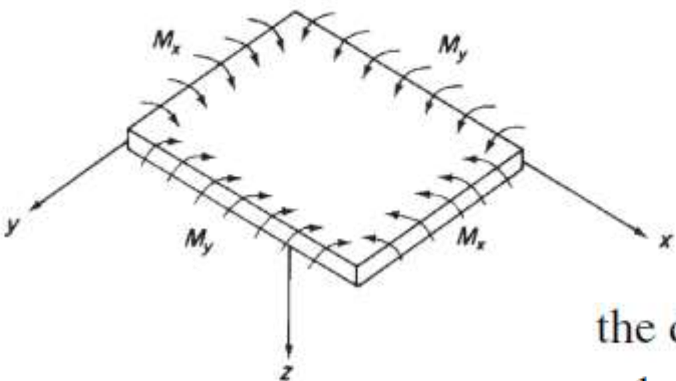
UNIT- I
Thin plate theory, Structural Instability:

Analysis of thin rectangular plates subject to bending, twisting, distributed transverse load, combined bending and in-plane loading

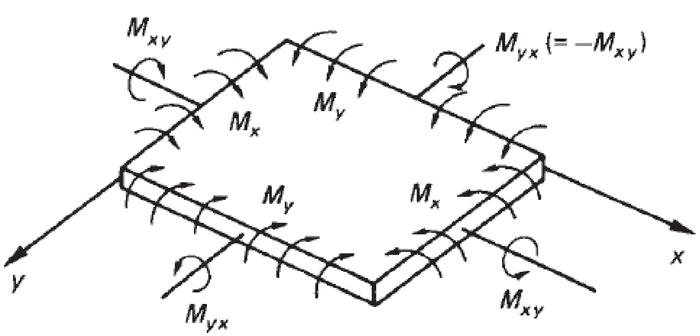


$$M_x = D\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right) \qquad M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = D\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

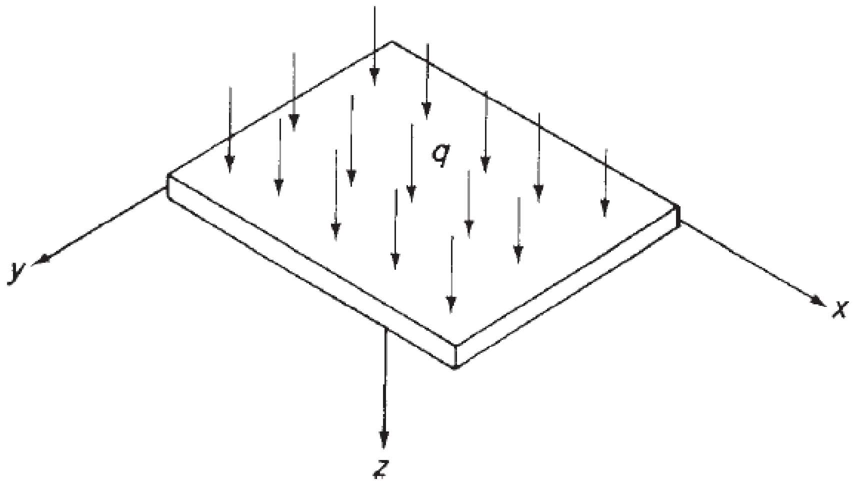
the deformed shape of the plate is spherical and of curvature

$$\frac{1}{\rho} = \frac{M}{D(1+\nu)} \qquad D = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2}\mathrm{d}z = \frac{Et^3}{12(1-\nu^2)}$$



$$M_{xy} = \frac{Et^3}{12(1+\nu)}\frac{\partial^2 w}{\partial x \partial y}$$

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$



$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

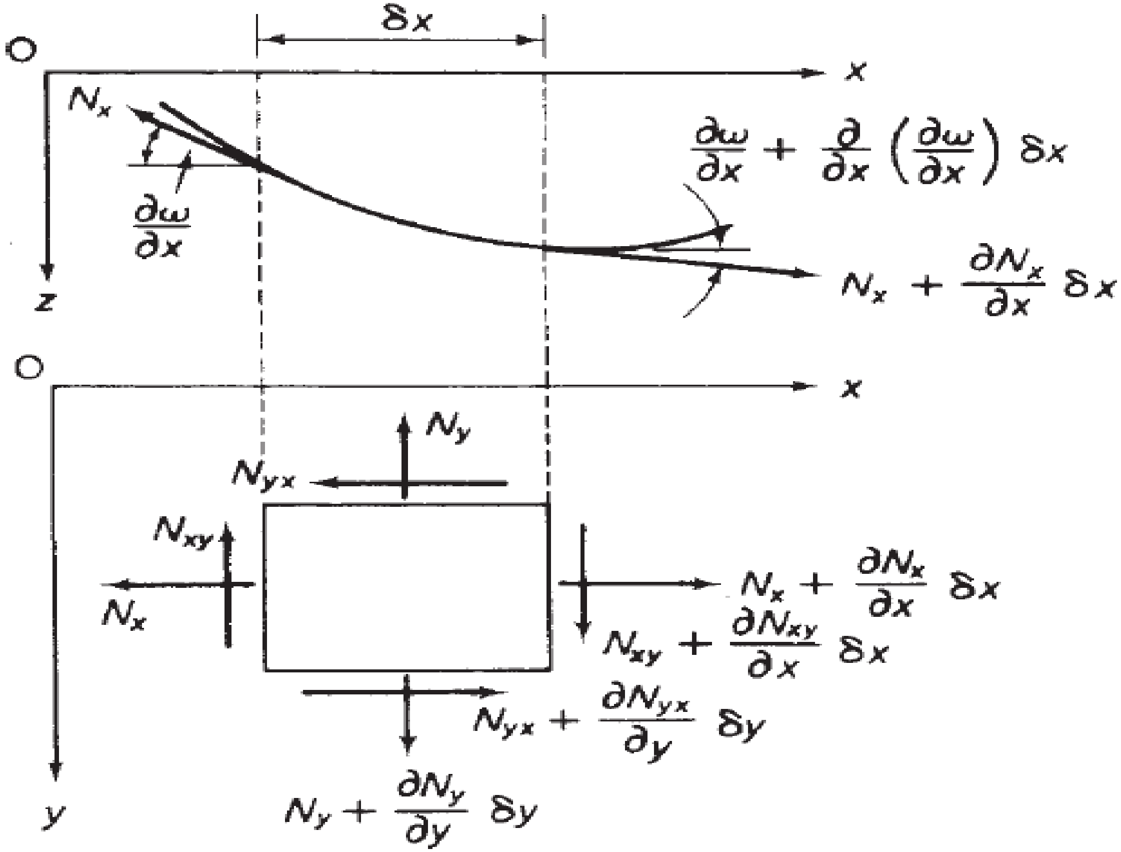
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w = \frac{q}{D}$$

The operator $(\partial^2/\partial x^2 + \partial^2/\partial y^2)$ is the well-known Laplace operator in two dimensions and is sometimes written as ∇^2. Thus

$$(\nabla^2)^2 w = \frac{q}{D}$$

$$Q_x = \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$



$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \qquad \frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial y} = 0$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(q + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\,\partial y}\right)$$

Thin plates having small initial curvature

thin plate has an initial curvature w_0
plate to deflect a further w_1

total deflection is then $w = w_0 + w_1$

$$\frac{\partial^4 w_1}{\partial x^4} + 2\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \frac{\partial^4 w_1}{\partial y^4} = \frac{1}{D}\left[q + N_x \frac{\partial^2 (w_0 + w_1)}{\partial x^2} + N_y \frac{\partial^2 (w_0 + w_1)}{\partial y^2} + 2N_{xy} \frac{\partial^2 (w_0 + w_1)}{\partial x \, \partial y}\right]$$

Assuming that the initial form of the deflected plate is

$$w_0 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

if N_x is compressive and $N_y = N_{xy} = 0$

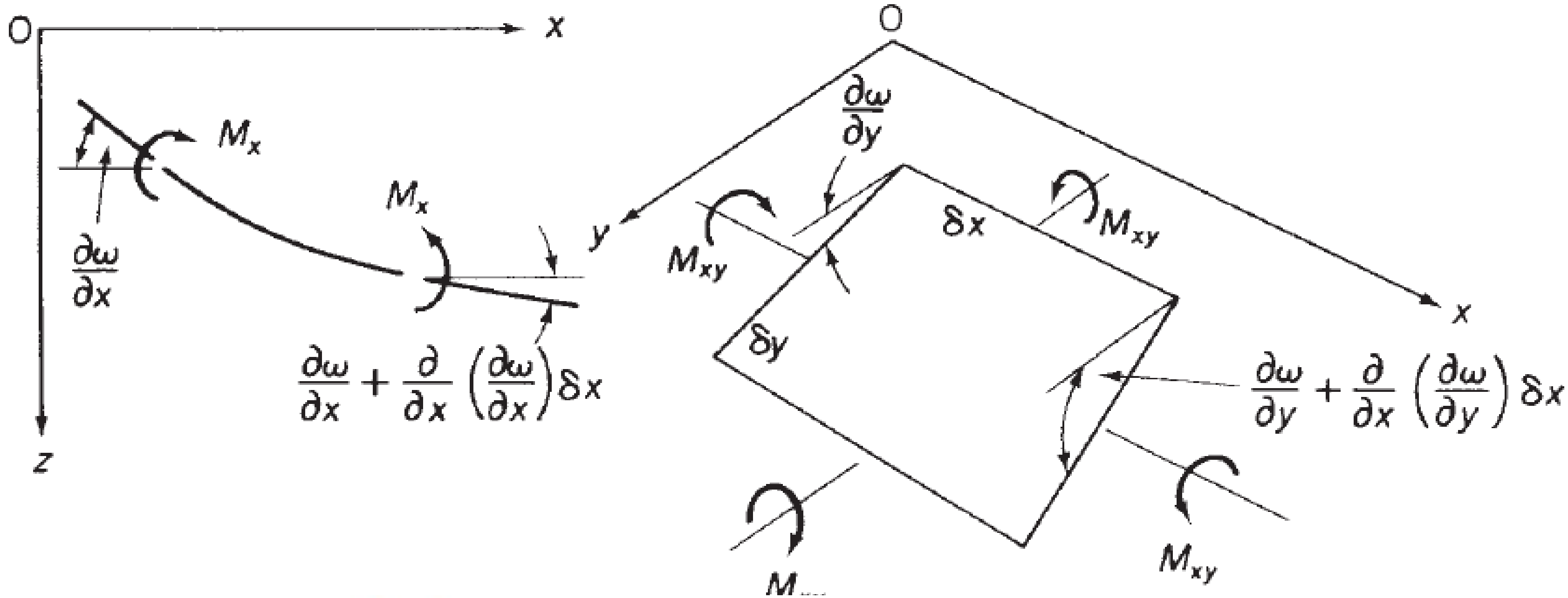
$$w_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

where

$$B_{mn} = \frac{A_{mn} N_x}{(\pi^2 D / a^2)[m + (n^2 a^2 / m b^2)]^2 - N_x}$$

Energy methods of analysis.

Strain energy produced by bending and twisting



Hence the total strain energy U of the rectangular plate $a \times b$ is

$$U = \frac{D}{2}\int_0^a \int_0^b \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y} \right)^2 \right] \right\} dx\,dy$$

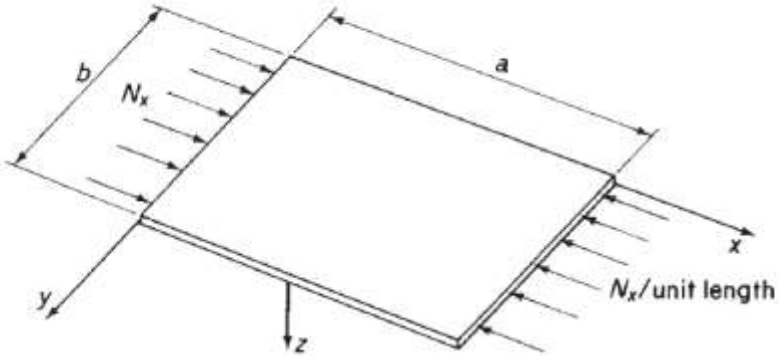
$$U = \frac{D}{2}\int_0^a \int_0^b \left[\left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \right] dx\,dy$$

$M_{xy} = 0$

$\partial^2 w / \partial x \partial y = 0,$

$$w = \frac{16 q_0}{\pi^6 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\sin(m\pi x/a)\sin(n\pi y/b)}{mn[(m^2/a^2) + (n^2/b^2)]^2}$$

Buckling of thin plates- elastic, inelastic



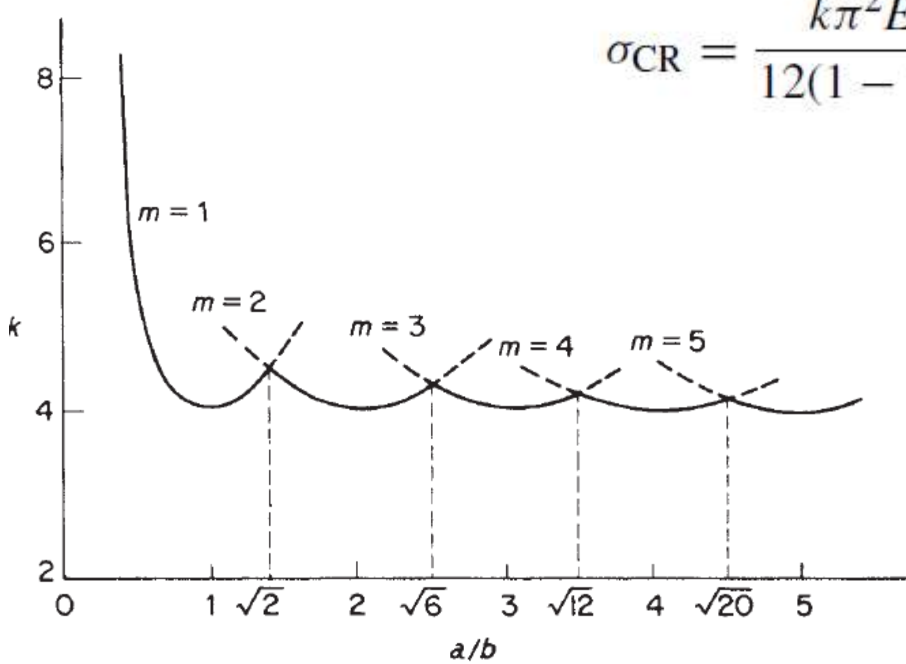
Buckling of a thin flat plate.

$$N_{x,\mathrm{CR}} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{1}{b^2}\right)^2$$

$$N_{x,\mathrm{CR}} = \frac{k\pi^2 D}{b^2}$$

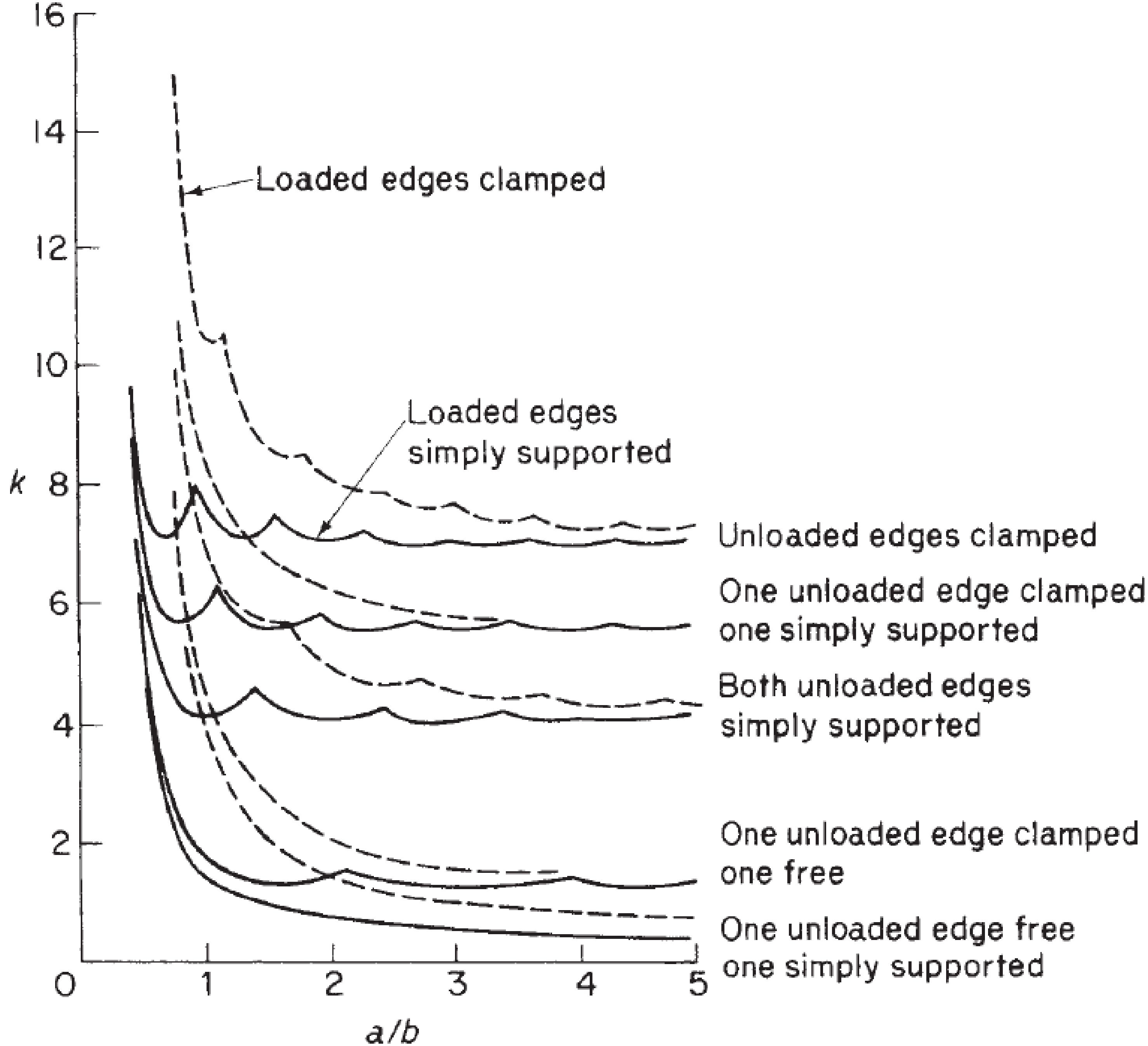
$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

$$\sigma_{\mathrm{CR}} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$



Inelastic buckling of plates

$$\sigma_{\mathrm{CR}} = \frac{\eta k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$



16
14
12
10
8
6
4
2
0
k
1
2
3
4
5
a/b
Loaded edges clamped
Loaded edges simply supported
Unloaded edges clamped
One unloaded edge clamped one simply supported
Both unloaded edges simply supported
One unloaded edge clamped one free
One unloaded edge free one simply supported

experimental determination of critical load for a flat plate

$$w_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$B_{mn} = \frac{A_{mn}N_x}{\frac{\pi^2 D}{a^2}\left(m + \frac{n^2a^2}{mb^2}\right)^2 - N_x}$$

$$w_1 = \frac{A_{11}N_x}{N_{x,\mathrm{CR}} - N_x}$$

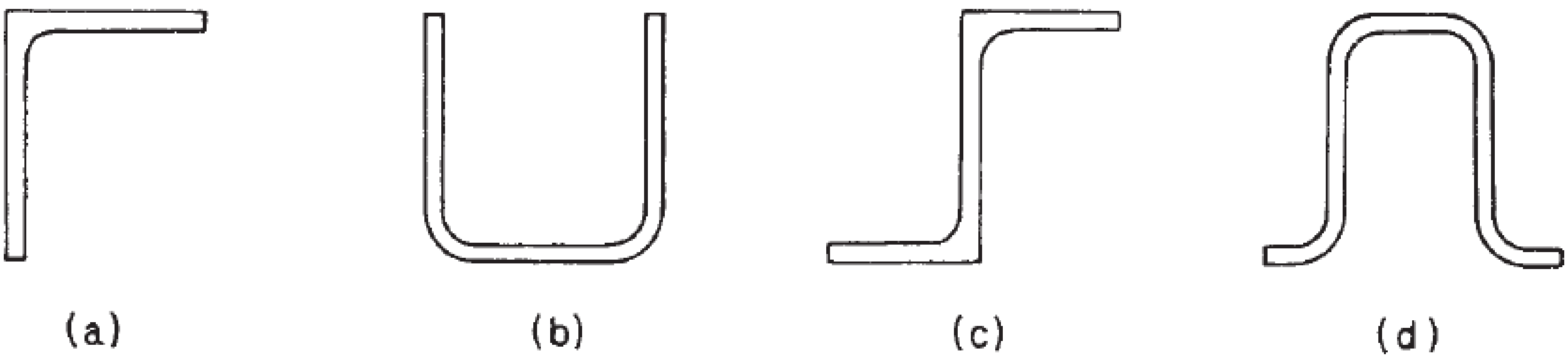
$$w_1 = N_{x,\mathrm{CR}}\frac{w_1}{N_x} - A_{11}$$

Thus, a graph of w_1 plotted against w_1/N_x will have a slope, in the region of the critical load, equal to $N_{x,\mathrm{CR}}$.

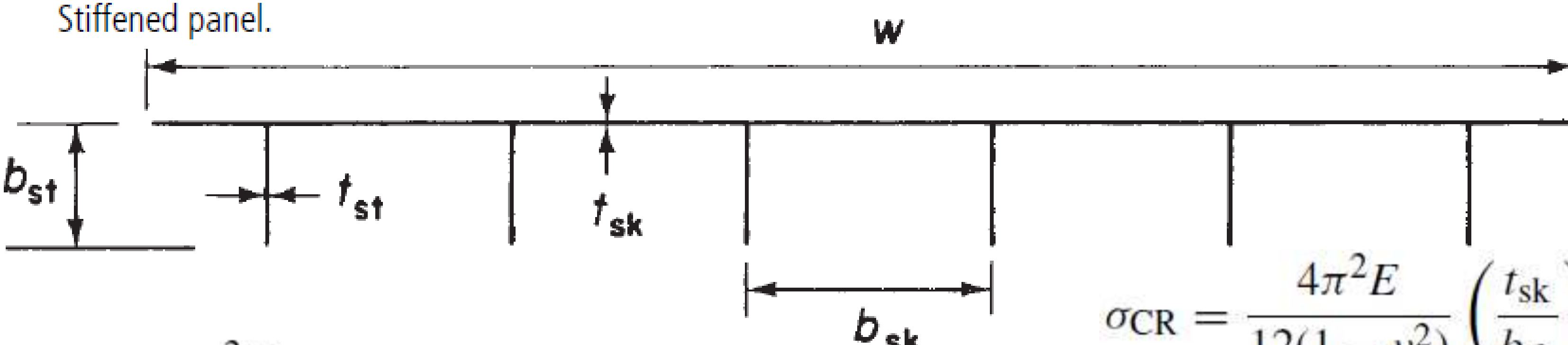
local instability, instability of stiffened panels,

$$\sigma_{CR} = 0.388E\left(\frac{t}{b}\right)^2 \quad (\nu = 0.3)$$

(a) Extruded angle; (b) formed channel; (c) extruded Z; (d) formed 'top hat'.



Stiffened panel.



$$\sigma_{CR} = \frac{4\pi^2 E}{12(1-\nu^2)}\left(\frac{t_{sk}}{b_{sk}}\right)^2$$

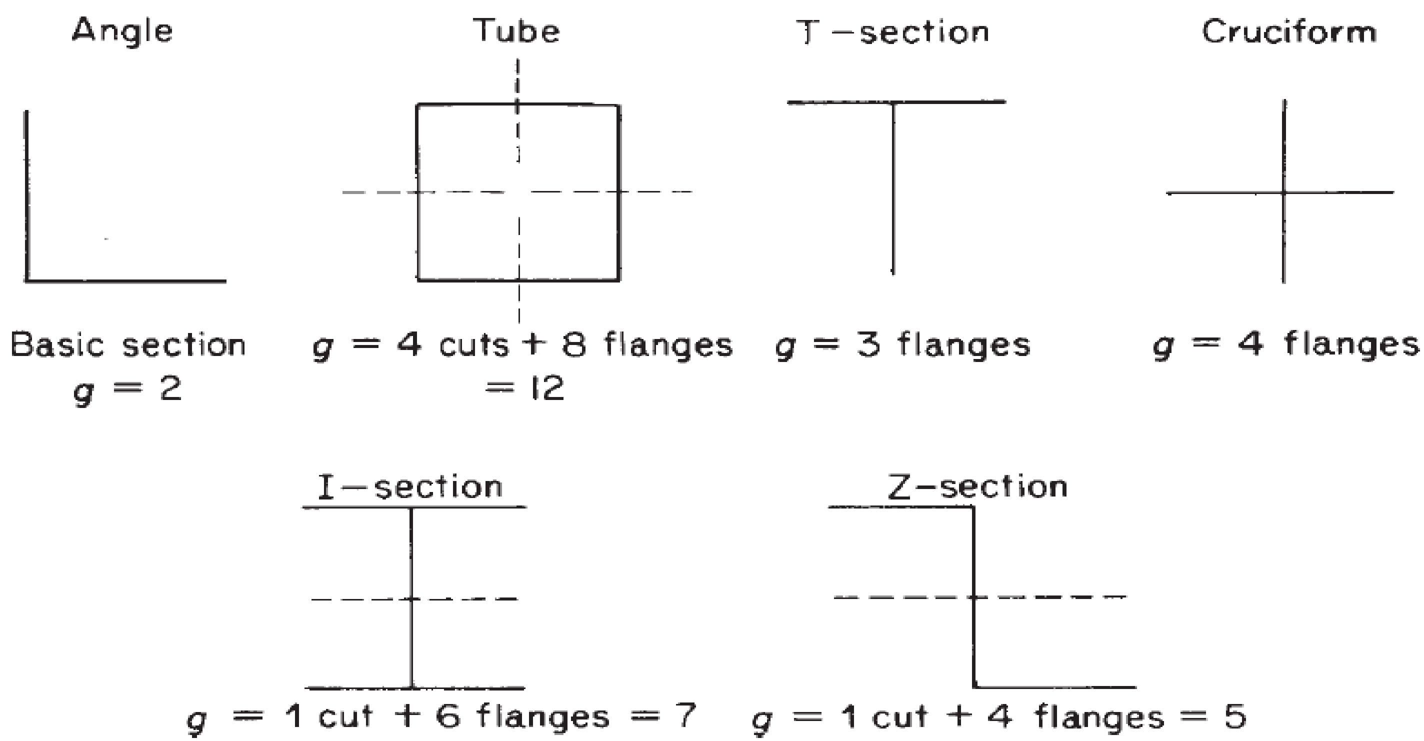
$$\sigma_{CR,E} = \frac{\pi^2 E}{(l_e/r)^2}$$

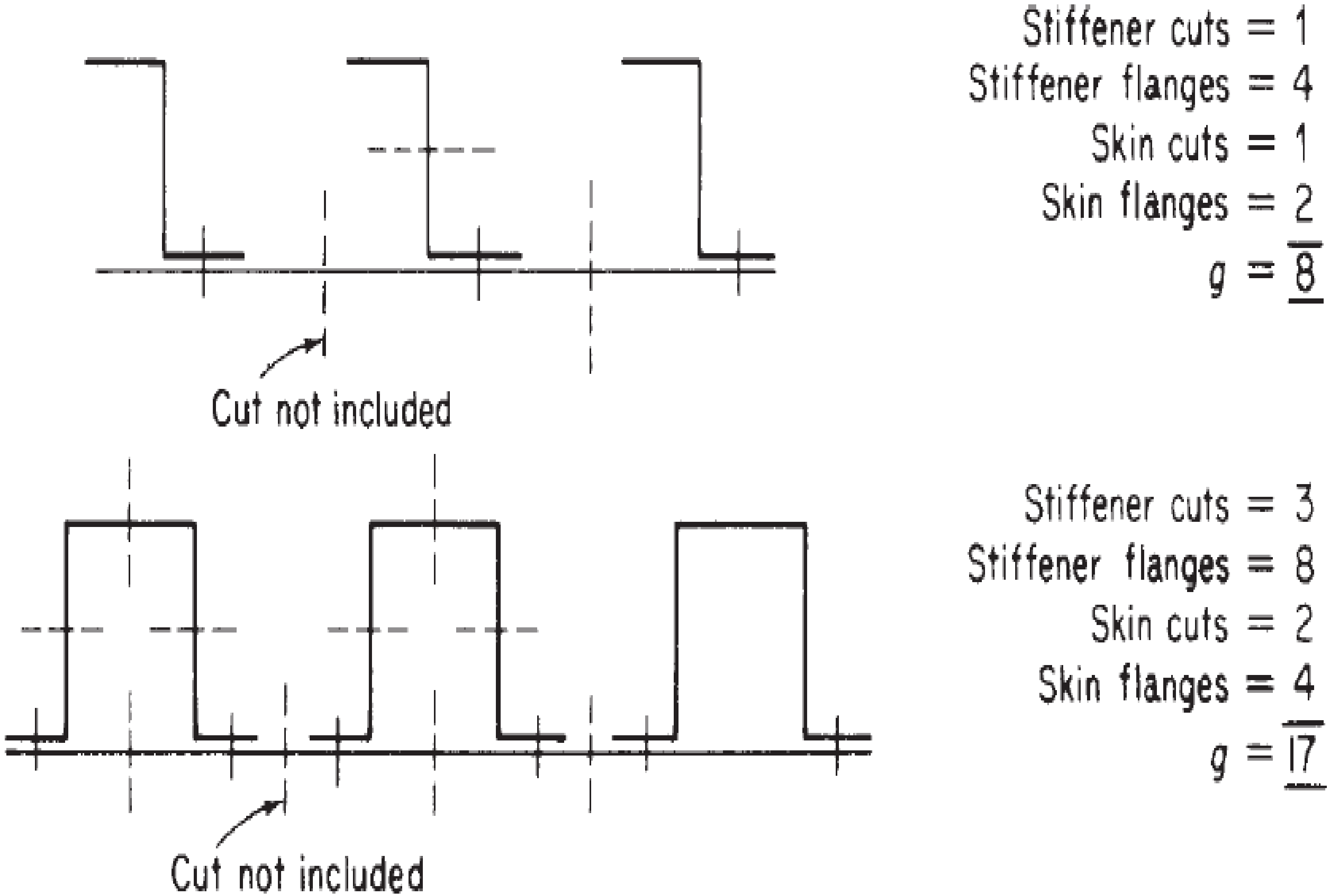
$$\sigma_{CR} = \frac{\eta k \pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

$$\sigma_{CR} = \frac{0.43\pi^2 E}{12(1-\nu^2)}\left(\frac{t_{st}}{b_{st}}\right)^2$$

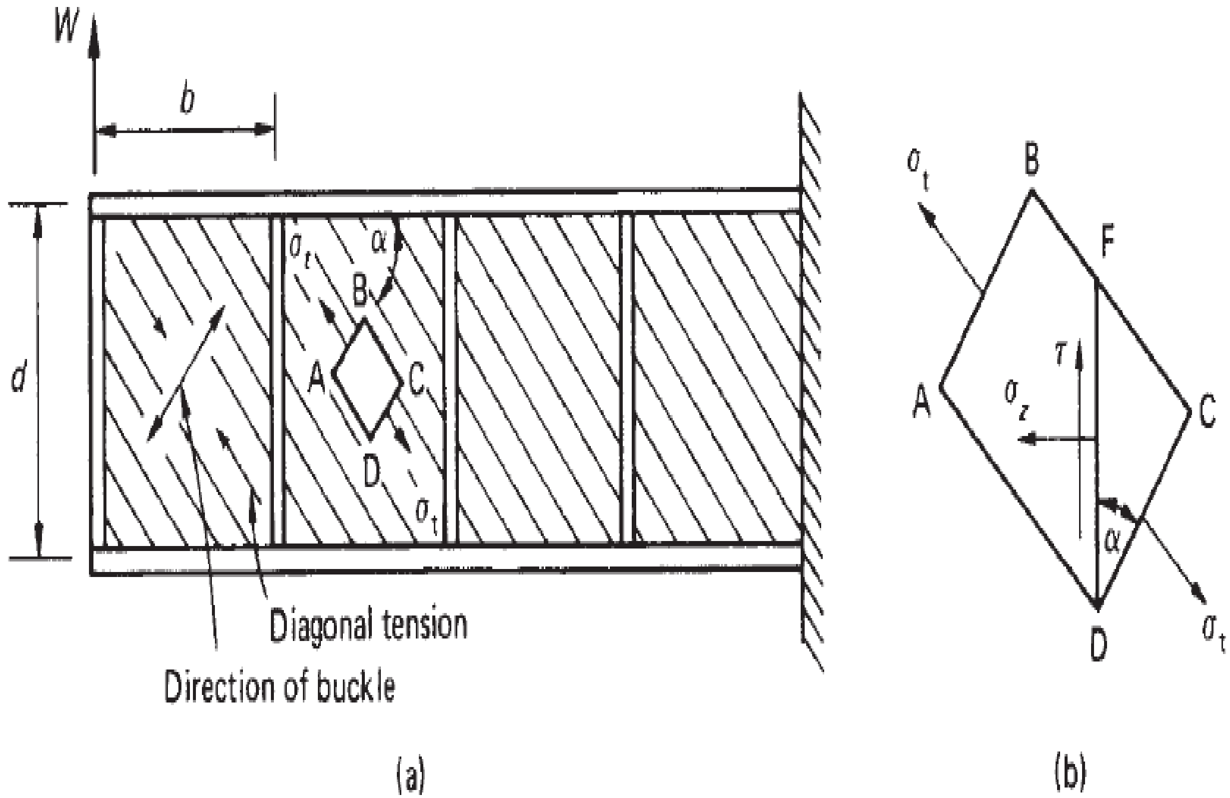
failure stresses in plates and stiffened panels.

$$\frac{\bar{\sigma}_a}{\sigma_{CR}} = \alpha_1 \left(\frac{\sigma_e}{\sigma_{CR}} \right)^n \qquad \sigma_{CR} = \frac{k\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \qquad \frac{\bar{\sigma}_f}{\sigma_{cy}} = \beta_g \left[\frac{g t_{sk} t_{st}}{A} \left(\frac{E}{\bar{\sigma}_{cy}} \right)^{\frac{1}{2}} \right]^m$$




Stiffener cuts = 1
Stiffener flanges = 4
Skin cuts = 1
Skin flanges = 2
g = 8
Cut not included
Stiffener cuts = 3
Stiffener flanges = 8
Skin cuts = 2
Skin flanges = 4
g = 17
Cut not included

Tension field beams- complete diagonal tension,



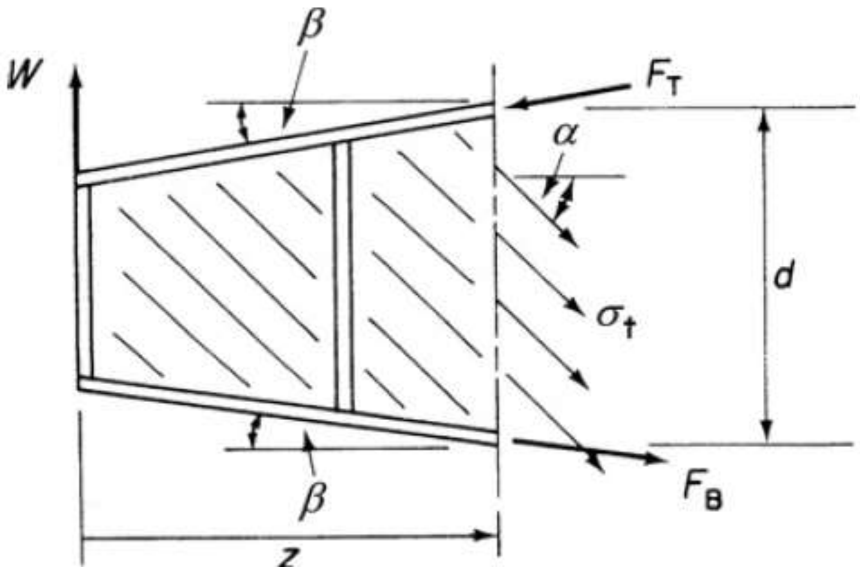
$$\tan^2 \alpha = \frac{\sigma_t + \sigma_F}{\sigma_t + \sigma_S}$$

$$\sigma_F = \frac{W}{2A_F \tan \alpha}$$

$$\sigma_S = \frac{Wb}{A_S d} \tan \alpha$$

$$\tan^4 \alpha = \frac{1 + td/2A_F}{1 + tb/A_S}$$

incomplete diagonal tension,



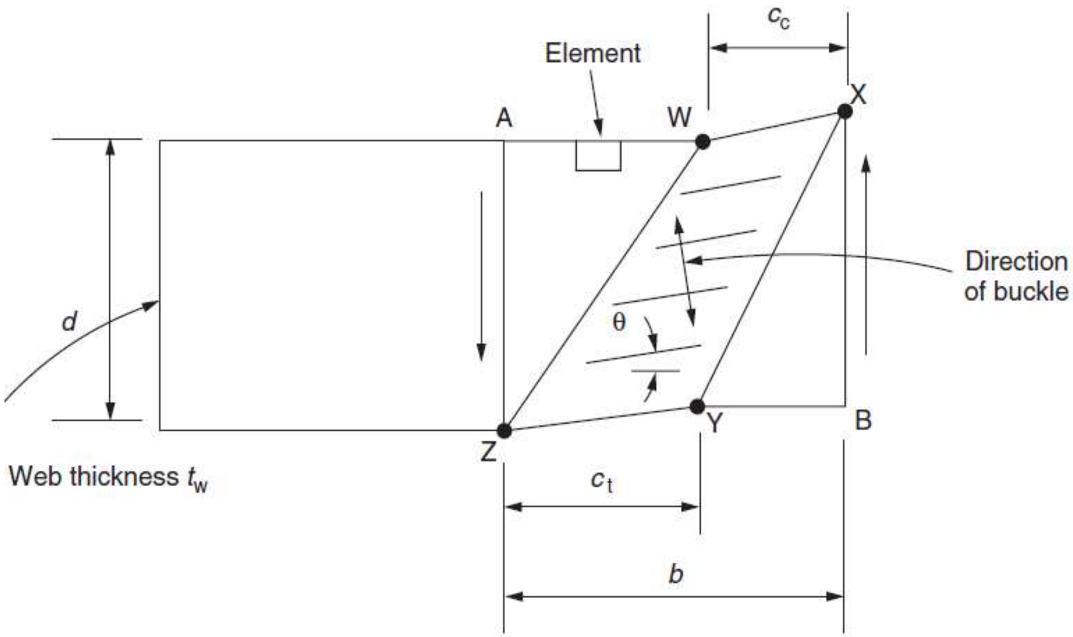
$$F_T = \frac{W}{d\cos\beta}\left[z + \frac{d\cot\alpha}{2}\left(1 - \frac{2z}{d}\tan\beta\right)\right]$$

$$F_B = \frac{W}{d\cos\beta}\left[z - \frac{d\cot\alpha}{2}\left(1 - \frac{2z}{d}\tan\beta\right)\right]$$

$$P = \frac{Wb}{d}\tan\alpha\left(1 - \frac{2z}{d}\tan\beta\right)$$

$$S = W\left(1 - \frac{2z}{d}\tan\beta\right)$$

post buckling behaviour.



$$\left(\frac{\sigma_{\mathrm{mb}}}{\sigma_{\mathrm{crb}}}\right)^2 + \left(\frac{\tau_{\mathrm{m}}}{\tau_{\mathrm{cr}}}\right)^2 = 1$$

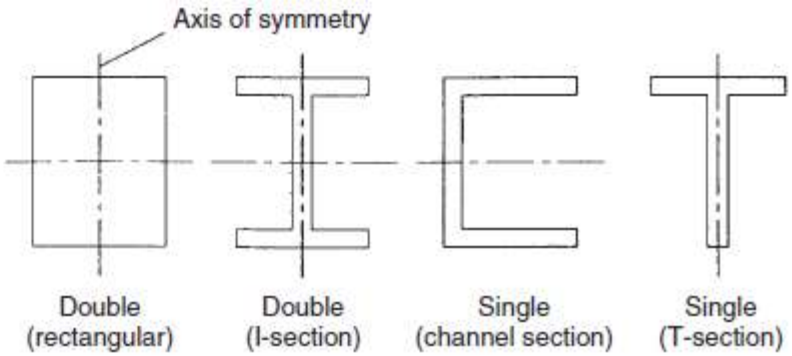
$$\sigma_\xi = -\sigma_{\mathrm{mb}} \cos^2\theta + \tau_{\mathrm{m}} \sin 2\theta$$

$$\tau_{\eta\xi} = -\frac{\sigma_{\mathrm{mb}}}{2} \sin 2\theta - \tau_{\mathrm{m}} \cos 2\theta$$

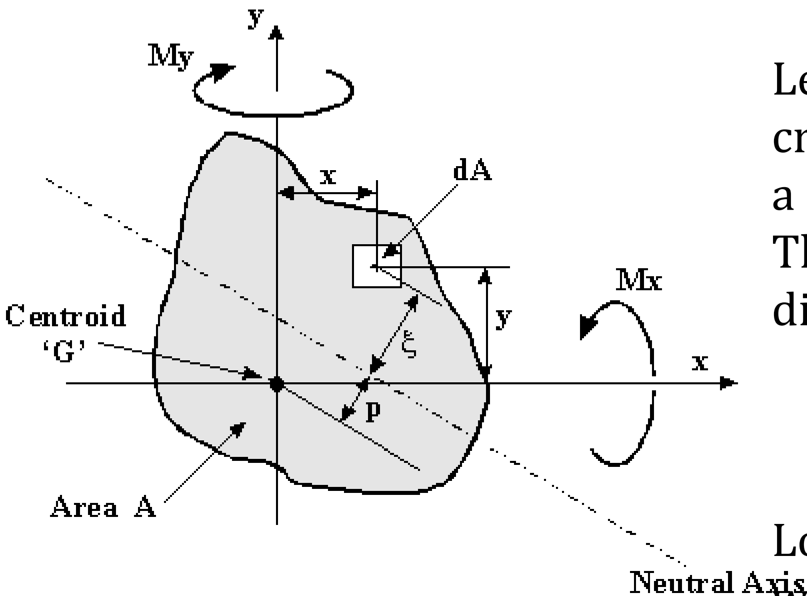
Unit-II

Bending and Shear and Torsion Of Thin Walled Beams:

Symmetrical bending arises in beams which have either singly or doubly symmetrical cross-sections



UNSYMMETRICAL BENDING

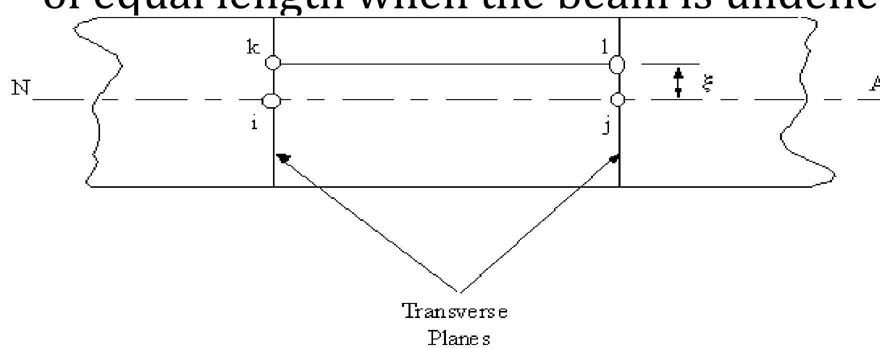


Let the axis origin coincide with the centroid G of the cross section, and that the neutral axis is a distance *p* from G.

The direct stress $\mathbf{s}_z$ on element $\mathbf{d_A}$ at point (x,y) and distance **x** from the neutral axis is:

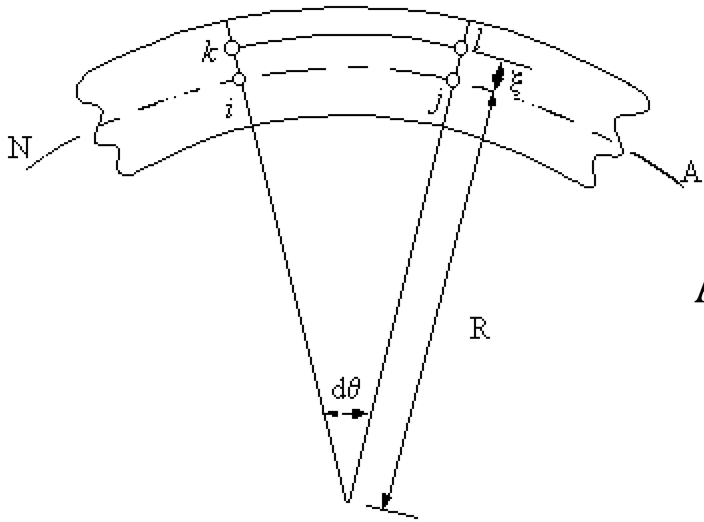
$$\boxed{\sigma_z = E\varepsilon_z}$$

Look at the beam in a plane parallel to the neutral axis with two segments ***ij*** and ***kl*** which are of equal length when the beam is undeflected:



Derivation of bending stress using general method

Once the beam has been deflected this section will look like this



where:

R = the radius of curvature

dq = angle between planes ***ik*** and ***jl***

The strain in plane ***kl*** can be defined as:

$$\varepsilon = \frac{\Delta L}{L} = \frac{kl - ij}{ij}$$

$$ij = R d\theta$$

$$kl = (R + \xi) d\theta$$

$$\varepsilon = \frac{\xi}{R}$$

$$\sigma_z = \frac{\xi E}{R}$$

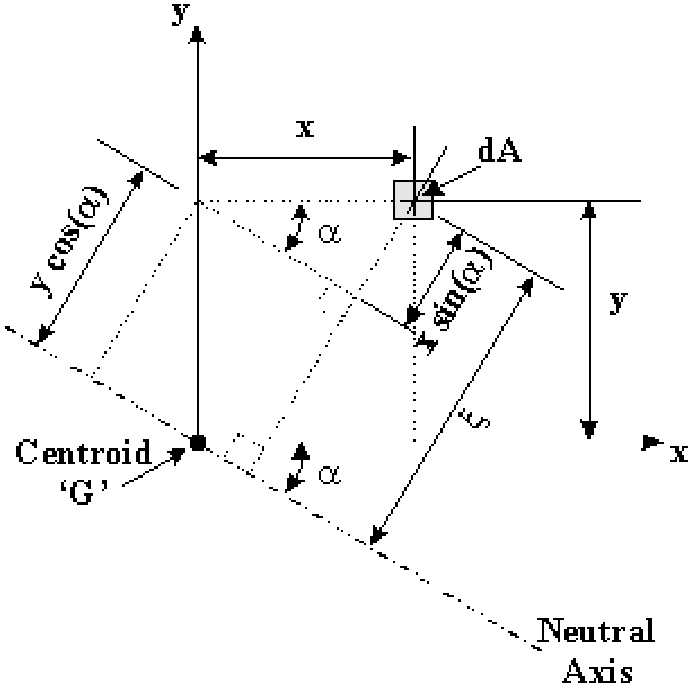
As the beam supports pure bending, the resultant load on the end section must be zero. Hence

$$\sum F_z = \int_A \sigma_z dA = 0$$

$$\int_A \frac{E\xi}{R} dA = \frac{E}{R}\int_A \xi dA = \int_A \xi dA = 0$$

$$\bar{x} = \frac{\sum \bar{x}_i * A_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum \bar{y}_i * A_i}{\sum A_i}$$



If the inclination of the Neutral Axis (N.A.) is at an angle from the x-axis then :
x = x sin(α) + y cos(α)

$$\sigma_z = \frac{E}{R}\left(x\sin(\alpha) + y\cos(\alpha)\right)$$

$$M_x = \int_A \sigma_z y\,dA \qquad M_y = \int_A \sigma_z x\,dA$$

$$\boxed{I_{xx} = \int_A y^2 dA \;, \quad I_{yy} = \int_A x^2 dA \;, \quad I_{xy} = \int_A xy\,dA}$$

$$\boxed{\bar{M}_x = \frac{M_x - M_y I_{xy}/I_{yy}}{1 - I_{xy}^2/I_{xx}I_{yy}}}$$

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)y$$

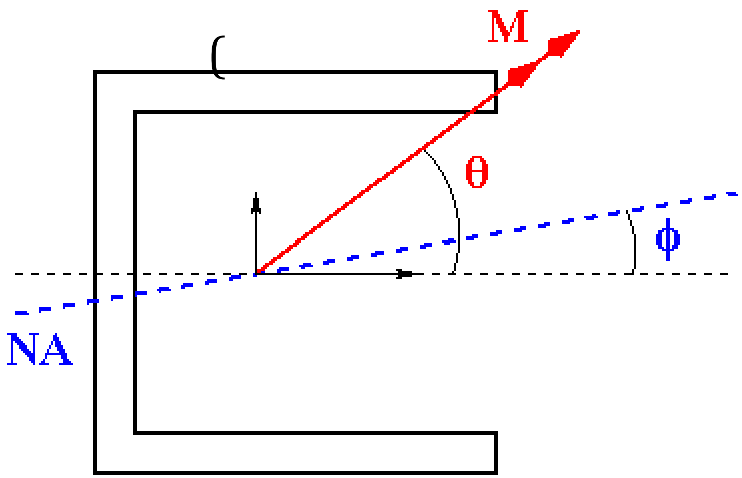
$$\boxed{\bar{M}_y = \frac{M_y - M_x I_{xy}/I_{xx}}{1 - I_{xy}^2/I_{xx}I_{yy}}}$$

$$\boxed{\sigma_z = \frac{\bar{M}_x}{I_{xx}}y + \frac{\bar{M}_y}{I_{yy}}x}$$

Neutral Axis:

- When a homogeneous beam is subjected to elastic bending, the neutral axis (NA) will pass through the centroid of its cross section, but the orientation of the NA depends on the orientation of the moment vector and the cross sectional shape of the beam.
- When the loading is unsymmetrical (at an angle) as seen in the figure below, the NA will also be at some angle **- NOT** necessarily the same angle as the bending moment.

$$\tan(\alpha) = -\frac{\bar{M}_y}{\bar{M}_x}\frac{I_{xx}}{I_{yy}}$$



- Real ... e zero, we can use the general ber Setting the stress to zero and so

SHEAR FLOW AND SHEAR CENTRE

Restrictions:

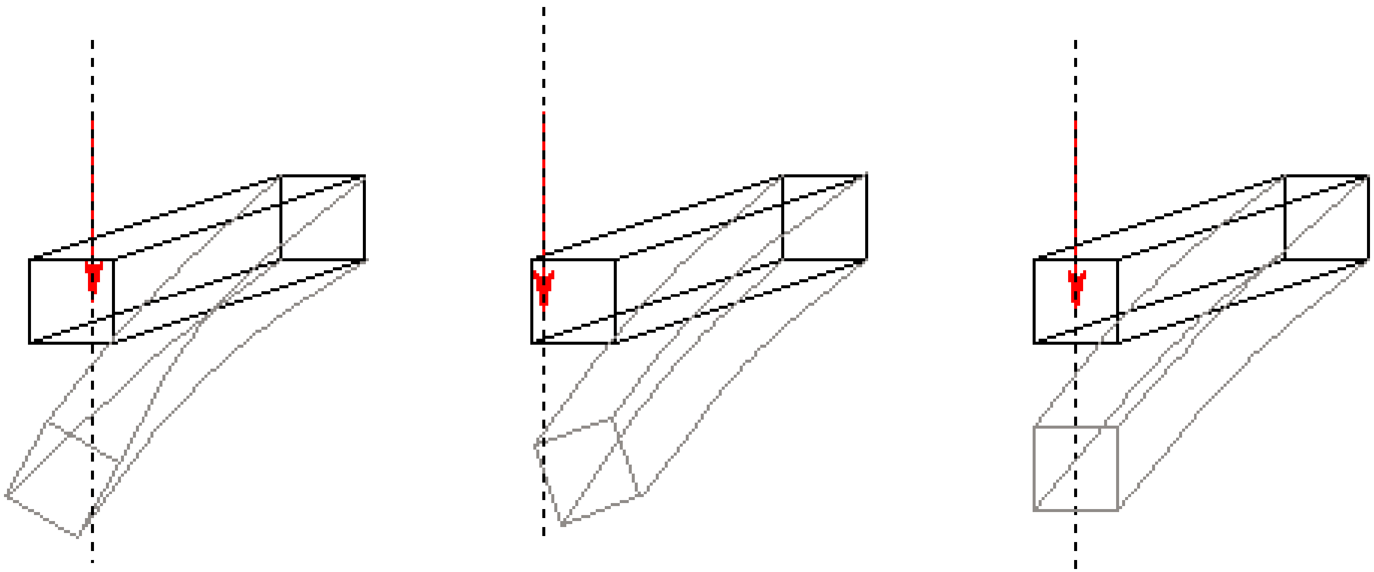
1. **Shear stress at every point in the beam must be less than the elastic limit of the material in shear.**
2. **Normal stress at every point in the beam must be less than the elastic limit of the material in tension and in compression.**
3. **Beam's cross section must contain at least one axis of symmetry.**
4. **The applied transverse (or lateral) force(s) at every point on the beam must pass through the elastic axis of the beam. Recall that elastic axis is a line connecting cross-sectional shear centers of the beam. Since shear center always falls on the cross-sectional axis of symmetry, to assure the previous statement is satisfied, at every point the transverse force is applied along the cross-sectional axis of symmetry.**
5. **The length of the beam must be much longer than its cross sectional dimensions.**
6. **The beam's cross section must be uniform along its length.**

Shear Center

If the line of action of the force passes through the **Shear Center** of the beam section, then the beam will only bend without any twist. Otherwise, twist will accompany bending.

The shear center is in fact the *centroid of the internal shear force system.* Depending on the beam's cross-sectional shape along its length, the location of shear center may vary from section to section. A line connecting all the shear centers is called the **elastic axis** of the beam. When a beam is under the action of a more general lateral load system, then to prevent the beam from twisting, the load must be centered along the elastic axis of the beam.

Shear Center



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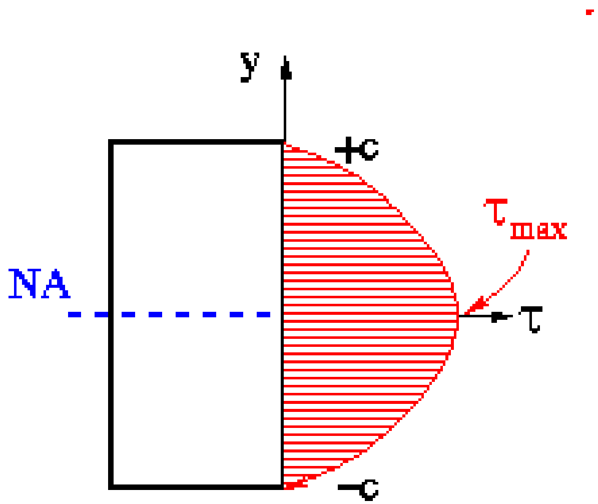
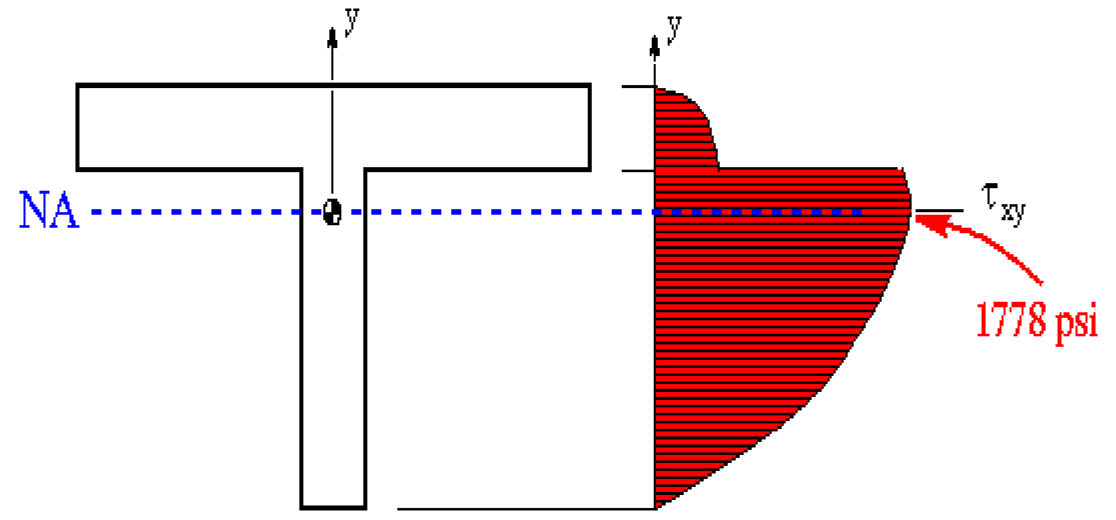
1. The shear center always falls on a cross-sectional axis of symmetry.
2. If the cross section contains two axes of symmetry, then the shear center is located at their intersection. Notice that this is the only case where shear center and centroid coincide.

SHEAR STRESS DISTRIBUTION

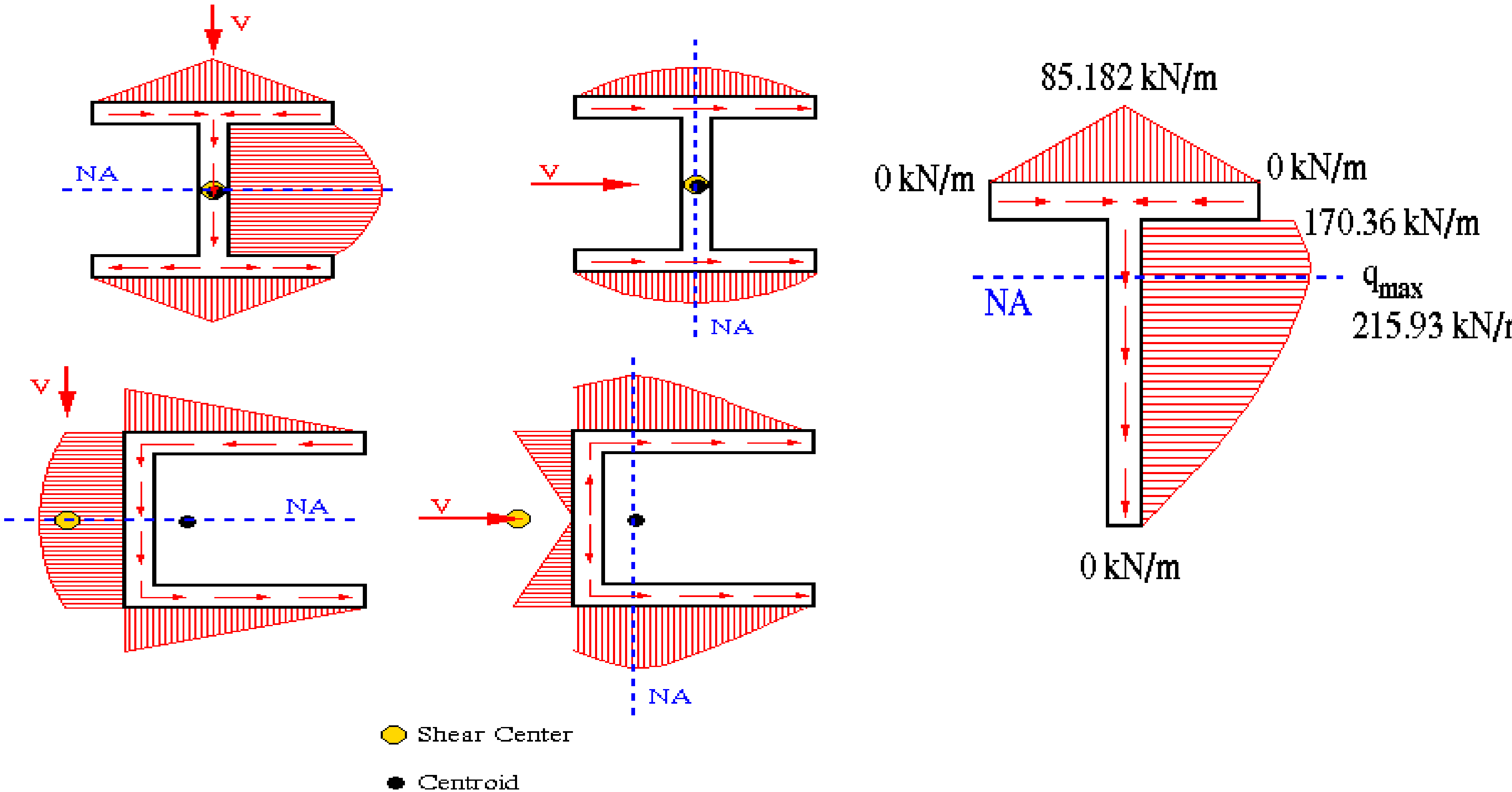
RECTANGLE



T-SECTION



SHEAR FLOW DISTRIBUTION



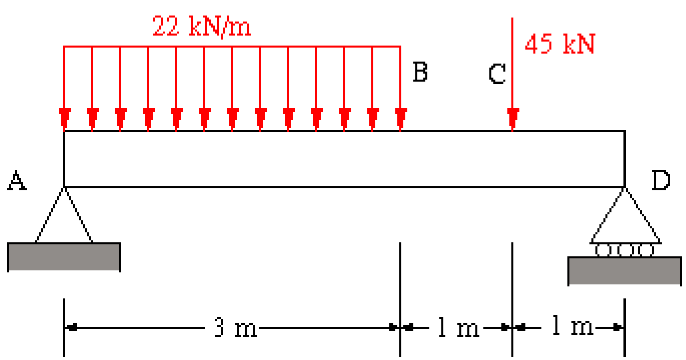
EXAMPLES

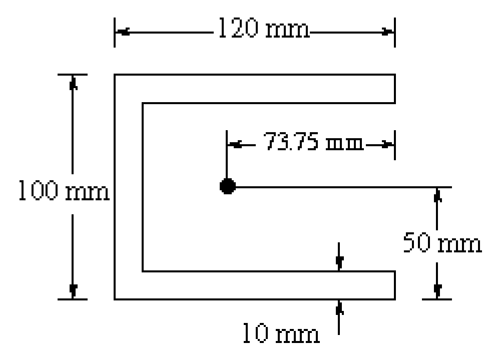
- For the beam and loading shown, determine:

(a) the location and magnitude of the maximum transverse shear force 'Vmax',
(b) the shear flow 'q' distribution due the 'Vmax',
(c) the 'x' coordinate of the shear center measured from the centroid,
(d) the maximun shear stress and its location on the cross section.

Stresses induced by the load do not exceed the elastic limits of the material.

NOTE: In this problem the applied transverse shear force passes through the centroid of the cross section, and not its shear center.





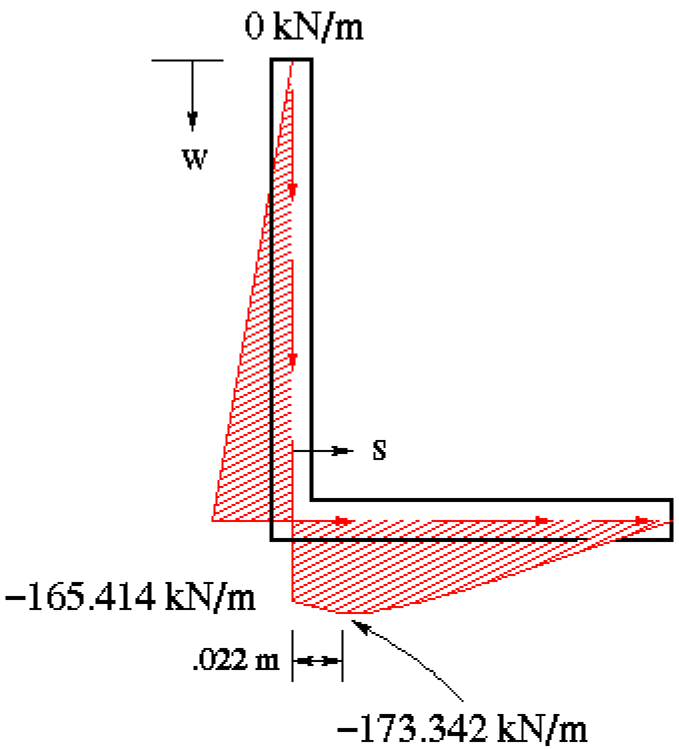
Shear Flow Analysis for Unsymmetric Beams

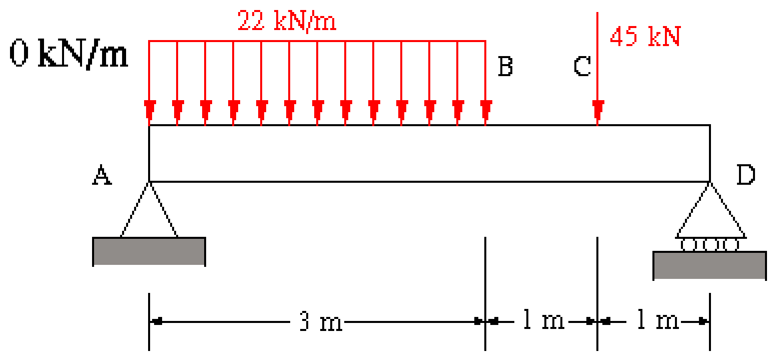
- SHEAR FOR EQUATION FOR UNSUMMETRIC SECTION IS

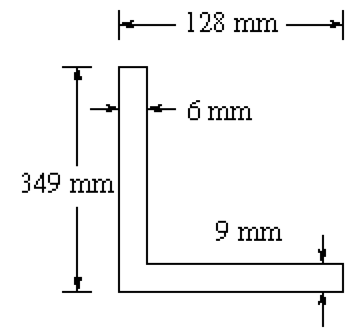
$$\boxed{q_s = -\frac{\bar{S}_y}{I_{xx}}\int_0^s ty\,ds - \frac{\bar{S}_x}{I_{yy}}\int_0^s tx\,ds}$$

SHEAR FLOW DISTRIBUTION

- For the beam and loading shown, determine:
- (a) the location and magnitude of the maximum transverse shear force,
- (b) the shear flow 'q' distribution due to 'Vmax',
- (c) the 'x' coordinate of the shear center measured from the centroid of the cross section.
- Stresses induced by the load do not exceed the elastic limits of the material. The transverse shear force is applied through the shear center at every section of the beam. Also, the length of each member is measured to the middle of the adjacent member.







Beams with Constant Shear Flow Webs

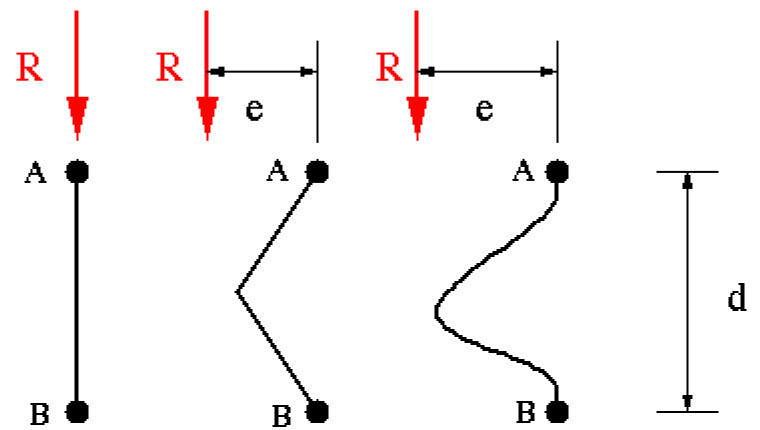
Assumptions:

1. Calculations of **centroid, symmetry, moments of area and moments of inertia** are based totally on the **areas and distribution** of beam stiffeners.
2. A web does not change the shear flow between two adjacent stiffeners and as such would be in the state of constant shear flow.
3. The stiffeners carry the entire bending-induced normal stresses, while the web(s) carry the entire shear flow and corresponding shear stresses.

Analysis

- Let's begin with a simplest thin-walled stiffened beam. This means a beam with two stiffeners and a web. Such a beam can only support a transverse force that is parallel to a straight line drawn through the centroids of two stiffeners. Examples of such a beam are shown below. In these three beams, the value of shear flow would be equal although the webs have different shapes.

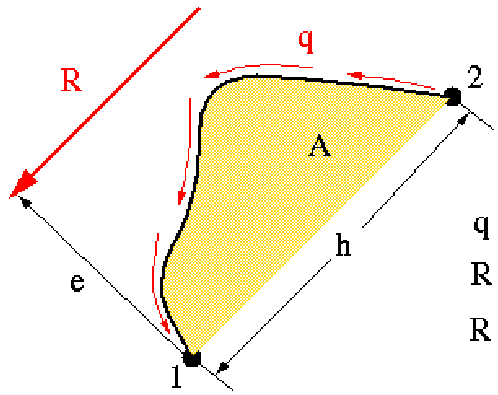
$$R = qd$$



- The reason the shear flows are e... nt stiffeners is shown to be 'd' in all ... equal to 'R' in all cases. The shear flow along the web can be determined by the following relationship

Important Features of Two-Stiffener, Single-Web Beams:

1. Shear flow between two adjacent stiffeners is constant.
2. The **magnitude** of the resultant shear force is only a function of the straight line between the two adjacent stiffeners, and is absolutely independent of the web shape.
3. The **direction** of the resultant shear force is parallel to the straight line connecting the adjacent stiffeners.
4. The **location** of the resultant shear force is a function of the enclosed area (between the web, the stringers at each end and the arbitrary point 'O'), and the straight distance between the adjacent stiffeners. This is the only quantity that depends on the shape of the web connecting the stiffeners.
5. The line of action of the resultant force passes through the **shear center** of the section.



q = constant

$R = qh$ (magnitude)

R is parallel to h (direction)

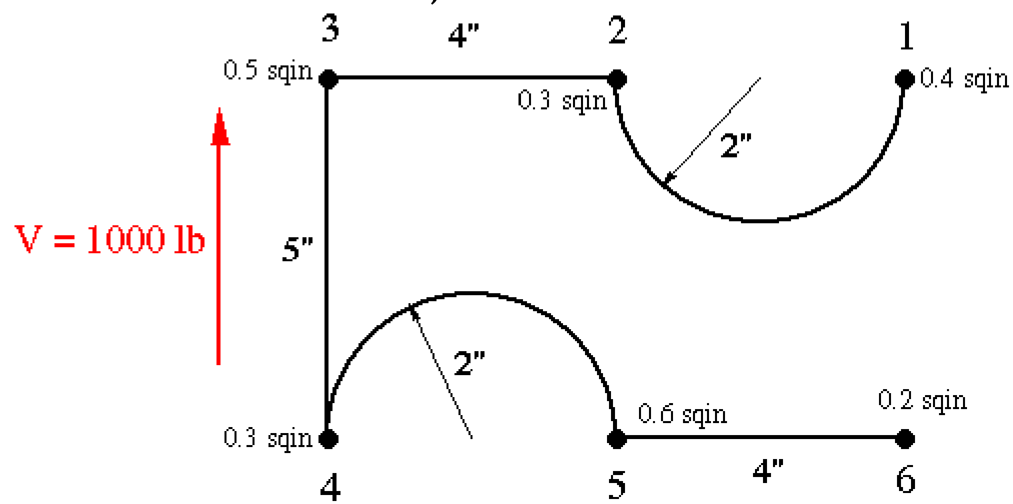
$e = \frac{2A}{h}$ (location)

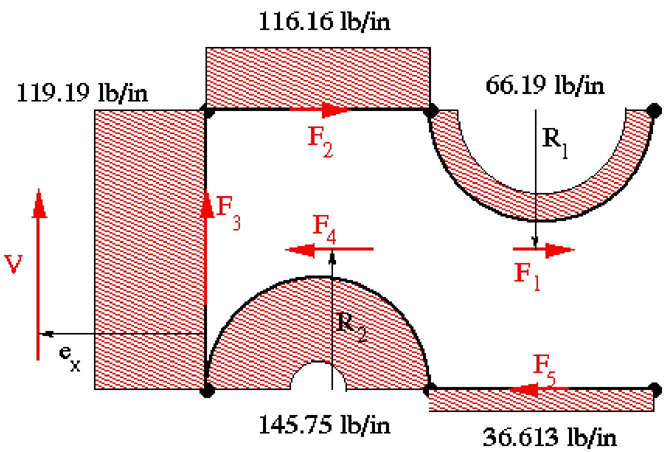
EXAMPLE

- For the multi-web, multi-stringer open-section beam shown, determine

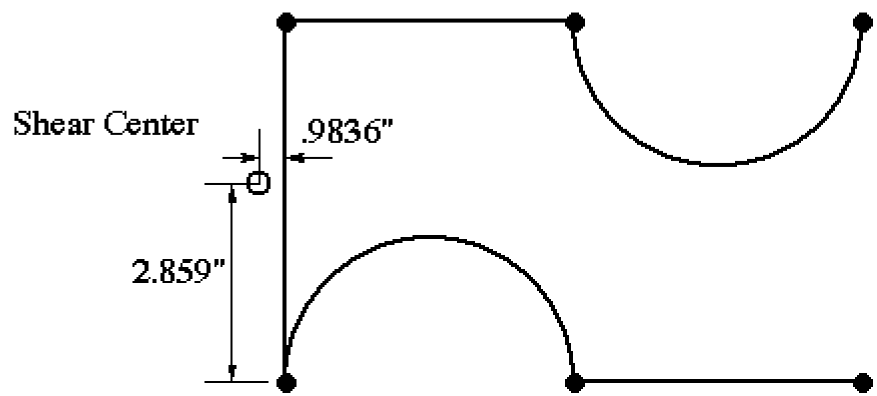
(a) the shear flow distribution,

(b) the location of the shear center





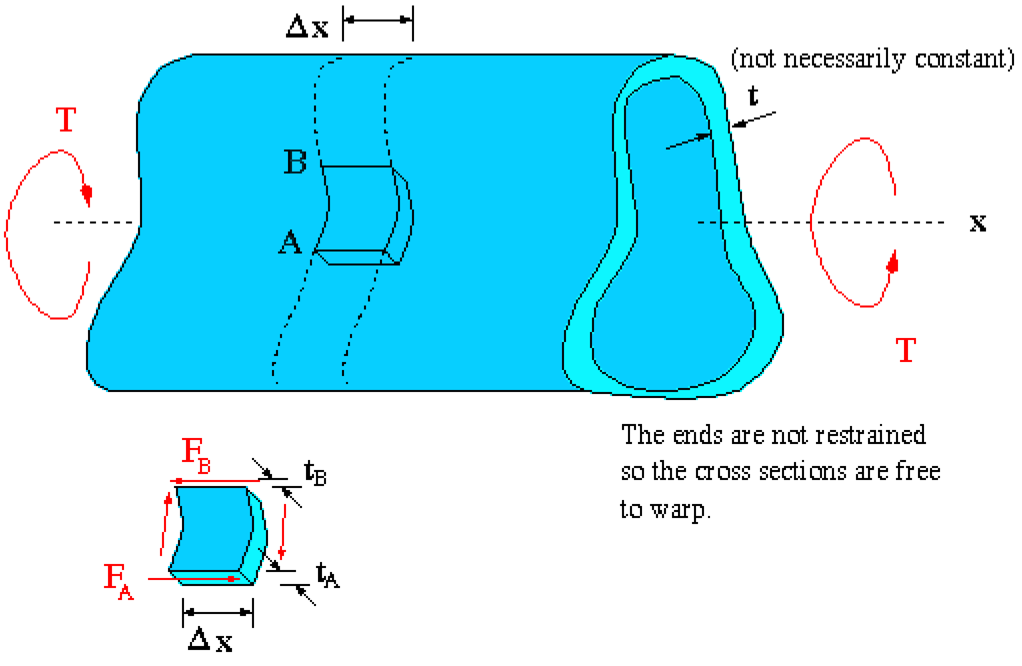
$$R_1 = R_2 = \frac{2A}{d} = \frac{2(\pi/2\,(2)^2)}{4}$$
$$= \pi$$



Torsion of Thin - Wall Closed Sections

- Derivation

Consider a thin-walled member with a closed cross section subjected to pure torsion.



Examining the equilibrium of a small cutout of the skin reveals that

$$\sum F_x = 0 \quad \Rightarrow \quad F_A - F_B = 0$$

Writing F_A and F_B in terms of the shearing stress at A and B yields

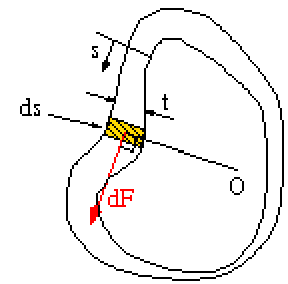
$$\tau_A (t_A \Delta x) - \tau_B (t_B \Delta x) = 0$$

$$\tau_A t_A = \tau_B t_b = \tau t \quad \textit{constant throughout the member}$$

Let $\tau t = q =$ constant
This is just like the product of VA (velocity x cross sectional area) is constant in a Venturi tube. 'q' is therefore called **shear flow**.

$dF = \tau(t\,ds) = q\,ds$

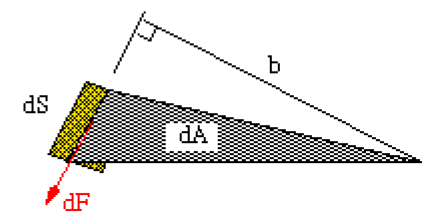
The moment of dF about an arbitrary point O is



$dM_O = b\,dF = b\,(q\,ds) = q\,(b\,ds)$
$dA = ½\,b\,ds \rightarrow b\,ds = 2\,dA$

now $dM = q\,(2dA)$

$$T = \oint dM_O = \oint q(2dA)$$



Since q is a constant we have

$$T = 2qA$$

Where A is the area bounded by the centerline of the wall cross section

$$\tau = \frac{T}{2At}$$

Where t is the thickness of the skin at the point considered in the shear stress calculation

Angle of Twist

By applying strain energy equation due to shear and Castigliano's Theorem the angle of twist for a thin-walled closed section can be shown to be

$$\frac{\delta U}{\delta T} = \frac{\phi}{L} = \frac{T}{4A^2G} \oint \frac{ds}{t}$$

Since T = 2qA, we have $\phi = \frac{qL}{2AG} \oint \frac{ds}{t}$

If the wall thickness is constant along each segment of the cross section, the integral can be replaced by a simple summation

$$\phi = \frac{qL}{2AG} \sum \frac{l}{t}$$

Torsion - Shear Flow Relations in Multiple-Cell Thin- Wall Closed Sections

- The torsional moment in terms of the internal shear flow is simply

$$T_o = 2q_1A_1 + 2q_2A_2 + ... + 2q_nA_n$$

Derivation

For equilibrium to be maintained at a exterior-interior wall (or web) junction point (point m in the figure) the shear flows entering should be equal to those leaving the junction $q_1 = q_2 + q_3$

Summing the moments about an arbitrary point O, and assuming clockwise direction to be positive, we obtain

$$\sum M_o = T_O = 2q_1(A_1 + A_{2b}) + 2q_2(A_{2a}) - 2q_3(A_{2b})$$

$$T_O = 2q_1A_1 + 2q_1A_{2b} + 2q_2A_{2a} - 2q_3A_{2b}$$

$$= 2q_1A_1 + 2q_1A_{2b} + 2q_2A_{2a} - 2(q_1 - q_2)A_{2b}$$

The moment equation above can be simplified to

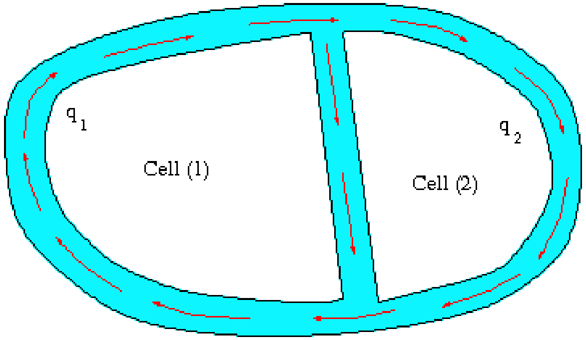
$$A_2 = A_{2a} + A_{2b}$$

$$T_o = 2q_1A_1 + 2q_2A_2$$

Shear Stress Distribution and Angle of Twist for Two-Cell Thin-Walled Closed Sections

- The equation relating the shear flow along the exterior wall of each cell to the resultant torque at the section is given as

$$T_o = 2q_1A_1 + 2q_2A_2$$



$$\phi_1 = \phi_2 = \phi$$

This is a statically indeterminate problem. In order to find the shear flows q1 and q2, the compatibility relation between the angle of twist in cells 1 and 2 must be used. The compatibility requirement can be stated as

where

$$\phi_1 = \frac{L}{2A_1G}\oint_{cell1}\frac{q}{t}ds$$

$$\phi_2 = \frac{L}{2A_2G}\oint_{cell2}\frac{q}{t}ds$$

$$q_1 = \frac{1}{2}\left[\frac{a_{20} A_1 + a_{12} A}{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}\right] T$$

$$q_2 = \frac{1}{2}\left[\frac{a_{10} A_2 + a_{12} A}{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}\right] T$$

$$A = A_1 + A_2$$

$$a_{10} = \int \frac{ds}{t} \quad \text{(along exterior wall of cell 1)}$$

$$a_{20} = \int \frac{ds}{t} \quad \text{(along exterior wall of cell 2)}$$

$$\tau = \frac{q}{t}$$

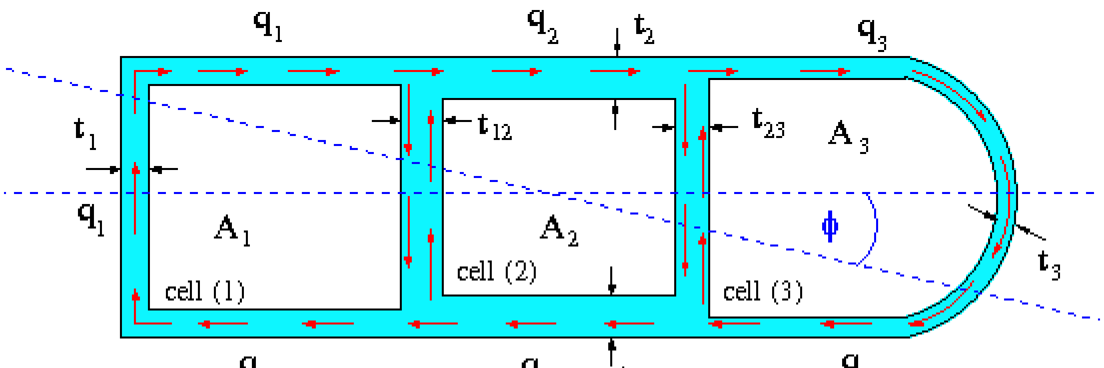
$$a_{12} = \int \frac{ds}{t} \quad \text{(along interior wall between cells 1 \& 2)}$$

- The shear stress at a point of interest is found according to the equation

$$\phi = \frac{TL}{JG} \qquad J = 4\left[\frac{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}{a_{10} a_{12} + a_{12} a_{20} + a_{10} a_{20}}\right]$$

- To find the angle of twist, we could use either of the two twist formulas given above. It is also possible to express the angle of twist equation similar to that for a circular section

Shear Stress Distribution and Angle of Twist for Multiple-Cell Thin-Wall Closed Sections



- In the figure above the area outside of the cross section will be designated as **cell (0)**. Thus to designate the exterior walls of cell (1), we use the notation **1-0**. Similarly for cell (2) we use **2-0** and for cell (3) we use **3-0**. The interior walls will be designated by the names of adjacent cells.
- the torque of this multi-cell member can be related to the shear flows in exterior walls as follows

$$q_1 = \tau_1 t_1 \quad , \quad q_2 = \tau_2 t_2 \quad , \quad q_3 = \tau_3 t_3$$

$$@\ wall\ 1-2 \qquad q_{12} = q_1 - q_2$$

$$@\ wall\ 2-3 \qquad q_{23} = q_2 - q_3$$

$$T = 2q_1 A_1 + 2q_2 A_2 + 2q_3 A_3$$

For elastic continuity, the angles of twist in all cells must be equal

$$\frac{\phi_1}{L} = \frac{\phi_2}{L} = \frac{\phi_3}{L} = \frac{q}{2AG}\oint\frac{ds}{t} = \frac{\phi}{L}$$

$$Let\ a = \oint\frac{ds}{t}$$

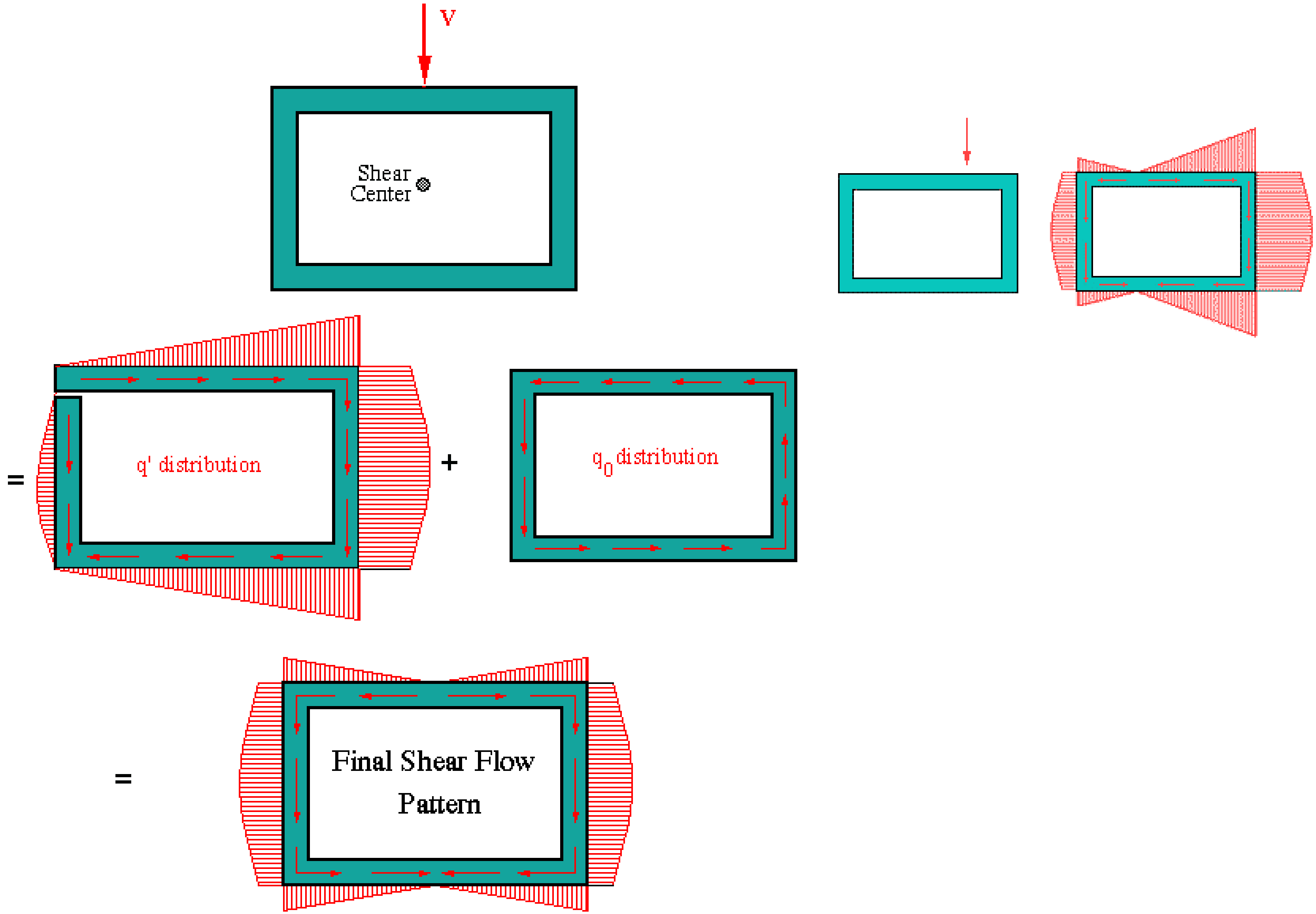
- The direction of twist chosen to be positive is clockwise.

$$\frac{2G\phi}{L} = \frac{1}{A_1}\left[q_1 a_{10} + (q_1 - q_2)a_{12}\right]$$

$$\frac{2G\phi}{L} = \frac{1}{A_2}\left[(q_2 - q_1)a_{12} + q_2 a_{20} + (q_2 - q_3)a_{23}\right]$$

$$\frac{2G\phi}{L} = \frac{1}{A_3}\left[(q_3 - q_2)a_{23} + q_3 a_{30}\right]$$

TRANSVERSE SHEAR LOADING OF BEAMS WITH CLOSED CROSS SECTIONS



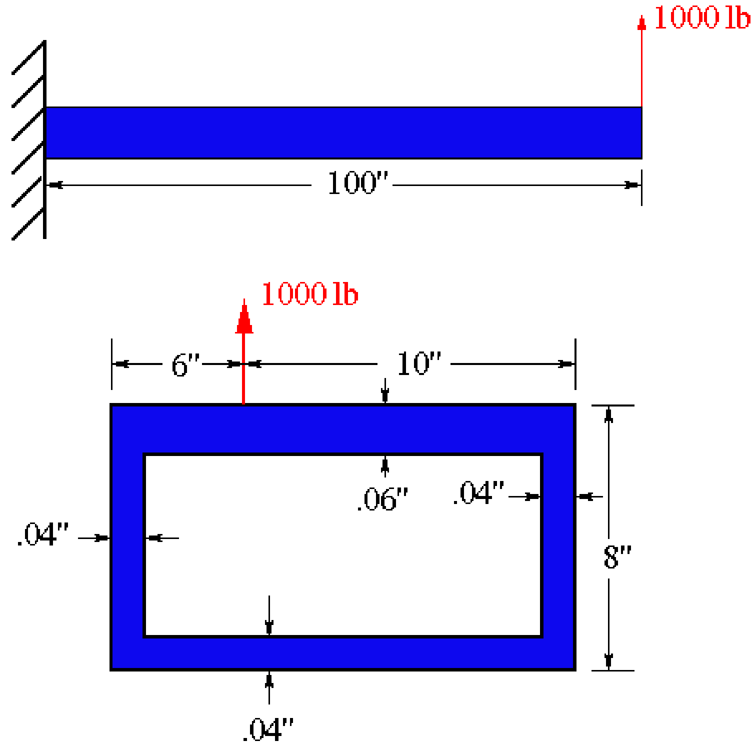
EXAMPLE

- For the thin-walled single-cell rectangular beam and loading shown, determine

  (a) the shear center location (ex and ey),

  (b) the resisting shear flow distribution at the root section due to the applied load of 1000 lb,

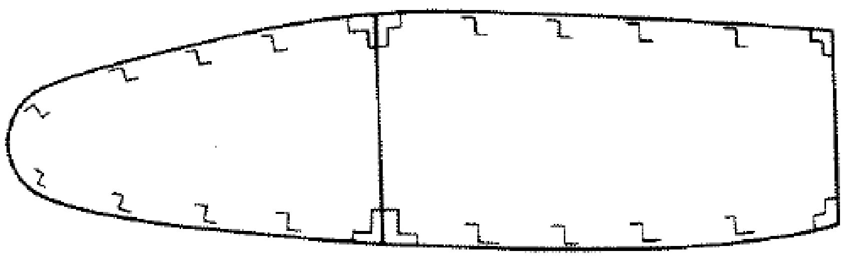
  (c) the location and magnitude of the maximum shear stress



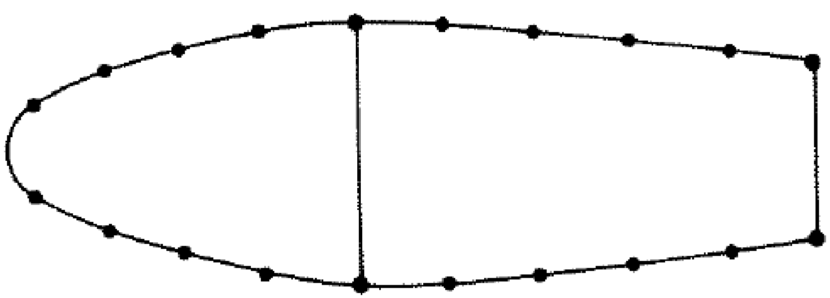
Unit-III

Structural Idealisation of Thin Walled Beams

Structural Idealization



- Consider the two-spar wing section shown. The stringers and spar carry most of the direct stresses while the skin carries the shear stresses.



- Since variation in stress over the stringer or spar flange due to bending of the wing would be small, it can be assumed to be constant.

$$(GJ)_{cl} = \frac{4A^2G}{\oint ds/t}$$

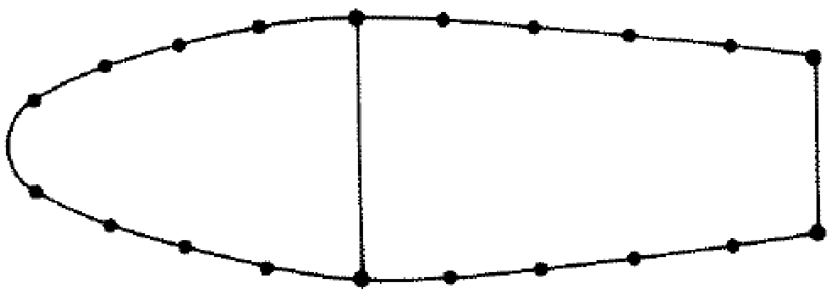
$$(GJ)_{op} = G\sum \frac{st^3}{3}$$

Structural Idealization

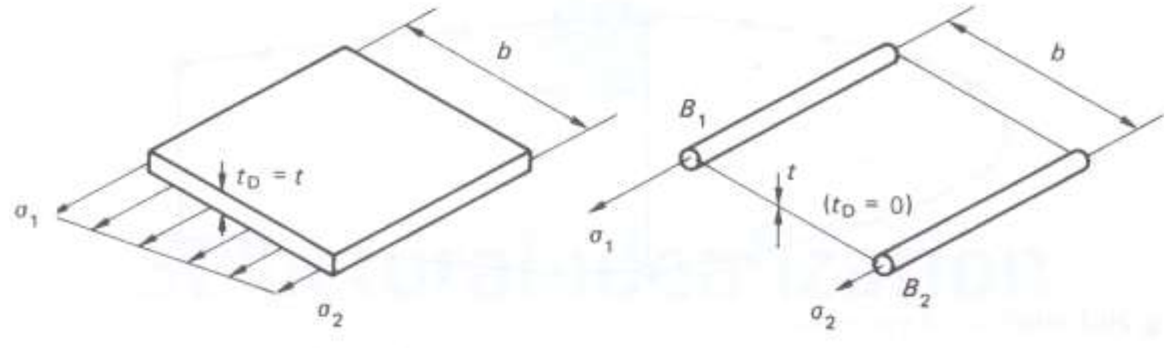
- Stingers and spar flanges can then be replaced by concentrations of areas known as booms.

- **It can be assumed that all direct stresses are carried by booms and the skin carries only shear.**

- Direct st :he skin may be accoı e boom cross-se



Structural Idealization



(a) Actual (b) Idealized

- If skin does carry direct stress, we idealize it as a section that carries only shear stress, and add effective area to the booms.

- ΣM @ Right Side

$$\sigma_2 t_D \frac{b^2}{2} + \frac{1}{2}(\sigma_1 - \sigma_2) t_D b \frac{2}{3} b = \sigma_1 B_1 b$$

- Similarly:

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

$$B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

Effective boom area due to skin carrying direct stress – these add to the boom areas of flanges, strings, spars, etc..

Shear of open-section beams

- In the expression for shear flow in the cross-section:

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D \cdot x \cdot ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s t_D \cdot y \cdot ds$$

  t_D is the direct stress carrying thickness of the skin

  $t_D = t$, if the skin is fully effective in carrying direct stress

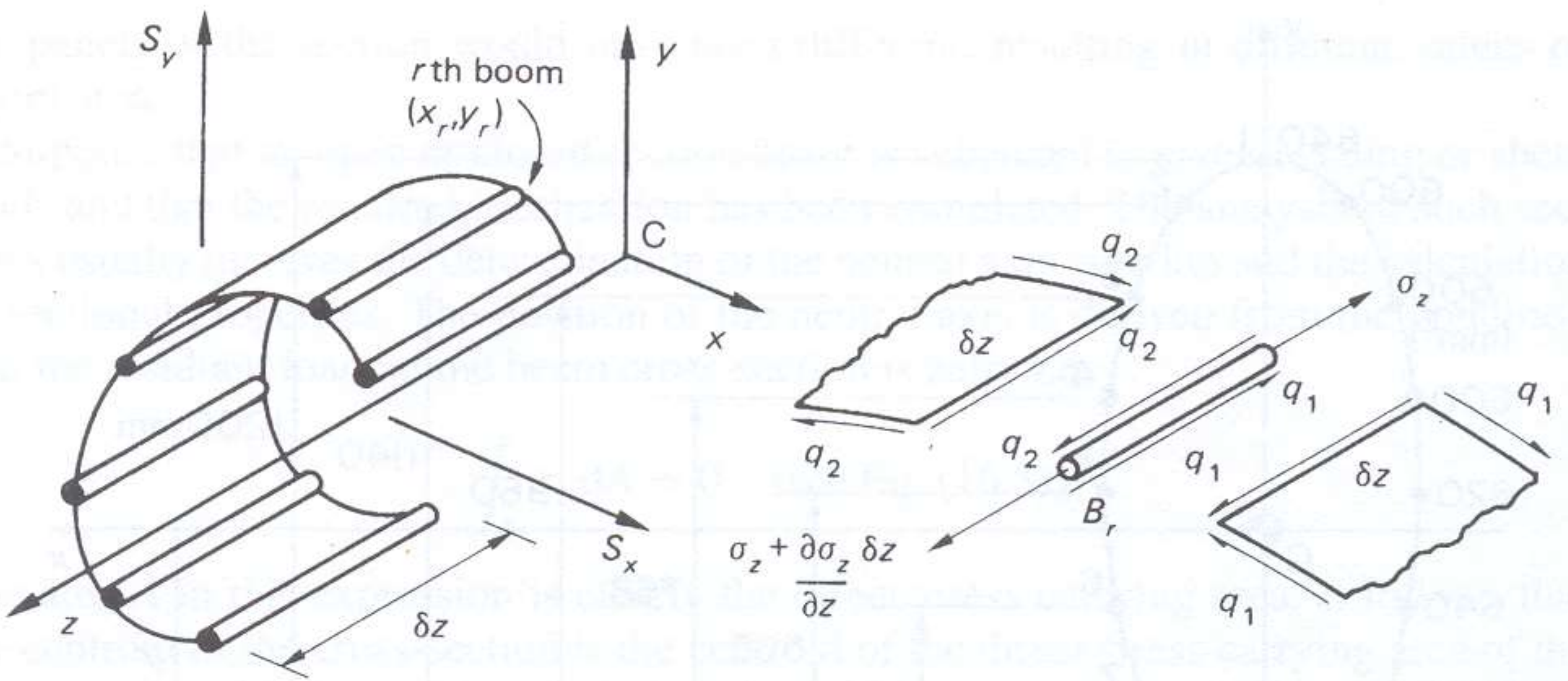
  $t_D = 0$, if the skin is assumed to carry only shear stress

- If we idealize skin as shown previously, then shear flow in the skin due to bending of the skin = 0.

- The above expression does not account for booms. How can we deal with booms that cause discontinuity in the skin and therefore interrupt the shear flow?

Shear of open-section beams

- Sx and Sy produce direct stresses due to bending in the booms (and skin) and shear stresses in the skin



Shear of open-section beams

- r^{th} boom has a cross-sectional area B_r
- Shear flows in skin adjacent to it are q_1 and q_2.

- Equilibrium in z direction of previous figure →

$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z}\delta z\right) B_r - \sigma_z \cdot B_r + q_2 \delta z - q_1 \delta z = 0$$

- This gives: $$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} \cdot B_r$$

Shear of open-section beams

- Recall:

$$\frac{\partial \sigma_z}{\partial z} = \left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) x + \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) y$$

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} \cdot B_r$$

$$\boxed{\therefore q_2 - q_1 = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) B_r x_r - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) B_r y_r}$$

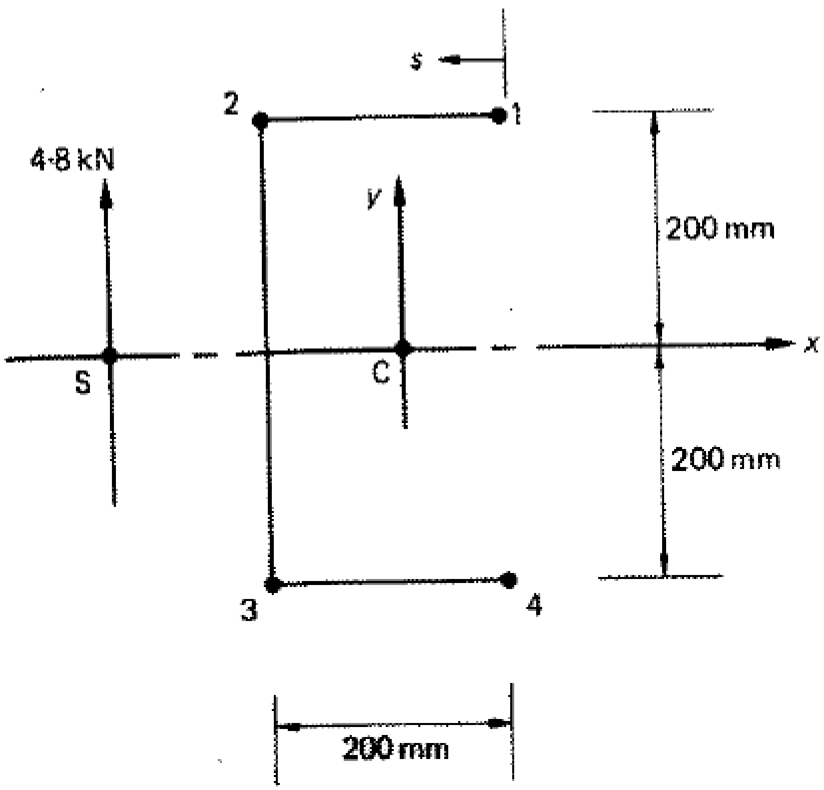
Shear of open-section beams

$$q_2 - q_1 = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) B_r x_r - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) B_r y_r$$

- This gives the change in shear flow induced by a boom.
- Each time a boom is encountered, the shear flow is
- If
  h

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D \cdot x \cdot ds + \sum_{r=1}^{n} B_r x_r\right) - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)\left(\int_0^s t_D \cdot y \cdot ds + \sum_{r=1}^{n} B_r y_r\right)$$

Open C/S Sample Problem



- Calculate the shear flow distribution in channel due to a 4.8 kN vertical shear load acting through the shear center.
- Booms carry all the direct stresses (B_r = 300 mm^2)

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r$$

Open C/S Sample Problem

Calculate I_{xx}: (Only consider direct stress carrying areas) ➔ I.e. Booms

$$I_{xx} = \sum Ad^2 = 4 \times 300 \times 200^2 = 48 \times 10^6 \text{ mm}^2$$

$$q_s = -\frac{4.8 \times 10^3}{48 \times 10^6} \sum_{r=1}^{n} B_r y_r = -10^{-4} \sum_{r=1}^{n} B_r y_r$$

At the outside of boom 1, $q_s = 0$. As boom 1 is crossed, the shear flow changes to:

$$q_{12} = 0 \underbrace{-10^{-4} \times 300 \times 200}_{q_1} = -6 \text{ N/mm}$$

Open C/S Sample Problem

There will be no further changes in shear flow until the next boom (2) is crossed.

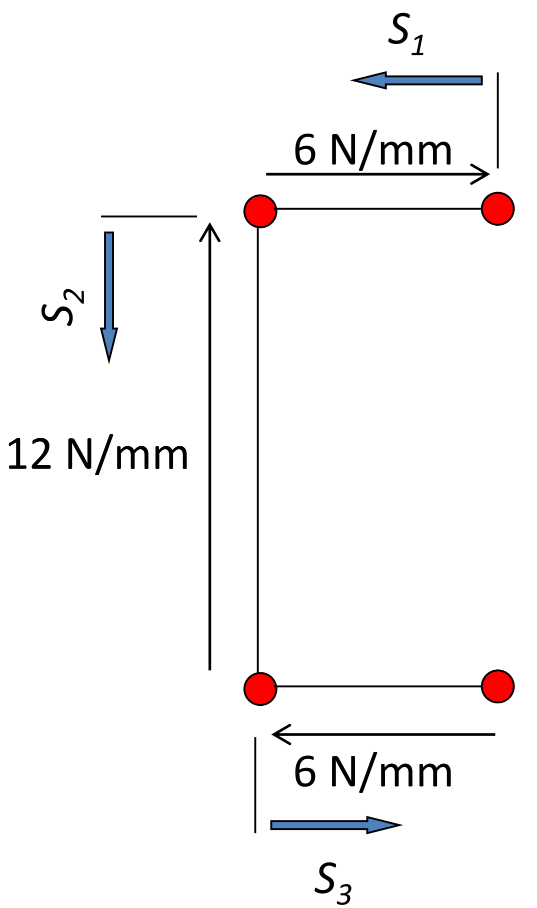
$$q_{23} = -6 - \underbrace{10^{-4} \times 300 \times 200}_{q_2} = -12 \text{ N/mm}$$

$$q_{34} = -12 - \underbrace{10^{-4} \times 300 \times \left(-200\right)}_{q_3} = -6 \text{ N/mm}$$

At the outside of boom 4, the shear flow is zero ($q_s = 0$)➔ as expected

$$-6 - 10^{-4} \times 300 \times \left(-200\right) = 0 \text{ N/mm}$$

Open C/S Sample Problem



$$q_{12} = -6 \text{ N/mm}$$

$$q_{23} = -12 \text{ N/mm}$$

$$q_{34} = -6 \text{ N/mm}$$

How come all the signs are negative?

Closed C/S Sample Problem

- For the single cell beam, the booms carry the direct stresses and the walls carry only the shear stress. A vertical shear load of 10 kN acts through the vertical plane between booms 3 and 6. Calculate the shear flow distribution

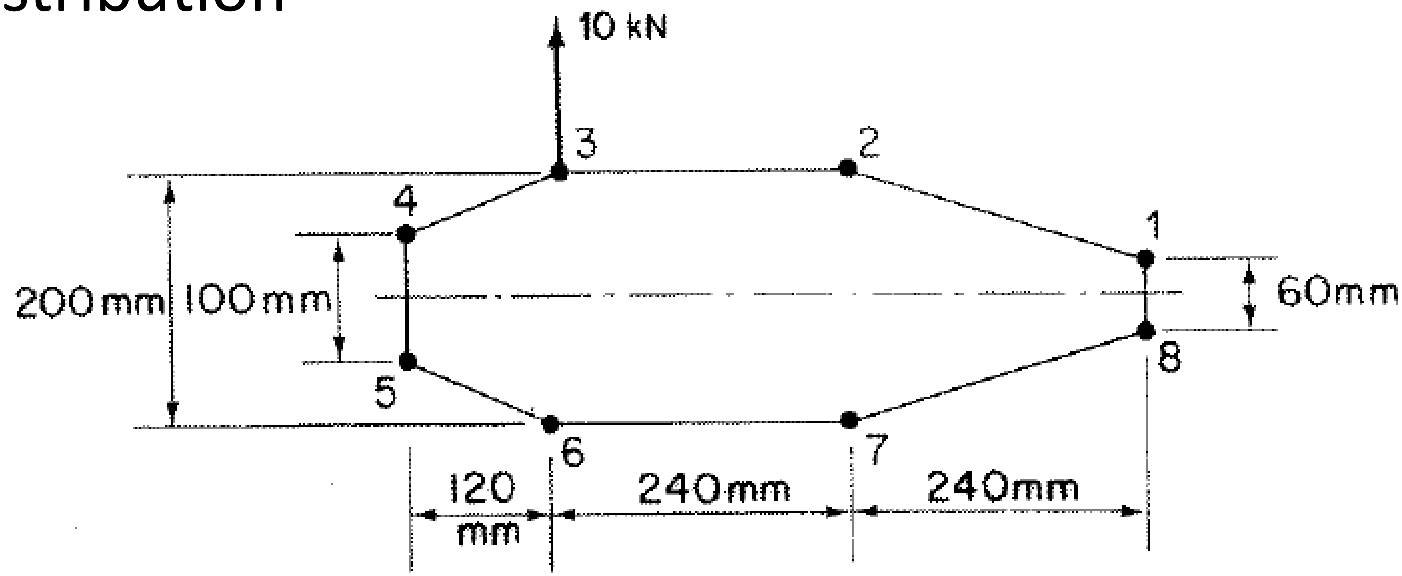
$$q_s = q_b + q_{s,o}$$

$$B_1 = B_8 = 200mm^2$$

$$B_2 = B_7 = 250mm^2$$

$$B_3 = B_6 = 400mm^2$$

$$B_4 = B_5 = 100mm^2$$



Closed C/S Sample Problem

Centroid on Horizontal Axis of Symmetry. $I_{xy} = 0$
Also $S_x = 0$, $t_D = 0$

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

I_{xx} can be calculated from the direct stress carrying area of the booms

$$I_{xx} = 2\left(B_1 \times 30^2 + B_2 \times 100^2 + B_3 \times 100^2 + B_4 \times 50^2\right)$$

Substituting $B_1, \ldots B_4$ gives $I_{xx} = 13.86 \times 10^6$ mm^4

$$q_s = -\frac{10 \times 10^3}{13.86 \times 10^6} \sum_{r=1}^{n} B_r y_r + q_{s,0} = -7.22 \times 10^{-4} \sum_{r=1}^{n} B_r y_r + q_{s,0}$$

Closed C/S Sample Problem

Introduce a cut in the wall 23 and calculate the basic shear flow around the walls

Since the $t_D = 0$

$$q_{b,23} = 0$$

$$q_{b,34} = -7.22\times10^{-4}\times\left(\overset{B_3}{400}\times\overset{y_3}{100}\right) = -28.9\ \text{N/mm}$$

$$q_{b,45} = -28.9 - 7.22\times10^{-4}\times\left(\overset{B_4}{100}\times\overset{y_4}{50}\right) = -32.5\ \text{N/mm}$$

$$q_{b,56} = q_{b,34} = -28.9\ \text{N/mm (by symmetry)}$$

$$q_{b,67} = q_{b,23} = 0\ \text{(by symmetry)}$$

$$q_{b,21} = -7.22\times10^{-4}\times\left(\overset{B_2}{250}\times\overset{y_2}{100}\right) = -18.1\ \text{N/mm}$$

$$q_{b,18} = -18.1 - 7.22\times10^{-4}\times\left(\overset{B_1}{200}\times\overset{y_1}{30}\right) = -22.4\ \text{N/mm}$$

$$q_{b,87} = q_{b,21} = -18.1\ \text{N/mm (by symmetry)}$$

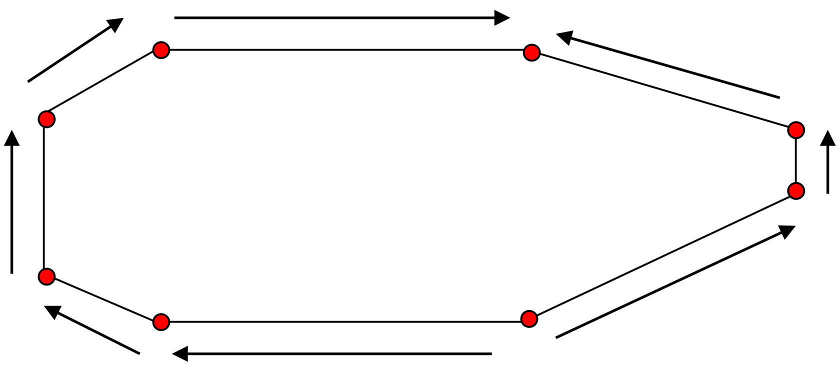
Closed C/S Sample Problem

- Taking moments about the intersection of the line of action of shear load and horizontal axis:

$$0 = \oint p q_b ds + 2Aq_{s,0}$$ ⟹ Solve for $q_{s,0}$

$\oint$ is broken up into segments where each q_b is constant

Draw out the shear flow distribution to determine the sign of the moment generated by the shear flow on each segment



Closed C/S Sample Problem

q *p* *s*

$$[q_{b,81}\times 480\times 60+2q_{b,12}\times 240\times 170+2q_{b,23}\times 100\times 240-$$

$$-\,2q_{b,43}\times 120\times 100-q_{b,54}\times 120\times 100]+2\times 97{,}200\,q_{s,0}=0$$

Substituting for the basic shear flow gives:

Enclosed Area, *A*

$$q_{s,0}=-5.4\ \text{N/mm}$$

Add $q_{s,0}$ to the basic shear flow to get the total shear flow in every wall.
In any wall the final shear flow is given by $q_s = q_b + q_{s,0}$ so that

$$q_{21}=-18.1+5.4=-12.7\ \text{N/mm}=q_{87}$$

$$q_{23}=-5.4\text{N/mm}=q_{67}$$

$$q_{34}=-34.3\ \text{N/mm}=q_{56}$$

$$q_{45}=-37.9\ \text{N/mm}$$

$$q_{81}=17\ \text{N/mm}$$

UNIT- IV
Structural and Loading Discontinuities in Thin Walled Beams

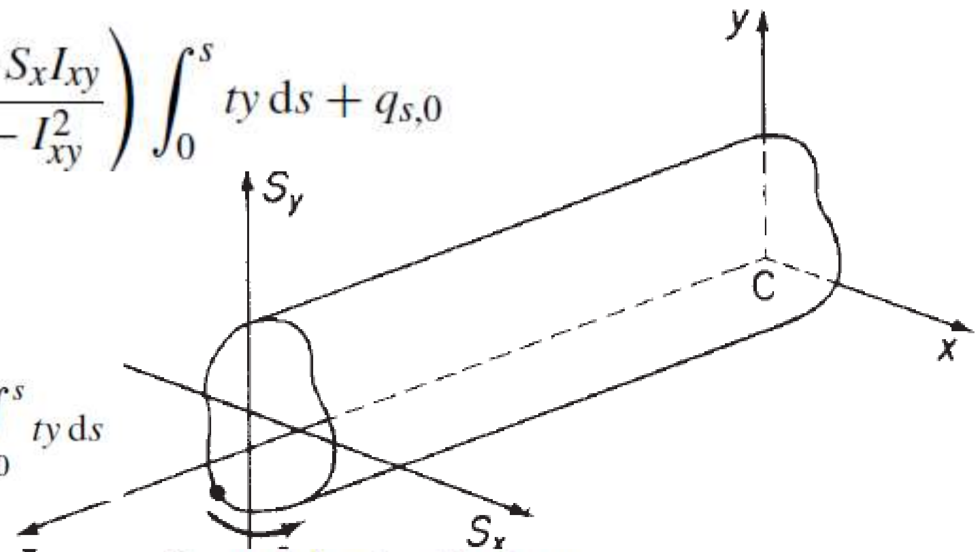
Closed section beams- shear stress distribution of a closed section beam built in at one end under bending, shear and torsion loads.

$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s + q_{s,0}$$

$$q_s = q_\mathrm{b} + q_{s,0}$$

$$q_\mathrm{b} = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{xy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s$$



Shear of closed section beams.

$$S_x\eta_0 - S_y\xi_0 = \oint pq\,\mathrm{d}s = \oint pq_\mathrm{b}\,\mathrm{d}s + q_{s,0}\oint p\,\mathrm{d}s$$

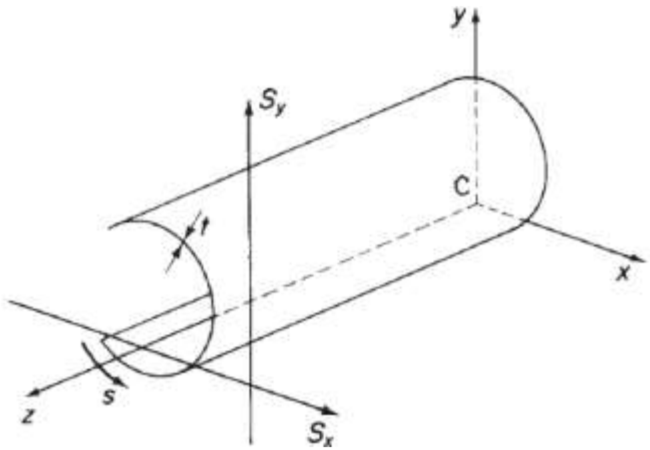
$$S_x\eta_0 - S_y\xi_0 = \oint pq_\mathrm{b}\mathrm{d}s + 2Aq_{s,0}$$

If the moment centre is chosen to coincide with the lines of action of *Sx and Sy then*

$$0 = \oint pq_\mathrm{b}\,\mathrm{d}s + 2Aq_{s,0}$$

Open section beams

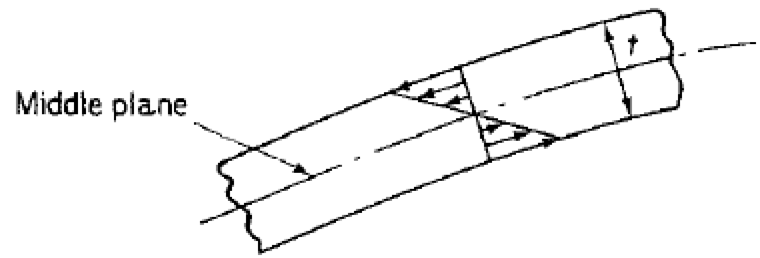
$$\frac{\partial q}{\partial s} + t\frac{\partial \sigma_z}{\partial z} = 0$$



$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s tx\,\mathrm{d}s - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)\int_0^s ty\,\mathrm{d}s$$

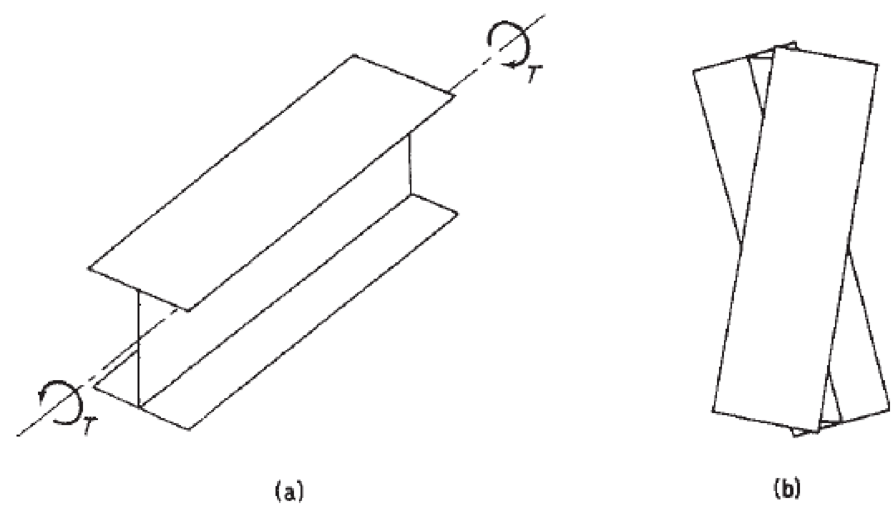
If such a beam is axially unconstrained and loaded by a pure torque T the rate of twist is constant along the beam and is given by

$$T = GJ\frac{\mathrm{d}\theta}{\mathrm{d}z}$$



Shear stress distribution across the wall of an open section beam subjected to torsion.

$$T = GJ\frac{\mathrm{d}\theta}{\mathrm{d}z} - EI_{\mathrm{F}}\frac{h^2}{2}\frac{\mathrm{d}^3\theta}{\mathrm{d}z^3}$$



Torsion of I-section beam; (b) plan view of beam showing undistorted shape of flanges.

- Shear lag- effect of shearing strains in beams- redistribution of bending stresses due to restraining of warping, limitation of elementary bending theory, effect of accounting for shear lag on the estimated strength.

$$q_s = Gt\left(\frac{\partial w}{\partial s} + \frac{\partial v_t}{\partial z}\right)$$

$$\frac{d\theta}{dz} = \frac{1}{2A}\oint \frac{q_s}{Gt} ds$$

$$w_s - w_0 = \int_0^s \frac{q_s}{Gt} ds - \frac{A_{Os}}{A}\oint \frac{q_s}{Gt} ds - y_R \frac{d\theta}{dz}(x_s - x_0) + x_R \frac{d\theta}{dz}(y_s - y_0)$$

$$w_s - w_0 = \int_0^s \frac{q_s}{Gt} ds - \frac{A_{Os}}{A}\oint \frac{q_s}{Gt} ds$$

$$w_0 = \frac{\oint w_s t \, ds}{\oint t \, ds}$$

UNIT- V
Stress Analysis of Aircraft Components- Wing, Fuselage:

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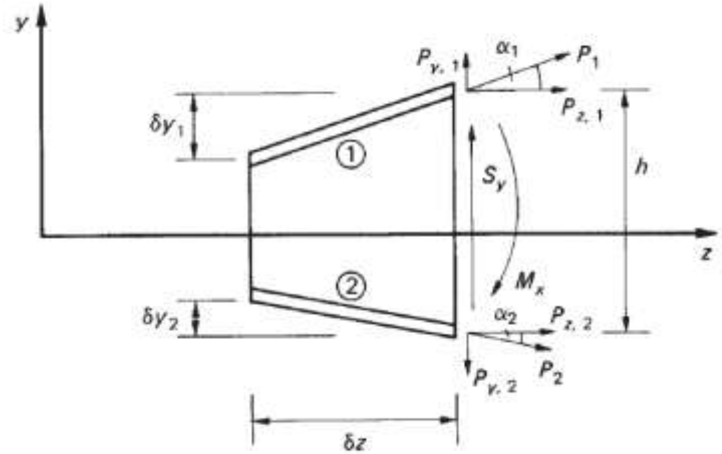
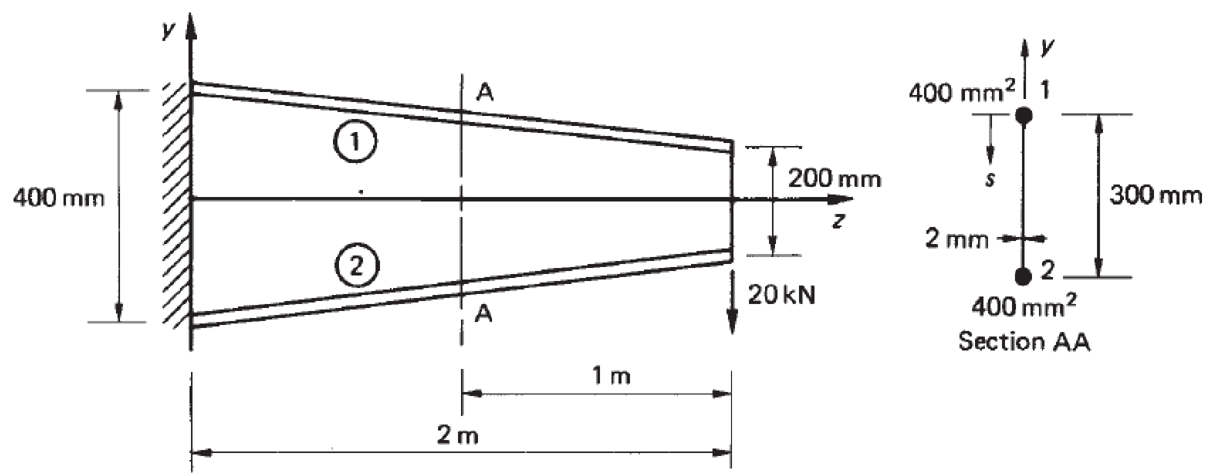
Wing spars and box beams- tapered wing spar, open and closed section beams, beams having variable stringer areas. Wings- Three-boom shell in bending, torsion, shear, tapered wings, deflections, cut-outs in wings.
Bending, shear, torsion, cut-outs in fuselages. Fuselage frames and wing ribs- principles of stiffener/ web construction, fuselage frames, wing ribs

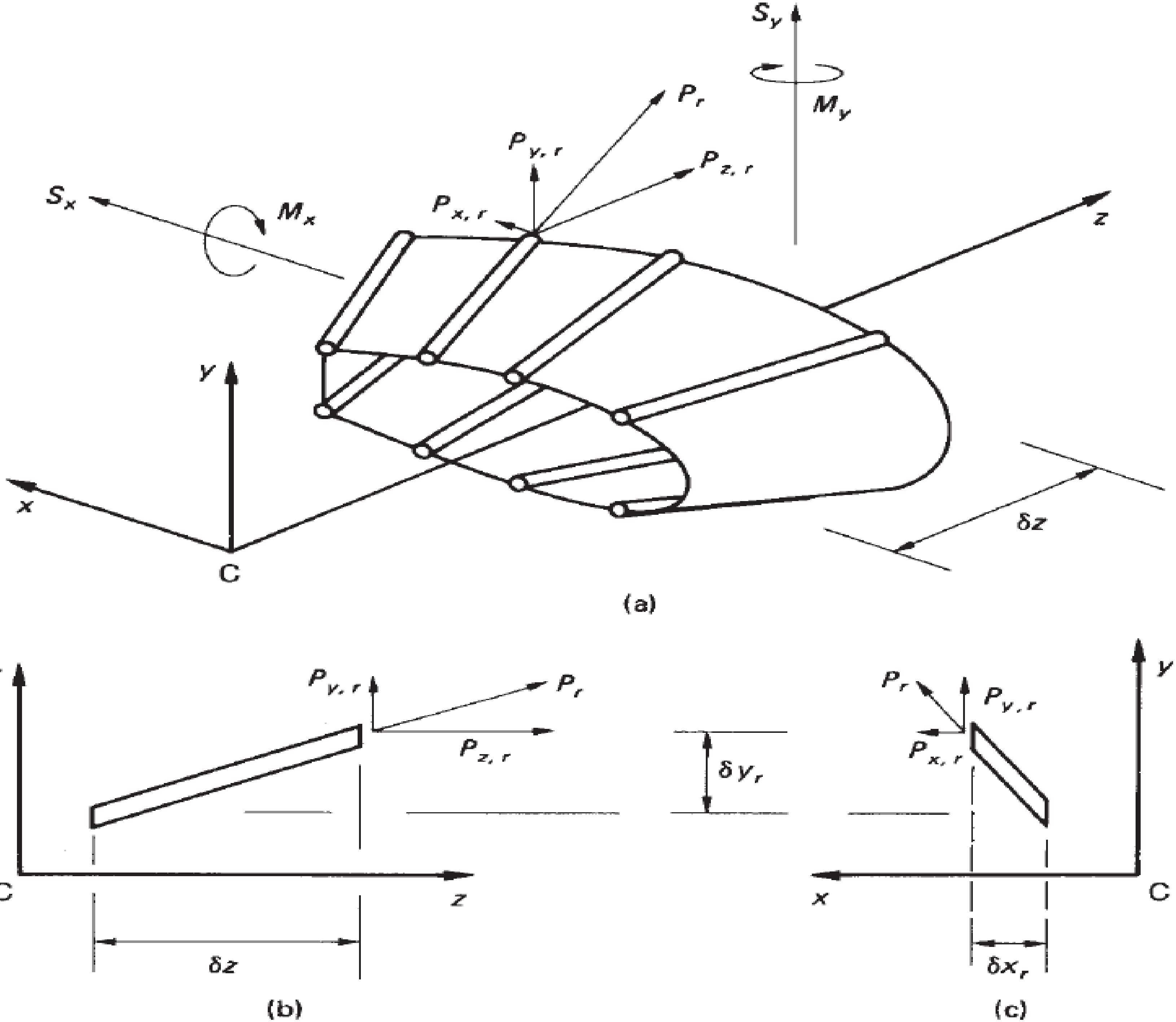
$$P_{y,1} = P_{z,1}\frac{\delta y_1}{\delta z} \quad P_{y,2} = -P_{z,2}\frac{\delta y_2}{\delta z}$$



$$q_s = -\frac{S_{y,w}}{I_{xx}}\left(\int_0^s t_D y\, ds + B_1 y_1\right)$$

$$q_s = -\frac{S_{y,w}}{I_{xx}}\left(\int_0^s t_D y\, ds + B_2 y_2\right)$$



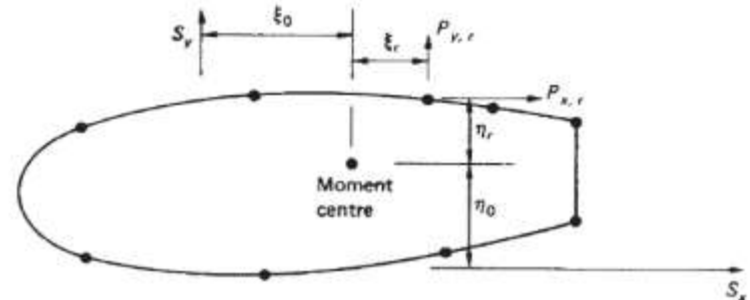


S_y
M_y
P_r
$P_{y,r}$
$P_{z,r}$
S_x
M_x
$P_{x,r}$
z
y
x
C
δz
(a)
y
$P_{y,r}$
P_r
$P_{z,r}$
C
z
δz
(b)
δy_r
P_r
$P_{y,r}$
$P_{x,r}$
y
x
C
δx_r
(c)

$$P_r = P_{z,r}\frac{(\delta x_r^2 + \delta y_r^2 + \delta z^2)^{1/2}}{\delta z}$$

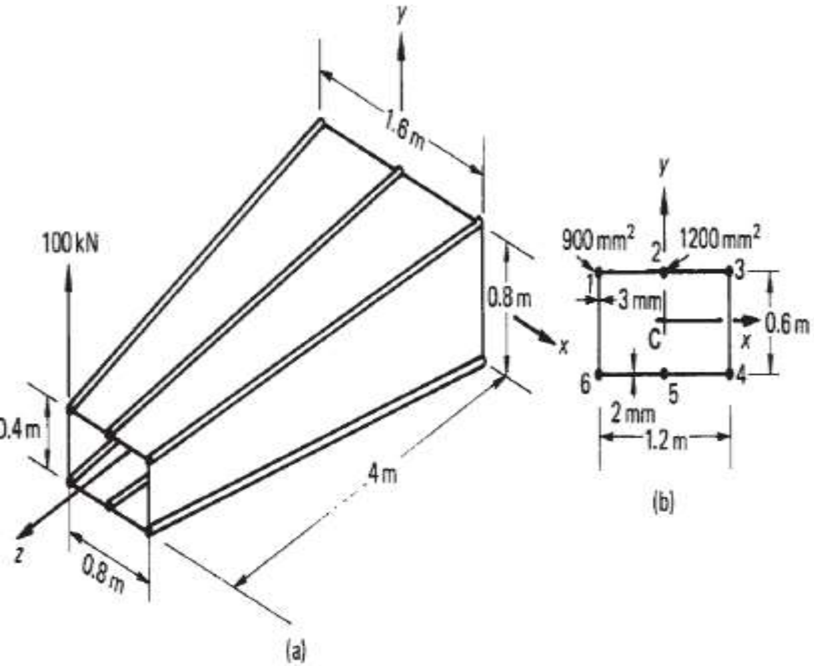
$$S_x = S_{x,w} + \sum_{r=1}^{m} P_{x,r} \quad S_y = S_{y,w} + \sum_{r=1}^{m} P_{y,r}$$

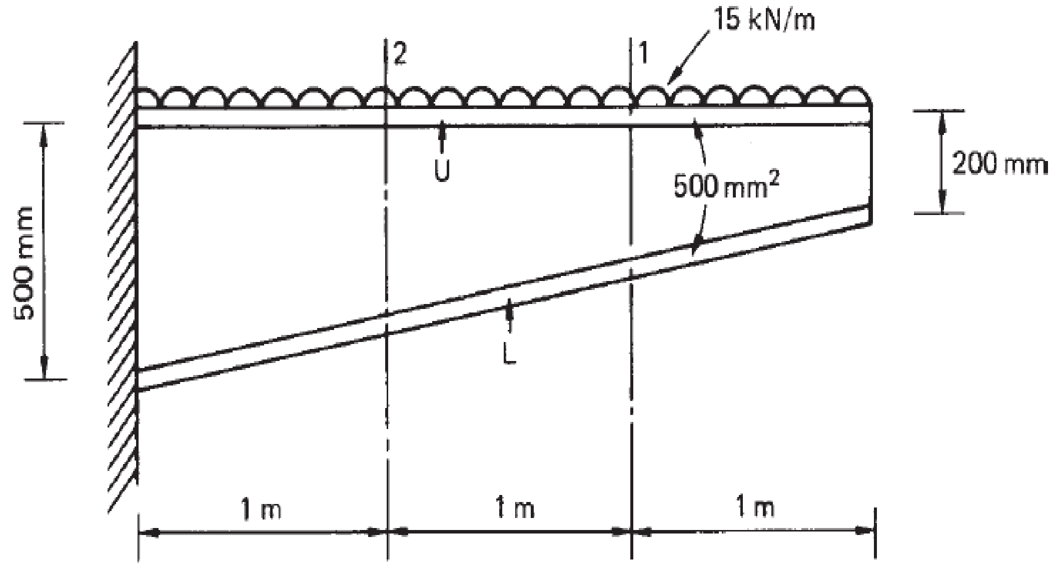
$$S_x = S_{x,w} + \sum_{r=1}^{m} P_{z,r}\frac{\delta x_r}{\delta z} \quad S_y = S_{y,w} + \sum_{r=1}^{m} P_{z,r}\frac{\delta y_r}{\delta z}$$



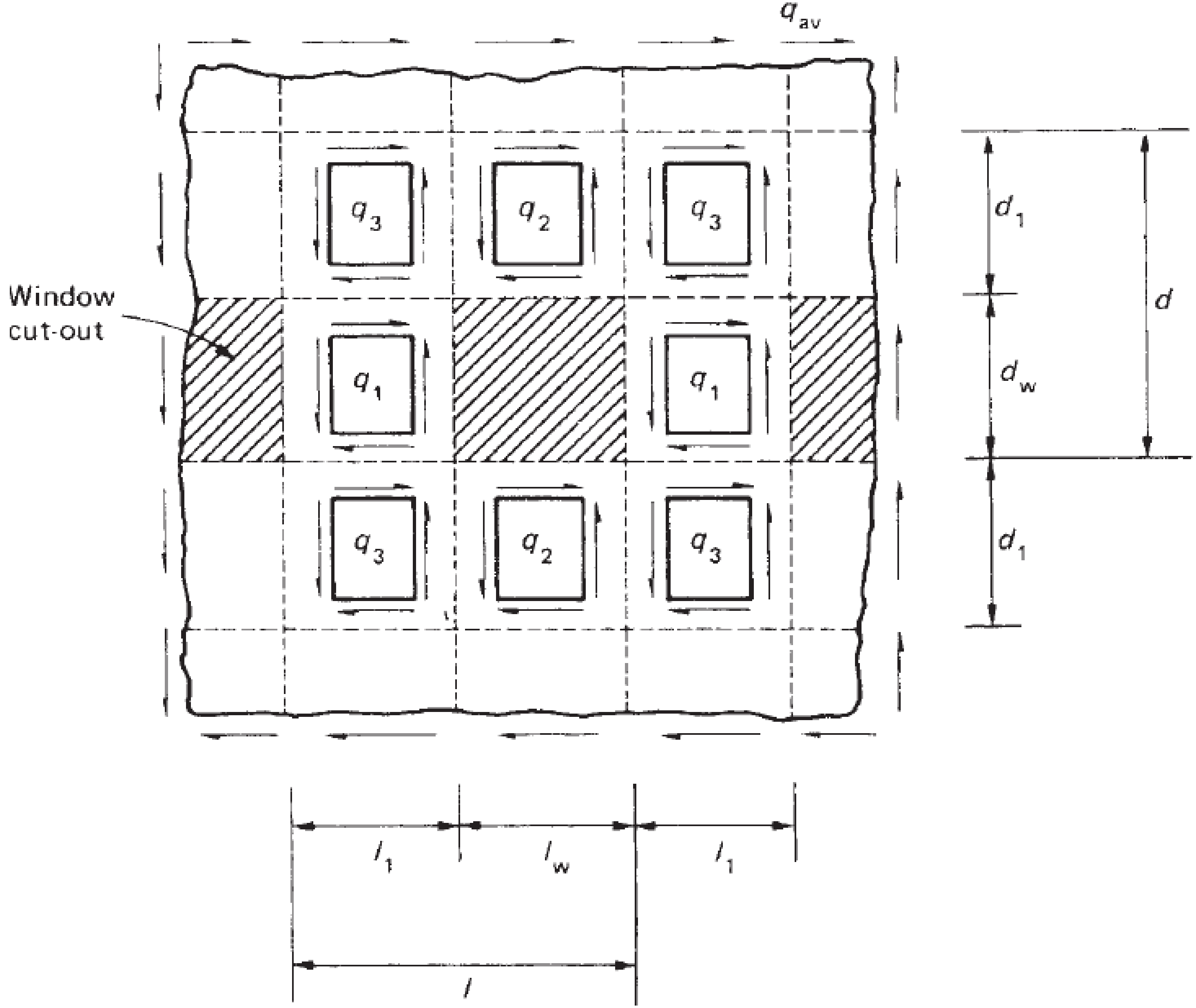
$$S_{x,w} = S_x - \sum_{r=1}^{m} P_{z,r}\frac{\delta x_r}{\delta z} \quad S_{y,w} = S_y - \sum_{r=1}^{m} P_{z,r}\frac{\delta y_r}{\delta z}$$

$$S_x\eta_0 - S_y\xi_0 = \oint q_b p\,\mathrm{d}s + 2Aq_{s,0} - \sum_{r=1}^{m} P_{x,r}\eta_r + \sum_{r=1}^{m} P_{y,r}\xi_r$$

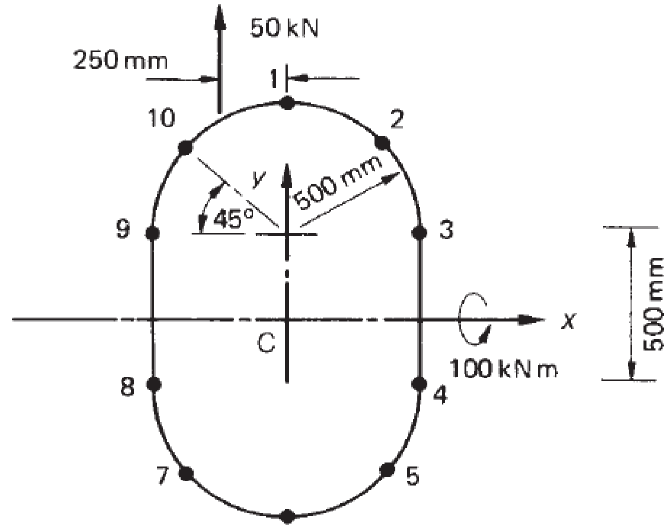


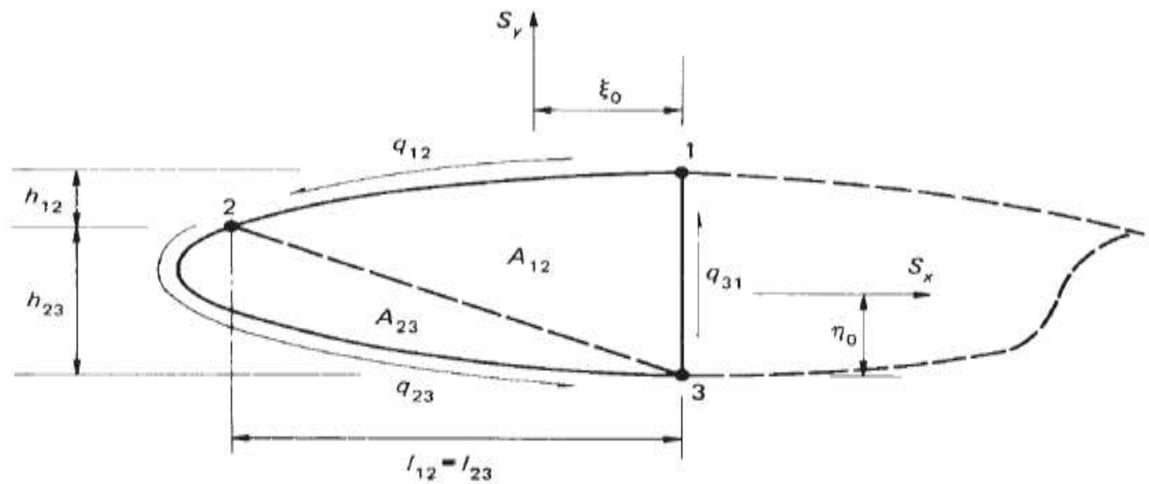


① Boom	② $P_{z,r}$ (kN)	③ $\delta x_r/\delta z$	④ $\delta y_r/\delta z$	⑤ $P_{x,r}$ (kN)	⑥ $P_{y,r}$ (kN)	⑦ P_r (kN)	⑧ ξ_r (m)	⑨ η_r (m)	⑩ $P_{x,r}\eta_r$ (kN m)	⑪ $P_{y,r}\xi_r$ (kN m)
1	−100	0.1	−0.05	−10	5	−101.3	0.6	0.3	3	−3
2	−133	0	−0.05	0	6.7	−177.3	0	0.3	0	0
3	−100	−0.1	−0.05	10	5	−101.3	0.6	0.3	−3	3
4	100	−0.1	0.05	−10	5	101.3	0.6	0.3	−3	3
5	133	0	0.05	0	6.7	177.3	0	0.3	0	0
6	100	0.1	0.05	10	5	101.3	0.6	0.3	3	−3



q_{av}
q_3
q_2
q_3
Window cut-out
q_1
q_1
q_3
q_2
q_3
d_1
d
d_w
d_1
l_1
l_w
l_1
l

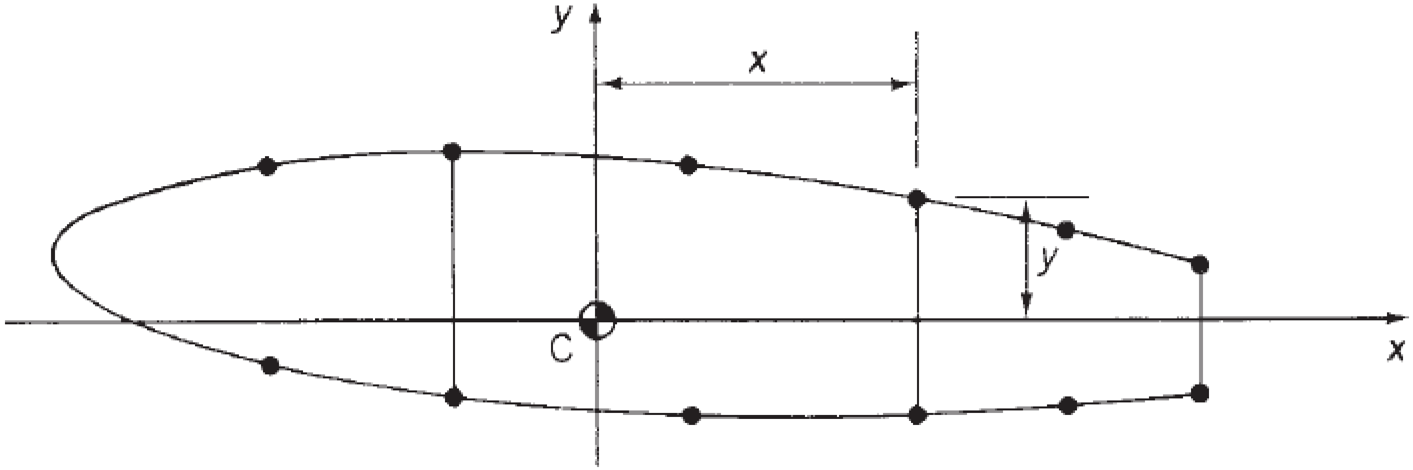




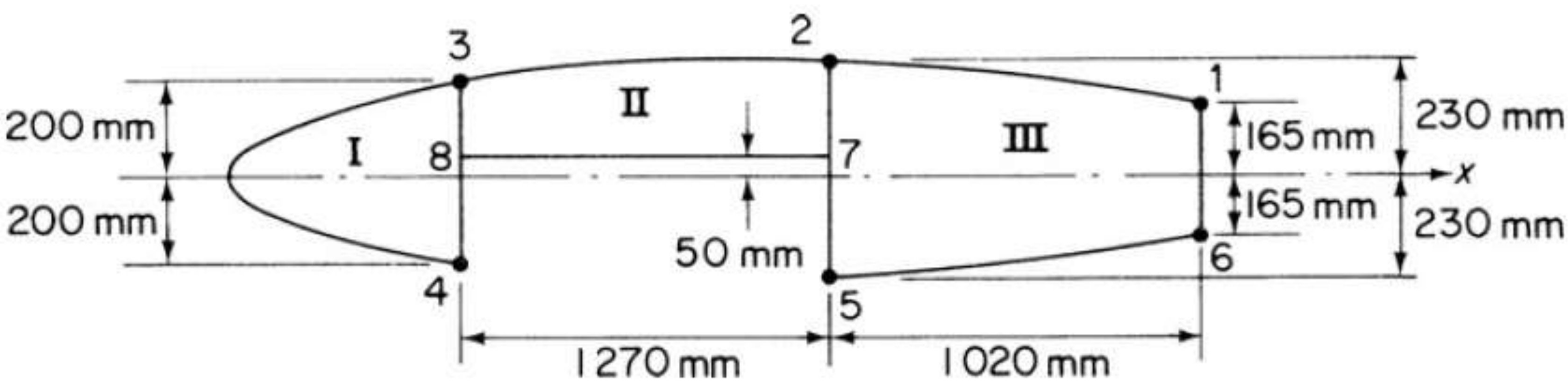
$$S_x = -q_{12}l_{12} + q_{23}l_{23}$$

$$S_y = q_{31}(h_{12} + h_{23}) - q_{12}h_{12} - q_{23}h_{23}$$

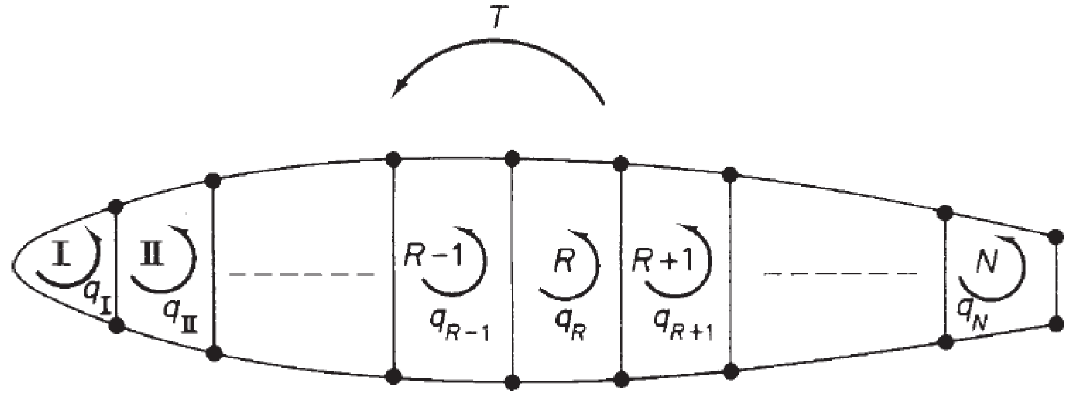
$$S_x\eta_0 + S_y\xi_0 = -2A_{12}q_{12} - 2A_{23}q_{23}$$



Idealized section of a multicell wing.



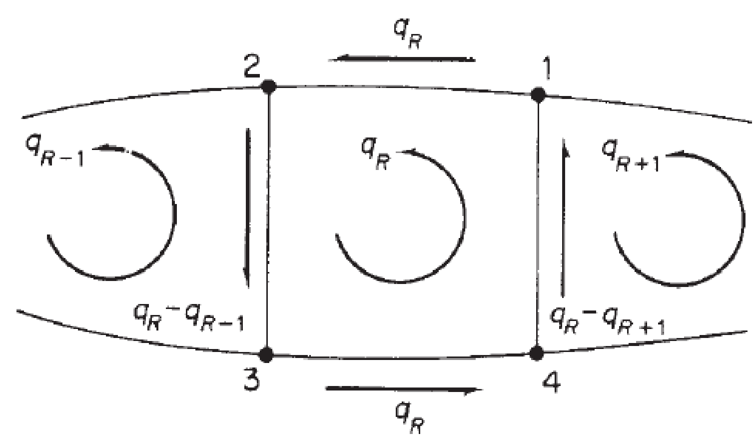
$$T = \sum_{R=1}^{N} 2A_R q_R$$



$$\frac{d\theta}{dz} = \frac{1}{2A_R G} \oint_R q \frac{ds}{t}$$

$$\frac{d\theta}{dz} = \frac{1}{2A_R G}[q_R\delta_{12} + (q_R - q_{R-1})\delta_{23} + q_R\delta_{34} + (q_R - q_{R+1})\delta_{41}]$$

$$\frac{d\theta}{dz} = \frac{1}{2A_R G}(-q_{R-1}\delta_{R-1,R} + q_R\delta_R - q_{R+1}\delta_{R+1,R})$$



$$\frac{d\theta}{dz} = \frac{1}{2A_R G_{REF}} \oint_R q \frac{ds}{t^*}$$

Shear flow distribution in the *R*th cell of an *N*-cell wing section.

$$t^* = \frac{G}{G_{REF}} t$$