INSTITUTE OF AERONAUTICAL ENGINEERING

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AERONAUTICAL ENGINEERING

IV B. Tech I semester (JNTUH-R15)

CONTROL THEORY – APPLICATION TO FLIGHT CONTROL SYSTEMS

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UNIT-I
Control Systems-Modeling-Feedback Control

- Dynamical systems are mathematical objects used to model physical phenomena whose state (or instantaneous description) changes over time.
- The models are widely used in control systems.
- The system state at time t is an instantaneous description of the system which is sufficient to predict the future states of the system without recourse to states prior to t.
- Physical model accurately describes the behavior of the physical process in so far as we are concerned.

The basic components of a control system are:

a) Input or Objective of control
b) Plant or control system components
c) Outputs or Results.

System – An interconnection of elements and devices for a desired purpose.

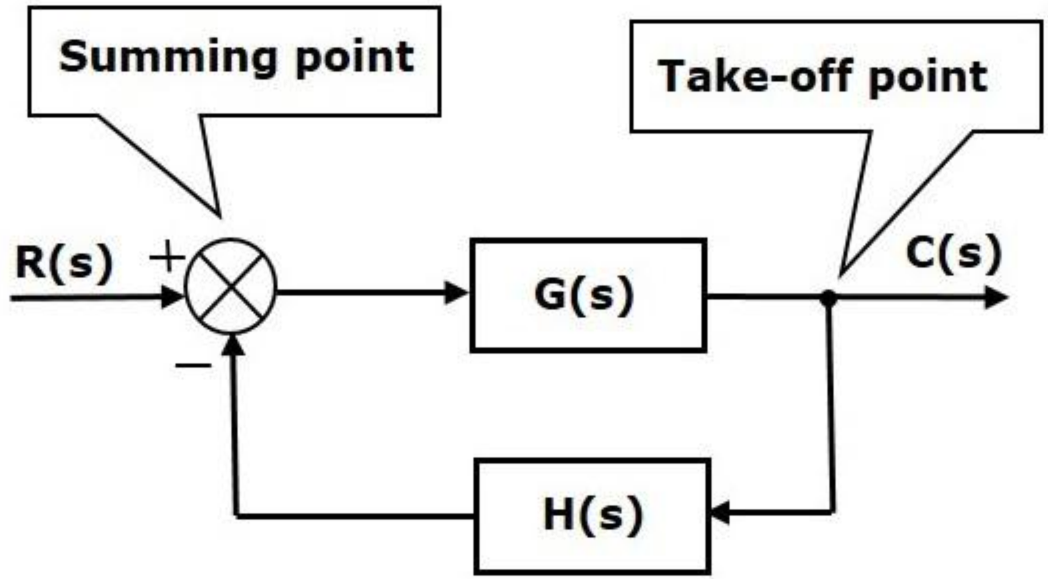
Control System – An interconnection of components forming a system configuration that will provide a desired response.

Process – The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.

Process to be controlled.

Block Diagram Representation

- ✓ A control system may consist of a number of components.
- ✓ To show the function performed by each component, in control engineering, we commonly use a diagram called the block diagram.
- ✓ In block diagram all system variables are linked to each other through functional blocks.
- ✓ The functional block is a symbol for the mathematical operation on the input signal to the block that produces the output.
- ✓ The transfer function of the components is usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals.



Block Diagram Representation

Noise

- A noise is a signal that tends to adversely affect the value of output of a system.
- It is undesired signal.
- Source of noise can be internal or external to the control system.
- Example - All electrical components generate electrical noise at various frequencies.

Function of Control as Regulation

- A control system can be used to keep the output constant irrespective of the variation in input.
- Example - Voltage regulator
  Cyclo converter

Function of Control as Tracking

- A control system can be used for tracking the input.
- Example - Guided air to air missile,
  Command guidance system of surface to air missile

Sensitivity

$$S_G^M = \frac{\partial M / M}{\partial G / G} = \frac{percentage\ change\ in\ M}{percentage\ change\ in\ G}$$

Robustness

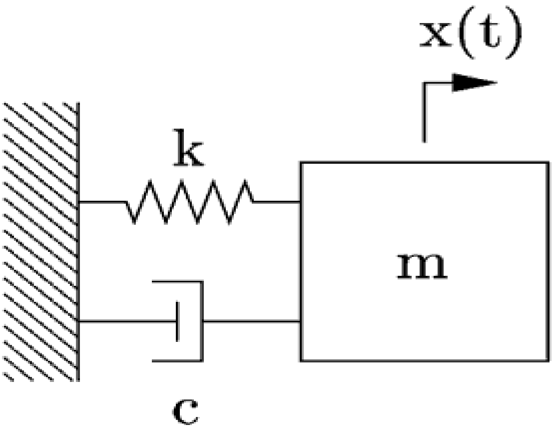
a) It is very sensitive to input command.
b) It is insensitive to system parameter variations due to aging, temperature variations and other environmental conditions.
c) It is insensitive to noise.
d) It is insensitive to external disturbance.
e) It has good tracking capability.
f) It has small errors.

Need for stable, effective (responsive), Robust Control:

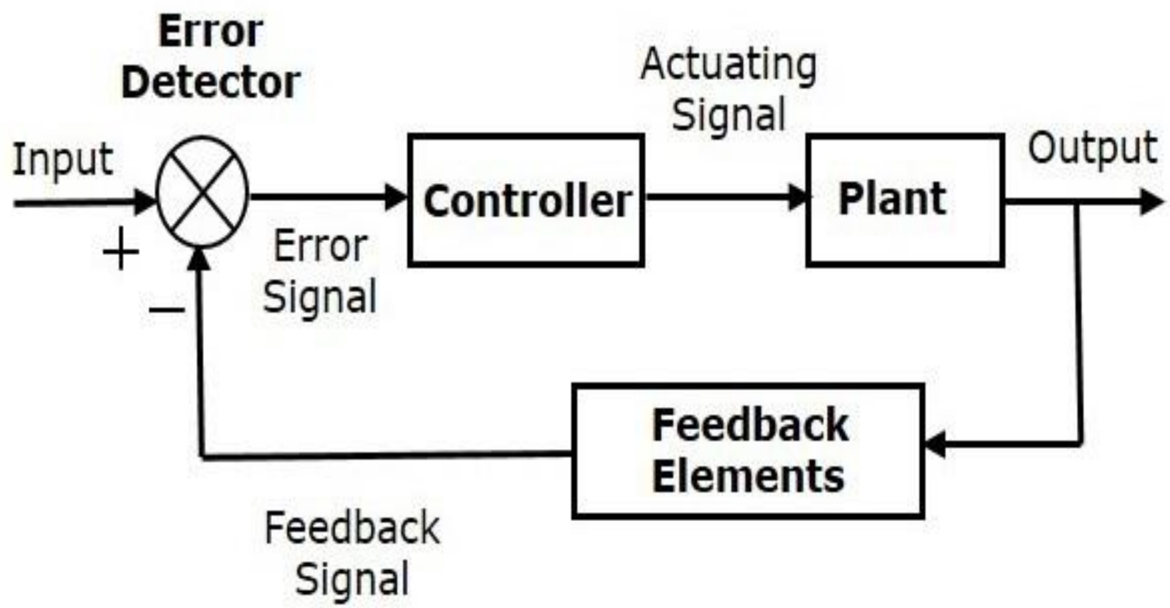
- To be useful a control system should be stable.
- A stable system may be defined as one that will have a bounded response for all possible bounded input.
- A linear system will be stable if and only if all the poles of its transfer function are located on the left side of imaginary ($j\omega$) axis.
- An unstable system is of no use as output keeps increasing with time even when input is constant or zero.
- Response is characterized in terms of its rise time, settling time.
- It is an indication of how fast the system responds to input.
- For example in aircraft control when pilot pulls the control stick he expects aircraft nose to go quickly up.
- Robust control system properties have already been explained above.

Modeling of Dynamical system by Differential Equation- system parameters, order of the system

A dynamical system can be modeled using the differential equations. The differential is derived by finding the relation between input and output using mathematical equations governing the system.



Mass, spring, damper system

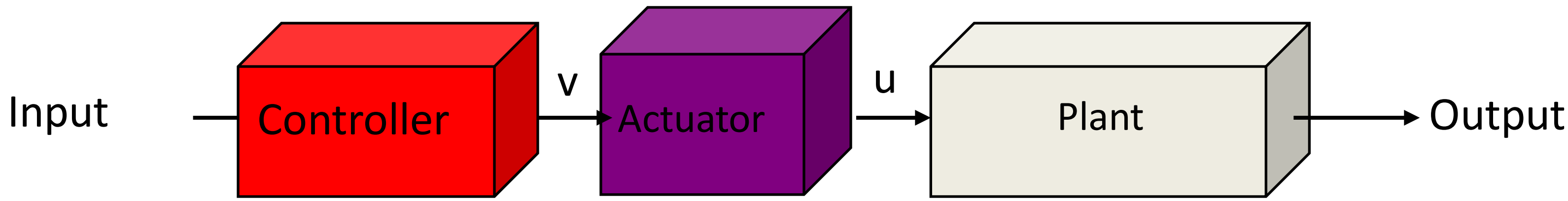


BLOCK DIAGRAM SISO CLOSED LOOP SYSTEM

Linear and Non Linear System:

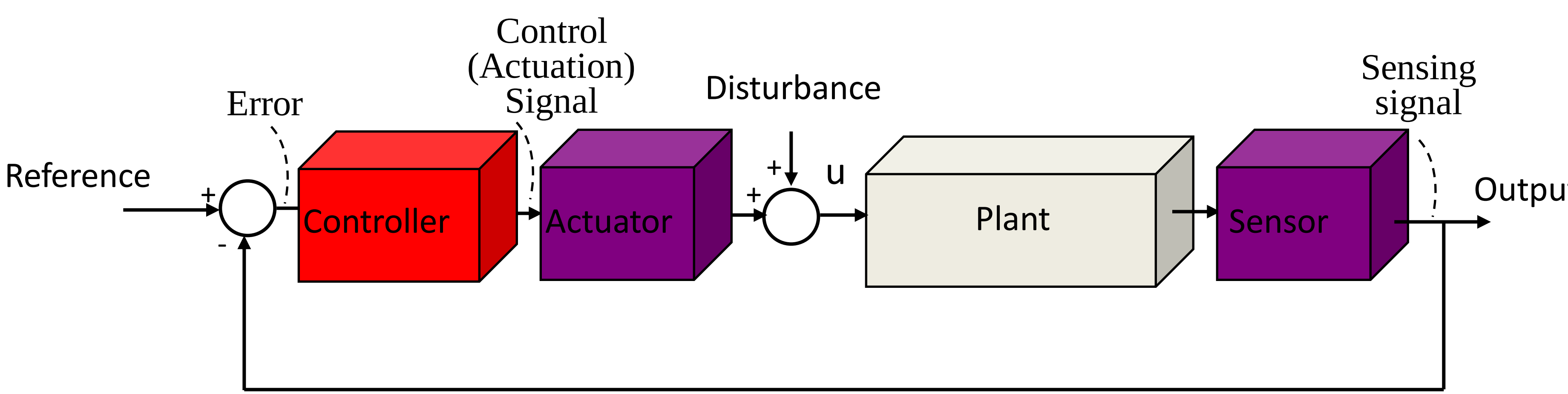
- A system is called linear if the principle of superposition applies.
- The principle of superposition states that the response produced by the simultaneous application of the different forcing functions is the sum of two individual responses.
- Hence for the linear system, the response to several inputs can be calculated by treating one input at a time and adding the results.
- It is this principle that allows one to build up complicated solutions to the linear differential equations from simple solutions.
- In an experimental investigation of a dynamical system, if cause and effect are proportional, thus implying that the principle superposition holds, then the system can be considered linear.

Open Loop Control Systems:



Sensitive to changing in parameters and disturbance.

Closed Loop Control Systems: (Feedback Systems)



Error signal= Reference – Sensing signal

Application of feedback in Stability Augmentation System, Control Augmentation, Automatic control-Examples:

The FCS of an aircraft generally consists of three important parts.

- Stability Augmentation system (SAS)
- Control Augmentation system (CAS)
- Automatic Control System
- Lead Compensator/High Frequency Filter
- Lag Compensator/High Frequency Filter
- Lead Lag filter
- Washout Filter

Closed-Loop Transfer Functions

The block diagrams considered so far have been specifically developed for the stirred-tank blending system. The more general block diagram in Fig. 11.8 contains the standard notation:

Y = controlled variable

U = manipulated variable

D = disturbance variable (also referred to as *load variable)*

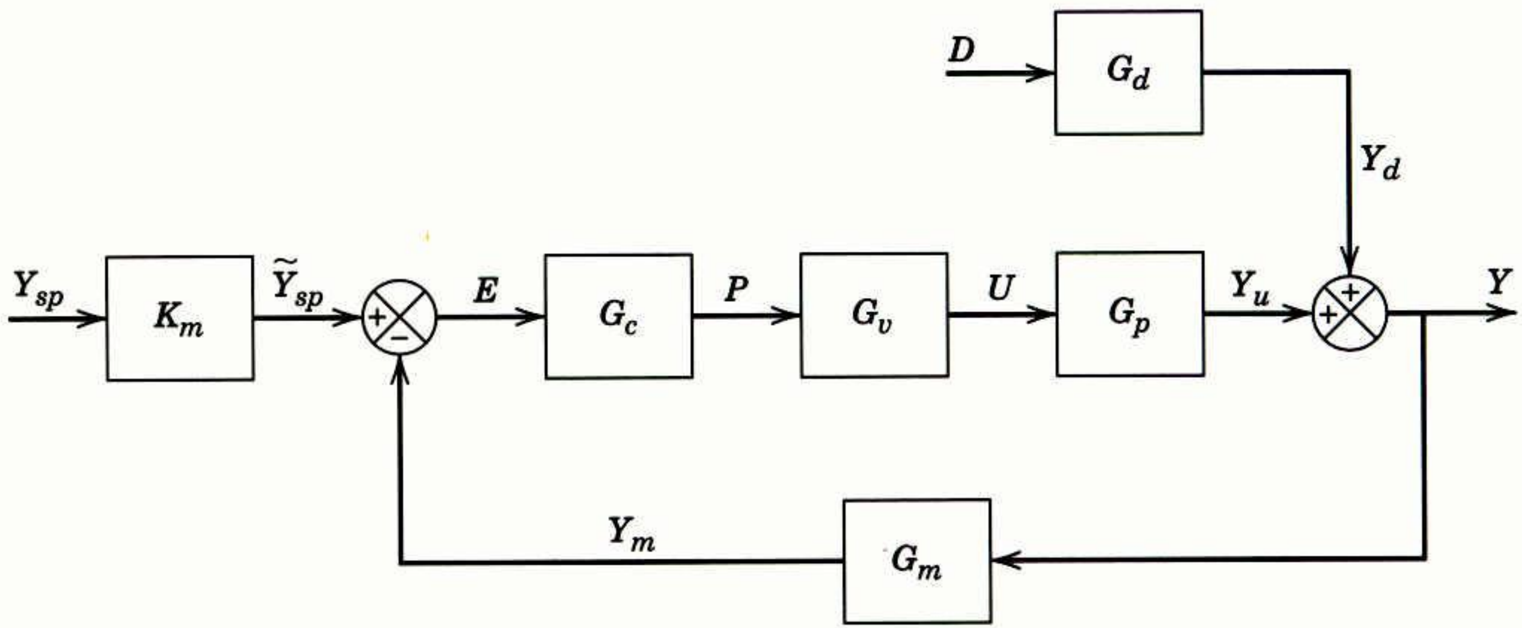
P = controller output

E = error signal

Y_m = measured value of Y

Y_{sp} = set point

$\tilde{Y}_{sp}$ = internal set point (used by the controller)



Standard block diagram of a feedback control system

Y_u = change in Y due to U

Y_d = change in Y due to D

G_c = controller transfer function

G_v = transfer function for final control element (including K_{IP}, if required)

G_p = process transfer function

G_d = disturbance transfer function

G_m = transfer function for measuring element and transmitter

K_m = steady-state gain for G_m

Block Diagram Reduction

In deriving closed-loop transfer functions, it is often convenient to combine several blocks into a single block. For example, consider the three blocks in series in Fig. 11.10. The block diagram indicates the following relations:

$$X_1 = G_1 U$$

$$X_2 = G_2 X_1 \tag{11-11}$$

$$X_3 = G_3 X_2$$

By successive substitution,

$$X_3 = G_3 G_2 G_1 U \tag{11-12}$$

or

$$X_3 = G U \tag{11-13}$$

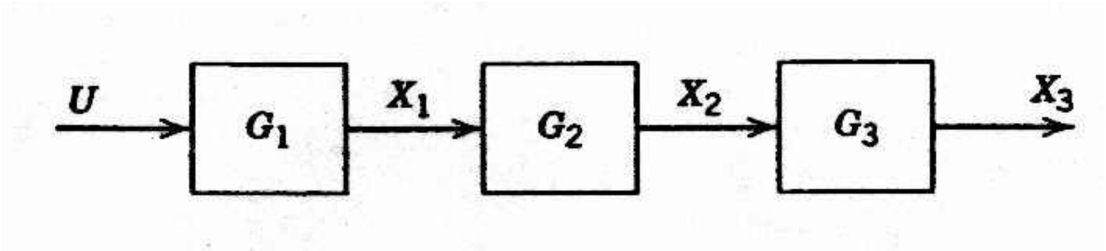


Figure 11.10 Three blocks in series.

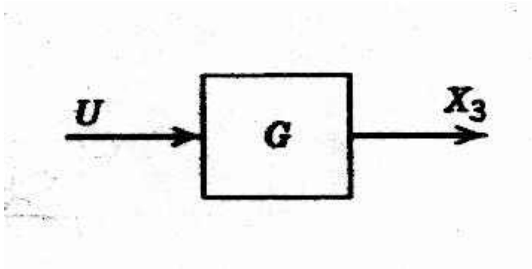


Figure 11.11 Equivalent block diagram.

General Expression for Feedback Control Systems

Closed-loop transfer functions for more complicated block diagrams can be written in the general form:

$$\frac{Z}{Z_i} = \frac{\Pi_f}{1 + \Pi_e} \qquad (11\text{-}31)$$

where:

Z =is the output variable or any internal variable within the control loop

Z_i =is an input variable (e.g., Y_{sp} or D)

Π_f =product of the transfer functions in the *forward* path from Z_i to Z

Π_e =product of *every* transfer function in the feedback loop

The Concept of Stability

A stable system is a dynamic system with a bounded response to a bounded input

In terms of linear systems, stability requirement may be defined in terms of the location of the poles of the closed-loop transfer function.

The closed-loop transfer function is written as

$$T(s) = \frac{p(s)}{q(s)} = \frac{K \prod_{i=1}^{M} (s + z_i)}{s^N \prod_{k=1}^{Q} (s + \sigma_k) \prod_{m=1}^{R} [s^2 + 2\alpha_m s + (\alpha_m^2 + \omega_m^2)]}$$

The Routh-Hurwitz Stability Criterion

This a necessary and sufficient criterion for the stability of linear systems

$$\Delta(s) = q(s) = a_n s^n + a_{n-1} s^{n-1} + .. + a_1 s + a_0 = 0$$

$$a_n (s - r_1)(s - r_2)..(s - r_n) = 0$$

$$q(s) = a_n s^n - a_n (r_1 + r_2 + .. + r_n) s^{n-1} +$$

$$a_n (r_1 r_2 + r_2 r_3 + r_1 r_3 + ..) s^{n-2}$$

$$- a_n (r_1 r_2 r_3 + r_1 r_2 r_4 ..) s^{n-3} + ..$$

$$+ a_n (-1)^n r_1 r_2 r_3 .. r_n = 0$$

The Routh-Hurwitz criterion is based on ordering the coefficients of the characteristic equation. **It states that the number of roots of q(s) with positive real parts is equal to the number of changes in sign of the first column of the Routh array.**

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_1 s + a_0 = 0$$

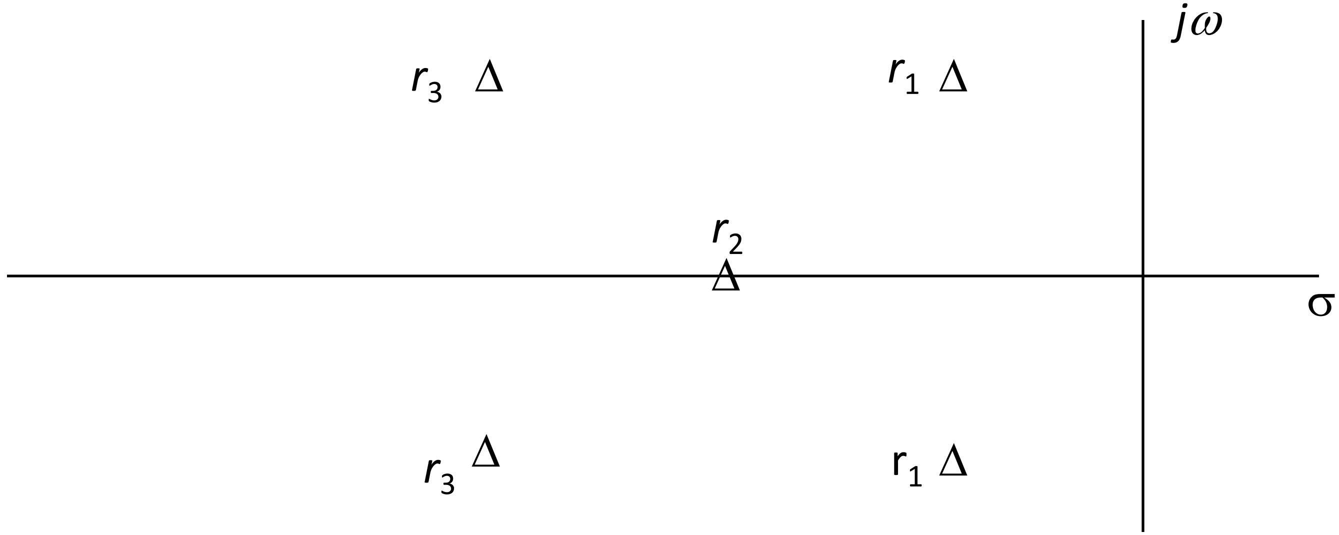
$$\begin{array}{lllll}
s^n & a_n & a_{n-2} & a_{n-4} & \cdot\cdot \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdot\cdot \\
s^{n-2} & b_{n-1} & b_{n-3} & b_{n-5} & \\
s^{n-3} & C_{n-1} & C_{n-3} & C_{n-5} &
\end{array}$$

$$b_{n-1} = \frac{(a_{n-1})(a_{n-2}) - a_n(a_{n-3})}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}$$

$$b_{n-3} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}; C_{n-1} = \frac{-1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & \mathrm{b}_{n-3} \end{vmatrix}$$

The Relative Stability of Feedback Control Systems

- If the system satisfies the Routh-Hurwitz criterion and is absolutely stable, it is desirable to determine the relative stability that is, it is necessary to investigate the relative damping of each root of the characteristic equation.
- The relative stability of a system may be defined as the property that is measured by the relative real part of each root or pair of roots.



The root r_2 is relatively more stable than the roots r_1

The Stability of State Variable Systems

- The stability of a system modeled by a state variable flow graph model may be readily ascertained.
- The stability of a system with an input-output transfer function $T(s)$ may be determined by examining the denominator polynomial of $T(s) = p(s) / q(s)$.
- The polynomial $q(s)$, when set equal to zero, is called the characteristic equation.
- The stability of the system may be evaluated with the characteristic equation associated with the system matrix **A**

Stability

- Most industrial processes are stable without feedback control. Thus, they are said to be *open-loop stable* or *self-regulating*.
- An open-loop stable process will return to the original steady state after a transient disturbance (one that is not sustained) occurs.
- By contrast there are a few processes, such as exothermic chemical reactors, that can be *open-loop unstable*.

Definition of Stability. An unconstrained linear system is said to be stable if the output response is bounded for all bounded inputs. Otherwise it is said to be unstable.

Root Locus Diagrams

Example 11.13

Consider a feedback control system that has the open-loop transfer function,

$$G_{OL}(s) = \frac{4K_c}{(s+1)(s+2)(s+3)} \qquad (11\text{-}108)$$

Plot the root locus diagram for $0 \le K_c \le 20.$

Solution

The characteristic equation is 1 + G_{OL} = 0 or

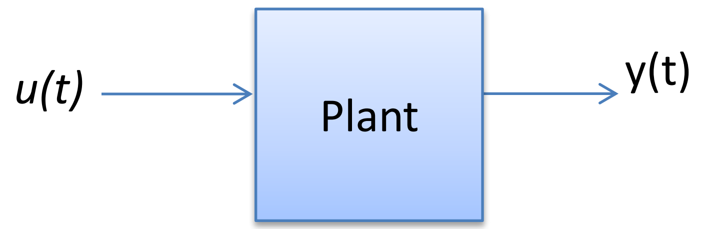
$$(s+1)(s+2)(s+3) + 4K_c = 0 \qquad (11\text{-}109)$$

- The root locus diagram in Fig. 11.27 shows how the three roots of this characteristic equation vary with K_c.
- When $K_c = 0$, the roots are merely the poles of the open-loop transfer function, -1, -2, and -3.

UNIT- II
PERFORMANCE-TIME, FREQUENCY AND S-DOMAIN DESCRIPTION

Transfer Function

- Transfer Function is the ratio of Laplace transform of the output to the Laplace transform of the input. Consider all initial conditions to zero.



$$If \qquad \ell u(t) = U(S) \qquad and$$

$$\ell y(t) = Y(S)$$

Where ℓ is the Laplace operator.

Transfer Function

- The transfer function G(S) of the plant is given as

$$G(S) = \frac{Y(S)}{U(S)}$$

Why Laplace Transform?

- Using Laplace transform, we can convert many common functions into algebraic function of complex variable *s*.
- For example

$$\ell \sin \omega t = \frac{\omega}{s^2 + \omega^2}$$

- Where *s* is a complex variable (complex frequency) and is given as

$$\ell e^{-at} = \frac{1}{s + a}$$

$$s = \sigma + j\omega$$

Laplace Transform of Derivatives

- Not only common function can be converted into simple algebraic expressions but calculus operations can also be converted into algebraic expressions.
- For example

$$\ell \frac{dx(t)}{dt} = sX(s) - x(0)$$

$$\ell \frac{d^2 x(t)}{dt^2} = s^2 X(s) - s \cdot x(0) - \frac{dx(0)}{dt}$$

Laplace Transform of Derivatives

In general

$$\ell \frac{d^n x(t)}{dt^n} = s^n X(s) - \sum_{k=1}^{n} s^{n-k} x^{(k-1)}(0)$$

Where $x(0)$ is the initial condition of the system.

Transfer Function

- Transfer function can be used to check
  - The stability of the system
  - Time domain and frequency domain characteristics of the system
  - Response of the system for any given input

Stability of Control System

- There are several meanings of stability, in general there are two kinds of stability definitions in control system study.
  - Absolute Stability
  - Relative Stability

Stability of Control System

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

- Roots of denominator polynomial of a transfer function are called ‘poles’.

- The roots of numerator polynomials of a transfer function are called ‘zeros’.

Stability of Control System

- Poles of the system are represented by ‘x’ and zeros of the system are represented by ‘o’.
- System order is always equal to number of poles of the transfer function.
- Following transfer function represents n^{th} order plant (i.e., any physical object).

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

Stability of Control System

- Poles is also defined as “it is the frequency at which system becomes infinite”. Hence the name pole where field is infinite.

$$\frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

- Zero is the frequency at which system becomes 0.

Examples

- Consider the following transfer functions.
  - Determine whether the transfer function is proper or improper
  - Calculate the Poles and zeros of the system
  - Determine the order of the system
  - Draw the pole-zero map
  - Determine the Stability of the system

i) $$G(s) = \frac{s+3}{s(s+2)}$$

ii) $$G(s) = \frac{s}{(s+1)(s+2)(s+3)}$$

iii) $$G(s) = \frac{(s+3)^2}{s(s^2+10)}$$

iv) $$G(s) = \frac{s^2(s+1)}{s(s+10)}$$

UNIT - III
SPECIFICATION OF CONTROL SYSTEM PERFORMANCE REQUIREMENTS , SYSTEM SYNTHESIS, CONTROLLERS, COMPENSATION TECHNIQUE

Control system specifications and design involves the following steps.

- Determine what the system should do and how to do it (Performance and design specification).
- Determine the controller or compensator configuration relative to how it is connected to the controlled process.
- Determine the parameter values of the controller to achieve the design.

Zeros and poles of a transfer function

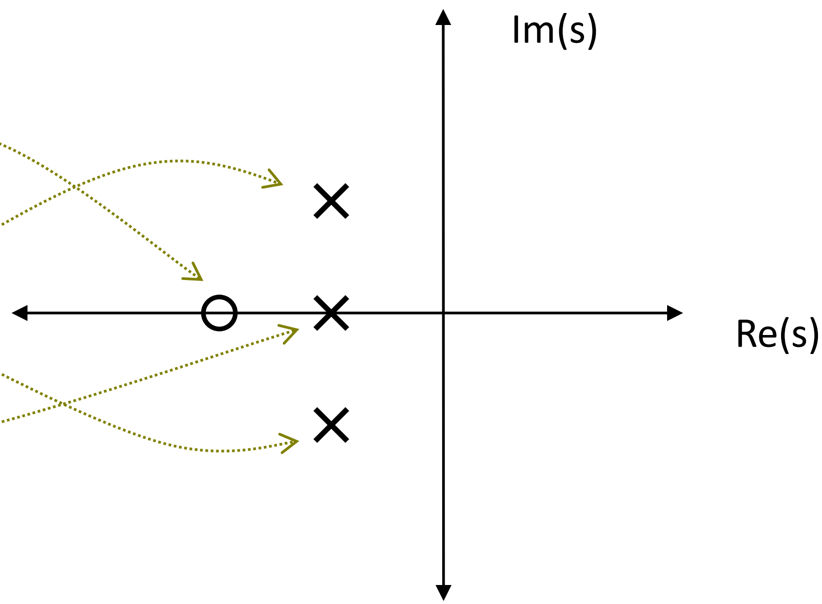
- Let G(s)=N(s)/D(s), then
  - Zeros of G(s) are the roots of N(s)=0
  - Poles of G(s) are the roots of D(s)=0

$$G(s) = \frac{(s+2)}{(s+1+j)(s+1-j)(s+1)}$$

$$z_1 = -2$$

$$p_{1,2} = -1 \pm j$$

$$p_3 = -1$$



Theorems

- Initial Value Theorem

$$\lim_{t \to 0} x(t) = \lim_{s \to \infty} s\mathcal{L}(x(t))$$

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s\mathcal{L}(x(t))$$

- Final Value Theorem
  - If all poles of X(s) are in the left half plane (LHP), then

First order system

- If the initial condition was not zero, then

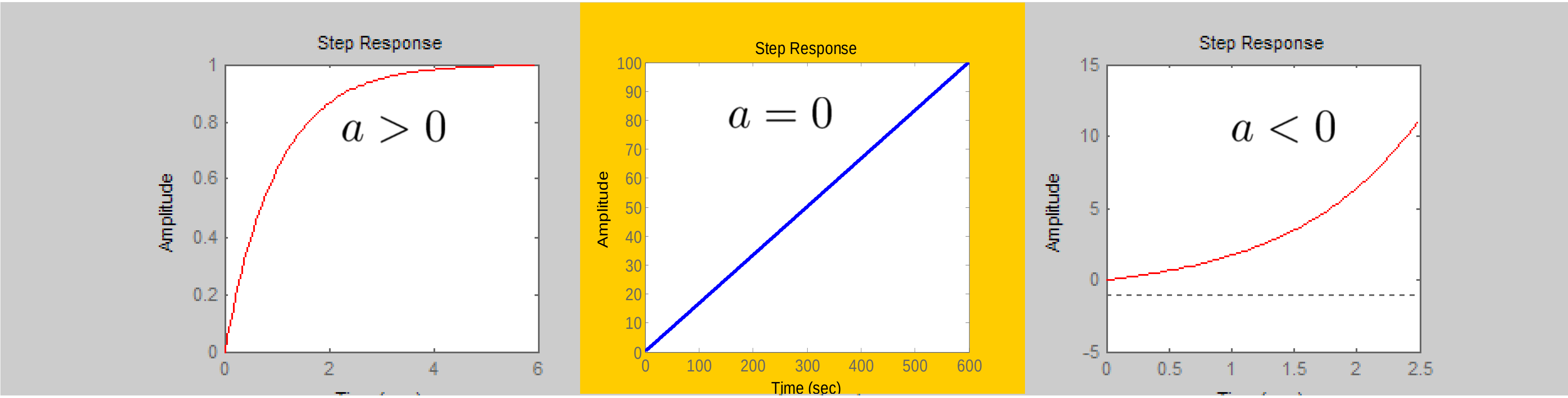
$$\tau \dot{y}(t) + y(t) = Ku(t) \xrightarrow{\mathcal{L}} \tau s Y(s) - \underbrace{y(0)}_{\neq 0} + Y(s) = KU(s)$$

$$\begin{aligned} y(t) &= \mathcal{L}^{-1}\left[(y(0)/K + U(s))\frac{K}{(\tau s + 1)}\right] \\ &= \mathcal{L}^{-1}\left[(y(0)/K + U(s))G(s)\right] \\ &= \underbrace{\frac{y(0)}{K}\mathcal{L}^{-1}(G(s))}_{\text{from the initial condition}} + \underbrace{\mathcal{L}^{-1}(G(s)U(s))}_{\text{from the input}} \end{aligned}$$

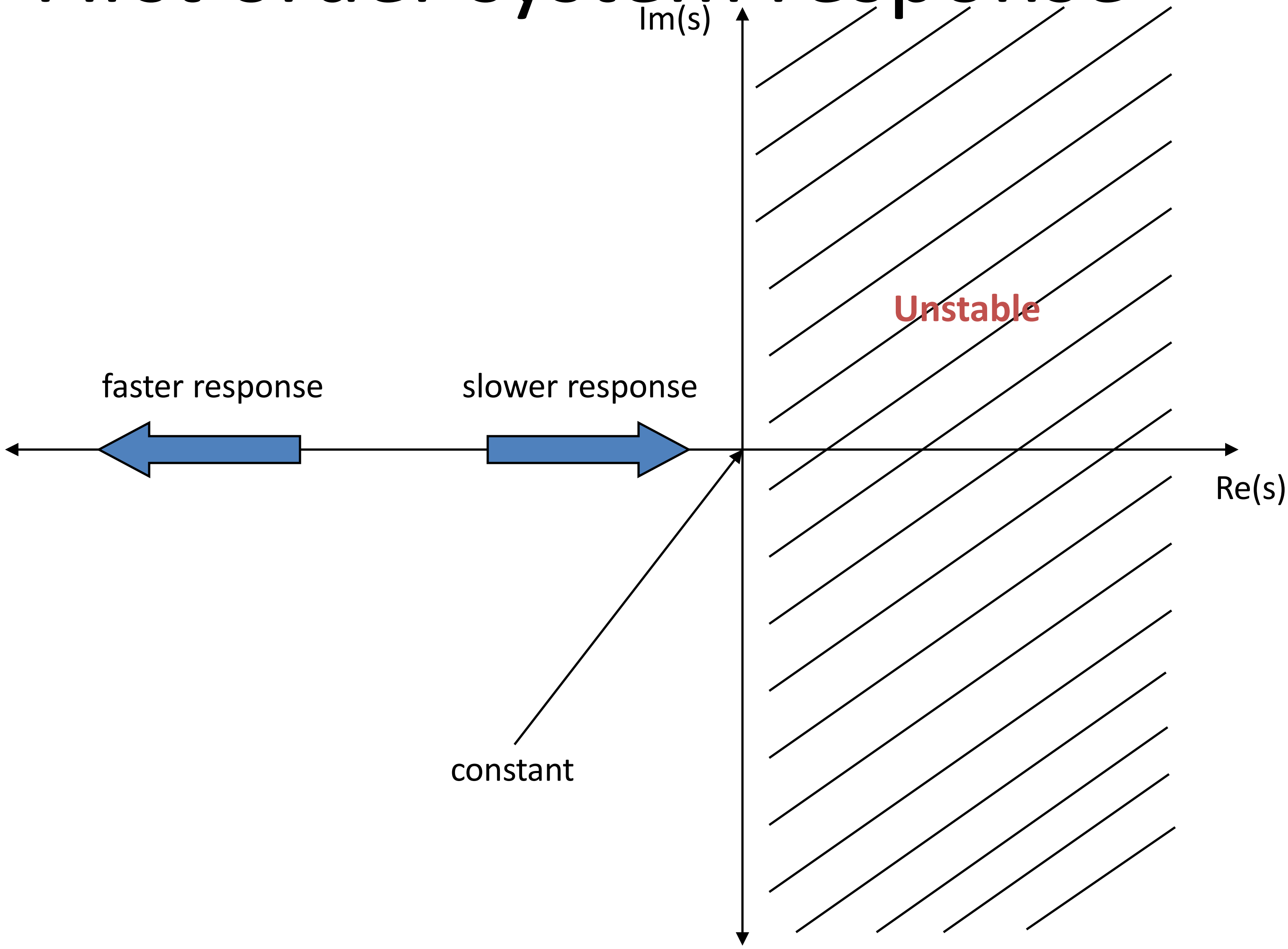
Physical meaning of the impulse response

First order system response

- System transfer function : $$H(s) = \frac{b}{s+a}$$
- Impulse response : $$h(t) \;=\; \mathcal{L}^{-1}[H(s)] \;=\; b\,e^{-at}\,1(t)$$
- Step response : $$y_{step}(t) \;=\; \mathcal{L}^{-1}[H(s)/s] \;=\; \frac{b}{a}(1-e^{-at})1(t)$$



First order system response



First order system – Time specifications.

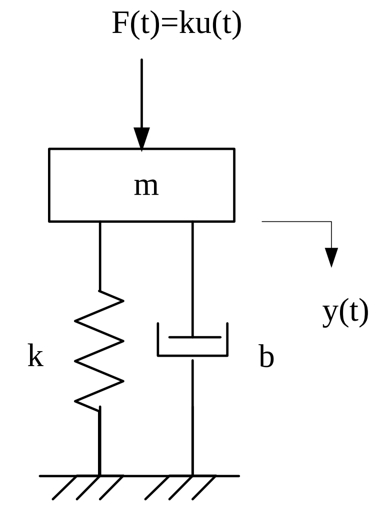
Time specs:

$$H(s) = \frac{b}{s+a}$$

- Steady state value : $y_{ss} = \lim_{t\to\infty} y_{step}(t) = \frac{b}{a}$
- Time constant : $T = \frac{1}{a}$ $\quad y_{step}(T) = 0.63\, y_{ss}$
- Rise time : $T_r = \frac{2.2}{a}$ $\quad$ Time to go from $0.1\, y_{ss}$ to $0.9\, y_{ss}$
- Settling time : $T_s = \frac{4}{a}$ $\quad y_{step}(T_s) = 0.98\, y_{ss}$

Second order system (mass-spring-damper system)

$$\ddot{y}(t) + \frac{b}{m}\dot{y}(t) + \frac{k}{m}y(t) = \frac{k}{m}u(t)$$



ζ : dampling ratio, ω_n : natural frequency

$$2\zeta\omega_n = b/m,\ \omega_n^2 = k/m$$

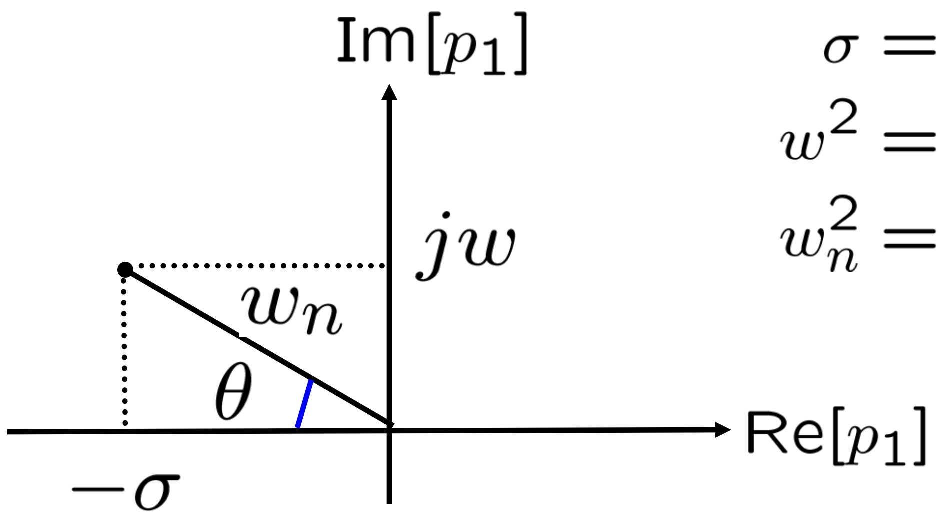
$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 u(t)$$

$$\frac{Y(s)}{U(s)} = H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n^2 s + \omega_n^2}$$

Underdamped second order system

$$p_1 = -\sigma + jw = w_n e^{j(\pi-\theta)} = -w_n e^{-j\theta}$$

$$\theta = \tan^{-1}\left(\frac{w}{\sigma}\right) = \tan^{-1}\left((w_n\sqrt{1-\zeta^2})/\ (\zeta w_n)\right)$$
$$= \tan^{-1}\left(\sqrt{1-\zeta^2}/\zeta\right)$$

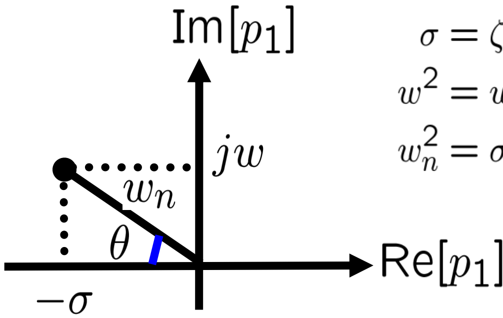


$$\sigma = \zeta w_n$$
$$w^2 = w_n^2(1-\zeta^2)$$
$$w_n^2 = \sigma^2 + w^2$$

Unit step response of undamped

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\begin{aligned}
&\Rightarrow \left[\cos(wt) + \frac{\sigma}{w}\sin(wt)\right] \\
&= \frac{w_n}{w}\left[\frac{w}{w_n}\cos(wt) + \frac{\sigma}{w_n}\sin(wt)\right] \\
&= \frac{w_n}{w}\left[\sin(\theta)\cos(wt) + \cos(\theta)\sin(wt)\right] \\
&= \frac{w_n}{w}\sin(wt+\theta)
\end{aligned}$$

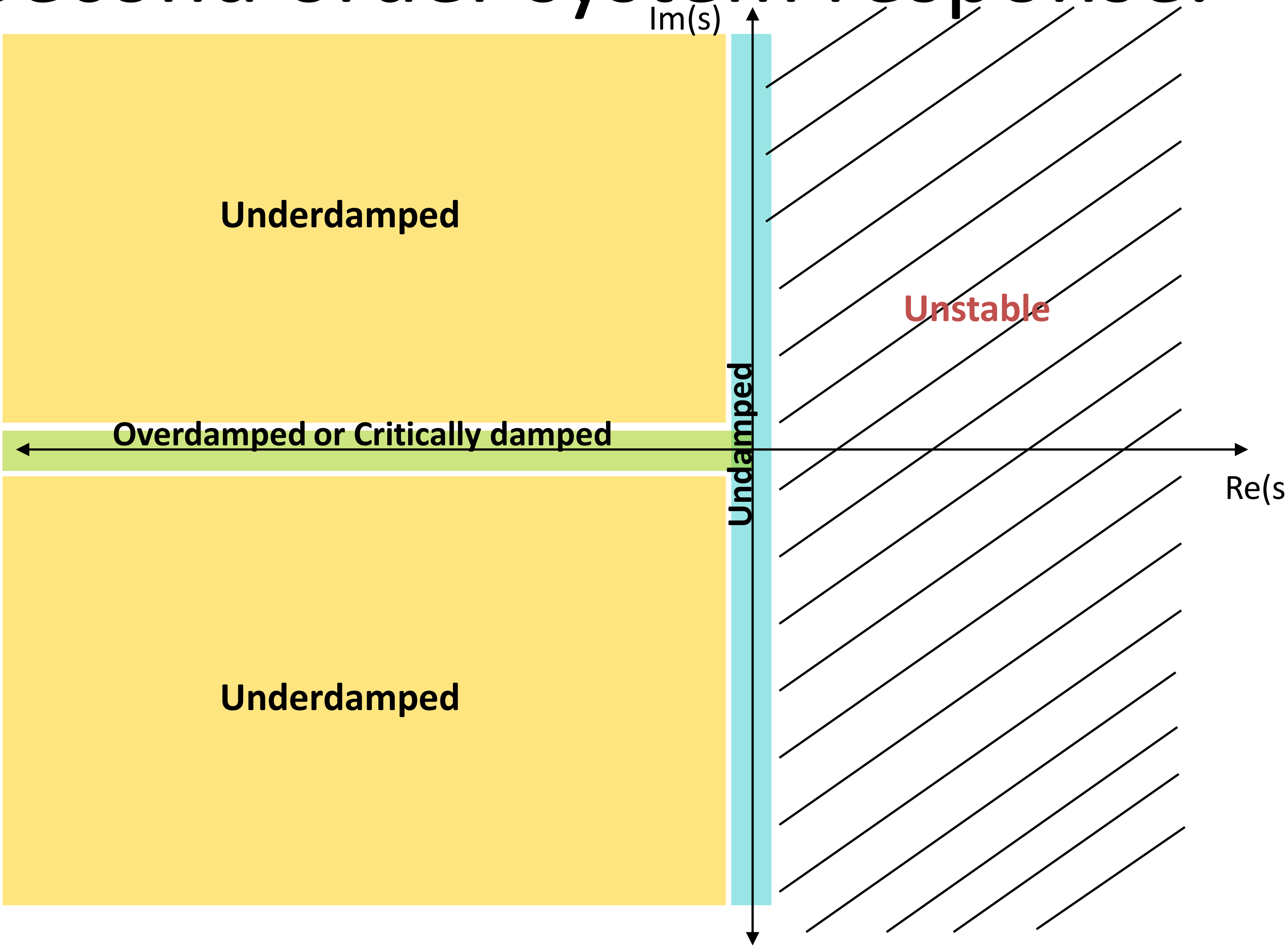


$$\sigma = \zeta w_n$$
$$w^2 = w_n^2(1-\zeta^2)$$
$$w_n^2 = \sigma^2 + w^2$$

$$\theta = \tan^{-1}\left(\frac{w}{\sigma}\right) = \tan^{-1}\left(\sqrt{1-\zeta^2}/\zeta\right)$$

$$\begin{aligned}
y_{step}(t) &= \left(1 - e^{-\sigma t}\frac{w_n}{w}\sin(wt+\theta)\right)1(t) \\
&= \left(1 - \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}w_n t + \theta)\right)1(t)
\end{aligned}$$

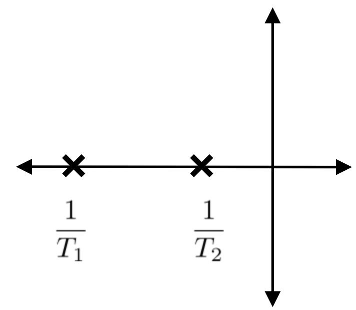
Second order system response.



Overdamped system response

- System transfer function :

$$H(s) = \frac{K}{(T_1 s + 1)(T_2 s + 1)}$$



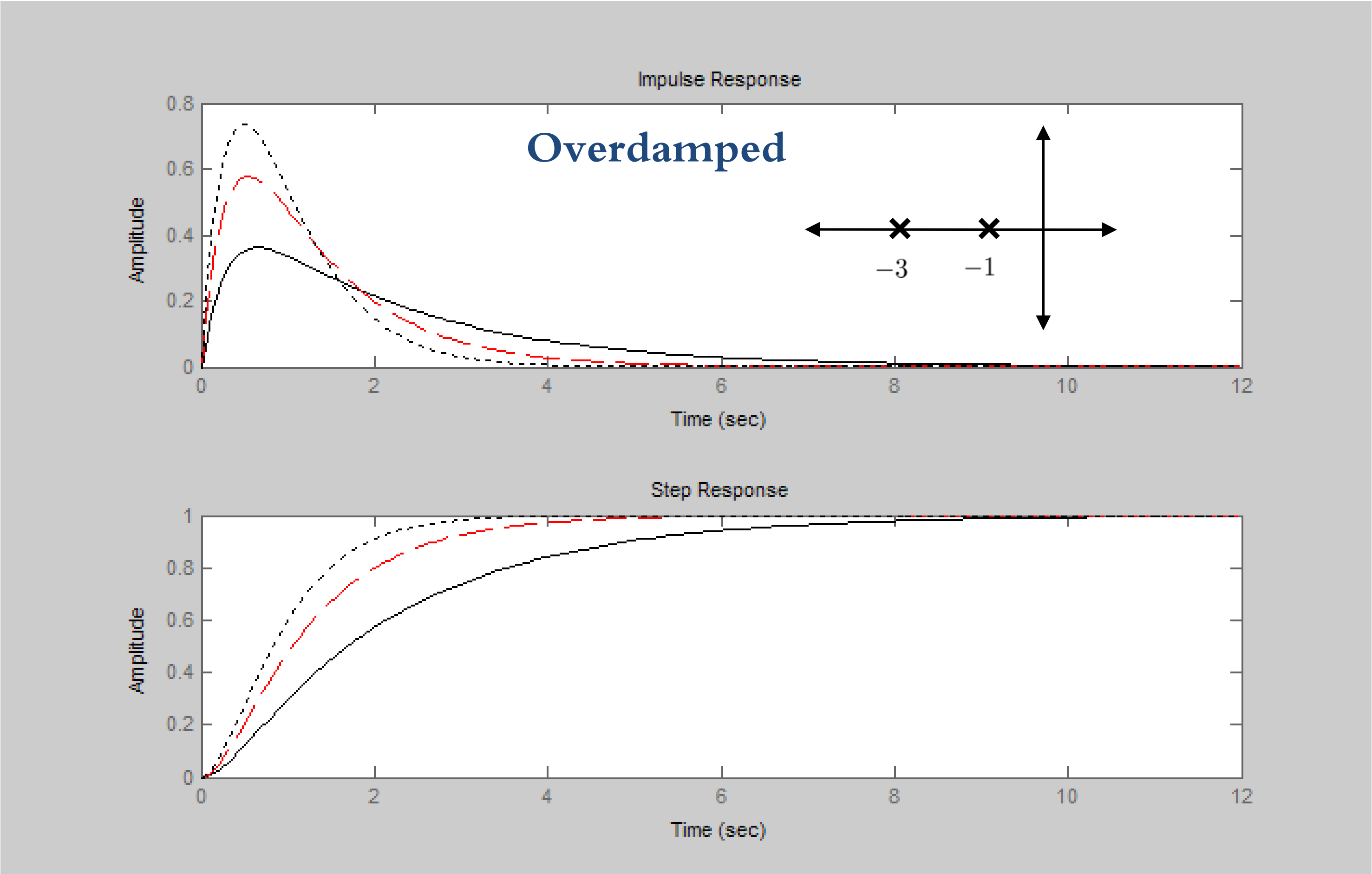
- Impulse response :

$$h(t) \;=\; \mathcal{L}^{-1}[H(s)] \;=\; \frac{K}{T_2 - T_1}\left(e^{-t/T_2} - e^{-t/T_1}\right) 1(t)$$

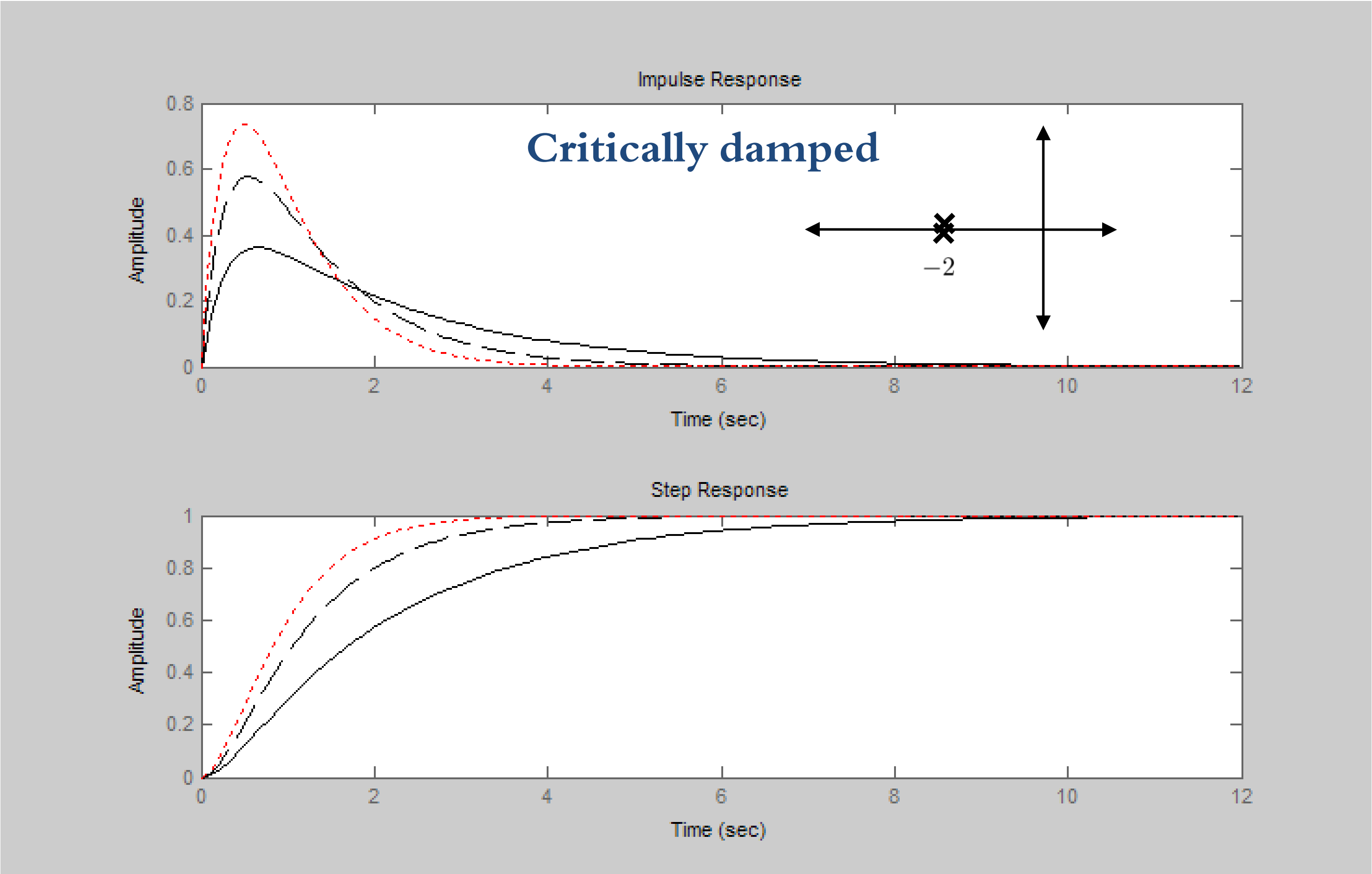
- Step response :

$$y_{step}(t) \;=\; \mathcal{L}^{-1}[H(s)/s] \;=\; K\left(1 + \frac{T_1}{T_2 - T_1}e^{-t/T_2} - \frac{T_2}{T_2 - T_1}e^{-t/T_1}\right) 1(t)$$

Overdamped and critically damped system response.



Overdamped and critically damped system response.

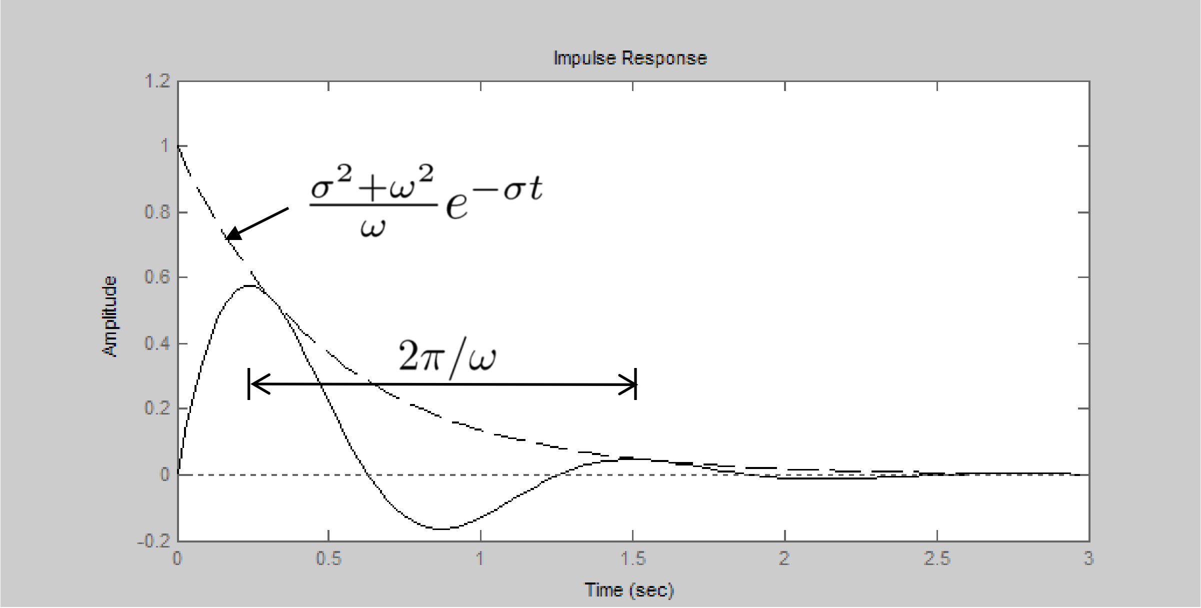


Second order impulse response – Underdamped and Undamped

$$H(s) = \underbrace{\frac{\omega_n^2}{s^2 + 2\zeta\omega_n\, s + \omega_n^2}}_{\text{Polar}} = \underbrace{\frac{\sigma^2 + \omega^2}{(s+\sigma)^2 + \omega^2}}_{\text{Cartesian}}$$

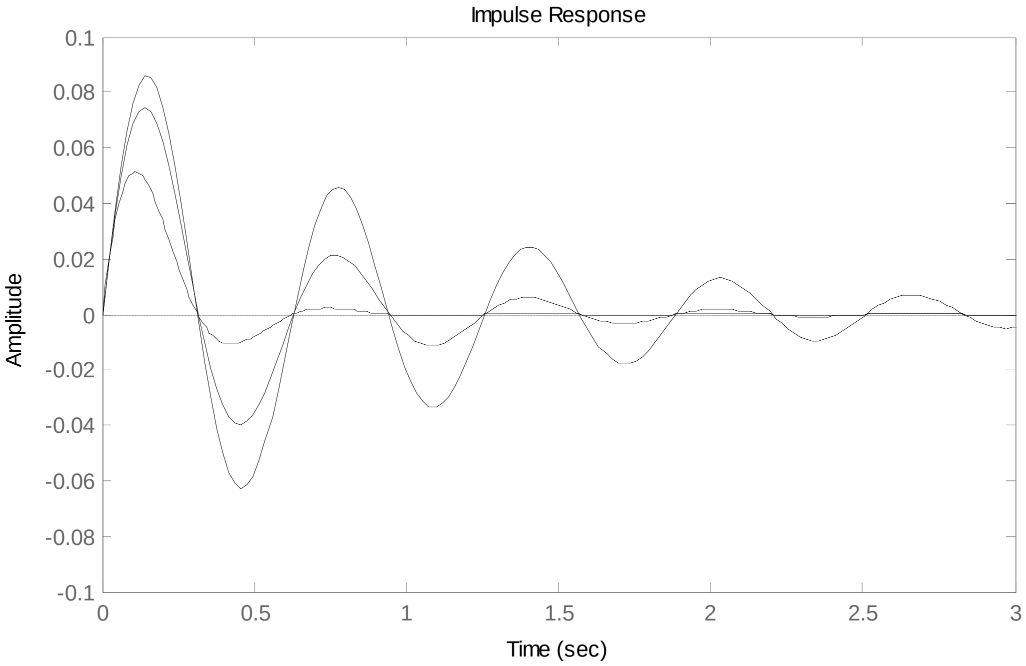
- Impulse response :

$$h(t) = \mathcal{L}^{-1}\Big[\frac{\sigma^2 + \omega^2}{(s+\sigma)^2 + \omega^2}\Big]$$
$$= \frac{\sigma^2 + \omega^2}{\omega} e^{-\sigma t} \sin(\omega t)\; 1(t)$$

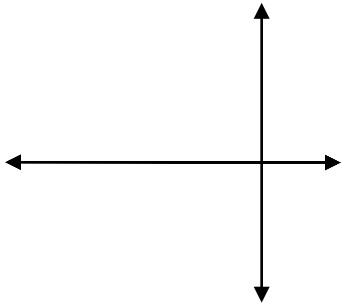


Second order impulse response – Underdamped and Undamped

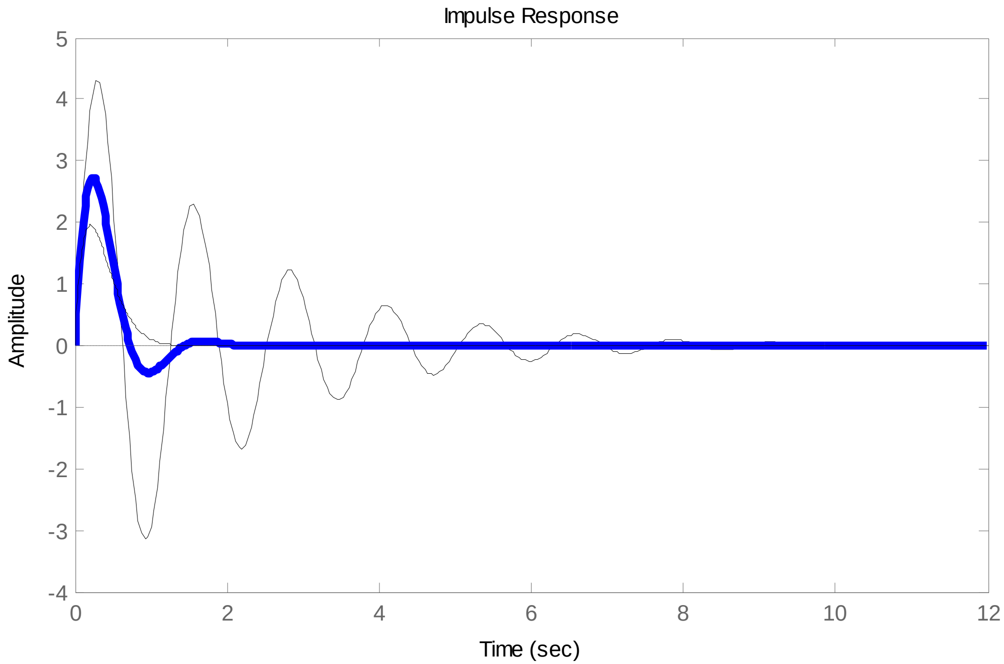
$$H(s) = \frac{\sigma^2 + \omega^2}{(s+\sigma)^2 + \omega^2}$$



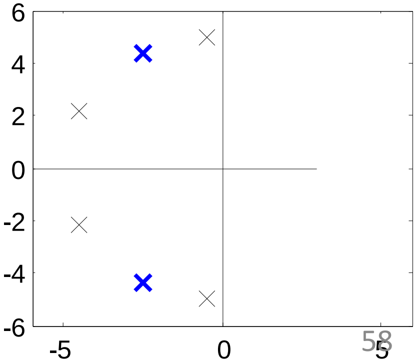
$$\boxed{h(t) = \frac{\sigma^2 + \omega^2}{\omega} e^{-\sigma t} \sin(\omega t)\ 1(t)}$$



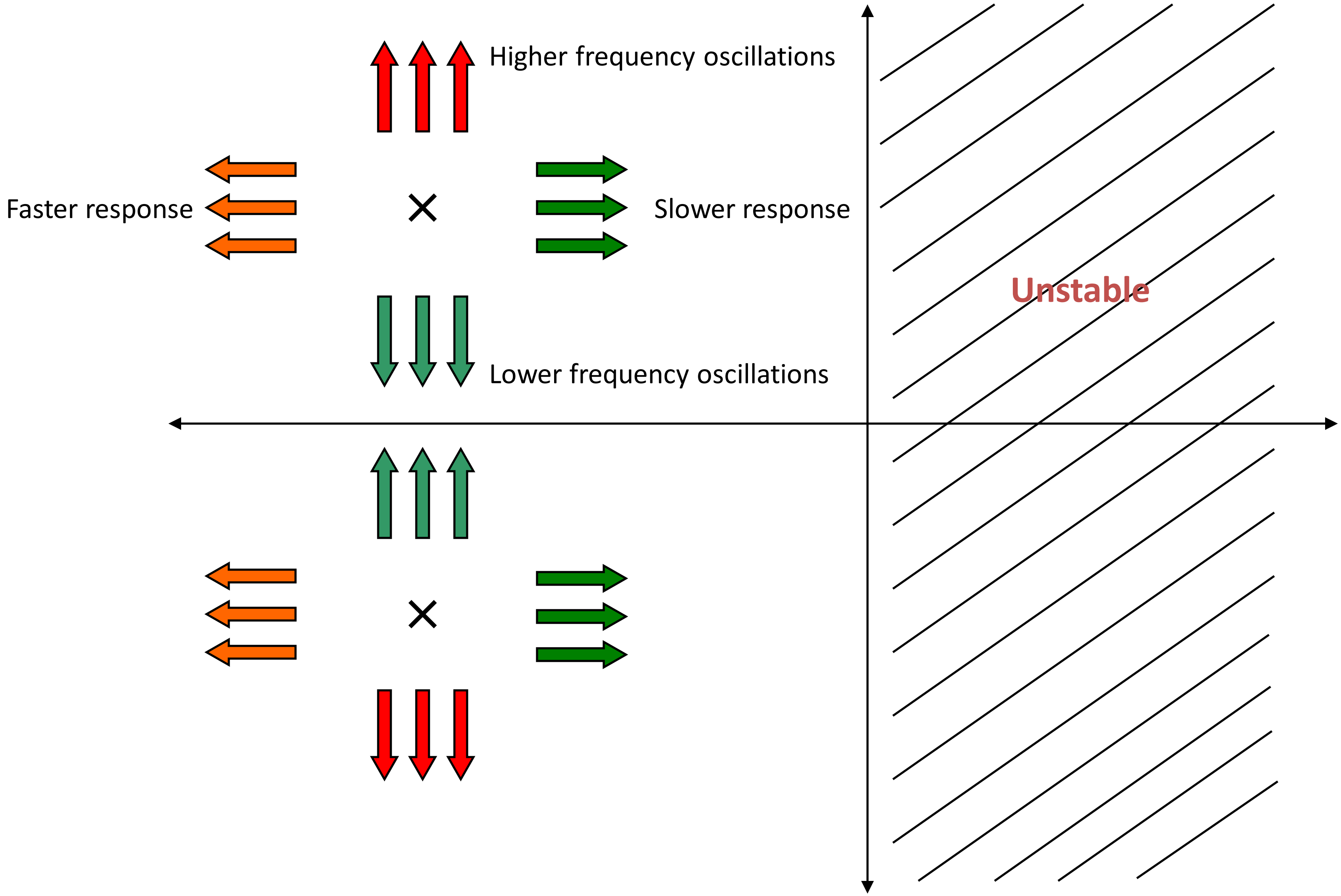
Second order impulse response – Underdamped and Undamped



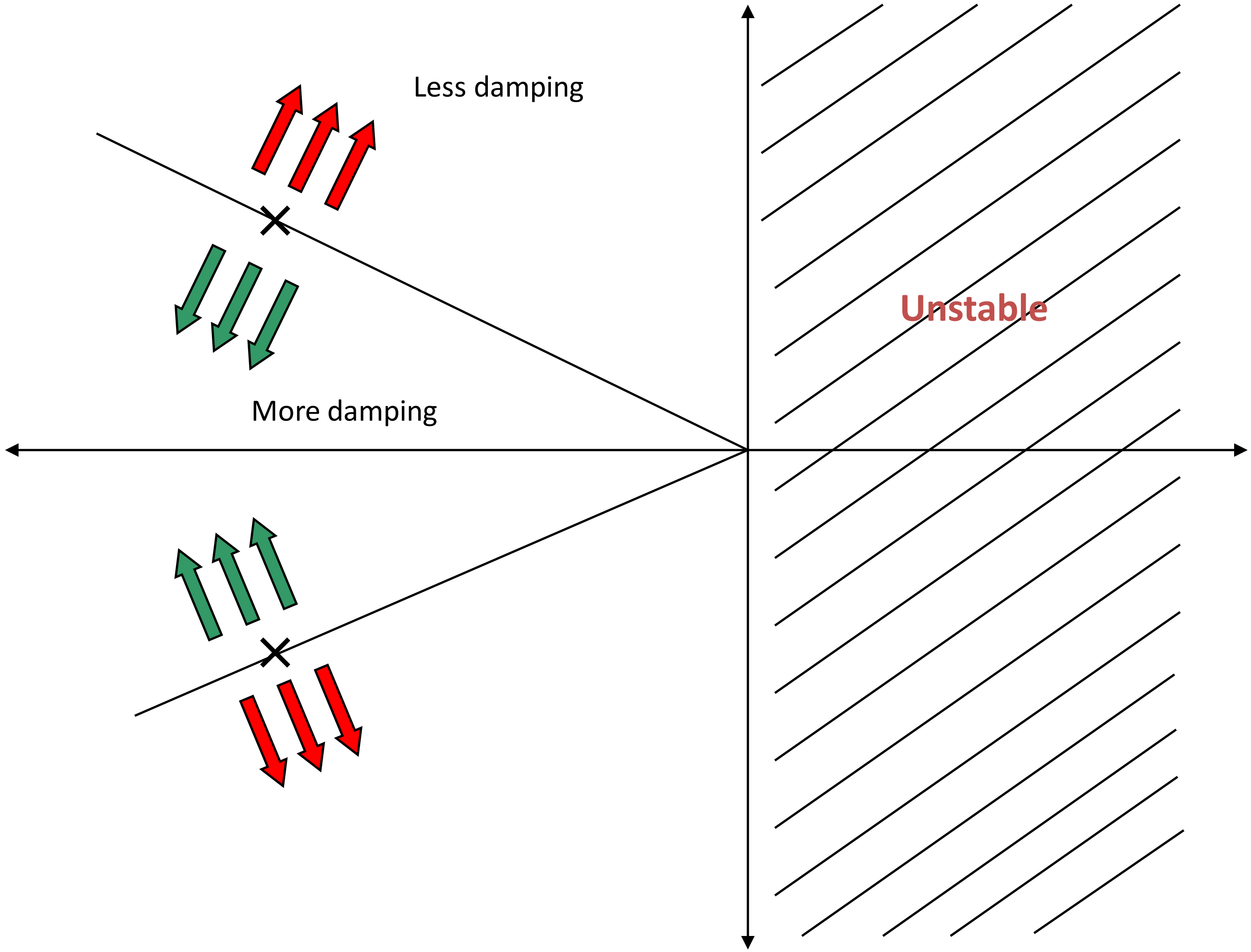
$$h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t) 1(t)$$



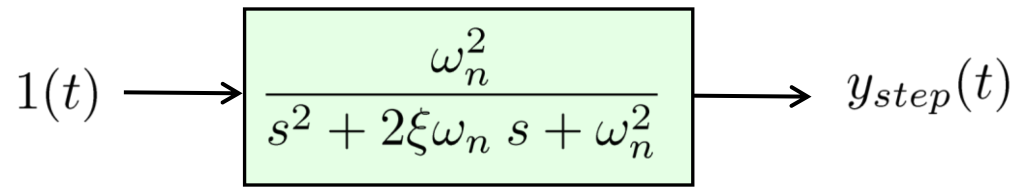
Second order impulse response – Underdamped and Undamped

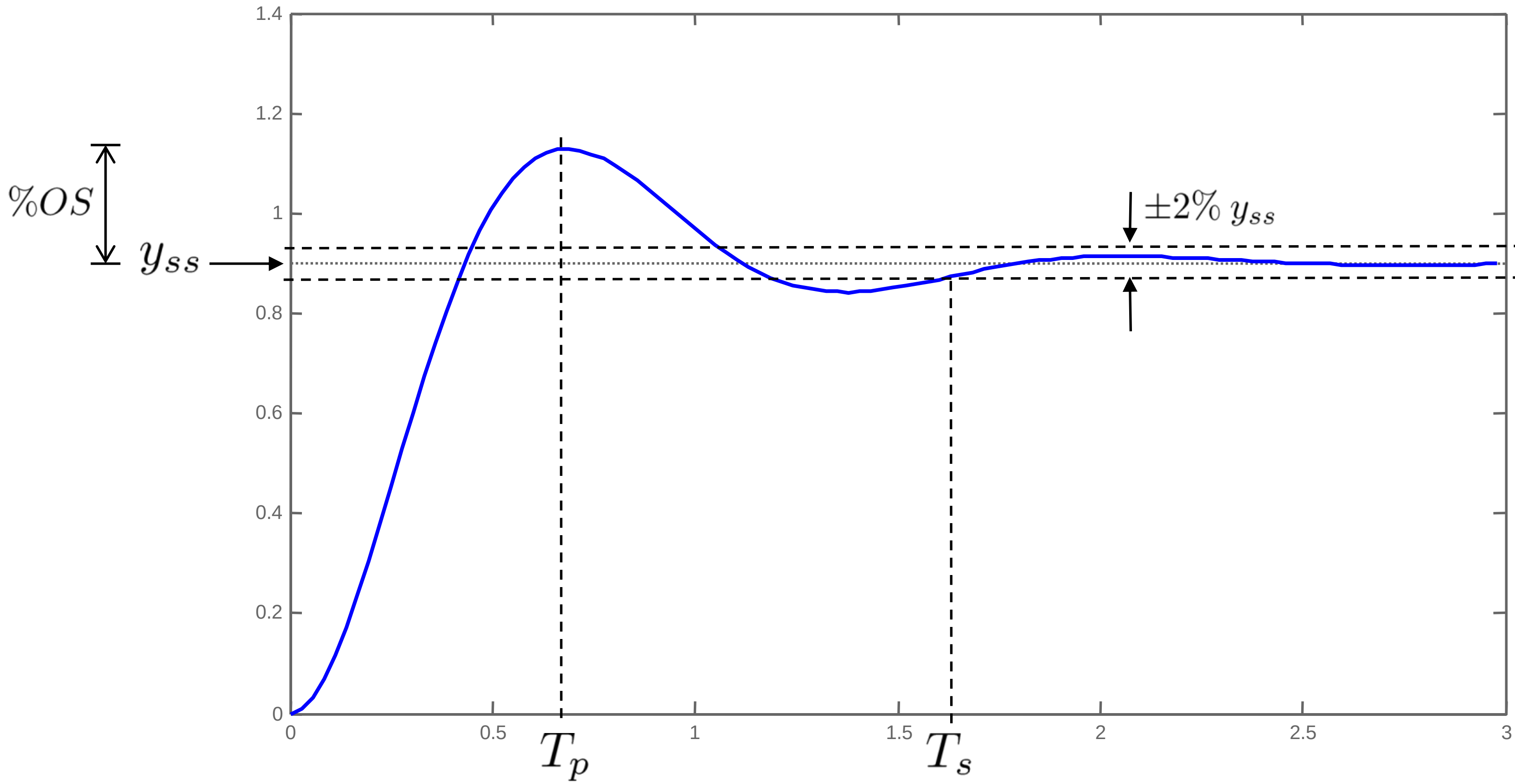


Second order impulse response – Underdamped and Undamped



Second order step response – Time specifications.





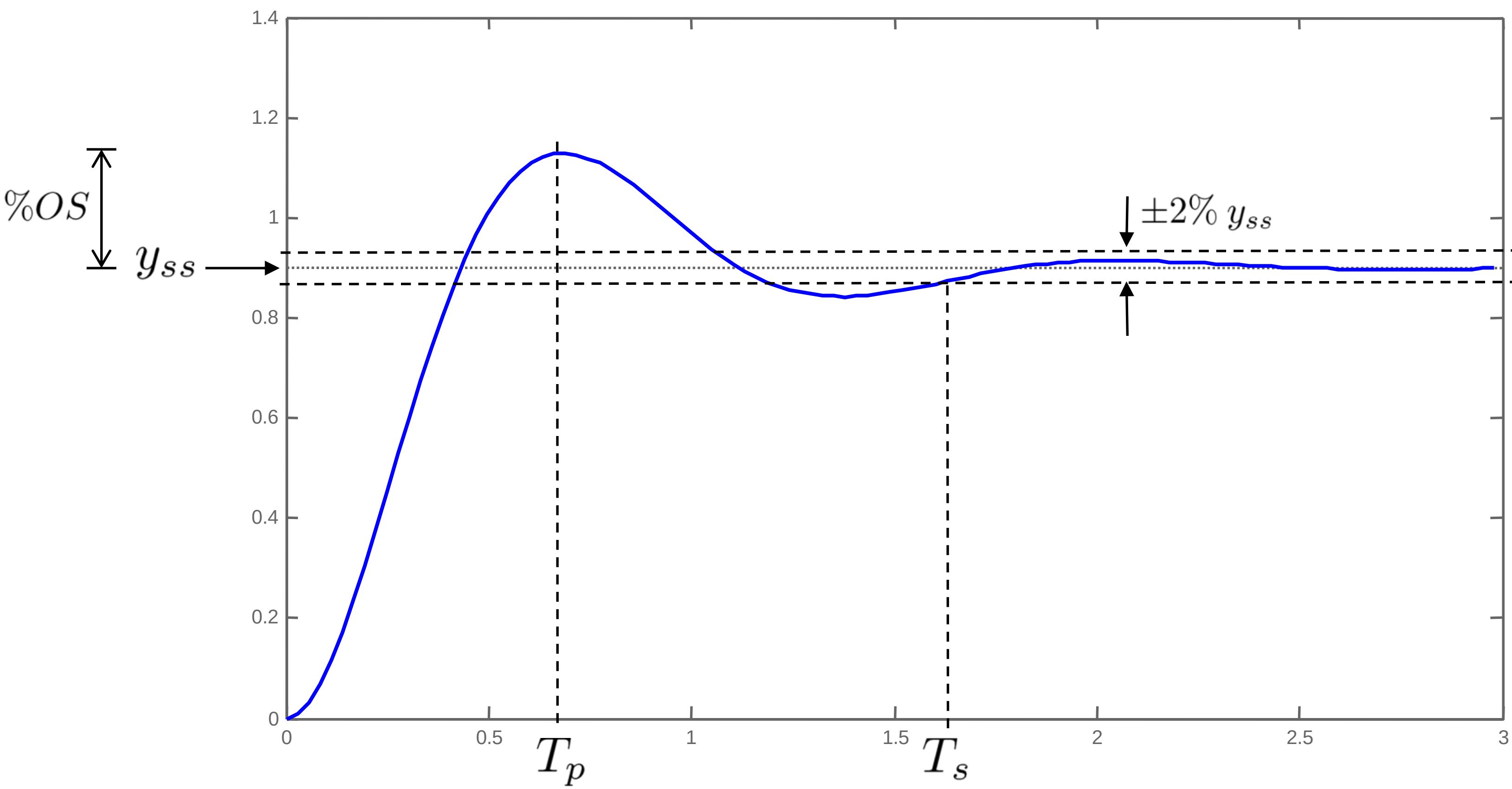
Second order step response – Time specifications.

y_{ss} ... Steady state value.

T_p ... Time to reach first peak (undamped or underdamped only).

$\%OS$ ... % of excess of y_{ss}.

T_s ... Time to reach and stay within 2% of y_{ss}



Tips for Designing a PID Controller

1. Obtain an open-loop response and determine what needs to be improved
2. Add a proportional control to improve the rise time
3. Add a derivative control to improve the overshoot
4. Add an integral control to eliminate the steady-state error
5. Adjust each of Kp, Ki, and Kd until you obtain a desired overall response.

Lag or Phase-Lag Compensator Using Root Locus

A first-order lag compensator can be designed using the root locus. A lag compensator in root locus form is given by

$$G_c(s) = \frac{(s + z)}{(s + p)}$$

where the magnitude of z is greater than the magnitude of p. A phase-lag compensator tends to shift the root locus to the right, which is undesirable. For this reason, the pole and zero of a lag compensator must be placed close together (usually near the origin) so they do not appreciably change the transient response or stability characteristics of the system.

Lag or Phase-Lag Compensator using Frequency Response

A first-order phase-lag compensator can be designed using the frequency response. A lag compensator in frequency response form is given by

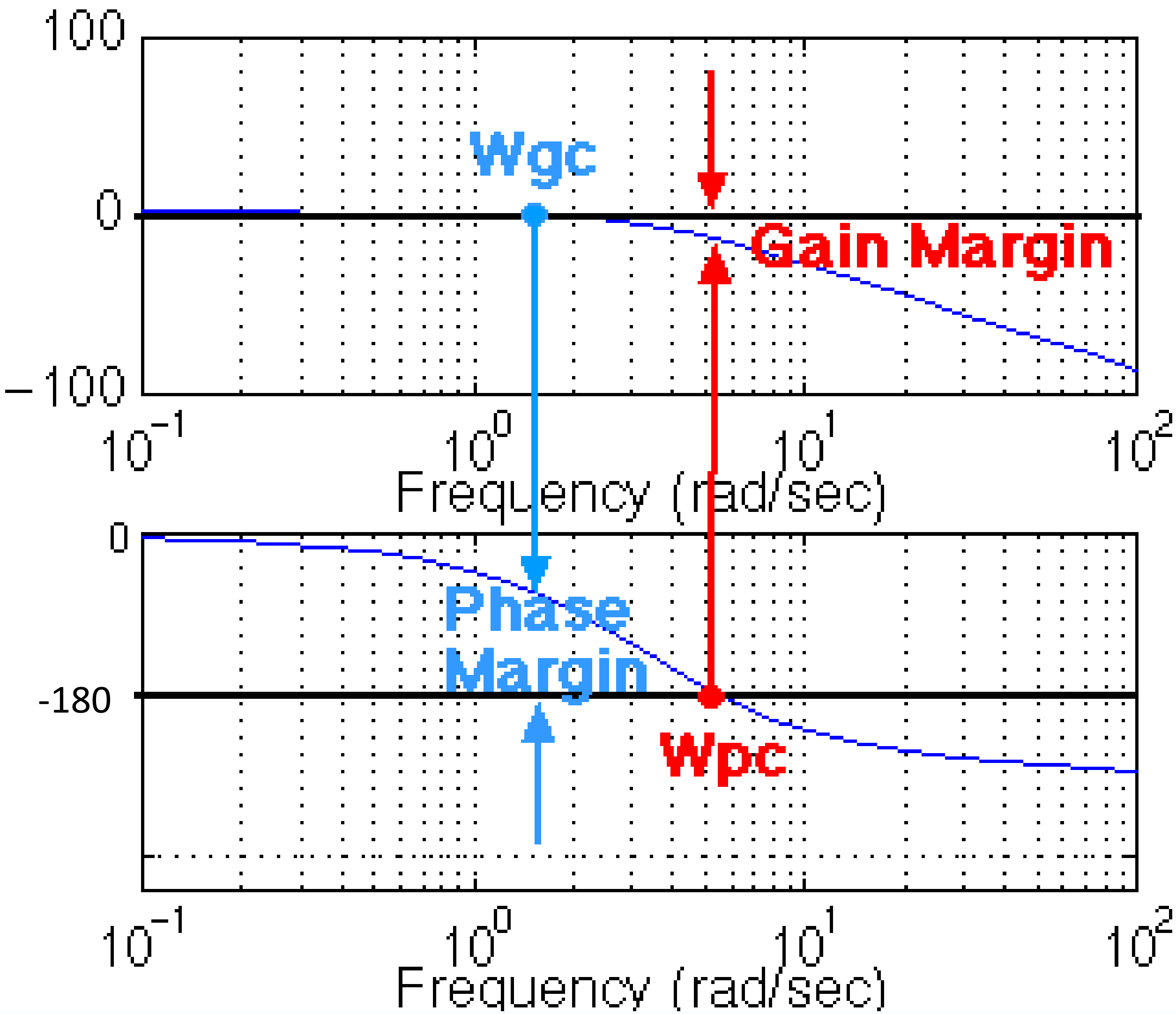
$$G_c(s) = \frac{(1 + \alpha \cdot \tau \cdot s)}{\alpha \cdot (1 + \tau \cdot s)}$$

The phase-lag compensator looks similar to a phase-lead compensator, except that a is now less than 1. The main difference is that the lag compensator adds negative phase to the system over the specified frequency range, while a lead compensator adds positive phase over the specified frequency. A bode plot of a phase-lag compensator looks like the following

Lead-lag Compensator using either Root Locus or Frequency Response

A lead-lag compensator combines the effects of a lead compensator with those of a lag compensator. The result is a system with improved transient response, stability and steady-state error. To implement a lead-lag compensator, first design the lead compensator to achieve the desired transient response and stability, and then add on a lag compensator to improve the steady-state response

Gain and Phase Margin



Gain and Phase Margin

Magnitude:

$$db(G,\omega) := 20\cdot\log(|G(j\cdot\omega)|)$$

Phase shift:

$$ps(G,\omega) := \frac{180}{\pi}\cdot\arg(G(j\cdot\omega)) - 360\cdot(\mathrm{if}(\arg(G(j\cdot\omega)) \geq 0, 1, 0))$$

Assume

$$K := 2 \qquad G(s) := \frac{K}{s\cdot(1+s)\cdot\left(1+\frac{s}{3}\right)}$$

Next, choose a frequency range for the plots (use powers of 10 for convenient plotting):

lowest frequency (in Hz): $\omega_{start} := .01$ number of points: $N := 50$

highest frequency (in Hz): $\omega_{end} := 100$

step size: $r := \log\left(\frac{\omega_{start}}{\omega_{end}}\right)\cdot\frac{1}{N}$

range for plot: $i := 0 .. N$ range variable: $\omega_i := \omega_{end}\cdot 10^{i\cdot r}$ $s_i := j\cdot\omega_i$

Gain and Phase Margin

Guess for **crossover frequency**: $\omega_c := 1$

Solve for the gain crossover frequency:

$$\omega_c := \mathrm{root}\left(\mathrm{db}\left(G, \omega_c\right), \omega_c\right) \qquad \omega_c = 1.193$$

Calculate the **phase margin**:

$$pm := \mathrm{ps}\left(G, \omega_c\right) + 180 \qquad pm = 18.265 \quad \text{degrees}$$

Gain Margin

Now using the phase angle plot, estimate the frequency at which the phase shift crosses 180 degrees:

$$\omega_{gm} := 1.8$$

Solve for $\boldsymbol{\omega}$ at the phase shift point of 180 degrees:

$$\omega_{gm} := \mathrm{root}\left(\mathrm{ps}\left(G, \omega_{gm}\right) + 180, \omega_{gm}\right)$$

$$\omega_{gm} = 1.732$$

Calculate the **gain margin**:

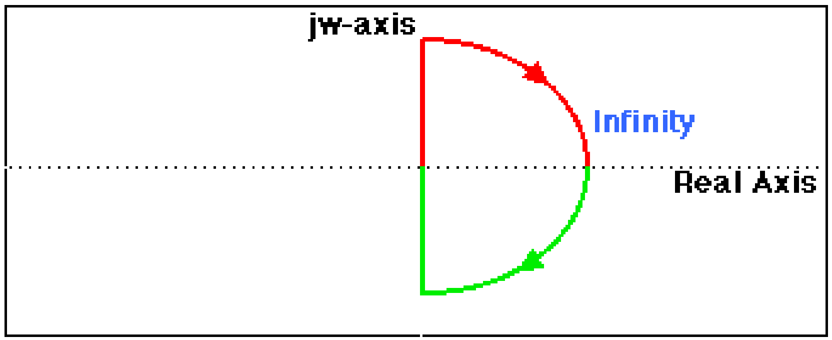
$$gm := -\mathrm{db}\left(G, \omega_{gm}\right) \qquad gm = 6.021$$

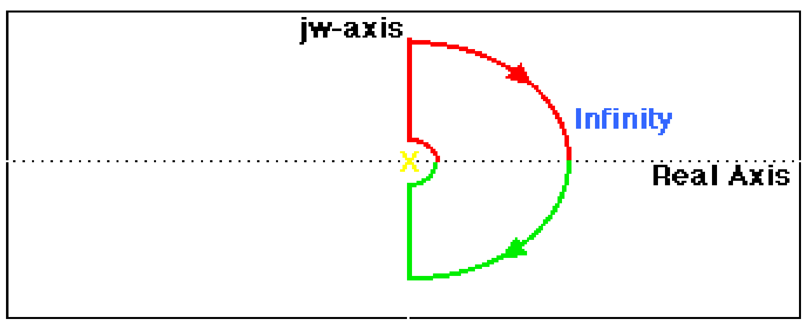
The Nyquist Stability Criterion

- The Nyquist plot allows us also to predict the stability and performance of a closed-loop system by observing its open-loop behavior.
- The Nyquist criterion can be used for design purposes regardless of open-loop stability (Bode design methods assume that the system is stable in open loop).
- Therefore, we use this criterion to determine closed-loop stability when the Bode plots display confusing information.
-
- The Nyquist diagram is basically a plot of G(j* w) where G(s) is the open-loop transfer function and w is a vector of frequencies which encloses the entire right-half plane.
- In drawing the Nyquist diagram, both positive and negative frequencies (from zero to infinity) are taken into account.

In the illustration below we represent positive frequencies in red and negative frequencies in green.

The frequency vector used in plotting the Nyquist diagram usually looks like this (if you can imagine the plot stretching out to infinity)

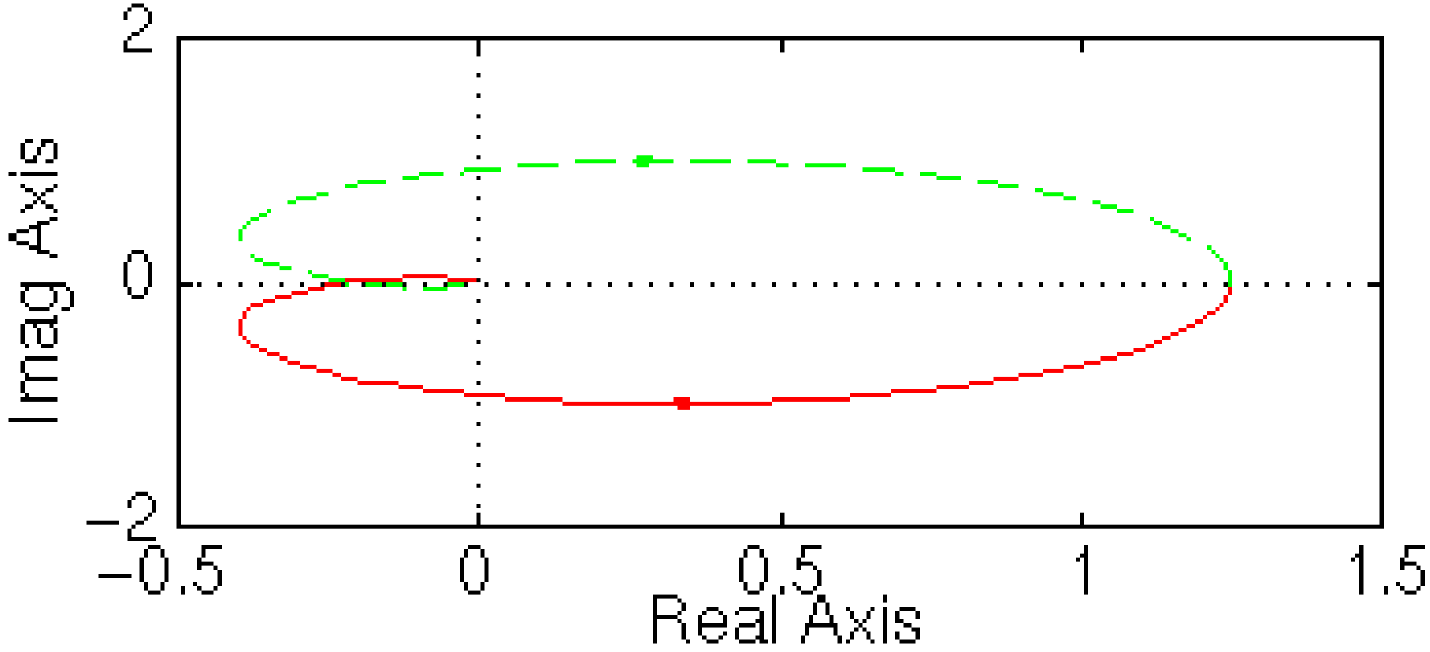




The Nyquist Stability Criterion

And that the Nyquist diagram can be viewed by typing:

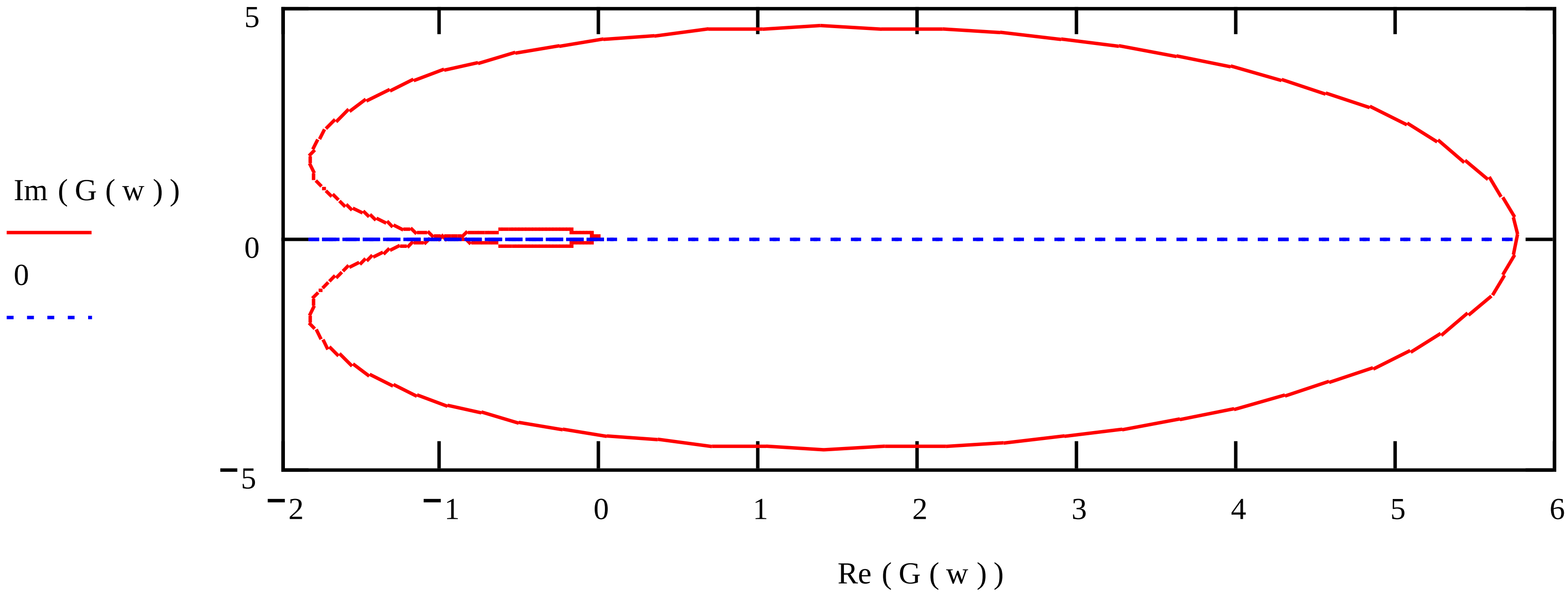
nyquist (50, [1 9 30 40])



The Nyquist Stability Criterion

$w := -100, -99.9 .. 100$ $j := \sqrt{-1}$ $s(w) := j \cdot w$ $f(w) := -1$

$$G(w) := \frac{50 \cdot 4.6}{s(w)^3 + 9 \cdot s(w)^2 + 30 \cdot s(w) + 40}$$



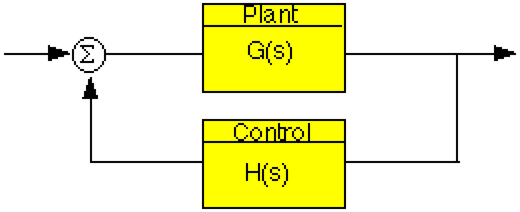
Consider the Negative Feedback System

Remember from the Cauchy criterion that the number N of times that the plot of G(s)H(s) encircles -1 is equal to the number Z of zeros of 1 + G(s)H(s) enclosed by the frequency contour minus the number P of poles of 1 + G(s)H(s) enclosed by the frequency contour (N = Z - P).

Keeping careful track of open- and closed-loop transfer functions, as well as numerators and denominators, you should convince yourself that:

The zeros of 1 + G(s)H(s) are the poles of the closed-loop transfer function

The poles of 1 + G(s)H(s) are the poles of the open-loop transfer function.



The Nyquist criterion then states that:

P = the number of open-loop (unstable) poles of G(s)H(s)

N = the number of times the Nyquist diagram encircles -1

clockwise encirclements of -1 count as positive encirclements

counter-clockwise (or anti-clockwise) encirclements of -1 count as negative encirclements

Z = the number of right half-plane (positive, real) poles of the closed-loop system

The important equation which relates these three quantities is:

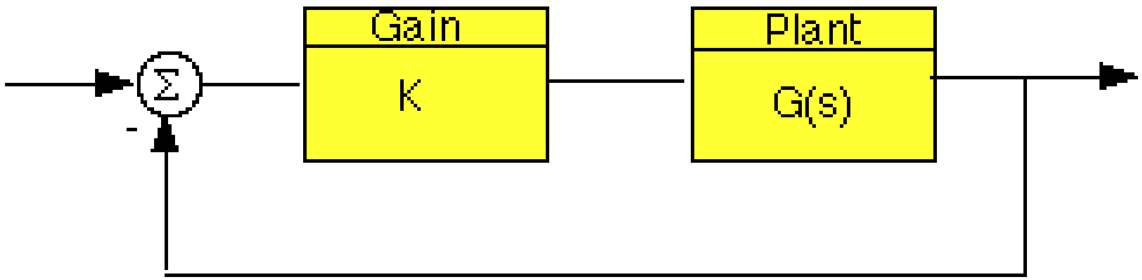
Z = P + N

The Nyquist Stability Criterion - Application

Knowing the number of right-half plane (unstable) poles in open loop (P), and the number of encirclements of -1 made by the Nyquist diagram (N), we can determine the closed-loop stability of the system.

If Z = P + N is a positive, nonzero number, the closed-loop system is unstable.

We can also use the Nyquist diagram to find the range of gains for a closed-loop unity feedback system to be stable. The system we will test looks like this:



Time-Domain Performance Criteria Specified In The Frequency Domain

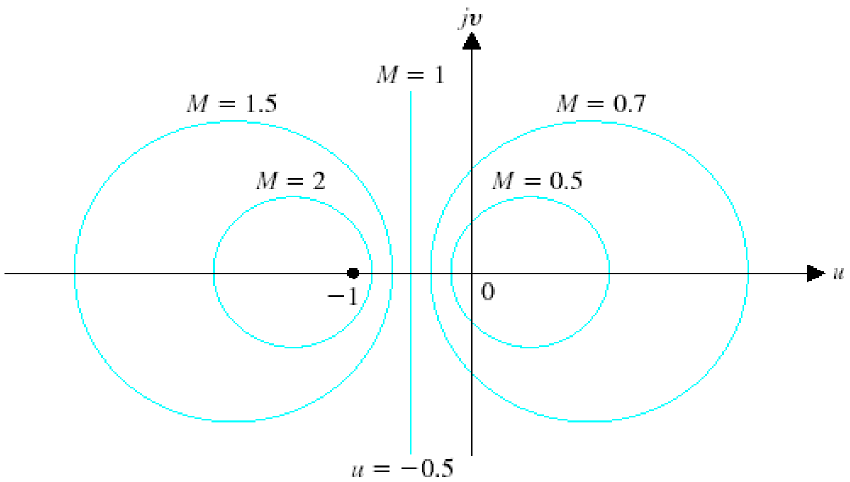
Open and closed-loop frequency responses are related by:

$$T(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)}$$

$$M_{pw} = \frac{1}{2 \cdot \zeta \cdot \sqrt{1 - \zeta^2}} \qquad \zeta < 0.707$$

$$G(\omega) = u + j \cdot v \qquad M = M(\omega)$$

$$M(\omega) = \left| \frac{G(j\omega)}{1 + G(j\omega)} \right| = \left| \frac{u + jv}{1 + u + jv} \right| = \frac{\sqrt{u^2 + v^2}}{\sqrt{(1 + u)^2 + v^2}}$$



Constant M circles.

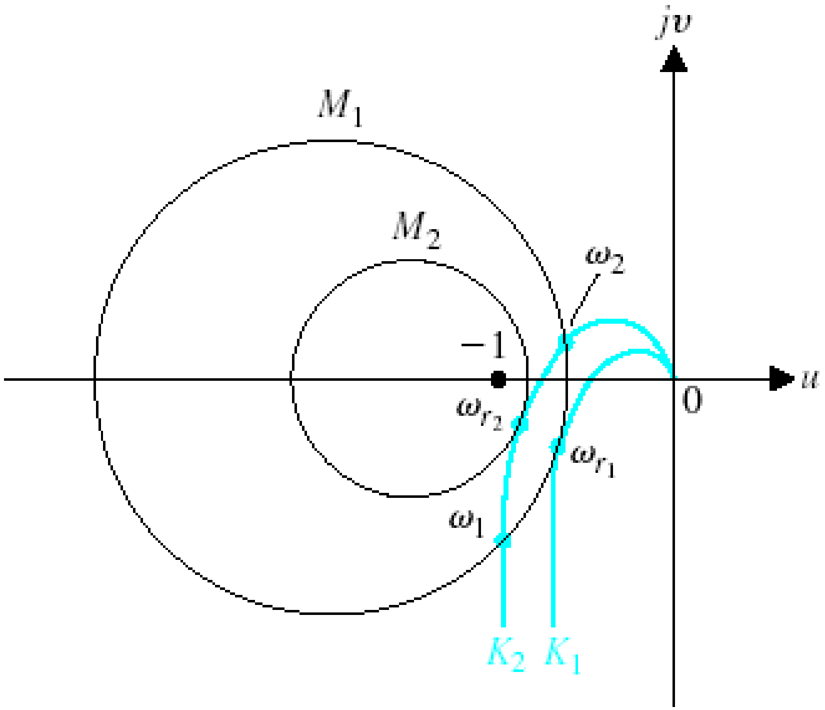
Squaring and rearrenging

$$\left(u - \frac{M^2}{1 - M^2} \right)^2 + v^2 = \left(\frac{M}{1 - M^2} \right)^2$$

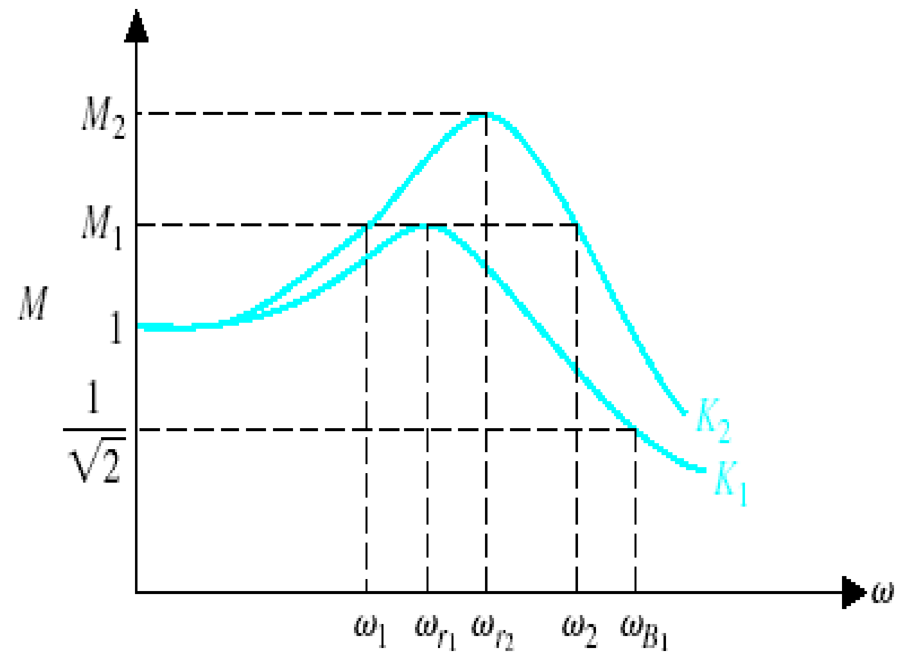
which is the equation of a circle on u-v planwe with a center at

$$u = \frac{M^2}{1 - M^2} \qquad v = 0$$

Time-Domain Performance Criteria Specified In The Frequency Domain



Polar plot of $G(j\omega)$ for two values of a gain ($K_2 > K_1$).



Closed-loop frequency response of $T(j\omega) = G(j\omega)/1 + G(j\omega)$. Note that $K_2 > K_1$.

Robustness Analysis

- In order for a control system to function properly, it should not be unduly sensitive to small changes in the process or to inaccuracies in the process model, if a model is used to design the control system.
- A control system that satisfies this requirement is said to be *robust* or *insensitive*.
- It is very important to consider robustness as well as performance in control system design.
- First, we explain why the S and T transfer functions in Eq. 14-15 are referred to as "sensitivity functions".

UNIT-IV
Aircraft Response to Controls-Flying Qualities-Stability and Control Augmentation- Autopilots

Approximation to aircraft transfer functions: The longitudinal and lateral equations of motion are described by a set of linear differential equations. The transfer function gives the relationship between the output and input to a system. The transfer function is defined as the Laplace transform of output to the Laplace transform of input, with all initial conditions set to zero. Following assumptions are made in **approximation to aircraft transfer functions**.

a. We assume that aircraft motion consists of small deviations from its equilibrium flight conditions.
b. We assume that the motion of the aircraft can be analyzed by separating the equation into Longitudinal and Lateral motion (later consists of yawing motion and roll motion).

Long-Period Approximation

Assume angle-of-attack and pitch-rate have stabilized when the aircraft is excited in long-period mode.

That is set : $\Delta \hat{\alpha} = 0 \qquad \Delta \hat{\bar{q}} = 0$

in the linearized level flight equations of motion:

$$\underbrace{\begin{bmatrix} 1-\cancel{R_{x,\hat{\mu}}} & -R_{x,\hat{\alpha}} & 0 & 0 & 0 & 0 \\ -R_{z,\hat{\mu}} & 1-R_{z,\hat{\alpha}} & 0 & 0 & 0 & 0 \\ -R_{m,\hat{\mu}} & -R_{m,\hat{\alpha}} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{computed\ about\ reference\ point!} \begin{bmatrix} \Delta \hat{\mu} \\ 0 \\ 0 \\ \Delta \hat{\xi}_x \\ \Delta \hat{\xi}_z \\ \Delta \hat{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{x,\mu} & R_{x,\alpha} & R_{x,\bar{q}} & 0 & 0 & -R_{gx} \\ R_{z,\mu} & R_{z,\alpha} & 1+R_{z,\bar{q}} & 0 & 0 & 0 \\ R_{m,\mu} & R_{m,\alpha} & R_{m,\bar{q}} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}}_{computed\ about\ reference\ point\ \theta_0 = 0!} \begin{bmatrix} \Delta \mu \\ \Delta \alpha \\ \Delta \bar{q} \\ \Delta \xi_x \\ \Delta \xi_z \\ \Delta \theta \end{bmatrix}$$

Second-order approximation for the *long-period*

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}\begin{bmatrix} \Delta\hat{\mu} \\ \Delta\hat{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{x,\mu} & -R_{gx} \\ -R_{z,\mu} & 0 \end{bmatrix}}_{computed\ about\ reference\ po\text{int}!}\begin{bmatrix} \Delta\mu \\ \Delta\theta \end{bmatrix}$$

Characteristic equation for eigenvalues:

$$\left|\underbrace{\begin{bmatrix} R_{x,\mu} & -R_{gx} \\ -R_{z,\mu} & 0 \end{bmatrix}}_{computed\ about\ reference\ po\text{int}!} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

Or:

$$\lambda^2 - \left(R_{x,\mu}\right)\lambda + \left(-R_{gx}R_{z,\mu}\right) = 0$$

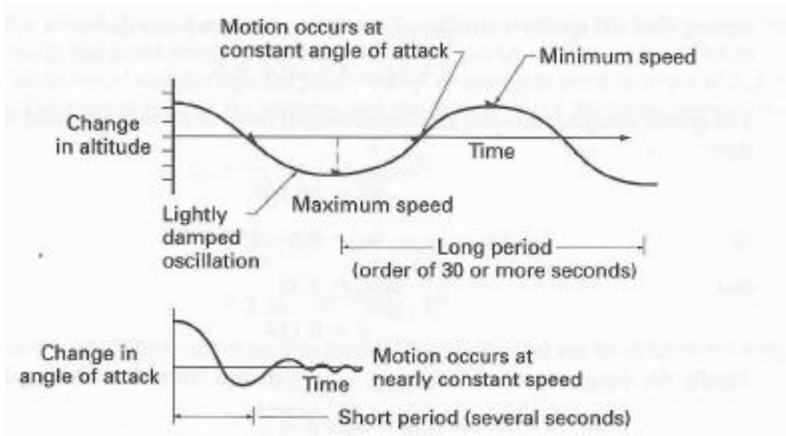
Notice that the change in axial force due to axial velocity is small, which means

$$\lambda^2 - \cancel{\left(R_{x,\mu}\right)}\lambda + \left(-R_{gx}R_{z,\mu}\right) = 0$$

$$\lambda^2 - R_{gx}R_{z,\mu} = 0$$

BUT: In general the Phugoid approximation is not nearly as accurate as the short-period approximation. This is because of the restrictive assumption of no changes in angle-of-attack.

Short-Period Approximation: An approximation to the short period mode of motion can be obtained by assuming $\Delta u = 0$ and dropping the X-force equation



Phugoid and Short Period Oscillations

LATERAL APPROXIMATION OF AIRCRAFT TRANSFER FUNCTION

The characteristic equation of aircraft lateral motion is characterized by the following equation.

$A\lambda^4 + B\,\lambda^3 + C\lambda^2 + D\,\lambda + E = 0$

Where A, B, C, D & E are the functions of stability derivative, mass and inertia characteristic of the airplane.

In general we find that the roots of the characteristic equation to be composed of two real roots and fair of complex roots.

The roots will be such that the airplane response can be characterized by the following motions.

a.A slowly convergent or divergent motion, called the spiral mode.

b.A highly convergent motion, called the rolling mode.

c.A lightly dumped oscillating motion having a low frequency, called the Dutch roll.

Standard Forms for System Models

- **State Space Model Representation**
  - Basic concepts
  - Example
  - General form

- **Input/Output Model Representation**
  - General Form
  - Example

- **Comments on the Difference between State Space and Input/Output Model Representations**

Basic Concepts related to State Space

State Variables

- **State**

  The smallest set of variables $\{q_1, q_2, \ldots, q_n\}$ such that the knowledge of these variables at time $t = t_0$, together with the knowledge of the input for $t\ ^{3}\ t_0$ completely determines the behavior (the values of the state variables) of the system for time $t\ ^{3}\ t_0$.

- **State Vector**

  All the state variables $\{q_1, q_2, \ldots, q_n\}$ can be looked on as components of state vector.

- **State Space**

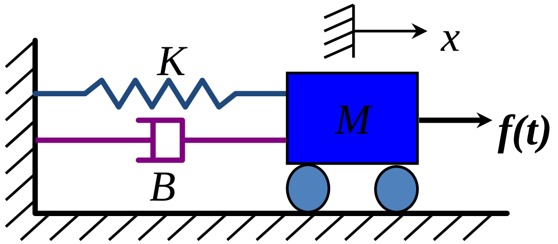
  A space whose coordinates consist of state variables is called a state space. Any state can be represented by a point in state space.

One Example

Example:

EOM:

$$M\ddot{x} + B\dot{x} + Kx = f(t)$$



Q: What information about the mass do we need to know to be able to solve for $x(t)$ for $t \geq t_0$?

Input: $f(t), \quad t \geq t_0$

Initial Conditions (ICs):

$x(t_0) \qquad q_1 = x(t)$

$\dot{x}(t_0) \qquad q_2 = \dot{x}(t)$

Rule of Thumb

Number of state variables = Sum of orders of EOMs

State Space Representation

Two parts:

- A set of first order ODEs that represents the derivative of each state variable q_i as an algebraic function of the set of state variables $\{q_i\}$ and the inputs $\{u_i\}$.
- A set of equations that represents the output variables as algebraic functions of the set of state variables $\{q_i\}$ and the inputs $\{u_i\}$.

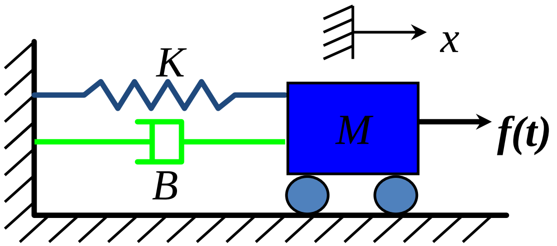
$$\begin{cases} \dot{q}_1 = f_1(q_1, q_2, q_3, \ldots, q_n, u_1, u_2, u_3, \ldots, u_m) \\ \dot{q}_2 = f_2(q_1, q_2, q_3, \ldots, q_n, u_1, u_2, u_3, \ldots, u_m) \\ \quad \vdots \\ \dot{q}_n = f_n(q_1, q_2, q_3, \ldots, q_n, u_1, u_2, u_3, \ldots, u_m) \end{cases}$$

State Space Model Representation

- Example

EOM

$$M\ddot{x} + B\dot{x} + Kx = f(t)$$



State Variables:

$$q_1 = x,$$

$$q_2 = \dot{x}$$

Outputs:

$$y_1 = x, \quad y_2 = -B\dot{x}$$

State Space Representation:

State equation

$$\begin{cases} \dot{q}_1 = \dot{x} = q_2 \\ \dot{q}_2 = \ddot{x} = \dfrac{1}{M}(-Bq_2 - Kq_1) + \dfrac{1}{M} f(t) \end{cases}$$

Output equation

$$\begin{cases} y_1 = x = q_1 \\ y_2 = -B\dot{x} = -Bq_2 \end{cases}$$

Matrix Form

$$\underbrace{\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\dfrac{K}{M} & -\dfrac{B}{M} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{M} \end{bmatrix}}_{\mathbf{B}} \underbrace{[f]}_{\mathrm{u}},$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathrm{y}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -B \end{bmatrix}}_{\mathbf{C}} \cdot \underbrace{\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{0_{2\times 1}}_{\mathrm{D}} \cdot \mathrm{u}$$

State Space Model Representation

- **Obtaining State Space Representation**
  - Identify State Variables
    - Rule of Thumb:
      - Nth order ODE requires N state variables.
      - Position and velocity of inertia elements are natural state variables for translational mechanical systems.
  - Eliminate all algebraic equations written in the modeling process.
  - Express the resulting differential equations in terms of state variables and inputs in coupled first order ODEs.
  - Express the output variables as algebraic functions of the state variables and inputs.
  - For linear systems, put the equations in matrix form.

Input/Output Models vs State-Space Models

- **State Space Models:**
  - consider the internal behavior of a system
  - can easily incorporate complicated output variables
  - have significant computation advantage for computer simulation
  - can represent multi-input multi-output (MIMO) systems and nonlinear systems

- **Input/Output Models:**
  - are conceptually simple
  - are easily converted to frequency domain transfer functions that are more intuitive to practicing engineers
  - are difficult to solve in the time domain (solution: Laplace transformation)

Lateral Approximation of aircraft transfer function

a. Spiral Approximation
b. Roll Approximation
c. Dutch Roll approximation

Control surface Actuators:

- An example of a controller for an aircraft system is a hydraulic actuator used to move to the control surface.
- A control valve on the actuator is positioned by either a mechanical or electrical input, the control valve ports hydraulic fluid under pressure to the actuator, and the actuator piston moves until the control valve shuts off the hydraulic fluid.

Response of aircraft to pilot's control inputs, to atmosphere.

Response of aircraft to Pilot's control input:

- Response of an aircraft to control input or atmosphere can be done by considering step input and sinusoidal input.
- The step and sinusoidal input functions are important for two reasons.
- First, the input to many physical systems takes the form of either a step change or sinusoidal signal.
- Second, an arbitrary function can be represented by a series of step changes or a periodic function can be decomposed by means of Fourier analysis into a series of sinusoidal waves.
- If we know the response of a linear system to either a step or sinusoidal input, then we can construct the system's response to an arbitrary input by the principle of superposition.

Aircraft Response to Atmosphere:

- The atmosphere is in a continuous state of motion.
- The winds and gusts created by the movement of the atmospheric air masses can degrade the performance and flying qualities of an airplane.
- In addition, the atmospheric gusts impose structural loads that must be accounted for in the structural design of an airplane.
- The velocity field within the atmosphere varies in both space and time in a random manner.
- This random velocity field is called atmospheric turbulence.
- The velocity field can be decomposed into a mean part and a fluctuating part.
- Because atmospheric turbulence is a random phenomenon it can be described only in a statistical term.

The control task of the pilot:

- The control task of the pilot is to fly the aircraft safely in the assigned mission of the aircraft.
- For a passenger aircraft mission profile will consist of take-off, cruise and landing at the designated airport.
- Similarly a military aircraft being a weapon delivery platform should be able to strike the designated target accurately.
- To accomplish these missions pilot should be able to control and fly the aircraft accurately and maintain the designated route without fatigue.
- The aircraft should be controllable even when it is disturbed from its equilibrium position either by pilot’s action or by atmospheric turbulence.
- An airplane must have sufficient stability such that the pilot does not become fatigued by constantly having to control the airplane owing to external disturbance

FLYING QUALITIES OF AIRCRAFT-RELATION TO AIRFRAME TRANSFER FUNCTION

Flying Qualities of an Aircraft:

- The flying qualities of an airplane are related to the stability and control characteristics and can be defined as those stability and control characteristics important in forming the pilot's impression of the aircraft.
- The pilot forms a subjective opinion about the ease or difficulty of controlling the airplane in steady and maneuvering flight.
- In addition to the longitudinal dynamics, the pilot's impression of the airplane is influenced by the feel of the airplane, which is provided by the stick force and stick force gradients.

Classification of airplanes:

Airplane can be placed in one of the following classes:
Class I: Small, light airplanes
Class II: Medium weight, low-to-medium maneuverability airplane
Class III: Large, heavy, low-to-medium maneuverability airplanes.
Class IV: High maneuverability airplanes

Flight Phase Category:

Flight Phases descriptions of most military airplane mission are:

Category A

Category B

Category C

Level of flying qualities:

The Levels are:

Level 1: Flying qualities clearly adequate for the mission Flight Phase

Level 2: Flying qualities adequate to accomplish the mission Flight Phase, but some increase in pilot work load or degradation in mission effectiveness, or both, exists.

Level 3: Flying qualities such that the airplane can be controlled safely, but pilot work load is excessive or mission effectiveness is inadequate, or both.

Longitudinal flying qualities- relation to airframe transfer function

a. Phugoid stability
b. Short period damping ratio limits

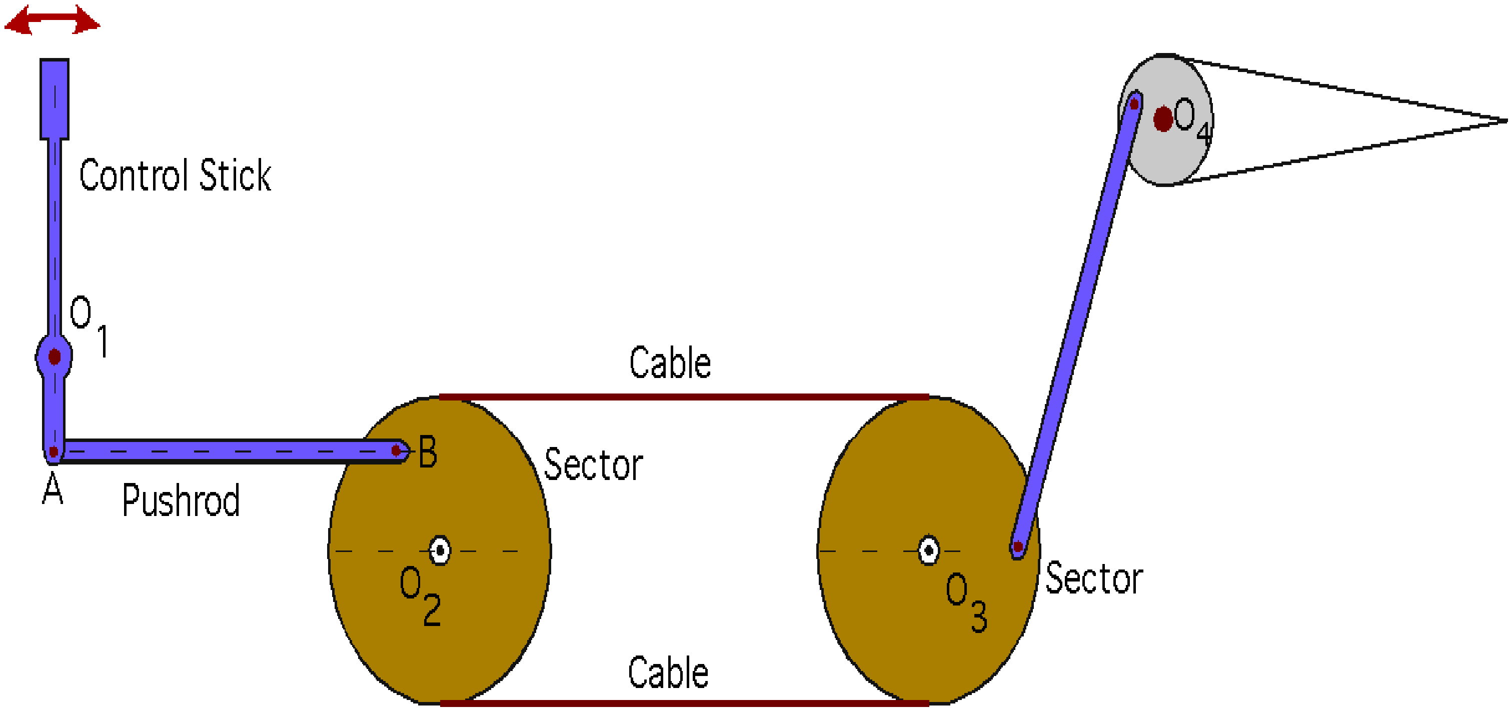
Reversible and irreversible flight control systems

Reversible flight Control System: In a reversible flight control system (FCS), the cockpit controls are directly connected to the aircraft flight control surfaces through mechanical linkages such as cables, push rods and bell cranks. There is no hydraulic actuator in this path and the muscle to move the control surface is provided directly by the pilot. With no hydraulic power on the aircraft, a reversible FCS will have the following characteristics

(a) Movement of the stick and rudder will move the respective control surface, and hand movement of each control surface will result in movement of the respective cockpit control, hence the name “reversible”.

(b)Reversible flight controls are used on light general aviation aircraft such as Cessna, Piper. They have the advantage of being relatively simple and “pilot feel” is provided directly by the air loads on each control surface being transferred to the stick or rudder pedals. They have the disadvantage of increasing stick and rudder forces as the speed of the aircraft increases. As a result, the control forces present may exceed the pilot’s muscular capabilities if the aircraft is designed to fly at high speed.

(c)Two types of static stability must be considered with reversible FCS. Stick fixed stability implies that the control surfaces are held in a fixed position by the pilot during a perturbation. Stick free stability implies that the stick & rudder pedals are not held in fixed position by the pilot but rather left to seek their own position during a flight perturbation. The stick free stability is lower in magnitude than stick fixed stability.



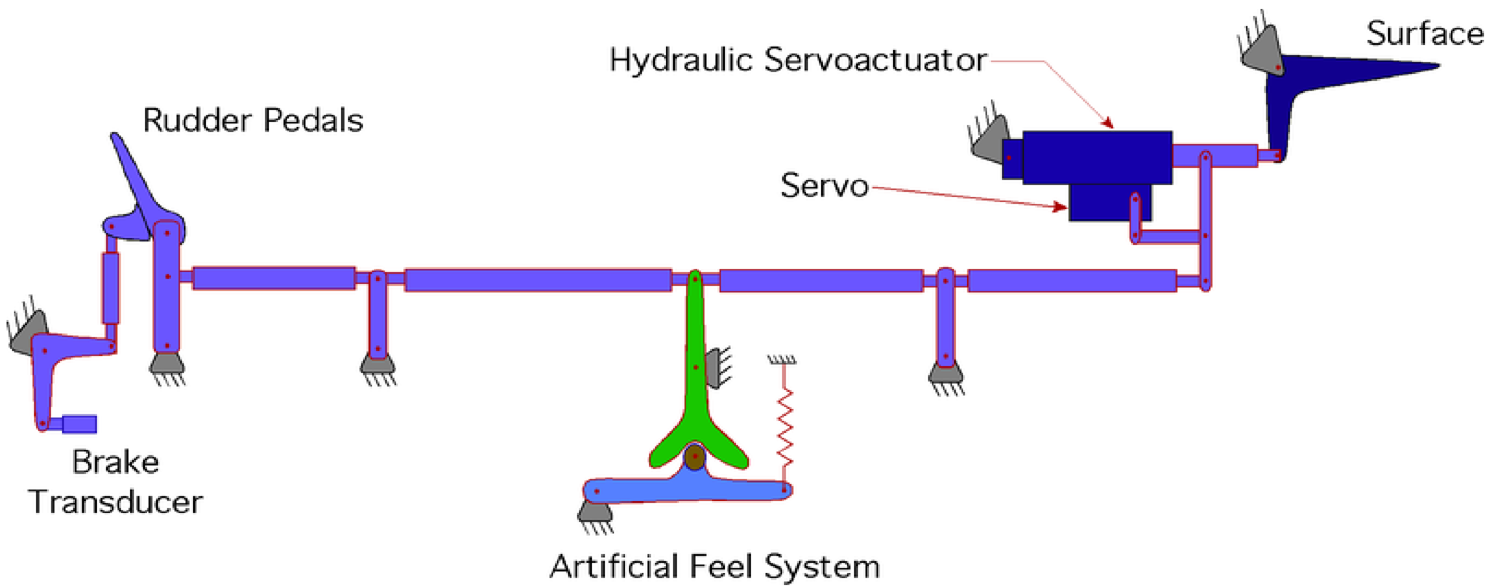
REVERSIBLE FLIGHT CONTROL SYSTEM

Irreversible Flight control System

In an irreversible FCS, the cockpit controls are either directly or indirectly connected to a controller that transforms the pilot's input into a commanded position for a hydraulic or electromechanical actuator.

The most common form of an irreversible FCS connects the pilot's displacement or force command from the stick or rudder pedals to a control valve on a hydraulic actuator.

The control valve positions the hydraulic actuator that, in turn, moves the flight control surface.



Following are the characteristics of irreversible FCS.

a) Such a system is called irreversible because manual movement of a control surface will not be transferred to movement of the stick or rudder pedals.

b) Irreversible control systems behave essentially a stick fixed system when the aircraft undergoes a perturbation because the hydraulic actuator holds the control surface in the commanded position even if the pilot let go off the stick.

c) Irreversible FCS is also ideal for incorporation of AFCS functions such as inner loop stability & outer loop auto pilot m modes.

d) A disadvantage of irreversible FCS is that artificial pilot feel must be designed into the stick and rudder pedals because the air loads on the control surface are not transmitted back. Artificial feel may be provided using spring system on the stick.

PILOT'S OPINION RATING:

- Flying qualities of an airplane is assessed by test pilot's comment obtained from simulations and test flying of the aircraft.
- A structured rating scale for aircraft handling qualities was developed by NASA in the late 1960s called the Cooper-Harper rating scale.
- This rating applies to specific pilot-in-loop tasks such as air-to-air tracking, formation flying, and approach.
- It does not apply to open-loop aircraft characteristics such as yaw response to a gust.
- Aircraft controllability, pilot compensation (workload), and task performance are key factors in the pilot's evaluation.
- A Cooper-Harper rating of "one" is highest or best and a rating of "ten" is the worst, indicating the aircraft cannot be controlled during a portion of the task and that improvement is mandatory.

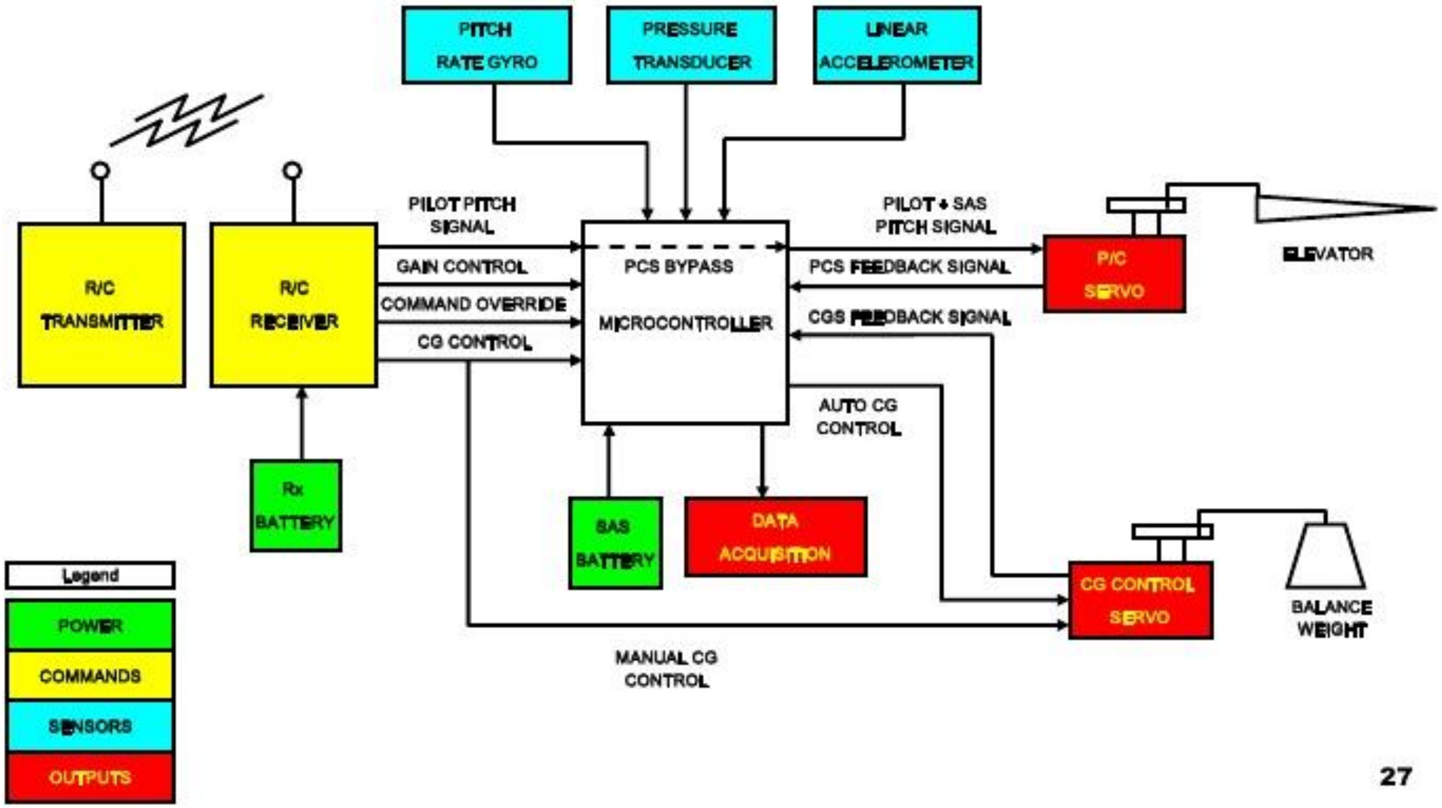
Flying quality requirements:

- Pole-zero specifications
- Frequency response
- Time-response specifications

Stability Augmentation System- displacement and rate feedback:

- Stability Augmentation Systems (SAS) were generally the first feedback control systems intended to improve dynamic stability characteristic.
- They were also referred to as dampers, stabilizers and stability augmenters.
- These systems generally fed back an aircraft motion parameter, such as pitch rate, to provide a control deflection that opposed the motion and increased damping characteristics.
- The SAS has to be integrated with primary flight control system of the aircraft consisting of the stick, pushrod, cables, and bellcranks leading to the control surface or the hydraulic actuator that activated the control surface.

SAS Block Diagram



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- Displacement (Position) feedback as a tool in SAS
- Rate feedback System
- Acceleration feedback
- Determination of Gain, Conflict with the pilot inputs-Resolution

Control Augmentation system:

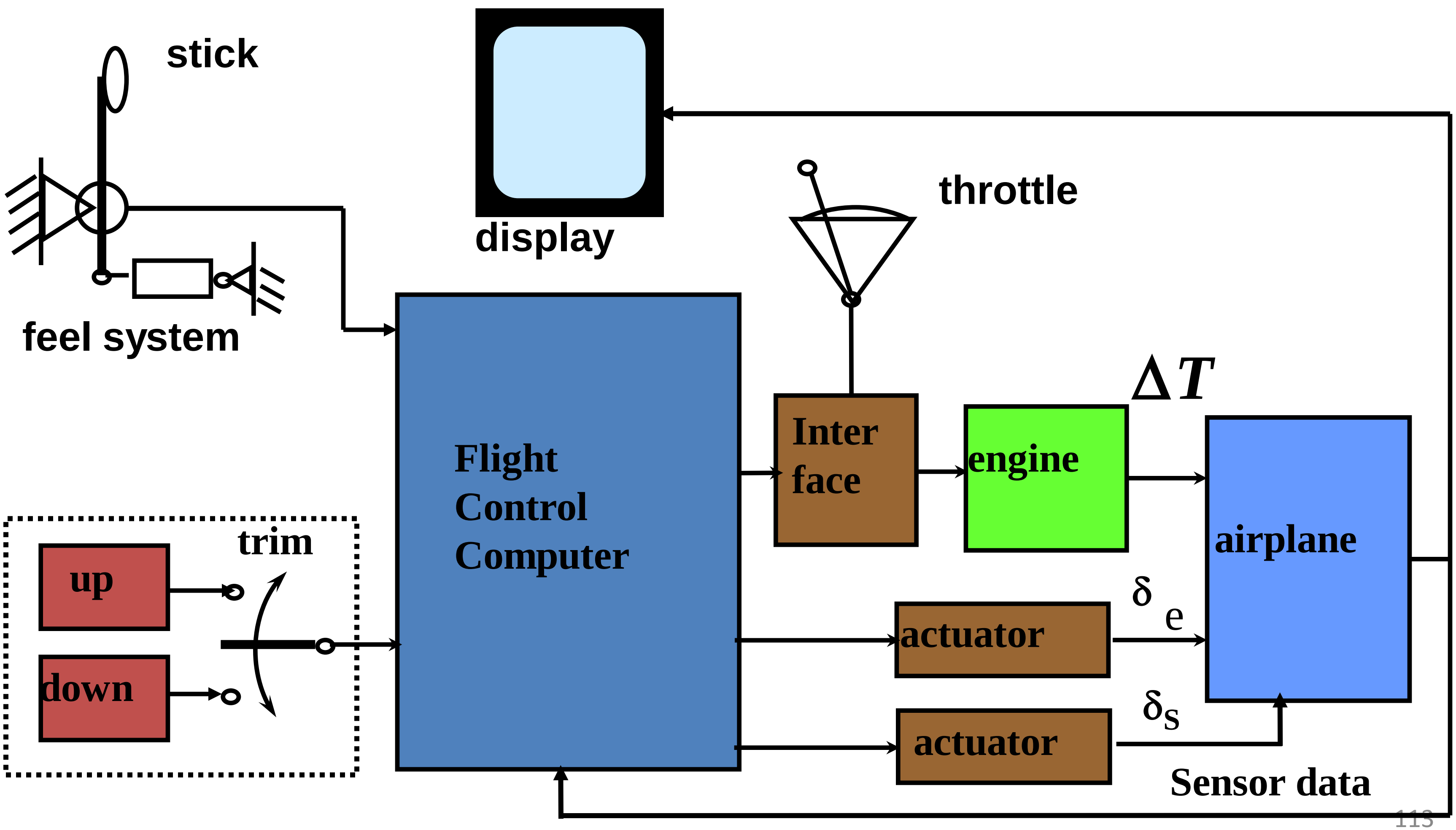
- Control augmentation system (CAS) added a pilot command input into the flight control computer.
- A force sensor on the control stick was usually used to provide this command input.
- With CAS, a pilot stick input is provided to FCS in two ways- through the mechanical system and through the CAS electrical path.
- The CAS design eliminated the SAS problem of pilot inputs being opposed by the feedback.

FULL AUTHORITY FLY-BY-WIRE CONTROL

Functions and operation:

- Full authority fly-by-wire (FBW) system has no mechanical link from the control stick to actuator system.
- Basically FBW systems are CAS system without mechanical control system and provide the CAS full authority.
- The input from control stick, pedal and from motion sensors are converted into electrical signals and sent to FBW computer.
- Software inside the FBW computer contains the control law which will command the control surfaces to move.
- However to improve the reliability, triple and quad redundancy in system components along with self-test software is used.
- Aircraft such as F-16, Mirage-2000 and Tejas have FBW FCS.
- The full authority provided by FBW allows significant tailoring of stability and control characteristics.
- This ability has led to FBW systems with several feedback parameters and weighting of feedback gains based on flight condition and other parameter.

FBW Control System Architecture



Advantages of FBW Control:

(a) Increased performance
(b) Reduced weight
(c) FBW control stick
(d) Automatic stabilization
(e) Carefree Maneuvering
(f) Ability to integrate additional controls.
(i) Leading and trailing edge flaps for maneuvering and not just for take-off and landing
(ii) Variable wing sweep
(iii) Thrust vectoring
(g) Ease of integration of the autopilot.
(h) Aerodynamics versus 'Stealth'

- Displacement autopilots-Pitch, yaw, bank, altitude and velocity hold-purpose
- Relevant Simplified Aircraft Transfer Functions
- Feedback signals
- Control actuators-Operation, analysis, Performance
- Maneuvering auto-pilots: pitch rate, normal acceleration, turn rate.

Autopilot design by displacement & rate feedback-iterative methods, design by displacement feedback and series PID compensator-Zeigler & Nichols method

Design of autopilot by displacement& rate feedback using iterative methods:

- Design of an autopilot by displacement & rate feedback is explained with an example of a pitch attitude hold auto pilot of a transport aircraft.
- This design reference the reference pitch angle is compared with the actual pith angle measured by the pitch gyro to produce an error signal to activate the control surface actuator to deflect the control surface.
- Movement of the control surface causes the aircraft to achieve a new pitch orientation, which is feedback to close the loop.
- To design the control system for this auto pilot we need the transfer function of each component.

Continuous Cycling Method

- Ziegler and Nichols (1942) introduced the *continuous cycling method* for controller tuning.
- based on the following trial-and-error procedure:

Step 1. After the process has reached steady state (at least approximately), eliminate the integral and derivative control action by setting:

τ_I = zero

τ_D = the largest possible value.

Need for Robust Control:

- ✓ No mathematical system can exactly model a physical system. Uncertainty due to un-modeled dynamics and uncertain parameters is always present.
- ✓ Two additional causes of inconsistencies between the mathematical model and the physical system are intentional model simplification, such as linearization and model reduction and incomplete data from the model identification experiment.
- ✓ Robust control theory deals with the design and synthesis of controllers for plants with uncertainty.
- ✓ A robust control system deals with the various control and stability specifications of plant in the presence of uncertainty.

- ✓ Stability and control is one of the technical major challenges in the design of an aircraft.
- ✓ Aircraft control system must work satisfactorily in all flight conditions without any flight safety concerns.
- ✓ Aircraft is a very complex system having lot of uncertainty in un-modeled dynamics, non-linearity, sensor noise, actuator error, uncertain parameters.
- ✓ Also aircraft control system has to deal with the environment such as turbulence, wind shear, and wind gust.

Typical autopilots of civil and military aircraft-description of design, construction, operation, performance.

The basic modes of autopilot in military/civil aircraft are:
(a) Height hold.
(b) Heading hold.
(c) Velocity hold.
(d) VOR/ILS coupled approach and landing.
(e) Bank hold mode.

UNIT V
Modern Control Theory-State Space Modeling, Analysis

Limitations of Classical methods of control modeling, analysis and design, applied to complex, MIMO system:

a) Transfer function models are used for linear time invariant (LTI) continuous time systems. These are called frequency response models due mainly to the interpretation of the Laplace transform variables s as complex frequency in contrast with differential equation models, which are time-domain models.
b) In classical control design of feedback control is accomplished using the root locus technique and Bode methods. These techniques are very useful in designing many practical control problems. However design of control system using root locus or Bode technique is trial & error procedure.

(c) With rapid development of high speed computers during the recent decade, a new approach to control system design has evolved. This new approach is called modern control theory. This theory permits a more systematic approach to control system design. In modern control theory, the control system is specified as a system of first-order differential equations.

State Equations

- **Let us define the state of the system by an *N*-element column vector, *x*(*t*):**

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_N(t) \end{bmatrix}^t$$

Note that in this development, *v*(*t*) will be the input, *y*(*t*) will be the output, and *x*(*t*) is used for the state variables.

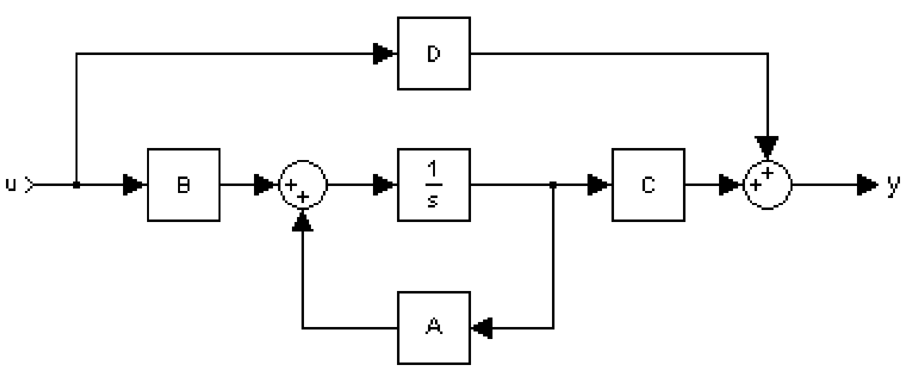
- **Any system can be modeled by the following state equations:**

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{v}(t) \qquad \mathbf{x} : Nx1 \quad \mathbf{A} : NxN \quad \mathbf{B} : Nxp \quad p : \text{number of inputs}$$

$$\mathbf{y}(t) = \mathbf{C}\,x(t) + \mathbf{D}\mathbf{v}(t) \qquad \mathbf{y} : qx1 \quad \mathbf{C} : qxN \quad \mathbf{D} : qxp \quad q : \text{number of outputs}$$

- **This system model can handle single input/single output systems, or multiple inputs and outputs.**

- **The equations above can be implemented using the signal flow graph shown to the right.**



STATE VARIABLE MODELS

We consider physical sytems described by nth-order ordinary differential equation. Utilizing a set of variables, known as state variables, we can obtain a set of first-order differential equations. We group these first-order equations using a compact matrix notation in a model known as the state variable model.

The time-domain state variable model lends itself readily to computer solution and analysis. The Laplace transform is utilized to transform the differential equations representing the system to an algebraic equation expressed in terms of the complex variable ***s***. Utilizing this algebraic equation, we are able to obtain a transfer function representation of the input-output relationship.

With the ready availability of digital computers, it is convenient to consider the time-domain formulation of the equations representing control system. The time domain techniques can be utilized for nonlinear, time varying, and multivariable systems.

A time-varying control system is a system for which one or more of the parameters of the system may vary as a function of time.

For example, the mass of a missile varies as a function of time as the fuel is expended during flight. A multivariable system is a system with several input and output.

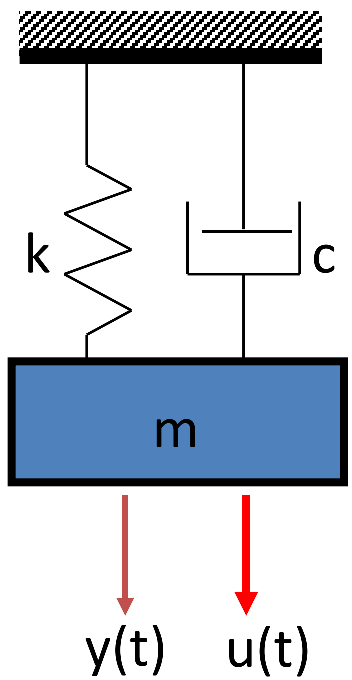
The State Variables of a Dynamic System:

The time-domain analysis and design of control systems utilizes the concept of the state of a system.

The state of a system is a set of variables such that the knowledge of these variables and the input functions will, with the equations describing the dynamics, provide the future state and output of the system. The state variables describe the future response of a system, given the present state, the excitation inputs, and the equations describing the dynamics.

A simple example of a state variable is the state of an on-off light switch. The switch can be in either the on or the off position, and thus the state of the switch can assume one of two possible values. Thus, if we know the present state (position) of the switch at t_0 and if an input is applied, we are able to determine the future value of the state of the element.

The concept of a set of state variables that represent a dynamic system can be illustrated in terms of the spring-mass-damper system shown in Figure 2. The number of state variables chosen to represent this system should be as small as possible in order to avoid redundant state variables. A set of state variables sufficient to describe this system includes the position and the velocity of the mass.



Dof system

In an actual system, there are several choices of a set of state variables that specify the *energy stored in a system* and therefore adequately describe the dynamics of the system.

The state variables of a system characterize the dynamic behavior of a system. The engineer's interest is primarily in physical, where the variables are voltages, currents, velocities, positions, pressures, temperatures, and similar physical variables.

The State Differential Equation: The state of a system is described by the set of first-order differential equations written in terms of the state variables [x_1 x_2 ... x_n]. These first-order differential equations can be written in general form as

$$
\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n + b_{11}u_1 + \cdots b_{1m}u_m \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n + b_{21}u_1 + \cdots b_{2m}u_m \\
&\vdots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots a_{nn}x_n + b_{n1}u_1 + \cdots b_{nm}u_m
\end{aligned}
$$

Thus, this set of simultaneous differential equations can be written in matrix form as follows:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \cdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \cdots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{bmatrix}\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}$$

n: number of state variables, m: number of inputs.

The column matrix consisting of the state variables is called the **state vector** and is written as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The vector of input signals is defined as u. Then the system can be represented by the compact notation of the state differential equation as

$$\dot{x} = A\,x + B\,u$$

This differential equation is also commonly called the state equation. The matrix **A** is an nxn square matrix, and **B** is an nxm matrix. The state differential equation relates the rate of change of the state of the system to the state of the system and the input signals. In general, the outputs of a linear system can be related to the state variables and the input signals by the output equation

$$y = C\,x + D\,u$$

Where **y** is the set of output signals expressed in column vector form. The state-space representation (or state-variable representation) is comprised of the state variable differential equation and the output equation.

We can write the state variable differential equation for the RLC circuit as

$$\dot{x} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} x + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} u(t)$$

and the output as

$$y = \begin{bmatrix} 0 & R \end{bmatrix} x$$

The solution of the state differential equation can be obtained in a manner similar to the approach we utilize for solving a first order differential equation. Consider the first-order differential equation

$$\dot{x} = ax + bu$$

Where x(t) and u(t) are scalar functions of time. We expect an exponential solution of the form $e^{at.}$ Taking the Laplace transform of both sides, we have

$$sX(s) - x_0 = aX(s) + bU(s)$$

therefore,

$$X(s) = \frac{x(0)}{s-a} + \frac{b}{s-a}U(s)$$

The inverse Laplace transform of X(s) results in the solution

$$x(t) = e^{at}x(0) + \int_0^t e^{a(t-\tau)} b\,u(\tau)\,d\tau$$

We expect the solution of the state differential equation to be similar to x(t) and to be of differential form. The **matrix exponential function** is defined as

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \cdots + \frac{A^kt^k}{k!} + \cdots$$

which converges for all finite t and any A. Then the solution of the state differential equation is found to be

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}B\,u(\tau)\,d\tau$$

$$X(s) = [sI - A]^{-1}x(0) + [sI - A]^{-1}B\,U(s)$$

where we note that $[sI-A]^{-1}=\varphi(s)$, which is the Laplace transform of $\varphi(t)=e^{At}$. The matrix exponential function $\varphi(t)$ describes the unforced response of the system and is called the fundamental or state transition matrix.

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)B\,u(\tau)\,d\tau$$

THE TRANSFER FUNCTION FROM THE STATE EQUATION

The transfer function of a single input-single output (SISO) system can be obtained from the state variable equations.

$$\dot{x} = A\,x + B\,u$$

$$y = C\,x$$

where y is the single output and u is the single input. The Laplace transform of the equations

$$sX(s) = AX(s) + B\,U(s)$$

$$Y(s) = CX(s)$$

where B is an nx1 matrix, since u is a single input. We do not include initial conditions, since we seek the transfer function. Reordering the equation

$$[sI - A]X(s) = BU(s)$$

$$X(s) = [sI - A]^{-1}BU(s) = \phi(s)BU(s)$$

$$Y(s) = C\phi(s)BU(s)$$

Therefore, the transfer function G(s)=Y(s)/U(s) is

$$G(s) = C\phi(s)B$$

Example:

Determine the transfer function G(s)=Y(s)/U(s) for the RLC circuit as described by the state differential function

$$\dot{x} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} x + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} u \quad , \quad y = \begin{bmatrix} 0 & R \end{bmatrix} x$$

$$[sI - A] = \begin{bmatrix} s & \frac{1}{C} \\ -\frac{1}{L} & s + \frac{R}{L} \end{bmatrix}$$

$$\phi(s) = [sI - A]^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} s + \frac{R}{L} & -\frac{1}{C} \\ \frac{1}{L} & s \end{bmatrix}$$

$$\Delta(s) = s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Then the transfer function is

$$G(s) = [0 \quad R] \begin{bmatrix} \frac{s + \frac{R}{L}}{\Delta(s)} & -\frac{1}{C\Delta(s)} \\ \frac{1}{L\Delta(s)} & \frac{s}{\Delta(s)} \end{bmatrix} \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix}$$

$$G(s) = \frac{R/LC}{\Delta(s)} = \frac{R/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

THE DESIGN OF STATE VARIABLE FEEDBACK SYSTEMS

The time-domain method, expressed in terms of state variables, can also be utilized to design a suitable compensation scheme for a control system. Typically, we are interested in controlling the system with a control signal, u(t), which is a function of several measurable state variables. Then we develop a state variable controller that operates on the information available in measured form.

State variable design is typically comprised of three steps. In the first step, we assume that all the state variables are measurable and utilize them in a **full-state feedback control law**. Full-state feedback is not usually practical because it is not possible (in general) to measure all the states. In paractice, only certain states (or linear combinations thereof) are measured and provided as system outputs. The second step in state varaible design is to construct an **observer** to estimate the states that are not directly sensed and available as outputs.

CONTROLLABILITY:

Full-state feedback design commonly relies on **pole-placement techniques**. It is important to note that a system must be completely controllable and completely observable to allow the flexibility to place all the closed-loop system poles arbitrarily. The concepts of controllability and observability were introduced by Kalman in the 1960s.

A system is completely controllable if there exists an unconstrained control $u(t)$ that can transfer any initial state $x(t_0)$ to any other desired location $x(t)$ in a finite time, $t_0 \leq t \leq T$.

For the system

$$\dot{x} = Ax + Bu$$

we can determine whether the system is controllable by examining the algebraic condition

$$\text{rank}\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} = n$$

The matrix A is an nxn matrix an B is an nx1 matrix. For multi input systems, B can be nxm, where m is the number of inputs.

For a single-input, single-output system, the controllability matrix P_c is described in terms of A and B as

$$P_c = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}$$

which is nxn matrix. Therefore, if the determinant of P_c is nonzero, the system is controllable.

OBSERVABILITY:

All the poles of the closed-loop system can be placed arbitrarily in the complex plane if and only if the system is **observable** and **controllable**. Observability refers to the ability to estimate a state variable.

A system is completely observable if and only if there exists a finite time T such that the initial state x(0) can be determined from the observation history y(t) given the control u(t).

Consider the single-input, single-output system

$$\dot{x} = Ax + Bu \quad \text{and} \quad y = Cx$$

where C is a 1xn row vector, and x is an nx1 column vector. This system is completely observable when the determinant of the **observability matrix P_0** is nonzero.

The observability matrix, which is an nxn matrix, is written as

$$P_O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Example:

Consider the previously given system

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$CA = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad , \quad CA^2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Thus, we obtain

$$P_O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The det P_0=1, and the system is completely observable. Note that determination of observability does not utility the B and C matrices.

Example: Consider the system given by

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \quad \text{and} \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} x$$

We can check the system controllability and observability using the P_c and P_0 matrices.

From the system definition, we obtain

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and} \quad AB = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

Therefore, the controllability matrix is determined to be

$$P_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

det P_c=0 and rank(P_c)=1. Thus, the system is not controllable.

From the system definition, we obtain

$$C = [1 \quad 1] \quad \text{and} \quad CA = [1 \quad 1]$$

Therefore, the observability matrix is determined to be

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

det P_O=0 and rank(P_O)=1. Thus, the system is not observable.

If we look again at the state model, we note that

$$y = x_1 + x_2$$

However,

$$\dot{x}_1 + \dot{x}_2 = 2x_1 + (x_2 - x_1) + u - u = x_1 + x_2$$

An example of a canonical transformation

$$F_1 = \sum_{m=1}^{M} q_m Q_m \qquad p_m = \frac{\partial F}{\partial q_m}; P_m = -\frac{\partial F}{\partial Q_m}; K - H = \frac{\partial F}{\partial t}$$

$$\boxed{Q_m = p_m; P_m = -q_m; K = H}$$

- Generalized coordinates are indistinguishable from their conjugate momenta, and the nomenclature for them is arbitrary

- Bottom-line: generalized coordinates and their conjugate momenta should be treated equally in the phase space

Criterion for canonical transformations

$$Q = Q(q, p); \; P = P(q, p) \qquad q = q(Q, P); \; p = p(Q, P)$$

- How to make sure this transformation is canonical?

$$\dot{Q} = \dot{q}\frac{\partial Q}{\partial q} + \dot{p}\frac{\partial Q}{\partial p} = \frac{\partial H}{\partial p}\frac{\partial Q}{\partial q} - \frac{\partial H}{\partial q}\frac{\partial Q}{\partial p}$$

- On the other hand

$$\frac{\partial H}{\partial P} = \frac{\partial H}{\partial p}\frac{\partial p}{\partial P} + \frac{\partial H}{\partial q}\frac{\partial q}{\partial P}$$

- If

$$\frac{\partial Q}{\partial q} = \frac{\partial p}{\partial P}; \frac{\partial Q}{\partial p} = -\frac{\partial q}{\partial P}$$

- Then

$$\dot{Q} = \frac{\partial H}{\partial P}$$

Criterion for canonical transformations

- **Similarly,** $$\dot{P} = \frac{\partial P}{\partial q}\dot{q} + \frac{\partial P}{\partial p}\dot{p} = \frac{\partial H}{\partial p}\frac{\partial P}{\partial q} - \frac{\partial H}{\partial q}\frac{\partial P}{\partial p}$$

$$\frac{\partial H}{\partial Q} = \frac{\partial H}{\partial p}\frac{\partial p}{\partial Q} + \frac{\partial H}{\partial q}\frac{\partial q}{\partial Q}$$

- **If** $$\frac{\partial P}{\partial q} = -\frac{\partial p}{\partial Q};\ \frac{\partial P}{\partial p} = \frac{\partial q}{\partial Q}$$

- **Then** $$\dot{P} = -\frac{\partial H}{\partial Q}$$

Criterion for canonical transformations

- **So,**

$$\dot{q} = \frac{\partial H}{\partial p};\ \dot{p} = -\frac{\partial H}{\partial q} \to \dot{Q} = \frac{\partial H}{\partial P};\ \dot{P} = -\frac{\partial H}{\partial Q};$$

- **If**

$$\boxed{\frac{\partial P}{\partial p} = \frac{\partial q}{\partial Q};\ \frac{\partial Q}{\partial q} = \frac{\partial p}{\partial P};\ \frac{\partial P}{\partial q} = -\frac{\partial p}{\partial Q};\ \frac{\partial Q}{\partial p} = -\frac{\partial q}{\partial P}}$$

DIGITAL CONTROL-OVERVIEW, ADVANTAGES AND DISADVANTAGES:

Digital Control Overview and Implementation:

- A digital control takes an analog signal, samples it with an analog to digital converter (A/D), processes the information in the digital domain, and the converts the signal to analog with a digital-to-analog converter.
- The key here is to provide redundant paths in the event of hard ware failure. Here the signal comes from a sensing device, such as gyro.
- Next, it is fed in parallel along multiple paths to an analog to digital (A/D) converter.
- After the signal is in the digital form, the flight control computer executes the control algorithms.
- The output from the flight control computers is then fed to a digital-to-analog (D/A) converter, which in turn operate an actuator.

Digital Control Advantages

1. They are more versatile than analog because they can be easily programmed without changing the hardware.
2. It is easy to implement gain scheduling to vary flight control gains as the aircraft dynamics change with flight conditions.
3. Digital components in the form of electronic parts, transducers and encoders are often more reliable, more rugged, and more compact than analog equipments.
4. Multi mode and more complex digital control laws can be implemented because of fast, light, and economical micro-processors.
5. It is possible to design "Robust" controller that can control the aircraft for various flight conditions including some mechanical failures.
6. Improved sensitivity with sensitive control elements that require relatively low energy levels.

Disadvantages of Digital Control

1. The lag associated with sampling process reduces the system stability.
2. The mathematical analysis and system design of a sampled data system is more complex.
3. The signal information may be lost because it must be digitally reconstructed from an analog signal.
4. The complexity of the control process is in the software implemented control algorithm that may contain error.
5. Software verification becomes critical because of the safety of flight issue. Software errors can cause the aircraft to crash.