer program on account of large number of variables and the involved interrelationships among them.
- Simulation is comparatively costlier, time consuming method in many situations.

TYPES OF SIMULATION MODELS

- Deterministic models

  In this type of models, the input and output variables cannot be random variables and can be described by exact functional relationships.

- Probabilistic models

  In these models, method of random sampling is used. This technique is called ‘Monte-Carlo Technique.

- Static Models:

  In these types of models, the variable time cannot be taken into account consideration.

- Dynamic Models:

  These models deal with time varying interaction.

PHASES OF SIMULATION MODEL

Phase-1: Data collection

Data generation involves the sample observation of variables and can be carried with the help of following methods.

- Using random number tables.
- Resorting to mechanical devices.)example: roulettes wheel)
- Using electronic computers

Phase -2

Phase -2. Book-keeping

Book-keeping phase of a simulation model deals with updating the system when new events occur, monitoring and recording the system states as and when they change, and keeping track of quantities of our interest (such as idle time and waiting time) to compute the measure of effectiveness.

GENERATION OF RANDOM NUMBERS

For clear understanding, the following perameters are be defined

- Random variable:

it refers to a particular outcome of an experiment.

- Random Number:

it refers to a uniform random variable or numerical value assigned to a random variable following uniform probability density function. (ie., normal, poison , exponential, etc).

- Pseudo-random Numbers:

Random numbers are called pseudo-random numbers when they are generated by aome deterministic process but they qualify the predetermined statistical test for randomness.

MONTE-CARLO SIMULATION

STEPS INVOLVED

1) First define the problem by Identifying the objectives of the problem
2) Identifying the main factors having the greatest effect on the objective of the problem
3) Construct an appropriate model by Specifying the variables and parameters of the model
4) Formulating the suitable decision rules.
5) Identifying the distribution that will be used.

- Specifying the number in which time will change,
- Defining the relationship between the variables and parameters.

3) Prepare the model for experimentation

- Defining the starting conditions for the simulation
- Specifying the number of runs of simulation.

4) Using step1 to step 3, test the model by

- Defining a coding system that will correlate the factors defined in step1 with random numbers to be generated for simulation.
- Selecting a random generator and creating the random numbers to be used in the simulation.
- Associating the generated random numbers with the factors as identified in step1 and coded in step-4

MONTE-CARLO SIMULATION STEPS INVOLVED . ... Contd.

5) Summarize and examine the results as obtained in.

6) Evaluate the results of the simulation

7) Formulate proposals for advice to management on the course of action to be adopted and modify the model, if required.

APPLICATION OF SIMULATION

- Simulation model can be applied for Solving Inventory problems
- Simulation model can be applied for solving Queuing problems

THANK YOU