BASIC ELECTRICAL ENGINEERING

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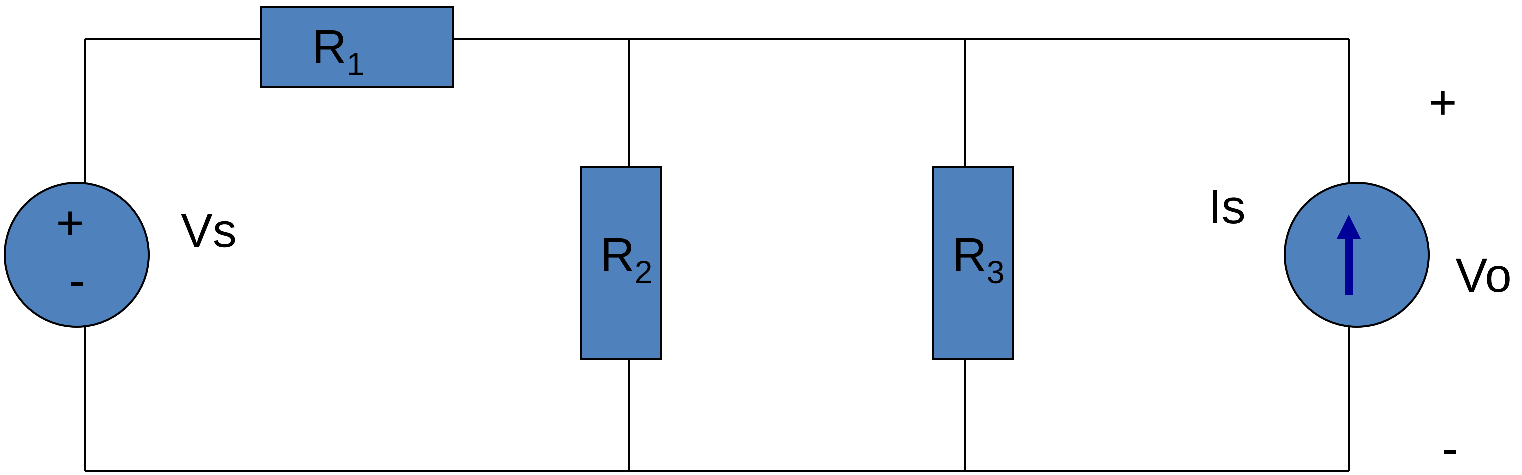
UNIT-I
Introduction to Electrical Engineering

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

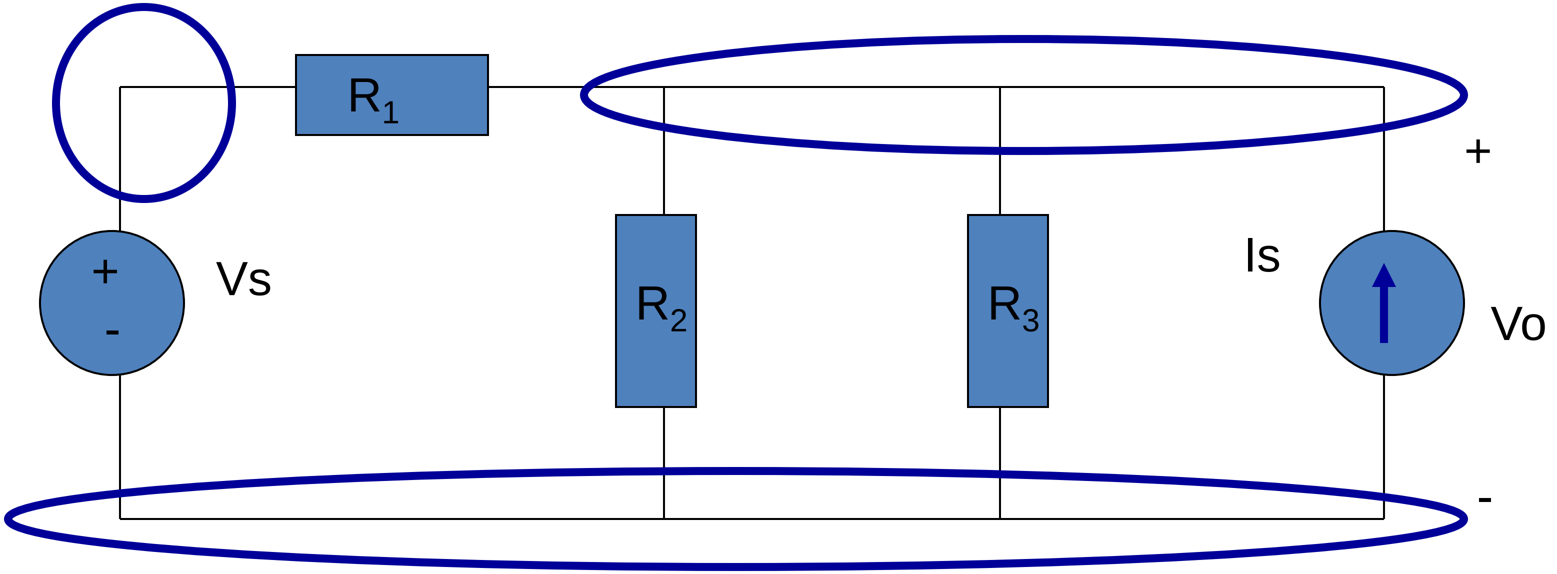
Example

- How many nodes, branches & loops?



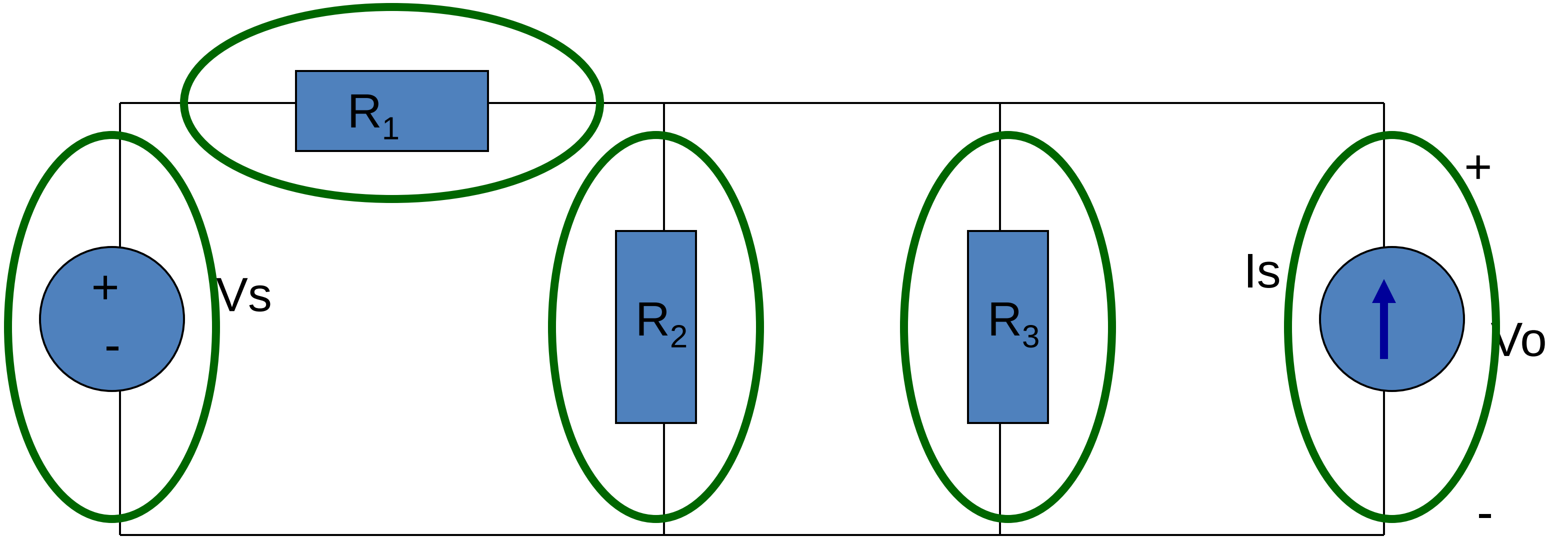
Example

- Three nodes



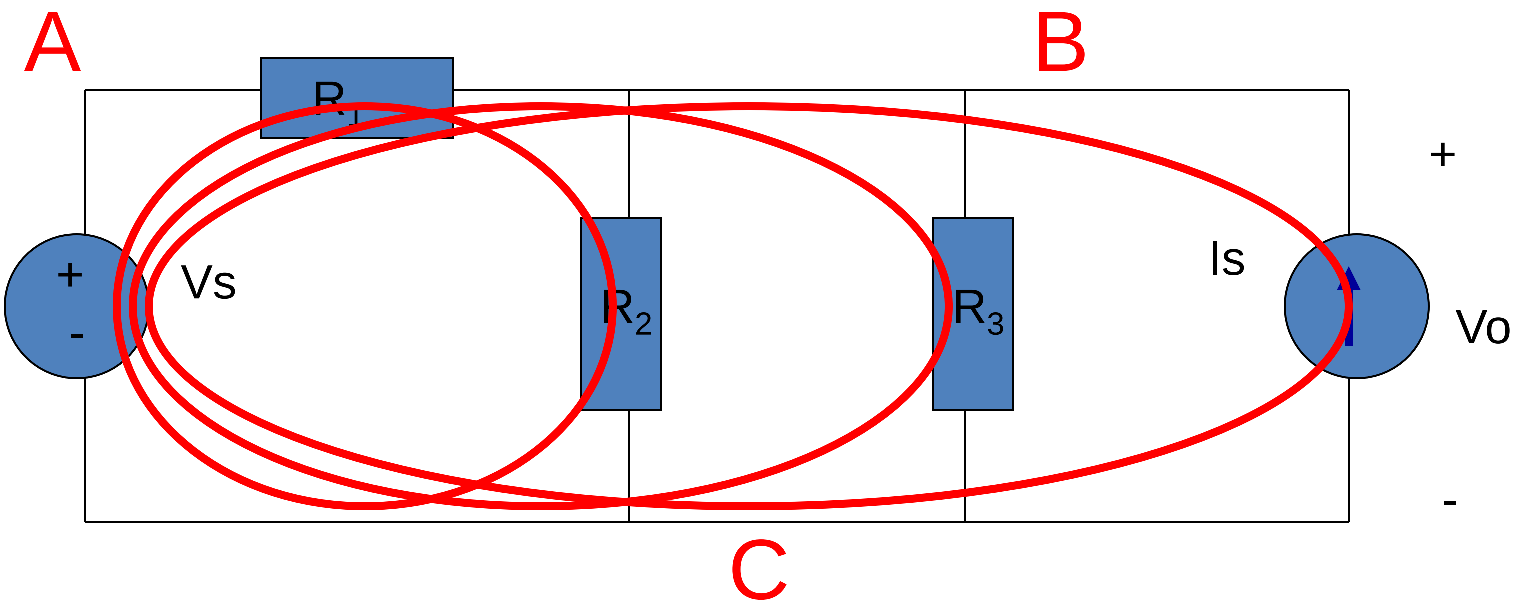
Example

- 5 Branches



Example

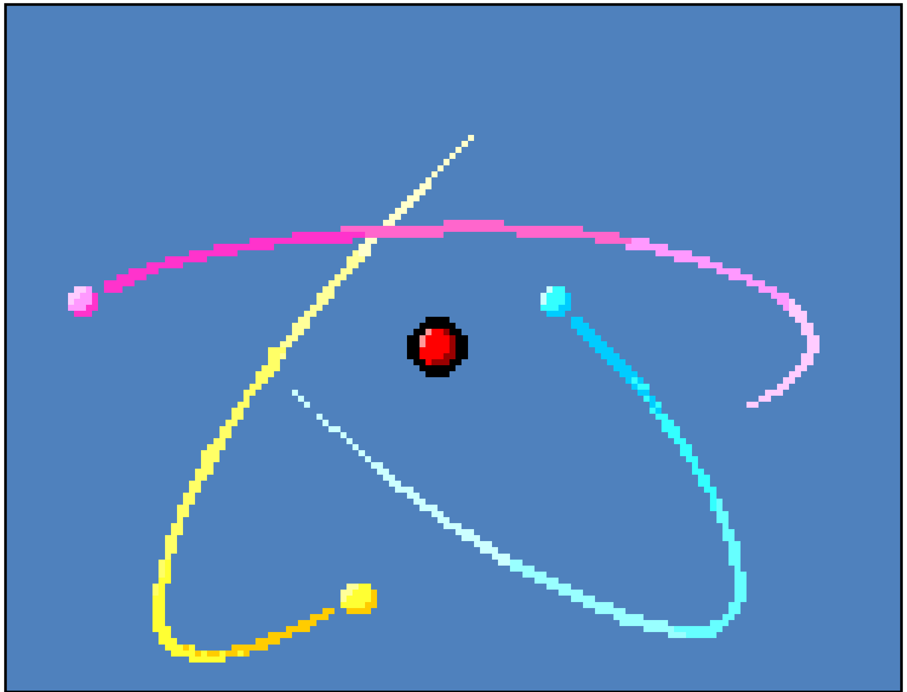
- Three Loops, if starting at node A



Basic definitions

- ✓ voltage
- ✓ Current
- ✓ Power
- ✓ Charge
- ✓ Work

Circuit Elements



Active vs. Passive Elements

- *Active elements* can generate energy
  – Voltage and current sources
  – Batteries
- *Passive elements* cannot generate energy
  – Resistors
  – Capacitors and Inductors (but CAN store energy)

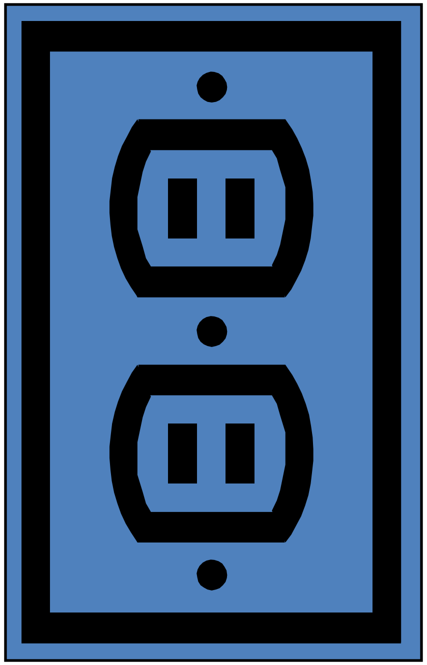
The 5 Basic Circuit Elements

There are 5 basic circuit elements:

1. Voltage sources
2. Current sources
3. Resistors
4. Inductors
5. Capacitors

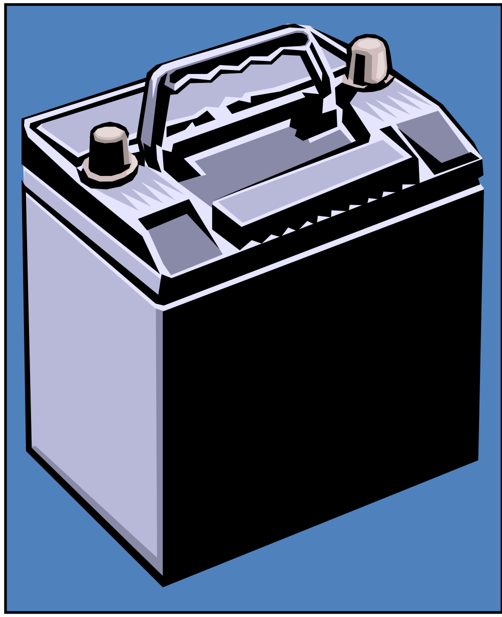
Voltage Sources

- A voltage source is a two-terminal circuit element that maintains a voltage across its terminals.
- The value of the voltage is the defining characteristic of a voltage source.
- Any value of the current can go through the voltage source, in any direction. The current can also be zero. The voltage source does not "care about" current. It "cares" only about voltage.



Voltage Sources – Ideal and Practical

- A voltage source maintains a voltage across its terminals no matter what you connect to those terminals.
- We often think of a battery as being a voltage source. For many situations, this is fine. Other times it is not a good model. A real battery will have different voltages across its terminals in some cases, such as when it is supplying a large amount of current. As we have said, a voltage source should not change its voltage as the current changes.
- We sometimes use the term <u>ideal voltage source</u> for our circuit elements, and the term <u>practical voltage source</u> for things like batteries. We will find that a more accurate model for a battery is an ideal voltage source in series with a resistor. More on that later.



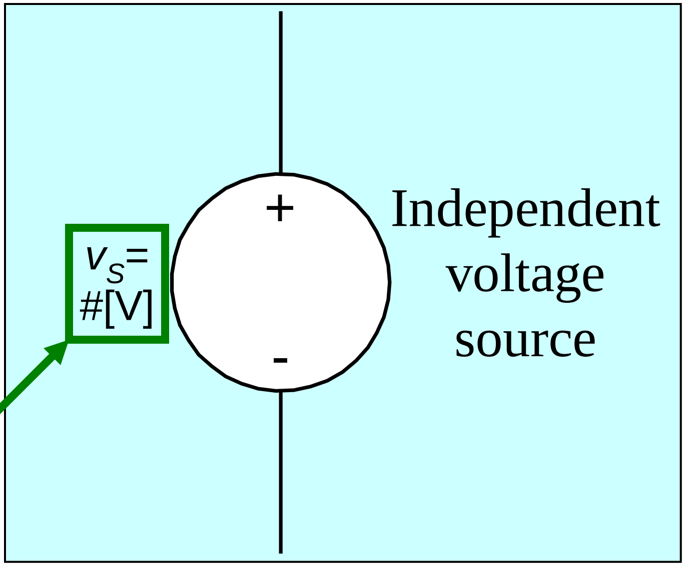
Voltage Sources – 2 kinds

There are 2 kinds of voltage sources:

1. Independent voltage sources
2. Dependent voltage sources, of which there are 2 forms:
   i. Voltage-dependent voltage sources
   ii. Current-dependent voltage sources

Voltage Sources – Schematic Symbol for Independent Sources

The schematic symbol that we use for independent voltage sources is shown here.



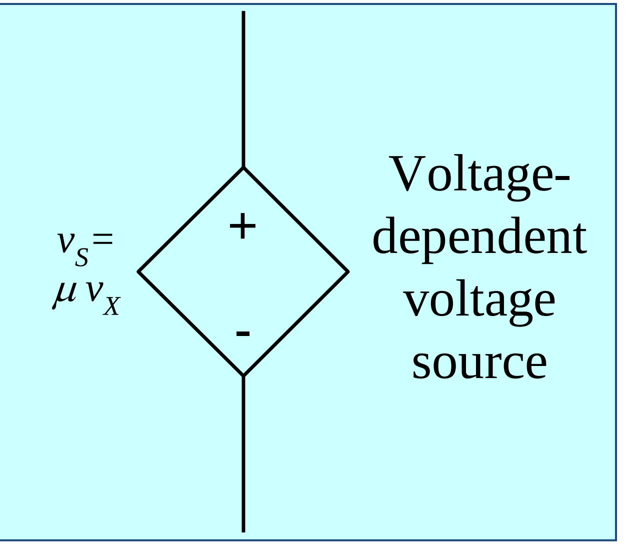
This is intended to indicate that the schematic symbol can be labeled either with a variable, like v_S, or a value, with some number, and units. An example might be 1.5[V]. It could also be labeled with both.

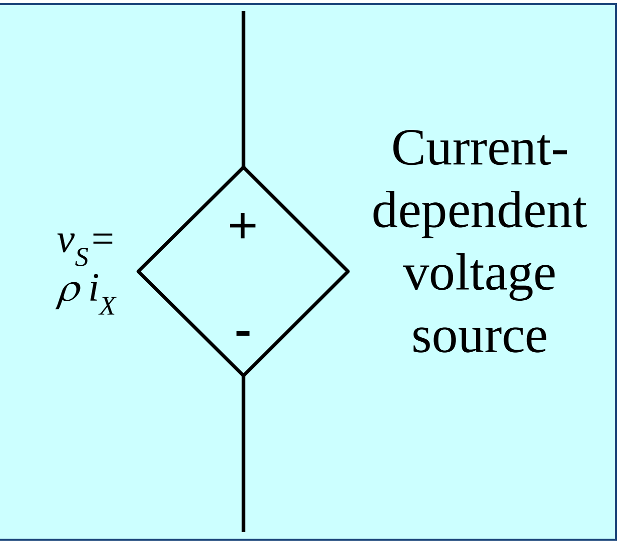
Voltage Sources – Schematic Symbols for Dependent Voltage Sources

The schematic symbols that we use for dependent voltage sources are shown here, of which there are 2 forms:

i. Voltage-dependent voltage sources

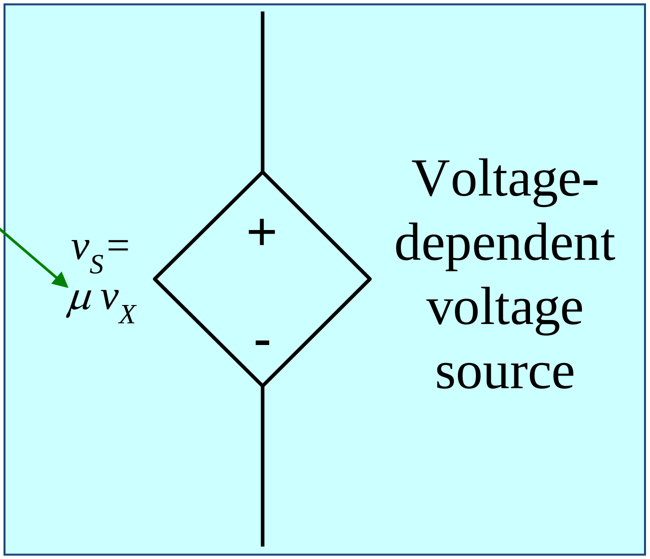
ii. Current-dependent voltage sources





Notes on Schematic Symbols for Dependent Voltage Sources

The symbol μ is the coefficient of the voltage v_X. It is dimensionless. For example, it might be 4.3 v_X. The v_X is a voltage somewhere in the circuit.



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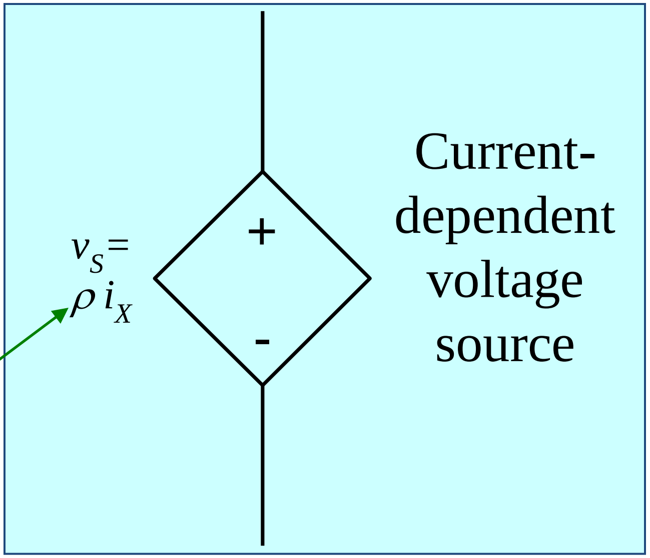
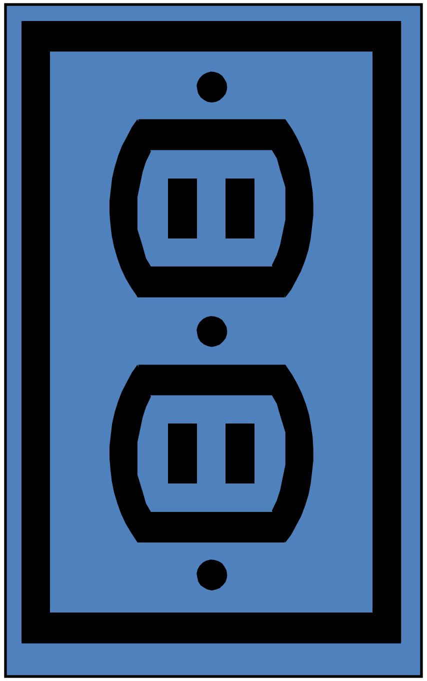
i. Voltage-dependent voltage sources
ii. Current-dependent voltage sources



The symbol ρ is the coefficient of the current i_X. It has dimensions of [voltage/current]. For example, it might be 4.3[V/A] i_X. The i_X is a current somewhere in the circuit.

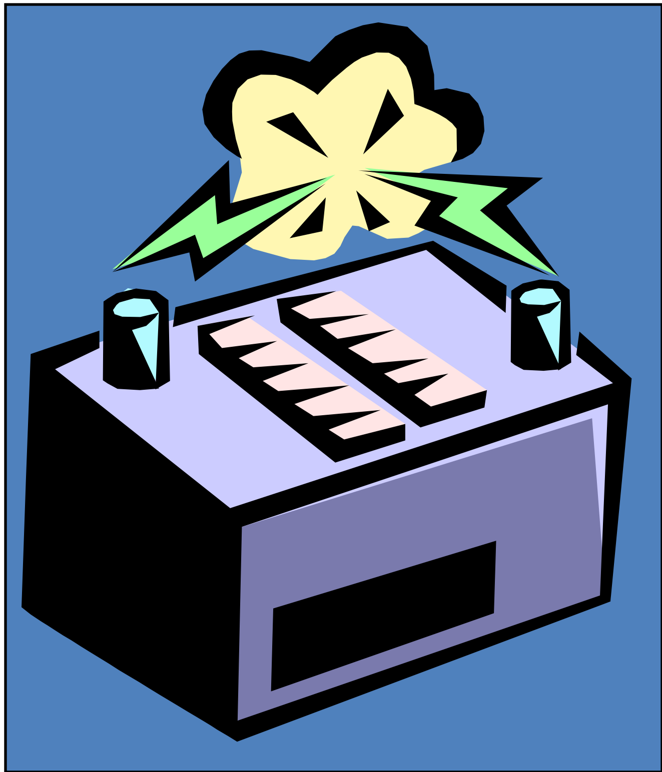
Current Sources

- A current source is a two-terminal circuit element that maintains a current through its terminals.
- The value of the current is the defining characteristic of the current source.
- Any voltage can be across the current source, in either polarity. It can also be zero. The current source does not “care about” voltage. It “cares” only about current.



Current Sources - Ideal

- A current source maintains a current through its terminals no matter what you connect to those terminals.
- While there will be devices that reasonably model current sources, these devices are not as familiar as batteries.
- We sometimes use the term <u>ideal current source</u> for our circuit elements, and the term <u>practical current source</u> for actual devices. We will find that a good model for these devices is an ideal current source in parallel with a resistor. More on that later.



Current Sources – 2 kinds

There are 2 kinds of current sources:

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2. Dependent current sources, of which there are 2 forms:
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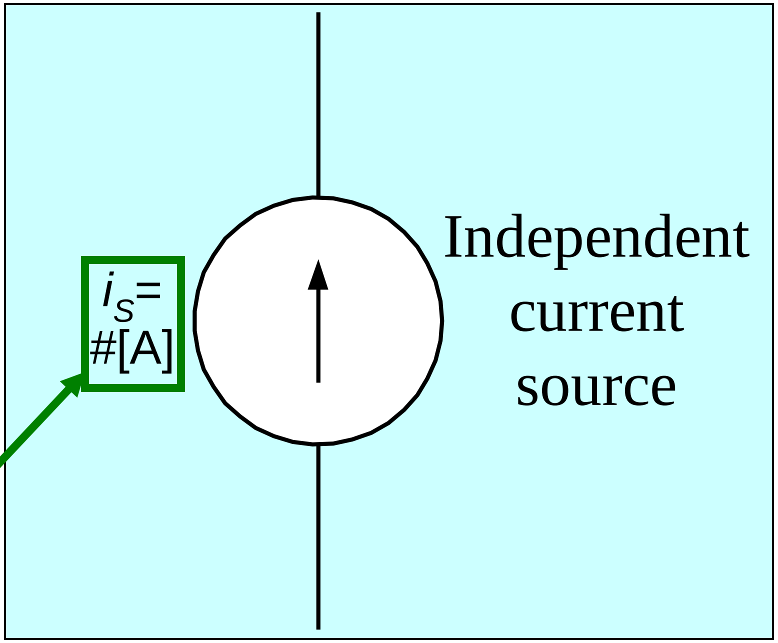
Current Sources – Schematic Symbol for Independent Sources

The schematic symbols that we use for current sources are shown here.



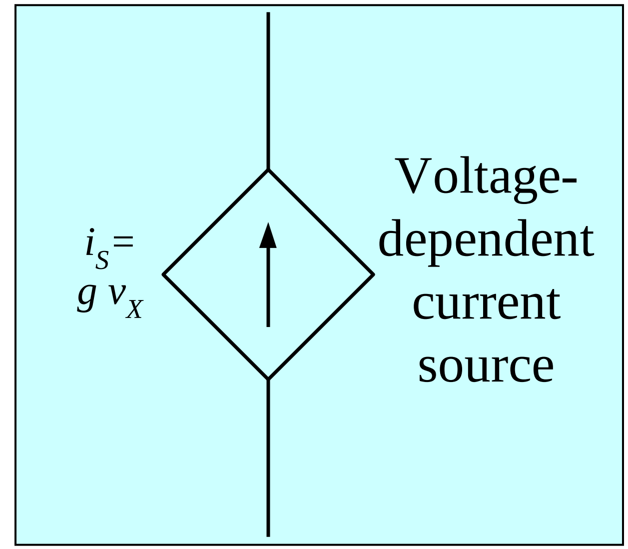
This is intended to indicate that the schematic symbol can be labeled either with a variable, like i_S, or a value, with some number, and units. An example might be 0.2[A]. It could also be labeled with both.

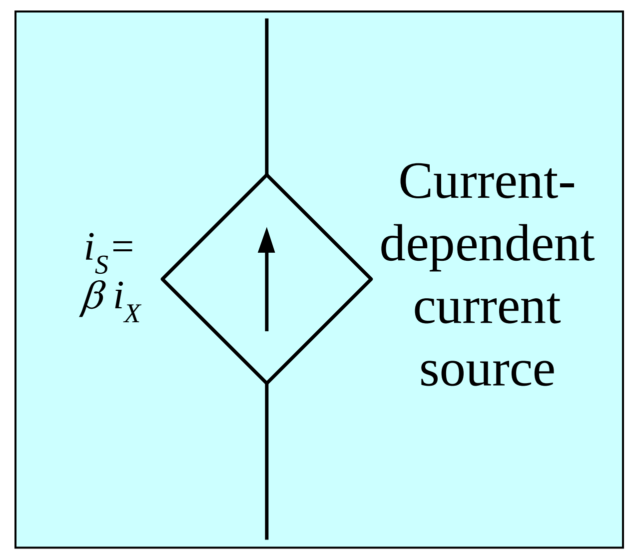
Current Sources – Schematic Symbols for Dependent Current Sources

The schematic symbols that we use for dependent current sources are shown here, of which there are 2 forms:

i. Voltage-dependent current sources

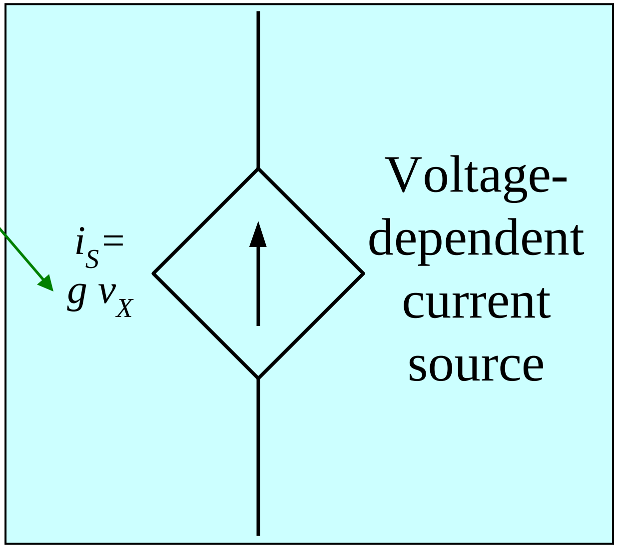
ii. Current-dependent current sources





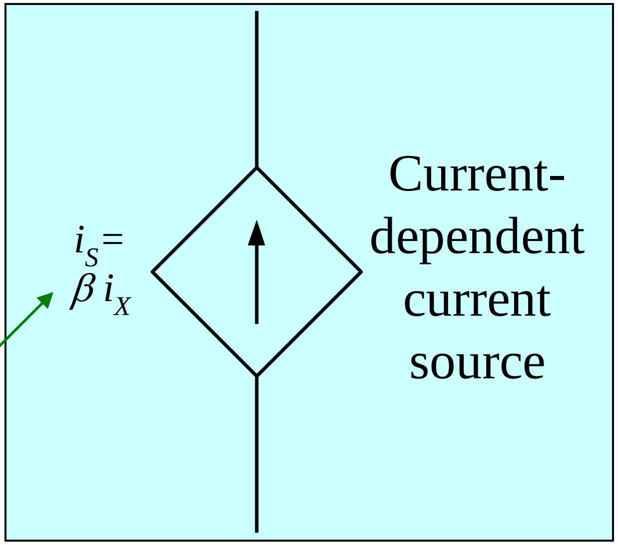
Notes on Schematic Symbols for Dependent Current Sources

The symbol g is the coefficient of the voltage v_X. It has dimensions of [current/voltage]. For example, it might be 16[A/V] v_X. The v_X is a voltage somewhere in the circuit.



The schematic symbols that we use for dependent current sources are shown here, of which there are 2 forms:

i. Voltage-dependent current sources
ii. Current-dependent current sources



The symbol β is the coefficient of the current i_X. It is dimensionless. For example, it might be 53.7 i_X. The i_X is a current somewhere in the circuit.

Kirchhoff’s Laws

Overview of this Part

In this part of the module, we will cover the following topics:

- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)

Kirchhoff's Current Law (KCL)

- With these definitions, we are prepared to state Kirchhoff's Current Law:

<u>The algebraic (or signed) summation of currents through a closed surface must equal zero.</u>

Kirchhoff's Current Law (KCL) – Some notes.

<u>The algebraic (or signed) summation of currents through any closed surface must equal zero.</u>

This definition essentially means that charge does not build up at a connection point, and that charge is conserved.

This definition is often stated as applying to nodes. It applies to any closed surface. For any closed surface, the charge that enters must leave somewhere else. A node is just a <u>small</u> closed surface. A node is the closed surface that we use most often. But, we can use any closed surface, and sometimes it is really necessary to use closed surfaces that are not nodes.

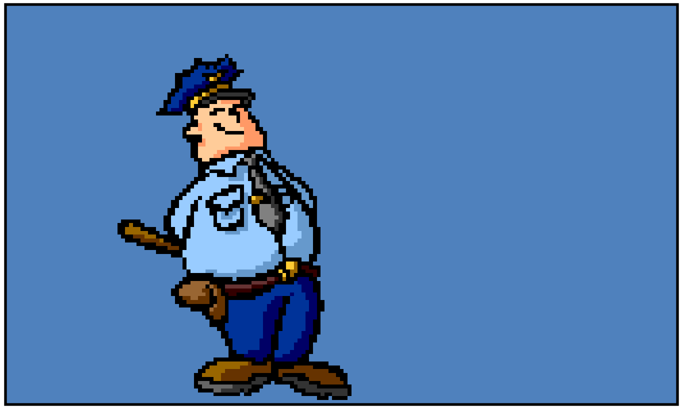
Kirchhoff's Current Law (KCL) – a Systematic Approach

<u>The algebraic (or signed) summation of currents through any closed surface must equal zero.</u>

For most students, it is a good idea to choose one way to write KCL equations, and just do it that way every time. The idea is this: If you always do it the same way, you are less likely to get confused about which way you were doing it in a certain equation.

For this set of material, we will always assign a positive sign to a term that refers to a reference current that leaves a closed surface, and a negative sign to a term that refers to a reference current that enters a closed surface.

Kirchhoff’s Voltage Law (KVL)



- Now, we are prepared to state Kirchhoff’s Voltage Law:

  The algebraic (or signed) summation of voltages around a closed loop must equal zero.

Kirchhoff’s Voltage Law (KVL) – Some notes.

The algebraic (or signed) summation of voltages around a closed loop must equal zero.

This definition essentially means that energy is conserved. If we move around, wherever we move, if we end up in the place we started, we cannot have changed the potential at that point.

This applies to all closed loops. While we usually write equations for closed loops that follow components, we do not need to. The only thing that we need to do is end up where we started.

Kirchhoff's Voltage Law (KVL) – a Systematic Approach

<u>The algebraic (or signed) summation of voltages around a closed loop must equal zero.</u>

For most students, it is a good idea to choose one way to write KVL equations, and just do it that way every time. The idea is this: If you always do it the same way, you are less likely to get confused about which way you were doing it in a certain equation.

(At least we will do this for planar circuits. For nonplanar circuits, clockwise does not mean anything. If this is confusing, ignore it for now.)

For this set of material, we will always go around loops clockwise. We will assign a positive sign to a term that refers to a reference voltage drop, and a negative sign to a term that refers to a reference voltage rise.

Ohm's Law

Georg Simon Ohm (1787-1854)

$$I = V / R$$

I = Current (Amperes) (amps)

V = Voltage (Volts)

R = Resistance (ohms)

Kirchhoff’s Laws

Kirchhoff laws

- Present Kirchhoff’s Current and Voltage Laws.
- Demonstrate how these laws can be used to find currents and voltages in a circuit.
- Explain how these laws can be used in conjunction with Ohm’s Law.

Kirchhoff’s Current Law

- Or KCL for short
  - Based upon conservation of charge – the algebraic sum of the charge within a system can not change.

$$\sum_{n=1}^{N} i_n = 0$$

Where N is the total number of branches connected to a node.

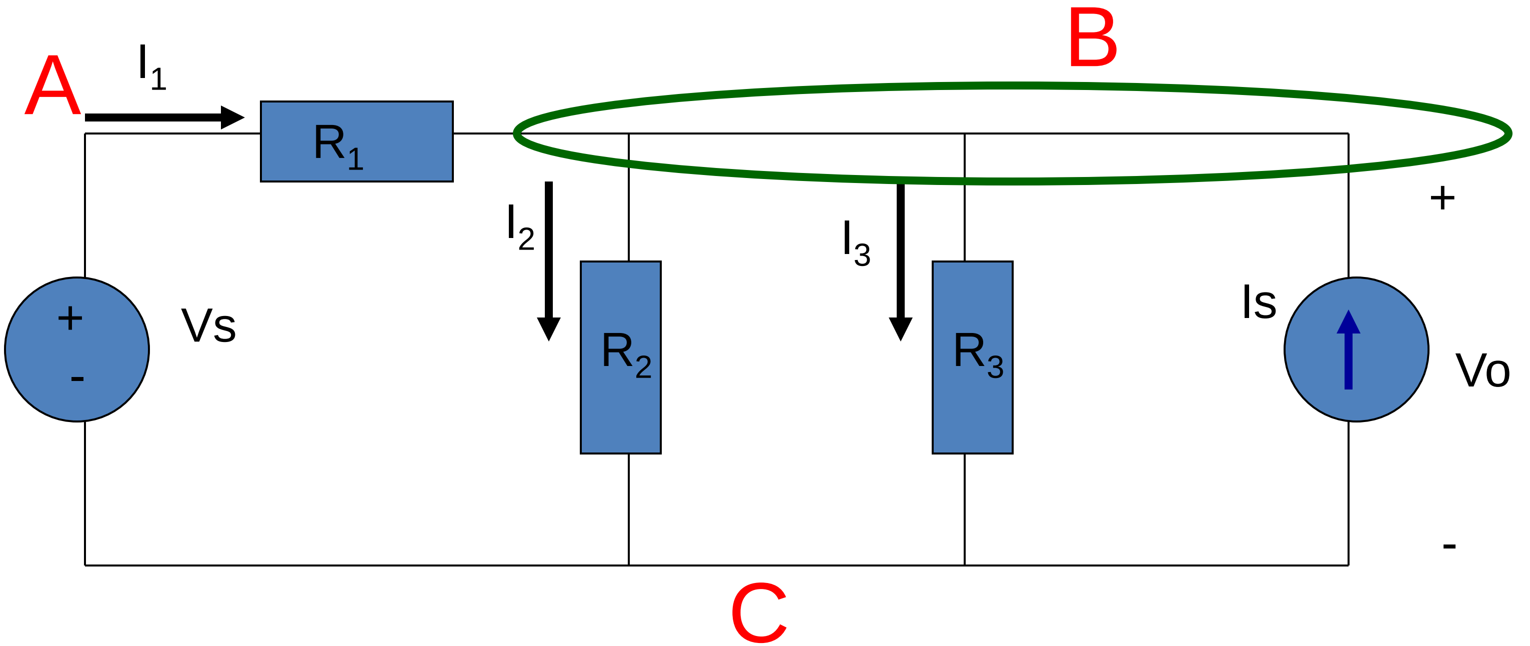
$$\sum_{\text{node}} i_{enter} = \sum_{\text{node}} i_{leave}$$

Kirchoff's Current Law (KCL)

- The algebraic sum of currents entering a node is zero
  - Add each branch current entering the node and subtract each branch current leaving the node
- Σ currents in - Σ currents out = 0
- Or Σ currents in = Σ currents out

Example

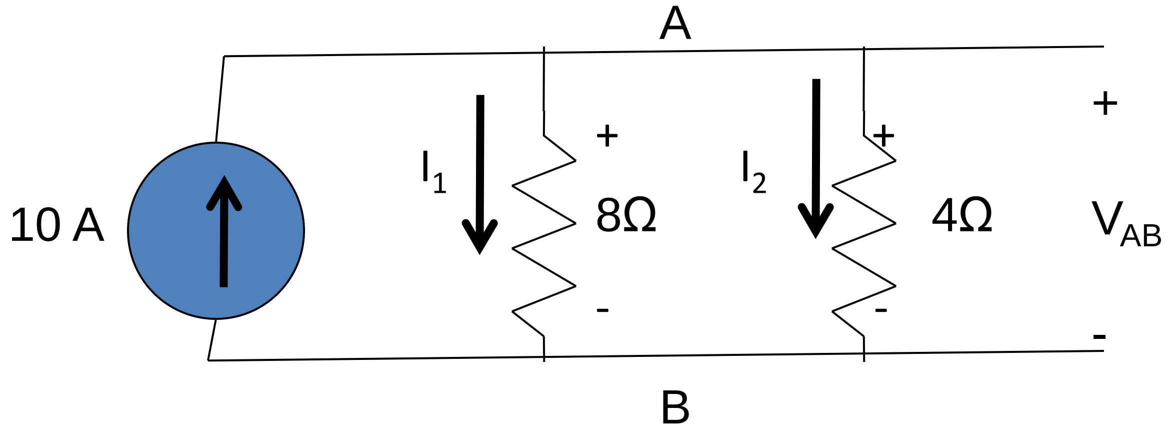
- Kirchoff's Current Law at B



Assign current variables and directions

Add currents in, subtract currents out: $I_1 - I_2 - I_3 + Is = 0$

Circuit Analysis



By KVL: $-\ I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$

Solving: $I_2 = 2 \cdot I_1$

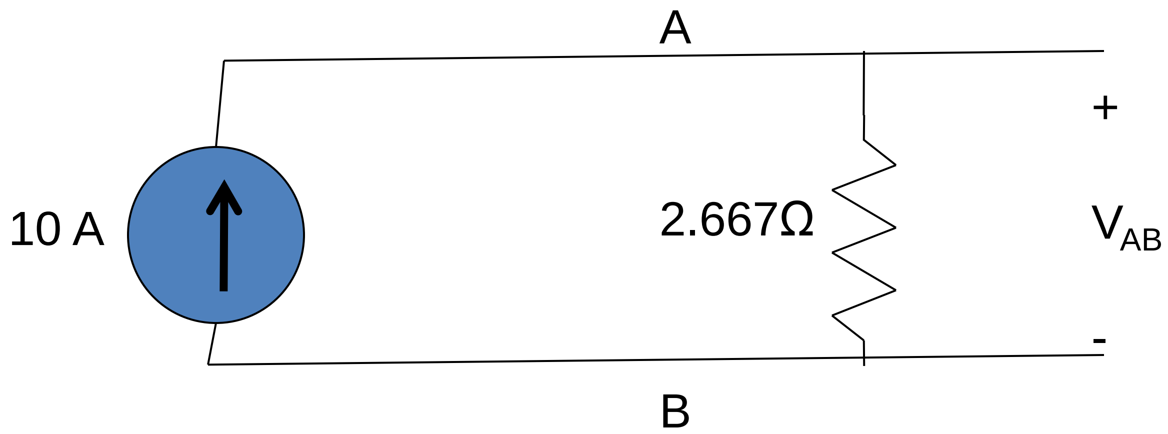
By KCL: $10A = I_1 + I_2$

Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$

So $I_1 = 3.33$ A and $I_2 = 6.67$ A

And $V_{AB} = 26.33$ volts

Circuit Analysis



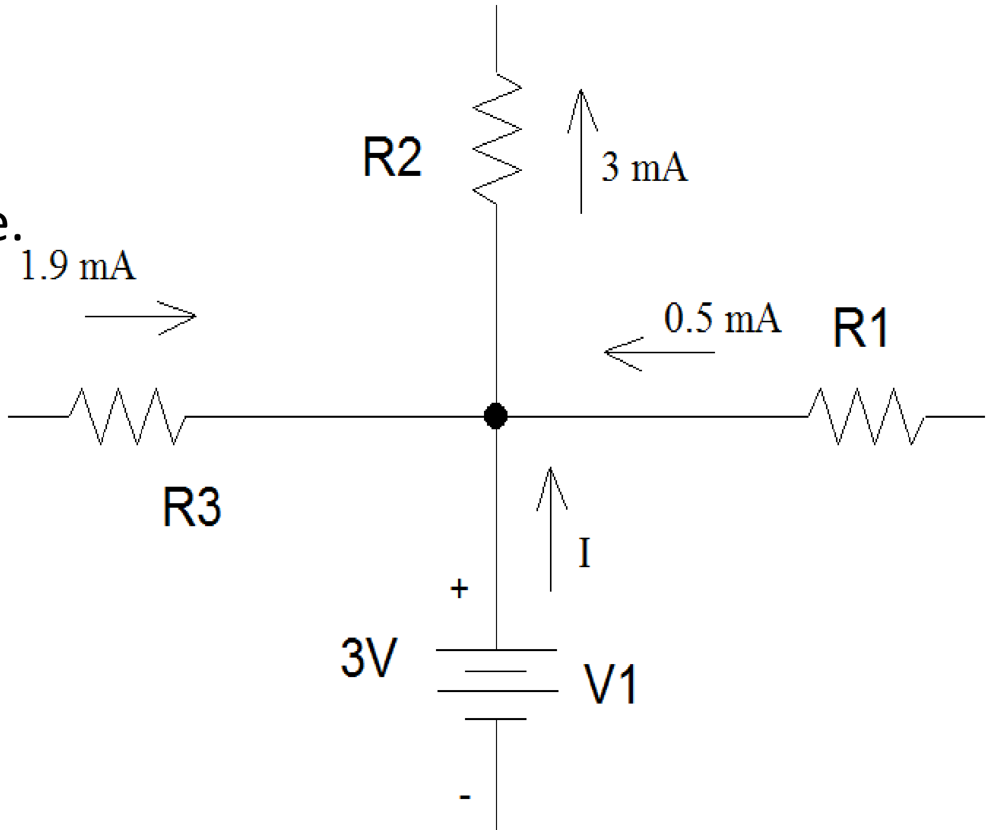
By Ohm's Law: $V_{AB} = 10\ A \cdot 2.667\ \Omega$

So $V_{AB} = 26.67$ volts

Replacing two parallel resistors (8 and 4 Ω) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.

Example 1

- Determine I, the current flowing out of the voltage source.
  - Use KCL
    - 1.9 mA + 0.5 mA + I are entering the node.
    - 3 mA is leaving the node.



$$1.9mA + 0.5mA + I = 3mA$$

$$I = 3mA - (1.9mA + 0.5mA)$$

$$I = 0.6mA$$

V1 is generating power.

Example 2

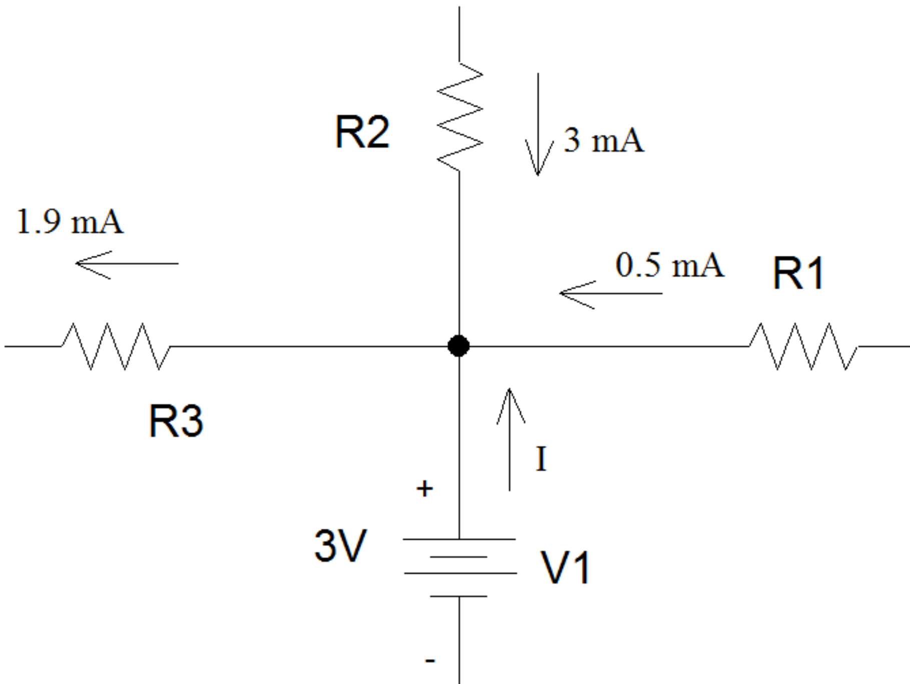
- Suppose the current through R2 was entering the node and the current through R3 was leaving the node.
  - Use KCL
    - 3 mA + 0.5 mA + I are entering the node.
    - 1.9 mA is leaving the node.

$$3mA + 0.5mA + I = 1.9mA$$

$$I = 1.9mA - (3mA + 0.5mA)$$

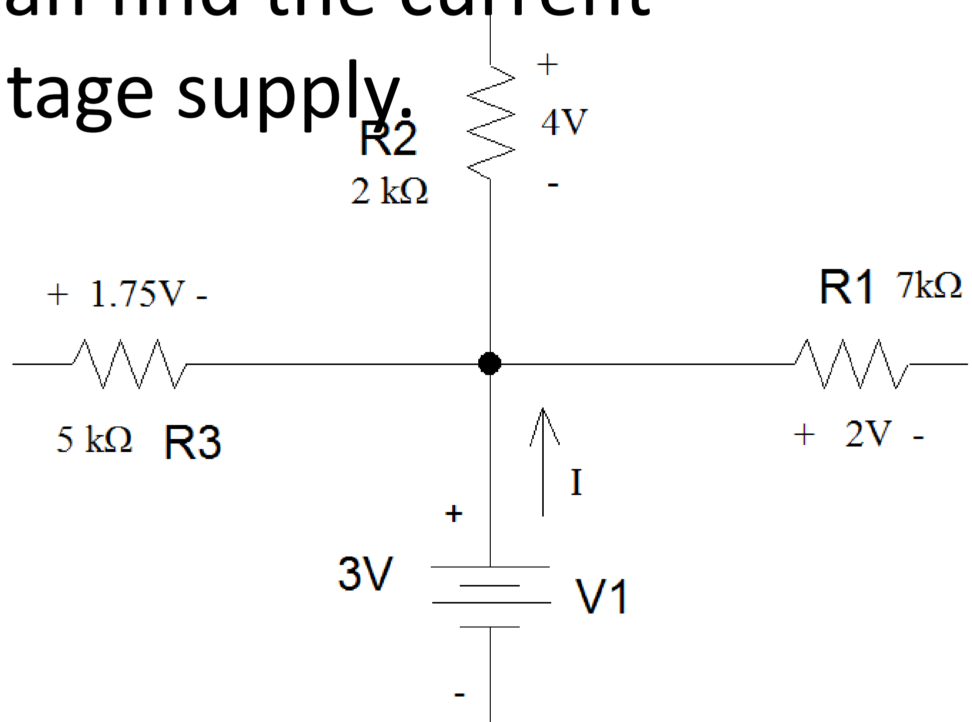
$$I = -1.6mA$$

V1 is dissipating power.



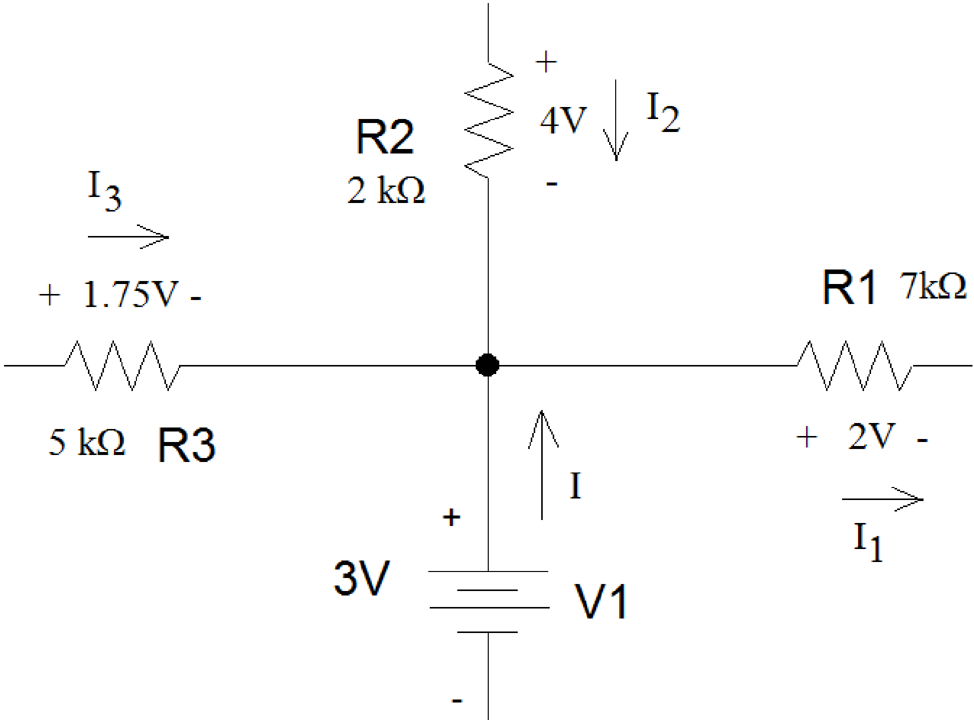
Example 3

- If voltage drops are given instead of currents, you need to apply Ohm's Law to determine the current flowing through each of the resistors before you can find the current flowing out of the voltage supply.



Example 3 (con’t)

- For power dissipating components such as resistors, passive sign convention means that current flows into the resistor at the terminal has the + sign on the voltage drop and leaves out the terminal that has the – sign.

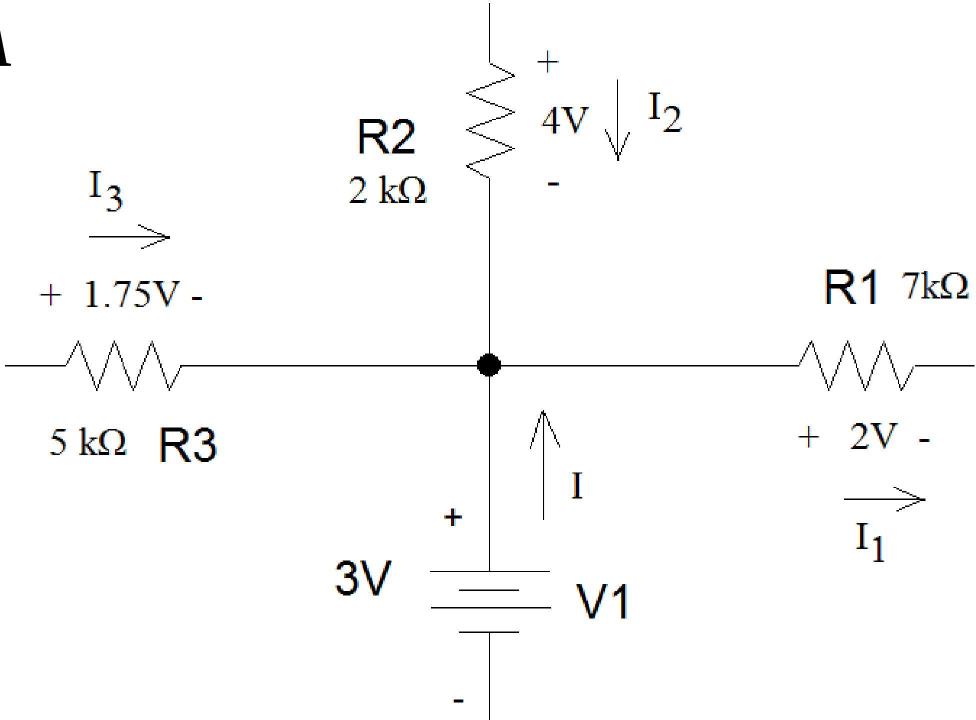


Example 3 (con't)

$$I_1 = 2V / 7k\Omega = 0.286mA$$

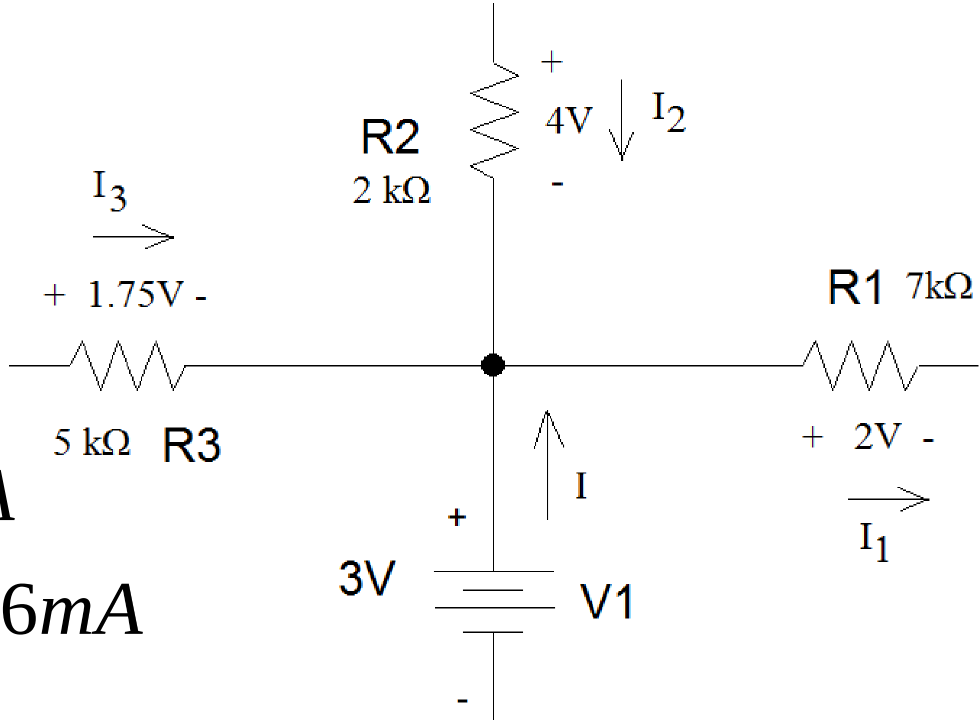
$$I_2 = 4V / 2k\Omega = 2mA$$

$$I_3 = 1.75V / 5k\Omega = 0.35mA$$



Example 3 (con’t)

- I1 is leaving the node.
- I2 is entering the node.
- I3 is entering the node.
- I is entering the node.



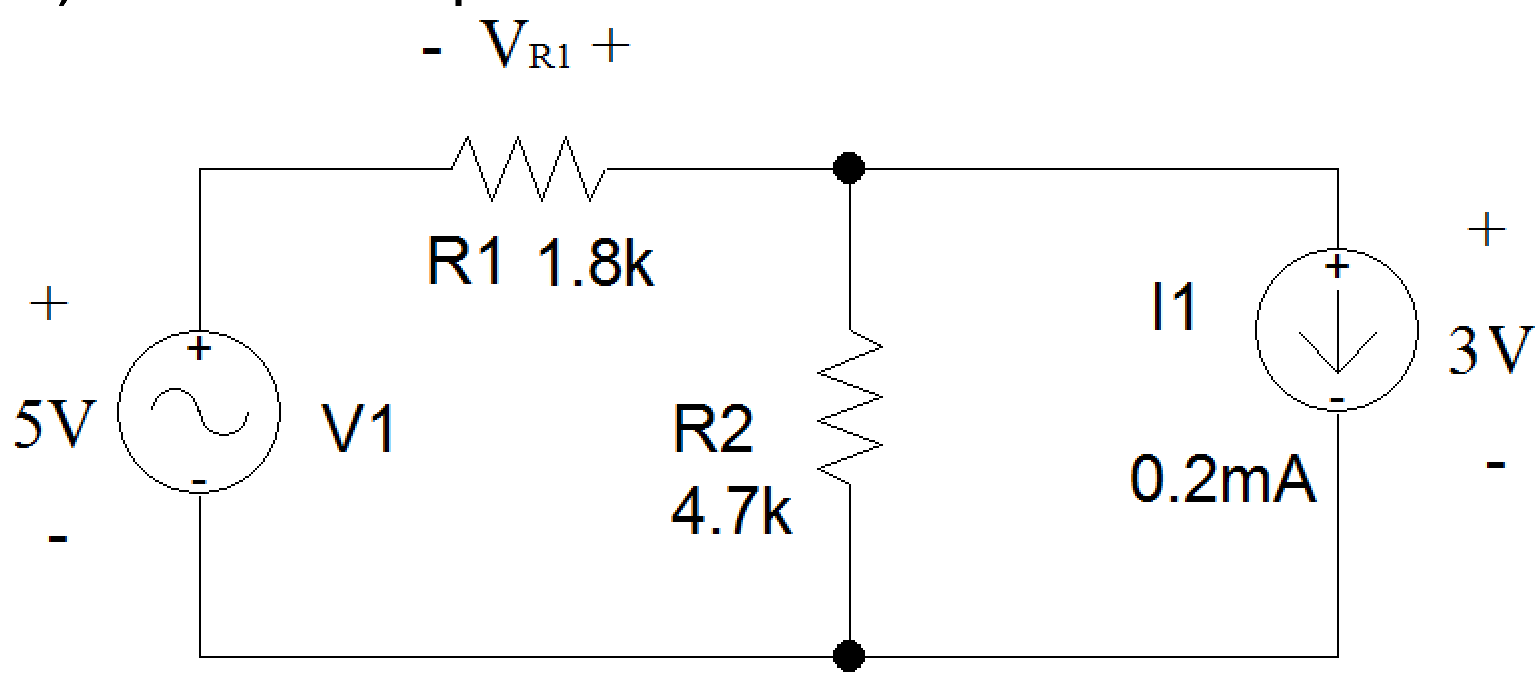
$$I_2 + I_3 + I = I_1$$

$$2mA + 0.35mA + I = 0.286mA$$

$$I = 0.286mA - 2.35mA = -2.06mA$$

Example 4

- Find the voltage across R1. Note that the polarity of the voltage has been assigned in the circuit schematic.
  - First, define a loop that include R1.



Kirchhoff’s Voltage Law

- Or KVL for short
  - Based upon conservation of energy – the algebraic sum of voltages dropped across components around a loop is zero.

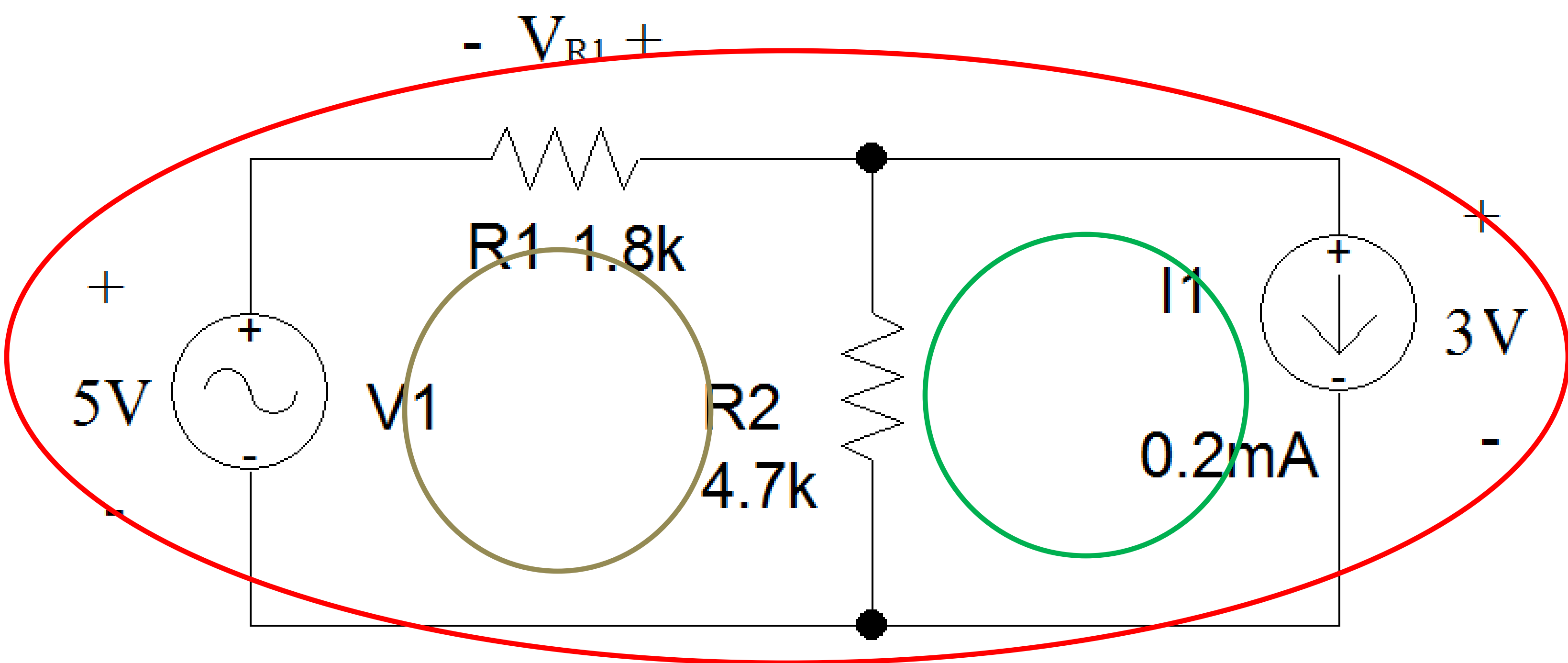
$$\sum_{m=1}^{M} v = 0$$

Where M is the total number of branches in the loop.

$$\sum v_{drops} = \sum v_{rises}$$

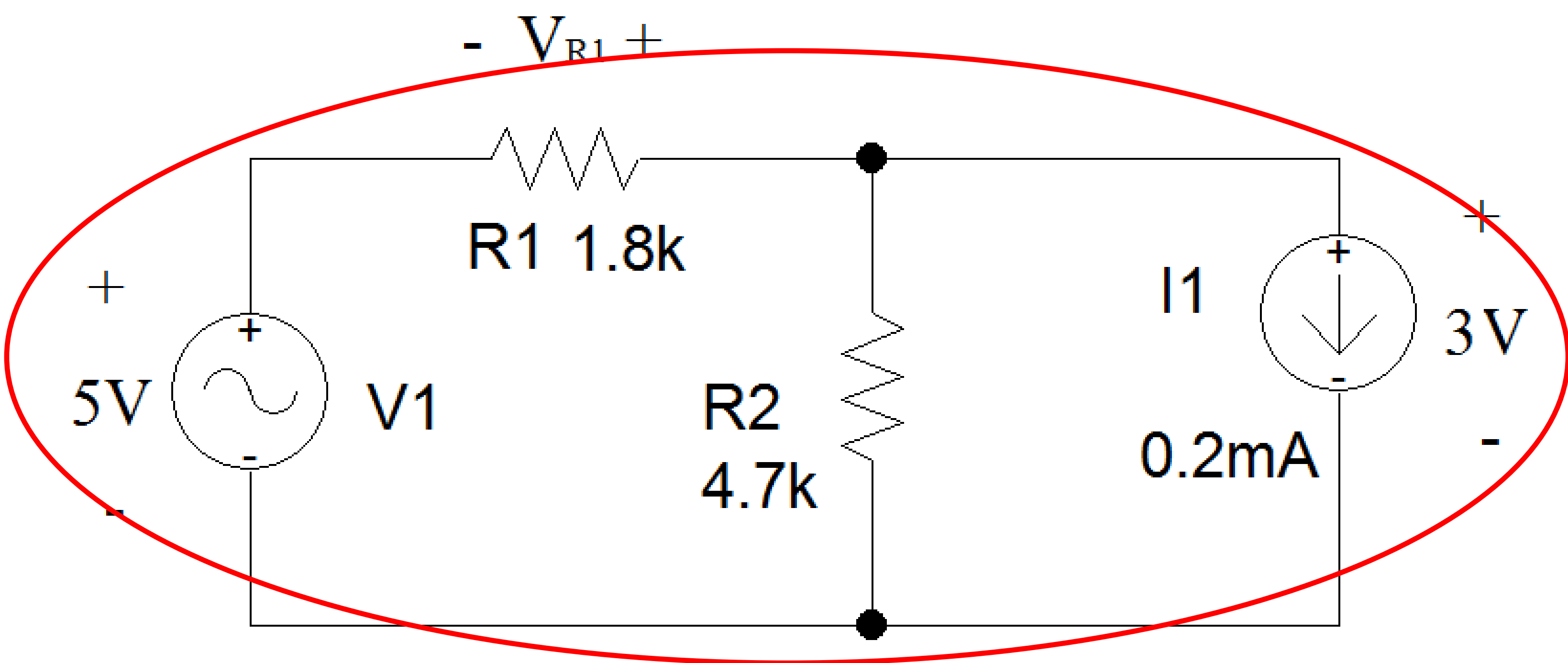
Example 4 (con't)

- There are three possible loops in this circuit – only two include R1.
  - Either loop may be used to determine V_{R1}.



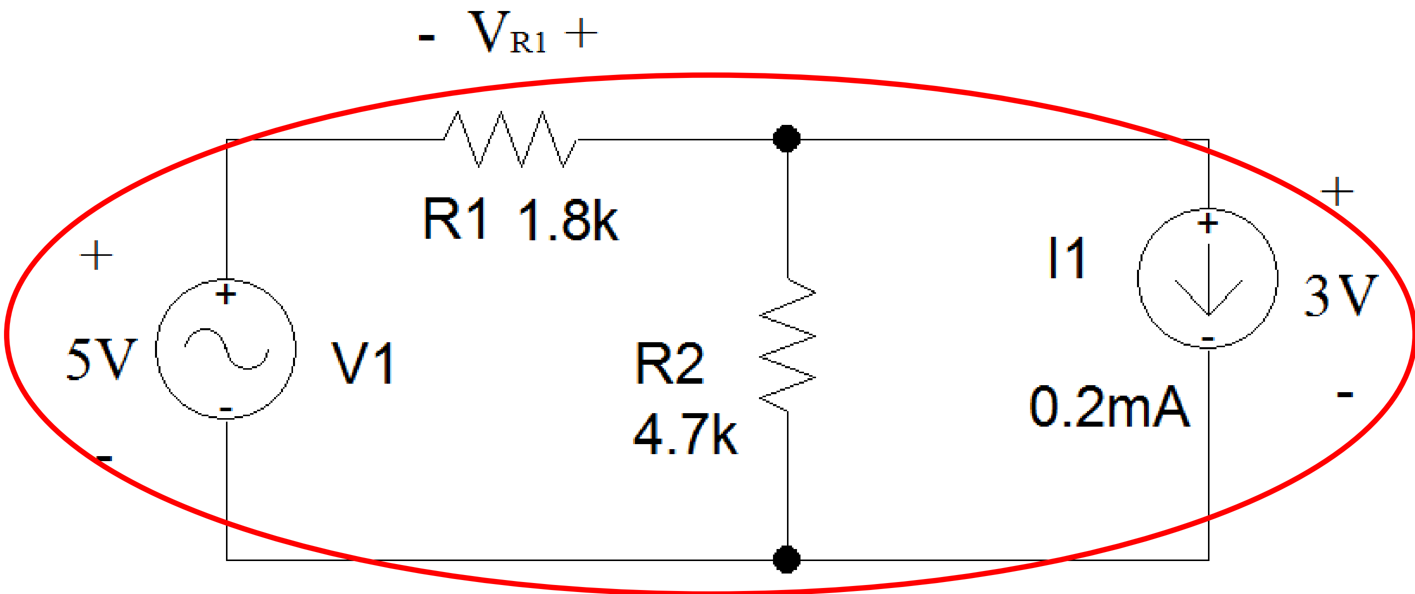
Example 4 (con't)

- If the outer loop is used:
  - Follow the loop clockwise.



Example 4 (con't)

- Follow the loop in a clockwise direction.
- The 5V drop across V1 is a voltage rise.
- V_{R1} should be treated as a voltage rise.
- The loop enters R2 on the positive side of the voltage drop and exits out the negative side. This is a voltage drop as the voltage becomes less positive as you move through the component.

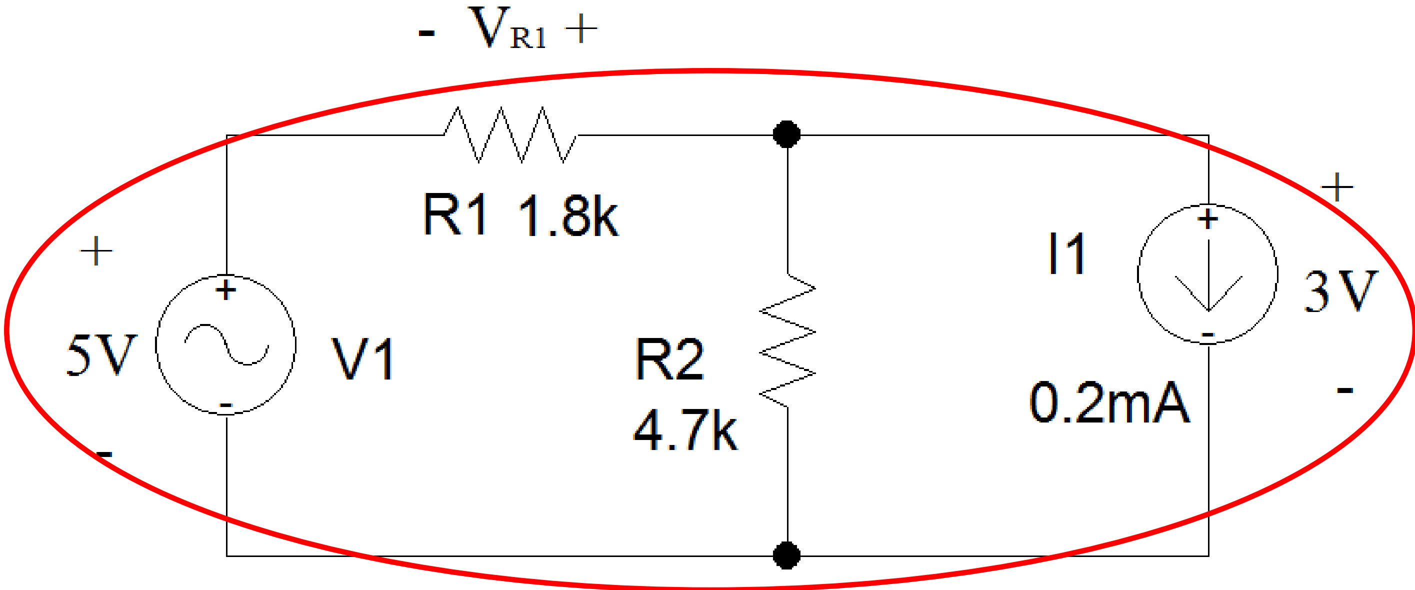


Example 4 (con't)

- By convention, voltage drops are added and voltage rises are subtracted in KVL.

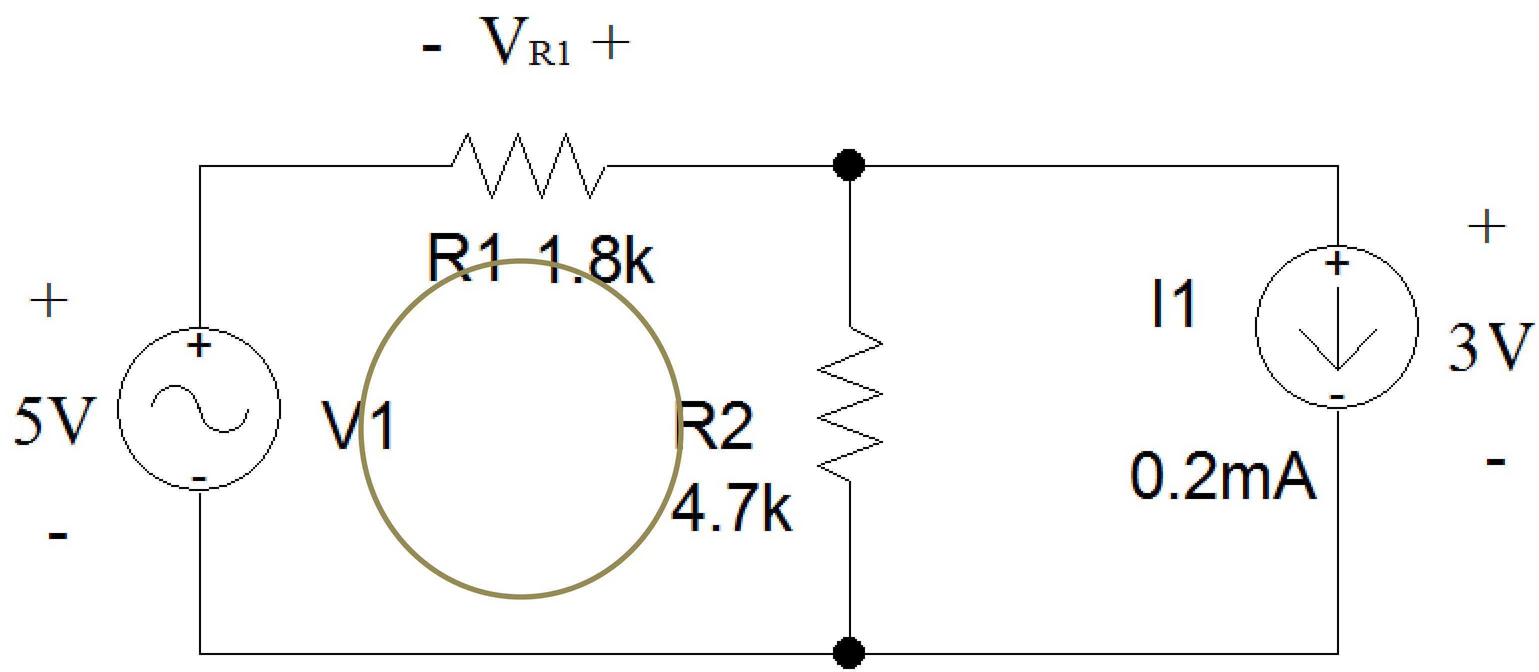
$$-5V - V_{R1} + 3V = 0$$

$$V_{R1} = 2V$$



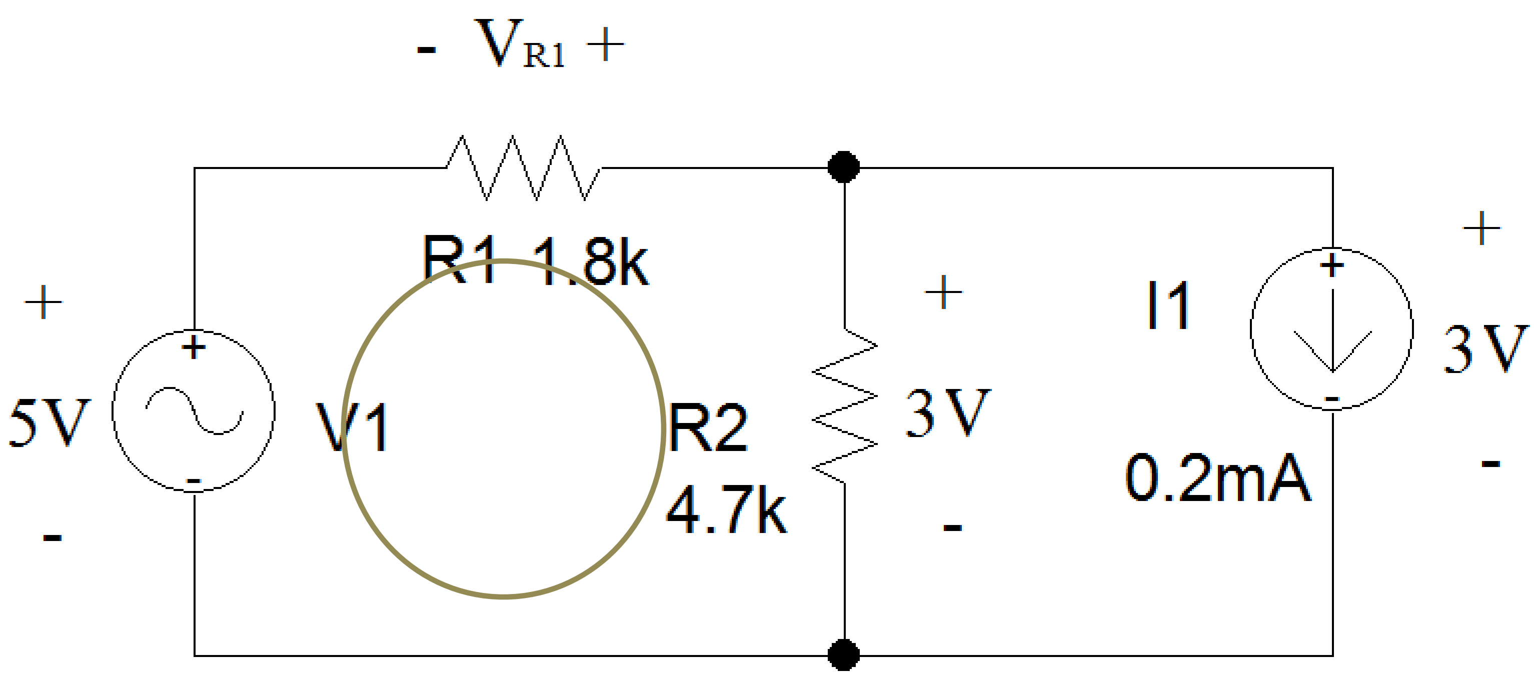
Example 4 (con't)

- Suppose you chose the blue loop instead.
  - Since R2 is in parallel with I1, the voltage drop across R2 is also 3V.



Example 4 (con't)

- The 5V drop across V1 is a voltage rise.
- V_{R1} should be treated as a voltage rise.
- The loop enters R2 on the positive side of the voltage drop and exits out the negative side. This is a voltage drop as the voltage becomes less positive as you move through the component.

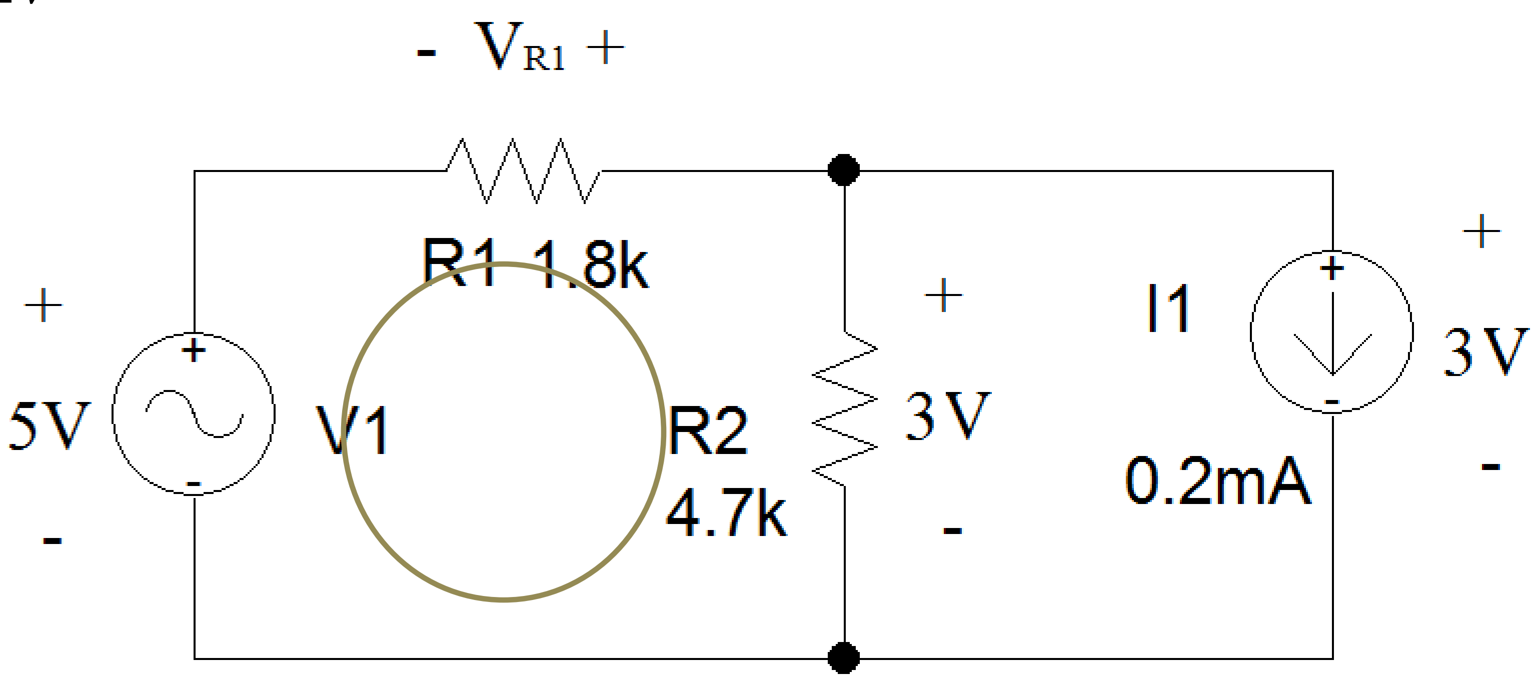


Example 4 (con't)

- As should happen, the answer is the same.

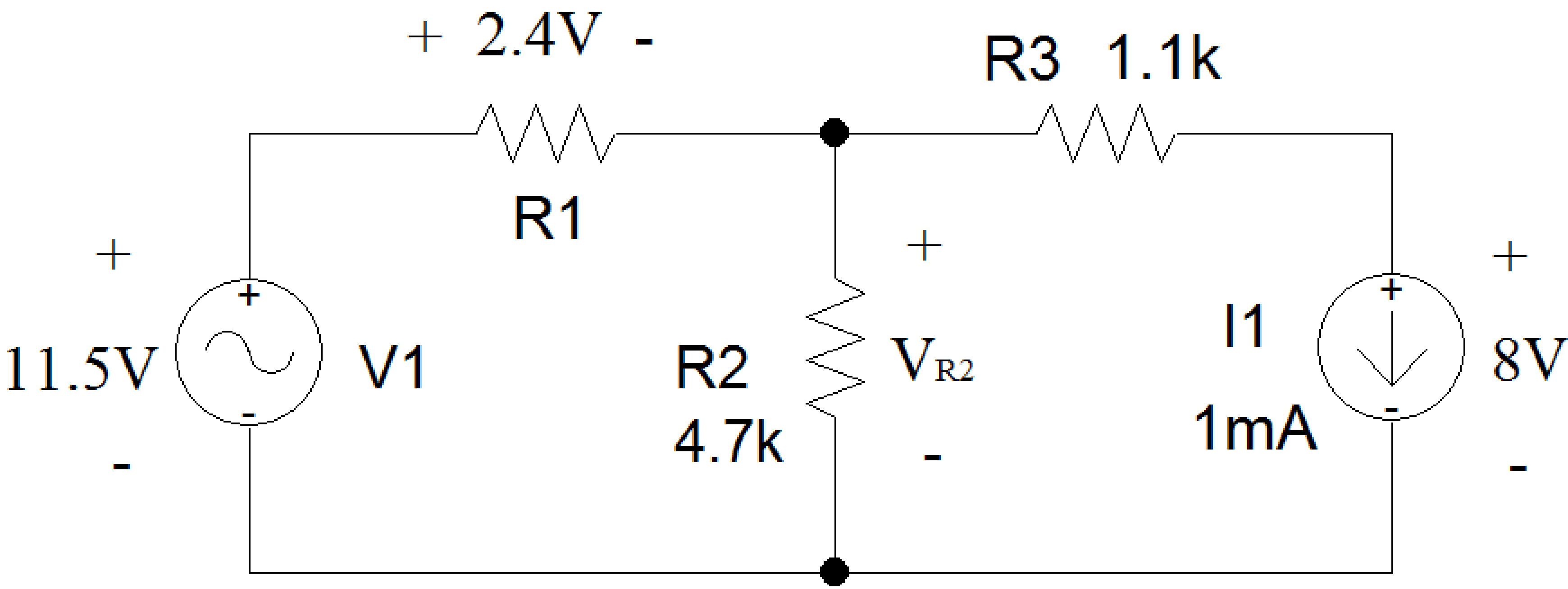
$$-5V - V_{R1} + 3V = 0$$

$$V_{R1} = 2V$$



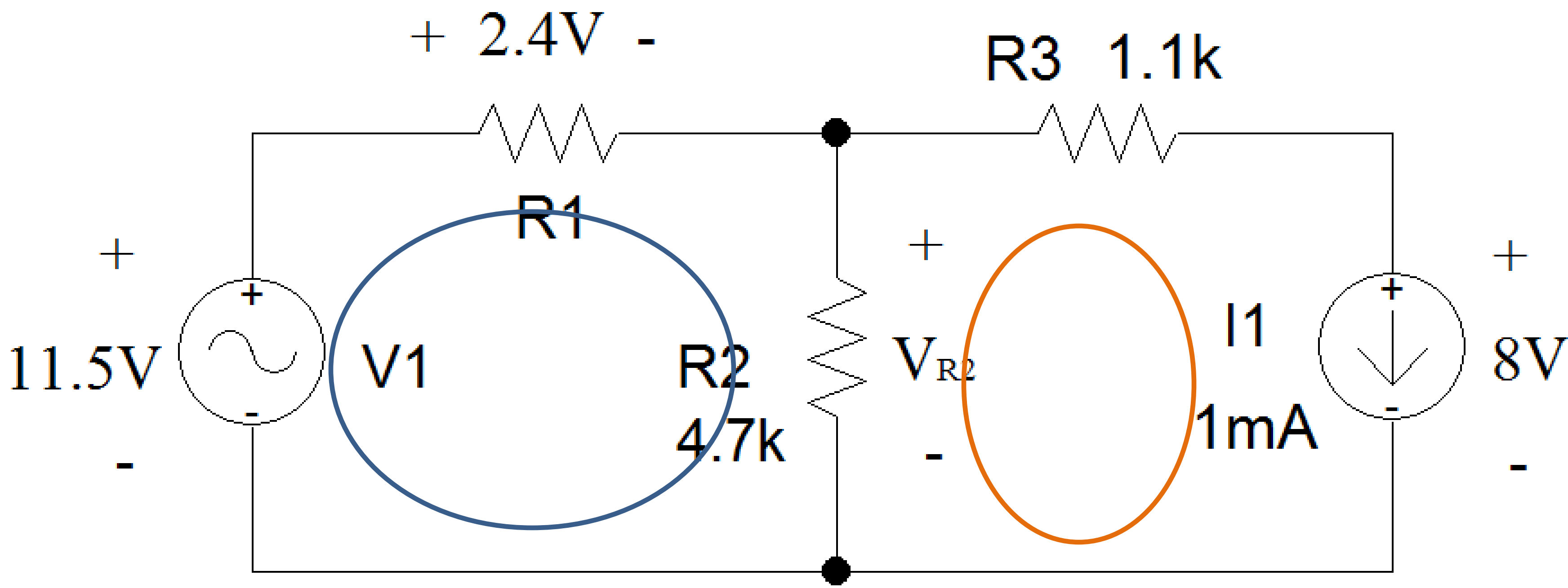
Example 5

- Find the voltage across R2 and the current flowing through it.
  - First, draw a loop that includes R2.



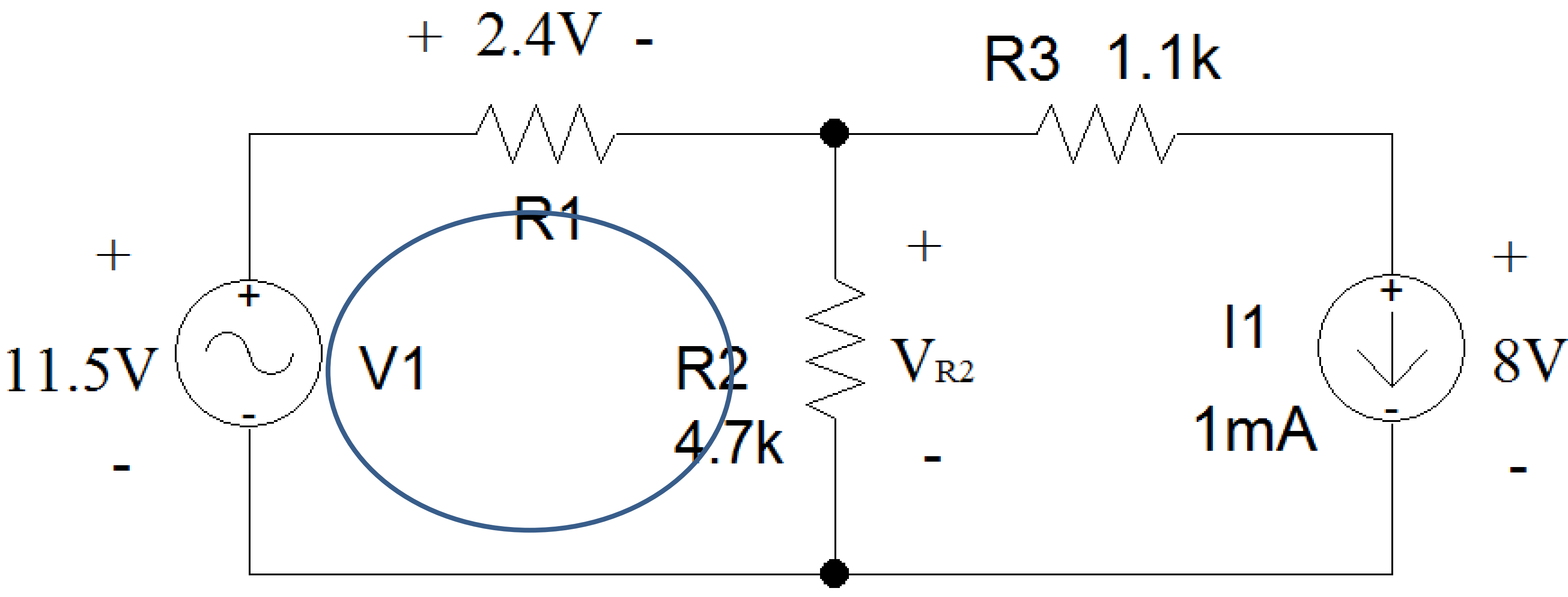
Example 5 (con't)

- There are two loops that include R2.
  - The one on the left can be used to solve for V_{R2} immediately.



Example 5 (con't)

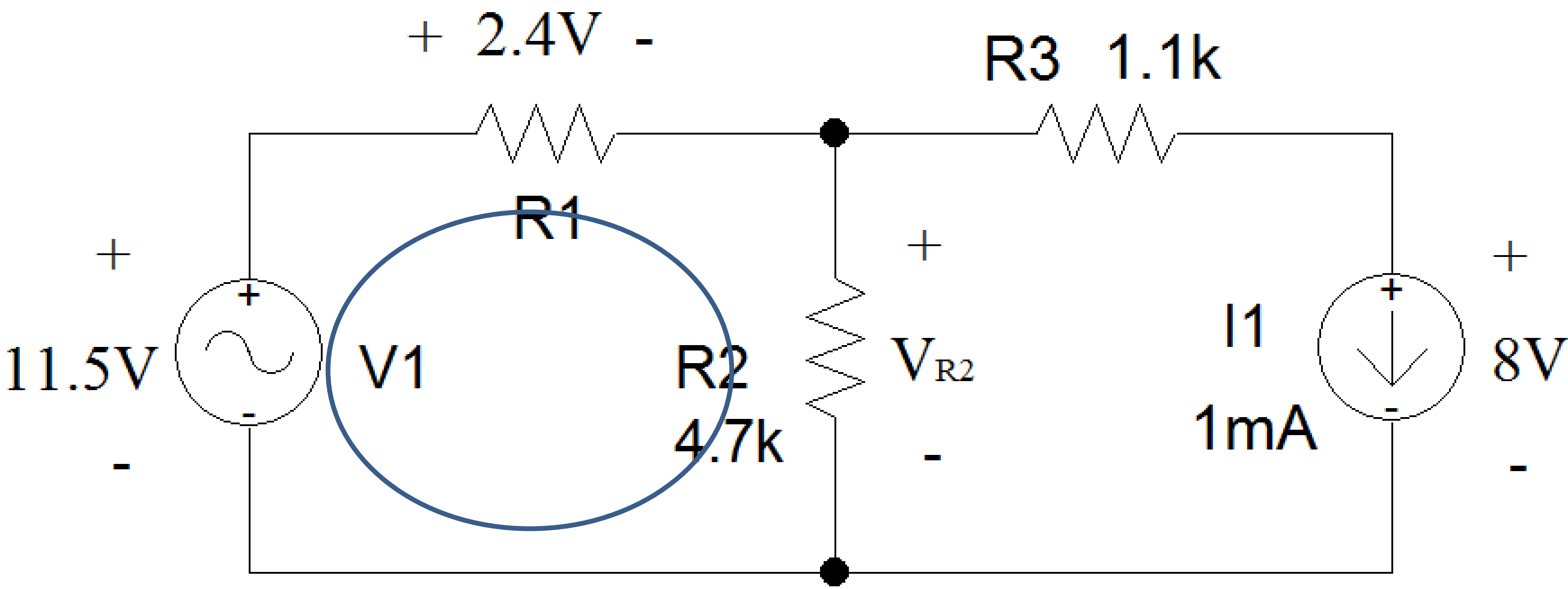
- Following the loop in a clockwise direction.
  - The 11.5V drop associated with V1 is a voltage rise.
  - The 2.4V associated with R1 is a voltage drop.
  - V_{R2} is treated as a voltage drop.



Example 5 (con't)

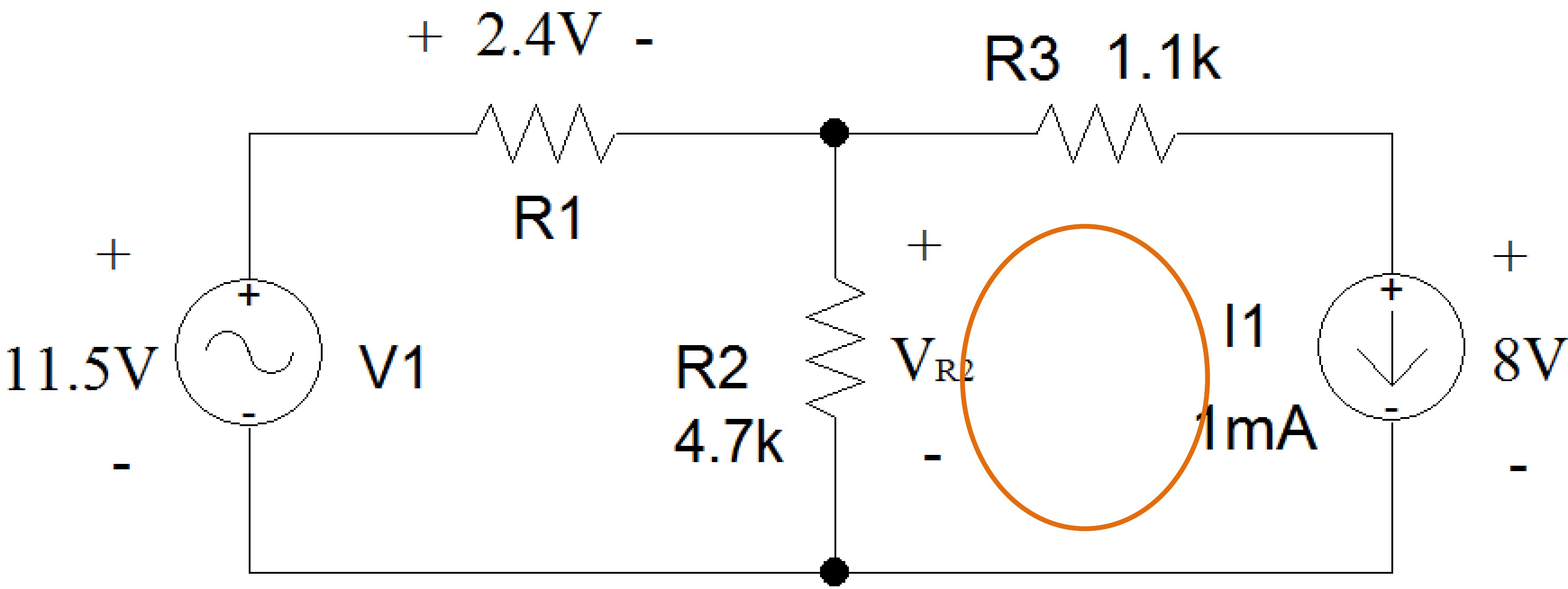
$$-11.5V + 2.4V + V_{R2} = 0$$

$$V_{R2} = 9.1V$$



Example 5 (con't)

- If you used the right-hand loop, the voltage drop across R3 must be calculated using Ohm's Law.



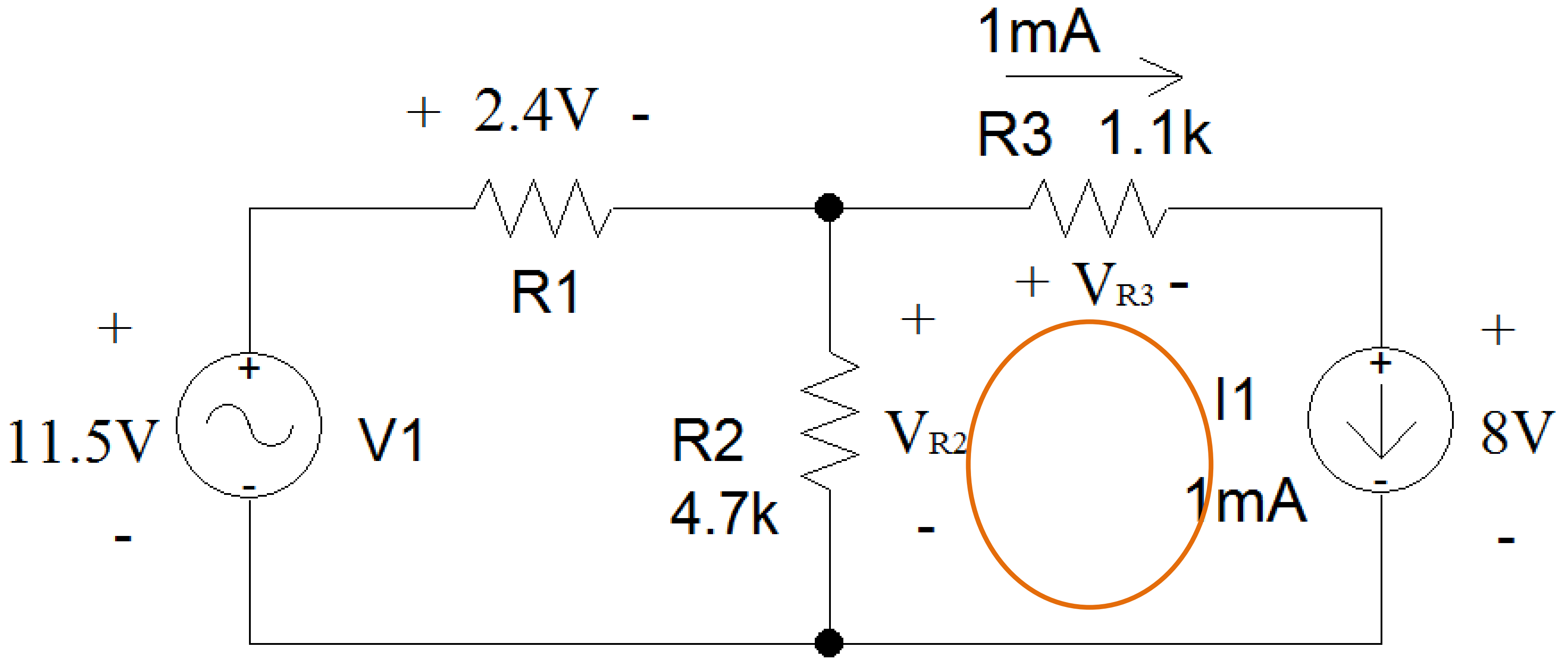
Example 5 (con't)

- Since R3 is a resistor, passive convention means that the positive sign of the voltage drop will be assigned to the end of R3 where current enters the resistor.
- As I1 is in series with R3, the direction of current through R3 is determined by the direction of current flowing out of the current source.
- Because I1 and R3 are in series, the magnitude of the current flowing out of I1 must be equal to the magnitude of the current flowing out of R3.

Example 5 (con't)

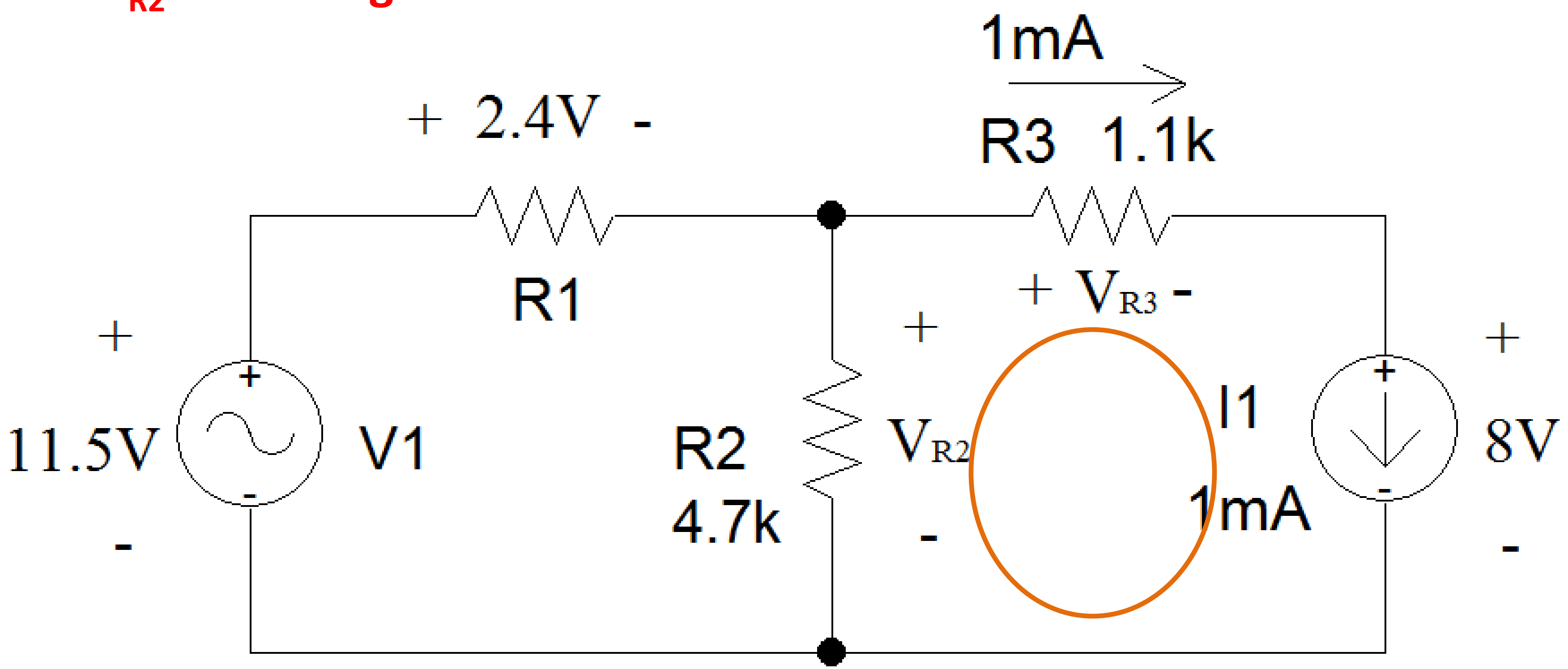
- Use Ohm's Law to find V_{R3}.

$$V_{R3} = 1mA(1.1k\Omega) = 1.1V$$



Example 5 (con't)

- Moving clockwise around the loop:
  - V_{R3} is a voltage drop.
  - The voltage associated with I1 is a voltage drop.
  - **V_{R2} is a voltage rise.**

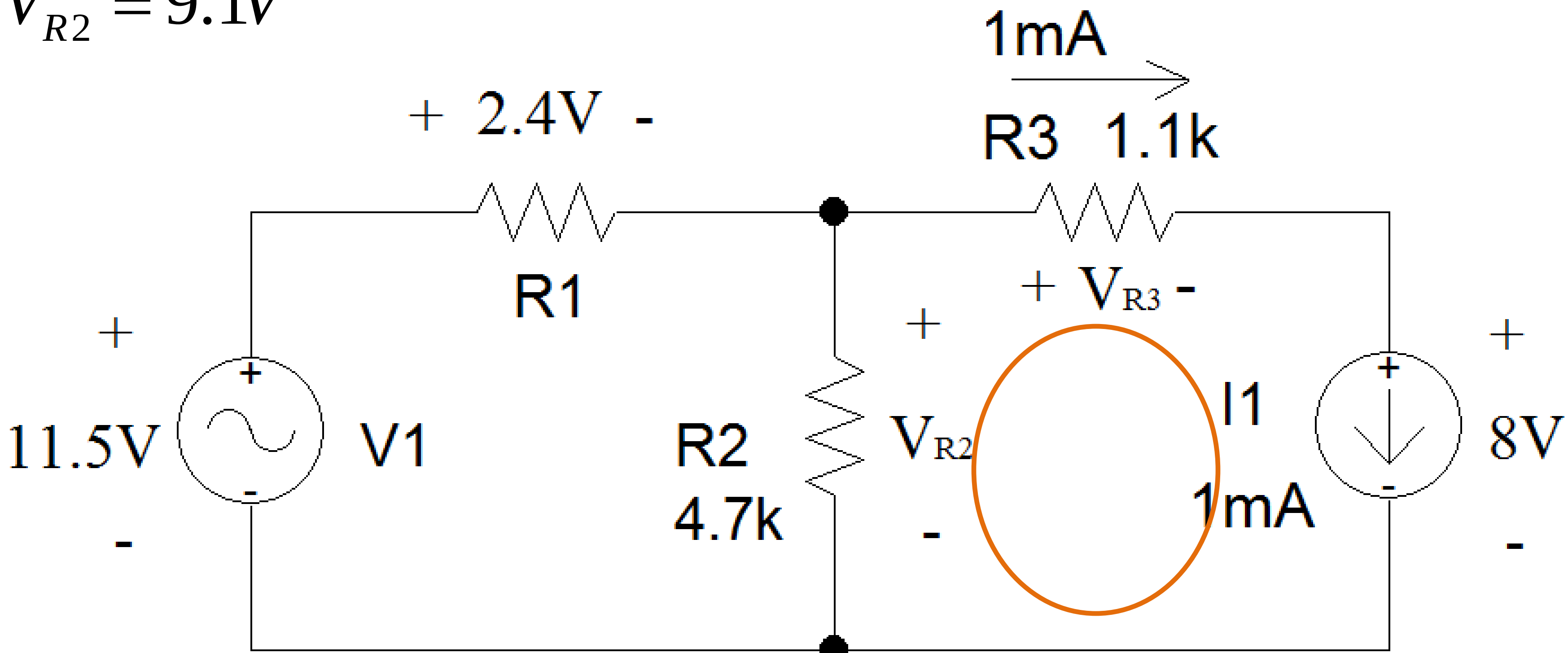


Example 5 (con't)

- Again, the same answer is found.

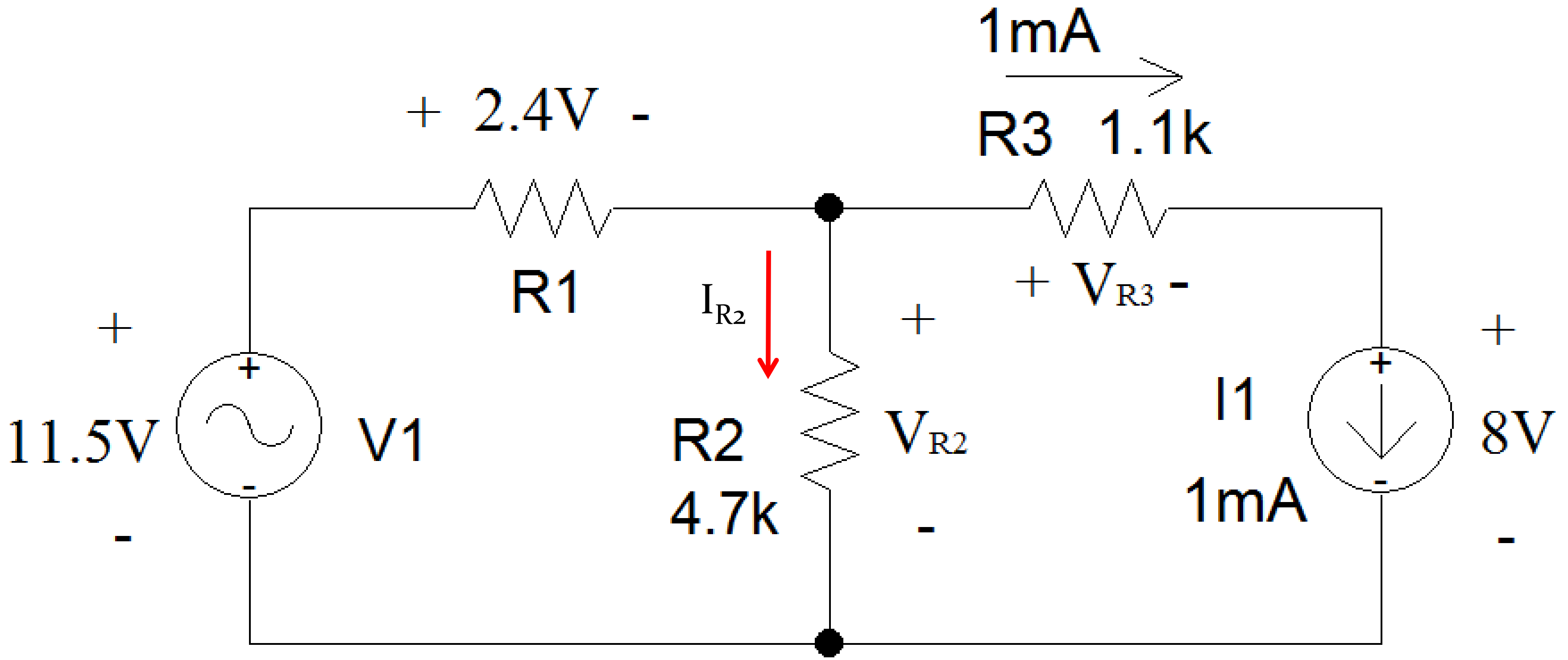
$$1.1V + 8V - V_{R2} = 0$$

$$V_{R2} = 9.1V$$



Example 5 (con't)

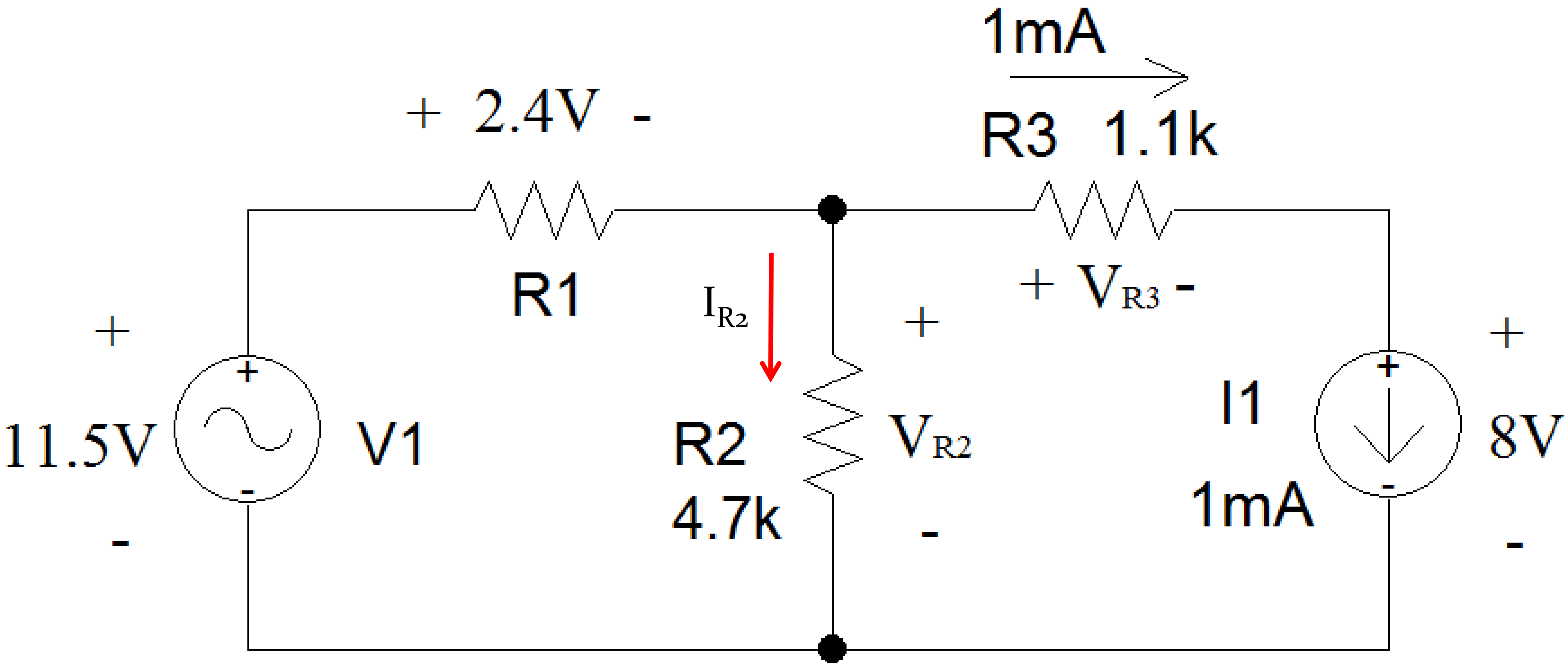
- Once the voltage across R2 is known, Ohm's Law is applied to determine the current.
  - The direction of positive current flow, based upon passive sign convention is shown in red.



Example 5 (con't)

$$I_{R2} = 9.1V / 4.7k\Omega$$

$$I_{R2} = 1.94mA$$

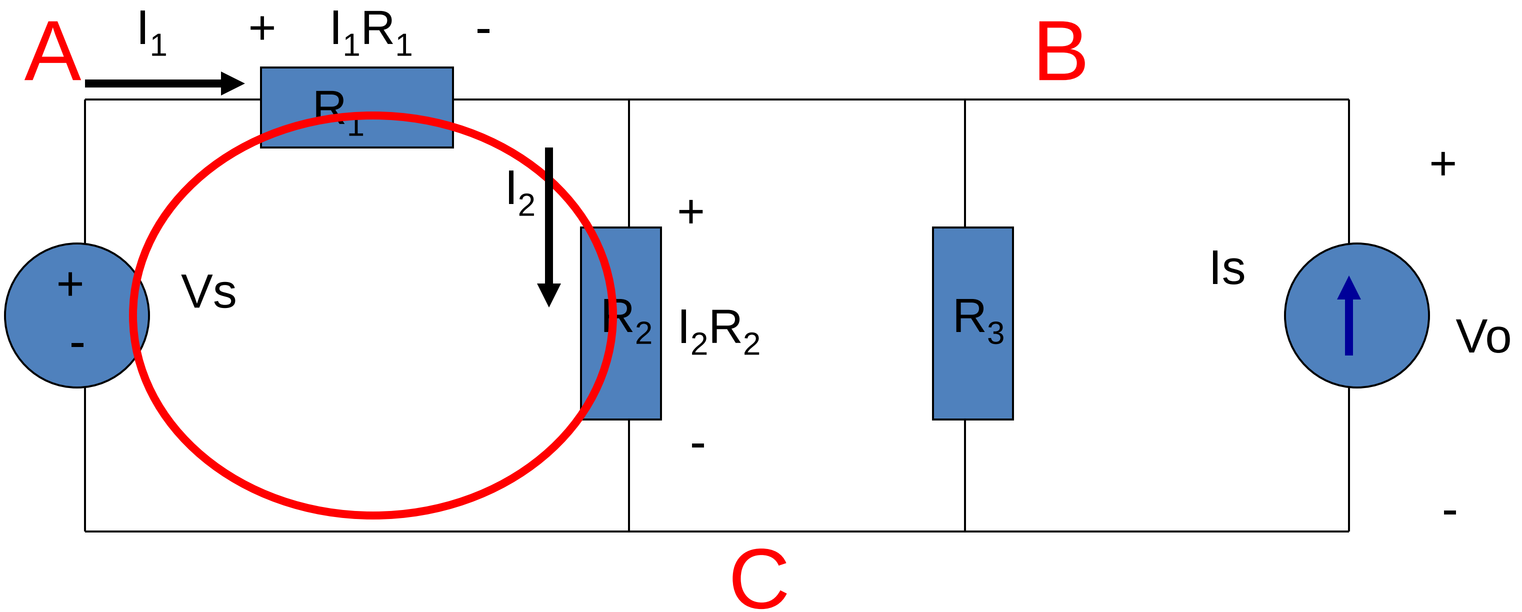


Kirchoff's Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
  - Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or Σ voltage drops = Σ voltage rises

Example

- Kirchoff's Voltage Law around 1st Loop

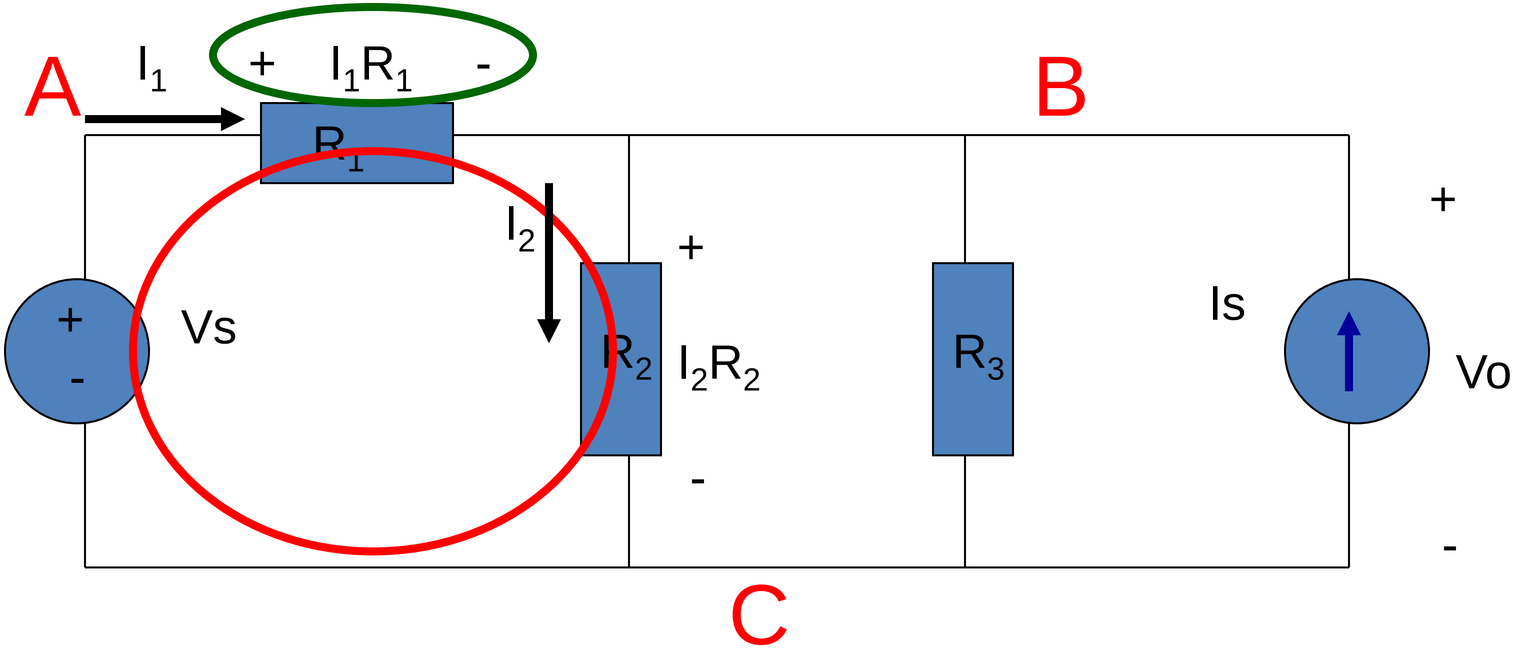


Assign current variables and directions

Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

Example

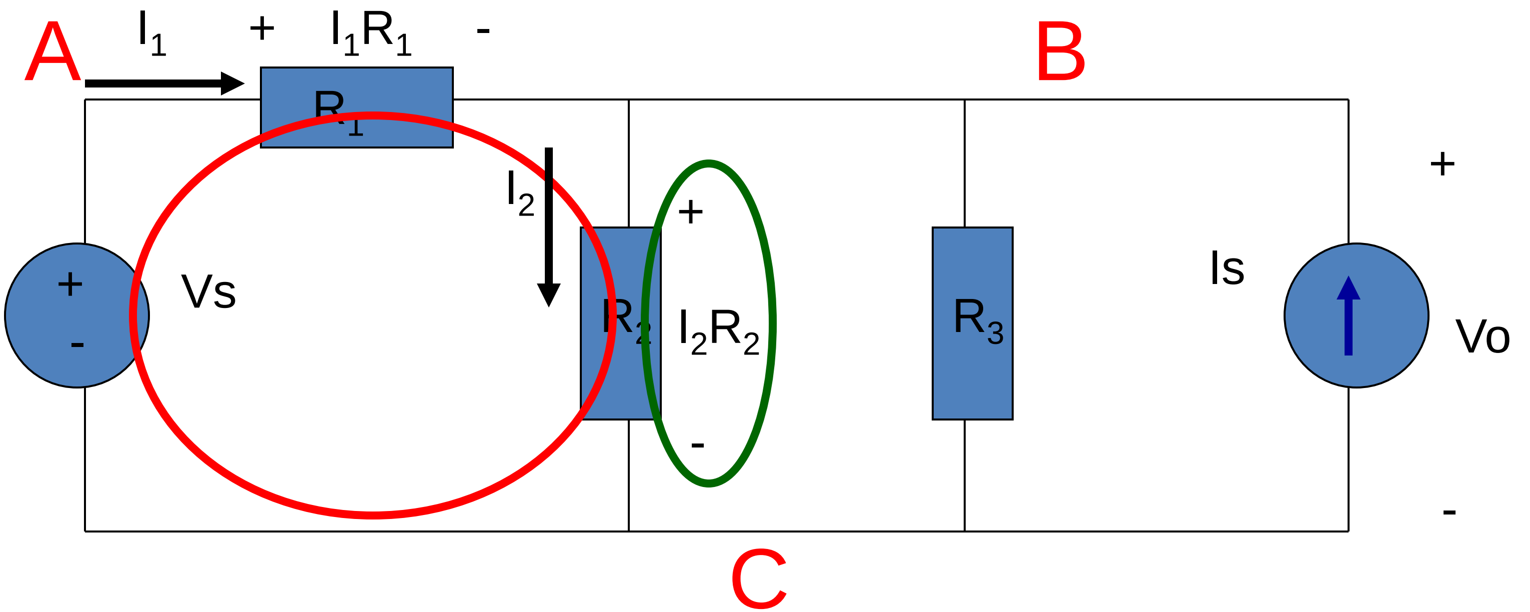
- Kirchoff's Voltage Law around 1st Loop



Starting at node A, add the 1st voltage drop: $+ I_1R_1$

Example

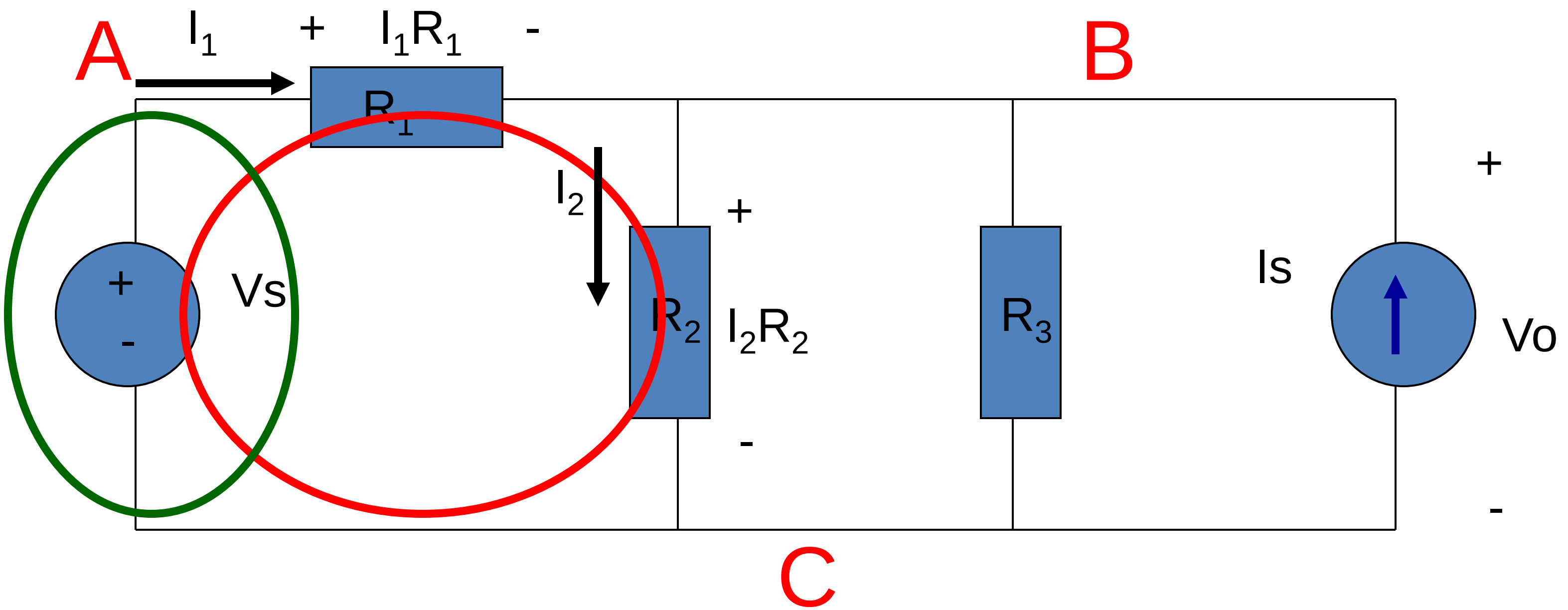
- Kirchoff's Voltage Law around 1st Loop



Add the voltage drop from B to C through R_2: $+ I_1R_1 + I_2R_2$

Example

- Kirchoff's Voltage Law around 1st Loop

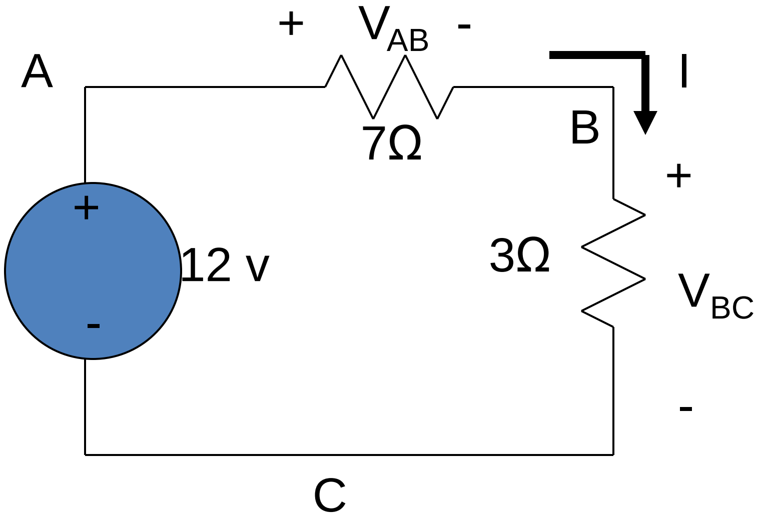


Subtract the voltage rise from C to A through Vs: $+ I_1R_1 + I_2R_2 – Vs = 0$

Notice that the sign of each term matches the polarity encountered 1st

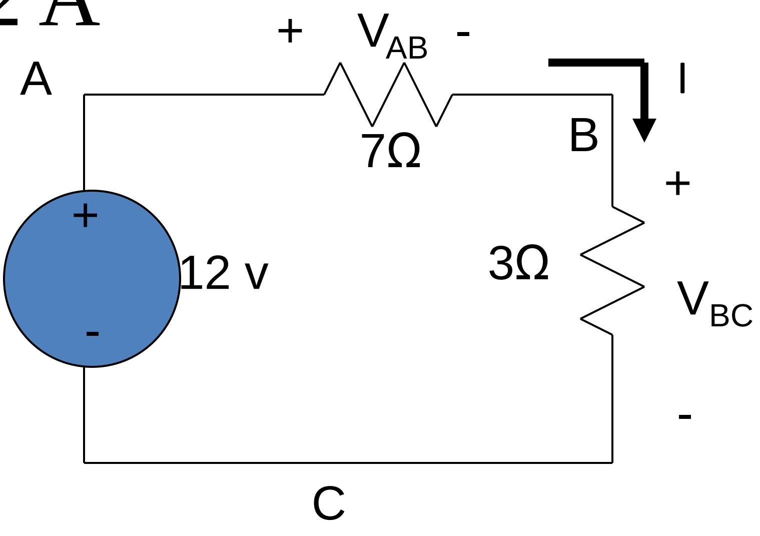
Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchoff’s two laws and Ohm’s law to determine all branch voltages and currents



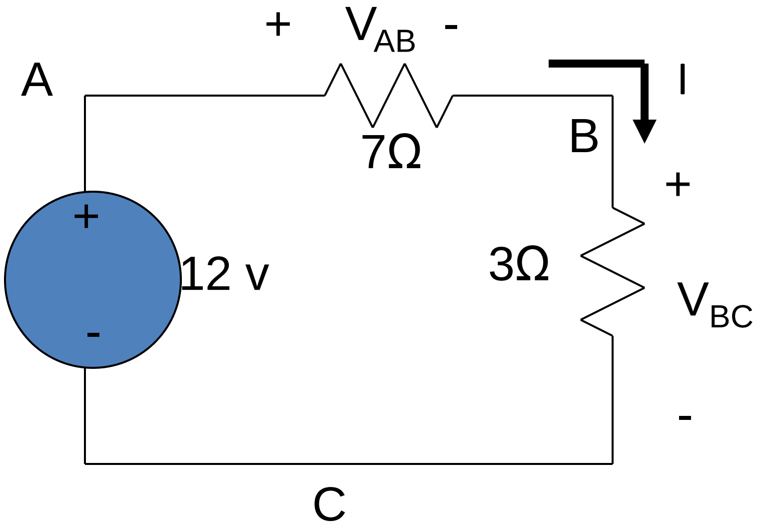
Circuit Analysis

- By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- By KVL: $V_{AB} + V_{BC} - 12\text{ v} = 0$
- Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\text{ v} = 0$
- Solving: $I = 1.2\text{ A}$



Circuit Analysis

- Since V_{AB} = $I \cdot 7\Omega$ and V_{BC} = $I \cdot 3\Omega$
- And $I = 1.2\ A$
- So V_{AB} = $8.4\ v$ and V_{BC} = $3.6\ v$



Note:

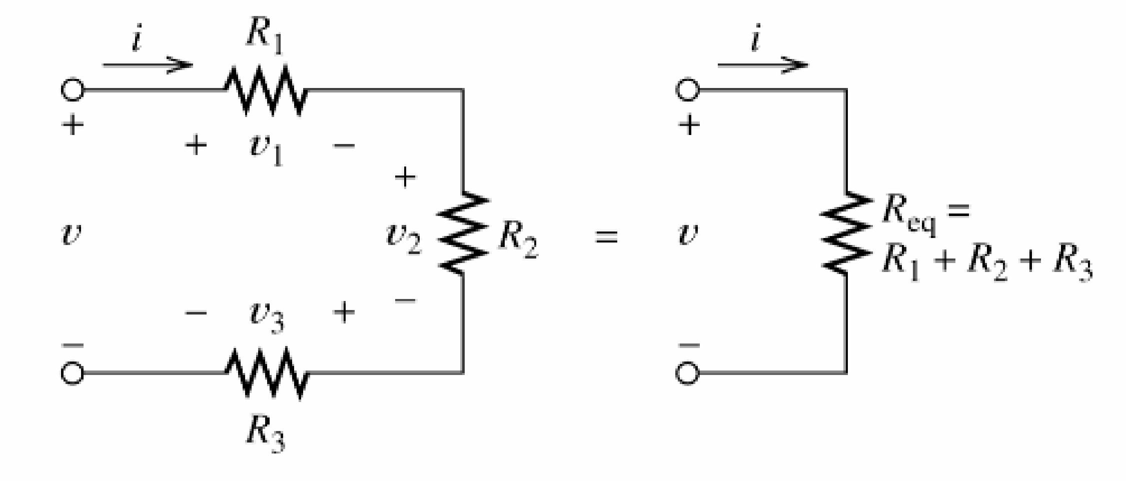
- If you use KCL and Ohm's Law, you could find out what the value of R1 is in Example 5.

- The currents at a node can be calculated using Kirchhoff’s Current Law (KCL).
- The voltage dropped across components can be calculated using Kirchhoff’s Voltage Law (KVL).
- Ohm’s Law is used to find some of the needed currents and voltages to solve the problems.

Chapter 2 Resistive Circuits

2.1 Series and parallel Resistances

2.1.1 Series Resistances



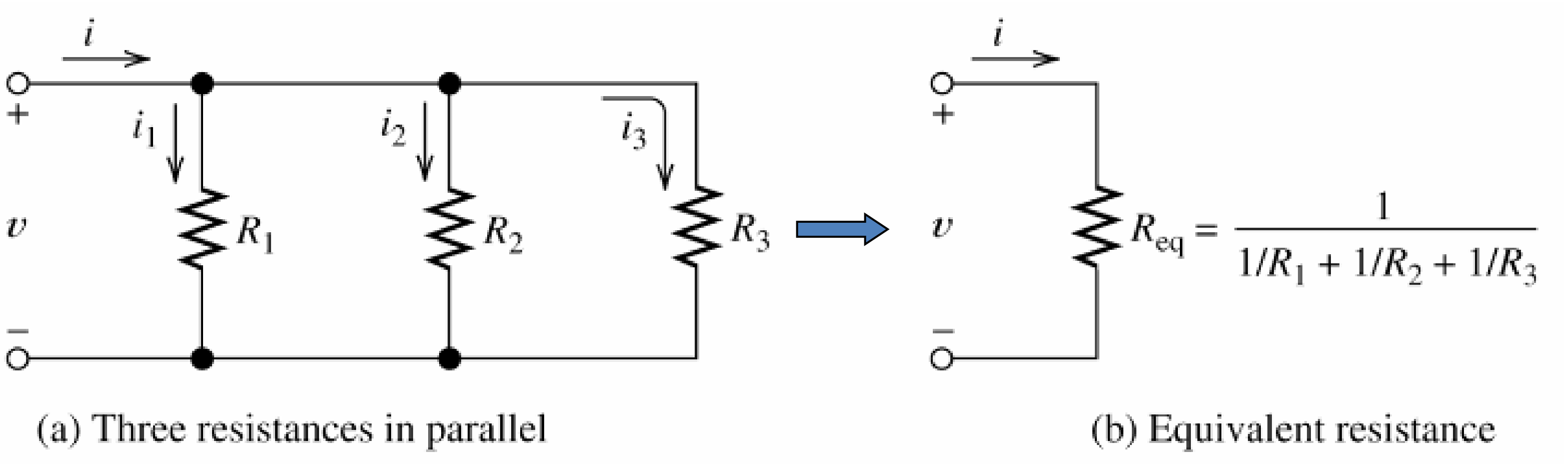
(a) Three resistances in series

(b) Equivalent resistance

$$
\begin{aligned}
KVL : v &= v_1 + v_2 + v_3 (+ \cdots + v_n) \\
&= R_1 i + R_2 i + R_3 i (+ \cdots + R_n i) \\
&= R_{eq} i
\end{aligned}
$$

$$\Rightarrow \boxed{R_{eq} = R_1 + R_2 + R_3 (+ \cdots R_n)}$$

2.1.2 Parallel Resistances



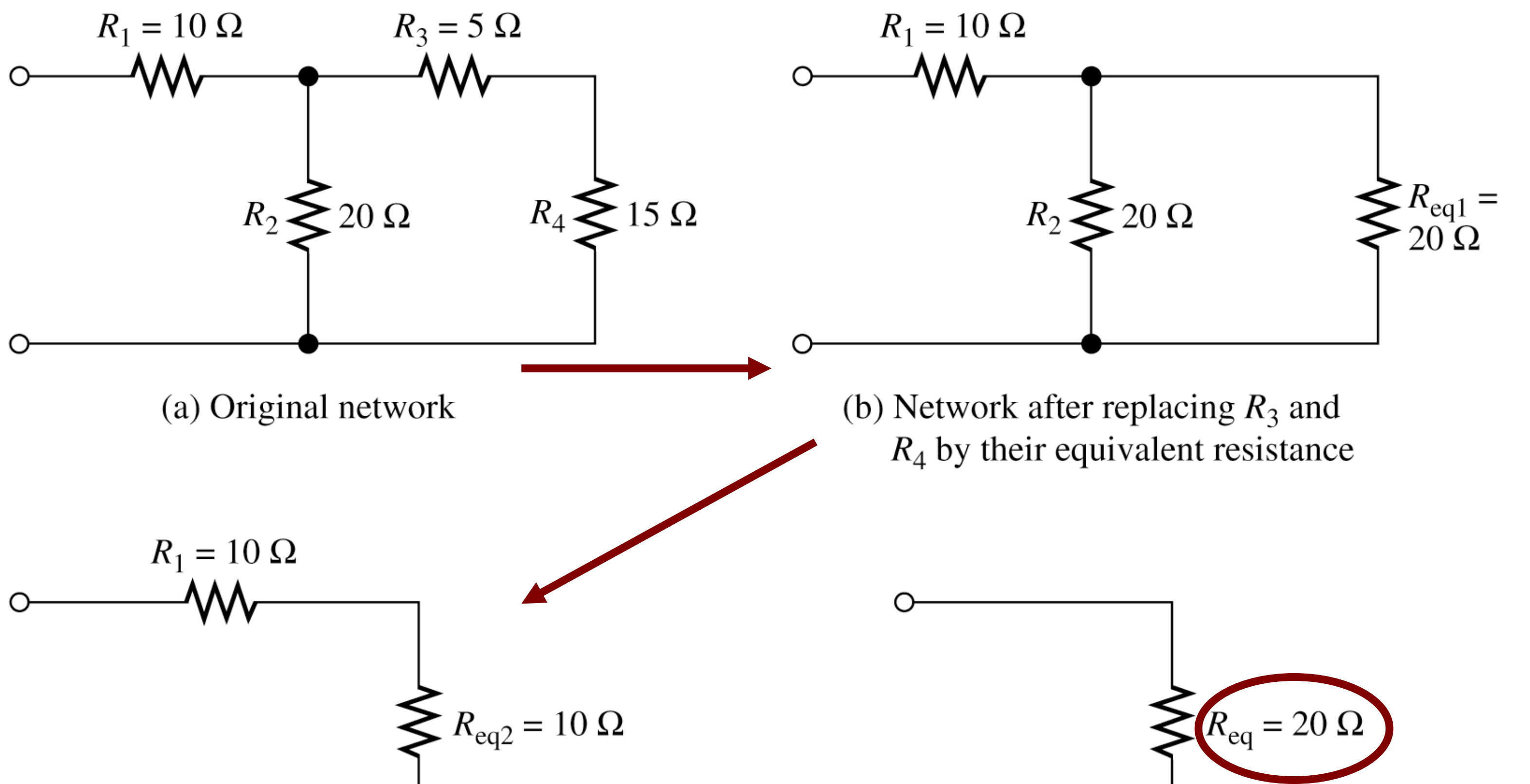
$$KCL : i = i_1 + i_2 + i_3 (+ \cdots + i_n)$$

$$= \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} (+ \cdots + \frac{v}{R_n})$$

$$= \frac{v}{R_{eq}}$$

$$\Rightarrow \boxed{R_{eq} = 1/(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n})}$$

Example 2.1 – Find equivalent resistance



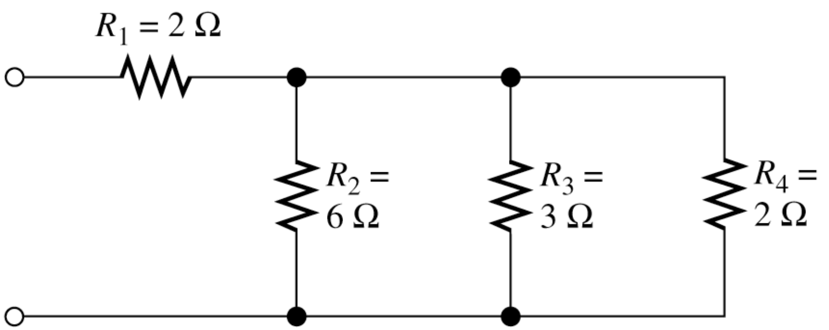
(a) Original network

(b) Network after replacing R_3 and R_4 by their equivalent resistance

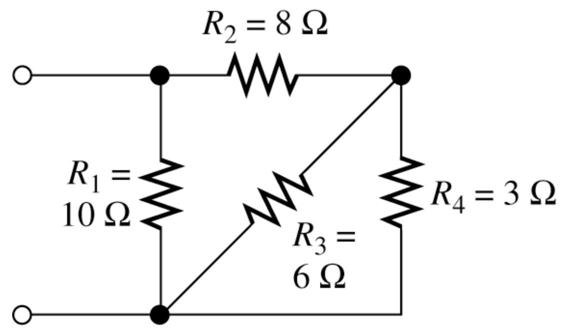
(c) Network after replacing R_2 and R_{eq1} by their equivalent

(d) Combining R_1 and R_{eq2} in series yields the equivalent resistance of the entire network

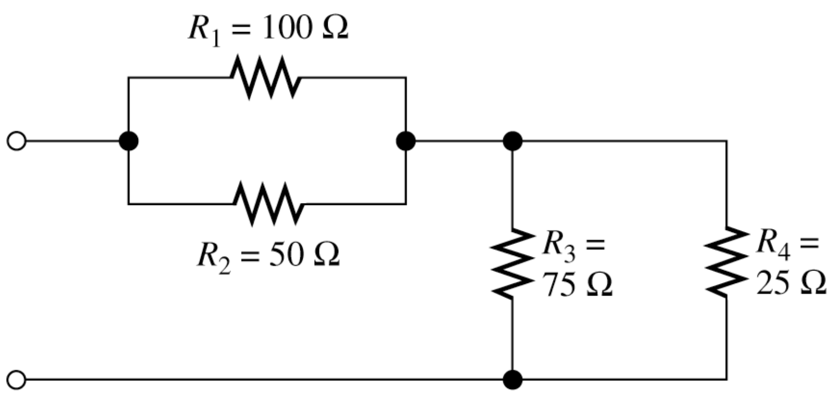
Exercise 2.1 – Find equivalent resistance



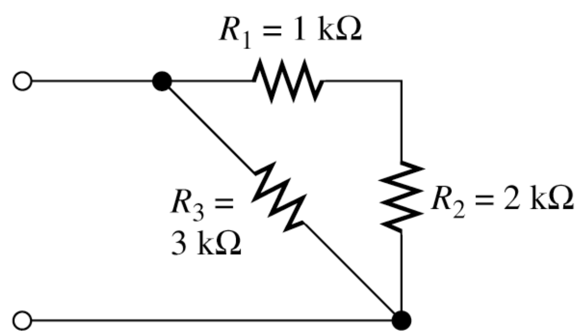
$$R_{eq} = R_1 + \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 3\,\Omega$$



$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5\,\Omega$$



$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} + \frac{1}{1/R_3 + 1/R_4} = 52.1\,\Omega$$



$$R_{eq} = \frac{1}{1/R_3 + 1/(R_1 + R_2)} = 1.5\,\mathrm{k}\Omega$$

Chapter 2 Resistive Circuits – Additional Example

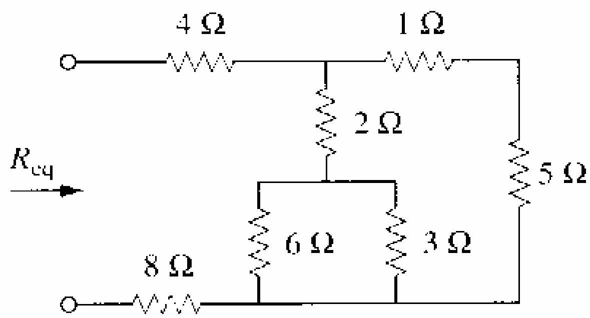
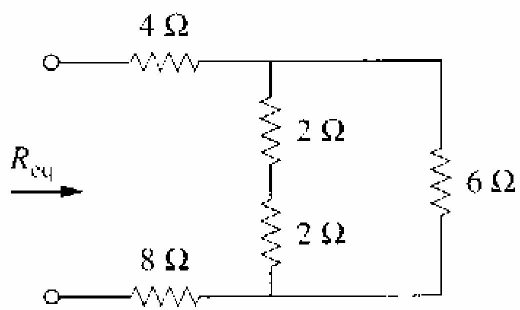
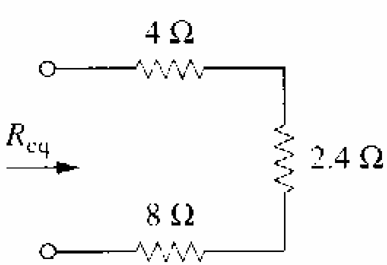


Figure 2.34 For Example 2.9.

Find R_{eq} for the circuit shown in Fig. 2.34.



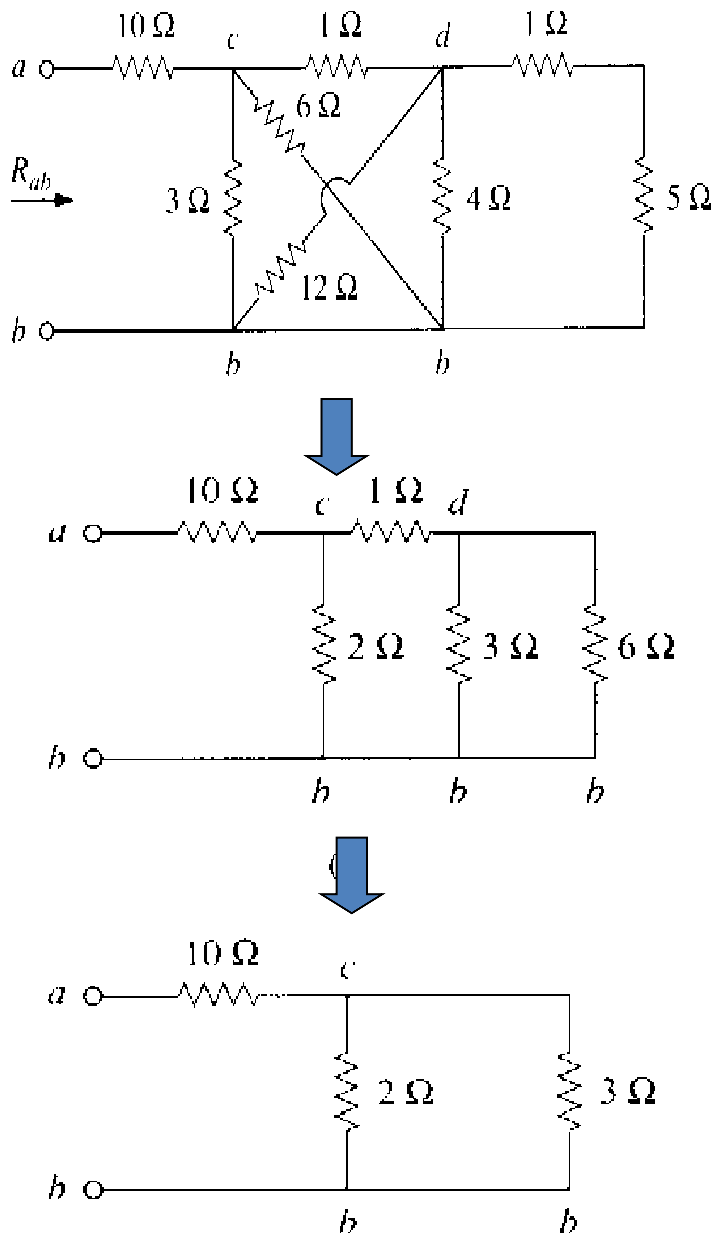
(a)



(b)

$$\boxed{R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega}$$

– Quiz: Find equivalent resistance



$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega$$

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega$$

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

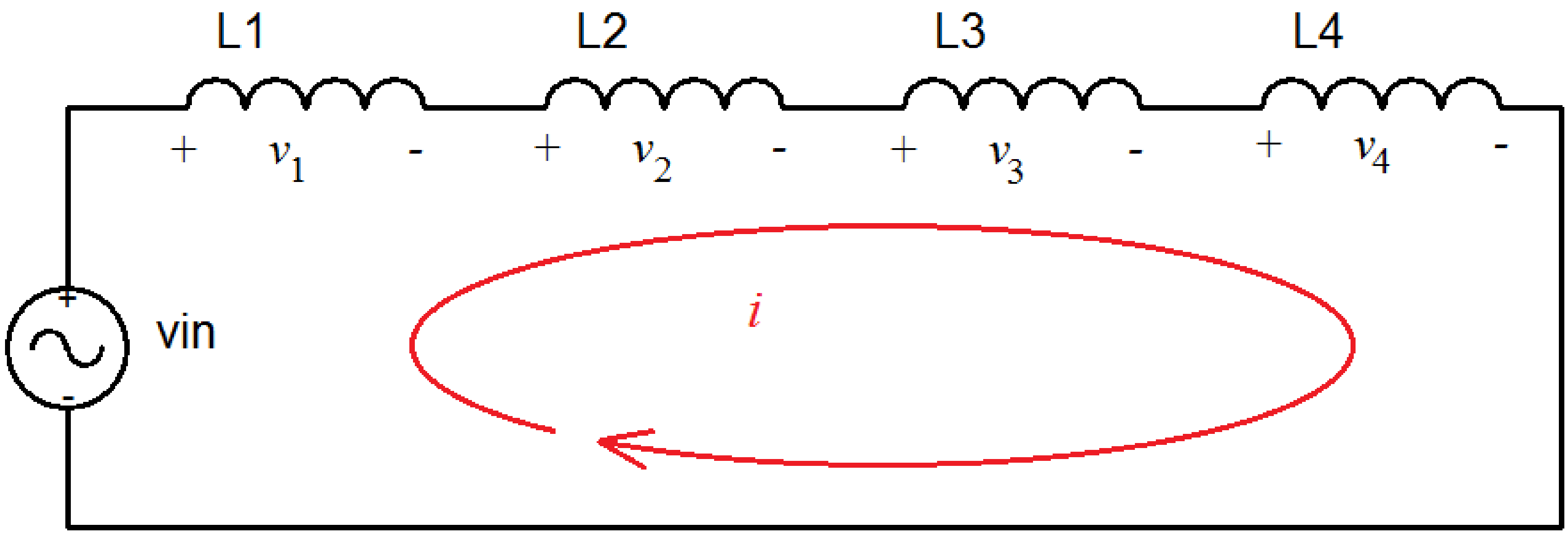
$$\boxed{R_{ab} = 10 + 1.2 = 11.2\ \Omega}$$

Series and Parallel Circuits

- Series Circuits
  - only one end of each component is connected
  - e.g. Christmas tree lights

- Parallel Circuits
  - both ends of a component are connected
  - e.g. household lighting

Inductors in Series



L_{eq} for Inductors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = L_1 \frac{\mathrm{di}}{\mathrm{dt}} \qquad v_2 = L_2 \frac{\mathrm{di}}{\mathrm{dt}}$$

i

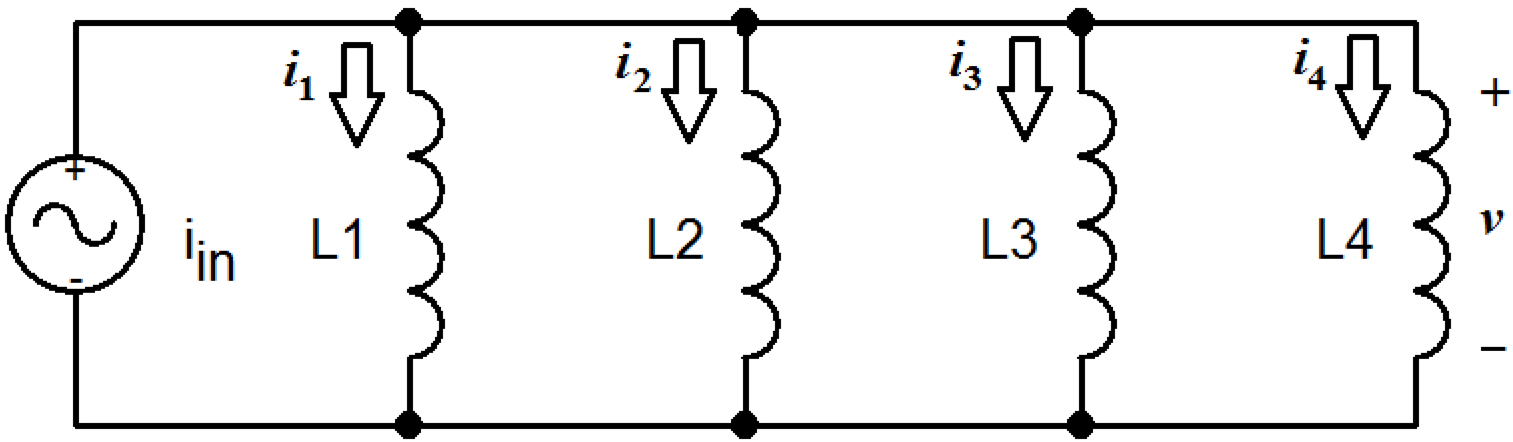
$$v_3 = L_3 \frac{\mathrm{di}}{\mathrm{dt}} \qquad v_4 = L_4 \frac{\mathrm{di}}{\mathrm{dt}}$$

$$v_{in} = L_1 \frac{\mathrm{di}}{\mathrm{dt}} + L_2 \frac{\mathrm{di}}{\mathrm{dt}} + L_3 \frac{\mathrm{di}}{\mathrm{dt}} + L_4 \frac{\mathrm{di}}{\mathrm{dt}}$$

$$v_{in} = L_{eq} \frac{\mathrm{di}}{\mathrm{dt}}$$

$$\mathrm{L}_{\mathrm{eq}} = L_1 + L_2 + L_3 + L_4$$

Inductors in Parallel



L_{eq} for Inductors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = \frac{1}{L_1}\int_{t_o}^{t_1} \mathrm{vdt} \qquad i_2 = \frac{1}{L_2}\int_{t_o}^{t_1} \mathrm{vdt}$$

i

$$i_3 = \frac{1}{L_3}\int_{t_o}^{t_1} \mathrm{vdt} \qquad i_4 = \frac{1}{L_4}\int_{t_o}^{t_1} \mathrm{vdt}$$

$$i_{in} = \frac{1}{L_1}\int_{t_o}^{t_1} \mathrm{vdt} + \frac{1}{L_2}\int_{t_o}^{t_1} \mathrm{vdt} + \frac{1}{L_3}\int_{t_o}^{t_1} \mathrm{vdt} + \frac{1}{L_4}\int_{t_o}^{t_1} \mathrm{vdt}$$

$$i_{in} = \frac{1}{L_{eq}}\int_{t_o}^{t_1} \mathrm{vdt}$$

$$\mathrm{L}_{\mathrm{eq}} = \left[\left(1/L_1\right) + \left(1/L_2\right) + \left(1/L_3\right) + \left(1/L_4\right)\right]^{-1}$$

General Equations for L_{eq}

Series Combination

- If S inductors are in series, then

$$L_{eq} = \sum_{s=1}^{S} L_s$$

Parallel Combination

- If P inductors are in parallel, then:

$$L_{eq} = \left[\sum_{p=1}^{P} \frac{1}{L_p} \right]^{-1}$$

- Inductors are energy storage devices.
- An ideal inductor act like a short circuit at steady state when a DC voltage or current has been applied.
- The current through an inductor must be a continuous function; the voltage across an inductor can be discontinuous.
- The equation for equivalent inductance for

inductors in series

$$L_{eq} = \sum_{s=1}^{S} L_s$$

inductors in parallel

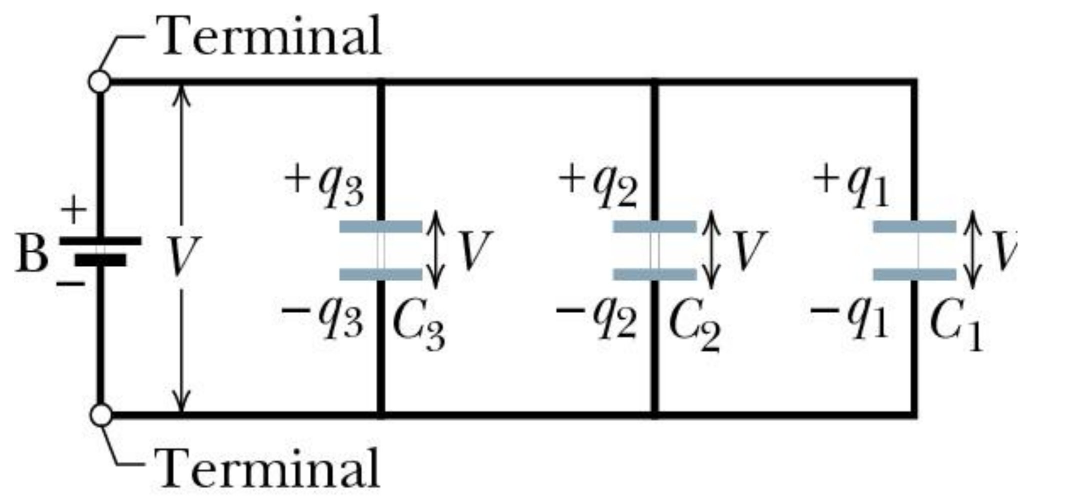
$$L_{eq} = \left[\sum_{p=1}^{P} \frac{1}{L_p} \right]^{-1}$$

Combinations of Capacitors

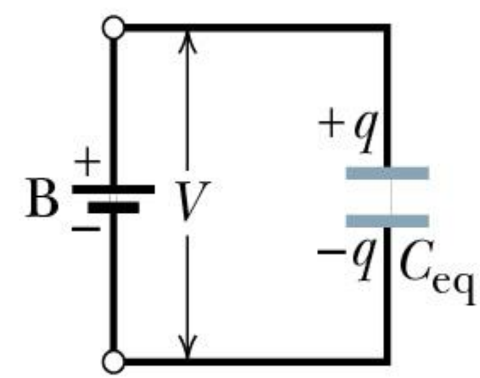
Parallel and Series Combinations

Capacitors in Parallel

- Three capacitors (C_1, C_2, and C_3) are connected in parallel to a battery B.
- All the capacitor plates connected to the positive battery terminal are positive.
- All the capacitor plates connected to the negative battery terminal are negative.



(a)



(b)

Capacitors in Parallel

- When the capacitors are first connected in the circuit, electrons are transferred through the battery from the plate that becomes positively charged to the plate that becomes negatively charged.
- The energy needed to do this comes from the battery.
- The flow of charge stops when the voltage across the capacitor plates is equal to that of the battery.
- The capacitors reach their maximum charge when the flow of charge stops.

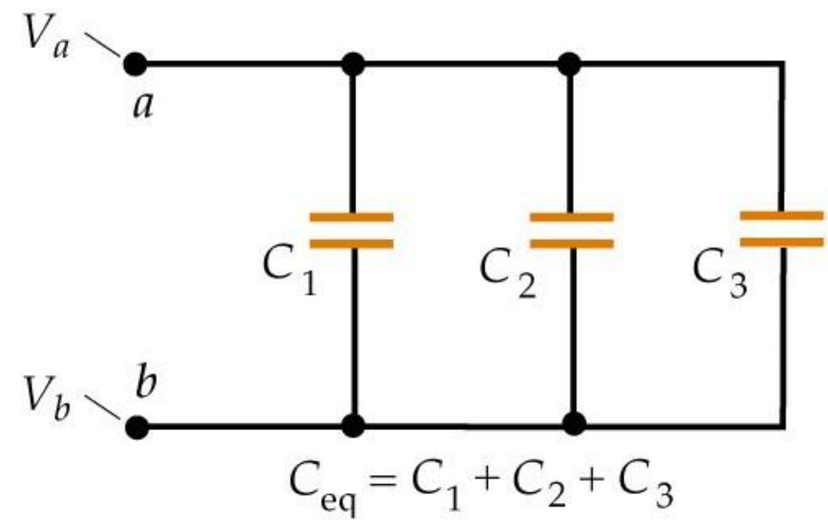
Capacitors in Parallel

- In the parallel circuit, the voltage (joules/coulomb) is constant.

$$V_{ab} = V_1 = V_2 = V_3$$

- The total charge stored on the capacitor plates is equal to the charge on each plate.

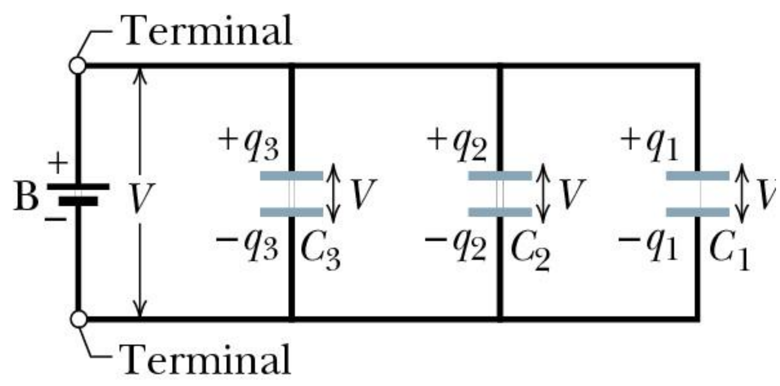
$$Q = Q_1 + Q_2 + Q_3$$



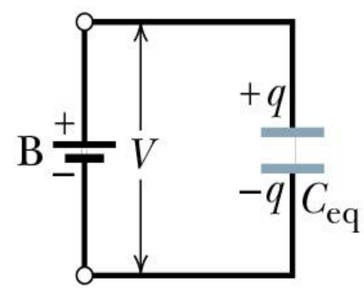
Capacitors in Parallel

- In order to make problem solving easier, we replace the three capacitors with a single capacitor that has the same effect on the circuit as the three single capacitors.
- In parallel:

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$



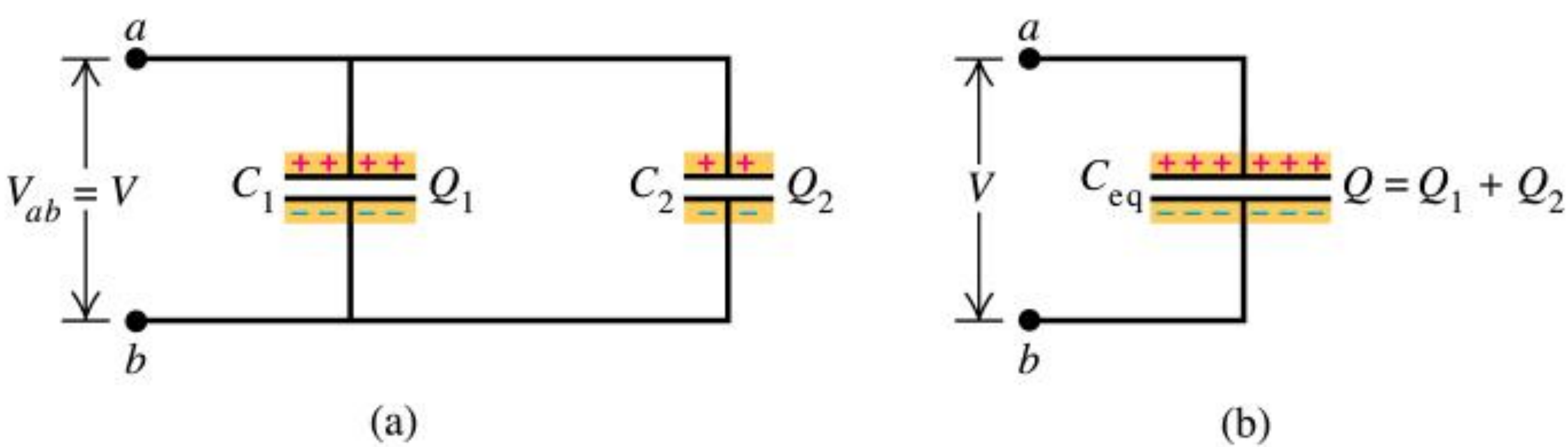
(a)



(b)

Capacitors in Parallel

- C_{eq} will be equal to the total capacitance of the circuit C_T.
- Increasing the number of capacitors increases the capacitance.



Capacitors in Parallel

- Problem solving involves reducing the circuit components to one total charge, one total voltage, and one total capacitance:

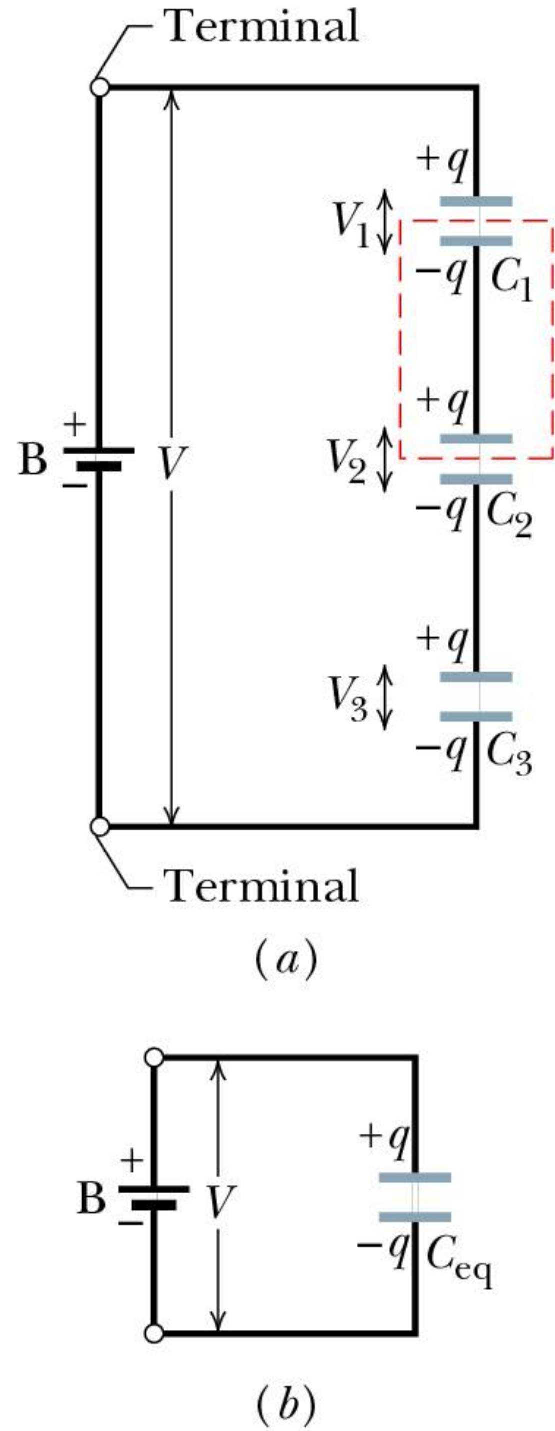
$$C_T = \frac{Q_T}{V}$$

- In parallel circuits, you will probably find the voltage first and then use this to determine the charge found on each capacitor.

$$Q_1 = C_1 \cdot V \qquad Q_2 = C_2 \cdot V$$

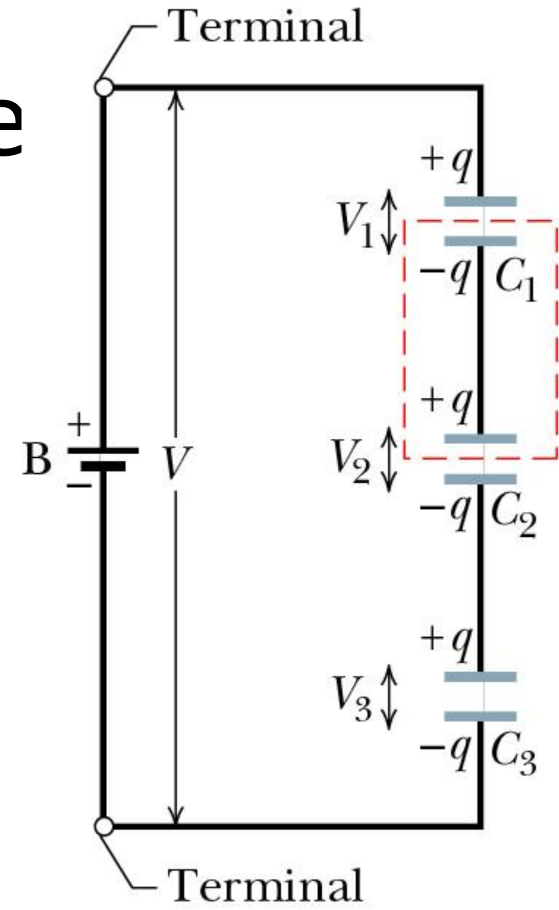
Capacitors in Series

- Three capacitors (C_1, C_2, and C_3) are connected in series to a battery B.
- When the capacitors are first connected in the circuit, electrons are transferred through the battery from the plate of C_1 that becomes positively charged to the plate of C_3 that becomes negatively charged.

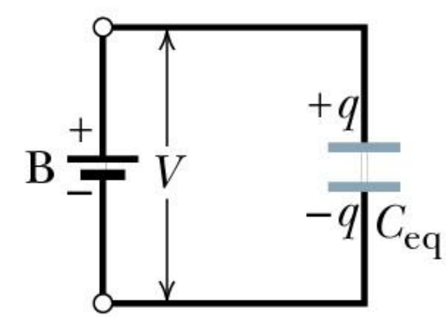


Capacitors in Se

- As the negative charge increases on the negatively charged plate of C_3, an equal amount of negative charge is forced off the plate of C_3 that becomes positive onto the plate of C_2 that becomes negative.
- The same amount of negative charge is also moved between C_2 and C_1.
- The energy needed to do this comes from the battery.



(a)

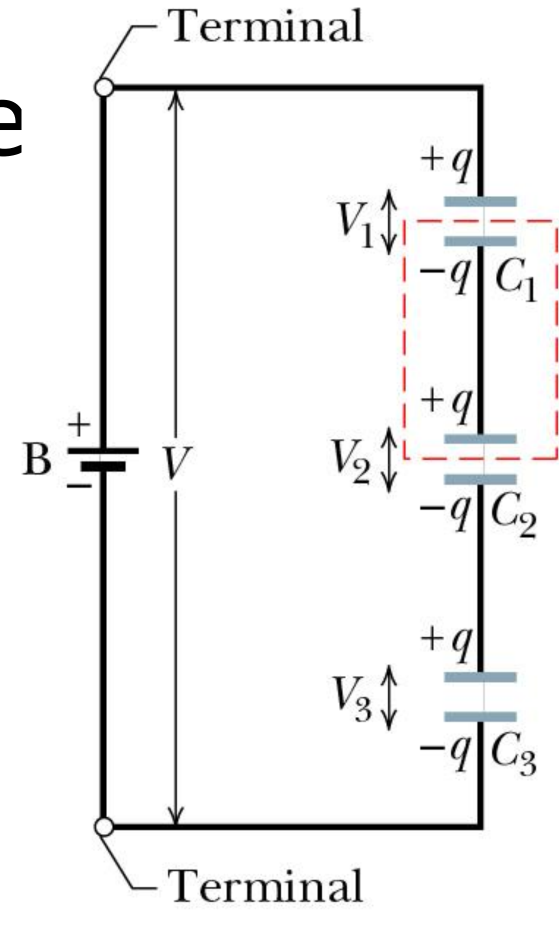


(b)

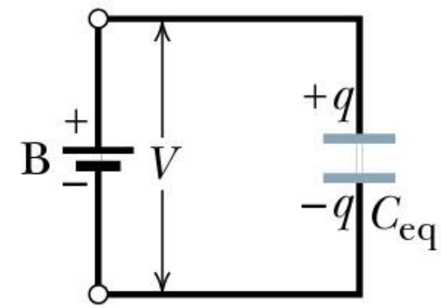
Capacitors in Se

- In the figure shown, all of the upper capacitor plates will have a charge of +Q and all of the lower capacitor plates will have a charge of –Q.
- For capacitors in series, the amount of charge on each plate is the same:

$$Q_T = Q_1 = Q_2 = Q_3 = \ldots$$



(a)

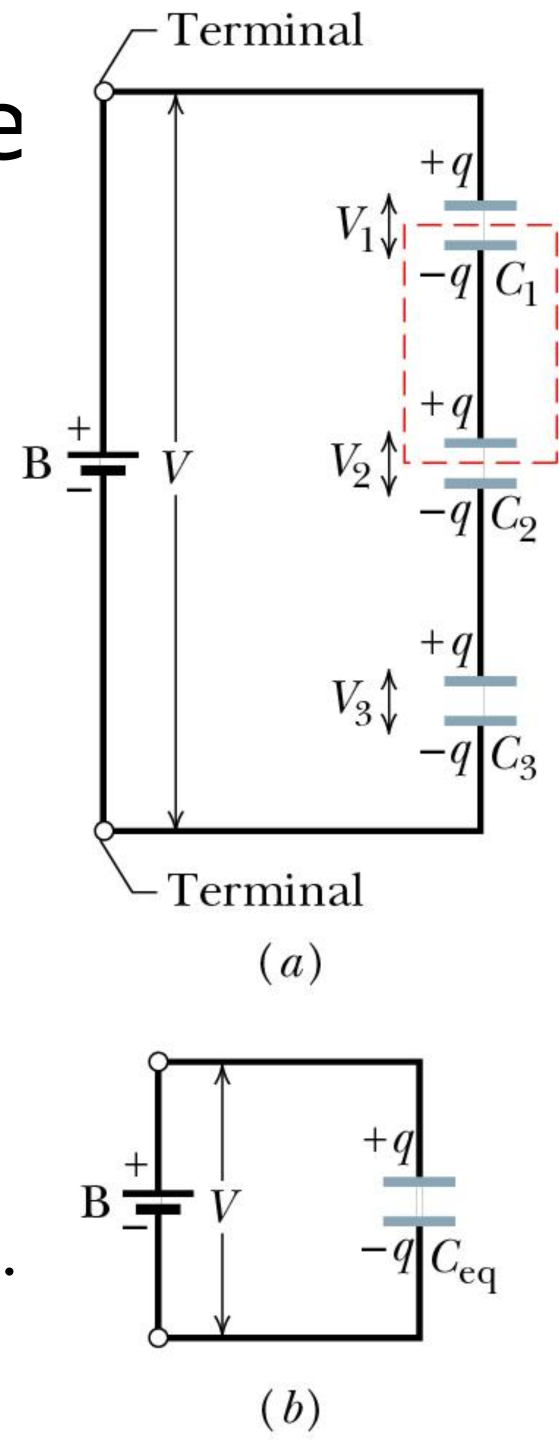


(b)

Capacitors in Se

- In order to make problem solving easier, we replace the three capacitors with a single capacitor that has the same effect on the circuit as the three single capacitors.
- In series, the reciprocal of the total capacitance is the sum of the reciprocals of the separate capacitors:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

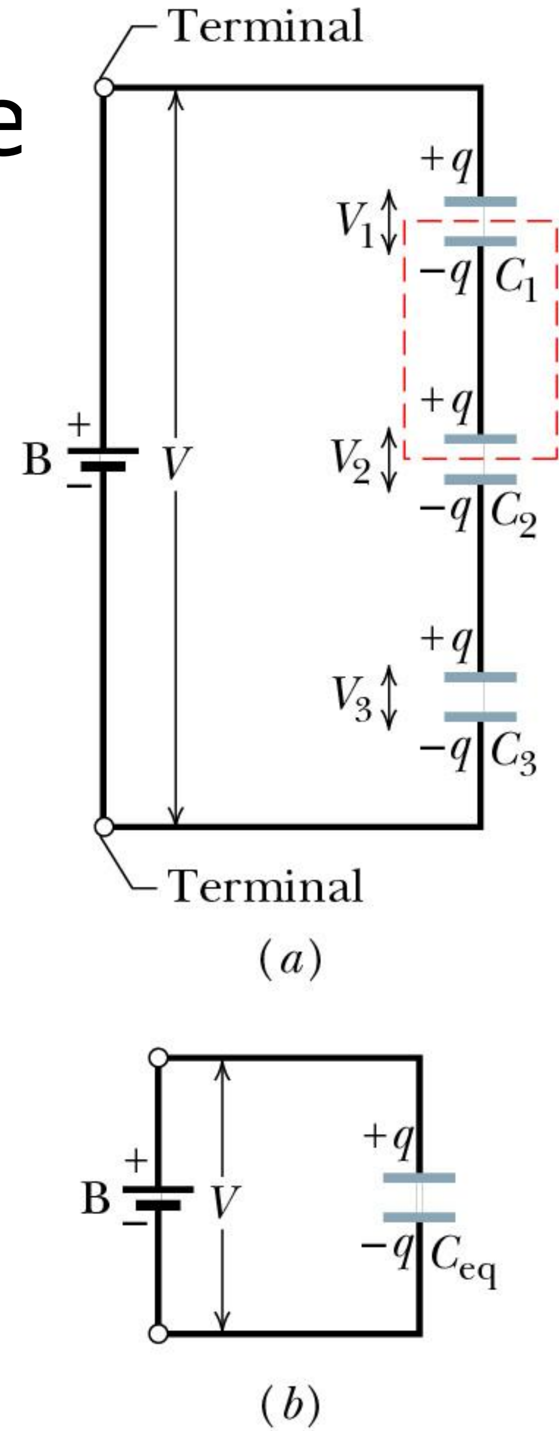


Capacitors in Se

- It is easier to use the reciprocal key (x^{-1} or $1/x$) on your calculator:

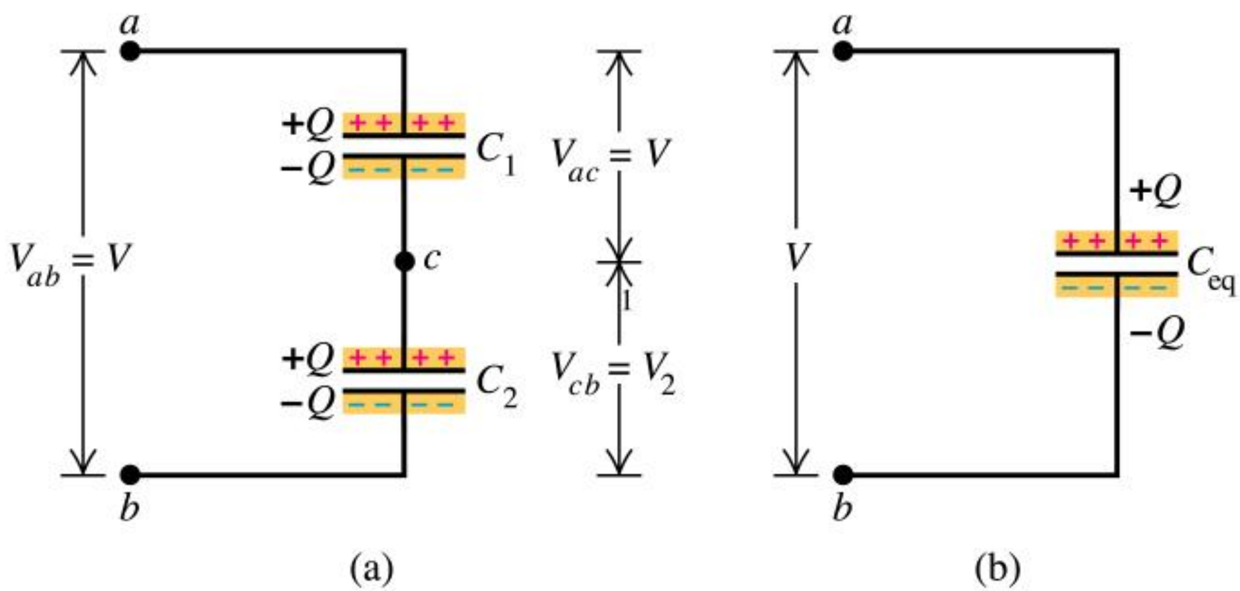
  $C_{eq} = (C_1^{-1} + C_2^{-1} + C_3^{-1} + \ldots)^{-1}$
- In series, the total voltage is equal to the combined voltage of each capacitor:

$$V_T = V_1 + V_2 + V_3 + \ldots$$



Capacitors in Series

- C_{eq} will be equal to the total capacitance of the circuit C_T.
- Increasing the number of capacitors decreases the capacitance.



Capacitors in Series

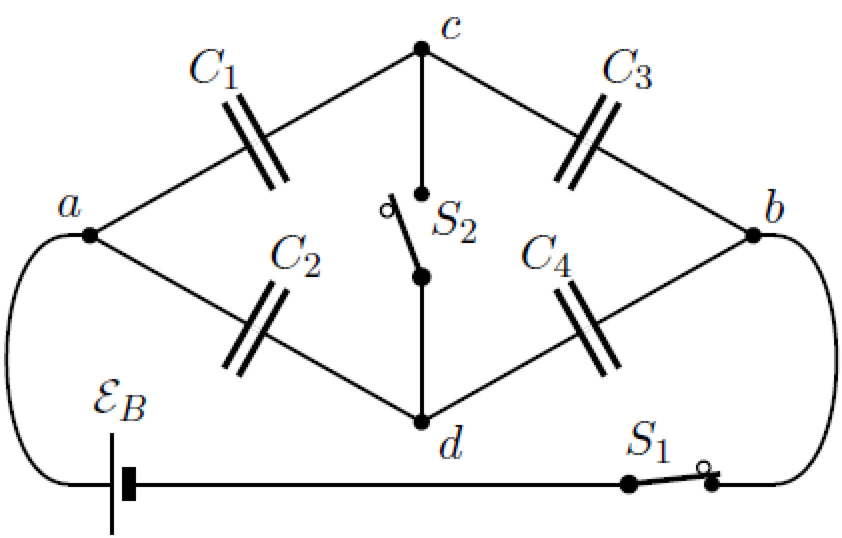
- Problem solving involves reducing the circuit components to one total charge, one total voltage, and one total capacitance:

$$C_T = \frac{Q_T}{V}$$

- In series circuits, you will probably find the charge first and then use this to determine the voltage across each capacitor.

$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2}$$

Capacitors In Parallel and In Series



- A circuit as shown on the left when both S1 and S2 are closed is actually 2 sets of capacitors in parallel with the 2 parallel combinations arranged in series.

Delta-Star Network

- Three branches in an electrical network can be connected in numbers of forms but most common among them is either star or delta form.
- In delta connection three branches are so connected that they form a closed loop that is they are mesh connected. As these three branches are connected nose to tail they forms an triangular closed loop, this configuration is referred as delta connection.

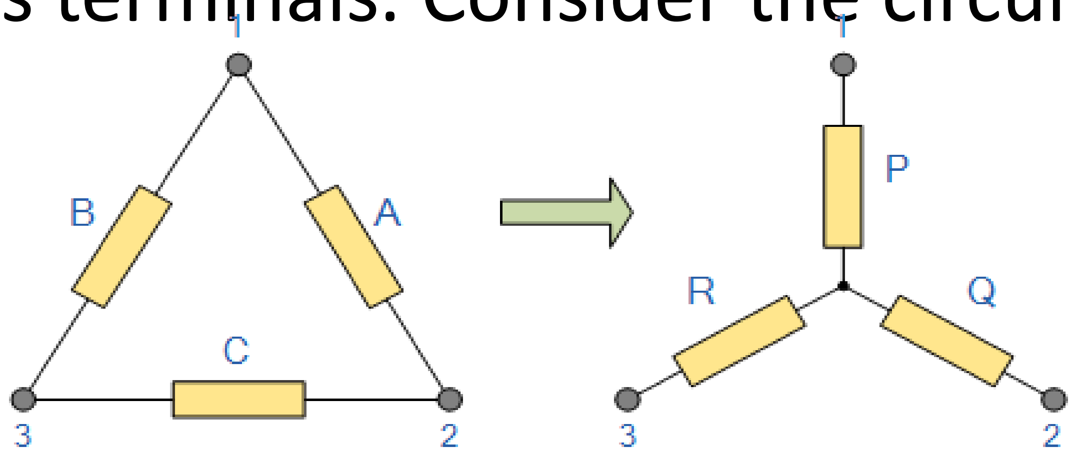
- On the other hand when either terminal of three branches are connected to a common point to form a Y like pattern is known as star connection.
- But these star and delta connections can be transformed from one form to other. For simplifying complex network, it is often required delta to star or star to delta transformation.

Delta - Star Transformation

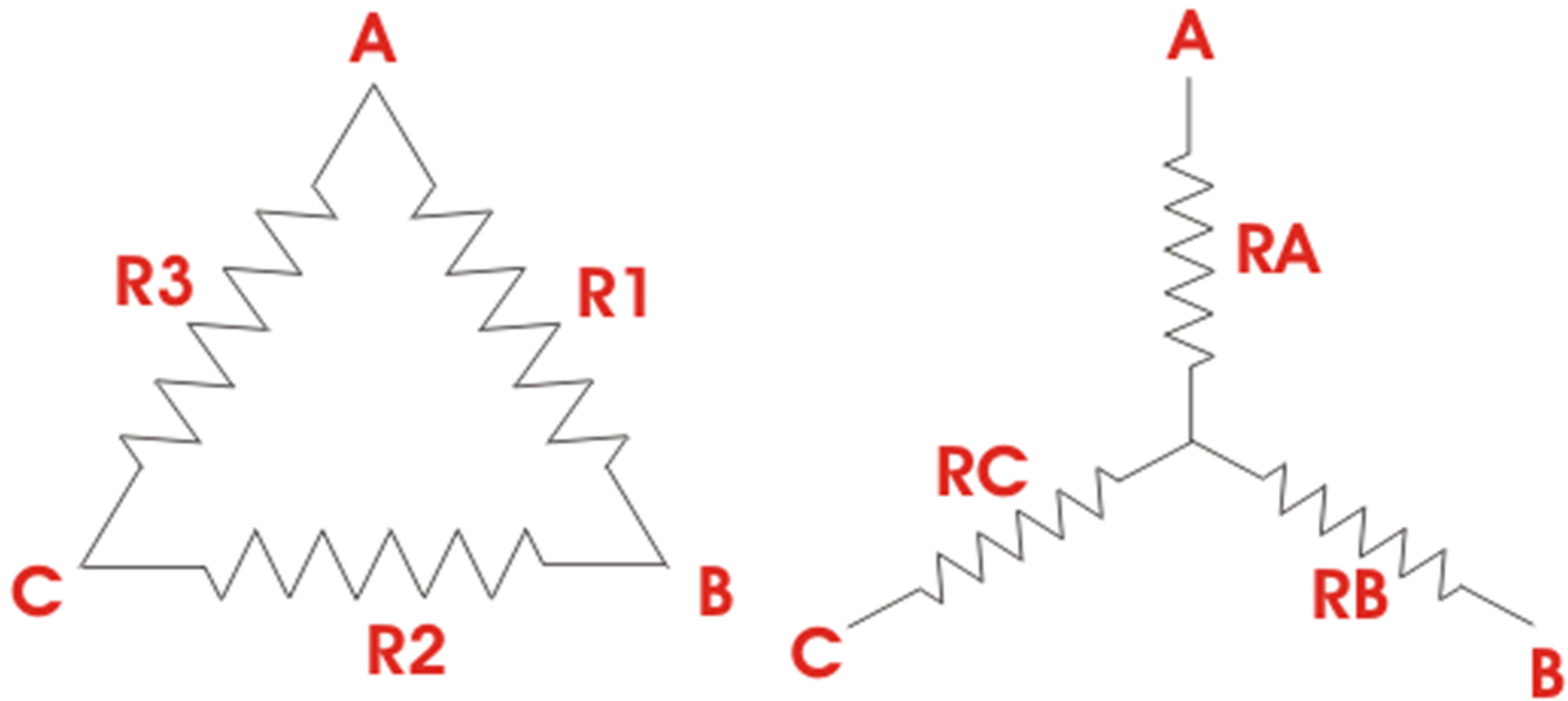
- The replacement of delta or mesh by equivalent star connection is known as **delta - star transformation**.
- The two connections are equivalent or identical to each other if the impedance is measured between any pair of lines.
- That means the value of impedance will be same if it is measured between any pair of lines irrespective of whether the delta is connected between the lines or its equivalent star is connected between that lines.

Derivation (Δ-Y)

- To convert a delta network to an equivalent star network we need to derive a transformation formula for equating the various resistors to each other between the various terminals. Consider the circuit below.



DELTA AND STAR CONNECTED RESISTORS



- Consider a delta system whose three corner points are A, B and C as shown in the figure. Electrical resistance of the branch between points A & B, B & C and C & A are R_1, R_2 and R_3 respectively. The resistance between the points A & B will be

$$\boxed{R_{AB} = R_1 \parallel (R_2 + R_3) = \frac{R_1.(R_2 + R_3)}{R_1 + R_2 + R_3}}$$

- Now, one star system is connected to these points A, B, and C as shown in the figure. Three arms R_A, R_B and R_C of the star system are connected with A, B and C respectively. Now if we measure the electrical resistance value between points A and B, we will get

$$\mathbf{R_{AB} = R_A + R_B}$$

- Since the two systems are identical, resistance measured between terminals A and B in both systems must be equal.

$$\boxed{R_A + R_B = \frac{R_1.(R_2 + R_3)}{R_1 + R_2 + R_3} \text{ ....(I)}}$$

Similarly resistance between points B and C being equal in the two system

$$\boxed{R_B + R_C = \frac{R_2.(R_3 + R_1)}{R_1 + R_2 + R_3} \text{ ....(II)}}$$

And resistance between points C and A being equal in the two system

$$\boxed{R_C + R_A = \frac{R_3.(R_1 + R_2)}{R_1 + R_2 + R_3} \text{ ....(III)}}$$

Adding equations (I), (II) and (III) we get,

$$2(R_A + R_B + R_C) = \frac{2(R_1.R_2 + R_2.R_3 + R_3.R_1)}{R_1 + R_2 + R_3}$$

$$R_A + R_B + R_C = \frac{R_1.R_2 + R_2.R_3 + R_3.R_1}{R_1 + R_2 + R_3} \quad \text{............(IV)}$$

Subtracting equations (I), (II) and (III) from equation (IV) we get,

$$R_A = \frac{R_3.R_1}{R_1 + R_2 + R_3} \quad \text{............(V)}$$

$$R_B = \frac{R_1.R_2}{R_1 + R_2 + R_3} \quad \text{............(VI)}$$

$$R_C = \frac{R_2.R_3}{R_1 + R_2 + R_3} \quad \text{............(VII)}$$

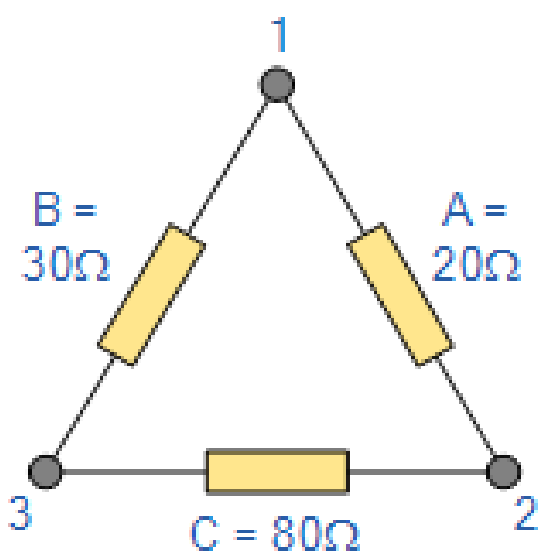
- The relation of delta - star transformation can be expressed as follows
- The equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta connected resistances.
- If the delta connected system has same resistance R at its three sides then equivalent star resistance r will be:

$$\boxed{r = \frac{R.R}{R + R + R} = \frac{R}{3}}$$

Example (Δ-Y)

Q. Convert the following Delta Resistive Network into an equivalent Star Network.

Ans:

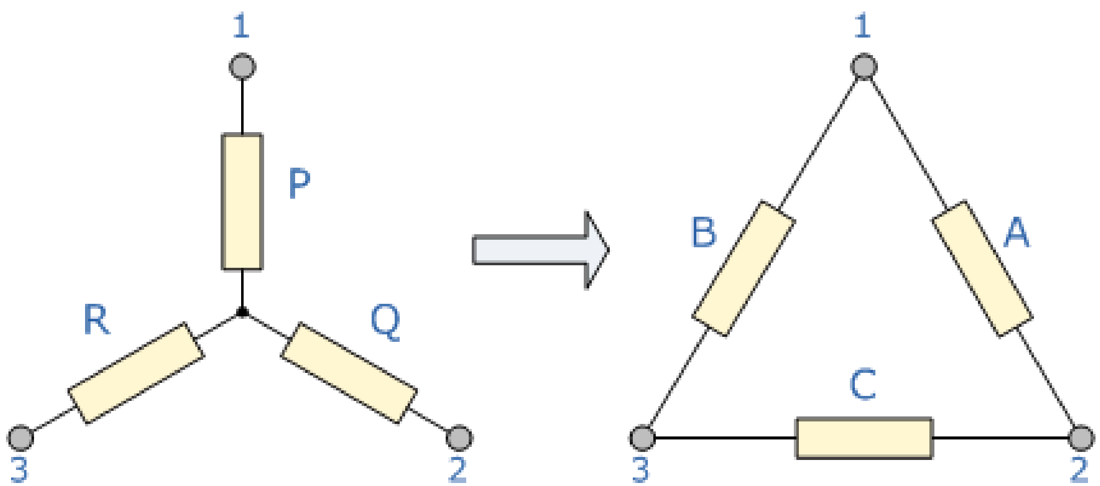


$$Q = \frac{AC}{A+B+C} = \frac{20\times 80}{130} = 12.31\Omega$$

$$P = \frac{AB}{A+B+C} = \frac{20\times 30}{130} = 4.61\Omega$$

$$R = \frac{BC}{A+B+C} = \frac{30\times 80}{130} = 18.46\Omega$$

Star-Delta Network



Star - Delta Transformation

- For **star - delta transformation** we just multiply equations (v), (VI) & (VI), (VII) & (VII),(V) that is by doing (v)X(VI) + (VI)X(VII) + (VII)X(V) we get

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1.R_2^2.R_3 + R_1.R_2.R_3^2 + R_1^2.R_2^2.R_3}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1.R_2.R_3\,(R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$= \frac{R_1.R_2.R_3}{R_1 + R_2 + R_3} \text{ ....(VIII)}$$

Now dividing equation (VIII) by equations (V), (VI) and equations (VII) separately we get,

$$\boxed{\begin{aligned} R_3 &= \frac{R_A R_B + R_B R_C + R_C R_A}{R_A} \\ R_1 &= \frac{R_A R_B + R_B R_C + R_C R_A}{R_B} \\ R_2 &= \frac{R_A R_B + R_B R_C + R_C R_A}{R_C} \end{aligned}}$$

ADVANTAGES

Advantages of Star Delta Connection

- The primary side is star connected. Hence fewer numbers of turns are required. This makes the connection economical for large high voltage step down power transformers.
- The neutral available on the primary can be earthed to avoid distortion.
- The neutral point allows both types of loads (single phase or three phases) to be met.
- Large unbalanced loads can be handled satisfactory.
- The Y-D connection has no problem with third harmonic components due to circulating currents inD. It is also more stable to unbalanced loads since the D partially redistributes any imbalance that occurs.

- The delta connected winding carries third harmonic current due to which potential of neutral point is stabilized. Some saving in cost of insulation is achieved if HV side is star connected. But in practice the HV side is normally connected in delta so that the three phase loads like motors and single phase loads like lighting loads can be supplied by LV side using three phase four wire system.
- As Grounding Transformer: In Power System Mostly grounded Y- Δ transformer is used for no other purpose than to provide a good ground source in ungrounded Delta system. Take, for example, a distribution system supplied by **Δ** connected (i.e., ungrounded) power source.

DISADVANTAGES

Disadvantages of Star-Delta Connection

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.
- One problem associated with this connection is that the secondary voltage is shifted by $30^0$with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts.
- If secondary of this transformer should be paralleled with secondary of another transformer without phase shift, there would be a problem

Superposition

- The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.
- Turn off, killed, inactive source:
  - independent voltage source: 0 V (short circuit)
  - independent current source: 0 A (open circuit)
- Dependent sources are left intact.

Superposition

- Steps to apply superposition principle:
  1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
  2. Repeat step 1 for each of the other independent sources.
  3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

How to turn off independent sources

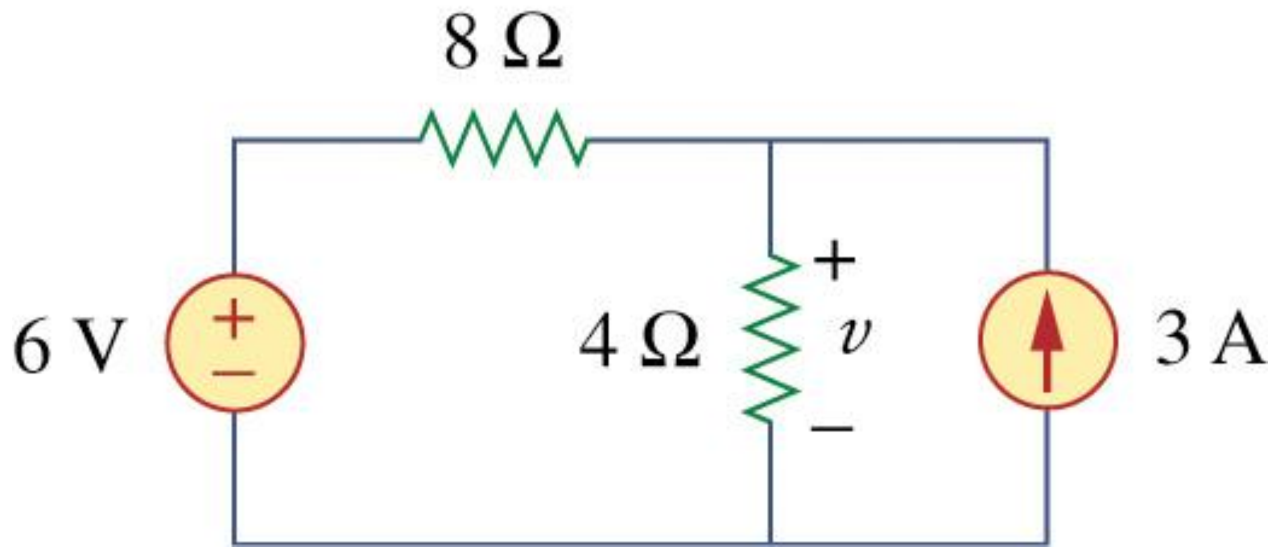
- Turn off voltages sources = short voltage sources; make it equal to zero voltage
- Turn off current sources = open current sources; make it equal to zero current

Superposition

- Superposition involves more work but simpler circuits.
- Superposition is not applicable to the effect on power.

Example

- Use the superposition theorem to find   in the circuit in Fig.



Example

Since there are two sources, let $V = V_1 + V_2$

Voltage division to get

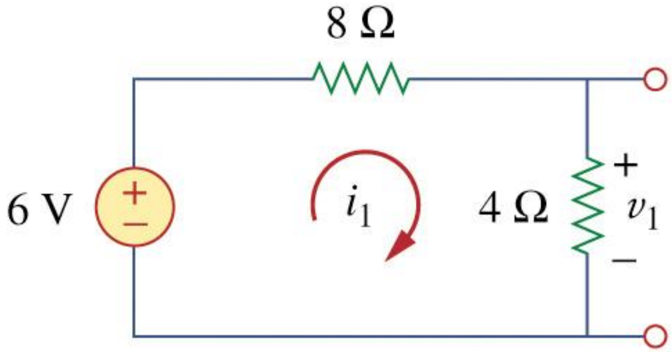
$$V_1 = \frac{4}{4+8}(6) = 2\text{V}$$

Current division, to get

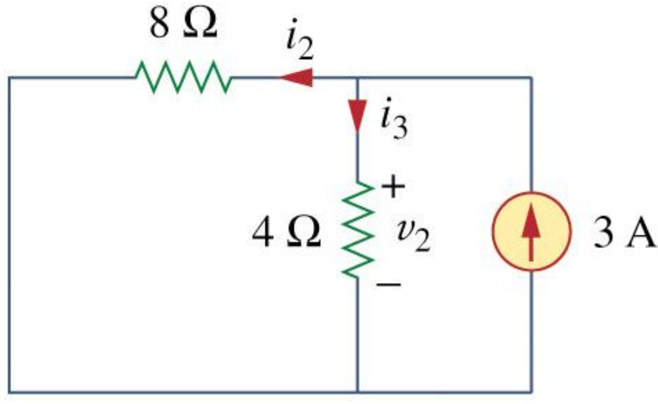
$$i_3 = \frac{8}{4+8}(3) = 2\text{A}$$

Hence $v_2 = 4i_3 = 8\text{V}$

And we find $v = v_1 + v_2 = 2 + 8 = 10\text{V}$



(a)

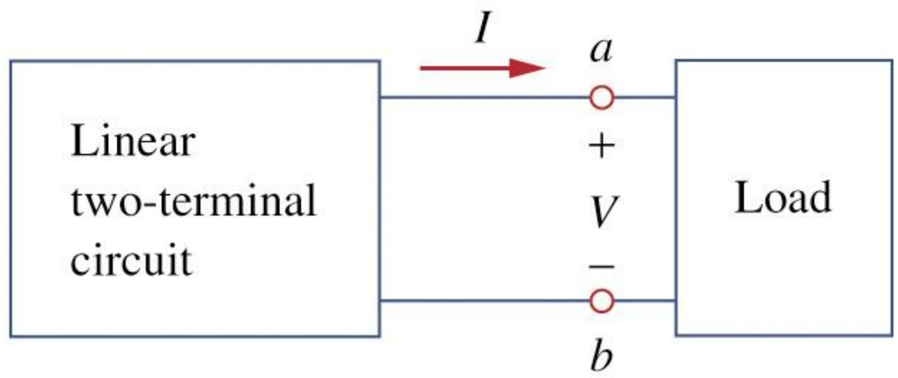


(b)

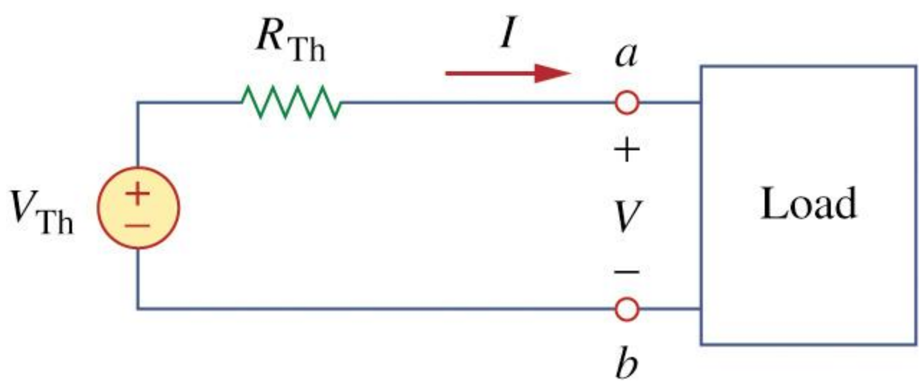
Thevenin's Theorem

- Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent source are turn off.

Thevenin's Theorem



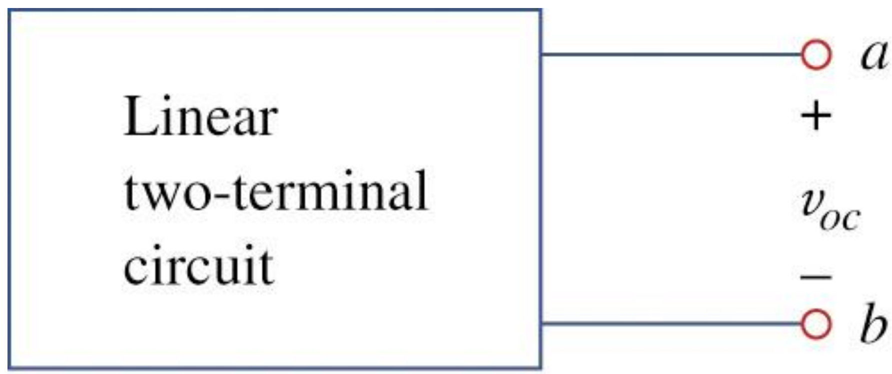
(a)



(b)

How to Find Thevenin's Voltage

- Equivalent circuit: same voltage-current relation at the terminals.
- $V_{\mathrm{Th}} = v_{oc}$:   open circuit voltage at $a-b$

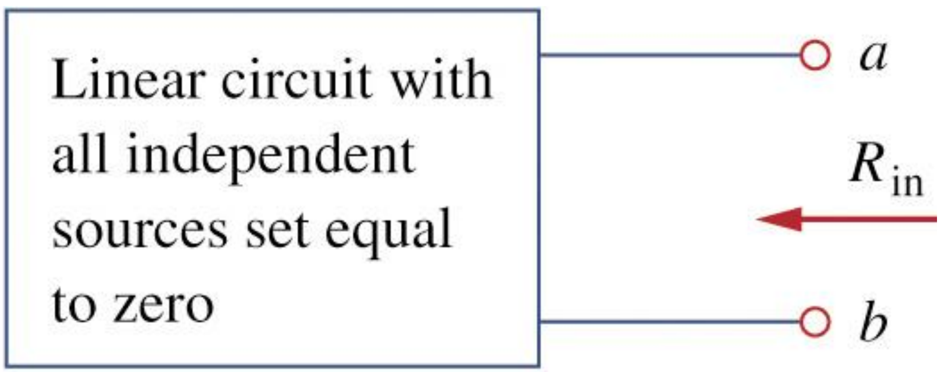


$V_{\mathrm{Th}} = v_{oc}$

(a)

How to Find Thevenin's Resistance

- $R_{\text{Th}} = R_{\text{in}}$:
  input $-$ resistance of the dead circuit at $a-b$.
  - $a-b$ open circuited
  - Turn off all independent sources



$R_{\text{Th}} = R_{\text{in}}$

(b)

Thevenin's Theorem

CASE 1

- If the network has no dependent sources:
  - Turn off all independent source.
  - R_{TH}: can be obtained via simplification of either parallel or series connection seen from a-b

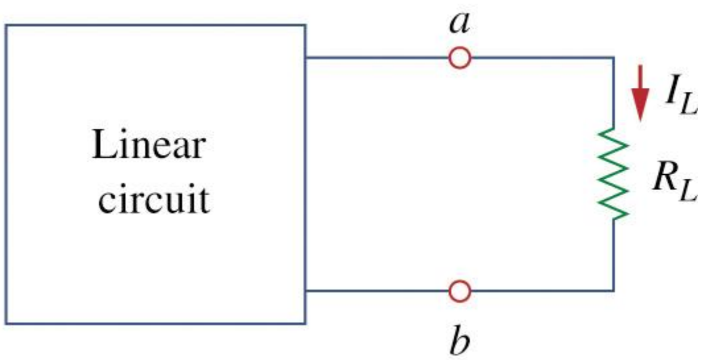
Fig. 4.26

Simplified circuit

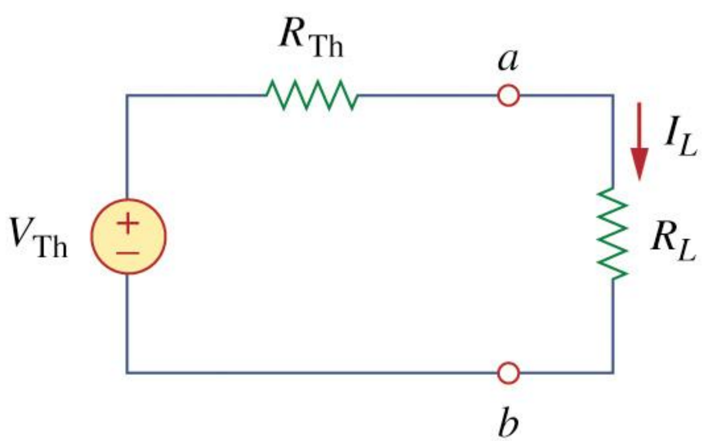
$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}}$$

Voltage divider



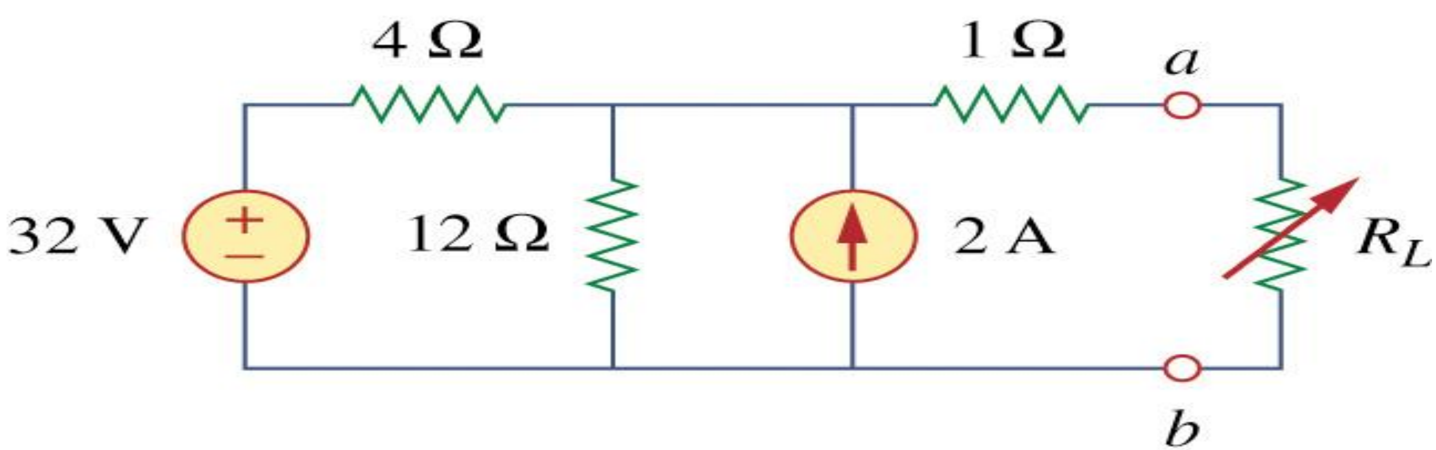
(a)



(b)

Example

- Find the Thevenin's equivalent circuit of the circuit shown in Fig to the left of the terminals *a*-*b*. Then find the current through R_L = 6,16,and 36 Ω.

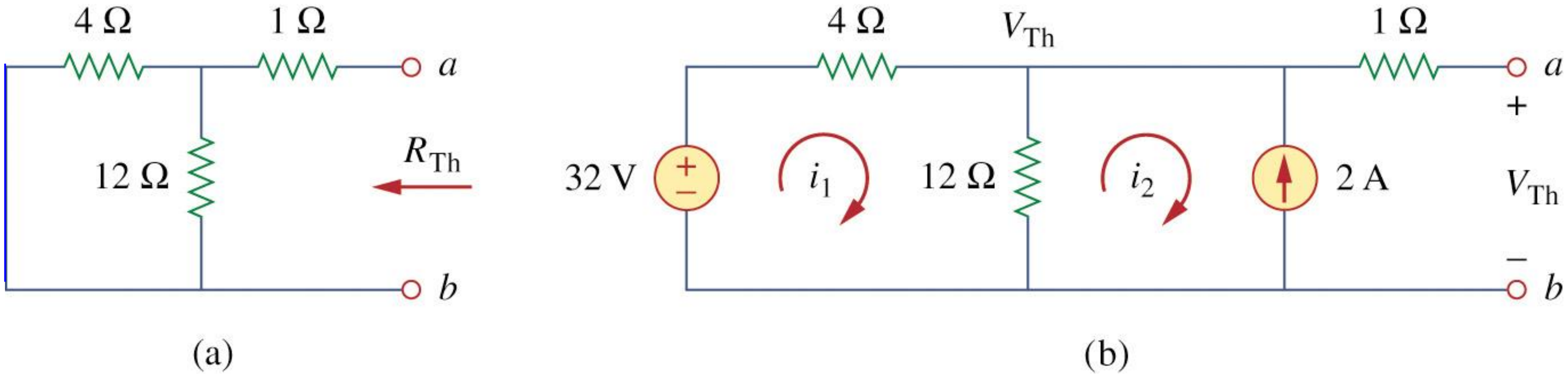


Find R_{th}

$$R_{\text{Th}} : \text{32V voltage source} \rightarrow \text{short}$$

$$\text{2A current source} \rightarrow \text{open}$$

$$R_{\text{Th}} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega$$



Find V_{th}

V_{Th} :

(1) Mesh analysis

$$-32 + 4i_1 + 12(i_1 - i_2) = 0\ ,\ \ i_2 = -2\mathrm{A}$$

$$\therefore i_1 = 0.5\mathrm{A}$$

$$V_{\mathrm{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\mathrm{V}$$

(2) Alternatively, Nodal Analysis

$$(32 - V_{\mathrm{Th}})/4 + 2 = V_{\mathrm{Th}}/12$$

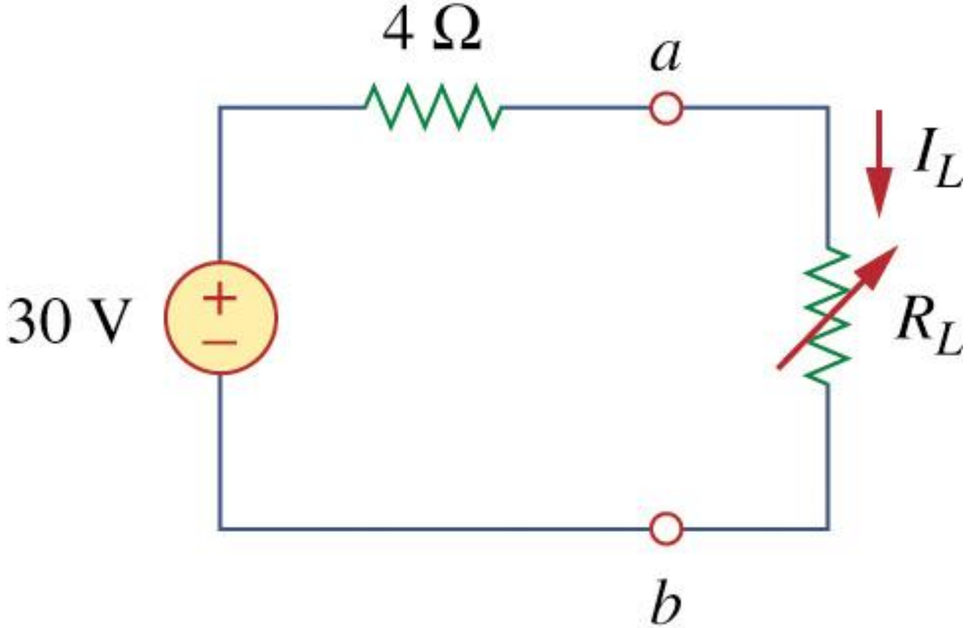
$$\therefore V_{\mathrm{Th}} = 30\mathrm{V}$$

Example

(3) Alternatively, source transform

$$\frac{32 - V_{\mathrm{TH}}}{4} + 2 = \frac{V_{\mathrm{TH}}}{12}$$

$$96 - 3V_{\mathrm{TH}} + 24 = V_{\mathrm{TH}} \Rightarrow V_{\mathrm{TH}} = 30\mathrm{V}$$



Example

To get i_L :

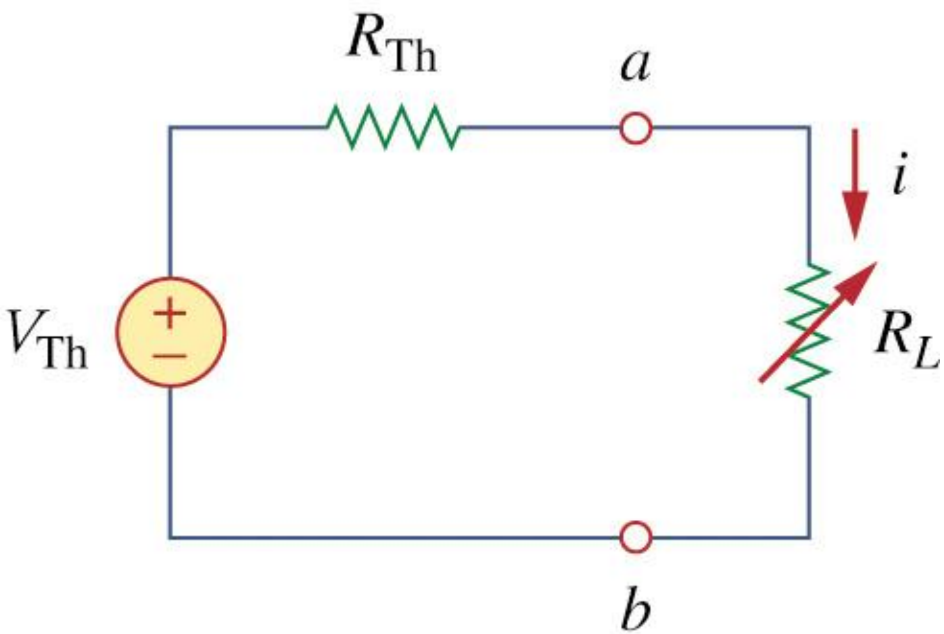
$$i_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$

$$R_L = 6 \rightarrow I_L = 30/10 = 3\text{A}$$

$$R_L = 16 \rightarrow I_L = 30/20 = 1.5\text{A}$$

$$R_L = 36 \rightarrow I_L = 30/40 = 0.75\text{A}$$

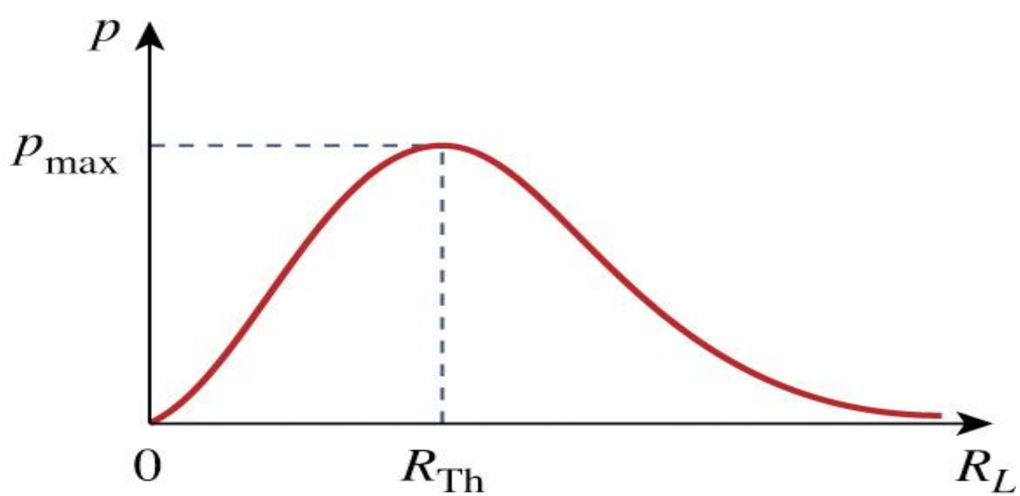
Maximum Power Transfer



$$p = i^2 R_L = \left(\frac{V_{\mathrm{TH}}}{R_{\mathrm{TH}} + R_{\mathrm{L}}} \right)^2 R_L$$

Fig.

- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen the load ($R_L = R_{TH}$).



Maximum Power Transfer

$$\frac{dp}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right]$$

$$= V_{TH}^2 \left[\frac{(R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} \right] = 0$$

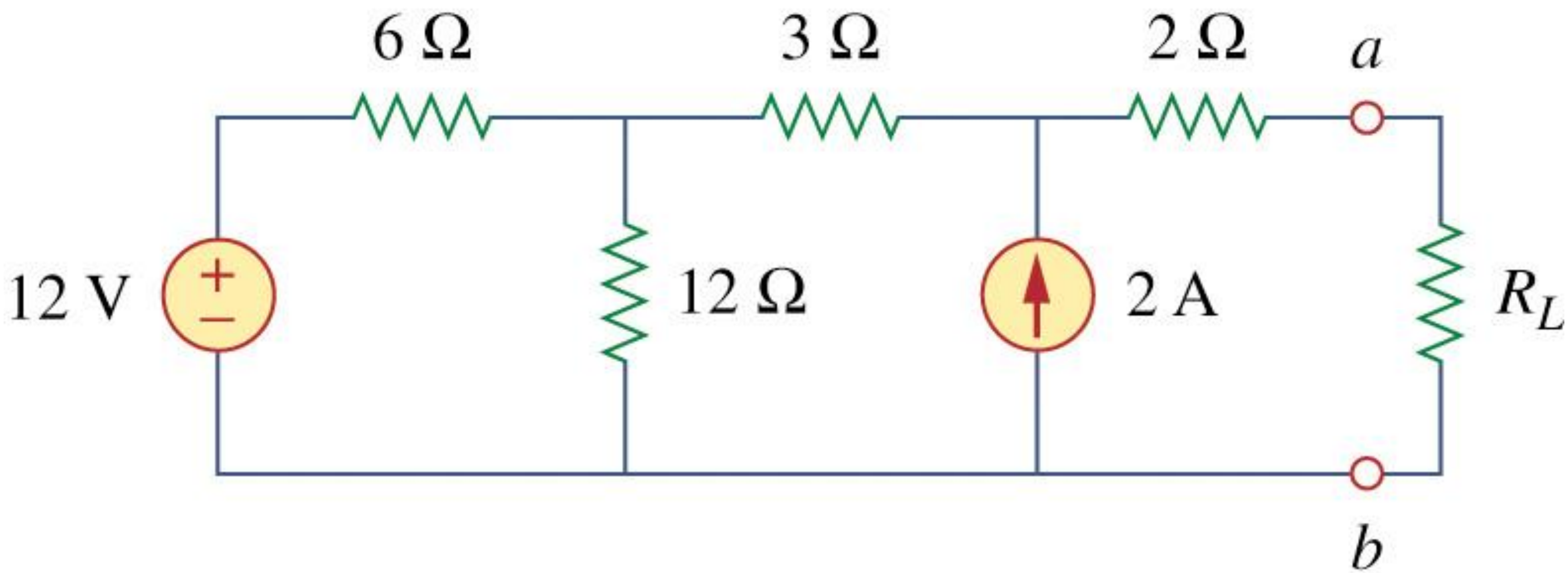
$$0 = (R_{TH} + R_L - 2R_L) = (R_{TH} - R_L)$$

$$R_L = R_{TH}$$

$$p_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

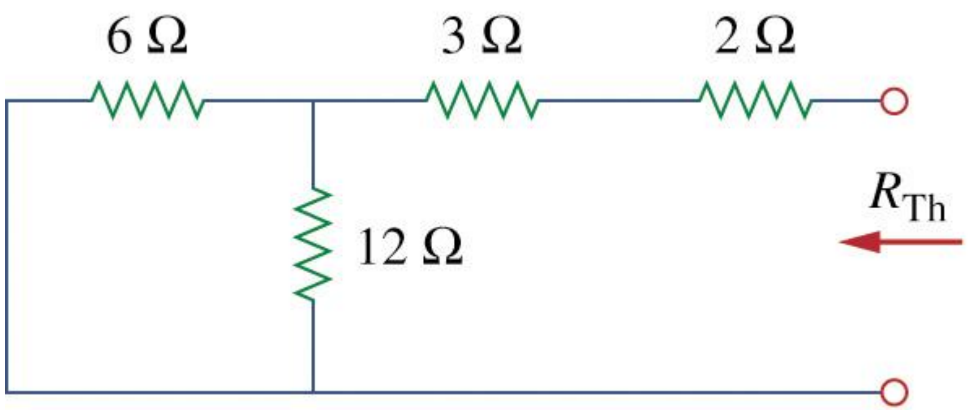
Example

- Find the value of R_L for maximum power transfer in the circuit of Fig.. Find the maximum power.



Example

$$R_{TH} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\Omega$$



(a)

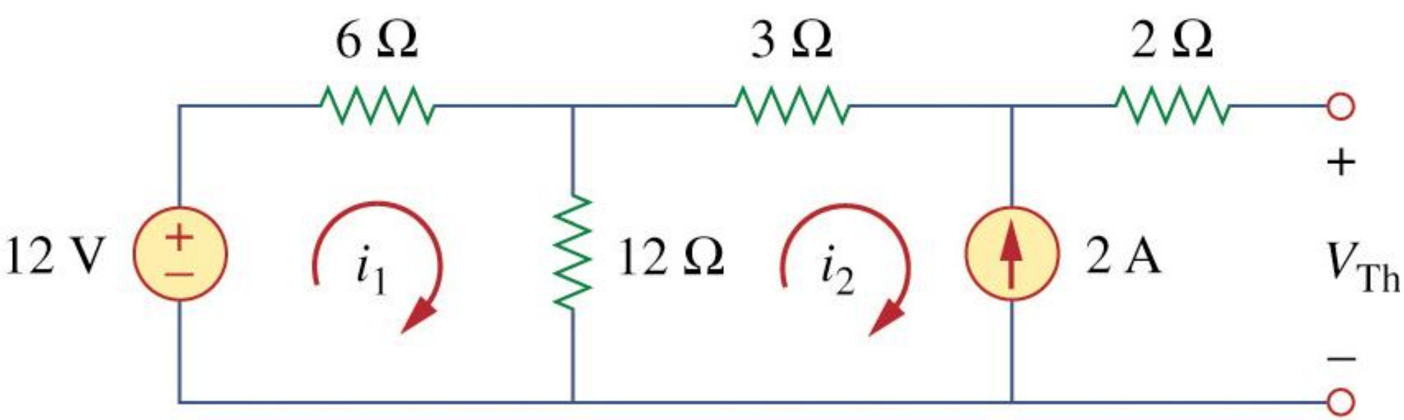
Example

$$-12+18i_1-12i_2, \quad i_2=-2A$$

$$-12+6i_i1+3i_2+2(0)+V_{TH}=0 \Rightarrow V_{TH}=22V$$

$$R_L=R_{TH}=9\Omega$$

$$p_{\max}=\frac{V_{TH}^2}{4R_L}=\frac{22^2}{4\times 9}=13.44W$$



(b)

UNIT-II
AC QUANTITIES

Introduction to Alternating Quantities

Objectives

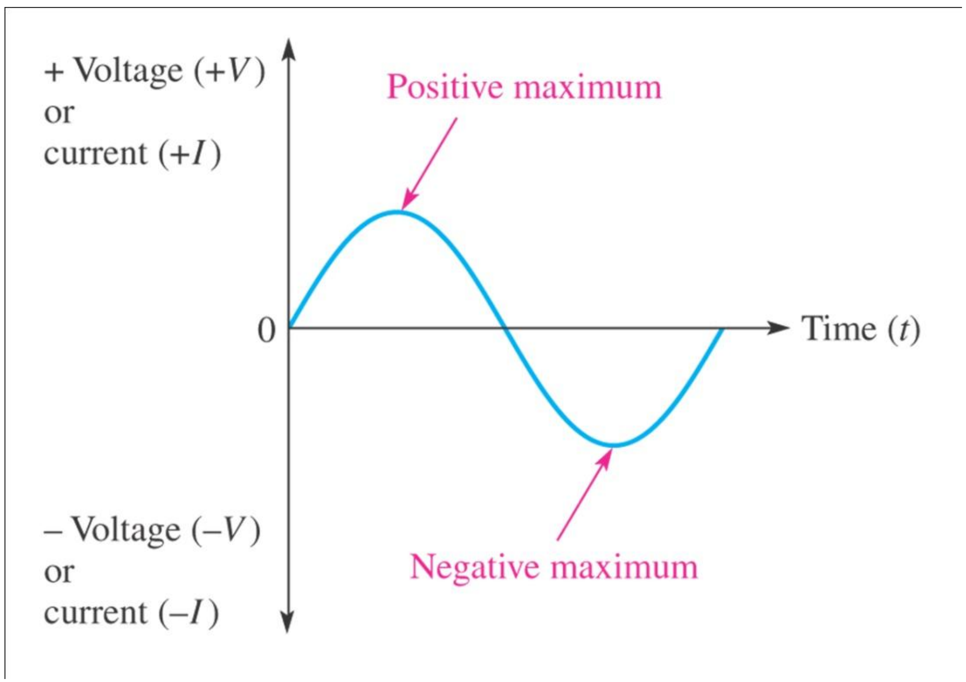
- Identify a sinusoidal waveform and measure its characteristics
- Describe how sine waves are generated
- Determine the various voltage and current values of sine waves
- Describe angular relationships of sine waves
- Mathematically analyze a sinusoidal waveform
- Apply the basic circuit laws to ac resistive circuits

Objectives

- Determine total voltages that have both ac and dc components
- Identify the characteristics of basic non-sinusoidal waveforms

Sine Wave

- The sine wave is a common type of alternating current (ac) and alternating voltage



Period of a Sine Wave

- The time required for a sine wave to complete one full cycle is called the period (T)
  - A cycle consists of one complete positive, and one complete negative alternation
  - The period of a sine wave can be measured between any two corresponding points on the waveform

Frequency of a Sine Wave

- Frequency (f) is the number of cycles that a sine wave completes in one second
  - The more cycles completed in one second, the higher the frequency
  - Frequency is measured in **hertz** (Hz)
- Relationship between frequency (f) and period (T) is:

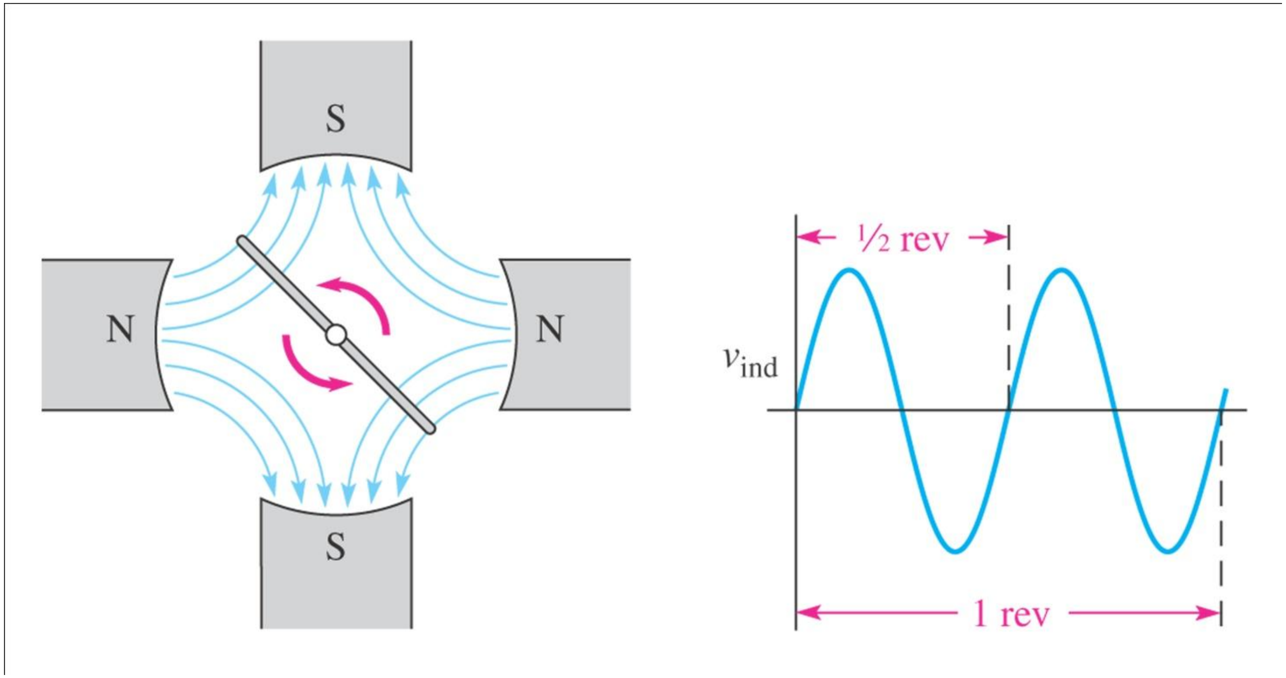
$$\boldsymbol{f = 1/T}$$

AC Generator

- The ac generator has slip rings that pick up the induced voltage through a complete rotation cycle
- The induced voltage is related to the number of lines of flux being cut. When the loop is moving parallel with the lines of flux, no voltage is induced. When the loop is moving perpendicular to the lines of flux, the maximum voltage is induced

Multi-pole ac Generator

- By increasing the number of poles, the number of cycles per revolution can be increased



Instantaneous Values of Sine Waves

- The **instantaneous values** of a sine wave voltage (or current) are different at different points along the curve, having negative and positive values

- Instantaneous values are represented as:

$$v \text{ and } i$$

Peak Values of Sine Waves

- The **peak value** of a sine wave is the value of voltage or current at the positive or negative maximum with respect to zero

- Peak values are represented as:

$$V_p \text{ and } I_p$$

Peak-to-Peak Values

- The **peak-to-peak value** of a sine wave is the voltage or current from the positive peak to the negative peak

- The peak-to-peak values are represented as:

$$V_{pp} \text{ and } I_{pp}$$

<u>where</u>: $V_{pp} = 2V_p$ and $I_{pp} = 2I_p$

RMS Value of a Sine Wave

- The **rms** (root mean square) value, or **effective value**, of a sinusoidal voltage is equal to the dc voltage that produces the same amount of heat in a resistance as does the sinusoidal voltage

$$V_{rms} = 0.707V_p$$
$$I_{rms} = 0.707I_p$$

Average Value of a Sine Wave

- The **average value** is the total area under the half-cycle curve divided by the distance in radians of the curve along the horizontal axis

$$V_{avg} = 0.637V_{p}$$

$$I_{avg} = 0.637I_{p}$$

Form factor

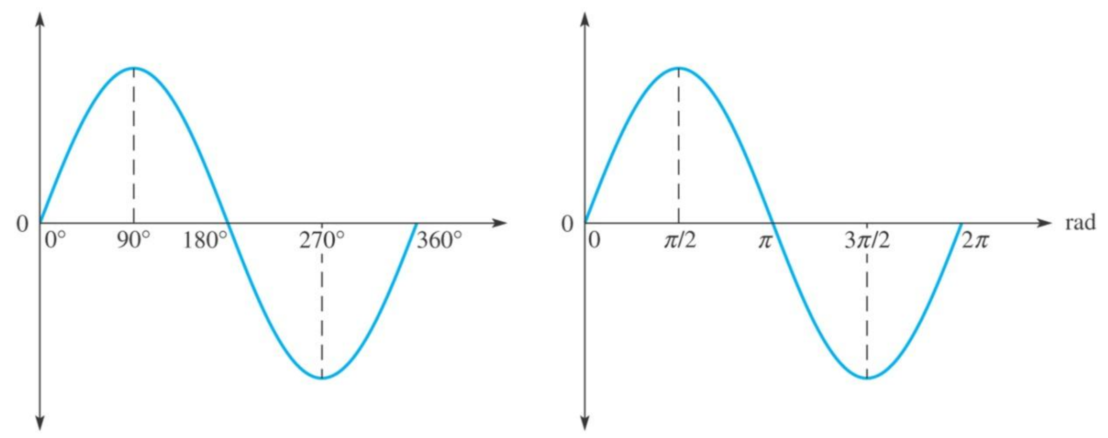
- Form factor is defined as ratio of

Angular Measurement of a Sine Wave

- A **degree** is an angular measurement corresponding to 1/360 of a circle or a complete revolution
- A radian (rad) is the angular measure along the circumference of a circle that is equal to the radius of the circle
- There are 2π radians or 360° in one complete cycle of a sine wave

Phase of a Sine Wave

- The phase of a sine wave is an angular measurement that specifies the position of a sine wave relative to a reference
- When a sine wave is shifted left or right with respect to this reference, there is a phase shift



(a) Degrees (b) Radians

Sine Wave Formula

The general expression for a sine wave is:

$$\mathbf{y = A \sin} \ \theta$$

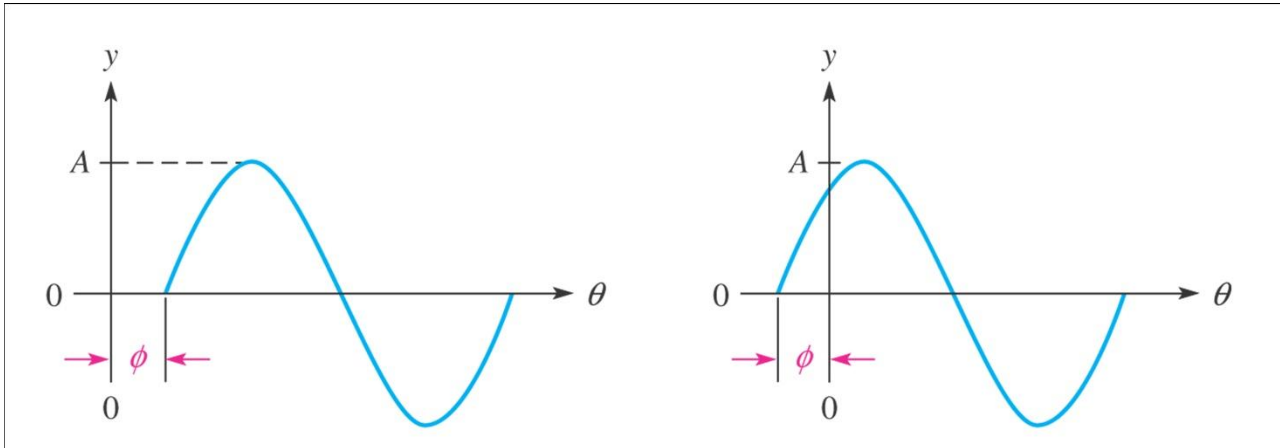
Where: y = an instantaneous value (*v* or *i*)

A = amplitude (maximum value)

θ = angle along the horizontal axis

Expressions for Shifted Sine Waves

- When a sine wave is shifted to the right of the reference by an angle ϕ, it is termed lagging
- When a sine wave is shifted to the left of the reference by an angle ϕ, it is termed leading



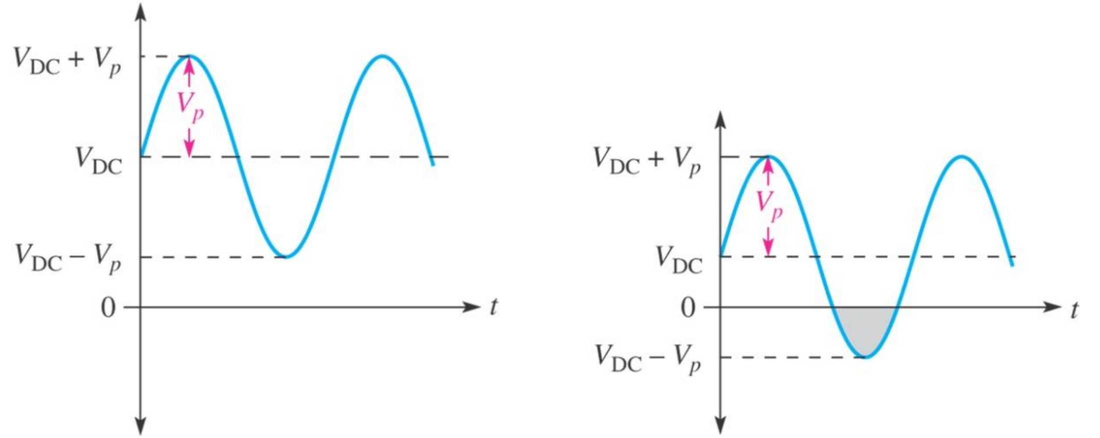
(a) $y = A \sin(\theta - \phi)$ (b) $y = A \sin(\theta + \phi)$

Ohms’s Law and Kirchhoff’s Laws in AC Circuits

- When time-varying ac voltages such as a sinusoidal voltage are applied to a circuit, the circuit laws that were studied earlier still apply
- Ohm’s law and Kirchhoff’s laws apply to ac circuits in the same way that they apply to dc circuits

Superimposed dc and ac Voltages

- DC and ac voltages will add algebraically, to produce an ac voltage "riding" on a dc level

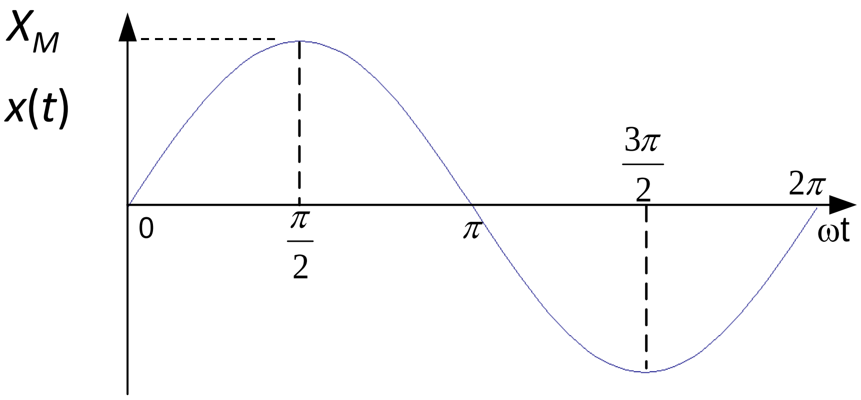


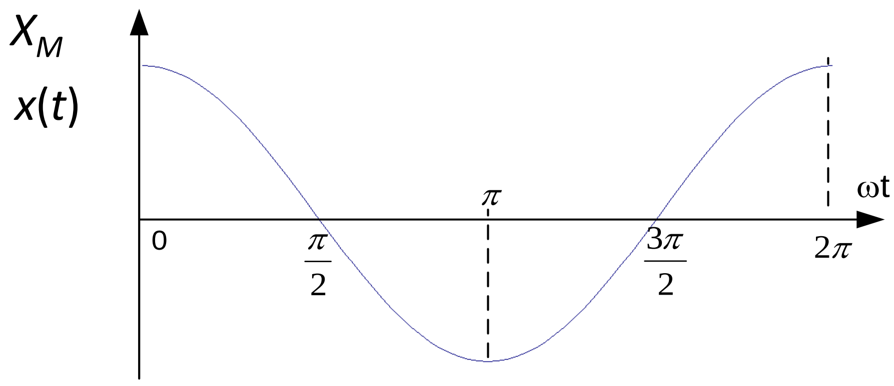
(a) $V_{DC} > V_p$. The sine wave never goes negative.

(b) $V_{DC} < V_p$. The sine wave reverses polarity during a portion of its cycle.

Sinusoids

A sinusoid is a signal that has the form of the sine or cosine function.





$$x(t) = X_M \sin \omega t$$

$$x(t) = X_M \cos \omega t$$

$$\text{period} \;\; T = 2\pi/\omega$$

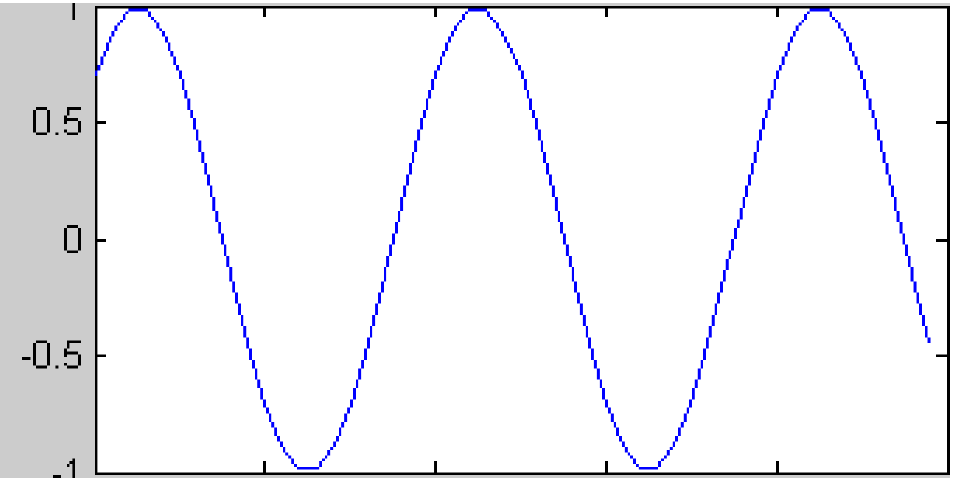
$$\sin\frac{\pi}{2} = 1 \qquad \cos\frac{\pi}{2} = 0$$

Sinusoids

In general:

$$x(t) = X_M \sin\left(\omega t + \phi\right)$$

$$x(t) = X_M \cos\left(\omega t + \phi\right)$$



Note trig identities:

$$\sin\left(\omega t \pm \phi\right) = \sin \omega t \cos \phi \pm \cos \omega t \sin \phi$$

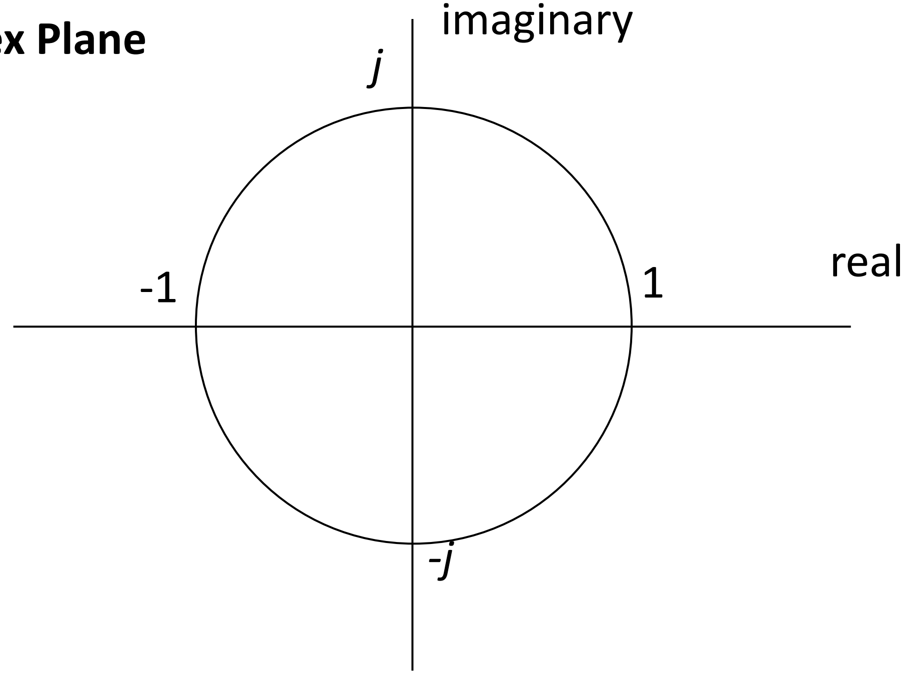
$$\cos\left(\omega t \pm \phi\right) = \cos \omega t \cos \phi \mp \sin \omega t \sin \phi$$

Complex Numbers

What is the solution of
X² = -1

$$X = \pm\sqrt{-1} = \pm j \qquad j = \sqrt{-1} \qquad j^2 = -1$$

Complex Plane



Note:

$$\frac{1}{j} = \frac{j}{jj} = \frac{j}{j^2} = -j$$

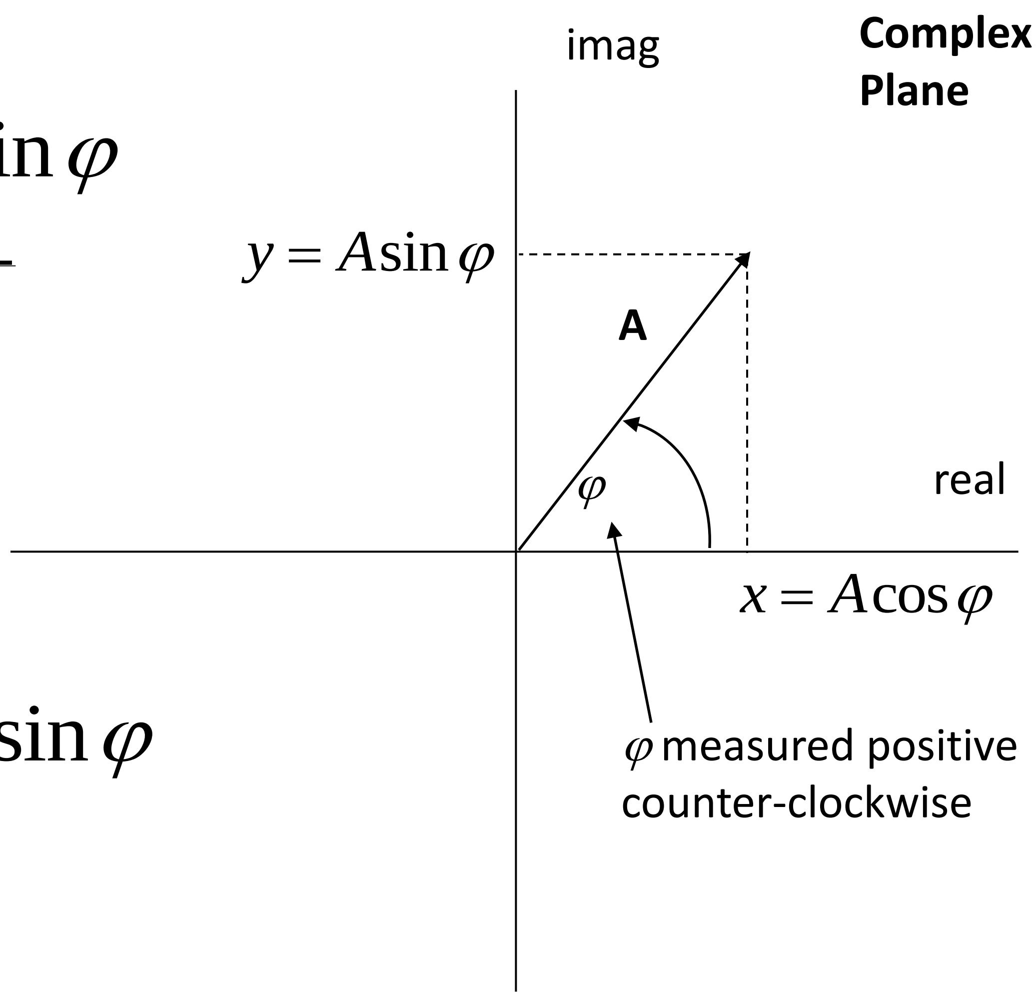
Complex Numbers

$$\mathbf{A} = x + jy$$

$$\mathbf{A} = A\cos\varphi + jA\sin\varphi$$

$$A = \sqrt{x^2 + y^2}$$

$$\varphi = \tan^{-1}\frac{y}{x}$$



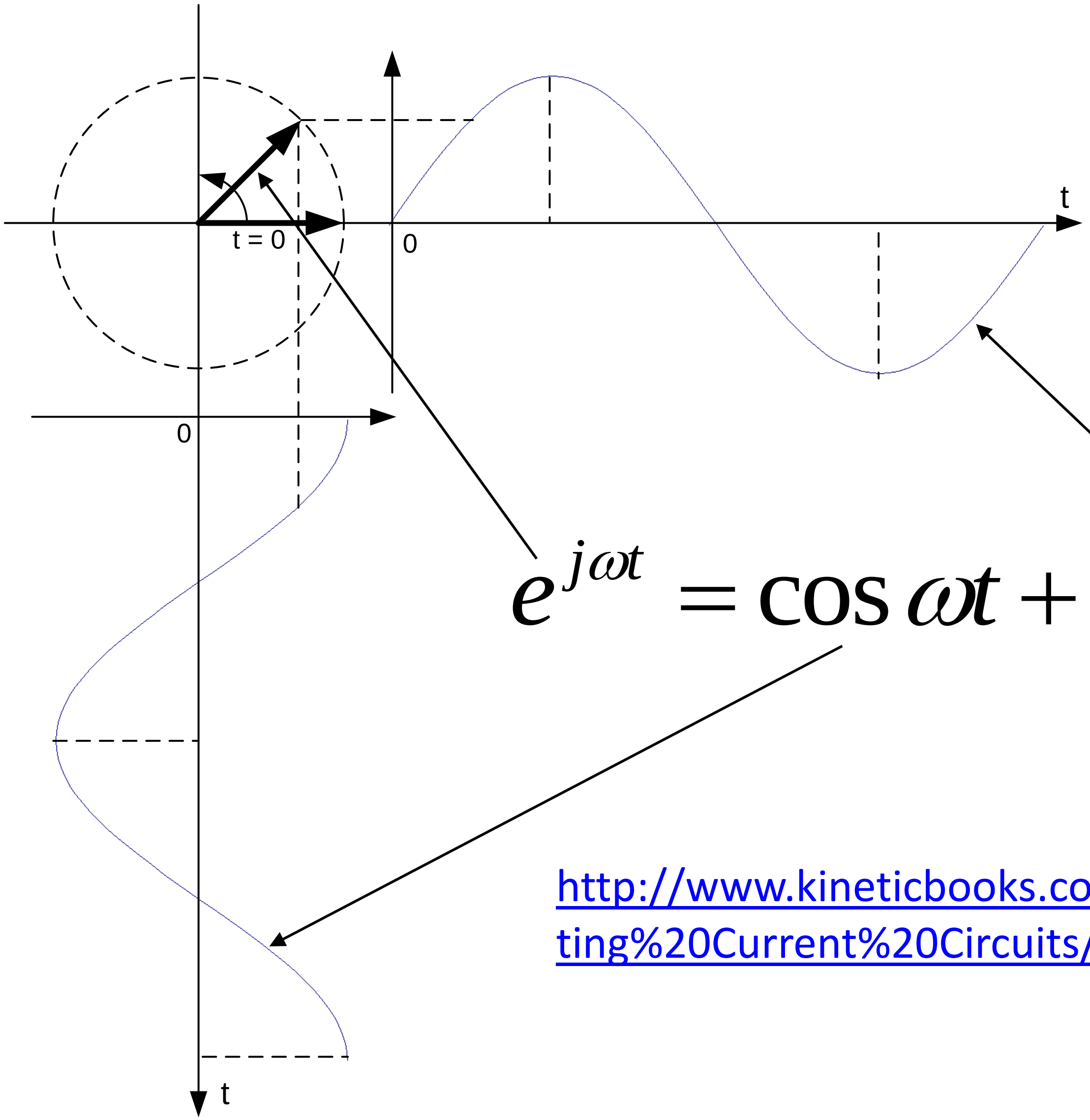
Euler's equation:

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\therefore \quad \mathbf{A} = Ae^{j\varphi}$$

Note: $$e^{j(\varphi+\theta)} = e^{j\varphi}e^{j\theta} = \cos(\varphi+\theta) + j\sin(\varphi+\theta)$$

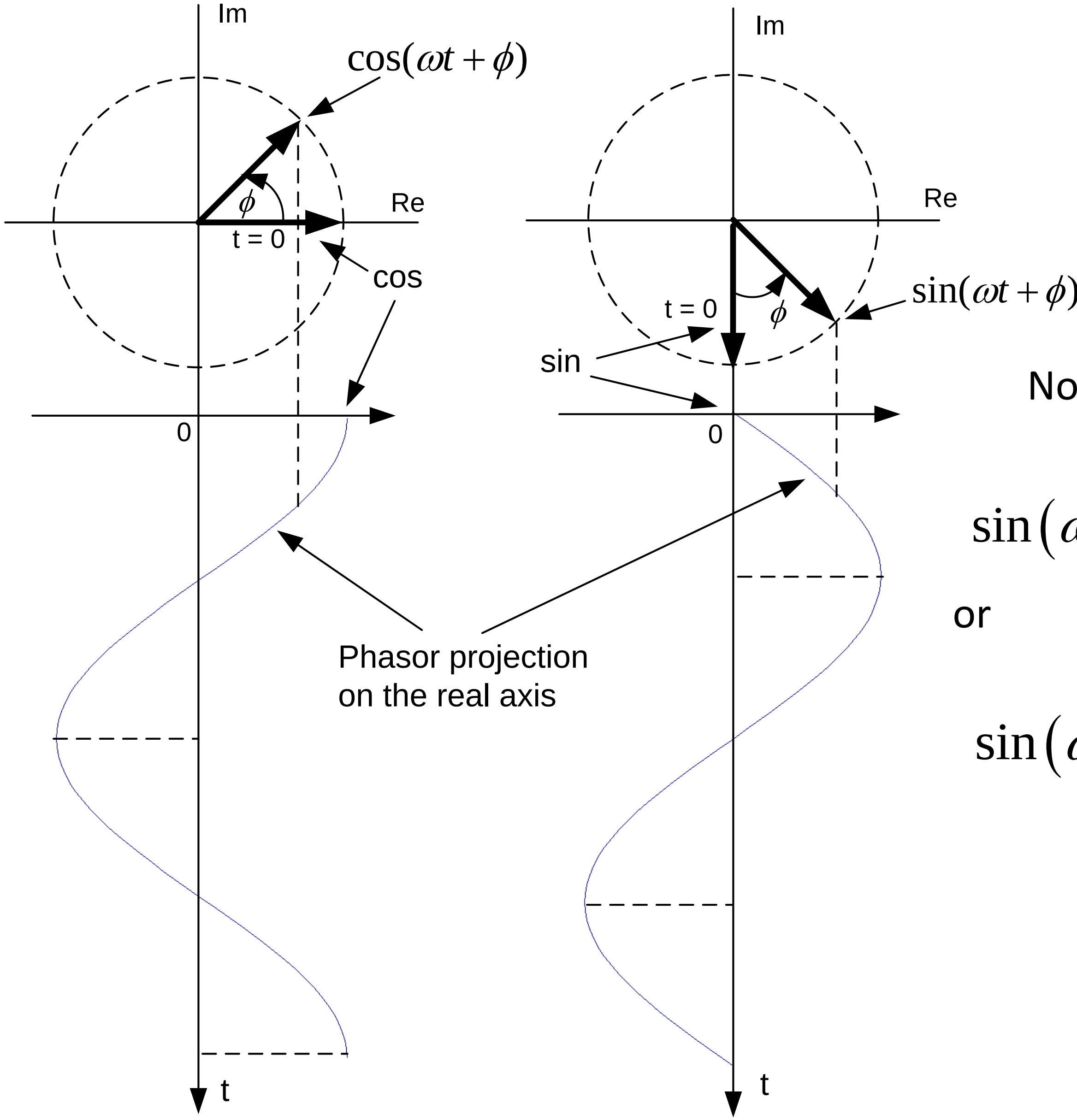
Sinusoidal Functions and Phasors



$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

http://www.kineticbooks.com/physics/trialpse/33_Alternating%20Current%20Circuits/13/sp.html

Relationship between *sin* and *cos*



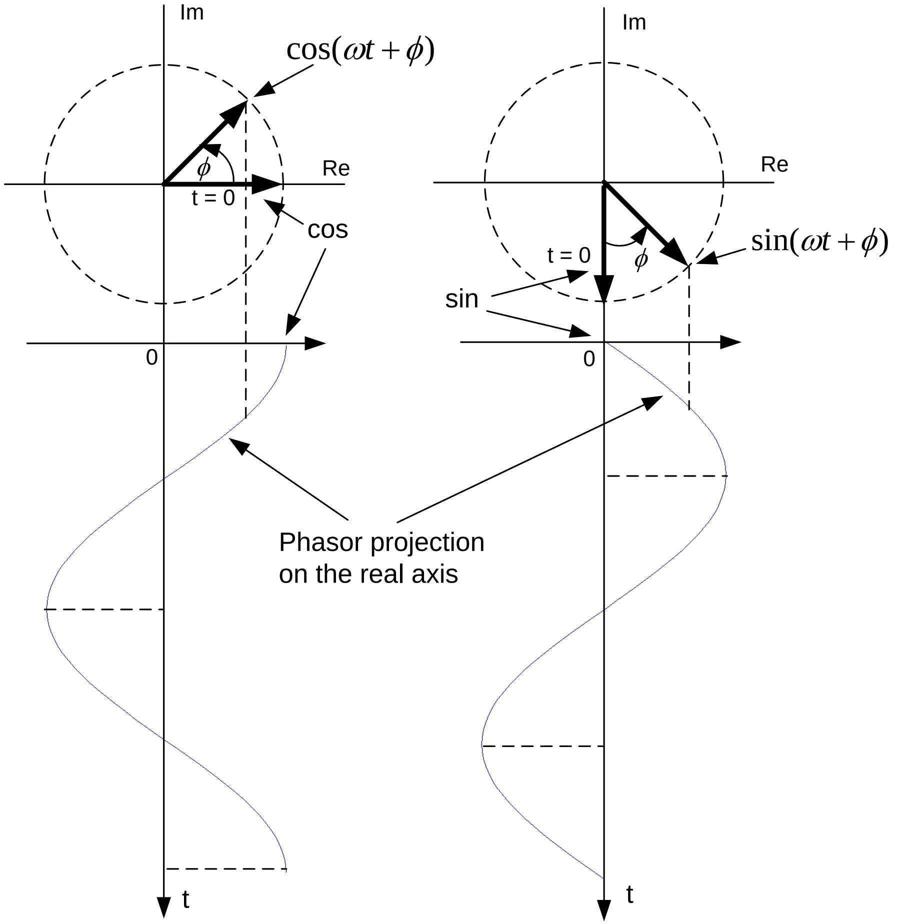
Note that

$$\sin(\omega t+\phi)=\cos\left(\omega t-\left[\frac{\pi}{2}-\phi\right]\right)$$

or

$$\sin(\omega t+\phi)=\cos\left(\omega t+\phi-\frac{\pi}{2}\right)$$

Relationship between *sin* and *cos*



$$\sin\left(\omega t \pm \frac{\pi}{2}\right) = \pm\cos\omega t$$

$$\sin(\omega t \pm \pi) = -\sin\omega t$$

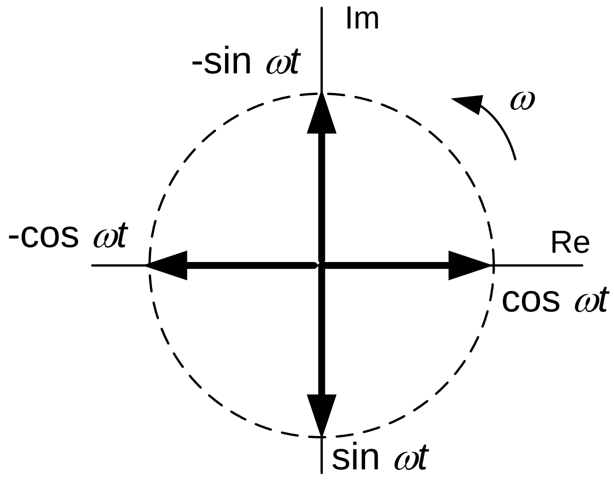
$$\cos\left(\omega t \pm \frac{\pi}{2}\right) = \mp\sin\omega t$$

$$\cos(\omega t \pm \pi) = -\cos\omega t$$

$$\sin(\omega t \pm \phi) = \sin\omega t\cos\phi \pm \cos\omega t\sin\phi$$

$$\cos(\omega t \pm \phi) = \cos\omega t\cos\phi \mp \sin\omega t\sin\phi$$

Comparing Sinusoids



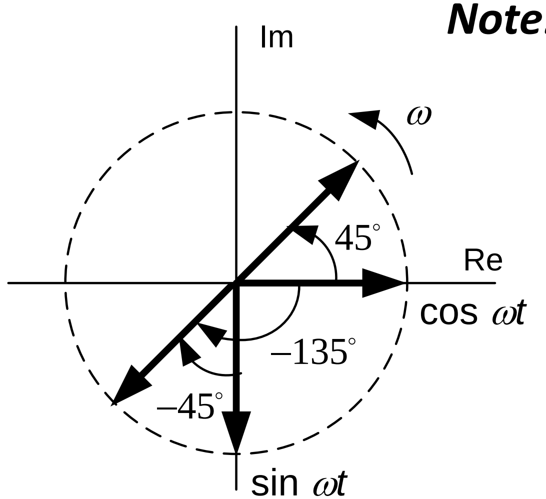
$$\cos(\omega t \pm \pi) = -\cos \omega t$$

$$\sin(\omega t \pm \pi) = -\sin \omega t$$

$$\sin\left(\omega t \pm \frac{\pi}{2}\right) = \pm\cos \omega t$$

$$\cos\left(\omega t \pm \frac{\pi}{2}\right) = \mp\sin \omega t$$

Note: positive angles are counter-clockwise



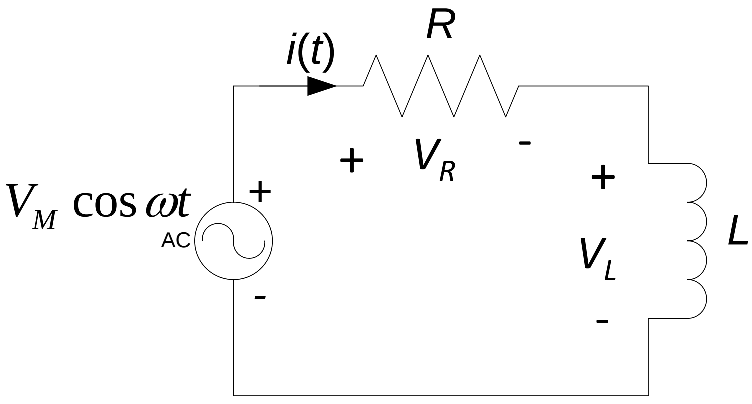
$\cos \omega t$ **leads** $\sin \omega t$ by 90°

$\cos \omega t$ **lags** $-\sin \omega t$ by 90°

$$\sin(\omega t - 45^\circ) = \cos(\omega t - 135^\circ)$$

$\cos(\omega t + 45^\circ)$ **leads** $\cos \omega t$ by 45° and **leads** $\sin \omega t$ by 135°

KVL



$$V_M \cos\omega t = Ri(t) + L\frac{di(t)}{dt}$$

This is a **differential equation** we must solve for *i*(*t*).

How? Guess a solution and try it!

Note that

$$\mathrm{Re}\left[V_M e^{j\omega t}\right] = \mathrm{Re}\left[V_M \cos\omega t + jV_M \sin\omega t\right] = V_M \cos\omega t$$

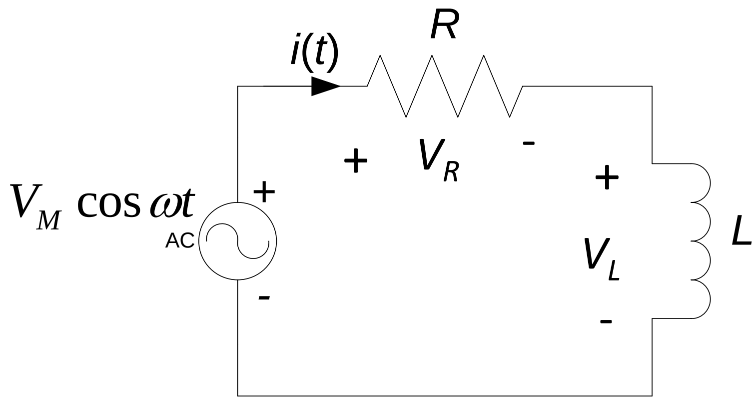
It turns out to be easier to use $V_M e^{j\omega t}$ as the forcing function rather than $V_M \cos\omega t$ and then take the real part of the solution.

This is because

$$\frac{de^{j\omega t}}{dt} = j\omega e^{j\omega t}$$

which will allow us to convert the differential equation to an algebraic equation. Let's see how.

Solve the **differential equation** for $i(t)$.



$$V_M \cos \omega t = Ri(t) + L\frac{di(t)}{dt}$$

Instead, solve

$$V_M e^{j\omega t} = Ri(t) + L\frac{di(t)}{dt} \qquad (1)$$

and take the real part of the solution

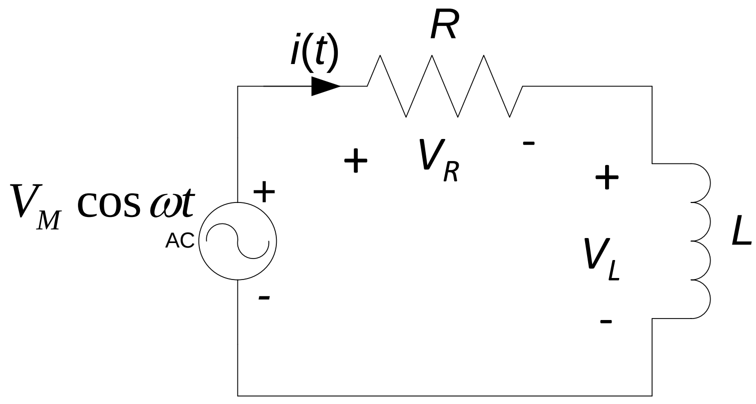
Guess that $i(t) = I_M e^{j(\omega t+\phi)}$ and substitute in (1)

$$V_M e^{j\omega t} = RI_M e^{j(\omega t+\phi)} + j\omega LI_M e^{j(\omega t+\phi)} = e^{j\omega t}\left[RI_M e^{j\phi} + j\omega LI_M e^{j\phi}\right]$$

Divide by $e^{j\omega t}$

$$V_M = RI_M e^{j\phi} + j\omega LI_M e^{j\phi}$$

Solve the **differential equation** for *i*(*t*).



$$V_M e^{j\omega t} = Ri(t) + L\frac{di(t)}{dt} \qquad (1)$$

$$i(t) = I_M e^{j(\omega t+\phi)} \qquad (2)$$

$$V_M = RI_M e^{j\phi} + j\omega L I_M e^{j\phi} \qquad (3)$$

Rearrange (3)

$$I_M e^{j\phi} = \frac{V_M}{R + j\omega L} \qquad (4)$$

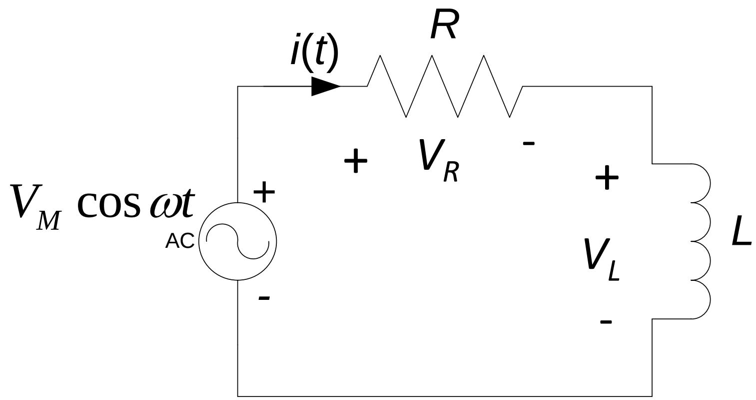
Recall that

$$R + j\omega L = \sqrt{R^2 + \omega^2 L^2}\, e^{j\tan^{-1}\left(\frac{\omega L}{R}\right)}$$

Therefore (4) can be written as

$$I_M e^{j\phi} = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} e^{j\left[-\tan^{-1}\left(\frac{\omega L}{R}\right)\right]} \qquad (5)$$

Solve the **differential equation** for $i(t)$.



$$V_M e^{j\omega t} = Ri(t) + L\frac{di(t)}{dt} \qquad (1)$$

$$i(t) = I_M e^{j(\omega t+\phi)} \qquad (2)$$

$$I_M e^{j\phi} = \frac{V_M}{\sqrt{R^2+\omega^2 L^2}} e^{j\left[-\tan^{-1}\left(\frac{\omega L}{R}\right)\right]} \qquad (5)$$

Therefore, from (5) $$I_M = \frac{V_M}{\sqrt{R^2+\omega^2 L^2}} \qquad \phi = -\tan^{-1}\left(\frac{\omega L}{R}\right) \qquad (6)$$

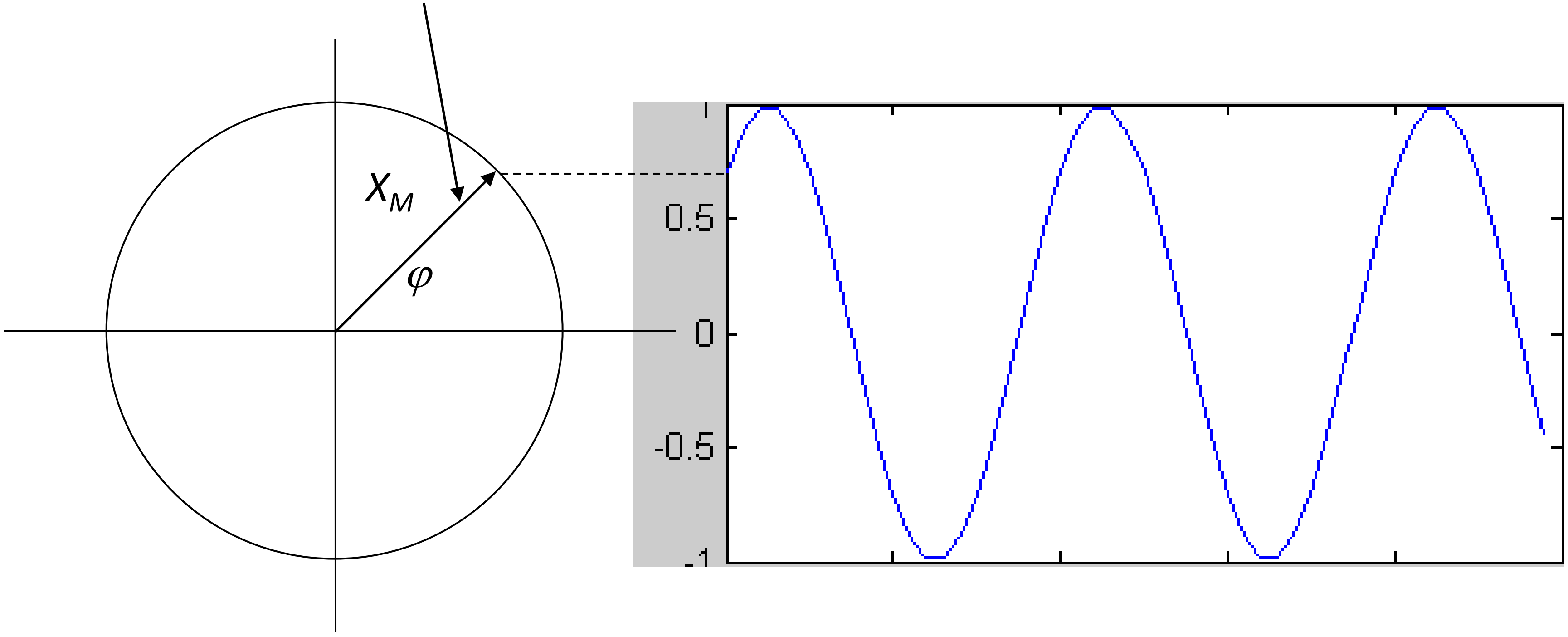
Substituting (6) in (2) and taking the real part

$$i(t) = \frac{V_M}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

Phasors

A phasor is a complex number that represents <u>the amplitude and phase</u> of a sinusoid.

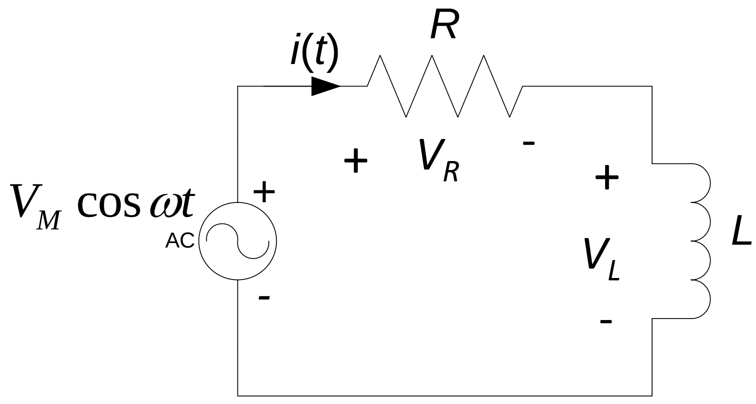
$$X_M e^{j\varphi} = X_M \angle \varphi = \mathbf{X}$$



Recall that when we substituted $i(t) = I_M e^{j(\omega t + \phi)}$ in the differential equations, the $e^{j\omega t}$ cancelled out.

We are therefore left with just the **<u>phasors</u>**

Solve the **differential equation** for $i(t)$ using phasors.



$$V_M e^{j\omega t} = Ri(t) + L\frac{di(t)}{dt} \qquad (1)$$

$$i(t) = I_M e^{j(\omega t+\phi)} = \mathbf{I}e^{j\omega t} \qquad (2)$$

$$\mathbf{V} = V_M \angle 0^\circ \qquad (3)$$

Substitute (2) and (3) in (1)

$$\mathbf{V}e^{j\omega t} = R\mathbf{I}e^{j\omega t} + j\omega L\mathbf{I}e^{j\omega t} \qquad (4)$$

Divide by $e^{j\omega t}$ and solve for **I**

$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L} = I_M \angle \phi = \frac{V_M}{\sqrt{R^2+\omega^2 L^2}} \angle \left[-\tan^{-1}\frac{\omega L}{R} \right]$$

$$i(t) = \frac{V_M}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R} \right) \qquad (5)$$

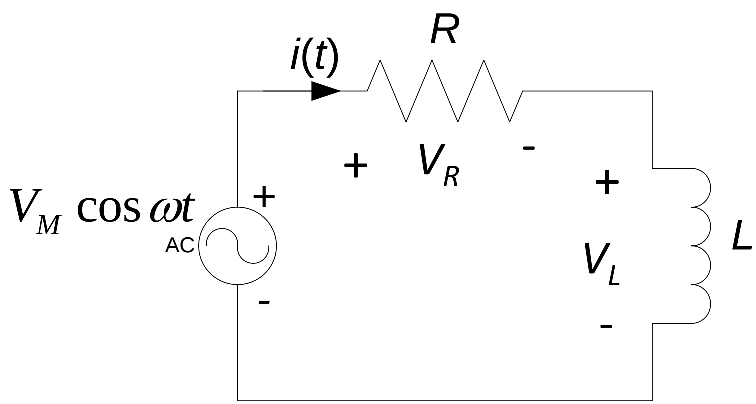
By using **phasors** we have transformed the problem from solving a set of differential equations in the **time domain** to solving a set of algebraic equations in the **frequency domain.**
The phasor solutions are then transformed back to the time domain.

$$A\cos\left(\omega t \pm \theta\right) \longleftrightarrow A\angle \pm \theta$$

$$A\sin\left(\omega t \pm \theta\right) \longleftrightarrow A\angle\left(\pm\theta - 90^\circ\right)$$

Impedance

Impedance $$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\text{phasor voltage}}{\text{phasor current}}$$ Units = ohms



$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = R + j\omega L = Z\angle\theta_z$$

$$\mathbf{Z} = R + jX$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\theta_z = \tan^{-1}\frac{\omega L}{R}$$

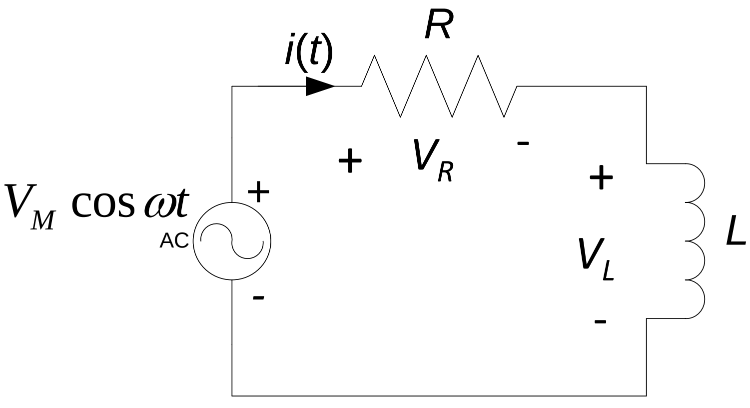
$R =$ resistance

$X =$ reactance

Note that impedance is a **complex number** containing a real, or **resistive** component, and an imaginary, or **reactive**, component.

Admittance

Admittance $$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{\text{phasor current}}{\text{phasor voltage}}$$ Units = siemens



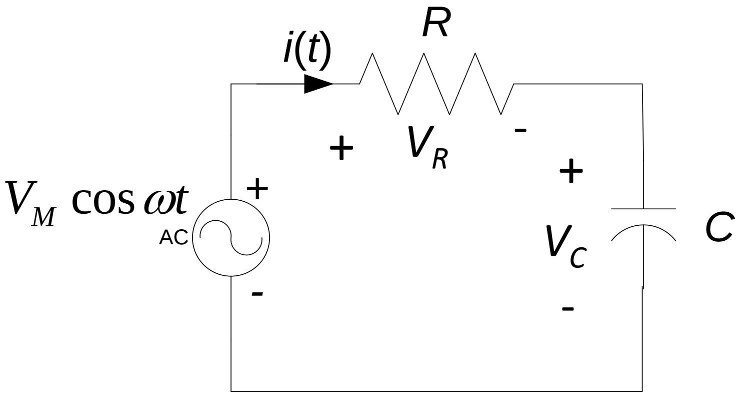
$$\mathbf{I} = \frac{\mathbf{V}}{R + j\omega L}$$

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R + j\omega L} = G + jB - \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

conductance $$G = \frac{R}{R^2 + \omega^2 L^2}$$

susceptance $$B = \frac{-\omega L}{R^2 + \omega^2 L^2}$$

Capacitor Circuit



$$V_M e^{j\omega t} = Ri(t) + \frac{1}{C}\int i(t)dt \qquad (1)$$

$$i(t) = I_M e^{j(\omega t + \phi)} = \mathbf{I}e^{j\omega t} \qquad (2)$$

$$\mathbf{V} = V_M \angle 0^\circ \qquad (3)$$

Substitute (2) and (3) in (1)

$$\mathbf{V}e^{j\omega t} = R\mathbf{I}e^{j\omega t} + \frac{1}{j\omega C}\mathbf{I}e^{j\omega t} \qquad (4)$$

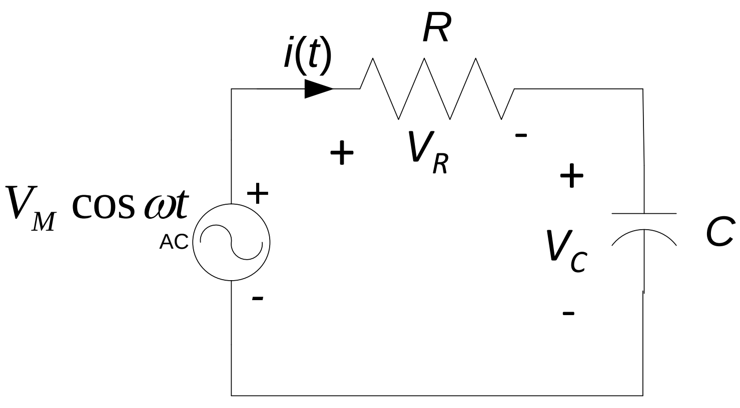
Divide by $e^{j\omega t}$ and solve for I

$$\mathbf{I} = \frac{\mathbf{V}}{R + \frac{1}{j\omega C}} = I_M \angle \phi = \frac{V_M}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \angle \left[\tan^{-1} \frac{1}{\omega RC} \right]$$

$$i(t) = \frac{V_M}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos\left(\omega t + \tan^{-1} \frac{1}{\omega RC} \right) \qquad (5)$$

Impedance

Impedance $$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\text{phasor voltage}}{\text{phasor current}}$$



$$\mathbf{I} = \frac{\mathbf{V}}{R + \dfrac{1}{j\omega C}}$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C} = Z\angle\theta_z$$

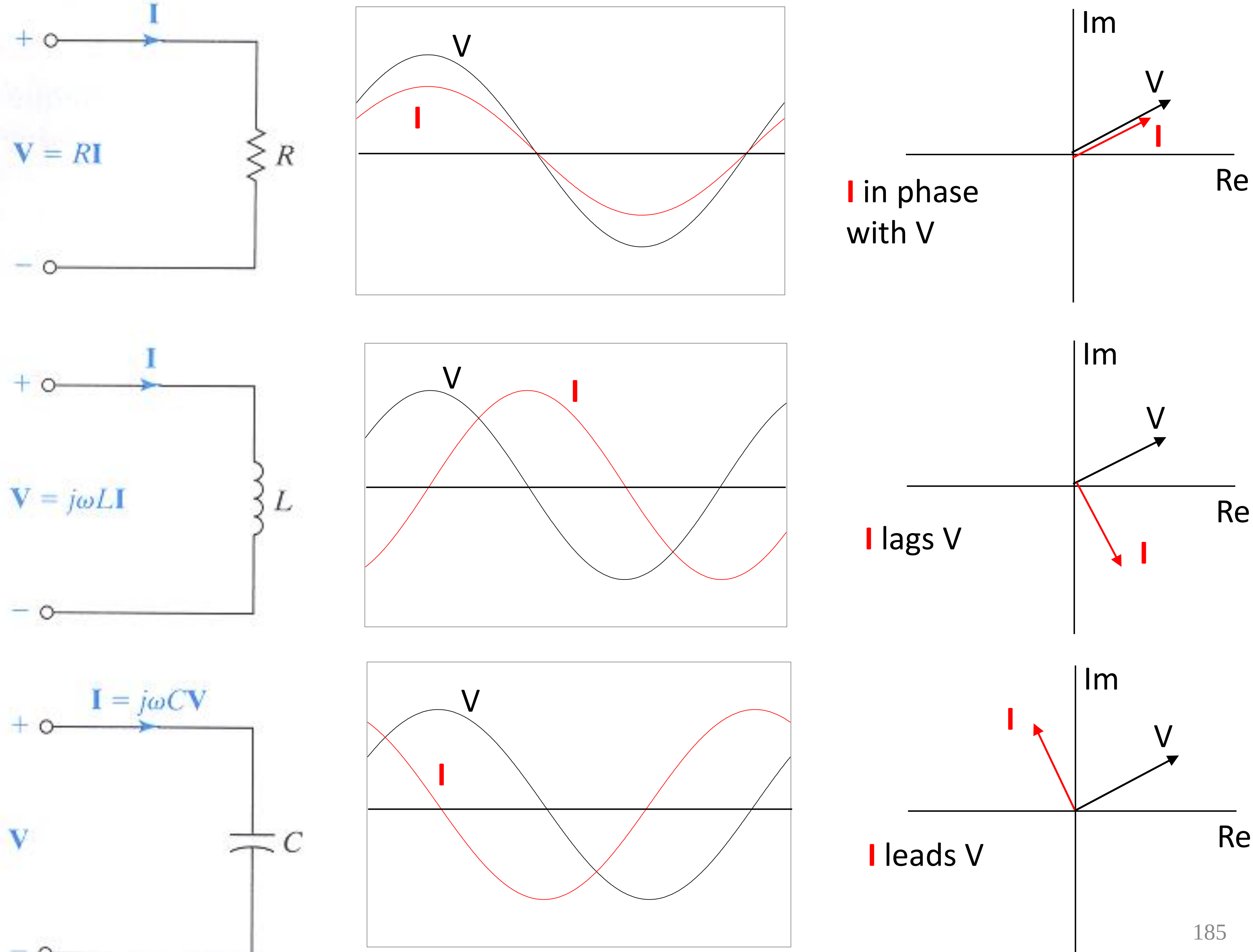
$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\mathbf{Z} = R + jX$$

$R =$ resistance

$$\theta_z = -\tan^{-1}\frac{1}{\omega RC}$$

$X =$ reactance



I
V = RI
R
V
I
Im
V
I
Re
I in phase with V
I
V = jωLI
L
V
I
Im
V
Re
I
I lags V
I = jωCV
V
C
V
I
Im
I
V
Re
I leads V

Expressing Kirchoff's Laws in the Frequency Domain

KVL: Let v_1, v_2,…v_n be the voltages around a closed loop.
KVL tells us that:

$$v_1 + v_2 + \ldots v_n = 0$$

Assuming the circuit is operating in sinusoidal steady-state at frequency ω we have:

$$V_{M1}\cos\left(\omega t+\theta_1\right)+V_{M2}\cos\left(\omega t+\theta_2\right)+\ldots+V_{Mn}\cos\left(\omega t+\theta_n\right)=0$$

or

$$\mathrm{Re}\left[V_{M1}e^{j\theta_1}e^{j\omega t}\right]+\mathrm{Re}\left[V_{M2}e^{j\theta_2}e^{j\omega t}\right]+\ldots+\mathrm{Re}\left[V_{Mn}e^{j\theta_n}e^{j\omega t}\right]=0$$

Phasor $\mathbf{V}_k = V_{Mk}e^{j\theta_k}$

$$\mathrm{Re}\left[\left(\mathbf{V}_1+\mathbf{V}_2+\ldots+\mathbf{V}_n\right)e^{j\omega t}\right]=0$$

Since $e^{j\omega t} \neq 0$

$$\mathbf{V}_1+\mathbf{V}_2+\ldots+\mathbf{V}_n=0$$

Which demonstrates that KVL holds for phasor voltages.

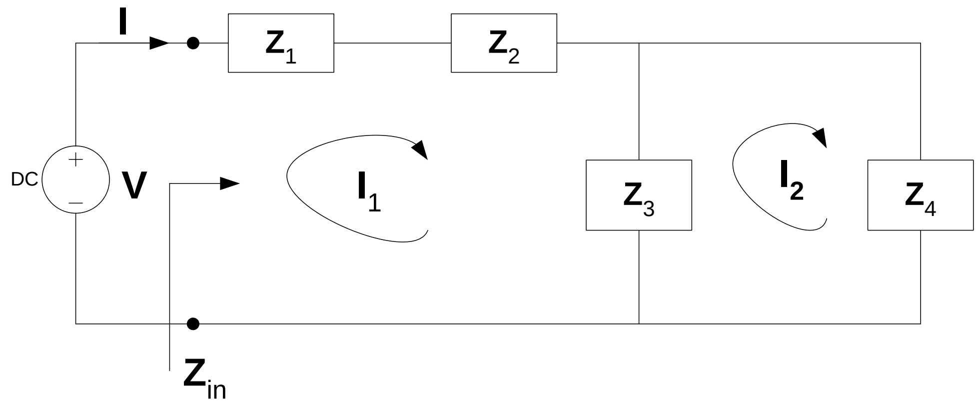
KCL: Following the same approach as for KVL, we can show that

$$\mathbf{I}_1 + \mathbf{I}_2 + \ldots + \mathbf{I}_n = 0$$

Where $\mathbf{I}_k$ is the phasor associated with the k^{th} current entering a closed surface in the circuit.

Thus**, both KVL and KCL hold when working with phasors in circuits operating in sinusoidal steady-state. This implies that all of the circuit analysis methods (mesh and nodal analysis, source transformations, voltage & current division, Thevenin equivalent, combining elements, etc,) work in the same way we found for resistive circuits. The only difference is that we must work with phasor currents & voltages and the impedances &/or admittances of the elements.**

Find $\mathbf{Z}_{in}$



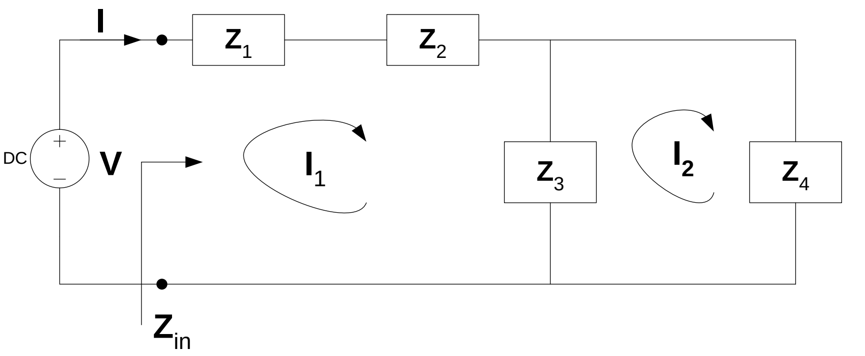
$$\begin{bmatrix} \mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3 & -\mathbf{Z}_3 \\ -\mathbf{Z}_3 & \mathbf{Z}_3+\mathbf{Z}_4 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}=\begin{bmatrix} \mathbf{V} \\ 0 \end{bmatrix}$$

$$\left(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3\right)\mathbf{I}_1-\mathbf{Z}_3\mathbf{I}_2=\mathbf{V}$$

$$-\mathbf{Z}_3\mathbf{I}_1+\left(\mathbf{Z}_3+\mathbf{Z}_4\right)\mathbf{I}_2=0$$

$$\left(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3\right)\mathbf{I}_1-\mathbf{Z}_3\mathbf{I}_2=\mathbf{V}$$

$$-\mathbf{Z}_3\mathbf{I}_1+\left(\mathbf{Z}_3+\mathbf{Z}_4\right)\mathbf{I}_2=0$$



$$\mathbf{I}_2=\frac{\mathbf{Z}_3}{\mathbf{Z}_3+\mathbf{Z}_4}\mathbf{I}_1$$

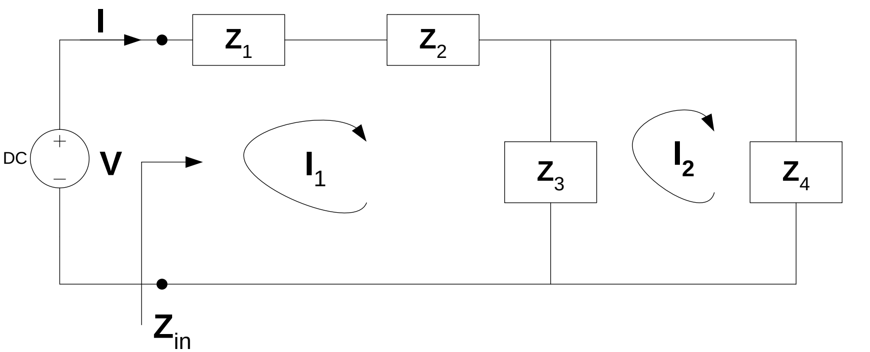
$$\mathbf{I}=\mathbf{I}_1$$

$$\left(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3\right)\mathbf{I}_1-\frac{\mathbf{Z}_3^2}{\mathbf{Z}_3+\mathbf{Z}_4}\mathbf{I}_1=\mathbf{V}$$

$$\frac{\left(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3\right)\left(\mathbf{Z}_3+\mathbf{Z}_4\right)-\mathbf{Z}_3^2}{\mathbf{Z}_3+\mathbf{Z}_4}\mathbf{I}_1=\mathbf{V}$$

$$\mathbf{Z}_{in}=\frac{\mathbf{V}}{\mathbf{I}}=\mathbf{Z}_1+\mathbf{Z}_2+\frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3+\mathbf{Z}_4}$$

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 + \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}$$

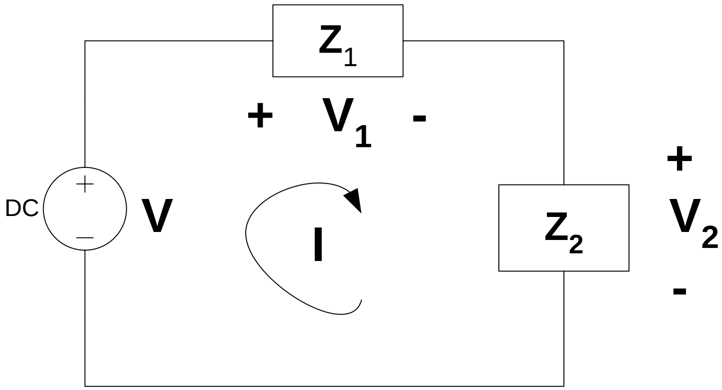


We see that if we replace **Z** by *R* the impedances add like resistances.

Impedances in series add like resistors in series

Impedances in parallel add like resistors in parallel

Voltage Division



$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

But

$$\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I}$$

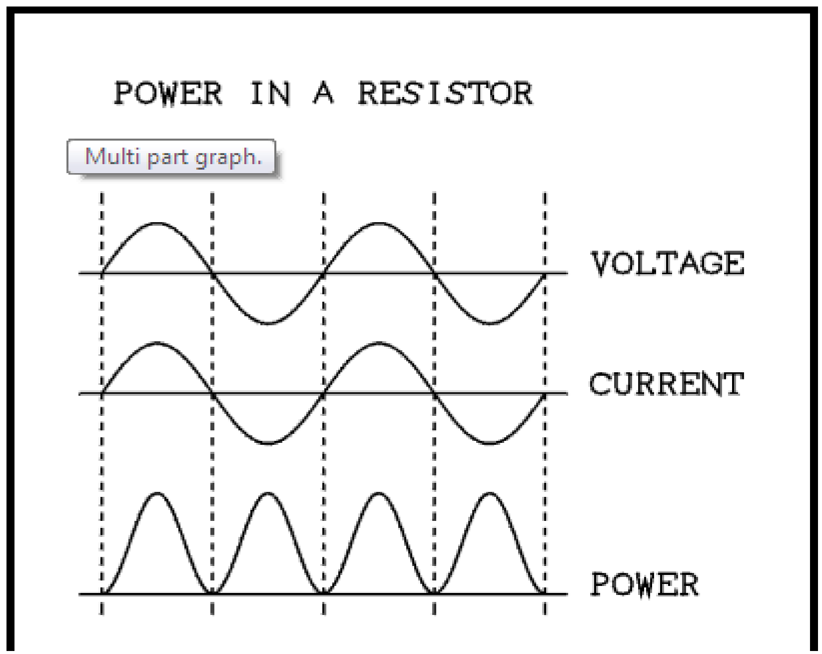
$$\mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}$$

Therefore

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V} \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$$

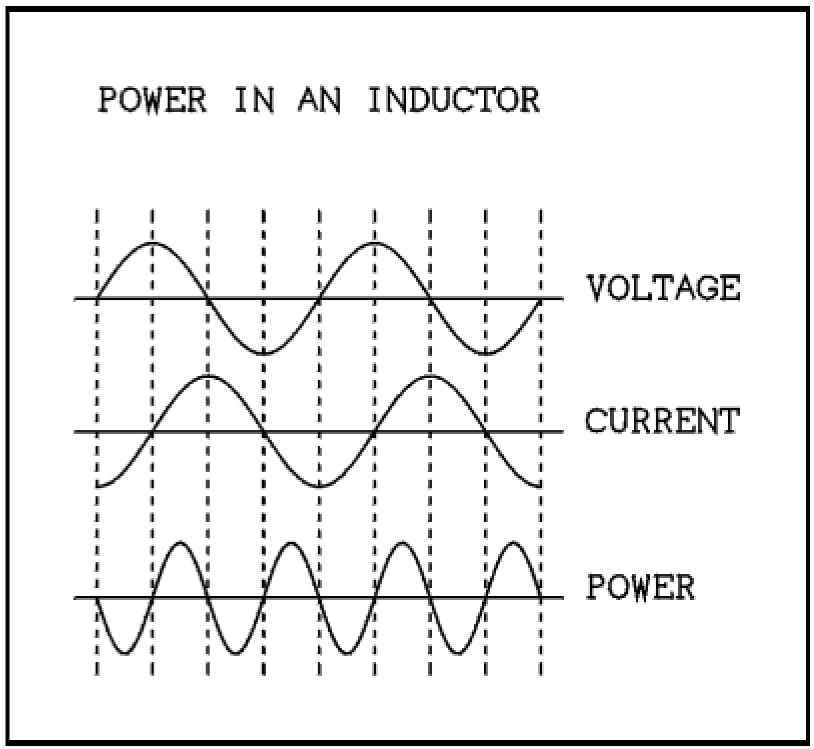
Resistive circuits

- In a pure resistor the current and voltage are in phase



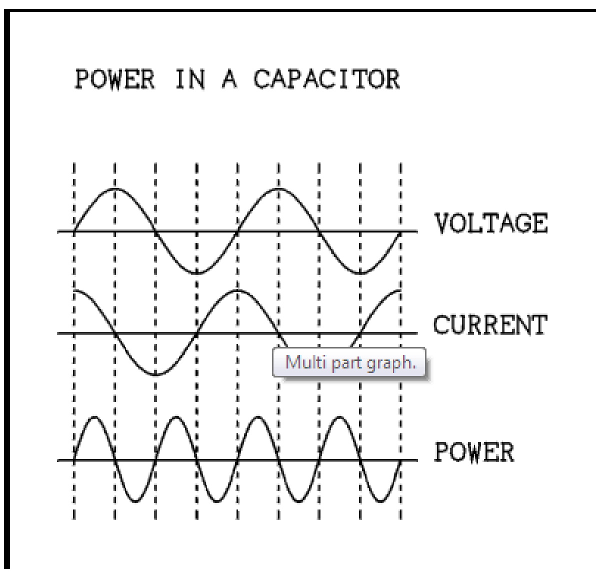
Inductive circuits

- In a pure inductor the current lags the voltage by 90 degrees. The same situation as a capacitor applies here. The power is zero as shown by the graph below



Capacitive circuits

- In a pure inductor the current leads the voltage by 90 degrees.



UNIT-III
TRANSFORMERS

Transformers

Introduction

- A transformer is a device that changes ac electric power at one voltage level to ac electric power at another voltage level through the action of a magnetic field.
- There are two or more stationary electric circuits that are coupled magnetically.
- It involves interchange of electric energy between two or more electric systems
- Transformers provide much needed capability of changing the voltage and current levels easily.
  - They are used to step-up generator voltage to an appropriate voltage level for power transfer.
  - Stepping down the transmission voltage at various levels for distribution and power utilization.

Transformer Classification

- In terms of number of windings
  - Conventional transformer: two windings
  - Autotransformer: one winding
  - Others: more than two windings

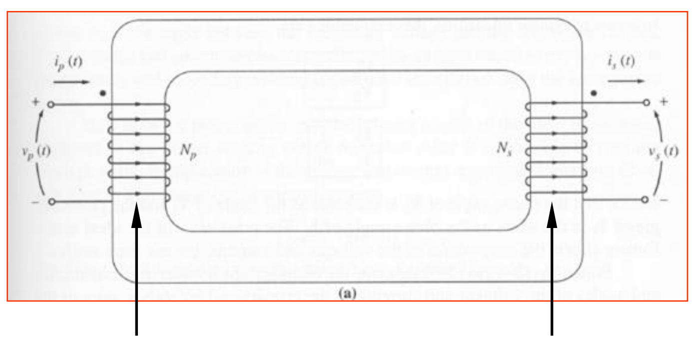
- In terms of number of phases
  - Single-phase transformer
  - Three-phase transformer

- Depending on the voltage level at which the winding is operated
  - Step-up transformer: primary winding is a low voltage (LV) winding
  - Step-down transformer : primary winding is a high voltage (HV) winding

Primary and Secondary Windings

A two-winding transformer consists of two windings interlinked by a mutual magnetic field.

- Primary winding – energized by connecting it to an input source
- Secondary winding – winding to which an electrical load is connected and from which output energy is drawn.



Primary winding

Secondary winding

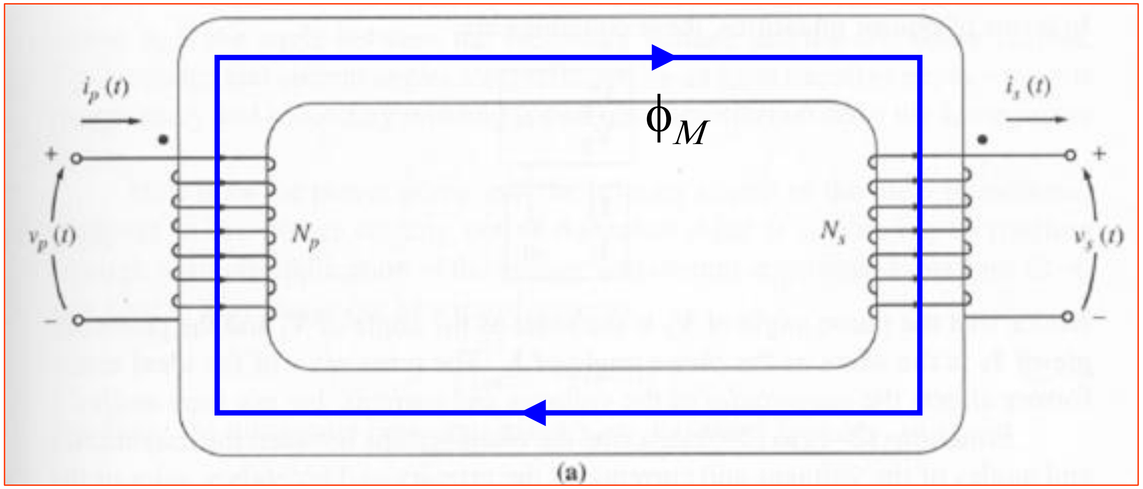
Ideal Transformers

An ideal transformer is a lossless device with an input winding and an output winding. It has the following properties:

- No iron and copper losses
- No leakage fluxes
- A core of infinite magnetic permeability and of infinite electrical resistivity
- Flux is confined to the core and winding resistances are negligible

Ideal Transformers

An ideal transformer is a lossless device with an input winding and an output winding.



The relationships between the input voltage and the output voltage, and between the input current and the output current, are given by the following equations.

In instantaneous quantities

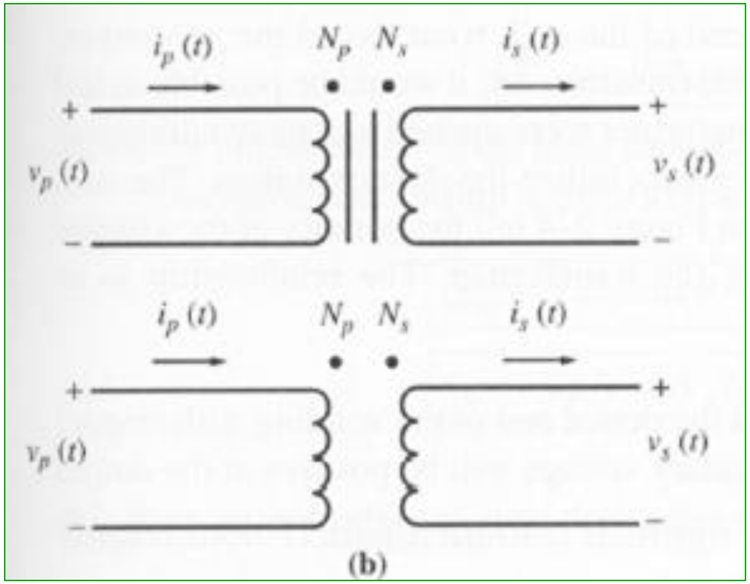
$$\frac{v_p(t)}{v_s(t)} = \frac{i_s(t)}{i_p(t)} = a$$

Ideal Transformers

$$\frac{v_p(t)}{v_s(t)} = \frac{i_s(t)}{i_p(t)} = \frac{N_p}{N_s} = a$$

In rms quantities

$$\frac{V_p}{V_s} = \frac{I_s}{I_p} = a$$



N_p: Number of turns on the primary winding
N_s: Number of turns on the secondary winding
$v_p(t)$: voltage applied to the primary side
$v_s(t)$: voltage at the secondary side
a: turns ratio
$i_p(t)$: current flowing into the primary side
$i_s(t)$: current flowing into the secondary side

Derivation of the Relationship

$$v_p(t) = \frac{d\lambda_p(t)}{dt} = N_p \frac{d\phi_M(t)}{dt} \qquad \dots\dots\dots\dots\dots\dots\dots \text{(1)}$$

$$v_s(t) = \frac{d\lambda_s(t)}{dt} = N_s \frac{d\phi_M(t)}{dt} \qquad \dots\dots\dots\dots\dots\dots\dots \text{(2)}$$

Dividing (1) by (2)

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(3)}$$

From Ampere's law

$$N_p i_p(t) = N_s i_s(t)$$

$$\frac{i_s(t)}{i_p(t)} = \frac{N_p}{N_s} = a \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(4)}$$

Equating (3) and (4)

$$\frac{v_p(t)}{v_s(t)} = \frac{i_s(t)}{i_p(t)} = \frac{N_p}{N_s} = a \qquad \dots\dots\dots\dots\dots\dots\dots\dots \text{(5)}$$

Power in an Ideal Transformer

Real power P supplied to the transformer by the primary circuit

$$P_{in} = V_p I_p \cos\theta_p$$

$$\theta_p = \theta_s = \theta$$

Real power coming out of the secondary circuit

$$P_{out} = V_s I_s \cos\theta_s = \left(\frac{V_p}{a}\right)(a I_p)\cos\theta = V_p I_p \cos\theta = P_{in}$$

Thus, *the output power of an ideal transformer is equal to its input power*.

The same relationship applies to reactive Q and apparent power S:

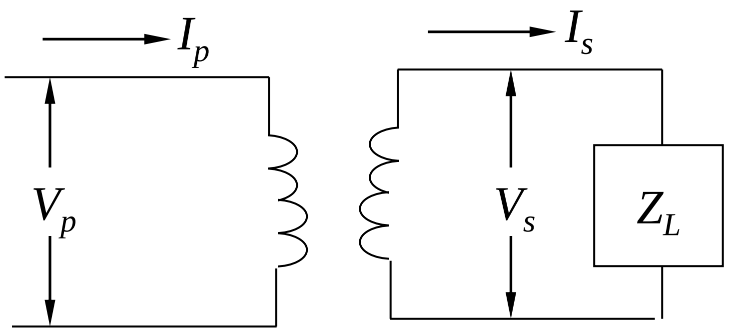
$$Q_{in} = V_p I_p \sin\theta = (a V_s)\left(\frac{I_s}{a}\right)\cos\theta = V_s I_s \sin\theta = Q_{out}$$

$$S_{in} = V_p I_p = V_s I_s = S_{out}$$

Impedance Transformation through a Transformer

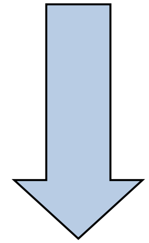
Impedance of the load:

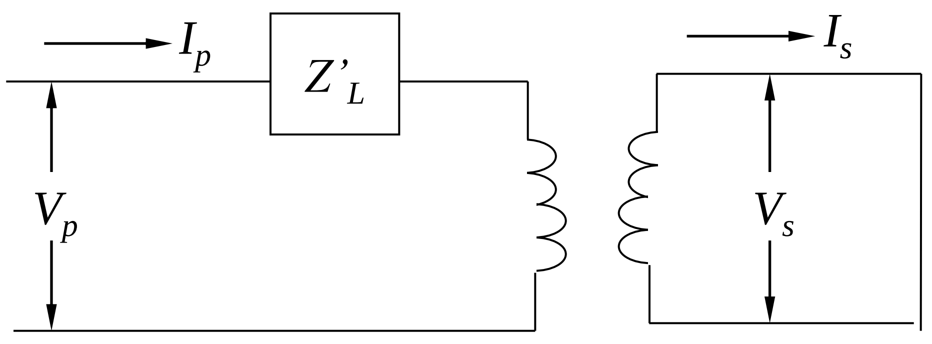
$$Z_L = V_s/I_s$$



The impedance of the primary circuit:

$$\begin{aligned} Z'_L &= V_p/I_p \\ &= (aV_s)/(I_s/a) \\ &= a^2 (V_s / I_s) \\ &= a^2 Z_L \end{aligned}$$





Example 1

A 100-kVA, 2400/240-V, 60-Hz step-down transformer (ideal) is used between a transmission line and a distribution system.

a) Determine turns ratio.
b) What secondary load impedance will cause the transformer to be fully loaded, and what is the corresponding primary current?
c) Find the load impedance referred to the primary.

Solution to Example 1

a) Turns ratio, $a = 2400 / 240 = 10$

b) $I_s = 100{,}000/240 = 416.67$ A

$I_p = I_s / a = 416.67 / 10 = 41.67$ A

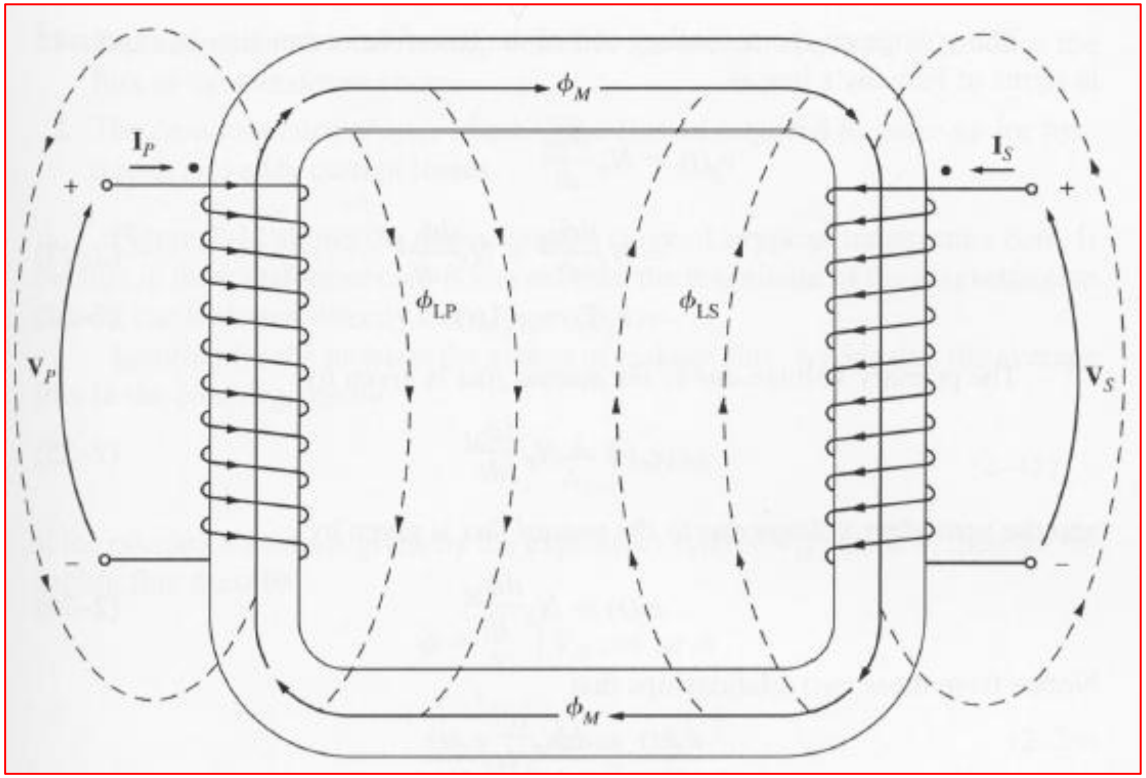
Magnitude of the load impedance

$= V_s / I_s = 240/416.7 = 0.576$ ohm

c) Load impedance referred to the primary

$= a^2 * 0.576 = 57.6$ ohm

Theory of Operation of Single-Phase Real Transformers

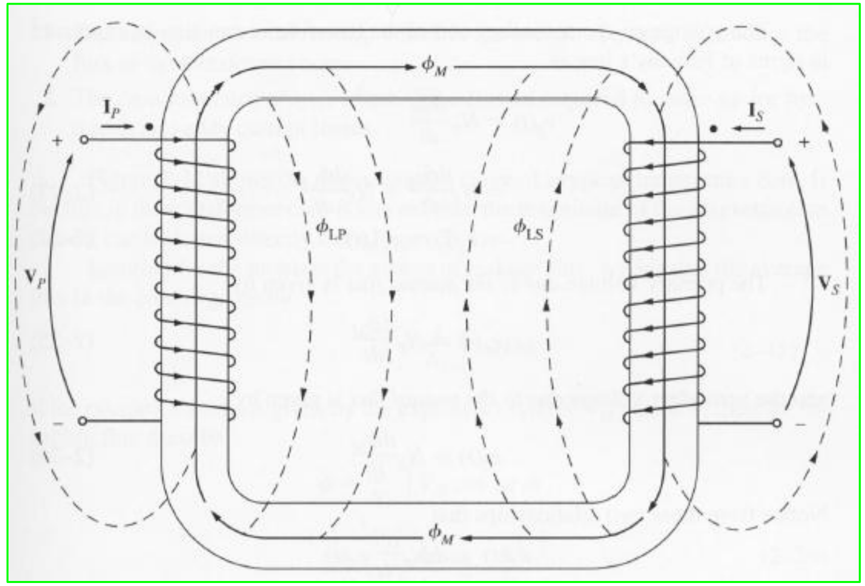


Leakage flux: flux that goes through one of the transformer windings but not the other one

Mutual flux: flux that remains in the core and links both windings

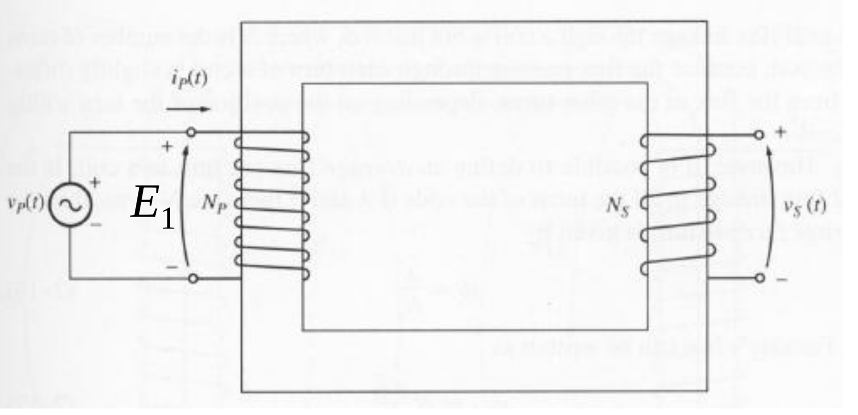
Theory of Operation of Single-Phase Real Transformers

$$\phi_P = \phi_M + \phi_{LP}$$
$$\phi_S = \phi_M + \phi_{LS}$$



ϕ_p: total average primary flux
ϕ_M : flux linking both primary and secondary windings
ϕ_{LP}: primary leakage flux
ϕ_S: total average secondary flux
ϕ_{LS}: secondary leakage flux

Magnetization Current



When an ac power source is connected to a transformer, a current flows in its primary circuit, even when the secondary circuit is open circuited. This current is the current required to produce flux in the ferromagnetic core and is called *excitation current*. It consists of two components:

1. The *magnetization current* I_m, which is the current required to produce the flux in the transformer core

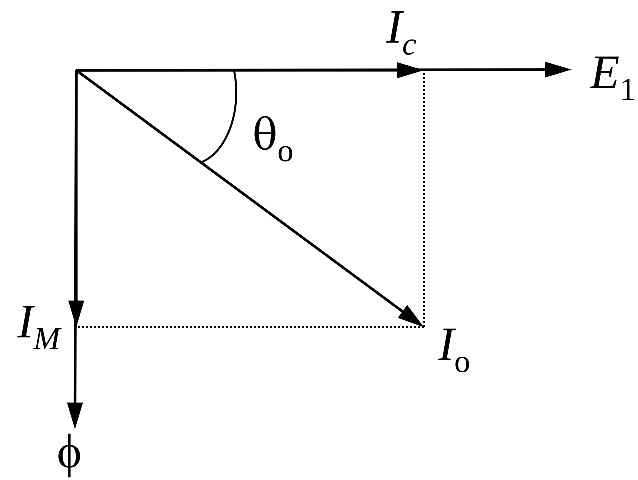
2. The *core-loss current* I_{h+e}, which is the current required to make up for hysteresis and eddy current losses

The Magnetization Current in a Real Transformer

When an ac power source is connected to the primary of a transformer, a current flows in its primary circuit, even when there is no current in the secondary. The transformer is said to be on no-load. If the secondary current is zero, the primary current should be zero too. However, when the transformer is on no-load, excitation current flows in the primary because of the core losses and the finite permeability of the core.

Excitation current, I_o

- Magnetization current I_M
  (current required to produce flux in the core)
- Core-loss current I_{h+e}
  (current required to make up for hysteresis and eddy current losses)



I_M is proportional to the flux ϕ

$I_c = I_{h+e} = \text{Core loss}/E_1$

The Equivalent Circuit of a Transformer

The losses that occur in transformers have to be accounted for in any accurate model of transformer behavior.

1. *Copper (I^2R) losses*. Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.

2. *Eddy current losses*. Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.

3. *Hysteresis losses*. Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle. They are a complex, nonlinear function of the voltage applied to the transformer.

4. *Leakage flux*. The fluxes which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a self-inductance in the primary and secondary coils, and the effects of this inductance must be accounted for.

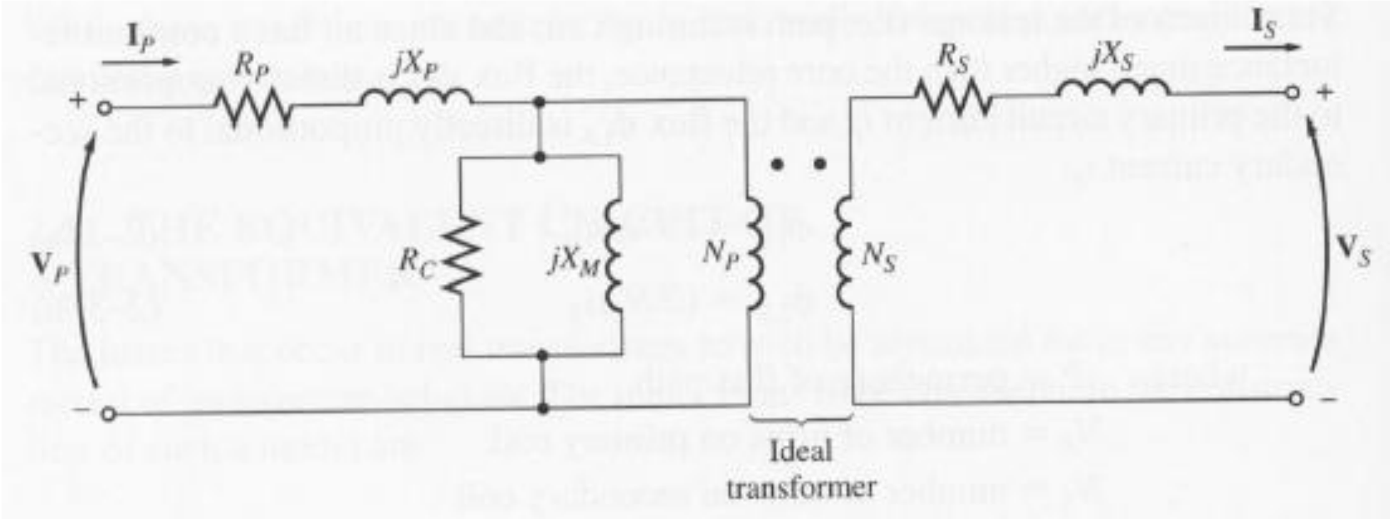
The Exact Equivalent Circuit of a Transformer

Modeling the copper losses: resistive losses in the primary and secondary windings of the core, represented in the equivalent circuit by R_P and R_S.

Modeling the leakage fluxes: primary leakage flux is proportional to the primary current I_P and secondary leakage flux is proportional to the secondary current I_S, represented in the equivalent circuit by X_P ($=\phi_{\mathrm{LP}}/I_P$) and X_S ($=\phi_{\mathrm{LS}}/I_S$).

Modeling the core excitation: I_m is proportional to the voltage applied to the core and lags the applied voltage by 90^o. It is modeled by X_M.

Modeling the core loss current: I_{h+e} is proportional to the voltage applied to the core and in phase with the applied voltage. It is modeled by R_C.



The Exact Equivalent Circuit of a Transformer

Although the previous equivalent circuit is an accurate model of a transformer, it is not a very useful one. To analyze practical circuits containing transformers, it is normally necessary to convert the entire circuit to an equivalent circuit at a single voltage level. Therefore, the equivalent circuit must be referred either to its primary side or to its secondary side in problem solutions.

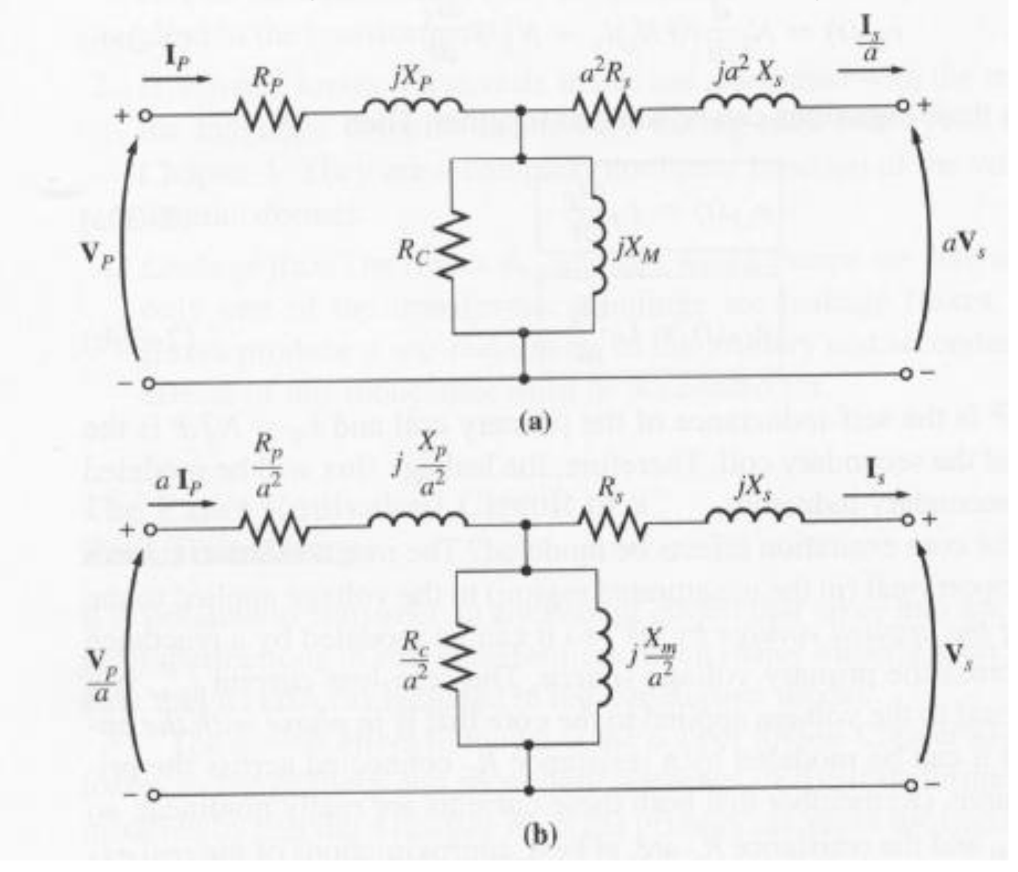


Figure (a) is the equivalent circuit of the transformer referred to its primary side.

Figure (b) is the equivalent circuit referred to its secondary side.

Approximate Equivalent Circuits of a Transformer

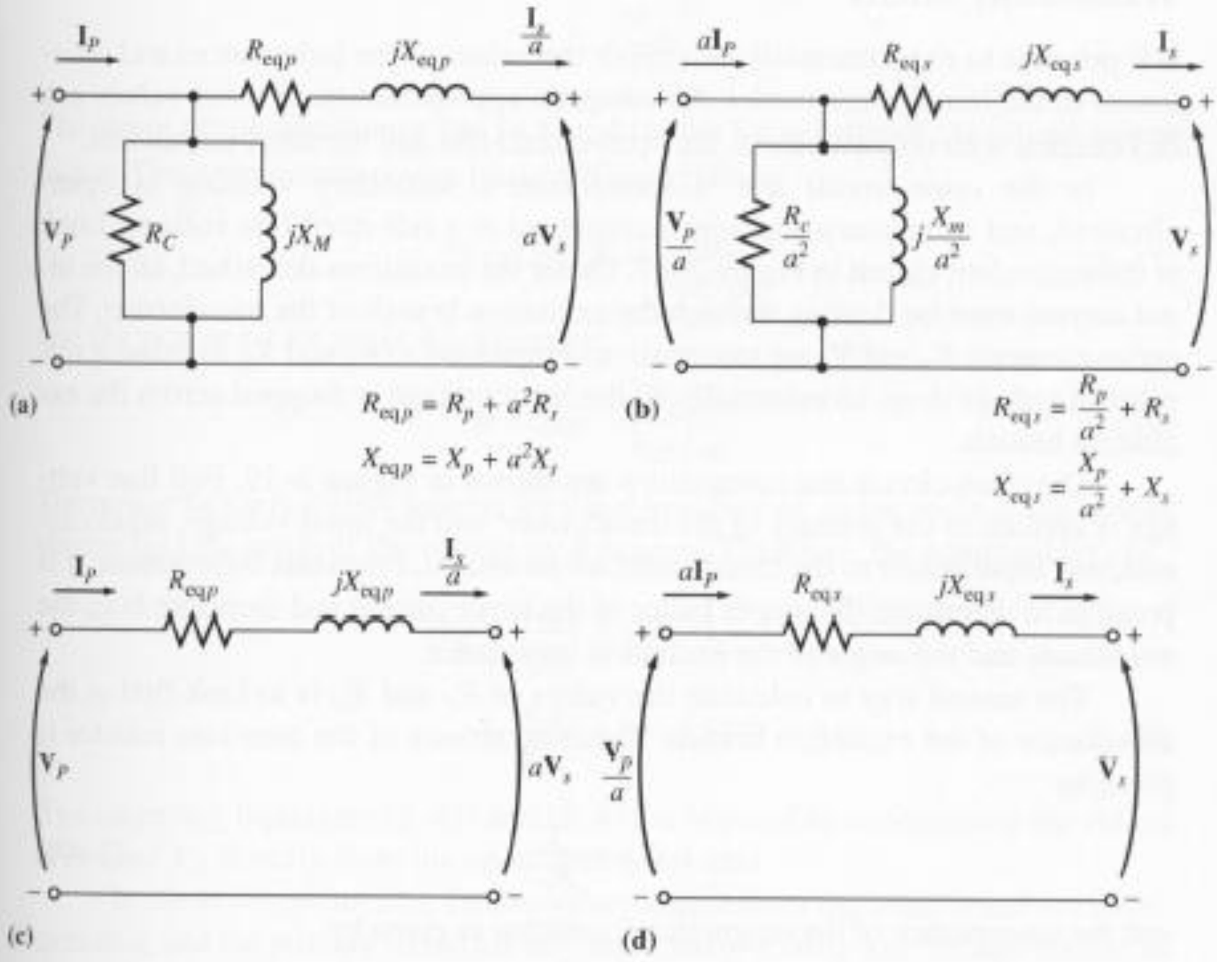


FIGURE 2–18

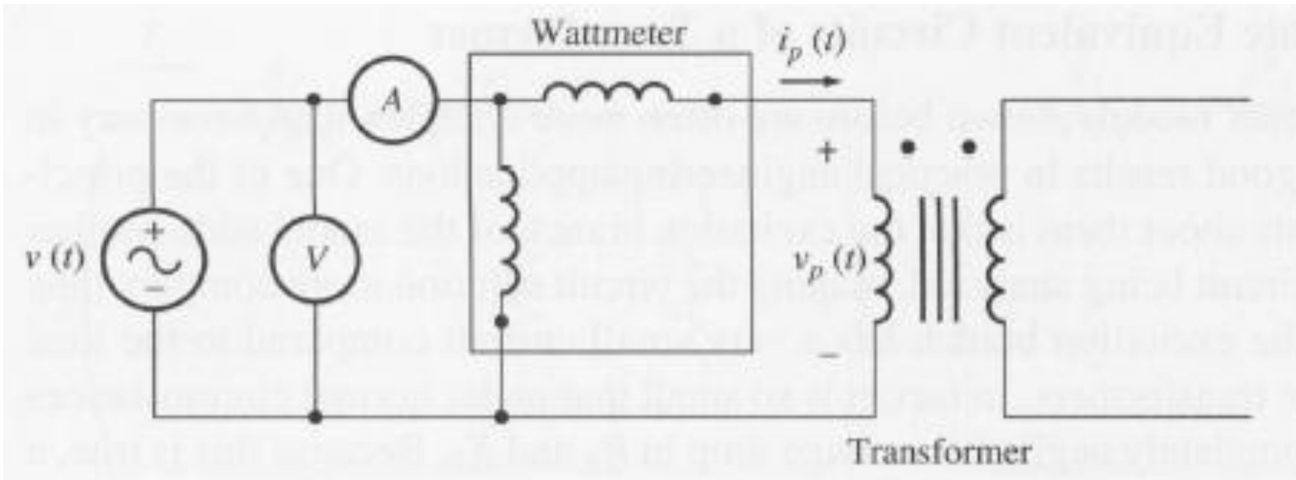
Approximate transformer models. (a) Referred to the primary side; (b) referred to the secondary side; (c) with no excitation branch, referred to the primary side; (d) with no excitation branch, referred to the secondary side.

Determining the Values of Components in the Transformer Model

It is possible to experimentally determine the parameters of the approximate the equivalent circuit. An adequate approximation of these values can be obtained with only two tests….

- *open-circuit test*
- *short-circuit test*

Circuit Parameters: Open-Circuit Test



- Transformer's secondary winding is open-circuited
- Primary winding is connected to a full-rated line voltage. All the input current must be flowing through the excitation branch of the transformer.
- The series elements R_p and X_p are too small in comparison to R_C and X_M to cause a significant voltage drop, so essentially all the input voltage is dropped across the excitation branch.
- Input voltage, input current, and input power to the transformer are measured.

Circuit Parameters: Open-Circuit Test

The magnitude of the excitation admittance:

$$|Y_E| = \frac{I_{oc}}{V_{oc}}$$

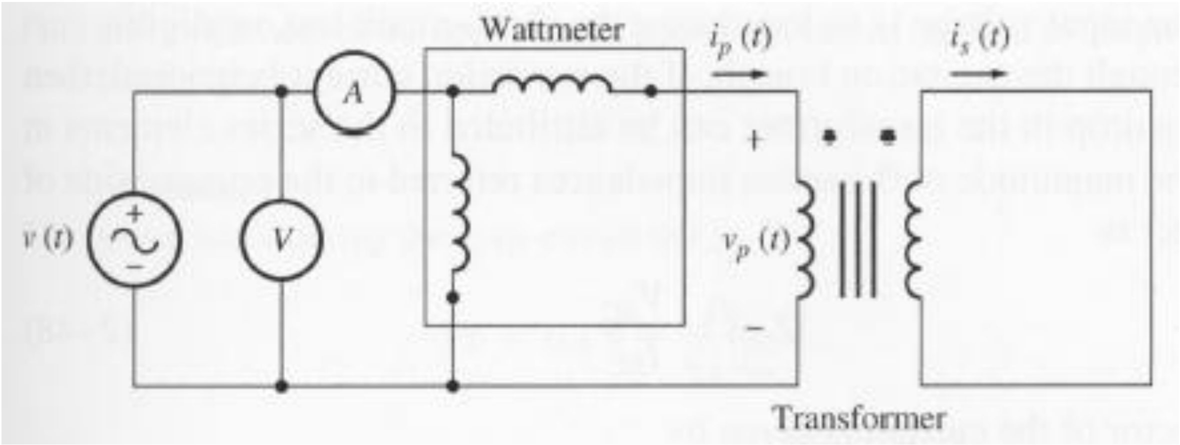
The open-circuit power factor and power factor angle:

$$PF = \cos\theta = \frac{P_{oc}}{V_{oc}\, I_{oc}} \quad or,\ \theta = \cos^{-1}\left[\frac{P_{oc}}{V_{oc}\, I_{oc}}\right]$$

The power factor is always lagging for a transformer, so the current will lag the voltage by the angle θ. Therefore, the admittance Y_E is:

$$Y_E = \frac{1}{R_C} - j\frac{1}{X_M} = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1}(PF)$$

Circuit Parameters: Short-Circuit Test



- Transformer's secondary winding is short-circuited
- Primary winding is connected to a fairly low-voltage source.
- The input voltage is adjusted until the current in the short-circuited windings is equal to its rated value.
- Input voltage, input current, and input power to the transformer are measured.
- Excitation current is negligible, since the input voltage is very low. Thus, the voltage drop in the excitation branch can be ignored. All the voltage drop can be attributed to the series elements in the circuit.

Circuit Parameters: Short-Circuit Test

The magnitude of the series impedance:

$$|Z_{SE}| = \frac{V_{sc}}{I_{sc}}$$

The short-circuit power factor and power factor angle:

$$PF = \cos\theta = \frac{P_{sc}}{V_{sc}\, I_{sc}} \quad or,\ \theta = \cos^{-1}\left[\frac{P_{sc}}{V_{sc}\, I_{sc}}\right]$$

Therefore the series impedance is:

$$\begin{aligned} Z_{SE} &= R_{eq} + jX_{eq} \\ &= \left(R_p + a^2 R_s\right) + j\left(X_p + a^2 X_s\right) = \frac{V_{sc}}{I_{sc}} \angle \cos^{-1}(PF) \end{aligned}$$

It is possible to determine the total series impedance, but there is no easy way to split the series impedance into the primary and secondary components. These tests were performed on the primary side, so, the circuit impedances are referred to the primary side.

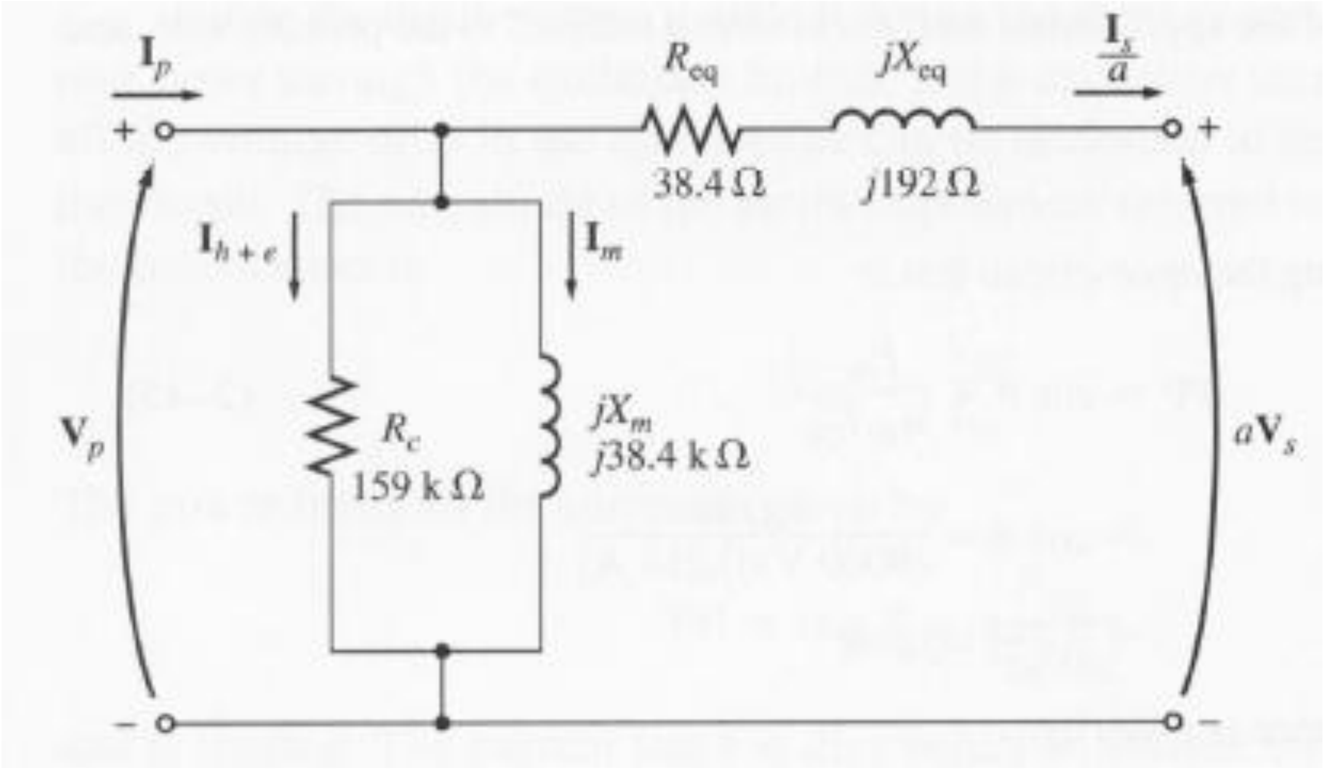
Example 2 (Example 2-2, page 92 of your text)

The equivalent circuit impedances of a 20-kVA, 8000/240-V, 60-Hz transformer are to be determined. The open-circuit test and the short-circuit test were performed on the primary side of the transformer, and the following data were taken:

Open-circuit test (on primary)	Short-circuit test (on primary)
$V_{oc} = 8000$ V	$V_{sc} = 489$ V
$I_{oc} = 0.214$ A	$I_{sc} = 2.5$ A
$P_{oc} = 400$ W	$P_{sc} = 240$ W

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch the circuit.

Answer to Example 2



Transformer Voltage Regulation

Because a real transformer has series impedance within it, the output voltage of a transformer varies with the load even if the input voltage remains constant. The voltage regulation of a transformer is the change in the magnitude of the secondary terminal voltage from no-load to full-load.

$$\%Voltage\ Regulation = \frac{V_s[no-load]-V_s[full-load]}{V_s[full-load]}\times 100$$

$$\boxed{\approx \frac{V_p[no-load]-V_p[full-load]}{V_p[full-load]}\times 100}$$

Referred to the primary side

Transformer Efficiency

$$\eta = \frac{Power\ Output}{Power\ Input}$$

$$= \frac{Power\ Input - Losses}{Power\ Input}$$

$$= 1 - \frac{Losses}{Power\ Input}$$

$$= 1 - \frac{P_{copper\ loss} + P_{core\ loss}}{P_{copper\ loss} + P_{core\ loss} + V_s I_s \cos\theta}$$

Usually the efficiency for a power transformer is between 0.9 to 0.99. The higher the rating of a transformer, the greater is its efficiency.

Example 3

A single-phase, 100-kVA, 1000:100-V, 60-Hz transformer has the following test results:

Open-circuit test (HV side open): 100 V, 6 A, 400 W

Short-circuit test (LV side shorted): 50 V, 100 A, 1800 W

- Draw the equivalent circuit of the transformer referred to the high-voltage side. Label impedances numerically in ohms and in per unit.
- Determine the voltage regulation at rated secondary current with 0.6 power factor lagging. Assume the primary is supplied with rated voltage
- Determine the efficiency of the transformer when the secondary current is 75% of its rated value and the power factor at the load is 0.8 lagging with a secondary voltage of 98 V across the load

PU System

Per unit system, a system of dimensionless parameters, is used for computational convenience and for readily comparing the performance of a set of transformers or a set of electrical machines.

$$\boldsymbol{PU\,Value = \frac{Actual\;\; Quantity}{Base\;\; Quantity}}$$

Where 'actual quantity' is a value in volts, amperes, ohms, etc.
$[VA]_{base}$ and $[V]_{base}$ are chosen first.

$$I_{base} = \frac{[VA]_{base}}{[V]_{base}}$$

$$P_{base} = Q_{base} = |S_{base}| = [VA]_{base} = [V]_{base}[I]_{base}$$

$$R_{base} = X_{base} = |Z_{base}| = \frac{[V]_{base}}{[I]_{base}} = \frac{[V]^2_{base}}{S_{base}} = \frac{[V]^2_{base}}{[VA]_{base}}$$

$$Y_{base} = \frac{[I]_{base}}{[V]_{base}}$$

$$|Z|_{PU} = \frac{|Z|_{ohm}}{|Z_{base}|}$$

$$\left[[VA]_{base}\right]_{pri} = \left[[VA]_{base}\right]_{sec}$$

$$\frac{\left[[V]_{base}\right]_{pri}}{\left[[V]_{base}\right]_{sec}} = turns\;\; ratio$$

Example 4 (Problem No. 2-2, page 144 of your text)

A 20-kVA, 8000:480-V distribution transformer has the following resistances and reactances:

$R_P = 32$ ohm	$R_S = 0.05$ ohm
$X_P = 45$ ohm	$X_S = 0.06$ ohm
$R_C = 250{,}000$ ohm	$X_M = 30{,}000$ ohm

The excitation branch impedances are referred to the high-voltage side.

a) Find the equivalent circuit of the transformer referred to the high-voltage side.

b) Find the per unit equivalent circuit of this transformer.

c) Assume that the transformer is supplying rated load at 480 V and 0.8 power factor lagging. What is this transformer's input voltage? What is its voltage regulation?

d) What is this transformer's efficiency under the conditions of part (c)?

UNIT-IV
DC MACHINES

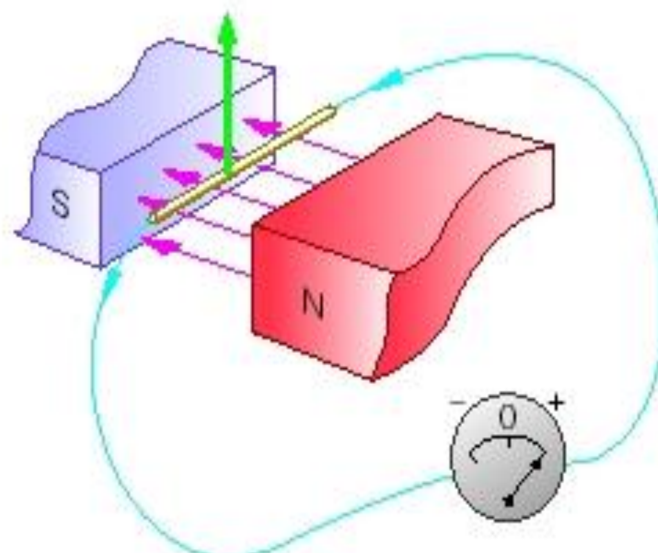
DC MACHINES

DC Generator

Mechanical energy is converted to electrical energy

Three requirements are essential

1. Conductors
2. Magnetic field
3. Mechanical energy



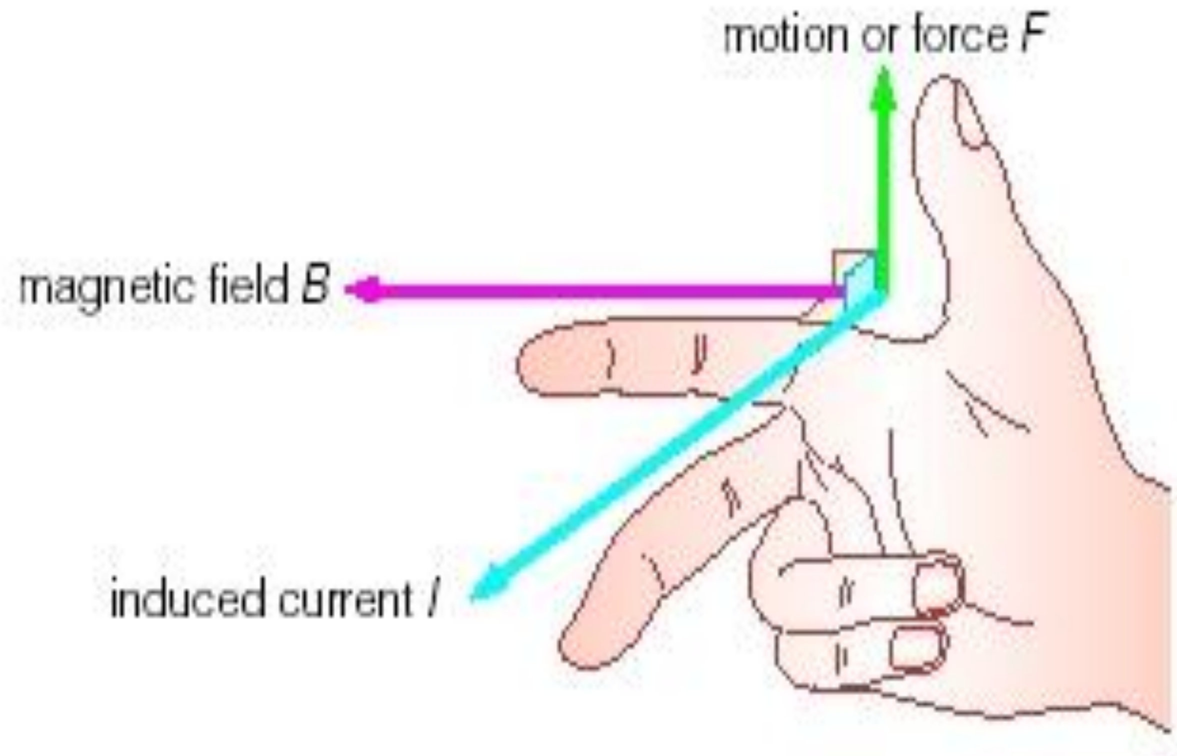
Working principle

- A generator works on the principles of Faraday's law of electromagnetic induction

- Whenever a conductor is moved in the magnetic field , an emf is induced and the magnitude of the induced emf is directly proportional to the rate of change of flux linkage.

- This emf causes a current flow if the conductor circuit is closed .

Fleming’s Right hand rule



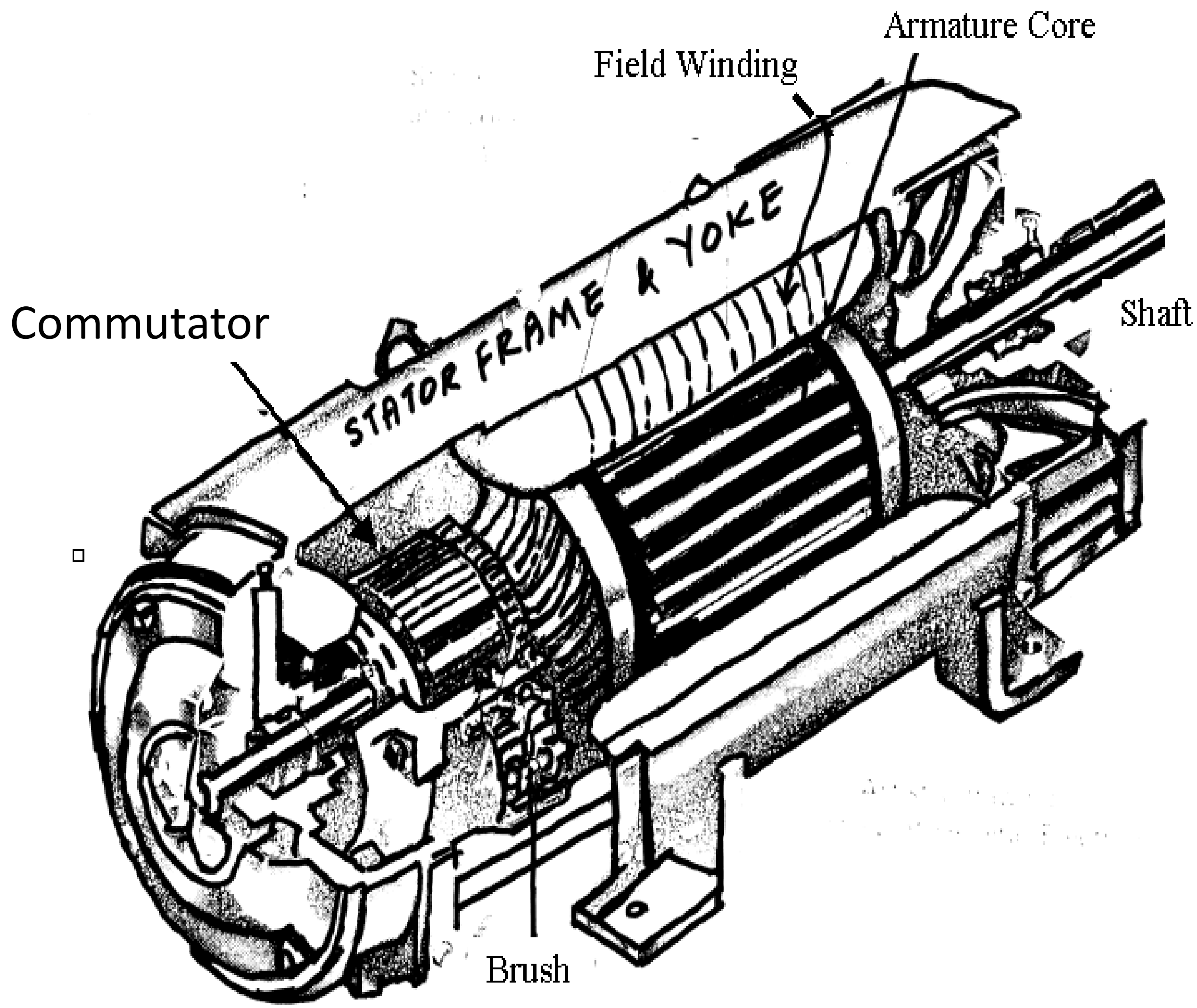
Fleming's Right hand rule

- Used to determine the direction of emf induced in a conductor for **DC Generators**
- The middle finger , the fore finger and thumb of the left hand are kept at right angles to one another.

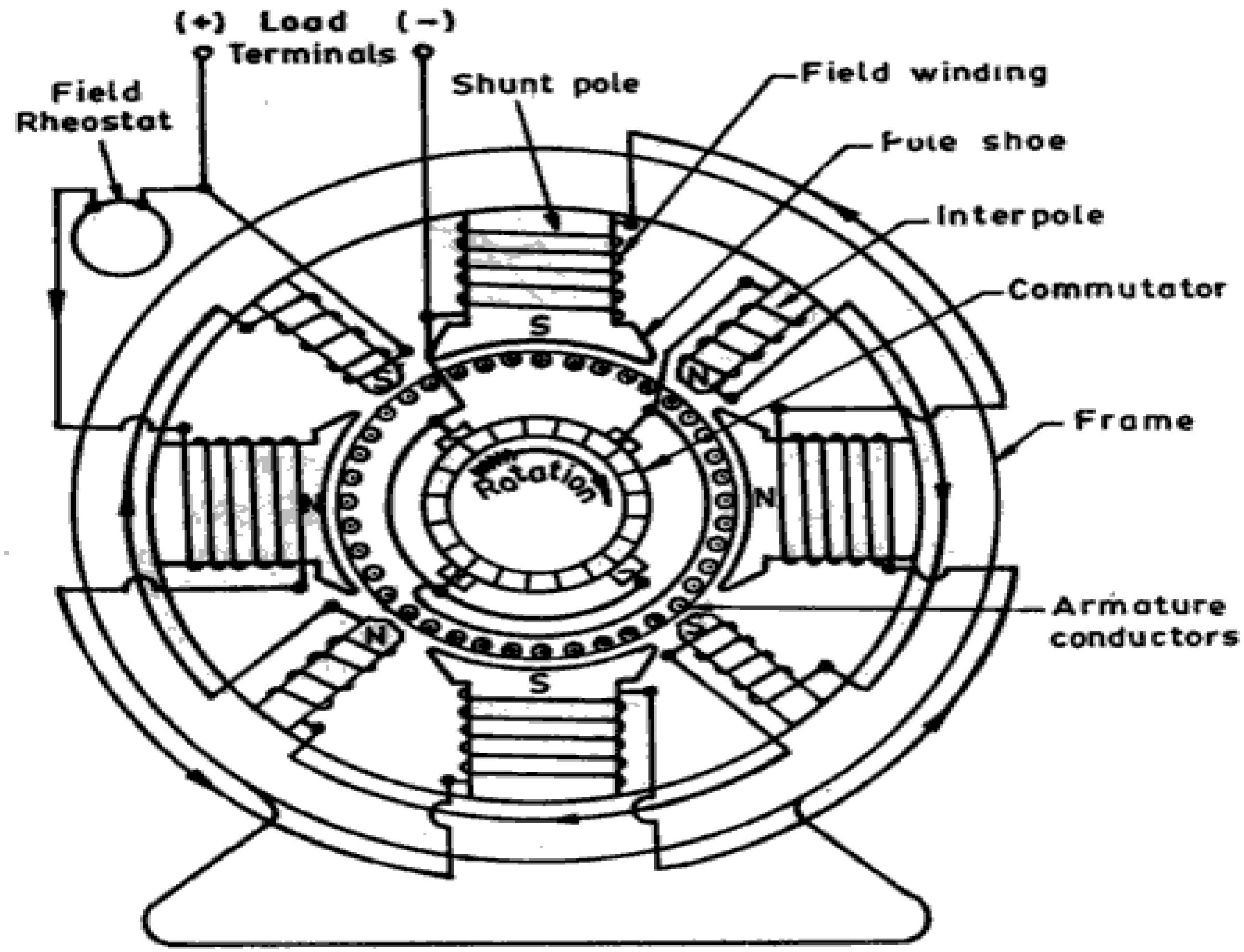
  - The fore finger represent the direction of magnetic field
  - The thumb represent the direction of motion of the conductor
  - The middle finger will indicate the direction of the inducted emf .

  This rule is used in DC Generators

DC Machine

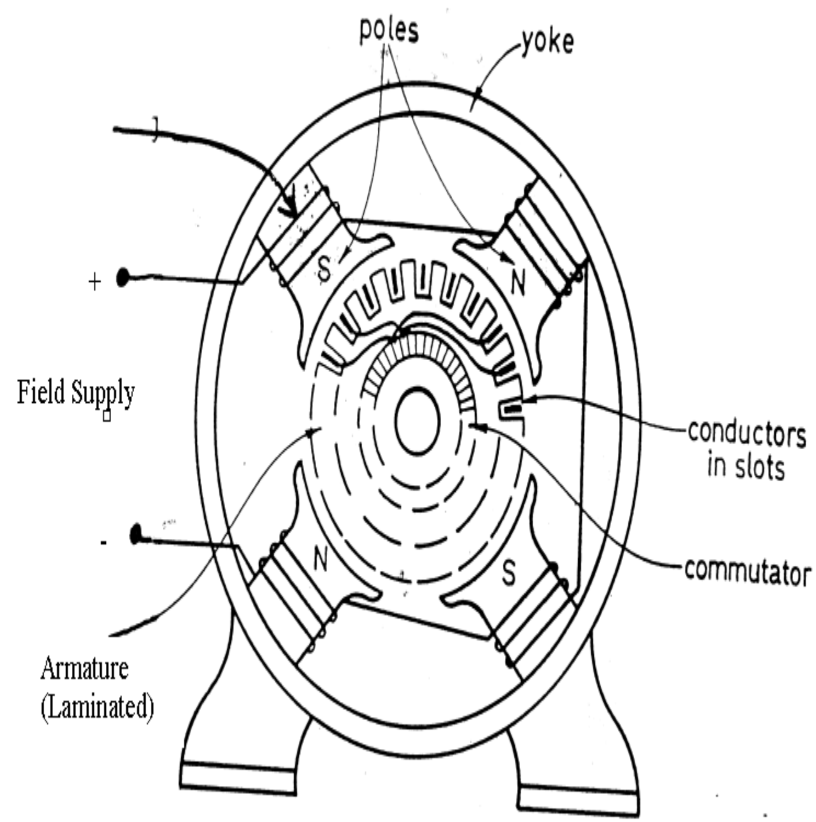

Armature Core
Field Winding
STATOR FRAME & YOKE
Commutator
Shaft
Brush

Sectional view of a DC machine

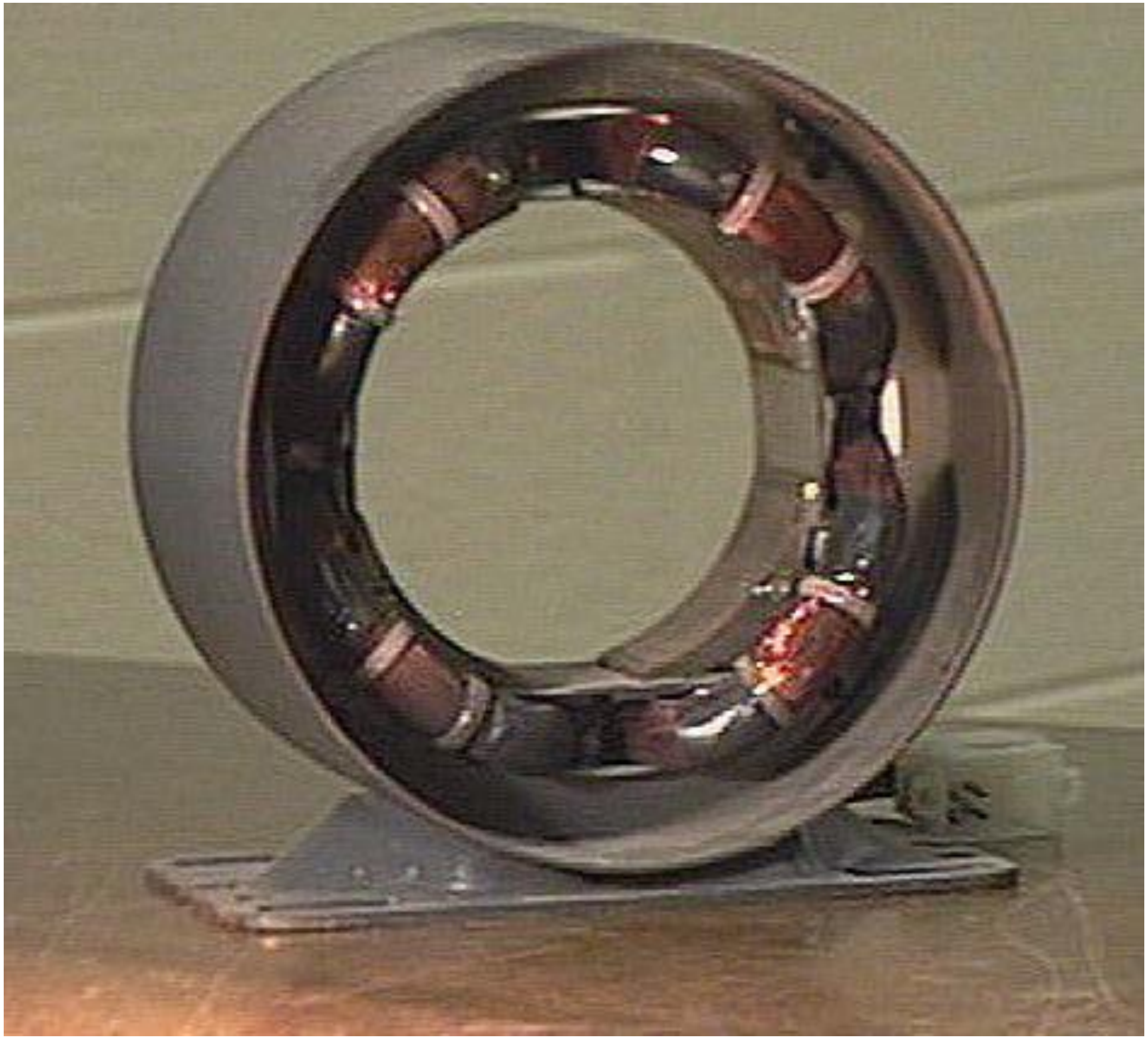


Construction of DC Generator

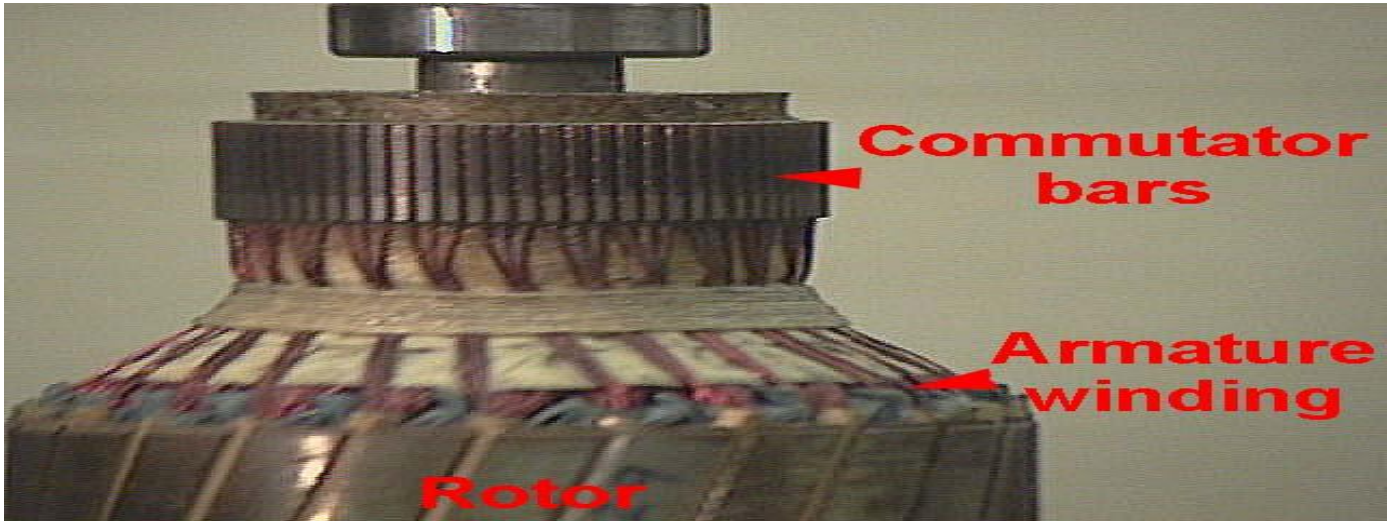
- Field system
- Armature core
- Armature winding
- Commutator
- Brushes



Field winding



Rotor and rotor winding



Armature winding

There are 2 types of winding

Lap and Wave winding

Lap winding

- **A = P**
- The armature windings are divided into no. of sections equal to the no of poles

Wave winding

- **A = 2**
- It is used in low current output and high voltage.
- 2 brushes

Field system

- It is for uniform magnetic field within which the armature rotates.
- Electromagnets are preferred in comparison with permanent magnets
- They are cheap , smaller in size , produce greater magnetic effect and
- Field strength can be varied

Field system consists of the following parts

- Yoke
- Pole cores
- Pole shoes
- Field coils

Armature core

- The armature core is cylindrical
- High permeability silicon steel stampings
- Impregnated
- Lamination is to reduce the eddy current loss

Commutator

- Connect with external circuit
- Converts ac into unidirectional current
- Cylindrical in shape
- Made of wedge shaped copper segments
- Segments are insulated from each other
- Each commutator segment is connected to armature conductors by means of a cu strip called riser.
- No of segments equal to no of coils

Carbon brush

- Carbon brushes are used in DC machines because they are soft materials
- It does not generate spikes when they contact commutator
- To deliver the current thro armature
- Carbon is used for brushes because it has negative temperature coefficient of resistance
- Self lubricating , takes its shape , improving area of contact

Brush rock and holder

Carbon brush

- Brush leads (pig tails)
- Brush rocker (brush gear)
- Front end cover
- Rear end cover
- Cooling fan
- Bearing
- Terminal box

EMF equation

Let,

- Ø= flux per pole in weber
- Z = Total number of conductor
- P = Number of poles
- A = Number of parallel paths
- N =armature speed in rpm
- Eg = emf generated in any on of the parallel path

EMF equation

Flux cut by 1 conductor
in 1 revolution $= P * \phi$
Flux cut by 1 conductor in
60 sec $= P \phi N / 60$
Avg emf generated in 1
conductor $= P\phi N/60$
Number of conductors in
each parallel path $= Z / A$

$$E_g = P\phi NZ/60A$$

Types of DC Generator

- Separately excited DC generator
- Self excited D C generator

Types of DC Machines

Both the armature and field circuits carry direct current in the case of a DC machine.

Types:

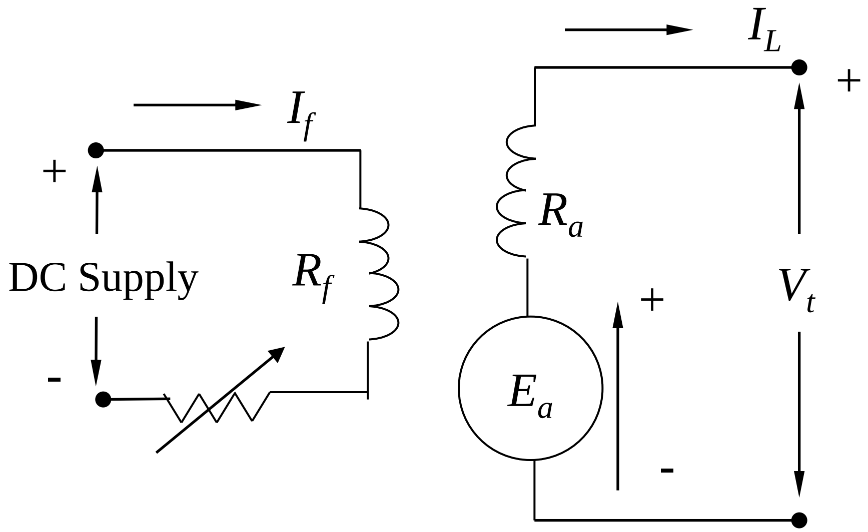
Self-excited DC machine: when a machine supplies its own excitation of the field windings. In this machine, residual magnetism must be present in the ferromagnetic circuit of the machine in order to start the self-excitation process.

Separately-excited DC machine: The field windings may be separately excited from an eternal DC source.

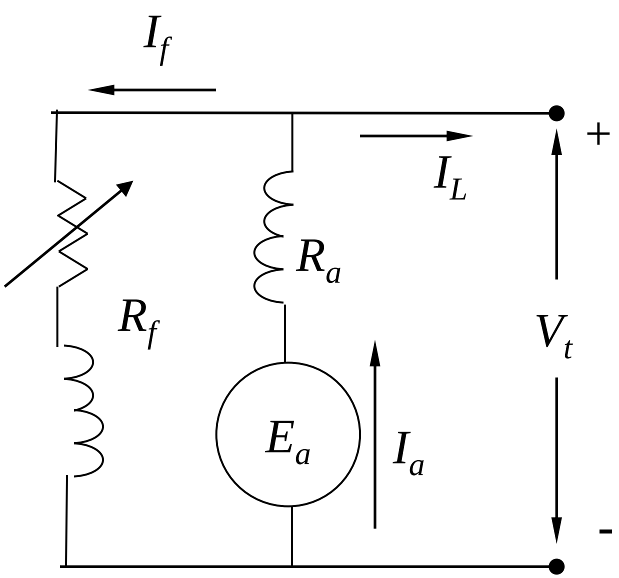
Shunt Machine: armature and field circuits are connected in parallel. Shunt generator can be separately-excited or self-excited.

Series Machine: armature and field circuits are connected in series.

<u>Separately-Excited and Self-Excited DC Generators</u>



Separately-Excited



Self-Excited

Further classification of DC Generator

- Series wound generator
- Shunt wound generator
- Compound wound generator
  - Short shunt & Long shunt
  - Cumulatively compound

    &

    Differentially compound

Applications

Shunt Generators:

a. in electro plating

b. for battery recharging

c. as exciters for AC generators.

Series Generators :

A. As boosters

B. As lighting arc lamps

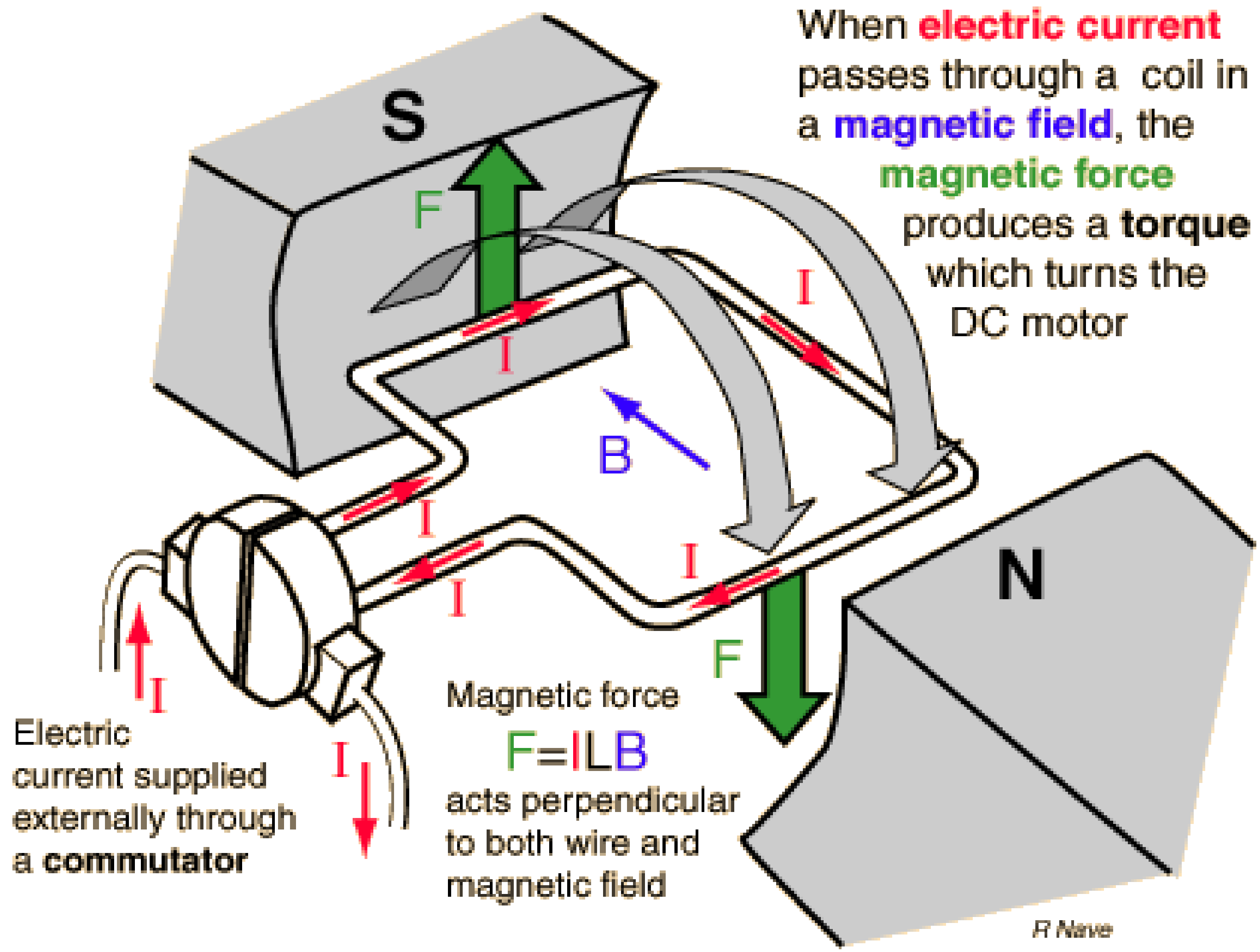
DC Motors

<u>Converts Electrical energy into Mechanical energy</u>

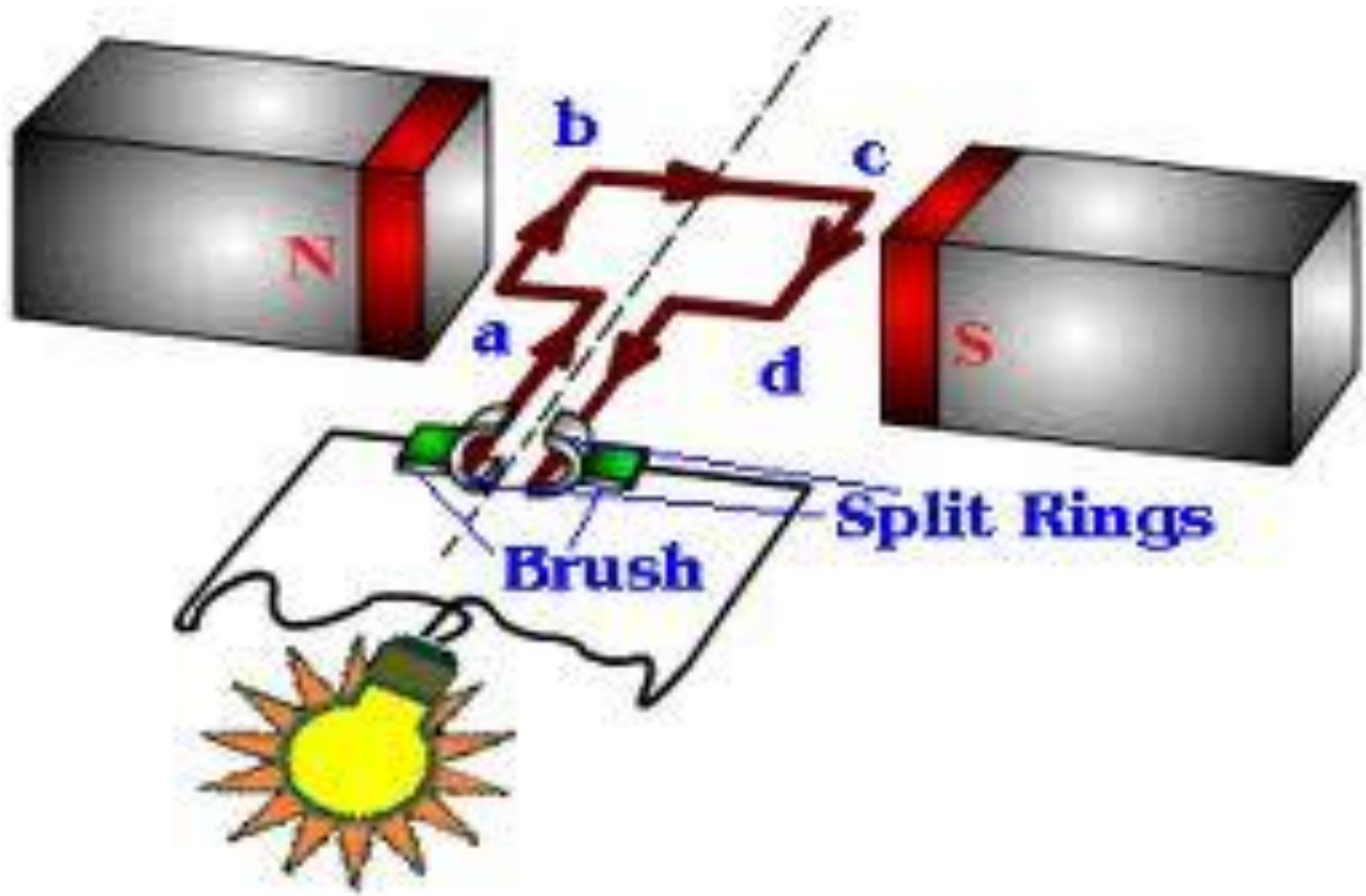
Construction : Same for Generator and motor

Working principle : Whenever a current carrying conductor is placed in the magnetic field , a force is set up on the conductor.

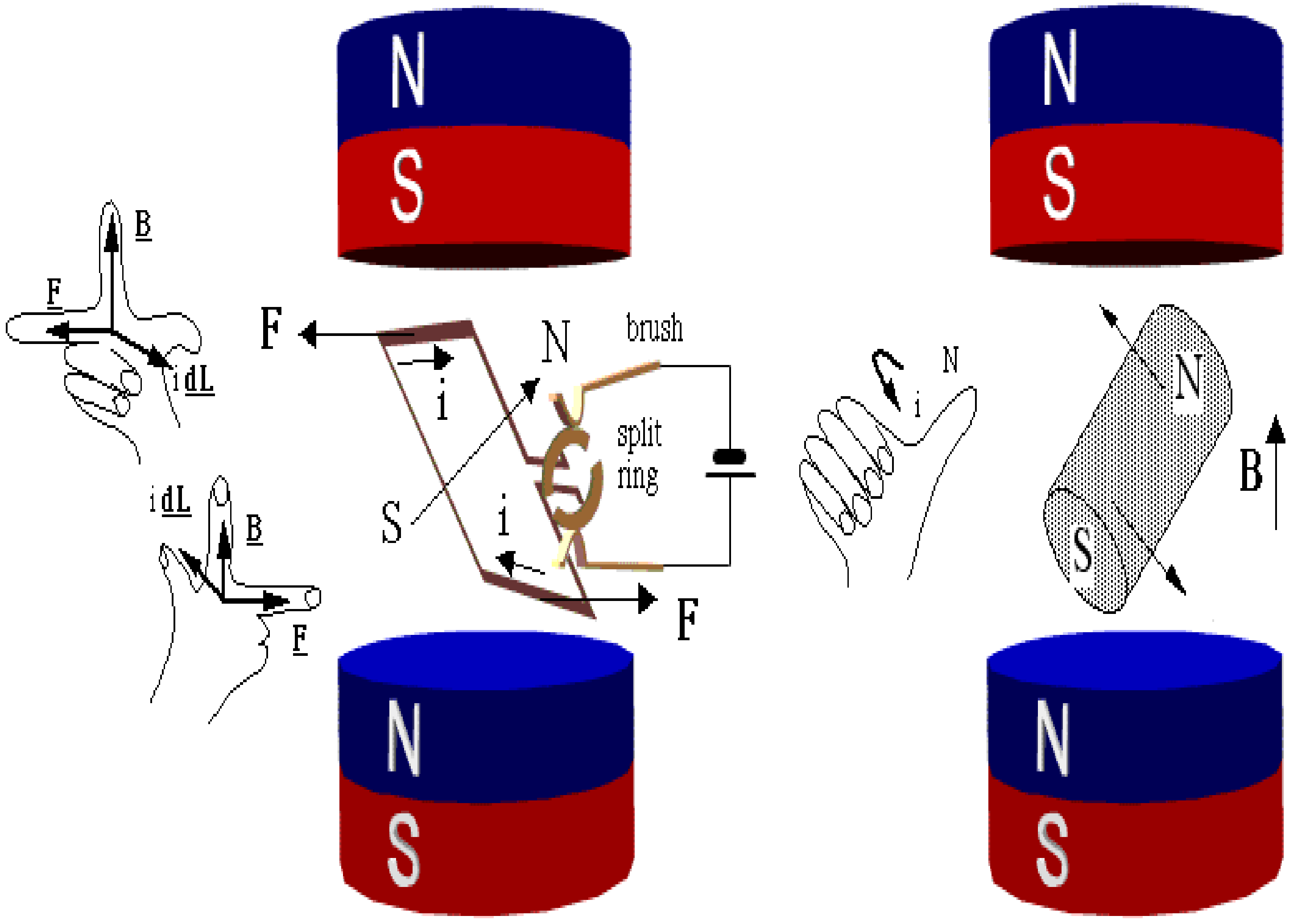
Working principle of DC motor



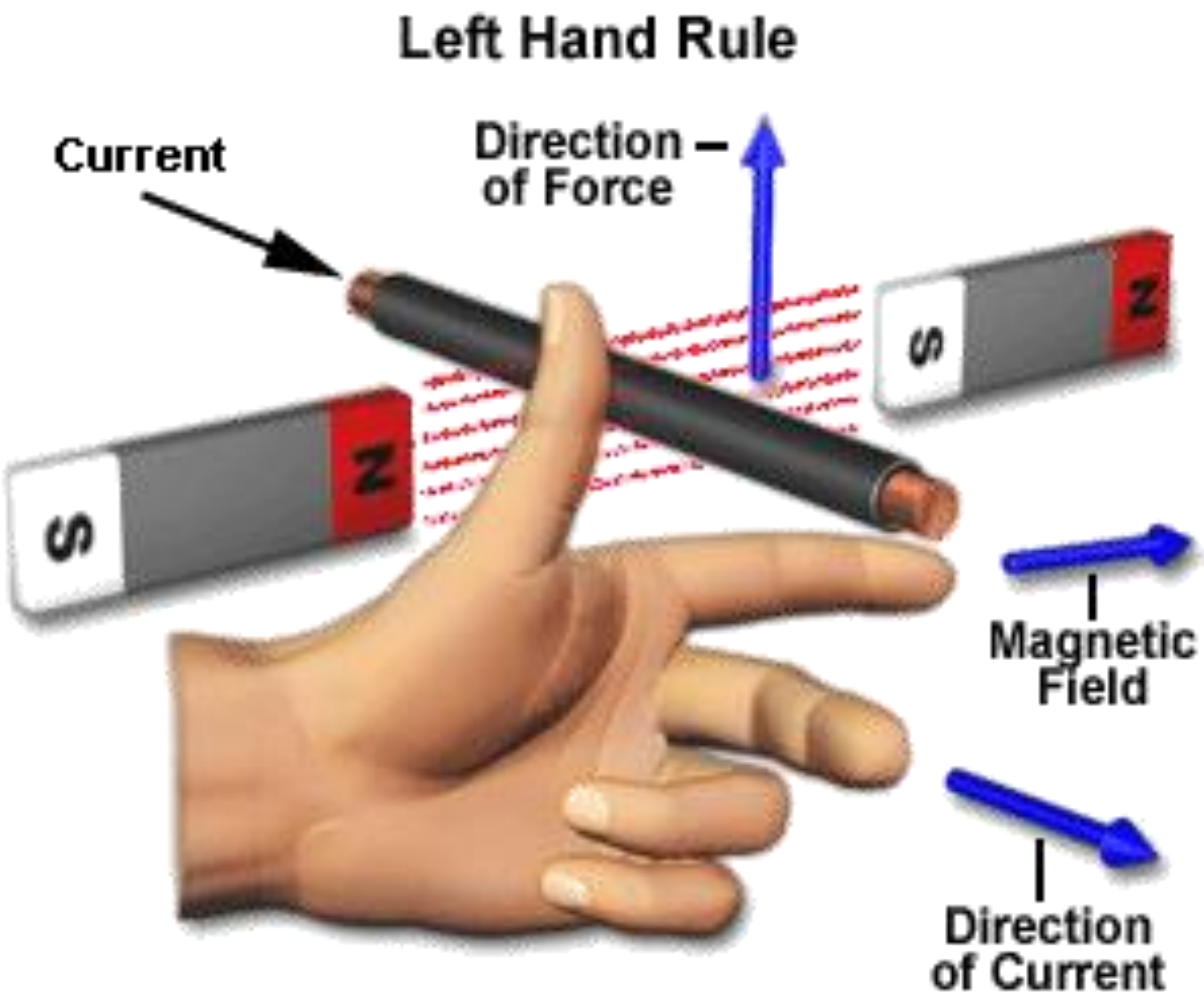
Working principle of DC motor



Force in DC motor



Fleming's left hand rule



Fleming's left hand rule

- Used to determine the <u>direction of force acting on a current carrying conductor </u>placed in a magnetic field .
- The middle finger , the fore finger and thumb of the left hand are kept at right angles to one another .
  - The middle finger represent the direction of current
  - The fore finger represent the direction of magnetic field
  - The thumb will indicate the direction of force acting on the conductor .

This rule is used in motors.

Back emf

The induced emf in the rotating armature conductors always acts in the opposite direction of the supply voltage .

According to the Lenz's law, the direction of the induced emf is always so as to oppose the cause producing it .

In a DC motor , the supply voltage is the cause and hence this induced emf opposes the supply voltage.

Len’s Law

The direction of induced emf is given by Lenz’s law .

According to this law, the induced emf will be acting in such a way so as to oppose the very cause of production of it .

- $e = -N\,(d\varnothing/dt)$ volts

Classification of DC motors

DC motors are mainly classified into three types as listed below:

- Shunt motor
- Series motor
- Compound motor
  - Differential compound
  - Cumulative compound

Torque

The turning or twisting force about an axis is called torque .

- $P = T * 2 \pi N / 60$
- $E_b I_a = T_a * 2 \pi N / 60$
- $T \infty \phi I_a$
- $T_a \infty I2a$

Characteristic of DC motors

- T/ Ia characteristic

- N/ I a characteristic

- N/T characteristic

Starters for DC motors

Needed to limit the starting current .

1. Two point starter
2. Three point starter
3. Four point starter

Testing of DC machines

determine the efficiency of as DC motor , the output and input should be known.

There are two methods.

- The load test or The direct method
- The indirect method

<u>Direct method:</u> In this method , the efficiency is determined by knowing the input and output power of the motor.

<u>Indirect method:</u> Swinburne's test is an indirect method of testing DC shunt machines to predetermine the effficency , as a motor and as a Generator. In this method, efficiency is calculated by determining the losses .

Applications:

- Blowers and fans
- Centrifugal and reciprocating pumps
- Lathe machines
- Machine tools
- Milling machines
- Drilling machines

Applications:

- Cranes
- Hoists , Elevators
- Trolleys
- Conveyors
- Electric locomotives

Applications:

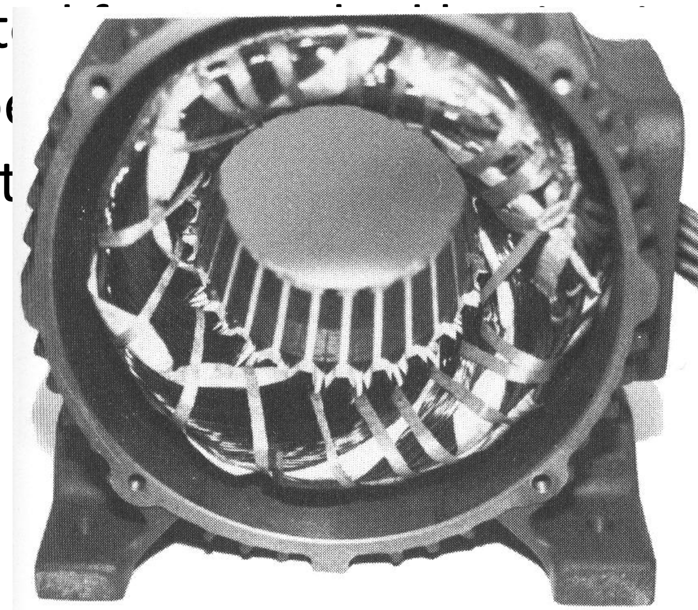
- Rolling mills
- Punches
- Shears
- Heavy planers
- Elevators

Three-phase induction motor

- Three-phase induction motors are the most common and frequently encountered machines in industry
  - simple design, rugged, low-price, easy maintenance
  - wide range of power ratings: fractional horsepower to 10 MW
  - run essentially as constant speed from no-load to full load
  - Its speed depends on the frequency of the power source
    - not easy to have variable speed control
    - requires a variable-frequency power-electronic drive for optimal speed control

Construction

- An induction motor has two main parts
  - a stationary stator
    - consisting of a steel frame that supports a hollow, cylindrical core
    - core, construct ... s (why?), having a numbe ... roviding the space for the st ...



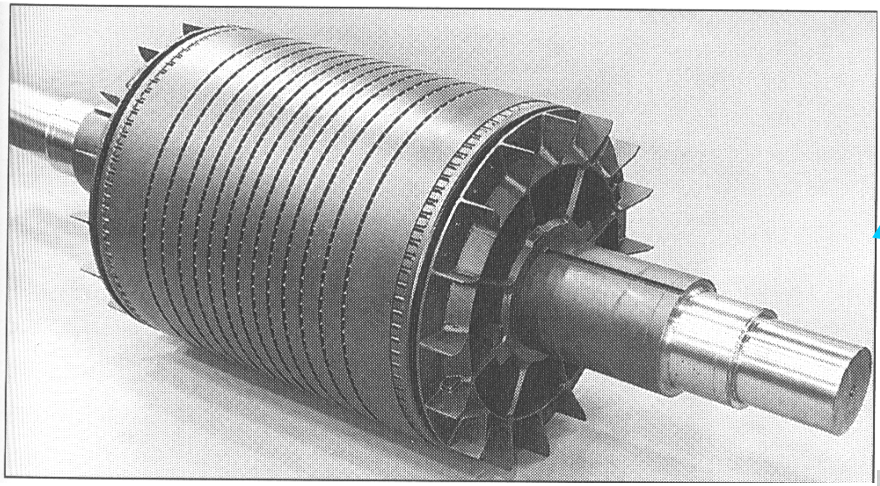
Stator of IM

Construction

- a revolving rotor
    - composed of punched laminations, stacked to create a series of rotor slots, providing space for the rotor winding
    - one of two types of rotor windings
    - conventional 3-phase windings made of insulated wire (wound-rotor) » similar to the winding on the stator
    - aluminum bus bars shorted together at the ends by two aluminum rings, forming a squirrel-cage shaped circuit (squirrel-cage)

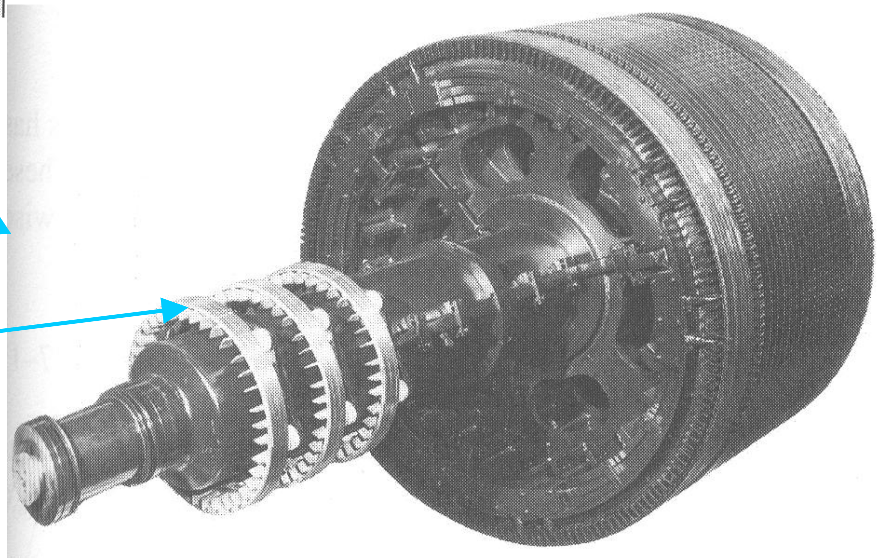
- Two basic design types depending on the rotor design
    - squirrel-cage: conducting bars laid into slots and shorted at both ends by shorting rings.
    - wound-rotor: complete set of three-phase windings exactly as the stator. Usually Y-connected, the ends of the three rotor wires are connected to 3 slip rings on the rotor shaft. In this way, the rotor circuit is accessible.

Construction



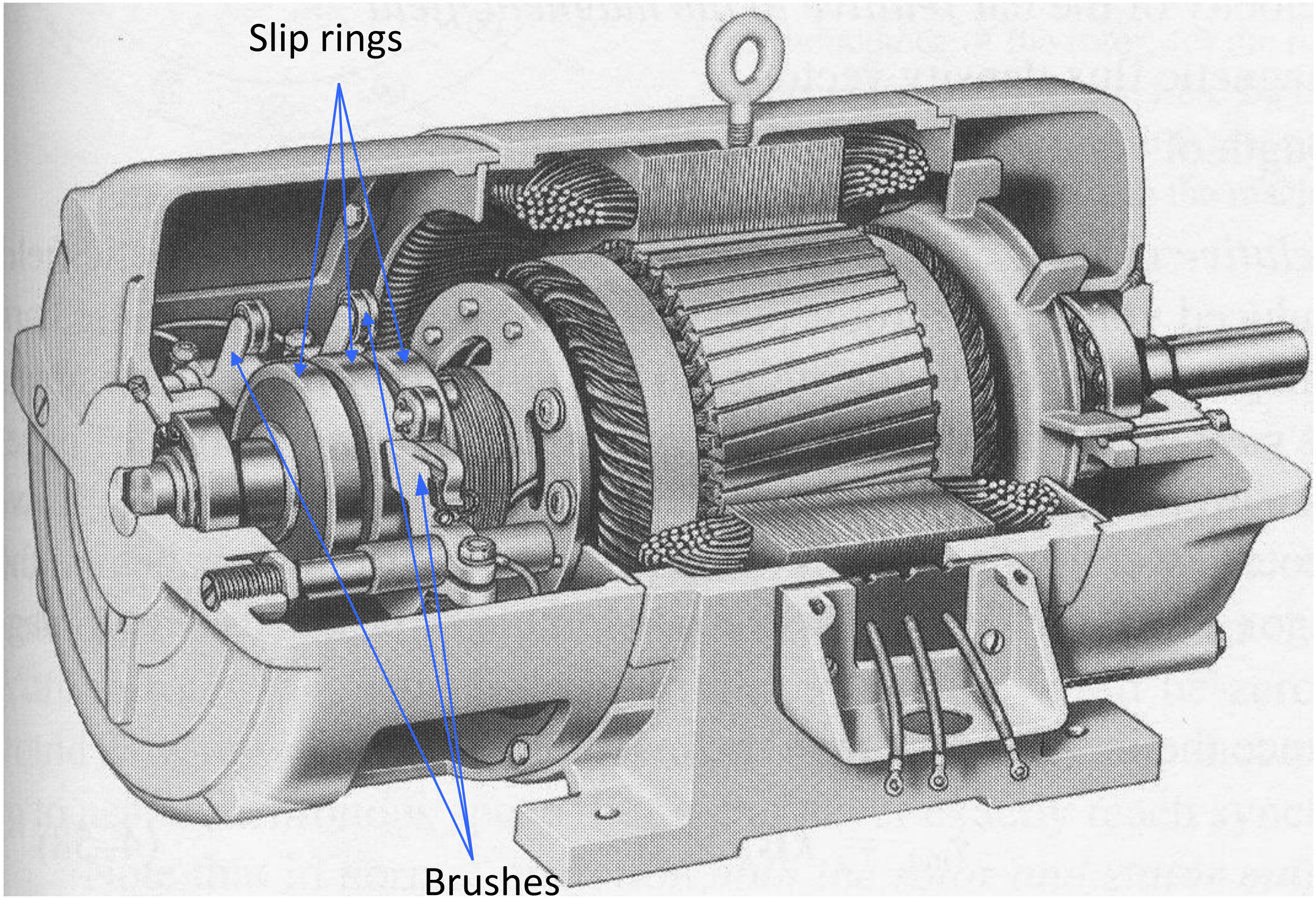
Squirrel cage rotor

Wound rotor



Notice the slip rings

Construction



Cutaway in a typical wound-rotor IM. Notice the brushes and the slip rings

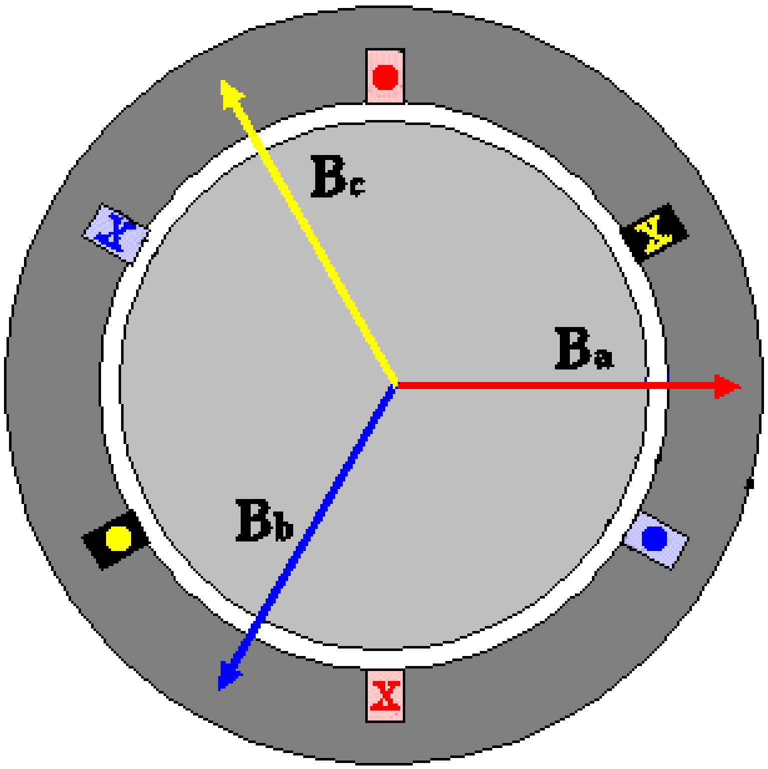
Rotating Magnetic Field

- Balanced three phase windings, i.e. mechanically displaced 120 degrees form each other, fed by balanced three phase source
- A rotating magnetic field with constant magnitude is produced, rotating with a speed

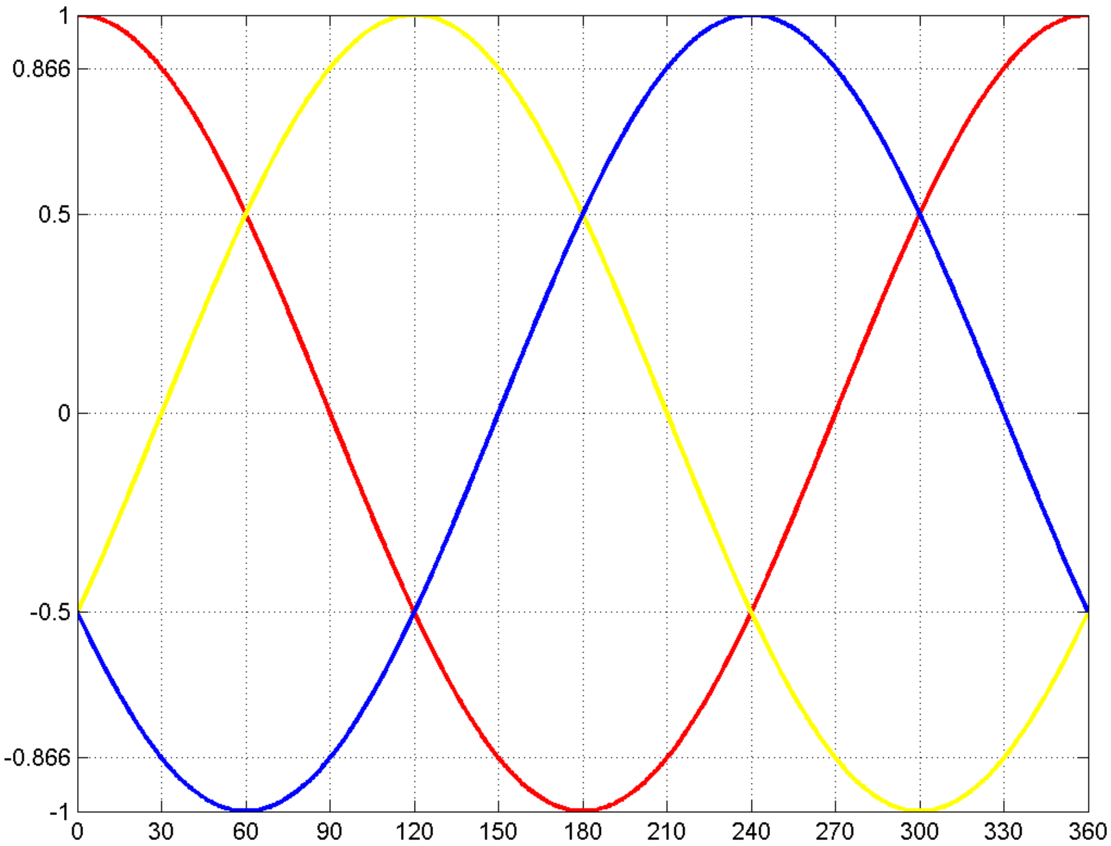
$$\boxed{n_{sync} = \frac{120 f_e}{P} \quad rpm}$$

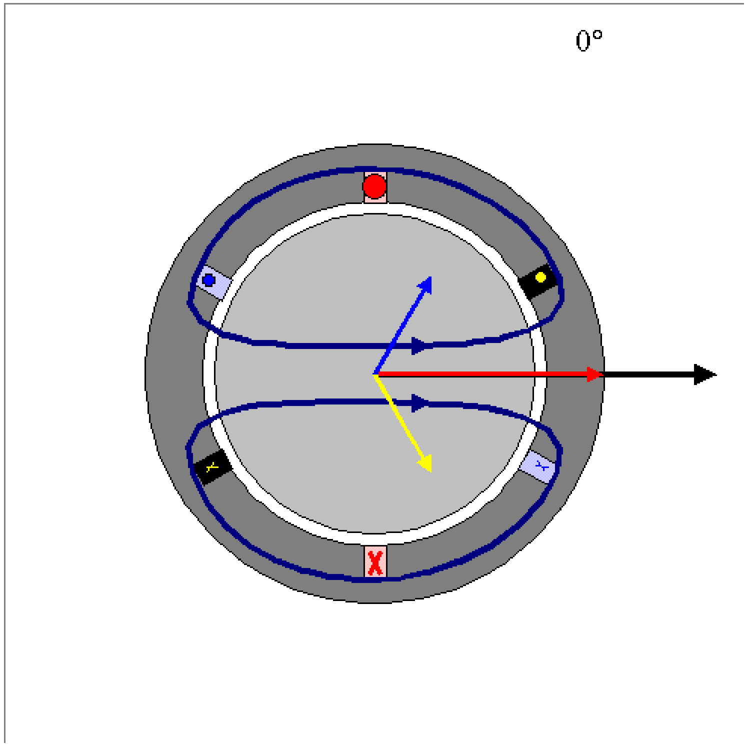
Where f_e is the supply frequency and

P is the no. of poles and n_{sync} is called the synchronous speed in *rpm* (revolutions per minute)

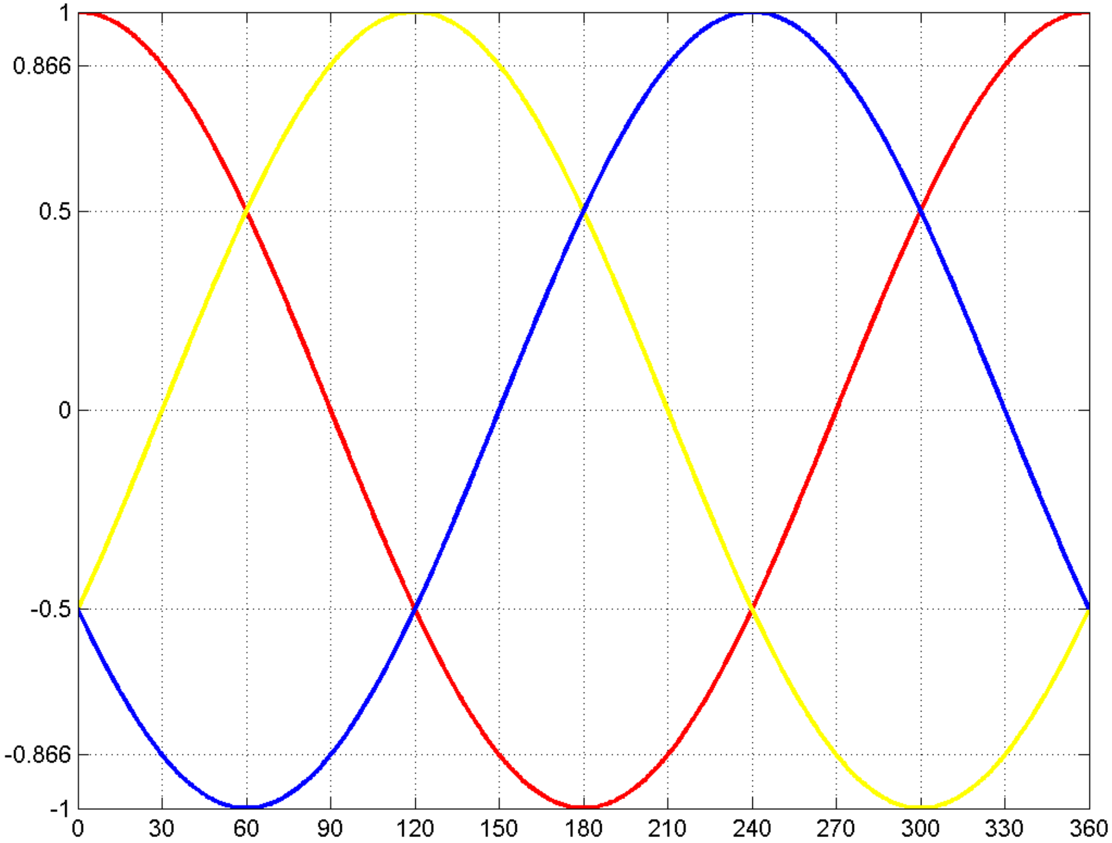


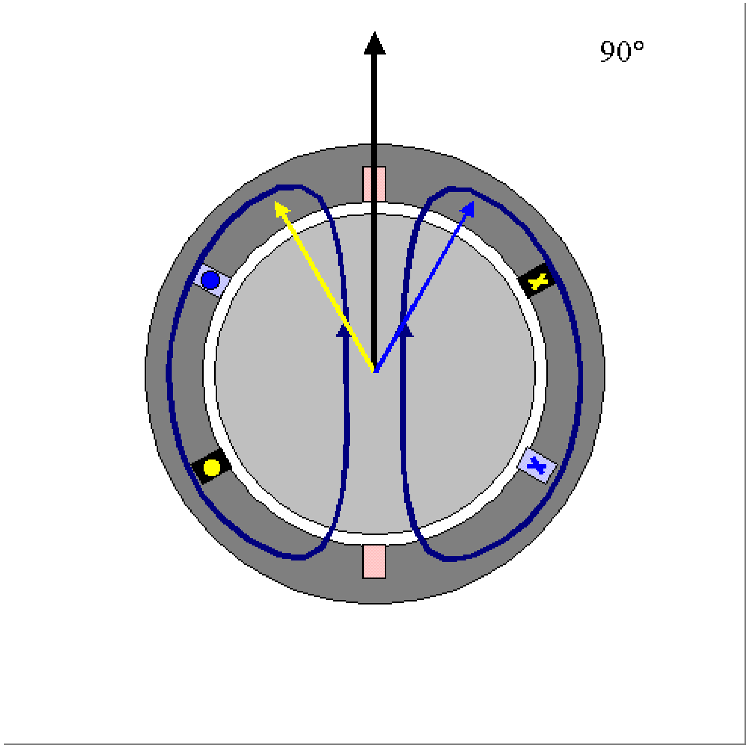
Rotating Magnetic Field





Rotating Magnetic Field





Rotating Magnetic Field

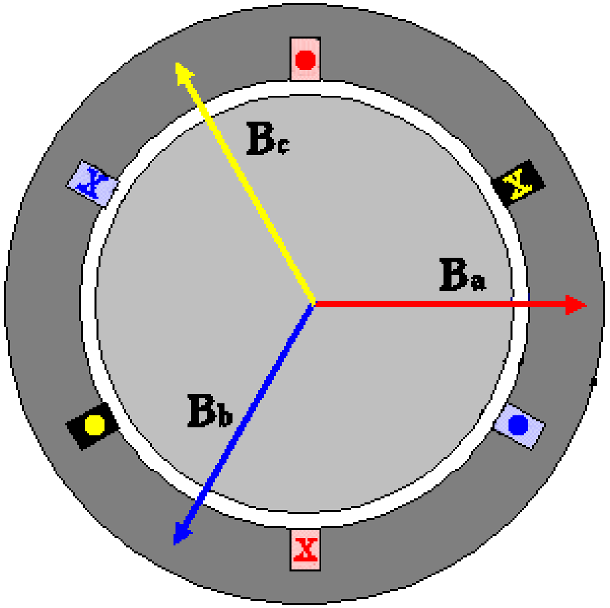
$$B_{net}(t) = B_a(t) + B_b(t) + B_c(t)$$

$$= B_M \sin(\omega t)\angle 0° + B_M \sin(\omega t - 120°)\angle 120° + B_M \sin(\omega t - 240)\angle 240°$$

$$= B_M \sin(\omega t)\hat{\mathbf{x}}$$

$$-[0.5B_M \sin(\omega t - 120°)]\hat{\mathbf{x}} - [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)]\hat{\mathbf{y}}$$

$$-[0.5B_M \sin(\omega t - 240°)]\hat{\mathbf{x}} + [\frac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)]\hat{\mathbf{y}}$$



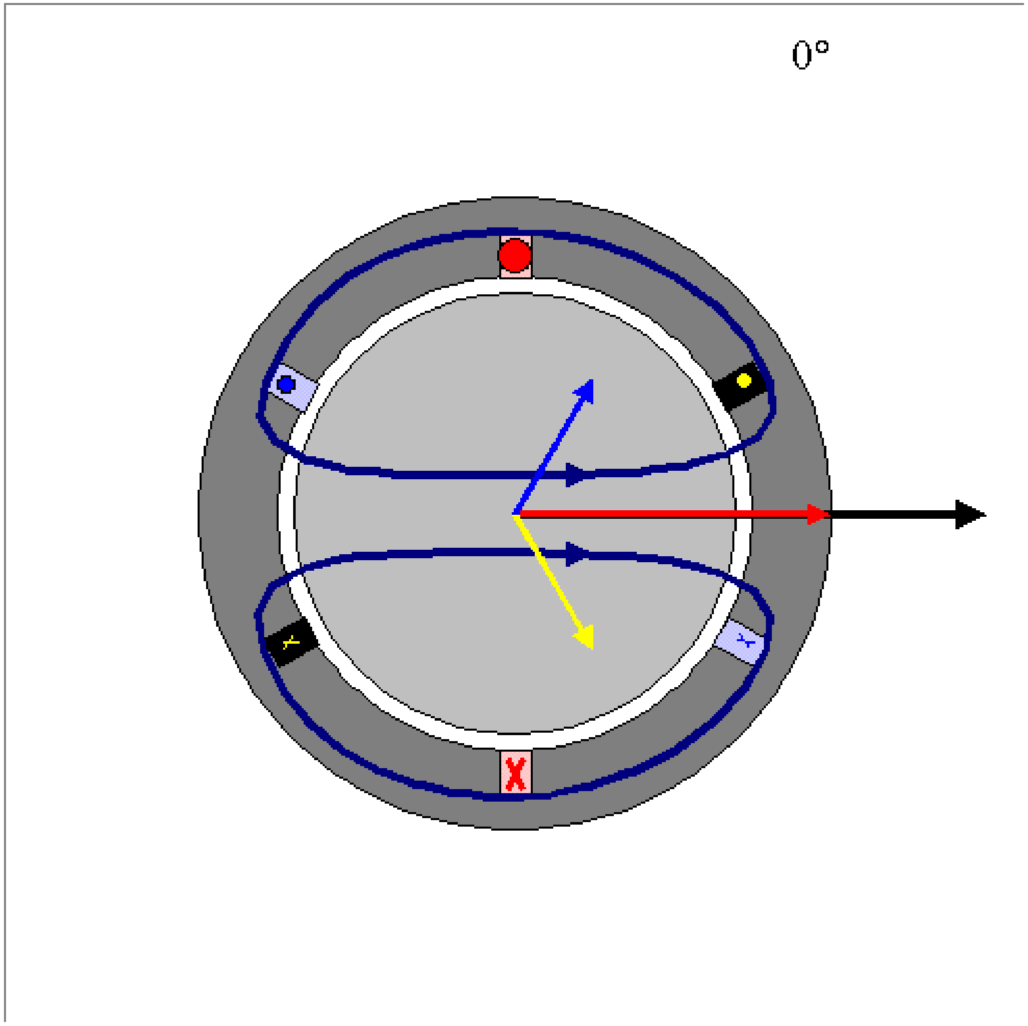
Rotating Magnetic Field

$$B_{net}(t) = [B_M \sin(\omega t) + \frac{1}{4} B_M \sin(\omega t) + \frac{\sqrt{3}}{4} B_M \cos(\omega t) + \frac{1}{4} B_M \sin(\omega t) - \frac{\sqrt{3}}{4} B_M \cos(\omega t)]\hat{\mathbf{x}}$$

$$+[-\frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t) + \frac{\sqrt{3}}{4} B_M \sin(\omega t) - \frac{3}{4} B_M \cos(\omega t)]\hat{\mathbf{y}}$$

$$= [1.5 B_M \sin(\omega t)]\hat{\mathbf{x}} - [1.5 B_M \cos(\omega t)]\hat{\mathbf{y}}$$

Rotating Magnetic Field



Principle of operation

- This rotating magnetic field cuts the rotor windings and produces an induced voltage in the rotor windings
- Due to the fact that the rotor windings are short circuited, for both squirrel cage and wound-rotor, and induced current flows in the rotor windings
- The rotor current produces another magnetic field
- A torque is produced as a result of the interaction of those two magnetic fields

$$\tau_{ind} = kB_R \times B_s$$

Where τ_{ind} is the induced torque and B_R and B_S are the magnetic flux densities of the rotor and the stator respectively

Induction motor speed

- So, the IM will always run at a speed lower than the synchronous speed
- The difference between the motor speed and the synchronous speed is called the *Slip*

$$\boxed{n_{slip} = n_{sync} - n_m}$$

Where n_{slip}= slip speed

n_{sync}= speed of the magnetic field

n_m = mechanical shaft speed of the motor

The Slip

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}}}$$

Where *s* is the *slip*

Notice that : if the rotor runs at synchronous speed

$s = 0$

if the rotor is stationary

$s = 1$

Slip may be expressed as a percentage by multiplying the above eq. by 100, notice that the slip is a ratio and doesn't have units

Induction Motors and Transformers

- Both IM and transformer works on the principle of induced voltage
  - Transformer: voltage applied to the primary windings produce an induced voltage in the secondary windings
  - Induction motor: voltage applied to the stator windings produce an induced voltage in the rotor windings
  - The difference is that, in the case of the induction motor, the secondary windings can move
  - Due to the rotation of the rotor (the secondary winding of the IM), the induced voltage in it does not have the same frequency of the stator (the primary) voltage

Frequency

- The frequency of the voltage induced in the rotor is given by

$$f_r = \frac{P \times n}{120}$$

Where f_r = the rotor frequency (Hz)

P = number of stator poles

n = slip speed (rpm)

$$f_r = \frac{P \times (n_s - n_m)}{120}$$

$$= \frac{P \times s n_s}{120} = s f_e$$

Frequency

- What would be the frequency of the rotor's induced voltage at any speed n_m?

$$\boxed{f_r = s\, f_e}$$

- When the rotor is blocked (s=1) , the frequency of the induced voltage is equal to the supply frequency
- On the other hand, if the rotor runs at synchronous speed (s = 0), the frequency will be zero

Torque

- While the input to the induction motor is electrical power, its output is mechanical power and for that we should know some terms and quantities related to mechanical power
- Any mechanical load applied to the motor shaft will introduce a Torque on the motor shaft. This torque is related to the motor output power and the rotor speed

$$\boxed{\tau_{load} = \frac{P_{out}}{\omega_m} \quad N.m}$$ and $$\boxed{\omega_m = \frac{2\pi n_m}{60} \quad rad/s}$$

UNIT-V

MEASURING INSTRUMENTS

MEASURING INSTRUMENTS

MEASURING INSTRUMENTS

"The device used for comparing the unknown quantity with the unit of measurement or standard quantity is called a Measuring Instrument."

OR

"An instrument may be defined as a machine or system which is designed to maintain functional relationship between prescribed properties of physical variables & could include means of communication to human observer."

CLASSIFICATION OF INSTRUMENTS

Electrical instruments may be divided into two categories, that are;

1. Absolute instruments,
2. Secondary instruments.

- Absolute instruments gives the quantity to be measured in term of instrument constant & its deflection.

- In Secondary instruments the deflection gives the magnitude of electrical quantity to be measured directly. These instruments are required to be calibrated by comparing with another standard instrument before putting into use.

CLASSIFICATION OF INSTRUMENTS

CLASSIFICATION OF INSTRUMENTS

Electrical measuring instruments may also be classified according to the kind of quantity, kind of current, principle of operation of moving system.

CLASSIFICATION OF SECONDARY INSTRUMENTS

- Secondary instruments can be classified into three types;

  i. Indicating instruments;

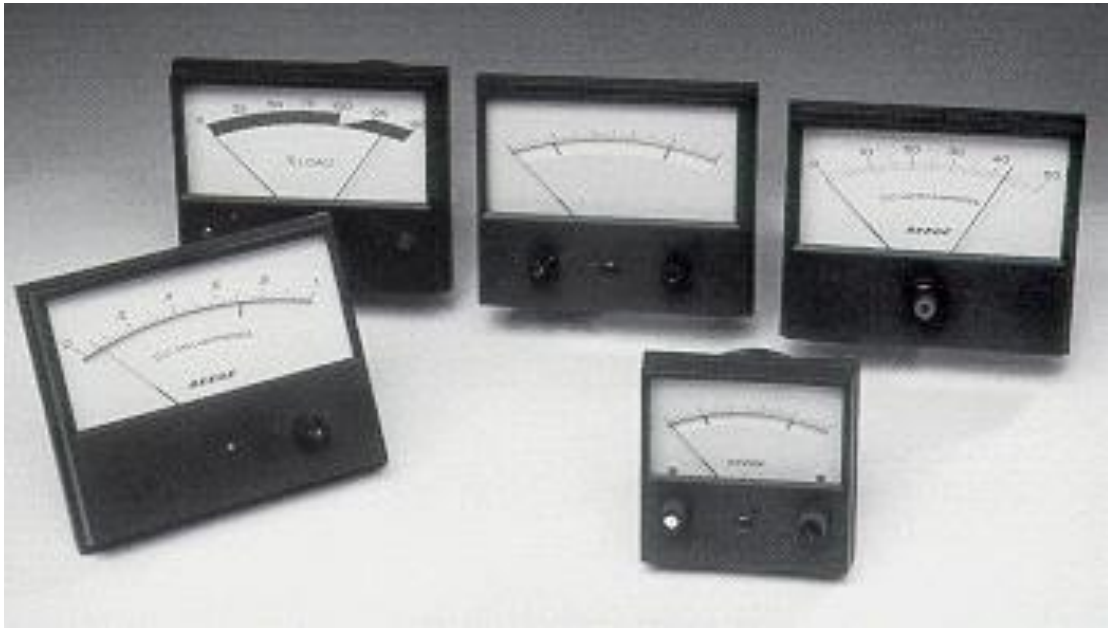
  ii. Recording instruments;

  iii. Integrating instruments.

CLASSIFICATION OF SECONDARY INSTRUMENTS

- Indicating Instruments:

It indicate the magnitude of an electrical quantity at the time when it is being measured. The indications are given by a pointer moving over a graduated dial.



CLASSIFICATION OF SECONDARY INSTRUMENTS

- Recording Instruments:

The instruments which keep a continuous record of the variations of the magnitude of an electrical quantity to be observed over a defined period of time.

CLASSIFICATION OF SECONDARY INSTRUMENTS

- Integrating Instruments:

  The instruments which measure the total amount of either quantity of electricity or electrical energy supplied over a period of time. For example energy meters.

ESSENTIALS OF INDICATING INSTRUMENTS

A defined above, indicating instruments are those which indicate the value of quantity that is being measured at the time at which it is measured. Such instruments consist essentially of a pointer which moves over a calibrated scale & which is attached to a moving system pivoted in bearing. The moving system is subjected to the following three torques:

1. A deflecting (or operating) torque;
2. A controlling (or restoring) torque;
3. A damping torque.

DEFLECTING TORQUE

- The deflecting torque is produced by making one of the magnetic, heating, chemical, electrostatic and electromagnetic induction effect of current or voltage and cause the moving system of the instrument to move from its zero position.
- The method of producing this torque depends upon the type of instrument.

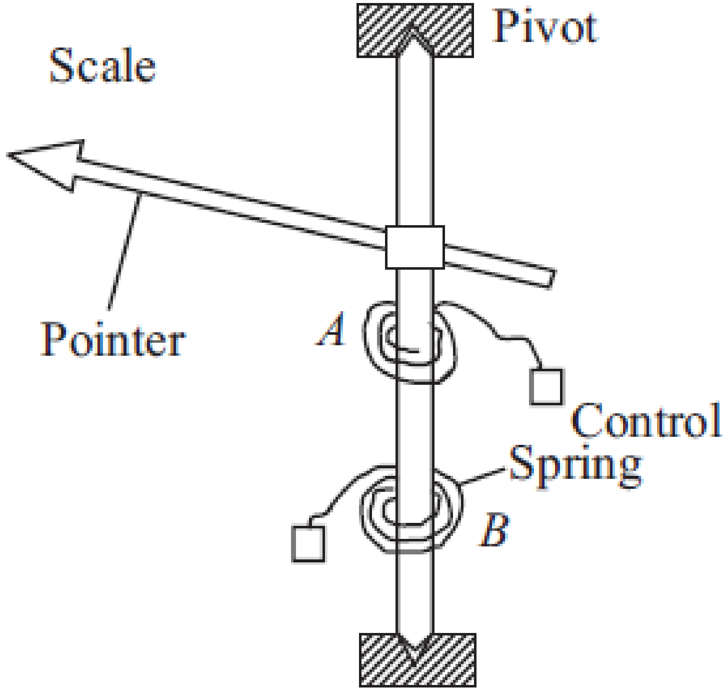
CONTROLLING TORQUE

- The magnitude of the moving system would be some what indefinite under the influence of deflecting torque, unless the controlling torque existed to oppose the deflecting torque.
- It increases with increase in deflection of moving system.
- Under the influence of controlling torque the pointer will return to its zero position on removing the source producing the deflecting torque.
- Without controlling torque the pointer will swing at its maximum position & will not return to zero after removing the source.

- Controlling torque is produced either by spring or gravity control.

Spring Control:

- When the pointer is deflected one spring unwinds itself while the other is twisted. This twist in the spring produces restoring (controlling) torque, which is proportional to the angle of deflection of the moving systems.



Spring Control

$$T_c \alpha \ \theta$$

$$T_c = K_s\theta$$

$$T_d \propto I$$

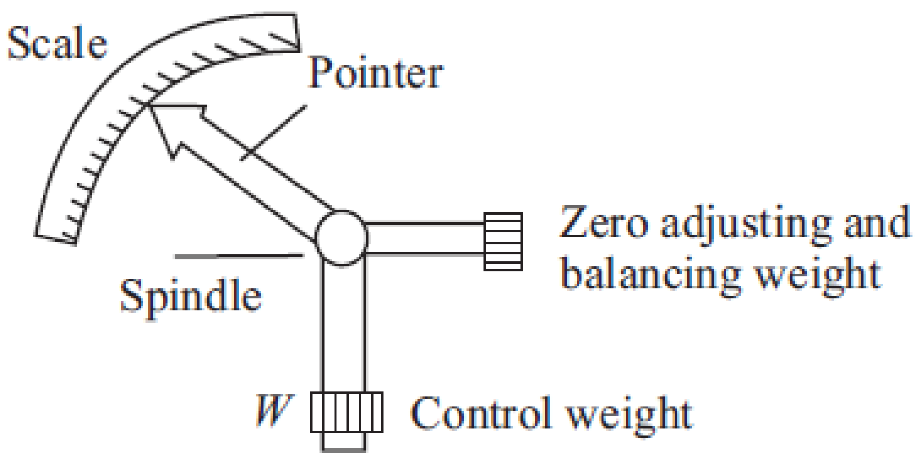
$$T_d = K_s I$$

$$T_c = T_d$$

$$\theta = I$$

Gravity Control

- In gravity controlled instruments, *a small adjustable weight is attached to the* spindle of the moving system such that the deflecting torque produced by the instrument has to act against the action of gravity.
- Thus a controlling torque is obtained. This weight is called the *control weight. Another adjustable weight is also attached is the moving system for zero adjustment and balancing* purpose. This weight is called *Balance weight.*



DAMPING TORQUE

- We have already seen that the moving system of the instrument will tend to move under the action of the deflecting torque.
- But on account of the control torque, it will try to occupy a position of rest when the two torques are equal and opposite.
- However, due to inertia of the moving system, the pointer will not come to rest immediately but oscillate about its final deflected position as shown in figure and takes appreciable time to come to steady state.
- To overcome this difficulty a damping torque is to be developed by using a damping device attached to the moving system.

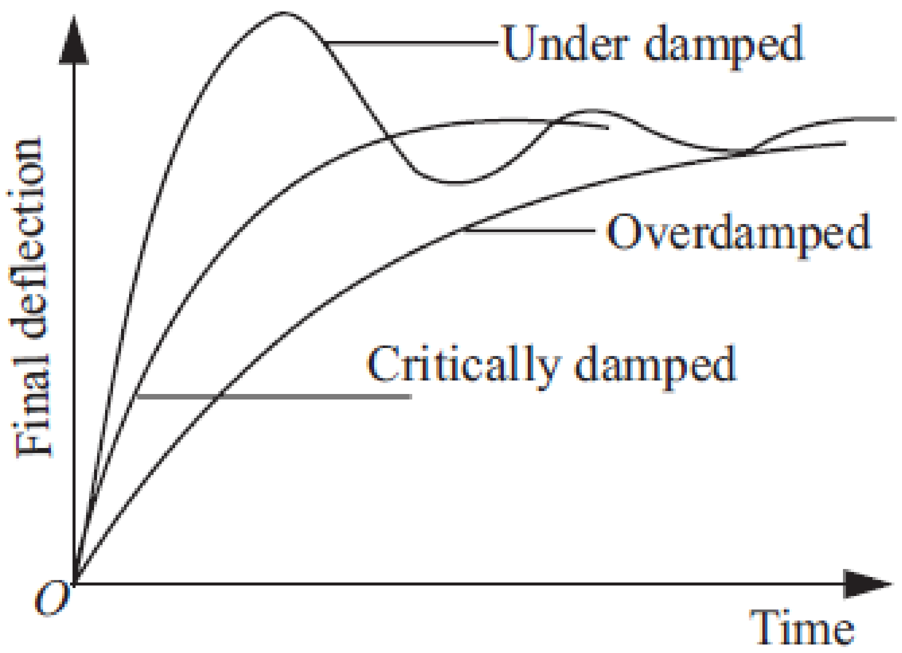
DAMPING TORQUE

- The damping torque is proportional to the speed of rotation of the moving system, that is

$$T_v = k_v \frac{d\theta}{dt}$$

where k_v = damping torque constant

$\frac{d\theta}{dt}$ = speed of rotation of the moving system



- Depending upon the degree of damping introduced in the moving system, the instrument may have any one of the

DAMPING TORQUE

1. **Under damped condition:**

   The response is oscillatory

2. **Over damped condition:**

   The response is sluggish and it rises very slowly from its zero position to final position.

3. **Critically damped condition:**

   When the response settles quickly without any oscillation, the system is said to be critically damped.

The damping torque is produced by the following methods:

1.Air Friction Damping

2.Fluid Friction Damping

3.Eddy Current Damping

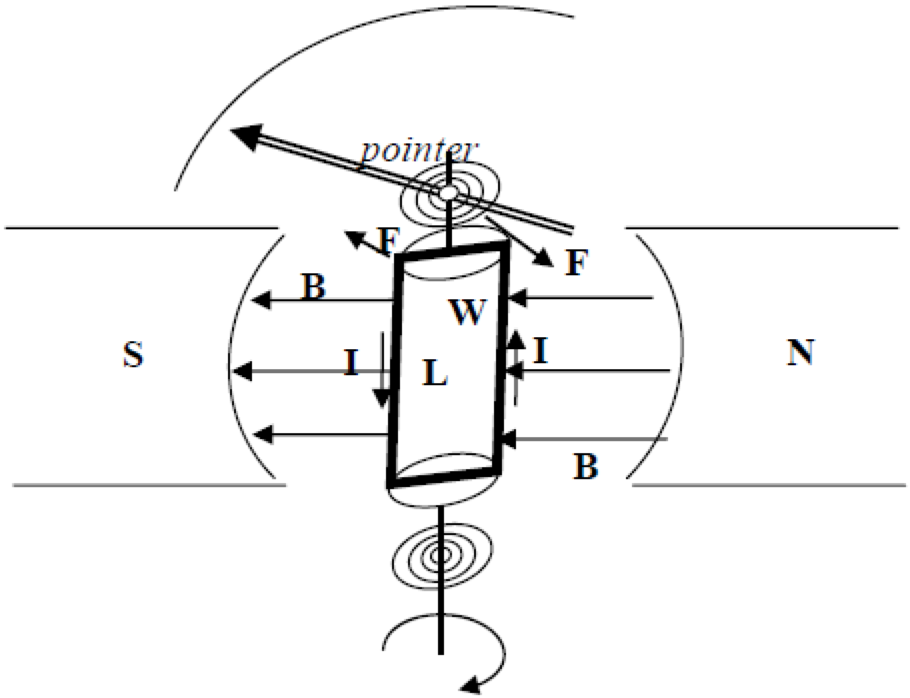
4.Electromagnetic

Moving-Coil instrument

- There are two types of moving coil instruments namely, permanent magnet moving coil type which can only be used for direct current, voltage measurements.
- The dynamometer type which can be used on either direct or alternating current, voltage measurements.

PERMANENT MAGNET MOVING COIL

"The principle operation of PMMC is based upon the principle of current carrying conductor is placed in a magnetic field it is acted upon by force which tends to move it."

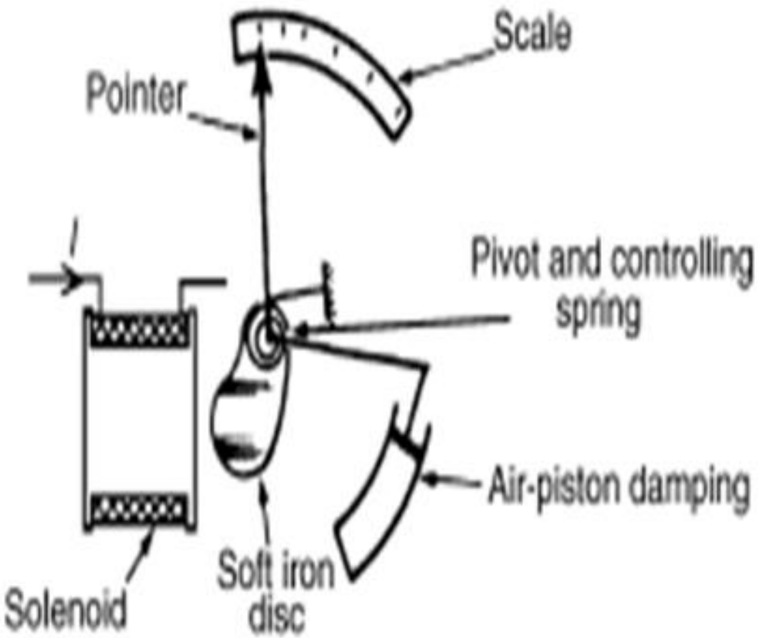


Moving-iron instrument

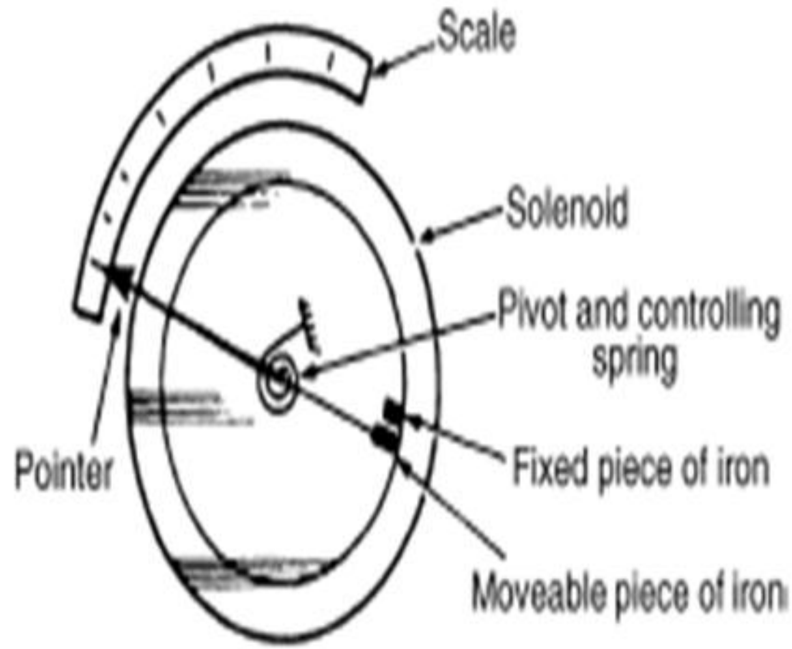
- An attraction type of moving-iron instrument is shown diagrammatically in Figure. When current flows in the solenoid, a pivoted soft-iron disc is attracted towards the solenoid and the movement causes a pointer to move across a scale.

- In the repulsion type moving-iron instrument shown diagrammatically in Figure, two pieces of iron are placed inside the solenoid, one being fixed, and the other attached to the spindle carrying the pointer.

Moving-iron instrument



(a) ATTRACTION TYPE



(b) REPULSION TYPE