INSTITUTE OF AERONAUTICAL ENGINEERING

STRUCTURAL ANALYSIS – I

By

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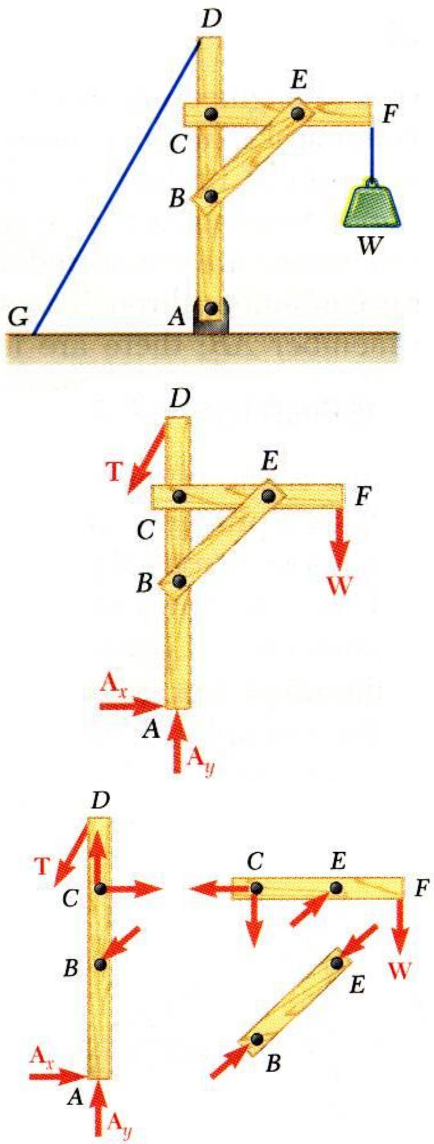
Unit – 1
Analysis of Perfect Frames

Contents

Introduction

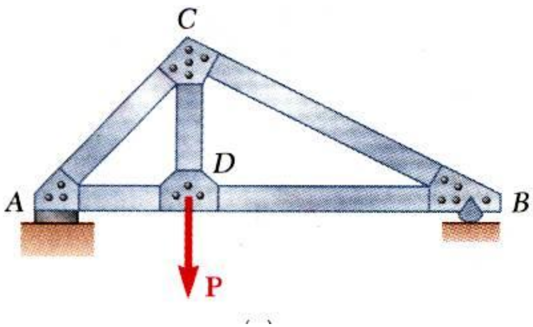


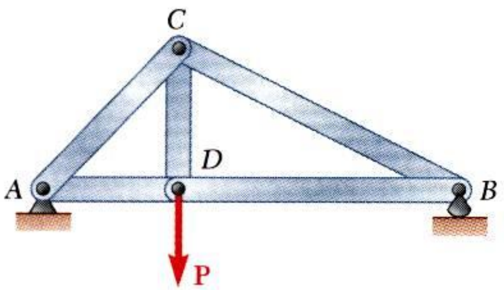
- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.

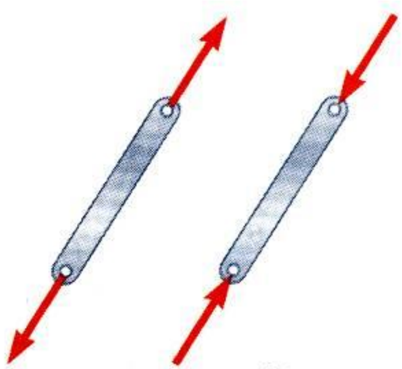
- In the interaction between connected parts, Newton's 3rd Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.

- Three categories of engineering structures are considered:
  - a) *Frames*: contain at least one one multi-force member, i.e., member acted upon by 3 or more forces.
  - b) *Trusses*: formed from *two-force members*, i.e., straight members with end point connections
  - c) *Machines*: structures containing moving parts designed to transmit and modify forces.

Definition of a Truss







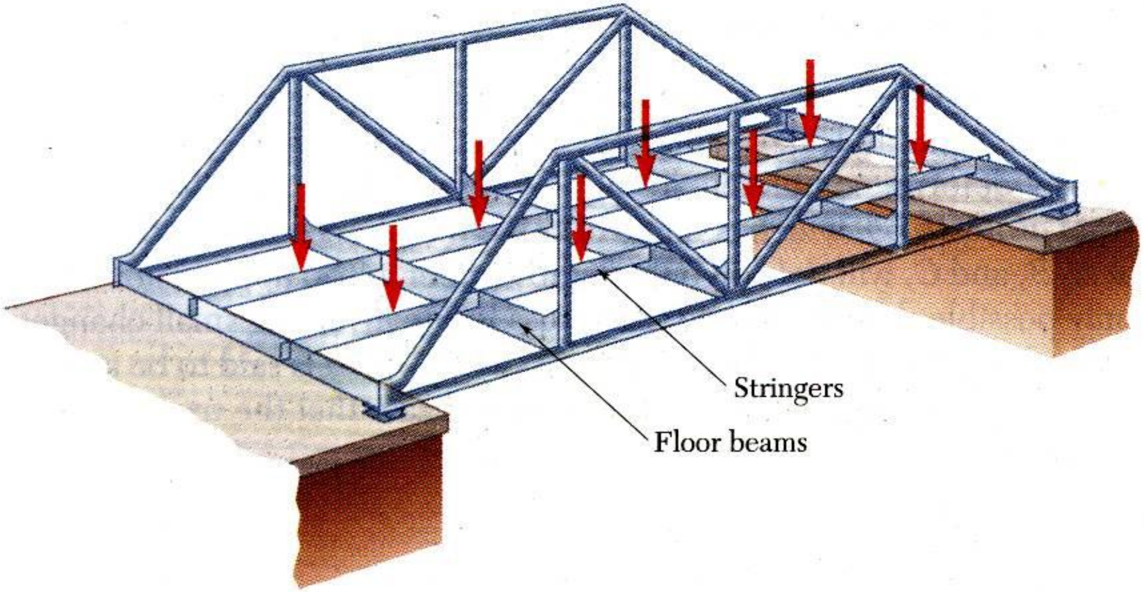
- A truss consists of straight members connected at joints. No member is continuous through a joint.

- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.

- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.

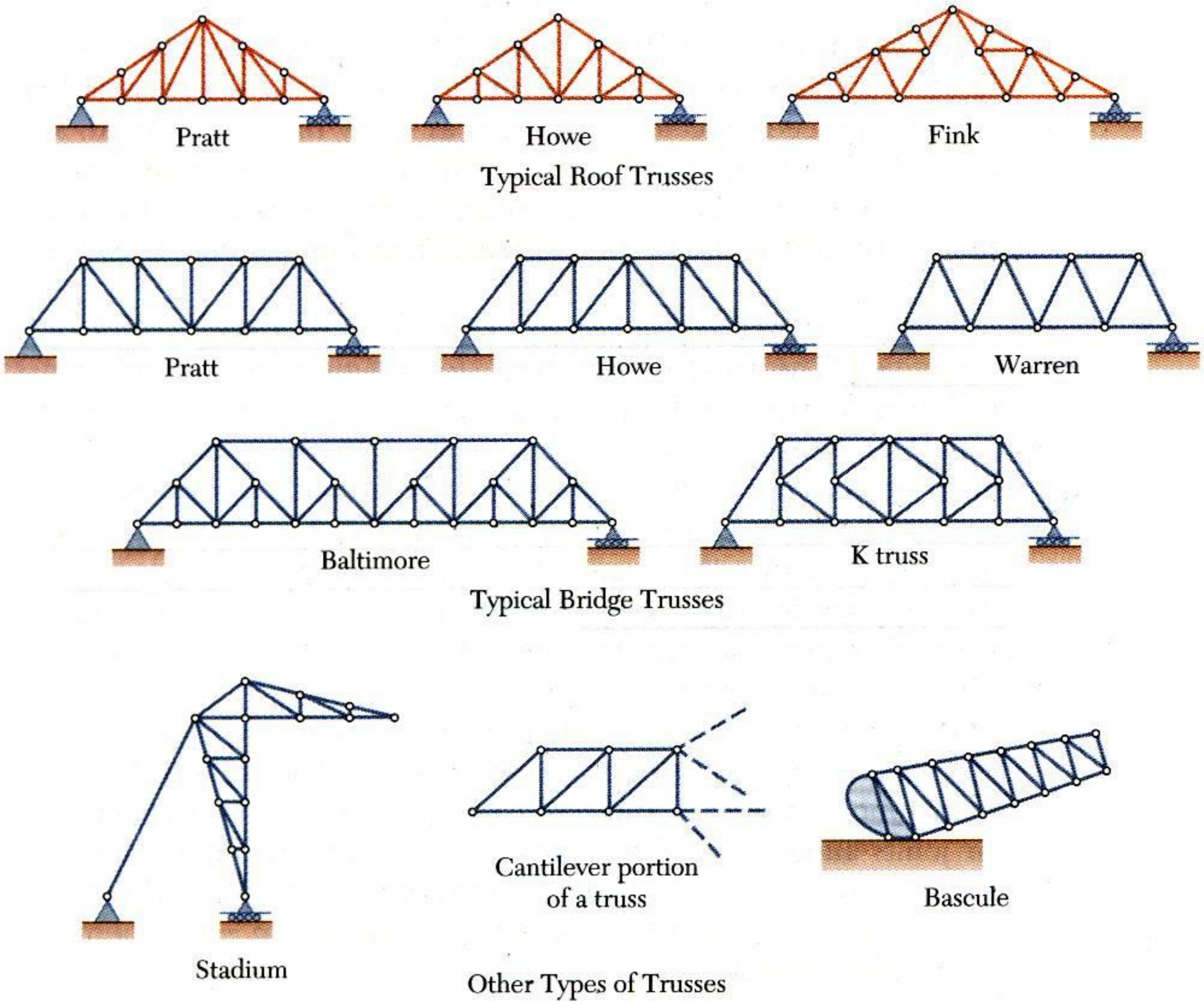
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

Definition of a Truss

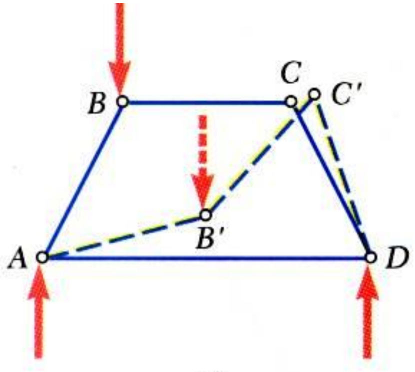


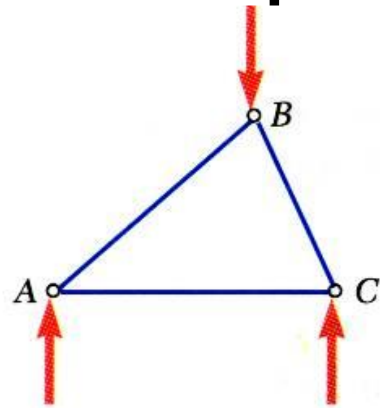
Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.

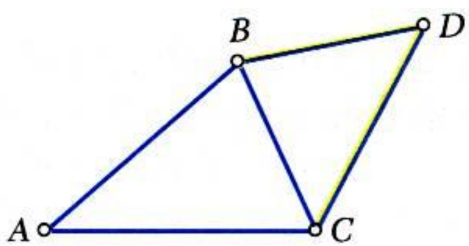
Definition of a Truss

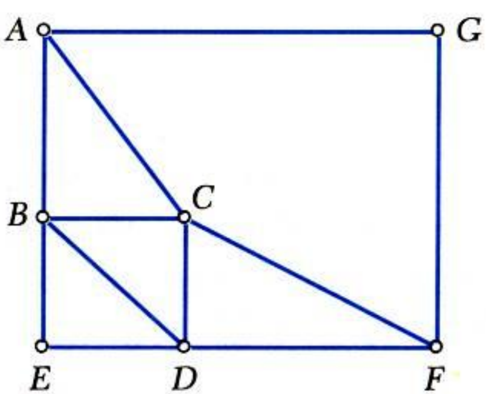


Simple Trusses







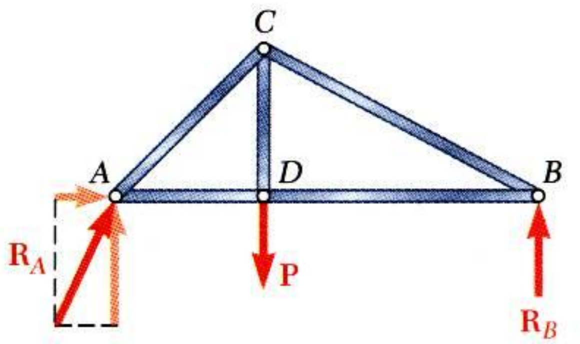


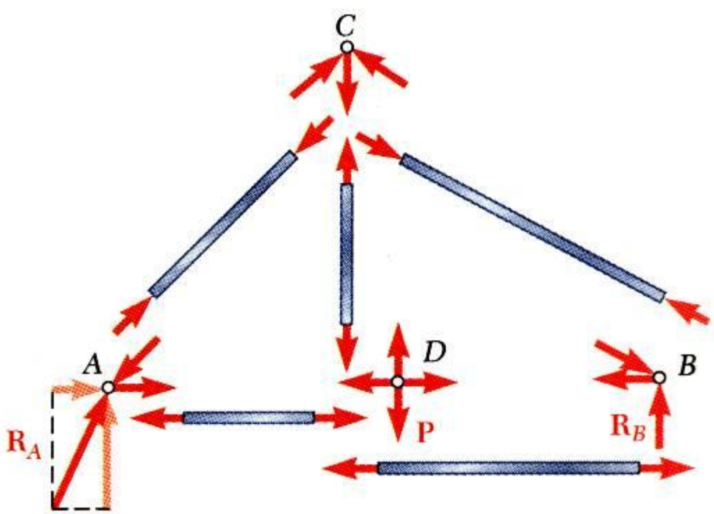
- A *rigid truss* will not collapse under the application of a load.

- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.

- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.

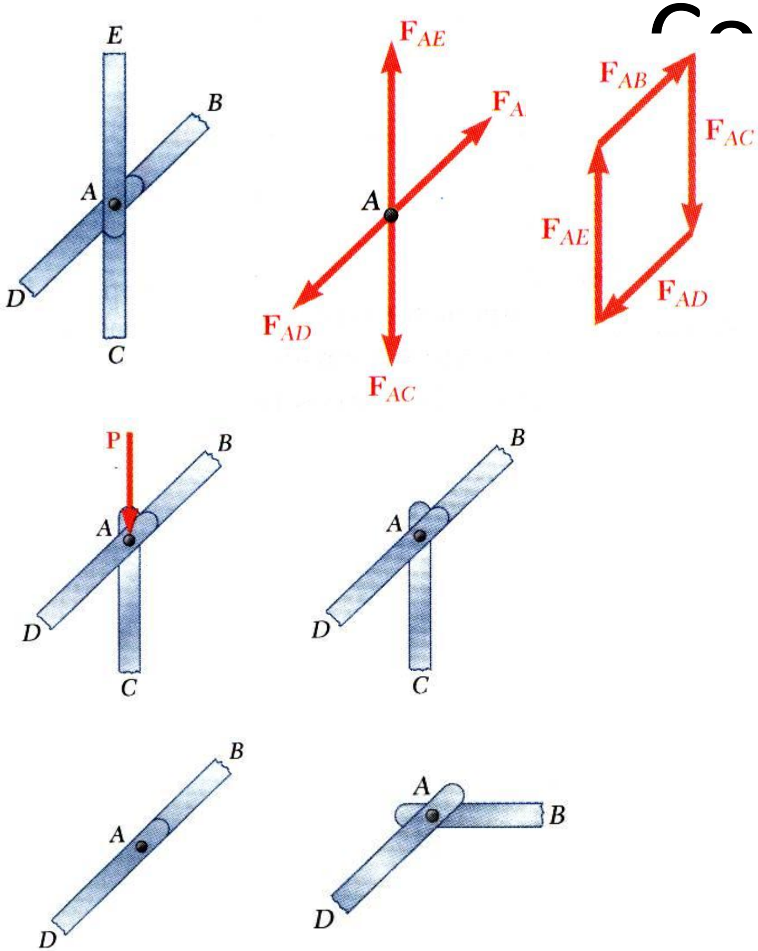
Analysis of Trusses by the Method of Joints



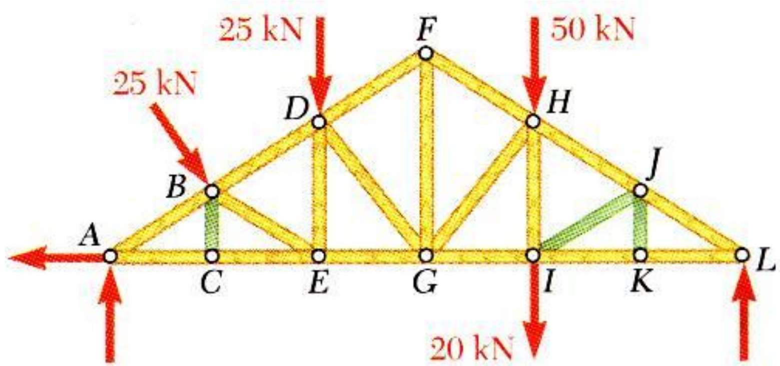


- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces at the supports.
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

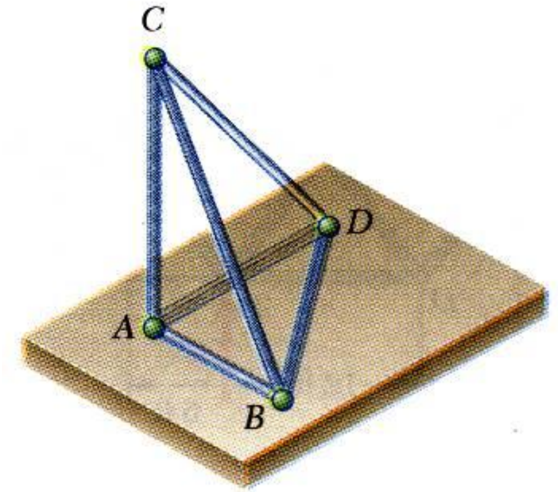
Joints Under Special Loading Conditions

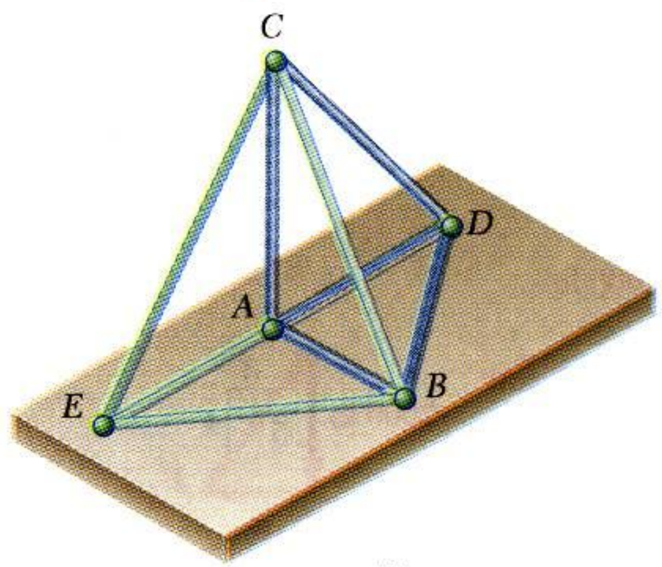


- Forces in opposite members intersecting in two straight lines at a joint are equal.
- The forces in two opposite members are equal when a load is aligned with a third member. The third member force is equal to the load (including zero load).
- The forces in two members connected at a joint are equal if the members are aligned and zero otherwise.
- Recognition of joints under special loading conditions simplifies a truss analysis.



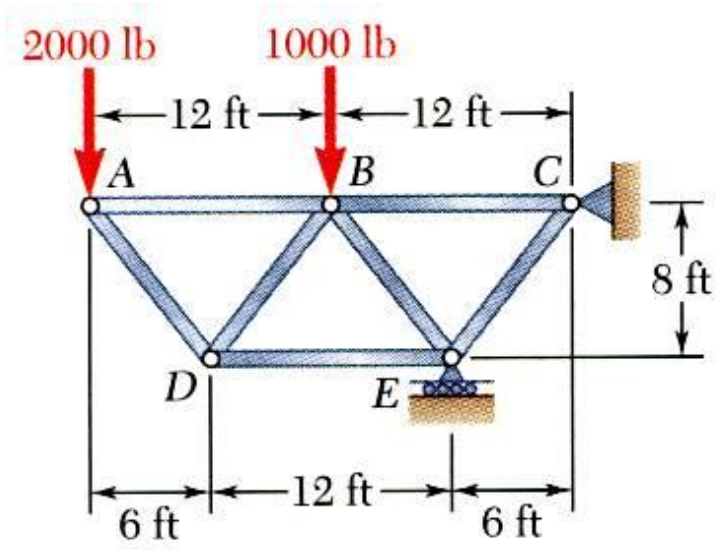
Space Trusses





- An *elementary space truss* consists of 6 members connected at 4 joints to form a tetrahedron.
- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.

Sample Problem 6.1

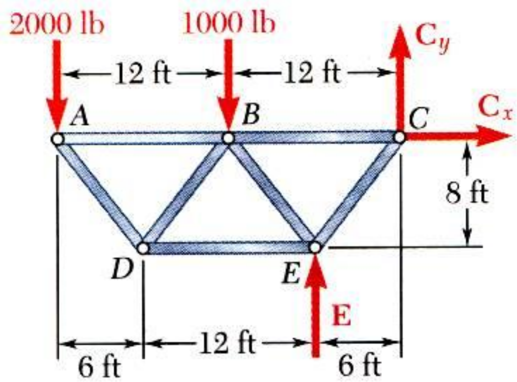


Using the method of joints, determine the force in each member of the truss.

<u>SOLUTION</u>:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

Sample Problem 6.1



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.

$$\sum M_C = 0$$
$$= (2000\text{ lb})(24\text{ ft}) + (1000\text{ lb})(12\text{ ft}) - E(6\text{ ft})$$

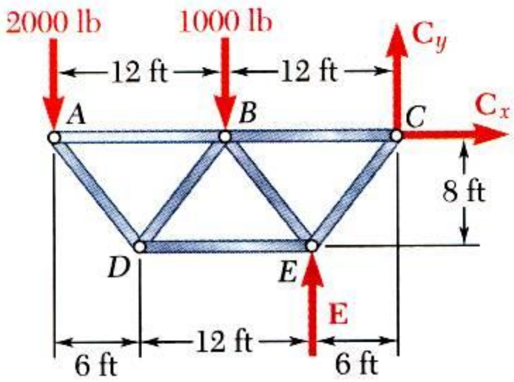
$$\boxed{E = 10{,}000\text{ lb} \uparrow}$$

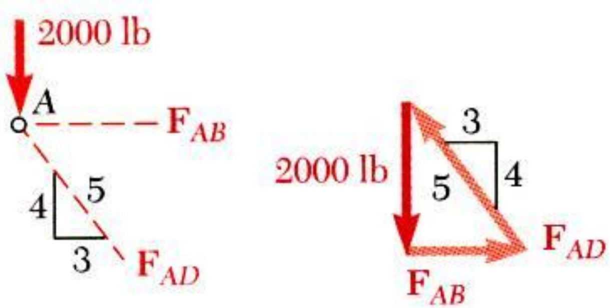
$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -2000\text{ lb} - 1000\text{ lb} + 10{,}000\text{ lb} + C_y$$

$$\boxed{C_y = 7000\text{ lb} \downarrow}$$

Sample Problem 6.1

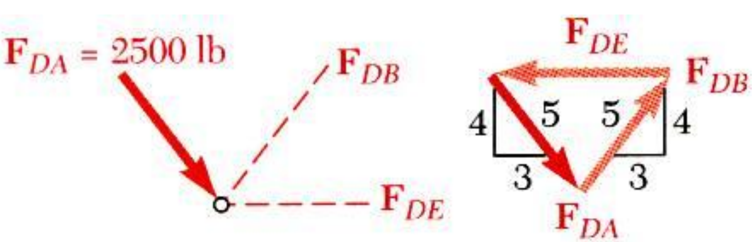




- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{2000\,\text{lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 1500\,\text{lb}\ \ T$$
$$F_{AD} = 2500\,\text{lb}\ \ C$$

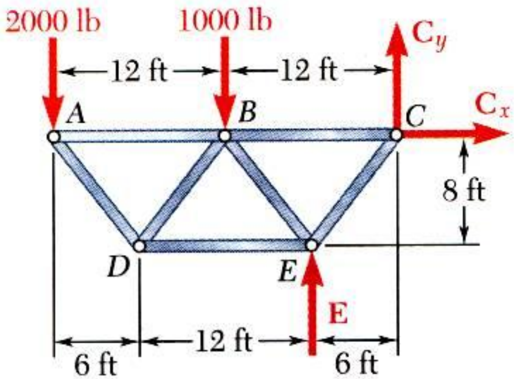


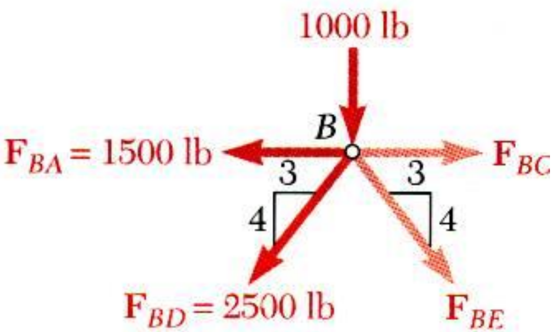
- There are now only two unknown member forces at joint D.

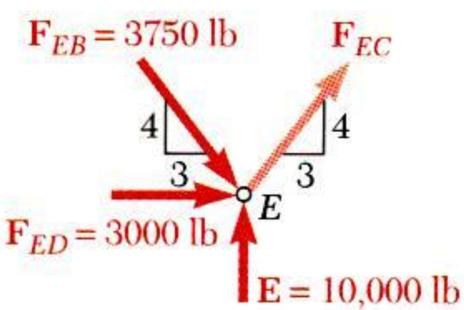
$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$F_{DB} = 2500\,\text{lb}\ \ T$$
$$F_{DE} = 3000\,\text{lb}\ \ C$$

Sample Problem 6.1







- There are now only two unknown member forces at joint B. Assume both are in tension.

$$\sum F_y = 0 = -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5} F_{BE}$$

$$F_{BE} = -3750\text{ lb} \qquad \boxed{F_{BE} = 3750\text{ lb } C}$$

$$\sum F_x = 0 = F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750)$$

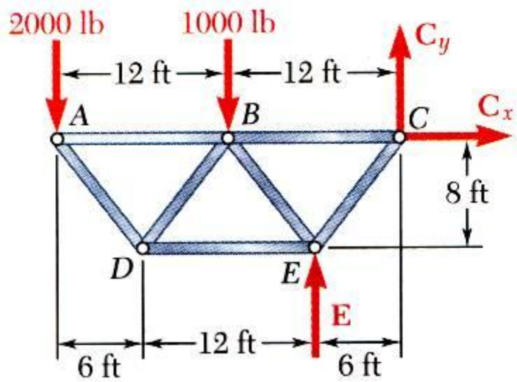
$$F_{BC} = +5250\text{ lb} \qquad \boxed{F_{BC} = 5250\text{ lb } T}$$

- There is one unknown member force at joint *E*. Assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5} F_{EC} + 3000 + \tfrac{3}{5}(3750)$$

$$F_{EC} = -8750\text{ lb} \qquad \boxed{F_{EC} = 8750\text{ lb } C}$$

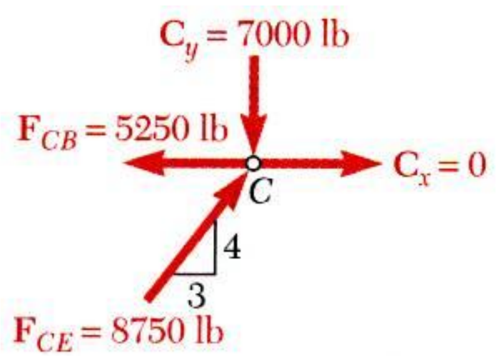
Sample Problem 6.1



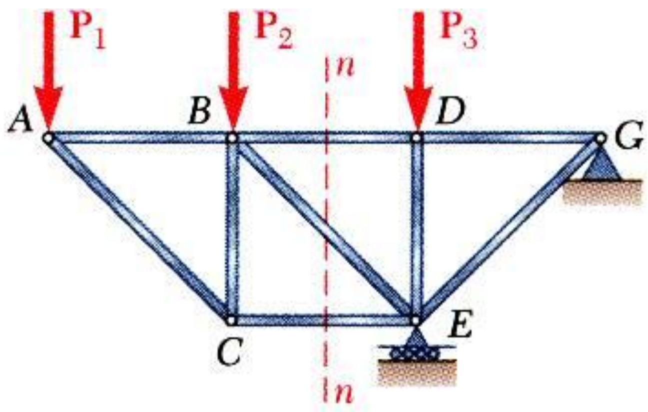
- All member forces and support reactions are known at joint C. However, the joint equilibrium requirements may be applied to check the results.

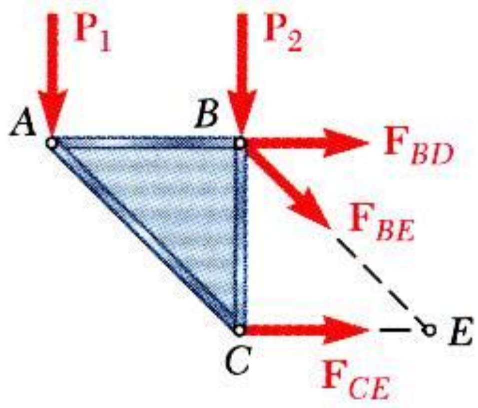
$$\sum F_x = -5250 + \tfrac{3}{5}(8750) = 0 \quad (\text{checks})$$

$$\sum F_y = -7000 + \tfrac{4}{5}(8750) = 0 \quad (\text{checks})$$



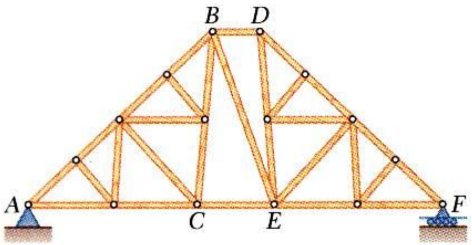
Analysis of Trusses by the Method of Sections

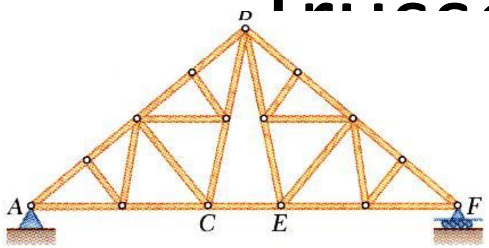




- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD*, *pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.

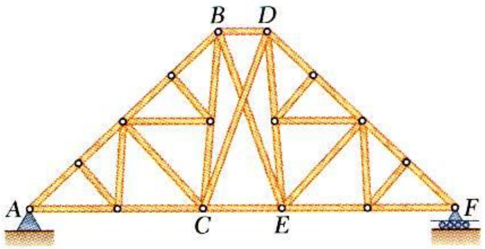
Trusses Made of Several Simple Trusses





- *Compound trusses* are statically determinant, rigid, and completely constrained.

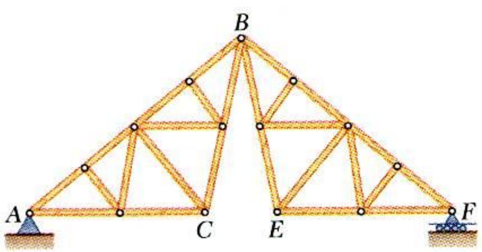
$$m = 2n - 3$$



- Truss contains a *redundant member* and is *statically indeterminate*.

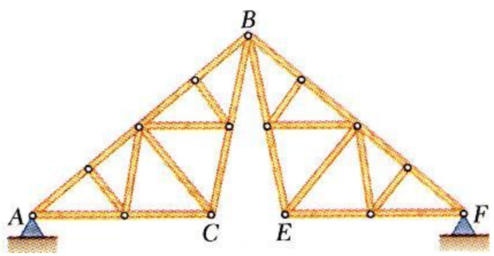
$$m > 2n - 3$$

- Additional reaction forces may be necessary for a rigid truss.



non-rigid

$$m < 2n - 3$$



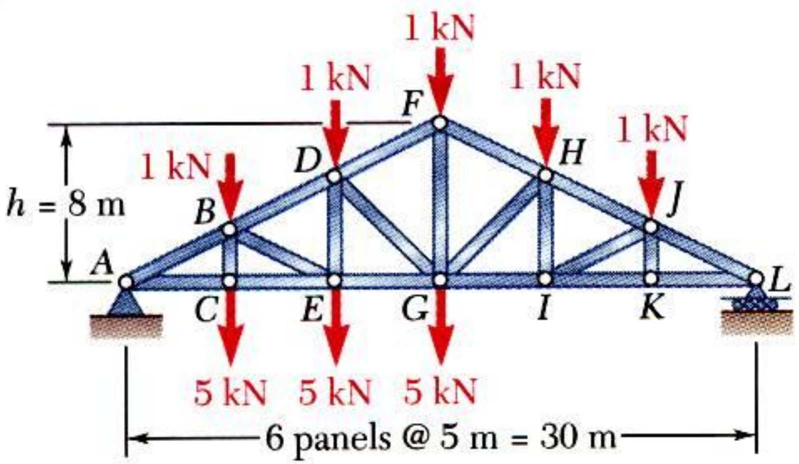
rigid

$$m < 2n - 4$$

- Necessary but insufficient condition for a compound truss to be statically determinant, rigid, and completely constrained,

$$m + r = 2n$$

Sample Problem 6.3

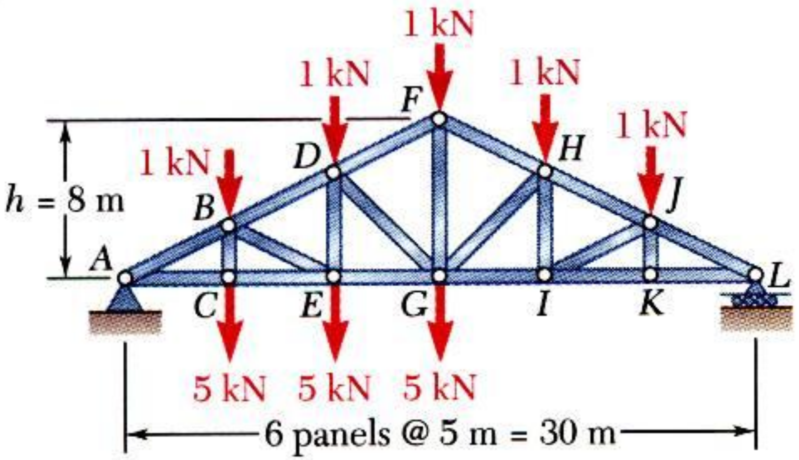


Determine the force in members *FH*, *GH*, and *GI*.

<u>SOLUTION</u>:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.
- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Sample Problem 6.3



SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.

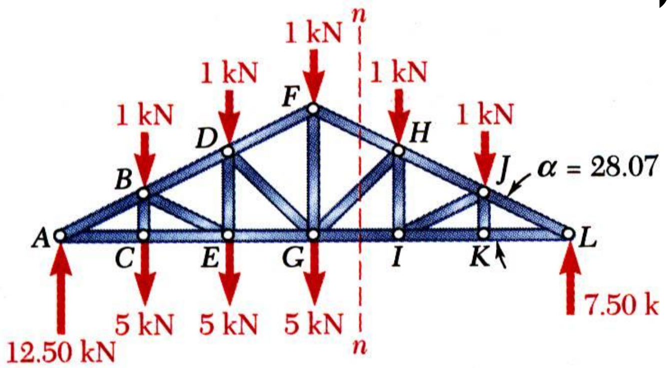
$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN}) - (20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$

$$L = 7.5\text{ kN} \uparrow$$

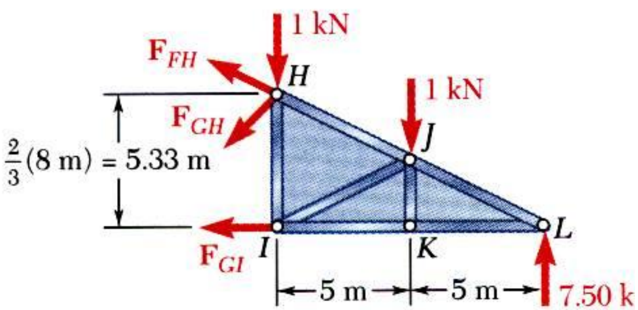
$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

Sample Problem 6.3



- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.



- Apply the conditions for static equilibrium to determine the desired member forces.

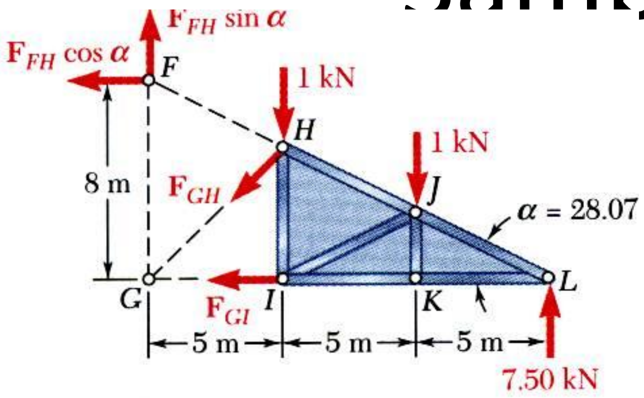
$$\sum M_H = 0$$

$$(7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) = 0$$

$$F_{GI} = +13.13\text{ kN}$$

$$\boxed{F_{GI} = 13.13\text{ kN}\ \ T}$$

Sample Problem 6.3



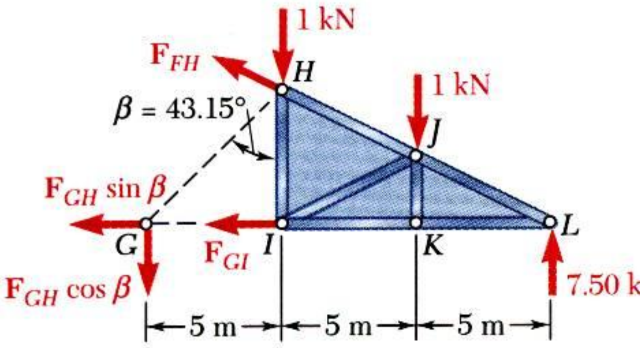
$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.82\text{ kN}$$

$$\boxed{F_{FH} = 13.82\text{ kN} \;\; C}$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

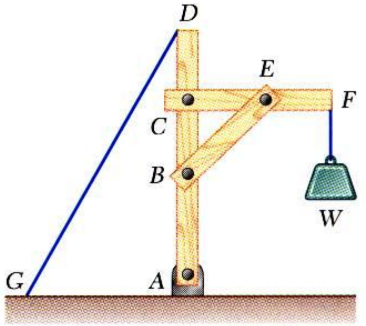
$$\sum M_L = 0$$

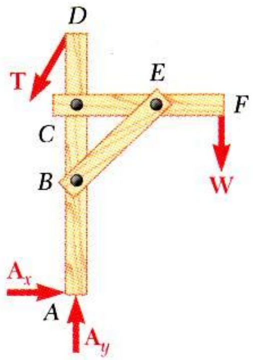
$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(10\text{ m}) = 0$$

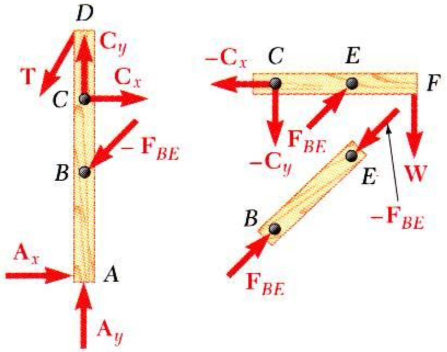
$$F_{GH} = -1.371\text{ kN}$$

$$\boxed{F_{GH} = 1.371\text{ kN} \;\; C}$$

Analysis of Frames

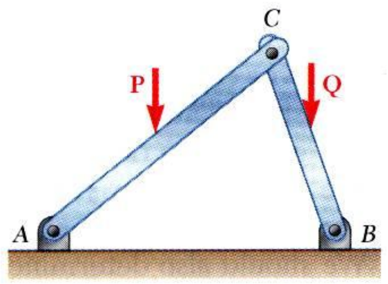


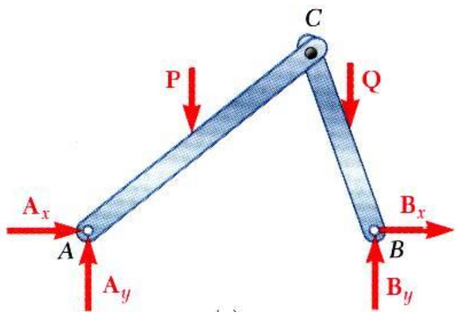


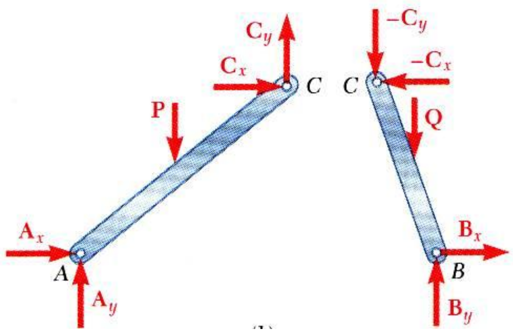


- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.

Frames Which Cease To Be Rigid When Detached From Their Supports

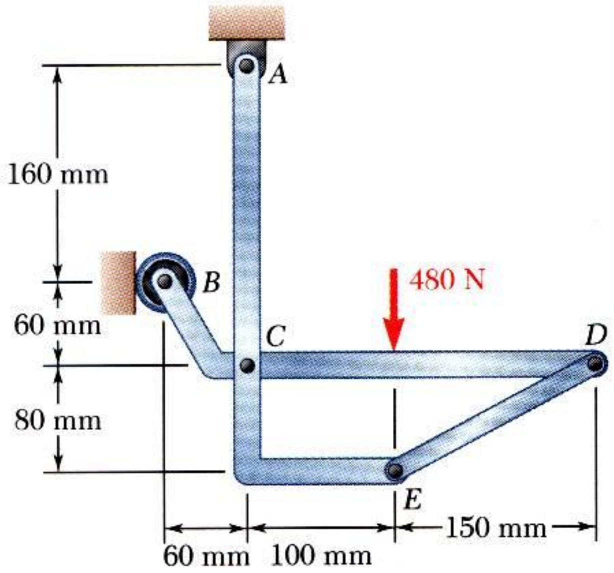






- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates four unknown force components which can not be determined from the three equilibrium conditions.
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams indicate 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations.

Sample Problem 6.4

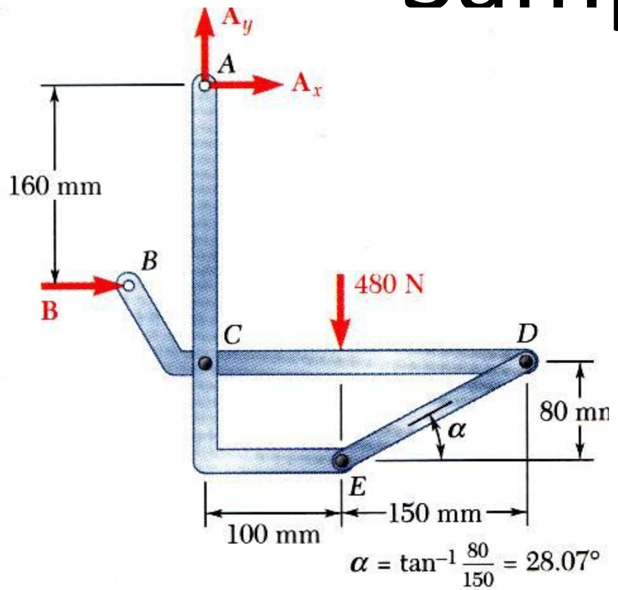


Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.
- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.
- With the force on the link *DE* known, the sum of forces in the *x* and *y* directions may be used to find the force components at *C*.
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

Sample Problem 6.4



SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\text{ N} \qquad \boxed{A_y = 480\text{ N} \uparrow}$$

$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

$$\boxed{B = 300\text{ N} \rightarrow}$$

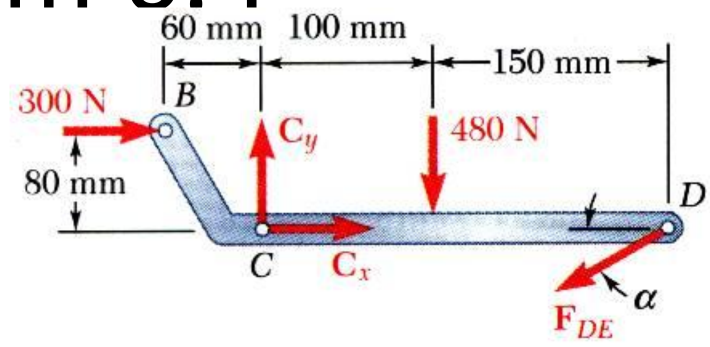
$$\sum F_x = 0 = B + A_x \qquad \boxed{A_x = -300\text{ N} \leftarrow}$$

Note:

$$\alpha = \tan^{-1}\frac{80}{150} = 28.07^\circ$$

Sample Problem 6.4

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.



$$\sum M_C = 0 = (F_{DE} \sin\alpha)(250\text{ mm}) + (300\text{ N})(60\text{ mm}) + (480\text{ N})(100\text{ mm})$$

$$F_{DE} = -561\text{ N}$$

$$\boxed{F_{DE} = 561\text{ N}\ C}$$

- Sum of forces in the *x* and *y* directions may be used to find the force components at *C*.

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$

$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N}$$

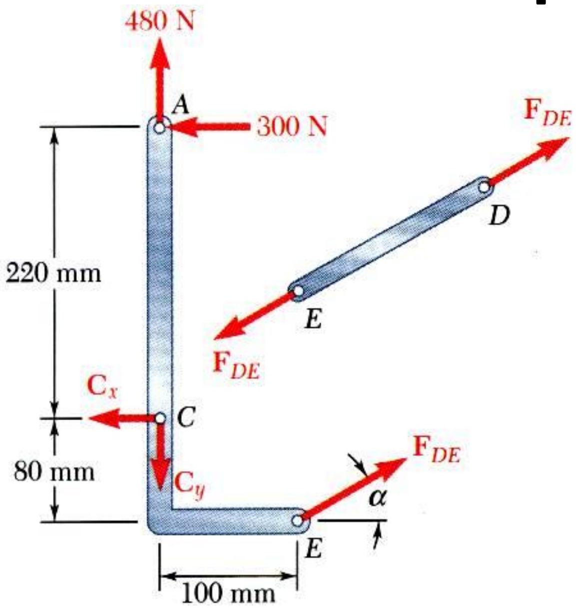
$$\boxed{C_x = -795\text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$

$$0 = C_y - (-561\text{ N})\sin\alpha - 480\text{ N}$$

$$\boxed{C_y = 216\text{ N}}$$

Sample Problem 6.4

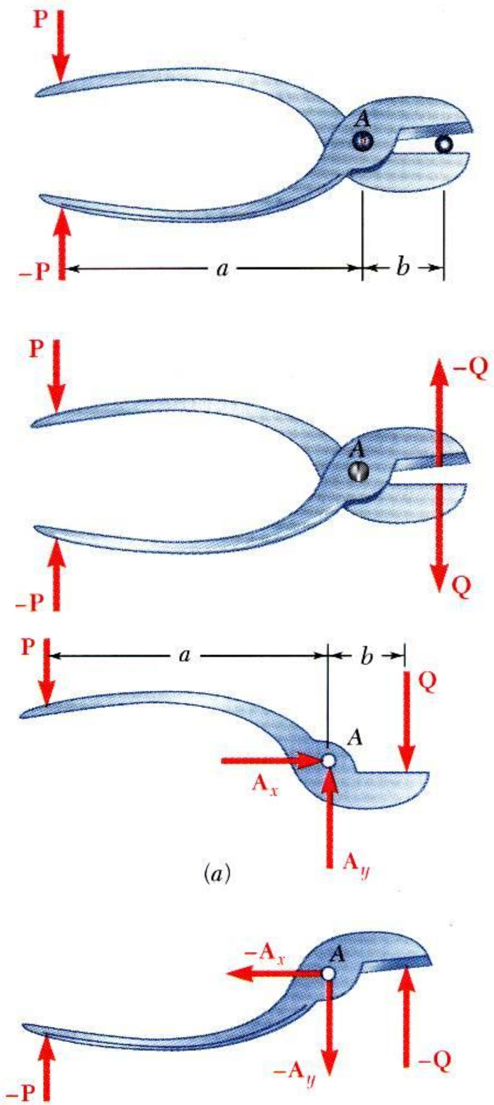


- With member *ACE* as a free-body, check the solution by summing moments about *A*.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

(checks)

Machines



- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*.
- Given the magnitude of ***P***, determine the magnitude of ***Q***.
- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.
- The machine is a nonrigid structure. Use one of the components as a free-body.
- Taking moments about *A*,

$$\sum M_A = 0 = aP - bQ \qquad Q = \frac{a}{b}P$$

Unit -2
Energy theorems & Three Hinged Arches

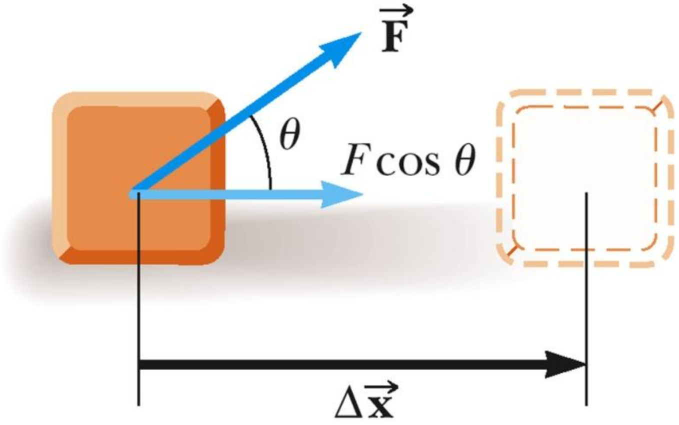
Potential Energy and Energy Conservation

- Gravitational Potential Energy
- Elastic Potential Energy
- Work-Energy Theorem
- Conservative and Non-conservative Forces
- Conservation of Energy

Definition of Work *W*

- The work, *W*, done by a constant force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement

$$W \equiv (F\cos\theta)\Delta x$$



- F is the magnitude of the force
- Δ x is the magnitude of the object's displacement
- θ is the angle between $\vec{F}$ and $\Delta\vec{x}$

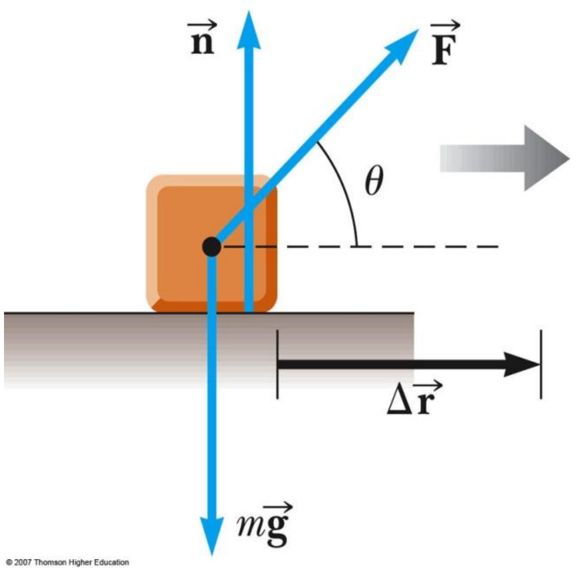
Work Done by Multiple Forces

- If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{\text{net}} = \sum W_{\text{by individual forces}}$$

  - Remember work is a scalar, so this is the algebraic sum

$$W_{net} = W_g + W_N + W_F = (F\cos\theta)\Delta r$$



Kinetic Energy and Work

- Kinetic energy associated with the motion of an object

$$KE = \frac{1}{2}mv^2$$

- Scalar quantity with the same unit as work
- Work is related to kinetic energy

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2} = (F_{net}\cos\theta)\Delta x$$

$$= \int_{x_i}^{x_f} \mathbf{F} \cdot d\mathbf{r}$$

Units: N-m or J

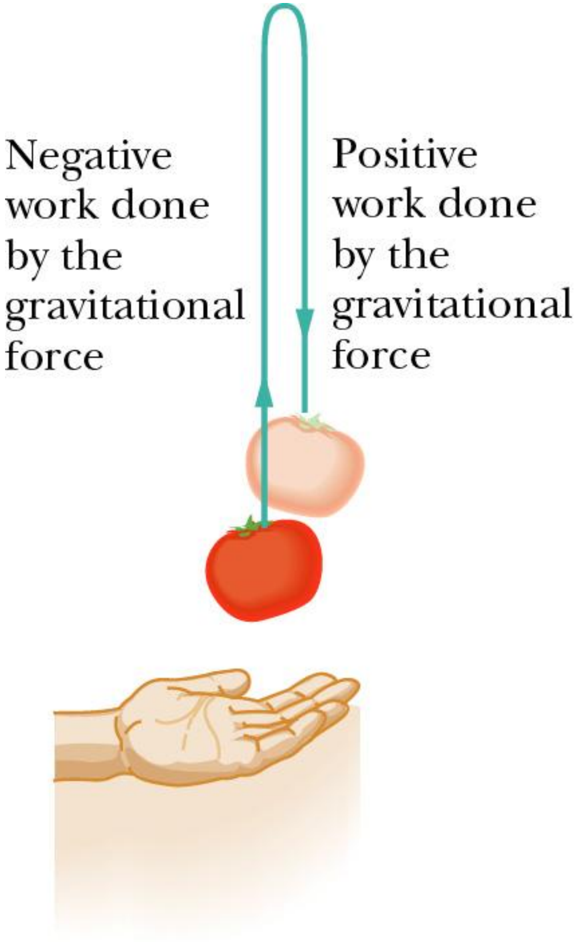
$$W_{net} = KE_f - KE_i = \Delta KE$$

Work done by a Gravitational Force

- Gravitational Force
  - Magnitude: mg
  - Direction: downwards to the Earth's center
- Work done by Gravitational Force

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}$$

$$W_g = mg\Delta r\cos\theta$$

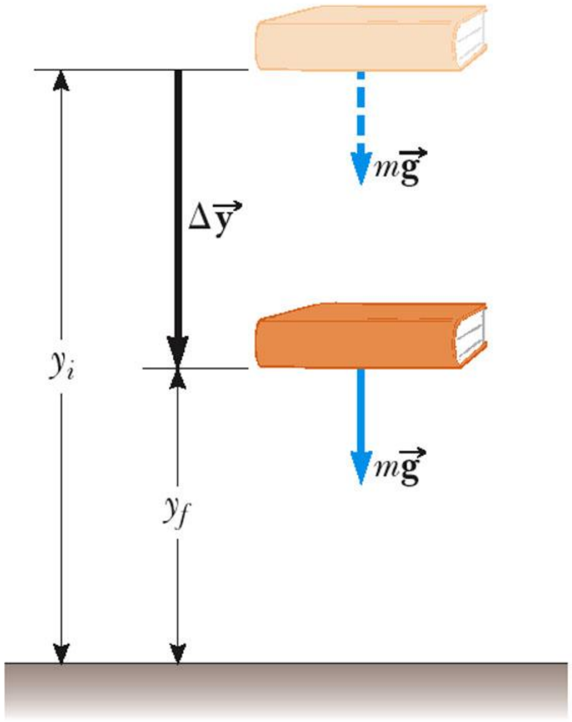


Potential Energy

- Potential energy is associated with the position of the object
- Gravitational Potential Energy is the energy associated with the relative position of an object in space near the Earth's surface
- The gravitational potential energy

$$PE \equiv mgy$$

  - m is the mass of an object
  - g is the acceleration of gravity
  - y is the vertical position of the mass relative the surface of the Earth
  - SI unit: joule (J)



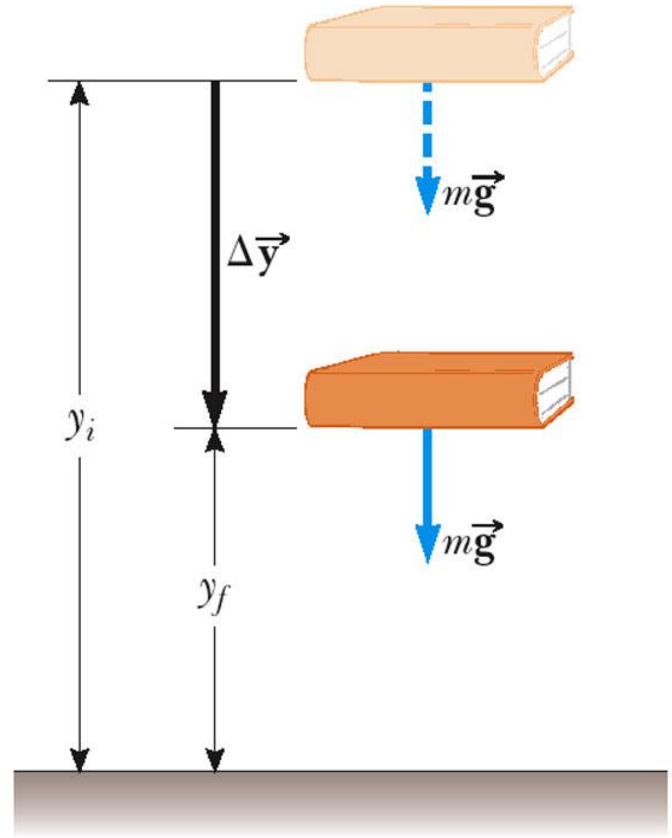
Reference Levels

- A location where the gravitational potential energy is zero must be chosen for each problem
  - The choice is arbitrary since the change in the potential energy is the important quantity
  - Choose a convenient location for the zero reference height
    - often the Earth's surface
    - may be some other point suggested by the problem
  - Once the position is chosen, it must remain fixed for the entire problem

Work and Gravitational Potential Energy

- $PE = mgy$
- $$W_g = F\Delta y \cos\theta = mg(y_f - y_i)\cos 180$$
  $$= -mg(y_f - y_i) = PE_i - PE_f$$
- Units of Potential Energy are the same as those of Work and Kinetic Energy

$$W_{gravity} = \Delta KE = -\Delta PE = PE_i - PE_f$$



Extended Work-Energy Theorem

- The work-energy theorem can be extended to include potential energy:

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_{gravity} = PE_i - PE_f$$

- If we only have gravitational force, then $W_{net} = W_{gravity}$

$$KE_f - KE_i = PE_i - PE_f$$

$$KE_f + PE_f = PE_i + KE_i$$

- The sum of the kinetic energy and the gravitational potential energy remains constant at all time and hence is a conserved quantity

Extended Work-Energy Theorem

- We denote the total mechanical energy by

$$E = KE + PE$$

- Since

$$KE_f + PE_f = PE_i + KE_i$$

- The total mechanical energy is conserved and remains the same at all times

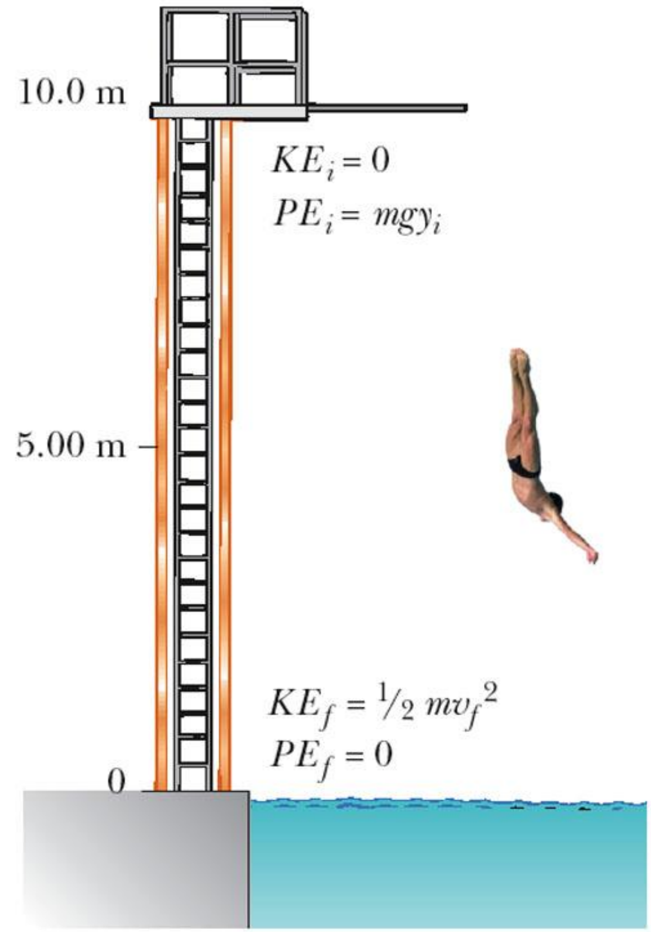
$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

Problem-Solving Strategy

- Define the system
- Select the location of zero gravitational potential energy
  - Do *not* change this location while solving the problem
- Identify two points the object of interest moves between
  - One point should be where information is given
  - The other point should be where you want to find out something

Platform Diver

- A diver of mass m drops from a board 10.0 m above the water's surface. Neglect air resistance.
- (a) Find is speed 5.0 m above the water surface
- (b) Find his speed as he hits the water



Platform Diver

- (a) Find his speed 5.0 m above the water surface

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$0 + gy_i = \frac{1}{2}v_f^2 + mgy_f$$

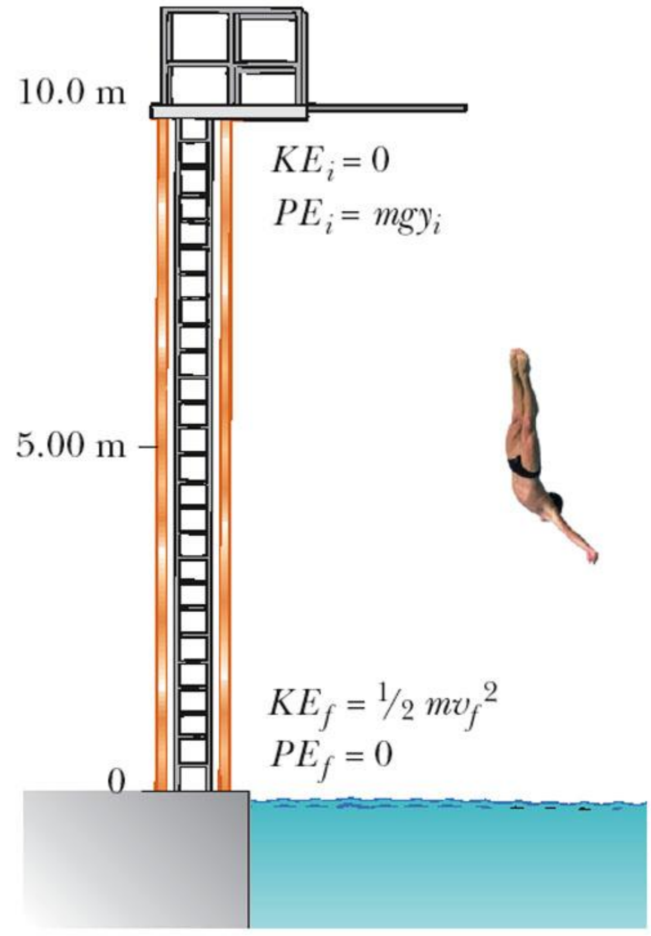
$$v_f = \sqrt{2g(y_i - y_f)}$$

$$= \sqrt{2(9.8m/s^2)(10m - 5m)} = 9.9m/s$$

- (b) Find his speed as he hits the water

$$0 + mgy_i = \frac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gy_i} = 14m/s$$

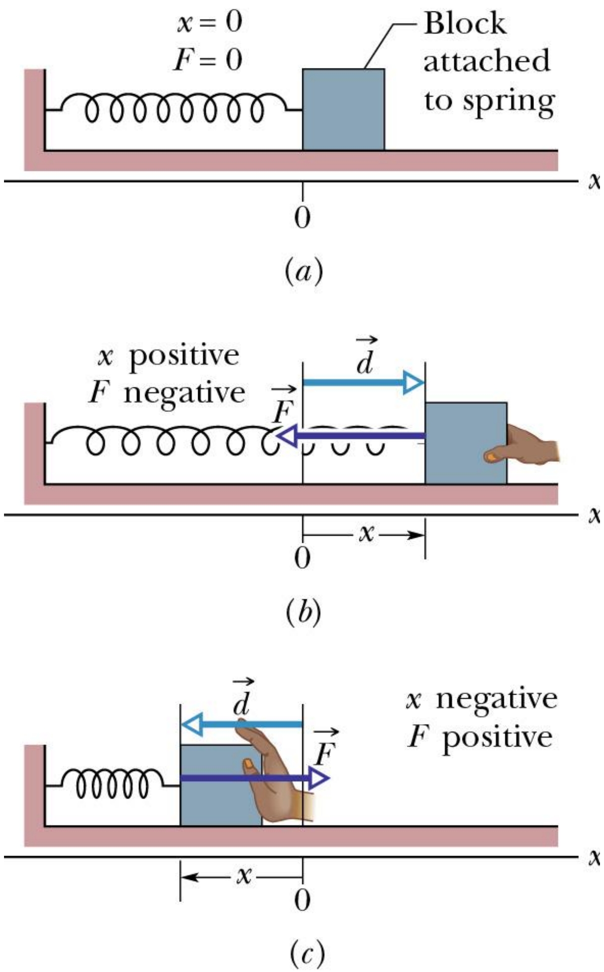


Spring Force

- Involves the *spring constant*, k
- Hooke's Law gives the force

$$\vec{F} = -k\vec{d}$$

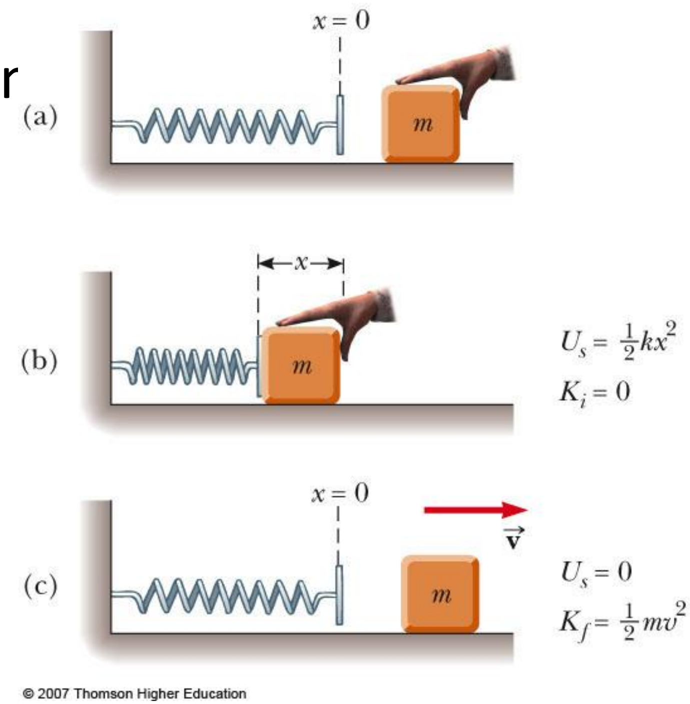
  - F is in the opposite direction of displacement d, always back towards the equilibrium point.
  - k depends on how the spring was formed, the material it is made from, thickness of the wire, etc. Unit: N/m.



Potential Energy in a Spring

- Elastic Potential Energy:
  - SI unit: Joule (J)
  - related to the work required to compress a spring from its equilibriur position to some final, arbitrary, position x

$$PE_s = \frac{1}{2}kx^2$$



- Work done by the spring

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_s = PE_{si} - PE_{sf}$$

Extended Work-Energy Theorem

- The work-energy theorem can be extended to include potential energy:

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_{gravity} = PE_i - PE_f \qquad W_s = PE_{si} - PE_{sf}$$

- If we include gravitational force and spring force, then

$$W_{net} = W_{gravity} + W_s$$

$$(KE_f - KE_i) + (PE_f - PE_i) + (PE_{sf} - PE_{si}) = 0$$

$$KE_f + PE_f + PE_{sf} = PE_i + KE_i + KE_{si}$$

Extended Work-Energy Theorem

- We denote the total mechanical energy by

$$E = KE + PE + PE_s$$

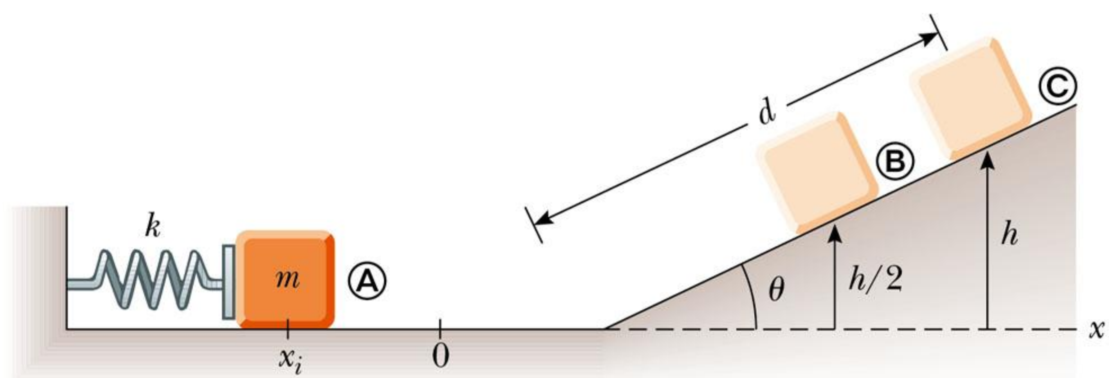
- Since $$(KE + PE + PE_s)_f = (KE + PE + PE_s)_i$$

- The total mechanical energy is conserved and remains the same at all times

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

A block projected up a incline

- A 0.5-kg block rests on a horizontal, frictionless surface. The block is pressed back against a spring having a constant of k = 625 N/m, compressing the spring by 10.0 cm to point A. Then the block is released.
- (a) Find the maximum distance d the block travels up the frictionless incline if θ = 30° .
- (b) How fast is the block going when halfway to its maximum height?



A block projected up a incline

- Point A (initial state): $v_i = 0, y_i = 0, x_i = -10cm = -0.1m$
- Point B (final state): $v_f = 0, y_f = h = d\sin\theta, x_f = 0$

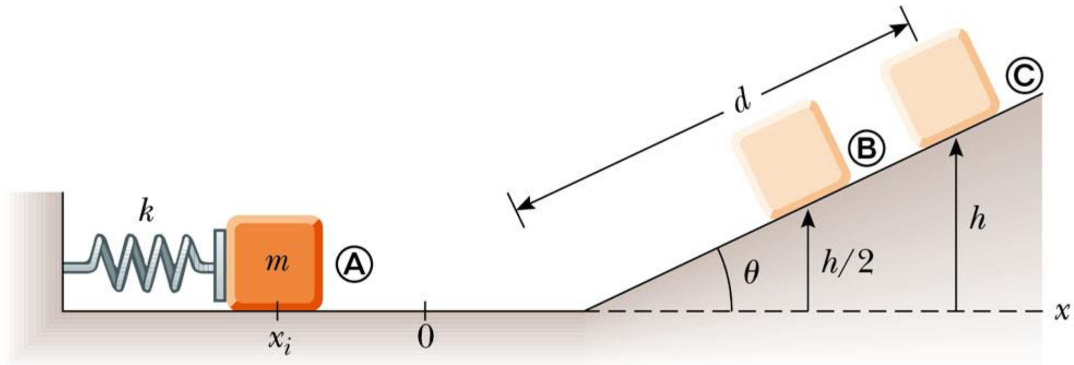
$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}kx_i^2 = mgy_f = mgd\sin\theta$$

$$d = \frac{\frac{1}{2}kx_i^2}{mg\sin\theta}$$

$$= \frac{0.5(625N/m)(-0.1m)^2}{(0.5kg)(9.8m/s^2)\sin 30^\circ}$$

$$= 1.28m$$



A block projected up a incline

- Point A (initial state): $v_i = 0, y_i = 0, x_i = -10cm = -0.1m$
- Point B (final state): $v_f = ?, y_f = h/2 = d\sin\theta/2, x_f = 0$

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

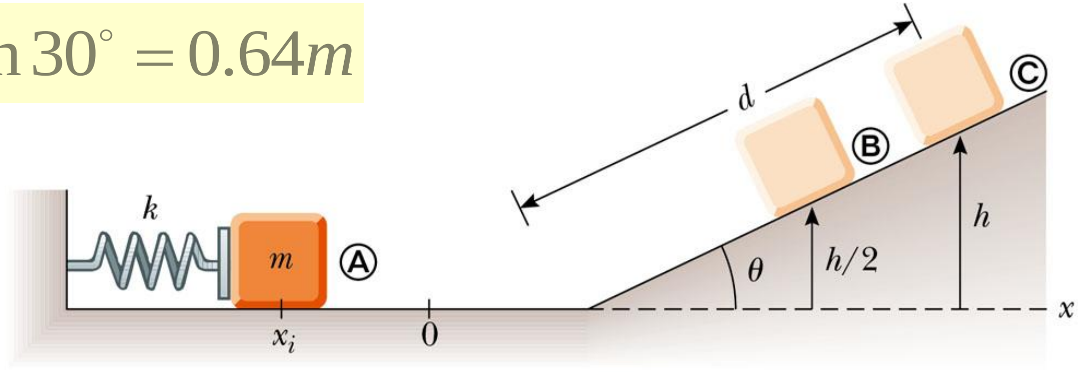
$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mg(\frac{h}{2})$$

$$\frac{k}{m}x_i^2 = v_f^2 + gh$$

$$h = d\sin\theta = (1.28m)\sin 30^\circ = 0.64m$$

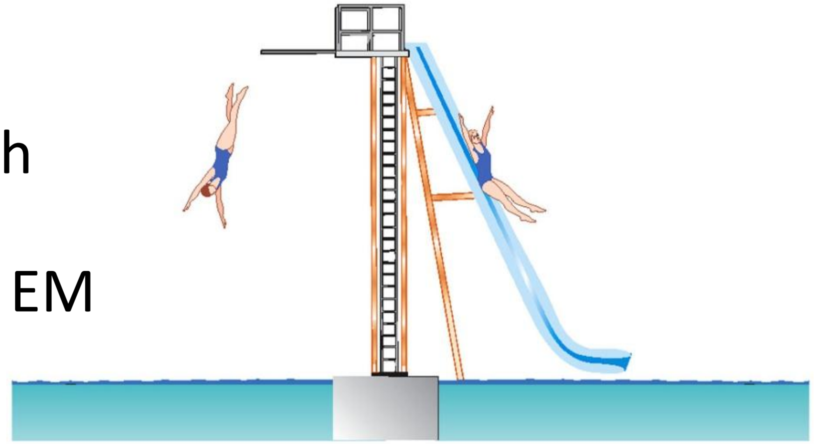
$$v_f = \sqrt{\frac{k}{m}x_i^2 - gh}$$

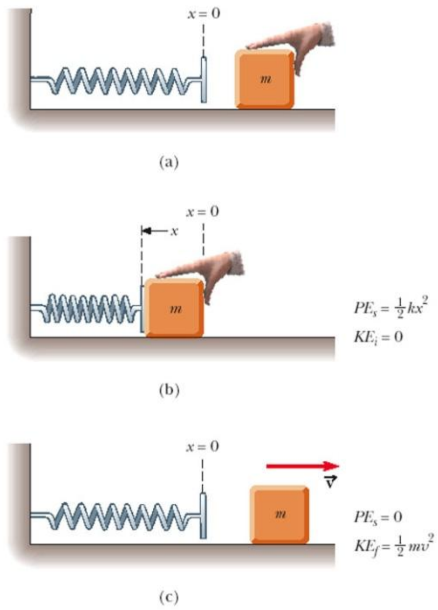
$$= \ldots\ldots = 2.5m/s$$



Types of Forces

- Conservative forces
  - Work and energy associated with the force can be recovered
  - Examples: Gravity, Spring Force, EM forces
- Nonconservative forces
  - The forces are generally dissipative and work done against it cannot easily be recovered
  - Examples: Kinetic friction, air drag forces, normal forces, tension forces, applied forces …





Conservative Forces

- A force is conservative if the work it does on an object moving between two points is independent of the path the objects take between the points

$$W_g = PE_i - PE_f = mgy_i - mgy_f$$

  - The work depends only upon the initial and final positions of the object
  - Any conservative force can have a potential energy function associated with it
  - Work done by gravity
  - Work done by spring force

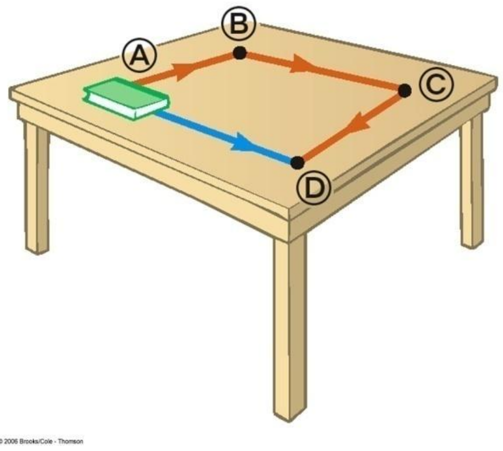
$$W_s = PE_{si} - PE_{sf} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Nonconservative Forces

- A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.
  - The work depends upon the movement path
  - For a non-conservative force, potential energy can NOT be defined
  - Work done by a nonconservative force

$$W_{nc} = \sum \vec{F} \cdot \vec{d} = -f_k d + \sum W_{otherforces}$$

  - It is generally dissipative. The dispersal of energy takes the form of heat or sound



Extended Work-Energy Theorem

- The work-energy theorem can be written as:

$$W_{net} = KE_f - KE_i = \Delta KE$$

$$W_{net} = W_{nc} + W_c$$

  - W_{nc} represents the work done by nonconservative forces
  - W_c represents the work done by conservative forces
- Any work done by conservative forces can be accounted for by changes in potential energy

$$W_c = PE_i - PE_f$$

  - Gravity work $$W_g = PE_i - PE_f = mgy_i - mgy_f$$
  - Spring force work $$W_s = PE_i - PE_f = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Extended Work-Energy Theorem

- Any work done by conservative forces can be accounted for by changes in potential energy

$$W_c = PE_i - PE_f = -(PE_f - PE_i) = -\Delta PE$$

$$W_{nc} = \Delta KE + \Delta PE = (KE_f - KE_i) + (PE_f - PE_i)$$

$$W_{nc} = (KE_f + PE_f) - (KE_i + PE_i)$$

- Mechanical energy includes kinetic and potential energy

$$E = KE + PE = KE + PE_g + PE_s = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

$$W_{nc} = E_f - E_i$$

Problem-Solving Strategy

- Define the system to see if it includes non-conservative forces (especially friction, drag force …)
- Without non-conservative forces

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

- With non-conservative forces

$$W_{nc} = (KE_f + PE_f) - (KE_i + PE_i)$$

$$-fd + \sum W_{otherforces} = (\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2) - (\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2)$$

- Select the location of zero potential energy
  - Do *not* change this location while solving the problem
- Identify two points the object of interest moves between
  - One point should be where information is given
  - The other point should be where you want to find out something

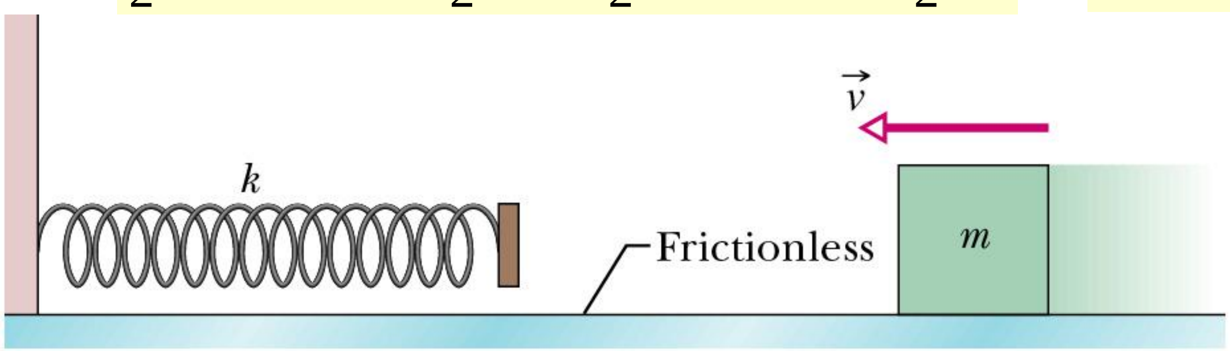
Conservation of Mechanical Energy

A block of mass m = 0.40 kg slides across a horizontal frictionless counter with a speed of v = 0.50 m/s. It runs into and compresses a spring of spring constant k = 750 N/m. When the block is momentarily stopped by the spring, by what distance d is the spring compressed?

$$W_{nc} = (KE_f + PE_f) - (KE_i + PE_i)$$

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

$$0 + 0 + \frac{1}{2}kd^2 = \frac{1}{2}mv^2 + 0 + 0$$



$$\sqrt{\frac{m}{k}v^2} = 1.15cm$$

Changes in Mechanical Energy for conservative forces

A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate starts from rest at the top. The surface friction can be negligible. Use energy methods to determine the speed of the crate at the bottom of the ramp.

$$-fd+\sum W_{otherforces}=(\frac{1}{2}mv_f^2+mgy_f+\frac{1}{2}kx_f^2)-(\frac{1}{2}mv_i^2+mgy_i+\frac{1}{2}kx_i^2)$$

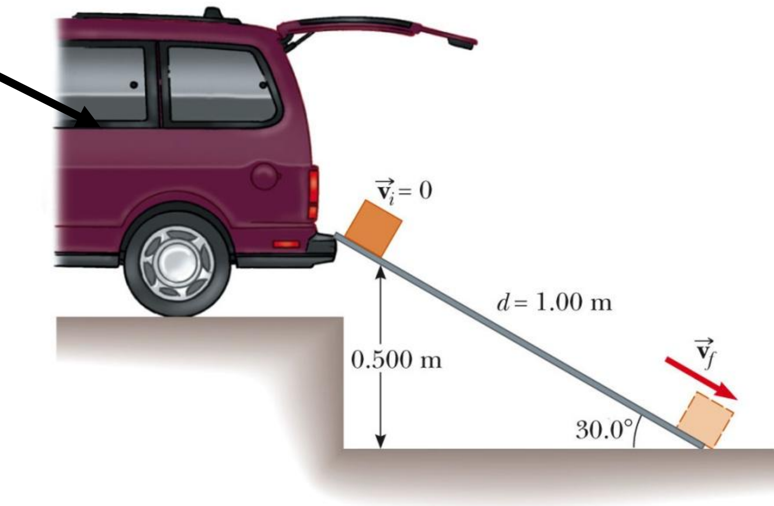
$$(\frac{1}{2}mv_f^2+mgy_f+\frac{1}{2}kx_f^2)=(\frac{1}{2}mv_i^2+mgy_i+\frac{1}{2}kx_i^2)$$

$$d=1m, y_i=d\sin 30^\circ=0.5m, v_i=0$$

$$y_f=0, v_f=?$$

$$(\frac{1}{2}mv_f^2+0+0)=(0+mgy_i+0)$$

$$v_f=\sqrt{2gy_i}=3.1m/s$$



Changes in Mechanical Energy for Non-conservative forces

A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate starts from rest at the top. The surface in contact have a coefficient of kinetic friction of 0.15. Use energy methods to determine the speed of the crate at the bottom of the ramp.

$$-fd+\sum W_{otherforces} = \left(\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2\right) - \left(\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2\right)$$

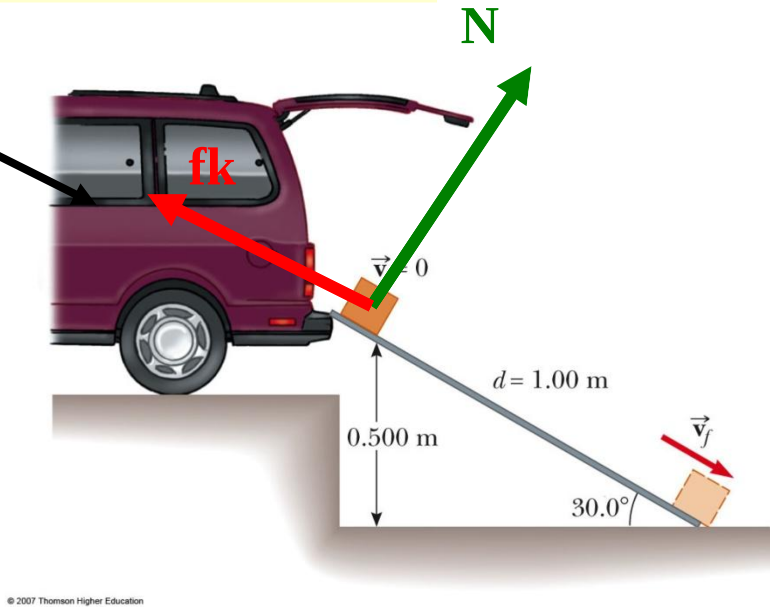
$$-\mu_k Nd + 0 = \left(\frac{1}{2}mv_f^2 + 0 + 0\right) - (0 + mgy_i + 0)$$

$$\mu_k = 0.15, d = 1m, y_i = d\sin 30^\circ = 0.5m, N = ?$$

$$N - mg\cos\theta = 0$$

$$-\mu_k dmg\cos\theta = \frac{1}{2}mv_f^2 - mgy_i$$

$$v_f = \sqrt{2g(y_i - \mu_k d\cos\theta)} = 2.7m/s$$



Changes in Mechanical Energy for Non-conservative forces

A 3-kg crate slides down a ramp. The ramp is 1 m in length and inclined at an angle of 30° as shown. The crate starts from rest at the top. The surface in contact have a coefficient of kinetic friction of 0.15. How far does the crate slide on the horizontal floor if it continues to experience a friction force.

$$-fd+\sum W_{otherforces}=(\frac{1}{2}mv_f^2+mgy_f+\frac{1}{2}kx_f^2)-(\frac{1}{2}mv_i^2+mgy_i+\frac{1}{2}kx_i^2)$$

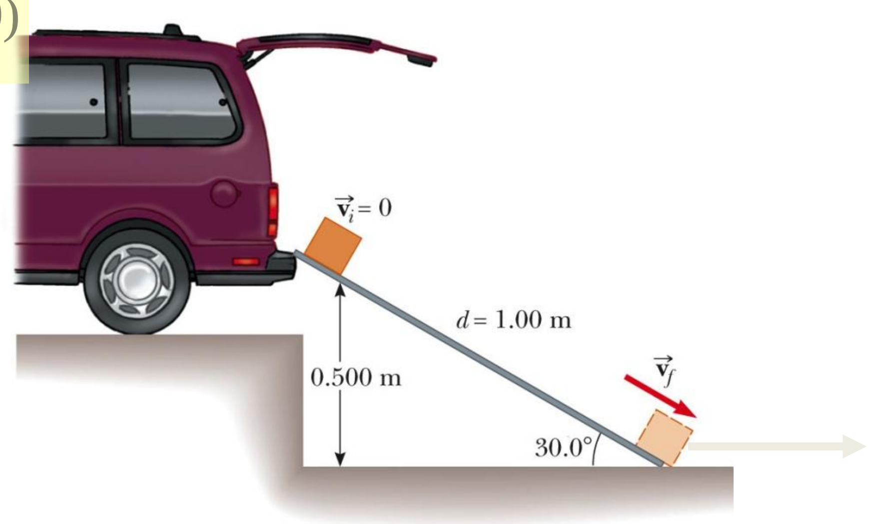
$$-\mu_k Nx+0=(0+0+0)-(\frac{1}{2}mv_i^2+0+0)$$

$$\mu_k=0.15, v_i=2.7m/s, N=?$$

$$N-mg=0$$

$$-\mu_k mgx=-\frac{1}{2}mv_i^2$$

$$x=\frac{v_i^2}{2\mu_k g}=2.5m$$



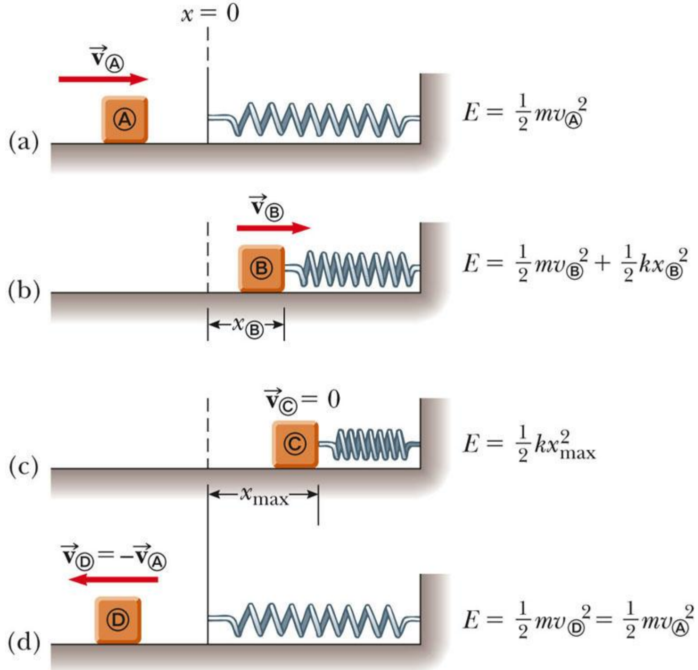
Block-Spring Collision

- A block having a mass of 0.8 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring whose mass is negligible and whose force constant is k = 50 N/m as shown in figure. Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

$$\frac{1}{2}mv_{max}^2 + 0 + 0 = \frac{1}{2}mv_A^2 + 0 + 0$$

$$x_{max} = \sqrt{\frac{m}{k}}v_A = \sqrt{\frac{0.8kg}{50N/m}}(1.2m/s) = 0.15m$$



Block-Spring Collision

- A block having a mass of 0.8 kg is given an initial velocity v_A = 1.2 m/s to the right and collides with a spring whose mass is negligible and whose force constant is k = 50 N/m as shown in figure. Suppose a constant force of kinetic friction acts between the block and the surface, with μ_k = 0.5, what is the maximum compression x_c in the spring.

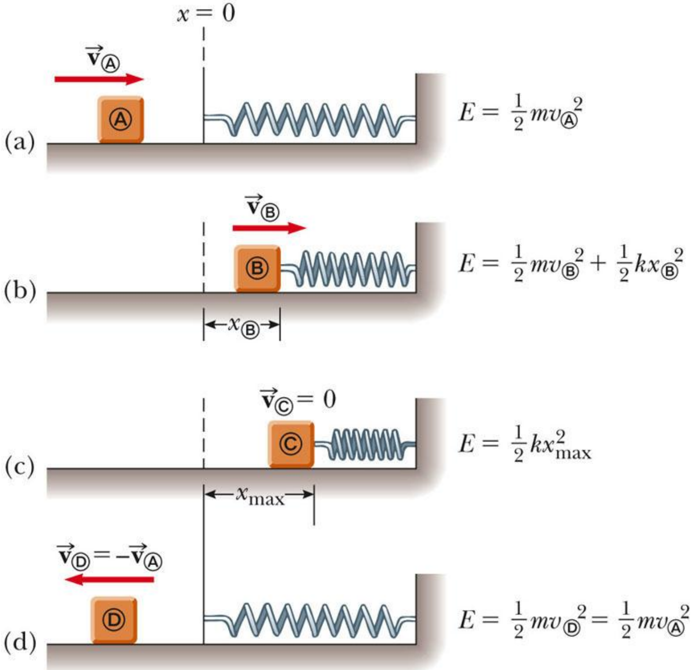
$$-fd + \sum W_{otherforces} = (\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2) - (\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2)$$

$$-\mu_k Nd + 0 = (0 + 0 + \frac{1}{2}kx_c^2) - (\frac{1}{2}mv_A^2 + 0 + 0)$$

$$N = mg \qquad \text{and} \qquad d = x_c$$

$$\frac{1}{2}kx_c^2 - \frac{1}{2}mv_A^2 = -\mu_k mgx_c$$

$$25x_c^2 + 3.9x_c - 0.58 = 0 \qquad x_c = 0.093m$$



Energy Review

- Kinetic Energy
  - Associated with movement of members of a system
- Potential Energy
  - Determined by the configuration of the system
  - Gravitational and Elastic
- Internal Energy
  - Related to the temperature of the system

Conservation of Energy

- ***Energy is conserved***
  - This means that energy cannot be created nor destroyed
  - If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer




Ways to Transfer Energy Into or Out of A System

- ***Work*** – transfers by applying a force and causing a displacement of the point of application of the force
- ***Mechanical Waves*** – allow a disturbance to propagate through a medium
- ***Heat*** – is driven by a temperature difference between two regions in space
- ***Matter Transfer*** – matter physically crosses the boundary of the system, carrying energy with it
- ***Electrical Transmission*** – transfer is by electric current
- ***Electromagnetic Radiation*** – energy is transferred by electromagnetic waves

Connected Blocks in Motion

- Two blocks are connected by a light string that passes over a frictionless pulley. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

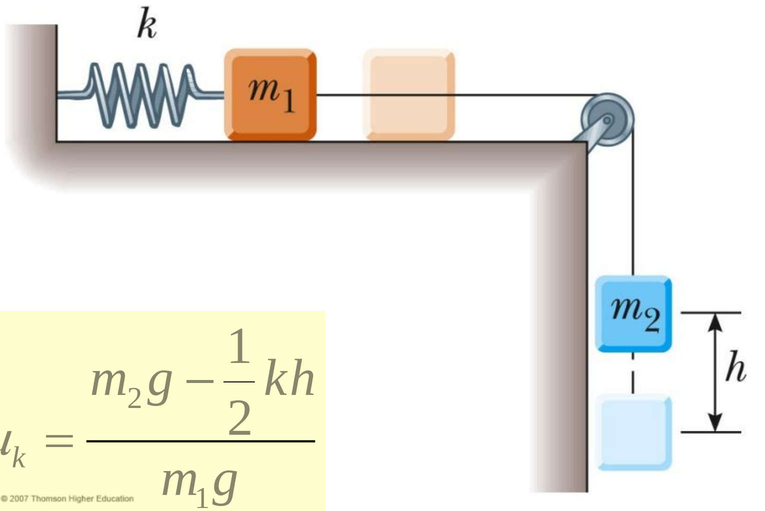
$$-fd + \sum W_{otherforces} = \Delta KE + \Delta PE$$

$$\Delta PE = \Delta PE_g + \Delta PE_s = (0 - m_2 gh) + (\frac{1}{2}kx^2 - 0)$$

$$-\mu_k N x + 0 = -m_2 gh + \frac{1}{2}kx^2$$

$$N = mg \quad \text{and} \quad x = h$$

$$-\mu_k m_1 gh = -m_2 gh + \frac{1}{2}kh^2$$



$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

Power

- Work does not depend on time interval
- The rate at which energy is transferred is important in the design and use of practical device
- The time rate of energy transfer is called power
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

  – when the method of energy transfer is work

Instantaneous Power

- Power is the time rate of energy transfer. Power is valid for any means of energy transfer
- Other expression

$$\overline{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\overline{v}$$

- A more general definition of instantaneous power

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^3
- A unit of power in the US Customary system is horsepower
  - 1 hp = 550 ft · lb/s = 746 W
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 $\times 10^6$ J

FORM ACTIVE STRUCTURE SYSTEM

- Non rigid, flexible matter, shaped in a certain way & secured at the ends which can support itself and span space.

- Form active structure systems develop at their ends horizontal stresses.

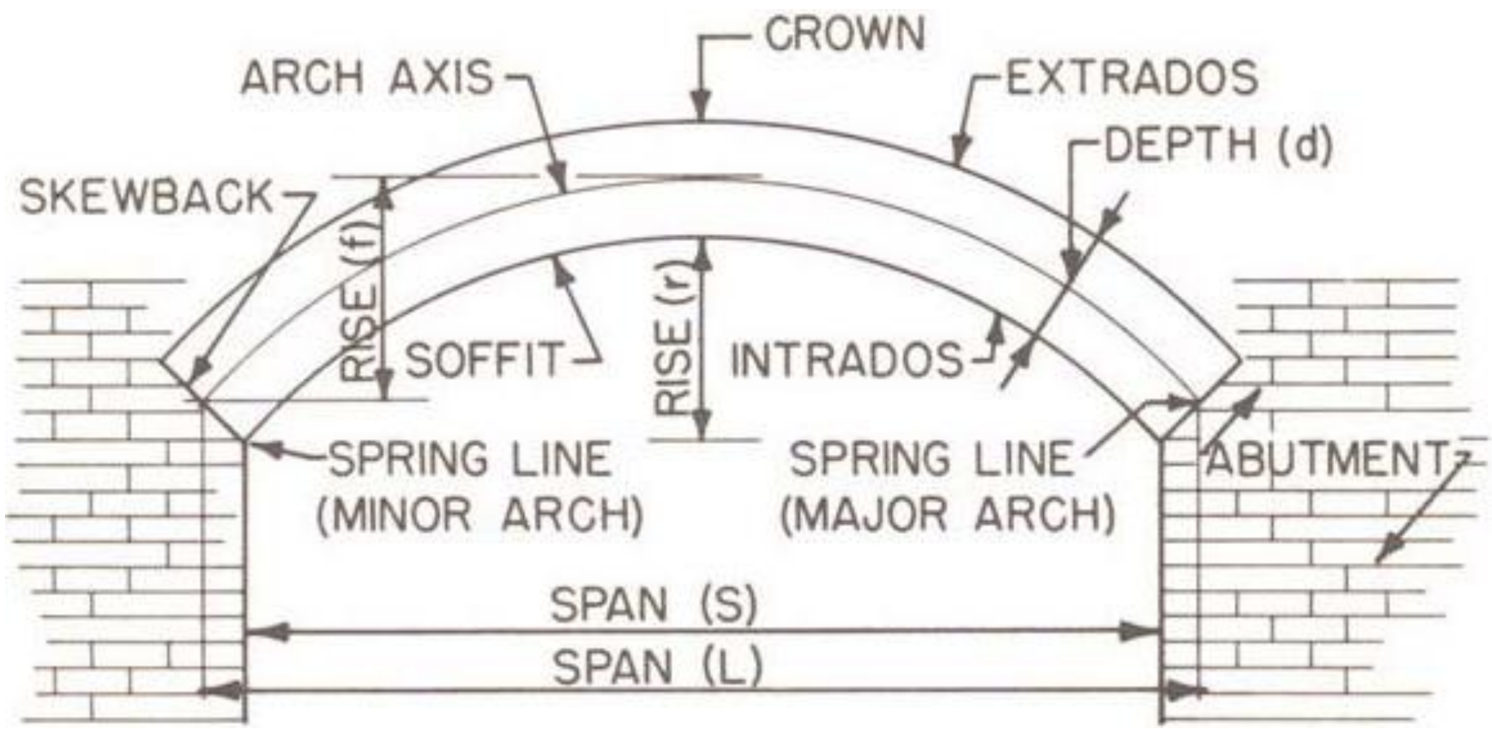
- The bearing mechanism of a form active systems rests essentially on the material form.

Arch

- A curved structure designed to carry loads across a gap mainly by compression.
- The mechanical principle of the arch is precisely the same as that of the portal frame. The straight pieces of material joined by sharp bends are smoothened into a continuous curve. This increases the cost of construction but greatly reduces the stresses.

- The geometry of the curve further affects the cost and stresses. The circular arch is easiest to construct, the catenary arch is the most efficient.
- Arches can be three pinned, two pinned or rigid.

Arch Terminology



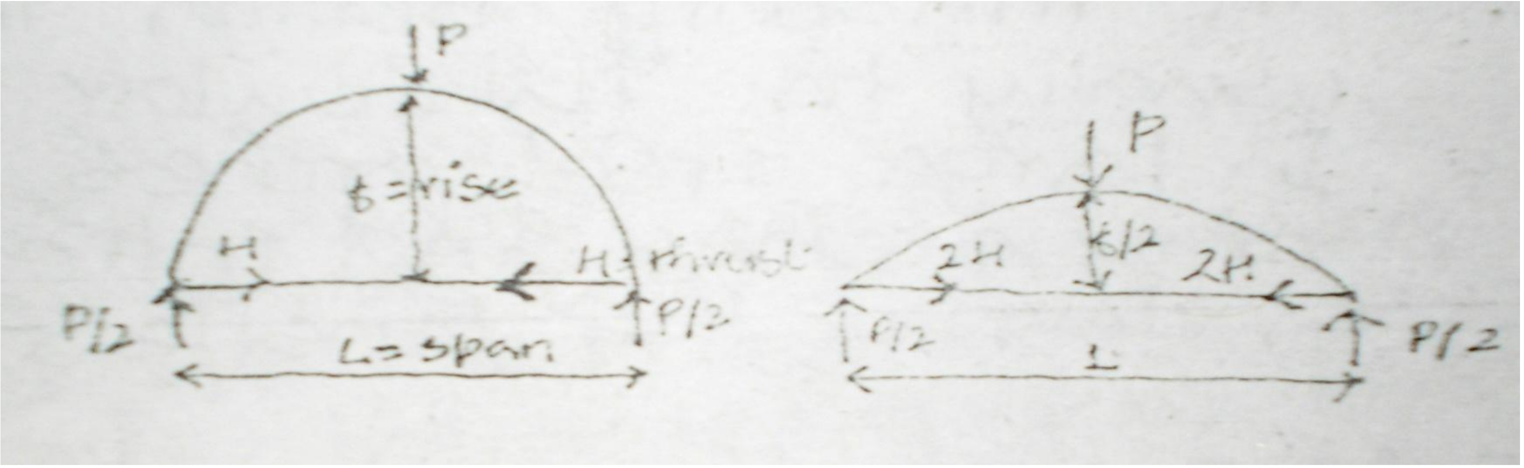
- It is important to minimize the arch THRUST so as to reduce the dimensions of the tie rod, or to ensure that the soil will not move under the pressure of the abutments.

- The THRUST is proportional to the total LOAD & to the SPAN, and inversely proportional to the RISE of the arch.

- In arches rise to span ratio should not be less than 1/8

- Riser minimum should be 1/8 of the span & 2/3rd maximum.

- Lesser rise takes compression but not tensile load.



- In masonry design the arch is heavy & loaded by the weight of walls, its shape is usually the funicular of the dead load, & some bending is introduced in it by live loads.

- In large steel arches, the live load represents a greater share of the total load & introduces a large amount of bending but it is seldom in view of the tensile strength of steel.

- The SHAPE of the arch may be chosen to be as close as possible to the FUNICULAR of the heaviest loads, so as to minimize BENDING.

- The arch thrust is absorbed by a tie-rod whenever the foundation material is not suitable to resist it.

- When it must allow the free passage of traffic under it, its thrust is asorbed either by buttresses or by tie-rods buried under ground.

- The stationary or moving loads carried by the arch are usually supported on a horizontal surface.

- This surface may be above or below the arch, connected to it by compression struts or tension hangers.

MATERIALS USED

STEEL-takes more tension

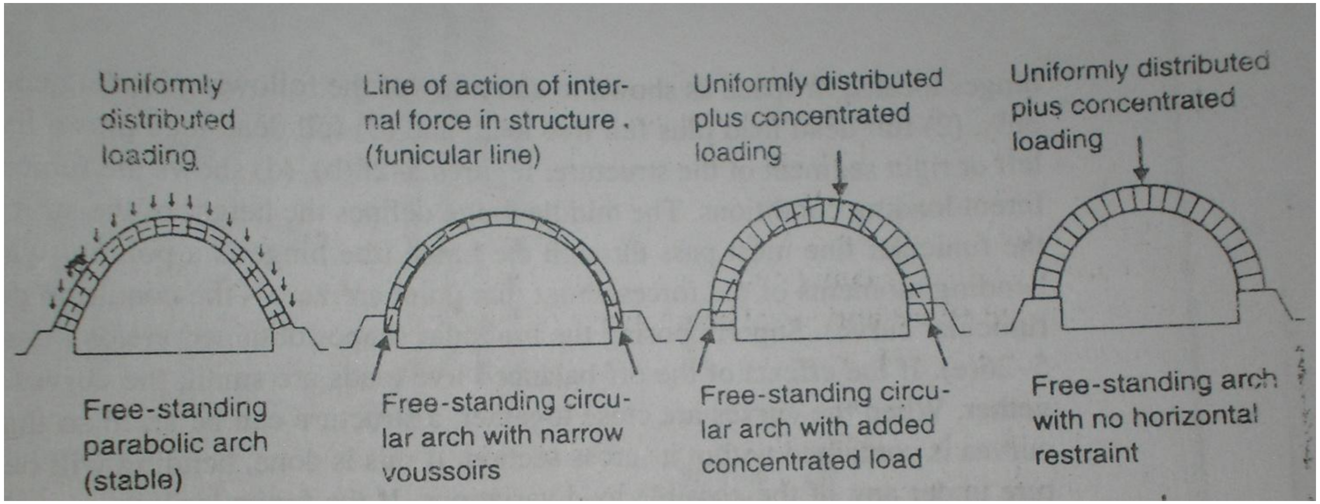
WOOD-both evenly

CONCRETE-takes more compression

LOAD APPLICATIONS

FUNICULAR ARCHES – CONCENTRATED LOADS

- ❖ The sum total of all rotational effects produced about any such location by the external and internal forces must be zero. In three hinged arch having a non-funicular shape, this observation is true only at three hinged conditions.
- ❖ The external shear at a section is balanced by an internal resisting shear force that is provided by vertical component of the internal axial force.



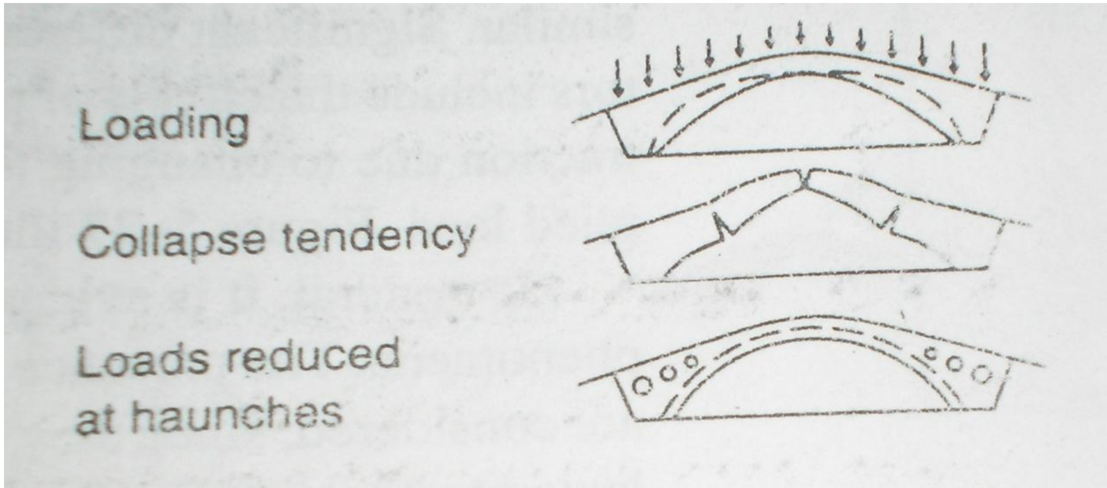
DESIGN OF ARCH STRUCTURES

The first important consideration when designing a brick arch is whether the arch is structural or non-structural. That is, will the arch be required to transfer vertical loads to abutments or will it be fully supported by a steel angle. While this may seem obvious, confusion often develops because of the many configurations of arch construction. To answer this question, one must consider the two structural requirements necessary for a brick arch to adequately carry vertical loads. First, vertical loads must be carried by the arch and transferred to the abutments. Second, vertical load and lateral thrust from the arch must be resisted by the abutments.

If either the arch or the abutment is deficient, the arch must be considered as non-structural and the arch and its tributary load must be fully supported by a steel angle or plates. Alternately, reinforcement may be used to increase the strength of either or both the arch and the abutments.

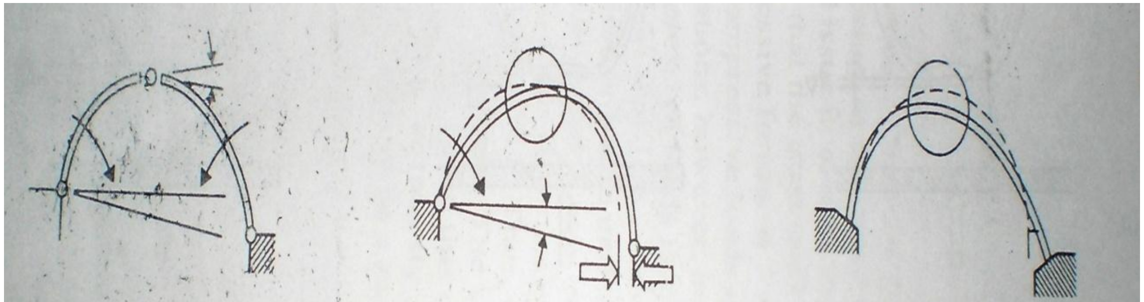
[A] DESIGNING FOR LOAD VARIATIONS

- One of the most significant aspect of the modern arch is that it can be designed to sustain some amount of variation in load without either changing shape or experiencing damage.
- The shape of an arch is initially determined as a response to its primary loading condition (e.g.: parabolic for uniformly distributed loads)



[B] SUPPORT ELEMENTS

- A basic issue is that whether or not to absorb the horizontal thrusts by some interior element (a tie rod or by the foundations). When it is functionally possible the rods are frequently used.
- The rod is a tension element and highly efficient to take up the outward arch thrusts.
- Usually there is less need to support an arch on the top of vertical elements, the use of buttressing elements is generally preferable as head room has to be maintained.



The three-hinged arch is relatively unaffected by support settlements since the two arch segments merely rotate with respect to one another (the hinges allow the structure to flex freely).

The two-hinged arch is relatively unaffected by vertical settlements since the hinges allow the structure to simply rotate as a unit. Horizontal spreading of the foundations, however, induces destructive bending at the crown of the arch.

The fixed-ended arch is severely affected by any type of foundation settlement. The absence of hinges does not allow the structure to flex freely, and destructive bending moments are consequently induced in the structure.

[C] CHOICE OF END CONDITIONS

- There are 3 primary types of arches used that are normally described in terms of end conditions :-

Three hinged arch

Fixed end arch

Two hinged arch

- ❑Different end conditions are preferable with respect to different phenomenon.
- ❑The presence of hinges is very important when supports, settlements and thermal expansions are considered.

Lateral Behavior Of Arches

- To deal with behaviour of arch in the lateral direction, there are two methods-
- Provide fixed base connections
- Commonly used is by relying on members placed transversely to the arch.

a pair of arches is stabilized through use of diagonal elements.

interior arches are stabilized by being connected to the end arches by connecting transverse members

- Lateral buckling can be solved by laterally bracing arches with other elements.

Flashing

- In residential construction, the presence of eaves, overhangs and small wall areas above openings will reduce the potential for water penetration at arch locations. However, flashing at an arch is just as important as over any other wall opening.

- Flashing an arch can be difficult, depending on the type of arch and the type of flashing material. Jack arches are the easiest to flash because they are flat.

- Flashing may be placed below the arch on the window framing for structural arches or above the steel lintel for non-structural arches.

- Alternately, flashing may be placed in the mortar joint above the arch or keystone. Attachment of the flashing to the backing and end dams should follow standard procedures.

- A segmental or semi-circular arch is more difficult to flash properly. This is because flashing materials such as metal flashings are very rigid and may be hard to work around a curved arch.

Construction Concerns

Both structural and non-structural arches must be properly supported throughout construction. Premature removal of the temporary support for a structural arch may result in a collapse of the arch. This is most often due to the introduction of lateral thrust on the abutment before proper curing has occurred. Out-of-plane bracing is required for all arches. In veneer construction, it is provided by the backup material through the wall ties. Arches that are not laterally braced may require increased masonry thickness or reinforcements to carry loads perpendicular to the arch plane. Arches may be constructed of special shapes or regular units. Mortar joints may be tapered with uncut regular units.

Alternately, regular units may be cut to maintain uniform joint thickness. In general, use of specially shaped brick that result in uniform joint thickness will be more aesthetically pleasing. Many brick manufacturers offer such specially-shaped arch units.

FAILURE MODES

1. **Rotation of the arch about the abutment-**
Rotation occurs when tension develops in the arch. Tension can be reduced by increasing the depth or rise of the arch. If tension develops in the arch, reinforcement can be added to resist the tensile forces.

2. **Sliding of the arch at the skewback-**

Sliding of the arch will depend on the angle of skewback (measured from horizontal) and the vertical load carried by the arch. Reinforcement can be added to avoid sliding at the skewback, as the reinforcement acts as a shear key.

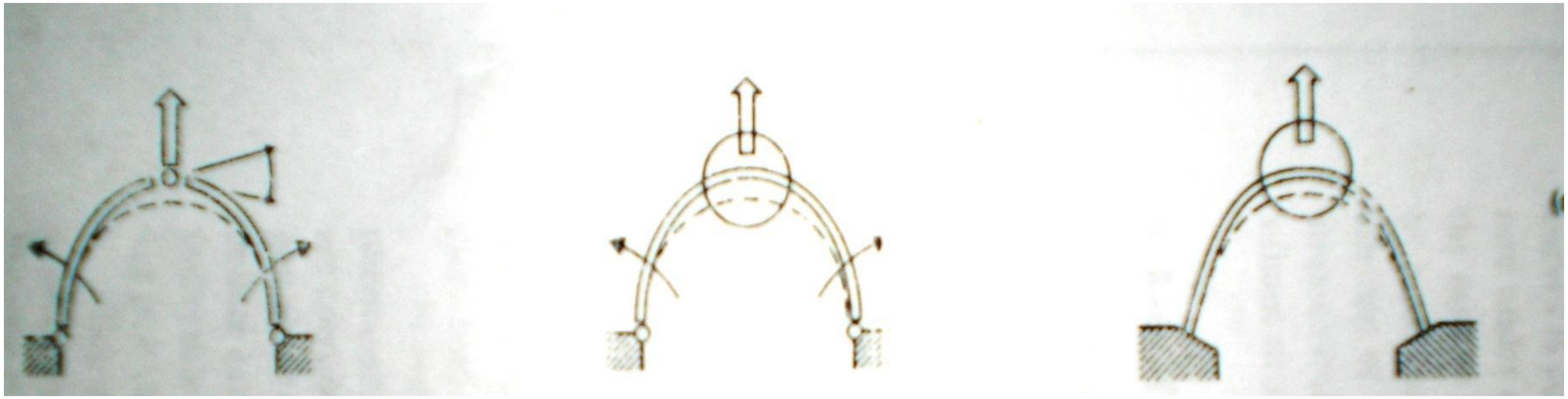
3. **Crushing of the masonry-**

Crushing will occur when compressive stresses in the arch exceed the compressive strength of the brick masonry. If compressive stresses are too large, the arch must be redesigned with a shorter

CORRECTIVE MEASURES AND DESIGN CHANGES

- Arches have horizontal restraints and these are responsible for their superior structural performance.

- During the night the arch shortens and during the day, it elongates. Similar problems are created by moisture movement in concrete as the concrete absorbs water and then dries out again. The stresses caused by temperature and moisture movement in arches are often much greater than the stresses caused by the live load, and thus they cannot be ignored.

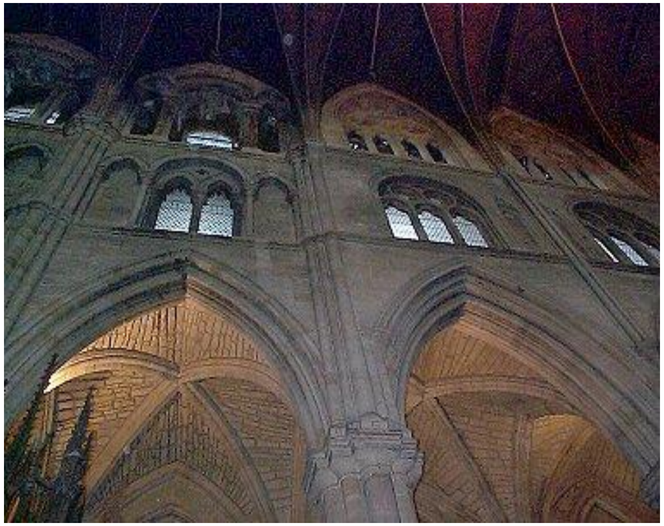


EARLY CURVED ARCHES

- Structure was often made more stable by the superimposition of additional weight on its top, thus firming up the arch.

- SHAPE OF ARCH is not chosen for purely structural reasons. The HALF CIRCLE, used by the Romans, has convenient construction properties that justify its use.

- Similarly, the POINTED gothic arch has both visual & structural advantages, while the arabic arch, typical of the mosques & of some venetian architecture is ‘incorrect’ from a purely structural viewpoint.



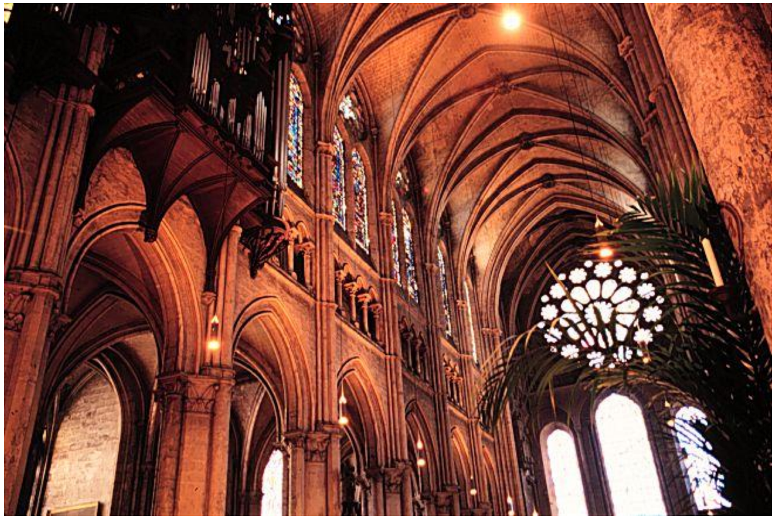
Notre-Dame Cathedral- Fine example of Gothic architecture, built in mid-13th century. Ornate west entrance shows the use of arches in early building construction. (Chartres, France)

Notre-Dame Cathedral- (South entrance) Note the use of heavy ornate pinnacles to increase the stability of the piers against overturning from horizontal thrust component of the arch. (Chartres, France)

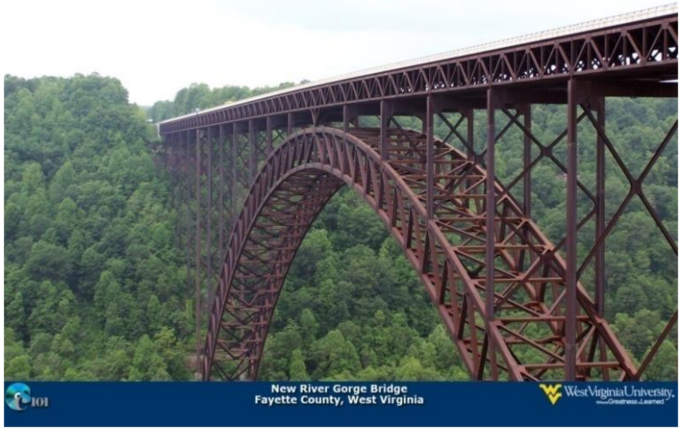
King's College Chapel- One of the finest examples of medieval architecture in England. Built in 1446-1515, Fan vaulting in the ceiling is essentially a series of pointed arches that require external buttresses to react to the horizontal thrust. (Cambridge, England)

APPLICATIONS & ADVANTAGES

- Roman & romanesque architecture are immediately recognized by the circular arch motif. Romans were pioneers in the use of arches for bridges, buildings, and aqueducts. This bridge, the Ponte Fabricio in Rome, spans between the bank of the River Tiber and Tiber Island. Built in 64 B.C. (Rome, Italy.)
- The gothic high rise arch & the buttresses required to absorb its thrust are typical of one of the greatest achievements in architectural design.
- Roman circular arches spanned about 100’ & medieval stone bridges up to 180’.

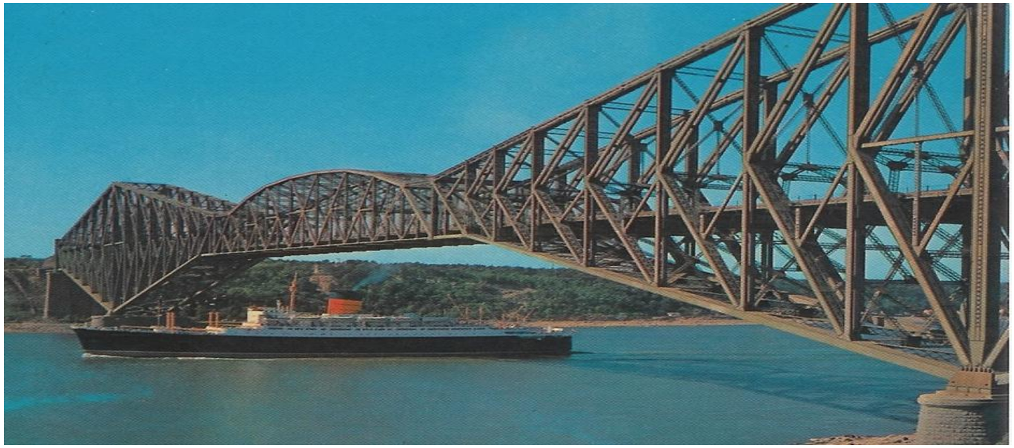


- The NEW RIVER GORGE BRIDGE in west virginia, the longest steel arch spans 1700’ (1986).
- The largest single arch span in reinforced concrete built to date is the 1280feet span KRK BRIDGE , yugoslavia.



- Combinations of trussed arches with cantilevered half arches connected by trusses were built to span as much as 1800feet in THE QUEBEC BRIDGE in 1917.

- To this day no other structural element is as commonly used to span large distances as the arch.



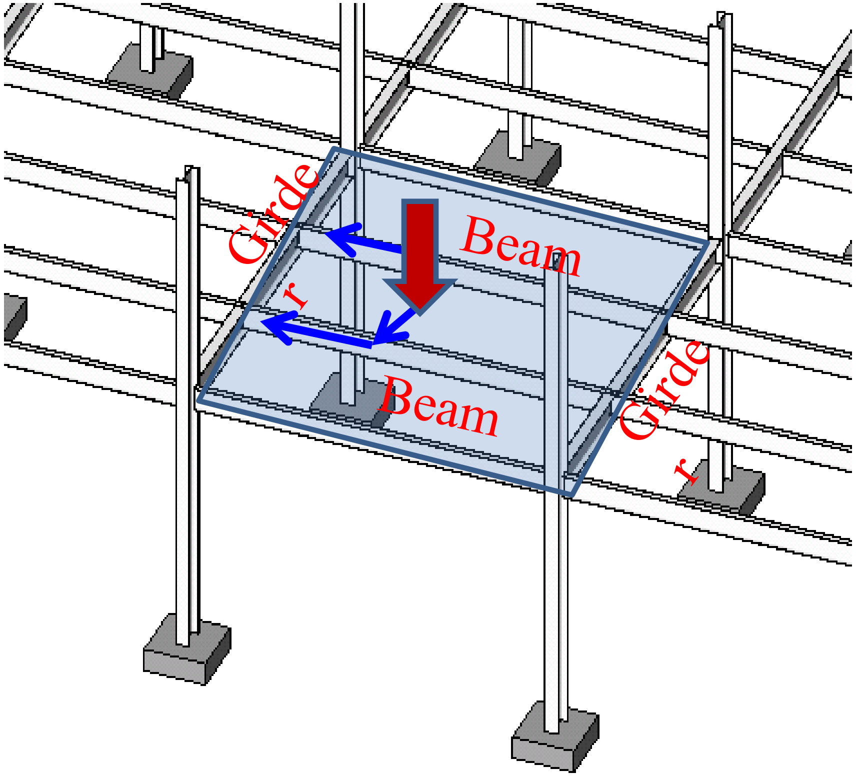
Unit – 3
Propped Cantilever and Fixed Beams

Beam

- Structural member that carries a load that is applied transverse to its length
- Used in floors and roofs
- May be called floor joists, stringers, floor beams, or girders

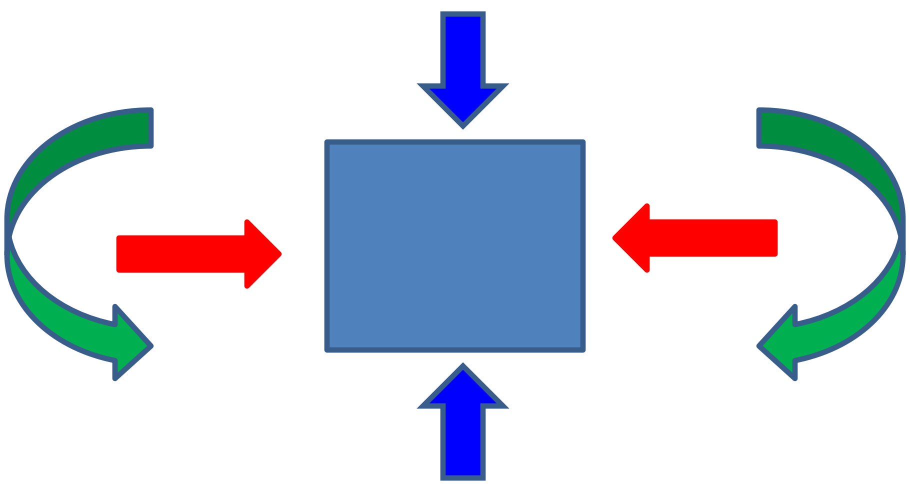
Chasing the Load

- The loads are initially applied to a building surface (floor or roof).
- Loads are transferred to beams which transfer the load to another building component.



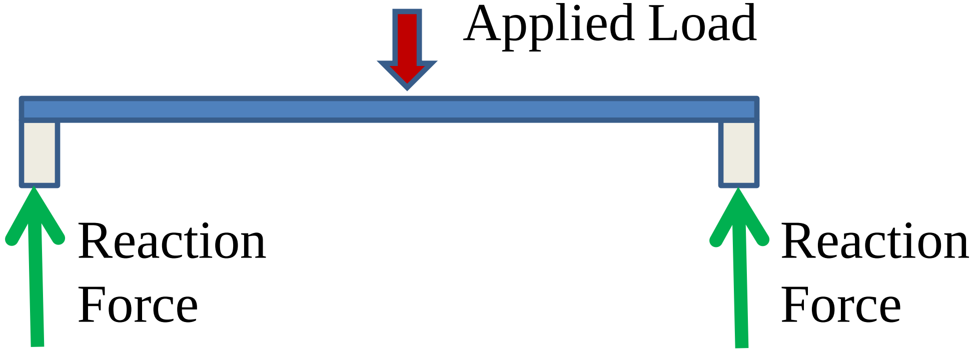
Static Equilibrium

- The state of an object in which the forces counteract each other so that the object remains stationary
- A beam must be in static equilibrium to successfully carry loads



Static Equilibrium

- The loads applied to the beam (from the roof or floor) must be resisted by forces from the beam supports.
- The resisting forces are called reaction forces.

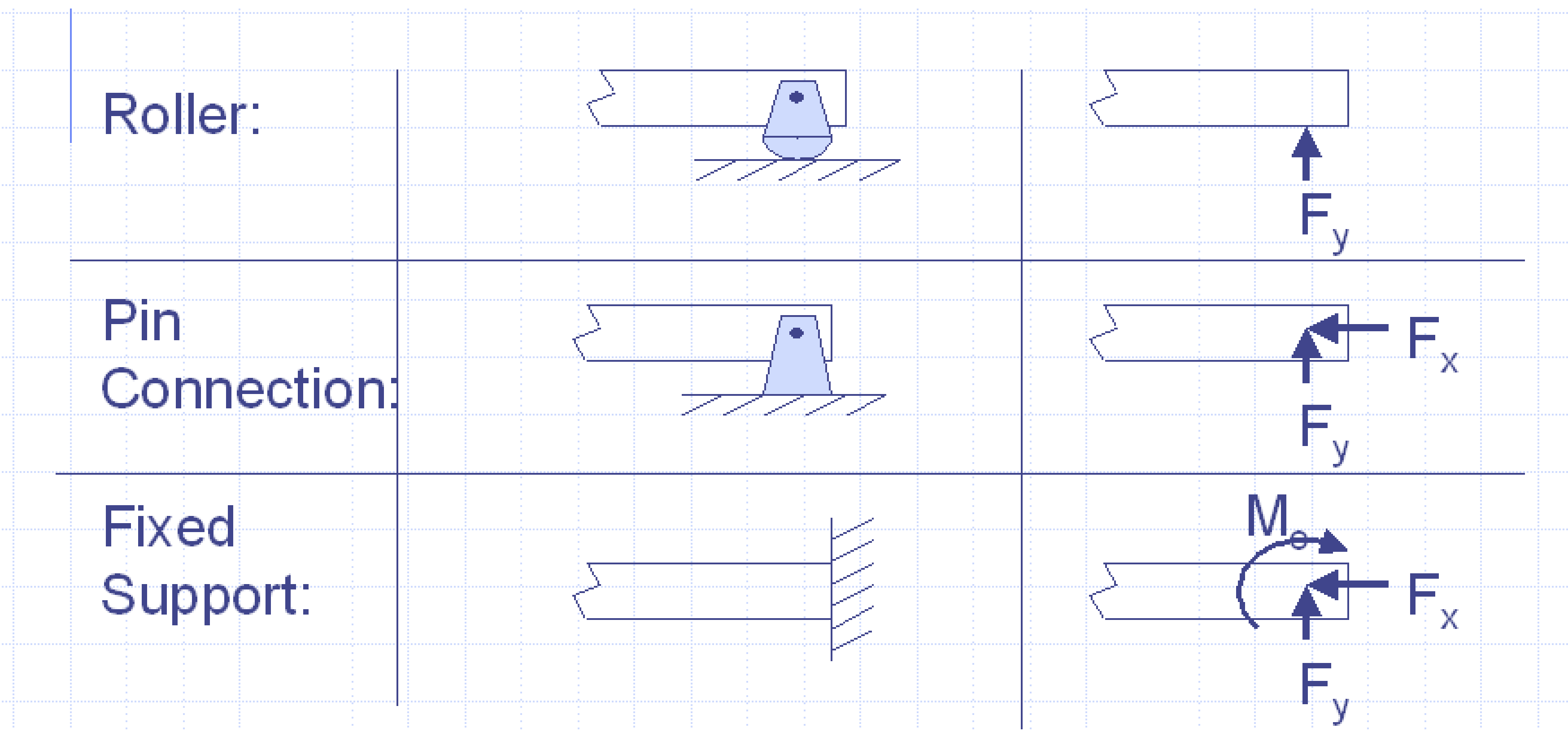


Reaction Forces

- Reaction forces can be linear or rotational.
  - A linear reaction is often called a shear reaction (F or R).
  - A rotational reaction is often called a moment reaction (M).
- The reaction forces must balance the applied forces.

Beam Supports

The method of support dictates the types of reaction forces from the supporting members.



Beam Types

Simple

Continuous

Cantilever

Moment

(fixed at one end)

Beam Types

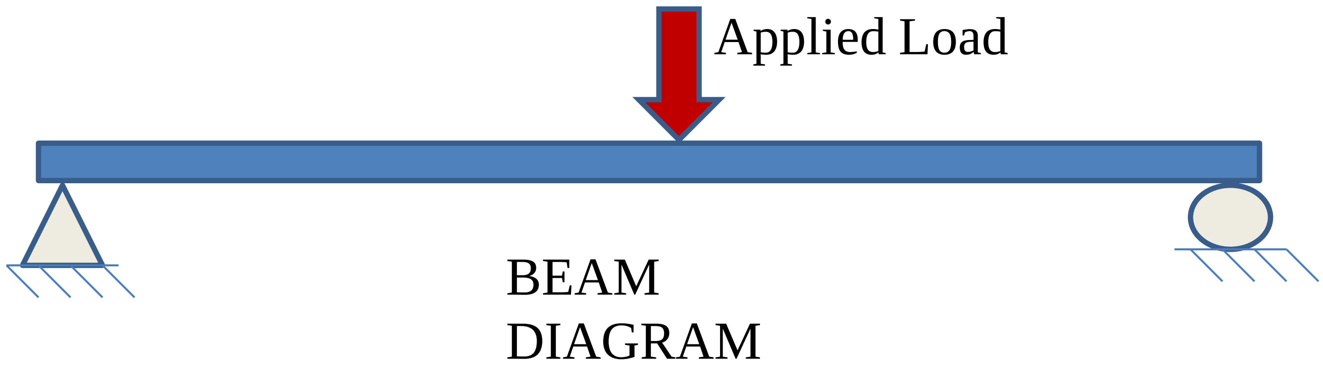
Fixed

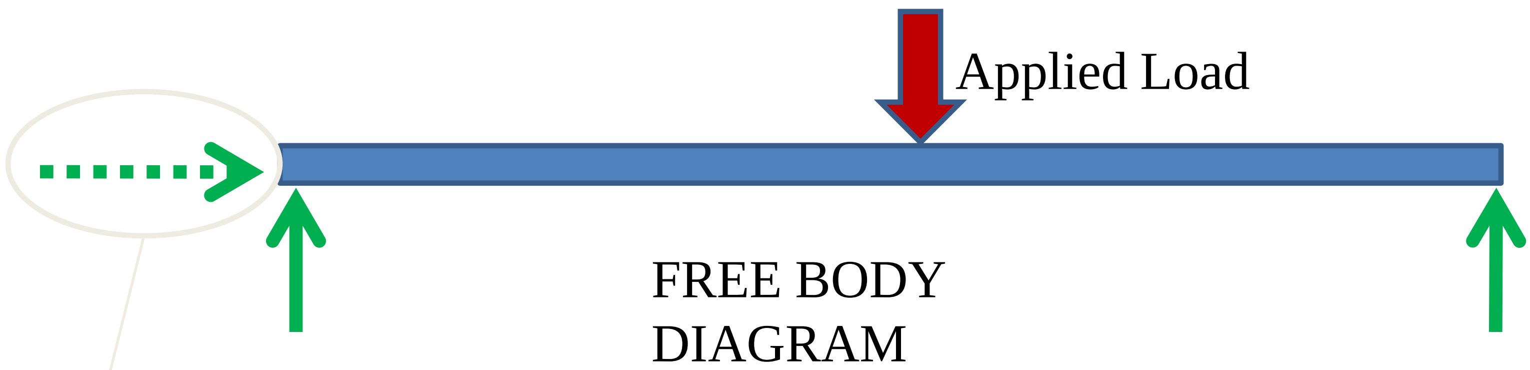
Moments at each end

Propped – Fixed at one end; supported at other

Overhang

Simple Beams





Note: When there is no applied horizontal load, you may ignore the horizontal reaction at the pinned connection.

Fundamental Principles of Equilibrium

$$\sum F_y = 0$$

The sum of all vertical forces acting on a body must equal zero.

$$\sum F_x = 0$$

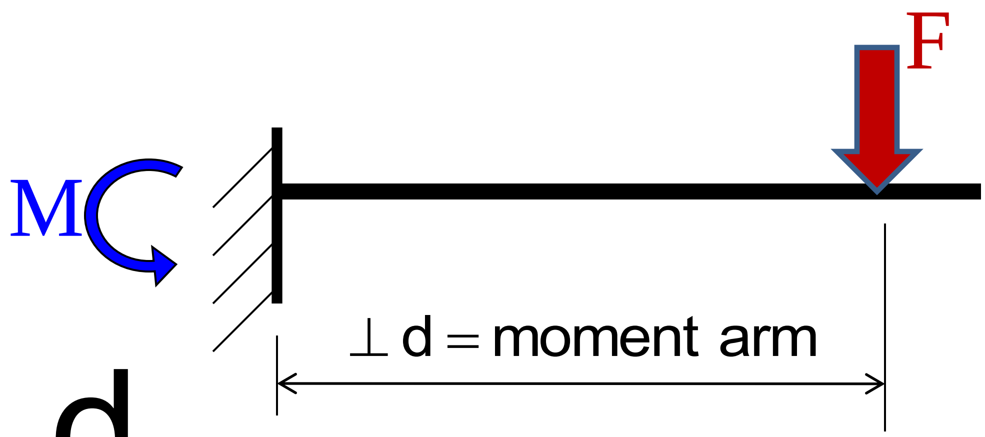
The sum of all horizontal forces acting on a body must equal zero.

$$\sum M_p = 0$$

The sum of all moments (about any point) acting on a body must equal zero.

Moment

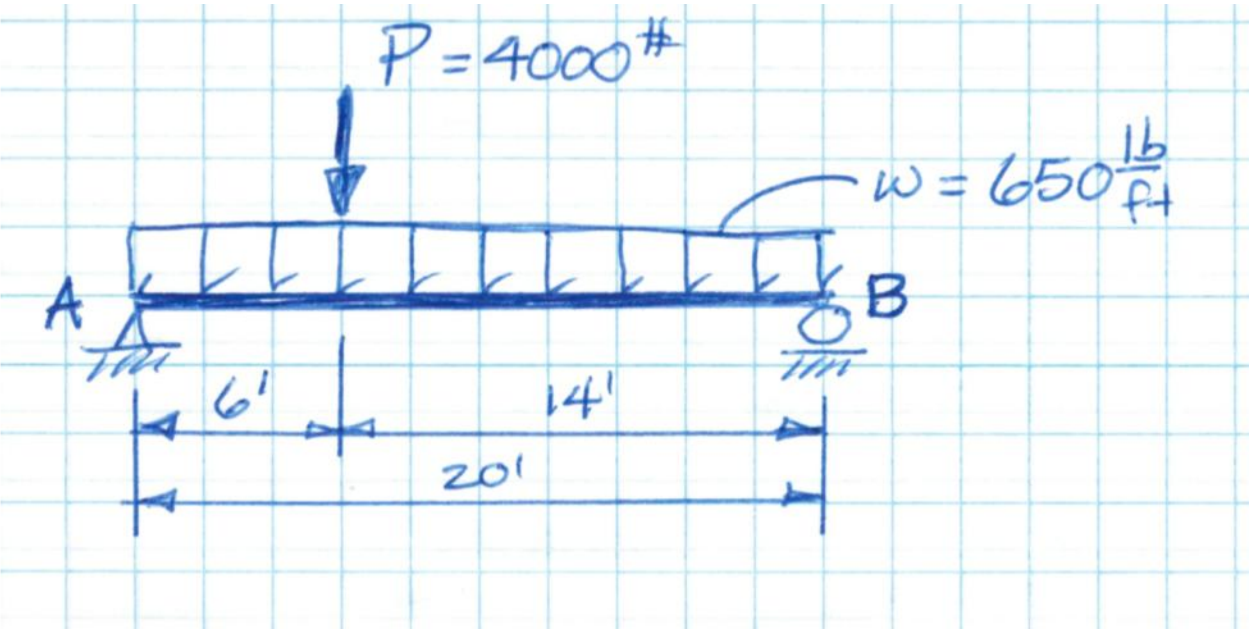
- A moment is created when a force tends to rotate an object.
- The magnitude of the moment is equal to the force times the perpendicular distance to the force (moment arm).



$$M = F \cdot \perp d$$

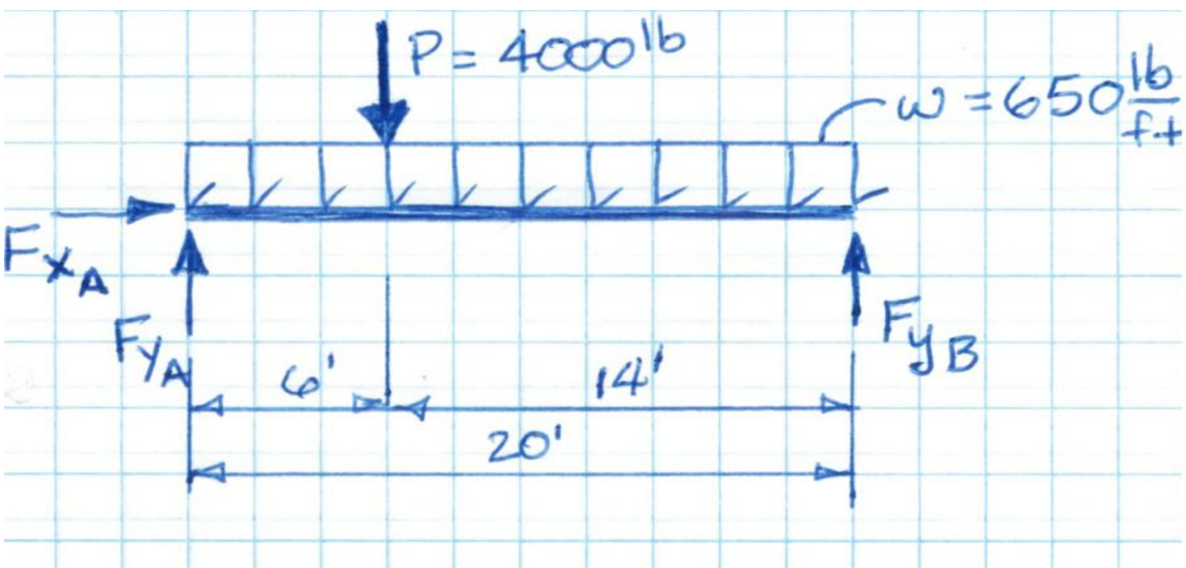
Calculating Reaction Forces

Sketch a beam diagram.



Calculating Reaction Forces

Sketch a free body diagram.



Calculating Reaction Forces

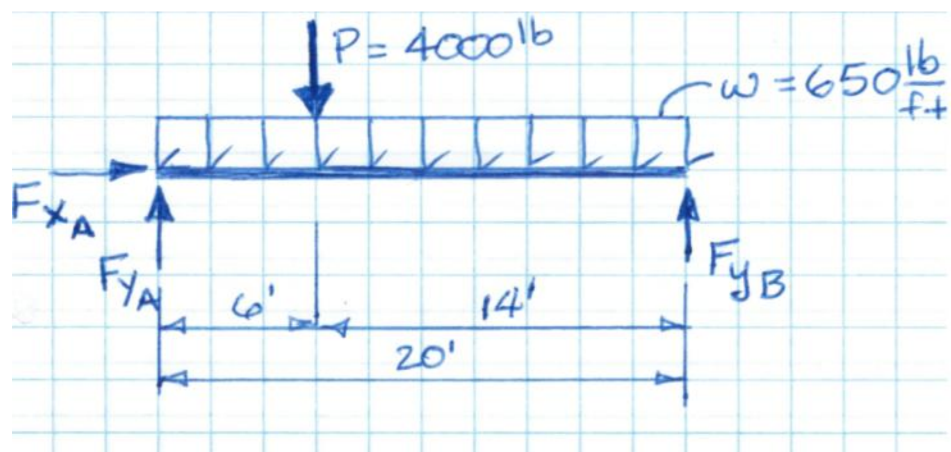
Use the equilibrium equations to find the magnitude of the reaction forces.

– **Horizontal Forces**
– Assume to the right is positive

$\overset{+}{\longrightarrow}$

$$\sum F_x = 0$$

$$F_{xA} = 0$$



Calculating Reaction Forces

- **Vertical Forces**
- Assume up is positive $+\uparrow$

$$\sum F_y = 0$$

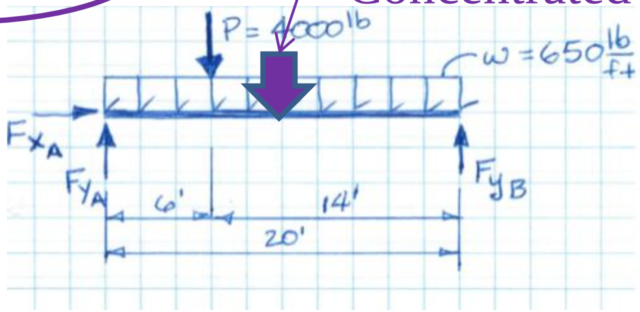
Equivalent Concentrated Load

$$F_{yA} + F_{yB} - 4000lb - \left(650\frac{lb}{ft}\right)(20ft) = 0$$

$$F_{yA} + F_{yB} = 4000lb + 13{,}000lb$$

$$F_{yA} + F_{yB} = 17{,}000lb$$

Equivalent Concentrated Load



Calculating Reaction Forces

- **Moments**
- Assume counter clockwise rotation is positive

$$\sum M_A = 0 \quad \circlearrowleft +$$

$$(F_{yB} \cdot 20\ ft) - (4000\ lb \cdot 6\ ft) - (13{,}000\ lb \cdot 10\ ft) + (F_{yA} \cdot 0) = 0$$

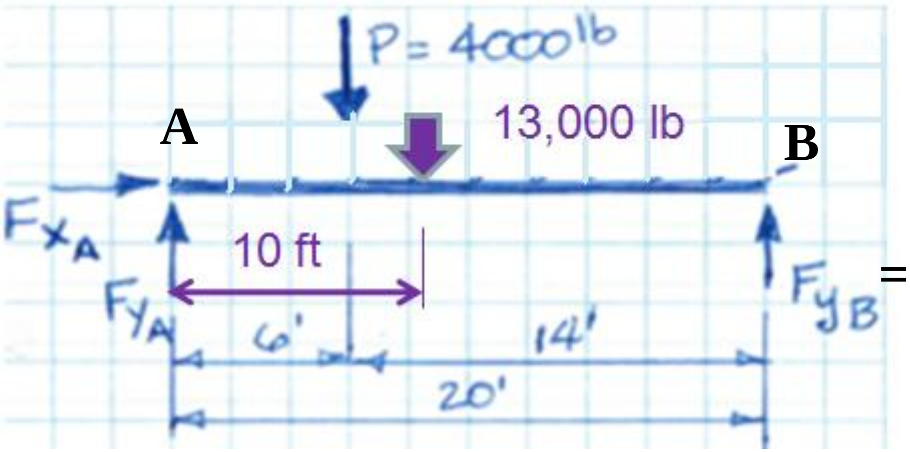
$$(20\ ft)F_{yB} - 24{,}000\ ft \cdot lb - 130{,}000\ ft \cdot lb + 0 = 0$$

$$(20\ ft)F_{yB} = 154{,}000\ ft \cdot lb$$

$$F_{yB} = \frac{154{,}000\ ft \cdot lb}{20\ ft}$$

$$\boxed{F_{yB} = 7{,}700\ lb}$$

0 =



= 7700 lb

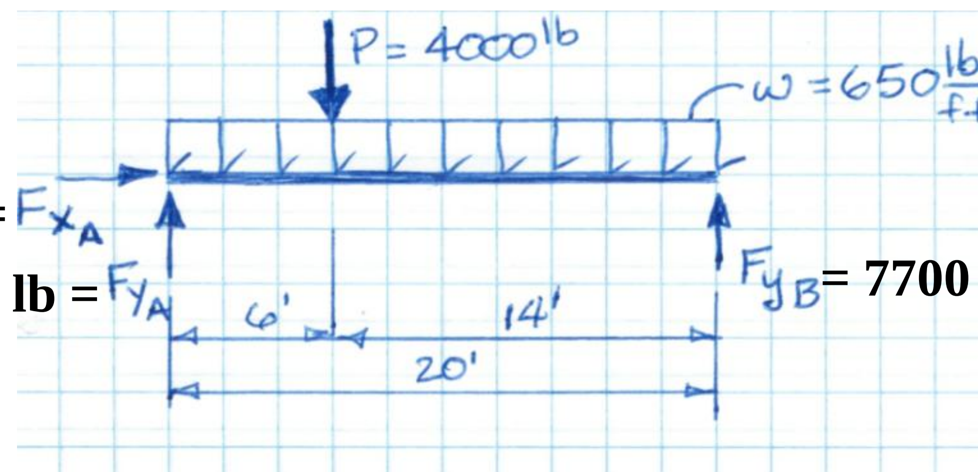
Calculating Reaction Forces

- Now that we know F_{yB}, we can use the previous equation to find F_{yA}.

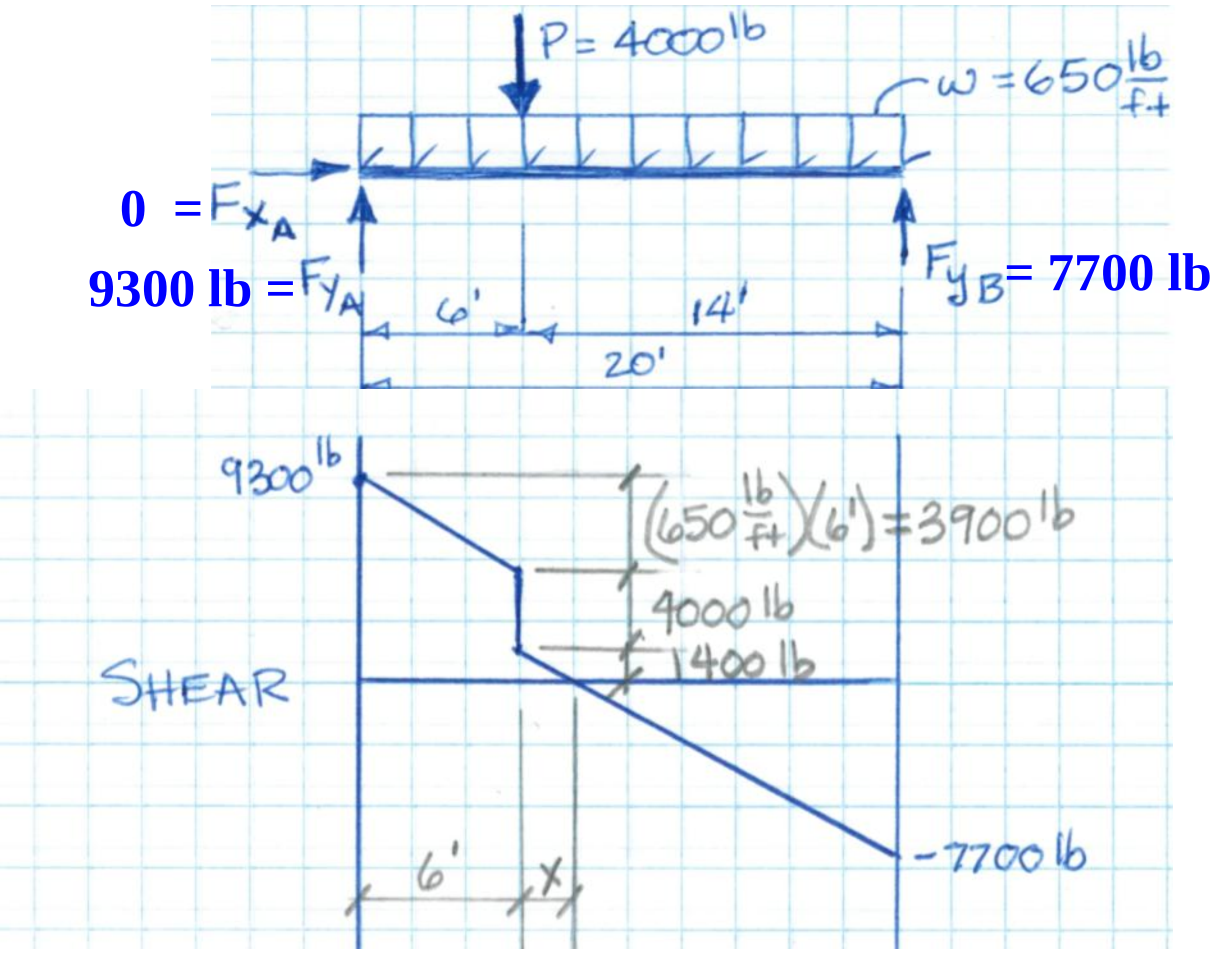
$$F_{yA} + F_{yB} = 17{,}000 lb$$

$$F_{yA} + 7700\ lb = 17{,}000\ lb$$

$$\boxed{F_{yA} = 9300\ lb}$$

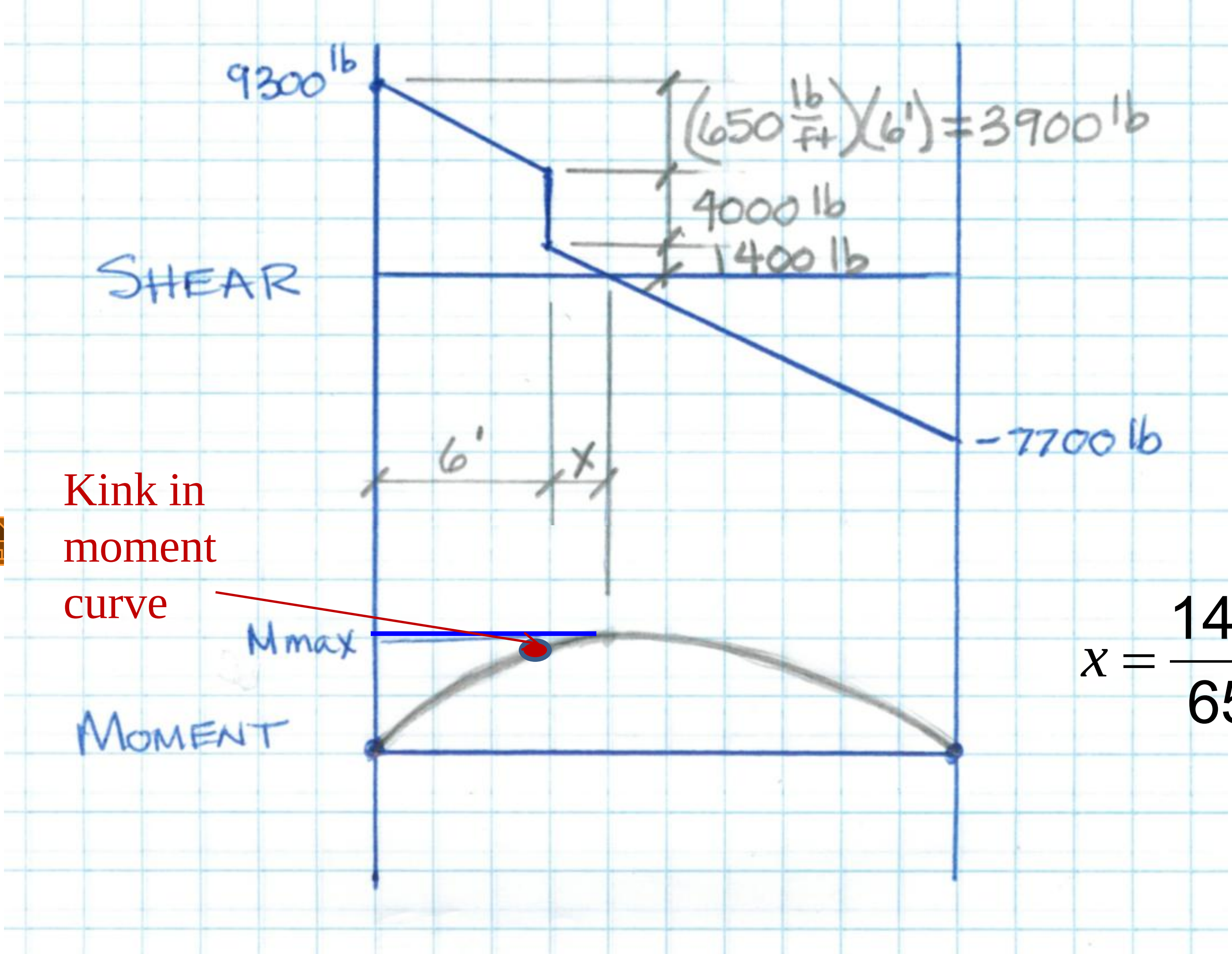


Shear Diagram



Shear at a point along the beam is equal to the reactions (upward) minus the applied loads (downward) to the left of that point.

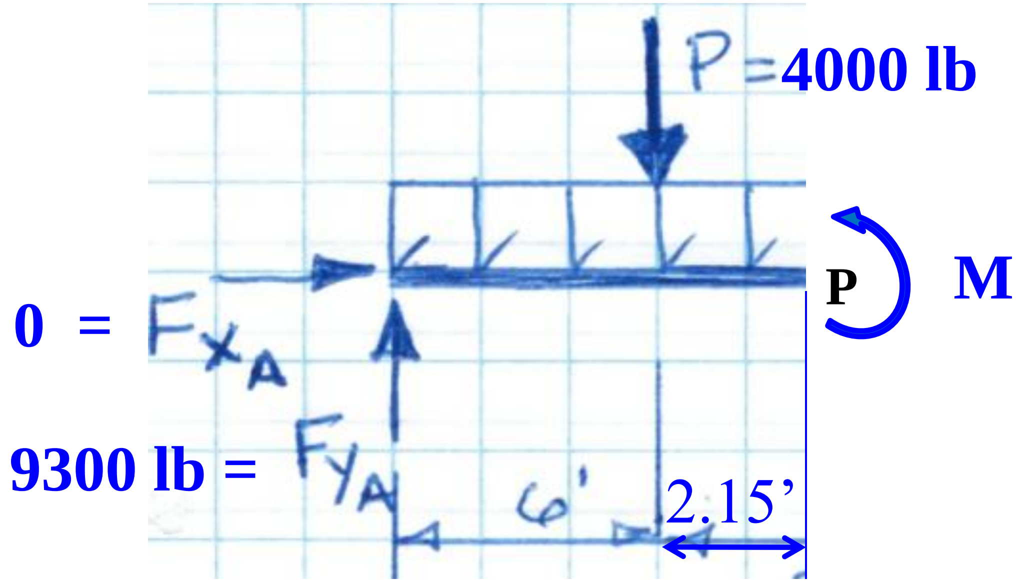
Moment Diagram



Kink in moment curve

$$x = \frac{1400 lb}{650 \frac{lb}{ft}} = 2.15\ ft$$

Moment Diagram

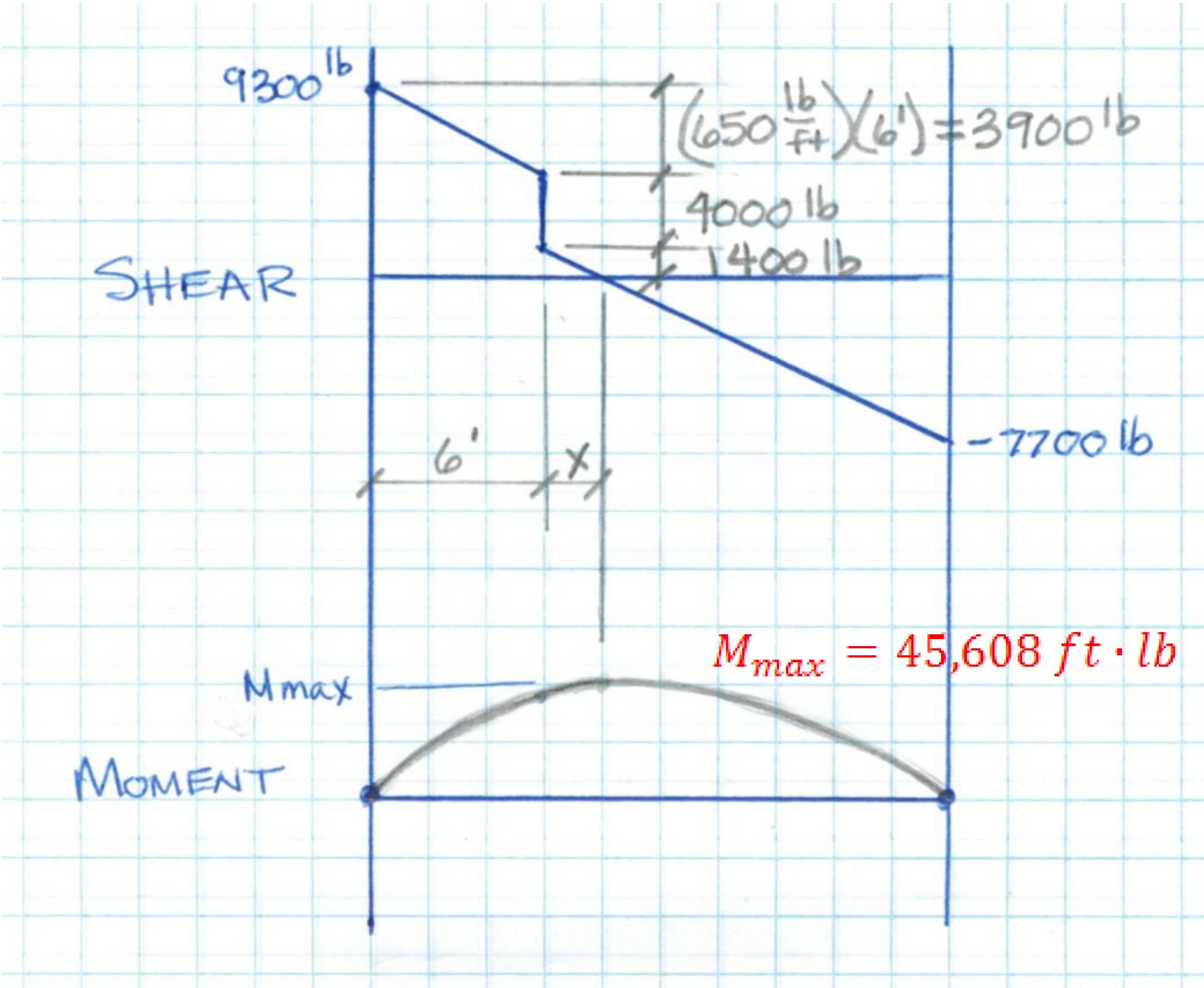


$$\sum M_p = 0$$

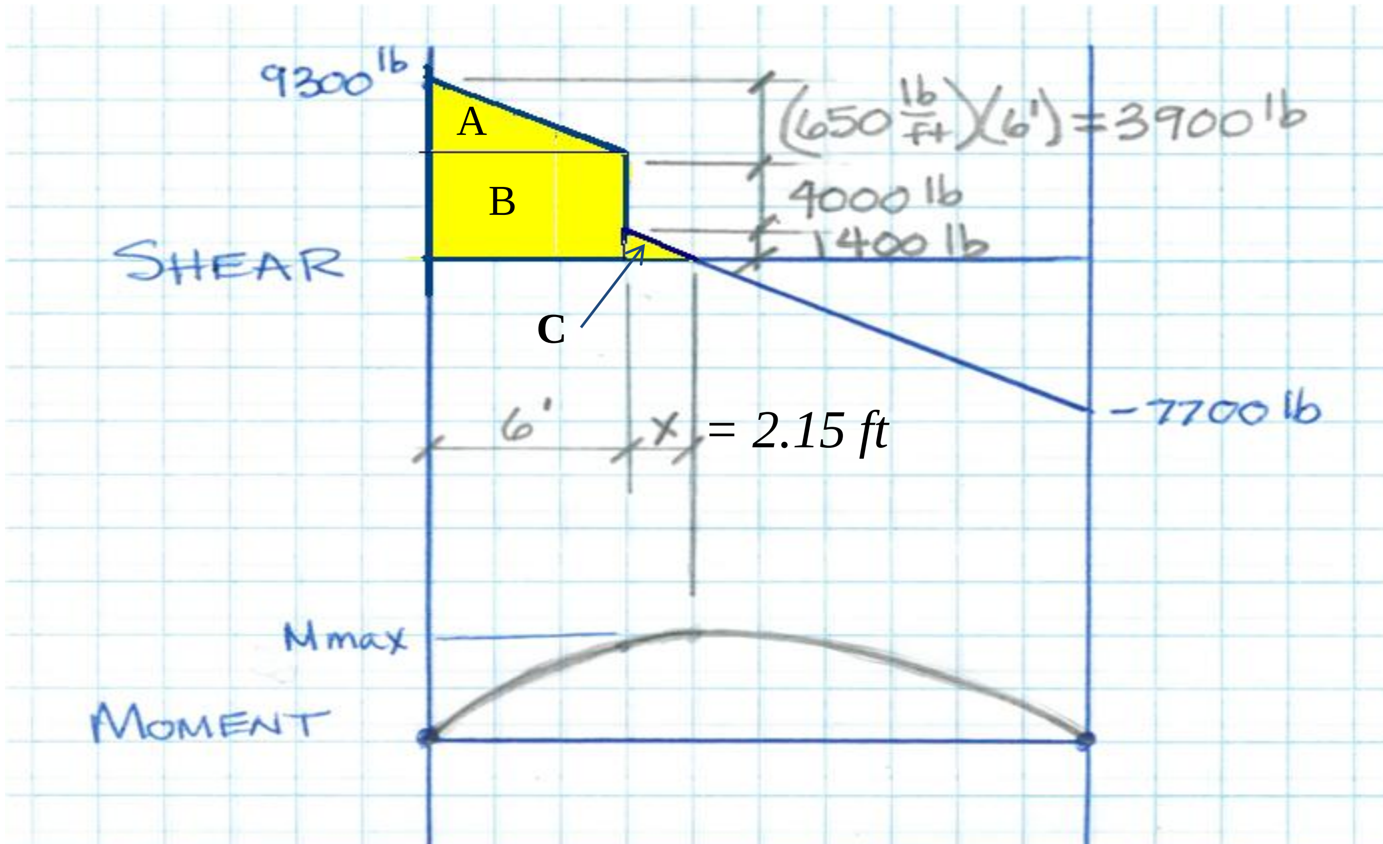
$$M + (4000 lb)(2.15\,ft) + (650\,\tfrac{lb}{ft})(8.15\,ft)\cdot\left(\frac{8.15\,ft}{2}\right) - (9300 lb)(8.15\,ft) = 0$$

$$\boxed{M = M_{max} = 45608\,ft \cdot lb}$$

Moment Diagram



Moment Diagram



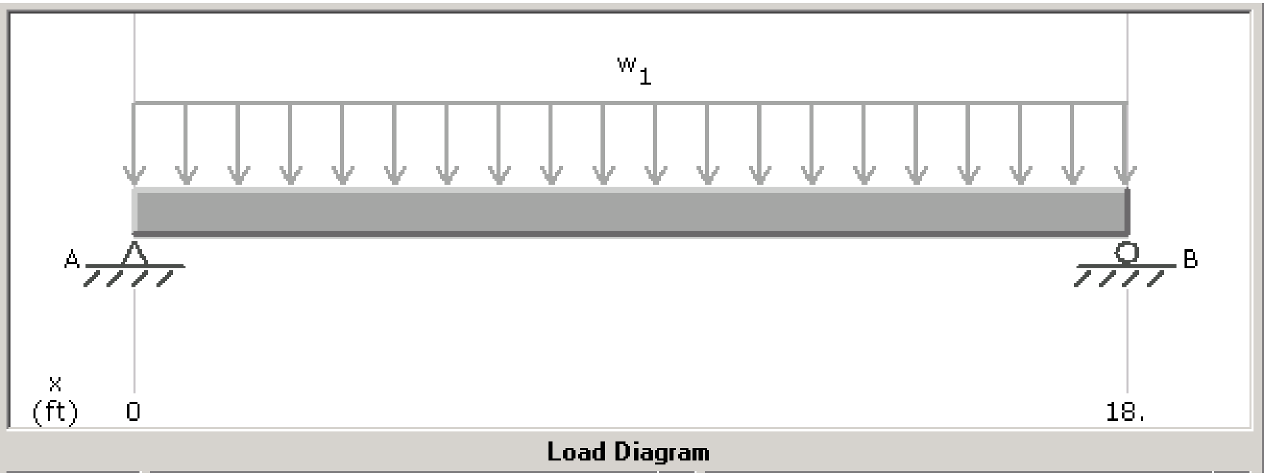
$$M_{max} = Area\ A + Area\ B + Area\ C$$

$$= \tfrac{1}{2}(6\ ft)(3900\ lb) + (6\ ft)(5400\ lb) + \tfrac{1}{2}(2.15\ ft)(1400\ lb)$$

$$= 45{,}605\ ft \cdot lb$$

Beam Analysis

- Example : simple beam with a uniform load, w_1= 1090 lb/ft
- Span = 18 feet

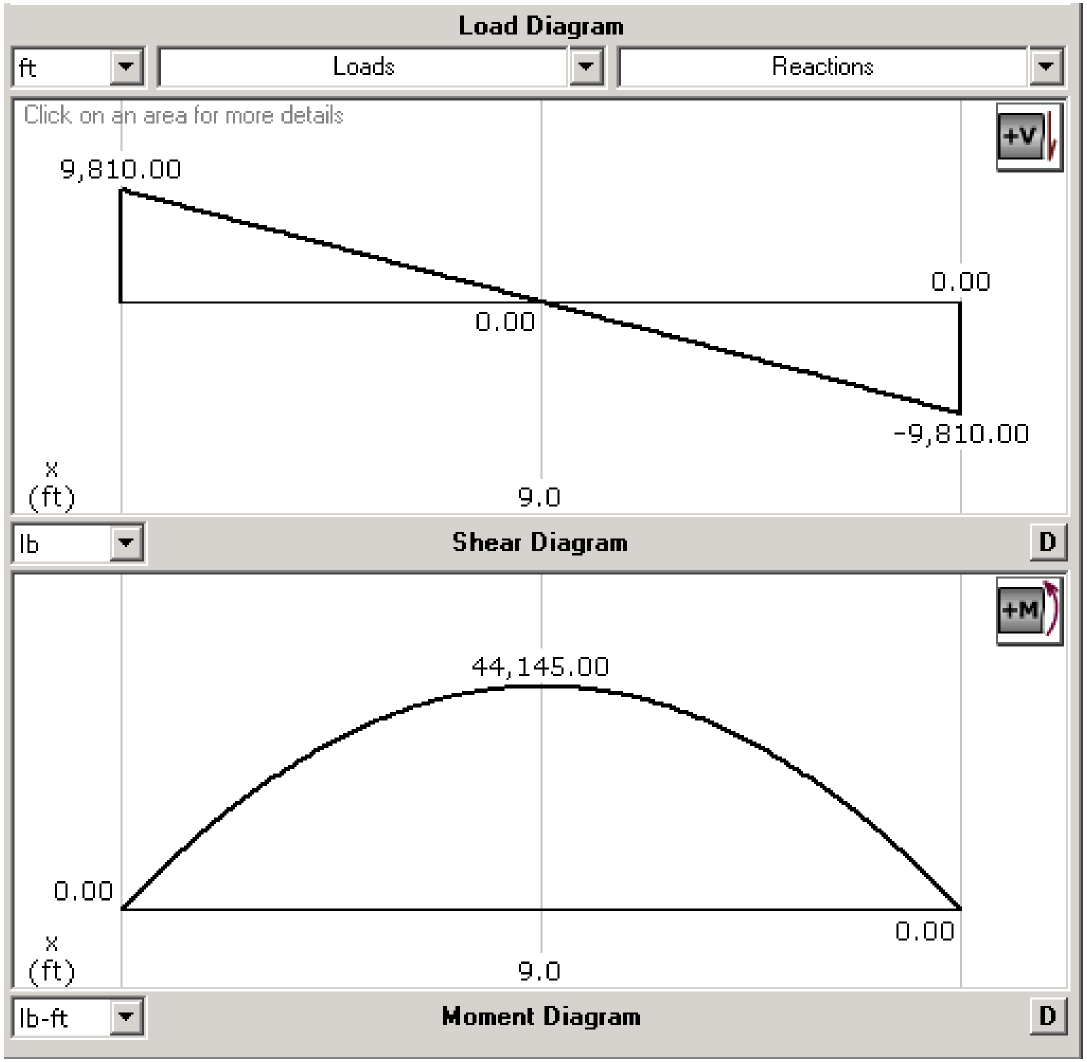


Test your understanding: Draw the shear and moment diagrams for this beam and loading condition.

Shear and Moment Diagrams

Shear

Moment



Max. Moment = 44,145l ft-lb   Max. Shear = 9,810 lb

Unit – 4
Slope Deflection &
Moment Distribution Method

MOMENT DISTRIBUTION METHOD - AN OVERVIEW

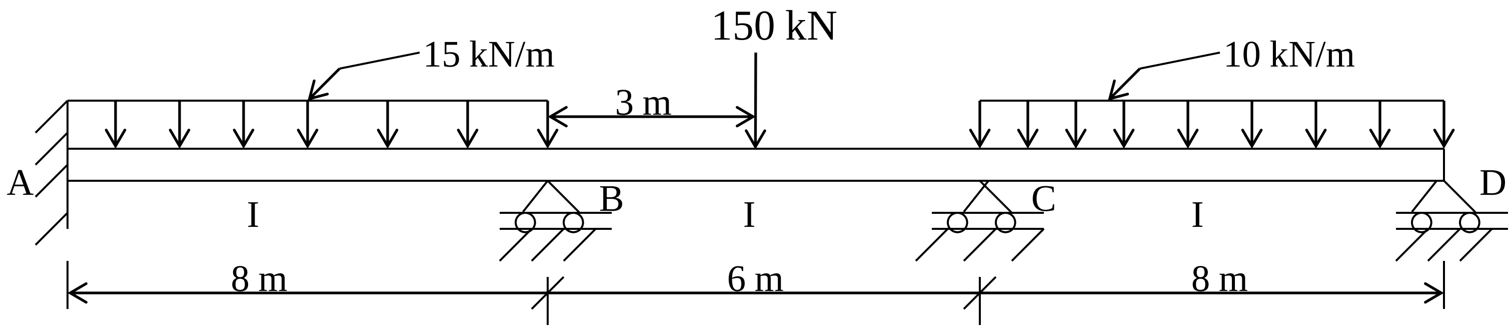
7.2 MOMENT DISTRIBUTION METHOD - INTRODUCTION AND BASIC PRINCIPLES

7.1 Introduction

(Method developed by Prof. Hardy Cross in 1932)
The method solves for the joint moments in continuous beams and rigid frames by successive approximation.

7.2 Statement of Basic Principles

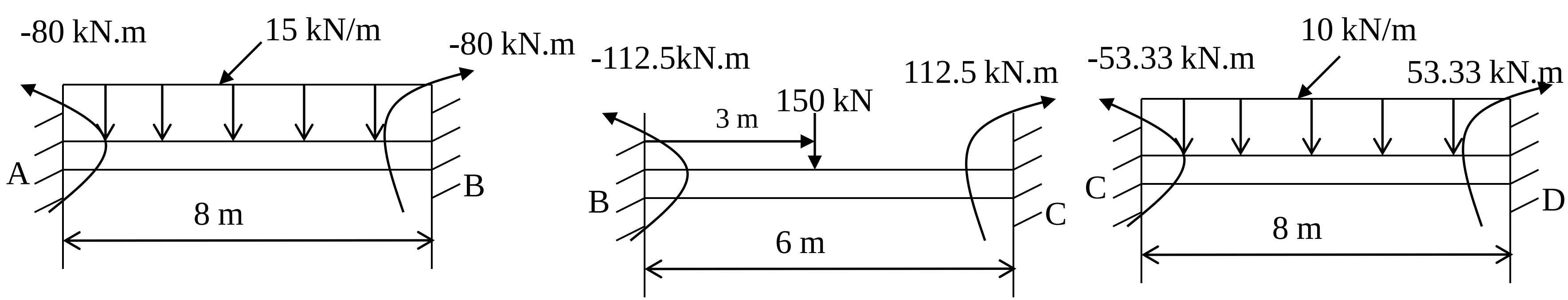
Consider the continuous beam ABCD, subjected to the given loads, as shown in Figure below. Assume that only rotation of joints occur at B, C and D, and that no support displacements occur at B, C and D. Due to the applied loads in spans AB, BC and CD, rotations occur at B, C and D.



In order to solve the problem in a successively approximating manner, it can be visualized to be made up of a continued two-stage problems viz., that of locking and releasing the joints in a continuous sequence.

7.2.1 Step I

The joints B, C and D are locked in position before any load is applied on the beam ABCD; then given loads are applied on the beam. Since the joints of beam ABCD are locked in position, beams AB, BC and CD acts as individual and separate fixed beams, subjected to the applied loads; these loads develop fixed end moments.



In beam AB

Fixed end moment at A = $-wl^2/12$ = - (15)(8)(8)/12 = - 80 kN.m

Fixed end moment at B = $+wl^2/12$ = +(15)(8)(8)/12 = + 80 kN.m

In beam BC

Fixed end moment at B = $-(Pab^2)/l^2 = -(150)(3)(3)^2/6^2$

= -112.5 kN.m

Fixed end moment at C = $+(Pab^2)/l^2 = +(150)(3)(3)^2/6^2$

= + 112.5 kN.m

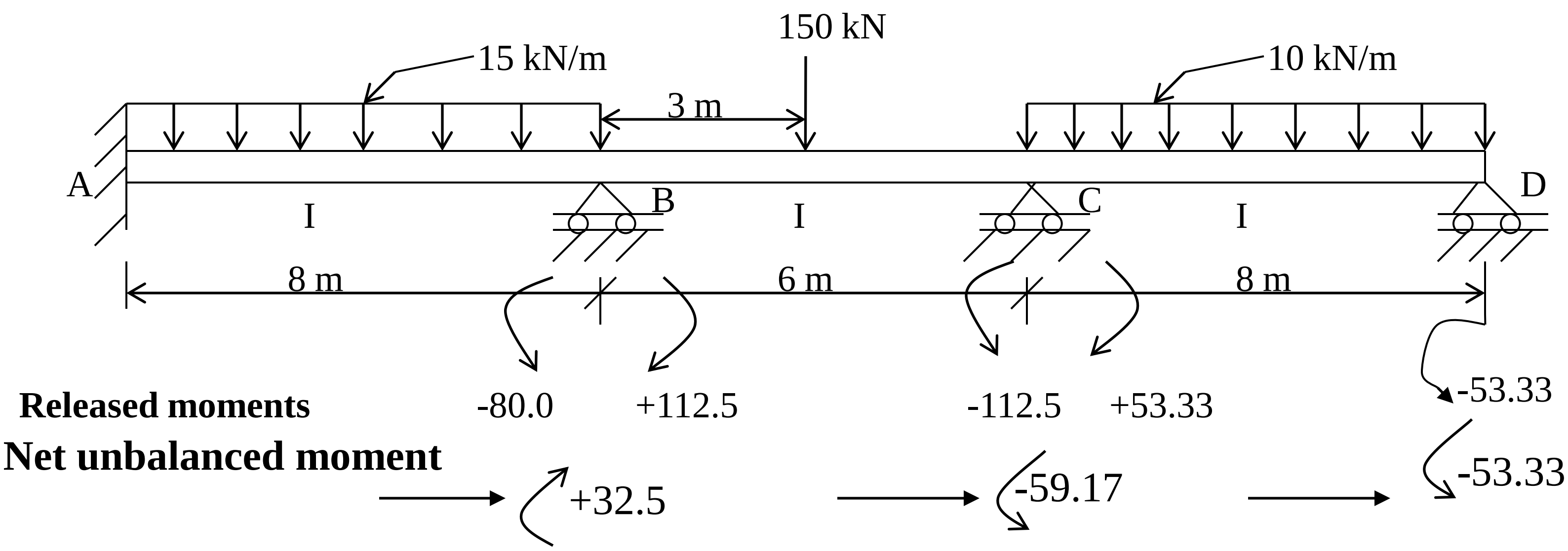
In beam AB

Fixed end moment at C = $-wl^2/12$ = - (10)(8)(8)/12 = - 53.33 kN.m

Fixed end moment at D = $+wl^2/12$ = +(10)(8)(8)/12 = + 53.33kN.m

7.2.2 Step II

Since the joints B, C and D were fixed artificially (to compute the the fixed-end moments), now the joints B, C and D are released and allowed to rotate. Due to the joint release, the joints rotate maintaining the continuous nature of the beam. Due to the joint release, the fixed end moments on either side of joints B, C and D act in the opposite direction now, and cause a net unbalanced moment to occur at the joint.



7.2.3 Step III

These unbalanced moments act at the joints and modify the joint moments at B, C and D, according to their relative stiffnesses at the respective joints. The joint moments are distributed to either side of the joint B, C or D, according to their relative stiffnesses. These distributed moments also modify the moments at the opposite side of the beam span, viz., at joint A in span AB, at joints B and C in span BC and at joints C and D in span CD. This modification is dependent on the carry-over factor (which is equal to 0.5 in this case); **when this carry over is made, the joints on opposite side are assumed to be fixed.**

7.2.4 Step IV

The carry-over moment becomes the unbalanced moment at the joints to which they are carried over. Steps 3 and 4 are repeated till the carry-over or distributed moment becomes small.

7.2.5 Step V

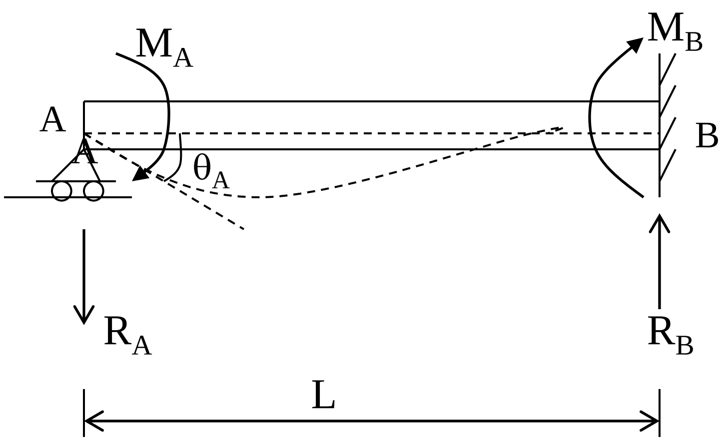
Sum up all the moments at each of the joint to obtain the joint moments.

7.3 SOME BASIC DEFINITIONS

In order to understand the five steps mentioned in section 7.3, some words need to be defined and relevant derivations made.

7.3.1 Stiffness and Carry-over Factors

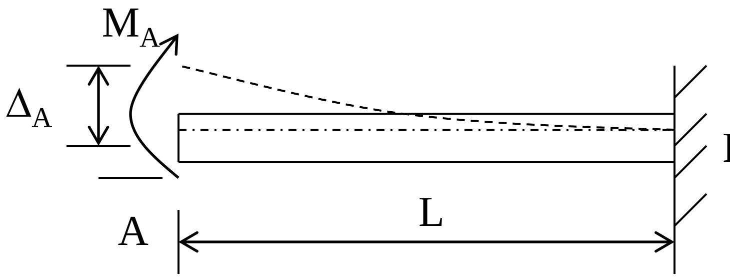
Stiffness = Resistance offered by member to a unit displacement or rotation at a point, for given support constraint conditions



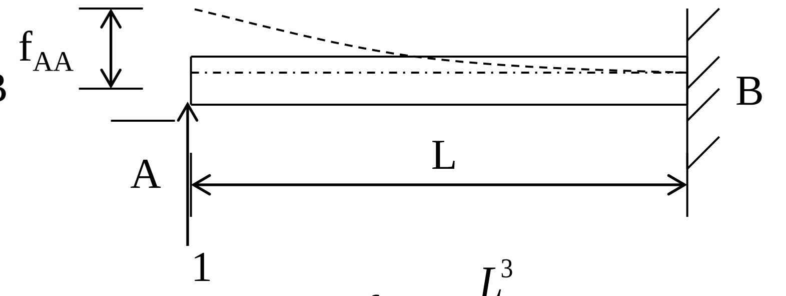
E, I – Member properties

A clockwise moment M_A is applied at A to produce a +ve bending in beam AB. Find θ_A and M_B.

Using method of consistent deformations



$$\Delta_A = +\frac{M_A L^2}{2EI}$$



$$f_{AA} = \frac{L^3}{3EI}$$

Applying the principle of consistent deformation,

$$\Delta_A + \mathbf{R}_A \mathbf{f}_{AA} = 0 \quad \rightarrow \mathbf{R}_A = -\frac{3\mathbf{M}_A}{2\mathbf{L}} \downarrow$$

$$\theta_A = \frac{M_A L}{EI} + \frac{R_A L^2}{2EI} = \frac{M_A L}{4EI} \qquad \therefore M_A = \frac{4EI}{L}\theta_A; \quad hence \quad k_\theta = \frac{M_A}{\theta_A} = \frac{4EI}{L}$$

Stiffness factor = k_θ = 4EI/L

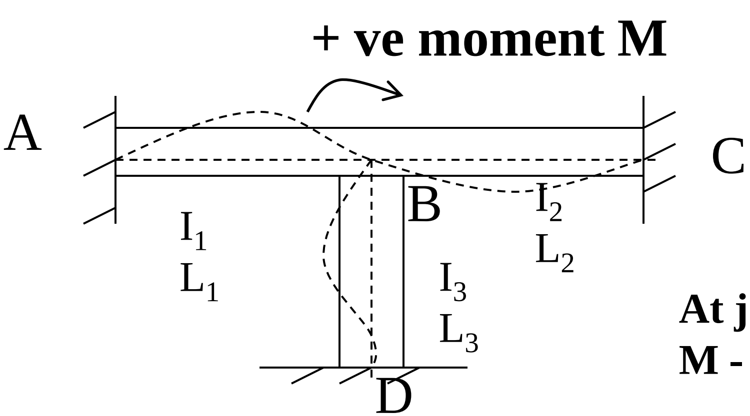
Considering moment $M_{B,}$

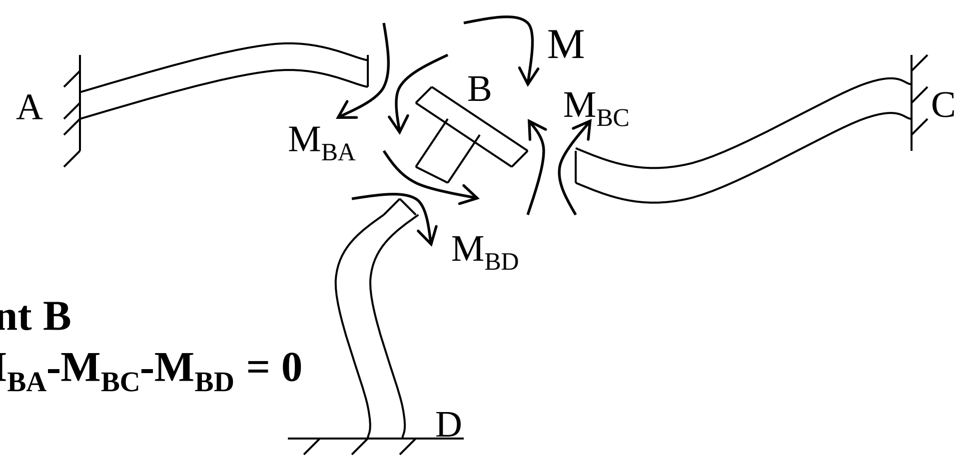
$$M_B + M_A + R_A L = 0$$
$$\therefore M_B = M_A/2 = (1/2)M_A$$

Carry - over Factor = 1/2

7.3.2 Distribution Factor

Distribution factor is the ratio according to which an externally applied unbalanced moment M at a joint is apportioned to the various members mating at the joint





At joint B

$$M - M_{BA} - M_{BC} - M_{BD} = 0$$

i.e., $\mathbf{M = M_{BA} + M_{BC} + M_{BD}}$

$$= \left[\left(\frac{4E_1 I_1}{L_1} \right) + \left(\frac{4E_2 I_2}{L_2} \right) + \left(\frac{4E_3 I_3}{L_3} \right) \right] \theta_B$$

$$= \left(K_{BA} + K_{BC} + K_{BD} \right) \theta_B$$

$$\therefore \quad \theta_B = \frac{M}{\left(K_{BA} + K_{BC} + K_{BD} \right)} = \frac{M}{\sum K}$$

$$M_{BA} = K_{BA} \theta_B = \left(\frac{K_{BA}}{\sum K} \right) M = (D.F)_{BA} M$$

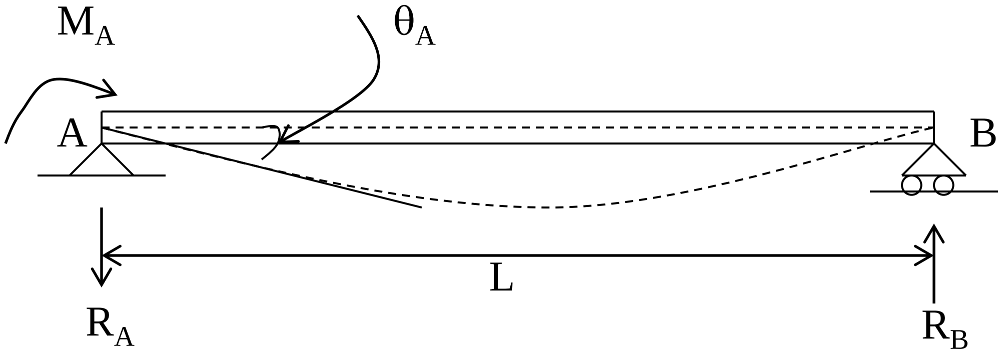
Similarly

$$M_{BC} = \left(\frac{K_{BC}}{\sum K} \right) M = (D.F)_{BC} M$$

$$M_{BD} = \left(\frac{K_{BD}}{\sum K} \right) M = (D.F)_{BD} M$$

7.3.3 Modified Stiffness Factor

The stiffness factor changes when the far end of the beam is simply-supported.



As per earlier equations for deformation, given in Mechanics of Solids text-books.

$$\theta_A = \frac{M_A L}{3EI}$$

$$K_{AB} = \frac{M_A}{\theta_A} = \frac{3EI}{L} = \left(\frac{3}{4}\right)\left(\frac{4EI}{L}\right)$$

$$= \frac{3}{4}(K_{AB})_{fixed}$$

7.4 SOLUTION OF PROBLEMS -

7.4.1 Solve the previously given problem by the moment distribution method

7.4.1.1: Fixed end moments

$$M_{AB} = -M_{BA} = -\frac{wl^2}{12} = -\frac{(15)(8)^2}{12} = -80\ kN.m$$

$$M_{BC} = -M_{CB} = -\frac{wl}{8} = -\frac{(150)(6)}{8} = -112.5\ kN.m$$

$$M_{CD} = -M_{DC} = -\frac{wl^2}{12} = -\frac{(10)(8)^2}{12} = -53.333\ kN.m$$

7.4.1.2 Stiffness Factors (Unmodified Stiffness)

$$\mathbf{K}_{AB} = \mathbf{K}_{BA} = \frac{4\mathbf{EI}}{\mathbf{L}} = \frac{(4)(\mathbf{EI})}{8} = 0.5\mathbf{EI}$$

$$\mathbf{K}_{BC} = \mathbf{K}_{CB} = \frac{4\mathbf{EI}}{\mathbf{L}} = \frac{(4)(\mathbf{EI})}{6} = 0.667\mathbf{EI}$$

$$\mathbf{K}_{CD} = \left[\frac{4\mathbf{EI}}{8}\right] = \frac{4}{8}\mathbf{EI} = 0.5\mathbf{EI}$$

$$\mathbf{K}_{DC} = \frac{4\mathbf{EI}}{8} = 0.5\mathbf{EI}$$

7.4.1.3 Distribution Factors

$$\mathbf{DF_{AB}} = \frac{\mathbf{K_{BA}}}{\mathbf{K_{BA}} + \mathbf{K_{wall}}} = \frac{0.5\mathbf{EI}}{0.5 + \infty\ (\mathbf{wall\ stiffness})} = 0.0$$

$$\mathbf{DF_{BA}} = \frac{\mathbf{K_{BA}}}{\mathbf{K_{BA}} + \mathbf{K_{BC}}} = \frac{0.5\mathbf{EI}}{0.5\mathbf{EI} + 0.667\mathbf{EI}} = 0.4284$$

$$\mathbf{DF_{BC}} = \frac{\mathbf{K_{BC}}}{\mathbf{K_{BA}} + \mathbf{K_{BC}}} = \frac{0.667\mathbf{EI}}{0.5\mathbf{EI} + 0.667\mathbf{EI}} = 0.5716$$

$$\mathbf{DF_{CB}} = \frac{\mathbf{K_{CB}}}{\mathbf{K_{CB}} + \mathbf{K_{CD}}} = \frac{0.667\mathbf{EI}}{0.667\mathbf{EI} + 0.500\mathbf{EI}} = 0.5716$$

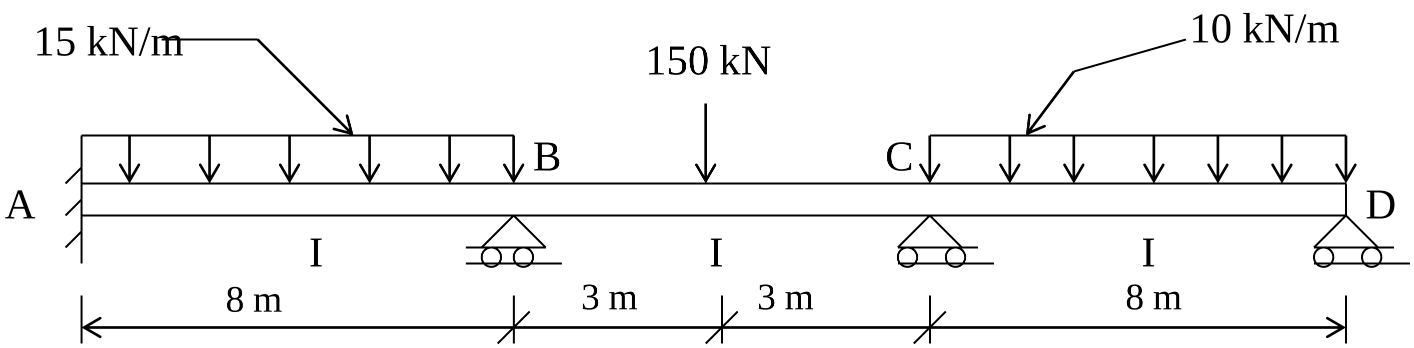
$$\mathbf{DF_{CD}} = \frac{\mathbf{K_{CD}}}{\mathbf{K_{CB}} + \mathbf{K_{CD}}} = \frac{0.500\mathbf{EI}}{0.667\mathbf{EI} + 0.500\mathbf{EI}} = 0.4284$$

$$\mathbf{DF_{DC}} = \frac{\mathbf{K_{DC}}}{\mathbf{K_{DC}}} = 1.00$$

7.4.1.4 Moment Distribution Table

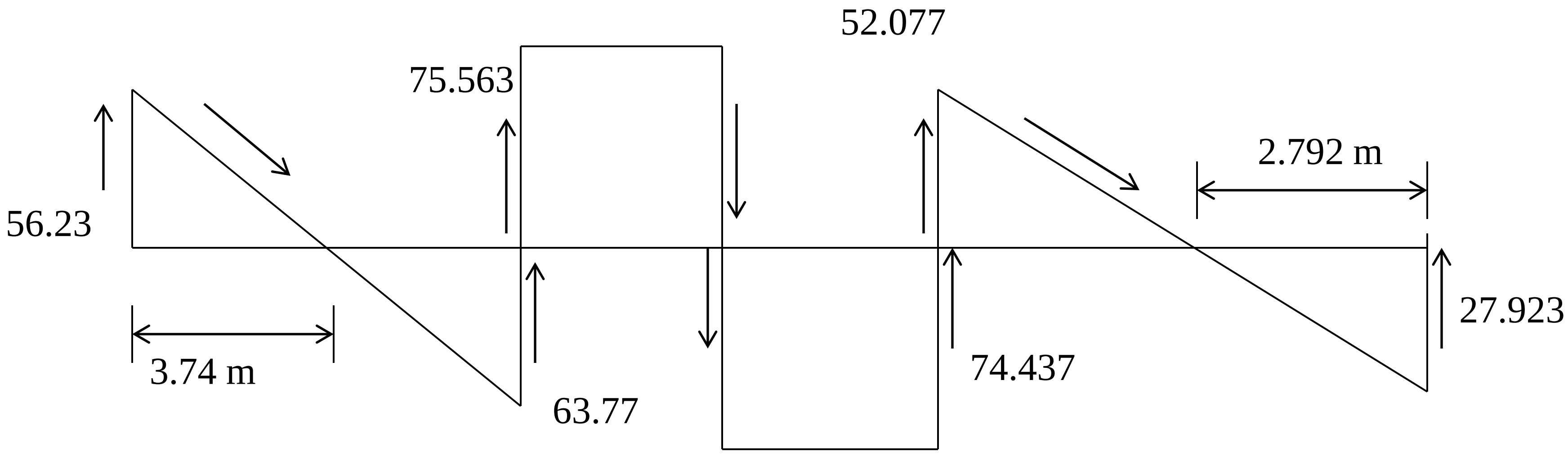
Joint		A	B		C		D
Member		AB	BA	BC	CB	CD	DC
Distribution Factors		0	0.4284	0.5716	0.5716	0.4284	1
Cycle 1	Computed end moments	-80	80	-112.5	112.5	-53.33	53.33
	Distribution		13.923	18.577	-33.82	-25.35	-53.33
Cycle 2	Carry-over moments	6.962		-16.91	9.289	-26.67	-12.35
	Distribution		7.244	9.662	9.935	7.446	12.35
Cycle 3	Carry-over moments	3.622		4.968	4.831	6.175	3.723
	Distribution		-2.128	-2.84	-6.129	-4.715	-3.723
Cycle 4	Carry-over moments	-1.064		-3.146	-1.42	-1.862	-2.358
	Distribution		1.348	1.798	1.876	1.406	2.358
Cycle 5	Carry-over moments	0.674		0.938	0.9	1.179	0.703
	Distribution		-0.402	-0.536	-1.187	-0.891	-0.703
	Summed up moments	-69.81	99.985	-99.99	96.613	-96.61	0

7.4.1.5 Computation of Shear Forces

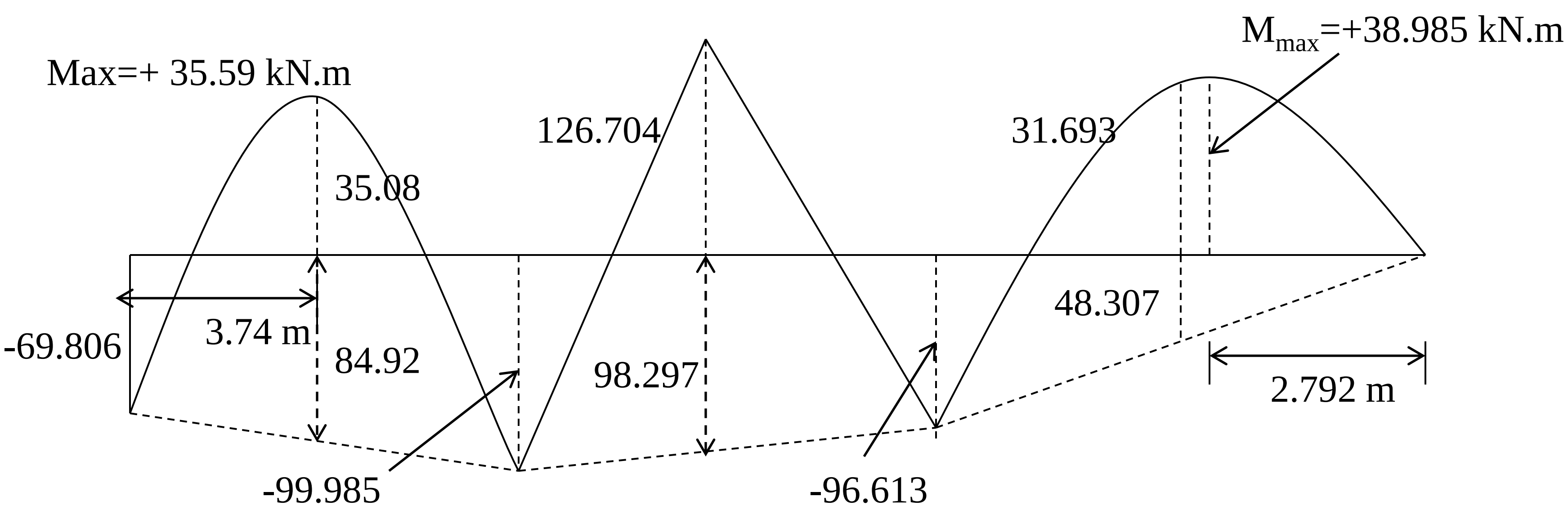


Simply-supported reaction	60	60	75	75	40	40
End reaction due to left hand FEM	8.726	-8.726	16.665	-16.67	12.079	-12.08
End reaction due to right hand FEM	-12.5	12.498	-16.1	16.102	0	0
Summed-up moments	56.228	63.772	75.563	74.437	53.077	27.923

7.4.1.5 Shear Force and Bending Moment Diagrams



S. F. D.



B. M. D

Simply-supported bending moments at center of span

M_{center} in AB = $(15)(8)^2/8 = +120$ kN.m

M_{center} in BC = $(150)(6)/4 = +225$ kN.m

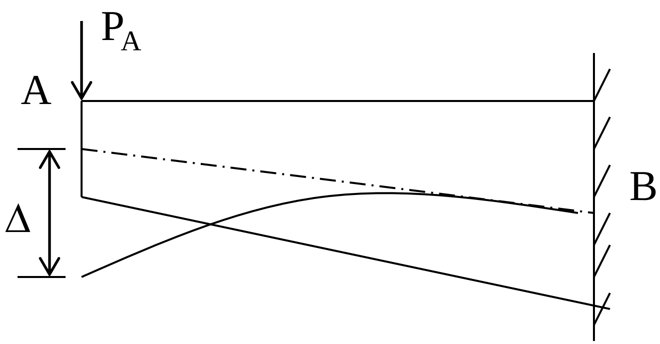
M_{center} in AB = $(10)(8)^2/8 = +80$ kN.m

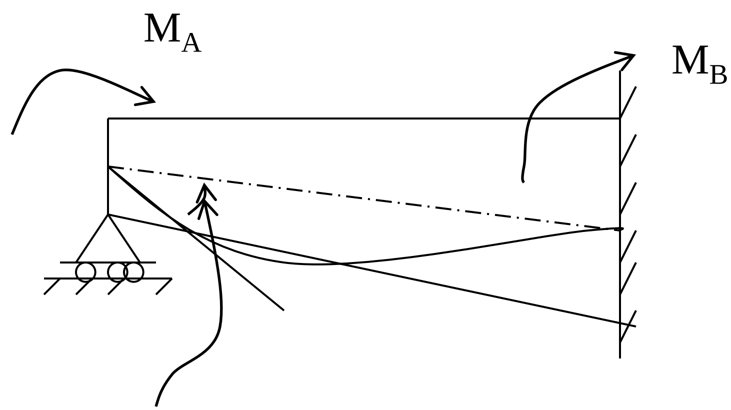
7.5 MOMENT DISTRIBUTION METHOD FOR NONPRISMATIC MEMBER (CHAPTER 12)

The section will discuss moment distribution method to analyze beams and frames composed of nonprismatic members. First the procedure to obtain the necessary carry-over factors, stiffness factors and fixed-end moments will be outlined. Then the use of values given in design tables will be illustrated. Finally the analysis of statically indeterminate structures using the moment distribution method will be outlined

7.5.1 Stiffness and Carry-over Factors

Use moment-area method to find the stiffness and carry-over factors of the non-prismatic beam.



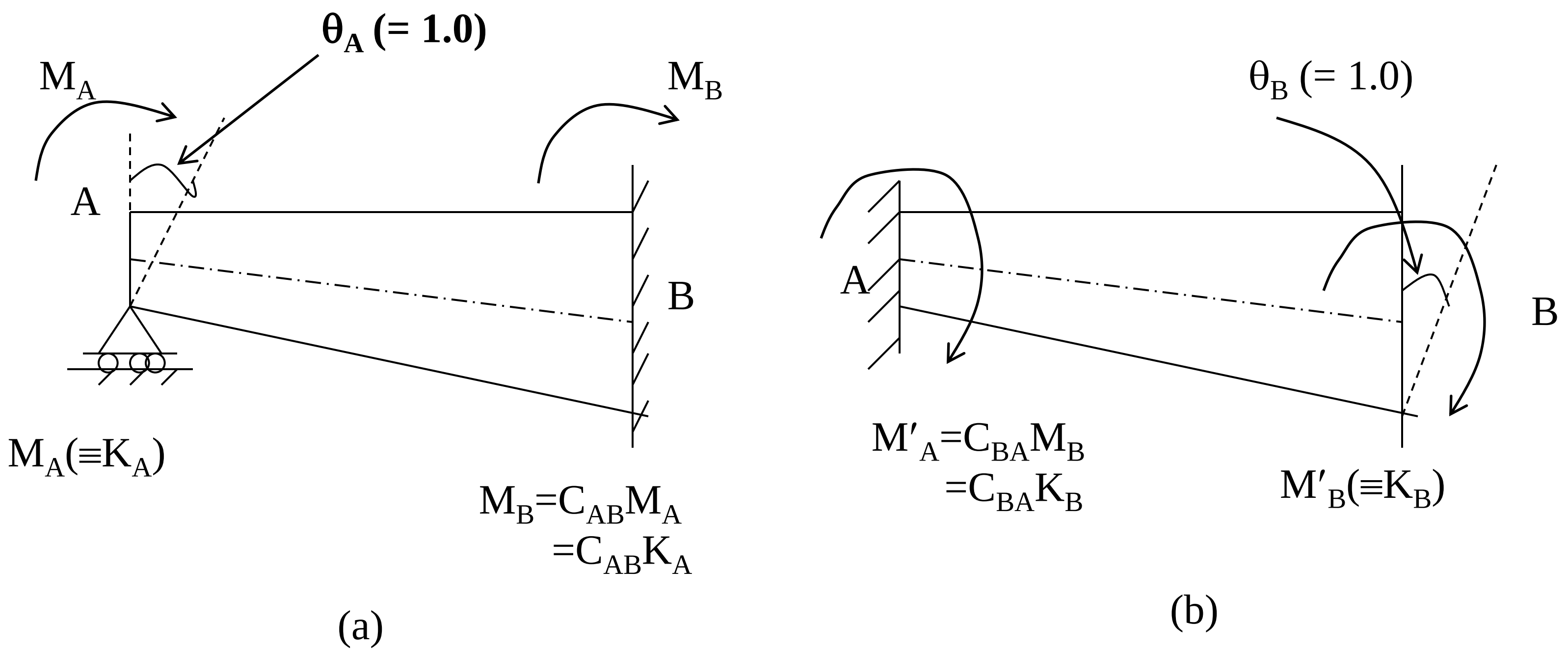


$$P_A = (K_A)_{AB}\Delta_A$$

$$M_A = (K_\theta)_{AB}\theta_A$$

$$M_B = C_{AB}M_A$$

C_{AB}= Carry-over factor of moment M_A from A to B



<u>Use of Betti-Maxwell's reciprocal theorem</u> requires that the work done by loads in case (a) acting through displacements in case (b) is equal to work done by loads in case (b) acting through displacements in case (a)

$$\left(M_A\right)(0)+\left(M_B\right)(1)=\left(M'_A\right)(1.0)+\left(M'_B\right)(0.0)$$

$$\mathrm{C_{AB}}K_A=\mathrm{C_{BA}}K_B$$

7.5.2 Tabulated Design Tables

Graphs and tables have been made available to determine fixed-end moments, stiffness factors and carry-over factors for common structural shapes used in design. One such source is the Handbook of Frame constants published by the Portland Cement Association, Chicago, Illinois, U. S. A. A portion of these tables, is listed here as Table 1 and 2

Nomenclature of the Tables

a_A a_b = ratio of length of haunch (at end A and B to the length of span

b = ratio of the distance (from the concentrated load to end A) to the length of span

h_A, h_B= depth of member at ends A and B, respectively

h_C = depth of member at minimum section

I_c = moment of inertia of section at minimum section = $(1/12)B(h_c)^3$, with B as width of beam

k_{AB}, k_{BC} = stiffness factor for rotation at end A and B, respectively

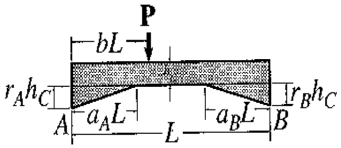
L = Length of member

M_{AB}, M_{BA} = Fixed-end moments at end A and B, respectively; specified in tables for uniform load w or concentrated force P

$$r_A = \frac{h_A - h_C}{h_C} \qquad r_B = \frac{h_B - h_C}{h_C}$$

Also $$K_A = \frac{k_{AB} E I_C}{L}, \quad K_B = \frac{k_{BA} E I_C}{L}$$

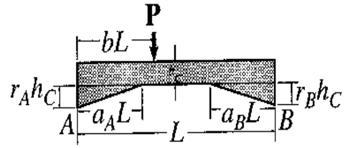
Table 12-1 Straight Haunches—Constant Width



Note: All carry-over factors are negative and all stiffness factors are positive.

Right Haunch		Carry-over Factors		Stiffness Factors		Unif. Load FEM Coef. × wL^2		Concentrated Load FEM—Coef. × PL, b = 0.1		b = 0.3		b = 0.5		b = 0.7		b = 0.9		Haunch Load at Left, FEM Coef. × $w_A L^2$		Haunch Load at Right, FEM Coef. × $w_B L^2$	
a_B	r_B	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
$a_A = 0.3$ a_B = *variable* $r_A = 1.0$ r_B = *variable*																					
0.2	0.4	0.543	0.766	9.19	6.52	0.1194	0.0791	0.0935	0.0034	0.2185	0.0384	0.1955	0.1147	0.0889	0.1601	0.0096	0.0870	0.0133	0.0008	0.0006	0.0058
	0.6	0.576	0.758	9.53	7.24	0.1152	0.0851	0.0934	0.0038	0.2158	0.0422	0.1883	0.1250	0.0798	0.1729	0.0075	0.0898	0.0133	0.0009	0.0005	0.0060
	1.0	0.622	0.748	10.06	8.37	0.1089	0.0942	0.0931	0.0042	0.2118	0.0480	0.1771	0.1411	0.0668	0.1919	0.0047	0.0935	0.0132	0.0011	0.0004	0.0062
	1.5	0.660	0.740	10.52	9.38	0.1037	0.1018	0.0927	0.0047	0.2085	0.0530	0.1678	0.1550	0.0559	0.2078	0.0028	0.0961	0.0130	0.0012	0.0002	0.0064
	2.0	0.684	0.734	10.83	10.09	0.1002	0.1069	0.0924	0.0050	0.2062	0.0565	0.1614	0.1645	0.0487	0.2185	0.0019	0.0974	0.0129	0.0013	0.0001	0.0065
0.3	0.4	0.579	0.741	9.47	7.40	0.1175	0.0822	0.0934	0.0037	0.2164	0.0419	0.1909	0.1225	0.0856	0.1649	0.0100	0.0861	0.0133	0.0009	0.0022	0.0118
	0.6	0.629	0.726	9.98	8.64	0.1120	0.0902	0.0931	0.0042	0.2126	0.0477	0.1808	0.1379	0.0747	0.1807	0.0080	0.0888	0.0132	0.0010	0.0018	0.0124
	1.0	0.705	0.705	10.85	10.85	0.1034	0.1034	0.0924	0.0052	0.2063	0.0577	0.1640	0.1640	0.0577	0.2063	0.0052	0.0924	0.0131	0.0013	0.0013	0.0131
	1.5	0.771	0.689	11.70	13.10	0.0956	0.1157	0.0917	0.0062	0.2002	0.0675	0.1483	0.1892	0.0428	0.2294	0.0033	0.0953	0.0129	0.0015	0.0008	0.0137
	2.0	0.817	0.678	12.33	14.85	0.0901	0.1246	0.0913	0.0069	0.1957	0.0750	0.1368	0.2080	0.0326	0.2455	0.0022	0.0968	0.0128	0.0017	0.0006	0.0141
$a_A = 0.2$ a_B = *variable* $r_A = 1.5$ r_B = *variable*																					
0.2	0.4	0.569	0.714	7.97	6.35	0.1166	0.0799	0.0966	0.0019	0.2186	0.0377	0.1847	0.1183	0.0821	0.1626	0.0088	0.0873	0.0064	0.0001	0.0006	0.0058
	0.6	0.603	0.707	8.26	7.04	0.1127	0.0858	0.0965	0.0021	0.2163	0.0413	0.1778	0.1288	0.0736	0.1752	0.0068	0.0901	0.0064	0.0001	0.0005	0.0060
	1.0	0.652	0.698	8.70	8.12	0.1069	0.0947	0.0963	0.0023	0.2127	0.0468	0.1675	0.1449	0.0616	0.1940	0.0043	0.0937	0.0064	0.0002	0.0004	0.0062
	1.5	0.691	0.691	9.08	9.08	0.1021	0.1021	0.0962	0.0025	0.2097	0.0515	0.1587	0.1587	0.0515	0.2097	0.0025	0.0962	0.0064	0.0002	0.0002	0.0064
	2.0	0.716	0.686	9.34	9.75	0.0990	0.1071	0.0960	0.0028	0.2077	0.0547	0.1528	0.1681	0.0449	0.2202	0.0017	0.0975	0.0064	0.0002	0.0001	0.0065
0.3	0.4	0.607	0.692	8.21	7.21	0.1148	0.0829	0.0965	0.0021	0.2168	0.0409	0.1801	0.1263	0.0789	0.1674	0.0091	0.0866	0.0064	0.0002	0.0020	0.0118
	0.6	0.659	0.678	8.65	8.40	0.1098	0.0907	0.0964	0.0024	0.2135	0.0464	0.1706	0.1418	0.0688	0.1831	0.0072	0.0892	0.0064	0.0002	0.0017	0.0123
	1.0	0.740	0.660	9.38	10.52	0.1018	0.1037	0.0961	0.0028	0.2078	0.0559	0.1550	0.1678	0.0530	0.2085	0.0047	0.0927	0.0064	0.0002	0.0012	0.0130
	1.5	0.809	0.645	10.09	12.66	0.0947	0.1156	0.0958	0.0033	0.2024	0.0651	0.1403	0.1928	0.0393	0.2311	0.0029	0.0950	0.0063	0.0003	0.0008	0.0137
	2.0	0.857	0.636	10.62	14.32	0.0897	0.1242	0.0955	0.0038	0.1985	0.0720	0.1296	0.2119	0.0299	0.2469	0.0020	0.0968	0.0063	0.0003	0.0005	0.0141

Table 12–2 Parabolic Haunches—Constant Width



Note: All carry-over factors are negative and all stiffness factors are positive.

Right Haunch		Carry-over Factors		Stiffness Factors		Unif. Load FEM Coef. × wL^2		Concentrated Load FEM—Coef. × PL: b = 0.1		0.3		0.5		0.7		0.9		Haunch Load at Left: FEM Coef. × w_AL^2		Haunch Load at Right: FEM Coef. × w_BL^2	
a_B	r_B	C_{AB}	C_{BA}	k_{AB}	k_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}	M_{AB}	M_{BA}
$a_A = 0.2$		a_B = *variable*			$r_A = 1.0$			r_B = *variable*													
	0.4	0.558	0.627	6.08	5.40	0.1022	0.0841	0.0938	0.0033	0.1891	0.0502	0.1572	0.1261	0.0715	0.1618	0.0073	0.0877	0.0032	0.0001	0.0002	0.0030
	0.6	0.582	0.624	6.21	5.80	0.0995	0.0887	0.0936	0.0036	0.1872	0.0535	0.1527	0.1339	0.0663	0.1708	0.0058	0.0902	0.0032	0.0001	0.0002	0.0031
0.2	1.0	0.619	0.619	6.41	6.41	0.0956	0.0956	0.0935	0.0038	0.1844	0.0584	0.1459	0.1459	0.0584	0.1844	0.0038	0.0935	0.0032	0.0001	0.0001	0.0032
	1.5	0.649	0.614	6.59	6.97	0.0921	0.1015	0.0933	0.0041	0.1819	0.0628	0.1399	0.1563	0.0518	0.1962	0.0025	0.0958	0.0032	0.0001	0.0001	0.0032
	2.0	0.671	0.611	6.71	7.38	0.0899	0.1056	0.0932	0.0044	0.1801	0.0660	0.1358	0.1638	0.0472	0.2042	0.0017	0.0971	0.0032	0.0001	0.0000	0.0033
	0.4	0.588	0.616	6.22	5.93	0.1002	0.0877	0.0937	0.0035	0.1873	0.0537	0.1532	0.1339	0.0678	0.1686	0.0073	0.0877	0.0032	0.0001	0.0007	0.0063
	0.6	0.625	0.609	6.41	6.58	0.0966	0.0942	0.0935	0.0039	0.1845	0.0587	0.1467	0.1455	0.0609	0.1808	0.0057	0.0902	0.0032	0.0001	0.0005	0.0065
0.3	1.0	0.683	0.598	6.73	7.68	0.0911	0.1042	0.0932	0.0044	0.1801	0.0669	0.1365	0.1643	0.0502	0.2000	0.0037	0.0936	0.0031	0.0001	0.0004	0.0068
	1.5	0.735	0.589	7.02	8.76	0.0862	0.1133	0.0929	0.0050	0.1760	0.0746	0.1272	0.1819	0.0410	0.2170	0.0023	0.0959	0.0031	0.0001	0.0003	0.0070
	2.0	0.772	0.582	7.25	9.61	0.0827	0.1198	0.0927	0.0054	0.1730	0.0805	0.1203	0.1951	0.0345	0.2293	0.0016	0.0972	0.0031	0.0001	0.0002	0.0072
$a_A = 0.5$		a_B = *variable*			$r_A = 1.0$			r_B = *variable*													
	0.4	0.488	0.807	9.85	5.97	0.1214	0.0753	0.0929	0.0034	0.2131	0.0371	0.2021	0.1061	0.0979	0.1506	0.0105	0.0863	0.0171	0.0017	0.0003	0.0030
	0.6	0.515	0.803	10.10	6.45	0.1183	0.0795	0.0928	0.0036	0.2110	0.0404	0.1969	0.1136	0.0917	0.1600	0.0083	0.0892	0.0170	0.0018	0.0002	0.0030
0.2	1.0	0.547	0.796	10.51	7.22	0.1138	0.0865	0.0926	0.0040	0.2079	0.0448	0.1890	0.1245	0.0809	0.1740	0.0056	0.0928	0.0168	0.0020	0.0001	0.0031
	1.5	0.571	0.786	10.90	7.90	0.1093	0.0922	0.0923	0.0043	0.2055	0.0485	0.1818	0.1344	0.0719	0.1862	0.0035	0.0951	0.0167	0.0021	0.0001	0.0032
	2.0	0.590	0.784	11.17	8.40	0.1063	0.0961	0.0922	0.0046	0.2041	0.0506	0.1764	0.1417	0.0661	0.1948	0.0025	0.0968	0.0166	0.0022	0.0001	0.0032
	0.4	0.554	0.753	10.42	7.66	0.1170	0.0811	0.0926	0.0040	0.2087	0.0442	0.1924	0.1205	0.0898	0.1595	0.0107	0.0853	0.0169	0.0020	0.0042	0.0145
	0.6	0.606	0.730	10.96	9.12	0.1115	0.0889	0.0922	0.0046	0.2045	0.0506	0.1820	0.1360	0.0791	0.1738	0.0086	0.0878	0.0167	0.0022	0.0036	0.0152
0.5	1.0	0.694	0.694	12.03	12.03	0.1025	0.1025	0.0915	0.0057	0.1970	0.0626	0.1639	0.1639	0.0626	0.1970	0.0057	0.0915	0.0164	0.0028	0.0028	0.0164
	1.5	0.781	0.664	13.12	15.47	0.0937	0.1163	0.0908	0.0070	0.1891	0.0759	0.1456	0.1939	0.0479	0.2187	0.0039	0.0940	0.0160	0.0034	0.0021	0.0174
	2.0	0.850	0.642	14.09	18.64	0.0870	0.1275	0.0901	0.0082	0.1825	0.0877	0.1307	0.2193	0.0376	0.2348	0.0027	0.0957	0.0157	0.0039	0.0016	0.0181

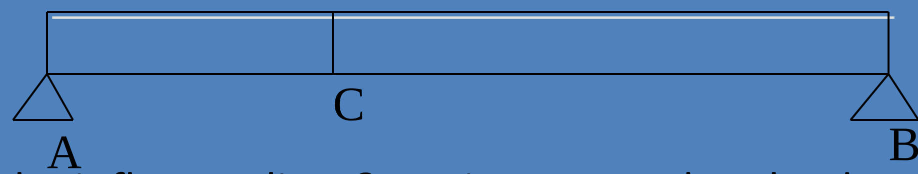
Unit - 5
Influence Lines For Statically Determinate Structures

3. INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES - AN OVERVIEW

- Introduction - What is an influence line?
- Influence lines for beams
- Qualitative influence lines - Muller-Breslau Principle
- Influence lines for floor girders
- Influence lines for trusses
- Live loads for bridges
- Maximum influence at a point due to a series of concentrated loads
- Absolute maximum shear and moment

3.1 INTRODUCTION TO INFLUENCE LINES

- **Influence lines describe the variation of an analysis variable** (reaction, shear force, bending moment, twisting moment, deflection, etc.) **at a point** (say at C in Figure 6.1) ..
  … …



- Why do we need the influence lines? For instance, when loads pass over a structure, say a bridge, one needs to know when the maximum values of shear/reaction/bending-moment will occur at a point so that the section may be designed

- **Notations:**
  - **Normal Forces** - +ve forces cause +ve displacements in +ve directions
  - **Shear Forces** - +ve shear forces cause clockwise rotation & - ve shear force causes anti-clockwise rotation
  - **Bending Moments:** +ve bending moments cause "cup holding water" deformed shape

3.2 INFLUENCE LINES FOR BEAMS

- **Procedure:**

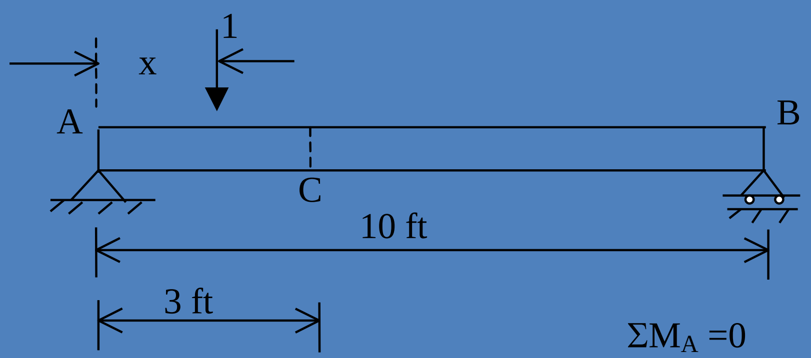
  (1) **Allow a unit load** (either 1b, 1N, 1kip, or 1 tonne) **to move over beam from left to right**

  (2) **Find the values** of shear force or bending moment, **at the point under consideration**, as the unit load moves over the beam from left to right

  (3) **Plot the values** of the shear force or bending moment, **over the length of the beam, computed for the point under consideration**

3.3 MOVING CONCENTRATED LOAD

3.3.1 Variation of Reactions R_A and R_B as functions of load position



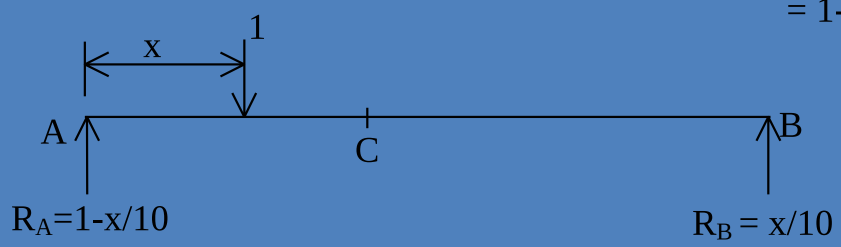
$\Sigma M_A = 0$

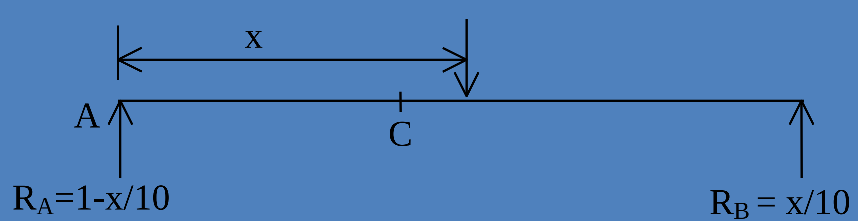
$(R_B)(10) - (1)(x) = 0$

$R_B = x/10$

$R_A = 1 - R_B$

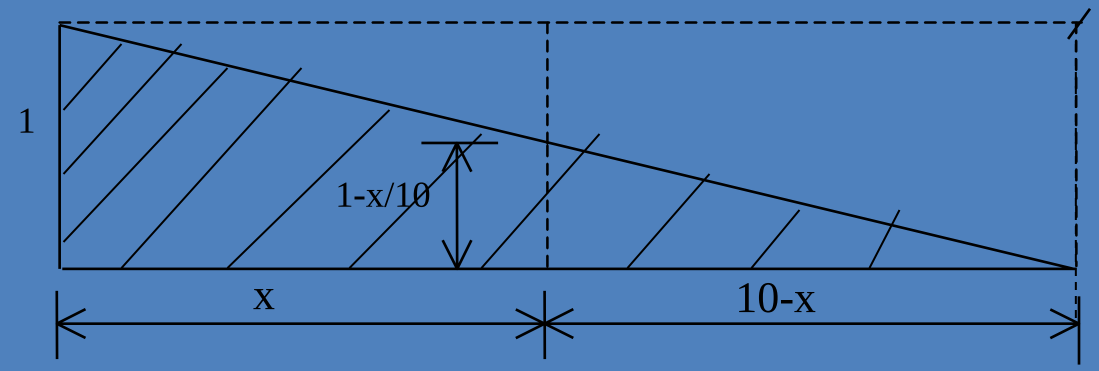
$= 1 - x/10$



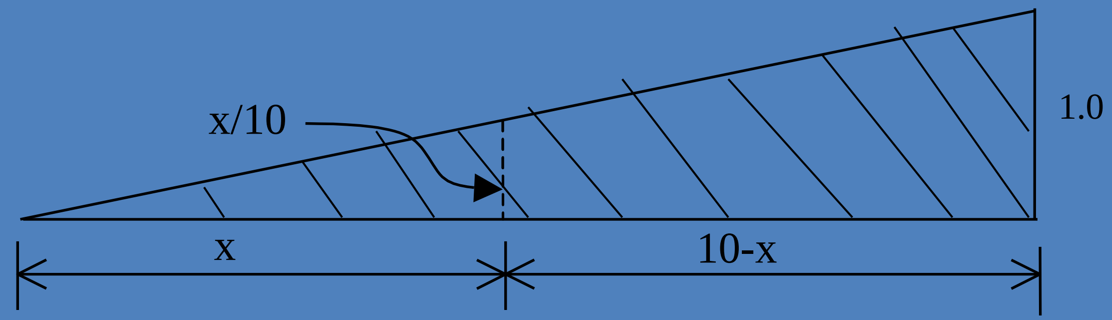


R_A occurs only at A; R_B occurs only at B

Influence line for R_A

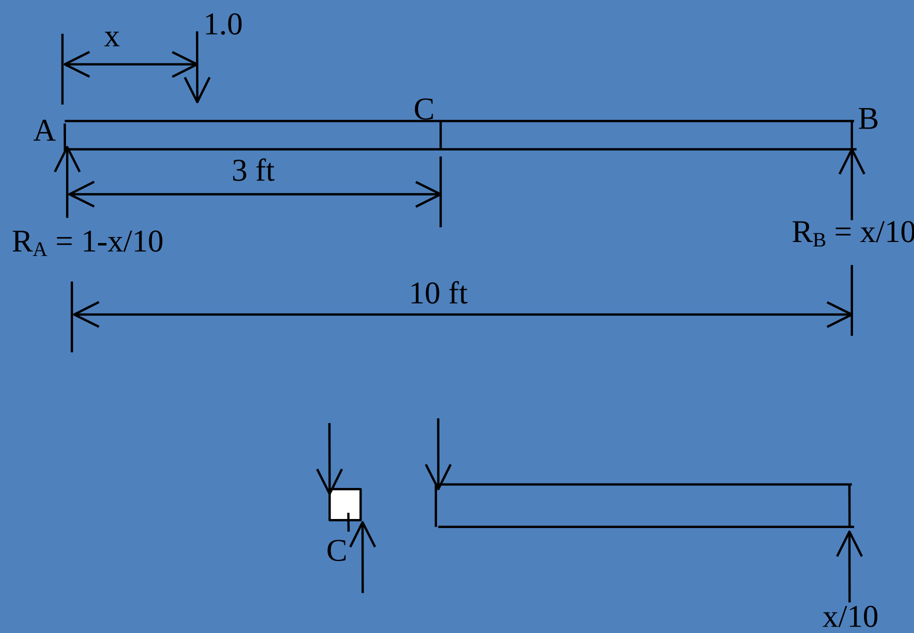


Influence line for R_B



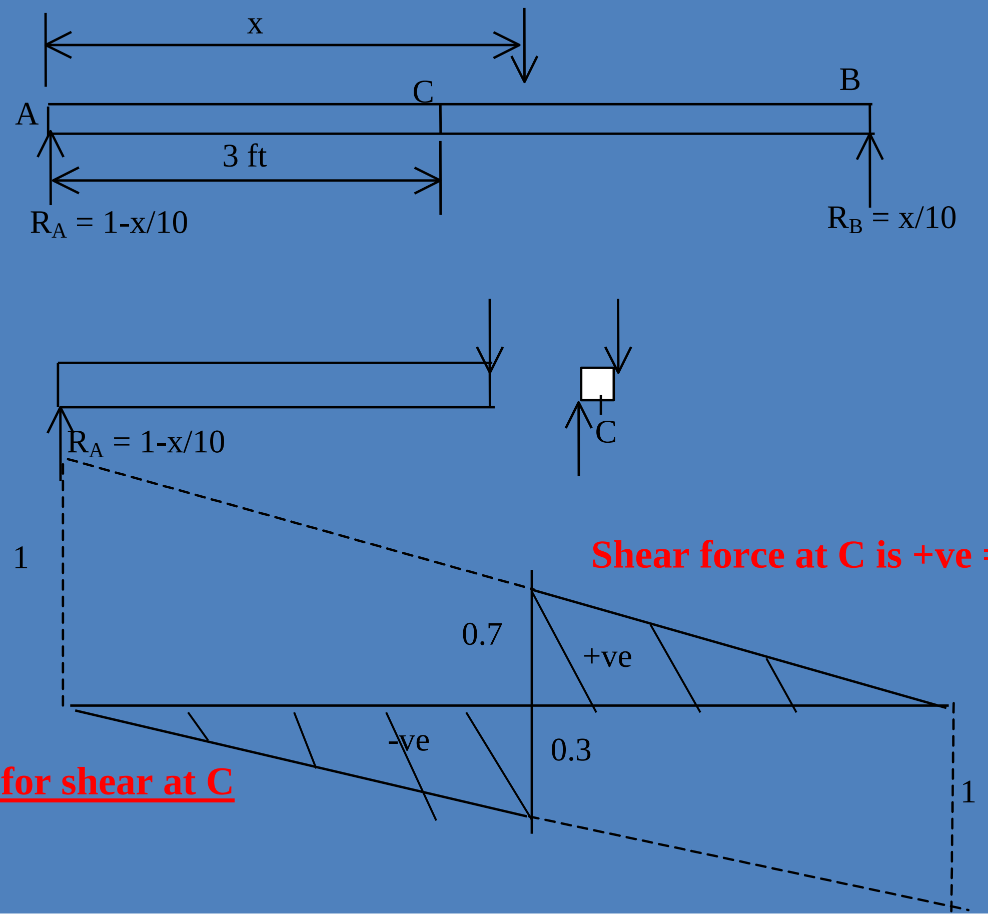
3.3.2 Variation of Shear Force at C as a function of load position

0 < x < 3 ft (unit load to the left of C)



Shear force at C is –ve, V_C =-x/10

3 < x < 10 ft (unit load to the right of C)

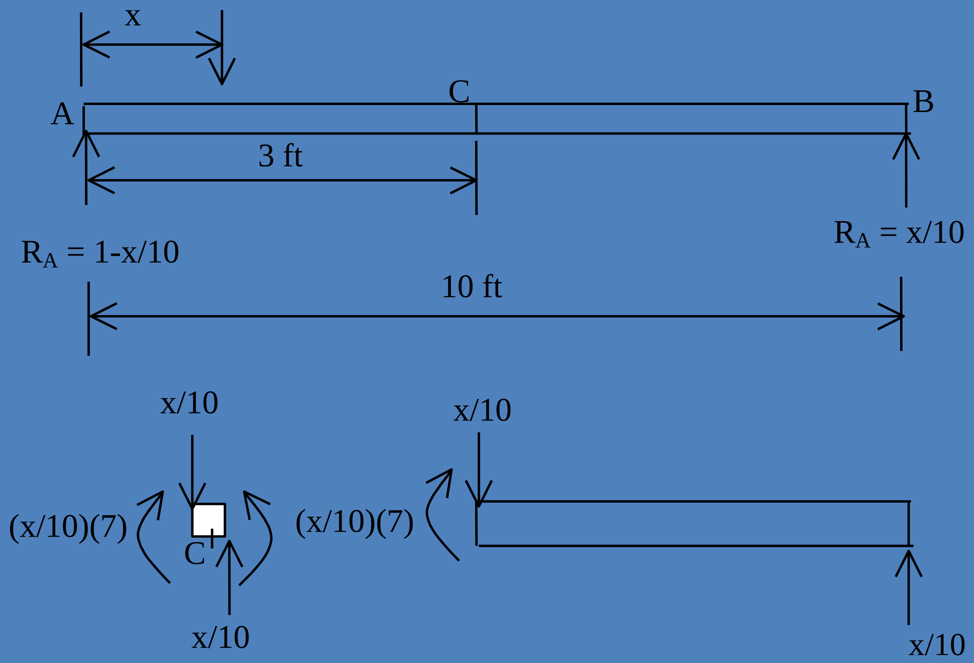


Shear force at C is +ve = 1-x/10

Influence line for shear at C

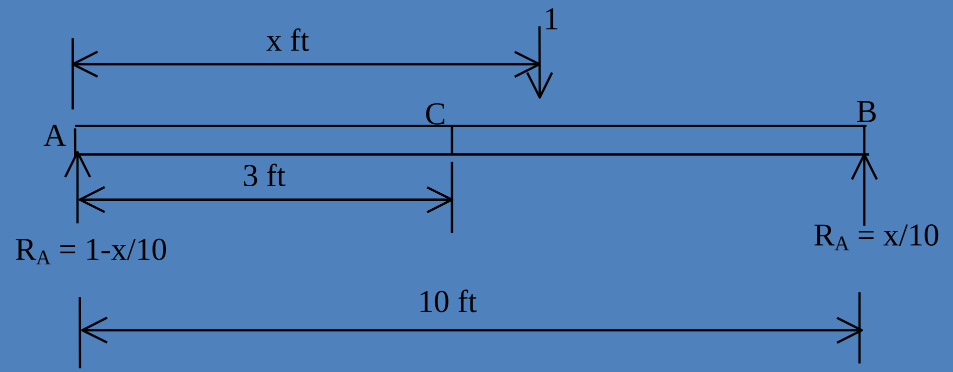
3.3.3 Variation of Bending Moment at C as a function of load position

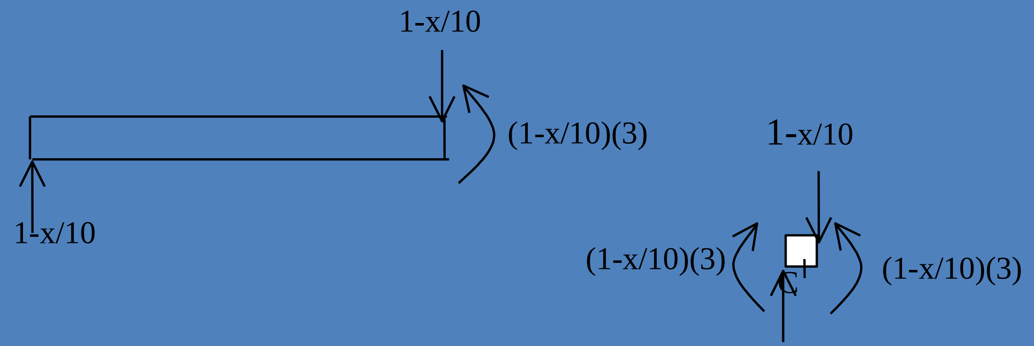
0 < x < 3.0 ft (Unit load to the left of C)



Bending moment is +ve at C

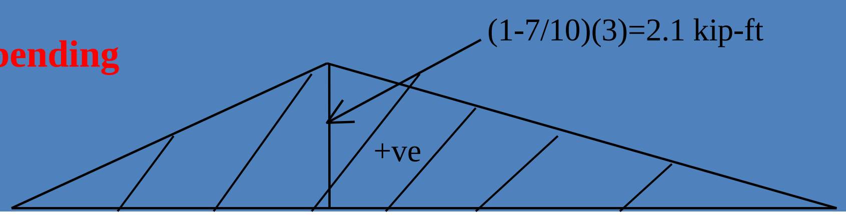
3 < x < 10 ft (Unit load to the right of C)





Moment at C is +ve

Influence line for bending Moment at C



3.4 QUALITATIVE INFLUENCED LINES - MULLER-BRESLAU'S PRINCIPLE

- The principle **gives only a procedure** to determine of the influence line of a parameter for a determinate or an indeterminate structure
- But **using the basic understanding of the influence lines**, the **magnitudes** of the influence lines also **can be computed**
- In order **to draw the shape of the influence lines properly**, the **capacity of the beam to resist the parameter investigated** (reaction, bending moment, shear force, etc.), **at that point, must be removed**
- The principle states that: **The influence line for a parameter** (say, reaction, shear or bending moment), at a point, **is to the same scale as the deflected shape of the beam**, when the beam is acted upon by that parameter.
  - The **capacity of the beam to resist that parameter**, at that point, **must be removed.**
  - **Then allow the beam to deflect under that parameter**
  - Positive **directions of the forces are the same** as before

3.5 PROBLEMS - 3.5.1 Influence Line for a Determinate Beam by Muller-Breslau's Method

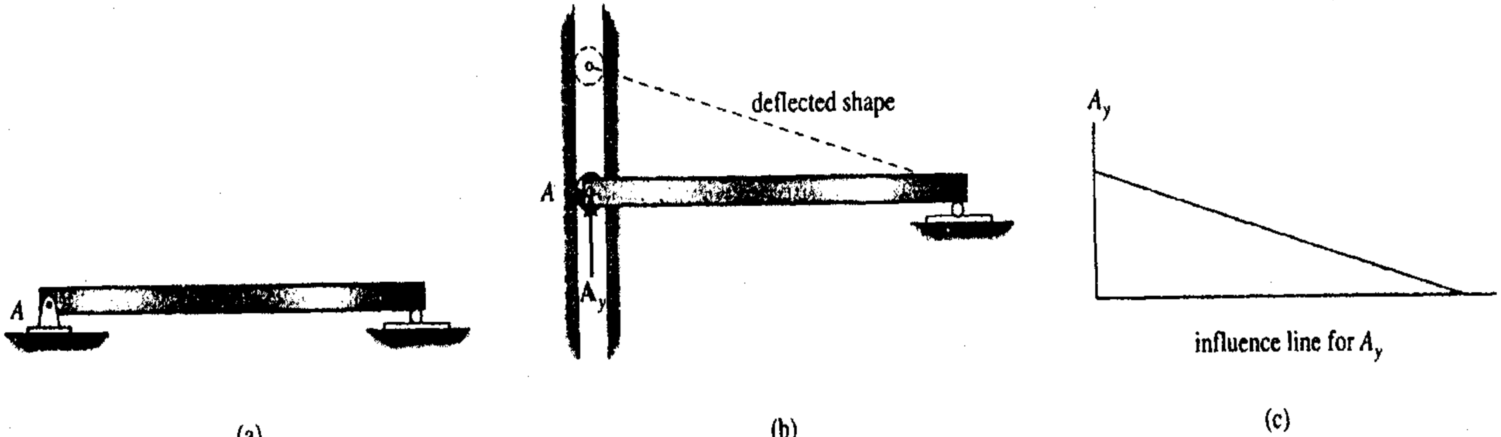


Fig. 6–12

Influence line for Reaction at A

3.5.2 Influence Lines for a Determinate Beam by Muller-Breslau's Method

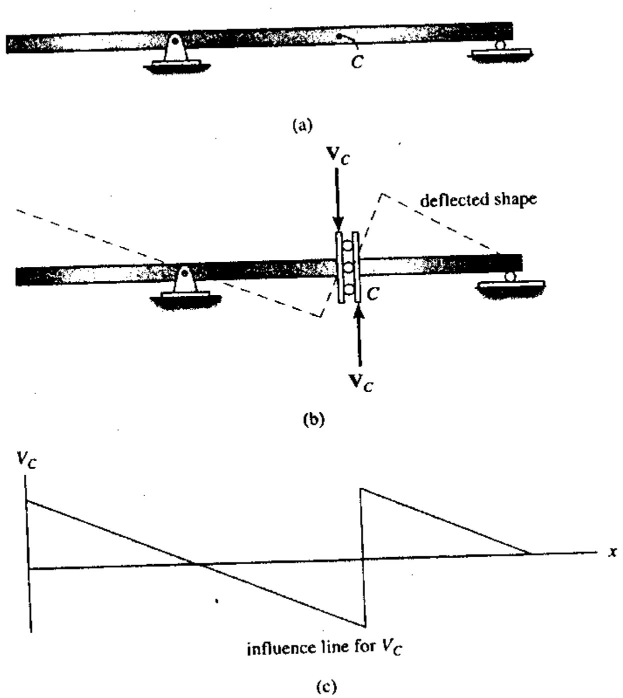


Fig. 6–13

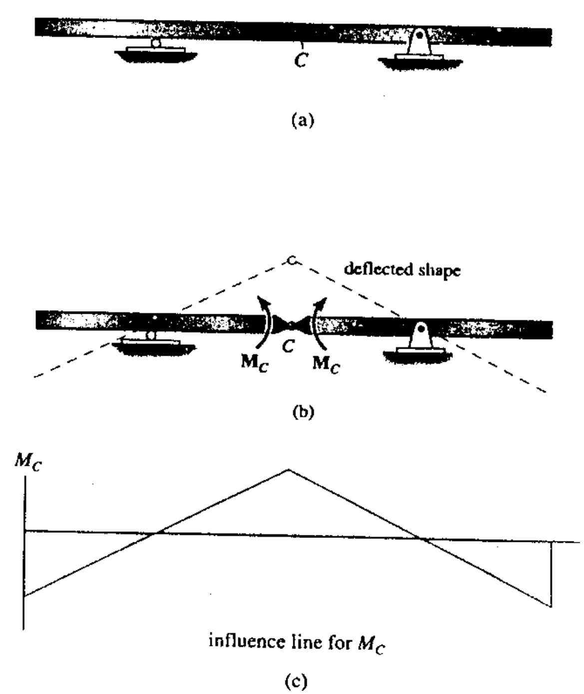


Fig. 6–14

Influence Line for Shear at C

Influence Line for Bending Moment at C

3.5.3 Influence Lines for an Indeterminate Beam by Muller-Breslau's Method

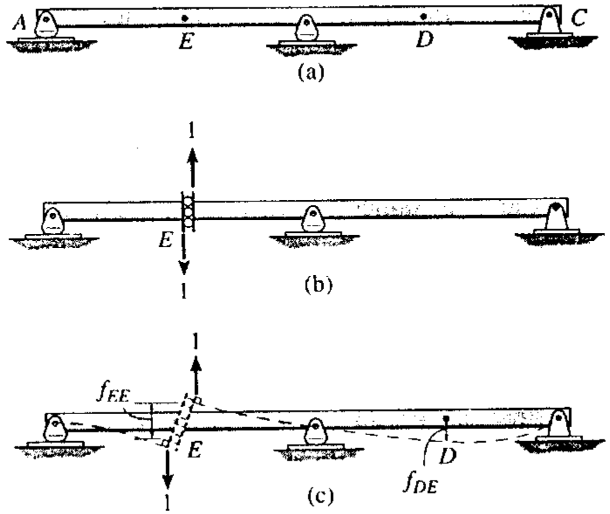


Fig. 9–24

Influence Line for Shear at E

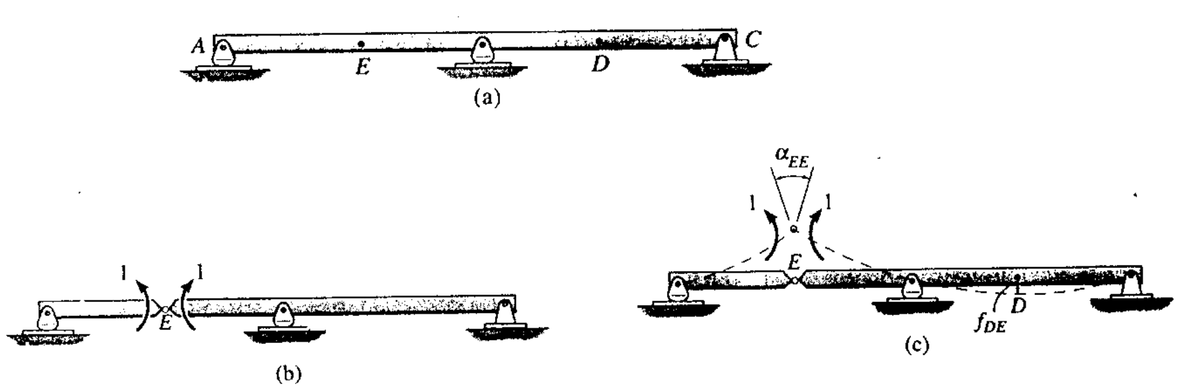
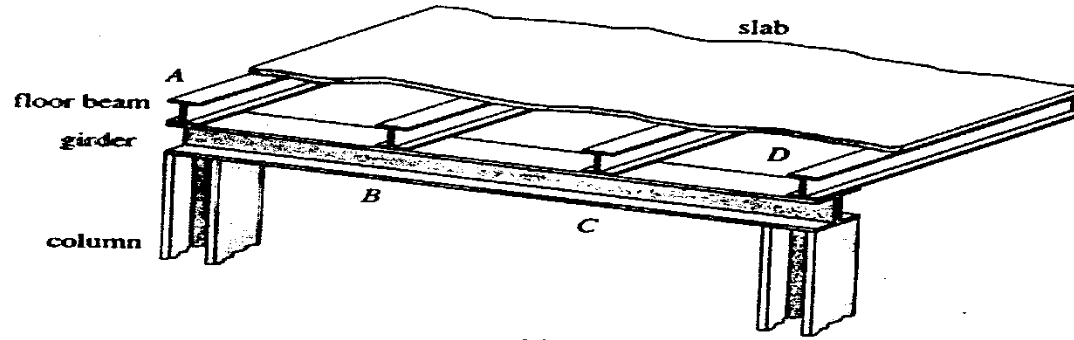


Fig. 9–25

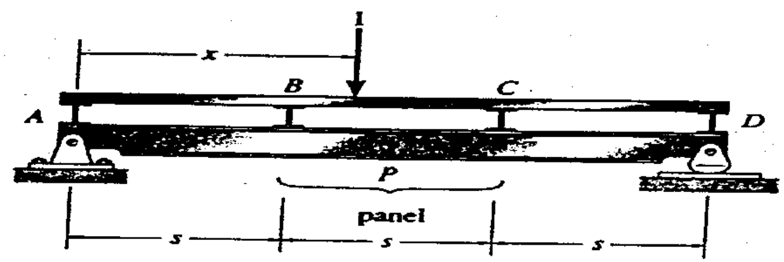
Influence Line for Bending Moment at E

3.6 INFLUENCE LINE FOR FLOOR GIRDERS

Floor systems are constructed as shown in figure below,



(a)



(b)

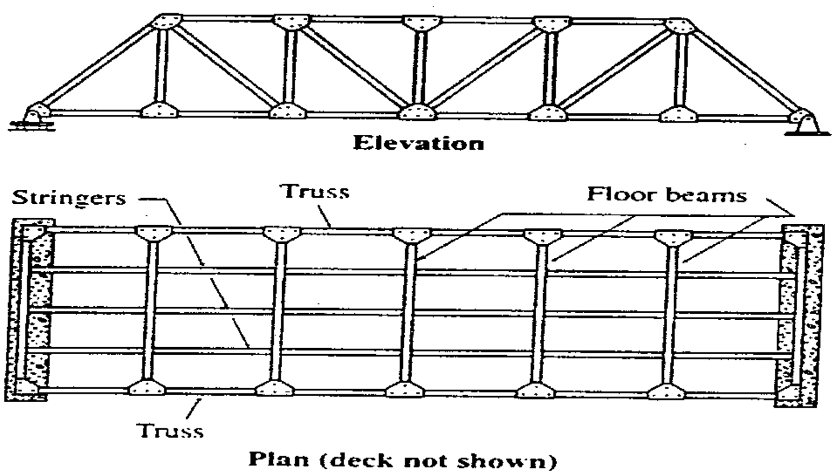


Fig. 8.14

3.6 INFLUENCE LINES FOR FLOOR GIRDERS (Cont'd)

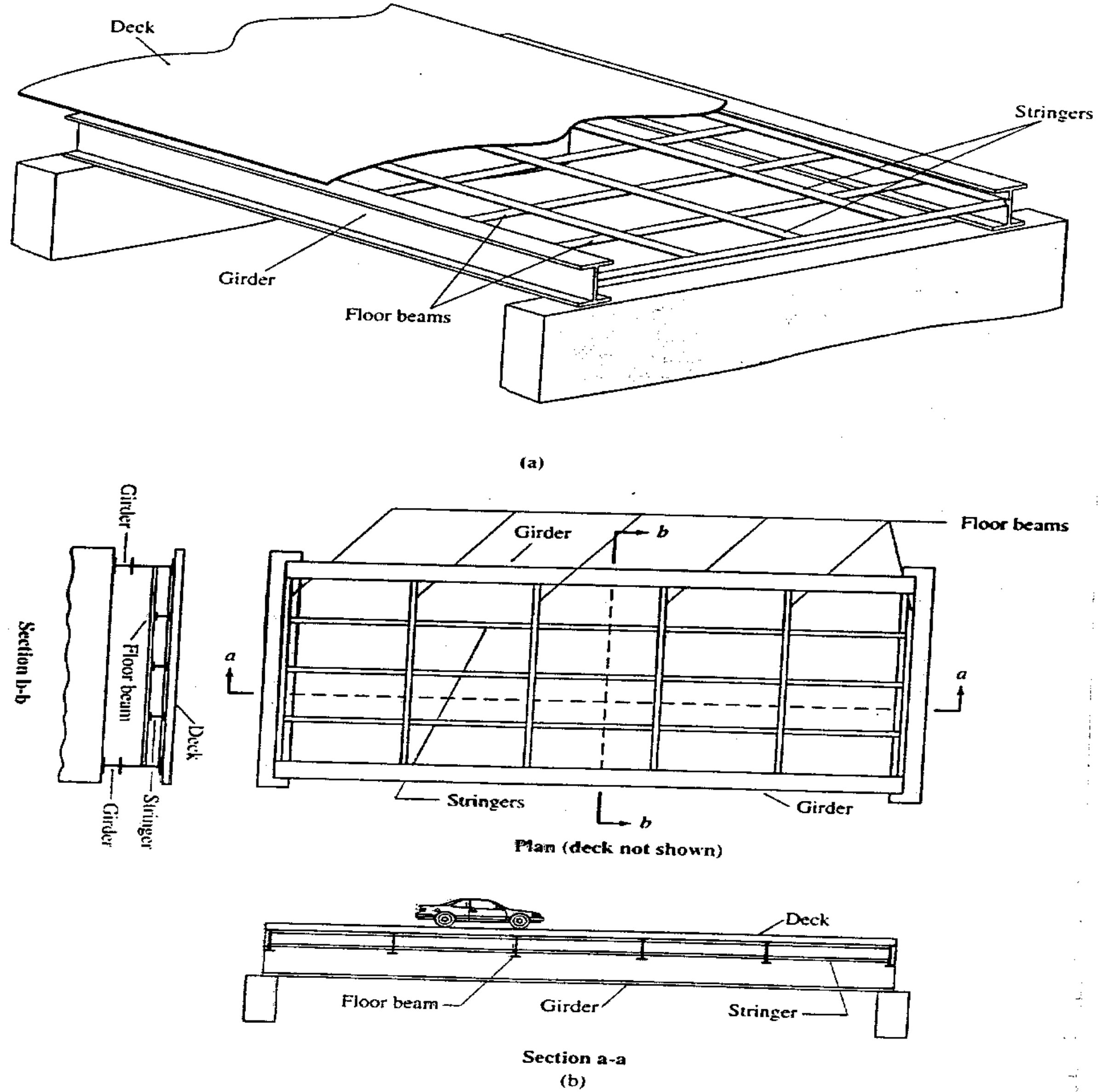
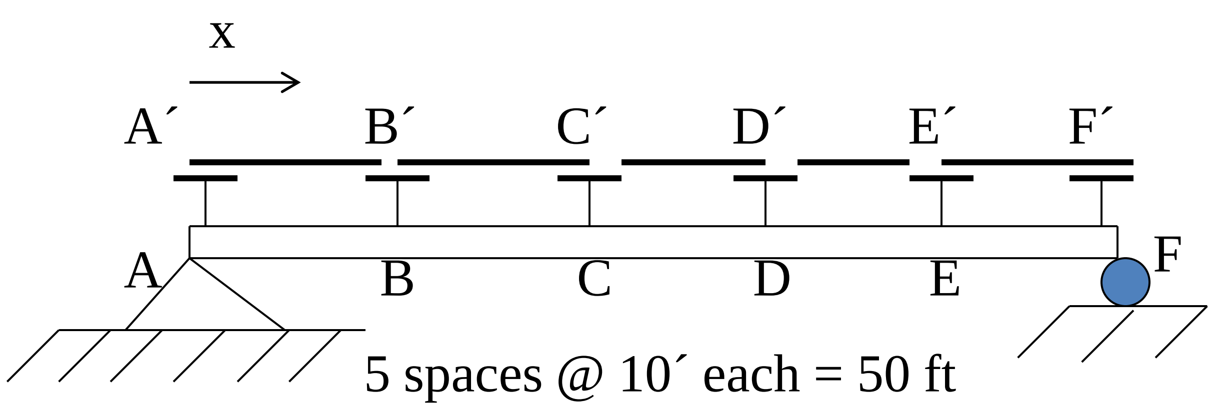


Fig. 8.10

3.6.1 Force Equilibrium Method:

Draw the Influence Lines for: (a) Shear in panel CD of the girder; and (b) the moment at E.



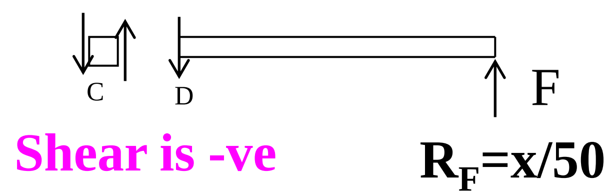
3.6.2 Place load over region A´B´ (0 < x < 10 ft)

Find the shear over panel CD

V_{CD}= - x/50

At x=0, V_{CD} = 0

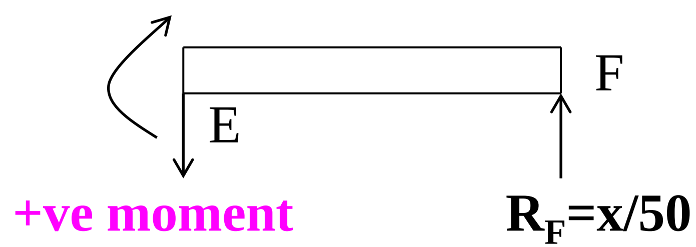
At x=10, V_{CD} = -0.2



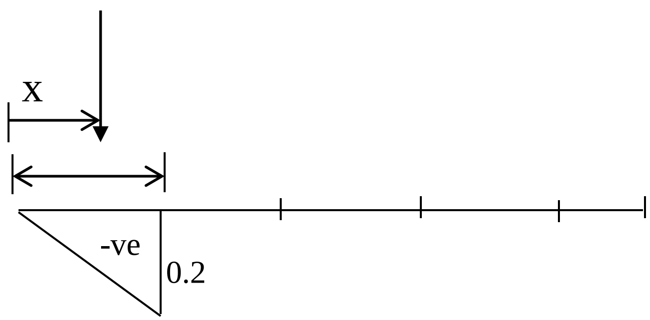
Find moment at E = +(x/50)(10)=+x/5

At x=0, M_E=0

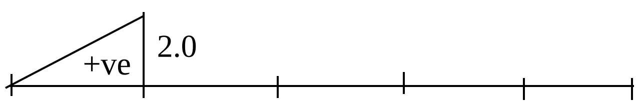
At x=10, M_E=+2.0



Continuation of the Problem



I. L. for V_{CD}



I. L. for M_E

Problem Continued -

3.6.3 Place load over region B´C´ (10 ft < x < 20ft)

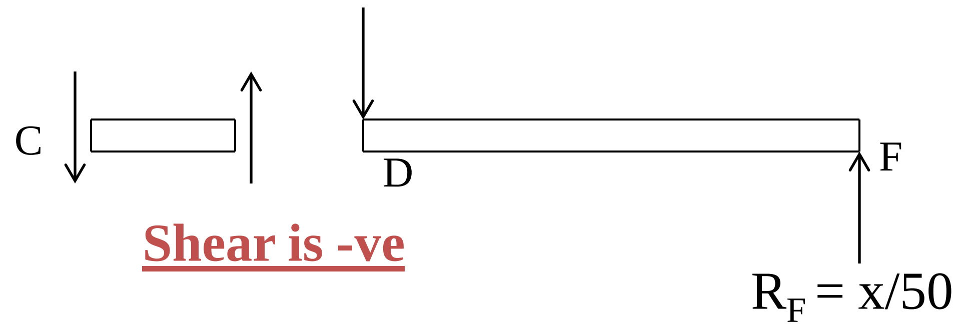
V_{CD} = -x/50 kip

At x = 10 ft

V_{CD} = -0.2

At x = 20 ft

V_{CD} = -0.4

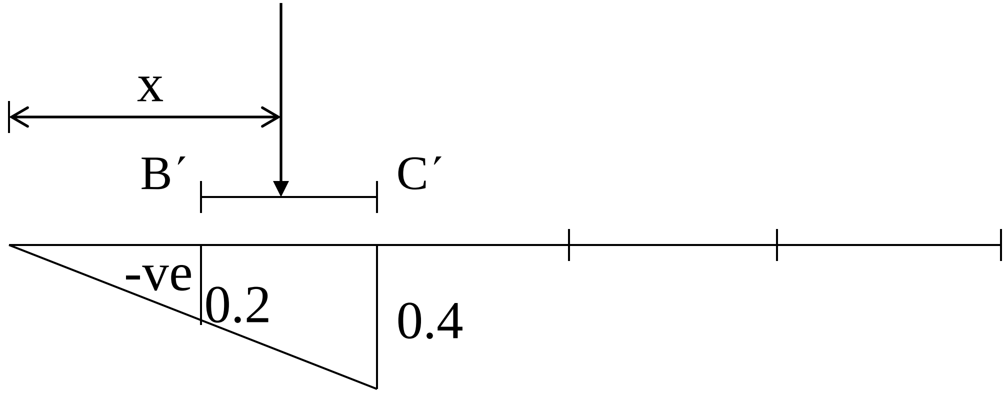


$M_E = +(x/50)(10)$

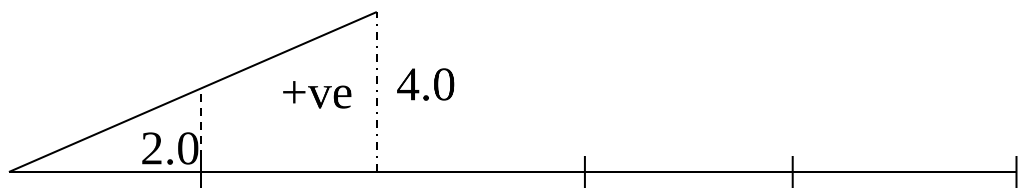
$= +x/5$ kip.ft

At x = 10 ft, M_E = +2.0 kip.ft

At x = 20 ft, M_E = +4.0 kip.ft



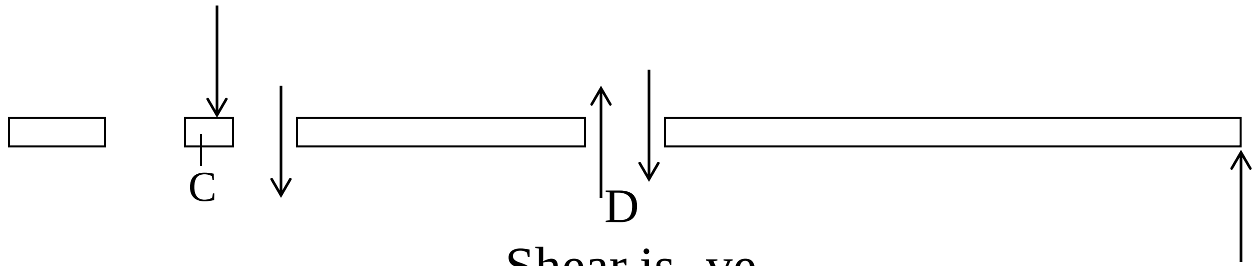
I. L. for V_{CD}



I. L. for M_E

3.6.4 Place load over region C´D´ (20 ft < x < 30 ft)

When the load is at C' (x = 20 ft)

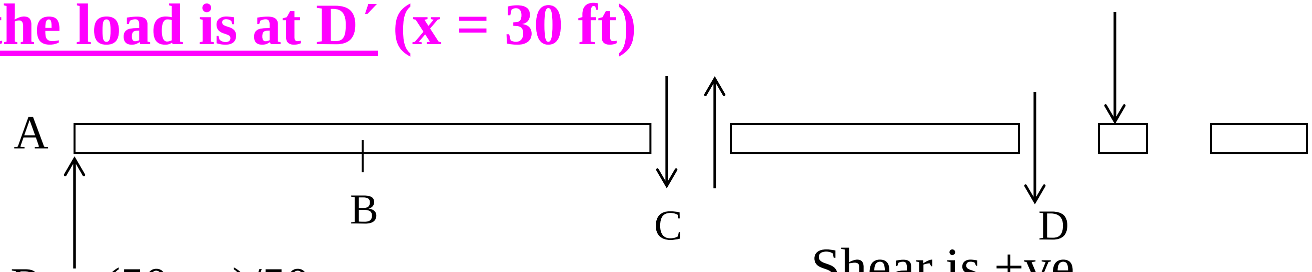


Shear is -ve

V_{CD} = -0.4 kip

R_F=20/50
=0.4

When the load is at D´ (x = 30 ft)

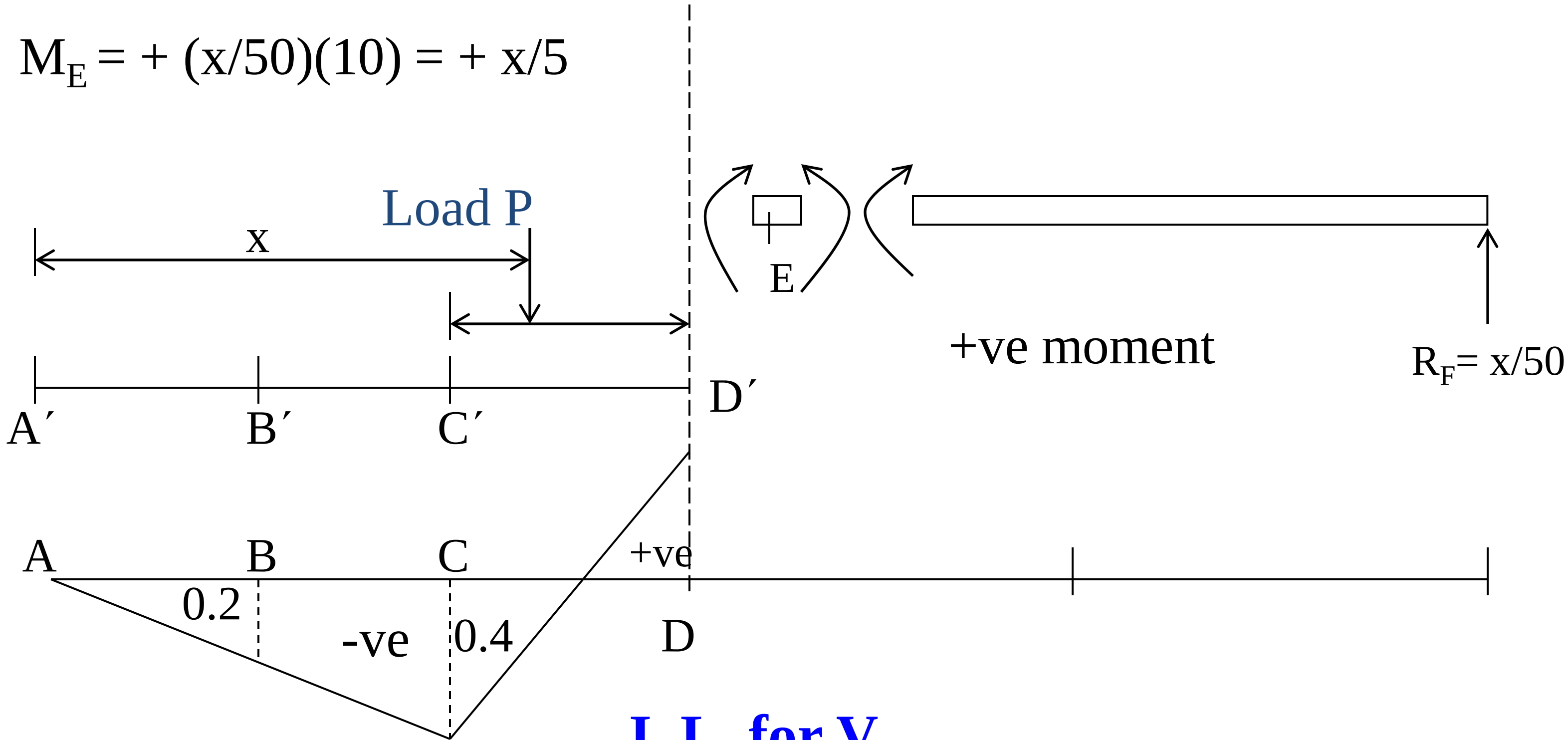


Shear is +ve

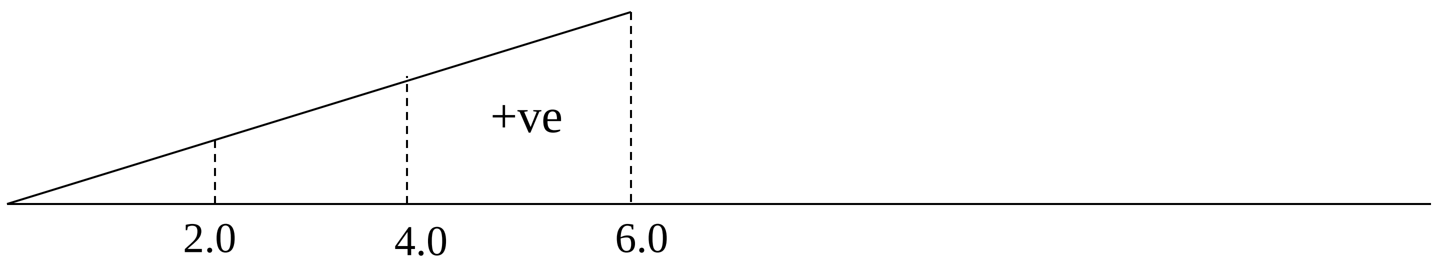
R_A= (50 - x)/50

V_{CD}= + 20/50
= + 0.4 kip

$M_E = +\ (x/50)(10) = +\ x/5$



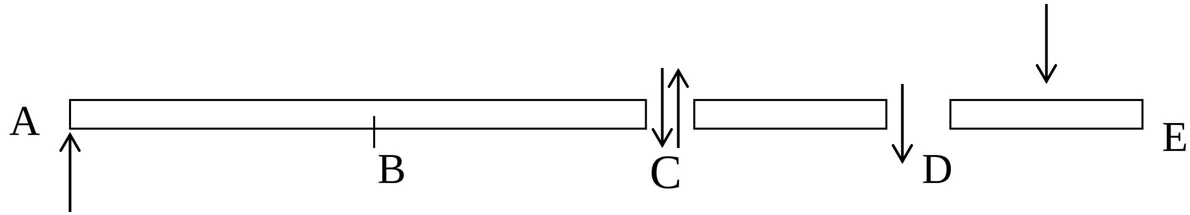
I. L. for V_{CD}



I. L. for M_E

3.6.5 Place load over region D′E′ (30 ft < x < 40 ft)

$V_{CD} = +(1 - x/50)$ kip



$R_A = (1 - x/50)$

Shear is +ve

$M_E = +(x/50)(10)$
$= +x/5$ kip.ft

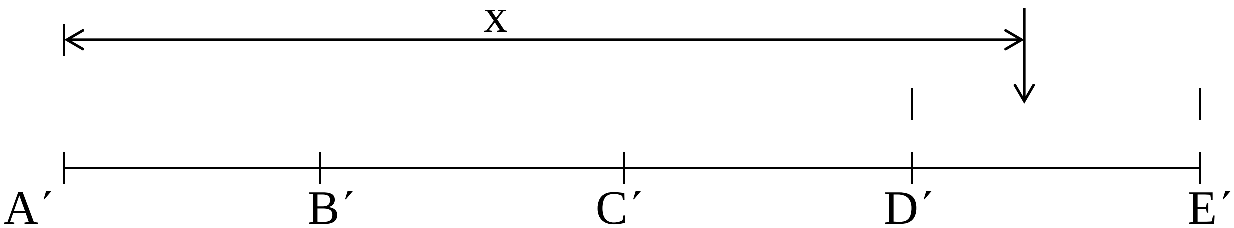
Moment is +ve

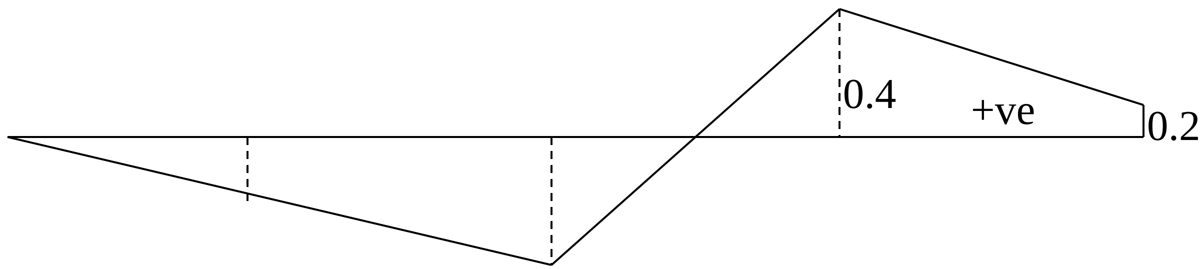
$R_F = x/50$

At x = 30 ft, $M_E = +6.0$

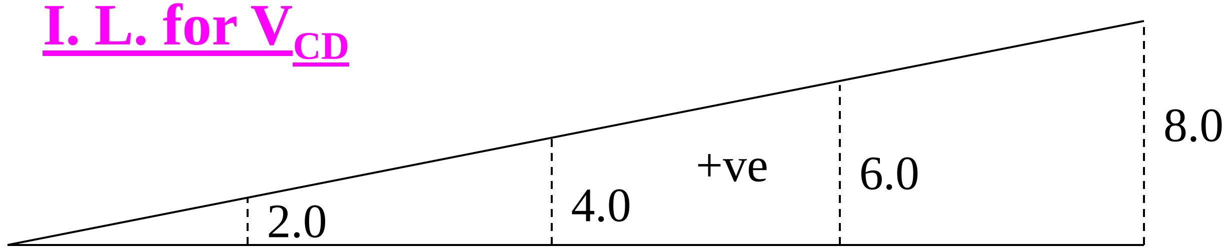
At x = 40 ft, $M_E = +8.0$

Problem continued





I. L. for V_{CD}

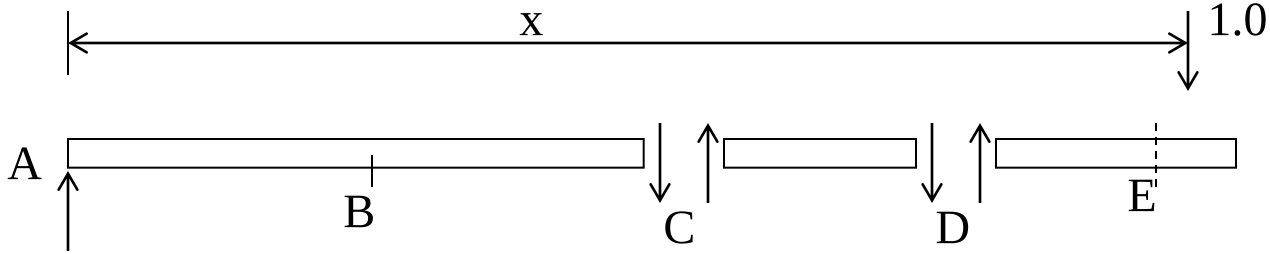


I. L. for M_E

3.6.6 Place load over region E′F′ (40 ft < x < 50 ft)

$V_{CD} = +\ 1\text{-}x/50$ At $x = 40$ ft, $V_{CD} = +\ 0.2$

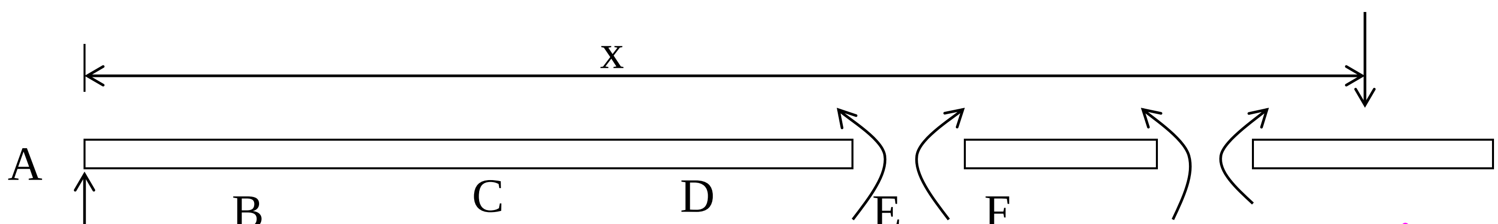
At $x = 50$ ft, $V_{CD} = 0.0$



$R_A = 1\text{-}x/50$

Shear is +ve

$M_E = +\ (1\text{-}x/50)(40) = (50\text{-}x)*40/50 = +(4/5)(50\text{-}x)$

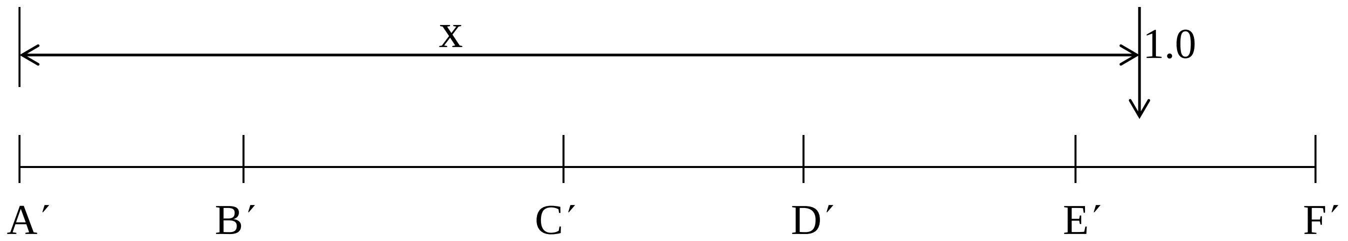


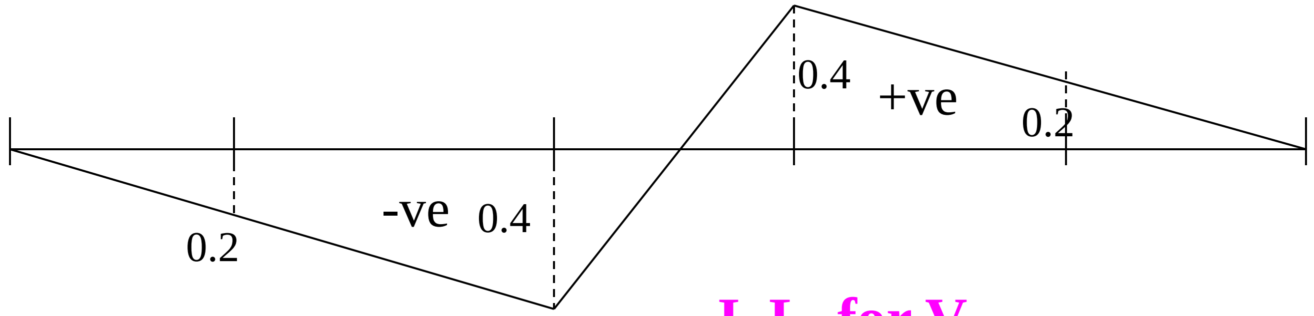
Moment is +ve

$R_A = 1\text{-}x/50$

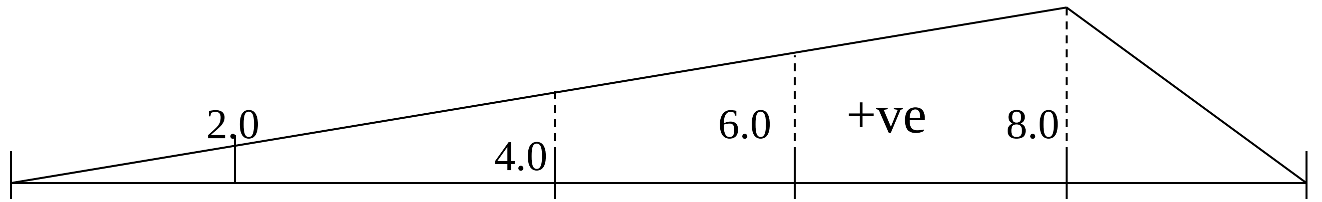
At $x = 40$ ft, $M_E = +\ 8.0$ kip.ft

At $x = 50$ ft, $M_E = 0.0$





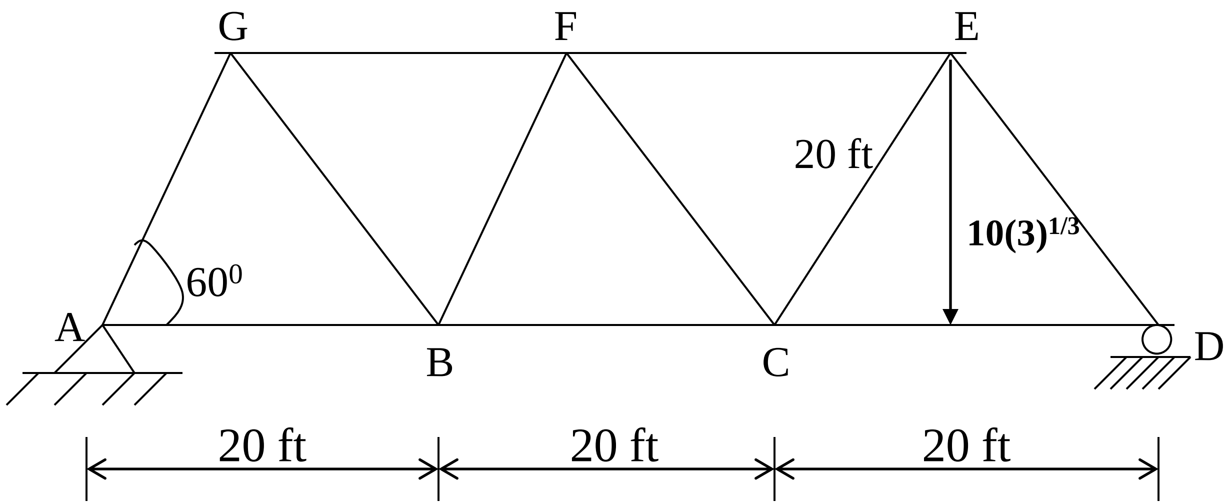
I. L. for V_{CD}



I. L. for M_E

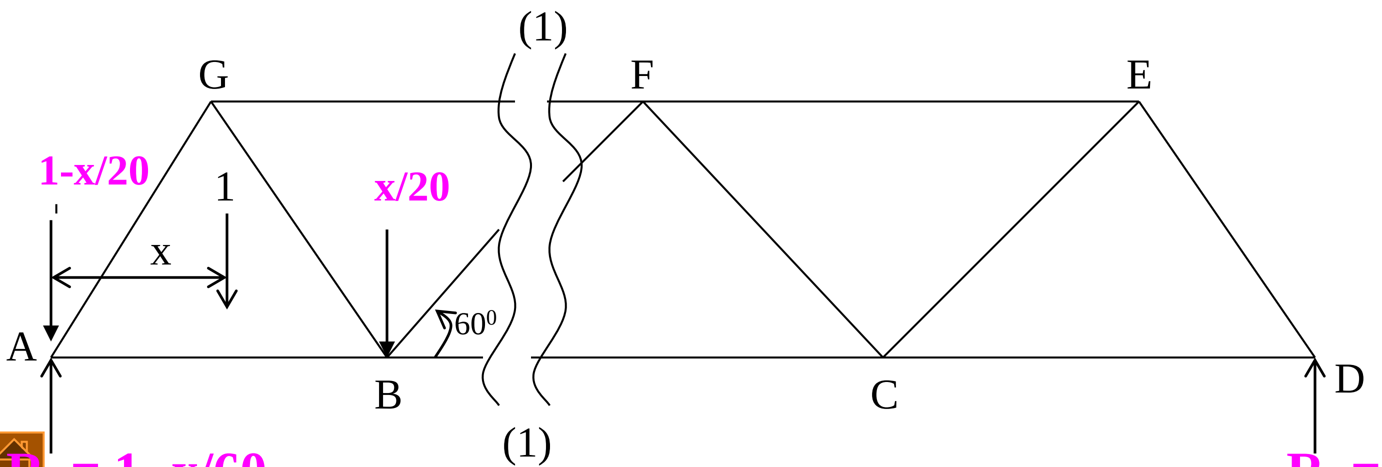
3.7 INFLUENCE LINES FOR TRUSSES

Draw the influence lines for: (a) Force in Member GF; and (b) Force in member FC of the truss shown below in Figure below



Problem 3.7 continued -
3.7.1 Place unit load over AB

(i) To compute GF, cut section (1) - (1)



$R_A = 1 - x/60$

$R_D = x/60$

At x = 0,
$F_{GF} = 0$
At x = 20 ft
$F_{GF} = -0.77$

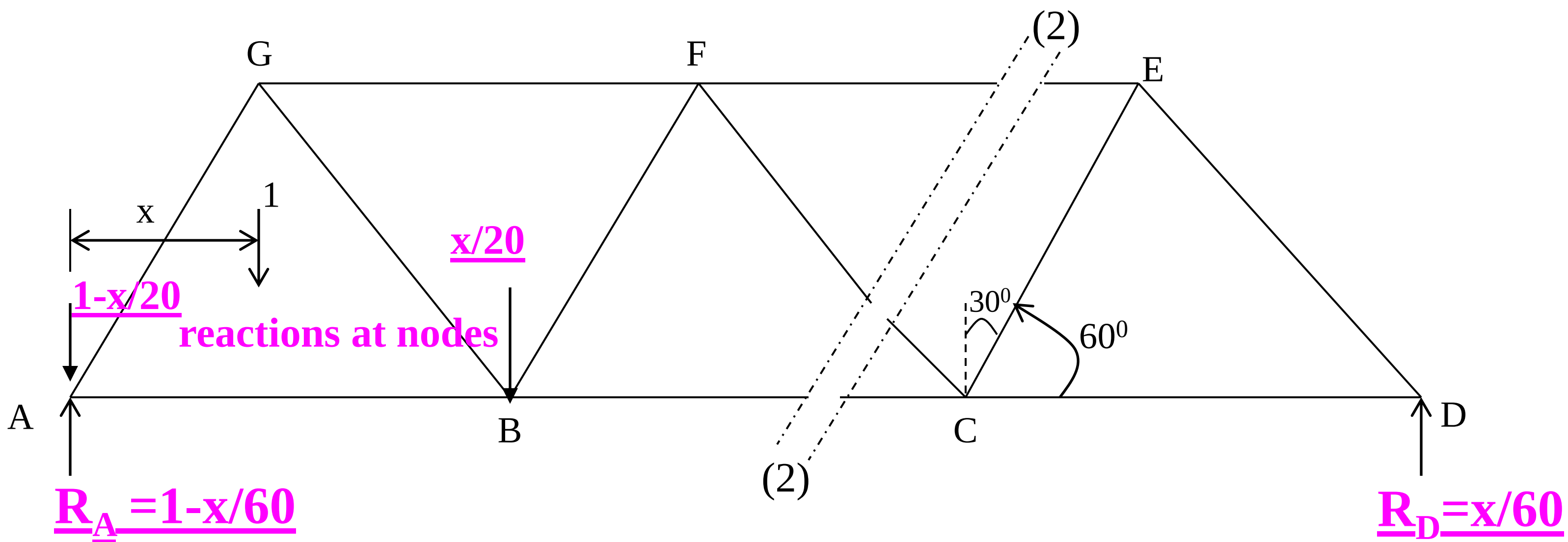
Taking moment about B to its right,

$(R_D)(40) - (F_{GF})(10\sqrt{3}) = 0$

$F_{GF} = (x/60)(40)(1/10\sqrt{3}) = x/(15\sqrt{3})$ (-ve)

PROBLEM 3.7 CONTINUED -

(ii) To compute F_{FC}, cut section (2) - (2)



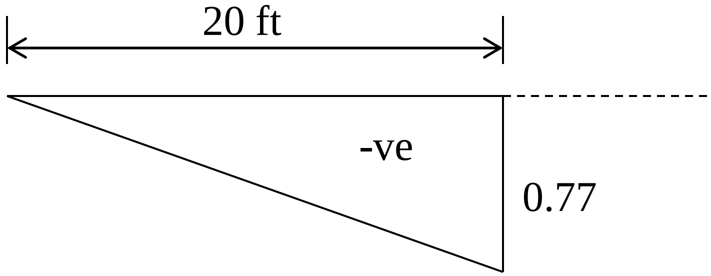
Resolving vertically over the right hand section

$$F_{FC} \cos 30^0 - R_D = 0$$

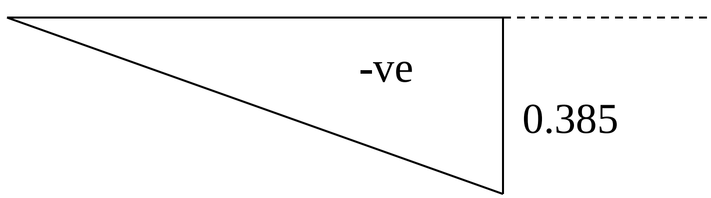
$$F_{FC} = R_D/\cos 30 = (x/60)(2/\sqrt{3}) = x/(30\sqrt{3}) \text{ (-ve)}$$

At x = 0, $F_{FC} = 0.0$

At x = 20 ft, $F_{FC} = -0.385$



I. L. for F_{GF}

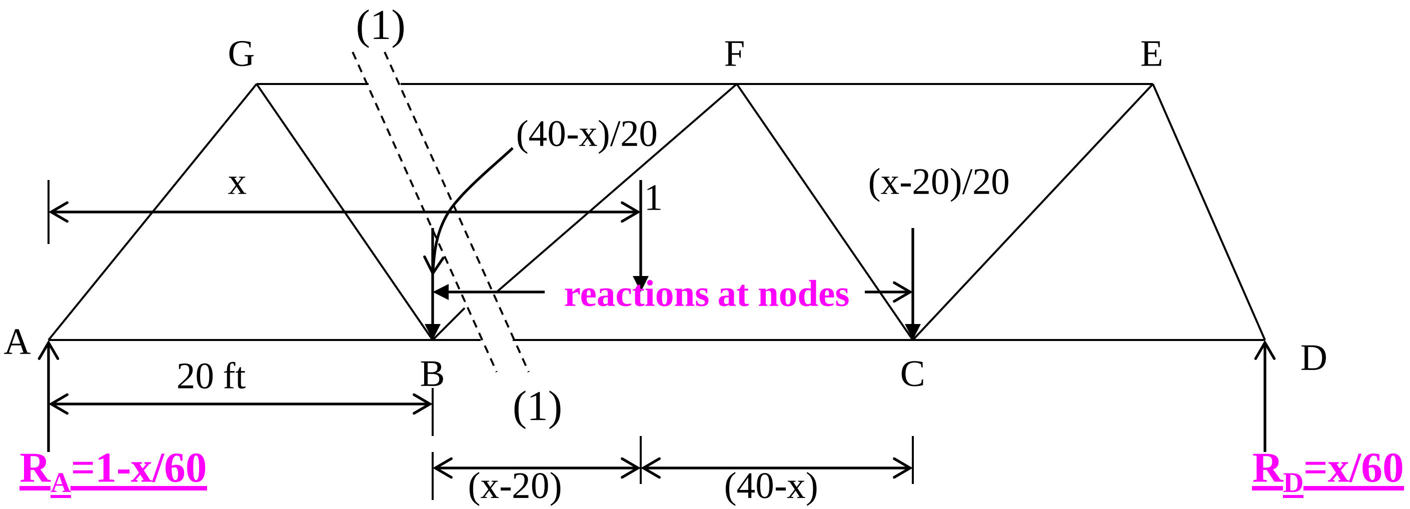


I. L. for F_{FC}

PROBLEM 3.7 Continued -
3.7.2 Place unit load over BC (20 ft < x <40 ft)

[Section (1) - (1) is valid for 20 < x < 40 ft]

(i) To compute F_{GF} use section (1) -(1)



Taking moment about B, to its left,

$$(R_A)(20) - (F_{GF})(10\sqrt{3}) = 0$$

$$F_{GF} = (20R_A)/(10\sqrt{3}) = (1-x/60)(2/\sqrt{3})$$

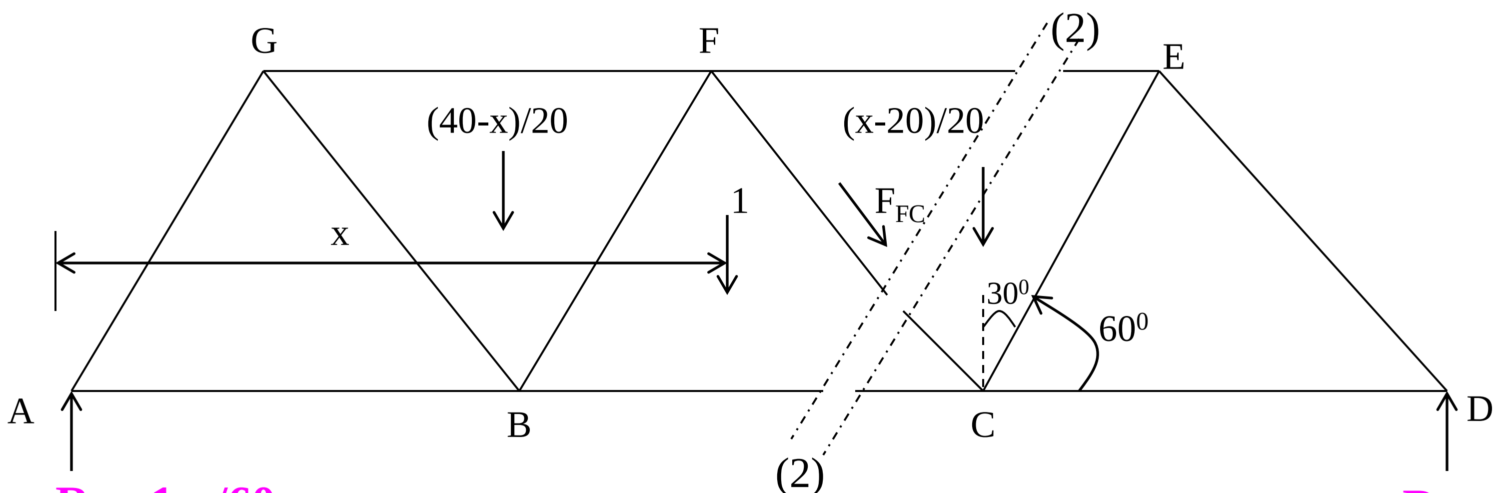
At x = 20 ft, F_{FG} = 0.77 (-ve)

At x = 40 ft, F_{FG} = 0.385 (-ve)

PROBLEM 6.7 Continued -

(ii) To compute F_{FC}, use section (2) - (2)

Section (2) - (2) is valid for 20 < x < 40 ft



R_A =1-x/60

R_D=x/60

Resolving force vertically, over the right hand section,

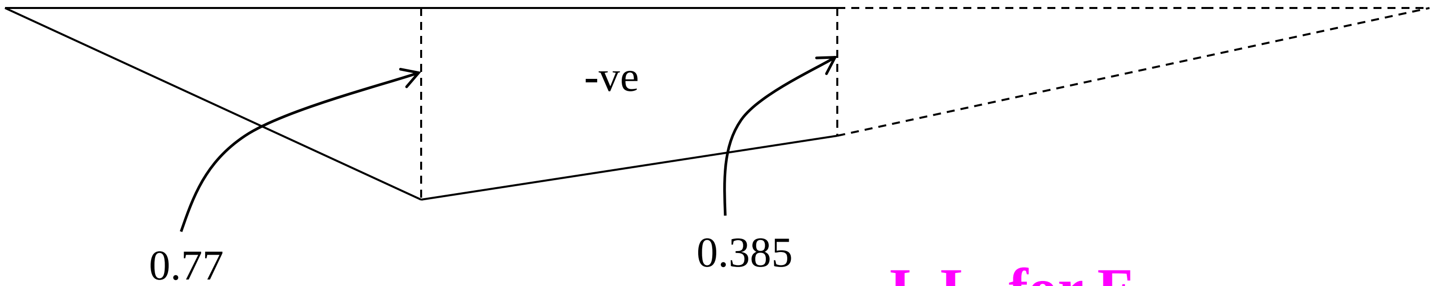
$$F_{FC}\cos30 - (x/60) + (x-20)/20 = 0$$

$$F_{FC}\cos30 = x/60 - x/20 + 1 = (1-2x)/60 \text{ (-ve)}$$

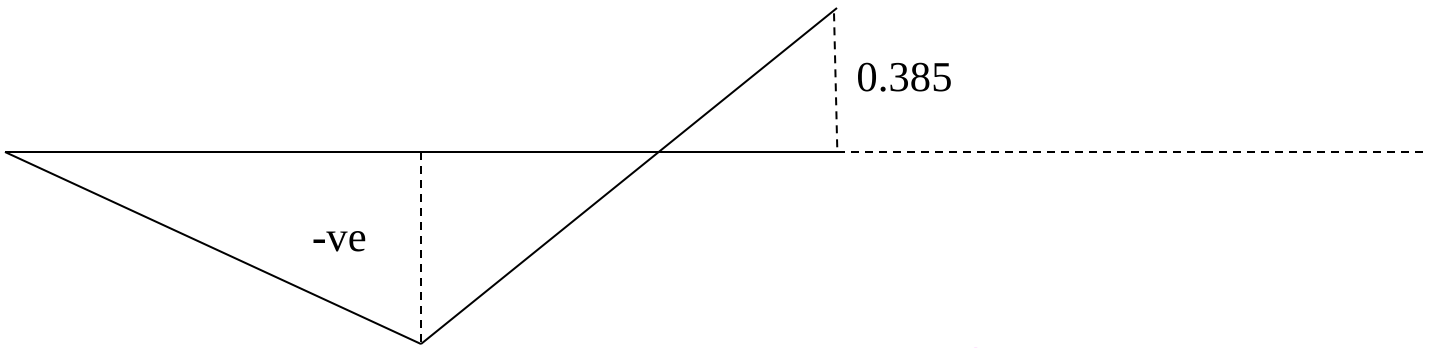
$$F_{FC} = ((60 - 2x)/60)(2/\sqrt{3}) \text{ -ve}$$

At x = 20 ft, $F_{FC} = (20/60)(2/\sqrt{3}) = 0.385$ (-ve)

At x = 40 ft, $F_{FC} = ((60-80)/60)(2/\sqrt{3}) = 0.385$ (+ve)



I. L. for F_{GF}

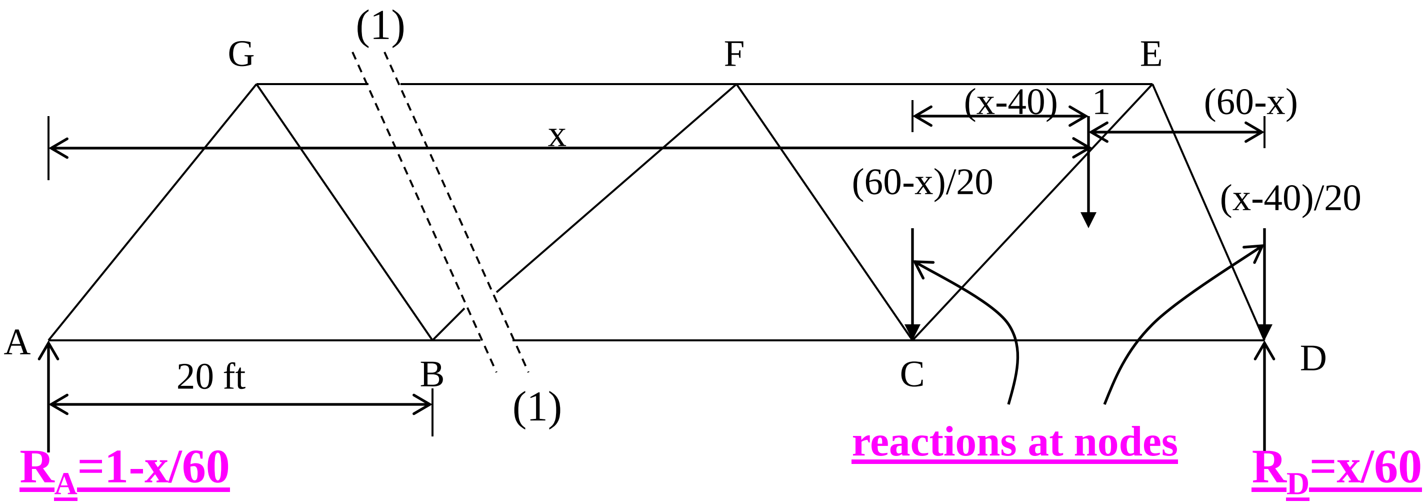


I. L. for F_{FC}

PROBLEM 3.7 Continued -

3.7.3 Place unit load over CD (40 ft < x <60 ft)

(i) To compute F_{GF}, use section (1) - (1)



Take moment about B, to its left,

$$(F_{FG})(10\sqrt{3}) - (R_A)(20) = 0$$

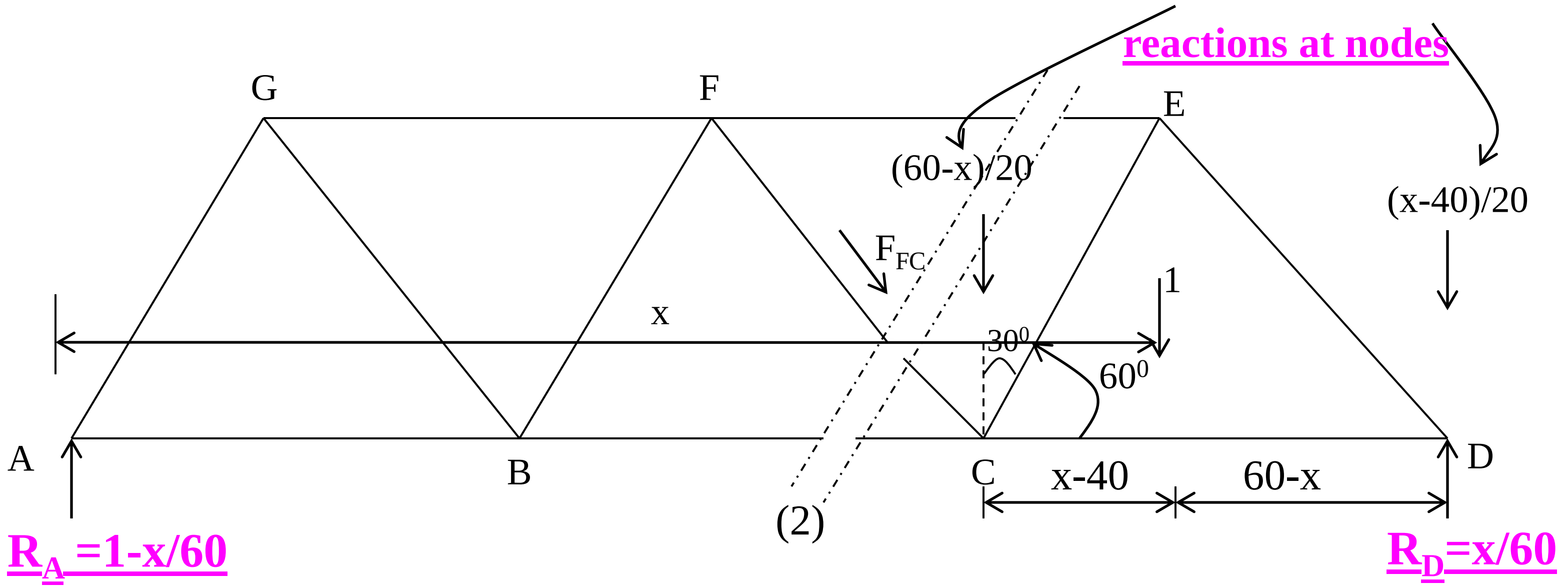
$$F_{FG} = (1-x/60)(20/10\sqrt{3}) = (1-x/60)(2/\sqrt{3}) \text{ -ve}$$

At x = 40 ft, F_{FG} = 0.385 kip (-ve)

At x = 60 ft, F_{FG} = 0.0

PROBLEM 3.7 Continued -

(ii) To compute F_{FG}, use section (2) - (2)



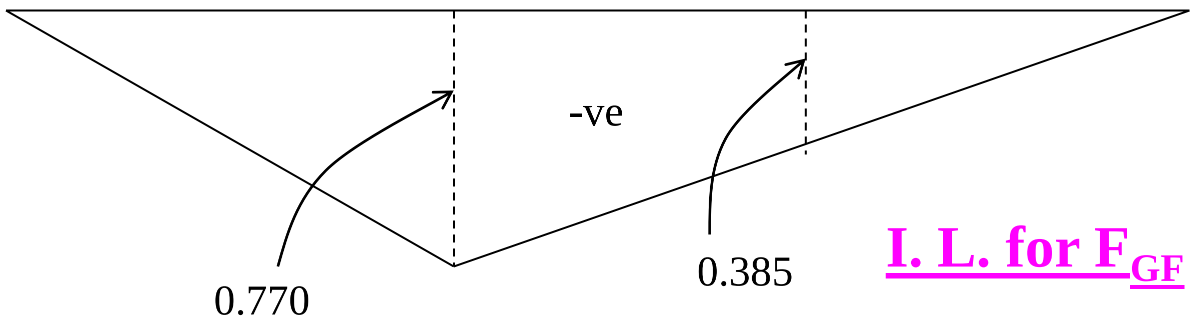
Resolving forces vertically, to the left of C,

$$(R_A) - F_{FC} \cos 30 = 0$$

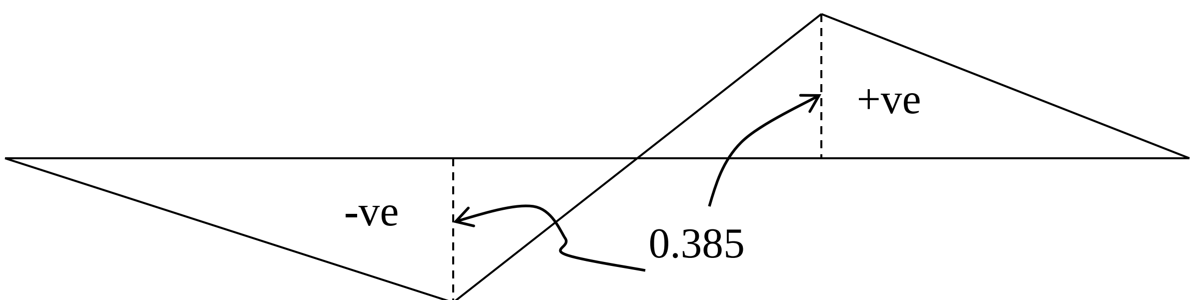
$$F_{FC} = R_A/\cos 30 = (1-x/10)\,(2/\sqrt{3}) \text{ +ve}$$

At x = 40 ft, F_{FC} = 0.385 (+ve)

At x = 60 ft, F_{FC} = 0.0

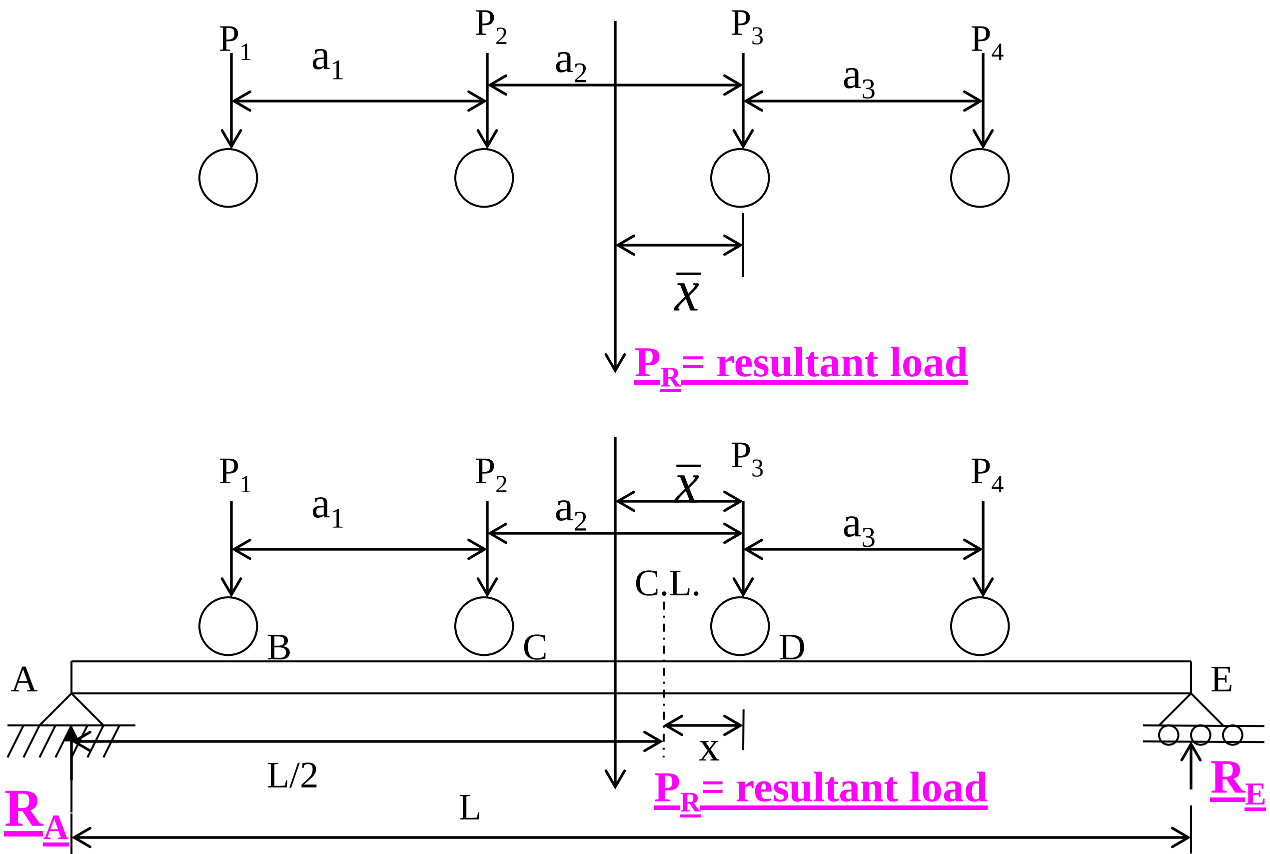


I. L. for F_{GF}



I. L. for F_{FC}

3.8 MAXIMUM SHEAR FORCE AND BENDING MOMENT UNDER A SERIES OF CONCENTRATED LOADS



Taking moment about A,

$R_E \times L = P_R \times[L/2 - (\bar{x} - x)]$

$$R_E = \frac{P_R}{L}(L/2 - \bar{x} + x)$$

Taking moment about E,

$$R_A \times L = P_R \times [L/2 + (\bar{x} - x)]$$

$$R_A = \frac{P_R}{L}(L/2 + \bar{x} - x)$$

$$M_D = R_A \times (L/2 + x) - P_1(a_1 + a_2) - P_2 \times a_2$$

$$= \frac{P_R}{L}(L/2 + \bar{x} - x)(L/2 + x) - P_1(a_1 + a_2) - P_2 \times (a_2)$$

$$\frac{dM_D}{dx} = 0$$

$$0 = \frac{P_R}{L}(L/2 + \bar{x} - x) + \frac{P_R}{L}(L/2 + x)(-1)$$

$$= \frac{P_R}{L}[(L/2) + \bar{x} - x - (L/2) - x]$$

$$i.e., \quad \bar{x} - 2x = 0$$

$$\bar{x} = 2x$$

$$x = \frac{\bar{x}}{2}$$

The centerline must divide the distance between the resultant of all the loads in the moving series of loads and the load considered under which maximum bending moment occurs.