Institute of Aeronautical Engineering
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APPLIED PHYSICS

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UNIT – I
DIELECTRICS AND MAGNETISM

Introduction

- Dielectrics are insulation materials.
- In dielectrics, all the electrons are bound to their parent molecules and there are no free charges.
- Even with normal voltage or thermal energy electrons are not released.
- Dielectrics are non metallic materials of high specific resistance and have negative temperature coefficient of resistance.

Fundamental definitions

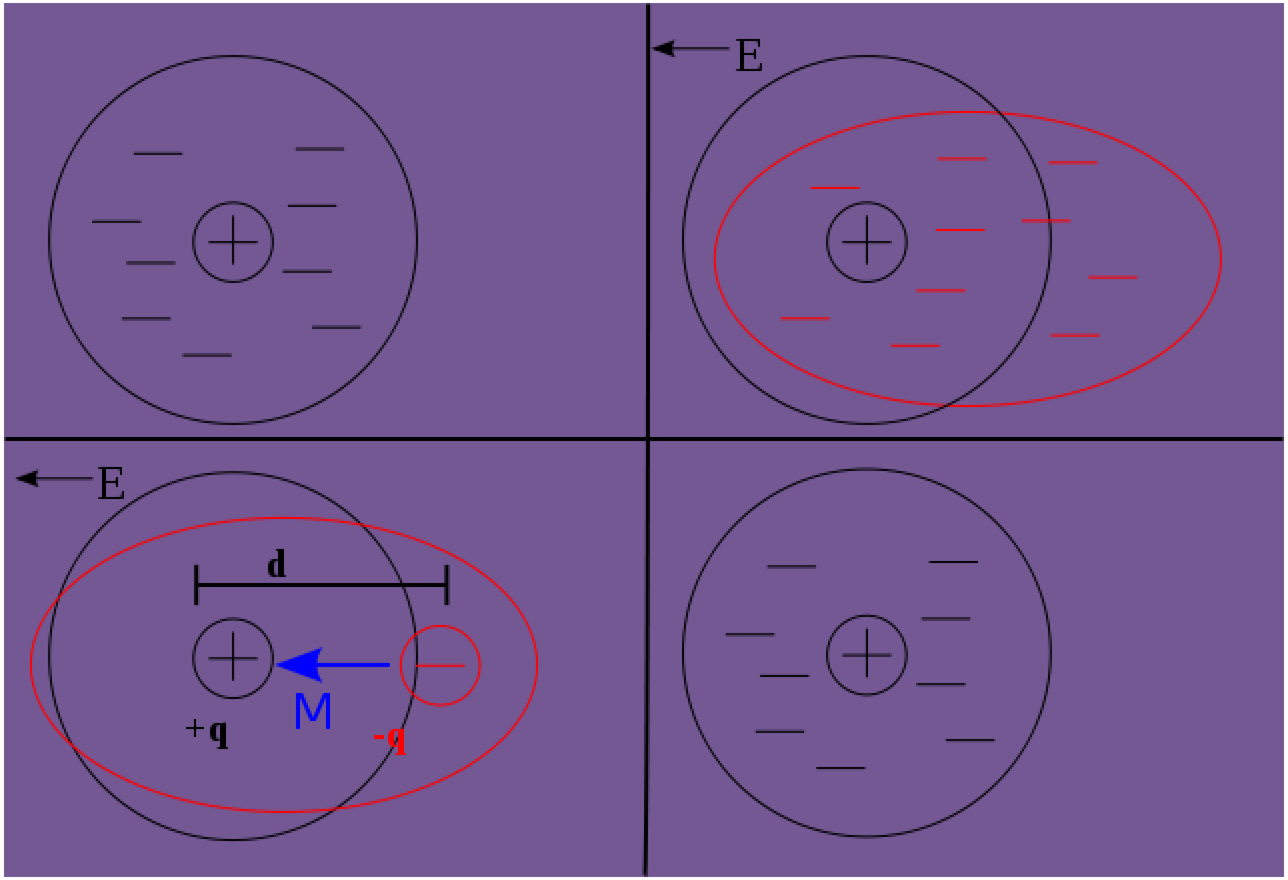
- **Dielectric constant (ε_r) :** The ratio between the permittivity of the medium (ε) and the permittivity of free space (ε_0).

$$\varepsilon_r = \varepsilon / \varepsilon_0$$

- The dielectric characteristics of a material are determined by the dielectric constant or relative permittivity ε_r of the material.
- Since it is a ratio of same quantity. ε_r has no unit. It is a measure of polarization in the dielectric material.

Fundamental definitions

- **Electric Polarization:**



Fundamental definitions

- **Electric Dipole** : Let us consider an atom placed inside an electric field. The centre of positive charge is displaced along the applied field direction while the centre of negative charge is displaced in the opposite direction. Thus a **dipole** is produced.
- When a dielectric material is placed inside an electric field such dipoles are created in all the atoms inside.

Fundamental definitions

- **This process of producing electric dipoles which are oriented along the field direction is called polarization in dielectrics.**
- **Polarizability (α): When the strength of the electric field E is increased the strength of the induced dipole μ also increases. Thus the induced dipole moment is proportional to the intensity of the electric field.**

i.e. $\mu=\alpha E$

Fundamental definitions

- **Where α, the constant of proportionality is called Polarizability. It can be defined as *induced dipole moment per unit electric field*.**
- **Polarization vector P: *The dipole moment per unit volume of the dielectric material is called Polarization vector P***
- If μ is the average dipole moment per molecule and N is the number of molecules per unit volume the polarization vector $P = N\mu$

Fundamental definitions

- The dipole moment per unit volume of the solid is the sum of all the individual dipole moments within that volume and is called the polarization P of the solid.

Fundamental definitions

- **Electric flux density (or) Electric displacement D:**

$$\mathbf{D = \varepsilon_r \varepsilon_0 E}$$

When E = electric field,

ε_r= Dielectric constant or permittivity of the material

ε_o= Dielectric constant or permittivity of the free space.

Fundamental definitions

- As the polarization measures the additional flux density arising from the presence of the material as compared to free space, it has the same units as D and is related to it as

$$D = \varepsilon_o E + P$$

Since $D = \varepsilon_r \varepsilon_0 E$

$$\therefore \varepsilon_r \varepsilon_0 E = \varepsilon_o E + P$$

$$P = \varepsilon_o(\varepsilon_r - 1)E$$

Electric flux density D is similar to magnetic induction B in magnetism

Fundamental definitions

- **Electric Susceptibility χ_e :.**

The polarization vector P is proportional to the total electric flux density E and is in the same direction of E.

Therefore the polarization vector can be written as

$$\mathbf{P} = \varepsilon_o \chi_e \mathbf{E}$$

Where the constant χ_e is the electric susceptibility

Fundamental definitions

$$\chi_e = P / \varepsilon_o E$$

$$= \varepsilon_o(\varepsilon_r-1)E / \varepsilon_o E$$

$$\chi_e = \varepsilon_r-1$$

Polarization

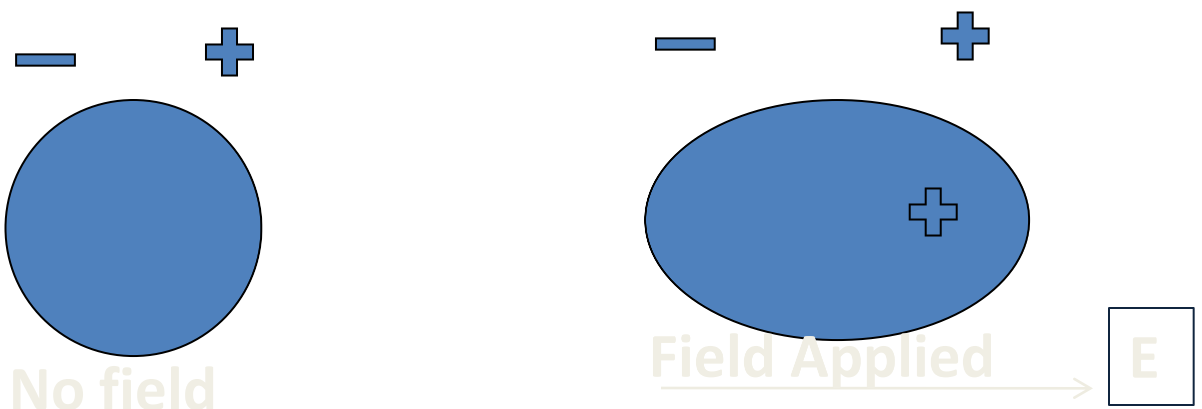
- **The specimen is placed inside a d.c. electric field, polarization is due to four types of process**
- **Electronic polarization**
- **Ionic polarization**
- **Orientation polarization**

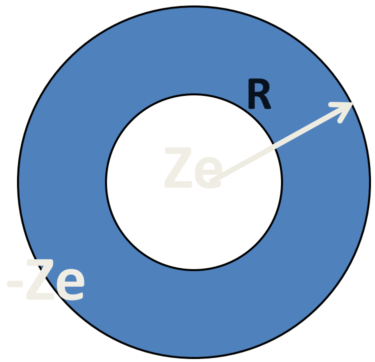
- **Electronic Polarization:** **The displacement of the positively charged nucleus and the (negative) electrons of an atom in opposite directions, on application of an electric field, result in electronic polarization.**

Induced dipole moment μ α E

$$\mu = \alpha_e E$$

Where α_e is the electronic Polarizability.





Here the Nucleus of charge Ze is surrounded by an electron cloud of charge – Ze distributed in a sphere of radius R. The charge density ρ is given by

$$\rho = \frac{-Ze}{{}^{4}/_{3}\,\pi R^{3}} = \frac{-3}{4}\left(\frac{Ze}{\pi R^{3}}\right)$$

When an external field of intensity E is applied, the nucleus and the electrons experience Lorentz forces of magnitude ZeE in opposite directions. Hence the nucleus and electron cloud are pulled apart.

- **When they are separated a coulomb force develops between them, which tends to oppose the displacement.**
- **When these forces namely Lorentz force and Coulomb force are equal and opposite, equilibrium is reached.**
- **Let x be the displacement under the condition**
- **Since nucleus is much heavier than the electron cloud it is assumed that only the electron cloud is displaced when the external field is applied**

Lorentz force = -ZeE

Coulomb force

$$= Ze \times \frac{\text{Charge enclosed in the sphere of radius } x}{4\pi\varepsilon_o x^2}$$

$$\text{The charge enclosed} = {}^{4}\!/_{3}\,\pi x^3 \rho$$

$$= {}^{4}\!/_{3}\,\pi x^3 \left[\frac{3}{4}\left(\frac{Ze}{\pi R^3}\right)\right]$$

$$= \frac{Zex^3}{R^3}$$

Hence coulomb force is

$$\frac{Ze}{4\pi\varepsilon_0 x^2}\left(\frac{Zex^3}{R^3}\right) = \frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}$$

In the equilibrium position

$$-ZeE = \frac{Z^2e^2x}{4\pi\varepsilon_0 R^3}$$

$$E = \frac{Zex}{4\pi\varepsilon_0 R^3}$$

Or

$$x = \frac{4\pi\varepsilon_0 R^3 E}{Ze}$$

- Thus the displacement of the electron cloud is proportional to the applied field.
- Thus the two electric charges +Ze and –ze are separated by a distance x under the action of the applied field thus consisting induced electric dipoles.
- Induced electric dipole moment

$$\mu_e = Zex = \frac{Ze 4\pi\varepsilon_0 R^3 E}{Ze}$$

$$= 4\pi\varepsilon_0 R^3 E$$

i.e., $\mu_e = \alpha_e E$

- Where $\alpha_e = 4\pi\varepsilon_0 R^3$ is called electronic Polarizability. **The dipole moment per unit volume is called electronic polarization.** It is independent of temperature.

$$P_e = N\mu_e = N\alpha_e E$$

Where N is the number of atoms/m^3

From $P_e = \varepsilon_0 E(\varepsilon_r - 1) = N\alpha_e E$

Or $$(\varepsilon_r - 1) = \frac{N\alpha_e}{\varepsilon_0}$$

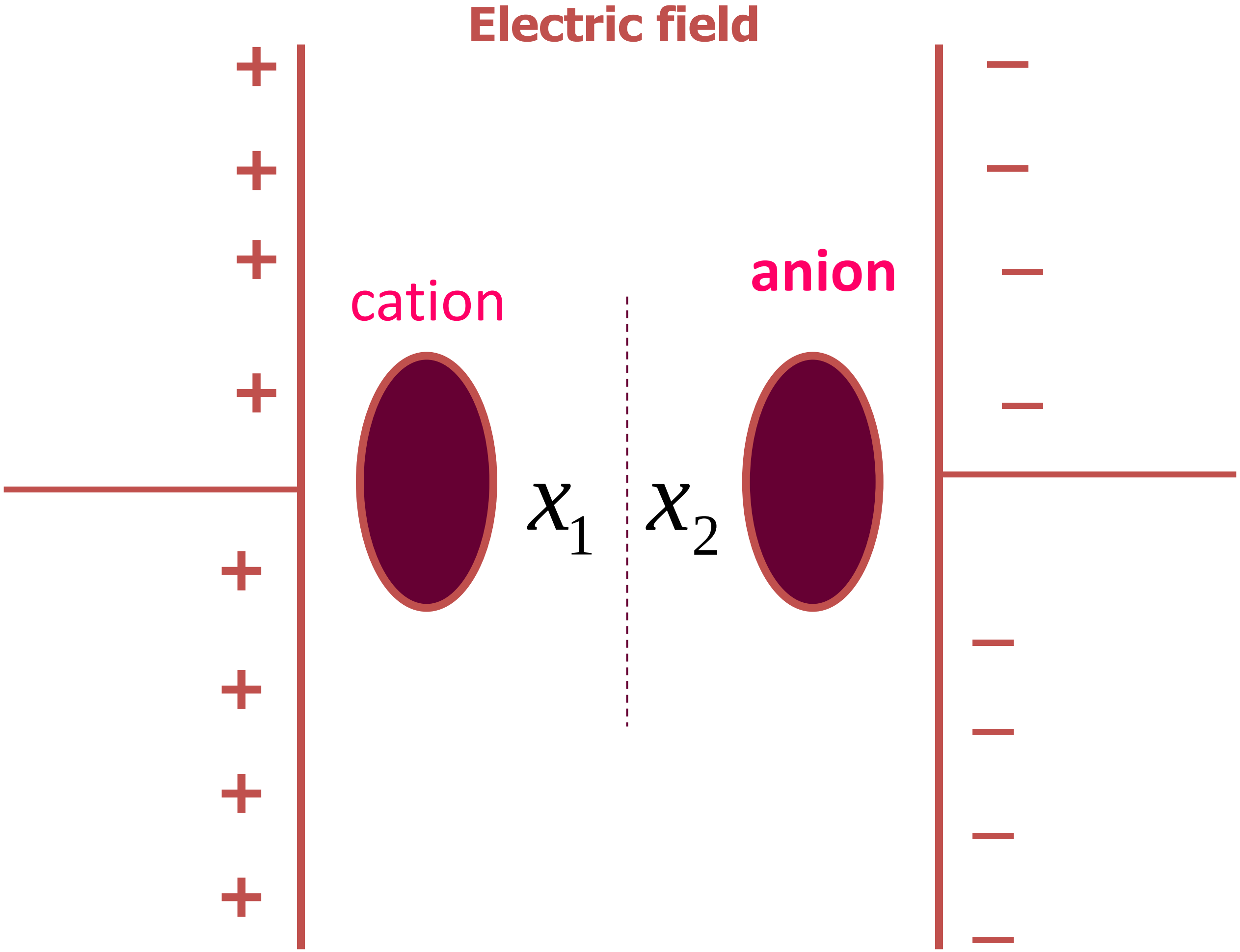
$$\alpha_e = \frac{\varepsilon_0(\varepsilon_r - 1)}{N}$$

Ionic polarization

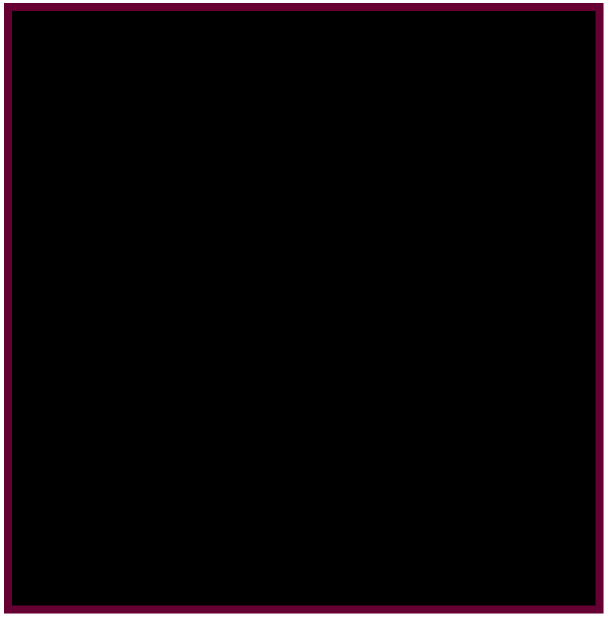
- The ionic polarization occurs, when atoms form molecules and it is mainly due to a relative displacement of the atomic components of the molecule in the presence of an electric field.

- When a EF is applied to the molecule, the positive ions displaced by X_1 to the negative side electric field and negative ions displaced by X_2 to the positive side of field.

- The resultant dipole moment $\mu = q\,(X_1 + X_2)$..


Electric field
cation
anion
x_1
x_2

Restoring force constant depend upon the mass of the ion and natural frequency and is given by



Where 'M' mass of anion and 'm' is mass of cat ion

[redacted]

This polarization occurs at frequency 10^{13} Hz (IR).

It is a slower process compared to electronic polarization.

It is independent of temperature.

Orientational Polarization

It is also called dipolar or molecular polarization. The molecules such as H_2 , N_2,O_2,Cl_2 ,CH_4,CCl_4 etc., does not carry any dipole because centre of positive charge and centre of negative charge coincides. On the other hand molecules like CH_3Cl, H_2O,HCl, ethyl acetate (polar molecules) carries dipoles even in the absence of electric field.

How ever the net dipole moment is negligibly small since all the molecular dipoles are oriented randomly when there is no EF. In the presence of the electric field these all dipoles orient them selves in the direction of field as a result the net dipole moment becomes enormous.

- It occurs at a frequency 10^6 Hz to 10^{10}Hz.
- It is slow process compare to ionic polarization.
- It greatly depends on temperature.

Expression for orientation polarization

$$P_o = N.\vec{\mu}_{orie} \Rightarrow \frac{N.\mu_{orie}^2.E}{3kT} = N.\alpha_o.E$$

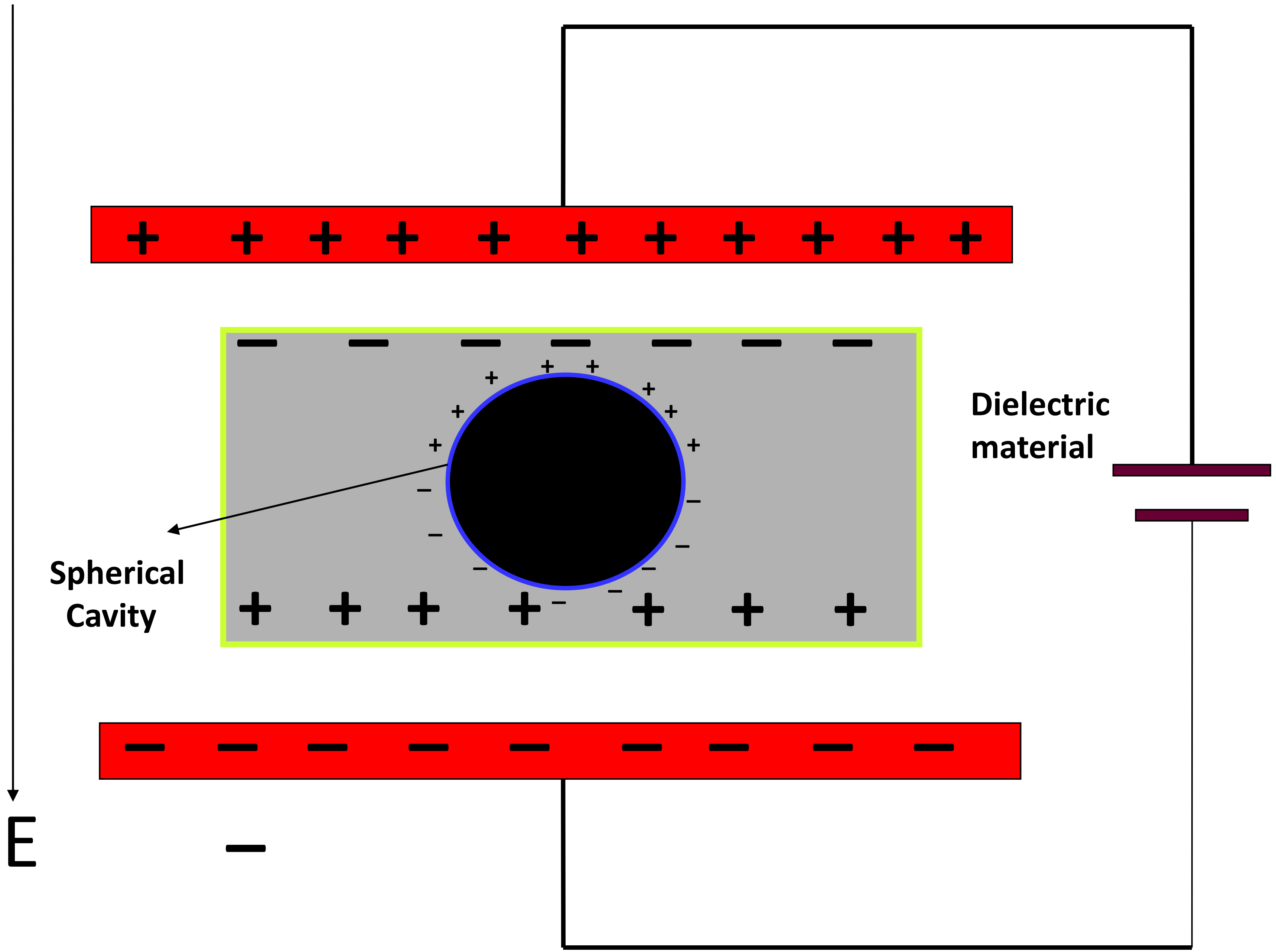
$$\alpha_o = \frac{\mu_{orie}^2}{3kT}$$

$$\therefore \alpha = \alpha_{elec} + \alpha_{ionic} + \alpha_{ori} = 4\pi\varepsilon_o R^3 + \frac{e^2}{w_0^2}\left[\frac{1}{M} + \frac{1}{m}\right] + \frac{\mu_{ori}^2}{3kT}$$

This is called Langevin – Debye equation for total Polaris ability in dielectrics.

Internal fields or local fields in Solids

Local field or internal field in a dielectric is the space and time average of the electric field intensity acting on a particular molecule in the dielectric material.


Dielectric
material
Spherical
Cavity
E

Evaluation of internal field

Consider a dielectric be placed between the plates of a parallel plate capacitor and let there be an imaginary spherical cavity around the atom A inside the dielectric.

The internal field at the atom site 'A' can be made up of four components E_1 ,E_2, E_3 & E_4.

Evaluation of internal field

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Field E_1:

E_1 is the field intensity at A due to the charge density on the plates

$$E_1 = \frac{D}{\varepsilon_0}$$

$$D = \varepsilon_0 E + P$$

$$E_1 = \frac{\varepsilon_0 E + P}{\varepsilon_0}$$

$$E_1 = E + \frac{P}{\varepsilon_0} \text{..........}(1)$$

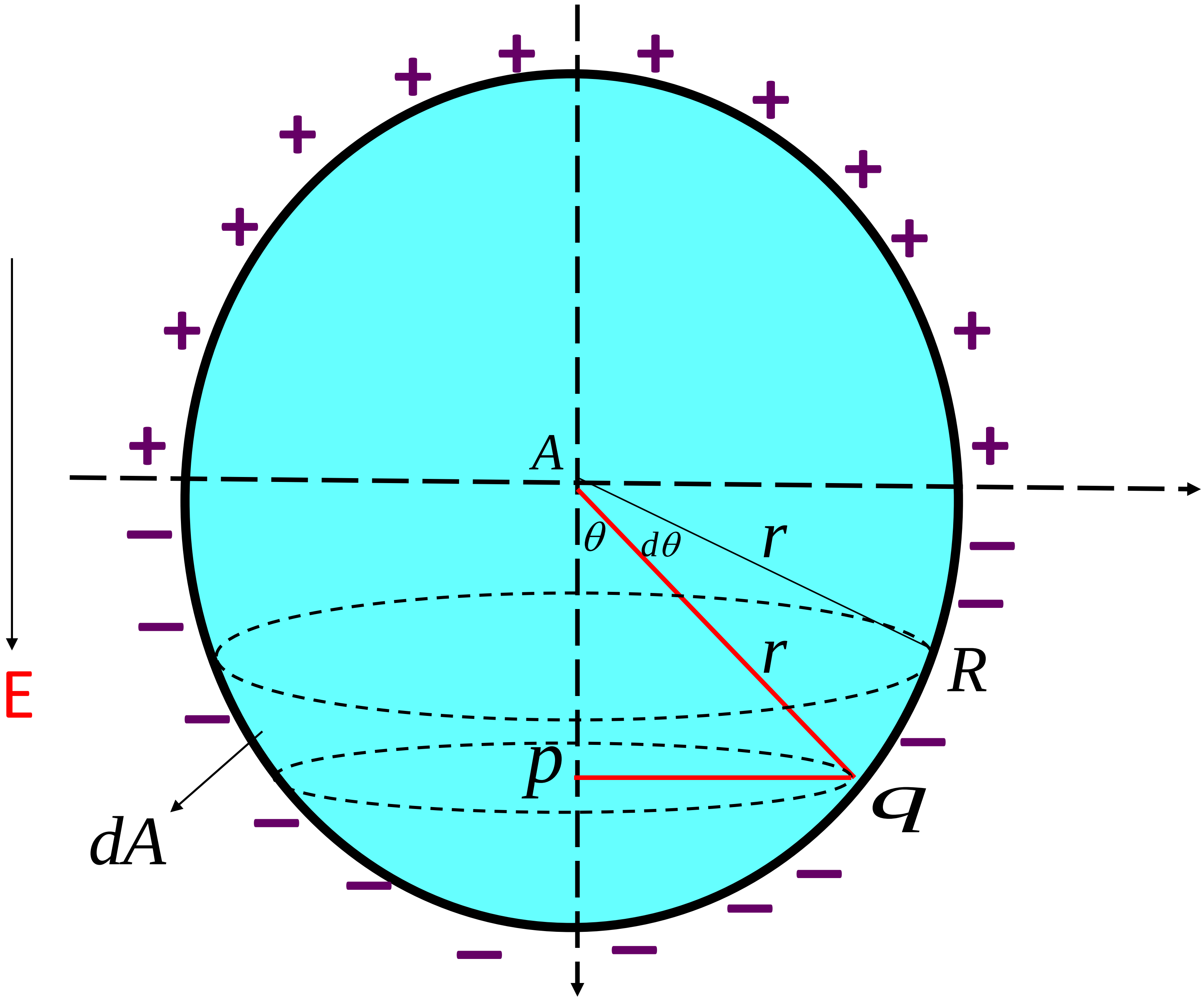
Field E_2:

E_2 is the field intensity at A due to the charge density induced on the two sides of the dielectric.

$$E_2 = \frac{-P}{\varepsilon_0} \quad \text{...........(2)}$$

Field E_3:

E_3 is the field intensity at A due to the atoms contained in the cavity, we are assuming a cubic structure, so $E_3 = 0$.



E
A
θ
dθ
r
r
R
p
q
dA

Field E_4:

1.This is due to polarized charges on the surface of the spherical cavity.

$$dA = 2\pi.pq.qR$$

$$dA = 2\pi.r\sin\theta.rd\theta$$

$$dA = 2\pi.r^2\sin\theta d\theta$$

Where dA is Surface area between θ & θ+dθ…

2.The total charge present on the surface area dA is…

dq = (normal component of polarization) X (surface area)

$$dq = p\cos\theta \times dA$$

$$dq = 2\pi r^2 p\cos\theta.\sin\theta.d\theta$$

3.The field due to this charge at A, denoted by dE_4 is given by

$$dE_4 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The field in θ = 0 direction

$$dE_4 = \frac{1}{4\pi\varepsilon_0}\frac{dq\cos\theta}{r^2}$$

$$dE_4 = \frac{1}{4\pi\varepsilon_0 r^2}(2\pi r^2 p\cos\theta.\sin\theta.d\theta)\cos\theta$$

$$dE_4 = \frac{P}{2\varepsilon_0}\cos^2\theta.\sin\theta.d\theta$$

4.Thus the total field E_4 due to the charges on the surface of the entire cavity is

$$E_4 = \int_0^{\pi} dE_4$$

$$= \int_0^{\pi} \frac{P}{2\varepsilon_0} \cos^2\theta.\sin\theta.d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_0^{\pi} \cos^2\theta.\sin\theta.d\theta$$

$$let..x = \cos\theta \rightarrow dx = -\sin\theta d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_1^{-1} x^2.dx$$

$$= \frac{-P}{2\varepsilon_0} \left(\frac{x^3}{3}\right)_1^{-1} \Rightarrow \frac{-P}{2\varepsilon_0} \left(\frac{-1-1}{3}\right)$$

$$E_4 = \frac{P}{3\varepsilon_0}$$

The internal field or Lorentz field can be written as

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$E_i = (E + \frac{p}{\varepsilon_o}) - \frac{p}{\varepsilon_o} + 0 + \frac{p}{3\varepsilon_o}$$

$$E_i = E + \frac{p}{3\varepsilon_o}$$

MAGNETIC PROPERTIES

Introduction to magnetic properties

- Magnetic materials play a prominent role in modern technology.
- They are widely used in industrial electronics and computer industry.
- The traditional methods of information storage and retrieval are rapidly replaced by magnetic storage.

- The magnetism of materials is mainly a consequence of interactions of uncompensated magnetic moments of constituent atoms and molecules.

- Basing on the response of materials in external magnetic field, and on the alignment of magnetic moments in the materials, they are classified into five types.

- Magnetic field : The space around a magnet where its influence is felt is called magnetic field.

- Magnetic induction or Magnetic flux density (B):
- In any material is the number of lines of magnetic force passing through unit area perpendicularly.
- Units of B : Tesla (or) Weber/m^2

Intensity of magnetic field (H)

- At any point in the magnetic field is the force experienced by an unit north pole placed at that point. Or
- It is defined as the field that induces magnetism in a magnetic material.
- H is measured in Ampere/meter.
- When a medium is exposed to magnetic field of intensity H it causes an induction B in the medium.

Magnetic permeability:

- It is defined as the ability of a medium to allow the magnetic lines of force to pass through it.
- $\mu = B/H$
- $B = \mu_o (H+M)$
- Relative permeability $\mu_r = \mu / \mu_0$
- $B = \mu_o \mu_r H$.
- $\mu_r = \mu / \mu_0 = \frac{B/H}{B/H+M} = 1 + M/H$
- Where $\mu_r = 1 + \chi$
- Which is called relative permeability.

Intensity of magnetization :

- It is the magnetic moment per unit volume or pole strength per unit area.
- $I = M/V = (2l.m)/(2l.a)$

a= area of cross section.

It is measured in Weber/m^2

- **Magnetic flux(Φ) Magnetic flux(Φ):**
- It is the total number of lines of induction passing normal to the cross section.
- Φ is a scalar.
- S I unit : Weber.

Magnetic moment

- It is a product of Magnetic length and pole strength of a magnet .
- Magnetic moment M=2l.m.
- S.I unit of Magnetic moment is **Weber-meter.**

Magnetic susceptibility

- Magnetic susceptibility is defined as the ratio of intensity of magnetization (I) to intensity of magnetizing field.
- Magnetic susceptibility(χ): $\chi = I/H$.

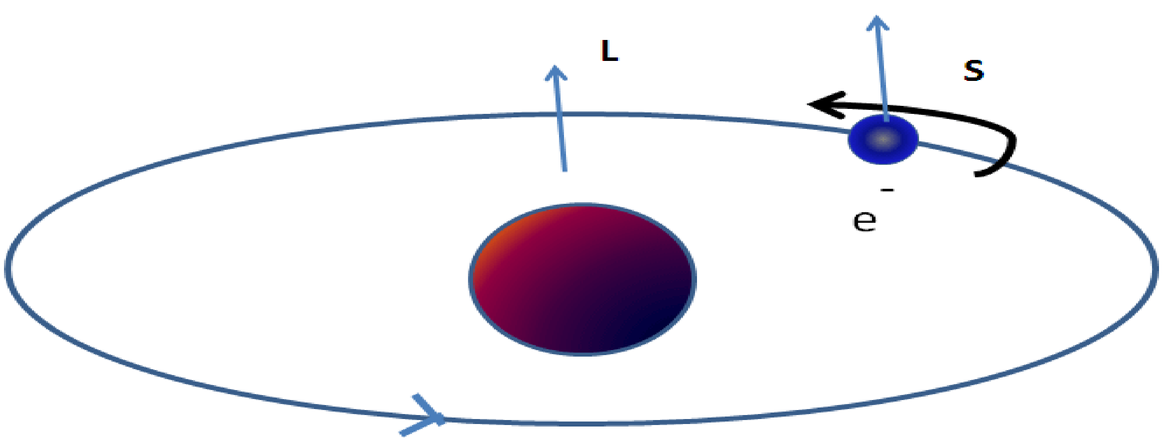
Susceptibility(χ) has no units.

Origin of Magnetic moment and Bohr magneton

- Magnetism originates from the spin and orbital magnetic moment of an electron.

- Spin and Orbital motion of an electron

- The orbital motion of an electron around the nucleus is analogous to the current in a loop of wire.



- The magnetic moment of a current carrying conductor is given by

$$\mu = I.A$$

where I is the current and A is the area in m^2.

Orbital magnetic moment

- The magnetic moment of an electron in orbit is given by

$$\mu = \pi r^2 (-ev/2\pi r)$$

$$= -evr/2 \text{------} (1)$$

where r is the radius of orbit, e -charge and v is the velocity

- The angular momentum of an electron must be an integral multiple of Planck's const.

$$mvr = nh/2\pi \qquad \text{-- (2)}$$

where m is the mass and h is the Planck's const.

- If the electron revolves in the first orbit then n=1
- Therefore orbital magnetic moment of an electron is given by from (1) and (2)

$$\mu = eh/4\pi m \qquad \text{-- (3)}$$

which is known as Bohr magneton, the smallest possible orbital magnetic moment

Spin magnetic moment

- Similarly the smallest possible magnetic moment due to spin of the electron is

$$\mu = eh/4\pi m$$

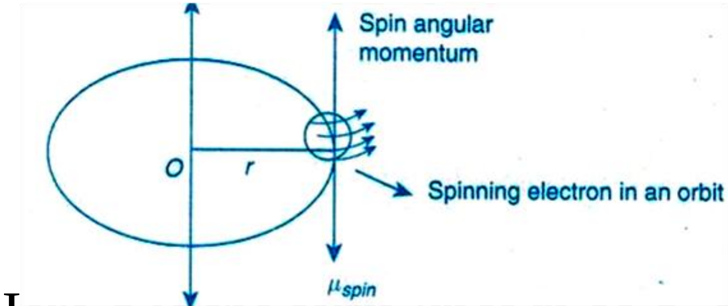
- According to quantum theory the spin of electrons have only two possibilites +1/2 or -1/2.

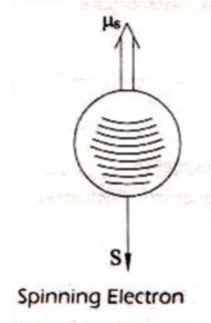
- Similar to eqn (3) we can write in the form

$$\mu = (e/2m)\ S \qquad \text{--(4)}$$

where S is the spin quantum number here given by (1/2).(h/2π)

$$\mu = g.(e/2m).S \qquad \text{--(5)}$$





Here g is the term known as g- factor.

- When g=2, the spin contribution arises and when g=1 the orbital contribution arises.

- The mass of the nucleus is so large that the magnetic moment contribution can be neglected compared to the electronic magnetic moment.

- The gryomagnetic ratio is proportional to the g-factor and 'g' arises due to the precession of the electrons similar to the precession of a top in a gravitational force. the value of g tells us whether the origin of magnetic moment is spin or orbital motion of electrons.

Magnetic materials

- These are the substances, which upon which being introduced into the external magnetic field, change so that they themselves become sources of an additional magnetic field.
- And they are classified into 3 groups.
- Diamagnetic, Paramagnetic, Ferromagnetic,

Diamagnetic materials

- The materials which when placed in magnetic field acquire feeble magnetism in the direction opposite to that of field are known as Diamagnetic substances.
- Diamagnetic materials exhibit negative magnetic susceptibility.
- The magnetization in diamagnetic materials is directed in opposite direction of the field applied.

- The relative permeability of a diamagnetic substance is slightly less than unity.
- $\mu_r < 1$; which implies that substances are repelled by a magnetic field.
- The magnetic susceptibility of diamagnetic materials is practically independent of temperature.
- Examples: Hydrogen, air, water, gold silver.

Paramagnetic materials

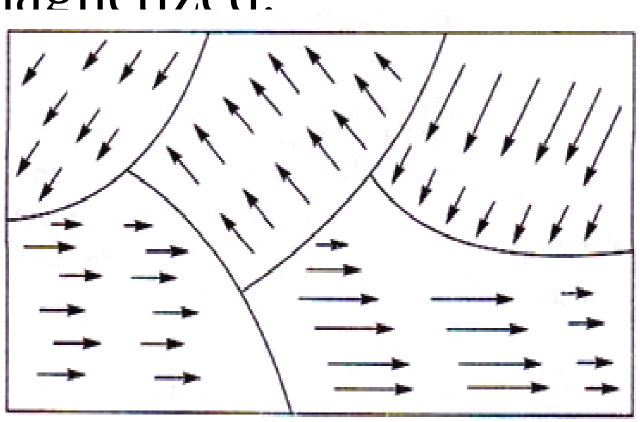
- These are the substances which when placed in magnetic field acquire feeble magnetism in the direction of magnetic field.
- It is the property of the material which has weak attractive force.
- The property is due to spin of electrons
- Spin is random
- These materials have permanent dipoles
- They possess permanent magnetic dipole moment.
- But there is no spontaneous magnetization in the absence of external field. Due to random spin.
- The relative permeability $\mu_r > 1$

Ferromagnetic materials

- It is property of the material which has strong attractive force.
- The property is due to spin of electrons
- Spin is parallel
- They have permanent magnetic dipoles
- They possess permanent magnetic dipole moment. Also in the absence of field they have spontaneous magnetization even in the absence of external field due to parallel
- The relative permeability $\mu_r >> 1$
  - Susceptibility is large and positive

Domain theory (or)
Weiss theory of Ferromagnetism

- According to Weiss, Ferromagnetic material consists of a number of regions called "Domains" [~10^{-6} mts] which are spontaneously magnetized.

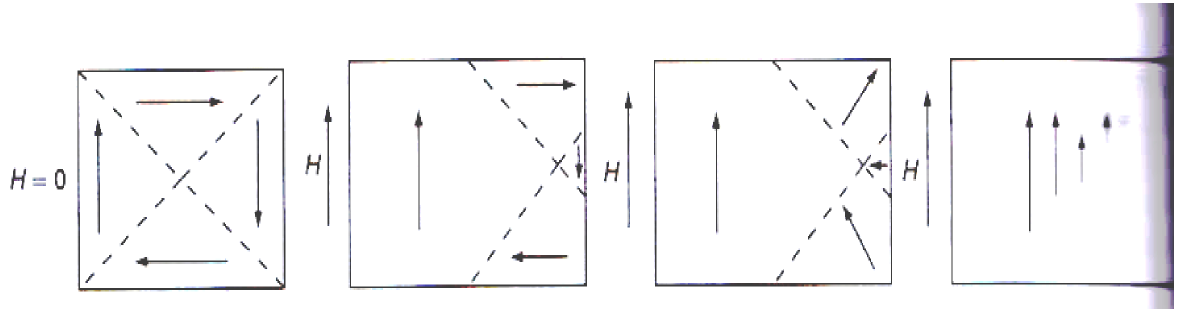


- Spontaneous magnetization is due to parallel alignment of all magnetic dipoles (in each domain) even when no external field is applied.

- Spontaneous magnetization is due to parallel alignment of all magnetic dipoles (in each domain) even when no external field is applied.
- Different domains possess different orientations hence net magnetization is zero.
- When an external field is applied there are two possible ways of alignment of domains. They are

- 1.By motion of domain walls
- 2.By rotation of domain walls

- Domains arise to minimize the energy of the material. The total internal energy is minimum. The alignment of domains, parallel to field is discussed as follows:

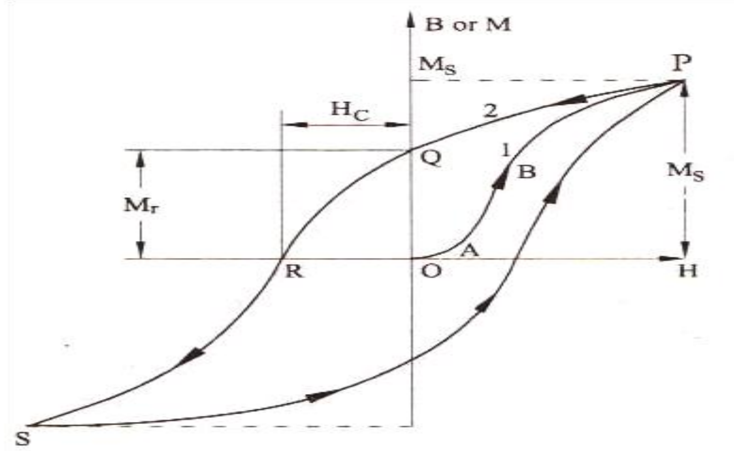


(a) (b) (c) (d)

- The domains which are parallel to the direction of applied magnetic field will grow in size than other domains. This is called “Motion of domain walls”. Also other domains which are opposite to the field direction are reduced.
- As the magnetic field is strong, the magnetic moments of the domains can rotate in the applied field direction. This is called “rotation of domain walls”.

Hysteresis loop

- **Definition:** Hysteresis means the lagging of magnetization "B" behind the applied magnetizing field "H". The energy supplied to the specimen during magnetization is not fully used. The balance of energy left in the material is produced as heat i.e. loss of heat called" Hysteresis Loss".
- This phenomenon of magnetic Hysteresis is an "Irreversible" characteristic of ferromagnetic material. The loop (or) area refers to the hysteresis loop. Hysteresis loss occurs in ferromagnetic materials below Curie temperature.



- When the magnetic field is applied on a ferromagnetic material the magnetization increases slowly and reaches a constant M_s called saturation magnetization
- In figure from point O to A, the displacement of domain walls takes place. When the field is suddenly off, the domains again go for original position.
- From point A to B, as the field is further increased, the magnetization also increases. Here when the field is made off the domain displacement does not return back to original condition.

- For higher fields the magnetization reaches maximum ie saturation magnetization M_s due to rotation of domain walls.
- In this case at the region B to P, if the field is suddenly made off, the domains does not return back to original direction. But there is some magnetic field remained inside the specimen.
- The point M_s is called saturation of magnetization. When the field is off, the curve does not go back to ‘O’[as shown in fig] but creates a new path to a point M_r called “retentivity” (or) residual (or) remanence Magnetization
- To reduce the residual magnetism to zero, a negative field ‘Hc’ has to be applied. When the sufficient negative field is applied, the residual; magnetization becomes zero and this field is known as “negative coercive field” ($-H_c$) or coercively. Further again if the negative field is applied then magnetization increases but in negative direction. This is known as negative saturation magnetization ($-M_s$).

- If the negative field is decreased back to zero the negative saturation of magnetization will not reach the initial path at ‘0’ but creates a new path and reaches a point called negative residual magnetism ‘-M_r’ know as negative receptivity.
- To decrease the negative residual magnetism to zero some positive field is applied. The amount of magnetic field required to bring residual magnetization to zero is known as positive coercive field (H_c).
- Further the increase of positive magnetic field the magnetization reaches again to positive saturation (M_s) and this is a cyclic process.
- The final conclusion is that when the magnetization vector is started from origin ‘O’ will not reach back to that point. “The magnetization lags behind H”. This is called magnetic hysteresis loss measured in the area of the loop (or) curve. If the loop area of a ferromagnetic material is large, more energy is wasted. This is also called as “dielectric loss” for one complete cyclic operation.

UNIT – II

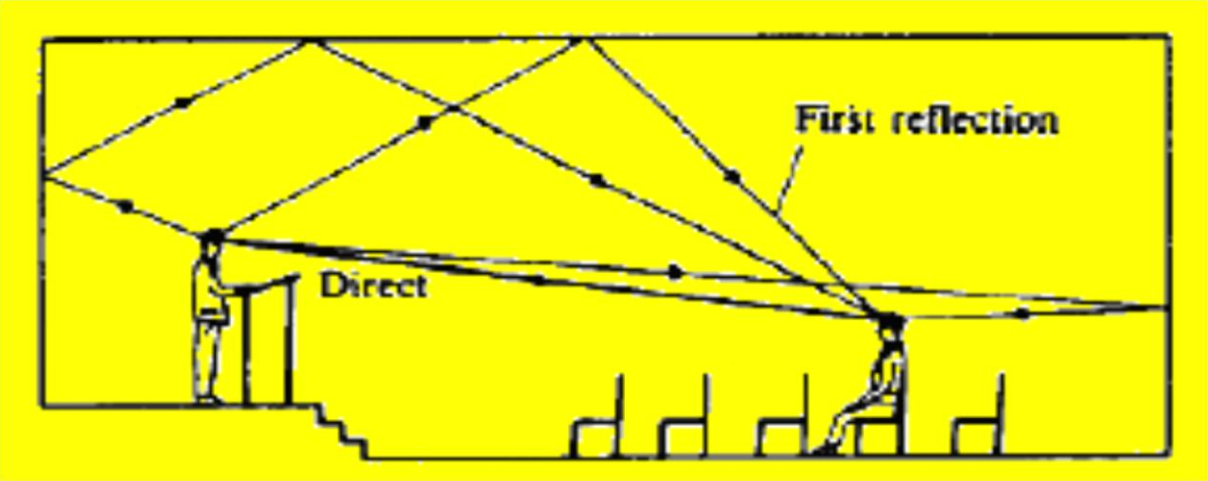
ACOUSTICS AND ULTRASONICS

ACOUSTICS

Acoustics is the branch of physics that deals with process of generation , propagation and reception of sound in room , it may be small rooms, auditorium, music rooms, sound recording rooms and movie theaters, what ever may be where the audience receive the best quality of sound In 1911 , Wallace C. Sabine ,professor of physics at Harvard university laid foundation of Acoustic engineering.

REVERBERATION

A sound produced inside a hall propagate in all directions .Sound waves incident on the surfaces of walls ,floors ceiling and furniture inside a hall will be multiply reflected .As shown in figure a listener inside the hall will receive the sound waves directly from speaker (source) ,as well as the reflected waves . As the source of sound is turned off ,the listener hears the sound with gradually reducing intensity for some time due to persistence of by multiple reflections at different places in the room. The persistence of audible sound even after the Source of sound turned off is called REVERBERATION.



REVERBERATION TIME

The reverberation time for room is the time required for the intensity to drop to one millionth (**10^{-6}**) of its initial intensity.

Reverberation time can be expressed interims of sound level(in dB) rather than Intensity .If initial intensity of sound level is I_i and the final level is I_f , ,then we can write

$$dB_i = 10 \log \frac{I_i}{I} \quad \text{(standerd)}$$

$$dB_f = 10 \log \frac{I_f}{I} \quad \text{(standerd)}$$

since $\frac{I_i}{I_f} = 10^6$

$$dB_i - dB_f = 10^{-6}$$

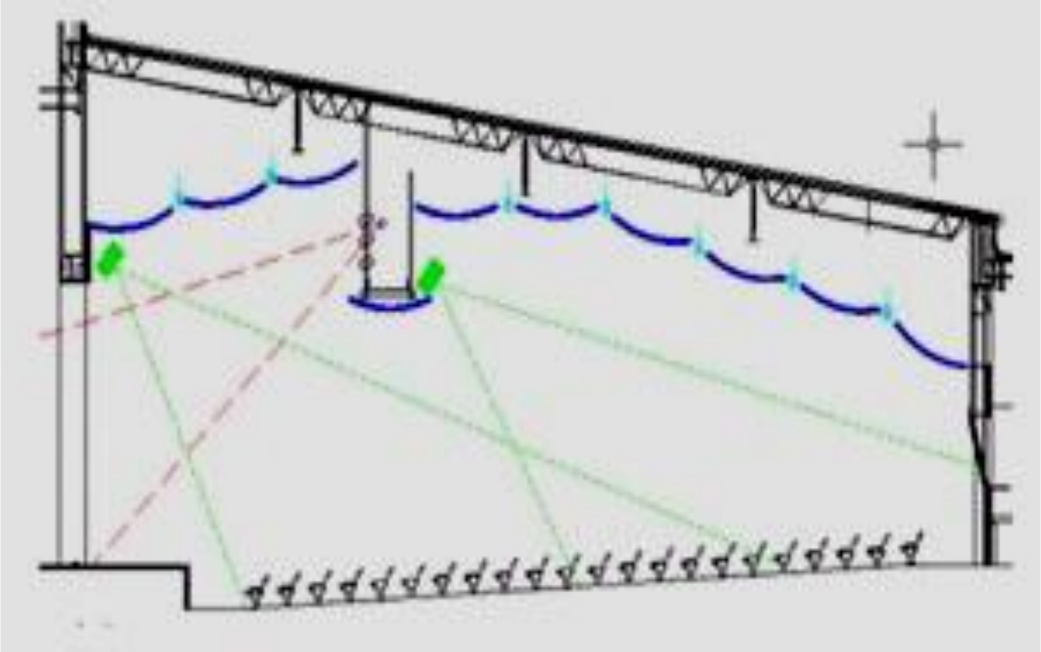
$$dB_i - dB_f = 10 \log 10^6 = 6\times 10 = 60 \text{ dB}$$

Thus, *the reverberation time is the time required for sound intensity to drop by 60 decibels (dB) after the sound source is turn off*.

BASIC REQUIRMENT OF ACOUSTICALLY GOOD HALL

The following conditions for acoustically good hall

1)Shape of the ceiling and walls should be so as to provide uniform distribution of sound throughout the hall as shown in below figure ,they should not be irregular surfaces.



2). The sound heard must be sufficiently loud in every part of hall and no echoes should be present.

3). There should not be concentration of sound any part, no resonance ,no Echelon effect, the reverberation time should be neither too large nor too small ,the quality of music of the speech and music unchanged.

4). The hall must be full of audience as shown in picture.

SABINE'S EMPIRICAL FORMULA FOR REVERBERTION TIME

Sabine given a relation between , reverberation time (T), Volume of the hall (V) and absorption of sound of all materials(a) which are present in the hall.

The following are Sabine's conclusions:

i). The reverberation time(T) is directly proportional to the volume(V) of the hall

ii). The reverberation time(T) is inversely proportional to the coefficient of absorption of different materials and surfaces inside the hall.

iii). The reverberation time(T) depends on the frequency of sound waves ,because absorption coefficient for most of the materials increases with frequency.

$$\text{The reverberation time(T)} \propto \frac{\text{Volume of the hall, V}}{\text{Absorption of sound, A}}$$

$$T \propto \frac{V}{A}$$

or

$$T = K\frac{V}{A}$$

Where **K** is known as proportionality constant and its value is 0.165

$$T = \frac{0.165V}{A} \quad \rightarrow \quad (4)$$

The above equation is known as **SABINE'S** formula for reverberation time.

If the hall consists of different materials of surface areas of $s_1, s_2, s_3, \ldots\ldots\ldots s_n$ and absorption coefficients respectively $a_1, a_2, a_3 \ldots\ldots\ldots, a_n$ then the Sabine's formula can be written as follows,

$$T = \frac{0.165V}{a_1 s_1 + a_2 s_2 + a_3 s + \ldots\ldots\ldots + a_n s_n}$$

or

$$T = \frac{0.165V}{\Sigma a S}$$

Sound Absorption Coefficient

- ***The effectiveness of a surface in absorbing sound energy is expressed with the help of absorption coefficient.***
- ***The coefficient of absorption `a' of a materials is defined as the ratio of sound energy absorbed by its surface to that of the total sound energy incident on the surface.***

$$a = \frac{Sound\ energy\ absorbed\ by\ the\ surface}{Total\ sound\ energy\ incident\ on\ the\ surface}$$

- ***A unit area of open window is selected as the standard. All the sound incident on an open window is fully transmitted and none is reflected. Therefore, it is considered as an ideal absorber of sound.***
- ***Thus the unit of absorption is the open window unit (O.W.U.), which is named a "sabine" after the scientist who established the unit.***
- ***A 1m² sabin is the amount of sound absorbed by one square metre area of fully open window.***

SOUND ABSORPTION COEFFINET OF SOME MATERIALS

MATERIAL	Absorption Coefficient O.W.U.
Open window	1.0
Marble	0.01
Carpets	0.30
Human body	0.50
Heavy curtains	0.50
Fiber glass	0.75

DETEMINATION OF ABSORPTION COEFFICIENT OF A MATERIAL

In order to find out the absorption coefficient of a material .Let us consider a hall of volume 'V'. the reverberation time of hall in absence of the material is T_1 According to Sabine's formula

The reverberation time $$T_1 = \frac{0.165V}{\Sigma aS}$$

Where , 'S' is the surface area of the hall ;
'a' is the absorption coefficient of the surface of the hall

or $$\frac{1}{T_1} = \frac{\Sigma aS}{0.165V} \longrightarrow (1)$$

To find out the absorption coefficient of the material, now keep the material inside the hall then the reverberation time (T_2) is given by,

$$\frac{1}{T_2} = \frac{\Sigma aS + a_1 S_1}{0.165V} \longrightarrow (2)$$

Where a_1 is the absorption coefficient of the material and s_1 is the surface area of the material.

Subtracting equation (2) from equation (1) we get

$$\frac{a_1}{0.165V} = \left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

or

$$a_1 = \frac{0.165V}{S_1}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

Hence knowing the terms on the right hand side of this above equation the absorption coefficient 'a' of the material can be calculated.

FACTORS AFFECTING THE ARCHITECTURAL ACOUSTICS AND THEIR REMEDIES

S.No.	FACTOR	SIGNIFICANCE	REMEDY
1.	Reverberation	Neither too long nor short	Providing heavy curtains with folds; covering floor with carpets, providing acoustic tiles; by providing windows and ventilators opened or closed.
2.	loudness	Due to large absorption this create confusion between the different Syllables	Quality of speakers; polished reflecting surface behind source facing the level of listener; low ceilings.
3.	Focusing	Concentration at focused regions Interference of direct & refracted waves	Avoiding curved surfaces(if present covered by a sound absorbent materials) ,providing parabolic surfaces behind the source.

FACTORS AFFECTING THE ARCHITECTURAL ACOUSTICS AND THEIR REMEDIES

S.No.	FACTOR	SIGNIFANCE	REMEDY
4.	Resonance	Due to interference of sound the original sound gets distorted	By hanging large number of curtains in the hall
5.	Echoes	A musical note is produced due to combination of echoes having regular phase Differences	Covering distant wall and ceiling with absorbent materials; make roughness of surface of wall.
6.	Focusing	Concentration at focused regions Interference of direct & refracted waves	Avoiding curved surfaces(if present covered by a sound absorbent materials),providing parabolic surfaces behind the source.

Another factor affecting architectural acoustics is **NOISE.**

Generally there are three types.
They are,

(a) Air – borne noise
(b) Structure-bore noise
(c) Inside noise

1.AIR-BORNE NOISE: The noise that enters the hall from outside through open windows ,doors and ventilators is known as **air-borne noise**

REMEDY: This can be reduced by using heavy glass doors, windows and ventilators ,by fixing windows proper places ,forming double wall construction.

FACTORS AFFECTING THE ARCHITECTURAL ACOUSTICS AND THEIR REMEDIES

2.STRUCTURE-NOISE: The noise that reaches through the structure of buildings are known as **structure –noise .** This nose is produced due to sound of foot steps, street traffic, moving furniture , operating machines etc.

REMEDY: This can be reduced by, using double walls with air space between them, keeping carpets on the floor etc.

3.INSIDE NOISE: The noises are produced inside big halls or offices due to equipment such as air conditioners, type writers, fans etc. are called **inside noise.**

REMEDY: This noise is reduced by covering the floor with carpets the walls ,floors and ceilings should be provided with sound absorbing materials, the machinery like type writers etc. should be placed on sound absorbent pads.

ACOUSTIC QUIETING

Acoustic quieting is the process of silencing the noise from machinery i.e., making machinery quieter by damping vibrations. When machinery vibrates, it generates sound waves in air, hydro acoustic waves in water and mechanical stresses in solid matter. By absorbing the irrational energy or minimizing the source of the vibration, acoustic quieting is achieved.

Aspects of Acoustic quieting

To achieve acoustic quieting, a number of different aspects might be considered with an aim to minimize the noise heard by the observer. They are,

1. Reducing the noise generation at its source.

2. Acoustic decoupling to reduce sympathetic vibrations.

3. Increasing acoustic damping or changing the size of the resonator to avoid resonance.

4. Reducing transmission using many methods, depending whether the transmission is through air, liquid or solid.

5. By limiting the reflection by using acoustic absorption materials, trapping the sound, opening windows to let sound out etc;

METHODS OF QUIETING

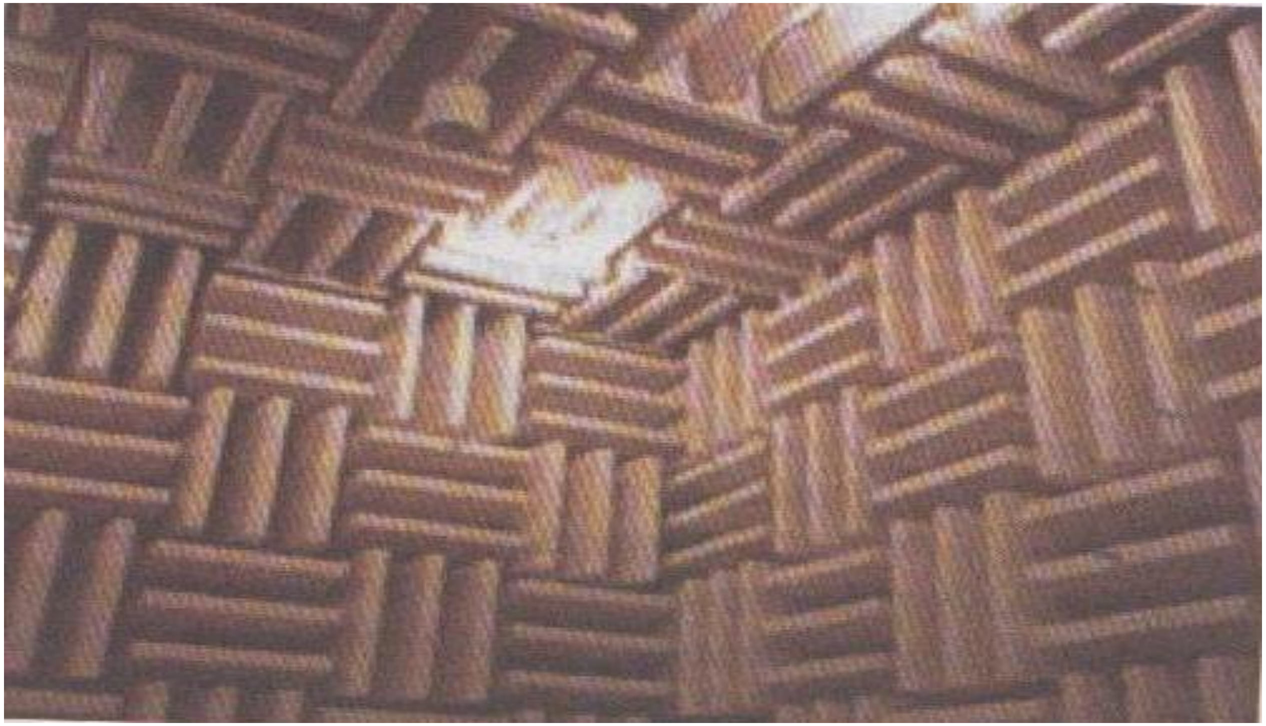
Mechanical acoustic quieting

1.Noise isolation: Noise isolation is the method of isolating noise by using barriers like deadening materials to trap sound and vibrational energy. For example, in home and office construction, sound control barriers such as fiber glass or synthetic rubber are placed in the walls to stop the transmission of noise through them.

2.Noise absorption: The unwanted sounds or noises that are produced inside a room can be made to be absorbed by suitable materials instead of being reflected towards the listener. Thus, the listener receives only the direct sound but not echo reflections. For example, sound proofing rooms are constructed using acoustic tiles for recording studios

3. **Acoustic damping:** Damping mounts are used to suppress the vibrations. The damping materials prevent the vibrations from being transferred from one material to the other. Motors and rotating shafts are fitted with damping mounts.

METHODS OF QUIETING



A sound proof room, showing acoustic damping tiles used for noise absorption

METHODS OF QUIETING

4).Acoustic decoupling: Certain parts of a machine such as frame, chasis or external shaft are decoupled from receiving unwanted vibrations from a moving part.

5).Preventing stalls: Whenever a machine undergoes an aero dynamic stall, it abruptly vibrates and hence this has to be avoided.

6).Preventing cavitations: The sounds of gas bubbles imploding is the source of noise, called cavitation (Rapid formation and vanishing of bubbles produce noise). When a machine is in contact with a fluid, it may be susceptible to cavitation. Ships and submarines have screws that cavitate and this may facilitate their detection through the sonar.

7).Preventing water hammer: In hydraulics and plumbing, water hammer is a known cause for the failure of piping systems. A valve that abruptly opens or shuts is the most common cause for water hammer. It also generates considerable noise.

METHODS OF QUIETING

8). Shock absorption: The shock absorbers present in vehicles, prevent mechanical shocks from reaching the passengers. They quieten the shocks.

9). Reduction of resonance: Every object vibrates with its natural frequency. Many parts of a machine vibrate and produce waves. These waves resonate and form noise.

10).Material selection: The transmission of sound and vibrations can be minimized by using non-metallic components (ex: flexible plastic pipe fittings) in place of metal components. This reduces noise. In some cases, air can be removed from the machine and sealed. The vacuum inside the machine arrests the noise transmission.

ULTRASONICS

- To explore the knowledge on high frequency sound waves, its generation and
- applications

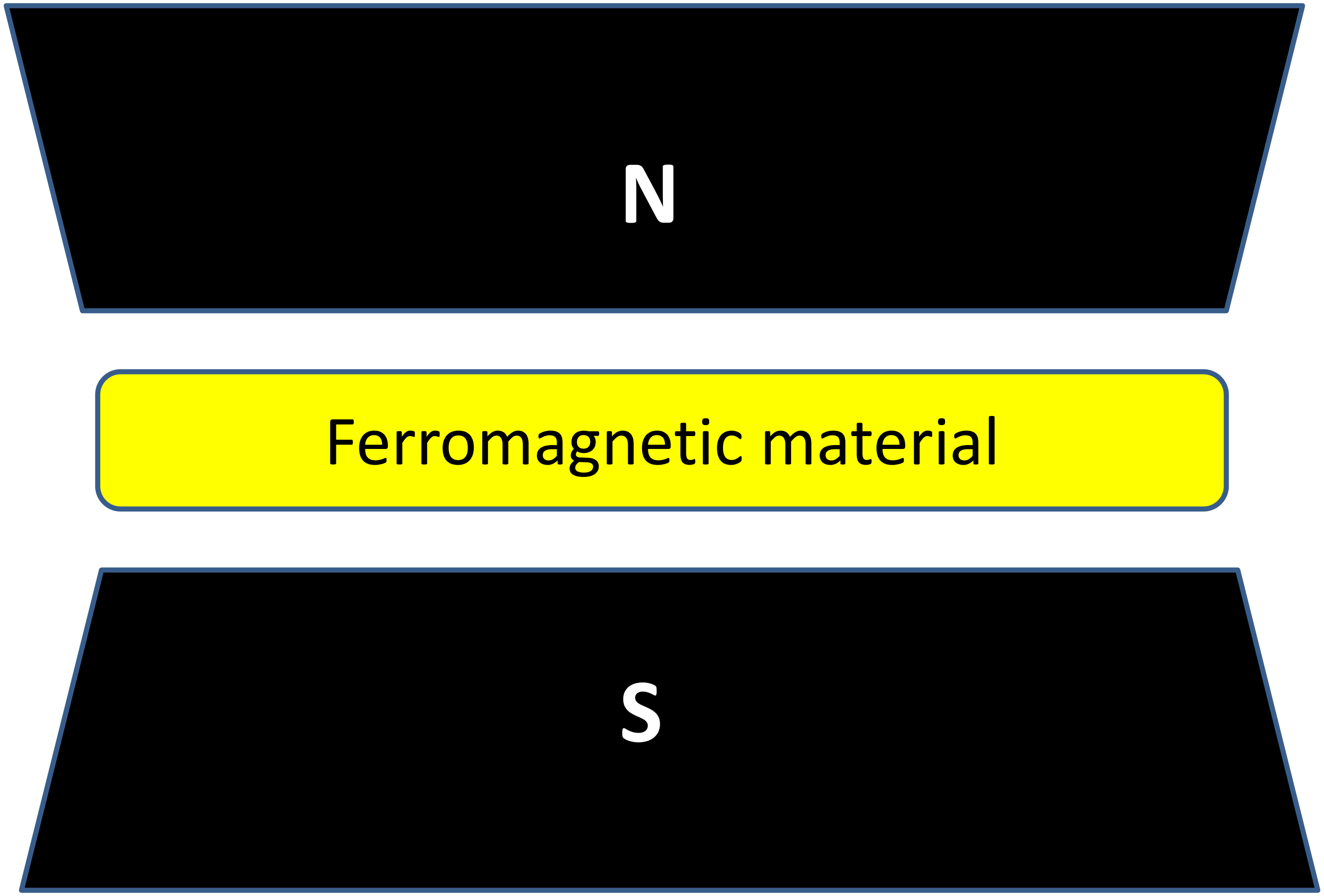
Introduction

- Sound waves whose frequencies lie above the audible frequencies (20,000 Hz) are called ultrasonic waves.
- The word ultrasonic combines the Latin roots *ultra*, meaning ‘beyond’ and *sonic*, or sound.
- Generally these waves are called as high frequency waves.
- The field of ultrasonics have applications for imaging, detection and navigation.
- They exist in solids, liquids, and gases.

GENERATION OF ULTRASONIC WAVES

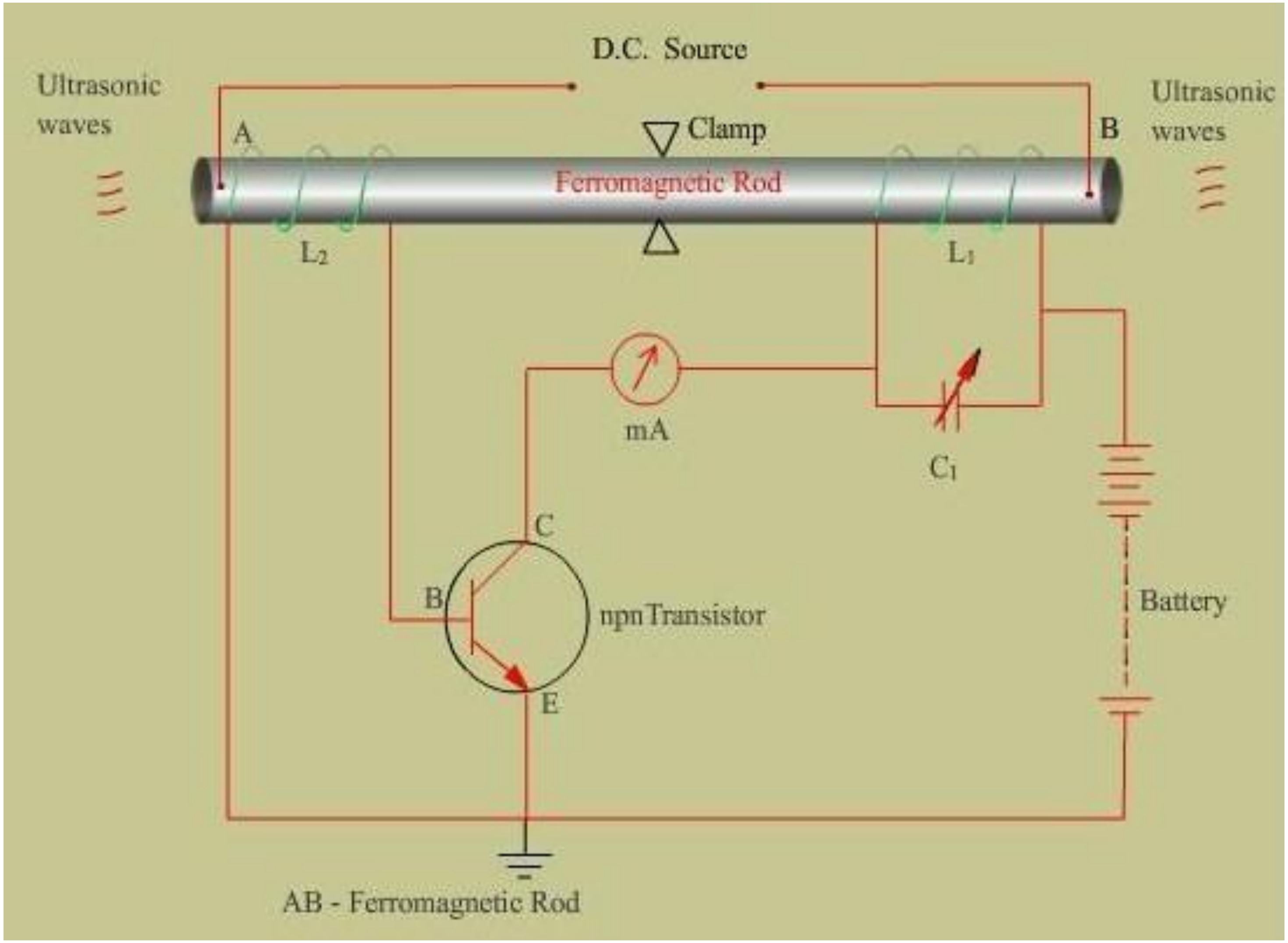
Magnetostriction Method

- When a ferromagnetic material in the form of a bar is subjected to an alternating magnetic field as shown in Figure, the bar undergoes alternate contractions and expansions at a frequency equal to the frequency of the applied magnetic field. This phenomena is called Magnetostriction effect.
- Such effect produces ultrasonic waves from bar to the surroundings.
- Such type of ferromagnetic materials which are used for production of ultrasonic waves are called Magnetostriction materials.



Magnetostriction effect in ferromagnetic material

The ultrasonic waves are produced by using Magnetostriction effect.
The arrangement is shown if figure below.



Magnetostriction Oscillator

- The ferromagnetic bar AB is clamped in the middle. The coils L1 & L2 are wounded at the ends of the bar.
- The coil L1 and capacitor C1 connected in parallel and the combination is connected between the anode and cathode circuit, through a milli-ammeter, while L2 is connected between the grid and cathode.
- The bar gets magnetized by the plate current passing through coil L1.
- Any change in plate current brings about a change in magnetization and consequently the length of the rod.
- By adjusting the capacitor value C1, the frequency of the tank circuit is varied.

- If the frequency of the tank circuit matches the natural frequency of the rod due to resonance, the rod vibrates vigorously and hence produce ultrasonic waves.
- The milli-ammeter reading shows maximum value of resonance condition.

The frequency of the ultrasonic waves produced by this method depends upon length l , density ρ and Young's modulus Y of the bar.

$$f = 1/2\, l \sqrt{\frac{Y}{\rho}}$$

Thus by varying the length and Y of the bar, ultrasonic waves can be generated at any desired frequency.

Advantages

1. The design of this oscillator is very simple and its production cost is low
2. At low ultrasonic frequencies, the large power output can be produced without the risk of damage of the oscillatory circuit.

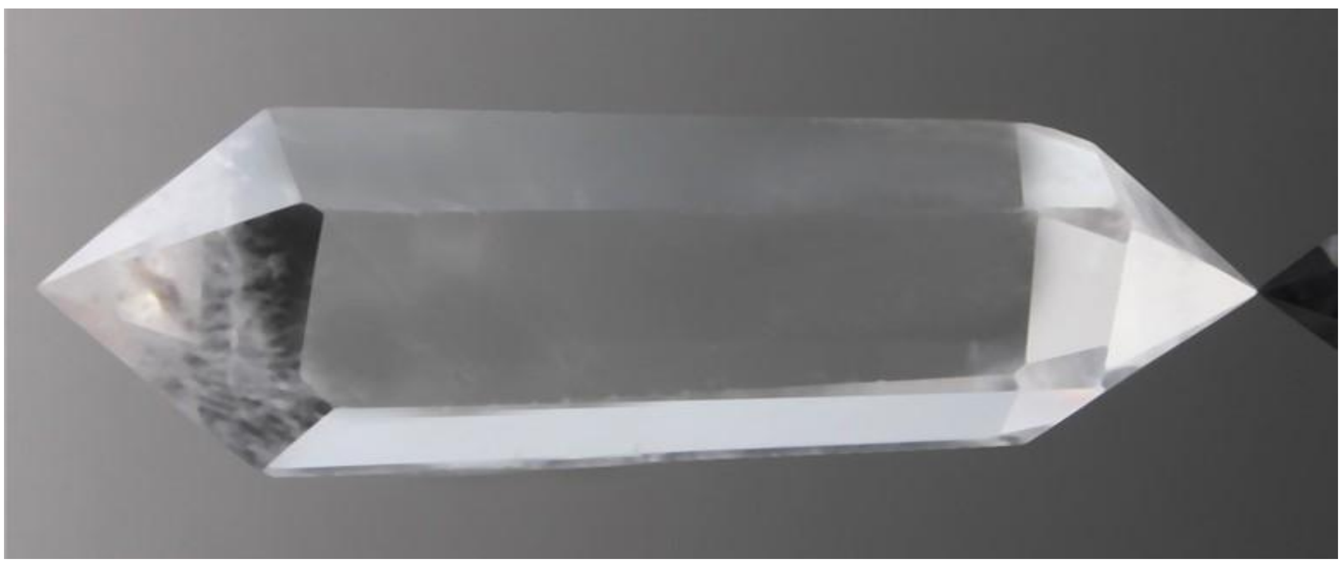
Disadvantages

1. It has low upper frequency limit and cannot generate ultrasonic frequency above 3000 kHz (ie. 3MHz).
2. The frequency of oscillations depends on temperature.
3. There will be losses of energy due to hysteresis and eddy current.

Piezoelectric Method

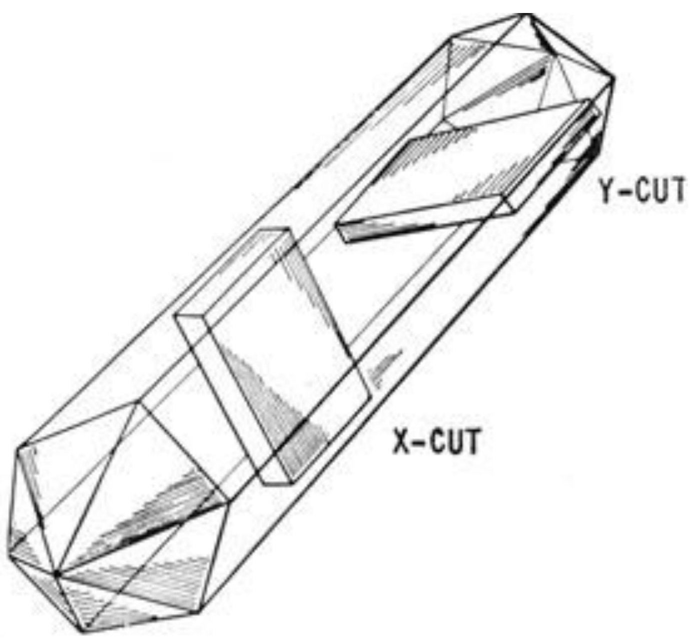
- When the opposite faces of a crystal, like quartz, tourmaline, rochelle salt, etc. are subjected to squeezing (crush), twisting or bending, a potential difference is developed across the perpendicular opposite faces.
- The magnitude of the potential difference developed across the crystal is proportional to extent of deformation produced.
- The effect is known as piezoelectric effect. The converse of piezoelectric effect is also true and is called inverse piezoelectric effect.

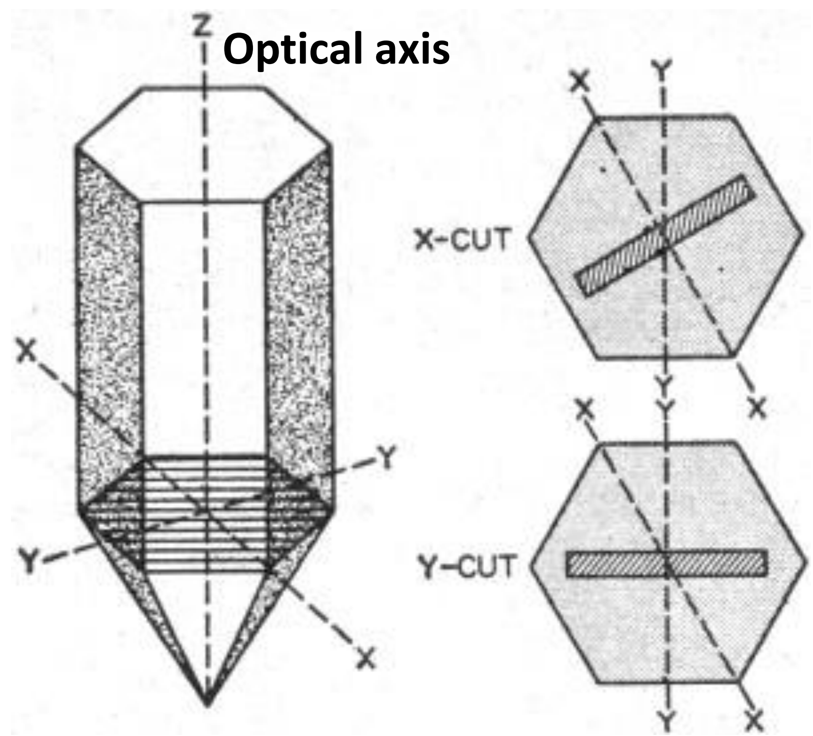
- According to this effect, if an ac voltage is applied to one pair of faces of a crystal, alternatively mechanical contractions and expansions are produced in crystal and starts to vibrate.
- If the frequency of the applied ac voltage equal to the vibrating frequency of the crystal, then the crystal will be in resonance state producing ultrasonic waves.



Quartz Crystal

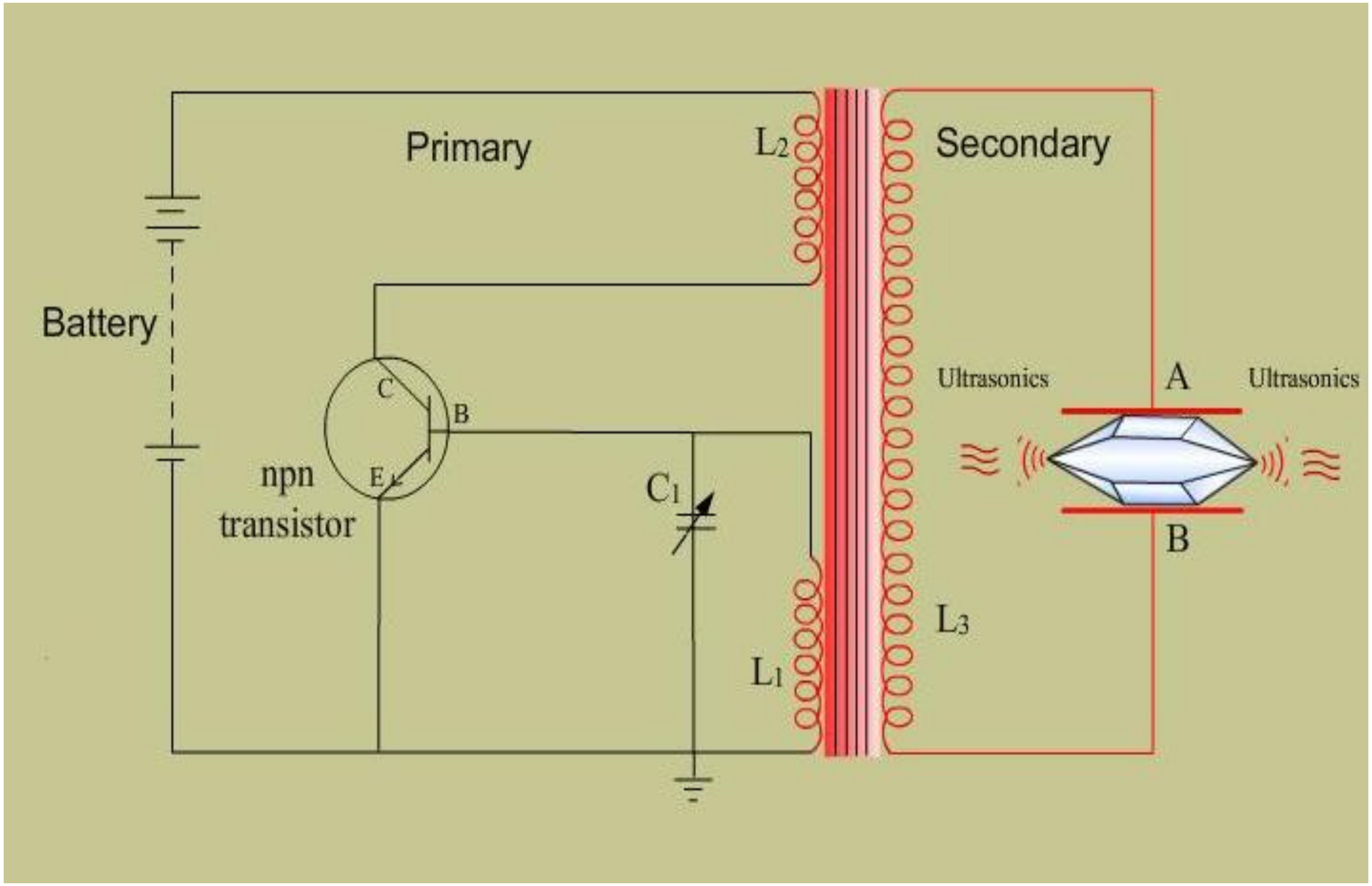
- When the two opposite faces of a quartz crystal are cut perpendicular to the electric axis (x-axis) or x-cut plate as shown in figure.
- Similarly, when the two opposite faces of a quartz crystal are also cut perpendicular to the mechanical axis (y-axis) or y-cut plate as shown in figure.





Piezoelectric Oscillator

- The experimental arrangement of the piezoelectric oscillator is as shown in below figure



- A thin slice of quartz crystal is placed between two metal plate A & B. The plates A & B are connected to coil L3.
- The coils present in circuit L1, L2. L3 are inductively coupled.
- The coil L2 connected to plate circuit. While L1 connected between the grid and cathode by tank capacitor C1.
- When the switch is closed, the oscillator produces oscillations of frequency

$$f = \frac{1}{2\pi\sqrt{L_1 C}}$$

- The frequency of the oscillation can be controlled by capacitor C1.
- Due to transformer action, an emf is induced in the secondary coil L3. This emf excites the quartz crystal into vibrations.
- Thus, the vibrating crystal produces longitudinal ultrasonic waves in the surrounding air.
- The frequency of the vibration is given by

$$f = \frac{P}{2l}\sqrt{\frac{Y}{\rho}}$$

(Using this equation, the ultrasonic waves of any frequency can be produced.)

Where Y is Young's Modulus, ρ the density of the material and P= 1, 2, 3 etc. And 'l' the length of the crystal.

- The velocity of the longitudinal waves in the crystal is given by

$$\upsilon = \sqrt{\frac{Y}{\rho}}$$

Advantages

- Ultrasonic frequencies as high as 5 x 108Hz or 500 MHz can be obtained with this arrangement.
- The output of this oscillator is very high.
- It is not affected by temperature and humidity.

Disadvantages

- The cost of piezo electric quartz is very high
- The cutting and shaping of quartz crystal are very complex.

Properties of Ultrasonic Waves

- The frequency of the ultrasonic waves is greater than 20 kHz.
- It travels a longer distances in the medium without any loss.
- It travels as a well defined sonic beam.
- It's velocity is constant for a homogenous medium.
- It has many modes of vibrations such as longitudinal, shear, surface and plate vibrations.
- These waves interacts with materials and plays a vital role in the study of molecular interactions and material characterizations.
- At higher frequencies, the wavelength is smaller and hence produces high resolution in flaw detection.
- It undergoes reflection and refraction at the interface, due to change in elastic and physical properties of the medium.

Applications

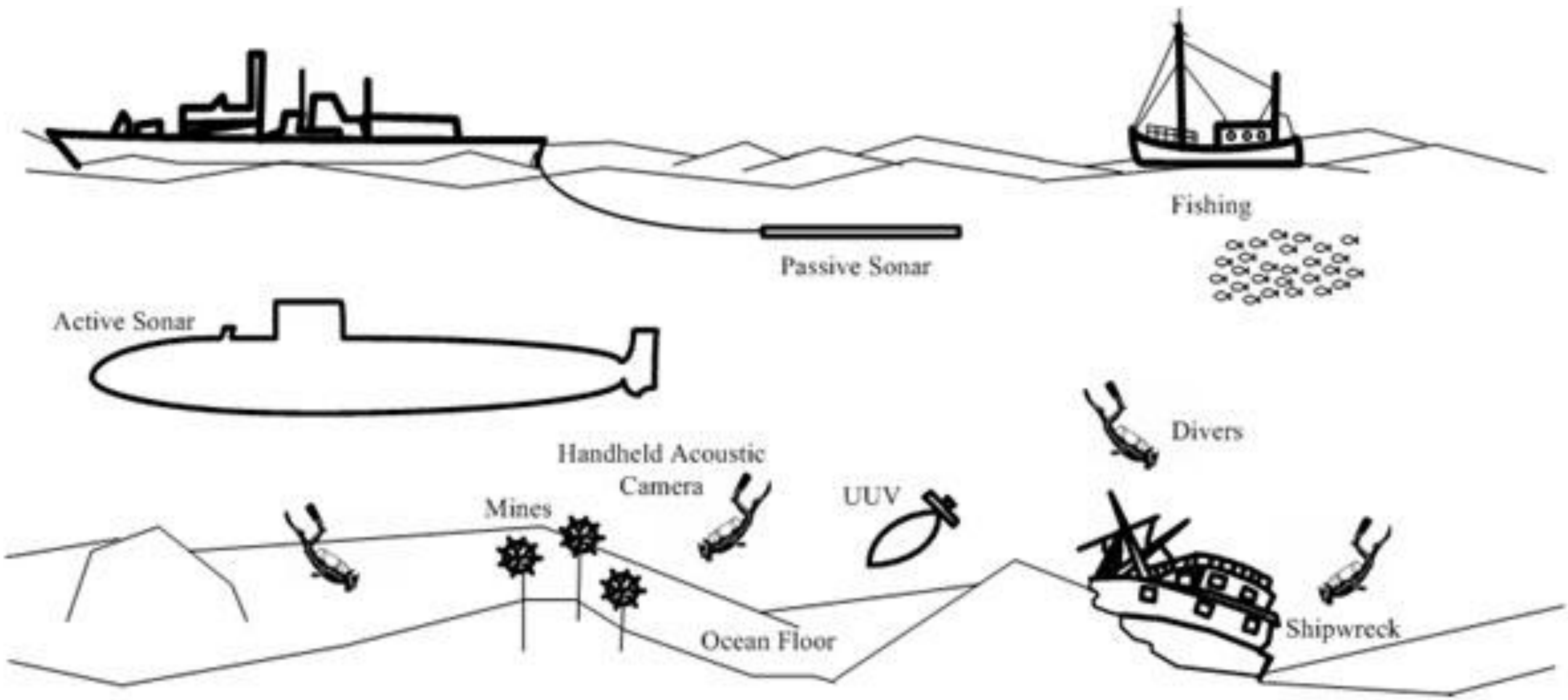
(1)Detection of flaws in metals (Non Destructive Testing –NDT)

- Ultrasonic waves are used to detect the presence of flaws or defects in the form of cracks, blowholes porosity etc., in the internal structure of a material
- By sending out ultrasonic beam and by measuring the time interval of the reflected beam, flaws in the metal block can be determined.

SONAR

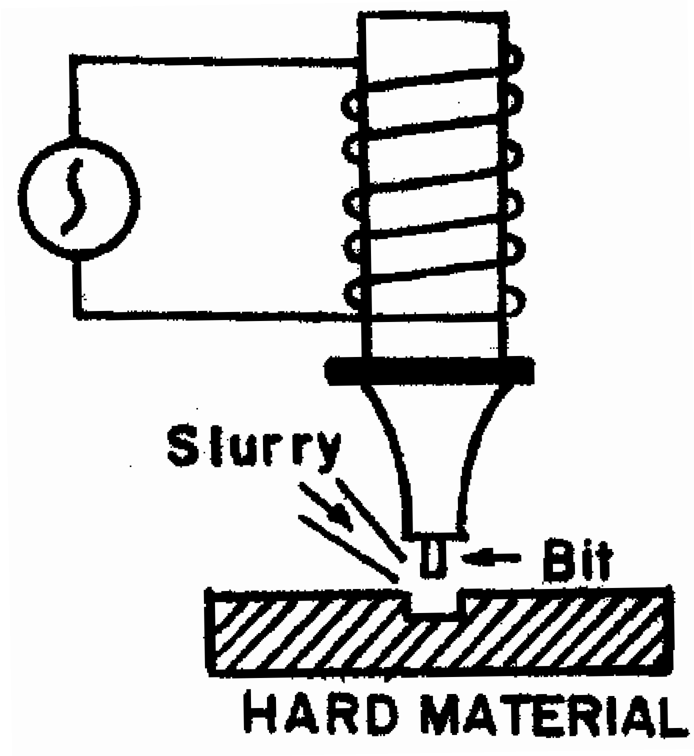
- SONAR is a technique which stands for ***Sound Navigation and Ranging***.
- It uses ultrasonics for the detection and identification of under water objects.
- The method consists of sending a powerful beam of ultrasonics in the suspected direction in water.
- By noting the time interval between the emission and receipt of beam after reflection, the distance of the object can be easily calculated.
- The change in frequency of the echo signal due to the Dopper effect helps to determine the velocity of the body and its direction.

1. Sonar is used in the location of shipwrecks and submarines on the bottom of the sea.
2. It is used for fish-finding application .
3. It is used for seismic survey.



Ultrasonic Drilling

- Ultrasonics are used for making holes in very hard materials like glass, diamond etc.
- For this purpose, a suitable drilling tool bit is fixed at the end of a powerful ultrasonic generator.
- Some slurry (a thin paste of carborundum powder and water) is made to flow between the bit and the plate in which the hole is to be made
- Ultrasonic generator causes the tool bit to move up and down very quickly and the slurry particles below the bit just remove some material from the plate.
- This process continues and a hole



Ultrasonic cleaning (SONICATOR)

It is the most cheap technique employed for cleaning various parts of the machine, electronic assembles, armatures, watches etc., which cannot be easily cleaned by other methods

Applications of Ultrasonics in Medicine

Diagnostic sonography

- Medical sonography (ultrasonography) is an ultrasound-based diagnostic medical imaging technique used to visualize muscles, tendons, and many internal organs, their size, structure and any pathological lesions.
- They are also used to visualize the foetus during routine and emergency prenatal care. Ultrasound scans are performed by medical health care professionals called sonographers. Obstetric sonography is commonly used during pregnancy.

Obstetric ultrasound is primarily used to:

- Date the pregnancy
- Check the location of the placenta
- Check for the number of fetuses
- Check for physical abnormities
- Check the sex of the baby
- Check for fetal movement, breathing, and heartbeat.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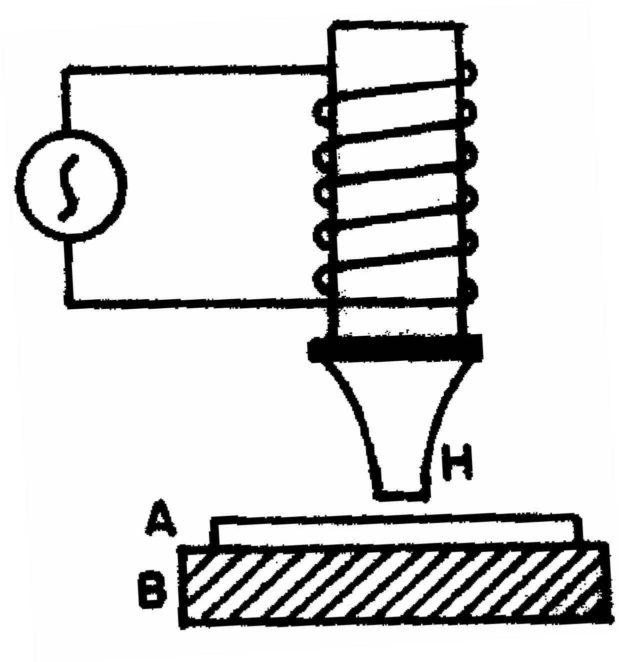
Ultrasound in research

- Scientists often use in research, for instant to break up high molecular weight polymers, thus creating new plastic materials.
- Indeed, ultrasound also makes it possible to determine the molecular weight of liquid polymers, and to conduct other forms of investigation on the physical properties of materials.
- Ultrasonic can also speed up certain chemical reactions. Hence it has gained application in agriculture, that seeds subjected to ultrasound may germinate more rapidly and produce higher yields.

Ultrasonic welding

- The properties of some metals change on heating and therefore, such metals cannot be welded by electric or gas welding.
- In such cases, the metallic sheets are welded together at room temperature by using ultrasonic waves.
- Thus, the two sheets get welded without heating. This process is known as ***cold welding***.



UNIT-III

Equilibrium of System of Forces

Basic Concepts

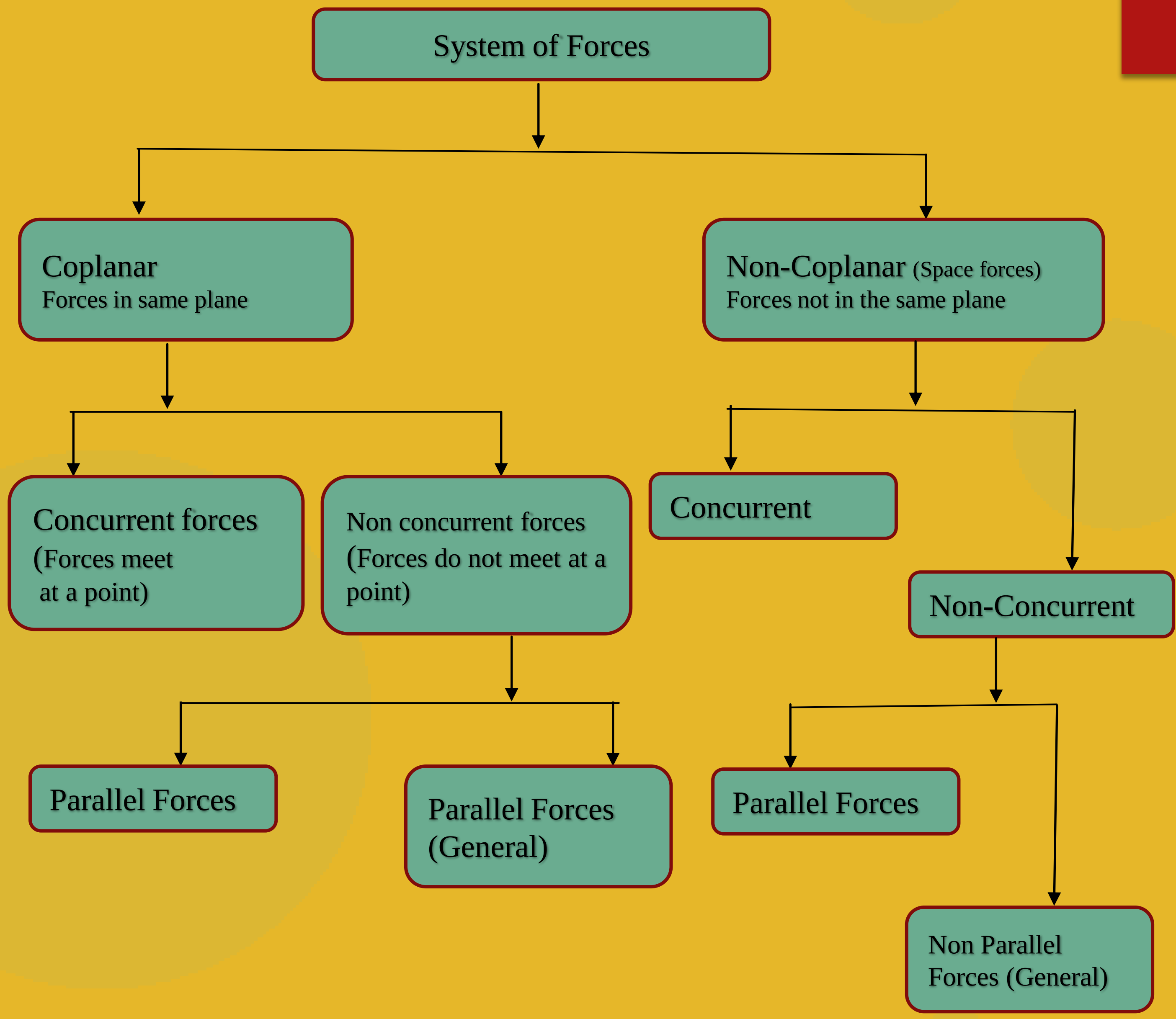
Particle: Point size mass (m ~ 0) of material which has negligible dimensions in the study of motion & equilibrium.
Ex: Earth & Moon in case of whole universe, electron in case of atomic structure

System of particles: the system of particles which is constituted with two or more bodies are dealt together.

Rigid Body: A rigid body is one in which the distance between any two arbitrary points is always constant.

- **Space**: it is geometric region containing bodies. Position of the bodies are described by linear and angular measurements relative coordinate system.
- Point: Just an exact location in space has no space at all, doesn't occupy any space having coordinates (x, y, z) (r, ϴ) (r, l, ϴ)
- **Length**: the measurements between two independent points measured in [L], cm, m. It is Scalar Quantity.
- **Mass:** It is quantity of matter. Matter refers to substance contained in the physical bodies. It is measured in [M]. g. Kg. It is Scalar quantity.
- **Time:** Measure of sequence of events. It is time related concept, before and after simultaneous occurrence. It is basic quantity in dynamics. It is not evolved with statics. It is measured in [T]. Units Seconds. It is Scalar quantity.

System of Forces



Fundamental and Derived Units

- Fundaments units are seven (7)
- M – Mass; Kg, g K – Temperature; Kelvin
- L – Length; m, cm A – Electric Current; Ampere
- T – Time; Seconds Cd –Luminous Intensity; Candella
- Mol – Amount of Substance; mole

- **Derived units**: The units which are derived from fundamental units are derived units.

- Momentum: mass ∗ Velocity Kg ms^{-2} $[MLT^{-2}]$

- Work: force ∗ distance Nm $[ML^{2} T^{-2}]$

- Pressure: Force/ Area N/m^{2} $[ML^{-1} T^{-1}]$

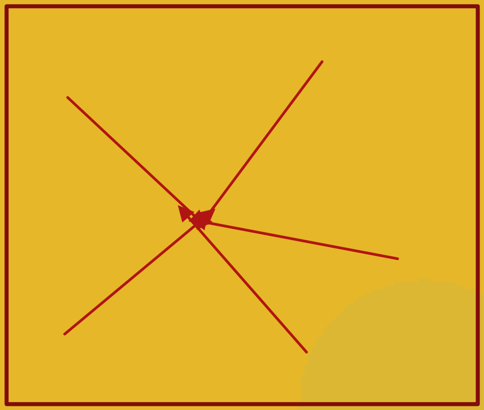
- Frequency: Number of vibrations/ Second Hz $[T^{-1}]$

- Angular acceleration: rad/ S^{2} $[T^{-2}]$

- Angular Momentum: Kg m^{2}/ second $[ML^{2} T^{-1}]$

Concurrent forces in a plane

Various forces meeting at a point is called concurrent forces in plane.

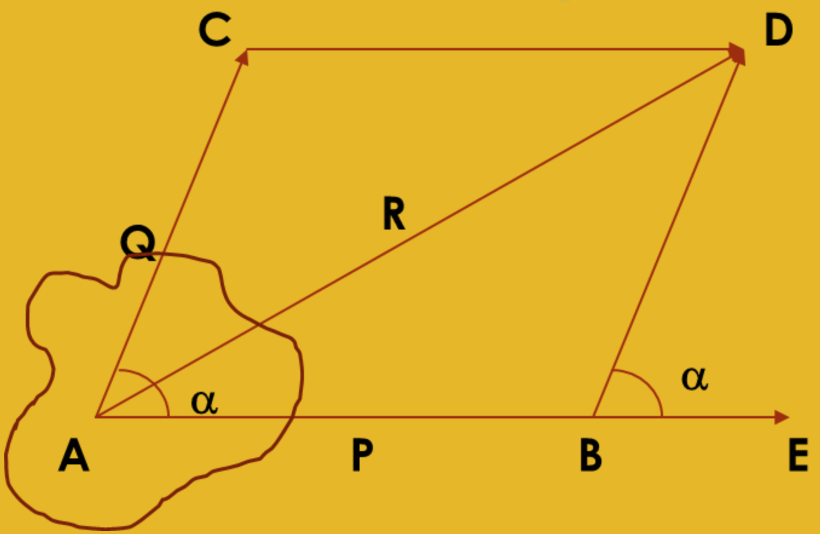


Composition of two forces in a plane

Force is vector quantity

$\vec{P} + \vec{Q} = \vec{R}$ (Resultant)

$\vec{AB}$, $\vec{AC}$ are components $\vec{P}$, $\vec{Q}$ are constructing a parallelogram ABCD as shown in Figure. And let α be the angle between both the components P and Q.



BE= Q cosα

DE= Q sinα

$\overrightarrow{AE} = \vec{P} + \vec{Q}$ Cosα

$\overrightarrow{R^2} = AE^2 + ED^2$

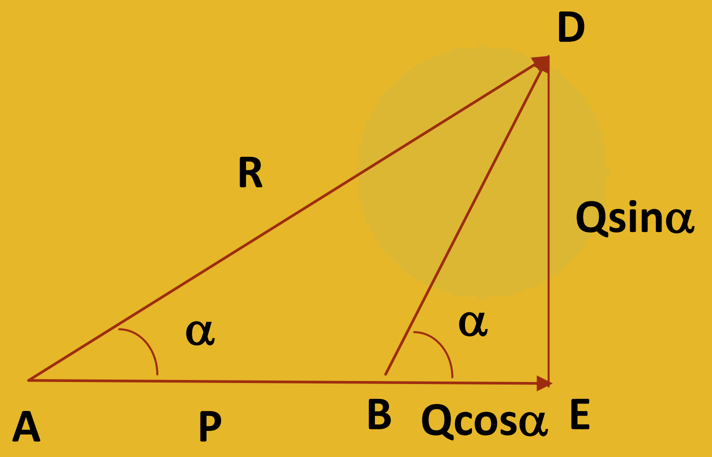
$= (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$

$= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha$

$= P^2 + Q^2(\cos^2\alpha + \sin^2\alpha) + 2PQ\cos\alpha$

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$R = P\sqrt{^2 + Q^2 + 2PQ\cos\alpha}$



Special cases:

When $\alpha = 0°$;

$$R = \sqrt{P^2 + Q^2 + 2PQ}$$

$$R = P + Q$$

When $\alpha = 180°$;

$$R = \sqrt{P^2 + Q^2 + 2PQ}\,(-1)$$

$$R = \sqrt{P^2 + Q^2 - 2PQ}$$

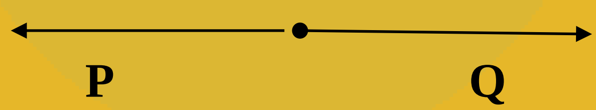
$$R = P - Q$$

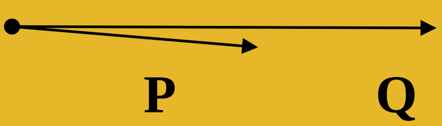
When $\alpha = 90°$;

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$

$$R = \sqrt{P^2 + Q^2}$$

$|R|$ is magnitude





Case study

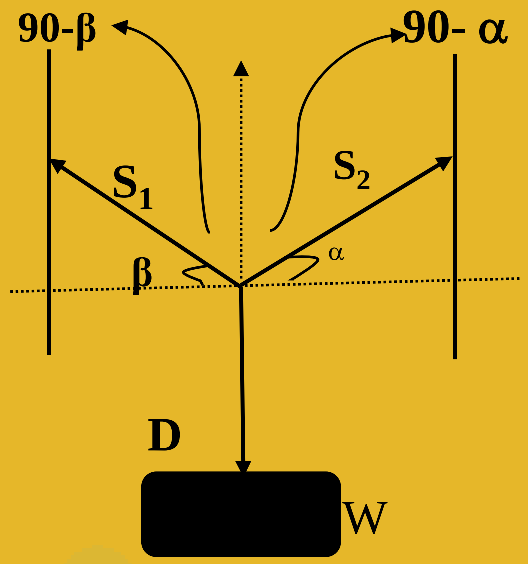
- **Force along BC = S_2 cos (90-β)**

  = S_2 sinβ

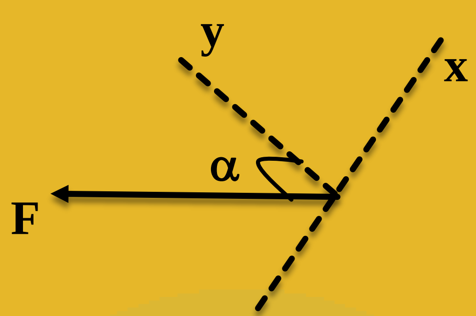
Force along BC = S_1 sinα

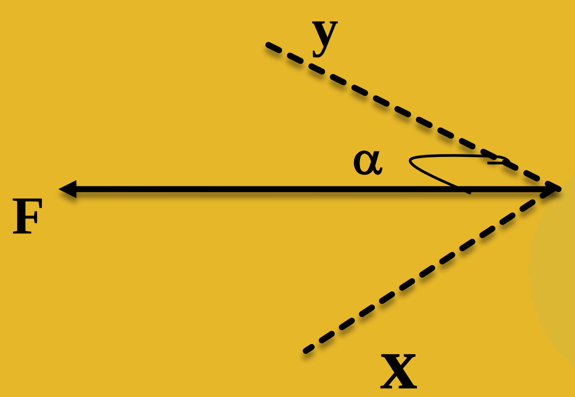
The total force along BC = S_1 sinα + S_2 sinβ

- **Resultant force, action & reaction are equal and opposite to keep the ring constant in equilibrium position.**



Resolving Force into Rectangular components

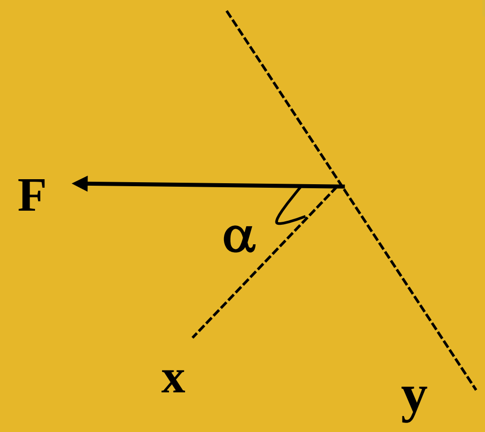


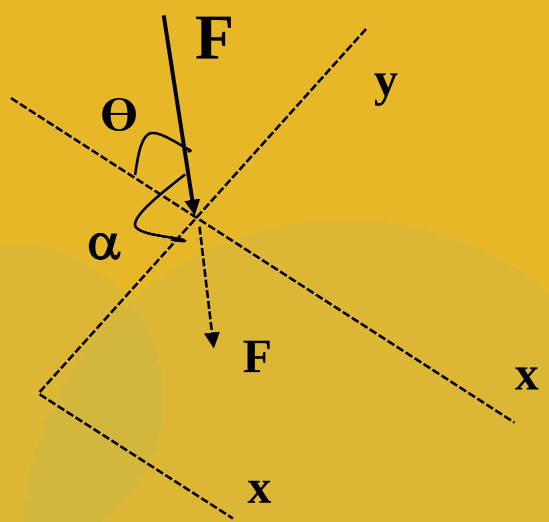


$$F_x = -F \sin \alpha$$
$$F_y = F \cos \alpha$$

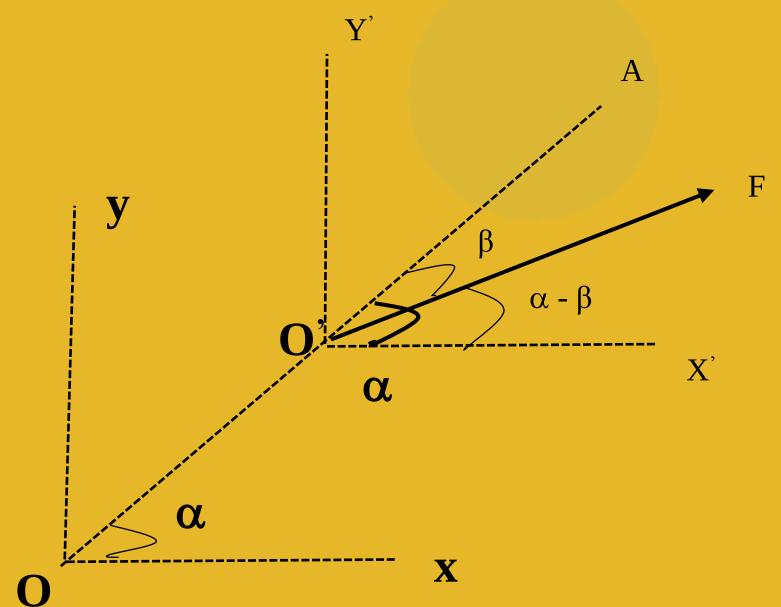
$$F_x = F \sin \alpha$$
$$F_y = -F \cos \alpha$$

$$F_x = F \cos\alpha$$
$$F_y = - F \cos\alpha$$





$$F_y = F \cos(180 - \alpha)$$
$$= - F \cos\alpha$$
$$\Theta = 90 - (180 - \alpha)$$
$$= - 90 + \alpha \;=\; - (90 - \alpha)$$
$$F_x = - F \cos \Theta$$
$$= - F \cos [(- (90 - \alpha))]$$
$$= F \cos (90 - \alpha) = F \sin\alpha$$

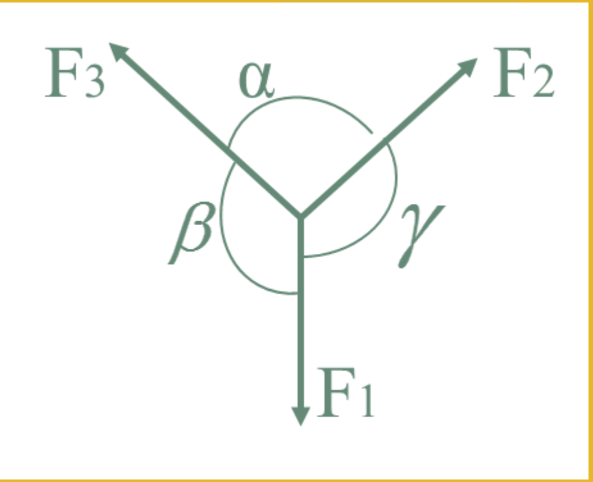


$$F_x = F \cos(\alpha - \beta)$$
$$F_y = F \sin(\alpha - \beta)$$

Lami's Theorem

If a system of Three forces is in equilibrium, then, each force of the system is proportional to sine of the angle between the other two forces (and constant of proportionality is the same for all the forces). Thus, with reference to Fig(2), we have,

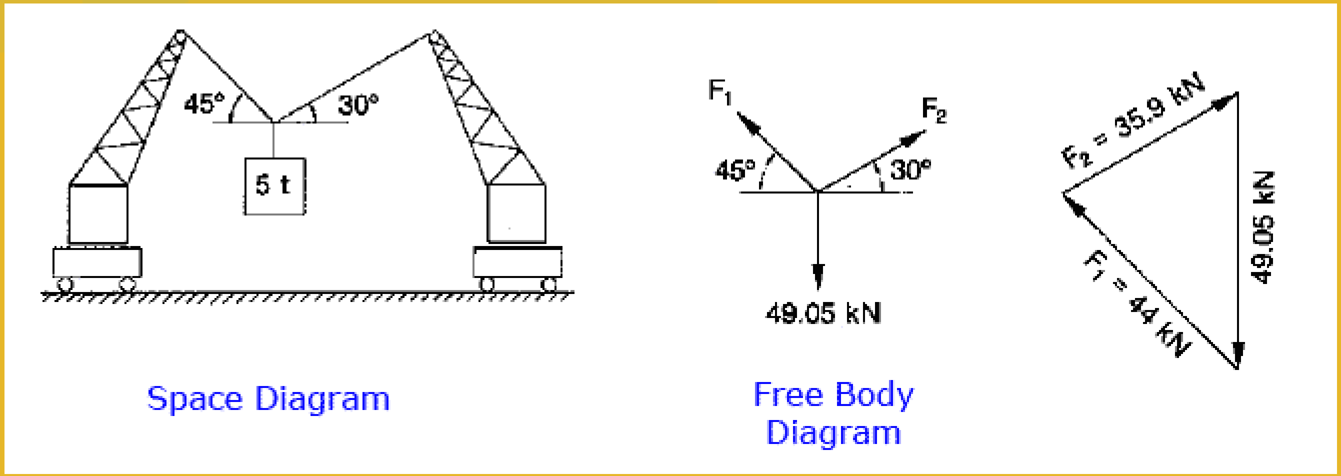
$$\frac{F_1}{Sin\,\alpha} = \frac{F_2}{Sin\,\beta} = \frac{F_3}{Sin\,\gamma}$$

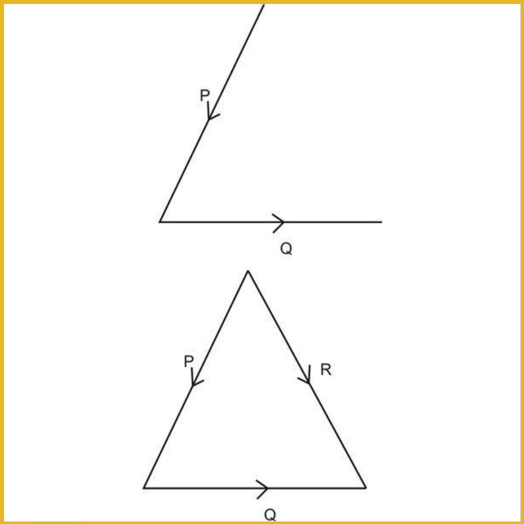


Note: While using Lami's theorem, all the three forces should be either directed away or all directed towards the point of concurrence.

Triangle law of forces

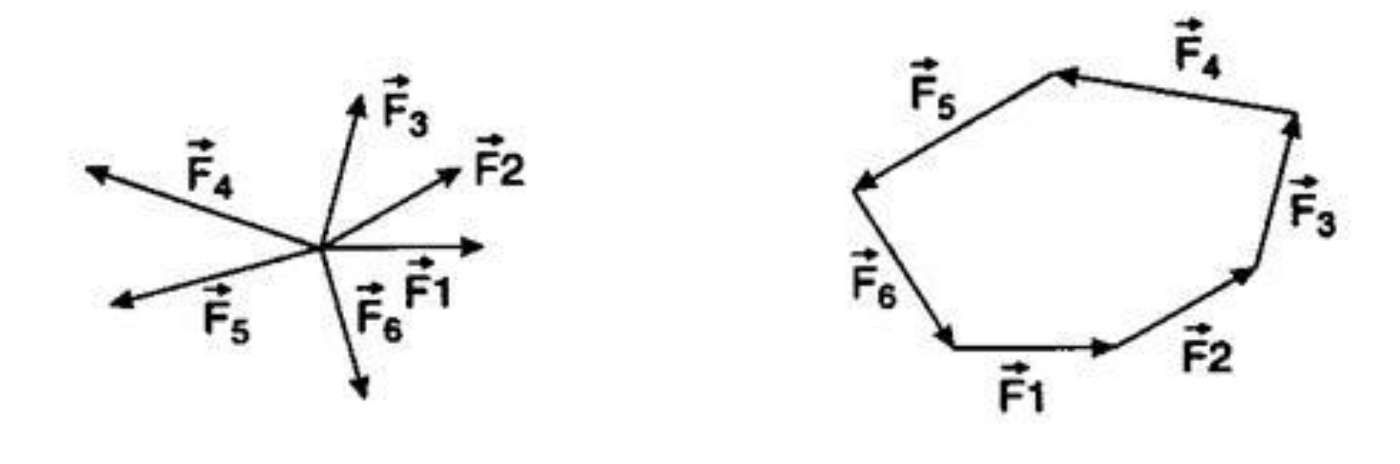
- Triangle law of forces states that, If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then the closing side of the triangle taken in the reversed order represents the resultant of the forces in magnitude and direction.
- The converse of this is also true, that is: “If three forces are in equilibrium their vectors can be put together to form a triangle





Polygon Law of forces

- **Polygonal Law:** If more than three forces are in equilibrium, then, they form a closed polygon when represented in a Tip to Tail arrangement, as shown in Fig.



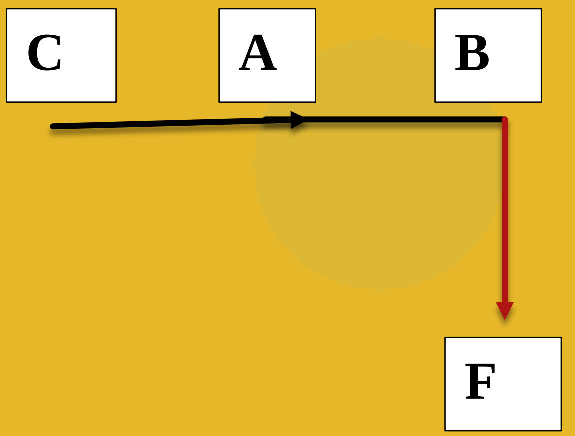
(a) Concurrent coplanar forces (b) Polygon of force vectors

Moment of Force:

Moment of force (M) about a point is a measure of rotational effect at that point. Moment of force about any point is defined as the product of magnitude of force (F) and perpendicular distance of line of force to that point.

Moment of force about point A= F ∗ (AB)

Moment of force about point C = F ∗ (BC)

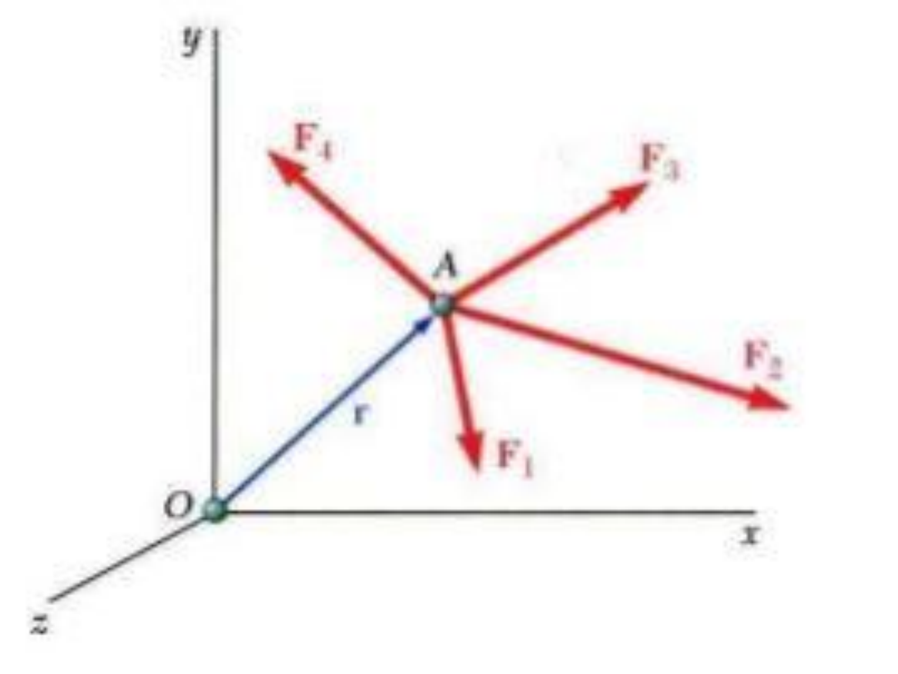


Varignon's Theorem

The Algebraic sum of moments of a system of coplanar forces about any point is equal to moment of their resultant forces about the same centre.

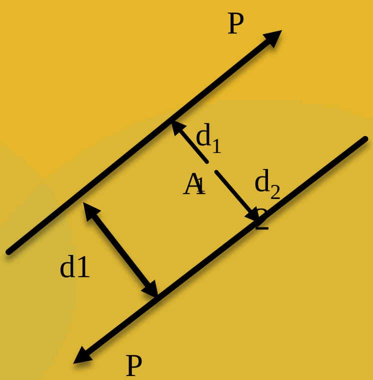
Then According to Varigon's Theorem

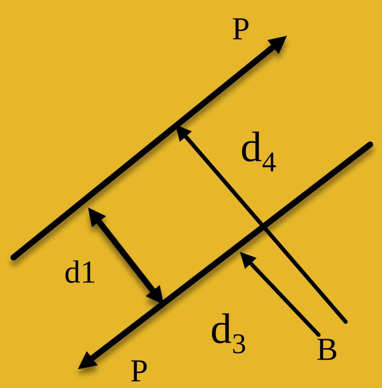
Resultant force = r ∗ (F_1 +F_2 +F_3 +F_4)

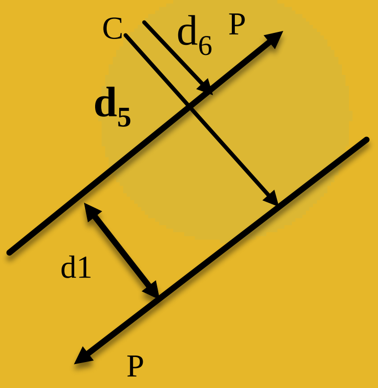


Couple:

Two parallel forces equal in magnitude and opposite in direction forms a couple.
The rotational effect is considered as positive in clockwise direction and negative in anti-clock wise direction.







$$M_A = Pd_1 + Pd_2 = P(d_1 + d_2) = Pd$$

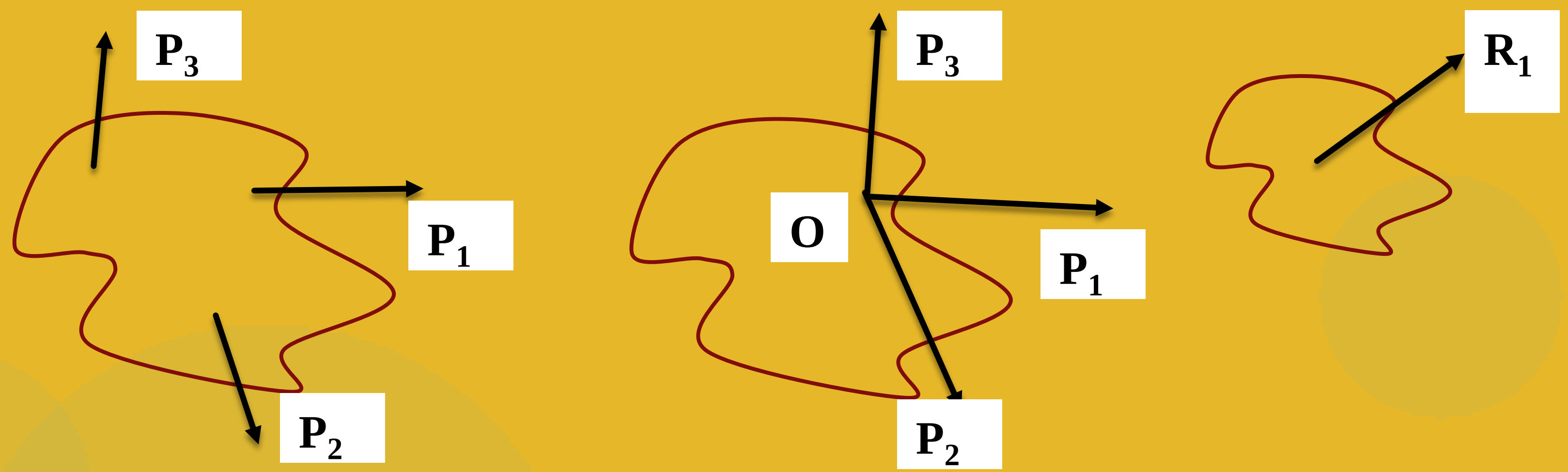
$$M_B = Pd_3 - Pd_4 = P(d_3 - d_4) = Pd$$

$$M_c = Pd_5 - Pd_6 = P(d_5 - d_6) = Pd$$

Characteristics of Couple

1. **Consists of pair of equal and opposite forces separated by a definite distance.**
2. **Translatory effect of couple on the body is zero.**
3. **Rotational effect (moment) of a couple about any point is constant and it is equal to product of magnitude of force and perpendicular distances between the forces.**
4. **The moment of couple is same, the effect is unchanged**
    a. **When couple rotated through any angle**
    b. **When couple shifted to any other position**
    c. **When is couple is replaced by any other pair of forces**

Resultant of System of Forces



$R_x = P_{1x} + P_{2x} + P_{3x}$, $R_y = P_{1y} + P_{2y} + P_{3y}$

$$R = \sqrt{(\sum x_i)^2 + (\sum y_i)^2}$$

$$\tan\alpha = \frac{\sum y}{\sum x} \quad ; \quad d = \frac{\sum M_O}{R}$$

$\sum x$ = algebraic sum of all components in x- direction

$\sum y$ = algebraic sum of all components in y- direction

α = Inclination of R with x-direction

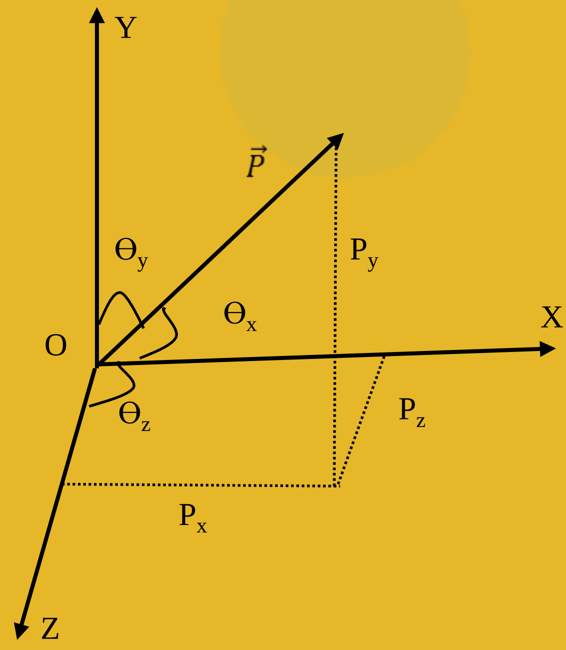
d = Perpendicular distance from R to O

Resultant of Concurrent forces in space

Let x, y, z are co-ordinate axes
P is a line of force from origin
Θ_x is the angle made by P with X-axis
Θ_y is the angle made by P with Y-axis
Θ_z is the angle made by P with Z-axis
Resolving P into X, Y and Z components
$P_x = P \cos \Theta_x$
$P_y = P \cos \Theta_y$
$P_z = P \cos \Theta_z$
Let A_i , B_j be two points on the line of action of $\vec{P}$ with Coordinates (x_i , y_i , z_i) and (x_J , y_j , z_j) respectively.

From co-ordinate Geometry. The distance between A_i , B_j is given by

$$L = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$



From trigonometric relations

$$\text{Cos } \Theta_x = \frac{(x_j - x_i)}{L}$$

$$\text{Cos } \Theta_y = \frac{(y_j - y_i)}{L}$$

$$\text{Cos } \Theta_z = \frac{(x_j - x_i)}{L}$$

The resultant of several concurrent forces P_1, P_2 , P_3……… Can be determined using their component forces P_{1x}, P_{1y}, P_{1z}, P_{2x} P_{2y} P_{2z}, P_{3x} P_{3y} P_{3z} ………….. etc.

x component of resultant R, $R_x = \sum P_x = P_{1x} + P_{2x} + P_{3x} + \ldots\ldots\ldots\ldots$

y component of resultant R, $R_y = \sum P_y = P_{1y} + P_{2y} + P_{3y} + \ldots\ldots\ldots\ldots$

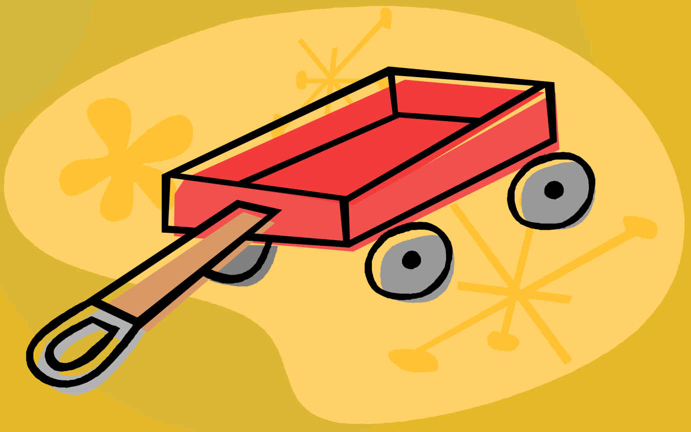
z component of resultant R, $R_z = \sum P_z = P_{1z} + P_{2z} + P_3 + \ldots\ldots\ldots\ldots$

And Resultant $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$$R = \sqrt{(\sum P_x)^2 + \left(\sum P_y\right)^2 + (\sum P_z)^2}$$

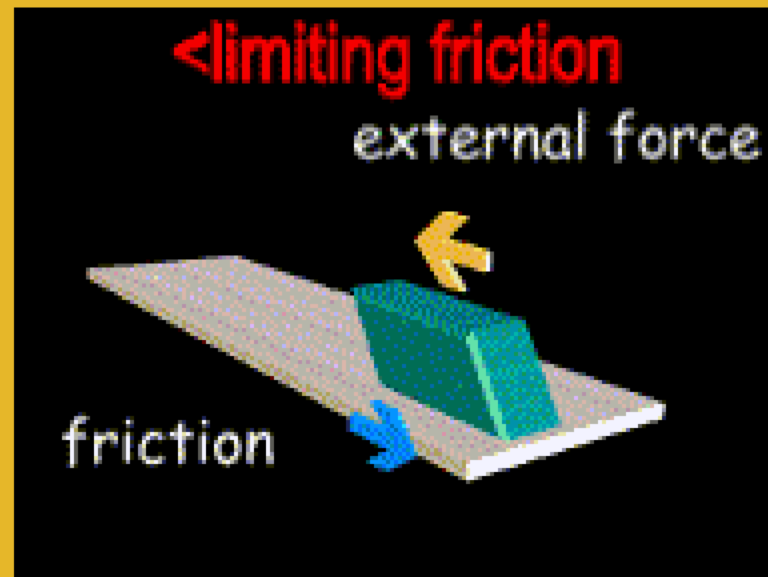
$\Theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$; $\Theta_y = \cos^{-1}\left(\frac{R_y}{R}\right)$; $\Theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$

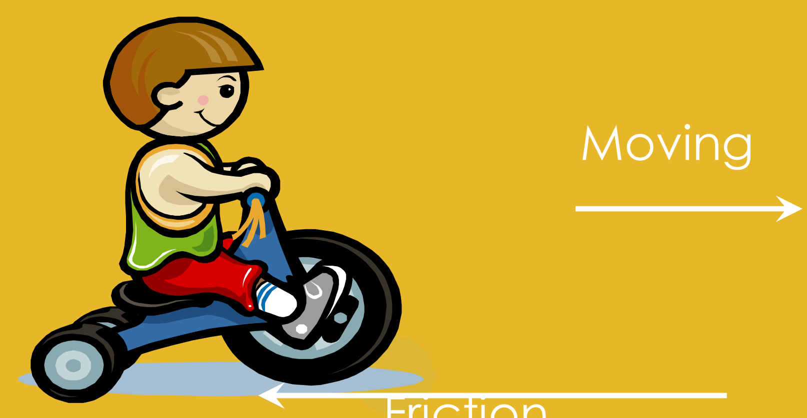
UNITY-IV
Friction



What is Friction?

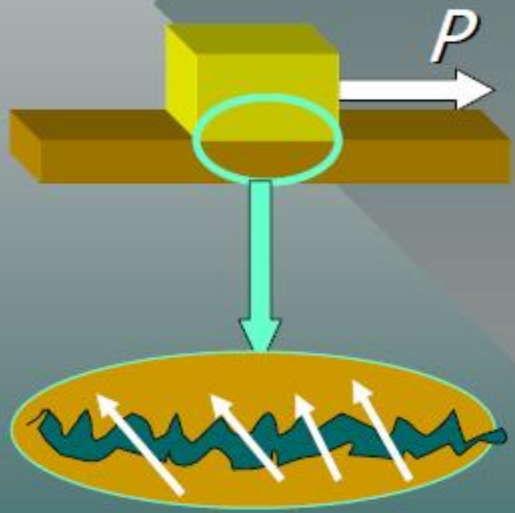
- Force that acts oppose the relative motion of two surfaces
- High for dry and rough surfaces
- Low for smooth and wet surfaces





Friction Forces

When two surfaces are in contact, friction forces oppose relative motion or impending motion.

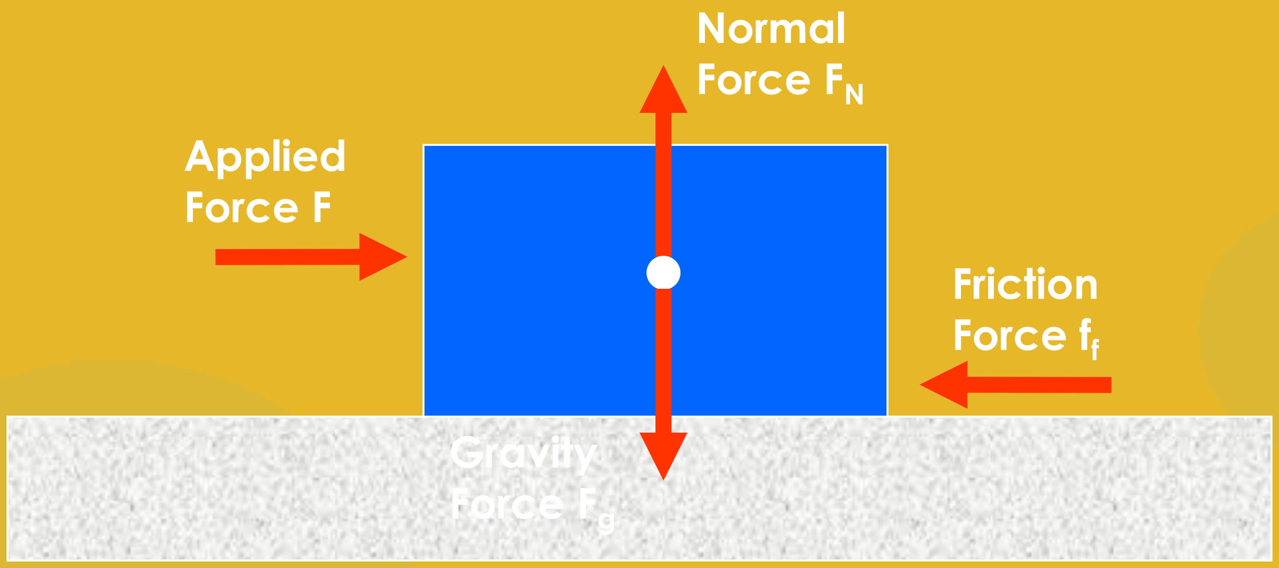


Friction forces are parallel to the surfaces in contact and oppose motion or impending motion.

Static Friction: No relative motion.

Kinetic Friction: Relative motion.

Free Body Diagram



$F_g = mg$

$F_N = F_g$

$f_f = F$

Types of Friction

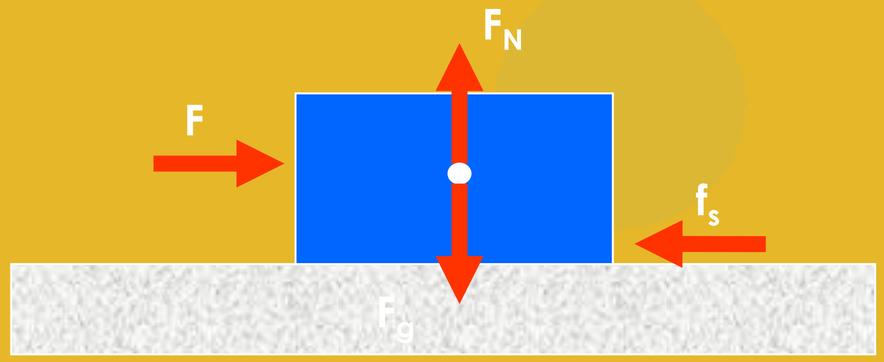
- Static Friction
- Rolling Friction
- Sliding Friction
- Fluid Friction

Static Friction

The Force of Static Friction keeps a stationary object at rest!

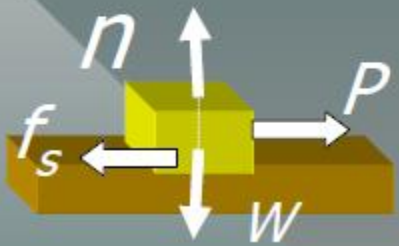
$$f_s = F_N \times \mu_s$$

$$\mu_s = coefficient\ of\ static\ friction$$



The Static Friction Force

When an attempt is made to move an object on a surface, static friction slowly increases to a MAXIMUM value.



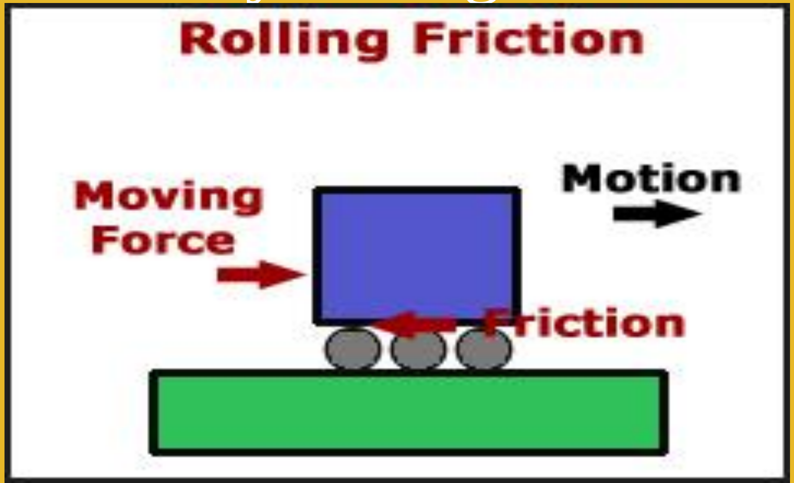
$$f_s \leq \mu_s n$$

In this module, when we use the following equation, we refer only to the maximum value of static friction and simply write:

$$f_s = \mu_s n$$

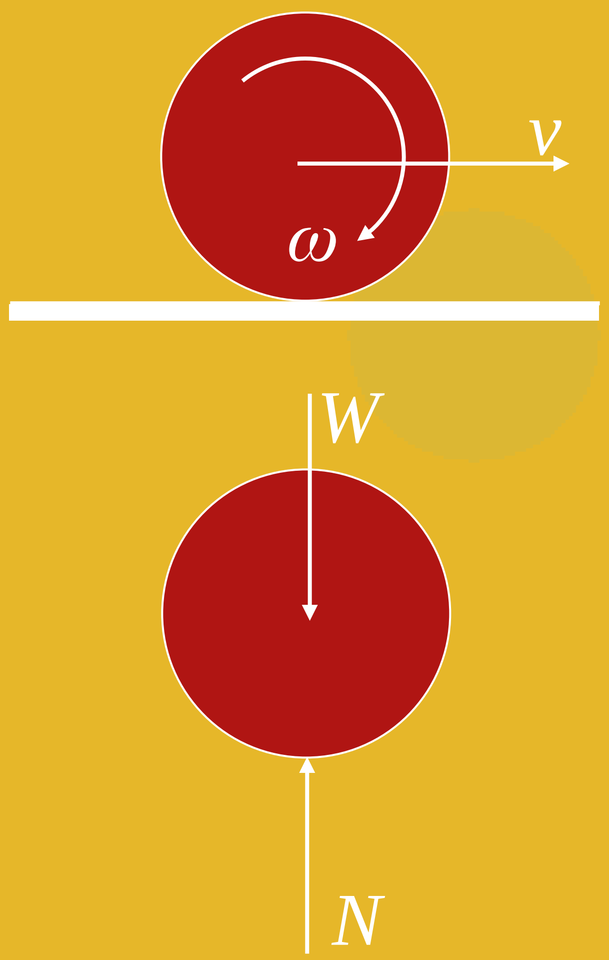
What is Rolling Friction

- Occurs when an object rolls across a surface
- Easier to overcome than sliding friction for similar materials
- Ball bearings are in moving objects such as skates, skateboards, and bicycles
- Ball bearings reduce friction by rolling between moving parts



I: Free Rolling or Inertial Rolling

- Continuum assumption
- Rigid Cylindrical Roller
- Rigid Horizontal Surface
- Velocity remains constant
- FBD gives:
  - $N = W$
  - No Frictional Force
  - No Rolling Friction



Free body diagram

Sliding Friction

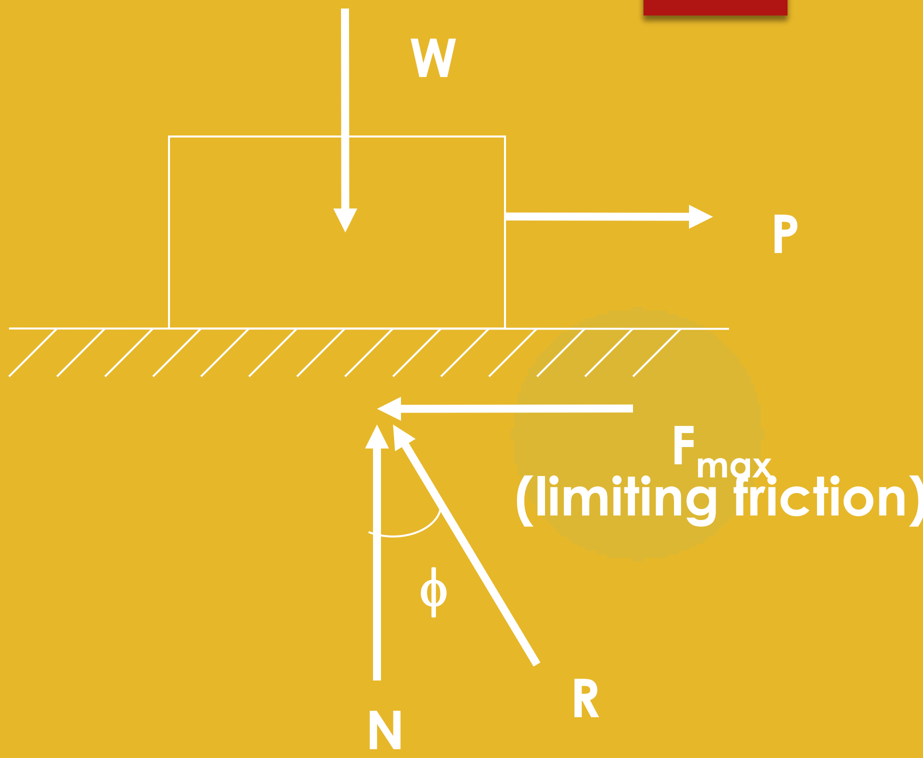
- Occurs when 2 solid surfaces slide over each other
- Examples: spreading sand on icy path
- Ballet dancers applying resin to ballet slippers so they don't slip
- Bicycle brakes
- Skinning your knee

Fluid Friction

- Occurs when solid objects move through a fluid
- Easier to overcome than sliding friction
- Reason why moving parts are often bathed in oil or other lubricants
- Fluid friction occurs between a bicyclist and the air. Bicyclists often wear streamlined helmets and clothes Fluids, such as water, oil, or air, are materials that flow easily.

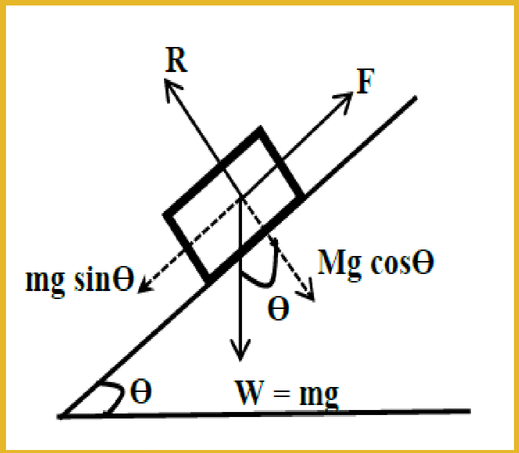
Angle of Friction

The angle between N & R depends on the value of F. This angle θ, between the resultant R and the normal reaction N is termed as angle of friction. As F increases, θ also increases and will reach to a maximum value of ϕ when F is F_{max} (limiting friction)



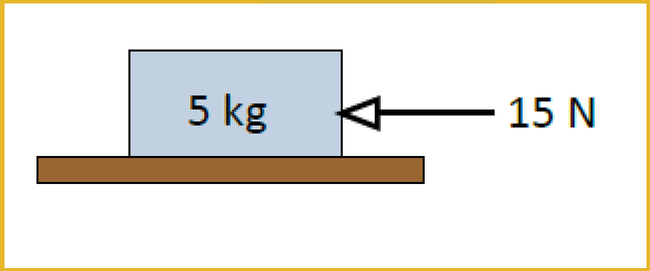
i.e. $\tan\phi = (F_{max})/N = \mu$

Angle ϕ is known as Angle of limiting Friction.



Problems

1) 5 kg box on a horizontal table is pushed by a horizontal force of 15 N as shown on the right. .If the coefficient of friction is 0.4, will the box move?



Ans:The sketch shows the forces acting on the box.

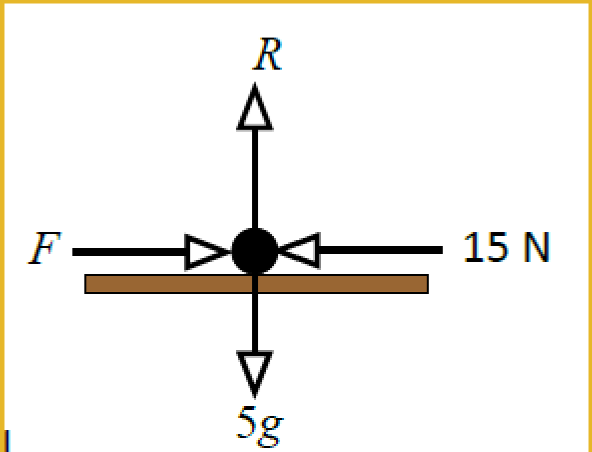
Note that the weight of a box of mass 5 kg is $5g$ where $g = 9.8\ ms^{-2}$

Since the vertical forces are in equilibrium, $R = 5g$

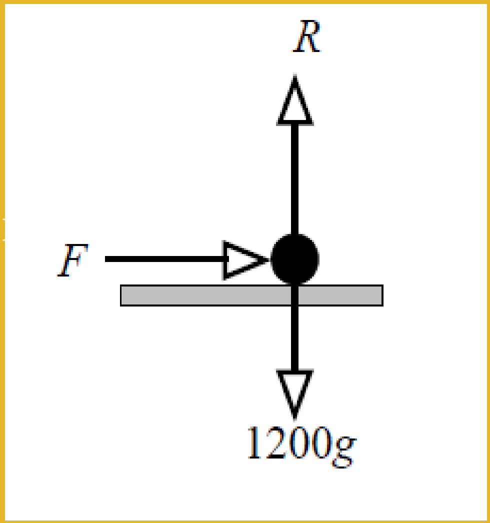
Therefore the maximum possible friction is $F = \mu R = 0.4 \times 5g = 19.6$ N

The pushing force, 15 N, is less than this and so cannot overcome the friction.

The box will not move.



2) A car of mass 1.2 tonnes is travelling along a straight horizontal road at a speed of 20 ms^{-1} when it brakes sharply then skids.
Friction brings the car to rest.
If the coefficient of friction between the tyres and road is 0.8, calculate
a) the deceleration
b) the distance travelled by the car before it comes to rest
a The sketch shows the forces acting on the car when it is skidding.



Since the vertical forces are in equilibrium $R = 1200g$
Since the car is skidding $F = \mu R = 0.8 \times 1200g = 9408$ N
The friction F causes deceleration a which is given by Newton's Seco
$-9408 = 1200a$ so $a = -7.84$
The deceleration is **7.84 ms^{-2}**
. **b)** Using with initial speed $u = 20$ ms^{-1}, final speed $v = 0$ ms^{-1}
and acceleration $a = -7.84$ ms^{-2}
$0^2 = 20^2 - 2 \times 7.84\, s$, $15.68\, s = 400$ $s = = 25.5$
The car travels **25.5 m** (to 3 sf) before coming to rest.

A car is taking a corner with radius of 100 m at a speed of 56 mph (25 m/s). On dry pavement the rubber/asphalt coefficient of static friction is 0.8. Will the car hold the line or start skidding?

So far we have found $n = mg$ and $f = \frac{mv^2}{r}$

We also know that for static friction: $f_s \le \mu_s n$

The maximum possible friction force is $\mu_s n$. Is this sufficient to keep the car going around the curve?

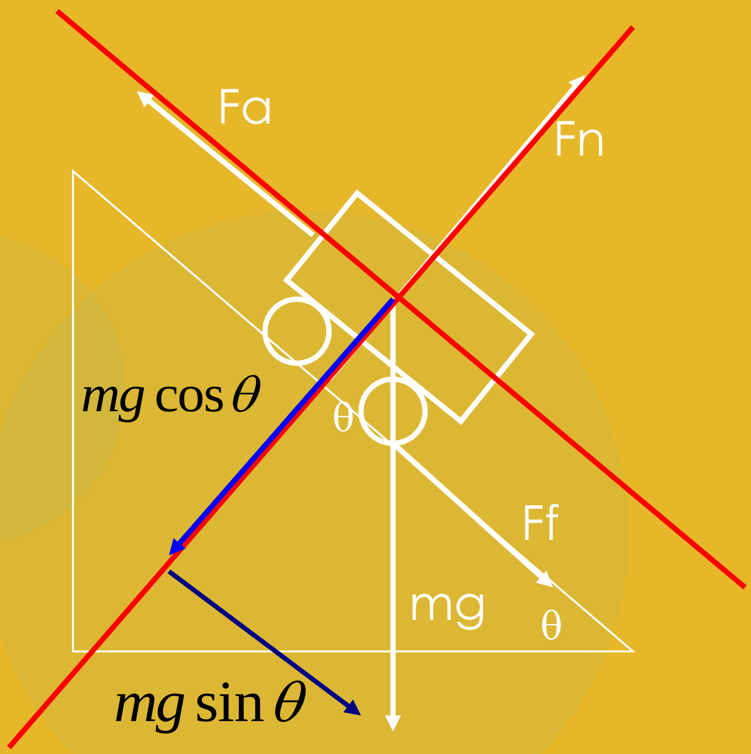
$f_{\max,s} = \mu_s n = \mu_s mg = 0.8 \cdot m \cdot 9.8\ \mathrm{m/s^2} = m \cdot 7.84\ \mathrm{m/s^2}$ (max friction force)

$f = \frac{mv^2}{r} = \frac{m \cdot (25\ \mathrm{m/s})^2}{100\ \mathrm{m}} = m \cdot 6.25\ \mathrm{m/s^2}$ (actual friction force needed)

So $f \le \mu_s n$ and therefore the car will hold the corner.

Friction & Inclines

A person pushes a 30-kg shopping cart up a 10 degree incline with a force of 85 N. Calculate the coefficient of friction if the cart is pushed at a ***constant speed.***



$$F_a = F_f + mg\sin\theta \quad F_f = \mu_k F_N$$

$$F_a = \mu_k F_N + mg\sin\theta \quad F_N = mg\cos\theta$$

$$F_a = \mu_k mg\cos\theta + mg\sin\theta$$

$$F_a - mg\sin\theta = \mu_k mg\cos\theta$$

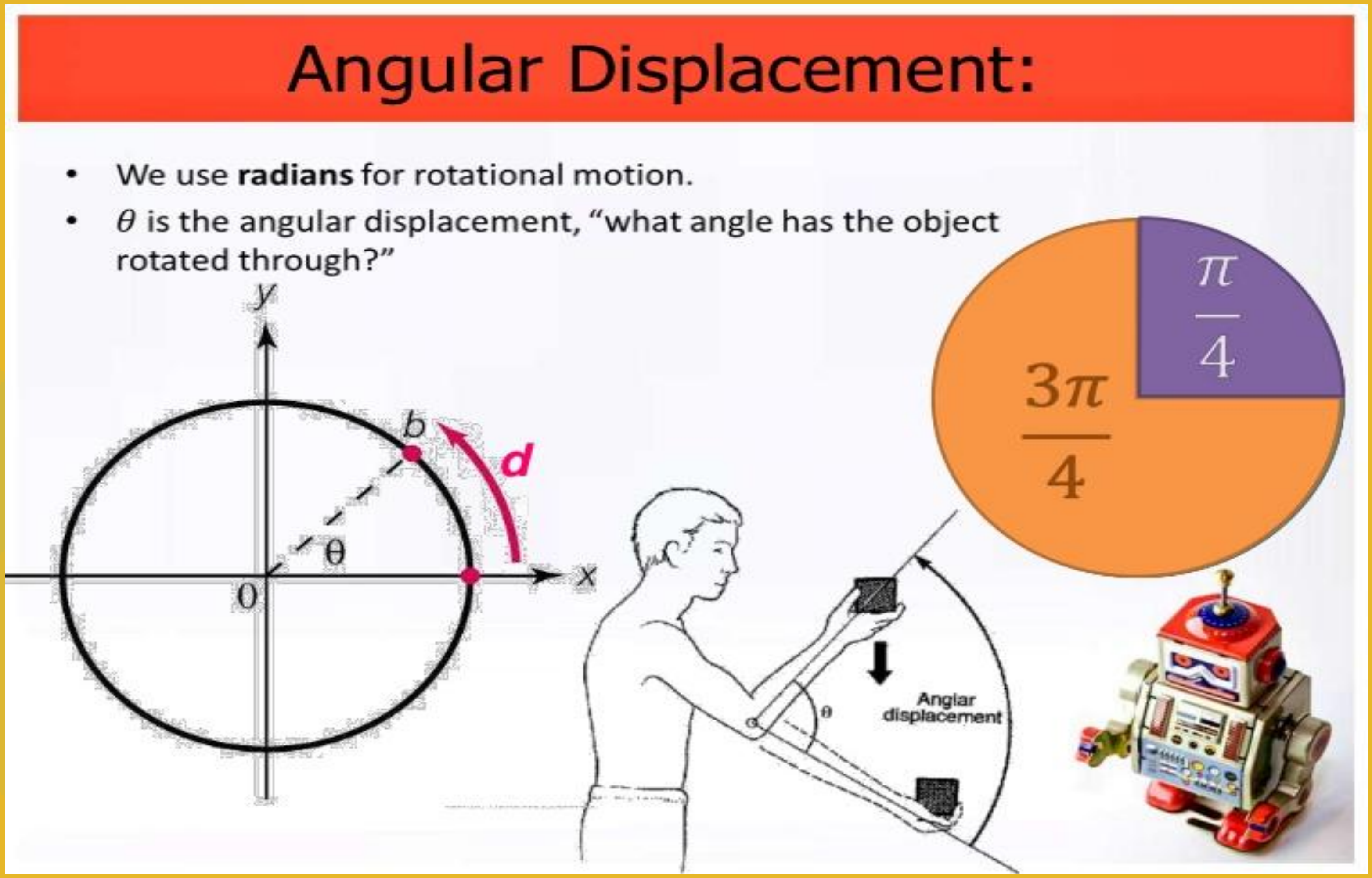
$$\mu_k = \frac{F_a - mg\sin\theta}{mg\cos\theta}$$

$$\mu_k = \frac{85 - (30)(9.8)(\sin 10)}{(30)(9.8)(\cos 10)} = \mathbf{0.117}$$

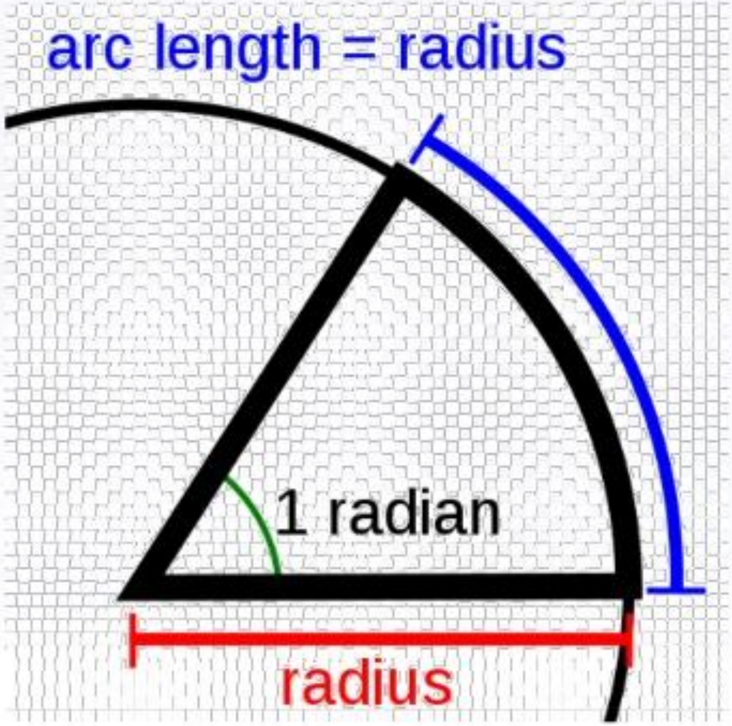
UNIT - V

DYNAMICS OF RIGID BODIES - MOMENT OF INERTIA

ROTATIONAL MOTION



What is a radian?



1 radian is about a sixth of a whole circle

(about 57°)

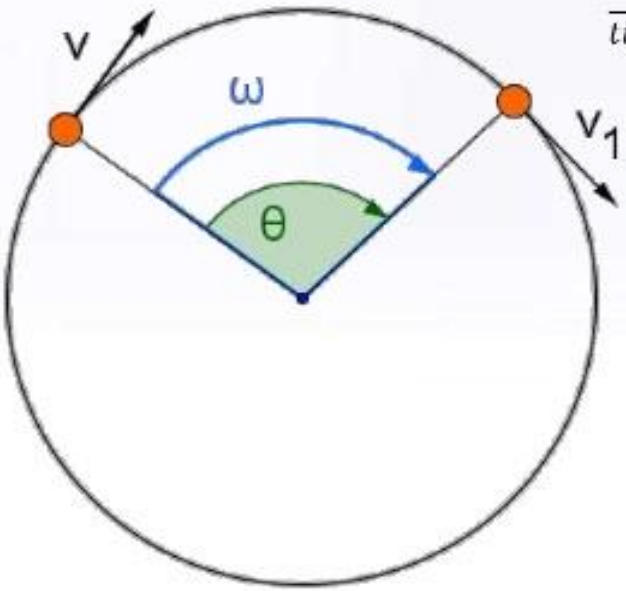
So…

There are just over 6 radians in a circle.

Angular Velocity:

- ω is the angular velocity, the angle the object has turned through divided by the time taken to do it, $\boldsymbol{\omega = \frac{\Delta\theta}{\Delta t}\ (rads^{-1})}$

- α is the angular acceleration, $\boldsymbol{\alpha = \frac{\Delta\omega}{\Delta t}\ (rads^{-2})}$



$$\frac{One\ revolution}{time\ taken\ for\ one\ revolution}$$

$$f = \frac{1}{T}$$

Other useful formulae:

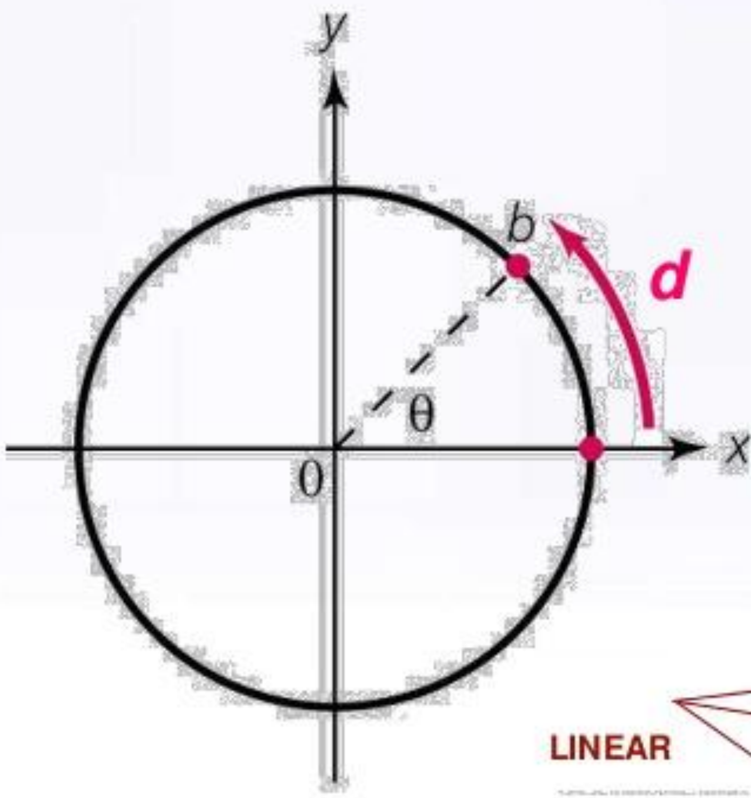
$$\boldsymbol{\omega = \frac{2\pi}{T}}$$

$$\boldsymbol{\omega = 2\pi f}$$

where T = period,

f = frequency.

Finding Linear Quantities:



Circumfrence = $2\pi r$

So the distance will be:

$$\frac{d}{2\pi r} = \frac{\theta}{2\pi}$$

$$\frac{d}{r} = \theta$$

$$d = r\theta$$

Similarly for ω and α

LINEAR

$$d = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

ANGULAR

Kinematics

- All our old mates the familiar kinematics for linear motion still hold for rotational motion although with new symbols.

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_f^2 + 2\alpha\theta$$

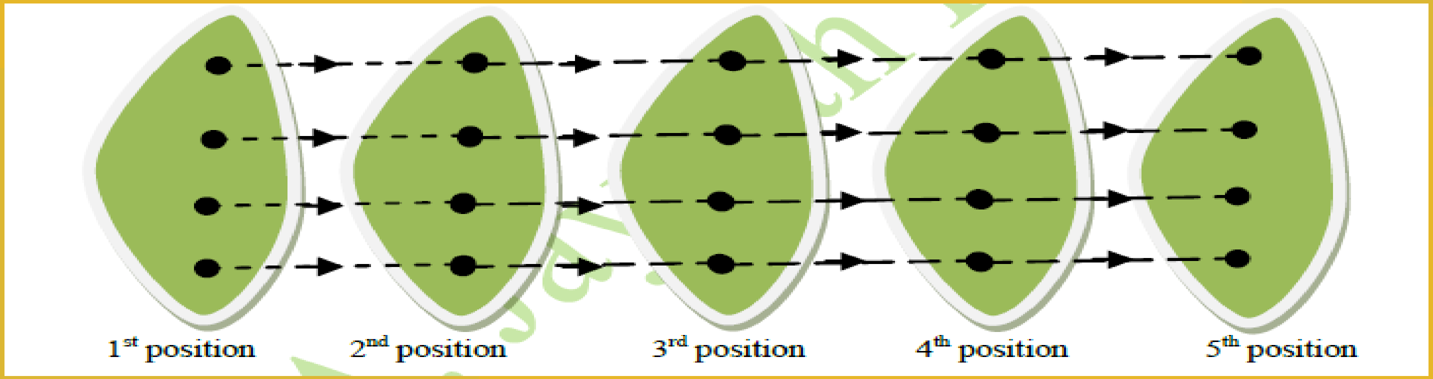
$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Rigid body:

Definition: A body which retains its shape and distance between any two particles remain
constant irrespective of any amount of force applied.
Examples: iron frames, wheels, doors, granite stones etc.

Translatory motion:

Definition: A body is said to have translatory motion if all particles in it move with the same velocity and their paths are parallel.



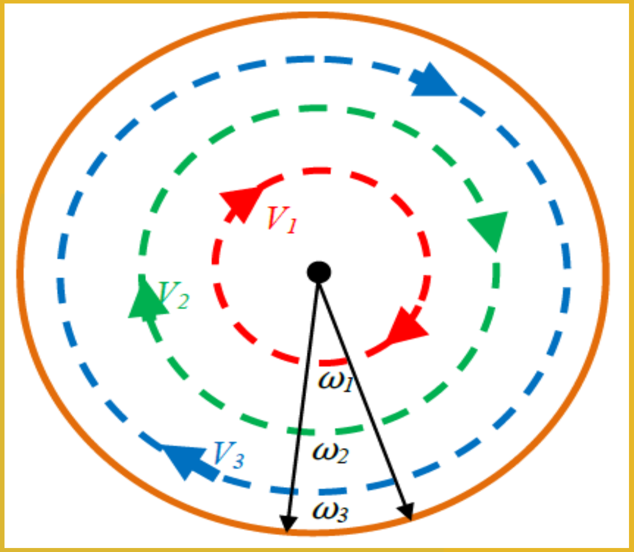
All the particles in the body move with equal velocity, They travel equal distances in equal intervals of time. The paths of the particles are parallel as shown in the above diagram.

Examples: 1. Motion of train.
2. Freely falling body without spin.
3. The motion of motor car on a straight road .

Rotatory motion:

Definition: **A body is said to be in circular motion when it moves in a circular path about a fixed point or about an axis.**

Different particles in the body move with different velocities. The linear velocities of the particles vary directly with their distances from the axis of rotation. The particles of same



The linear velocities V1 < V2 < V3

Linear velocity increases with radius.

The angular velocities ω1 = ω2 = ω3

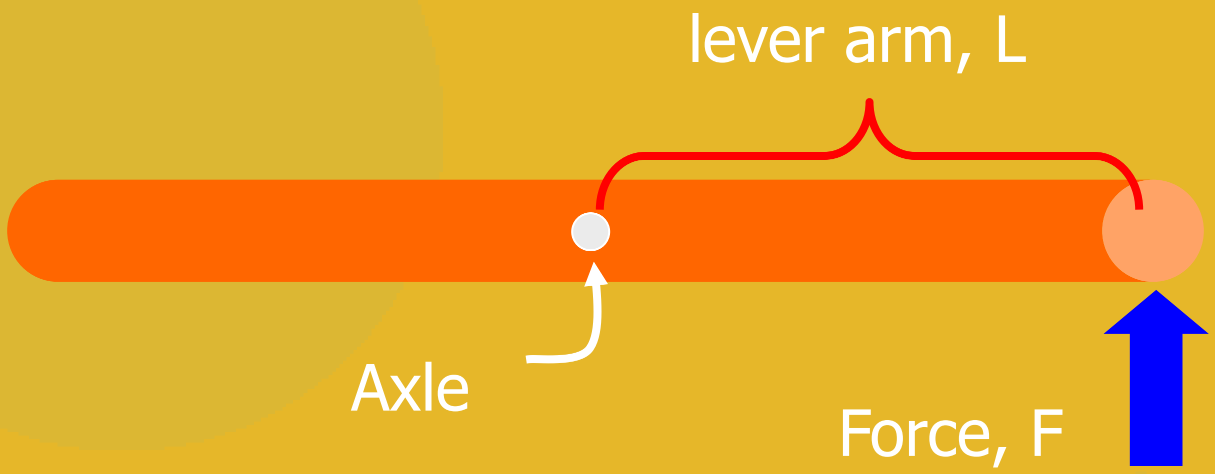
The angular velocities are same.

Examples:

1. Motion of wheel.
2. Rotatory motion of earth about its own axis.
3. Motion of bicycle wheel or grinding wheel.

TORQUE

- To make an object rotate, a force must be applied in the right place.
- the combination of force and point of application is called **TORQUE**



Torque = force times lever arm

Torque = $F \times L$

Units: SI units - Newton-m (N-m), CGS unit - Dyne-cm

Dimensional formula: Torque = Force x Distance

$= (MLT^{-2})(L)$

$= (ML^{2}T^{-2})$

Moment of inertia:

According to Newton's first law or law of inertia; every body continues its state of rest or motion unless an external force acts on it. This law is called the law of inertia. The property of the body to maintain its state of rest or motion is called "inertia".

When a rotating body about an axis keeps on rotating with the same angular velocity unless an external force acts on it in order to increase or decrease or to stop the motion depending on the magnitude and direction of applied force.

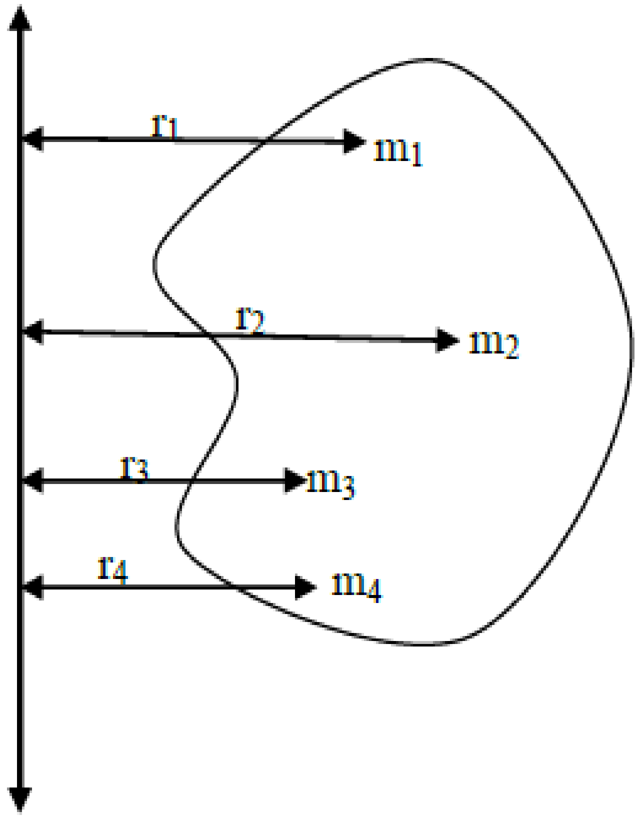
Definition: The moment of inertia of a rigid body about an axis is the sum of products of particle
and the square of the distance between the axis of rotation and the particle.
If ' m ' is the mass of a particle and ' r ' is radius of rotation (the perpendicular distance from axis of
rotation to the particle; the moment of inertia, i = m r2 .
A rigid body contains a large number of particles with masses m1. m2, m3, ..., at distances r1, r2, r3, ..., respectively.

$$I = i_1 + i_2 + i_3 + i_4 + \ldots\ldots$$
$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + \ldots\ldots$$
$$= \sum m r^2 \quad or$$
$$I = M K^2$$

M is the total mass of the body and

K is called the RADIUS OF GYRATION

The whole mass of the body is expected to be situated at its center of mass. Radius of gyration is the distance from the center of mass of the body to the axis of rotation.



Radius of gyration is the root mean square distance of all the particles of the body from the axis of rotation.

Units: MKS units: kg-m^2 CGS units: gram-cm^2

Dimensional formula: Mass × $(\text{Radius of gyration})^2 = [M] . [L]^2 = [ML^2]$

PERPENDICULAR AXIS THEOREM

The moment of inertia of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis.

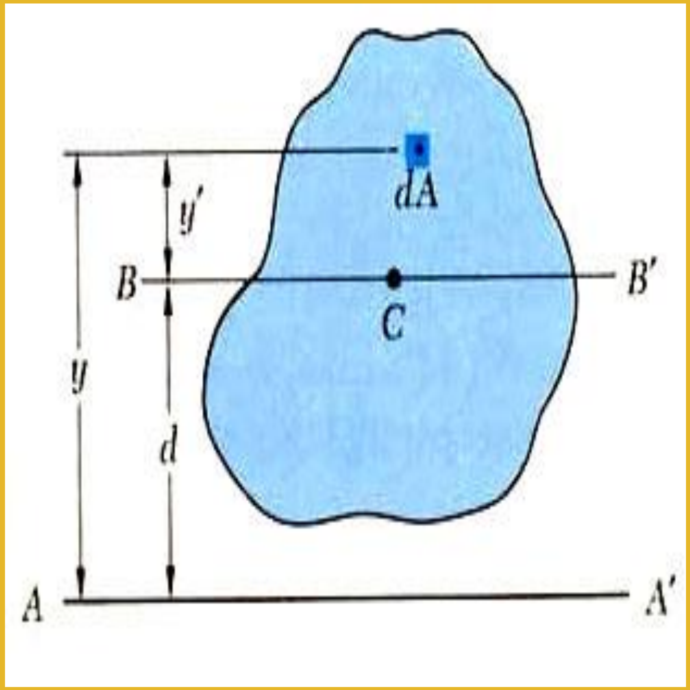
Moment of Inertia: $I_z = I_x + I_y$

PARALLEL (TRANSFER)AXIS THEOREM

- ***THE MOMENT OF AREA OF AN OBJECT ABOUT ANY AXIS PARALLEL TO THE CENTROIDAL AXIS IS THE SUM OF MI ABOUT IT'S CENTRODAL AXIS AND THE PRODUCT OF AREA WITH THE SQUARE OF DISTANCE OF CG FROM THE REF AXIS***

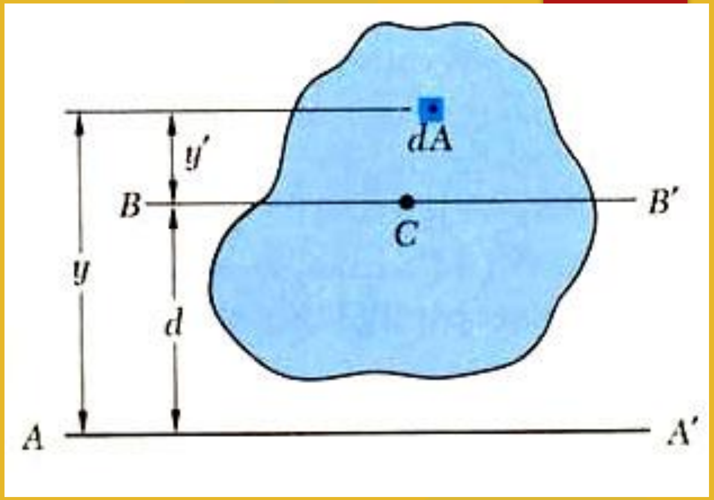
- $I_{XX} = I_G + Ad^2$

- A is the cross-sectional area.
  : is the perpendicuar distance between the centroidal axis and the parallel axis.



Parallel axis theorem:

Consider the moment of inertia Ix of an area A with respect to an axis AA'. Denote by y the distance from an element of area dA to AA'.



$$I_x = \int y^2 dA$$

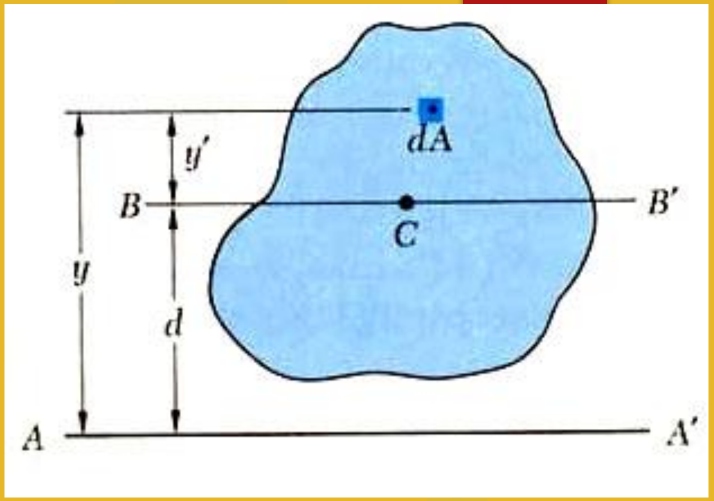
Consider an axis BB' parallel to AA' through the centroid C of the area, known as the centroidal axis. The equation of the moment inertia becomes

$$I_x = \int y^2 dA = \int (y' + d)^2 dA$$

$$= \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

The first integral is the moment of inertia about the centroid.

$$\overline{I_x} = \int y'^2 dA$$



The second component is the first moment area about the centroid

$$\overline{y'}A = \int y' dA \Rightarrow \overline{y'} = 0$$

$$\Rightarrow \int y' dA = 0$$

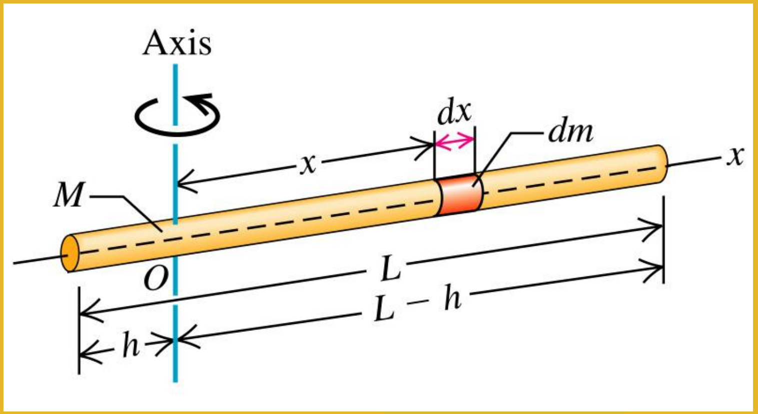
Modify the equation obtained with the parallel axis theorem.

$$I_x = \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

$$= \overline{I_x} + d^2 A$$

MOMENT OF INERTIA OF VARIOUS BODIES

UNIFORM THIN ROD, AXIS ⊥ TO LENGTH



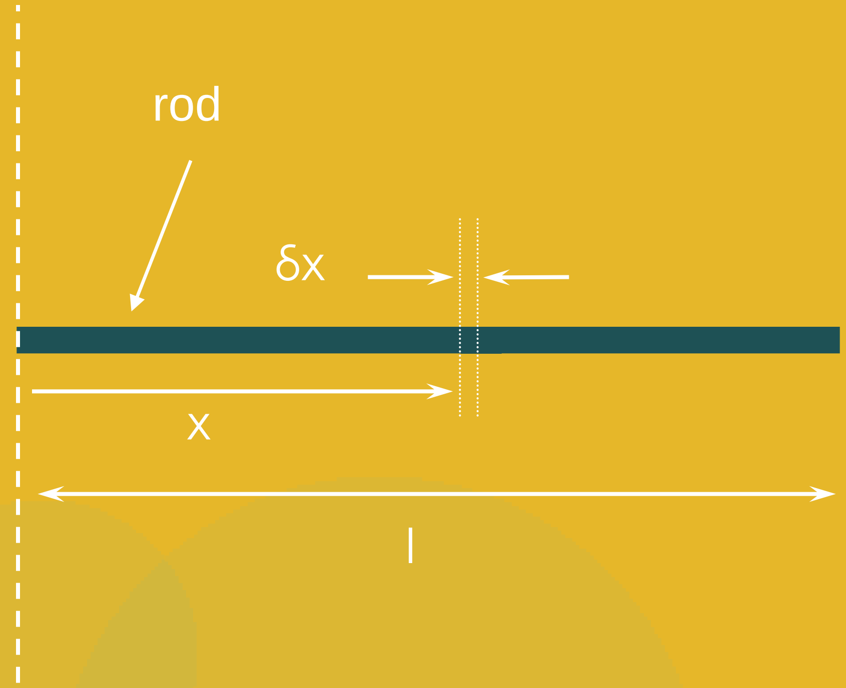
- Slender uniform rod with mass ***M*** and length **L**.
- Compute its moment of inertia about an axis through ***O***, at an arbitrary distance **h** from the end.
- Choose as an element of mass a short section of rod with length d***x*** at a distance ***x*** from ***O***. The ratio of the mass d***m*** of this element to the total mass ***M*** is equal to the ratio of its length d***x*** to the total length ***L***:

$$\frac{dm}{M} = \frac{dx}{L}$$

$$dm = \frac{M}{L} dx$$

$$\int x^2 dm = \frac{M}{L} \int_{-h}^{L-h} x^2 dx = \left[\frac{M}{L} \left(\frac{x^3}{3} \right) \right]_{-h}^{L-h} = \frac{1}{3} M (L^2 - 3Lh + 3h^2)$$

MOMENT OF INERTIA OF A ROD OF LENGTH '*l*' ABOUT AN AXIS THROUGH ONE END PERPENDICULAR TO THE ROD.



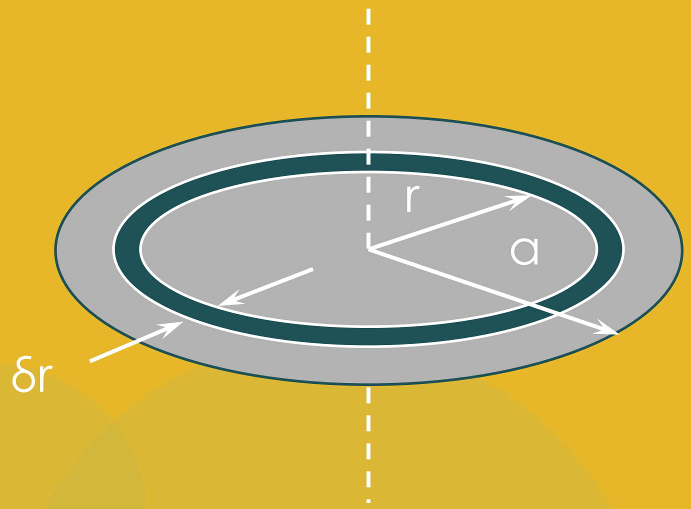
Let the rod's density be ρ per unit length. Consider an element of the rod, length δx, at distance x from one end. The mass of the element is δm = ρ δx. The contribution to the total moment of inertia from this element is δI, given by:

$$\delta I = x^2 \delta m = x^2 \rho \delta x$$

$$\therefore \quad I = \int_{x=0}^{l} \rho\, x^2 dx = \frac{1}{3} \rho\, l^3$$

But $M = \rho\, l$, so $I = \frac{1}{3} M l^2$

THE MOMENT OF INERTIA OF A SOLID DISC OF MASS M AND RADIUS A ABOUT AN AXIS THROUGH ITS CENTRE PERPENDICULAR TO THE PLANE OF THE DISC.



Let the disc have a surface density of σ per unit area. Consider a radial element of the disc at radius r, thickness δr. This has mass δm which is equal to 2πrδrσ. It makes a contribution, δI, to the total moment of inertia given by:

$$\therefore \quad \delta I = r^2 \delta m = 2\pi\sigma r^3 \delta r$$

$$\therefore \quad I = \int_{r=0}^{a} 2\pi\sigma r^3 dr = \frac{1}{2}\pi\sigma a^4$$

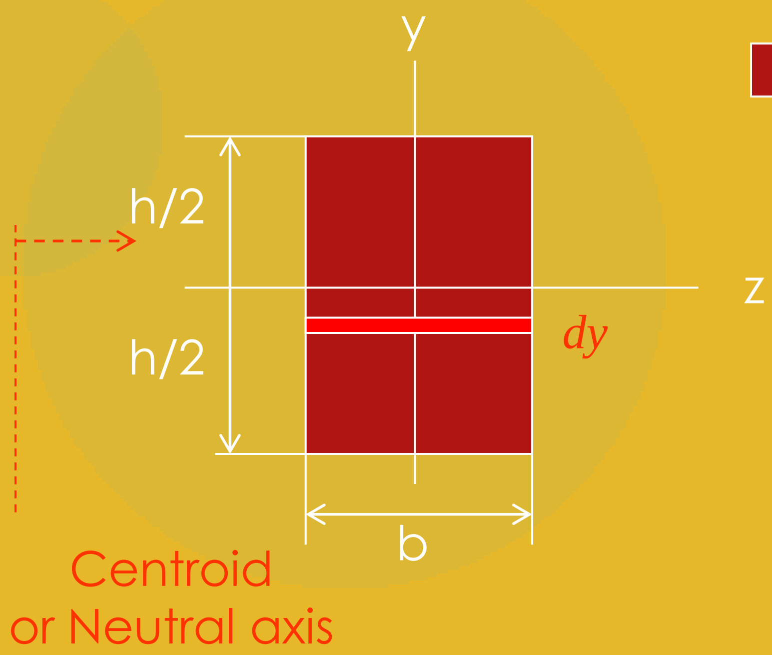
But $M = \pi\sigma a^2$, so $I = \frac{1}{2}Ma^2$

Note that this formula also applies to a cylinder.

Moment of Inertia simple rectangular shape

$$I_z = \int y^2 dA$$

$$dA = bdy$$



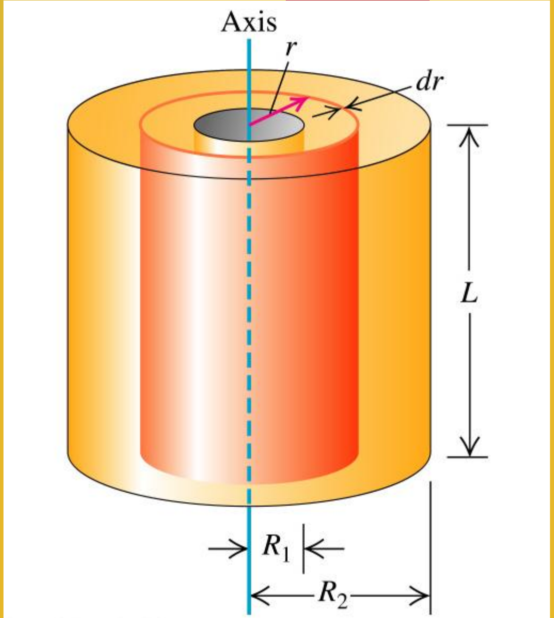
$$I_z = \int_{\frac{-h}{2}}^{\frac{h}{2}} y^2 bdy$$

$$= b\frac{y^3}{3}\Big|_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$= \frac{b}{3}\left[\frac{h^3}{8} - \left(-\frac{h^3}{8}\right)\right]$$

$$= \frac{bh^3}{12}$$

- **Hollow or solid cylinder, rotating about axis of symmetry**
- Hollow, uniform cylinder with length **L**, inner radius ***R_1***, outer radius ***R_2***. Compute its moment of inertia about the axis of symmetry.
- Choose as a volume element a thin cylindrical shell of radius ***r***, thickness d***r***, and length ***L***. All parts of this element are at very nearly ***the same distance*** from the axis. The volume of this element:



$$dm = \rho dV = \rho(2\pi rLdr)$$

$$\int r^2 dm = \int_{R_1}^{R_2} r^2 \rho(2\pi rLdr) = 2\pi\rho L \int_{R_1}^{R_2} r^3 dr = \frac{2\pi\rho L}{4}(R_2^4 - R_1^4) =$$

$$= \frac{\pi\rho L}{2}(R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

$$V = \pi L(R_2^2 - R_1^2)$$

$$I = \frac{1}{2}M(R_2^2 + R_1^2)$$

- **Hollow or solid cylinder, rotating about axis of symmetry**

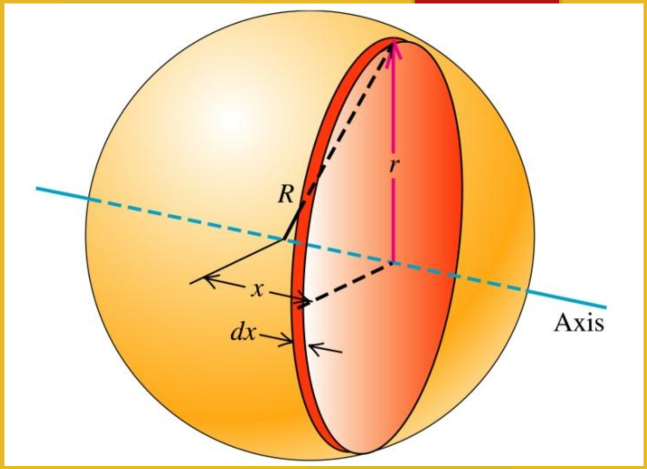
$$I = \frac{1}{2} M (R_2^2 + R_1^2)$$

- If cylinder is solid, R_1=0, R_2=R:

- If cylinder has a very thin wall, R_1 and R_2 are very nearly equal:

- ***Note***: moment of inertia of a cylinder about an axis of symmetry depends on its mass and radii, but ***not*** on its ***length***!

- **Uniform sphere, axis through center**
- Uniform sphere with radius **R**. the axis is through its center. Find the moment of inertia about the axis is through the center of this sphere.
- Divide sphere into thin disks of thickness d***x***, whose moment of inertia we already know. The radius ***r*** of the disk is

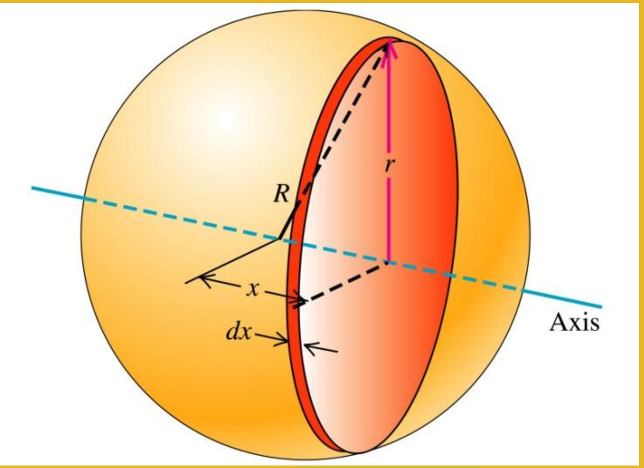


$$r = \sqrt{R^2 - x^2}$$

- The volume is $dV = \pi r^2 dx = \pi(R^2 - x^2)dx$
- The mass is $dm = \rho dV = \pi r^2 dx = \pi\rho(R^2 - x^2)dx$
- The moment of inertia for the disk of radius ***r*** and mass d***m*** is

$$dI = \frac{1}{2} r^2 dm = \frac{1}{2}\left(\sqrt{R^2 - x^2}\right)^2 [\pi\rho(R^2 - x^2)dx] = \frac{\pi\rho}{2}(R^2 - x^2)^2 dx$$

- Integrating from ***x***=0 to **x**=*R* gives the moment of inertia of the right hemisphere.
- From symmetry, the total ***I*** for the entire sphere is just twice this:



$$I = (2)\frac{\pi\rho}{2}\int_0^R (R^2 - x^2)^2\,dx$$

$$I = \frac{8\pi\rho}{15}R^5$$

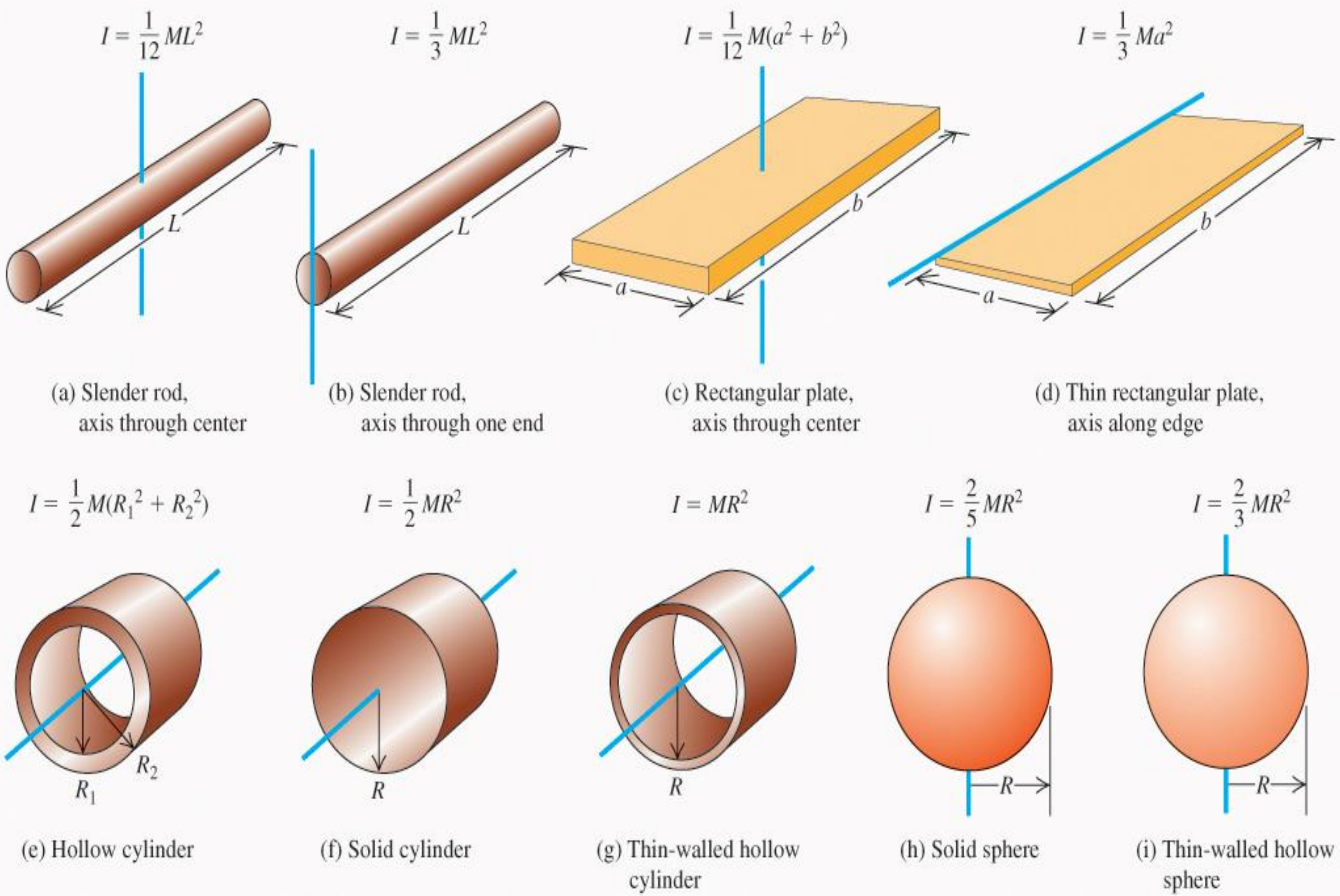
$$I = \frac{2}{5}MR^2$$

Volume of the sphere $V = \frac{4\pi R^3}{3}$

The mass ***M*** of the sphere $M = \rho V = \frac{4\pi\rho}{3}R^3$

- ***Note***: moment of inertia of a solid sphere is less than the moment of inertia of a solid cylinder of the same mass and radius! (*Reason is that more of the sphere's mass is located close to the axis*)

Table 9.2 Moments of Inertia of Various Bodies



Thank you

Quiz