INSTITUTE OF AERONAUTICAL ENGINEERING
(Autonomous)
Dundigal, Hyderabad - 500043

## MODEL QUESTION PAPER-I

## B. Tech IV Semester End Examinations (Regular), May-2020 Regulation: IARE-R18 <br> COMPLEX ANALYSIS AND PROBABILITY DISTRIBUTIONS

(EEE)
Max Marks: 70
Time: 3 Hours

Answer ONE Question from each Module<br>All Questions Carry Equal Marks<br>All parts of the question must be answered in one place only

## MODULE-1

1. (a) Define the term Continuity of a complex function $f(z)$.Justify whether every differentiable function is continuous or not. Give a valid example.
(b) Examine the complex variable function $\mathrm{f}(\mathrm{z})=\frac{x-i y}{x^{2}+y^{2}}$ for analyticity in Cartesian form.
2. (a) Define the term Analyticity and Differentiability of a complex function f(z).Prove that an analytic function with constant real part is always constant.
(b) Find an analytic function $f(\mathrm{z})$ whose real part of an analytic function is $\mathrm{u}=\frac{\sin 2 x}{\cos \operatorname{tin} y-\cos 2 x}$ by Milne-Thompson method.

## MODULE-2

3. (a) Define the term power series expansion of a complex functions. Write the Cauchy's integral and general integral formulae for multiple connected regions.
(b) Verify Cauchy's theorem for the integral of $z^{3}$ taken over the boundary of the rectangle formed with the vertices $-1,1,1+\mathrm{i},-1+\mathrm{i}$.
(a)

Define the term Line integral. Estimate the value of line integral to $\int_{z=0}^{z=1+1}\left[x^{2}+2 x y+i\left(y^{2}-z\right)\right] d z$ along the curve $y=x^{2}$.
(b) Estimate the value of line integral to $\int_{c} \frac{z^{4}-3 z^{2}+6}{(z+1)^{3}} d z$ where $c$ is the circle $|z|=2$ using Cauchy's integral formula.

## MODULE-3

5. (a) State Taylor's and Laurent's series theorem of complex power series. State Cauchy's Residue theorem of an analytic function $\mathrm{f}(\mathrm{z})$ within and on the closed curve.
(b) Obtain Laurent's series expansion of $\mathrm{f}(\mathrm{z})=\frac{z^{2}-4}{z^{2}+5 z+4}$ valid in $1<\mathrm{z}<4$.
6. 

(a) Define the following terms
(i) The Isolated singularity of an analytic function $\mathrm{f}(\mathrm{z})$
(ii) Pole of order m of an analytic function $\mathrm{f}(\mathrm{z})$
(iii) Essential and Removable singularities of an analytic function $\mathrm{f}(\mathrm{z})$
(b) Estimate the value of $\int_{0}^{\pi} \frac{d \theta}{(a+b \cos \theta)}$ using Residue theorem.

## MODULE-4

7. (a) Define Discrete random variables and Mass function, density function of a probability distribution.
(b) Let X denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once. Determine (i) Discrete probability distribution (ii) Expectation (iii) Variance
8. (a) Define Mathematical expectation. Moment about origin, Central moments, Moment generating function of probability distribution
(b) If $\mathrm{f}(\mathrm{x})=\mathrm{k} e^{-|x|}$ is probability density function in the interval, $-\infty<x<\infty$, then find i) $k$ ii) Mean iii) Variance iv) $\mathrm{P}(0<\mathrm{x}<4)$

## MODULE-5

9. (a) Describe about the Binomial distributions and their properties
(b) Out of 20 tape recorders 5 are defective. Find the standard deviation of defective in the sample of 10 randomly chosen tape recorders. Find (i) $\mathrm{P}(\mathrm{X}=0)$ (ii) $\mathrm{P}(\mathrm{X}=1)$ (iii) $\mathrm{P}(\mathrm{X}=2)$ (iv) $\mathrm{P}(1<\mathrm{X}<4)$.

10 (a) Describe about the Normal distribution .
(b) The marks obtained in Statistics in a certain examination found to be normally distributed. If $15 \%$ of the students greater than or equal to 60 marks, $40 \%$ less than 30 marks. Find the mean and standard deviation.

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## COURSE OBJECTIVES:

The course should enable the students to:

| I | Understand the basic theory of complex functions to express the power series. |
| :---: | :--- |
| II | Evaluate the contour integration using Cauchy residue theorem. |
| III | Enrich the knowledge of probability on single random variables and probability distributions. |

## COURSE OUTCOMES (COs):

| CO 1 | Discuss about continuity/differentiability/analyticity of a Complex function using Cauchy-Riemann <br> Equations. Estimate complex conjugate using Milne Thomson method and understand the concept of <br> Bilinear transformation. |
| :---: | :--- |
| CO 2 | Recognize and apply the Cauchy's integral formula and the generalized Cauchy's integral formula. <br> Evaluate complex functions as power series and radius of convergence of power series |
| CO 3 | Establish the contour integral with an integrand which has singularities lying inside or outside the <br> simple closed contour. Expand complex function as power series using Taylor's and Laurent series.. |
| CO 4 | Enrich the knowledge of Probability to discrete and continuous random variables. |
| CO 5 | Analyze probability distributions for Binomial, Poisson and normal distributions and study its <br> properties. |

## COURSE LEARNING OUTCOMES (CLOs):

| AHSB06.01 | Recall continuity, differentiability, analyticity of a function using limits. |
| :--- | :--- |
| AHSB06.02 | Interpret the conditions for a complex variable to be analytic and/or entire function. |
| AHSB06.03 | Interpret the concepts of Cauchy-Riemann relations and harmonic functions. |
| AHSB06.04 | Analyze the Bilinear transformation by cross ratio property. |
| AHSB06.05 | Identify the conditions of fixed and critical point of Bilinear Transformation. |
| AHSB06.06 | Demonstrate the area under a curve using the concepts of indefinite integration. |
| AHSB06.07 | Interpret the concepts of the Cauchy's integral formula and the generalized Cauchy's <br> integral formula. |
| AHSB06.08 | Demonstrate complex functions as power series and radius of convergence of power series. |
| AHSB06.09 | Interpret the concept of complex integration to the real-world problems of flow with <br> circulation around a cylinder. |
| AHSB06.10 | Asses the Taylor's and Laurent series expansion of complex functions. |
| AHSB06.11 | Interpret the concept of different types of singularities for analytic function. |


| AHSB06.12 | Identify the poles, residues and solve integrals using Cauchy's residue theorem. |
| :--- | :--- |
| AHSB06.13 | Interpret the concept of Cauchy's residue theorem to the real-world problems of Quantum <br> Mechanical scattering and Quantum theory of atomic collisions. |
| AHSB06.14 | Demonstrate an understanding of the basic concepts of probability and random variables. |
| AHSB06.15 | Classify the types of random variables and calculate mean, variance. |
| AHSB06.16 | Estimate moment about origin, central moments, moment generating function of probability <br> distribution. |
| AHSB06.17 | Recognize where the Binomial distribution could be appropriate model of the distributions. |
| AHSB06.18 | Recognize where the Poisson distribution could be appropriate model of the distributions. |
| AHSB06.19 | Recognize where the Binomial distribution and Poisson distribution could be appropriate to find <br> mean, variance of the distributions. |
| AHSB06.20 | Apply the inferential methods relating to the means of normal distributions. |
| AHSB06.21 | Interpret Binomial distribution to the phenomena of real-world problem like sick versus healthy. |
| AHSB06.22 | Identify the mapping of Normal distribution in real-world problem to analyze the stock market. |
| AHSB06.23 | Use Poisson distribution in real-world problem to predict soccer scores. |
| AHSB06.24 | Possess the knowledge and skills for employability and to succeed in national and international <br> level competitive examinations. |

## MAPPING OF SEMESTER END EXAMINATIONS TO COURSE LEARNING OUTCOMES:

| SEE <br> Question <br> No. |  | CLO Code | Course learning Outcomes | CO | Blooms Taxonomy Level |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | a | AHSB06.01 | Recall continuity, differentiability, analyticity of a function using limits. | CO 1 | Remember |
|  | b | AHSB06.02 | Interpret the conditions for a complex variable to be analytic and/or entire function. | CO 1 | Understand |
| 2 | a | AHSB06.01 | Recall continuity, differentiability, analyticity of a function using limits. | CO 1 | Remember |
|  | b | AHSB06.03 | Interpret the concepts of Cauchy-Riemann relations and harmonic functions. | CO 1 | Understand |
| 3 | a | AHSB06.07 | Interpret the concepts of the Cauchy's integral formula and the generalized Cauchy's integral formula. | CO 2 | Remember |
|  | b | AHSB06.06 | Demonstrate the area under a curve using the concepts of indefinite integration. | CO 2 | Understand |
| 4 | a | AHSB06.06 | Demonstrate the area under a curve using the concepts of indefinite integration. | CO 2 | Remember |
|  | b | AHSB06.07 | Interpret the concepts of the Cauchy's integral formula and the generalized Cauchy's integral formula. | CO 2 | Understand |
| 5 | a | AHSB06.10 | Asses the Taylor's and Laurent series expansion of complex functions. | CO 3 | Remember |
|  | b | AHSB06.10 | Asses the Taylor's and Laurent series expansion of complex functions. | CO 3 | Understand |
| 6 | a | AHSB06.11 | Interpret the concept of different types of singularities for analytic function. | CO 3 | Remember |


| SEE <br> Question No. |  | CLO Code | Course learning Outcomes | CO | Blooms Taxonomy Level |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | b | AHSB06.12 | Identify the poles, residues and solve integrals using Cauchy's residue theorem. | CO 3 | Understand |
| 7 | a | AHSB06.14 | Demonstrate an understanding of the basic concepts of probability and random variables. | CO 4 | Remember |
|  | b | AHSB06.15 | Classify the types of random variables and calculate mean, variance. | CO 4 | Understand |
| 8 | a | AHSB06.16 | Estimate moment about origin, central moments, moment generating function of probability distribution. | CO 4 | Remember |
|  | b | AHSB06.16 | Classify the types of random variables and calculate mean, variance. | CO 4 | Understand |
| 9 | a | AHSB06.17 | Recognize where the binomial distribution and Poisson distribution could be appropriate model and find mean, variance of the distributions. | CO 5 | Remember |
|  | b | AHSB06.17 | Recognize where the binomial distribution and Poisson distribution could be appropriate model and find mean, variance of the distributions. | CO 5 | Understand |
| 10 | a | AHSB06.20 | Apply the inferential methods relating to the means of normal distributions. | CO 5 | Remember |
|  | b | AHSB06.20 | Apply the inferential methods relating to the means of normal distributions. | CO 5 | Understand |

Signature of Course Coordinator

