Antennas &Wave Propagation

Prepared By

A. Usha Rani Assistant Professor
J. Shiva Ramakrishna Assistant Professor
G. Nagendra Prasad, Assistant Professor
K. Ravi Assistant Professor

Basic Antenna Theory Purpose

- to refresh basic concepts related to the antenna physics
  - needed to understand better the operation and design of microwave links and systems

Outline

- Introduction
- Review of basic antenna types
- Radiation pattern, gain, polarization
- Equivalent circuit & radiation efficiency
- Smart antennas
- Some theory
- Summary

Quiz

We use a transmitting antenna to radiate radio wave and a receiving antenna to capture the RF energy carried by the wave.

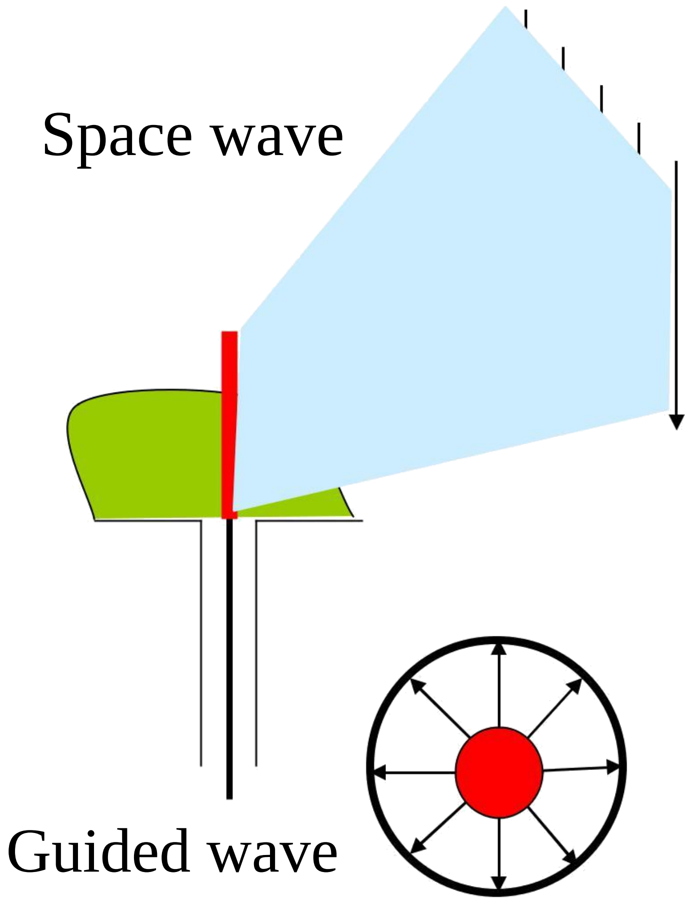
Somebody told that the receiving antenna also radiates radio waves during the reception.

Is it a true fact or a slip of the tongue?

Intended & unintended radiators

- Antennas intended to produce specified EM field
  - Radiocommunication antennas; Measuring antennas; EM sensors, probes; EM applicators (Industrial, Medical, Scientific)
- Radiators not intended to generate any EM field, but producing it as an unintended side-effect
  - Any conductor/ installation with varying electrical current (e.g. electrical installation of vehicles)
  - Any slot/ opening in the screen of a device/ cable carrying RF current
  - Any discontinuity in transmission medium (e.g. conducting structures/ installations) irradiated by EM waves
    - Stationary (e.g. antenna masts or power line wires); Time-varying (e.g. windmill or helicopter propellers); Transient (e.g. aeroplanes, missiles)

Antenna purpose

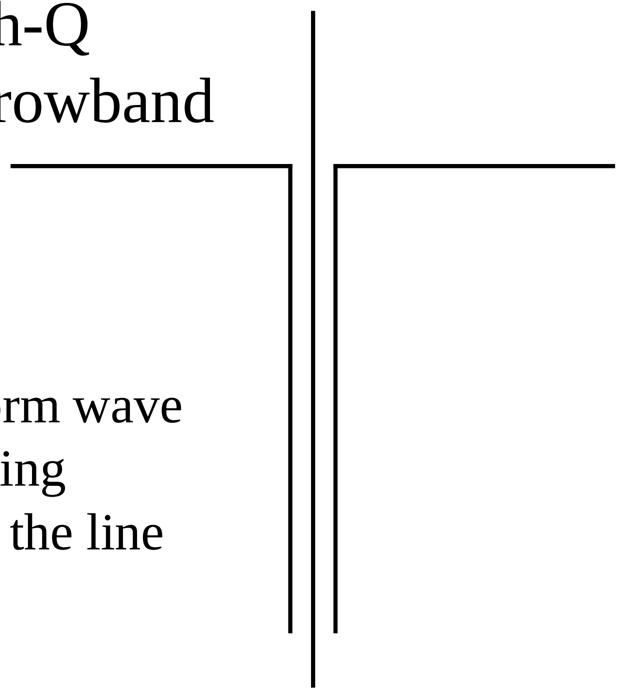


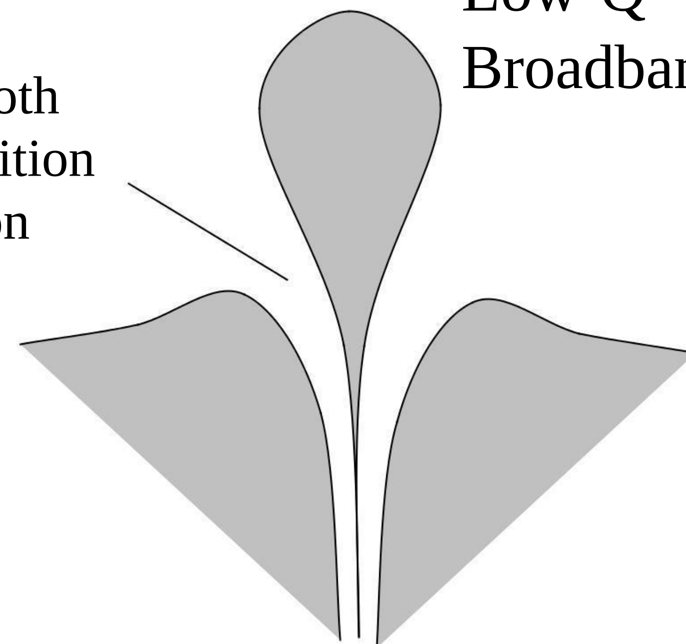
- Transformation of a guided EM wave in transmission line (waveguide) into a freely propagating EM wave in space (or vice versa) with specified directional characteristics
  - Transformation from time-function in one-dimensional space into time-function in three dimensional space
  - The specific form of the radiated wave is defined by the antenna structure and the environment

Antenna functions

- Transmission line
  - Power transport medium - must avoid power reflections, otherwise use matching devices
- Radiator
  - Must radiate efficiently – must be of a size comparable with the half-wavelength
- Resonator
  - Unavoidable - for broadband applications resonances must be attenuated

Monopole (dipole over plane)





- If there is an inhomogeneity (obstacle) a reflected wave, standing wave, & higher field modes appear
- With pure standing wave the energy is stored and oscillates from entirely electric to entirely magnetic and back
- Model: a resonator with high Q = (energy stored) / (energy lost) per cycle, as in LC circuits
- Kraus p.2

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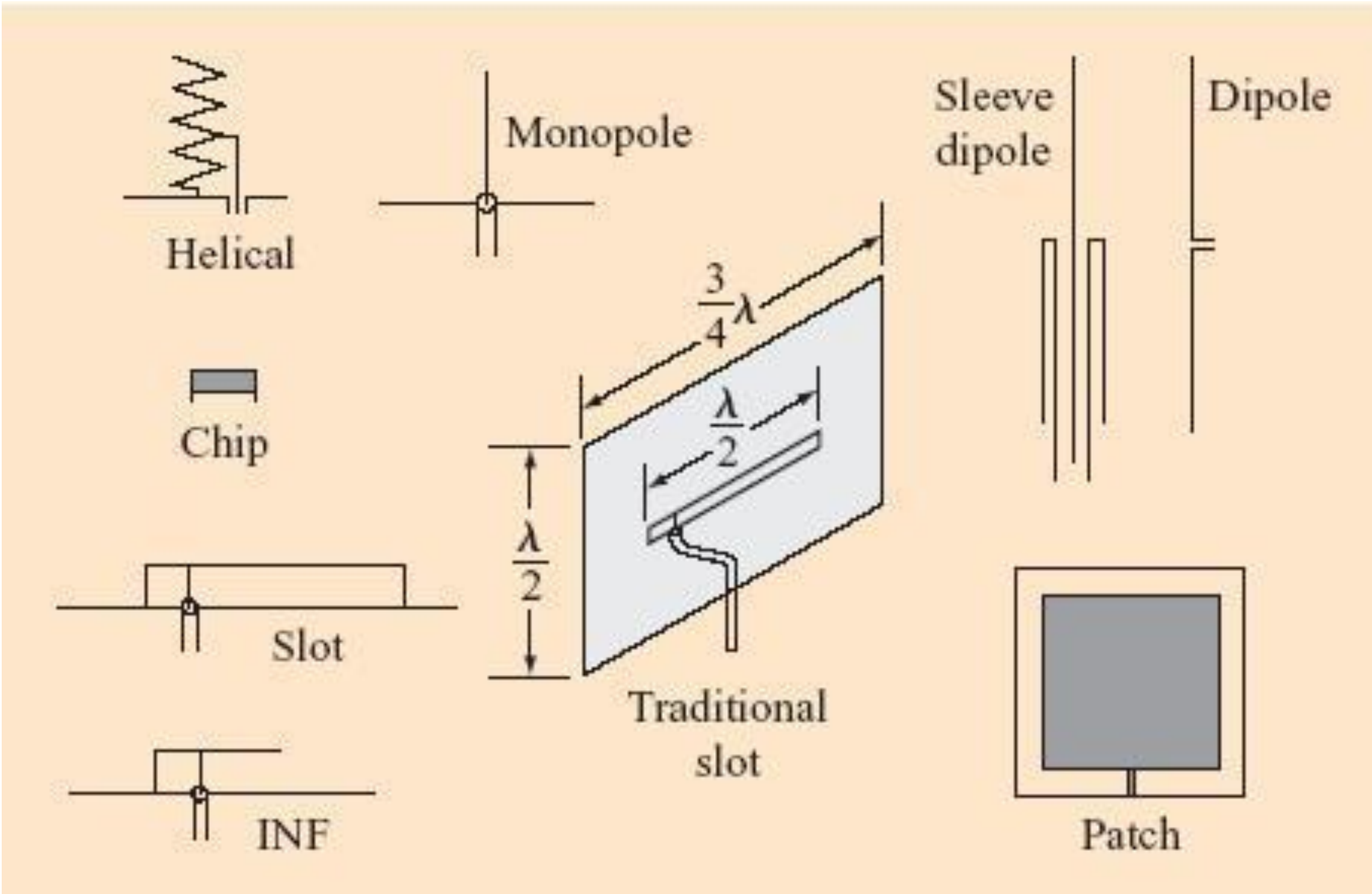
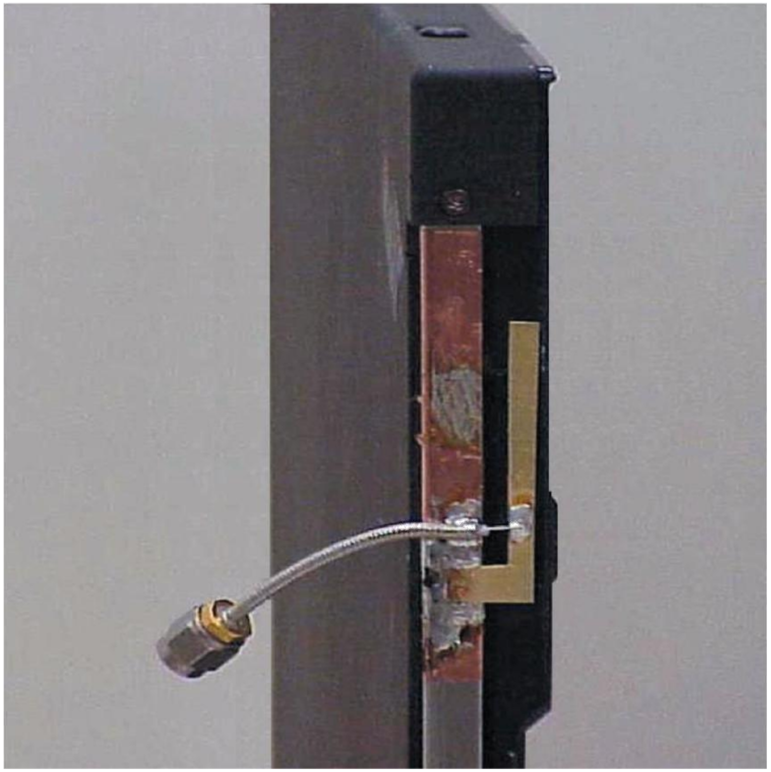
Antennas for laptop applications



Source: D. Liu et al.: Developing integrated antenna subsystems for laptop computers; IBM J.RES.& DEV. VOL. 47 NO. 2/3 MARCH/MAY 2003 p. 355-367



- Patch and slot antennas derived from printed-circuit and micro-strip technologies
- Ceramic chip antennas are typically helical or inverted-F (INF) antennas, or variations of these two types with high dielectric loading to reduce the antenna size

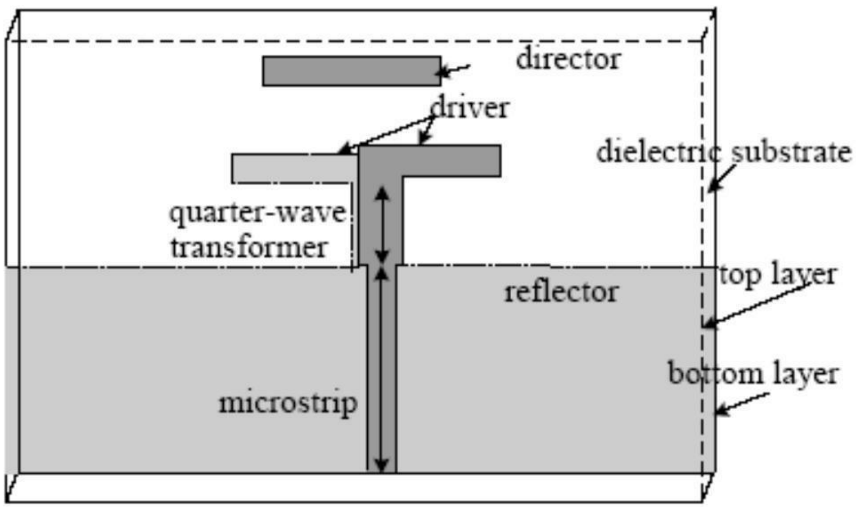
Source: D. Liu et al.: Developing integrated antenna subsystems for laptop computers; IBM J. RES. & DEV. VOL. 47 NO. 2/3 MARCH/MAY 2003 p. 355-367

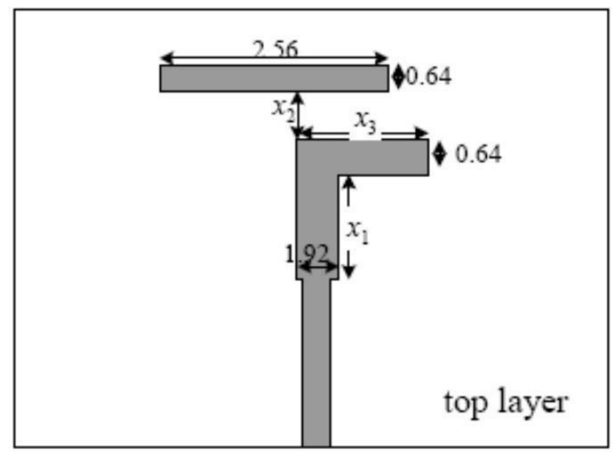
Slot & INF antennas

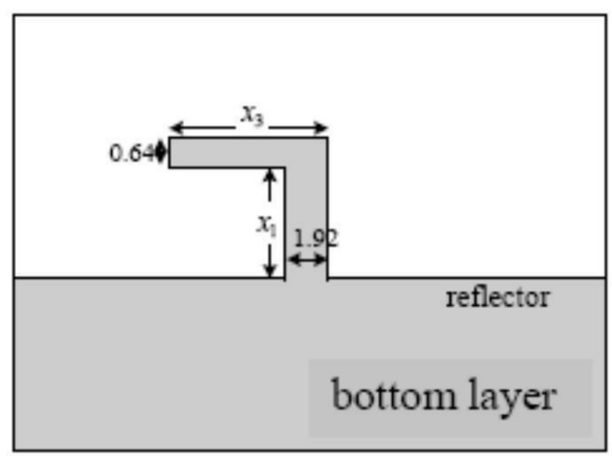
- Slot antenna: a slot is cut from a large (relative to the slot length) metal plate.
  - The center conductor of the feeding coaxial cable is connected to one side of the slot, and the outside conductor of the cable - to the other side of the slot.
- The slot length is some ($\lambda/2$) for the slot antenna and ($\lambda/4$) long for the INF antenna.
- The slot and INF antennas behave similarly.
  - The slot antenna can be considered as a loaded version of the INF antenna. The load is a quarter-wavelength stub, i.e. a narrowband device.
  - When the feed point is moved to the short-circuited end of the slot (or INF) antenna, the impedance decreases. When it is moved to the slot center (or open end of the INF antenna), the impedance increases

Example

double-layer printed Yagi antenna





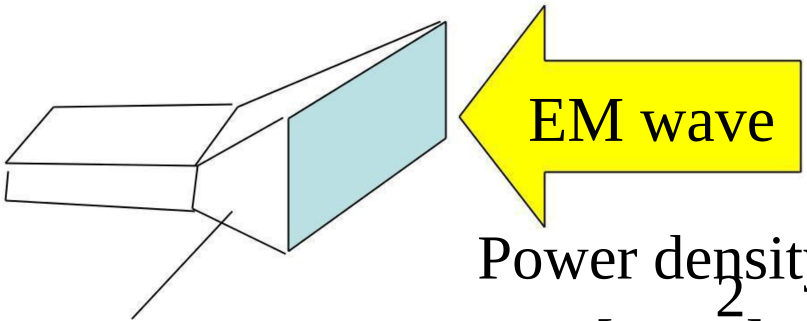


Note: no galvanic contact with the director

Source: N Gregorieva

- Patch and slot antennas are
  - Cheap and easy to fabricate and to mount
  - Suited for integration
  - Light and mechanically robust
  - Have low cross-polarization
  - Low-profile - widely used in antenna arrays
    - spacecrafts, satellites, missiles, cars and other mobile applications

Aperture-antenna



Power density:
PFD [w/m^2]

Power
absorbed: P [watt]

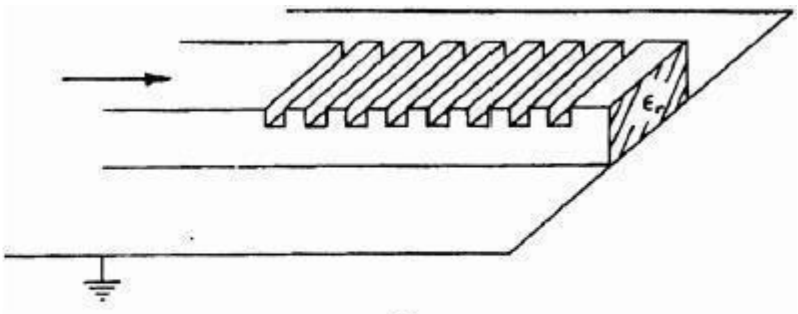
A = A*PFD

Effective
aperture: A[m^2]

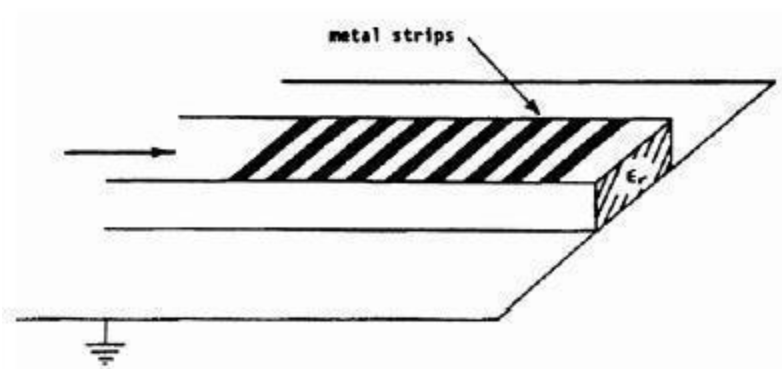
Note: The aperture concept is applicable also to wired antennas. For instance, the max effective aperture of linear $\lambda/2$ wavelength dipole antenna is $\lambda^2/8$

- Aperture antennas derived from waveguide technology (circular, rectangular)
- Can transfer high power (magnetrons, klystrons)
- Above few GHz
- Will be explored inprace during the school

Leaky-wave antennas



(a)



(b)

- Derived from millimeter-wave guides (dielectric guides, microstrip lines, coplanar and slot lines).
- For frequencies > 30 GHz, including infrared
- Subject of intensive study.
  - Note: Periodical discontinuities near the end of the guide lead to substantial radiation leakage (radiation from the dielectric surface).

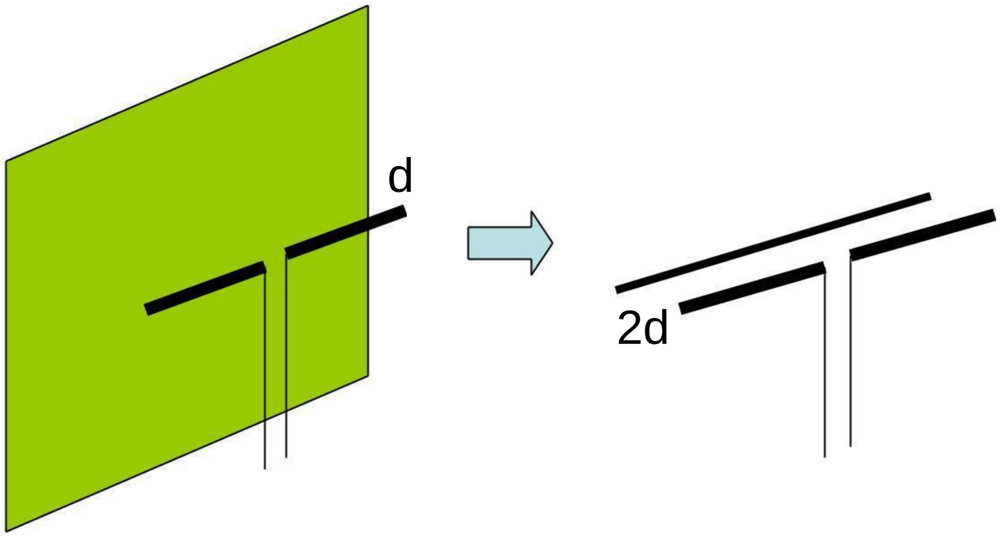
Source: adapted from N Gregorieva

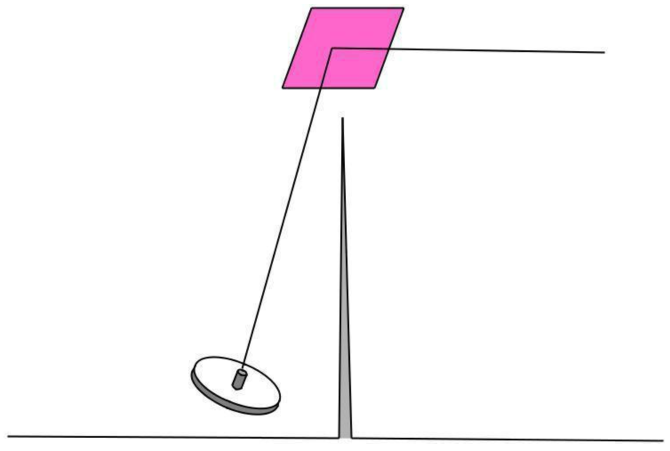
Reflector antennas

- Reflectors are used to concentrate flux of EM energy radiated/ received, or to change its direction
- Usually, they are parabolic (paraboloidal).
  - The first parabolic (cylinder) reflector antenna was used by Heinrich Hertz in 1888.
- Large reflectors have high gain and directivity
  - Are not easy to fabricate
  - Are not mechanically robust
  - Typical applications: radio telescopes, satellite telecommunications.

Source: adapted from N Gregorieva

Planar reflectors

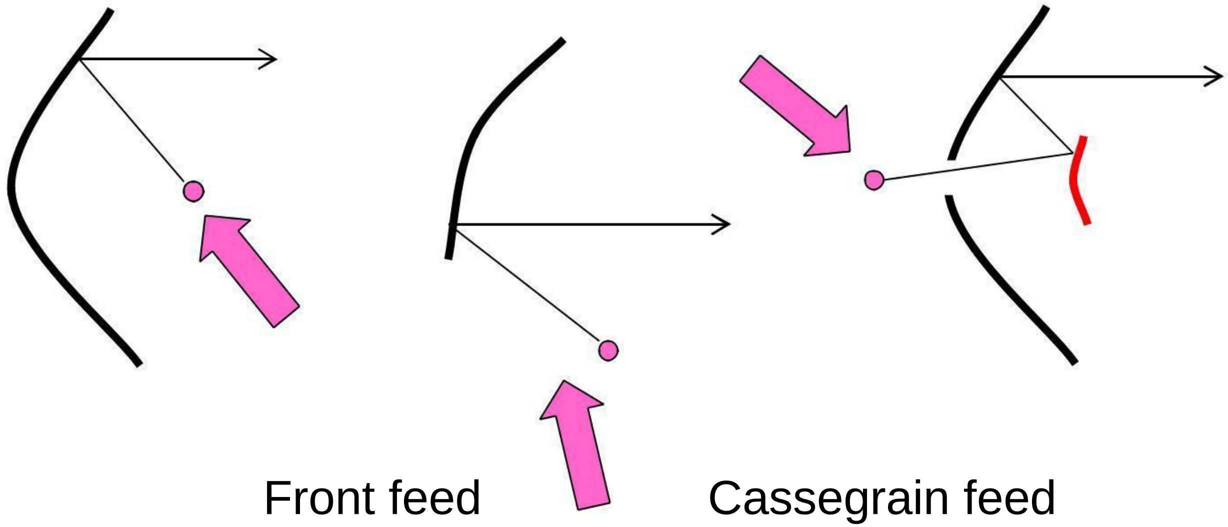




- Uda-Yagi, Log-periodic antennas

- Intended reflector antenna allows maintaining radio link in non-LOS conditions (avoiding propagation obstacles)
- Unintended antennas create interference

Paraboloidal reflectors



The largest radio telescopes

- Max Plank Institüt für Radioastronomie radio telescope, Effelsberg (Germany), 100-m paraboloidal reflector
- The Green Bank Telescope (the National Radio Astronomy Observatory) – paraboloid of aperture 100 m

Source: adapted from N Gregorieva

The Arecibo Observatory Antenna System

The world's largest single radio telescope

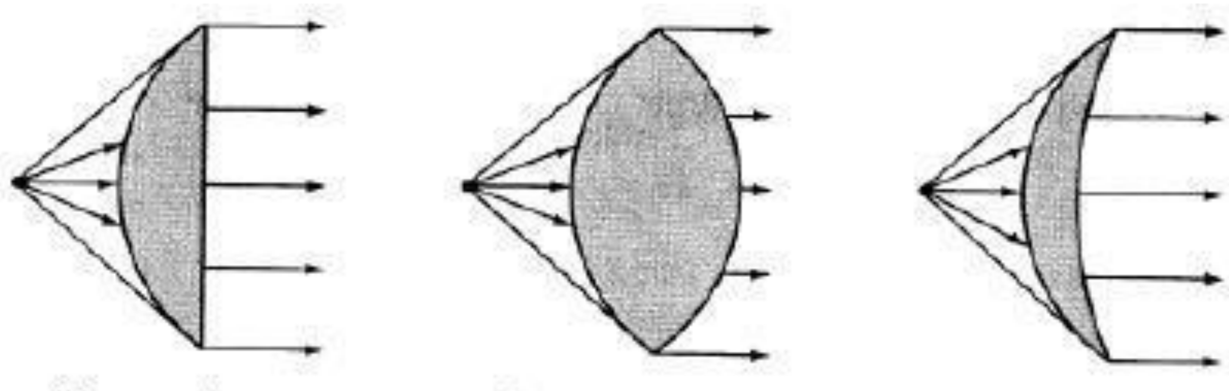
304.8-m spherical reflector

National Astronomy and Ionosphere Center (USA), Arecibo, Puerto Rico

The Arecibo Radio Telescope

[Sky & Telescope
Feb 1997 p. 29]

Lens antennas



(a) Lens antennas with index of refraction $n > 1$

Lenses play a similar role to that of reflectors in reflector antennas: they collimate divergent energy
Often preferred to reflectors at frequencies > 100 GHz.

Source: Kraus p.382, N Gregorieva

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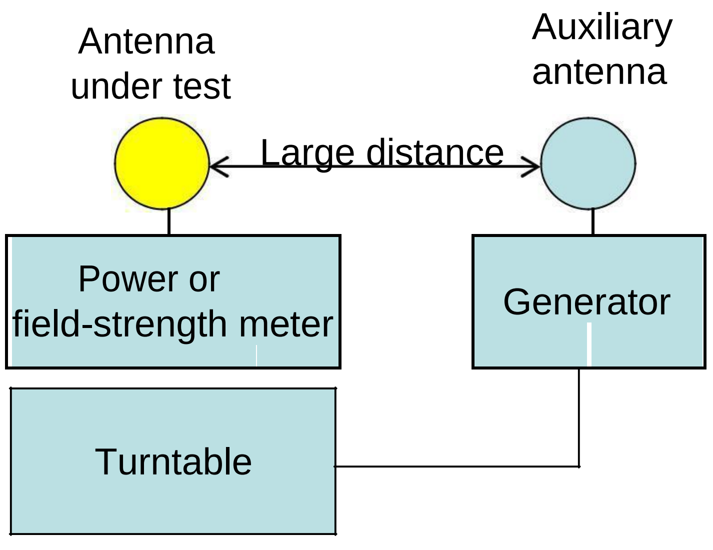
Radiation pattern

- The *radiation pattern of antenna* is a representation (pictorial or mathematical) of the distribution of the power out-flowing (radiated) from the antenna (in the case of transmitting antenna), or inflowing (received) to the antenna (in the case of receiving antenna) as a function of direction angles from the antenna
  - Antenna radiation pattern (*antenna pattern)*:
    - is defined for large distances from the antenna, where the spatial (angular) distribution of the radiated power does not depend on the distance from the radiation source
    - is independent on the power flow direction: it is the same when the antenna is used to transmit and when it is used to receive radio waves
    - is usually different for different frequencies and different polarizations of radio wave radiated/ received

Power pattern vs. Field pattern



- The power pattern and the field patterns are inter-related:

  $P(\theta, \phi) = (1/\eta)*|E(\theta, \phi)|^2 = \eta*|H(\theta, \phi)|^2$

P = power

- The *power pattern* is the measured (calculated) and plotted received power: $|P(\theta, \phi)|$ at a constant (large) distance from the antenna
- The *amplitude field pattern* is the measured (calculated) and plotted electric (magnetic) field intensity, $|E(\theta, \phi)|$ or $|H(\theta, \phi)|$ at a constant (large) distance from the antenna

E = electrical field component vector

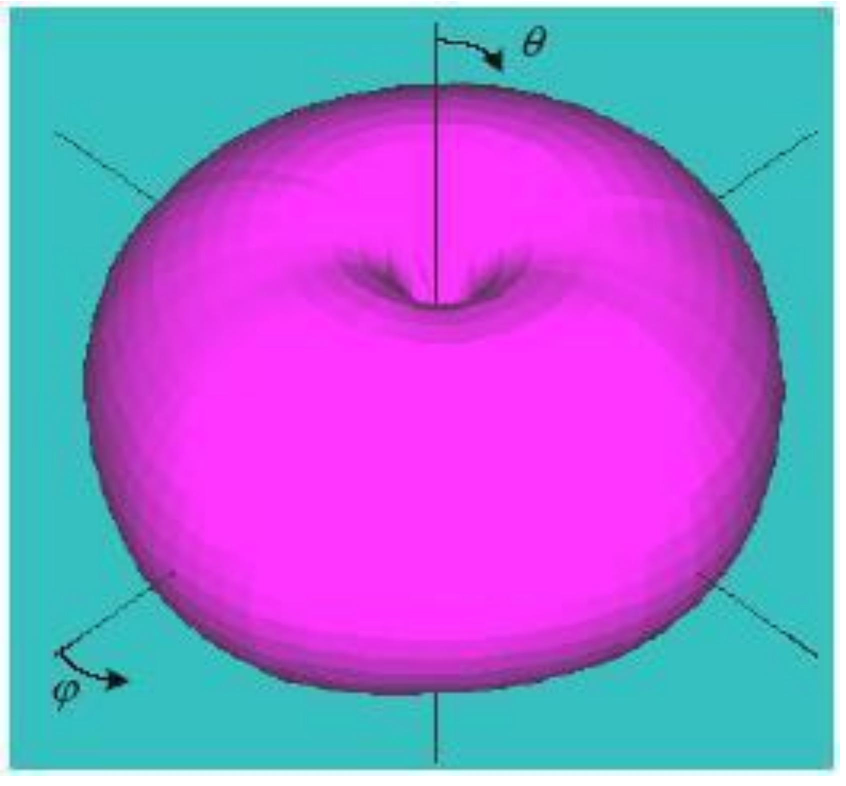
H = magnetic field component vector

η = 377 ohm (free-space, plane wave impedance)

Normalized pattern

- Usually, the pattern describes the *normalized* field (power) values with respect to the maximum value.
  - Note**:** The power pattern and the amplitude field pattern are the same when computed and when plotted in dB.

3-D pattern

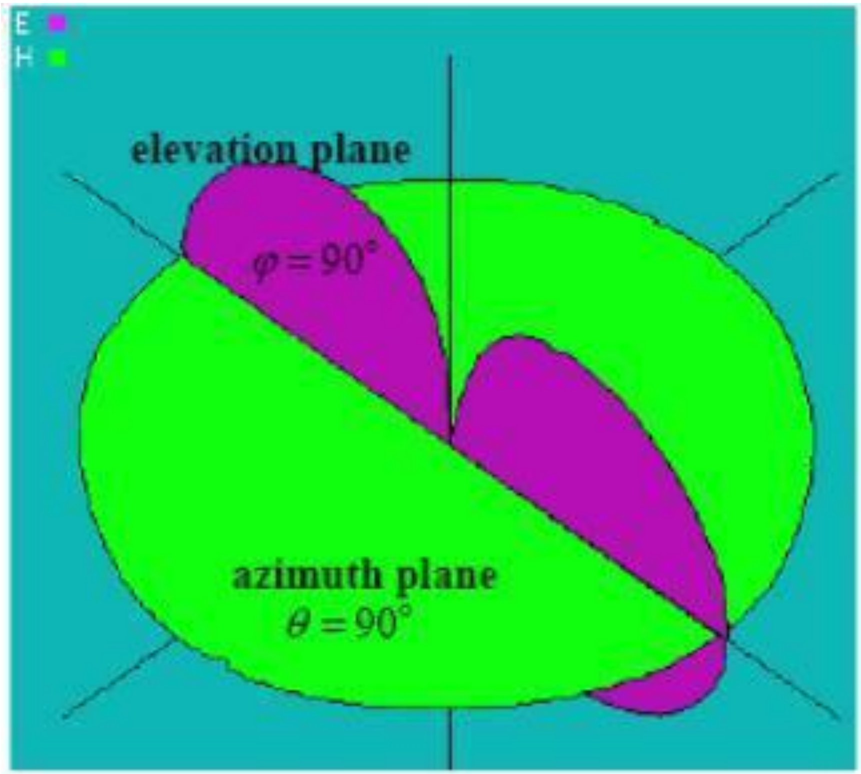


3-D pattern

- Antenna radiation pattern is 3-dimensional
- The 3-D plot of antenna pattern assumes both angles θ and ϕ varying, which is difficult to produce and to interpret

Source: NK Nikolova

2-D pattern



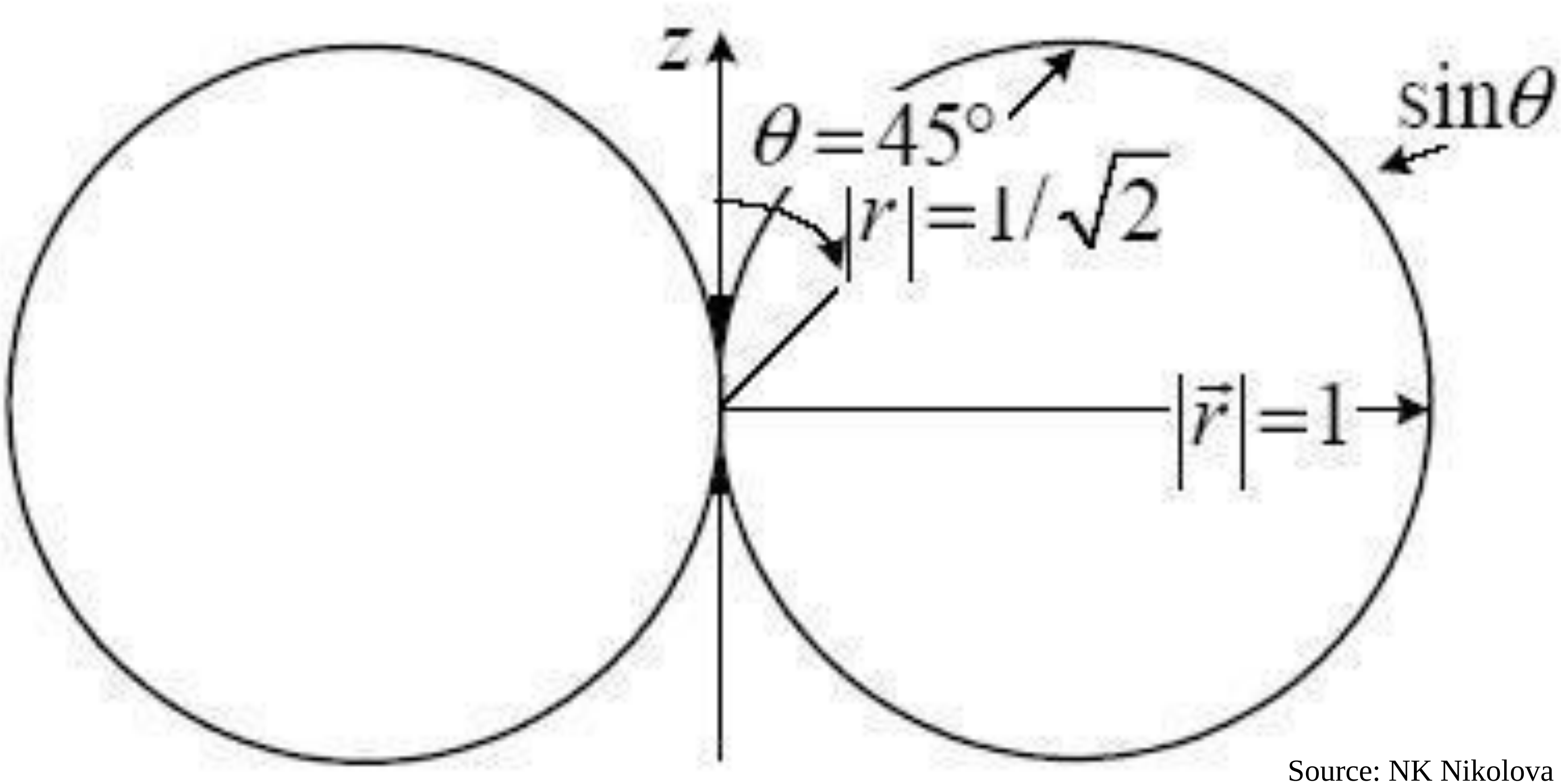
Two 2-D patterns

- Usually the antenna pattern is presented as a 2-D plot, with only one of the direction angles, θ or ϕ varies
- It is an intersection of the 3-D one with a given plane
  - usually it is a θ = *const* plane or a ϕ= *const* plane that contains the pattern's maximum

Source: NK Nikolova

Example: a short dipole on z-axis

Elevation plane: $\varphi = const$



Source: NK Nikolova

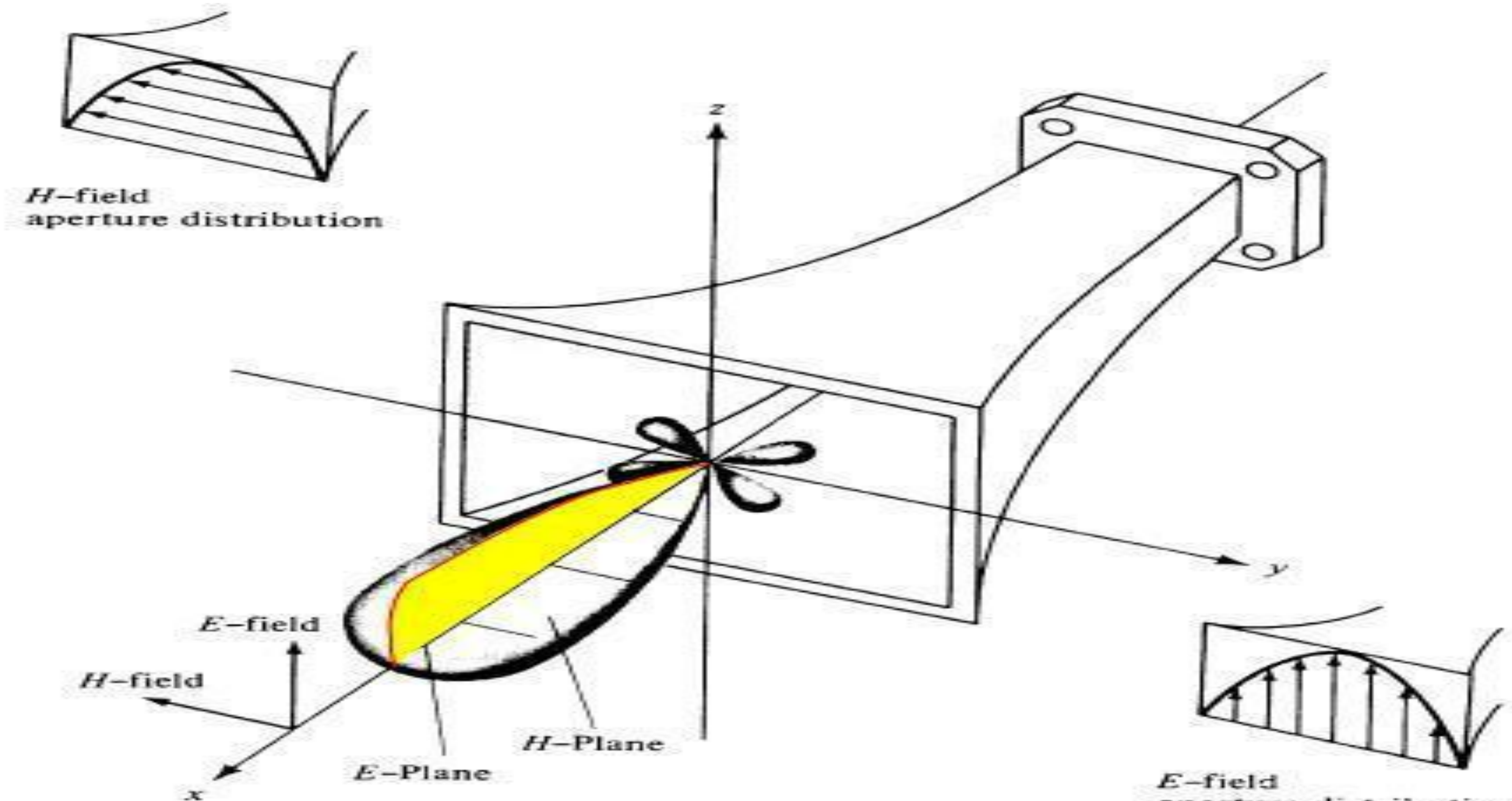
Principal patterns

- Principal patterns are the 2-D patterns of linearly polarized antennas, measured in 2 planes

1. the ***E-plane:*** a plane parallel to the *E* vector and containing the direction of maximum radiation, and
2. the ***H-plane:*** a plane parallel to the *H* vector, orthogonal to the *E*-plane, and containing the direction of maximum radiation

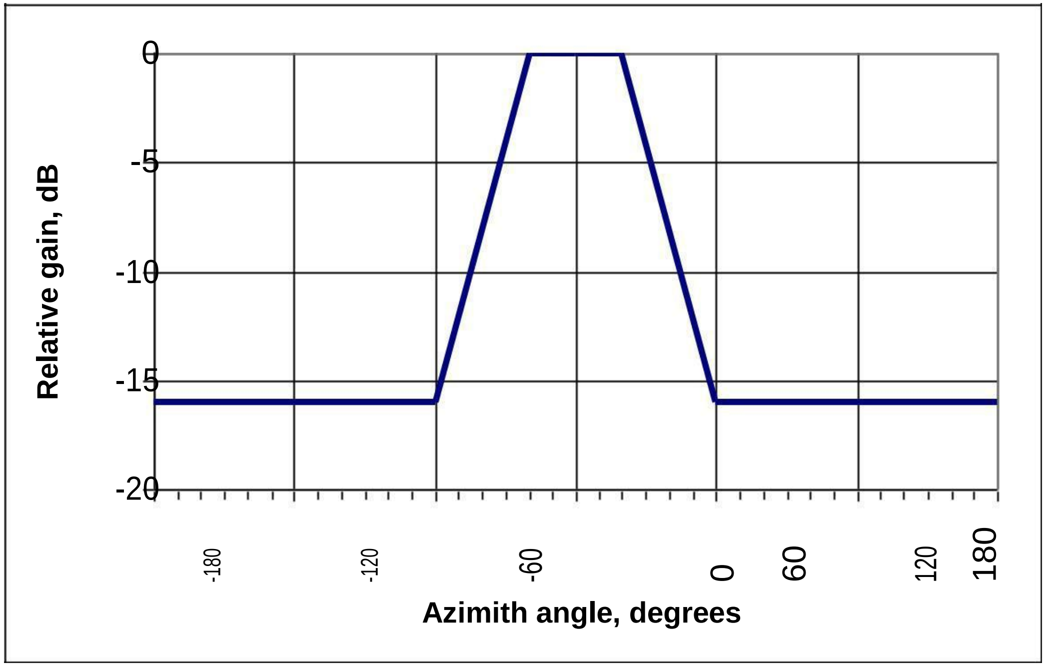
Source: NK Nikolova

Example



Source: NK Nikolova

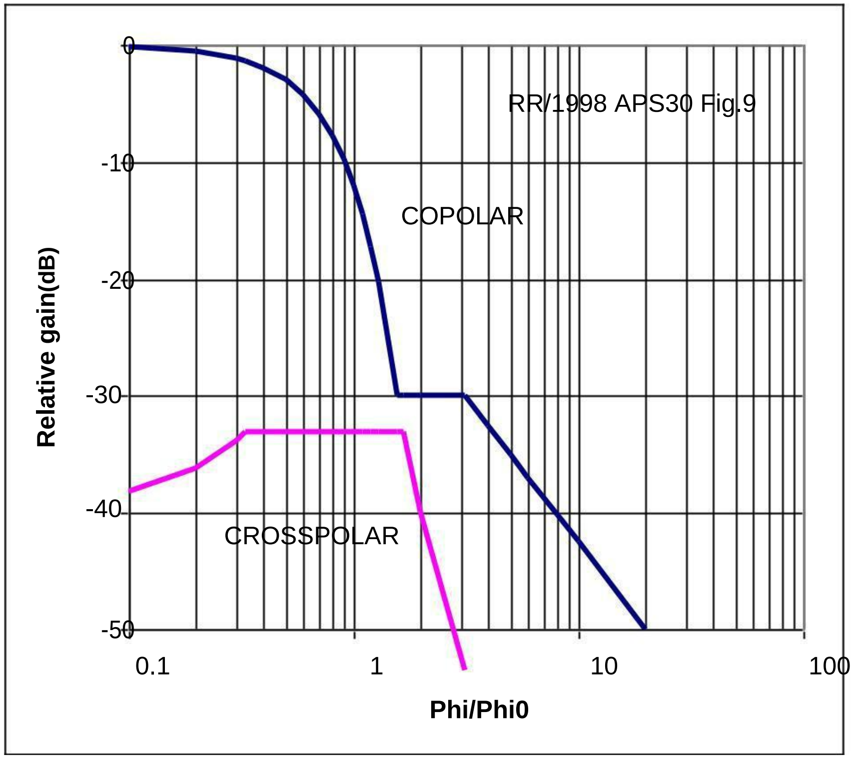
Antenna Mask (Example 1)

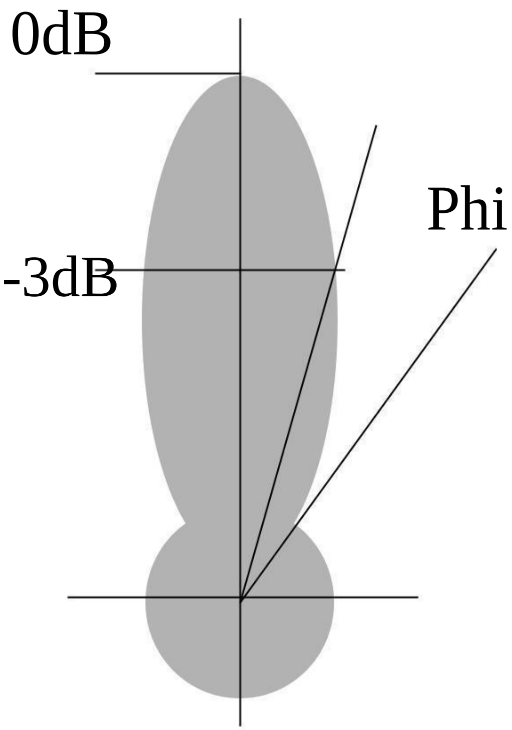


Typical relative directivity- mask of receiving antenna (Yagi ant., TV dcm waves)

[CCIR doc. 11/645, 17-Oct 1989)

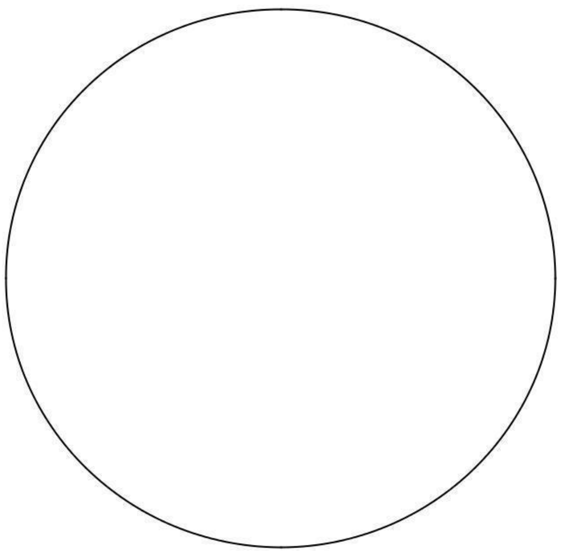
Antenna Mask (Example 2)





Reference pattern for co-polar and cross-polar components for satellite transmitting antennas in Regions 1 and 3 (Broadcasting ~12 GHz)

Isotropic antenna



- ***Isotropic antenna or isotropic radiator*** is a hypothetical (not physically realizable) concept, used as a useful reference to describe real antennas.
- Isotropic antenna radiates equally in all directions.
  - Its radiation pattern is represented by a sphere whose center coincides with the location of the isotropic radiator.

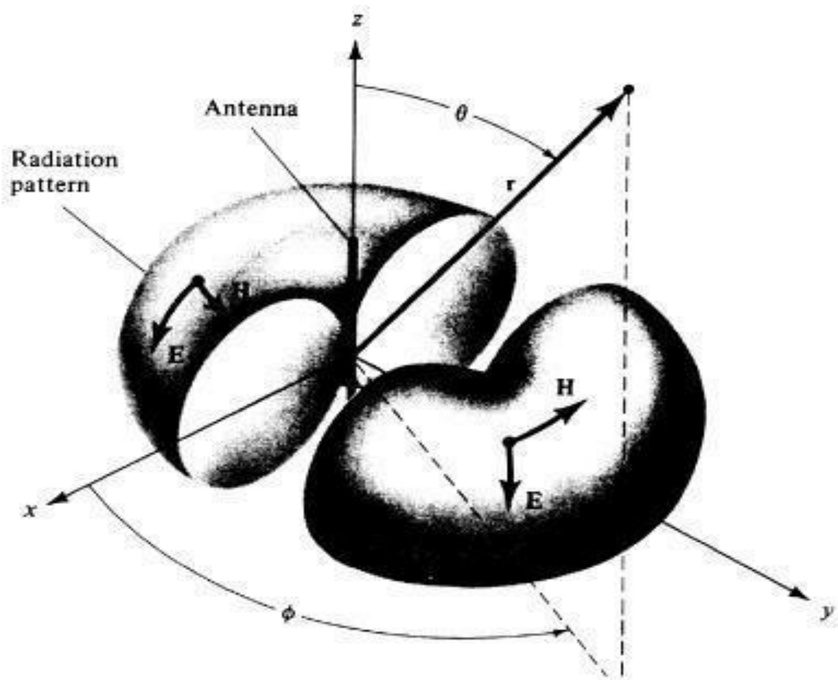
Source: NK Nikolova

Directional antenna

- ***Directional antenna*** is an antenna, which radiates (or receives) much more power in (or from) some directions than in (or from) others.
  - – Note: Usually, this term is applied to antennas whose directivity is much higher than that of a half-wavelength dipole.

Source: NK Nikolova

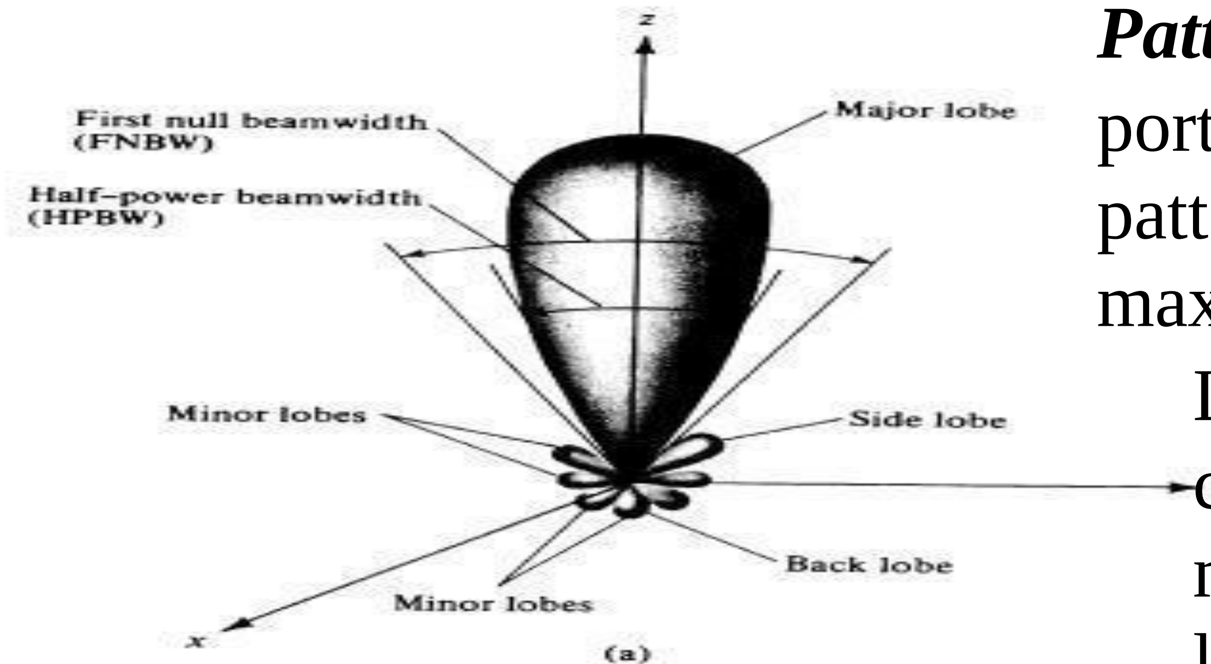
Omnidirectional antenna



- An antenna, which has a non-directional pattern in a plane
  - It is usually directional in other planes

Source: NK Nikolova

Pattern lobes

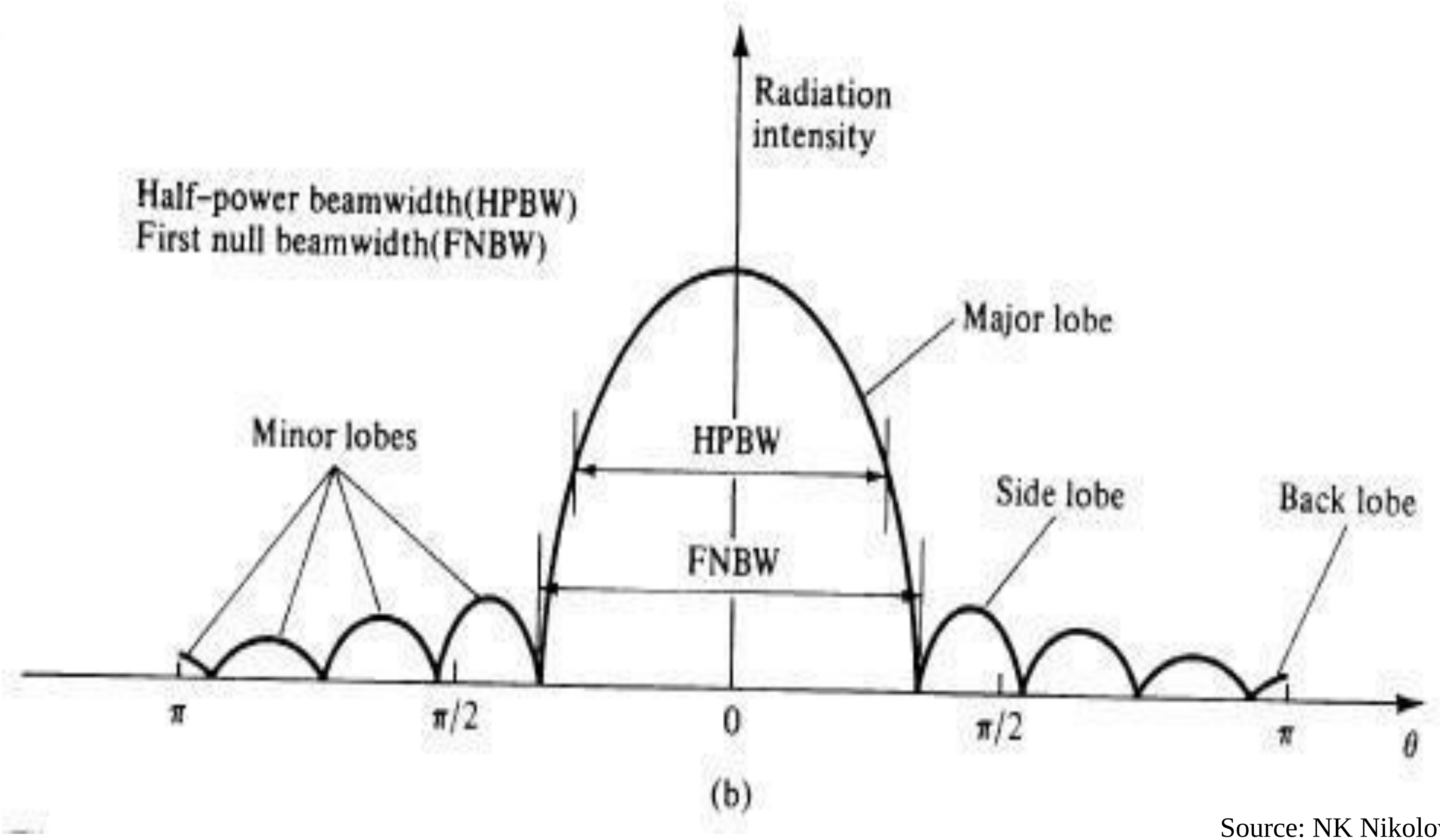


Pattern lobe is a portion of the radiation pattern with a local maximum

Lobes are classified as: major, minor, side lobes, back lobes.

Source: NK Nikolova

Pattern lobes and beam widths

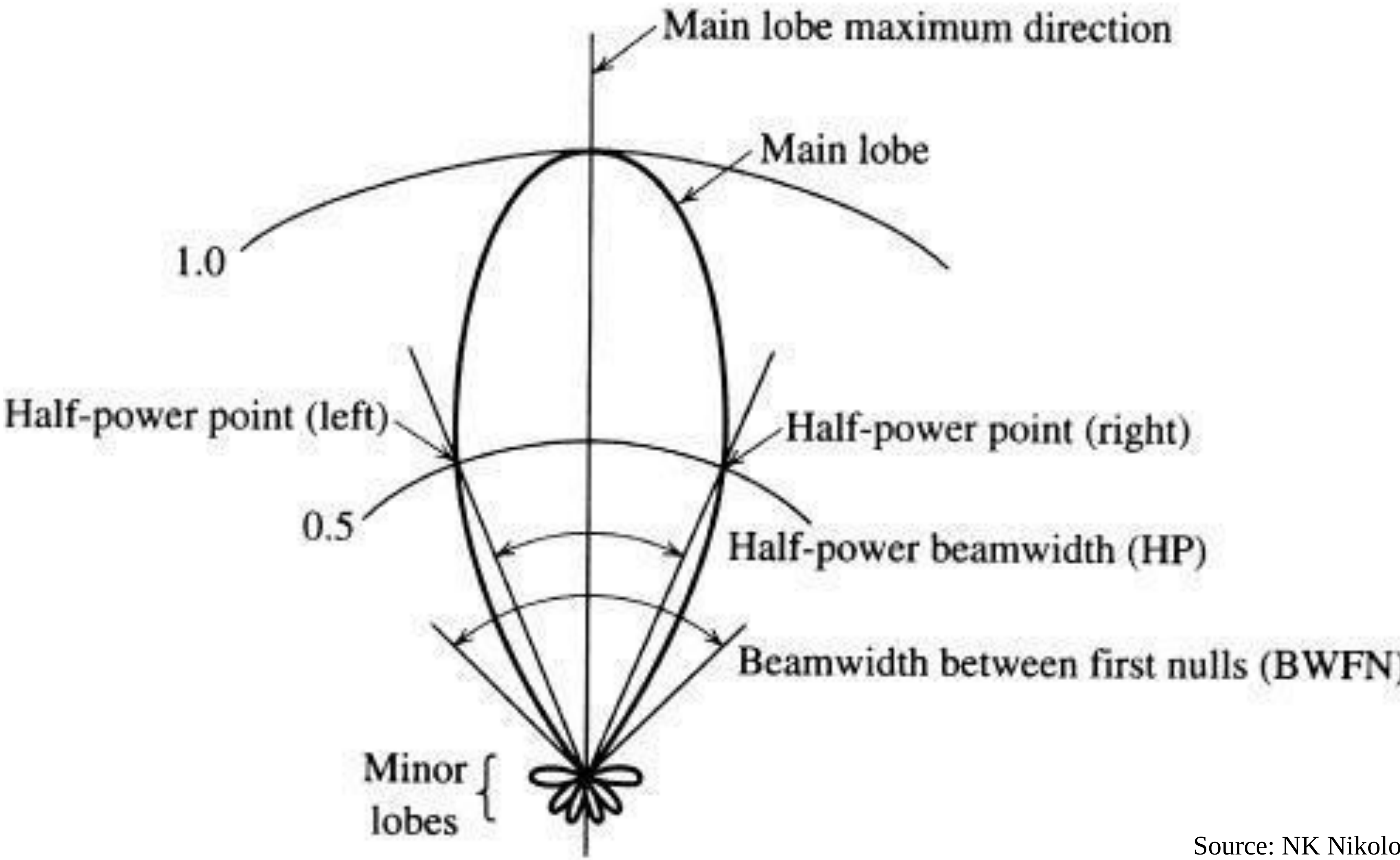


Source: NK Nikolova

Beamwidth

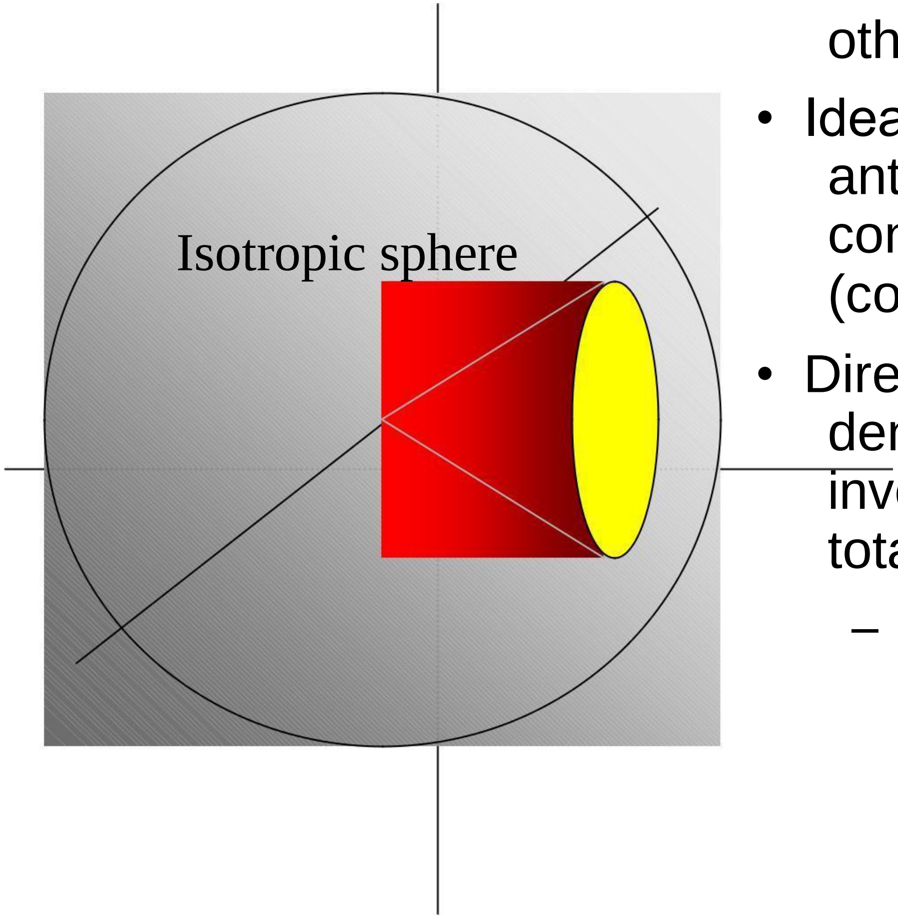
- ***Half-power beamwidth*** (HPBW) is the angle between two vectors from the pattern‘s origin to the points of the major lobe where the radiation intensity is half its maximum
  - Often used to describe the antenna resolution properties
    - » Important in radar technology, radioastronomy, etc.
- ***First-null beamwidth*** (FNBW) is the angle between two vectors, originating at the pattern‘s origin and tangent to the main beam at its base.
    - » Often FNBW ≈ 2*HPBW

Example



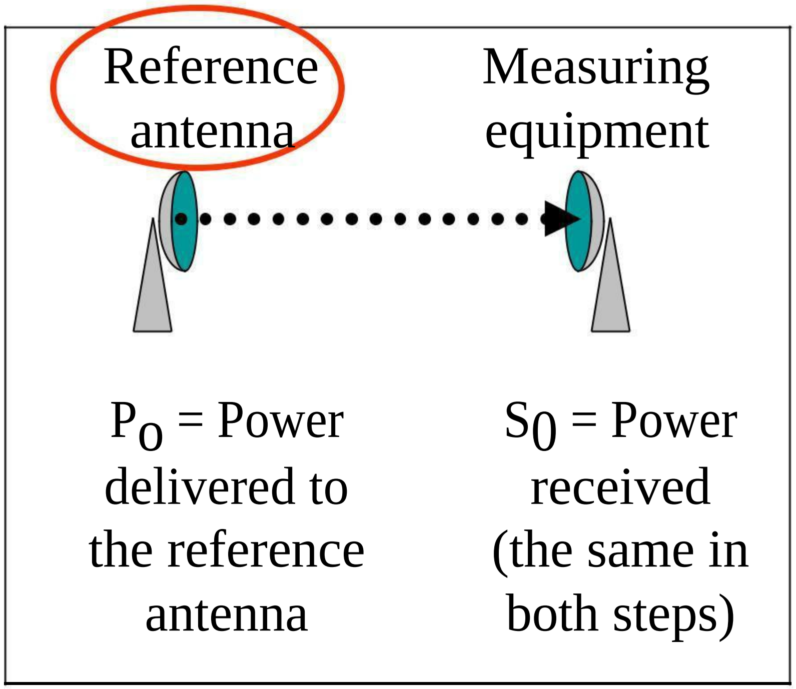
Source: NK Nikolova

Anisotropic sources: gain

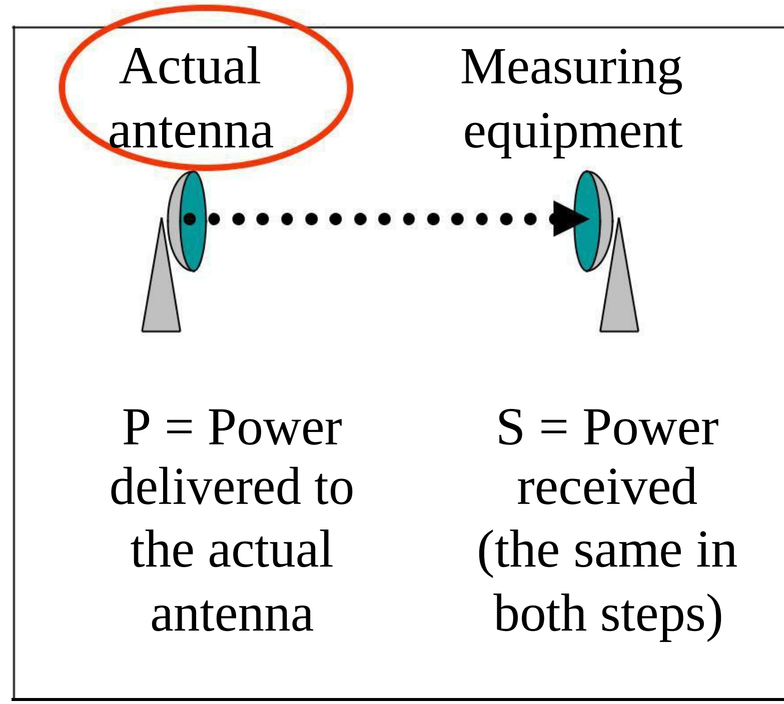


- Every real antenna radiates more energy in some directions than in others (i.e. has directional properties)
- Idealized example of directional antenna: the radiated energy is concentrated in the yellow region (cone).
- Directive antenna gain: the power flux density is increased by (roughly) the inverse ratio of the yellow area and the total surface of the isotropic sphere
  - Gain in the field intensity may also be considered - it is equal to the square root of the power gain.

Antenna gain measurement



Step 1: reference



Step 2: substitution

Antenna Gain = $(P/P_0)_{S=S0}$

Antenna Gains G_i, G_d

- Unless otherwise specified, the gain refers to the direction of maximum radiation.
- Gain is a dimension-less factor related to power and usually expressed in decibels
- G_i —Isotropic Power Gain‖ – theoretical concept, the reference antenna is isotropic
- G_d - the reference antenna is a half-wave dipole

Typical Gain and Beamwidth

Type of antenna	G_i [dB]	BeamW.
Isotropic	0	360^{0} x360^{0}
Half-wave Dipole	2	360^{0} x120^{0}
Helix (10 turn)	14	35^{0} x35^{0}
Small dish	16	30^{0} x30^{0}
Large dish	45	1^{0} x1^{0}

Antenna gain and effective area

- Measure of the effective absorption area presented by an antenna to an incident plane wave.
- Depends on the antenna gain and wavelength

$$A_e = \eta \frac{\lambda^2}{4\pi} G(\theta, \varphi) \ [\mathrm{m}^2]$$

Aperture efficiency: $\eta_a = A_e / A$

A: physical area of antenna's aperture, square meters

Power Transfer in Free Space

$$P_R = PFD \cdot A_e$$

$$= \left(\frac{G_T P_T}{4\pi r^2}\right)\left(\frac{\lambda^2 G_R}{4\pi}\right)$$

$$= P_T G_T G_R \left(\frac{\lambda}{4\pi r}\right)^2$$

- λ: wavelength [m]
- P_R: power available at the receiving antenna
- P_T: power delivered to the transmitting antenna
- G_R: gain of the transmitting antenna in the direction of the receiving antenna
- G_T: gain of the receiving antenna in the direction of the transmitting antenna
- Matched polarizations

e.i.r.p.

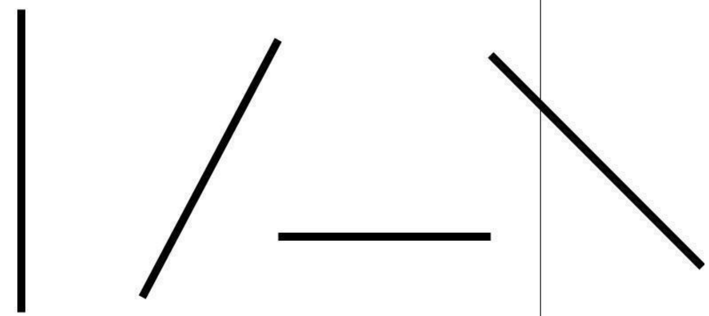
- Equivalent Isotropically Radiated Power (in a given direction):

$$e.i.r.p. = PG_i$$

- The product of the power supplied to the antenna and the antenna gain (relative to an isotropic antenna) in a given direction

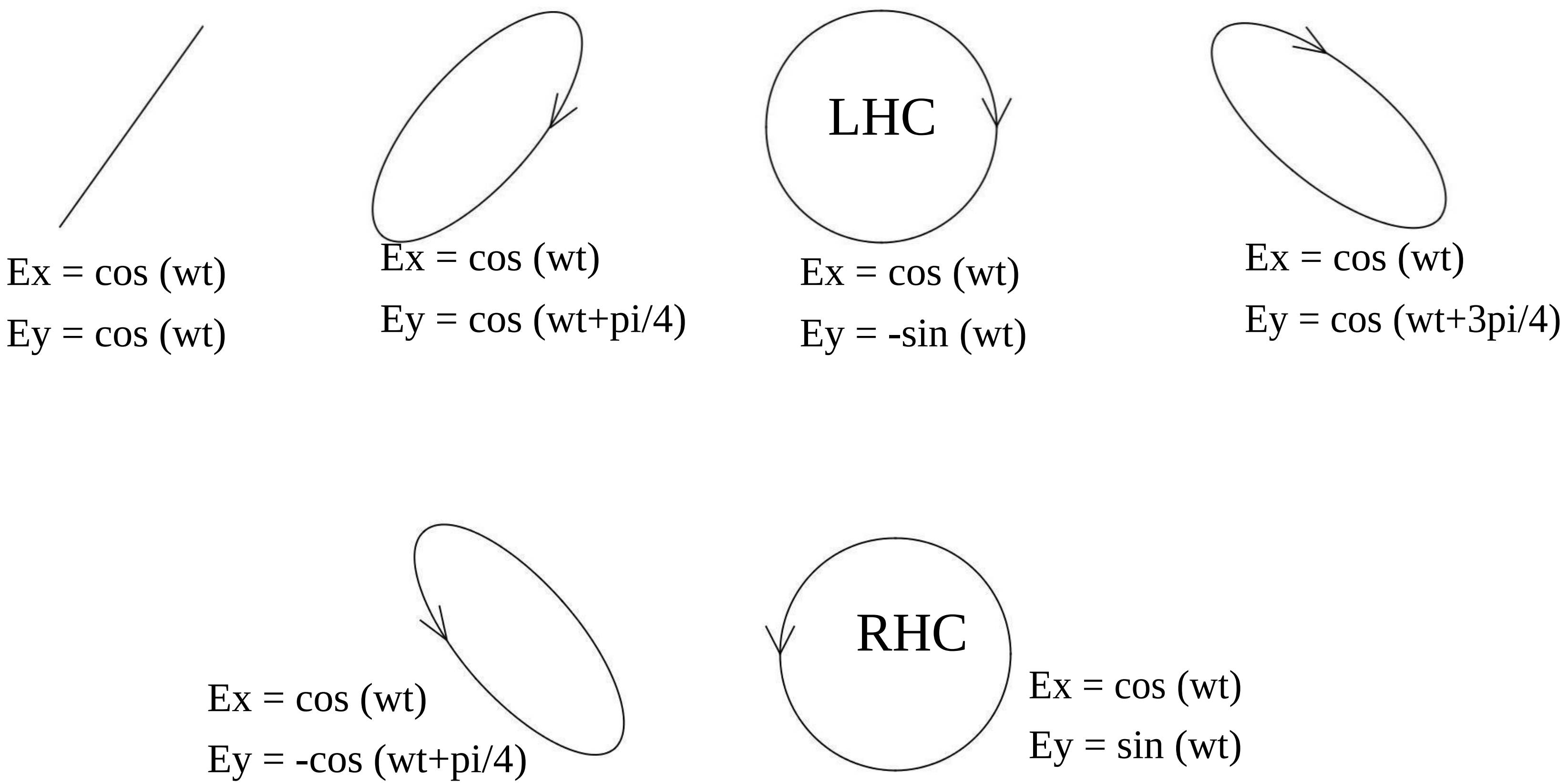
Linear Polarization

- In a linearly polarized plane wave the direction of the E (or H) vector is constant.

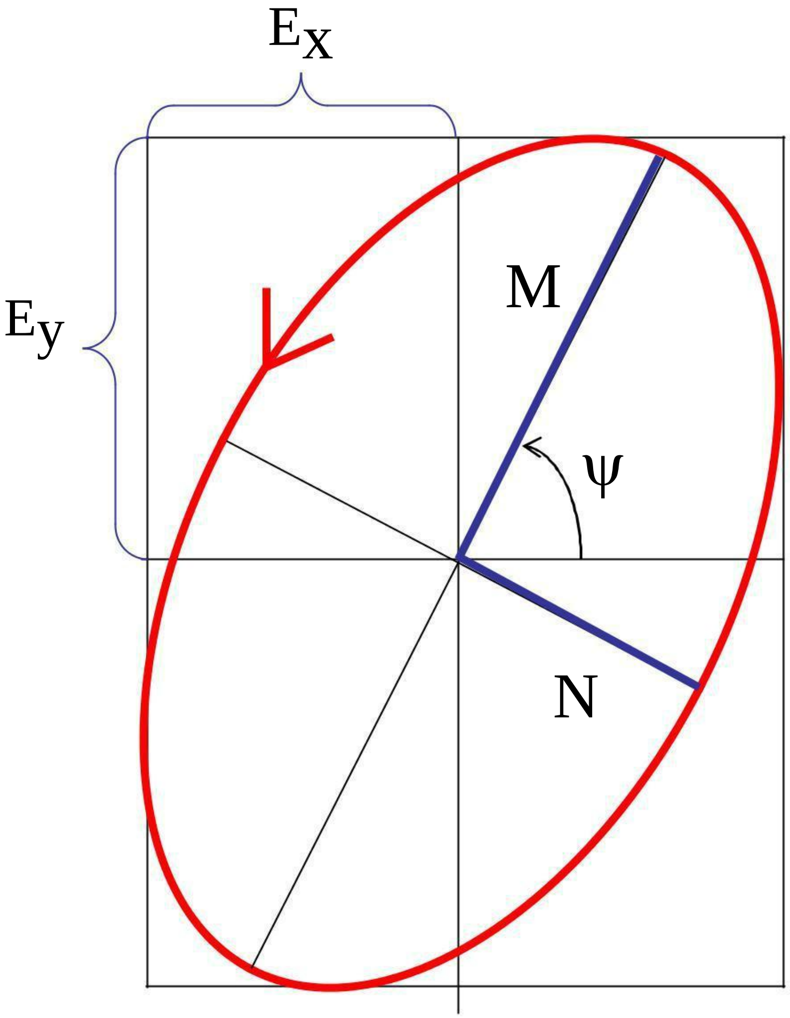


- http://www.amanogawa.com/archive/wavesA.html

Elliptical Polarization



Polarization ellipse



- The superposition of two plane-wave components results in an elliptically polarized wave
- The polarization ellipse is defined by its axial ratio N/M (ellipticity), tilt angle ψ and sense of rotation

Polarization states

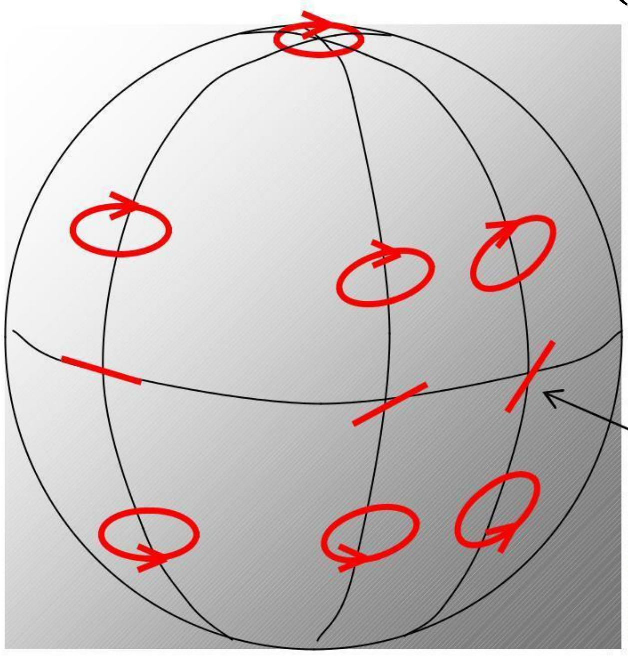
LHC

(Poincaré sphere)

UPPER HEMISPHERE:
ELLIPTIC POLARIZATION
LEFT_HANDED SENSE

EQUATOR:
LINEAR POLARIZATION

LOWER HEMISPHERE:
ELLIPTIC POLARIZATION
RIGHT_HANDED SENSE



LATTITUDE:
REPRESENTS
AXIAL RATIO

45^{0} LINEAR

LONGITUDE:
REPRESENTS
TILT ANGLE

RHC

POLES REPRESENT
CIRCULAR POLARIZATIONS

Comments on Polarization

- At any moment in a chosen reference point in space, there is actually a single electric vector E (and associated magnetic vector H).
- This is the result of superposition (addition) of the instantaneous fields E (and H) produced by all radiation sources active at the moment.
- The separation of fields by their wavelength, polarization, or direction is the result of ‗filtration'.

Antenna Polarization

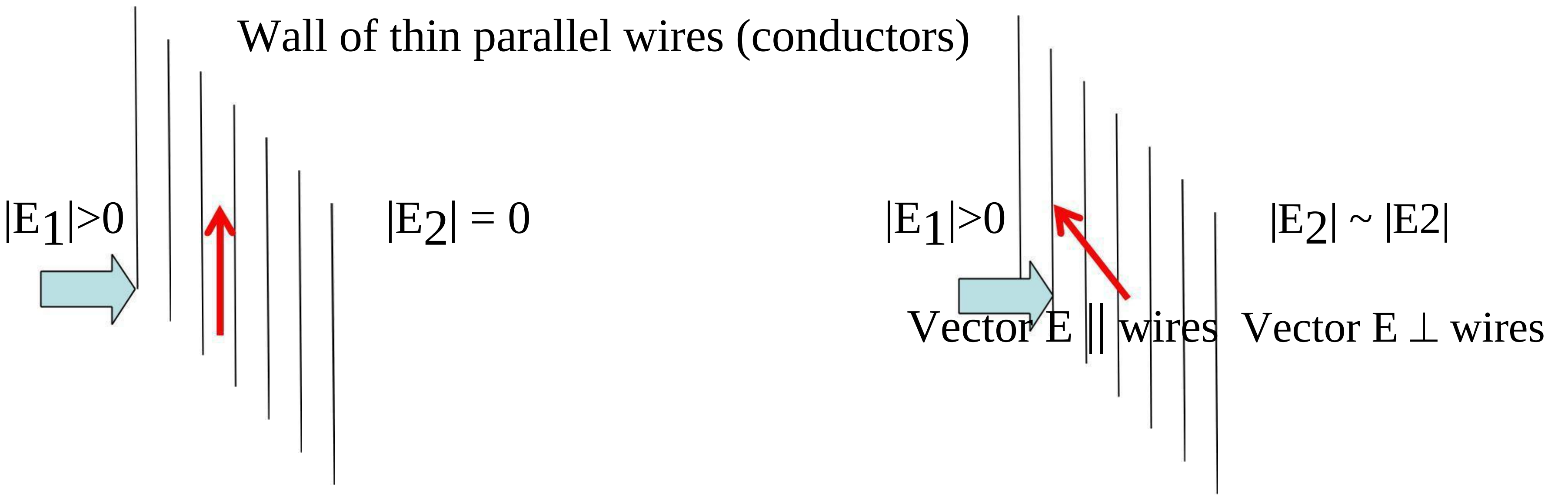
- The polarization of an antenna in a specific direction is defined to be the polarization of the wave produced by the antenna at a great distance at this direction

Polarization Efficiency

- The power received by an antenna from a particular direction is maximal if the polarization of the incident wave and the polarization of the antenna in the wave arrival direction have:
  - the same axial ratio
  - the same sense of polarization
  - the same spatial orientation

.

Polarization filters/ reflectors



Wire distance ~ 0.1λ

- At the surface of ideal conductor the tangential electrical field component = 0

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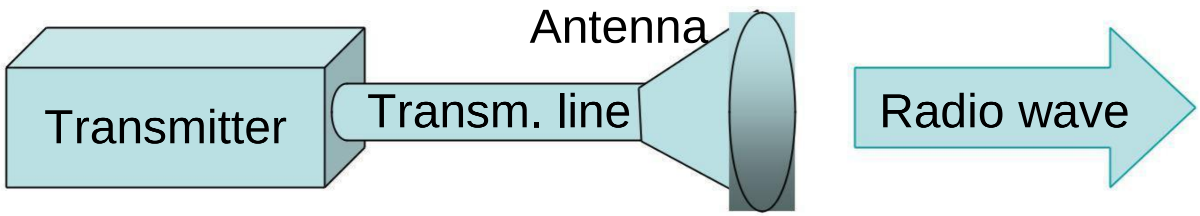
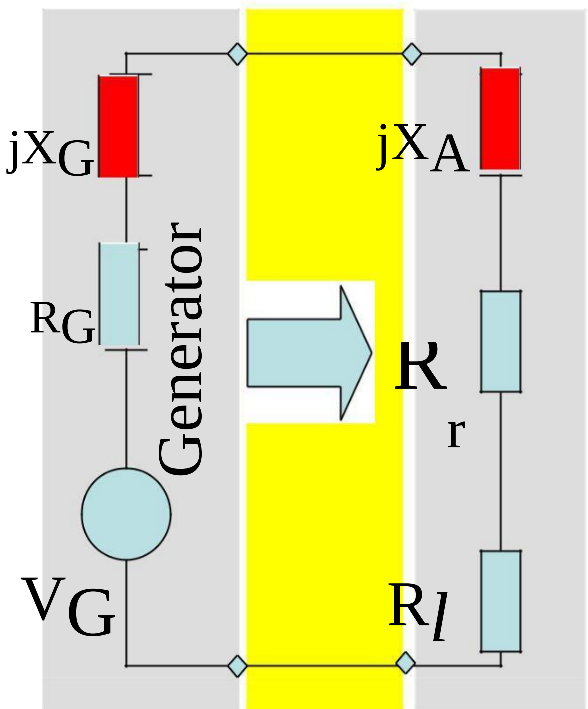
Transmitting antenna equivalent circuit





The transmitter with the transmission line is represented by an (Thevenin) equivalent generator

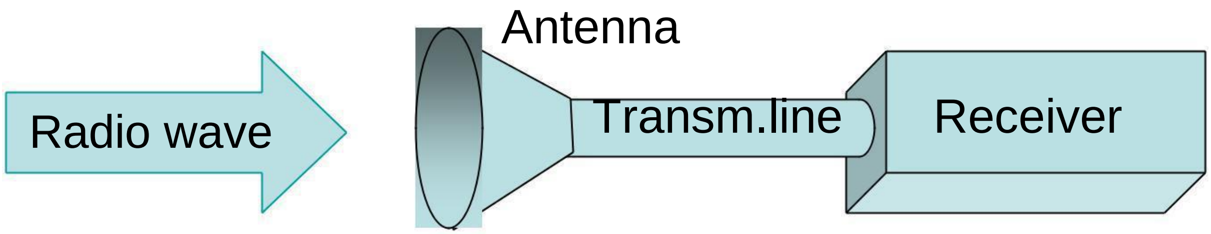
The antenna is represented by its input impedance (which is frequency-dependent and is influenced by objects nearby) as seem from the generator

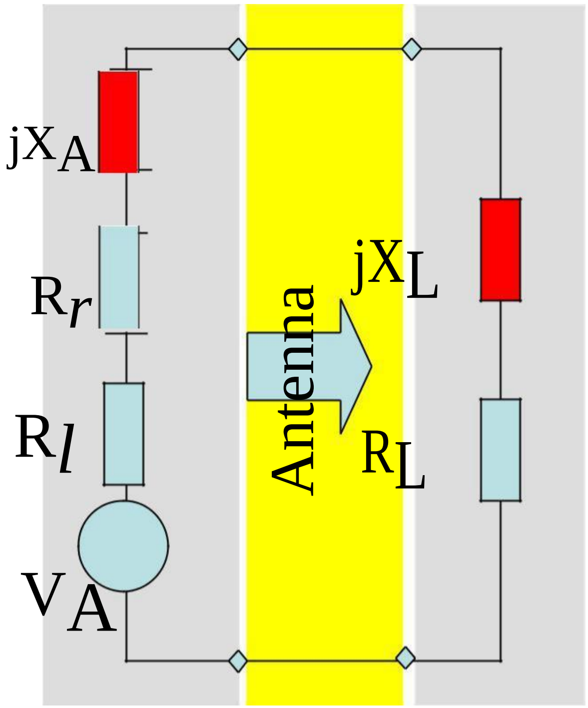
jX_A represents energy stored in electric (E_e) and magnetic (E_m) near-field components; if $|Ee| = |E_m|$ then $X_A = 0$ (antenna resonance)

R_r represents energy radiated into space (far-field components)

R_l represents energy lost, i.e. transformed into heat in the antenna structure

Receiving antenna equivalent circuit





Thevenin equivalent

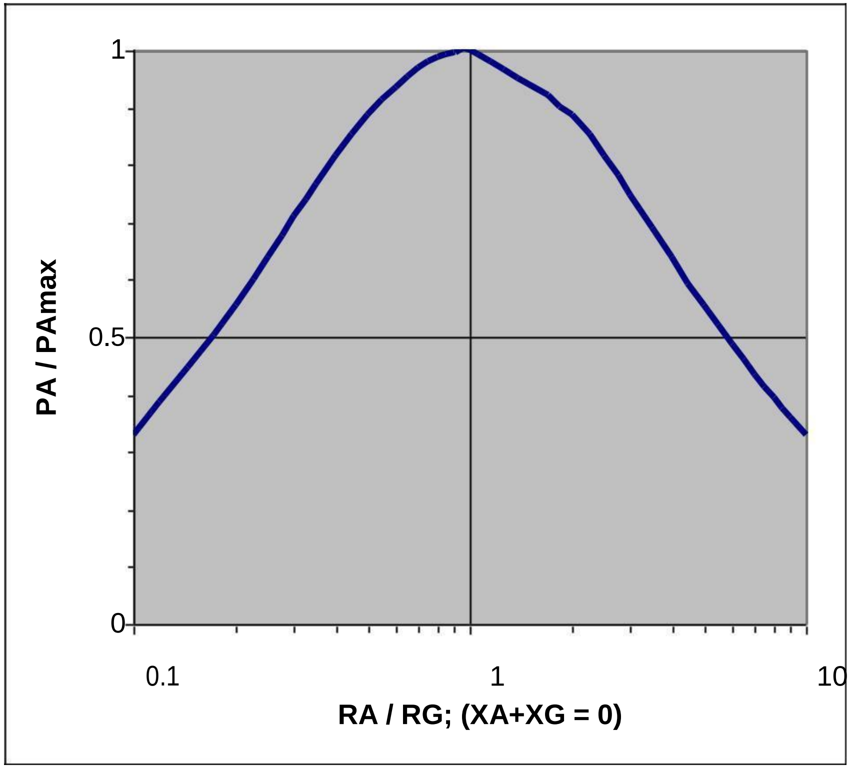
The antenna with the transmission line is represented by an (Thevenin) equivalent generator

The receiver is represented by its input impedance as seen from the antenna terminals (i.e. transformed by the transmission line)

V_A is the (induced by the incident wave) voltage at the antenna terminals determined when the antenna is open circuited

Note: The antenna impedance is the same when the antenna is used to radiate and when it is used to receive energy

Power transfer



- The maximum power is delivered to (or from) the antenna when the antenna impedance and the impedance of the equivalent generator (or load)

  are matched

- When the impedances are matched
  - Half of the source power is delivered to the load and half is dissipated within the (equivalent) generator as heat
  - In the case of receiving antenna, a part (P_l) of the power captured is lost as heat in the antenna elements, , the other part being reradiated (scattered) back into space
    - Even when the antenna losses tend to zero, still only half of the power captured is delivered to the load (in the case of conjugate matching), the other half being scattered back into space

- When the antenna impedance is not matched to the transmitter output impedance (or to the receiver input impedance) or to the transmission line between them, impedance-matching devices must be used for maximum power transfer

- Inexpensive impedance-matching devices are usually narrow-band
- Transmission lines often have significant losses

Radiation efficiency

- The radiation efficiency *e* indicates how efficiently the antenna uses the RF power
- It is the ratio of the power radiated by the antenna and the total power delivered to the antenna terminals (in transmitting mode). In terms of equivalent circuit parameters:

$$e = \frac{R_r}{R_r + R_l}$$

Outline

- Introduction
- Review of basic antenna types
- Radiation pattern, gain, polarization
- Equivalent circuit & radiation efficiency
- Smart antennas
- Some theory
- Summary

Antenna arrays

- Consist of multiple (usually identical) antennas (elements) ‗collaborating' to synthesize radiation characteristics not available with a single antenna. They are able
  - to match the radiation pattern to the desired coverage area
  - to change the radiation pattern electronically (electronic scanning) through the control of the phase and the amplitude of the signal fed to each element
  - to adapt to changing signal conditions
  - to increase transmission capacity by better use of the radio resources and reducing interference
- Complex & costly
  - Intensive research related to military, space, etc. activities
    - » Smart antennas, signal-processing antennas, tracking antennas, phased arrays, etc.

Source: adapted from N Gregorieva

Satellite antennas (TV)

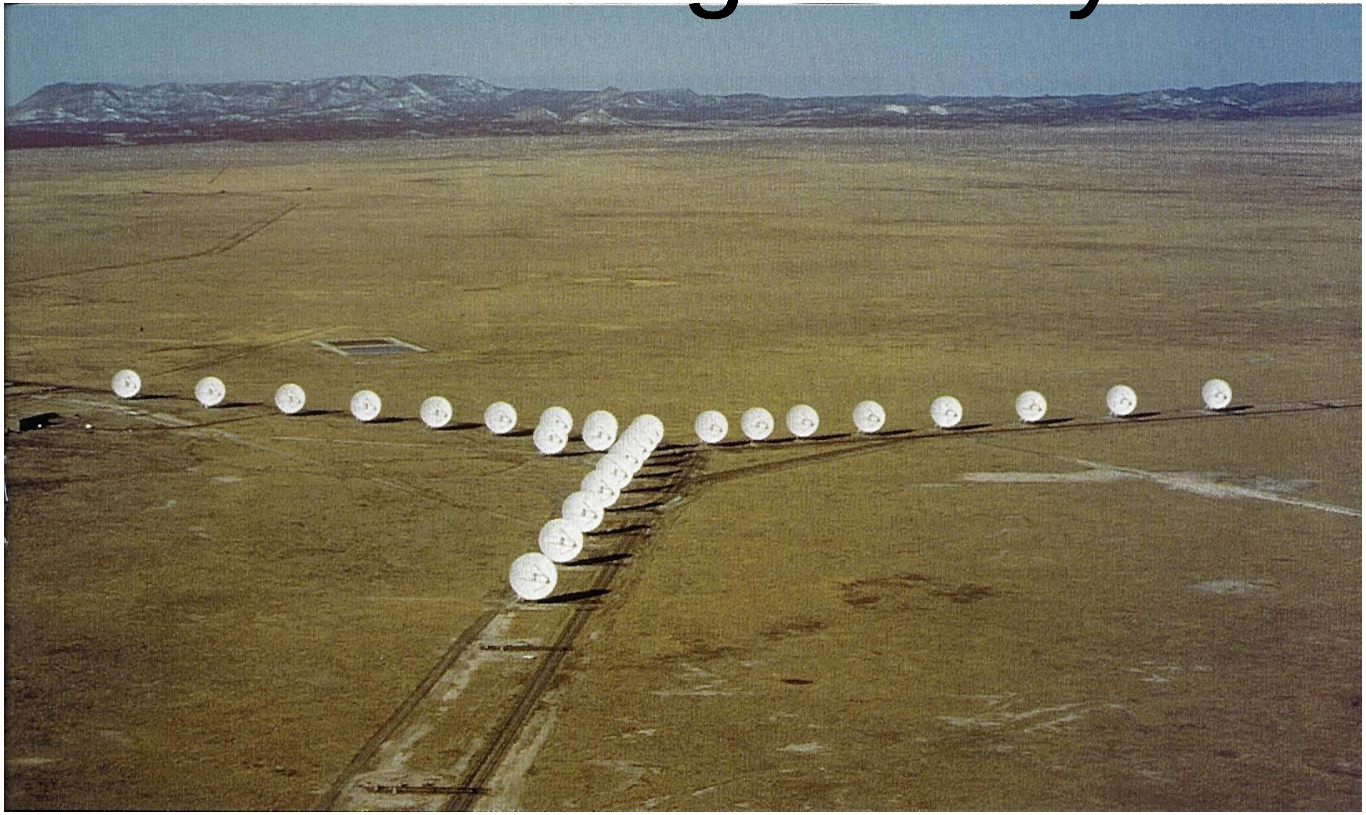
- Not an array!

Owens Valley Radio Observatory

The Earth's atmosphere is transparent in the narrow visible-light window (4000-7000 angstroms) and the radio band between 1 mm and 10 m.

[Sky & Telescope Feb 1997 p.26]

The New Mexico Very Large Array



[Sky & Telescope
Feb 1997 p. 30]

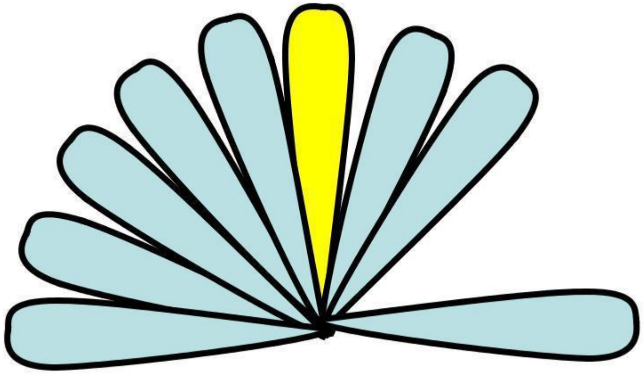
27 antennas along 3 railroad tracks provide baselines up to 35 km. Radio images are formed by correlating the signals garnered by each antenna.

2 GHz adaptive antenna

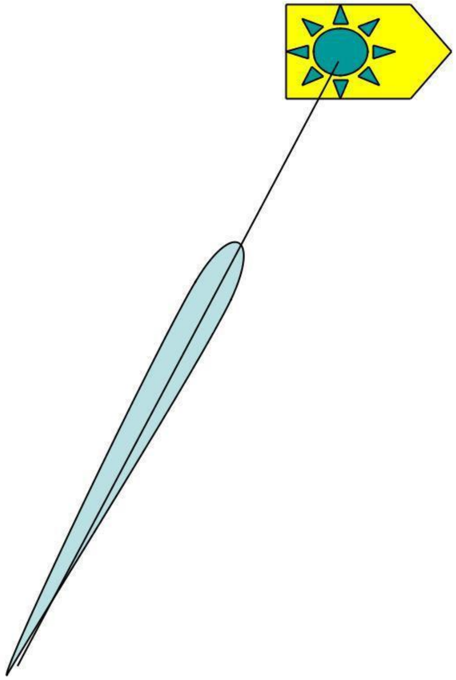
- A set of 48 2GHz antennas
  - Source: Arraycomm

Phased Arrays

- Array of N antennas in a linear or two-dimensional configuration + beam-forming & control device
- The amplitude and phase excitation of each individual antenna controlled electronically (—software-defined‖)
  - – Diode phase shifters
  - – Ferrite phase shifters
- Inertia-less beam-forming and scanning (μsec) with fixed physical structure

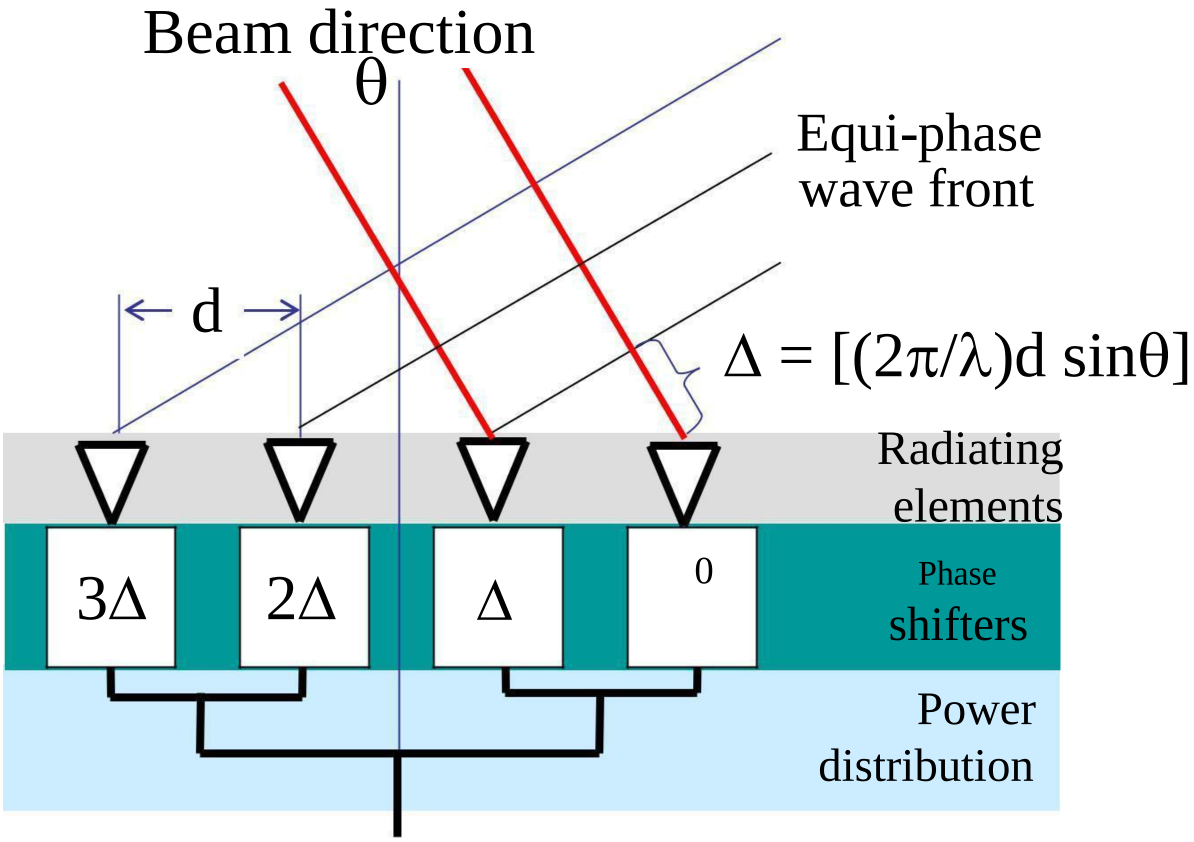


- *Switched beam antennas*
  - Based on switching function between separate directive antennas or predefined beams of an array
- *Space Division Multiple Access* (SDMA) = allocating an angle direction sector to each user
  - In a TDMA system, two users will be allocated to the same time slot and the same carrier frequency
  - They will be differentiated by different direction angles



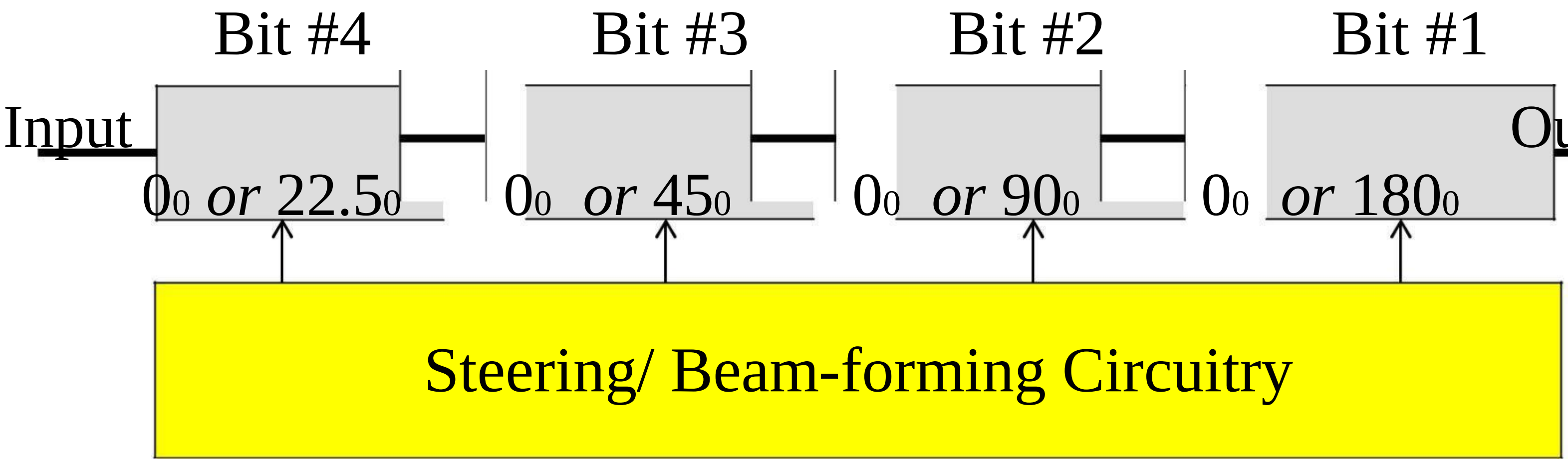
- *Dynamically phased array (PA)*:
  - A generalization of the switched lobe concept
  - The radiation pattern continuously track the designated signal (user)
  - Include a *direction of arrival* (DoA) tracking algorithm

Beam Steering



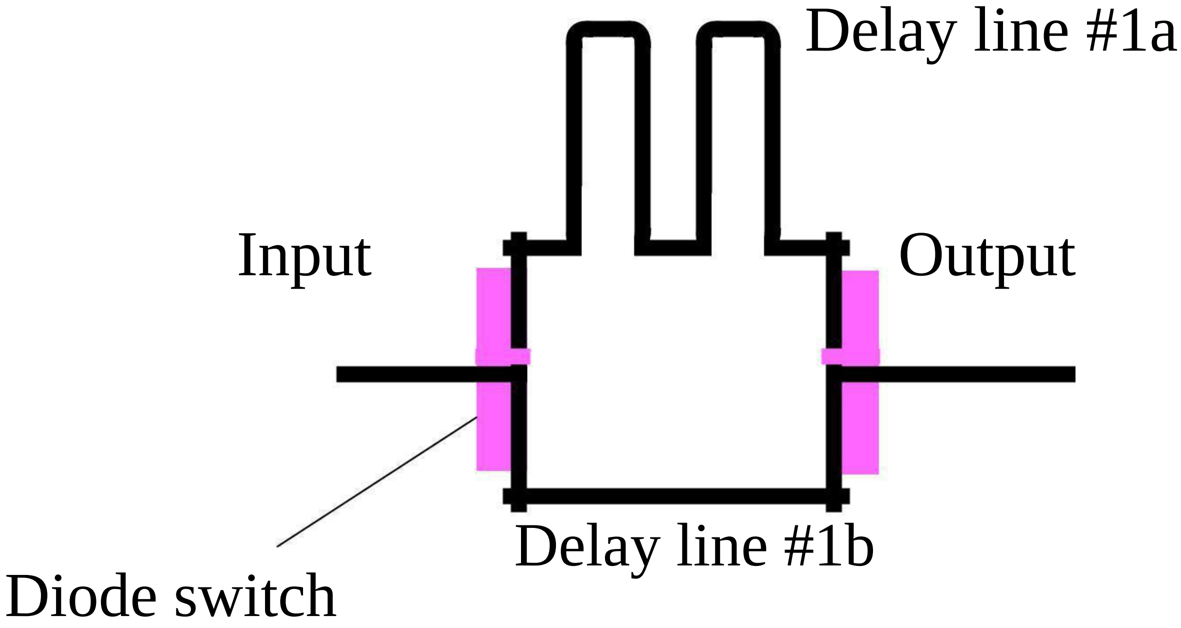
- Beam-steering using phase shifters at each radiating element

4-Bit Phase-Shifter (Example)



Alternative solution: Transmission line with controlled delay

Switched-Line Phase Bit

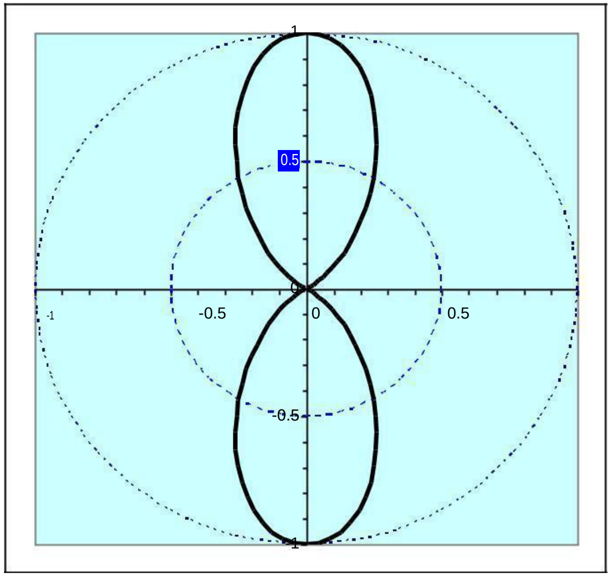


Phase bit = delay difference

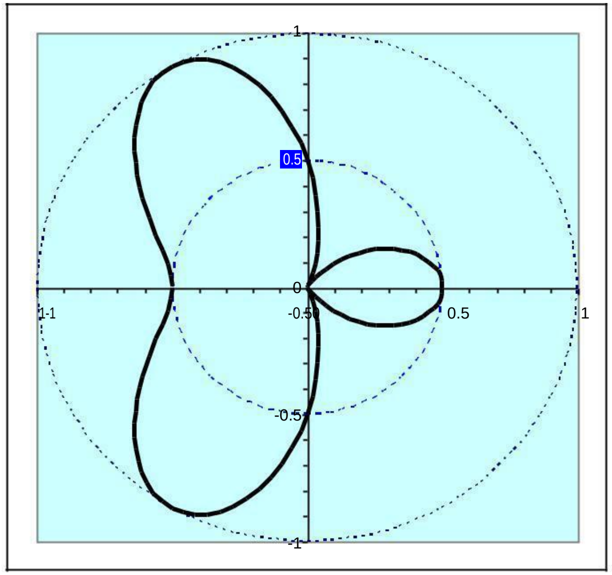
Simulation

- 2 omnidirectional antennas (equal amplitudes)
  - Variables
    - distance increment
    - phase increment
- N omnidirectional antennas
  - Group factor (N omnidirectional antennas uniformly distributed along a straight line, equal amplitudes, equal phase increment)
- http://www.amanogawa.com/archive/TwoDipole/Antenna2-2.html (more details)

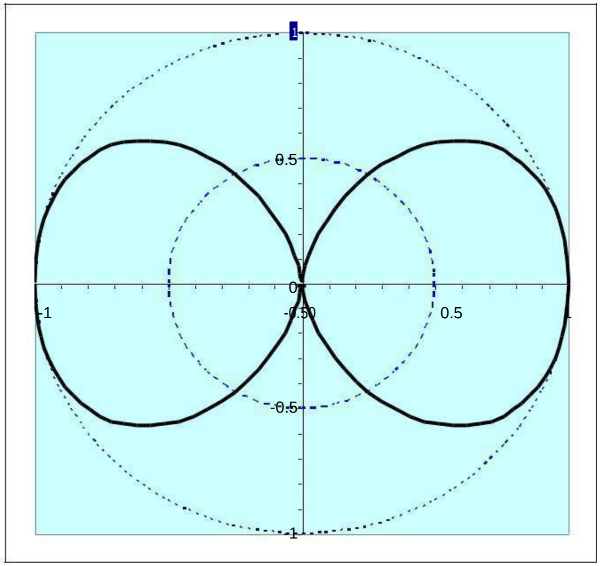
2 omnidirectional antennas



$D = 0.5\lambda, \theta = 0^{0}$

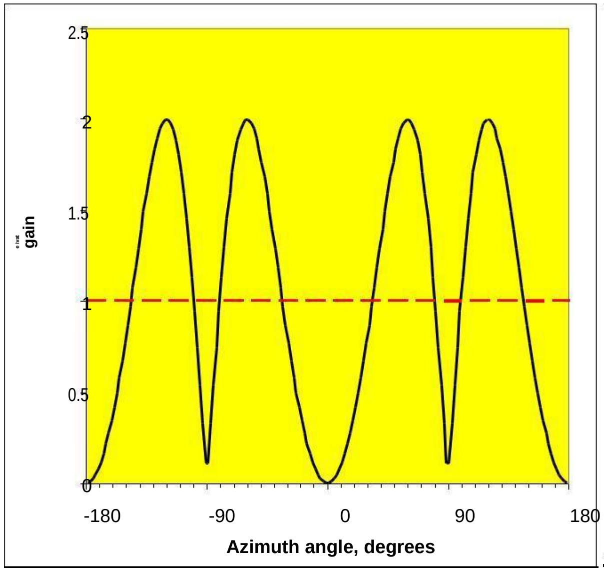


$D = 0.5\lambda, \theta = 90^{0}$

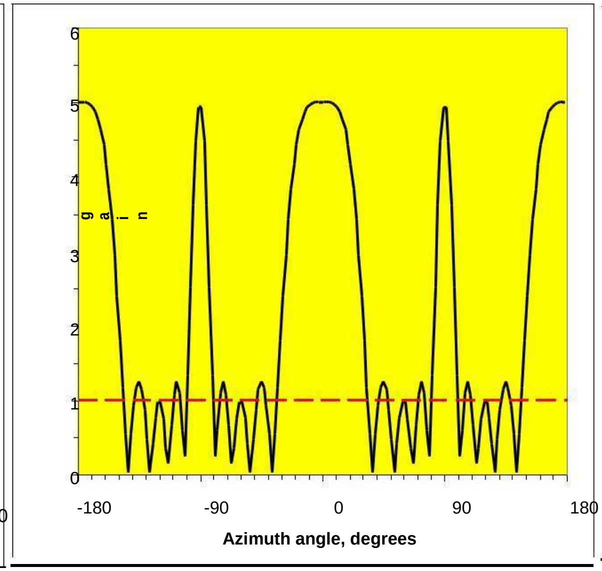


$D = 0.5\lambda, \theta = 180^{0}$

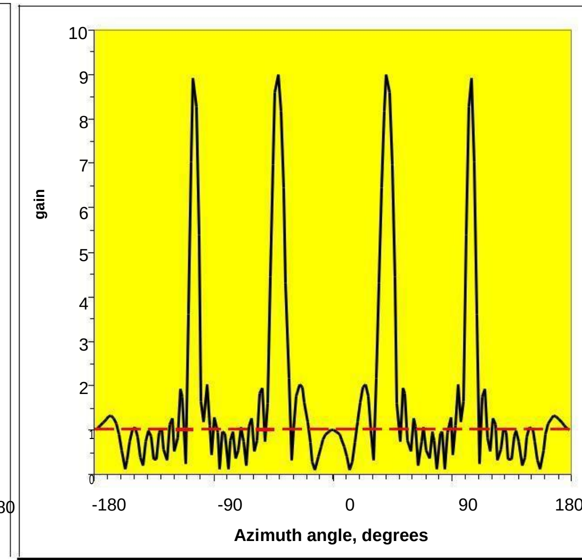
N omnidirectional antennas



$N = 2, \theta = 90^0$



$N = 5, \theta = 180^0$



$N = 9, \theta = 45^0$

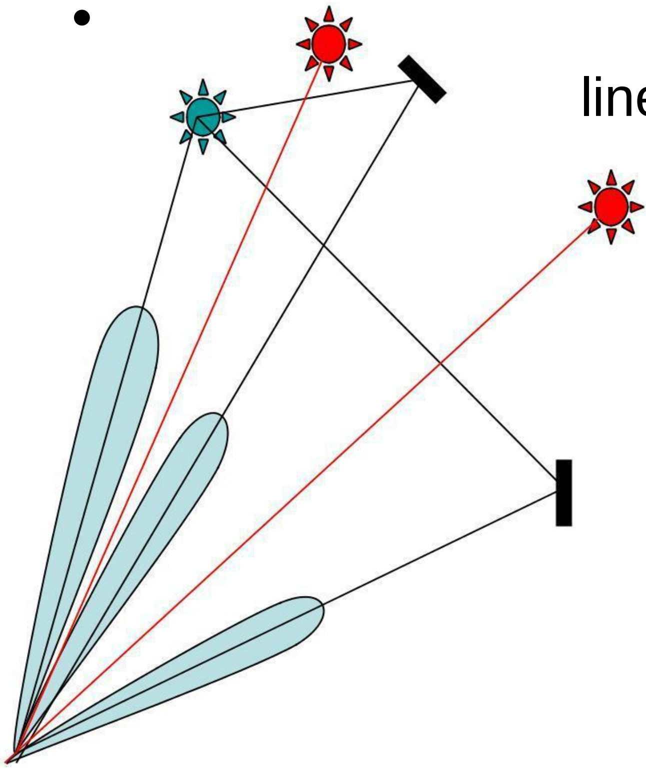
- Array gain (line, uniform, identical power)

Antenna Arrays: Benefits

- Possibilities to control electronically
  - Direction of maximum radiation
  - Directions (positions) of nulls
  - Beam-width
  - Directivity
  - Levels of sidelobes

  using standard antennas (or antenna collections) independently of their radiation patterns
- Antenna elements can be distributed along straight lines, arcs, squares, circles, etc.

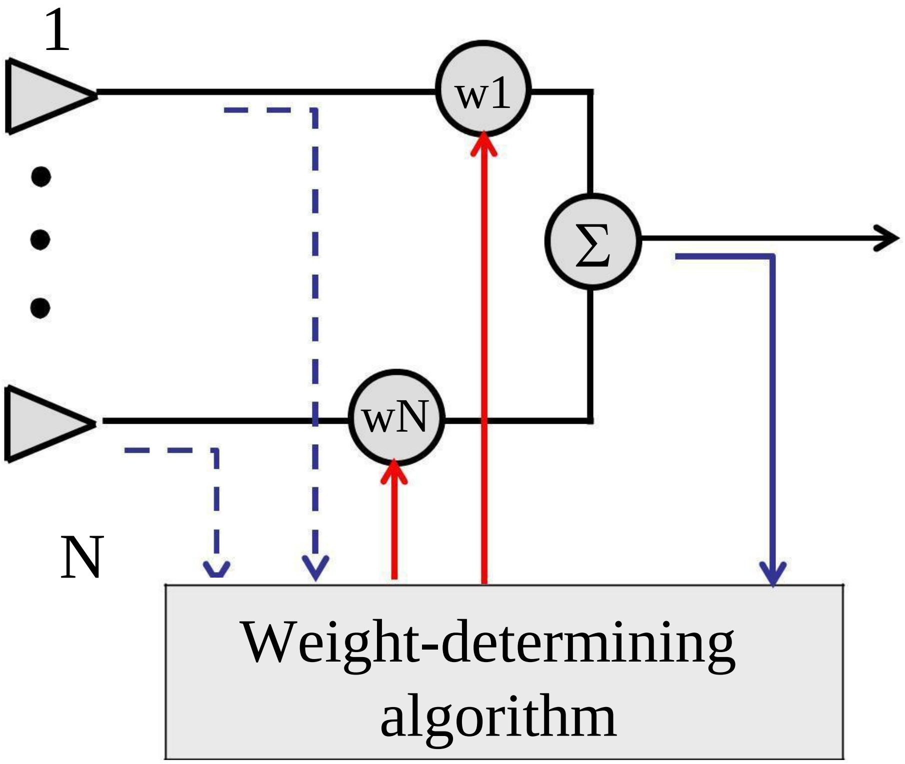
Adaptive (—Intelligentll)Antennas

- 

Array of N antennas in a linear,circular, or planar configuration

- Used for selection signals from desired sources and suppress incident signals from undesired sources
- The antenna pattern track the sources
- It is then adjusted to null out the interferers and to maximize the signal to interference ratio (SIR)
- Able to receive and combine constructively multipath signals

- The amplitude/ phase excitation of each antenna controlled electronically (―software-defined‖)
- The weight-determining algorithm uses a-priori and/ or measured information to adapt antenna to changing environment
- The weight and summing circuits can operate at the RF and/ or at an intermediate frequency



Antenna sitting

- Radio horizon
- Effects of obstacles & structures nearby
- Safety
  - – operating procedures
  - – Grounding
    - lightning strikes
    - static charges
  - – Surge protection
    - lightning searches for a second path to ground

Outline

- Introduction
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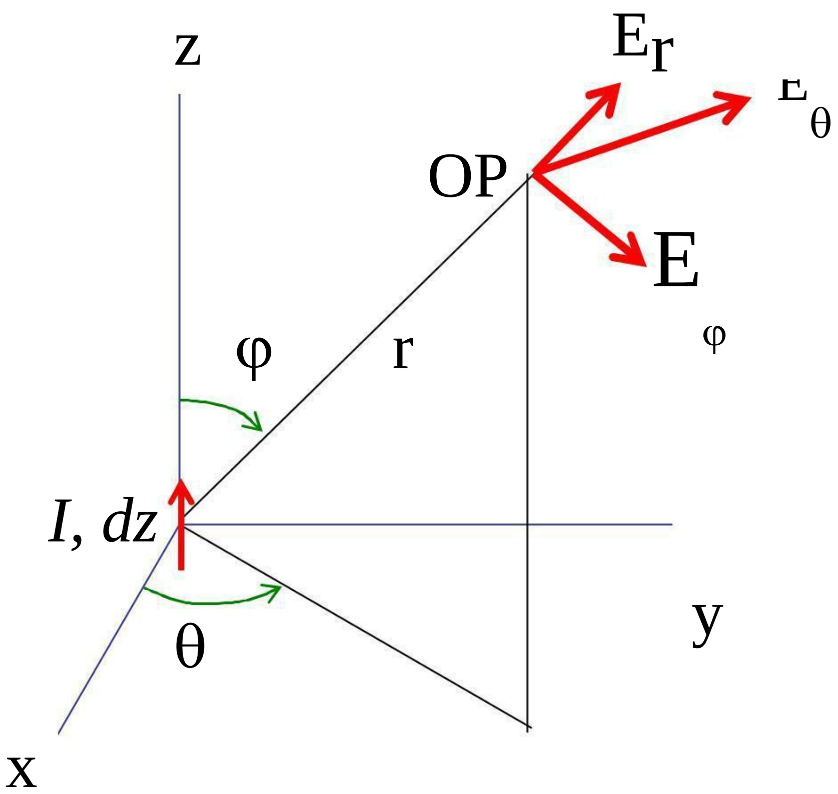
Maxwell‘s Equations

- EM field interacting with the matter
  - 2 coupled vectors E and H (6 numbers!), varying with time and space and satisfying the boundary conditions (see http://www.amanogawa.com/archive/docs/EM1.pdf; http://www.amanogawa.com/archive/docs/EM7.pdf; http://www.amanogawa.com/archive/docs/EM5.pdf)
- Reciprocity Theorem
  - Antenna characteristics do not depend on the direction of energy flow. The impedance & radiation pattern are the same when the antenna radiates signal and when it receives it.
  - Note: This theorem is valid only for linear passive antennas (i.e. antennas that do not contain nonlinear and unilateral elements, e.g. amplifiers)

EM Field of Current Element



$$E = E_r + E_\vartheta + E_\varphi$$

$$H = H_r + H_\vartheta + H_\varphi$$

$$|E| = \sqrt{|E_r|^2 + |E_\vartheta|^2 + |E_\varphi|^2}$$

$$|H| = \sqrt{|H_r|^2 + |H_\vartheta|^2 + |H_\varphi|^2}$$

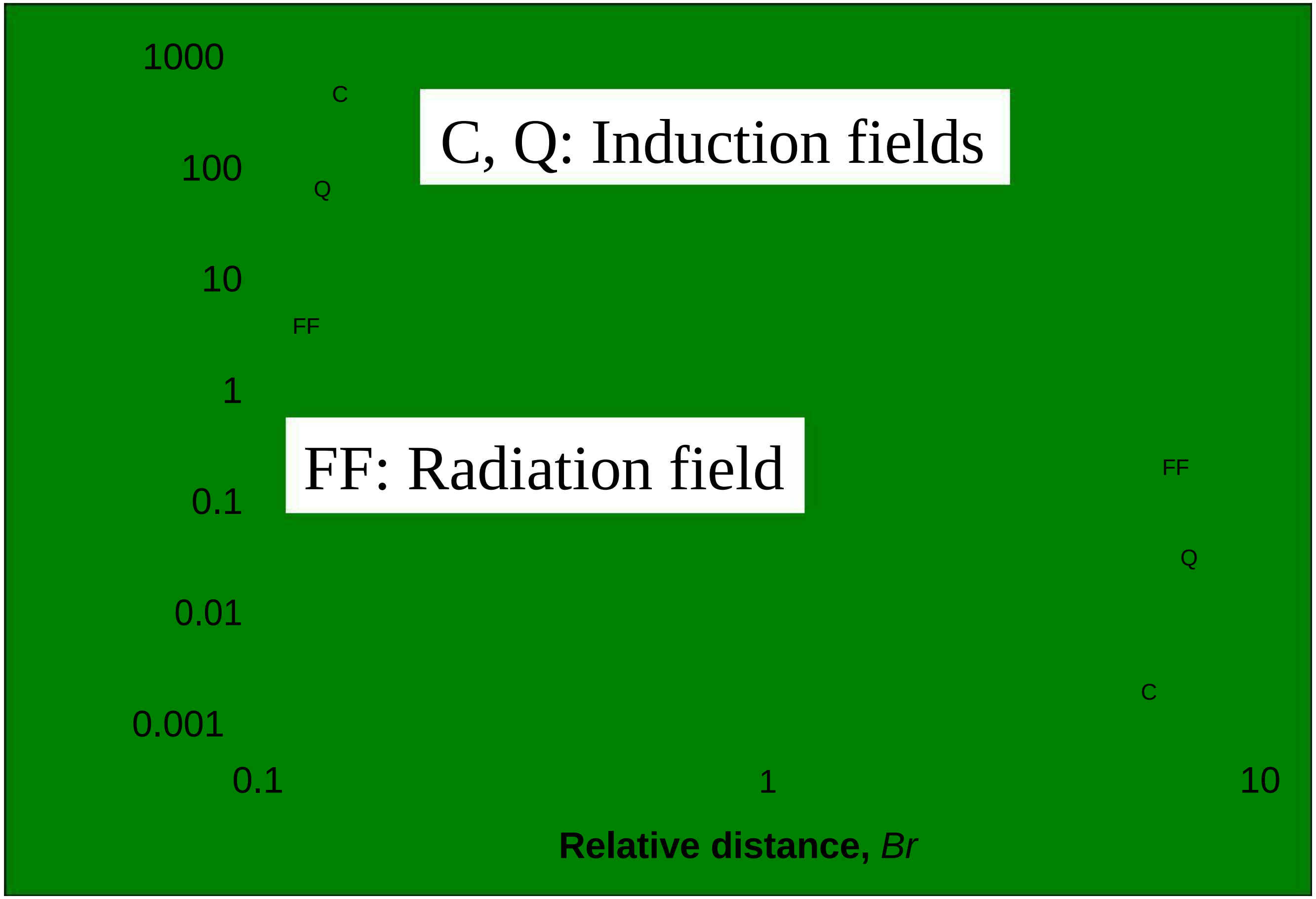
I: current (monochromatic) [A]; *dz*: antenna element (short) [m]

Short dipole antenna: summary

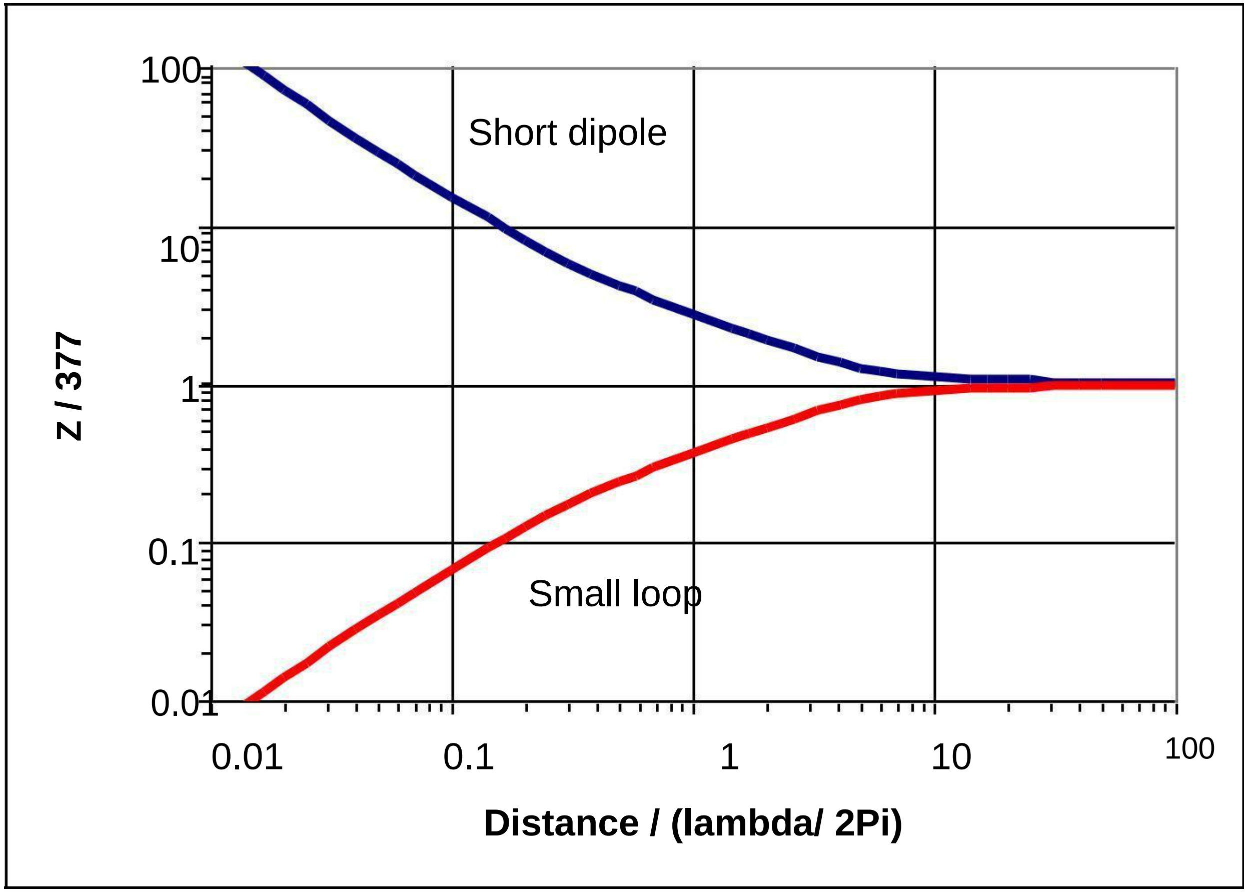
- E_θ & H_θ are maximal in the equatorial plane, zero along the antenna axis
- E_r is maximal along the antenna axis *dz, z*ero in the equatorial plane
- All show axial symmetry
- All are proportional to the current moment *Idz*
- Have 3 components that decrease with the distance-to-wavelength ratio as
  - $(r/\lambda)^{-2}$ & $(r/\lambda)^{-3}$: near-field, or induction field. The energy oscillates from entirely electric to entirely magnetic and back, twice per cycle. Modeled as a resonant LC circuit or resonator;
  - $(r/\lambda)^{-1}$: far-field or radiation field
  - These 3 component are all equal at $(r/\lambda) = 1/(2\pi)$

β

Field components



Field impedance



Field impedance

Z = E/H depends

on the antenna type and

on distance

Far-Field, Near-Field

- Near-field region:
  - Angular distribution of energy depends on distance from the antenna;
  - Reactive field components dominate (L, C)
- Far-field region:
  - Angular distribution of energy is independent on distance;
  - Radiating field component dominates (R)
  - The resultant EM field can locally be treated as uniform (TEM)

Poynting vector

- The time-rate of EM energy flow per unit area in free space is the *Poynting vector*

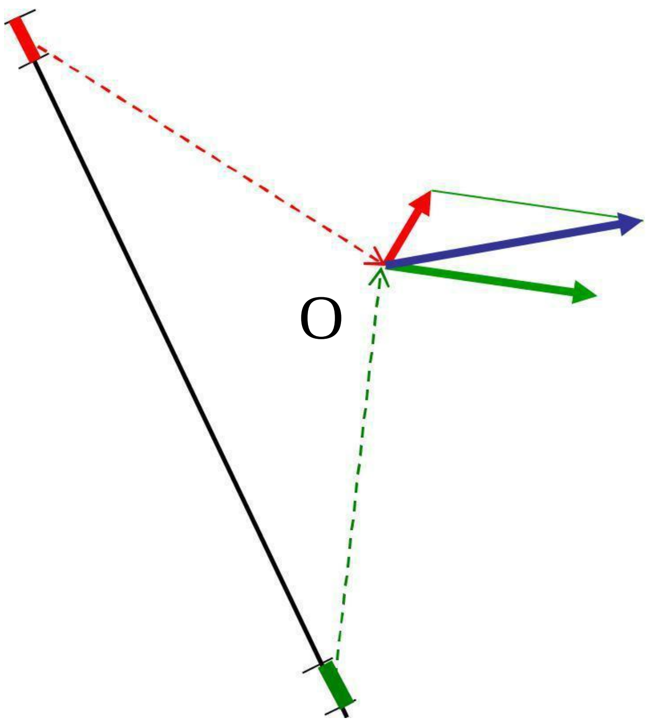
  (see http://www.amanogawa.com/archive/docs/EM8.pdf).
- It is the cross-product (vector product, right-hand screw direction) of the electric field vector (E) and the magnetic field vector (H): P = E x H.
- For the elementary dipole $E_\theta \perp H_\theta$ and only

  $E_\theta \times H_\theta$ carry energy into space with the speed of light.

Power Flow

- In free space and at large distances, the radiated energy streams from the antenna in radial lines, i.e. the Poynting vector has only the radial component in spherical coordinates.
- A source that radiates uniformly in all directions is an *isotropic source (radiator, antenna)*.
  For such a source the radial component of the Poynting vector is independent of θ and φ.

Linear Antennas



- Summation of all vector components E (or H) produced by each antenna element

$$E = E_1 + E_2 + E_3 + ...$$

$$H = H_1 + H_2 + H_3 + ...$$

- In the far-field region, the vector components are parallel to each other
- Phase difference due to
  - Excitation phase difference
  - Path distance difference
- Method of moments

Simulation: Linear dipole antenna

- http://www.amanogawa.com/archive/DipoleAnt/DipoleAnt-2.html
  - Linear dipole antenna
- http://www.amanogawa.com/archive/Anten na1/Antenna1-2.html
  - Detailed analysis

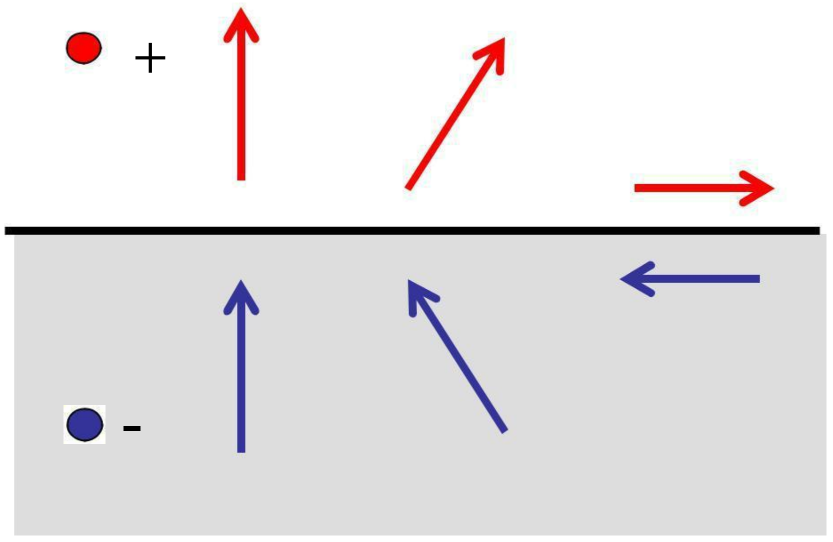
Point Source

- For many purposes, it is sufficient to know the direction (angle) variation of the power radiated by antenna at large distances.
- For that purpose, any practical antenna, regardless of its size and complexity, can be represented as a point-source.
- The actual field near the antenna is then disregarded.

- The EM field at large distances from an antenna can be treated as originated at a point source - fictitious volume-less emitter.
- The EM field in a homogenous unlimited medium at large distances from an antenna can be approximated by an uniform plane TEM wave

Image Theory

- Antenna above perfectly conducting plane surface
- Tangential electrical field component = 0
  - – vertical components: the same direction
  - – horizontal components: opposite directions
- The field (above the ground) is the same as if the ground is replaced by an mirror image of the antenna
- http://www.amanogawa.com/archive/wavesA.html



Elliptical polarization:
change of the rotation sense!

Summary

- Introduction
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Selected References

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  - Pozar D. —*Antenna Design Using Personal Computers*‖
  - NEC Archives www.gsl.net/wb6tpu /swindex.html ()

Any questions?

Thank you for your attention

HISTORY

- The first antennas were built in 1888 by German physicist Heinrich Hertz in his pioneering experiments to prove the existence of electromagnetic waves predicted by the theory of James Clerk Maxwell.

- Hertz placed dipole antennas at the focal point of parabolic reflectors for both transmitting and receiving. He published his work in Annalen der Physik und Chemie (vol. 36, 1889).

INTRODUCTION

- An antenna is an electrical device which converts electric currents into radio waves, and vice versa. It is usually used with a radio transmitter or radio receiver.
- In transmission, a radio transmitter applies an oscillating radio frequency electric current to the antenna's terminals, and the antenna radiates the energy from the current as electromagnetic waves (radio waves).

- Transmitting Antenna: Any structure designed to efficiently radiate electromagnetic radiation in a preferred direction is called a *transmitting antenna*.
- In reception, an antenna intercepts some of the power of an electromagnetic wave in order to produce a tiny voltage at its terminals, that is applied to a receiver to be amplified. An antenna can be used for both transmitting and receiving.
- Receiving Antenna: Any structure designed to efficiently receive electromagnetic radiation is called a receiving antenna

BASIC STRUCTURE

- It is a metallic conductor system capable of radiating and receiving em waves.
- Typically an antenna consists of an arrangement of metallic conductors (“elements"), electrically connected (often through a transmission line) to the receiver or transmitter.
- An oscillating current of electrons forced through the antenna by a transmitter will create an oscillating magnetic field around the antenna elements, while the charge of the electrons also creates an oscillating electric field along the elements.

- These time-varying fields radiate away from the antenna into space as a moving electromagnetic field wave.
- Conversely, during reception, the oscillating electric and magnetic fields of an incoming radio wave exert force on the electrons in the antenna elements, causing them to move back and forth, creating oscillating currents in the antenna.
- Antenna reciprocity : can be used as transmitter and receiver.In two way communication same antenna can be used as transmitter and receiver.

- Antennas may also contain reflective or directive elements or surfaces not connected to the transmitter or receiver, such as parasitic elements, parabolic reflectors or horns, which serve to direct the radio waves into a beam or other desired radiation pattern.
- Antennas can be designed to transmit or receive radio waves in all directions equally (omnidirectional antennas), or transmit them in a beam in a particular direction, and receive from that one direction only (directional or high gain antennas).

WHY ANTENNAS ?

- Need of antenna arisen when two person wanted to communicate between them when separated by some distance and wired communication is not possible.
- Antennas are required by any radio receiver or transmitter to couple its electrical connection to the electromagnetic field.
- Radio waves are electromagnetic waves which carry signals through the air (or through space) at the speed of light with almost no transmission loss.

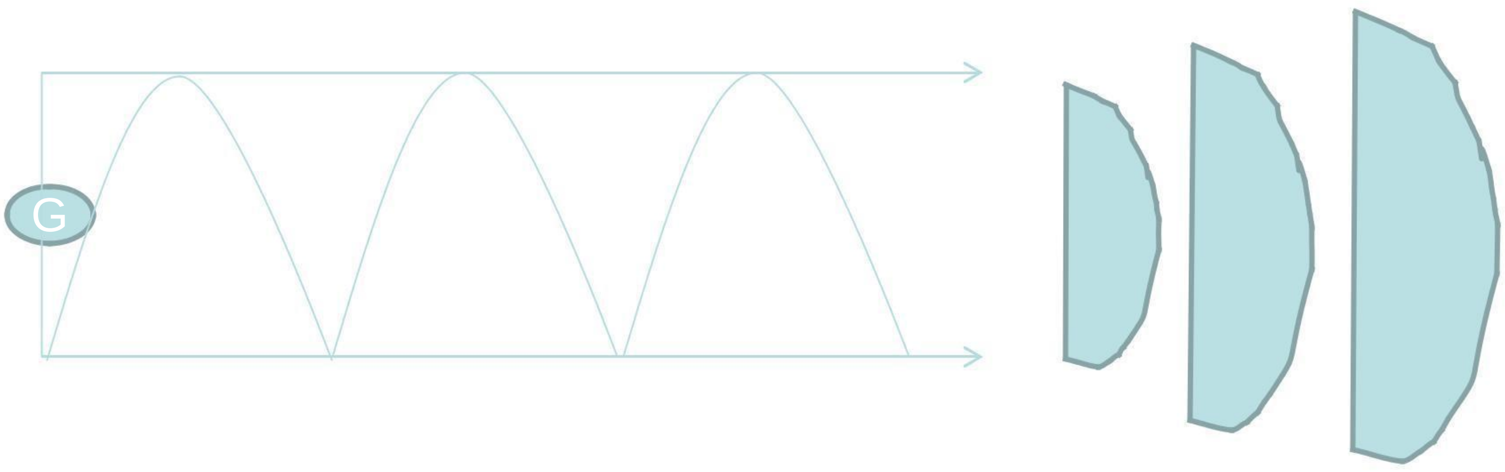
- Radio transmitters and receivers are used to convey signals (information) in systems including broadcast (audio) radio, television, mobile telephones , point-to-point communications links (telephone, data networks), satellite links.
- Radio waves are also used directly for measurements in technologies including Radar, GPS, and radio astronomy.
- In each and every case, the transmitters and receivers involved require antennas, although these are sometimes hidden (such as the antenna inside an AM radio or inside a laptop computer equipped with wi-fi).

WHERE USED?

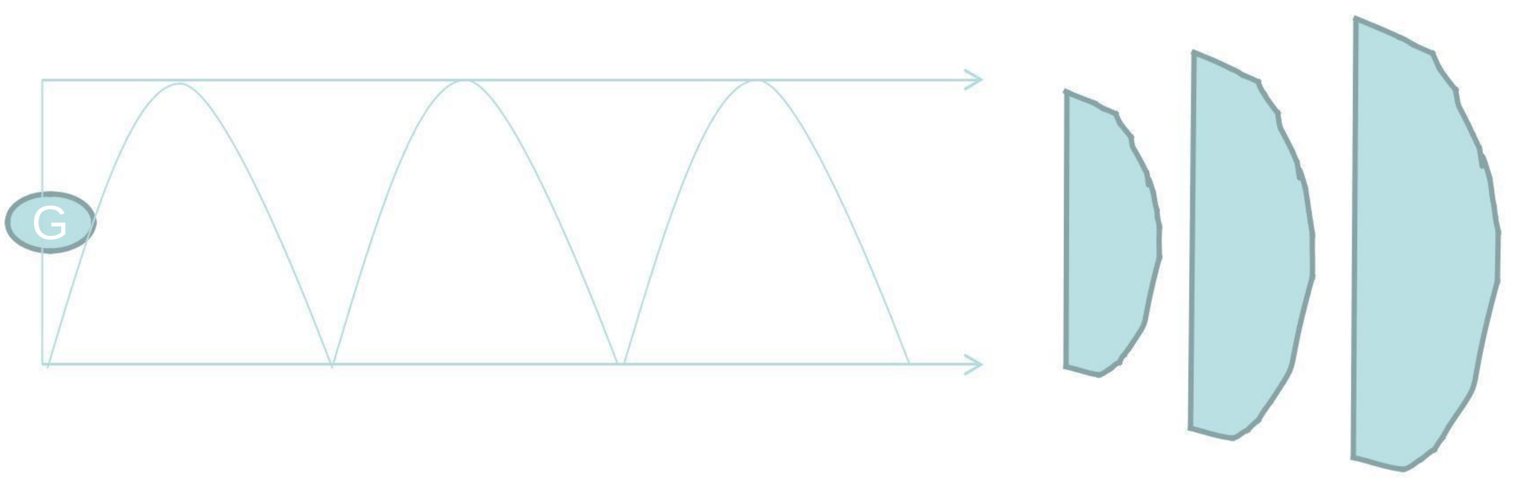
- Antennas are used in systems such as radio and television broadcasting, point to point radio communication, wireless LAN, radar and space exploration
- Antennas are most utilized in air or outer space
- But can also be operated under water or even through soil and rock at certain frequencies for short distances

RADIATION MECHANISM

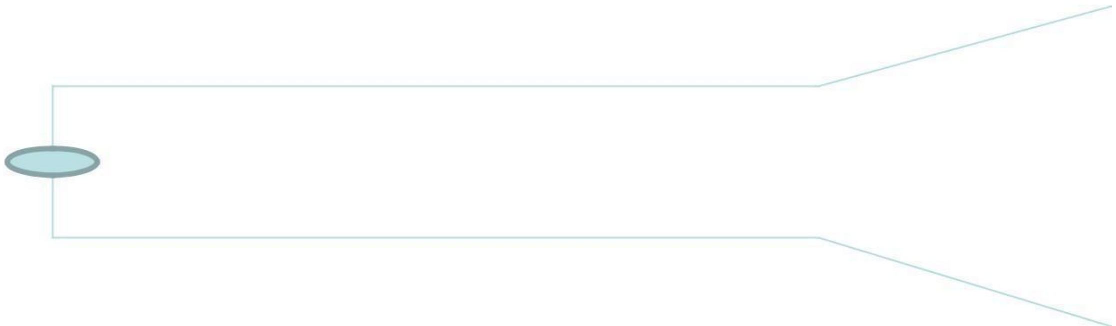
- Ideally all incident energy must be reflected back when open circuit. But practically a small portion of electromagnetic energy escapes from the system that is it gets radiated.
- This occurs because the line of force don't undergo complete phase reversal and some of them escapes.



- The amount of escaped energy is very small due to mismatch between transmission line and surrounding space.
- Also because two wires are too close to each other, radiation from one tip will cancel radiation from other tip.(as they are of opposite polarities and distance between them is too small as compared to wavelength)



- To increase amount of radiated power open circuit must be enlarged , by spreading the two wires.
- Due to this arrangement, coupling between transmission line and free space is improved.
- Also amount of cancellation has reduced.
- The radiation efficiency will increase further if two conductors of transmission line are bent so as to bring them in same line.



TYPES OF ANTENNAS

- According to their applications and technology available, antennas generally fall in one of two categories:

1. Omnidirectional or only weakly directional antennas which receive or radiate more or less in all directions. These are employed when the relative position of the other station is unknown or arbitrary. They are also used at lower frequencies where a directional antenna would be too large, or simply to cut costs in applications where a directional antenna isn't required.

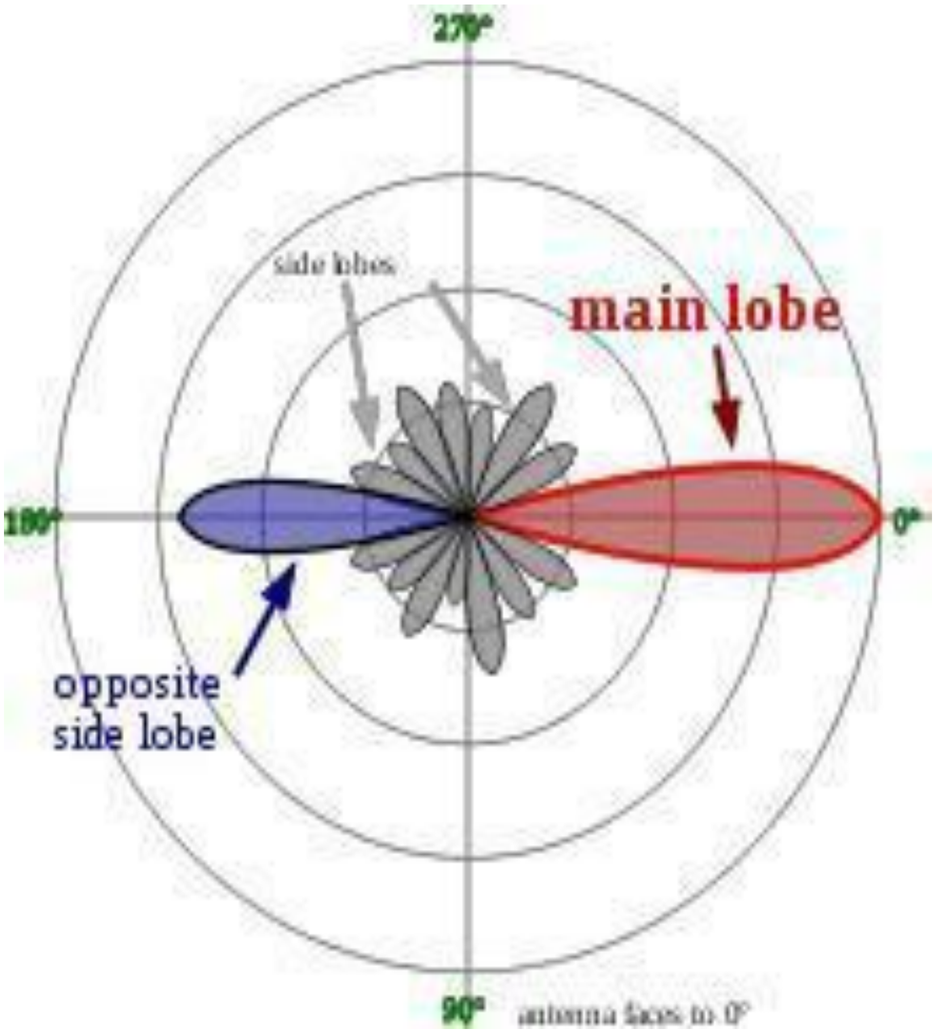
2. Directional or *beam* antennas which are intended to preferentially radiate or receive in a particular direction or directional pattern.

- According to length of transmission lines available, antennas generally fall in one of two categories:

1. Resonant Antennas – is a transmission line, the length of which is exactly equal to multiples of half wavelength and it is open at both ends.

2. Non-resonant Antennas – the length of these antennas is not equal to exact multiples of half wavelength. In these antennas standing waves are not present as antennas are terminated in correct impedance which avoid reflections. The waves travel only in forward direction .Non-resonant antenna is a unidirectional antenna.

RADIATION PATTERN



- The radiation pattern of an antenna is a plot of the relative field strength of the radio waves emitted by the antenna at different angles.
- It is typically represented by a three dimensional graph, or polar plots of the horizontal and vertical cross sections. It is a plot of field strength in V/m versus the angle in degrees.
- The pattern of an ideal isotropic antenna , which radiates equally in all directions, would look like a sphere.
- Many non-directional antennas, such as dipoles, emit equal power in all horizontal directions, with the power dropping off at higher and lower angles; this is called an omni directional pattern and when plotted looks like a donut.

- The radiation of many antennas shows a pattern of maxima or "*lobes*" at various angles, separated by “*nulls*", angles where the radiation falls to zero.
- This is because the radio waves emitted by different parts of the antenna typically interfere, causing maxima at angles where the radio waves arrive at distant points in phase, and zero radiation at other angles where the radio waves arrive out of phase.
- In a directional antenna designed to project radio waves in a particular direction, the lobe in that direction is designed larger than the others and is called the "*main lobe*".
- The other lobes usually represent unwanted radiation and are called “*sidelobes*". The axis through the main lobe is called the "*principle axis*" or “*boresight axis*".

ANTENNA GAIN

- Gain is a parameter which measures the degree of directivity of the antenna's radiation pattern. A high-gain antenna will preferentially radiate in a particular direction.
- Specifically, the *antenna gain*, or *power gain* of an antenna is defined as the ratio of the intensity (power per unit surface) radiated by the antenna in the direction of its maximum output, at an arbitrary distance, divided by the intensity radiated at the same distance by a hypothetical isotropic antenna.

- The gain of an antenna is a passive phenomenon - power is not added by the antenna, but simply redistributed to provide more radiated power in a certain direction than would be transmitted by an isotropic antenna.
- High-gain antennas have the advantage of longer range and better signal quality, but must be aimed carefully in a particular direction.
- Low-gain antennas have shorter range, but the orientation of the antenna is relatively inconsequential.

- For example, a dish antenna on a spacecraft is a high-gain device that must be pointed at the planet to be effective, whereas a typical Wi-Fi antenna in a laptop computer is low-gain, and as long as the base station is within range, the antenna can be in any orientation in space.
- In practice, the half-wave dipole is taken as a reference instead of the isotropic radiator. The gain is then given in **dBd** (decibels over **d**ipole)

ANTENNA EFFICIENCY

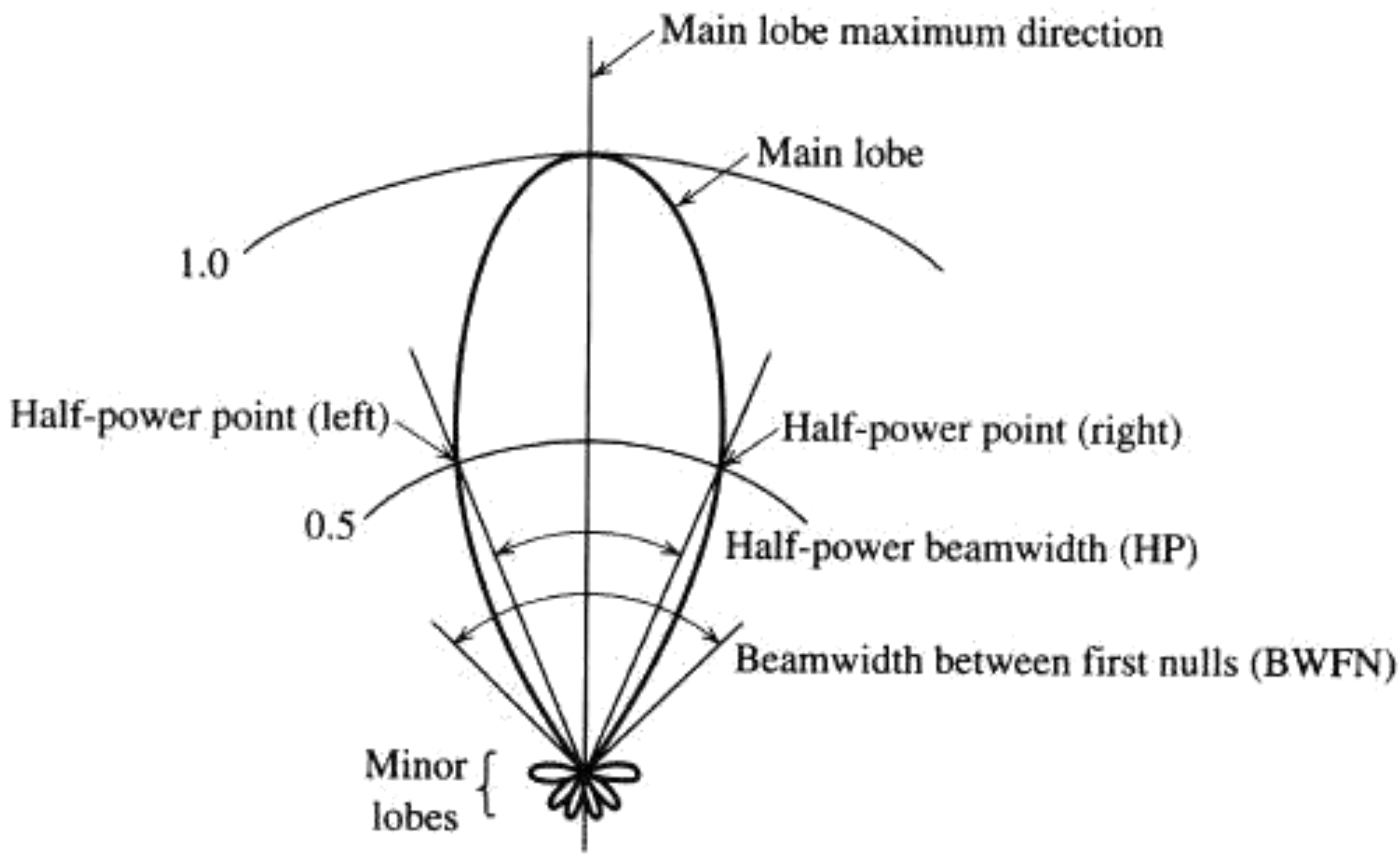
- Efficiency of a transmitting antenna is the ratio of power actually radiated (in all directions) to the power absorbed by the antenna terminals.
- The power supplied to the antenna terminals which is not radiated is converted into heat. This is usually through loss resistance in the antenna's conductors, but can also be due to dielectric or magnetic core losses in antennas (or antenna systems) using such components.

POLARIZATION

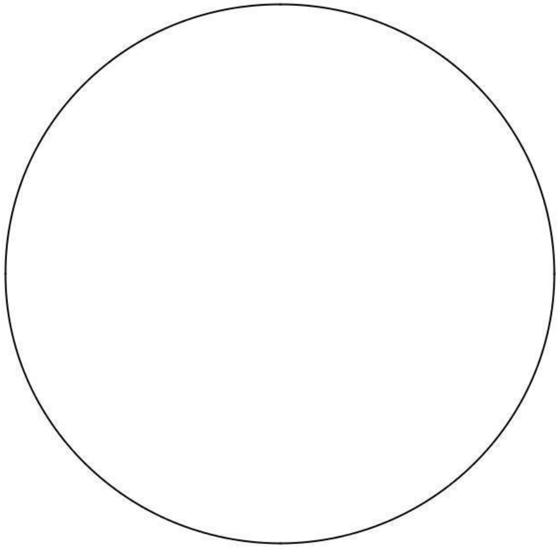
- The polarization of an antenna is the orientation of the electric field (E-plane) of the radio wave with respect to the Earth's surface and is determined by the physical structure of the antenna and by its orientation.
- A simple straight wire antenna will have one polarization when mounted vertically, and a different polarization when mounted horizontally.
- Reflections generally affect polarization. For radio waves the most important reflector is the ionosphere - signals which reflect from it will have their polarization changed
- LF,VLF and MF antennas are vertically polarized

BEAM-WIDTH

- Beam-width of an antenna is defined as angular separation between the two half power points on power density radiation pattern OR
- Angular separation between two 3dB down points on the field strength of radiation pattern
- It is expressed in degrees


Main lobe maximum direction
Main lobe
1.0
Half-power point (left)
Half-power point (right)
0.5
Half-power beamwidth (HP)
Beamwidth between first nulls (BWFN)
Minor
lobes

ISOTROPIC ANTENNA



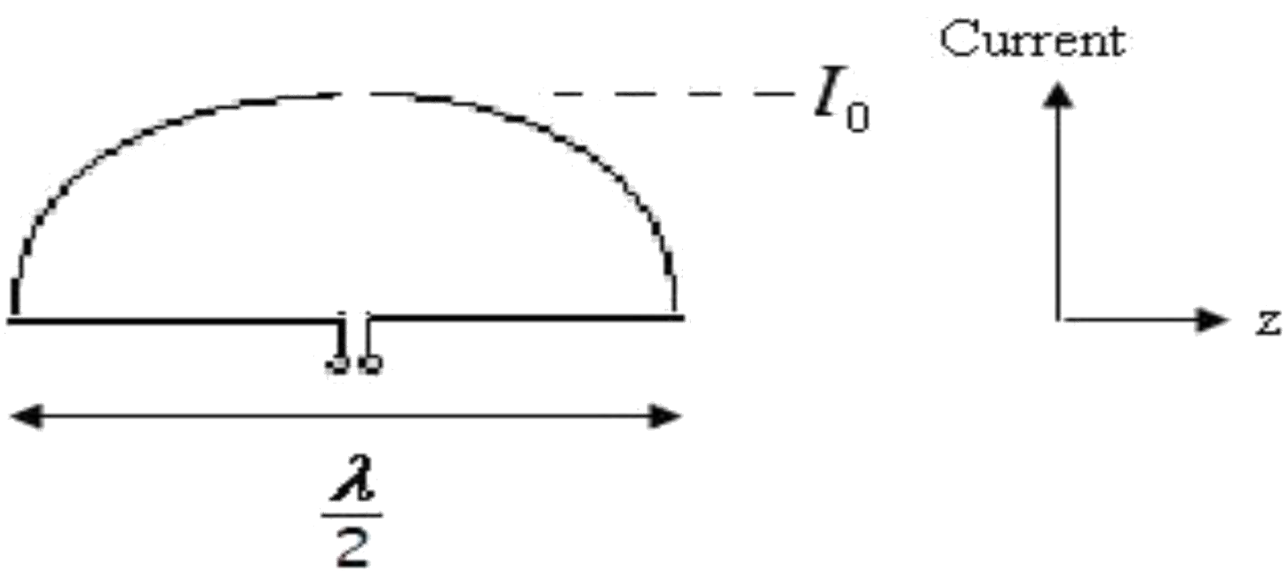
- ***Isotropic antenna or isotropic radiator*** is a hypothetical (not physically realizable) concept, used as a useful reference to describe real antennas.
- Isotropic antenna radiates equally in all directions.
  - Its radiation pattern is represented by a sphere whose center coincides with the location of the isotropic radiator.

- It is considered to be a point in space with no dimensions and no mass. This antenna cannot physically exist, but is useful as a theoretical model for comparison with all other antennas.
- Most antennas' gains are measured with reference to an isotropic radiator, and are rated in dBi (decibels with respect to an isotropic radiator).

HALF WAVE DIPOLE ANTENNA

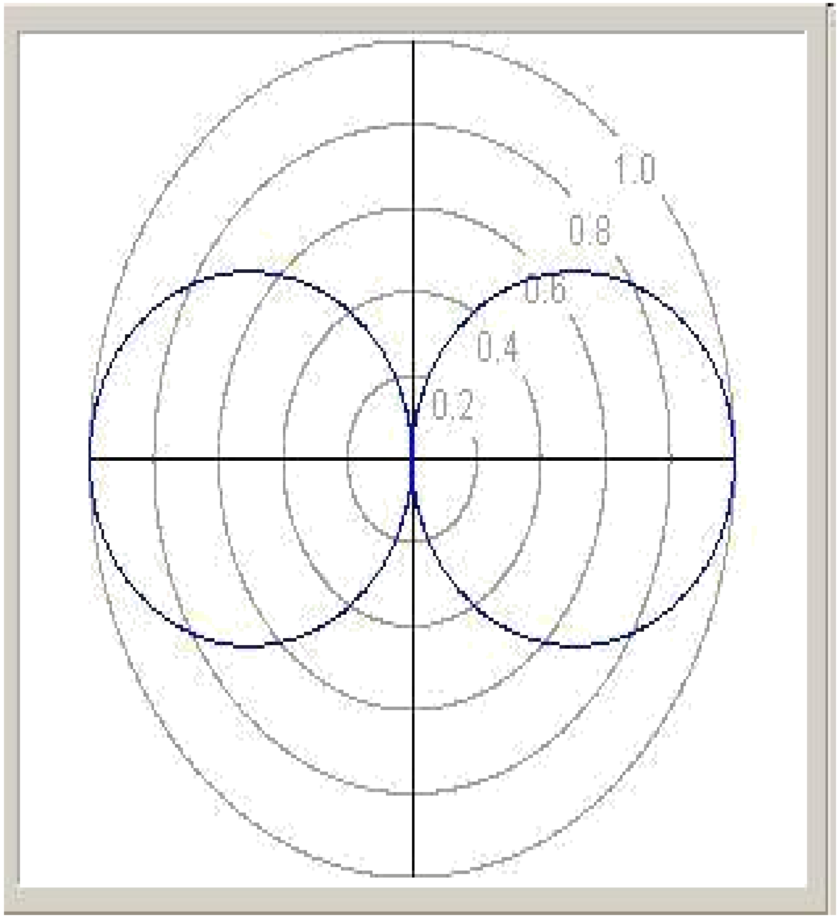
- The **half-wave dipole antenna** is just a special case of the dipole antenna.
- Half-wave term means that the length of this dipole antenna is equal to a half-wavelength at the frequency of operation.
- The dipole antenna, is the basis for most antenna designs, is a balanced component, with equal but opposite voltages and currents applied at its two terminals through a balanced transmission line.

- To make it crystal clear, if the antenna is to radiate at 600 MHz, what size should the half-wavelength dipole be?
- One wavelength at 600 MHz is = c / f = 0.5 meters. Hence, the half-wavelength dipole antenna's length is 0.25 meters.
- The half-wave dipole antenna is as you may expect, a simple half-wavelength wire fed at the center as shown in Figure



Current
I_0
z
$\frac{\lambda}{2}$

- Dipoles have an radiation pattern, doughnut symmetrical about the axis of the dipole. The radiation is maximum at right angles to the dipole, dropping off to zero on the antenna's axis.



FOLDED DIPOLE

- Folded antenna is a single antenna but it consists of two elements.
- First element is fed directly while second one is coupled inductively at its end.
- Radiation pattern of folded dipole is same as that of dipole antenna i.e figure of eight (8).

Advantages

- Input impedance of folded dipole is four times higher than that of straight dipole.
- Typically the input impedance of half wavelength folded dipole antenna is 288 ohm.
- Bandwidth of folded dipole is higher than that of straight dipole.

HERTZ ANTENNA

- The Hertzian dipole is a theoretical short dipole (significantly smaller than the wavelength) with a uniform current along its length.
- A true Hertzian dipole cannot physically exist, since the assumed current distribution implies an infinite charge density at its ends, and significant radiation requires a very high current over its very short length.

LOOP ANTENNA

- A **loop antenna** is a radio antenna consisting of a loop of wire with its ends connected to a balanced transmission line
- It is a single turn coil carrying RF current through it.
- The dimensions of coil are smaller than the wavelength hence current flowing through the coil has same phase.
- Small loops have a poor efficiency and are mainly used as receiving antennas at low frequencies. Except for car radios, almost every AM broadcast receiver sold has such an antenna built inside of it or directly attached to it.

- A technically small loop, also known as a magnetic loop, should have a circumference of one tenth of a wavelength or less. This is necessary to ensure a constant current distribution round the loop.
- As the frequency or the size are increased, a standing wave starts to develop in the current, and the antenna starts to have some of the characteristics of a folded dipole antenna or a self-resonant loop.
- Self-resonant loop antennas are larger. They are typically used at higher frequencies, especially VHF and UHF, where their size is manageable. They can be viewed as a form of folded dipole and have somewhat similar characteristics. The radiation efficiency is also high and similar to that of a dipole.

- Radiation pattern of loop antenna is a doughnut pattern.

- Can be circular or square loop
- No radiation is received normal to the plane of loop and null is obtained in this direction.
- Application: Used for direction finding applications

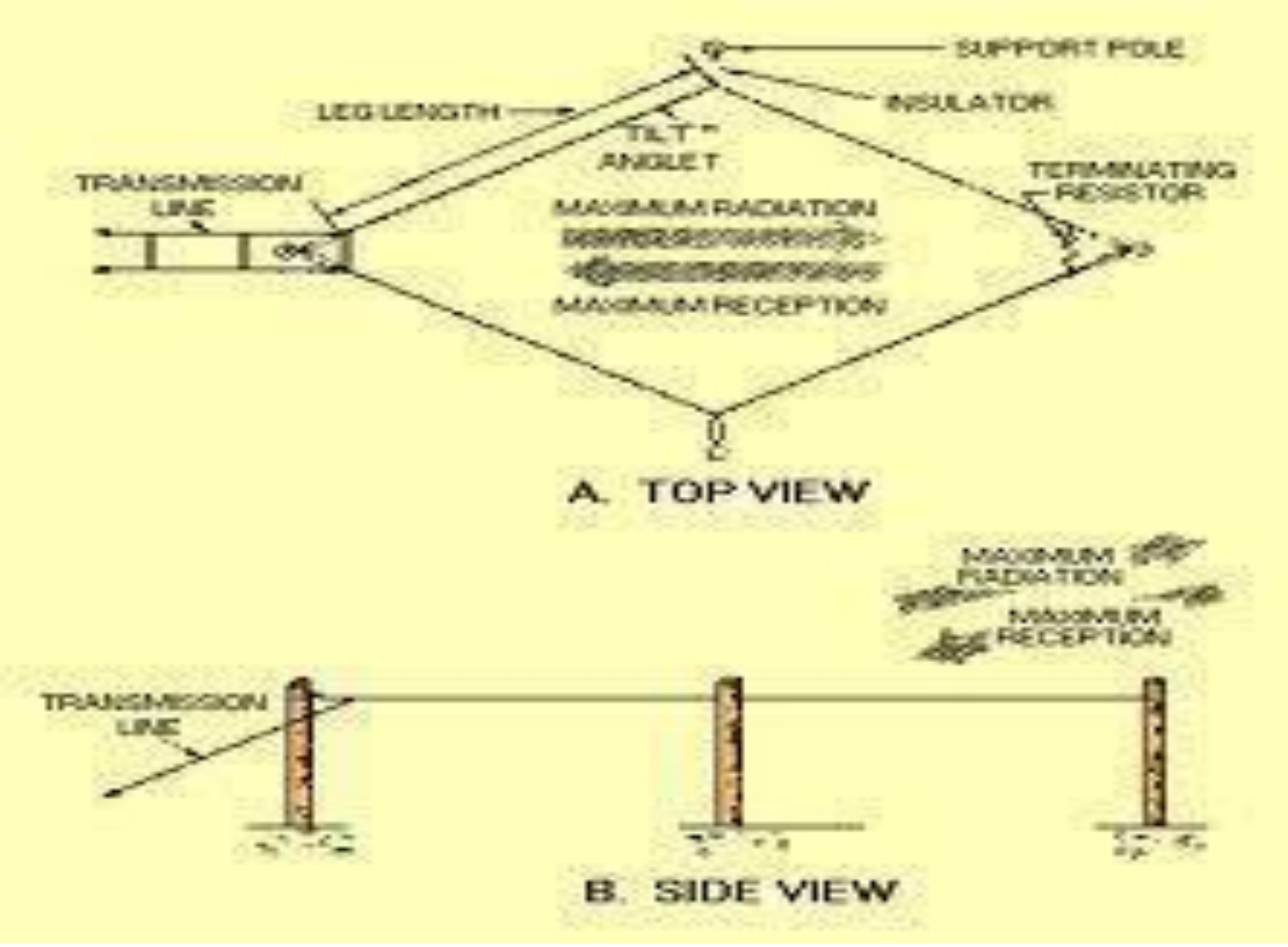
TURNSTILE ANTENNA

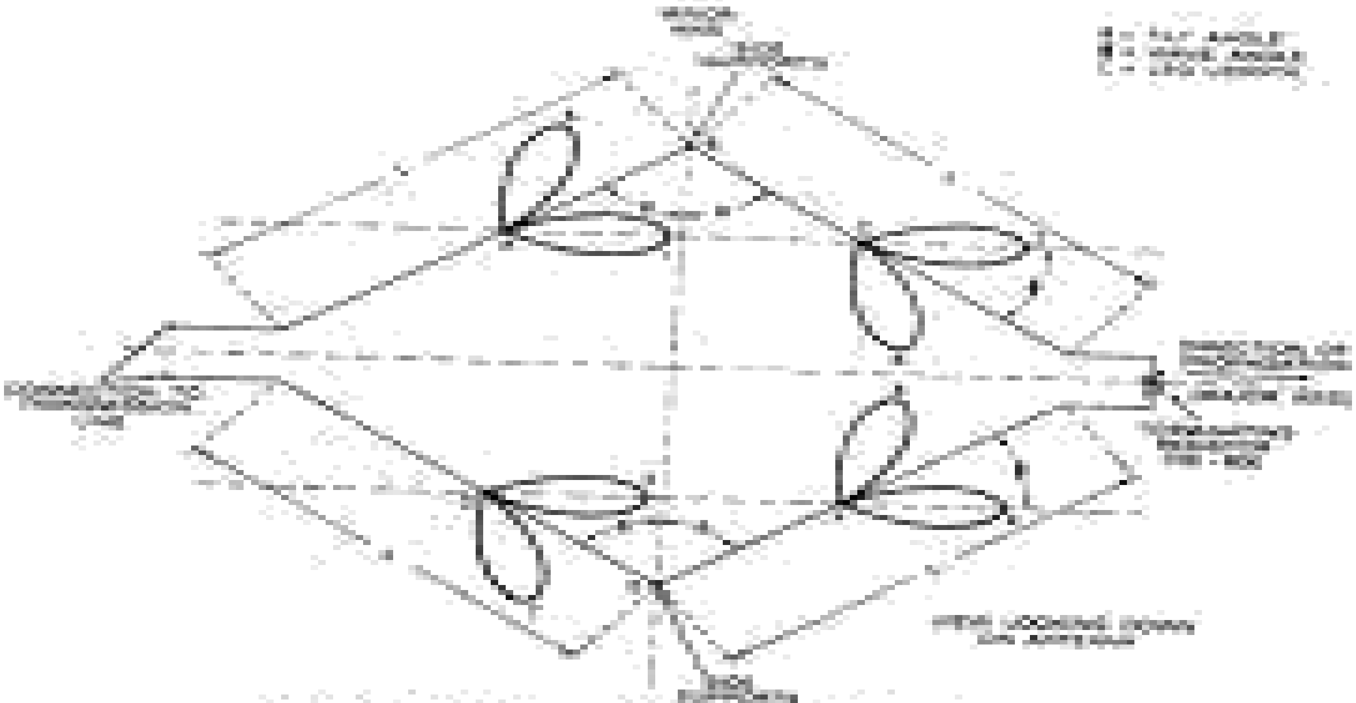
- A **turnstile antenna** is a set of two dipole antennas aligned at right angles to each other and fed 90 degrees out-of-phase.
- The name reflects that the antenna looks like a turnstile when mounted horizontally.
- When mounted horizontally the antenna is nearly omnidirectional on the horizontal plane.

- When mounted vertically the antenna is directional to a right angle to its plane and is circularly polarized.
- The turnstile antenna is often used for communication satellites because, being circularly polarized, the polarization of the signal doesn't rotate when the satellite rotates.

RHOMBIC ANTENNA

- Structure and construction
  - 4 wires are connected in rhombic shape and terminated by a resistor.
  - Mounted horizontally and placed > ^/2 from ground.
- Highest development of long wire antenna is rhombic antenna.


SUPPORT POLE
INSULATOR
LEG LENGTH
TILT ANGLE
TRANSMISSION LINE
TERMINATING RESISTOR
MAXIMUM RADIATION
MAXIMUM RECEPTION
A. TOP VIEW
MAXIMUM RADIATION
MAXIMUM RECEPTION
TRANSMISSION LINE
B. SIDE VIEW



A. FULL - RHOMBIC TRANSMITTING ANTENNA

B. NON - RESONANT HORIZONTAL THREE - WIRE RHOMBIC

- Advantages
  - Easier to construct
  - Its i/p impedance and radiation pattern are relatively constant over range of frequencies.
  - Maximum efficiency
  - High gain can be obtained.
- Disadvantages
  - Large site area and large side lobes.

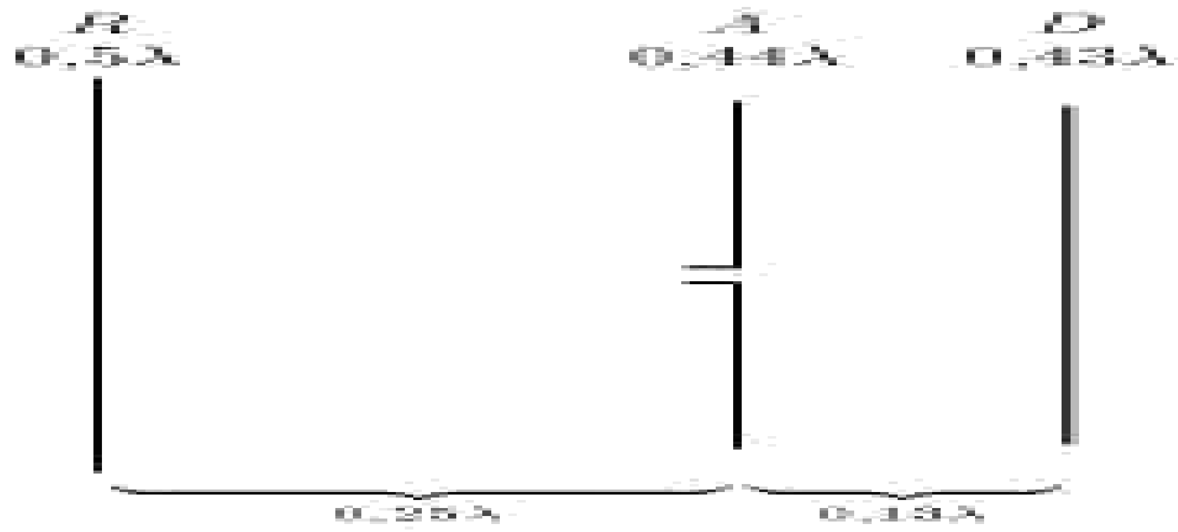
- Application
  - Long distance communication, high frequency transmission and reception.
  - Point to point communication.
  - Radio communication.
  - Short wave radio broadcasting.

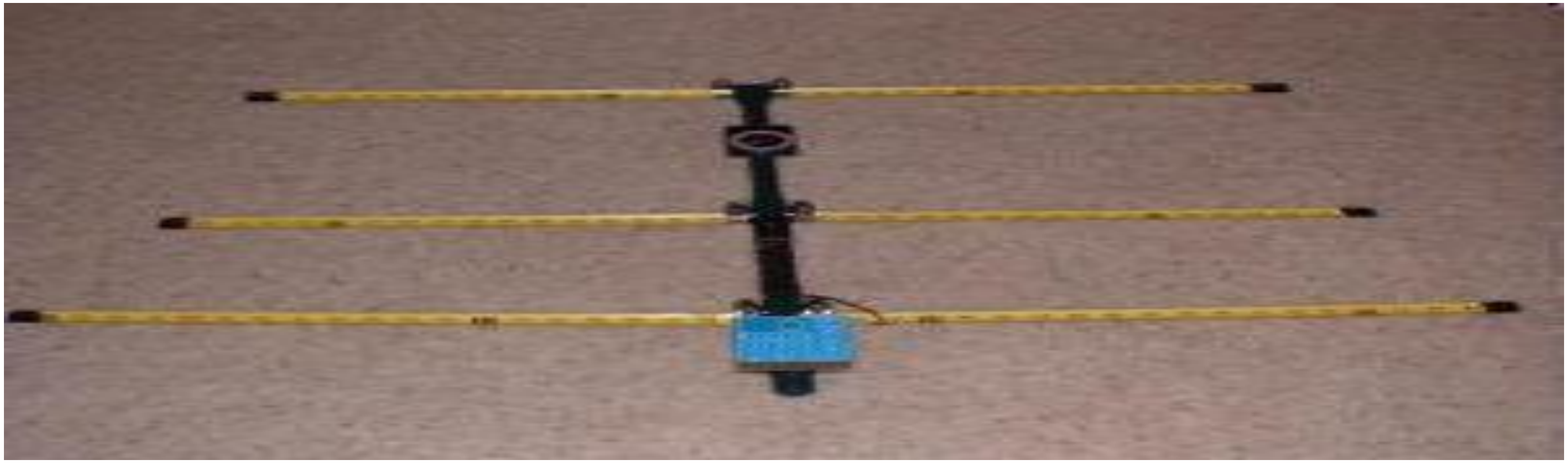
ANTENNA ARRAYS

- Antenna arrays is group of antennas or antenna elements arranged to provide desired directional characteristics.
- Generally any combination of elements can form an array.
- However equal elements of regualar geometry are usually used.

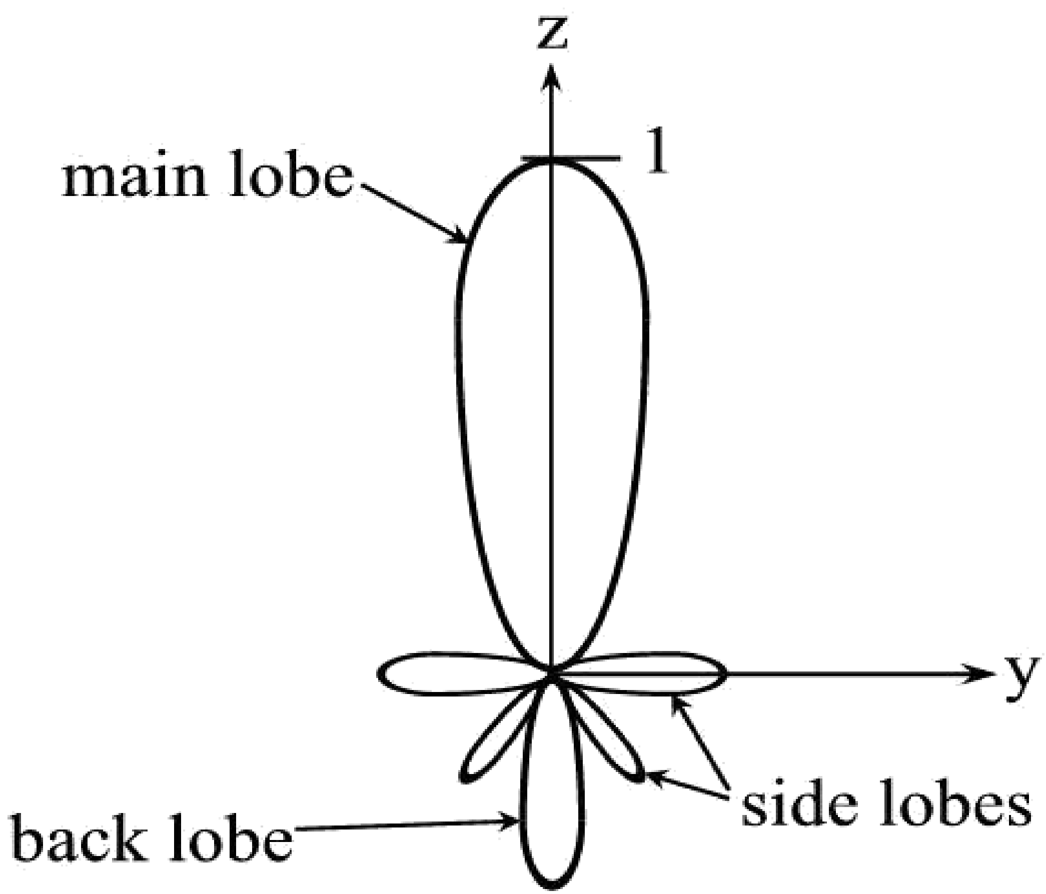
YAGI-UDA ANTENNA

- It is a directional antenna consisting of a driven element (typically a dipole or folded dipole) and additional parasitic elements (usually a so-called *reflector* and one or more *directors*).
- All the elements are arranged collinearly and close together.
- The reflector element is slightly longer (typically 5% longer) than the driven dipole, whereas the so-called directors are a little bit shorter.
- The design achieves a very substantial increase in the antenna's directionality and gain compared to a simple dipole.


R
0,5λ
A
0,44λ
D
0,43λ
0,25λ
0,13λ



- Typical spacing between elements vary from about 1/10 to 1/4 of a wavelength, depending on the specific design.
- The elements are usually parallel in one plane.
- Radiation pattern is modified figure of eight
- By adjusting distance between adjacent directors it is possible to reduce back lobe
- Improved front to back ratio



(a)

ANTENNA APPLICATIONS

They are used in systems such as

- Radio broadcasting
- Broadcast television
- Two-way radio
- Communication receivers
- Radar
- Cell phones
- Satellite communications.

ANTENNA CONSIDERATIONS

- The space available for an antenna
- The proximity to neighbors
- The operating frequencies
- The output power
- Money

Antenna Fundamentals (1)

Summary Slide

- Introduction
- PFD
- Directivity and Gain
- EIRP

Introduction

Radio Link

Antennas: important elements of any radio link

Photographs
of
Various Antenna Types

T-Antenna

- Transmitting antenna transforms
  power in the form of time-dependent electrical current
  into
  time-and-space-dependent electro-magnetic (EM) wave.

R-Antenna

- Receiving antenna transforms
  time-and-space-dependent EM
  wave into
  time-dependent electrical current (power)

Intended Antennas

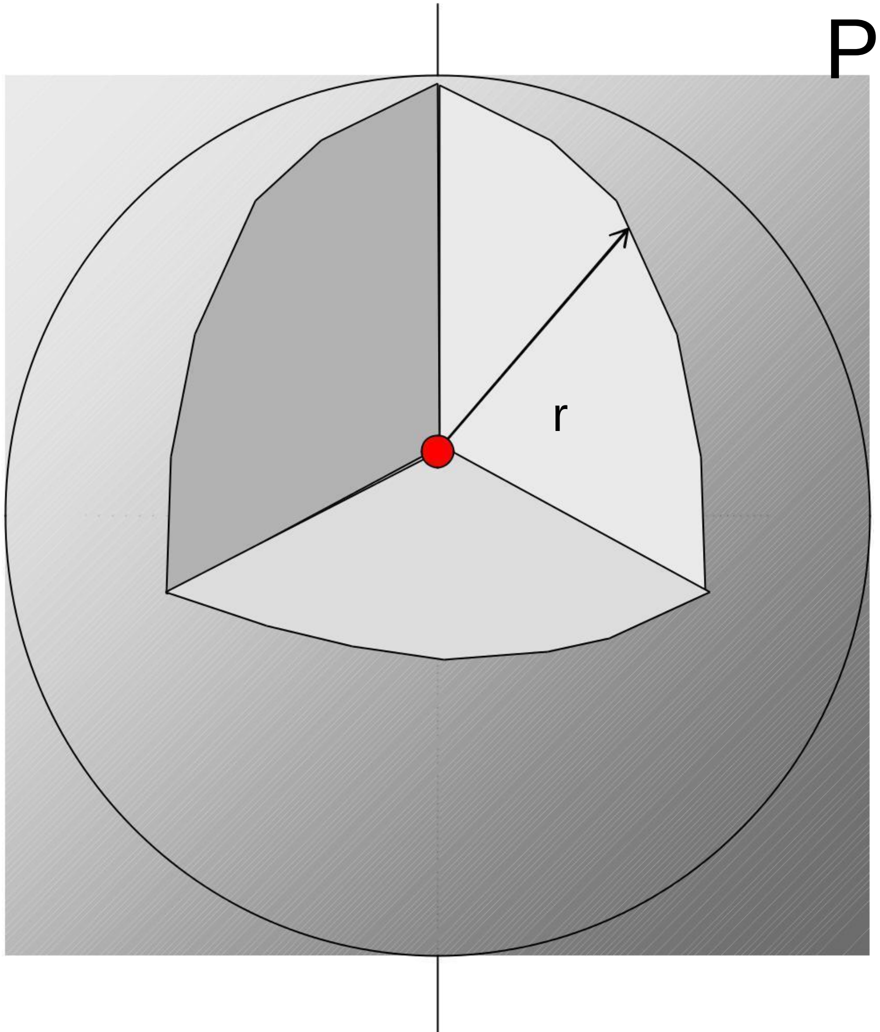
- Radiocommunication antennas
  - Transmitting
  - Receiving
- EM applicators
  - Industrial
  - Medical
- Measuring antennas

Unintended Antennas

- Any conductor/ installation carrying electrical current
  - (e.g. electrical installation of vehicles)
- Any conducting structure/ installation irradiated by EM waves
  - Permanent (e.g. Antenna masts, or power network)
  - Time-varying (e.g. Windmills, or helicopter propellers)
  - Transient (e.g. Re-radiating aeroplane)

PFD

PFD: Isotropic Radiator



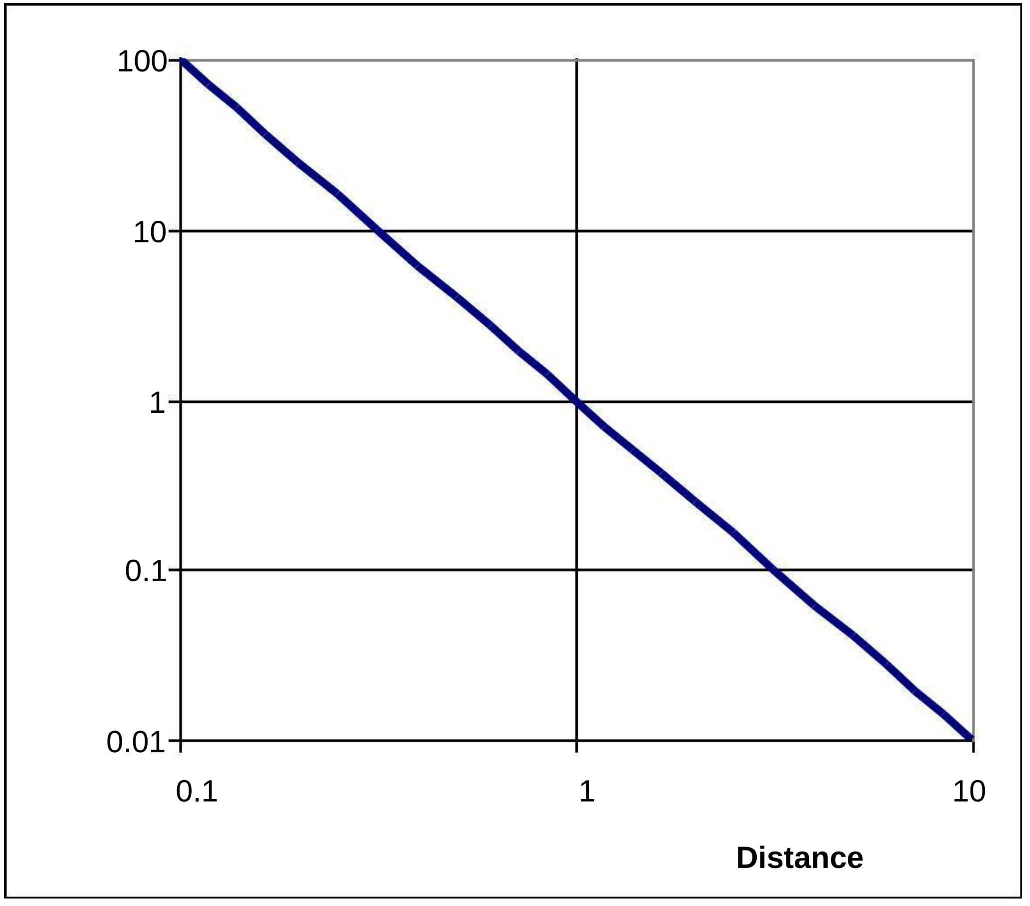
Power Flux Density (PFD)

$$PFD = \frac{P_T}{4\pi r^2}$$

Notes

- Loss-less propagation medium assumed
- Isotropic radiator cannot be physically realized
- PFD does not depend on frequency/ wavelength

PFD: Distance Dependence



PFD: Example 1

- What is the PFD from TV broadcast GEO satellite at Trieste?
- EIRP = 180 kW (52.5 dB(W))
- Distance: ~38'000 km
- Free space

$$PFD = \frac{1.8 \bullet 10^2 \bullet 10^3}{4 \bullet \pi \bullet (38 \bullet 10^6)^2}$$

$$\approx \frac{1.8 \bullet 10^5}{1.8 \bullet 10^{16}}$$

$$= 1 \bullet 10^{-11} \text{ Wm}^{-2}$$

$$= -100 \text{ dB(Wm}^{-2}\text{)}$$

PFD: Example 2

- What is the PFD from a hand-held phone at the head?
- EIRP = 180 mW
- Distance = ~3.8 cm
- Free space

$$PFD = \frac{1.8 \bullet 10^{-1}}{4 \bullet \pi \bullet (3.8 \bullet 10^{-2})^{2}}$$

$$\approx \frac{1.8 \bullet 10^{-1}}{1.8 \bullet 10^{-2}}$$

$$= 10 \text{ Wm}^{-2}$$

$$= 10 \text{ dB(Wm}^{-2}\text{)}$$

PFD: Example 3

- What is the ratio of the powers required to produce the same power flux density at a GEO-satellite and at a LEO-satellite.?
- Distances:
  - GEO: 38 000 km
  - LEO: 1 000 km

$$\frac{PFD_{GEO}}{PFD_{LEO}} = \left(\frac{P_{GEO}}{P_{LEO}}\right)\left(\frac{Dist_{LEO}}{Dist_{GEO}}\right)^2$$

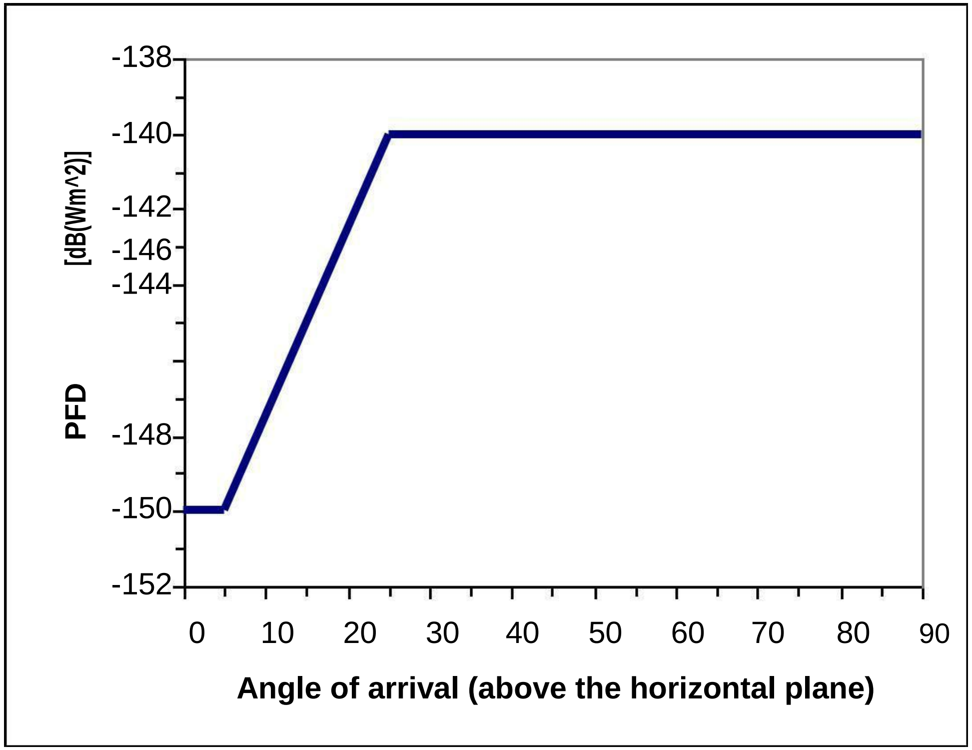
$$\frac{P_{GEO}}{P_{LEO}} = \left(\frac{38000}{1000}\right)^2 = 1444$$

PFD concept

- Used often in the management/ regulating the use of the radio frequency spectrum
- To define the restrictions imposed on radiocommunication systems
- To assure electromagnetic compatibility
- Relates to the field-strength of plane wave

PFD Limits

- The WRC 2000 decided that the PFD at the Earth's surface produced by emission from a space station in Fixed-satellite service shall not exceed the limit shown in the figure.
- The figure is valid for stations at the geostationary orbit in frequency band 10.7-11.7 GHz and reference band 4 kHz. For other cases see RR Table S21-4.

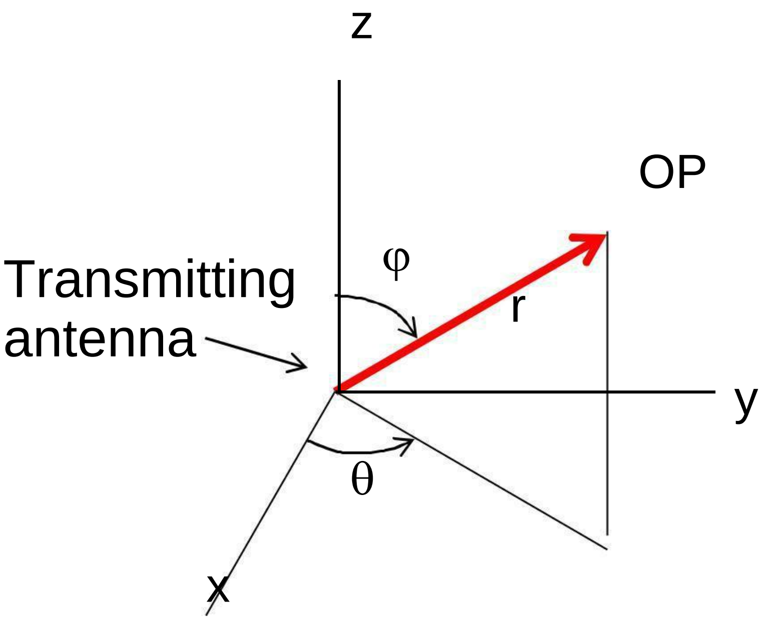


PFD: Real Antenna

- PFD produced by physically realizable antennas depends on
  - power and distance (as isotropic source)
  - horizontal direction angle (θ)
  - vertical direction angle (φ)

Directivity and Gain

Radiation Intensity



Distance (r) is very large

- measure of the ability of an antenna to concentrate radiated power in a particular direction
- Radiation intensity = Power per steradian = $= \Phi(\theta, \varphi)$ [watts/steradian]

Antenna Directivity

Total power radiated

$$P_0 = \int_0^{2\pi} \int_0^{\pi} \Phi(\theta, \varphi) \sin\vartheta \, d\vartheta \, d\varphi$$

Average radiation intensity

$$\Phi_{avg} = \frac{P_0}{4\pi}$$

$$D(\vartheta, \varphi) = \frac{\Phi(\vartheta, \varphi)}{\Phi_{avg}} = \frac{\Phi(\vartheta, \varphi)}{P_0 / 4\pi}$$

- *D* Has no units
- Note:

  P_0 = power radiated

Antenna Gain

- The directivity and gain are measures of the ability of an antenna to concentrate power in a particular direction.
- Directivity – power radiated by antenna (P_0)
- Gain – power delivered to antenna (P_T)

$$G(\vartheta,\varphi) = \eta D(\vartheta,\varphi)$$

$$\eta = \frac{P_T}{P_0}$$

- η: radiation efficiency (50% - 75%)
- G has no units
  - Usually relates to the peak directivity of the main radiation lobe
  - Often expressed in dB
  - Known as ―Absolute Gain‖ or ―Isotropic Gain‖

PFD vs. Antenna Gain

$$S(\vartheta,\varphi) = \frac{\Phi(\vartheta,\varphi)\Delta\vartheta\Delta\varphi}{(r\Delta\vartheta)(r\Delta\varphi)} = \frac{\Phi(\vartheta,\varphi)}{r^2}$$

$$= G(\vartheta,\varphi)\frac{P_0}{4\pi r^2}$$

$$= G(\vartheta,\varphi)S_0$$

S_0 = PFD produced by a loss-less isotropic radiator

Other Definitions of Gain

- For practical purposes, the antenna gain is defined as the ratio (usually in dB), of the power required at the input of a loss-free reference antenna to the power supplied to the input of the given antenna to produce, in a given direction, the same field strength or the same power flux-density at the same distance.
- When not specified otherwise, the gain refers to the direction of maximum radiation.
- The gain may be considered for a specified polarization. [RR 154]

Antenna Gain

Actual antenna

Measuring equipment

P = Power Delivered to the antenna

S = Power received at a great distance

Reference antenna

Measuring equipment

P_o = Power Delivered to the antenna

S = Power received at a great distance

Antenna Gain (in the specific direction) = P / P_o

Reference Antennas

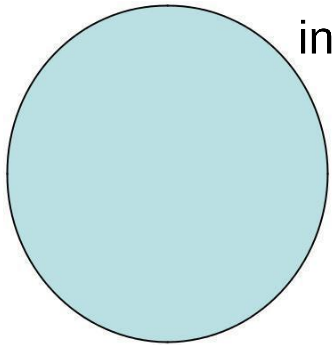
- Isotropic radiator
  - isolated in space (G_i, absolute gain, or isotropic gain)
- Half-wave dipole
  - isolated in space, whose equatorial plane of symmetry contains the given direction (G_d)
- Short vertical antenna
  - (much shorter than $\lambda/4$), close to, and normal to a perfectly conducting plane which contains the given direction (G_v)

Reference Antennas (1)

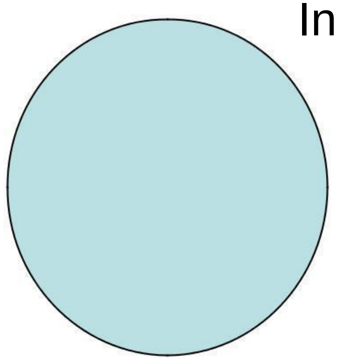
Isotropic antenna

- Sends (receives) energy equally in (from) all directions
- Gain = 1 (= 0 dB)
- When supplied by P, produces at distance r power flux density = $P/(4\pi r^2)$
- Theoretical concept, cannot be physically realized

Radiation pattern
in vertical plane



Radiation pattern
In horizontal plane

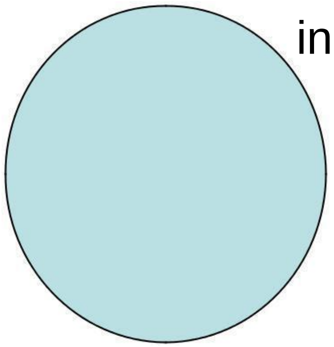


Reference Antennas (2)

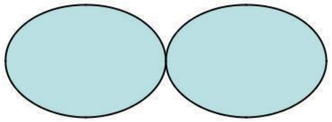
Half-Wave Dipole

- Linear antenna, realizable
- Gain = 1.64 (= 2,15 dB) in the direction of maximum radiation
- Figure-eight-shaped radiation pattern in the dipole plane, omnidirectional (circular) in the orthogonal plan

Radiation pattern in vertical plane



Radiation pattern In horizontal plane



Typical radiation pattern

- Omnidirectional
  - Broadcasting
  - Mobile telephony
- Pencil-beam
  - Microwave links
- Fan-beam (narrow in one plane, wide in the other)
- Shaped-beam
  - Satellite antennas

Typical Gain and Beam-width

Type of antenna	G_i [dB]	HPBW [ ]
Isotropic	0	360x360
Dipole	2	360x120
Helix (10 turn)	14	35x35
Small dish	16	30x30
Large dish	45	1x1

Gain and Beam-width

- Gain and beam-width of directive antennas are inter-related
- G ~ 30000 / (θ_1*θ_2)
- θ_1 and θ_2 are the 3-dB beam-widths (in degrees) in the two orthogonal principal planes of antenna radiation pattern.

EIRP

e.i.r.p.

- Equivalent Isotropically Radiated Power (in a given direction):
- The product of the power supplied to the antenna and the antenna gain relative to an isotropic antenna in a given direction

e.i.r.p.: Example 1

- What is the maximum e.i.r.p. of a GEO satellite station if RR impose PFD limits of (-160) dB ($W/(m^2*4kHz)$) at the earth surface in Equator (distance 35900 km) ?

$PFD = e.i.r.p./(4\pi d^2)$

-160 dB $\rightarrow 10^{-16}$ $W/(m^2*4kHz)$

$d^2 \sim 1.29*10^{15}\ m^2$

$4\pi d^2 \sim 4*10^{15}\ m^2$

e.i.r.p. ~ 0.4 W/4kHz

e.r.p.

- Effective Radiated Power (in a given direction):
- The product of the power supplied to the antenna and its gain relative to a half-wave dipole in a given direction

Chapter Sixteen: Antennas

Introduction

- The antenna is the interface between the transmission line and space
- Antennas are passive devices; the power radiated cannot be greater than the power entering from the transmitter
- When speaking of *gain* in an antenna, gain refers to the idea that certain directions are radiated better than others
- Antennas are reciprocal - the same design works for receiving systems as for transmitting systems

Simple Antennas

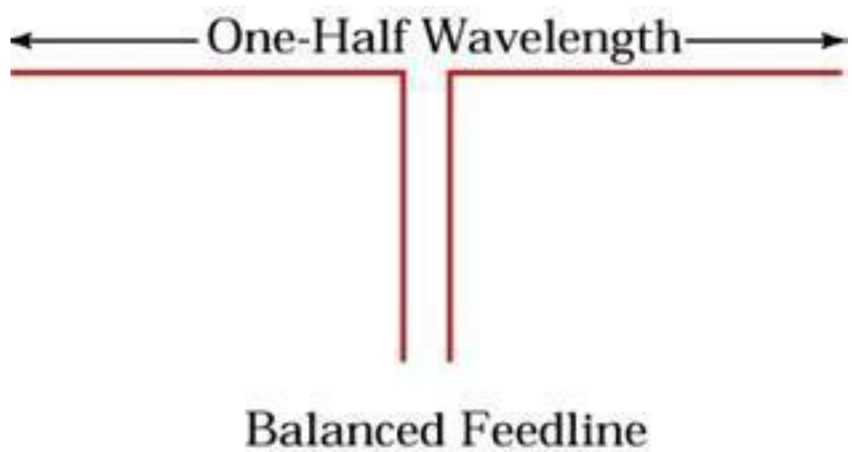
- The Isotropic Radiator would radiate all the power delivered to it and equally in all directions
- The isotropic radiator would also be a *point source*

The Half-Wave Dipole

- A more practical antenna is the half-wave dipole
- Dipole simply means it is in two parts
- A dipole does not have to be one-half wavelength, but that length is handy for impedance matching
- A half-wave dipole is sometimes referred to as a *Hertz* antenna

Basics of the Half-Wave Dipole

- Typically, the length of a half-wave dipole is 95% of one-half the wavelength measured in free space: $\lambda = \frac{c}{f}$



- The halfRadiation-wavedipoledoesResistancenotdissipatepower,

  assuming lossless material

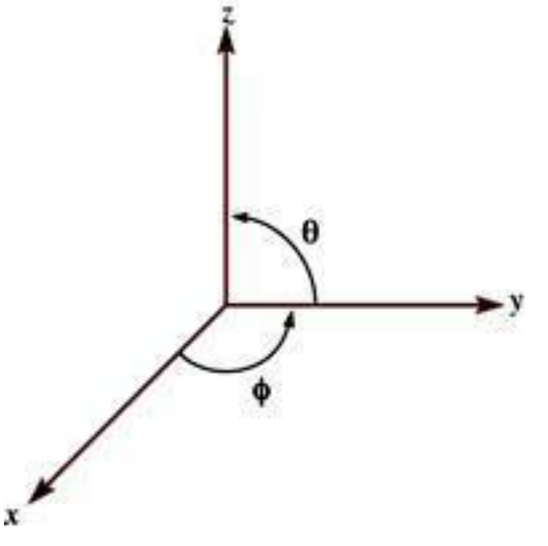
- It will radiate power into space
- The effect on the feedpoint resistance is the same as if a loss had taken place
- The half-wave dipole looks like a resistance of 70 ohms at its feedpoint
- The portion of an antenna‘s input impedance that is due to power radiated into space is known as **radiation resistance**

Antenna Characteristics

- It should be apparent that antennas radiate in various directions
- The terms applied to isotropic and half-wave dipole antennas are also applied to other antenna designs

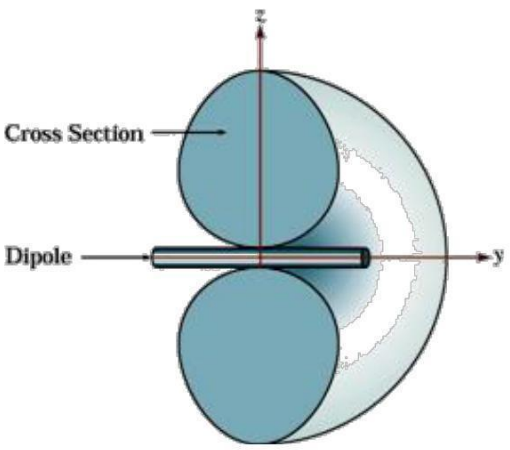
Radiation Patterns

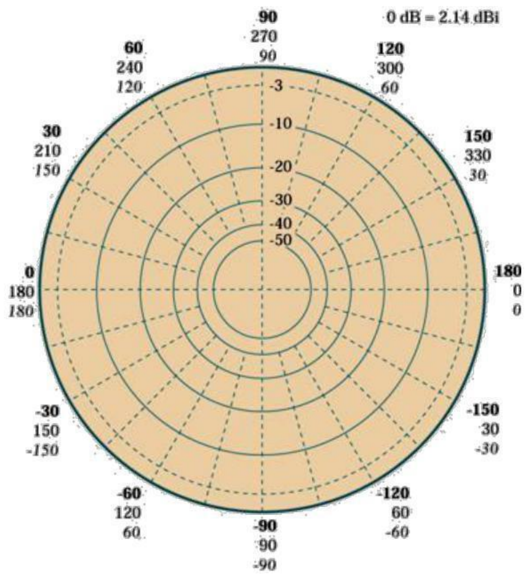
- Antenna coordinates are shown in three-dimensional diagrams
- The angle ϕ is measured from the *x* axis in the direction of the *y* axis
- The *z* axis is vertical, and angle θ is usually measured from the horizontal plane to the zenith

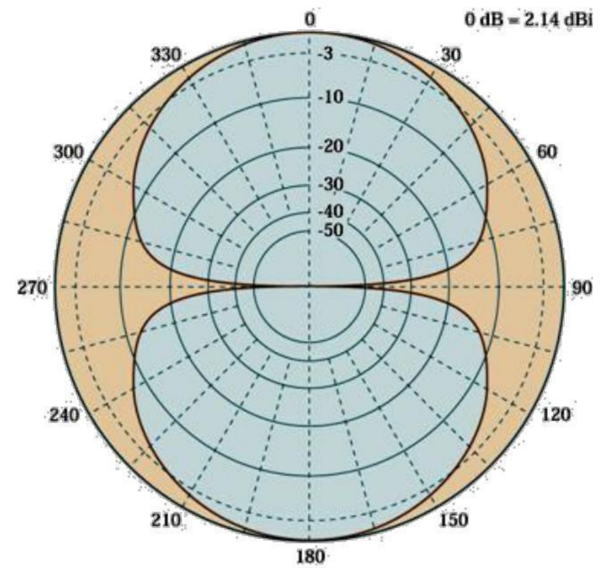


Plotting Radiation Patterns

- Typical radiation patters are displayed in a polar plot

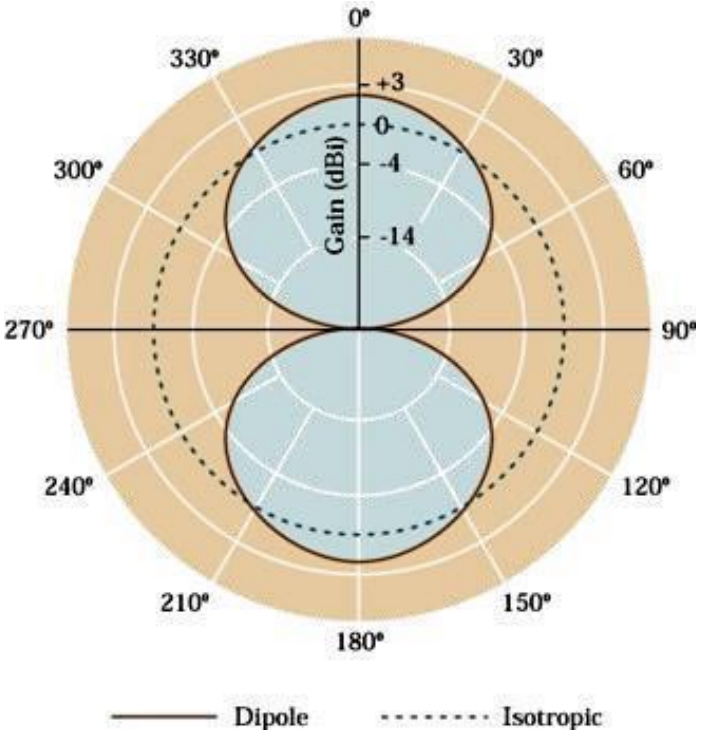






Gain and Directivity

- In antennas, power gain in one direction is at the expense of losses in others
- Directivity is the gain calculated assuming a lossless antenna

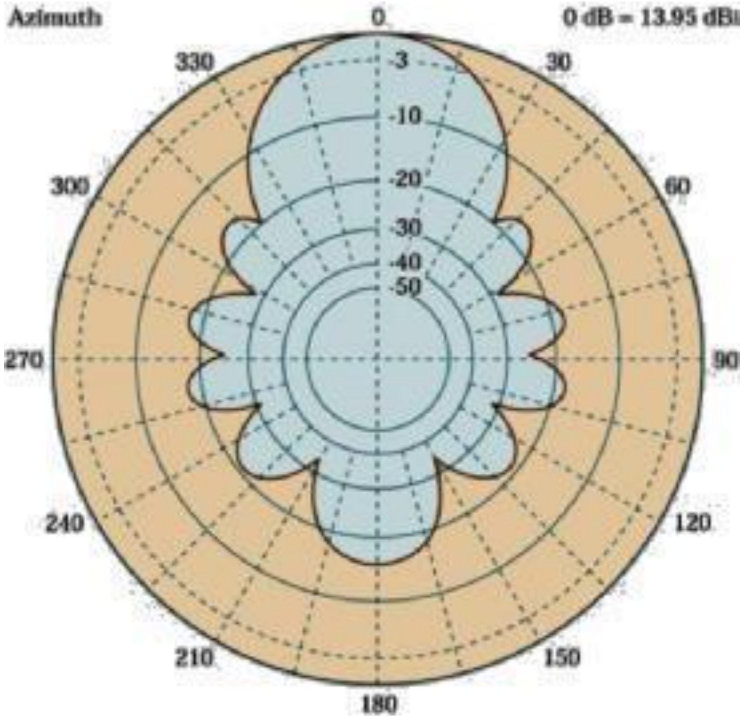


Beamwidth

- A directional antenna can be said to direct a beam of radiation in one or more directions
- The width of this bean is defined as the angle between its half-power points
- A half-wave dipole has a beamwidth of about 79º in one plane and 360º in the other
- Many antennas are far more directional than this

Front-to-Back Ratio

- The direction of maximum radiation is in the horizontal plane is considered to be the *front* of the antenna, and the back is the direction 180º from the front
- For a dipole, the front and back have the same radiation, but this is not always the case



Major and Minor Lobes

- In the previous diagram, the antenna has one *major lobe* and a number of minor ones
- Each of these lobes has a gain and a beamwidth which can be found using the diagram

Effective Isotropic Radiated Power and Effective Radiated Power

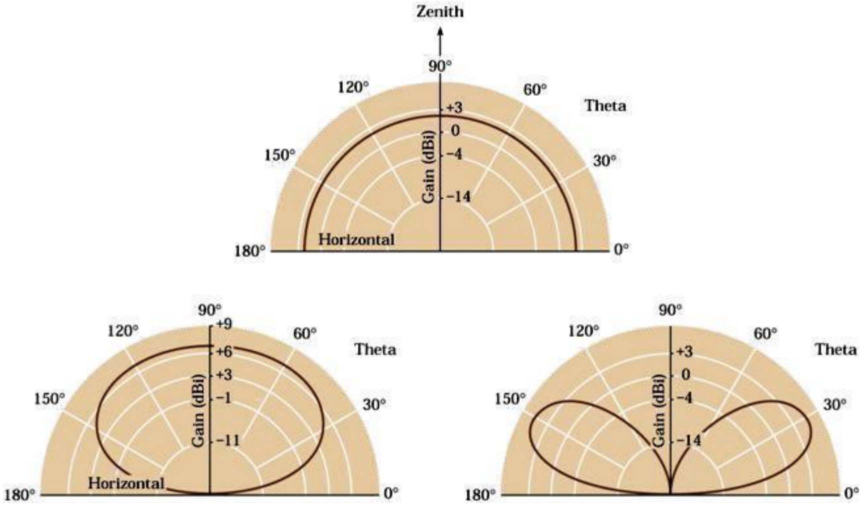
- In practical situations, we are more interested in the power emitted in a particular direction than in total radiated power
- Effective Radiated Power represents the power input multiplied by the antenna gain measured with respect to a half-wave dipole
- An Ideal dipole has a gain of 2.14 dBi; EIRP is 2.14 dB greater than the ERP for the same antenna combination

Impedance

- The radiation resistance of a half-wave dipole situated in free space and fed at the center is approximately 70 ohms
- The impedance is completely resistive at resonance, which occurs when the length of the antenna is about 95% of the calculated free-space, half-wavelength value
- If the frequency is above resonance, the feedpoint impedance has an inductive component; if the frequency is below resonance, the component is capacitive

Ground Effects

- When an antenna is installed within a few wavelengths of the ground, the earth acts as a reflector and has a considerable influence on the radiation pattern of the antenna
- Ground effects are important up through the HF range. At VHF and above, the antenna is usually far enough above the earth that reflections are not significant
- Ground effects are complex because the characteristics of the ground are variable

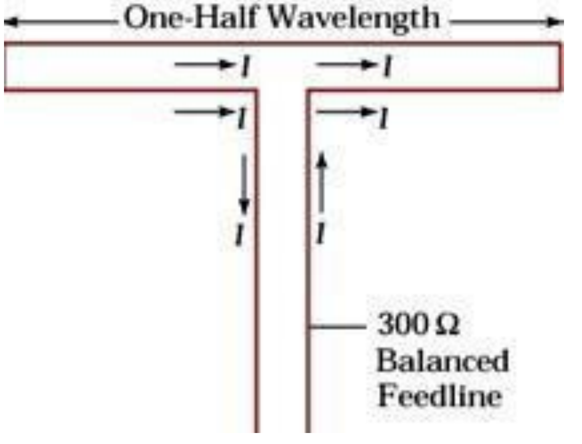


Other Simple Antennas

- Other types of simple antennas are:
  - The folded dipole
  - The monopole antenna
  - Loop antennas
  - The five-eighths wavelength antenna
  - The Discone antenna
  - The helical antenna

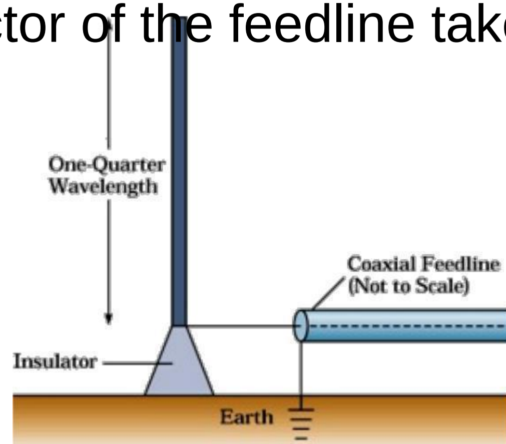
The Folded Dipole

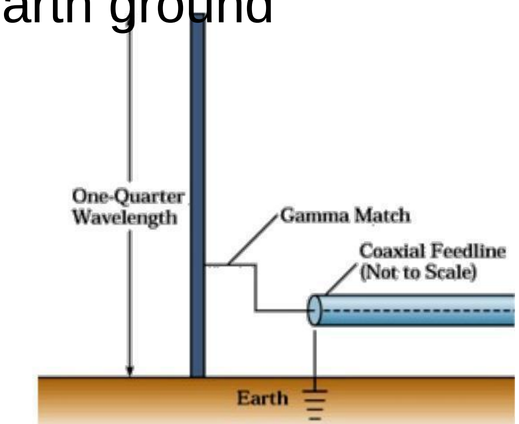
- The folded dipole is the same length as a standard dipole, but is made with two parallel conductors, joined at both ends and separated by a distance that is short compared with the length of the antenna
- The folded dipole differs in that it has wider bandwidth and has approximately four times the the feedpoint impedance of a standard



The Monopole Antenna

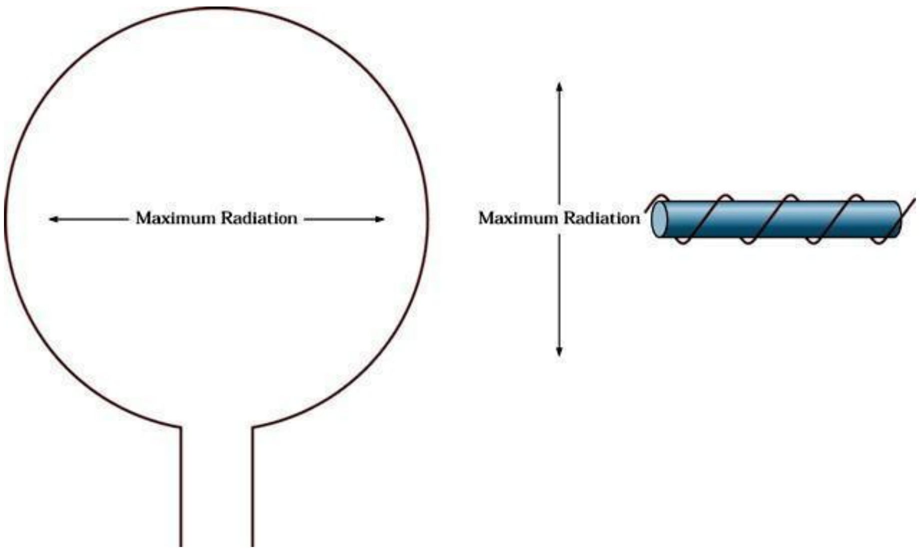
- For low- and medium-frequency transmissions, it is necessary to use vertical polarization to take advantage of ground-wave propagation
- A vertical dipole would be possible, but similar results are available from a quarter-wavelength monopole antenna
- Fed at one end with an unbalanced feedline, with the ground conductor of the feedline taken to earth ground





Loop Antennas

- Sometimes, smaller antennas are required for certain applications, like AM radio receivers
- These antennas are not very efficient but perform adequately
- Two types of loop antennas are:
  - Air-wound loops
  - Ferrite-core loopsticks



The Five-Eighths Wavelength Antenna

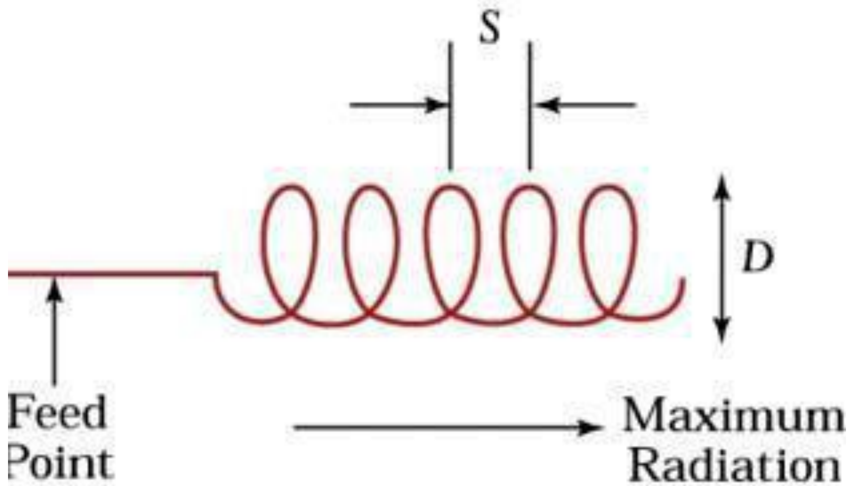
- The five-eighths wavelength antenna is used vertically either as a mobile or base antenna in VHF and UHF systems
- It has omnidirectional response in the horizontal plane
- Radiation is concentrated at a lower angle, resulting in gain in the horizontal direction
- It also has a higher impedance than a quarter-wave monopole and does not require as good a ground

The Discone Antenna

- The discone antenna is characterized by very wide bandwidth, covering a 10:1 frequency range
- It also has an omnidirectional pattern in the horizontal plane and a gain comparable to that of a dipole
- The feedpoint resistance is typically 50 ohms
- Typically, the length of the surface of the cone is about one-quarter wavelength at the lowest operating frequency

The Helical Antenna

- Several types of antennas are classified as *helical*
- The antenna in the sketch has its maximum radiation along its long axis
- A quarter-wave monopole can be shortened and wound into a helix— common in *rubber ducky* antenna



Antenna Matching

- Sometimes a resonant antenna is too large to be convenient
- Other times, an antenna may be required to operate at several widely different frequencies and cannot be of resonant length all the time
- The problem of mismatch can be rectified by matching the antenna to the feedline using an *LC* matching network

Antenna Arrays

- Simple antenna elements can be combined to form **arrays** resulting in reinforcement in some directions and cancellations in others to give better gain and directional characteristics
- Arrays can be classified as *broadside* or *end-fire*
  - Examples of arrays are:
  - The Yagi Array
  - The Log-Periodic Dipole Array
  - The Turnstile Array
  - The Monopole Phased Array
  - Other Phased Arrays

Reflectors

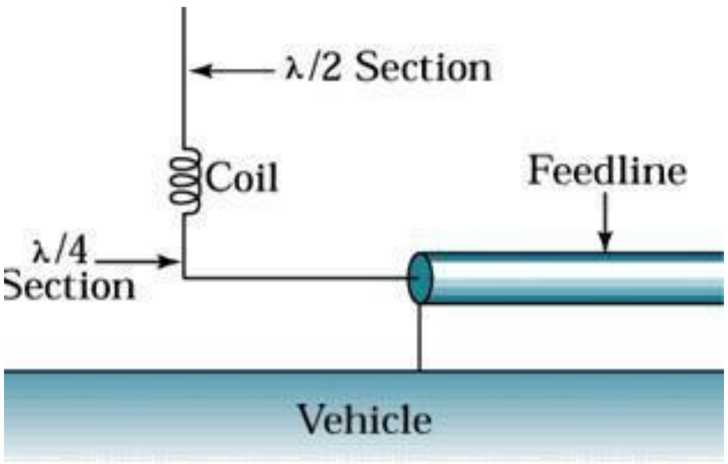
- It is possible to construct a conductive surface that reflects antenna power in the desired direction
- The surface may consist of one or more planes or may be parabolic
- Typical reflectors are:
  - Plane and corner Reflectors
  - The Parabolic Reflector

Cell-Site Antenna

- For cellular radio systems, there is a need for omnidirectional antennas and for antennas with beamwidths of 120º, and less for sectorized cells
- Cellular and PCS base-station receiving antennas are usually mounted in such a way as to obtain space diversity
- For an omnidirectional pattern, typically three antennas are mounted on a tower with a triangular cross section and the antennas are mounted at 120º intervals

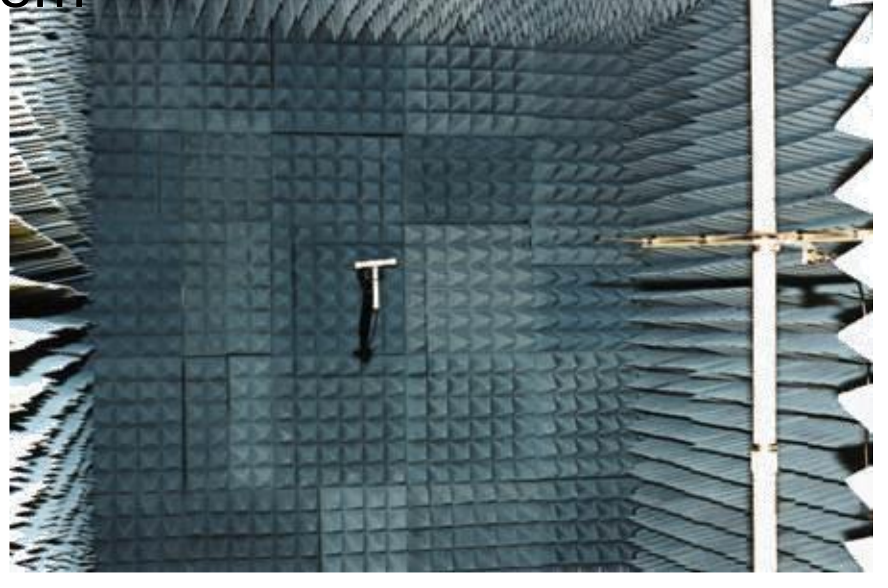
Mobile and Portable Antenna

- Mobile and portable antennas used with cellular and PCS systems have to be omnidirectional and small
- The simplest antenna is the quarter-wavelength monopole are these are usually the ones supplied with portable phones
- For mobile phones, and common configuration is the quarter-wave antenna with a half-wave antenna mounted collinearly above it



Test Equipment: The Anechoic Chamber

- The anechoic chamber is used to set up antennas in a location that is free from reflections in order to evaluate them



Chapter 10 Potentials and Fields

10.1 The Potential Formulation

10.2 Continuous Distributions

10.3 Point Charges

10.1 The Potential Formulation

10.1.1 Scalar and vector potentials

10.1.2 Gauge transformation

10.1.3 Coulomb gauge and Lorentz gauge

10.1.1 Scalar and Vector Potentials

$\left.\begin{matrix}\rho(\mathbf{r},t)\\ \mathbf{J}(\mathbf{r},t)\end{matrix}\right\} \rightarrow \left\{\begin{matrix}\mathbf{E}(\mathbf{r},t)\\ \mathbf{B}(\mathbf{r},t)\end{matrix}\right.$ field formulism

or

$\left\{\begin{matrix}V(\mathbf{r},t)\\ \mathbf{A}(\mathbf{r},t)\end{matrix}\right.$ potential formulism

Maxwell's eqs

MS

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{B} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A})$$

ES $\quad \nabla \times \mathbf{E} = 0 \quad \mathbf{E} = -\nabla V$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A}}$$

$$\nabla \times \left(\mathbf{E} + \frac{\partial}{\partial t}\mathbf{A}\right) = 0$$

$$\boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}}$$

10.1.1 (2)

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

↓

$$\nabla \cdot \left[-\nabla V - \frac{\partial A}{\partial t}\right] = \frac{\rho}{\varepsilon_0}$$

$$\boxed{\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\frac{\rho}{{}_0}}$$

$$\nabla \times (\nabla \times A) = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left[-\nabla V - \frac{\partial}{\partial t} A\right]$$

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

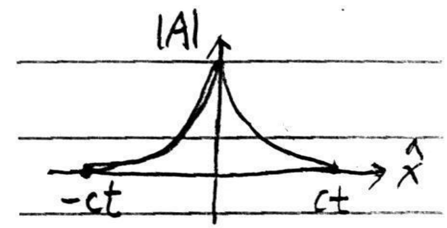
$$\boxed{\left(\nabla^2 A - \mu_0 \varepsilon_0 \frac{\partial^2 A}{\partial t^2}\right) - \nabla\left(\nabla \cdot A + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 J}$$

10.1.1 (3)

Ex.10.1

$$V = 0 \qquad \mathbf{A} = \begin{cases} \dfrac{\mu_0 k}{4c}(ct - |x|)^2\,\hat{z} & \text{for } |x| < ct \\ 0 & \text{for } |x| > ct \end{cases} \qquad \rho, \mathbf{J} = ?$$

where k is a constant, $c = (\mu_0 \varepsilon_0)^{-\frac{1}{2}}$



Sol:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 k}{2}(ct - |x|)\,\hat{z}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 k}{4c}\frac{\partial}{\partial x}(ct - |x|)^2\,\hat{y} = \pm\frac{\mu_0 k}{2c}(ct - |x|)\,\hat{y}$$

for $|x| < ct$, $(\mathbf{E} = \mathbf{B} = 0 \text{ for } |x| > ct)$

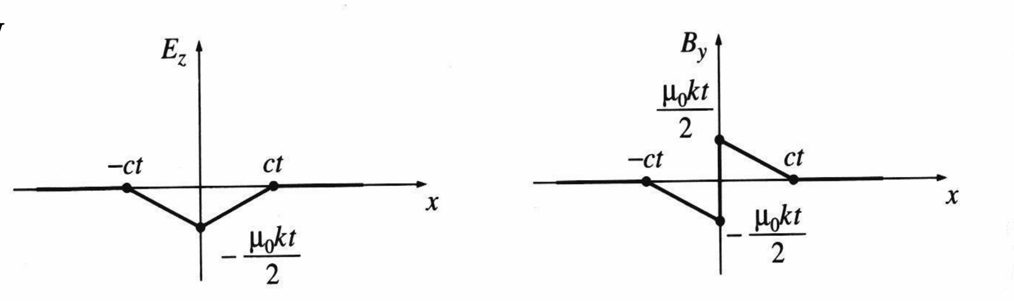
$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\mu_0 k}{2}\frac{\partial}{\partial x}(ct - |x|)(-\hat{y}) = \frac{\mu_0 k}{2}\hat{y}$$

$$\nabla \times \mathbf{B} = \pm\frac{\mu_0 k}{2c}\frac{\partial}{\partial x}(ct - |x|)(\hat{z}) = -\frac{\mu_0 k}{2c}\hat{z}$$

10.1.1 (4)

$$\frac{\partial E}{\partial t} = -\frac{\mu_0 kc}{2}\hat{z} \qquad \frac{\partial B}{\partial t} = \pm\frac{\mu_0 k}{2}\hat{y}$$



$$\rho = \epsilon_0 \nabla \cdot E = 0$$

$$J = \frac{1}{\mu_0}(\nabla \times B) - \epsilon_0 \frac{\partial E}{\partial t} = -\frac{k}{2c}\hat{z} - \epsilon_0 \frac{\mu_0 kc}{2}\hat{z} = 0 \qquad (\mu_0\epsilon_0 = \frac{1}{c^2})$$

$$\text{at } x = 0, E_z\big|_{x=0^-} = E_z\big|_{x=0^+} \quad \text{as } E_1^{\parallel} = E_2^{\parallel}, \quad E\big|_{x=0}$$

$$B_y\big|_{x=0^-} \neq B_y\big|_{x=0^+} \qquad \frac{1}{\mu_0}\left(B_y\big|_{x=0^-} - B_y\big|_{x=0^+}\right) = K_f \times \hat{x}$$

$$[(-\frac{kt}{2}) - (-\frac{kt}{2})]\hat{y} = K_f \underbrace{(\hat{z} \times \hat{x})}_{-\hat{y}}$$

$$\boxed{K_f = kt\,\hat{z}}$$

10.1.2 Gauge transformation

$$\| \qquad \partial t \qquad , \qquad ,$$

$$\nabla V \text{ for } V = V + \beta,\ \beta(r, t) = \beta(t) \nabla\beta = 0$$

How about $A' = A + \alpha$?

$$B = \nabla \times A = \nabla \times A' \text{ for } A' = A + \alpha\ ,\alpha = \nabla\lambda(r, t) \qquad \nabla \times \nabla\lambda = 0$$

$$E = -\nabla V - \frac{\partial A}{\partial t} = -\nabla V' + \nabla\beta - \frac{\partial A'}{\partial t} \quad \frac{\partial \alpha}{\partial t}$$

$$= -\nabla V' - \frac{\partial A'}{\partial t} + \nabla(\beta + \overline{}\ ) \qquad \underbrace{}_{k(t)} \partial t \quad \beta = -\frac{}{\partial t} + k(t) = -\frac{}{\partial t}$$

$$\frac{\partial A'}{}$$

$$\lambda' \to = \lambda - \frac{\partial \lambda}{\partial t}$$

10.1.2 (2)

When

$$\left.\begin{array}{l} \mathbf{A}' = \mathbf{A} + \nabla\lambda \\ V' = V - \dfrac{\partial\lambda}{\partial t} \end{array}\right\} \text{Gauge transformation}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla \times \mathbf{A}'$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\nabla V' - \frac{\partial \mathbf{A}'}{\partial t}$$

The fields are independent of the gauges.

(note: physics is independent of the coordinates.)

10.1.3 Coulomb gauge and Lorentz gauge

Potential formulation

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot A) = -\frac{\rho}{\varepsilon_0}$$

$$(\nabla^2 A - \mu_0 \varepsilon_0 \frac{\partial^2 A}{\partial t^2}) \quad + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t}) = -\mu_0 J$$

Sources: $\rho, J \rightarrow V, A \rightarrow \begin{cases} \quad \partial t \\ B = \nabla \times A \end{cases}$

Coulomb gauge:

$$\nabla \cdot A = 0$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad V(r, t) = \frac{1}{\quad 0} \int \frac{\rho(r', t)}{\quad} d\tau'$$

easy to solve V

$$\nabla^2 \vec{A} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \mu_0 \varepsilon_0 \nabla\left(\frac{\partial V}{\partial t}\right)$$

difficult to solve A

10.1.3 (2)

Lorentz gauge:

$$\nabla \cdot \mathbf{A} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$$

$$\left.\begin{aligned} \nabla^2 \mathbf{A} - \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\mu_0 \mathbf{J} \\ \nabla^2 V - \mu_0\varepsilon_0 \frac{\partial^2 V}{\partial t^2} &= -\frac{\rho}{\varepsilon_0} \end{aligned}\right\}$$

$$\Box^2 \mathbf{A} = -\mu_0 \mathbf{J}$$ inhomogeneous wave eq.

$$\Box^2 V = -\frac{\rho}{\varepsilon_0}$$

the d‘Alembertion $\Box^2 \equiv \nabla^2 - \mu_0\varepsilon_0 \frac{\partial^2}{\partial t^2}$

$$\Box^2 f = 0$$ wave equation

[Note:Since $\Box^2$ is with ,the potentials with both $+t$ and $-t$ are solutions.]

10.1.3 (3)

Gauge transformation

$$\mathbf{A}' = \mathbf{A} + \nabla\lambda$$

$$V' = V - \frac{\partial \lambda}{\partial t}$$

Coulomb gauge :

$$\nabla \cdot \mathbf{A} = 0$$

If you have a $\mathbf{A}$ and $\nabla \cdot \mathbf{A} \neq 0$, $\nabla \cdot \mathbf{A}' = \nabla \cdot \mathbf{A} + \nabla^2 \lambda$

Find λ, $\nabla^2 \lambda = \nabla \cdot \mathbf{A}'$

Then, you have a solution and $\mathbf{A}$ $\nabla \cdot \mathbf{A} = 0$

10.1.3 (4)

Lorentz gauge : $\nabla \cdot A = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

If you have a set of A' and V', and $\nabla \cdot A' \neq -\mu_0 \epsilon_0 \frac{\partial V'}{\partial t}$

$$\nabla \cdot A' \quad \mu_0 \epsilon_0 \frac{\partial V'}{\partial t} \quad A \quad \mu_0 \epsilon_0 \frac{\partial V}{\partial t} \quad {}^2 \quad \mu_0 \epsilon_0 \frac{\partial^2 \lambda}{\partial t^2}$$

Find λ, $\nabla^2 \lambda - \mu_0 \epsilon_0 \frac{\partial^2 \lambda}{\partial t^2} = \nabla \cdot A' + \mu_0 \epsilon_0 \frac{\partial V'}{\partial t}$

Then ,you have a set if solutions and A, and V

$\nabla \cdot A = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

10.2 Continuous Distributions :

With the Lorentz gauge $\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \dfrac{\partial V}{\partial t}$,

$$\Box^2 V = -\frac{1}{\epsilon_0}\rho$$

where $\Box^2 = \nabla^2 - \mu_0 \epsilon_0 \dfrac{\partial^2}{\partial t^2}$

$$\Box^2 \mathbf{A} = -\mu_0 \mathbf{J}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

In the static case $\Box^2 \to \nabla^2$,

$$\left.\begin{aligned} \nabla^2 V &= -\frac{1}{\epsilon_0}\rho \\ \nabla^2 \mathbf{A} &= -\mu_0 \mathbf{J} \end{aligned}\right\}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{R}\, d\tau'$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}')}{R}\, d\tau'$$

10.2 (2)

For nonstatic case, the above solutions only valid when

$$|\mathbf{r}-\mathbf{r}'| << (t-t_r)c \qquad (\ ',\quad)$$

for , and due to and ,

where $A(\mathbf{r}, t)$ $\rho(\mathbf{r}', t$) of $\mathbf{J}\ \mathbf{r}\ \ t_r$

is $V(\mathbf{r}, t)$ retarded time. Because the message

the pensencet of and must travel a distance

the delay is ; that is , $R = |\mathbf{r}-\mathbf{r}'|$,

$\mathbf{J}$

R / c

$$t_r \equiv t - \frac{R}{c} \qquad \text{(Causality)}$$

10.2 (3)

The solutions of retarded potentials for nonstatic sources are

$$V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}',t_r)}{R}\,d\tau'$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}',t_r)}{R}\,d\tau'$$

Proof:

$$\nabla V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int \left[(\nabla\rho)\frac{1}{R} + \rho\nabla\left(\frac{1}{R}\right) \right] d\tau'$$

$$\nabla\frac{1}{R} = -\frac{1}{R^2}\nabla R = -\frac{\hat{\mathbf{R}}}{R^2} \qquad R = |\mathbf{r}-\mathbf{r}'|$$

$$\nabla\rho(\mathbf{r}',t_r) \qquad t_r = t - \frac{R}{c}) = \frac{\partial\rho}{\partial t_r}\nabla t_r = \dot{\rho}\left(-\frac{1}{c}\nabla R\right) = -\frac{\dot{\rho}}{c}\hat{\mathbf{R}}$$

10.2 (4)

$$\nabla V = \frac{1}{4\pi\varepsilon_0}\int\left[-\frac{\rho}{c}\frac{\hat{R}}{R} - \rho\frac{\hat{R}}{R^2}\right]d\tau'$$

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0}\int\left\{-\frac{1}{c}\left[\frac{\hat{R}}{R}\cdot(\nabla\rho) + \rho\nabla\cdot\frac{\hat{R}}{R}\right] - \left[\frac{\hat{R}}{R^2}\cdot(\nabla\rho) + \rho\nabla\cdot\left(\frac{\hat{R}}{R^2}\right)\right]\right\}d\tau'$$

$$\nabla\rho = \frac{\partial\rho}{\partial t_r}\nabla t_r = -\frac{\rho}{c}\hat{R}$$

$$\nabla\cdot\left(\frac{\hat{R}}{R}\right) = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{1}{R}\right) = \frac{1}{R^2} \qquad \nabla\cdot\left(\frac{\hat{R}}{R^2}\right) = 4\pi\delta^3(R)$$

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0}\int\left\{\left[\frac{1}{c^2}\frac{\rho}{R} - \frac{1}{c}\frac{\rho}{R^2}\right] + \left[\frac{1}{c}\frac{\rho}{R^2} + \rho\,4\pi\delta^3(R)\right]\right\}d\tau'$$

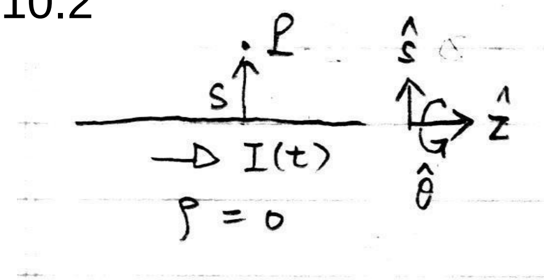
$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r', t_r)}{R}d\tau'\right) - \frac{\rho(r, t_r = t)}{\varepsilon_0}$$

$$= \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} - \frac{1}{\varepsilon_0}\rho(r, t)$$

The same procedure is for proving A.

10.2 (5)

Example 10.2



$$I(t) = \begin{cases} 0, & \text{for} \quad t \le 0 \\ I_0, & \text{for} \quad t > 0 \end{cases}$$

$E(s, t) = ?$

$B(s, t) = ?$

Solution:

$$\rho = 0 \Rightarrow V = 0$$

$$A(s, t) = \frac{\ _0}{4\pi}\hat{z}\int_{-\infty}^{\infty} \frac{\ _r\, dz}{R}$$

for $t < \frac{s}{c}$, $A(s, t) = 0$ $\quad E(s, t) = B(s, t) = 0$

for $\quad t > \frac{s}{c}$, only $\quad |z| \le \sqrt{(ct\ ) \ - s}$ $\quad$ contribute s

10.2 (6)

$$A(s,t) = \left(\frac{\mu_0 I_0}{4\pi}\hat{z}\right) 2 \int_0^{\sqrt{(ct)^2 - s^2}} \frac{dz}{\sqrt{s^2 + z^2}}$$

$$= \frac{\mu_0 I_0}{2\pi} \hat{z} \ln\left(\sqrt{s^2 + z^2} + z\right)\Big|_0^{\sqrt{(ct)^2 - s^2}}$$

$$= \frac{\mu_0 I_0}{2\pi} \hat{z} \left[\ln\left(ct + \sqrt{(ct)^2 - s^2}\right) - \ln s\right]$$

$$= \frac{\mu_0 I_0}{2\pi} \ln\left(\frac{ct + \sqrt{(ct)^2 - s^2}}{s}\right)\hat{z}$$

$$\frac{d}{dz} \ln\left(\sqrt{s^2 + z^2} + z\right)$$

$$= \frac{\frac{1}{2}\frac{2z}{\sqrt{s^2+z^2}} + 1}{\sqrt{s^2 + z^2} + z}$$

$$= \frac{1}{\sqrt{s^2 + z^2}}$$

$$E(s,t) = -\frac{\partial A}{\partial t} = -\frac{\mu_0 I_0 c}{2\pi\sqrt{(ct)^2 - s^2}}\hat{z}$$

$$B(s,t) = \nabla \times A = -\frac{\partial A_z}{\partial s}\hat{\phi}$$

$$= -\frac{\mu_0 I_0}{2\pi} \frac{s}{ct + \sqrt{(ct)^2 - s^2}} \left\{ -\frac{1}{s^2}\left(ct + \sqrt{(ct)^2 - s^2}\right) + \frac{1}{s}\left(\frac{1}{2}\right)\frac{(-2s)}{\sqrt{(ct)^2 - s^2}} \right\}\hat{\phi}$$

$$2\pi\, ct + \sqrt{(ct)^2 - s^2} \quad \left(s \;\; 2 \sqrt{(ct)^2 - s^2} \right.$$

10.2 (7)

$$= \frac{\mu_0 I_0}{2\pi s}\,\frac{1}{ct+\sqrt{(ct)^2-s^2}}\left[\frac{s^2+ct\sqrt{(ct)^2-s^2}+(ct)^2-s^2}{\sqrt{(ct)^2-s^2}}\right]$$

$$B(s,t) = \frac{\mu_0 I_0}{2\pi s}\,\frac{ct}{\sqrt{(ct)^2-s^2}}\,\hat{\phi}$$

Note:

$$D = \frac{ct+\sqrt{(ct)^2-s^2}}{s} = \alpha+\sqrt{\alpha^2-1} \qquad \alpha = \frac{ct}{s}$$

$$\frac{\partial}{\partial t}\ln D = \frac{c}{s}\frac{\partial}{\partial \alpha}\ln D = \frac{c}{s}\frac{1}{D}\left[1+\frac{2\alpha}{2\sqrt{\alpha^2-1}}\right]$$

$$= \frac{c}{s}\frac{1}{D}\frac{\alpha+\sqrt{\alpha^2-1}}{\sqrt{\alpha^2-1}} = \frac{c}{s}\frac{1}{\sqrt{\alpha^2-1}}$$

$$= \frac{c}{s}\frac{1}{\sqrt{\left(\frac{ct}{s}\right)^2-1}} = \frac{c}{\sqrt{(ct)^2-s^2}}$$

s

10.2 (8)

$$\frac{\partial}{\partial s}\ln D = \frac{\partial \alpha}{\partial s}\frac{\partial}{\partial \alpha}\ln D = \frac{\partial \alpha}{\partial s}\cdot\frac{\partial t}{\partial \alpha}\cdot\frac{\partial}{\partial t}\ln D$$

$$= ct\left(-\frac{1}{s^2 c}\right)\cdot s\cdot\frac{\partial}{\partial t}\ln D$$

$$= -\frac{t}{s}\,\frac{c}{\sqrt{(ct)^2 - s^2}}$$

$t \rightarrow \infty,$

$$\left.\begin{aligned} &E = 0 \\ &B = \frac{\mu_0 I_0}{2\pi s}\hat{\phi} \end{aligned}\right\}$$ recover the static case

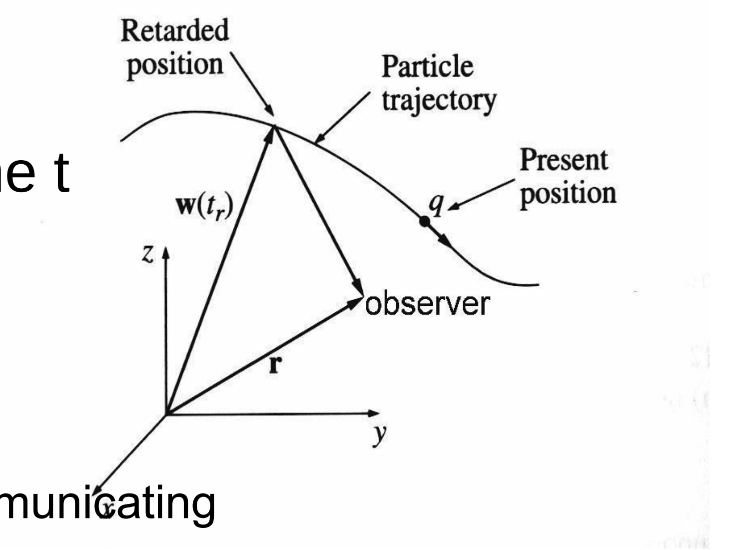
10.3 Point Charges

10.3.1 Lienard-Wiechert Potentials

10.3.2 The Fields of a Moving Point Charge

10.3.1 Lienard-Wiechert potentials

Consider a point charge q moving on a trajectory W(t)



retarded position $R = r - w(t_r)$

location of the observer at time t

$$t_r = t - \frac{R}{c}$$

Two issues

•There is at most one point on the trajectory communicating with r at any time t.

Suppose there are two points:

$$R_1 = c(t - t_1) \quad R_2 = c(t - t_2),$$

$$R_1 - R_2 = c(t_2 - t_1)$$

$$\rightarrow \langle V \rangle = \frac{R_1 - R_2}{t_2 - t_1} = c$$

Since q can not move at the speed of light,
there is only one point at meet.

10.3.1 (2)

- the point chage

$$\int \rho(\mathbf{r}', t_r)\, d\tau' = \frac{q}{1 - \hat{R} \cdot V / c}$$

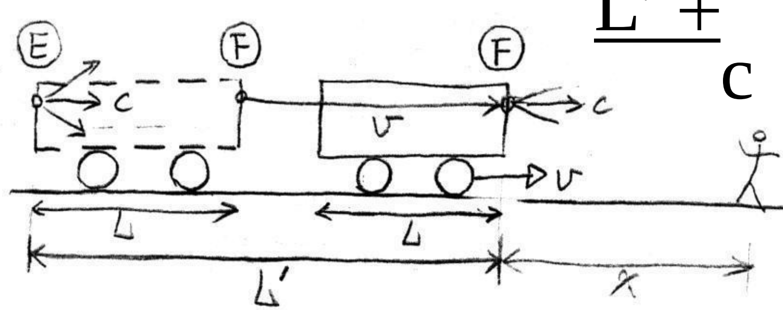
due to Doppler –shift effect as the point charge is considered as an extended charge.

Proof.

consider the extended charge has a length L as a

train (a) moving directly to the observer

time for the light to arrive the observer.



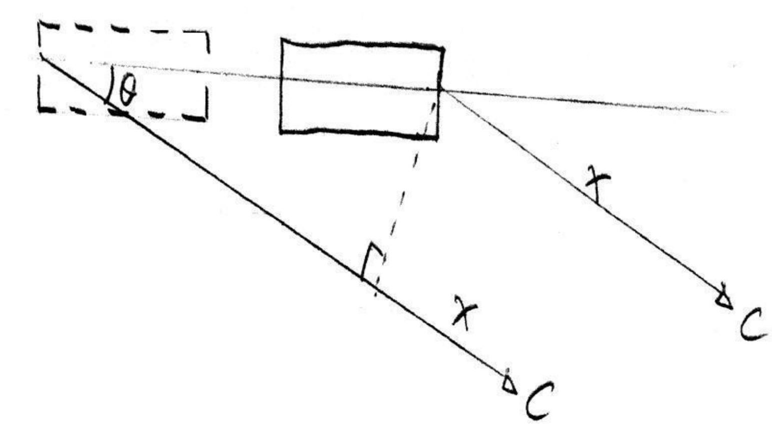
$$\frac{L' + x}{c} = \frac{L' - L}{v} + \frac{x}{c}$$

(E) (F)

$$L' = \frac{L}{1 - v / c}$$

10.3.1 (3)

(b) moving with an angle θ to the observer



$$\frac{L'\cos\theta + x}{c} = \frac{L' - L}{v} + \frac{x}{c}$$

$$L' = \frac{L}{1 - v\cos\theta / c}$$

∴ The apparent volume

$$\tau' = \frac{\tau}{1 - \hat{R}\cdot v / c}$$

τ ← actual volume

$$\therefore \int \rho(r', t_r)\, d\tau' = \frac{q}{1 - \hat{R}\cdot v / c}$$

10.3.1 (4)

$$V(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{R} d\tau' = \frac{1}{4\pi\varepsilon_0} \frac{q}{R\left(1 - \frac{\hat{\mathbf{R}} \cdot \mathbf{v}}{c}\right)}$$

$$\boxed{V(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0} \frac{qc}{Rc - \mathbf{R} \cdot \mathbf{v}}}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\rho(\mathbf{r}', t_r)\mathbf{v}}{R} d\tau' = \frac{\mu_0}{4\pi} \frac{\mathbf{v}}{R} \int \rho(\mathbf{r}', t_r)\, d\tau'$$

$$\boxed{\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{qc\mathbf{v}}{(Rc - \mathbf{R} \cdot \mathbf{v})} = \frac{\mathbf{v}}{c^2} V(\mathbf{r}, t)}$$

→ Lienard-Wiechert Potentials for a moving point charge

10.3.1 (5)

Example 10.3

$V(r, t) = ?$ (

$,) = ?$

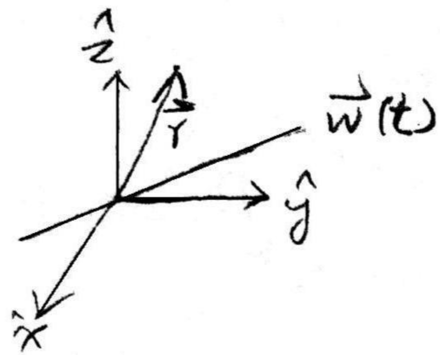
v

cons

$A\ r\ t$

t

Solution:



let $\quad w(t = 0) = 0$

$\therefore \quad w(t) = vt$

$$R = |r - vt_r| = c(t - t_r)$$

$$r^2 - 2r \cdot vt_r + v^2\ {}_r^2 = c^2(t^2 - 2\,t\,t_r + t_r^2)$$

$$t_r = \frac{(c^2 t - r \cdot v) \pm \sqrt{(c^2 t - r \cdot v)^2 + (c^2 - v^2)(r^2 - c^2 t^2)}}{c^2 - v^2} \quad 1$$

consider $v \to 0, \quad t_r = t \pm \frac{r}{c} \to t - \frac{r}{c}$ retarded

$$\left\lfloor \therefore \text{choose} - \text{sign} \right\rfloor$$

10.3.1 (6)

$$R = c(t - t_r) \qquad \hat{R} = \frac{\mathbf{r} - \mathbf{v}t_r}{c(t - t_r)}$$

$$\therefore\ R\left(1 - \frac{\hat{R}\cdot\mathbf{v}}{c}\right) = c(t - t_r)\left[1 - \frac{\mathbf{v}}{c}\cdot\frac{\mathbf{r} - \mathbf{v}t_r}{c(t - t_r)}\right]$$

$$= c(t - t_r) - \frac{\mathbf{v}\cdot\mathbf{r}}{c} - \frac{v^2}{c}t_r$$

$$① = \frac{1}{c}\sqrt{(c^2 t - \mathbf{r}\cdot\mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2 t^2)}$$

$$V(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0}\frac{qc}{Rc\left(1 - \frac{\hat{R}\cdot\mathbf{v}}{c}\right)} = \frac{1}{4\pi\varepsilon_0}\frac{qc}{\sqrt{(c^2 t - \mathbf{r}\cdot\mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2 t^2)}}$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mathbf{v}}{c^2}V(\mathbf{r}, t) = \frac{\mu_0}{4\pi}\frac{qc\mathbf{v}}{\sqrt{(c^2 t - \mathbf{r}\cdot\mathbf{v})^2 + (c^2 - v^2)(r^2 - c^2 t^2)}}$$

$$\frac{1}{c^2} = \mu_0\varepsilon_0$$

10.3.2 The Fields of a Moving Point Charge

Lienard-Weichert potentials:

$$V(r, t) = \frac{1}{4\pi\varepsilon_0} \frac{qc}{(Rc - R \cdot v)}$$

$$A(r, t) = \frac{v}{c^2} V(r, t)$$

$$E = -\nabla V - \frac{\partial A}{\partial t}$$

$$B = \nabla \times A$$

$$R = r - w(t_r), \quad R = c(t - t_r), \quad \text{and} \quad v = w(t_r)$$

Math., Math., and Math,…. are in the following:

$$\nabla V = \frac{qc}{4\pi\varepsilon_0} \frac{-1}{(Rc - R \cdot v)^2} \nabla(Rc - R \cdot v)$$

10.3.2 (2)

$$\nabla R = -c\nabla t_r$$

$$\nabla(R\cdot v) = (R\cdot\nabla)v + (v\cdot\nabla)R + R\times(\nabla\times v) + v\times(\nabla\times R)$$

$$(R\cdot\nabla)v = R_i\,\partial_i v_j(t_r)\,\hat{j} = R_i\,\frac{\partial t_r}{\partial i}\frac{\partial v_j}{\partial t_r}\,\hat{j} = a(R\cdot\nabla t_r)$$

$a = \dot{v}$

$$(v\cdot\nabla)R = (v\cdot\nabla)r - (v\cdot\nabla)w$$

$$(v\cdot\nabla)r = v_i\partial_i r_j\hat{j} = v_i\delta_{ij}\hat{j} = v$$

$$(v\cdot\nabla)w = v_i\,\partial_i w_j(t_r)\,\hat{j} = v_i\,\frac{\partial w_j}{\partial t_r}\frac{\partial t_r}{\partial i}\,\hat{j} = v(v\cdot\nabla t_r)$$

$$\nabla\times v = \partial_i v_j(t_r)\hat{k} = \frac{\partial v_j}{\partial t_r}\frac{\partial t_r}{\partial i}\hat{k} = a_j\partial_i t_r\hat{k} = -a\times\nabla t_r$$

$$\nabla\times R = \nabla\times r - \nabla\times w(t_r) = -(-v\times\nabla t_r) = v\times\nabla t_r$$

$$\nabla(R\cdot v) = a(R\cdot\nabla t_r) + v + v(v\cdot\nabla t_r) - R\times(a\times\nabla t_r) + v\times(v\times\nabla t_r)$$

0 =

0

$$a(R\cdot\nabla t_r) - \nabla t_r(R\cdot a)$$

10.3.2 (4)

$$\nabla V = \frac{qc}{4\pi\varepsilon_0}\frac{-1}{(Rc - \mathbf{R}\cdot\mathbf{v})^2}\nabla(Rc - \mathbf{R}\cdot\mathbf{v})$$

$$= \frac{qc}{4\pi\varepsilon_0}\frac{-1}{(Rc - \mathbf{R}\cdot\mathbf{v})^2}\left[-c^2\nabla t_r - \mathbf{v} - (\mathbf{R}\cdot\mathbf{a} - v^2)\nabla t_r\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qc}{(Rc - \mathbf{R}\cdot\mathbf{v})^2}\left[\mathbf{v} + (c^2 - v^2 + \mathbf{R}\cdot\mathbf{a})\frac{-\mathbf{R}}{Rc - \mathbf{R}\cdot\mathbf{v}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qc}{(Rc - \mathbf{R}\cdot\mathbf{v})^3}\left[(Rc - \mathbf{R}\cdot\mathbf{v})\mathbf{v} - (c^2 - v^2 + \mathbf{R}\cdot\mathbf{a})\mathbf{R}\right]$$

$$\frac{\partial\mathbf{A}}{\partial t} \overset{\text{Prob.10.17}}{=\!=} \frac{1}{4\pi\varepsilon_0}\frac{qc}{(Rc - \mathbf{R}\cdot\mathbf{v})^3}\left[(Rc - \mathbf{R}\cdot\mathbf{v})\left(-\mathbf{v} + \frac{R\mathbf{a}}{c}\right) + \frac{R}{c}(c^2 - v^2 + \mathbf{R}\cdot\mathbf{a})\mathbf{v}\right]$$

10.3.2 (3)

$$0 = \mathbf{v} \times (\mathbf{v} \times \nabla t_r) = \mathbf{v}(\mathbf{v} \cdot \nabla t_r) - \nabla t_r (\mathbf{v} \cdot \mathbf{v})$$

$$\therefore \mathbf{v}(\mathbf{v} \cdot \nabla t_r) = v^2 \nabla t_r$$

$$\nabla(\mathbf{R} \cdot \mathbf{v}) = \mathbf{v} + (\mathbf{R} \cdot \mathbf{a} - v^2) \nabla t_r$$

$$-c \nabla t_r = \nabla R = \nabla(\sqrt{\mathbf{R} \cdot \mathbf{R}}) = \frac{1}{2\sqrt{\mathbf{R} \cdot \mathbf{R}}} \nabla(\mathbf{R} \cdot \mathbf{R})$$

$$= \frac{1}{R}\left[(\mathbf{R} \cdot \nabla)\mathbf{R} + \mathbf{R} \times (\nabla \times \mathbf{R})\right]$$

$\mathbf{R} - \mathbf{v}(\mathbf{R} \cdot \nabla t_r)$; $\mathbf{v} \times \nabla t_r$

$$= \frac{1}{R}\left[\mathbf{R} - \mathbf{v}(\mathbf{R} \cdot \nabla t_r) + \mathbf{R} \times (\mathbf{v} \times \nabla t_r)\right]$$

$\mathbf{v}(\mathbf{R} \cdot \nabla t_r) - \nabla t_r (\mathbf{R} \cdot \mathbf{v})$

$$= \frac{1}{R}\left[\mathbf{R} - (\mathbf{R} \cdot \mathbf{v}) \nabla t_r\right]$$

$-R$

$$\nabla t\, r = \frac{}{Rc - R \cdot v}$$

10.3.2 (5)

$$\mathbf{E}(\mathbf{r},t) = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\frac{1}{4\pi\varepsilon_0}\frac{qc}{(Rc - \mathbf{R}\cdot\mathbf{v})^3}\Big[(Rc - \mathbf{R}\cdot\mathbf{v})\mathbf{v} - (c^2 - v^2 + \mathbf{R}\cdot\mathbf{a})\frac{\mathbf{R}}{c} + (Rc - \mathbf{R}\cdot\mathbf{v})\Big(-\mathbf{v} + \frac{R\mathbf{a}}{c}\Big) + \frac{R}{c}(c^2 - v^2 + \mathbf{R}\cdot\mathbf{a})\mathbf{v}\Big]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qR}{(Rc - \mathbf{R}\cdot\mathbf{v})^3}\Big[(c^2 - v^2 + \mathbf{R}\cdot\mathbf{a})(c\hat{\mathbf{R}} - \mathbf{v}) - \mathbf{R}\cdot(c\hat{\mathbf{R}} - \mathbf{v})\mathbf{a}\Big]$$

define $\mathbf{u} = c\hat{\mathbf{R}} - \mathbf{v} \rightarrow \mathbf{R}\cdot\mathbf{u} = Rc - \mathbf{R}\cdot\mathbf{v}$

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qR}{(\mathbf{R}\cdot\mathbf{u})^3}\Big[(c^2 - v^2)\mathbf{u} + \mathbf{u}(\mathbf{R}\cdot\mathbf{a}) - \mathbf{a}(\mathbf{R}\cdot\mathbf{u})\Big]$$

$$\boxed{\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\frac{qR}{(\mathbf{R}\cdot\mathbf{u})^3}\Big[(c^2 - v^2)\mathbf{u} + \mathbf{R}\times(\mathbf{u}\times\mathbf{a})\Big]}$$

$\mathbf{u} = c\hat{\mathbf{R}} - \mathbf{v}$

generalized Coulomb field radiation field or acceleration field dominates at large R

if $\mathbf{a} = 0,\ \mathbf{v} = 0,$ $\mathbf{E}(\mathbf{r},t) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R^2}\hat{\mathbf{R}}$ Electrostatic field

10.3.2 (6)

$$\mathbf{A} = \frac{\mathbf{v}}{c^2} V$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{1}{c^2} \nabla \times (V\mathbf{v}) = \frac{1}{c^2} \left[V(\nabla \times \mathbf{v}) - \mathbf{v} \times (\nabla V) \right]$$

$$\nabla \times \mathbf{v} = -\mathbf{a} \times \nabla t_r = \frac{\mathbf{a} \times \mathbf{R}}{Rc - \mathbf{R} \cdot \mathbf{v}}$$

$$\nabla V = \frac{1}{4\pi\varepsilon_0} \frac{qc}{(Rc - \mathbf{R} \cdot \mathbf{v})^3} \left[(Rc - \mathbf{R} \cdot \mathbf{v})\mathbf{v} - (c^2 - v^2 + \mathbf{R} \cdot \mathbf{a})\mathbf{R} \right]$$

$$\mathbf{B} = \frac{1}{c^2} \left[\frac{1}{4\pi\varepsilon_0} \frac{qc}{(Rc - \mathbf{R} \cdot \mathbf{v})} \frac{\mathbf{a} \times \mathbf{R}}{(Rc - \mathbf{R} \cdot \mathbf{v})} + \frac{1}{4\pi\varepsilon_0} \frac{qc}{(Rc - \mathbf{R} \cdot \mathbf{v})^3} (c^2 - v^2 + \mathbf{R} \cdot \mathbf{a})\, \mathbf{v} \times \mathbf{R} \right]$$

10.3.2 (7)

$$= \frac{1}{c}\frac{q}{4\pi\varepsilon_0}\frac{1}{(\mathbf{R}\cdot\mathbf{u})^3}\mathbf{R}\times\left[-\mathbf{a}(\mathbf{R}\cdot\mathbf{u})-\mathbf{v}(\mathbf{R}\cdot\mathbf{a})-\mathbf{v}(c^2-v^2)\right]$$

$$= \frac{1}{c}\hat{\mathbf{R}}\times\left\{\frac{1}{4\pi\varepsilon_0}\frac{q\mathbf{R}}{(\mathbf{R}\cdot\mathbf{u})^3}\left[(\mathbf{R}-\mathbf{v})(c^2-v^2)+(\mathbf{R}-\mathbf{v})(\mathbf{R}\cdot\mathbf{a})-\mathbf{a}(\mathbf{R}\cdot\mathbf{u})\right]\right\}$$

$$= \frac{1}{c}\hat{\mathbf{R}}\times\left\{\frac{1}{4\pi\varepsilon_0}\frac{q\mathbf{R}}{(\mathbf{R}\cdot\mathbf{u})^3}\left[\mathbf{u}(c^2-v^2)+\mathbf{u}(\mathbf{R}\cdot\mathbf{a})-\mathbf{a}(\mathbf{R}\cdot\mathbf{u})\right]\right\}$$

$$= \frac{1}{c}\hat{\mathbf{R}}\times\left\{\frac{1}{4\pi\varepsilon_0}\frac{q\mathbf{R}}{(\mathbf{R}\cdot\mathbf{u})^3}\left[\mathbf{u}(c^2-v^2)+\mathbf{R}\times(\mathbf{u}\times\mathbf{a})\right]\right\}$$

$$= \frac{1}{c}\hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)$$

$$\boxed{\mathbf{B}(\mathbf{r},t)=\frac{1}{c}\hat{\mathbf{R}}\times\mathbf{E}(\mathbf{r},t)}$$

10.3.2 (8)

The force on a test charge Q with velocity $\mathbf{V}$ due to a moving charge q with velocity $\mathbf{v}$ is

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

$$= \frac{qQ\mathcal{R}}{4\pi\varepsilon_0 (\boldsymbol{\mathcal{R}} \cdot \mathbf{u})^3} \left\{ \left[(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathcal{R}} \times (\mathbf{u} \times \mathbf{a}) \right] + \frac{\mathbf{V}}{c} \times \left[\hat{\boldsymbol{\mathcal{R}}} \times \left[(c^2 - v^2)\mathbf{u} + \boldsymbol{\mathcal{R}} \times (\mathbf{u} \times \mathbf{a}) \right] \right] \right\}$$

Where

R, u, v, and a are all evaluated at t_r

10.3.2 (9)

$(\ ,\ ) = ?$

$E\ r\ t$

$B\ r\ t$

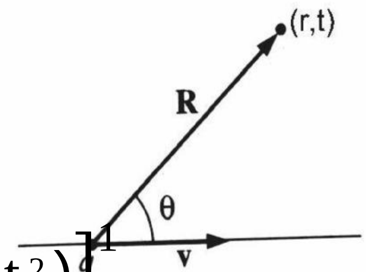
Example 10.4

q ● ⟶ $v = \text{const}$

Solution:

$a = 0 \qquad w = v\,t \qquad t = 0 \quad w \ \text{at origin}$

$$E(r,t) = \frac{q}{4\pi\varepsilon_0} \frac{(c^2 - v^2)R}{(R \cdot u)^3}\, u$$



$Ru = cR - Rv = c(r - Vt_r) - c(t - t_r)v = c(r - vt)$

Ex.10.3

$$R \cdot u = Rc - R \cdot v \ \overset{\text{Ex.10.3}}{=\!=} \ \left[(c^2 t - r \cdot v)^2 + (c^2 - v^2)(r^2 - c^2 t^2)\right]^{\frac{1}{2}}$$

$R = r - vt$

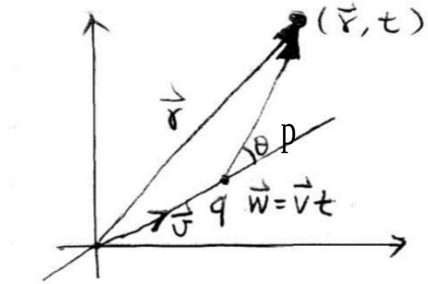
Prob.10.14

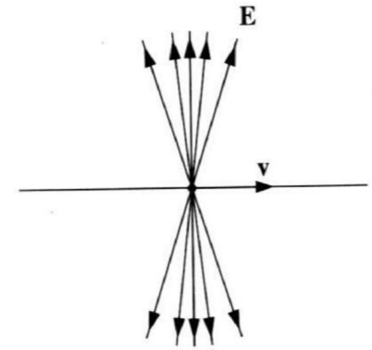
$$\overset{\text{Prob.10.14}}{=\!=} \ Rc\sqrt{1 - v^2 \sin^2\theta / c^2}$$

$$E(r,t) = \frac{q}{4\pi\varepsilon_0}(c^2 - v^2)\frac{cR}{\left[Rc(1 - v^2 \sin^2\theta / c^2)^{\frac{1}{2}}\right]^3}$$

10.3.2 (10)

$$E(r,t) = \frac{q}{\quad} \frac{1 - v^2/c^2}{\quad} \frac{\hat{R}}{\quad}$$



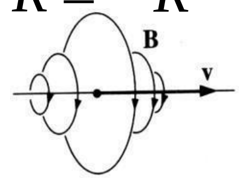


$$B(r,t) = \frac{1}{c}\left[\hat{R} \times E(r,t)\right]$$

$$B(r,t) = \frac{1}{c^2}\left[v \times E(r,t)\right]$$

E point to $\hat{p}$ by coincidence

$$\hat{R} = \frac{r - vt_r}{R} = \frac{(r - vt) + (t - t_r)v}{R} = \frac{P}{R} + \frac{v}{c}$$



when $v^2 << c^2$,

$$E(r,t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{P^2}\hat{R}, \quad B(r,t) = \frac{\mu_0}{4\pi}\frac{q}{P^2}(v \times \hat{P})$$

Coulomb`s law —Biot-savart Law for a point charge.‖

Introduction to antennas

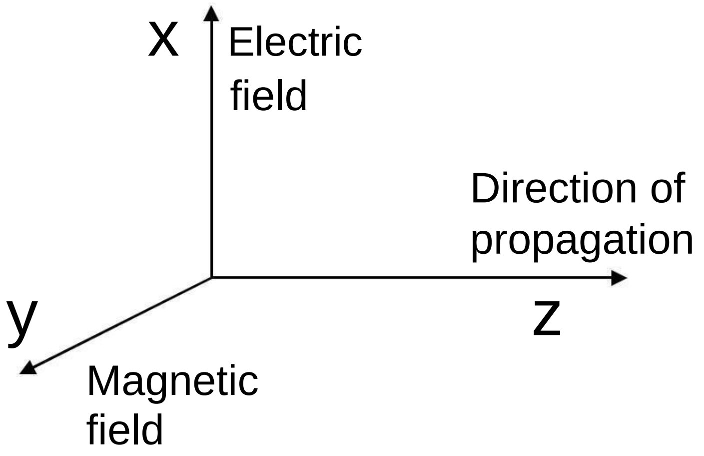
Michel Anciaux / APEX

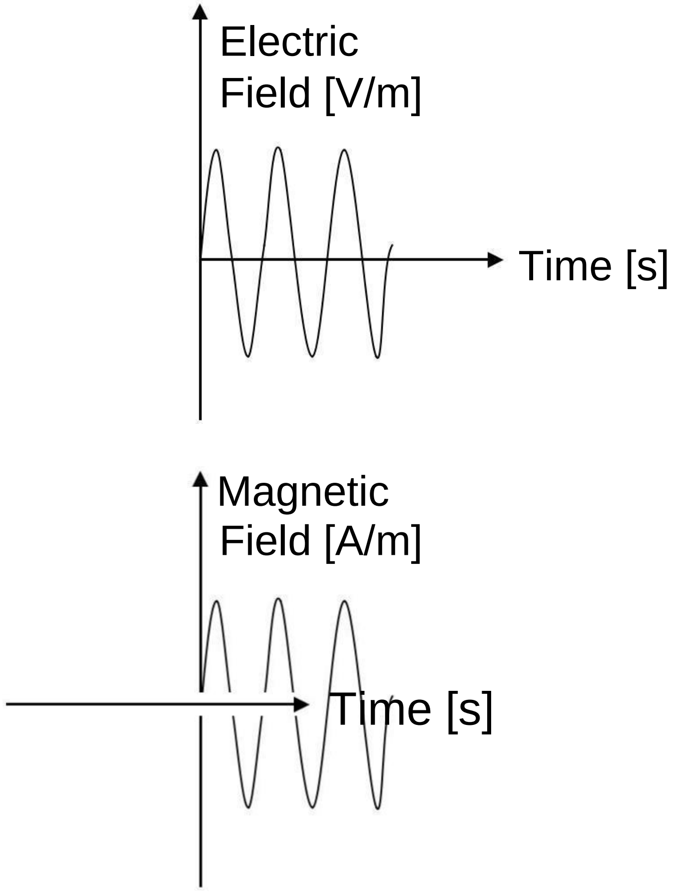
November 2004

What is an antenna?

- Region of transition between guided and free space propagation
- Concentrates incoming wave onto a sensor (receiving case)
- Launches waves from a guiding structure into space or air (transmitting case)
- Often part of a signal transmitting system over some distance
- Not limited to electromagnetic waves (e.g. acoustic waves)

Free space electromagnetic wave





- Disturbance of EM field
- Velocity of light (~300 000 000 m/s)
- E and H fields are orthogonal
- E and H fields are in phase
- Impedance, Z_0: 377 ohms

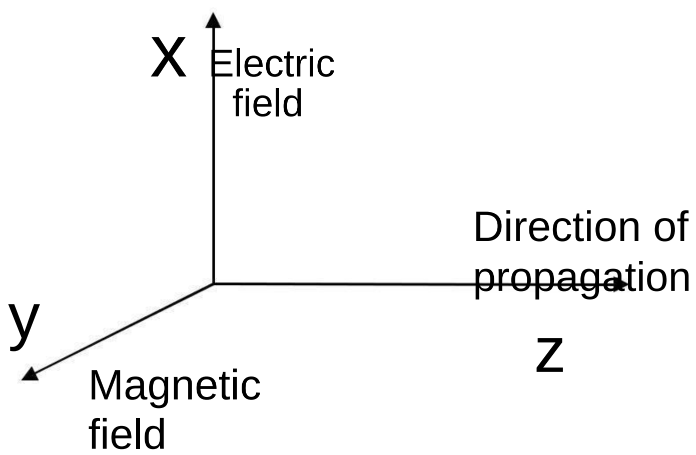
EM wave in free space

$$\frac{\partial^2 E_x}{\partial t} = \frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 E_x}{\partial z}$$

$$\frac{\partial^2 H_y}{\partial t} = \frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 H_y}{\partial z}$$

$$E_x = E_0 e^{j(\omega t \pm \beta z)}$$

$$H_y = H_0 e^{j(\omega t \pm \beta z)}$$



frequency $f = \frac{\omega}{2\pi}$

wavelength $\lambda = \frac{1}{\sqrt{\mu_0 \varepsilon_0}\, f}$

Phase constant $\beta = \frac{2\pi}{\lambda}$

$$Z_0 = \frac{E_0}{H_0} \qquad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Wave in lossy medium

$$E_x = E_0 e^{-\gamma z} e^{j\omega t} = E_0 \cdot e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{j\omega t}$$

Attenuation increases with z

Phase varies with z

Periodic time variation

$\gamma = \alpha + j\beta$ Propagation constant

α Attenuation constant

β Phase constant

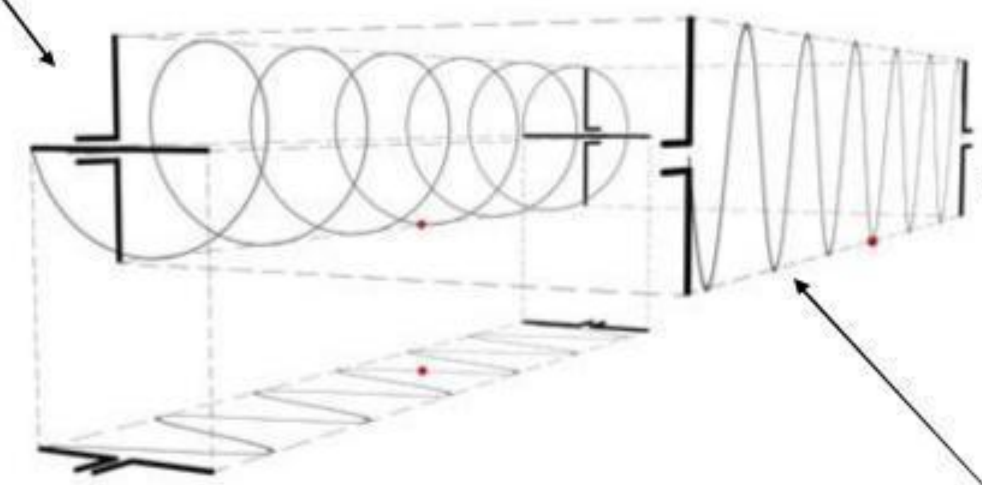
Power flow

Poynting vector $$S = E \times H$$

Average power density $$S_{av} = \frac{1}{2}\left|E_x\right|^2 \frac{1}{Z_0} = \frac{1}{2}\left|H_y\right|^2 Z_0$$

Polarisation of EM wave

circular



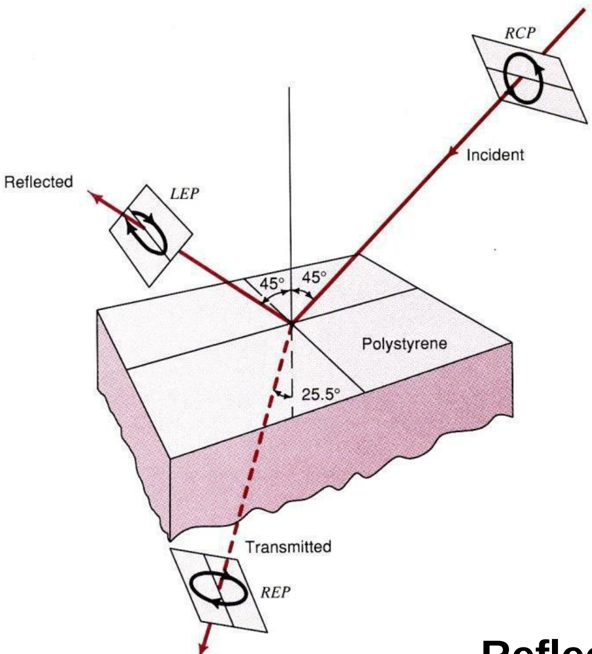
vertical

Electrical field, E horizontal

Reflection, refraction

Reflection $\theta_r = \theta_i$

Reflection coefficient: $\rho = \dfrac{E_r}{E_i}$ Depends on media, polarisation of incident wave and angle of incidence.



Refraction $\sin(\theta_t) = \dfrac{\eta_1}{\eta_2}\sin(\theta_i)$

if both media are lossless $\sin(\theta_t) = \sqrt{\dfrac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\sin(\theta_i)$

Reflection and refraction affect polarisation

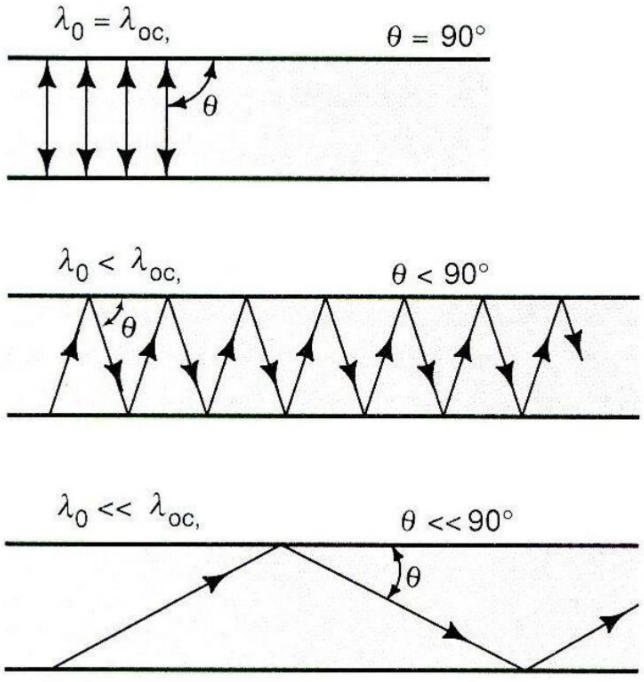
Guided electromagnetic wave

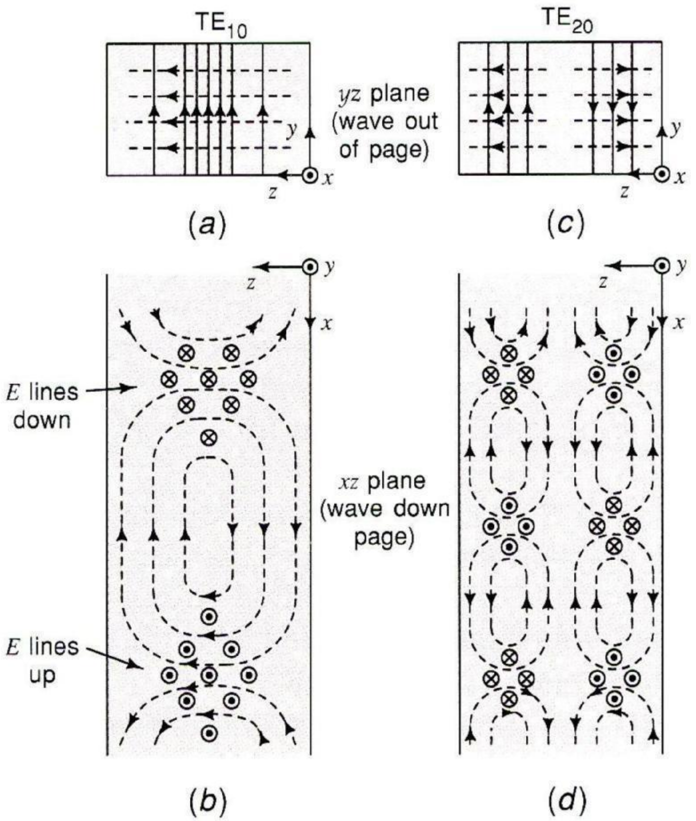
- Cables
  - Used at frequencies below 35 GHz
- Waveguides
  - Used between 0.4 GHz to 350 GHz
- Quasi-optical system
  - Used above 30 GHz

Guided electromagnetic wave (2)

- TEM wave in cables and quasi-optical systems (same as free space)
- TH,TE and combinations in waveguides
  - E or H field component in the direction of propagation
  - Wave bounces on the inner walls of the guide
  - Lower and upper frequency limits
  - Cross section dimensions proportional to wavelength

Rectangular waveguide





Launching of EM wave

Open up the cable and separate wires

↓

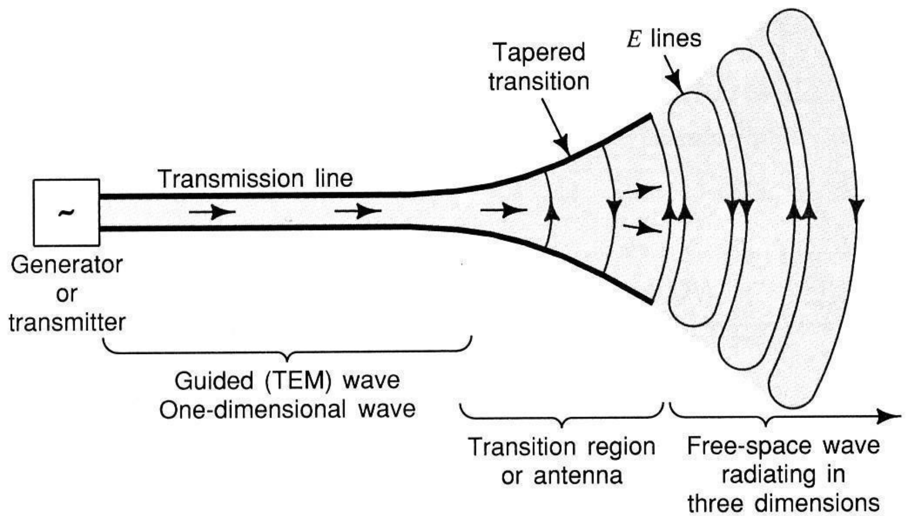
Dipole antenna

Open and flare up wave guide

↓

Horn antenna

Transition from guided wave to free space wave



Reciprocity

- Transmission and reception antennas can be used interchangeably
- Medium must be linear, passive and isotropic
- Caveat: Antennas are usually optimised for reception or transmission not both !

Basic antenna parameters

- Radiation pattern
- Beam area and beam efficiency
- Effective aperture and aperture efficiency
- Directivity and gain
- Radiation resistance

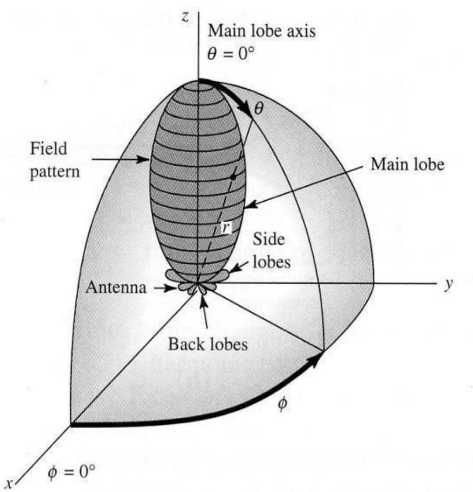
Radiation pattern

- Far field patterns
- Field intensity decreases with increasing distance, as 1/r
- Radiated power density decreases as $1/r^2$
- Pattern (shape) independent on distance
- Usually shown only in principal planes

$$\text{Far field}: r > \frac{2D^2}{\lambda}$$

D : largest dimension of the antenna e.g. r > 220 km for APEX at 1.3 mm !

Radiation pattern (2)



Field patterns

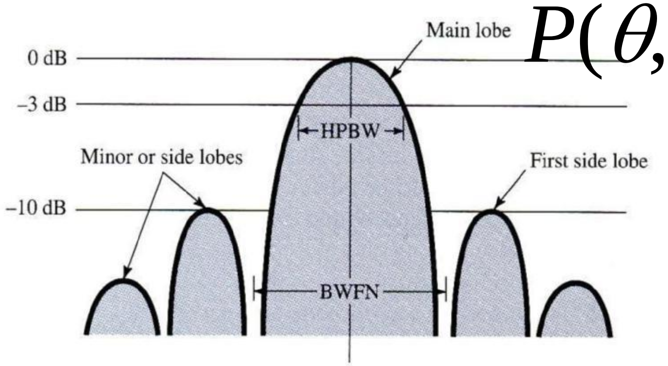
$$E_\theta(\theta,\phi) \qquad E_\phi(\theta,\phi)$$

+ phase patterns

$$\varphi_\theta(\theta,\phi) \qquad \varphi_\phi(\theta,\phi)$$

$$P(\theta,\phi) = \frac{E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)}{Z_0} r^2$$

$$P_n(\theta,\phi) = \frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}$$



HPBW: half power beam width

Beam area and beam efficiency

Beam area

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi)\cdot\sin(\theta)\,d\theta\,d\phi = \iint_{4\pi} P_n(\theta,\phi)\,d\Omega$$

Main beam area

$$\Omega_M = \iint_{\substack{Main\\beam}} P_n(\theta,\phi)\,d\Omega$$

Minor lobes area

$$\Omega_m = \iint_{\substack{\min or\\lobes}} P_n(\theta,\phi)\,d\Omega$$

$$\Omega_A = \Omega_M + \Omega_m$$

Main beam efficiency

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A}$$

Effective aperture and aperture efficiency

Receiving antenna extracts power from incident wave

$$P_{rec} = S_{in} \cdot A_e$$

Aperture and beam area are linked: $A_e = \frac{\lambda^2}{\;}$

For some antennas, there is a clear physical aperture and an aperture efficiency can be defined

$$\varepsilon_{ap} = \frac{A_e}{A_p}$$

Directivity and gain

Directivity
$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{average}}$$

From pattern
$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\,d\Omega} = \frac{4\pi}{\Omega_A}$$

From aperture $D = 4\pi \frac{A_e}{\lambda^2}$ Isotropic antenna: $\Omega_A = 4\pi \quad D = 1$

Gain $G = k_g D$

k_g = efficiency factor ($0 < k_g < 1$)

G is lower than D due to ohmic losses only

Radiation resistance

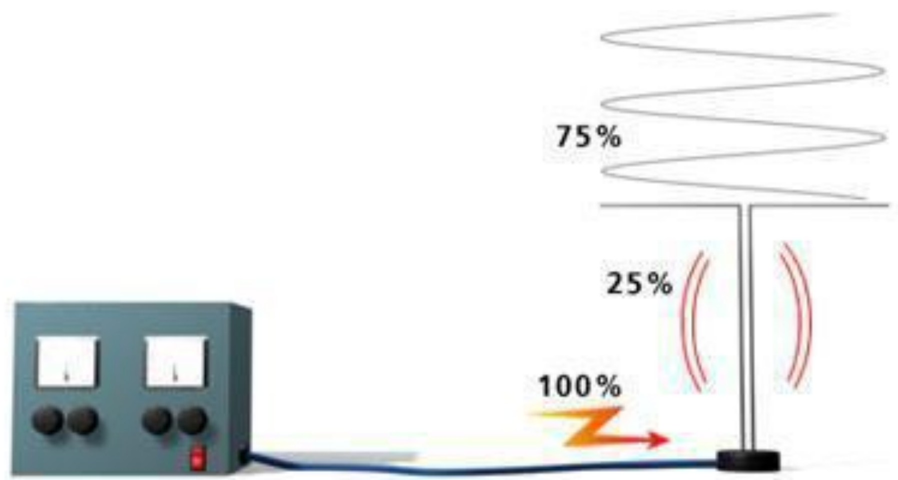
- Antenna presents an impedance at its terminals

$$Z_A = R_A + jX_A$$

- Resistive part is radiation resistance plus loss resistance

$$R_A = R_R + R_L$$

The radiation resistance does not correspond to a real resistor present in the antenna but to the resistance of space coupled via the beam to the antenna terminals.

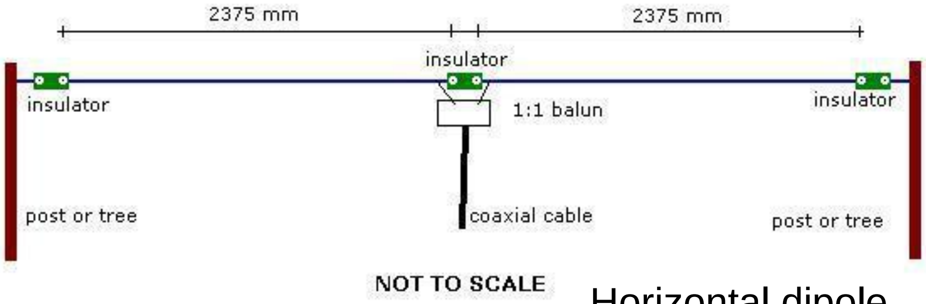


Types of Antenna

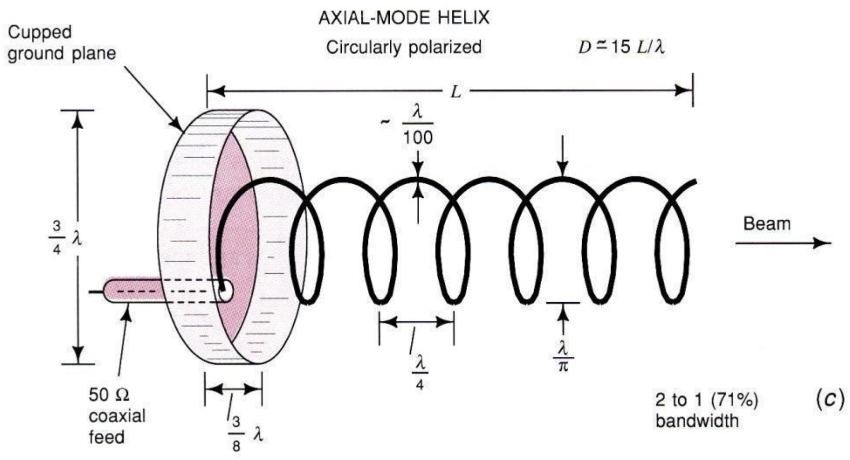
- Wire
- Aperture
- Arrays

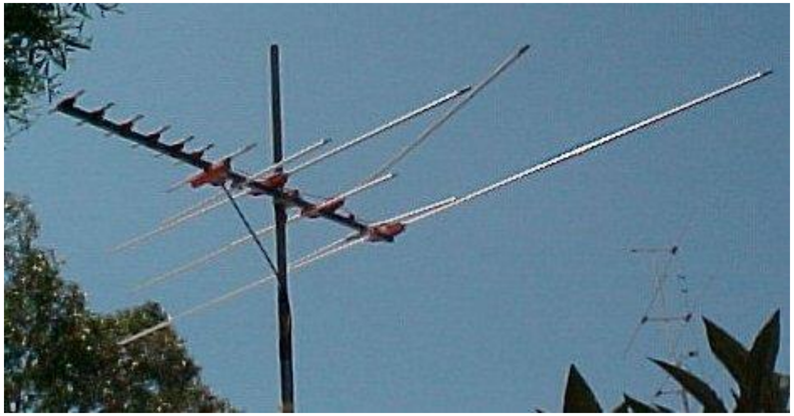
Wire antenna

- Dipole
- Loop
- Folded dipoles
- Helical antenna
- Yagi (array of dipoles)
- Corner reflector
- Many more types



Horizontal dipole

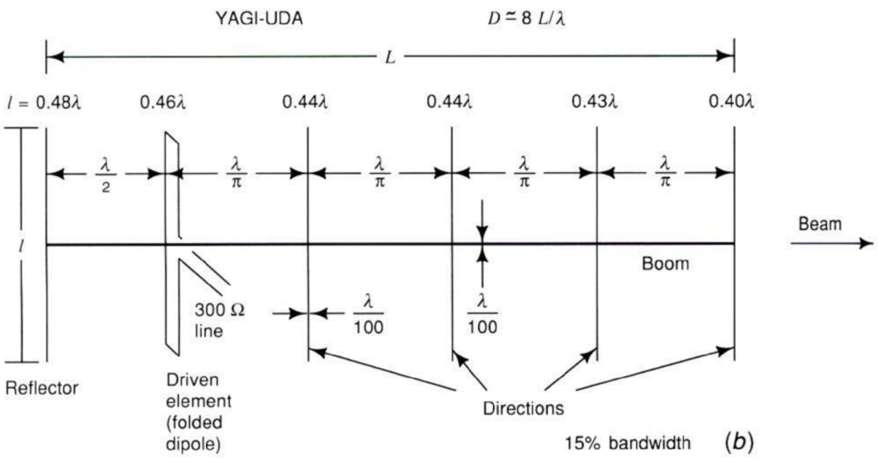


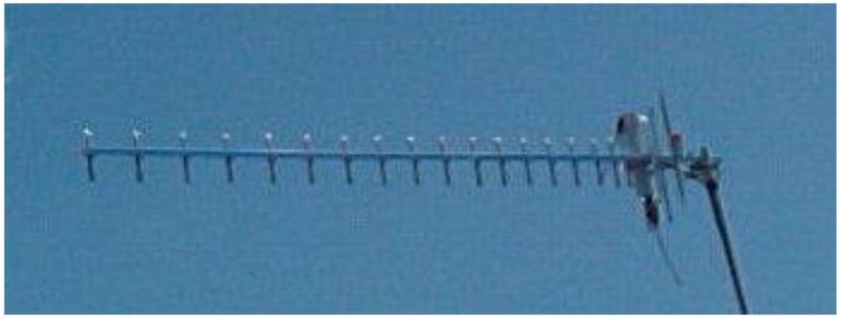


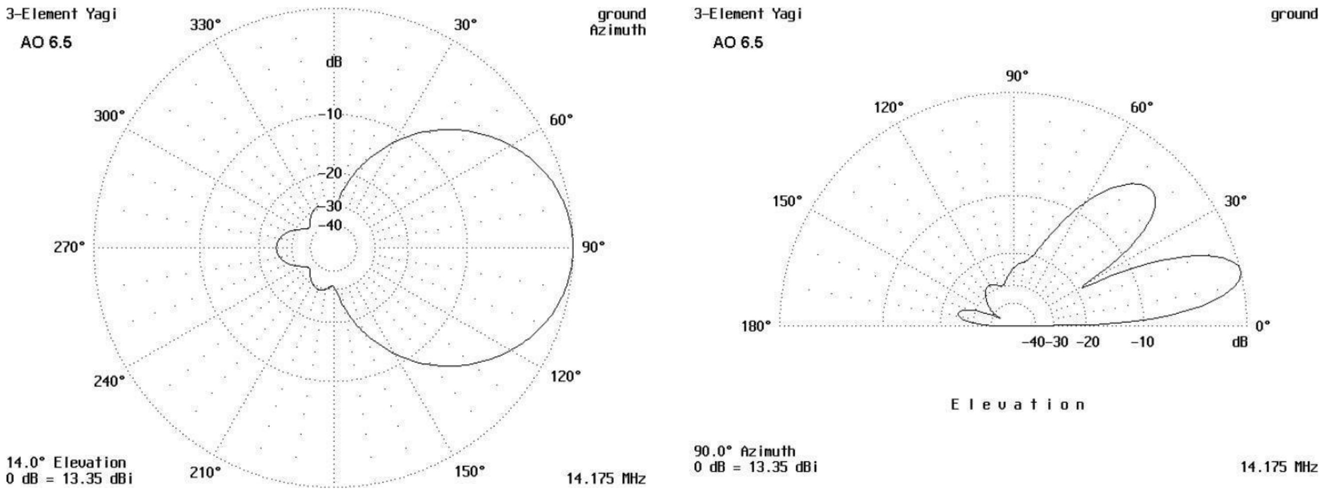
Wire antenna - resonance

- Many wire antennas (but not all) are used at or near resonance
- Some times it is not practical to built the whole resonant length
- The physical length can be shortened using loading techniques
  - Inductive load: e.g. center, base or top coil (usually adjustable)
  - Capacitive load: e.g. capacitance ―hats‖ (flat top at one or both ends)

Yagi-Uda







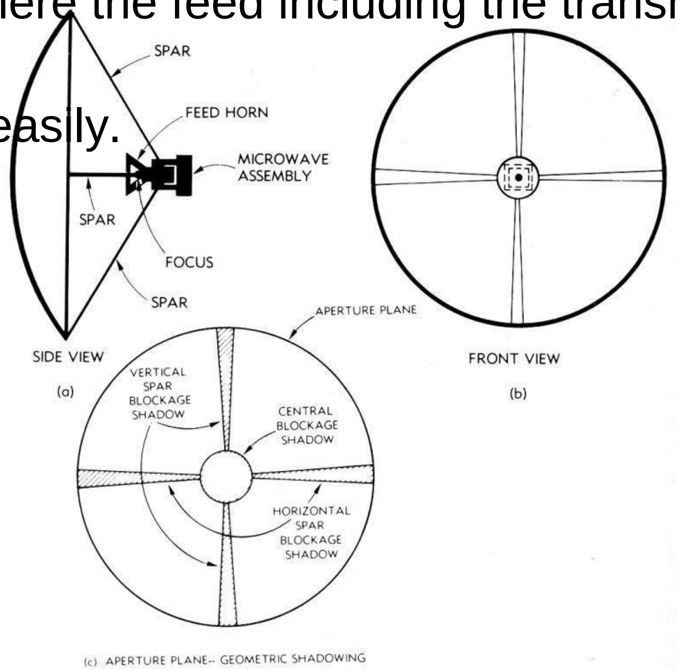
Elements	Gain dBi	Gain dBd
3	7.5	5.5
4	8.5	6.5
5	10	8
6	11.5	9.5
7	12.5	10.5
8	13.5	11.5

Aperture antenna

- Collect power over a well defined aperture
- Large compared to wavelength
- Various types:
  - Reflector antenna
  - Horn antenna
  - Lens

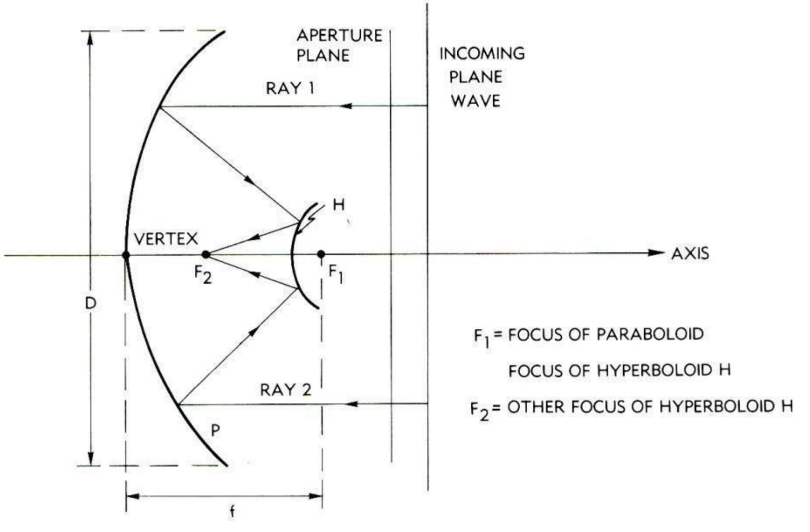
Reflector antenna

- Shaped reflector: parabolic dish, cylindrical antenna …
  - – Reflector acts as a large collecting area and concentrates power onto a focal region where the feed is located
- Combined optical systems: Cassegrain, Nasmyth …
  - – Two (Cassegrain) or three (Nasmyth) mirrors are used to bring the focus to a location where the feed including the transmitter/receiver can be installed more easily.



Cassegrain antenna

- Less prone to back scatter than simple parabolic antenna
- Greater beam steering possibility: secondary mirror motion amplified by optical system
- Much more compact for a given f/D ratio



Cassegrain antenna (2)

- Gain depends on diameter, wavelength, illumination
- Effective aperture is limited by surface accuracy, blockage
- Scale plate depends on equivalent focal length
- Loss in aperture efficiency due to:
  - Tapered illumination
  - Spillover (illumination does not stop at the edge of the dish)
  - Blockage of secondary mirror, support legs
  - Surface irregularities (effect depends on wavelength)

$$K_g = \cos\left(4\pi\frac{\delta}{\lambda}\right)^2$$

δ = rms of surface deviation

At the SEST:

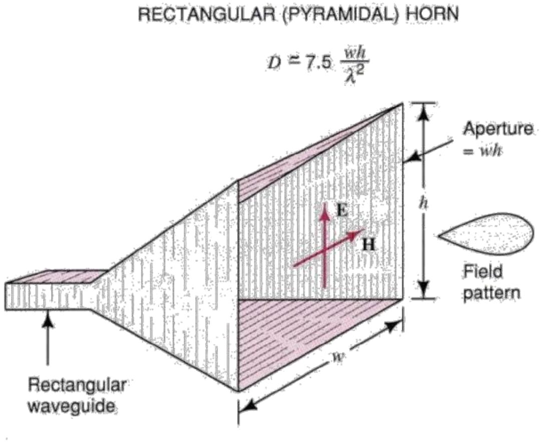
taper efficiency : $\varepsilon_t = 0.87$

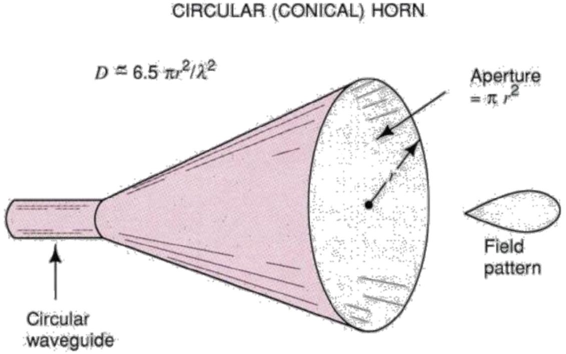
spillover efficiency : $\varepsilon_s = 0.94$

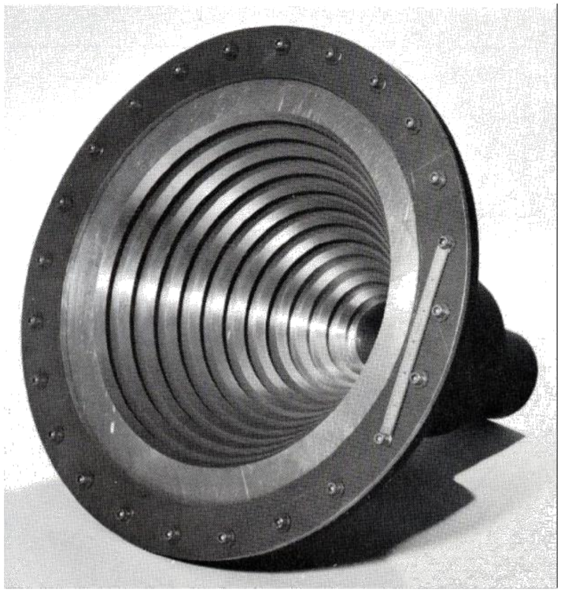
blockage efficiency : $\varepsilon_b = 0.96$

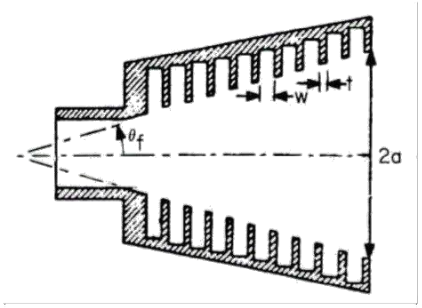
Horn antenna

- Rectangular or circular waveguide flared up
- Spherical wave fronts from phase centre
- Flare angle and aperture determine gain

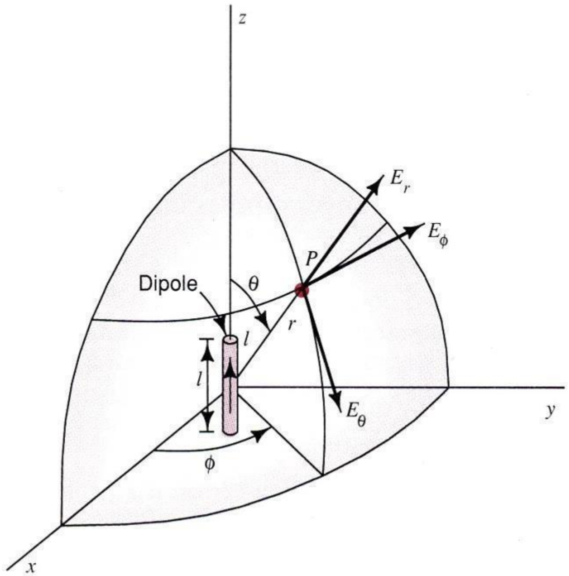








Short dipole



$$E_r = \frac{I_0 l e^{j(\omega t - \beta r)} \cos(\theta)}{2\pi\varepsilon_0} \left(\frac{1}{cr} + \frac{1}{j\omega r}\right)$$

$$E_\theta = \frac{I_0 l e^{j(\omega t - \beta r)} \sin(\theta)}{4\pi\varepsilon_0} \left(\frac{j\omega}{c r} + \frac{1}{cr} + \frac{1}{j\omega r}\right)$$

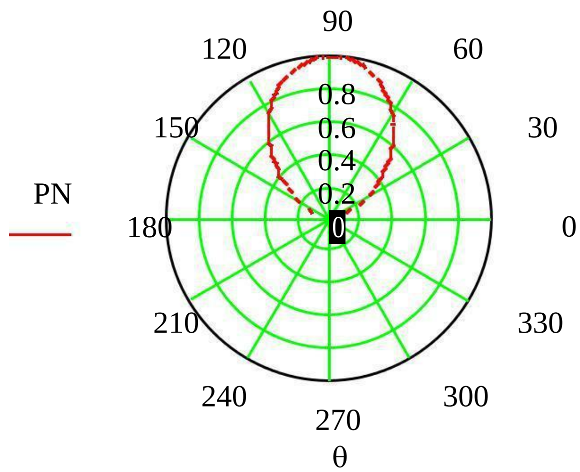
$$H_\phi = \frac{I_0 l e^{j(\omega t - \beta r)} \sin(\theta)}{4\pi} \left(\frac{j\omega}{cr} + \frac{1}{r_2}\right)$$

- Length much shorter than wavelength
- Current constant along the length
- Near dipole power is mostly reactive
- As r increases E_r vanishes, E and H gradually become in phase

$for\ r >> \frac{\lambda}{2\pi}$, $E\theta$ *and* $H\phi$ vary as $\frac{1}{r}$ ⟶ $$E_\theta = \frac{j60\pi I_0 e^{j(\omega t - \beta r)} \sin(\theta)}{r} \frac{l}{\lambda}$$

P varies as $\frac{1}{r^2}$

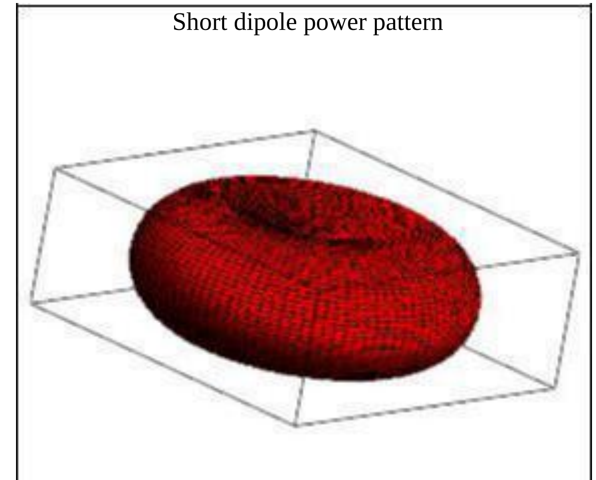
Short dipole pattern

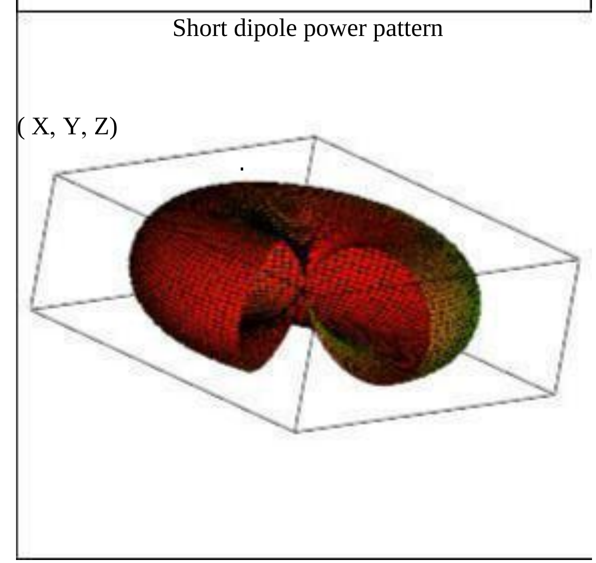


$$\Omega_A = \frac{8\pi}{3}$$

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$$

$$D = 1.5$$

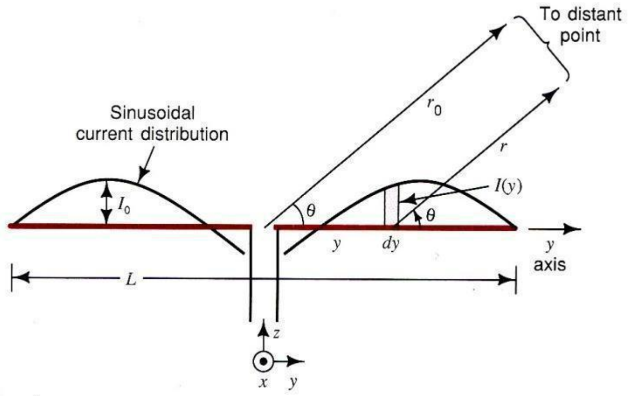




(X, Y, Z)

Thin wire antenna

•Wire diameter is small compared to wavelength •Current distribution along the wire is no longer constant



$$\text{e.g. } I(y) = I_0 \sin\left(\frac{2\pi}{\lambda}\left(\frac{L}{2} \pm y\right)\right)$$

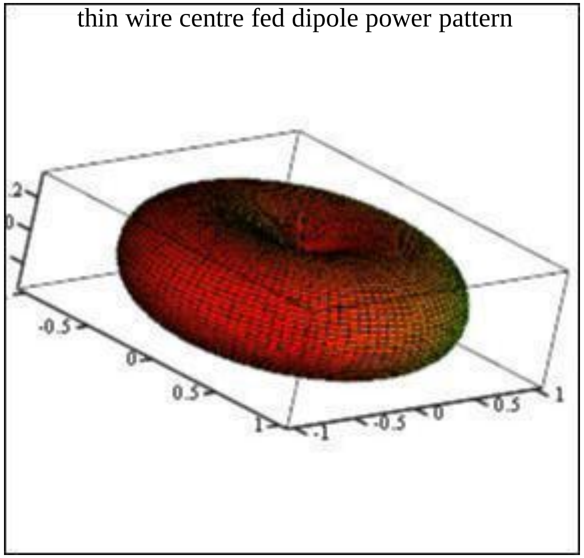
centre - fed dipole

•Using field equation for short dipole,
replace the constant current with actual distribution

$$E_\theta = \frac{j60 I_0 e^{j(\omega t - \beta r)}}{r}\left(\frac{\cos\left(\frac{\beta L \cos(\theta)}{2}\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin(\theta)}\right)$$

centre - fed dipole, I_0 = current at feed point

Thin wire pattern

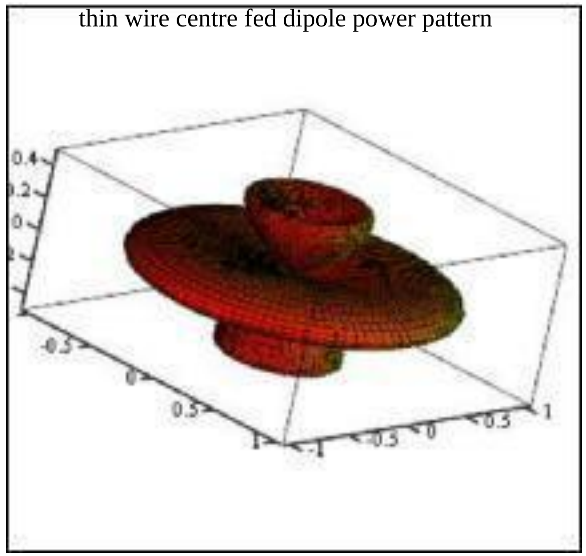


(X, Y, Z)

$l = 1\,\frac{\lambda}{2}$

$\Omega_A = 7.735$

D = 1.625

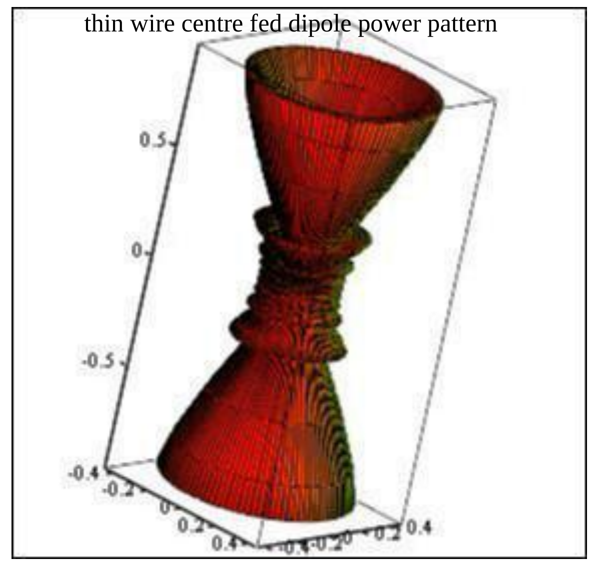


(X, Y, Z)

$l = 1.395\ \lambda$

$\Omega_A = 5.097$

D = 2.466



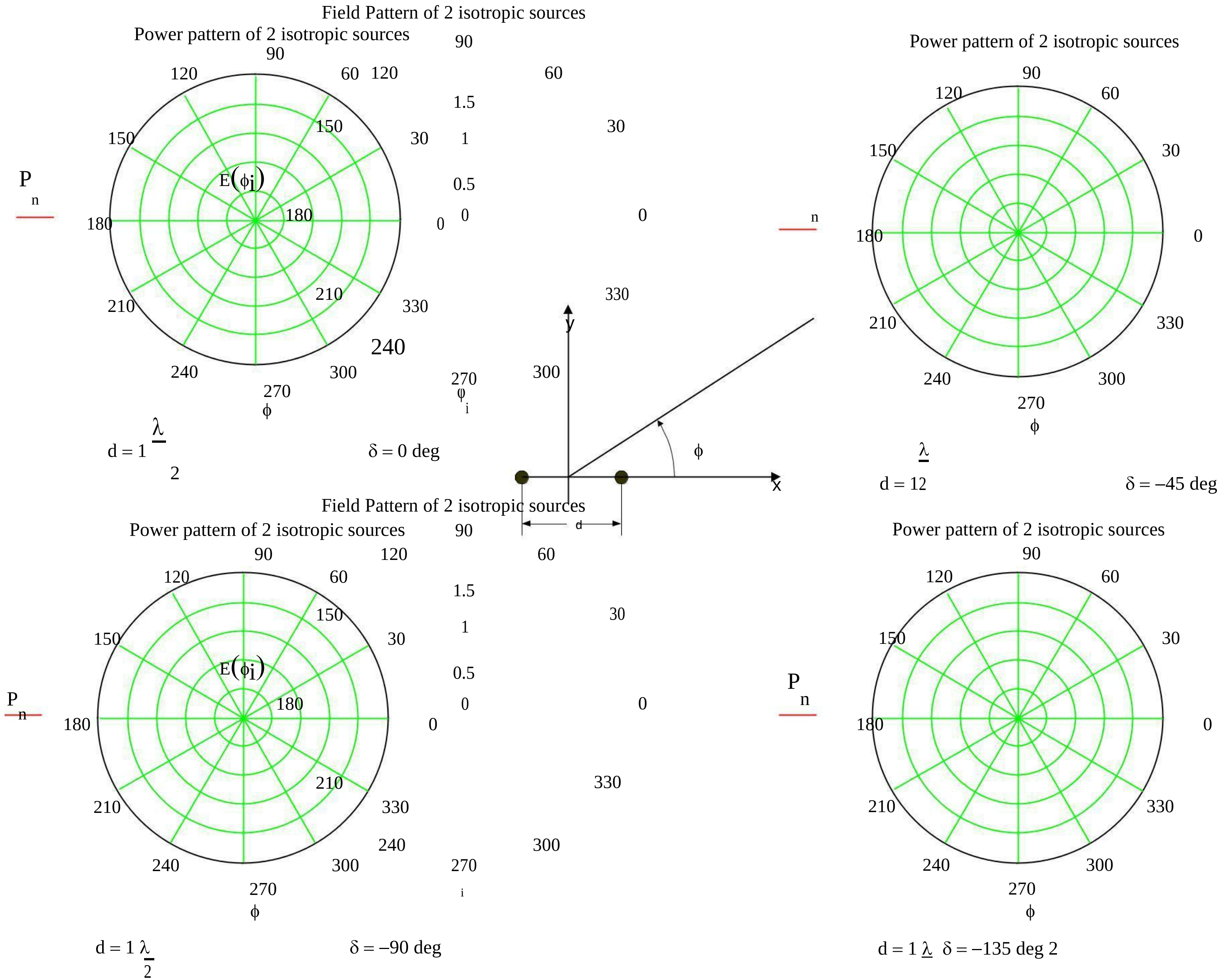
(X, Y, Z)

$l = 10\ \lambda$

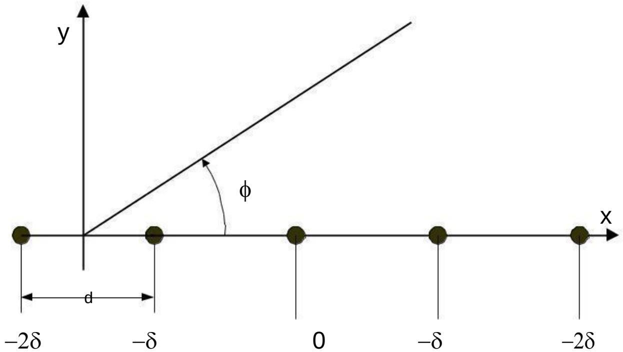
$\Omega_A = 1.958$

D = 6.417

Array of isotropic point sources – beam shaping

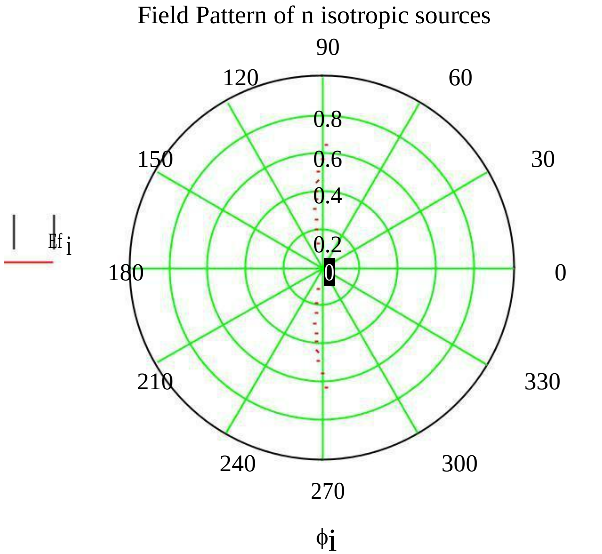


Array of isotropic point sources – centre-fed array

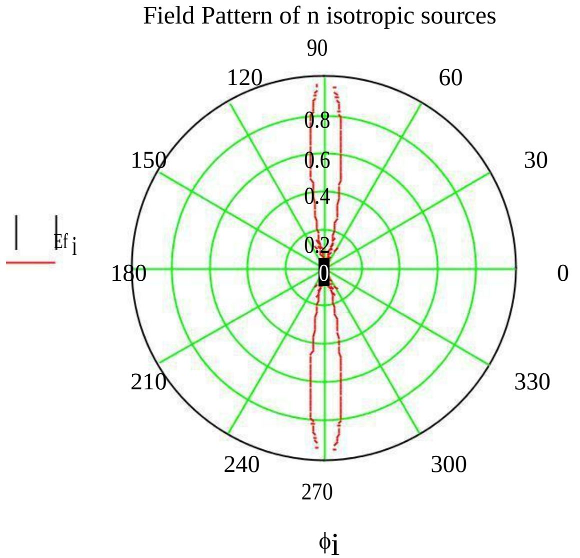


$$\psi(\phi) = \frac{2\pi d}{\lambda}\cos(\phi) + \delta$$

$$E_n(\psi) = \frac{1}{n}\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin(\psi/2)}$$



n = 3 δ = −67.5 deg d = 0.5 λ

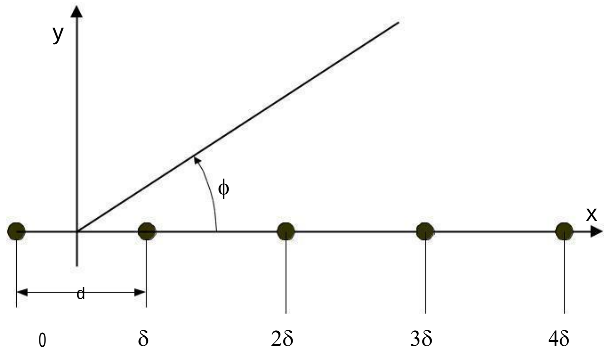


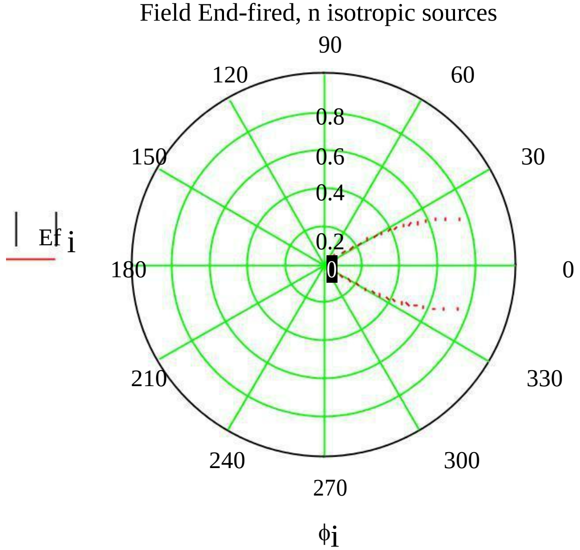
n = 8 δ = 0 deg d = 0.5 λ

Array of isotropic point sources – end-fired

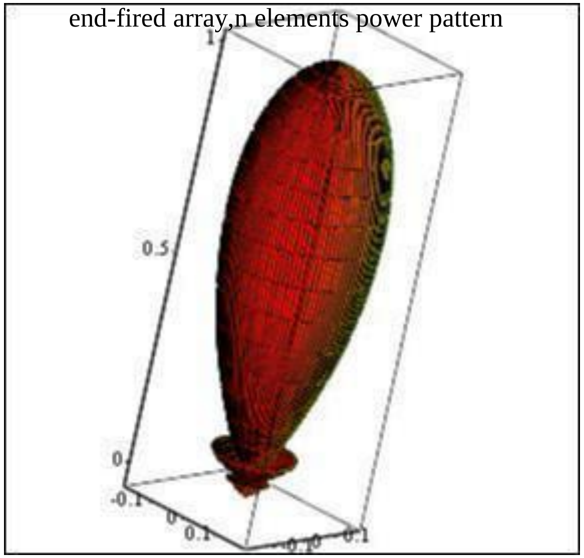
$$\psi(\phi) = \frac{2\pi d}{\lambda}(\cos(\phi) - 1) - \frac{\pi}{n}$$

$$E_n(\psi) = \sin\left(\frac{\pi}{2n}\right)\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$





n = 10 δ = −108 deg $d = 1\frac{\lambda}{4}$



(X, Y, Z)

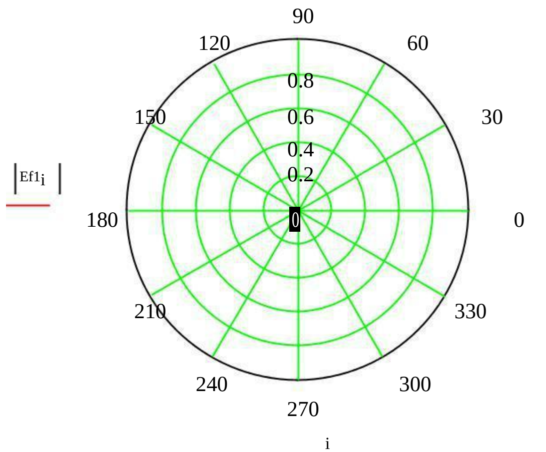
n = 10 d = 0.25 λ

$\Omega_A = 0.713$ D = 17.627

Pattern multiplication

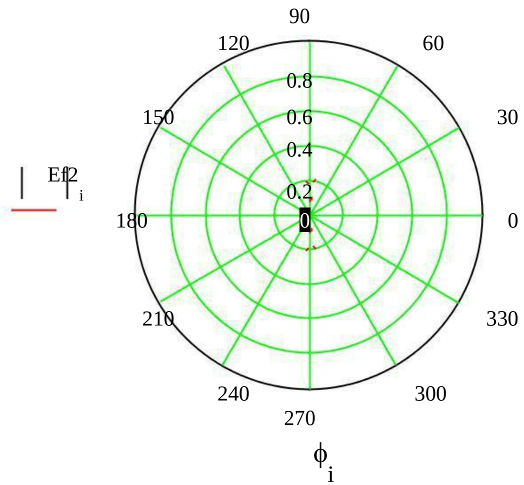
The total field pattern of an array of non-isotropic but similar point sources is the product of the individual source pattern and the pattern of an array of isotropic point sources having the same locations,relative amplitudes and phases as the non-isotropic point sources.

Primary field pattern



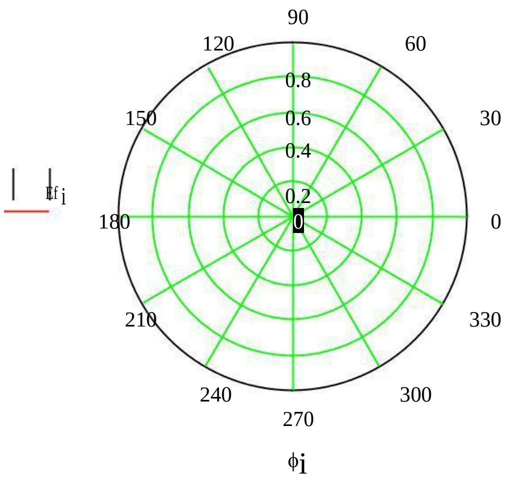
n = 2 $\delta 1 = -104$ deg d1 = 0.3 λ

Secondary field pattern



n = 2 $\delta 2 = 180$ deg d2 = 0.6 λ

Total field pattern



Total pattern of two primary sources (each an array of two isotropic sources) replacing two isotropic sources (4 sources in total).

Patterns from line and area distributions

•When the number of discrete elements in an array becomes large, it may be easier to consider the line or the aperture distribution as continuous.

• line source:

$$E(u) = \frac{l}{2}\int_{-1}^{+1} f(x)e^{jux}dx \qquad u = \frac{\pi l}{\lambda}\sin(\phi)\text{ , l = length, } \phi = \text{angle from normal to line}$$

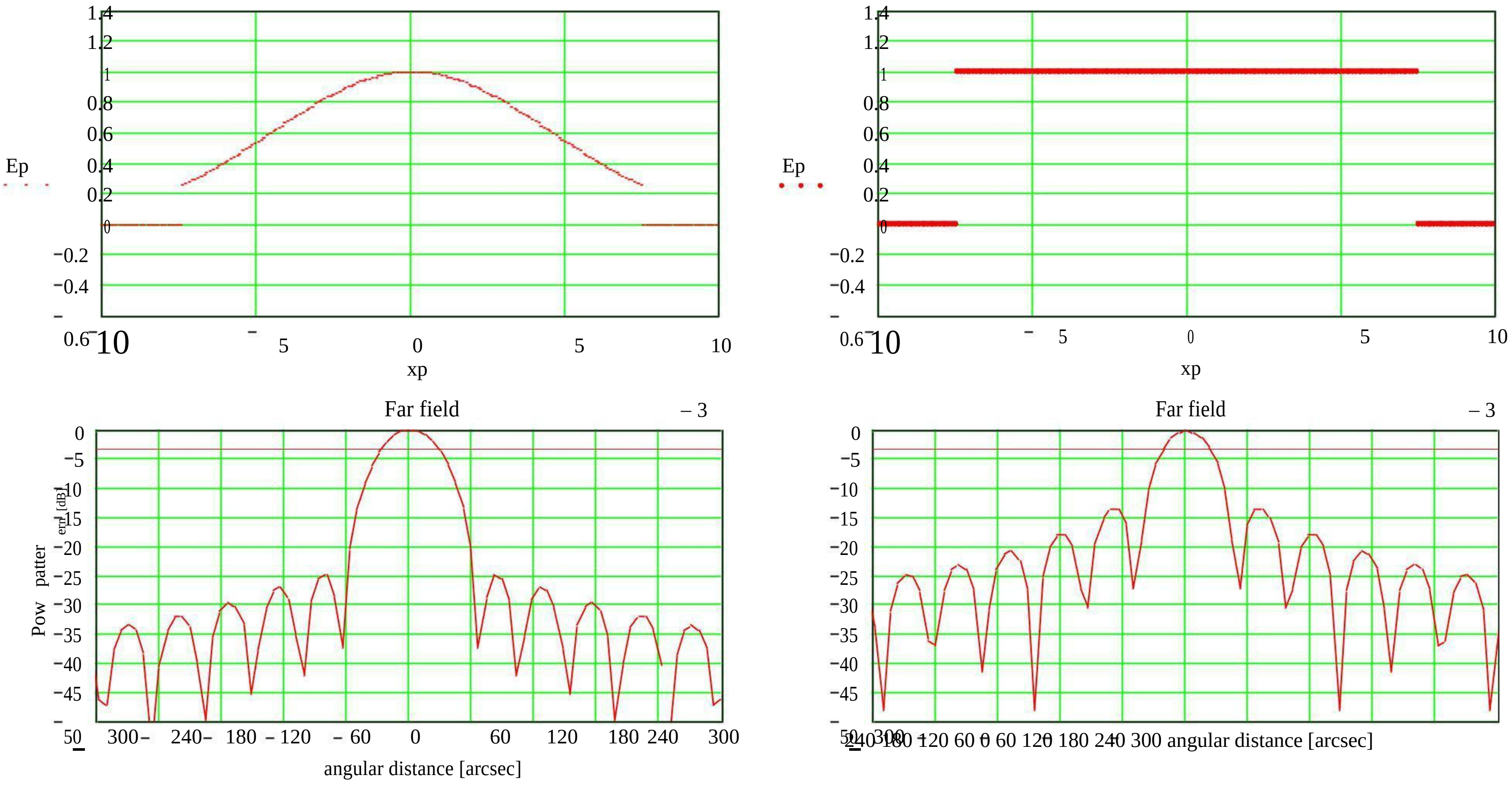
•2-D aperture source:

$$E(\theta,\phi) = \iint_{aperture} f(x,y)e^{j\beta \sin(\theta)(x\cos(\phi) + y\sin(\phi))}dx\,dy$$

$$f(x,y) = \text{aperture field distribution}$$

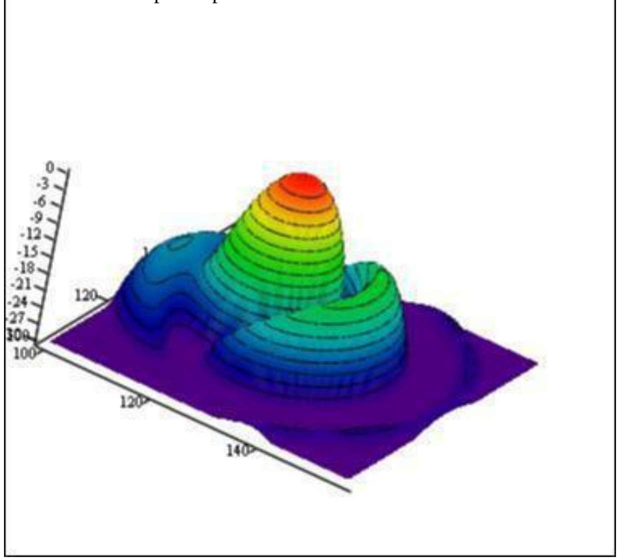
Fourier transform of aperture illumination
Diffraction limit

$$HPBW = \frac{\lambda}{D}$$ rough estimate only



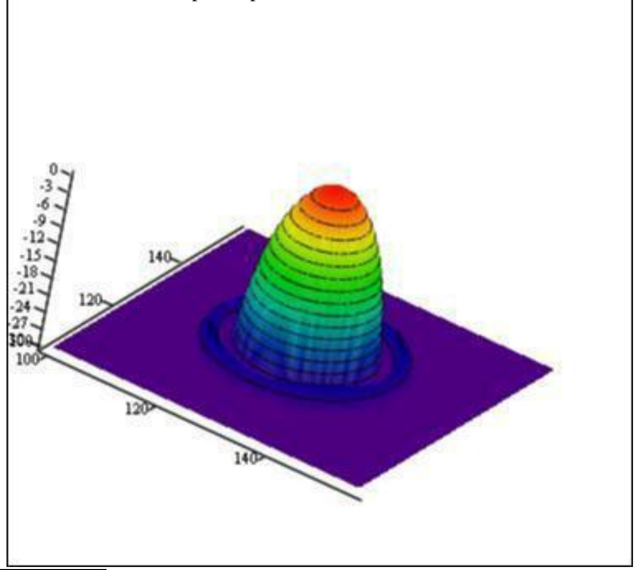
Far field pattern from FFT of Aperture field distribution

Predicted power pattern - SEST 1.3 mm - off axis 130 mm

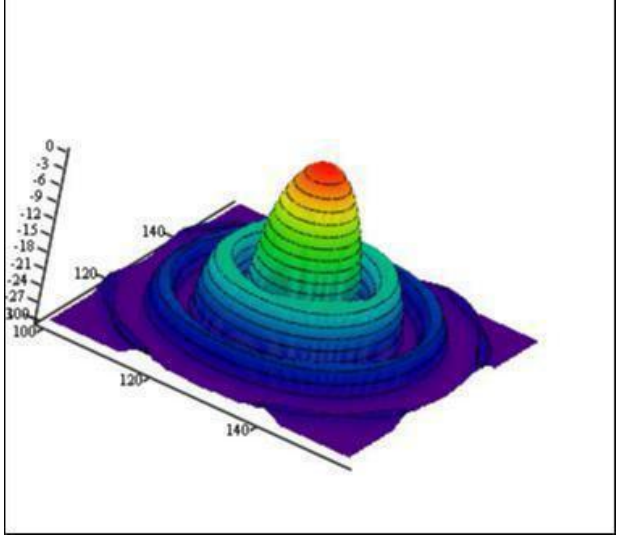


EFN

Predicted power pattern - SEST 1.3 mm - on axis



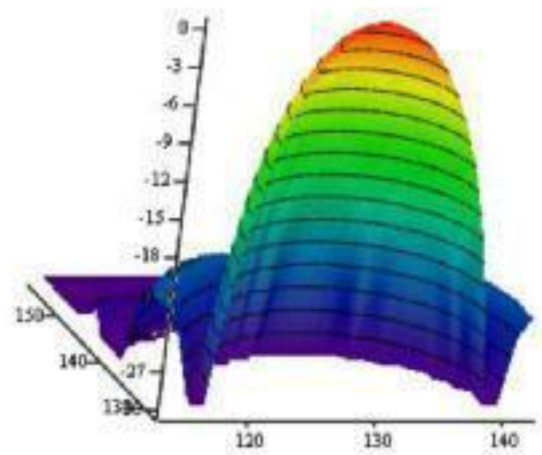
Predicted power pattern - flat illumination
EFN



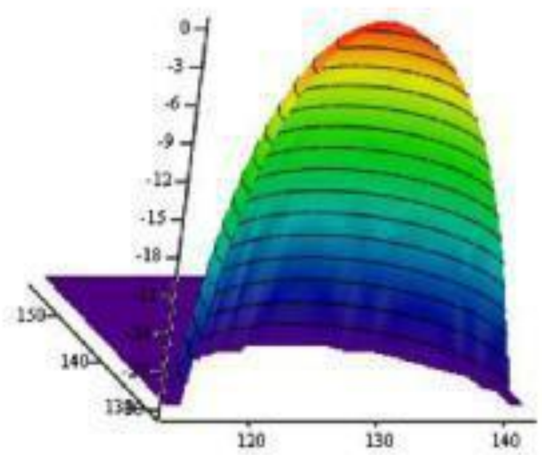
EFN

Effect of edge taper

Predicted power pattern -8dB taper



Predicted power pattern -16dB taper



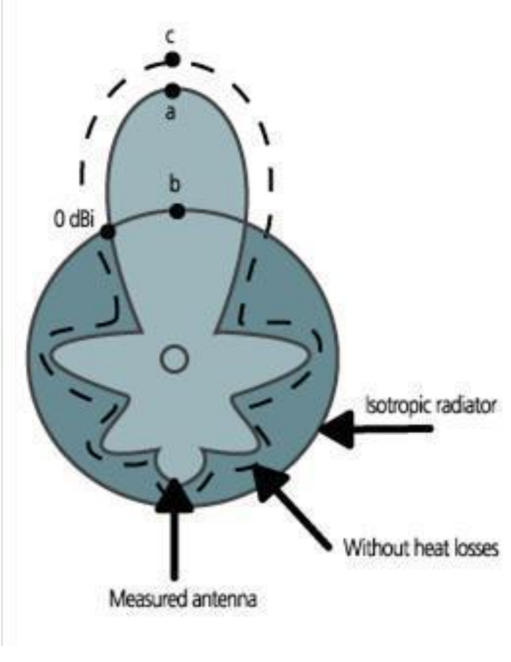
EFN

EFN

dBi versus dBd

- **dBi** indicates gain vs. isotropic antenna
    - Isotropic antenna radiates equally well in all directions, spherical pattern

- **dBd** indicates gain vs. reference half-wavelength dipole
    - Dipole has a doughnut shaped pattern with a gain of 2.15 dBi



$$dBi = dBd + 2.15\ dB$$

Feed and line matching

•The antenna impedance must be matched by the line feeding it if maximum power transfer is to be achieved
•The line impedance should then be the complex conjugate of that of the antenna
•Most feed line are essentially resistive

Signal transmission, radar echo

- Transmitting antenna $A_{et}, P_t, G_t, \lambda$
- Receiving antenna A_{er}, P_r, G_r

$$P_r = \frac{G_t P_t}{4\pi \cdot r^2} \frac{\lambda^2 G_r}{4\pi} = \left(\frac{\lambda}{4\pi \cdot r}\right)^2 G_t G_r P_t$$

S, power density Effective receiving area

Radar return

$$P_r = \frac{G_t P_t}{4\pi \cdot r^2} \frac{\sigma}{4\pi \cdot r^2} \frac{G_r \lambda^2}{4\pi} = P_t G_t G_r \frac{\lambda^2}{(4\pi)^3 r^4} \sigma$$

S, power density

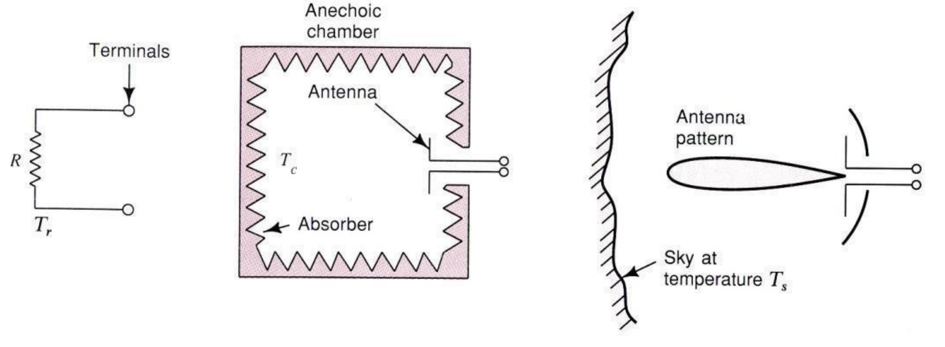
Reflected power density

Effective receiving area

σ = radar cross section (area)

Antenna temperature

- Power received from antenna as from a black body or the radiation resitance at temperature Ta



The end

Lecture 10: Antennas

Radiation fundamentals

Recall, that using the Poynting's theorem, the total power radiated from a source can be found as:

$$_{rad} = \int_{s} E \times H \cdot ds \qquad (10.2.1)$$

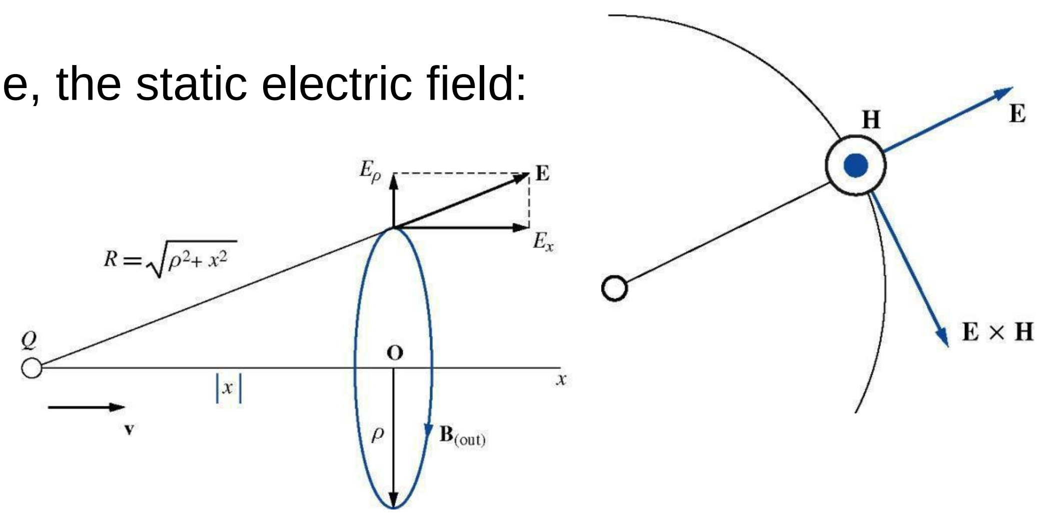
Which suggests that both electric and magnetic energy will be radiated from the region.

A stationary charge will NOT radiate EM waves, since a zero current flow will cause no magnetic field.

In a case of uniformly moving charge, the static electric field:

$$E = \frac{Q}{\quad} \frac{1}{\quad} u_{\rho}$$

The magnetic field is:



$$H = \frac{Q}{4}\,\frac{1}{\pi\rho^2} + \chi^2\left(v \times u_\rho\right) \tag{10.2.3}$$

Radiation fundamentals

In this situation, the Poynting vector does not point in the radial direction and represent a flow rate of electrostatic energy – does not contribute to radiation!

A charge that is accelerated radiates EM waves. The radiated field is:

$$E_t = \frac{Q\mu_0 [a] \sin\theta}{4\pi R} \tag{10.3.1}$$

Where θ is the angle between the point of observation and the velocity of the accelerated charge and [a] is the acceleration at the earliest time (retarded acceleration). Assuming that the charge is moving in vacuum, the magnetic field can be found using the wave impedance of the vacuum:

$$H_t = \frac{Q[a]\sin\theta}{4\pi cR} \tag{10.3.2}$$

And the Poynting vector directed radially outward is:

$$S_t = \frac{Q^2 \mu_0 [a]^2 \sin^2\theta}{16\pi^2 cR^2} \tag{10.3.3}$$

Radiation fundamentals

A current with a time-harmonic variation (AC current) satisfies this requirement.

Example 10.1: Assume that an antenna could be described as an ensemble of *N* oscillating electrons with a frequency ω in a plane that is orthogonal to the distance *R*. Find an expression for the electric field $E_\perp$ that would be detected at that location.

The maximum electric field is when $\theta = 90^0$:

$$E_\perp = \frac{NQ\mu_0}{4\pi R}\left[\frac{dv}{dt}\right] = \frac{\mu_0}{4\pi R}\left[\frac{dJ}{dt}\right] \qquad (10.4.1)$$

Where we introduce the electric current density $J = NQv$ of the oscillating current. Assuming that the direction of oscillation in the orthogonal plane is *x*, then

$$x(t) = x_m \sin \omega t \qquad (10.4.2)$$

$$v(t) = \frac{dx}{dt} = \omega x_m \cos \omega t \qquad (10.4.3)$$

Radiation fundamentals

The current density will become:

$$J(t) = \omega NQx_m \cos \omega t \qquad (10.5.1)$$

Finally, the transverse electric field is

$$E_{\perp}(R, t) = \omega^2 \frac{NQx_m \mu_0}{4\pi R} \sin \omega t \qquad (10.5.2)$$

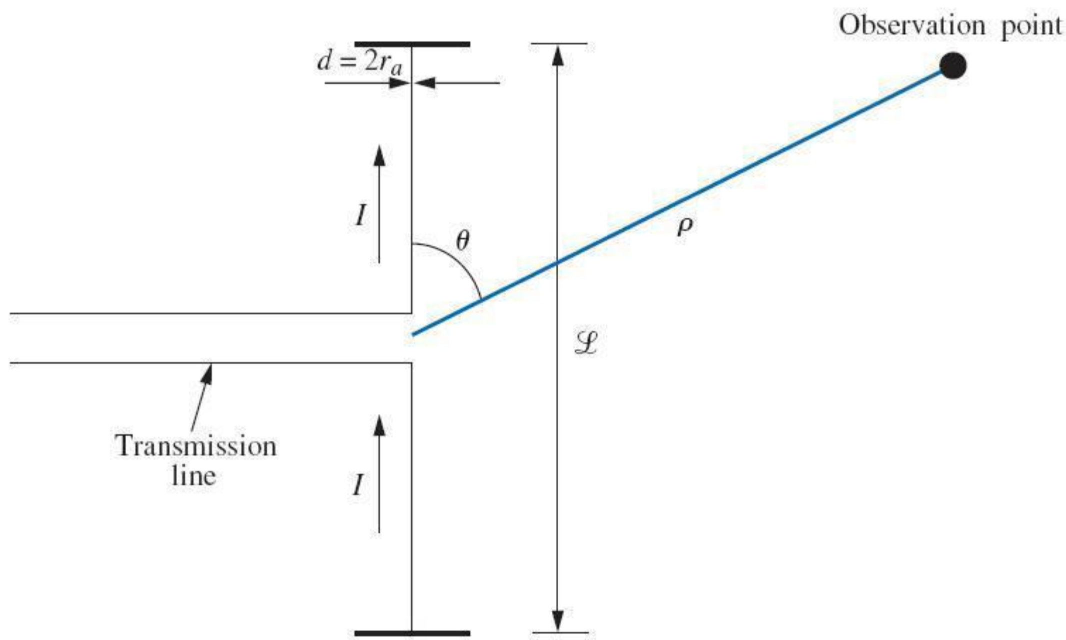
The electric field is proportional to the square of frequency implying that radiation of EM waves is a high-frequency phenomenon.

Infinitesimal electric dipole antenna

We assume the excitation as a signal at the frequency ω, which results in a time-harmonic radiation.

The length of the antenna L is assumed to be much less than the wavelength:
. Typically: $L < \lambda/50$. The antenna is also assumed as very thin:
.

The current along the antenna is assumed as :



$$I = \frac{dQ}{}$$ (10.6.1)

For a time-harmonic excitation:

$$I(r) = j\omega Q(r)$$ (10.6.2)

Infinitesimal electric dipole antenna

The vector potential can be computed as:

$$\nabla^2 A(r,t) - \frac{1}{c^2}\frac{\partial^2 A(r,t)}{\partial t^2} = -\mu_0 J(r,t) \quad (10.7.1)$$

With the solution that can be found in the form:

$$A(r,t) = \frac{\mu_0}{4\pi}\int_V \frac{J(r', t - R/c)\, dv'}{R} \quad (10.7.2)$$

Assuming a time-harmonic current density:

$$J\left(r', t - R/c\right) = J(r')e^{-j(\omega t - k\cdot R)} \quad (10.7.3)$$

The distance from the center of the dipole *R* = *r* and *k* is the wave number.
The volume of the dipole antenna can be approximated as *dv'* = *Lds'*.

Infinitesimal electric dipole antenna

Considering the mentioned assumptions and simplifications, the vector potential becomes:

$$A(r) = u_Z \frac{\mu_0 IL}{4\pi}\left(\frac{e^{-jkr}}{r}\right) \qquad (10.8.1)$$

This infinitesimal antenna with the current element *IL* is also known as a

.

Assuming that the distance from the antenna to the observer is much greater than the wavelength ( ), i.e. $r >> \lambda$, let us find the components of the field generated by the antenna. Using the spherical coordinates:

$$u_z = \cos\theta\, u_r - \sin\theta\, u_\theta \qquad (10.8.2)$$

Infinitesimal electric dipole antenna

The components of the vector potential are:

$$A_r = A_z(r)\cos\theta = \frac{\mu_0 IL}{4\pi}\frac{e^{-jkr}}{r}\cos\theta \qquad (10.9.1)$$

$$A_\theta = -A_z(r)\sin\theta = -\frac{\mu_0 IL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta \qquad (10.9.2)$$

$$A_\phi = 0 \qquad (10.9.3)$$

The magnetic field intensity can be computed from the vector potential using the definition of the curl in the SCS:

$$H(r) = \frac{1}{\mu_0}\nabla\times A = \frac{1}{\mu_0 r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]u_\phi = -\frac{I(z)}{4\pi}k^2\sin\theta\left[\frac{1}{jkr} + \frac{1}{(jkr)^2}\right]e^{-jkr}u_\phi \qquad (10.9.4)$$

Infinitesimal electric dipole antenna

Which can be rewritten as

$$H_r = 0 \qquad (10.10.1)$$

$$H_\phi \approx \frac{jkIL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta \qquad (10.10.2)$$

$$H_\theta = 0 \qquad (10.10.3)$$

Note: the equations above are approximates derived for the far field assumptions.
The electric field can be computed from Maxwell's equations:

$$E(r) = \frac{1}{j\omega\varepsilon_0}\nabla\times H(r) = \frac{1}{j\omega\varepsilon_0}\left[\frac{1}{r\sin\theta}\frac{\partial\left(H_\phi\sin\theta\right)}{\partial\theta}u_r - \frac{1}{r}\frac{\partial\left(rH_\phi\right)}{\partial r}u_\theta\right] \qquad (10.10.4)$$

Infinitesimal electric dipole antenna

The components of the electric field in the far field region are:

$$E_r \approx 0 \tag{10.11.1}$$

$$E_\theta \approx \frac{jZ_0 kIL}{4\pi}\frac{e^{-jkr}}{r}\sin\theta \tag{10.11.2}$$

$$E_\phi = 0 \tag{10.11.3}$$

where

$$Z_0 = \frac{E_\theta(r)}{H_\phi(r)} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377\,\Omega \tag{10.11.4}$$

is the wave impedance of vacuum.

Infinitesimal electric dipole antenna

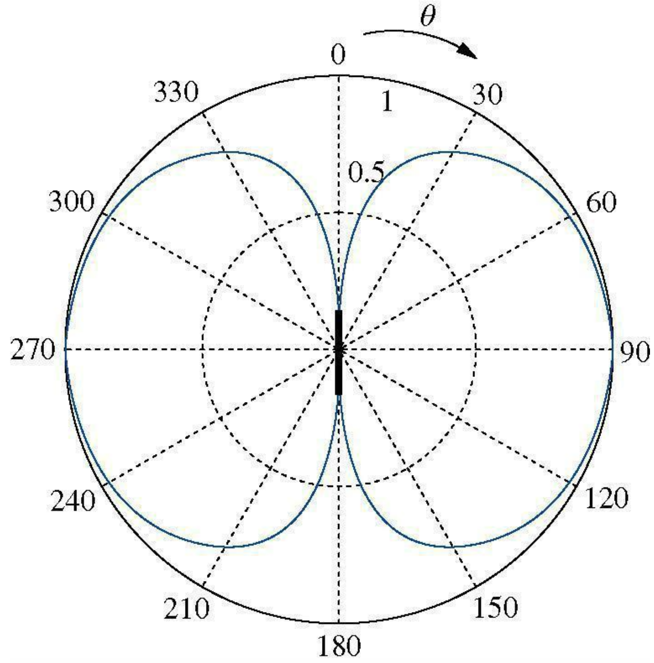
The angular distribution of the radiated fields is called the

.

Both, electric and magnetic fields depend on the angle and have a maximum when $\theta = 90^0$ (the direction perpendicular to the dipole axis) and a minimum when $\theta = 0^0$.

The blue contours depicted are called lobes. They represent the antenna's radiation pattern. The lobe in the direction of the maximum is called the , while any others are called .

A is a minimum value that occurs between two lobes.



For the radiation pattern shown, the main lobes are at 90^0 and 270^0 and nulls at 0^0 and 180^0.

Lobes are due to the constructive and destructive interference.

Infinitesimal electric dipole antenna

Every null introduces a 180^0 phase shift.

In the far field region (traditionally, the region of greatest interest) both field components are transverse to the direction of propagation. The radiated power:

$$P_{rad} = \frac{1}{2}\mathrm{Re}\left\{\int_s \left(E(r)\times H^*(r)\right)_{av}\cdot ds\right\} = \frac{1}{2}Z_0\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left|H_\phi(r)\right|_{av}^2 r^2\sin\theta\, d\theta\, d\phi$$

$$= \frac{Z_0 k^2 (I_{av}L)^2}{16\pi}\int_0^\pi \sin^3\theta\, d\theta = -\frac{Z_0 k^2 (I_{av}L)^2}{16\pi}\int_0^\pi \left(1-\sin^2\theta\right) d(\cos\theta)$$

$$= \frac{Z_0 k^2 I_{av}^2 L^2}{12\pi} \qquad (10.13.1)$$

We have replaced the constant current by the averaged current accounting for the fact that it may have slow variations in space.

Infinitesimal electric dipole antenna

Example 10.2: A small antenna that is 1 cm in length and 1 mm in diameter is designed to transmit a signal at 1 GHz inside the human body in a medical experiment. Assuming the dielectric constant of the body is approximately 80 (a value for distilled water) and that the conductivity can be neglected, find the maximum electric field at the surface of the body that is approximately 20 cm away from the antenna. The maximum current that can be applied to the antenna is 10 μA. Also, find the distance from the antenna where the signal will be attenuated by 3 dB.

The wavelength within the body is:

$$\lambda = \frac{c}{f\sqrt{\varepsilon_r}} = \frac{3 \cdot 10^8}{10^9\sqrt{80}} \approx 3.3\ cm$$

The characteristic impedance of the body is:

$$Z_c = \frac{Z_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{80}} \approx 42\ \Omega$$

Infinitesimal electric dipole antenna

Since the dimensions of the antenna are significantly less than the wavelength, we can apply the far field approximation for $\theta = 90^0$, therefore:

$$\left|E_\theta\right| = \frac{I\ L}{4\pi} Z_c\, k\, \frac{1}{r} = \frac{10^{-5} \cdot 10^{-2}}{4\pi} \cdot 42 \cdot \frac{2\pi}{0.033} \cdot \frac{1}{0.2} \approx 320\ \mu V/m$$

An attenuation of 3 dB means that the power will be reduced by a factor of 2. The power is related to the square of the electric field. Therefore, an attenuation of 3 dB would mean that the electric field will be reduced by a square root of 2. The distance will be

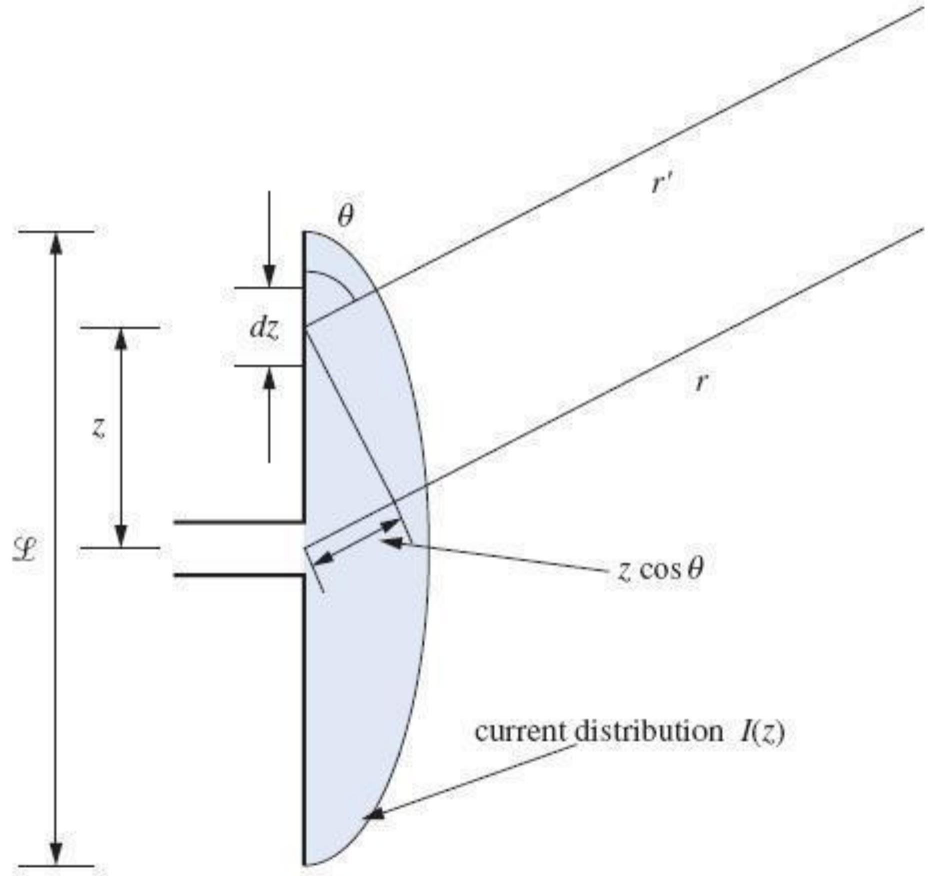
$$r_1 = \sqrt{2} r \approx 1.41 \cdot 0.2 = 0.28\ m$$

Finite electric dipole antenna

Finite electric dipole consists of two thin metallic rods of the total length L, which may be of the order of the free space wavelength.

Assume that a sinusoidal signal generator working at the frequency ω is connected to the antenna. Thus, a current $I(z)$ is induced in the rods.

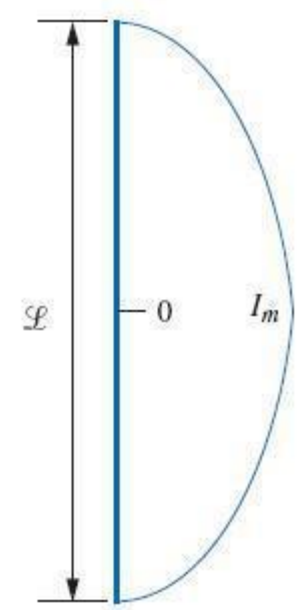
We assume that the current is zero at the antenna's ends ($z = \pm L/2$) and that the current is symmetrical about the center ($z = 0$).
The actual current distribution depends on antenna's length, shape, material, surrounding,…



Finite electric dipole antenna

A reasonable approximation for the current distribution is

$$I(z) = I_m \sin\left[k\left(L/2 - |z|\right)\right] \qquad (10.17.1)$$



Far field properties, such as the radiated power, power density, and radiation pattern, are not very sensitive to the choice of the current distribution. However, the near field properties are very sensitive to this choice.

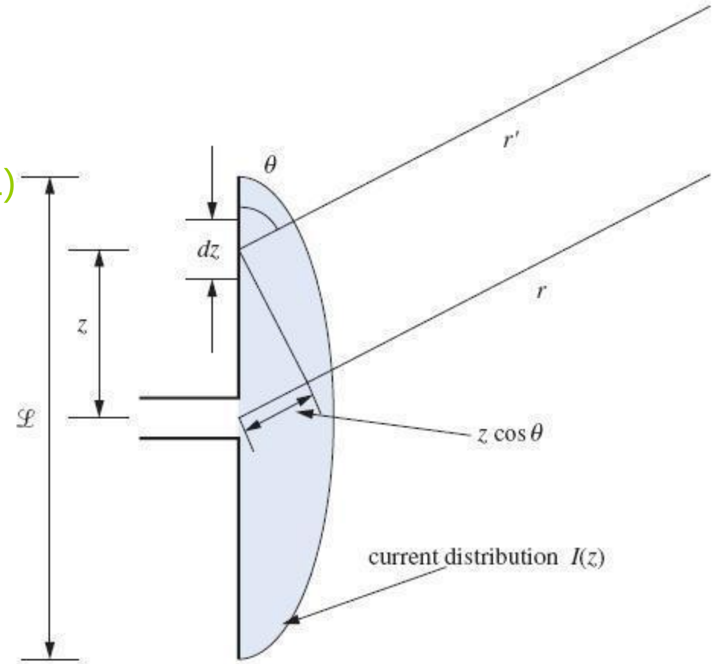
Deriving the expressions for the radiation pattern of this antenna, we represent the finite dipole antenna as a linear combination of infinitesimal electric dipoles. Therefore, for a differential current element *I*(*z*)*dz*, the differential electric field in a far zone is

$$dE_\theta = \frac{jZ_0 k}{4\pi} I(z)\, dz \frac{e^{-jkr'}}{r'} \sin\theta \qquad (10.17.2)$$

Finite electric dipole antenna

The distance can be expressed as:

$$r' = \sqrt{r^2 + z^2 - 2rz\cos\theta} \approx r - z\cos\theta \quad (10.18.1)$$



This approximation is valid since *r* >> *z*

Replacing *r’* by *r* in the amplitude term will have a very minor effect on the result. However, the phase term would be changed dramatically by such substitution! Therefore, we may use the approximation *r’* ≅ *r* in the amplitude term but not in the phase term.

The EM field radiated from the antenna can be calculated by selecting the appropriate current distribution in the antenna and integrating (11.17.2) over *z*.

$$E_\theta = Z_0 H_\phi = j\frac{Z_0 I_m k \sin\theta e^{-jkr}}{4\pi r}\int_{-L/2}^{L/2}\left[\left(\frac{L}{2} - |z|\right)\right] e^{jkz\cos\theta} dz \quad (10.18.2)$$

Finite electric dipole antenna

Since $$e^{jkz\cos\theta} = \cos(kz\cos\theta) + j\sin(kz\cos\theta) \quad (10.19.1)$$

and the limits of integration are symmetric about the origin, only a —non-odd‖ term will yield non-zero result:

$$E_\theta = j2\frac{Z_0 I_m k\sin\theta e^{-jkr}}{4\pi r}\int_0^{L/2}\left[\sin\left(k\left(\frac{L}{2}-|z|\right)\right)\right]\cos(kz\cos\theta)dz \quad (10.19.2)$$

The integration results in:

$$E_\theta = j60 I_m \frac{e^{-jkr}}{r}F(\theta) \quad (10.19.3)$$

Where $F(\theta)$ is the radiation pattern:

$$F(\theta) \equiv F_1(\theta)F_a(\theta) = \sin\theta\cdot\frac{\cos\left(\frac{kL}{2}\cos\theta\right)-\cos\left(\frac{kL}{2}\right)}{\sin^2\theta}$$

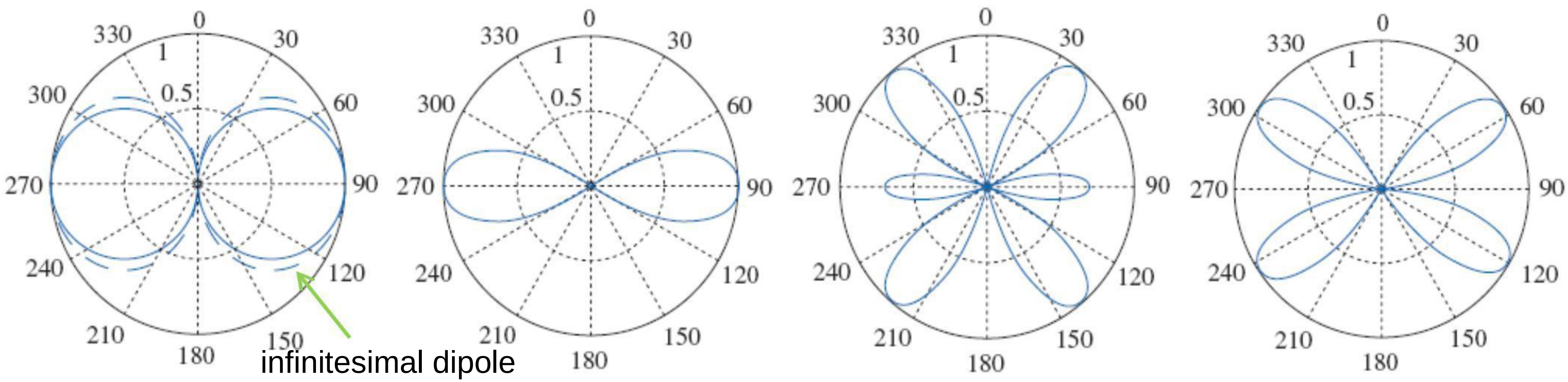
$$= \frac{\cos\left(\frac{kL}{2}\cos\theta\right)-\cos\left(\frac{kL}{2}\right)}{\sin\theta} \quad (10.19.4)$$

Finite electric dipole antenna

The first term, $F_1(\theta)$ is the radiation characteristics of one of the elements used to make up the complete antenna – .

The second term, $F_a(\theta)$ is the array (or space) factor – the result of adding all the radiation contributions of the various elements that form the antenna array as well as their interactions.

The *E*-plane radiation patterns for dipoles of different lengths.



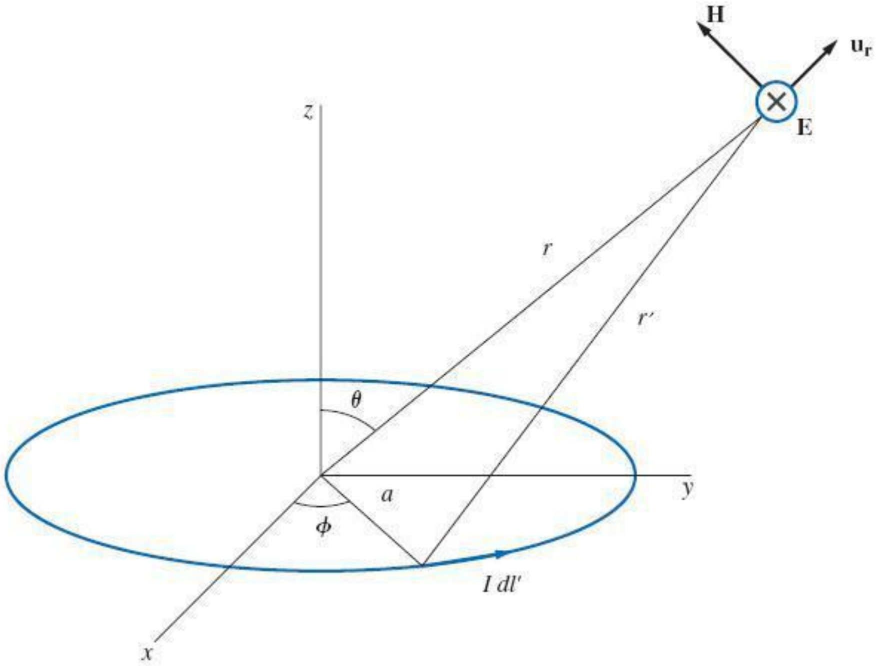
$L = \lambda/2$ $L = \lambda$ $L = 3\lambda/2$ $L = 2\lambda$

If the dipole length exceeds wavelength, the location of the maximum shifts.

Loop antenna

A loop antenna consists of a small conductive loop with a current circulating through it.

We have previously discussed that a loop carrying a current can generate a magnetic dipole moment. Thus, we may consider this antenna as equivalent to a magnetic dipole antenna.



If the loops circumference $C < \lambda/10$
The antenna is called . If C is in order of λ or larger, the antenna is . Commonly, these antennas are used in a frequency
band from about 3 MHz to about 3 GHz. Another application of loop antennas is in magnetic field probes.

Loop antenna

Assuming that the antenna carries a harmonic current:

$$i(t) = I\cos\omega t$$

and that $$ka \equiv \frac{2\pi}{\lambda}a << 1$$

The retarded vector potential can be found as:

$$A(r) = \frac{\mu_0 I u_\phi}{4\pi}\oint_L \frac{e^{-jkr'}}{r'}dl'$$

If we rewrite the exponent as:

$$e^{-jkr'} = e^{-jkr}e^{-jk(r'-r)} \approx e^{-jkr}\left[1 - jk(r'-r)\right]$$

where we assumed that the loop is small: i.e. *a* << *r*, we arrive at

$$A(r) = \mu_0 I e^{-jkr}\left[(1+jkr)\oint\frac{dl'}{r'} - ik\oint dl'\right]u_\phi$$

(10.22.1)

(10.22.2)

(10.22.3)

(10.22.4)

(10.22.5)

Loop antenna

Evaluating the integrals, we arrive at the following expression:

$$A(r) = \frac{\mu_0 I a^2}{4\pi}\left(\frac{1+jkr}{r^2}\right) e^{-jkr} \sin\theta \cdot u_\phi \approx j\frac{\mu_0 I a^2 k}{4\pi} \frac{e^{-jkr}}{r} \sin\theta \cdot u_\phi \qquad (10.23.1)$$

Recalling the magnetic dipole moment:

$$m = I \pi a^2 u_z \qquad (10.23.2)$$

Therefore, the electric and magnetic fields are found as

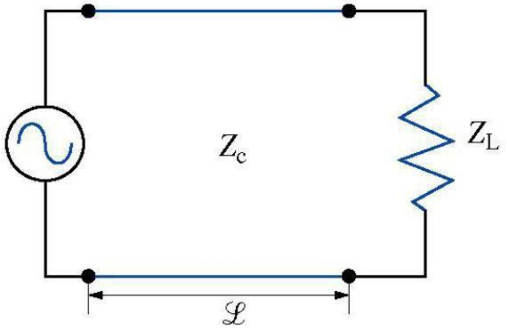
$$H_\theta \approx -\frac{\omega \mu_0 m k e^{-jkr}}{4\pi Z_0 r} \sin\theta \qquad (10.23.3)$$

$$E_\phi = -Z_0 H_\theta \approx -\frac{\omega \mu_0 m k e^{-jkr}}{4\pi r} \sin\theta \qquad (10.23.4)$$

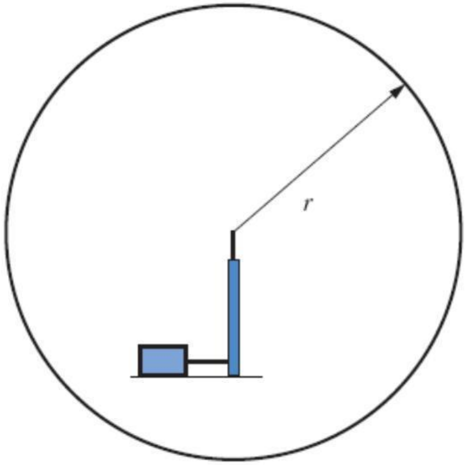
We observe that the fields are similar to the fields of short electric dipole. Therefore, the radiation patterns will be the same.

Antenna parameters

In addition to the radiation pattern, other parameters can be used to characterize antennas. Antenna connected to a transmission line can be considered as its load, leading to:



We consider the antenna to be a load impedance Z_L of a transmission line of length L with the characteristic impedance Z_C. To compute the load impedance, we use the Poynting vector…



If we construct a large imaginary sphere of radius r (corresponding to the far region) surrounding the radiating antenna, the power that radiates from the antenna will pass trough the sphere. The sphere‘s radius can be approximated as $r \cong L^2/2\lambda$.

Antenna parameters

The total radiated power is computed by integrating the time-average Poynting vector over the closed spherical surface:

$$P_{rad} = \frac{1}{2}\text{Re}\left\{\oint_S \mathbf{E}(r)\times\mathbf{H}^*(r)\cdot d\mathbf{s}\right\} = \frac{1}{2}\text{Re}\left\{\int_0^{2\pi} d\phi \int_0^{\pi} r^2 \sin\theta \left(E_\theta H_\phi^*\right) d\theta\right\} \quad (10.25.1)$$

Notice that the factor ½ appears since we are considering power averaged over time. This power can be viewed as a ―lost power‖ from the source‘s concern. Therefore, the antenna is ―similar‖ to a resistor connected to the source:

$$R_{rad} = \frac{2P_{rad}}{I_0^2} \quad (10.25.2)$$

where I_0 is the maximum amplitude of the current at the input of the antenna.

Antenna parameters: Example

Example 10.3: Find the radiation resistance of an infinitesimal dipole. The radiated power from the Hertzian dipole is computed as:

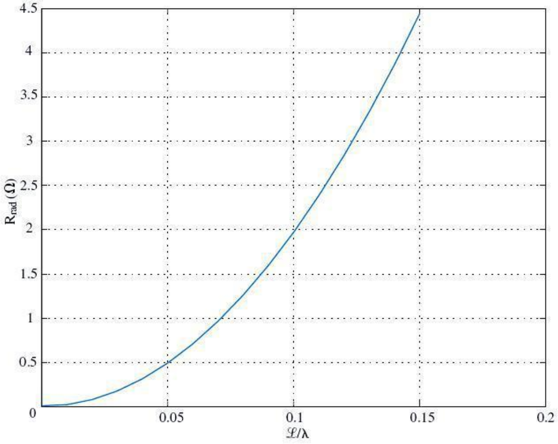
$$P_{rad} = \frac{Z_0 k^2 I_{av}^2 L^2}{12\pi} \qquad (10.26.1)$$

Using the free space impedance and assuming a uniform current distribution:

$$R_{rad} = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 \left(\frac{I_{av}}{I_0}\right)^2 = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 \qquad (10.26.2)$$

Assuming a triangular current distribution, the radiation resistance will be:

$$R_{rad} = 20\pi^2 \left(\frac{L}{\lambda}\right)^2 \qquad (10.26.3)$$



Small values of radiation resistance suggest that this antenna is not very efficient.

Antenna parameters

For the small loop antennas, the antennas radiation resistance, assuming a uniform current distribution, will be:

$$R_{rad} = 20\pi^2 (ka)^4 \qquad (10.27.1)$$

For the large loop antennas *(ka* >> 1*)*, no simple general expression exists for antennas radiation resistance.

Example 10.4: Find the current required to radiate 10 W from a loop, whose circumference is $\lambda/5$.

We can use the small loop approximation since $ka = 2\pi a/\lambda = 0.2$. The resistance:

$$R_{rad} = 20\pi^2 \cdot 0.2^4 = 0.316\ \Omega$$

The radiated power is:

$$P_{rad} = \frac{1}{2} R_{rad} \left| I(\phi) \right|^2$$

$$\Rightarrow \quad I(\phi) = \sqrt{\frac{_{rad}}{R_{rad}}} = \sqrt{\frac{2 \cdot 10}{0.316}} = 7.95\ A$$

Antenna parameters

The equation (10.25.1) for a radiated power can also be written as an integral over a solid angle. Therefore, we define the as

$$I(\theta,\phi) = r^2 S_r(\theta,\phi) \qquad (10.28.1)$$

Time-averaged radial component of a Poynting vector

The power radiated is then:

$$P_{rad} = \int_{4\pi} I(\theta,\phi)d\Omega \qquad (10.28.2)$$

Introducing the as

$$I_n(\theta,\phi) = \frac{I(\theta,\phi)}{I(\theta,\phi)_{max}} \qquad (10.28.3)$$

The of the antenna is

$$\Omega_A \equiv \int_{4\pi} I_n(\theta,\phi)\,d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi} I_n(\theta,\phi)\sin\theta\, d\theta \qquad (10.28.4)$$

Antenna parameters

It follows from the definition that for an isotropic (directionless – radiating the equal amount of power in any direction) antenna, $I_n(\theta,\phi) = 1$ and the beam solid angle is $\Omega_A = 4\pi$.

We introduce the of the antenna:

$$D = \frac{I(\theta,\phi)_{max}}{P_{rad}/4\pi} = \frac{4\pi}{\int_{4\pi} I_n(\theta,\phi) d\Omega} = \frac{4\pi}{\Omega_A} \qquad (10.29.1)$$

Note: since the denominator in (10.29.1) is always less than 4π, the directivity .

Antenna parameters: Example

Example 10.5: Find the directivity of an infinitesimal (Hertzian) dipole. Assuming that the normalized radiation pattern is

$$I_n(\theta, \phi) = \sin^2\theta$$

the directivity will be

$$D = \frac{4\pi}{2\pi \int_0^{\pi} \sin^2\theta \sin\theta \, d\theta} = \frac{2}{\int_0^{\pi} \left(\cos^2\theta - 1\right) d(\cos\theta)} = \frac{2}{-2/3 + 2} = 1.5$$

Note: this value for the directivity is approximate. We conclude that for the short dipole, the directivity is D = 1.5 = 10lg(1.5) = 1.76 dB.

Antenna parameters

The is related to directivity and is defined as

$$G = \eta D \qquad (10.31.1)$$

Here η is the . For the lossless antennas, $\eta = 1$, and gain equals directivity. However, real antennas always have losses, among which the main types of loss are losses due to energy dissipated in the dielectrics and conductors, and reflection losses due to impedance mismatch between transmission lines and antennas.

Antenna parameters

is associated with the lobes in the antenna pattern. It is defined as the angular separation between two identical points on the opposite sides of the main lobe.

The most common type of beamwidth is the (HPBW). To find HPBW, in the equation, defining the radiation pattern, we set power equal to 0.5 and solve it for angles.

Another frequently used measure of beamwidth is the
, which is the angular separation between the first nulls on either sides of the main lobe.

Beamwidth defines the resolution capability of the antenna: i.e., the ability of the system to separate two adjacent targets.

For antennas with rotationally symmetric lobes, the directivity can be approximated:

$$D \approx \frac{4\pi}{\theta \phi} \quad (10.32.1)$$

HP *HP*

Antenna parameters: Example

Example 10.6: Find the HPBW of an infinitesimal (Hertzian) dipole.

Assuming that the normalized radiation pattern is

$$I_n(\theta, \phi) = \sin^2 \theta$$

and its maximum is 1 at $\theta = \pi/2$. The value $I_n = 0.5$ is found at the angles $\theta = \pi/4$ and $\theta = 3\pi/4$. Therefore, the HPBW is $\theta_{HP} = \pi/2$.

Antenna parameters

Antennas exhibit a property of : the properties of an antenna are the same whether it is used as a transmitting antenna or receiving antenna.

For the receiving antennas, the can be loosely defined as a ratio of the power absorbed by the antenna to the power incident on it.

More accurate definition: —in a given direction, the ratio of the power at the antenna terminals to the power flux density of a plane wave incident on the antenna from that direction. Provided the polarization of the incident wave is identical to the polarization of the antenna.‖

The incident power density can be found as:

$$S_{av} = \frac{E^2}{2Z_0} = \frac{E^2}{240\pi} \qquad (10.34.1)$$

Antenna parameters

Assuming that the antenna is matched with the transmission line, the power received by the antenna is

$$P_L = S_{av} A_e \tag{10.35.1}$$

where A_e is the of the antenna.

Maximum power can be delivered to a load impedance, if it has a value that is complex conjugate of the antenna impedance: $Z_L = Z_A^*$. Replacing the antenna with an equivalent generator with the same voltage V and impedance Z_A, the current at the antenna terminals will be:

$$I_0 = \frac{V}{Z_A + Z_L} \tag{10.35.2}$$

Since $Z_A + Z_A^* = 2R_A$, the maximum power dissipated in the load is

$$P_L = \frac{1}{2} I_0^2 R_L = \frac{1}{2}\left(\frac{V}{Z_A + Z_A^*}\right)^2 R_L = \frac{V^2}{8R_A} \tag{10.35.3}$$

Antenna parameters

For the Hertzian dipole, the maximum voltage was found as *EL* and the antenna resistance was calculated as $20\pi^2 (L/\lambda)^2$. Therefore, for the Hertzian dipole:

$$P_L = \frac{(EL)^2}{8 \cdot 80\pi^2 (L/\lambda)^2} = \frac{E^2 \lambda^2}{640\pi^2} \quad (10.36.1)$$

Therefore, for the Hertzian dipole:

$$A_e = \frac{\lambda^2}{4\pi}\frac{3}{2} = \frac{3\lambda^2}{8\pi} \quad (10.36.2)$$

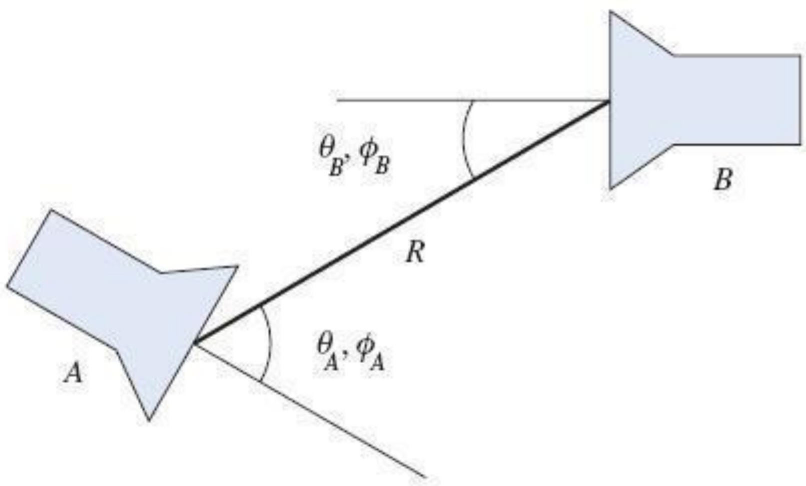
In general, the effective area of the antenna is:

$$A_e = \frac{\lambda^2}{4\pi} D \quad (10.36.3)$$

$$A_e = \frac{\lambda^2}{4\pi} G \quad (10.36.4)$$

Antenna parameters

Assuming that both antennas are in the far field region and that antenna A transmit to antenna B. The gain of the antenna A in the direction of B is G_t, therefore the average power density at the receiving antenna B is



$$S_{av} = \frac{P_t}{4\pi R^2} G_t \qquad (10.37.1)$$

The power received by the antenna B is:

$$P_r = S_{av} A_{e,r} = \frac{P_t}{4\pi R^2} G_t \frac{\lambda^2}{4\pi} G_r = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2} \qquad (10.37.2)$$

The (ignoring polarization and impedance mismatch) is:

$$\frac{P_r}{P_t} = \frac{G_t G_r \lambda^2}{(4\pi R)^2} = \frac{A_{e,t} A_{e,r}}{\lambda^2 R^2} \qquad (10.37.3)$$

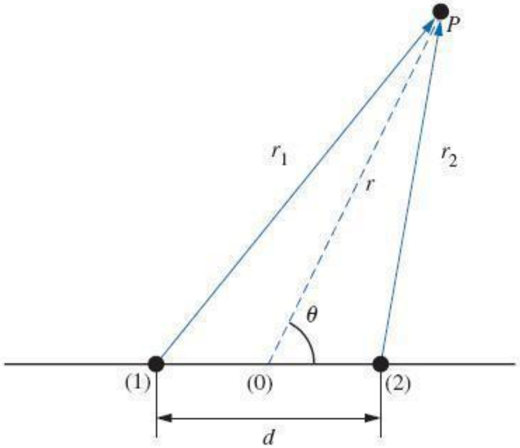
Antenna arrays

It is not always possible to design a single antenna with the radiation pattern needed. However, a proper combination of various types of antennas might yield the required pattern.

An is a cluster of antennas arranged in a specific physical configuration (line, grid, etc.). Each individual antenna is called an

. We initially assume that all array elements (individual antennas) are identical. However, the excitation (both amplitude and phase) applied to each individual element may differ. The far field radiation from the array in a linear medium can be computed by the superposition of the EM fields generated by the array elements.

We start our discussion from considering a (elements are located in a straight line) consisting of two elements excited by the signals with the same amplitude but with phases shifted by δ.



Antenna arrays

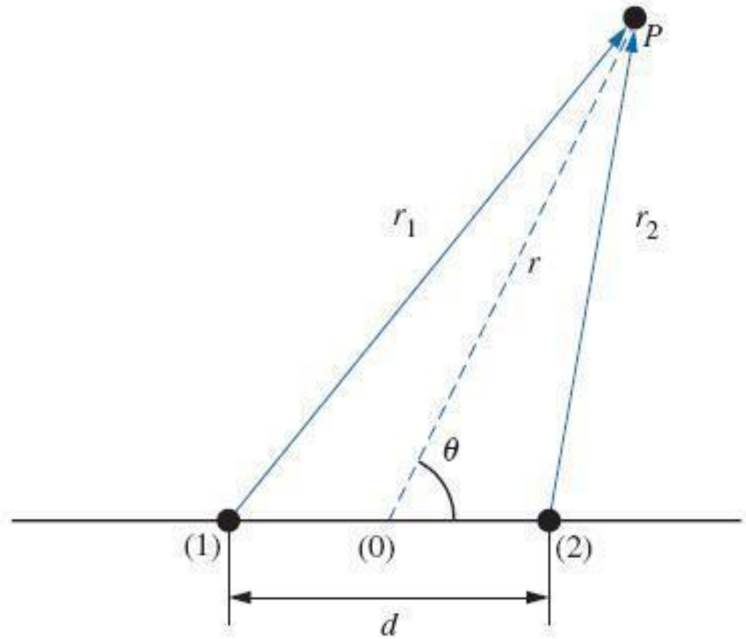
The individual elements are characterized by their element patterns $F_1(\theta,\phi)$.

At an arbitrary point P, taking into account the phase difference due to physical separation and difference in excitation, the total far zone electric field is:

$$E(r) = E_1(r)\,e^{j\psi/2} + E_2(r)\,e^{-j\psi/2} \quad (10.39.1)$$

Field due to antenna 1 Field due to antenna 2

Here: $\psi = kd\cos\theta + \delta$ (10.39.2)



The phase center is assumed at the array center. Since the elements are identical

$$E(r) = 2E_1(r)\frac{e^{j\psi/2} + e^{-j\psi/2}}{2} = 2E_1(r)\cos\frac{\psi}{2} \quad (10.39.3)$$

Relocating the phase center point only changes the phase of the result but not its amplitude.

Antenna arrays

The can be written as a product of the radiation pattern of an individual element and the radiation pattern of the array (array pattern):

$$F(\theta, \phi) = F_1(\theta, \phi) \cdot F_a(\theta, \phi) \qquad (10.40.1)$$

where the array factor is:

$$F_a(\theta, \phi) = \cos\left(\frac{kd\cos\theta + \delta}{2}\right) \qquad (10.40.2)$$

Here δ is the phase difference between two antennas. We notice that the array factor depends on the array geometry and amplitude and phase of the excitation of individual antennas.

Antenna arrays: Example

Example 10.7: Find and plot the array factor for 3 two-element antenna arrays, that differ only by the separation difference between the elements, which are isotropic radiators. Antennas are separated by 5, 10, and 20 cm and each antenna is excited in phase. The signal's frequency is 1.5 GHz.

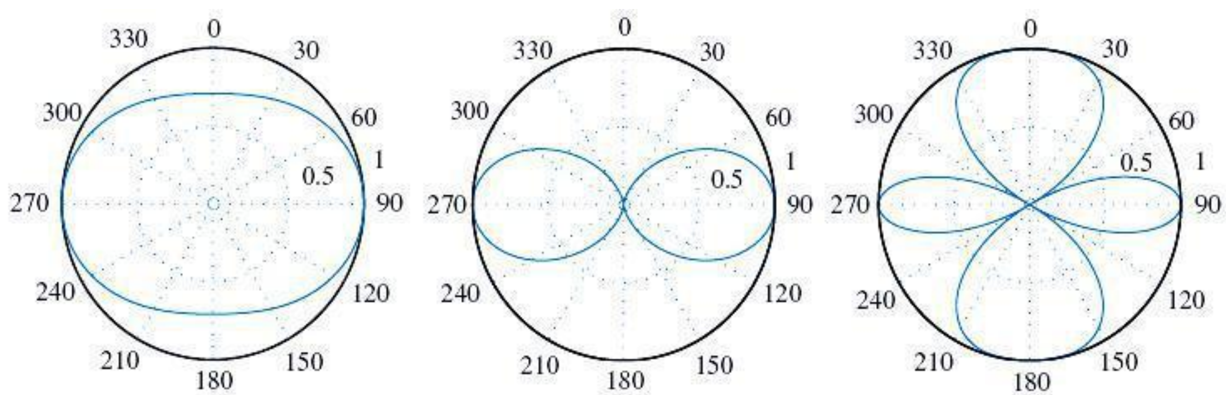
The separation between elements is normalized by the wavelength via

$$\xi = kd/2 = \pi d/\lambda$$

The free space wavelength:

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{1.5 \cdot 10^9} = 20\ cm$$

Normalized separations are $\lambda/4$, $\lambda/2$, and λ. Since phase difference is zero ($\delta = 0$) and the element patterns are uniform (isotropic radiators), the total radiation pattern $F(\theta) = F_a(\theta)$.



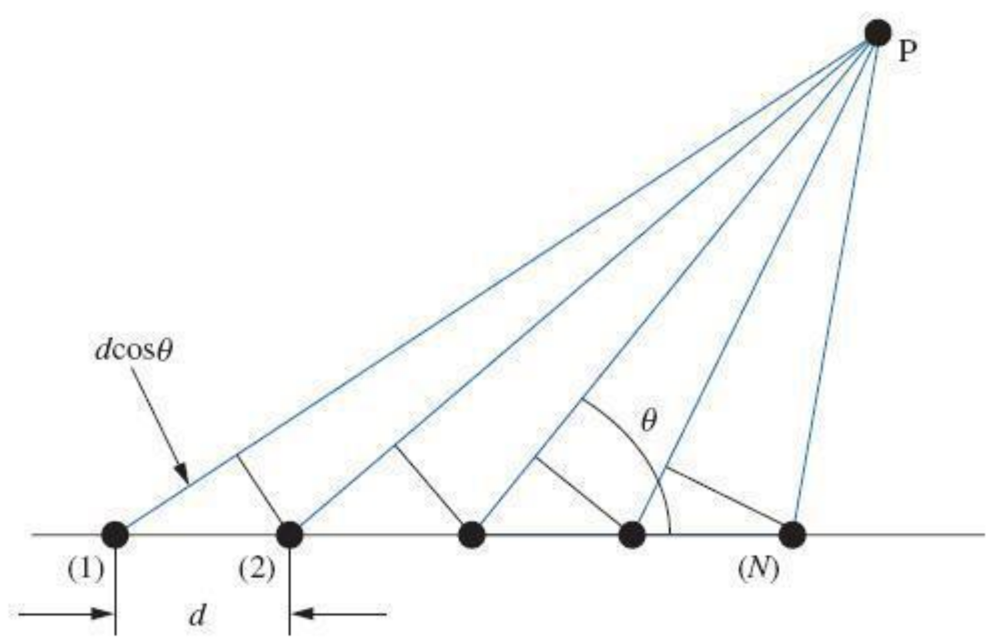
Antenna arrays

Another method of modifying the radiation pattern of the array is to change electronically the phase parameter δ of the excitation. In this situation, it is possible to change direction of the main lobe in a wide range: the antenna is scanning through certain region of space. Such structure is called a

.

We consider next an antenna array with more identical elements.

There is a linearly progressive phase shift in the excitation signal that feeds N elements.



The total field is:

$$E(r) = E_0(r)\left[1 + e^{j\psi} + \ldots + e^{j(N-1)\psi}\right] \quad (10.42.1)$$

Utilizing the following relation:

$$\sum_{n=0}^{N-1} q^n = \frac{1-q^N}{1-q} \quad (10.42.2)$$

Antenna arrays

the total radiated electric field is

$$E = E_0 \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \tag{10.43.1}$$

Considering the magnitude of the electric field only and using

$$\left|1 - e^{j\xi}\right| = \left|2 j e^{j\xi/2} \sin\frac{\xi}{2}\right| = 2 \sin\frac{\xi}{2} \tag{10.43.2}$$

we arrive at

$$E(\theta) = E_0 \sin\left(\frac{N\psi}{2}\right) \Big/ \sin\left(\frac{\psi}{2}\right) \tag{10.43.3}$$

where

$$\psi = kd \cos\theta + \delta \tag{10.43.4}$$

δ is the progressive phase difference between the elements. When $\psi = 0$:

$$E(\theta) = E_{max} = NE_0 \tag{10.43.5}$$

Antenna arrays

The :

$$F_a(\theta) = \frac{\sin\left(\frac{N\psi}{2}\right)}{N \sin\left(\frac{\psi}{2}\right)} \qquad (10.44.1)$$

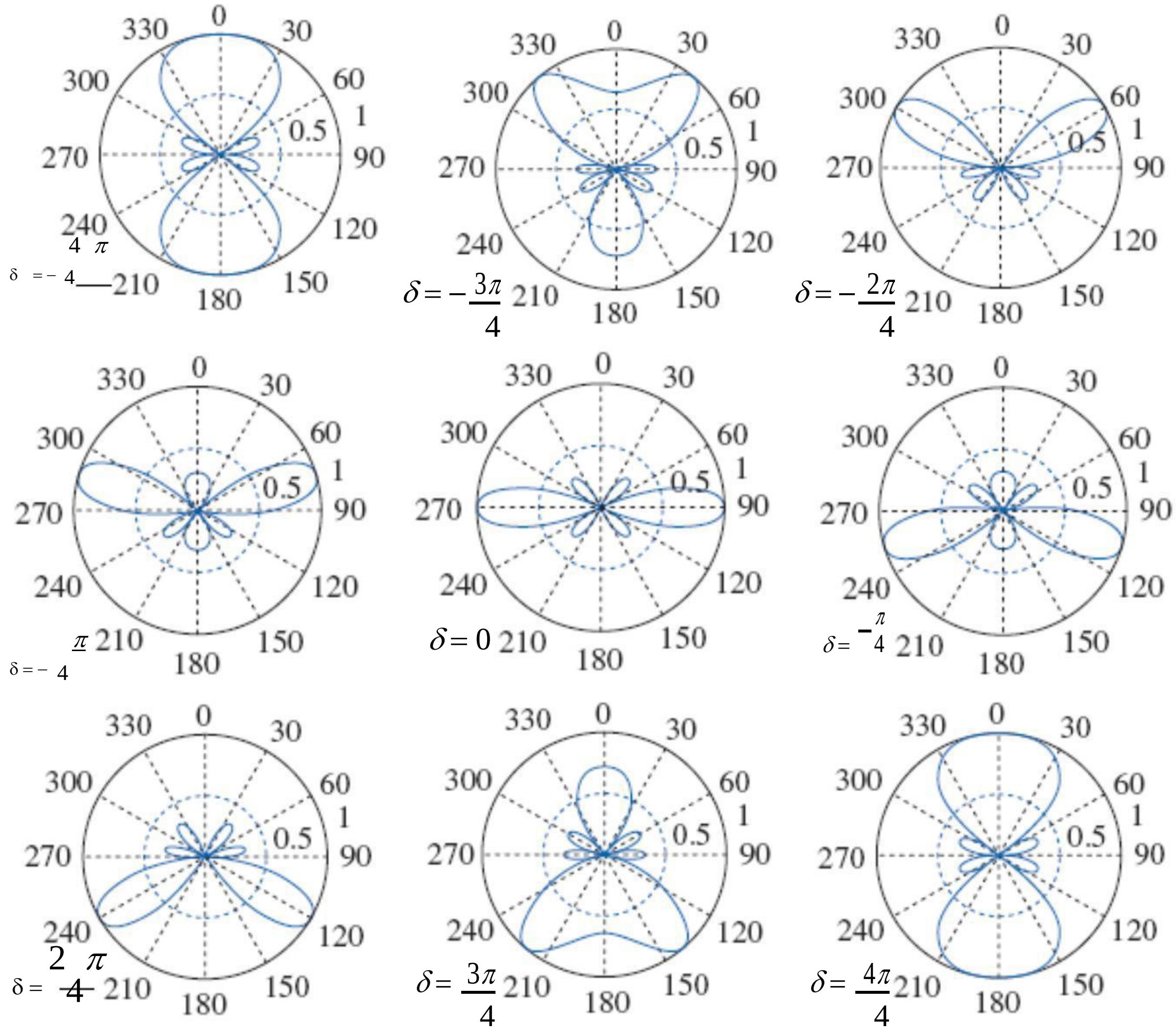
The angles where the first null occur in the numerator of (10.43.1) define the beamwidth of the main lobe. This happens when

$$\psi = \pm k\, 2\pi / N, \qquad k \text{ is integer} \qquad (10.44.2)$$

Similarly, zeros in the denominator will yield maxima in the pattern.

Antenna arrays

Field patterns of a four-element (N = 4) phased-array with the physical separation of the isotropic elements $d = \lambda/2$ and various phase shift.



Antenna arrays

Another method to analyze behavior of a phase-array is by considering a
of its elements.

Let us consider a three-element array shown. The elements are excited in phase (δ = 0) but the excitation amplitude for the center element is twice the amplitude of the other elements. This system is called a .

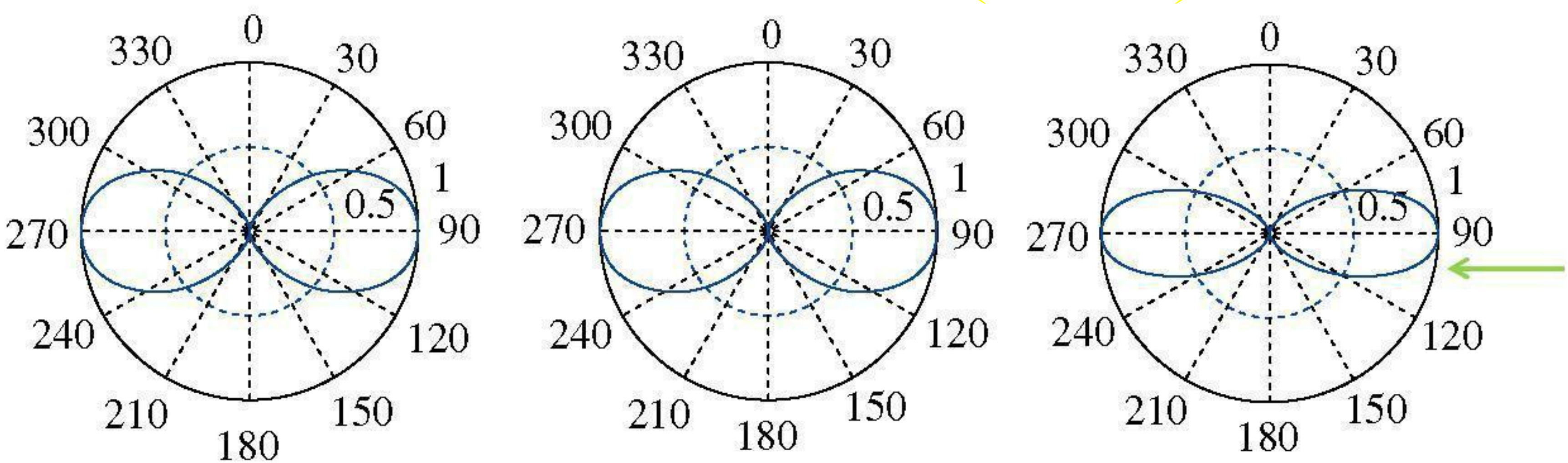
Because of this type of excitation, we can assume that this three-element array is equivalent to 2 two-element arrays (both with uniform excitation of their elements) displaced by λ/2 from each other. Each two-element array will have a radiation pattern:

$$F(\theta) = \cos\left(\frac{\pi}{2}\cos\theta\right) \qquad (10.46.1)$$

Antenna arrays

Next, we consider the initial three-element binomial array as an equivalent two-element array consisting of elements displaced by $\lambda/2$ with radiation patterns (10.46.1). The array factor for the new equivalent array is also represented by (10.46.1). Therefore, the magnitude of the radiated field in the far-zone for the considered structure is:

$$F(\theta) = F_1(\theta)\, F_A(\theta) = \cos^2\left(\frac{\pi}{2}\cos\theta\right) \qquad (10.47.1)$$



Element pattern $F_1(\theta)$ Array factor $F_A(\theta)$ Antenna pattern $F(\theta)$

Antenna arrays (Example)

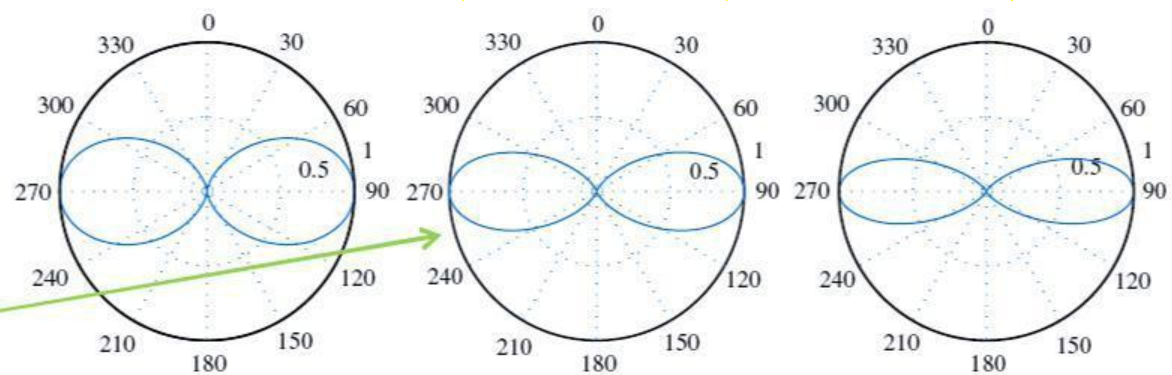
Example 10.8: Using the concept of multiplication of patterns (the one we just used), find the radiation pattern of the array of four elements shown below.

This array can be replaced with an array of two elements containing three sub-elements (with excitation 1:2:1). The initial array will have an excitation 1:3:3:1 and will have a radiation pattern, according to (10.40.1), as:

$$F(\theta) = \cos\left(\frac{\pi}{2}\cos\theta\right)\cos^2\left(\frac{\pi}{2}\cos\theta\right) = \cos^3\left(\frac{\pi}{2}\cos\theta\right)$$

Array factor

Element pattern



Antenna array pattern

Antenna arrays

Continuing the process of adding elements, it is possible to synthesize a radiation pattern with arbitrary high directivity and no sidelobes if the excitation amplitudes of array elements correspond to the coefficients of binomial series.

This implies that the amplitude of the k^{th} source in the *N*-element binomial array is calculated as

$$I_k = \frac{N!}{k!(N-k)!}, \qquad k = 0,1,\ldots, N \tag{10.49.1}$$

It can be seen that this array will be symmetrically excited:

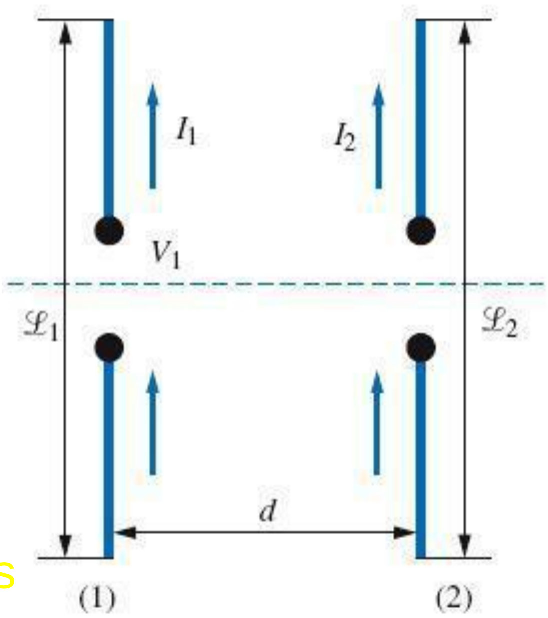
$${}_{N-k} = I_k \tag{10.49.2}$$

Therefore, the resulting radiation pattern of the binomial array of *N* elements separated by a half wavelength is

$$F(\theta) = \cos^{N-1}\left(\frac{\pi}{2}\cos\theta\right) \tag{10.49.3}$$

Antenna arrays

During the analysis considered so far, the effect of between the elements of the antenna array was ignored. In the reality, however, fields generated by one antenna element will affect currents and, therefore, radiation of other elements.



Let us consider an array of two dipoles with lengths L_1 and L_2. The first dipole is driven by a voltage V_1 while the second dipole is passive. We assume that the currents in both terminals are I_1 and I_2 and the following circuit relations+hold: =

$$Z_{11} I_1 \quad Z_{12} I_2 \quad V_1$$

(10.50.1)

$$Z_{21} I_1 + Z_{22} I_2 = 0$$

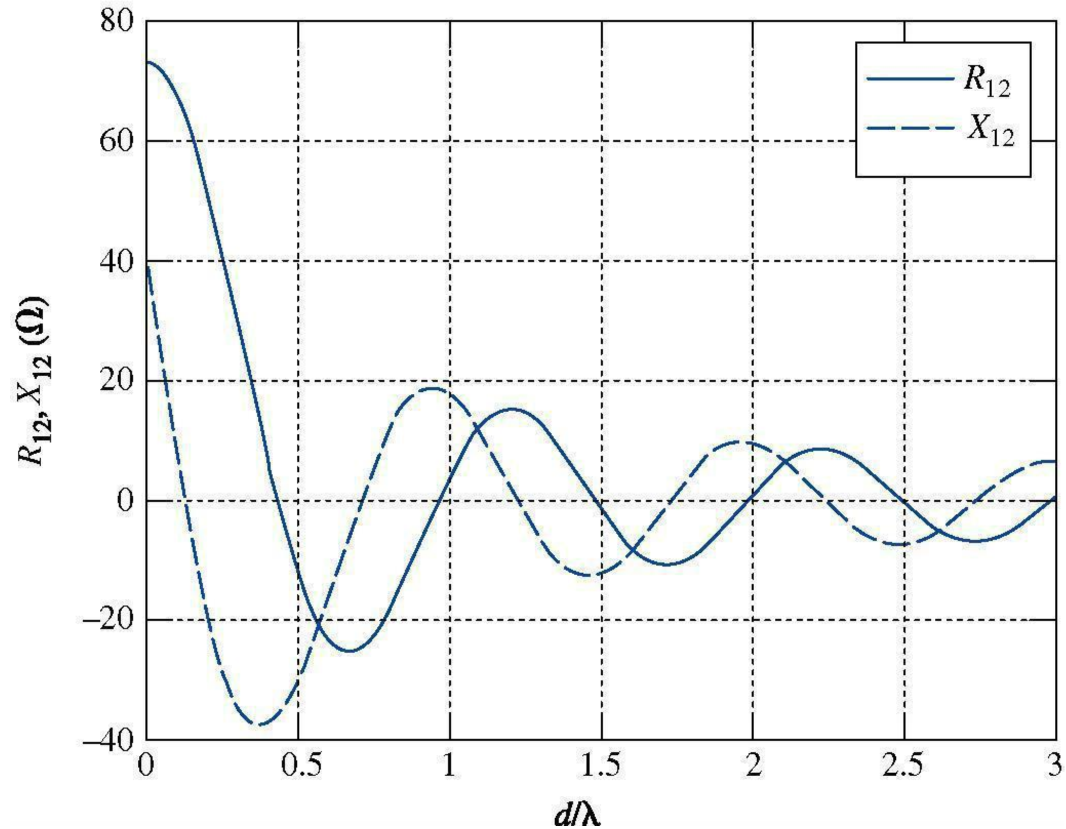
where Z_{11} and Z_{22} are the self-impedances of antennas (1) and (2) and $Z_{12} = Z_{21}$ are the mutual impedances between the elements. If we further assume that the dipoles are equal, the self-impedances will be equal too.

Antenna arrays

In the case of thin half-wavelength dipoles, the self-impedance is

$$Z_{11} = 73.1 + j\,42.5\ \ \Omega$$

The dependence of the mutual impedance between two identical thin half-wavelength dipoles is shown. When separation between antennas *d* $\rightarrow 0$, mutual impedance approaches the self-impedance.



For the 2*M*+1 identical array elements separated by λ/2, the directivity is:

$$\left(\quad \right)^2 \Big/ \sum_{n=-M}^{M} I_n^{\ 2} \qquad (10.51.1)$$

Antenna arrays: Example

Example 10.9: Compare the directivities of two arrays consisting of three identical elements separated by a half wavelength for the:

a) Uniform array: $I_{-1} = I_0 = I_1 = 1A$;

b) Binomial array: $I_{-1} = I_1 = 1A$; $I_0 = 2A$.

We compute from (10.51.1):

Uniform array:

$$D = \frac{(1+1+1)^2}{1+1+1} = 3 \approx 4.77\ dB$$

Binomial array:

$$D = \frac{(1+2+1)^2}{1+4+1} = \frac{16}{6} \approx 4.26\ dB$$

The directivity of a uniform array is higher than of a binomial array.